EFFECTS OF CONDUCTION AND CONVECTION ON MAGNETOHYDRODYNAMIC FLOW WITH VISCOUS DISSIPATION AND JOULE HEATING FROM A VERTICAL FLAT PLATE

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I here by declare that this work which is being presented in this thesis entitled "EFFECTS OF CONDUCTION AND CONVECTION ON MAGNETOHYDRODYNAMIC FLOW WITH VISCOUS DISSIPATION AND JOULE HEATING FROM A VERTICAL FLAT PLATE" submitted in partial fulfillment of the requirements for the award of the degree of Master of Philosophy in Mathematics in the department of Mathematics, Bangladesh University of Engineering and Technology (BUET), Dhaka – 1000 is an authentic record of my own work.

It has not been submitted elsewhere (Universities or Institutions) for the award of any other degree.

(Md. Bellal Hossain)

Date: 21-03-07

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Abstract

In this thesis, the combined effect of conduction and convection on magneto hydrodynamic (MHD) boundary layer flow with viscous dissipation and joule heating from a vertical flat plate has been investigated. By using the appropriate transformations, the basic equations are transformed to dimensionless boundary layer equations which are solved numerically by using the finite difference method known as Keller box method. The effects of the magnetic parameter M, the dissipation parameter N, the Prandtl number Pr and the joule heating parameter J have been examined on the flow field. The analysis has shown that the flow field is appreciably influenced by the effects of conduction and convection. Similarity solutions of the momentum and energy equations are derived by introducing the same transformations. Here we have focused our attention on the evolution of the skin friction coefficient, the surface temperature distribution, velocity as well as temperature distribution for a selection of parameters set consisting of magnetic parameter M, the dissipation parameter N , the Prandtl number Pr and the joule heating parameter J. The dimensionless skin friction coefficient, the surface temperature distribution, velocity as well as temperature distribution over the whole boundary layer are shown graphically for different values of the magnetic parameter M, the dissipation parameter N, the Prandtl number Pr and the joule heating parameter J.

Nomenclature

b	:Plate thickness
C_p	:Specific heat
d	: $(T_b - T_\infty)/T_\infty$
f	:Dimensionless stream function
g	:Acceleration due to gravity
h	: Dimensionless temperature
H_0	:Applied magnetic field
L	:Reference length, $\nu^{2/3}/g^{1/3}$
l	:Length of the plate
M	:Magnetic parameter
N	:Viscous dissipation parameter
J	:Joule heating parameter
p	:Coupling parameter
Pr	:Prandtl number
T	:Temperature of the flow fluid
T_b	:Temperature at outside of the plate
T_{s0}	:Solid temperature
T_∞	:Temperature of the ambient fluid
u	:Velocity component in the x-direction
v	:Velocity component in the y-direction
x	: Stream wise co-ordinate
y	: Transverse co-ordinate

Greek Symbols

β	: Co-efficient of thermal expansion
ψ	: Stream function
τ	: Skin friction
η	: Dimensionless similarity variable
ρ	: Density of the fluid inside the boundary layer
ν	: Kinematic viscosity
μ	: Viscosity of the fluid
θ	: Dimensionless temperature
σ	: Electrical conductivity
κ	: Thermal conductivity
κ_f	: Thermal conductivity of the ambient fluid
κ_s	: Thermal conductivity of the ambient solid

Table of Contents

Page No.

Chapter - 1

Introduction

Fluid dynamics or hydrodynamics is that branch of science which is concerned with the study of the motion of fluids or that of bodies in contact with fluids .The state of matter which is capable of flowing or of changing shape is called fluid. Fluids are classified as liquids and gasses. Gasses have no definite volume and shape but liquids have a definite volume but no shape. Density, volume, temperature, viscosity and pressure are five characteristics of actual fluids. Viscosity is the property of a fluid which gives rise to shearing stress. A perfect fluid is frictionless and incompressible. In the motion of such a perfect fluid, two contacting layers experience no tangential forces (shearing stress) but act on each other with normal forces (pressures) only. A perfect fluid offers no internal resistance to a change in shape. The motion of perfect fluid is very far developed and supplies in many cases a satisfactory description of real motions. On the other hand, in real fluids, the existence of inter molecular attractions causes the fluid to adhere to a solid wall and thia gives rise to shearing stress. The inner layers of a real fluid transmit tangential and normal stresses. On the boundary between a perfect fluid and a solid wall there exists a difference in relative tangential velocities i,e there is slip. The existence of tangential (shearing) stresses and the condition of no slip near solid walls constitute the essential differences between a perfect and a real fluid. In fluid dynamics, the study of individual molecule is neither necessary nor appropriate from the point of view of use of mathematical methods. Hence we consider the macroscopic behavior of fluid by supposing the fluid to be continuously distributed in a given space. This assumption is known as the continuum hypothesis.

The branch of science which incorporates with the motion of a highly conducting fluid in presence of a magnetic field is called magneto hydrodynamics. The motion of the conducting fluid across the magnetic field produces electric currents which change the magnetic field. The effect of the magnetic field on these currents give rise to mechanical forces which modify the fluid. If the current is parallel to the magnetic field, it is possible to attain equilibrium in a conducting fluid .For then the magnetic forces vanish and the equilibrium of the fluid is the same as in the absence of magnetic fields. These types of magnetic fields are called force free. For poor conductive fluid, electromagnetic forces which will be produced may be of the same order of magnitude as the hydrodynamical and inertial forces. However by ionizing some gases, it is possible to make them very highly conducting, the gas must be very hot at temperature upwards of 5000^0 K or so for ionization. Such ionized gases are called plasmas. The material within a star is plasma of very high conductivity and it exists within a strong magnitude. MHD incorporated mainly on the study of severely incompressible fluid but at present terminology is applied also to the studies of ionized gases. Significant applications have been reported such as the MHD generator, MHD flow meter, MHD pump and MHD marine propulsion. Some other quite promising applications are in the field of metallurgy such as MHD stirring of molten metal and magnetic-levitation casting. A very useful proposed application which involves MHD is the lithium cooling blanket in a nuclear fusion reactor. With the high temperature plasma contained in the reactor by means of a toroidal magnetic field, liquid lithium flows in channels (blankets) between the plasma and magnetic windings to absorb the thermal energy released by the fusion reaction. The proximity of the lithium channels (blankets) to the field coils means that the flow will be acted upon by extremely strong magnetic fields. Consequently, knowledge of fundamental MHD interactions is necessary in order to determine pressure drops, heat transfer etc in channels or pipes situated at different angles to a magnetic field. The laminar flow considered here is also the limiting case for turbulent flow at high Reynolds number (even 10^6 or higher) where high magnetic fields can dump out turbulent fluctuations and luminaries the flow. Magneto hydrodynamic was originally applied to astrophysical and geophysical problems where it is still important but more recently to the problem of fusion power where the application is the creation and containment of hot plasmas by electromagnetic forces since material walls would be destroyed. The primary geophysical problem is planetary magnetism, produced by currents deep in the planet, a problem that has not been solved to any degree of satisfaction.

On heat transfer studies, conduction, transfer of heat or electricity through a substance, results from a difference in temperature between different parts of the substance. Since heat is energy associated with the motions of the particles, it is transferred by such motions, shifting from regions of higher temperature to regions of lower temperature. The rate of heat flow between two regions is proportional to the temperature difference between them and the heat conductivity of the substance. In solids, the molecule themselves are bound and contribute to conduction of heat mainly by vibrating against neighboring molecules. Metals which have a high free electron density are good conductors of heat, while nonmetals, such as wood or glass, have few free electrons and don not conduct as well. Liquids and gasses have their molecules farther apart and are generally poor conductors of heat. In liquids and gasses, current consists not only in the flow of electrons but also in that of ions. A highly ionized liquid solution, e.g., salt water, is a good conductor. Gasses at high temperatures tend to become ionized and thus become good conductors.

Only in the presence of a fluid medium, the convection is possible. If a fluid flows in the interior of a channel or over a solid body while temperatures of the fluid and the solid surface are different, heat between the fluid and the solid surface takes place as an outcome of the motion of fluid comparative to the surface; this type of mechanism of heat transfer is termed as convection. There are two basic procedures of the convective sort of heat transfer. Free convection is convection in which motion of the fluid arises solely due to temperature differences existing within the fluid. The basic premise behind free convection is that heated fluid becomes more buoyant and rises, while cooler fluid sinks. Free convection occurs in any liquid or gas which expands or contracts in response to change in temperatures in an acceleration field such as gravity. In natural convection procedures are governed essentially by three features namely the body force, the temperature difference in the flow field and the fluid density discrepancy with temperature. In the way of life, free convection is the most important style of heat transfer from pipes, transmission lines, refrigerating coils, burning radiators and various realistic situations. When with the help of a pump or a fan the fluid motion is unnaturally stimulated that forces the fluid flow over the surface, the heat transfer is termed forced convection. When a person blows on their food to cool it, he/she is using forced convection. Though in a lot of cases of sensible attention, both procedures are significant and heat transfer is by mixed convection in which neither approach is really most important. The manipulation of natural convection of the heat transfer can be deserted in the case of Reynolds number and Grashof number. The natural convection should be the governing aspect for immense Grashof oumber and miniature Reynolds number. We observe in nature some situations where forced and free convection is of analogous order. The phenomena may be termed as the mixed or combined convection flows. The main difference between free and forced convection lies in the nature of the fluid generation. In forced convection the externally imposed flow is generally known, whereas in free convection it results from an interaction between the density difference and the gravitational field and is therefore invariably linked with and depend on the temperature field.

It is evident that the formation of boundary layer over a surface is due to the viscous nature of the fluid flowing over the surface. The energy equation for two dimensional boundary layer includes the terms $\rho\nu(\frac{\partial u}{\partial y})^2$, which accounts for the heat generation due to fluid friction. In order to explain the effect of viscous dissipation, we shall first take a grossly simplified view that the compressibility effect may be neglected and the properties such as viscosity, thermal conductivity and specific heat are constant and there consider the flow between parallel flat plates on of which is stationary and the other moving in its own plane with a constant velocity U_∞.

In electronics and in physics more broadly, joule heating or ohmic heating refers to the increase in temperature of a conductor as a result of resistance to an electrical current flowing though it. At an atomic level, joule heating is the result of moving electrons collitling with atoms in a conductor, whereupon momentum is transferred to the atom, increasing its kinetic energy. When similar collisions cause a permanent structural change, rather than an elastic response, the result is known as electro migration.

The problems of free convection boundary layer flow over or on bodies of various shapes were discussed by many mathematicians, versed engineers and researchers. Amongst them are Merkin and Mahmood [1], Hossain et al. [2], Yao [3], Nazar et al. [4,5], Huang and Chen [6]. The problem of free convection boundary layer on a vertical plate with prescribed surface heat flux investigated by Merkin and Mahmood [1]. Hossain et al. [2] also discussed the same problems but with the temperature dependent viscosity and thermal conductivity. Yao [3] have studied the problem of natural convection flow along a vertical wavy surface. Nazar et at. [4,5] consider the free convection boundary layer on an isothermal sphere and on an isothermal horizontal circular cylinder. The effect of laminar free convection from a sphere with blowing and suction studied by Huang and Chen [6]. The interaction of the magnetic field and the moving electric charge carried by the flowing fluid induces a force which tends to oppose the fluid motion. Near the leading edge, the velocity is very small so that the magnetic force which is proportional to the magnitude of the longitudinal velocity and acts in the opposite direction is also very small. Consequently the effect of the magnetic field on the boundary layer is exerted only through induced forces with in the boundary layer itself, with no additional influences arising from the free stream pressure gradient. The effect of magnetic field on free convection heat transfer has been studied by Sparrow and Cess [7]. Raptis and Kafousius [8] have investigated the problem of magneto hydrodynamic free convection flow and mass transfer through a porous medium bounded by an infinite vertical porous plate with constant heat flux. Kuiken [9] studied the problem of magneto hydrodynamic free convection in a strong cross field. Elbashbeshy [10] discussed the effect of free convection flow with variable viscosity and thermal diffusivity along a vertical plate in the presence of magnetic field. MHD free convection flow of visco-elastic fluid past an infinite porous plate was investigated by Chowdhury and Islam [11]. Hossain [12] introduce the viscous and joule heating effects on MHD free convection flow with variable plate temperature. Very recently Ahmad and Zaidi [13] investigated the magnetic effect on over back convection through a vertical stratum. Hossain et al. [14] also have investigated the heat transfer response of MHD free convection flow along a vertical plate to surface temperature oscillation. Hossain et al. [15,16] discussed the both forced and free convection boundary layer flow of an electrically conducting fluid in presence of magnetic field. Moreover the effect of conjugate natural convection flow on or from various heated shapes studied by Bynnycky and Kimura [17], Yu and Lin [18], Merkin and Pop [19] and Hossain et al. [20]. Hossain and Pop [21] investigated the magneto hydrodynamic boundary layer flow and heat transfer from a continuous moving wavy surface while the problem of free convection flow from wavy vertical surface in presence of a transverse magnetic field was studied by Alam et al. [22]. On the other hand Rees and Pop [23-25] discussed the free convection boundary layer induced by vertical and horizontal surface exhibiting small amplitude waves embedded in porous medium. Natural convection heat and mass transfer near a vertical wavy surface with constant wall temperature and concentration in a porous medium was studied by Cheng [26]. Kafousius and Williams [27] and Kafousius et al. [28] have studied the effects of variable viscosity on the free and mixed convection flow from a vertical flat plate in the region near the leading edge. The combined forced and free convection in boundary layer flow of a micropolar fluid above a horizontal plate was investigated by

Hassanien [29]. Similarity solutions are acquired in this work for the case of wall temperature which is inversely proportional to the square root of the distance from the leading edge. The influence of the magnetic field on the laminar free convection of liquid metals was investigated by Cramer [30] above a vertical flat plate and between two parallel plates. For liquid metals he got an analytical solution. Shiralkar and Tein [31] studied numerically natural convection in an enclosure with temperature gradients imposed in both the horizontal and vertical direction simultaneously. The coupling of conduction with laminar natural convection along a flat plate was investigated by Pozzi and Lupo [32]. Miyamoto et al. [33] discussed the effect of axial heat conduction in a vertical flat plate on free convection heat transfer. Effect of conduction and coovection on magneto hydrodynamic flow from a vertical flat plate was studied by Mamun [34]. Taher [35] introduced the magneto hydrodynamic natural convection flow on a sphere. The governing partial differential equations are reduced to non-similar partial differential forms by adopting appropriate transformations. The transformed boundary layer equations are solved numerically by implicit finite difference method together with the Keller box method by Keller [36] and later by Cebeci and Bradshaw [37].

In chapter – 1available information regarding MHD heat and mass transfer flows are studied from both analytical and numerical point of view. In chapter-2 we have considered a steady laminar MHD free convective boundary layer flow with viscous dissipation and joule heating from a vertical flat plate. In chapter-3 a two dimensional laminar free convection flow with viscous dissipation from a vertical flat plate is considered. The above two problems have been solved numerically by using a most practical, an efficient and accurate solution method, known as implicit finite difference method together with Keller box scheme.

Chapter – 2

Combined Effect of Conduction and Convection on Magneto Hydrodynamic Flow with Viscous Dissipation and Joule Heating From a Vertical Flat Plate

2.1 Introduction:

In the present chapter we shall describe the effect of conduction and convection on magneto hydrodynamic flow with viscous dissipation and joule heating in the entire region from up stream to down stream of a viscous incompressible and electrically conducting fluid from a vertical flat plate. The governing boundary layer equations are transformed into a non dimensional form and the resulting non linear system of partial differential equations are reduced to local non similarity equations which are solved numerically by very efficient implicit finite difference method together with Keller box method. Numerical results are presented by skin friction coefficient, surface temperature distribution, velocity and temperature distribution for a wide range of magnetic parameter M = (0.2, 0.5, 0.8, 1.0). The Prandtl number Pr is to be taken 0.05, 0.73, 1.0 which correspond to sodium, air, electrolyte solutions such as salt water. In the following section detailed derivations of the governing equations for the flow and heat transfer and the method of solutions along with the results and discussions are presented. All the investigations for the fluid with low Prandtl number appropriates for the liquid metals are carried out.

2.2 Governing equations of the flow:

The mathematical statement of the basic conservation laws of mass, momentum and energy for the steady viscous incompressible and electrically conducting fluid with joule heating effect are given by

$$\nabla.\underline{q} = 0 \tag{2.1}$$

$$\rho\left(\underline{q}.\nabla\right)\underline{q} = -\nabla p + \mu\nabla^2\underline{q} + \underline{F} + \underline{J}\times\underline{B} \tag{2.2}$$

$$\rho c_p^2\left(\underline{q}.\nabla\right)T = kc_p\nabla^2 T + u(\underline{J}\times\underline{B}) \tag{2.3}$$

Where $\underline{q} = (u, v)$, u and v are the velocity components along the x and y axes respectively, F is the body force per unit volume which is defined as $-\rho g$, the terms J and B are respectively the current density and magnetic induction vector and the term $\underline{J} \times \underline{B}$ is the force on the fluid per unit volume produced by the interaction of current and magnetic field in the absence of excess charges. T is the temperature of the fluid in the boundary layer, g is the acceleration due to gravity, k is the thermal conductivity and C_p is the specific heat at constant pressure and μ is the viscosity of the fluid. Here $B = \mu_e H_0$, μ_e being the magnetic permeability of the fluid, H_0 is the applied magnetic field and ∇ is the vector differential operator and is defined by

$$\nabla = \hat{i}_x \frac{\partial}{\partial x} + \hat{j}_y \frac{\partial}{\partial y}$$

Where $\hat{i}_x$ and the $\hat{j}_y$ are the unit vector along x and y axes respectively.

When the external electric field is zero and the induced electric field is negligible, the current density is related to the velocity by Ohm's law as follows

$$\underline{J} = \sigma(\underline{q} \times \underline{B}) \qquad (2.4)$$

Where σ denotes the electric conductivity of the fluid. Next under the conduction that the magnetic Reynold's number is small, induced magnetic field is negligible compared with applied field. This condition is usually well satisfied in terrestrial applications, especially so in (low velocity) free convection flows. So we can write

$$\underline{B} = \hat{j}_y H_0 \qquad (2.5)$$

Bringing together equations (2.4) and (2.5) the force per unit volume $\underline{J} \times \underline{B}$ acting along the x axis takes the form:

$$\underline{J} \times \underline{B} = \sigma H_0^2 u \qquad (2.6)$$

Under the Boussinesq approximation, the variation of ρ is taken into only in the term $\underline{F}$ in equation (2.2) and the variation of ρ in the inertia term is neglected. We then can write:

$$\rho = \rho_\infty [1 - \beta(T - T_\infty)] \qquad (2.7)$$

Where ρ_∞ and T_∞ are the density and temperature respectively outside the boundary layer, β is the co-efficient of thermal expansion.

We consider the steady two dimensional laminar free convection boundary layer flow of a viscous incompressible and electrically conducting fluid along a side of a vertical flat plate of thickness 'b' insulated on the edges with temperature T_b maintained on the other side. The flow configuration and the coordinates system are shown in figure 2.

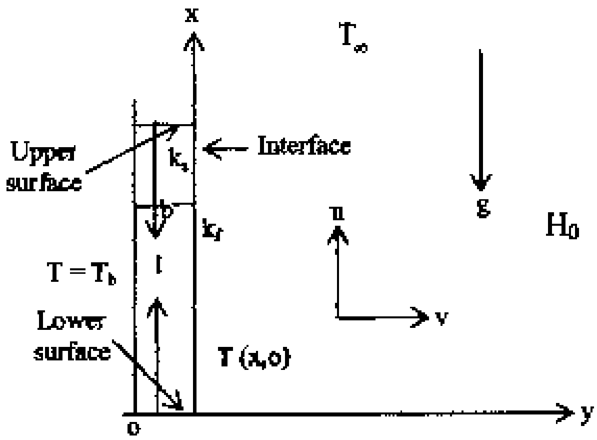

Fig. 2: Physical model and coordinate systems

Using the equations (2.4) to (2.6) with respect to our above considerations into the basic equations (2.1) – (2.3), the steady two dimensional laminar free convection boundary layer flow of a viscous incompressible and electrically conducting fluid with viscosity and also constant thermal conductivity past a vertical flat plate take the following form:

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \tag{2.8}$$

$$u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = \nu\frac{\partial^2 u}{\partial y^2} + g\beta(T - T_\infty) - \frac{\sigma H_0^2 u}{\rho} \tag{2.9}$$

$$u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y} = \frac{k_f}{\rho c_p}\frac{\partial^2 T}{\partial y^2} + \frac{\nu}{c_p}\left(\frac{\partial u}{\partial y}\right)^2 + \frac{\sigma H_0^2 u^2}{\rho c_p} \tag{2.10}$$

The appropriate boundary conditions to be satisfied by the above equations are

$$u = 0,\ v = 0 \ at\ y = 0 \tag{2.11a}$$
$$u \longrightarrow 0,\ T \longrightarrow T_\infty \ as\ y \longrightarrow \infty \tag{2.11b}$$

The temperature and the heat flux are required continuous at the interface for the coupled conditions and at the interface we must have

$$\frac{k_s}{k_f}\frac{\partial T_{so}}{\partial y} = k_f(\frac{\partial T}{\partial y})_{y=0} \tag{2.12}$$

Where k_s and k_f are the thermal conductivity of the solid and the fluid respectively. The temperature T_{so} in the solid is given by

$$T_{so} = T(x,o) - \{T_b - T(x,o)\}\frac{y}{b} \tag{2.13}$$

Where T(x,o) is the unknown temperature at the interface to be determined from the solutions of the equations. We observed that the equations (2.8) – (2.10) together with the boundary conditions (2.11) – (2.13) are non-linear partial differential equations. In the following sections the solutions methods of these equations are discussed in details.

2.3 Transformation of the governing equations

Equations (2.8) – (2.13) may now be nondimensionalized by using the following dimensionless dependent and independent variables:

$$\bar{x} = \frac{x}{L}, \bar{y} = \frac{y}{L}d^{\frac{1}{4}}, u = \frac{\upsilon}{L}d^{\frac{1}{2}}\bar{u}, v = \frac{\upsilon}{L}d^{\frac{1}{4}}\bar{v}, \frac{T - T_\infty}{T_b - T_\infty} = \theta, L = \frac{\upsilon^{\frac{2}{3}}}{g^{\frac{1}{3}}}, d = \beta(T_b - T_\infty) \tag{2.14}$$

As the problem of natural convection, its parabolic character has no characteristic length; L has been defined in terms of v and g which is the intrinsic properties of the system. The reference length along the 'y' direction has been modified by a factor $d^{-1/4}$ in order to eliminate this quantity from the dimensionless equations and the boundary conditions.

The magneto hydrodynamic field in the fluid is governed by the boundary layer equations, which is the nondimensional form obtained by introducing the dimensionless variables described in (2.14) may be written as

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \tag{2.15}$$

$$u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} + Mu = \frac{\partial^2 u}{\partial y^2} + \theta \tag{2.16}$$

$$u\frac{\partial \theta}{\partial x} + v\frac{\partial \theta}{\partial y} = \frac{1}{P_r}\frac{\partial^2 \theta}{\partial y^2} + N(\frac{\partial u}{\partial y})^2 + Ju^2 \tag{2.17}$$

Where $N = \dfrac{\upsilon^2 d}{L^2 c_p (T_b - T_\infty)}$, the dimensionless viscous dissipation parameter

$J = \dfrac{\sigma H_o^2 \upsilon d^{\frac{1}{2}}}{\rho c_p (T_b - T_\infty)}$, the dimensionless joule heating parameter

$M = \dfrac{\sigma H_o^2 L^2}{\mu d^{1/2}}$, the dimensionless magnetic parameter

and $Pr = \dfrac{\mu c_p}{k_f}$, the Prandtl number

The corresponding boundary conditions (2.11) – (2.13) take the following form:

$$u = v = o, \theta - 1 = p\frac{\partial \theta}{\partial y} \quad at \quad y = o \tag{2.18a}$$

$$u \to o, v \to o \quad as \quad y \to \infty \tag{2.18b}$$

Where p is the conjugate conduction parameter given by $p = (k_f / k_s)(b/L)d^{1/4}$

Here the coupling parameter *'p'* governs the described problem. The order of magnitude of 'p' depends actually on *(b/L)*, (k_f/k_s) and $d^{1/4}$ being the order of unity. The term *(b/L)* attains values much greater than one because of *L* being small. In case of air, (k_f/k_s) becomes very small when the vertical plate is highly conductive i.e. $k_s >> 1$ different but not always a smal*l* such as glass. Therefore in different cases *'p'* is different but not always a small number. In the present investigation we have considered $p = 1$ which is accepted for *(b/L)* of $O(k_f/k_s)$.

To solve the equations (2.15) – (2.17) subject to the boundary conditions (2.18), the following transformations are introduced for the flow region starting from up stream to down stream;

$$\psi = x^{4/5}(1+x)^{-1/20} f(\eta, x), \eta = yx^{-1/5}(1+x^{-1/20}), \theta = x^{1/5}(1+x)^{-1/5} h(\eta, x) \tag{2.19}$$

Here η is the dimensionless similarity variable and Ψ is the stream function which satisfies the equation of continuity and $u = \dfrac{\partial \psi}{\partial y}, v = -\dfrac{\partial \psi}{\partial x}$ and h(η,x) is the dimensionless temperature. Substituting equation (2.19) into equations (2.16) and (2.17) we get the following transformed nondimensional equations

$$f''' + \frac{16+15x}{20(1+x)} ff'' - \frac{6+5x}{10(1+x)} f'^2 - Mx^{2/5}(1+x)^{1/10} f' + h = x\left(f'\frac{\partial f'}{\partial x} - f''\frac{\partial f}{\partial x}\right) \tag{2.20}$$

$$\frac{1}{P_r}h'' + \frac{16+15x}{20(1+x)}fh' - \frac{1}{5(1+x)}f'h + Nxf''^2 + Jx^{7/5}(1+x)^{1/10}f'^2$$
$$= x\ (f'\frac{\partial h}{\partial x} - h'\frac{\partial f}{\partial x}) \qquad (2.21)$$

In the above equations the primes denote differentiation with respect to η.

The boundary conditions (2.18) then take the following form:

$$\left.\begin{array}{l} f(x,o) = f'(x,o) = o, h'(x,o) = -(1+x)^{1/4} + x^{1/5}(1+x)^{1/20}h(x,o) \\ f'(x,\infty) = o, h'(x,\infty) = o \end{array}\right\} \qquad (2.22)$$

2.4 Method of Solution

To get the solutions of the parabolic differential equations (2.20) and (2.21) along with the boundary condition (2.22), we shall employ a most practical, an efficient and accurate solution technique, known as implicit finite difference method together with Keller- box elimination technique.

To apply the aforementioned method, we first convert the equations (2.20) and (2.21) and their boundary conditions into the system of first order equations. For this reason we introduce new dependent variables $u(\xi,\eta)$, $v(\xi,\eta)$ and $p(\xi,\eta)$ so that the transformed momentum and energy equations can be written as

$$\left.\begin{array}{l} f' = u \\ u' = v \\ g' = p \end{array}\right\} \qquad (2.23)$$

$$v' + p_1 fv - p_2 u^2 - p_4 u + g = \xi(u\frac{\partial u}{\partial \xi} - v\frac{\partial f}{\partial \xi}) \qquad (2.24)$$

$$\frac{1}{P_r}p' + p_1 fp - p_3 ug + p_5 v^2 + p_6 u^2 = \xi(u\frac{\partial g}{\partial \xi} - p\frac{\partial f}{\partial \xi}) \qquad (2.25)$$

where $\xi = x$, $h = g$ and

$$\frac{16+5x}{20(1+x)} = p_1,\ \frac{6+5x}{10(1+x)} = p_2,\ \frac{1}{5(1+x)} = p_3, Nx = p_5, Jx^{7/5}(1+x)^{1/10} = p_6$$

$$p_4 = Mx^{2/5}(1 + x)^{1/10}$$

and the boundary cooditions (2.20) are

$$f(\xi,0)=0,\ u(\xi,0)=0,\ p(\xi,0)=-(1+\xi)^{1/4}+\xi^{1/5}(1+\xi)^{1/20}g(\xi,0)$$
$$u(\xi,\infty)=0,\ g(\xi,\infty)=0 \tag{2.26}$$

We now consider the net rectangle on the (ξ ,η) plane shown in the figure (2.1) and denote the net points by

$$\xi^0=0,\quad \xi^n=\xi^{n-1}+k_n,\ n=1,2,\ldots N$$
$$\eta_0=0,\ \eta_j=\eta_{j-1}+h_j,\quad j=1,2,\ldots J$$

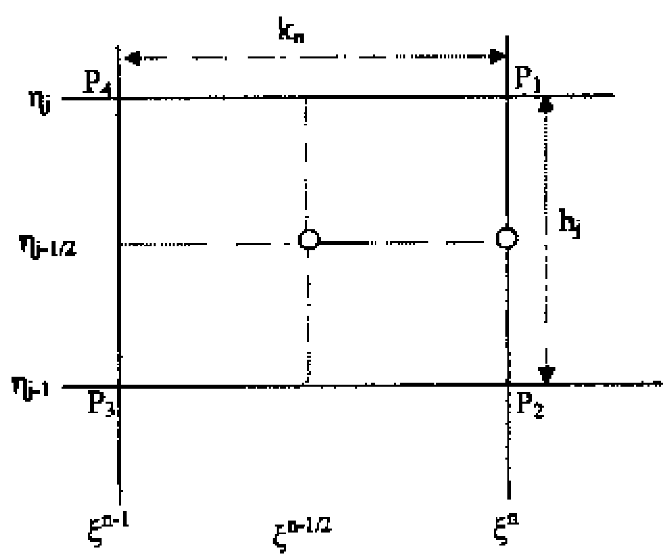

Figure 2.1 : Net rectangle for difference approximations for the Box scheme.

Here 'n' and 'j' are just sequence of numbers on the (ξ , η) plane , k_n and h_j are the variable mesh widths.

We approximate the quantities f, u, v, p at the points (ξ^n , η_j) of the net by f^n_j , u^n_j , v^n_j, p^n_j which we call net function . We also employ the notation g^n_j for the quantities midway between net points shown in figure (2.1) and for any net function as

$$\xi^{n-1/2}=\frac{1}{2}(\xi^n+\xi^{n-1}) \tag{2.27}$$

$$\eta_{j-1/2}=\frac{1}{2}(\eta_j-\eta_{j-1}) \tag{2.28}$$

$$g_j^{n-1/2}=\frac{1}{2}(g_j^{\,n}+g_j^{\,n-1}) \tag{2.29}$$

$$g_{j-1/2}^{n} = \frac{1}{2}(g_j^{\,n} + g_{j-1}^{n}) \tag{2.30}$$

The finite difference approximations according to Box method to the three first order ordinary differential equations (2.23) are written for the mid point (ξ^n, $\eta_{j-1/2}$) of the segment P_1P_2 shown in the figure (2.1) and the finite difference approximations to the two first order differential equations (2.24) and (2.25) are written for the mid point ($\xi^{n-1/2}$, $\eta_{j-1/2}$) of the rectangle $P_1P_2P_3P_4$. This procedure yields.

$$h_j^{-1}(f_j^n - f_{j-1}^n) = u_{j-1/2}^n = \frac{u_{j-1}^n + u_j^n}{2} \tag{2.31}$$

$$h_j^{-1}(u_j^n - u_{j-1}^n) = v_{j-1/2}^n = \frac{v_{j-1}^n + v_j^n}{2} \tag{2.32}$$

$$h_j^{-1}(g_j^n - g_{j-1}^n) = p_{j-1/2}^n = \frac{p_{j-1}^n - p_j^n}{2} \tag{2.33}$$

$$\begin{aligned} &\frac{1}{2}\left(\frac{v_j^n - v_{j-1}^n}{h_j} + \frac{v_j^{n-1} - v_{j-1}^{n-1}}{h_j}\right) + (p_1 f v)_{j-1/2}^{n-1/2} - (p_2 u^2)_{j-1/2}^{n-1/2} - (p_4 u)_{j-1/2}^{n-1/2} \\ &+ g_{j-1/2}^{n-1/2} = \xi_{j-1/2}^{n-1/2}\left(u_{j-1/2}^{n-1/2}\frac{u_{j-1/2}^n - u_{j-1/2}^{n-1}}{k_n} - v_{j-1/2}^{n-1/2}\,\frac{f_{j-1/2}^n - f_{j-1/2}^{n-1}}{k_n}\right) \end{aligned} \tag{2.34}$$

$$\begin{aligned} &+\frac{1}{2P_r}\left(\frac{p_j^n - p_{j-1}^n}{h_j} + \frac{p_j^{n-1} - p_{j-1}^{n-1}}{h_j}\right) + (p_1 f p)_{j-1/2}^{n-1/2} - (p_3 u g)_{j-1/2}^{n-1/2} + (p_5 v^2)_{j-1/2}^{n-1/2} + (p_6 u^2)_{j-1/2}^{n-1/2} \\ &= \xi_{j-1/2}^{n-1/2}\left(u_{j-1/2}^{n-1/2}\frac{g_{j-1/2}^n - g_{j-1/2}^{n-1}}{k_n} - p_{j-1/2}^{n-1/2}\,\frac{f_{j-1/2}^n - f_{j-1/2}^{n-1}}{k_n}\right) \end{aligned} \tag{2.35}$$

Now from the equation (2.34) we get

$$\begin{aligned} &\frac{1}{2}\left(\frac{v_j^n - v_{j-1}^n}{h_j}\right) + \frac{1}{2}\left(\frac{v_j^{n-1} - v_{j-1}^{n-1}}{h_j}\right) + \frac{1}{2}\left\{(p_1 f v)_{j-1/2}^n + (p_1 f v)_{j-1/2}^{n-1}\right\} \frac{1}{2}\left\{(p_4 u)_{j-1/2}^n + (p_4 u)_{j-1/2}^{n-1}\right\} \\ &- \frac{1}{2}\left\{(p_2 u^2)_{j-1/2}^n + (p_2 u^2)_{j-1/2}^{n-1}\right\} + \frac{1}{2}\left\{g_{j-1/2}^{n-1/2} + g_{j-1/2}^{n-1/2}\right\} \\ &= \frac{1}{2k_n}\xi_{j-1/2}^{n-1/2}(u_{j-1/2}^n + u_{j-1/2}^{n-1})(u_{j-1/2}^n - u_{j-1/2}^{n-1}) \\ &- \frac{1}{2k_n}\xi_{j-1/2}^{n-1/2}(v_{j-1/2}^{n-1/2} + v_{j-1/2}^{n-1/2})(f_{j-1/2}^n - f_{j-1/2}^{n-1}) \end{aligned}$$

$$\Rightarrow h_j^{-1}(v_j^n - v_{j-1}^n) + h_j^{-1}(v_j^{n-1} - v_{j-1}^{n-1}) + (p_1)_{j-1/2}^n (fv)_{j-1/2}^n + (p_1)_{j-1/2}^{n-1}(fv)_{j-1/2}^{n-1}$$
$$-(p_2)_{j-1/2}^n (u^2)_{j-1/2}^n - (p_2)_{j-1/2}^{n-1}(u^2)_{j-1/2}^{n-1} + g_{j-1/2}^n + g_{j-1/2}^{n-1/2} - (p_4)_{j-1/2}^n (u)_{j-1/2}^n - (p_4)_{j-1/2}^{n-1}(u)_{j-1/2}^{n-1}$$
$$= \alpha_n \{(u^2)_{j-1/2}^n - (u)_{j-1/2}^n (u)_{j-1/2}^{n-1}\}$$
$$+\alpha_n \left\{ \begin{array}{l} (u)_{j-1/2}^n (u)_{j-1/2}^{n-1} - (u^2)_{j-1/2}^{n-1} - (fv)_{j-1/2}^n + v_{j-1/2}^n f_{j-1/2}^{n-1} \\ - v_{j-1/2}^{n-1} f_{j-1/2}^n + (fv)_{j-1/2}^{n-1} \\ \end{array} \right\}$$

$$where\, \alpha_n = \frac{1}{k_n}\xi_{j-1/2}^{n-1/2}$$

$$\Rightarrow h_j^{-1}(v_j^n - v_{j-1}^n) + \{(p_1)_{j-1/2}^n + \alpha_n\}(fv)_{j-1/2}^n - \{(p_2)_{j-1/2}^n + \alpha_n\}(u^2)_{j-1/2}^n + g_{j-1/2}^n - (p_4)_{j-1/2}^n + \alpha_n (u)_{j-1/2}^n$$
$$= \alpha_n \left\{ -(u^2)_{j-1/2}^{n-1} + v_{j-1/2}^n f_{j-1/2}^{n-1} - v_{j-1/2}^{n-1} f_{j-1/2}^n + (fv)_{j-1/2}^{n-1} \right\}$$
$$-(p_1)_{j-1/2}^{n-1}(fv)_{j-1/2}^{n-1} + (p_2)_{j-1/2}^{n-1}(u^2)_{j-1/2}^{n-1} - g_{j-1/2}^{n-1} - h_j^{-1}(v_j^{n-1} - v_{j-1}^{n-1})$$

$$\Rightarrow h_j^{-1}(v_j^n - v_{j-1}^n) + \{(p_1)_{j-1/2}^n + \alpha_n\}(fv)_{j-1/2}^n - \{(p_2)_{j-1/2}^n + \alpha_n\}(u^2)_{j-1/2}^n$$
$$+ g_{j-1/2}^n + \alpha_n (v_{j-1/2}^{n-1} f_{j-1/2}^n - v_{j-1/2}^n f_{j-1/2}^{n-1}) - (p_4)_{j-1/2}^n + \alpha_n (u)_{j-1/2}^n$$
$$= \alpha_n \{(fv)_{j-1/2}^{n-1} - (u^2)_{j-1/2}^{n-1}\} - (p_1)_{j-1/2}^{n-1}(fv)_{j-1/2}^{n-1}$$
$$+ (p_2)_{j-1/2}^{n-1}(u^2)_{j-1/2}^{n-1} - h_j^{-1}(v_j^{n-1} - v_{j-1}^{n-1}) - (p_3)_{j-1/2}^{n-1}(u)_{j-1/2}^{n-1}$$
$$- g_{j-1/2}^{n-1}$$

$$\Rightarrow h_j^{-1}(v_j^n - v_{j-1}^n) + \{(p_1)_{j-1/2}^n + \alpha_n\}(fv)_{j-1/2}^n - \{(p_2)_{j-1/2}^n + \alpha_n\}(u^2)_{j-1/2}^n$$
$$+ g_{j-1/2}^n + \alpha_n (v_{j-1/2}^{n-1} f_{j-1/2}^n - v_{j-1/2}^n f_{j-1/2}^{n-1}) - (p_4)_{j-1/2}^n + \alpha_n (u)_{j-1/2}^n$$
$$= -L_{j-1/2}^n + \alpha_n \{(fv)_{j-1/2}^{n-1} - (u^2)_{j-1/2}^{n-1}\}$$

$$L_{j-1/2}^{n-1} = (p_1)_{j-1/2}^{n-1}(fv)_{j-1/2}^{n-1} - (p_2)_{j-1/2}^{n-1}(u^2)_{j-1/2}^{n-1} +$$
$$h_j^{-1}(v_j^{n-1} - v_{j-1}^{n-1}) + (p_3)_{j-1/2}^{n-1}(u)_{j-1/2}^{n-1} + g_{j-1/2}^{n-1}$$

$$\Rightarrow h_j^{-1}(v_j^n - v_{j-1}^n) + \{(p_1)_{j-1/2}^n + \alpha_n\}(fv)_{j-1/2}^n - \{(p_2)_{j-1/2}^n + \alpha_n\}(u^2)_{j-1/2}^n - (p_4)_{j-1/2}^n + \alpha_n (u)_{j-1/2}^n$$
$$+ g_{j-1/2}^n + \alpha_n (v_{j-1/2}^{n-1} f_{j-1/2}^n - v_{j-1/2}^n f_{j-1/2}^{n-1}) = R_{j-1/2}^{n-1} \qquad (2.36)$$
$$where,\, R_{j-1/2}^{n-1} = -L_{j-1/2}^n + \alpha_n \{(fv)_{j-1/2}^{n-1} - (u^2)_{j-1/2}^{n-1}\}$$

Again from the equation (2.35) we get

$$\frac{1}{P_r}h_j^{-1}(p_j^n - p_{j-1}^n) + \{(p_1)_{j-1/2}^{n-1/2} + \alpha_n\}(fp)_{j-1/2}^n - \{(p_3)_{j-1/2}^{n-1/2} + \alpha_n\}(ug)_{j-1/2}^n$$
$$+ (p_5)_{j-1/2}^n (v^2)_{j-1/2}^n + (p_6)_{j-1/2}^n (u^2)_{j-1/2}^n$$
$$+ \alpha_n (u_{j-1/2}^n g_{j-1/2}^{n-1} - u_{j-1/2}^{n-1} g_{j-1/2}^n - p_{j-1/2}^n f_{j-1/2}^{n-1} + p_{j-1/2}^{n-1} f_{j-1/2}^n)$$
$$= - M_{j-1/2}^{n-1} + \alpha_n \{(fp)_{j-1/2}^{n-1} - (ug)_{j-1/2}^{n-1}\}$$
$$where, M_{j-1/2}^{n-1} = \frac{1}{P_r}h_j^{-1}(p_j^{n-1} - p_{j-1}^{n-1}) + (p_1)_{j-1/2}^{n-1/2}(fp)_{j-1/2}^{n-1} - (p_3)_{j-1/2}^{n-1/2}(ug)(ug.$$

$$\Rightarrow \frac{1}{P_r}h_j^{-1}(p_j^n - p_{j-1}^n) + \{(p_1)_{j-1/2}^{n-1/2} + \alpha_n\}(fp)_{j-1/2}^n - \{(p_3)_{j-1/2}^{n-1/2} + \alpha_n\}(ug)_{j-1/2}^n$$
$$+ (p_5)_{j-1/2}^n (v^2)_{j-1/2}^n + (p_6)_{j-1/2}^n (u^2)_{j-1/2}^n$$
$$+ \alpha_n (u_{j-1/2}^n g_{j-1/2}^{n-1} - u_{j-1/2}^{n-1} g_{j-1/2}^n - p_{j-1/2}^n f_{j-1/2}^{n-1} + p_{j-1/2}^{n-1} f_{j-1/2}^n) = T_{j-1/2}^{n-1} \tag{2.37}$$
$$where, T_{j-1/2}^{n-1} = - M_{j-1/2}^{n-1} + \alpha_n \{(fp)_{j-1/2}^{n-1} - (ug)_{j-1/2}^{n-1}\}$$

The corresponding boundary conditions (2.26) become

$$\left.\begin{aligned} &f_0^n = 0 \quad u_0^n = 0 \quad p_0^n = -(1+\xi)^{1/4} + \xi^{1/5}(1+\xi)^{1/20} g_0^n \\ &u_J^n = 0 \quad g_J^n = 0 \end{aligned}\right\} \tag{2.38}$$

If we assume $f_j^{n-1}, u_j^{n-1}, v_j^{n-1}, g_j^{n-1}, p_j^{n-1}$ to be known for $0 \le j \le J$, equations (2.27) to (2.35) and (2.36) – (2.37') form a system of 5J + 5 non linear equations for the solutions of the 5J + 5 unknowns $(f_j^n, u_j^n, v_j^n, g_j^n, p_j^n)$, j = 0,1,2 ...J. These non linear systems of algebric equations are to be linearized by Newton's Quassy linearization method. We define the iterates $[f_j^n, u_j^n, v_j^n, g_j^n, p_j^n]$, i = 0, 1, 2 ...N with initial values equal those at the previous x-station (which is usually the best initial available). For the higher iterates we set

$$f_j^{(i+1)} = f_j^{(i)} + \delta f_j^{(i)} \tag{2.39}$$

$$u_j^{(i+1)} = u_j^{(i)} + \delta u_j^{(i)} \tag{2.40}$$

$$v_j^{(i+1)} = v_j^{(i)} + \delta v_j^{(i)} \tag{2.41}$$

$$g_j^{(i+1)} = g_j^{(i)} + \delta g_j^{(i)} \tag{2.42}$$

$$p_j^{(i+1)} = p_j^{(i)} + \delta p_j^{(i)} \tag{2.43}$$

Now by substituting the right hand sides of the above equations in place of f_j^n, u_j^n, v_j^n and g_j^n in equations (2.31) – (2.33) and in equations (2.36) – (2.37) dropping the terms that are quadratic in $\delta f_j^i, \delta u_j^i, \delta v_j^i, \delta p_j^i$. This procedure yields the following linear system of algebraic equations:

$$f_j^{(i)} + \delta f_j^{(i)} - f_{j-1}^{(i)} - \delta f_{j-1}^{(i)} = \frac{h_j}{2}\left\{u_j^{(i)} + \delta u_j^{(i)} + u_{j-1}^{(i)} + \delta u_{j-1}^{(i)}\right\}$$

$$\delta f_j^{(i)} - \delta f_{j-1}^{(i)} - \frac{h_j}{2}(\delta u_j^{(i)} + \delta u_{j-1}^{(i)}) = (r_1)_j \qquad (2.44)$$

Where $(r_1)_j = f_{j-1}^{(i)} - f_j^{(i)} + h_j u_{j-1/2}^{(i)}$

$$\delta u_j^{(i)} - \delta u_{j-1}^{(i)} - \frac{h_j}{2}(\delta v_j^{(i)} + \delta v_{j-1}^{(i)}) = (r_4)_j \qquad (2.45)$$

$where, (r_4)_j = u_{j-1}^{(i)} - u_j^{(i)} + h_j v_{j-1/2}^{(i)}$

$$\delta g_j^{(i)} - \delta g_{j-1}^{(i)} - \frac{h_j}{2}(\delta p_j^{(i)} + \delta p_{j-1}^{(i)}) = (r_5)_j \qquad (2.46)$$

Where $(r_5)_j = g_{j-1}^{(i)} - g_j^{(i)} + h_j p_{j-1/2}^{(i)}$

$$h_j^{-1}(v_j^{(i)} + \delta v_j^{(i)} - v_{j-1}^{(i)} - \delta v_{j-1}^{(i)}) + \left\{(p_1)_{j-1/2}^n + \alpha_n\right\}\left\{(fv)_{j-1/2}^{(i)} + \delta(fv)_{j-1/2}^{(i)}\right\}$$
$$-\left\{(p_2)_{j-1/2}^n + \alpha_n\right\}\left\{(u^2)_{j-1/2}^{(i)} + \delta(u^2)_{j-1/2}^{(i)}\right\} + g_{j-1/2}^{(i)} + \delta g_{j-1/2}^{(i)}$$
$$+\alpha_n(f_{j-1/2}^{(i)} + \delta f_{j-1/2}^{(i)})v_{j-1/2}^{n-1} - \alpha_n(v_{j-1/2}^{(i)} + \delta v_{j-1/2}^{(i)})f_{j-1/2}^{n-1} = R_{j-1/2}^{n-1}$$

$$\Rightarrow h_j^{-1}(v_j^{(i)} + \delta v_j^{(i)} - v_{j-1}^{(i)} - \delta v_{j-1}^{(i)}) + \left\{(p_1)_{j-1/2}^n + \alpha_n\right\} - \frac{1}{2}\left\{(p_4)_{j-1/2}^n + \alpha_n\right\}(\delta u_j^i + \delta u_{j-1}^i)$$
$$\left\{(fv)_{j-1/2}^{(i)} + \frac{1}{2}(f_j^{(i)}\delta(v)_j^{(i)} + v_j^{(i)}\delta(f)_j^{(i)} + f_{j-1}^{(i)}\delta(v)_{j-1}^{(i)} + v_{j-1}^{(i)}\delta(f)_{j-1}^{(i)}\right\}$$
$$-\left\{(p_2)_{j-1/2}^n + \alpha_n\right\}\left\{(u^2)_{j-1/2}^{(i)} + u_j^{(i)}\delta(u)_j^{(i)} + u_{j-1}^{(i)}\delta(u)_{j-1}^{(i)}\right\} + g_{j-1/2}^{(i)} + \frac{1}{2}(\delta g_j^{(i)} + \delta g_{j-1}^{(i)})$$
$$+\alpha_n\left\{v_{j-1/2}^{n-1}(f_{j-1/2}^{(i)} + \frac{1}{2}(\delta f_j^{(i)} + \delta f_{j-1}^{(i)}) - (v_{j-1/2}^{(i)} + \delta v_{j-1/2}^{(i)})f_{j-1/2}^{n-1}\right\} = R_{j-1/2}^{n-1}$$

$$\Rightarrow (s_1)_j\,\delta v_j^{(i)} + (s_2)_j\,\delta v_{j-1}^{(i)} + (s_3)_j\,\delta f_j^{(i)} + (s_4)_j\,\delta f_{j-1}^{(i)} + (s_5)_j\,\delta u_j^{(i)}$$
$$+(s_6)_j\,\delta u_{j-1}^{(i)} + (s_7)_j\,\delta g_j^{(i)} + (s_8)_j\,\delta g_{j-1}^{(i)} + (s_9)_j.0 + (s_{10})_j.0 = (r_2)_j \qquad (2.47)$$

$$\text{where, } (r_2)_j = R_{j-1/2}^{n-1} - \left\{h_j^{-1}(v_j^{(i)} - v_{j-1}^{(i)}) + ((p_1)_{j-1/2}^n + \alpha_n)(fv)_{j-1/2}^{(i)}\right\}$$
$$+((p_2)_{j-1/2}^n + \alpha_n)(u^2)_{j-1/2}^{(i)} - \alpha_n(f_{j-1/2}^{(i)}v_{j-1/2}^{n-1} - f_{j-1/2}^{n-1}v_{j-1/2}^{(i)}) - g_{j-1/2}^{(i)}$$

Thus the coefficient of momentum equation is

$$(s_1)_j = h_j^{-1} + \frac{(p_1)_{j-1/2}^n + \alpha_n}{2} f_j^{(i)} - \frac{1}{2}\alpha_n f_{j-1/2}^{n-1} \tag{2.48}$$

$$(s_2)_j = -h_j^{-1} + \frac{(p_1)_{j-1/2}^n + \alpha_n}{2} f_{j-1}^{(i)} - \frac{1}{2}\alpha_n f_{j-1/2}^{n-1} \tag{2.49}$$

$$(s_3)_j = \frac{(p_1)_{j-1/2}^n + \alpha_n}{2} v_j^{(i)} + \frac{1}{2}\alpha_n v_{j-1/2}^{n-1} \tag{2.50}$$

$$(s_4)_j = \frac{(p_1)_{j-1/2}^n + \alpha_n}{2} v_{j-1}^{(i)} + \frac{1}{2}\alpha_n v_{j-1/2}^{n-1} \tag{2.51}$$

$$(s_5)_j = -\frac{(p_2)_{j-1/2}^n + (p_4)_{j-1/2}^n}{2} u_j^i - \alpha_n u_j^i \tag{2.52}$$

$$(s_6)_j = -\frac{(p_2)_{j-1/2}^n + (p_4)_{j-1/2}^n}{2} u_{j-1}^i - \alpha_n u_{j-1}^i \tag{2.53}$$

$$(s_7)_j = 0.5 \tag{2.54}$$

$$(s_8)_j = 0.5 \tag{2.55}$$

$$(s_9)_j = 0 \tag{2.56}$$

$$(s_{10})_j = 0 \tag{2.57}$$

Here the coefficients $(s_9)_j$ and $(s_{10})_j$, which is zero in this case, are included here for the generality.

Similarly by using the equations (2.39) - (2.43) we get the equation (2.38) in the following form:

$$\begin{aligned}&(t_1)_j \delta p_j^{(i)} + (t_2)_j \delta p_{j-1}^{(i)} + (t_3)_j \delta f_j^{(i)} + (t_4)_j \delta f_{j-1}^{(i)} + (t_5)_j \delta u_j^{(i)} \\ &+ (t_6)_j \delta u_{j-1}^{(i)} + (t_7)_j \delta v_j^{(i)} + (t_8)_j \delta v_{j-1}^{(i)} + (t_9)_j \delta g_j^{(i)} + (t_{10})_j \delta g_{j-1}^{(i)} = (r_3)_j\end{aligned} \tag{2.58}$$

where,

$$\begin{aligned}(r_3)_j &= T_{j-1/2}^{n-1} - \frac{1}{P_r} h_j^{-1}(p_j^{(i)} - p_{j-1}^{(i)}) - \frac{(p_1)_{j-1/2}^n + \alpha_n}{2}(fp)_{j-1/2}^{(i)} \\ &+ \frac{(p_3)_{j-1/2}^n + \alpha_n}{2}(ug)_{j-1/2}^{(i)} - (p_5)_{j-1/2}^n (v^2)_{j-1/2}^{(i)} - (p_6)_{j-1/2}^n (u^2)_{j-1/2}^{(i)} \\ &- \alpha_n (u_{j-1/2}^{(i)} g_{j-1/2}^{n-1} - u_{j-1/2}^{n-1} g_{j-1/2}^{(i)} - p_{j-1/2}^{(i)} f_{j-1/2}^{n-1} + p_{j-1/2}^{n-1} f_{j-1/2}^{(i)})\end{aligned}$$

Again the coefficient of energy equation is

$$(t_1)_j = \frac{1}{P_r} h_j^{-1} + \frac{(p_1)_{j-1/2}^n + \alpha_n}{2} f_j^{(i)} - \frac{1}{2}\alpha_n f_{j-1/2}^{n-1} \tag{2.59}$$

$$(t_2)_j = -\frac{1}{P_r} h_j^{-1} + \frac{(p_1)_{j-1/2}^n + \alpha_n}{2} f_j^{(i)} - \frac{1}{2}\alpha_n f_{j-1/2}^{n-1} \tag{2.60}$$

$$(t_3)_j = \frac{(p_1)_{j-1/2}^n + \alpha_n}{2} p_j^{(i)} + \frac{1}{2}\alpha_n p_{j-1/2}^{n-1} \tag{2.61}$$

$$(t_4)_j = \frac{(p_1)_{j-1/2}^n + \alpha_n}{2} p_{j-1}^{(i)} + \frac{1}{2}\alpha_n p_{j-1/2}^{n-1} \tag{2.62}$$

$$(t_5)_j = -\frac{(p_3)_{j-1/2}^n + \alpha_n}{2} g_j^{(i)} + (p_6)_{j-1/2}^n u_j^i + \frac{1}{2}\alpha_n g_{j-1/2}^{n-1} \tag{2.63}$$

$$(t_6)_j = -\frac{(p_3)_{j-1/2}^n + \alpha_n}{2} g_{j-1}^{(i)} + (p_6)_{j-1/2}^n u_{j-1}^i + \frac{1}{2}\alpha_n g_{j-1/2}^{n-1} \tag{2.64}$$

$$(t_7)_j = (p_5)_{j-1/2}^n v_j^i \tag{2.65}$$

$$(t_8)_j = (p_5)_{j-1/2}^n v_{j-1}^i \tag{2.66}$$

$$(t_9)_j = -\frac{(p_3)_{j-1/2}^n + \alpha_n}{2} u_j^{(i)} - \frac{1}{2}\alpha_n u_{j-1/2}^{n-1} \tag{2.67}$$

$$(t_{10})_j = -\frac{(p_3)_{j-1/2}^n + \alpha_n}{2} u_{j-1}^{(i)} - \frac{1}{2}\alpha_n u_{j-1/2}^{n-1} \tag{2.68}$$

The boundary conditions (2.38) become

$$\begin{aligned} &\delta f_0^n = 0,\ \delta u_0^n = 0,\ \delta p_0^n = \delta\left[-(1+\xi)^{1/4} + \xi^{1/5}(1+\xi)^{1/20} g_0^n\right] \\ &\delta u_J^n = 0,\ \delta g_J^n = 0 \end{aligned} \tag{2.69}$$

Which just express the requirement for the boundary conditions to remain during the iteration process.

Now the system of linear equations (2.44) - (2.47), (2.58) together with the boundary conditions (2.69) can written in a block matrix from a coefficient matrix, which are solved by modified 'Keller Box' methods especially introduced by Keller[36] . Later, this method has been used most efficiently by Cebeci and Bradshaw [37] and recently by Hossain[12].Hossain et al .[15],taking the initial iteration to be given by convergent solution at $\zeta = \zeta_{i-1}$. Results are shown in graphical form by using the numerical values obtained from the above technique. The solutions of the above equations (2.20) and (2.21) together with the boundary conditions (2.22) enable us to calculate the skin friction τ and the rate of heat transfer θ at the surface in the boundary layer from the following relations:

$$\tau = x^{2/5}(1+x)^{-3/20} f''(0,x) \tag{2.70}$$

$$\theta = x^{1/5}(1+x)^{-1/5} h(0,x) \tag{2.71}$$

2.5 Results and Discussion:

Equations (2.20) and (2.21) subject to the boundary conditions (2.22) are solved numerically by using a very efficient implicit finite difference method together with Keller-box method. Numerical solutions are obtained for the fluid having Prandtl number Pr = (0.05, 0.7, 1.0, 1.74, 5.4, 4.24, 2.55) and for wide range of values of the magnetic parameter M = (0.2,0.5,0.8,1.0), dissipation parameter N = (0.1, 0.3, 0.5, 0.9, 0.02, 0.04, 0.08, 0.2) and the joule heating parameter J = (0.2, 0.5, 0.8, 1.0, 0.02, 0.05, 0.08, 0.1). If we know the values of the functions $f(\eta,x)$, $h(\eta,x)$ and their derivatives for different values of the Prandtl number Pr , the magnetic parameter M, the viscous dissipation parameter N and the joule heating parameter J. We may calculate the numerical values of the surface temperature $\theta(0,x)$ and the velocity gradient $f''(0,x)$ at the surface that are important from the physical point of view. Numerical values of the surface temperuture $\theta(0,x)$ and the velocity gradient $f''(0,x)$ are illustrated in fig. 2.2 and fig. 2.3 respectively against the axial distance x in the interval [0,30] for different values of the viscous dissipation parameter N = (0.02, 0.04, 0.08, 0.2) for the fluid having Prandtl number Pr = 0.73 and the magnetic parameter M = 0.5 and the joule heating parameter J = 0.05. In fig. 2.4 and 2.5, the share stress coefficient $f''(0,x)$ and the surface temperature $\theta(0,x)$ are depicted graphically for different values of Prandtl number Pr = (0.7, 1.0, 1.74, 5.4, 4.24, 2.55) when the value of the magnetic parameter M = 0.5, dissipation parameter N = 0.05 and joule heating parameter J = 0.05. In fig. 2.6 and 2.7, the velocity gradient $f''(0,x)$ and the surface temperature $\theta(0,x)$ are shown graphically for different values of magnetic parameter M = (0.2, 0.5, 0.8, 1.0) when the value of Prandtl number Pr = 0.73, dissipation parameter N = 0.5 and the joule heating parameter J = 0.05. Fig. 3.8 and 3.9 shows the effect of joule heating parameter J = (0.02, 0.05, 0.08, 0.10) for Prandtl number Pr = 0.73, dissipation parameter N = 0.05 and for the magnetic parameter M = 0.05 on the skin friction and the surface temperature distribution. Fig. 2.2 shows that increase in the value of the dissipation parameter N leads to increase of the value of the skin friction coefficient $f''(0,x)$. Again from fig. 2.3 it can be observed that the increase of the viscous dissipation parameter N leads to increase of the surface temparture $\theta(0,x)$. From fig. 2.4, it is observed that the share stress coefficient $f''(0,x)$ decreases monotonically with the increase of the Prandtl number Pr' = (0.7, 1.0, 1.74) and from the fig. 2.5, the same result is observed on the surface temperature due to increase of the Prandtl number Pr.

Further from fig. 2.6 it is clear that the velocity gradient $f''(0,x)$ decreases with the increasing value of the magnetic parameter M = (0.2, 0.5, 0.8, 1.0) when Prandtl number Pr = 0.73, dissipation parameter N = 0.5 and the joule heating parameter J = 0.005 and from fig. 2.7 the same result is obtained on the surface temperature distribution due to increase of magnetic parameter. From fig. 2.8 we observe that the skin friction coefficient $f''(0,x)$ increases with the increase of joule heating parameter J = (0.02, 0.05, 0.08, 0.10) and from fig. 2.9 the same result is observed on the surface temperature distribution due to increase of the joule healing parameter J. Fig. 2.10 and Fig. 2.11 deal with the effect of the viscous dissipation parameter N = (0.1, 0.3, 0.5, 0.9) when Prandtl number Pr = 0.73, magnetic parameter M= 0.5 and the joule heating parameter J= 0.05 on the velocity profile $f'(\eta,x)$ and the temperature profile $\theta(\eta,x)$. Fig. 2.12 illustrated graphically the velocity profile for different values of the Prandtl number Pr = (0.05, 0.70, 1.0) while the magnetic parameter M = 0.5, dissipation parameter N = 0.05 and the joule heating parameter J = 0.05, corresponding distribution of the temperature profile $\theta(\eta,x)$ is shown in fig. 2.13. Fig. 2.14 depicts the velocity profile for different values of the magnetic parameter M = (0.2, 0.5, 0.8, 1.0) when Prandtl number Pr = 0.73, dissipation parameter N = 1.0 and the joule heating parameter J = 0.05 and the corresponding temperature distribution $\theta(\eta,x)$ is shown in fig. 2.15.

In fig. 2.16 and fig. 2.17 we have shown the effect of the joule heating parameter J = (0.2, 0.5, 0.8, 1.0) for Prandtl number Pr = 0.73, dissipation parameter N = 0.05 and magnetic parameter M = 0.05 on the velocity profile $f'(\eta,x)$ and temperature profile $\theta(\eta,x)$. From fig. 2.10 we see that the velocity profile $f'(\eta,x)$ increases slowly with the increase of the dissipation parameter N and in fig. 2.11 , small increment is shown on the temperature profile $\theta(\eta,x)$ for increasing values of N. From fig. 2.12, it is revealed that if the Prandtl number Pr increases the velocity of the fluid decreases. On the other hand, from fig. 2.13, we see that the same result is hold for temperature profile within the boundary layer due to increase of the Prandtl number Pr. From fig. 2.14, we observe that the velocity profile decreases monotonically with the increase of the magnetic parameter M while the of the Prandtl number Pr , dissipation parameter N and the joule heating parameter J are respectively 0.73, 1.0 and 0.05. Opposite result is shown in fig. 2.15 for temperature distribution for the same values of the magnetic parameter M. Again in fig. 2.16, we see that the velocity of the fluid $f'(\eta,x)$ increases with the increase of the values of the joule heating parameter J when Prandtl number Pr = 0.73, dissipation parameter N = 0.05 and the magnetic parameter M = 0.05. The same result is obtained in fig. 2.17 for temperature distribution for the same values of the joule heating parameter J.

Conclusions:

The effect of different values of dissipation parameter N on the skin friction coefficient and surface temperature distribution, velocity and temperature profiles while magnetic parameter M = 0.5, Prandtl number Pr =0.73 and joule heating parameter J =0.05 and also the effect of different values of magnetic parameter M, Prandtl number Pr and joule heating parameter J on the skin friction coefficient, surface temperature distribution and also on the velocity distribution as well as temperature distribution have been investigated theoretically. The transformed non similar boundary layer equations together with the boundary condition based on conduction and convection are solved numerically by the very efficient implicit finite difference method known as Keller box method. From the present investigation it may be drawn the following conclusions:

1. The skin friction coefficient, the surface temperature distribution, the velocity and the temperature distribution increase for increasing value of the viscous dissipation parameter.
2. An increase in the values of the Prandtl number Pr leads to decrease the skin friction coefficient, the surface temperature distribution, the velocity and the temperature distribution over the whole boundary layer.
3. The skin friction coefficient, the surface temperature distribution and the velocity profile decrease while the temperature profile increases for the increased values of the magnetic parameter M.
4. It has been observed that skin friction coefficient, the surface temperature distribution, the velocity and the temperature distribution increase with the increase of the joule heating parameter J.

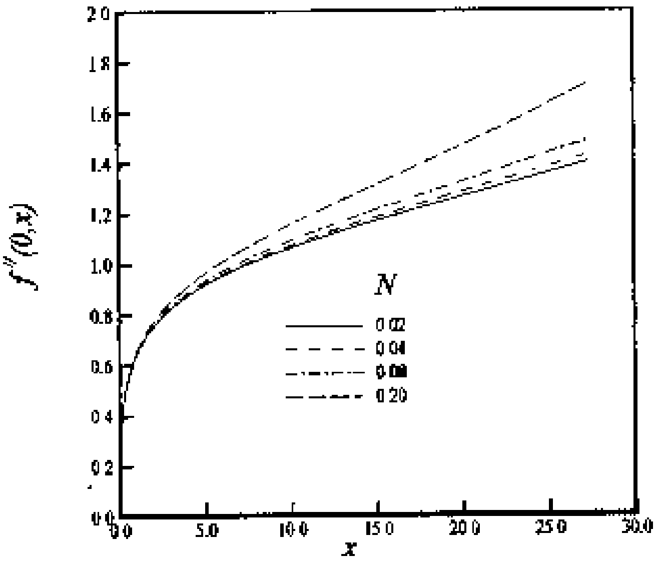

Fig. 2.2: Skin friction for different values of Dissipation Parameter when Prandtl number Pr = 0.73, magnetic parameter $M = 0.5$ and Joule Heating Parameter $J = 0.05$

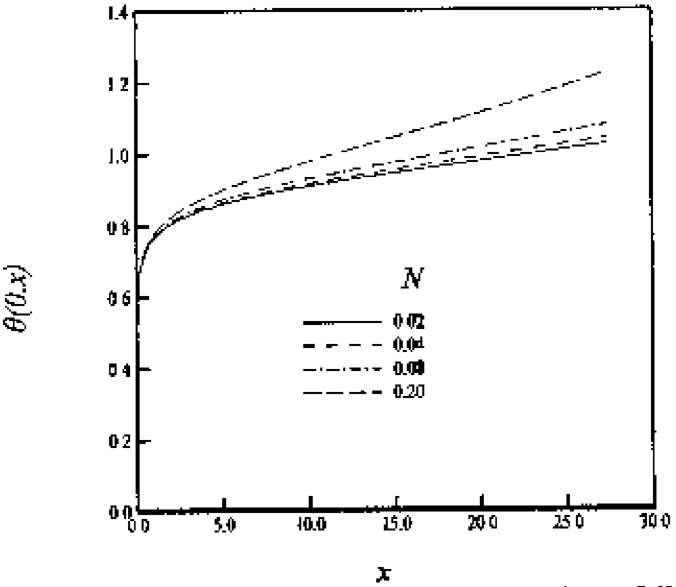

Fig 2.3: Surface temperature distribution for different values of dissipation parameter when Prandtl number Pr = 0.73, magnetic parameter $M = 0.5$ and Joule Heating Parameter $J = 0.05$

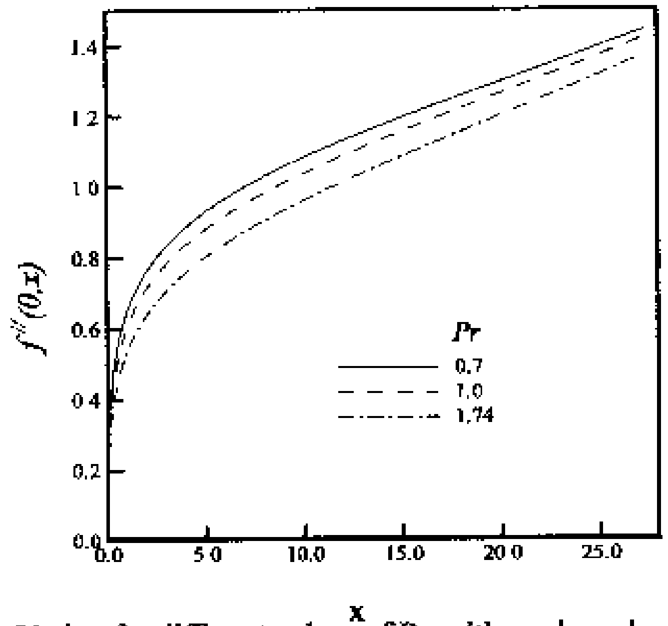

Fig. 2.4: Skin friction for different values of Prandtl number when magnetic parameter $M = 0.5$, dissipation parameter $N = 0.05$ and Joule heating Paramcter $J = 0.05$

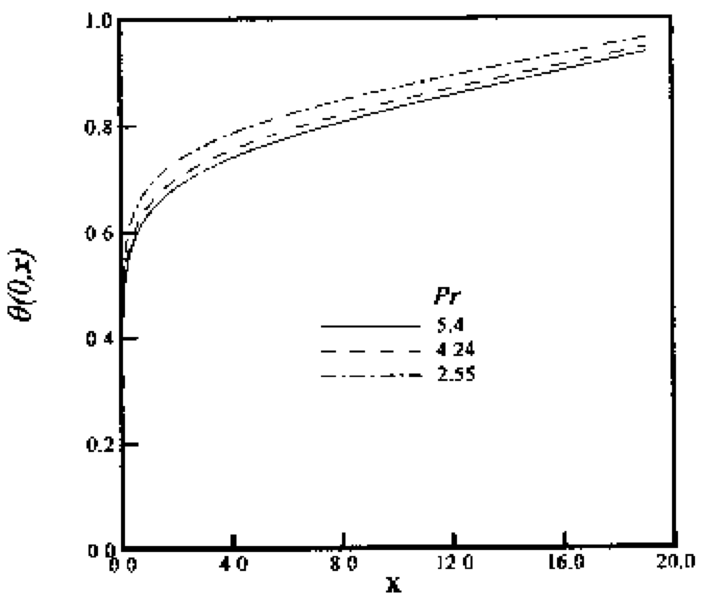

Fig. 2.5 Surface temperature distribution for different values of Prandtl number when magnetic parameter $M = 0.5$, dissipation parameter $N = 0.05$ and Joule heating Parameter $J = 0.05$

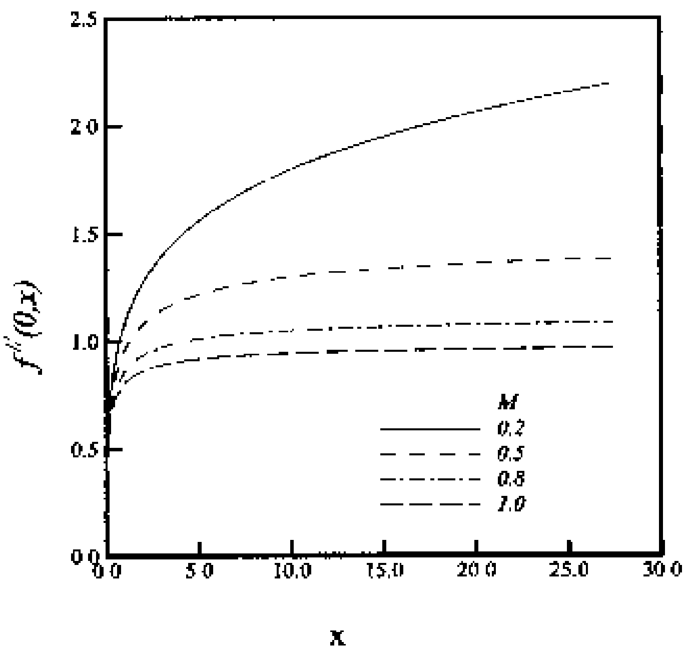

Fig. 2.6: Skin friction for different values of magnetic parameter when dissipation parameter $N = 0.5$ Prandtl number $Pr = 0.05$ and Joule Heating Parameter $J = 0.005$

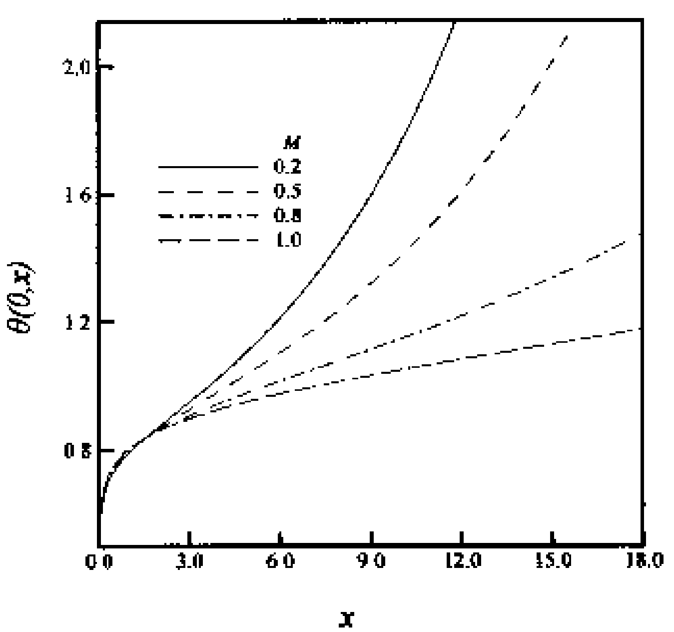

Fig. 2.7: Surface temperature distribution for different values of magnetic parameter when dissipation parameter $N = 0.5$ Prandtl number $Pr = 0.73$ and Joule Heating Parameter $J = 0.05$

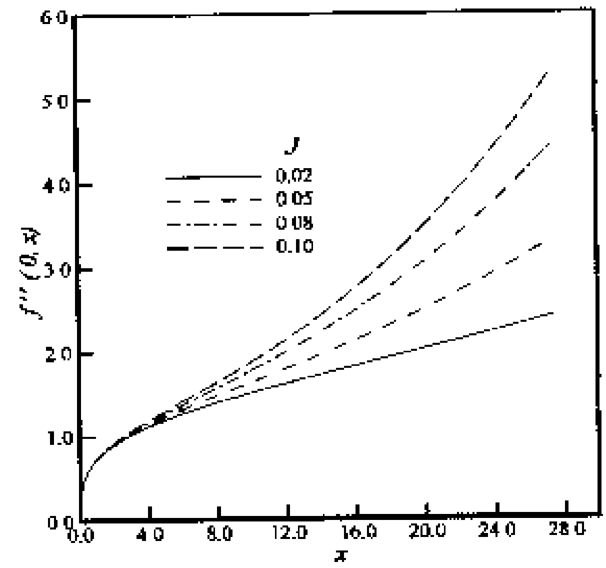

Fig. 2.8: Skin friction for different values of Joule Heating Parameter when Pr =0.73 and Dissipation Parameter N = 0.05 and magnetic parameter M = 0.05

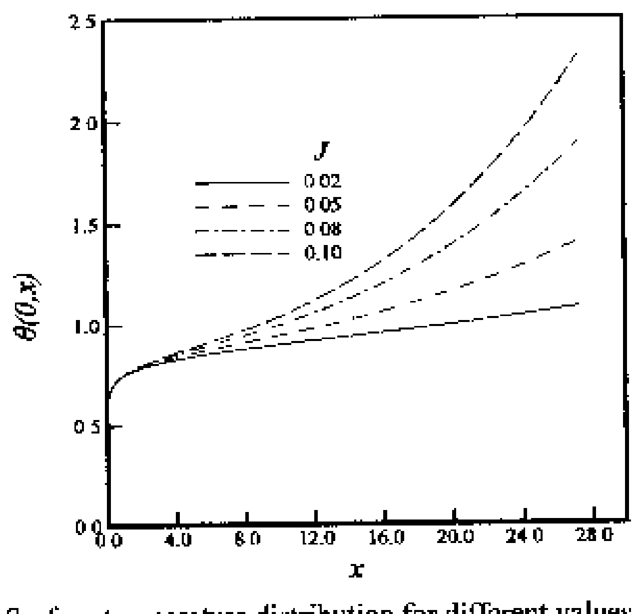

Fig. 2.9: Surface temperature distribution for different values of Joule Heating Parameter when Pr =0.73 and Dissipation Parameter N = 0.05 and magnetic parameter M = 0.05

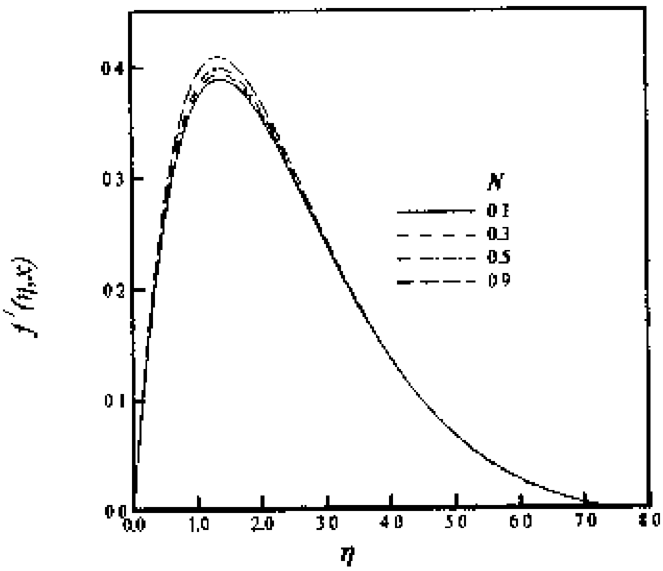

Fig. 2.10: Velocity profile for different values of dissipation parameter when Prandtl number Pr = 0.73 , magnetic parameter M = 0.5 and Joule Heating Parameter J = 0.05

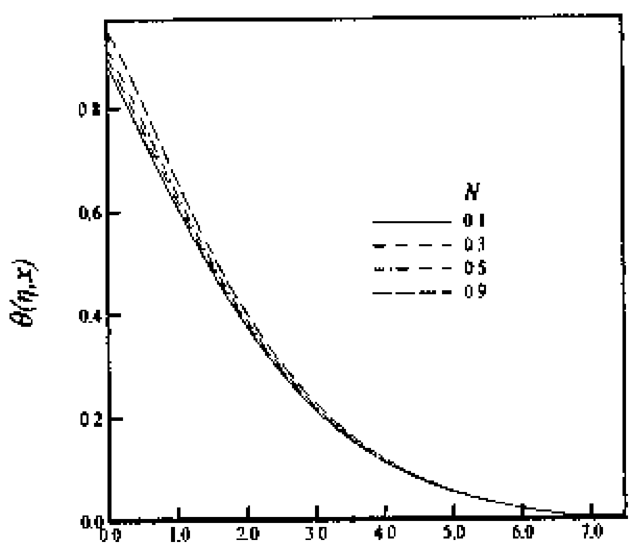

Fig. 2.11: Temperature distribution for different values dissipation parameter when Prandtl number Pr = 0.73, magnetic parameter M = 0.5 and Joule Heating Parameter J = 0.05

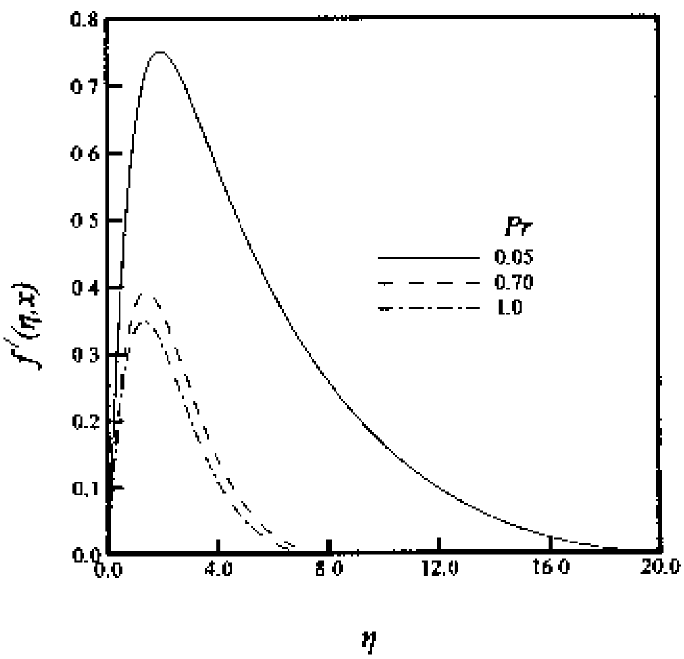

Fig. 2.12: Velocity profile for different values of Prandt number when magnetic parameter $M = 0.5$, dissipation parameter $N = 0.05$ and Joule Heating Parameter $J = 0.05$

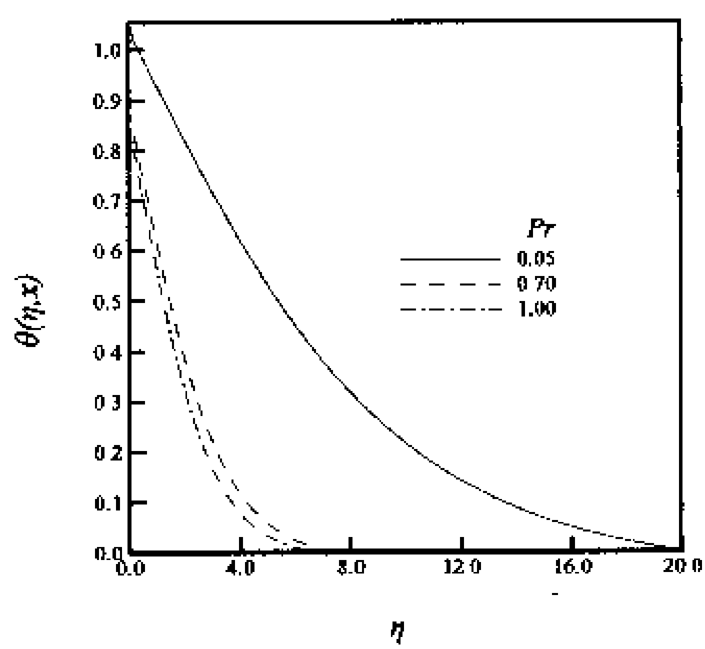

Fig. 2.13: Temperature distribution for different values of Prandt number when magnetic parameter $M = 0.5$, dissipation parameter $N = 0.05$ and Joule Heating Parameter $J = 0.05$

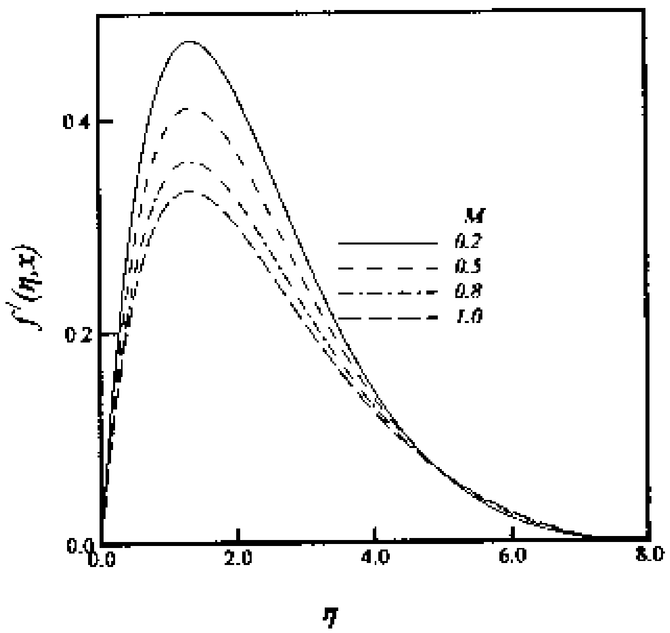

Fig. 2.14: Velocity profile for different values of magnetic parameter when dissipation parameter $N = 1.0$ Prandtl number $Pr = 0.73$ and Joule Heating Parameter $J = 0.05$

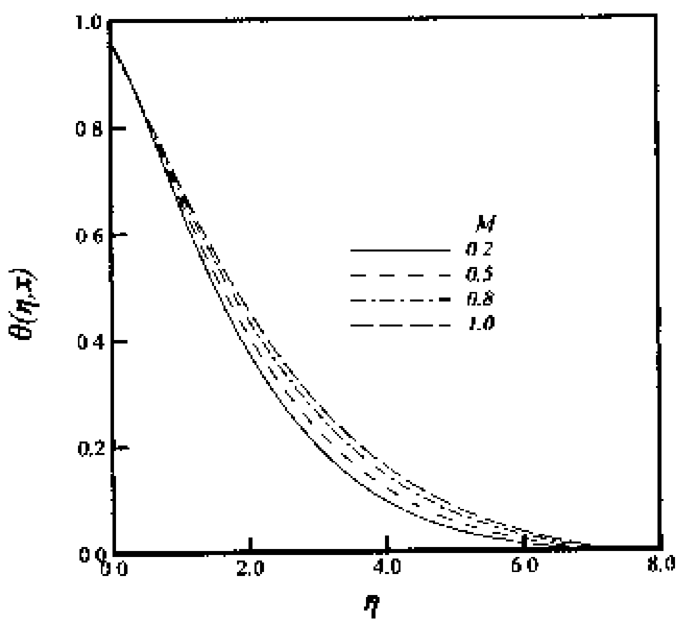

Fig. 2.15: Temperature distribution for different values of magnetic parameter when dissipation parameter $N = 1.0$, Prandtl number $Pr = 0.73$ and Joule Heating Parameter $J = 0.05$

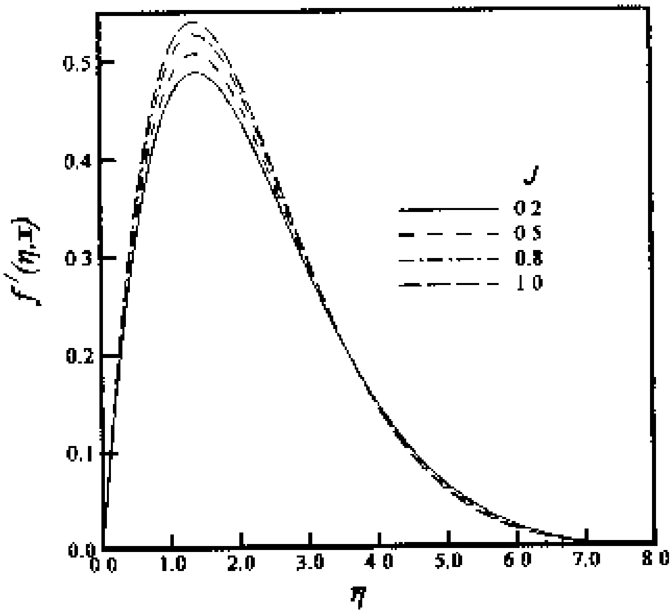

Fig. 2.16: Velocity profile for different values of Joule Heating Parameter when Pr =0.73 and Dissipation Parameter N = 0.05 and magnetic parameter M = 0.05

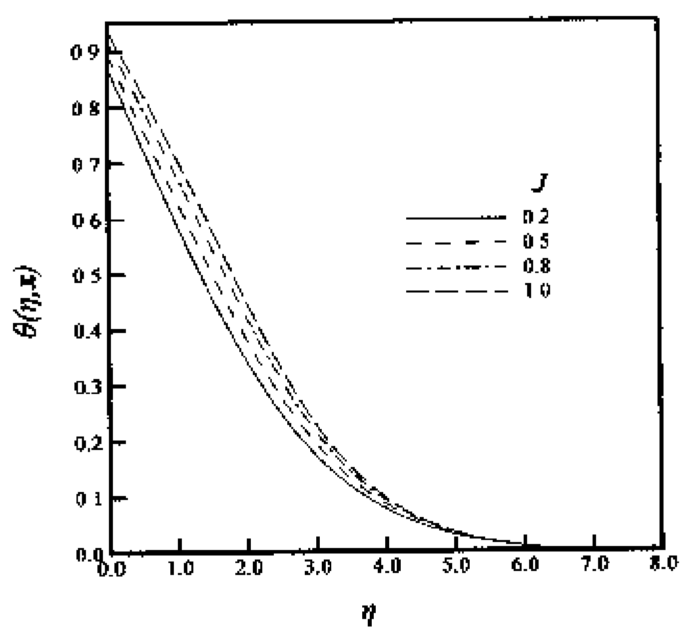

Fig. 2.17: Temperature distribution for different values of Joule Heating Parameter when Pr = 0.73 and Dissipation Parameter N = 0.05 and magnetic parameter M = 0.05

Chapter – 3

Effects of Conduction and Convection on Magneto hydrodynamic Flow with Viscous Dissipation from a Vertical Flat Plate

3.1 Introduction

This chapter describes the effect of conduction and convection on magneto hydrodynamic flow with viscous dissipation from a vertical flat plate. Using the appropriate transformations, the governing boundary layer equations are transformed into a non-dimensional form which are solved numerically by using a finite difference method known as Keller box method. We have represented the effect of the dissipation parameter N, Prandtl number Pr and the magnetic parameter M on the velocity and temperature including the skin friction coefficient and surface temperature coefficient. The complete derivations of the governing equation for the natural convection flow and heat transfer including the method of solutions together with the results and discussions are presented.

3.2 Governing equations of the flow:

We consider the steady two dimensional laminar free convection boundary layer flow of a viscous incompressible and electrically conducting fluid along a side of a vertical flat plate of thickness 'b' insulated on the edges with temperature T_b maintained on the other side. The flow configuration and the coordinates system are shown in figure 3.1.

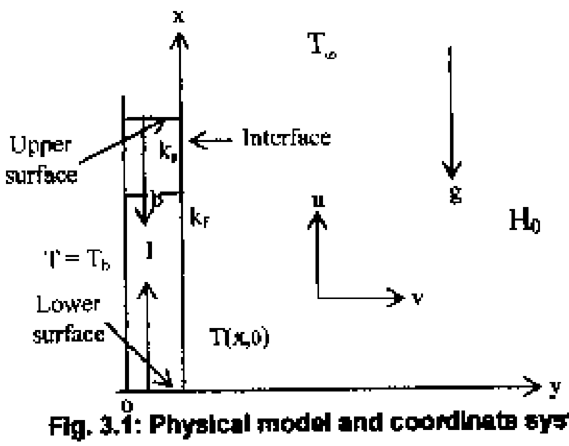

Fig. 3.1: Physical model and coordinate systems

The equations governing the flow are

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0 \tag{3.1}$$

$$u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}=\nu\frac{\partial^2 u}{\partial y^2}+g\beta(T-T_\infty)-\frac{\sigma H_o^2 u}{\rho} \tag{3.2}$$

$$u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}=\frac{k_f}{\rho c_p}\frac{\partial^2 T}{\partial y^2}+\frac{\nu}{c_p}\left(\frac{\partial u}{\partial y}\right)^2 \tag{3.3}$$

The appropriate boundary conditions to be satisfied by the above equations are

$$u=0,\ v=0\ at\ y=0 \tag{3.4a}$$
$$u\longrightarrow 0,\ T\longrightarrow T_\infty\ as\ y\longrightarrow\infty \tag{3.4b}$$

The temperature and the heat flux are required continuous at the interface for the coupled conditions and at the interface we must have

$$\frac{k_s}{k_f}\frac{\partial T_{so}}{\partial y}=k_f\left(\frac{\partial T}{\partial y}\right)_{y=o} \tag{3.5}$$

Where k_s and k_f are the thermal conductivity of the solid and the fluid respectively The temperature T_{so} in the solid is given by

$$T_{so}=T(x,o)-\{T_b-T(x,o)\}\frac{y}{b} \tag{3.6}$$

Where T(x,o) is the unknown temperature at the interface to be determined from the solutions of the equations. We observed that the equations (3.1) – (3.3) together with the boundary conditions (3 4) – (3.5) are non-linear partial; differential equations. In the following sections the solutions methods of these equations are discussed in details.

3.3 Transformation of the governing equations:

Equations (3.1) – (3.3) may now be non-dimensionalized by using the following dimensionless dependent and independent variables:

$$\bar{x}=\frac{x}{L},\bar{y}=\frac{y}{L}d^{\frac{1}{4}},u=\frac{\upsilon}{L}d^{\frac{1}{2}}\bar{u},v=\frac{\upsilon}{L}d^{\frac{1}{4}}\bar{v},\frac{T-T_\infty}{T_b-T_\infty}=\theta,L=\frac{\upsilon^{\frac{2}{3}}}{g^{\frac{1}{3}}},d=\beta(T_b-T_\infty) \tag{3.7}$$

As the problem of natural convection, its parabolic character has no characteristic length; L has been defined in terms of v and g which is the intrinsic properties of the system. The reference length along the 'y' direction has been modified by a factor $d^{-1/4}$ in order to eliminate this quantity from the dimensionless equations and the boundary conditions.

The magneto hydrodynamic field in the fluid is governed by the boundary layer equations, which is the non-dimensional form obtained by introducing the dimensionless variables described in (3.7) may be written as

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0 \tag{3.8}$$

$$u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}+Mu=\frac{\partial^2 u}{\partial y^2}+\theta \tag{3.9}$$

$$u\frac{\partial \theta}{\partial x}+v\frac{\partial \theta}{\partial y}=\frac{1}{P_r}\frac{\partial^2 \theta}{\partial y^2}+N(\frac{\partial u}{\partial y})^2 \tag{3.10}$$

Where $N=\dfrac{\upsilon^2 d}{L^2 c_p (T_b - T_\infty)}$, the dimensionless viscous dissipation parameter

$M=\dfrac{\sigma H_o^{\,2} L^2}{\mu d^{1/2}}$, the dimensionless magnetic parameter

and $Pr=\dfrac{\mu c_p}{k_f}$, the Prandtl number

The corresponding boundary conditions (3.4) – (3.6) take the following form:

$$u=v=o, \theta-1=p\frac{\partial \theta}{\partial y} \quad at \quad y=o \tag{3.11a}$$

$$u\to o, v\to o \quad as \quad y\to\infty \tag{3.11b}$$

Where p is the conjugate conduction parameter given by $p=(k_f/k_s)(b/L)d^{1/4}$

Here the coupling parameter *'p'* governs the described problem. The order of magnitude of 'p' depends actually on *(b/L)*, (k_f/k_s) and $d^{1/4}$ being the order of unity. The term *(b/L)* attains values much greater than one because of L being small. In case of air, (k_f/k_s) becomes very small when the vertical plate is highly conductive i.e. $k_s >> 1$ different but not always a small such as glass. Therefore in different cases *'p'* is different but not always a small number. In the present investigation we have considered $p = 1$ which is accepted for *(b/L)* of $O(k_f/k_s)$.

To solve the equations (3.8) – (3.10) subject to the boundary conditions (3.11), the following transformations are introduced for the flow region starting from up stream to down stream;

$$\psi = x^{4/5}(1+x)^{-1/20} f(\eta, x), \eta = yx^{-1/5}(1+x^{-1/20}), \theta = x^{1/5}(1+x)^{-1/5} h(\eta, x) \quad (3.12)$$

Here η is the dimensionless similarity variable and Ψ is the stream function which satisfies the equation of continuity and $u = \frac{\partial \psi}{\partial y}, v = -\frac{\partial \psi}{\partial x}$ and h(η,x) is the dimensionless temperature. Substituting equation (3.12) into equations (3.9) and (3.10) we get the following transformed non dimensional equations

$$f''' + \frac{16+15x}{20(1+x)} ff'' - \frac{6+5x}{10(1+x)} f'^2 - Mx^{2/5}(1+x)^{1/10} f' + h = x \left(f' \frac{\partial f'}{\partial x} - f'' \frac{\partial f}{\partial x} \right) \quad (3.13)$$

$$\frac{1}{P_r} h'' + \frac{16+15x}{20(1+x)} fh' - \frac{1}{5(1+x)} f'h + Nxf''^2 = x \left(f' \frac{\partial h}{\partial x} - h' \frac{\partial f}{\partial x} \right) \quad (3.14)$$

In the above equations the primes denote differentiation with respect to η.
The boundary conditions (3.11) then take the following form:

$$\left.\begin{array}{l} f(x,o) = f'(x,o) = o, h'(x,o) = -(1+x)^{1/4} + x^{1/5}(1+x)^{1/20} h(x,o) \\ f'(x,\infty) = o, h'(x,\infty) = o \end{array}\right\} \quad (3.15)$$

3.4 Method of Solution

To get the solutions of the parabolic differential equations (3.13) and (3.14) along with the boundary condition (3.15), we shall employ a most practical, an efficient and accurate solution technique, known as implicit finite difference method together with Keller box method. Since a good description of this method has been discussed in details in chapter-2, further discussion is disregarded here. Numerical results obtained are presented in the following section.

3.5 Results and discussions

Here we have investigated the effect of conduction and convection on magneto hydrodynamic flow with viscous dissipation and joule heating from a vertical flat plate of thickness b insulated on the edges with temperature T_b maintained on the other side in the presence of a uniformly distributed transverse magnetic field. Solutions are obtained for the fluid having Prandtl number Pr = (0.05, 0.73, 1.0) and for a wide range of the values of the values of the viscous dissipation parameter N = (0.2, 0.5, 0.8, 1.0) and the magnetic parameter M = (0.2,20.5,0.8,1.0). If we know the values of the functions $f(\eta,x)$, $h(\eta,x)$ and their derivatives for different values of the Prandtl number Pr, the viscous dissipation parameter N and the magnetic parameter M, we may calculate the numerical values of the surface temperature $\theta(0,x)$ and the shear stress co-efficient $f''(o,x)$ at the surface that are important from the physical point of view. Numerical values of the velocity gradient $f''(o,x)$ and the surface temperature $\theta(0,x)$ are illustrated graphically in fig. 3.2 and fig. 3.3 respectively against the axial distance x in the interval [0, 30] for different values of the viscous dissipation parameter N = (0.2, 0.5, 0.8, 1.0) for the fluid having Prandtl number Pr = 0.05 and the magnetic parameter M = 0.05. In fig. 3.4 and 3.5, the share stress co-efficient $f''(o,x)$ and the surface temperature $\theta(0,x)$ are depicted graphically for different values of the Prandtl number Pr = (0.05, 0.7, 1.0) when this viscous dissipation parameter N = 0.005 the magnetic parameter M = 0.005. The values of the Prandtl number Pr are taken to be 0.05 that corresponds physically the sodium, 0.7 that corresponds to air and 1.0 corresponding to electrolyte solutions such as salt water. In fig. 3.6 and fig. 3.7, the skin friction $f''(o,x)$ and the surface temperature distribution $\theta(0,x)$ are shown graphically for different values of the magnetic parameter M = (0.2,0.5,0.8,1.0) when Prandtl number Pr = 0.05 and the viscous dissipation parameter N = 1.05. From fig. 3.2 it can be observed that increase in the value of the viscous dissipation parameter N leads to increase of the skin friction $f''(o,x)$. Again fig. 3.3 shows that the increase of the viscous parameter N leads to increase of the surface temperature $\theta(0,x)$. From fig. 3.4, it is shown that the skin friction $f''(o,x)$ decreases monotonically with the increase of the Prandtl number Pr = (0.05, 0.7, 1.0) and from the fig. 3.5 the same result is observed on the surface temperature distribution $\theta(0,x)$ due to increase of the value of the Prandtl number when the value of the dissipation parameter N = 0.005 and the value of the magnetic parameter M= 0.005. Further from fig. 3.6 it is clear that the shear stress co-efficient f'' (0,x) decreases with the increase of the magnetic parameter M = (0.2,0.5,0.8,1.0) and from fig. 3.7 the same result is observed on the surface temperature distribution due to increase of the value of the magnetic parameter. Fig. 3.8 and fig. 3.9 deal with the effect of the viscous dissipation parameter N = (0.2, 0.5, 0.8, 1.0) for Prandtl number Pr = 0.73 and for the magnetic parameter M = 0.05 on the velocity profile $f'(\eta,x)$ and the temperature profile for θ (η,x). Fig. 3.10 depicts the velocity profile for different values of the Prandtl number Pr = (0.05, 0.7, 1.0) when the viscous dissipation parameter N = 0.2 and the magnetic parameter M = 1.0. Corresponding distribution of the temperature profile $\theta(\eta,x)$ in the fluid is shown in fig. 3.11. Again in fig. 3.12 and in fig. 3.13, the velocity profile and the temperature profile are shown graphically for different values of the magnetic parameter M = (0.2, 0.5, 0.8, 1.0) when Prandtl number Pr = 0.7 and the dissipation parameter N = 0.2.

From fig. 3.8, it is revealed that the velocity profile $f'(\eta,x)$ increases slowly with the increase of the viscous dissipation N and the same result is observed in fig. 3.9 for temperature for the same values of the viscous dissipation parameter N. From fig. 3.10, it can be seen that if the Prandtl number increases, the velocity of the fluid decreases. On the other hand, from fig. 3.11 we observe that the temperature profile decreases within the boundary layer due to increase of the Prandtl number Pr. From fig. 3.12, it is clear that the velocity profile decreases monotonically with the increase of the magnetic parameter M when the values of the Prandtl number Pr and the dissipation parameter N are respectively 0.7 and 0.2. Opposite result is shown in fig. 3.13 for temperature distribution for the same values of the magnetic parameter M.

3.5 Conclusion

The effect of viscous dissipation and the magnetic parameter N and M respectively for small prandtl number Pr = (0.05, 0.7, 1.0) on the convective natural convection boundary layer flow have been investigated. The transformed dimensionless boundary layer equations governing the flow are solved numerically by using the very efficient implicit finite difference method known as Keller box scheme. The coupled effect of natural convection and conduction required that the temperature and the heat flux be continuous at the interface. From present investigation, the following conclusions may be drawn:

1. The skin friction coefficient, the surface temperature distribution, the velocity distribution and the temperature distribution increase for increasing value of the viscous dissipation parameter N.

2. It has been observed that the skin friction coefficient ,the surface temperature distribution ,the velocity distribution and the temperature distribution over the whole boundary layer decrease with the increase of the prandtl number Pr.

3. Increased value of the magnetic parameter M leads to decrease the skin friction coefficient, the surface temperature distribution and the velocity distribution while the temperature distribution increases.

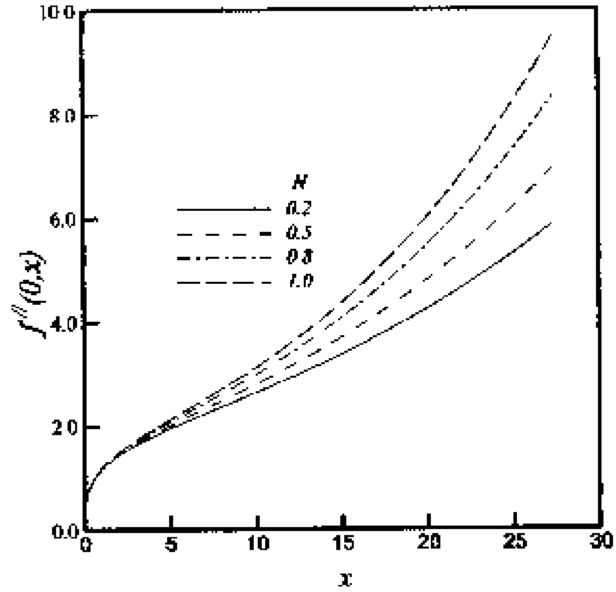

Fig. 3.2: Skin friction for different values of Dissipation Parameter when Prandtl number $Pr = 0.05$ and Magnetic parameter M = 0.05

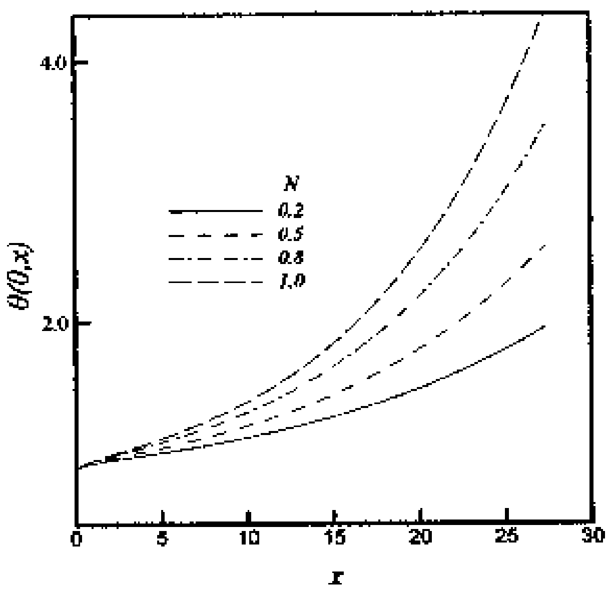

Fig. 3.3: Surface temperature distribution for different values of Dissipation parameter when Prandtl number $Pr = 0.05$ and Magnetic parameter M = 0.05

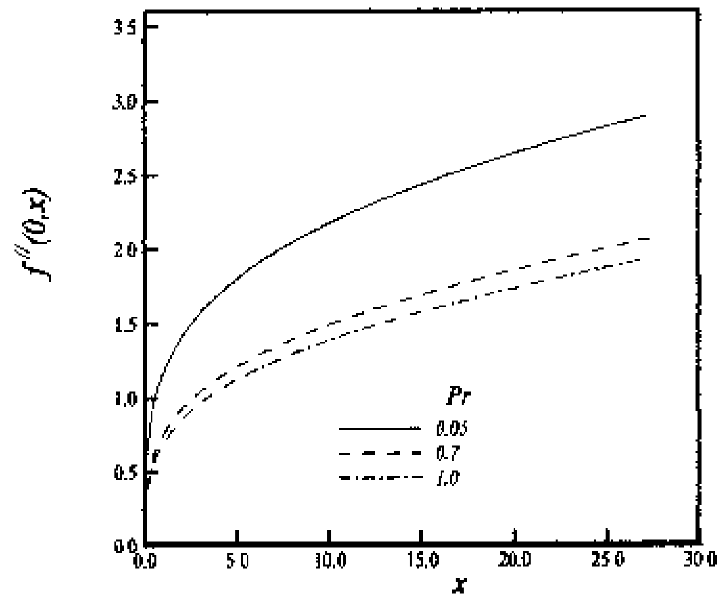

Fig.3.4: Skin friction for different values of Prandtl number when and Magnetic parameter M = 0.005 and Dissipation Parameter N = 0.005

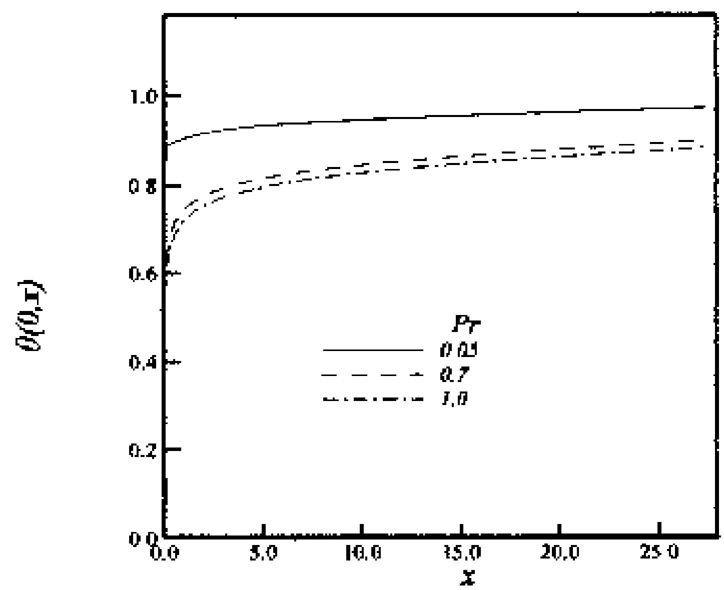

Fig. 3.5: Surface temperature distribution for different values of Prandtl number when Magnetic parameter M = 0.005 and Dissipation Parameter N = 0.005

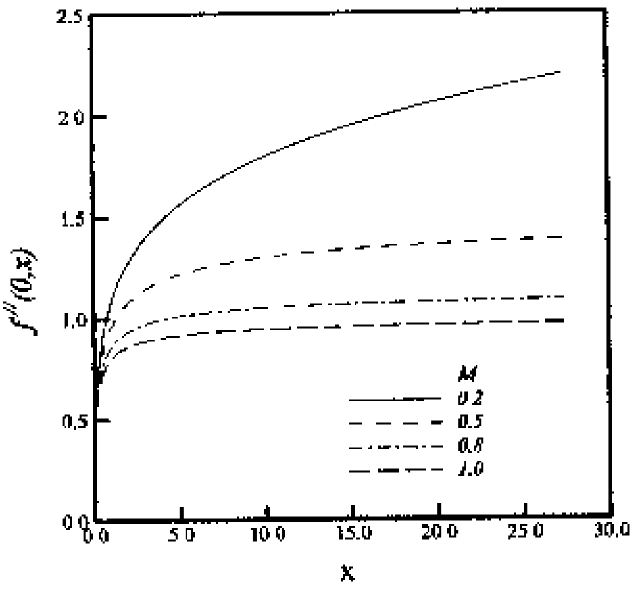

Fig. 3.6: Skin friction for different values of and Magnetic parameter when $Pr = 0.05$ and Dissipation Parameter $N = 1.05$

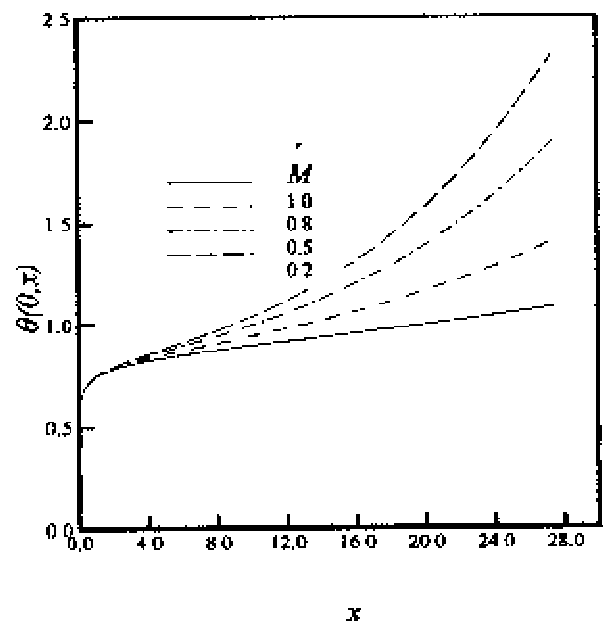

Fig. 3.7 Surface temperature distribution for different values of Magnetic parameter M when P_r =0.05 and Dissipation Parameter $N = 1.05$

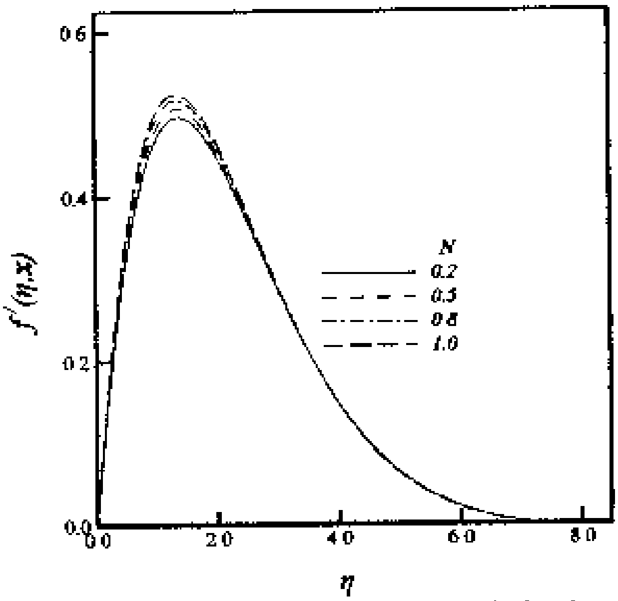

Fig. 3.8: Velocity profile for different values of Dissipation parameter when Prandtl number $Pr = 0.73$ and Magnetic parameter M = 0.05

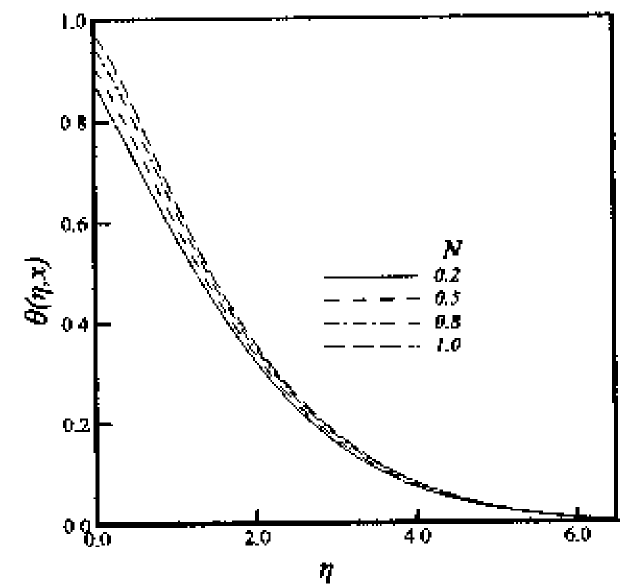

Fig. 3.9: Temperature distribution for different values of Dissipation parameter when Prandtl number $Pr = 0.73$ and Magnetic parameter M = 0.05

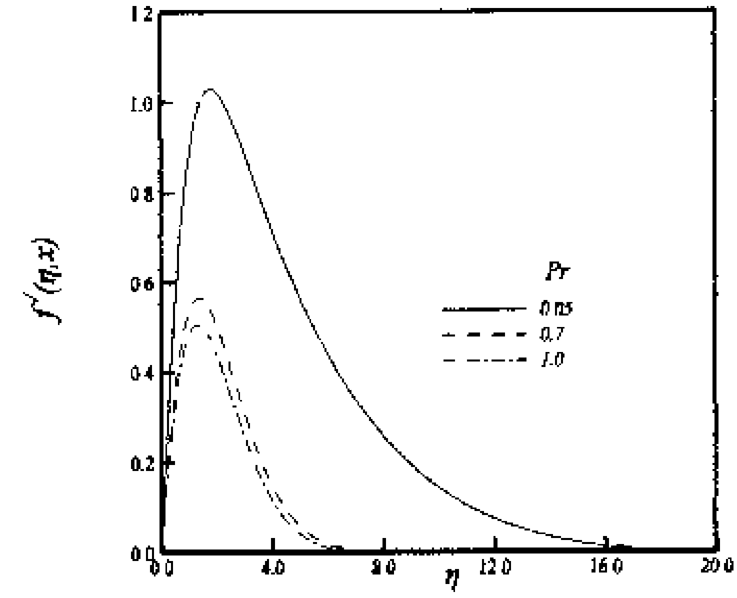

Fig. 3.10: Velocity profile for different values of Prandtl number when and Magnetic parameter M = 1.0 and Dissipation Parameter N = 0.2

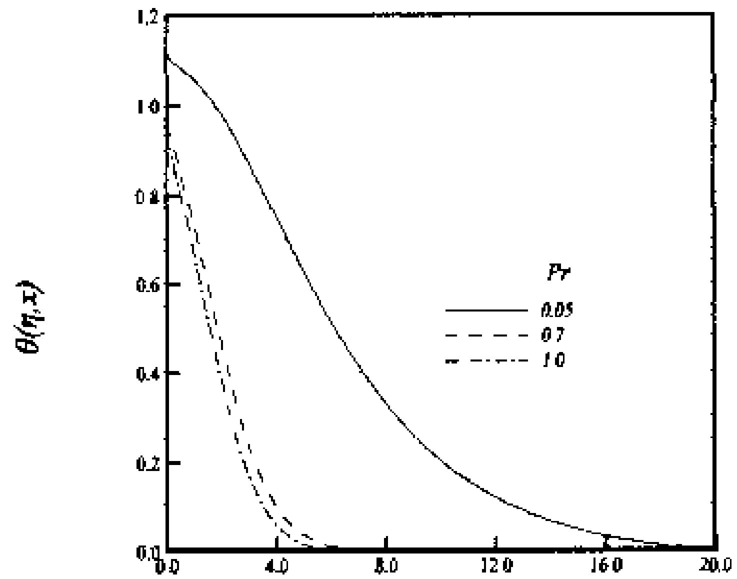

Fig 3.11: Temperature distribution for different values of Prandtl number when Magnetic parameter M = 0.05 and Dissipation Parameter N = 0.2

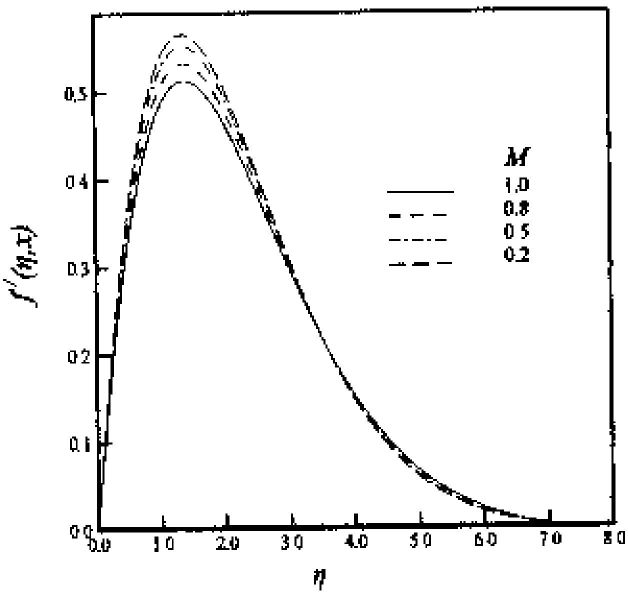

Fig. 3.12: Velocity profile for different values of and Magnetic parameter when Pr = 0.7 and Dissipation Parameter N = 0.2

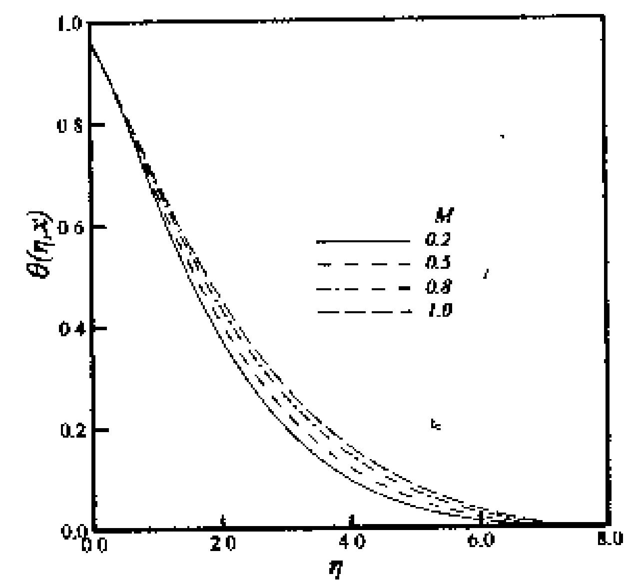

Fig. 3.13: Temperature distribution for different values of Magnetic parameter when Pr = 0.7 and dissipation Parameter N = 0.2

Chapter – 4

Conclusions

In this theses, conjugate effect of conduction and convection with natural convection flow on steady two dimensional magneto hydrodynamic flows with viscous dissipation and joule heating have been investigated. The basic equations are transformed to non similar boundary layer equations by using the appropriate transformations which have been solved numerically using a very efficient implicit finite difference method known as Keller box method. Here we have focused our attention on the evolution of the skin friction, the surface temperature distribution, velocity distribution as well as temperature distribution for a selection of parameters set consisting of the Prandtl number Pr, magnetic parameter M, the viscous dissipation parameter N and the joule heating parameter J.

From chapter 2

The effect of different values of dissipation parameter N on the skin friction coefficient and surface temperature distribution, velocity and temperature profiles while magnetic parameter M = 0.5, Prandtl number Pr =0.73 and joule heating parameter J =0.05 and also the effect of different values of magnetic parameter M, Prandtl number and joule heating parameter J on the skin friction coefficient, surface temperature distribution and also on the velocity distribution as well as temperature distribution have been investigated. The transformed non similar boundary layer equations together with the boundary conditions based on conduction and convection are solved numerically by the very efficient implicit finite difference method known as Keller box method. From the present investigation the following conclusions may be drawn:

1. The skin friction coefficient, the surface temperature distribution, the velocity and the temperature distribution increase for increasing value of the viscous dissipation parameter.
2. An increase in the values of the Prandtl number Pr leads to decrease the skin friction coefficient, the surface temperature distriburion, the velocity and the temperature distribution over the whole boundary layer.

3. The skin friction coefficient, the surface temperature distribution and the velocity profile decrease while the temperature profile increases for the increased values of the magnetic parameter M.
4. It has been observed that skin friction coefficient, the surface temperature distribution, the velocity and the temperature distribution increase with the increase of the joule heating parameter J.

From chapter 3

The effect of viscous dissipation and the magnetic parameter N and M respectively for small prandtl number Pr = (0.05, 0.7, 1.0) on the convective natural convection boundary layer flow have been investigated. The transformed dimensionless boundary layer equations governing the flow are solved numerically by using the very efficient implicit finite difference method known as Keller box scheme. The coupled effect of natural convection and conduction required that the temperature and the heat flux be continuous at the interface. From present investigation, the following conclusions may be drawn:

1. The skin friction coefficient, the surface temperature distribution, the velocity distribution and the temperature distribution increase for increasing value of the viscous dissipation parameter N.
2. It has been observed that the skin friction coefficient, the surface temperature distribution, the velocity distribution and the temperature distribution over the whole boundary layer decrease with the increase of the prandtl number Pr.
3. Increased value of the magnetic parameter M leads to decrease the skin friction coefficient, the surface temperature distribution and the velocity distribution while the temperature distribution increases.

REFERENCES

[1] Merkin, J. H and Mahmood, T.: On the free convection boundary layer on a vertical plate with prescribed surface heat flux. J. Engg. Math., vol. 24, pp. 95-107 (1990).

[2] Hossain, M. A., Molla, M. M. and Chowdhury, M. M. K.: Natural convection flow with natural convection flow with temperature dependent viscosity and thermal conductivity along a vertical wavy surface. 2nd BSME – ASME International Conference on Thermal Engineering, vol. 1, pp. 230-239 (2004).

[3] Yao, L. S.: Natural convection along a vertical wavy surface. ASME J. Heat Transfer, vol. 105, pp. 465-468 (1983).

[4] Nazer, R., Amin, N. and Pop, I.: Free convection boundary layer on an isothermal horizontal circular cylinder in a micropolar fluid. Heat Transfer, Proceeding of the Twelfth International Heat Transfer Conference (2002).

[5] Nazar, R., Amin, N., Grosa, T. and Pop, I.: Free convection boundary layer on an isothermal sphere in a micropolar fluid. Int. Comm. Heat Mass Transfer, vol. 29, No. 3, pp. 377-386 (2002).

[6] Huang, M. J. and Chen, C. K.: Laminar free convection from a sphere with blowing and suction. J. Heat Transfer, vol. 109, pp. 529-532 (May, 1987).

[7] Sparrow, E. M. and Cess, R. D.: The effect of a magnetic field on free convection heat transfer. Int. J. Heat Mass Transfer, vol. 3, pp. 267-274 (1961).

[8] Raptis, A. and Kafousius, N. G.: Magneto hydrodynamic free convection flow and mass transfer through a porous medium bounded by an infinite vertical porous plate with constant heat flux. Canadian Journal of Physics, vol. 60, No. 12, pp. 1725-1729 (1982).

[9] Kuiken, H. K.: Magneto hydrodynamic free convection in a strong cross field. Journal of Fluid Mechanics, vol. 4, part – 1, pp. 21-38 (1970).

[10] Elbashbeshy, E. M. A.: Free convection flow with variable viscosity and thermal diffusivity along a vertical plate in the presence of magnetic field. Int. J. Engg. Science, vol. 38, pp. 207-213 (2000).

[11] Chowdhury, M. K. and Islam, M. N.: MHD free convection flow of visco-elastic fluid past and infinite porous plate. Heat and Mass Transfer, vol. 36, pp. 439-447 (2000).

[12] Hossain, M. A.: Viscous and joule heating effects on MHD free convection flow with variable plate temperature. Int. J. Heat Mass Transfer, vol. 35, No. 12, pp. 3485-3487 (1992).

[13] Ahmad, N. and Zaidi, H. N.: Magnetic effect on overback convection through vertical stratum. 2nd BSME – ASME International Conference on Thermal Engineering, vol. 1, pp. 157-168 (2004).

[14] Hossain, M. A., Das, S. K. and Pop, I.: Heat transfer response of MHD free convection flow along a vertical plate to surface temperature oscillation. Int. J. Non-linear Mechanics, vol. 33, No. 3, pp. 541-553 (1998).

[15] Hossain, M. A., Alam, K. C. A. and Rees, D. A. S.: MHD forced and free convection boundary layer flow along a vertical porous plate. Applied Mechanics and Engineering, vol. 2, No. 1, pp. 33-51 (1997).

[16] Hossain, M. A. and Ahmad, M.: MHD forced and free convection boundary layer flow near the leading edge. Int. J. Heat Mass Transfer, vol. 33, No. 3, pp. 571-575 (1990).

[17] Bynnycky, M. and Kimura, S.: Conjugate free convection due to heated vertical plate. Int. J. Heat Mass Transfer, vol. 38, No. 5, pp. 1067-1080 (1982).

[18] Yu, W-S. and Lin, H. T.: Conjugate problems of conduction and free convection on vertical and horizontal flat plate. Int. J. Heat Mass Transfer, vol. 36, No. 5, pp. 1303-1313 (1993).

[19] Merkin, J. H. and Pop, I.: Conjugate free convection on a vertical surface. Int. J. Heat Mass Transfer, vol. 39, No. 7, pp. 1527-1534 (1982).

[20] Hossain, M. A., Molla, M. M. and Gorla, R. S. R.: Conjugate effect of heat and mass transfer in natural convection flow from on isothermal sphere with chemical reaction. Int. J. Fluid Mech. Research, vol. 31, pp. 104-117 (2004).

[21] Hossain, M. A. and Pop, I.: Magneto hydrodynamic boundary layer flow and heat transfer on a continuous moving wavy surface. Arch. Mech. Vol. 48, pp. 813-823 (1996).

[22] Alam, K. C. A., Hossain, M. A., and Rees, D. A. S.: Magneto hydrodynamic free convection along a vertical wavy surface. Appl. Mech. Engg. Vol. 1, pp. 555-566 (1997).

[23] Rees, D. A. S. and Pop, I.: A note on free convection along a vertical wavy surface in a porous medium. Int. J. Heat Transfer, vol. 116, pp. 505-508 (1994).

[24] Rees, D. A. S. and Pop, I.: Free convection induced by a horizontal wavy surface in a porous medium. Fluid Dynamic Res. vol. 14, pp. 151-166 (1994).

[25] Rees, D. A. S. and Pop, I.: Free convection induced by a vertical wavy surface with uniform heat flux in a porous medium. J. Heat Transfer, vol. 117, pp. 545-550 (1995).

[26] Cheng, C. Y.: Natural convection heat and mass transfer near a vertical wavy surface with constant wall temperature and concentration in a porous medium. Int. Comm. Heat Mass Transfer, vol. 27, No. 8, pp. 1143-1154 (2000).

[27] Kafousius, N. G., Williams, E. M.: The effect of temperature dependent viscosity on the free convective laminar boundary layer flow past a vertical isothermal flat plate. Acta Mechanica, vol. 110, pp. 123-137 (1995).

[28] Kafousius, N. G., Rees, D. A. S.: Numerical study of the combined free and forced convective laminar boundary layer flow past a vertical isothermal flat plate with temperature dependent viscosity. Acta Mechanica, vol. 127, pp. 39-50 (1998).

[29] Hassanien, I. A.: Combined forced and free convection in boundary layer flow of a micropolar fluid over horizontal plate. ZAMP, vol. 48, No. 4, pp. 571 (1977).

[30] Cramer, K. R.: Several magneto hydrodynamic free convection solutions. ASME Journal of Heat Transfer, vol. 85, pp. 35-40 (1963).

[31] Shiralkar, G. and Tein, C.: A numerical study of the effect of a vertical temperature difference imposed on a horizontal enclosure. Numerical Heat Transfer, vol. 5, pp. 185-197 (1982).

[32] Pozzi, A. and Lupo, M.: The coupling of conduction with laminar natural convection along a flat plate. Int. J. Heat Mass Transfer, vol. 31, No. 9, pp. 1807-1814 (1988).

[33] Miyamoto, M., Sumikawa, J., Akiyoshi, T. and Nakamura, T.: Effect of axial heat conduction in a vertical flat plate on free convection heat transfer. Int. J. Heat Mass Transfer, vol. 23, pp. 1545-1553 (1980).

[34] Mamun, A. A.: Effects of conduction and convection on magneto hydrodynamic flow from a vertical flat plate. M. Phil Thesis (2005).

[35] Taher, M. A.: Magneto hydrodynamic natural convection flow on a sphere. M. Phil Thesis (2005).

[36] Keller H. B.: Numerical methods in boundary layer theory, Annual Rev. Fluid Mechanics. Vol.10, pp.417-433 (1978).

[37] Cebici T., Bradshaw P.: Physical and Computational Aspects of Convective Heat Transfer, Springer, New york (1984).