Economic Scheduling of Generation for Power Systems
in
Bangladesh

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CERTIFICATE

THIS IS TO CERTIFY THAT THIS WORK WAS DONE BY ME AND IT HAS NOT BEEN SUBMITTED ELSEWHERE FOR THE AWARD OF ANY DEGREE OR DIPLOMA.

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ABSTRACT

In determining the economic distribution of load between plants in a large power system we encounter the need to take losses in transmission lines into account and to express the total transmission losses of a power system as a function of plant loadings. Eastern Grid Network of Bangladesh Power Development Board (BPDB) with four major generating plants, namely at Kaptai, Siddhirganj, Ashuganj and Shahjibazar and eight major load centres is considered to be the basis of this study. The system is simulated in an A.C. Network Analyzer to determine loss-formula coefficient matrix, which is utilized to find the transmission losses as a function of the plant outputs.

On the basis of the plant heat rate data, input-output curves, incremental fuel rate, fuel cost versus power output curves for various plant, supplied by BPDB, optimum generation schedules have been obtained for cases of (i) neglecting and (ii) including transmission losses in scheduling formulations.

It has been shown that a considerable economy in fuel cost may be achieved in the case of generation schedule by including transmission losses over that when the losses have been neglected in generation schedules, and also over any other random methods of generation schedules.

ACKNOWLEDGEMENTS

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LIST OF SYMBOLS

a_m	Parameter as given by Eqn. (B.12)
B	Loss-formula coefficient matrix
B_{mn}	Loss-formula coefficients as given by Eqn. (3.8)
b_n	Parameters as given by Eqn. (B.12)
C	Transformation tensor
C_k^j	Transformation tensor from reference frame j to k
C^*	Conjugate of C
d_m	Parameter as given by Eqn. (B.24)
∂	Partial derivative
E	Voltage symbol
E_k	Voltage quantities with respect to feference frame k
E_{G_m}, E_{G_n}	Voltages of generators m, n
E_L	Load voltage
E_{L_j}	Voltage of load j
E_R	Voltage of reference point
F_t	Total fuel input to a system
F_n	Fuel input to plant n
$\mathcal{F}$	Parameter indicated by Eqn. (A.3)
G	Symbol indicating generator or plant
G_m	Generator number m
I_j	Current with respect to reference frame j
i	Symbol representing currant
i_C	Charging current
i_G	Generator current

i_{G_m}	Current in generator m
i_{d_m}	Real part of generator m current
i_{q_m}	Imaginary part of generator m current
i_k	Current in line k
$\|i_k\|$	Magnitude of current in line k
i_L	Total load current
i_{L_j}	Current of line or Load j
i_{LD_j}	Current of load j
j	Operator for imaginary term
L	Symbol for load point
N	Current distribution factor
N_k	Current distribution factor of line k
$N_{k(n)}$	Current distribution factor of line k when supplied by generator n only
P	Symbol for real power
P_n	Power supplied by generator n, load upon plant n
P_L	Transmission losses in line
P_R	Received load
pf_m	Power factor of generator m
Q	Symbol for reactive power
R_k	Resistance of line k
R_{m-n}	Real part of Z_{m-n} as indicated by Eqn. (B.18)
$\Re$	Real part of
T	Transformation tensor as given in Eqn. (B.16)

V_m	Voltage of generator m
$\lvert V_m \rvert$	Magnitude of voltage of generator m
X	Symbol of line reactance
Z	Symbol of line impedance
Z_{kk}	Parameter as indicated by Eqn. (B.5)
Z_{m-n}	Parameter as indicated by Eqn. (B.18)

Greck Symbols

θ	Symbol representing angle
ω	Parameter as indicated by Eqn. (B.12)
ω'	Real part of
σ_m	Angle of generator m with respect to some reference
ψ	Function arising from Lagrangian type of multipliers as given in Eqn. (A.1)
λ	Incremental cost of received powr

CHAPTER I

INTRODUCTION

1.1 General

An electric power system consists of three principal components: the generating stations, the transmission lines, and the distribution systems. The transmission lines are the connecting links between all the generating stations and the distribution systems. Well-coordinated, flexible and reliable transmission systems, with per unit cost of production of electrical energy as low as possible, are the most desirable requirements to the power generating company. As the system grows, more and more energy sources must be exploited to satisfy the increasing demand, and more transmission lines must be built to link the new generating stations to each other; the economic aspect of system operation becomes more and more important under the growing conditions.

The economic operation of power stations in an interconnected system is a complex problem to the power system engineer. This is usually achieved by optimum scheduling of the generating units to share the total load within the usual constraints of voltage, spinning reserve and stability limits of the system.

In an interconnected system where electric load is supplied from more than one generating plant, one plant with minimum fuel cost is normally chosen to supply part or full of the base load of the system. First, Hydro Stations, next Nuclear Plants, with high 'plant factor' are normally selected to do so. The rest of the load of the system has to be shared by the remaining generating stations. The problem is: which generator should generate what amount of power to meet the total demand of load such that maximum overall economy is achieved. Because of the following factors, the need to operate the system economically has become indispensable.

First, the generating plants which are normally situated near the fuel sources such as stored water, or fossil fuel reserves, may not necessarily be located adjecent to the load centres. The transfer of power over long distances from the generating plants to the load centre becomes unavoidable.

Second, the bulk power generated in a given area has to be transmitted to another area when the generating power is in excess of the consumption capacity of the generating area.

Third, even if the generating area has the capacity to consume the total of the locally generated power, power systems are to be interconnected with another area for purposes of economy interchange and spinning reserve capacity.

Because of its relative importance, the economic distribution of loads among different plants is a great problem to the load despatch and scheduling engineer.

1.2 Transmission Loss Considerations

One of the major problems involved in the operation of a power system is the scheduling of generation for the most economic system operation. (Much effort has been expended in the analysis of fuel costs and thermal performance of generating units at equal incremental fuel costs[1]. However, with the development of integrated power systems and interconnection of operating companies for purposes of economy interchange, it is necessary to consider not only the incremental fuel costs but also the incremental transmission losses for optimum economy.)

The need to take the transmission losses into account in determining the economic distribution of load between plants is thus great. (Although the incremental fuel cost at one plant bus may be lower than that of another plant for a given distribution of load between the plants, the plant with the lower incremental cost at its bus may be much farther from the load centre. The losses in transmission from the plant having the lower incremental cost may be so great that economy may dictate lowering the load at the plant with lower incremental cost and increasing it at the plant with the higher incremental cost. The coordination of incremental fuel cost and incremental transmission losses results in a considerable amount of fuel savings in terms of money.)

Another aspect of the determination of transmission losses is in connection with billing for various interconnection transactions for the operation of interconnected systems. The revenue to be gained by properly billing for losses involved during interconnection transactions may be indeed a very large.

Transmission loss considerations have often proved to be important in the planning of future systems with particular regard to location of plants and the design of transmission lines.

1.3 Review of Published Literature

A transmission-loss-formula expressing the total transmission losses in terms of source powers was first presented by E.E. George[2] in 1943. The formula was of the following form:

$$\begin{aligned} P_L &= \text{total transmission losses} \\ &= B_{11}P_1^2 + B_{22}P_2^2 + B_{33}P_3^2 + \dots\dots\dots + B_{nn}P_n^2 \\ &\quad + 2\,B_{12}P_1P_2 + 2B_{13}P_1P_3 + \dots \\ &\quad + 2B_{23}P_2P_3 + \dots\dots + 2B_{mn}P_mP_n \\ &= \sum_m \sum_n P_m B_{mn} P_n \qquad \dots \qquad \dots (1.1) \end{aligned}$$

where P_m, P_n = source powers,

B_{mn} = transmission-loss-formula coefficients.

The determination of the B_{mn} coefficients was based on a longhand procedure which required two to three week's work by two men for a system of eight to ten generators.

The application of the network analyser to determine a similar loss formula was developed later by Ward, Eaton and Hale[3] of Purdue University and published in 1950.

At the 1951 AIEE Summer Convention, G. Kron, in conjunction with G.W. Stagg and L.K. Kirchmayer, presented companion papers[4,5] which described an improved method of deriving a total transmission loss formula requiring considerably less network analyzer measurements and arithmatic calculations. Reference 4, in addition, evaluated the discrepancies introduced by the assumptions made in obtaining a loss formula.

The application of automatic digital computers to calculate a loss formula was presented by A.F. Glimn, R. Habermann, Jr., L.K. Kirchmayer, and G.W. Stagg in the summer of 1953[6]. An improved digital-computer method of calculating loss-formula coefficients is given in Reference 7.

W.R. Brownlee[8] has indicated a method of expressing transmission losses in terms of generator voltages and angles and the X/R ratios of the transmission circuits. Also, loss formulas involving linear terms and a constant term, in addition to the quadratic terms indicated by Eqn. (1.1), have been recently described[9,10]. The form of the loss equation is then

$$P_L = \sum_m \sum_n P_m B_{mn} P_n + \sum_n B_{no} P_n + B_{oo} \quad . \; . \; . \quad (1.2)$$

The first major step in the development of a method of coordinating incremental fuel costs and incremental transmission losses was presented in 1949 by E.E. George, H.W Page, and J.B. Ward[11] in their use of the network analyzer to prepare plant loading schedules for a power system. At the same time the electrical engineering staff of the American Gas and Electric Service Corporation, also with the aid of the network analyzer, developed a method of modifying the incremental fuel costs of the various plants on an incremental slide rule in order to account for transmission losses. Next, the American Gas and Electric Service Corporation, in cooperation with the General Electric Company, successfully employed transmission-loss formulae and punched-card machines for the preparation of penalty-factor charts to be used in the economic scheduling of generation[12]. The incremental production cost of a given plant multiplied by the penalty factor for that plant gives the incremental cost of power delivered to the system load from that plant. Optimum economy with the effect of transmission losses considered is then obtained when the incremental cost of delivered power is the same from all sources.

In 1952, L.K. Kirchmayer, G.W. Stagg[13] presented a mathematical analysis of various methods of coordinating incremental fuel costs and incremental transmission losses and an evaluation of the savings to be obtained by coordinating incremental fuel costs and incremental transmission losses. Progress in the analysis of the economic operation of a combined thermal and hydro-electric power system was reported by W.G. Chandler, P.L. Dandeno, A.F. Glimn, and L.K. Kirchmayer[14].

An interative method of calculating generation schedules suitable for the use of a high-speed automatic digital computer has been described by A.F. Glimn, R. Habermann, Jr., L.K. Kirchmayer, and R.W. Thomas[15]. For a given total load, the computer calculates and tabulates incremental cost of received power, total transmission losses, total fuel input, penalty factors, and received load, along with the allocation and summation of generation.

The American Gas and Electric Service Corporation early in 1955 installed an incremental transmission loss computer[16,17]. This computer calculates incremental transmission losses and penalty factors for various system operating conditions. The coordinated operation of this computer and an incremental slide rule furnishes a flexible and accurate method of taking into account the changing system conditions in the plant and on the transmission system.

With respect to interconnected systems energy accounting, methods[18,19,20,21,22,23] recently developed permit determination of loss formulae for each seperate company, and analytical interpretation of the loss-formulae of these companies for study of losses in interconnected operation. Loss-formulae can be derived which express losses in a given area in terms of the source and interconnection loadings of that area or in terms of source loadings in all areas and the scheduled interchange between areas. These formulas can also be used to express change in losses from a given condition in terms of the changes in source loadings and scheduled flows from a given condition.

1.4 Scope of Thesis

The scope of this thesis lies in the determination of transmission loss equations for the Eastern Grid Network of Bangladesh Power Development Board (BPDB)* using a G.E.A.C. Network Analyzer, and in the determination of the most economic generation schedules for the generating plants of the same system, including transmission losses. As far as is known , optimum generation schedules for the generating plants of the system under investigation, including transmission losses has not been attempted so far, in the existing system, although

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* Previously known as East Pakistan Water and Power Development Authority (EPWAPDA).

considered most desirable on all accounts. Economic operation of combined Eastern and Western Grid System has not been attempted since the two grids have not yet been interconnected and moreover, the generating capacity of the Western Grid is quite small in comparison with that of the Eastern Grid.

In order to achieve the above objectives, the necessary studies and experimental works were as follows:-

(i) to develop a simple method of determining transmission losses of the system under study.

(ii) to obtain loss-formula-coefficient matrix for the system from network analyzer study.

(iii) to ascertain the assumptions to be made in determining losses.

(iv) to obtain separate generation schedules by including transmission losses and by neglecting transmission losses.

(v) to estimate the transmission losses under the two methods of scheduling.

(vi) to investigate the difference of fuel input for the above two cases and also to compare these with the inputs obtainable by arbitrary loading of generating plants.

(vii) to calculate annual savings achieved under the most desirable schedulings.

1.5 Summary of the Remaining Chapters

In Chapter 2 a theoretical discussion of the economic operation of generating plants has been made for the method of neglecting transmission losses in generation schedules. The different characteristic curves involved have been obtained and drawn properly.

Chapter 3 gives a theoretical analysis of the development of transmission loss equations for the adopted system and the assumptions involved.

Chapter 4 presents the practical procedure of determination of loss-formula coefficients by means of network analyzer for the system under investigation.

In Chapter 5, incremental production costs of the plants have been coordinated with the transmission losses of the system to find the generation schedules. Individual plant generation against the total generation and total received load has been found for various values of incremental cost of received power. The schedules thus obtained have been compared with those obtained by the method of neglecting transmission losses.

In Chapter 6, a brief discussion is made upon the amount of annual savings, as a result of including transmission losses in generation schedules. A comparative study of fuel inputs has been given, with other arbitrary methods of loading the generating plants.

Conclusions of this study are presented in Chapter 7. This concluding chapter also contains a few possible extensions of the work in the future.

CHAPTER 2

ECONOMIC OPERATION WITHOUT CONSIDERING LINE LOSSES

2.1 Introduction

For a power system to return a profit on the capital invested, proper operation is very important. The power delivering companies, to this end, always try to achieve maximum efficiency of operation and to improve efficiency continually in order to maintain a reasonable relation between cost of kilowatthour to a consumer and the cost to the company of delivering a kilowatthour in the face of constantly rising prices of fuel, labour, supplies and maintanance.

In operating the system for any load condition the contribution from each plant and from each unit within a plant must be determined so that the cost of delivered power is a minimum. How the engineer has met and solved this challanging problem is the subject of this chapter.

An early method of attempting to minimize the cost of delivered power called for supplying power from only the most efficient plant at light loads. As load increased, power would be supplied by the most efficient plant until the point of maximum efficiency of that plant was reached. Then for further increase in load, the next most efficient plant would start to feed power to the system, and a third plant would not be called upon until the point of maximum afficiency of the second plant was reached. Even with transmission losses neglected this method fails to minimize.

We shall study here the most economic distribution of the output of a plant between the generator units within the plant. Since system generation is often expanded by adding units to existing plants, the various units within a plant and hence the various plants usually have different characteristics. The method that will be developed is also applicable to economic scheduling of various plant outputs for a given loading of the system without consideration of transmission losses.

2.2 Characteristics Curves of Plants

We have taken the Eastern Grid System of BPDB for economic study. Full description of this system will be found in Sec.3.2. Four large generating plants at Kaptai, Siddhirganj, Ashuganj and Shahjibazar mainly supply the system load. The Kaptai plant is a hydro plant which has a minimum fuel cost and may be employed to carry the base load, as usually happens. The rest of the plants will have different characteristics. To determine the economic distribution of load between these plants (or between units considting of a turbine, generator and boiler within a plant), the fuel input in Btu per hour must be known as a function of the power output in megawatts. Typical input-output curves* for Ashuganj, Siddhirganj and Shahjibazar plants are shown in Fig. 2.1, with the fuel input in Btu per hour

* Obtained by the courtesy of Efficiency Engineer, BPDB.

against the outputs in megawatts. The corresponding 'heat rate' which is obtained by dividing the input by the corresponding output, is given in Fig. 2.2. It is to be noted that units associated with heat rate are Btu per KW-hr.

Another consideration is to find 'incremental fuel rate' data. It is given by the following definition:

$$\text{incremental fuel rate} = \frac{\Delta\,\text{input}}{\Delta\,\text{output}} \quad \cdots\cdots \qquad (2.1)$$

In other words, the incremental fuel rate is equal to a small change in the input divided by the corresponding small change in the output. As the Δ quantities become progressively smaller, it is seen that the

$$\text{incremental fuel rate} = \frac{d(\text{input})}{d(\text{output})} \quad \cdots\cdots \qquad (2.2)$$

The units associated with the incremental fuel rate are the same as the heat rate (that is, in Btu per KWh.). Fig. 2.3 gives the incremental fuel rate versus station output graph for the three plants.

Again, Fig. 2.4* shows specific fuel cost in paisa per KW-hr. with megawatt output for Siddhirganj, Ashuganj and Shahjibazar plants. The fuel costs of the above plants in Taka per million Btu are given in Tables 2.1, 2.2 and 2.3. These costs may be used to convert the units of input-output curve

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* Obtained by the courtesy of Efficiency Engineer, BPDB.

to input in Takas per hour versus output in megawatts. Figs. 2.5(a) and 2.5(b) show such input-output characteristics for Ashuganj, Siddhirganj and Shahjibazar plants.

Again, the incremental fuel rate is converted to incremental fuel cost by multiplying the incremental fuel rate in Btu per KW-hr. by the cost in Taka per million Btu. The incremental fuel cost may also be obtained from the slope of input (Taka per hr.) versus output (MW) curves. The unit of incremental fuel cost is usually Taka per MW-hr. The incremental fuel cost graphs for the three plants are given in Fig. 2.6.

TABLE 2.1

Fuel cost Data of Ashuganj Power Station

Power output (MW)	Specific fuel cost (paisa/KW-hr)	Fuel input (Thousand Btu/KW-hr.)	Fuel cost (paisa/Thousand Btu)	Fuel cost (Taka/million Btu)
10	1.78	11.1	.16	1.6 → 3.72
15	1.72	10.8	"	"
20	1.66	10.45	"	"
25	1.62	10.1	"	"
30	1.58	9.85	"	"
35	1.54	9.60	"	"
40	1.52	9.46	"	"
45	1.49	9.38	"	"
50	1.48	9.30	"	"
55	1.48	9.30	"	"

TABLE 2.2

Fuel cost Data of Siddhirganj Power Station

Power output (MW)	Specific fuel cost (paisa/MW-hr)	Fuel input (Thousand Btu/KW-hr.)	Fuel cost (paisa/ Thousand Btu)	Fuel cost (Taka/million Btu)
10	2.06	18.10	.16	1.6 → 3.72
15	1.96	12.35	.16	"
20	1.89	11.90	"	"
25	1.86	11.68	"	"
30	1.87	11.69	"	"
35	1.88	11.70	"	"
40	1.89	11.78	"	"
45	1.895	11.82	"	"
50	1.90	11.90	"	"

TABLE 2.3

Fuel cost Data of Shahjibazar Power Station

Power output (MW)	Specific fuel cost (paisa/KW-hr.)	Fuel input (Thousand Btu/KW-hr)	Fuel cost (paisa/ Thousand Btu)	Fuel cost (Taka/million Btu)
6	1.80	19.0	.09	.9 → 2.90
7	1.60	17.8	"	"
8	1.50	17.0	"	"
9	1.44	16.0	"	"
10	1.38	15.2	"	"
12	1.30	14.4	"	"
15	1.22	13.65	"	"
20	1.37	15.00		
25	1.28	14.25		
30	1.22	13.65		
35	1.30	14.4		
40	1.26	14.0		
45	1.22	13.65		
50	1.27	14.20		
60	1.22	13.65		

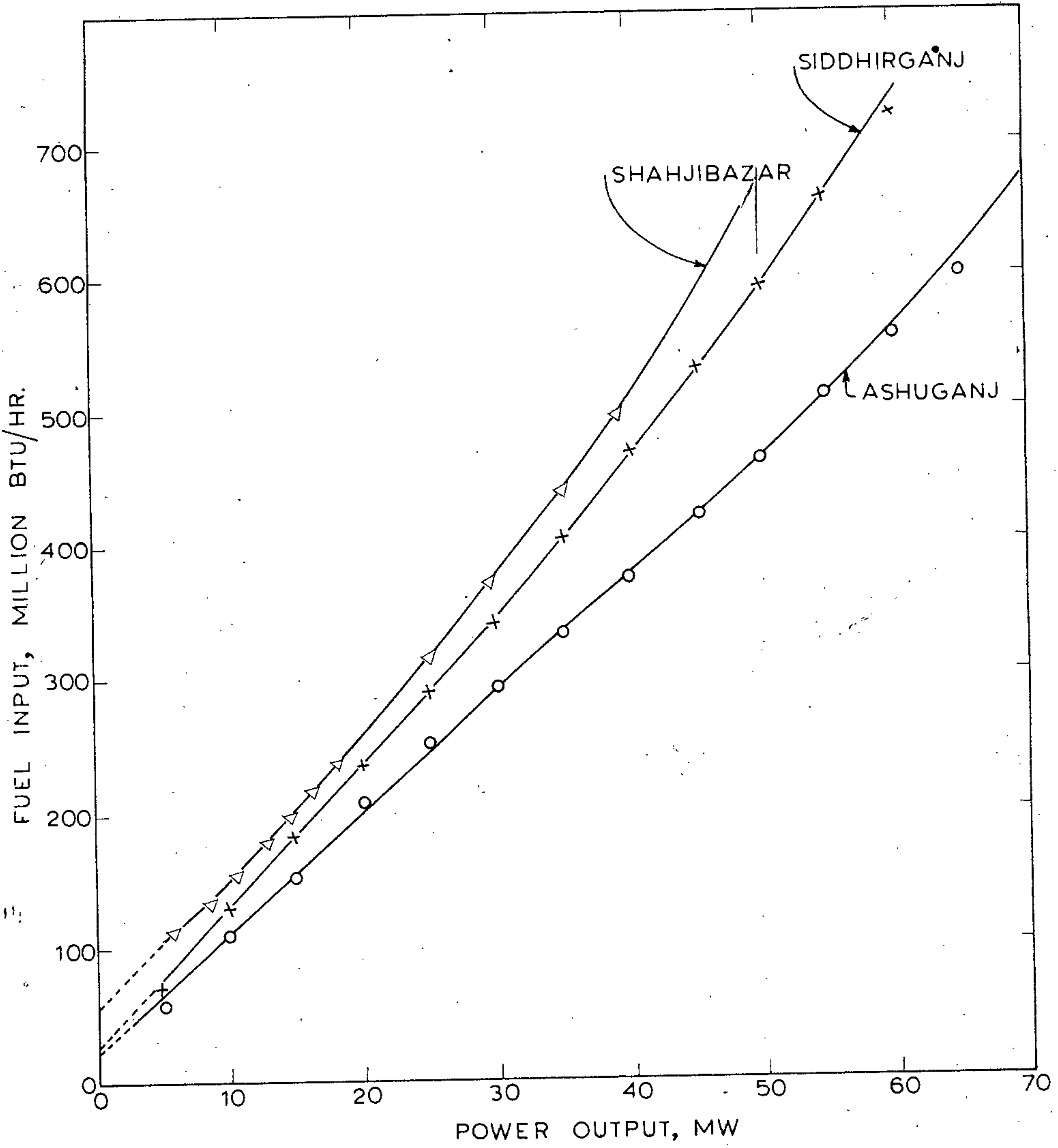

Fig. 2.1-Input - output curves for various plants showing fuel Input versus power output

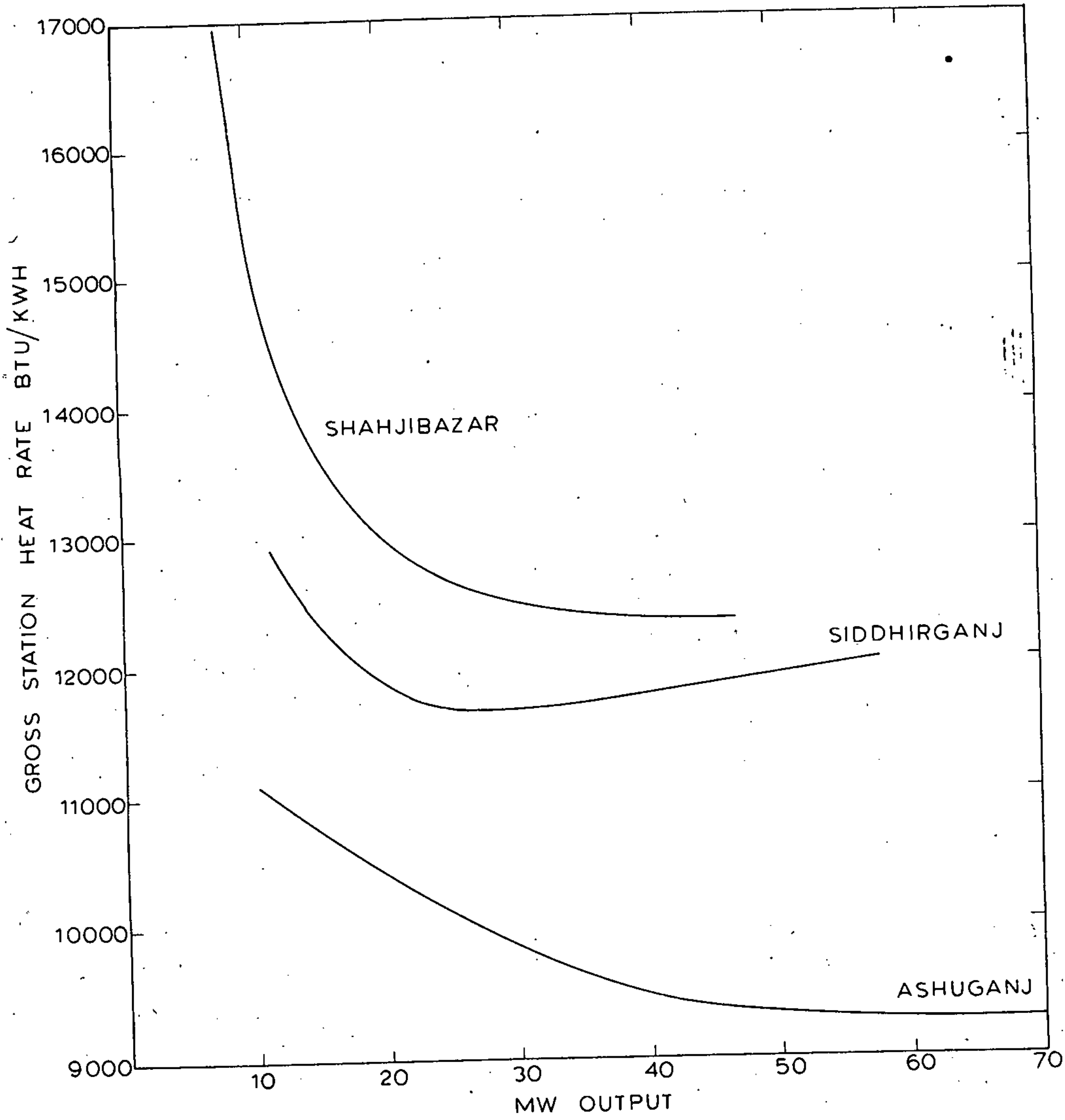

Fig. 2.2 - Heat rate versus power output curves for various plants

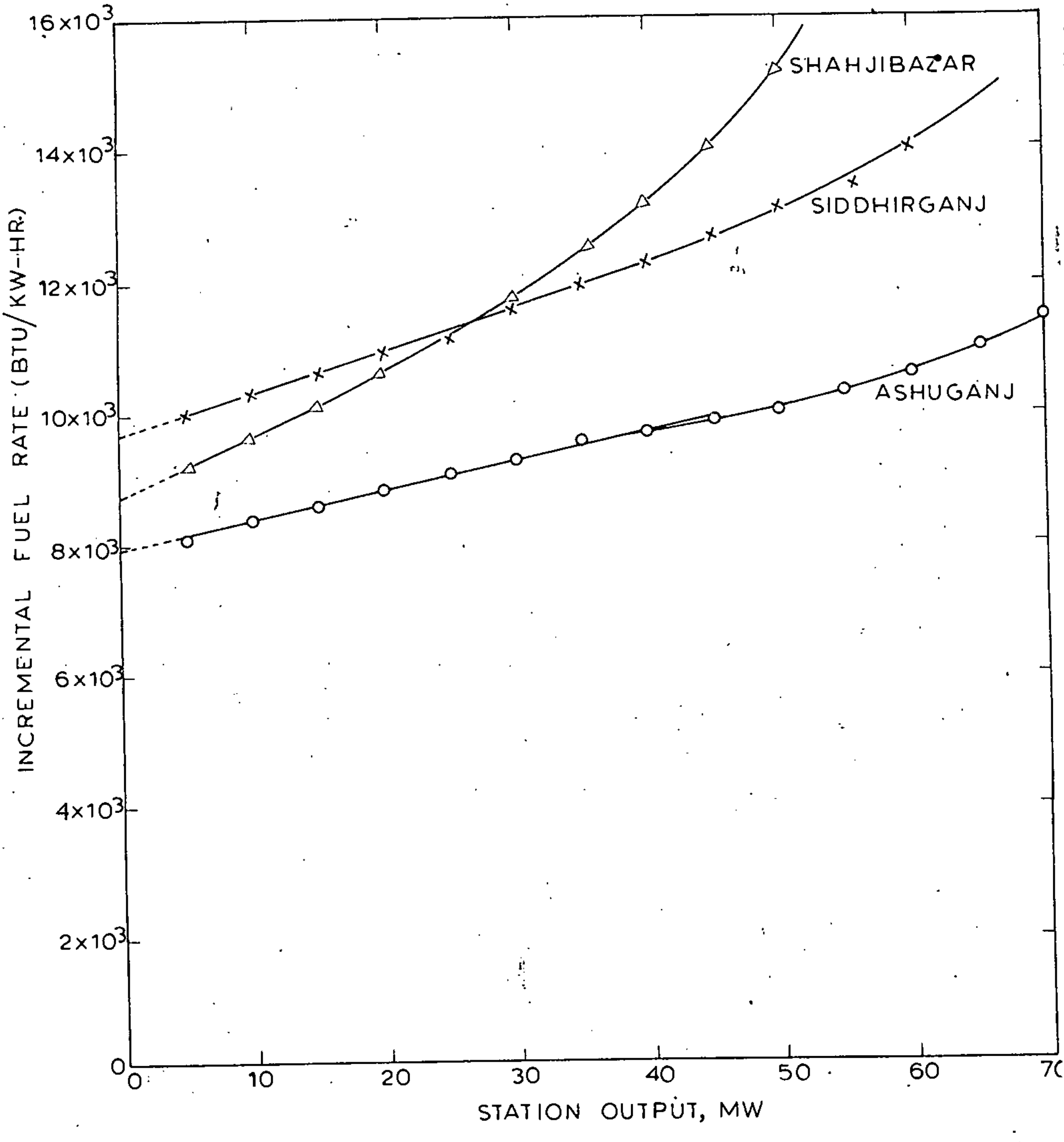

Fig. 2.3 - Incremental fuel rate versus power output curves for various plants whose input-output curves are given in Fig 2.1

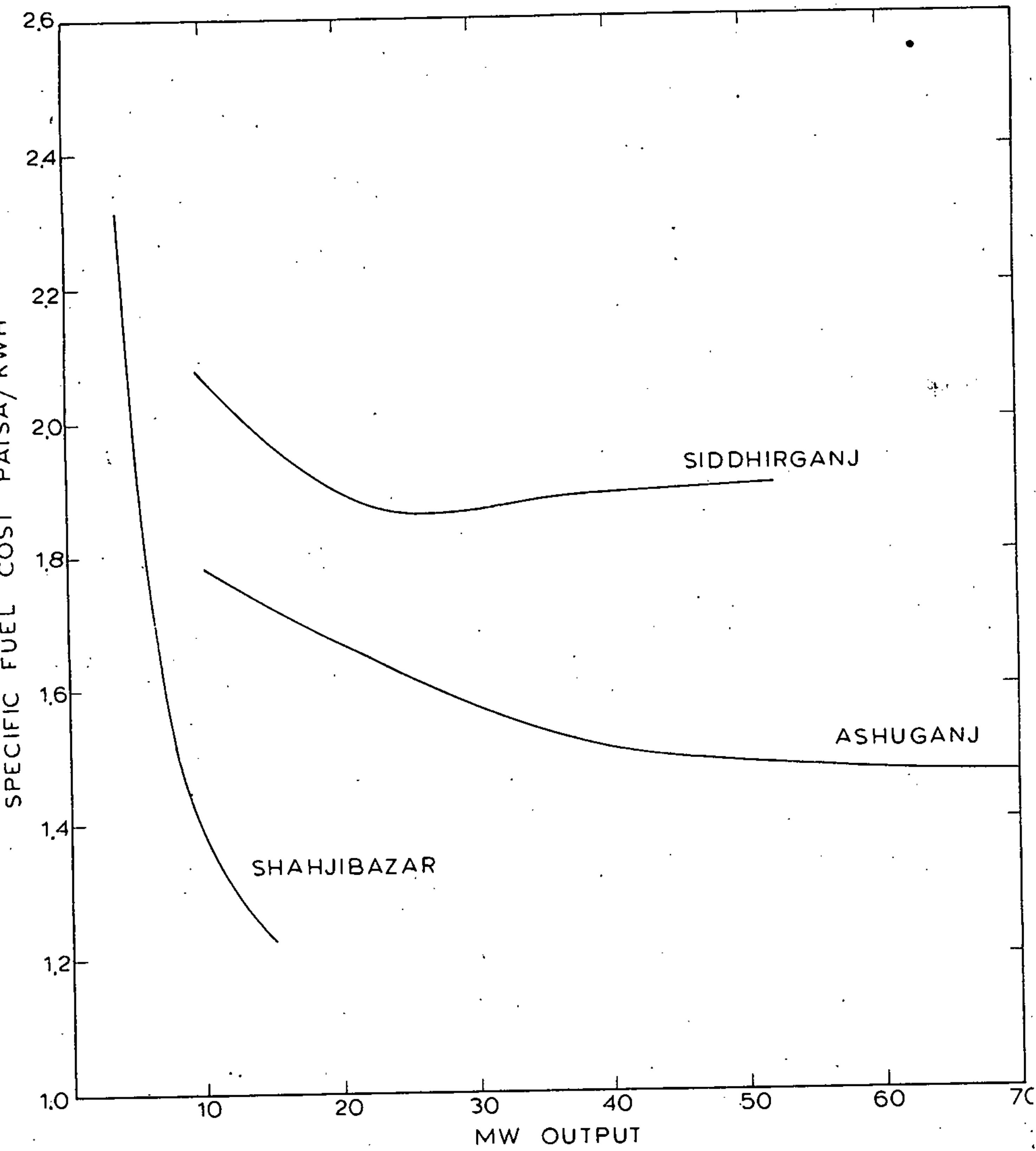

Fig. 2.4 - Fuel cost versus power output for the plants whose inpu -output curves are shown in Fig. 2.1

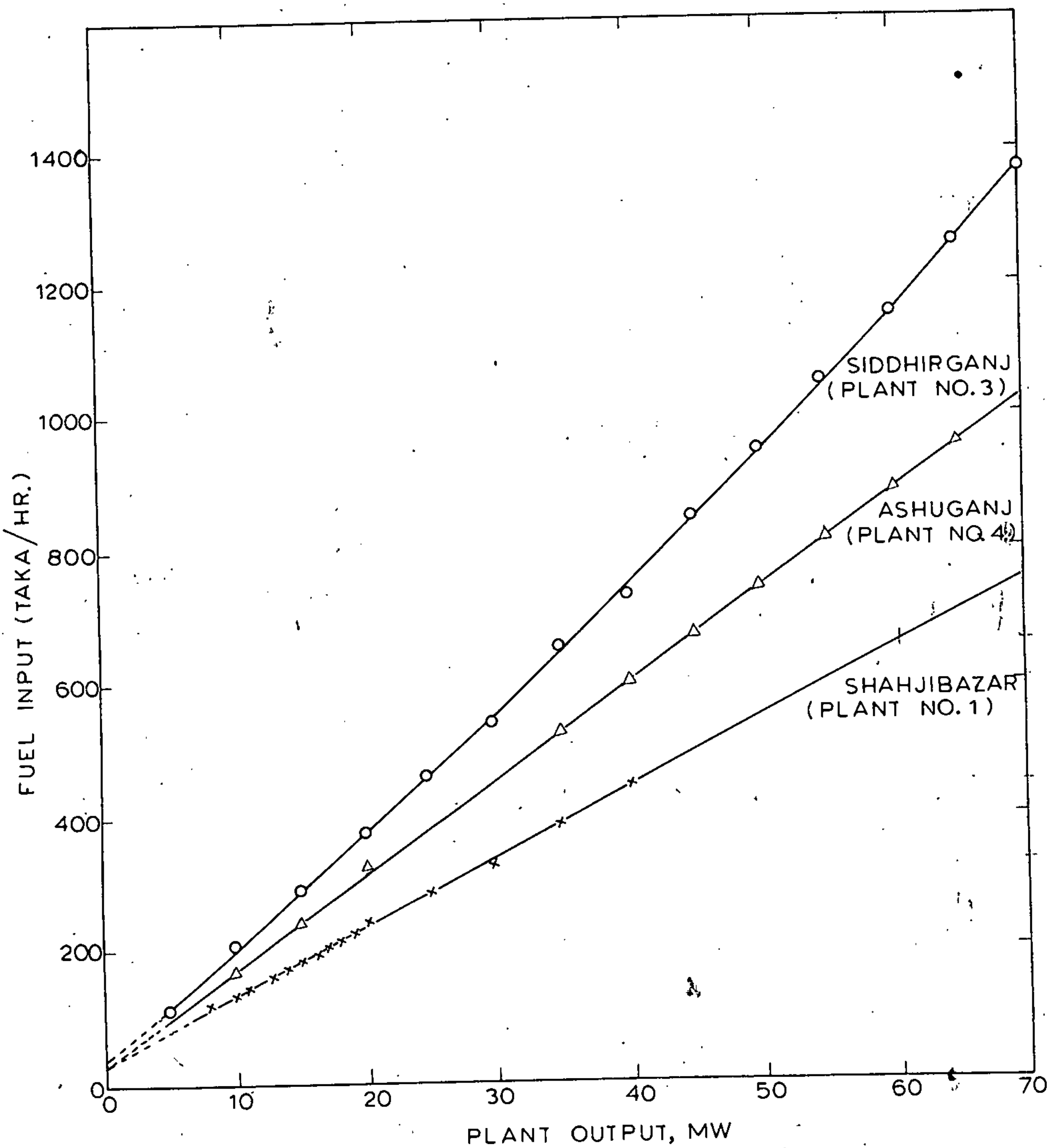

Fig. 2.5 (a) - Fuel input versus power output curves for the variou plants under consideration

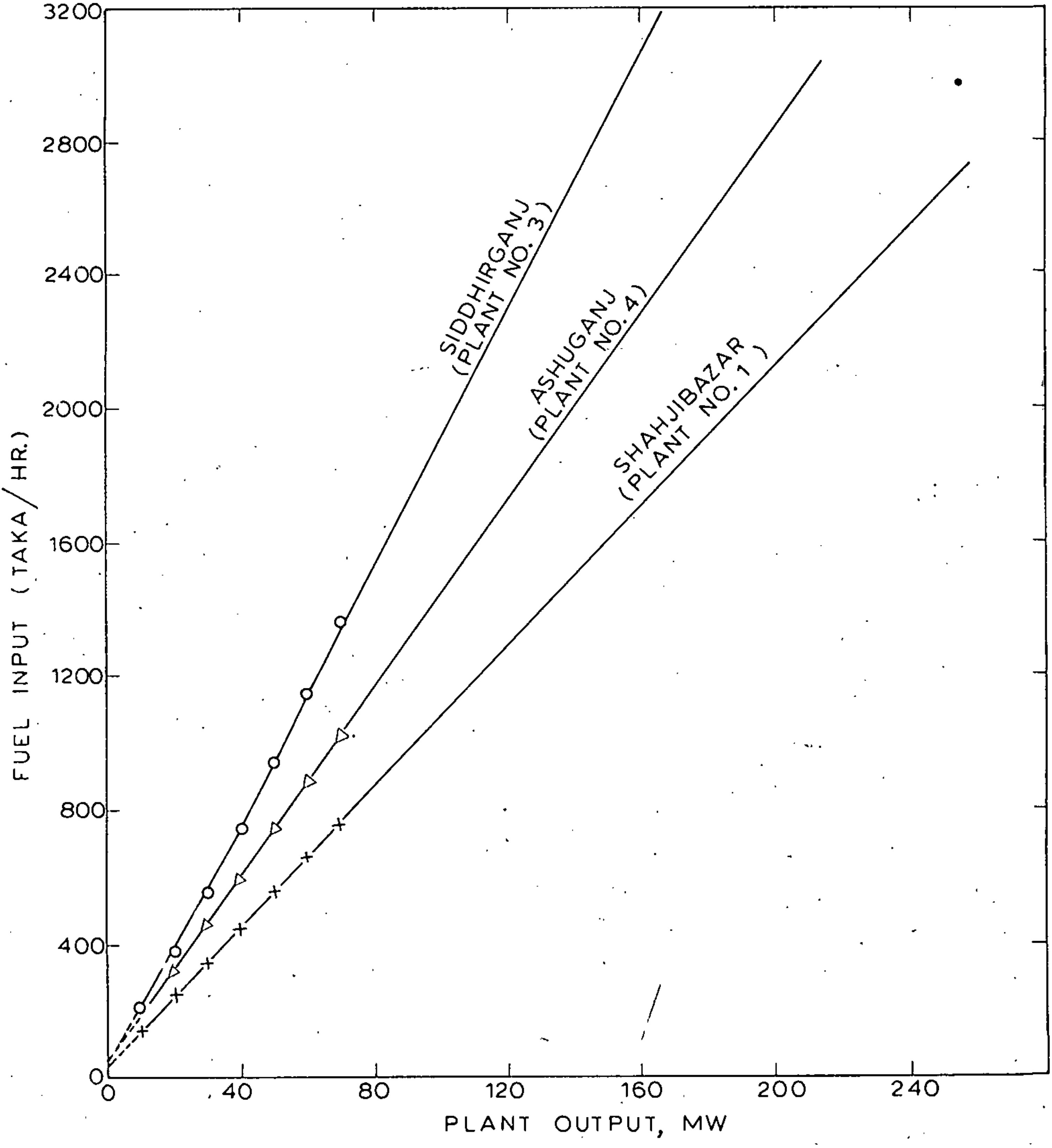

Fig. 2.5 (b) - Fuel input versus power output curves for the various plants under consideration

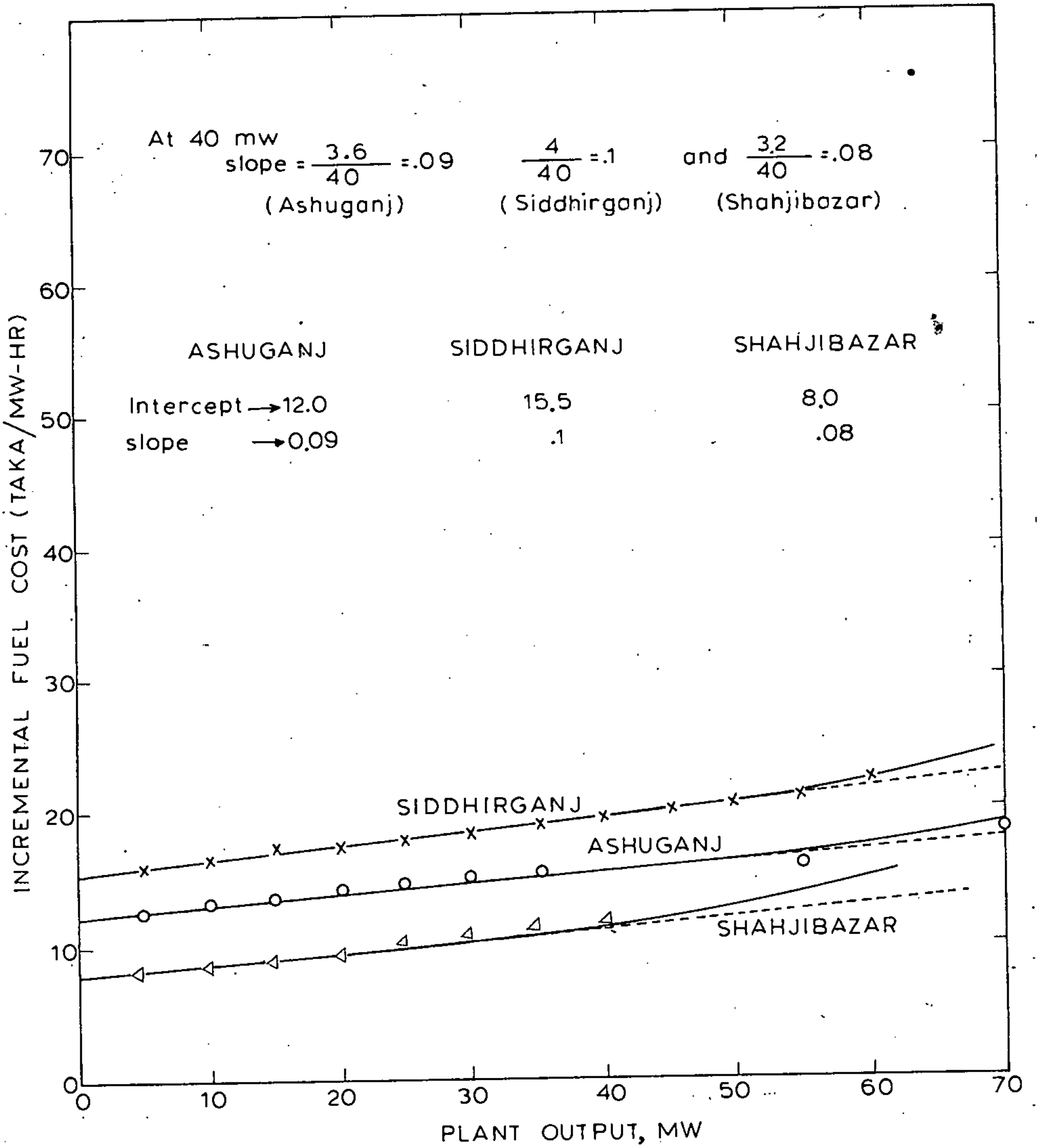

Fig. 2.6 - Incremental fuel cost versus power output for the plants whose input-output curves are shown in Fig. 2.1

2.3 Incremental Production Costs

The incremental production cost of a given unit or plant is made up of incremental fuel cost plus the incremental cost of such items as labour, supplies, maintenance and water. It is necessary for a rigorous analysis to be able to express the costs of these production items as a function of instantaneous output. However, no methods are presently available for expressing the cost of labour, supplies or maintenance accurately as a function of output. Arbitrary methods of determining incremental costs of labour, supplies, and maintenance are used, the commonest of which is to assume these costs to be a fixed percentage of the incremental fuel costs. In many systems, for purposes of scheduling generation, the incremental production cost is assumed to be equal to the incremental fuel cost. We shall assume, for our purpose, the incremental production cost as equal to the incremental fuel cost[24].

2.4 Optimum Generation Scheduling by Neglecting Transmission Losses.

We can now discuss the guiding principle of the most economic scheduling of generation for distributing the load among the units within a plant or among the plants without considering transmission losses of a system. For instance, let us suppose that the total load is supplied by two units of a plant and that the division of load between these units is such that the incremental production cost of one is higher than that of the other. Now, let us suppose that some of the load is transferred from the unit with the higher incremental cost to the unit with the lower incremental cost. Reducing the load on the unit with the higher incremental cost will result in a greater reduction of cost than the increase in cost for adding that same amount of load to the unit with the lower incremental cost. The transfer of load from one to the other can be continued with a reduction in total production cost until the incremental production costs of the two units are equal. The same reasoning can be applied to a plant with more than two units or more than one plants (without considering transmission losses) in a system. Thus the criterion for economic division of load between units within a plant or between plants in a system without considering transmission losses is that all units or plants must operate at the same incremental fuel cost.

The above statements may be justified in the following manner:-

Let F_n = input to nth unit or plant in Takas per hour.

F_t = total input to system in Takas per hour.

Then $F_t = \sum_n F_n$

It is desired to schedule generation such that

$$F_t = \text{Minimum} \quad \ldots \quad \ldots \qquad (2.3)$$

with the restriction that

$$\sum_n P_n = P_R = \text{received load} \quad \ldots \qquad (2.4)$$

where P_n = output of unit or plant n.

As explained above and also shown in Appendix-A, conditions 2.3 and 2.4 are satisfied when

$$\frac{dF_n}{dP_n} = \lambda \quad \ldots \quad \ldots \qquad (2.5)$$

where $\frac{dF_n}{dP_n}$ = incremental production cost of unit or plant n in Takas per MW-hr.

λ = incremental cost of received power in Takas per MW-hr.

Stated in words, the minimum input in Takas per hour for a given total load is obtained when all generating units or plants are operated at the same incremental production cost.

Again, the incremental production cost of a plant over a limited range may be represented by

$$\frac{dF_n}{dP_n} = F_{nn}P_n + f_n \quad \ldots\ldots \quad \ldots \qquad (2.6)$$

where F_{nn} = slope of incremental production cost curve.

f_n = intercept of incremental production cost curve.

Economic generation schedules are obtained by solution of the following equation:

$$F_{nn}P_n + f_n = \lambda \qquad \ldots \qquad \ldots \qquad (2.7)$$

for various values of λ. In Eqn. (2.7) increasing λ results in increase in total generation; decreasing λ results in a decrease in total generation.

2.5 Approximation of Incremental Production Costs

Incremental production cost for a plant or for a unit for any given power output is the limit of the ratio of the increase in cost of fuel input in Takas per hour to the corresponding increase in power output in megawatts as the increase in power output approaches zero. Practices vary greatly among companies in the representation of the incremental production costs. Some companies have refined the curves of Fig. 2.6 to include discontinuities due to valves, whereas others use a block representation.

In the interest of obtaining an economic design it is necessary to discover the simplest straight-line approximation of the incremental cost data that may be made without incurring appreciable loss in operating economy. In analytical work, the curve is often approximated by one or more straight lines. Fig. 2.6 shows that incremental production cost is quite linear with respect to power output over an appreciable range. Thus the dashed straight-lines in the figure is a good approximation of the curves representing Siddhirganj, Ashuganj and Shahjibazar incremental production costs with megawatt output.

2.6 Different Considerations and Limitations to Scheduling

Although the criterion of equal incremental production costs will result in the optimum economic scheduling of generation, the following methods of scheduling are sometimes still found in use[25].

1. Base Loading to Capacity.

The turbine-generator units are successively loaded to capacity in order of their efficiencies.

2. Base Loading to Most Efficient Load.

The turbine-generator units are successively loaded, in ascending order of their heat rates, to their most efficient loads. When all units are operating at their most efficient loads they are loaded to capacity in the same order.

3. Proportional to Capacity.

The loads on the units or plants are scheduled in proportion to their rated capacity.

The discussion thus far has considered the optimum allocation of generation for a given connected capacity. A problem which is not answered by inspection of incremental cost data is the determination of the maximum capacity to which a plant can be operated for a given total amount of load. Determination of this capacity is based upon such considerations as

1. Economic evaluation.
2. Reserve requirements.
3. Stability limitations.
4. Voltage limitations.
5. Ability to pick up load quickly.

Very frequently, and in particular, in extended systems, conditions 2 to 5 overrule condition 1.

The determination of the most economic combination of capacity to be operated at a given time is accomplished by inspection of the total fuel input to the system for various assumed combinations of capacity. Of course, for any assumed capacity in operation, the economic allocation of generation is given by equal incremental cost loading.

In general, in a given station the units are placed in service in ascending order of their heat rates assuming the cost per Btu to be the same. To determine the most economic combination of units for a given station load, it is necessary to plot total station heat rate curves of successive combinations and to note the combination providing the lowest heat rate for a given station load.

Another problem of importance is to determine the economic advisability of taking units off the line for relatively short periods of time, such as between the morning and evening peaks. This determination is based upon calculating the total fuel input in Takas to the system during this period of time with the units in question both on and off the line. This calculation should include cost of restoring the units under consideration back in service and losses involved in banking the boilers.

An analysis of the effects of errors in the economic despatching of power systems[26] is important in understanding and choosing the accuracy requirements of the components of a despatching system.

Deviations from the most economic schedule are obtained if

(1) The representation of the incremental production cost curve is in error.

(2) The servomechanism loop which matches the desired generation with the actual generation is inaccurate. In case of manual operation the station operator represents the servomechanism loop.

The two types of error indicated may occur in an automatic despatching system as well as in the manual despatching of a power system.

From a study of the effect of simultaneously displacing the incremental cost of one source by $(1+\epsilon)$ and the incremental cost of another source by $(1-\epsilon)$ the following conclusions may be drawn[24]:

(a) The loss in hourly economy varies as the square of the per unit error $\in$ in the representation of the incremental production cost.

(b) For a given value of $\in$ the loss of hourly economy varies directly as the square of the incremental cost level and inversely as the average slope of the incremental cost characteristics of the sources in question.

From a study of the effect of displacing the output of one source by + ΔP MW and the output of another source by $-\Delta P$ MW from the desired economic value it is noted that the loss of hourly economy varies as the square of the ΔP MW deviation from optimum schedule[24].

2.7 Systems with Transmission Losses

In the general case all sources of generation are not located at the same bus but are connected by means of a transmission network to the various loads. Some plants will be favourably located with respect to the load than others. Also, if the criterion of equal incremental production costs is applied, there will, in general, be transmission of power from low-cost to high-cost areas. Again, in determining the economic distribution of load between plants, it will be necessary to recognize that transmission losses occur in this operation and to modify the incremental production costs of all plants to take these line losses into account. To coordinate transmission losses in the problem we need to express the total transmission losses of a system as a function of plant output.

Chapter 3 will deal with the development of transmission loss equation which will be necessary to find optimum generation schedules with transmission losses.

CHAPTER 3

DEVELOPMENT OF TRANSMISSION LOSS EQUATION

3.1 Introduction

To include transmission losses in scheduling of plant output the basic problem involved is the determination of an expression for the transmission losses in terms of plant outputs. For this purpose, it is desired to proceed from a circuit in which the various sources are connected by an arbitrary transmission network to the individual loads, as shown in Fig. 3.1, to an equivalent circuit, as suggested in Fig. 3.2.

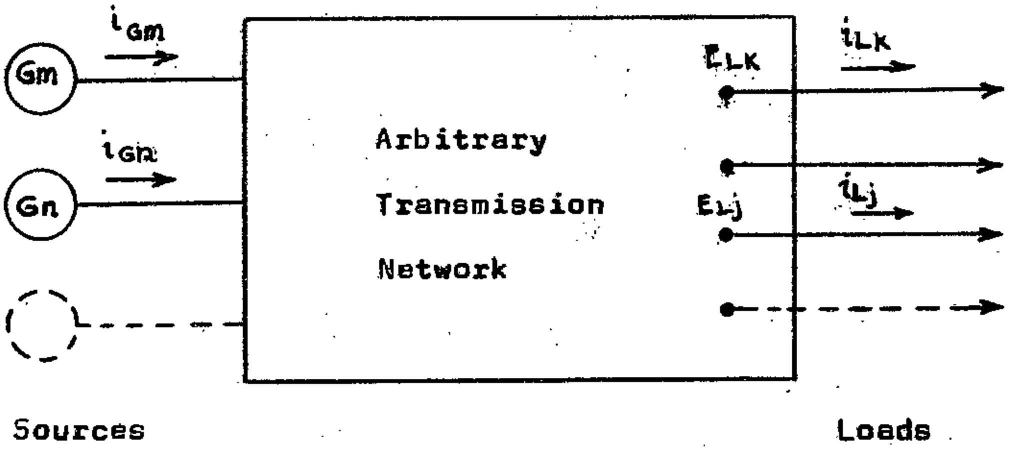

Fig. 3.1 Schematic diagram of a system connecting sources and loads by arbitrary transmission network.

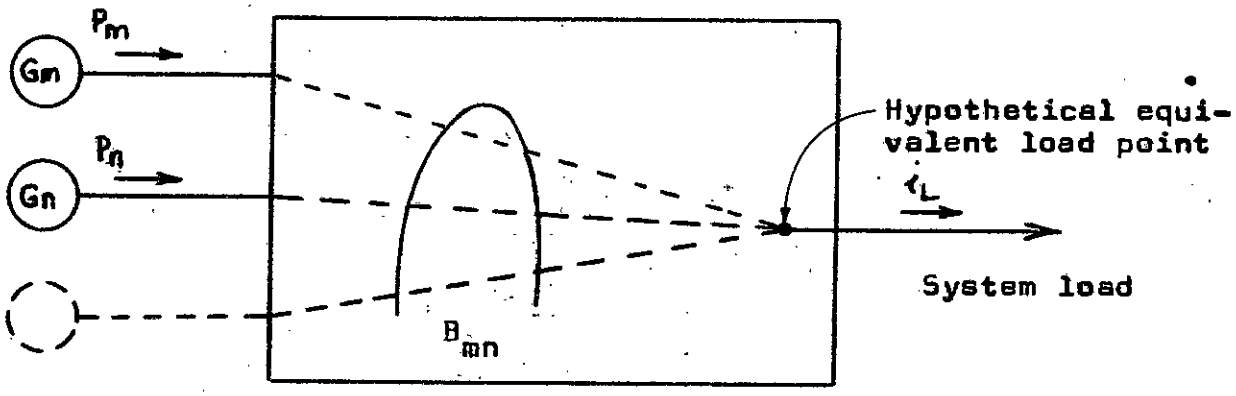

Fig. 3.2 Equivalent circuit representation of Fig. 3.1 with impressed generator powers supplying a single hypothetical load.

The transmission losses[4] of both Figs. 3.1 and 3.2 are to be identical and may be expressed in the following manner:

For Fig. 3.1

$$P_L = \sum_K i_k^2 R_k \text{ (transmission losses in terms of line parameters)} \quad . . . \quad (3.1)$$

where i_k = scalar line current in line k

R_k = resistance of line k

For Fig. 3.2

$$P_L = \sum_m \sum_n P_m B_{mn} P_n \text{ (transmission losses in terms of source powers)} \quad . . . \quad (3.2)$$

where P_m, P_n = outputs of sources m, n.

B_{mn} = loss-formula-coefficient matrix to be determined.

The method of transformation of Fig. 3.1 to 3.2, developed by Kron[5] is shown in Appendix B. In this chapter, we shall develop equations expressing system losses in terms of loss-formula- coefficients and source powers.

3.2 Description of Eastern Grid System of Bangladesh Power Development Board[27].

A general geographic layout of Bangladesh with the electric network as existing at present is shown in Fig. 3.3. The basic problem for the electric high voltage network of this country is created by the fact that the country is divided into an eastern and a western region by the Jamuna river. Therefore, the basic concept of the Power Development Board foresees two independent networks, an eastern and a western one. The generating capacity of the western region is small in comparison with that of the eastern part. Hence, eastern grid system or network has been taken up as the basis of this study.

A map of 132 KV system of the Eastern Grid showing major generating plants and principal interconnections (at present in operation) is indicated in Fig. 3.4. The interconnected system includes four major generating stations, namely, Kaptai, Siddhirganj, Ashuganj and Shahjibazar and eight major load centres. The system covers an approximate distance of 300 circuit miles. For the purposes of developing techniques and to determine the accuracy of transmission loss formula, a simplified impedance diagram of the system is shown in Fig. 3.5.

Physical simplifications made were the following facts:

1. Multicircuit lines were paralleled.
2. Small tapping stations were omitted.
3. Generators were directly bussed to the high voltage line.

3.3 Assumptions

The loss-formula coefficients may be considered as an equivalent transmission loss circuit from each generating source to the hypothetical load point as shown in Fig. 3.2.

An attempt to develop general loss-formula coefficients and consequently the transmission-loss-formula, as will be obtained presently, requires following simplifying assumptions:

1. The ratio X/R is considered to be the same for all branches of the network.

2. All load currents have the same phase angle and maintain a constant ratio to the total current.

3. The voltage at every source bus remains constant in magnitude.

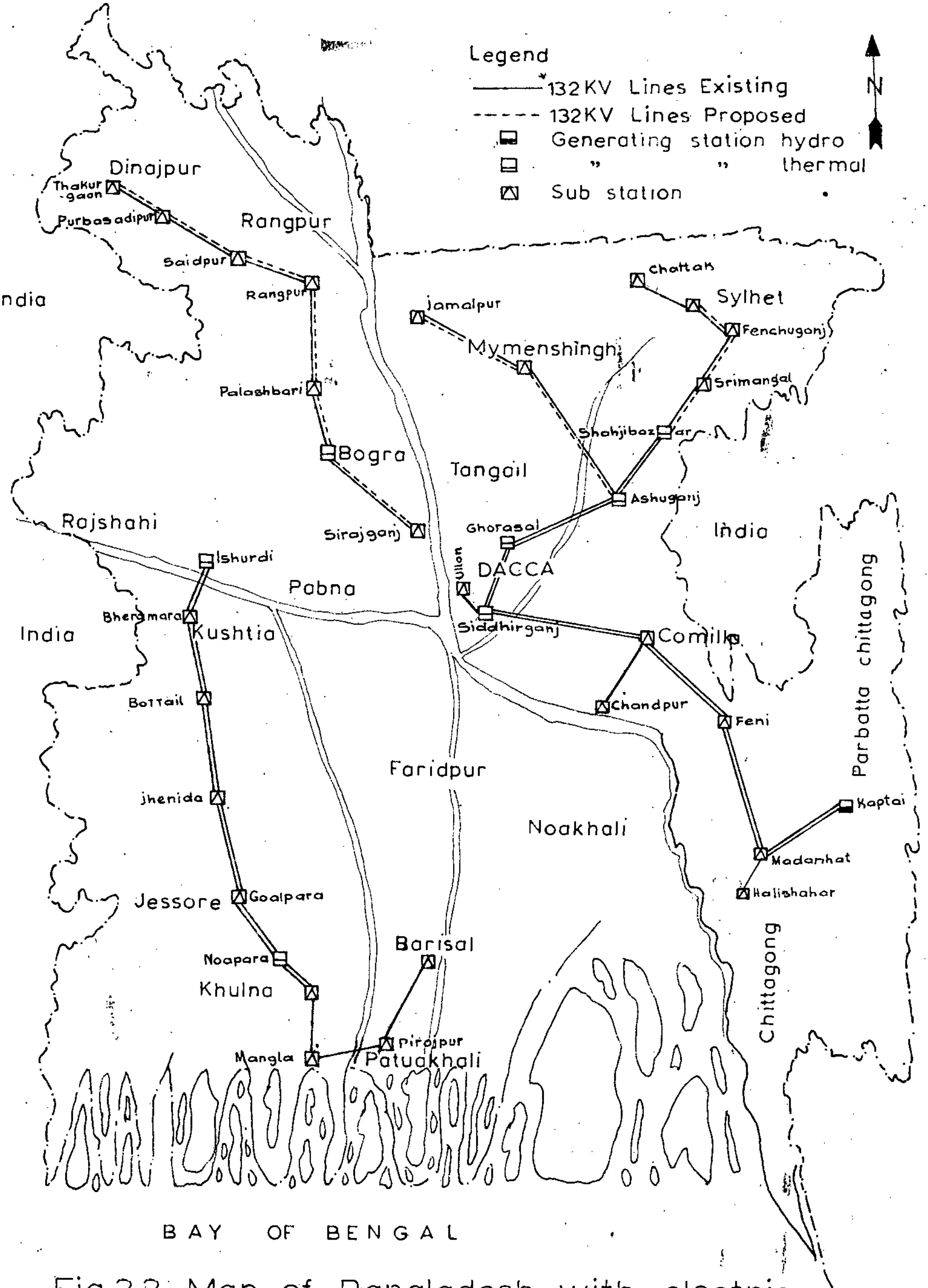

Fig. 3.3- Map of Bangladesh with electric network of BPDB.

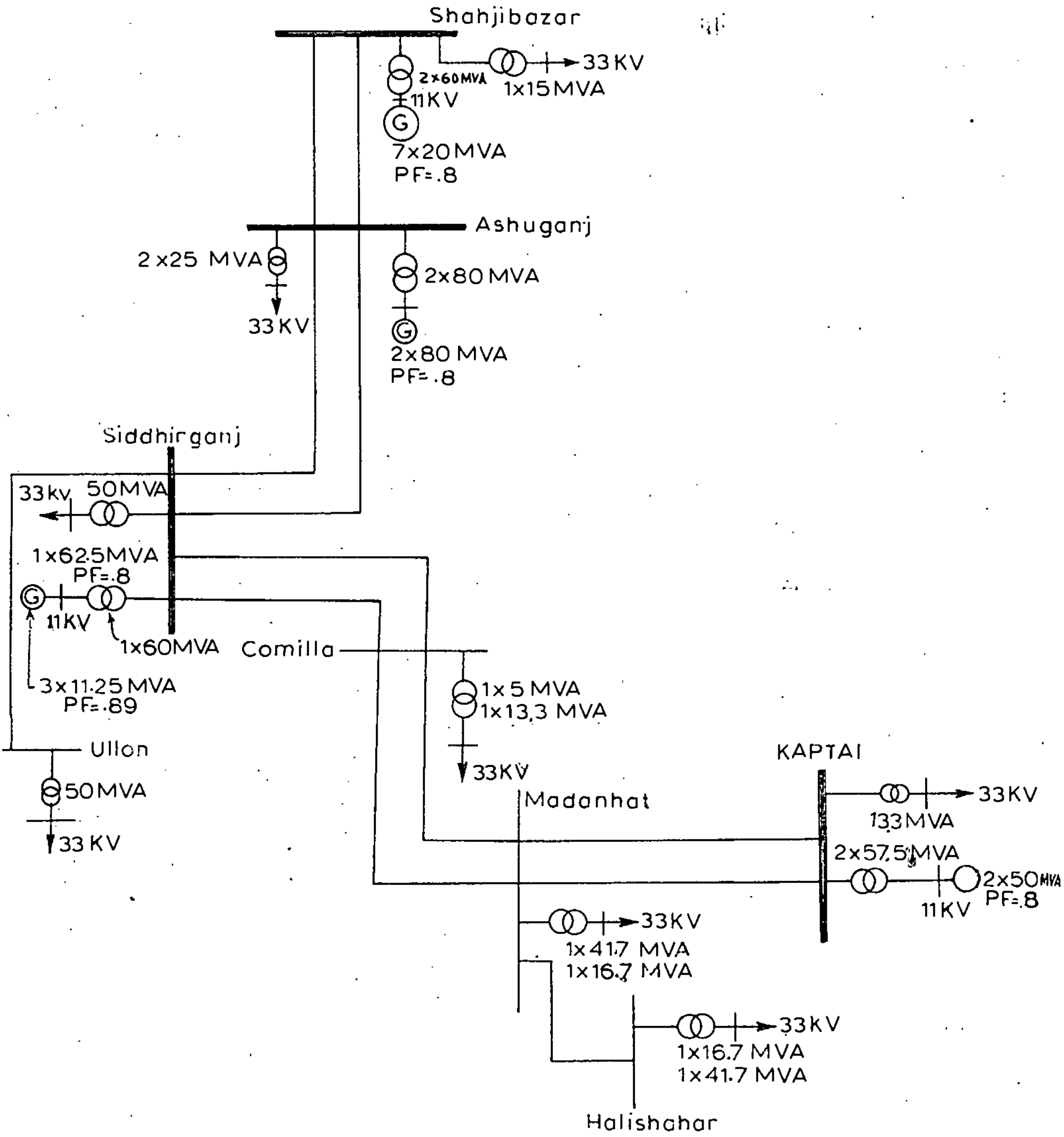

Fig. 3.4 - Electrical transmission system, eastern zone, Bangladesh power development board (Single line diagram)

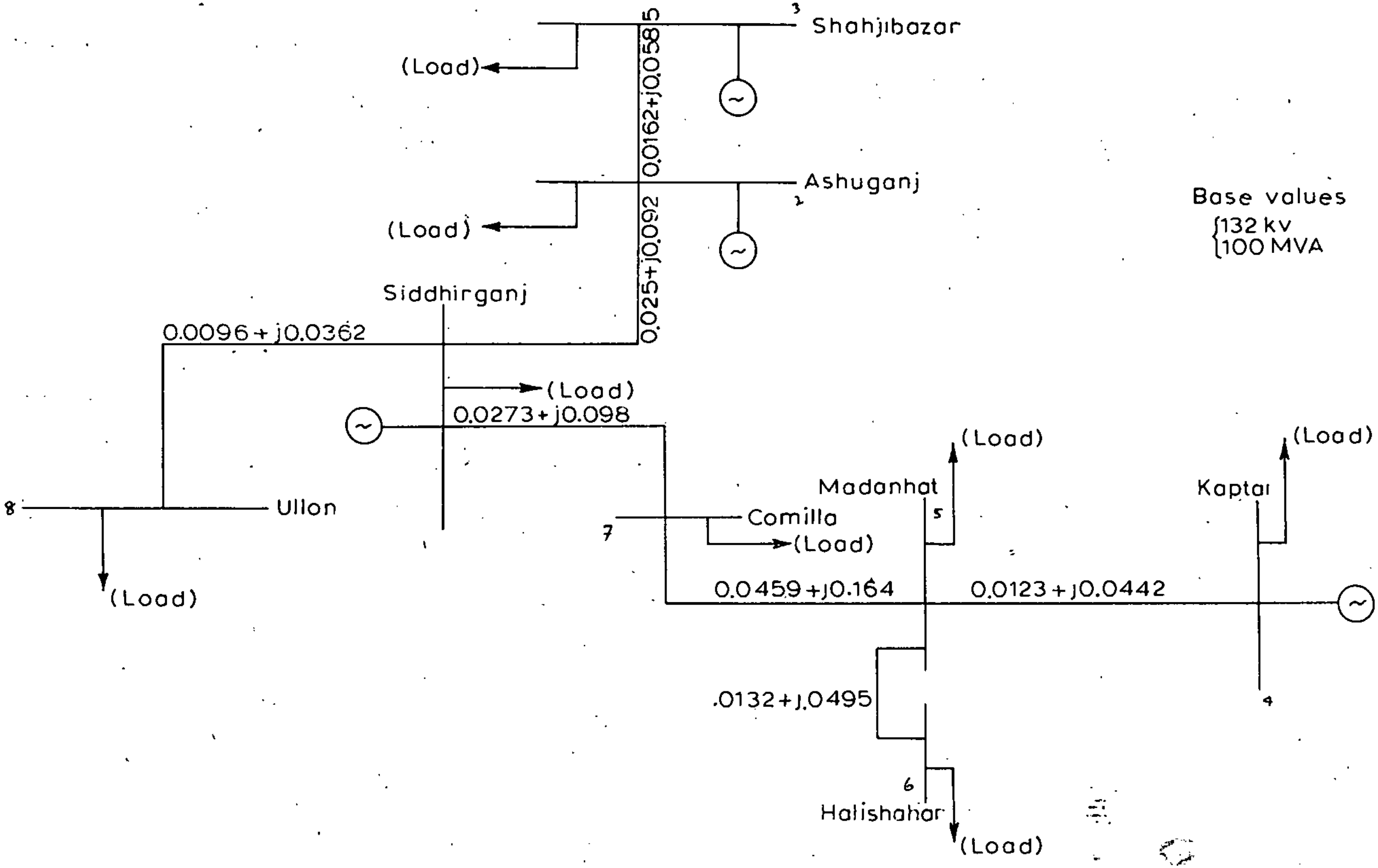

Fig. 3.5 - Simplified impedance diagram of the eastern grid of Bangladesh power development board.

4. Power factor at each source remains constant.

5. Voltage phase angles at source bus remain constant. This assumption is equivalent to assuming that source currents maintain constant phase angles with respect to a common reference since source power factors are assumed constant.

The assumption that load currents maintain a constant ratio to the total load current is reasonbly valid for most loads. Thus assumption 2 implies that the ratio of individual load current to the total load current (called current distribution factor) can be treated as real numbers. This assumption could be avoided if the availability of a large digital computer made the use of more elaborate method feasible.

Where extreme variations in operating conditions occur so that the assumptions cause appreciable errors in loss calculations, one or two additional sets of loss coefficients may be determined to apply to widely different circumstances. Many power companies, however, obtain sufficiently accurate results based on just one set of coefficients calculated for a typical operating condition.

3.4 Application of Superposition Principle to find Current Distribution Factors.

Let us consider the system of Fig. 3.1 which is redrawn with two generating plants and indefinite number of loads as shown in Fig. 3.6. The technique which will be followed[28] can be extended easily to suit a system having any number of sources.

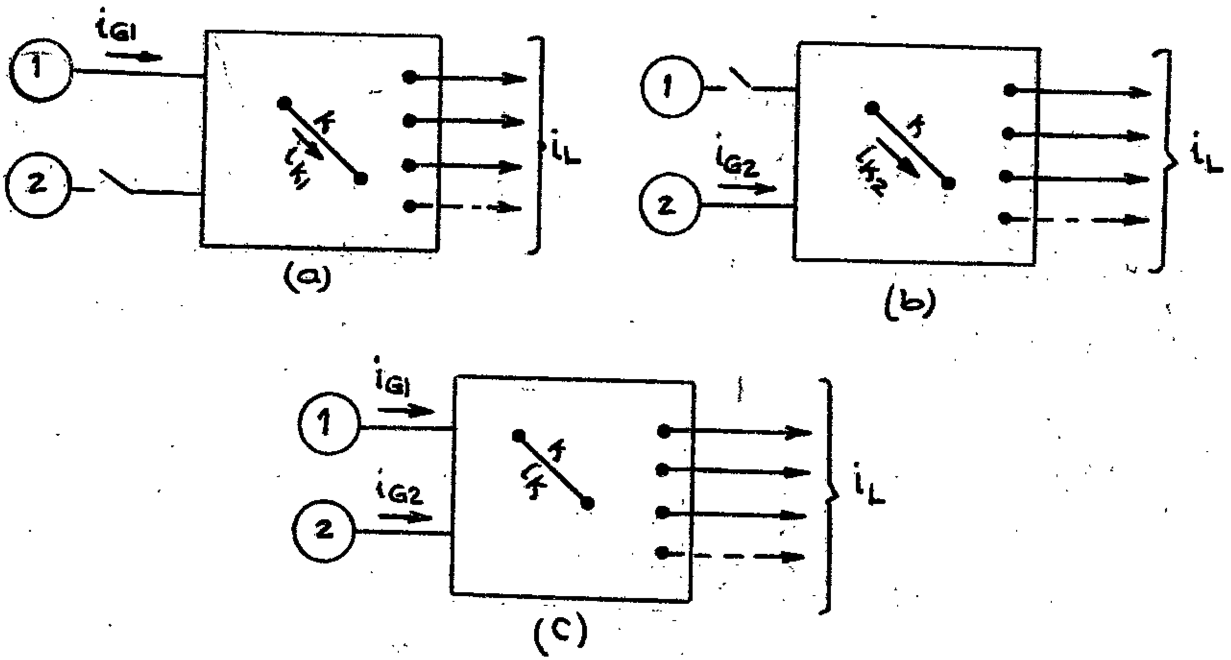

Fig. 3.6 Schematic diagram showing two generating plants connected through an arbitrary network to any number of loads. One branch k of the network is indicated.

One three-phase line within the system is designated as branch k. In Fig. 3.6(a), only source 1 is feeding the system, but all loads are connected. The total load current i_L is supplied by source 1, and the current in line k due to source 1 is $i_{k(1)}$. Let

$$N_{k(1)} = \frac{i_{k(1)}}{i_L} = \frac{i_{k(1)}}{i_{G_1}} \qquad \ldots \quad \ldots \quad (3.3)$$

Similarly, with source 2 supplying the entire load as indicated in Fig. 3.6(b), let

$$N_{k(2)} = \frac{i_{k(2)}}{i_L} = \frac{i_{k(2)}}{i_{G_2}} \quad \ldots \quad \ldots \quad (3.4)$$

By the superposition principle with both sources connected, as shown in Fig. 3.6(c), the current in line k is

$$i_k = i_{k(1)} + i_{k(2)}$$

$$= N_{k(1)}\, i_{G_1} + N_{k(2)}\, i_{G_2} \quad \ldots \quad \ldots \quad (3.5)$$

where i_{G_1} and i_{G_2} are the currents from plants 1 and 2, respectively. The term $N_{k(1)}$ and $N_{k(2)}$ are called the current distribution factors due to source 1 and 2 respectively. For n number of sources, the current in line k is

$$i_k = N_{k(1)} i_{G_1} + N_{k(2)} i_{G_2} + \; . \; . \; . \; . \; + N_{k(n)} i_{G_n} \quad \ldots \quad (3.6)$$

The current distribution factors $N_{k(n)}$ can be readily found when the system is set up on an a-c calculating board. The values of these factors for the system considered in Sec. 3.2 are found in Chapter 4. Under the assumptions made in Sec. 3.3, the current distribution factors are real, rather than complex.

3.5 General Expression for Loss-formula-coefficients and Transmission Loss Equation

Derivation of the general form of the transmission loss equation for any number of sources is shown in Appendix C. It is the same as given in Eqn.(2.2) and is repreated as

$$P_L = \sum_m \sum_n P_m B_{mn} P_n \quad \ldots \quad \ldots \quad (3.2)$$

where $\sum_m$ and $\sum_n$ indicate independent summations to include all sources.

For instance, for three sources,

$$P_L = P_1^2 B_{11} + P_2^2 B_{22} + P_3^2 B_{33} + 2P_1 P_2 B_{12} + 2P_1 P_3 B_{13}$$
$$+ 2P_2 P_3 B_{23} \quad \ldots \quad \ldots \quad (3.7)$$

Again, as shown in Appendix C, the general expression for the loss-formula-coefficients is

$$B_{mn} = \frac{\mathrm{Cos}(\sigma_m - \sigma_n)}{|V_m||V_n|(pf_m)(pf_n))} \sum_K N_{k(m)} N_{k(n)} R_k \quad \ldots \quad (3.8)$$

where σ_m = phase angle of current of generator m with respect to some common reference.

σ_n = phase angle of current of generator n with respect to some common reference.

pf_m = power factor of generator m.

pf_n = power factor of generator n.

$N_{k(m)}$ = current distribution factor of line k when all the load is supplied by generator m alone.

$N_{k(n)}$ = current distribution factor of line k when all the load is supplied by generator n alone.

R_k = resistance of line k.

In matrix form Eqn. (3.8) may be written (for a total of n sources) as :

$$B_{mn} = \begin{bmatrix} B_{11} & B_{12} & B_{13} & \cdots & \cdots & B_{1n} \\ B_{21} & B_{22} & B_{23} & \cdots & \cdots & B_{2n} \\ \cdot & & & & & \\ \cdot & & & & & \\ \cdot & & & & & \\ \cdot & & & & & \\ B_{n1} & B_{n2} & B_{n3} & \cdots & \cdots & B_{nn} \end{bmatrix} \quad ..(3.9)$$

The numerical values of the B matrix and the transmission losses for our considered system are given in Chapter 4.

CHAPTER 4

THE PRACTICAL DETERMINATION OF LOSS FORMULA COEFFICIENTS

4.1 Introduction

This chapter deals with the step by step procedure showing how the loss-formula coefficients for the system considered in Sec. 3.2 have been determined from actual operating conditions. One G.E. A.C. Network Analyzer having the capacity of representing 4 generator units, 30 transmission line units, 10 load units, 15 buss units, 12 capacitor units and 4 autotransformer units has been utilized for this study (Fig. 4.1). The procedure outlined in Chapter 3 has been followed in this chapter.

For the studies and calculations on the network analyzer, the system was represented as closely as possible to the existing network. Yet a few common reductions were made in order to simplify the procedure.

The reductions are:

(1) Generators in the same station and on the same high voltage bus were represented as one unit.

(2) Double circuit and parallel lines were represented as one common line.

(3) The generators were connected directly to the high voltage bus without consideration of the unit transformer.

(4) Loads were connected directly to the high voltage bus.

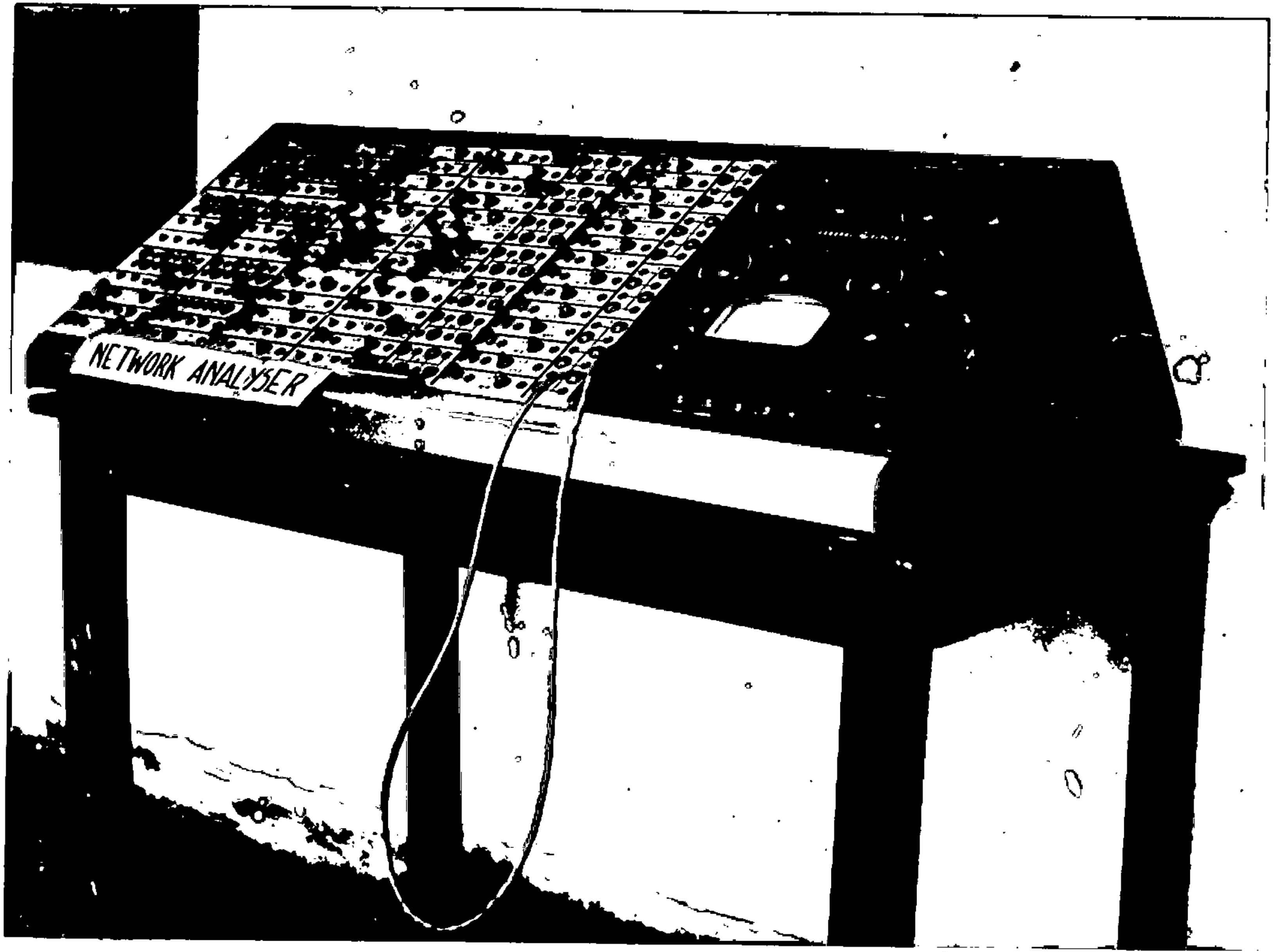

Fig. 4.1 General Photographic view of the G.E.A.C. Network Analyzer utilized in the study.

4.2 Simulation of the Circuit in Network Analyzer

Fig. 4.2 represents the Eastern Grid System of Bangladesh Power Development Board as simulated in the G.E.A.C. Network Analyzer. The numbers identifying generators, loads, lines, and capacitors are given in accordance with the particular unit number used in the analyzer. Fig. 4.3 shows the photographic view of the experimental set up as the system has been simulated in the G.E.A.C. Network Analyzer.

The setting up of the line impedances of Fig. 3.5 requires selection of proper range of the units available with the analyzer. The schedule of line impedances in this analyzer is given in Table 4.1.

A normal operating day load (date 27.8.70), a typical day in BPDB System is shown in Fig. 4.4. The data of the daily variation of this load encountered at different busses such as Kaptai, Madanhat, Halishahar, Comilla, Siddhirganj, Ullon, Ashuganj and Shahjibazar are given in Table 4.2. The hourly variations have been averaged and used in the network analyzer study. The average distribution of load on different busses is given in Table 4.3. The schedule of these loads in the Analyzer is given in Table 4.4.

The purpose of using average load is to make the resulting loss-formula coefficients consistent with the variation of daily load cycle. The schedule of line capacitances in the analyzer is given in Table 4.5.

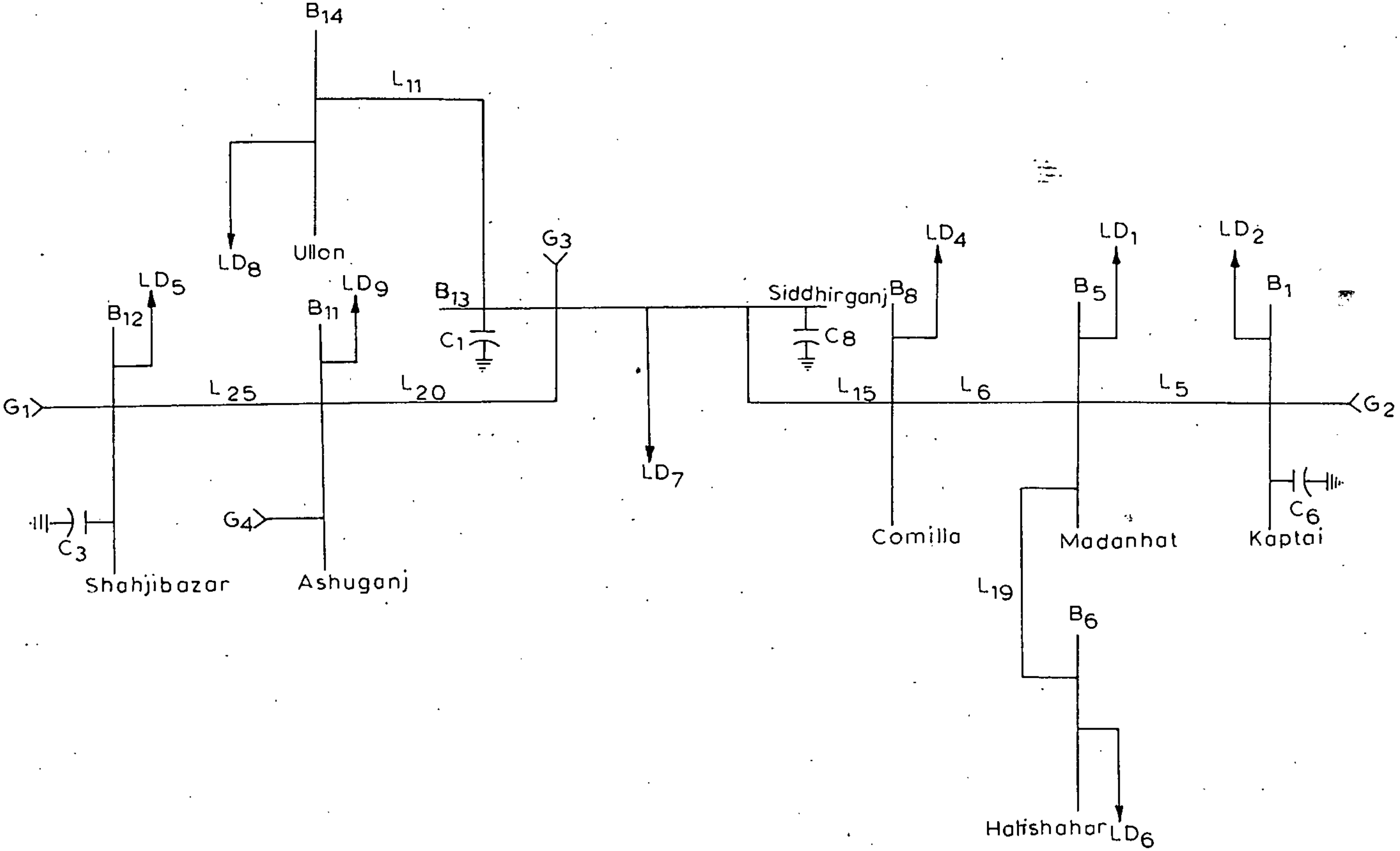

Fig. 4.2- Representation of eastern grid system in network analyzer

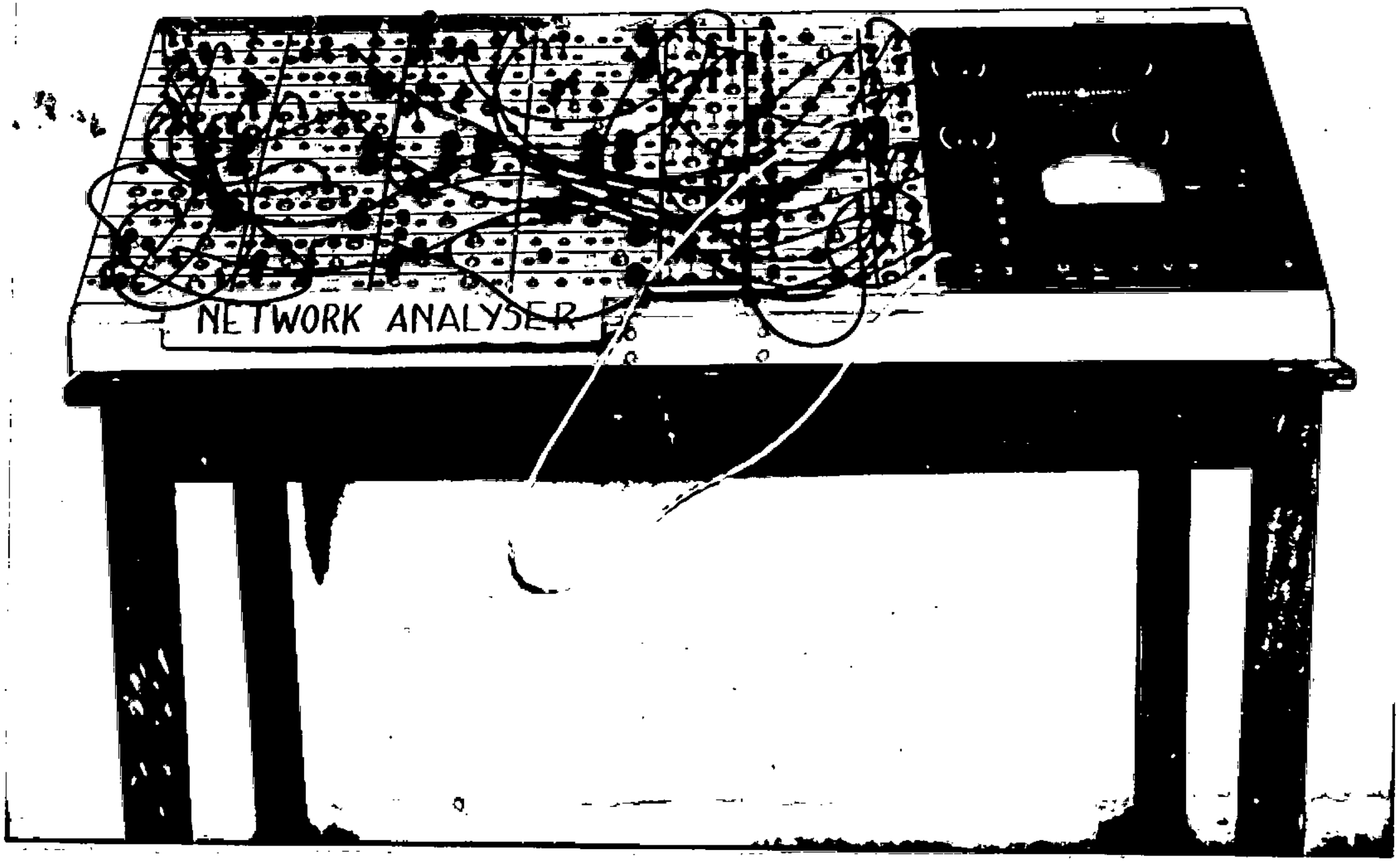

Fig. 4.3 Photograph: System as simulated in the A.C. Analyzer.

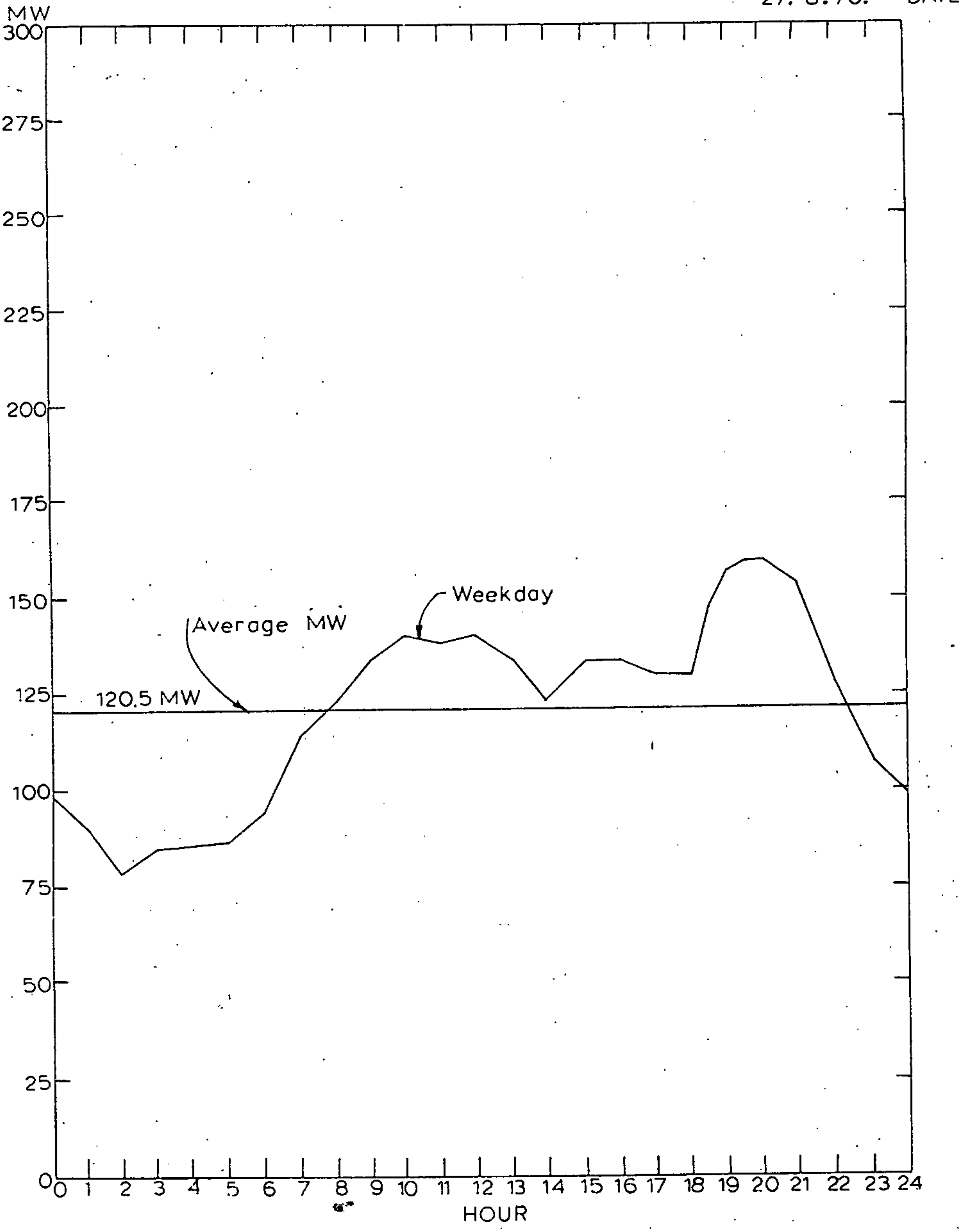

Fig. 4.4 - Actual daily load cycle of BPDB

TABLE 4.1

Base KV=132

Schedule of Line Impedances

Base MVA=100

Circuit name	$Z=R+jX_L$	Line unit No.	E^2/X_L (Q, set by L)	Z^2	E^2X_L/Z^2 (Q, set by R)	E^2R/Z^2 (P, checked)
A	0.0123+j0.0442	L_5	.226(E=.1)	.0021	.21	.0586
B	0.0459+j0.164	L_6	6.1(E=1)	.0289	5.67	1.59
C	0.0273+j0.096	L_{15}	10.4(E=1)	.00994	9.66	2.75
D	0.0132+j0.0495	L_{19}	.202(E=.1)	0.00262	.189	.0504
E	0.0096+j0.0362	L_{11}	.276(E=.1)	.00140	.259	.0685
F	0.025+j0.092	L_{20}	.109(E=.1)	.00907	.1015	.0276
G	0.0162+j.0585	L_{25}	.171(E=.1)	.00368	.158	.044

TABLE 4.2

Daily Variation of Load on Different Busses

Hour	Load on Busses (MW)							
	Kaptai	Madanhat	Halishahar	Comilla	Siddhirganj	Ullon	Ashuganj	Shahjibazar
00	3.2	8.0	23.0	5.2	31.0	20.0	2.2	.96
01	3.0	6.5	22.0	4.7	29.7	18.0	1.9	.80
02	3.0	5.5	21.0	4.4	28.8	18.0	1.8	.80
03	2.5	6.0	21.0	4.4	26.6	16.5	1.8	.80
04	2.1	6.0	20.0	4.4	25.6	16.5	1.8	.80
05	2.4	6.5	24.0	4.9	25.6	16.0	1.7	.75
06	2.5	7.0	23.0	5.9	37.36	16.0	2.0	.75
07	2.7	9.0	24.0	6.4	46.15	19.0	2.1	.75
08	3.0	10.0	28.0	6.7	51.70	20.5	2.4	.85
09	2.9	11.5	26.0	6.9	55.38	21.0	3.1	.97
10	3.0	11.5	27.0	6.8	59.23	22.0	3.8	1.20
11	3.1	11.0	25.0	6.8	58.53	22.5	3.5	1.50

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TABLE 4.2 (Continued)

12	2.9	11.0	25.0	6.7	59.27	22.0	3.6	1.40
13	3.0	10.0	26.0	5.9	57.20	21.0	3.5	1.30
14	3.5	10.0	27.0	6.0	55.80	20.0	3.5	1.30
15	3.6	10.5	26.0	6.9	55.90	20.1	3.4	1.26
16	4.1	11.0	25.0	6.5	56.80	22.5	3.4	1.50
17	4.0	11.0	26.0	6.9	57.0	27.2	3.6	1.55
18	4.62	12.0	29.0	8.0	60.10	32.0	3.9	1.65
19	4.7	12.5	30.0	8.5	65.0	32.0	4.0	1.70
20	4.85	13.5	30.0	8.0	63.9	31.0	4.0	1.60
21	4.5	12.0	30.0	7.6	57.50	30.0	3.6	1.45
22	4.2	11.0	24.0	6.5	45.90	27.8	2.8	1.20
23	4.0	9.0	24.0	5.6	33.9	24.0	2.4	1.0
24	3.0	8.0	23.0	5.2	31.0	21.0	2.2	.92
Average Load (MW)	4.0	9.0	25.0	6.0	50.0	22.0	3.0	1.5

TABLE 4.3

Average Distribution of Load

Load at Buss	MW	P.U. MW*	MVAR	P.U. MVAR*
Kaptai	4.0	.04	3.0	.03
Madanhat	9.0	.09	6.8	.068
Halishahar	25.0	.25	18.0	.18
Comilla	6.0	.06	4.5	.045
Siddhirganj	50.0	.50	38.0	.38
Ullon	22.0	.22	16.5	.165
Ashuganj	3.0	.03	2.27	.0227
Shahjibazar	1.5	.015	1.25	.0125

* Base MVA = 100

TABLE 4.4

Schedule of Buss Loads (for a normal weekday)

Location of Load at Buss	Buss Unit No.	Load Unit			Master Meter Multiplier Setting		Desired Reading (as per Table 4.3)	
		No.	S*/P*	Hi**/Lo**	Current	Voltage	Watt(P.U.)	Var(P.U.)
Kaptai	B_1	LD_2	P	Hi	.3	1.0	.04	.03
Madanhat	B_5	LD_1	"	"	.3	1.0	.09	.068
Halishahar	B_6	LD_6	"	"	1.0	1.0	.25	.18
Comilla	B_8	LD_4	"	"	.3	1.0	.06	.045
Siddhirganj	B_{13}	LD_7	"	Lo	1.0	1.0	.50	.38
Ullon	B_{14}	LD_8	"	Hi	1.0	1.0	.22	.165
Ashuganj	B_{11}	LD_9	"	"	.3	1.0	.03	.0277
Shahjibazar	B_{12}	LD_5	"	"	.3	1.0	.015	.0125

* S indicates series arrangement of load.

P indicates parallel arrangement of load.

** Hi indicates up position of load switch.

Lo indicates low position of load switch.

Base values: 132 KV
100 MVA

TABLE 4.5

Schedule of Line Capacitances

Line Section	Total Susceptance b(P.U.)	$\frac{b}{2}$ (P.U.)	$E^2\frac{b}{2}=Q$ (P.U.Var) ($E^*=1$)	Cap. Unit No.	Cap. Switch Position	Remarks
Kaptai to Siddirganj	0.168	0.084	0.084	C_6 C_8	3	Set C_6 Cap. at Kaptai bus and C_8 at Siddhirganj buss
Siddhirganj to Shahjibazar	0.0656	0.0328	0.0328	C_1 C_3	2	Set C_1 Cap. at Siddhirganj bus and C_3 at Shahjibazar bus.

E^* is the per unit voltage

4.3 Load Flow Study

For the system represented in Fig. 4.2 following load flow data were taken:

(1) Generator megawatts, megavars, line charging megavars, line megawatts, megavars and load megawatts, megavars (Fig.4.5).

(2) Scalar values of all line currents, load currents and voltage magnitude of each buss (Fig. 4.6).

Kaptai generator buss has been taken as the reference point and a detailed calculations were performed in Appendix D for verification and checking of load flow data in Figs. 4.5 and 4.6. A summary of generator quantities as obtained in load flow study of Figs. 4.5 and 4.6 and verified in Appendix D is shown in Table 4.6. A summary of load current components along with charging current is given in Table 4.7.

Because of the limitation of the Master-meter of the G.E.A.C. Network Analyzer, the generator voltage angles were obtained through calculations as given in Appendix D.

4.4 Current Distribution Factors (N_k) from Network Analyzer Study.

As already outlined in Sec. 3.4 that principle of superposition has to be applied to find current distribution factors. Each of Shahjibazar (Plant no. 1), Kaptai (Plant no.2), Siddhirganj (Plant no.3), and Ashuganj (Plant no.4) generating plants

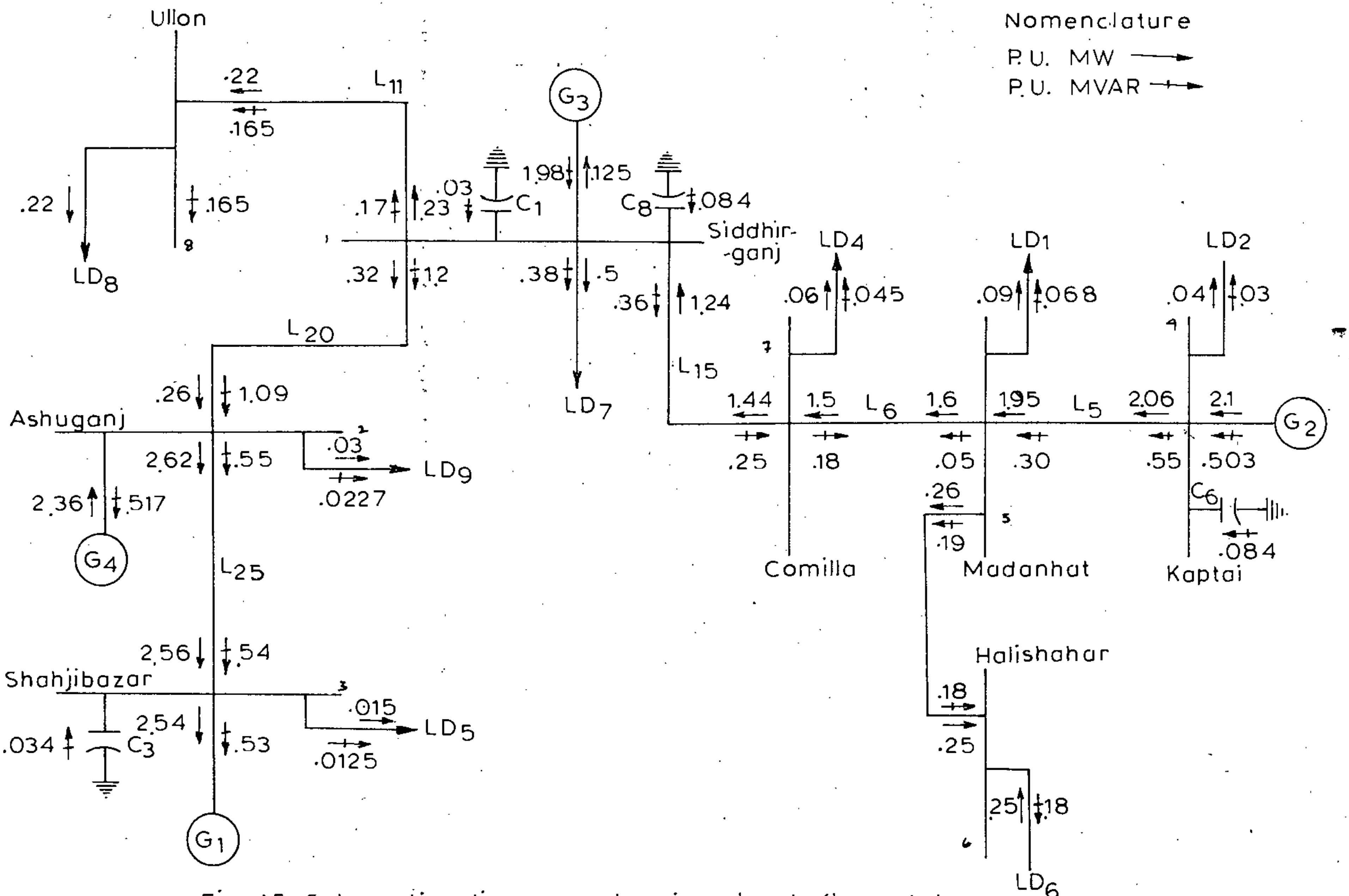


Fig. 4.5-Schematic diagram showing load flow data for a typical loading condition given in Fig. 4.4

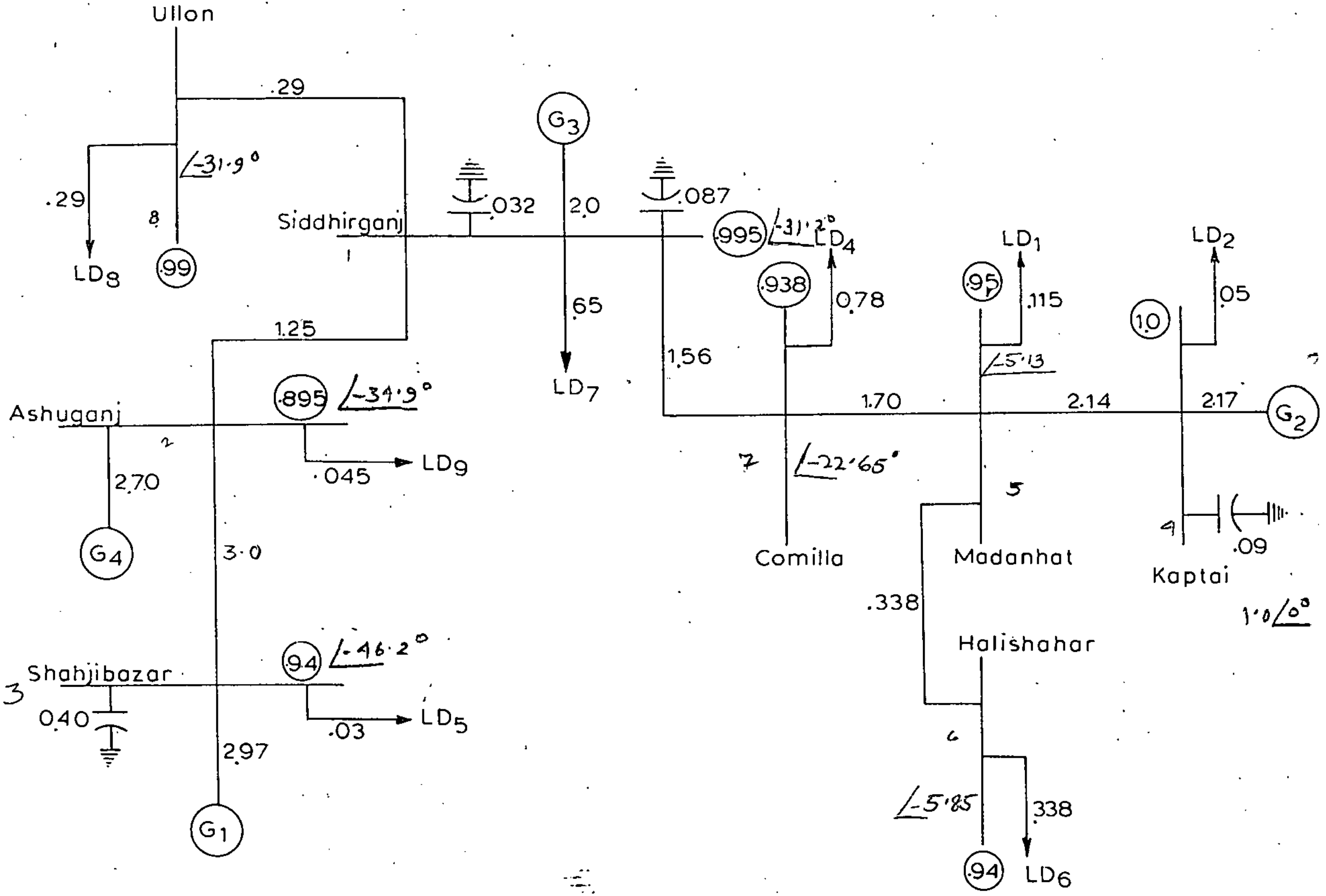


Fig. 4.6-Schematic diagram showing magnitudes of buss voltages and scalar values of line, load and charging current for the loading condition given in Fig. 4.4

TABLE 4.6

Summery of Generator Quantities

Generator No.	Generator Current $i_G = \|i_G\|\angle\theta = \|i_G\|(\cos\theta + j\sin\theta)$ (per unit)	P (per unit)	Q (per unit)	V (per unit)
G_1	$2.97\angle 169^\circ$ $= -2.906 + j.566$	2.54 (in)	.532 (in)	$.94\angle -46.2^\circ$
G_2^*	$2.17\angle -13.45^\circ$ $= 2.10 - j.504$	2.1 (out)	.503 (out)	$1.0\angle 0^\circ$
G_3	$2.0\angle -93.6^\circ$ $= -.47 - j1.935$	.125 (in)	1.98 (out)	$.995\angle -31.2^\circ$
G_4	$2.70\angle 12.4^\circ$ $= 2.63 + j.58$	2.36 (out)	.517 (in)	$.895\angle -34.9^\circ$

* reference generator

TABLE 4.7

Load Current Components

Load No.	P	Q	θ°	$i_{LD}\angle{-\theta}$*	$i_{LD}(\cos\theta - j\sin\theta)$	$i_{LD}\cos\theta - ji_{L\theta}\sin\theta$	$i_{capacitor}$ (charging current)	i_{bus} (load current + charging current)
i_{LD_2}	.04	.03	36.8	$.05\angle{-36.8^{\circ}}$	$.05(.8-j.6)$	$.04-j.03$	$j.09$	$.04+j.06$
i_{LD_1}	.09	.068	37.0	$.115\angle{-37.0^{\circ}}$	$.115(.798-j.603)$	$.0917-j.0609$	-	$.0917-j.0609$
i_{LD_6}	.25	.18	35.7	$.338\angle{-35.7^{\circ}}$	$.338(.81-j.585)$	$.272-j.198$	-	$.272-j.198$
i_{LD_4}	.06	.045	36.8	$.078\angle{-36.8^{\circ}}$	$.078(.8-j.6)$	$.0624-j.0608$	-	$.0624-j.0608$
i_{LD_7}	.5	.38	37.2	$.65\angle{-37.2^{\circ}}$	$.65(.795-j.605)$	$.516-j.393$	$j.119$	$.516-j.274$
i_{LD_8}	.22	.165	36.8	$.29\angle{-36.8^{\circ}}$	$.29(.8-j.6)$	$.232-j.174$	-	$.232-j.174$
i_{LD_9}	.03	.0227	36.8	$.045\angle{-36.8^{\circ}}$	$.045(.8-j.6)$	$.036-j.027$	-	$.036-j.027$
i_{LD_5}	.015	.0125	36.8	$.03\angle{-36.8^{\circ}}$	$.03(.8-j.6)$	$.024-j.018$	$j.04$	$.024+j.022$

* loads considered lagging ones.

Total = $1.27-j.708$

was considered separately in the G.E.A.C. Network Analyzer to supply the entire load of the system. Figs. 4.7, 4.8, 4.9 and 4.10 show the components of line and load currents (these values are real according to the assumptions made in Sec.3.3) as the entire system load was supplied by Plant 1, Plant 2, Plant 3 and Plant 4 respectively. The current distribution factors were found from the following equations:

$$N_{k(1)} = \frac{i_{L_{k(1)}}}{i_L} \quad \dots \quad (4.1)$$

$$N_{k(2)} = \frac{i_{L_{k(2)}}}{i_L} \quad \dots \quad (4.2)$$

$$N_{k(3)} = \frac{i_{L_{k(3)}}}{i_L} \quad \dots \quad (4.3)$$

$$N_{k(4)} = \frac{i_{L_{k(4)}}}{i_L} \quad \dots \quad (4.4)$$

where $i_{L_{k(n)}}$ = current in line k (real values) when supplied by plant n only.

$$i_L = \sum_{k}^{i} i_{L_{k(n)}} = \text{total current} \quad \dots \quad (4.5)$$

$N_{k(n)}$ = current distribution factor of line k when suppling plant n only.

Considering the proper direction of flow of currents when all the generating plants are connected to the system (Fig.4.5), the values of N_k's as obtained are summarized in Table 4.8. For a check of the values of current distribution factors, the line currents are calculated by using N_k's [as given by Eq.(3.6)] and given in Appendix E.

TABLE 4.8

Summary of Calculation of Current Distribution Factors (N_k)

Only plant no.1 supplying				Only plant no. 2 supplying			
Line No. L_k	Line current $i_{L_{k(1)}}$	Total current $i_L=\sum_k i_{L_{k(1)}}$	current distribution factor $N_{k(1)}=\frac{i_{L_{k(1)}}}{i_L}$	Line No. L_k	Line current $i_{L_{k(2)}}$	Total current $i_L=\sum_k i_{L_{k(2)}}$	current distribution factor $N_{k(2)}=\frac{i_{L_{k(2)}}}{i_L}$
L_5	.04	1.27	-.0315	L_5	1.23	1.27	+.973
L_{19}	.272	"	+.214	L_{19}	.272	"	+.214
L_6	.403	"	-.317	L_6	.87	"	+.685
L_{15}	.466	"	-.367	L_{15}	.808	"	+.635
L_{11}	.232	"	+.183	L_{11}	.232	"	+.183
L_{20}	1.21	"	-.958	L_{20}	.06	"	+.0471
L_{25}	1.25	"	-.984	L_{25}	.024	"	+.0189

Contd. to next page

TABLE 4.8 (Contd.)

Summary of Calculation of Current Distribution Factors (N_k)

Only plant no. 3 supplying				Only plant no. 4 supplying			
Line No. L_k	Line current $i_{L_{k(3)}}$	Total current $i_L=\sum_k i_{L_{k(3)}}$	Current distribution factor $N_{k(3)}=\frac{i_{L_{k(3)}}}{i_L}$	Line No. L_k	Line current $i_{L_{k(4)}}$	Total current $i_L=\sum_k i_{L_{k(4)}}$	Current distribution factor $N_{k(4)}=\frac{i_{L_{k(4)}}}{i_L}$
L_5	.04	1.27	-.0315	L_5	.04	1.27	-.0315
L_{19}	.272	"	+.214	L_{19}	.272	"	+.214
L_6	.403	"	-.317	L_6	.403	"	-.317
L_{15}	.466	"	-.367	L_{15}	.466	"	-.367
L_{11}	.232	"	+.183	L_{11}	.232	"	+.183
L_{20}	.06	"	+.0471	L_{20}	1.21	"	-.958
L_{25}	.024	"	+.0189	L_{25}	.024	"	+.0189

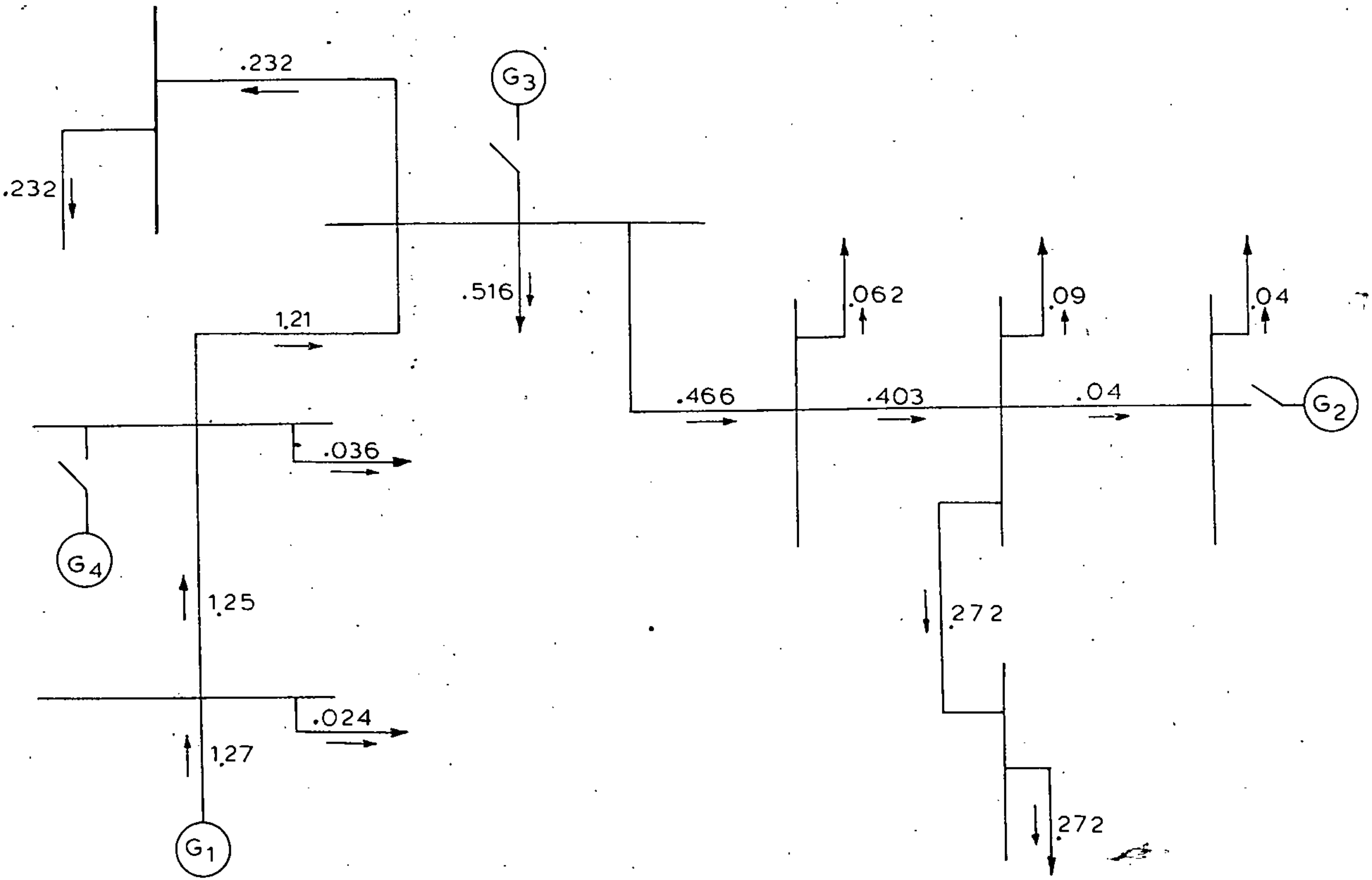


Fig. 4.7-Schematic diagram showing distribution of real components of line and load currents when supplied by generator G_1 only

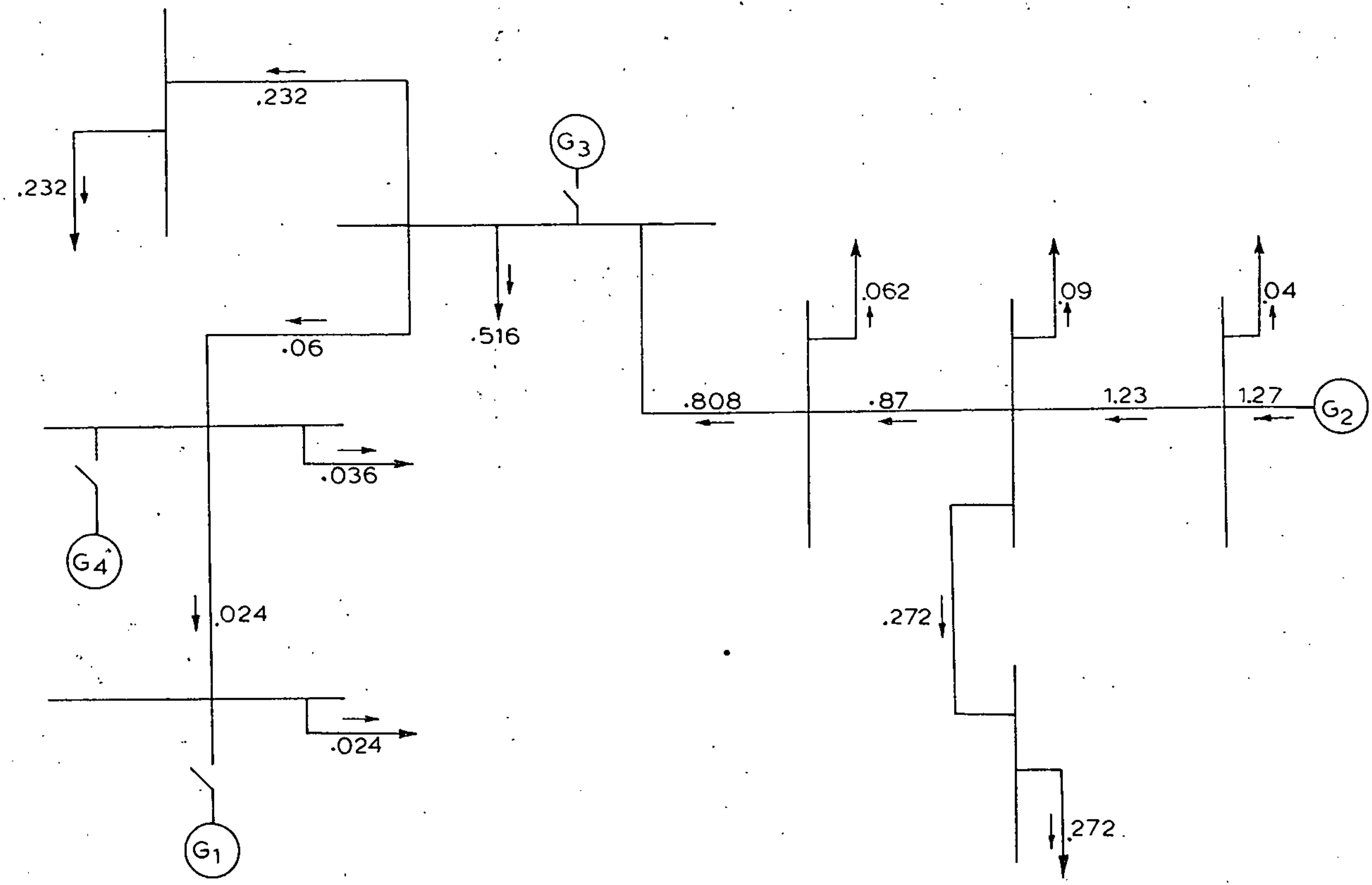


Fig. 4.8- Schematic diagram showing real components of different line and load currents when supplied by generator G_2 only

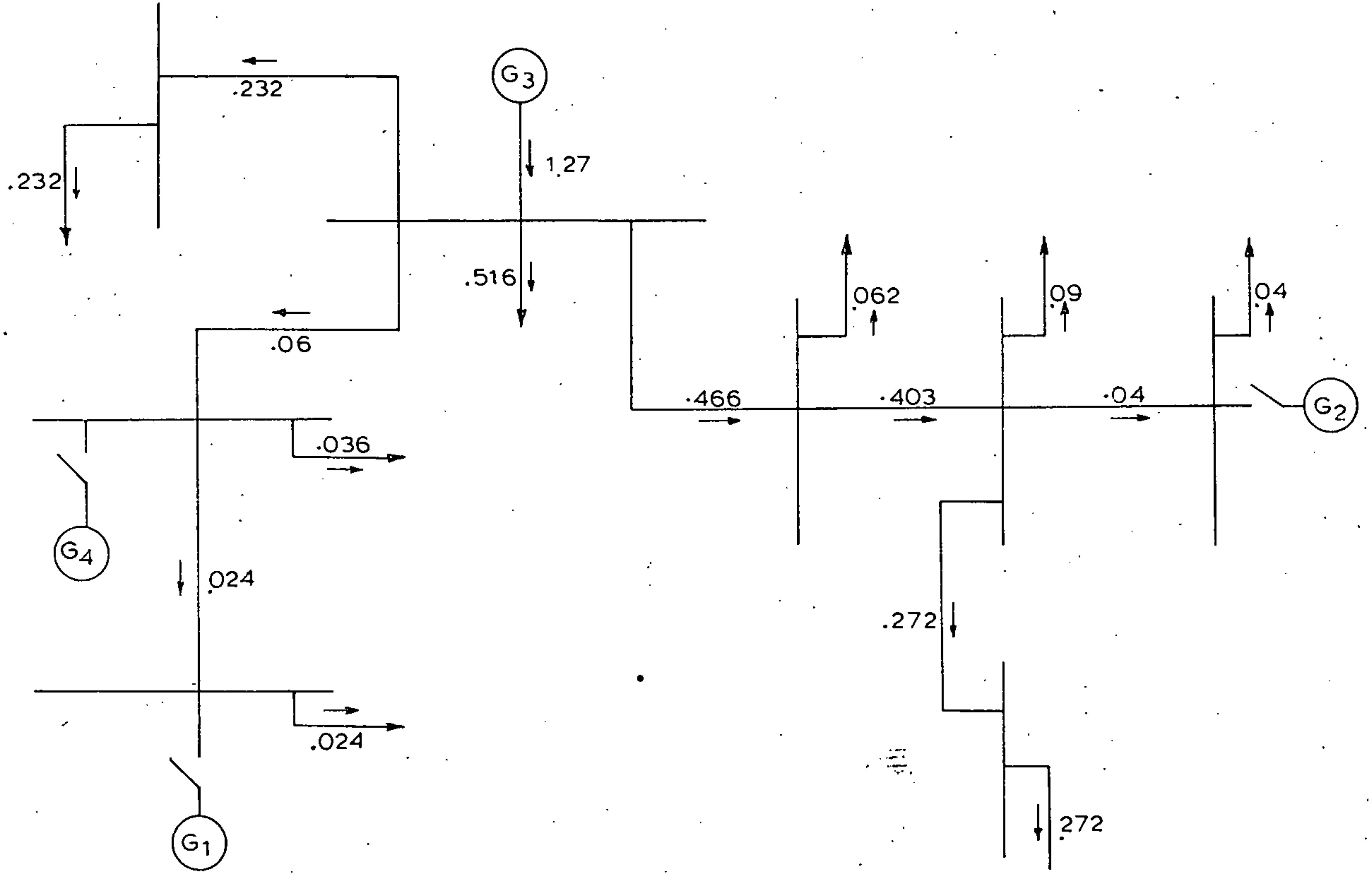


Fig. 4.9 - Schematic diagram showing distribution of real components of line and load currents when supplied by generator G_3 only

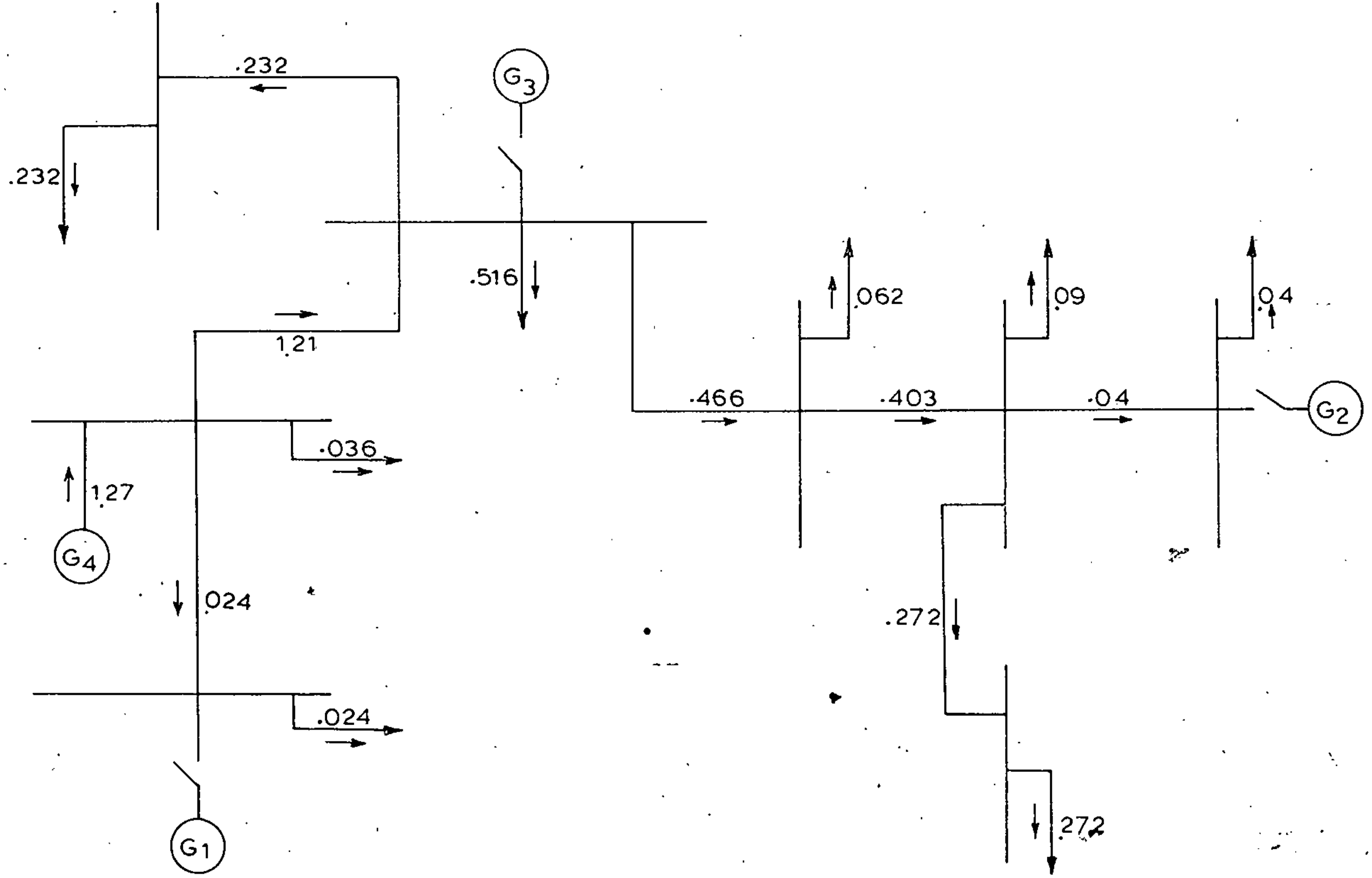


Fig. 4.10 Schematic diagram showing real components of line and load currents when supplied by generator G_4 only

4.5 Loss-formula Coefficient Matrix

The general expression for the loss formula coefficients is given in Eqn.(3.8). The values of the current distribution factors (N_k's) which will be utilized in this equation are given in Table 4.8. Other values of Eqn.(3.8) as obtained from the Network Analyzer study may be summarized in the following table (Table 4.9):

TABLE 4.9

B_{mn} Matrix Equation Quantities

Generator No.	Generator Voltage $\lvert V_m \rvert$	Generator current $\lvert i_{G_m} \rvert$	Generator current phase angles with respect to reference	Power factor of generator m pf_m $(\cos\theta_m)$
1	.94	2.97	122.8°	.98
2*	1.0	2.17	-13.45°	.97
3	.995	2.0	-124.8°	.233
4	.895	2.7	-22.5°	.975

* Reference generator.

With the above mentioned values, the B_{mn} matrix has been determined from the study and is given in Table 4.10.

TABLE 4.10

B_{mm} Matrix

m → n	1	2	3	4
↓ 1	.0564	+.00938	-.0102	-.0325
2		.0560	+.0289	-.0191
3			.1710	-.0105
4				.0558

The matrix is symmetrical

A sample calculation of determination of B_{mn} matrix is given in Appendix F(A) and the accuracy of the B_{mn} matrix values thus obtained is analyzed in the next section.

4.6 Checking of B_{mn} Matrix Values

To test the correctness of the values of the B_{mn} matrix given in Table 4.10, it is required to find the transmission losses given by Eqn.(3.2) and to compare this value with that found by using Eqn.(3.1) for the same system.

The agreement of the two sets of results for transmission losses justifies the correctness of the B_{mn} matrix values. The check of the elements of the B_{mn} matrix is shown in Appendix F(B).

CHAPTER 5

CO-ORDINATION OF INCREMENTAL PRODUCTION COSTS AND INCREMENTAL TRANSMISSION LOSSES FOR OPTIMUM ECONOMY

5.1 Introduction

A major problem involved in the operation of large integrated power systems is the determination of generation schedules for optimum economy, including effects of both incremental production costs and incremental transmission losses. This chapter discusses the use of loss-formula (such as described in Chapters 3 and 4) in co-ordinating incremental production costs and incremental transmission losses and is intended to be the basis for formulating a method which will allow system dispatchers to determine plant loading schedules quickly for normal operating conditions in a large power network.

Considering the large amount of saving which may be achieved by economical scheduling of generation, this chapter will give a detailed mathematical analysis of the exact method of co-ordinating incremental fuel costs and incremental transmission losses[13].

5.2 Exact Method of Co-ordination Equations

In order to combine incremental production costs and incremental transmission losses it is first necessary to express the incremental transmission losses in terms of incremental costs.

The mathematical treatment is similar to that of scheduling units within a plant except that we shall now include transmission loss as an additional constraint. As shown in Appendix G, the incremental transmission losses should be charged at a rate equal to the incremental cost of received power.

The minimum fuel input in Takas per hour for a given received load is obtained by solution of the following simultaneous equations[13]:

$$\frac{dF_n}{dP_n} + \lambda \frac{\delta P_L}{\delta P_n} = \lambda \qquad \ldots \qquad (5.1)$$

where F_n = input to plant n in Taks per hour.

P_n = output of plant n in megawatts.

$\frac{dF_n}{dP_n}$ = incremental production cost of plant n in Takas per megawatt-hour.

P_L = total transmission losses

$\frac{\delta P_L}{\delta P_n}$ = incremental transmission loss at plant n in megawatts per megawatt.

λ = incremental cost of received power in Takas per megawatt-hour.

In general, the incremental transmission loss at plant n may be expressed by

$$\frac{\delta P_L}{\delta P_n} = \frac{\delta}{\delta P_n}\left[\sum_m \sum_n P_m B_{mn} P_n\right]$$

$$= \sum_{m} 2B_{mn}P_m \quad .. \quad .. \quad (5.2)$$

where B_{mn} = transmission loss-formula coefficients.

Again, the incremental production cost of a plant over a limited range may be represented by the equations

$$\frac{dF_n}{dP_n} = F_{nn}P_n + f_n \quad .. \quad .. \quad (5.3)$$

where F_{nn} = slope of incremental production cost curve.

f_n = intercept of incremental production cost curve.

Then equation (5.1) becomes

$$F_{nn}P_n + f_n + \lambda \sum_{m} 2B_{mn}P_m = \lambda \quad .. \quad .. \quad (5.4)$$

The simultaneous equations thus obtained may be solved by choosing appropriate values of λ for different total loads.

5.3 Physical Interpretation of Co-ordination Equations

The physical interpretation of the co-ordination Eqn.(5.1) may be visualized by inspection of Fig. 5.1. The incremental production cost of a given plant n is measured at the plant bus and is denoted by dF_n/dP_n. A given plant n incurs an incremental transmission loss $\partial P_L/\partial P_n$ in supplying the next increment of system load. It is desired that the incremental cost of the power received from each plant be the same at the receiver point R.

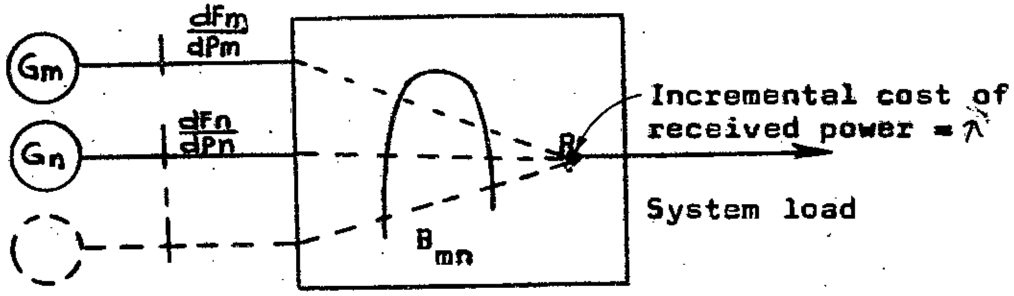


Fig. 5.1 Schematic representation of cost relations.

Let us suppose that the load increases by an amount ΔP_R and that this load change is first taken up by plant 1 only by increasing the output of plant 1 by ΔP_1. Then the cost of this increment of power at the receiver is given by

$$\lambda = \frac{dF_1}{dP_1} \quad \frac{\Delta P_1}{\Delta P_R}$$

The expression $\Delta P_1 / \Delta P_R$ may be thought of as the reciprocal of 'the incremental efficiency of the transmission system'.

By similar reasoning we can express the incremental cost of received power from plant n as :

$$\lambda = \frac{dF_n}{dP_n} \quad \frac{\Delta P_n}{\Delta P_R}$$

and the incremental transmission loss incurred at this plant for supplying the increment of load is charged at a rate equal to the incremental cost of its received power.

5.4 Generation Schedules

For the system considered in our Fig. 3.4 we have designated Shahjibazar Generator as plant no.1, Kaptai Generator as plant no.2, Siddhirganj Generator as plant no.3 and Ashuganj Generator as plant no.4. The generation schedules will be obtained for all plants except plant no.2. For all methods of solution, plant 2 will be considered to carry the base load of 80 MW.

From the incremental production cost curves of Fig. 2.6, the values of the coefficients of Eqns. (5.3) are given in Table 5.1.

TABLE 5.1

Values of F_{nn} and f_n

Plant n	F_{nn}	f_n
1	8.0	0.08
3	15.5	0.10
4	12.0	0.09

Table 5.2 gives the values of the elements of loss-formula-coefficient matrix (on 100 MVA base) as obtained in Sec. 4.5.

TABLE 5.2

Transmission Loss-formula coefficients (B_{mn})

m	n	B_{mn}
1	1	0.000564
3	3	0.00171
4	4	0.000558
1	3	-0.000102
1	4	-0.000325
3	4	-0.000105

The equations for optimum scheduling of generation neglecting transmission losses may be written from Eqns. (2.5) and (5.3) as follows:

$$\left.\begin{aligned} F_{11}P_1 + f_1 &= \lambda \\ F_{33}P_3 + f_3 &= \lambda \\ F_{44}P_4 + f_4 &= \lambda \end{aligned}\right\} \quad \ldots \quad (5.5)$$

The exact co-ordination equations for optimum generation schedule may be written from Eqns.(5.1) as follows:

$$\begin{aligned} F_{11}P_1 + 2P_1B_{11} + 2P_3B_{13} + 2P_4B_{14} &= \lambda - f_1 \\ F_{33}P_3 + 2P_3B_{33} + 2P_1B_{13} + 2P_4B_{34} &= \lambda - f_3 \quad \ldots \quad \ldots \quad (5.6) \\ F_{44}P_4 + 2P_4B_{44} + 2P_1B_{14} + 2P_3B_{34} &= \lambda - f_4 \end{aligned}$$

With the given data of Tables 5.1 and 5.2, Eqns. (5.5) and (5.6) may be written respectively in the following forms:

$$\left.\begin{aligned} 0.08P_1 + 8.0 &= \lambda \\ 0.10P_3 + 15.5 &= \lambda \\ 0.09P_4 + 12.0 &= \lambda \end{aligned}\right\} \quad \ldots \quad (5.7)$$

and

$$\left.\begin{aligned} &\left[0.08 + \lambda\,(0.001128)\right]P_1 - (0.000204\,\lambda)P_3 - (0.00065\,\lambda)P_4 = \lambda - 8.0 \\ &-(0.000204\,\lambda)P_1 + \left[0.10 + \lambda\,(0.00342)\right]P_3 - (0.00021\,\lambda)P_4 = \lambda - 15.5 \\ &-(0.00065\,\lambda)P_1 - (0.00021\,\lambda)P_3 + \left[0.09 + \lambda\,(0.001116)\right]P_4 = \lambda - 12.0 \end{aligned}\right\} \quad \ldots (5.8)$$

Solutions of Eqns. (5.7) and (5.8) for different values of λ give the required generation schedules. Solution of these simultaneous equations by Digital Computer (IBM 1620) and the programming for obtaining the solution are shown in Appendix J.

5.5 Graphs and Results

Fig. 5.2 gives a plot of generation schedules with individual plant generation plotted against the total generation. The calculations of generation schedules for various values of λ are given in Appendix H. Fig. 5.3 gives a plot of individual generation of plant against the total received load.

The abscissa of this graph is obtained by subtracting the transmission losses from the corresponding total generation of Fig. 5.2 for different values of λ. The magnitudes of total transmission losses as obtained from generation schedules based on the two methods [i.e., (i) including and (ii) neglecting transmission losses] are plotted in Fig. 5.4 as a function of the received load. The details of calculation for these transmission losses are given in Appendix I. To facilitate a quick understanding of the general nature of the various generation schedules (as given in Figs. 5.2 and 5.3), charts of individual plant generation against the total generation and against the total received load (for both types of scheduling)are given in Table 5.3, Table 5.4 and Table 5.5 respectively.

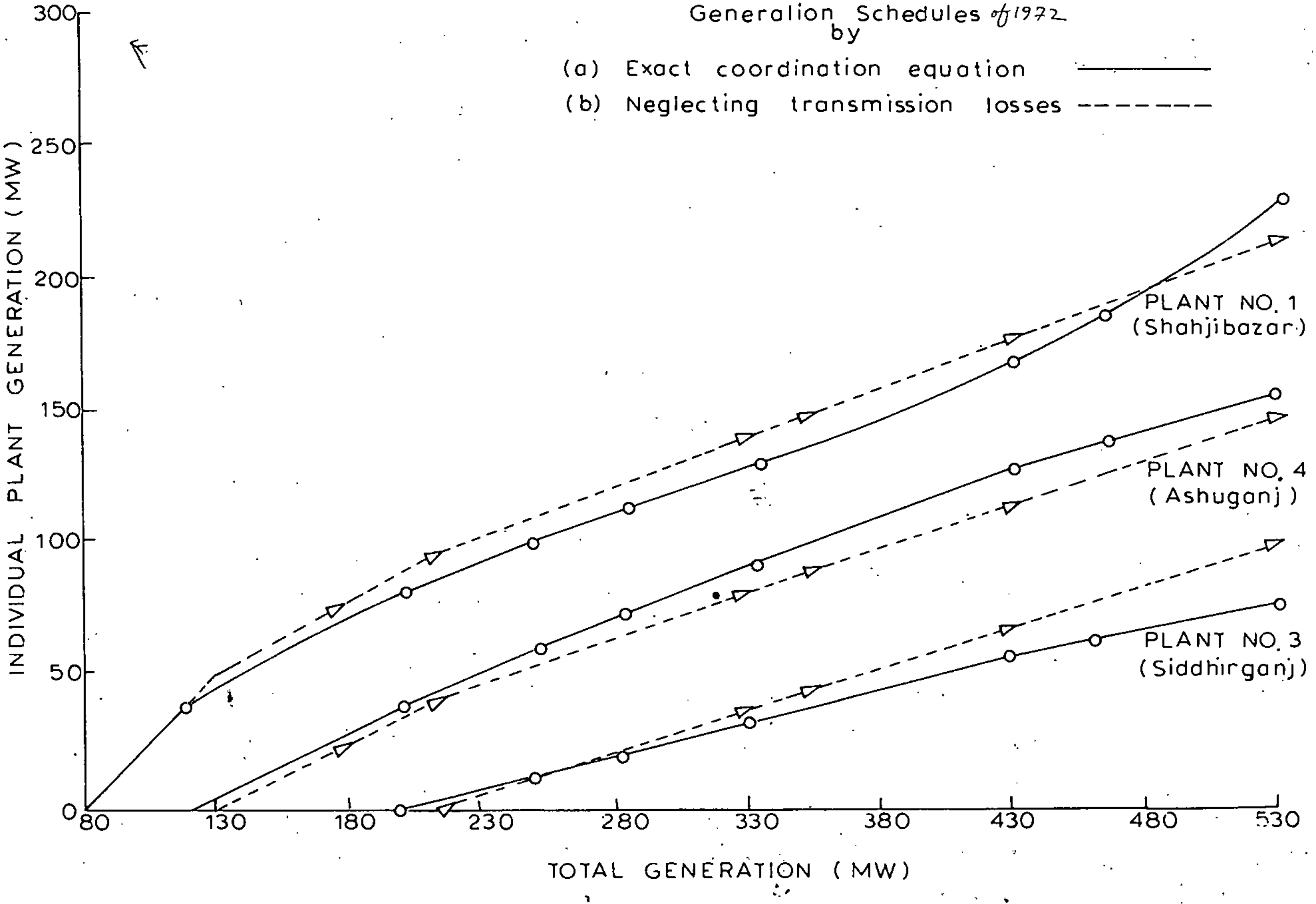

Generalion Schedules of 1972
by
(a) Exact coordination equation ————
(b) Neglecting transmission losses ------
PLANT NO. 1
(Shahjibazar)
PLANT NO. 4
(Ashuganj)
PLANT NO. 3
(Siddhirganj)
INDIVIDUAL PLANT GENERATION (MW)
0
50
100
150
200
250
300
80
130
180
230
280
330
380
430
480
530
TOTAL GENERATION (MW)

FIG. 5.2

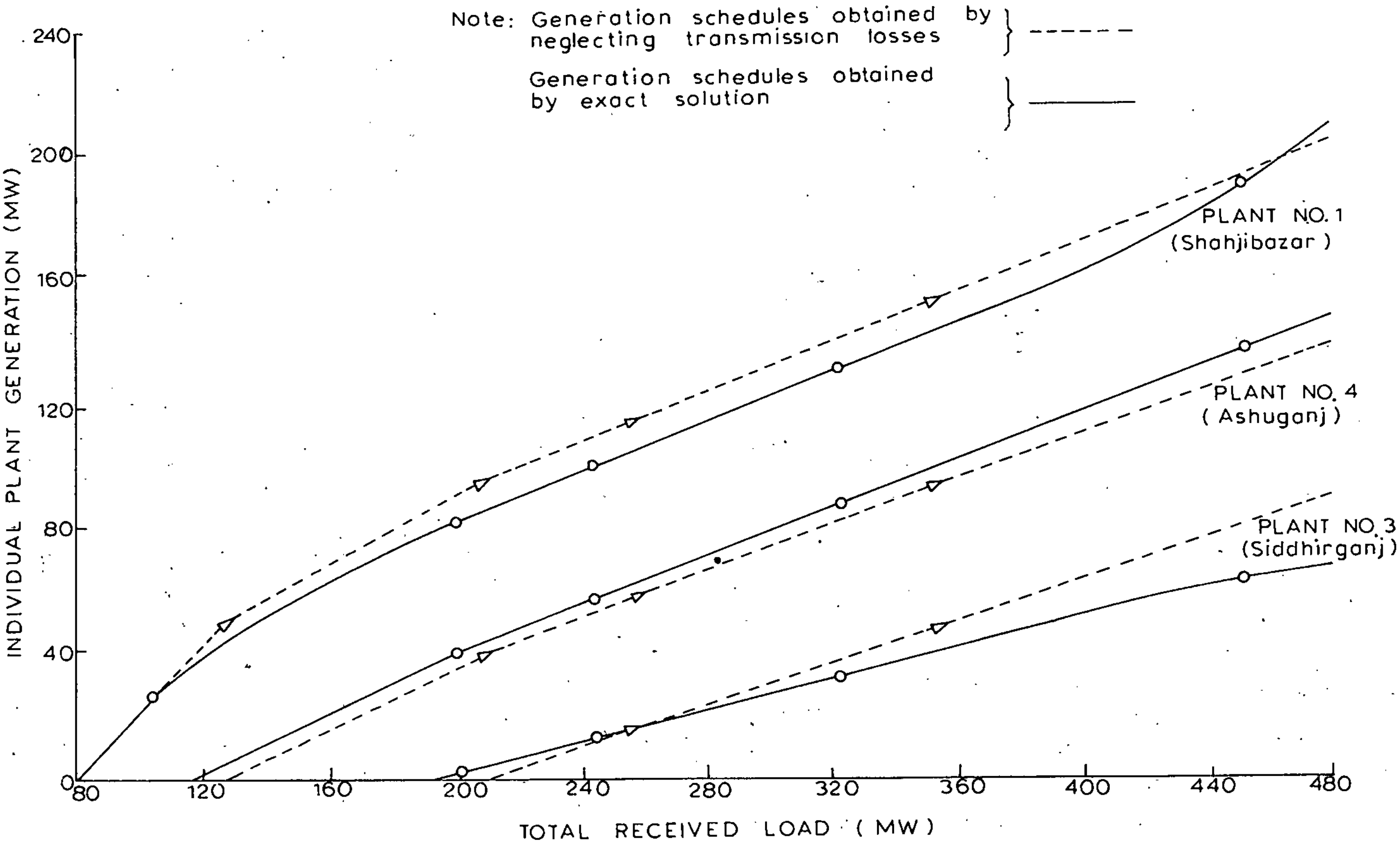

Note: Generation schedules obtained by neglecting transmission losses
Generation schedules obtained by exact solution
PLANT NO. 1 (Shahjibazar)
PLANT NO. 4 (Ashuganj)
PLANT NO. 3 (Siddhirganj)
INDIVIDUAL PLANT GENERATION (MW)
0
40
80
120
160
200
240
80
120
160
200
240
280
320
360
400
440
480
TOTAL RECEIVED LOAD (MW)

FIG. 5.3

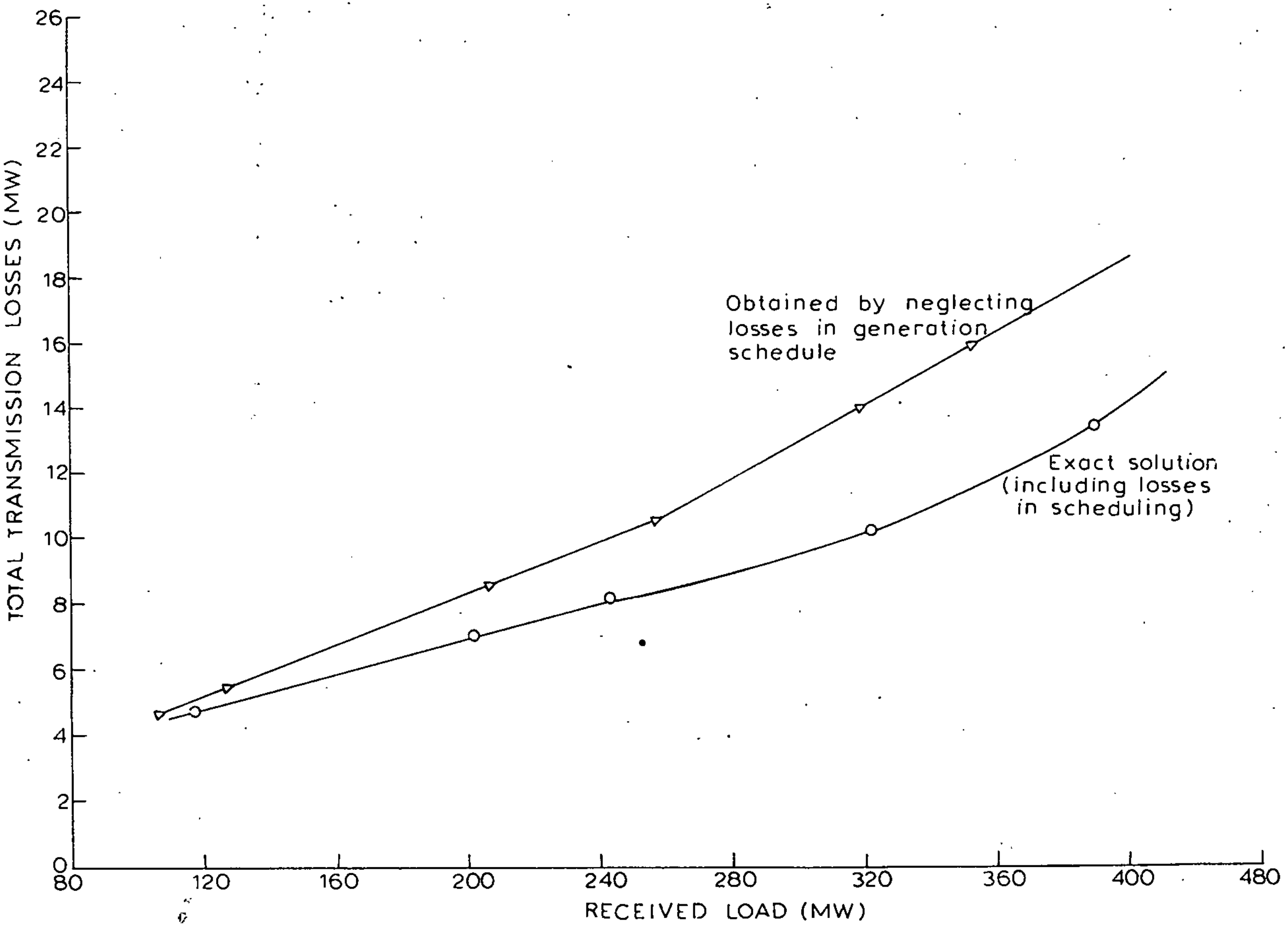


Fig. 5.4 - Transmission losses plotted as a function of received load

TABLE 5.3

Chart of Individual Plant Generation against Total Generation

Total Generation (MW)	Plant Identification	Individual Generation (MW)	
		Exact Solution (including transmission losses)	By neglecting transmission losses
80	1	–	–
	2	80	80
	3	–	–
	4	–	–
130	1	46	50
	2	80	80
	3	–	–
	4	4	–
180	1	72	77
	2	80	80
	3	–	–
	4	28	23
230	1	92	100
	2	80	80
	3	7	5
	4	51	45
280	1	110	119
	2	80	80
	3	19	19
	4	71	62
330	1	128	135
	2	80	80
	3	30	35
	4	92	80
380	1	147	156
	2	80	80
	3	45	50
	4	108	94
430	1	169	174
	2	80	80
	3	56	66
	4	125	110
480	1	192	192
	2	80	80
	3	66	80
	4	142	128
530	1	222	210
	2	80	80
	3	73	95
	4	155	145

TABLE 5.4

Generation Schedules for Exact Method of Solution

(including transmission losses)

Values of λ	Total generation, P_t (MW)	Individual Generation of Plants				Total Loss P_L (MW)	Received Load P_R (MW)
		P_1 (MW)	P_2 (MW)	P_3 (MW)	P_4 (MW)		
-	84.0	-	84	-	-	4.0	80.0
10	102.21	18.21	84	-	-	4.21	98.0
11.8	121.7	37.7	84	-	0	4.70	117.0
12.0	126.62	40.0	84	0	2.62	4.84	121.78
15.5	208.0	81.5	84	2.5	40.0	7.0	201.0
17.0	251.1	98.0	84	13.0	56.1	8.1	243.0
20.0	333.0	130.0	84	31.80	87.5	10.2	322.8
25.0	470.5	187.0	84	61.50	138.0	20.5	450.0

TABLE 5.5

Generation Schedules by Neglecting Transmission Losses

Values of λ	Total generation, P_t (MW)	Individual Generation of Plants				Total Loss P_L (MW)	Received Load P_R (MW)
		P_1 (MW)	P_2 (MW)	P_3 (MW)	P_4 (MW)		
10.0	109.0	25.0	84	-	-	4.4	104.6
12.0	134.0	50.0	"	-	0	5.5	128.5
15.5	216.65	93.75	"	0	38.9	8.65	208.0
17.0	268.1	112.5	"	15	56.6	10.6	257.5
20.0	370.9	150.0	"	46	90.9	15.89	354.01
25.0	536.0	212.5	"	95	144.5	30.2	505.8

CHAPTER 6

EVALUATION OF SAVINGS

6.1 Introduction

The technique of including transmission losses in generation schedule must have certain outstanding value in the field of economic study of power generation. The saving in terms of money is considered to be an important factor that is associated with such technique. The two types of scheduling curves (including and neglecting transmission losses) for each plant have distinctly different natures causing significant differences in total fuel consumption costs. The difference is the amount of saving (Taka per hour) that is obtainable for a particular value of received load. The saving, however small at one time, accumulates into a huge amount when taken into account throughout the year.

6.2 Calculation of Fuel Input (Taka per hour) for Various Received Load[31].

Fuel input versus megawatt output graphs [Figs.2.5(a) and 2.5(b)] and individual plant generation against the received load graphs (Fig. 5.3) are utilized in this section. For any given value of received load, the individual plant generation and hence the individual plant input (in Taka per hour) are found out by the use of the above graphs. Table I of Appendix K gives the total fuel input for particular values of received load for both types of scheduling along with the difference of fuel inputs involved in the two cases. Fig.6.1 shows the nature of the total amount of fuel inputs (Taka per hour) as plotted against the received load for the two cases of scheduling (including and neglecting transmission losses). A comparative study of the fuel costs for these schedules and also for other arbitrary methods of scheduling of plant generations is given in the next section.

6.3 Comparative Study

We shall now make a comparative study of the fuel inputs for various cases. As shown in Fig. 6.1, a comparison is made at the received load of 200 MW. The total fuel inputs for various cases and the annual losses incurred at different cases as compared to the case of including transmission losses in generation schedules (reference case) are given in Table 6.1.

TABLE 6.1

Comparative Study of Fuel Inputs and Losses Incurred at Different Cases (for a received load of 200 MW)

Case no.	Types of Scheduling	Total fuel input (Taka/hr.)	Difference of fuel inputs as compared to the reference case (Taka/hr.)	Annual Loss* as compared to the reference case (Taka in lacs)
'a' (reference case)	Generation Schedules obtained by exact solution (including transmission losses)	1,570	-	-
'b'	Generation Schedule obtained considering equal incremental production costs for each generating station (neglecting transmission losses)	1,625	55	4.818
'c'	The plants (Siddhirganj, Ashuganj and Shahjibazar) are scheduled to supply the load equally i.e., 40 MW each.	1,800	230	20.148
'd'	The plants are scheduled to supply proportional to their rated capacity i.e., Siddhirganj supplies 30 MW, Ashuganj and Shahjibazar Supply 45.5 MW each.	1,740	170	14.892
'e'	Siddhirganj supplies 80 MW, Ashuganj supplies 40 MW and Shahjibazar, 0 MW.	2,120	550	48.18
'f'	Siddhirganj supplies 80 MW, Shahjibazar, 40 MW and Ashuganj, 0 MW.	1,970	400	35.04

* Considering 24 hours of operation of plants.

6.4 Remarks

Table 6.1 shows that different arbitrary methods of scheduling as given by cases 'c', 'd', 'e' and 'f' cause annual loss of about 20, 16, 48 and 36 lacs of Taka respectively as compared to the case of including transmission losses in generation schedules for a received load of 200 MW for continuous operation of plants. Even if we consider the plants to operate 18 hours a day or 12 hours a day, the net annual saving in generation schedules obtained by including transmission losses is noteworthy.

Another important point is that inclusion of transmission losses in the generation schedules saves a total of 4,81800 Takas annually (considering contnuous operation) over the schedules when the losses are not considered. With the expansion of the system, further transmission losses should have to be included in scheduling formulations and this will certainly play an important part in economic scheduling of generation for the entire system.

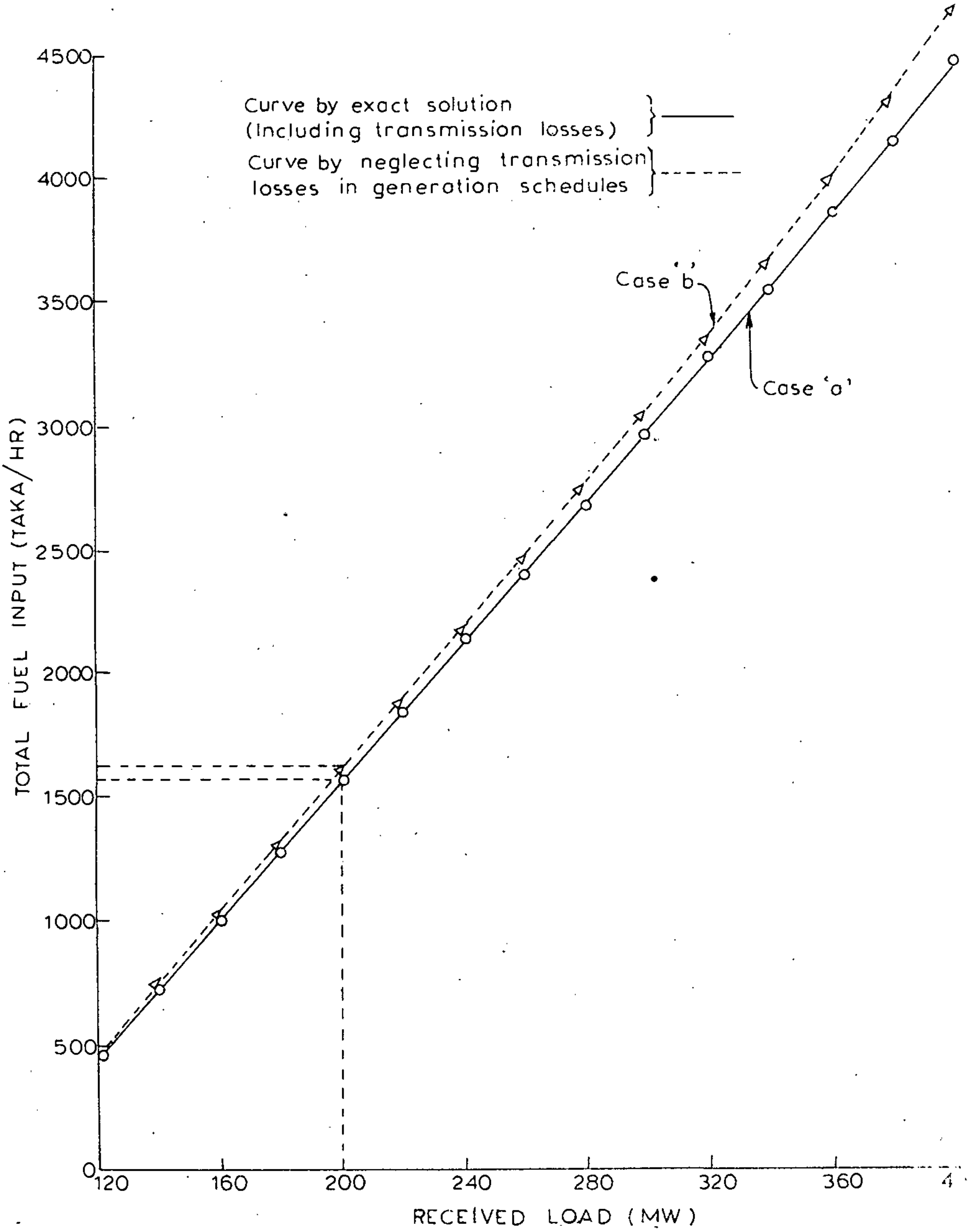


Fig. 6.1 - Graph showing fuel inputs for various received load

CHAPTER 7

DISCUSSION AND CONCLUSION

7.1 General

Economic operation is the most important consideration in determining whether a power system is being run and managed most efficiently. It is based on scheduling of generation to minimize the cost of supplying all the loads on the system. Several methods are available for the economic scheduling of generating plants, such as, (i) economic scheduling by neglecting transmission losses, (ii) economic scheduling by including transmission losses, and (iii) economic scheduling by penalty factor method. For all the methods except (i) determination of system transmission losses is necessary for the optimum scheduling of power plants and hence, of the economic operation of a given system. The law of conservation of energy obviously requires that the sum of the plant outputs minus the system losses be equal to the received load. Economic operation is obtained when the production cost is minimized subject to the restriction forced on the system by the law of conservation of energy.

In this thesis, economic scheduling of generations of Eastern Grid Network of Bangladesh Power Development Board has been obtained when the incremental production cost (dF_n/dP_n) plus the incremental transmission losses $(\partial P_L \partial P_n)$ which is charged at the rate of incremental cost of received power (λ) is considered to be the same for all the plants and is equal to the incremental cost of received power (λ). The main endeavour in this work has been to compare the optimum scheduling obtained by the 'exact' method, as mentioned above, with that obtained when the transmission losses are not taken into consideration while scheduling generation.

As the system grows larger and larger, the importance of including transmission losses in economic and optimum scheduling of generations becomes more and more important and indispensable. The transmission losses have been correlated with the production costs at the generator to solve the problem of the most economical loading of each generating plant.

7.2 Main Results from the Present Investigation

The various chapters of this thesis contain valuable date and results, which may constitute guide-lines for the economic generation of electrical power in our country. Specifically, some of the 'contributions' that this work may be considered to have made in the field of establishing a procedure for determining the optimum scheduling in a power system, may be listed as follows:-

(i) A set of transmission loss coefficients has been found to utilize them to determine the system transmission losses.

(ii) Correlation between the measured and the calculated values of transmission losses has been established as a proof of correctness of the procedure for calculating the coefficients.

(iii) Generation schedules have been obtained for all the generating plants except that of Kaptai, both by including and neglecting transmission losses.

(iv) The economic consequences of scheduling based on an approach which includes transmission losses as compared to a partially correct method of scheduling without considering losaes have been shown.

(v) The economic consequences of 'random' or 'arbitrary' operation of generating plants, as is most likely to happen if no consideration is given to the question of optimization of the system economy, with or without consideration of the transmission losses, are also shown under an assumed loading of the system.

In short, this work may be taken to be an attempt, undertaken for the first time in regard to the power system of this country which has grown up to be considerably large and complex network in the past two decades, to establish by an exact method of analysis, the economic losses incurred by the management, because of a lack of either any 'method' in the scheduling of generators, or of a 'method' which is actually based on erroneous ideas.

7.3 Suggestions for Improvement

An improvement of the present research could be done by following digital method of finding the general loss-formula coefficients and hence the general loss formula. The digital method could avoid some of the simplifying assumptions made in obtaining loss-formula coefficients. An improved method of determining loss-formula coefficients and hence the incremental transmission loss factors ($\partial P_L/\partial P_n$) could be obtained from power system admittances and voltage[32]. Another suggestion for finding the coefficients is to determine the second partial derivatives of the system losses with respect to plant outputs[33].

7.4 Future Research Areas

The present analysis of the economic generation schedules of Eastern Grid Network of BPDB has merely provided the ground work for several stimulating areas for future research work. Some of them are:

(i) Economic control of power for interconnected areas considering transmission losses.

(ii) Digital method of optimizing both hydro and steam stations, accounting transmission losses.

(iii) Penalty factor approach in economic interchange of electrical energy.

(iv) Economic operation of power systems with losses as a function of voltage phase angles.

(v) Computer control of power system operation, for securing maximum economy.

In view of the fact that in Bangladesh the cost of both industrial as well as domestic power are rather prohibitively high at present, future research work in the field of power system should be undertaken along the lines suggested above, so as to find out ways and means for bringing down the cost of power available to all types of consumers.

APPENDIX A

Optimum Scheduling by Neglecting Transmission Losses

This derivation follows directly from the method of Lagrangian multipliers[29,30]. Let

F_t = total input to system in Takas per hour

$= \sum_n F_n$

where F_n = input to unit or plant n in Takas per hour. It is desired to minimize the total input (F_t) in Takas per hour for a given received load (P_R). Let

P_R = given received load.

By application of the method of Lagrangian multipliers, the equation of constraint is given by

$$\psi(P_1, P_2, P_3, \ldots \ldots P_n) = \sum_n P_n - P_R = 0 \ldots \qquad \text{(A.1)}$$

Then minimum fuel input for a given received load is obtained when

$$\frac{\partial \mathcal{F}}{\partial P_n} = 0, \text{ for all values of } n \ldots \qquad \text{(A.2)}$$

where $\mathcal{F}$ is a new expression of the type:

$$\mathcal{F} = F_t - \lambda \psi \ldots \qquad \text{(A.3)}$$

and λ = Lagrangian type of multiplier.

$$\text{So, } \frac{\partial \mathcal{F}}{\partial P_n} = \frac{\partial F_t}{\partial P_n} - \lambda \frac{\partial \psi}{\partial P_n} = 0 \ldots \qquad \text{(A.4)}$$

$$\text{Or, } \frac{\partial F_t}{\partial P_n} - \lambda \frac{\partial}{\partial P_n}\left[\sum_n P_n - P_R\right] = 0$$

Or, $\frac{\partial F_t}{\partial P_n} - \lambda\left[1 - 0\right] = 0$

Or, $\frac{\partial F_t}{\partial P_n} = \lambda$ ... (A.5)

But $\frac{\partial F_t}{\partial P_n} = \frac{\partial\left(\sum_n F_n\right)}{\partial P_n} = \frac{\partial F_n}{\partial P_n} = \frac{dF_n}{dP_n}$ . . . (A.6)

Then Eqn.(A.5) becomes

$$\frac{dF_n}{dP_n} = \lambda \qquad \text{. . .} \qquad (2.5)$$

The units of λ are Takas per megawatt hour when the fuel cost is expressed in Takas per hour and output is in megawatt.

APPENDIX B

Summary of Transformation

In the following analysis, the lower case indices m,n,j and k are tensor indices, and the capitals G and L are identification indices. Whenever a repeated tensor index appears in a product (one upper and the other a lower index) a summation on that index is identified.

Transformation to Reference Frame 1:

Let us consider Fig. 3.1. If any point R in this network is chosen as a reference point (Fig. 3.7),

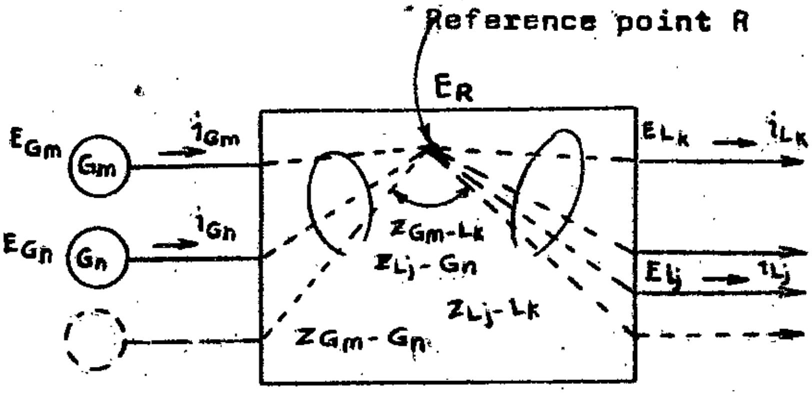


Fig. 3.7 Self and mutual impedances for transmission network (reference frame 1).

the following set of equations may be written in terms of all the generator and load self and mutual impedances with respect to the reference point:

$$\begin{array}{cc} G_m & E_{G_m}-E_R \\ L_j & E_{L_j}-E_R \end{array} = \begin{array}{ccc} & G_n & L_k \\ & i_{G_n} & i_{L_k} \\ G_m & Z_{G_m-G_n} & Z_{G_m-L_k} \\ L_j & Z_{L_j-G_n} & Z_{L_j-L_k} \end{array} \quad \ldots \quad (B.1)$$

where m, n = number of sources.

j, k = number of loads.

$Z_{G_m-G_n}$ represents the self and mutual impedances between the generators.

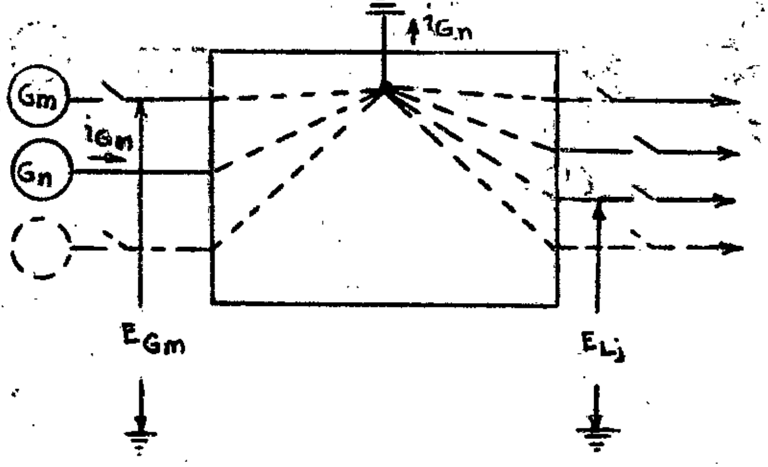
$Z_{L_j-L_k}$ represents the self and mutual impedances between the loads.

$Z_{G_m-L_k}$ and $Z_{L_j-G_n}$ represent the mutual impedances between the generators and the loads.

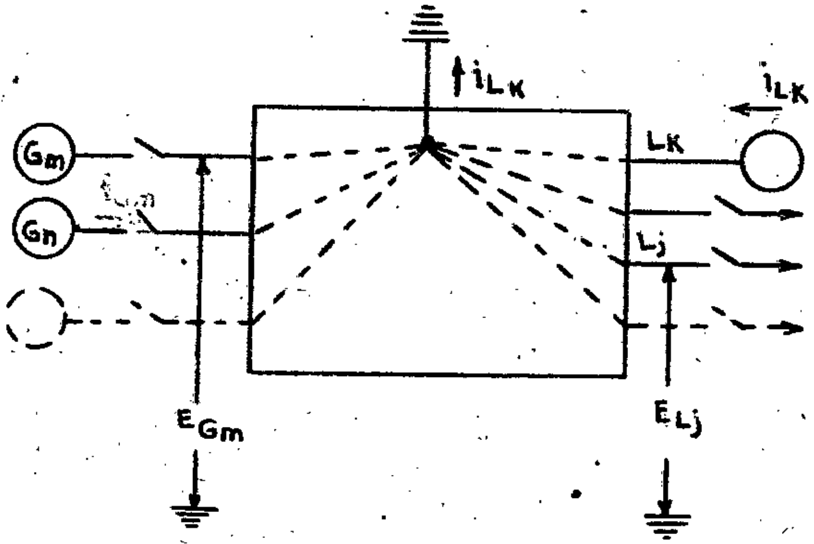
Since Eqns. (B.1) refer to reference frame 1 quantities, they may be written of the form:

$$E_1 = Z_{11}\, I_1 \quad \ldots \quad (B.2)$$

The Z_{11} impedances are defined as shown in Fig. 3.8



$E_{G_m} = Z_{G_m-G_n} i_{G_n}$ (a) $E_{L_j} = Z_{L_j-G_n} i_{G_n}$



$E_{G_m} = Z_{G_m-L_k} i_{L_k}$ (b) $E_{L_j} = Z_{L_j-L_k} i_{L_k}$

Fig. 3.8 Definition of impedances.

Transformation to Reference Frame 2:

It is known in general that if a set of currents I_j is related to a set of currents I_k by a tensor C_k^j such that

$$I_j = C_k^j I_k \quad \dots \quad (8.3)$$

and if the power remains invariant, then

$$E_k = C_k^{j*} E_j \quad \dots \quad (8.4)$$

and $Z_{kk} = C_k^{j*} Z_{jj} C_k^j$ . . . (8.5)

Let us define

$$i_L = \sum_j i_{L_j} \quad \dots \quad (8.6)$$

Under the assumptions given in Section 3.3, we may write

$$i_{L_j} = N_j i_L \quad \dots \quad (8.7)$$

where N_j is the current distribution factor. Then we have for Eqns. (B.7),

$$\begin{array}{c|c|} & \\ \hline G_n & i_{G_n} \\ \hline L_k & i_{L_k} \\ \hline \end{array} = \begin{array}{c|c|c|} & G_n & L \\ \hline G_n & 1 & 0 \\ \hline L_k & 0 & N_k \\ \hline \end{array} \begin{array}{c|c|} \hline G_n & i_{G_n} \\ \hline L & i_L \\ \hline \end{array} \quad \ldots \quad (B.8)$$

i.e., $I_1 = C_2^1 I_2$ . . . (B.9)

$$\text{where} \quad C_2^1 = \begin{array}{c|c|c|} & G_n & L \\ \hline G_n & 1 & \\ \hline L_k & & N_k \\ \hline \end{array} \quad \ldots \quad (B.10)$$

Applying the transformations indicated by Eqns. (B.3),(B.4) and (B.5), the resulting voltages, impedances and currents are given by

$$\begin{array}{c|c|} \hline G_m & E_{G_m} - E_R \\ \hline L & E_L - E_R \\ \hline \end{array} = \begin{array}{c|c|c|} & G_n & L \\ \hline G_m & Z_{G_m - G_n} & a_m \\ \hline L & b_n & \omega \\ \hline \end{array} \begin{array}{c|c|} \hline G_n & i_{G_n} \\ \hline L & i_L \\ \hline \end{array} \quad \ldots \quad (B.11)$$

$$\left.\begin{aligned} \text{where } a_m &= Z_{G_m-L_k}N_k \\ b_n &= N_j^{*}Z_{L_j-G_n} \\ \omega &= N_j^{*}Z_{L_j-L_k}N_k \\ E_L &= N_j^{*}E_{L_j} \end{aligned}\right\} \quad \dots \qquad (B.12)$$

By means of the above transformation, the circuit of Fig. 3.7 has been arranged to the circuit as given in Fig. 3.9.

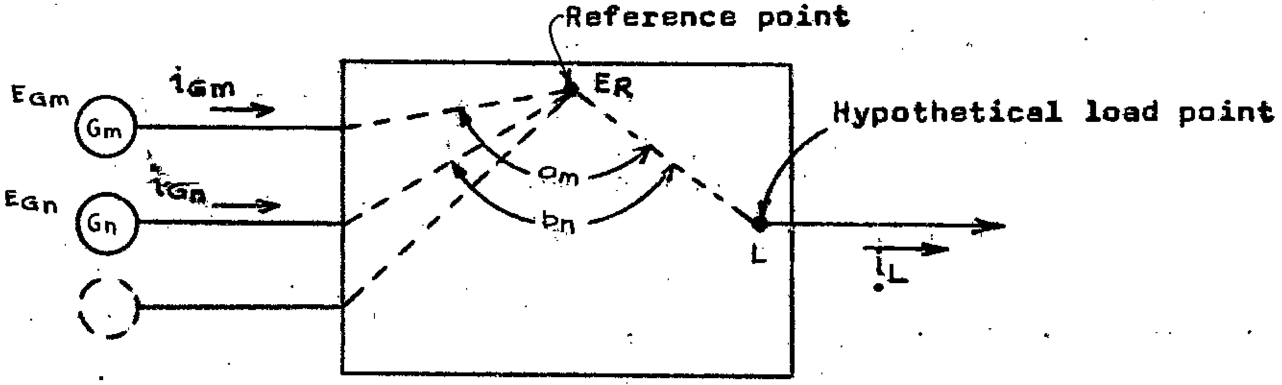


Fig. 3.9 Equivalent circuit with respect to reference frame 2.

Transformation to Reference Frame 3:

$$\text{Since } i_L = \sum_n i_{G_n} \quad \dots \qquad (B.13)$$

the load current may be eliminated by the transformation

$$I_2 = C_3^2 I_3 \quad \dots \qquad (B.14)$$

$$\text{where } I_3 = \begin{array}{c|c|} \cline{2-2} G_n & i_{G_n} \\ \cline{2-2} \end{array} \qquad I_2 = \begin{array}{c|c|} \cline{2-2} G_n & i_{G_n} \\ \cline{2-2} L & i_L \\ \cline{2-2} \end{array} \quad \dots \qquad (B.15)$$

and

$$\begin{array}{c|c|} \cline{2-2} G_n & i_{G_n} \\ \cline{2-2} L & i_L \\ \cline{2-2} \end{array} = \begin{array}{c|c|} \cline{2-2} G_n & 1 \\ \cline{2-2} L & T_n \\ \cline{2-2} \end{array} \; \begin{array}{c|c|} \cline{2-2} G_n & i_{G_n} \\ \cline{2-2} \end{array} \quad \dots \qquad (B.16)$$

where $T_n = + 1$ for all values of n. Thus the transformation matrix is given by

$$C_3^2 = \begin{array}{c|c|} & \\ \cline{2-2} G_n & 1 \\ \cline{2-2} L & T_n \\ \cline{2-2} \end{array} \quad \cdot \cdot \qquad (B.17)$$

By application of Eqns. (B.3), (B.4) and (B.5), we obtain

$$\boxed{E_{G_m} - E_L} = \boxed{Z_{G_m - G_n} + a_m + b_n + \omega} \quad \boxed{i_{G_n}}$$

$$= \boxed{Z_{m-n}} \quad \boxed{i_{G_n}} \quad \cdot \cdot \cdot \qquad (B.18)$$

The circuit of reference frame 2 is modified as given by Eqns. (B.18) to that given in Fig. 3.10. The power losses in this

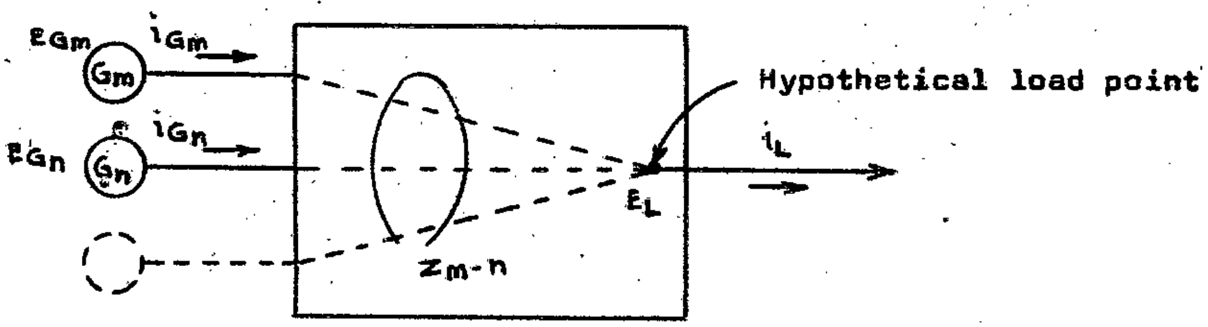


Fig. 3.10 Equivalent circuit with respect to reference frame 3.

equivalent circuit are:

$$P_L = \Re I_3^* E_3 \quad \cdot \cdot \cdot \qquad (B.19)$$
$$= \Re I_3^* Z_3 I_3$$

where E_3, I_3 and Z_3 denote reference frame 3 quantities.

Since, $I_3 = i_{G_n} = i_{d_n} + ji_{q_n}$

and $Z_3 \; I_3 = Z_{m-n}(i_{d_n} + ji_{q_n})$ .. (B.20)

we have

$$P_L = i_{d_m} R_{m-n} i_{d_n} + i_{q_m} R_{n-m} i_{q_n} + i_{d_m} (X_{n-m} - X_{m-n}) i_{q_m} \quad .. \text{(B.21)}$$

where i_{d_m} = real part of generator m current.

i_{q_m} = imaginary part of generator m current.

In general, $X_{m-n} - X_{n-m}$ is negligibly small. Thus

$$P_L = i_{d_m} R_{m-n} i_{d_n} + i_{q_m} R_{n-m} i_{q_n} \quad .. \text{(B.22)}$$

The asymmetrical R_{m-n} in Eqn.(B.22) may be replaced by the symmetrical R_{m-n}. Then from Eqns.(B.12) and (B.18),

Symmetrical R_{m-n}

$$= \text{real part of } \frac{Z_{m-n}+Z_{n-m}}{2}$$

$$= R_{G_m-G_n} + d_m + d_n + \omega' \quad .. \text{(B.23)}$$

where d_m = real part of $\frac{a_m + b_m}{2}$ .. (B.24)

and ω' = real part of ω .. (B.25)

Hence, expression for the power loss may be written as:

$$P_L = (i_{d_m} i_{d_n} + i_{q_m} i_{q_n}) R_{m-n} \quad .. \text{(B.26)}$$

where R_{m-n} is the real symmetric part of Eqn. (B.23).

Transformation from Reference Frame 3 to Reference Frame 6.

It is next necessary to obtain the equivalent circuit with impressed powers instead of impressed currents as indicated in Fig. 3.11. That is, it is necessary to express the generator currents $i_{d_m} + j i_{q_m}$ in interms of generator powers P_m.

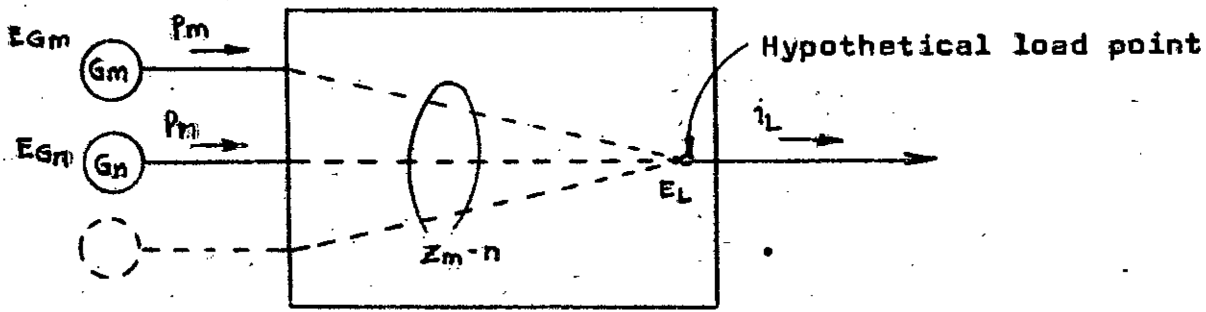


Fig. 3.11 Equivalent circuit with respect to reference frame 6.

Let σ_m be the angle of generator current i_{G_m} with respect to some reference. Then

$$\left.\begin{aligned} i_{d_m} &= \left|i_{G_m}\right| \cos \sigma_m \\ i_{q_m} &= \left|i_{G_m}\right| \sin \sigma_m \end{aligned}\right\} \quad \cdot\cdot \qquad (B.27)$$

Substituting Eqs.(B.27) in Eqn. (B.26), we have

$$P_L = \left(\left|i_{G_m}\right| \left|i_{G_n}\right| \cos \sigma_m \cos \sigma_n + \left|i_{G_m}\right| \left|i_{G_n}\right| \sin \sigma_m \sin \sigma_n \right) R_{m-n}$$

$$= \left|i_{G_m}\right| \left|i_{G_n}\right| \cos\left(\sigma_m - \sigma_n\right) R_{m-n} \quad \cdot\cdot\cdot \qquad (B.28)$$

Again,
$$\left.\begin{aligned} \left|i_{G_m}\right| &= \frac{P_m}{\sqrt{3}\left|V_m\right|(pf_m)} \\ \left|i_{G_n}\right| &= \frac{P_n}{\sqrt{3}\left|V_n\right|\ (pf_n)} \end{aligned}\right\} \quad \cdot\ \cdot \qquad (B.29)$$

Hence Eqn.(B.28) becomes

$$P_L = P_m \left\{ \frac{\cos\left(\sigma n - \sigma n\right)}{\left|V_m\right|\left|V_n\right|\ (pf_m)(pf_n)} \qquad \frac{R_{m-n}}{3} \right\} P_n$$

$$= P_m B_{mn} P_n \qquad \ldots \qquad \ldots \qquad (3.2)$$

where
$$B_{mn} = \frac{\cos\left(\sigma m - \sigma n\right)}{\left|V_m\right|\left|V_n\right|\ (pf_m)(pf_n)} \qquad \frac{R_{m-n}}{3} \qquad (B.30)$$

APPENDIX C

Derivation of General Expression for Loss-formula Coefficients and Transmission Loss Equations.

Let us consider the circuit of Fig. 3.6 where the current in line k is given by

$$i_k = N_{k(1)} i_{G_1} + N_{k(2)} i_{G_2} \quad \ldots \quad (3.5)$$

If we let

$$i_{G_1} = \left|i_{G_1}\right| \cos \sigma_1 + j \left|i_{G_1}\right| \sin \sigma_1$$

and $$i_{G_2} = \left|i_{G_2}\right| \cos \sigma_2 + j \left|i_{G_2}\right| \sin \sigma_2$$

where δ_1 and δ_2 are the phase angles of i_{G_1} and i_{G_2} with respect to some common reference, we obtain from Eqn. (3.5) [with $N_{k(1)}$ and $N_{k(2)}$ real, according to assumption made in Sec. 3.3]

$$\left|i_k^2\right| = \left[N_{k(1)} \left|i_{G_1}\right| \cos \sigma_1 + N_{k(2)} \left|i_{G_2}\right| \cos \sigma_2\right]^2$$

$$+ \left[N_{k(1)} \left|i_{G_1}\right| \sin \sigma_1 + N_{k(2)} \left|i_{G_2}\right| \sin \sigma_2\right]^2 \quad \ldots \quad (C.1)$$

from where

$$\left|i_k\right|^2 = N_{k(1)}^2 \left|i_{G_1}\right|^2 + N_{k(2)}^2 \left|i_{G_2}\right|^2 + 2N_{k(1)} N_{k(2)} \left|i_{G_1}\right| \left|i_{G_2}\right| \cos (\sigma_1 - \sigma_2)$$

Since, $\left|i_{G_1}\right| = \dfrac{P_1}{\sqrt{3}\left|V_1\right|(pf_1)}$ and $\left|i_{G_2}\right| = \dfrac{P_2}{\sqrt{8}\left|V_2\right|(pf_2)}$

we obtain total transmission loss as (for k lines)

$$P_L = \sum_K 3\,|i_k|^2 R_k = \frac{P_1^2}{|V_1|^2 (pf_1)^2} \sum_K N_{k(1)}^2 R_k$$
$$+ \frac{2P_1P_2 \cos\,(\sigma_1 - \sigma_2)}{|V_1||V_2|(pf_1)(pf_2))} \sum_K N_{k(1)}\, N_{k(2)}\, R_k$$
$$+ \frac{P_2^2}{|V_2|^2\,(pf_2)^2} \sum_K N_{k(2)}^2\, R_k \quad \cdot\cdot \qquad (C.2)$$

This may be written in the form:

$$P_L = P_1^2 B_{11} + 2P_1P_2B_{12} + P_2^2 B_{22} \quad \cdot\cdot \qquad (C.3)$$

where the loss coefficients are

$$\left.\begin{aligned} B_{11} &= \frac{1}{|V_1|^2 (pf_1)^2} \sum_K N_{k(1)}^2\, R_k \\ B_{12} &= \frac{\cos(\sigma_1 - \sigma_2)}{|V_1||V_2|(pf_1)(pf_2)} \sum_K N_{k(1)}\, N_{k(2)} R_k \\ B_{22} &= \frac{1}{|V_1|^2 (pf_2)^2} \sum_K N_{k(2)}^2 R_k \end{aligned}\right\} \cdot\cdot \qquad (C.4)$$

The units of loss coefficients are per megawatt.

Upon looking Eqns. (C.3) and (C.4) the general form of transmission loss equations and loss formula coefficients may be written respectively as follows:

$$P_L = \sum_m \sum_n P_m B_{mn} P_n \quad \cdot\cdot \qquad (3.2)$$

$$\text{and } B_{mn} = \frac{\cos(\sigma_m - \sigma_n)}{|V_m||V_n|(pf_m)(pf_n))} \sum_K N_{k(m)} N_{k(n)} R_k \quad \cdot\cdot \qquad (3.8)$$

APPENDIX D

Detailed Calculation for Checking of Load Flow Data

This appendix deals with the step by step verification of the different values of line currents, load currents and charging currents and bus voltages as obtained from the network analyzer reading with the calculated values which in turn, will be obtained by using known values of the corresponding real and reactive power readings from the analyzer as shown in Fig. 4.3 and Fig. 4.4.

(a) Calculation of line currents and bus voltages

(i) i_{L_5} (current from Kaptai to Madanhat)

$P = 2.06$ (out)

$Q = .55$ (out) $\qquad V_{Kaptai} = 1 \angle 0^\circ$

So, $\theta = \angle -15.3^\circ$

$P = VI \cos\theta$

Or, $2.06 = 1.0 \times i_{L_5} \times \cos 15.3^\circ$

$i_{L_5} = 2.14 \angle -15.3^\circ$

$V_{Madanhat} = V_{Kaptai} - (i_{L_5})(Z_{L_5})$

$= 1. \angle 0^\circ - (2.14) \angle -15.3^\circ \times (.0123+j.0442)$

$= .95 \angle -5.13^\circ$

(ii) $i_{L_{19}}$ (current from Madanhat to Halishahar)

$P = .26$ (out) $\qquad V_{Madanhat} = .95 \angle -5.13^\circ$

$Q = .19$ (out)

So, $\theta = -36.0^\circ$

So, $.26 = .95 \times \cos 36.0^\circ \times i_{L_{19}}$

Hence $i_{L_{19}} = .338\angle -36^\circ$

$$V_{Helishahar} = V_{Madanhat} - i_{L_{19}} Z_{L_{19}}$$

$$= .95\angle -5.13^\circ - (.338\angle -36^\circ)(.0132+j.0495)$$

$$= .94\angle -5.85^\circ$$

(iii) i_{L_6} (current from Madanhat to Comilla)

$P = 1.6$ (out)

$Q = .05$ (out) $\qquad V_{Madanhat} = .95\angle -5.13^\circ$

So, $\theta = -1.78^\circ$

$1.6 = .95 \times \cos 1.78 \times i_{L_6}$

Hence $i_{L_6} = 1.70\angle -1.78^\circ$

$$V_{Comilla} = V_{Madanhat} - (i_{L_6})(Z_{L_6})$$

$$= .95\angle -5.13^\circ - (1.70\angle -1.78^\circ)(.0459+j.164)$$

$$= .938\angle -22.65^\circ$$

(iv) $i_{L_{15}}$ (current from Comilla to Siddhirganj)

$P = 1.44$ (out)

$Q = .25$ (in) $\qquad V_{Comilla} = .938\angle -22.65^\circ$

So, $\theta = 9.8^\circ$

Now $1.44 = .938 \times \cos 9.8 \times i_{L_{15}}$

Hence $i_{L_{15}} = 1.56 \angle 9.8^{\circ}$

$V_{\text{Siddhirganj}} = V_{\text{Comilla}} - i_{L_{15}} Z_{L_{15}}$

$= .938 \angle -22.65 - (1.56 \angle 9.8^{\circ})(.0273+j.096)$

$\doteq .995 \angle -31.2^{\circ}$

(v) $i_{L_{11}}$ (current from Siddhirganj to Ullon)

$P = .23$ (out)

$Q = .17$ (out)

$V_{\text{Siddhirganj}} = .995 \angle -31.2^{\circ}$

So, $\theta = -36.4^{\circ}$

Now $.23 = .995 \times \cos 36.4 \times i_{L_{11}}$ so $i_{L_{11}} = .29 \angle -36.4^{\circ}$

$V_{\text{Ullon}} = V_{\text{Siddhirganj}} - i_{L_{11}} Z_{L_{11}}$

$= .995 \angle -31.2 - (.29 \angle -36.4)(.0096+j.0362)$

$= .99 \angle -31.9^{\circ}$

(vi) $i_{L_{20}}$ (current from Siddhirganj to Ashuganj)

$P = .32$ (out)

$Q = 1.2$ (out)

$V = .995 \angle -31.2^{\circ}$

So, $\theta = -76^{\circ}$

$P = VI \cos\theta$ (per unit)

Hence $.32 = .995 \times \cos 76^{\circ} \times i_{L_{20}}$

Or, $i_{L_{20}} = 1.25 \angle 376^{\circ}$

$V_{Ashuganj} = V_{Siddhirganj} - i_{L_{20}} \; Z_{L_{20}}$

$= .995 \quad -31.2 - (1.25 \angle -76^{o})(.025+j.092)$

$= .895 \angle -34.9^{o}$

(vii) $i_{L_{25}}$ (current from Ashuganj to Shahjibazar)

$P = 2.62$ (out) $\qquad V = .895 \angle -34.9^{o}$

$Q = .55$ (out)

So, $\theta = -12^{o}$

Hence, $2.62 = .895 \times \cos 12^{o} \times i_{L_{25}}$

Or, $i_{L_{25}} = 3.0 \angle -12^{o}$

$V_{Shahjibazar} = V_{Ashuganj} - i_{L_{25}} \; Z_{L_{25}}$

$= .895 \angle -34.9 - (3 \angle -13^{o})(.0162+j.0585)$

$= .94 \angle -46.2^{o}$

The above values of currents and voltages closely agree with the those obtained from analyzer readings as given in Fig. 4.3 and Fig. 4.4.

(b) Calculating of Load currents

(i) i_{LD_2} (Kaptai load)

$P = .04$ (out $\qquad V_{Kaptai} = 1.0 \angle 0^{o}$

$Q = .03$ (out

$P = VI \cos\theta$ (per unit)

So, $.04 = 1.0 \times \cos 36.8 \times i_{LD_2} \quad \therefore i_{LD_2} = .05$

(ii) i_{LD_1} (Madanhat load)

$P = .09$ (out)

$Q = .068$ (out) $V_{Madanhat} = .95$

Hence, $.09 = .95 \times \cos 37^\circ \times i_{LD_1}$

Or, $i_{LD_1} = .115$

(iii) i_{LD_6} (Halishahar load)

$P = .25$ (out

$Q = .18$ (out) $V_{Halishahar} = .94$

$\theta = -35.7^\circ$

So, $.25 = .94 \times \cos 35.7 \times i_{LD_6}$

Or, $i_{LD_6} = .34$

(iv) i_{LD_4} (Comilla load)

$P = .06$ (out)

$Q = .045$ (out) $V_{Comilla} = .938, \theta = -36.8^\circ$

Hence, $.06 = .938 \times \cos 36.8 \times i_{LD_4}$

Or, $i_{LD_4} = .078$

(v) i_{LD_7} (Siddhirganj load)

$P = .5$ (out)

$Q = .38$ (out $V_{Siddhirganj} = .995$

$\theta = -37.2^\circ$

Hence, $.5 = .99 \times \cos 37.2 \times i_{LD_7}$

Or, $i_{LD_7} = .65$

(vi) i_{LD_8} (Ullon load)

$P = .22$(Out) $\qquad V_{Ullon} = .99$

$Q = .165$ (Out)

$\theta = -\,36.8^{o}$

Hence, $.22 = .99 \times \cos 36.8 \times i_{LD_8}$

Or, $i_{LD_8} = .29$

(vii) i_{LD_9} (Ashuganj load)

$P = .03$ (out) $\qquad V_{Ashuganj} = .895$

$Q = .0227$ (out)

$\theta = -\,36.8^{o}$

Hence, $.03 = .895 \times \cos 36.8 \times i_{LD_9}$

Or, $i_{LD_9} = .044$

(viii) i_{LD_5} (Shahjibazar load)

$P = .015$ $\qquad V_{Shahjibazar} = .94$

$Q = .0125$

$\theta = -\,36.8^{o}$

Hence, $.015 = .94 \times \cos 36.8 \times i_{LD_5}$ Or, $i_{LD_5} = .028$

The above calculated values of load currents closely resemble the values obtained from network analyzer studies as shown in Fig.4.4.

(c) Calculation of charging currents.

(i) i_{C_6} (current through the capacitor at Kaptai bus)

$Q = .084,\ \theta = 90^{\circ},\ V = 1.0$

$Q = VI \sin\theta$ (per unit)

So, $i_{C_6} = .084$

(ii) i_{C_8} (current through the capacitor at Siddhirganj bus)

$Q = .084,\ \theta \doteq 90^{\circ},\ V = .995$

$Q = VI \sin\theta$

Sor, $i_{C_8} = .085$

(iii) i_{C_1} (current through the capacitor at Siddhirganj bus)

$Q = .03$

$V = .995$

So, $i_{C_1} = .031$

(iv) i_{C_3} (current through the capacitor at Shahjibazar bus)

$Q = .032$

$V = .94$

So, $i_{C_3} = .036$

The values of the charging currents thus calculated closely agree with the values obtained by means of analyzer readings as given in fig. 4.4.

APPENDIX E

Check of Current Distribution Factors (N_k)

To check the values of current distribution factors, line currents as given by Eqns. (3.6) have to be calculated by using these factors. These are done as follows:

$$i_{L_5} = N_{L_{5(1)}} i_{G_1} + N_{L_{5(2)}} i_{G_2} + N_{L_{5(3)}} i_{G_3} + N_{L_{5(4)}} i_{G_4}$$

$$= (-.0315)(-2.906 + j.566) + (.973)(2.1 - j.504)$$

$$+ (-.0315)(-.47 - j1.935) + (-.0315)(2.63 + j.58)$$

$$= 2.06 - j.47 = 2.10 \angle -13^\circ$$

$$i_{L_{19}} = N_{L_{19(1)}} i_{G_1} + N_{L_{19(2)}} i_{G_2} + N_{L_{19(3)}} i_{G_3} + N_{L_{19(4)}} i_{G_4}$$

$$= N_{L_{19(1)}} (i_{G_1} + i_{G_2} + i_{G_3} + i_{G_4})$$

$$= (.214)(-.74 - j.789 + 2.1 - j.504)$$

$$= .284 - j.273 = .4 \angle -43.2^\circ$$

$$i_{L_6} = N_{L_{6(1)}} (i_{G_1} + i_{G_3} + i_{G_4}) + N_{L_{6(2)}} i_{G_2}$$

$$= (-.317)(-.74 - j.789) + (.685)(2.1 - j.504$$

$$= 1.67 - j.097 = 1.69 \angle -3.32^\circ$$

$$i_{L_{15}} = N_{L_{15(1)}} (i_{G_1} + i_{G_3} + i_{G_4}) + N_{L_{15(2)}} i_{G_2}$$

$$= (-.367)(-.74 - j.789) + (.635)(2.1 - j.504)$$

$$= 1.6 - j.032 = 1.6 \angle -.9^\circ$$

$$i_{L_{11}} = N_{L_{11(1)}} (i_{G_1} + i_{G_2} + i_{G_3} + i_{G_4})$$

$$= (.183)\ (1.36 - j1.28)$$

$$= .33 \angle 43.2^{\circ}$$

$$i_{L_{20}} = N_{L_{20(1)}} i_{G_1} + N_{L_{20(2)}} i_{G_2} + N_{L_{20(3)}} i_{G_3} + N_{L_{20(4)}} i_{G_4}$$

$$= (-.958)\ (-2.906 + j.566 + 2.63 + j.58)$$

$$+ (.0471)\ (2.1 - j.504 - .47 - j\ 1.935)$$

$$= .348 - j1.21$$

$$= 1.25 \angle -74^{\circ}$$

$$i_{L_{25}} = N_{L_{25(1)}} i_{G_1} + N_{L_{25(2)}} (i_{G_2} + i_{G_3} + i_{G_4})$$

$$= (-.984)\ (-2.906 + j.566) + .0189)\ (4.26 - j1.86)$$

$$= 2.93 - j.591 = 3.0 \angle -11.4^{\circ}$$

The values of the line currents (L_k) thus obtained closely agree with the values obtained in Appendix D. This proves a check for the correctness of the current distribution factors given in Table 4.8.

APPENDIX F(A)

A Sample Calculation of Loss-formula-coefficients (B_{mn})

Using the values of Table 4.8 and Table 4.9, the B_{mn} matrix elements may be calculated for the system of Fig. 3.4. Calculation of B_{11} is shown below.

$$B_{11} = \frac{\cos(\rho_1 - \sigma_1)}{|V_1||V_1|(pf_1)(pf_1)} \Big[{}^{N}L_{5(1)}\,{}^{N}L_{5(1)}\,{}^{R}L_{5}$$

$$+{}^{N}L_{19(1)}\,{}^{N}L_{19(1)}\,{}^{R}L_{19} + {}^{N}L_{6(1)}\,{}^{N}L_{6(1)}\,{}^{R}L_{6} + {}^{N}L_{15(1)}\,{}^{N}L_{15(1)}\,{}^{R}L_{15}$$

$$+ {}^{N}L_{11(1)}\,{}^{N}L_{11(1)}\,{}^{R}L_{11} + {}^{N}L_{20(1)}\,{}^{N}L_{20(1)}\,{}^{R}L_{20} + {}^{N}L_{25(1)}\,{}^{N}L_{25(1)}\,{}^{R}L_{25}\Big]$$

$$= \frac{1}{(.94)^2(.98)^2}\Big[(-.0315)^2(.0123) + (.214)^2(.0132)$$

$$+ (-.317)^2(.0459) + (-.367)^2\,(.0273) + (.183)^2\,(.0096)$$

$$+ (-.958)^2\,(.025) + (-.984)^2\,(.0162)\Big]$$

$$= .0564$$

In a similar way other values of B_{mn} may be found as given in Table 4.10.

APPENDIX F(B)

Check of B_{mn} Matrix Values

(a) Line losses (P_L) from $\sum_m \sum_n P_m B_{mn} P_n$

For the system having 4 numbers of plants,

$$P_L = P_1^2 B_{11} + P_2^2 B_{22} + P_3^2 B_{33} + P_4^2 B_{44} + 2P_1 P_2 B_{12} + 2P_1 P_3 B_{13}$$
$$+ 2P_1 P_4 B_{14} + 2P_2 P_3 B_{23} + 2P_2 P_4 B_{24} + 2P_3 P_4 B_{34}$$

With the values given in Table 4.6 and Table 4.10,

$$P_L = (2.54)^2(.0564) + (2.1)^2(.056) + (.125)^2(.171) + (2.36)^2(.0558)$$
$$+ 2(2.54)(2.1)(.00938) + 2(2.54)(.125)(-.102) + 2(2.54)$$
$$(2.36)(-.0325) + 2(2.1)(.125)(.0289) + 2(2.1)(2.36)(-.0191)$$
$$+ 2(.125)(2.36)(-.0105)$$
$$= 1.0348 - .5924 = .4424$$

(b) Line losses (P_L) from $\sum_k i_k^2 R_k$.

Here, $P_L = i_{L_5}^2 R_{L_5} + i_{L_{19}}^2 R_{L_{19}} + i_{L_6}^2 R_{L_6} + i_{L_{15}}^2 R_{L_{15}}$
$+ i_{L_{11}}^2 R_{L_{11}} + i_{L_{20}}^2 R_{L_{20}} + i_{L_{25}}^2 R_{L_{25}}$

With the values of Fig. 4.4 for the system of Fig. 3.4 the loss becomes

$$P_L = (2.14)^2(.0123) + (.338)^2(.0132) + (1.7)^2(.0459)$$
$$+ (1.56)^2(.0273) + (.29)^2(.0096) + (1.25)^2(.025) + (3.0)^2(.0162)$$
$$= .4432$$

The agreement between the two sets of losses thus calculated out checks the correctness of the elements of B_{mn} matrix.

APPENDIX G

Determination of Coordination Equations

The derivation of these equations follows directly from the method of Lagrangian multipliers[29,30]. Let

F_t = total fuel input to system in Takas per hour.

$= \sum_n F_n$

where F_n = input to plant n is Takas per hour.

Let P_L = total transmission losses in megawatts.

$= \sum_m \sum_n P_m B_{mn} P_n$

where P_n = loading of plant n

B_{mn} = transmission loss-formula coefficients.

It is desired to minimize the total fuel input (F_t) in Takas per hour for a given received load (P_R). Let

P_R = given received load

By application of the method of Lagrangian multipliers the equation of constraint is given by

$$\psi(P_1, P_2, P_3 \; . \; . \; . \; . \; P_n) = \sum_n P_n - P_L - P_R = 0 \quad . \; . \quad (G.1)$$

Then minimum input for a given received load is obtained when

$$\frac{\partial \mathcal{F}}{\partial P_n} = 0 \quad . \; . \quad (A.2)$$

where $\mathcal{F}$ is a new expression and is given by

$$\mathcal{F} = F_t - \lambda\psi \qquad .. \qquad .. \qquad (A.3)$$

where λ = Lagrangian type multiplier

Hence, $$\frac{\partial \mathcal{F}}{\partial P_n} = \frac{\partial F_t}{\partial P_n} - \lambda \frac{\partial}{\partial P_n} = 0 \qquad .. \qquad (A.4)$$

Or, $$\frac{\partial F_t}{\partial P_n} - \lambda \frac{\partial}{\partial P_n}\left[\sum_n P_n - P_L - P_R\right] = 0$$

Or, $$\frac{\partial F_t}{\partial P_n} - \lambda \left[1 - \frac{\partial P_L}{\partial P_n}\right] = 0$$

Or, $$\frac{\partial F_t}{\partial P_n} + \lambda \frac{\partial P_L}{\partial P_n} = \lambda \qquad .. \qquad (G.2)$$

But $$\frac{\partial F_t}{\partial P_n} = \frac{\partial \left(\sum_n F_n\right)}{\partial P_n} = \frac{\partial F_n}{\partial P_n} = \frac{dF_n}{dP_n}$$

Then Eqn. (G.2) becomes

$$\frac{dF_n}{dP_n} + \lambda \frac{\partial P_L}{\partial P_n} = \lambda \qquad .. \qquad .. \qquad (5.1)$$

APPENDIX H

A. Calculation of Generation Schedules for Exact Equations (Transmission Loss Included)

Case(i) $\lambda = 10.0$

Then Eqns. (5.8) may be written a follows:

$$\left.\begin{aligned} .09128P_1 - .00204P_3 - .0065P_4 &= 2.0 \\ -.00204P_1 + .1342P_3 - .0021P_4 &= -5.5 \\ -.0065P_1 - .0021P_3 + .10116P_4 &= -2.0 \end{aligned}\right\} \quad \ldots \quad (H.1)$$

Solving for P_1, P_3 and P_4 we get

$P_1 = 18.21$ MW

$P_3 = -41.25$ MW

$P_4 = -19.4$ MW

Since plants 3 and 4 are giving negative generation, they should not generate and plant 1 only should generate.

Case (ii) $\lambda = 11.8$

Eqns. (5.8) for this case becomes

$$\left.\begin{aligned} .0933P_1 - .0024P_3 - .00766P_4 &= 3.8 \\ -.0024P_1 + .1404P_3 - .00248P_4 &= -3.7 \\ -.00766P_1 - .00248P_3 + .1031P_4 &= -.2 \end{aligned}\right\} \quad \ldots \quad (H.2)$$

Solving these equations we get

$P_1 = 37.7$ MW

$P_3 = -26.36$ MW

$P_4 = .013$ MW

That means plant 1 is only generating, plant 3 remaining idle, and plant 4 output very near to zero (in terms of MW).

Case (iii) $\lambda = 12.0$

Solvable equations are:

$$\left.\begin{aligned} 0.0934P_1 - .00244P_3 - .0078P_4 &= 4.0 \\ -.00244P_1 + .1410P_3 - .0025P_4 &= -3.5 \\ -.0078P_1 - .0025P_3 + .1033P_4 &= 0 \end{aligned}\right\} \quad . \quad .. \quad (H.3)$$

Solving the equations,

$P_1 = 40$ MW

$P_3 = -24.28$ (not generating)

$P_4 = 2.62$ MW.

Case (iv) $\lambda = 15.5$

Eqns. (5.8) become

$$\left.\begin{aligned} .0973P_1 - .00316P_3 - .0101P_4 &= 7.5 \\ -.00316P_1 + .153P_3 - .00325P_4 &= 0 \\ -.0101P_1 - .00325P_3 + .1073P_4 &= 3.5 \end{aligned}\right\} \quad .. \quad (H.4)$$

After solving, we get

$P_1 = 81.5$ MW, $P_3 = 2.50$ MW, $P_4 = 40$ MW.

Case (v) $\lambda = 17.0$

Eqns. (5.8) becomes

$$\left.\begin{array}{r} .09904P_1 - .00346P_3 - .01105P_4 = 9.0 \\ -.00346P_1 + .1581P_3 - .00357P_4 = 1.5 \\ -.01105P_1 - .00357P_3 + .1089P_4 = 5.0 \end{array}\right\} \quad .. \quad (H.5)$$

Solving these equations, we have

$P_1 = 98.0$ MW

$P_3 = 13.0$ MW

$P_4 = 56.1$ MW

Case (vi) $\lambda = 20.0$

Here, Eqns. (5.8) become

$$\left.\begin{array}{r} .1024P_1 - .00408P_3 - .0130P_4 = 12.0 \\ -.00408P_1 + .1684P_3 - .0042P_4 = 4.5 \\ -.013P_1 - .0042P_3 + .1123P_4 = 8.0 \end{array}\right\} \quad .. \quad (H.6)$$

Solving these equations, we have

$P_1 = 130$ MW, $P_3 = 31.50$ MW, $P_4 = 87.5$ MW

Case (vii) $\lambda = 25.0$

Here, Equations (5.8) become

$$\left.\begin{array}{r} .1080P_1 - .0051P_3 - .01625P_4 = 17.0 \\ -.0051P_1 + .1855P_3 - .00525P_4 = 9.5 \\ -.01625P_1 - .00525P_3 + .1179P_4 = 13.0 \end{array}\right\} \quad .. \quad (H.7)$$

Solution of these equations gives

$P_1 = 187.0$ MW, $P_3 = 61.5$ MW, $P_4 = 138$ MW.

B. Calculation of Generation Schedules for the Case of neglecting Transmission Losses.

Case (i) $\lambda = 10.0$

From Eqns. (5.7), it is evident

$P_1 = 25$ MW, plants 3 and 4 are not generating

Case (ii) $\lambda = 12.0$

From Eqns. (5.7), we have

$$\left.\begin{aligned} 0.08P_1 &= 12.0 - 8.0 = 4 \\ 0.1P_3 &= 12 - 15.5 = -3.5 \\ 0.09P_4 &= 12 - 12.0 = 0 \end{aligned}\right\} \qquad .\,. \qquad \text{(H.8)}$$

Solving these

$P_1 = 50$ MW, plant 3 is not generating, $P_4 = 0$

Case (iii) $\lambda = 15.5$

Eqns. (5.7) become

$$\left.\begin{aligned} 0.08P_1 &= 7.5 \\ 0.1P_3 &= 0 \\ 0.09P_4 &= 3.5 \end{aligned}\right\} \qquad .. \qquad .. \qquad \text{(H.9)}$$

from which

$P_1 = 93.75$ MW

$P_3 = 0$

$P_4 = 38.9$ MW

Case (iv) $\lambda = 17.0$

Here we have,
$$\left.\begin{aligned} .08P_1 &= 9.0 \\ 0.1P_3 &= 1.5 \\ 0.09P_4 &= 5.0 \end{aligned}\right\} \qquad .\,. \qquad \text{(H.10)}$$

Solving Eqns. (H.10), we get

$P_1 = 112.5$ MW, $P_3 = 15$ MW, $P_4 = 56.6$ MW

Case (v) $\lambda = 20$

Eqns. (5.7) become

$$\left.\begin{aligned} .08P_1 &= 12.0 \\ .1P_3 &= 4.5 \\ .09P_4 &= 8.0 \end{aligned}\right\} \quad .. \tag{H.11}$$

and we have from here

$P_1 = 150$ MW, $P_3 = 46$ MW, $P_4 = 90.9$ MW

Case (vi) For $\lambda = 22.0$, Eqns. (5.7) become:

$$\left.\begin{aligned} 0.08P_1 &= 14.0 \\ 0.1P_3 &= 6.5 \\ 0.09P_4 &= 10 \end{aligned}\right\} \quad .. \tag{H.12}$$

from where we get

$P_1 = 175$ MW

$P_3 = 65$ MW

$P_4 = 111.1$ MW

Case (vii) For, $\lambda = 25$, we get

$P_1 = 212.5$ MW

$P_3 = 95$ MW

$P_4 = 144.5$ MW.

APPENDIX I

Calculation of Transmission Losses.

A. Exact Method of Solution:

(i) $P_2 = 84.0$

$P_L = B_{22}P_2^2 = (.00056)(84)^2 = 4.0$ MW

$P_R = 84 - 4 = 80.0$ MW

Here, Plant 2 is operated to carry 80 MW base load of the system. Eventually it has to generate 84 MW with a transmission loss of 4.0 MW to supply the required load.

(ii) $\lambda = 10.0$ $P_1 = 18.21$ MW

P_3 and P_4 = remaining idle

$P_L = P_1^2 B_{11} = (18.21)^2 (5.64\text{X}10^{-4}) = .21$ MW

$P_R = 18.21 - .21 = 18.0$ MW

Total received load = 80.0 + 18.0 = 98.0 MW

(iii) $\lambda = 11.8$ $P_1 = 37.7$ MW

P_3 = not generating

$P_4 = 0$

$P_L = (37.7)^2 (5.64\text{X}10^{-4}) = .70$ MW

$P_R = 37.7 - .70 = 37.0$ MW

Total received load = 37.0 + 80 = 117.0 MW

(iv) $\lambda = 12.0$ $\quad P_1 = 40$ MW

P_3 = not generating

$P_4 = 2.62$ MW

$$P_L = (40)^2\,(5.64\text{X}10^{-4}) + (2.62)(5.58\text{X}10^{-4})$$

$$= .84 \text{ MW}$$

$$P_R = 42.62 - .84 = 41.78 \text{ MW}$$

Total received load = 80 + 41.78 = 121.78 MW

(v) $\lambda = 15.5$ $\quad P_1 = 81.5$ MW

$P_3 = 2.50$ MW

$P_4 = 40$ MW

$$P_L = P_1^2B_{11} + P_3^2B_{33} + P_4^2B_{44} + 2P_1P_3B_{13} + 2P_1P_4B_{14} + 2P_3P_4B_{34}$$

$$= (81.5)^2(5.64\text{X}10^{-4}) + (2.5)^2(1.71\text{X}10^{-3}) + (40)^2(5.58\text{X}10^{-4})$$

$$- 2(81.5)(40)(3.25\text{X}10^{-4}) - 2(81.5)(2.5)(1.02\text{X}10^{-4})$$

$$- 2(2.5)(40)(1.05\text{X}10^{-4})$$

$$= 3.0 \text{ MW} \qquad \sum_n P_n = 124.0 \text{ MW}$$

$$P_R = 124.0 - 3.0 = 121.0 \text{ MW}$$

Total received load = 80 + 121.0 = 201.0 MW.

(vi) $\lambda = 17.0$ $\quad P_1 = 98.0$ MW

$P_3 = 13.0$ MW

$P_4 = 56.1$ MW

$$P_L = (98.0)^2(5.64X10^{-4}) + (13.0)^2(1.71X10^{-3})$$

$$+ (56.1)^2(5.58X10^{-4}) - 2(98.0)(13.0(1.02X10^{-4})$$

$$- 2(98.0)(56.1)(3.25X10^{-4}) - 2(13.0)(56.1)(1.05X10^{-4})$$

$$= 4.1 \text{ MW} \quad \sum_n P_n = 167.1 \text{ MW}$$

$$P_R = 167.1 - 4.1 = 163.0 \text{ MW}$$

Total received load = 163.0 + 80.0 = 243.0 MW

(vii) $\lambda = 20.0$

$P_1 = 130$ MW

$P_3 = 31.5$ MW

$P_4 = 87.$ MW

$$P_L = (130)^2(5.64X10^{-4}) + (31.5)^2(1.71X10^{-3})+(87.5)^2(5.58X10^{-4})$$

$$-2(130)(31.5)(1.02X10^{-4}) -2(130)(87.5)(3.25X10^{-4})$$

$$-2(31.5)(87.5)(1.05X10^{-4})$$

$$= 6.20 \text{ MW}, \quad \sum_n P_n = 249.0 \text{ MW}$$

$$P_R = 249 - 6.2 = 242.8 \text{ MW}$$

Total received load = 242.8 + 80.0 = 322.8 MW

(viii) $\lambda = 25.0$ $P_1 = 187.0$ MW, $P_3 = 61.5$ MW, $P_4 = 138$ MW.

$$P_L = (187)^2(5.64X10^{-4})+(61.5)^2(1.71X10^{-3})+(138)^2(5.58X10^{-4})$$

$$-2(187(61.5)(1.02X10^{-4})-2(187)(138)(3.25X10^{-4})$$

$$-2(61.5)(138)(1.05X10^{-4})$$

$$= 16.5 \text{ MW}, \quad \sum_n P_n = 386.5 \text{ MW}, \; P_R = 386.5 - 16.5 = 370.0 \text{ MW}.$$

Total received load = 370.0 + 80 = 450 MW.

B. Solution for Scheduling when Transmission Losses are Neglected.

(i) $\lambda = 10.0$

$P_1 = 25$ MW, P_3 and P_4 = remaining idle, $P_2 = 84$ MW

$P_L = (25)^2(5.64.X10^{-4})$

$= .4$ MW $\quad P_L$(total) $= 4 + .4 = 4.4$ MW

$\sum_n P_n = 109$ MW, $P_R = 109 - 4.4 = 104.6$ MW.

(ii) $\lambda = 12.0$

$P_1 = 50$ MW, $P_2 = 84$ MW, P_3 = remaining idle, $P_4 = 0$

$P_L = (50)^2(5.64X10^{-4})$

$= 1.5$ MW.

P_L(total $= 5.5$ MW .

$\sum_n P_n = 134$ MW $\quad P_R = 134 - 5.5 = 128.5$ MW

(iii) $\lambda = 15.5$

$P_1 = 93.75$ MW, $P_3 = 0$, $P_4 = 38.9$ MW, $P_2 = 84$ MW

$P_L = (98.75)^2(5.64X10^{-4}) + 0 + (38.9)^2(5.58X10^{-4})$

$-0 - 2(93.75)(38.9)(3.25X10^{-4}) - 0$

$= 4.65$ MW

P_L (total) $= 8.65$ MW.

$\sum_n P_n = 216.65$ MW $\quad P_R = 216.65 - 8.65 = 208.0$ MW.

(iv) $\lambda = 17.0$

$P_1 = 112.5$ MW, $P_2 = 84$ MW, $P_3 = 15$ MW, $P_4 = 56.6$ MW.

$$P_L = (112.5)^2(5.64\text{X}10^{-4})+(15)^2(1.71\text{X}10^{-3})$$
$$+ (56.6)^2(5.58\text{X}10^{-4}) -2(112.5)(15)(1.02\text{X}10^{-4})$$
$$-2(112.5)(56.6)(3.25\text{X}10^{-4})-2(15)(56.6)(1.05\text{X}10^{-4})$$
$$= 6.6 \text{ MW}, \quad P_L(\text{total}) = 4 + 6.6 = 10.6 \text{ MW}$$

$\sum_n P_n = 68.1$ MW $P_R = 268.1 - 10.6 = 257.5$ MW

(v) $\lambda = 20.0$

$P_1 = 150$ MW, $P_2 = 84$ MW, $P_3 = 46$ MW, $P_4 = 90.9$ MW.

$$P_L = (150)^2(5.64\text{X}10^{-4}) +(46)^2(1.71\text{X}10^{-3})$$
$$+(90.9)^2(5.58\text{X}10^{-4}) - 2(150)(46)(1.02\text{X}10^{-4})$$
$$-2(150)(90.9)(3.25\text{X}10^{-4}) -2(46)(90.9)(1.05\text{X}10^{-4})$$
$$= 11.89 \text{ MW}$$

P_L (total) $= 4.0 + 11.89 = 15.89$ MW

$\sum_n P_n = 370.9$ MW, $P_R = 354.01$ MW

(vi) $\lambda = 25.0$

$P_1 = 212.5$ MW, $P_2 = 84$ MW, $P_3 = 95$ MW, $P_4 = 144.5$ MW.

$$P_L = (212.5)^2(5.64\text{X}10^{-4}) +(95)^2(1.71\text{X}10^{-3})$$
$$+ (144.5)^2(5.58\text{X}10^{-4}) -2(212.5)(95)(1.02\text{X}10^{-4})$$
$$-2(212.5)(144.5)(3.25\text{X}10^{-4})-2(95)(144.4)(1.05\text{X}10^{-4})$$
$$= 26.2 \text{ MW.} \qquad \sum_n P_n = 536.0 \text{ MW}$$

P_L (total) $= 4.0 + 26.2 = 30.2$ MW

$P_R = 505.8$ MW.

APPENDIX J

Digital Method of Solution of Coordination Equations

a. Executing Form of Equations:

The exact coordination equations as given by equations (5.4) may be rewritten in the following form:

$$F_{nn}P_n + f_n + \lambda \sum_{m} 2B_{mn}P_m = \lambda \quad \ldots \quad (5.4)$$

Collecting all coefficients of P_n, we obtain

$$P_n(F_{nn} + \lambda 2B_{nn}) = -\lambda\left(\sum_{m\neq n} 2B_{mn}P_m\right) - f_n + \lambda \quad \ldots \quad (J.1)$$

Solving for P_n

$$P_n = \frac{\lambda - f_n - \lambda\left(\sum_{m\neq n} 2B_{mn}P_m\right)}{F_{mn} + 2B_{nn}} \quad \ldots \quad (j.2)$$

$$= \frac{1 - \frac{f_n}{\lambda} - \sum_{m\neq n} 2B_{mn}P_m}{\frac{F_{nn}}{\lambda} + 2B_{mn}} \quad \ldots \quad (J.3)$$

Putting the numerical values from Tables 5.1 and 5.2, Eqns. (J.3) become

$$P_1 = \frac{1 -(8.0/\lambda) + .000204P_3 + .00065P_4}{0.08/\lambda + .001128} \quad \ldots \quad (J.4)$$

$$P_2 = \frac{1 - (15.5/\lambda) + .000204P_1 + .00021P_4}{0.1/\lambda + .00242} \quad \ldots \quad (J.5)$$

$$P_4 = \frac{1 - (12.0/\lambda) + .00065P_1 + .00021P_3}{0.09/\lambda + .001116} \quad \ldots \quad (J.6)$$

b. Computer Program

```
C  SOLUTION OF NONLINEAR SIMULTANEOUS EQUATIONS
       P1 = 0.0
       P3 = 0.0
       P4 = 0.0
       AMD = 10.0
       PRINT 2, AMD
 2     FORMAT (1F5.2)
       DD1 = 8.0/AMD
 200   D1 = 1.0 - DD1 + 0.000204 * P3 + 0.00065 * P4
       BB1 = 0.08/AMD
       B1 = BB1 + 0.001128
       PN1 = D1/B1
       DD3 = 15.5/AMD
       D3 = 1.0 - DD3 + 0.000204 * PN1 + 0.00021 * P4
       BB3 = 0.1/AMD
       B3 = BB3 + 0.00342
       PN3 = D3/B3
       DD4 = 12.0/AMD
       D4 = 1.0 - DD4 + 0.00065 * PN1 + 0.00021 * PN3
       BB4 = 0.09/AMD
       B4 = BB4 + 0.001116
       PN4 = D4/B4
```

```
      PRINT 5, PN1, PN3, PN4
5     FORMAT (3F13.4)
      IF ((PN1 - P1) - 0.1) 10, 10, 11
10    IF ((PN3 - P3) - 0.1) 12, 12, 11
12    IF ((PN4 - P4) - 0.1) 14, 14, 11
14    GO TO 100
11    P1 = PN1
      P3 = PN3
      P4 = PN4
      GO TO 200
100   CALL EXIT
      END
```

C Result

```
 ‡ ‡ JOB 5
 ‡ ‡ FORX5
 01950 CORES USED
 59999 NEXT COMMON
 END OF COMPULATION
 EXECUTION
 ER F8 .10000000E + 02
          0.0000         0.0000         0.0000
           .2191         -.4098         -.1976
           .2188         -.4098         -.1976
```

The above results are closely near to the calculated values given in Appendix H for λ =10.0. In a similar way, other values of P_1 , P_3 and P_4 are obtainable by putting different values of λ in the same programming.

APPENDIX K

TABLE I

Table showing the Difference of Fuel Input for Two Schedules

Received Load P_R (MW)	Total fuel input (Taka/hr.) when losses neglected, F_t	Total fuel input (Taka/hr.) when losses included. F_t	Difference of fuel input (Taka/hr.)
80	-	-	-
100	240	240	0
120	470	460	10
140	755	735	20
160	1,020	985	35
180	1,310	1,265	45
200	1,625	1,570	55
220	1,910	1,850	60
240	2,190	2,120	70
260	2,470	2,390	80
280	2,770	2,680	90
300	3,062	2,960	102
320	3,380	3,270	110
340	3,690	3,565	125
360	4,000	3,855	145
380	4,330	4,150	180
400	4,690	4,460	230

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