ON COMPUTER SOLUTIONS OF SOME MICROWAVE PROBLEMS

By

B. M. AZIZUR RAHMAN

A THESIS

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CERTIFICATES

This is to certify that this work has been done by me and this has not been submitted elsewhere for the award of any degree or diploma .

COUNTERSIGNED :

B.M. Azizur Rahman

(Supervisor) (Signature of the candidate)

ACCEPTED AS SATISFACTOTY FOR THE PARTIAL FULFILMENT OF THE REQUIREMENTS FOR THE DEGREE OF M.SC.ENGG. IN ELECTRICAL ENGINEERING.

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Dated-

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DR. SHAMSUDDIN AHMED

Professor,Elect. Engg. Deptt.,BUET,Dacca.

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ABSTRACT

Propagation of electromagnetic wave through waveguides of arbitrary cross section has been analysed for TM_{11} mode. Four different methods, namely, Finite difference five point, Finite difference nine point, Finite element triangular shaped, Finite element rectangular shaped, have been used to calculate cut-off frequencies, field distributions inside the waveguides. Effect of assumed trial values has been analysed with iteration counts. Effect of discretizational error has been tabulated by changing the mesh sizes. These have been calculated and compared for all the four above mentioned methods for rectangular waveguide of different aspect ratios. For single ridge and double ridge waveguides variation of cutoff-frequency with channel depth and width variation have been calculated.

Scattering of electromagnetic wave impinging on an obstacle has been calculated by direct method. Radar cross section and field distribution outside the circular cylindrical obstacle have also been computed for different incident angles and different obstacle dimensions.

Scattering of electromagnetic waves from the class of circular cylindrical obstacles have also been calculated by approximate methods, namely, finite element and finite difference methods. Several variations of finite element method have been tried for those problems. Radar cross sections and field distribution outside the scatterers have also been computed .

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1.1. General

From the point of view of classical physical theory, the scattering and diffraction of waves from obstacles have long been fully understood in the sense that the underlying differential equations and boundary conditions of the relevant variables are known. In principle, it is merely necessary to solve the equations s subject to the boundary conditions appropriate to any particular source and obstacle in order to determine the complete picture of the diffracted field. IN practice, this has proved to be a very formidable problem even under simple and idealized conditions. Approximate mathematical solutions and experimental techniques that serve very well in the study of optical phenomena related to scattering and diffraction are inadequate to deal with more general electromagnetic problems that are encountered, for example, in radar It is to the investigation of these problems that attention is directed. New mathematical techniques and new experimental methods are required in order to permit a reduction to numbers and graphs of information that is already implicit in the general wave equation and the associated boundary conditions.

The introduction of an obstacle in the path of a plane electromagnetic wave produces scattering. The incident electromagnetic field induces time-varying distributions of oscilating charges and currents in the obstacle. These, in turn maimtain an electromagnetic field that is known as the scattered ,reflected, or reradiated field. The solutions to this scattered field, radar cross section, and the current distribution on scatterers of different geometrical shapes and sizes have long been the aim of scientists and engineers.

The starting point for the exact solution of any scattering problem is Maxwell's Equations. For regions having no divergence, these reduce to the scalar wave equation. Any admissible solution to this equation must satisfy the appropriate boundary conditions along with Sommerfeld's condition of radiation at infinity. Moreover, in case of scatterers with edges and sharp corners, it must also satisfy Bouwkamp-Meixner edge condition.

From the mathematical point of view, geometrical simplicity is not always synonymous with analytical simplicity. Of primary importance in yhe mathematical formulation are those aspects of an obstacle and the field in which it is immersed that determine whether the problem can be reduced to the scalar rather than the vector wave equations. Insofar as the shapes of the boundaries are concerned, any shape which is readily expressed in terms of suitable coordinates is mathematically simple, especially if the appropriate coordinate system is one in terms of which the wave equation is separable.

Exact solutions to the wave equations are possible by the classical method of separation of variables for only in a very few geometries, such as circular cylinder[1], elliptical cylinder[2,3], parabolic cylinder[4], strip[5,6], infinite wedge[7,8,9,10], sphere[11], prolate spheroid[13], circular plate[14], and semiinfinite cone[15]. The wave equation in two dimensions and three dimensions can be separated in any coordinate system which is a conformal transform from the rectangular system. They are separable in all the 11 coordinates, listed below[16]: rectangular, circular cylinder, elliptic cylinder, parabolic cylinder, spherical, conical, parabolic, prolatespheroidal, oblate spheroidal, ellipsoidal, and paraboloidal corodinates. Therefore, the number of scattering problems that can be solved by this classical method is severely limited. For other configurations, various approximate methods such as pertubbation, variational, geometrical and physical optics and the geometric theory of diffraction are used.

The whole class of problems of scattering has been divided according to the wavelength of the electromagnetic wave and the dimensions of the scatterer into three regions: the low frequency or Rayleigh region, the mid-frequency or resonance region and the high-frequency or geometrical optics region. The regions are not watertight compartments, but are just convenient designations. Again no one solution, not even the harmonic solutions of separation of variables method is satisfactory throughout the whole range of the frequency spectrum. In the mid-frequency or resonance region, the dimensions of the scatterers are comparable to the wavelength of the incident e-m wave. Neither the low-frequency approximations used in the Rayleigh region nor the asymptotic methods used in the high-frequency region are quite valid in the mid-frequency range. With the rapid development in the radar techniques and technology, with an ever growing emphasis on 'weapons systems' design in which all aspects of the poperational use of an aircraft or missile are considered in the conceptual stage and with the ever-increasing effort in space explorations by sending vehicles into outer space, it has become all the more urgent to find solutions to scattering problems not only in the whole of the frequency spectrum but also for scatterers of arbitrary size and shape.The availability of high-speed digital computers has made possible some revolutionary improvement in the analysis of scattering and diffraction problems.

In the electromagnetic theory of light,two distinct disciplines are often used to find the scattered field.One is the theory developed from the Maxwellian field equationswhich are supposed to be valid for all frequencies. The other is the theory of geometrical optics,valid only for very high frequencies,i.e., for wavelengths that are short compared with the size of the obstacle.

The geometrical-optics solution of a problem is of importance for the radio engineer in that it represents the high-frequency limit of the more rigorous field problem and often yields quick approximate answers when more elaborate methods are unworkable. The basic assumption of geometric optics is that an incident ray is reflected by the scatterer as if the latter's surface were plane at the reflection point.The diffracted rays are produced by incident rays which hit edges,corners,or vertices of boundary surfaces or which graze such surfaces.Keller has proposed an extension of geometrical optics which includes the diffracted rays in addition to the usual rays of geometrical optics.A more general approach to the low-frequency-scattering has been proposed by Stevenson.

Variational and quasi-static solutions have shown considerable success for scatterers of various shapes,but these techniques have been limited to bodies which are smallin comparison with the wavelength or are on the order of one wavelength in maximum diameter.Large scatterers are han handled with the aid of physical optics,geometric optics, and the geometric theory of diffraction.These optical solutions provide reliable data only when the scatterer has a diameter or width whish is large in comparison with the wavelength.

The linear-equation technique is valid for scatterers of any convex or concane shape, and the exact solution can be approached simply by enforcing the boundary conditions at a sufficiently large number of points.The computation time is least for small scatterers but it is reasonable even for bodies of resonant size or larger size,depending on the capacity of the computer.Solutions can readily be obtained for prrfectly conductind,imperfectly conducting,and dielectric bodies. If the body is placed in the near-zone field of a source,the solution proceeds in the same straight forward manner as in the plane-wave case. Thus, the linear-equation solution shows promise for the accurate,systematic calculations for bodies of any material having arbitrary size and shape.

In the past years, wirh the widespread availability of high speed digital computers,attention has been given to a distinct approach to the scattering problem. First, a system of linear equations is obtained by enforcing the boundary conditions at many points within the scatterer or on its surface.Next wieh thw aid of a digital computer,this system of equations is solved to determine the currents distribution on the surface or the coefficients in the mode expansion for the scattered field. Finally,the distant scattering pattern can be computed.

Since its conception,radar has been a useful tool, helpful in providing solutions to numerous problems.While one of radar's earliest applications was that of simple target detector as used by the military, radar has since evolved into a sophisticated tool often used for probing and interrogation remote objects and scenes.Presently radar is being seriously considered as a potential tool for remotely sensing croplands and forests from satellite.Upon the development of radar during World War II, radar cross section(RCS) reduction was and continues to be persued as a passive technique for reducing detecaability.The two major tools used have been radar absorbent material(RAM) and target shaping.The target can be oriented so as to minimise the probability of the radar lying along these sensitive angles,target shaping remains a useful tool and the combination of RAM and shaping can often be explained when neither can satisfy the desird objective alone.

Radio signals of decameter wavelength resonantly scattered from waves on the sea surface are used to measure precisely the wavelength,frequency,and direction of travel of these waves.These measurements are not only important in themselves, but are also used to deduce currents,winds, and perhaps wind stress at the sea surface.Microwave radars can provide reliable measurement of low wind speed.An altimeter incorporated into Skylab programme is being used to deduce the geoid to an accuracy approaching $\pm$ 10 cm by measuring the round trip delay time of pulse transmitted from orbiting satellite to the surface of the ocean. A two dimensional image of terrain can be constructed by cross-track temporal processing and along-track Doppler processing of the radar return using synthetic aperture(SAR) with broad beam antennas. These will enable us to better understand our planet and to obtain tangible benefits such as routing ships away from damaging seas, and to construct more reliable long range weather prediction model.

In the line-of-sight radio communication on very short waves the field at receiving point may be broken up into that of the direct ray and the ray reflected from the surface of the earth. In the case of a smooth earth the field may be accurately computed; normally its phase is such that the total field is diminished by it. The terrain between the terminals of a communication path, however, is not usually smooth and absence of a reliable theory of rough-surface scatterer the field strength at the receiving point cannot be accurately predicted. If the communication path lies above the surface of the sea, the scattering surface will not only be rough, but in addition time-varying; the received signal will therefore fade and the character of the fading will be affected by the form and movement of the water waves. In tropospheric propagation, very short radio waves are scattered beyond the horizon by the rough layers in the atmosphere; as they are in constant movement, the instantaneous field strength beyond the horizon will rapidly fluctuate. Similarly, short waves are often scattered by irregularities in the ionosphere F layer instead of being reflected by a smooth ionosphere.

In radio astronomy, the surface roughness of the Moon estimated by studing the distortion of the radar pulse back-scattered from the Moon to the Earth .

In acoustics, the sound engineer often endeavours to make acoustic field diffuse by scattering sound waves from rough surfaces. Analytically, the case of sound wave is simpler than that of electromagnetic waves because the field of the latter is given by the solution of the vector wave equation whilst the acoustic field is given by the solution of a scalar wave equation

A large number of papers have been published on the subject of scattering,specially in the last 25 years Many experimental [illegible]

1.2 Brief Literature Review

On October 30, 1861, A. Clebsch, one of the greatest mathematicians of nineteenth century(1833-1872) completed a 68 page memoir[17], in which he developed the mathematical theory required to solve by the method of separation of variables, the class of boundary-value problems in which a wave propagating in a elastic medium impinges upon a spherical surface. However, the mathematical ingenuity of this master craftsman was doomed to lie buried within the pages of one of the leading mathematical journals of the middle of the nineteenth century. This is a forerunner to the famous electromagnetic theory of Maxwell[18].

In 1871, Lord Rayleigh (1842-1919) was interested in the scattering of light by particles in the earth's atmosphere, andconceived of a sphere as being the simplest model for such scatterers. The oroginal studies of Rayleigh,were based upon the elastic-wave equations.Rayleigh[19] pointed out the exact solution for the scattering of sound by a sphere. Some of the mathematical tools which he needed were taken from the source.of a paperpublished earlier by Stokes[20]. Some of the materials from the 1872 paper were included in the first edition of Theory of Sound[21].This treatise is also important in the history of the scattering of waves by a circular cylinder. In this book, Rayleigh lays the foundation for this geometry by showing how to separate the wave equation in circular-cylinder co-ordinates.

L.Lorentz was a brilliant mathematical physicist whose work in diffraction theory is a superb example of the mathematical prowess of the analysts who tackled problems in mathematical physics during the last half of the 19 th century. His memoir of 1883[22] represents an attempt to study the propagation of light in concentric spherical shells. In 1890, the year before his death, he published his classic memoir 'Upon the reflection and refraction of light by a transparent sphere'. In this ... 23 ...

There were othre scientific workers such as Lamb[24], Thompson[25], Walker[26], Nicholson[27,28,29,30], Bromwich[31], Mie[32], Debye[33], Watson[34,35,36], and White[31]. Mie was the first to compute complete numerical values from the complex analytical expressions. Debye developed the method of steepest descent which was earlier originated by Riemann[38]. White extended the method of Watson transformation and obtained a solution in terms of a contour integral containing of the reflected awve plus a residue series which contained that are later known as the 'Creeping waves'. The classic computations of proudman, Doodson and Kennedy[39] in the case of a sphere are worth mention because of the superhuman labour they had to undergo as they did not have access even to a desk-type calculator, not to speak of modern high speed computer, which could do all their calculations in a few seconds.

A very significant study was conducted on electromagnetic wave scattering by integral equation method and integral/variational method by A.M. Patwary in his Ph.D. thesis[40] which has immensly contributed to this field. It contains brief literature review on the subject upto 1967 having 139 important references.

A bibliography[41] consists of 1420 references from over 275 journals and other sources in the open literature, and represents contribution from over 1225 authors compiled at the Georgia Institute of Technology for the time period 1957-1964.

During the early fifties, two new developments gave an added impetus to the solution of problems in scattering and diffraction. One of these is the celebrated variational formulation by Levine and Schwinger, and other is Keller's development of the geometrical theory of diffraction. The fact that an integral equation can be formulated as a variational principle was stated as early as 1884 by Volterra[67] but it has not been used for practical calculation in scattering problems until when Schwinger discovered that the amplitude of the scattered is closely related to the quantity whose variation has to be considered. In a series of papers[42,43] Levine and Schwinger demonstrated the power of the variational method in solving the problem of diffraction by an aperture in an aperture in an infinite plane screen, using very simple trial functions.They expanded the field in a set of functions and solved the simultaneous linear equations resulting from the variational principle for the coefficients. Their variational solution of the circular aperture problem agrees quite well with an earlier exact solution due to C.J.Bouwkamp[44]. Following Levine and Schwinger's formulation, Huang et al[45] gave a variational solution for circular and elliptical apertures. In 1950, Papas[46] published his paper on diffraction by an infinitely long circular cylinderusing variational method. The most extensive work on circular conducting cylinders has been done by Kodis using variational method[47,48]. Kouyonmjian[49,50] used a variational method to determine an approximate formula for the back-scattering cross-sections of the thin wire loops. In 1955, Cohen[51] found out that solution in scattering problems using the stationary property of the 'Reaction Concept'.

One of the earliest and perhaps easiest method of computing the scattered field and scattering cross-section of objects that are large compared to the wavelength, is the method of geometric optics[52,53]. It makes use of reflected and refracted rays in describing the behaviour of electromagnetic waves. The basic assumption of geometric optics is that an incident ray is reflected by the scatterer as if the latter's surface were plane at the reflection point. The diffracted rays are produced by incident rays which hit edges,corners, or vertices of boundary surfaces or which graze such surfaces. Schensted[54] obtained two correction terms of the Luneberg-Kline series in axial backscatter from a perfectly conducting semi-infinite body of revolution. Bremmer[55] ,Siegal et al[56] and others have utilized physical optics approximation to solve several diffraction problems at high frequency end of the spectrum. Keller[57] has proposed an extension of geometrical optics which includes the diffracted rays in addition to the usual rays of geometrical optics. The geometrical-optics solution of a problem is of importance for the radio engineer in that it represents the high-frequency limit of more rigorous field problem and often yields quick approximate answers when more elaborate methods are unworkable. A more general approach to the low-frequency problem has been proposed by Stevenson[58].

In the last two decades, the widespread availability of high speed digital computers revolutionized the method and scope of handling scattering problems. The scattering problems for scatterers of arbitrary shape and size, and an accurate solution for the mid-frequency range which were thought to be beyond the range of exact theoritical analysis, can now be numerically solved substantially in an exact manner, using integral equation techniques. In 1963 Mei and Van Bladel[59,60] calculated the surface current density and scatterin cross-section of perfectly conducting infinilely long rectangular cylinders. Banaugh and Goldsmith[61] solved the problem of diffraction of steady acoustic waves by surfaces of arbitrary shape using integral equation method. In 1964, Andreasen[62,63] gave a numerical solution for the associated problems of e.m. wave scattering by infinite parallel cylinders of arbitrary cross section. Richmond[64] obtained solutions for arrays of infinitely long thin parallel wires and also calculated[65] the field distribution induced in a dilectric cylinder of infinite length using numerical methods. Waterman[66] has published a matrix formulation of electromagnetic scattering for solving the surface current density and the scattered field of a smooth perfectl conducting axially symmetric obstacle. In electromagnetics, the discretization for transforming an integral equation to a matrix equation is commonly accomplished using pulse function as a basis[69,70]. Two new methods termed 'plane-wave correction' and cylindrical-cell correction' are developed for improving the convergence of moment-method solution in electromagnetics[71]. Mullin, Sandberg and Velline[72] calculated the monostatic and bistatic cross-section for perfectly conducting infinite cylinders of arbitrary shape using point matching technique. They obtained a fourier series expansion of the scattered field in terms of Hankel functions, and then satisfied the appropriate boundary conditions at a finite number of points. The Fourier coefficients were then obtained by matrix inversion.

There are several numerical methods available for the problem of scattering by dielectric bodies of arbitrary shape[65,68]. Among these methods the extended boundary condition (EBC) method [72] originally developed by Waterman is particularly effective when applied to a homogeneous and isotropic dielectric object of convex and relatively smooth surface. Barber has applied the EBC method to an investigation of the differential scattering characteristics[73],and the internal resonance absorption of dielectric bodies[74]. As for the cylindrical dielectric objects, the relatively simple integral equations can be obtained from the usual surface boundary conditions New integral equations developed by Morita[75,76] which is applicable in the exterior region of the circumscribed circle.Radiation pattern from dielectric rod antennas[77,78], composite dielectric bodies[79],and lossy multilayer bodies[80] have been investigated.

Electromagnetic scattering from a moving body generally is very difficult problem to solve. Due to the motion of the scatterer, the constitutive relationships at a fixed position in space will then vary with time, thus introducing modulation in the scattered fields. To make thing worse, the boundary conditions for a moving body are more complicated than those encountered in problems involving stationary bodies only. The scattering of linearly polarized electromagnetic waves by a slowly rotating rectangular metal plate is investigated theoretically and experimentally. Recently, similar problems have been discussed from the point of view of radar signature applications. Back scattering of electromagnetic signals by rotating or vibrating short wires are discussed theoretically [81,82]. Limited theoretical and experimental results on bistatic and monostatic scattering of plane electromagnetic waves by a slowly rotating conducting wire of longer length have been reported[83].The scattering of electromagnetic waves by linearly oscillating conducting objects is analyzed[84]. As long ago as World War II, signal modulation by the rotating blades of an aircraft were observed and studied experimentally[85]. Recently a quasi-stationary method in conjunction with geometrical theory of diffraction (GTD) techniques is used for calculating the backscattered field from a large rotating conducting cylinder with arbitrary cross section[86].

Asymptotic approximation is very useful tool in studing the scattering and diffraction of an electromagnetic wave, since it is rather insensitive to the shape of the scatterer, its expression is usually simple,easy to evaluate numerically, and it sometimes admits simple physical interpretation. Zitron and Karp[87] proposes a higher order approximation method in multiple scattering by two cylindrical obstacle with an arbitrary cross section (ZK method) . Complete asymptotic expansion, which is a convergent series, was given by Twersky[88] and was applied by Young and Bertrand[89] for two circular cylinders. Another method to derive an approximate solution for multiple scattering has been developed by Karp and Russak (KR method)[90]. It is found from the numerical results for the scattering cross section for E-polarization, the ZK method gives very precise results even for rather large ka and small kd, where a is the radius of the cylinder, and the KR method is extremely accurate when applied to a thin scatterer, and can only be significantly improved by the ZK method[91].

The records of these years is something to be proud of in scientific applications, in engineering and in business enterprise. After all, we were able to build well on the foundations of Hertz and Maxwell. Thetechnical papers demonstrated that great stride have been made in the past years and provided a clear picture of how various infant aspects of microwave technology have matured. At the same time, however, they also illustrated the enormity of work that yet remain. In effect, old challenges have been met and conquered, but new ones are constantly being created. One may consider this situation as the existence of a 'Technical Medusa'.

1.3 CONTENT OF THE THESIS

This thesis contains computer solutions of microwave problems. Under the heading of microwave problems two most common phenomenon had been discussed. One is the electromagnetic wave propagation through waveguide of different uniform hollow waveguide of arbitrary cross section . The other one is the scattering of electromagnetic wave illuminating a cylindrical obstacle of different radius and having different incident angles of the impinging e.m. waves.

Chapter 2 describes concisely the scalar wave equation starting from Maxwell's equations .Applying proper boundary conditions equation for the scattered field has been formulated. These simultaneous equations are conveniently represented by the matrix.This matrix has been solved by Gaussian method of elimination.Solving that matrix we obtained the coefficients of the Hankel series.From that series Radar cross section and field pattern outside the scatterer has been obtained for direct method.

Chapter 3 contains the scattering problem using finite element method of different forms and the finite difference method. There we have found the radar cross section variation with the incident angles,nature of variation of Hankel's coefficient with the length of the Hankel series.Field variation in space outside the obstacle are shown with phases and amplitudes.

In chapter 4 waveguide problems had been discussed by different methods namely 9 point finite difference method,5 point finite difference method,finite element method triangular shaped and finite element method rectangular shaped.Here effect of mesh size, iteration number,initial values are compared for rectangular shaped waveguide of different aspect ratios.These are also calculated for single ridge waveguide and double ridge waveguide for all those four methods.

Chapter 5 contains the conclusions and recommendations and Chapter 6 contains the bibliography.

CHAPTER 2 SOLUTION OF SCATTERING FROM INFINITE CIRCULAR CYLINDERS BY DIRECT METHODS

2.1 Preliminaries

Bistatic radar cross section has been calculated for perfectly conducting circular cylinders with different diameter and various incident angles of the impinging electromagnetic wave.The two-dimensional scalar wave equation is derived starting from Maxwell's equations. A set of linear equations is then obtained by setting proper boundary conditions.The matrix is solved by Gauss elimination method to find the coefficients of the Hankel series.Then radar cross-section and electromagnetic field pattern outside the scatterer is calculated .

2.2 Derivation of the Scalar Wave Equation

The starting point for the solution of any electromagnetic boundary value problem is the Maxwell's equations.These are given by

$$\nabla \times \underline{E} = -\partial \underline{B} / \partial t \qquad (2.1)$$

$$\nabla \times \underline{H} = -\partial \underline{D} / \partial t + \underline{J} \qquad (2.2)$$

$$\nabla \cdot \underline{D} = \rho \qquad (2.3)$$

$$\nabla \cdot \underline{B} = 0 \qquad (2.4)$$

where $\underline{E}$ is the Electric field intensity, $\underline{H}$ is the Magnetic field intensity, $\underline{D}$ is the electric displacement, $\underline{B}$ is the magnetic induction, $\underline{J}$ is the current density. $\nabla \times$ and $\nabla \cdot$ are respectively the curl and divergence operators. All field quantities are considered to be varying sinusoidally in time according to e^{-jwt} where $w=2\pi f$ f being the frequency, t, the time and j is the imaginary unit vector, has the property that $i^2 = -1$. The term e^{-jwt} is suppressed but is understood throughout. It is further assumed that the parameter, μ and ε , are respectively the permeability and the permittivity of the medium are independent of time. Further assuming, a free space with zero conductivity and constitutive parameters μ, ε are independent of position. Thus, equation (2.1) to (2.4) reduces to

$$\nabla \times \nabla \times \underline{E} = k^2 \underline{E} \tag{2.5}$$

$$\nabla \times \nabla \times \underline{H} = k^2 \underline{H} \tag{2.6}$$

where k is the propagation constant, has the property $k = \omega/c = 2\pi/\lambda = \sqrt{\omega^2 \mu\varepsilon}$ for free-space propagation.

The solution of general scattering problems consists in solving equations (2.6) and (2.5) , together with the appropriate boundary conditions on the surface of the scatterer. The solution must also satisfy Sommerfeld's radiation condition at infinity for proper behaviour. The boundary conditions for perfectly conducting scatterers are given by :

At the junction between two media, the boundary conditions are, for an E wave in the absence of surface magnetic currents,

1. e_t continuous
2. $(1/\mu)(\partial h_t/\partial n)$ continuous

At the surface of a perfect conductor, the boundary condition reduces to $e_t = 0$. Then the surface carries a z-oriented electric current under time-harmonic conditions.

The boundary condition for an H wave at the surface of a perfect conductor produced by transverse electric currents are, for time-harmonic phenomena, the external field satisfies the condition $\partial H_n/\partial n = 0$.

2.2 Scalar wave equation in two-dimensions

A two-dimensional problem is defined to be one in which there is no variation in one coordinate direction. The geometry of the scatterers under consideration is shown in Fig.2.1. The cylinder is infinitely long and the axis of the cylinder coincide with the z-axis. Under these assumptions, therefore, there is no variation in z-direction, i.e.,

$$\frac{\partial}{\partial z} = \frac{\partial^2}{\partial z^2} = 0 \qquad (2.7)$$

and the reduced wave equation for two dimensional problem is given by

$$\nabla_t^2\phi + k^2\phi = 0 \qquad (2.8)$$

where ∇_t is the two-dimensional Laplacian operator. Hence forward, we shall drop the subscript 't' from ∇_t for the sake of simplicity

The part of the boundary condition which is common to all scattering problems is that, at large distances from the scattering region, the scattered field approaches a wave diverging from a point source located in the scattering region. And this Sommerfeld's radiation condition for two-dimensional problem reduces to

$$\lim_{R\to\infty} \sqrt{R} \left| \frac{\partial\phi}{\partial R} - ik\phi \right| = 0 \qquad (2.9)$$

2.3 Formulation for the Scattered Fields

When a plane electromagnetic wave impinges on a scattering body, the simple relation between the electric and magnetic fields of the plane wave in free space is modified due to currents induced in the scattering body. These induced currents will radiate scattered fields both interior to and exterior to the body, in such a way that the well known boundary condition for the tangential electric and magnetic fields at the surface of the scattering body will be satisfied.

Considering an incident electromagnetic wave $E_z^{\,i}$ impinging on the closed, perfectly conducting surface Γ of Fig.2.2 in otherwise free space, it is assumed throughout that Γ is sufficiently regular that Green's theorem is applicable, and that Γ possesses a continuous, single-valued normal $\underline{n}$ at each point.

For parallel polarization, all field components can be generated from the electric field E_z along the cylinder axis. This field component must satisfy the homogeneous scalar wave equation (2.8) with Dirichlet boundary condition, $E_z = 0$ on the contour of the scattering cylinder. Since the cylinder is assumed to be infinitely long in the z-direction and perfectly conducting, the consequent scattered electric field has also only a z-directed component, E_z^s . Therefore, the nature of the field is scalar. The total field, E_z^t at any point on the scattering contour or outside is given by

$$E_z^t = E_z^i + E_z^s \qquad (2.10)$$

where E_z^i and E_z^s are, respectively, the incident and scattered fields. At any point (r,ϕ) on the contour of the cylinder or outside (Fig. 2.2) , the incident field for a unit plane wave is given by,

$$E_z^i = e^{jkr \operatorname{Cos}(\phi+\psi_o)} \qquad (2.11)$$

Its circumscribing circle touching the conducting cylinder is identical with its contour. We can obtain an infinite expansion for the scattered field E_z^s, valid on the circumscribing circle and outside it, in terms of Hankel functions of the first kind and exponential functions as follows

$$E_z^s = \sum_{n=-\infty}^{+\infty} \bar{a}_n H_n^{(1)} (kr)\, e^{jn\phi} \qquad (2.12)$$

This expansion guarantees that the correct radiation condition at infinite is satisfied, and that E_z^s also satisfies the scalar wave equation (2.8).

Now the total scattered field is

$$E_z^t (r,\phi) = e^{jkr \operatorname{Cos}(\phi+\psi_o)} + \sum_{n=-\infty}^{+\infty} \bar{a}_n H_n^{(1)}(kr) e^{jn\phi} \qquad (2.13)$$

From the Dirichlet boundary condition $E_z=0$ on the contour of the conducting cylinder, we get

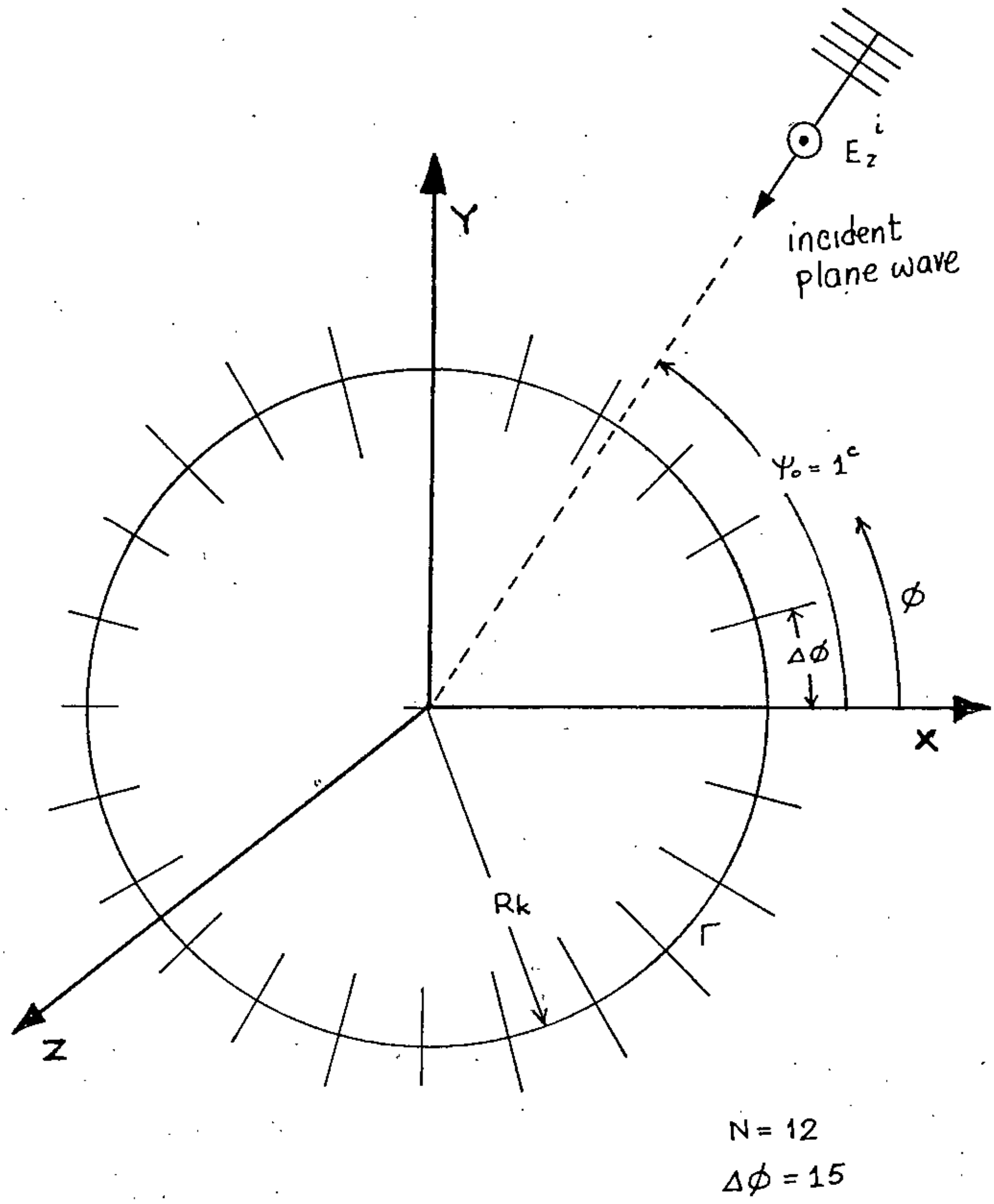


Fig. 2.6 INCIDENT ELECTROMAGNETIC WAVE ON A CONDUCTING INFINITE CIRCULAR CYLINDRICAL OBSTACLE.

$$E_z^t(r_o, \phi_p) = e^{jkr_o \mathrm{Cos}(\phi_p + \psi_o)} + \sum_{n=-\infty}^{+\infty} \bar{a}_n H_n^{(1)}(kr_o) e^{jn\phi_p} = 0 \qquad (2.13)$$

Solving the linear equation (2.13) for different points ϕ_p on the boundary we can complete the Fourier expansion coefficients in equation (2.13). But to solve these infinite numbers of unknown coefficients we have to solve a matrix of infinite dimension. For practical purpose we can truncate the infinite series and thus making that matrix equation less complex, requiring less memory and computing time.

$$E_z^t(r_o, \phi_p) = e^{jkr_o \mathrm{Cos}(\phi_p + \psi_o)} + \sum_{-N+1}^{N} \bar{a}_n H_n^{(1)}(kr_o) e^{jn\phi_p} \qquad (2.14)$$

In this way some error is being introduced but the converging nature of the series introduces reasonable amount of error depending upon the shape and size of the scatterer and the number N.

To solve this 2N unknown complex coefficients we need to solve 2N numbers of complex equations, that is we have to take 2N numbers of points on the contour Γ of the scatterer. Solving this complex matrix of order 2N x 2N we can find the complex coefficients of the fourier coefficients of the Hankel function.

Once these coefficients can be obtained we can calculate scattered field distribution outside the scatterer. Now we can also calculate scattering cross section of the obstacle in any direction , for any incidence angle of incident field ψ_o. The back scattering or monostatic cross section, σ_B is given by

$$\sigma_B = \frac{4}{k} \left| \sum_{-N+1}^{N} \bar{a}_n(\psi_o)\, e^{jn(\pi/2 - \psi_o)} \right|^2 \qquad (2.15)$$

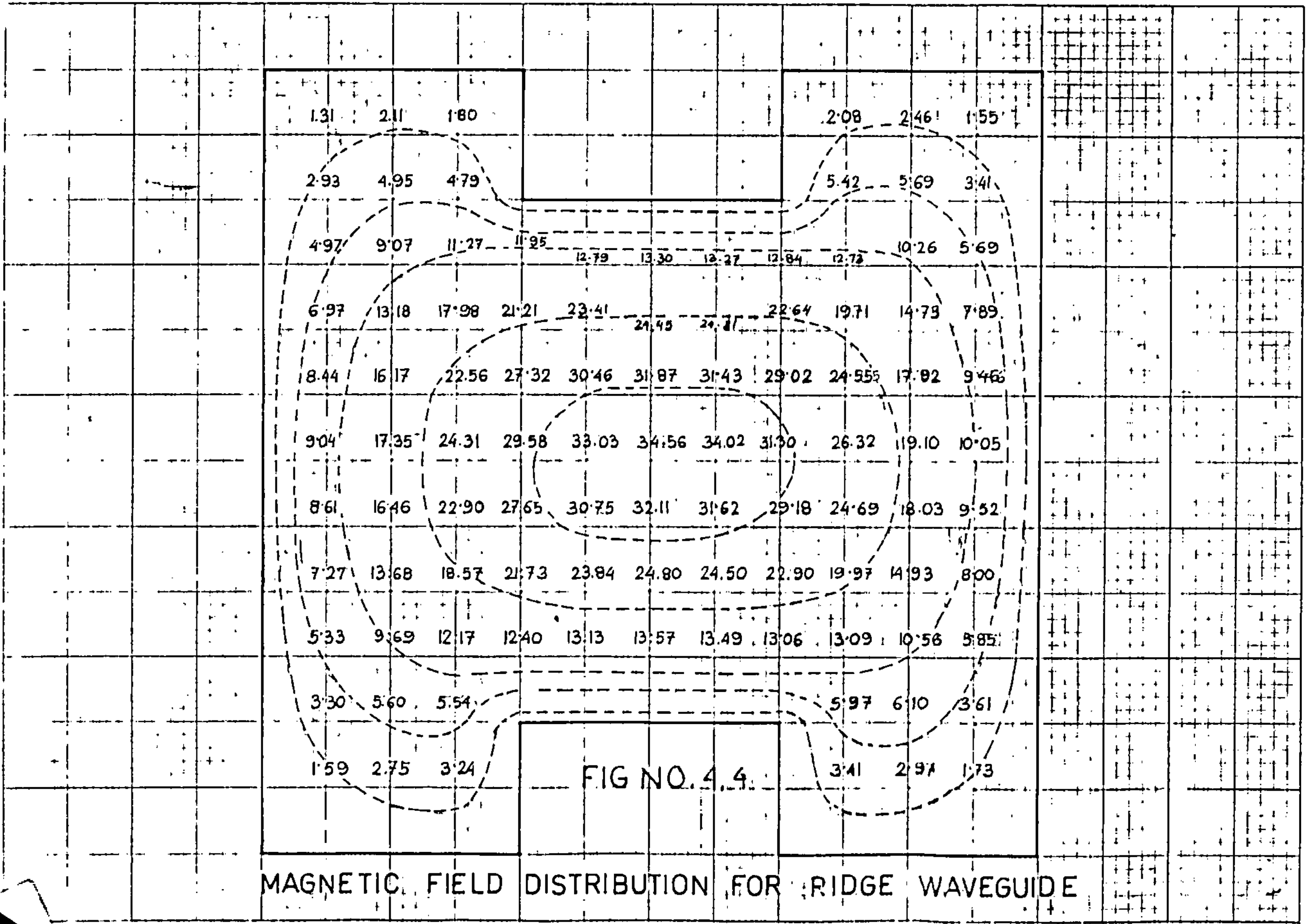
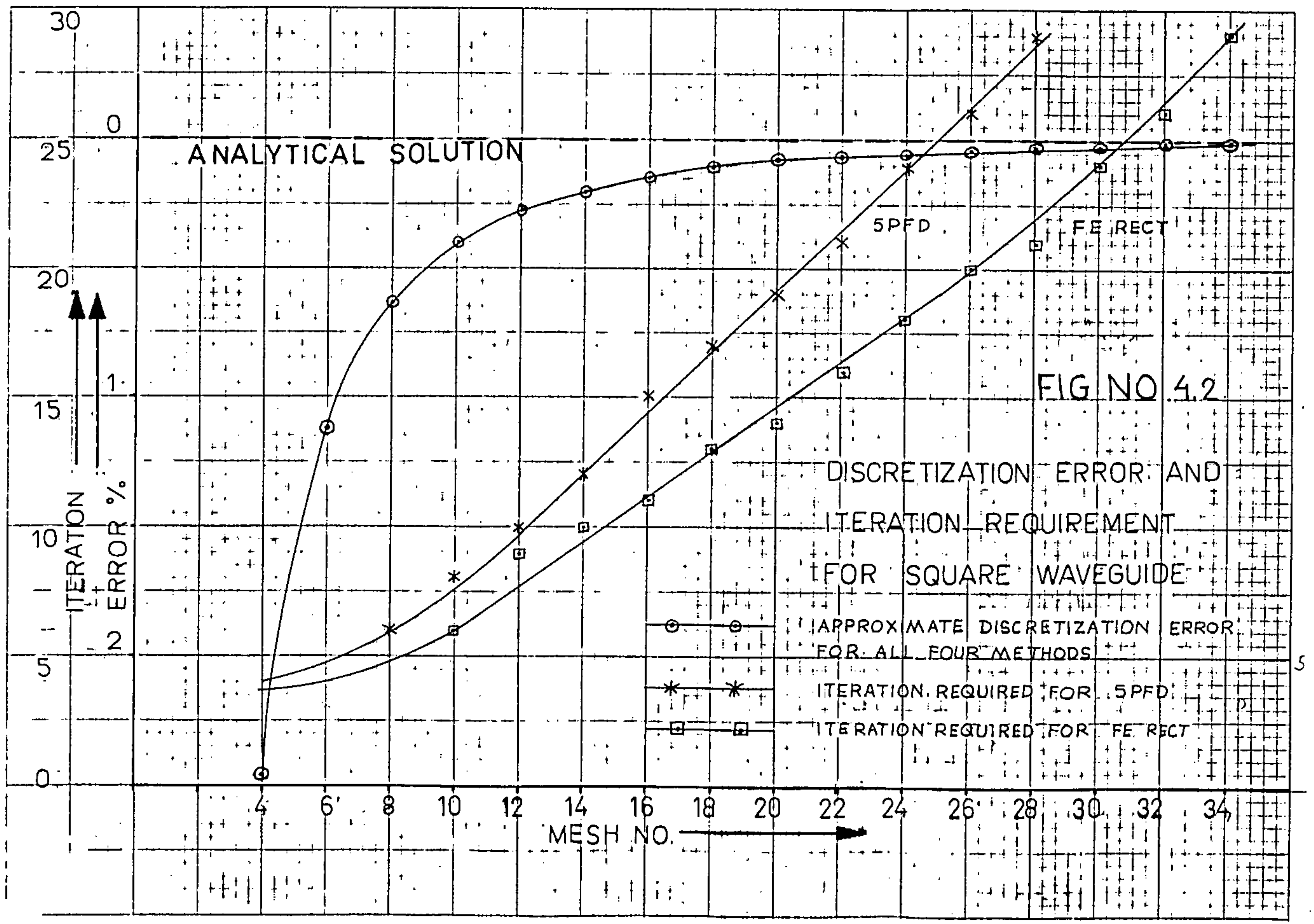


FIG NO. 4.4.

MAGNETIC FIELD DISTRIBUTION FOR RIDGE WAVEGUIDE


F. L. MOSELEY COMPANY
ANALYTICAL SOLUTION
5PFD
FE RECT
FIG NO 4.2
DISCRETIZATION ERROR AND
ITERATION REQUIREMENT
FOR SQUARE WAVEGUIDE
APPROXIMATE DISCRETIZATION ERROR
FOR ALL FOUR METHODS
ITERATION REQUIRED FOR 5PFD
ITERATION REQUIRED FOR FE RECT
ITERATION
ERROR %
MESH NO.
30
25
20
15
10
5
0
1
2
4
6
8
10
12
14
16
18
20
22
24
26
28
30
32
34

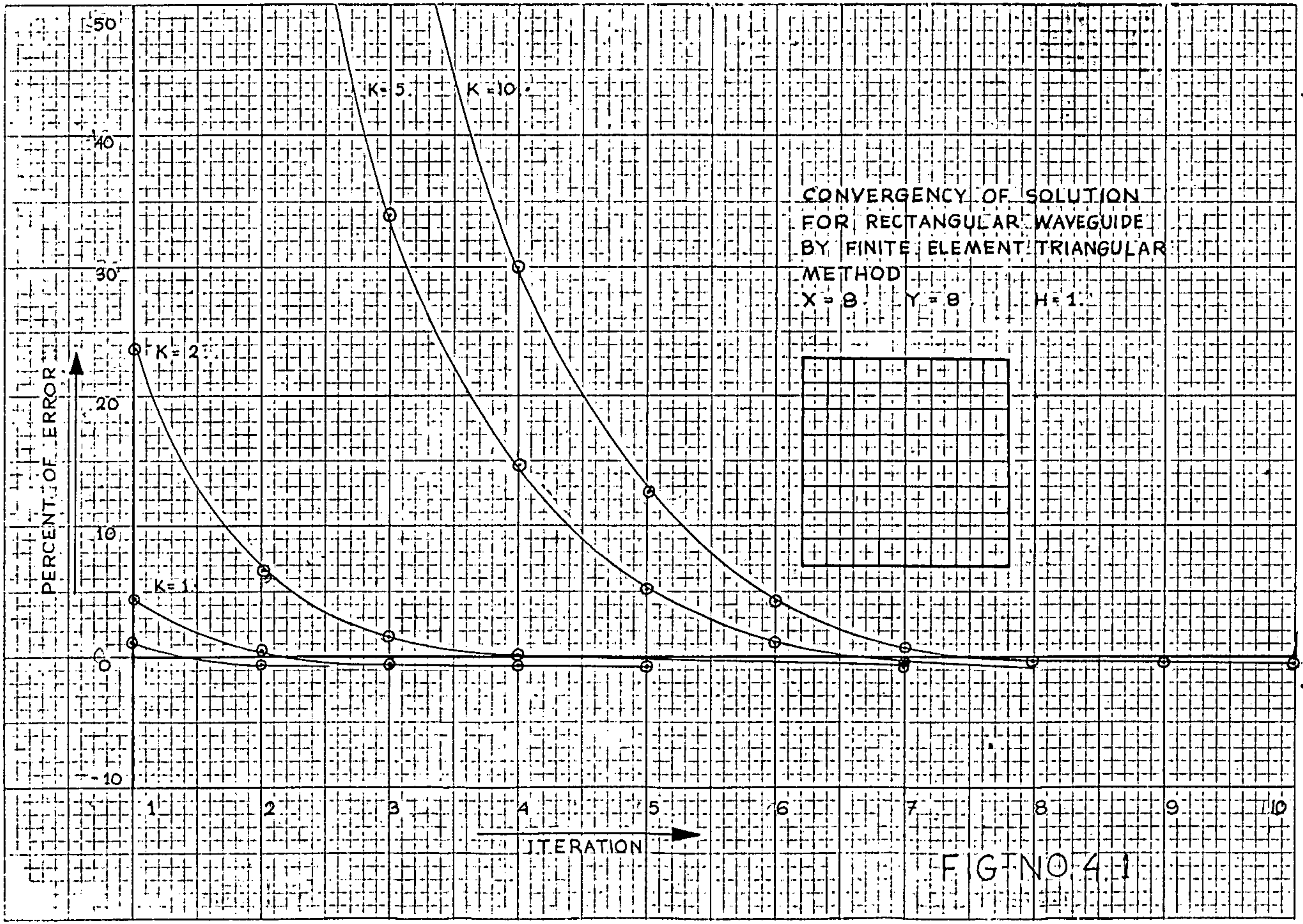

CONVERGENCY OF SOLUTION
FOR RECTANGULAR WAVEGUIDE
BY FINITE ELEMENT TRIANGULAR
METHOD
X = 8   Y = 8   H = 1
K = 5
K = 10
K = 2
K = 1
PERCENT OF ERROR
ITERATION
50
40
30
20
10
0
-10
1
2
3
4
5
6
7
8
9
10

FIG NO 4.1

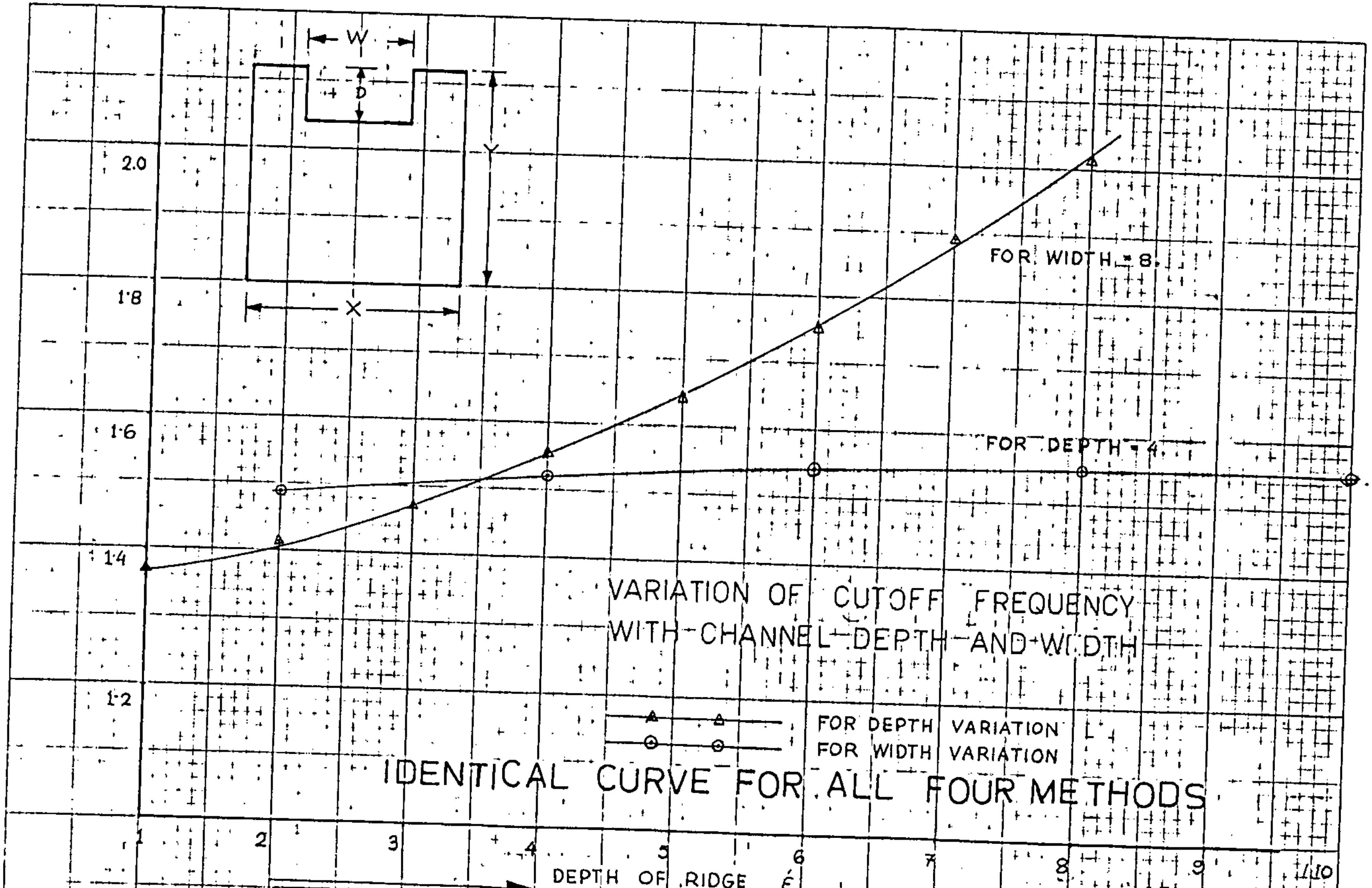

W
D
Y
X
2.0
1·8
1·6
1·4
1·2
FOR WIDTH = 8.
FOR DEPTH = 4
VARIATION OF CUTOFF FREQUENCY
WITH CHANNEL DEPTH AND WIDTH
FOR DEPTH VARIATION
FOR WIDTH VARIATION
IDENTICAL CURVE FOR ALL FOUR METHODS
1
2
3
4
5
6
7
8
9
10
DEPTH OF RIDGE
WIDTH OF RIDGE

FIG NO. 4.3

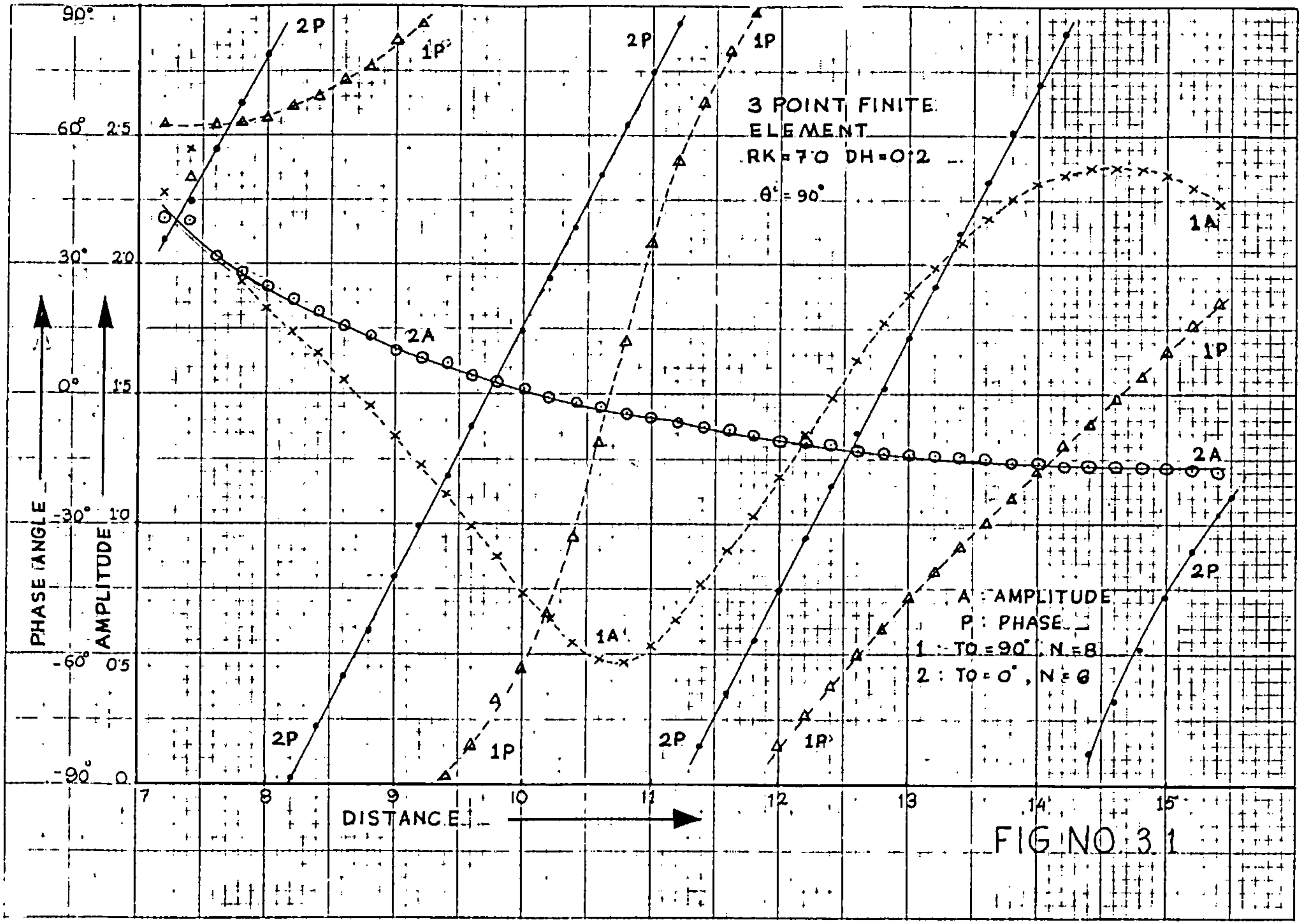

3 POINT FINITE ELEMENT
RK = 7.0 DH = 0.2
θ = 90°
A : AMPLITUDE
P : PHASE
1 : TO = 90°, N = 8
2 : TO = 0°, N = 6
2P
1P
1A
2A
PHASE ANGLE
AMPLITUDE
90°
60°
30°
0°
-30°
-60°
-90°
2.5
2.0
1.5
1.0
0.5
0
7
8
9
10
11
12
13
14
15
DISTANCE

FIG NO. 31

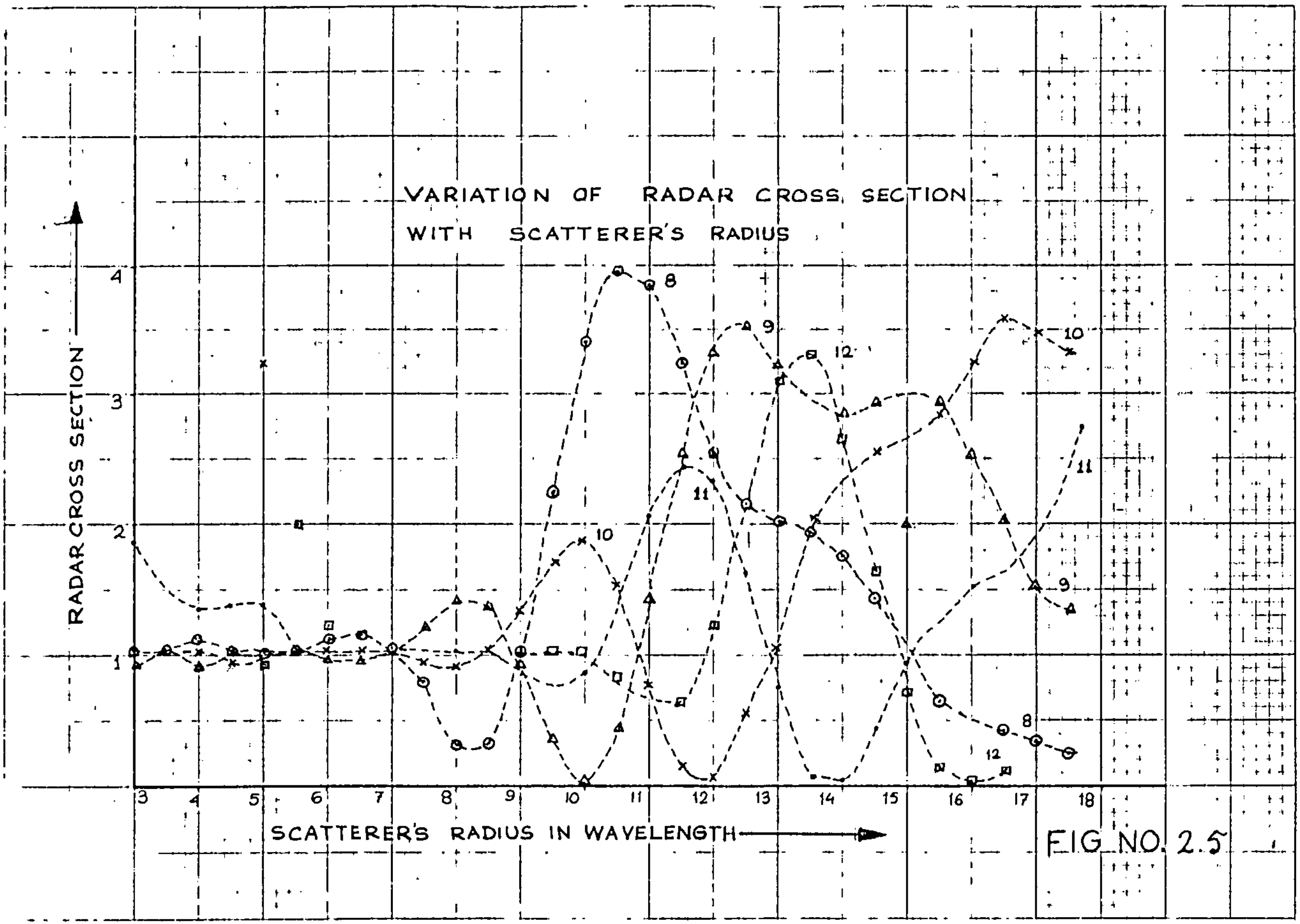

VARIATION OF RADAR CROSS SECTION
WITH SCATTERER'S RADIUS
RADAR CROSS SECTION
SCATTERER'S RADIUS IN WAVELENGTH
1
2
3
4
3
4
5
6
7
8
9
10
11
12
13
14
15
16
17
18
8
9
10
11
12

FIG NO. 2.5

2.5 Development of the Matrix

Approximation of the operator ∇ over a finite set of points at which the field is to be computed as in finite differences or approximation of the field estimate as in variational methods, or in direct method, causes the problem to be modeled by simultaneous equations. These equations are conveniently represented by matrices. Basically we obtained a 2N x 2N complex matrix to solve the problem. It was converted to 4N x 4N real matrix by separating real and imaginary components of the fourier coefficients and also separating real and imaginary part of the linear equations.

2.6 Solution of the Matrix

To obtain a solution, we must employ a process that produce the required accuracy in a finite number of steps performed in a finite time upon machines having finite word length and store.There are various methods for solving a matrix equation in a digital computer.They can be broadly classified into two categories: direct and indirect methods.Among the direct methods, there are Gaussian method,method of principal elements, inversion of matrix, square root method, scheme of Khaletsky and many others. Among the iterative methods, we can name Jacobi iterative, Gauss-Seidel, method of successive over relaxation and many others.For this particular problem, the Gaussian method of elimination has been used to solve the problem.

This is one of the most popular algorithms, a sequence of mathematical operations, for the solution of systems of linear equations .It cosists of two parts - elimination or traingularization and back substitution. In this method only $n^3/3$ multiplications are required to solve a system of n real, linear equations. This simple procedure glosses over certain difficulties that may occasionally occur. If one of the pivots is zero, it is impossible to employ it in clearing out a column. The cure is to rearrange the sequance of equations such that the pivot is non zero . One can appreciate that even if the pivot is nonzero, but is very small by comparison with other numbers in its column,numerical problems will cause error in the solution because of the limited word-length store

TABLE – 2.1

Variation of Radar Cross-Section with Obstacle radius

RK	N= 6 TO=180^0	N= 6 TO=0^o	N= 6 TO= 90^o	N= 8 TO= 0^o
3.0	1.05212	1.05149	1.05135	1.03825
3.5	1.04738	1.04525	1.04547	1.04019
4.0	0.99272	0.99336	0.99263	1.11838
4.5	0.86443	0.86491	0.86464	1.01497
5.0	0.77368	0.77386	0.77402	1.01128
5.5	1.01232	1.01149	1.01029	1.03878
6.0	1.71422	1.71408	1.71403	1.12021
6.5	2.60004	2.60464	2.60194	1.19100
7.0	3.16102	3.16586	3.16450	1.07939
7.5	3.05585	3.06151	3.05963	0.70421
8.0	2.34831	2.35207	2.35059	0.30842
8.5	1.41745	1.41927	1.41852	0.33726
9.0	0.66846	0.67115	0.66942	1.04829
9.5	0.28536	0.28520	0.28520	2.25263
10.0	0.18588	0.18114	0.18221	3.40503
10.5	0.17066	0.17099	0.17071	3.99490
11.0	0.11890	0.12062	0.12017	3.85347
11.5	0.04738	0.04711	0.04734	3.22386
12.0	0.03783	0.03744	0.03765	2.54738
12.5	0.13138	0.13114	0.13129	2.14151
13.0	0.27720	0.27723	0.27729	2.01185
13.5	0.38656	0.38654	0.38644	1.97112
14.0	0.42175	0.42129	0.42162	1.78276
14.5	0.43408	0.43312	0.43362	1.41172
15.0	0.51286	0.51143	0.51231	x
15.5	0.70137	0.70074	0.70127	0.63415
16.0	0.95694	0.95699	0.95717	x
16.5	1.18561	1.18501	1.18535	0.40103
17.0	1.31880	1.31571	1.31868	0.35730
17.5	1.35563	1.35625	1.35564	0.26256

TABLE - 2.1 (Continued)

Variation of Radar Cross Section with Obstacle radius

RK	N= 9 TO= 0°	N= 10 TO = 0°	N= 11 TO = 0°	N= 12 TO = 0°
3.0	0.97231	1.03419	1.89369	1.03234
3.5	1.06251	1.04013	1.01263	1.08495
4.0	0.90921	1.01676	1.33479	-
4.5	1.02738	0.90212	1.39458	0.92494
5.0	1.02593	3.24800	1.40885	0.95280
5.5	1.01245	1.01303	1.00771	2.02292
6.0	0.99400	1.01783	1.01340	1.21181
6.5	0.98081	1.01733	1.01340	1.01794
7.0	1.04319	1.00209	1.01232	1.00688
7.5	1.22993	0.94510	1.02261	0.97254
8.0	1.42958	0.90767	1.03690	1.00186
8.5	1.38255	1.01091	1.01309	1.02517
9.0	0.93815	1.33660	0.87294	1.04506
9.5	0.35043	1.74792	0.77898	1.07681
10.0	0.06769	1.89351	0.89271	1.02883
10.5	0.47667	1.52425	1.37415	0.84183
11.0	1.44218	0.79303	2.08504	0.57706
11.5	2.58430	0.14541	2.49438	0.67564
12.0	3.35284	0.04666	2.34519	1.22604
12.5	3.53377	0.57906	1.62251	2.13931
13.0	3.24509	1.40383	0.73672	3.13336
13.5	x	2.20679	0.08792	3.30784
14.0	2.83106	x	0.04952	2.65829
14.5	2.93066	2.58918	0.46754	1.66685
15.0	1.99925	x	0.96042	0.70143
15.5	2.93836	2.83993	1.27040	0.11738
16.0	2.53684	3.24370	1.51218	0.02139
16.5	2.02528	3.60374	1.64530	0.11709
17.0	1.59528	3.49762	-	x
17.5	1.35982	3.34540	2.78728	x

TABLE - 2.2

Variation of Radar Cross Section with Incidence for RK=3. ,N=6

θ^i	BSC	θ^i	BSC	θ^i	BSC
0	1.05137	35	1.04887	75	1.03631
5	1.04936	40	1.04074	90	1.05135
10	1.04115	45	1.03638	105	1.03660
15	1.03575	50	1.03902	120	1.05113
20	1.03868	55	1.04675	135	1.03355
25	1.04680	60	1.05161	150	1.04810
30	1.05111			180	1.05212

θ^i is the incidence angle in degree

BSC is the Radar Cross Section

TABLE - 2.3

Variation of Radar Cross Section with N , for RK = 3.0

N	$\theta^i = 0^\circ$	$\theta^i = 90^\circ$
6	1.05137	1.05135
7	1.04772	1.04551
8	1.03825	1.05251
9	0.97231	0.94952
10	1.03419	1.08841
11	1.89369	1.19673
12	1.03234	1.01411

θ^i and N identifies ~~mesh~~ incident angle and the limit of Hankel series

TABLE - 2.4

Hankel coefficients for RK=5. with incidence angle -1. radians and N= 8

	Real part				Imaginary part		
a_0	0.24900			a_0	-0.43277		
a_1	0.49627	a_{-1}	0.90146	a_1	0.76433	a_{-1}	-0.13334
a_2	0.11991	a_{-2}	-0.10677	a_2	-0.03762	a_{-2}	-0.06612
a_3	0.22038	a_{-3}	-0.46341	a_3	0.90176	a_{-3}	-0.80434
a_4	-0.82593	a_{-4}	-0.22695	a_4	0.35099	a_{-4}	-0.86808
a_5	-0.36152	a_{-5}	-0.11639	a_5	-0.34460	a_{-5}	-0.48570
a_6	0.01759	a_{-6}	-0.07933	a_6	-0.17736	a_{-6}	-0.15972
a_7	0.03451	a_{-7}	-0.03701	a_7	-0.03101	a_{-7}	-0.02799
a_8	-0.00001			a_8	-0.00190		

TABLE - 2.6

Hankel coefficients for RK=4. with incidence angle 1. radian and N= 12

	Real part				Imaginary part		
a_0	0.21016			a_0	0.41321		
a_1	0.83845	a_{-1}	0.35618	a_1	0.00594	a_{-1}	-0.76379
a_2	-0.23101	a_{-2}	0.67083	a_2	0.69019	a_{-2}	-0.26888
a_3	0.28782	a_{-3}	-0.32436	a_3	0.15186	a_{-3}	-0.06419
a_4	-0.40884	a_{-4}	-0.80092	a_4	0.87390	a_{-4}	-0.53314
a_5	-0.86132	a_{-5}	-0.62012	a_5	-0.18277	a_{-5}	-0.62243
a_6	-0.11708	a_{-6}	-0.36007	a_6	-0.48509	a_{-6}	-0.34648
a_7	0.11922	a_{-7}	-0.16715	a_7	-0.14979	a_{-7}	-0.09366
a_8	0.05098	a_{-8}	-0.05006	a_8	-0.00909	a_{-8}	-0.00585
a_9	0.00919	a_{-9}	-0.00914	a_9	0.00359	a_{-9}	0.00368
a_{10}	0.00050	a_{-10}	-0.00079	a_{10}	0.00096	a_{-10}	0.00104
a_{11}	0.00003	a_{-11}	-0.00002	a_{11}	0.00013	a_{-11}	0.00014
a_{12}	0.00049			a_{12}	0.00002		

TABLE – 2.5

General data for Radar Cross Section

RK	N	Incident Angle θ^i in radians 0.0	0.5	1.0	– 0.5	– 1.0
3.0	6	1.05137	1.04	1.04	1.05110	1.05472
3.0	8	1.04416	1.04405	1.04716	1.05250	1.04456
3.0	12	1.05010	1.04247	1.03794	1.04318	1.04601
3.0	16	1.12700	1.14640	1.07490	1.00700	1.04001
4.0	6	0.99397	0.98851	1.01641	0.98714	1.00168
4.0	8	1.03941	1.02390	1.03049	1.01954	1.02000
4.0	12	1.03060	1.03481	1.02998	1.01915	1.02392
4.0	16	1.02450	1.02262	1.02170	1.02112	1.02651
5.0	6	0.77351	0.89965	0.73367	0.82674	0.74280
5.0	8	1.01217	1.03073	1.02693	1.01550	1.02567
5.0	16	1.07389	0.90692	0.96068	0.66068	0.88547
6.0	6	1.71377	1.58		1.68	
6.0	8	1.11986	0.85786	0.85931	0.96952	0.95733
6.0	12	1.01290	1.01620	0.99964	1.01929	1.01439
6.0	16	1.02301	1.01367	0.98787	1.02714	1.01952

RK is the obstacle radius in terms of wavelength and
N identifies mesh angle and the limit of the Hankel series .

FIELD AMPLITUDE VARIATION IN SPACE FOR RK = 3 θ = 180°

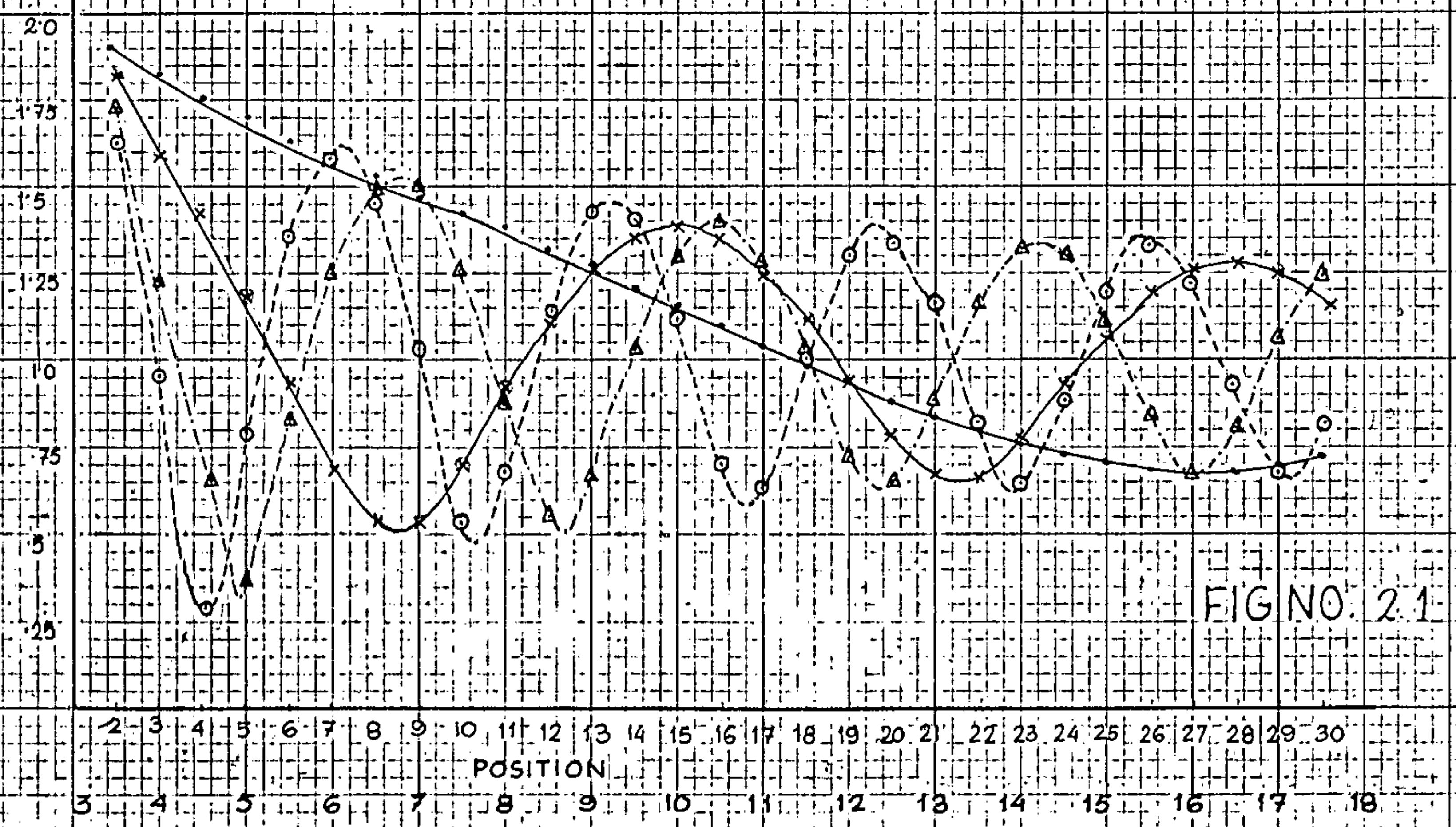


FIG NO. 21

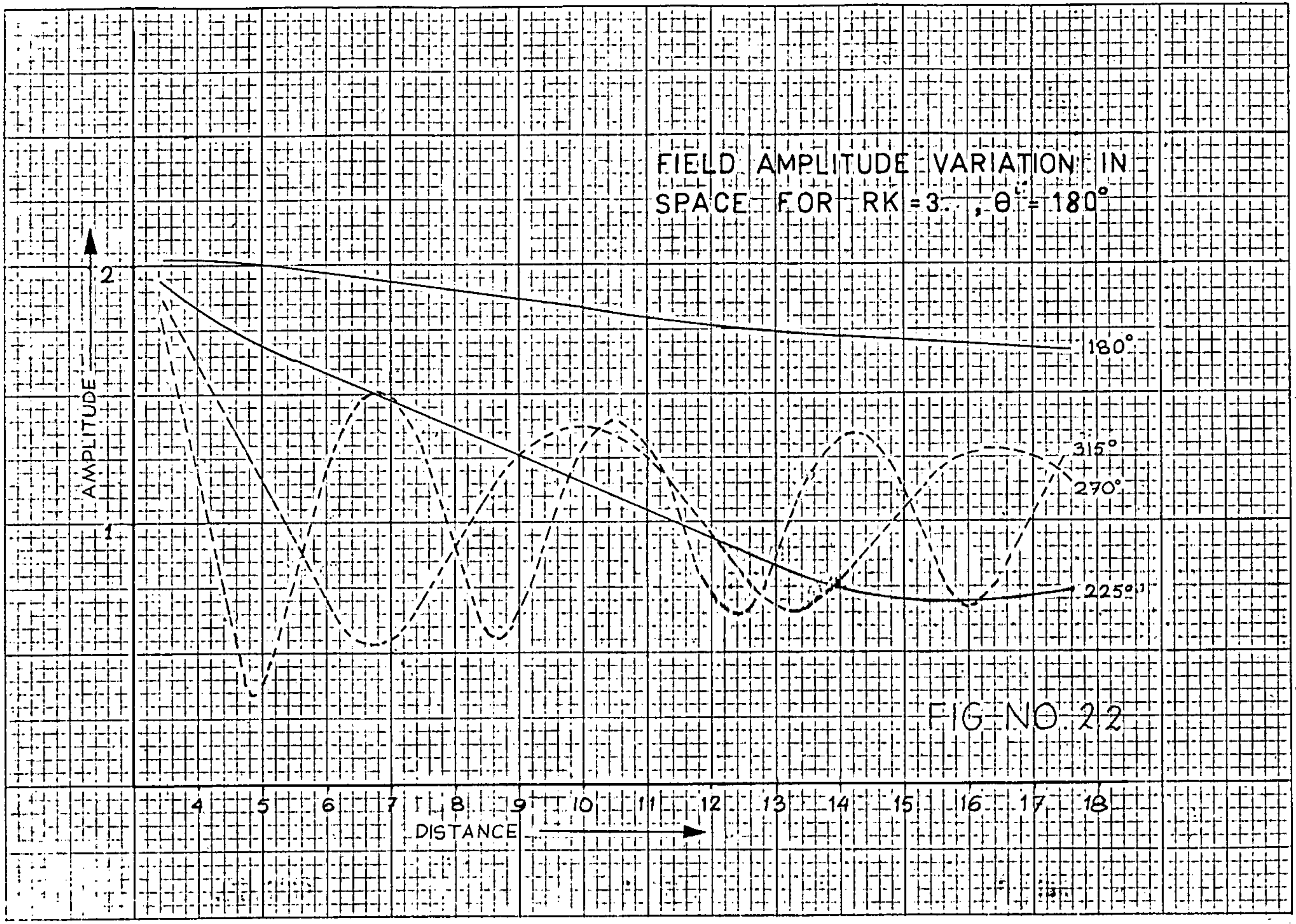

FIELD AMPLITUDE VARIATION IN
SPACE FOR RK = 3 , θ = 180°
AMPLITUDE
2
1
180°
315°
270°
225°
4
5
6
7
8
9
10
11
12
13
14
15
16
17
18
DISTANCE

FIG NO. 22

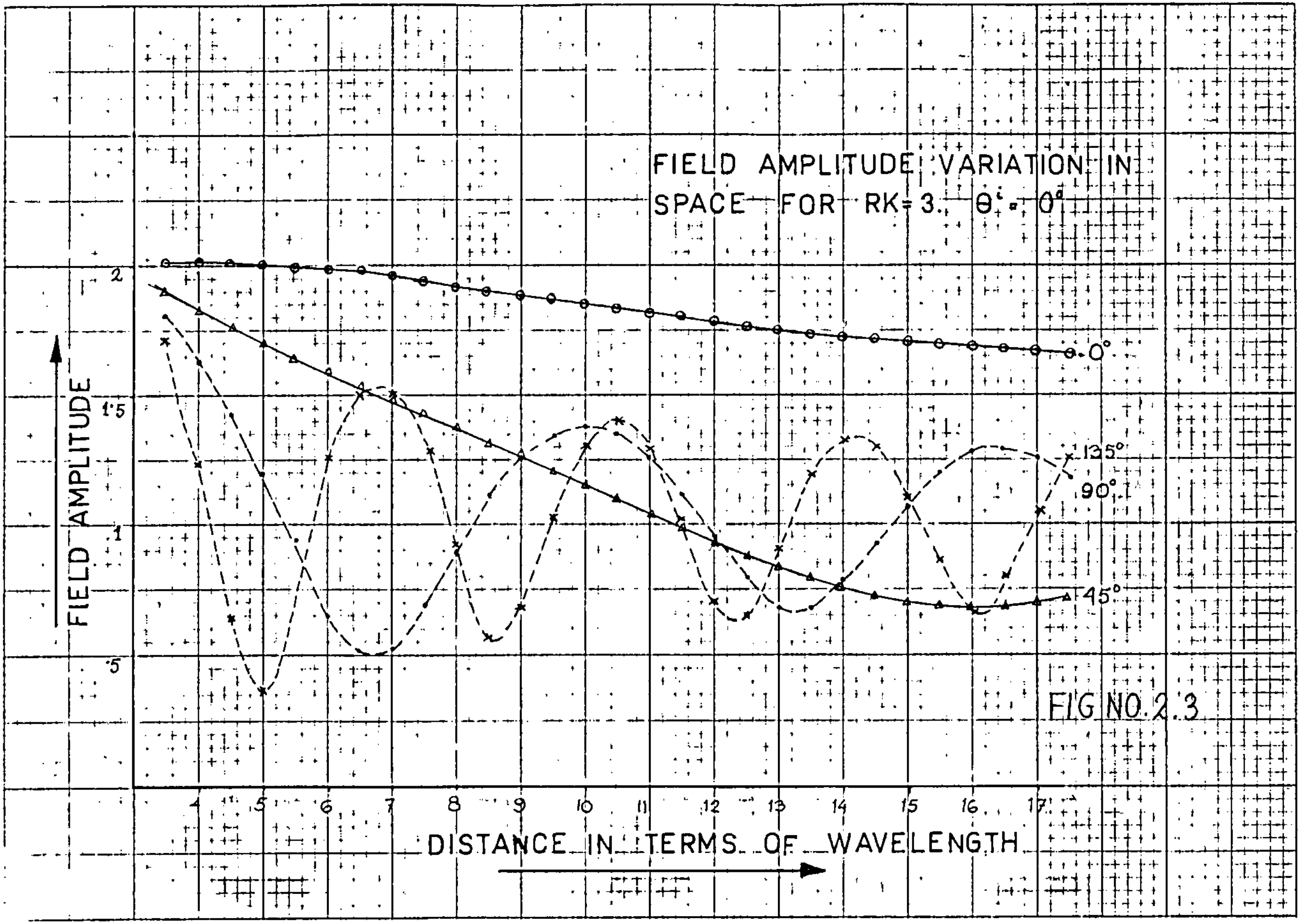

FIELD AMPLITUDE VARIATION IN
SPACE FOR RK = 3, θi = 0°
FIELD AMPLITUDE
2
1·5
1
·5
4
5
6
7
8
9
10
11
12
13
14
15
16
17
DISTANCE IN TERMS OF WAVELENGTH
0°
135°
90°
45°

FIG NO. 2.3

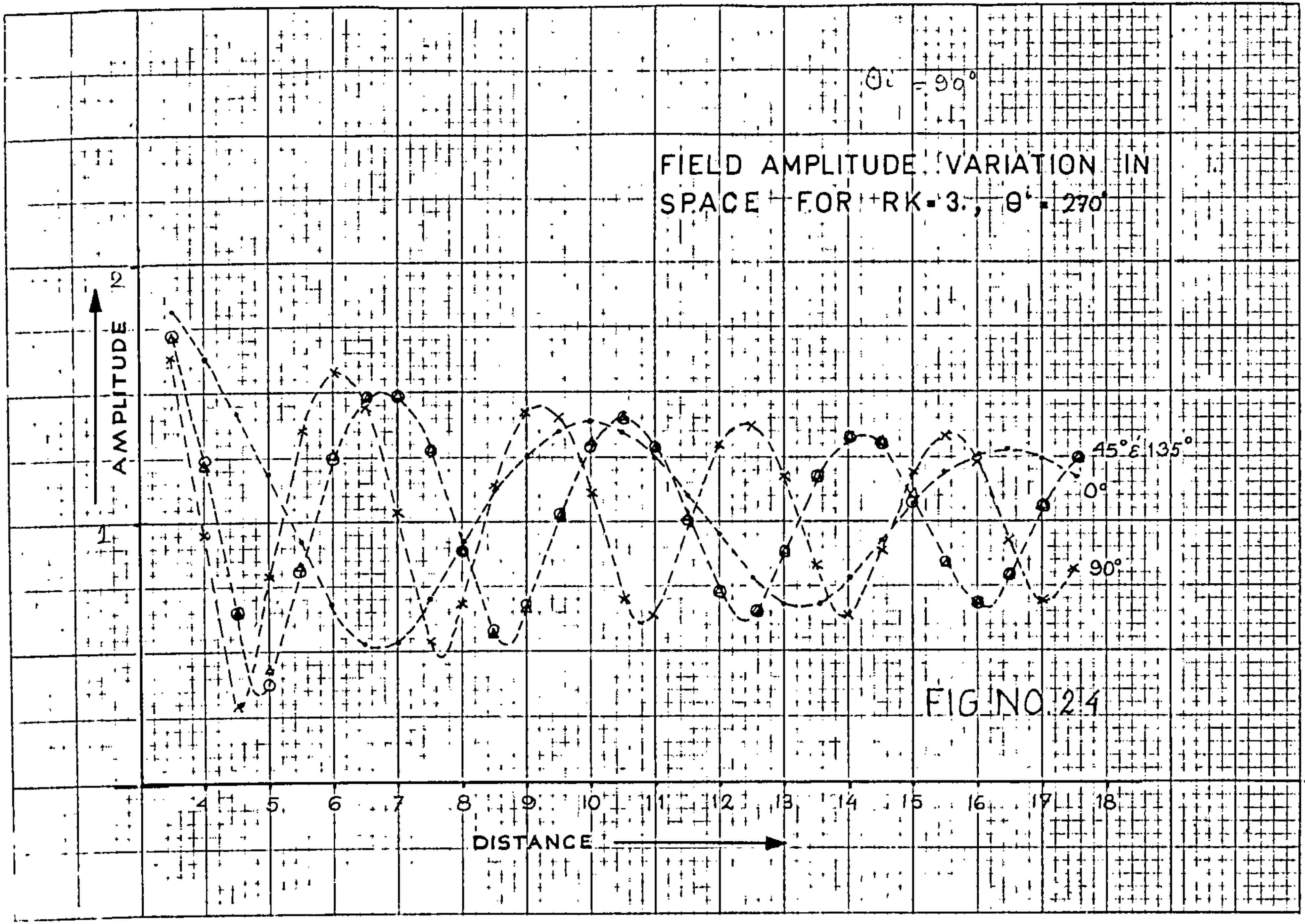

θi = 90°
FIELD AMPLITUDE VARIATION IN SPACE FOR RK = 3., θ' = 270°
AMPLITUDE
2
1
45° & 135°
0°
90°
4 5 6 7 8 9 10 11 12 13 14 15 16 17 18
DISTANCE

FIG NO. 24

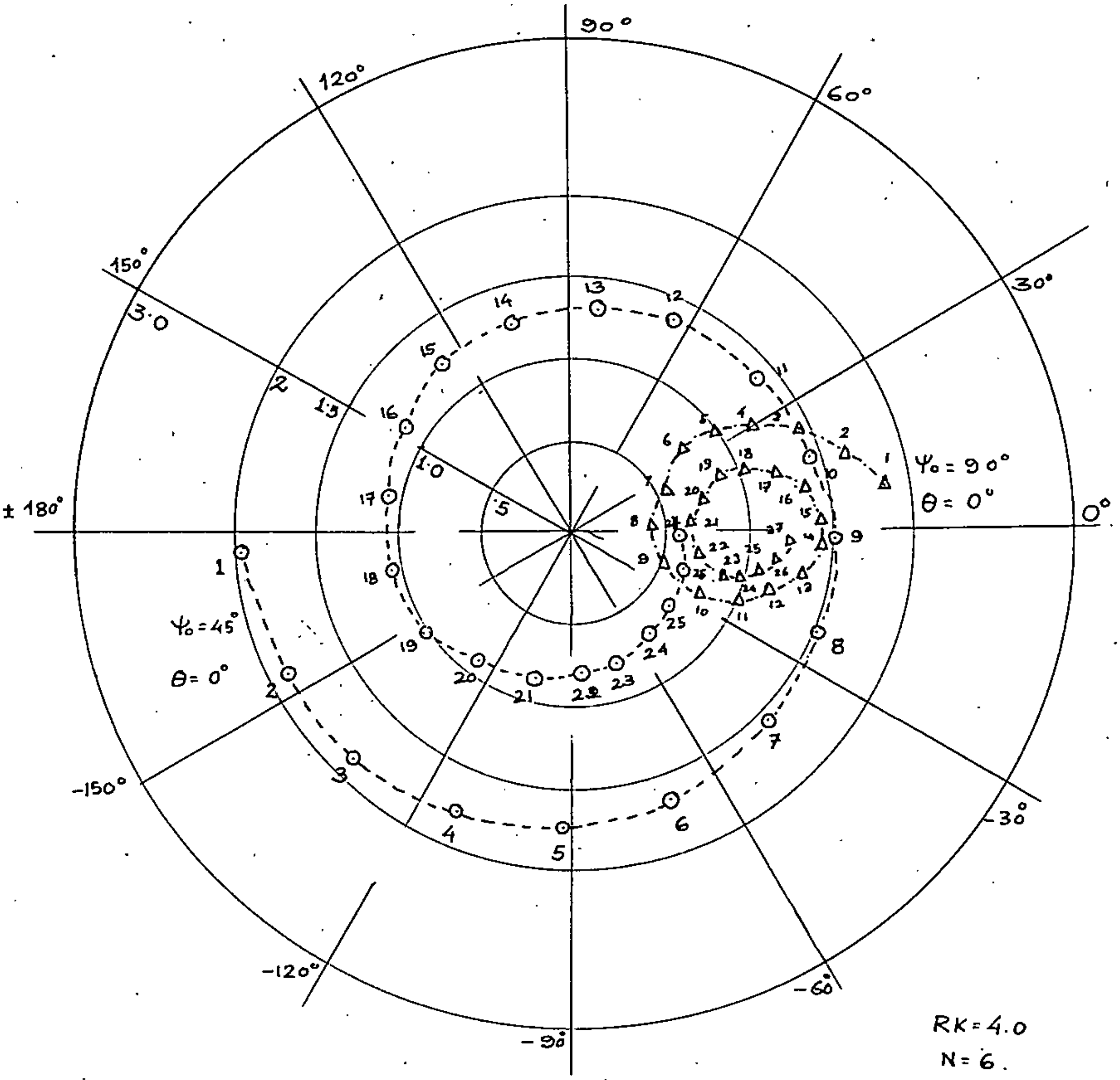


Fig. 2·7 . Phase and Amplitude of field variation outside the scatterer.

2.9 Discussion

Radar cross sections calculated by Direct method are very much satisfactory. These results are shown in Table 2.1. Variation of radar cross section with incidence angle is shown in Table. 2.2. There it can be observed that, radar cross section does not vary with incidence angle which is true from theoretical point of view. Variation of radar cross section with the number of Hankel series upper limitation taken, does not vary much, which is also reasonable for converging solutions. These are shown in Table. 2.3. Converging nature of Hankel series is shown in Table. 2.4. Variation of field amplitude and its phase angle in for different incident angles and different scatterer's radius are also had been calculated. Some of these are shown in Fig. 2.1 to Fig. 2.4. These results are very much satisfactory. Variation of radar cross section with scatterer's radius are shown in Table 2.1, Table 2.5, and Fig. 2.5. Here it can be observed that for larger scatterer's radius the result is oscillating in nature, but when higher number of Hankel series was taken steady state zone increased and peak variation reduced. But we could not calculate with a Hankel series larger than 16 as the memory capacity of IBM system 360 in Statistics Bureau exceeded. And also for higher order of Hankel functions for smaller argument, Neumann function increases exponentially, this increases the error in computation because of finite word length. Using Double Precision mode in computer this error can be minimized, but it will require double memory space. We can conclude that, within limited computer capacity and facility, this method of finding scattered field and radar cross section for circular cylindrical obstacles is very much satisfactory.

CHAPTER 3

SOLUTION OF SCATTERING FROM INFINITE CIRCULAR CYLINDERS BY FINITE ELEMENT AND FINITE DIFFERENCE METHODS.

3.1 Preliminaries

Scattered field and radar cross section of a circular cylindrical obstacle was calculated by different approximate methods.At first, Finite element method was tried.Then its simplier version was also tried.Next, some different versions were initiated, one having the Hankel variational form and other one is the concept of minimizing energy.At last, the finite difference version aws Tried.

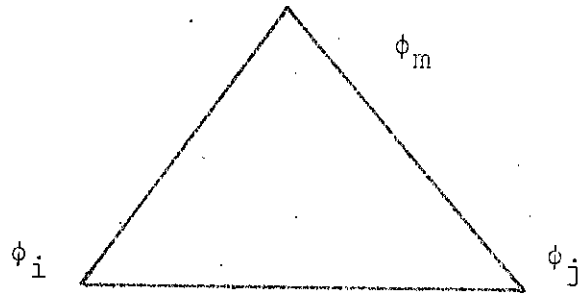
3.2 Finite Element Method

In finite element method, the differential operator equations which describe the physical problem are replaced by an appropriate extremum functional which is variational for the desired quantity and is written in Euler density form. A discretised set of linear algebric equations is then derived from this variational expression. In this scheme of grading, a number of vertices is generated and a set of field values are assigned to these vertices. Within the domain described by an element, the actual field function is approximated by a set of algebric polynomials which are uniquely defined over the particular element under consideration and which are linearly dependent on the values of fields assigned to the vertices of the element and which are reduced to zero outside the element.Thus,in this method, the actual field over the entire domain is approximated by a finite number of trial sets of algebric functions which are uniquely defined and differentiable.When the integrations involved in the extremum functional are carried out over each element separately, the resulting expression becomes a function of the vertex values of the fields. This is then minimised to the fields at each of the vertices so as to produce the optimum values for fields at the vertices. This type of minimization generates a set of linear algebric equations which can be written in matrix form.

A triangular shaped subregion or element is considered.This typical element is described by the vertices i , j , m in cylic order. Let ϕ_i, ϕ_j, ϕ_m be the values of ϕ at these vertices. Over the element 'e' the functional dependence of $\phi(x,y)$ can be written as

$$\phi^e(x,y) = \alpha_o + \alpha_1 x + \alpha_2 y \qquad (3.1)$$

where α_o, α_1 and α_2 are to be determined



If (x_i, y_i), (x_j, y_j) and (x_m, y_m) are the co-ordinates of the vertices i,j,m then the coefficients α_o, α_1 and α_2 be determined from the following set of equations.

Substituting for α_o, α_1 and α_2 in terms of ϕ_i, ϕ_j and ϕ_m we obtain

$$\phi^e(x,y) = \frac{1}{2\Delta_e} \left(a_i + b_i x + c_i y)\phi_i^e + (a_j + b_j x + c_j y)\phi_j^e + (a_m + b_m x + c_m y)\phi_m^e \right. \qquad (3.2)$$

where, $a_i = x_j y_m - y_j x_m$

$$b_i = y_j - y_m \qquad (3.3)$$

$$c_i = x_m - x_j$$

and Δ_e is the area of the triangular element. The equation can be written in compact form using matrix notation as :

$$\phi^e(x,y) = |N^e| \; |\phi^e| \qquad (3.4)$$

Appling this to wave equation and integrating over the entire cross section [91]

$$2J(\phi) = 2 \sum_{e=1}^{P} J^e(\phi_i, \phi_j, \phi_m)$$

$$= 2 \sum_{e=1}^{P} \left[\iint_{\Delta e} |\nabla_t \phi|^2 \, dx\, dy - k_o^2 \iint_{\Delta e} \phi^2 \, dx\, dy \right] \quad (3.5)$$

Each element contains only three values of ϕ_i and each ϕ_i common to those elements for which the vertex i is common. If we minimise the functional $J(\phi)$ with respect to each ϕ_i,

$$\frac{\partial J}{\partial \phi_i} = \sum_{e=1}^{R} \left\{ \iint_{\Delta_e} \left(\left[\frac{\partial N^e}{\partial x}\right] \left[\phi^e\right] \frac{\partial N_i}{\partial x} + \left[\frac{\partial N^e}{\partial y}\right] \left[\phi^e\right] \frac{\partial N_i}{\partial y} \right) dx\, dy - k_o^2 \iint_{\Delta_e} \left(\left[N^e\right] \left[\phi^e\right] N_i \right) dx\, dy \right\} \quad (3.6)$$

Evaluating this matrix equation we can get

$$S_{ij} = \frac{1}{4\Delta_e} (b_i b_j + c_i c_j) \quad (3.7)$$

and

$$\underline{F}^e = \frac{\Delta e}{6} \begin{vmatrix} 1 & \frac{1}{2} & \frac{1}{2} \\ \frac{1}{2} & 1 & \frac{1}{2} \\ \frac{1}{2} & \frac{1}{2} & 1 \end{vmatrix} \quad (3.8)$$

irespective of the shape of the triangular elements where

$$\frac{\partial J^e}{\partial \phi e} = \underline{S}^e \, \phi^e - k_o^2 \, \underline{F}^e \, \phi^e \quad (3.9)$$

For the element number a

$$b_1 = y_2 - y_o \qquad b_2 = y_o - y_1 \qquad b_o = y_1 - y_2$$
$$c_1 = x_o - x_2 \qquad c_2 = x_1 - x_o \qquad c_o = x_2 - x_1 \qquad (3.10)$$

$$S_{oo} = b_o b_o + c_o c_o = (y_1 - y_2)^2 + (x_2 - x_1)^2$$

and so on. Similarly other values can be calculated.

Taking $x_1 = R$, $x_o = R + H$, $x_2 = R + 2H$, $x_1 = x_3 = (R+H)\cos\theta$ and $y_o = y_2 = y_4 = 0$, $y_3 = -y_1 = (R+H)\sin\theta$

Evaluating the whole matrix we can come to the point

$$|4R^2 + 8HR + 6H^2 - 2(R+H)(2R+2H)\cos\theta|\phi_o + |(R+H)(2R+H)\cos\theta$$
$$-(2R^2 + 3RH + H^2)|\ \phi_1 - H^2\phi_2 - H^2\phi_4 + |(R+H)(2R+3H)\cos\theta -$$
$$(2R^2 + 5RH + 4H^2)|\ \phi_3 = 0 \qquad (3.11)$$

From the above equation we can compute the field values at four nodes in term of other 4 nodes.The field values at nodes 0,1,2,3 are known in terms of Hankel function with its unknown coefficients.Applying the boundary condition to the computed field value at node we can calculate the unknown Hankel coefficients.Then with computed Hankel coefficients ,we calculated the scattered field in the space outside the scatterer for different incidence angles and for different dimension of the scatterers.

TABLE 3.1.

Bistatic radar cross section by Finite element method (5 point) with DH = .1.

RK	N	TO in radians: 0.	.5	1.	-.5	-1.
3	6	.33982	.34952	7.95707	.36747	.36215
	8	7.95854	7.85395	8.06007	7.94636	7.88248
	12	—	—	7.99377	—	8.07354
	16	[illegible]	8.86103	—	8.29198	8.52874
	4	—	10.91244	—	—	—
	8	10.00994	10.86497	10.79770	10.79285	10.75671
	12	—	10.96772	—	—	11.00184
	16	—	—	—	11.22963	11.43346
5	6	4.92713	4.73110	4.55219	5.09619	5.24127
	8	4.81774	—	—	—	—
	12	—	—	—	—	5.56906
	16	—	4.43353	4.32378	4.97129	4.93748
6	6	10.51527	9.95465	9.26036	10.90397	11.09036
	8	10.35500	10.43840	10.04362	9.98558	10.19745
	12	—	—	10.34317	—	10.21534
16	16	11.61350	10.86292	—	10.03122	7.75414

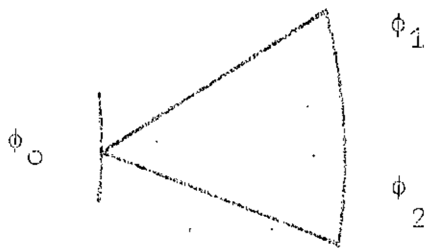
TABLE 3.2

Biatatic radar cross section by Finite element method (5 Point) with DH = .02

RK	N	TO in Radians 0.	.5	1.	-.5	-1.
3	6	8.05518	8.04956	–	–	–
3	8	7.88510	7.94695	7.96574	7.98888	7.92665
3	12	7.92596	7.86360	7.85003	–	7.94038
3	16	–	7.91610	–	7.90427	7.91224
4	6	10.83409	10.78099	10.72734	10.89261	–
4	8	10.66451	10.61567	10.63530	10.61878	10.65364
4	12	10.72848	10.34698	10.25447	10.80742	10.48604
4	16	10.54366	10.55204	10.53987	10.51884	10.51291
5	12	4.83107	–	4.37645	–	–
5	16	–	4.55209	5.93521	9.52879	7.74636
6	16	–	–	–	10.47573	10.53542

3.5 Three point Finite element method

Using the simplemost element, 3 point Finite element method has been tried.



Konwing the field at point ϕ_1 and ϕ_2 it has been tried to calculate field at ϕ_o. One of its advantage would be that, it can be applied to any arbitrary shaped scatterer most effectively because its simplest shape. In this method, for all radius size and incident angle the field outside the scatterer and the radar cross section was calculated. When the $n\Delta\phi$ was 180^0 the matrix coefficients became zero. For this reason some times it was the cause of large errors. After several attempt, the column having large number of zeros was shifted outside the matrix then to make it a square matrix some of the rows was deleted.

Considering the Hankel coefficients as real, some changed method was applied to solve the scattering problem. In both of these methods the matrix was solved by Gauss method of elimination.

TABLE - 3.3

Hankel coefficient for 3 point Finite Element method, RK =6. , N= 6, TO = -1.0 radians and DH = 0.10

Real part				Imaginary part			
a(0)	0.3039			a(0)	0.4620		
a(1)	0.8678	a(-1)	0.3046	a(1)	-0.0187	a(-1)	-0.6585
a(2)	-0.2401	a(-2)	1.0072	a(2)	1.0983	a(-2)	-0.4993
a(3)	-0.0133	a(-3)	-0.2934	a(3)	-0.2085	a(-3)	-0.1560
a(4)	-0.0569	a(-4)	-0.7384	a(4)	1.0889	a(-4)	-0.8022
a(5)	1.2626	a(-5)	-2.9551	a(5)	1.2718	a(-5)	-0.6846
a(6)	-25.0000			a(6)	10.3850		

TABLE - 3.4

Hankel coefficients for 3 point Finite Element method, RK=5., N = 6, TO = -1.0 radians and DH = 0.10

a(o)	0.1414			a(o)	-0.3980		
a(1)	0.6037	a(-1)	1.0131	a(1)	0.5730	a(-1)	-0.3694
a(2)	0.0651	a(-2)	-0.0688	a(2)	-0.0408	a(-2)	-0.0343
a(3)	0.1480	a(-3)	-0.5099	a(3)	0.8277	a(-3)	-0.9071
a(4)	-0.9495	a(-4)	-0.4518	a(4)	0.3257	a(-4)	-0.8964
a(5)	-0.7960	a(-5)	0.9150	a(5)	-1.6702	a(-5)	0.5398
a(6)	2.0000			a(6)	1.5625		

TABLE - 3.5

Hankel coefficients for 3 point Finite Element method, RK=4., N=8, TO = 0.5 radian and DH = 0.02

Real part				Imaginary part			
a(o)	0.9958	a		a(o)	-0.0625		
a(1)	-0.0414	a(-1)	0.2836	a(1)	-0.1098	a(-1)	-0.1139
a(2)	-0.7637	a(-2)	-0.0156	a(2)	-0.3652	a(-2)	0.8463
a(3)	0.8343	a(-3)	1.0230	a(3)	0.0601	a(-3)	0.2785
a(4)	-0.6128	a(-4)	0.7081	a(4)	0.4698	a(-4)	-0.3076
a(5)	-0.0231	a(-5)	-0.3853	a(5)	0.1900	a(-5)	-0.3248
a(6)	0.0099	a(-6)	-0.0081	a(6)	0.0237	a(-6)	0.0243
a(7)	-0.0000	a(-7)	0.0081	a(7)	-0.0222	a(-7)	0.0186
a(8)	0.0469			a(8)	-0.1328		

Table 3.6

Radar cross section obtained by 5 point finite element method with complex Hankel coefficients , showing the last coefficients of the Hankel series

RK	DH	TO	N	BSC	$\bar{a}_N$ Real	Imaginary
7.	.2	1.57080	6	3.12894	-2.87	-0.69
7.	.2	0	6	2.93827	2.43	-.54
7.	.2	0	8	156.54111	-5.0	29.14
7.	.2	1.57080	8	.93675	-4.0	-.707
7.	.2	-1.57080	8	19.40097	-4.0	11.46
6.	.1	-1.	6	130.77318	-25.	10.37
6.	.1	-1.	8	ff**	-1833.	1067.
6.	.1	-.5	8	*****	1825	-1010.
5.	.1	-1.	6	3.82398	2.00	1.56
5.	.1	-1.	5	1.10666	-.5	-2.56
5.	.1	.5	8	1.41727	.1250	-3.25
4.	.02	0	6	1.86615	2.37	-1.73
4.	.02	.5	8	1.01996	0.0469	-.1328

Radar cross section, showing the effect of column having large zero coefficients

TABLE 3.7

N	RK	TO	DH	BCS	Corrected BCS
6	3	-.5	.1	2.03945	1.66623
6	3	0.	.1	1.85886	8.70921
6	3	-1.	.1	2.28696	.38192
6	4.	.5	.02	.34011	1.30908
6	5.	0.	.1	.34384	3.42163
6	5	-.5	.1	4.29361	5.25418
6	5	1.	.1	.47936	1.88734
6	5	.5	.1	1.11255	2.28020
6	6.	.5	.1	8201.52734	
8	6.	-.5	.1	2170.41040	12.99412
8	3.	-.5	.1	1.26133	1.626117
8	4.	1.	.1	1.95686	1.14312
6	4.	.5	.02	1.19938	2.81145
6	5	0.	.1	1.32335	1.84922
6	5	1.	.1	4.12202	.47824
6	5	.5	.1	11.80625	7.85836
6	6.	.5	.1	14.87997	1.71[illegible]63
6.	6.	0	.1	3.53462	.48180

TABLE 3.8

Radar cross section, after deleting the column having large numer of zeros in 3 point FE Method N =6

RK	TO	DH	BSC
4.	1.	.1	4.03215
4	.5		1.47530
4	-.5		6.11323
4	-.1		4.03415
4.	-.5	.02	4.96412
4	1.	.02	00.70328
6	-.1	.02	899.18872
6	-.5	.02	148.58513
6.	1.	.02	3.92899
6	.5	.02	163.33885
6	0	.02	881.87158
5	-1.	.02	2.39041
5	-.5	.02	3.72677
5	1.	.02	.31819
5	.5	.02	.21223
5	0	.02	12.69241
3	-1.	.02	1.74314
3	-.5	.02	2.33640
3	1.	.02	1.02213
3	.5	.02	1.09671
3	0.	.02	1.71756

TABLE 3.9

Hankel Function coefficients in 3 point FE method, after deleting columns having large number of zeros

for N = 6, TO = 1.radian RK = 3 and DH = .02

AX(1)	.3511	AX(12)	.4674
AX(2)	-.4212	AX(13)	.5453
AX(3)	.6330	AX(14)	-.6834
AX(4)	-.2277	AX(15)	-.3593
AX(5)	.2136	AX(16)	-.4084
AX(6)	.0010	AX(17)	.0048
AX(7)	-.8601	AX(18)	-.1261
AX(8)	.1462	AX(19)	.9374
AX(9)	1.1332	AX(20)	-.0274
AX(10)	-.3162	AX(21)	-.3192
AX(11)	.0083	AX(22)	-.0356

BSC = 1.02213

TABLE 3.10

AX(1)	.2700	AX(7)	-.7289	AX(12)	.5259	AX(18)	-.4095
AX(2)	-.0544	AX(8)	-.8987	AX(13)	.4636	AX(19)	.6597
AX(3)	-.2099	AX(9)	.6313	AX(14)	-1.885	AX(10)	.6069
AX(4)	.4172	AX(10)	.2512	AX(15)	.1016	AX(21)	.0790
AX(5)	-.2494	AX(11)	.0022	AX(16)	.2309	AX(22)	.0001
AX(6)	-.0001			AX(17)	-.0069		

BSC = 1.09671

TABLE 3.11

For N = 6 TO = 1 RK = 3 DH = .1

AX(1)	.4665	AX(7)	-.8784	AX(12)	.4541	AX(18)	-.1099
AX(2)	-.3817	AX(8)	.2490	AX(13)	.5532	AX(19)	.9679
AX(3)	.5573	AX(9)	1.1168	AX(14)	-.7159	AX(20)	-.1323
AX(4)	-.2992	AX(10)	-.3360	AX(15)	-.3269	AX(21)	-.3171
AX(5)	.2337	AX(11)	-.0150	AX(16)	-.4708	AX(22)	-.0090
AX(6)	.0011			AX(17)	.0464		

BSC = 1.05924

Table. 3.12

Radar cross section by 3 point finite element method with real Hankel coefficients, showing the $a_{N/2}$ coefficient .

RK	TO	DH	N	RSC	$a_{N/2}$
7.	1.57080	.2	4	20.42696	3.13
			5	1.19877	
			6	233.38860	-24.20
			7	331.95068	
			8	40.43901	12.62
			9	36.88184	
			10	243.58777	-27.00
			11	3.12678	
			12	46.65254	-12.00
5.	1.57080	.2	4	181.31271	20.40
			5	0.04377	
			6	5.29580	-5.63
			7	0.82834	
			8	2.28512	-2.55
			9	4.83322	
			10	10.22503	2.21
			11	2.78335	
			12	2.54224	-0.498

TABLE 3.13

Scattered Feld amplitude and its phase angle variation in space for RK=7, N=7, TO=90 degree, DH=0.2

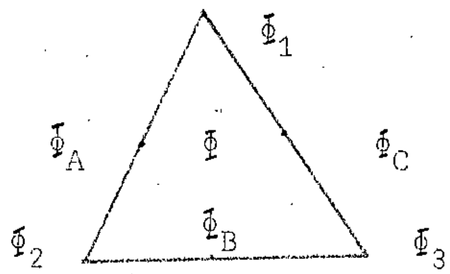
Position	Field Amplitude	Phase Angle	Position	Field Amp.	Phase Angle
7.2	2.00006	-.002	12.2	1.06480	-11.50
7.4	1.88588	3.32	12.4	1.02663	-13.28
7.6	1.77362	-9.37	12.6	.97986	-14.68
7.8	1.65342	-13.19	12.8	.92628	-15.57
8.0	1.53132	-16.33	13.0	.86814	-15.74
8.2	1.40999	-1871	13.2	.80835	-14.98
8.4	1.29281	-20.18	13.4	.75077	-13.03
8.6	1.18343	-20.67	13.6	.70034	-9.69
8.8	1.08554	-20.02	13.8	.66248	-4.92
9.0	1.00358	-18.22	14.0	.64312	1.02
9.2	.94124	-15.39	14.2	164609	7.47
9.4	.90114	-11.80	14.4	.67242	13.56
9.6	.88371	-7.95	14.6	.71993	18.54
9.8	.88704	-4.36	14.8	.78390	22.05
10.0	.90690	-1.44	15.0	.85928	24.07
10.2	.93800	.56	15.2	.94094	24.80
10.4	97508	1.64	15.4	1.02505	24.46
10.6	1.01346	1.84			
10.8	1.04927	1.32			
11.0	1.07958	.2			
11.2	1.10222	-1.31			
11.4	1.11565	-3.17			
11.6	1.11895	-5.22			
11.8	1.11152	-7.35			
12.0	1.09333	-9.49			

3.8 Hankel Variational Form

We can assume the field variation within a small element as

$$\Phi = \bar{a}_1 H_1(kr)\, e^{j\phi} + \bar{a}_o H_o(kr) + \bar{a}_{-1} H_{-1}(kr)\, e^{-j\phi} \quad (3.\ \ )$$

Here only three coefficients are taken as for a triangular shaped element, only three unknown coefficients can be evaluated.



This equation should aslo satisfy the boundary conditions on the three vertices.

$$\Phi_1 = \bar{a}_1 H_1(1) e^{j\phi_1} + \bar{a}_o H_o(1) + \bar{a}_{-1} H_{-1}(1) e^{-j\phi_1}$$

$$\Phi_2 = \bar{a}_1 H_1(2) e^{j\phi_2} + \bar{a}_o H_o(2) + \bar{a}_{-1} H_{-1}(2) e^{-j\phi_2}$$

$$\Phi_3 = \bar{a}_1 H_1(3) e^{j\phi_3} + \bar{a}_o H_o(3) + \bar{a}_{-1} H_{-1}(3) e^{-j\phi_3}$$

Now $\bar{a}_1$ can be calculated in terms of Φ_1, Φ_2, Φ_3

$$\bar{a}_1 = \frac{\begin{vmatrix} \Phi_1 & H_o(1) & H_{-1}(1)e^{-j\phi_1} \\ \Phi_2 & H_o(2) & H_{-1}(2)e^{-j\phi_2} \\ \Phi_3 & H_o(3) & H_{-1}(3)e^{-j\phi_3} \end{vmatrix}}{\begin{vmatrix} H_1(1)e^{j\phi_1} & H_o(1) & H_{-1}(1)e^{-j\phi_1} \\ H_1(2)e^{j\phi_2} & H_o(2) & H_{-1}(2)e^{-j\phi_2} \\ H_1(3)e^{j\phi_3} & H_o(3) & H_{-1}(3)e^{-j\phi_3} \end{vmatrix}}$$

Similarly $\bar{a}_o$ and $\bar{a}_{-1}$ can also be calculated .

Once these coéfficients are calculated then field values at any point inside the element can be calçulated.A programme was made for this trial function,ran it in IBM System 360 computer for some arbitrary shaped triangular element and after calculation of these coefficients,field values at the middle point of the line joining was calculated.It was found that field values at the middlepoint of the line joining two vertices are depends on the field value of the third vertix.This violates the implied condition of finite element formulation,hence this variational form was discarded.

3.9 Minimization of Energy

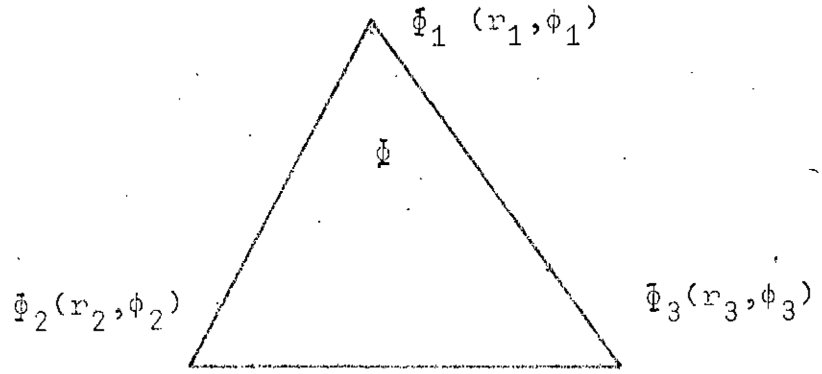
Assuming the field variation inside an element as

$$E(r,\phi) = \bar{a}_1 H_1^2(kr)\, e^{j\phi} + \bar{a}_o\, H_o^2(kr) + \bar{a}_{-1} H_{-1}^2(kr) e^{-j\phi} \quad (3.15)$$

then the energy content in the element is given by

$$W_E = \frac{\varepsilon}{4} \int_S |E|^2 \, dS$$

but to integrate the field over the whole element of an arbitrary shape,it is required to generate Hankel functions at every points within the element,which at present within the limited time and facilities seems to be involved with too much labour, so this attempt was also discarded.



3.10 Finite Difference Formulation

In order to obtain a finite difference expression, we express the scalar wave equation (2.8) in circular cylindrical coordinates as follows :

$$(\nabla^2 + k^2)\Phi = \frac{1}{r}\left|\frac{\partial}{\partial r}\left(\frac{1}{r}\frac{\partial \Phi}{\partial r}\right) + \frac{\partial}{\partial \phi}\left(r\frac{\partial \Phi}{\partial \phi}\right)\right| + k^2\Phi$$

$$= \frac{\partial \Phi^2}{\partial r^2} + \frac{1}{r}\frac{\partial \Phi}{\partial r} + \frac{1}{r}\frac{\partial^2 \Phi}{\partial \phi^2} + k^2\Phi = 0 \qquad (3.17)$$

where Φ is a function of only r and ϕ . In place of function $\Phi(r,\phi)$ we now consider the discrete values of at the mesh points only , and replace the partial derivatives of equation (3.17) by their finite difference approximation[92]. Figure 3.2 shows the detailed parameters involved in a five-point finite difference formulation. Since we sweep in anticlockwise direction, the direction of increasing ϕ is indicated by arrow. We obtain the expression for Φ_1, Φ_2, Φ_3 and Φ_4 in terms of the value Φ_0 at node 0, by means of a Taylor's series.

$$\Phi_1 = \Phi_0 + (\Delta\phi)\left(\frac{\partial \Phi}{\partial \phi}\right)_0 + \frac{1}{2!}(\Delta\phi)^2\left(\frac{\partial^2 \Phi}{\partial \phi^2}\right)_0 + \frac{1}{3!}(\Delta\phi)^3\left(\frac{\partial^2 \Phi}{\partial \phi^2}\right)_0 + \dots \qquad (3.18)$$

$$\Phi_2 = \Phi_0 + h\left(\frac{\partial \Phi}{\partial r}\right)_0 + \frac{1}{2!}h^2\left(\frac{\partial^2 \Phi}{\partial r^2}\right)_0 + \frac{1}{3!}h^3\left(\frac{\partial^3 \Phi}{\partial r^3}\right)_0 + \dots\dots \qquad (3.19)$$

$$\Phi_3 = \Phi_0 - (\Delta\phi)\left(\frac{\partial \Phi}{\partial \phi}\right)_0 + \frac{(\Delta\phi)^2}{2!}\left(\frac{\partial^2 \Phi}{\partial \phi^2}\right)_0 - \frac{(\Delta\phi)^3}{3!}\left(\frac{\partial^3 \Phi}{\partial \phi^3}\right)_0 + \dots\dots \qquad (3.20)$$

$$\Phi_4 = \Phi_0 - h\left(\frac{\partial \Phi}{\partial r}\right)_0 + \frac{h^2}{2}\left(\frac{\partial^2 \Phi}{\partial r^2}\right)_0 - \frac{h^3}{3}\left(\frac{\partial^3 \Phi}{\partial r^3}\right)_0 + \dots\dots \qquad (3.21)$$

Adding equations (3.19) and (3.21),

$$\Phi_2 + \Phi_4 - 2\Phi_0 = h^2\left(\frac{\partial^2 \Phi}{\partial r^2}\right)_0 + \frac{2h^4}{4!}\left(\frac{\partial^4 \Phi}{\partial r^4}\right)_0 + \frac{2h^6}{6!}\left(\frac{\partial^6 \Phi}{\partial r^6}\right) + \dots \qquad (3.22)$$

Again ,substructing equations (3.19) and (3.21), we obtain

$$\Phi_2 - \Phi_4 = 2h\left(\frac{\partial \Phi}{\partial r}\right)_0 + \frac{2h^3}{3!}\left(\frac{\partial^3 \Phi}{\partial r^3}\right)_0 + \frac{2h^5}{5!}\left(\frac{\partial^5 \phi}{\partial r^5}\right)_0 + \dots \qquad (3.23)$$

Neglecting the terms containing third, fourth and higher order derivatives which also involve the product by higher power of mesh size h, we obtain from equations (3.22) and (3.23).

$$\left(\frac{\partial^2\phi}{\partial r^2}\right)_o = \frac{\phi_2+\phi_4-2\phi_o}{h^2} \qquad (3.24)$$

and
$$\left(\frac{\partial\phi}{\partial r}\right)_o = \frac{\phi_2 - \phi_4}{2h} \qquad (3.25)$$

Similarly, adding equations (3.18) and (3.20), we obtain

$$\phi_1+\phi_3-2\phi_o = (\Delta\phi)^2\left(\frac{\partial^2\phi}{\partial\phi^2}\right)_o + \frac{2(\Delta\phi)^4}{4!}\left(\frac{\partial^4\phi}{\partial\phi^4}\right)_o + \frac{2(\Delta\phi)^6}{6!}\left(\frac{\partial^6\phi}{\partial\phi^6}\right)_o \qquad (3.26)$$

Neglecting fourth and higher order derivatives, we have,

$$\left(\frac{\partial^2\phi}{\partial\phi^2}\right)_o = \frac{\phi_1 + \phi_3 - 2\phi_o}{(\partial\phi)^2} \qquad (3.27)$$

Using equations (3.24),(3.25) and (3.27), the finite difference equivalent of scalar wave equation in (2.8) reduces to the following in circular cylindrical coordinates;

$$\phi_2\left(1+\frac{h}{r}\right)+\phi_4\left(1-\frac{h}{r}\right)+\left(\frac{h}{R\Delta\phi}\right)^2(\phi_1+\phi_3) = \phi_o\left(2+2\left(\frac{h}{R\Delta\phi}\right)^2 - k^2h^2\right) \qquad (3.28)$$

Any one of the ϕ's can be calculated from the above equation if the remaining four neighbouring points are known,

For a circulat cylinder the scatterer perfectly match to the inner most circle Γ. Two outer circles Γ_1 and Γ_2 are considered with radius R and R+h. Here the radius of the obstacle is R-h. Here fields at different mesh points on circle Γ_1 and Γ_2 are known as

$$E_{zp}^{\,i} = e^{jkr_p} \qquad (3.29)$$

$$E_{zp}^{\,s} = \sum_{-N+1}^{N} \bar{a}_n H_n^{(1)}(kr_p)\, e^{jn\phi_p} \qquad (3.30)$$

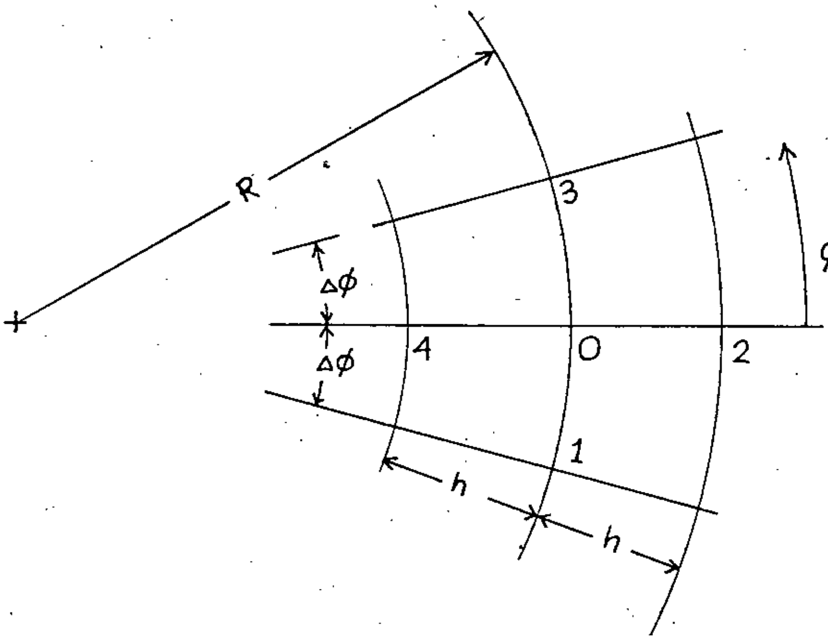


Fig 3.2

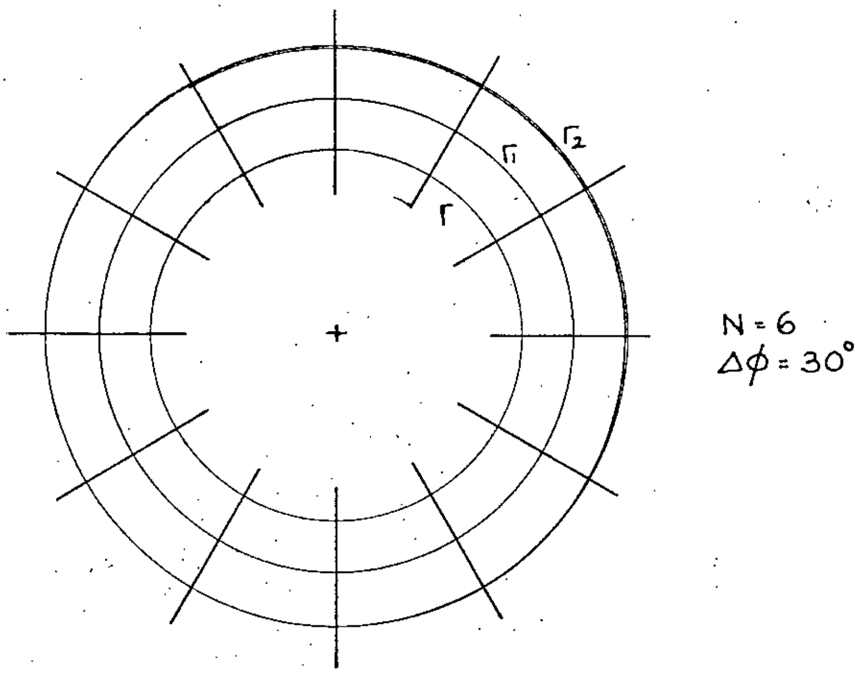


Fig. 3.3

Here for thr different points p both r_p and ϕ_p is known. So in terms of fields at points 1,2,3 and 0 , fields at point can be calculated. Then applying the boundary condition on circle Γ,

$$E_{zp}^{t} = E_{zp}^{i} + E_{zp}^{s} = 0 \qquad (3.30)$$

a set of linear equations is developed like direct method. Solving the matrix, the coefficients of the Hankel function can be obtained. Once these are known, total field at any point outside the obstacle and its radar cross section can be obtained.

TABLE 3.14

Radar cross section by Finite difference method having DH=.1

To incidinace Angle in Radians

RK	N	0.	.5	1.	-.5	-1.
3	6	.66637	.74264	.75854	.73829	.93137
3	8	14.43606	14.39312	14.20973	14.24136	14.027748
4	6	26.03322	25.95175	25.88425	26.15720	26.29330
4	8	26.59669	26.52765	26.60736	26.47316	26.54422
5	6	13.22999	12.85002	12.56574	13.64329	14.07441
5	8	13.93036	14.00657	14.00024		14.01364
6	6	17.375	16.34	15.18362	18.18452	18.72
6	8	18.39314	18.88068	18.40131	18.05283	18.66745

TABLE 3.15

Radar cross section by Finite difference method having DH = .02

To incidinace Angle in Radians

RK	N	0.	.5	1.	-.5	-1.
3	8	13.74199	13.69152	13.74122	13.77714	-
4	6		24.43269	24.34819	24.64073	
	8	24.72987	24.69954	24.71091	24.67633	24.76247

3.12 Discussion

Radar cross section obtained by Finite element method are shown in in Table 3.1 and 3.2 .In those tables variations of radar cross section are shown with respect to the scatterer's radius,mesh size incidence angle and radius increment.Finite element method involving three points had been tried also.Results obtained in this method are shown in Table 3. .Looking at this table we can observe that some of the results are satisfactory.Analysing the nature of the Hankel coefficients we can see that some of the coefficients have large values.These are shown in Table 3.3 and 3.4 and Table 3.5 Later it was found that two columns of the total matrix were zero or near zero. Those caused very large coefficients of the Hankel function.This also introduced some error in the calculation for radar cross section. At the first attempt radar cross sections were calculated ignoring the Hankel coefficients for the columns which have large number of zeros. Table 3.7 shows some of those results. In some cases results were improved but in other cases they deterioted. In the next attempt, we shifted those columns having large number of zeros,outside the matrix and also simultaneously deleted two rows to have a square matrix. Then the matrix was solved. The results obtained in this method are shown in Table 3.8 .Here some of the results are acceptable.but the method is not very much satisfactory.Coefficients of Hankel functions obtained in this method are shown in Table 3.9 Table 3.10 and Table 3.11 .Finally some calculations were done by considering the Hankel coefficients as real. Results obtained in this method are shown in Table 3. . Some of the Hankel coefficients for each case are troubled one.Those are shown at the righmost column of Table 3. .Fields at different points were also calculated and those are shown in Fig.3.1 .Then considering field variation inside an elementary area as a series of Hankel function,a new method was tried to develop.But it failed some basic boundary conditions.Another method considering minimization of energy was thought of.But But it involved continuous Hankel integration over an arbitrary shaped element.Considering the amount of labour and time involved for this trial method it was shelfed off. Finite difference method was also tried.Results obtained in this meth are shown in Table 3.14.

When the scattering obstacle's dimension is comparable with its wavelength i.e. in the high-frequency region, electromagnetic theory approach is not very much converging in nature. It can be seen in chapter 2 ,that, with increasing obstacle's radius the results were more oscillating in nature. The error can be minimized by taking a larger Hankel series. This is true for the approximate methods. Finite difference method has been tried by A.M.Patwary[118] using matrix of the order of 200 x 200 to get acceptable results. For the present work it was not possible to solve a matrix larger than 64 x 64 as larger one exceeded the memory requirement of the computer in Statistics Bereau.

CHAPTER 4

NUMERICAL SOLUTIONS OF UNIFORM HOLLOW WAVEGUIDE PROBLEMS.

CHAPTER - 4

NUMERICAL SOLUTION OF UNIFORM HOLLOW WAVEGUIDE PROBLEMS

4.1 Introduction

The most fundamental equations those describe the phenomenon of electromagnetic wave propagation in any medium, such as homogeneous or inhomogeneous, isotropic or anisotropic, are the Maxwell's equations. As a result of the rapid growth of telecommunication engineering, microwaves is becoming an increasingly important field of study. Modern engineering applications require a clear understanding of the wave propagation in waveguides of complex shapes.

Hollow conducting waveguides are easily analysed in terms of tabulated functions when their cross-sections are rectangular, circular or elliptical. Exact solutions to the wave equation are possible in 11 coordinate systems where the equation is separable[16]. However, a knowledge of propagation behaviour and characteristics are required for other-shaped cross-sections for considering bandwidth, power-handling, mechanical and other reasons. All these problems may be solved by finite difference, finite element, point matching and integral-equation approach, conformal transformation and many others[104-10

The objective of this work is to use some of these methods in different form for different shaped waveguides and compare those results with different parameters such as initial error, final error mesh size, waveguide size, error limitation, iterations etc.

4.2 Theory

We are concerned here with methods of finding numerical solutions to the equation

$$\nabla^2\phi + k^2\phi = 0 \qquad (4.1)$$

valid over a region R, the waveguide cross-section subject to boundary conditions

$$\phi = 0 \qquad (4.2)$$

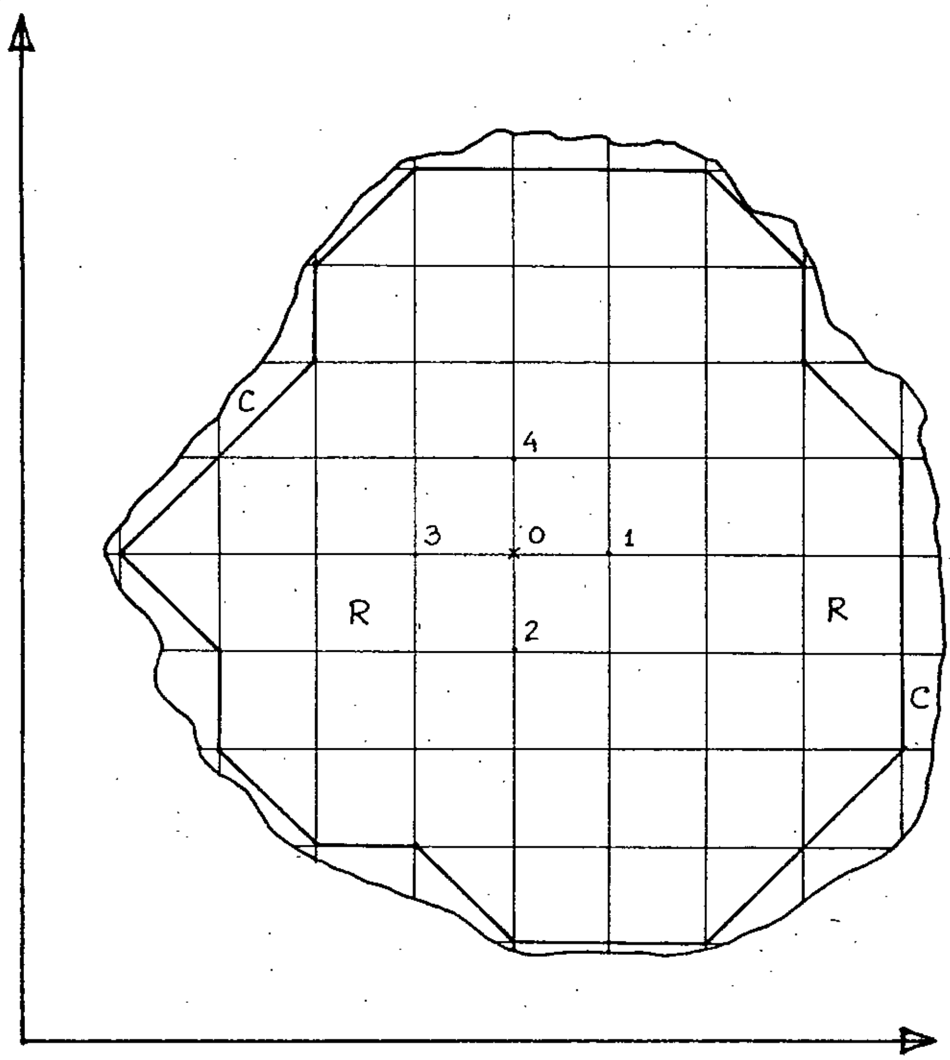


Fig 4.6 CROSS SECTIONAL MESH SHOWING INSIDE APPROXIMATION TO THE BOUNDARY C AND NUMBERING SYSTEM FOR INTERIOR NODES

on C for E modes or

$$\frac{\partial \phi}{\partial n} = 0 \qquad (4.3)$$

on C for H modes, where R is bounded by the closed curve C.

4.3 Finite Difference Method

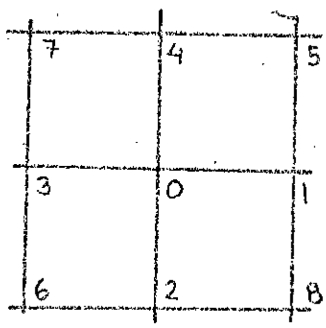
The finite difference method[93] is the oldest and perhaps the most commonly used technique for the solution of boundary value problems. Its application to waveguides and cavities was shown by Motz[94], and some 20 years later[95], was used to develop a computer programme for waveguides of arbitrary shapes. By the use of automatic mesh halving and estimation of the over relaxation factor, the computing time is quite short. Since then a lot of work has been done using finite difference methods[96-99].

For five point finite difference method wave equation over the waveguide cross section is approximated in terms of the scalar field at discrete points. It can be shown that[96],

$$\phi_o = (\phi_1 + \phi_2 + \phi_3 + \phi_4) / (4 - k^2h^2) \qquad (4.4)$$

In the case of nine point finite difference method, the wave equation can be written as[98],

$$8\phi_o = \sum_{i=1}^{8} \phi_i + 3k^2h^2\phi_o \qquad (4.5)$$



For detailed discussion on finite difference method for waveguide problem, reader can be referred to the referrence 96 and 98 .

4.4 Finite Element Method

An alternative and almost parallel approach to that of finite differences is to use a continuous piecewise linear approximation . This has been applied to the waveguide problem by Zienkiewiez[99,100] Silvester[101], Ahmed[102], and others[110-116] under the name of 'Finite element' and by Harrington[103], as a particular 'moment' solution using 'triangular function'. The simplest version considers field values at discrete points, regards the points as vertices of many adjacent triangles and explicitly takes the field over each triangle to be the unique linear interpolation between the three vertex values This field can be used as a trial function in the straight forward Rayleigh-Ritz procedure. Field values at the vertices become linear Ritz parameters, so that the resulting eigenvalue equation has the following standard matrix form :

$$A x = k^2 B x \qquad (4.6)$$

A and B are both symmetric and positive-definite (A is semidefinite for H modes) , so allowing standard methods of solution.

For triangular shaped elements it can be shown that[102]

$$4\,\phi_o - \sum_{i=1}^{4} \phi_i = \frac{k^2 h^2}{12} \left(6\,\phi_o + \sum_{i=1}^{6} \phi_i \right) \qquad (4.7)$$

and for square mesh

$$8\,\phi_o - \sum_{i=1}^{8} \phi_i = \frac{k^2 h^2}{6} \left(8\,\phi_o + 2 \sum_{i=1}^{4} \phi_i + \tfrac{1}{2} \sum_{i=5}^{8} \phi_i \right) \qquad (4.8)$$

To solve the matrix problem in both Finite difference and Finite element Gauss-Seidel iterative method has been used. First, a guess has been made at all variables x_i and successively corrected them, using an updated estimate, just as soon as required, and variables are immediately overwritten in computation.

$$x_i^{m+1} = - \sum_{j=1}^{i-1} \frac{a_{ij}}{a_{ii}} x_j^{m+1} - \sum_{j=i+1}^{n} \frac{a_{ij}}{a_{ii}} x_j^{m} + \frac{b_i}{a_{ii}} \qquad (4.9)$$

with $1 \leq i \leq n$, $m > 0$, m denotes the iteration count.

TABLE 4.1

N	Aki	5PFD ITRE	5PFD ERROR	FE Tr. ITRE	FE Tr. ERROR
8	80	5	-.63252	11	-.63239
8	40	8	-	10	-.63134
8	16	8	-.62981	7	-.62919
8	8	6	-.63041	6	-.63181
8	4	5	-.63234	5	-.63125
8	2	4	-.62756	4	-.62758
8	8	4	-.63473	4	-.63176
8	4	4	-.63632	4	-.63262
16	160	8	-.12263	20	2.42460
16	80	14	-	20	1.733108
16	32	20	-.02591	17	-.05175
16	16	12	-.03810	11	-.03668
16	8	10	-.06978	10	-.07871
16	1.2	9	-.08881	9	-.09114
16	.8	9	-.90033	9	-.09307
16	.32	9	-.09264	9	-.09526
16	160	3	-.12622	9	-.06019
16	64	8	-.12622	20	2.41613

TABLE 4.3

N	M	AKi	9 PFD ITRE	9 PFD ERROR	FE Rc. ITRE	FE Rc. ERROR
9	4	4.2	3	-.2.17769	3	2.18023
18	8	8.4	5	-.42722	5	-.47527
18	8	4.4	4	-.42240	4	-.43229
36	16	8.8	8	+.83980	7	1.16797
36	16	4	7	1.06527	7	1.06627
36	16	8.8	6	1.32586	7	.98746
36	16	4.4	7	1.04391	7	1.04664
36	16	8	7	1.06514	7	1.06630
18	8	4	4	-.44904	4	-.44978
18	8	2	4	-.45176	4	-.45139

TABLE 4.2

N	AKi	9 PFD ITRE	9 PFD ERROR	FE Rc. ITRE	FE Rc. ERROR
32	8	M+	1.05064	11	1.05465
32	4	11	1.05205	11	1.05519
32	2	11	1.05205	11	1.05650
32	8.4	10	1.05053	10	1.05517
32	16	11	1.08520	11	1.08288
32	8	11	1.05351	11	1.05939
16	32	11	6.18926	12	.45790
16	8	5	.08004	5	.06980
16	32	5	.29437	5	.23060
32	16	11	1.05353	11	1.05939
32	8	11	1.05064	11	1.05529

TABLE 4.4

Effect of mesh on discretizational error for rectangular waveguide for all four methods.

Mesh	9PFD ITRE	9PFD ERROR	FE Rc ITRE	FE Rc ERROR	E T ITRE	E T ERROR	5PFD ITRE	5PFD ERROR
4	6	-2.34379	4	-2.46657	4	-2.30773	4	-2.55045
4	6	-1.12677	3	-1.13128	4	-1.10580	4	-1.13806
8	5	-.63823	5	-.64022	6	-.63335	6	-.64005
10	7	-.40891	6	-.40988	8	-.40649	8	-.40773
12	8	-.28186	8	-.28373	10	-.27984	10	-.27958
14	10	-.20562	10	-.20701	12	-.20121	12	-.20049
16	11	-.15255	11	-.15391	14	-.14847	15	-.15118
18	13	-.11782	13	-.11890	16	-.11091	17	-.11396
20	14	-.08655	14	-.08769	18	-.08071	19	-.08427
22	16	-.06716	16	-.06783	21	-.06229	21	-.06182
24	18	-.05210	18	-.05255	23	-.04398	24	-.04851
26	20	-.03982	20	-.03993	26	-.03291	26	-.03364
28	22	-.03089	21	-.02537	29	-.02391	29	-.02383
30	24	-.02291	24	-.02215				
32			26	-.01315				
34			29	-.01213				

TABLE 4.5

Convergency with iteration for different initial trial values for FE Tr method

X2EN, YLEN = 8, H = 1.

ITRE	10.	5.	2.	1.	.5
1.	288.90063	159.334	23.298	4.161	1.236
2.	144.59090	71.474	6.501	.614	-.289
3.	65.90072	34.025	1.734	-.267	-.541
4.	30.20605	14.865	0.054	-.542	-.614
5.	12.81074	5.187	- .403	-.616	-.637
6.	4.26441	1.166	-.600	-.631	
7.	.83998	-.163	-.629		
8.	-.25763	-.528			
9.	-.55176	-.614			
10.	-.6195	-.631			
11.	-.63239				

TABLE 4.7

Convergency with iterationforfor different initial trial values for XCEN=8, YLEN=8, DH=1.

ITRE	10.0	2.0	1.0	.5	.25
1.	60.595	54.147	6.293	1.436	.856
2.	.715	18.234	1.263	-.205	-.467
3.	-.403	6.555	-.03735	-.509	-.591
4.	-.594	1.810	-.46829	-.605	-.627
5.	-.632	0.086	-.59673	-.632	-
6.	-	-.448	-.63041	-	-
7.	-	-.593	-	-	-
8.	-	-.629	-	-	-

TABLE 4.8

Variation of cutoff frequency with channel depth for a double ridge waveguide by finite element (rectangular) method. X = 16, Y = 16, W = 8 S = 1, DEL = .9999

D	IT	Dev.
1	14	3.02782
2	13	6.88527
3	12	11.52757
4	11	17.11088
5	11	23.89159
6	10	32.24316
7	9	42.69005

TABLE 4.9

Variation of cutoff frequency with channel width for a double ridge waveguide by finite element (rectangular) method X = 16, Y = 16, W = 8, S = 1, DEL = .9999

W	ITRE	Dev
2	11	12.64248
4	11	14.89491
6	11	16.32658
8	11	17.11044
10	12	17.45450

TABLE 4.10

Comparison between four methods for the calculation of cutoff frequency in GHz for Ridge waveguide, dimension in cm.

Mesh N	W	D	Dimension	5PFD	9PFD	FE Tr.	FE Rc.
16	8	1	16	1.3657	1.3658	1.3657	1.3660
16	8	2	16	1.4169	1.41771	1.4169	1.4171
16	8	3	16	1.4789	1.4786	1.4784	1.4787
16	8	4	16	1.5524	1.5527	1.5524	1.5527
16	8	5	16	1.6422	1.6426	1.6422	1.6426
16	8	6	16	1.7528	1.7533	1.7528	1.7533
16	8	7	16	1.8911	1.8917	1.8912	1.8918
16	8	8	16	2.0674	2.0688	2.0674	2.0684
16	10	4	16	1.5572	1.5572	1.5571	1.5572
16	6	4	16	1.5418	1.5423	1.54418	1.5423
16	4	4	16	1.52226	1.5233	1.5226	1.5233
16	2	4	16	1.4924	1.4934	1.4924	1.4934
16	8	4	16	17.09052	17.10860	17.08902	17.110044
12	8	4	12	16.93025	16.96614	16.93384	16.97162
20	8	4	10	17.15727	17.34604	17.15646	
24	8	4	12	17.18698	17.34555	17.18654	
28	8	4	12	17.20578	17.34805	17.20561	
32	8	4	16	17.21721	17.34872	17.21684	
36	8	4	18	17.22380	17.34613	17.22362	
40	8	4	10	-	17.53168	17.23660	
44	8	4	11		17.52429	17.24174	

TABLE 4.11

Comparision of time required for different methods to solve ridge waveguide problem in terms of iteration number.

Variables			Methods			
Depth	Width	N	5PFD	9PFD	FE tr	FE Re.
1	8	16	18	16	17	14
2	8	16	18	15	17	13
3	8	16	17	13	16	12
4	8	16	16	12	15	11
5	8	16	16	11	15	11
6	8	16	15	11	14	10
7	8	16	14	10	13	9
8	8	16	14	9	13	9
4	10	16	16	13	15	12
4	6	16	16	12	15	11
4	4	16	17	12	16	11
4	2	16	17	12	16	11
4	8	16	16	12	15	11
4	8	12	12	9	11	8
4	8	20	16	12	16	-
4	8	24	21	16	21	-
4	8	28	26	19	26	-
4	8	32	32	23	32	-
4	8	36	39	28	39	-
4	8	40	-	33	45	-
4	8	44	-	39	53	-

4.8 Discussion

The importance of finite difference lies in the ease with which many logically complicated operations and functions may be discretized. Operations are then performed not upon continuous functions but rather, approximately, in terms of values over a discrete point set. It is hoped that as the distance between points is made sufficiently small, the approximation becomes increasingly accurate. The great advantage of this approach is that operations may be reduced to simple arithmatic forms and then be conveniently programmed for automatic digital computation.In short, complexity is exchanged for labor.

Variational method is relatively new, some times, it is relatively easy to formulate the solution of certain differential and integral equations in variational terms. The solution is found by selecting a field which minimizes a certain integral. The integral is often proportional to the energy contained in the system and so the method embodies a close correspondence with the real world.

Four separate method has been used to calculate the waveguide propagation for TM_{11} mode in Rectangular waveguide, ridge waveguide and double ridge waveguide. All the four methods are quite satisfactory. Variation of cutoff frequency with initial values,mesh size has been calculated for rectangular waveguide of different aspect ratios. It can be concluded that whatever be the initial error in initial values of k had been assumed, solution always converges to the analytical solution.Effect of mesh numbers on the discretization error and iteration required is shown in Table 4.4 and Fig. 4.1.It can be noticed that, increasing the mesh number the discretization error reduces exponentially to zero.But time required for finer mesh problems increases for all methods.Comparing the four methods we can say that discretization error for all these methods for a particular mesh number are more or less same.In this respect they dont have any comparable advantage over one another.But Iteration or the time required to come to a particular error limitation for finite element rectangular and nine point finite difference are more or less identical. And time required by finite element triangular and finite difference 5 point are more or less identical but 20-30 % more than previous two methods.In some cases time required by 5 PFD is slightly more than FETr.Field values and cutoff frequency had been calculated for ridge waveguide by all four methods.Variation of cutoff frequency has been shown for varying channel depth and width in Table-4.8 and Table.4.9 and Fig.4.3.The nature of variation are identical for all methods,and are quite satisfactory in nature.Effect of mesh size on discretization error had been also calculated for all the four methods and they have identical nature and identical error.But for this type of waveguide, finite element rectangular method took least time for computation followed by nine point finite difference method, then by finite element triangular shaped element and 5 point finite difference came out last. 5 point finite difference takes 30-50 % more time than Finite element rectangular method.Computer solution was also obtained for double ridge waveguide and its magnetic field distribution is shown in Fig.4.4 .

Near singularities, such as the corners of conductors,the usual finite difference formulas become inaccurate because of the unbounded nature of the field derivatives in these regions. There is a concentration of energy near the singularity and any inaccuracy in the approximating potential function has a relatively serious effect on the overall energy computation.By using a finely but finitely divided graded mesh in the vicinity of the singularity, errors due to singularity by finite difference method may be reduced.

In the author's opinion, the difference between finite-element and finite difference method is very small.One fact that comes to light from the above analysis is that in general those methods in which larger number of adjacent points are utilized for the calculation of the fields at successive mesh points , better results are achived .

CHAPTER 5

CONCLUSIONS AND RECOMMENDATIONS.

CONCLUSIONS AND RECOMMENDATIONS

Scattering of electromagnetic wave impinging on an obstacle and propagation of electromagnetic wave through an arbitrary shaped waveguide have been analysed by different numerical methods in IBM computer system 360 at Bureau of Statistics, Dacca.

Four different methods were used to solve the electromagnetic wave propagation problems in rectangular waveguides of different aspect ratios. Starting with the trial assumed values of field distribution and propagation constant, final field distribution and cutoff frequencies have been computed. Effects of assumed trial values for the variation of propagation constants withiteration count have been shown in Tables. Effects of increasing the mesh numbers for the variation of discretizational error had been shown in a figure. Solutions obtained by all the four methods have been compared. Cutoff frequency and field variation for a ridge waveguide have also been calculated. The effect of increasing the channel depth and width for a single ridge waveguide or for a double ridge waveguide have been computed and those are shown in Tables and also in in figure.

For the solution of electromagnetic wave scattering problems, direct method has been applied to find the radar cross sections and field distributions in space outside the scatterers. These problems have been solved for different incident angles, for different obscle's dimensions, and varing the length of the Hankel series taken. Effect of all those variations have been shown in tables and in figures.

For the solution of electromagnetic wave scattering problems, for circular cylidrical obstacles, have also been calculated by different approximate methods. As far as the author's knowledge is concerned, finite element method has been applied for electromagnetic wave scattering problems for the first time in this work. Its several variations have been tried to solve the problems. Finite difference method has also been tried for these class of problems.

Solutions obtained for electromagnetic wave propagation problems through the waveguides of arbitrary shape and size, for all the four numerical methods, are very much satisfactory.Among those four methods, the finite element Rectangular method requires least time for a particular problem and finite difference five point takes the longest time for the same problem.Solutions obtained for the electromagnetic wave scattering problems by direct method were satisfactory.When the obstacle's dimension was very large compared to the wave length of the impinging wave then the results were not very much satisfactory for the poor convergency nature of the Hankel series.The range of satisfactory results zone can be extended by taking a longer Hankel series.Because of the limited memory capacity of the IBM system 360 computer at Bureau of Statistics, the upper limit of the Hankel series taken was 16.For that problem , a matrix of order 64 x 64 was solved. Similarly problems solved by approximate numerical methods for electromagnetic wave scattering problems,were limited by memory capacity.Electromagnetic wave scattering problems were previously solved by A.M.Patwary by finite difference method required matrix order 200 x 200 for good solutions.

Electromagnetic wave progation problems can be solvedfor TE modes and higher modes for arbitrary shaped waveguide .The waveguides may be hollow or may be filled,its cross section may be uniform or tapered These types of practical problems can be solved by different numerical methods.

As now BUET has its own computer IBM system 370 with 160K real store and 6M bytes virtual store,larger matrix problems associated with approximate numerical methods of scattering problems can be solved numerically.Here it can be mentioned that,Integral and Integral/Variational methods are still the most convenient methods for solving this class of problems.So some important works can also be done applying these two methods.

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