THE EFFECT OF SIGNAL POWER VARIATION AND PATTERN CONSTRAINTS ON AN ADAPTIVE ANTENNA ARRAY SYSTEM

BY

FARHAD FUAD ISLAM

A THESIS

Submitted to the Department of Electrical and Electronic Engineering, Bangladesh University of Engineering and Technology, Dhaka, in partial fulfillment of the requirement for the degree

OF

MASTER OF SCIENCE IN ELECTRICAL AND ELECTRONIC ENGINEERING

MARCH 1988.

DECLARATION

I do hereby declare that neither this thesis nor any part thereof has been submitted or is being concurrently submitted in candidature for any degree at any other university.

Candidate

CERTIFICATE OF RESEARCH

Certified that the work presented in this thesis is the result of the investigation under the supervision of Dr. Kazi Mohiuddin Ahmed at the Department of Electrical and Electronic Engineering, BUET, Dhaka.

Candidate

Supervisor

Accepted as satisfactory for partial fulfillment of the requirements for the degree of M.Sc. Engg. in Electrical and Electronic Engineering.

BOARD OF EXAMINERS

(i) ______ 5/4/88 — Chairman & Supervisor

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(Prof. Shamsuddin Ahmed)
Department of Electrical and
Electronic Engg., ICTVTR, Dhaka.

ACKNOWLEDGMENT

The author expresses his profound gratitude to his supervisor Dr. Kazi Mohiuddin Ahmed, Associate Professor of the Department of Electrical and Electronic Engineering, BUET, for his constant guidance, continuous encouragement and valuable suggestions and supervision, throughout the entire progress of this work.

The author also wishes to express his thanks to Dr. Md. Mujibur Rahman, Professor and Head of the Department of Electrical and Electronic Engineering, BUET, for his support to successfully complete this work. Thanks are also due to Dr. Saiful Islam, Associate Professor in the Department of Electrical and Electronic Engineering, BUET, for his constructive cooperation in providing the computer facilities of the department.

The author gratefully recalls the valuable cooperation obtained from Mr. Dipak Bin Qasem Chowdhury and Mr. Javed Sabir Barkatullah, both belonging to the

Computer Engineering Department for their valuable suggestions specially regarding efficient use of computers. Mr. Tahsin Asker, Hardware Chief of Beximco Computers Ltd., deserves praise and appreciation for his help in providing computing facilities.

Finally the author would like to thank all the staff of the Faculty of Electrical and Electronic Engineering, BUET, for their spontaneous assistance and friendly cooperation.

ABSTRACT

Adaptive array antenna systems are currently the subject of intense interest and investigation / development for radar and communications applications. The principal reason for the use of adaptive arrays is in their ability to automatically steer nulls onto undesired sources of interference, thereby reducing output noise and enhancing the detection of desired signals. These systems consist of an array of antenna elements and an adaptive receiver processor which has feedback control over the element weights. The concept of the adaptive system works on the principle of minimizing the output power under the constrained response to specified directions.

The present work studies the behaviour of the adaptive array system depending on the signal power of the desired signal and the interference sources, and also on the placement of directional constraints in the system. A brief theoretical analysis and intensive computer simulation on this directional constrained system are presented. The results confirm that the algorithm used is able to iteratively adapt variable weights on the tap of the sensor array to minimize noise power at the array output subject to certain constraints on the mainlobe as well as on the sidelobes of the array response pattern. A number of interesting observations regarding the behaviour of this adaptive antenna system in presence of the desired signal and interfering noise of varying power levels are also revealed. Detailed discussion is held for all types of simulations and inferences are drawn about the response of the array under different signal environment as well as pattern constraints.

LIST OF PRINCIPAL SYMBOLS AND ABBREVIATIONS

SYMBOLS

Capital letters

c	Velocity of light
$\underline{C}$	Constraint matrix
E[.]	Expected value
K	Number of antenna elements in an array
L	Number of tap points in each channel
$\underline{N}$	Noise vector
$\underline{P}$	Projection vector
P_s	Total signal power
P_{tn}	Total noise power
P_{in}	Interfering noise power
P_{sn}	System noise power
$\underline{R}$	Covariance or correlation matrix
$\underline{S}$	Signal vector
$\underline{W}$	Weight vector
W_{KL}	Tap-weight value due to KL-th tap
$\underline{X}$	Composite (signal and noise combined) signal vector

Small letters

$\underline{d}$ Desired output vector

$\underline{e}$ Error signal for least mean square (LMS) algorithm; Tolerance band of desired response in Primal-dual algorithm

f Frequency of desired signal

m Number of iterations

n_i Noise signal in i-th antenna element

s_i Desired signal at i-th antenna element

y Output signal

Greek letters

θ Angle made by the signal arrival direction (look direction) with axis of the linear array

$\Psi_K(\theta)$ Phase of the signal arriving from the direction at K-th element with phase reference taken at the centre of the array

α Small scalar step

β Small scalar step

λ	Wave length
λ_1, λ_2	Lagrange multipliers
Δ	Time delay between two adjacent tap-points

<u>Abbreviations</u>

ECM	Electronic Counter Measure
LMS	Least Mean Square
RF	Radio Frequency
RFI	Radio Frequency Interference
SINR	Signal to Interference and Noise Ratio
SNR	Signal to System Noise Ratio

CONTENTS

CHAPTER 1

INTRODUCTION

1.1 ADAPTIVE ANTENNA ARRAY PROCESSING

Signal processing using an array of sensors has long been an attractive solution to many detection and estimation problems. An array offers the means of overcoming the directivity and beam width limitations of a single sensor element thus reducing the vulnerability of the reception of desired signals in the presence of interference signals in radar, sonar, seismic and communication systems. Such conventional arrays are mostly built with fixed weights designed to produce a pattern that is a compromise between resolution, gain and low side lobes. The versatility of the array antenna, however, invites the use of more sophisticated techniques for array weighting. Particularly attractive are adaptive schemes that can sense and respond to a time varying environment. Array antennas that incorporate this characteristic adaptive property of automatically steering nulls onto undesired sources of interference thereby reducing output noise and enhancing the capability of detection of desired signal are called adaptive antenna arrays.

Such an adaptive array is a system consisting of an array of antenna elements and a real time adaptive receiver-processor which, given a beam steering command, samples its current environment and then automatically proceeds to adjust its element control weights. This weight adjustment is carried on by means of some feed back scheme and all this is done in accordance with a selected algorithm. The ultimate objective is, of course, to improve output signal to noise ratio.

Conventional communications and radar antenna systems are susceptible to a degradation in SNR performance caused by undesired "noise" which intrudes via the antenna side lobes. The noise may consist of deliberate electronic counter measure (ECM), friendly RF interference (RFI), clutter scatter returns and natural noise sources. This degradation is often aggravated by movement of the antenna, poor siting conditions, multipath, and a changing interference environment. Adaptive array techniques offer possible solutions to these serious interference problems via their flexible capabilities of automatic null steering. In fact, adaptive nulling can be considered to be a principal benefit of adaptive techniques at the present time. For these reasons, adaptive array antenna systems are currently the subject of intense investigation / development for radar and communications applications. Immediate advantages of such an array can thus be grouped as follows [1] :

(i) Automatic adjustment of the antenna response to give minimum interference.

(ii) No need for an a priori knowledge of the bearing of the interference.

(iii) Ability to handle multiple sources of interference upto a limit which can be defined.

(iv) Ability to track the changing apparent direction of interference whether resulting from antenna movement, source movement or propagation effect.

(v) It does not need to know the bearing of the interference, the array can handle new sources of interference or jamming not previously predicted , and

(vi) There is no longer a severe mounting and tolerance problem.

1.2 BACKGROUND / HISTORY

Adaptive arrays have their roots [2] in a number of different fields, including retrodirective and self phasing RF antenna arrays, sidelobe cancellers, adaptive filters [3], acoustic or sonar arrays [4], and seismic arrays.

The first real contribution in the RF antenna field was the retrodirective array invented by L.C. Van Atta in the 1950's. Another major step was the phase-locked loop theory and practice, which made possible self steering arrays.

Then in the early 1960's, two groups independently developed schemes for achieving the key capability of adaptive interference nulling. One group arrived via radar sidelobe canceller development as represented by the patent of P.W. Howells [16] for an IF side-lobe canceller. A subsequent analysis by S.P. Applebaum [5] established the associated control law theory, an algorithm that maximizes a generalized signal to noise ratio. The other group arrived largely via self-training or self-optimizing array processor control systems which operate upon sampled signals, as described by B. Widrow in his report on adaptive filters [3], [17]. Widrow and his co-workers [6] subsequently applied their approach to adaptive antenna systems and firmly established the least mean square error algorithm, based upon the method of steepest descent.

The LMS algorithm was developed further by Griffith and Frost [7], with the result that one can maintain a chosen frequency characteristic for the array in a desired direction, while discriminating against noises coming from other directions. Rieglar and Compton [18] provided timely experimental performance verification, utilizing a laboratory array system which was oriented toward communications applications. Compton [19] noted a power equalization phenomenon, which was also addressed by Zahm [20] and shown to permit acquisition of weak signals in presence of strong jamming.

The maximum signal-to-noise (MSN) algorithm was also developed further, with open literature contributions primarily by Brennan and Reed [21]-[23].

Other algorithms and techniques have also been under investigation. Reed et al. [24] have found that rapid convergence can be obtained in all cases via a direct method of adaptive weight computation, based on a sample covariance matrix of the noise field. Search techniques [25] combined with simple array performance monitoring have been receiving increasing attention in an effort to achieve implementations which are less complicated / costly.

Young and Howard [26], [27] have described a parallel-processing array system concept which adapts optimally on the basis of decision theory.

1.3 THESIS OUTLINE

This thesis is concerned with the study of the behaviour of an adaptive array antenna system under varying signal environment conditions and response pattern constraints. Adaptive array antenna system with directional constraints was first put forward by Takao et. al [8]. In this type of adaptive processor, it is assumed that there is a priori knowledge of the desired signal direction. The algorithm iteratively adapts the element weights and finds the optimum weights which steer the response pattern main-lobe in the desired signal direction and put nulls in the noise (interference) signal directions. This algorithm in doing so requires a "directional constraint" to place the main lobe of the response pattern in the desired signal direction (look direction).

In our thesis, we use this "directional constraint" idea and introduce a number of constraints on the response pattern of the adaptive array antenna system to investigate its behaviour under such condition. Apart from the main lobe, there are a number of side-lobes in the response pattern of an adaptive array like any other antenna system. The set of optimum weights which controls the main lobe direction and level do not have any control on the side-lobe directions and levels. For an array antenna designer it would be quite interesting as well as important to have more or less complete control on the response pattern of the array. In this work, we show that it is possible to control the response of the array antenna at these side-lobe directions by imposing proper side-lobe constraints. The upper limit of such controllable side-lobes is however limited by the degrees of freedom -- a phenomena discussed elaborately in a later chapter. The chief advantage that we may achieve by controlling these side lobe levels in the response pattern, is apparent by the fact that reduced side lobe levels tend to improve the overall output signal to noise ratio for unaccounted noise and also allow us to exercise greater authority on the overall antenna response. In this thesis, this side lobe constraint phenomena was given a particular importance and its different aspects were thoroughly investigated by computer simulation techniques.

The signal environment is the most important factor for consideration in adaptive array antenna processing. The capability of the array antenna may not be similar in any input signal to noise (interference) ratio condition. In certain occasions we may have strong desired signal condition compared to weak interference / jamming signals, sometimes it can be otherwise. The internal system noise may have considerable effect on the overall output SINR (signal to interference and system noise ratio). The effect of signal power variation i.e. variation in the strength of the desired signal, interference signal as well as the effect of system noise on the overall behaviour of the adaptive array system is investigated in this thesis.

In chapter 2 of this thesis, a general discussion on the various techniques employed on adaptive array antenna processing is given. Chapter 3 gives in brief the mathematical tools required for solving the optimization schemes employed for adaptive arrays. Chapter 4 is devoted to problem formulation and to the development of the algorithm used. The response patterns generated after computer simulation for various signal environments and constraints, are embodied in chapter 5.

Finally, a conclusion is drawn based on the various observations of the behavior of the narrowband adaptive array antenna for various signal / jamming conditions and sidelobe constraints.

CHAPTER 2

GENERAL REVIEW ON ADAPTIVE ANTENNA ARRAY PROCESSING

2.1 INTRODUCTION

This chapter deals with the general discussions regarding antenna arrays, their capability of adaptive response to some signal environment, as well as the basic principles which give rise to their adaptive nature. Some control criteria dependent on signal quality (e.g., signal strength etc.) are also discussed. The chapter ends by citing some methods used for achieving desired response from adaptive antenna arrays.

2.2 DEFINITION

An adaptive array is an antenna-array system which adapts to its signal environment to satisfy some criteria, such as to maximize the signal to noise ratio. The set of antenna elements may be evenly or unevenly spaced. They may be monopoles, dipoles, dipole-arrays, or feeds into highly directional parabolic reflectors. They may be placed in a line, on a circle, or even randomly over a surface or within a volume of space. But in all cases the arrays are designed so that they automatically adapt their responses to give low gain to unwanted signals while maintaining adequate gain to a wanted signal.

If we assume an array A to consist of L omnidirectional elements and the cartesian co-ordinates (Fig.2.2.1) of the elements to be denoted by L triples { x(i), y(i), z(i), i=1,2,....,L } then we can have the following definitions for linear, planar and general arrays [9]:-

Linear array : The array A is said to be a linear array if its L points, { x(i), y(i), z(i), i=1,2,....,L } are collinear.

Planar array : The array A is said to be a planar array if the L points of this array are coplanar but not collinear. Thus only non-degenerate planar arrays are called planar arrays.

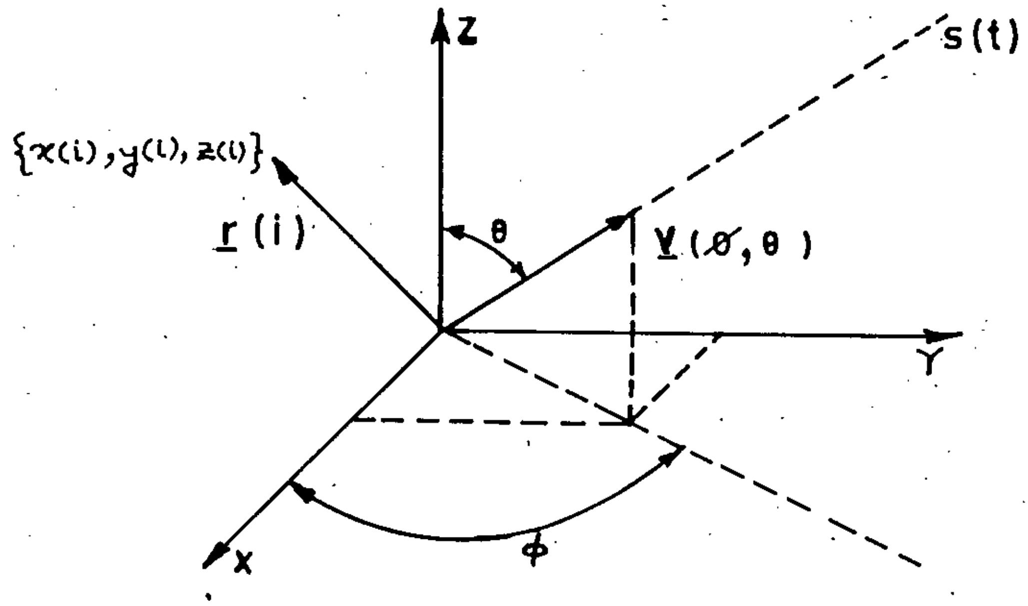

Fig. 2.2.1 : DEFINITION OF CO-ORDINATE SYSTEM.

General array : The array A is a general array if it is neither a linear nor a planar array.

The three classes of arrays defined above are mutually exclusive and exhaustive.

In Fig.2.2.1, $\hat{\underline{V}}(\phi, \theta)$ is called the steering vector which denotes the direction of the signal s(t) with angle θ and ϕ. The vector $\tilde{\underline{r}}(i)$ denotes the position of an antenna element. It is to be noted that, for a linear array, the direction of the signal source is given by the angle θ taking the axis of the array as the Z axis and the element position is given by only Z co-ordinates.

2.3 BASIC PRINCIPLES

In general, the antenna elements of an adaptive array have an associated electronic control unit or processor. The array itself is illustrated in Fig.2.3.1. The outputs of the N elements are weighted (W_1 to W_n) and then summed to give the array output [10].

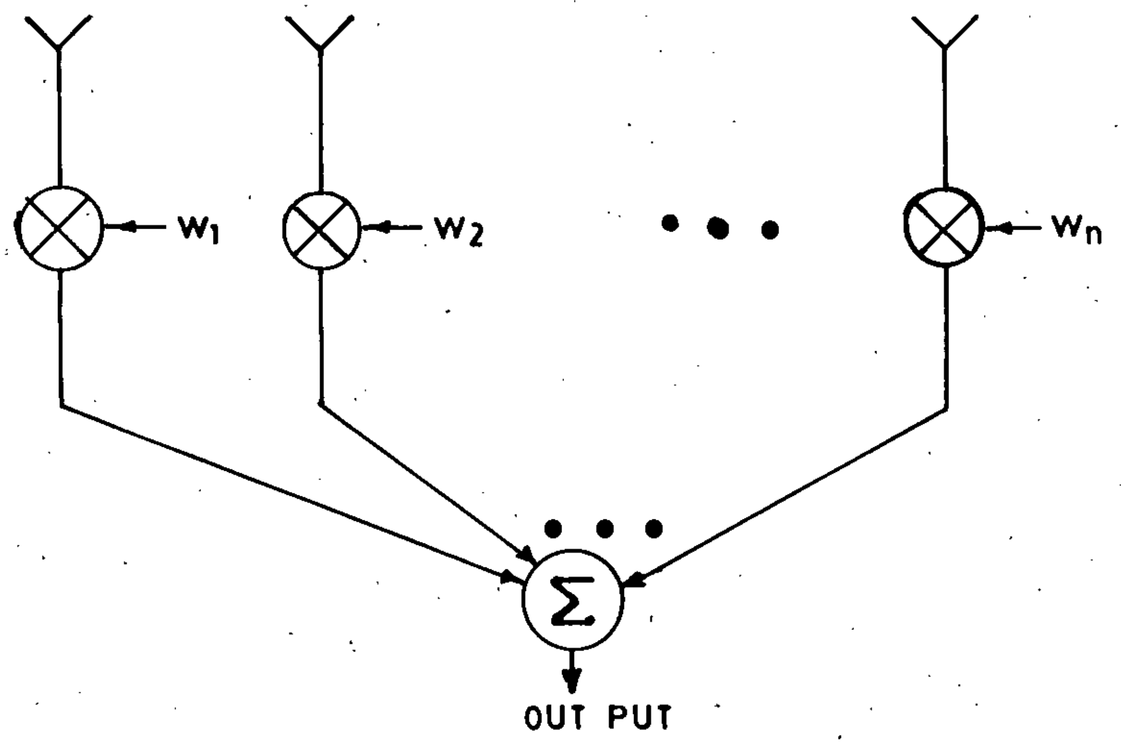

Fig. 2.3.1 : BASIC ARRAY.

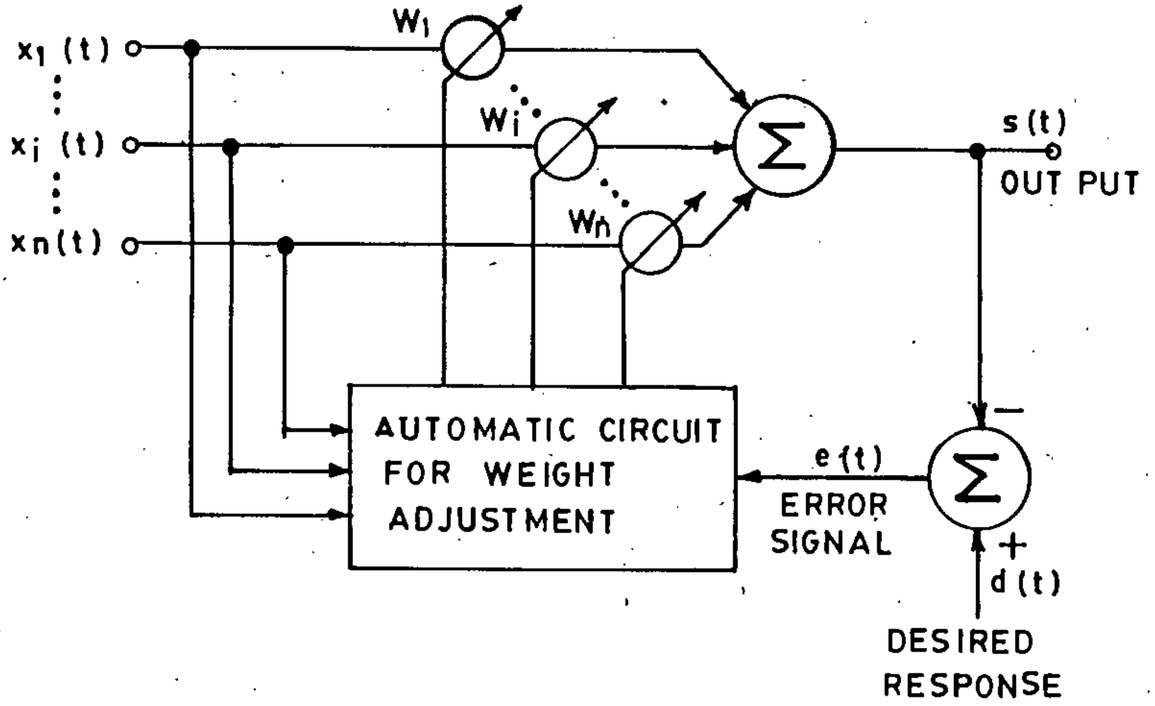

Fig. 2.3.2 : BASIC ADAPTIVE ARRAY.

The general form of an adaptive system is shown in Fig.2.3.2. Here x_1, x_2,....,x_n are the input signals to the array elements. The weights (W_1, W_2,....,W_n) are determined by a control unit (often called the adaptive processor). The control unit requires, in most systems, an input from each of the elements, and in many cases from the array output. A reference signal which defines in some way the wanted signal may also be required.

A simple example follows (Fig.2.3.3) which illustrates the existence and calculation of a set of weights which will cause a signal from a desired direction to be accepted while a "noise" from a different direction is rejected [6]. Let the signal arriving from the desired direction $\theta=0$ be called the "pilot" signal $p(t)=P_m \sin w_o t$; where P_m is the amplitude, $w_o = 2\pi f_o$ and f_o is the frequency of the pilot signal. The other unwanted signal or noise is chosen as $n(t)=N_m \sin w_o t$ and is incident on the receiving array at an angle $\theta=\pi/6$ radians. At a point in space midway between the antenna array elements, the signal and noise are assumed to be in phase. The omni-directional array elements are spaced $\lambda_0/2$ apart. The signals received by each element are fed to two variable weights, one weight being preceded by a quarter wave time delay of $1/4f_0$. The four weighted signals are then summed to form the array output.

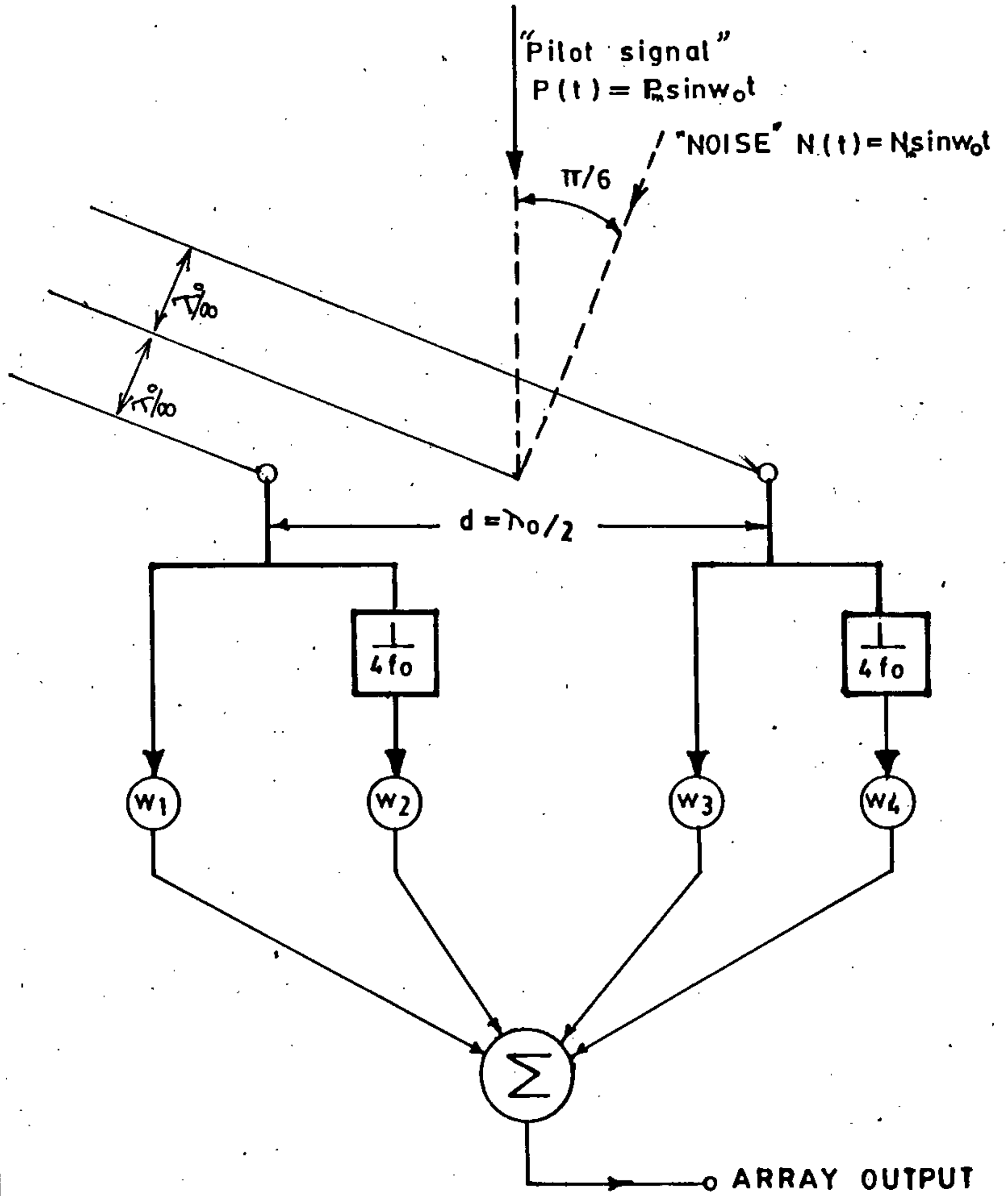

Fig. 2.3.3 ARRAY CONFIGURATION FOR NOISE ELIMINATION EXAMPLE.

The problem of obtaining a set of weights to accept p(t) and reject n(t) can now be studied. It is to be noted that with any set of non-zero weights, the output is of the form $A_{m}\sin(w_{o}t+\phi)$, and a number of solutions exist which will make the output be p(t). However, the output of the array must be independent of the amplitude and phase of the noise signal if the array is to be regarded as rejecting the noise. Satisfaction of this constraint leads to a unique set of weights determined as follows.

The array output due to pilot signal is :

$$P_{m}[(W_{1}+W_{3})\sin w_{o}t + (W_{2}+W_{4})\sin(w_{o}t-\pi/2)t] \quad \text{.....(2.3.1)}$$

For this output to be equal to the desired output of $p(t)=P_{m}\sin w_{o}t$ (which is the pilot signal itself), it is necessary that:

$$W_{1} + W_{3} = 1 \quad \text{..................................(2.3.2)}$$

$$W_{2} + W_{4} = 0$$

With respect to the mid-point between the antenna elements, the relative time delays of the noise at the two antenna elements are:

$$\pm[(1/4f_o)]\sin \pi/6 = \pm(1/8f_o) = \pm\lambda_o/(8c) \quad \text{....(2.3.3)}$$

which corresponds to a phase shift of $\pm\pi/4$ at frequency f_o. The array output due to the incident noise at $\theta=\pi/6$ is then:

$$N_m[W_1 \sin(w_o t - \pi/4) + W_2 \sin(w_o t - 3\pi/4)$$
$$+ W_3 \sin(w_o t + \pi/4) + W_4 \sin(w_o t - \pi/4)] \quad \text{......(2.3.4)}$$

For this response to equal zero, it is necessary that

$$W_1 + W_4 = 0 \quad \text{............................(2.3.5)}$$
$$W_2 - W_3 = 0$$

Thus the set of weights that satisfies the signal and noise response requirements can be found by solving (2.3.2) and (2.3.5) simultaneously. The solution is :

$W_1 = 1/2, \quad W_2 = 1/2, \quad W_3 = 1/2, \quad W_4 = -1/2$

With these weights, the array will have the distinctive property of accepting a signal from a desired direction. Thus a noise, which may even be of the same frequency f_0 as the signal, will be rejected. This rejection is possible as long as the noise comes from a different direction than does the signal.

The directivity pattern, i.e., the relative sensitivity of response to signals from various directions, is plotted in Fig.s 2.3.4, 2.3.5, 2.3.6, and 2.3.7. Each figure is plotted in a plane over an angular range of $-\pi/2 < \theta < \pi/2$ for frequency f_0.

Due to introduction of time delay, the directivity pattern of Fig.2.3.5 has its main lobe at an angle of ψ radians, where

$$\Psi = \sin^{-1} \{\lambda_0 \delta f_0 / d\} = \sin^{-1} \{c \delta / d\}$$

and f_0 = frequency of received signal

λ_0 = wave-length at frequency f_0

δ = time delay difference between neighboring element outputs

d = spacing between antenna elements

c = signal propagating velocity = $\lambda_0 f_0$

The directivity pattern of Fig.2.3.7, with its associated weight sets, has its main lobe almost unchanged from that shown in Fig.s 2.3.4 and 2.3.6. However, the particular sidelobe that previously intercepted a sinusoidal noise in Fig. 2.3.6 has been shifted so that a null is now placed in the direction of that noise.

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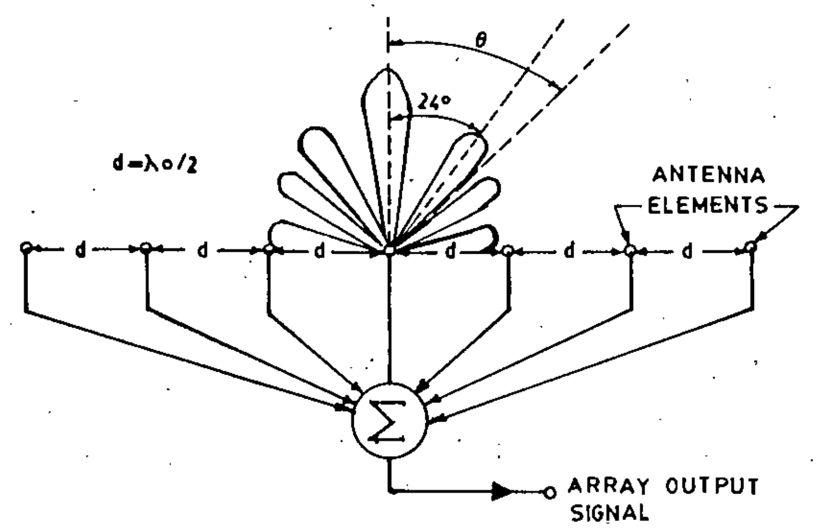


Fig. 2.3.4 : DIRECTIVITY PATTERN FOR A SIMPLE LINEAR ARRAY.

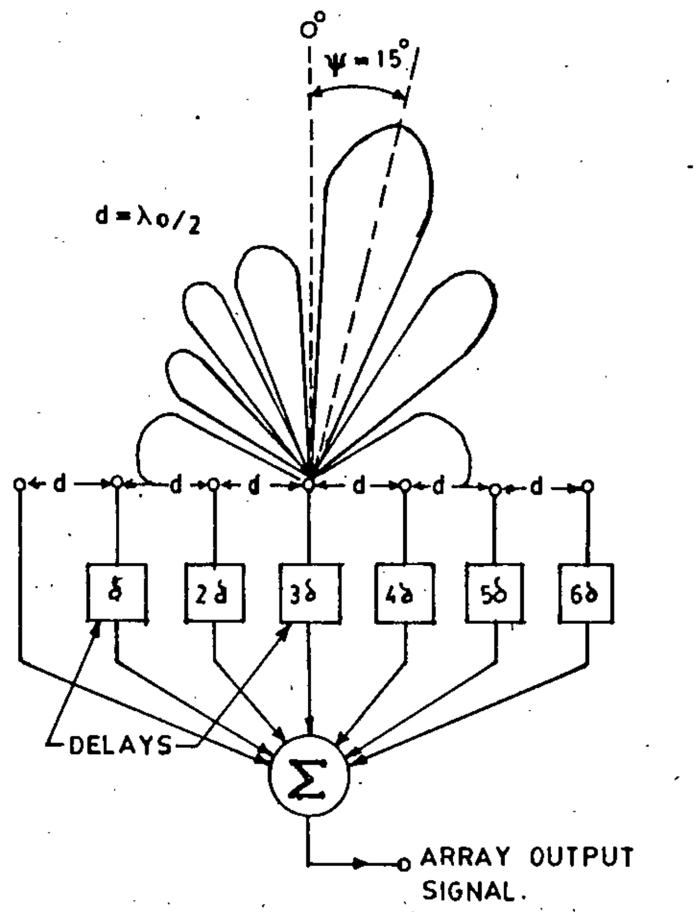


Fig. 2.3.5 : DIRECTIVITY PATTERN FOR A LINEAR ARRAY WITH DELAYS ADDED.

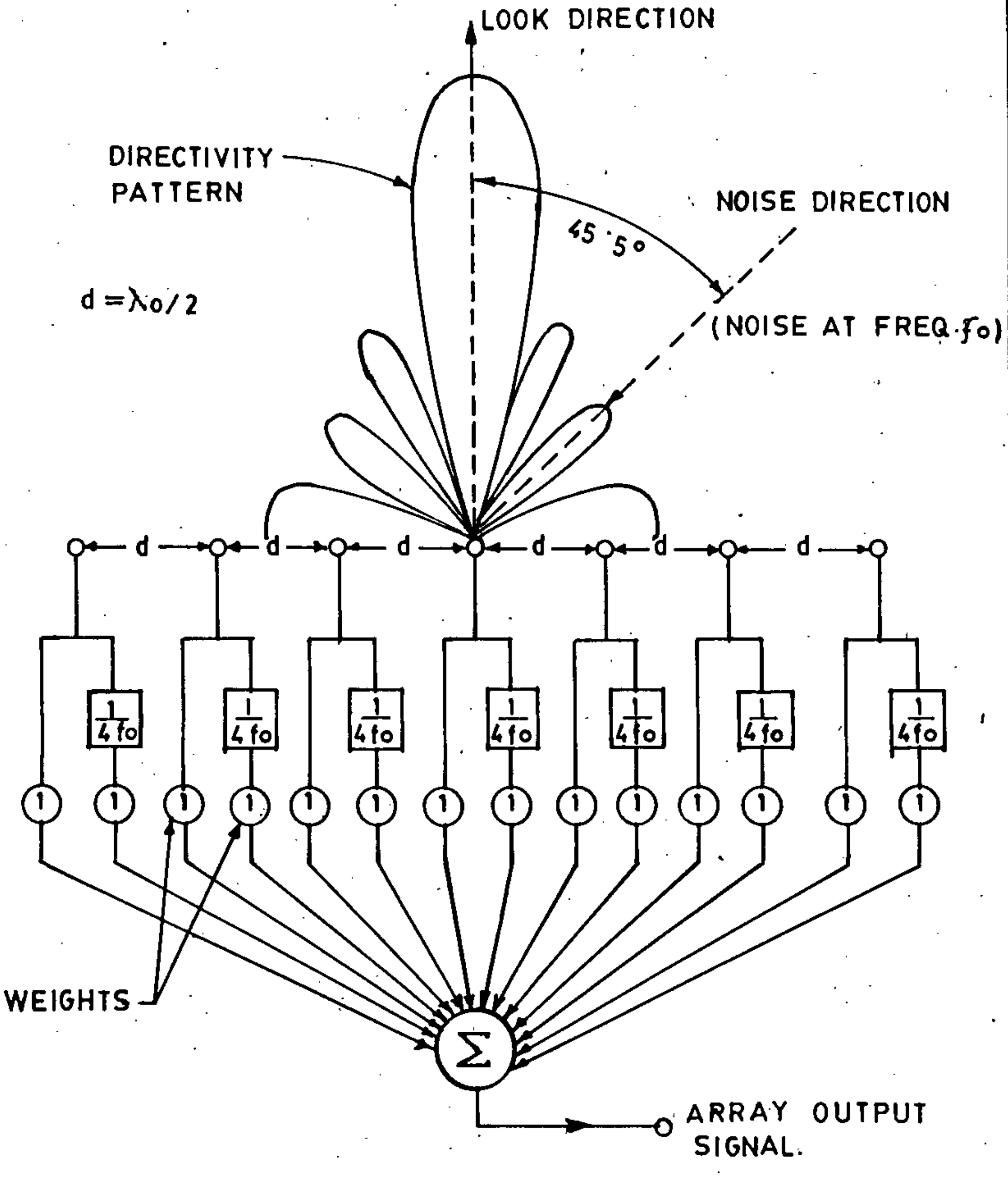


Fig. 2.3.6 : DIRECTIVITY PATTERN FOR LINEAR ARRAY WITH EQUAL WEIGHTING.

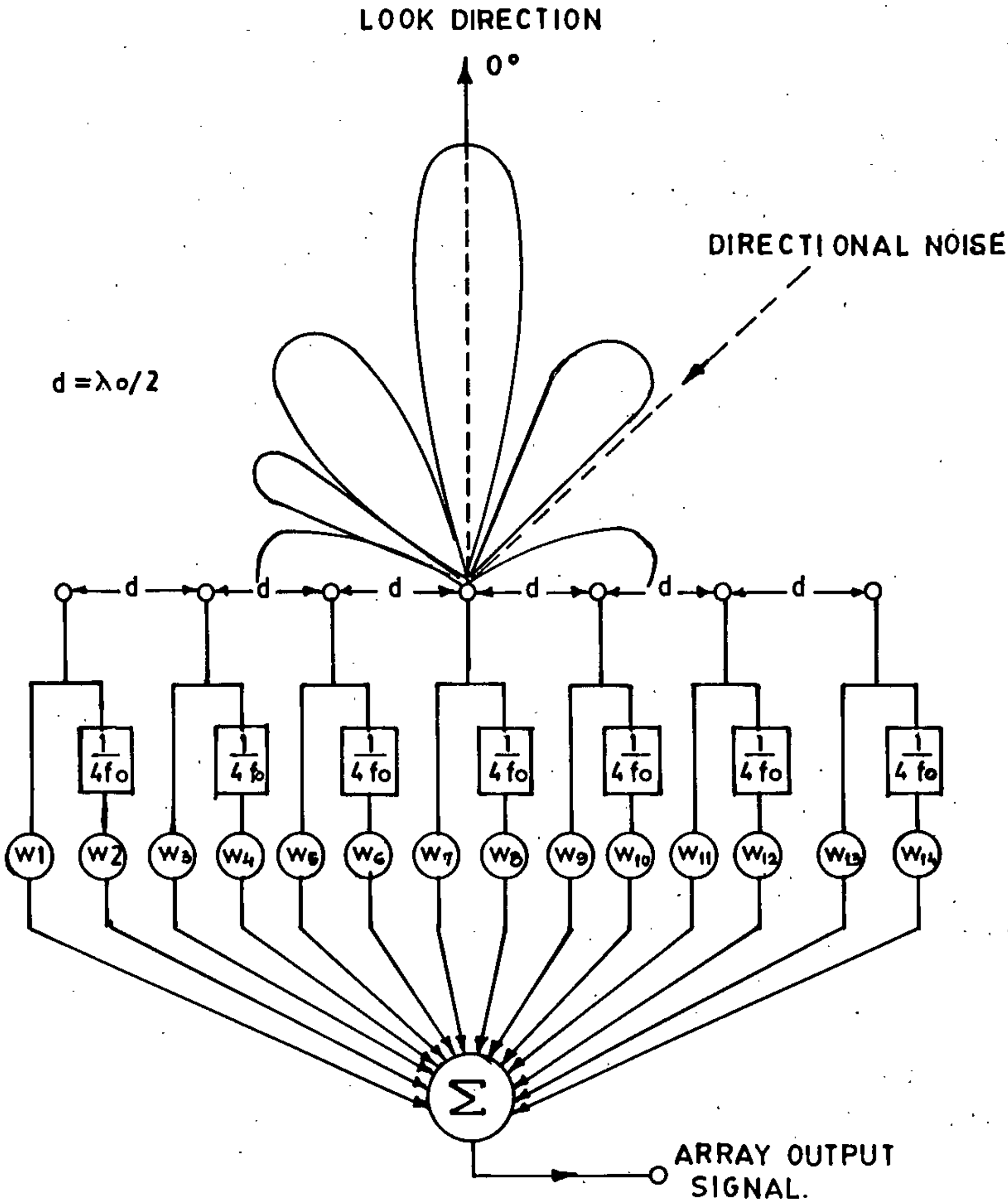


Fig. 2.3.7 : DIRECTIVITY PATTERN OF LINEAR ARRAY WITH WEIGHTING FOR NOISE ELIMINATION.

An adaptive array system can be either of the feedback (i.e.,closed loop) type or of the open loop type [10]. In the feedback system the array output (possibly as well as the signal from the array elements) is required for the processor. Whereas in the open loop system this is not required. In the later case (open loop system), the required weights must be calculated precisely from the input waveforms and then accurately applied, and there is no feedback to confirm that the correct performance has been achieved. The objection to the feedback systems is their limited speed of response. However the advantage is that the weight multipliers need not be accurate, or even linear, because the feedback nature of the system will set whatever values are needed to obtain the required output. The main advantage of the open loop system over the feedback system is, their fast response, but, of course there is a trade-off between speed of response and degree of freedom. Because of the high accuracy required, an open loop system must really be implemented digitally, whereas a closed loop system may be either digital or analogue.

2.4 WANTED SIGNAL AND CONTROL CRITERIA

Adaptive arrays can be configured to receive narrowband or broadband signal. A narrowband signal may be characterized by its centre frequency, say f_o. Such an array configuration is shown in Fig. 2.4.1.

The narrowband signal is received by the antenna element and is thus weighted by a complex gain factor $Ae^{j\theta}$. Any phase angle $\theta = -\tan^{-1}(W_2/W_1)$ can be chosen by setting the two weight values, and the magnitude of this complex gain factor $A=\sqrt{(W_1)^2 + (W_2)^2}$ can take on a wide range of values limited only by the range limitations of the two individual weights. Thus this two tap arrangement in each channel of the array configuration provide complete adjustable linear processing for narrowband signals received by each antenna element.

When one is interested in receiving signals over a wide range of frequencies (i.e., broadband signal), each of the phase shifters in Fig. 2.4.1 can be replaced by a tapped-delay-line network as shown in Fig.2.4.2 . This tapped delay line permits adjustment of gain and phase as desired at a number of frequencies over the band of interest.

In our analysis, we will consider a narrowband signal that can be expressed by its centre frequency and slowly varying envelope.

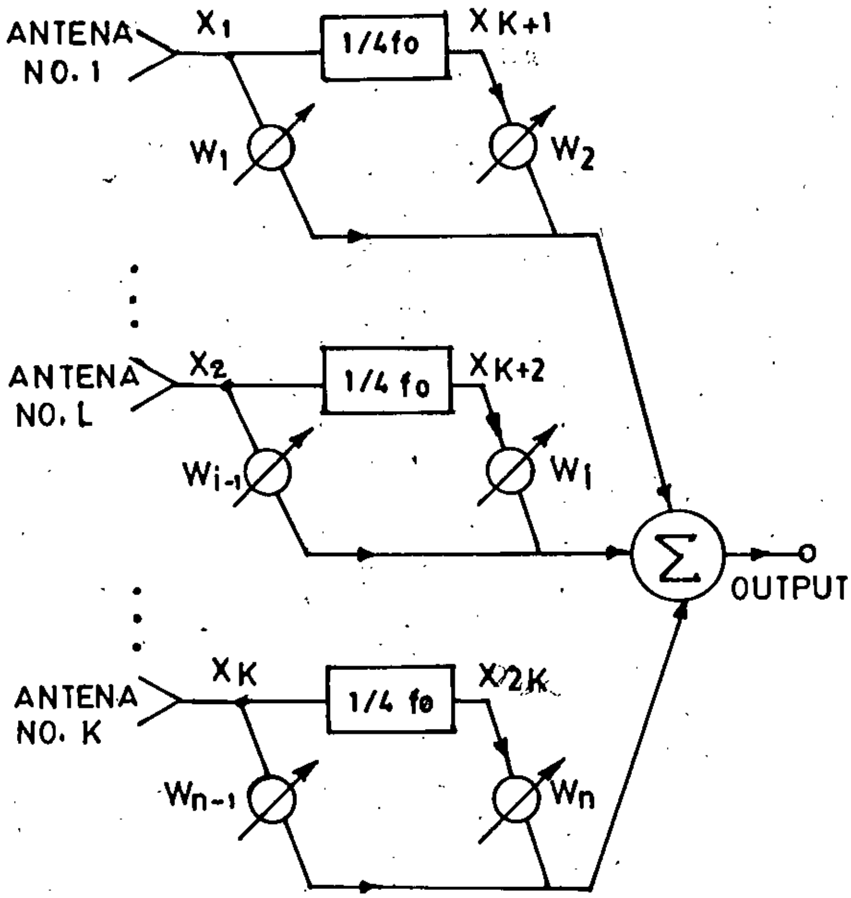


Fig.2.4.1 : ADAPTIVE ARRAY CONFIGURATION FOR RECEIVING NARROWBAND SIGNAL.

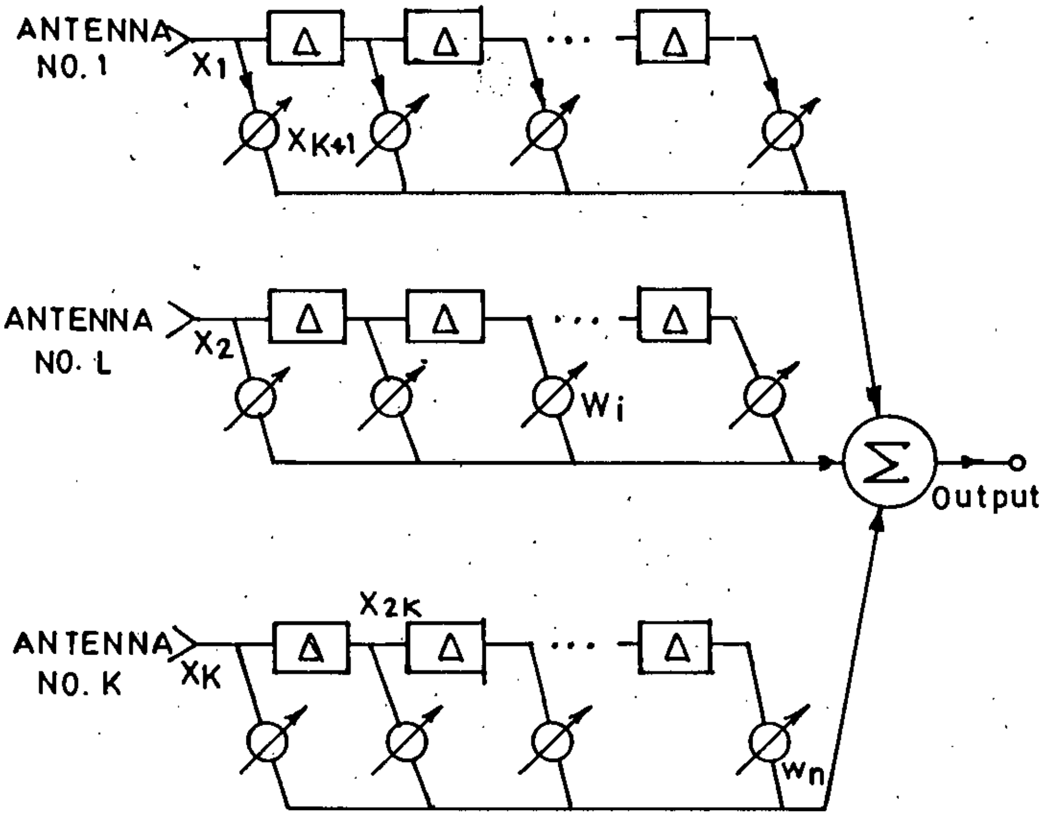


Fig. 2.4.2 : ADAPTIVE ARRAY CONFIGURATION FOR RECEIVING BROADBAND SIGNALS.

In cases where the wanted signal is of extremely low mean noise power (well below receiver noise level in general), the control system is designed to minimize the total output power from the array [10], subject of course, to a constraint on the weights to prevent them all being set to zero. This criteria is specially suited for radar sidelobe cancellation.

However in the communications case, the mean wanted signal power is generally much greater, and if above criteria is followed will be susceptible to cancellation. So the control criteria is modified to protect this signal. One approach is to use the constraint (or constraints) [discussed later] to specify a fixed gain in the direction of the wanted signal, if known. The output power is then minimized subject to this constraint, which ensures that the wanted signal is still received. This approach, more suitable for open loop systems, requires knowledge of the wanted signal direction and also of the relationship between the weight vector and the array gain pattern.

Another approach, not requiring this information, is to minimize the array output power excluding the wanted signal. This can be achieved by preventing the control system from seeing the wanted signal, so that there is no tendency to cancel it. This method is particularly appropriate for closed-loop systems, as it is only necessary to exclude the wanted signal from the feedback path. This criteria may be broadly termed a maximum-signal-to-noise (MSN) criteria, because, for a nonzero gain to the wanted signal, the unwanted power is minimized.

A closely related approach is the minimum-mean-square-error criteria. Here a reference signal is provided which, ideally, matches the expected wanted signal. The adaptive system then minimizes the power (or mean square value) of the error signal defined as the difference between the reference signal and the array output.

2.5 GENERAL CONTROL METHODS

Some general approaches used to improve the signal to noise ratio (SNR) may be outlined as follows [10]:-

(i) Matrix inversion methods

(ii) Correlation feedback systems

(iii) Perturbation or random search

(iv) Gram-Schmidt

These methods are briefly described below:-

(i) Matrix inversion method: The basic adaptive array problem is to determine and apply the set of weights which optimize the system performance according to the chosen direction. The optimizing condition which most systems attempt to meet gives the set of weights (i.e., the weight vector $\underline{W}$) as the solution of an equation of the form $\underline{R}\ \underline{W} = \underline{b}$, where $\underline{R}$ is a matrix of cross correlations of the signals from the array elements (ideally excluding the wanted signal). $\underline{b}$ is a vector depending on the wanted signal or the array steered direction. Open loop systems, for which matrix inversion method (Fig. 2.5.1) is specially suited calculate the required weight directly from the equation $\underline{W} = \underline{R}^{-1}\ \underline{b}$ using digital techniques. This results in a very swift determination of the desired optimum weight vector.

(ii) Correlation Feedback Systems : In these systems (Fig 2.5.2 and 2.5.3) the optimizing weights are determined by using correlation between the signals from the array elements and a feedback signal derived from the array output. The feedback loops operate to reduce the feedback signal power to a low level, ideally approaching zero as the loop gain is increased. The Howell-Applebaum MSN processor [2] maximizes a generalized signal/noise ratio in the case where the wanted signal power is negligible (as in the radar case). For communications, the wanted signal may have to be reduced (indicated by a generalized filter in

Fig. 2.5.2) in the feedback path to the processor to prevent it being cancelled. The LMS system [6] (Fig.2.5.3) uses such a processor which minimizes the power (i.e., obtains the least mean square value) of the error or difference between the array output and a reference signal.

(iii) <u>Perturbation or Random Search</u> : This is also a feedback system (Fig.2.5.4) but is relatively simpler. This does not require signals from the array-elements and does not perform any correlation. The principle is that the element weights are altered (or perturbed), initially at random, by small steps, and the effect on the output power is observed. Logical decisions are made on the observations, and perturbations are made to maintain the output at a low level. The performance of this method is very modest, mainly in speed of response, which is very much below the LMS system, let alone the DMI system.

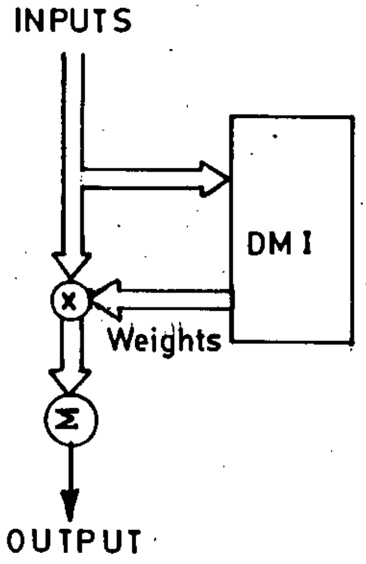


Fig. 2.5.1 : MATRIX INVERSION.

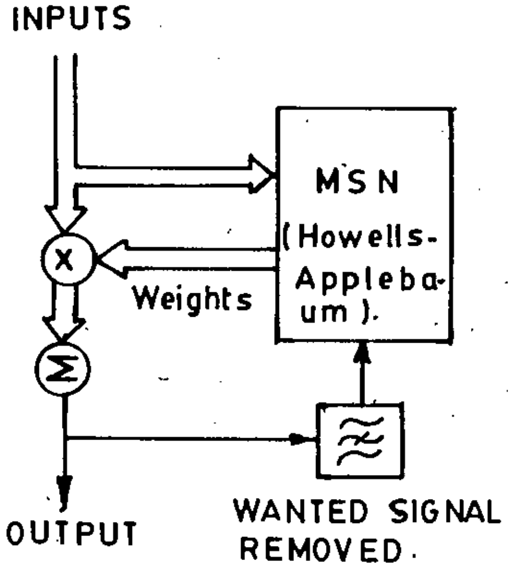


Fig. 2.5.2 : MSN PROCESSOR.

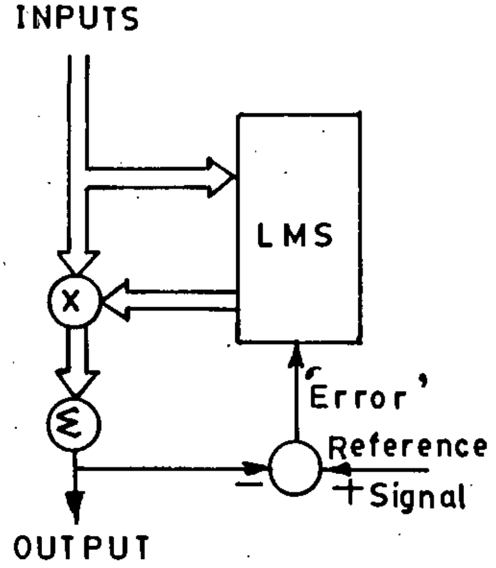


Fig. 2.5.3 : LMS PROCESSOR.

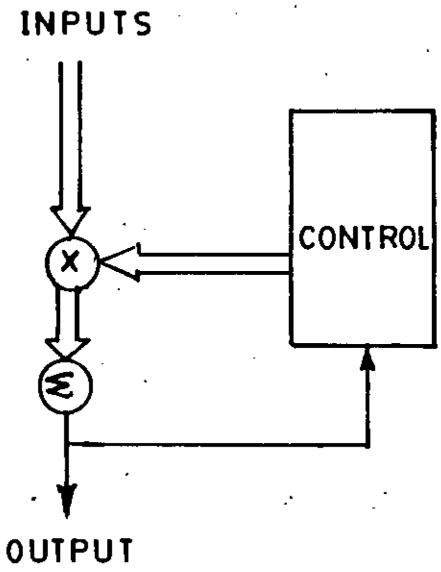


Fig. 2.5.4 : PERTURBATION METHOD.

(iv) <u>Gram-Schmidt</u> : This is a sequential-decorrelation process where separation of signals is achieved by certain cascade processor circuits (Fig. 2.5.5), which may then be followed by an MSN or LMS processor. It consists of a cascade of elementary correlation-loop cancellers, each using two inputs with one adaptive weight. There are N-1 cancellers in the first row of the cascade. (where N is the number of antenna elements), N-2 in the next, and so on. This system is flexible in that the individual cancelling units may be either closed or open loop and realized either in the digital or analogue circuitry.

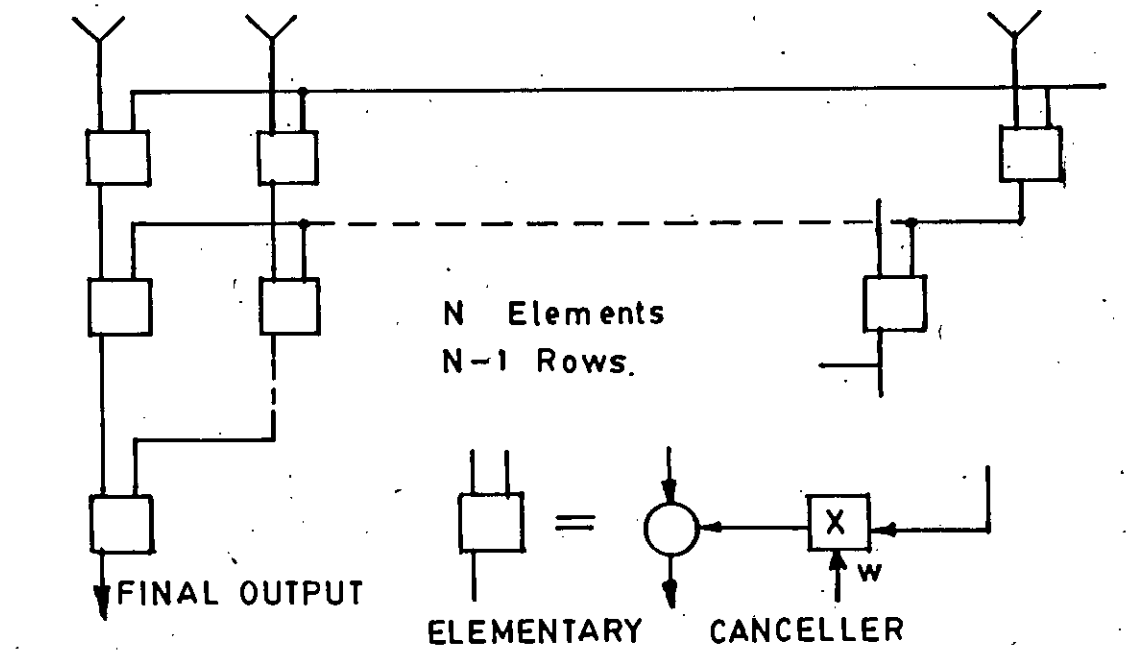


Fig. 2.5.5: GRAM-SCHMIDT OR SEQUENTIAL DECORRELATION.

In our problem formulation and analysis, we use the general idea of the correlation feedback system control method. Before going into detail of our approach later, we discuss briefly in the next chapter some concepts about constrained optimization techniques relevant to our problem.

CHAPTER 3

CONSTRAINED OPTIMIZATION TECHNIQUES

3.1 INTRODUCTION

Antenna array processing with minimization of output power with certain constraints make use of constrained optimization technique. In this chapter we describe in a general framework, the optimization problem of convex cost functional over a convex set which is defined by linear constraints. We explain in general terms the concepts of equality and inequality constraints, their general solution approach, the idea behind the Lagrange multipliers and the principle of the iterative methods used to solve these optimization problems.

3.2 GENERAL APPROACH TO THE DEVELOPMENT OF THE SOLUTION TO THE OPTIMIZATION PROBLEM

The optimization problem, interpreted from a general point of view, is to locate from within a given subset of a vector space that particular vector which minimizes a given functional [11]. The particular vector may be termed as the optimized quantity and the minimized functional is the cost functional. In constrained optimization problems, the subset of admissible vectors competing for the optimum is defined implicitly by a set of constraint relations. These implicit constraints may again be defined by linear equality or inequality conditions.

A typical optimization problem with equality constraint may be defined as follows :

$$\text{minimize } f(x) \qquad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(3.2.1)$$

$$\text{subject to } H(x) = \theta$$

where f is a real valued functional on a Banach space X and H is a mapping from X into a Banach space Z.

Problem (3.2.1) can be visualized geometrically in the space X in terms of the tangent space of the constraint surface. This is shown in Fig. 3.2.1, where the contours of the cost functional f, as well as the constraint surface for a single functional constraint, h(x) = 0, are drawn.

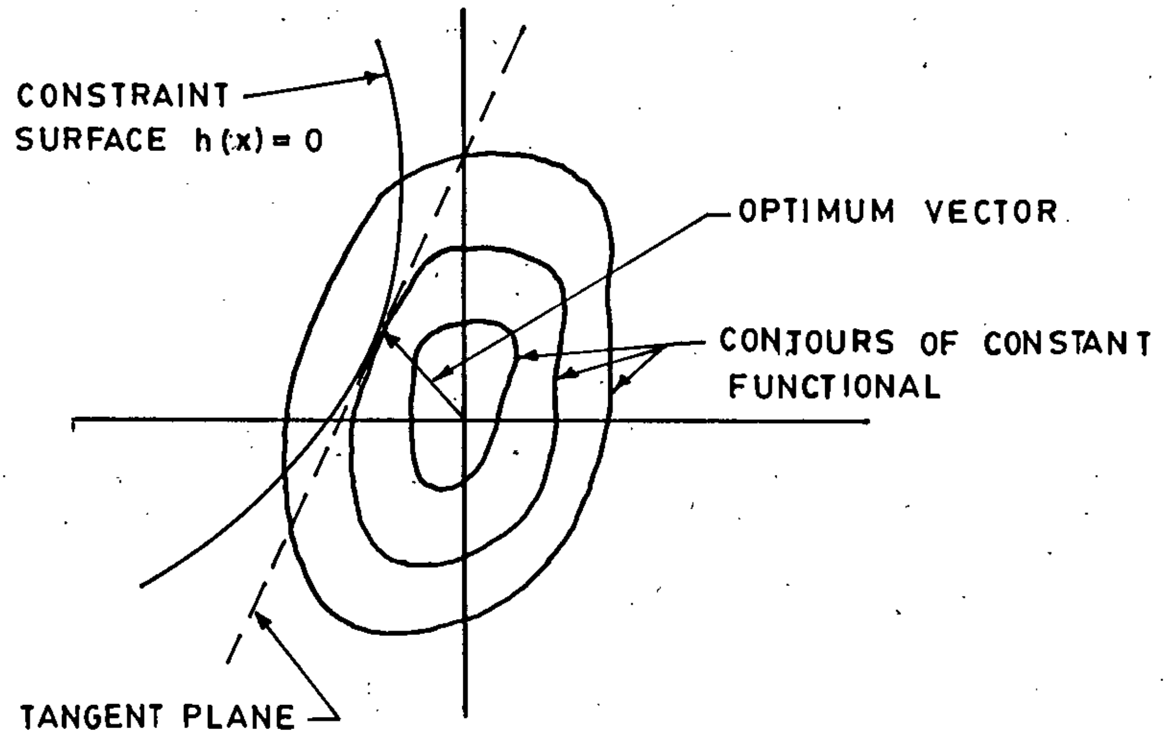


Fig. 3.2.1 : CONSTRAINED OPTIMIZATION.

As an example of constrained optimization problem with inequality constraints, let us consider a basic problem, which can be referred to as the primal problem, that requires to :

$$\text{minimize} \quad f(x) \qquad \text{(3.2.2)}$$

$$\text{subject to} \quad G(x) \leqslant \theta \quad , \quad x \in \Omega$$

where Ω is a convex subset of a vector space X, f is a real valued functional on Ω , and G is a convex mapping from Ω into a normal space Z having positive cone P.

In general, these constrained optimization problems are quite difficult to solve if some additional parameters are not used. And these additional parameters are nothing other than the Lagrange multipliers which are found to somehow almost always unscramble a difficult constrained problem. By interpreting the Lagrange multiplier as a hyper plane (discussed in section 3.3 next), it is natural to expect that its use will be elegant and more effective with problem involving convex functionals.

3.3 INTRODUCTION OF LAGRANGE MULTIPLIERS IN CONSTRAINED OPTIMIZATION PROBLEM

The introduction of Lagrange multiplier in the optimization problem can be understood from the following discussion [11].

We consider the minimization problem (3.2.2) which can be analyzed by essentially embedding it in the following general type of problem :

minimize $f(x)$

subject to $G(x) \leq z, \quad x \in \Omega$

where z is an arbitrary vector in Z.

The solution of this type of problem depends on Z.

Let us now define a set $\Gamma \subset Z$ as

$$\Gamma = \{z : \text{there is an } x \in \Omega \text{ with } G(x) \leq z\}$$

It can be shown that the set will be convex.

On the set Γ, we define the primal function w (which may be finite), as

$$w(z) = \inf \{ f(x) : x \in \Omega , G(x) \leq z \}$$

The original problem (3.2.2) can be regarded as determining the angle value $w(\Theta)$. A typical w for z one dimensional is shown in Fig.3.3.1.

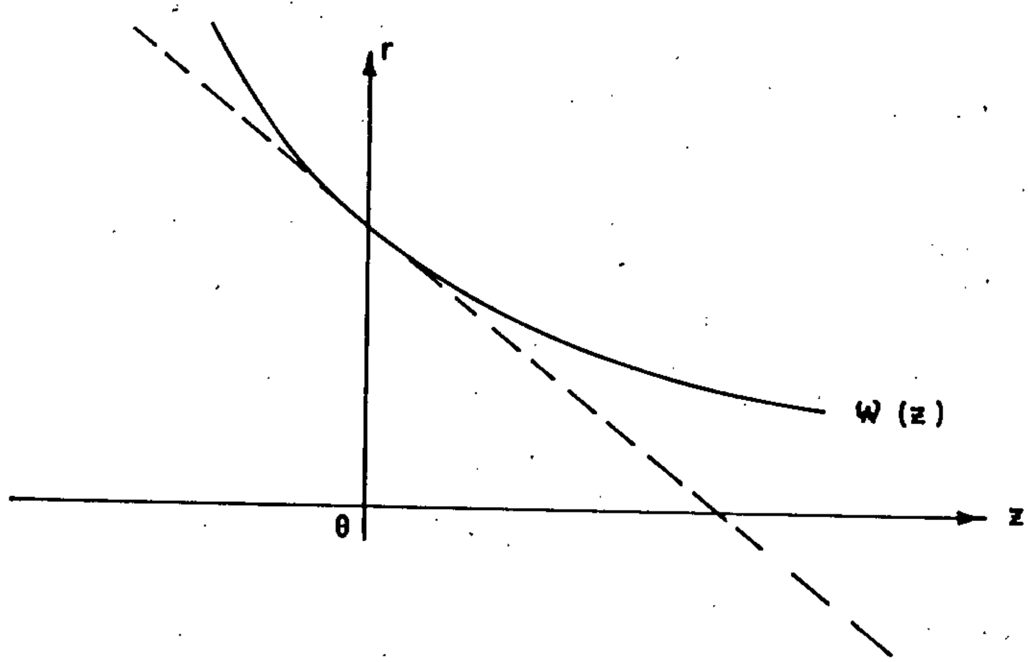


Fig. 3.3.1 : THE PRIMAL FUNCTION.

Conceptually the Lagrange multiplier theorem follows from the simple observation that, since w is convex, there is a hyper plane tangent to w at $z = \theta$ and lying below w through out its region of definition. If one were to tilt his head so that the tangent hyper plane became the new horizontal, it would appear that w was minimized at θ, or, said another way, by adding an appropriate linear functional $\langle z, z_0^* \rangle$ to $w(z)$, the resulting combination $w(z)+\langle z, z_0^* \rangle$ is minimized at $z=\theta$. The functional z_0^* is the Lagrange multiplier for the problem.

To see the use of Lagrange multiplier technique in the optimization problem defined by (3.2.2) in more detail, let us consider a problem [11] in two dimensions having three scalar equations $g_i(x) \leq 0$ as constraints. Fig.3.3.2 (a) shows the constraint region. In Fig.3.3.2(b) where it is assumed that the minimum occurs at a point x_0 in the interior of the region, it is apparent that $f'(x_0)=0$. In Fig.3.3.2(a), where it is assumed that the minimum occurs on the boundary $g_1(x)=0$, it is clear that $f'(x_0)$ must be orthogonal to the boundary and point inside. Therefore, in this case, $f'(x_0)+\lambda_1 g_1'(x_0)=\theta$ for some $\lambda_1 \geqslant 0$. Similarly, in Fig.3.3.2(b), where it is assumed that the minimizing point x_0 satisfies both $g_1(x_0)=0$ and $g_2(x_0)=0$, we must have

$$f'(x_0) + \lambda_1 g_1'(x_0) + \lambda_2 g_2'(x_0) = \theta \quad \text{.............(3.3.1)}$$

with $\lambda_1 \geqslant 0$, $\lambda_2 \geqslant 0$

All of these cases can be summarized by the general statement

$$f'(x_0) + \lambda^* G'(x_0) = 0 \quad \text{.........................(3.3.2)}$$

where $\lambda^* \geqslant \theta$ and $\lambda_i g_i(x_0) = 0$, $i=1,2,3$.

The equality $\lambda_i g_i(x_0)=0$ merely says that if $g_i(x_0)<0$, then the corresponding Lagrange multiplier is absent from the necessary condition.

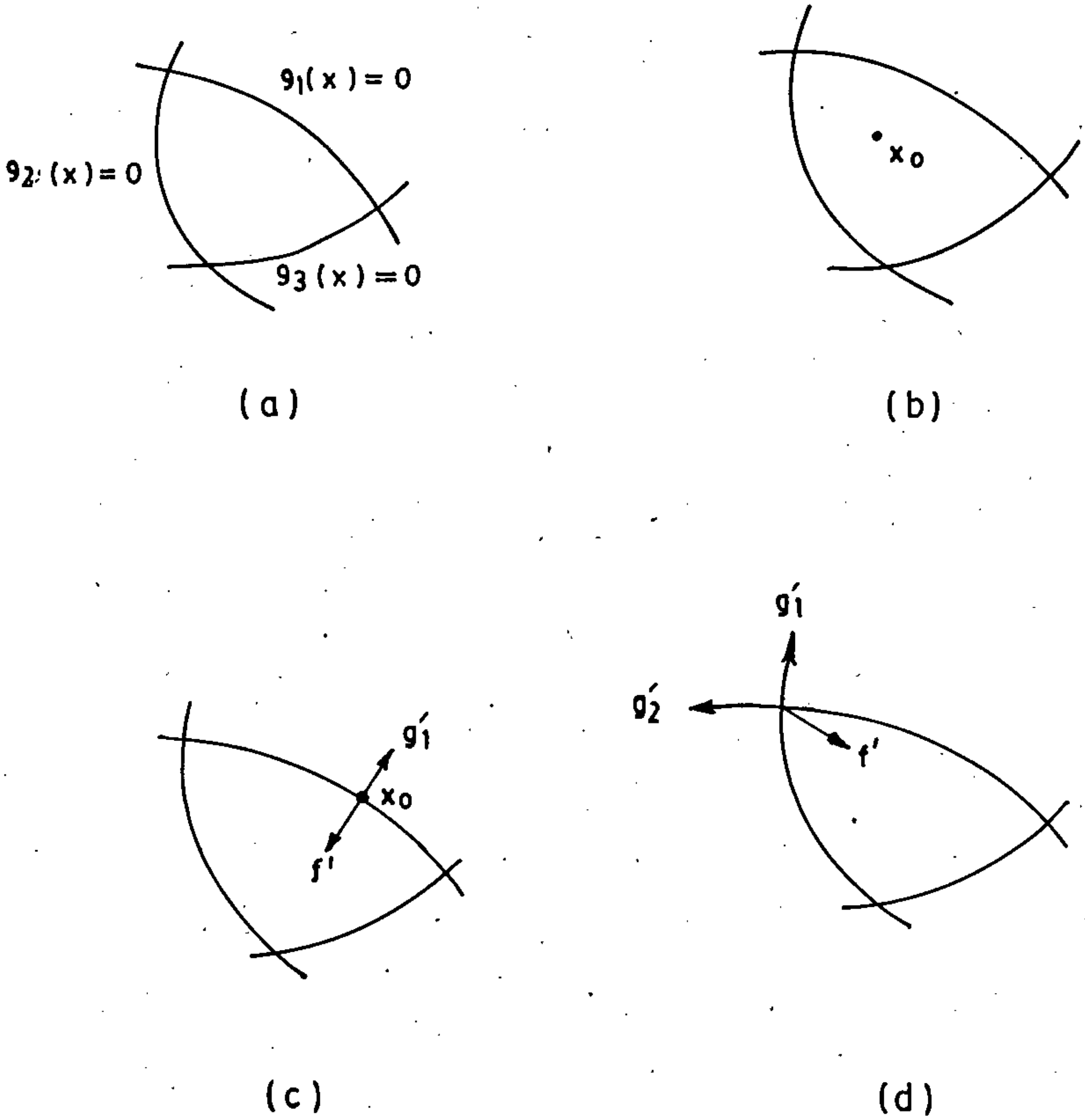


Fig. 3.3.2 : INEQUALITY CONSTRAINTS.

Equation (3.3.2) is in fact a modified version of the Generalized Kuhn-Tucker Theorem which is given below:

Theorem : Let X be a vector space and Z a normed space having positive cone P which contains an interior point. Let f be a Gauteaux differentiable real-valued functional on X and G a Gauteaux differentiable mapping from X into Z. Let it be assumed that the Gauteaux differentials are linear in their increments. Let it further be supposed that x_0 minimizes f subject to $G(x) \leqslant \theta$ and that x_0 is a regular point of the inequality $G(X) \leqslant \theta$. Then there is a $z_0^* \in Z$, $z_0^* \geqslant \theta$ such that the Lagrangian

$$f(x) + \langle G(x), z_0^* \rangle$$

is stationary at x_0; furthermore,

$$\langle G(x_0); z_0^* \rangle = 0$$

The proof of this theorem is provided in [11].

3.4 THE STEEPEST DESCENT METHOD EMPLOYED TO ARRIVE AT THE OPTIMUM VALUE

A direct approach for solving non-linear equations in optimization problems is to iterate in such a way as to decrease the cost functional continuously from one step to the next. In this way global convergence, convergence from an arbitrary starting point, may be achieved. As a general frame work for the method, let us assume that we seek to minimize a functional f and that an initial point x_1 is given [11]. The iterations are constructed according to an equation of the form :

$$x_{n+1} = x_n + \alpha_n p_n \qquad (3.4.1)$$

where α_n is a scalar and p_n is a (direction) vector. The procedure for selecting the vector p_n varies from technique to technique but, ideally, once it is chosen the scalar α_n is selected to minimize $f(x_n + \alpha p_n)$, regarded as a function of the scalar α. Generally, things are arranged (by multiplying p_n by -1 if necessary) so that $f(xn + \alpha p_n) < f(x_n)$ for small positive α. The scalar α_n is then often taken as the smallest positive root of the equation

$$\frac{d}{d\alpha} f(x_n + \alpha p_n) = 0$$

In practice, of course, it is rarely possible to evaluate the minimizing α exactly. Instead some iterative search or approximation is required. The essential point, however, is that after an α_n is selected, $f(x_n + \alpha_n p_n)$ is evaluated to verify that the objective has in fact decreased from $f(x_n)$. If f has not decreased, a new value of α_n is chosen. The Primal dual method (discussed in section 3.6) was selected as the above mentioned iterative search method.

The descent process can be visualized in the space X where the functional f is represented by its contours. This f is in fact the cost function we mentioned in section 3.2. Starting from a point x_1, one moves along the direction vector P_1, until reaching, as illustrated in Fig. 3.4.1, the first point where the line $x_1 + \alpha P_1$ is tangent to a contour f. Alternatively, the method can be visualized, as illustrated in Fig.3.4.2, in the space

R x X, the space containing the graph of f [here R is the real line consisting of points of the form (r;θ)].

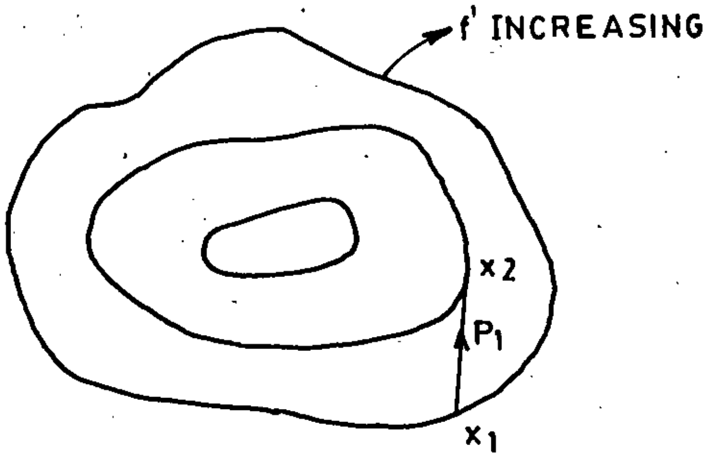


Fig. 3.4.1 : THE DESCENT PROCESS IN X

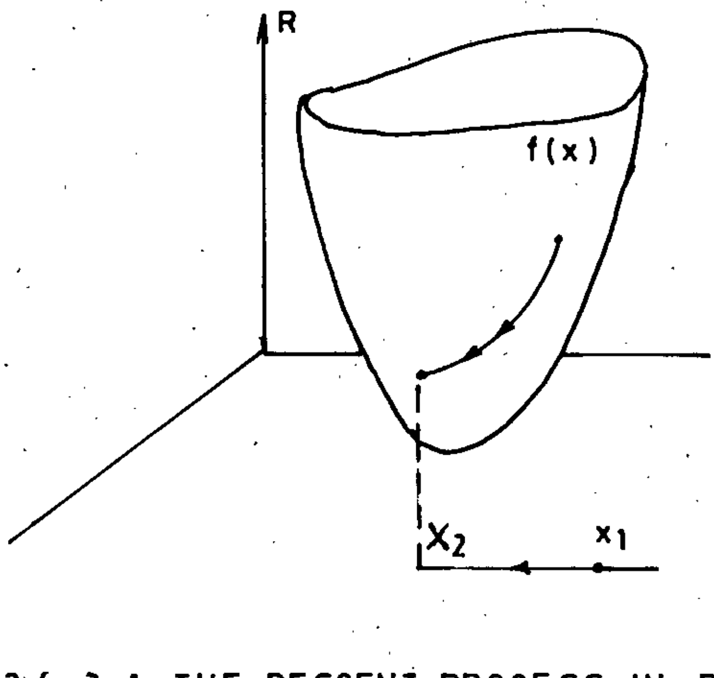


Fig. 3.4.2 : THE DESCENT PROCESS IN Rx X.

If f is bounded below, it is clear that the descent process defines a bounded decreasing sequence of functional values and hence the objective values tend toward a limit f_0.

The Primal-Dual method is employed for solving the algorithm (dealt in chapter 4). This iterative method is again based on the Duality theorem which is described below.

3.5 DUALITY

There are several duality principles in optimization theory that relate a problem expressed in terms of vectors in a space to a problem expressed in terms of hyperplanes in the space. Many of these duality principles are based on the geometric relation illustrated in Fig. 3.5.1. The essence of duality can be understood from following statement. The shortest distance from a point to a convex set is equal to the maximum of the distances from the point to a hyperplane separating the point from the convex set. Thus the original minimization over vectors can be converted to maximization over hyperplanes.

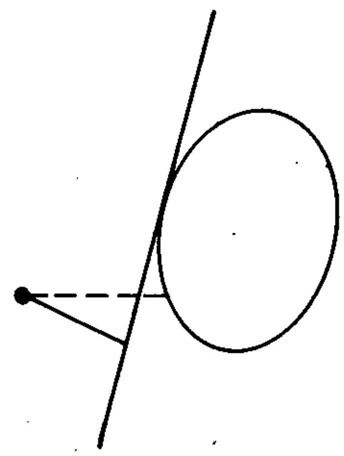


Fig. 3.5.1 : DUALITY

3.6 PRIMAL-DUAL METHOD EXPLAINED FROM A GENERAL POINT OF VIEW

Let us consider a dual method for the convex problem (referred to as the primal problem) given by [11]:

$$\text{minimize } f(x) \quad \ldots\ldots\ldots\ldots\ldots\ldots(3.6.1)$$

$$\text{subject to } G(x) \leqslant \theta \ , \ x \in \Omega$$

Assuming that the constraint is regular, this problem is equivalent to :

$$\max_{z^* \geqslant \theta} \ \inf_{x \in \Omega} \{ f(x) + \langle G(x), z^* \rangle \} \ldots\ldots\ldots\ldots(3.6.2)$$

or, defining the dual functional

$$\phi(z^*) = \inf_{x \in \Omega} \{ f(x) + \langle G(x), z^* \rangle \} \ldots\ldots\ldots(3.6.3)$$

the problem is equivalent to the dual problem :

$$\text{maximize } \phi(z^*) \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(3.6.4)$$

$$\text{s.t. } z^* \geqslant \theta$$

The dual problem (3.6.4) has only the constraint $z^* \geqslant \theta$, hence, assuming that the gradient of ϕ is available, the dual problem can be solved in a rather routine fashion. Once the dual problem is solved yielding an optimal z_0^*, the primal problem (3.6.1) can be solved by minimizing the corresponding Lagrangian.

CHAPTER 4

PROBLEM FORMULATION FOR DIRECTIONALLY CONSTRAINED ADAPTIVE ARRAYS

4.1 INTRODUCTION

This chapter deals with formulation of the array processing optimization problem to be solved under inequality constraints. A basic linear antenna array is selected and the sequential steps involved to formulate the cost functional for optimization, as well as the constraint relations, are shown. Necessary notations to deal with the problem mathematically are also introduced in the process of formulation.

4.2 FORMULATION OF THE OPTIMIZATION PROBLEM

We consider a typical linear antenna array system. Fig.4.2.1 shows the general configuration of such an array processor where for convenience it is assumed that the K isotropic antenna elements are located linearly and are equally spaced. Each channel is provided with L tap points. x, y, W and Δ denote input signal, output signal, weight and time delay respectively.

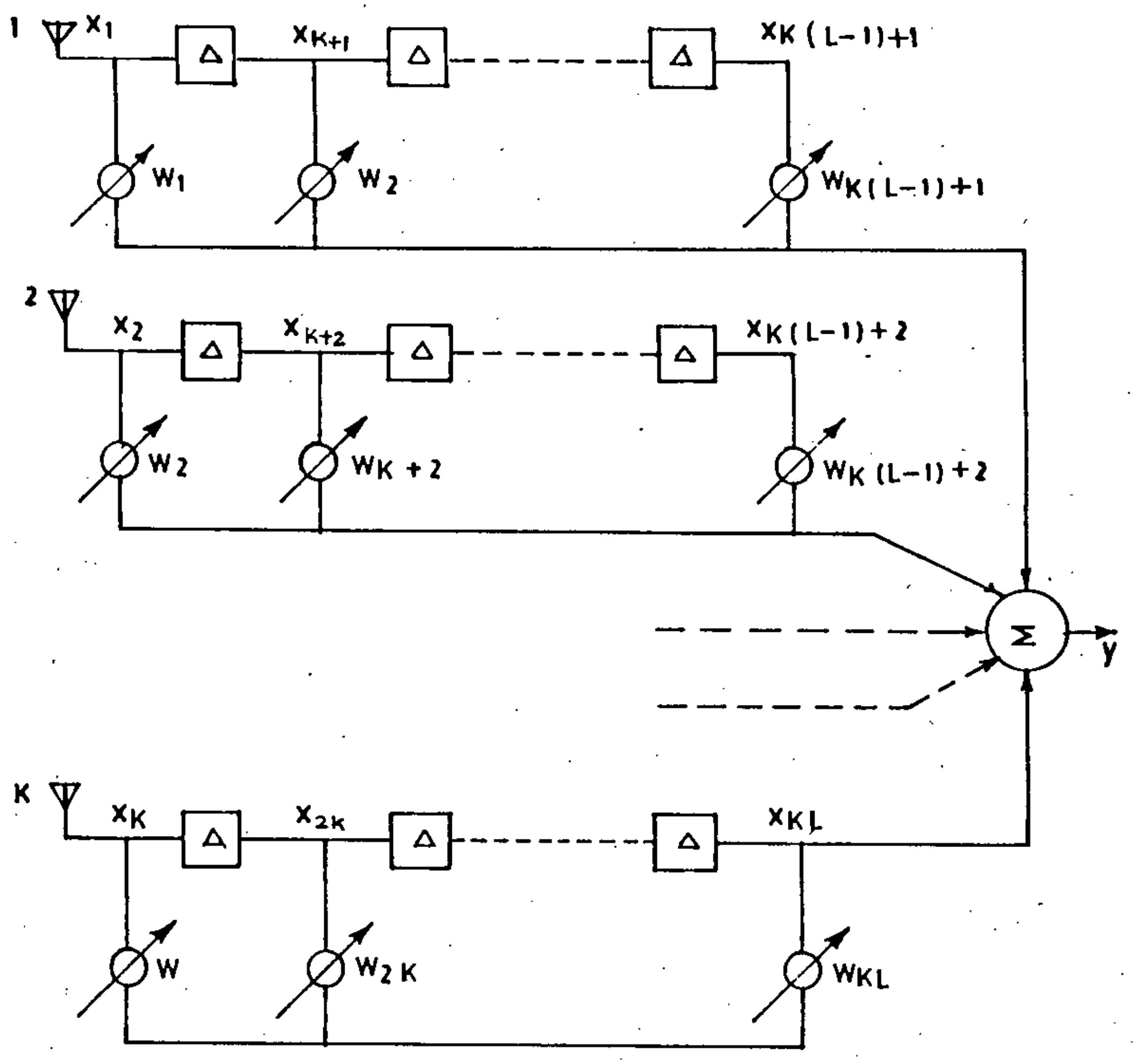


Fig. 4.2.1 : GENERAL CONFIGURATION OF ADAPTIVE ARRAYS.

The weight W_k (k=1,2,...,KL), are usually the gains of the amplifiers attached to each tap point which is controlled by a feed back circuitry, although the later is not shown in this figure. For convenience of following formulation, weights are treated in a vector form as follows :

$$\underline{W}^T = (W_1 , W_2 , \ldots , W_{KL}) \ldots\ldots\ldots\ldots\ldots\ldots (4.2.1)$$

which will be called the weight vector here after. Here the superscript T denotes the transpose. The signals x_k (k=1,2,...,KL) at each tap point to which a weight is connected are also expressed in a vector form as :

$$\underline{X}^T = (x_1 , x_2 , \ldots , x_{KL}) \ldots\ldots\ldots\ldots\ldots\ldots (4.2.2)$$

Thus y, the output of the array system, can be expressed by the inner product of two vectors [8] as follows :

$$y = \underline{X}^T . \underline{W} = \underline{W}^T \underline{X} \ldots\ldots\ldots\ldots\ldots\ldots (4.2.3)$$

The antenna array system is exposed to a signal environment created by wanted signal (desired signal) and a number of noise signals (unwanted or interfering signals). Consequently, the signals at each tap point consist of a desired signal component and the unwanted noise signal component, i.e., s_k and n_k respectively at the kth tap point. Thus

$$x_k = s_k + n_k \ , \quad k = 1,2,\ldots,KL \qquad (4.2.4)$$

In general the undesired noise component consist of two parts; the interference coming from external sources and the internal noise generated in the system.

Again vector symbols are employed as follows :

$$\underline{S}^T = (s_1, s_2, \ldots, s_{KL}) \qquad (4.2.5)$$

$$\underline{N}^T = (n_1, n_2, \ldots, n_{KL}) \qquad (4.2.6)$$

Thus (4.2.4) can be written in a vector form as :

$$\underline{X} = \underline{S} + \underline{N} \qquad (4.2.7)$$

It should be noted that though the signal component x_k may contain, in theory, more than one desired signal but it is rare in practice. On the other hand, the unwanted noise signals are usually more than one in number.

Under the assumption that the desired signal is uncorrelated with the noise signal, we can define the expected cross spectral density matrix or correlation matrix as it is usually called, of $\underline{X}$ as :

$$\underline{R} = E[\,\underline{X}\ \underline{X}^T] \quad \ldots\ldots(4.2.8)$$

$$= E[\,\underline{S}\ \underline{S}^T\,] + E[\,N\ N^T\,] \quad \ldots\ldots(4.2.9)$$

where the operator E[] represent the expectation.

It is to be noted that [12] $\underline{R}$ is a positive semidefinite real symmetric matrix having KL X KL dimension for a K element array with L tap points in each channel. That is,

$$\underline{R}^T = \underline{R}$$

and $$\underline{W}^T\ \underline{R}\ \underline{W} > 0 \quad \forall\ \underline{W} \quad \ldots\ldots(4.2.10)$$

From equation (4.2.8) and (4.2.9), the correlation matrix $\underline{R}$ has the form :

$$R_{xx} = R_{ss} + R_{nn} \quad \ldots\ldots(4.2.11)$$

where :

$$\underline{R}_{ss} = \begin{bmatrix} s_1 s_1 & s_1 s_2 & \cdots & s_1 s_{KL} \\ s_2 s_1 & s_2 s_2 & \cdots & s_2 s_{KL} \\ \cdot & & & \\ \cdot & & & \\ s_{KL} s_1 & s_{KL} s_2 & \cdots & s_{KL} s_{KL} \end{bmatrix} \qquad \ldots\ldots\ldots\ldots\ldots(4.2.12)$$

and

$$\underline{R}_{nn} = \begin{bmatrix} n_1 n_1 & n_1 n_2 & \cdots & n_1 n_{KL} \\ n_2 n_1 & n_2 n_2 & \cdots & n_2 n_{KL} \\ \cdot & & & \\ \cdot & & & \\ n_{KL} n_1 & n_{KL} n_2 & \cdots & n_{KL} n_{KL} \end{bmatrix} \qquad \ldots\ldots\ldots\ldots\ldots(4.2.13)$$

Finally by adding equations (4.2.13) and (4.2.14) we have the correlation matrix as :

$$\underline{R}_{xx} = \begin{bmatrix} x_1 x_1 & x_1 x_2 & \cdots & x_1 x_{KL} \\ x_2 x_1 & x_2 x_2 & \cdots & x_2 x_{KL} \\ \cdot & & & \\ \cdot & \cdot & & \\ & & & \\ x_{KL} x_1 & x_{KL} x_2 & \cdots & x_{KL} x_{KL} \end{bmatrix} \quad \ldots\ldots\ldots\ldots\ldots\ldots (4.2.15)$$

where x_1, x_2, ... , x_{KL} are the tap point signal values consisting of desired signal values s_1, s_2, ... , s_{KL} and interference signal values n_1, n_2, ... , n_{KL}

Finally, from (4.2.3) the array output power may be given by :

$$y^2 = (\underline{X}^T \underline{W})^2$$

$$= \underline{W}^T \underline{X} \underline{X}^T \underline{W} \quad \ldots\ldots\ldots\ldots\ldots\ldots (4.2.16)$$

Thus the expected output power of the array can be expressed as :

$$E [y^2] = \underline{W}^T \underline{R} \underline{W} \quad \ldots\ldots\ldots\ldots\ldots\ldots (4.2.17)$$

In array processing problem, the most straight forward quantity to be optimized for maximizing the desired signal reception and minimizing the noise signal turns out to be the array output power [7]. Thus, equation (4.2.17) gives the expected cost functional of our optimization problem.

4.3 ARRAY PROCESSING OPTIMIZATION WITH DIRECTIONAL CONSTRAINTS

Any one of the many parameters related with antenna array processor may be chosen as the quantity to be optimized. For example, instead of considering the output power, we could have taken signal to noise ratio as the optimizing quantity. Now, if by optimizing the output power we mean to minimize it, then without the presence of any other guiding parameter, we would lose the desired signal along with the noise, because, the output power would then be reduced to zero. Thus to receive our desired signal with satisfactory power level, and at the same time reject all noise that may be present, a good approach would be to optimize the pattern with such a constraint which will enable the array to maintain a pre-assigned response in the desired signal direction. Such an optimization technique would then minimize noise power and place nulls in the response pattern along the noise directions. Since by utilizing the a-priori knowledge of the signal direction we are assigning a response pattern in this direction, we are in fact making use of a "directional constraint" in dealing with our optimization problem. The directional constraint we just talked about aims to guarantee the response of the array system to the desired signal input that is coming from a specified direction at a certain frequency; i.e., by virtue of this constraint, the main lobe of the antenna response pattern is placed along the desired signal direction. So it may be termed as the "Main lobe constraint". However, in the process of generating the main lobe, a number of side lobes are also created in the response pattern of the array.The optimization technique, with its minimum output power criterion, however, adjusts the side lobes in such a manner so that their peaks do not coincide with any noise direction. This is however, strictly true, as long as the system can successfully utilize the available "degrees of freedom" (this term is elaborately discussed in the next chapter) in nullifying all noise. Now, if the side lobe levels are of significant magnitude, then the system will be vulnerable to any "unaccounted for noise" which may coincide with the peak of such a side lobe. Thus the array system could achieve greater flexibility and utility if its sidelobe levels could also be controlled; and fortunately this is possible by assigning "sidelobe constraints". The mainlobe constraint must however remain to guarantee reception of our desired signal. Thus we have a number of directional constraints which we may use to shape the response of the array to our advantage. The aim of our present work is to minimize the cost functional, which is the output power, by utilizing such directional constraints which account for the mainlobe as well as the sidelobes.

Following [8] we can write the array output resulting from an incoming narrowband signal of wave length λ (frequency f), direction θ, and an array element spacing of $\lambda/2$, as

$$Y_\theta = \sum_{k=1}^{K} \{ W_k \exp[j\Psi_k(\theta)] + W_{k+K} \exp[j(\Psi_k(\theta) - 2\pi f \Delta)] + W_{k+2K} \exp[j(\Psi_k(\theta) - 4\pi f \Delta)] + \cdots \cdots + W_{k+(L-1)K} \exp[j(\Psi_k(\theta) - (L-1)2\pi f \Delta] \} \quad \cdots\cdots\cdots (4.3.1)$$

where

$$\Psi_k(\theta) = \pi\left(\frac{K+1}{2} - k\right)\cos\theta, \quad k = 1, 2, \cdots, K \quad \cdots\cdots\cdots (4.3.2)$$

$\Psi_k(\theta)$ denotes the phase of the signal arriving from the direction θ at k-th element with the phase reference taken at the centre of the array. The angle θ is considered with reference to the axis of the array, i.e., the broadside direction of the array will be θ =90 degrees.

The phase of the signal $\Psi_k(\theta)$ can be represented in different ways depending upon the reference point taken. For example, if we consider the number one element of the array as our reference, then

$$\Psi_k(\theta) = \pi(k-1)\cos\theta \quad \dots\dots\dots\dots\dots\dots \quad (4.3.3)$$
$$k = 1, 2, \dots, K$$

Here and in our numerical studies, the phase expression of (4.3.2) has been used.

Imposing constraints on the response y in N directions is equivalent to ensuring that

$$\text{Re}\,[y_{\theta_i^\circ}] = \gamma_i \;, \quad i = 1, 2, \ldots, N \quad \ldots\ldots\ldots(4.3.4)$$

$$\text{Im}\,[y_{\theta_i^\circ}] = \mu_i \;, \quad i = 1, 2, \ldots, N \quad \ldots\ldots\ldots(4.3.5)$$

After simple manipulation it can be shown that this can be rewritten as :

$$\underline{D}\ \underline{W} = \underline{\gamma} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(4.3.6)$$

$$\underline{B}\ \underline{W} = \underline{\mu} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(4.3.7)$$

where the N X KL matrices $\underline{D}$ and $\underline{B}$ are defined by :

$$\underline{D} = \begin{bmatrix} a_{11}^{0} \cdots a_{K1}^{0}, & a_{11}^{1} \cdots a_{K1}^{1}, & \cdots, & a_{11}^{(L-1)} \cdots a_{K1}^{(L-1)} \\ a_{12}^{0} \cdots a_{K2}^{0}, & a_{12}^{1} \cdots a_{K2}^{1}, & \cdots, & a_{12}^{(L-1)} \cdots a_{K2}^{(L-1)} \\ \bullet & & & \\ \bullet & & & \\ \bullet & & & \\ a_{1N}^{0} \cdots a_{KN}^{0}, & a_{1N}^{1} \cdots a_{KN}^{1}, & \cdots, & a_{1N}^{(L-1)} \cdots a_{KN}^{(L-1)} \end{bmatrix}$$

$$\ldots\ldots\ldots\ldots(4.3.8)$$

where

$$a_{ij}^{\ell} = \cos[\Psi_i(\theta_j) - \ell\Delta] \quad \ldots\ldots\ldots\ldots\ldots(4.3.9)$$

$$\ell = 0, 1, \ldots, (L-1)$$

$$\underline{B} = \begin{bmatrix} b_{11}^{0} \cdots b_{K1}^{0}, & b_{11}^{1} \cdots b_{K1}^{1}, & \cdots, & b_{11}^{(L-1)} \cdots b_{K1}^{(L-1)} \\ b_{12}^{0} \cdots b_{K2}^{0}, & b_{12}^{1} \cdots b_{K2}^{1}, & \cdots, & b_{12}^{(L-1)} \cdots b_{K2}^{(L-1)} \\ \vdots & & & \\ b_{1N}^{0} \cdots b_{KN}^{0}, & b_{1N}^{1} \cdots b_{KN}^{1}, & \cdots, & b_{1N}^{(L-1)} \cdots b_{KN}^{(L-1)} \end{bmatrix} \quad \ldots\ldots\ldots (4.3.10)$$

where

$$b_{ij}^{\ell} = \sin[\Psi_i(\theta_j) - \ell\Delta], \quad \ldots\ldots\ldots (4.3.11)$$

$$\ell = 0, 1, \ldots, (L-1)$$

The formulation gives us a constraint matrix which can thus be expressed in a convenient fashion as :

$$\underline{C} = \begin{bmatrix} \underline{D} \\ \cdots \\ \underline{B} \end{bmatrix} \quad \ldots\ldots\ldots (4.3.12)$$

For example, for a 4 element 2 tap (i.e., K=4 and L=2) array with a delay of $\pi/2$ (i.e., $\Delta = \pi/2$) and a look direction (which is also the constraint direction), constraint matrix $\underline{C}$ is given by:

$$\underline{C} = \begin{bmatrix} \cos\Phi_1 & \cos\Phi_2 & \cos\Phi_2 & \cos\Phi_1 & \sin\Phi_1 & \sin\Phi_2 & -\sin\Phi_2 & -\sin\Phi_1 \\ \sin\Phi_1 & \sin\Phi_2 & -\sin\Phi_2 & -\sin\Phi_1 & -\cos\Phi_1 & -\cos\Phi_2 & -\cos\Phi_2 & -\cos\Phi_1 \end{bmatrix}$$

where

$$\Phi_1 = \frac{3\pi}{2}\cos\theta$$

and $$\Phi_2 = \frac{\pi}{2}\cos\theta$$

Here θ is the constraint (Look) direction.

The desired output vector can be rewritten as :

$$\underline{d} = \begin{bmatrix} \Upsilon \\ \cdots \\ \mu \end{bmatrix} \qquad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(4.3.13)$$

Hence the equality directional constraint is given by :

$$\underline{C}\ \underline{W} = \underline{d} \ \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(4.3.14)$$

Note that the above system constrains the real and imaginary parts of y_e separately and it is normal practice to set the imaginary part to zero [8], i.e., $\underline{\mu} = 0$.

The desired output vector can be rewritten as :

$$\underline{d} = \begin{bmatrix} \gamma \\ \cdots \\ \mu \end{bmatrix} \quad \text{.......................(4.3.13)}$$

Hence the equality directional constraint is given by :

$$\underline{C}\ \underline{W} = \underline{d} \quad \text{..................................(4.3.14)}$$

Note that the above system constrains the real and imaginary parts of y_e separately and it is normal practice to set the imaginary part to zero [8], i.e., $\underline{\mu} = 0$.

A problem of considerable interest is to

$$\text{minimize} \quad E[\,y^2\,], \quad \underline{W} \in \mathbb{R}^{KL} \quad \text{...........(4.3.15)}$$

subject to $\underline{C}\,\underline{W} = \underline{d}$

The optimum weight vector is given [8] as

$$\underline{W}_{opt} = \underline{R}^{-1}\,\underline{C}^{T}(\,\underline{C}\,\underline{R}^{-1}\,\underline{C}^{T}\,)^{-1}\,\underline{d} \quad \text{...........(4.3.16)}$$

writing the cost functional as

$$y^2 = \underline{W}^{T}\,\underline{X}\,\underline{X}^{T}\,\underline{W} \quad \text{...........................(4.3.17)}$$

and applying the standard gradient projection algorithm we obtain an adaptive algorithm which can be shown to converge to $\underline{W}$ under various conditions [7],[8],

$$\underline{W}(m+1) = \underline{W}(m) - \;(m)\,\underline{P}\,\underline{X}\,(m)\,y\,(m) \quad \text{........(4.3.18)}$$

where

$$\underline{P} = \underline{I} - \underline{C}^{T}\,(\underline{C}\,\underline{C}^{T}\,)^{-1}\,\underline{C} \quad \text{...................(4.3.19)}$$

and $\alpha(m)$ is a small scalar step and m being the number of iteration. In equation (4.3.19), $\underline{P}$ and $\underline{I}$ are projection vector and identity matrix respectively.

The following primal-dual algorithm can also be shown [13] to adaptively find $\underline{W}_{opt}$ under fairly general circumstances :

$$\underline{W}(m+1) = \underline{W}(m) + \frac{1}{2}\alpha(m)\left[\underline{X}(m)y(m) + \underline{C}\,\underline{\lambda}(m)\right] \quad \text{.................(4.3.20)}$$

$$\underline{\lambda}(m+1) = \underline{\lambda}(m) - \alpha(m)\left[\underline{C}\,\underline{W}(m) - \underline{d}\right] \quad \text{................(4.3.21)}$$

where λ is the Lagrange multiplier.

The algorithm we describe later in this chapter is based on the primal-dual idea.

We note in passing that the weight vector which solves the mean squared error cost in the well known LMS algorithm is given by :

$$\underline{W}_{opt} = \underline{R}^{-1} E[\underline{X}\,\underline{d}] \quad \text{..................(4..3.22)}$$

where

$$E[\underline{X}\ \underline{d}] \triangleq E\begin{bmatrix} x_1 d \\ x_2 d \\ \cdot \\ \cdot \\ \cdot \\ x_{KL} d \end{bmatrix} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(4.3.23)$$

Here E[] is the set of cross correlations between the input signals and the desired response signal. The LMS adaption algorithm which determines $\underline{W}_{opt}$ is the simple gradient algorithm :

$$\underline{W}(m+1) = \underline{W}(m) - \alpha(m)\underline{e}(m)\underline{X}(m) \quad \ldots\ldots\ldots\ldots(4.3.24)$$

where

$$\underline{e}(m) = \underline{d}(m) - \underline{y}(m) \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(4.3.25)$$

is the error signal.

The algorithm we will present shortly aims at minimizing $E[y^2]$ subject to the inequality directional constraints.

Thus in our case, the problem (4.3.15) can be expressed mathematically as

$$\text{minimize} \quad E[y^2] \quad \underline{W} \in \mathbb{R}^{KL} \quad \ldots\ldots\ldots\ldots\ldots(4.3.26)$$

$$\text{subject to} \quad | \underline{C}\, \underline{W} - \underline{d} | \leqslant \underline{e}$$

and find an optimum set of weights to be denoted by, say $\underline{W}_{opt}$. Here $\underline{e}$ is the tolerance band of the desired response.

4.4 ALGORITHM

In order to describe the algorithms which will adaptively determine the optimal tap weight vector $\underline{W}_{opt}$ [13] we rewrite the problem (4.3.26) as :

$$\text{minimize } \underline{W}^T \underline{R}\, \underline{W}, \quad \underline{W} \in \mathbb{R}^{KL}$$

$$\text{s.t.} \quad \underline{C}\, \underline{W} - \underline{d} \leqslant \underline{e} \quad \ldots\ldots\ldots\ldots\ldots\ldots(4.4.1)$$

$$-\underline{C}\, \underline{W} + \underline{d} \leqslant \underline{e}$$

where $\underline{e}$ is the tolerance band as mentioned before.

The Lagrangian [14] for this problem is :

$$L(\underline{W}, \underline{\lambda}_1, \underline{\lambda}_2) = \underline{W}^T \underline{R} \underline{W} + \underline{\lambda}_1^T (\underline{C} \underline{W} - \underline{d} - \underline{e}) - \underline{\lambda}_2^T (\underline{C} \underline{W} - \underline{d} + \underline{e}) \qquad (4.4.2)$$

A useful algorithm for determining the optimal weight vector $\underline{W}_{opt}$ can be based on the primal-dual idea established in chapter 3. The algorithm can be derived by differentiating the Lagrangian with respect to $\underline{W}$, λ_1 and λ_2 and applying a constrained steepest descent procedure as follows :-

$$\underline{W}^{K+1} = \underline{W}^{K} - \alpha^{K} \frac{\partial L}{\partial W^{K}} \{ \underline{W}^{K}, \underline{\lambda}_1^{K}, \underline{\lambda}_2^{K} \}$$

$$\lambda_1^{K+1} = \max \left[0, \underline{\lambda}_1^{K} + \alpha^{K} \frac{\partial L}{\partial \underline{\lambda}_1^{K}} \{ \underline{W}^{K}, \underline{\lambda}_1^{K}, \underline{\lambda}_2^{K} \} \right]$$

$$\lambda_2^{K+1} = \max \left[0, \lambda_2^{K} + \alpha^{K} \frac{\partial L}{\partial \lambda_2^{K}} \{ \underline{W}^{K}, \underline{\lambda}_1^{K}, \underline{\lambda}_2^{K} \} \right]$$

$$\ldots\ldots\ldots\ldots\ldots (4.4.3)$$

The final algorithm can be written as:

$$\underline{W}(K+1) = \underline{W}(K) - \alpha(K) \left[2 \underline{R}\, \underline{W}(K) + \underline{C}^{T} \underline{\lambda}_{-}(K) \right]$$

$$\lambda_1(K+1) = \underline{P} \left[\underline{\lambda}_1(K) + \beta(K) \{ \underline{C}\, \underline{W}(K) - e^{+} \} \right]$$

$$\lambda_2(K+1) = \underline{P} \left[\underline{\lambda}_2(K) - \beta(K) \{ \underline{C}\, \underline{W}(K) - e^{-} \} \right]$$

$$\ldots\ldots\ldots\ldots\ldots (4.4.4)$$

where

$$\left[\underline{P}[x] \right]_i = \begin{cases} x_i, & \text{if } x_i \geq 0 \\ 0, & \text{otherwise} \end{cases} \quad \ldots\ldots\ldots (4.4.5)$$

and

$$\underline{\lambda}_{-} = \lambda_1 - \lambda_2$$

$$\underline{e}^{+} = \underline{d} + \underline{e} \quad \ldots\ldots\ldots\ldots\ldots (4.4.6)$$

$$\underline{e}^{-} = \underline{d} - \underline{e}$$

The problem of minimizing a convex cost functional over a convex set which is defined by some linear inequality constrains is described in Appendix A. In the said appendix, some definitions and results from the theory of real finite dimensional constrained optimization problem are represented.

The scalar step sizes α^K and β^K, used in the algorithm represented by (4.4.4.) are subject to certain conditions which ensure that these algorithms converge to $\underline{W}_{opt}$ as $K \longrightarrow \infty$. The details of these convergence properties are discussed in Appendix B.

Finally in appendix C, the flow chart of the algorithm and the corresponding program are provided.

CHAPTER 5

SIMULATION STUDY ON THE BEHAVIOUR OF DIRECTIONALLY CONSTRAINED ADAPTIVE ARRAY

5.1 INTRODUCTION

In this chapter we give detailed numerical simulation results of our formulated problem for directionally constrained antenna array processor. In the simulations we investigate the behavior of a narrowband directionally constrained antenna array processor with respect to sidelobe constraints and signal power variations. In the process of our study we first observe the noise cancelling capability of the narrowband antenna array processor.

Our analysis of adaptive antenna arrays deals with the narrowband type, i.e., one that responds to narrowband signals which can be expressed by their centre frequency and slowly varying envelope. The antenna system we considered is assumed to have only two taps for each channel [L=2 in Fig.4.2.1], and the time delay between them is taken to be a quarter period of the desired signal. The antenna elements are placed half wave lengths apart. It should be noted, however, that these assumptions are made in order to simplify the physical meaning of the results, and is not expected to affect the validity of the analysis. Such a 4-element narrowband receiving array is shown in Fig.5.1.1. In course of our numerical simulations the element number is not always limited to four, but is rather allowed to vary to higher numbers as well.

The algorithm (4.4.4) is tested on an IBM personal computer using FORTRAN. The sections that follow, refer to the simulated results thus obtained.

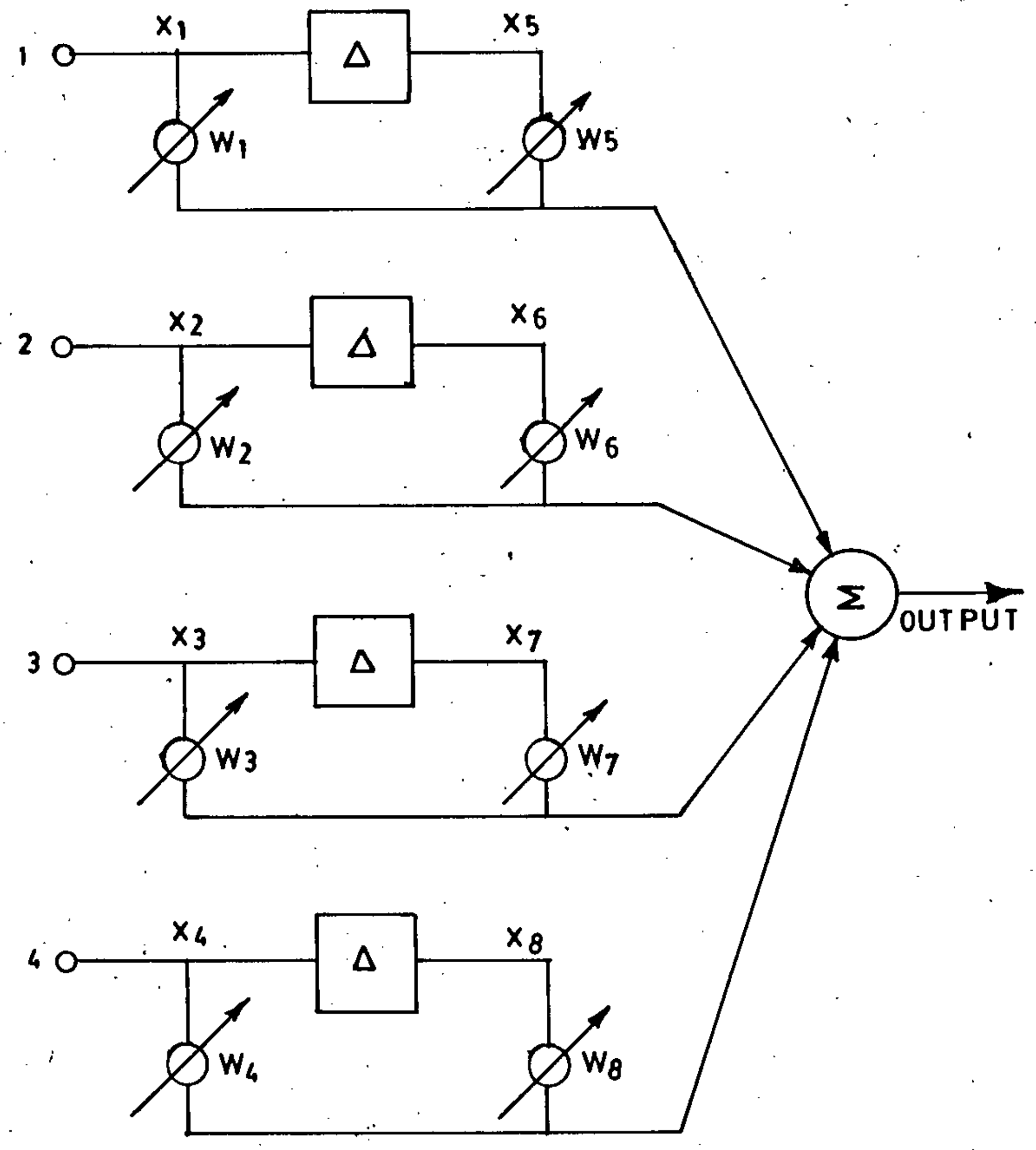


Fig. 5.1.1 : A 4-ELEMENT NARROWBAND RECEIVING ARRAY.

Before beginning discussion on our simulation results, we clarify certain terms related with the measurement of power contained in desired signal and in noise. Two such terms are SINR and SNR which give an idea of the relative power between desired signal and noise by considering their power ratios. We now discuss briefly about SINR and SNR since they frequently occur in our discussions that is to follow shortly.

The SINR stands for "signal to interference plus system noise ratio" and is defined by :

$$SINR = 10 \operatorname{Log}_{10} (P_s / P_{tn}) \quad \text{..................(5.1.1)}$$

where

P_s = Total signal power

and P_{tn} = Total noise power

It should be noted, that the total noise of the antenna array consists of the interference (or "jammer") signal and the noise that is generated within the system. i.e.,

$$P_{tn} = P_{in} + P_{sn} \quad \text{.............................(5.1.2)}$$

where

P_{in} = Interfering signal power

and P_{sn} = System noise power

In fact this categorization of noise power has given rise to the other form of power ratio which is expressed by

$$SNR = 10\ Log_{10}\ (P_s\ /\ P_{sn}) \quad \text{..................(5.1.3)}$$

where SNR stands for "Signal to system noise ratio".

If P_s in equation (5.1.1) and (5.1.3) expresses the output signal power, then the corresponding SINR and SNR are called "Output SINR" and "Output SNR" respectively. On the other hand, if Ps denotes the input signal power then by using this value we can get the "Input SINR" as well as the "Input SNR".

Generally at any particular instant both the noise forms (interference and system noise) are expected to be present in the array system. However, for convenience of our discussion, from now on, we will use the term "Noise" to indicate the interfering signal only. The other form of noise will be explicitly denoted by the term "System noise" whenever required. Moreover by the term "Signal" from now on we will understand the desired signal which is coming from the look direction.

We now proceed to describe our simulation results.

5.2 SIMULATION RESULTS

The first simulation was carried out to check the capability of the algorithm as to whether it can successfully utilize the degrees of freedom in order to achieve its goal of placing nulls onto noise directions, and provide pre-assigned response to the look direction. The degrees of freedom mentioned above is defined [7] to be equal to the number of antenna elements and the look direction is the desired signal direction.

Thus a K element antenna array possesses K degrees of freedom, but when there is one look direction, one degree of freedom is used for the look direction. Remaining (K-1) degrees of freedom, therefore, are available to succesfully tackle a maximum of (K-1) noise sources by placing nulls in the processor response pattern along the noise directions. This implies, however, that other than the main-lobe constraint in the look direction there is no side-lobe constraint put in the antenna response pattern.If N_d represents the number of noise directions (where $N_d < K$), then the maximum number of side-lobe constraints that may be assigned to this array processor is given by $(K-1-N_d)$. Fig.s 5.2.1 through 5.2.3 represent antenna response patterns which are obtained for a four element array antenna, each with a single look direction at 90 degrees. The noise directions are arbitrarily chosen. In each case, the look direction response is found to satisfy the pre-assigned constrained desired value taking tolerance into account , and nulls are also placed along the noise directions. However, with increase of noise directions, the output SINR is found to decrease. But upto the previously defined maximum permissible number of noise sources (=K-1), the degradation of SINR is not very significant. This is because, as long as the upper limit of the number of noise sources is not reached, deep nulls could be placed in each response pattern along the noise directions;therefore, the number of noise sources had little influence on the output SINR. This fact is evident from the curve for a 4 element array in Fig.5.2.13.

The same simulation is repeated for 5 and 6 element arrays and the resulting patterns are given in Fig.5.2.4 through 5.2.12. The curve for output SINR and number of corresponding noise sources for these two type of arrays are also given in Fig.5.2.13. It is evident from this result that increase in number of elements gives more control on the response pattern thereby increasing the SINR.

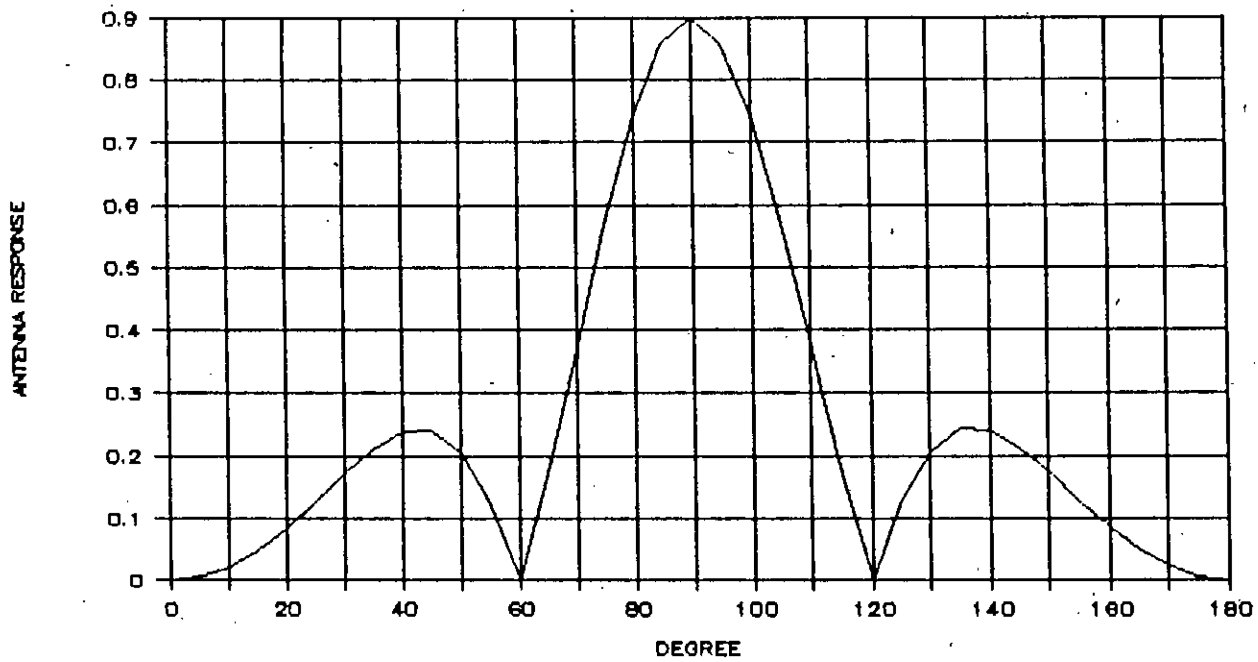


Fig.5.2.1: Response pattern of a 4 element array antenna with the look direction at 90 degrees and a noise direction at 0 degree. (**Antenna Response = $\underline{C}\ \underline{W}$**)

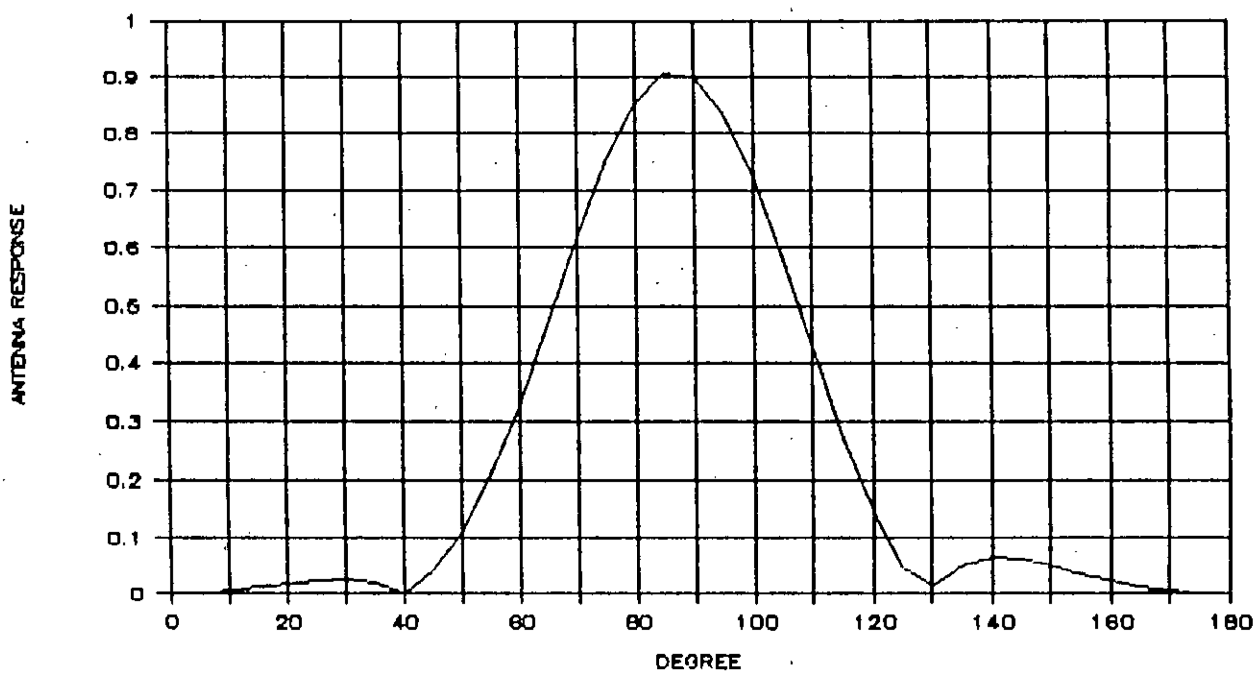


Fig.5.2.2: Response pattern of a 4 element array antenna with the look direction at 90 degrees and two noise directions at 0 and 40 degrees.

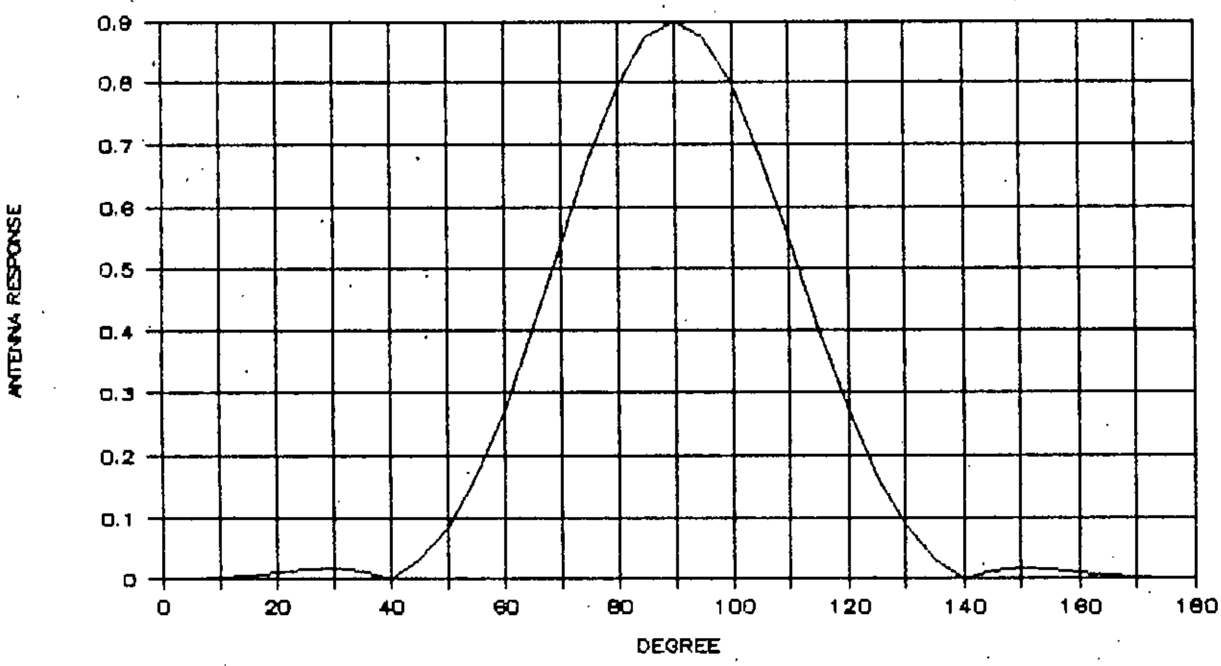


Fig.5.2.3: Response pattern of a 4 element array antenna with the look direction at 90 degrees and three noise directions at 0, 40 and 140 degrees.

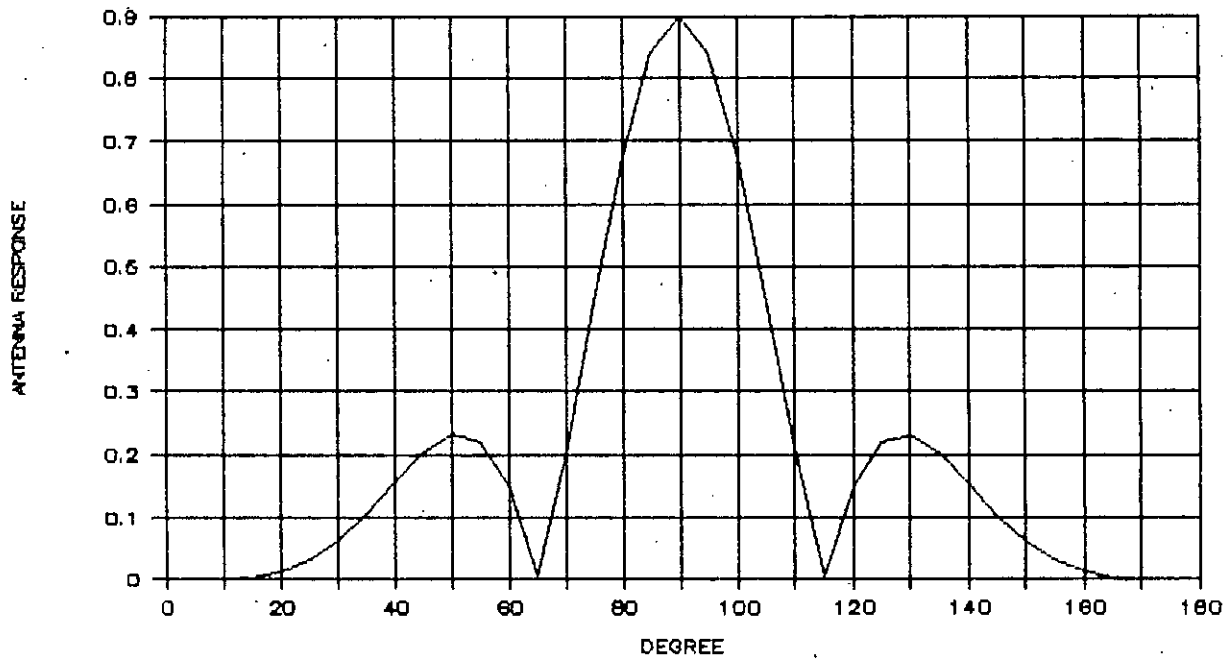


Fig.5.2.4: Response pattern of a 5 element array antenna with the look direction at 90 degrees and a noise direction at 0 degree.

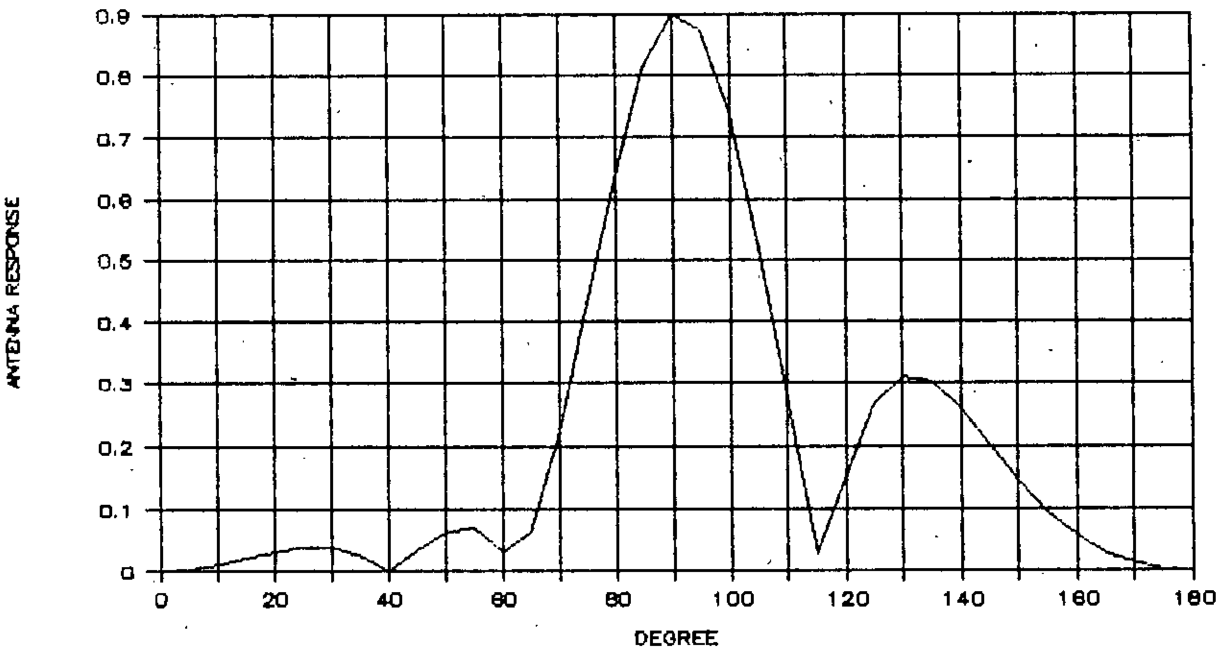


Fig.5.2.5: Response pattern of a 5 element array antenna with the look direction at 90 degrees and two noise directions at 0 and 40 degrees.

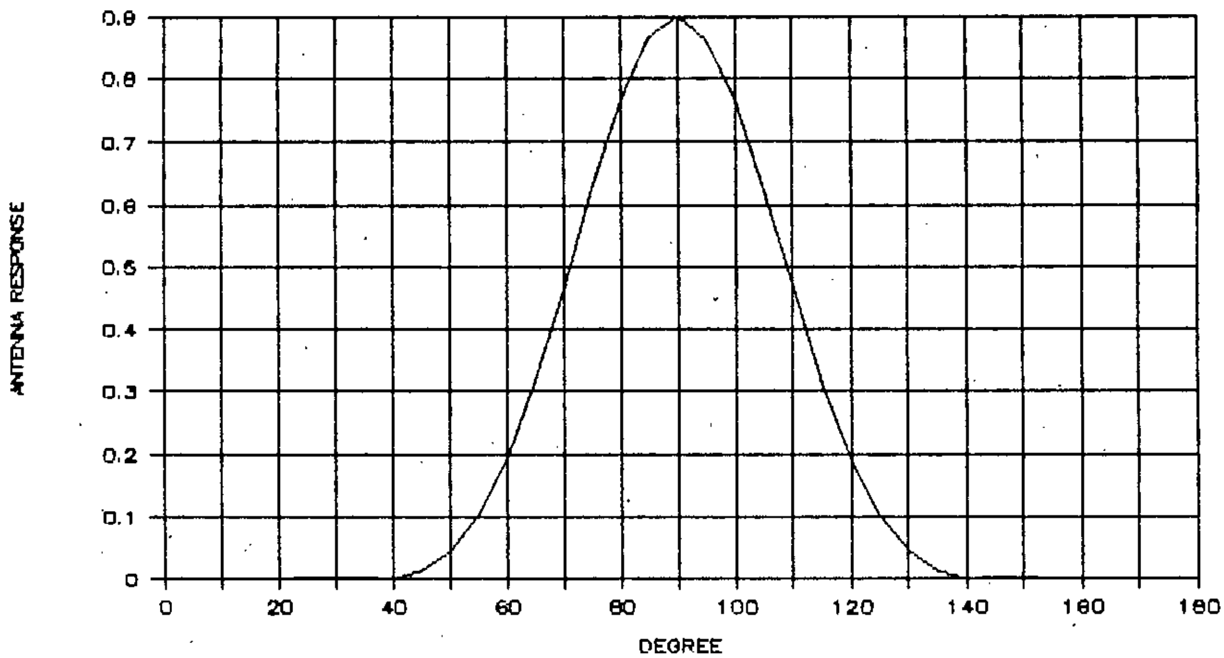


Fig.5.2.6: Response pattern of a 5 element array antenna with the look direction at 90 degrees and three noise directions at 0, 40 and 140 degrees.

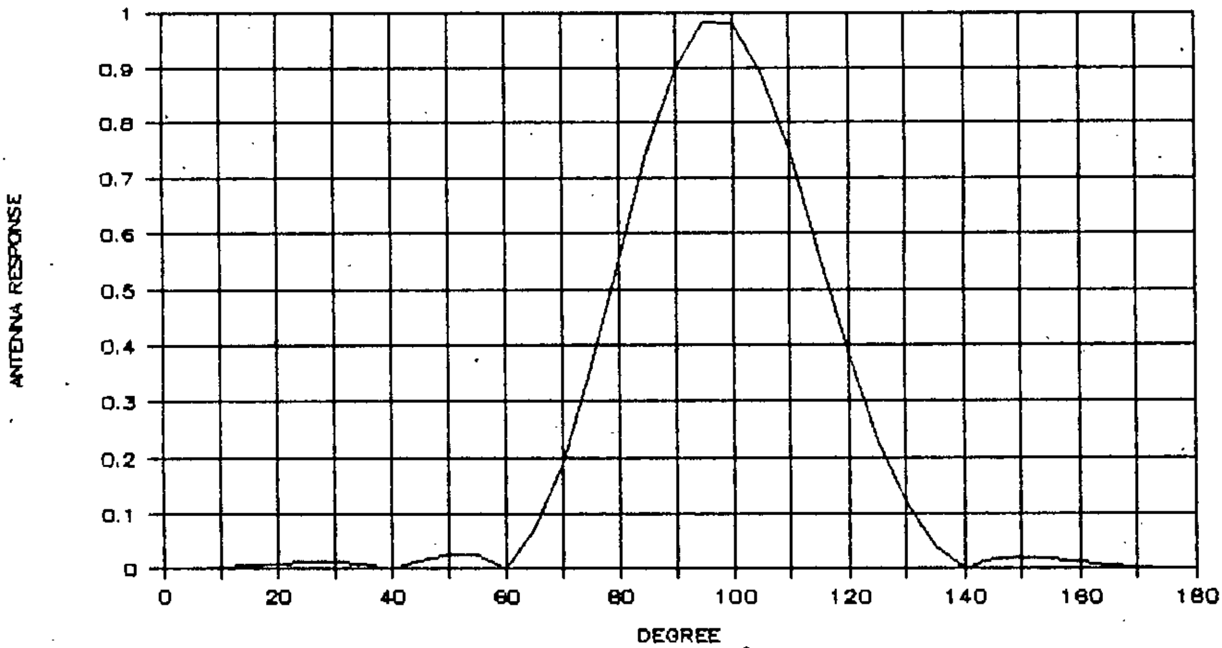


Fig.5.2.7: Response pattern of a 5 element array antenna with the look direction at 90 degrees and four noise directions at 0, 40, 140 and 60 degrees.

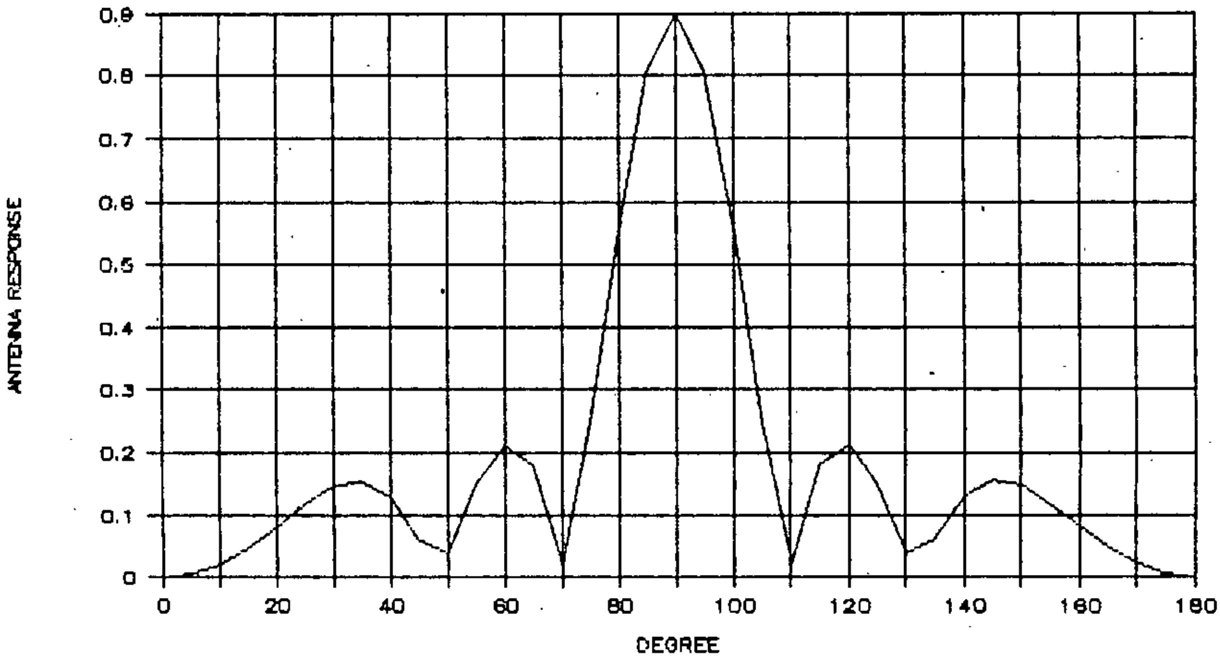


Fig.5.2.8: Response pattern of a6 element array antenna with the look direction at 90 degrees and a noise direction at 0 degree.

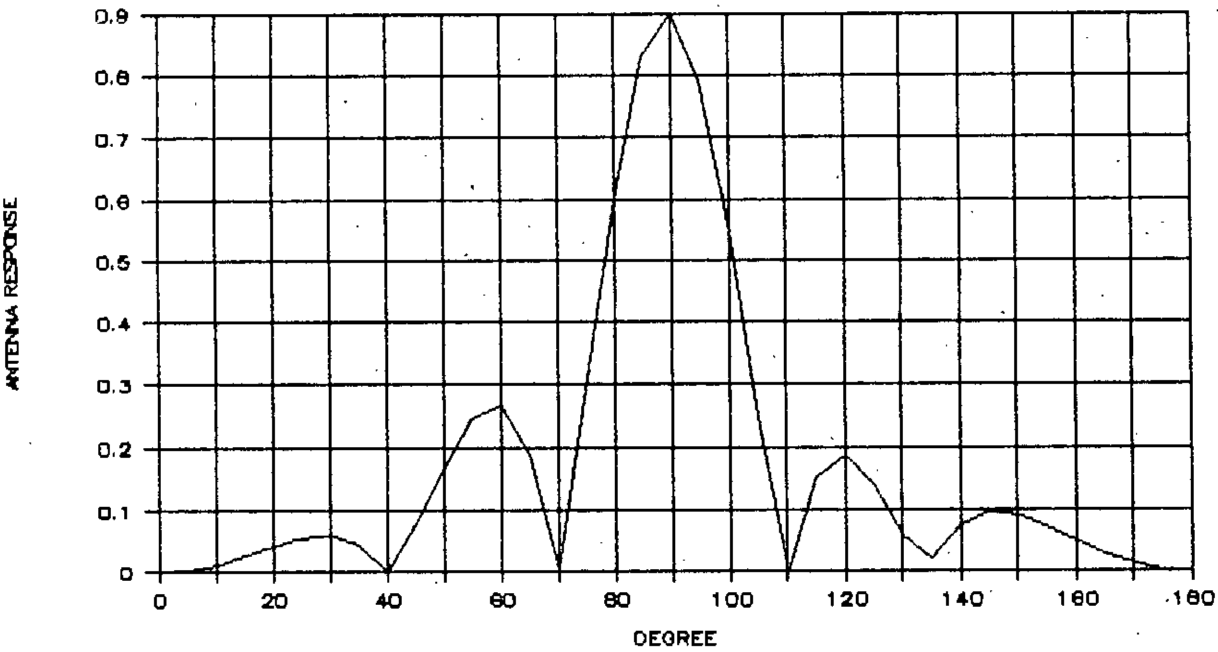


Fig.5.2.9: Response pattern of a 6 element array antenna with the look direction at 90 degrees and two noise directions at 0 and 40 degrees.

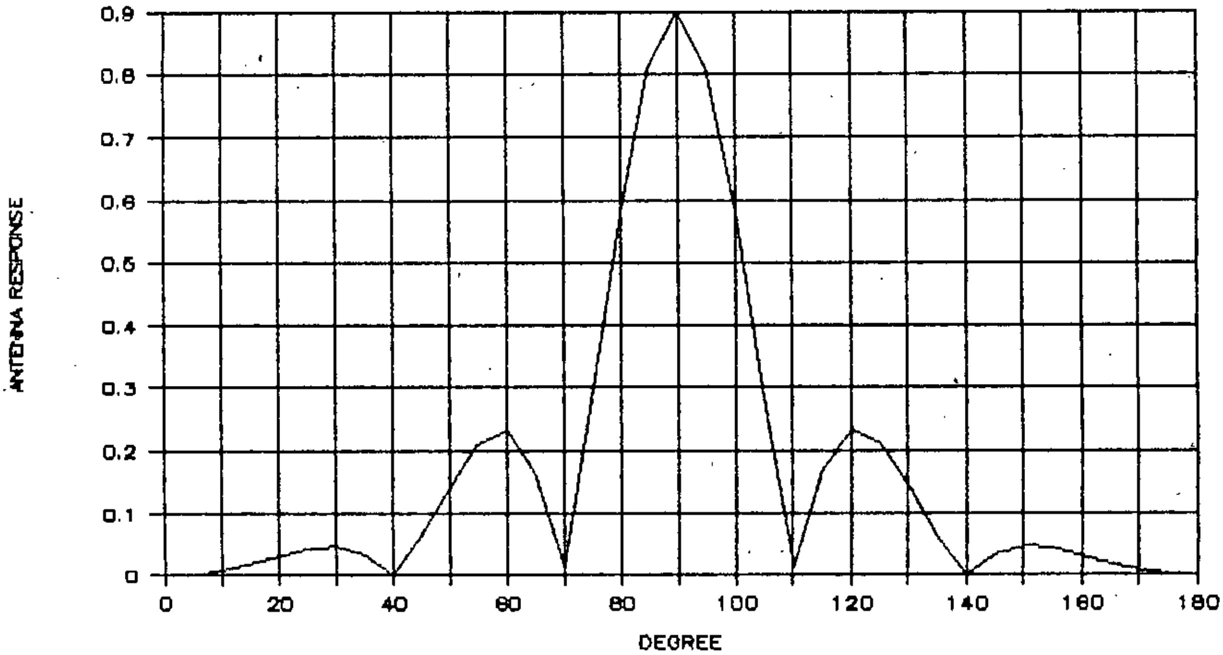


Fig.5.2.10: Response pattern of a 6 element array antenna with the look direction at 90 degrees and three noise directions at 0, 40 and 140 degrees.

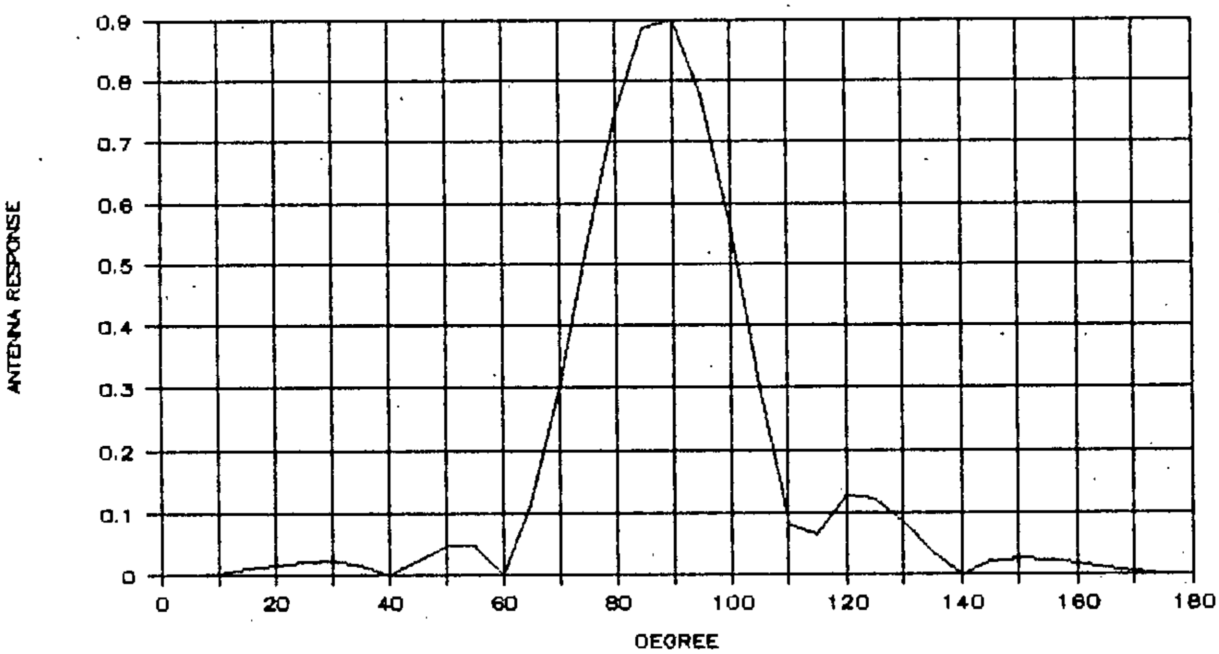


Fig.5.2.11: Response pattern of a 6 elemenet array antenna with the look direction at 90 degrees 4 noise directions at 0, 40, 140 and 60 degrees.

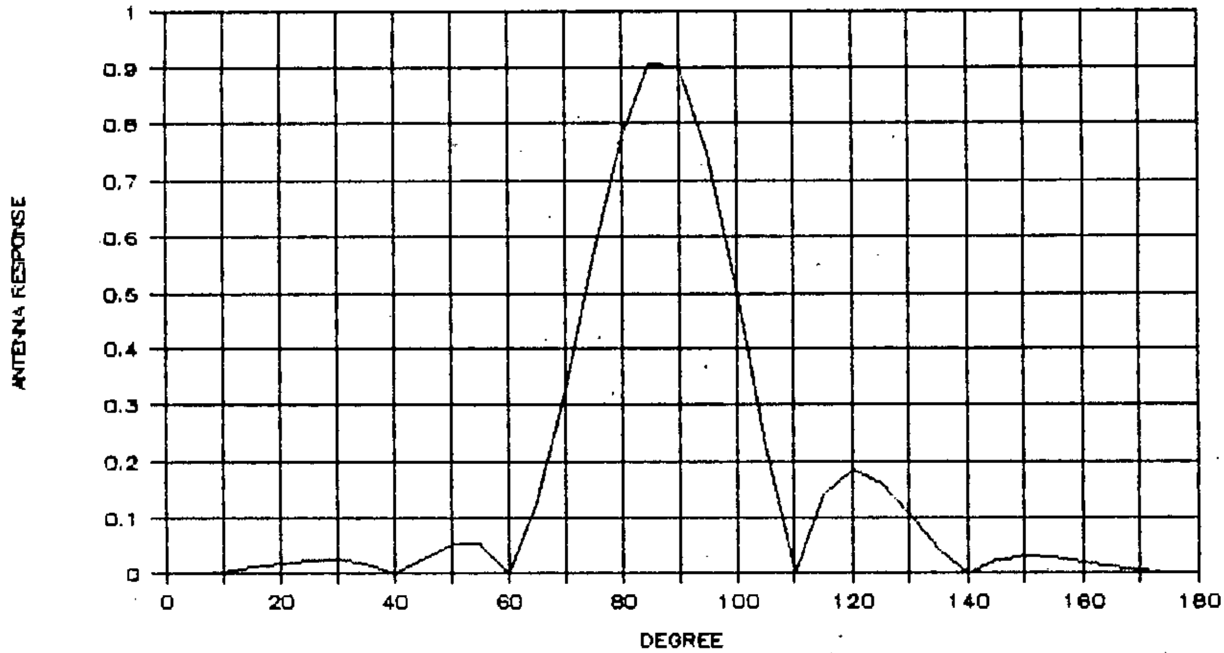


Fig.5.2.12: Response pattern of a 6 element array antenna with the look direction at 90 degrees and 5 noise directions at 0, 40, 140, 60 and 110 degrees.

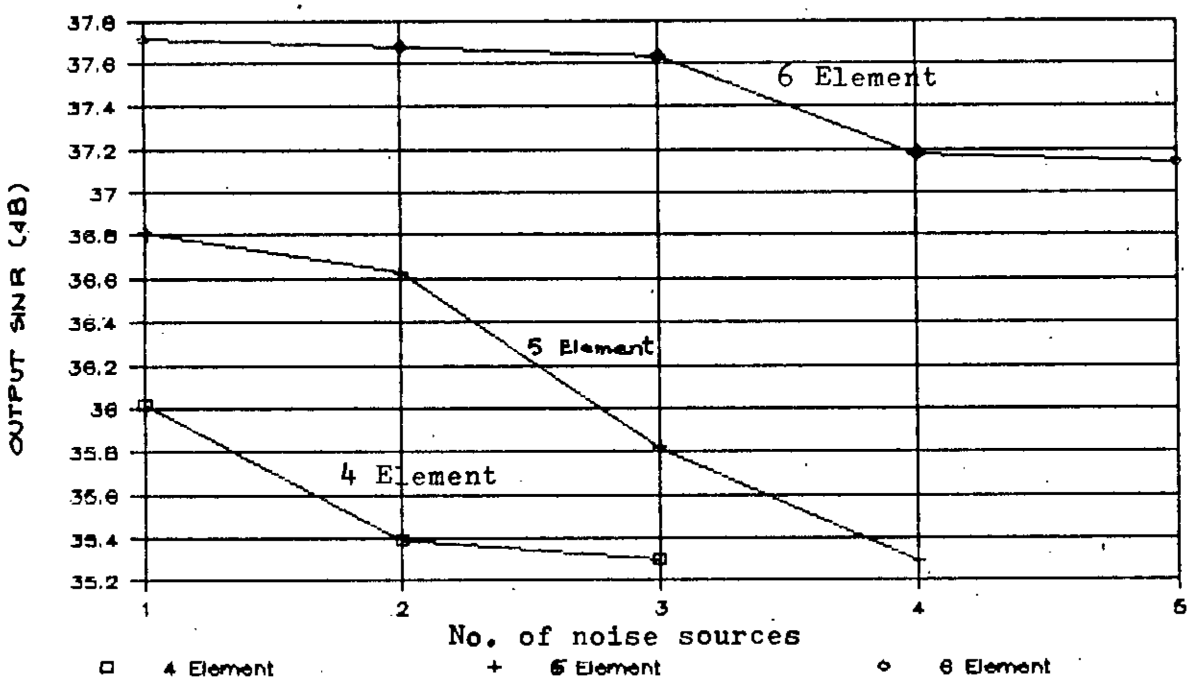


Fig.5.2.13: Variation of output SINR with change in number of noise sources for 4, 5 and 6 element array antenna.

Before proceeding on to the next simulation, we observe the nature of the weight values corresponding to 4 element array having a look direction at 90 degrees and a noise direction at 40 degrees. This is shown in the graph of Fig.5.2.14. Fig.5.2.15 represents weight values when there are three noise sources at 0, 40 and 140 degrees with the signal direction fixed at 90 degrees for the same array. The weight values can differ from each other radically depending on the look direction and the noise directions. This observation is evident from Fig.5.2.16 Where the look direction is at 60 degrees with one noise at 120 degrees.

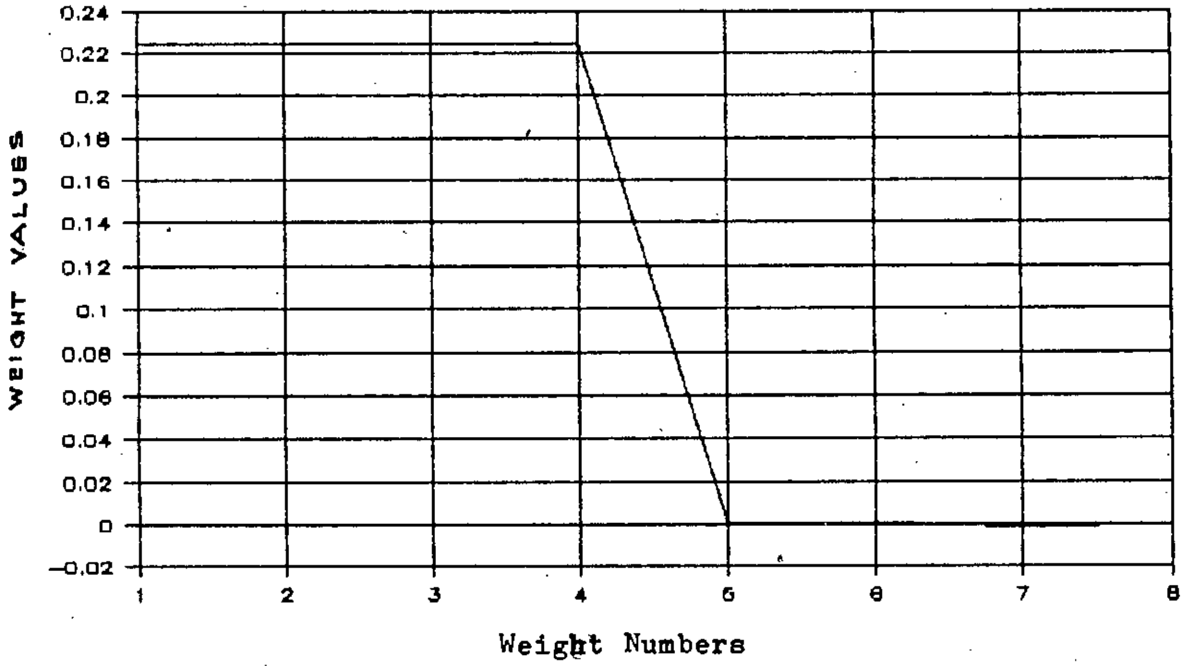


Fig.5.2.14: Tap-weight values for a 4 element array antenna having the look direction at 90 degrees and a noise direction at 40 degree.

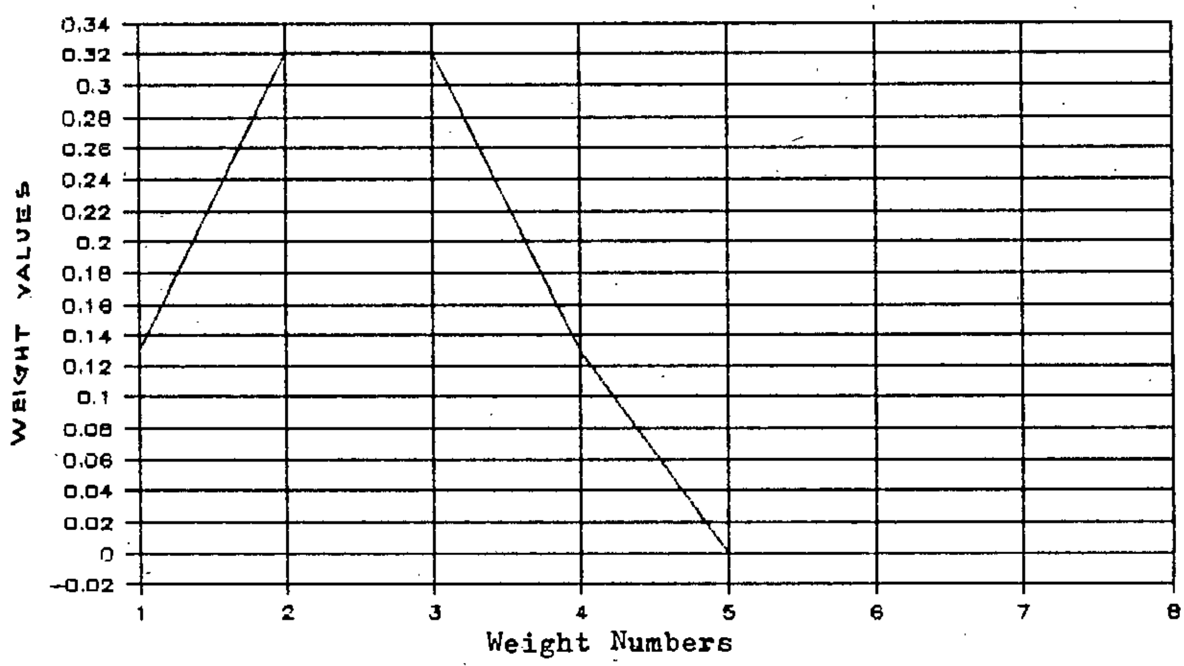


Fig.5.2.15: Tap-weight values for a 4 element array antenna having the look direction at 90 degrees and three noise directions at 0, 40 and 140 degrees.

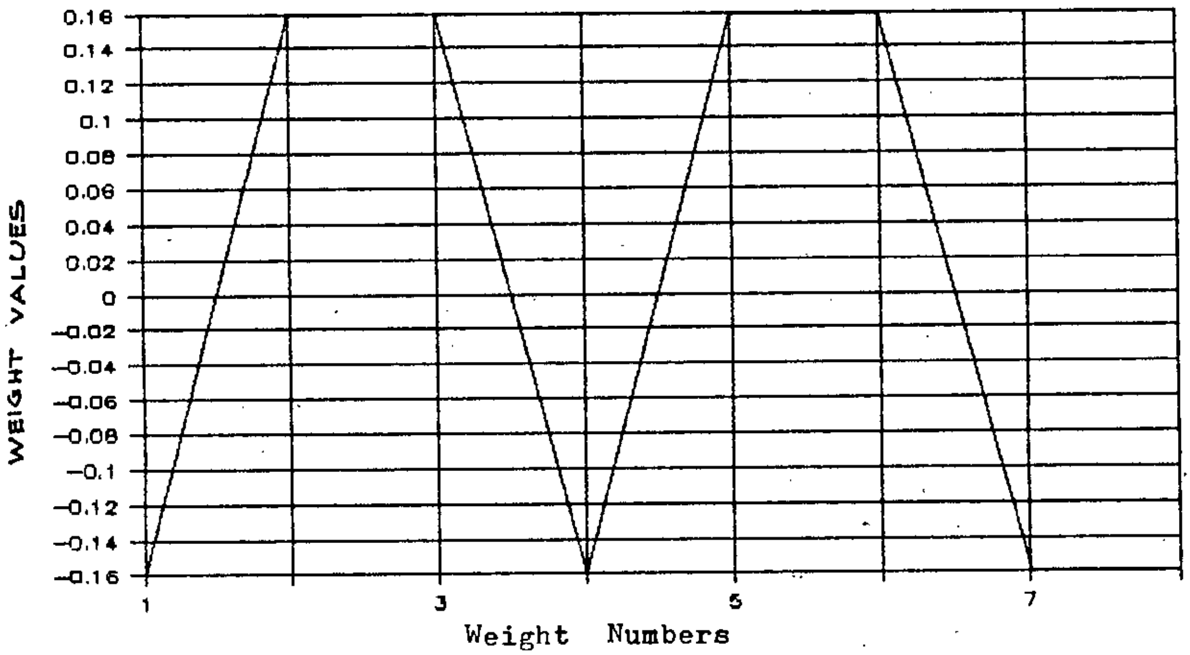


Fig.5.2.16: Tap-weight values for a 4 element array antenna having the look direction at 60 degrees and a noise direction at 120 degrees.

Our next study is aimed at investigating the effect the "distance" of the noise from the look direction has on the output SINR. The "distance" is measured in terms of the angle which the noise source makes with the signal source. Both the source angles are denoted in terms of degrees which the respective sources make with some reference axis. The reference axis in our case is the array axis. Thus the broadside direction of the array is at ninety degrees. For the present simulations, the look direction for a 4 element array is taken at the broadside direction. One noise source is gradually shifted from zero degrees towards the look direction. The variation of output SINR, with distance of noise from the look direction, for a 4 element array is given in Fig.5.2.18. It can be seen from this Fig., that as long as the noise is "outside" the mainlobe, there is no significant variation in SINR. Though in these cases, there is a slight gradual degradation in SINR as the noise moved nearer to the mainlobe. But when the noise is close enough to the look direction, such that it could be considered to "enter inside" the mainlobe, a drastic degradation in SINR is observed. The rate of the gradual reduction of SINR, with the nearing of the noise, is found to be much greater, after the noise entered the mainlobe.

The variation in weight values coresponding to distance of noise from the main lobe is plotted in Fig. 5.2.19.

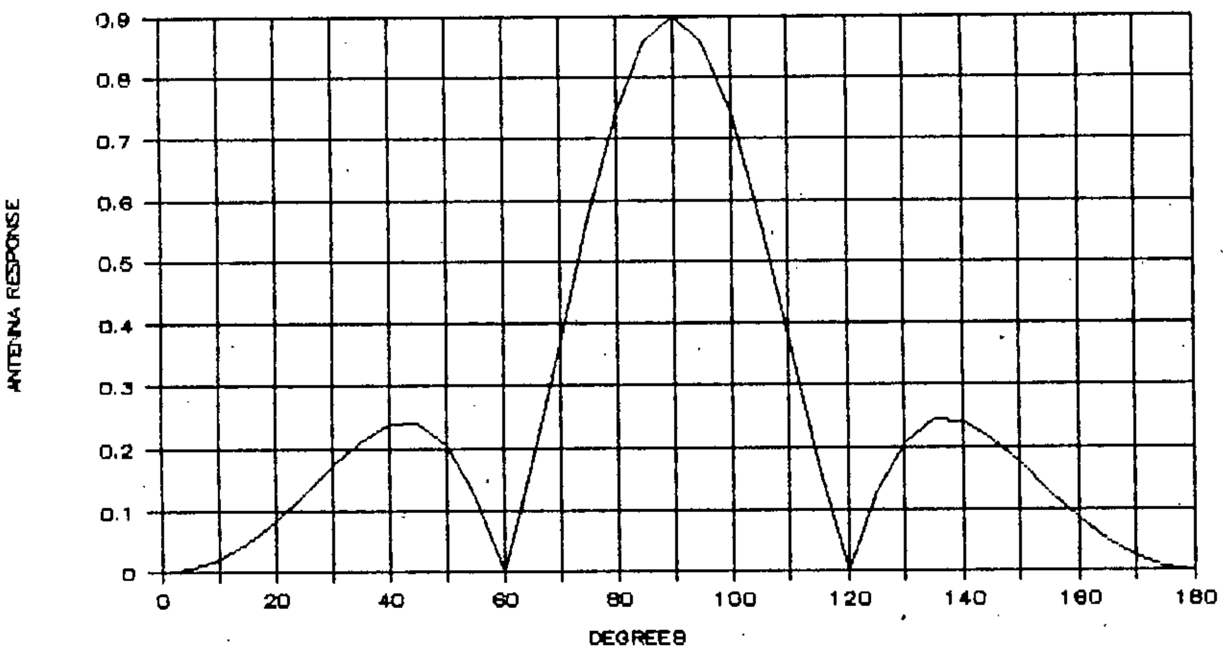


Fig.5.2.17: Response pattern of a 4 element array antenna with the look direction at 90 degrees and a noise direction at 0 degrees.

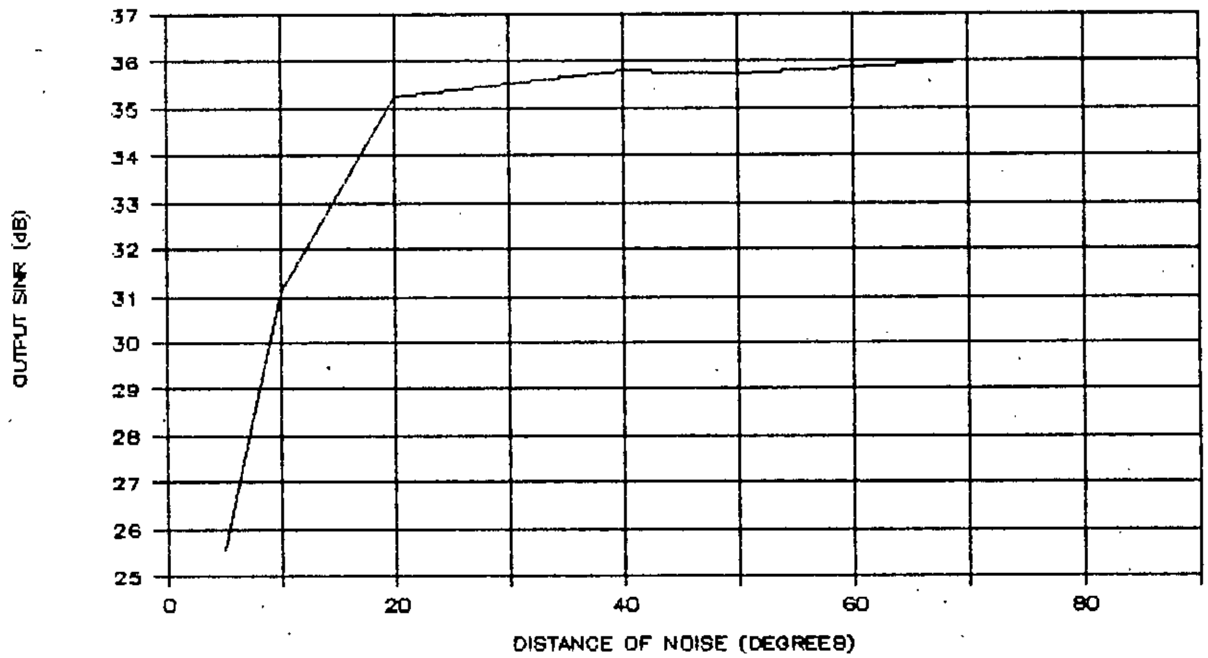


Fig.5.2.18: Variation of output SINR with change in distance of noise from the mainlobe constraint direction for a 4 element array antenna.

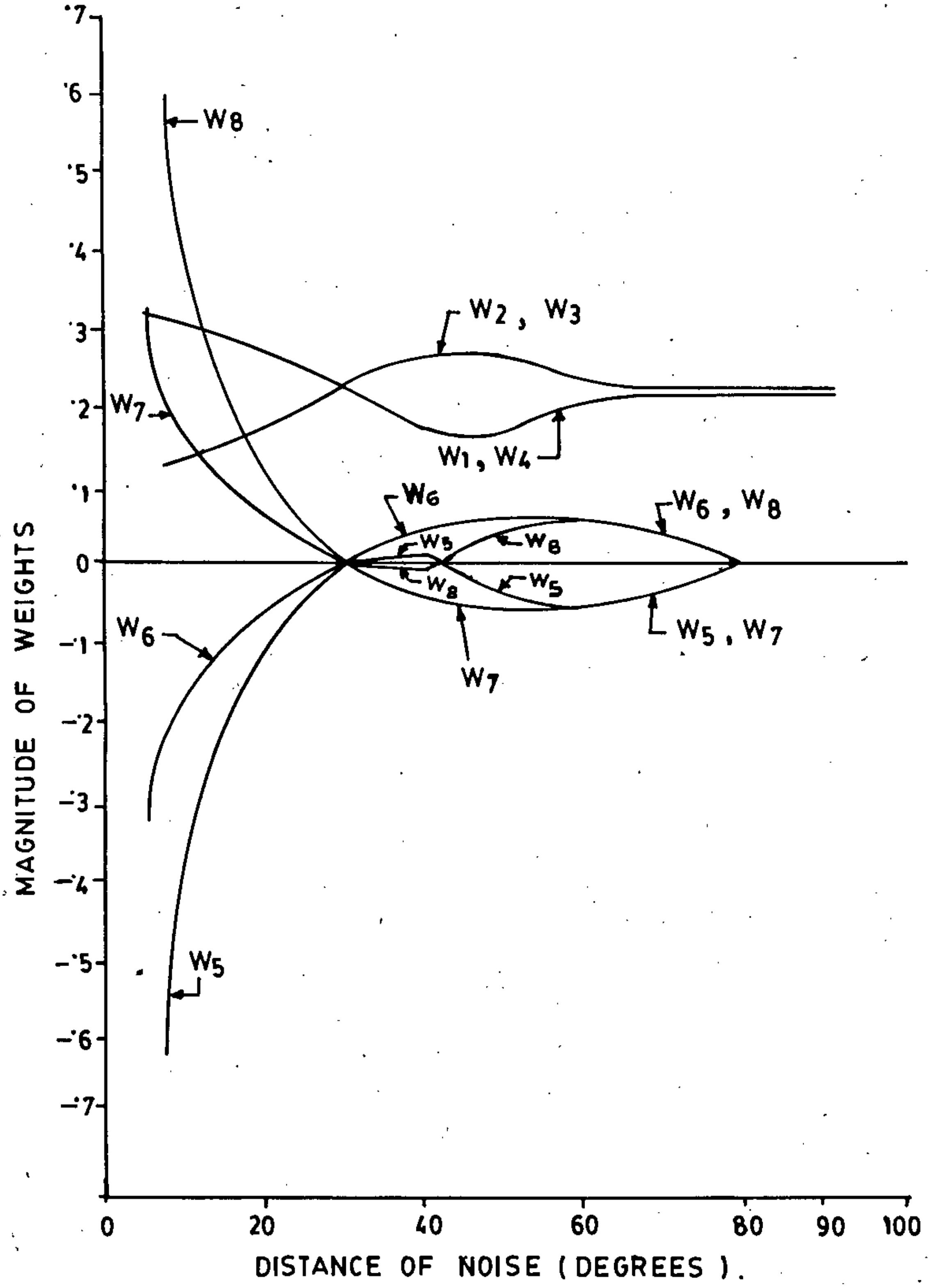


Fig. 5.2.19 : VARIATION OF WEIGHT VALUES WITH DISTANCE NOISE FOR A 4 ELEMENT ARRAY.

After our observation of noise cancelling capability of the antenna array, we proceed to investigate the behaviour of the antenna under pattern constraints. In our previous simulations we used only the main-lobe constraint for achieving the desired response pattern in the look direction. Now we intend to put some more constraints in the side-lobe directions and observe the response pattern. The simulations are carried out with four, five and six element antenna array with varying number of side-lobe constraints in each case. The results are shown in the response patterns plotted in Fig.5.2.20 to 5.2.31 It is observed from these results that the algorithm can generate effectively pattern constraints apart from the main-lobe constraint. These side-lobe constraints are successfully met without any major degradation in the noise cancelling capability. However, increase in number of side-lobe constraints sometimes produces some degradation in output SINR. This is obviously due to the fact that the algorithm is more concerned with meeting the constraints put rather than minimizing the noise power. Thus in some cases where the side-lobe constraints may be quite stringent, the array processor sacrifices deep null in the noise directions in favour of satisfying the constraints. Note that the number of side-lobe constraints that can be satisfied depends on the degrees of freedom available after considering the look and the noise directions. In Fig. 5.2.22, a four element array utilizes all its degrees of freedom in the look direction, noise direction and two side-lobe constraint directions. Similar is the case with a six element array as shown in Fig.5.2.33 where four side-lobe constraint directions are met apart from the look direction and noise direction utilizing thus all degrees of freedom. It is interesting to know that what happens if there is one more noise direction or one more side-lobe constraint direction. In Fig.5.2.27 and Fig.5.2.28 these two cases are shown for a 5 element array.

We also plot here the weight values to observe the changing pattern in their values. Fig.5.2.34 and Fig.5.2.35 are two such plots drawn for a four element and a six element array.

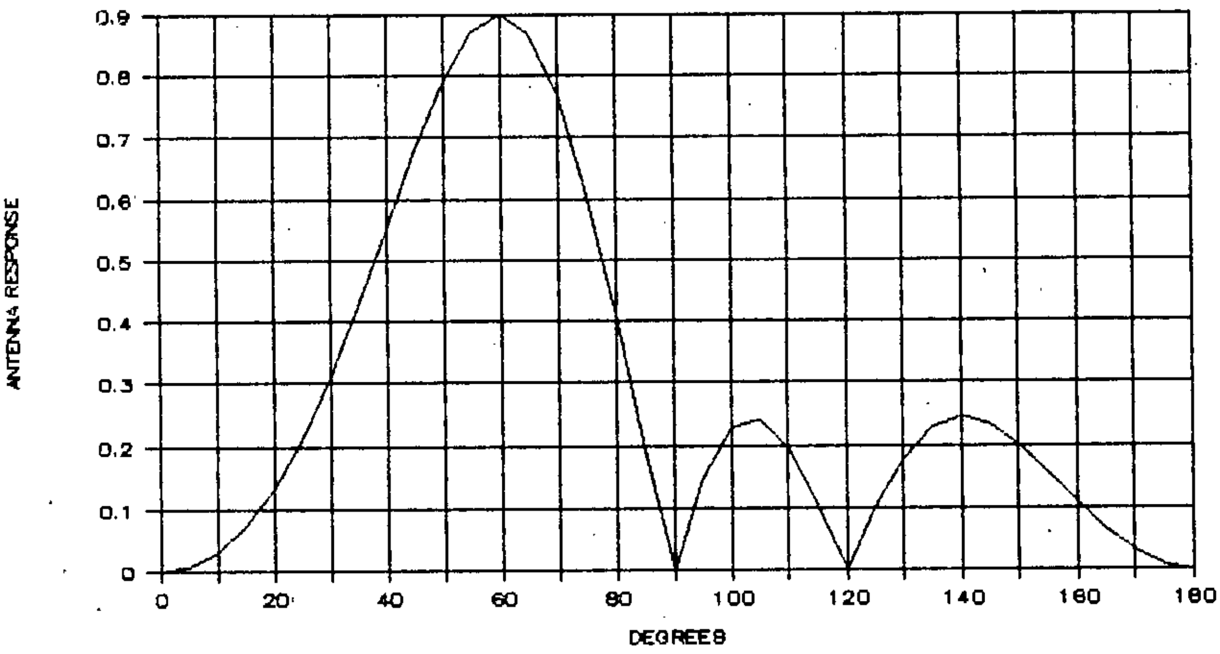


Fig.5.2.20: Response pattern of a 4 element array antenna with the look direction at 60 degrees and a noise direction at 120 degrees. No sidelobe constraints are assigned.

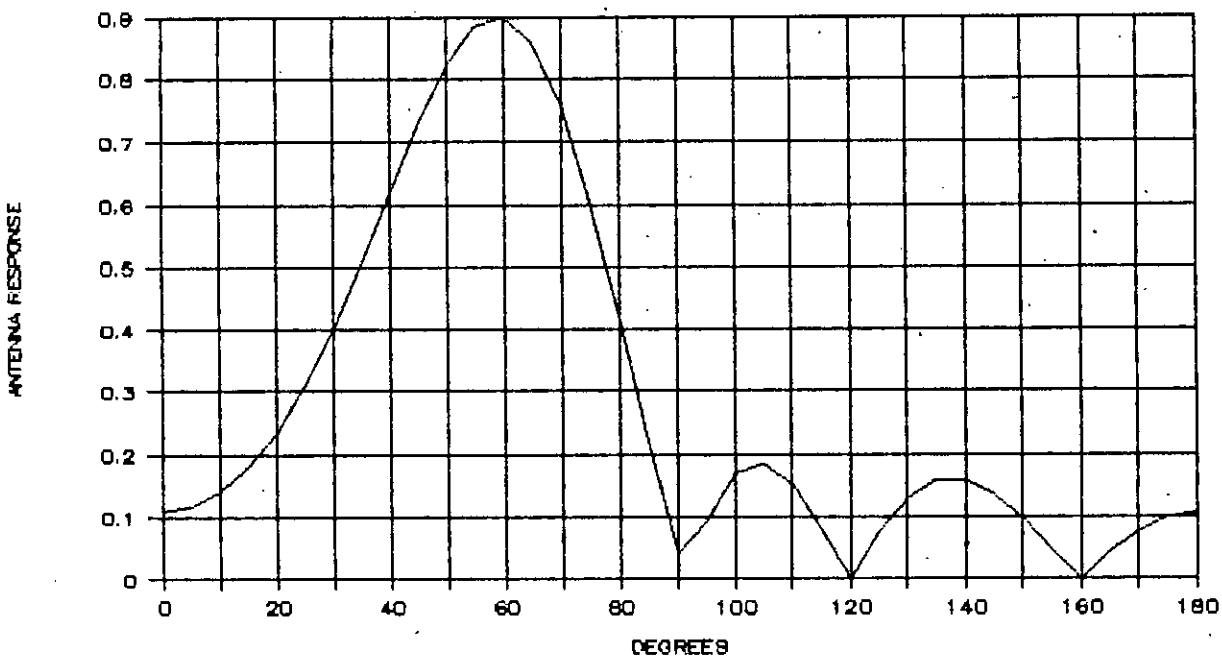


Fig.5.2.21: Response pattern of a 4 element array antenna with the look direction at 60 degrees and a noise direction at 120 degrees. A sidelobe constraint level of 0.15 (tolerance ±.01) is assigned at 140 degrees.

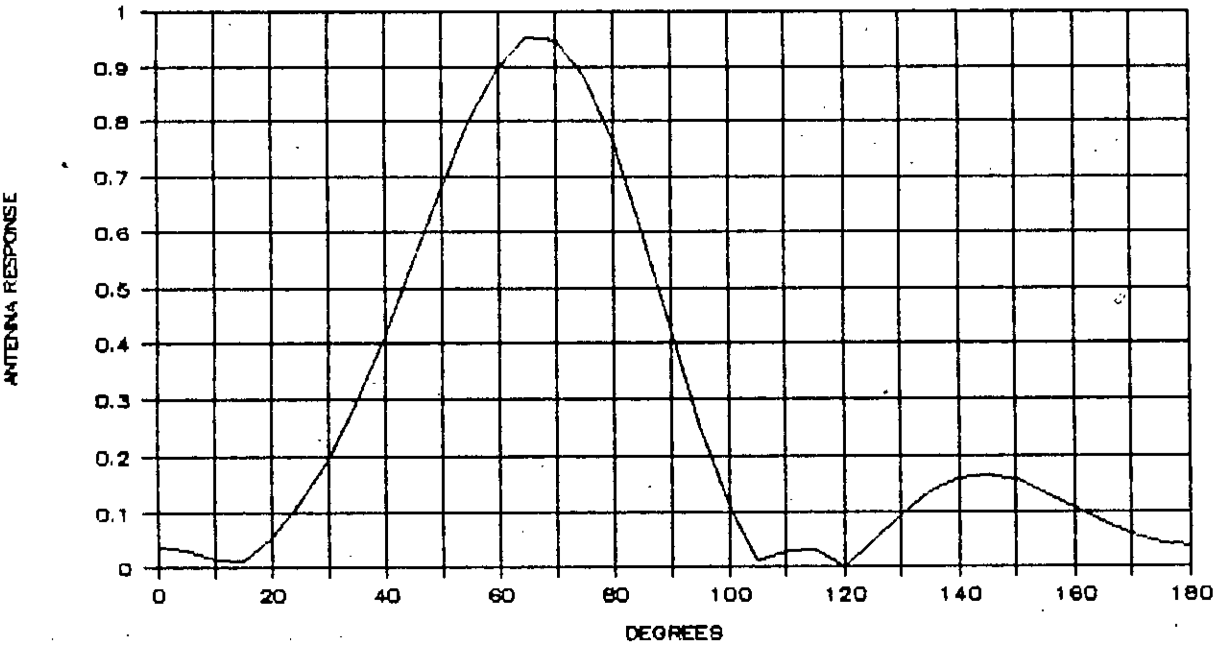


Fig.5.2.22: Response pattern of a 4 element array antenna with the look direction at 60 degrees and the noise direction at 120 degrees. Two sidelobe constraint levels of .15 and .05 are assigned at 140 and 105 degrees respectively; the tolerance level was assigned ± .01 for both the cases.

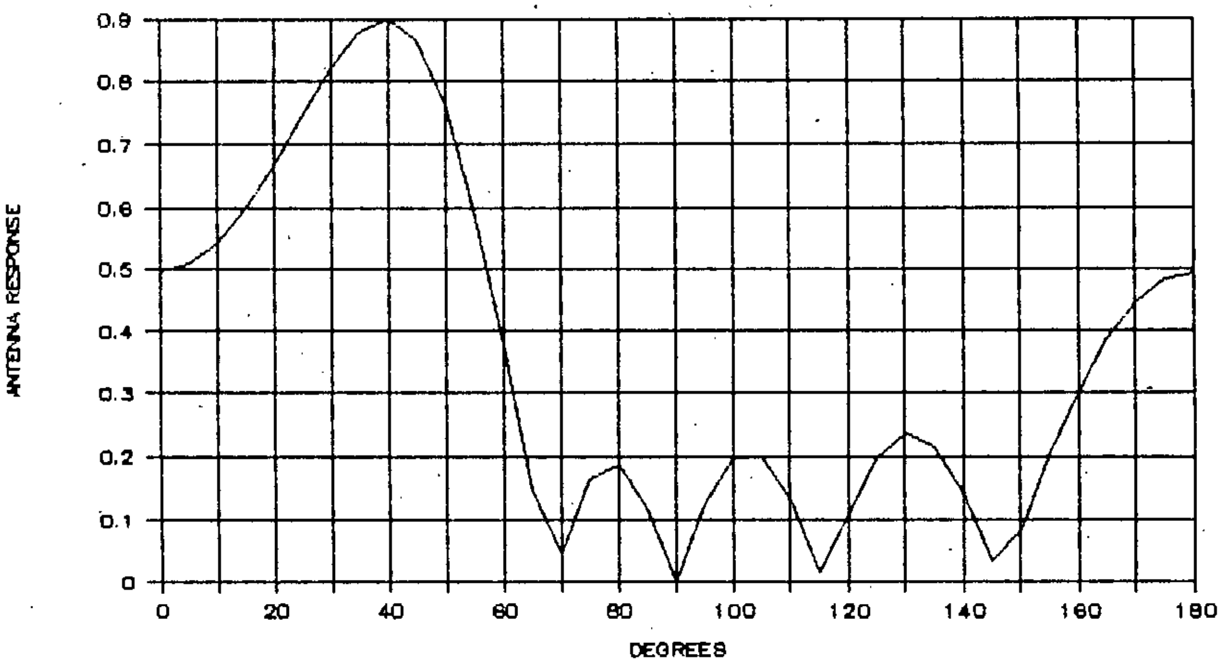


Fig.5.2.23: Response pattern of a 5 element array antenna with the look direction at 40 degrees and the noise direction at 90 degrees. No sidelobe constraint is assigned.

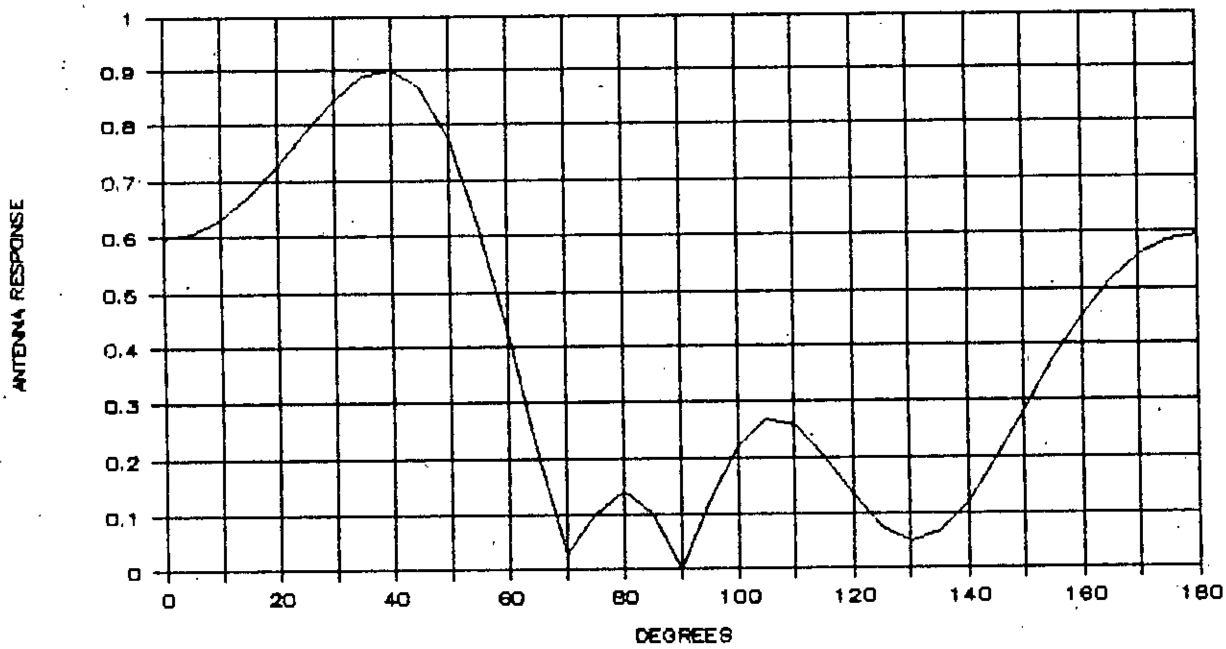


Fig.5.2.24: Response pattern of a 5 element array antenna with the look direction at 40 degrees and the noise direction at 90 degrees. One sidelobe constraint level of .1 (tolerance ± .05) is assigned at 130 degrees.

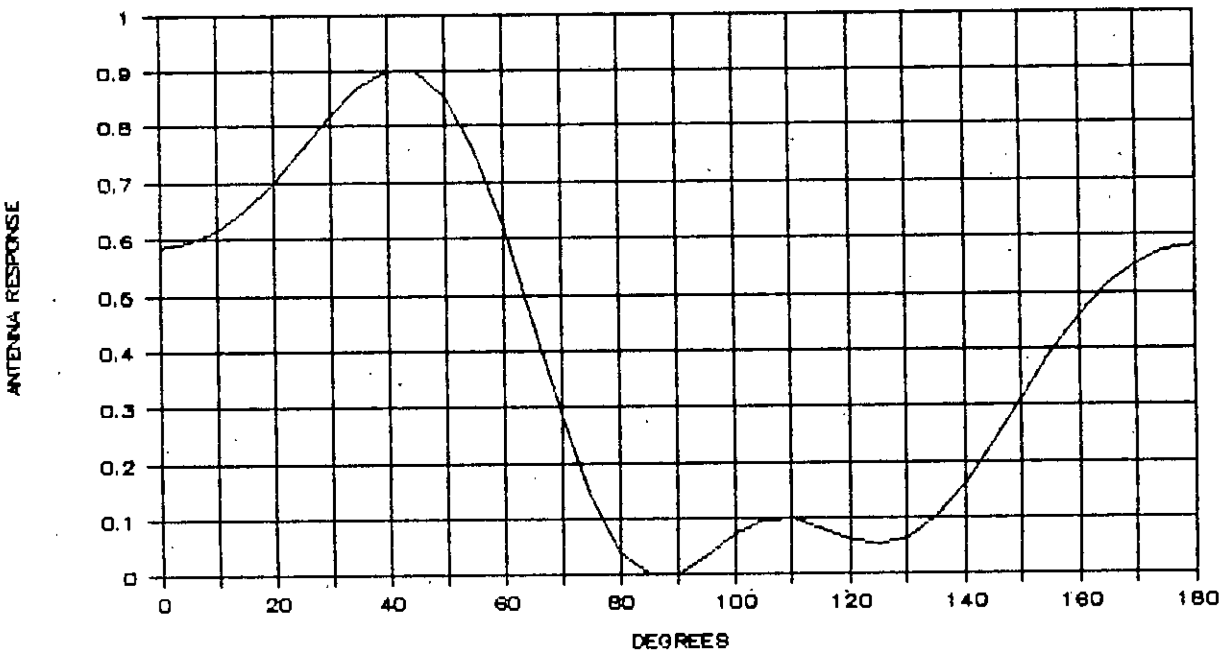


Fig.5.2.25: Response pattern of a 5 element array antenna with the look direction at 40 degrees and the noise direction at 90 degrees. Two sidelobe constraints of level 0.1 (tolerance ± .05) and .05(tolerance ± .01) are assigned at 130 and 80 degrees respectively.

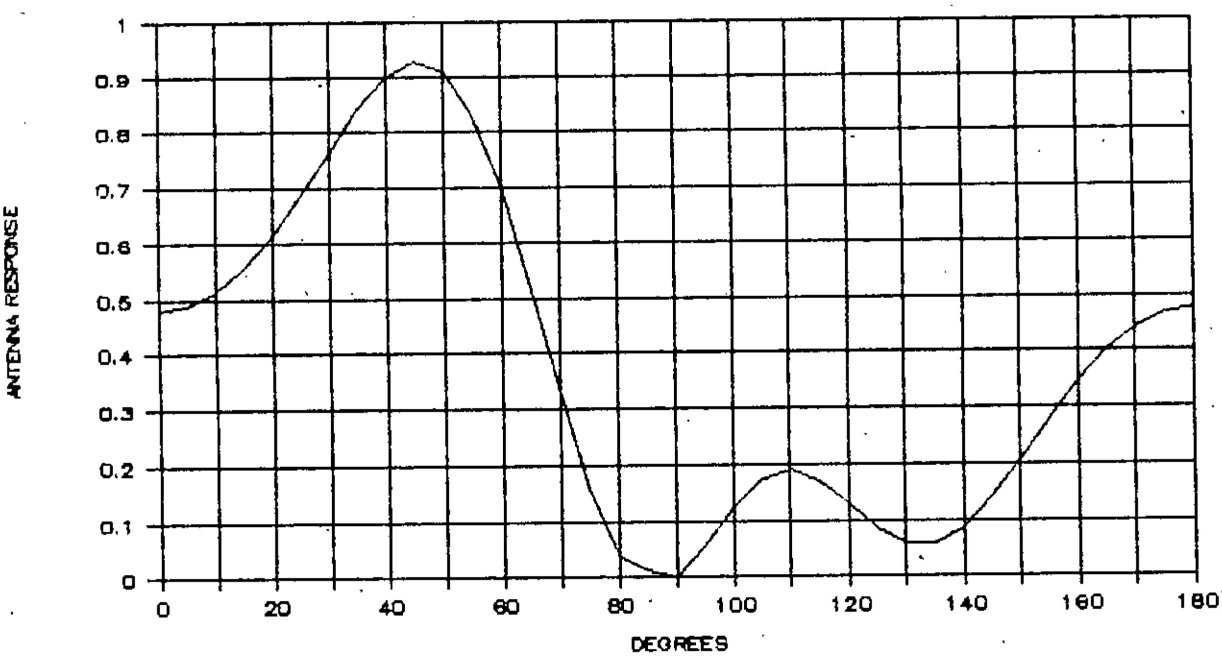


Fig.5.2.26: Response pattern of a 5 element array antenna with the look direction at 40 degrees and a noise direction at 90 degrees. Three sidelobe constraint levels of .1 (tolerance ± .05), .05(tolerance ±.01), and .4(tolerance ±.05) are assigned at 130, 80, and 170 degrees respectively.

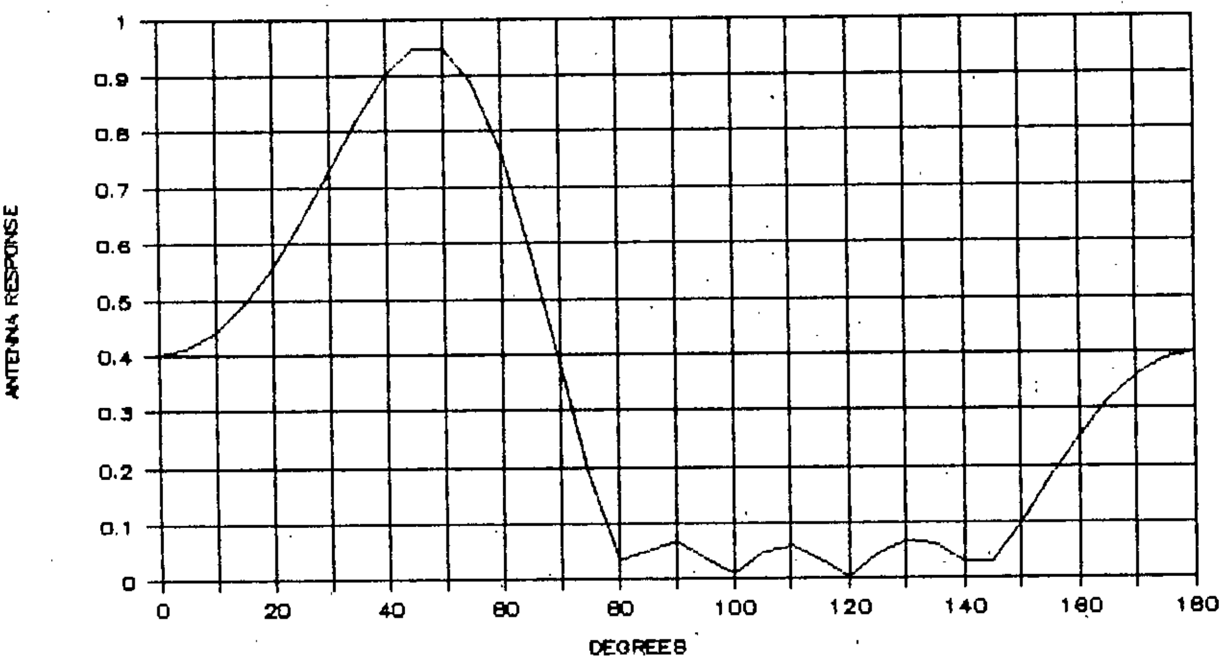


Fig.5.2.27: Response pattern of a 5 element array antenna with the look direction at 40 degrees and two noise directions at 90 and 110 degrees. Three sidelobe constraint levels of .1(tolerance ± .05),.05(tolerance ± .01) and .4 (tolerance ± .05) are assigned at 130, 80 and 170 degrees respectively.

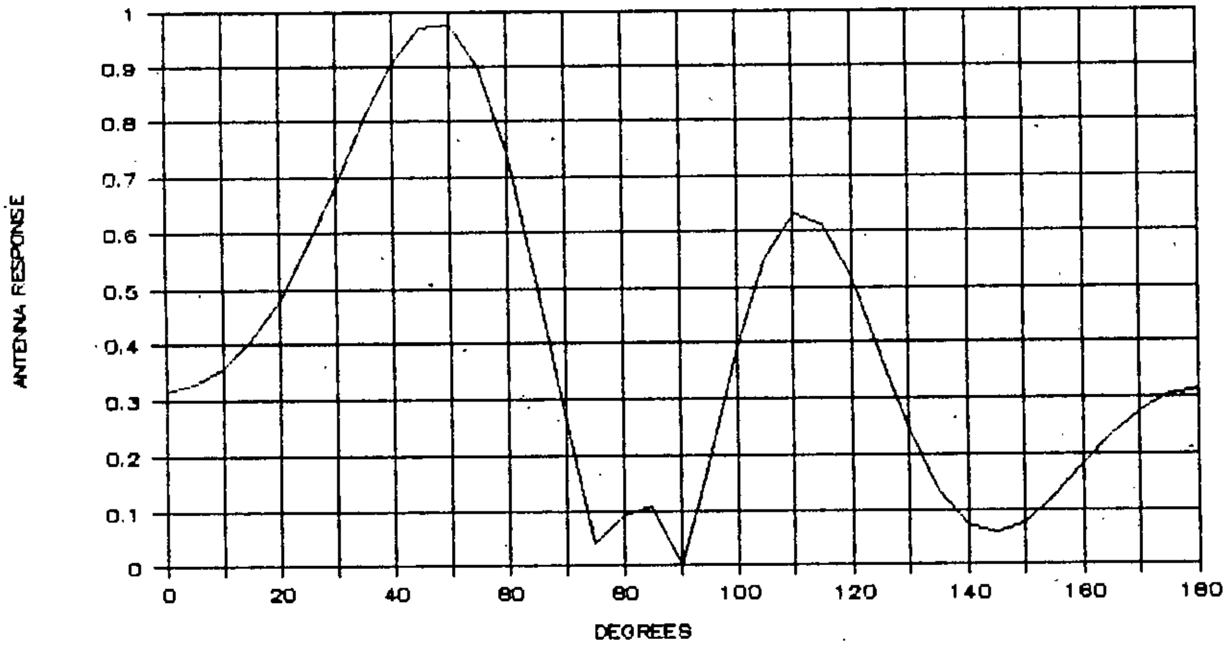


Fig.5.2.28: Response pattern of a 5 element array antenna with the look direction at 40 degrees and a noise direction at 90 degrees. 4 sidelobe constraint levels of 0.1 (tol.±.05), .05(tol.± .01), .4(tol. ±.05) and .1(tol. ± .01) are assigned at 130, 80, 170 and 110 degrees respectively.

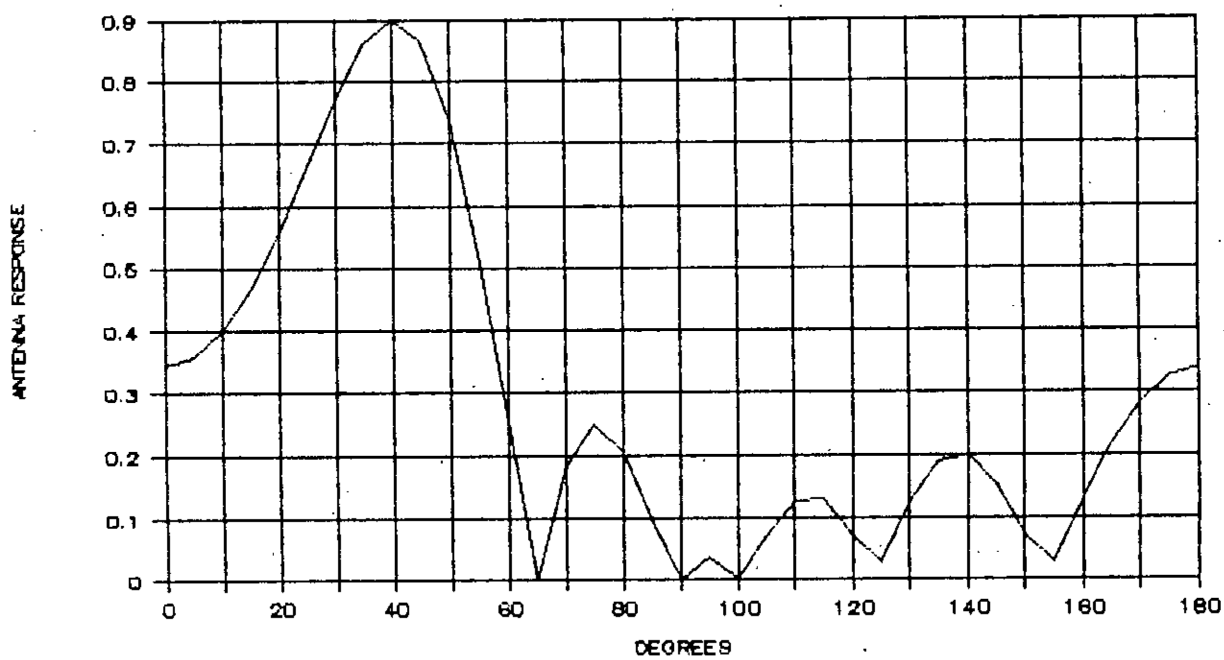


Fig.5.2.29: Response pattern of a 6 element array antenna with the look direction at 40 degrees and a noise derection at 90 degrees.

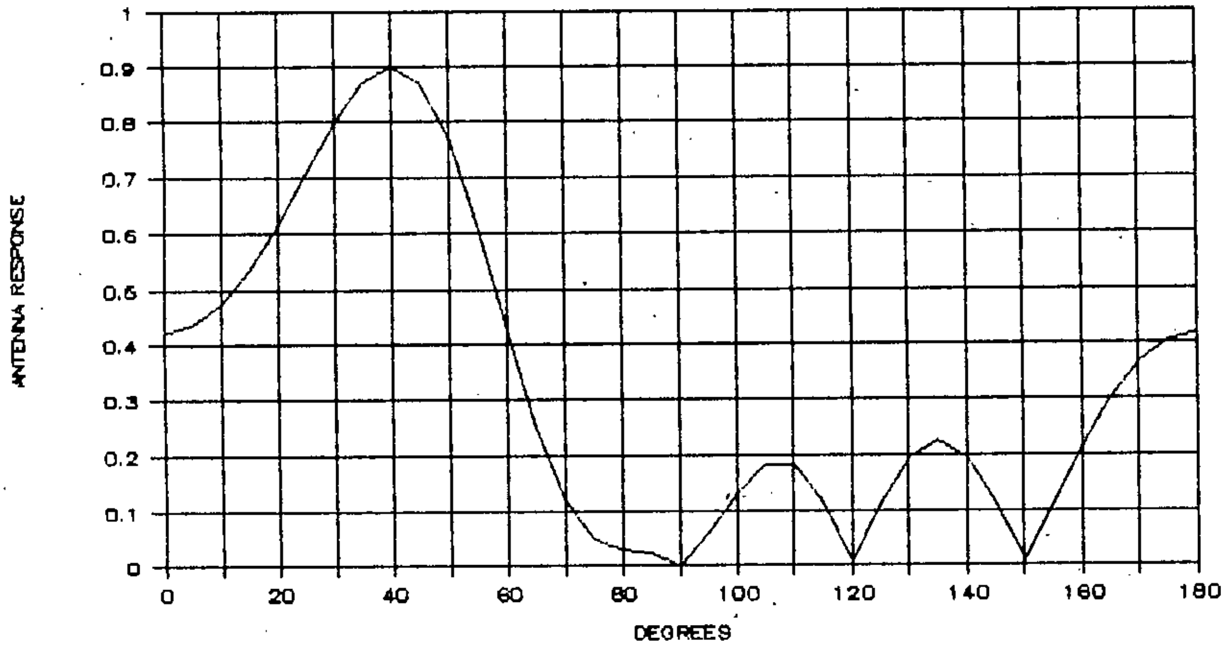


Fig.5.2.30: Response pattern of a 6 element array antenna with the look direction at 40 degrees and a noise direction at 90 degrees. One sidelobe constraint level of .1 (tolerance ± .05) is assigned at 75 degrees.

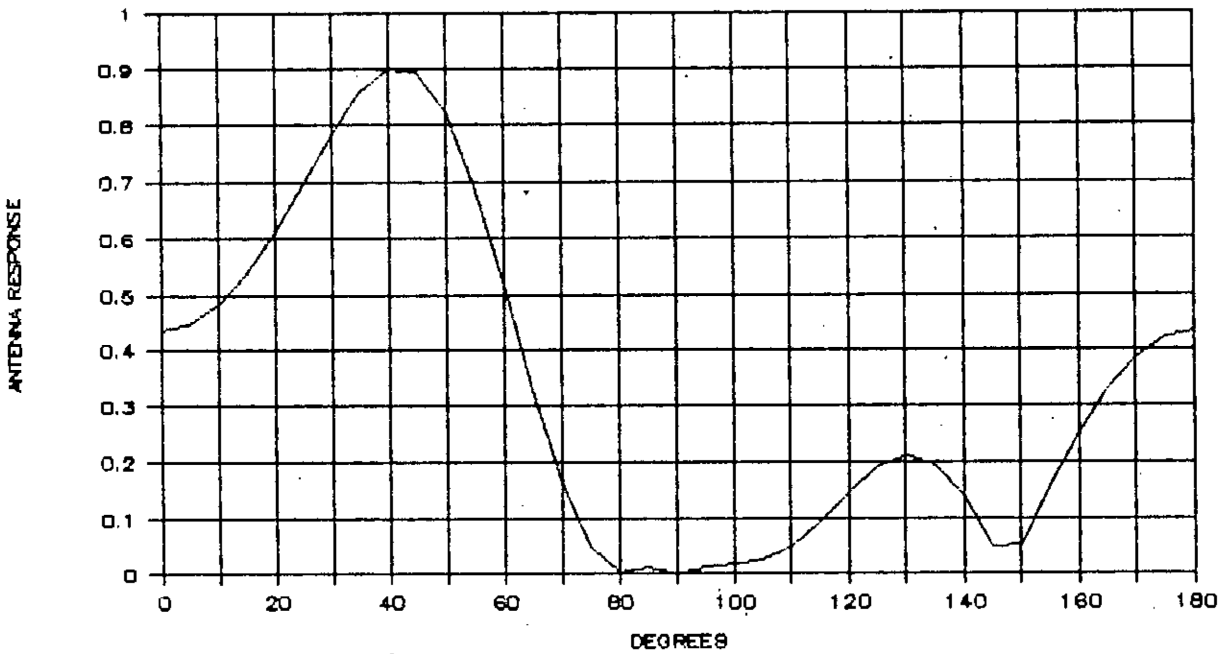


Fig.5.2.31: Response pattern of a 6 element array antenna with the look direction at 40 degrees and a noise direction at 90 degrees. Two sidelobe constraint levels of 0.1 (tolerance ± .05) and .08 (tolerance ± .05) are assigned at 75 and 108 degrees respectively.

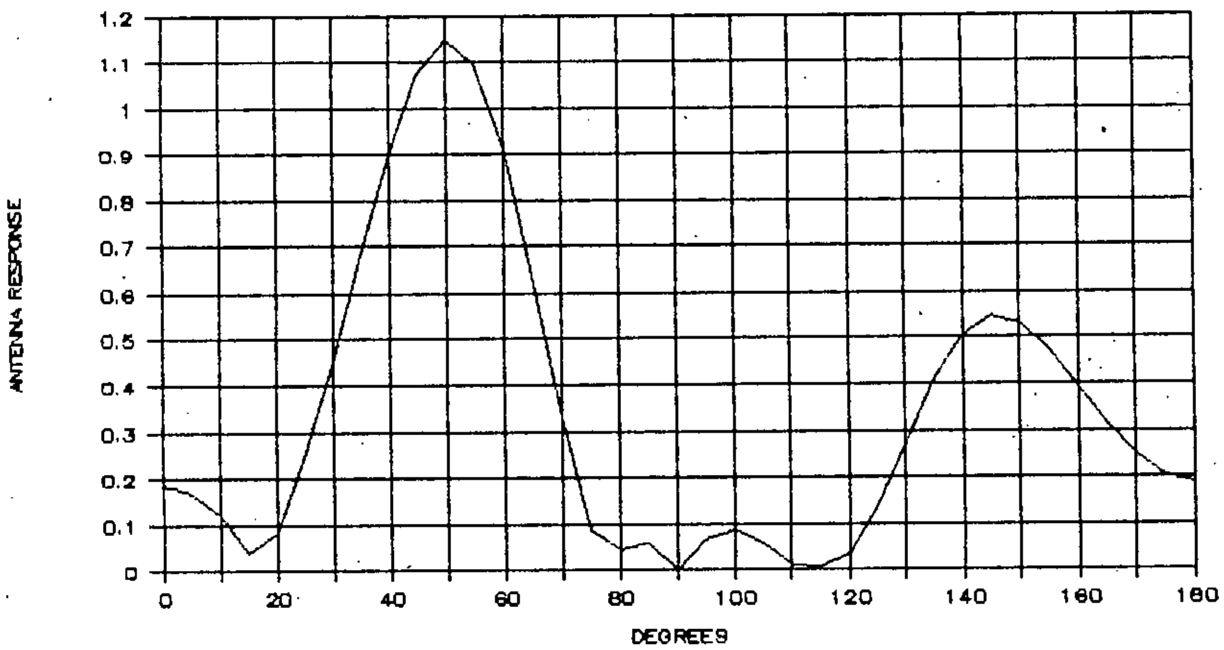


Fig.5.2.32: Response pattern of a 6 element array antenna with the look direction at 40 degrees and a noise direction at 90 degrees. Three sidelobe constraint levels of .1 (tol ± .05), .08(tol ±.05), .3(tol ±.05) are assigned at 75 108 and 170 degrees respectively.

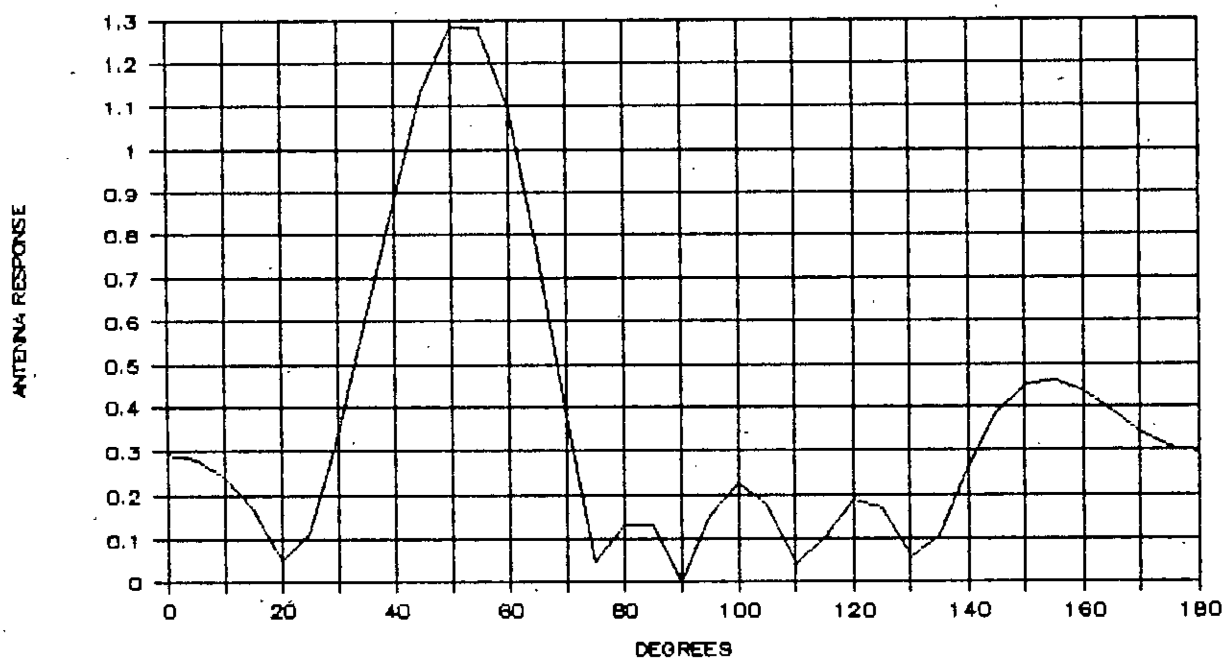


Fig.5.2.33: Response pattern of a 6 element array antenna with the look direction at 40 degrees and a noise direction at 90 degrees. Four sidelobe constraint levels of .1 (tol±.05), .08(tol ± .05), .3(tol ± .05) and .4(tol ±.02) are assigned at 75, 108, 170 and 150 degrees respectively.

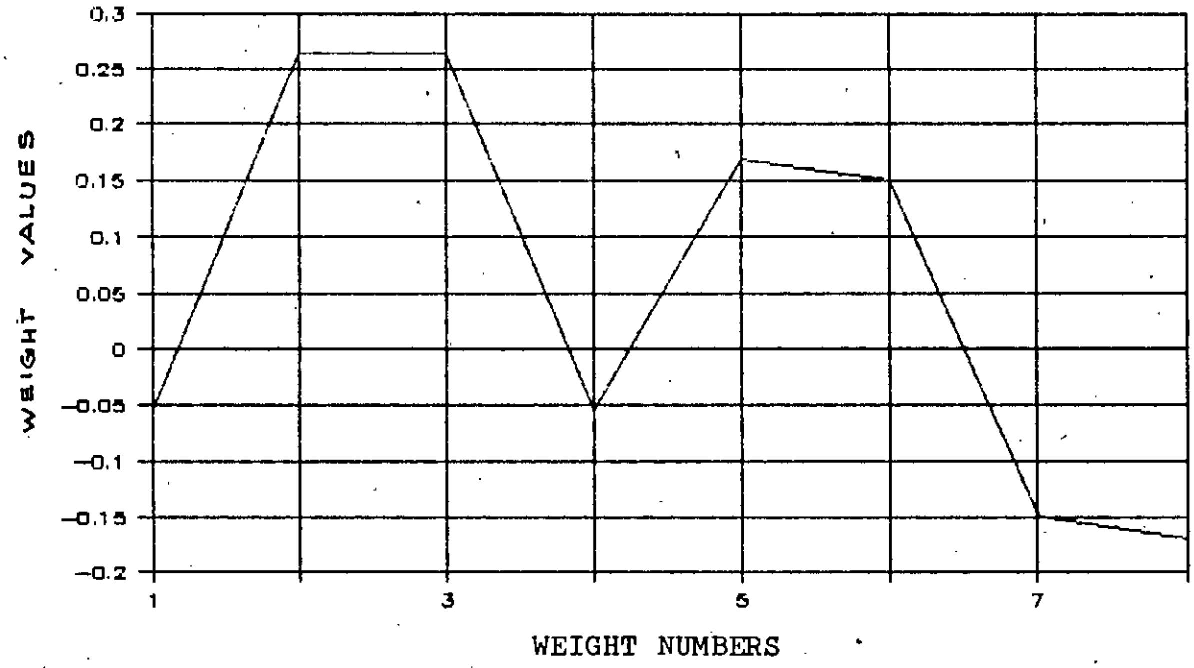


Fig.5.2.34: Tap-weight values for a 4 element array antenna having the look direction at 60 degrees, a noise direction at 120 degrees and two sidelobe constraints at 140 and 105 degrees.

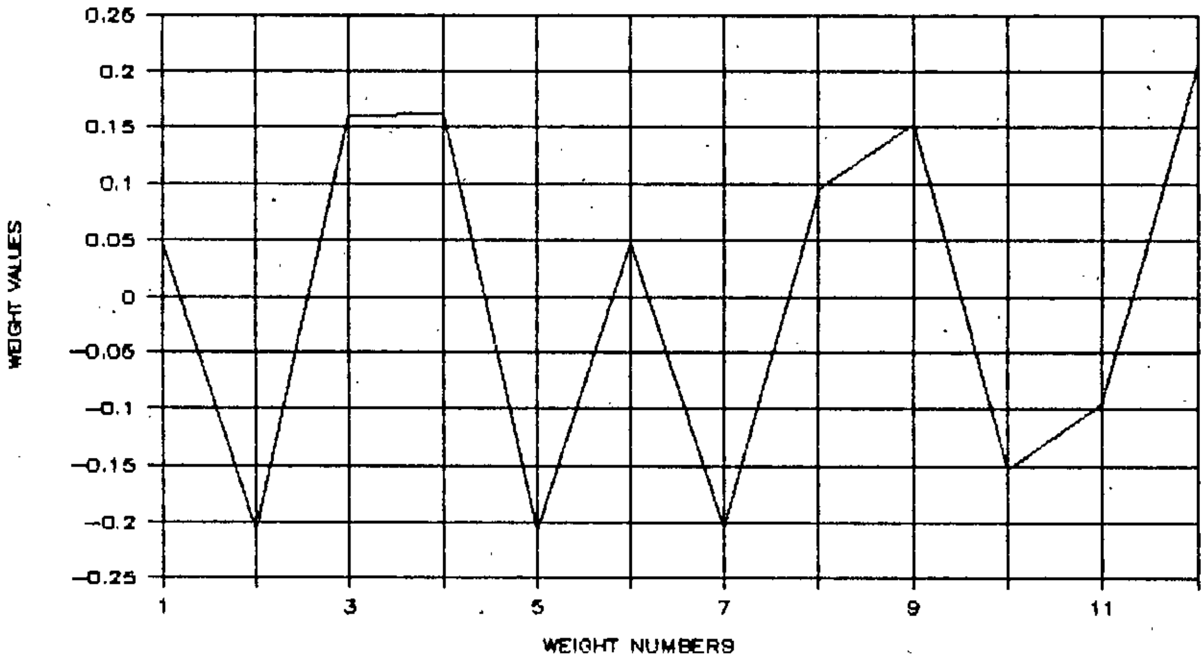


Fig.5.2.35: Tap-weight values for a 6 element array antenna having a look direction, a noise direction and 4 sidelobe constraints.

In all of our side-lobe constraint studies we put the desired values in the constraint directions less than the normal response values. We now see that the algorithm can give a satisfactory response pattern even if we desire a bigger response value than the normal value in the constrained directions. The corresponding simulation results are provided in Fig.5.2.36 through Fig.5.2.41.

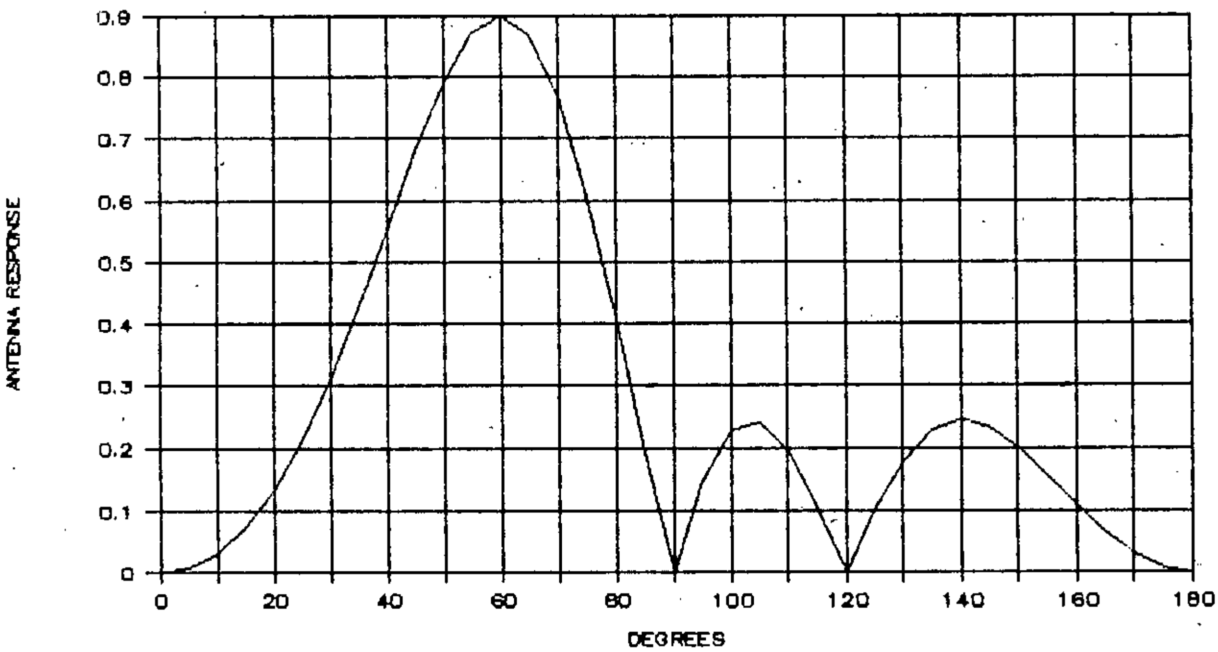


Fig.5.2.36: Response pattern of a 4 element array antenna with the look direction at 60 degrees and a noise direction at 120 degrees.

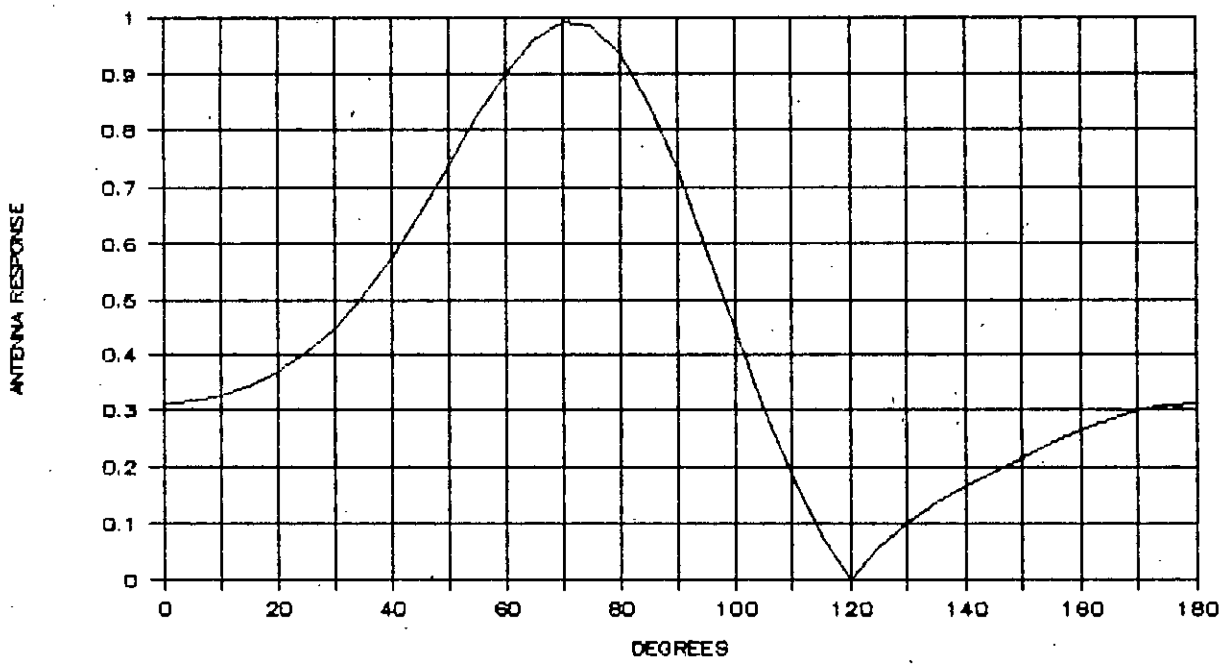


Fig.5.2.37: Response pattern the same 4 element array with a sidelobe constraint level of .3(tolerance ± .01) at 105 degrees; the previous response level of .24 is thus enhanced to the current value of .3 at 105 degrees.

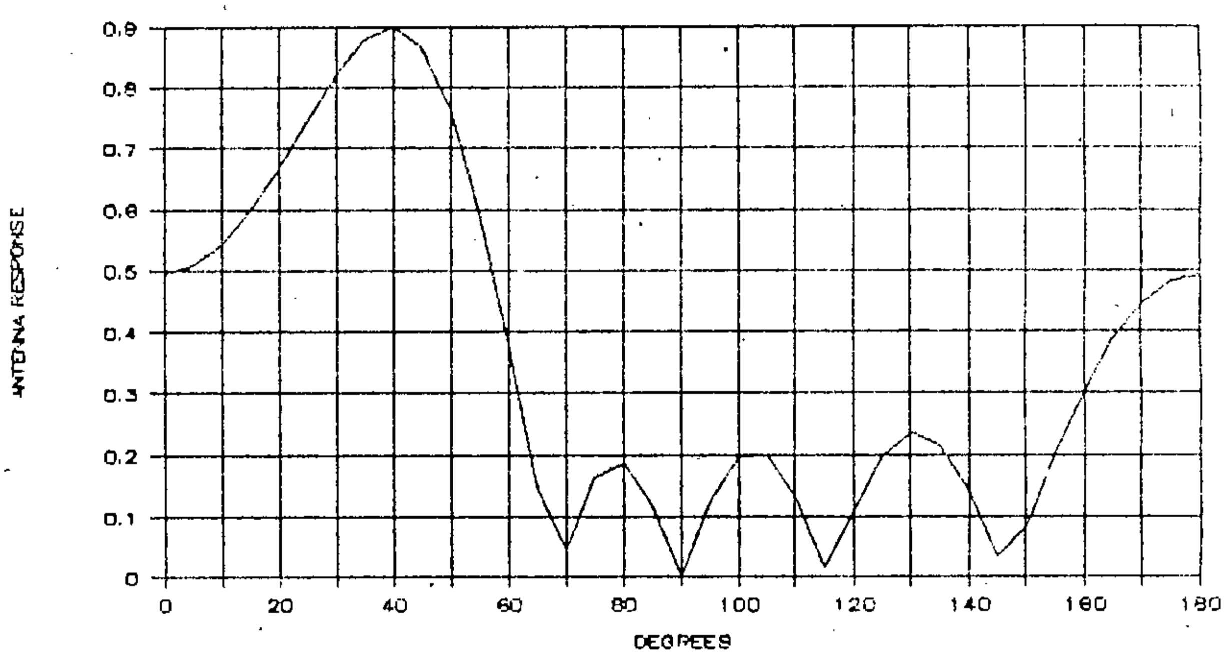


Fig.5.2.38: Response pattern of a 5 element array antenna with the look direction at 40 degrees and the noise direction at 90 degrees. No sidelobe constraint is assigned.

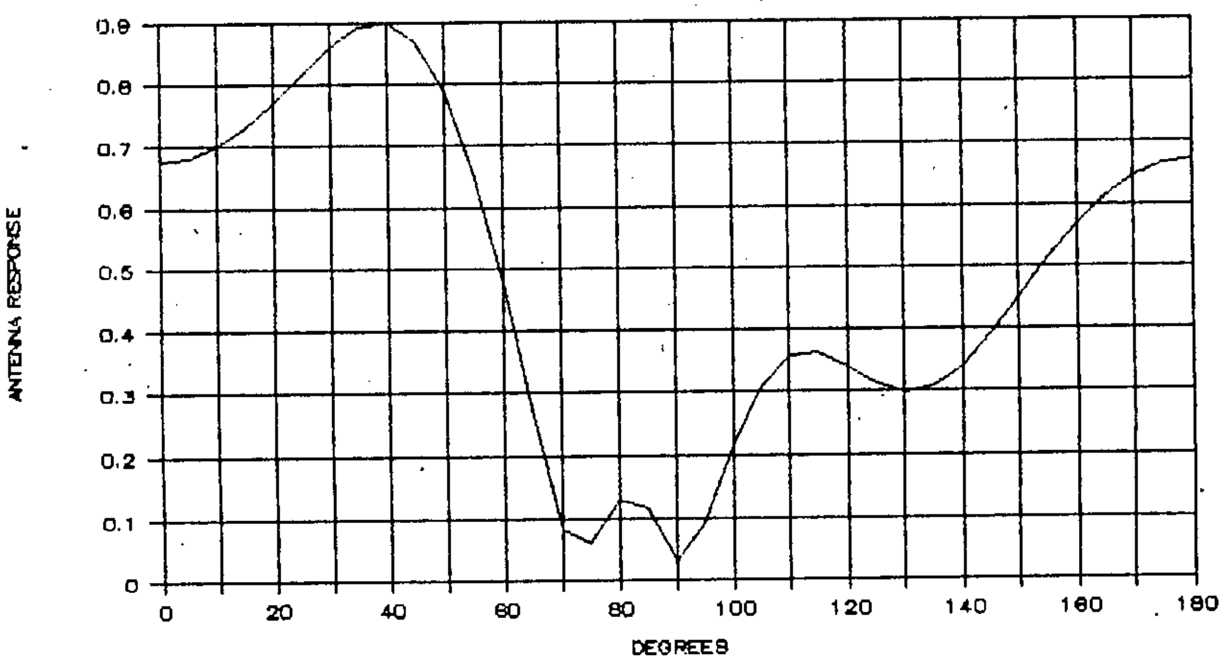


Fig.5.2.39: Response pattern of the same 5 element array with a sidelobe constraint level of 0.3(tolerance ±.01) at 130 degrees. The previous response level of 0.2 is thus enhanced to the current value of 0.3 at 105 degrees.

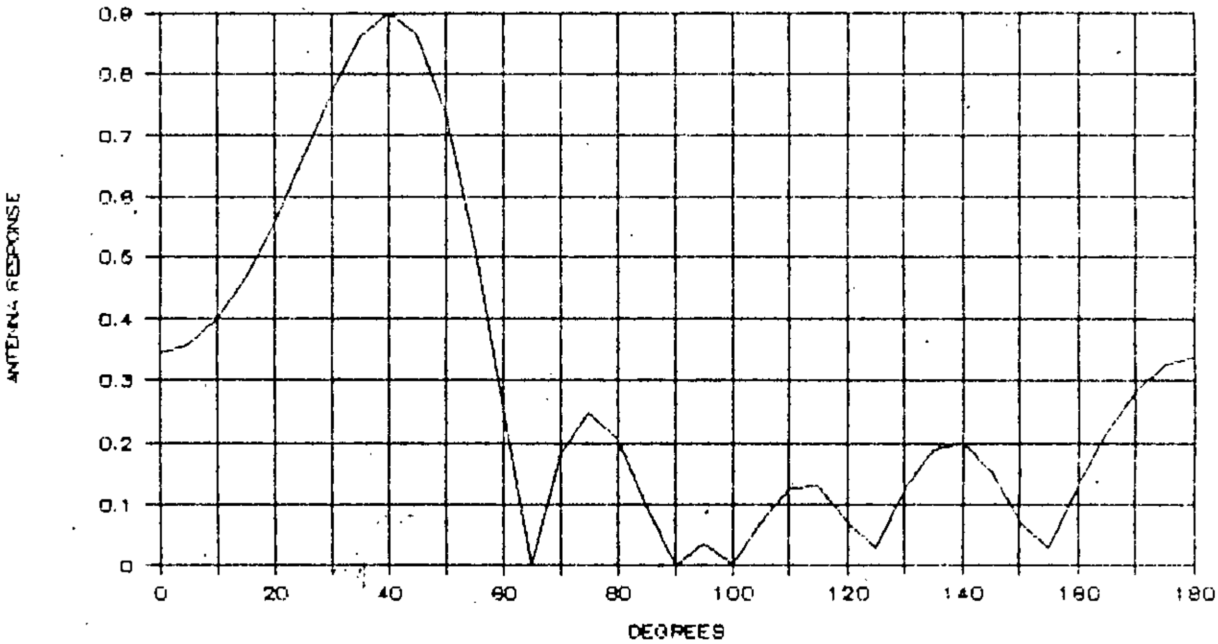


Fig.5.2.40 : Response pattern of a 6 element array antenna with the look direction at 40 degrees and a noise derection at 90 degrees.

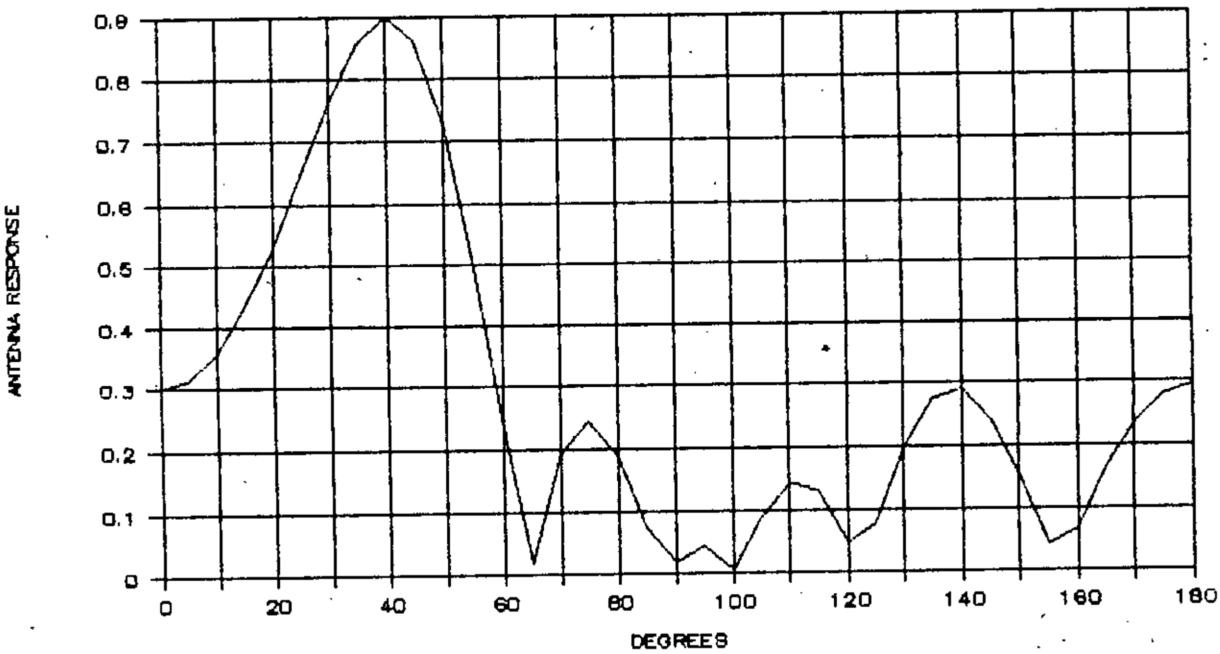


Fig.5.2.41: Response pattern of the same 6 element array with a sidelobe constraint level of 0.3(tolerance ±.01) assigned at 140 degrees. The previous response level of 0.2 is thus enhanced to the current value of .25 at 140 degrees.

. The convergence of the algorithm is checked through some simulations. To accomplish this, the output power ($=W^TRW$) is chosen as the quantity to determine the amount of convergence. The number of sidelobe constraints are varied for a specific number of antenna elements; The output power is then plotted for corresponding number of iterations with number of sidelobes as a parameter. Fig.s 5.2.42 through 5.2.44 represent such plots corresponding to four, five and six element antenna arrays.

It is observed that irrespective of the number of side-lobe constraints (of course within the degrees of freedom) and number of elements in the antenna array, the algorithm is stable and globally convergent.

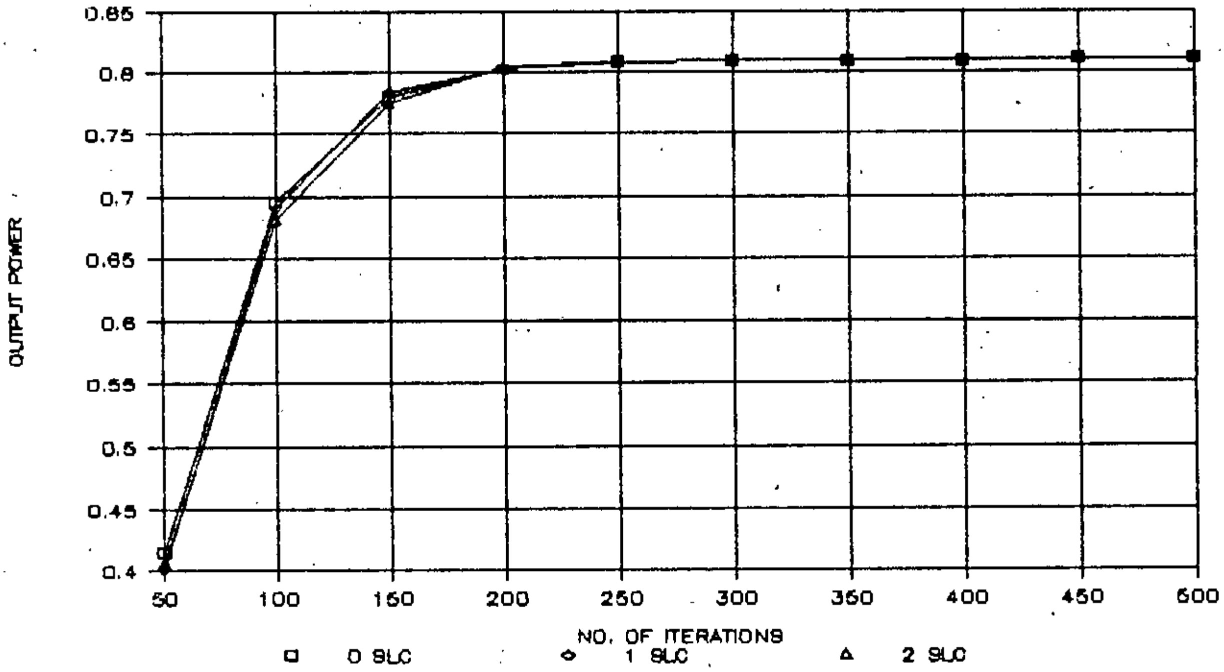


Fig.5.2.42: Output power vs number of iterations for a 4 element array antenna with 0, 1 and 2 sidelobe constraints.

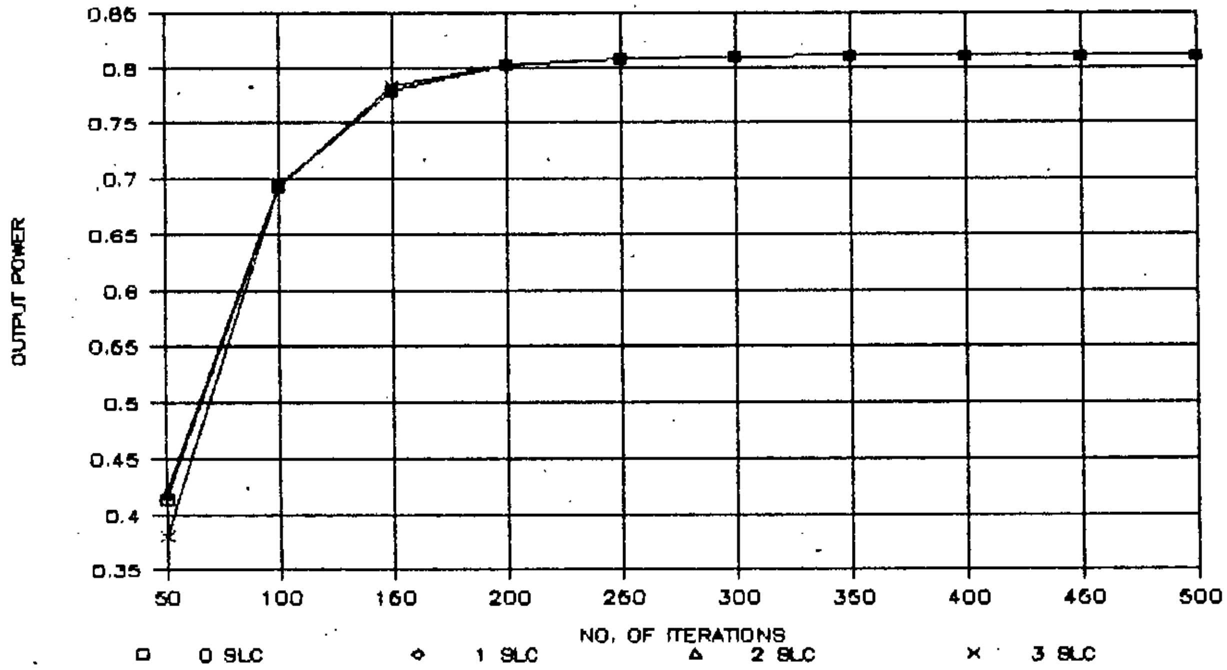


Fig.5.2.43: Output power vs number of iterations for a 5 element array antenna with 0, 1, 2, and 3 sidelobe constraints.

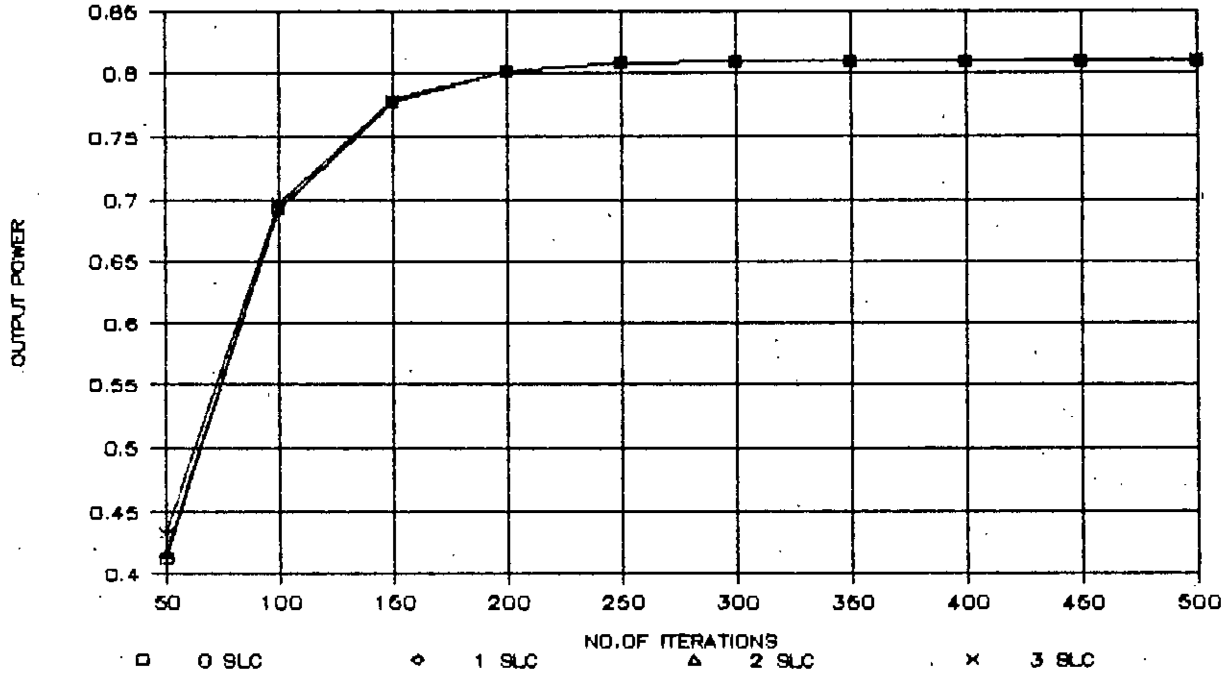


Fig.5.2.44: Output power vs number of iterations for a 6 element array antenna with the number of sidelobes varying between zero to three.

Uptil now we were concerned with the behaviour of the antenna array under pattern constraints i.e., putting some constraints on the side-lobe directions apart from the main-lobe in the response pattern of the antenna. In our study we intend to look into the antenna array processor behaviour under varying signal environment i.e., under the changing input signal/noise ratio condition.

The algorithm used for simulation is expected to produce the desired response by placing deep nulls in the noise directions and at the same time maintaining a pre-assigned level at the signal direction or look direction. However in doing so, the response of array is expected to be negligibly affected by the level of input SINR. Because, due to the presence of the distinct nulls at the noise directions (Fig.5.2.45 to Fig.5.2.47), the influence of incoming noise power level on the output SINR is simply insignificant. Thus, with both high and low input SINR s, the output SINR should remain more or less the same. The simulations that follow are carried for low and high input SINR s. The resulting patterns and their corresponding output SINR s (Fig.5.2.48) are seen to tally with our expected performance. One question in this regards however remains to be answered. And this is, "How does low and high input SINR s affect the output SINR if the actual signal is slightly deviated from the assumed signal direction?" . This phenomena is investigated in a later simulation.

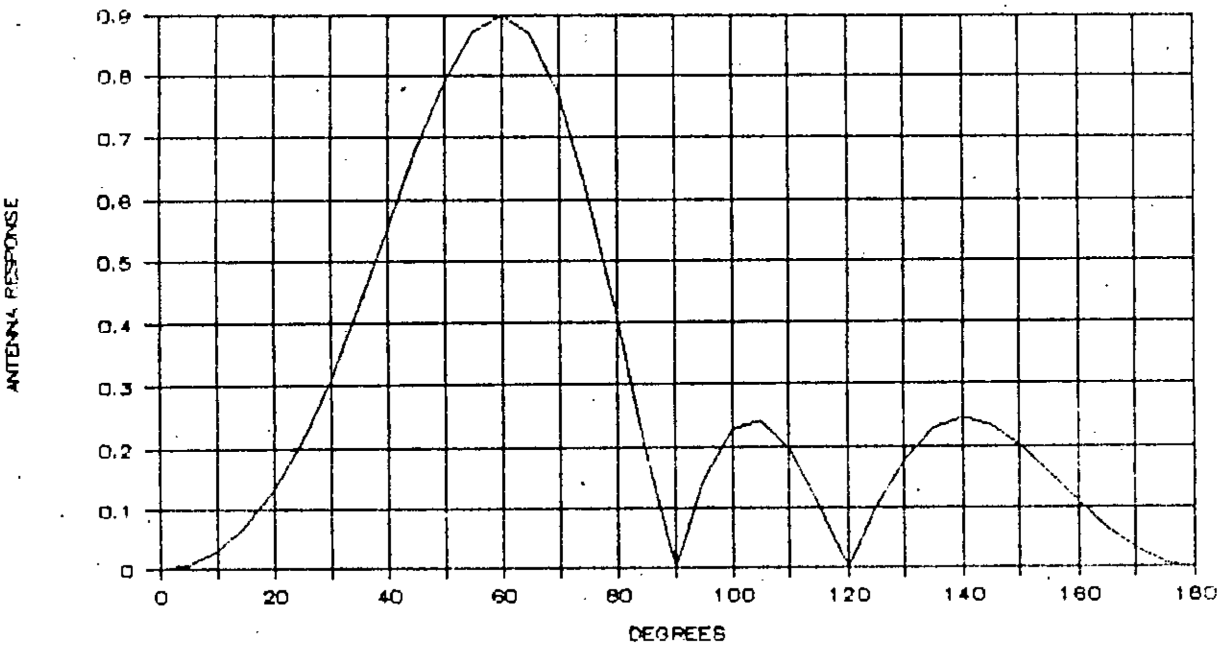


Fig.5.2.45: Response pattern of a 4 element array antenna with the look direction at 60 degrees and a noise direction at 120 degrees. No sidelobe constraints are assigned.

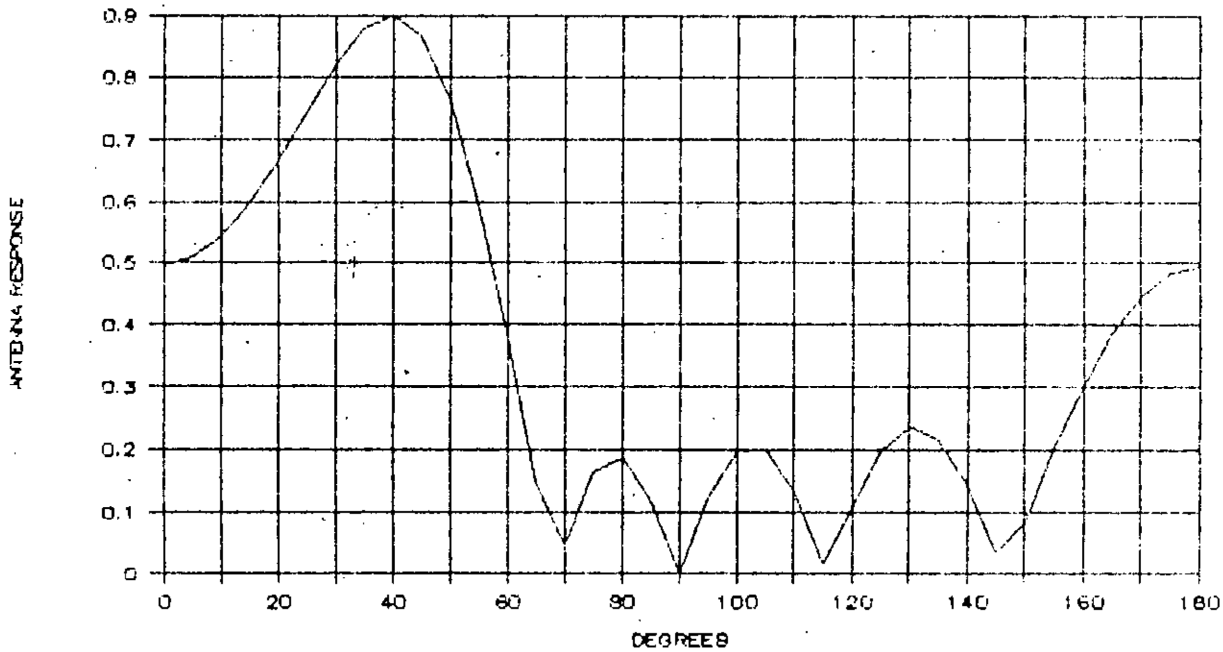


Fig.5.2.46 : Response pattern of a 5 element array antenna with the look direction at 40 degrees and the noise direction at 90 degrees. No sidelobe constraint is assigned.

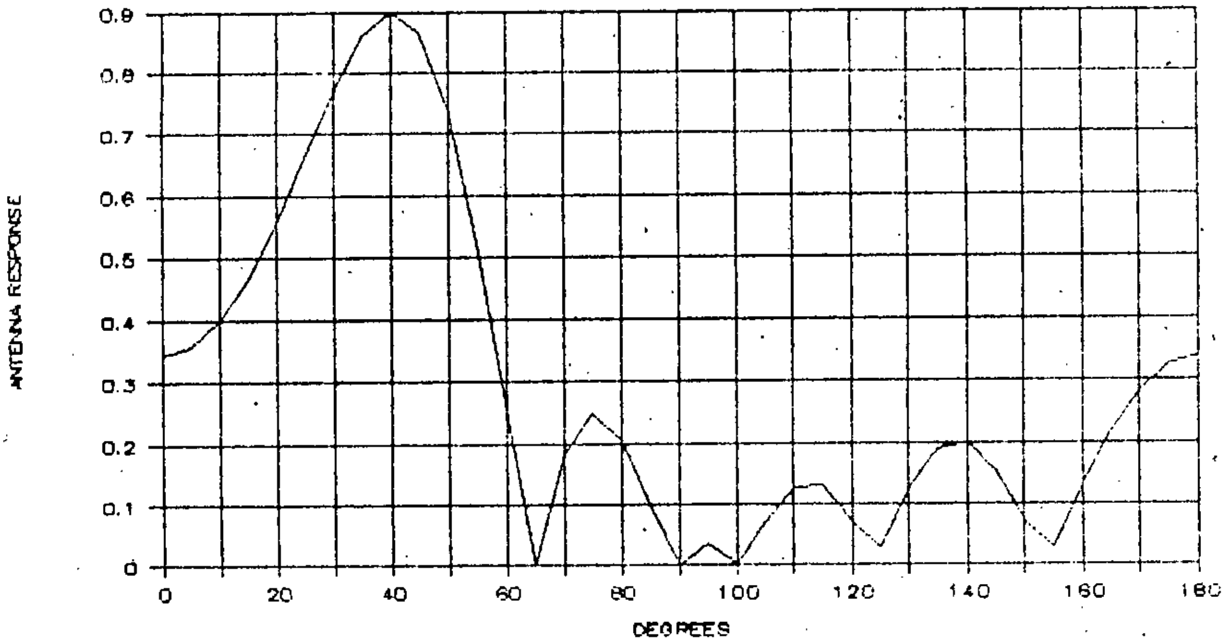


Fig.5.2.47: Response pattern of a 6 element array antenna with the look direction at 40 degrees and a noise derection at 90 degrees.

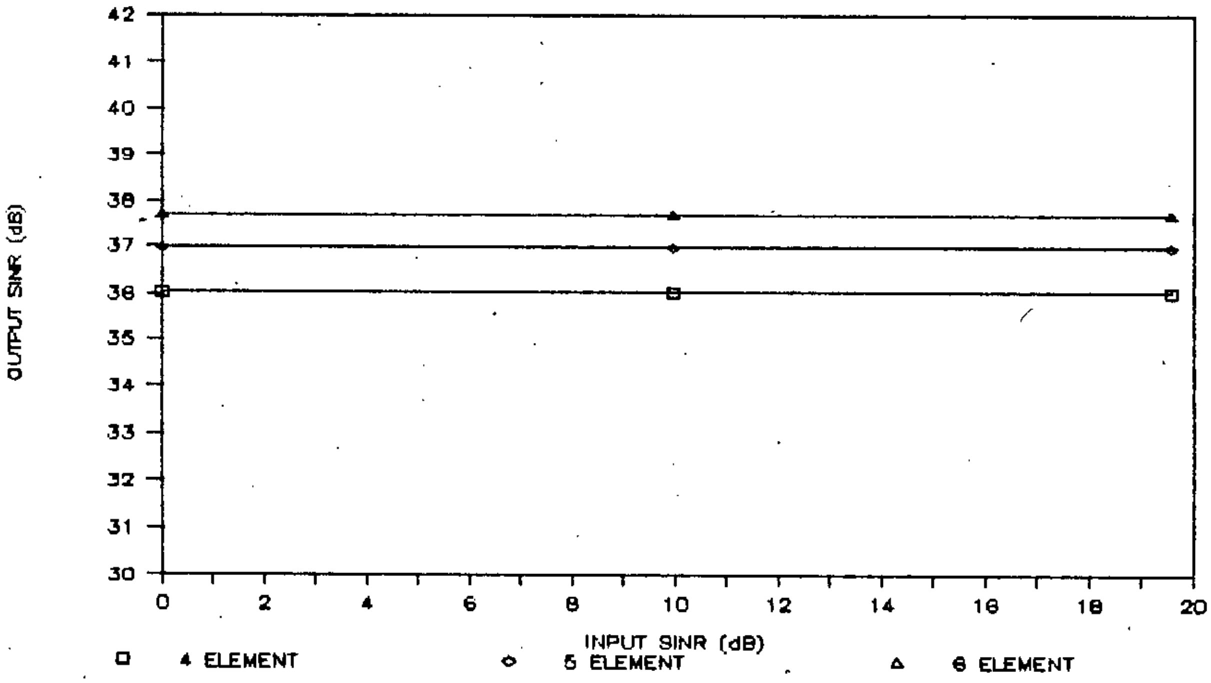


Fig.5.2.48: Output SINR vs input SINR for 4, 5 and 6 element array antenna.

The simulations that follow are used to study the situation when the signal arrival direction does not match with the assumed constrained direction of desired signal arrival. We call this mismatch of angles as "Pointing error" and denote it by δ . From the curves of Fig.5.2.49, it can be seen that in the absence of any mismatch between constrained main-lobe direction and the actual direction of the desired signal arrival (i.e., when δ =0), there is no variation of output SINR with change in input SINR. This is because, when a deep null is placed in the response pattern along a noise direction [Fig.5.2.50], noise level can have little or no effect on the output SINR.

But it can be seen from Fig.5.2.49, that there is a significant degradation in output SINR when the pointing error has a non-zero value. Due to the mismatch between the actual direction of the desired signal and the constrained direction of the main lobe, the algorithm treats the signal as another interfering noise and strives to cancel its influence by placing an additional null in its direction. This fact is evident from the response pattern of Fig.5.2.51) which is drawn for a 4 element array with the main-lobe constrained direction at 60 degrees, noise direction at 120 degrees, and the actual direction of desired signal arrival at 59 degrees. In this Fig. it can be seen that due to a mismatch of 1 degree, a null is approached at around 59 degrees. However, deep null could not be achieved due to the constrained main lobe response at 60 degrees.

It can further be seen from Fig.5.2.49, that in the presence of angle deviation, the output SINR gradually decreases with the increase of input SINR or with the relative weakening of the noise. This seems to be puzzling at first sight but is evident from the fact that, with the decrease of input interference power level the algorithm attaches less importance in cancelling it and thus deep null is no longer placed in this direction. This can be further verified from the response patterns of Fig.5.2.51 and 5.2.52 which are drawn for 1 degree pointing error with interference levels of 1.0 and 0.01 respectively; the interference direction is fixed at 120 degrees for both the cases. Due to the presence of a weaker noise the deep null which is present at 120 degrees in Fig.5.2.51 is no longer produced in Fig.5.2.52 Thus the output SINR also decreases from 15.507 dB to 10.873dB and the corresponding curve in Fig.5.2.49 has a downward trend with increasing input SINR.

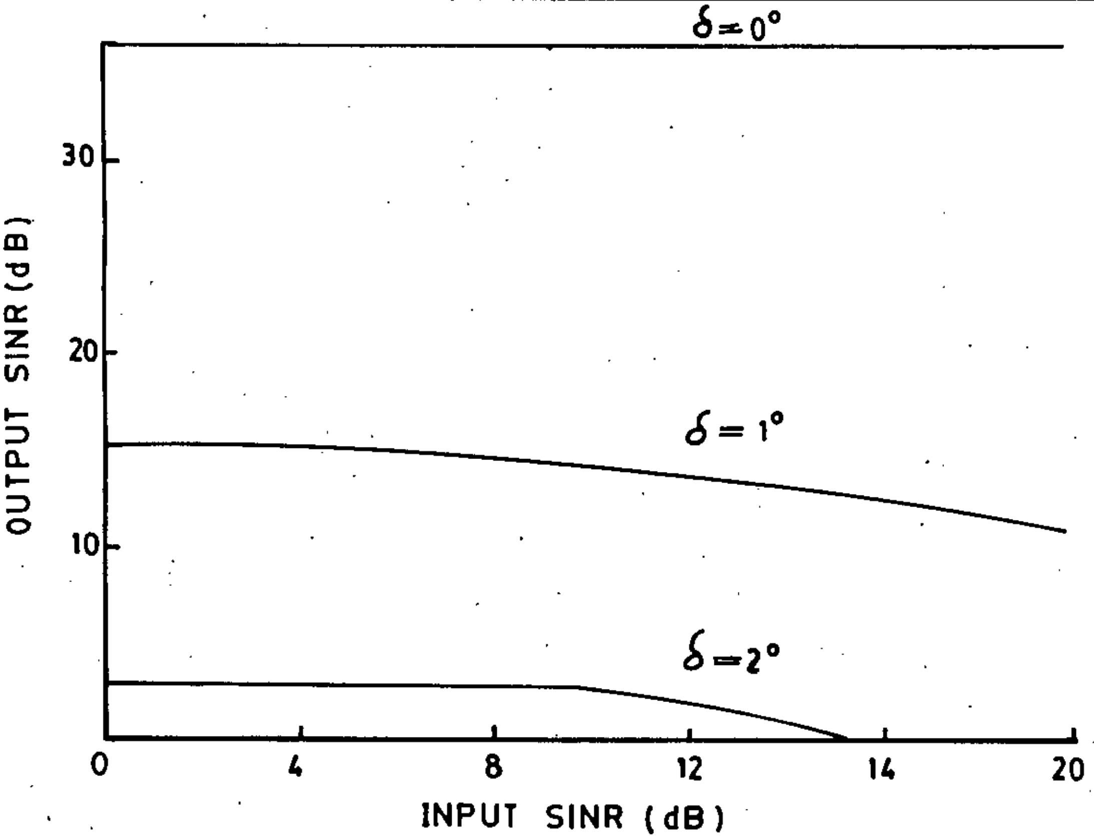


Fig. 5.2.49.: VARIATION OF OUTPUT SINR WITH CHANGE IN INPUT SINR IN PRESENCE OF 0,1 AND 2 DEGREES POINTING ERROR.

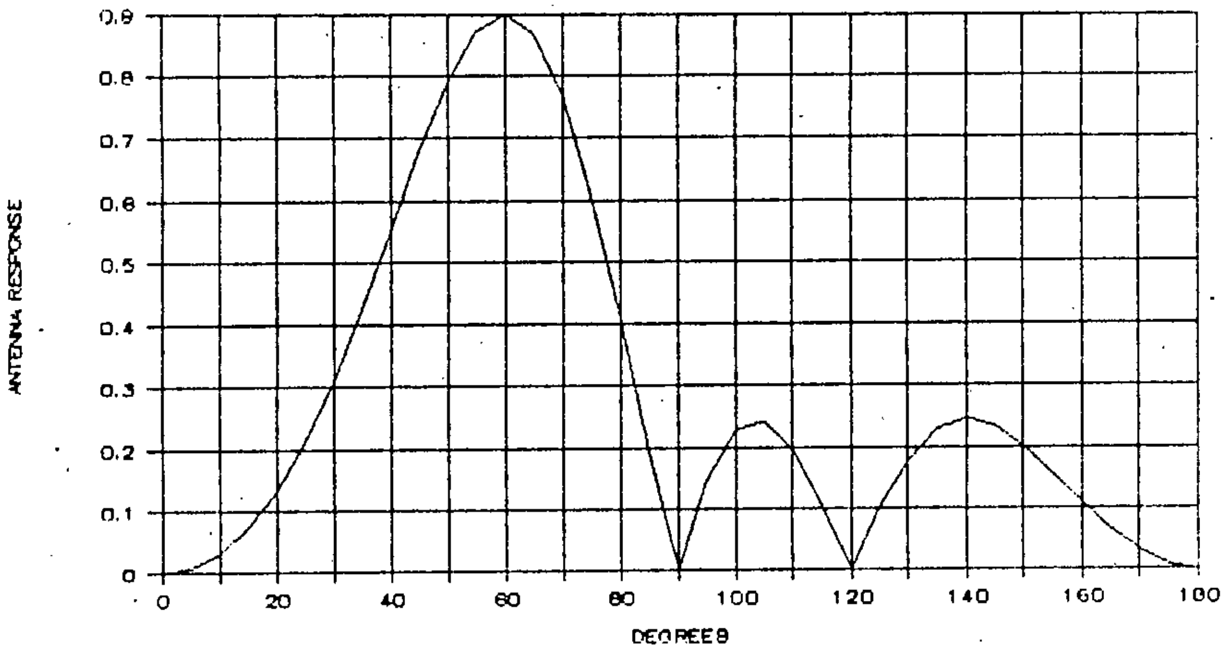


Fig.5.2.50: Response pattern of a 4 element array antenna with the look direction at 60 degrees and a noise direction at 120 degrees. No sidelobe constraints are assigned.

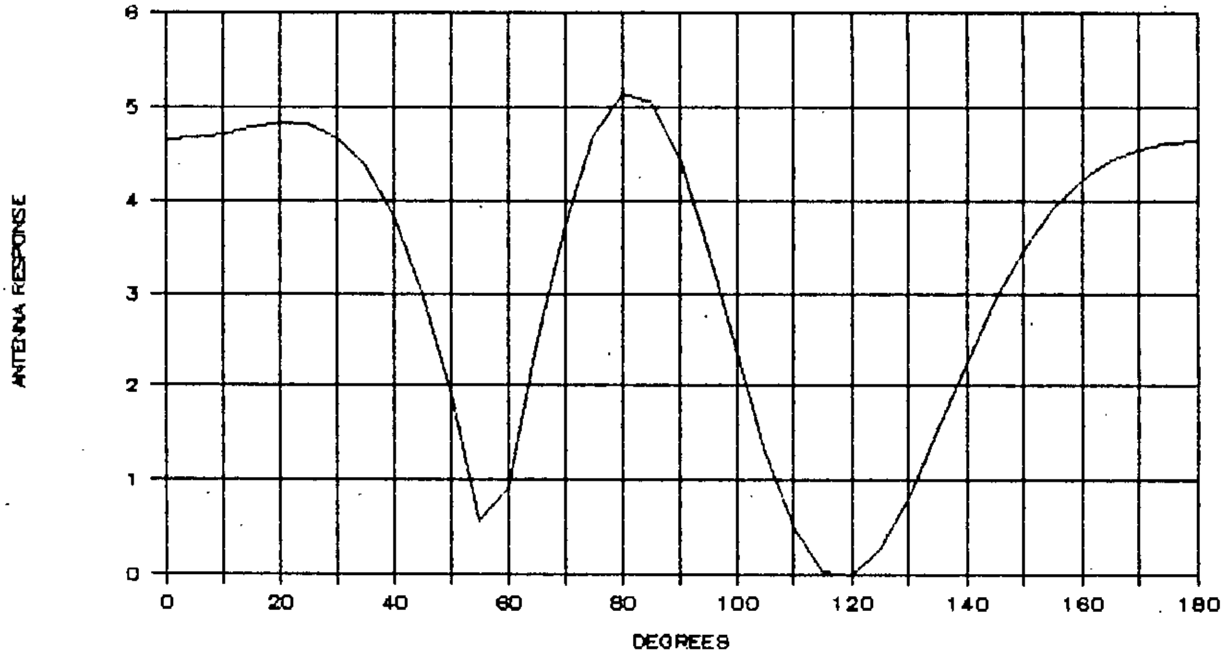


Fig.5.2.51: Response pattern of the same 4 element array with the look direction remaining fixed at 60 degrees and the noise direction at 120 degrees. The actual signal arrival direction is howvere 59 degrees (i.e., $\delta = 1^{\circ}$); the signal and the noise power levels are both assigned equal values of 1.0.

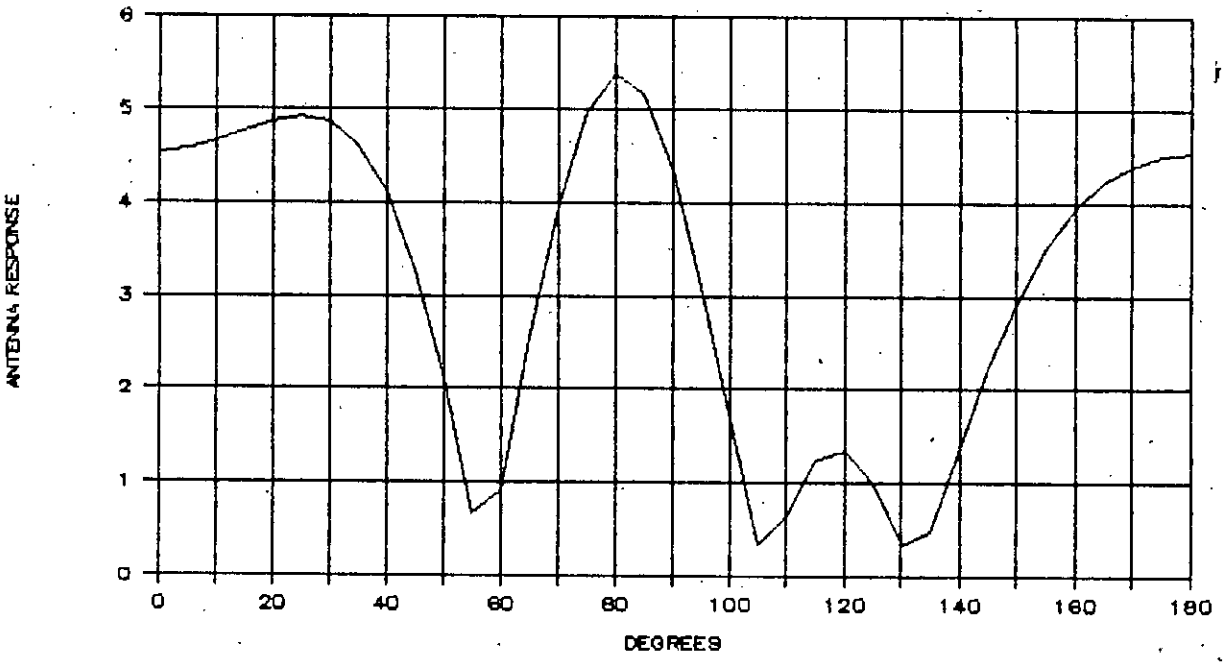


Fig.5.2.52: Response pattern of the 4 element array having look direction at 60 degrees, noise at 120 degrees with the pointing error being 1 degree. The signal power level is 1.0 and the noise power level is 0.01.

Using the same 4 element, two tap system, we now calculate the effect of input SNR i.e., signal to system noise ratio, on the output SINR while varying the pointing error δ . From Fig.5.2.53 it can be seen that, with a high input SNR, i.e., weak system noise, the output SINR is sensitively affected by the error of the constraint direction. With stronger system noise, however, the decrease in output SINR is more or less the same for various pointing error conditions.

Another obvious fact reveals itself from the graph of Fig.5.2.53 When δ =0, the output SINR is seen to continuously increase with the gradual decrease of input system noise. This behaviour is rather expected, since due to zero mismatch, the desired signal is properly accepted by the array and the output SINR should improve or deteriorate with the decrease or increase of system noise level as the interference signal has already been rejected due to placement of deep null in its direction. This sharp dependence of output SINR with the system noise level for zero mismatch condition becomes even more vivid when we draw a graph having the above two quantities as ordinate and abscissa respectively. This is shown in Fig.5.2.54.

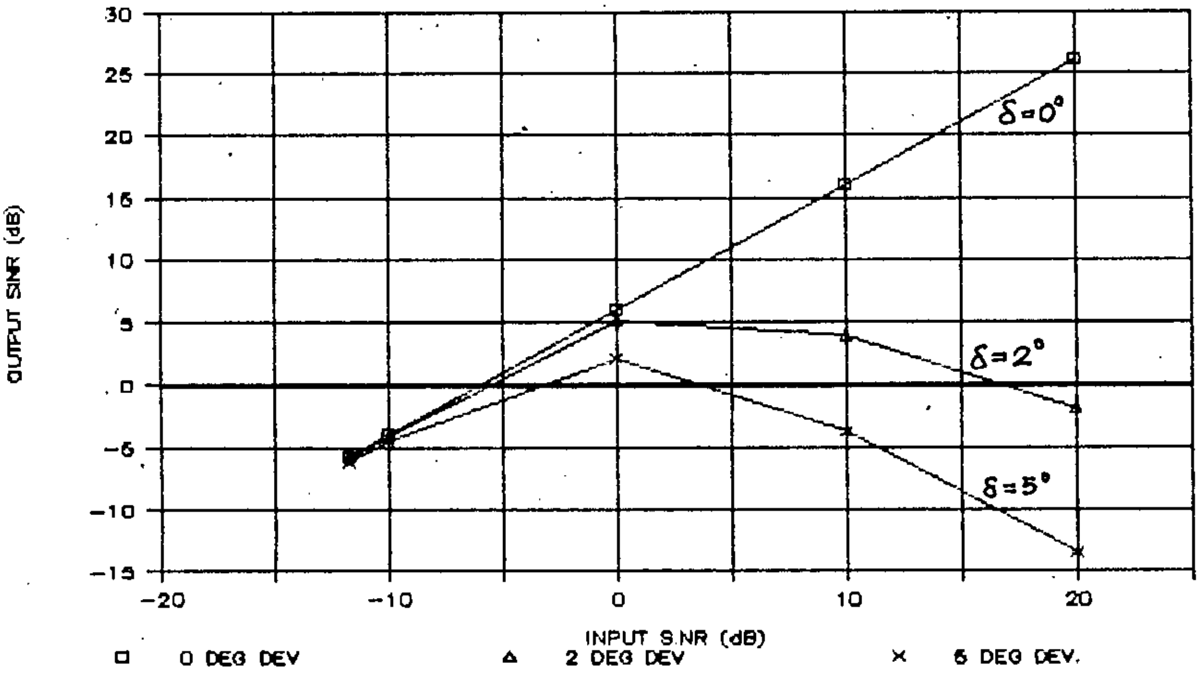


Fig.5.2.53: Variation of output SINR with change in input SINR for a 4 element array antenna. Pointing error is also seen to be present.

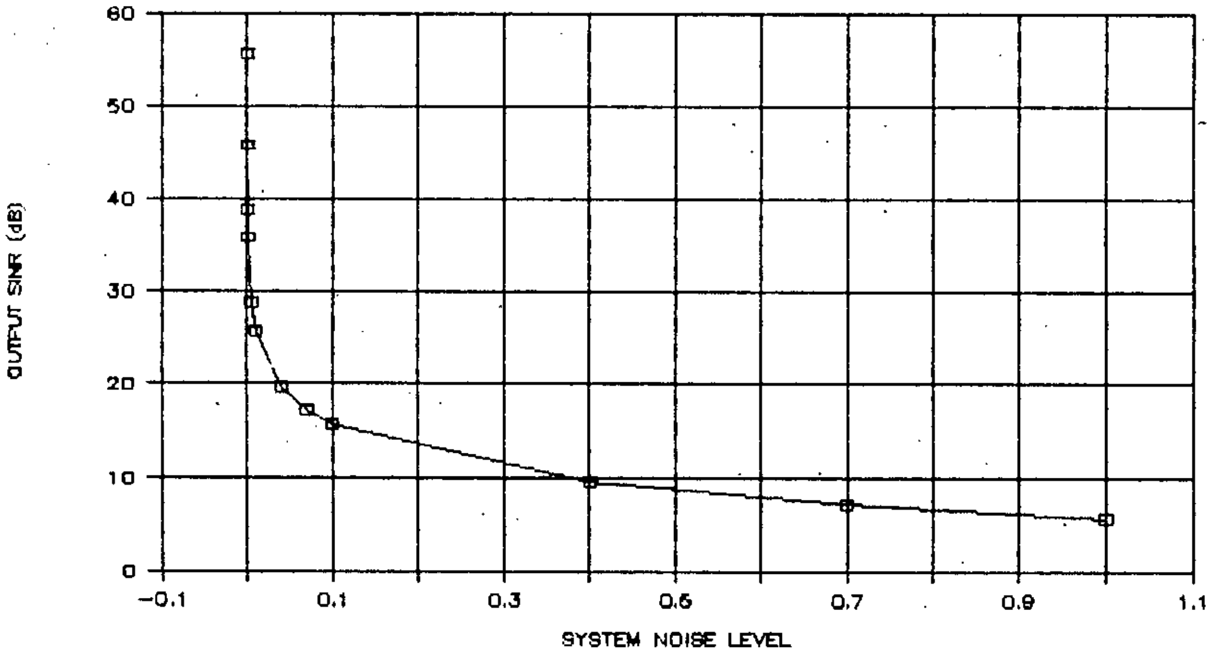


Fig.5.2.54: Variation of output SINR with change in system noise level for a 4 element array antenna.

CHAPTER 6

CONCLUSIONS

In this thesis we concentrated on the study of the behaviour of a narrowband adaptive array under some pattern constraints and varying signal strength environment. For carrying out our investigation we used a primal-dual algorithm with inequality constraints for minimizing the output power so that the antenna array processor generates deep nulls in the interference signal directions and produces desired response in the wanted signal direction. This desired response direction or the mainlobe direction as it is called, is constrained to the desired response value, but the algorithm allows the processor to generate sidelobes with uncontrolled levels of response. Our investigation thus, is centered around the possibility of introducing some constraints in the response patterns of this adaptive antenna array in the sidelobe directions so that the response levels at these directions can be controlled.

In course of our study, we found that the adaptive antenna array processor can process out the desired signal very satisfactorily by placing the response pattern mainlobe at the desired signal direction and putting nulls in the interference directions even though the processor was put under some pattern constraints in the sidelobe directions. The array processor satisfactorily met those sidelobe constraints to their given desired levels with some tolerance band. It was found, however, that in some cases, the output SINR undergoes some degradation due to some stringent conditions in the constraints. The number of sidelobe constraints that could be placed was dictated by the degrees of freedom available as was expected. The sidelobe levels could be decreased or increased (as a matter of fact, increase in sidelobe level is not wanted), showing the controlling capability of the processor. Finally, the processor was found to be stable and globally convergent in generating a set of optimum weight values irrespective of the number of array elements and pattern constraints.

Regarding our study on the effect of signal strength variation, it was observed that the antenna array processor behaves equally good in placing nulls in the interference directions and producing desired response in the wanted signal direction irrespective to the variation in the input signal power to interference power ratio. But this is not true when the constrained response direction does not match the assumed direction of the desired signal. The output SINR starts degrading with increasing input SINR and this degradation is more with increasing mismatch. It was observed that with strong interference signal, the processor does not notice the signal direction error, but with weaker interference signal, the processor concentrates more on the signal direction mismatch. Some interesting behaviour were observed for the processor when the system noise varies. The results showed some optimum value of the system noise when the output SINR was maximum for signal direction error. With other values the output SINR decreases drastically. With perfect match of the constrained direction and the signal arrival direction, the output SINR increases linearly with decreasing value of system noise which should be the obvious behaviour.

Concluding our discussion, we can summarize the contribution of the investigation as follows:

--A narrow band linear adaptive array antenna processor using primal-dual algorithm with inequality constraint behaves satisfactorily with pattern constraints in the sidelobe directions, the number of which is, however, restricted by the degrees of freedom. In Some cases, slight degradation in output SINR is observed. With pattern constraints, the algorithm remains globally convergent.

-- Variation in the strength of the input signal environment as well as the system noise play a significant role in determining the behaviour of the narrowband linear array antenna, specially when there is error in assumed signal arrival direction. The output SINR is quite dependent on this situation.

The present work does not complete the investigation on the behaviour of the adaptive antenna array. For example, as a future extension of this work it would be very interesting as well as important to be able to control the degradation of the output SINR for the case when the interference signal is on the mainlobe. We have observed in our study that in this situation the output SINR drastically degrades. Another is the case when there are multipath arrival of the desired signal making multipath components correlated with the desired component. The pattern constraint idea could be of help in attacking both the problems.

APPENDIX - A

Here we present some definitions and results concerning optimization problem relevant to our work. Since the results are well known [11],[14],[15], the presentation here is very brief.

Below we consider the optimization problem

minimize $f(\underline{u})$, $\underline{u} \in \mathbb{R}^n$ ....(A.1)

subject to $g(\underline{u}) \leq \underline{0}$

where $f: \mathbb{R}^n \to \mathbb{R}$ is a convex functional of $\underline{u} \in \mathbb{R}^n$ and $g: \mathbb{R}^n \to \mathbb{R}^m$ is a convex mapping.

Convex function: A function f defined on a convex set $\mathbb{R}$ is said to be convex if, for every $\underline{u}$, $\underline{v} \in \mathbb{R}^n$ and every α, $0 \leq \alpha \leq 1$ we have

$$f(\alpha \underline{u} + (1-\alpha)\underline{v}) \leq \alpha f(\underline{u}) + (1-\alpha) f(\underline{v})$$

If, for every $0<\alpha<1$ and $\underline{u} \neq \underline{v}$, there holds

$$f(\alpha \underline{u} + (1-\alpha)\underline{v}) < \alpha f(\underline{u}) + (1-\alpha) f(\underline{v}),$$

then f is said to be strictly convex.

Regular Point: Let $\underline{u}^*$ be a point which satisfies $g(u) \leq \underline{0}$ and let J be the set of indices j for which $g_j(\underline{u}^*)=0$. Then $\underline{u}^*$ is a regular point of the constraints if the gradient vectors $\nabla g_j(\underline{u}^*)$, $1 \leq j \leq m$, $j \in J$ are linearly independent.

we can define the Lagrangian for problem (A.1) as

$$L(\underline{u}, \underline{\lambda}) \triangleq f(\underline{u}) + \underline{\lambda}' g(\underline{u}) \quad \text{(A.2)}$$

where $\underline{\lambda} \in \mathbb{R}^m$ is the Lagrange multiplier vector.

The dual function for (A.1) is defined as

$$\phi(\underline{\lambda}) \triangleq \min_{\underline{u} \in \mathbb{R}^n} L(\underline{u}, \underline{\lambda}) \quad \text{(A.3)}$$

where ϕ is concave.

For the problem we consider in this thesis, f is strictly convex, continuous and differentiable with respect to $\underline{u}$. Further f is bounded below, and $f(\underline{u}) \to +\infty$ as $\| \underline{u} \| \to +\infty$, where $\| \underline{u} \| = \sqrt{\underline{u}'\underline{u}}$.

In the case of the constraint function $g(\underline{u})$, we consider the case:

$g(\underline{u})$ is of the form

$$\underline{\varepsilon}^- \leq p(\underline{u}) \leq \underline{\varepsilon}^+ \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \quad (A.4)$$

where p is linear, continuous and differentiable.

We now consider the problem (A.1)

$$\begin{aligned} &\text{minimize } f(\underline{u}),\ \underline{u} \in \mathbb{R}^n \\ &\text{s.t.} \quad g(\underline{u}) \leq \underline{0} \end{aligned} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \quad (A.5)$$

where $g(\underline{u})$ is of the type $\underline{\varepsilon}^- \leq p(\underline{u}) \leq \underline{\varepsilon}^+$

<u>Theorem A.1</u>: $\underline{u}^* \in \mathbb{R}^n$ solves problem (A.5) if and only if there exists a Lagrange multiplier vector $0 \leq \underline{\lambda}^* \in \mathbb{R}^m$ such that

(i) $\underline{\lambda}^{*\prime} g(\underline{u}^*) = 0$

(ii) $\nabla_u L(\underline{u}^*, \underline{\lambda}^*) = 0$.

The proof of above theorem follows directly from standard Kuhn-Tucker theory [11]

<u>Theorem A.2 (Duality)</u>: The Lagrange multiplier vector in Theorem A.1, $\underline{0} \leq \underline{\lambda}^*$ satisfies

(i) $\Phi(\underline{\lambda}^*) \geq \Phi(\underline{\lambda})$, $\underline{\lambda} \geq \underline{0}$

(ii) $\min f(\underline{u}) = \max \Phi(\underline{\lambda})$

$g(\underline{u}) \leq 0 \qquad \lambda_i \geq 0$

The proof of this theorem is provided in [11]

<u>Theorem A.3 (Uniqueness)</u>: The problem A.5 has a unique solution.

<u>Theorem A.4 (Existence)</u>; A solution to the problem (A.5) exists.

Proof of theorem A.3 and A.4 are given in [13]

APPENDIX-B

Let us define for convenience the following state vector comprising the the primal and dual variables of the algorithm (4.4.4)

$$\underline{x}^K \triangleq \begin{bmatrix} \underline{W}^K \\ \underline{\lambda}_1^K \\ \underline{\lambda}_2^K \end{bmatrix} \in \mathbb{R}^{n+N+N} \quad \text{.............. (B.1)}$$

Thus the optimal solution can be denoted by

$$\underline{x}^* \triangleq \begin{bmatrix} \underline{W}^* \\ \underline{\lambda}_1^* \\ \underline{\lambda}_2^* \end{bmatrix} \quad \text{.......................... (B.2)}$$

where $(\underline{W}^*; \underline{\lambda}_1^*, \underline{\lambda}_2^*)$ satisfy the saddle point conditions

$$L(\underline{W}; \underline{\lambda}_1^*, \underline{\lambda}_2^*) \geq L(\underline{W}^*; \underline{\lambda}_1^*, \underline{\lambda}_2^*) \geq L(\underline{W}^*, \underline{\lambda}_1; \underline{\lambda}_2) \quad \text{........................ (B.3)}$$

Finally we can define the step matrix

$$\underline{\Delta}^{K} \triangleq \mathrm{diag}\,(\alpha_1^{K}, \dots, \alpha_n^{K}, \beta_1^{K}, \dots, \beta_N^{K}, \gamma_1^{K}, \dots, \gamma_N^{K}) \quad \cdots\cdots (B.4)$$

The above definitions allow the algorithm (4.4.3) to be put in to the slightly more general form:

$$\underline{x}^{K+1} = \underline{P}\,(\underline{x}^{K} + \underline{\Delta}^{K} \cdot \underline{g}\,(\underline{x}^{K})) \quad \cdots\cdots\cdots\cdots (B.5)$$

where $\underline{P}(.)$ and $\underline{g}(.)$ are appropriately defined vector valued functions

$$\underline{g} = \begin{bmatrix} \dfrac{\partial L}{\partial \underline{W}} \\[2ex] \dfrac{\partial L}{\partial \underline{\lambda}_1} \\[2ex] \dfrac{\partial L}{\partial \underline{\lambda}_2} \end{bmatrix}$$

$$[P(\underline{x})]_i = \begin{bmatrix} W_i \\ \max(0, \underline{\lambda}_1)_i \\ \max(0, \underline{\lambda}_2)_i \end{bmatrix}$$

After defining algorithm in its more general and simple form of (B.5), the convergence of the algorithm can be proved with the help of the following theorems:

Theorem B.1 : If the step matrix defined in (B.4) is such that $\|\underline{\Delta}^K\| \leq K < \infty$ and the following conditions

$$\underline{\Delta}^K = \rho^K \underline{\Delta} \quad \text{for} \quad \underline{\Delta} > \underline{0} \qquad \text{(B.6)}$$

where

$$\sum_{K=1}^{\infty} \rho^K = \infty, \quad \sum_{K=1}^{\infty} (\rho^K)^2 < \infty \quad \text{for} \quad \rho^K > 0$$

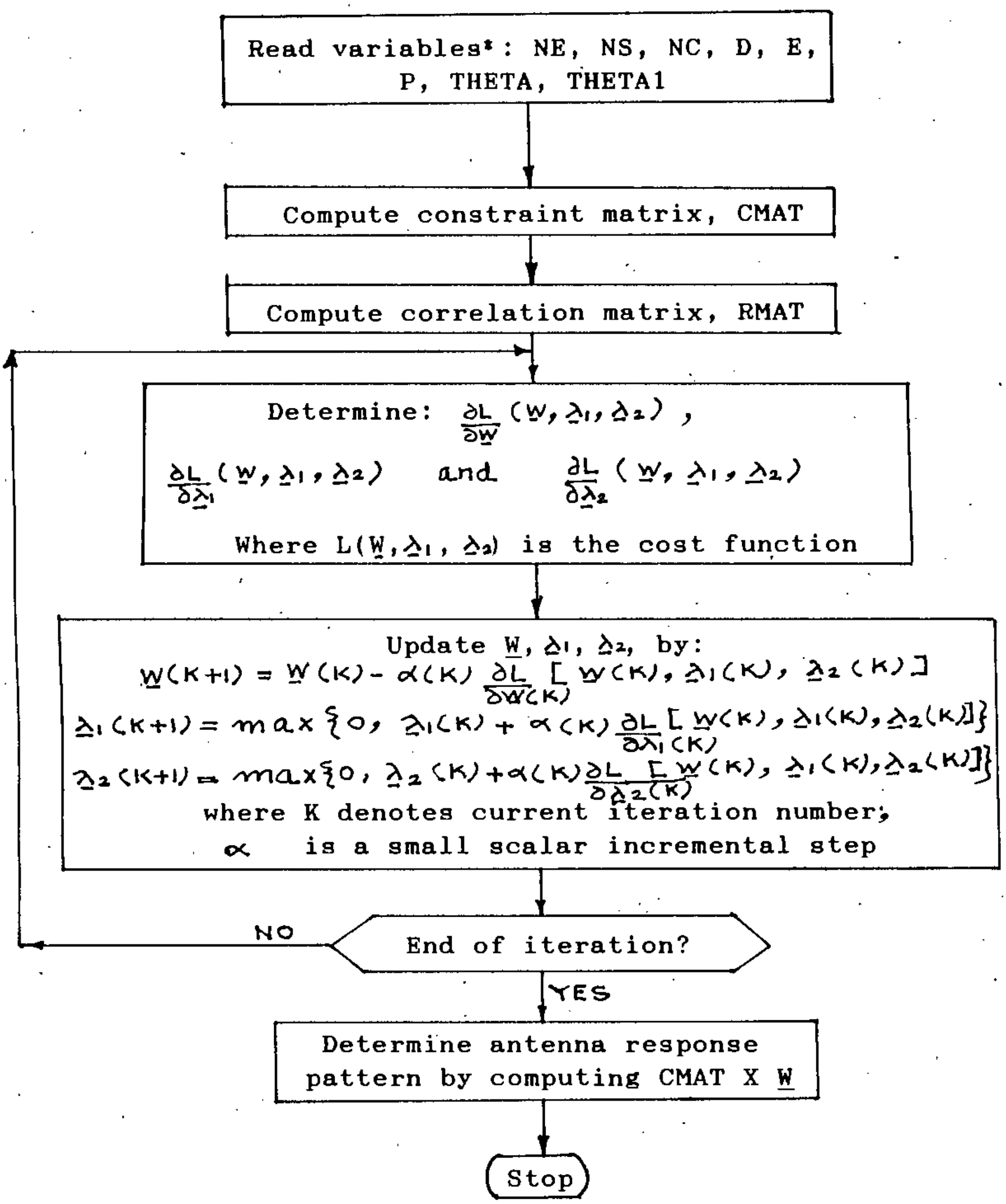
then

$$\lim_{K \to \infty} \underline{x}^K = \underline{x}^* \qquad \text{(B.7)}$$

The proof of this theorem is given in [13]

APPENDIX C

FLOW CHART OF THE ALGORITHM



* A description of the variables is given below:

NE	Number of antenna elements
NS	Number of sources
NC	Number of constraints
D	Desired response
E	Tolerance of response levels
P	Signal and noise power levels
THETA	Signal and noise arrival directions
THETA1	Constraint directions

```
C     THIS IS THE MAIN PROGRAM TO DESIGN ADAPTIVE ARRAY ANTENNA
      DIMENSION W (20), CM (20,20), CMW (10), DM (20,20), SI (20,20)
      DIMENSION BM (20,20), ALAM1 (10), ALAM2 (10), X (20,20), Y (20)
      DIMENSION ALAMP (10), ALAMN (10), TEM1 (20), TEM2 (20), SPROD (40)
      DIMENSION TEM3 (20), GAMA (20,20), R (20,20), F(20), D(10), E(10)
      DIMENSION TOU (10), THETA (10), P (10), THETA1 (10), THETAP (10),
     +X1(20,20),Y1(20,20),Z1(20,20),SI1(20,20),Z1W(10),OPMAT(40)
      OPEN(6,FILE='DATA')
      CALL XREAD(XSZ,YSZ,PI,RAD,LP,JL,NE,NT,NS,NC,J1,STEP,SIGMA,D,E,P,
     +THETA,THETA1,ELSPA,OMEGA,GAMA,NC2,NENT,I1,NIT)
      CALL DISPLY(LP,JL,NE,NT,NS,NC,J1,STEP,SIGMA,D,E,P,THETA,THETA1,
     +ELSPA,OMEGA,NC2)
      FREQ=OMEGA/(2*PI)
      DELTA=1/(4*FREQ)
      DO 42 I=1,NC2
      ALAM1(I)=0.0
42    ALAM2(I)=0.0
      CALL CMAT(DM,BM,CM,THETA1,X,NE,NC,PI,RAD,DELTA)
      WRITE(5,'(50X,A/)') 'CMAT'
      WRITE(5,421)((CM(I,J),J=1,NENT),I=1,NC2)
421   FORMAT(/,2X,8(2X,F10.6),/,2X)
      CALL RMAT(R,TOU,THETA,NS,NT,NE,P,NENT,FREQ,ELSPA,
     +OMEGA,RAD,DELTA,SIGMA)
      WRITE(5,491)
491   FORMAT(/,50X,'RMAT'/)
      WRITE(5,492)((R(I,J),J=1,NENT),I=1,NENT)
      492   FORMAT(/,2X,8(2X,F10.6),/,2X)
      DO 43 I=1,NENT
43    W(I)=0.0
      K1=1
      AK=0.0
      K2=1
44    DO 45 I=1,NC2
      ALAMP(I)=ALAM1(I)+ALAM2(I)
45    ALAMN(I)=ALAM1(I)-ALAM2(I)
      IF(JL.EQ.0)GO TO 450
      IF(JL.EQ.1)GO TO 451
      IF(JL.EQ.2)GO TO 452
      450   ALPHA=STEP
      GO TO 460
451   ALPHA=STEP/(AK+1.0)
      GO TO 460
452   ALPHA=STEP/(AK+5.0)
460   CALL ANUPD(TEM1,TEM2,TEM3,GAMA,ALAMP,ALAMN,CM,R,
     +W,F,NC2,NENT,ALPHA)
      CALL ANDIF(CMW,TEM1,TEM2,CM,GAMA,ALAM1,ALAM2,ALPHA,
     +NC2,NENT,D,E)
      IF(LP.EQ.0)GO TO 57
      CALL CONVRG(K2,NIT,K1,NENT,R,TEM1,I1)
      IF(K2.EQ.500)GO TO 46
      K2=K2+1
      GO TO 44
46    K2=1
      WRITE(3,999)K1
999   FORMAT(2X,'ITER=',I2,/)
      WRITE(3,47)
47    FORMAT(/,/,2X,'WEIGHT VECTOR',/)
      DO 48 I=1,NENT
      WRITE(3,49)TEM1(I)
```

```
48     CONTINUE
49     FORMAT(2X,F10.6)
       WRITE(5,50)
50     FORMAT(2X,'PRODUCT',/)
       DO 51 I=1,NENT
       Y(I)=0.0
       DO 51 J=1,NENT
51     Y(I)=Y(I)+R(I,J)*TEM1(J)
       PROD=0.0
       DO 52 I=1,NENT
52     PROD=PROD+TEM1(I)*Y(I)
       SPROD(K1)=0.0
       SPROD(K1)=PROD+SPROD(K1)
       WRITE(5,53)PROD
53     FORMAT(F10.6)
       WRITE(5,54)
54     FORMAT(2X,'WRITE ALAM1,ALAM2,CMW'/)
       DO 55 I=1,NC2
       WRITE(5,56)ALAM1(I),ALAM2(I),CMW(I)
55     CONTINUE
56     FORMAT(3(F10.6,2X))
57     IF(K1.EQ.J1)GO TO 563
       K1=K1+1
       AK=AK+1.0
       GO TO 44
563    CALL ANPAT(X1,Y1,Z1,THETAP,SI1,NE,NC,PI,RAD,DELTA,
      +NC2,NENT,Z1W,TEM1,DEG)
       CALL SINR(TEM1,THETA,NS,NT,NE,P,NENT,FREQ,ELSPA,OMEGA,
      +RAD,DELTA,SIGMA)
       STOP
       END
C      ____________________________________________________________
C
       SUBROUTINE CMAT(X,Y,Z,ANGLE,SI,NE,IC,PI,RAD,DELTA)
       DIMENSION X(20,20),Y(20,20),Z(20,20),ANGLE(10),SI(20,20)
       DO 20 I=1,IC
       ANGLE(I)=ANGLE(I)*RAD
       DO 20 J=1,NE
       EX=NE
       EJ=J
       EXJ=(EX+1.0)/2.0
       EJ=EJ-EXJ
       EJ=PI*EJ
       SI(I,J)=COS(ANGLE(I))*EJ
       X(I,J)=COS(SI(I,J))
       Y(I,J)=SIN(SI(I,J))
       X(I,NE+J)=COS(SI(I,J)+DELTA)
       Y(I,NE+J)=SIN(SI(I,J)+DELTA)
       Z(I,J)=X(I,J)
       Z(IC+I,J)=Y(I,J)
       Z(I,NE+J)=X(I,NE+J)
20     Z(IC+I,NE+J)=Y(I,NE+J)
       RETURN
       END
C      ____________________________________________________________
C
       SUBROUTINE RMAT(R, TOU,THETA,NS,NT,NE,P,NENT,
      +FREQ,ELSPA,OMEGA,RAD,DELTA,SIGMA)
       DIMENSION R(20,20),TOU(10),THETA(10),P(10)
       DO 11 I=1,NENT
```

```
      DO 11 J=1,NENT
11    R(I,J)=0.0
      DO 12 L=1,NS
      TOU(L)=ELSPA*COS(THETA(L)*RAD)/FREQ
      DO 12 M=1,NT
      DO 12 N=1,NT
      DO 12 I=1,NE
      DO 12 J=1,NE
      I1=(M-1)*NE+I
      J1=(N-1)*NE+J
12    R(I1,J1)=R(I1,J1)+P(L)*COS(OMEGA*((J-I)*TOU(L)+(N-M)*
     +DELTA))
      DO 13 I=1,NENT
13    R(I,I)=R(I,I)+SIGMA
      RETURN
      END
C     ------------------------------------------------------------
C
      SUBROUTINE ANUPD(TEM1,TEM2,TEM3,GAMA,ALAMP,ALAMN,
     +CM,R,W,F,NC2,NENT,ALPHA)
      DIMENSION TEM1(20),TEM2(20),TEM3(20),GAMA(20,20),W(20)
      DIMENSION ALAMP(10),ALAMN(10),CM(20,20),R(20,20),F(20)
      DO 15 I=1,NENT
      TEM1(I)=0.0
      TEM2(I)=0.0
      DO 15 J=1,NC2
      TEM1(I)=TEM1(I)+GAMA(J,I)*ALAMP(J)
15    TEM2(I)=TEM2(I)+CM(J,I)*ALAMN(J)
      DO 16 I=1,NENT
      TEM3(I)=0.0
      DO 16 J=1,NENT
16    TEM3(I)=TEM3(I)+R(I,J)*W(J)
      DO 60 I=1,NENT
      IF(W(I).GT.0.001)GO TO 30
      IF (W(I).LT.-0.001)GO TO 40
      IF((2.0*TEM3(I)+TEM2(I)+TEM1(I)).GT.0.001)GO TO 30
      IF((2.0*TEM3(I)+TEM2(I)+TEM1(I)).LT.0.001)GO TO 40
      IF(ABS(2.0*TEM3(I)+TEM2(I)).LE.TEM1(I))GO TO 50
      WRITE(5,35)
35    FORMAT(2X,'NO SOLUTION IN UPD'/)
      GO TO 50
30    F(I)=TEM1(I)
      GO TO 60
40    F(I)=-TEM1(I)
      GO TO 60
50    F(I)=0.0
60    CONTINUE
      DO 70 I=1,NENT
70    TEM1(I)=W(I)
      DO 80 I=1,NENT
80    W(I)=W(I)-ALPHA*(2.0*TEM3(I)+TEM2(I)+F(I))
      RETURN
      END
C     ------------------------------------------------------------
C
      SUBROUTINE ANDIF(CMW,TEM1,TEM2,CM,GAMA,ALAM1,
     +ALAM2,ALPHA,NC2,NENT,D,E)
      DIMENSION CMW(10),TEM1(20),TEM2(20),CM(20,20)
      DIMENSION GAMA(20,20),ALAM1(10),ALAM2(10),D(10),E(10)
      DO 17 I=1,NENT
```

```
17      TEM2(I)=0.0
        DO 18 I=1,NC2
        CMW(I)=0.0
        DO 18 J=1,NENT
        CMW(I)=CMW(I)+CM(I,J)*TEM1(J)
18      TEM2(I)=TEM2(I)+GAMA(I,J)*ABS(TEM1(J))
        DO 40 I=1,NC2
        ALAM1(I)=ALAM1(I)+ALPHA*(CMW(I)+TEM2(I)-D(I)-E(I))
        IF(ALAM1(I).GT.0.0)GO TO 30
        ALAM1(I)=0.0
30      ALAM2(I)=ALAM2(I)-ALPHA*(CMW(I)-TEM2(I)-D(I)+E(I))
        IF(ALAM2(I).GT.0.0)GO TO 40
        ALAM2(I)=0.0
40      CONTINUE
        RETURN
        END
C       ------------------------------------------------------------
C
        SUBROUTINE ANPAT(X1,Y1,Z1,THETAP,SI1,NE,NC,PI,RAD,DELTA,
       +NC2,NENT,Z1W,TEM1,DEG)
        DIMENSION X1(20,20),Y1(20,20),Z1(20,20)
        DIMENSION SI1(20,20),Z1W(10),TEM1(20),THETAP(10),DEG(1)
        WRITE(1,110)
110     FORMAT(/,/,2X,'ANTENNA RESPONSE PATTERN',/)
        NC1=1
        NC12=2
        DO 11 ITHETA=5,185,5
        THETAP(1)=ITHETA-5
        DEG(1)=THETAP(1)
        CALL CMAT(X1,Y1,Z1,THETAP,SI1,NE,NC1,PI,RAD,DELTA)
        DO 13 I=1,NC12
        Z1W(I)=0.0
        DO 13 J=1,NENT
13      Z1W(I)=Z1W(I)+Z1(I,J)*TEM1(J)
        WRITE(1,14)DEG(1),(Z1W(I),I=1,NC12)
11      CONTINUE
14      FORMAT(2X,10(F10.6,1X))
        RETURN
        END
C       ------------------------------------------------------------
C
        SUBROUTINE XREAD(XSZ,YSZ,PI,RAD,LP,JL,NE,NT,NS,NC,J1,STEP,
       +SIGMA,D,E,P,THETA,THETA1,ELSPA,OMEGA,GAMA,NC2,NENT,I1,NIT)
        DIMENSION D(10),E(10),P(10),THETA(10),THETA1(10),GAMA(20,20)
        READ(6,*)NIT,LP,JL,NE,NT,NS,NC
        READ(6,*)J1,STEP,SIGMA,ELSPA,OMEGA
        READ(6,*)I1,XSZ,YSZ,PI,RAD
        NC2=2*NC
        NENT=NE*NT
        DO 10 I=1,NC2
10      READ(6,*)D(I),E(I)
        DO 20 I=1,NS
20      READ(6,*)P(I),THETA(I)
        DO 30 I=1,NC
30      READ(6,*)THETA1(I)
        DO 40 I=1,NC2
        DO 40 J=1,NENT
40      GAMA(I,J)=0.0
        RETURN
        END
```

```
C     ------------------------------------------------------------------
C
      SUBROUTINE DISPLY(LP,JL,NE,NT,NS,NC,J1,STEP,SIGMA,D,E,P,
     +THETA,THETA1,ELSPA,OMEGA,NC2)
      DIMENSION D(10),E(10),P(10),THETA(10),THETA1(10)
      WRITE(2,36)
36    FORMAT(2X,'DATA READ'/)
      WRITE(2,37)LP,NE,NT,NS,NC,J1,STEP,SIGMA,ELSPA,OMEGA
37    FORMAT(/,2X,'LP=',I1,3X,'NE=',I2,3X,'NT=',I2,3X,
     +'NS=',I2,3X,'NC=',I2,3X,'J1=',I5,3X,'STEP=',F10.6,
     +3X,'SIGMA=',F10.6,3X,'ELSPA=',F10.6,3X,'OMEGA=',F10.6,/)
      WRITE(2,38)(D(I),I=1,NC2)
38    FORMAT(/,2X,'D=',10(F10.6,1X))
      WRITE(2,39)(E(I),I=1,NC2)
39    FORMAT(/,2X,'E=',10(F10.6,1X))
      WRITE(2,40)(P(I),I=1,NS)
40    FORMAT(/,2X,'P=',10(F10.6,1X))
      WRITE(2,41)(THETA(I),I=1,NS)
41    FORMAT(/,2X,'THETA=',10(F10.6,1X))
      WRITE(2,410)(THETA1(I),I=1,NC)
410   FORMAT(/,2X,'THETA1=',10(F10.6,1X))
      RETURN
      END
C     ------------------------------------------------------------------
C
      SUBROUTINE SINR(TEM1,THETA,NS,NT,NE,P,NENT,FREQ,ELSPA,OMEGA,
     +RAD,DELTA,SIGMA)
      DIMENSION RNN(20,20),RSS(20,20),RXX(20,20),Y(20),TEM1(20),
     +TOU(10),THETA(10),P(10)
      DO 11  I=1,NENT
      DO 11  J=1,NENT
11    RNN(I,J)=0.0
      DO 12  L=1,NS
      TOU(L)=ELSPA*COS(THETA(L)*RAD)/FREQ
      DO 14  M=1,NT
      DO 14  N=1,NT
      DO 14  I=1,NE
      DO 14  J=1,NE
      I1=(M-1)*NE+I
      J1=(N-1)*NE+J
      RNN(I1,J1)=RNN(I1,J1)+P(L)*COS(OMEGA*((J-I)*TOU(L)+(N-M)*DELTA))
14    CONTINUE
      IF(L.NE.1)GO TO 12
      DO 13  I2=1,NENT
      DO 13  J2=1,NENT
      RSS(I2,J2)=RNN(I2,J2)
13    RNN(I2,J2)=0.0
12    CONTINUE
      WRITE(5,*)'RSS IS GIVEN BY FOLLOWING MATRIX:-'
      WRITE(5,60)((RSS(I,J),J=1,NENT),I=1,NENT)
60    FORMAT(/,2X,8(2X,F10.6),/,2X)
      DO 15  I=1,NENT
15    RNN(I,I)=RNN(I,I)+SIGMA
      DO 16  I=1,NENT
      DO 16  J=1,NENT
16    RXX(I,J)=RNN(I,J)+RSS(I,J)
      DO 17  I=1,NENT
      Y(I)=0.0
      DO 17  J=1,NENT
17    Y(I)=Y(I)+RSS(I,J)*TEM1(J)
```

```
      WRITE(5,*)'    RSS*W  IS GIVEN BY FOLLOWING MATRIX:-'
      WRITE(5,70)(Y(I),I=1,NENT)
70    FORMAT(/,2X,8(2X,F10.6),/,2X)
      SPOWER=0.0
      DO 18 I=1,NENT
18    SPOWER=SPOWER+TEM1(I)*Y(I)
      WRITE(2,75)
75    FORMAT(/,1X,80('*')//)
      WRITE(2,*)'    SIGNAL POWER=',SPOWER
      DO 19 I=1,NENT
      Y(I)=0.0
      DO 19 J=1,NENT
19    Y(I)=Y(I)+RNN(I,J)*TEM1(J)
      XPOWER=0.0
      DO 20 I=1,NENT
20    XPOWER=XPOWER+TEM1(I)*Y(I)
      WRITE(2,*)'    TOTAL NOISE POWER=',XPOWER
      XSINR=SPOWER/XPOWER
      YSINR=10.0*ALOG10(XSINR)
      WRITE(2,*)'    SIGNAL TO NOISE RATIO IS GIVEN BY:'
      WRITE(2,*)'    SINR=',XSINR
      WRITE(2,21)YSINR
21    FORMAT(4X,'IN DECIBELS SINR =',F8.3)
      RETURN
      END
C     ________________________________________________________________
C
      SUBROUTINE CONVRG(K2,NIT,K1,NENT,R,TEM1,I1)
      DIMENSION Y(20),R(20,20),TEM1(20),OPMAT(40)
      IF(I1.NE.1)GO TO 10
      NIT1=NIT
10    K2NEW=(K1-1)*500+K2
      IF(K2NEW.NE.NIT1)GO TO 60
      ITNUM=K2NEW
      DO 51 I=1,NENT
      Y(I)=0.0
      DO 51 J=1,NENT
51    Y(I)=Y(I)+R(I,J)*TEM1(J)
      PROD=0.0
      DO 52 I=1,NENT
52    PROD=PROD+TEM1(I)*Y(I)
      OPMAT(I1)=PROD
      WRITE(7,14)ITNUM,OPMAT(I1)
14    FORMAT(4X,I8,4X,F10.6)
      I1=I1+1
      NIT1=NIT*I1
60    RETURN
      END
```

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