EQUIVALENT POTENTIAL FOR ALPHA-DEUTERON MODEL OF ^{6}Li

BY

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A Thesis submitted to the Department of Mathematics ,Bangladesh University of Engineering and Technology,Dhaka in partial fulfilment for the Degree of Master of Philosophy in Mathematics.

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FEBRUARY ,1992

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ACKNOWLEDGEMENTS

I would like to express my sincerest gratitude and indebtedness to Dr.Syed Ali Afzal, Professor, Department of Mathematics,BUET for his invaluable suggestion , constant inspiration ,careful supervision and helpful advice throughout this research work.

I am indebted to all my colleagues, especially Dr.Md. Zakerullah ,Professor and Md. Ali Ashraf,Associate Professor, Department of Mathematics,BUET for their valuable suggestion and encouragement throughout this work.

I am also grateful to Md. Amanullah, scieutific officer,Atomic energy ceuter,Dhaka who helped me much in procuring many Journals from Atomic energy center library.

Finally thanks to Md. Sohrabuddin aud Md. Hossain Ali for typing this thesis.

ABSTRACT

We have presented the formalism of α+d cluster model of ^{6}Li following resonating group method. In this procedure, a non-local, non-separable Kernel $K(r,r')$ appears. The appearance of this kernel broadly involves:
First, the calculation of analytical expression of the Kernel,
Second, solving the integro-differential equation containing kernel by numerical method.
In the first the calculation of the kernel involves huge (complicated) work. In the present work we have shown the formalism in a bit detail deliberately only to show how big and laborious the calculation are for six-nucleon system of ^{6}Li, not to speak of higher nuclei. That is why although the resonating group formalism has been in application for a long time and the results obtained are good, the detailed work following this method hardly passes the nuclei of 15-20 nucleons.

In our present study we have replaced the non-local non-separable kernel by a simple non-local separable kernel keeping the other terms namely direct poteutial part and coulomb part as obtained from the resonating group method into

account.The results obtained agree well with the experimental results.This shows that resonating group formalism can be extended to any higher mass number in a modified way.

CONTENTS

CHAPTER ONE

INTRODUCTION

Numerous experimental observations and theoretical calculations have confirmed that the nucleus is built up of protons an neutrons. One of our main pursuits in nuclear physics is to understand the nature of the force that holds the protons and neutrons together. Experimental investigations have shown that the nuclei exhibit varieties of complex phenomena. For an explanation of these phenomena different types of nuclear models have been proposed. Some of them are liquid drop model, compound nucleus model, shell model, alpha particle model, cluster model and resonating group model etc.

The liquid drop model is perhaps the simplest of all nuclear models. This model was proposed by Bohr and Calker who compares the nucleus to a liquid drop, the nucleons corresponding to the molecules of the liquid due to several points of similarity. This model has been utilized with a certain amount of success in the interpretation of intra-nuclear forces and of nuclear transformations and in particular of nuclear fission.

The compound nucleus model was proposed by Bohr.This was

specifically introduced to describe the complexity shown by the energy spectra of many medium-heavy and heavy nuclei. It was based on the belief that, since nucleons in nuclei interact strongly with one another, their motion must be correlated to a very large extent.

The possibility that atomic nuclei might exhibit shell structure was considered in the early 1930's but the real success of the shell model dates from the year 1948. The basic assumption of the shell model of the nucleus is that a single nucleon travels within a complex nucleus in a smoothly varying average field of force generated by all the other nucleons in the nucleus and that each particle moves essentially undisturbed in its own closed orbit. In the shell model the nuclear potential is taken as static and spherically symmetric and it generates a complete orthonormal set of single particle functions $\Psi^{\nu_i}(r_i)$ where the r_i indicates the position, spin and charge variables of nucleons and ν_i denotes the quantum numbers of the state. Any function obeying the same boundary conditions as these eigenfunctions can be uniquely expressed as a linear combination of them. This model has been successfully applied to explain many nuclear phenomena but is not true for many other properties.

Wheeler et. al have proposed the alpha-particle model, according to which the nucleus contains alpha-particle at least as substructures although they cannot maintain their

identity for a very long time inside the condensed nuclear matter, but will dissolve into more elementary particles. The two protons and two neutrons which make up an alpha-particle can go into the same spatial state with the four different spin-charge combinations $p\uparrow p\downarrow n\uparrow n\downarrow$ without violating the Pauli exclusion principle. The wave function of the nucleons will thus overlap completely, giving rise to a large binding energy. The basic assumption of the alpha-particle model is that alpha-particles can be regarded as stable subunits of nuclei. At the very least this requires that there is a pronounced clustering of nucleons close together in such subunits i.e. there is a large probability of finding a group of nucleons close together and well separated from other such groups. The first and the most obvious success of the alpha-particle model is in the prediction of the binding energies of nuclei which can be formed out of an integral number of alpha-particles.

It has been known that there are certain properties of nuclei which suggest that nucleons tend to cluster in groups within a nucleus. This means that the binding forces between nucleons in the group have more effect than the binding forces between these nucleons and the other ones in the nucleus. The basic assumption of the cluster model is that the nucleons in the nucleus form a group of cluster among them instead of independent entity of each nucleon.

The resonating group model was proposed in 1937 by

Wheeler [1] which suggests that the neutrons and protons in the nucleus as being divided into various groups which do not maintain their identity forever but undergo continual changes, redistributing themselves into new groups. It treats correctly the motion of the total center of mass. The wave function of the composite nucleus is written as a totally antisymmetrized combination of the wave functions for the various possible groups nucleons. In the resonating group method one obtains for the relative motion of two groups an integral equation in which appears an interaction generated from two-nucleon forces. This consists of two parts - a direct part which involves no particle exchange between the two groups and another part appearing in the form of a non-local non-separable kernel interaction containing terms corresponding to the exchange of one, two, or more nucleons between the groups, Resonating group method was employed by Wildermuth [2] Van der spuy[3], Okai and Park [4] and Thompson and Tang [5,6] to study the problems of nuclear scattering and reaction for the case of ^{8}Be, ^{9}Be,^{7}Be, ^{8}Li,^{8}He etc. The result thus obtained agreed generally quite well with the experiment. All these cases deal with light nuclei, because of computational difficulties, in particular in handling the non-local, non-separable, large size Kernel.

The nuclear cluster structure was also described by Margenau [7]. This cluster structure was extended by Bloch and Brink [8] who formulated the so-called alpha-cluster model

through the use of harmonic oscillator shell model wave function. The properties of nuclei such as ^{12}C,^{16}O, ^{20}Ne etc. have been investigated [9,10,11,12] by utilizing alpha-cluster model. This alpha-cluster model has been further extended by Hill-Griffin Wheeler [13,14] by introducing generator coordinate method which is equivalent to the resonating group method. The purpose of these extensions was the dynamic description of clustering phenomena in nuclei. With this method it is possible to study nuclear bound state structures, scattering phenomena and reaction problems.

The formation of the microscopic cluster theory was described by Wildermuth [15] which is based on a variational principle, with the Hilbert space spanned by a set of non-orthogonal basis wave functions. Microscopic cluster theory has certain important characteristics which distinguish itself from other methods. These characteristics are:

It utilizes a N-N potential which explains the two-nucleon low energy scattering data.

It treats correctly the motion of the total center of mass.

It considers nuclear boundstate, scattering and reaction problems from a unified point of view.

It can be used to study cases where the particles involved in the incoming and outgoing channels are arbitrary composite nuclei.

The formulation of the microscopic cluster theory has been determined by the resonating group method or by generator

coordinate method. These two methods are equivalent but may not be the same. They are founded on exactly the same physical viewpoint but only difference between the choice of set of non-orthogonal basis wave functions. The microscopic cluster theory has been found to be successful in the case of light nuclei. Because of above features, the calculations are generally not so easy to perform. In spite of this, many calculations have been performed by using resonating group method and generator coordinate method. The elastic scattering of various nuclei has been studied by Kamimura [16] and Thompson and Tang [17] with single-channel resonating group method formulation. And also scattering has been studied by Fujiwara [18] by using generator coordinate method. These results were in good agreement with experimental findings.

In the earlier studies the resonating group method matrix elements are computed by the so-called cluster coordinate technique [15]. But this technique was a laborious procedure. Because of computational difficulties, the cluster coordinate technique has been replaced by another technique, the complex-generator coordinate technique [15]. This technique is especially useful for reactiou calculation.

The orthogonality condition method was proposed by Saito [19] which avoids the derivation of the complicated Kernel function that appears in the effects of the Pauli principle. Siuce then, it has been extensively applied to treat especially multi-cluster structure problems, where a

straightforward application of the resonating group method approach would be quite difficult.

Like Saito in our present study we are also trying to avoid the involvement of the kernel by another way. We know that, when two body fundamental N-N interaction is employed to generate resonating group model calculation, then a non-local non-separable kernel expression appears in the calculation. As this expression is non-local in character it is not easy to handle. Furthermore the terms become lengthy and complicated. Moreover for higher nuclei the calculation of this expression is troublesome.So it is necessary to devise way to include the effect of the non-local kernel and at the same time it is possible to handle the calculation for higher nuclei. Keeping this in mind we shall try to replace the non-local potential term by a simple term in our calculation, although the direct interaction terms (nuclear and coulomb) obtained from the resonating group consideration is fully taken into account.

CHAPTER TWO

ALPHA–DEUTERON MODEL OF ^{6}Li

The nucleus of ^{6}Li contains three protons and three neutrons. Inside the nucleus the nucleons behave differently - sometimes like independent particles and sometimes collectively. The famous shell model is based on the independent particle concept. It is also well known that there are certain properties of nuclei which suggest that the nucleons tend to cluster structnre within a nucleus. The pattern of cluster structure which is preferred depends on the form and strength of the interaction forces, the type of nucleus and excitation levels of this nucleus. From the experimental observation we know that the nuclear force between two nucleons acts in a short range (2fm) and this force is strongly attractive within this range but at a very short distance (0.5 fm) it becomes strongly repulsive. The effect of Pauli's exclusion principle in a system of nuclear dimension is to allow low energy nucleons to move relatively undisturbed throughout the nuclear volume, because these nucleons may not be scattered into other already occupied energy levels.

The nucleons move completely correlated with one another, otherwise they will be outside the range of each other's attractive forces and there will be little mutual potential energy among them. Alpha particle consists of two protons and two neutrons. These four nucleons can occupy the lowest level position to the maximum. It means that the alpha particle is a very stable nucleus. So we should expect that the four of the six nucleons of ^{6}Li are to correlate as an alpha-cluster. Next comes what happens when two or more nucleons are added to the strongly tight alpha particle. The fact that ^{5}He and ^{5}Li are not stable implies that one nucleon will not be bound with the first closed shell of nucleons. Hence there can be two assumptions consistent with the facts:

i)That a proton and neutron simultaneously joining alpha-cluster leads to the destruction of the alpha-cluster and the formation of a complex system of six nucleons.

ii)an alpha-cluster is not destroyed in forming the nucleus of ^{6}Li and binds effectively not with each nucleon but with a system of a proton and a neutron as a whole.

In the first case the nucleus of ^{6}Li consisting of six independent particles is studied from the stand point of shell model theory. And in the second case we can say that the proton and neutron above the closed shell of the nuclens of ^{6}Li must form a bound system. If they are not bound system, then these nucleons will be independent which implies that the formation of ^{6}Li nucleus is impossible. This contradicts the

existence of stable nucleus of ^{6}Li. Hence the proton and the neutron in this nucleus are not independent but are correlated strongly with each other forming a deuteron cluster. Thus the lowest states of ^{6}Li can be described as a system of alpha and deuteron clusters. In the α+d structure, the two clusters are bound together by only 1.47 MeV, which means that they are on the average rather far apart and behave more or less like free particles. Now we shall study the ^{6}Li nucleus as a cluster of alpha and deuteron particles.

Two distinct approaches are generally pursued in case of employing the alpha-deuteron cluster model. In one case antisymmetrization effect is assumed to be negligible. Main argument in favor of this assumption is that as the cluster binding energy between alpha and deuteron is small, both the particles move rather freely inside the ^{6}Li nucleus. As a result there is no appreciable effect of exchange force between them so that the exchange terms between the two clusters have little overlapping with the direct potential term. Further there is an argument that the α+d model of ^{6}Li has a meaning only and only when there is clearly expressed isolation of the alpha particle and the deuteron. On the other side those who take antisymmetrization effect into account have the argument in their favor that although the alpha deuteron relative energy is small but it is not that small for the two clusters in ^{6}Li so that they may be treated as free particles. Since the generalized cluster model is the

generalization of shell model such that in the limiting case both are same. As such the cluster model should naturally incorporate appropriate antisymmetrisation effect which is so vitally important in the case of shell model studies. Along this line many works have been done in the alpha-deuteron cluster model by the different authors [33,34,4].

The experimental calculation for the phase shifts of α+d scattering was performed by different groups, e.g. Senhouse and Tombrello [20] in the energy range of 2.9 to 11.5 Mev. Another experiment was performed by McIntyre and Haeberli [22] in the energy range of 2 to 10 Mev. In these two groups studies the value of S and D wave phase shifts are quite similar in he energy range of 2 to 6.7 Mev. but the value of the P-wave phase shifts are very different. Thus while the P-wave phase shifts of Senhouse [20] are small and show no such behavior. Darriulat and his collaborators [22] extended the experimental measurement of phase shifts for α+d system from 10 to 27 MeV.

The elastic scattering of deuteron by alpha-particle has been studied by Thompson and Tang [5] with the resonating group method in one channel approximation. In that study the calculation was simplified by introducing the assumption that the deuteron cluster can be described by single Gaussian function and obtained good fitting to the α+d phase shifts [20,21,22]. In a subsequent study [6] the same authors introduced more representative wave function which can be

described by a sum of two Gaussian functions for deuteron mainly to clear up the ambiguities faced earlier to large ratio of Serber force. Wildermuth and his collaborators [23] calculated the α+d S-wave phase shift by introducing distortion effect in the deuteron wave function and obtained good result. Thompson and Tang [24] calculated distortion effects in α+d system by using microscopic procedure and obtained good fitting to the α+d phase shifts [20,21,22]. Kanada et.al studied the α+d scattering system with multi-channel resonating group method.They have used the multi-channel resonating group R-matrix theory together with a variational method and obtained good result to the experiment.

In the next chapter we shell describe in detail the mathematical formalism which is pursued in the calculation of alpha-deuteron model of ^{6}Li in the resonating group formalism.

CHAPTER THREE

MATHEMATICAL FORMULATION

3.1 Differential Equation

^{6}Li nucleus consists of six nucleons - three neutrons and three protons. We shall consider that they can be divided into alpha-cluster (α-cluster) and deuteron-cluster (d-cluster). Let the nucleons 1,2,3,4 form the structure of an α-particle and 5,6 that of a deuteron where 1,2,5 are neutrons and 3,4,6 are protons.

Starting with the Hamiltonian

$$H = T+V \tag{3.1.1}$$

where the kinetic energy T and the potential energy V are respectively.

$$T = -\frac{\hbar^2}{2M}\sum_{i=1}^{6}\nabla_i^2 \tag{3.1.2}$$

and

$$V = \sum_{i<j}^{6} V(ij) \qquad (3.1.3)$$

Nucleon-Nucleon interaction V(ij) is given by

$$V(ij) = [\omega + mP_x(ij) + bP_\sigma(ij) + hH(ij)]\, v(ij) \qquad (3.1.4)$$

where ω , m, b, h are coefficients of Wigner, Majorana, Bartlett and Heisenberg exchange operators respectively satisfying the relation.

$$\omega + m + b + h = 1$$
$$\text{and} \qquad (3.1.5)$$
$$\omega + m - b - h = 0.63$$

the operators Px(ij), P_σ(ij) being the space and spin exchange operators for ith and jth particles. The operator H(ij) is the Heisenberg exchange operator, exchanging both space and spin coordinates of particles i and j and v(ij) the common radial dependence of the central exchange type Nucleon-Nucleon force.

The six-body Schrodinger equation is

$$H\Psi = E\Psi$$
$$\text{or, } [-\frac{\hbar^2}{2M}\sum_{i=1}^{6}\nabla_i^2 + \sum_{i<j=1}^{6} V(ij)]\,\Psi = E\Psi \qquad (3.1.6)$$

where Ψ is the total wave function of the six particle system. We define independent internal coordinates ρ_α , the relative coordinate r and the center of mass coordinate R_c

in the following way:

$$\vec{\rho}_1 = \vec{r}_1 - \vec{r}_2$$

$$\vec{\rho}_2 = \vec{r}_3 - \frac{\vec{r}_1 + \vec{r}_2}{2}$$

$$\vec{\rho}_3 = \vec{r}_4 - \frac{\vec{r}_1 + \vec{r}_2 + \vec{r}_3}{3}$$

$$\vec{\rho} = \vec{r}_5 - \vec{r}_6$$

$$\vec{r} = \frac{\vec{r}_1 + \vec{r}_2 + \vec{r}_3 + \vec{r}_4}{4} - \frac{\vec{r}_5 + \vec{r}_6}{2}$$

$$\vec{R}_c = \frac{\vec{r}_1 + \vec{r}_2 + \vec{r}_3 + \vec{r}_4 + \vec{r}_5 + \vec{r}_6}{6} \qquad (3.1.7)$$

By using these relations, the kinetic energy part of Hamiltonian H can be written as

$$-\frac{\hbar^2}{2M}\sum_{i=1}^{6}\nabla_i^2 = -\frac{\hbar^2}{2M}\left[2\nabla_{\rho_1}^2 + \frac{3}{2}\nabla_{\rho_2}^2 + \frac{4}{3}\nabla_{\rho_3}^2 + 2\nabla_{\rho}^2 + \frac{3}{4}\nabla_r^2 + \frac{1}{6}\nabla_{R_c}^2\right]$$

We are not interested in the center of mass motion which generally gives rise to spurious state. So the kinetic energy of the c.m motion $\nabla_{R_c}^2$ is neglected. Then the Schrodinger equation for six particles becomes

$$\left[-\frac{\hbar^2}{2M}\left(2\nabla_{\rho_1}^2 + \frac{3}{2}\nabla_{\rho_2}^2 + \frac{4}{3}\nabla_{\rho_3}^2 + 2\nabla_{\rho}^2 + \frac{3}{4}\nabla_r^2\right) + \sum_{i<j} V(ij)\right]\Psi = E\Psi \qquad (3.1.8)$$

$$or, \quad \left[-\frac{\hbar^2}{M}(T_\alpha + T_d) - \frac{3\hbar^2}{8M}\nabla_r^2 - E + \sum_{i\neq j=1}^{6} V(ij)\right]\Psi = 0 \qquad (3.1.9)$$

where

$$T_\alpha = \nabla^2_{\rho_1} + \frac{3}{4}\nabla^2_{\rho_2} + \frac{2}{3}\nabla^2_{\rho_3} \qquad (3.1.10)$$

and

$$T_d = \nabla^2_{\rho} \qquad (3.1.11)$$

are kinetic energy operator in the c.m system of α-cluster and d-cluster respectively.

3.2 The wave function and the detailed calculation:

Following the resonating group formalism the overall antisymmetrized wave function Ψ for six-nucleon system may be written as,

$$\Psi = A[\Psi_\alpha(12,34)\Psi_d(56)F(\vec{r})] \qquad (3.2.1)$$

where A is an antisymmetrization operator and is given by

$$A = [1-H(15)-H(25)][1-H(36)-H(46)] \qquad (3.2.2)$$

Here H(ij) is the Heisenberg exchange operator, exchanging both space and spin coordinates of particles i and j. Then $\Psi_\alpha(12,34)$ is the internal wave functions of the α-particles and have the from

$$\Psi_\alpha(12,34) = \chi(12,34)\phi_\alpha(12,34)$$

where $\chi(12,34)$ is the antisymmetric spin part and $\Phi_\alpha(12,34)$ the symmetric space part of Ψ_α.

$$\chi(12,34) = \frac{1}{2}[\alpha(1)\beta(2)-\beta(1)\alpha(2)][\alpha(3)\beta(4)-\beta(3)\alpha(4)]$$

is the spin wave function with α,β corresponding to np and down spin respectively. Similarly,

$$\Psi_d(56) = \chi(56)\phi_d(56)$$

is the internal wave function of the d-cluster, $F(\vec{r})$ describes the relative motion of alpha cluster and deuteron cluster. From the eqnations (3.2.1) and (3.2.2) the wave function Ψ can be rewritten as

$$\Psi = \Psi_\alpha\Psi_d F(\vec{r}) + [-H(15)-H(25)-H(36)-H(46)+\{H(15)+H(25)\}\{H(36)+H(46)\}]\Psi_\alpha\Psi_d F(\vec{r}) \qquad (3.2.3)$$

Since 1,2 and 5 are neutrons, the exchange operators H(15) and H(25) effectively represent the same operation and 3,4 and 6 are protons exchange operators H(36) and H(46) effectively represent the same operation. Hence one operator can be replaced by the other. i.e the operators H(25) and H(46) can be replaced by H(15) and H(36) respectively. The equation (3.2.3) becomes,

$$\Psi = \Psi_\alpha\Psi_d F(\vec{r}) + [-2H(15)-2H(36)+4H(15)H(36)]\Psi_\alpha\Psi_d F(\vec{r}) \qquad (3.2.4)$$

To obtain an equation for the relative wave function $F(\vec{r})$, the nucleon coordinates must be integrated out. Thus (3.1.9) is multiplied by $\Psi_\alpha^*\Psi_d^*$ and integrating over the space and spin variables of alpha and deuteron particles, we get

$$\int \Psi_\alpha^*\Psi_d^*\left[-\frac{\hbar^2}{M}(T_\alpha+T_d)-\frac{3\hbar^2}{8M}\nabla_r^2+\sum V(ij)-E\right]\Psi\, d\mathbf{T} = 0 \qquad (3.2.5)$$

(where dT is a differential volume element in the combined space and spin coordinates of alpha and deuteron particles.)

$$\text{or, } \int \Psi_\alpha^*\Psi_d^*\left[-\frac{\hbar^2}{M}(T_\alpha+T_d)-\frac{3}{8}\frac{\hbar^2}{M}\nabla_r^2-E_\alpha-E_d-(E-E_\alpha-E_d)\right.$$

$$\left.+\sum_{1234} V(ij)+\sum_{56} V(ij)+\sum_{1234,56} V(ij)\right]\Psi\, d\mathbf{T} = 0 \qquad (3.2.6)$$

where E_α and E_d are binding energy of alpha and d-cluster respectively. Combining equations (3.2.4) and (3.2.6) we get,

$$\int \Psi_\alpha^*\Psi_d^*\left[-\frac{\hbar^2}{M}(T_\alpha+T_d)-\frac{3}{8}\frac{\hbar^2}{M}\nabla_r^2-E_\alpha-E_d-(E-E_\alpha-E_d)+\sum_{i=1..6} V(ij)\right]$$

$$\Psi_\alpha\Psi_d F(\vec{r})\, d\mathbf{T}+\int \Psi_\alpha^*\Psi_d^*\left[-\frac{\hbar^2}{M}(T_\alpha+T_d)-\frac{3}{8}\frac{\hbar^2}{M}\nabla_r^2-E+\sum_{i=1..6} V(ij)\right]$$

$$\left[-2H(15)-2H(36)+4H(15)H(36)\right]\Psi_\alpha\Psi_d F(\vec{r})\, d\mathbf{T} = 0$$

$$\left[\sum_{1..6} V(ij) = \sum_{1234} V(ij) + \sum_{56} V(ij) + \sum_{1234,56} V(ij)\right]$$

$$\text{or, } \int \Psi_\alpha^* \Psi_d^* [-\frac{\hbar^2}{M} T_\alpha - E_\alpha + \sum_{1234} V(ij)] \Psi_\alpha \Psi_d F(\vec{r})\, d\mathbf{T}$$

$$+\int \Psi_\alpha^* \Psi_d^* [-\frac{\hbar^2}{M} T_d - E_d + \sum_{56} V(ij)] \Psi_\alpha \Psi_d F(\vec{r})\, d\mathbf{T}$$

$$+\int \Psi_\alpha^* \Psi_d^* [-\frac{3}{8}\frac{\hbar^2}{M} \nabla_r^2 - (E - E_\alpha - E_d)] \Psi_\alpha \Psi_d F(\vec{r})\, d\mathbf{T}$$

$$+\int \Psi_\alpha^* \Psi_d^* \sum_{1234,56} V(ij) \Psi_\alpha \Psi_d F(\vec{r})\, d\mathbf{T}$$

$$+\int \Psi_\alpha^* \Psi_d^* [-\frac{\hbar^2}{M}(T_\alpha + T_d) - \frac{3}{8}\frac{\hbar^2}{M} \nabla_r^2 - E + \sum_{12..6} V(ij)]$$

$$[-2H(15) - 2H(36) + 4H(15)H(36)] \Psi_\alpha \Psi_d F(\vec{r})\, d\mathbf{T} = 0 \qquad (3.2.7)$$

Then the variational alpha and deuteron equations are respectively

$$\int \Psi_\alpha^* [-\frac{\hbar^2}{M} T_\alpha - E_\alpha + \sum_{1234} V(ij)] \Psi_\alpha\, d\mathbf{T} = 0 \qquad (3.2.8)$$

$$\int \Psi_d^* [-\frac{\hbar^2}{M} T_d - E_d + \sum_{56} V(ij)] \Psi_d\, d\mathbf{T} = 0 \qquad (3.2.9)$$

From equations (3.2.7),(3.2.8) and (3.2.9) we get

$$[\frac{3}{8}\frac{\hbar^2}{M} \nabla_r^2 + E - E_r - E_d] F(\vec{r}) = \int \Psi_\alpha^* \Psi_d^* [-\frac{\hbar^2}{M}(T_\alpha + T_d) - \frac{3}{8}\frac{\hbar^2}{M} \nabla_r^2 - E]$$

$$[-2H(15)-2H(36)+4H(15)H(36)]\Psi_\alpha\Psi_d F(\vec{r})\,d\tau$$

$$+\int\Psi_\alpha^*\Psi_d^*[\sum_{1..6}V(ij)][-2H(15)-2H(36)$$

$$+4H(15)H(36)]\Psi_\alpha\Psi_d F(\vec{r})\,d\tau+\int\Psi_\alpha^*\Psi_d^*[\sum_{1234,56}$$

$$V(ij)]\Psi_\alpha\Psi_d F(\vec{r})\,d\tau \qquad (3.2.10)$$

After finding the value of spin operator, the equation (3.2.10) becomes,

$$[\frac{3}{8}\frac{\hbar^2}{M}\nabla_r^2+E-E_\alpha-E_d]\;F(\vec{r})=\int\Phi_\alpha^*\Phi_d^*[-\frac{\hbar^2}{M}(T_\alpha+T_d)$$

$$-\frac{3}{8}\frac{\hbar^2}{M}\nabla_r^2-E][-2P_x(15)+P_x(15)P_x(36)]\Phi_\alpha\Phi_d F(\vec{r})\,dP$$

$$+\int\Psi_\alpha^*\Psi_d^*[\sum_{1234,56}V(ij)][-4H(15)+4H(15)H(36)]$$

$$\Psi_\alpha\Psi_d F(\vec{r})\,d\tau+\int\Psi_\alpha^*\Psi_d^*[\sum_{1234,56}V(ij)]\Psi_\alpha\Psi_d F(\vec{r})\,d\tau \qquad (3.2.11)$$

where dP is differential volume element in the space coordinates of alpha and deuteron particles.Also we consider that the exchange effect between two protons and two neutrons is same and hence one can be replaced by the other. In that case

$$V = \int \Psi_\alpha^* \Psi_d^* \{ [\sum_{i<j=1} V(ij)] [-4H(15) + 4H(15) H(36)]$$

$$\Psi_\alpha \Psi_d F(\vec{r}) + [\sum_{1234,56} V(ij)] \Psi_\alpha \Psi_d F(\vec{r}) \} d\tau \qquad (3.2.12)$$

From equation (3.1.4) and (3.2.12) we get

$$V = V_0 \int \Psi_\alpha^* \Psi_d^* (\sum_{i<j=1} [\omega + mP_x(ij) + bP_\sigma(ij) + hP_x(ij) P_\sigma(ij)]$$

$$[-4H(15) + 4H(15) H(36)] v(ij) \Psi_\alpha \Psi_d F(\vec{r}) + \sum_{1234,56} [\omega$$

$$+ mP_x(ij) + bP_\sigma(ij) + hP_x(ij) P_\sigma(ij)] v(ij) \Psi_\alpha \Psi_d F(\vec{r}) \} d\tau$$

$$\text{or, } V = V_0 \int \Psi_\alpha^* \Psi_d^* (\sum_{i<j=1} v(ij) [\{ \omega + bP_\sigma(ij) \} + \{ m + hP_\sigma(ij) \}$$
$$P_x(ij)] [-4H(15) + 4H(15) H(36)] \Psi_\alpha \Psi_d F(\vec{r}) + 8v(15)$$
$$[\omega + mP_x(15) + bP_\sigma(15) + hP_x(15) P_\sigma(15)] \Psi_\alpha \Psi_d F(\vec{r}) \} d\tau$$

Integrating over the spin variables of alpha and deuteron particles and simplifying we obtain

$$V = -2V_0 (4\omega - m + 2b - 2h) \int \Phi_\alpha^* \Phi_d^* v(15) \Phi_\alpha \Phi_d F(\vec{r}) dP$$

$$+ V_0 \int \Phi_\alpha^* \Phi_d^* [-6 (\omega + m) v(12) + 2 (4m + 5h - 2b - \omega) v(15)$$
$$-2 (\omega + m + b + h) v(16) - 6 (\omega + m) v(23) + 2 (2m - b - 3\omega)$$
$$v(26) - 6 (\omega + m) v(25) - 2 (\omega + m + b + h) v(56)] P_x(15)$$
$$\Phi_\alpha \Phi_d F(\vec{r}) dP + V_0 \int \Phi_\alpha^* \Phi_d^* [2 (2\omega + 2m - b - h) v(12) +$$
$$(\omega + m + b + h) v(13) + 2 (\omega - 3m - h + 2b) v(15) + 2 (b + \omega)$$
$$v(16) + (\omega + b + m + h) v(24) + \omega - 3m - h + b) v(56)]$$
$$P_x(15) P_x(36) \Phi_\alpha \Phi_d F(\vec{r}) dP \qquad (3.2.13)$$

Then the equation (3.2.11) can be written as

$$[\frac{3}{8}\frac{\hbar^2}{M}\nabla_r^2 - V_D(r) + (E - E_a - E_d)]F(\vec{r}) = \int K(\vec{r},\vec{r}')F(\vec{r}')\,d\vec{r}' \qquad (3.2.14)$$

Where $V_D(r)$ is the direct interaction between the clusters originating from the identity element of the antisymmetrization operator A and is given by

$$V_D(r) = 3V_0(4\omega - m + 2b - 2h)\int \Phi_a^* \Phi_d^* v(15)\,\Phi_a \Phi_d \qquad (3.2.15)$$

The kernel $K(\vec{r},\vec{r}')$ represents the non-local interaction between the clusters and is given by

$$K(\vec{r},\vec{r}') = \int \Phi_a^* \Phi_d^* [-\frac{\hbar^2}{M}(T_a + T_d) - \frac{3}{8}\frac{\hbar^2}{M}\nabla_r^2 - E][-2P_x(15) + P_x(15)P_x(36)]\Phi_a \Phi_d F(\vec{r})\,dP + V_0 \int \Phi_a^* \Phi_d^* [-6(\omega + m)$$
$$v(12) - 2(\omega + 2b - 4m - 5h)V(15) - 2(\omega + m + b + h)$$
$$v(16) - 6(\omega + m)v(23) - 2(3\omega + b - 2m)v(26) - 6(\omega + m)$$
$$v(25) - 2(\omega + m + b + h)v(56)]P_x(15)\,\Phi_a \Phi_d F(\vec{r})\,dP + V_0 \int$$
$$\Phi_a^* \Phi_d^* [2(2\omega + 2m - b - h)v(12) + (\omega + m + b + h)v(13) + 2$$
$$(\omega - 3m - h + 2b)v(15) + 2(b + \omega)v(16) + (\omega + m + b + h)$$
$$v(24) + 2(2\omega + 2m - b - h)v(25) + (\omega + m + b + h)v(56)]$$
$$\Phi_a \Phi_d F(\vec{r})\,dP \qquad (3.2.16)$$

we consider the radial dependence of the Nucleon-Nucleon interaction v(ij) as

$$v(ij) = -V_0 e^{-\beta r_{ij}^2} \qquad (3.2.17)$$

The direct part $V_c(r)$ of the coulomb interaction is also added to $V_D(r)$ to give a total direct Part. Thus total direct part is

$$V'_D(r) = V_D(r) + V_c(r) \qquad (3.2.17)$$

From the equation (3.2.14) it is clear that in order to know $F(\vec{r})$ at the point r in space, it is necessary to know $F(\vec{r}')$ at all other points r' in space. Thus equation (3.2.14) describes a nonlocal process. The kernel of the integral equation $K(\vec{r},\vec{r}')$, involving both r and r' represents a nonlocal interaction and is symmetric. We can separate the partial waves by making the following expression in terms of legendre polynomials

$$F(\vec{r}) = \frac{1}{r}\sum_l f_l(r) P_l(\cos\theta) \qquad (3.2.19)$$

and

$$K(\vec{r},\vec{r}') = \frac{1}{4\pi r r'}\sum_{l=0}^{\infty}(2l+1) k_l(r,r') p_l(\cos\theta)$$

Thus

$$k_l(r,r') = 2\pi r r' \int_{-1}^{1} K(\vec{r},\vec{r}')\, P_l(\cos\theta)\, d(\cos\theta) \qquad (3.2.20)$$

Where θ is the angle between $\vec{r}$ and $\vec{r}'$. Now combining equation (3.2.14), (3.2.18), (3.2.19) and (3.2.20) we obtain

$$\left[\frac{3\hbar^2}{8M}\left(\frac{d^2}{dr^2} - \frac{l(l+1)}{r^2}\right) + E - E_\alpha - E_d - V_D(r) - V_c(r)\right] f_l(r)$$

$$= \int_0^\infty k_l(r,r')\, f_l(r')\, dr' \qquad (3.2.21)$$

Where M is the reduced mass. The wave functions for alpha particle and deuteron are respectively.

$$\Psi_\alpha(1234) = \exp\left[-\frac{\alpha}{2}\sum_{i=1}^{4}(\vec{r}_i - \vec{R}_\alpha)^2\right] \qquad (3.2.22)$$

and

$$\Psi_d(56) = \exp\left[-\frac{\alpha_1}{2}\sum_{i=5}^{6}(\vec{r}_i - \vec{R}_d)^2\right] + c\,\exp\left[-\frac{\alpha_2}{2}\sum_{i=5}^{6}(\vec{r}_i - \vec{R}_d)^2\right] \qquad (3.2.23)$$

Where R_α and R_d are the position vectors of the center of mass of alpha and deuteron cluster respectively.The direct potential V_0 (r) [6] in equation (3.2.21) is given by

$$V_D(r) = -\frac{V_0}{G_0}(8\omega - 2m + 4b - 4h)\sum_{i=1}^{4} K_i \left(\frac{\pi^3}{4\alpha^3}\right)^{\frac{3}{2}} \left(\frac{\pi}{2\lambda_i}\right)^{\frac{3}{2}}$$

$$\left[\frac{4\alpha\lambda_i}{4\alpha\lambda_i + \beta(2\alpha+3\lambda_i)}\right]^{\frac{3}{2}} \exp\left[-\frac{4\alpha\beta\lambda_i r^2}{4\alpha\lambda_i + \beta(2\alpha+3\lambda_i)}\right] \qquad (3.2.24)$$

where

$$G_0 = \sum_{i=1}^{4} K_i \left(\frac{\pi^3}{4\alpha^3}\right)^{\frac{3}{2}} \left(\frac{\pi}{2\lambda_i}\right)^{\frac{3}{2}} \tag{3.2.25}$$

and

$$K_1 = 1, K_2 = K_3 = C, K_4 = C^2$$
$$\lambda_1 = \alpha_1, \lambda_2 = \lambda_3 = \frac{\alpha_1+\alpha_2}{2}, \lambda_4 = \alpha_2 \tag{3.2.26}$$

The coulomb potential $V_c(r)$ [6] in (3.2.21) is given by

$$V_c(r) = \frac{zz'e^2}{rG_0} \sum_{i=1}^{4} K_i \left(\frac{\pi^3}{4\alpha^3}\right)^{\frac{3}{2}} \left(\frac{\pi}{2\lambda_i}\right)^{\frac{3}{2}} \Psi\left[\left(\frac{4\alpha\lambda_i}{2\alpha+3\lambda_i}\right)^{\frac{1}{2}} r\right] \tag{3.2.27}$$

Where z and z' being the atomic numbers of the alpha and deuteron clusters respectively and

$$\Psi(u) = \frac{2}{\sqrt{\pi}} \int_0^u \exp(-t^2)\, dt \tag{3.2.28}$$

The kernel [6] $k_1(r,r')$ is written as

$$k_1(r,r') = \frac{1}{G_0} \sum_{i=1}^{4} K_i \left(-\frac{\hbar^2}{2M} X_i - V_0 Y_i + E' Z_i\right) \tag{3.2.29}$$

Where E' is given as

$$E' = E + E_\alpha + E_d \tag{3.2.30}$$

with

$$E_a = \frac{\hbar^2}{2M}\frac{9}{2}\alpha - 6(m+\omega)V_0\left(\frac{\alpha}{\alpha+2\beta}\right)^{\frac{3}{2}} + \frac{z(z-1)e^2}{2}\left(\frac{2\alpha}{\pi}\right)^{\frac{1}{2}} \qquad (3.2.31)$$

and

$$E_g = \frac{1}{G_0}\left(\frac{\pi^3}{4\alpha^3}\right)^{\frac{3}{2}}\sum_{i=1}^{4} K_i\left(\frac{\pi}{2\lambda_i}\right)^{\frac{3}{2}}\left[\frac{\hbar^2}{2M}(3\nu_i - \frac{3\nu_i^2}{2\lambda_i}\right.$$

$$\left. -(\omega+m+b+h)V_0\left(\frac{\lambda_i}{\lambda_i+2\beta}\right)^{\frac{3}{2}} + \frac{z'(z-1)e^2}{2}\left(\frac{2\lambda_i}{\pi}\right)^{\frac{1}{2}}\right] \qquad (3.2.32)$$

and the quantities X_i, Y_i and Z_i are defined as follows

$$X_i = 2e_1\left[\left\{6\alpha+3\nu_i - \frac{6\alpha^2+9\nu_i^2}{4\alpha+6\lambda_i} + 9a_1 - \left(\frac{3}{2}T_a + 4a_1^2\right)r^2\right.\right.$$

$$\left. -\left(\frac{2}{3}T_b + \frac{1}{3}c_1^2\right)r^2\right\}S_1\left(-\frac{3}{2}c_1\right) + \left(\frac{3}{2}T_c + \frac{4}{3}a_1c_1\right)rr'$$

$$\left. T_1\left(-\frac{3}{2}c_1\right)\right]\exp\left\{-\frac{2}{3}\left(a_1r^2+b_1r^2\right)\right] - e_2\left[\left\{\frac{25}{2}\alpha\right.\right.$$

$$\left. -\frac{3\alpha^2}{3\lambda_i+\alpha-\nu_i} + \frac{3\alpha\nu_i}{\alpha+\nu_i} - \frac{112}{27}\alpha^2(r^2+r^2)\right\}S_1\left(\frac{-2}{3}c_2\right)$$

$$\left. -\frac{208}{27}\alpha^2 rr'T_1\left(\frac{-2}{3}c_2\right)\right]\exp\left[-\frac{2}{3}\left(a_2r^2+b_2r^2\right)\right] \qquad (3.2.33)$$

$$Y_i = e_1\left[e_{23}^1 - (6\omega-6m)S_1\left(\frac{-2}{3}c_1\right)\exp\left[\frac{-2}{3}\left(a_1r^2+b_1r^\beta\right)\right]\right.$$

$$e_{15}^1(-2\omega+8m-4b+4h)S_1\left(\frac{-2}{3}c'_{15}\right)\exp\left[-\frac{2}{3}\left(a'_{15}r^2+b'_{15}r^\beta\right)\right]$$

$$+e_{26}^1(-6\omega+4m-2b+6h)S_1\left(-\frac{2}{3}c'_{26}\right)\exp\left[-\frac{2}{3}\left(a'_{26}r^2+b'26r^\beta\right)\right]$$

$$\left. +e_{16}^1(-2\omega-2m-2b-2h)S_1\left(-\frac{2}{3}c'_{16}\right)\left(\exp\left[-\frac{2}{3}\left(a'_{16}r^2+b'_{16}r^\beta\right)\right]\right.\right.$$

$$+\exp\left[-\frac{2}{3}\left(b'_{16}r^2+a'_{16}r^{\beta}\right)\right]\Big)+e^1_{12}\left(-6\omega-6m\right)S_1\left(-\frac{2}{3}c'_{12}\right)$$

$$\left(\exp\left[-\frac{2}{3}\left(a'_{12}r^2+b'_{12}r^{\beta}\right)\right]+\exp\left[-\frac{2}{3}\left(b'_{12}r^2+a'_{12}r^{\beta}\right)\right]\right)\Big)$$

$$e_2\Big\{\left[e^2_{12}\left(2\omega+2m+2b+2h\right)+e^2_{34}\left(\omega+m+b+h\right)\right]S_1\left(-\frac{2}{3}c_2\right)$$

$$\exp\left[-\frac{2}{3}\left(a_2r^2+b_2r^{\beta}\right)\right]+e^2_{15}\left(4\omega-6m-6b-2h\right)S_1\left(-\frac{2}{3}c^2_{15}\right)$$

$$\exp\left[-\frac{2}{3}\left(a^2_{15}r^2+b^2_{15}r^{\beta}\right)\right]+e^2_{13}\left(4\omega+4m-2b-2h\right)S_1\left(-\frac{2}{3}c^2_{13}\right)$$

$$\left(\exp\left[-\frac{2}{3}\left(a^3_{13}r^2+b^3_{13}r^{\beta}\right)\right]+\exp\left[-\frac{2}{3}\left(b^3_{13}r^2+a^3_{13}r^{\beta}\right)\right]\right)\Big\} \qquad (3.2.34)$$

$$Z_1=2e_1S_1\left(-\frac{2}{3}c_1\right)\exp\left[-\frac{2}{3}\left(a_1r^2+b_1r^{\beta}\right)\right]$$

$$-e_2S_1\left(-\frac{2}{3}c_2\right)\exp\left[-\frac{2}{3}\left(a_2r^2+b_2r^{\beta}\right)\right] \qquad (3.2.35)$$

with $S_1(\omega)=\frac{4\pi}{\omega}J_{1+\frac{1}{2}}(\omega rr')$ and

$$T_1(\omega)=\frac{4\pi}{\omega}\left[J_{1+\frac{3}{2}}(\omega rr')-\frac{1}{\omega rr'}J_{1+\frac{1}{2}}(\omega rr')\right]$$

Where $J_{\beta}(x)$ is a hyperbolic spherical Bessel's function. Also in equations (3.2.29),(3.2.30), (3.2.31), (3.2.32), (3.2.33), (3.2.34) and (3.2.35) the following definitions have been made:

$$\nu_1=\nu_2=\alpha_1,\ \nu_3=\nu_4=\alpha_2$$

$$e_1=\left(\frac{4}{3}\right)^3\left(\frac{\pi}{\alpha}\right)^3\left(\frac{\pi}{4\alpha+6\lambda_i}\right)^{\frac{3}{2}}$$

$$e_2=\left(\frac{4}{3}\right)^3\left(\frac{\pi}{2\alpha}\right)^{\frac{3}{2}}\left(\frac{\pi}{\alpha+\nu_i}\right)^{\frac{3}{2}}\left(\frac{\pi}{2\lambda_i+\alpha-\nu_i}\right)^{\frac{3}{2}}$$

$$a_1=\frac{e_1}{6\alpha+9\lambda_i},\quad b_1=\frac{g_1}{6\alpha+9\lambda_i},$$

$$c_1=\frac{h_1}{6\alpha+9\lambda_i}$$

$$a_2=b_2=\frac{5}{8}\alpha,\qquad c_2=-\frac{8}{3}\alpha,$$

$$T_a=\frac{2\alpha^2}{(2\alpha+3\lambda_i)^2}(\alpha^2-2\alpha\lambda_i+2\alpha\nu_i-2\nu_i\lambda_i+\nu_i^2+\lambda_i^2)$$

$$+\frac{\nu_i^2}{3(2\alpha+3\lambda_i)^2}(\alpha^2-12\alpha\lambda_i+6\alpha\nu_i-36\nu_i\lambda_i+9\nu_i^2+36\lambda_i^2)$$

$$T_b=\frac{2\alpha^2}{(2\alpha+3\lambda_i)^2}(\alpha^2+4\alpha\nu_i+4\nu_i\lambda_i+\nu_i^2+4\lambda_i^2)$$

$$+\frac{\nu_i^2}{3(2\alpha+3\lambda_i)^2}(25\alpha^2+60\alpha\lambda_i-30\alpha\nu_i-36\nu_i\lambda_i+9\nu_i^2+36\lambda_i^2)$$

$$T_c=\frac{2\alpha^2}{(2\alpha+3\lambda_i)^2}(2\alpha^2+2\alpha\lambda_i+4\alpha\nu_i+2\nu_i\lambda_i+2\nu_i^2-4\lambda_i^2)+$$

$$\frac{\nu_i^2}{3(2\alpha+3\lambda_i)^2}(-10\alpha^2+48\alpha\lambda_i-24\alpha\nu i-72\nu_i\lambda_i+18\nu_i^2+72\lambda_i^2)$$

$$a_{15}^2 = b_{15}^2=\frac{5\alpha(\alpha+\nu_i)(2\lambda_i+\alpha-\nu_i)+\beta(12\alpha^2+14\alpha\lambda_i+4\nu_i\lambda_i-2\nu_i^2)}{3(\alpha+\nu_i)(2\lambda_i+\alpha-\nu_i)+3\beta(2\alpha+2\lambda_i)}$$

$$c_{15}^2 = -\frac{8\alpha(\alpha+\nu_i)(2\lambda_i+\alpha-\nu_i)+\beta(12\alpha^2+8\lambda_i-8\lambda_i\nu_i+4\nu_i^2)}{3(\alpha+\nu_i)(2\lambda_i+\alpha-\nu_i)+3\beta(2\alpha+2\lambda_i)}$$

$$e_{15}^2 = \left[\frac{2\alpha(\alpha+\nu_i)}{2\alpha(\alpha+\nu_i)+\beta(3\alpha+\nu_i)}\right]^{\frac{3}{2}}$$

$$e_{23}^{1} = \left[\frac{\alpha}{\alpha+2\beta}\right]^{\frac{3}{2}}$$

$$e_{15}^{1} = 1$$

$$a_{15}^{1} = a_1+\frac{8}{3}\beta$$

$$b_{15}^{1} = b_1+\frac{8}{3}\beta$$

$$c_{15}^{1} = c_1+\frac{16}{3}\beta$$

$$e_{26}^{1} = \left[\frac{2\alpha^2+3\alpha\lambda_1}{2\alpha+3\alpha\lambda_1+\beta(2\alpha+2\lambda_1)}\right]^{\frac{3}{2}}$$

$$a_{26}^{1} = \frac{\alpha f_1+2\beta(6\alpha^2+7\alpha\lambda_1+2\nu_1\lambda_1-\nu_1^2)}{\alpha(6\alpha+9\lambda_1)+\beta(6\alpha+6\lambda_1)}$$

$$b_{26}^{1} = \frac{\alpha g_1+2\beta(6\alpha^2+7\alpha\lambda_1+2\nu_1\lambda_1-\nu_1^3)}{\alpha(6\alpha+9\lambda_1)+\beta(6\alpha+6\lambda_1)}$$

$$c_{26}^{1} = \frac{\alpha h_1-2\beta(6\alpha^2-4\alpha\lambda_1-4\lambda_1\nu_1+2\nu_1^2)}{\alpha(6\alpha+9\lambda_1)+\beta(6\alpha+6\lambda_1)}$$

$$e_{16}^{1} = \left(\frac{2\alpha+3\lambda_1}{2\alpha+3\lambda_1+6k}\right)^{\frac{3}{2}}$$

$$a_{16}^{1} = \frac{f_1+\beta(10\alpha+24\lambda_1-12\nu_1)}{6\alpha+9\lambda_1+18\beta}$$

$$b_{16}^{1} = \frac{g_1+\beta(34\alpha+24\lambda_1-12\nu_1)}{6\alpha+9\lambda_1+18\beta}$$

$$c_{16}^{1} = \frac{h_1+\beta(8\alpha+48\lambda_1-24\nu_1)}{6\alpha+9\lambda_1+18\beta}$$

$$e_{12}^{1} = \left[\frac{2\alpha^2+3\alpha\lambda_i}{2\alpha^2+3\alpha\lambda_i+\beta(4\alpha+2\lambda_i)}\right]^{\frac{3}{8}}$$

$$a_{12}^{1} = \frac{\alpha f_i+\beta(10\alpha^2+54\alpha\lambda_i-20\alpha v_i+4v_i\lambda_i-2v_i^2)}{\alpha(6\alpha+9\lambda_i)+\beta(12\alpha+6\lambda_i)}$$

$$b_{12}^{1} = \frac{\alpha g_i+\beta(10\alpha^2+6\alpha\lambda_i+4\alpha v_i+4v_i\lambda_i-2v_i^2)}{\alpha(6\alpha+9\lambda_i)+\beta(12\alpha+6\lambda_i)}$$

$$c_{12}^{1} = \frac{\alpha h_i+\beta(20\alpha^2+24\alpha\lambda_i-16\alpha v_i+8v_i\lambda_i-4v_i^2)}{\alpha(6\alpha+9\lambda_i)+\beta(12\alpha+6\lambda_i)}$$

$$e_{12}^{2} = \left[\frac{\alpha+v_i}{\alpha+v_i+4\beta}\right]^{\frac{3}{2}}, \quad e_{34}^{2} = \left[\frac{\alpha}{\alpha+2\beta}\right]^{\frac{3}{2}}$$

$$e_{15}^{2} = \left[\frac{(\alpha+v_i)(2\lambda_i+\alpha-v_i)}{(\alpha+v_i)(2\lambda_i+\alpha-v_i)+\beta(2\alpha+2\lambda_i)}\right]^{\frac{3}{2}}$$

$$a_{13}^{2} = \frac{10\alpha^2(\alpha+v_i)+\alpha\beta(31\alpha+21vi)}{6\alpha(\alpha+v_i)+3\beta(3\alpha+v_i)}$$

$$b_{13}^{2} = \frac{10\alpha^2(\alpha+v_i)+\alpha\beta(19\alpha+9v_i)}{6\alpha(\alpha+v_i)+3\beta(3\alpha+v_i)}$$

$$c_{13}^{2} = -\frac{16\alpha^2(\alpha+v_i)+\alpha\beta(40\alpha+24v_i)}{6\alpha(\alpha+v_i)+3\beta(3\alpha+v_i)}$$

with

$$f_i = 3\alpha^2+17\alpha\lambda_i-6\alpha v_i+6\lambda_i v_i-3v_i^2$$

$$g_i = 3\alpha^25\alpha\lambda_i+6\alpha v_i+6\lambda_i v_i-3v_i^2$$

$$h_i = 6\alpha^2+4\alpha\lambda_i+12v_i\lambda_i-6v_i^2$$

CHAPTER FOUR

PRESENT WORKS

Now if one follows the resonating group formalism then one is to solve the equation (3.2.21) namely

$$[\frac{3}{8}\frac{\hbar^2}{M}\{\frac{d^2}{dr^2}-\frac{l(l+1)}{r^2}\}+E-E_\alpha-E_d-V_D(r)-V_c(r)]f_l(r)$$
$$=\int_0^\infty k_l(r,r')f_l(r')dr'$$

numerically to obtain the desired result. In the first, the Kernel $k_l(r,r')$ is non-local and non-separable in character. Further for the tiny ^{6}Li nucleus the term $k_l(r,r')$ contains a number of terms as can be seen from (3.2.29) and after words. The numerical calculation of the above equation may not be too difficult for this case. But for nuclei of higher mass number where more and more nucleons will be involved the analytical calculation of the Kernel will be very lengthy and complicated. In that case numerical calculation will get unmanageable. That is why although the resonating group formalism has been in application for a long time and the results following this approach are good, the detailed calculation following this method hardly passes the nuclei of

15-20 nucleons.

In that case we see that we can not apply this beautiful, elaborate and systematic method of resonating group formalism except to a handful of light nuclei for nuclear study. So in our present study we are trying to test how this non-local, non-separable part namely, Kernel $k_1(r,r')$ can be replaced by a simple non-local separable potential namely $Ae^{\gamma_1 r}e^{\gamma_2 r'}$ (A, γ_1, γ_2, are parameters). So that not only the calculation could be made manageable but also nuclei of any number of nucleons can be studied with this resonating group formalism. We have fully taken into account of other effects of resonating group such as direct potential term and coulomb energy term obtained from resonating group formalism. In our present study we are following the middle path. As stated earlier in the cluster model approach one group totally ignores antisymmetrization effect while another group takes full account of it resulting to inability in calculation of higher nuclei. In our present study we have taken the direct part of nuclear potential and coulomb energy from antisymmetrization and replace the non-local part by phenomenological potential.

In the present study we shall take the wave functions for ^{4}He and deuteron as

$$\Phi_\alpha = \exp\left[-\frac{\alpha}{2}\sum_{i=1}^{4}(\vec{r}_i-\vec{R}_\alpha)^2\right] \qquad (4.1)$$

and

$$\Phi_d = \exp[-\frac{\alpha_1}{2}\sum_{I=5}^{6}(\vec{r}_I-\vec{R}_d)^2] + C\exp[-\frac{\alpha_2}{2}\sum_{I=5}^{6}(\vec{r}_I-\vec{R}_d)^2] \qquad (4.2)$$

for alpha-cluster and d-cluster respectively with

$$\alpha = 0.514\ F^{-2} \qquad (4.3)$$

and

$$\alpha_1 = 0.157\ F^{-2},\quad \alpha_2 = 1.137\ F\text{-}2 \text{ and } C = 2.747 \qquad (4.4)$$

N-N Potential is given by [6],

$$V_{ij} = -V_0(\omega+mP^r_{ij}+bP^\sigma_{ij}+hP^\tau_{ij})\exp[-\beta r^2] \qquad (4.5)$$

$$V_0 = 72.98\ \text{MeV and } \beta = 0.46\ F^{-2} \qquad (4.6)$$

From the expression of $V_0(r)$ and $V_c(r)$ one notes that the constants ω, m, b, h occur only in the combination of (3.1.5) and

$$V_{ij} = yV_{\text{serber}} + (1-y)V_{\text{symmetric}} \qquad (4.7)$$

where V_{serber} is given by potential (4.5) with ω = m and b=h and $V_{\text{symmetric}}$ given by potential (4.5) with m=2b and h=2ω.

The value of y is 1.004 as given by Thompson[6]. With the above consideration, the integro differential equation becomes,

$$[\frac{3}{8}\frac{E^2}{M}\{\frac{d^2}{dr^2}-\frac{l(l+1)}{r^2}\}+E-V_c(r)-V_b(r)]f_l(r) = \int_0^\infty Ae^{-\gamma_1 r}e^{-\gamma_2 r'}f_l(r')dr' \qquad (4.8)$$

where the expression for $V_0(r)$, $V_2(r)$ are given by equations (3.2.24) and (3.2.27) respectively and E is the relative energy of α+d system. We shall calculate the phase shift for different energy levels by employing the numerical method which we have described in detail in the next chapter.

CHAPTER FIVE

NUMERICAL CALCULATION

Generally the kernel $k_l(r,r') = Ae^{-\gamma_1 r} e^{-\gamma_2 r'}$ on the right side of equation (4.8) decreases rapidly as r or r' increases to enable us to replace the infinite upper limit in the integral by some finite value R_0. To solve the integrodifferential equation (4.8), this equation can be converted into a set of simultaneons-linear-algebraic equations following the method of Robertson[28] in the region $r \leq R_0$. This method is described as follows:

The equation (4.8) can be written as

$$\left[\frac{3}{8}\frac{\hbar^2}{M}\left\{\frac{d^2}{dr^2}-\frac{l(l+1)}{r^2}\right\}+E'-V_D(r)-V_c(r)\right]f_l(r)$$
$$= \int_0^\infty k_l^1(r,r')\,f_1(r')\,dr' + E\int_0^\infty k_l^2(r,r')\,f_1(r')\,dr' \qquad (5.1)$$

where E is the total energy of the α + d system i, e $E=E'+E_\alpha+E_d$ and

$$k_1^1(r,r') = Ae^{-\gamma_1 r}\, e^{-\gamma_1 r'}$$

$$k_1^2(r,r') = Be^{-\beta_1 r}\, e^{-\beta_1 r'} \qquad (5.2)$$

At the point r = r_n, the integral and derivative are replaced by sum and difference respectively as follows:

$$\int_0^{R_n} k_1^1(r_n,r')\, f_1(r')\, dr' = \sum_{m=0}^{N} T_m^1 k_1^1(r_n,r'_m)\, f_1(r'_m) \qquad (5.3)$$

$$\int_0^{R_n} k_1^2(r_n,r')\, f_1(r')\, dr' = \sum_{m=0}^{N} T_m^2 k_1^2(r_n,r'_m)\, f_1(r'_m) \qquad (5.4)$$

and

$$\delta^2 f_n = h^2\left(1+\frac{\delta^2}{12}-\frac{\delta^4}{240}+ \text{-------}\right) f''_n \qquad (5.5)$$

Substituting (5.3), (5.4) in equation (5.1) we obtain,

$$\frac{3}{8}\frac{\hbar^2}{M} f''_n + \left[E'-Y_n-\frac{3\hbar^2}{8M}\frac{l(l+1)}{r^2}\right] f_n$$

$$= \sum_{m=0}^{N} T_m k_1^1(r_n,r'_m)\, f_m + E\sum_{m=0}^{N} T_m^1 k_m^2(r_n,r'_m)\, f_n$$

where $Y_n = V_D(r_n) + V_c(r_n)$

This equation can be written as

$$f''_n + u_n f_n = \sum_{m=0}^{N} T_m k_{n,m} f_m \qquad (5.6)$$

where $u_n = \frac{8M}{3\hbar^2}\left[E'-Y_n-\frac{3\hbar^2}{8M}\frac{l(l+1)}{r_n^2}\right]$ and $T_m^1 = T_m^2 = T_n$

and

$$k_{n,m} = [k_1^1(r_n, r'_m) + Ek_1^2(r_n, r'_m)]\,\frac{8M}{3R^2}$$

Also the equation (5.5) can be written as

$$\delta^2 f_n = h^2\left(1+\frac{\delta^2}{12}\right) f''_n + e \qquad (5.7)$$

Combining (5.6) and (5.7) we get,

$$\delta^2 f_n = h^2\left(1+\frac{\delta^2}{12}\right)\left(-u_n f_n + \sum_{m=0}^{N} T_m k_{n,m} f_m\right)$$

[neglecting other terms]

$$\text{or, } f_{n+1} - 2f_n + f_{n-1} = -\frac{h^2}{12}\left(u_{n+1}f_{n+1} + 10u_n f_n + u_{n-1}f_{n-1}\right) + \frac{h^2}{12}\sum_{m=0}^{N}\left(T_m k_{n+1,m} f_m + 10k_{n,m}f_m + T_m k_{n-1,m} f_m\right)$$

$$\text{or, } \left(1+\frac{h^2}{12}u_{n+1}\right) f_{n+1} - \left(2-\frac{10}{12}h^2 u_n\right) f_n + \left(1+\frac{h^2}{12}u_{n-1}\right) f_{n-1} = \frac{h^2}{12}\sum_{m=0}^{N} T_m\left(k_{n+1,m} + 10k_{n,m} + k_{n-1,m}\right) f_m$$

Then by applying the boundary conditions $f_0 = 0$ and $f_{N+1} = 0$ the above equation becomes

$$\frac{1}{DH}\left[DH - \left(Y_{n-1} + \frac{DA}{r_{n-1}^2} - E'\right)\right] f_{n-1} - \frac{1}{DH}\left[2DH + 10\left(Y_n + \frac{DA}{r_n^2} - E'\right)\right] f_n + \frac{1}{DH}\left[DH - \left(Y_{n+1} + \frac{DA}{r_{n+1}^2} - E'\right)\right] f_{n+1}$$

$$= \frac{T_1}{DH}\left[k_{n-1,1}^1 + 10k_{n,1}^1 + k_{n+1,1}^1 + E\left(k_{n-1,1}^2 + 10k_{n,1}^2 + k_{n+1,1}^2\right)\right] f_1$$

$$+ \frac{T_2}{DH}\left[k^1_{n-1,2}+10k^1_{n,2}+k^1_{n+1,2}+E\left(k^2_{n-1,2}+10k^2_{n,2}+k^2_{n+1,2}\right)\right] f_2$$

$$+ \frac{T_3}{DH}\left[k^1_{n-1,3}+10k^1_{n,3}+k^1_{n+1,3}+E\left(k^2_{n-1,3}+10k^2_{n,3}+k^2_{n+1,3}\right)\right] f_3$$

$$+ \quad --- \quad --- \quad --- \quad --- \quad --- \quad --- \quad --- \quad ---$$

$$+ \quad --- \quad --- \quad --- \quad --- \quad --- \quad --- \quad --- \quad ---$$

$$+ \frac{T_N}{DH}\left[k^1_{n-1,N}+10k^1_{n,N}+k^1_{n+1,N}+E\left(k^2_{n-1,N}+10k^2_{n,N}+k^2_{n+1,N}\right)\right] f_N \qquad (5.8)$$

Where $D=\frac{3\hbar^2}{8M}$, $A = l(l+1)$ *and* $H = \frac{12}{h^2}$

Substituting n = 1, 2, 3.....*in equation* (5.8) *we get a set linear algebraic equations*

$$\frac{1}{DH}\left[-T_1\left\{10k^1_{1,1}+k^1_{2,1}+E\left(10k^2_{1,1}+k^2_{2,1}\right)\right\}-2DH-10\left(Y_1+\frac{DA}{r_1^2}-E'\right)\right] f_1$$

$$+ \frac{1}{DH}\left[-T_2\left\{10k^1_{1,2}+k^1_{2,2}+E\left(10k^2_{1,2}+k^2_{2,2}\right)\right\}+DH-\left(Y_2+\frac{DA}{r_1^2}-E'\right)\right] f_2$$

$$+ \frac{1}{DH}\left[-T_3\left\{10k^1_{1,3}+k^1_{2,3}+E\left(10k^2_{1,3}+k_{3,3^2}\right)\right\}\right] f_3$$

$$+ \frac{1}{DH}\left[-T_4\left\{10k^1_{1,4}+k^1_{2,4}+E\left(10k^2_{1,4}+k^2_{2,4}\right)\right\}\right] f_4$$

$$+ \quad --- \quad --- \quad --- \quad --- \quad --- \quad --- \quad ---$$

$$+ \quad --- \quad --- \quad --- \quad --- \quad --- \quad --- \quad ---$$

$$+ \frac{1}{DH}\left[-T_N\left\{10k^1_{1,N}+k^1_{2,N}+E\left(10k^2_{1,N}+k^2_{2,N}\right)\right\}\right] f_N = 0$$

$$\frac{1}{DH}\{-T_1(k^1_{1,1}+10k^1_{2,1}+k^1_{3,1}+E(k^2_{1,1}+10k^2_{2,1}+k^2_{3,1})\}+DH-(Y_1+\frac{DA}{x_1^2}-E')]\,f_1$$

$$+\ \frac{1}{DH}[-T_2(k^1_{1,2}+10k^1_{2,2}+k^1_{3,2}+E(k^2_{1,2}+10k^2_{2,2}+k^2_{3,2})\}-2DH-10(Y_2+\frac{DA}{x_2^2}-E')]\,f_2$$

$$+\ \frac{1}{DH}[-T_3(k^1_{1,3}+10k^1_{2,3}+k^1_{3,3}+E(k^2_{1,3}10k^2_{2,3}+k^2_{3,3})\}+DH-(Y_3+\frac{DA}{x_3^2}-E')]\,f_3$$

$$+\ \frac{1}{DH}[-T_4(k^1_{1,4}+10k^1_{2,4}k^1_{3,4}+E(k^2_{1,4}+10k^2_{2,4}+k^2_{3,4})]\,f_4$$

$$+\quad --- \quad --- \quad --- \quad --- \quad --- \quad --- \quad ---$$

$$+\quad --- \quad --- \quad --- \quad --- \quad --- \quad --- \quad ---+$$

$$\frac{1}{DH}[-T_N(k^1_{1,N}+10k^1_{2,N}+k^1_{3,N}+E(k^2_{1,N}+10k^2_{2,N}+k^2_{3,N})]\,f_N=0$$

$$\frac{1}{DH}[-T_1(k^1_{2,1}+10k^1_{3,1}+k^1_{4,1}+E(k^2_{2,1}+10k^2_{3,1}+k^2_{4,1})]\,f_1$$

$$+\frac{1}{DH}[-T_2(k^1_{2,2}+10k^1_{3,2}+k^1_{4,2}+E(k^2_{2,2}+10k^2_{3,2}+k^2_{4,2})\}+DH-(Y_2+\frac{DA}{x_2^2}-E')]\,f_2$$

$$+\ \frac{1}{DH}[-T_3(k^1_{2,3}+10k^1_{3,3}+k^1_{4,3}+E(k^2_{2,3}+10k^2_{3,3}+k^2_{4,3})\}-2DH-10(Y_3+\frac{DA}{x_3^2}-E')]\,f_3$$

$$+\ \frac{1}{DH}[-T_4(k^1_{2,4}+10k^1_{3,4}+k^1_{4,4}+E(k^2_{2,4}+10k^2_{3,4}+k^2_{4,4})\}+DH-(Y_4+\frac{DA}{x_4^2}-E')]\,f_4$$

$$+\quad --- \quad --- \quad --- \quad --- \quad --- \quad --- \quad ---$$

$$+\quad --- \quad --- \quad --- \quad --- \quad --- \quad --- \quad ---$$

$$+\frac{1}{DH}[-T_N(k^1_{2,N}+10k^1_{3,N}+k^1_{4,N}+E(k^2_{2,N}+10k^2_{3,N}+k^2_{4,N})\}]\,f_N\ =\ 0$$

and so on

The integral involving the kernel function is evaluated by Simson formula. In the region $r>R_m$ is obtained by solving the equation (4.8) with a method given by Fox and Goodwin [29]. The function $f_l(r)$ will be matched to coulomb function at a distance which is large enough to fulfil the requirement of a simple method of calculation for coulomb function given by Frogberg[30].

Numerical calculation of phase shift is performed from 2 to 18 Mev for the incident energy of α+d system in the C.M. System of each partial waves for l = 0,2 and 4 by adjusting the parameters A, B, γ_1, γ_2, β_1 and β_2.

CHAPTER SIX

RESULTS AND CONCLUSIONS

Using the value of width parameters α_1 =0.157 α_2 =1.137 F^{-2} and C=2.747 in equation (3.2.3.2) the binding energy of deuteron is -2.04 Mev and corresponding deuteron rms radius is 1.68F. These values are in good agreement with the experimental values -2.22 Mev and 1.93 F.

Scattering calculation for the α+d system will be made in the range of the bombarding energy upto 18 MeV in the c.m system. As was mentioned in the previous section, the parameters $A, B, \gamma_1, \gamma_2, \beta_1$ and β_2 of the kernel function are adjusted to yield a best agreement with experimental results. The results are given by the curves in figs. (1), (2) and (3). In this figs, we have also given the experimental values of the phase shift as obtained by McIntyre and Haeberli [21] in the energy range of 1.3 to 6.3 MeV and by Darriulatel [22] in the energy ranges of 6.7 to 18.2 MeV. The calculated phase shifts for 1_{S0} is shown by the three Solid curves in fig-1 which indicated by curve a, curve b, and curve c, where the parameters $A, B, \gamma_1, \gamma_2, \beta_1$ and β_2 are Chosen to have the

following values

Curve	A	B	γ_1	γ_2	β_1	β_2
a	-0.88	-42.49	0.236	0.144	0.795	0.8025
b	-0.88	-42.49	0.125	0.124	0.695	0.762
c	86.00	94.00	0.247	0.588	0.795	0.557

Here one sees that, a-curve is in good agreement with experimental results in the high energy region but poor agreement in the low-energy region, c-curve is in best agreement in the low-energy region. Among these curves b-curves is the best fit with experimental results over the entire energy ranges considered. The calculated phase shifts for l=2 is shown by the solid curve in fig. 2, where the set of parameters of kernel are given below:

Curve	A	B	γ_1	γ_2	β_1	β_2
a	-0.948	0.145	0.255	0.343	0.256	0.249
b	-0.948	0.145	0.265	0.333	0.256	0.269
c	-0.948	0.075	0.165	0.183	0.201	0.239

Here it is seen that, the phase shifts for l=2 agree quite well with the experimental results in the low-energy region but poor agreement in the high energy region which goes down ward sharply. In the l=4 case, the results is shown by the solid curve in fig-3, where the value of kernel parameters

$A, B, \gamma_1, \gamma_2, \beta_1$ *and* β_2 are given below:

Curve	A	B	γ_1	γ_2	β_1	β_2
a	98.0	88.0	0.228	0.883	0.985	0.597
b	98.0	88.0	0.228	0.879	0.976	0.593
c	98.0	88.0	0.204	0.784	0.895	0.537

It is seen that, the agreement between the calculated and experimental values is satisfactory.In the figures (1),(2), and(3) we have also given the comparative results of ours with those of Tang and Thompson by dashed curve for l = o, 2, 4.

So we see that in our present study we have fully taken into account of the direct part of the interaction (both nuclear and coulomb calculated from the exchange of particles and replaced the non-local non-separable kernel by a simple non-local separable interaction. This simple separable interaction gives us good fit to the experimental results. So it is an encouraging sign that such a simple separable non-local, interaction can do the needful for the non-local,non-separable kernel. In that case this sort of calculation following resonating group method can be extended to any nucleons of higher mass number.

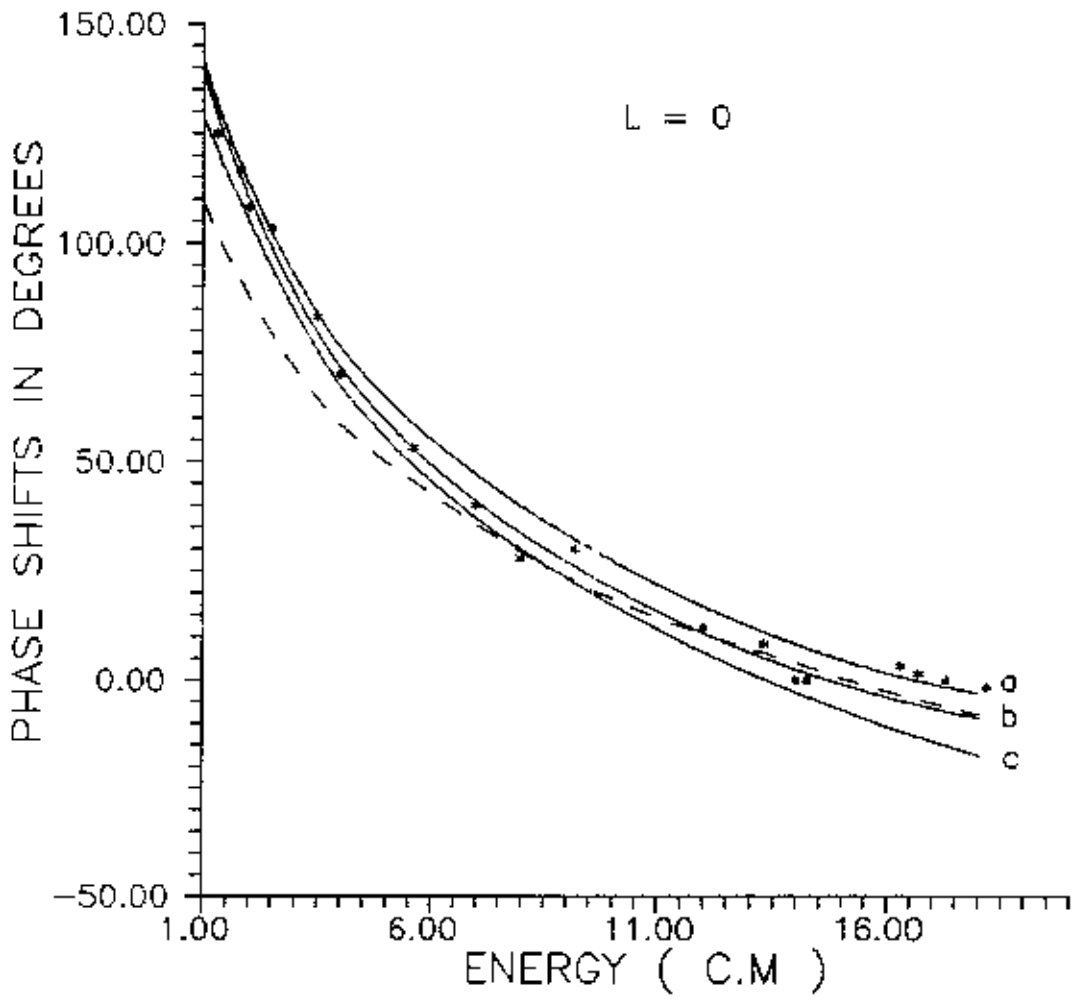

Fig.1 Calculated phase shifts of alpha–deuteron scatting are shown by solid curves , the phase shift calculated by Thompson and Tang is shown by dashed curve.The experimental phase shifts are shown by asterisks(*).

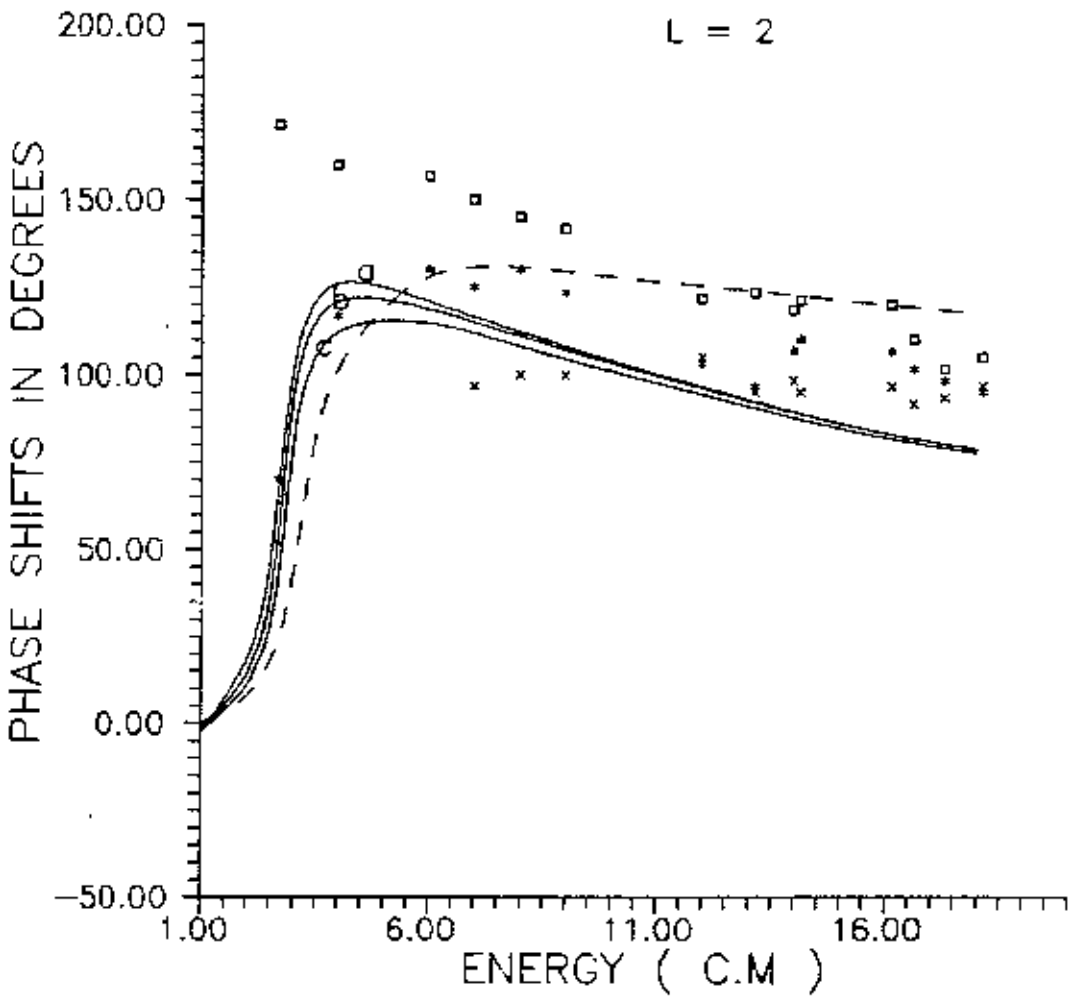

Fig.2 Calculated phase shifts of alpha–deuteron scatting are shown by solid curves ,the phase shift calculated by Thompson and Tang is shown by dashed curve.The experimental phase shifts are shown by asterisks,crosses and squares.

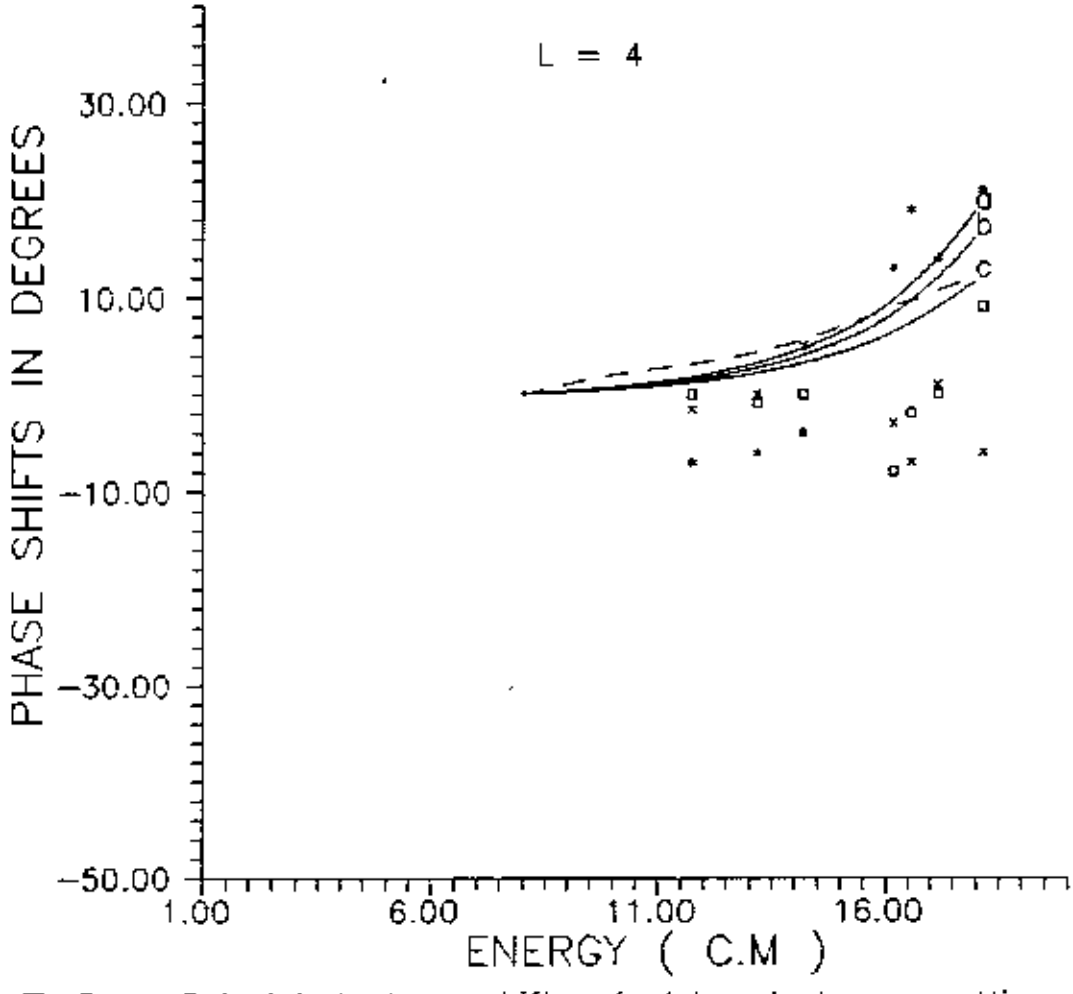

Fig.3 Calculated phase shifts of alpha−deuteron scatting are shown by solid curves ,the phase shift calculated by Thompson ond Tong is shown by dashed curve.The experimental phase shifts are shown by asterisks,crosses and squares.

REFERENCES

1. J. A. Wheeler, Phys. Rev. 52, (1937), 1083, 1107;
2. k.Wildermuth and T.Kamellopoulos, Nucl. Phys 7,(1958) 150;Nucl. Phys 9(1958)449;
3. E. Van der spuy, Nucl. Phys 11(1959) 615.
4. S. Okai and S.C. Park, Phys. Rev. 145 (1966) 787.
5. D.R. Thompson and Y.C. Tang, Phys. Letters 26B, (1968), 194;
6. D.R. Thompson and Y.C. Tang, Phys. Rev. 179, (1969), 971;
7. H. Margenau, Phys. Rev. 59(1941), 37;
8. D.M. Brink, Proc. of the Intern. School of Physics, edited by C. Bloch (Academic Press, New York, (1966) p247.
9. C. Abulaffio and J.M. Irvine, Phys. Lett. 38B, (1972) 492.
10. D.M. Brink, H. Friedrich, A. Weiguny and C.W. Wong Phys. Lett. 33B, (1970), 143.
11. H. Friedrich and A. Weiguny, Phys. LeH. 35B, (1971), 105;
12. H. Friedrich, H. Husken and A.Weiguny, Phys. Lett. 38B, (1972)199;
13. D.L. Hill and J.A. Wheeler, Phys. Rev. 89(1953)1102.

14. J. Griffin and J.A. Wheeler, Phys. Rev. 108, (1957), 311.

15. K. Wildermuth and Y.C. Tang. A unified Theory of the nucleus (Vieweg, Braunschweig, Germany 1977).

16. M. Kamimura, Proc. Intern. Symposium on nuclear collisions and their Microscopic description, edited by M. Poljsak, 1977 p-159.

17. D.R. Thompson, M. LeMere and Y.C. Tang, Nucl. phys A286(1977),53;

18. Y. Fujiwara, Prog. Theor. Phys 62(1979)122, 138;

19. S. Saito, Prog. Theor. Phys. 41 (1969) 705;

20. L.S. Senhouse, Jr. and T.A.Tombrello, Nucl. Phys, 57, (1964), 624.

21. L.C. McIntyre and W. Haeberli, Nucl. Phys. A91(1967), 382.

22. P. Darriulat, D. Garreta, A. Tarrats, and J. Arvieux, Nucl. Phys A94, (1967) 653;

23. H. Jacobs, K. Wildermuth and E.J. Wurster, Phys. letters 29B, (1969), 194;

24. D.R. Thompson and Y.C. Tang, Phys. Rev. C8, (1973), 1649;

25. H. Kanada, T. Kaneko and Y.C. Tang, Nucl. Phys. A389, (1982) 285;

26. H. Kanada, T. Kaneko, and Y.C. Tang, Nucl. Phys A380(1982), 87;

27. H. Kanada, T. Kaneko, S.Saito, and Y.C. Tang Nucl. Phys. A444, (1985), 209.

28. H. Robertson, Proc. Cambridge Phil. Soc. 52, (1956). 538;

29. L.Fox and E.T. Goodwin. Proc. Cambridge Phil. Soc. 45, (1949), 373.

30. C.E. Frogberg, Rev. Mod. Phys 27, (1955), 399;

31. Y.C. Tang, E. Schmid and K. Wildermuth, Phys. Rev. 131, (1963), 2631.

32. S.A. Afzal, A.A. Z. Ahmad and S. Ali Rev. mod. Phys. Vol. 41, (1969), 247.

33. S.Ali and S.A.Afzal Nuovo cimento 49A (1967) 103

34. Y.C.Tang and K.Wildermuth Phys. Rev. 123 (1961) 548