DETERMINATION OF TRANSFER FUNCTIONS OF DIFFERENT ELECTRICAL CIRCUITS USING COMPLEX CURVE FITTING METHOD.

BY

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A THESIS

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DECLARATION

This is to certify that the work has not already been accepted in substance for any degree, nor is it being concurrently submitted in candidature for any degree.

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This is to certify that the work presented in this thesis is the result of the investigation carried out by the candidate under the supervision of Dr. M. Zaman at the Department of Electrical & Electronic Engineering, Bangladesh University of Engineering and Technology, Dhaka. Except where the reference is made to other authors, the work presented is original.

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NOTATION

Sections and subsections in this thesis are labelled with a decimal system, each new level of subsection being represented by a decimal point in the lebel, i.e. subsections to Chapter 5 are 5.1, 5.2, etc. No. of pages, figures and tables are related with the Chapters, i.e. first figure of Chapter 7 is leveled as Fig. 7.1, fifth page of Chapter 7 is lebelled as 7.5, first table in Chapter 2 is lebelled as Table 2.1 etc.

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ABSTRACTS

Different electrical circuits of different orders have been constructed using resistors and capacitors. Their frequency responses have been determined experimentally using Lissajou's figure. With these frequency responses their transfer functions have been determined using complex curve fitting method . These fitted transfer functions have been compared with their theoritical transfer functions. To calculate the theoritical transfer function the values of the resistors, capacitors and the internal resistances of the capacitors have been determined from experimental data.

An one-band radio-receiver has been taken. The frequency responses of the R.F. Section (including the I.F. Sections) of the receiver has been obtained experimentally using Lissajou's figure. Then the transfer function of this Section has been determined using complex curve fitting method . The frequency responses of the fitted transfer function gives satisfactory result. Complex curve fitting method developed by Levy and Zaman was not fully complete. Some changes have been

made to improve its facilities.

Previous method of determination of the exact order of a system did not always give the correct results. Emperical relation have been developed for obtaining the actual order of a system for use in complex curve fitting method. This has been done analysing the errors at different orders. This formulation has been tested for different systems.

A computer Programme has been developed for the generalised complex curve fitting method. The advantages of this program over the previous program are -

1) There is no necessity for previous estimation about the maximum possible order of the system.

II) Computer itself will determine the actual order and will print it out.

III) If the data is so noisy that good fit is not possible, the computer will give message about it.

In Levy's method transfer function of higher type system was not possible to determine practically without having any idea about the exact type. In this work a technique has been developed to overcome the above problem. The computer program which has been developed in this work is also applicable for higher type system. This method has been tested for systems of different type.

To get an idea about the critical frequency band, frequency responses of several systems have been studied. Also the effect of frequency interval and the effect of error in the sampled frequency response on the fitted transfer function have been studied.

LIST OF SYMBOLS

W	Anglar frequency in radian
F(jw)	Ideal frequency response
S	jw, where j= $\sqrt{-1}$
R	Real part of frequency response
Q	Quadrature part of frequency response
G(jw)	Transfer function of a linear dynamic system
A_i, B_i	where i = 0,1,2,3 .. ..are the constant coefficients of transfer function.
α	Real part of the numerator of G(jw)
$\omega\beta$	Imaginary part of the numerator of G(jw)
σ	Real part of the demominator of G(jw)
$\omega\tau$	Imaginary part of the denominator of the G(jw)
P(jw)	$\alpha + j\omega\beta$
I(jw)	$\sigma + j\omega\tau$
e(w)	F(jw) - G(jw)
e'(w)	e(w) I(jw)

$e'(W_k)$ $C(W_k) + jd(W_k)$

W_k, R_k, Q_k, α_k etc. denote any specific values of W, R, Q, α etc.

E sum of squares of $e'(W_k)$ for all specific values of W_k.

m Any integer number

λ_i $\sum_{K=1}^{m} w_k^i$

S_i $\sum_{K=1}^{m} w_k^i R_k$

T_i $\sum_{K=1}^{m} w_k^i Q_k$

U_i $\sum_{K=1}^{m} w_k^i (R_k^2 + Q_k^2)$

V_{kL} $\left| \frac{1}{I(jw_k)_{L-1}} \right|^2$ where L is the No. of iteration where

λ_i' $\sum_{K=1}^{m} w_k^i V_{kL}$

S_i' $\sum_{K=1}^{m} w_k^i R_k V_{kL}$

T_i' $\sum_{K=1}^{m} w_k^i Q_k V_{kL}$

U_i' $\sum_{K=1}^{m} w_k^i (R_k^2 + Q_k^2) V_{kL}$

E' $\sum_{K=1}^{m} \left| e'(w_k) \right|^2 V_{kL}$

E_R	Error E' normalized with respect to each of datum points.
I	Row of a matrix
M	Column of a metrix
O	order of transfer function
N	2.0 + I
N_1	$\frac{N+1}{2}$
N_3	$\frac{N+1}{2}$
N_5	$\frac{N+5}{2}$

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CHAPTER - 1

INTRODUCTION

1.1 <u>Introduction</u> : The transfer function is a type of mathematical model necessary for designing a system or using a system as a part of a big system. This can be determined analytically if the circuit parameters are known.

In determining the transfer function of an electrical system it is not always possible to know the inner details of the systems, or it may be very complicated. More over manufacturer's ratings of circuit parameters may vary from their actual values and may not be that much accurate as they are theoretically considered. Some times it became necessary to determine the transfer function of a circuit considering it as a black box. Hence for these cases complex curve fitting method is necessary for the determination of the transfer function. In this method transfer function is obtained from sampled frequency response.

Work has been done on finding the transfer function from frequency response by Levy[1] and Sanathanan et al[2]. Levy's[1] method does not give a good fit in the low frequency range when the transfer function is to be determined over several decades. Sanathananat al's work is an improvement development of Levy's[1] work which gives a good fit in the low frequency range.

The main disadvantage of both methods are that

a) Some estimation of the exact order of the systems has to be made from the frequency response.

b) The computer programme has to be changed if the assumed order of the system is changed.

Zaman's[3] work is an improvement development of Levy[1] and Sanathanan et al[2]'s method in which only a guess about the maximum possible order is necessary. The computer programme has not to be changed whatever the order may be. The order of the system has been taken here as that order at which the normalised square error first drops to a very small value compared to the previous values. But these methods have the following disadvantage -

a) Here a guess about the maximum possible order of the system is necessary.

b) The exact order is not always that order at which the normalised mean square errors first drops to a very small value with compared to the previous values.

c) It is not practically possible to obtain transfer function of higher type[5] systems (systems having infinite gain at zero frequency).

However in this work, methods have been developed where the above deficiencies are eliminated.

Transfer function of different electrical circuits of different orders and the R.F. Section (including the I.F. Section) of a radio receiver have been determined considering them as black box using the complex curve fitting method. In this work the frequency responses have been obtained from Lissajou's Patterns.

CHAPTER 2

METHOD TO PROCESS TRANSFER FUNCTION FROM FREQUENCY RESPONSES

2.1. <u>Introduction:</u> E.C. Levy[1] developed a method to process transfer functions from frequency responses of linear dynamic systems. However his method does not give a good fit at the low frequency range when the transfer function is to be determined for frequencies extending over several decades. Santhanan et al[2]s work is an improvement development of Levy's[1] work which gives a reasonably good fit at all frequencies. In both these methods the order of the system has to be ascertained accurately from the frequency response before proceeding to evaluate transfer function, which is not always possible. Zaman[3] in his work developed a generalised method by which the transfer function can be evaluated by Sanathanan et al's method, but without ascertaining accurately the order of the system before hand. Here only an idea about the maximum possible order of the system is necessary . Here in this work an improvement of Zaman's generalized method has been done where no idea about the maximum possible order will be necessary.

Levy[1], Sanathanan et al[2], and Zaman's[3] methods are described below for ready reference.

2.2 <u>Levy's complex curve fitting method</u>: Transfer function, G(jw), of a linear dynamic system can be expressed as a ratio of two frequency dependent polynomials as shown below :

$$G(jw) = \frac{A_0 + A_1 (jw) + A_2 (jw)^2 + A_3(jw)^3 + \dots}{1 + B_1(jw) + B_2 (jw)^2 + B_3 (jw)^3 + \dots} \quad \dots (2.1)$$

Where A_0, A_1 A_2, ..., B_1 B_2, ... are constant coefficients of the transfer function.

Equation (2.1) can be rewritten as

$$G(jw) = \frac{(A_0 - A_2w^2 + A_4w^4 - \dots) + jw\,(A_1 - A_3w^2 + A_5w^4 - \dots)}{(1 - B_2w^2 + B_4w^4 - \dots) + jw(B_1 - B_3w^2 + B_5w^4 - \dots)} \quad \dots (2.2)$$

$$= \frac{\alpha + j\omega\beta}{\sigma + j\omega\tau} \quad \dots \quad (2.3)$$

$$= \frac{P(jw)}{I(jw)} \quad \dots \quad (2.4)$$

But function F(jw) is an ideal function, one which represents the data exactly and which can be obtain experimentally.

F(jw) is a complex quantity.It has real and imaginary components, R and Q respectively.

$$F(jw) = R + jQ \quad \dots\dots\dots \quad (2.5)$$

The numerical difference between the two functions G(jw) and F(jw) represents the error in fitting,that is

$$e(w) = F(jw) - G(jw) \quad \dots \quad (2.6)$$

$$= F\,(jw) - \frac{P(jw)}{I(jw)} \quad \dots \quad (2.7)$$

$$e'(w) = e(w).\ I(jw) = F(jw)\ I(j\omega) - P(j\omega) \cdots \dots \quad (2.8)$$

At any specific value of frequency w_k

$$e'(w_k) = F(jw_k).\ I(jw_k) - P(jw_k) \quad \ldots \qquad (2.9)$$

$$\text{or } e'(w_k) = C(w_k) + jd(w_k) \quad \ldots \qquad (2.10)$$

$$\left[e'(w_k)\right]^2 = c^2(w_k) + d^2(w_k) \quad \ldots \qquad (2.11)$$

Now E is defined as being the function given in (2.11) summed over all the sampling frequencies w_k. Hence

$$E = \sum_{k=1}^{m} \left[c^2(w_k) + d^2(w_k)\right] \quad \ldots. \qquad (2.12)$$

where m is the no of data.

The unknown polynomial coefficients A_i and B_i, i=0,1,2,3 are evaluated on the basis of minimising the error function E.

Using equation (2.3) and (2.5) we get

$$E = \sum_{k=1}^{m} \left[(R_k \sigma_k - W_k \tau_k Q_k - \alpha_k)^2 + (W_k \tau_k R_k + \sigma_k Q_k - W_k \beta_k)^2\right] \quad \ldots \qquad (2.13)$$

Following the usual mathematical procedure equation(2.13) is now partially differentiated with respect to each of the unknown coefficients A_i and B_i and the results are set equal to zero. In this way it is possible to obtain as many equations as there are unknown. The results are condensed by the following relationships.

$$\lambda_i = \sum_{k=1}^{m} W_k^i \quad .. \qquad (2.14)$$

$$S_i = \sum_{k=1}^{m} W_k^i \; R_k \quad .. \qquad (2.15)$$

$$T_i = \sum_{k=1}^{m} W_k^i \; Q_k \quad .. \qquad (2.16)$$

$$U_i = \sum_{k=1}^{m} W_k^i \; (R_k^2 + Q_k^2) \quad .. \qquad (2.17)$$

The final equations in the matrix notation, due to partial differentiations of E, are as follows (detail procedure is given in the Appendix A)

$$\left[P \right] \qquad \left[D \right] = \left[C \right] \ldots \qquad (2.18)$$

where,

$$\left[P \right] = \left[\begin{array}{cccccccc} \lambda_0 & 0 & -\lambda_2 & 0 & \lambda_4 & \cdots\; T_1 & S_2 & -T_3 \\ 0 & \lambda_2 & 0 & -\lambda_4 & 0 & \cdots\; -S_2 & T_3 & S_4 \\ \lambda_2 & 0 & -\lambda_4 & 0 & \lambda_6 & \cdots\; T_3 & S_4 & -T_5 \\ 0 & \lambda_4 & 0 & -\lambda_6 & 0 & \cdots\; -S_4 & T_5 & S_6 \\ \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots \\ T_1 & -S_2 & -T_3 & S_4 & T_5 & \cdots\; U_2 & 0 & -U_4 \\ S_2 & T_3 & -S_4 & -T_5 & S_6 & \cdots\; 0 & U_4 & 0 \\ T_3 & -S_4 & -T_5 & S_6 & T_7 & \cdots\; U_4 & 0 & -U_6 \\ \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots \end{array} \right] \quad \ldots (2.9$$

$$[D] = \begin{bmatrix} A_0 \\ A_1 \\ A_2 \\ \cdot \\ \cdot \\ \cdot \\ B_1 \\ B_2 \\ B_3 \\ \cdot \\ \cdot \\ \cdot \end{bmatrix} \quad \ldots\ldots\ldots\ldots\ldots(2.20) \qquad [C] = \begin{bmatrix} S_0 \\ T_1 \\ S_2 \\ T_3 \\ \cdot \\ \cdot \\ \cdot \\ 0 \\ U_2 \\ 0 \\ \cdot \\ \cdot \end{bmatrix} \quad \ldots(2.21)$$

The unknown materix $[D]$ can be obtained after calculating $[P]$ and $[C]$ matrices from experimental data.

2.3 Development of Levy's method by Sanathanan et al.: Serious deficiency of this technique is that it does not give a good fit at lower frequencies when the transfer function is to be determined for frequencies extending over several decades. This deficiency can be overcome by an iteration process as described below and this is the contribution of Santhanan et al[2].

$$e''(w_k) = \frac{F(jw_k) \cdot I(jw_k)_L}{I(jw_k)_{L-1}} - \frac{F(jw_k)_L}{I(jw_k)_{L-1}} \quad \ldots\ldots \quad (2.22)$$

Where the subscript L corresponds to the iteration number. As $I(jw_k)$ is not known initially, it is assumed to be equal to 1. The subsequent iteration tends to converge rapidly and the coefficients evaluated become effictively those obtained by minimising the sum of $\left|e(w_k)\right|^2$ at all data points. From equation (2.22)

$$\left|e''(w_k)\right| = \frac{\left|F(jw_k)\ I(jw_k)_L - P(jw_k)_L\right|}{\left|I(jw_k)_{L-1}\right|} \quad \ldots\ldots\ldots \quad (2.23)$$

Let, $$V_{kL} = \frac{1}{\left|I(jw_k)_{L-1}\right|^2} \quad \ldots\ldots \quad (2.24)$$

Hence $$E' = \sum_{k=1}^{m} \left|e''(w_k)\right|^2$$

$$= \sum_{k=1}^{m} \left|e'(w_k)\right|^2 . V_{KL} \quad \ldots\ldots \quad (2.25)$$

Equation 2.25 is now partially differentiated with respect to each of the polynomial co-efficients and equated to Zero. Method is the same as above ,but all equtions are to be multiplied by V_{kL}. They would yield the same form of matrix equations as that in (2.18) but here λ_i, S_i, T_i, U_i will be replaced by λ'_i, S'_i, T'_k, U'_i respectively, where

$$\lambda'_i = \sum_{k=1}^{m} w_k^i\ V_{KL} \quad \ldots \quad (2.26)$$

$$S_i' = \sum_{k=1}^{m} w_k^i \quad R_k \quad V_{kL} \quad ;\ldots (2.27)$$

$$T_i' = \sum_{k=1}^{m} w_k^i \quad Q_k \quad V_{kL} \quad \ldots (2.28)$$

$$U_i' = \sum_{k=1}^{m} w_k^i \quad (R_k^2 + Q_k^2).\ V_{kL} \ldots (2.29)$$

The coefficients $A_0, A_1\ A_2, \ldots\ldots B_1,\ B_2,\ \ldots\ldots$ evaluated at iteration (L-1) are used to evaluate V_{kL} for the L-th iteration.

2.4: Brief description of Zaman's[3] Generalised Method :

In actual use of the above method to evaluate the transfer function from the frquency response, the order of the system has to be ascertained . That is, the function in the form given in equation (2.1) has to be assumed accurately which is to be curve fitted with the experimental frequency response. This was generally done by the Bode plot of the frequency response[5]. This may not always be accurate and feasible. Observing the symmetry in the [P] and [C] matrices, Zaman developed a generalised method in which he formulated 15 equations. These equations generate [P] and [C] matrices exactly whatever the order of the system may be. The equations are given below :

I = row of a matrix

M = column of matrix

O = order of the system

$N = (2.O + 1)$, $N_1 = (N+1)/2$,

$N_3 = (N+3)/2$, $N_5 = (N+5)/2$,

$[P']$ - Matrix

$$p'(I,M) = (-1)^{(M-1)/2} \chi'(M+I-2), \quad \begin{bmatrix} I=1, N_1, 2 \\ M=1, N_1, 2 \end{bmatrix}$$

$$p'(I,M) = (-1)^{(M-2)/2} \chi'(M+I-2) \quad \begin{bmatrix} I=2, N_1, 2 \\ M=2, N_1, 2 \end{bmatrix}$$

$$p'(I,M) = (-1)^{(M-1)/2} T'(M+I-N_3) \quad \begin{bmatrix} I=N_3, N, 2 \\ M=1, N_1, 2 \end{bmatrix}$$

$$*p'(I,M) = (-1)^{(M-1)/2} S'(M+I-N_3), \quad \begin{bmatrix} I=N_5, N, 2 \\ M=1, N_1, 2 \end{bmatrix}$$

$$p'(I,M) = (-1)^{(M/2)} S'(M+I-N_3), \quad \begin{vmatrix} I=N_3, N, 2 \\ M=2, N_1, 2 \end{vmatrix}$$

$$*p'(I,M) = (-1)^{(M+2)/2}\ T'(M+I-N_3), \quad \begin{bmatrix} I = N_5, N, 2 \\ M = 2, N_1, 2 \end{bmatrix}$$

$$p'(I,M) = (-1)^{(M-N_3)/2}\ T'(M+I-3), \quad \begin{bmatrix} I = 1, N_1, 2 \\ M = N_3, N, 2 \end{bmatrix}$$

$$p'(I,M) = (-1)^{(M-(N-1)/2)/2}\ S'(M+I-N_3), \quad \begin{bmatrix} I = 2, N_1, 2 \\ M = N_3, N, 2 \end{bmatrix}$$

$$*\ p'(I,M) = (-1)^{(M-N_5)/2}\ S'(M+I-N_3), \quad \begin{bmatrix} I = 1, N_1, 2 \\ M = N_5, N, 2 \end{bmatrix}$$

$$*\ p'(I,M) = (-1)^{(M-N_5)/2}\ T'(M+I-N_3), \quad \begin{bmatrix} I = 2, N_1, 2 \\ M = N_5, N, 2 \end{bmatrix}$$

$$p'(I,M) = (-1)^{(M-N_3)/2}\ U'(M+I-N-1), \quad \begin{bmatrix} I = N_3, N, 2 \\ M = N_3, N, 2 \end{bmatrix}$$

$$*p'(I,M) = (-1)^{(M-N_5)/2}\ U'(M+I-N-1), \quad \begin{bmatrix} I = N_5, N, 2 \\ M = N_5, N, 2 \end{bmatrix}$$

Rest of the $\left[p' \right]$ -Matrix elements are zero.

$[c']$ - Matrix

$C'(M) = S'_{(M-1)}$, $M = 1, N_1, 2$

$C'(M) = T'_{(M-1)}$, $M = 2, N_1, 2$

* $C'(M) = U'_{(M-N_1)}$, $M = N_5\ N, 2$

* These equations will not exist for 1st order system.

Emperical formulae were also developed for finding out the frequency response from fitted transfer function of any order. These equations are necessary for determination of frequency response from fitted transfer function and also error in complex curve fitting .They are

** $F = D(1) + (-1)^{(M-1)/2} D(M)\, W_k^{(M-1)}$, $M = 3, N_1, 2$

$T = (-1)^{(M-2)/2} D(M)\, W_k^{(M-1)}$, $M = 2, N_1, 2$

** $X = 1 + (-1)^{(M-N_1)/2} D(M)\, W_k^{(M-N_1)}$, $M = N_5, N, 2$

$$Y = (-1)^{(M-N_3)/2} \quad D(M)\, W_k^{(M-N_1)} , \quad M= N_3, \quad N, 2$$

$$Z_k = \sqrt{\frac{F^2 + T^2}{X^2 + Y^2}}$$

$$H_k = \left[\tan^{-1} T/F - \tan^{-1} Y/X \right]$$

$$Z_{1k} = Z_k \text{ Cos } H_k$$

$$Z_{2k} = Z_k \text{ Sin } H_k$$

where $Z_k \angle H_k$ is the frequency response of the evaluated transfer function.

$$E' = \sum_{k=1}^{m} \left[(R_k - Z_{1k})^2 + (Q_k - Z_{2k})^2 \right]$$

$$E_r = \sum_{k=1}^{m} \left[((R_k - Z_{1k})/R_k)^2 + ((Q_k - Z_{2k})/Q_k)^2 \right]$$

where E_r is the normalised error of E'.

$$V_{kL} = 1/ (x^2 + y^2)_{L-1}$$

** For 1-st order system X= 1 only and F= D (1) only.

When actually using Zaman's[3] method, it is necessary to make a guess about the maximum possible order of the system, this can be ascertained from the results and can be modified accordingly. The experimental frequency response is then used to find the transfer function initially assuming a first order system and then subsequently raising the order untill the order of the guess is reached. In each case the transfer function is evaluated and the error between the frequency response from this transfer function and the experimental frequency responses are calculated, normalised, squared and added for all data points. For noise free data, the correct transfer function is one which gives the minimum error. The minimum error should be very small with respect to the normalisation criterion. If a reasonable minimum error is not obtained then it is to be understood that the guess was not right and still a higher order guess in necessary. On the other hand, the correct answer can be obtained even if the guess of the order is too high. For noisy data the correct order of transfer function is one at which the error drops to a very small value compared with previous values.

2.5 Results showing the accuracy of complex curve fitting method:

The generalised complex curve fitting method has been checked extensively to varify its accuracy. To do this the following examples of different order system have been selected

A program has been developed to evalute the frequency response from these transfer functions. Then from these frequency responses their transfer functions have been evaluated using the 'Generalised Complex curve fitting Method'. The results which are given below were found very accurate.

i) First Order System.

The transfer function was selected as $G(jw) = \dfrac{1 + 0.09s}{1 + 0.1s}$

The fitted transfer function was found as 1st order as it was.
The values of the coefficients of the fitted transfer function are given in table 2.1.

Table 2.1

Showing the accuracy for 1st order system

Constant coefficients	From Original transfer function	From Fitted transfer function
A_0	1.0	1.0
A_1	0.09	0.09002
B_0	1.0	1.0
B_1	0.1	0.1

ii) Second order system : The transfer function was selected as $G(jw) = \dfrac{1+0.09S + 0.00079S^2}{1+ 0.1S + 0.0008S^2}$

The fitted transfer function was found very accurate at 2nd order as it was. The values of the coefficients of the fitted transfer function were given Table 2.2 below.

Table 2.2

Showing the accuracy for 2nd order system

Constant coefficients	From original transfer function	From Fitted Transfer function
A_o	1.0	0.99987
A_1	0.09	0.09
A_2	0.00079	0.00079
B_o	1.0	1.0
B_1	0.1	0.09999
B_2	0.0008	0.0008

iii) 3rd order system : The transfer function was selected as

$$G(jw) = \frac{1+ 0.3097X10^{-3}S + 0.283x10^{-7}S^2}{1+ 0.312 x10^{-1}S + 0.3758x10^{-4}S^2 \; 1 \; 0.104x10^{-7}S^3}$$

The fitted transfer function was found very accurate at 3rd ord[e]
as it was. The values of the constant coefficients of the fitted
transfer function are given in table 2.3.

Table 2.3

Showing the accuracy for 3rd order system

Constant coefficients	From original Transfer function	From Fitted Transfer function
A_o	1.0	0.9999999
A_1	0.3097×10^{-3}	0.3098×10^{-3}
A_2	0.283×10^{-7}	0.283×10^{-7}
A_3	0	0
B_o	1.0	1.0
B_1	0.312×10^{-1}	0.312×10^{-1}
B_2	0.3758×10^{-4}	0.3758×10^{-4}
B_3	0.104×10^{-7}	0.104×10^{-7}

CHAPTER — 3

DEVELOPMENT TO COUNTER THE DEFICIENCY OF ZAMANS METHOD.

3.1. Deficiency of Zaman's method:

In Zamans[3] method it is necessary to guess a highest possible order of the system. If a reasonable minimum error is not found, it is to be understand that the guess was not right and a still higher order guess is necessary, In his method, the actual order of the system was supposed to be that order at which the error first drops to a small value compaired with previous values.

But it has been found that the above method of deciding the actual order does not always give the right order of the system. As an example, a system having a transferfunction of

$$\frac{1 + 0.312 \times 10^{-3}s + 0.286 \times 10^{-7}s^2}{1 + 0.312 \times 10^{-1}s + 0.379 \times 10^{-4}s^2 + 0.105 \times 10^{-7}s^3}$$

generates the following errors at different orders.

In thise case 20sets of data within the frequency band 5 Hz to 575 Hz were used.

a) At 1st order, Error = 0.2097×10^{0}

Fitted transfer function at this order is

$$\frac{1.0046 - 0.3308 \times 10^{-3} S}{1 + 0.30929 \times 10^{-1} S}$$

b) At 2nd order , Error = 0.200×10^{-4}

Fitted transfer function at this order is

$$\frac{0.99978 - 0.2128 \times 10^{-4} S + 0.203576 \times 10^{-7} S^2}{1 + 0.3085 \times 10^{-1} S + 0.2779 \times 10^{-4} S^2}$$

c) At 3rd order, Error= 0.598×10^{-12}

Fitted transfer function at this order was found very accurate and it was:-

$$\frac{0.999999 + 0.311768 \times 10^{-3}s + 0.28599 \times 10^{-7}s^2}{1 + 0.311997 \times 10^{-1}s + 0.3789 \times 10^{-4}s^2 + 0.1049 \times 10^{-7}s^3}$$

d) At 4th order ,Error = 0.81722×10^{-12}

fitted transfer function at this order is

$$\frac{0.99999 + 0.933 \times 10^{-3}s + 0.22246 \times 10^{-6}s^2 + 0.17768 \times 10^{-10}s^3 + 0.568 \times 10^{-18}s^4}{1 + 0.318 \times 10^{-1}s + 0.5728 \times 10^{-4}s^2 + 0.3404c10^{-7}s^3 + 0.652 \times 10^{-11}s^4}$$

According to the previous method the order of the system should be 2nd order . Because it first drops from 0.209×10^0 to a reasonable minimum value of 0.2×10^{-4}. But the system is actually 3rd order . So it is evident that the above concept of deciding actual order of a system is som time confusing.

3.2 Development to counter the deficiency of Zaman's method:

To eleminate the above difficulties three emperical relations are developed observing the error of different systems.

The actual order of a system is decided here considering the following conditions.

a) The error should be reasonably small,

b) If x is the actual order then

$$\frac{E_{(x-2)} - E_{(x-1)}}{E_{(x-2)}} < \frac{E_{(x-1)} - E_x}{E_{(x-1)}} > \frac{E_x - E_{(x+1)}}{E_x} \qquad \ldots\ldots\ldots(3.1)$$

Where E_x — error at x-th order.

In the previous example errors at different orders were

$E_1 = 0.2097 \times 10^0$

$E_2 = 0.200 \times 10^{-4}$

$E_3 = 0.598 \times 10^{-12}$

$E_4 = 0.817 \times 10^{-12}$

Here ,

$$\frac{E_{(3-2)} - E_{(3-1)}}{E_{(3-2)}} = 0.9999$$

$$\frac{E_{(3-1)} - E_3}{E_{(3-1)}} = 0.9999999$$

$$\frac{E_3 - E_{(3+1)}}{E_3} = -0.366$$

Therefore it is found that

$$\frac{E_{(3-2)} - E_{(3-1)}}{E_{(3-2)}} < \frac{E_{(3-1)} - E_3}{E_{(3-1)}} > \frac{E_3 - E_{(3+1)}}{E_3}$$

According to this method the result indicates that the system is 3rd order and actually it is of 3rd order. Hence supports the above emperical relation for the detection of of the actual order.

The relation (3.1) is applicable for systems higher than 2nd order. If the system satisfies the relation

$$\frac{E_r - E_1}{E_r} > \frac{E_1 - E_2}{E_1} \quad \ldots\ldots\ldots\ldots\ldots(3.2)$$

Where E_r = maximum acceptable error limit to consider as good fitting, then the system will be of 1st order.

For 2nd order system the relation (3.1) can be used only replacing $E_{(x-2)}$ by E_r.

i.e. $$\frac{E_r - E_1}{E_r} < \frac{E_1 - E_2}{E_1} > \frac{E_2 - E_3}{E_2}$$

$$\ldots\ldots\ldots\ldots(3.3)$$

To support the above emperical relations several more examples are given below.

1) Transfer function is taken as

$$\frac{0.0 + 0.3097\times10^{-3}S + 0.283\times10^{-7}S^2}{1 + 0.312\times10^{-1}S + 0.375776\times10^{-4}S^2 + 0.3\times10^{-5}S^3}$$

At 3rd order the fitted transfer fumction was found very accurate 39 data sets between 5Hz to 765 Hz were used. The fitted transfer function was

$$\frac{0.180\times10^{-5} + 0.30964\times10^{-3}S + 0.28289\times10^{-7}S^2}{1 + 0.31488\times10^{-1}S + 0.37589\times10^{-4}S^2 + 0.2999\times10^{-5}S^3}$$

The errors at different orders are

Order	Error
1st	0.8859×10^4
2nd	0.168×10^3
* 3rd	0.554×10^{-11}
4th	0.759×10^{-12}

Here

$$\frac{E_{(3-2)} - E_{(3-1)}}{E_{(3-2)}} = 0.981$$

$$\frac{E_{(3-1)} - E_3}{E_{(3-1)}} \approx 1$$

$$\frac{E_3 - E_{(3+1)}}{E_3} = 0.8629$$

Therefore

$$\frac{E_{(3-2)} - E_{(3-1)}}{E_{(3-2)}} < \frac{E_{(3-1)} - E_3}{E_{(3-1)}} > \frac{E_3 - E_{(3+1)}}{E_3}$$

According to this method the system is of 3rd order and actually it is. Hence the above example supports this method.

II) Transfer function was taken as

$$\frac{1+0.3\times10^{-3}s+0.3\times10^{-7}s^2}{1+0.312\times10^{-1}s+0.4\times10^{-4}s^2+0.3\times10^{-5}s^3}$$

At 3rd order the transfer function was found very accurate 39 sets of data between 5 Hz to 765 Hz were taken . The fitted transfer function at this order was

$$\frac{0.99999+0.299999\times10^{-3}s+0.29999\times10^{-7}s^2}{1+0.31199\times10^{-1}s+0.40\times10^{-4}s^2+0.29999\times10^{-5}s^3}$$

Erors at different orders were —

Order	Error
1st	0.5774×10^{8}
2nd	0.4839×10^{-1}
* 3rd	0.216×10^{-9}
4th	0.2056×10^{-12}

Here ,

$$\frac{E_{(3-2)} - E_{(3-1)}}{E_{(3-2)}} \simeq 1$$

$$\frac{E_{(3-1)} - E_3}{E_{(3-1)}} \approx 1$$

$$\frac{E_3 - E_{(3+1)}}{E_3} = 0.999$$

Therefore for 3rd order the system satisfies the relation (3.1) .Again actually the system is of 3rd order. Hence the example supports this method.

III) Transfer function was taken as

$$\frac{1+0.03S+0.0004S^2}{1+0.02S+0.0005S^2}$$

Frequency response was taken from 5 Hz to 100 Hz. At 2nd order the system was found very accurate and it was

$$\frac{1+0.029999S+0.00039999S^2}{1+0.0199999S+0.49999S^2}$$

Errors at different orders were

Order	Error
1st	0.353×10^{1}
2nd	0.180×10^{-10}
3rd	0.256×10^{-10}

Here

$$\frac{E_r - E_{(2-1)}}{E_r} = -35299$$

For noise free data error E_r is taken as 10^{-4}

$$\frac{E_{(2-1)} - E_2}{E_{(2-1)}} \approx 1$$

$$\frac{E_2 - E_{(2-1)}}{E_2} = -0.422$$

Therefore for 2nd order

$$\frac{E_r - E_{(2-1)}}{E_r} < \frac{E_{(2-1)} - E_1}{E_{(2-1)}} > \frac{E_2 - E_{(2+1)}}{E_2}$$

Hence it supports the relation (4. 3)

iv) Transfer function was taken as

$$\frac{1 + 0.9\times10^{-1}S + 0.4\times 10^{-3} S^2}{1 + 0.1S + 0.41\times 10^{-3}S^2}$$

Frequency response were taken between 5 Hz to 100 Hz . At 2nd order the transfer function was found accurate and it was

$$\frac{1{\cdot}001 + 0.91\times 10^{-1}S + 0.40 \times 10^{-3} S^2}{1 + 0.10 S + 0.417 \times 10^{-3} S^2}$$

The errors at different orders are

Order	Error
1st	0.128×10^{-2}
2nd	0.275×10^{-6}
3rd	0.111×10^{-8}

Here

$$\frac{E_r - E_{(2-1)}}{E_r} = -127999$$

$$\frac{E_{(2-1)} - E_2}{E_{(2-1)}} = 0.9999999$$

$$\frac{E_2 - E_{(2+1)}}{E_2} = 0.99596$$

$$\frac{E_r - E_{(2-1)}}{E_r} < \frac{E_{(2-1)} - E_2}{E_{(2-1)}} > \frac{E_2 - E_{(2+1)}}{E_2}$$

Hence it shows that the system is 2nd order and actually it is .

v) Transfer function was taken as

$$\frac{1 + 0.13\,S}{1 + 0.44\,S}$$

Frequency response was taken between 5 Hz to 100 Hz

At 1st order fitted transfer function was found accurate and it was

$$\frac{0.9979 + 0.1296S}{1.0 + 0.438S}$$

Error at different orders

Order	Error
1st	0.964×10^{-11}
2nd	0.286×10^{-10}

Here

$$\frac{E_r - E_1}{E_r} = 0.99999$$

$$\frac{E_1 - E_{(1+1)}}{E_1} = -1.956$$

Therefore $$\frac{E_r - E_1}{E_r} > \frac{E_1 - E_{(1+1)}}{E_1}$$

Hence the system is 1st order

and actually it is

vi) Transfer function was taken as

$$\frac{1 + 0.09S}{1 + 0.1S}$$

At 1st order the fitted transfer function was found very accurate. Frequency response was taken between 5 Hz to 100 Hz The fitted transfer function at 1st order was

$$\frac{1 + 0.09S}{1 + 0.1S}$$

Error at different orders were

Order	Error
1st	0.3145×10^{-8}
2nd	0.3948×10^{-8}

Here

$$\frac{E_r - E_1}{E_r} = 0.9999685$$

$$\frac{E_1 - E_{(1+1)}}{E_1} = -0.255$$

Therefore $$\frac{E_r - E_1}{E_r} > \frac{E_1 - E_{(1+1)}}{E_1}$$

Hence the system will be of 1st order and actual it is.

All the above examples support the emperical relations of 3.1, 3.2 & 3.3.

Hence the above error analysis gives the correct order of the system.

3.3. Method to process transfer function of higher Type systems (systems having infinite gain at zero frequency)

In developing the complex curve fitting method Levy[1] considered B_o as 1. For this reason transfer function of higher Type[5] system is not possible to evaluate directly. If this method has to be applied for a system having infinite gain at zero frequency the transfer function to be multiplied by $(jw)^n$, n being large enough to reduce the absolute magnitude of the function at zero frequency to a finite value. Then from the modified frequency response a transfer function is obtained. Dividing this fitted transfer function by $(jw)^n$ the actual transfer function can be obtained.

But for practical system having no idea about the type of the system it is very difficult to obtain the actual transfer function. A technique has been developed in this work to overcome this problem.

Let us consider a transfer function of the following form:

$$G(jw) = \frac{A_o + A_1 S + A_2 S^2}{B_1 S^n + B_2 S^{1+n} + B_3 S^{2+n}} \quad \ldots\ldots\ldots (3\cdot4)$$

$$= \frac{A_o + A_1 S + A_2 S^2}{S^n (B_1 + B_2 S + B_3 S^2)}$$

When n= 1, it is Type one system

When n= 2, it is Type two system

According to Levy's method it not directly possible to determine the transfer function from the sampled frequency responses. Let us suppose at W_k the sampled frequency respons is $F_k \angle \varphi_K$.

Then the frequency response of $\frac{1}{F_k} \angle -\varphi_K$

will corresponds to the transfer function of the system $G'(j\omega)$.

Taking n= 1

$$G'(jw) = \frac{1}{G(jw)} = \frac{B_1 S + B_2 S^2 + B_3 S^3}{A_o + A_1 S + A_2 S^2} \qquad (3.5)$$

Or

$$G'(jw) = \frac{B_1/A_o\, S + B_2/A_o\, S^2 + B_3/A_o S^3}{1 + A_1/A_o\, S + A_2/A_o\, S^2} \qquad (3.6)$$

Then the transfer function of the system corresponds to equation (3.6) can be obtained directly using Zaman's[3] generalised method, if its frequency response is known. But this frequency response can be easily obtained taking the inverse of the frequency response of the actual higher type system. By this inverse frequency response the transfer function $G'(jw)$ can be obtained. The actual transfer function $G(jw)$ can be obtained by simply inverting the fitted transfer function $G'(jw)$.

Computer program has been developed in this work on the basis of the above concept to determine the transfer function of higher type systems. This method has been tested for different systems and satisfactory results have been found.

Three examples have been given below for higher type systems.

1) System was taken as

$$\frac{1 + 0.2235 \times 10^{-1} S + 0.54 \times 10^{-4} S^2}{0.269 \times 10^{-3} S + 0.187 \times 10^{-7} S^2}$$

using the above method the fitted transfer function was found as

$$\frac{1+ 0.223499 \times 10^{-1} s + 0.539999 \times 10^{-4} s^2}{0.268999 \times 10^{-3} S + 0.187 \times 10^{-7} S^2}$$

Frequency response was taken from 5 Hz to 260 Hz

No of date sets were

II) Transfer function was taken as

$$\frac{1+ 0.3 \times 10^{-1} S + 0.375 \times 10^{-4} S^2 + 0.3 \times 10^{-6} S^3}{0.3 \times 10^{-3} S + 0.283 \times 10^{-7} S^2}$$

using the deserve method the fitted transfer function was found as

$$\frac{1+ 0.3 \times 10^{-1} S + 0.375 \times 10^{-4} S^2 + 0.3 \times 10^{-6} S^3}{0.3 \times 10^{-3} S + 0.28299 \times 10^{-7} S^2}$$

Frequency response was taken between 5Hz to 195 Hz. No.of data sets were

III) Transfer function was taken as

$$\frac{1 + 0.022355S + 0.54\times10^{-4}S^2 + 0.143\times10^{-6}S^3}{0.345\times10^{-5}S^2 + 0.145\times10^{-7}S^3}$$

Frequency response was taken from 6Hz to 385 Hz. No of data sets were 39. The fitted tran fer function using the above method was found as

$$\frac{1 + 0.0223496S + 0.53999\times10^{-4}S^2 + 0.14299\times10^{-7}S^3}{0.34499\times10^{-5}S^2 + 0.14499\times10^{-7}S^3}$$

CHAPTER 4

EFFECT OF FREQUENCY RANGE AND FREQUENCY INTERVAL OF DATA SETS ON THE FITTED TRANSFER FUNCTION

4.1. Introduction: In determination of transfer function from frequency response using complex curve fitting method, it is necessary to know the range of exact frequency band over which the sampled data should be taken. The selection of incorrect frequency band may change the fitted transfer function by a great deal. In this chapter the effect of the frequency range on the fitted transfer function has been discussed with example. An attempt has been made to get an idea about the critical frequency band observing the results at various conditions. Finally the effect of frequency interval of data sets within a definite frequency band has been studied.

4.2. Effect of the frequency range : The most important factor of determination of transfer function from the frequency response using the complex curve fitting method is the frequency range, within which the data sets are taken. The values of the coefficients as well as the order of the fitted transfer function may vary as the frequency band within which the data is taken is narrowed. As an example, a 3rd order practical circuit is taken whose transfer function is as follows .

$$\frac{1+ 0.3097 \times 10^{-3} S + 0.283 \times 10^{-7} S^2}{1 + 0.312 \times 10^{-1} S + 0.3758 \times 10^{-4} S^2 + 0.104 \times 10^{-7} S^3}$$

The constant coefficients are -

$A_o = 1.0$, $A_1 = 0.3097 \times 10^{-3}$ $A_2 = 0.283 \times 10^{-7}$

$B_o = 1.0$, $B_1 = 0.312 \times 10^{-1}$, $B_2 = 0.3758 \times 10^{-4}$

$B_3 = 0.104 \times 10^{-7}$

a) When the frequency range of the sampled data sets were between 5 Hz to 100 Hz (No. of data sets were 20), the transfer function was found 2nd order and the constant coeffieients of the fitted transfer function were -

$A_0 = 0.99999$, $A_1 = 0.693 \times 10^{-4}$

$A_2 = 0.3827 \times 10^{-7}$,

$B_0 = 1.0$, $B_1 = 0.3082 \times 10^{-1}$

$B_2 = 0.2588 \times 10^{-4}$

b) When the frequency band of the sampled data sets were between 5 Hz to 175 Hz (No. of data sets were 35), the fitted transfer function was found 2nd order again. The constant coefficients of the fitted transfer function were

$A_0 = 0.9999$, $A_1 = 0.6072 \times 10^{-4}$, $A_2 = 0.35306 \times 10^{-7}$

$B_0 = 1.000$, $B_1 = 0.30826 \times 10^{-1}$, $B_2 = 0.261556 \times 10^{-4}$

c) Frequency range was between 5 Hz to 385 Hz (no. data sets were 39).

The order of the system was found 3rd, and the constant coefficients of the fitted transfer function were

$A_0 = 0.9999997$, $A_1 = 0.3099079 \times 10^{-3}$, $A_2 = 0.2829 \times 10^{-7}$

$A_3 = 0.59 \times 10^{-14}$

$B_0 = 1.0$, $B_1 = 0.312021 \times 10^{-1}$, $B_2 = 0.37583 \times 10^{-4}$,

$B_3 = 0.10405 \times 10^{-7}$

The above results were found to be acceptable .

d) Frequency range was between 5 Hz to 765 Hz (39 sets of data were taken).

The order of the system was found 3rd and the constant coefficients of the fitted transfer function were -

$A_0 = 0.9999999$, $A_1 = 0.3098 \times 10^{-3}$, $A_2 = 0.2830 \times 10^{-7}$,

$A_4 = 0.18 \times 10^{-14}$

$B_0 = 1.00$, $B_1 = 0.312 \times 10^{-1}$, $B_2 = 0.3758 \times 10^{-4}$,

$B_3 = 0.104 \times 10^{-4}$.

This result is found to be very accurate.

4.3. Detection of critical band : To get an idea about the the selection of critical frequency band, three arbitrary transfer functions of 1st, 2nd and 3rd order have been taken. Using these transfer functions frequency responses are generated and at different conditions their fitted transfer function have been determined using complex curve fitting method. The conditions are—No. of data sets, starting frequency of the sets and the frequency interval of the data sets. Observing these results an attempt has been made to reach a conclusion about the critical frequency band.

a) Transfer function was taken as $\frac{1 + 0.09S}{1 + 0.1S}$. Frequency response has been generated from the transfer function and is given in the Table 4.1.

Table 4.1

Frequency response for the 1st order system

$$\frac{1 + 0.09S}{1 + 0.1S}$$

FREQUENCY	F(L)	PHI(L) (RAD.)
1	0.972739	-0.046
2	0.940036	-0.052
3	0.922892	-0.045
4	0.914313	-0.038
5	0.909659	-0.032
10	0.902604	-0.017
15	0.901174	-0.012
20	0.900664	-0.009
30	0.900295	-0.006
40	0.900167	-0.004
50	0.900107	-0.004
60	0.900074	-0.003
80	0.900042	-0.002

Contd...

Table 4.1 (Continued from last page)

FREQUENCY	F(L)	PHI (L) (RAD.)
100	0.900027	-0.002
120	0.900019	-0.001
140	0.900013	-0.001
160	0.900010	-0.001
180	0.900008	-0.001
200	0.900007	-0.001
220	0.900005	-0.001
240	0.900005	-0.001

N.B. Where F(L) is magnitude,
PHI(L) is phase in radian.

The following results have been found at different conditions :

Table - 4.2
Results at different conditions for the system

(1. + 0.09S) / (1 + 0.1S).

No. of sets of Data	Starting frequency Hz	Frequency interval Hz	Remark on the fitted transfer function.
15	5	5	gives accurate result
15	100	5	Does not give accurate result
15	50	5	"
15	02	02	Gives accurate result
15	25	02	Does not give actual result

From the data in Table 4.1 it is found that the maximum rate of change of magnitude and phase difference is at the lower frequency region (nearly upto 45 Hz) and it decrease as frequency increases. Again from the table 4.2

it is seen that only those data sets whose frequency band covers the relatively large changing (in magnitude and angle) region give the accurate results .

b) Second order system :

The transfer function is taken as :

$$\frac{1 + 0.09S + 0.00079S^2}{1 + 0.1S + 0.008S^2}$$

The following results are found at different conditions :

Table 4.4
Results at different conditions for the system

$(1 + 0.09S + 0.00079S^2) / (1 + 0.1S + 0.008S^2)$

No of sets of data	Starting frequency Hz	Frequency interval Hz	Remark on the fitted transfer function.
14	25	5	Does not give accura result
14	15	5	"
25	15	5	"
15	1	1	gives exact result
15	15	1	Does not give
20	10	1	gives nearby exact
10	10	5	"
14	5	5	gives exact result

Table 4.3

Frequency response for the 2nd order system

$$\frac{1 + 0.09S + 0.00079S^2}{1 + 0.1S + 0.008S^2}$$

FREQUENCY	F(L)	PHI(L) (RAD.)
1	1.207619	-0.215
2	1.113841	-0.865
3	0.699077	-1.175
4	0.485776	-1.234
5	0.374261	-1.221
10	0.193417	-1.010
15	0.147115	-0.820
20	0.127913	-0.678
30	0.112536	-0.493
40	0.106701	-0.383
50	0.103902	-0.312
60	0.102353	-0.263
80	0.100791	-0.199
100	0.100060	-0.160
120	0.099662	-0.134

Contd...

Table 4.3 (Continued from last page)

FREQUENCY	F(L)	PHI(L) (RAD.)
140	0.099421	-0.115
160	0.099264	-0.101
180	0.099156	-0.089
200	0.099079	-0.081
220	0.099022	-0.073
240	0.098979	-0.067

From the data of Table 4.3 it has been seen that the change in magnitude and angle is greater in the lower frequency range than that of the higher, and from the above statistics of Table 4.4 it is evident that only those data sets which lies in that region give accurate results.

c) Third order system : The transfer function is taken as

$$\frac{1 + 0.3097 \times 10^{-3}s + 0.283 \times 10^{-7} s^2}{1 + 0.312 \times 10^{-1}s + 0.3758 \times 10^{-4} s^2 + 0.104 \times 10^{-7}s^3}$$

The Following results are observed at different conditions

Table 4.6

Results at different conditions for the system

$$(1 + 0.3079 \times 10^{-3} s + 0.283 \times 10^{-7} s^2) / (1 + 0.312 \times 10^{-1}s + 0.3758 \times 10^{-4} s^2 + 0.104 \times 10^{-7} s^3)$$

No of sets of data	Starting frequency	Frequency interval	Remark on the fitted transfer function
20	5	5	Does not gives accurate result
35	5	5	"
39	5	10	gives exact result
20	100	10	Does not give accurate result
30	100	10	gives exact result
30	200	10	Does not give exact result

Table 4.5

Frequency response for the system

$$\frac{1 + 0.3097 \times 10^{-3}S + 0.283 \times 10^{-7}S^2}{1 + 0.312 \times 10^{-1}S + 0.3758 \times 10^{-4}S^2 + 0.104 \times 10^{-7}S^3}$$

FREQUENCY	F(L)	PHI(L) (RAD.)
5	0.727925	-0.784
10	0.468424	-1.141
15	0.332664	-1.318
20	0.255103	-1.428
30	0.171910	-1.570
40	0.128393	-1.670
50	0.101629	-1.752
60	0.083452	-1.824
70	0.070261	-1.889
80	0.060228	-1.949
100	0.045940	-2.059
120	0.036250	-2.156
140	0.029271	-2.242
160	0.024042	-2.319
180	0.020015	-2.387

Contd.

Table 4.5 (Continued from last page)

FREQUENCY	F(L)	PHI(L) (RAD.)
200	0.016848	-2.447
220	0.014318	-2.499
240	0.012271	-2.545
260	0.010597	-2.584
280	0.009215	3.665
300	0.008064	3.636
320	0.007100	3.611
340	0.006285	3.590
360	0.005593	3.573
380	0.005002	3.559
400	0.004494	3.549
420	0.004056	3.541
440	0.003675	3.535
460	0.003344	3.531
480	0.003053	3.529
500	0.002798	3.529

Contd....

Table 4.5 (Continue from last page)

FREQUENCY	F(L)	PHI(L) (RAD.)
520	0.002573	3.531
540	0.002374	3.534
560	0.002197	3.538
580	0.002039	3.543
600	0.001898	3.550
620	0.001772	3.557
640	0.001658	3.564
660	0.001555	3.573

From the data sets in Table 4.5 it is seen that in the lower frequency range (about upto 300 Hz) both magnitude and angle changes relatively rapidly than that of the higher frequency range. Now from the above statistics it can be assumed that if the data sets cover the relatively rapid changing region will give accurate result.

Finally it can be said roughly that to get actual result the frequency range should be as large as possible and it must cover the relatively rapid changing (both magnitude and angle) region.

4.4. Effect of frequency interval : If the number of data sets are increased within a definite frequency band, it has been found that the result does not improve remarkably. As an example- the following result was observed . Frequency range was between 5Hz to 770Hz. Three frequency intervals of data were taken (10 Hz, 15 Hz, 20 Hz) .The transfer function was taken as

$$\frac{1+0.3097\times10^{-3}S+0.283\times10^{-7}S^2}{1+0.312\times10^{-1}S+0.35776\times10^{-4}S+0.104\times10^{-7}S^3}$$

Table 4.7

Showing the effect of frequency interval.

Constant Coefficients	Actual Value of constants	Constant at different frequency interval		
		10Hz	15Hz	20Hz
A_0	1.00	0.999999	0.999999	1.000005
A_1	0.30971×10^{-3}	0.30972×10^{-3}	0.30972×10^{-3}	0.30976×10^{-3}
A_2	0.283×10^{-7}	0.283×10^{-7}	0.283×10^{-7}	0.283×10^{-7}
A_3	0.1	0.424×10^{-15}	0.45×10^{-15}	0.18×10^{-14}
B_0	1.00	1.00	1.00	1.00
B_1	0.312×10^{-1}	0.3120×10^{-1}	0.3120×10^{-1}	0.3120×10^{-1}
B_2	0.35776×10^{-4}	0.37577×10^{-4}	0.37577×10^{-4}	0.3757×10^{-4}
B_3	0.104×10^{-7}	0.104×10^{-7}	0.104×10^{-7}	0.104×10^{-7}

CHAPTER 5

EFFECT OF ERROR IN THE SAMPLED FREQUENCY RESPONSE

5.1. Introduction : The effect of different levels of error in the sampled frequency response on the fitted transfer function has been studied in this chapter with the help of few examples. The frequency responses are generated from the transfer functions and errors have been introduced randomely within a certain level. Here three error levels $\pm$ 1%, $\pm$ 2% and $\pm$ 3% have been studied. The results are given in tabular form.

5.2. EXAMPLES :

a) The transfer function was taken as

$$\frac{1+0.312\times10^{-3}s+0.286\times10^{-7}s^2}{1+0.312\times10^{-1}s+0.379\times10^{-4}s^2+0.105\times10^{-7}s^3}$$

The constant coefficients of the fitted transfer functions at different error levels are shown in the Table 5.1. 20 sets of data within the frequency band 5 Hz to 585 Hz were taken

Table 5.1

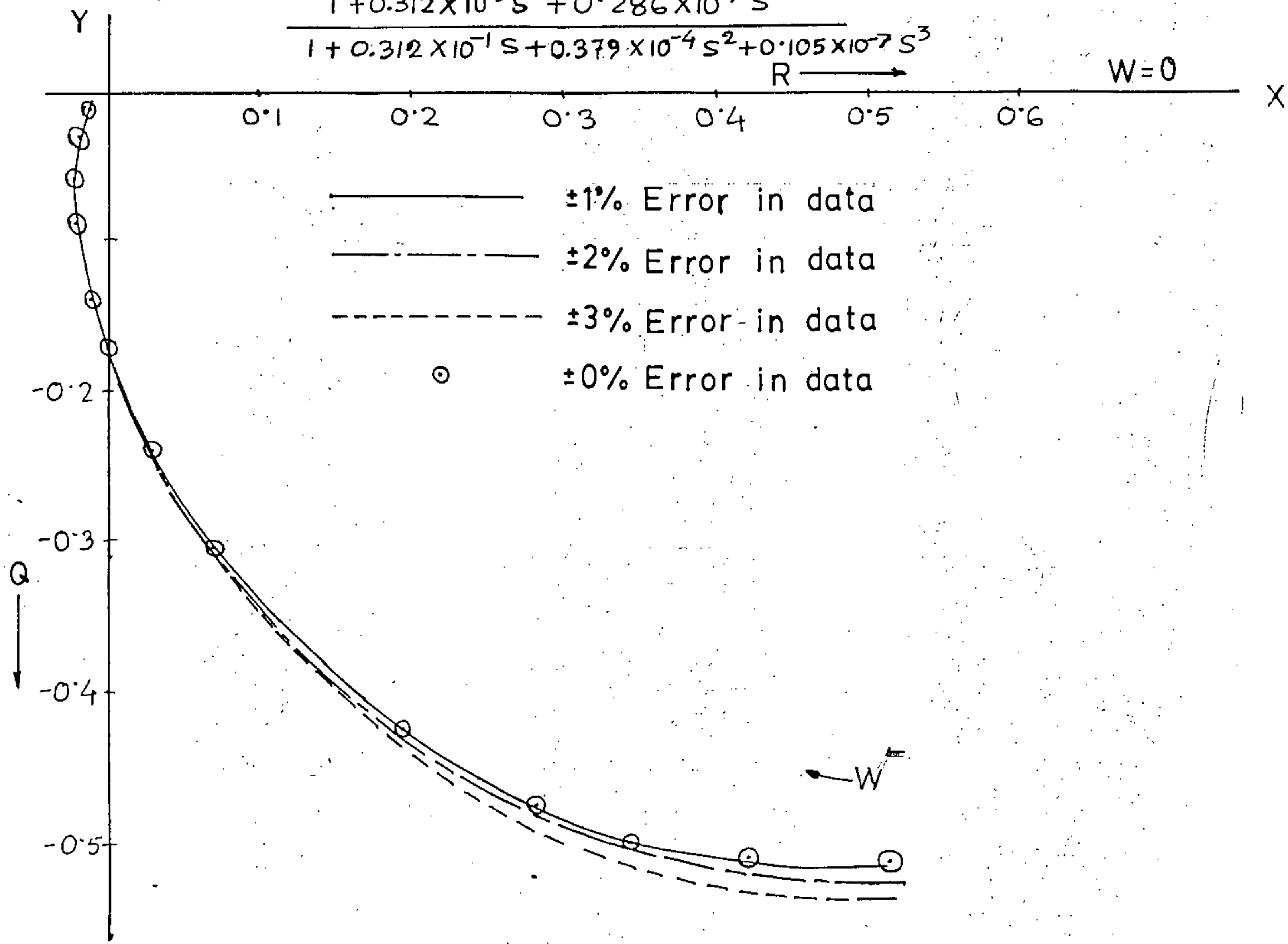
Results showing the effect of error in the sampled for the transfer function

$$\frac{1+0.312\times10^{-3}s+0.286\times10^{-7}s^{2}}{1+0.312\times10^{-1}s+397\times10^{-4}s^{2}+0.105\times10^{-7}s^{3}}$$

Constant coefficients	From original Transfer function	0% error	From 1% error	From 2% error	From 3% error
A_0	1.0	0.999999	1.0052	1.0278	1.0455
A_1	0.312×10^{-3}	0.31177×10^{-3}	0.2103×10^{-4}	-0.21479×10^{-4}	-0.10938×10^{-4}
A_2	0.286×10^{-7}	0.28599×10^{-5}	0.16456×10^{-7}	0.3095×10^{-7}	0.27924×10^{-7}
A_3	0.0	0.0	0.0	0.0	0.0
B_0	1.0	1.0	1.0	1.0	1.0
B_1	0.312×10^{-1}	0.311997×10^{-1}	0.3066×10^{-1}	0.3136×10^{-1}	0.318949×10^{-1}
B_2	0.379×10^{-1}	0.3789×10^{-4}	0.29159×10^{-4}	0.2762×10^{-4}	0.2858×10^{-4}
B_3	0.105×10^{-7}	0.1049×10^{-7}	0.0	0.0	[illegible]

Fig 5.1 Polar plot of frequency respose showing the effect of different levels of error in the frequency response on the transfer function

$$\frac{1+0.312\times10^{-3}s+0.286\times10^{-7}s^2}{1+0.312\times10^{-1}s+0.379\times10^{-4}s^2+0.105\times10^{-7}s^3}$$



From the above Table 5.1 it is found that for all the error (±1%, ± 2%, ± 3%) the fitted transfer function are of 2nd order whether in the original transfer function it was of 3rd order. Though these fitted transfer functions are quantitively incorrect but from the polar plot of the frequency response of fig. 5.1 it is found that the results are qualititively acceptable

b) Transfer function was taken as

$$\frac{1+0.3\times10^{-2}s+0.3\times10^{-5}s^{2}}{1+0.3\times10^{-1}s\;\;0.4\times10^{-4}s^{2}+0.5\times10^{-5}s^{3}}$$

The constant coefficients of the fitted transfer functions at different error levels are shown in the Table 5.2. 20 set of data within the frequency band 5 Hz to 585 Hz were taken.

TABLE 5.2

Results showing the effect of error in the sampled data for the transfer function

$$\frac{1+ 0.3\times10^{-2}S + 0.3\times10^{-5}S^2}{1+ 0.3\times10^{-1}S + 0.4\times10^{-4}S^2 + 0.5\times10^{-5}S^3}$$

Constant coefficient	From original Transfer function	From ±0% error	From ±1% error	From ± 2% error	From ± 3% error
A_o	1.0	1.0	0.99936	1.01487	1.1042
A_1	0.3×10^{-5}	0.30×10^{-2}	0.3012×10^{-2}	0.2871×10^{-2}	0.349967×10^{-2}
A_2	0.3×10^{-5}	0.30×10^{-5}	0.302×10^{-5}	0.3×10^{-5}	0.348338×10^{-5}
A_3	0.0	0.0	0.0	0.0	0.0
B_o	1.0	1.0	1.0	1.0	1.0
B_2	0.4×10^{-4}	0.4×10^{-4}	0.4824×10^{-4}	0.45794×10^{-4}	-0.1567×10^{-4}
B_3	0.5×10^{-5}	0.5×10^{5}	0.502×10^{-3}	0.494×10^{-5}	0.5691×10^{-5}

From the Table 5.2 it is seen that the fitted transfer functions are of 3rd order as they were. Upto $\pm$ 2% error the fitted transfer function is quantitively acceptable. It is also observed that as the magnitude of the constant coefficients increases, the effect of error on the fitted transfer function is lesser.

c) Transfer function was taken as

$$\frac{1 + 0.03S + 0.0004S^2}{1 + 0.02\,S + 0.0005\,S^2}$$

The constant coefficients of the fitted transfer function at different error levels are shown in Table 5.3, 20 sets of data within the frequency band 5 Hz to 100 Hz were taken.

Table 5.3

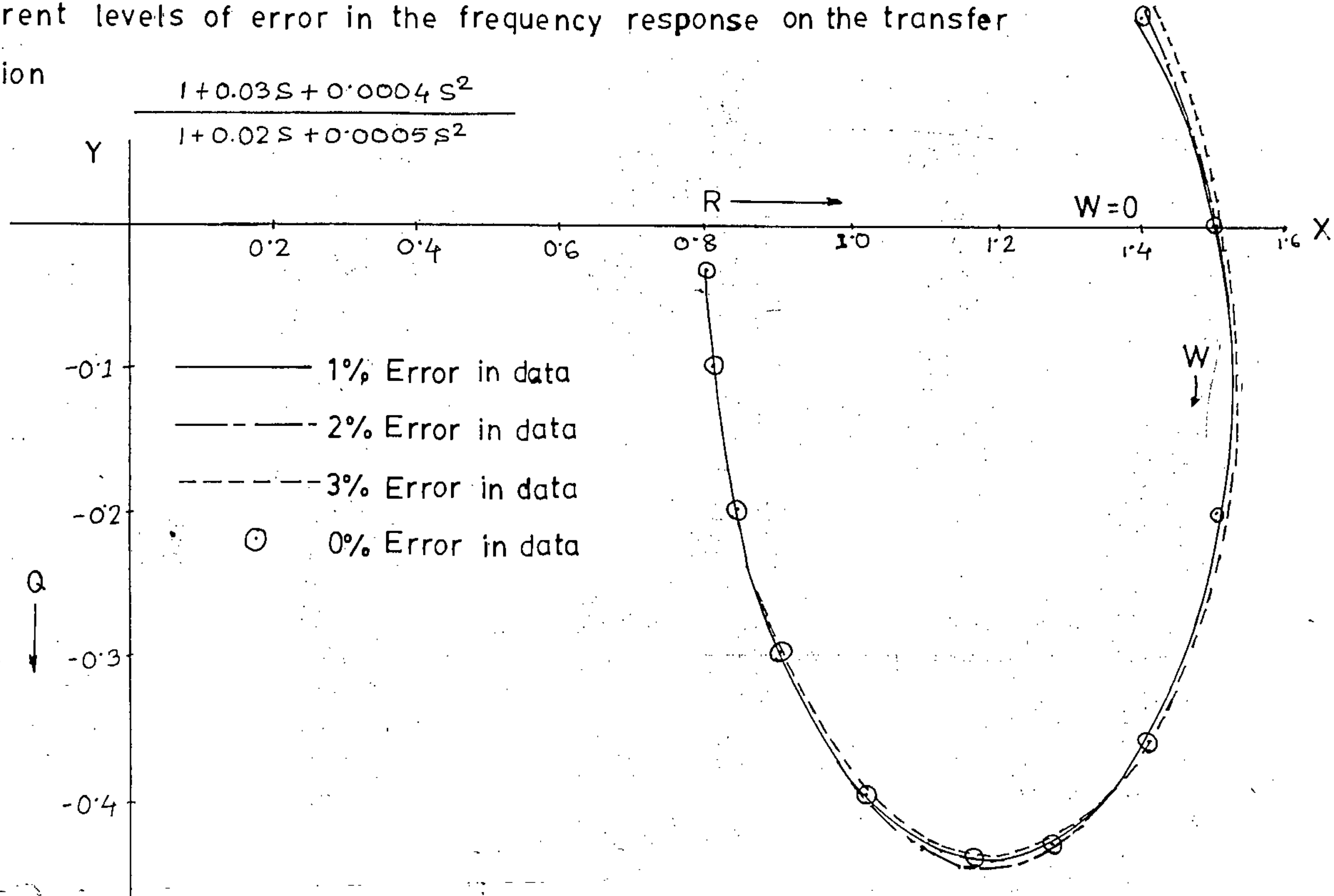
Results showing the effect of error in the sampled data for the transfer function

$$\frac{1 + 0.03S + 0.0004S^2}{1 + 0.02S + 0.0005S^2}$$

Constant coefficients	From original transfer function	From 0% error	From ± 1% error	From ± 2% error	From ± 3% error
A_o	1.0	1.0	0.9934	0.99588	0.9777
A_1	0.03	0.03	0.03088	0.03091	0.0327
A_2	0.0004	0.004	0.000413	0.000413	0.00044
B_o	1.0	1.0	1.0	1.0	1.0
B_1	0.02	0 02	0.02049	0.02037	0.0215
B_2	0.005	0.005	0.000517	0.0005159	0.0005505

Fig 5.2 Polar plot of frequency respose showing the effect of different levels of error in the frequency response on the transfer function

$$\frac{1+0.03S+0.0004S^2}{1+0.02S+0.0005S^2}$$



From the above Table 5.3 it is seen that the results are acceptable . The Polar plot of the frequency responses for these fitted transfer function is given in fig. 5.3

5.3. Conclusion : From the above examples it is clear that though ± 3% error even ± 1% error may change order of the fitted transfer function, but still the result is qualititively acceptable. The deviation of the values of the constants of the fitted transfer function with error from that of the noise free case depends on the error level as well as the relative magnitudes of the constants.

CHAPTER 6

MEASUREMENT OF DATA

6.1. <u>Selection of circuits:</u> Different electrical circuits of different orders were constructed using resistors and capacitors their circuit diagrams and the theoritical callulation procedure of transfer function are given in the Appendix B

6.2. <u>Instrument used :</u> The instruments used for carrying out the experiment were- Phasemeter, Oscilloscope, Digital multimeter, frequency generator, frequency counter.

6.3. <u>Difficulties of the phase meter:</u> The Phasemeter was of model Type 2971. It can measure phase difference in both radians and degrees. It has two voltage selective circuit -1) 5V to 10mv and other was 2) 15 V to 30mv. The general procedure of measuring phase difference is to keep the voltage level of the points from where phase difference will be measured as high as possible within the selective voltage range.

It is found that the voltage level effects the phase reading badly. The higher the voltage level, the higher the phase reading (the more accurate reading). This voltage characteristics of the phase- meter made the experiment impossible for two reasons:

1) In higher frequency (about 100 Hz to above depending on loading condition) the frequency generator could not effort the minimum required voltage level (10mV to 5V range), so the phase reading found was lower than that of the actual value.

2) The output voltage of a circuit is generally much less than the input voltage. Though in some case the input voltage could be kept in the required level, but the output voltage was much less. So obviously error in reading occurs.

Owing to these reasons, to determine phase difference 'Lissajou's figure' has been used.

6.4. Lissajou's pattern: A Lisajou's pattern[4] is the figure created on an Oscilloscope screen when sine-wave potentials are applied to both the horizontal and vertical deflecting plates. If the frequency of these two component potentials are the same but they differs in phase, the resulting pattern is a measure of the phase difference between the two waves and the ratio of the output and input voltages (gain) at that point.

To see that this is so, let us suppose that the potential across the horizontal deflecting plates of the Oscilloscope is denoted as

$$e_x = E_1 \text{Sin}\,(\omega t + \theta_1) \cdots\cdots \quad (6.1)$$

and that accross the vertical deflecting plate is given by:

$$e_y = E_2 \text{ Sin } (wt + \theta_2) \quad \ldots\ldots \quad (6.2)$$

i.e. ,

$$\frac{e_x}{E_1} = \text{Sin wt Cos } \theta_1 + \text{Cos wt Sin } \theta_1 \quad \ldots \quad (6.3)$$

and $\frac{e_y}{E_2} = \text{Sin wt Cos}\,\theta_2 + \text{Cos wt Sin}\,\theta_2$ ..... (6.4)

To el minate the time factor wt, let us multiply the first equation by Cos θ_2 and the second equation by Cos θ_1. Then we get -

$$\frac{e_x}{E_1}\,\text{Cos}\,\theta_2 = \text{Sin wt. Cos}\,\theta_1 \text{Cos}\,\theta_2 + \text{Cos wt Cos}\,\theta_2\,\text{Sin}\,\theta_1 \quad (6.5)$$

and $$\frac{e_y}{E_2}\,\text{Cos}\,\theta_1 = \text{Sin wt Cos}\,\theta_1\,\text{Cos}\,\theta_2 + \text{Cos wt Cos}\,\theta_1\,\text{Sin}\theta_2 \quad (6.6)$$

Subtracting equation (6.6) to (6.5) we get

$$\frac{e_x}{E_1}\,\text{Cos}\,\theta_2 - \frac{e_y}{E_2}\,\text{Cos}\,\theta_1 = \text{Cos wt(Sin}\,\theta_1\,\text{Cost}\,\theta_2 - \text{Cos}\theta_1\,\text{Sin}\theta_2 \quad (6.7)$$

Similarly, multiplying the first by Sin θ_2 and the second by Sin θ_1 and subtracting we get -

$$\frac{e_x}{E_1}\,\text{Sin}\theta_2 - \frac{e_y}{E_2}\,\text{Sin}\,0_1 = (\text{Cos}\,\theta_2 - \text{Sin}\,\theta_1\,\text{Cos}\,\theta_2).\,\text{Sin wt} \quad \text{....}(6.8)$$

equation (6.7) and (6.8) can be written as

$$\frac{e_x}{E_1}\,\text{Cos}\,\theta_2 - \frac{e_y}{E_2}\,\text{Cos}\,\theta_1 = -\,\text{Cos wt Sin}\,(\theta_2 - \theta_1) \quad \text{....}(6.9)$$

$$\frac{e_x}{E_1} \sin\theta_2 - \frac{e_y}{E_2} \sin\theta_1 = \sin wt \sin(\theta_2 - \theta_1) \quad ....(6.10)$$

Squaring and adding equation (6.9) and (6.10) we get -

$$\frac{e_x^2}{E_1^2} + \frac{e_y^2}{E_2^2} - \frac{2e_x e_y}{E_1 E_2} \cos(\theta_2 - \theta_1) = \sin^2(\theta_2 - \theta_1) \quad ....(6.11)$$

This is the equation of an ellipse whose principal axes coincide with the coordinate axes when $\theta_2 - \theta_1 = \pi/2$. Hence in most cases the result is an ellipse (except $\Delta\theta = 0$ & $\Delta\theta = \pi$), the orientation of which depends upon the phase difference between the two waves.

The results have the form illustrated in Fig. 6.1

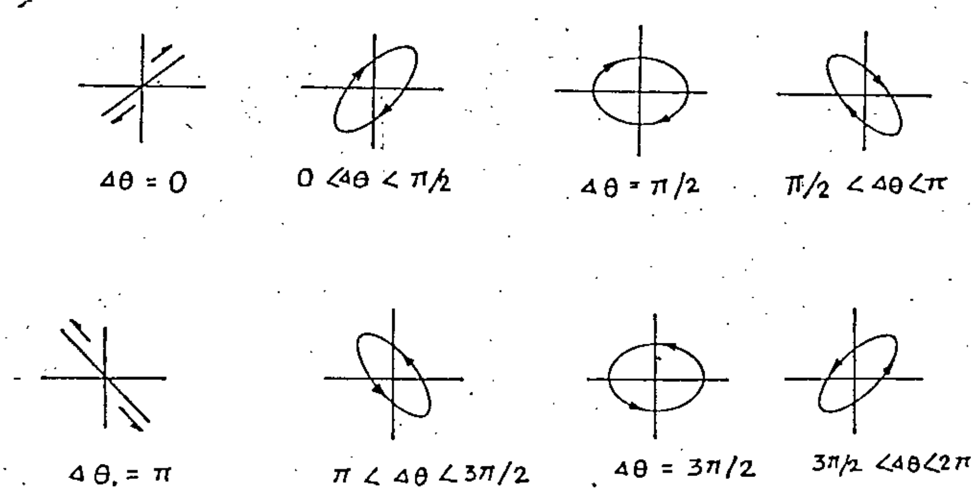


Fig. 6.1 : Form of ellipse at different phase difference.

If the amplitudes of the potential applied to the vertical and horizontal deflecting plates are equal then the pattern at phases $\Delta\theta = \pi/2$ and $\Delta\theta = 3\pi/2$ will be circular.

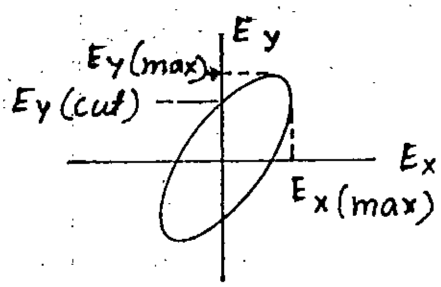


Fig. 6.2. General form of an ellipse for a Particular considion .

The experimental procedure necessary for measuring the phase difference and gain between the two potential is quite direct and consists in measuring the three distances $E_{y(cut)}$, $E_{y\,(Max)}$ and $E_{x(max)}$.

Obviously, gain = $\dfrac{E_y(max)}{E_x(max)}$ ... (6.12)

$E_{y(cut)}$ is the value of e_y at the point when $e_x = 0$. Then from equation (6.11)

$$E_{y(cut)} = E_2 \mathrm{Sin}(\theta_2 - \theta_1) \quad .... \quad (6.13)$$

Again E_2 is the maximum value of e_y

$$E_2 = E_{y\,(max)}$$

$$\sin(\theta_2 - \theta_1) = \frac{E_{y(cut)}}{E_{y(max)}}$$

$$\theta_2 - \theta_1 = \sin^{-1} \frac{E_{y(cut)}}{E_{y\,(max)}} \quad \ldots\ldots\ldots (6.14)$$

The shape and rotation of the ellipse illustrated in Fig. 6.1 can be explained graphically[6]. when two sinusoidal voltages of equal frequency which are in phase with each other are applied to the horizontal and vertical diflecting plates, the pattern appearing on the screen is a straight line as is clear from Fig- 6.3.

Thus when two equal voltages of equal frequency but with 90° phase displacement are applied to a CRO, the trace on the sereen is a circle. This is shown in Fig. 6.4.

When two equal voltage of equal frequency but with a phase shift (Not equal to 0° or 90°) are applied to a CRO we obtain an elipse as shown in Fig 6.5. An ellipse is also obtained when unequal voltages of same frequency are applied to the CRO.

A number of conclusions can be drawn from the above discussions when two sinusoidal voltages of same frequency are applied:

i) A straight line results when the two volteges are equal and are either in phase with each other ,or 180 out of phase

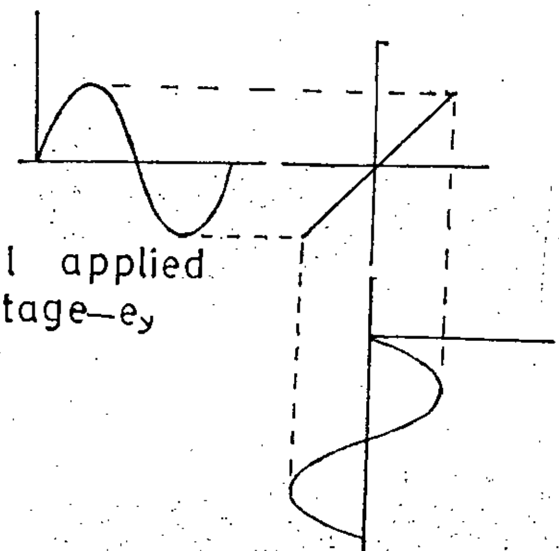


Fig 6·3 Lissajou's pattern when phase difference is zero and equal voltage is applied.

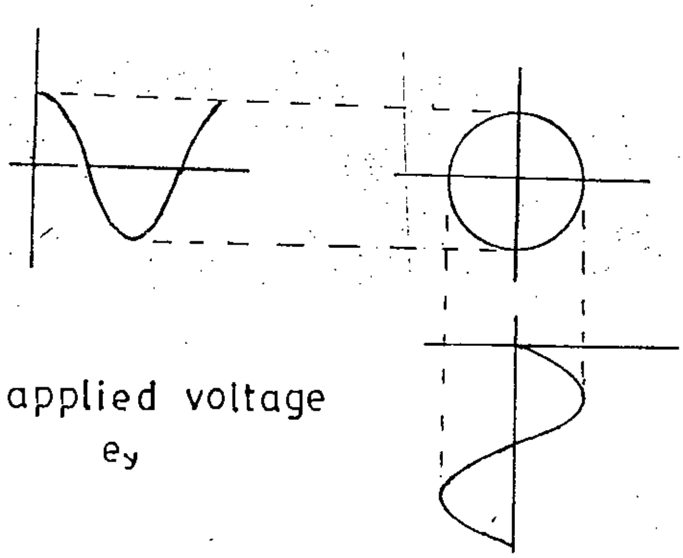


Fig 6·4 Form of Lissajou's pattern when phase difference is $\pi/2$ and equal voltage is applied with each other . The angle formed with the horizontal is 45° when the magnitudes of voltages are equal. An increase in the vertical deflection voltage causes the line to have an angle greater than 45° with the horizontal.

ii) Two sinusoidal waveforms of the same frequency produce a Lissajou's pattern, which may be a straight line, a circle or an ellipse depending upon the phase and magnitude of the voltages.

A circle can be formed only when the magnitude of the two signals are equal and the phase difference between them is either 90° or 270°. However, if the two voltages are not equal and/or out of phase an ellipse is formed .If the Y voltage is is larger, an ellipse with vertical major axis is formed while if the x plate voltage has a greater magnitude, the major axis of the ellipse lies along horizontal axis.

iii) It is clear from Fig. 6.6 that for equal voltages of same frequency progressive variation of phase voltage causes the pattern to vary from a straight diagonal line to ellipses f different ecentricities and then to a circle, after that through another series of ellipses and finally a diagonal straight line again .

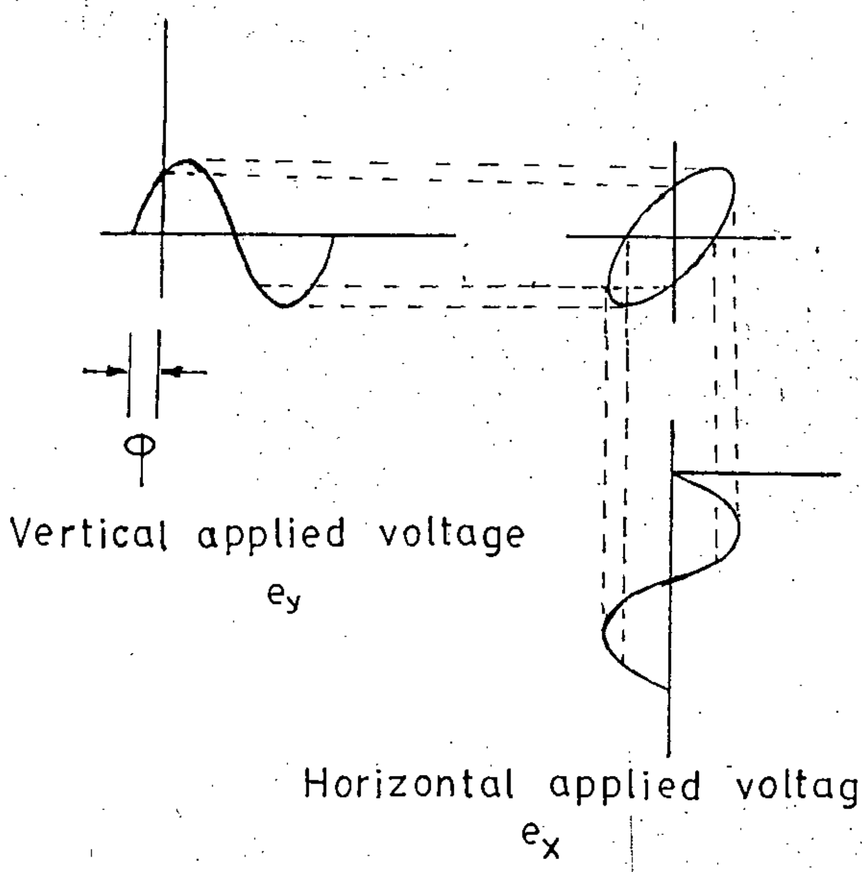


Fig 6.5 Shape of an ellipse at a phase difference φ

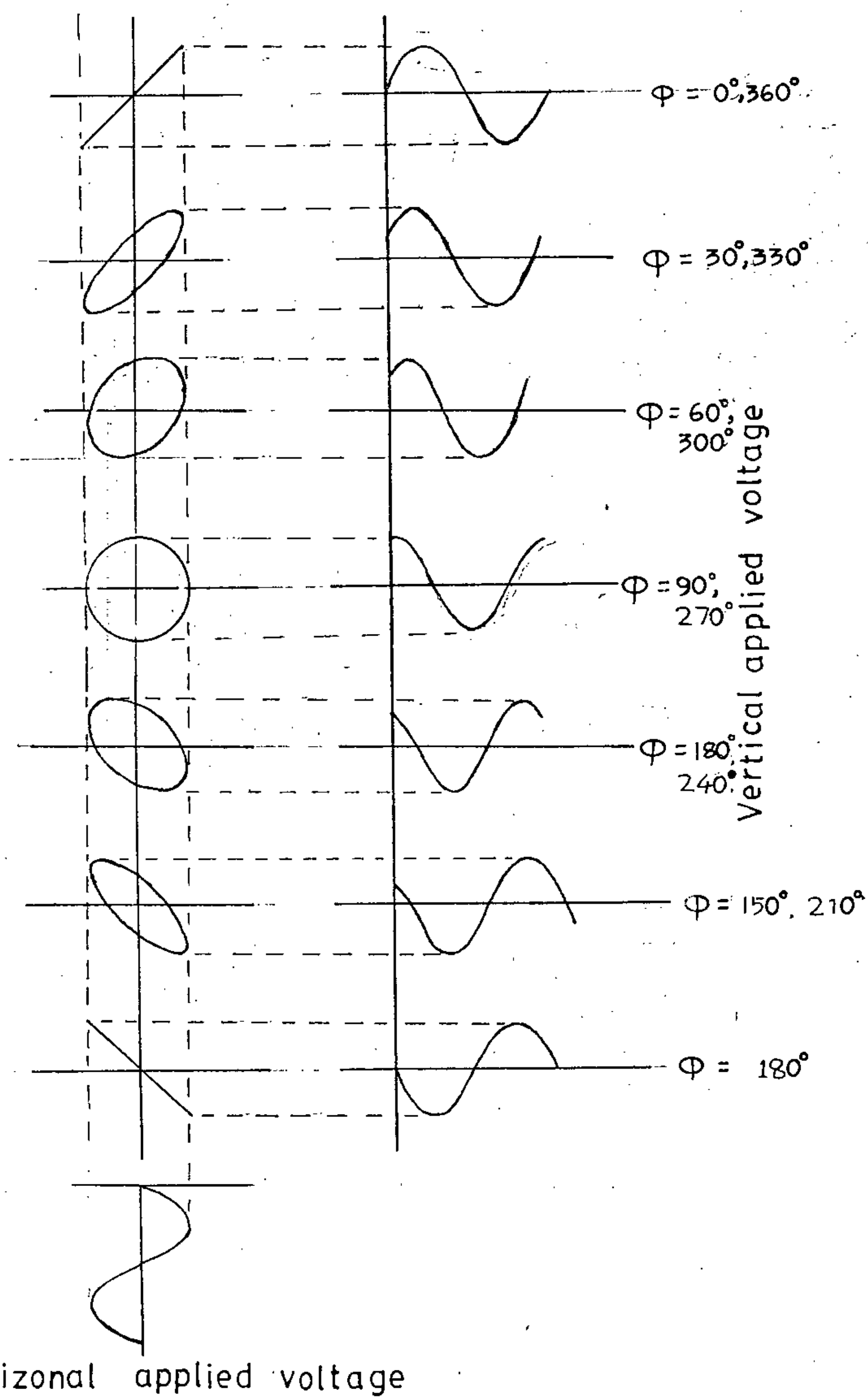


Fig 6.6 Forms of Lissajou's patterns at different conditions when equal voltages are applied.

The rotation of figure 6.1 can be explain easily from fig. 6.6.

When $0 < \Delta\theta < \pi/2$ and the voltage at horizontal axis is zero., then voltage on vertical axis has some +Ve value corresponds to the phase difference. As voltage on X axis increases, then the resultant voltage voltage vector starts to move towards the +ve X- axis . So rotation takes place as indicated in fig. 6.1 Similarly when $3\pi/2 < \Delta\theta < 2\pi$ the same form of ellipse occurs but the rotation is in opposite direction that of for $0 < \Delta\theta < \pi/2$. In this case when voltage on X-axis is zero then voltage on y-axis has some -ve voltage. As voltage on X -axis increase the resultant voltage vector starts to move towards the +ve x-axis, and form the rotation as in fig. 6.1. for all conditions the rotation of the ellipse follows the rotation of Fig. 6.1

6.5 <u>Measurement procedure</u> : The input was connected to the frequency generator and the horizontal axis of the Oscilloscope. The out-put was connected to the vertical axis of the Oscilloscope properly. Then varying the input frequency and voltage level, data were taken. The Lissajou's figures were made as large as possible on the Oscilloscope screen within its callibration so that full scale readings could be taken. Voltage readings were also taken by digital multimeter as well as in the Oscilloscope.

6.6. Errors in measurement :The possible reasons for which error may include in the data were -

i) The data was taken by graphical method.So personal error may be encounteed.

ii) The frequency generator generates higher harmonics especially at low frequency range. The use of filter lowered the input voltage by such a low level that the output voltage was very diffiult to measure. So without using filter at low frequencies average of several sets of readings were taken.

iii) At some frequencies the attern of Lissajou's figure was not steady. At these conditions reading were taken estimating the midpoint. To minimize this error, different sets of readings were taken at different occasion and if steady readings were not found, an average of those sets had been taken.

6.7. Measurement of cricuit parameters: To check the accuracy of the fitted transfer functions of different electrical circuits obtained from their frequency responses using generalized complex curve fitting method,it is necessary to know their theorecal transfer functions. To obtain the theoretical transfer function it is necessary to know the values of the circuit parameters accurately. The parameters were resistors and capacitors. The resistance, capacitance and the internal resistance of a capacitor were obtained by the following procedure . It has been observed that the rated values especially in the case of a capacitor varies greatly.

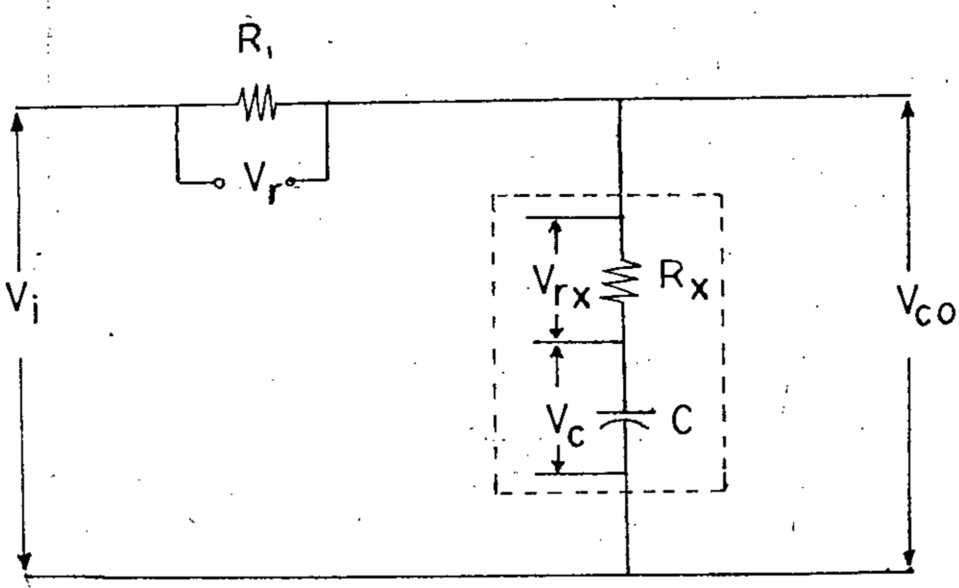


Fig. 6.7. : R-C circuit showing the internal resistance of a capacitor.

V_i = input voltage

V_r = voltage accross the resistance

V_{co} = Output voltage
= voltage accross the terminals of the capactor.

V_{rx} = Voltage drop due to the internal resistance of the capacitor

V_c = Actual voltage drop for the capacitor.

If there is no internal resistance in the capacitor then.

$$V_i^2 = V_r^2 + V_{co}^2 \qquad (6.15)$$

But it was found that

$$V_i^2 > V_r^2 + V_{co}^2. \qquad (6.16)$$

Then it is evident that there is an internal resistance component in the capacitor. The vector diagram of the voltage drops at different sections is shown in Fig 6.8.

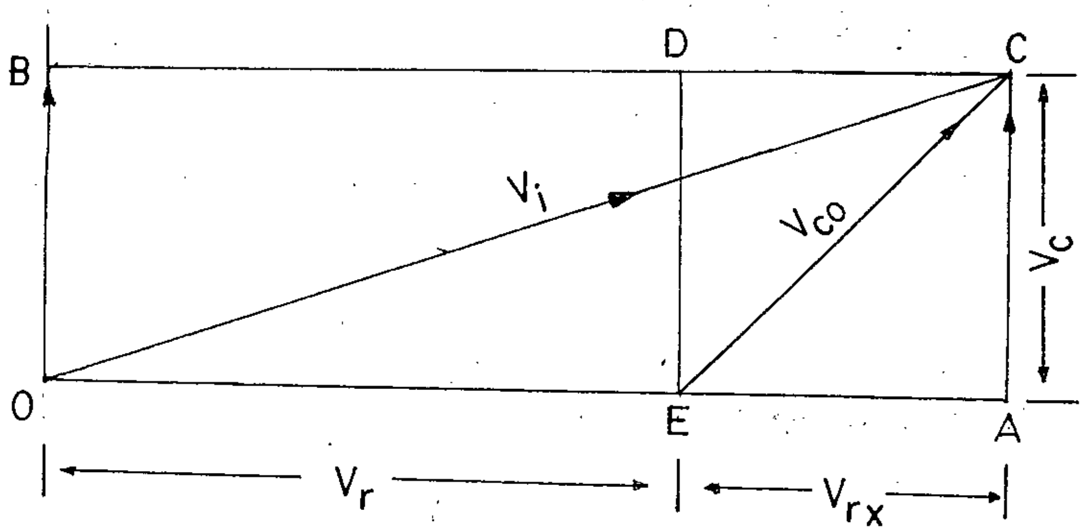


Fig 6.8 Vector diagram showing the different voltage drops at different sections of a R-C circuit.

Here,

$OC = V_i$ = input voltage

$EC = V_{co}$ = Voltage accross the terminals of the capacitor.

$OB = AC = V_c$ actual voltage drop for the capacitor

EA = Voltage drop due to the internal resistance of the capacitor.
$= V_{rx}$

The resistive drop V_{rx} can be isolated from V_{co} by the following trial and error method.

$$V_{co}^2 = V_c^2 + V_{rx}^2 \qquad (6.17)$$

let, $$V_x = \sqrt{V_r^2 + V_{co}^2} \qquad (6.18)$$

i.e. $$V_r^2 + V_{co}^2 = V_x^2 \qquad (6.19)$$

Since there is resistive drop component in V_{co}, an small amount of voltage E is subtrated from V_{co} vectorically and added with V_r algebrically .

For conveniant of operation E is taken as

$$E = \frac{V_i - V_x}{2} \qquad (6.20)$$

let, $V'_{co}(1) = \sqrt{V_{co}^2 - E^2}$ (6.21)

and $V'_{r(1)} = V_r + E$ (6.22)

Then again it is checked that whether

$V_i^2 > V_{co}^2(1) + V_r'^2(1)$

If it is so, the same produre will be repeated until $(V_i - \sqrt{V'^2_{co(n)} + V'^2_{r(n)}})$ falls behind a predetermined small value (s 0. 0001) . Where n is the iteration no.

$$\text{Then } V_{rx} = V'_{r(n)} - V_r \quad (6.23)$$

$$V_c = V'_{co(n)} \quad (6.24)$$

Now, R, R_x and C can be obtained if the current, I and the corresponding frequency, f is known.

$$R = \frac{V_r}{I} \quad (6.25)$$

$$R_x = \frac{V_{rx}}{I} \quad (6.26)$$

$$C = \frac{1}{2\pi f V_c} \quad (6.27)$$

CHAPTER 7

DETERMINATION OF TRANSFER FUNCTIONS OF DIFFERENT ELECTRICAL CIRCUITS

7.1 Introduction: Different circuits of 1st order to 3rd order have been constructed using resistors and capacitors of different values . The values of the resistores, capacitores and the internal resistances of the capacitors have been calculated from experimental data using the technique explained in section 6.7. With these values, their theoretical transfer functions were calculated (The calculation procedures have been given in Appendix B).

The frequency response of these circuits have been obtained experimentally using Lissajou's pattern and they are given in tabular form in Table 7.1, to Table 7.5.With these frequency responses fitted transfer functions have been determined using the generalized complex curve fitting method.

7.2. Practical Examples :

1) 1st example (1st order system):

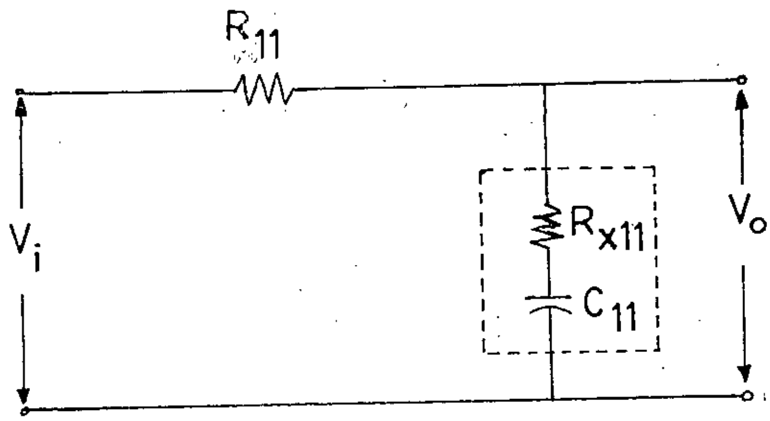


Fig. 7.1 1st order circuit

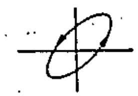
$R_{11} = 97.53\,\Omega$, $R_{x11} = 1.47\,\Omega$, $C_{11} = 83.92\,\mu f$

The theoretical transfer function was calculated as

$$\frac{1.0 + 0.000123S}{1.0 + 0.00831S}$$

Experimental data for the 1st order system from Lissajous patterns

$$\frac{1 + 0.000123S}{1 + 0.00831S}$$

No. of Obs.	Frequency	Vertical Axis Readings			Horizontal Axis Readings		Voltage Readings volts		Rotation	Gain	Phse difference
	Hz	S/D	Y cut	Y max	S/D	X max	Outpur (V_o)	Inpur (V_i)			
1	5	2V/D	1.1	4.0	2V/D	4.2	-	-		0.9524	-15.96
2	6	+	1.34	4.0	"	4.3	-	-	,,	0.9302	-19.57
3	7	"	1.5	4.0	"	4.4	-	-	,,	0.9091	-22.04
4	8	"	1.25	3.0	"	3.35	-	-	,,	0.8955	-24.62
5	9	IV/D	1.8	4.0	IV/D	4.5	-	-	,,	0.8889	-26 74
6	10	11	1.96	4.0	"	4.6	-	-	,,	0.8696	-29.34
7	12	"	2.17	4.0	"	4.8	-	-	,,	0.8333	-32.85
8	15	"	1.85	3.0	"	3.9	-	-	,,	0.7692	-38.07
9	20	"	2.88	4.0	-	-	3.00	4.425	,,	0.6780	-4605
10	25	0.5V/D	3.1	4.0	-	-	1.463	2.433	,,	0.6013	-50.81
11	30	"	3.3	4.0	-	-	1.485	2.783	,,	0.5336	-55.59
12	40	"	3.55	4.0	-	-	1.448	3.368	,,	0.4299	-6256
13	50	0.2V/D	3.65	4.0	-	-	0.575	1.61	,,	0.3571	-65.85

contd....

TABLE 7.1 (continued from last page)

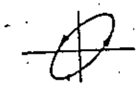
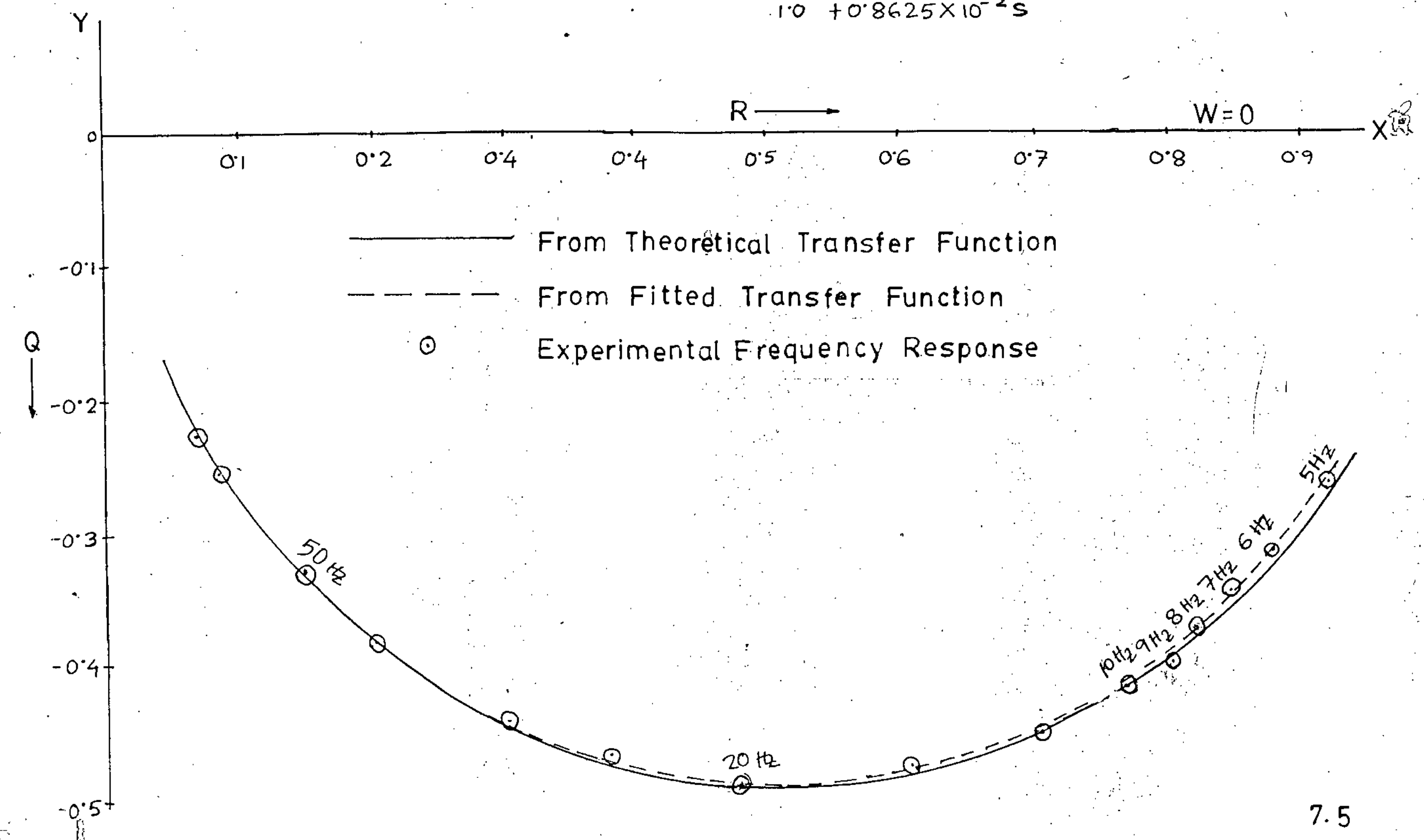
No. of obs.	Frequency	Vertical Axis Reading			Horizontal Axis Reading		Voltage reading volts		Rotation	Gain	Phase difference in degrees
	Hz	S/D	Y cut	Y max	S/D	X max	out put V_o	Input V_i			
14	60	0°sV/D	3.75	4.0	-	-	0.574	1.867		0.3074	-69.64
15	70	"	3.8	4.0	-	-	0.567	2.103	"	0.2696	-71.81
16	80	"	3.83	4.0	-	-	0.571	2.395	"	0.2384	-73.24
17	100	"	3.86	4.0	-	-	0.560	2.898	"	0.1932	-74.80
18	120	"	3.90	4.0	-	-	0.562	3.460	"	0.1624	-77.16
19	150	"	3.92	4.0	-	-	0.282	2.163	"	0.1304	-78.52
20	200	"	3.93	4.0	-	-	0.284	2.858	"	0.0994	-79.27

Fig 7.2 Polar plot of frequency response showing the accuracy of the fitted transfer function $\frac{0.99 + 0.1279 \times 10^{-3} s}{1.0 + 0.8625 \times 10^{-2} s}$



The fitted transfer function by this method was found as -

$$\frac{0.990 + 0.0001279S}{1.0 + 0.008625S}$$

The result is found to be quite satisfactory ,

The polar plot of frequency response has been shown in Fig. 7.2.

ii) 2nd Example (1st order system):

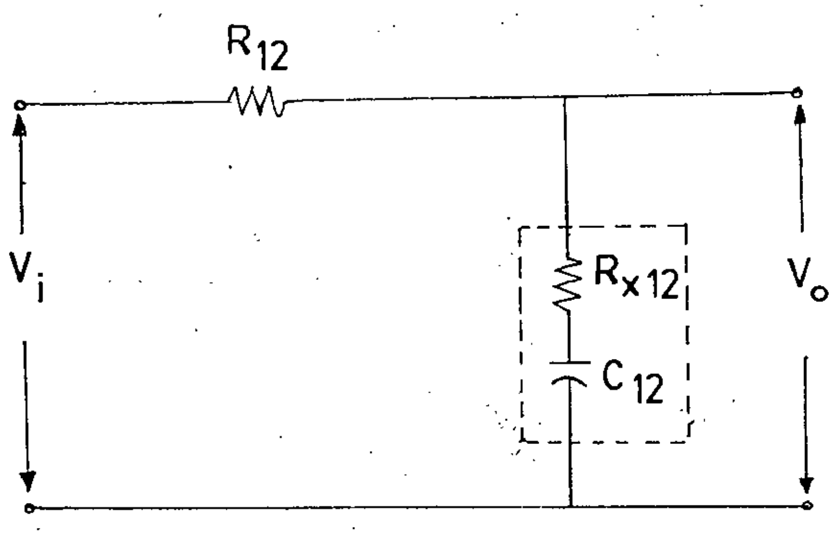
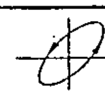


Fig 7.3 1-st order circuit .

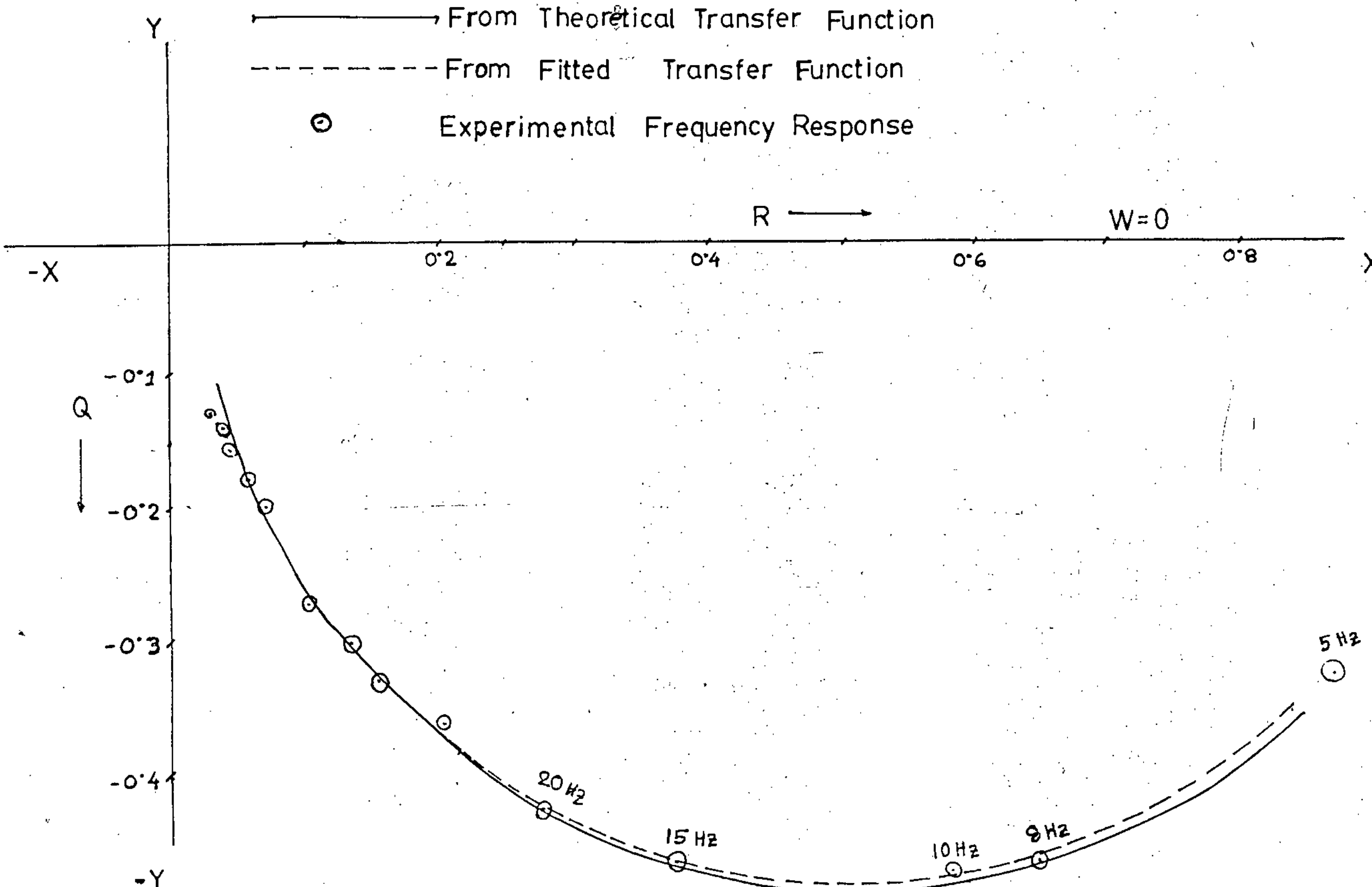
Table 7.2

Experimental data from Lissajous pattern for the 1st order system

$$\frac{1+0.000363S}{1+0.01385S}$$

No. of obs.	Frequency Hz	Vertical Axis Readings S/D	Vertical Axis Readings Y cut	Vertical Axis Readings Y max	Horizonal axis readings S/D	Horizonal axis readings X max	Rotation	Gain	Phase difference Degree
1	5	2V/D	1.4	4	2V/D	4.3		0.930	-20.5
2	8	"	2.21	4	"	5	,,	0.800	-35.5
3	10	"	2.35	3.7	"	5	,,	0.740	-39.4
4	15	"	2.32	3	"	5	,,	0.600	-50.7
5	20	"	2.0	2.4	"	4.7	,,	0.511	-56.4
6	25	IV/D	3.47	4.0	"	4.8	,,	0.417	-60.2
6	30	"	3.3	3.7	"	5.0	,,	0.370	-63.1
8	35	"	3.0	3.3	"	5.0	,,	0.330	-65.4
9	40	"	2.71	2.9	"	5.0	,,	0.290	-69.1
10	50	"	2.3	2.46	"	5.0	,,	0.246	-69.2
11	60	0.5V/D	3.75	4.0	"	4.8	,,	0.208	-69.6
12	70	"	3.5	3.7	"	5.0	,,	0.185	-71.1
13	80	"	3.08	3.2	"	5.0	,,	0.160	-74.3
14	90	"	2.8	2.9	"	5.0	,,	0.145	-74.9
15	100	"	2.58	2.65	"	5.0	,,	0.133	-76.8

fitted transfer function $\dfrac{0.9857 + 0.0003325S}{1 + 0.01331S}$



Here

$$R_{12} = 967.5\,\Omega \;,\; R_{x12} = 26.089\,\Omega \;,\; C_{12} = 13.93\,\mu f$$

The theoretical transfer function is calculated as

$$\frac{1 + 0.000363s}{1 + 0.01385s}$$

The fitted transfer function with this method was found as

$$\frac{0.9857 + 0.0003325s}{1.0 + 0.01331s}$$

The polar plot of the frequency response of these transfer functions have been shown in fig. 7.4.

The result is found to be quite satisfactory.

iii) 3rd Example (2nd order system):

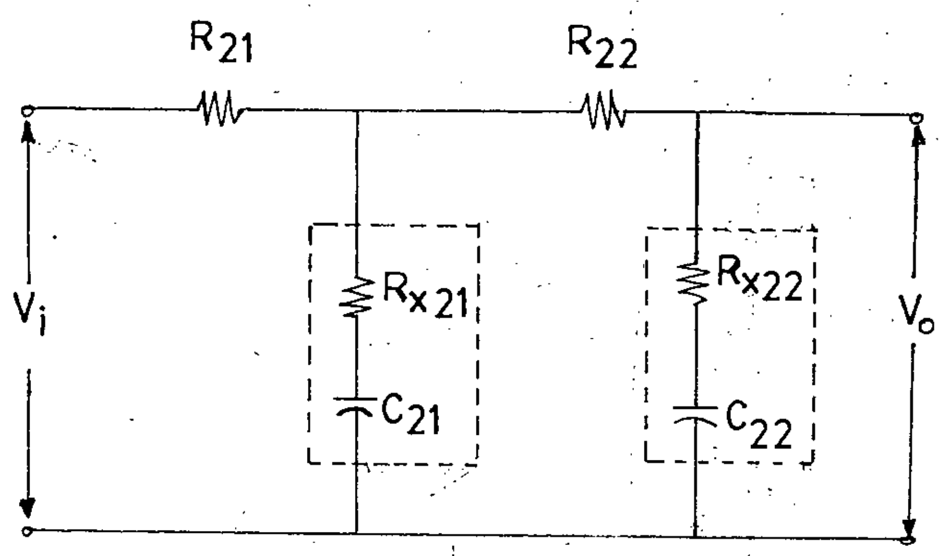


Fig. 7.5 : 2nd order circuit

a) In this case the parameters were as follows :

$R_{21} = 92.53\,\Omega$, $R_{x21} = 1.47\,\Omega$, $C_{21} = 83.92\,\mu f$

$R_{22} = 97.00\,\Omega$, $R_{x22} = 2.27\,\Omega$, $C_{22} = 65.8\,\mu f$

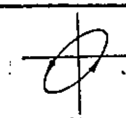
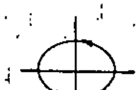
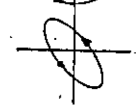
The theoretical transfer function is calculated as

$$\frac{1 + 0.27272\times 10^{-3}S + 0.184\times 10^{-7}S^2}{1 + 0.212575\times 10^{-1}S + 0.5506\times 10^{-4}S^2}$$

TABLE 7.3

Experimental data from Lissajou's patterns for the 2nd order system

$$\frac{1+0.27272\times10^{-3}S+0.184\times10^{-7}S^{2}}{1+0.212575\times10^{-1}S+0.5506\times10^{-4}S^{2}}$$

No. of obs.	Freqeency	Vertical Axis Readings			Horizontal Axis readings		Rotation	Gain	Phase different
	Hz	S/D	Y cut	Y max	S/D	X amx			Degree
1	5	0.2V/D	2.5	4.0	0.2V/D	4.8		0.8333	- 38.68
2	6	"	2.7	4.0	"	5	"	0.8000	- 42.45
3	7	"	2.96	4.0	"	5.4	"	0.7407	- 47.73
4	8	"	3.18	4.0	0.5V/D	2.25	"	0.7111	- 52.66
5	9	"	3.35	4.0	"	2.4	"	0.6667	- 58.88
6	10	"	3.5	4.0	"	2.55	"	0.6275	-61.04
7	12	"	3.65	4.0	"	2.82	"	0.5674	-65.85
8	15	"	3.85	4.0	"	3.4	"	0.4706	-74.26
9	22	"	4.0	4.0	"	4.7		0.3404	-90.00
10	30	"	3.9	4.0	1V/D	3.32		0.2462	-102.84
11	40	"	3.68	4.0	"	4.66		0.1717	-113.07
12	50	0.1V/D	3.45	4.0	"	3.2	"	0 1250	-120.40
13	60	"	3.2	4.0	"	4.8	"	0.0833	-126.87
14	70	"	3.00	4.0	"	5.15	"	0.0777	- 131.41

contd.

Table 7.3 (continued from last page)

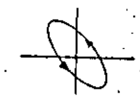
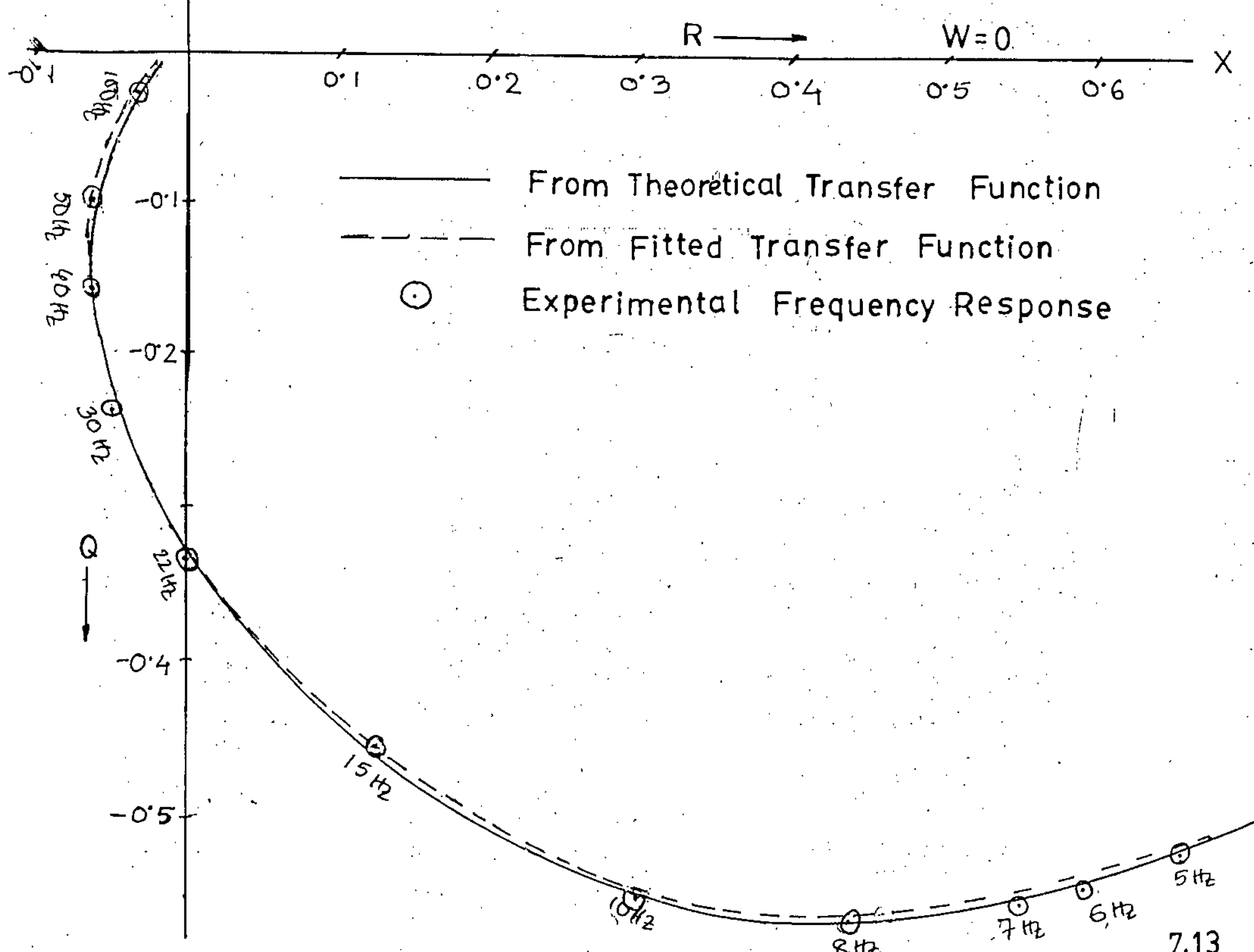
No. of Obs.	Frequency	Vertical Axis Readings			Horizontal Axis Readings		Rotation	Gain	Phase different
	Hz	S/D	Y cut	Y max	S/D	X max			Degree
14	70	0.1V/D	3.00	4.0	1V/D	5.15		0.0777	- 131.41
15	80	50mv/D	2.8	4.0	"	3.3	,,	0.0606	-135.57
16	90	"	2.6	4.0	"	4.0	,,	0.0500	- 139.46
17	100	"	2.45	4.0	"	4.8	,,	0.0417	-142.23
18	120	"	1.65	3.0	"	5.0	,,	0.0300	- 146.63
19	140	20mv/D	2.1	4.0	"	3.45	,,	0.0232	-148.33
20	160	"	1.9	4.0	"	4.4	,,	0.0182	-151.64
21	180	"	1.6	3.6	"	5.0	,,	0.0144	-153.61
22	200	"	1.26	3.0	"	5.0	,,	0.0120	-155.17

Fig 7-6 Polar plot of frequency response showing the accuracy of the fitted transfer function

$$\frac{1.0079 - 0.8\times10^{-4}s - 0.374\times10^{-7}s^2}{1.0 + 0.224\times10^{-1}s + 0.508\times10^{-4}s^2}$$



The fitted transfer function was found as

$$\frac{1.0079 - 0.8 \times 10^{-4}s - 0.374 \times 10^{-7}s^2}{1 + 0.224 \times 10^{-1}s + 0.508 \times 10^{-4}s^2}$$

The result is not very accurate . The order is found to be same as the theoretical transfer function and the constant coefficients of the denominator is nearly accurate. But from the polar plot of the frequency response of fig 7.6 it is seen that the result is qualitatively satisfactory

iv) 4th example (2nd order system)

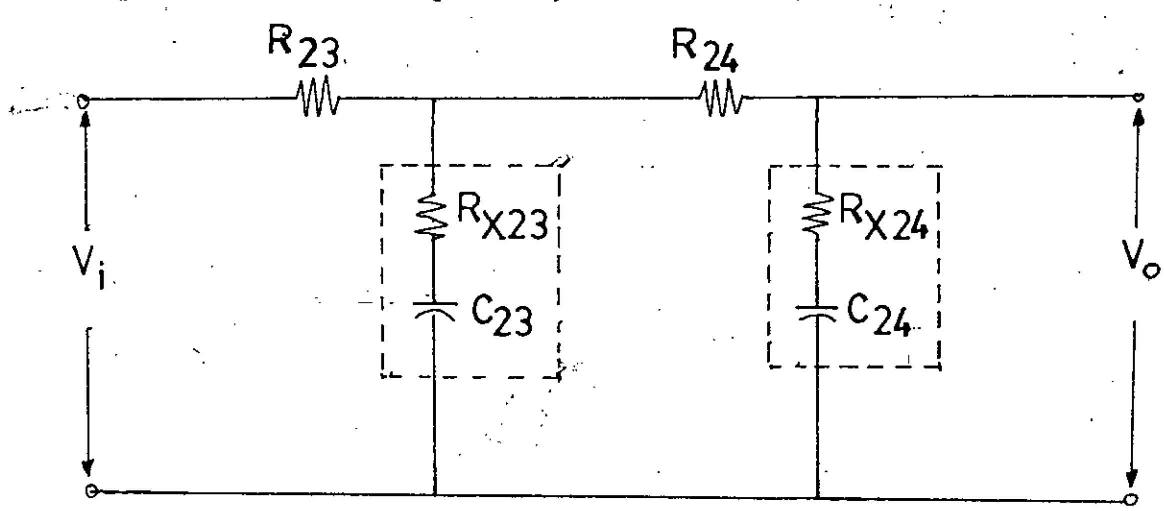


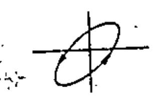
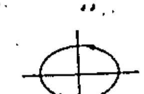
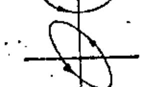
Fig 7.10: 2nd order circuit

$R_{23} = 967.98\,\Omega$, $R_{x23} = 26.09\,\Omega$, $C_{23} = 13.93\,\mu f$

$R_{24} = 974.86\,\Omega$, $R_{x24} = 39.38\,\Omega$, $C_{24} = 12.74\,\mu f$

TABLE 7.4

Experimental data from Lissajou's pattern for the 2nd order system $\dfrac{1+0.865\times10^{-3}S+0.1823\times10^{-6}S^2}{1+0.391\times10^{-1}S+0.1834\times10^{-3}S^2}$

No. of obs.	Frequency	Vertical Axis Readings			Horizontal Axis Readings		Voltage Readings in volts		Rotation	Gain	Phase Difference
	Hz	S/D	Y cut	Y max	S/D	X max	output (V_o)	Input (V_i)			Degree
1	5	IV/D	3.3	4.0	2V/D	3.2	-	-		0.625	-56.4
2	6	"	3.55	4.0	"	3.5	-	-	"	0.571	-62.6
3	7	"	3.7	4.0	"	3.9	-	-	"	0.513	-67.7
4	8	"	3.8	4.0	"	4.3	-	-	"	0.465	-71.8
5	9	"	3.86	4.0	"	4.7	-	-	"	0.426	-74.8
6	10	"	3.9	4.0	-	-	3.345	8.8	"	0.380	-77.2
7	11	"	3.98	4.0	-	-	5.16	14.7	"	0.351	-84.3
8	13	"	3.0	3.0	-	-	2.375	8.173		0.291	-90.0
9	15	"	3.98	4 0	-	-	3.036	12.0		0.253	-95.7
10.	20	0.5V/D	3.92	4.0	-	-	1.496	7,875	"	0.190	-101.5

contd...

TABLE 7.4 (continued from last page)

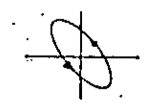
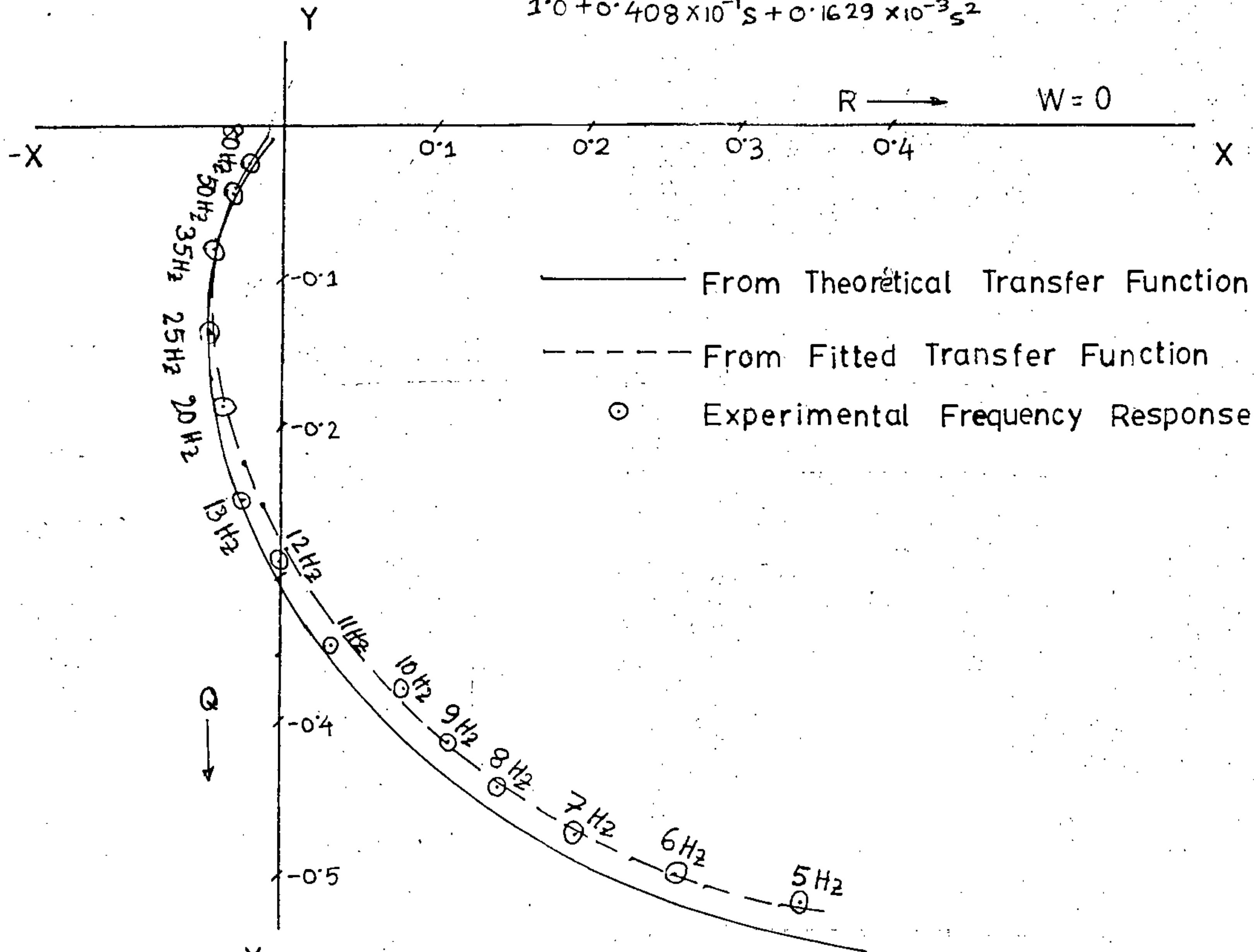
No. of obs.	Frequency	Vertical Axis readings			Horizontal Axis Readings		Voltage readings in volts		Rotation	Gain	Phase difference
	Hz	S/D	Ycut	Y max	S/D	X max	Output (V_o)	Input (V_i)			Degree
11	25	0.5V/D	3.8	4.0	-	-	1.455	9.947		0.146	-108.2
12	30	"	3.67	4.0	-	-	1.432	12.51	"	0.114	-113.4
13	35	"	3.5	4.0	-	-	1.417	15.255	"	0.093	-119.0
14	40	0.2V/D	3.42	4.0	-	-	1.092	14.273	"	0.077	-121.2
15	50	"	3.2	4.0	-	-	0.564	10.30	"	0.055	-126.9
16	60	"	3.0	4.0	-	-	0.556	13.335	"	0.042	-131.4
17	70	"	2.9	4.0	-	-	0.280	8.465	"	0.033	-133.5
18	80	"	2.75	4.0	-	-	0.277	10.294	"	0.027	-136.6

Fig 7.8 Polar plot of frequency response showing the accuracy of the fitted transfer function $\frac{0{\cdot}977+0{\cdot}6508\times10^{-3}s-0{\cdot}1081\times10^{-5}s^{2}}{1{\cdot}0+0{\cdot}408\times10^{-1}s+0{\cdot}1629\times10^{-3}s^{2}}$



The theoretical transfer function was calculated as

$$\frac{1+ 0.865 \times 10^{-3}s + 0.1823\times10^{-6}s^2}{1 + 0.391 \times 10^{-1}s + 0.1834 \times10^{-3}s^2}$$

The fitted transfer function was found as

$$\frac{0.977 + 0.6508 \times10^{-3}s - 0.1081\times10^{-5}s}{1+ 0.408 \times 10^{-1}s + 0.1629 \times 10^{-3}s^2}$$

The fitted transfer function is not very accurate. The order is same as theoretically it was . The constant coefficien-ts at the denominator are nearer to that of theoretical values. Though the fitted transfer function is not quantitatively very accurate,but from the frequency response plot of fig. 7.8 the result is found satisfactory.

v) 5th example (3rd order system) :

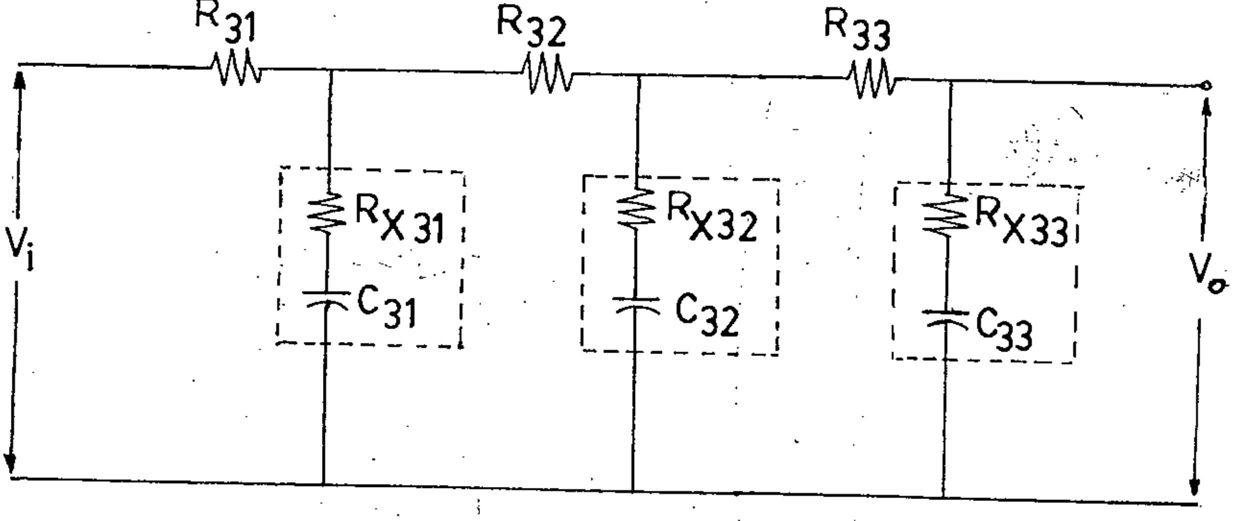


Fig. 7.9 3rd order circuit.

Here

$R_{31} = 100.26\,\Omega$, $R_{x31} = 13.13\,\Omega$, $C_1 = 13.31\,\mu f$

$R_{32} = 98.08\,\Omega$, $R_{x32} = 0.75\,\Omega$, $C_{32} = 81.45\,\mu f$

$R_{33} = 10.11\,\Omega$, $R_{33} = 1.18\,\Omega$, $C_{33} = 64.38\,\mu f$

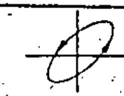
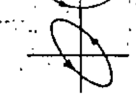
The theoritical transfer function was calculated as

$$\frac{1 + 0.311949 \times 10^{-3} s + 0.286 \times 10^{-7} s^2}{1 + 0.312212 \times 10^{-1} s + 0.379 \times 10^{-4} s^2 + 0.105 \times 10^{-7} s^3}$$

Table 7.2

Experimental data from Lissajou's pattern for the 3rd order system

$$\frac{1+0.311949\times10^{-3}S+0.286\times10^{-7}S^2}{1+0312212\times10^{-1}S+0.379\times10^{-4}S^2+0.105\times10^{-7}S^3}$$

No. of obs.	Frequency	Vertical Axis Readings			Horizontal Axis readings		Rotation	Gain	Phase difference
	Hz	S/D	Y cut	X max	S/D	X max			Degree
1	5	0.5v/D	2.95	4.0	IV/D	2.9		0.6897	- 47.52
2	10	"	3.68	4.0	"	4.5	"	0.4444	- 66.93
3	15	"	3.9	4.0	2v/D	3.2	"	0.3125	-77.16
4	20	0.2v/D	3.975	4.0	IV/D	3.3	"	0.2424	-83.59
5	25	"	3.99	4.0	"	4.1	"	0.1951	-85.95
6	30	"	4.0	4.0	"	4.75		0.1684	-90.00
7	40	"	3.99	4.0	2V/D	3.25		0.1230	-94.05
8	50	20mv/D	3.93	4.0	0.2v/D	4.07	"	0.09828	-100.73
9	60	"	3.85	4.0	"	4.95	"	0.08081	-105-74
10.	80	"	2.775	3.0	"	5.1	"	0 05882	-112.35
11	100	"	3.55	4.0	0.5v/D	3.5	"	0.04571	-117.44
12	120	"	3.35	4.0	"	4.42	"	0.03620	-123.12
13	150	10mv/D	3.0	4.0	"	2.99	"	0.02676	- 131.41
14	200	"	2.4	4.0	"	4.45	"	0.01798	- 143.13
15	250	"	1.95	4.0	IV/D	3.25	"	0.01231	-150.82
16	300	"	1.45	4.0	"	4.5	"	0.00889	-158.75

contd....

Table 7.5 (continued from last page)

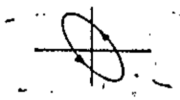
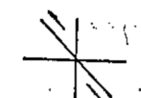
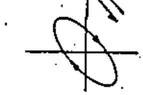
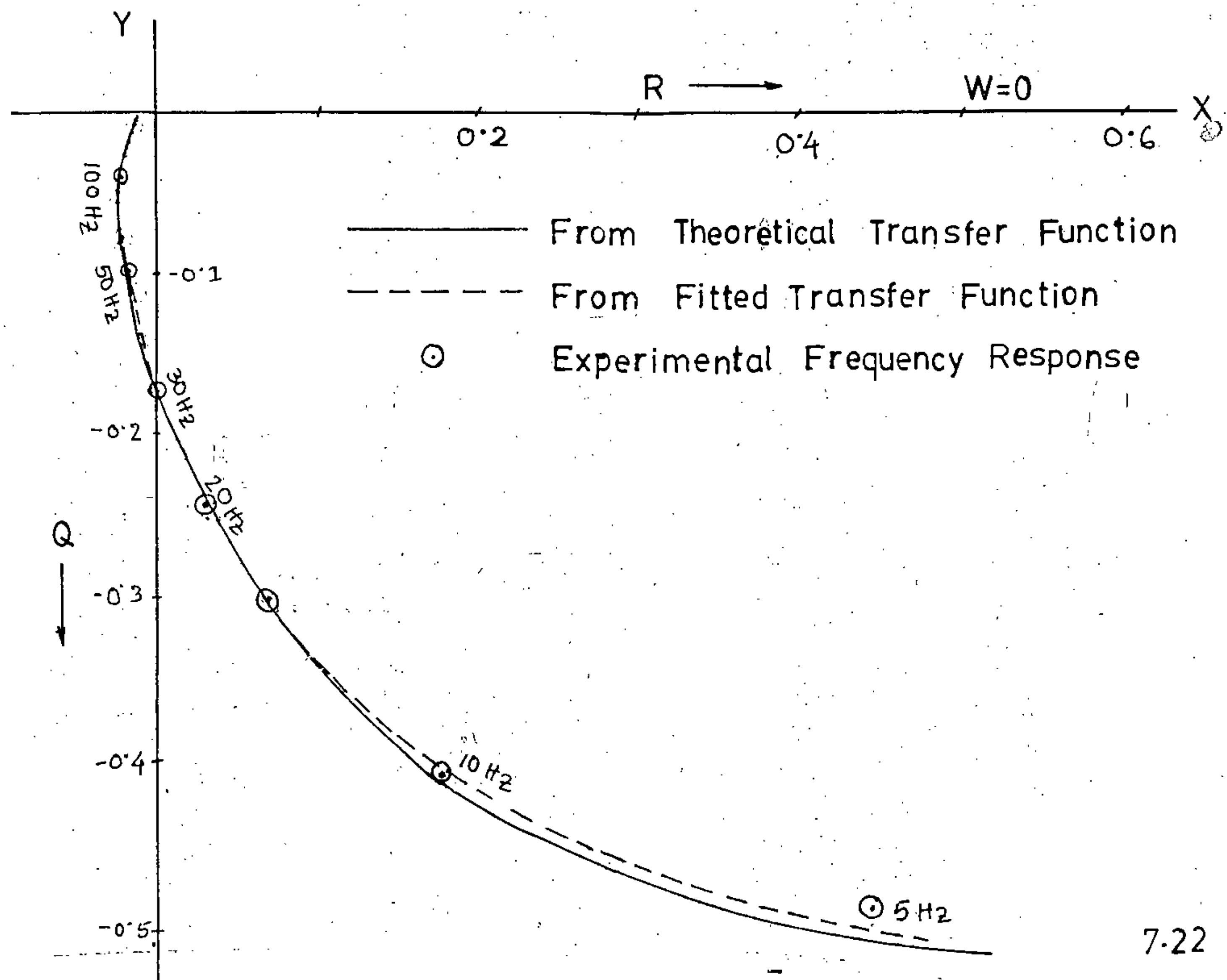
No. of Obs.	Frequency	Vertical Axis Readings			Horizontal Axis readings		Rotation	Gain	Phase difference
	Hz	S/D	Y cut	X mac	S/D	X max			Degree
17.	350	10mv/D	1.06	4.0	2v/D	3.0		0.00667	- 164.63
18.	400	5mv/D	0.75	4.0	IV/D	4.0	„	0.00500	-169.19
19.	450	"	0.46	3.95	"	5.0	„	0.00395	- 173.31
20.	500	"	0.20	3.15	"	5.0	„	0.00315	-176.36
21	574	"	0·0	2.35	"	5.0		0.00235	-180.00
22	690	"	0.10	1.5	"	5.0		0.00150	185.82
23	815	"	0.10	1.0	"	5.0	„	0.00100	-185.74

Fig 7.10 Polar plot of frequency response showing the accuracy of the fitted transfer function —

$$\frac{0.9928 - 0.19286\times10^{-3}s + 0.1056\times10^{-7}s^2}{1.0 + 0.3285\times10^{-1}s + 0.235\times10^{-4}s^2}$$



The fitted transfer function was found as 2nd order and it was

$$\frac{0.9928-0.19286 \times 10^{-3}s + 0.1036 \times 10^{-7}s^2}{1.0 + 0.3285 \times 10^{-1}s + 0.235 \times 10^{-4}s^2}$$

At 3rd order it was

$$\frac{0.99-0.1252 \times 10^{-2}s + 0.1331 \times 10^{-6}s^2}{1 + 0.3176 \times 10^{-1}s - 0.1075 \times 10^{-4}s^2 - 0.2277 \times 10^{-7}s^3}$$

Quantitatively the result is found to be unsatisfactory. But from the frequency response plot of fig. 7.10 it is found that the result is quite satisfactory.

7.3. <u>Reasons for variation from theoretical values</u> : It has found that ,in 2nd order system, specially in the case of 3rd order the fitted transfer functions differ from that of the theoretical.

It is not possible to take 100% accurate data by graphical method. Moreover the values of the parameters were found different at different frequencies. If we look into the the equivalent circuit of a capacitor[7] we can see a series resistance r_s, a series inductance L and a parallel resistance r_p.

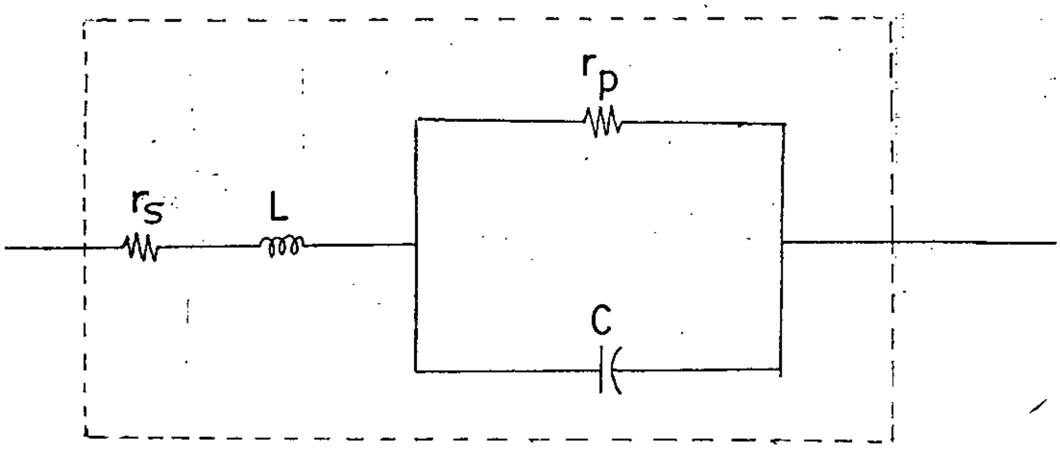


Fig. 7.11 Equivalent circuit of capacitor.

In calculating the capacitance and the internal resistance, the effect of inductance and parallel resistance is neglected So the values of internal resistance R_x and capacitor c which are measured are not very accurate.

It has been observed in the chapter 5 that in most of the cases $\pm$ 1% error gives satisfactory results .Hence error in measuring the frequency response should not be greater than $\pm$ 1% . But so much accurancy cannot be expected when frequency response is measured from Lissajou's pattern.Hence better instrument for measuring frequency response is necessary which is not available in our department at present.

CHAPTER 8

DETERMINATION OF TRANSFER FUNCTION OF THE R.F. SECTION OF A RADIO RECEIVER.

8.1. Introduction: An one band radio receiver was taken. Tracing the whole circuit of the receiver its circuit diagram has been constracted. The R.F. section including the I.F. sections is selected as a circuit whose transfer function is to be determined.

The frequency response of the circuit has been obtained using Lissajou's patterns considering the whole system as a black box. Then from this frequency response transfer function of the whole section has been determined using generalized complex curve fitting method.

8.2. Measurement procedure : The transformers of the converter and I.F. stages are designed for very low voltage level. But the signal generator requires a minimum voltage level to generate any desired frequency perfectly. This level is much higher than that of the saturation level of the transformers. More over the gain was so high that it was almost impossible to take the frequency response using oscilloscope.

To overcome these difficulties a high resistance was connected to the circuit at input.

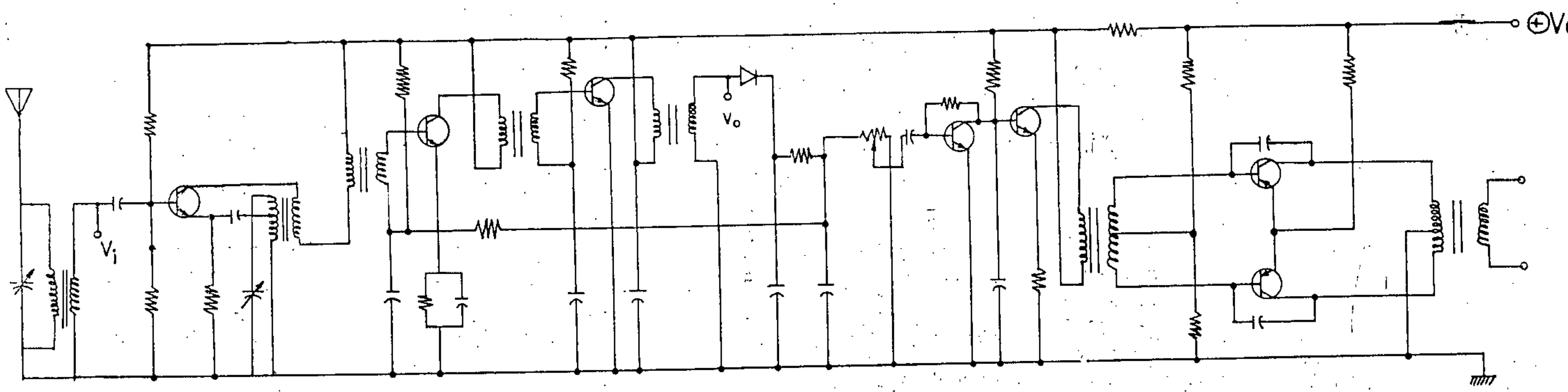


Fig 8.1 CIRCUIT DIAGRAM OF A ONE-BAND RADIO RECEIVER.

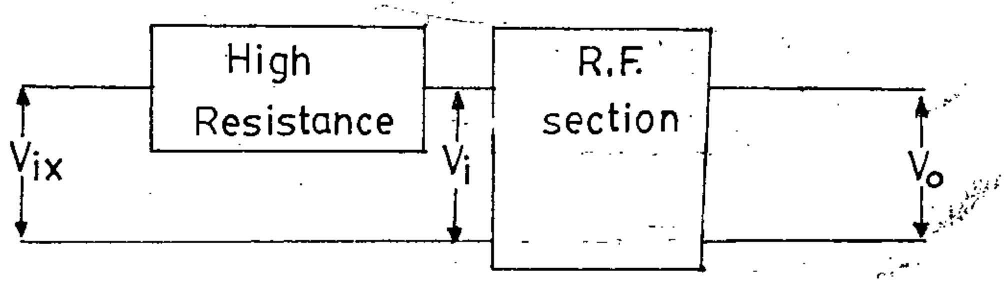


Fig. 8.2 Block diagram of the R.F section with High Resistance

The high resistance acts as an <u>attenuator</u> which lowered the voltage level sufficiently at the input of the R.F. section to keep it in the required voltage level. Moreover this time the gain, V_o/V_{ix} is possible to obtain from oscilloscope.

The transfer function of the R.F. section = $\dfrac{V_o}{V_i}$ ...(8.1)

Transfer function of the high resistance section

$$= \frac{V_i}{V_{ix}} \quad \ldots\ldots\ldots (8.2)$$

The transfer function of the whole system

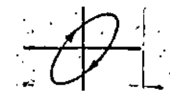
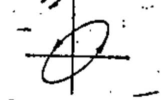
$$= \frac{V_o}{V_{ix}} \quad \ldots\ldots\ldots\ldots (8.3)$$

Therefore,

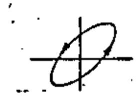
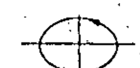
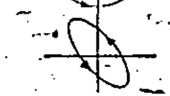
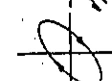
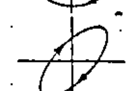
$$\frac{V_o}{V_i} = \frac{V_o}{V_{ix}} \Big/ \frac{V_i}{V_{ix}} \quad \ldots\ldots (.8.4)$$

TABLE 8.1

Experimental data from Lissajou's pattern for the ' High Resistance' Section for determination of transfer function of the R.F. Section

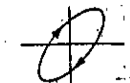
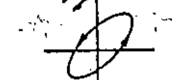
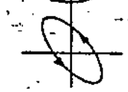
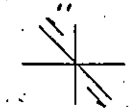
No. of obs.	Frequency	Vertical Axis Readings			Horizontal Axis Readings		Rotation	Gain	Phase Difference
	Hz	S/D	Y cut	Y max	S/D	X max			
1	400300	20mv/D	1.1	2.3	5v/D	5.0		0.00184	28.57
2	407200	"	1.0	2.4	"	5.0	"	0.00192	24.62
3	414135	"	1.05	2.35	"	5.0	"	0.00188	26.54
4	421030	"	1.0	2.4	"	5.0	"	0.00192	24,62
5	428000	"	1.0	2.4	"	5.0	"	0.00192	24.62
6	434780	"	0.9	2.4	"	5.0	"	0.00192	22.02
7	442010	"	0.6	2.5	"	5.0	"	0.00200	13.87
8	450250	"	0.0	1.4	"	5.0		0.00112	0.0
9	453138	"	0.4	1.1	"	5.0		0.00088	21.32
10	460150	"	1.0	1.7	"	5.0	"	0.00136	-36.03
11	468900	"	1.1	1.9	"	5.0	"	0.00152	-35.3
12	475980	"	1.15	2.1	"	5.0	"	0.00168	-33.20
13	485660	"	1.2	2.15	"	5.0	"	0.00172	-33.93
14	493850	"	1.2	2.25	"	5.0	"	0.00180	-32.32
15	502290	"	1.2	2.25	"	5.0	"	0.00180	-32.23

Experimental data from Lissajou's pattern for the ' RF + High Reisstance' section for determination of the transfer function or the R.F. Section.

No of obs.	Frequency	Vertical axis Readings			Horizontal Axis Readings		Rotation	Gain	Phase difference
	Hz	S/D	Y cut	X max	S/D	X max			Degree
1	300000	51mv/D	0.0	0.001	IV/D	5.0		0.0	0.0
2	400000	"	0.0	0.001	"	5.0		0.0	0.0
3	423745	50mv/D	1.4	1.58	20mv/D	5.0		0.7900	-62.38
4	425298	"	1.8	1.92	"	5.0	,,	0.960	-69.64
5	427563	"	2.6	2.75	"	5.0	,,	1.375	-70.99
6	429564	"	3.8	3.9	"	5.0	,,	1.95	-79.00
7	432252	0.1V/D	3	3	"	5.2		2.885	-90.00
8	434452	"	3.8	3.95	"	5.0		3.950	-105.84
9	436904	0.2v/D	2.0	2.7	"	5.1	,,	5.294	-132.21
10.	438518	"	1.4	3.1	"	5.	,,	6.200	-153.15
11	439812	"	0	3.4	"	5		6.800	-180.0
12	440395	"	1.0	3.5	"	5.0		7.000	-196.60
13	441064	"	2.0	3.6	"	5.0		7.200	-213.75
14	441793	"	3.0	3.66	"	5.0	,,	7.320	-235.65
15	443030	"	3.75	3.75	"	5.0		7.500	-270.00
16	444193	"	3.0	3.78	"	5.0		7.560	-307.47
17	444925	"	2.0	3.75	"	5.0	,,	7.500	-327.78

contd...

Table 8.2 (contd. from last page)

No. of obs.	Frequency	Vertical Axis Readings			Horizontal Axis Readings		Rotation	Gain	Phase difference
	Hz	S/D	Y cut	Y max	S/D	X max			Degree
18.	445462	0.2v/D	1.0	3.7	20mv/D	5.0		7.400	-344.32
19	446003	"	0.0	3.6	"	5.0		7.200	0.0
20	446624	"	1.0	3.55	"	5.0		7.100	-16.36
21	447487	"	2.0	3.4	"	5.0	,,	6.800	-36.03
22	451974	"	2.15	"	"	5.0		4.300	-90.00
23	454658	0.1v/D	2.8	2.87	"	5.0		2.870	-102.68
24	456725	"	2.0	2.1	"	5.0	,,	2.100	-107.75
25	461228	"	1.0	1.13	"	5.0	,,	1.130	-117.75
26	468273	"	0.4	1.075	"	5.0	,,	1.0750	-158.15
27	498438	5mv/D	0.0001	0.0001	1v/D	5.0		0.0	-180.00
28	500999	"	0.0001	0.0001	"	5.0	,,	0.0	-180.00
29	600000	"	0.001	0.0001	"	5.0	,,	0.0	-100.00

8.3. Results and the errors: Using the above measuring technique and using equation (8.4) the transfer function of the system was found as

$$\frac{V_o}{V_{ix}} = \frac{0.744969\times10^{-2} - 0.2302449\times10^{-8}s + 0.167377\times10^{-14}s^2 - 0.16317\times10^{-21}s^3 + 0.7668\times10^{-28}s^4}{1 + 0.26177\times10^{-8}s + 0.25683\times10^{-12}s^2 + 0.3380\times10^{-21}s^3 + 0.16485\times10^{-25}s^4}$$

$$\frac{V_i}{V_{ix}} = \frac{0.1494\times10^{-2} - 0.2817\times10^{-9}s}{1 + 0.67768\times10^{-9}s}$$

$$\frac{V_o}{V_i} = \frac{0.744969\times10^{-2} - 0.2297\times10^{-8}s + 0.1672\times10^{-14}s^2 - 0.16204\times10^{-21}s^3 + 0.7657\times10^{-28}s^4 + 0.51966\times10^{37}s^5}{0.1494 - 0.2426\times10^{-10}s + 0.3837\times10^{-15}s^2 - 0.673079\times10^{-23}s^3 + 0.2462\times10^{-28}s^4 - 0.4644\times10^{-36}s^5}$$

The freqency responses were taken using Lissajou's figure. To get exact results from the Lissajou's figure the input output frequency must be same. Since there are active elements in the circuit, the input and output frequency were obviously different. The I.F. amplifers are tuned .So it attenuates all the frequencies except its tuning frequency . Therefore the effect of the frequences

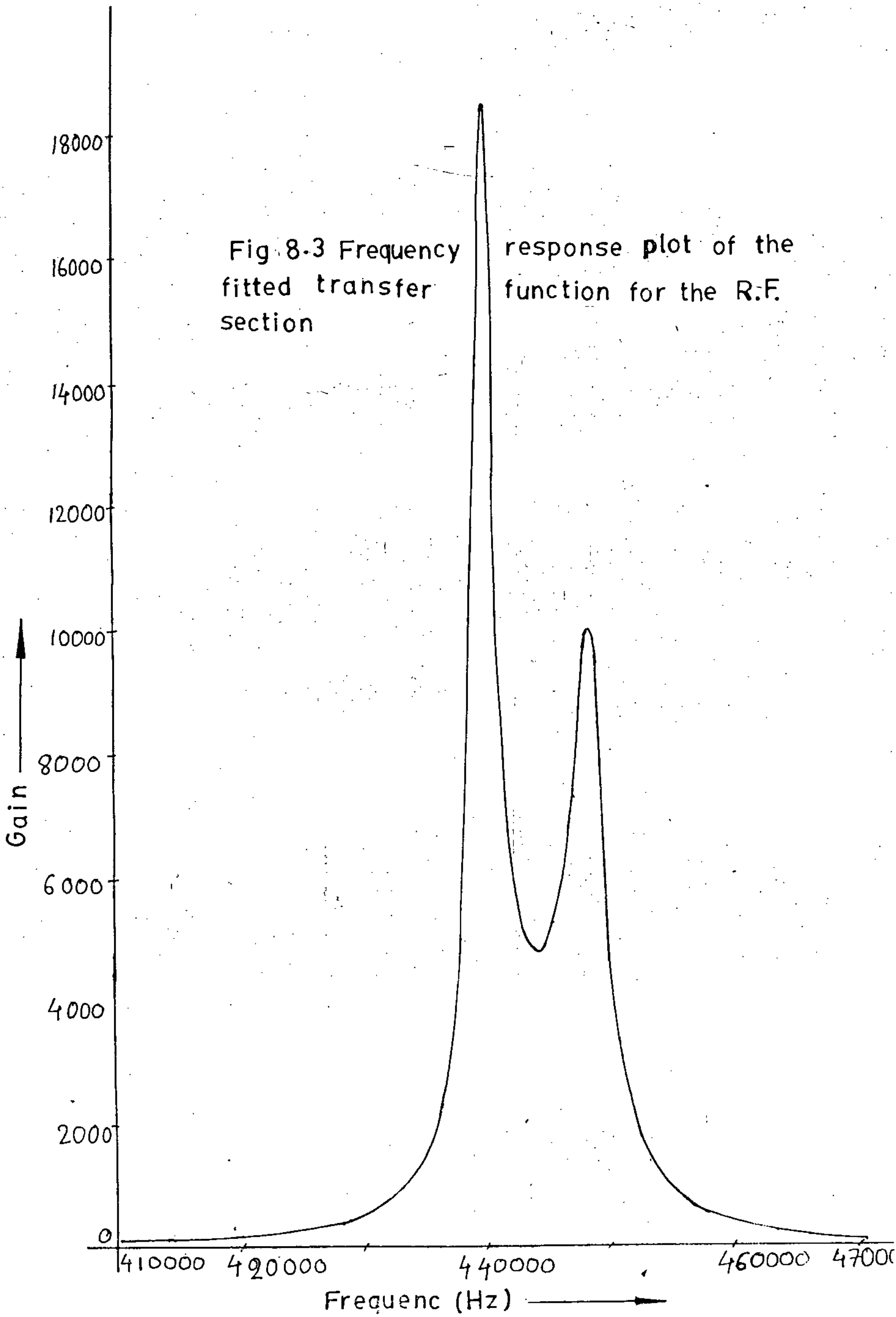


Fig 8-3 Frequency response plot of the fitted transfer function for the R.F. section

outside the tuned band is eliminated. Though for this reason the effect due to the different frequencies on the Lissajou's figure was partially eliminated , the figure was found unsymmetrical. For the high resistance section it was never found distinct. The data was taken for this case on the basis of an approximate figure.

The phase difference and gain both had a great variation if the input voltage level was changed. The readings were taken at the maximum input voltage level at which the Lissajou's pattern was most nearer to its symmetry.

Though the exact frequency response was not possible to obtain for the above reasons, still the fitted transfer function was found satisfactory. The frequency response obtained from the fitted transfer function is plotted in fig. 8.3.

CHAPTER 9

SUGGESTIONS AND CONCLUSIONS

9.1. Summary and conclusion : The complex curve fitting method for obtainmng transfer function from frequency response gives very accurate result if the frequency response is noise free and the corresponding error in fitting the frequency response to the transfer function is very small, i.e, of the order of 10^{-4} or smaller[3]. But if the frequency response is noisy then the above error is larger and may be of the order of 10^{-1}, Because of the noisy data a better fitting cannot be obtained.

Some times it may become difficult to ascertain the correct order of the transfer function using the previous method[3]. The emperical relations developed in section 3.2 eliminates the problem.

In previous method[1] determination of transfer functions of higher type systems were not practically possible. The technique which has been developed in this work in section 3.4

on the concept of inverting the data gives excellent fittings for higher type systems.

The frequency range over which the data sets are taken is very important factor. A smaller range may even change the order of the transfer function. To get accurate result the frequency range should be taken as large as possible and the critical frequency band has to be included. The critical frequency band is that region where both the magnitude and angle changes relatively rapidly than that of the other frequencies.

To get the actual transfer function from the frequency response, data should be very accurate. It has been found that in some cases ± 1% error may change the fitted transfer function quantitively and even the order of the transfer function.

Finally the transfer function of six practical electrical systems of different order including the R.F. Section of a Radio receiver have been obtained using generalised complex curve fitting method. It has been found that in some cases the fitted transfer function is quantitatively different and for the case of 3rd order system the order of the transfer function has been found different from that of the theoretical . From error analysis in Chapter 5 it has been observed that for obtaining accurate result, the error should be less than ± 1% for some cases. But it is not practically possible to get so much accurate data from oscilloscope using Lissajou's pattern. So the variation of the fitted transfer functions from that of their theretical values is obvious .But from the frequnecy response plots of the fitted transfer functions the results were found reasonably acceptable.

9.2. Scope of Future Research : In this work the ritical frequency band has not been determined accurately ,only an idea about the critical frequency band has been given. Further work is necessary to determine the actual critical frequency band.

The computer programme is developed in Fortran IV language. Fortrna IV compiler can handle data of maximum magnitude of about 10^{75}. If the frequency is higher like R.F. frequency the magnitude of the data inside the calculation procedure some times become larger than that of the maximum handling capacity of the Fortran IV compiler. To overcome this problem program should have to develop in other languages (such as combination of Fortran and Assembly).

The programme which have been developed is not optimised. Optimised program should be developed for the saving of valuable C.P.U time of the computer.

The measurement of frequency response for the R.F. Section with **Lissajou's** pattern was not very accurate .Since the input output frequencies were not the same. More accurate result will be obtain if better measurement technique is used.

In this work Lissajou's figure has been used to measure frequency response. It has been observed that ± 1% error may change the fitted transfer function. To get accurate result error should be less than ± 1% .But it is not possible to obtain so much accurate result using Lissajou's figure. So better instrument for measuring frequency response is necessary to work practically.

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APPENDIX

APPENDIX - A

DETAILED CALCULATIONS OF THE COMPLEX CURVE FITTING METHOD

Transfer function, G(jw), of a linear dynamic system can be expressed as a ratio of two frequency dependent polynomials as shown below:

$$G(jw) = \frac{A_0+A_1(jw) + A_2(jw)^2+A_3(jw)^3+ \ldots\ldots}{1+B_1(jw)+B_2(jw)^2+B_3(jw)^3\ldots\ldots\ldots\ldots} \quad \ldots\ldots \quad (1)$$

where A_0, A_1, A_2, ..., B_1, B_2, ... are constant coefficients of the transfer function. Equation (1) can be written as -

$$G(jw) = \frac{(A_0-A_2W^2+A_4W^4\ldots\ldots) + jw(A_1 - A_3w^2+A_5w^4 \ldots\ldots)}{(1-B_2W^2 + B_4W^4\ldots\ldots) + jw(B_1 - B_3w^2+B_5w^4 \ldots\ldots)} \quad \ldots\ldots \quad (2)$$

$$= \frac{\alpha + jw.\beta}{\sigma + jw.\gamma} \quad \ldots\ldots \quad \ldots \quad (3)$$

$$= \frac{P(jw)}{I(jw)} \quad \ldots \quad \ldots \quad (4)$$

F(jw) is the ideal function, i.e., one which represents the data exactly. It has real and imajinary components.

$$F(jw) = R(w) + jQ(w) \quad \ldots\ldots \quad (5)$$

The numerical difference between the two functions

F(jw) and G(jw) represents the error in fitting, that is

$$e(w) = F(jw) - G(jw) \quad \ldots \quad (6)$$

$$= F(jw) - \frac{P(jw)}{I(jw)} \quad \ldots \quad (7)$$

Now, $e'(w) = e(w)\, I(jw) = F(jw).\, I(jw) - P(jw)$ ..... (8)

At any specific value of frequency w_k

$$e'(w_k) = F(jw_k)\, I(jw_k) - P(jw_k) \quad \ldots. \quad (9)$$

$$= C(w_k) + jd(w_k) \quad \ldots \ \ldots \quad (10)$$

The magnitude of the function $e'(w_k)$ is given by -

$$|e'(w_k)| = |C(w_k) + jd(w_k)| = \sqrt{C^2(w_k) + d^2(w_k)} \quad \ldots. \quad (11)$$

$$\text{Or, } |e'(w_k)|^2 = C^2(w_k) + d^2(w_k) \quad \ldots.. \quad (12)$$

Now, E is defined as being the function given in (12) summed over all the sampling frequencies w_k. Hence

$$E = \sum_{k=1}^{m} \left[C^2(w_k) + d^2(w_k)\right] \quad \ldots. \quad (13)$$

The unknown polynomial coefficients A_i and B_i, $i = 0, 1, 2, \ldots.$ are evaluated on the basis of minimising the function E. Using equations (3) and (5) -

$$E = \sum_{k=1}^{m} \left[(R_k \sigma_k - w_k \tau_k Q_k - \alpha_k)^2 + (w_k \tau_k R_k + \sigma_k Q_k - w_k \beta_k)^2\right] \ldots (14)$$

Now,

the following relationships are defined:

$$\lambda_i = \sum_{k=1}^{m} w_k^i \quad \ldots\ldots \qquad \ldots \qquad (15)$$

$$S_i = \sum_{k=1}^{m} w_k^i R_k \qquad \ldots \qquad (16)$$

$$T_i = \sum_{k=1}^{m} w_k^i Q_k \qquad \ldots \qquad (17)$$

$$U_i = \sum_{k=1}^{m} w_k^i (R_k^2 + Q_k^2) \qquad \ldots \qquad (18)$$

Now, partially differentiating E with respect to each of Ais and B_is and substituting λ_i, S_i, T_i, U_i,:-

$$1)\ \frac{\delta E}{\delta A_0} = \sum_{k=1}^{m} \left[-2(\sigma_k R_k - w_k \tau_k Q_k - \alpha_k) \right] = 0$$

$$\text{or, } \sum_{k=1}^{m} \left[-2 \left\{ R_k(1-B_2 w_k^2 + B_4 w_k^4 \ldots\ldots) - w_k Q_k (B_1 - B_3 w_k^2 \right.\right.$$

$$\left.\left. + B_5 w_k^4 \ldots\ldots) - (A_0 - A_2 w_k^2 + A_4 w_k^4 \ldots\ldots) \right\} \right] = 0 \ \ldots \ (19)$$

$$\text{or, } \lambda_0 A_0 - \lambda_2 A_2 + \lambda_4 A_4 - \ldots\ldots + B_1 T_1 + B_2 S_2 - B_3 T_3 - B_4 S_4 +$$

$$\ldots\ldots\ldots = S_0 \ \ldots\ldots \qquad (20)$$

ii) $\frac{\delta E}{\delta A_1} = \sum_{k=1}^{m} \left[- 2w_k(w_k \tau_k R_k + \sigma_k Q_k - w_k \beta_k) \right] = 0$

$$\text{or, } \sum_{k=1}^{m} \Big[-2w_k \Big\{ w_k R_k (B_1 - B_3 w_k^2 + B_5 w_k^4 \ldots\ldots) + Q_k (1 - B_2 w_k^2$$

$$+ B_4 w_k^4 - \ldots\ldots) - w_k (A_1 - A_3 w_k^2 + A_5 w_k^4 \ldots\ldots) \Big\} \Big] \ldots \qquad (21)$$

$$\text{Or, } A_1 \lambda_2 - A_3 \lambda_4 + A_5 \lambda_6 - A_7 \lambda_8 + \ldots\ldots - B_1 S_2 + B_2 T_3 + B_3 S_4 - B_4 T_5$$

$$- B_5 S_6 + \ldots\ldots = T_1 \quad \ldots\ldots \qquad (22)$$

iii) $\frac{\delta E}{\delta A_2} = \sum_{k=1}^{m} \left[2w_k^2 \, (\sigma_k R_k - w_k \tau_k Q_k - \alpha_k) \right] = 0$

$$\text{or, } \sum_{k=1}^{m} \Big[2w_k^2 \;\; R_k (1 - B_2 w_k^2 + B_4 w_k^4 - \ldots\ldots) + w_k Q_k (B_1$$

$$- B_3 w_k^2 + B_5 w_k^4 - \ldots\ldots) - (A_0 - A_2 w_k^2 + A_4 w_k^4 - \ldots\ldots) \Big] \qquad (23)$$

$$\text{or, } A_0 \lambda_2 - A_2 \lambda_4 + A_4 \lambda_6 - A_6 \lambda_8 + \ldots\ldots\ldots + B_1 T_3 + B_2 S_4$$

$$- B_3 T_5 - B_4 S_6 + B_5 T_7 + \ldots\ldots = S_2 \ldots\ldots \qquad (24)$$

iv) $\frac{\delta E}{\delta A_3} = \sum_{k=1}^{m} \left[2w_k^3 \, (w_k \tau_k R_k + \sigma_k Q_k - w_k \beta_k) \right] = 0$

$$\text{or,} \quad \sum_{k=1}^{m} 2w_k^3 \Big[w_k R_k (B_1 - B_3 w_k^2 + B_5 w_k^4 \ldots\ldots) + Q_k(1 - B_2 w_k^2$$

$$+ B_4 w_k^4 - \ldots\ldots) - w_k (A_1 - A_3 w_k^2 + A_5 w_k^4 - \ldots\ldots) \Big] = 0 \ \ldots\ldots \quad (25)$$

$$\text{or,} \quad A_1 \lambda_4 - A_3 \lambda_6 + A_5 \lambda_8 - A_7 \lambda_{10} + \ldots\ldots - B_1 S_4 + B_2 T_5$$

$$+ B_3 S_6 - B_4 T_7 - B_5 S_8 + \ldots = T_3 \ldots\ldots \quad (26)$$

$$\text{v)} \quad \frac{\delta E}{\delta B_1} = \sum_{k=1}^{m} \Big[-2w_k Q_k (\sigma_k R_k - w_k \gamma_k Q_k - \alpha_k)$$

$$+ 2w_k R_k (w_k \gamma_k R_k + \sigma_k Q_k - w_k \beta_k) \Big] = 0$$

$$\text{or,} \quad \sum_{k=1}^{m} \Big[-2w_k Q_k (R_k (1 - B_2 w_k^2 + B_4 w_k^4 - \ldots\ldots) - w_k Q_k (B_1 -$$

$$B_3 w_k^2 + B_5 w_k^4 - \ldots\ldots) - (A_0 - A_2 w_k^2 + A_4 w_k^4 - \ldots\ldots)) +$$

$$2w_k R_k (w_k R_k (B_1 - B_3 w_k^2 + B_5 w_k^4 - \ldots\ldots) + Q_k (1 -$$

$$B_2 w_k^2 + B_4 w_k^4 - \ldots\ldots) - w_k (A_1 - A_3 w_k^2 + A_5 w_k^4 - \ldots\ldots)) \Big] \ldots \quad (27)$$

$$\text{or,} \quad A_0 T_1 - A_1 S_2 - A_2 T_3 + A_3 S_4 + A_4 T_5 - \ldots\ldots + B_1 U_2$$

$$- B_3 U_4 + B_5 U_6 - B_7 U_8 + \ldots = 0 \quad \ldots\ldots \quad (28)$$

$$\text{vi)} \quad \frac{\delta E}{\delta B_2} = \sum_{k=1}^{m} \Big[-2w_k^2 R_k (\sigma_k R_k - w_k \gamma_k Q_k - \alpha_k) -$$

$$2w_k^2 \quad {}_k(w_k \gamma_k R_k + \sigma_k Q_k - w_k \beta_k)\Big] \quad = 0$$

$$\text{or,} \quad \sum_{k=1}^{m} \Big[-2w_k^2 R_k (R_k(1-B_2 w_k^2 + B_4 w_k^4 - \ldots.) - w_k Q_k (B_1$$

$$- B_3 w_k^2 + B_5 w_k^4 - \ldots.) - (A_0 - A_2 w_k^2 + A_4 w_k^4 \ldots..))$$

$$-2w_k^2 Q_k (w_k R_k (B_1 - B_3 w_k^2 + B_5 w_k^4 - \ldots..) + Q_k (1-$$

$$B_2 w_k^2 + B_4 w_k^4 - \ldots..) - w_k (A_1 - A_3 w_k^2 + A_5 w_k^4 - \ldots.))\Big]$$

$$\ldots\ldots\ldots. \qquad (29)$$

$$\text{or,} \ A_0 S_2 + A_1 T_3 - A_2 S_4 - A_3 T_5 + A_4 S_6 + \ldots\ldots\ldots. + B_2 U_4$$

$$- B_4 U_6 + B_6 U_8 - B_8 U_{10} + \ldots.. = U_2 \ldots.. \qquad (30)$$

$$\text{vii)} \quad \frac{\delta E}{\delta B_3} = \sum_{k=1}^{m} \Big[2w_k^3 Q_k (\sigma_k R_k - w_k \gamma_k Q_k - \alpha_k) - w_k^3 R_k (w_k \gamma_k R_k$$

$$+ \sigma_k Q_k - w_k \beta_k)\Big] = 0$$

$$\text{Or,} \quad \sum_{k=1}^{m} \Big[2w_k^3 Q_k (R_k (1-B_2 w_k^2 + B_4 w_k^4 - \ldots...) - w_k Q_k (B_1$$

$$-B_3 w_k^2 + B_5 w_k^4 - \ldots.) - (A_0 - A_2 w_k^2 + A_4 w_k^4 - \ldots..))$$

$$-2w_k^3 R_k (w_k R_k (B_1 - B_3 w_k^2 + B_5 w_k^4 - \ldots... \quad)$$

$$+ Q_k (1 - B_2 w_k^2 + B_4 w_k^4 - \ldots..) - \ldots.) - w_k (A_1 - A_3 w_k^2 + A_5 w_k^4$$

$$- \ldots.))\Big] \quad = 0 \ \ldots \ \ldots \qquad (31)$$

Or, $A_0T_3 - A_1S_4 - A_2T_5 + A_3S_6 + A_4T_7 - \ldots\ldots + B_1U_4$

$$- B_3U_6 + B_5U_8 - B_7U_{10} + \ldots\ldots = 0 \quad \ldots\ldots \qquad (32)$$

From equations (20), (22), (24), (26), (28), (30) and (32) a matrix equation can be written as

$$[P]\,[D] = [C] \quad \ldots\ldots \qquad (33)$$

Where,

$$[P] = \begin{bmatrix} \lambda_0 & 0 & -\lambda_2 & 0 & \lambda_4 & \cdots & T_1 & S_2 & -T_3 & -S_4 & T_5 \\ 0 & \lambda_2 & 0 & -\lambda_4 & 0 & \cdots & -S_2 & T_3 & S_4 & -T_5 & -S_6 \\ \lambda_2 & 0 & -\lambda_4 & 0 & \lambda_6 & \cdots & T_3 & S_4 & -T_5 & -S_6 & T_7 \\ 0 & \lambda_4 & 0 & -\lambda_6 & 0 & \cdots & -S_4 & T_5 & S_6 & -T_7 & -S_8 \\ \vdots & \vdots & \vdots & \vdots & \vdots & & \vdots & \vdots & \vdots & \vdots & \vdots \\ T_1 & -S_2 & -T_3 & S_4 & T_5 & \cdots & U_2 & 0 & -U_4 & 0 & U_6 \\ S_2 & T_3 & -S_4 & -T_5 & S_6 & \cdots & 0 & U_4 & 0 & -U_6 & 0 \\ T_3 & -S_4 & -T_5 & -S_6 & T_7 & \cdots & U_4 & 0 & -U_6 & 0 & U_8 \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \end{bmatrix}$$

$$\ldots\ldots\ldots\ldots \quad (34)$$

$$[D] = \begin{bmatrix} A_0 \\ A_1 \\ A_2 \\ \vdots \\ B_1 \\ B_2 \\ \vdots \end{bmatrix} \qquad \cdots\cdots (35)$$

$$[C] = \begin{bmatrix} S_0 \\ T_1 \\ S_2 \\ T_3 \\ \vdots \\ 0 \\ U_2 \\ 0 \\ U_4 \\ \vdots \end{bmatrix} \qquad \cdots\cdots (36)$$

APPENDIX - B

THEORITICAL CALCULATION OF TRANSFER FUNCTION

a) First order system:

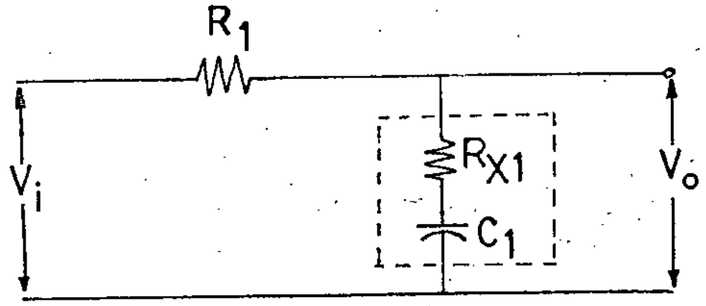

Fig.B1: 1st order R.C. circuit

$$V_i = (R_1 + R_{x1})\, I + \frac{1}{C_1 S}$$

$$I = \frac{V_i}{R_1 + R_{x1} + 1/C_1 S}$$

$$\frac{V_o}{V_i} = \frac{R_{x1} + 1/\,C_1 S}{R_1 + R_{x1} + 1/\,C_1 S} = \frac{1 + R_{x1} C_1 S}{1 + (R_1 + R_{x1})\, C_1 S}$$

b) Second Order system :

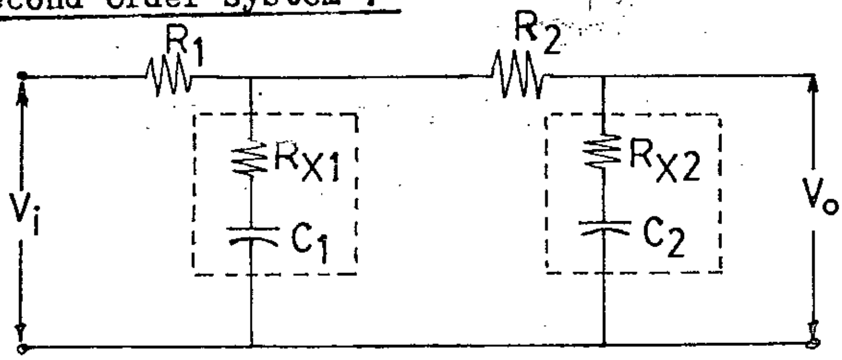

Fig.B2: 2nd order R.C. crcuit

$$V_i = (R_1 + R_{x1} + 1/C_1 s)\, I_1 - (R_{x1} + 1/C_1 S)\, I_2$$

$$0 = -(R_{x1} + 1/C_1 S)\, I_1 + \left(R_2 + R_{x1} + R_{x2} + \frac{1}{C_1 s} + \frac{1}{C_2 s}\right) I_2$$

$$I_2 = \frac{\begin{vmatrix} R_1 + R_{x1} + 1/C_1 s & V_i \\ -(R_{x1} + 1/C_1 s) & 0 \end{vmatrix}}{\begin{vmatrix} R_1 + R_{x1} + 1/C_1 s & -(R_{x1} + 1/C_1 s) \\ -(R_{x1} + 1/C_1 s) & R_2 + R_{x1} + R_{x2} + \frac{1}{C_1 s} + \frac{1}{C_2 s} \end{vmatrix}}$$

$$I_2 = \frac{(1 + R_{x1}\, C_1 s)\, V_i / C_1 s}{R_1\, R_2 + R_1\, (R_{x1} + R_{x2}) + R_{x1}\, (R_2 + R_{x2}) + R_1 (1/C_1 + 1/C_2) + R_{x1}/C_2 + R_{2/C_1} + R_{x2}/C_1 \quad 1/s + 1/(C_1\, C_2 s^2)}$$

$$\frac{V_0}{V_i} = \frac{1 + R_{x1}\, R_{x2}\, C_1\, C_2 s^2 + (R_{x1}\, C_1 + R_{x2}\, C_2)\, s}{1 + R_1 (C_1 + C_2) + R_{x1}\, C_1 + R_2\, C_2 + R_{x2}\, C_2) \quad s \; + \; R_1\, R_2 + R_1 (R_{x1} + R_{x2}) + R_{x1}\, (R_2 + R_{x2})\, C_1\, C_2 \; s^2}$$

c) 3rd Order system:

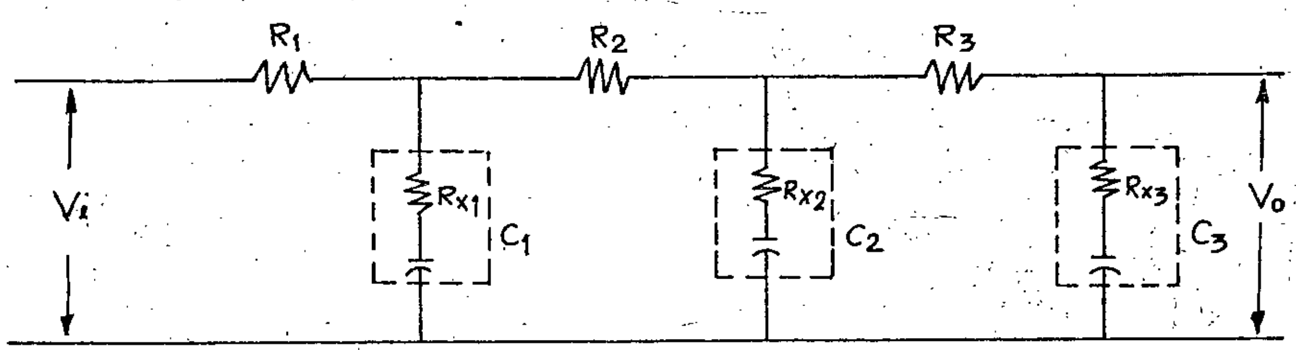

Fig B3: 3rd order circuit

$$V_i = (R_1 + R_{x1} + 1/C_1 s)\, I_1 - (R_{x1} + 1/C_1 s)\, I_2 + 0 \cdot I_3$$

$$0 = -(R_{x1} + 1/C_1 s)\, I_1 + (R_{x1} + R_2 + R_{x2} + 1/C_1 s + 1/C_2 s)\, I_2 - (R_{x2} + 1/C_2 s)\, I_3$$

$$0 = -0 \cdot I_1 - (R_{x2} + 1/C_2 s) I_2 + (R_{x2} + R_3 + R_{x3} + 1/C_2 s + 1/C_3 s) I_3$$

$$I_3 = \frac{\begin{vmatrix} (R_1 + R_{x1} + 1/C_1 s) & -(R_{x1} + 1/C_1 s) & V_i \\ -(R_{x1} + 1/C_1 s) & (R_{x1} + R_2 + R_{x2} + 1/C_1 s + 1/C_2 s) & 0 \\ 0 & -(R_{x2} + 1/C_2 s) & 0 \end{vmatrix}}{\begin{vmatrix} (R_1 + R_{x1} + 1/C_1 s) & -(R_{x1} + 1/C_1 s) & 0 \\ -(R_{x1} + 1/C_1 s) & (R_{x1} + R_2 + R_{x2} + 1/C_1 s + 1/C_2 s) & -(R_{x2} + 1/C_2 s) \\ 0 & -(R_{x2} + 1/C_2 s) & (R_{x2} + R_3 + R_{x3} + 1/C_2 s + 1/C_3 s) \end{vmatrix}}$$

$$V_0 = (R_{x3} + 1/C_3 s)\ I_3$$

Let, $R_1' = R_1 + R_{x1}$

$R_2' = R_2 + R_{x2}$

$R_3' = R_3 + R_{x3}$

$$\frac{V_0}{V_i} = \frac{1 + (R_{x1} C_1 + R_{x2} C_2 + R_{x3} C_3)s + (R_{x1} R_{x2} C_1 C_2 + R_{x1} R_{x3} C_1 C_3 + R_{x2} R_{x3} C_2 C_3) s^2 + R_{x1} R_{x2} R_{x3} C_1 C_2 C_3 s^3}{\begin{array}{l} 1 + R_1'(C_1 + C_2 + C_3) + R_2'(C_2 + C_3) + R_3' C_3 - R_{x1}(C_2 + C_3) \\ - R_{x2} C_3 \,|\, s + R_1' R_{x1} (C_1 C_2 + C_1 C_3) + R_2' (C_1 C_2 + C_1 C_3) \\ + R_{x2} C_2 C_3 + R_3' (C_1 C_3 + C_2 C_3) - R_{x2} C_1 C_3 + R_2' (R_{x2} \\ + R_3') C_2 C_3 - R_{x1}^2 (C_1 C_2 + C_1 C_3) - R_{x2}^2 C_2 C_3 - R_{x1} (R_{x2} \\ + R_3') C_2 C_3 s^2 + R_1' R_{x1} (R_{x2} + R_3') + R_2' (R_{x2} + R_3') \\ - R_{x2}^2 - R_{x1}^2 (R_{x2} + R_3') \; C_1 C_2 C_3 s^3 \end{array}}$$

APPENDIX - C

```
C         PROGRAMME FOR DETERMINATION OF TRANSFER FUNCTION FOR ANY
C                           TYPE OF SYSTEMS
C         ******* ** ******** ** ******** ** ******** ** ******** ** ***
C         ********************** ***** ***** ****************************
C           TO USE THIS PROGRANME THE FOLLOWING DATA HAVE TO BE ENTERED
C         ACCORDING TO THE FORMAT SPECIFIED.THEY ARE -
C           XX    = 0 OR 1
C                  WHERE, '0' STANDS FOR TYPE ZERO SYSTEMS
C                         '1' STANDS FOR HIGHER TYPE SYSTEMS
C           N     = NO. OF DATA SETS
C           L1    = NO. OF ITERATIONS WANTED
C           ER    = THE MAXIMUM ACCEPTABLE ERROR
C           IFRE(K) = FREQUENCY CORRESPONDS TO K-TH DATA SET
C           F(K)    = GAIN CORRESPONDS TO K-TH DATA SET
C           PHI(K)  = ANGLE IN DEGREE CORRESPONDS TO K-TH DATA SET
C         IN THE OUTPUT THERE WILL BE LIST OF ALL THE ABOVE DATA
C         WHICH HAVE BEEN GIVEN AS INPUT, VALUE OF CONSTANTS(A & B)
C         CORRESPONDS TO THE MINIMUM NORMALISED ERROR FOR DIFFERENT
C         ORDERS(UPTO THE (ACTUAL+1)TH ORDER) WHICH HAVEBEEN CALCULATED
C         AND THE ACTUAL ORDER OF THE SYSTEM. IF THE INPUT FREQUENCY
C         RESPONSE IS SO NOISY THAT GOOD FITTING IS NOT POSSIBLE THIS
C         PROGRAMME WILL ALSO GIVE MESSAGE ABOUT IT.
C         * BEFORE GO FOR A RUN DIMENSIONS SHOULD BE CHECKED CAREFULY.
C         ******** ** ******** ** ******** ** ******** ** ********
C           ******** ** ******** ** ******** ** ******** ** ********
          DIMENSION  F(60), W(60), R(60), Q(60), V(60,15),P(31,31),X(15),
         *H(2),T(2),AB(15),SB(15),TB(15),UB(15),A(10),B(10),PHI(60),
         * IFRE(60),EXX(10),PHX(60),FX(60),ZA(10),FA(60),HE(2),TE(2),
         *AA(10,10),BB(10,10),EN(10)
          DOUBLE PRECISION   R,Q,W,V,P,X,H,T,AP,SB,TB,UB,A,B,EXX,EX,AA,
         *BB,EN
C             *** READS AND WRITES THE INPUT DATA ***
          READ  (1,5) XX, N ,L1, ER,((IFRE(K),F(K),PHI(K)),  K=1,N)
5         FORMAT (I2 /I2/ I2 / F3.2 / (I3,1X,F5.4,1X,F6.2))
          WRITE (3,51) XX, N, L1,ER, ((IFRE(K),F(K),PHI(K)) ,K=1,N)
51        FORMAT  ('0', I2/1X,I2 / 1X, I2 / 1X, F4.2 / (1X,I5,1X,F8.6,
         * 1X,F7.2))

C             *** CONVERTS THE ANGEL FROM DEGREE INTO RADIAN AND INVERTS
C                          THE DATA IF NECESSARY ***
          DO 181 K=1,N
          W(K) = 2.0*3.1415927 *IFRE(K)
          PHI(K) = 3.1415927*PHI(K)/180.0
          IF (XX .EQ. 0) GO TO 182
          PHX(K) = - PHI(K)
          FX(K)  = 1 / F(K)
          GO TO 181
182       PHX(K) = PHI(K)
          FX(K) = F(K)
181       CONTINUE
C           *** CALCULATION FOR R(K) AND Q(K) ***
          DO  61  K = 1, N
          MF = 1
          IF (PHX(K) .LT. 0)  MF = -1
          PHX(K) = ABS(PHX(K))
          R(K) = FX(K) * COS (PHX(K))
          Q(K) = MF * FX(K) * SIN (PHX(K))
61        CONTINUE
C           *** CONSTRACTION OF P MATRIX ***
          J = 0
400       J = J + 1
          NR = (2 * J  + 1 )
          N1 = (NR + 1) / 2
          N3 = ( NR + 3) / 2
          N5 = ( NR + 5 ) / 2
          NC = NR + 1
          DO  380  K = 1, N
   380    V(K,1) = 1
          DO  390  L = 1, L1
          IF ( L .EQ. 1 )  GO TO 15
          DO 62 K = 1, N
          V(K,L) = V(K,L4)
    62    CONTINUE
    15    CONTINUE
          NA = 2 * J  + 1
          DO 20 I = 1, NA, 2
          AB(I) = 0.0
          SB(I) = 0.0
          TB(I) = 0.0
          UB(I+1) = 0.0
```

```
      DO 30 K = 1, N
      IF ( W(K) .EQ. 0 )  GO TO 30
      AB(I) = AB(I) + W(K) * * (I-1) * V(K.L)
      SB(I) = SB(I) + R(K) * W(K) * * (I-1) * V(K,L)
      TB(I) = TB(I) + Q(K) * W(K) * * I * V(K,L)
      UB(I+1) = UB(I+1) + ( R(K) * * 2 + Q(K) * * 2 ) * W(K) * * (I+1)
     * * V(K,L)
   30 CONTINUE
   20 CONTINUE
  112 DO 40 I = 1,N1,2
      DO 50 M = 1,N1,2
   50 P(I,M) = (-1) * * (M/2) * AB(I+M-1)
      DO 60 M = 2, N1, 2
   60 P(I,M) = 0.0
      DO 70 M = N3, NR, 2
   70 P(I,M) = (-1) * * ((M - N3 ) / 2 ) * TB(I+M-N3)
      DO 80 M = N5, NR, 2
   80 P(I,M) = (-1) * * (( M - N5 ) / 2 ) * SB(I+M-N1)
   40 CONTINUE
      DO 90 I = 2, N1, 2
      DO 100 M = 1, N1, 2
  100 P(I,M) = 0.0
      DO 110 M = 2, N1, 2
  110 P(I,M) = (-1) * * ((M-2)/2) * AB(I+M-1)
      DO 120 M = N5, NR, 2
  120 P(I,M) = (-1) * * ((M-N5)/2) * TB(I+M-N3)
      DO 130 M = N3, NR, 2
  130 P(I,M) = (-1) * *((M-N3+2)/2) * SB(I+M-N1)
   90 CONTINUE
      DO 140 I = N3, NR, 2
      DO 150 M = 1, N1, 2
  150 P(I,M) = (-1) * * (M/2) * TB(I+M-N3)
      DO 160 M = 2, N1, 2
  160 P(I,M) = (-1) * * (M/2) * SB(I+M-N1)
      DO 170 M = N3, NR, 2
  170 P(I.M) = (-1) * * ((M-N3)/2) * UB(I+M-2*N1)
      DO 180 M = N5, NR, 2
  180 P(I.M) = 0.0
  140 CONTINUE
      DO 190 I = N5, NR, 2
      DO 200 M = 2, N1, 2
  200 P(I,M) = (-1) * * ((M-2)/2) * TB(I+M-N3)
      DO 210 M= 1, N1, 2
  210 P(I,M) = (-1) * * (M/2) * SB(I+M-N1)
      DO 220 M = N5, NR, 2
  220 P(I,M) = (-1) * * ((M-N5)/2) * UB(I+M-2*N1)
      DO 230 M = N3, NR, 2
  230 P(I,M) = 0.0
  190 CONTINUE
C        *** CONSTACTION OF C MATRIX AS THE LAST COLUMN OF P MATRIX
C              FOR THE USE OF GAUSS-JARDAN METHOD OF DITERMINATION OF
C              UNKNOWN VARIABLES***
      DO 240 I = 1, N1, 2
  240 P(I,NC) = SB(I)
      DO 250 I = 2, N1, 2
  250 P(I,NC) = TB(I-1)
      DO 260 I = N3, NR, 2
  260 P(I,NC) = 0.0
      DO 270 I = N5, NR, 2
  270 P(I,NC) = UB(I-N1)
C        *** DETERMINATION OF THE VALUES OF 'A' AND 'B' USING
C              GAUSS-JARDAN METHOD ***
      DO 280 K = 1, NR
      S = P(K,K)
      DO 290 M = 1, NC
  290 P(K,M) = P(K,M) / S
      DO 300 I = 1, NR
      S = P(I,K)
      IF ( I .EQ. K ) GO TO 300
      DO 310 M = 1, NC
  310 P(I,M) = P(I,M) - P(K,M) * S
  300 CONTINUE
  280 CONTINUE
C        *** FITTED VALUES OF 'A' AND 'B' ***
      DO 320 I = 1, N1
  320 AA(L,I)    = P(I,NC)
      BB(L,I) = 1
      KX = 2
      DO 330 I = N3, NR
      BB(L,KX) = P(I,NC)
```

```
  330 KX = KX + 1
C       *** DETERMINATION OF ERROR AND V(K,L) ***
      EN(L) = 0.0
      DO 340 K = 1, N
      DO 350 JX = 1, 2
      CC = AA(L,1)
      IF ( JX .EQ. 2 ) CC = BB(L,1)
      D = 0.0
      DO 360 I = 2, N1
      X(I) = AA(L,I)
      IF ( JX .EQ. 2) X(I) = BB(L,I)
      IA = I - 1
      IF ((I/2) * 2 .EQ. I) GO TO 370
      IF ( W(K) .EQ. 0 ) GO TO 360
      CC = CC + (-1) * * (IA/2)* X(I) * W(K) * * IA
      GO TO 360
  370 D = D + (-1) * * ((IA-1)/2)* X(I) * W(K) * * IA
  360 CONTINUE
      H(JX) = CC
      T(JX) = D
      HE(JX) = SQRT(CC**2 + D**2)
      IF (CC .LT. 0) GO TO 166
      TE(JX) = ATAN(D/CC)
      GO TO 121
166   TE(JX) = ATAN(D/CC)+ 3.1415927
      IF (D .LT. 0) TE(JX) = ATAN(D/CC) - 3.1415927
121   CONTINUE
350   CONTINUE
      FF    = (HE(1) / HE(2))
      PH =  TE(1) - TE(2)
      MF = 1
      IF (PH. .LT. 0)   MF = -1
      PH = ABS(PH)
      RR = FF * COS (PH)
      QQ = MF * FF * SIN (PH)
      IF (R(K) .EQ. 0) GO TO 111
      IF (Q(K) .EQ. 0) GO TO 111
      RN = (R(K) - RR) / R(K)
      QN = (Q(K) - QQ) / Q(K)
111   CONTINUE
      LA = L
      EN(L) = EN(L) + (RN**2 + QN**2 )*V(K,L)
340   V(K,LA)= 1 / ( H(2) * * 2 + T(2) * * 2 )
C       *** DETERMINATION OF THE MINIMUM NORMALISED ERROR WITHIN
C            THE GIVEN ITERATION NUMBER. ***
      IF ( L .GT. 1 ) GO TO 430
      EXX(J) = EN(L)
      LX = 1
      GO TO 390
430   IF (EXX(J) .GT. EN(L)) EXX(J) = EN(L)
      IF (EXX(J) .EQ. EN(L)) LX = L
390   CONTINUE
C  ******    PRINTING OF CONSTANTS AND NORMALISED ERROR
      IF (XX .EQ. 0) GO TO 341
      DO 342 IX = 1,N1
      ZA(IX)= AA(LX,IX)
      AA(LX,IX) = BB(LX,IX)
      BB(LX,IX) = ZA(IX)
342   CONTINUE
341   CONTINUE
      WRITE (3,64) J, (AA(LX,I),I=1,N1),(BB(LX,I),I=1,N1),EN(LX)
64    FORMAT ('0'1X,I2,5X,4(5X,D25.16))
C    **** DETERMINATION OF THE ACTUAL ORDER OF THE SYSTEM.****
403   IF ( J .EQ. 1 ) GO TO 400
      IF ( ER .GT. EXX(J) ) GO TO 414
      IF ( EXX(J-1) .GT. EXX(J) ) GO TO 400
      WRITE (3,411)
411   FORMAT('0' 1X, 'DATA IS TOO NOISY. GOOD FIT IS NOT POSSIBLE
     * MORE ACCURATEDATA DATA IS REQUIRED')
      GO TO 420
414   IF ( J .GT. 2 ) GO TO 410
      IF (((ER - EXX(J-1)) / ER ) .GT. ((EXX(J-1) - EXX(J))
     * /EXX(J-1))) GO TO 412
      GO TO 400
410   IF (((EXX(J-2) - EXX(J-1)) / EXX(J-2)) .GT. ((EXX(J-1)
     * - EXX(J)) / EXX(J-1))) GO TO 412
      GO TO 400
412   JY = J-1
      IF(EXX(JY) .GT. ER) GO TO 400
      WRITE (3,413) JY
413   FORMAT('0' 1X,'ACTUAL ORDER OF THE SYSTEM = ',I3 ,1X,'ORDER')
420   STOP
      END
```