RADIATION FROM A SLOT ANTENNA IN AN ANISOTROPIC PLASMA

BY

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A THESIS

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CERTIFICATE

This to certify that this work was done by me and it has not been submitted elsewhere for the award of any degree or diploma.

Signature of the student

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ABSTRACT

Radiation from a slot antenna in an anisotropic plasma with the static magnetic field in arbitrary direction has been investigated. The field distributions at the slot aperture have been derived assuming transverse electric excitation at the feed point and are found to support plane waves of both ordinary and extraordinary modes. Using the aperture fields as equivalent electric and magnetic current sheets the fields outside the slot are derived in triple Fourier integral forms which are evaluated by residue theorem and saddle point method to obtain the radiation field. Numerical calculations of the time averaged Poynting vector were carried out for a half wavelength slot antenna and for plasma parameters corresponding to the region (a) of Clemmow-Mullali-Allis (CMA) diagram. The radial component of the power flow vector normalized by the maximum power density for a half-wave slot in free-space is plotted to obtain the directional pattern. Plots of the directional pattern show that two strong pencil beams are radiated - one along the ground plane in a direction perpendicular to the slot axis and the other perpendicular to the plane of the slot. The radiation is dominated by the ordinary-mode power with a comparatively negligible extraodinary-mode power. With the static magnetic field in the yz-plane of an xyz-system, the inclination of the static magnetic field does not significantly influence the radiation pattern of an x-directed slot but has considerable effect for y and z-directed slots.In the latter cases, the beam maxima are also slightly tilted from the z and y-axes due to the lack of propagation symmetry in the yz-plane.

INDEX

List of principal Symbols

e = electronic charge , 1.602×10^{-19} coulomb .

m = electronic mass , 9.1×10^{-31} Kg .

N = electron particle density , m^{-3} .

ϵ_0 = free space electric permittivity , 8.554×10^{-12} farad/m .

μ_0 = free space magnetic permeability , $4\pi \times 10^{-7}$ henry/m.

$\bar{H}_0$ = static magnetic field intensity , Amp/m .

$\bar{B}_0$ = $\mu_0 \bar{H}_0$, static magnetic flux density , Weber/m^2 .

α = angle of inclination of the static magnetic field with the z-axis in the yz-plane .

$\hat{\epsilon}_r'$ = relative permittivity tensor of the anisotropic plasma with the static magnetic field in the z'-direction in an xy'z'-system .

$\hat{\epsilon}_r$ = relative permitivitty tensor with reference to an xyz-system for arbitrary α .

ω = angular operating frequency , radian/sec .

f = operating frequency , Hz .

$\omega_p = \left(\frac{Ne^2}{m\epsilon_0}\right)^{1/2}$, angular plasma resonant frequency , radian/sec .

f_p = plasma resonant frequency , Hz .

ω_H = angular cyclotron resonant frequency , radian/sec .

f_H = cyclotron resonant frequency ,Hz .

$\bar{E}'$ = electric field intensity in the xy'z'-system , volt/m .

$\bar{E}$ = electric field intensity in the xyz-system for arbitrary α, volt/m .

$\bar{H}'$ = magnetic field intensity in the xy'z'-system , Amp/m .

$\bar{H}$ = magnetic field intensity in the xyz-system for arbitrary α, Amp/m .

k_o = $\omega\sqrt{\mu_o\epsilon_o}$, free space propagation constant , radian/m .

$\bar{k}'$ = complex propagation constant in the xy'z'-system , radian/m .

$\bar{k}$ = complex propagation constant in the xyz-system , radian/m

$\bar{E}^a$ = electric field distribution of the slot aperture .

$\bar{H}^a$ = magnetic field distribution of the slot aperture .

$\bar{n}$ = unit outward normal to the aperture plane .

$\bar{J}_e$ = equivalent electric current density on the aperture plane , Amp/m^2 .

$\bar{J}_m$ = equivalent magnetic current density on the aperture plane , Volt/m^2 .

$\hat{\lambda}$ = wave matrix .

P_r = radial component of Poynting vector .

CHAPTER – 1

General Introduction

The space shuttle program and the prospect in nuclear fusion provided increased motivation for research on properties of antennas in plasmas. Antennas are well-known not only as important parts of communication system but also as useful devices for diagonostics of the surrounding medium.These important properties of an antenna are determined by its physical shape, the properties of the surrounding medium and its feeding conditions.Again the presence of boundaries and obstacles in the vicinity of an antenna can greatly modify the transmitted and received signal pattern. When the medium is anisotropic,studies of antenna properties in the presence of these obstacles and boundaries are expected to have diverse applications in ionospheric research,thermonuclear fusion and ferrite technology.In this thesis a theoretical study has been given to the radiation from a slot antenna in an anisotropic plasma.

Radiation blackout has long been a problem to satellite communication.When a space satellite enters the ionosphere during its return a sheath of positive charges is formed around its hot body which causes a capacitance between the satellite and the surrounding plasma.At microwave frequencies which are chosen for communication with the satellite,this capacitance will be a short circuit for the transmitting signal blocking radiation from the satellite antenna.At frequencies in the range 3MHz to 12 MHz over which plasma effects are prominent,this capacitance effect will be much weaker and it can be investigated to see whether radiation is possible in this frequency range.Dipole and monopole type of antennas for this frequency range will be very long in size for effective radiation of the signal.Moreover the installation of such antennas on a high speed space vehicle is not mechanically feasible.Hence a slot antenna can be an alternative from the mechanical point of view.However, it is still remaining as a problem to investigate the electrical characteristics of a slot antenna in this frequency range where accounts must be taken of the influence of the anisotropy of the medium around the antenna.

Research on radiation from sources in anisotropic plasma has a wide past history,[1]-[18]. Most of these works were done on probes,monopole,dipole and cylindrical antennas.Very few works have been done on a slot antenna in anisotropic plasma.Wait [19] and Burman [20] formulated the theory of electromagnetic radiation into a continuously stratified magnetoplasma half space from an infinitely long slot in a perfectly conducting plane of infinite extent with the static magnetic field parallel to the slot axis. Radiation from a circular waveguide in an anisotropic plasma has been studied by Smirnov and Yablochkin [21] for the static magnetic field perpendicular to the aperture.A mathematical formulation of the problem has been given by Brandstatter [22] assuming a transverse electric excitation of a rectangular slot. As none of these papers illustrated either numerical or experimental plot of the radiation pattern,it is difficult to get a physical picture of radiation from a slot in an anisotropic plasma.Numerical plots of radiation for axially slotted plasma-clad cylinders were shown in [23] - [25] .Their analysis is restricted to a thin plasma layer around the cylinder. Slots on flat ground plane covered with a thin plasma layer with an axial magnetic field were studies in [26] - [29] .However,in the present study we shall consider that the plasma medium over the ground plane is of infinite extent and the direction of the static magnetic field is arbitrary. To this author's knowledge study of such a model has not yet been reported.

In the present work in studying the radiation from a slot antenna the arbitrary direction of the static magnetic field has been taken into consideration because of the fact that during revolution of the satellite the slot antenna will be arbitrarily inclined with respect to the earth's magnetic field.The medium properties corresponding to various combinations of plasma parameters have been discussed with the help of wave number surfaces.Fundamental relations between the field components for plane waves propagating in arbitrary directions with respect to the static magnetic field were derived.These derivations were used to obtain the field distribution in the slot aperture.Using these distributions as sources of radiation the fields outside the slot were derived from Maxwell's equations.Chapter-2 deals with the polarization and propagation of plane waves in arbitrary directions.Chapter-3 deals with the derivation of aperture field distribution of a slot.In chapter-4 the field solution outside the slot is given in Fourier integral form.In chapter-5 the far field evaluation of the integral is performed by residue calculation and saddle point method.In chapter-6 the radiation pattern of a half-wavelength slot antenna placed along different axes and planes with arbitrary inclination of the static magnetic field is calculated and plots of directional pattern are shown.The thesis is concluded with a general discussion in chapter-7.

CHAPTER – 2

A Perspective of Wave Propagation in Anisotropic Plasma

2.1 Introduction

The present chapter deals with the wave propagation in an anisotropic plasma.The propagation of a plane wave through a plasma in the presence of a static magnetic field is an appropriate model for propagation of wave through the ionosphere and through some laboratory plasma. A plasma medium contains enormous number of ions and electrons. It may be hot or cold . We shall consider the cold plasma model only.The distinguishing feature of cold plasma model is that ions and electrons in the unperturbed state are motionless by the zero temperature assumption. One essential property which distinguishes a plasma from an ordinary gas is that the almost free streaming of the particles is completely missing in a cold plasma model. Here each particle of the plasma oscillates about of a fixed position in space under the influence of the electromagnetic field of the wave. All the dissipation effects including collisions are neglected. The model does not include acoustic field, particle bunching, Landau damping and plasma shock waves [30] .

2.2 Anisotropy in Presence of a Static Magnetic Field

In presence of a static magnetic field the dielectric behaviour of a plasma depends on the movement of the charged particles. An electric field component transverse to the magnetic field can give rise to transverse electron motion. Electrons and ions in the ionosphere moving under the influence of the electric field of a passing electromagnetic wave, experience force because of their velocity in the presence of static magnetic field. The anisotropy of plasma can be described with reference to the principal axes represented by the $x\, y' z'$ co-ordinates as shown in fig.2.1 in which the static magnetic field $\bar{H}o$ is along the z'-axis at an angle α with respect to

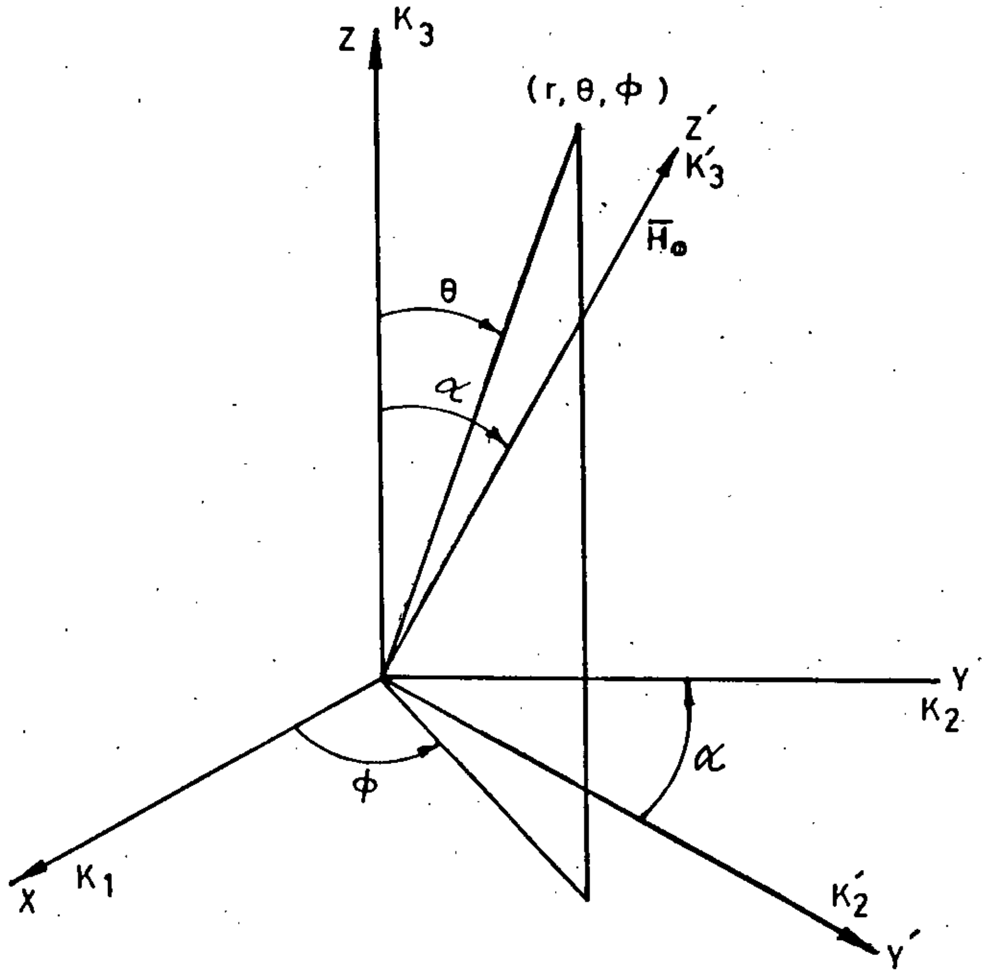


Fig.2.1 : Co-ordinate system for relative permittivity tensor in an anisotropic plasma.

the z-axis of a fixed co-ordinate system xyz.

Taking into account the effect of the static magnetic field $\bar{H}o$, but neglecting frictional forces due to collision between electrons and gas molecules, the equation of force on the particle can be written as[33]

$$F = e(\bar{E}' + \bar{v}' \times \bar{B}_o) = m\frac{d\bar{v}'}{dt} \qquad (2.1)$$

where

$\bar{E}' = \bar{a}_x E_x + \bar{a}_y' E_y' + \bar{a}_z' E_z'$

$\bar{v}' = \bar{a}_x v_x + \bar{a}_y' v_y' + \bar{a}_z' v_z'$

$\bar{B}_o = \bar{a}_z' B_o = \bar{a}_z' \mu_o H_o$

where $\bar{E}'$ is the electric field, $\bar{v}'$ is the electron velocity, $\bar{a}_x$, $\bar{a}_y'$, $\bar{a}_z'$ are unit vectors in the x, y' and z' directions respectively. Assuming all the field variables to be time-periodic with angular frequency ω, equation(2.1) becomes

$$E_x + v_y' B_o = \frac{j\omega m}{e} v_x \qquad (2.2)$$

$$E_y' - v_x B_o = \frac{j\omega m}{e} v_y' \qquad (2.3)$$

$$E_z' = \frac{j\omega m}{e} v_z' \qquad (2.4)$$

Solving (2.2), (2.3), and (2.4) simultaneously,

$$v_x = \frac{j\omega(e/m)E_x + (e/m)\omega_H E_y'}{\omega_H^2 - \omega^2} \qquad (2.5)$$

$$V_y' = \frac{-(e/m)\,\omega_H E_x + j\omega (e/m) E_y'}{\omega_H^2 - \omega^2} \tag{2.6}$$

$$V_z' = \frac{j(e/m)}{\omega} E_z' \tag{2.7}$$

where $\omega_H = (e/m)B_0$ is called the angular cyclotron frequency.

Maxwell's second equation for a region in which there is a convection current density $Ne\bar{v}'$ becomes.

$$\nabla \times \bar{H}' = j\omega \epsilon_0 \bar{E}' + Ne\bar{V}' \tag{2.8}$$

Substituting $\bar{v}'$ by the relations (2.5)-(2.7), equation (2.8) can be written as

$$\nabla \times \bar{H}' = j\omega \epsilon_0 \hat{\epsilon}_r' \bar{E}' \tag{2.9}$$

where $\hat{\epsilon}_r'$ is the relative permittivity tensor of the medium

$$\hat{\epsilon}_r' = \begin{bmatrix} \epsilon_1 & -j\epsilon_4' & 0 \\ j\epsilon_4' & \epsilon_1 & 0 \\ 0 & 0 & \epsilon_3' \end{bmatrix} \tag{2.10}$$

in which

$\epsilon_1 = 1 - X/(1-Y^2)$

$\epsilon_3' = 1 - X$

$\epsilon_4' = XY/(1-Y^2)$

$X = \omega_p^2/\omega^2$

$Y = \omega_H/\omega$

Now with reference to the xyz-system the relative permittivity of the medium takes the following form:

$$\hat{\epsilon}_r = \begin{bmatrix} \epsilon_1 & -j\epsilon_4 & j\epsilon_5 \\ j\epsilon_4 & \epsilon_2 & \epsilon_6 \\ -j\epsilon_5 & \epsilon_6 & \epsilon_3 \end{bmatrix} \qquad (2.11)$$

where

$\epsilon_2 = \epsilon_1 \cos^2\alpha + \epsilon_3' \sin^2\alpha$

$\epsilon_3 = \epsilon_1 \sin^2\alpha + \epsilon_3' \cos^2\alpha$

$\epsilon_4 = \epsilon_4' \cos\alpha$

$\epsilon_5 = \epsilon_4' \sin\alpha$

$\epsilon_6 = (\epsilon_3' - \epsilon_1) \sin\alpha \cos\alpha$

The derivation of (2.11) from (2.10) is shown in appendix-A. Relative permittivity of the form(2.11) has also been used by Matin et al. [2] .It is interesting to note that the permittivity tensor given by (2.10) and (2.11) are not equivalent unless $\alpha = 0^\circ$.This indicates that a rotation of the static magnetic field with respect to the z-axis can give different results in the medium permittivity. This is due to the anisotropic property of the medium. Such a situation is not encountered in the case of an isotropic medium, because in an isotropic medium the medium permittivity is invariant with any arbitrary transformation of the co-ordinate system

2.3. Dispersion into Ordinary and Extraordinary Modes of Propagation

In art 2.3 we have seen how anisotropic property is developed in a plasma in the presence of a static magnetic field. Let us now review the propagation characteristics in the medium.The electromagnetic field must satisfy Maxwell's equations in a source free region with $e^{j\omega t}$ variation suppressed,

$$\nabla \times \bar{E} = -j\omega\mu_0 \bar{H} \tag{2.12}$$

$$\nabla \times \bar{H} = j\omega\epsilon_0 \hat{\epsilon}_r \bar{E} \tag{2.13}$$

Taking curl of equation(2.12) and using (2.13) we get

$$\nabla \times \nabla \times \bar{E} - k_0^2 \hat{\epsilon}_r \bar{E} = 0 \tag{2.14}$$

where $k_0 = \omega\sqrt{\mu_0\epsilon_0}$ is the free space propagation constant. Equation (2.14) is called the vector wave equation.

For a plane wave of the form $\bar{E} = \bar{E}_0 e^{-jk_1x - jk_2y - jk_3z}$ the wave numbers k_1, k_2, k_3 can be obtained as the roots of the characteristic equation which follows by introducing equation (2.11) into equation(2.14) and setting the determinant of the wave matrix to zero (detailed analysis is shown in appendix-B):

$$(k_1^2 + k_2^2 + k_3^2 - \epsilon_1 k_0^2)(\epsilon_1 k_1^2 + \epsilon_2 k_2^2 + \epsilon_3 k_3^2 + 2\epsilon_6 k_2 k_3 - \eta_1 k_0^2/\epsilon_1) + \Delta(k_1, k_2, k_3) = 0 \tag{2.15}$$

where

$$\Delta(k_1, k_2, k_3) = k_0^2(c_1 k_1^2 + c_2 k_2^2 + c_3 k_3^2 + c_4 k_2 k_3)$$

$$c_1 = \epsilon_4'^2(1 - \epsilon_3'/\epsilon_1)$$

$$c_2 = \epsilon_4'^2(\cos^2\alpha - \epsilon_3'/\epsilon_1)$$

$$c_3 = \epsilon_4'^2(\sin^2\alpha - \epsilon_3'/\epsilon_1)$$

$C_4 = -2\sin\alpha\cos\alpha\,\epsilon_4'^2$

$\eta_1 = \epsilon_3'(\epsilon_1^2 - \epsilon_4'^2)$

Equation (2.15) is called the dispersion equation and with reference to the x y′z′ system it takes the following form:

$$\epsilon_3' k_3'^4 - (2\epsilon_1\epsilon_3' k_0^2 - (\epsilon_1+\epsilon_3')k'^2)k_3'^2 + (k'^2 - \epsilon_3' k_0^2)(\epsilon_1 k'^2 - (\epsilon_1^2 - \epsilon_4'^2)k_0^2) = 0 \tag{2.16}$$

where $k' = \sqrt{k_1^2 + k_2'^2}$

In the above k_1, k_2' and k_3' are the wave numbers assumed along x, y′ and z′ directions respectively of a plane wave, $\bar{E} = \bar{E}_0\, e^{-jk_1x - jk_2'y - jk_3'z}$. The dispersion equation has two pair of roots corresponding to each wave number. For example the solution of the roots with respect to k_3' is given by

$$k_3' = \pm\left[\frac{2\epsilon_1\epsilon_3' k_0^2 - (\epsilon_1+\epsilon_3')k'^2 \pm \sqrt{(\epsilon_1-\epsilon_3')^2 k'^4 - 4\epsilon_3'\epsilon_4'^2 k_0^2 (k'^2 - \epsilon_3' k_0^2)}}{2\epsilon_3'}\right]^{1/2} \tag{2.17}$$

The positive sign inside the bracket corresponds to the 'ordinary' or the O-mode and the negative sign corresponds to the 'extraordinary' or the X-mode of propagation. For studying radiation problems it is necessary to obtain the real roots of the characteristics equation. Because any complex root will cause attenuation of the wave. For this purpose the medium is assumed to be lossless by neglecting electron-ion collision. Resonances and cut-offs of wave propagation takes place for frequencies at which $\epsilon_1 = 0$ or ∞, $\epsilon_1 \pm \epsilon_4' = 0$, $\epsilon_3' = 0$ and $\epsilon_4' = \infty$. These phenomena can be shown in an X-Y plot as shown in Fig.2.2. Propagation modes will exist for X-Y values lying beyond any of the resonance lines. Thus there are eight regions of wave propagation in the X-Y plane [31] corresponding to these eight regions the plots of k_3' versus k' are shown in Fig.2.3. The plots are symmetrical about the axis of the static magnetic field

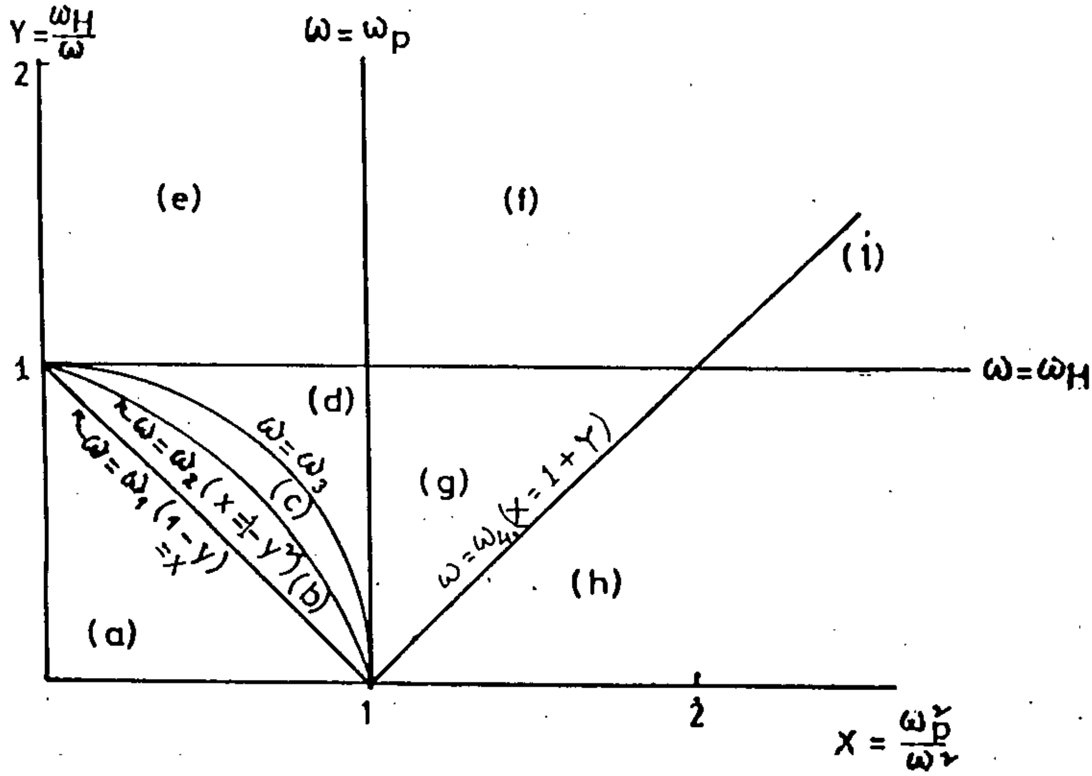
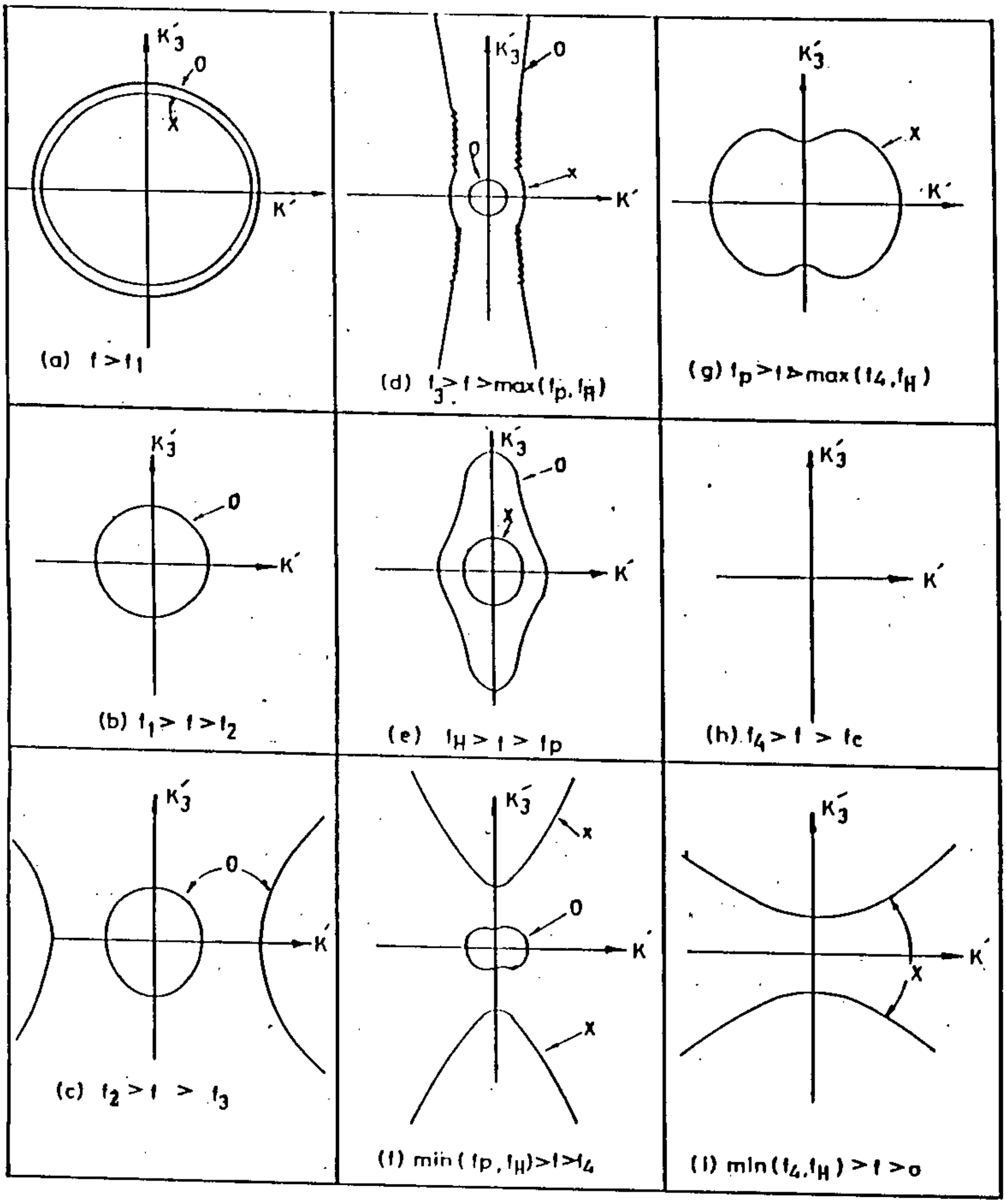


Fig.2.2 : Subdivision of a plasma parameter or X,Y plane corresponding to various wave numbers of a cold anisotropic plasma.



$$f_1 = \sqrt{f_H^2/4 + f_p^2} + f_H/2$$

$$f_2 = \sqrt{f_p^2 + f_H^2}$$

$$f_3 = \sqrt{f_H^2/2 + \sqrt{f_H^4/4 + f_p^4}}$$

$$f_4 = \sqrt{f_p^2 + f_H^2/4} - f_H/2$$

Fig.2.3 : Wave number surfaces for various ranges of plasma parameters shown in Fig.2.2 [Ref [31]]

and are called wave surfaces or dispersion surfaces in the wave-space. For an arbitrary direction of the static magnetic field the dispersion surfaces will be inclined with respect to the k_3 - axis which is the z-axis in real space. Fig.2.2 is called the Clemmow-Mullali-Allis (CMA) diagram.

The wave number surfaces are very useful for having a physical picture of radiation in such medium. Because the complex power flow vector or Poynting vector for a certain wave number can be shown to be normal to the wave number surface [5] . This helps tracing the rays in the real space isometrically from the normals on the wave number surfaces.

2.4. Plane Waves parallel to the Static Magnetic Field

For a uniform plane wave $\bar{E}'=\bar{E}_o' e^{-jk_3' z'}$ traveling along the static magnetic field we have $\frac{\partial}{\partial x}=0$ and $\frac{\partial}{\partial y'}=0$. Hence the fields are governed by the equations (details shown in appendix-C)

$$\frac{\partial E_y'}{\partial z'} = j\omega\mu_o H_x \tag{2.18}$$

$$\frac{\partial E_x}{\partial z'} = -j\omega\mu_o H_y' \tag{2.19}$$

$$0 = H_z' \tag{2.20}$$

$$-\frac{\partial H_y'}{\partial z'} = j\omega\epsilon_o(\epsilon_1 E_x - j\epsilon_4' E_y') \tag{2.21}$$

$$\frac{\partial H_x}{\partial z'} = j\omega\epsilon_o(j\epsilon_4' E_x + \epsilon_1 E_y') \tag{2.22}$$

$$0 = E_z' \tag{2.23}$$

Since $E'_z=0$ and $H'_z=0$, then the propagation along the static magnetic field is in the form of a TEM-wave . The wave number k_3 can be obtained from the above relations as:

$$k'_3 = \pm k_o (\epsilon_1 \pm \epsilon'_1)^{1/2} \quad (2.24)$$

The +ve and -ve signs inside the bracket correspond to the O-mode and the X-mode of propagation respectively .From appendix-C it can be noted that the following relations hold:

$E'_y=+jE_x$ for O-mode

$E'_y=-jE_x$ for X-mode

This indicates that the fields of the two modes are 90° out of phase and if one mode is polarized in the clockwise direction the other is polarized in the counter clockwise direction. The difference of wave numbers will create Faraday rotation of the field [32] .

2.5. Plane Waves perpendicular to the Static Magnetic Field

Let us consider a plane wave $\bar{E}' = \bar{E}'_o e^{-jk_1 x}$ traveling along the x-direction which is perpendicular to the static magnetic field. Then we have $\frac{\partial}{\partial y'}=0$ and $\frac{\partial}{\partial z'}=0$. Here the fields are governed by the equations (details shown in appendix-C):

$$H_x = 0 \quad (2.25)$$

$$\frac{\partial E'_z}{\partial x} = j\omega\mu_o H'_y \quad (2.26)$$

$$\frac{\partial E'_y}{\partial x} = -j\omega\mu_o H'_z \quad (2.27)$$

$$E_x = j(\epsilon_4'/\epsilon_1)\,E_y' \tag{2.28}$$

$$-\frac{\partial H_z'}{\partial x} = j\,\omega\epsilon_0\,(j\epsilon_4' E_x + \epsilon_1 E_y') \tag{2.29}$$

$$\frac{\partial H_y'}{\partial x} = j\,\omega\epsilon_0\epsilon_3' E_z' \tag{2.30}$$

From the above equations the electric field components are found to satisfy the wave equation

$$\frac{\partial^2 E_y'}{\partial x^2} = -k_0^2\left(\frac{\epsilon_1^2-\epsilon_4'^2}{\epsilon_1}\right)E_y' \tag{2.31}$$

$$\frac{\partial^2 E_z'}{\partial x^2} = -k_0^2\epsilon_3' E_z' \tag{2.32}$$

Since $H_x=0$, then the propagation perpendicular to the static magnetic field is in the form of a TM wave. The wave number k_1 is given by

$$k_{11} = \pm k_0\sqrt{\frac{\epsilon_1^2-\epsilon_4'^2}{\epsilon_1}} \qquad \text{for X-mode} \tag{2.33}$$

$$k_{12} = \pm k_0\sqrt{\epsilon_3'} \qquad \text{for O-mode} \tag{2.34}$$

The O-mode is a TEM wave having the field components E_z' and H_y'. The X-mode is a TM wave having the field components E_x, E_y', and H_z' [33].

2.6. Plane Waves in Arbitrary Directions

So far we have investigated plane waves propagating parallel and perpendicular to the static magnetic field. Now we shall consider plane waves propagating in arbitrary direction with respect to the static magnetic field. The mechanism of wave propagation in arbitrary direction with respect to the static magnetic field is more complicated than the two cases studied above.

2.6.1. Waves in the x-direction for Static Magnetic Field in the yz-Plane

For a plane wave $\bar{E}=\bar{E}_0 e^{-jk_1x}$ traveling along the x-direction the field equations are same as (2.24)-(2.29). By co-ordinate transformation we can find the field components in the y and z-directions.

$$E_y = E_y' \cos\alpha + E_z' \sin\alpha \tag{2.35}$$

$$E_z = E_z' \cos\alpha - E_y' \sin\alpha \tag{2.36}$$

$$H_y = H_y' \cos\alpha + H_z' \sin\alpha \tag{2.37}$$

$$H_z = H_z' \cos\alpha - H_y' \sin\alpha \tag{2.38}$$

2.6.2. Waves in the y-direction for Static Magnetic Field in the yz-Plane

For a uniform plane wave $\bar{E} = \bar{E}_0 e^{-jk_2y}$ traveling in the y-direction we have $\frac{\partial}{\partial x} = 0$ and $\frac{\partial}{\partial z} = 0$. Here the field components are governed by the equations (details shown in appendix-D):

$$H_y = 0 \tag{2.39}$$

$$\frac{\partial E_z}{\partial y} = -j\omega\mu_0 H_x \tag{2.40}$$

$$\frac{\partial E_x}{\partial y} = j\omega\mu_0 H_z \tag{2.41}$$

$$\frac{\partial H_z}{\partial y} = j\omega\epsilon_0 (\epsilon_1 E_x - j\epsilon_4 E_y + j\epsilon_5 E_z) \tag{2.42}$$

$$0 = (j\epsilon_4 E_x + \epsilon_2 E_y + \epsilon_6 E_z) \tag{2.43}$$

$$-\frac{\partial H_x}{\partial y} = j\omega\epsilon_0(-j\epsilon_5 E_x + \epsilon_6 E_y + \epsilon_3 E_z) \tag{2.44}$$

Since $H_y=0$, then the propagation along the y direction is in the form of a TM wave. The wave number k_2 can be obtained as

for X-mode

$$K_{21} = \pm\left[\frac{\epsilon_1(\epsilon_2+\epsilon_3') - \epsilon_4'^2\cos^2\alpha - \sqrt{[\epsilon_1(\epsilon_2+\epsilon_3') - \epsilon_4'^2]^2 - 4\epsilon_2\epsilon_3'(\epsilon_1^2-\epsilon_4'^2)}}{2\epsilon_2}\right]^{1/2} \tag{2.45}$$

for O-mode

$$K_{22} = \pm\left[\frac{\epsilon_1(\epsilon_2+\epsilon_3') - \epsilon_4'^2\cos^2\alpha + \sqrt{[\epsilon_1(\epsilon_1+\epsilon_3') - \epsilon_4'^2]^2 - 4\epsilon_2\epsilon_3'(\epsilon_1^2-\epsilon_4'^2)}}{2\epsilon_2}\right]^{1/2} \tag{2.46}$$

For $\alpha=0°$ these are the transverse wave numbers as obtained in Art. 2.5 and for $\alpha=90°$ these are the longitudinal wave numbers as obtained in Art. 2.6

From appendix -D the field components are:

For X-mode

$$E_z^X = \frac{j\epsilon_4'\epsilon_5'\sin\alpha\, K_{21}^2}{(\epsilon_1^2-\epsilon_4'^2)(K_0^2\epsilon_3'-K_{21}^2) + K_{21}^2(\epsilon_1^2-\epsilon_4'^2-\epsilon_1\epsilon_3')\sin^2\alpha}\, E_x^X \tag{2.47}$$

$$E_y^X = \frac{-j\epsilon_4'\cos\alpha\,(K_0^2\epsilon_3'-K_{21}^2)\, K_{21}^2}{K_0^2[(\epsilon_1^2-\epsilon_4'^2)(K_0^2\epsilon_3'-K_{21}^2) + K_{21}^2(\epsilon_1^2-\epsilon_4'^2-\epsilon_1\epsilon_3')\sin^2\alpha]}\, E_x^X \tag{2.48}$$

$$H_z^X = \frac{-j}{\omega\mu_0}\frac{\partial E_x^X}{\partial y} \tag{2.49}$$

$$H_x^X = \frac{-\epsilon_4'\epsilon_5'\sin\alpha\, K_2^2}{\omega\mu_0[(\epsilon_1^2-\epsilon_4'^2)(K_0^2\epsilon_3'-K_{21}^2) + K_{21}^2(\epsilon_1^2-\epsilon_4'^2-\epsilon_1\epsilon_3')\sin^2\alpha]}\frac{\partial E_x^X}{\partial y} \tag{2.50}$$

For O-mode

$$E_x^O = \frac{-j\epsilon_4'\sin\alpha\, K_{22}^2}{K_0^2(\epsilon_1^2-\epsilon_4'^2)-\epsilon_1 K_{22}^2}\, E_z^O \tag{2.51}$$

$$E_y^{O} = \frac{K_{zz}^2 \sin\alpha \cos\alpha}{K_0^2 \epsilon_3' (\epsilon_1^2 - \epsilon_4'^2)} \left[\frac{(\epsilon_1^2 - \epsilon_4'^2 - \epsilon_1 \epsilon_3')\{K_0^2(\epsilon_1^2 - \epsilon_4'^2) - \epsilon_1 K_{zz}^2\} - \epsilon_4'^2 \epsilon_3' K_{zz}^2}{K_0^2(\epsilon_1^2 - \epsilon_4'^2) - \epsilon_1 K_{zz}^2} \right] E_z^{O} \qquad (2.52)$$

$$H_x^{O} = \frac{j}{\omega\mu_0} \frac{\partial E_z^{O}}{\partial y} \qquad (2.53)$$

$$H_z^{O} = \frac{-\epsilon_4' \sin\alpha \; K_{zz}^2}{\omega\mu_0 [K_0^2(\epsilon_1^2 - \epsilon_4'^2) - \epsilon_1 K_{zz}^2]} \frac{\partial E_z^{O}}{\partial y} \qquad (2.54)$$

For $\alpha = 0^\circ$, $E_x^{O\,x} = 0$, $E_y^{O\,x} = 0$, $H_z^{O\,x} \neq 0$, so that the O-mode in this case consists of E_z and H_x only. Regarding the X-mode fields for $\alpha = 0^\circ$, we have $E_z = 0$, $H_x = 0$ and this mode consists of E_x, E_y and H_z only. These are consistent with the two simple cases described in Art. 2.4 and Art 2.5.

2.6.3. Waves in the z-direction for Static Magnetic Field in the yz -Plane

For a uniform plane wave $E = E_0$ traveling along the z-direction we have $\frac{\partial}{\partial x} = 0$ and $\frac{\partial}{\partial y} = 0$. Here the field components are governed by the equations given below (details shown in appendix-D):

$$H_z = 0 \qquad (2.55)$$

$$\frac{\partial E_y}{\partial z} = j\omega\mu_0 H_x \qquad (2.56)$$

$$\frac{\partial E_x}{\partial z} = -j\omega\mu_0 H_y \qquad (2.57)$$

$$-\frac{\partial H_y}{\partial z} = j\omega\epsilon_0(\epsilon_1 E_x - j\epsilon_4 E_y + j\epsilon_5 E_z) \qquad (2.58)$$

$$\frac{\partial H_x}{\partial z} = j\omega\epsilon_0(j\epsilon_4 E_x + \epsilon_2 E_y + \epsilon_6 E_z) \qquad (2.59)$$

$$0 = j\omega \epsilon_0 (-j\epsilon_5 E_x + \epsilon_6 E_y + \epsilon_3 E_z) \tag{2.60}$$

Since $H_z = 0$, then the propagation along z-direction is in the form of a TM wave. The wave number k_3 can be obtained as

$$\left.\begin{matrix} K_{31} \\ K_{32} \end{matrix}\right] = \pm \left[\frac{[\epsilon_1 \epsilon_3'(1+\cos^2\alpha) - (\epsilon_4'^2 - \epsilon_1^2)\sin^2\alpha] \mp \sqrt{\{(\epsilon_4'^2 - \epsilon_1^2)\sin^2\alpha - \epsilon_1\epsilon_3'(1+\cos^2\alpha)\}^2 - 4\epsilon_3\epsilon_3'(\epsilon_1^2 - \epsilon_4'^2)}}{2\epsilon_3} \right]^{1/2} \tag{2.61}$$

The +ve and +ve signs before the square root inside the 3rd bracket correspond to the X-mode and the O-mode of propagation respectively.

From appendix-D the field components are related as:

For X-mode

$$E_y^X = \frac{-j\epsilon_4'\epsilon_3'\cos\alpha\, K_{31}^2}{K_0^2\epsilon_3'(\epsilon_1^2 - \epsilon_4'^2) - K_{31}^2[(\epsilon_1^2 - \epsilon_4'^2)\sin^2\alpha + \epsilon_1\epsilon_3'\cos^2\alpha]} E_x^X \tag{2.62}$$

$$E_z^X = \frac{-j\sin\alpha[(\epsilon_1^2 - \epsilon_4'^2 - \epsilon_1\epsilon_3')\{K_0^2\epsilon_3'(\epsilon_1^2 - \epsilon_4'^2) - K_{31}^2\epsilon_1\epsilon_3'\} - K_{31}^2\epsilon_4'^2\epsilon_3'^2]}{K_0^2\epsilon_4'\epsilon_3'^2(\epsilon_1^2 - \epsilon_4'^2)} E_x^X \tag{2.63}$$

$$H_x^X = \frac{1}{\omega\mu_0}\left[\frac{-\epsilon_4'\epsilon_3'\cos\alpha\, K_{31}^2}{K_0^2\epsilon_3'(\epsilon_1^2 - \epsilon_4'^2) - K_{31}^2[(\epsilon_1^2 - \epsilon_4'^2)\sin^2\alpha + \epsilon_1\epsilon_3'\cos^2\alpha]}\right] \frac{\partial E_x^X}{\partial z} \tag{2.64}$$

$$H_y^X = \frac{j}{\omega\mu_0} \frac{\partial E_x^X}{\partial z} \tag{2.65}$$

For O-mode

$$E_x^O = \frac{j\epsilon_4'\cos\alpha\, K_{32}^2}{K_0^2(\epsilon_1^2 - \epsilon_4'^2) - K_{32}^2\epsilon_1} E_y^O \tag{2.66}$$

$$E_z^O = \frac{K_{32}^2 \sin\alpha\cos\alpha}{K_0^2\epsilon_3'} \left[\frac{K_{32}^2(\epsilon_3' - \epsilon_1) + K_0^2(\epsilon_1^2 - \epsilon_4'^2 - \epsilon_1\epsilon_3')}{K_0^2(\epsilon_1^2 - \epsilon_4'^2) - K_{32}^2\epsilon_1}\right] E_y^O \tag{2.67}$$

$$H_x^0 = \frac{j}{\omega\mu_0} \frac{\partial E_y^0}{\partial z} \tag{2.68}$$

$$H_y^0 = \frac{-\epsilon_q' \cos\alpha\, K_{32}^2}{\omega\mu_0 [K_0^2(\epsilon_1^2 - \epsilon_q'^2) - K_{32}^2 \epsilon_1]} \frac{\partial E_y^0}{\partial z} \tag{2.69}$$

For $\alpha = 0°$ and $90°$,the above relations are consistent with those discussed in Art. 2.4 and 2.5 .

2.7. Discussion

In the above a study has been given on the mechanism of plane wave propagation in a cold anisotropic plasma for an arbitrary direction of the static magnetic field. The dispersion characteristics of the mediums is such that plane wave is dispersed into an ordinary mode and an extra-ordinary mode advancing in different phase velocities. The wave number surfaces for the two modes were found to have diverse curvatures depending on the plasma parameters . Fundamental relations between the field components for plane waves propagating in arbitrary directions with respect to the static magnetic field were derived from Maxwell's equations. In the next chapter these results will be used to derive the aperture field distribution of a slot antenna excited by plane waves.

CHAPTER – 3

Plane Wave Excitation of the Aperture of a Slot Antenna in an Anisotropic Plasma

3.1 Introduction

To find the electric and magnetic field radiated from a slot antenna in the medium it is necessary to know the field distribution at the aperture. In this chapter we shall deduce the field distribution at the aperture of a narrow rectangular slot excited by a plane wave with the static magnetic field in an arbitrary direction. This will be carried out by making use of the results of plane wave polarization obtained in chapter-2.

3.2. A Narrow Rectangular Slot along the x-axis

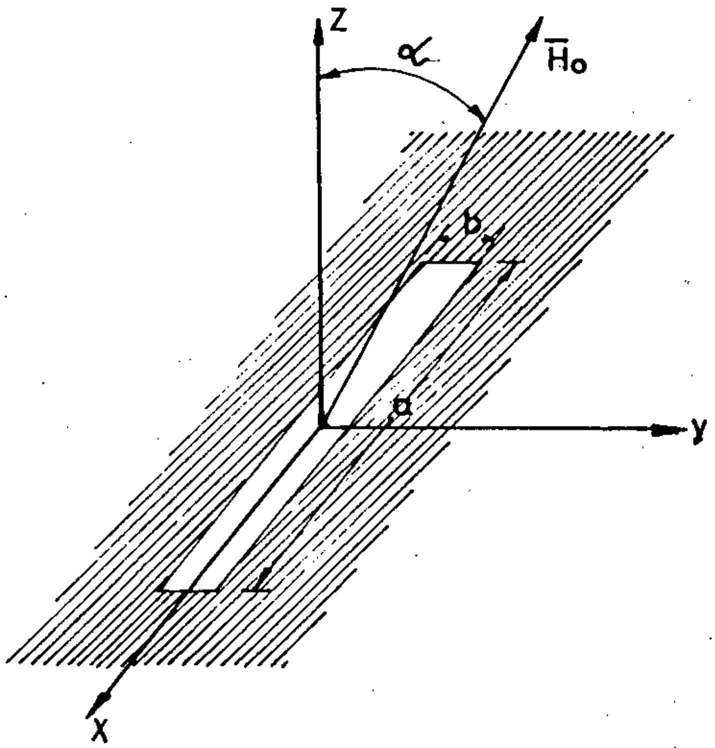


Fig 3.1: A narrow rectangular slot along the x-axis and lying in the xy-plane

As shown in Fig.3.1, let us consider a narrow rectangular slot along the x-axis on a ground plane of infinite extent lying in the xy-plane. The slot is excited by a plane wave traveling along the x-direction. Analysis similar to art 2.6.1 shows that the ordinary mode of TEM type and the extraordinary mode of TM type will be excited at the aperture. Let us designate the aperture field components as:

$$E_x^a = E_x^{ao} + E_x^{ax} \tag{3.1}$$

$$E_y^a = E_y^{ao} + E_y^{ax} \tag{3.2}$$

$$E_z^a = E_z^{ao} + E_z^{ax} \tag{3.3}$$

and

$$E_x^{ax} = j(\epsilon_2'/\epsilon_1)\, E_y^{a'} \tag{3.4}$$

$$E_y^{ax} = E_y^{a'} \cos\alpha \tag{3.5}$$

$$E_y^{ao} = E_z^{a'} \sin\alpha \tag{3.6}$$

$$E_z^{ax} = -E_y^{a'} \sin\alpha \tag{3.7}$$

$$E_z^{ao} = E_z^{a'} \cos\alpha \tag{3.8}$$

The superscripts 'a' refer to the aperture field and 'o' and 'x' refer to the O-mode and the X-mode fields respectively. In this case since the O-mode has no field component in the x-direction then $E_x^{ao}=0$. For the slot the field components must satisfy the boundary conditions $E_x^a = E_y^a = 0$ at $x = \pm a/2$ and also on the ground plane. Substituting these conditions in Maxwell's equations we find also that $E_z^a=0$ at $x=\pm a/2$.To meet these conditions the corresponding O-mode and the X-mode fields traveling at different phase velocities must separately vanish at the surface of the ground plane. From the superposition of positively and negatively traveling waves the aperture field components are found to have sinusoidal distribution of the forms:

$$E_y^a = A \sin k_1 (a/2 - |x|) \tag{3.9}$$

$$E_z^a = B \sin k_1 (a/2 - |x|) \tag{3.10}$$

where A and B are arbitrary constants. Resolving E_y^a and E_z^a into components along and transverse to the static magnetic field we find that the transverse field components travel with a wave number k_{11} and the longitudinal components travel with the wave number k_{12} , so that

$$E_y^{a'} = (A\cos\alpha - B\sin\alpha) \sin k_{11} (a/2 - |x|) \tag{3.11}$$

$$E_z^{a'} = (A\sin\alpha + B\cos\alpha) \sin k_{12} (a/2 - |x|) \tag{3.12}$$

Hence

$$\begin{aligned} E_z^a = & (A \sin\alpha \cos\alpha + B\cos^2\alpha) \sin k_{12} (a/2 - |x|) \\ & - (A \sin\alpha \cos\alpha - B \sin^2\alpha) \sin k_{11} (a/2 - |x|) \end{aligned} \tag{3.13}$$

Also it is assumed that the field component perpendicular to the slot i.e.,E_z^a is zero along the feed line i.e., at $x=0$. This assumption is quite reasonable because only at the feed point we can expect a purely transverse electric field excitation. In the case of isotropic medium this is true for all points in the slot. Hence from the condition $E_z^a=0$ at $x=0$ we get the relation between A and B,

$$\beta = \frac{\sin\alpha\cos\alpha\,[\sin(K_{11}a/2) - \sin(K_{12}a/2)]}{\sin^2\alpha\,\sin(K_{11}a/2) + \cos^2\alpha\,\sin(K_{12}a/2)}\,A \tag{3.14}$$

The magnetic field components are obtained as

$$H_y^{a'} = \pm\frac{jK_{12}}{\omega/\tau_0}\,(A\sin\alpha + \beta\cos\alpha)\cos K_{12}(a/2 - |x|),\quad x \gtrless 0 \tag{3.15}$$

$$H_z^{a'} = \mp\frac{jK_{11}}{\omega/\tau_0}\,(A\cos\alpha - \beta\sin\alpha)\cos K_{11}(a/2 - |x|),\quad x \gtrless 0 \tag{3.16}$$

3.3. A Narrow Rectangular Slot along the y-axis

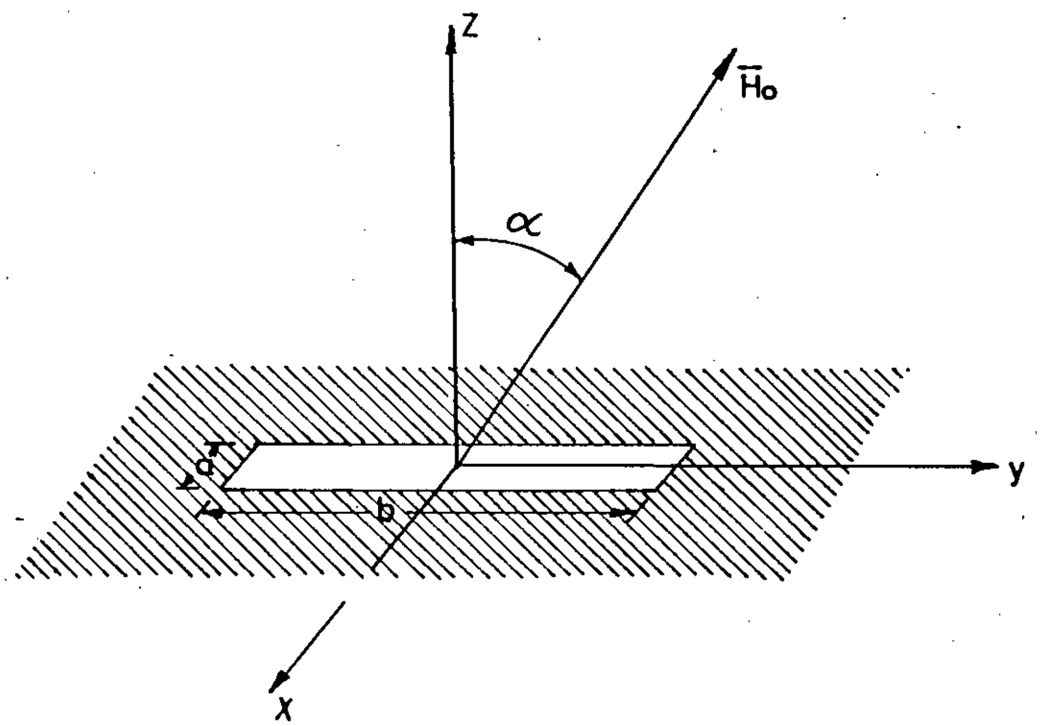


Fig 3.2: A narrow rectangular slot along the y-axis and lying in the xy-plane

As shown in Fig.3.2 let us consider a narrow rectangular slot along the y-axis excited by a plane wave traveling along the y-direction. Analysis similar to art 2.6.2 shows that ordinary and extraordinary modes of TM type will be excited at the aperture. Each field component can be represented as the summation of two mode as before.

For the slot along the y-axis lying in the xy-plane the field components must satisfy the boundary condition $E_x^a = E_y^a = 0$ at $y = \pm b/2$ as well as on the ground plane. To satisfy Maxwell's equations E_z^a will also be zero at $y = \pm b/2$.This requires that the corresponding O-mode and the X-mode fields traveling at different phase velocities must separately vanish at the surface of the ground plane so that from the superposition of positively and negatively traveling waves the field components at the aperture are found to have sinusoidal distribution. Accordingly the two field components from which the other field components can be found by using equations (2.46) and (2.53) are given by

$$E_x^{ax} = P \sin K_{21}(b/2 - |y|) \tag{3.17}$$

$$E_z^{ao} = Q \sin K_{22}(b/2 - |y|) \tag{3.18}$$

where P and Q are arbitrary constants.Substituting (3.17) into (2.46) and imposing the condition, $E_z^a = 0$ at $y = 0$, we find that P and Q are related as

$$Q = \frac{-j \epsilon_4' \epsilon_3' \sin\alpha \, K_{21}^2 \sin(K_{21} b/2)}{A \sin(K_{22} b/2)} P \tag{3.19}$$

where

$$A = (\epsilon_1^2 - \epsilon_4'^2)(K_0^2 \epsilon_3' - K_{21}^2) + K_{21}^2 (\epsilon_1^2 - \epsilon_4'^2 - \epsilon_1 \epsilon_3') \sin^2\alpha$$

Equation (3.19) establishes the relationship between the two modes of excitation at the slot aperture.

3.4. A Narrow Rectangular Slot along the z-axis

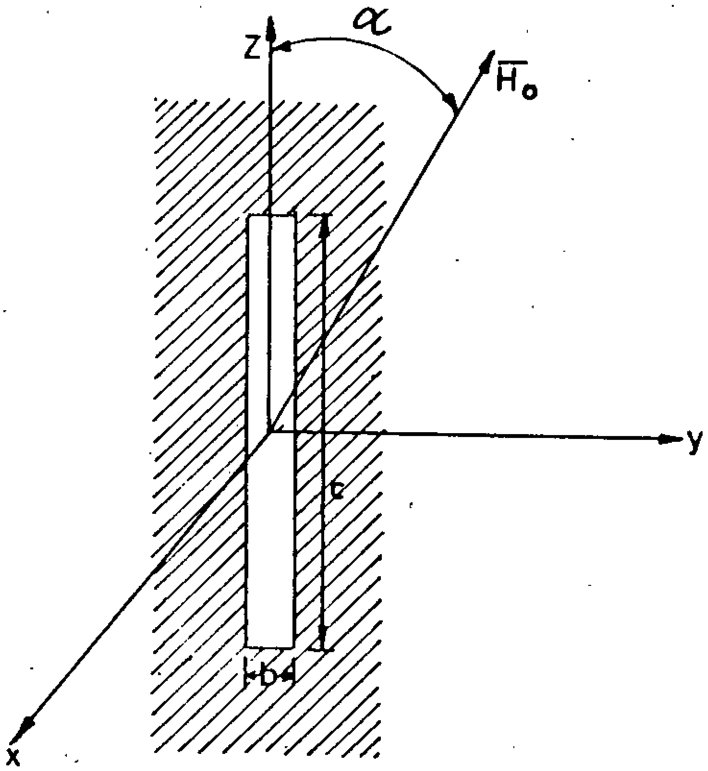


Fig 3.3: A narrow rectangular slot along the z-axis and lying in the xy-plane

As shown in Fig.3.3 Let us consider a narrow rectangular slot along the z-axis excited by a plane wave traveling along the z-direction. Analysis similar to art 2.6.3 shows that ordinary and extraordinary modes of TM type will be excited at the aperture. Each field component can be represented as the summation of the field components of two modes as before.

For the slot along the z-axis in the yz-plane the field components must satisfy the boundary condition $E_y^a = E_z^a = 0$ at $z = \pm c/2$ and also on the metal sheet. To satisfy Maxwell's equation E_x^a will also be zero at $z = \pm c/2$.This requires the that corresponding to the O-mode and the X-mode traveling at different phase velocities must separately vanish at the surface of the metal sheet and from the superposition of positively and negatively traveling waves the field components at the aperture can be shown to have sinusoidal distribution. Accordingly the two field components from which the other field components can be found by using equations (2.61)-(2.66) can be expressed as

$$E_y^{ao} = R \sin K_{32} (c/2 - |z|) \tag{3.20}$$

$$E_x^{ax} = S \sin K_{31} (c/2 - |z|) \tag{3.21}$$

where R and S are arbitrary constants. Substituting (3.20) into (2.63) and applying the condition $E_x^a = 0$ at $z=0$ we get

$$S = \frac{j \epsilon_q' \cos\alpha \, K_{32}^2 \sin(K_{32} c/2)}{C \sin(K_{31} c/2)} R \tag{3.22}$$

where

$$C = K_0^2 (\epsilon_q^2 - \epsilon_q'^2) - K_{32}^2 \epsilon_1$$

Equation(3.22) represents the relationship between the two modes of excitation in the slot aperture.

3.5. Discussion

In the above we derived the field distribution at the aperture of a narrow rectangular slot excited by a plane wave in an anisotropic medium by using the fundamental concepts of plane waves given in chapter-2. Here we made use of the boundary condition that the tangential field components must vanish at the slot edge and also on the ground plane. To obtain relationship between the two modes the electric field component perpendicular to the slot is considered to be zero at the feed point. With the knowledge of the aperture field distribution we are now in a position to find the fields outside the slot.

CHAPTER – 4

Field of a Narrow Rectangular Slot Antenna in an Anisotropic Plasma

4.1. Introduction

In the previous chapter we deduced the field distribution at the aperture of a slot excited by x,y and z directed traveling waves. In the present chapter we shall derive the field components produced outside the slot. We shall first obtain the general solution of the field components for aperture type of sources and then apply them to find the field components for narrow rectangular slots along x,y and z directions.

4.2 General Solution for Aperture Type of Sources

The electromagnetic field outside the slot is governed by

$$\nabla \times \bar{E} = -j\omega\mu_0 \bar{H} - \bar{J}_m \tag{4.1}$$

$$\nabla \times \bar{H} = j\omega\epsilon_0 \hat{\epsilon}_r \bar{E} + \bar{J}_e \tag{4.2}$$

where $\bar{J}_m$ and $\bar{J}_e$ are equivalent magnetic and electric current sheets in the slot.

$$\bar{J}_m = \bar{E}^a \times \bar{n} = \bar{a}_x J_{mx} + \bar{a}_y J_{my} + \bar{a}_z J_{mz} \tag{4.3}$$

$$\bar{J}_e = \bar{n} \times \bar{H}^a = \bar{a}_x J_{ex} + \bar{a}_y J_{ey} + \bar{a}_z J_{ez} \tag{4.4}$$

where $\bar{n}$ is a unit vector normal to the slot, $\bar{E}^a$ and $\bar{H}^a$ are aperture field distributions.

Taking the curl of equation(4.1) and substituting $\nabla \times \bar{H}$ by equation(4.2) we get

$$\nabla\times\nabla\times\bar{E} - K_0^2\epsilon_0\hat{\epsilon}_r\bar{E} = -j\,\omega\mu_0\,\bar{J}_e - \nabla\times\bar{J}_m \tag{4.5}$$

Let us write the field solution in Fourier transform pair.

$$\bar{E}(r) = \frac{1}{(2\pi)^3}\int_{-\alpha}^{\alpha} dK_1 \int_{-\alpha}^{\alpha} dK_2 \int_{-\alpha}^{\alpha} dK_3\, \bar{E}(K)\, e^{-j\bar{K}\cdot\bar{r}} \tag{4.6}$$

and

$$E(K) = \frac{1}{(2\pi)^3}\int_{-\alpha}^{\alpha} dx \int_{-\alpha}^{\alpha} dy \int_{-\alpha}^{\alpha} dz\, \bar{E}(r)\, e^{+j\bar{K}\cdot\bar{r}} \tag{4.7}$$

Now by Fourier transformation equation (4.5) can be arranged as

$$-\bar{K}\times\bar{K}\times\bar{E}(K) - K_0^2\epsilon_0\hat{\epsilon}_r\bar{E}(K) = j\left[\bar{K}\times\bar{J}_m(K) - \omega\mu_0\,\bar{J}_e(K)\right] \tag{4.8}$$

also

$$\bar{K}\times\bar{J}_m(K) - \omega\mu_0\,\bar{J}_e(K)$$

$$= \bar{a}_x M(K) + \bar{a}_y N(K) + \bar{a}_z O(K) \tag{4.9}$$

where

$$\left.\begin{aligned} M(K) &= K_2\,J_{mz}(K) - K_3\,J_{my}(K) - \omega\mu_0\,J_{ex}(K) \\ N(K) &= K_3\,J_{mx}(K) - K_1\,J_{mz}(K) - \omega\mu_0\,J_{ey}(K) \\ O(K) &= K_1\,J_{my}(K) - K_2\,J_{mx}(K) - \omega\mu_0\,J_{ez}(K) \end{aligned}\right\} \tag{4.10}$$

Introducing (4.9) into (4.8) and after several operations we can arrange equation(4.8) in the following form:

$$\begin{bmatrix} E_x(k) \\ E_y(k) \\ E_z(k) \end{bmatrix} = \frac{j}{\det \hat{\lambda}} \begin{bmatrix} A_{11} & A_{12} & A_{13} \\ A_{21} & A_{22} & A_{23} \\ A_{31} & A_{32} & A_{33} \end{bmatrix} \begin{bmatrix} M(k) \\ N(k) \\ O(k) \end{bmatrix} \tag{4.11}$$

The elements A_{ij} in the 3 X 3 matrix on the right hand side are as shown in appendix-B. Fourier transform of the electric field components are given by

$$E_x(k) = \frac{j}{\det \hat{\lambda}} X(k) \tag{4.12}$$

$$E_y(k) = \frac{j}{\det \hat{\lambda}} Y(k) \tag{4.13}$$

$$E_z(k) = \frac{j}{\det \hat{\lambda}} Z(k) \tag{4.14}$$

where

$$X(k) = A_{11} M(k) + A_{12} N(k) + A_{13} O(k) \tag{4.15}$$

$$Y(k) = A_{21} M(k) + A_{22} N(k) + A_{23} O(k) \tag{4.16}$$

$$Z(k) = A_{31} M(k) + A_{32} N(k) + A_{33} O(k) \tag{4.17}$$

In terms of the electric field components the magnetic field components are given by

$$H_x(K) = \frac{1}{\omega/k_o}\left[K_2 E_z(K) - K_3 E_y(K)\right] \tag{4.18}$$

$$H_y(K) = \frac{1}{\omega/k_o}\left[K_3 E_x(K) - K_1 E_z(K)\right] \tag{4.19}$$

$$H_z(K) = \frac{1}{\omega/k_o}\left[K_1 E_y(K) - K_2 E_x(K)\right] \tag{4.20}$$

The inverse Fourier transforms by equation (4.6) give the field components in the real space.

4.3 Solution for a Narrow Rectangular Slot along the x-axis

Field distribution at the aperture of a narrow rectangular slot along the x-axis excited by a plane wave will have all the field components except $H_x{}^a=0$ (chapter -2)

1) Slot on the xy-plane

we have,

$$\begin{aligned}\bar{J}_m &= \bar{E}^a \times \bar{n} \\ &= (\bar{a}_x E_x^a + \bar{a}_y E_y^a + \bar{a}_z E_z^a) \times \bar{a}_z\, \delta(z) \\ &= (\bar{a}_x E_y^a - \bar{a}_y E_x^a)\, \delta(z)\end{aligned} \tag{4.21}$$

where δ (z) is Dirac's delta function and indicates that the magnetic current sheets are along the xy-plane at z=0

$$\bar{J}_e = \bar{n} \times \bar{H}^a$$
$$= \bar{a}_z \times (\bar{a}_y H_y^a + \bar{a}_z H_z^a)\, \delta(z)$$
$$= -\bar{a}_x H_y^a \delta(z) \tag{4.22}$$

$$\left.\begin{aligned} J_{mx} &= E_y^a \delta(z) \\ J_{my} &= -E_x^a \delta(z) \\ J_{mz} &= 0 \\ J_{ex} &= -H_y^a \delta(z) \\ J_{ey} &= 0 \\ J_{ex} &= 0 \end{aligned}\right\} \tag{4.23}$$

Using relations (4.23) and (4.10) we get

$$\left.\begin{aligned} M(K) &= K_3 E_x^a(K_1, K_2) + \omega/\mu_0 H_y^a(K_1, K_2) \\ N(K) &= K_3 E_y^a(K_1, K_2) \\ O(K) &= -K_1 E_x^a(K_1, K_2) - K_2 E_y^a(K_1, K_2) \end{aligned}\right\} \tag{4.24}$$

II) Slot along the xz-plane

We have,

$$\bar{J}_m = \bar{E}^a \times \bar{n}$$
$$= (\bar{a}_x E_x^a + \bar{a}_y E_y^a + \bar{a}_z E_z^a) \times \bar{a}_y\, \delta(y)$$
$$= (-\bar{a}_x E_z^a + \bar{a}_z E_x^a)\, \delta(y) \tag{4.25}$$

and

$$\bar{J}e = \bar{n} \times \bar{H}^a$$
$$= \bar{a}_y \times (\bar{a}_y H_y^a + \bar{a}_z H_z^a)\, \delta(y)$$
$$= \bar{a}_x H_z^a\, \delta(y) \qquad (4.26)$$

$$\left.\begin{aligned} J_{ex} &= H_z^a\, \delta(y) \\ J_{mx} &= -E_z^a\, \delta(y) \\ J_{my} &= 0 \\ J_{mz} &= E_x^a\, \delta(y) \\ J_{ey} &= 0 \\ J_{ez} &= 0 \end{aligned}\right\} \qquad (4.27)$$

Using relations (4.27) and (4.10) we get

$$\left.\begin{aligned} M(k) &= k_2 E_x^a(k_1, k_3) - \omega\mu_0 H_z^a(k_1, k_3) \\ N(k) &= -k_3 E_z^a(k_1, k_3) - k_1 E_x^a(k_1, k_3) \\ O(k) &= k_2 E_z^a(k_1, k_3) \end{aligned}\right\} \qquad (4.28)$$

4.4. Solution for a Narrow Rectangular Slot along the y-axis

Field distribution at the aperture of a narrow rectangular slot along the y-axis excited by a plane wave will have all the field components except $H_y^a=0$ (chapter-2).

1) Slot along the xy-plane

we have

$$
\begin{aligned}
\bar{J}_m &= \bar{E}^a \times \bar{n} \\
&= \left(\bar{a}_x E_x^a + \bar{a}_y E_y^a + \bar{a}_z E_z^a\right) \times \bar{a}_z\, \delta(z) \\
&= \left(\bar{a}_x E_y^a - \bar{a}_y E_x^a\right) \delta(z)
\end{aligned}
\tag{4.29}
$$

and

$$
\begin{aligned}
\bar{J}_e &= \bar{n} \times \bar{H}^a \\
&= \bar{a}_z \times \left(\bar{a}_x H_x^a + \bar{a}_z H_z^a\right) \delta(z) \\
&= \bar{a}_y H_x^a\, \delta(z)
\end{aligned}
\tag{4.30}
$$

Therefore, we have

$$
\left.
\begin{aligned}
J_{mx} &= E_y^a\, \delta(z) \\
J_{my} &= -E_x^a\, \delta(z) \\
J_{mz} &= 0 \\
J_{ex} &= 0 \\
J_{ey} &= H_x^a\, \delta(z) \\
J_{ez} &= 0
\end{aligned}
\right\}
\tag{4.31}
$$

Using relations (4.31) and (4.10) we get

$$
\left.
\begin{aligned}
M(K) &= K_3\, E_x^a(K_1, K_2) \\
N(K) &= K_3\, E_y^a(K_1, K_2) - \omega/\kappa_0\, H_x^a(K_1, K_2) \\
O(K) &= -K_1\, E_x^a(K_1, K_2) - K_2\, E_y^a(K_1, K_2)
\end{aligned}
\right\}
\tag{4.32}
$$

II) Slot along the yz-plane

we have

$$
\begin{aligned}
\bar{J}_m &= \bar{E}^a \times \bar{n} \\
&= \left(\bar{a}_x E_x^a + \bar{a}_y E_y^a + \bar{a}_z E_z^a\right) \times \bar{a}_x\, \delta(x) \\
&= \left(\bar{a}_y E_z^a - \bar{a}_z E_y^a\right) \delta(x)
\end{aligned}
\tag{4.33}
$$

and

$$
\begin{aligned}
\bar{J}_e &= \bar{n} \times \bar{H}^a \\
&= \bar{a}_x \times \left(\bar{a}_x H_x^a + \bar{a}_z H_z^a\right) \delta(x) \\
&= -\bar{a}_y H_z^a\, \delta(x)
\end{aligned}
\tag{4.34}
$$

Therefore, we have

$$\left.\begin{aligned}
J_{mx} &= 0 \\
J_{my} &= E_z^a \delta(x) \\
J_{mz} &= -E_y^a \delta(x) \\
J_{ex} &= 0 \\
J_{ey} &= -H_z^a \delta(x) \\
J_{ez} &= 0
\end{aligned}\right\} \tag{4.35}$$

Using relations (4.35) and (4.10) we get

$$\left.\begin{aligned}
M(K) &= -K_2 E_y^a(K_2,K_3) - K_3 E_z^a(K_2,K_3) \\
N(K) &= K_1 E_y^a(K_2,K_3) + \omega\mu_0 H_z^a(K_2,K_3) \\
O(K) &= K_1 E_z^a(K_2,K_3)
\end{aligned}\right\} \tag{4.36}$$

4.5. Solution for a Narrow Rectangular Slot along the z-axis

Field distribution at the aperture of a narrow rectangular slot along z-axis excited by a plane wave will have all the field components except $H_z^a=0$ (chapter-2).

1) Slot along the yz -plane

we have

$$\begin{aligned}
\bar{J}_m &= \bar{E}^a \times \bar{n} \\
&= (\bar{a}_x E_x^a + \bar{a}_y E_y^a + \bar{a}_z E_z^a) \times \bar{a}_x \delta(x) \\
&= (\bar{a}_y E_z^a - \bar{a}_z E_y^a)\, \delta(x)
\end{aligned} \tag{4.37}$$

and

$$\begin{aligned}
\bar{J}_e &= \bar{n} \times \bar{H}^a \\
&= \bar{a}_x \times (\bar{a}_x H_x^a + \bar{a}_y H_y^a)\, \delta(x) \\
&= \bar{a}_z H_y^a \delta(x)
\end{aligned} \tag{4.38}$$

Therefore, we have

$$\left.\begin{aligned} J_{mx} &= 0 \\ J_{mz} &= -E_y^a\,\delta(x) \\ J_{my} &= E_z^a\,\delta(x) \\ J_{ex} &= 0 \\ J_{ey} &= 0 \\ J_{ez} &= H_y^a\,\delta(x) \end{aligned}\right\} \tag{4.39}$$

Using relations (4.39) and (4.10) we get

$$\left.\begin{aligned} M(K) &= -K_2 E_y^a(K_2,K_3) - K_3 E_z^a(K_2,K_3) \\ N(K) &= K_1 E_y^a(K_2,K_3) \\ O(K) &= K_1 E_z^a(K_2,K_3) - \omega/\kappa_0\, H_y^a(K_2,K_3) \end{aligned}\right\} \tag{4.40}$$

II) Slot along the xz-plane

we have

$$\begin{aligned} \bar{J}_m &= \bar{E}^a \times \bar{a}_y\,\delta(y) \\ &= (\bar{a}_x E_x^a + \bar{a}_y E_y^a + \bar{a}_z E_z^a) \times \bar{a}_y\,\delta(y) \\ &= (-\bar{a}_x E_z^a + \bar{a}_z E_x^a)\,\delta(y) \end{aligned} \tag{4.41}$$

and

$$\begin{aligned} \bar{J}_e &= \bar{n} \times \bar{H}_a \\ &= \bar{a}_y \times (\bar{a}_x H_x^a + \bar{a}_y H_y^a)\,\delta(y) \\ &= -\bar{a}_z H_x^a\,\delta(y) \end{aligned} \tag{4.42}$$

Therefore we have

$$\left.\begin{aligned} J_{mx} &= -E_z^a\,\delta(y) \\ J_{my} &= 0 \\ J_{mz} &= E_x^a\,\delta(y) \\ J_{ex} &= 0 \\ J_{ey} &= 0 \\ J_{ez} &= -H_x^a\,\delta(y) \end{aligned}\right\} \tag{4.43}$$

Using relations (4.43) and (4.10) we get

$$\left.\begin{aligned} M(k) &= k_2 E_x^a(k_1, k_3) \\ N(k) &= -k_3 E_z^a(k_1, k_3) - k_1 E_x^a(k_1, k_3) \\ O(k) &= k_2 E_z^a(k_1, k_3) + \omega\mu_0 H_x^a(k_1, k_3) \end{aligned}\right\} \quad (4.44)$$

4.6. Discussion

In the above we have found a solution in integral form of the field outside a slot assuming the sources as equivalent magnetic and electric current sheets in the slot. For various slot positions with respect to the static magnetic field the equivalent electric and magnetic currents were obtained. In the next chapter the field integrals will be evaluated by taking residues and saddle points.

CHAPTER – 5

Closed Form Evaluation of the Radiation Field

5.1. Introduction

In the previous chapter the field integrals were derived in triple Fourier Integral forms and can not be generally evaluated in closed forms. The integrands contain pole and branch point singularities which must be investigated separately. For pole-singularities it is necessary to employ residue evaluation. Asymptotic techniques should be applied for obtaining the radiation field or far field in closed form. One of the integrals is evaluated by taking residue with respect to the wave number parallel to the slot axis. The remaining field integrals are evaluated by two dimensional saddle points.

5.2. Residue Evaluation of an Integral

From the Fourier transforms of the field components given by(4.12)-(4-14),the inverse Fourier transformers are

$$E_x(x,y,z) = \frac{j}{(2\pi)^3}\iiint_{-\infty}^{\infty} \frac{X(k)}{\det\hat{\lambda}} e^{-jk_1x-jk_2y-jk_3z} \, dk_1\, dk_2\, dk_3 \tag{5.1}$$

$$E_y(x,y,z) = \frac{j}{(2\pi)^3}\iiint_{-\infty}^{\infty} \frac{Y(k)}{\det\hat{\lambda}} e^{-jk_1x-jk_2y-jk_3z} \, dk_1\, dk_2\, dk_3 \tag{5.2}$$

$$E_z(x,y,z) = \frac{j}{(2\pi)^3}\iiint_{-\infty}^{\infty} \frac{Z(k)}{\det\hat{\lambda}} e^{-jk_1x-jk_2y-jk_3z} \, dk_1\, dk_2\, dk_3 \tag{5.3}$$

One of the integrals can be evaluated by taking residue with respect to the wave number parallel to the slot axis. The residue evaluation requires that det $\hat{\lambda}$ be factorized in terms of the wave numbers parallel to the slot axis.

5.2.1. Residue Evaluation for Slot along the x-axis

In terms of the x-directed wave numbers det $\hat{\lambda}$ can be expressed as

$$\det \hat{\lambda} = -K_0^2 \epsilon_1 (K_1 - \rho_{11}^+)(K_1 - \rho_{11}^-)(K_1 - \rho_{12}^+)(K_1 - \rho_{12}^-) \tag{5.4}$$

where $\rho_{11}^{\pm}$ corresponds to the X-mode wave numbers in the $\pm$ x-directions and $\rho_{12}^{\pm}$ corresponds to the O-mode wave numbers in the $\pm$ x directions. Referring to equations (5.3), the k_1 - integral is given by taking residues at $k_1 \to \rho_{11}^{\pm}$ and $k_1 \to \rho_{12}^{\pm}$ remembering the facts that $\rho_{11}^+ = -\rho_{11}^-$ and $\rho_{12}^+ = -\rho_{12}^-$

$$E_x(x,y,z) = \frac{1}{4\pi^2 K_0^2 \epsilon_1} \iint_{-\alpha}^{\alpha} \left[\frac{X(\rho_{11}^+)\, e^{-j\rho_{11}^+|x|}}{D_1(\rho_{11}^+)} + \frac{X(\rho_{12}^+)\, e^{-j\rho_{12}^+|x|}}{D_2(\rho_{12}^+)} \right] e^{-jk_2 y - jk_3 z}\, dk_2\, dk_3 \tag{5.5}$$

$$E_y(x,y,z) = \frac{1}{4\pi^2 K_0^2 \epsilon_1} \iint_{-\alpha}^{\alpha} \left[\frac{Y(\rho_{11}^+)\, e^{-j\rho_{11}^+|x|}}{D_1(\rho_{11}^+)} + \frac{Y(\rho_{12}^+)\, e^{-j\rho_{12}^+|x|}}{D_2(\rho_{12}^+)} \right] e^{-jk_2 y - jk_3 z}\, dk_2\, dk_3 \tag{5.6}$$

$$E_z(x,y,z) = \frac{1}{4\pi^2 K_0^2 \epsilon_1} \iint_{-\alpha}^{\alpha} \left[\frac{Z(\rho_{11}^+)\, e^{-j\rho_{11}^+|x|}}{D_1(\rho_{11}^+)} + \frac{Z(\rho_{12}^+)\, e^{-j\rho_{12}^+|x|}}{D_2(\rho_{12}^+)} \right] e^{-jk_2 y - jk_3 z}\, dk_2\, dk_3 \tag{5.7}$$

where

$$D_1(\rho_{11}^+) = (\rho_{11}^+ - \rho_{11}^-)(\rho_{11}^+ - \rho_{12}^+)(\rho_{11}^+ - \rho_{12}^-) \tag{5.8}$$

$$D_2(\rho_{12}^+) = (\rho_{12}^+ - \rho_{11}^+)(\rho_{12}^+ - \rho_{11}^-)(\rho_{12}^+ - \rho_{12}^-) \tag{5.9}$$

5.2.2. Residue Evaluation for Slot along the y-axis

In terms of the y-directed wave numbers

$$\det\hat{\lambda} = -k_0^2\epsilon_2(k_2-\beta_{21}^{+})(k_2-\beta_{21}^{-})(k_2-\beta_{22}^{+})(k_2-\beta_{22}^{-}) \tag{5.10}$$

Where $\beta_{21}^{\pm}$ correspond to the χ-mode wave numbers in the $\pm$ y-direction and $\beta_{22}^{\pm}$ correspond to the ρ-mode wave numbers, in the $\pm$ y-direction. Referring to equations(5.1) -(5.3) the k_2 - integral is evaluated by taking residues at $k_2 \to \beta_{21}^{\pm}, k_2 \to \beta_{22}^{\pm}$ for $y \gtrless 0$

$$E_x(x,y,z) = \frac{1}{4\pi^2 k_0^2\epsilon_2}\iint_{-\alpha}^{\alpha}\left[\frac{X(\beta_{21}^{\pm})e^{-j\beta_{21}^{\pm}y}}{D_1(\beta_{21}^{\pm})} + \frac{X(\beta_{22}^{\pm})e^{-j\beta_{22}^{\pm}y}}{D_2(\beta_{22}^{\pm})}\right]e^{-jk_1x-jk_3z}\,dk_1\,dk_3 \qquad y \gtrless 0 \tag{5.11}$$

$$E_y(x,y,z) = \frac{1}{4\pi^2 k_0^2\epsilon_2}\iint_{-\alpha}^{\alpha}\left[\frac{Y(\beta_{21}^{\pm})e^{-j\beta_{21}^{\pm}y}}{D_1(\beta_{21}^{\pm})} + \frac{Y(\beta_{22}^{\pm})e^{-j\beta_{22}^{\pm}y}}{D_2(\beta_{22}^{\pm})}\right]e^{-jk_1x-jk_3z}\,dk_1\,dk_3 \qquad y \gtrless 0 \tag{5.12}$$

$$E_z(x,y,z) = \frac{1}{4\pi^2 k_0^2\epsilon_2}\iint_{-\alpha}^{\alpha}\left[\frac{Z(\beta_{21}^{\pm})e^{-j\beta_{21}^{\pm}y}}{D_1(\beta_{21}^{\pm})} + \frac{Z(\beta_{22}^{\pm})e^{-j\beta_{22}^{\pm}y}}{D_2(\beta_{22}^{\pm})}\right]e^{-jk_1x-jk_3z}\,dk_1\,dk_3 \qquad y \gtrless 0 \tag{5.13}$$

where

$$D_1(\beta_{21}^{\pm}) = (\beta_{21}^{+}-\beta_{21}^{-})(\beta_{21}^{\pm}-\beta_{22}^{+})(\beta_{21}^{\pm}-\beta_{22}^{-}) \tag{5.14}$$

$$D_2(\beta_{22}^{\pm}) = (\beta_{22}^{+}-\beta_{22}^{-})(\beta_{22}^{\pm}-\beta_{21}^{\pm})(\beta_{22}^{\pm}-\beta_{21}^{-}) \tag{5.15}$$

It can be noted that $\beta_{21}^{+} \neq -\beta_{21}^{-}$ and $\beta_{22}^{+} \neq \beta_{22}^{-}$ for arbitrary α ,indicating that there is no propagation symmetry between +y and -y directions.

5.2.3. Residue Evaluation for Slot along the z-axis

In terms of the z-directed wave numbers

$$\det \hat{\lambda} = -K_0^2\epsilon_3 (K_3-\beta_{31}^+)(K_3-\beta_{31}^-)(K_3-\beta_{32}^+)(K_3-\beta_{32}^-) \tag{5.16}$$

Where $\beta_{31}^{\pm}$ corresponds to the χ-mode wave numbers in the $\pm$ z-direction and $\beta_{32}^{\pm}$ corresponds to the θ-mode wave numbers , in the $\pm$ z-direction. Referring to equations(5.1)-(5.3), the k_3- integral is given by taking residues at $k_3 \to \beta_{31}^{\pm}, k_3 \to \beta_{32}^{\pm}$ for $z \gtrless 0$.

$$E_x(x,y,z) = \frac{1}{4\pi^2 K_0^2\epsilon_3}\iint_{-\infty}^{\infty}\left[\frac{X(\beta_{31}^{\pm})e^{-j\beta_{31}^{\pm}z}}{D_1(\beta_{31}^{\pm})} + \frac{X(\beta_{32}^{\pm})e^{-j\beta_{32}^{\pm}z}}{D_2(\beta_{32}^{\pm})}\right]e^{-jK_1x-jK_2y}\,dK_1\,dK_2 \qquad z\gtrless 0 \tag{5.17}$$

$$E_y(x,y,z) = \frac{1}{4\pi^2 K_0^2\epsilon_3}\iint_{-\infty}^{\infty}\left[\frac{Y(\beta_{31}^{\pm})e^{-j\beta_{31}^{\pm}z}}{D_1(\beta_{31}^{\pm})} + \frac{Y(\beta_{32}^{\pm})e^{-j\beta_{32}^{\pm}z}}{D_2(\beta_{32}^{\pm})}\right]e^{-jK_1x-jK_2y}\,dK_1\,dK_2 \qquad z\gtrless 0 \tag{5.18}$$

$$E_z(x,y,z) = \frac{1}{4\pi^2 K_0^2\epsilon_3}\iint_{-\infty}^{\infty}\left[\frac{Z(\beta_{31}^{\pm})e^{-j\beta_{31}^{\pm}z}}{D_1(\beta_{31}^{\pm})} + \frac{Z(\beta_{32}^{\pm})e^{-j\beta_{32}^{\pm}z}}{D_2(\beta_{32}^{\pm})}\right]e^{-jK_1x-jK_2y}\,dK_1\,dK_2 \qquad z\gtrless 0 \tag{5.19}$$

where

$$D_1(\beta_{31}^{\pm}) = (\beta_{31}^{+}-\beta_{31}^{-})(\beta_{31}^{\pm}-\beta_{32}^{+})(\beta_{31}^{\pm}-\beta_{32}^{-}) \tag{5.20}$$

$$D_2(\beta_{32}^{\pm}) = (\beta_{32}^{+}-\beta_{32}^{-})(\beta_{32}^{\pm}-\beta_{31}^{+})(\beta_{32}^{\pm}-\beta_{31}^{-}) \tag{5.21}$$

Note that $\beta_{31}^{+} \neq \beta_{31}^{-}$ and $\beta_{32}^{+} \neq \beta_{32}^{-}$ for arbitrary α indicating the lack of propagation symmetry between +z and -z directions.

5.3 Evaluation of the far field

The double Fourier integrals in art 5.2 can be evaluated asymptotically by the method of saddle points. The method is briefly described in appendix-E. An asymptotic evaluation of these field integrals requires evaluation of two dimensional saddle points such that the phase is minimum with respect to the transverse wave number i.e.

$$\frac{\partial \psi}{\partial k_t} = 0 \tag{5.22}$$

where $\psi = k_1 x + k_2 y + k_3 z$ is the phase of each wave and $k_t = (k_{t1}, k_{t2})$ is the transverse wave number which is (k_2, k_3), (k_3, k_1) and (k_1, k_2) for slots along x, y and z-directions respectively.

The contribution of the saddle points to the magnitude of each wave of type $e^{-j\psi}$ is given by Matin et al. [3] .

$$M = \frac{2\pi}{\left[\left(\frac{\partial^2 \psi}{\partial k_{t1s}^2}\right)\left(\frac{\partial^2 \psi}{\partial k_{t2s}^2}\right) - \left(\frac{\partial^2 \psi}{\partial k_{t1s}\, \partial k_{t2s}}\right)^2\right]^{1/2}} \tag{5.23}$$

where k_{t1s}, k_{t2s} are the two dimensional saddle points satisfying equation (5.22) for a stationary phase of the propagating waves.

The phase is determined by

$$\alpha = Exp\left[j\psi(k_{t1s}, k_{t2s}) + j\frac{\pi}{4}\left\{sgn\frac{\partial^2 \psi}{\partial k_{t1s}^2} + sgn\left[\frac{\partial^2 \psi}{\partial k_{t2s}^2} - \left(\frac{\partial^2 \psi}{\partial k_{t1s}\,.\,\partial k_{t2s}}\right)^2 \Big/ \frac{\partial^2 \psi}{\partial k_{t1s}^2}\right]\right\}\right] \tag{5.24}$$

The expressions for $\frac{\partial\psi}{\partial K_{\ell 1s}}$, $\frac{\partial\psi}{\partial K_{\ell 2s}}$, $\frac{\partial^2\psi}{\partial K_{\ell 1s}^2}$, $\frac{\partial^2\psi}{\partial K_{\ell 1s}\partial K_{\ell 2s}}$, $\frac{\partial^2\psi}{\partial K_{\ell 2s}^2}$ etc. for slots in different directions are shown in Appendix-E

The saddle points can be determined either by solving equation(5.22) or graphically from the wave surfaces. Only those saddle points for which the wave numbers are real, yield propagating wave contribution to the radiation field; the field associated with complex wave numbers is exponentially damped and will not be considered for the radiation field.

For plane wave analysis in the xy- and xz-planes the saddle points are determined by the real roots of the characteristic equation setting k_3 and k_2 equal to zero respectively in equation (2.15) . In the two cases there is a symmetry of wave surfaces for an arbitrary inclination of the static magnetic field. For studying waves in the yz-plane the determination of the saddle points from the characteristic equation is analytically difficult for arbitrary α due to the presence of the cross terms $k_2k_3, k_2^2k_3^2, k_2^3k_3, k_2k_3^3$ etc. Alternatively these can be determined graphically from the wave number surfaces in the k_3-k_2 plane for an arbitrary inclination of the static magnetic field.The field integrals are obtained asymptotically in closed form:

$$E_x = \frac{1}{4\pi^2 K_0^2 \epsilon_{1,2,3}} \sum_{n=1,2} \frac{X_{ns} M_{ns} \alpha_{ns}}{D_{ns}} \tag{5.25}$$

$$E_y = \frac{1}{4\pi^2 K_0^2 \epsilon_{1,2,3}} \sum_{n=1,2} \frac{Y_{ns} M_{ns} \alpha_{ns}}{D_{ns}} \tag{5.26}$$

$$E_z = \frac{1}{4\pi^2 k_0^2 \epsilon_{1,2,3}} \sum_{n=1,2} \frac{Z_{ns} M_{ns} \alpha_{ns}}{D_{ns}} \qquad (5.27)$$

where X_{ns}, Y_{ns}, Z_{ns} and D_{ns} are the values of $X(\rho)$, $Y(\rho)$, $Z(\rho)$ and $D(\rho)$ at saddle points respectively . The subscripts n=1,2 refer to mode no.1 and mode no.2 respectively and $\epsilon_{1,2,3}$ stand for slots along x,y, and z axes respectively.

5.4. Discussion

The field integrals of a slot antenna in anisotropic plasma as derived in chapter-4 have been evaluated by residue theorem and saddle point method. This evaluation have been done for plasma parameters corresponding to the region (a) in the CMA diagram where the wave number surfaces have regular shapes permitting the saddle point evaluation.The other regions of CMA diagram which contain turning points, saddle point method can not be applied in a straight forward manner. The magnetic field components can be obtained readily from the electric field components. In the next chapter the power density will be calculated by evaluating the Poynting vector.

CHAPTER – 6

Radiation Pattern of a Slot Antenna in an Anisotropic Plasma

6.1 Introduction

With the closed form evaluation of the radiation field given in the previous chapter we are now at a stage to calculate the radial component of the Poynting vector. In this chapter the computation of the Poynting power flow in various directions has been carried out to find the radiation pattern of a slot along different axis and planes at arbitrary angles with respect to the static magnetic field.

6.2 Calculation of the Radiation Pattern

Magnetic field components H_x, H_y, H_z are obtained from Maxwell's equations in terms of E_x, E_y, E_z as derived in chapter-5. The field components are then transformed into spherical co-ordinates using the transformations:

$$E_\theta = E_x \cos\theta \cos\phi + E_y \cos\theta \sin\phi - E_z \sin\theta \tag{6.1}$$

$$E_\phi = -E_x \sin\phi + E_y \cos\phi \tag{6.2}$$

$$H_\theta = H_x \cos\theta \cos\phi + H_y \cos\theta \sin\phi - H_z \sin\theta \tag{6.3}$$

$$H_\phi = -H_x \sin\phi + H_y \cos\phi \tag{6.4}$$

Then the time averaged poynting vector is calculated from :

$$P_r = \frac{1}{2} Re\left[E_\theta H_\phi^* - E_\phi H_\theta^*\right] \tag{6.5}$$

A computer program for the directional power pattern is run by an IBM-4331 VM at the BUET Computer Center for evaluating the radiation pattern of a

Slot placed along the x,y and z-axes. The computed power intensity is normalized by the maximum power density of a half-wavelength slot antenna in free space and expressed in decibel by

$$db = 10\log_{10} \frac{P}{P_{rmo}} \tag{6.6}$$

where P_{rmo} is the maximum power density for a half-wavelength slot radiating in free space.

6.3. Radiation Pattern of a Slot along the x-axis lying in the xy -plane

In this case the angle of radiation is calculated from the saddle points by

$$\tan\phi = -\left.\frac{\partial K_1}{\partial K_2}\right|_{K_3=0} \tag{6.7}$$

for xy plane pattern and

$$\cot\phi = -\left.\frac{\partial K_1}{\partial K_3}\right|_{K_2=0} \tag{6.8}$$

For xz-plane pattern

Using the procedure stated above normalized power in db is computed for a half wavelength slot placed along the x-axis lying in the xy-plane. Power pattern is plotted for xy and xz -plane for various inclination of the static magnetic field and is illustrated in figs. 6.1 - 6.2 .

It is found that the radiation is dominated by the O-mode with negligible radiation in the form of X-mode. A very sharp beam is radiated symmetrically

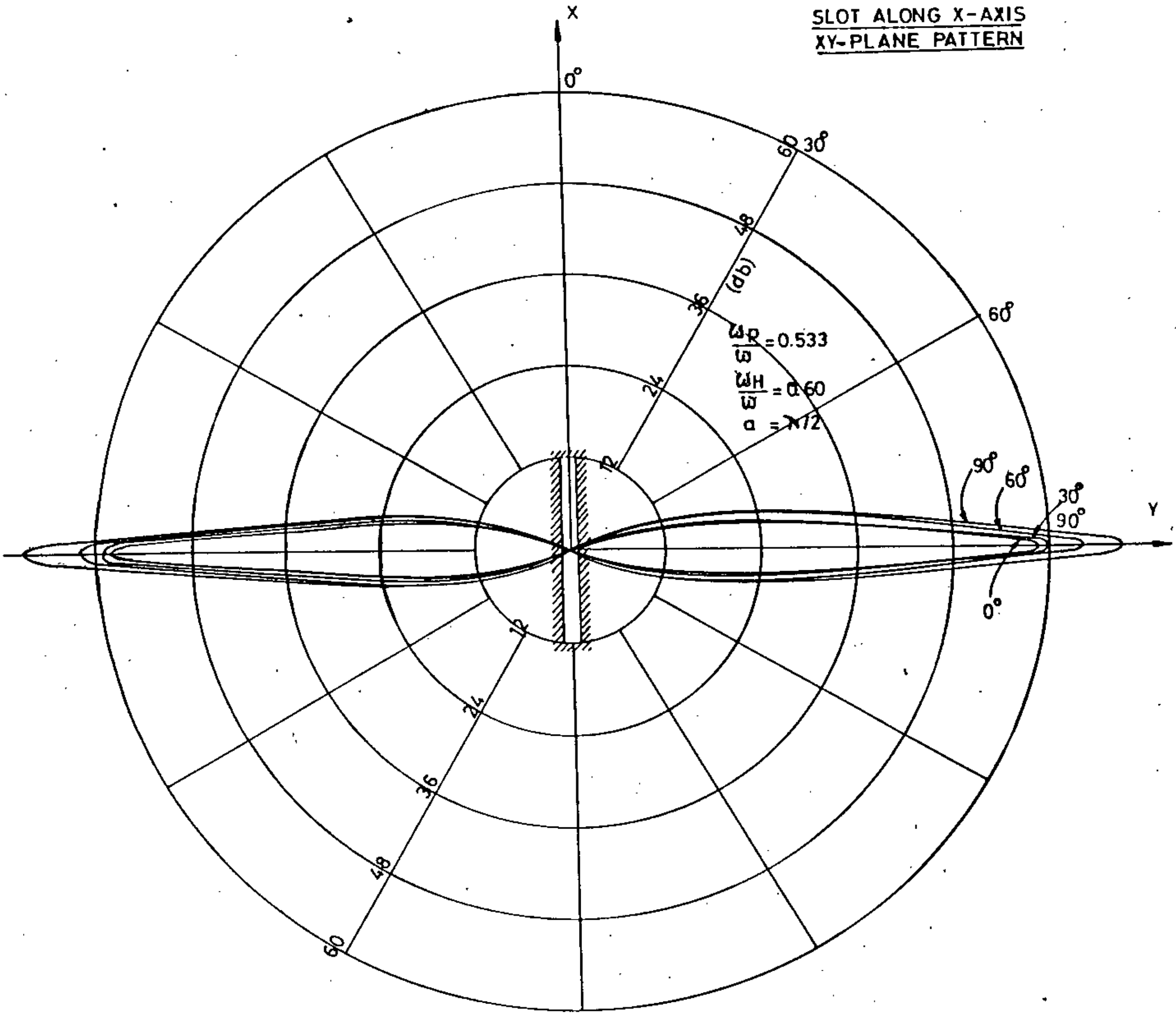


Fig.6.1 : Normalized radiation pattern in the xy-plane with α=0°,30°,60°,90° for a slot along the x-axis lying in the xy-plane.

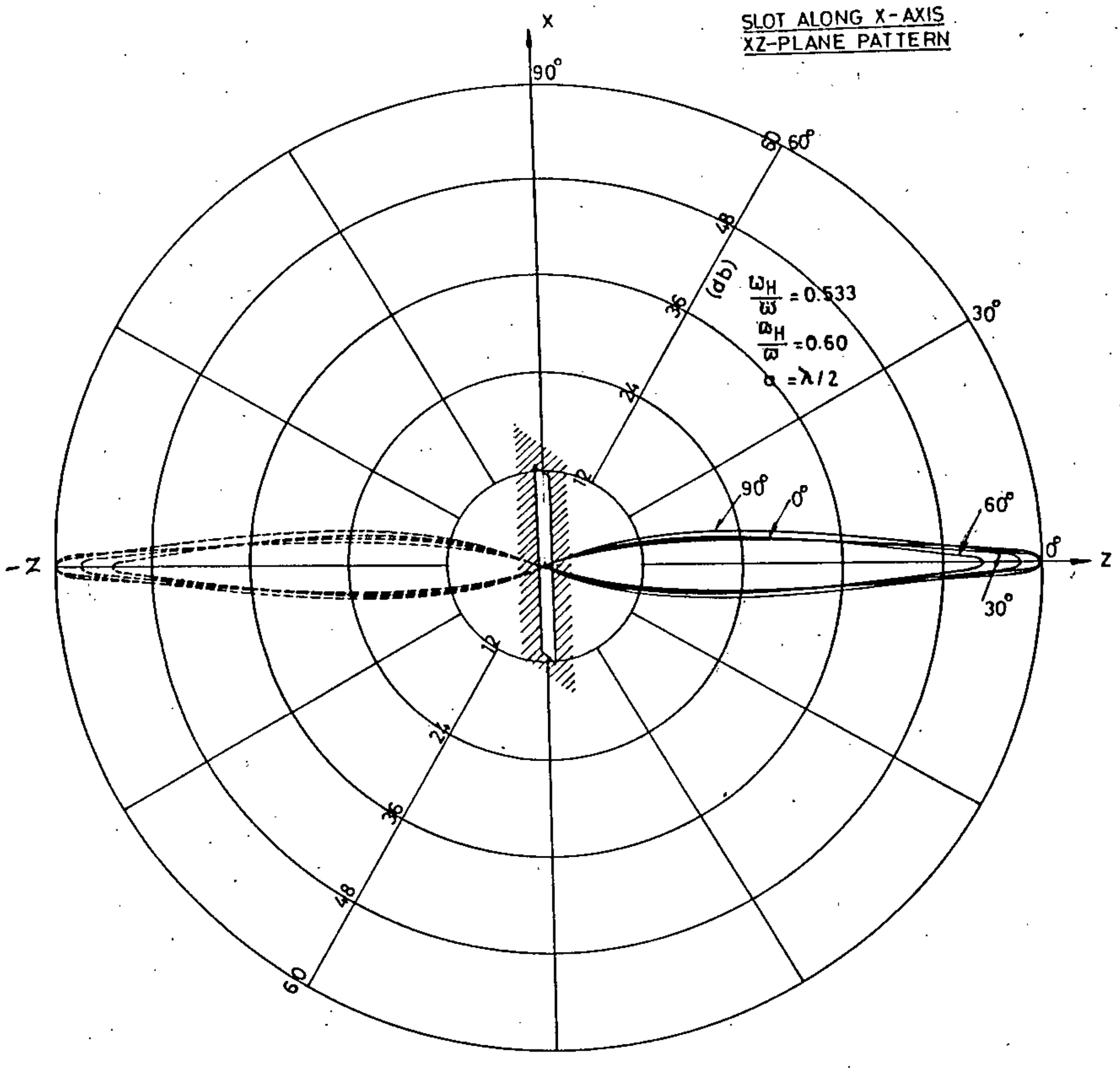


Fig.6.2 : Normalized radiation pattern in the xz-plane with α=0°,30°,60°,90° for a slot along the x-axis lying in the xy-plane. The dotted pattern is the image of the solid pattern since radiation is considered in the anisotropic half-space over the ground plane filling the region z>0.

about the y-axis in the xy-plane and about the z-axis in the xz-plane.The peaks are at perpendicular directions with respect to the plane wave excitation at the slot aperture. In both the cases there is a slight variation in db power with the change of inclination of the static magnetic field which is due to the fact that while the static magnetic field rotates in the yz -plane it remains always perpendicular to the slot axis. The slight variation in the radiation pattern can, therefore, be attributed to the variation of inclination of the static magnetic field with respect to the ground plane.

6.4. Radiation Pattern of a Slot along the y-axis lying in the xy plane

In this case the angle of radiation is calculated from the saddle points by

$$\cot\phi = -\left.\frac{\partial k_2}{\partial k_1}\right|_{k_3=0} \tag{6.9}$$

for xy plane pattern and

$$\cot\theta = -\left.\frac{\partial k_2}{\partial k_3}\right|_{k_1=0} \tag{6.10}$$

for yz plane pattern

The xy-plane pattern is shown in Fig.6.3 . For the yz-plane the saddle points were determined graphically from the plot of wave surfaces shown in Fig.6.4. The corresponding pattern is shown in Fig.6.5. Fig.6.3 indicates that a sharp beam is radiated symmetrically about the x-axis in the xy-plane. This tendency is also observed in the yz-plane pattern (Fig.6.5) for α=0° and 90°. But for α=30° and α=60° the beam maxima is slightly tilted from the z-axis

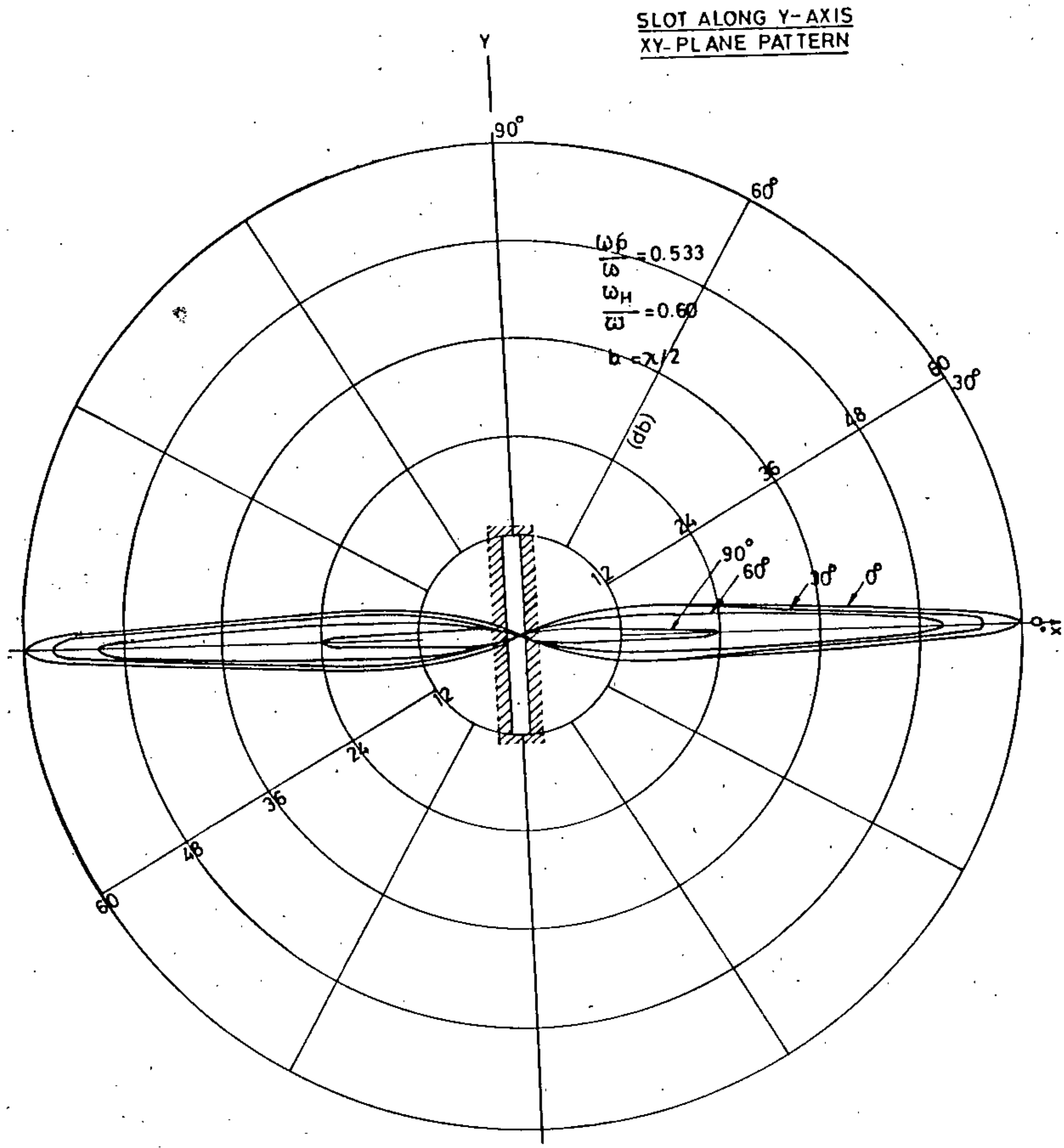


Fig.6.3 : Normalized radiation pattern in the xy-plane with α=0°,30°,60°,90° for slot along the y-axis lying in the xy-plane.

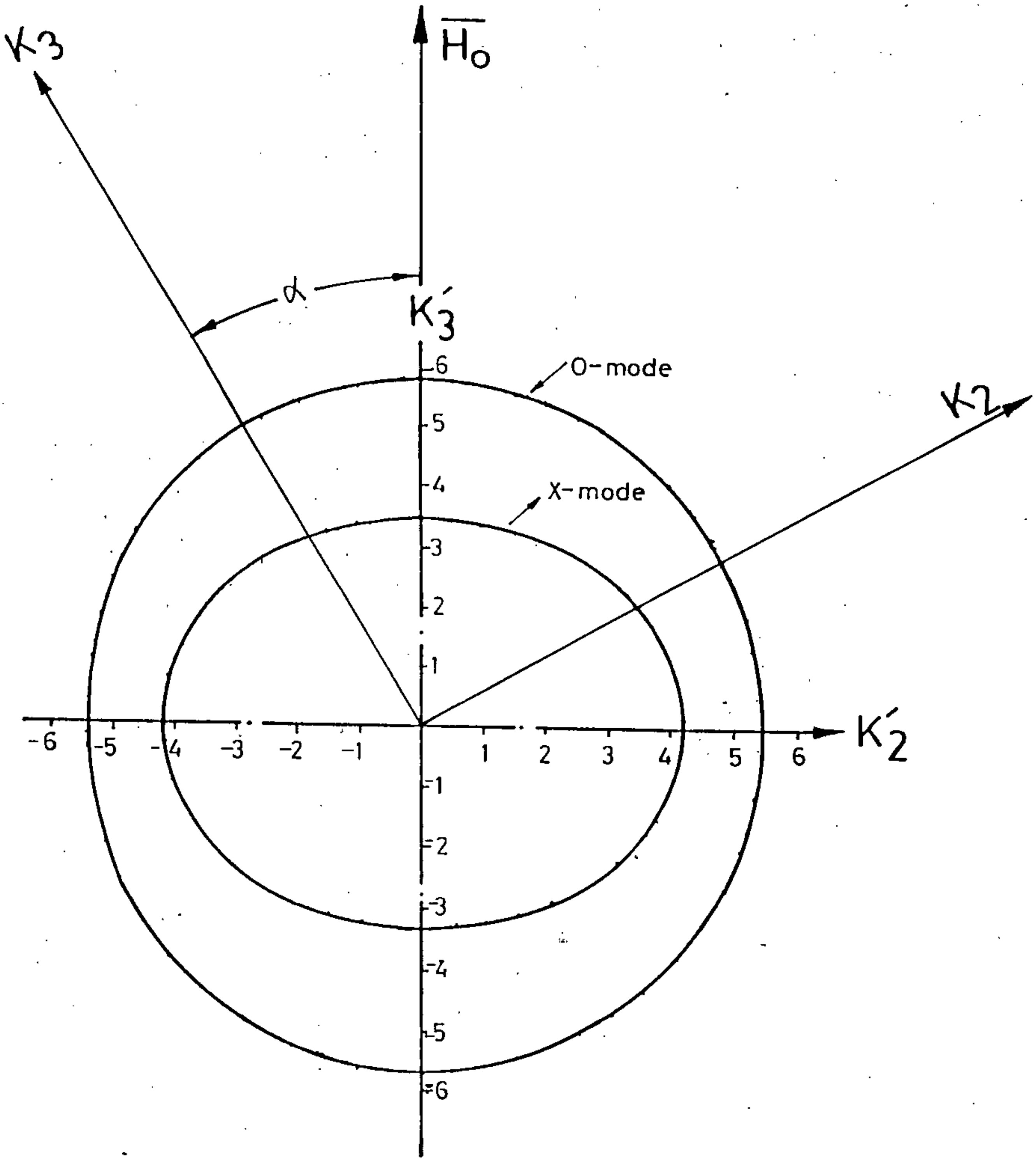


Fig.6.4 : Wave number surfaces in the yz-plane

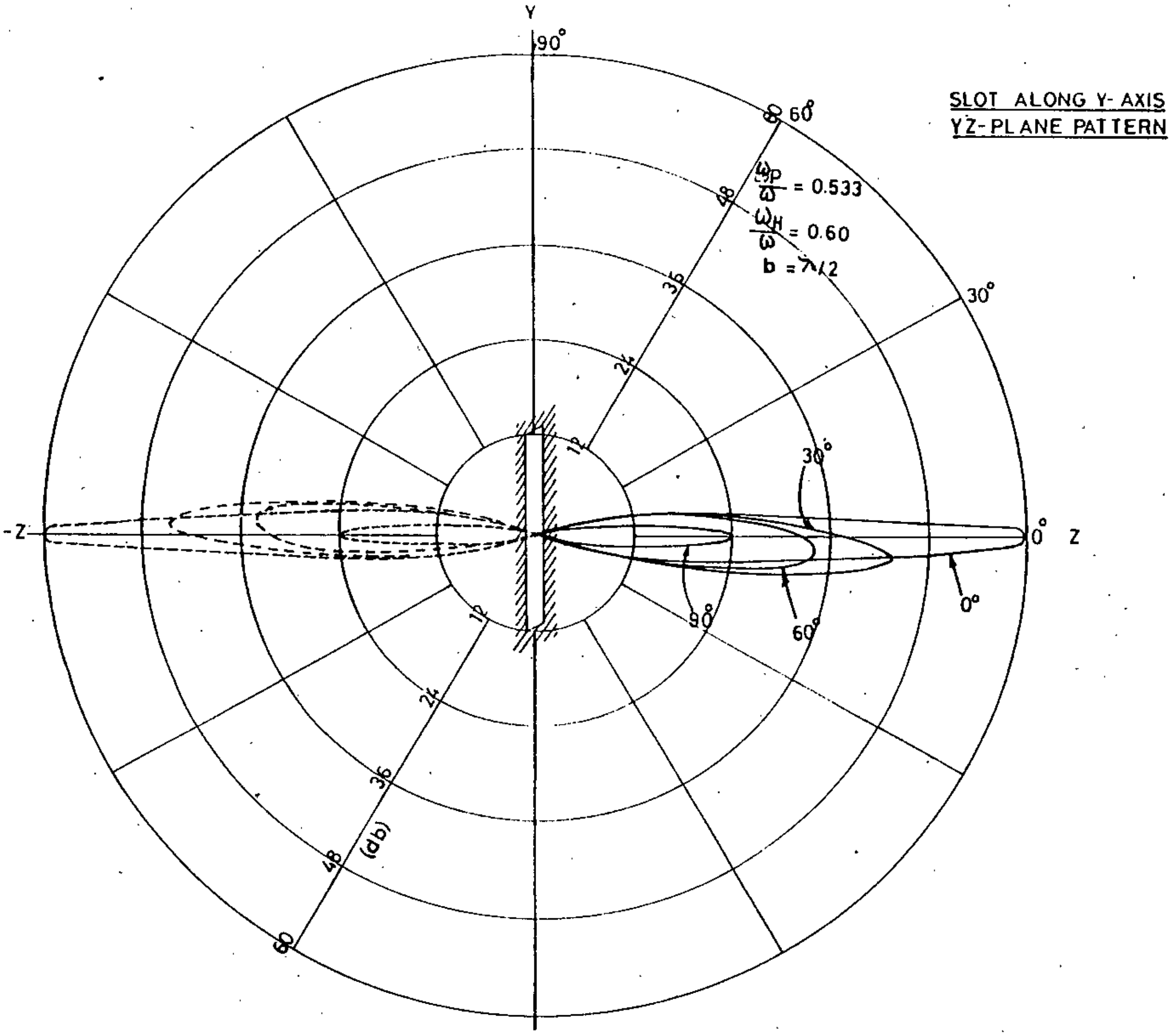


Fig.6.5 : Normalized power pattern in the yz-plane with α=0°,30°,60°,90° for slot along the y-axis lying in the xy-plane. The dotted pattern is the image of the solid pattern since radiation is considered in the anisotropic half-space over the ground plane filing the region z>0.

. This is due to the lack of propagation symmetry in the yz-plane for arbitrary α. Figs.6.3 and 6.5 indicate also that the power pattern is quite sensitive to the variation of the angle α. Along the x-axis the minimum power is radiated for $\alpha=90°$ and the maximum for $\alpha=0°$. Plot shows that the latter case is identical to the case of radiation along the y-axis from an x-directed slot with $\alpha=0°$.

6.5. Radiation Pattern of a Slot along the z-axis lying the yz-plane

In this case the angle of radiation in the yz plane is given by

$$\tan\theta = -\left.\frac{\partial k_3}{\partial k_2}\right|_{k_1=0} \tag{6.11}$$

and the angle of radiation in the xz-plane is given by

$$\tan\theta = -\left.\frac{\partial k_3}{\partial k_1}\right|_{k_2=0} \tag{6.12}$$

The xz-plane pattern for $\alpha = 0°$, $30°$, $60°$, and $90°$ is shown in Fig. 6.6 and the yz-plane pattern is illustrated in Fig. 6.7 for $\alpha = 0°, 30°, 60°$ and $90°$. For $\alpha=30°$ and $60°$ an analytical solution for the characteristic equation (2.15) could not be obtained because of the appearance of the cross terms involving $k_3{}^3$ and k_3. Hence the saddle points were obtained graphically from Fig.6.4. In this case also the arbitrary direction of the static magnetic field tilts the beam direction slightly away from the y-axis due to the lack of propagation symmetry in the yz-plane.

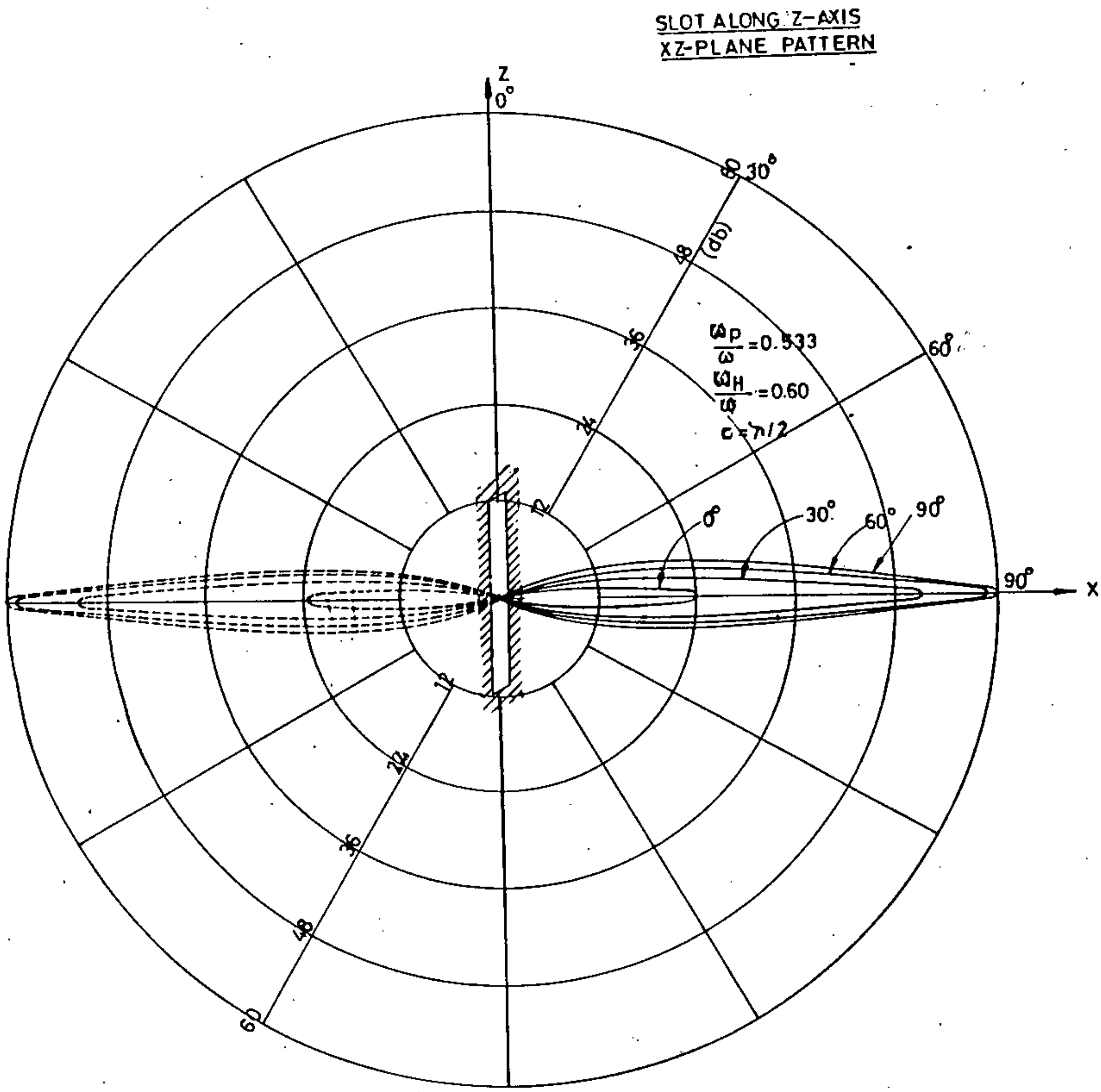


Fig.6.6 : Normalized power pattern in the xz-plane with α=0°,30°,60°,90° for slot along the z-axis lying in the yz-plane. The dotted pattern is the image of the solid pattern since radiation is considered in the anisotropic half space over the ground plane filling the region x>0.

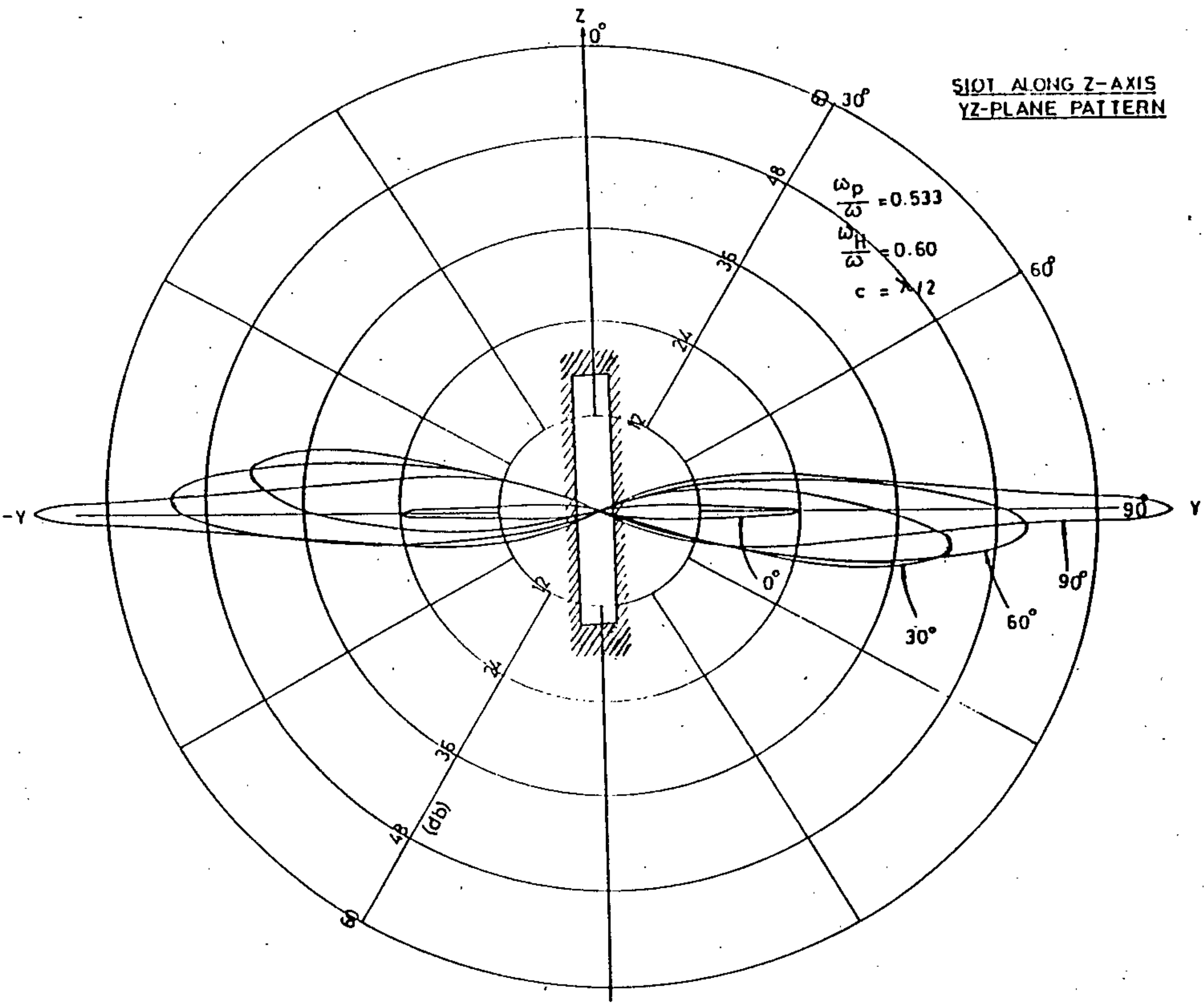


Fig.6.7 : Nornalized power pattern in the yz-plane with α=0°,30°,60°,90°
for slot along the z-axis lying in the yz-plane.

6.6. Discussion

Numerical results and plots of directional pattern show that two sharp beams of power is radiated from a narrow rectangular slot in an anisotropic medium. One beam is along the ground plane in a direction perpendicular to the slot axis and the other in a plane perpendicular to the plane of the slot. No power is radiated along the axis of the slot.This highly directive radiation is dominated by the O-mode power . The X-mode radiation is quite negligible compared to the O-mode radiation. The variation in inclination of the static magnetic field with respect to the ground plane of the slot does not effect the radiation pattern significantly for the x-directed slot with the static magnetic field in the yz-plane . For slots along y and z-axes considerable effect of the inclination of the static magnetic field on the radiation pattern can be observed . Also an arbitrary inclination of the static magnetic field can tilt the beam maxima slightly away from the z and y-directions for slots along y and z-axes respectively.

CHAPTER – 7

General Discussions and Conclusions

In the above an investigation has been carried out to find the radiation from a slot antenna in an anisotropic plasma with the static magnetic field in an arbitrary direction. For this purpose the polarization of plane waves at an arbitrary direction with respect to the static magnetic field has been studied. The results have been used to determine the plane wave excitation in narrow rectangular slots radiating in such medium.

The field distribution in the slot aperture has been derived assuming a transverse electric excitation. Both ordinary and extra-ordinary modes of excitation with sinusoidal electric field distribution were found to exist in the slot aperture. Derivation of the field distribution in the slot aperture enabled us to find the full-Maxwell field in the medium resulting from the plane wave excitation of the slot. The field is obtained in triple Fourier integral form. One of the integrals was tackled by residue evaluation with respect to the wave number parallel to the slot axis .It was found that the waves are dispersed into the ordinary and the extraordinary modes of propagation - a phenomenon that is predictable also from the characteristic equation of the medium. In an xyz co-ordinate system if the static magnetic field is arbitrarily inclined in the yz-plane, a lack of field symmetry takes place in this plane.

For obtaining the radiation field of the antenna the remaining double Fourier integral was asymptotically obtained by the saddle point method. The saddle points are the real solutions of the characteristic equation and were used to find the magnitude and direction of power flow from the antenna.

Using the far field components, the radial component of the power flow vector was obtained. The numerical results of the radial power flow were plotted to obtain the directional power pattern of a slot antenna placed at different angular positions with respect to the static magnetic field. Calculations were carried out for the (a) region of the CMA diagram where the asymptotic field theory by saddle point method is valid for both the ordinary and the extra-ordinary modes of propagation.

Some interesting electromagnetic phenomena were observed from the plot of the numerical radiation pattern of slot antennas radiating in an anisotropic plasma. For a narrow rectangular slot along the x-axis with the static magnetic field arbitrarily directed in the yz-plane , the variation in inclination of the static magnetic field does not significantly influence the radiation pattern. However, for an y-directed slot significant change in the radiation pattern can be observed with the change of inclination of the static magnetic field in the y-z plane. Similarly significant variation in the radiation pattern can be observed in the xz-plane pattern for a z-directed slot lying in the yz-plane.In both the cases the beam maxima are slightly tilted from the z and y-axes for arbitrary inclination of the static magnetic field in the yz-plane causing a lack of propagation symmetry in this plane.

The most interesting feature is that a strong pencil beam is radiated from the slot along the ground plane in a direction transverse to the slot axis and another pencil beam is radiated perpendicular to the plane of the slot. The maximum intensity of these pencil beams has been found to be about 66 db compared with the maximum intensity of a similar slot radiating in free space. These indicate that there may be concentrated field regions in the directions perpendicular to the slot aperture . If such type of phenomena occurs physically in a plasma environment it will be helpful in plasma heating as well as in communication using very low frequency during the re-entry blackout period of the space vehicle.

In conclusion it can be said that the results should be experimentally verified for confirmation of the theory. The impedance of the antenna also remains to be a major problem for further investigation.

APPENDIX -A

Derivation of the relative permittivity tensor $\hat{\epsilon}_r$ from the tensor $\hat{\epsilon}_r'$ by rotation of axis.

For static magnetic field in the z'-direction the relative permittivity tensor with reference to the $x\,y'z'$-system is given by :

$$\hat{\epsilon}_r' = \begin{bmatrix} \epsilon_1 & -j\epsilon_q' & 0 \\ j\epsilon_q' & \epsilon_1 & 0 \\ 0 & 0 & \epsilon_3' \end{bmatrix} \tag{A-1}$$

The xyz-system in Fig.2.1 is related to the $x\ y'\ z'$ system by the transformations:

$$y' = y\cos\alpha - z\sin\alpha \tag{A-2}$$

$$z' = z\cos\alpha + y\sin\alpha \tag{A-3}$$

$$\frac{\partial}{\partial y'} = \cos\alpha\frac{\partial}{\partial y} - \sin\alpha\frac{\partial}{\partial z} \tag{A-4}$$

$$\frac{\partial}{\partial z'} = \cos\alpha\frac{\partial}{\partial z} + \sin\alpha\frac{\partial}{\partial y} \tag{A-5}$$

Now we have

$$\nabla' \times H' = j\omega\epsilon_0 \hat{\epsilon}_r' \bar{E}'$$

$$\begin{bmatrix} \bar{a}_x & \bar{a}_y' & \bar{a}_z' \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y'} & \frac{\partial}{\partial z'} \\ H_x & H_y' & H_z' \end{bmatrix} = j\omega\epsilon_0 \begin{bmatrix} \epsilon_1 & -j\epsilon_4' & 0 \\ j\epsilon_4' & \epsilon_1 & 0 \\ 0 & 0 & \epsilon_3' \end{bmatrix} \begin{bmatrix} E_x \\ E_y' \\ E_z' \end{bmatrix} \qquad \text{(A-6)}$$

The co-factor of a_x is given by

$$\frac{\partial H_z'}{\partial y'} - \frac{\partial H_y'}{\partial z'} = j\omega\epsilon_0 (\epsilon_1 E_x - j\epsilon_4' E_y')$$

This by (A-2)-(A-5) becomes

$$\left(\cos\alpha\frac{\partial}{\partial y} - \sin\alpha\frac{\partial}{\partial z}\right)\left(H_z\cos\alpha + H_y\sin\alpha\right) - \left(\cos\alpha\frac{\partial}{\partial z} + \sin\alpha\frac{\partial}{\partial y}\right)\left(H_y\cos\alpha - H_z\sin\alpha\right)$$

$$= j\omega\epsilon_0 \left\{\epsilon_1 E_x - j\epsilon_4' (E_y \cos\alpha - H_z \sin\alpha)\right\}$$

or

$$\frac{\partial H_z}{\partial y} - \frac{\partial H_y}{\partial z} = j\omega\epsilon_0 \left(\epsilon_1 E_x - j\epsilon_4 E_y + j\epsilon_5 E_z\right) \qquad \text{(A-a)}$$

The co-factor of a_y' is given by

$$\frac{\partial H_x}{\partial z'} - \frac{\partial H_z'}{\partial x} = j\omega\epsilon_0 (j\epsilon_4' E_x + \epsilon_1 E_y')$$

This by (A-2) (A-5) become

$$\left(\cos\alpha\frac{\partial}{\partial z}+\sin\alpha\frac{\partial}{\partial y}\right)Hx-\frac{\partial}{\partial x}\left(Hz\cos\alpha+Hy\sin\alpha\right) = j\omega\epsilon_0\left(j\epsilon_q'E_x+\epsilon_1E_y'\right)$$

$$\cos\alpha\,\frac{\partial Hx}{\partial z}+\sin\alpha\,\frac{\partial Hx}{\partial y}-\cos\alpha\,\frac{\partial Hz}{\partial x}-\sin\alpha\,\frac{\partial Hy}{\partial x}$$

$$= j\omega\epsilon_0\left(j\epsilon_q'E_x+\epsilon_1\cos\alpha\,E_y-\epsilon_1\sin\alpha\,E_z\right) \qquad \text{(A-7)}$$

The co-factor of $a_z{'}$

$$\frac{\partial Hy'}{\partial x}-\frac{\partial Hx}{\partial y'}=j\omega\epsilon_0\epsilon_3'E_z'$$

By (A-2) - (A-5) this becomes

$$\frac{\partial}{\partial x}\left(Hy\cos\alpha-Hz\sin\alpha\right)-\left(\cos\alpha\cdot\frac{\partial}{\partial y}-\sin\alpha\frac{\partial}{\partial z}\right)Hx$$

$$=j\omega\epsilon_0\epsilon_3'\left(Ez\cos\alpha+Ey\sin\alpha\right)$$

or

$$\cos\alpha\,\frac{\partial Hy}{\partial x}-\sin\alpha\,\frac{\partial Hz}{\partial x}-\cos\alpha\,\frac{\partial Hx}{\partial y}+\sin\alpha\,\frac{\partial Hx}{\partial z} \qquad \text{(A-8)}$$

$$=j\omega\epsilon_0\epsilon_3'\left(E_z\cos\alpha+E_y\sin\alpha\right)$$

Multiplying (A-7) by $\sin\alpha$ and (A-8) by $\cos\alpha$ and adding

$$\left(\cos^2\alpha+\sin^2\alpha\right)\frac{\partial Hx}{\partial z}-\left(\cos^2\alpha+\sin^2\alpha\right)\frac{\partial Hz}{\partial x}$$

$$=j\omega\epsilon_0\left\{j\epsilon_q'\cos\alpha\,E_x+\left(\epsilon_1\cos^2\alpha+\epsilon_3'\sin^2\alpha\right)E_y+\left(\epsilon_3'-\epsilon_1\right)\sin\alpha\cos\alpha\right\}$$

$$\Rightarrow\ \frac{\partial Hx}{\partial z}-\frac{\partial Hz}{\partial x}=j\omega\epsilon_0\left(j\epsilon_q E_x+\epsilon_2E_y+\epsilon_6E_z\right) \qquad \text{(A-b)}$$

Again (A-8) x $\cos\alpha$ - (A-7) x $\sin\alpha$ gives

$$\frac{\partial H_y}{\partial x} - \frac{\partial H_x}{\partial y} = j\omega\epsilon_0 \left(-j\epsilon_5 E_x + \epsilon_6 E_y + \epsilon_3 E_z\right) \tag{A-c}$$

Equations (A-a), (A-b), (A-c) can be arranged in the following form

$$\begin{bmatrix} \bar{a}_x & \bar{a}_y & \bar{a}_z \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ H_x & H_y & H_z \end{bmatrix} = j\omega\epsilon_0 \begin{bmatrix} \epsilon_1 & -j\epsilon_4 & j\epsilon_5 \\ j\epsilon_4 & \epsilon_2 & \epsilon_6 \\ -j\epsilon_5 & \epsilon_6 & \epsilon_3 \end{bmatrix} \begin{bmatrix} E_x \\ E_y \\ E_z \end{bmatrix},$$

or

$$\nabla \times \bar{H} = j\omega\epsilon_0 \hat{\epsilon}_r \bar{E}$$

Hence the transformed dielectric tensor is

$$\hat{\epsilon}_r = \begin{bmatrix} \epsilon_1 & -j\epsilon_4 & j\epsilon_5 \\ j\epsilon_4 & \epsilon_2 & \epsilon_6 \\ -j\epsilon_5 & \epsilon_6 & \epsilon_3 \end{bmatrix} \tag{A-9}$$

where

$$\epsilon_2 = \epsilon_1 \cos^2\alpha + \epsilon_3' \sin^2\alpha$$

$$\epsilon_3 = \epsilon_1 \sin^2\alpha + \epsilon_3' \cos^2\alpha$$

$$\epsilon_4 = \epsilon_4' \cos\alpha$$

$$\epsilon_5 = \epsilon_4' \sin\alpha$$

$$\epsilon_6 = (\epsilon_3' - \epsilon_1) \sin\alpha \cos\alpha$$

APPENDIX - B

Derivation of Wave Matrix and Characteristic Equation

For a plane wave of the form $\bar{E} = \bar{E}_o e^{-j\bar{K}.\bar{r}}$ equation (2-14) can be written as

$$-j\bar{K} \times -j\bar{K} \times \bar{E}(K) - K_o^2 \hat{\epsilon}_r \bar{E}(K) = \hat{\lambda} E(K) = 0$$

or

$$\bar{K} \times \bar{K} \times E(K) + K_o^2 \epsilon_o \hat{\epsilon}_r \bar{E}(K) = -\hat{\lambda} E(K) = 0 \qquad \text{(B-1)}$$

where $\hat{\lambda}$ is the wave matrix

$$\bar{K} = \bar{a}_x K_1 + \bar{a}_y K_2 + \bar{a}_z K_3 \qquad \text{(B-2)}$$

$$\bar{E}(K) = \bar{a}_x E_x(K) + \bar{a}_y E_y(K) + \bar{a}_z E_z(K) \qquad \text{(B-3)}$$

$$\bar{r} = \bar{a}_x x + \bar{a}_y y + \bar{a}_z z \qquad \text{(B-4)}$$

Now

$$\bar{K} \times \bar{E}(K) = \begin{bmatrix} \bar{a}_x & \bar{a}_y & \bar{a}_z \\ K_1 & K_2 & K_3 \\ E_x(K) & E_y(K) & E_z(K) \end{bmatrix}$$

$$= \bar{a}_x\left(K_2 E_z(K) - K_3 E_y(K)\right) + \bar{a}_y\left(K_3 E_x(K) - K_1 E_z(K)\right) + \bar{a}_z\left(K_1 E_y(K) - K_2 E_x(K)\right)$$

Then

$$\bar{K}\times\bar{K}\times\bar{E}(K) = \begin{bmatrix} \bar{a}_x & \bar{a}_y & \bar{a}_z \\ K_1 & K_2 & K_3 \\ K_2E_z(K)-K_3E_y(K) & K_3E_x(K)-K_1E_z(K) & K_1E_y(K)-K_2E_x(K) \end{bmatrix}$$

$$= \bar{a}_x\left[K_1K_2E_y(K) - K_2^2E_x(K) - K_3^2E_x(K) + K_1K_3E_z(K)\right]$$

$$+ \bar{a}_y\left[K_2K_3E_z(K) - K_3^2E_y(K) - K_1^2E_y(K) + K_1K_2E_x(K)\right] \qquad \text{(B-5)}$$

$$+ \bar{a}_z\left[K_1K_3E_x(K) - K_1^2E_z(K) - K_2^2E_z(K) + K_3K_2E_y(K)\right]$$

Using relation (B-5), (B-1) can be written as

$$\begin{bmatrix} -K_2^2-K_3^2 & K_1K_2 & K_1K_3 \\ K_1K_2 & -K_1^2-K_3^2 & K_2K_3 \\ K_1K_3 & K_2K_3 & -K_1^2-K_2^2 \end{bmatrix}\begin{bmatrix} E_x(K) \\ E_y(K) \\ E_z(K) \end{bmatrix} + K_0^2\begin{bmatrix} \epsilon_1 & -j\epsilon_4 & j\epsilon_5 \\ j\epsilon_4 & \epsilon_2 & \epsilon_6 \\ -j\epsilon_5 & \epsilon_6 & \epsilon_3 \end{bmatrix}\begin{bmatrix} E_x(K) \\ E_y(K) \\ E_z(K) \end{bmatrix}$$

(B-6)

$$
\begin{bmatrix} -K_2^2 - K_3^2 + \epsilon_1 K_0^2 & K_1 K_2 - j\epsilon_4 K_0^2 & K_1 K_3 + j\epsilon_5 K_0^2 \\ K_1 K_2 + j\epsilon_4 K_0^2 & -K_1^2 - K_3^2 + \epsilon_2 K_0^2 & K_2 K_3 + \epsilon_6 K_0^2 \\ K_1 K_3 - j\epsilon_5 K_0^2 & K_2 K_3 + \epsilon_6 K_0^2 & -K_1^2 - K_2^2 + \epsilon_3 K_0^2 \end{bmatrix} \begin{bmatrix} E_x(K) \\ E_y(K) \\ E_z(K) \end{bmatrix} = -\hat{\lambda} E(K) = 0
$$

or

$$
\hat{\lambda} = \begin{bmatrix} \lambda_{11} & \lambda_{12} & \lambda_{13} \\ \lambda_{21} & \lambda_{22} & \lambda_{23} \\ \lambda_{31} & \lambda_{32} & \lambda_{33} \end{bmatrix}
$$

where

$$\lambda_{11} = -K_2^2 - K_3^2 + \epsilon_1 K_0^2 \qquad \lambda_{12} = K_1 K_2 - j\epsilon_4 K_0^2 \qquad \lambda_{13} = K_1 K_3 + j\epsilon_5 K_0^2$$

$$\lambda_{21} = K_1 K_2 + j\epsilon_4 K_0^2 \qquad \lambda_{22} = -K_1^2 - K_3^2 + \epsilon_2 K_0^2 \qquad \lambda_{23} = K_2 K_3 + \epsilon_6 K_0^2$$

$$\lambda_{31} = K_1 K_3 - j\epsilon_5 K_0^2 \qquad \lambda_{32} = K_2 K_3 + \epsilon_6 K_0^2 \qquad \lambda_{33} = -K_1^2 - K_2^2 - \epsilon_3 K_0^2$$

The inverse wave matrix can be computed by Cramer's rule

$$\hat{\lambda}^{-1} = \frac{\hat{A}}{\det \hat{\lambda}} \tag{B-7}$$

where $\hat{A}$ is the adjoint and det $\hat{\lambda}$ is the determinant of the wave matrix $\hat{\lambda}$

$$
\hat{A} = \begin{bmatrix} A_{11} & A_{12} & A_{13} \\ A_{21} & A_{22} & A_{23} \\ A_{31} & A_{32} & A_{33} \end{bmatrix}
$$

where

$$A_{11} = \lambda_{22}\lambda_{33} - \lambda_{23}\lambda_{32} \qquad A_{12} = \lambda_{13}\lambda_{32} - \lambda_{12}\lambda_{33} \qquad A_{13} = \lambda_{12}\lambda_{23} - \lambda_{13}\lambda_{22}$$

$$A_{21} = \lambda_{23}\lambda_{31} - \lambda_{21}\lambda_{33} \qquad A_{22} = \lambda_{11}\lambda_{33} - \lambda_{13}\lambda_{31} \qquad A_{23} = \lambda_{13}\lambda_{21} - \lambda_{11}\lambda_{23}$$

$$A_{31} = \lambda_{21}\lambda_{32} - \lambda_{22}\lambda_{31} \qquad A_{32} = \lambda_{12}\lambda_{31} - \lambda_{11}\lambda_{32} \qquad A_{33} = \lambda_{11}\lambda_{22} - \lambda_{12}\lambda_{21}$$

$$\det\hat{\lambda} = -K_0^2\left\{(K_1^2+K_2^2+K_3^2-\epsilon_1 K_0^2)(\epsilon_1 K_1^2+\epsilon_2 K_2^2+\epsilon_3 K_3^2+2\epsilon_8 K_2 K_3-\eta_1 K_0^2/\epsilon_1)+\Delta(K_1,K_2,K_3)\right\}$$

where

$$\Delta(K_1,K_2,K_3) = K_0^2(C_1 K_1^2 + C_2 K_2^2 + C_3 K_3^2 + C_4 K_2 K_3)$$

$$C_1 = \epsilon_4'^2(1-\epsilon_3'/\epsilon_1)$$

$$C_2 = \epsilon_4'^2(\cos^2\alpha - \epsilon_3'/\epsilon_1)$$

$$C_3 = \epsilon_4'^2(\sin^2\alpha - \epsilon_3'/\epsilon_1)$$

$$C_4 = -2\sin\alpha\cos\alpha\,\epsilon_4'^2$$

$$\eta_1 = \epsilon_3'(\epsilon_1^2 - \epsilon_4'^2)$$

The dispersion equation or the characteristic equation is given by

$$\det\hat{\lambda} = 0 .$$

APPENDIX - C

Plane Wave Propagation along and parallel to the Static Magnetic Field

With the static magnetic field $\overline{H_0}$ in z'- direction Maxwell's equations in the $x\,y'\,z'$ system with $e^{j\omega t}$ variation of field components become,

$$\begin{vmatrix} \bar{a}_x & \bar{a}_y' & \bar{a}_z' \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y'} & \frac{\partial}{\partial z'} \\ E_x & E_y' & E_z' \end{vmatrix} = -j\omega\mu_0 (\bar{a}_x H_x + \bar{a}_y' H_y' + \bar{a}_z' H_z') \qquad \text{(C-1)}$$

and

$$\begin{vmatrix} \bar{a}_x & \bar{a}_y' & \bar{a}_z' \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y'} & \frac{\partial}{\partial z'} \\ H_x & H_y' & H_z' \end{vmatrix} = j\omega\epsilon_0 \begin{bmatrix} \epsilon_1 & -j\epsilon_q' & 0 \\ j\epsilon_q' & \epsilon_1 & 0 \\ 0 & 0 & \epsilon_3' \end{bmatrix} \begin{bmatrix} E_x \\ E_y' \\ E_z' \end{bmatrix} \qquad \text{(C-2)}$$

<u>**For plane wave traveling along $\bar{H}_0$**</u>

$\frac{\partial}{\partial x} = 0$, $\frac{\partial}{\partial y'} = 0$ which gives

$$\begin{bmatrix} \bar{a}_x & \bar{a}_y' & \bar{a}_z' \\ 0 & 0 & \frac{\partial}{\partial z'} \\ E_x & E_y' & E_z' \end{bmatrix} = j\omega\mu_0 \left(\bar{a}_x H_x + \bar{a}_y' H_y' + \bar{a}_z' H_z' \right) \tag{C-3}$$

or

$$\frac{\partial E_y'}{\partial z'} = j\omega\mu_0 H_x \tag{C-4}$$

$$\frac{\partial E_x}{\partial z'} = -j\omega\mu_0 H_y' \tag{C-5}$$

$$0 = H_z' \tag{C-6}$$

Again

$$\begin{bmatrix} \bar{a}_x & \bar{a}_y' & \bar{a}_z' \\ 0 & 0 & \frac{\partial}{\partial z'} \\ H_x & H_y' & 0 \end{bmatrix} = j\omega\epsilon_0 \begin{bmatrix} \epsilon_1 & -j\epsilon_4' & 0 \\ j\epsilon_4' & \epsilon_1 & 0 \\ 0 & 0 & \epsilon_3' \end{bmatrix} \begin{bmatrix} E_x \\ E_y' \\ E_z' \end{bmatrix} \tag{C-7}$$

or

$$-\frac{\partial H_y'}{\partial z'} = j\omega\epsilon_0 \left(\epsilon_1 E_x - j\epsilon_4' E_y' \right) \tag{C-8}$$

$$\frac{\partial H_x}{\partial z'} = j\omega\epsilon_0 (j\epsilon_4' E_x + \epsilon_1 E_y') \tag{C-9}$$

$$0 = j\omega\epsilon_0 \epsilon_3' E_z' \tag{C-10}$$

Using C-4) to (C-9) we get

$$\frac{\partial^2 E_z'}{\partial z'^2} = -K_0^2 (j\epsilon_4' E_x + \epsilon_1 E_y') \tag{C-11}$$

$$\frac{\partial^2 E_x}{\partial z'^2} = -K_0^2 (\epsilon_1 E_x - j\epsilon_4' E_y') \tag{C-12}$$

The above equations can be written in the following form

$$\frac{K_3'^2}{K_0^2} \begin{bmatrix} E_x \\ E_y' \\ 0 \end{bmatrix} = \begin{bmatrix} \epsilon_1 & -j\epsilon_4' & 0 \\ j\epsilon_4' & \epsilon_1 & 0 \\ 0 & 0 & \epsilon_3' \end{bmatrix} \begin{bmatrix} E_x \\ E_y' \\ 0 \end{bmatrix} \tag{C-13}$$

which can be further arranged as

$$\begin{bmatrix} \epsilon_1 - K_3'^2/K_0^2 & -j\epsilon_4' & 0 \\ j\epsilon_4' & \epsilon_1 - K_3'^2/K_0^2 & 0 \\ 0 & 0 & \epsilon_3' \end{bmatrix} \begin{bmatrix} E_x \\ E_y' \\ 0 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} \tag{C-14}$$

Then the characteristic equation is

$$K_3'^2 = K_0^2(\epsilon_1 \pm \epsilon_4') \tag{C-15}$$

Then for O-mode $\quad K_3' = \pm K_0\sqrt{\epsilon_1 + \epsilon_4'}$ (C-16)

and for X-mode $\quad K_3' = \pm K_0\sqrt{\epsilon_1 - \epsilon_4'}$ (C-17)

From (C-14)-(C-17)

$$(\epsilon_1 - K_3'^2/K_0^2)\,E_x - j\epsilon_4' E_y' = 0$$

$$\Rightarrow E_y' = \frac{(K_0^2\epsilon_1 - K_3'^2)}{j\epsilon_4' K_0^2} E_x$$

or

$E_y' = jE_x$ for O-mode

(C-18)

and for X-mode

$E_y' = -jE_x$

For plane wave perpendicular to $\bar{H}_0$

For a plane wave along x-direction, $\frac{\partial}{\partial y'} = 0$, $\frac{\partial}{\partial z'} = 0$
Then from (c-1) and (c-2) we have

$$\begin{bmatrix} \bar{a}_x & \bar{a}_y' & \bar{a}_z' \\ \frac{\partial}{\partial x} & 0 & 0 \\ E_x & E_y' & E_z' \end{bmatrix} = -j\omega\mu_0 (\bar{a}_x H_x + \bar{a}_y H_y' + \bar{a}_z' H_z').$$

or

$$0 = H_x \tag{C-16}$$

$$-\frac{\partial E_z'}{\partial x} = -j\omega\mu_0 H_y' \tag{C-17}$$

$$\frac{\partial E_y'}{\partial x} = -j\omega\mu_0 H_z' \tag{C-18}$$

And

$$\begin{bmatrix} \bar{a}_x & \bar{a}_y' & \bar{a}_z' \\ \frac{\partial}{\partial x} & 0 & 0 \\ H_x & H_y' & H_z' \end{bmatrix} = j\omega\epsilon_0 \begin{bmatrix} \epsilon_1 & -j\epsilon_q' & 0 \\ j\epsilon_q' & \epsilon_1 & 0 \\ 0 & 0 & \epsilon_3' \end{bmatrix} \begin{bmatrix} E_x \\ E_y' \\ E_z' \end{bmatrix}$$

$$0 = j\omega\epsilon_0 (\epsilon_1 E_x - j\epsilon_4' E_y') \tag{C-19}$$

$$-\frac{\partial H_z'}{\partial x} = j\omega\epsilon_0 (j\epsilon_4' E_x + \epsilon_1 E_y') \tag{C-20}$$

$$\frac{\partial H_y'}{\partial x} = j\omega\epsilon_0\epsilon_3' E_z' \tag{C-21}$$

Using (C-17) and (C-21) we get

$$\frac{\partial^2 E_z'}{\partial x^2} = -K_0^2\epsilon_3' E_z' \tag{C-22}$$

Using (C-18), (C-19) and (C-20) we get

$$E_x = j(\epsilon_4'/\epsilon_1) E_y' \tag{C-23}$$

and

$$\frac{\partial^2 E_y'}{\partial x^2} = -K_0^2\left(\frac{\epsilon_1^2 - \epsilon_4'^2}{\epsilon_1}\right) E_y' \tag{C-24}$$

From (C-22) and (C-24) the wave numbers are given by

$K_1 = \pm K_0\sqrt{\epsilon_3'}$ which represent O-mode

$K_1 = \pm K_0\sqrt{\frac{\epsilon_1^2 - \epsilon_4'^2}{\epsilon_1}}$ which represent X-mode

APPENDIX - D

Plane Waves along Arbitrary Direction

With static magnetic field $\bar{H}_0$ along z'-direction in the yz plane making angle α with the z-direction of the x y z system (fig 2.1) we can write Maxwell's equation with $e^{-j\omega t}$ variation surpressed

$$\begin{bmatrix} \bar{a}_x & \bar{a}_y & \bar{a}_z \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ E_x & E_y & E_z \end{bmatrix} = -j\omega\epsilon_0(\bar{a}_x H_x + \bar{a}_y H_y + \bar{a}_z H_z) \qquad \text{(D-1)}$$

and

$$\begin{bmatrix} \bar{a}_x & \bar{a}_y & \bar{a}_z \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ H_x & H_y & H_z \end{bmatrix} = j\omega\epsilon_0 \begin{bmatrix} \epsilon_1 & -j\epsilon_4 & j\epsilon_5 \\ j\epsilon_4 & \epsilon_2 & \epsilon_6 \\ -j\epsilon_5 & \epsilon_6 & \epsilon_5 \end{bmatrix} \begin{bmatrix} E_x \\ E_y \\ E_z \end{bmatrix} \qquad \text{(D-2)}$$

For plane wave travelling along the y-dirction

Here $\frac{\partial}{\partial y} = 0$, $\frac{\partial}{\partial z} = 0$, so that

$$\begin{bmatrix} \bar{a}_x & \bar{a}_y & \bar{a}_z \\ 0 & \frac{\partial}{\partial y} & 0 \\ E_x & E_y & E_z \end{bmatrix} = -j\omega\mu_0 \left(\bar{a}_x H_x + \bar{a}_y H_y + \bar{a}_z H_z\right) \tag{D-3}$$

or

$$\frac{\partial E_z}{\partial y} = -j\omega\mu_0 H_x \tag{D-4}$$

$$0 = H_y \tag{D-5}$$

$$\frac{\partial E_x}{\partial y} = +j\omega\mu_0 H_z \tag{D-6}$$

From Maxwell's second curl equation

$$\begin{bmatrix} \bar{a}_x & \bar{a}_y & \bar{a}_z \\ 0 & \frac{\partial}{\partial y} & 0 \\ H_x & 0 & H_z \end{bmatrix} = j\omega\epsilon_0 \begin{bmatrix} \epsilon_1 & -j\epsilon_4 & j\epsilon_5 \\ j\epsilon_4 & \epsilon_2 & \epsilon_6 \\ -j\epsilon_5 & \epsilon_6 & \epsilon_3 \end{bmatrix} \begin{bmatrix} E_x \\ E_y \\ E_z \end{bmatrix} \tag{D-7}$$

or

$$\frac{\partial H_z}{\partial y} = j\omega\epsilon_0 \left(\epsilon_1 E_x - j\epsilon_4 E_y + j\epsilon_5 E_z\right) \tag{D-8}$$

$$0 = j\omega\epsilon_0 (j\epsilon_4 E_x + \epsilon_2 E_y + \epsilon_6 E_z) \tag{D-9}$$

$$-\frac{\partial H_x}{\partial y} = j\omega\epsilon_0 (-j\epsilon_5 E_x + \epsilon_6 E_y + \epsilon_3 E_z) \tag{D-10}$$

From equations (D-4),(D-6).(D-8) and (D-10) we get

$$\frac{\partial^2 E_x}{\partial y^2} = -K_0^2 (\epsilon_1 E_x - j\epsilon_4 E_y + j\epsilon_5 E_z) \tag{D-11}$$

$$\frac{\partial^2 E_z}{\partial y^2} = -K_0^2 (-j\epsilon_5 E_x + \epsilon_6 E_y + \epsilon_3 E_z) \tag{D-12}$$

Equations (D-9),(D-11) and (D-12) can be arranged as

$$K_2^2 \begin{bmatrix} E_x \\ 0 \\ E_z \end{bmatrix} = K_0^2 \begin{bmatrix} \epsilon_1 & -j\epsilon_4 & j\epsilon_5 \\ j\epsilon_4 & \epsilon_2 & \epsilon_6 \\ -j\epsilon_5 & \epsilon_6 & \epsilon_3 \end{bmatrix} \begin{bmatrix} E_x \\ E_y \\ E_z \end{bmatrix} \tag{D-13}$$

Which can be written as

$$\frac{K_2^2}{K_0^2}\begin{bmatrix} E_x \\ 0 \\ E_z \end{bmatrix} = \begin{bmatrix} \epsilon_1 & -j\epsilon_4 & j\epsilon_5 \\ j\epsilon_4 & \epsilon_2 & \epsilon_6 \\ -j\epsilon_5 & \epsilon_6 & \epsilon_3 \end{bmatrix}\begin{bmatrix} E_x \\ E_y \\ E_z \end{bmatrix} \qquad \text{(D-14)}$$

This can be written as

$$\begin{bmatrix} \epsilon_1 - K_2^2/K_0^2 & -j\epsilon_4 & j\epsilon_5 \\ j\epsilon_4 & \epsilon_2 & \epsilon_6 \\ -j\epsilon_5 & \epsilon_6 & \epsilon_3 - K_3^2/K_0^2 \end{bmatrix}\begin{bmatrix} E_x \\ E_y \\ E_z \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} \qquad \text{(D-15)}$$

The characteristic equation is given by

det [M]=0 which gives

$$K_2^2 = K_0^2 \frac{\epsilon_1(\epsilon_2+\epsilon_3') - \epsilon_4'^2\cos^2\alpha \pm \sqrt{[\epsilon_1(\epsilon_2+\epsilon_3') - \epsilon_4'^2\cos^2\alpha]^2 - 4\epsilon_2\epsilon_3'(\epsilon_4^2 - \epsilon_4'^2)}}{2\epsilon_2}$$

(D-16)

From (D-14) we can write

$$\begin{bmatrix} E_x \\ E_y \\ E_z \end{bmatrix} = \frac{k_2^2}{k_0^2} \begin{bmatrix} \epsilon_1 & -j\epsilon_4 & j\epsilon_5 \\ j\epsilon_4 & \epsilon_2 & \epsilon_6 \\ -j\epsilon_5 & \epsilon_6 & \epsilon_3 \end{bmatrix}^{-1} \begin{bmatrix} E_x \\ 0 \\ E_z \end{bmatrix} \tag{D-17}$$

$$\begin{bmatrix} \epsilon_1 & -j\epsilon_4 & j\epsilon_5 \\ j\epsilon_4 & \epsilon_2 & \epsilon_6 \\ -j\epsilon_5 & \epsilon_6 & \epsilon_3 \end{bmatrix} = \frac{1}{det[\hat{\epsilon}_r]} \begin{bmatrix} A_{11} & B_{21} & C_{31} \\ A_{12} & B_{22} & C_{32} \\ A_{13} & B_{23} & C_{33} \end{bmatrix}$$

det $[\hat{\epsilon}_r]$ = $(\epsilon_1^2 - \epsilon_4'^2)\,\epsilon_3'$

and

$A_{11} = \epsilon_1 \epsilon_3'$ $\qquad$ $A_{12} = -j\epsilon_4'\epsilon_3' \cos\alpha$ $\qquad$ $A_{13} = j\epsilon_4'\epsilon_3' \sin\alpha$

$B_{21} = j\epsilon_4'\epsilon_3' \cos\alpha$ $\qquad$ $B_{22} = (\epsilon_1^2 - \epsilon_4'^2)\sin^2\alpha + \epsilon_1\epsilon_3'\cos^2\alpha$ $\qquad$ $B_{23} = (\epsilon_1^2 - \epsilon_4'^2 - \epsilon_1\epsilon_3')\sin\alpha\cos\alpha$

$C_{31} = -j\epsilon_4'\epsilon_3' \sin\alpha$ $\qquad$ $C_{32} = (\epsilon_1^2 - \epsilon_4'^2 - \epsilon_1\epsilon_3')\sin\alpha\cos\alpha$ $\qquad$ $C_{33} = (\epsilon_1^2 - \epsilon_4'^2)\cos^2\alpha + \epsilon_1\epsilon_3'\sin^2\alpha$

equation (D-17) becomes

$$\begin{bmatrix} E_x \\ E_y \\ E_z \end{bmatrix} = \frac{(K_2^2/K_0^2)}{\epsilon_3'(\epsilon_1^2 - \epsilon_4'^2)} \begin{bmatrix} \epsilon_1\epsilon_3' & j\epsilon_4'\epsilon_3'\cos\alpha & -j\epsilon_4'\epsilon_3'\sin\alpha \\ -j\epsilon_4'\epsilon_3'\cos\alpha & (\epsilon_1^2-\epsilon_4'^2)\sin^2\alpha + \epsilon_1\epsilon_3'\cos^2\alpha & (\epsilon_1^2-\epsilon_4'^2-\epsilon_1\epsilon_3')\sin\alpha\cos\alpha \\ j\epsilon_4'\epsilon_3'\sin\alpha & (\epsilon_1^2-\epsilon_4'^2-\epsilon_1\epsilon_3')\sin\alpha\cos\alpha & (\epsilon_1^2-\epsilon_4'^2)\cos^2\alpha + \epsilon_1\epsilon_3'\sin^2\alpha \end{bmatrix} \begin{bmatrix} E_x \\ 0 \\ E_z \end{bmatrix} \tag{D-18}$$

or

$$E_x = \frac{k_2^2}{k_0^2 \epsilon_3' (\epsilon_1^2 - \epsilon_4'^2)} \left[\epsilon_1 \epsilon_3' E_x - j \epsilon_4' \epsilon_3' \sin\alpha \, E_z \right] \qquad \text{(D-19)}$$

$$E_y = \frac{k_2^2}{k_0^2 \epsilon_3' (\epsilon_1^2 - \epsilon_4'^2)} \left[-j \epsilon_4' \epsilon_3' \cos\alpha \, E_x + (\epsilon_1^2 - \epsilon_4'^2 - \epsilon_1 \epsilon_3') \sin\alpha \cos\alpha \, E_z \right] \qquad \text{(D-20)}$$

$$E_z = \frac{k_2^2}{k_0^2 \epsilon_3' (\epsilon_1^2 - \epsilon_4'^2)} \left[j \epsilon_4' \epsilon_3' \sin\alpha \, E_x + \left\{ (\epsilon_1^2 - \epsilon_4'^2) \cos^2\alpha + \epsilon_1 \epsilon_3' \sin^2\alpha \right\} E_z \right] \qquad \text{(D-21)}$$

From equation (D-21) we have

$$E_z = \frac{j \epsilon_4' \epsilon_3' \sin\alpha \, k_2^2 \, E_x}{(\epsilon_1^2 - \epsilon_4'^2)(k_0^2 \epsilon_3' - k_2^2) + k_2^2 (\epsilon_1^2 - \epsilon_4'^2 - \epsilon_1 \epsilon_3') \sin^2\alpha} \qquad \text{(D-22)}$$

Substituting equation (D-22) into equation (D-20) we get

$$E_y = \frac{-j \epsilon_4' k_2^2 \cos\alpha \, (k_0^2 \epsilon_3' - k_2^2) \, E_x}{k_0^2 \left[(\epsilon_1^2 - \epsilon_4'^2)(k_0^2 \epsilon_3' - k_2^2) + k_2^2 (\epsilon_1^2 - \epsilon_4'^2 - \epsilon_1 \epsilon_3') \sin^2\alpha \right]} \qquad \text{(D-23)}$$

Equation (D-19) gives

$$E_x = \frac{-j \epsilon_4' \sin\alpha \, k_2^2}{k_0^2 (\epsilon_1^2 - \epsilon_4'^2) - k_2^2 \epsilon_1} E_z \qquad \text{(D-24)}$$

Using equation (D-24) in equation (D-24) we have

$$E_y = \frac{k_2^2 \sin\alpha \cos\alpha}{k_0^2 \epsilon_3' (\epsilon_1^2 - \epsilon_4'^2)} \left[\frac{(\epsilon_1^2 - \epsilon_4'^2 - \epsilon_1 \epsilon_3') \left\{ k_0^2 (\epsilon_1^2 - \epsilon_4'^2) - k_2^2 \epsilon_1 \right\} - \epsilon_4'^2 \epsilon_3' k_2^2}{k_0^2 (\epsilon_1^2 - \epsilon_4'^2) - k_2^2 \epsilon_1} \right] E_z \qquad \text{(D-25)}$$

For Plane Wave in z-direction

$\frac{\partial}{\partial x} = 0$, $\frac{\partial}{\partial y} = 0$ which gives

$$\begin{bmatrix} \bar{a}_x & \bar{a}_y & \bar{a}_z \\ 0 & 0 & \frac{\partial}{\partial z} \\ E_x & E_y & E_z \end{bmatrix} = -j\omega\mu_0 (\bar{a}_x H_x + \bar{a}_y H_y + \bar{a}_z H_z) \tag{D-26}$$

or

$$\frac{\partial E_y}{\partial z} = j\omega\mu_0 H_x \tag{D-27}$$

$$\frac{\partial E_x}{\partial z} = -j\omega\mu_0 H_y \tag{D-28}$$

$$0 = H_z \tag{D-29}$$

and

$$\begin{bmatrix} \bar{a}_x & \bar{a}_y & \bar{a}_z \\ 0 & 0 & \frac{\partial}{\partial z} \\ H_x & H_y & 0 \end{bmatrix} = j\omega\epsilon_0 \begin{bmatrix} \epsilon_1 & -j\epsilon_4 & j\epsilon_5 \\ j\epsilon_4 & \epsilon_2 & \epsilon_6 \\ -j\epsilon_5 & \epsilon_6 & \epsilon_3 \end{bmatrix} \begin{bmatrix} E_x \\ E_y \\ E_z \end{bmatrix} \tag{D-30}$$

$$-\frac{\partial H_y}{\partial z} = j\omega\epsilon_0 (\epsilon_1 E_x - j\epsilon_4 E_y + j\epsilon_5 E_z) \tag{D-31}$$

$$\frac{\partial H_x}{\partial z} = j\omega\epsilon_0 (j\epsilon_4 E_x + \epsilon_2 E_y + \epsilon_6 E_z) \tag{D-32}$$

$$0 = j\omega\epsilon_0 (-j\epsilon_5 E_x + \epsilon_6 E_y + \epsilon_3 E_z) \tag{D-33}$$

From equations (D-27),(D-28),(D-31),(D-32) we can write

$$\frac{\partial^2 E_y}{\partial z^2} = -k_0^2 (j\epsilon_4 E_x + \epsilon_2 E_y + \epsilon_6 E_z)$$

$$\Rightarrow k_3^2 E_y = k_0^2 (j\epsilon_4 E_x + \epsilon_2 E_y + \epsilon_6 E_z) \tag{D-34}$$

$$\frac{\partial^2 E_x}{\partial z^2} = -k_0^2 (\epsilon_1 E_x - j\epsilon_4 E_y + j\epsilon_5 E_z)$$

$$\Rightarrow k_3^2 E_x = k_0^2 (\epsilon_1 E_x - j\epsilon_4 E_y + j\epsilon_5 E_z) \tag{D-35}$$

Equations (D-33),(D-34),(D-35) can be arranged as

$$\frac{k_3^2}{k_0^2} \begin{bmatrix} E_x \\ E_y \\ 0 \end{bmatrix} = \begin{bmatrix} \epsilon_1 & -j\epsilon_4 & j\epsilon_5 \\ j\epsilon_4 & \epsilon_2 & \epsilon_6 \\ -j\epsilon_5 & \epsilon_6 & \epsilon_3 \end{bmatrix} \begin{bmatrix} E_x \\ E_y \\ E_z \end{bmatrix} \tag{D-36}$$

or

$$
\begin{bmatrix} \epsilon_1 - K_3^2/K_0^2 & -j\epsilon_4 & j\epsilon_5 \\ j\epsilon_4 & \epsilon_2 - K_3^2/K_0^2 & \epsilon_6 \\ -j\epsilon_5 & \epsilon_6 & \epsilon_3 \end{bmatrix} \begin{bmatrix} E_x \\ E_y \\ E_z \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} \qquad \text{(D-37)}
$$

or [M][E] =0

Characteristics equation is given by

det [M]=0 which gives

$$
\epsilon_3 K_3^4 + K_3^2 K_0^2 \left[(\epsilon_4'^2 - \epsilon_1^2) \sin^2\alpha - \epsilon_1 \epsilon_3' (1+\cos^2\alpha) \right] + \epsilon_3' (\epsilon_1^2 - \epsilon_4'^2) K_0^4 = 0
$$

$$
K_3^2 = K_0^2 \; \frac{-\left[(\epsilon_4'^2 - \epsilon_1^2) \sin^2\alpha - \epsilon_1 \epsilon_3' (1+\cos^2\alpha) \right] \pm \sqrt{\left[(\epsilon_4'^2 - \epsilon_1^2) \sin^2\alpha - \epsilon_1 \epsilon_3' (1+\cos^2\alpha) \right]^2 - 4 \epsilon_3 \epsilon_3' (\epsilon_1^2 - \epsilon_4'^2)}}{2\,\epsilon_3}
$$

(D-38)

Equation (D-36) , may be written as

$$
\begin{bmatrix} E_x \\ E_y \\ E_z \end{bmatrix} = \frac{K_3^2}{K_0^2} \begin{bmatrix} \epsilon_1 & -j\epsilon_4 & j\epsilon_5 \\ j\epsilon_4 & \epsilon_2 & \epsilon_6 \\ -j\epsilon_5 & \epsilon_6 & \epsilon_3 \end{bmatrix}^{-1} \begin{bmatrix} E_x \\ E_y \\ 0 \end{bmatrix}
$$

or

$$\begin{bmatrix} E_x \\ E_y \\ E_z \end{bmatrix} = \frac{k_3^2}{k_0^2\epsilon_3'(\epsilon_1^2-\epsilon_4'^2)} \begin{bmatrix} \epsilon_1\epsilon_3' & j\epsilon_4'\epsilon_3'\cos\alpha & -j\epsilon_4'\epsilon_3'\sin\alpha \\ -j\epsilon_4'\epsilon_3'\cos\alpha & (\epsilon_1^2-\epsilon_4'^2)\sin^2\alpha+\epsilon_1\epsilon_3'\cos^2\alpha & (\epsilon_1^2-\epsilon_4'^2-\epsilon_1\epsilon_3')\sin\alpha\cos\alpha \\ j\epsilon_4'\epsilon_3'\sin\alpha & (\epsilon_1^2-\epsilon_4'^2-\epsilon_1\epsilon_3')\sin\alpha\cos\alpha & (\epsilon_1^2-\epsilon_4'^2)\cos^2\alpha+\epsilon_1\epsilon_3'\sin^2\alpha \end{bmatrix} \begin{bmatrix} E_x \\ E_y \\ 0 \end{bmatrix} \qquad \text{(D-39)}$$

From equation (D-39) we can write

$$E_x = \frac{k_3^2}{k_0^2\epsilon_3'(\epsilon_1^2-\epsilon_4'^2)}\left[\epsilon_1\epsilon_3' E_x + j\epsilon_4'\epsilon_3'\cos\alpha E_y\right] \qquad \text{(D-40)}$$

$$E_y = \frac{k_3^2}{k_0^2\epsilon_3'(\epsilon_1^2-\epsilon_4'^2)}\left[-j\epsilon_4'\epsilon_3'\cos\alpha E_x + \left\{(\epsilon_1^2-\epsilon_4'^2)\sin^2\alpha+\epsilon_1\epsilon_3'\cos^2\alpha\right\}E_y\right] \qquad \text{(D-41)}$$

$$E_z = \frac{k_3^2}{k_0^2\epsilon_3^2(\epsilon_1^2-\epsilon_4'^2)}\left[j\epsilon_4'\epsilon_3'\sin\alpha + (\epsilon_1^2-\epsilon_4'^2-\epsilon_1\epsilon_3')\sin\alpha\cos\alpha E_y\right] \qquad \text{(D-42)}$$

From equation (D-40) we get

$$E_x = \frac{j\epsilon_4'\cos\alpha\, k_3^2}{k_0^2(\epsilon_1^2-\epsilon_4'^2)-k_3^2\epsilon_1} E_y \qquad \text{(D-43)}$$

Using equation (D-43) in equation (D-42) we have

$$E_z = \frac{k_3^2 \sin\alpha \cos\alpha}{k_0^2 \epsilon_3'} \left[\frac{k_3^2(\epsilon_3' - \epsilon_1) + k_0^2(\epsilon_1^2 - \epsilon_q'^2 - \epsilon_1 \epsilon_3')}{k_0^2(\epsilon_1^2 - \epsilon_q'^2) - k_3^2 \epsilon_1} \right] E_y \qquad \text{(D-44)}$$

Again from equation (D-41) we have

$$E_y = \frac{-j \epsilon_q' \epsilon_3' \cos\alpha \, k_3^2}{k_0^2 \epsilon_3'(\epsilon_1^2 - \epsilon_q'^2) - k_3^2[(\epsilon_1^2 - \epsilon_q'^2)\sin^2\alpha + \epsilon_1 \epsilon_3' \cos^2\alpha]} E_x \qquad \text{(D-45)}$$

From equation (D-40) we can write

$$\cos\alpha \, E_y = \frac{k_0^2 \epsilon_3'(\epsilon_1^2 - \epsilon_q'^2) - k_3^2 \epsilon_1 \epsilon_3'}{j \epsilon_q' \epsilon_3' k_3^2} E_x \qquad \text{(D-46)}$$

Using equation (D-46) in equation (D-42) we have

$$E_z = \frac{-j \sin\alpha \left[(\epsilon_1^2 - \epsilon_q'^2 - \epsilon_1 \epsilon_3') \left\{ k_0^2 \epsilon_3'(\epsilon_1^2 - \epsilon_q'^2) - k_3^2 \epsilon_1 \epsilon_3' \right\} - \epsilon_q'^2 \epsilon_3'^2 k_3^2 \right]}{k_0^2 \epsilon_q' \epsilon_3'^2 (\epsilon_1^2 - \epsilon_q'^2)} E_x \qquad \text{(D-47)}$$

APPENDIX - E

E-1 Saddle Point Method of Integration

Under certain conditions complex integrals of the type

$$F(\rho) = \int_{-\alpha}^{\alpha} f(h)\, e^{j\rho\phi(h)}\, dh \tag{E-1}$$

can be evaluated approximately by the method of saddle-point integration. The key conditions are that ρ be a large number compared to unity and $\phi(h)$, whose magnitude is of the order of unity, have an extreme value at a certain point h_0, so that $\phi'(h_o) = o$. The function f(h) is assumed to be a slowly varying function in the neighborhood of h_0. We consider $\phi(h)$ to be an analytic function of the complex variable $h = \xi + j\eta$ so that

$$\phi(h) = u(\xi, \eta) + jv(\xi, \eta) \tag{E-2}$$

then u and v satisfy the Cauchy-Riemann relations

$$\frac{\partial u}{\partial \xi} = \frac{\partial v}{\partial \eta}, \quad \frac{\partial u}{\partial \eta} = -\frac{\partial v}{\partial \xi} \tag{E-3}$$

A three-dimensional plot of the surface $z = v(\xi, \eta)$ shows that in the neighborhood of the point $h = h_0$ or $\xi = \xi_o$ and $\eta = \eta_o$ the surface has the shape of a saddle because

$$\left[\frac{\partial^2 v}{\partial \xi^2} \cdot \frac{\partial^2 v}{\partial \eta^2} - \left(\frac{\partial^2 v}{\partial \xi \partial \eta}\right)^2\right]_{\xi=\xi_o,\ \eta=\eta_o} < 0$$

as a result of the Cauchy-Reimann relations. The family of curves described by

$$v(\xi,\eta) = c$$

for different values of c has the appearance shown in Fig. E-1 in the neighborhood of the saddle point, where $v_o = v(\xi_o, \eta_o)$

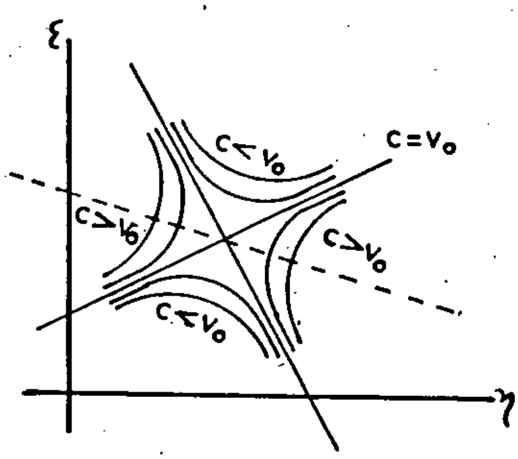


Fig. E-1 :The family of curves described by $v(\xi,\eta)$ =c.

The above description also applies to the function $u(\xi,\eta)$ and the family of curves corresponding to $u(\xi,\eta)$= const. are orthogonal to those shown in Fig.E.1 . We now change the path of integration such that the contour would pass through h_o and follow a path corresponding to

$$u(\xi,\eta) = u_o$$

in the neighborhood of the saddle point. A section of this path is shown in Fig.E.1 by the dotted line. Under that condition the original integral can be written in the form

$$F(\rho) = \int_c f(h)\, e^{j\rho(u_0 + jv)} dh = e^{j\rho u_0} \int_c f(h)\, e^{-\rho v} dh.$$

The function $e^{-\rho v}$ has a significant value along a small segment of the path near the saddle point. Thus we may approximate the function $f(h)$ by a series expansion in the neighborhood of h_0 by retaining only the first three terms, that is,

$$\phi(h) = \phi(h_0) + \phi'(h_0)(h-h_0) + \tfrac{1}{2}\phi''(h_0)(h-h_0)^2 = \phi(h_0) + \tfrac{1}{2}\phi''(h_0)(h-h_0)^2 \qquad \text{(E-4)}$$

For the slowly varying function f(h) it can be replaced by $f(h_0)$.The original integral, then, can be approximated by

$$F(\rho) = e^{j\rho\phi(h_0)} f(h_0) \int_{h_0-\Delta h}^{h_0+\Delta h} e^{j\frac{1}{2}\rho\phi''(h_0)(h-h_0)^2} dh \qquad \text{(E-5)}$$

This can be simplified if we let

$$h - h_0 = s e^{j\alpha}$$

$$\phi''(h_0) = |\phi''(h_0)| e^{j\beta}$$

then

$$j\tfrac{1}{2}\rho\phi''(h_0)(h-h_0)^2 = \tfrac{1}{2}\rho|\phi''(h_0)| s^2 e^{j(2\alpha+\beta+\pi/2)}$$

In order to confine the path of integration along the contour $u = u_0$, the angle α must be so chosen that

$$\alpha = \tfrac{1}{2}(\tfrac{\pi}{2} - \beta)$$

Under this condition, (E-5) reduces to

$$F(\rho) = e^{j[\rho\phi(h_0) - \frac{\beta}{2} + \frac{\pi}{4}]} f(h_0) \int_{-\tau}^{\tau} e^{-\frac{1}{2}\rho\phi''(h_0)s^2} ds \qquad \text{(E-6)}$$

where $\tau = |\Delta h|$. The last integral can be evaluated by a change of variable to

$$t = \left[\frac{\rho|\phi''(h_0)|}{2}\right]^{1/2} s$$

and in the limit as

$$\left[\frac{\rho|\phi''(h_0)|}{2}\right]^{1/2} \tau$$

becomes very large we obtain [34]

$$F(\rho) = \left[\frac{2\pi}{\rho|\phi''(h_0)|}\right] f(h_0)\, e^{j[\rho\phi(h_0) - \beta/2 + \pi/4]} \qquad \text{(E-7)}$$

Let us consider an integral of the form $\iint_{-\infty}^{\infty} L(k_1, k_2, \rho)\, e^{-jk_1x - jk_2y - j\rho z}\, dk_1 dk_2$. Starting with the integration of K_1, we have the saddle points determined by

$$\frac{\partial\psi}{\partial k_1} = 0 \quad , \quad \psi = -k_1x - k_2y - \rho z$$

$$\text{or} \quad -\frac{x}{z} = \left.\frac{\partial\rho}{\partial k_1}\right|_s \qquad \text{(E-8)}$$

and contribution to each saddle point k_{1s} is

$$\left[\frac{2\pi}{\left|\frac{\partial^2\psi}{\partial k_{1s}^2}\right|}\right]^{1/2} \tag{E-9}$$

The integration now becomes

$$\int_{-\alpha}^{\alpha} L(k_{1s}, k_2, \rho)\left[\frac{2\pi}{\left|\frac{\partial^2\psi}{\partial k_{1s}^2}\right|}\right]^{1/2} e^{-jk_{1s}x-jk_2y-j\rho z} \cdot e^{j\frac{\pi}{4}\operatorname{sgn}\left(\frac{\partial^2\psi}{\partial k_{1s}^2}\right)} dk_2 \tag{E-10}$$

For the k_2-integration ,the saddle points are determined by

$$-\frac{y}{z} = \frac{d\rho}{dk_{2s}} = \frac{\partial\rho}{\partial k_{2s}} + \frac{dk_{1s}}{dk_{2s}}\cdot\frac{\partial\rho}{\partial k_{1s}} \tag{E-11}$$

Differentiation of (E-11) gives

$$\frac{\partial}{\partial k_{1s}}\left(\frac{d\rho}{dk_{2s}}\right) = \frac{\partial^2\rho}{\partial k_{2s}\partial k_{1s}} + \frac{dk_{1s}}{dk_{2s}}\cdot\frac{\partial^2\rho}{\partial k_{1s}^2} = 0 \tag{E-12}$$

Again using (E-12) we can write

$$\frac{\partial}{\partial k_{2s}}\left(\frac{d\rho}{dk_{2s}}\right) = \frac{\partial^2\rho}{\partial k_{2s}^2} - \left(\frac{\partial^2\rho}{\partial k_{1s}\partial k_{2s}}\right)^2\Bigg/\frac{\partial^2\rho}{\partial k_{1s}^2} \tag{E-13}$$

Therefore, contribution from each two dimensional saddle point is given by

$$\left[\frac{2\pi}{\left|\frac{\partial^2\psi}{\partial k_{1s}^2}\right|}\right]^{1/2}\cdot\left[\frac{2\pi}{\left|\frac{\partial^2\psi}{\partial k_{2s}^2} - \left(\frac{\partial^2\psi}{\partial k_{1s}\partial k_{2s}}\right)^2\Big/\frac{\partial^2\psi}{\partial k_{1s}^2}\right|}\right]^{1/2}$$

$$= \frac{2\pi}{\left|\left(\frac{\partial^2\psi}{\partial k_{1s}^2}\right)\left(\frac{\partial^2\psi}{\partial k_{2s}}\right)^2 - \left(\frac{\partial^2\psi}{\partial k_{1s}\partial k_{2s}}\right)^2\right|^{1/2}}$$

and the integration becomes

$$L(k_{1s}, k_{2s}, \rho)\, M\, e^{-jk_{1s}x - jk_{2s}y - \rho z} \cdot \alpha$$

where

$$\alpha = Exp\left\{ j\,\pi/4 \left[sgn\left(\frac{\partial^2 \psi}{\partial k_{1s}^2}\right) + sgn\left\{\frac{\partial^2 \psi}{\partial k_{2s}^2} - \left(\frac{\partial^2 \psi}{\partial k_{1s}\,\partial k_{2s}}\right)^2 \Big/ \frac{\partial^2 \psi}{\partial k_{1s}^2}\right\}\right]\right\}$$

E-2 Some Useful Derivatives

E.2.a For slot along the x-axis

The phase of each wave is given by $\psi = \rho_{11}^{+}|x| + k_2 y + k_3 z$

Taking the first derivative of ψ with respect to k_3 ,the saddle points are determined by

$$\frac{\partial \psi}{\partial k_3} = |x| \frac{\partial \rho_{11}^{+}}{\partial k_3} + z = 0$$

$$\Rightarrow -\frac{\partial \rho_{11}^{+}}{\partial k_3} = \frac{z}{|x|} = \cot\theta$$

Now utilizing the characteristic equation (equation (2.15)) we find that

$$\left.\frac{\partial k_1}{\partial k_3}\right|_{\substack{k_1 \to \rho_{11}^{+} \\ k_3 \to k_{3s}}} = -\left.\frac{A}{B}\right|_{\substack{k_1 \to \rho_{11} \\ k_3 \to k_{3s}}}$$

Where

$$A = \epsilon_1\epsilon_3 k_1^2 k_3 + \epsilon_1\epsilon_6 k_1^2 k_2 + \epsilon_1\epsilon_3 k_2^2 k_3 + \epsilon_1\epsilon_6 k_2^3 + \epsilon_1^2 k_1^2 k_3 + \epsilon_1\epsilon_2 k_2^2 k_3 + 2\epsilon_1\epsilon_3 k_3^3$$
$$+ 3\epsilon_1\epsilon_6 k_2 k_3^2 - \eta_1 k_0^2 k_3 - \epsilon_1^2\epsilon_3 k_0^2 k_3 - \epsilon_1^2\epsilon_6 k_2 k_0^2 + C_3\epsilon_1 k_3 k_0^2 + C_4\epsilon_1 k_2 k_0^2/2$$

$$B = 2\epsilon_1^2 k_1^3 + \epsilon_1\epsilon_2 k_1 k_2^2 + \epsilon_1\epsilon_3 k_1 k_3^2 + 2\epsilon_1\epsilon_6 k_1 k_2 k_3 - \eta_1 k_0^2 k_1 + \epsilon_1^2 k_2^2 k_1$$
$$+ \epsilon_1^2 k_3^2 k_1 - \epsilon_1^3 k_1 k_0^2 + C_1\epsilon_1 k_0^2 k_1$$

and

$$\frac{\partial^2 \psi}{\partial k_3^2} = |x| \frac{\partial^2 \rho_{11}^+}{\partial k_3^2}$$

differentiating $\frac{\partial k_1}{\partial k_3}$ with respect to k_3 we have

$$\left.\frac{\partial^2 k_1}{\partial k_3^2}\right|_{\substack{k_1 \to \rho_{11}^+ \\ k_3 \to k_{3s}}} = -\left[E + D\left(\frac{\partial k_1}{\partial k_3}\right) + C\left(\frac{\partial k_1}{\partial k_3}\right)^2 \right] \Big/ B$$

where

$$E = \epsilon_1 \epsilon_3 k_1^2 + \epsilon_1 \epsilon_3 k_2^2 + \epsilon_1 k_1^2 + \epsilon_1 \epsilon_2 k_2^2 + 6 \epsilon_1 \epsilon_3 k_3^2 + 6 \epsilon_1 \epsilon_6 k_2 k_3$$
$$- \eta_1 k_0^2 - \epsilon_1^2 \epsilon_3 k_0^2 + c_3 \epsilon_1 k_0^2$$

$$D = 4 \epsilon_1 \epsilon_3 k_1 k_3 + 4 \epsilon_1 \epsilon_6 k_1 k_2 + 4 \epsilon_1^2 k_1 k_3$$

$$C = 6 \epsilon_1^2 k_1^2 + \epsilon_1 \epsilon_2 k_2^2 + \epsilon_1 \epsilon_3 k_3^2 + 2 \epsilon_1 \epsilon_6 k_2 k_3 - \eta_1 k_0^2 + \epsilon_1^2 k_2^2 + \epsilon_1^2 k_3^2 - \epsilon_1^3 k_0^2 + c_1 \epsilon_1 k_0^2$$

Taking the first derivative of ψ with respect to k_2 the saddle points are determined by

$$\frac{\partial \psi}{\partial k_2} = |x| \frac{\partial \rho_{11}^+}{\partial k_2} + y = 0$$

$$\Rightarrow -\frac{\partial \rho_{11}^+}{\partial k_2} = \frac{y}{|x|} = \tan \phi$$

Now from characteristic equation we get

$$\left.\frac{\partial k_1}{\partial k_2}\right|_{\substack{k_1 \to \rho_{11}^+ \\ k_2 \to k_{2s}}} = -\left.\frac{F}{B}\right|_{\substack{k_1 \to \rho_{11}^+ \\ k_2 \to k_{2s}}}$$

where

$$F = \epsilon_1 \epsilon_2 k_1^2 k_2 + \epsilon_1 \epsilon_6 k_1^2 k_3 + \epsilon_1^2 k_1^2 k_2 + 2 \epsilon_1 \epsilon_2 k_2^3 + \epsilon_1 \epsilon_3 k_2 k_3^2 + 3 \epsilon_1 \epsilon_6 k_2^2 k_3 - \eta_1 k_0^2 k_2$$
$$+ \epsilon_1 \epsilon_2 k_3^2 k_2 + \epsilon_1 \epsilon_6 k_3^3 - \epsilon_1^2 \epsilon_2 k_0^2 k_2 - \epsilon_1^2 \epsilon_6 k_3 k_0^2 + c_2 \epsilon_1 k_0^2 k_2 + c_9 \epsilon_1 k_3 k_0^2$$

and

$$\frac{\partial^2 \psi}{\partial k_2^2} = |x| \frac{\partial^2 \rho_{11}^+}{\partial k_2^2}$$

differentiating $\frac{\partial k_1}{\partial k_2}$ with respect to k_2 we have

$$\left.\frac{\partial^2 k_1}{\partial k_2^2}\right|_{\substack{k_1\to\rho_{11}^+\\ k_2\to k_{2s}}} = -\left[H + G\left(\frac{\partial k_1}{\partial k_2}\right) + C\left(\frac{\partial k_1}{\partial k_2}\right)^2\right] \Bigg/ \beta \,\Bigg|_{\substack{k_1\to\rho_{11}^+\\ k_2\to k_{2s}}}$$

where

$$G = 4\epsilon_1\epsilon_2 k_1 k_2 + 4\epsilon_1\epsilon_6 k_1 k_3 + 4\epsilon_1^2 k_1 k_2$$

$$H = \epsilon_1\epsilon_2 k_1^2 + \epsilon_1^2 k_1^2 + \epsilon_1\epsilon_3 k_3^2 + 6\epsilon_1\epsilon_6 k_2 k_3 - \eta_1 k_0^2 + \epsilon_1\epsilon_2 k_3^2$$
$$- \epsilon_1^2\epsilon_2 k_0^2 + c_2\epsilon_1 k_0^2 + 6\epsilon_1\epsilon_2 k_2^2$$

we know

$$\frac{\partial\psi}{\partial k_3} = |x|\,\frac{\partial\rho_{11}^+}{\partial k_3} + z$$

then

$$\frac{\partial^2\psi}{\partial k_3\,\partial k_2} = |x|\,\frac{\partial^2\rho_{11}^+}{\partial k_3\,\partial k_2}$$

Differentiating $\frac{\partial k_1}{\partial k_3}$ with respect to k_2 we have

$$\left.\frac{\partial^2 k_1}{\partial k_3\,\partial k_2}\right|_{\substack{k_1\to\rho_{11}\\ k_2\to k_{2s}\\ k_3\to k_{3s}}} = -\left[P + C\left(\frac{\partial k_1}{\partial k_2}\right)\left(\frac{\partial k_1}{\partial k_3}\right) + \frac{G}{2}\left(\frac{\partial k_1}{\partial k_3}\right) + \frac{D}{2}\left(\frac{\partial k_1}{\partial k_2}\right)\right] \Bigg/ \beta$$

where

$$P = \epsilon_1\epsilon_6 k_1^2 + 2\epsilon_1\epsilon_3 k_2 k_3 + 3\epsilon_1\epsilon_6 k_2^2 + 2\epsilon_1\epsilon_2 k_2 k_3 + 3\epsilon_1\epsilon_6 k_3^2$$
$$- \epsilon_1^2\epsilon_6 k_0^2 + c_4\epsilon_1 k_0^2$$

E.2.b. For slot along the y-axis

The phase of each wave is determined by $\psi = k_1 x + \rho_{21}^{\pm}\,(\text{or } \rho_{22}^{\pm})\,|y| + k_3 z$

Taking the first derivatives of ψ with respect to k_1, the saddle points are determined by

Now from characteristic equation (equation 2.15) we have

$$\left.\frac{\partial k_2}{\partial k_1}\right|_{\substack{k_2\to\rho \\ k_1\to k_{1s}}} = -\left.\frac{A_1}{B_1}\right|_{\substack{k_2\to\rho \\ k_1\to k_{1s}}}$$

where

$$A_1 = 2\epsilon_1^2 k_1^3 + \epsilon_1\epsilon_2 k_1 k_2^2 + \epsilon_1\epsilon_3 k_1 k_3^2 + 2\epsilon_1\epsilon_6 k_1 k_2 k_3 - \eta_1 k_0^2 k_1$$
$$+ \epsilon_1^2\epsilon_2 k_1 + \epsilon_1^2 k_1 k_3^2 - \epsilon_1^3 k_0^2 k_1 + C_1\epsilon_1 k_0^2 k_1$$

$$B_1 = \epsilon_1\epsilon_2 k_1^2 k_2 + \epsilon_1\epsilon_6 k_1^2 k_3 + \epsilon_1^2 k_1^2 k_2 + 2\epsilon_1\epsilon_2 k_2^3 + \epsilon_1\epsilon_3 k_3^2 k_2 + 3\epsilon_1\epsilon_6 k_3 k_2^2 - \eta_1 k_0^2 k_2$$
$$+ \epsilon_1\epsilon_2 k_3^2 k_2 + \epsilon_1\epsilon_6 k_3^3 - \epsilon_1^2\epsilon_2 k_0^2 k_2 - \epsilon_1^2\epsilon_6 k_3 k_0^2 - C_2\epsilon_1 k_0^2 k_2 + C_4\epsilon_1 k_3 k_0^2/2$$

and

$$\frac{\partial^2\psi}{\partial k_1^2} = |y|\frac{\partial^2\rho}{\partial k_1^2}$$

differentiating $\frac{\partial k_2}{\partial k_1}$ with respect to k_1 we have

$$\left.\frac{\partial^2 k_2}{\partial k_1^2}\right|_{\substack{k_2\to\rho \\ k_1\to k_{1s}}} = -\left[E_1 + D_1\left(\frac{\partial k_2}{\partial k_1}\right) + C_1\left(\frac{\partial k_2}{\partial k_1}\right)^2\right]\Big/B_1$$

where

$$C_1 = \epsilon_1\epsilon_2 k_1^2 + \epsilon_1^2 k_1^2 + 6\epsilon_1\epsilon_2 k_2^2 + \epsilon_1\epsilon_3 k_3^2 + 6\epsilon_1\epsilon_6 k_2 k_3 - \eta_1 k_0^2$$
$$+ \epsilon_1\epsilon_2 k_3^2 - \epsilon_1^2\epsilon_2 k_0^2 + C_2\epsilon_1 k_0^2$$

$$D_1 = 4\epsilon_1\epsilon_2 k_1 k_2 + 4\epsilon_1\epsilon_6 k_1 k_3 + 4\epsilon_1^2 k_1 k_2$$

$$E_1 = 6\epsilon_1^2 k_1^2 + \epsilon_1\epsilon_2 k_2^2 + \epsilon_1\epsilon_3 k_3^2 + 2\epsilon_1\epsilon_6 k_2 k_3 - \eta_1 k_0^2$$
$$+ \epsilon_1^2 k_2^2 + \epsilon_1^2 k_3^2 - \epsilon_1^3 k_0^2 + C_1\epsilon_1 k_0^2$$

Taking first derivative of ψ with respect to k_3, the saddle points are given by,

$$\frac{\partial\psi}{\partial k_3} = z + |y|\frac{\partial\rho}{\partial k_3} = 0 \quad \text{or} \quad -\frac{\partial\rho}{\partial k_3} = \frac{z}{|y|} = \cot\phi$$

Now characteristic equation gives

$$\left.\frac{\partial k_2}{\partial k_3}\right|_{\substack{k_2\to\rho \\ k_3\to k_{3s}}} = -\left.\frac{F_1}{B_1}\right|_{\substack{k_2\to\rho \\ k_3\to k_{3s}}}$$

where

$$F_1 = \epsilon_1 \epsilon_3 k_1^2 k_3 + \epsilon_1 \epsilon_6 k_1^2 k_2 + \epsilon_1 \epsilon_3 k_2^2 k_3 + \epsilon_1 \epsilon_6 k_2^3 + \epsilon_1^2 k_1^2 k_3 + \epsilon_1 \epsilon_2 k_1^2 k_3$$
$$+ 2\epsilon_1 \epsilon_3 k_3^3 + 3\epsilon_1 \epsilon_6 k_2 k_3^2 - \eta_1 k_0^2 k_3 - \epsilon_1^2 \epsilon_3 k_0^2 k_3 - \epsilon_1^2 \epsilon_6 k_2 k_0^2 + C_3 \epsilon_1 k_0^2 k_3 + C_9 \epsilon_1 k_2 k_0^2/2$$

and

$$\frac{\partial^2 \psi}{\partial k_3^2} = |Y| \frac{\partial^2 \rho}{\partial k_3^2}$$

Differentiation of $\frac{\partial k_2}{\partial k_3}$ with respect to k_3 gives

$$\left.\frac{\partial^2 k_2}{\partial k_3^2}\right|_{\substack{k_2 \to \rho \\ k_3 \to k_{3s}}} = -\left[H_1 + G_1\left(\frac{\partial k_2}{\partial k_3}\right) + C_1\left(\frac{\partial k_2}{\partial k_3}\right)^2 \right] \Big/ B_1$$

where

$$G_1 = 2\left[\epsilon_1 \epsilon_6 k_1^2 + 2\epsilon_1 \epsilon_3 k_2 k_3 + 3\epsilon_1 \epsilon_6 k_2^2 + 2\epsilon_1 \epsilon_2 k_2 k_3 + 3\epsilon_1 \epsilon_6 k_3^2 - \epsilon_1 \epsilon_6 k_0^2 + \frac{C_9}{2} \epsilon_1 k_0^2\right]$$
$$H_1 = \epsilon_1 \epsilon_3 k_1^2 + \epsilon_1 \epsilon_3 k_2^2 + \epsilon_1^2 k_1^2 + \epsilon_1 \epsilon_2 k_2^2 + 6\epsilon_1 \epsilon_3 k_3^2 + 6\epsilon_1 \epsilon_6 k_2 k_3$$
$$- \eta_1 k_0^2 - \epsilon_1^2 \epsilon_3 k_0^2 + C_3 \epsilon_1 k_0^2$$

we known

$$\frac{\partial \psi}{\partial k_1} = |Y| \frac{\partial \rho}{\partial k_1} + X$$

then

$$\frac{\partial^2 \psi}{\partial k_1 \partial k_3} = |Y| \frac{\partial^2 \rho}{\partial k_1 \partial k_3}$$

differentiation of $\frac{\partial k_2}{\partial k_1}$ with respect to k_3 gives

$$\frac{\partial^2 k_2}{\partial k_1 \partial k_3} = -\left[P_1 + C_1\left(\frac{\partial k_2}{\partial k_1}\right)\left(\frac{\partial k_2}{\partial k_3}\right) + \frac{G_1}{2}\left(\frac{\partial k_2}{\partial k_1}\right) + \frac{D_1}{2}\left(\frac{\partial k_2}{\partial k_3}\right)\right] \Big/ B_1$$

where

$$P_1 = 2\epsilon_1 \epsilon_3 k_1 k_3 + 2\epsilon_1 \epsilon_6 k_1 k_2 + 2\epsilon_1^2 k_1 k_3$$

E.2.c. For slot along the z-axis

The phase of each wave is given by $\psi = k_1 x + k_2 y + \rho(\beta_{31}^{\pm} \text{ or } \beta_{32}^{\pm}) z$

Taking the first derivative of ψ with respect to k_1, the saddle points are determined by

$$\frac{\partial \psi}{\partial k_1} = x + |z| \frac{\partial \rho}{\partial k_1}$$

$$\Rightarrow -\frac{\partial \rho}{\partial k_1} = \frac{x}{|z|} = \tan\theta$$

Now from characteristic equation

$$\left.\frac{\partial k_3}{\partial k_1}\right|_{\substack{k_3 \to \rho \\ k_1 \to k_{1s}}} = -\left.\frac{A_2}{B_2}\right|_{\substack{k_3 \to \rho \\ k_1 \to k_{1s}}}$$

where

$$A_2 = 2\epsilon_1^2 k_1^3 + \epsilon_1\epsilon_2 k_1 k_2^2 + \epsilon_1\epsilon_3 k_3^2 k_1 + 2\epsilon_1\epsilon_6 k_1 k_2 k_3 - \eta_1 k_0^2 k_1 + \epsilon_1^2 k_2^2 k_1 + \epsilon_1^2 k_3^2 k_1 - \epsilon_1^3 k_0^2 k_1 + C_1\epsilon_1 k_0^2\epsilon_1$$

$$B_2 = \epsilon_1\epsilon_3 k_1^2 k_3 + \epsilon_1\epsilon_6 k_1^2 k_2 + \epsilon_1\epsilon_3 k_2^2 k_3 + \epsilon_1\epsilon_6 k_2^3 + \epsilon_1 k_1^2 k_3 + \epsilon_1\epsilon_2 k_2^2 k_3 + 2\epsilon_1\epsilon_3 k_3^3 + 3\epsilon_1\epsilon_6 k_2 k_3^2 - \eta_1 k_0^2 k_3 - \epsilon_1^2\epsilon_3 k_0^2 k_3 - \epsilon_1^2\epsilon_6 k_2 k_0^2 + C_3\epsilon_1 k_0^2 k_3 + C_4\epsilon_1 k_2 k_0^2/2$$

and

$$\frac{\partial^2 \psi}{\partial k_1^2} = |z| \frac{\partial^2 \rho}{\partial k_1^2}$$

differentiating $\frac{\partial k_3}{\partial k_1}$ with respect to k_1 we get

$$\left.\frac{\partial^2 k_3}{\partial k_1^2}\right|_{\substack{k_3 \to \rho \\ k_1 \to k_{1s}}} = -\left[E_2 + D_2\left(\frac{\partial k_3}{\partial k_1}\right) + C_2\left(\frac{\partial k_3}{\partial k_1}\right)^2\right] \Big/ B_2$$

where

$$C_2 = \epsilon_1\epsilon_3 k_1^2 + \epsilon_1\epsilon_3 k_2^2 + \epsilon_1^2 k_1^2 + \epsilon_1\epsilon_2 k_2^2 + 6\epsilon_1\epsilon_3 k_3^2 + 6\epsilon_1\epsilon_6 k_2 k_3 - \eta_1 k_0^2 - \epsilon_1^2\epsilon_3 k_0^2 + C_3\epsilon_1 k_0^2$$

$$D_2 = 4\epsilon_1\epsilon_3 k_1 k_3 + 4\epsilon_1\epsilon_6 k_1 k_2 + 4\epsilon_1^2 k_1 k_3$$

$$E_2 = 6\epsilon_1^2 k_1^2 + \epsilon_1\epsilon_2 k_2^2 + \epsilon_1\epsilon_3 k_3^2 + 2\epsilon_1\epsilon_6 k_2 k_3 - \eta_1 k_0^2 + \epsilon_1^2 k_2^2 + \epsilon_1^2 k_3^2 - \epsilon_1^3 k_0^2 + C_1\epsilon_1 k_0^2$$

Taking the first derivative of ψ with respect to k_2, the saddle points are determined by

$$\frac{\partial \psi}{\partial k_2} = |z| \frac{\partial \rho}{\partial k_2} + y$$

$$\Rightarrow -\frac{\partial \rho}{\partial k_2} = \frac{y}{|z|} = \tan\theta$$

now using characteristic equation

$$\frac{\partial k_3}{\partial k_2} = -\frac{F_2}{B_2}$$

where

$$F_2 = \epsilon_1 \epsilon_2 k_1^2 k_2 + \epsilon_1 \epsilon_6 k_3 k_1^2 + \epsilon_1^2 k_1^2 k_2 + 2\epsilon_1 \epsilon_2 k_2^3 + \epsilon_1 \epsilon_3 k_2 k_3^2 + 3\epsilon_1 \epsilon_6 k_3 k_2^2$$
$$-\eta_1 k_0^2 k_2 + \epsilon_1 \epsilon_2 k_3^2 k_2 + \epsilon_1 \epsilon_6 k_3^3 - \epsilon_1^2 \epsilon_2 k_0^2 k_2 - \epsilon_1^2 \epsilon_6 k_3 k_0^2 + c_2 \epsilon_1 k_0^2 \epsilon_2 + c_4 \epsilon_1 k_3 k_0^2 / 2$$

and

$$\frac{\partial^2 \psi}{\partial k_2} = |z| \frac{\partial^2 \rho}{\partial k_2^2}$$

differentiation of $\frac{\partial k_3}{\partial k_2}$ with respect to k_2 gives

$$\left.\frac{\partial^2 k_3}{\partial k_2^2}\right|_{\substack{k_3 \to \rho \\ k_2 \to k_{2s}}} = -\left.\left[H_2 + G_2\left(\frac{\partial k_3}{\partial k_1}\right) + C_2\left(\frac{\partial k_3}{\partial k_1}\right)^2\right] \Big/ B_2 \right|_{\substack{k_3 \to \rho \\ k_2 \to k_{2s}}}$$

where

$$G_2 = 2\left[\epsilon_1 \epsilon_6 k_1^2 + 2\epsilon_1 \epsilon_3 k_2 k_3 + 3\epsilon_1 \epsilon_6 k_2^2 + 2\epsilon_1 \epsilon_2 k_2 k_3 + 3\epsilon_1 \epsilon_6 k_3^2 - \epsilon_1^2 \epsilon_6 k_0^2 + \epsilon_4 \epsilon_1 k_0^2/2\right]$$

$$H_2 = \epsilon_1 \epsilon_2 k_1^2 + \epsilon_1^2 k_1^2 + 6\epsilon_1 \epsilon_2 k_2^2 + \epsilon_1 \epsilon_3 k_3^2 + 6\epsilon_1 \epsilon_6 k_2 k_3$$
$$-\eta_1 k_0^2 + \epsilon_1 \epsilon_2 k_3^2 - \epsilon_1^2 \epsilon_2 k_0^2 + c_2 \epsilon_1 k_0^2$$

we know

$$\frac{\partial \psi}{\partial k_1} = x + |z| \frac{\partial \rho}{\partial k_1}$$

then

$$\frac{\partial^2 \psi}{\partial k \, \partial k_2} = |z| \frac{\partial^2 \rho}{\partial k_1 \partial k_2}$$

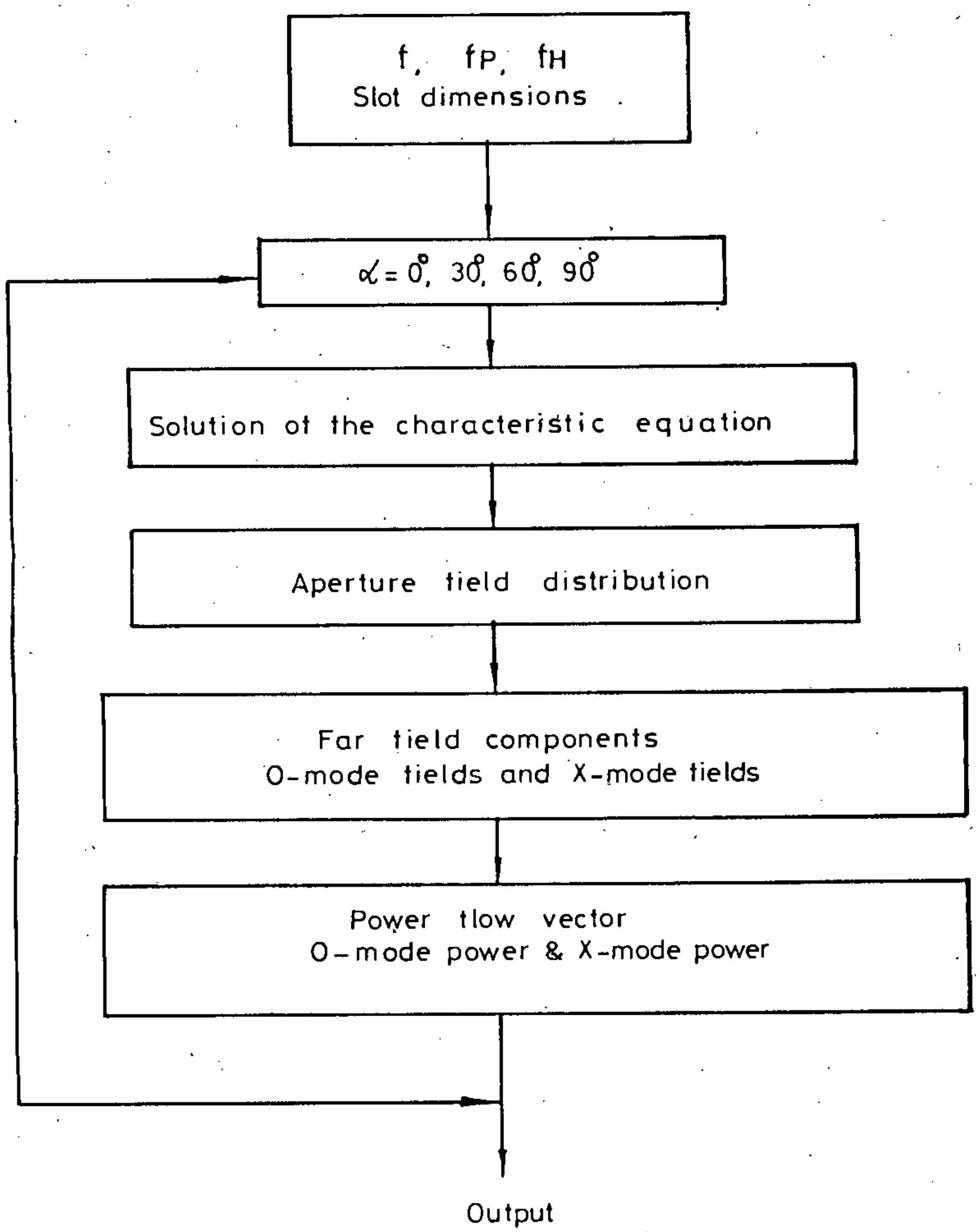
differentiation of $\frac{\partial k_3}{\partial k_1}$ with respect to k_2 gives

$$\frac{\partial^2 k_3}{\partial k_1 \partial k_2} = -\left[P_2 + \frac{D_2}{2}\left(\frac{\partial k_3}{\partial k_2}\right) + \frac{G_2}{2}\left(\frac{\partial k_3}{\partial k_1}\right) + C_2\left(\frac{\partial k_3}{\partial k_1}\right)\left(\frac{\partial k_3}{\partial k_2}\right)\right] \Big/ B_2$$

$$P_2 = 2\epsilon_1 \epsilon_2 k_1 k_2 + 2\epsilon_1 \epsilon_6 k_1 k_3 + 2\epsilon_1^2 k_1 k_2$$

COMPUTER PROGRAM

FLOW CHART OF CALCULATION



```
*      SLOT ALONG X-AXIS ON XY PLANE
*      FOR X-Y PLANE PATTERN AND X-Z PLANE PATTERN OF O & X MODE
*      STATIC MAGNETIC FIELD MAKES AN ARBITRARY ANGLE WITH Z-AXIS

       COMPLEX U,V,A1,A2,DETR,DT,AA1,AA2,F1,F2,C12,C13,C21,C31,A12,
      +A13,A21,A23,A31,A32,EXAO,HYAP,HZAP,HYAO,HYAX,HZAO,HZAX,HYA,HZA,
      +EXA,X1,Y1,Z1,EXO,EYO,EZO,HXO,A22,A33,
      +HYO,HZO,ETO,ESO,HTO,HSO,CHTO,CHSO,
      +CK3,CK4,FRC1,FRC2
       DIMENSION AK1(4)

       OPEN(UNIT=9,FILE='OUT',STATUS='NEW')
       PAI=3.141592654
       V=(1.0,0.0)
       U=(0.0,1.0)
       C=3.0E+08
       FR=300.0E+06
       AW=2.0*PAI*FR
       AKO=AW/C
       AWP=160.0E+06*2.0*PAI
       AWH=180.0E+06*2.0*PAI
       X=AWP*AWP/(AW*AW)
       Y=AWH/AW
       Z=1-Y*Y
       E1=1.0-X/Z
       E3=1.0-X
       E4=X*Y/Z
       DO 60 N=0,3
       ALPHA=N*PAI/6.0
       ALP=ALPHA*180.0/PAI
       WRITE(9,71) ALP
 71    FORMAT(4X,'ALPHA IN DEGREE=',F4.1)
       CA=COS(ALPHA)
       SA=SIN(ALPHA)
       G=E3/E1
       ESP2=E1*CA*CA+E3*SA*SA
       ESP3=E1*SA*SA+E3*CA*CA
       ESP4=E4*CA
       ESP5=E4*SA
       ESP6=(E3-E1)*SA*CA
       EE1=E1*E1
       EE4=E4*E4
       C1=EE4*(1.0-G)
       C2=EE4*(CA*CA-G)
       C3=EE4*(SA*SA-G)
       C4=-2.0*SA*CA*EE4
```

```
      ZETA=E3*(EE1-EE4)
      CD=(EE1-EE4)/E1
      AK11=AKO*SQRT(CD)
      AK12=AKO*SQRT(E3)
      AAK11=AK11*AK11
      AAK12=AK12*AK12

C     HERE I=1 FOR XY PLANE PATTERN
C     HERE I=2 FOR XZ PLANE PATTERN

      DO 50 I=1,2
      IF(I .EQ. 1) GO TO 73
      IF(I .EQ. 2) GO TO 74
 73   WRITE(9,81)
 81   FORMAT(//,4X,'X-Y PLANE PATTERN')
      GO TO 75
 74   WRITE(9,82)
 82   FORMAT(//,4X,'X-Z PLANE PATTERN')
 75   DO 40 J=1,3,2
      IF(J .EQ. 1) GO TO 76
      IF(J .EQ. 3) GO TO 77
 76   WRITE(9,83)
 83   FORMAT(4X,'FOR X-MODE')
      GO TO 78
 77   WRITE(9,84)
 84   FORMAT(4X,'FOR O-MODE')
      WRITE(9,121)
 121  FORMAT(6X,'POWER', 6X,'ANGLE',72X,'NOR.DB')
 78   DO 30 K=-140,140
      IF(K .EQ. 1) GO TO 30
      AK2=FLOAT(K-1)/20
      AK3=FLOAT(K-1)/20
      IF(I .EQ.1) GO TO 21
      IF(I .EQ.2) GO TO 22
 21   AK3=0.0
      GO TO 23
 22   AK2=0.0
 23   CONTINUE
      AAK2=AK2*AK2
      AAK3=AK3*AK3
      AAKO=AKO*AKO

      B1=E1*(E1+ESP2)*AAK2+(C1*E1-E1**3-ZETA)*AAKO+E1*(E1+ESP3)*AAK3
      A1=B1*V
      B2=E1*ESP2*AAK2*AAK2+(C2*E1-EE1*ESP2-ZETA)*AAKO*AAK2+
     +ZETA*E1*AAKO*AAKO+E1*ESP3*AAK3*AAK3+(C3*E1-EE1*ESP3-ZETA)*AAKO*
     +AAK3
```

```
      A2=B2*V
      DETR=A1*A1-4.0*EE1*A2
      DT=CSQRT(DETR)
      Q=2.0*EE1
      AA1=(-A1+DT)/Q
      AA2=(-A1-DT)/Q
      F1=CSQRT(AA1)
      F2=CSQRT(AA2)
      AK1(1)=REAL(F2)
      AK1(2)=-AK1(1)
      AK1(3)=REAL(F1)
      AK1(4)=-AK1(3)
      IF(AIMAG(F1) .EQ. 0.0 .OR. AIMAG(F2) .EQ. 0.0) GO TO 15
      GO TO 30
   15 CONTINUE
      IF (AK1(J) .EQ. 0.0)  GO TO 30

C     K1 FROM GIVEN VALUES OF K2 OR K3 IS DETERMINED

C     DETERMINATION OF FIRST DERIVATIVE

      AA=E1*ESP3*AK1(J)*AK1(J)*AK3+E1*ESP6*AK1(J)*AK1(J)*AK2+
     +E1*ESP3*AAK2*AK3+E1*ESP6*AAK2*AK2+EE1*AK1(J)*AK1(J)*AK3+
     +E1*ESP2*AAK2*AK3+2.0*E1*ESP3*(AK3**3)+3.0*E1*ESP6*AK2*AAK3-
     +ZETA*AAKO*AK3-EE1*ESP3*AAKO*AK3-EE1*ESP6*AK2*AAKO+
     +C3*E1*AK3*AAKO+C4*E1*AK2*AAKO*0.5

      BB=2.0*EE1*AK1(J)*AK1(J)*AK1(J)+E1*ESP2*AK1(J)*AAK2+E1*ESP3*
     +AK1(J)*AAK3+2.0*E1*ESP6*AK1(J)*AK2*AK3-ZETA*AAKO*AK1(J)+EE1*
     +AAK2*AK1(J)+EE1*AAK3*AK1(J)-(E1**3)*AK1(J)*AAKO+C1*E1*AAKO*AK1(J)

      CC=6.0*EE1*AK1(J)*AK1(J)+E1*ESP2*AAK2+E1*ESP3*AAK3+2.0*E1*ESP6*
     +AK2*AK3-ZETA*AAKO+EE1*AAK2+EE1*AAK3-EE1*E1*AAKO+C1*E1*AAKO

      DD=4.0*E1*ESP3*AK1(J)*AK3+4.0*E1*ESP6*AK1(J)*AK2+4.0*EE1*
     +AK1(J)*AK3
      EE=E1*ESP3*AK1(J)*AK1(J)+E1*ESP3*AAK2+E1*AK1(J)*AK1(J)+
     +E1*ESP2*AAK2+6.0*E1*ESP3*AAK3+6.0*E1*ESP6*AK2*AK3-ZETA*AAKO-
     +EE1*ESP3*AAKO+C3*E1*AAKO

      FF=E1*ESP2*AK1(J)*AK1(J)*AK2+E1*ESP6*AK1(J)*AK1(J)*AK3+EE1*AK1(J)*
     +AK1(J)*AK2+2.0*E1*ESP2*(AK2**3)+E1*ESP3*AK2*AAK3+3.0*E1*ESP6*AAK2*
     +AK3-ZETA*AAKO*AK2+E1*ESP2*AAK3*AK2+E1*ESP6*(AK3**3)-EE1*
     +ESP2*AAKO*AK2-EE1*ESP6*AK3*AAKO+C2*E1*AAKO*AK2+C4*E1*AK3*AAKO
      GG=4.0*E1*ESP2*AK1(J)*AK2+4.0*E1*ESP6*AK1(J)*AK3+4.0*EE1*
     +AK1(J)*AK2
      HH=E1*ESP2*AK1(J)*AK1(J)+EE1*AK1(J)*AK1(J)+E1*ESP3*AAK3+6.0*E1*
```

```
     +ESP6*AK2*AK3-ZETA*AAKO+E1*ESP2*AAK3-EE1*ESP2*AAKO+C2*E1*AAKO+
     +6.0*E1*ESP2*AAK2
      PP=E1*ESP6*AK1(J)*AK1(J)+2.0*E1*ESP3*AK2*AK3+3.0*E1*ESP6*AAK2+
     +2.0*E1*ESP2*AK2*AK3+3.0*E1*ESP6*AAK3-EE1*ESP6*AAKO+C4*E1*AAKO

      DT13=-AA/BB
      DT23=-(EE+DD*DT13+CC*DT13*DT13)/BB
      DT12=-FF/BB
      DT22=-(HH+GG*DT12+CC*DT12*DT12)/BB
      DTM=-(PP+CC*DT12*DT13+0.5*GG*DT13+0.5*DD*DT12)/BB

C     DETERMINATION OF ANGLE
      IF (I .EQ. 1) GO TO 31
      IF (I .EQ. 2) GO TO 32
 31   SHI=ATAN(-DT12)
      ANGLE=SHI*180.0/PAI
      GO TO 33
 32   THETA=ATAN(-1.0/DT13)
      ANGLE=THETA*180.0/PAI
 33   CONTINUE
      IF(J .EQ. 1) GO TO 11
      IF(J .EQ. 3) GO TO 12
 11   DD1=(AK1(1)-AK1(2))*(AK1(1)-AK1(3))*(AK1(1)-AK1(4))
      GO TO 13
 12   DD1=(AK1(3)-AK1(1))*(AK1(3)-AK1(2))*(AK1(3)-AK1(4))
 13   AAA=DT23*DT22-DTM*DTM
      AAA1=ABS(AAA)
      AAA2=SQRT(AAA1)
      AC1=2.0*PAI/AAA2
      AC2=4.0*PAI*PAI*AAKO*E1*DD1
      AC=AC1/AC2

C     DETERMINATION OF MATRIX ELEMENTS
      C11=-E1*AAKO+AAK2+AAK3
      C12=-AK1(J)*AK2+U*ESP4*AAKO
      C13=-AK1(J)*AK3-U*ESP5*AAKO
      C21=-U*ESP4*AAKO-AK1(J)*AK2
      C22=AK1(J)*AK1(J)+AAK3-ESP2*AAKO
      C23=-ESP6*AAKO-AK2*AK3
      C31=-AK1(J)*AK3+U*ESP5*AAKO
      C32=-ESP6*AAKO-AK2*AK3
      C33=AK1(J)*AK1(J)-ESP3*AAKO+AK2*AK2
      A11=C22*C33-C23*C32
      A12=C13*C32-C12*C33
      A13=C12*C23-C13*C22
      A21=C23*C31-C21*C33
      A22=C11*C33-C13*C31
```

```
      A23=C13*C21-C11*C23
      A31=C21*C32-C22*C31
      A32=C12*C31-C11*C32
      A33=C11*C22-C12*C21

C     MATRIX ELEMENTS ARE DETERMINED
C     DETERMINATION OF FOURIER TRANSFORM OF APERTURE FIELD
      AMIU=4.0*PAI*1.0E-07
      A=0.5
      B=0.02
      AM=AW*AMIU
      P1=AK11*A*0.5
      P2=AK12*A*0.5
      P3=AK1(J)*A*0.5
      S1=SIN(P1)
      S2=SIN(P2)
      CC1=COS(P1)
      CC2=COS(P2)
      CC3=COS(P3)
      BP=SA*CA*(S1-S2)/(CA*CA*S2+SA*SA*S1)
C     DETERMINATION OF FOURIER TRANSFORM OF SINE & COSINE ANGLE
      CK1=2.0*AK11*B/(AK1(J)*AK1(J)-AAK11)
      CK2=2.0*AK12*B/(AK1(J)*AK1(J)-AAK12)
      CK3=U*2.0*AK1(J)*B/(AK1(J)*AK1(J)-AAK11)
      CK4=U*2.0*AK1(J)*B/(AK1(J)*AK1(J)-AAK12)
      FRS1=CK1*(CC1-CC3)
      FRS2=CK2*(CC2-CC3)
      FRC1=CK3*(CC3-CC1)
      FRC2=CK4*(CC3-CC2)
C     DETERMINATION OF FOURIER TRANSFORM OF APERTURE FIELD
      EYAP=CA*FRS1-BP*SA*FRS1
      EZAP=BP*CA*FRS2+SA*FRS2
      HYAP=-U*(BP*CA+SA)*AK12*FRC2/AM
      HZAP=U*AK11*(CA-BP*SA)*FRC1/AM
      EXAO=U*E4*EYAP/E1
      EYAO=EYAP*CA
      EYAX=EZAP*SA
      EZAX=EZAP*CA
      EZAO=-EYAP*SA
      HYAX=HYAP*CA
      HYAO=HZAP*SA
      HZAX=-HYAP*SA
      HZAO=HZAP*CA
      EXA=EXAO
      EYA=EYAO+EYAX
      EZA=EZAO+EZAX
      HYA=HYAO+HYAX
```

```
      HZA=HZAO+HZAX
      X1=(A11*AK3-A13*AK1(J))*EXA+(A12*AK3-A13*AK2)*EYA+A11*AM*HYA
      Y1=(A21*AK3-A23*AK1(J))*EXA+(A22*AK3-A23*AK2)*EYA+A21*AM*HYA
      Z1=(A31*AK3-A33*AK1(J))*EXA+(A32*AK3-A33*AK2)*EYA+A31*AM*HYA

      EXO=AC*X1
      EYO=AC*Y1
      EZO=AC*Z1

      HXO=(AK2*EZO-AK3*EYO)/AM
      HYO=(AK3*EXO-AK1(J)*EZO)/AM
      HZO=(AK1(J)*EYO-AK2*EXO)/AM

      IF (I .EQ. 1) GO TO 41
      IF (I .EQ. 2) GO TO 42
41    THETA=PAI/2.0
      GO TO 43
42    SHI=0.0
43    ETO=EXO*COS(THETA)*COS(SHI)+EYO*COS(THETA)*SIN(SHI)-EZO*SIN(THETA)
      ESO=-EXO*SIN(SHI)+EYO*COS(SHI)

      HTO=HXO*COS(THETA)*COS(SHI)+HYO*COS(THETA)*SIN(SHI)-HZO*SIN(THETA)
      HSO=-HXO*SIN(SHI)+HYO*COS(SHI)
      CHTO=CONJG(HTO)
      CHSO=CONJG(HSO)
      AX=SIN(THETA)*COS(SHI)
      AY=AX*AX
      POO=0.5*REAL(ETO*CHSO-ESO*CHTO)
      PO=POO/AY
      SIKO=SIN(AKO*A/4)
      SSI=4.0*SIKO**4
      RH=SSI*B*B/(8.0*120*PAI*PAI*PAI)
      RPP=PO/RH
      RCA=ABS(RPP)
      IF (RCA .EQ. 0.0) GO TO 30
      RP=10.0*ALOG10(RCA)
      IF(RP .LT. 0.0) GO TO 30
      WRITE(9,28) PO,ANGLE,AK1(1),AK1(3),AK11,AK12,RH,RP
28    FORMAT(2X,E12.5,1X,F8.4,1X,6(1X,E12.5))
30    CONTINUE
      WRITE(9,29)
29    FORMAT(4X,'ONE MODE IS COMPLETE')
40    CONTINUE
50    CONTINUE
60    CONTINUE
      STOP
      END
```

ALPHA IN DEGREE= 0.0

X-Y PLANE PATTERN
FOR X-MODE
ONE MODE IS COMPLETE

FOR O-MODE

POWER	ANGLE						NOR.db
0.31006E-04	-85.6975	0.00000E+00	0.39873E+00	0.41084E+01	0.53150E+01	0.13438E-07	0.33631E+02
0.15506E-05	-81.0320	0.00000E+00	0.82850E+00	0.41084E+01	0.53150E+01	0.13438E-07	0.20622E+02
0.46731E-06	-78.0607	0.00000E+00	0.10995E+01	0.41084E+01	0.53150E+01	0.13438E-07	0.15413E+02
0.21475E-06	-75.6871	0.00000E+00	0.13140E+01	0.41084E+01	0.53150E+01	0.13438E-07	0.12036E+02
0.11994E-06	-73.6484	0.00000E+00	0.14963E+01	0.41084E+01	0.53150E+01	0.13438E-07	0.95063E+01
0.75032E-07	-71.8318	0.00000E+00	0.16572E+01	0.41084E+01	0.53150E+01	0.13438E-07	0.74691E+01
0.50566E-07	-70.1760	0.00000E+00	0.18025E+01	0.41084E+01	0.53150E+01	0.13438E-07	0.57552E+01
0.35925E-07	-68.6432	0.00000E+00	0.19356E+01	0.41084E+01	0.53150E+01	0.13438E-07	0.42705E+01
0.26553E-07	-67.2088	0.00000E+00	0.20589E+01	0.41084E+01	0.53150E+01	0.13438E-07	0.29577E+01
0.20240E-07	-65.8554	0.00000E+00	0.21740E+01	0.41084E+01	0.53150E+01	0.13438E-07	0.17786E+01
0.15813E-07	-64.5697	0.00000E+00	0.22823E+01	0.41084E+01	0.53150E+01	0.13438E-07	0.70664E+00
0.15813E-07	64.5697	0.00000E+00	0.22823E+01	0.41084E+01	0.53150E+01	0.13438E-07	0.70664E+00
0.20240E-07	65.8554	0.00000E+00	0.21740E+01	0.41084E+01	0.53150E+01	0.13438E-07	0.17786E+01
0.26553E-07	67.2088	0.00000E+00	0.20589E+01	0.41084E+01	0.53150E+01	0.13438E-07	0.29577E+01
0.35925E-07	68.6432	0.00000E+00	0.19356E+01	0.41084E+01	0.53150E+01	0.13438E-07	0.42705E+01
0.50566E-07	70.1760	0.00000E+00	0.18025E+01	0.41084E+01	0.53150E+01	0.13438E-07	0.57552E+01
0.75032E-07	71.8318	0.00000E+00	0.16572E+01	0.41084E+01	0.53150E+01	0.13438E-07	0.74691E+01
0.11994E-06	73.6484	0.00000E+00	0.14963E+01	0.41084E+01	0.53150E+01	0.13438E-07	0.95063E+01
0.21475E-06	75.6871	0.00000E+00	0.13140E+01	0.41084E+01	0.53150E+01	0.13438E-07	0.12036E+02
0.46731E-06	78.0607	0.00000E+00	0.10995E+01	0.41084E+01	0.53150E+01	0.13438E-07	0.15413E+02
0.15506E-05	81.0320	0.00000E+00	0.82850E+00	0.41084E+01	0.53150E+01	0.13438E-07	0.20622E+02
0.31006E-04	85.6975	0.00000E+00	0.39873E+00	0.41084E+01	0.53150E+01	0.13438E-07	0.33631E+02

ONE MODE IS COMPLETE

X-Z PLANE PATTERN
FOR X-MODE
ONE MODE IS COMPLETE

FOR O-MODE

POWER	ANGLE						NOR.db
-0.20200E-05	-7.6665	0.00000E+00	0.70708E+00	0.41084E+01	0.53150E+01	0.13438E-07	0.21770E+02
-0.44642E-06	-11.0035	0.00000E+00	0.10113E+01	0.41084E+01	0.53150E+01	0.13438E-07	0.15214E+02
-0.18163E-06	-13.5516	0.00000E+00	0.12410E+01	0.41084E+01	0.53150E+01	0.13438E-07	0.11308E+02
-0.94185E-07	-15.6999	0.00000E+00	0.14326E+01	0.41084E+01	0.53150E+01	0.13438E-07	0.84564E+01
-0.55654E-07	-17.5953	0.00000E+00	0.15998E+01	0.41084E+01	0.53150E+01	0.13438E-07	0.61716E+01
-0.35669E-07	-19.3127	0.00000E+00	0.17497E+01	0.41084E+01	0.53150E+01	0.13438E-07	0.42395E+01
-0.24155E-07	-20.8959	0.00000E+00	0.18863E+01	0.41084E+01	0.53150E+01	0.13438E-07	0.25467E+01
-0.17017E-07	-22.3733	0.00000E+00	0.20123E+01	0.41084E+01	0.53150E+01	0.13438E-07	0.10255E+01
0.17017E-07	22.3733	0.00000E+00	0.20123E+01	0.41084E+01	0.53150E+01	0.13438E-07	0.10255E+01
0.24155E-07	20.8959	0.00000E+00	0.18863E+01	0.41084E+01	0.53150E+01	0.13438E-07	0.25467E+01

0.35669E-07	19.3127	0.00000E+00	0.17497E+01	0.41084E+01	0.53150E+01	0.13438E-07	0.42395E+01
0.55654E-07	17.5953	0.00000E+00	0.15998E+01	0.41084E+01	0.53150E+01	0.13438E-07	0.61716E+01
0.94185E-07	15.6999	0.00000E+00	0.14326E+01	0.41084E+01	0.53150E+01	0.13438E-07	0.84564E+01
0.18163E-06	13.5516	0.00000E+00	0.12410E+01	0.41084E+01	0.53150E+01	0.13438E-07	0.11308E+02
0.44642E-06	11.0035	0.00000E+00	0.10113E+01	0.41084E+01	0.53150E+01	0.13438E-07	0.15214E+02
0.20200E-05	7.6665	0.00000E+00	0.70708E+00	0.41084E+01	0.53150E+01	0.13438E-07	0.21770E+02

ONE MODE IS COMPLETE

ALPHA IN DEGREE=30.0

X-Y PLANE PATTERN
FOR X-MODE
ONE MODE IS COMPLETE

FOR O-MODE

POWER	ANGLE						NOR.db
0.56951E-04	-85.9462	0.00000E+00	0.37017E+00	0.41084E+01	0.53150E+01	0.13438E-07	0.36272E+02
0.23439E-05	-81.0966	0.00000E+00	0.81050E+00	0.41084E+01	0.53150E+01	0.13438E-07	0.22416E+02
0.69717E-06	-78.0715	0.00000E+00	0.10825E+01	0.41084E+01	0.53150E+01	0.13438E-07	0.17150E+02
0.32050E-06	-75.6648	0.00000E+00	0.12968E+01	0.41084E+01	0.53150E+01	0.13438E-07	0.13775E+02
0.17961E-06	-73.6024	0.00000E+00	0.14787E+01	0.41084E+01	0.53150E+01	0.13438E-07	0.11260E+02
0.11277E-06	-71.7665	0.00000E+00	0.16391E+01	0.41084E+01	0.53150E+01	0.13438E-07	0.92387E+01
0.76255E-07	-70.0946	0.00000E+00	0.17838E+01	0.41084E+01	0.53150E+01	0.13438E-07	0.75393E+01
0.54323E-07	-68.5482	0.00000E+00	0.19162E+01	0.41084E+01	0.53150E+01	0.13438E-07	0.60664E+01
0.40228E-07	-67.1019	0.00000E+00	0.20389E+01	0.41084E+01	0.53150E+01	0.13438E-07	0.47619E+01
0.30698E-07	-65.7379	0.00000E+00	0.21535E+01	0.41084E+01	0.53150E+01	0.13438E-07	0.35878E+01
0.23993E-07	-64.4430	0.00000E+00	0.22611E+01	0.41084E+01	0.53150E+01	0.13438E-07	0.25174E+01
0.19121E-07	-63.2070	0.00000E+00	0.23629E+01	0.41084E+01	0.53150E+01	0.13438E-07	0.15318E+01
0.15489E-07	-62.0221	0.00000E+00	0.24594E+01	0.41084E+01	0.53150E+01	0.13438E-07	0.61676E+00
0.15489E-07	62.0221	0.00000E+00	0.24594E+01	0.41084E+01	0.53150E+01	0.13438E-07	0.61675E+00
0.19121E-07	63.2070	0.00000E+00	0.23629E+01	0.41084E+01	0.53150E+01	0.13438E-07	0.15318E+01
0.23993E-07	64.4430	0.00000E+00	0.22611E+01	0.41084E+01	0.53150E+01	0.13438E-07	0.25174E+01
0.30698E-07	65.7379	0.00000E+00	0.21535E+01	0.41084E+01	0.53150E+01	0.13438E-07	0.35878E+01
0.40228E-07	67.1019	0.00000E+00	0.20389E+01	0.41084E+01	0.53150E+01	0.13438E-07	0.47619E+01
0.54323E-07	68.5482	0.00000E+00	0.19162E+01	0.41084E+01	0.53150E+01	0.13438E-07	0.60664E+01
0.76255E-07	70.0946	0.00000E+00	0.17838E+01	0.41084E+01	0.53150E+01	0.13438E-07	0.75393E+01
0.11277E-06	71.7665	0.00000E+00	0.16391E+01	0.41084E+01	0.53150E+01	0.13438E-07	0.92387E+01
0.17961E-06	73.6024	0.00000E+00	0.14787E+01	0.41084E+01	0.53150E+01	0.13438E-07	0.11260E+02
0.32050E-06	75.6648	0.00000E+00	0.12968E+01	0.41084E+01	0.53150E+01	0.13438E-07	0.13775E+02
0.69717E-06	78.0715	0.00000E+00	0.10825E+01	0.41084E+01	0.53150E+01	0.13438E-07	0.17150E+02
0.23439E-05	81.0966	0.00000E+00	0.81050E+00	0.41084E+01	0.53150E+01	0.13438E-07	0.22416E+02
0.56951E-04	85.9462	0.00000E+00	0.37017E+00	0.41084E+01	0.53150E+01	0.13438E-07	0.36272E+02

ONE MODE IS COMPLETE

X-Z PLANE PATTERN
FOR X-MODE
ONE MODE IS COMPLETE

FOR O-MODE

POWER	ANGLE						NOR.db
-0.38352E-05	-6.9083	0.00000E+00	0.63356E+00	0.41084E+01	0.53150E+01	0.13438E-07	0.24554E+02
-0.68663E-06	-10.5046	0.00000E+00	0.96008E+00	0.41084E+01	0.53150E+01	0.13438E-07	0.17084E+02
-0.26647E-06	-13.1611	0.00000E+00	0.11987E+01	0.41084E+01	0.53150E+01	0.13438E-07	0.12973E+02
-0.13675E-06	-15.3731	0.00000E+00	0.13954E+01	0.41084E+01	0.53150E+01	0.13438E-07	0.10076E+02
-0.81160E-07	-17.3116	0.00000E+00	0.15659E+01	0.41084E+01	0.53150E+01	0.13438E-07	0.78100E+01
-0.52659E-07	-19.0605	0.00000E+00	0.17181E+01	0.41084E+01	0.53150E+01	0.13438E-07	0.59314E+01
-0.36286E-07	-20.6680	0.00000E+00	0.18565E+01	0.41084E+01	0.53150E+01	0.13438E-07	0.43141E+01
-0.26111E-07	-22.1645	0.00000E+00	0.19840E+01	0.41084E+01	0.53150E+01	0.13438E-07	0.28848E+01
-0.19409E-07	-23.5717	0.00000E+00	0.21025E+01	0.41084E+01	0.53150E+01	0.13438E-07	0.15967E+01
-0.14798E-07	-24.9043	0.00000E+00	0.22136E+01	0.41084E+01	0.53150E+01	0.13438E-07	0.41856E+00
0.14798E-07	24.9043	0.00000E+00	0.22136E+01	0.41084E+01	0.53150E+01	0.13438E-07	0.41856E+00
0.19409E-07	23.5717	0.00000E+00	0.21025E+01	0.41084E+01	0.53150E+01	0.13438E-07	0.15967E+01
0.26111E-07	22.1645	0.00000E+00	0.19840E+01	0.41084E+01	0.53150E+01	0.13438E-07	0.28848E+01
0.36286E-07	20.6680	0.00000E+00	0.18565E+01	0.41084E+01	0.53150E+01	0.13438E-07	0.43141E+01
0.52659E-07	19.0605	0.00000E+00	0.17181E+01	0.41084E+01	0.53150E+01	0.13438E-07	0.59314E+01
0.81160E-07	17.3116	0.00000E+00	0.15659E+01	0.41084E+01	0.53150E+01	0.13438E-07	0.78100E+01
0.13675E-06	15.3731	0.00000E+00	0.13954E+01	0.41084E+01	0.53150E+01	0.13438E-07	0.10076E+02
0.26647E-06	13.1611	0.00000E+00	0.11987E+01	0.41084E+01	0.53150E+01	0.13438E-07	0.12973E+02
0.68663E-06	10.5046	0.00000E+00	0.96008E+00	0.41084E+01	0.53150E+01	0.13438E-07	0.17084E+02
0.38352E-05	6.9083	0.00000E+00	0.63356E+00	0.41084E+01	0.53150E+01	0.13438E-07	0.24554E+02

ONE MODE IS COMPLETE

ALPHA IN DEGREE=60.0

X-Y PLANE PATTERN

FOR X-MODE

0.46611E-07	-20.7651	0.39131E+01	0.52319E+01	0.41084E+01	0.53150E+01	0.13438E-07	0.54015E+01
0.46611E-07	20.7651	0.39131E+01	0.52319E+01	0.41084E+01	0.53150E+01	0.13438E-07	0.54015E+01

ONE MODE IS COMPLETE

FOR O-MODE

POWER	ANGLE						NOR.db
0.13116E-04	-83.0918	0.00000E+00	0.63355E+00	0.41084E+01	0.53150E+01	0.13438E-07	0.29895E+02
0.23420E-05	-79.4954	0.00000E+00	0.96007E+00	0.41084E+01	0.53150E+01	0.13438E-07	0.22413E+02
0.90743E-06	-76.8390	0.00000E+00	0.11987E+01	0.41084E+01	0.53150E+01	0.13438E-07	0.18295E+02
0.46540E-06	-74.6270	0.00000E+00	0.13954E+01	0.41084E+01	0.53150E+01	0.13438E-07	0.15395E+02
0.27633E-06	-72.6884	0.00000E+00	0.15659E+01	0.41084E+01	0.53150E+01	0.13438E-07	0.13131E+02
0.17954E-06	-70.9395	0.00000E+00	0.17181E+01	0.41084E+01	0.53150E+01	0.13438E-07	0.11258E+02
0.12400E-06	-69.3320	0.00000E+00	0.18565E+01	0.41084E+01	0.53150E+01	0.13438E-07	0.96509E+01
0.89520E-07	-67.8355	0.00000E+00	0.19839E+01	0.41084E+01	0.53150E+01	0.13438E-07	0.82358E+01
0.66824E-07	-66.4283	0.00000E+00	0.21025E+01	0.41084E+01	0.53150E+01	0.13438E-07	0.69659E+01
0.51208E-07	-65.0956	0.00000E+00	0.22136E+01	0.41084E+01	0.53150E+01	0.13438E-07	0.58100E+01
0.40077E-07	-63.8259	0.00000E+00	0.23182E+01	0.41084E+01	0.53150E+01	0.13438E-07	0.47456E+01
0.31915E-07	-62.6105	0.00000E+00	0.24173E+01	0.41084E+01	0.53150E+01	0.13438E-07	0.37566E+01
0.25786E-07	-61.4425	0.00000E+00	0.25114E+01	0.41084E+01	0.53150E+01	0.13438E-07	0.28305E+01

0.21092E-07	-60.3161	0.00000E+00	0.26012E+01	0.41084E+01	0.53150E+01	0.13438E-07	0.19578E+01
0.17436E-07	-59.2270	0.00000E+00	0.26870E+01	0.41084E+01	0.53150E+01	0.13438E-07	0.11311E+01
0.14546E-07	-58.1712	0.00000E+00	0.27692E+01	0.41084E+01	0.53150E+01	0.13438E-07	0.34418E+00
0.14546E-07	58.1712	0.00000E+00	0.27692E+01	0.41084E+01	0.53150E+01	0.13438E-07	0.34418E+00
0.17436E-07	59.2270	0.00000E+00	0.26870E+01	0.41084E+01	0.53150E+01	0.13438E-07	0.11311E+01
0.21092E-07	60.3161	0.00000E+00	0.26012E+01	0.41084E+01	0.53150E+01	0.13438E-07	0.19578E+01
0.25786E-07	61.4425	0.00000E+00	0.25114E+01	0.41084E+01	0.53150E+01	0.13438E-07	0.28305E+01
0.31915E-07	62.6105	0.00000E+00	0.24173E+01	0.41084E+01	0.53150E+01	0.13438E-07	0.37566E+01
0.40077E-07	63.8259	0.00000E+00	0.23182E+01	0.41084E+01	0.53150E+01	0.13438E-07	0.47456E+01
0.51208E-07	65.0956	0.00000E+00	0.22136E+01	0.41084E+01	0.53150E+01	0.13438E-07	0.58100E+01
0.66824E-07	66.4283	0.00000E+00	0.21025E+01	0.41084E+01	0.53150E+01	0.13438E-07	0.69659E+01
0.89520E-07	67.8355	0.00000E+00	0.19839E+01	0.41084E+01	0.53150E+01	0.13438E-07	0.82358E+01
0.12400E-06	69.3320	0.00000E+00	0.18565E+01	0.41084E+01	0.53150E+01	0.13438E-07	0.96509E+01
0.17954E-06	70.9395	0.00000E+00	0.17181E+01	0.41084E+01	0.53150E+01	0.13438E-07	0.11258E+02
0.27633E-06	72.6884	0.00000E+00	0.15659E+01	0.41084E+01	0.53150E+01	0.13438E-07	0.13131E+02
0.46540E-06	74.6270	0.00000E+00	0.13954E+01	0.41084E+01	0.53150E+01	0.13438E-07	0.15395E+02
0.90743E-06	76.8390	0.00000E+00	0.11987E+01	0.41084E+01	0.53150E+01	0.13438E-07	0.18295E+02
0.23420E-05	79.4954	0.00000E+00	0.96007E+00	0.41084E+01	0.53150E+01	0.13438E-07	0.22413E+02
0.13116E-04	83.0918	0.00000E+00	0.63355E+00	0.41084E+01	0.53150E+01	0.13438E-07	0.29895E+02

ONE MODE IS COMPLETE

X-Z PLANE PATTERN

FOR X-MODE

-0.37203E-07	-73.1334	0.39522E+01	0.52176E+01	0.41084E+01	0.53150E+01	0.13438E-07	0.44224E+01
0.37203E-07	73.1334	0.39522E+01	0.52176E+01	0.41084E+01	0.53150E+01	0.13438E-07	0.44224E+01

ONE MODE IS COMPLETE

FOR O-MODE

POWER	ANGLE						NOR.db
-0.37522E-04	-4.0541	0.00000E+00	0.37021E+00	0.41084E+01	0.53150E+01	0.13438E-07	0.34460E+02
-0.15785E-05	-8.9035	0.00000E+00	0.81052E+00	0.41084E+01	0.53150E+01	0.13438E-07	0.20699E+02
-0.47906E-06	-11.9286	0.00000E+00	0.10825E+01	0.41084E+01	0.53150E+01	0.13438E-07	0.15520E+02
-0.22438E-06	-14.3352	0.00000E+00	0.12968E+01	0.41084E+01	0.53150E+01	0.13438E-07	0.12226E+02
-0.12793E-06	-16.3976	0.00000E+00	0.14787E+01	0.41084E+01	0.53150E+01	0.13438E-07	0.97863E+01
-0.81608E-07	-18.2336	0.00000E+00	0.16391E+01	0.41084E+01	0.53150E+01	0.13438E-07	0.78339E+01
-0.55989E-07	-19.9054	0.00000E+00	0.17838E+01	0.41084E+01	0.53150E+01	0.13438E-07	0.61977E+01
-0.40417E-07	-21.4518	0.00000E+00	0.19162E+01	0.41084E+01	0.53150E+01	0.13438E-07	0.47823E+01
-0.30292E-07	-22.8981	0.00000E+00	0.20389E+01	0.41084E+01	0.53150E+01	0.13438E-07	0.35299E+01
-0.23367E-07	-24.2620	0.00000E+00	0.21535E+01	0.41084E+01	0.53150E+01	0.13438E-07	0.24026E+01
-0.18439E-07	-25.5570	0.00000E+00	0.22611E+01	0.41084E+01	0.53150E+01	0.13438E-07	0.13741E+01
-0.14822E-07	-26.7930	0.00000E+00	0.23629E+01	0.41084E+01	0.53150E+01	0.13438E-07	0.42558E+00
0.14822E-07	26.7930	0.00000E+00	0.23629E+01	0.41084E+01	0.53150E+01	0.13438E-07	0.42558E+00
0.18439E-07	25.5570	0.00000E+00	0.22611E+01	0.41084E+01	0.53150E+01	0.13438E-07	0.13741E+01
0.23367E-07	24.2620	0.00000E+00	0.21535E+01	0.41084E+01	0.53150E+01	0.13438E-07	0.24026E+01
0.30292E-07	22.8981	0.00000E+00	0.20389E+01	0.41084E+01	0.53150E+01	0.13438E-07	0.35299E+01
0.40417E-07	21.4518	0.00000E+00	0.19162E+01	0.41084E+01	0.53150E+01	0.13438E-07	0.47823E+01
0.55989E-07	19.9054	0.00000E+00	0.17838E+01	0.41084E+01	0.53150E+01	0.13438E-07	0.61977E+01

0.81608E-07	18.2336	0.00000E+00	0.16391E+01	0.41084E+01	0.53150E+01	0.13438E-07	0.78339E+01
0.12793E-06	16.3976	0.00000E+00	0.14787E+01	0.41084E+01	0.53150E+01	0.13438E-07	0.97863E+01
0.22438E-06	14.3352	0.00000E+00	0.12968E+01	0.41084E+01	0.53150E+01	0.13438E-07	0.12226E+02
0.47906E-06	11.9286	0.00000E+00	0.10825E+01	0.41084E+01	0.53150E+01	0.13438E-07	0.15520E+02
0.15785E-05	8.9035	0.00000E+00	0.81052E+00	0.41084E+01	0.53150E+01	0.13438E-07	0.20699E+02
0.37522E-04	4.0541	0.00000E+00	0.37021E+00	0.41084E+01	0.53150E+01	0.13438E-07	0.34460E+02

ONE MODE IS COMPLETE

ALPHA IN DEGREE=90.0

X-Y PLANE PATTERN

FOR X-MODE

0.22358E-07	-21.5619	0.39141E+01	0.52460E+01	0.41084E+01	0.53150E+01	0.13438E-07	0.22110E+01
0.22358E-07	21.5619	0.39141E+01	0.52460E+01	0.41084E+01	0.53150E+01	0.13438E-07	0.22110E+01

ONE MODE IS COMPLETE

FOR O-MODE

POWER	ANGLE						NOR.db
0.11220E-04	-82.3335	0.00000E+00	0.70707E+00	0.41084E+01	0.53150E+01	0.13438E-07	0.29217E+02
0.25182E-05	-78.9965	0.00000E+00	0.10113E+01	0.41084E+01	0.53150E+01	0.13438E-07	0.22728E+02
0.10424E-05	-76.4484	0.00000E+00	0.12410E+01	0.41084E+01	0.53150E+01	0.13438E-07	0.18897E+02
0.55105E-06	-74.3001	0.00000E+00	0.14326E+01	0.41084E+01	0.53150E+01	0.13438E-07	0.16129E+02
0.33258E-06	-72.4046	0.00000E+00	0.15998E+01	0.41084E+01	0.53150E+01	0.13438E-07	0.13936E+02
0.21816E-06	-70.6873	0.00000E+00	0.17497E+01	0.41084E+01	0.53150E+01	0.13438E-07	0.12104E+02
0.15153E-06	-69.1041	0.00000E+00	0.18863E+01	0.41084E+01	0.53150E+01	0.13438E-07	0.10522E+02
0.10974E-06	-67.6267	0.00000E+00	0.20123E+01	0.41084E+01	0.53150E+01	0.13438E-07	0.91203E+01
0.82040E-07	-66.2352	0.00000E+00	0.21297E+01	0.41084E+01	0.53150E+01	0.13438E-07	0.78569E+01
0.62882E-07	-64.9153	0.00000E+00	0.22398E+01	0.41084E+01	0.53150E+01	0.13438E-07	0.67019E+01
0.49180E-07	-63.6561	0.00000E+00	0.23437E+01	0.41084E+01	0.53150E+01	0.13438E-07	0.56345E+01
0.39106E-07	-62.4495	0.00000E+00	0.24420E+01	0.41084E+01	0.53150E+01	0.13438E-07	0.46391E+01
0.31531E-07	-61.2888	0.00000E+00	0.25355E+01	0.41084E+01	0.53150E+01	0.13438E-07	0.37041E+01
0.25725E-07	-60.1686	0.00000E+00	0.26247E+01	0.41084E+01	0.53150E+01	0.13438E-07	0.28202E+01
0.21201E-07	-59.0844	0.00000E+00	0.27101E+01	0.41084E+01	0.53150E+01	0.13438E-07	0.19802E+01
0.17627E-07	-58.0327	0.00000E+00	0.27918E+01	0.41084E+01	0.53150E+01	0.13438E-07	0.11783E+01
0.14768E-07	-57.0103	0.00000E+00	0.28704E+01	0.41084E+01	0.53150E+01	0.13438E-07	0.40971E+00
0.14768E-07	57.0103	0.00000E+00	0.28704E+01	0.41084E+01	0.53150E+01	0.13438E-07	0.40971E+00
0.17627E-07	58.0327	0.00000E+00	0.27918E+01	0.41084E+01	0.53150E+01	0.13438E-07	0.11783E+01
0.21201E-07	59.0844	0.00000E+00	0.27101E+01	0.41084E+01	0.53150E+01	0.13438E-07	0.19802E+01
0.25725E-07	60.1686	0.00000E+00	0.26247E+01	0.41084E+01	0.53150E+01	0.13438E-07	0.28202E+01
0.31531E-07	61.2888	0.00000E+00	0.25355E+01	0.41084E+01	0.53150E+01	0.13438E-07	0.37041E+01
0.39106E-07	62.4495	0.00000E+00	0.24420E+01	0.41084E+01	0.53150E+01	0.13438E-07	0.46391E+01
0.49180E-07	63.6561	0.00000E+00	0.23437E+01	0.41084E+01	0.53150E+01	0.13438E-07	0.56345E+01
0.62882E-07	64.9153	0.00000E+00	0.22398E+01	0.41084E+01	0.53150E+01	0.13438E-07	0.67019E+01
0.82040E-07	66.2352	0.00000E+00	0.21297E+01	0.41084E+01	0.53150E+01	0.13438E-07	0.78569E+01
0.10974E-06	67.6267	0.00000E+00	0.20123E+01	0.41084E+01	0.53150E+01	0.13438E-07	0.91203E+01
0.15153E-06	69.1041	0.00000E+00	0.18863E+01	0.41084E+01	0.53150E+01	0.13438E-07	0.10522E+02
0.21816E-06	70.6873	0.00000E+00	0.17497E+01	0.41084E+01	0.53150E+01	0.13438E-07	0.12104E+02
0.33258E-06	72.4046	0.00000E+00	0.15998E+01	0.41084E+01	0.53150E+01	0.13438E-07	0.13936E+02

0.55105E-06	74.3001	0.00000E+00	0.14326E+01	0.41084E+01	0.53150E+01	0.13438E-07	0.16129E+02
0.10424E-05	76.4484	0.00000E+00	0.12410E+01	0.41084E+01	0.53150E+01	0.13438E-07	0.18897E+02
0.25182E-05	78.9965	0.00000E+00	0.10113E+01	0.41084E+01	0.53150E+01	0.13438E-07	0.22728E+02
0.11220E-04	82.3335	0.00000E+00	0.70707E+00	0.41084E+01	0.53150E+01	0.13438E-07	0.29217E+02

ONE MODE IS COMPLETE

X-Z PLANE PATTERN
FOR X-MODE
ONE MODE IS COMPLETE

FOR O-MODE

POWER	ANGLE						MOE.db
-0.46583E-04	-4.3031	0.00000E+00	0.39880E+00	0.41084E+01	0.53150E+01	0.13438E-07	0.35399E+02
-0.24360E-05	-8.9681	0.00000E+00	0.82852E+00	0.41084E+01	0.53150E+01	0.13438E-07	0.22583E+02
-0.76396E-06	-11.9394	0.00000E+00	0.10995E+01	0.41084E+01	0.53150E+01	0.13438E-07	0.17547E+02
-0.36390E-06	-14.3131	0.00000E+00	0.13140E+01	0.41084E+01	0.53150E+01	0.13438E-07	0.14326E+02
-0.20990E-06	-16.3516	0.00000E+00	0.14963E+01	0.41084E+01	0.53150E+01	0.13438E-07	0.11937E+02
-0.13515E-06	-18.1683	0.00000E+00	0.16573E+01	0.41084E+01	0.53150E+01	0.13438E-07	0.10025E+02
-0.93444E-07	-19.8240	0.00000E+00	0.18025E+01	0.41084E+01	0.53150E+01	0.13438E-07	0.84221E+01
-0.67908E-07	-21.3568	0.00000E+00	0.19356E+01	0.41084E+01	0.53150E+01	0.13438E-07	0.70358E+01
-0.51200E-07	-22.7913	0.00000E+00	0.20589E+01	0.41084E+01	0.53150E+01	0.13438E-07	0.58093E+01
-0.39709E-07	-24.1446	0.00000E+00	0.21741E+01	0.41084E+01	0.53150E+01	0.13438E-07	0.47055E+01
-0.31489E-07	-25.4304	0.00000E+00	0.22823E+01	0.41084E+01	0.53150E+01	0.13438E-07	0.36983E+01
-0.25426E-07	-26.6579	0.00000E+00	0.23846E+01	0.41084E+01	0.53150E+01	0.13438E-07	0.27693E+01
-0.20836E-07	-27.8353	0.00000E+00	0.24817E+01	0.41084E+01	0.53150E+01	0.13438E-07	0.19048E+01
-0.17289E-07	-28.9686	0.00000E+00	0.25742E+01	0.41084E+01	0.53150E+01	0.13438E-07	0.10944E+01
-0.14499E-07	-30.0626	0.00000E+00	0.26625E+01	0.41084E+01	0.53150E+01	0.13438E-07	0.32990E+00
0.14499E-07	30.0626	0.00000E+00	0.26625E+01	0.41084E+01	0.53150E+01	0.13438E-07	0.32990E+00
0.17289E-07	28.9686	0.00000E+00	0.25742E+01	0.41084E+01	0.53150E+01	0.13438E-07	0.10944E+01
0.20836E-07	27.8353	0.00000E+00	0.24817E+01	0.41084E+01	0.53150E+01	0.13438E-07	0.19048E+01
0.25426E-07	26.6579	0.00000E+00	0.23846E+01	0.41084E+01	0.53150E+01	0.13438E-07	0.27693E+01
0.31489E-07	25.4304	0.00000E+00	0.22823E+01	0.41084E+01	0.53150E+01	0.13438E-07	0.36983E+01
0.39709E-07	24.1446	0.00000E+00	0.21741E+01	0.41084E+01	0.53150E+01	0.13438E-07	0.47055E+01
0.51200E-07	22.7913	0.00000E+00	0.20589E+01	0.41084E+01	0.53150E+01	0.13438E-07	0.58093E+01
0.67908E-07	21.3568	0.00000E+00	0.19356E+01	0.41084E+01	0.53150E+01	0.13438E-07	0.70358E+01
0.93444E-07	19.8240	0.00000E+00	0.18025E+01	0.41084E+01	0.53150E+01	0.13438E-07	0.84221E+01
0.13515E-06	18.1683	0.00000E+00	0.16573E+01	0.41084E+01	0.53150E+01	0.13438E-07	0.10025E+02
0.20990E-06	16.3516	0.00000E+00	0.14963E+01	0.41084E+01	0.53150E+01	0.13438E-07	0.11937E+02
0.36390E-06	14.3131	0.00000E+00	0.13140E+01	0.41084E+01	0.53150E+01	0.13438E-07	0.14326E+02
0.76396E-06	11.9394	0.00000E+00	0.10995E+01	0.41084E+01	0.53150E+01	0.13438E-07	0.17547E+02
0.24360E-05	8.9681	0.00000E+00	0.82852E+00	0.41084E+01	0.53150E+01	0.13438E-07	0.22583E+02
0.46583E-04	4.3031	0.00000E+00	0.39880E+00	0.41084E+01	0.53150E+01	0.13438E-07	0.35399E+02

ONE MODE IS COMPLETE

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