A STUDY OF OVERFLOW TRAFFIC IN TELEPHONE NETWORKS WITH SPECIAL REFERENCE TO BANGLADESH

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A THESIS

SUBMITTED TO THE DEPARTMENT OF ELECTRICAL ENGINEERING, BANGLADESH UNIVERSITY OF ENGINEERING AND TECHNOLOGY DACCA, IN PARTIAL FULFILMENT OF THE REQUREMENTS FOR THE DEGREE OF MASTER OF SCIENCE IN ENGINEERING (ELECTRICAL).

DEPARTMENT OF ELECTRICAL ENGINEERING, BANGLADESH UNIVERSITY OF ENGINEERING AND TECHNOLOGY, DACCA

August, 1980.

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ACKNOWLEDGEMENT

The author expresses his profound gratitudes to his supervisor Dr. A.K.M. Mahfuzur Rahman Khan, Associate Professor of the Department of Electrical Engineering, Bangladesh University of Engineering and Technology(BUET) Dacca, for his invaluable guidance throughout the work and without his assistance and encouragement this work could not have been a success. Thanks are also conveyed to Professor A.M. Zahoorul Huq, Head of the Department of Electrical Enginnering, BUET, for kindly providing all facilities for this work in the Department.

The author also conveys thanks to all the staff of the Computer Centre for the help and co-operation they have extended during the progress of the work.

Finally, the author is also indebted to Professor A.M.Patwari, Director Computer Centre, BUET, for extending the use of different facilities of the Computer Centre.

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ABSTRACT

This thesis firstly deals with the technique of calculating mean and variance of traffic overflow from a primary group of trunks to a secondary group. Number of sources is assumed finite and comparable to the number of trunks. Exact analytical solution of statistical equilibrium equations by using generating function is attempted first. An approximate method is also employed for the same case. Numerical calculations show good agreement between the exact and approximate method.

Secondly, approximate method (Equivalent Random Theory method) developed with the help of the results obtained in the case of single primary group has been applied in several primary groups.

Finally, simulation techniques have been applied in the above network structures to determine different parameters of overflow traffic. Simulation results agree reasonably well with the results obtained from exact and approximate calculation methods.

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LIST OF SYMBOLS

Symbols	Name of the symbols
q	Number of sources
	Calling rate
n_1	Number of primary servers
n_2 & N_2	Number of secondary servers
R_1	Mean traffic rejected from the primary, hence offered to the secondary group.
R_2	Mean traffic rejected by the secondary and hence lost from the system.
V_1	Variance of overflow traffic
R_s	Mean traffic (composite)
V_s	Variance (composite)
N_e	Equivalent primary group.

CHAPTER I

INTRODUCTION

1.1 GENERAL :

Study of overflow traffic is important for efficient planning and management of telecommunication systems. Larger overall server efficiency is always desired in telephony System. Trunks are most efficiently used when heavily loaded but blocking is increased with rise of traffic. Overflow systems are economic solutions to this contrary demand. Overflow system helps the practice of concentrating traffic from less costly lines to more costly trunks. Within the limits of a fixed grade of service, concentration of traffic from two or more groups of sources can be realised with fewer equipment if they are made to share a common secondary group in addition to their primary or direct trunks, instead of, feeding individual groups of trunks. Traffic to the common group in such an arrangement are those overflowed from the different primary groups. Gradings, peak & top load finders, alternate routing arrangement are common examples of overflow systems in Telephone connection networks.

1.2 CHARACTERIZATION OF OVERFLOW TRAFFIC :

Overflow traffic differs from pure chance traffic in that it has more noticeable fluctuations than pure chance traffic. Statistical analysis shows that during a certain period in some group of sources there are large number of overflow calls, while in other groups idle servers are still available and have hardly any overflow calls. Unlike pure chance traffic this overflow traffic is described by two parameters, the mean and the variance;- variance is more than mean in the case of overflow traffice while in case of pure chance traffic variance equals mean.

1.3 OVERFLOW TRAFFIC WITH INFINITE NUMBER OF SOURCES.

For practical design purposes tables giving mean overflow traffic values with associated variance for different switch dimension are available. These tables are based on the assumption of infinite number of sources and infinite secondary group (i.e. a no loss system). Traffic variance method is employed for calculating full availability overflow trunk groups in loss systems. The method of calculating overflow trunk groups developed by Wilkinson called the Equivalent Random Theory is based on the same fundamental idea as the traffic variance method. The only difference is that Wilkinson used the variance $\sigma^2 = D+R$ instead of variance coefficient D where R is the mean traffic.

1.4 OVERFLOW TRAFFIC WITH FINITE NUMBER OF SOURCES.

In practical situations the number of sources are finite and comparable with the number of secondary trunks. No table or working charts are available for such cases. Tables prepared considering infinite sources are used for the said purpose.

1.5 MAIN IDEA OF THIS WORK.

This work is mainly concerned with the study of overflow traffic when the number of sources are finite. Exact formulae derived by Schehrer[2] which starts from the basic difference equation satisfying the statistical equilibrium condition of the network is employed in the case of one primary group to find different parameters of overflow traffic. Tables giving mean traffic and variance for different calling rate, and switch dimensions are prepared. Variation of mean traffic and variance with calling rate and number of sources are shown graphically. Variation of the ratio of variance to mean with the mean traffic is also shown graphically. Approximate method of calculating mean and variance is used to plot graph and found reasonably accurate with the exact calculation.

Exact calculation of overflow system with several primary groups leads to very complex set of equations. The numerical solution of the equations is only possible in case of small systems with the aid of large computer. An approximate method with the various primary groups is presented here. The various primary groups are replaced by one (fictitious) "equivalent primary group" which is chosen such that the overflow traffic offered to the secondary group has the same mean and variance as in the case of the various (really existimg) primary groups.

The exact and approximate methods of calculating over-flow traffic parameters in the case of single primary group, though possible, involves lengthy calculations and consumes lot of time. As mentioned above, numerical solution of the complicated equations arising in the case of several primary groups is difficult and the approximate method of calculating overflow traffic also takes a lengthy procedure to get solutions. Simulation techniques have proved successful in solving such practical problems. Therefore, simulation technique has been applied to similar network structures. The results obtained from such process compare with those obtained from exact and approximate methods of calculations.

CHAPTER 2

REVIEW OF WORKS ON OVERFLOW SYSTEM

2.1 INTRODUCTION

Overflow traffic is characterised by its mean and variance. There are available techniques to find the mean of purely random traffic. Number of available technique to find the mean and variance of non random and peaked overflow traffic is limited. A review of available works on these is made in the following sections.

2.2. WORK ON OVERFLOW TRAFFIC CALCULATION WITH INFINITE NUMBER OF SOURCES.

In 1923 Kendrick who was working with the American Telephone and Telegraph Company, undertook to solve the graded multiple problem through an application of Erlang's statistical equilibrium method. His principal contribution was to set up the equations for describing the existence of calls on a full access group of $x + y$, arranged so that arriving calls always seek service first in the x group and then in the y group when the x are all busy. Kendrick suggested solving the series of simultaneous equations by determinants and also by a method of continued fraction. However, little of this numerical work was actually undertaken until several years later. Early in 1935 Wyekoff of Bell Telephone Laboratory became interested in the solution of $(x + 1)$ $(y + 1)$ lost calls cleared simultaneous eqns. leading to all terms in the $f(m,n)$ distribution ($f(m,n)$ be the probability that at random instant m calls exist on the x paths and n calls on the y paths, when an average poisson load of a erlang is submitted to the x + y paths). She devised an order of substituting one equation in the next which provided an entirely practical and relatively rapid means for the numerical solution of almost any of these equations. By this method a considerable number of $f(m,n)$ distributions on x,y type multiples with varying load levels were calculated. From the complete m.n matrix of probabilities one easily obtains the distribution $\theta m(n)$ of overflow calls when exactly m are present on the lower group of x trunks; or by summing on m, the $\theta(n)$ distribution without regard to m is realized. A number of other procedures for obtaining the $f(m,n)$ values have been proposed. All involve lengthy computations, very tedious for solution by desk calculating machines and most do not have the ready checks of the Wyekoff method available at regular points through the calculations. In 1937 Kosten gave expression for $f(m,n)$. Equations involved in this expression are also very much laborious to calculate if the load and numbers of trunks are not small. The corresponding application of the statistical equilibrium equations to the graded multiple problem was visualized by Kendrick who, however, went only so far as to write out for the three-trunk case consisting of two sub-groups of one trunk each and one common overflow trunk. Wilkinson developed an ingenious method of solving overflow problems. It is called the Equivalent Random Theory(ERT). Wilkinson defined overflow traffic by two well-chosen parameters, oOne is mean and the other is variance which needed to be calculated in the ERT method. Riordan also gave expressions for mean and variance[3].

Kuczura[4] suggested "interrupted poisson process" where he narrated that traffic overflowing a first choice trunk group can be approximated accurately by a simple renewal process. This involves alternately turned on for an exponentially distributed time and then turned off for another (independent) exponentially distributed time. The approximation is obtained by matching either the first two or three moments of an interrupted poisson process to those of an overflow process. Numerical investigation of errors in the approximation and subsequent experience has shown that this method of generating overflow traffic is accurate and very useful in both simulations and analyses of traffic systems.

Schehrer[5] did some exact calculation on overflow systems considering two groups — primary and secondary. THe considered the primary group as well as the secondary group of an overflow system can either be a full available group, or an ideal grading, or a non ideal grading. Thus he counted 9 possible types of overflow systems. Schehrer derived exact solution for all of these types of systems assuming poisson input i.e. an infinite number of traffic sources.

Schehrer[6] also worked for the optional design of alternate routing systems. Most of the known methods for the economically optimal design of alternate routing systems neglect the variance of overflow traffic, especially in case of gradings (for reasons of simplicity). Therefore in some cases these methods yield results which differ remarkably from the actual economic optimum group sizes. Schehrer presented some methods— more accurate and laborious ones upto simplified and easier ones for the calculation of such networks. They regard the variance of overflow traffic, too, and are suitable for groups with full access as well as for gradings. German Research Society helped in Schbbrer's above investigation.

Schehrer[7] dealt with the calculation of higher order moments of traffic overflowing from trunk groups of full access with poisson input. Earlier Riordan determined the factorial moments of arbitrary order for overflow traffic behind groups of full access with offered poisson traffic. He determined all of these exact formulae by using generating functions. Schehrer derived these moments in an elementary way without employing a transformation by means of generating functions.

2.3. OVERFLOW CALCULATION WITH FINITE NUMBER OF SOURCES.

Schehrer[2] worked with the calculation of loss probabilities in overflow systems with finite number of sources and full available groups. He derived exact, analytic solution for overflow systems with only one primary group. He applied an exact numerical method in the case of small overflow systems with two primary groups. Schehrer also developed an approximate method which takes into account the variance of offered overflow traffic for overflow systems with an arbitrary number of primary groups. He showed that this approximate method yield results which are in good agreement with exact calculations and simulation results. Details of this simulation technique followed was not available with Schehrer's work.

CHAPTER 3

GRADING AS AN OVERFLOW SYSTEM

3.1 INTRODUCTION

Grading is a practical overflow arrangement. In a switch which hunts successively over the individual outlets assigned to a serving trunkgroup, with the hunting process always commencing at a certain start position, the individual hunting steps will of course be subjected to different loads. Whereas the first-choice hunting steps are used almost continuously, the late-choice steps will only have to carry the traffic not handled by the first choices. Measurements have shown that with an accessibility of k=10, ten serving trunks (full-access trunkgroup) and a probability of loss of 1%, 70% of the traffic will flow over the first five hunting steps. Allowance for this increased share of traffic can be made by assigning 70% of the serving trunks to the first five hunting steps. With 20 serving trunks, this would result in 14 serving trunks (Fig.3.1) which may be "graded" as follows : one serving trunks per outlet for the first two hunting steps, one serving trunk per two outlets for steps 3 through 6, and one serving trunk per four outlets for steps 7 through 10. Thus a graded multiple is a mixing pattern in which hunting steps of different subgroups are commoned in such a manner that the number of interconnected hunting steps increases in some way with the increasing ordinal numbers

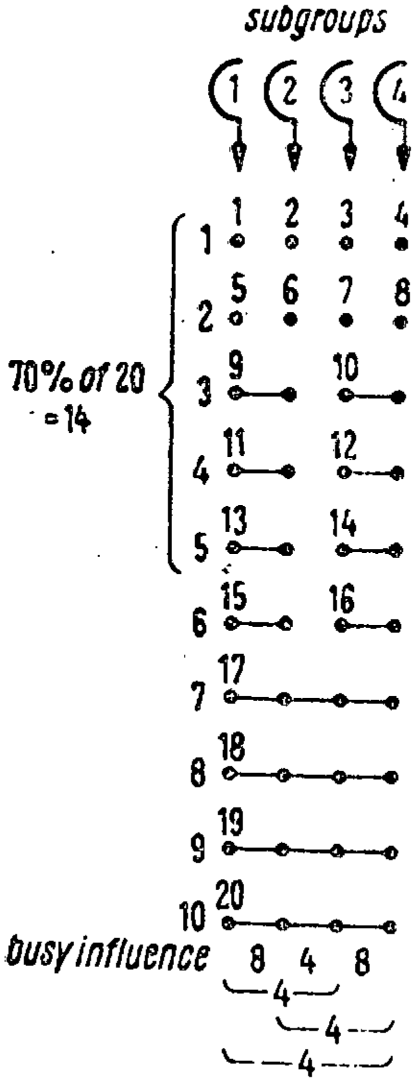


Fig. 3.1.

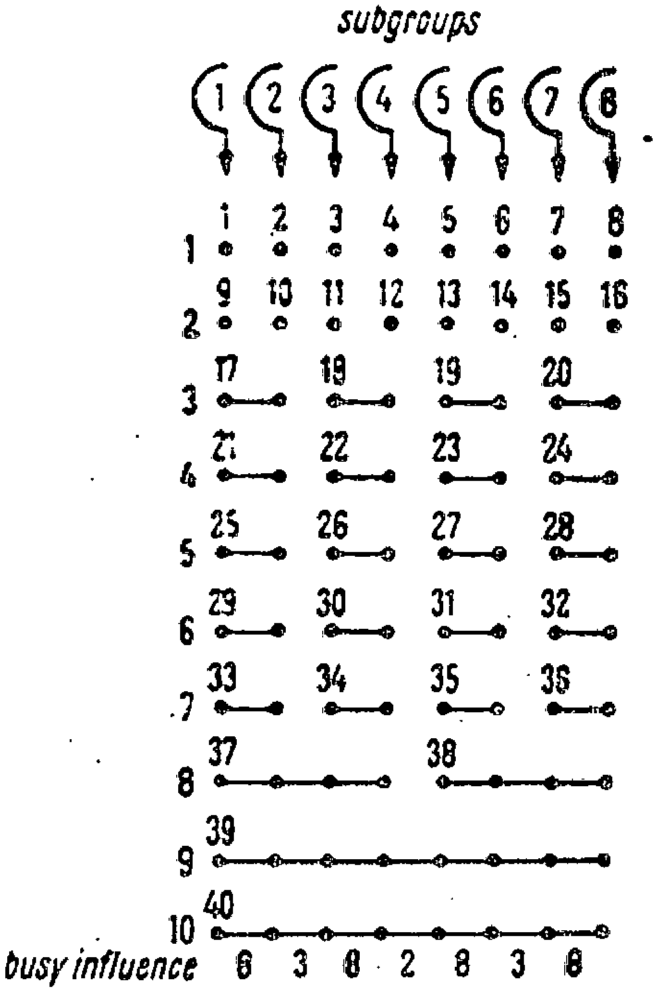


Fig. 3.2.

Examples of grading.

assigned to them.

At the same time, grading enables the busy influence between the individual subgroups to be balanced to a certain to a certain extent. The numerals under the mixing diagram indicate that a busy influence of 8 (eight common serving trunks) exists between subgroups 1 and 2 as well as between 3 and 4 . Between subgroups 1-3, 2-3, 2-4, and 1-4, the busy influence is now 4. In other word , subgroups 1 and 2 have access to twelve serving trunks whereas in the simple multiple they were only able to reach ten. Thus a free connecting path can be found more easily with this scheme even in the case of non-uniform traffic. The load-carrying capacity increases inspite of the unchanged number of serving trunks.

For comparison, Fig.3.2 shows an example of a straight grading. A straight grading is a grading in which only identically numbered hunting steps of adjoining subgroups are commoned (identically numbered outlets or hunting steps are outlets which are assigned the same ordinal number in different subgroups). For the sake of simplicity, only the busy influence of adjoining subgroups is shown in Fig. 3.2 .

3.2 CHOICE OF A GOOD GRADING SYSTEM:

There may be several arrangements of sharing circuits. All these arrangements are not equally efficient. One has to find out the particular grading which gives the best grade of service.

In general it is not necessary to calculate the grade of service of all alternative gradings to find out the best one. A grading which is very nearly the best can be found out by the method of sum of successive difference.

3.3. ERLANG'S IDEAL INTERCONNECTING FORMULA

Erlang's[11] Ideal interconnecting formula state that if the number of trunks required, N, is larger than the availability, K, then some form of interconnecting, in which some trunks are shared between two or more groups must be employed. An approximate formula applying to interconnecting schemes where both A (average traffic) and N are large and the traffic is offered to the trunks in a pure chance manner so that each call can test only K trunks out of the total of N is

$$B = (A/N)^K$$

$$\text{or } A/N = B^{1/K} \qquad (\text{Where } A/N = \text{average traffic/trunk})$$

O'Dell modified this ideal interconnecting formula, and stated that the actual increase in capacity obtained by grading was bnly 53% of the theoratical increase in capacity calculated from Erlang's Ideal formula. O'Dell used this estimate to calculate the traffic capacity of his gradings.

3.4 SIMPLE PRACTICAL OVERFLOW SYSTEM

In our country the telephone subscribers are considered in three groups :

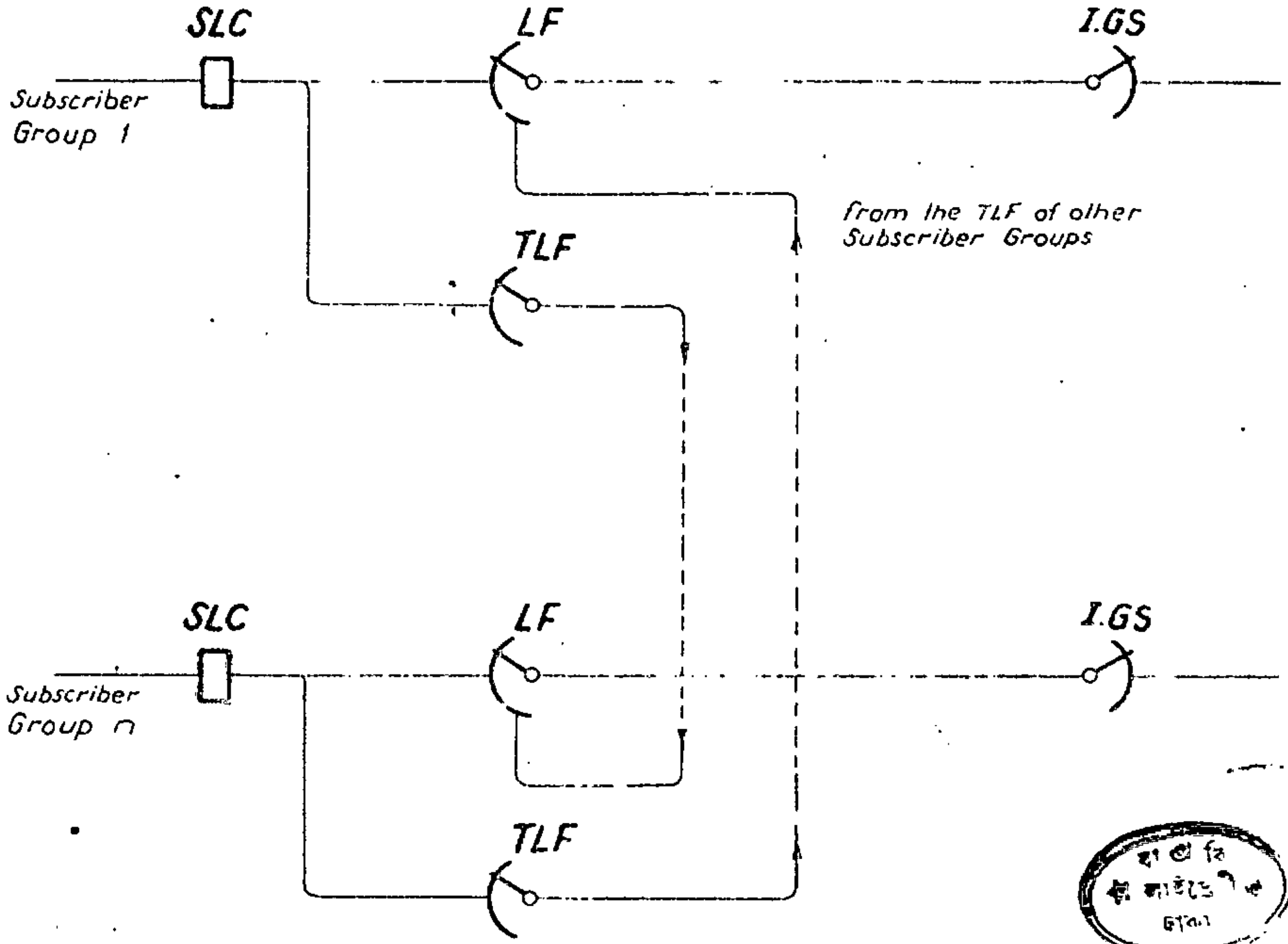
i) Normal group

ii) Heavy group

iii) PABX or PBX group

Normal group covers the residential part of the total subscriber while the heavy group is considered to be the commercial enterprises. PABX is used in office and are always fed with maximum load. Considering the traffic pattern of each group overflow facilities are incorporated . PABX does not employ any provision for overflow traffic. Hence its switching equipments are designed with a consideration of maximum offered traffic. In the case of the arrangement of subscriber line groups in the preselection stage, the traffic arriving from the subscriber line groups is concentrated in the preselection stage in such a way that switches are well-utilised in the group selection stage.

[redacted] *Single - Stage LF Arrangement*
Straight - Arrangement



Two - Stage LF - Arrangement
with Overflow for n Subscriber Groups.

Fig.3.3.

In the EMD exchanges the LF's (line finder) as a rule be either grouped in a straight forward trunking arrangement or in an overflow arrangement. In the straight forward trunking arrangement each line finder is permanently fixed to a 1st group selector. But in the overflow arrangement a 2-stage line finder (Fig. 3.3) is utilised which is known as LF/TLF (Top Load Finder) arrangement. In this arrangement only the LF's connected to one 1st GS (group selector) are seized first in each subscriber groups and are highly utilised. The line finders which carry only peak traffic are designated as top load line finder and are connected to the free inlets of the neighbouring line groups. This causes an equalisation of traffic within several subscriber line groups. In practical use LF/TLF may be 8,12,16,18 or 24 switches per 100 subscribers in number depending on the traffic intensity of each subscriber group. In designing an EMD exchange it is therefore necessary to indicate the number of LF/TLF/100 subscribers which will carry the subscribers traffic to the group selection stage. For example a normal group of 100 subscribers generating total traffic of 6.3 erlang will be equipped with 11 LF and 5TLF assuming a grade of service of 2% and a heavy group of 100 subscribers will have 16 LF and 8TLF.

CHAPTER 4

OVERFLOW SYSTEM WITH INFINITE NUMBER OF SOURCES

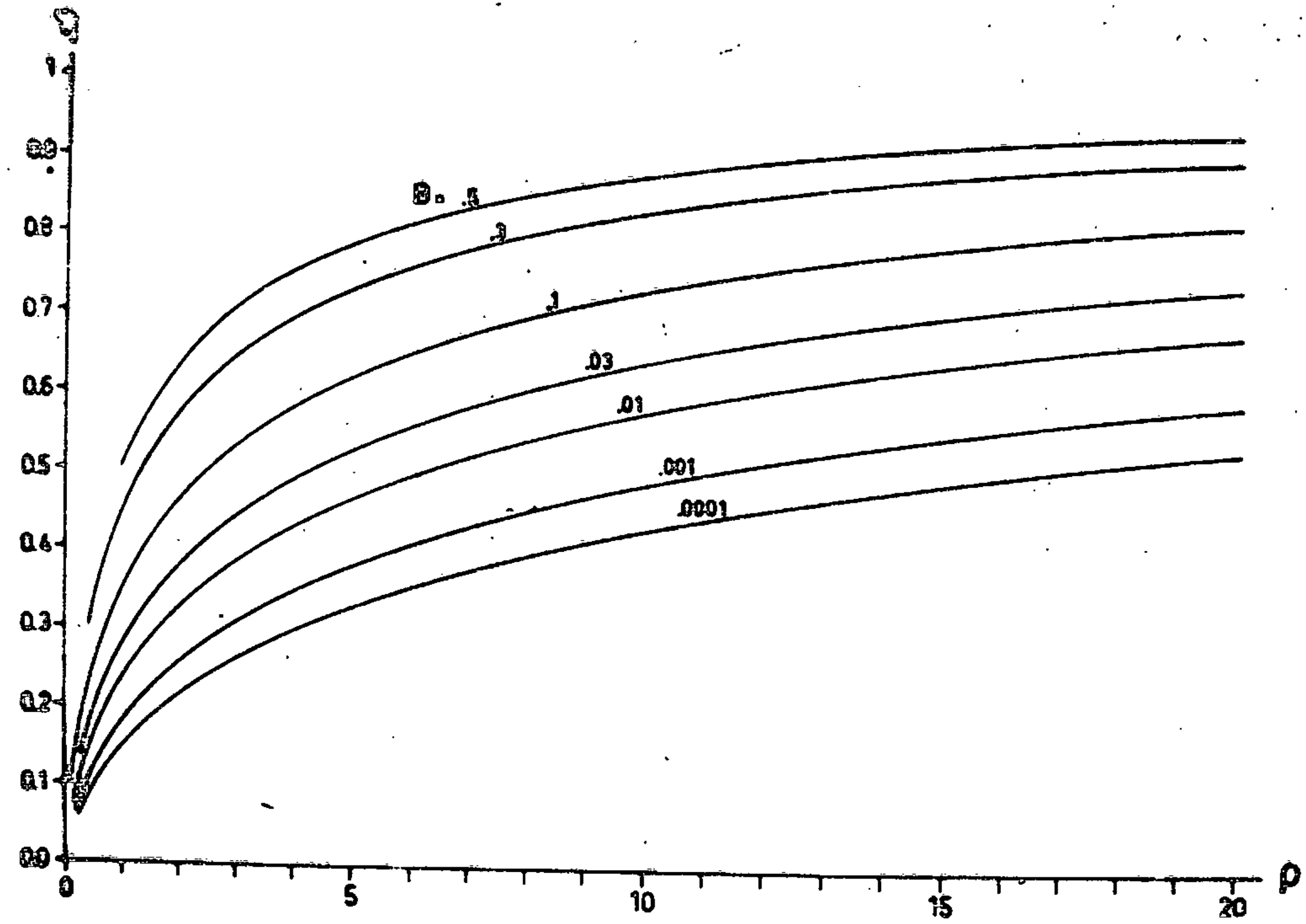


FIG. 4·1. Overall server efficiency.

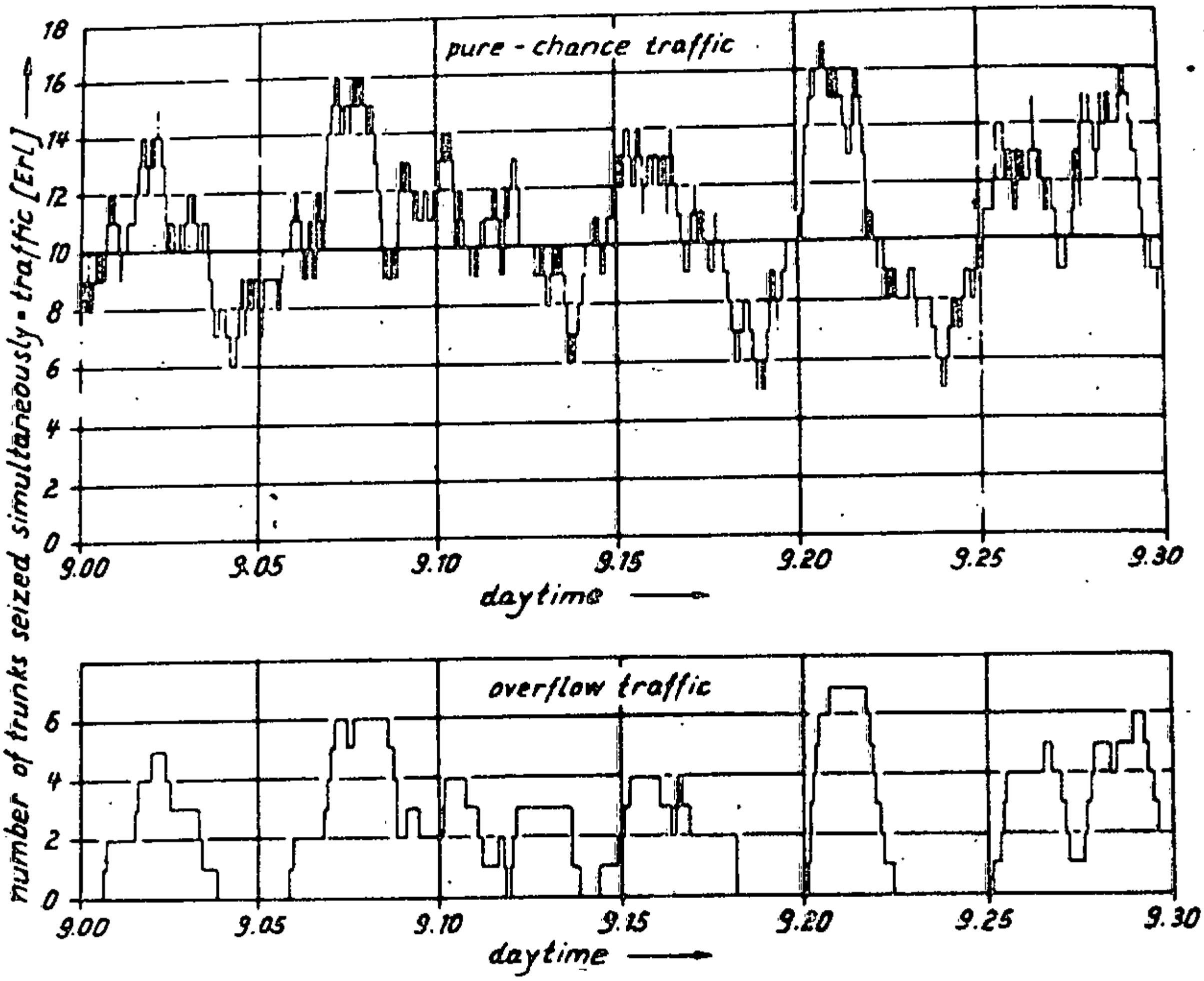


Fig.4.2.Production of peakedness in overflow traffic.

4.1 INTRODUCTION.

Before going to find the parameters of overflow traffic a clear idea about its creation and behavior should be formulated. As mentioned in the previous chapter, to realise maximum efficiency of a telephone system the system should be loaded to the maximum possible traffic.

4.2 RELATION BETWEEN EFFICIENCY AND BLOCKING.

Fig. 4.1 shows that if efficiency is to be raised by giving more load on the system blocking is also increased considerably. This is not desired. A compromise solution to this can be attained by providing some common equipment or trunks for the traffic rejected from a group of switches or trunks heavily loaded for the sake of efficiency. Traffic to the common group make the overflow traffic.

4.3 DIFFERENCE BETWEEN PURE CHANCE TRAFFIC(PCT) AND OVERFLOW TRAFFIC.

An idea of the difference between a pure-chance traffic and an overflow traffic can be formed with the aid of Fig.4.2.[8] It shows that in the observation period (1 hour) the pure-chance traffic fluctuates between +70% and -50% around the mean value 10. The overflow traffic, on the other hand, fluctuates between +250% and -100% around the mean value 2. (These values apply to the

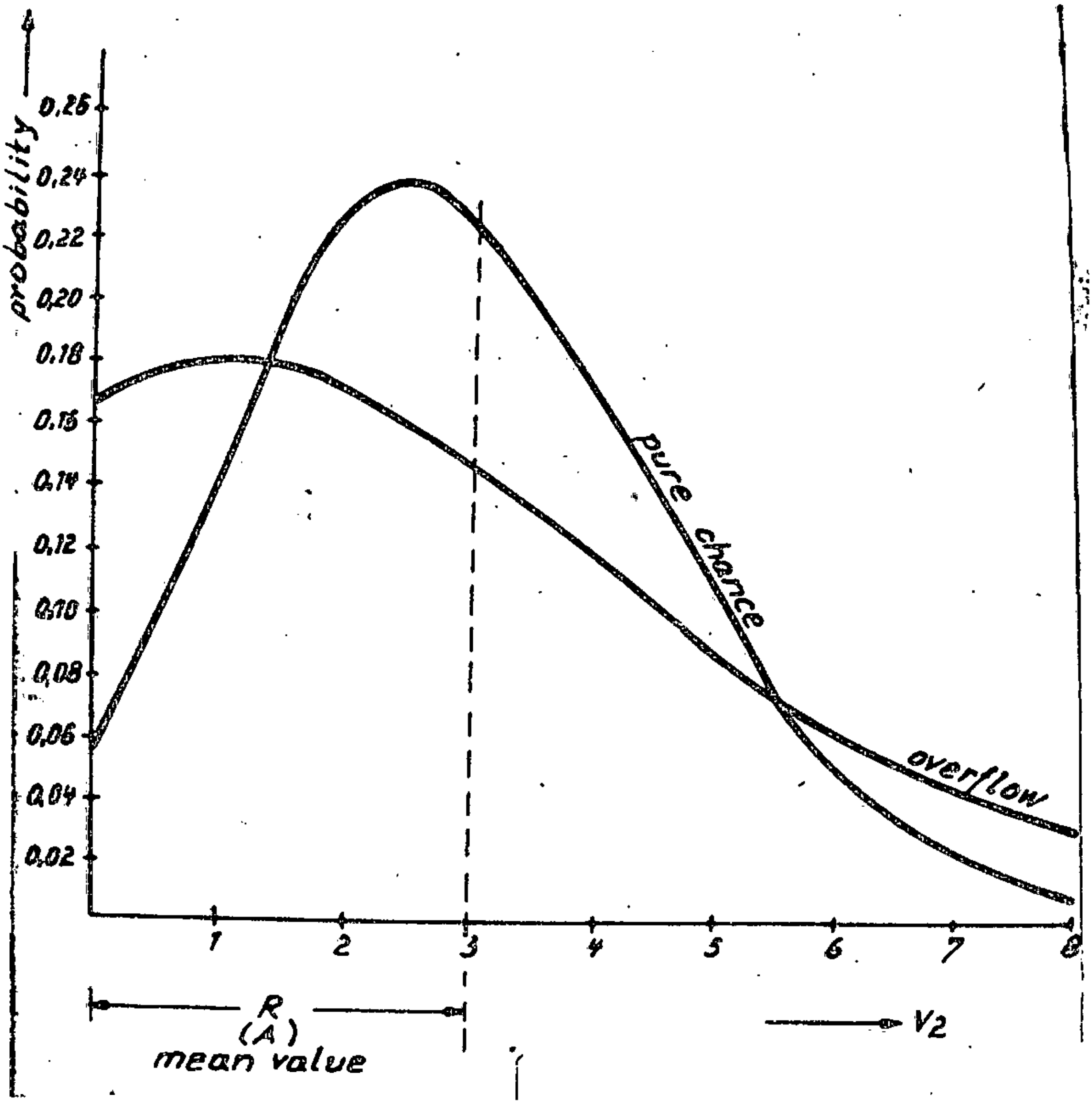


Fig.4.3.Frequency distribution curves.

above example). It will be noted that the fluctuations around a mean value are much larger in the case of overflow traffic. If a limited number of trunks is provided the peaks of fig. 4.2 will be clipped and the overflow traffic will either be lost or they may be handled on a subsequent set of paths. Traffic overflowing a first set of paths to which call may have been randomly offered is non-random in character.

Fig. 4.3[8] shows two frequency distribution curves. Both curves refer to 8 serving trunks to which a traffic load of approximately 3 Erlang is offered. The difference between the two curves consists in that, in the first case, these 3 Erlang are offered in the form of pure-chance traffic, while in the second case these 3 Erlang represent overflow traffic (from a trunk group comprising 12 trunks to which a pure-chance traffic of 12.882 is offered). It may be mentioned that in the case of the overflow traffic the danger time ascertained does not equal the loss, since the equation "danger time = loss" applies solely to pure-chance traffic which is described by an Erlang-type distribution. Fig. 4.3 shows that the two frequency distribution curves display considerable differences. They cannot be described solely by means of the mean value. Some other factor which is called variance is also required to explain the property of over-

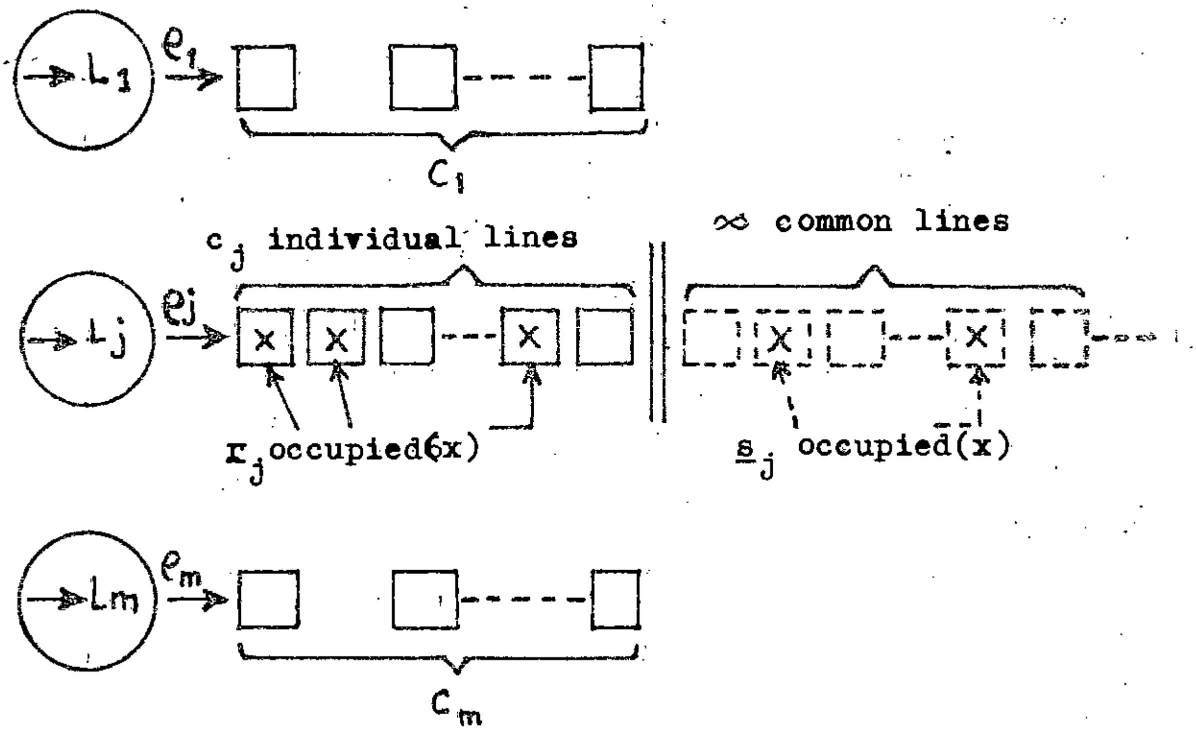


Fig.4.4. Characterisation of overflow traffic

flow traffic. Variance of overflow traffic indicates how much the number of simultaneously existing calls fluctuates around its average value.

4.4 ANALYSIS OF A SIMPLE OVERFLOW SYSTEM WITH INFINITE NUMBER OF SOURCES .

Wilkinson[1] developed an ingenious method dealing with overflow system. It is called the equivalent random method. Wilkinson characterised overflow traffic by two well chosen parameters. For the definition of those parameters the overflow traffic of the group($\rightarrow L_j$) is fictiously offered to an infinite group of common lines (Fig.4.4). The numbers $\underline{r}_j$ and $\underline{s}_j$ of simultaneously occupied direct and common lines (by demands stemming from the ($\rightarrow L_j$) traffic) are correlated stochastic variables. Now, the two parameters chosen by Wilkinson are the average and the variance of $\underline{s}_j$ when $\underline{r}_j$ is unknown. As there is no loss of traffic in the infinite group of common lines, the average of $\underline{s}_j$ must be $\alpha_j = \rho_j E_{1,c_j}(\rho_j)$ (where ρ_j, is the arrival rate and $E_{1,c_j}(\rho_j)$ is the congestion). Let it be assumed that $v_j := \mathrm{var}(\underline{s}_j)$ is also known. Let all overflow traffics be offered simultaneously to the group of common lines. The number $\underline{s}$ of simultaneous occupations in this group clearly is the sum of the number of occupations stemming from the different groups:

$$\underline{s} = \underline{s}_1 + \dots + \underline{s}_m .$$

Now, $\underline{s}_1, \ldots, \underline{s}_m$ obviously are mutually independent. Hence, the average and the variance v of $\underline{s}$ are :

$$; = E(s) = \sum_1^m \alpha_j$$

$$v := \mathrm{var}(\underline{s}) = \sum_1^m v_j \qquad (4.1)$$

So, if it is considered that and v are the characteristics of the total composite overflow traffic, those quantities are simply obtained by addition of those quantities for the m constituents. Suppose there is one fictitious Poisson traffic with arrival rate ρ^* that, after being skimmed by c* lines, yields an overflow traffic with the same characteristics α and v. Then Wilkinson conjectures that this overflow traffic and the original composite overflow do not only behave in the same way when offered to an infinite group of common lines, but also when the number of common lines is finite Fig. 4.5. Hence the total loss traffic l of the original configuration is supposed to be equal to the loss l^* of the fictitious traffic after having been skimmed by $c^* + c$ lines in total, i.e. $l = l^* = E_{1, c^*+c}(\rho^*)$. This loss then is distributed among the m groups proportional to $\alpha_1, \ldots, \alpha_m$. From the separate losses follow the m probabilities of blocking per group.

In order for the method to be workable one needs:

(i) a method for evaluating the average α and variance v of the overflow traffic of one group as a function of the arrival rate ρ and the number of direct lines c (the group index $_j$ has been dropped);

(ii) tables or graphs of those functions.

These requirements are met by Wilkinson. Moreover, the method has been experimentally verified.

Let the case of Fig. 4.4 be considered dropping the suffixes j. Let f_{rs} be the steady-state probability of the simultaneous occupation of r direct and s common lines. The birth-and death equations are :

$$0 = -(r+s+\rho)f_{rs}+(r+1)f_{r+1,s}+(s+1)f_{r,s+1}+\rho f_{r-1,s}$$

$$(r < c;\ s = 0,1,\ldots), \tag{4.2}$$

$$0 = -(c+s+\rho)f_{cs}+\rho f_{c,s-1}+(s+1)f_{c,s+1}+\rho f_{c-1,s}$$

$$(s = 0,1,\ldots) \tag{4.3}$$

and

$$\sum_{r=0}^{c}\sum_{s=0}^{\infty} f_{rs} = 1 . \tag{4.4}$$

Assuming (4.2) to be valid for $r \geq c$ too (defining fictitious quantities $f_{c+1,s},\ldots$). Let the following single and double generating functions be introduced:

$$f_{rs} \mathrel{\widehat{=}} F_r(y) \mathrel{\widehat{=}} F(x,y). \tag{4.5}$$

Then the equations given above yield

$$(1-x)\frac{\partial F}{\partial x}+(1-y)\frac{\partial F}{\partial y} = (1-x)F, \tag{4.6}$$

$$0=(-c-\rho+x)F_c+(1-y)\frac{\partial F_c}{\partial y}+\rho F_{c-1}, \tag{4.7}$$

$$F_c^1(1) = 1 \ . \tag{4.8}$$

The general solution of (4.6) is

$$F(x,y) = K\left(\frac{1-y}{1-x}\right) e^{-\rho(1-x)}, \tag{4.9}$$

where K is an arbitrary function. It is assumed that K is expandable in a power series:

$$K\left(\frac{1-y}{1-x}\right) = \sum_{i=0}^{\infty} \beta_i \left(\frac{1-y}{1-x}\right)^i . \tag{4.10}$$

Taking the arithmetic function generated by (4.9) yields:

$$F_r(y) = \sum_{i=0}^{\infty} \beta_i \varphi_r^i (1-y)^i . \tag{4.11}$$

Comparison of (4.2) and (4.3) yields $(c+1) f_{c+1,s} = f_{c,s-1}$ $(s = 0, 1, \ldots)$, which is equivalent to

$$(c+1) F_{c+1}(y) = \rho y F_c(y) . \tag{4.12}$$

When the series (4.11) is inserted here and the coefficients of $(1 - y)^i$ in the resulting equation are equated to zero, one obtains

$$(c+1) \beta_i \varphi_{c+1}^i = \rho \beta_i \varphi_c^i - \rho \beta_{i-1} \varphi_c^{i-1} \tag{4.13}$$

or

$$i \beta_i \varphi_c^{i+1} = -\rho \beta_{i-1} \varphi_c^{i-1} \quad (i = 1, 2, \cdots) \tag{4.14}$$

Multiple application of this relation yields

$$\beta_i = \frac{(-\rho)^i}{i!} \frac{\varphi_c' \varphi_c}{\varphi_c^{i+1} \varphi_c^i} \beta_0 \tag{4.15}$$

From (4.8) and (4.11) it follows that $\beta_0 = 1/\varphi_c'$. Now, all unknown quantities have been determined.

Then

$$\begin{aligned} \beta_1 &= -\rho \varphi_c / \varphi_c' \varphi_c^2 , \\ &= \tfrac{1}{2} \rho^2 \varphi_c / \varphi_c^2 \varphi_c^3 \end{aligned} \tag{4.16}$$

The generating function $H(y)$ of the probability h_s of $\underline{s} = s$ occupied common lines, irrespective of the number of occupied direct lines, is

$$h_s = H(y) = F_c^1(y) = \sum_{i=0}^{\infty} \beta_i \varphi_e^{i+1} (1-y)^i. \tag{4.17}$$

The average and the second factorial moment are

$$= E(\underline{s}) = H'(1) = -\beta_1 \varphi_e^2 = \rho \varphi_e / \varphi_e' = \rho E_{1,c}(\rho),$$

$$E\left\{\underline{s}(\underline{s} - 1)\right\} = H''(1) = 2\beta_2 \varphi_e^3 = \rho^2 \varphi_e / \varphi_e'^2 \tag{4.18}$$

Hence, the variance v is

$$v = \mathrm{var}(\underline{s}) = E\left\{\underline{s}(\underline{s} - 1)\right\} + E(\underline{s}) - E^2(\underline{s}) = \rho^2 \varphi_e / \varphi_e'^2 + \alpha - \alpha^2 \tag{4.19}$$

The result (4.18) is the known average. The derivation of v given above followeda suggestion made by Riordan[3].

CHAPTER 5

ANALYSIS OF OVERFLOW SYSTEM WITH FINITE NUMBER OF SOURCES

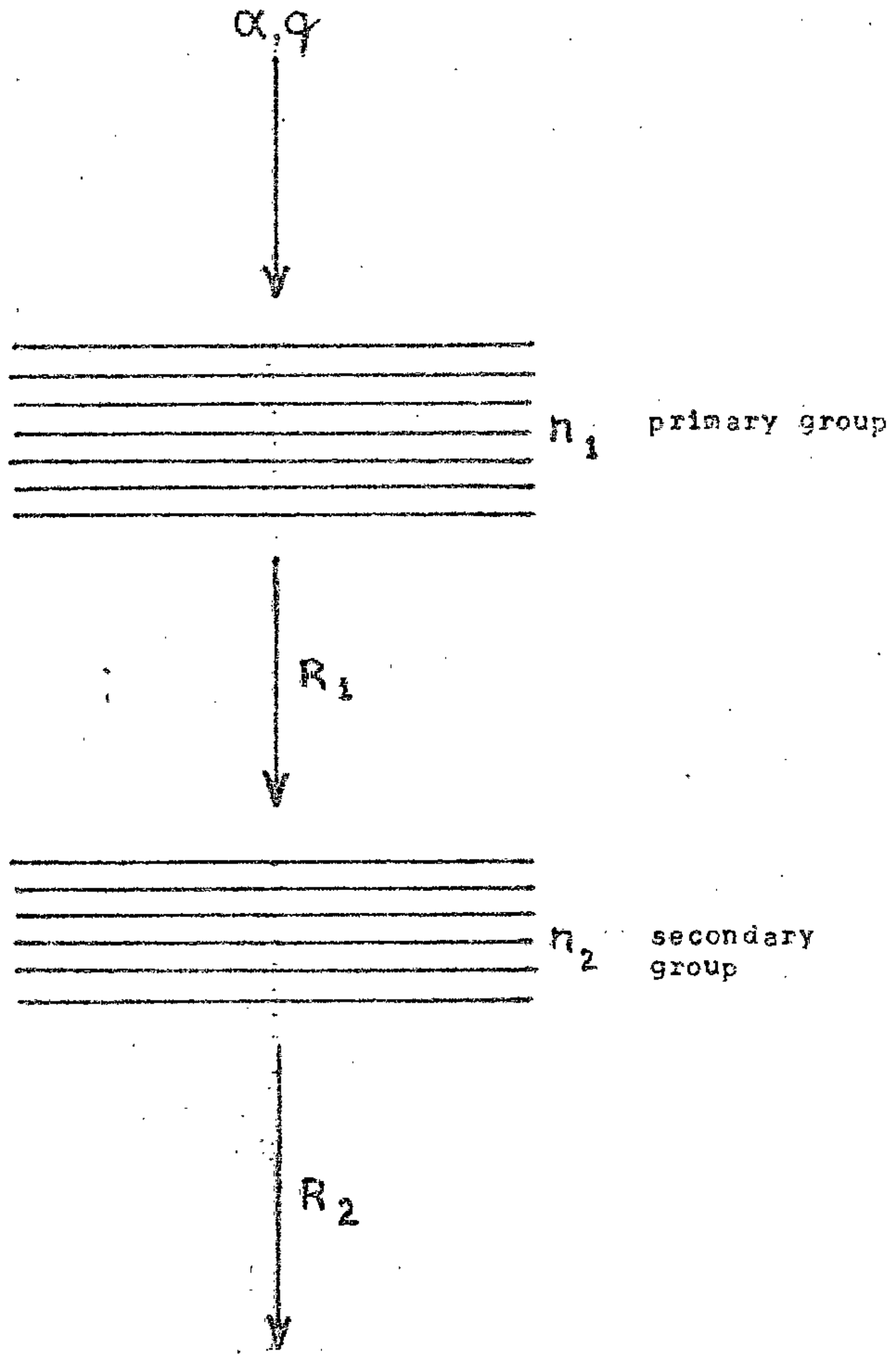


Fig.5.1 Overflow system with two groups of full availability.

5.1 INTRODUCTION

This chapter deals with the study of the overflow characteristics considering finite number of sources and full available groups. Technique of finding loss probabilities is also stressed here. An exact analytical solution derived by Schehrer[2] for overflow systems is applied in case of only one primary group. Approximate method of finding overflow traffic is also applied to one primary group. For several primary groups approximated formulae are first attempted. Simulation technique is lastly used to solve different problems.

5.2 EXACT CALCULATIONS ON OVERFLOW SYSTEMS WITH ONE PRIMARY GROUP.

5.2.1 THE SYSTEM AND THE EQUATION OF STATE.

Figure 5.1 shows a simple overflow system with one primary and one secondary group of trunks. Let a pure chance traffic is offered from a finite number of sources q calling at the rate of α to the system. Let R1 denote the mean traffic rejected from the primary, hence offered to the secondary group and R2 the mean traffic rejected by the secondary and hence lost from the system.

Let just x_1 trunks are busy in the primary group when x_2 trunks are busy in the secondary. From the principle of statistical equilibrium

$$p(x_1,x_2)[\alpha(q-x_1-x_2)+x_1+x_2] = p(x_1-1,x_2)\alpha[q-(x_1-1)-x_2]+p(x_1+1,x_2)(x_1+1)+p(x_1,x_2+1)(x_2+1) \quad (5.1)$$

$$(x_1=0,1,2,\ldots,n_1-1\ ; \quad x_2=0,1,2,\ldots,n_2),$$

$$p(n_1,x_2)[\gamma\alpha(q-n_1-x_2)+n_1+x_2]= p(n_1-1,x_2)\alpha[q-(n_1-1)-x_2]+p(n_1,x_2-1)[q-n_1-(x_2-1)]+p(n_1,x_2+1)(x_2+1) \quad (5.2)$$

$$(x_2= 0,1,2,\ldots,n_2),$$

where

$$\gamma = \begin{cases} 1, & x_2 < n_2, \\ 0, & \text{Otherwise}, \end{cases}$$

and

$$p(x_1,x_2)=0 \text{ for } \begin{array}{l} x_1 < 0, \\ x_2 < 0, \\ x_2 > n_2 . \end{array}$$

The sum of all probabilities $p(x_1,x_2)$ is equal to unity

$$\sum_{x_1=0}^{n_1} \sum_{x_2=0}^{n_2} p(x_1,x_2)= 1. \quad (5.3)$$

The set of equations (5.1,5.2, 5.3) has got (n_1+1) (n_2+1) unknowns. An analytic solution of the above can be obtained by reducing them to one dimensional form.

5.2.2 REDUCTION TO A ONE-DIMENSIONAL SYSTEM OF EQUATIONS.

The two dimensional set of equations (Eqns.(5.1,5.2)) is reduced to a one dimensional set of equations by replacing x_2 by n_2 in equation (5.1), then

$$p(x_1,n_2)\left[\alpha(q-x_1-n_2)+x_1+n_2\right] = p(x_1-1,n_2)\alpha\left[q-(x_1-1)-n_2\right] + p(x_1+1,n_2)(x_1+1) \tag{5.4}$$

For the solution of this second order difference equation it is convenient to use the generating function

$$F(t)=G\left\{p(x_1,x_2)\right\} = \sum_{x_1=0}^{\infty} p(x_1,x_2)t^{x_1}, \tag{5.5}$$

where $t(0 \leq t \leq 1)$ is a real parameter.

Using (5.5) the following relation can be derived :

a) $$G\left\{p(x_1-k,x_2)\right\} = \sum_{x_1=0}^{\infty} p(x_1-k,x_2)\ t^{x_1}$$

$$\begin{aligned}
\text{R.H.S.} &= \sum_{x_1=k}^{\infty} p(x_1-k,x_2)t^{x_1} + \sum_{x_1=0}^{k-1} p(x_1-k,x_2)\ t^{x_1} \\
&= t^k \sum_{x_1'=0}^{\infty} p(x_1',x_2)\ t^{x_1} + \sum_{\nu=0}^{k-1} p(-k+\nu,x_2)\ t^{\nu} \\
&= \quad (\text{putting } x_1' = x_1-k) \\
&= t^k\ G\left\{p(x_1,x_2)\right\} + \sum_{\nu=0}^{k-1} p(-k+\nu,x_2)\ t^{\nu} \qquad (5.6) \\
&\qquad (k= 1,2,3 \ldots\ldots x_1)
\end{aligned}$$

b) $$G\left\{c_1p_1(x_1,x_2) + c_2p_2(x_1,x_2)\right\} = c_1G\left\{p_1(x_1,x_2)\right\} + c_2G\left\{p(x_1,x_2)\right\} \tag{5.7}$$

c) $$G\left\{x_1p(x_1,x_2)\right\} = \sum_{x_1=0}^{\infty} x_1p(x_1,x_2)\ t^{x_1}$$

or $$\frac{d}{dt}G\ p(x_1,x_2) = \sum_{x_1=0}^{\infty} p(x_1,x_2)x_1t^{x_1-1}$$

$$= 1/t \sum_{x_1=0}^{\infty} p(x_1,x_2)x_1\,t^{x_1}$$

$$= 1/t\ G\left\{x_1p(x_1,x_2)\right\} \tag{5.8}$$

d) $\frac{d}{dt} G \left\{p(x_1, x_2)\right\} = \frac{d}{dt} \sum_{x_1=0}^{\infty} p(x_1, x_2)\ t^{x_1}$

$$\text{R.H.S.} = \frac{d}{dt}\Big[p(0,x_2)\ t^0 + p(1,x_2)t^1 + p(2,x_2)\ t^2 + p(3,x_3)\ t^3 + \ldots\ldots\ldots\Big]$$

$$= 1\ p(1,x_2) + 2p(2,x_2)\ t + 3p(3,x_3)t^2 + \ldots\ldots$$

$$= \sum_{x=0}^{\infty} (x+1)\ p(x_1+1,\ x_2)\ t^{x_1}$$

$$= G\left\{(x_1+1)\ p\ (x_1+1, x_2)\right\} \tag{5.9}$$

e) $G\left[(x_1-1)\ p(x_1-1, x_2)\right] = \sum_{x_1=0}^{\infty} (x_1-1)\ p(x_1-1, x_2) t^{x_1}$

then $\text{R.H.S.} = t \sum_{x_1'=0}^{\infty} x_1'\ p(x_1',\ x_2)\ t^{x_1'}$

$$\left[\because p(x_1, x_2) = 0 \text{ for } x_1 < 0\right]$$

(putting $x_1 - 1 = x_1'$)

$$= t\ G\left\{x_1\ p\ (x_1, x_2)\right\}$$

$$= t.t\ \frac{d}{dt} G\left\{p(x_1, x_2)\right\}$$

$$= t^2\ \frac{d}{dt} G\left\{p(x_1, x_2)\right\} \tag{5.10}$$

Eqns(5.6,5.7, 5.8, 5.9, 5.10) are inserted in equation (5.4) to yield

$$\alpha(q-n_2)\ F(t) - \alpha t\ \frac{d}{dt}\ F(t) + n_2 F(t) + t\ \frac{d}{dt}\ F(t)$$

$$= \alpha(q-n_2)\ \left[t\ F(t)\right] - \alpha t^2\ \frac{d}{dt}\ F(t) + \frac{d}{dt}\ F(t)$$

Therefore

$$\frac{d}{dt}\ F(t)\left[1 - \alpha t^2 + \alpha t - t\right] = F(t)\left[-\alpha t\ (q-n_2) + n_2 + \alpha(q-n_2)\right]$$

or $\frac{d}{dt}\ F(t)\left[(1-t)(1+\alpha t)\right] = F(t)\left[n_2 + \alpha(q-n_2)\ (1-t)\right]$

Integrating both sides

$$\int \frac{d\,F(t)}{F(t)} = \int \frac{n_2}{(1-t)(1+\alpha t)}\,dt + \int \frac{\alpha(q-n_2)}{(1+\alpha t)}\,dt$$

$$= n_2 \int \left[\frac{1}{(1+\alpha)} \cdot \frac{1}{(1-t)} + \frac{\alpha}{(1+\alpha)} \cdot \frac{1}{(1+t)} \right] dt$$

$$+ \alpha(q-n_2) \int \frac{1}{(1+\alpha t)}\,dt$$

$$= \frac{n_2}{1+\alpha} \int \frac{1}{1-t}\,dt + \alpha \left[\frac{n_2}{1+\alpha} + (q-n_2) \right] \int \frac{1}{1+\alpha t}\,dt$$

then

$$\log(F(t)) = \frac{-n_2}{1+\alpha} \log(1-t) + \alpha \left[q - \frac{\alpha n_2}{1+\alpha} \right] \cdot \frac{1}{\alpha} \log(1+t)$$

$$+ \log C n_2$$

$$\text{or} \quad F(t) = C n_2 \frac{(1+\alpha t)^{\left(q - \frac{n_2}{1+\alpha}\right)}}{(1-t)^{\frac{\alpha n_2}{1+\alpha}}} \tag{5.11}$$

For calculating the probability $p(x_1, n_2)$, i.e. for the inverse transformation, it is convenient to expand $G\left\{p(x_1, x_2)\right\}$ according to eqn. (5.11) in a power series of the form of equation (5.5). Then one obtains

$$p(x_1, n_2) = C n_2\, T_{n_2, x_1}, \tag{5.12}$$

where

$$T_{r,m}(\alpha, q) = \sum_{\eta=0}^{m} \binom{q - \frac{\alpha r}{1+\alpha}}{m-\eta} \alpha^{m-\eta} \binom{\frac{r}{1+\alpha} - 1 - \eta}{\eta} \tag{5.13}$$

and

$$T_{r,m} = 0, \tag{5.14}$$

for $m < 0$ or $r < 0$.

Inserting $x_1 = 0$ in Eqn.(5.12) one obtains

$$C_{n_2} = p(0,n_2) ,$$

and thus

$$p(x_1, n_2) = p(0, n_2)\ T_{n_2,x_1} \qquad (5.15)$$

In Eqn. (5.15) the probabilities $p(x_1,n_2)$ are expressed as a function of the probability $p(0,n_2)$ only.

Now the probabilities $p(x_1,n_2-1)$ are determined in a similar way. For $x_2 = n_2-1$, eqn.(5.1) reads

$$\begin{aligned} p(x_1,n_2-1)\Big[\alpha(q-x_1-n_2+1) + x_1+n_2 -1\Big] \\ = p(x_1-1,n_2-1)\alpha\Big[q-(x_1-1) -(n_2-1)\Big] \\ +p(x_1+1,n_2-1)\ (x_1+1) \\ +p(x_1,n_2)\ n_2 \end{aligned} \qquad (5.16)$$

Applying generating function according to Eqns.(5.5) and (5.6, 5.7, 5.8, 5.9, 5.10),

$$\begin{aligned} \frac{d}{dt} G\Big\{(p(x_1,n_2-1)\Big\}\Big[(1-t)\ (1+ t)\Big] \\ = G\Big\{p(x_1,n_2-1)\Big\}\Big[(n_2-1) + (q-(n_2-1))- \\ \alpha(q-(n_2-1))\ t\Big]-n_2\ G\Big\{p(x_1,n_2)\Big\} \end{aligned} \qquad (5.17)$$

In this inhomogeneous differential equation for the generating function $G\Big\{p(x_1,n_2-1)\Big\}$, the function $G\Big\{p(x_1,n_2)\Big\}$ is already known according to equation (5.11). The integration of the differential Eqn.(5.17) yields the generating function

$$G\{P(x_1,n_2-1)\} = C_{n_2-1}\frac{(1+\alpha t)^{q-\frac{\alpha(n_2-1)}{1+\alpha}}}{(1-t)^{\frac{n_2-1}{1+\alpha}}} - n_2 P(0,n_2)\, t\, \frac{(1+\alpha t)^{q-\frac{\alpha n_2}{1+\alpha}}}{(1-t)^{n_2/1+\alpha}}$$

Applying Equations (5.6) and (5.11) one obtains (after determining Cn_2-1 in the same way as Cn_2 above)

$$p(x_1,n_2-1) = p(0,n_2-1)\, T_{n_2-1,x_1} - n_2 p(0,n_2)\, T_{n_2,x_1-1} \tag{5.19}$$

In Eqn.(5.19) the probabilities $p(x_1,n_2-1)$ are expressed as a function of of the probabilities $p(0,n_2)$ and $p(0,n_2-1)$.

In the same way the probabilities $p(x_1,n_2-2)$, $p(x_1,n_2-3)$, $p(x_1,n_2-4)$,........, $p(x_1,0)$ can be determined successively. Then the following formula for the probabilities $p(x_1,x_2)$ is obtained.

$$p(x_1,x_2) = \sum_{\xi=x_2}^{n_2} (-1)^{-x_2}\binom{\xi}{x_2} T_{\xi,x_1+x_2-\xi}\, p(0,\xi) \tag{5.20}$$

In Eqn.(5.20), all probabilities $p(x_1,x_2)$ are expressed as a function of the probabilities $p(0, x_2)$ only.

5.2.3 RECURSIVE SOLUTION.

To find $p(0,x_2)$ in a very simple way the full available primary group and the full available secondary group together can be considered as one total group of full availability. The probability that $x=x_1+x_2$ trunks in this total group is given by

$$p_{tot}(x) = \sum_{x_1=0}^{x} p(x_1, x-1) \qquad (5.21)$$

On the other hand, $p_{tot}(x)$ can be calculated according to Erlang's Bernoulli Formula

$$p_{tot}(x) = \frac{\binom{q}{x}\alpha^{x}}{\sum_{\xi=0}^{n_1+n_2}\binom{q}{\xi}\alpha^{\xi}} \qquad (5.22)$$

If Eqns.(5.20) and (5.22) are inserted in Eqn.(5.21), one obtains the recursion formula

$$P(0,x_2) = \frac{1}{T_{x_2,n_1}}\left(P_{tot}(n_1+x_2) - x_2\sum_{\xi=x_2+1}^{n_2}\frac{1}{\xi}(-1)^{\xi-x_2}\binom{\xi}{x_2}T_{\xi,n_1+x_2-\xi}P(0,\xi)\right)$$

$$(x_2=0,1,2,\ldots\ldots, n_2) \qquad (5.23)$$

With the aid of Eqn. (5.23) the probabilities $p(0,n_2)$, $p(0,n_2-1$, $p(0,n_2-2)$, ..., $p(0,0)$ can be determined. Then the remaining probabilities $p(x_1,x_2)$ are determined according to Eqn.(5.20).

In this work $p(x_1, n_2)$ according to Eqn.(5.15) fulfil Eqn.(5.4). Inserting (5.15) in (5.4) yields

$$(m+1)\ T_{r,m+1} = \left[m+r+\alpha(q-m-r)\right]T_{r,m}-\alpha\left[q-r-(m-1)\right]T_{r,m-1} \qquad (5.24)$$

For the T terms. All offering terms $T_{r,m}$ can be evaluaed successively according to Eqn.(5.24), starting with the value $T_{r,-1} = 0$ and $T_{r,0} = 1$.

5.2.4 PROBABILITIES OF LOSS AND OVERFLOW TRAFFIC.

With the aid of Erlang's Bernoulli formula Eqn.(5.22) the overflow traffic

$$R_2 = p_{tot}(n_1+n_2)\,\alpha(q-n_1-n_2), \tag{5.25}$$

the offered traffic

$$A = \frac{\alpha}{1+\alpha}(q+R_2) \quad , \tag{5.26}$$

the total carried load

$$Y_{tot} = A - R_2 \quad , \tag{5.27}$$

and the total loss probability

$$B_{tot} = \frac{R_2}{A} \tag{5.28}$$

The probability $p_2(x_2)$, that x_2 trunks are busy in the secondary group, can be determined by means of the equation:

$$p_2(x_2) = \sum_{x_1=0}^{n_1} p(x_1, x_2), \tag{5.29}$$

or, more easily, with the equation

$$(x_2+1)\, p_2(x_2+1) = \alpha(q-n_1-x_2)\,p(n_1,x_2) \quad , \tag{5.30}$$

which is obtained by applying the principle of statistical equilibrium to the secondary group separately.

Inserting Eqn.(5.20) in Eqn.(5.30) leads to the formula

$$p_2(x_2+1) = \frac{\alpha(q-n_1-x_2)}{x_2+1}\left(T_{x_2,n_1}P(0,x_2)+\sum_{\xi=x_2+1}^{n_2}\binom{\xi}{x_2}T_{\xi,n_1+x_2-\xi}P(0,\xi)\right) \quad (x_2=0,1,\cdots n_2-1) \tag{5.31}$$

With these probabilities $p_2(x_2)$ the load

$$Y_2=\sum_{x_2=0}^{n_1}x_2p_2(x_2)\ , \tag{5.32}$$

carried in the secondary group can be evaluated easily. Furthermore the load

$$Y_1= Y_{tot}- Y_2\ , \tag{5.33}$$

carried in the primary group and the overflow traffic

$$R_1 = A - Y_1\ , \tag{5.34}$$

can be calculated.

Finally, the loss probability

$$B_1 = \frac{R_1}{A}\ , \tag{5.35}$$

of the primary group and the loss probability

$$B_2= \frac{R_2}{R_1}\ , \tag{5.36}$$

of the secondary group.

5.2.5 THE OVERFLOW TRAFFIC BEHIND A PRIMARY GROUP AND THE CORRESPONDING VARIANCE ACCORDING TO THE "LOST CALLS HELD"

The model "lost calls held" is based on the asumption that calls which cannot be switched in the primary group can always find a free trunk in the secondary group. Therefore this (assumed) secondary group must have

$$n_2 = q - n_1 , \tag{5.37}$$

trunks. Thus taking into account Eqn. (5.37), the overflow traffic for the model "lost calls held" can be calculated according to the Equations(5.20) and (5.23). The variance of this overflow traffic amounts to

$$V = \sum_{x_2=0}^{q-n_1} (x_2-y_2)^2 \; p_2(x_2) \; . \tag{5.38}$$

5.2.6 APPROXIMATE CALCULATION OF MEAN AND VARIANCE OF OVERFLOW TRAFFIC BEHIND ONE PRIMARY GROUP

For practical engineering purposes, the exact method derived earlier is sometimes too lengthy specially in case of overflow systems with a large number of trunks. Therefore, a simple approximation formula for the overflow traffic is developed in this section.

The traffic R_1 overflowing from a primary group as indicated in Fig. 5.2, can be calculated exactly according to the method derived in Section 5.2. The

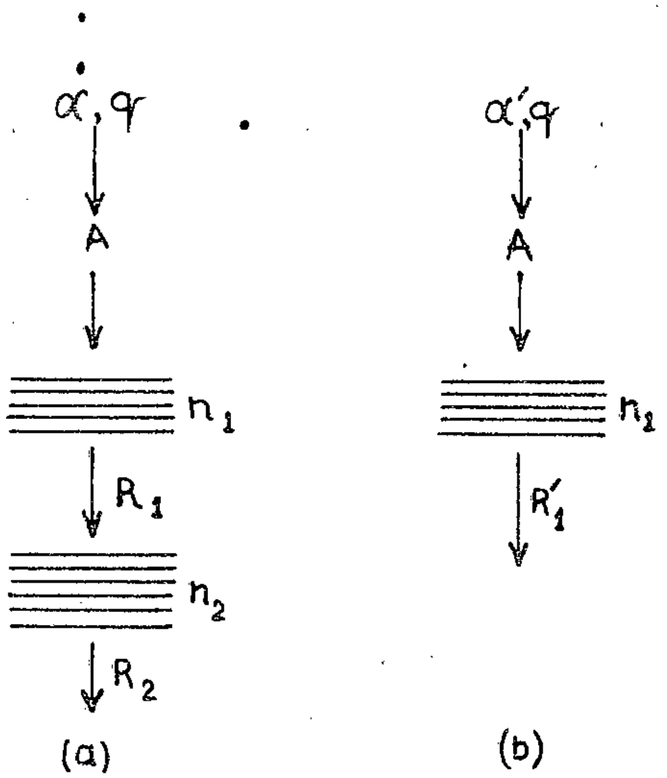


Fig.5.2 Approximate calculation of the overflow traffic R_1.

traffic A which is offered to the considered overflow system (shown in Fig.5.2a) can be calculated easily according to Eqns. (5.22),(5.25) and (5.26). For an approximate calculation of the overflow traffic R_1, a single group (as shown in Fig.5.2b) is considered which has the same number of trunks n_1 and the same number of sources q as the primary group shown in Fig. 5.2a. The sources of the single group have a calling rate α' which is chosen such that the offered traffic has the same mean A as the traffic offered to the overflow system.

$$A = \frac{\alpha}{1+\alpha}(q + R_2) = \frac{\alpha'}{1+\alpha'}(q+R_1') \tag{5.39}$$

For calculating the variance V_1 of an overflow traffic R_1 a group of n_1 trunks with offered Poisson traffic is considered. The mean A_v of this offered pure chance traffic (consideringg infinite sources)[12] is chosen such that the overflow traffic behind this group has the same mean $R_v = R_1$ as the actual overflow traffic in case of offered pure chance traffic considering finite sources. The variance corresponding to the overflow traffic R_v is denoted as V_v. If this variance V_v is multiplied by the factor $(q - A_v)/q$, an approximation

$$V_1' = V_v \frac{q - A_v}{q} \qquad (5.40)$$

for the actual variance V_1 is obtained. This formula yields values of high accuracy for calling rate upto about $\alpha = 1$.

5.3 AN ERT METHOD FOR SEVERAL PRIMARY GROUPS

5.3.1 GENERAL REMARKS

The exact calculation of overflow system with several primary groups leads to very large sets of equations. The numerical solution of these sets of equations is only possible in case of small systems even with the aid of very large digital computers.

Therefore, in this section an approximate method for the calculation of larger overflow systems with various primary groups is presented. In this method, offered overflow traffic is characterised by its first moment(mean) and its second moment (variance). Higher moments are not regarded. For systems with a large number of traffic sources (in the case of infinite sources), this method approaches the well-known equivalent random theory (ERT method)[1] which holds for an infinite number of sources.

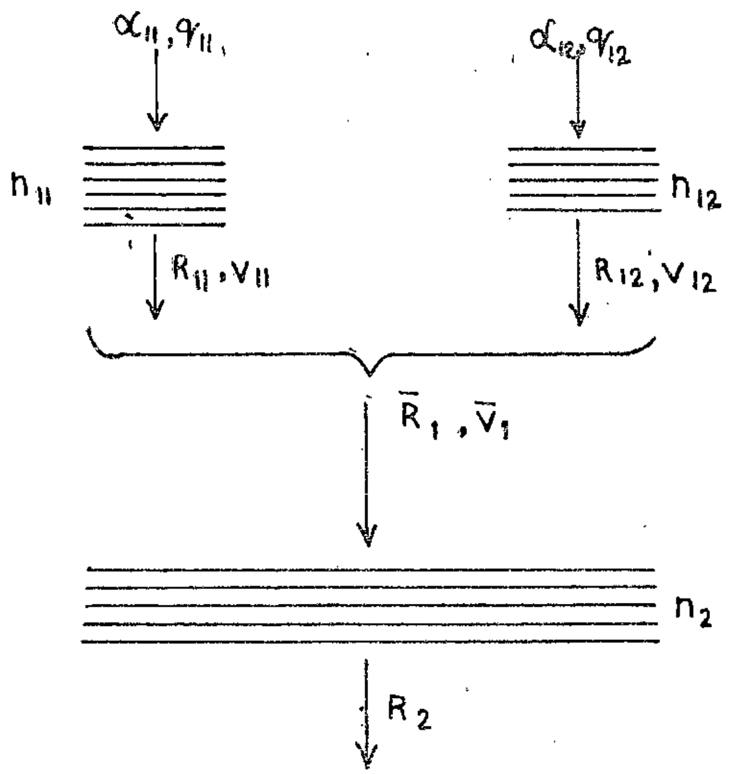


Fig.5.3 Overflow system with two primary groups.

In the new method, the ERT method is contained as a special (limiting) case. Therefore, the method presented here can be looked at as an "ERT method for finite source traffic".

For reasons of the finite number of sources, this method is a little more complicated that the ERT method, but the basic idea is the same. The various primary groups are replaced by one (fictitious) "equivalent primary group" which is chosen such that the overflow traffic offered to the secondary group has the same mean and variance as in case of the various (really existing) primary groups. Thus the calculation of overflow system with several primary groups is reduced to the case of only one primary group.

The approximation method shown here can be applied to overflow systems with an arbitrary number of primary groups. For reasons of simplicity, however, the following description of the various calculation steps of this method refers to an overflow system with only two primary groups (as shown in Fig.5.3).

5.3.2 MEAN AND VARIANCE OF THE OVERFLOWING TRAFFIC RESTS

As a first step the overflow traffic and the corresponding variances are determined. The variance of an overflow traffic is defined only if this overflow traffic is offered to a secondary group with zero loss probability. Thus it is necessary to determine at first the traffic R_{11} which would overflow to a separate secondary group with zero loss probability (i.e. with $\widetilde{n}_{21} = q_{11} - n_{11}$ trunks as indicated in Fig. 5.4) and the variance $\widetilde{V}_{11}$ corresponding to this overflow traffic $\widetilde{R}_{11}$. The overflow traffic $\widetilde{R}_{11}$ and the corresponding variance $\widetilde{V}_{11}$ can be calculated by means of the exact method derived in Section 5.2 or according to the approximation formula developed in Section 5.2.6 .

Analogously the overflow traffic $\widetilde{R}_{12}$, which is overflowing from primary group No. 2 to a secondary group with $\widetilde{n}_{22} = q_{12} - n_{12}$ trunks (as shown in Fig.5.4), and the corresponding variance $\widetilde{V}_{12}$ is determined.

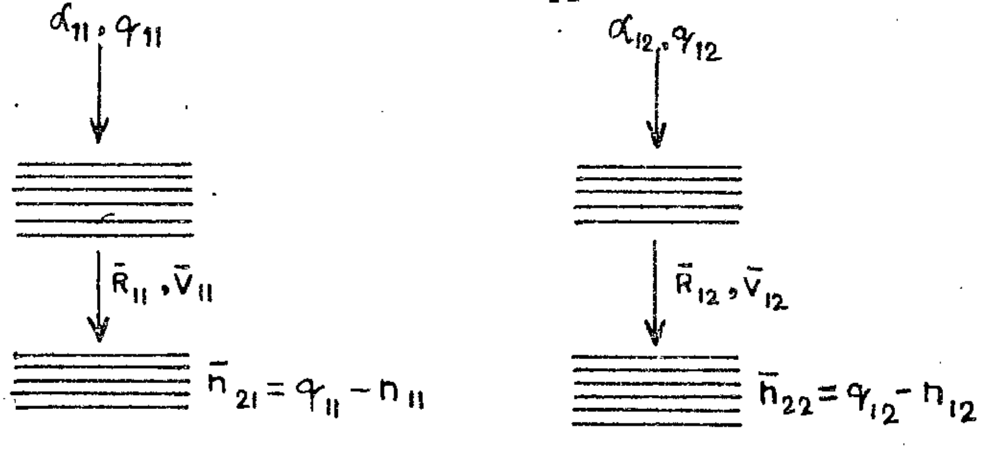


Fig.5.4 Mean and variance of the overflow traffics.

5.3.3 THE TOTAL OVERFLOW TRAFFIC

In a second step an overflow system as shown in Fig. 5.5 is considered. Here the overflot traffic

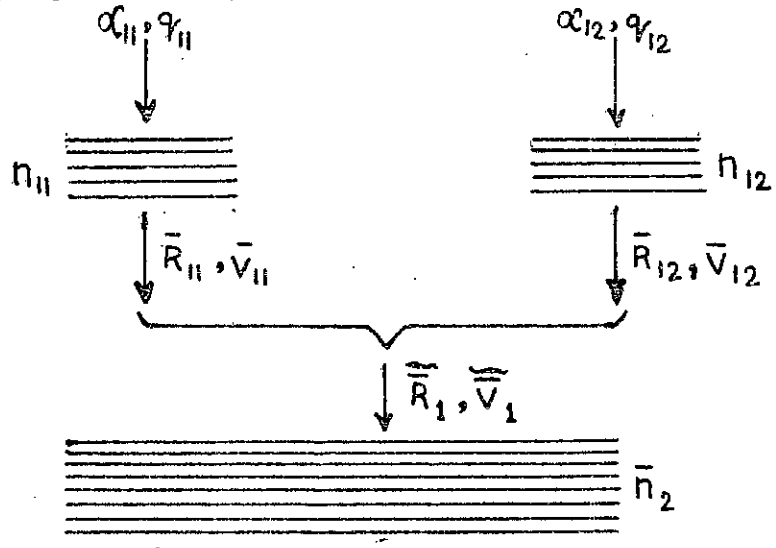


Fig. 5.5 The total overflow traffic.

R_{11} and the overflow traffic R_{12} are offered to a common secondary group with

$$\tilde{n}_2 = \tilde{n}_{21}+\tilde{n}_{22} = q_{11}+q_{12}-n_{11}-n_{12} \qquad (5.41)$$

trunks. As there is no loss in this common secondary group, $\tilde{R}_{11}$ and $\tilde{R}_{12}$ are independent of each other. Thus the total overflow traffic $\tilde{\bar{R}}_1$, which is offered to the secondary group, equals the sum of $\tilde{R}_{11}$ and $\tilde{R}_{12}$

$$\tilde{\bar{R}}_1 = \tilde{R}_{11} + \tilde{R}_{12} \qquad (5.42)$$

and the variance $\tilde{\bar{V}}_1$ of this total overflow traffic is given by the sum of the variances $\tilde{V}_{11}$ and $\tilde{V}_{12}$

$$\tilde{\bar{V}}_1 = \tilde{V}_{11}+\tilde{V}_{12} \qquad (5.43)$$

5.3.4 THE EQUIVALENT PRIMARY GROUP

Now the various, actually existing primary groups are replaced by one (fictitious) "equivalent primary group", as shown in Fig. 5.6. This equivalent group (with n^* trunks and q^* sources of calling rate α^*) is determined such that the traffic overflowing to the

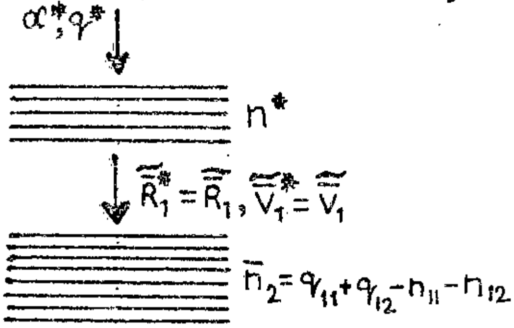


Fig. 5.6 The equivalent primary group.

secondary group (with $\tilde{n}_2$ trunks) has the same mean $\tilde{\bar{R}}_1^* = \tilde{\bar{R}}_1$ and the same variance $\tilde{\bar{V}}_1^* = \tilde{\bar{V}}_1$ as the total overflow traffic obtained in Section 5.3.3 .

The values (n^*, q^*, α^*) of the equivalent primary group can be determined in the following way.

Starting with an estimated number of trunks n^*, the number of sources q^* can be calculated according to the equation

$$q^* = n^* + \tilde{n}_2 \qquad (5.44)$$

(because the secondary group has zero loss probability). Then the calling rate α^* is (iteratively) determined such that the mean $\tilde{\bar{R}}_1^*$ of the overflow traffic equals the prescribed value R_1^*. The variance V_1^* which is corresponding to this overflow traffic R_1^* will, however, not have the prescribed value V_1. Therefore, in a further iteration the number of trunks n^* is determined such that the variance has the prescribed value $V_1^* = V_1$.

5.3.5 THE LOSS PROBABILITY IN THE SECONDARY GROUP

In the overflow system with two (or more) primary groups (as shown in Fig.5.5) as well as in the overflow

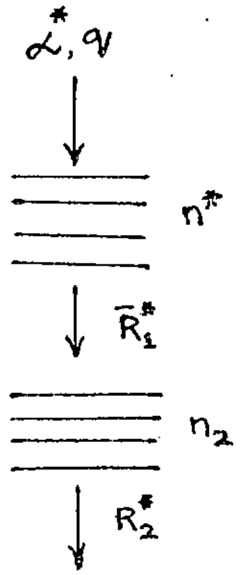


Fig. 5.7. Calculation of the loss probability in the secondary group.

system with only one equivalent primary group (as shown in Fig. 5.6) an overflow traffic with the mean $R_1^* = R_1$ and with the variance $V_1^* = V_1$ is offered to a secondary group with n_2 trunks and zero loss probability.

If this (fictitious) number of trunks n_2 (as shown in Fig.5.6) is reduced to the actual number of trunks n_2 of the secondary group (as shown in Fig. 5.7), an overflow traffic $R_2^* > 0$ arises behind the secondary group. This overflow traffic R_2^* is an approximation for the actual overflow traffic R_2 which is indicated in Figure 5.3. Similarly, the overflow traffic R_1^* (indicated in Fig.5.7) is an approximation for the actual overflow traffic R_1 which is offered to the secondary group. With these values, the loss probability B_2 in the secondary group can be calculated

$$B_2 \approx B_{22}^* = \frac{R_2^*}{R_1^*} \tag{5.45}$$

5.3.6 OFFERED TRAFFIC VALUES AND CARRIED LOADS

The offered traffic values of the various primary groups can be determined under the assumption that the loss probability B_{21} of the overflow traffic R_{11} (with respect to the secondary group) and the loss probability B_{22} of the overflow traffic R_{12} are equal.

$$B_{21} = B_{22} = B_2 \tag{5.46}$$

and that the proportion of the overflow traffic values R_{11} and R_{12} is the same as in the case of a secondary group with zero loss probability

$$\frac{R_{11}}{R_{12}} = \frac{\widetilde{R}_{11}}{\widehat{R}_{12}} \tag{5.47}$$

Then the following overflow traffic values are obtained

$$R_{11} = \frac{\widetilde{R}_{11}}{\widehat{R}_1} \bar{R}_1 , \tag{5.48}$$

$$R_{12} = \frac{\widetilde{R}_{12}}{\widetilde{R}_1} R_1 , \tag{5.49}$$

$$R_{21} = B_2 R_{11} = R_2 \frac{\widetilde{R}_{11}}{\widehat{R}_1} , \tag{5.50}$$

$$R_{22} = B_2 R_{12} = R_2 \frac{\widehat{R}_{12}}{\widehat{R}_1} , \tag{5.51}$$

Now the offered traffic A_{11} can be determined in analogy to Eqn.5.26

$$A_{11} = \frac{\alpha_{11}}{1+\quad_{12}} (q_{11} + R_{21}). \tag{5.52}$$

and similarly

$$A_{12} = \frac{\alpha_{12}}{1+\alpha_{12}} (q_{12} + R_{22}). \tag{5.53}$$

Finally the carried loads

$$Y_{11} = A_{11} - R_{11}, \qquad (5.54)$$

and

$$Y_{12} = A_{12} - R_{12}, \qquad (5.55)$$

in the primary groups and the load

$$Y_2 = \overline{R}_1 - R_2, \qquad (5.56)$$

carried in the secondary group can be calculated.

CHAPTER 6

SIMULATION TECHNIQUE OF ANALYSIS

6.1 INTRODUCTION

In most practical cases analytical solution of the traffic problems is difficult due to the complexity involved. For example, in the case of calculation of overflow systems with several primary groups the problem leads to large sets of equations. The numerical solution of these equations is only possible in case of small systems even with a large digital computer. For some large and complex system numerical solution are beyond the power of a large digital computers. In such cases a number of simulation can serve the purpose. Besides these, testing the performance of a complex connection system can thus be done without going to actual measurement.

6.2 GENERAL CONCEPT OF SIMULATION.

The general concept of simulation can be understood with the help of the following example.[1] There are two groups of sources (A and B), producing Poisson flows of demands with rates λ_a and λ_b, respectively. There are n servers. Demands from group B are allowed to seize any free server; those demands are lost for which no free server is available on their arrival. Demands stemming from group A cannot seize a server, unless they leave at least m(>0) servers unoccupied. Once the processing of a group A demand begins, however, this action is completed. Contrary to group B demands the demands of group A have a queueing facility. One can think of the following interpretation. Let the demands of group A and B be patients to be taken into a hospital. Group B are emergency cases that may "seize" any free bed; in the case where no bed is available the patient goes to another hospital (i.e. is "lost" for the hospital under consideration). Group A are patients on a waiting list (i.e. in a queue). Their admission to hospital is made possible only in the case where m beds are still left free for emergency cases. The model is completed by stating that the cumulative distribution function of holding-times for group A and B are $F_a(t)$ and $F_b(t)$, respectively. It is required that we determine the fraction of group B demands that is lost and the average waiting-time for group A demands. Needless to say this is a gross oversimplification of the situation at hand, but it is a good example.

6.3 REQUIREMENTS IN SIMULATION.

It is necessary to develop techniques for simulating traffic under controlled conditions. The main requirements for simulation are

First, a model of the system in sufficient detail to represent all states which are relevant to the investigation, and the transitions from one state to another.

Secondly, a means of generating the events which may alter the state of the system.

Thirdly, a set of rules describing how the system behaves in the case of any given event or combination of events. In general, the events and the rules may be either probabilistic or deterministic. In telephone traffic simulation, the events are usually probabilistic being dependent on the design of the system.

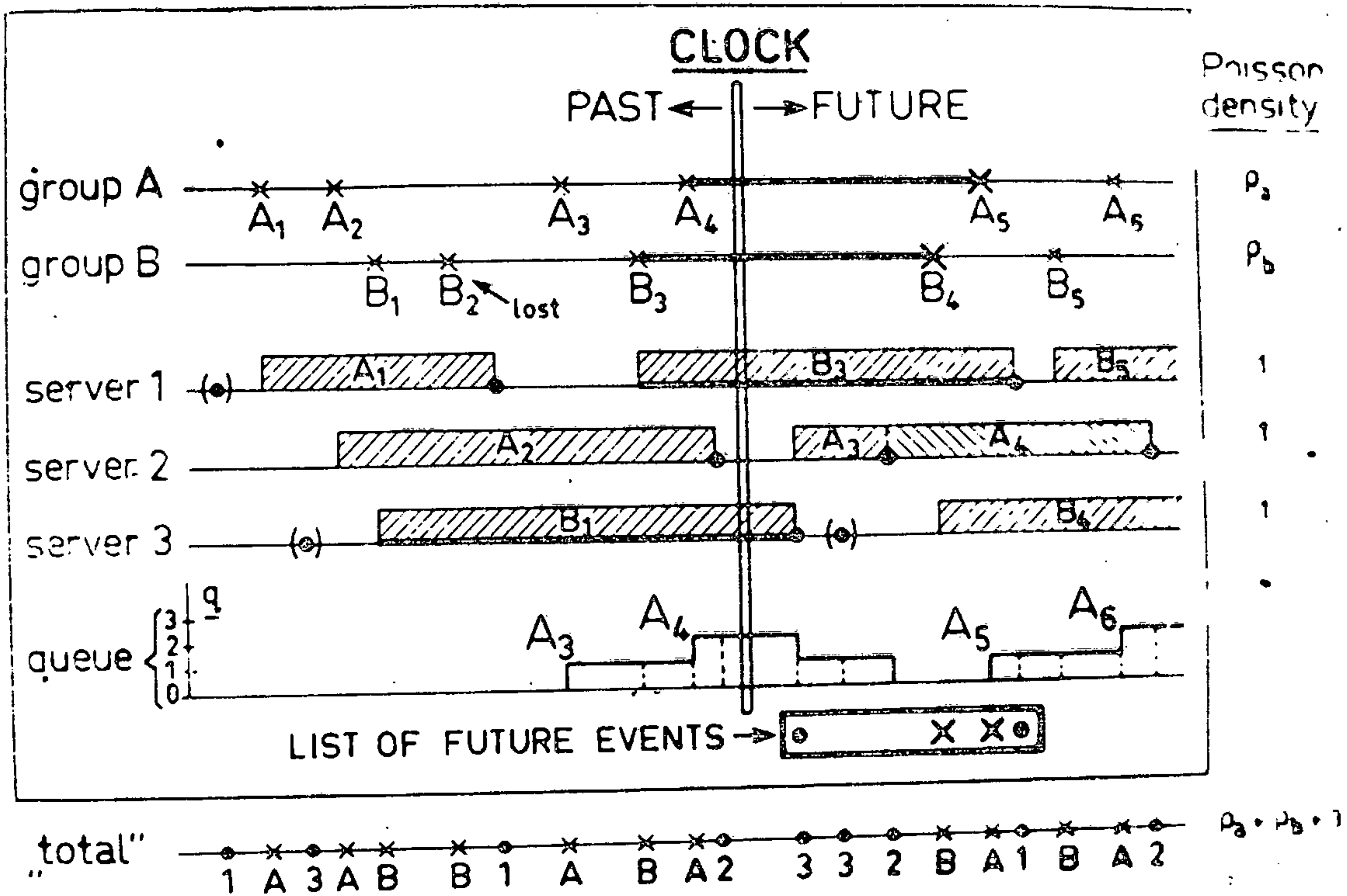


FIG. 6.1 Realization of the process of arrivals and occupations.

6.3.1 TYPES OF SIMULATION

Let it be considered that the stochastic process governing the flow of demands and the occupation of servers and of the queue. One realization of such a process — i.e. one possible development of events in the system — is depicted in Fig.6.1 for the case discussed in Section 6.2. Simulation is a way of reconstructing such a realization. There are mainly two types of simulation.

a) Time-true simulation :

The diagram of fig. 6.1 consists of lapses of time (holding-times and interarrival-times) that are "tied together" in instants, to be called events. The events here are arrivals and ends of occupations. The state of the system specifies which of the servers are busy at a certain moment as well as the number of items in the queue. The state can change at events only. The way in which the state changes at events is determined by (i) the type of event and (ii) the configuration of the facilities (servers and queues). Hence, the only stochastic elements are the holding- and interarrival-times. Now, in time-true simulation their durations will be determined at their beginning by sampling from the prescribed distributions. The process of reconstruction develops according to a fictitious time, called CLOCK (Fig. 6.1). It separates the PAST from the FUTURE. At

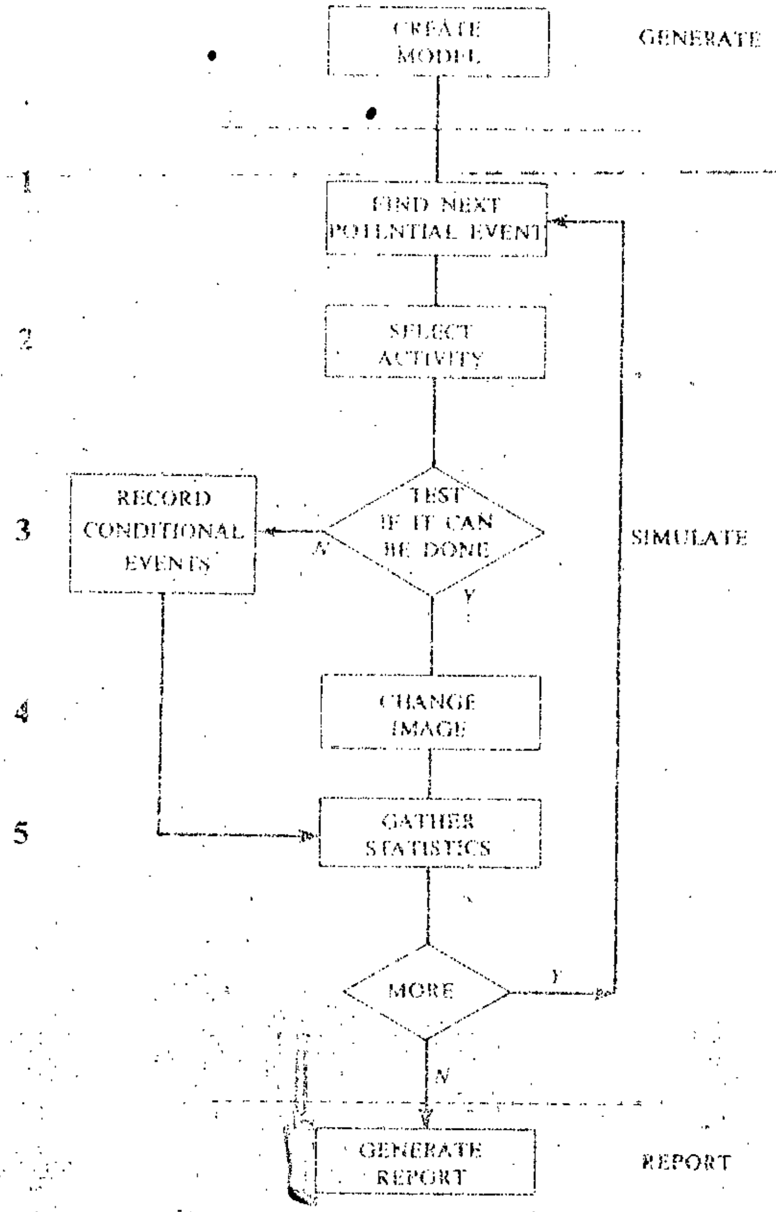


Fig. 6.2 Execution of a simulation algorithm

any CLOCK-time the state is supposed to be known as well as the endpoints of interarrival- and holding-time that at CLOCK have not yet completely ended (together called "current intervals", denoted by thick segments). At the beginning (CLOCK-time zero) this is satisfied, for example, by an empty system and known first arrivals in both groups. Nothing happens until CLOCK meets the earliest of there endpoints, the next event. The type of this event, the state and the configuration then determine the change of state, if any, at this event. Any such a change may entail the start of one or more new current intervals, the lengths of which are immediately determined by sampling. The ends of those new current intervals mark possible new events. Hence, every event that is passed by CLOCK, may generate new future events. Mostly, a chronological LIST OF FUTURE EVENTS is constructed. The future events, created at the passing of an event by CLOCK, have to be inserted on this list in the right chronological place. In Time-true simulation one can count the demands in group B as well as those that were lost. A simple division of those totals yields an estimate of the probability of loss in group B. The denominator (the total of all demands) can be replaced by its (known) expected value. As all durations between consecutive events are also known, it is relatively easy

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to keep a record of the total waiting-time of group A demands during a simulation run (the area under the q-line in Fig. 6.1). Dividing this total by the total of A demands (either observed or expected) yields an estimate for the expected waiting-time of group A demands. From waiting-times of individual demands— which can be obtained with slightly more effort— a histogram of waiting-times can be constructed. This then offers an approximation to the distribution of waiting-times.

In this thesis work time-true simulation run has been applied to find different parameters of overflow traffic.

The time-true simulation has three disadvantages:

(i) the implementation of the sampling from given distributions may be difficult and time-consuming;

(ii) when the system is large, much information about holding- and interarrival-times (the current intervals) should be memorized;

(iii) When the system is large, the filing of future events on their chronological list may be cumbersome or time-consuming.

b) Roulette simulation does not possess the disadvantages mentioned in the case of time-true simulation. In Fig. 6.1 the arriving demands in groups A and B are given by Poisson point-processes (the crosses). Now let the holding-times be exponentially distributed with average 1. Consequently, the arrival rates will be given in normed notation: ρ_a and ρ_b: respectively. Not only the interarrival-times, but also the holding-times, will now possess the property of forgetfulness. A statistically true picture of holding-times may be obtained by "chopping them off" by unit density Poisson point-processes: the dots on server-occupation lines in Fig. 6.1 When such breakdown points occur at instances at which the associated server is not occupied (dots between brakets) nothing happens. The five Poisson point-processes (two arrival processes and three breakdown processes) may be merged into one Poisson point-processes, called "total". Its density is $\rho_a + \rho_b + 3$. In each point of this "total" process the class of the point (A,B,1,2, or 3) may be obtained by drawing lots with probabilities proportional to $\rho_a : \rho_b : 1 : 1 : 1$, respectively. Let it be considered that the class indices of the consecutive points of the "total" process (marks A,B, 1,2 or 3 on the "total" line) is known. It is evident that one is able then to reconstruct the realization of the complete stochastic process in the system as far as the sequence of events is concerned ! The concept "time" disappears. What remains is a so-called sequence-true resume (Roulette simulation) of the realization. The class indices in question can be determined by the use of a roulette with positions marked A,B,1,2 and 3 in the correct proportion $\rho_a : \rho_b : 1 : 1 : 1$. In a general case the roulette should possess the following sets of differently marked equiprobable positions :

(i) breakdown positions: one set per server; the numbers of positions in those sets are equal;

(ii) build-up positions: one set per group of sources; when ρ is the arrival rate of such a group, the corresponding set of build-up positions should consist of ρ times as many positions as the sets in (i).

When the roulette stops at a breakdown position associated with a non-occupied server, this roulette-point simply is ineffective. The roulette simulation ceases to be applicable when the concept 'time' enters as an explicit variable. It is impossible, for example, to deal with the probability of a delay in excess of a certain prescribed duration. Furthermore, cases where the system's behaviour depends on some holding- or waiting-time exceeding some value are not covered by the roulette simulation principle. The roulette model does not need a list of future events, so there are no filing difficulties. There is no need for sampling from arbitrary distributions. Simple (pseudo-) random numbers are sufficient.

6.3.2 METHODS OF PSEUDO-RANDOM NUMBER GENERATION

In simulation the instants of call origination is decided first. Random numbers are generated and the start of a call is allocated to each interval corresponding to a random number. In current practice random numbers are replaced by pseudo-random numbers generated by mathematical formula. Usually pseudo-random numbers are taken to be non negative. The main conditions to be satisfied by a pseudo-random number generator are

(i) a large period

(ii) uniformity of distribution

(iii) freedom from correlation of successive outcomes.

The pseudo-random number generators that have found widespread use are :

(i) The mixed congruence generator:

This type of generator is defined by

$$x_n = ax_{n-1} + c(\text{mod } M) \quad (a,c>0).$$

The former number x_{n-1} is multiplied by a single length number a, a constant c is added and the "tail of the double length result is taken as x_n. The generator described has the greatest advantage that under favourable conditions the period is maximal (M), thus ensuring uniformity. In this thesis work the above method of generation has been used .

(ii) The Fibonacci generator:

This type of generator is defined by

$$x_n = x_{n-1} + x_{n-2} \ (\text{mod. } M)$$

This generator is very fast, as the operation of addition may be very much quicker on a computer than multiplication. Uniformity is met reasonably well.

6.3.3 CHOICE OF A TIME UNIT FOR THE NEGATIVE EXPONENTIAL PROCEDURE.

Call generation and call release are represented by random events generated by using a negative exponential distribution with parameter λ equal to the interarrival time and mean holding time respectively. In this work the mean holding time is considered to be 180 seconds and the interarrival time = (mean holding time/traffic).

A set of random numbers

$$\left\{u/v : 1 \leq u \leq w-1\right\}$$

When $w=2^{31}-1 = 2147483647$ and having fairly uniform distribution is generated by a linear congruence method producing the next value of u in the pseudo-random number sequence from the present value of u. With appropriate λ, the mean of the negative exponential distribution, the output

$$-\lambda \ln(u/w)$$

is rounded to the nearest integer which is a necessity in the simulation programme. This rounding, besides introducing error in the expected mean and variance, sets practical limitations to the value of λ. This is clear from the analysis below :

If $-\lambda \ln(u/w)$ is rounded to the nearest integer then

$$\left| -\lambda \ln(u/w) < 0.5 \right| = 0$$

So the output gives zero with a probability

$$\Pr\left\{-\lambda \ln(u/w) < 0.5\right\}$$
$$= 1-e^{-1/2\lambda}$$
$$\approx 1/2 \quad \text{for large value of } \lambda$$

for = 1000, approximately 1 in 2000 number produced will be zero which is not desirable. To avoid the production of a zero completely, λ needs to be more than 2^{30} when $|-\lambda \ln u/w|$ is too large for the computer maximum integer storage 8388607. For the generation of interarrival time and holding time in this work it is recommended that λ should be near albout $10{,}000^9$. To get this requirement fulfilled a time unit of 1000 is chosen in this work.

CHAPTER 7

RESULTS

7.1 GRAPHIC REPRESENTATION OF RESULTS

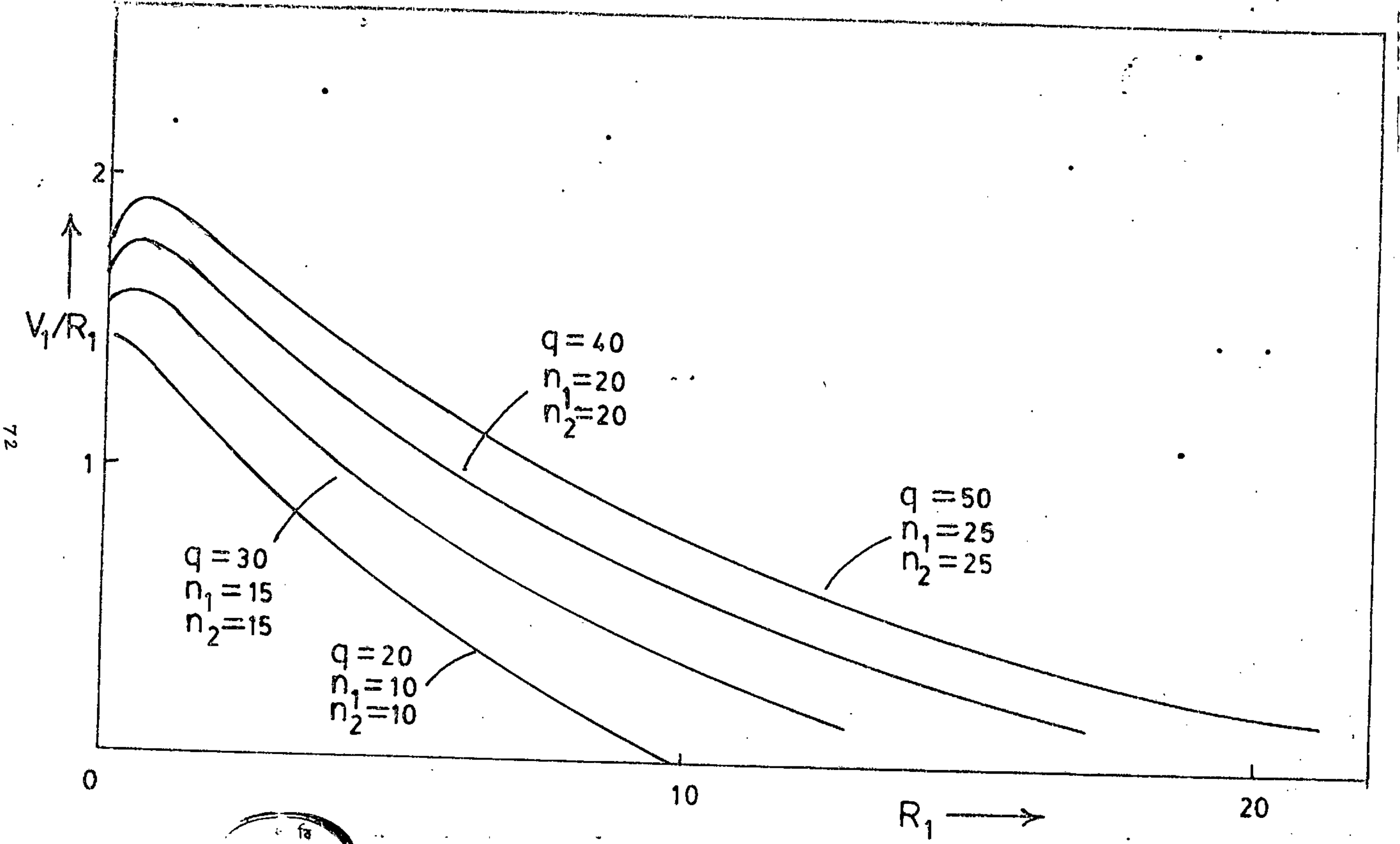


Fig. 7.i. The varianse to mean ratio of overflow traffic as a function of the mean.

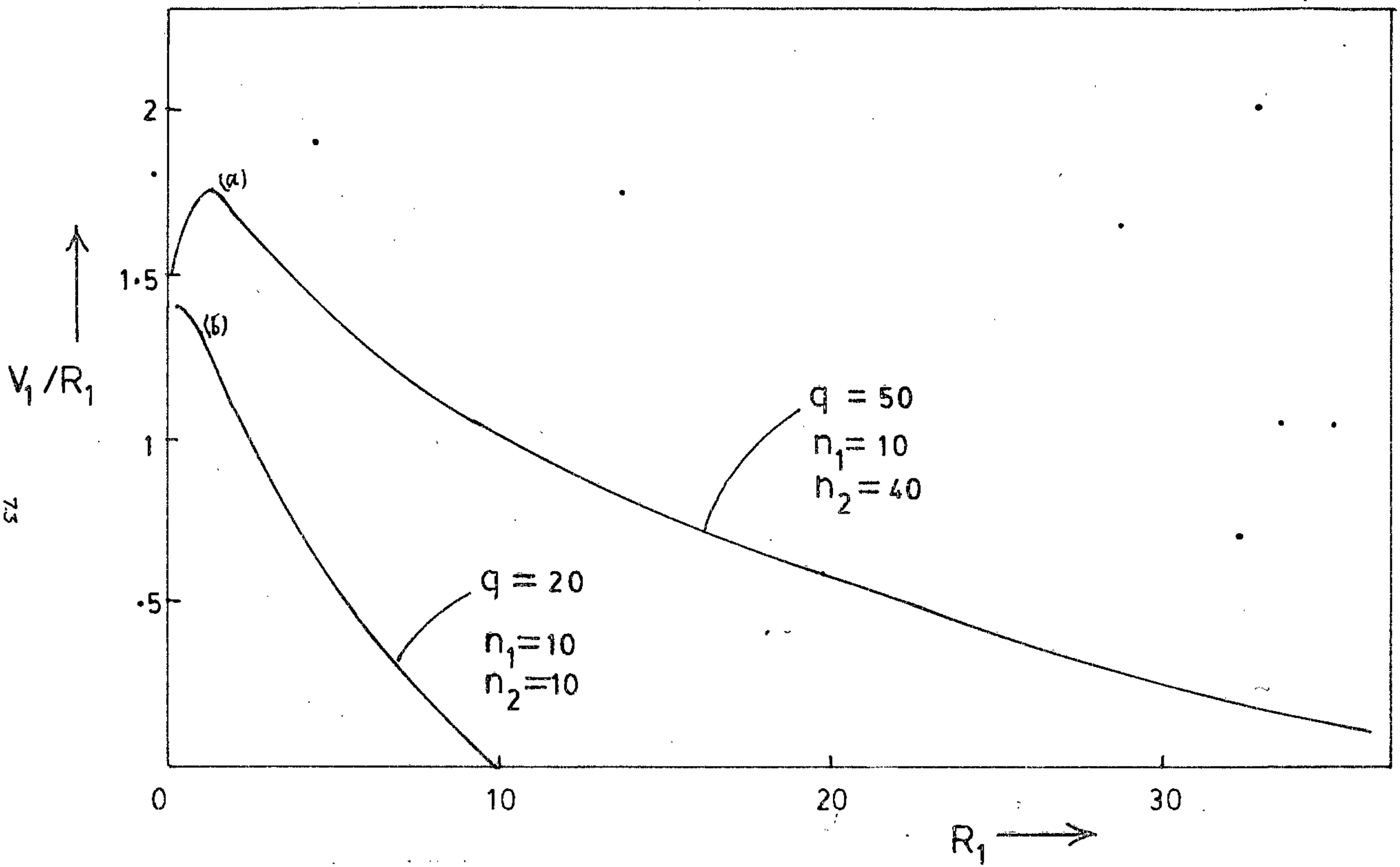


Fig.7.2.The variance to mean ratio of overflow traffic as a function of the mean.

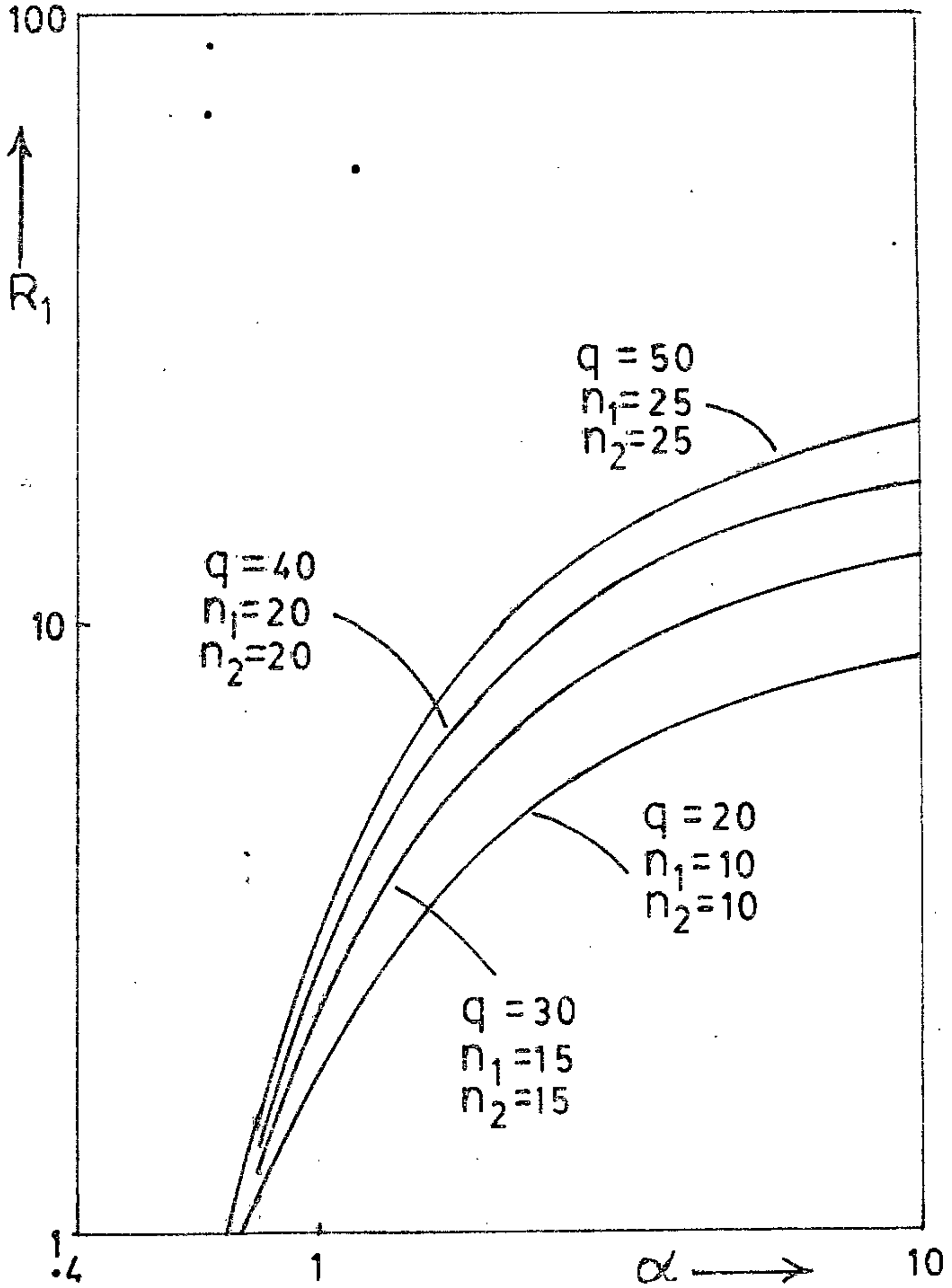


Fig. 7.3. The mean of overflow traffic as a function of the calling rate.

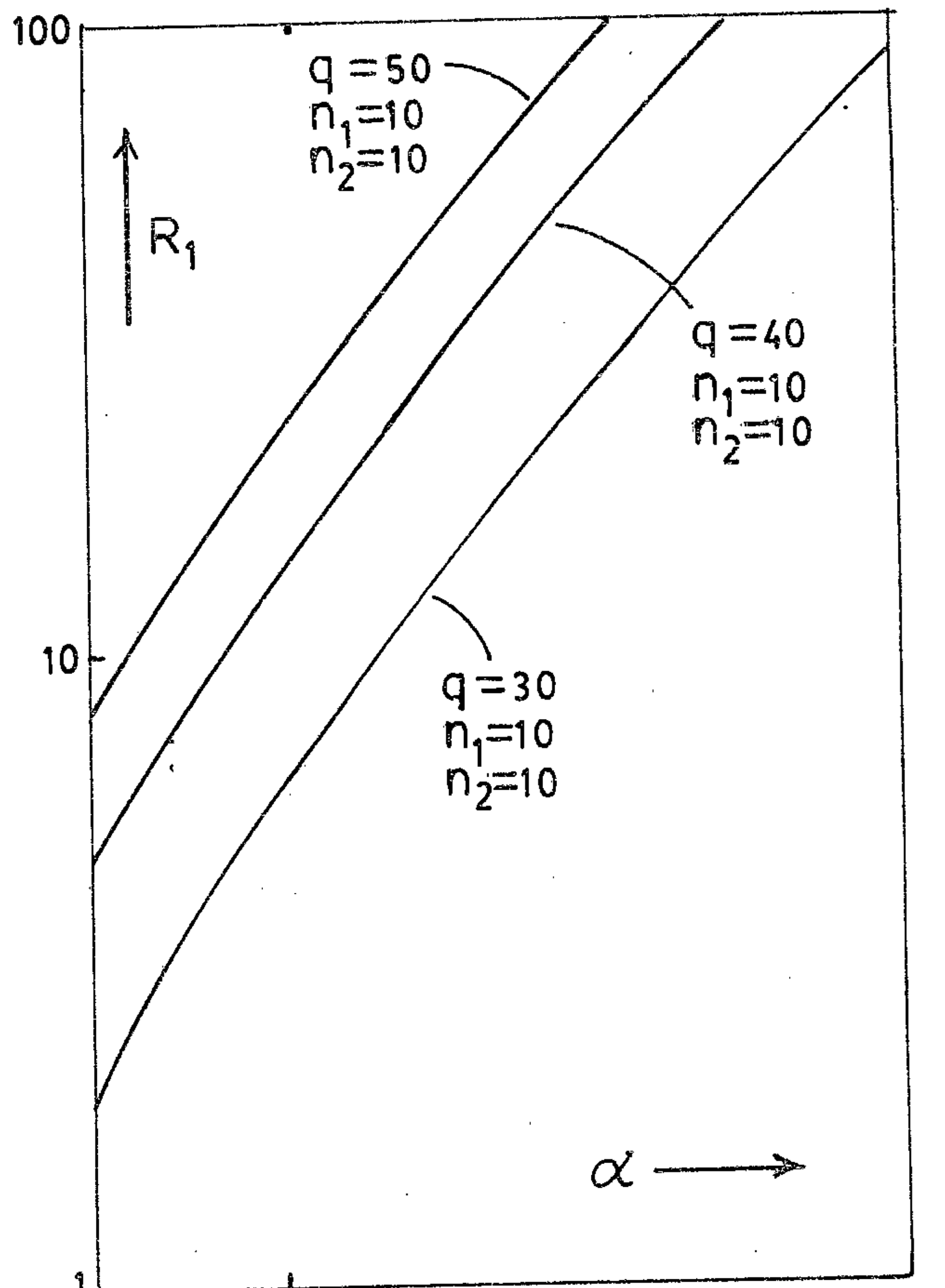


Fig.7.4.The mean of overflow traffic as a function of the calling rate.

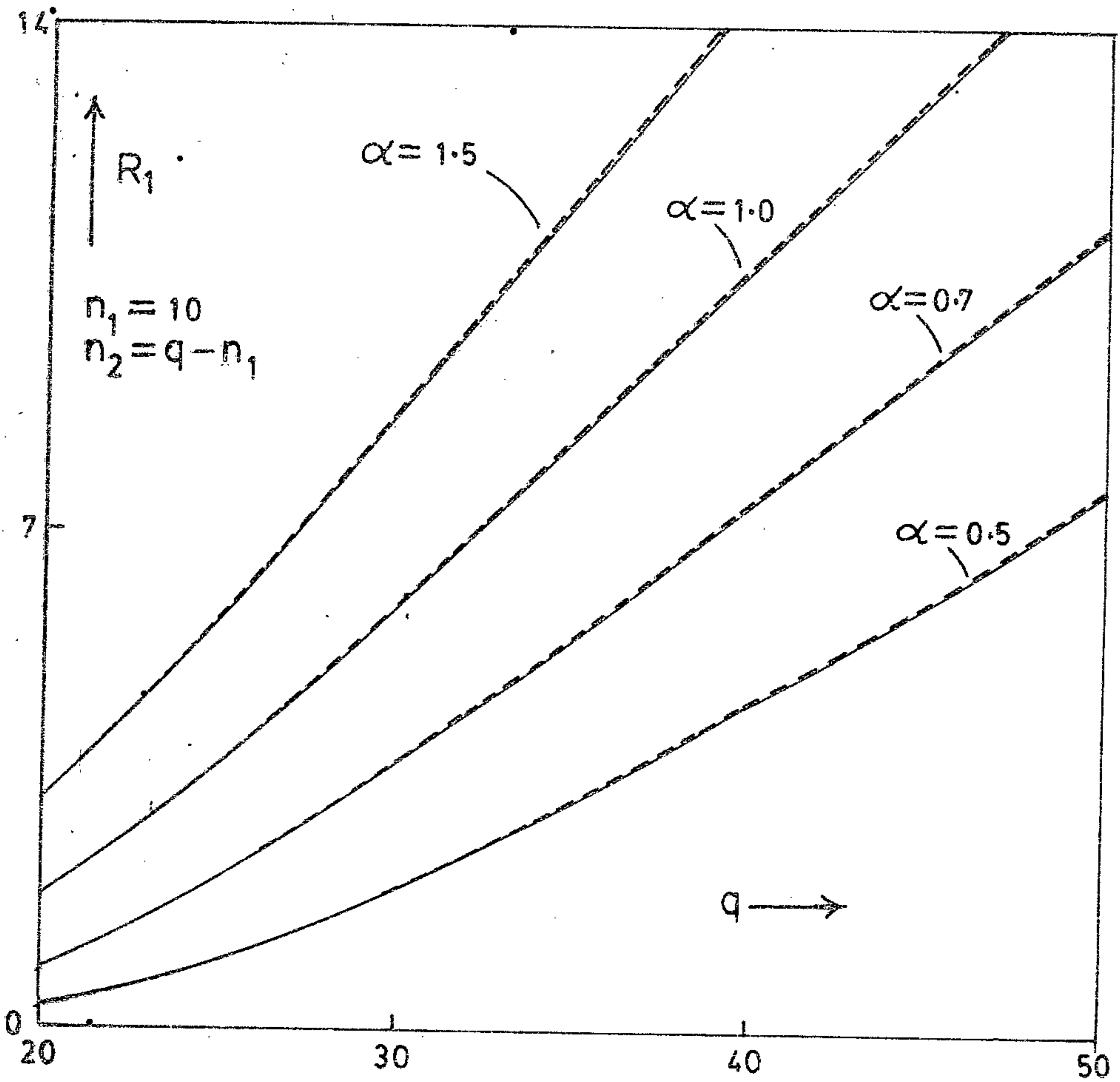


Fig.7.5. The overflow traffic as a function of number of sources(—— Exact values R_1,---- Approximate values R_1').

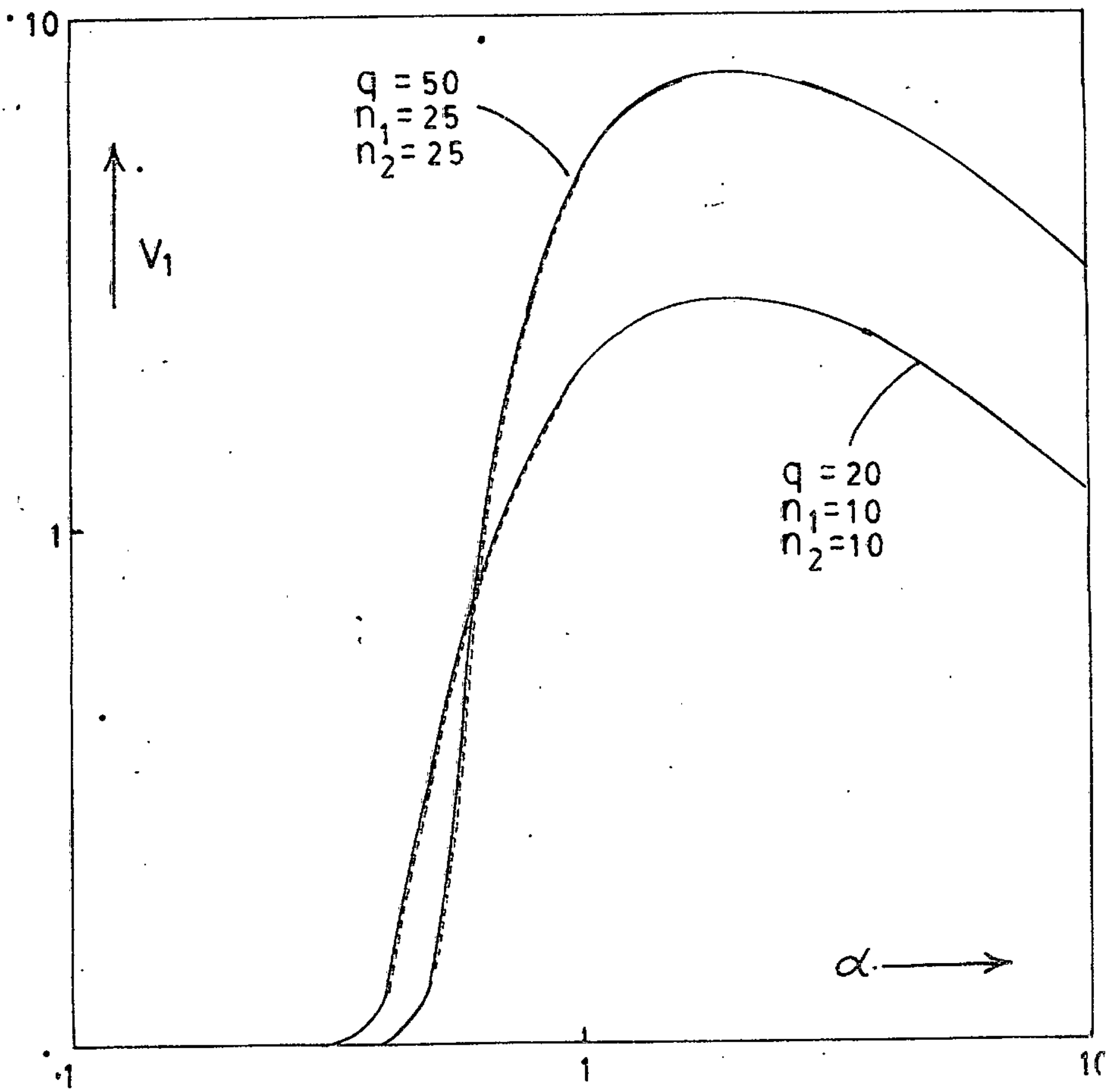


Fig.7.6.The variance as a function of the calling rate
(——— Exact values V_1, ----- approximate V_1').

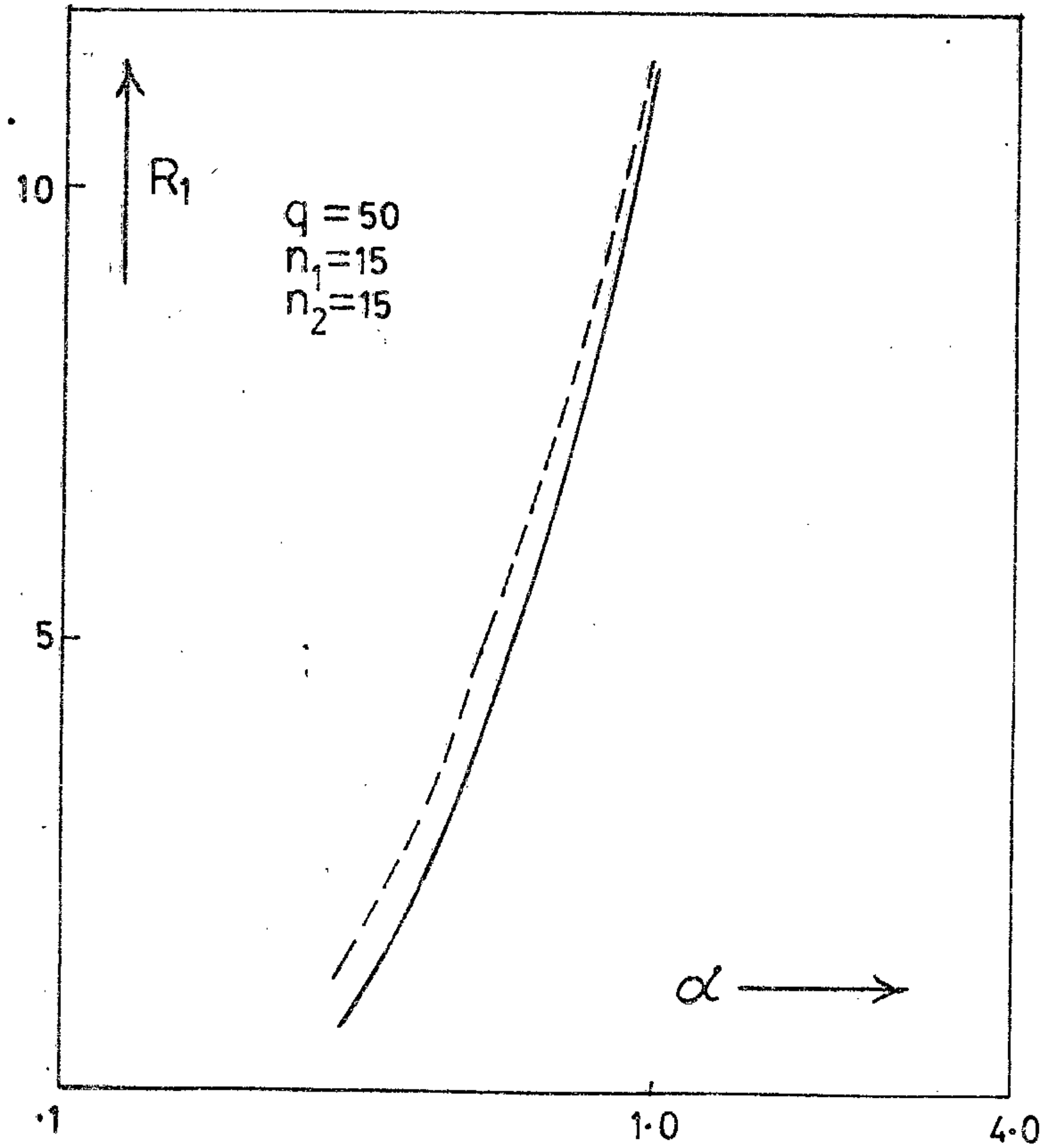


Fig. 7.7. The mean as as function of the calling rate
(——— Exact values , ----- Simulation values)

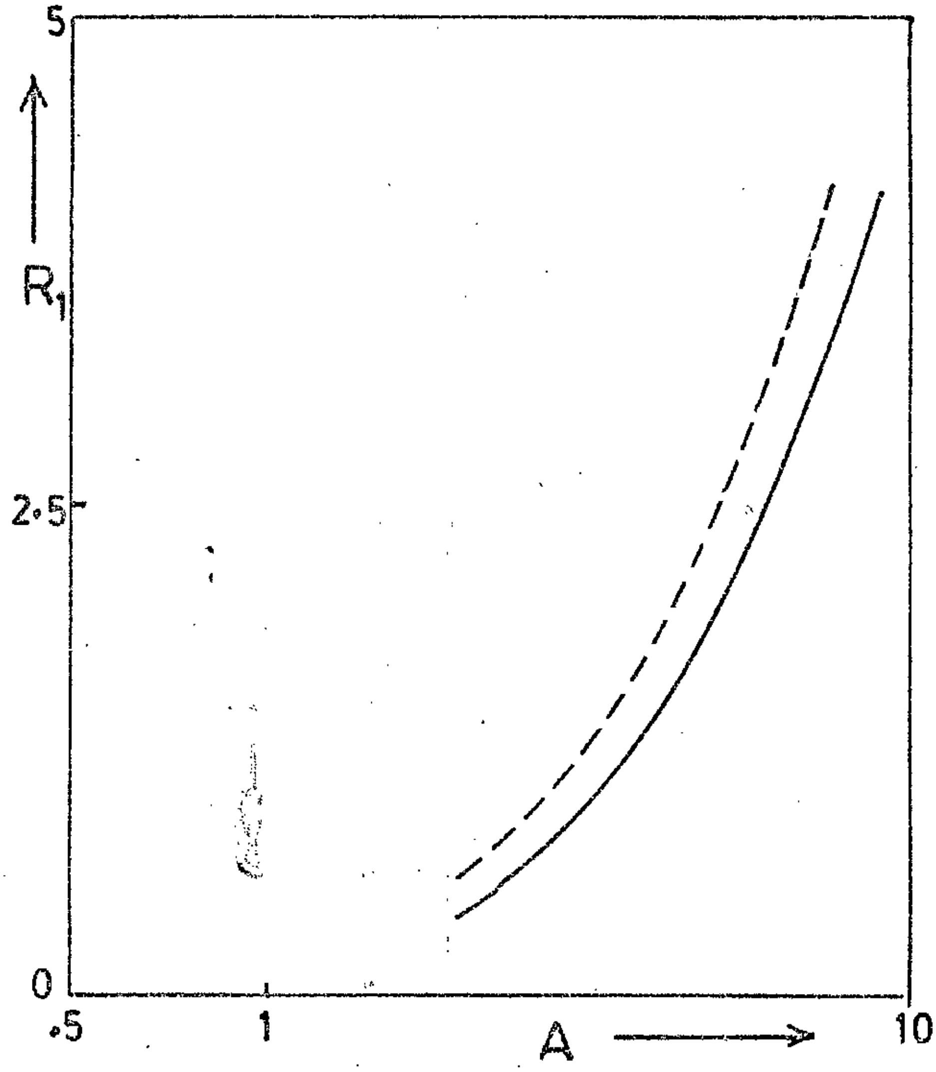


Fig. 7.8. The mean as a function of the offered traffic
(------ Approximation, ——— Simulation) for
the case of two primary groups.

7.2 OVERFLOW TABLES

Finite number of traffic sources

full availability

Symbols used

ALPHA	calling rate
q	Number of sources
n	Number of primary trunks

The top numbers in each row indicates the value of mean while the bottom numbers in each row is the value of variance.

VARIATION OF MEAN TRAFFIC R1 AND VARIANCE V FOR DIFFERENT
CALLING RATE ALPHA AND DIFFERENT SWITCH DIMENSI ON
NUMBER OF SOURCES Q=17

n

ALPHA	1	2	3	4	5	6	7	8	9	10
0.10	0.92555	0.46857	0.19045	0.05981	0.01433	0.00264	0.00038	0.00004	0.00000	0.00000
	0.99151	0.56119	0.23926	0.07469	0.01730	0.00307	0.00043	0.00005	0.00000	0.00000
0.20	2.08010	1.41542	0.86800	0.45294	0.20661	0.07459	0.02146	0.00487	0.00087	0.00012
	2.00651	1.55793	1.06178	0.60555	0.27630	0.09820	0.02712	0.00588	0.00101	0.00014
0.30	3.11251	2.35852	1.68025	1.10102	0.64455	0.32551	0.13684	0.04654	0.01258	0.00267
	2.72221	2.33773	1.86984	1.35122	0.84822	0.44280	0.18523	0.06088	0.01568	0.00317
0.40	4.01473	3.21293	2.46465	1.78680	1.20016	0.72700	0.38390	0.17026	0.06131	0.01748
	3.21725	2.89037	2.48270	2.00050	1.47203	0.95423	0.52211	0.23139	0.08069	0.02191
0.50	4.80395	3.97271	3.18219	2.44473	1.77632	1.19673	0.72744	0.38519	0.17073	0.06098
	3.55754	3.27494	2.92137	2.49346	1.99933	1.46701	0.95035	0.52013	0.22924	0.07921
0.60	5.49821	4.64707	3.82864	3.05224	2.33020	1.67850	1.11583	0.66643	0.34320	0.14597
	3.78825	3.53943	3.22931	2.85144	2.40507	1.90114	1.36999	0.86623	0.45384	0.19089
0.70	6.11286	5.24740	4.40924	3.60572	2.84659	2.14467	1.51658	0.98261	0.56395	0.27442
	3.94031	3.71770	3.44195	3.10592	2.70526	2.24218	1.73139	1.20766	0.71456	0.34959
0.80	6.66045	5.78420	4.93138	4.10801	3.32198	2.58376	1.90715	1.31000	0.81366	0.43893
	4.03520	3.83336	3.58503	3.28325	2.92236	2.50033	2.02289	1.50987	0.97145	0.52943
0.90	7.15120	6.26649	5.40239	4.56388	3.75757	2.99215	2.27918	1.63398	1.07615	0.62828
	4.08813	3.90311	3.67699	3.40324	3.07598	2.69119	2.24962	1.76150	1.20173	0.70401
1.00	7.59345	6.70193	5.82884	4.97841	4.15631	3.36987	2.62391	1.94655	1.34020	0.83170
	4.11010	3.93891	3.73106	3.48048	3.18149	2.82923	2.42180	1.96351	1.39827	0.85856
1.10	7.99391	7.09678	6.21638	5.35634	4.52161	3.71848	2.95543	2.24379	1.59984	1.04069
	4.10904	3.94942	3.75685	3.52562	3.25947	2.92628	2.54977	2.12172	1.56233	0.99020
1.20	8.35825	7.45640	6.56994	5.70196	4.85689	4.04023	3.25936	2.52418	1.84811	1.24932
	4.09091	3.94106	3.76144	3.54655	3.29165	2.99165	2.64257	2.24311	1.69805	1.10256
1.30	8.69112	7.78522	6.89366	6.01900	5.16533	4.33747	3.54195	2.78750	2.08598	1.45370
	4.06009	3.91862	3.75012	3.54917	3.31163	3.03251	2.70757	2.33433	1.80975	1.20073

4.01989	3.86555	3.72553	3.55789	[illegible]	3.05435	[illegible]	2.40107	[illegible]	[illegible]

NUMBER OF SOURCES Q=18

ALPHA	1	2	3	4	5	6	7	8	9	10
0.10	1.00342	0.52648	0.22513	0.07549	0.01954	0.00392	0.00062	0.00008	0.00001	0.00000
	1.07395	0.63201	0.28523	0.09559	0.02398	0.00463	0.00071	0.00009	0.00001	0.00000
0.20	2.23675	1.55618	0.98538	0.54997	0.26101	0.10199	0.03214	0.00810	0.00163	0.00026
	2.15108	1.70616	1.20349	0.72262	0.35358	0.13685	0.04161	0.01003	0.00194	0.00030
0.30	3.33514	2.56876	1.87209	1.26670	0.77578	0.41679	0.18997	0.07137	0.02167	0.00526
	2.90657	2.53060	2.06910	1.54720	1.02199	0.57284	0.26249	0.09604	0.02789	0.00645
0.40	4.29349	3.48151	2.71847	2.01951	1.40356	0.89191	0.50317	0.24363	0.09797	0.03189
	3.42816	3.11072	2.71373	2.23954	1.70896	1.16916	0.69030	0.33804	0.13309	0.04146
0.50	5.13114	4.29117	3.48832	2.73348	2.04079	1.42805	0.91558	0.52206	0.25553	0.10356
	3.78624	3.51287	3.17090	2.75521	2.26965	1.73438	1.19306	0.71102	0.35201	0.13928
0.60	5.86765	5.00883	4.17973	3.38857	2.64610	1.96631	1.36674	0.85749	0.48698	0.23265
	4.02872	3.78858	3.48982	3.12528	2.69200	2.19595	1.65884	1.12461	0.65700	0.31286
0.70	6.51950	5.64710	4.79950	3.98312	3.20640	2.48041	1.81954	1.24176	0.76748	0.41439
	4.18822	3.97369	3.70868	3.38587	2.99983	2.54983	2.04469	1.50937	0.99074	0.53934
0.80	7.10011	6.21749	5.35616	4.52133	3.71984	2.96064	2.25551	1.61973	1.07230	0.63408
	4.28745	4.09315	3.85485	3.56570	3.21958	2.81281	2.34730	1.83557	1.30758	0.77989
0.90	7.62041	6.72979	5.85736	5.00898	4.18884	3.40483	2.66573	1.98742	1.38364	0.87582
	4.34248	4.16448	3.94771	3.68587	3.37294	3.00401	2.57748	2.09846	1.58447	1.00472
1.00	8.08918	7.19212	6.31177	5.45185	4.51720	3.81404	3.05064	2.33794	1.69042	1.12678
	4.36484	4.20024	4.00111	3.76171	3.47637	3.13983	2.74873	2.30395	1.81482	1.20107
1.10	8.51365	7.61130	6.72410	5.85526	5.00899	4.19065	3.40733	2.66841	1.98644	1.37809
	4.36298	4.20956	4.02513	3.80438	3.54214	3.23318	2.87331	2.46105	2.00058	1.36817
1.20	8.89983	7.99302	7.10013	6.22394	5.36816	4.53750	3.73813	2.97818	2.26861	1.62416
	4.34313	4.19918	4.02716	3.82212	3.57941	3.29396	2.96131	2.57868	2.14707	1.51013
1.30	9.25258	8.34197	7.44429	6.56190	5.69823	4.85739	4.04481	3.26768	2.53562	1.86168
	4.30992	4.17406	4.01268	3.82101	3.59495	3.32968	3.02072	2.66459	2.26036	1.63149
1.40	9.57503	8.66216	7.76041	6.87274	6.00238	5.15295	4.32935	3.53793	2.78725	2.08893
	4.26685	4.13798	3.98584	3.80567	3.59395	3.34611	3.05779	2.72519	2.34621	1.73572

NUMBER OF SOURCES Q=19

ALPHA	1	2	3	4	5	6	7	8	9	10
0.10	1.08216	0.58646	0.26257	0.09345	0.02596	0.00564	0.00097	0.00013	0.00002	0.00000
	1.15700	0.70520	0.33513	0.11989	0.03236	0.00677	0.00112	0.00015	0.00002	0.00000
0.20	2.39421	1.69909	1.10682	0.64299	0.32221	0.13494	0.04620	0.01280	0.00287	0.00052
	2.29569	1.85527	1.34868	0.84728	0.44116	0.18445	0.06122	0.01623	0.00348	0.00061
0.30	3.55845	2.78083	2.06757	1.43856	0.91607	0.51920	0.25390	0.10410	0.03505	0.00958
	3.09073	2.72330	2.26924	1.74729	1.20569	0.71872	0.35684	0.14372	0.04649	0.01210
0.40	4.57285	3.75156	2.97539	2.25763	1.61555	1.06917	0.63789	0.33283	0.14720	0.05374
	3.63876	3.33044	2.94423	2.47947	1.95079	1.39637	0.87923	0.46919	0.20537	0.07228
0.50	5.45886	4.61102	3.79714	3.02688	2.31281	1.67085	1.11987	0.67901	0.36105	0.16281
	4.01460	3.75000	3.41928	3.01611	2.54106	2.00796	1.45117	0.92806	0.50542	0.22542
0.50	6.23759	5.37130	4.53320	3.72898	2.96860	2.26432	1.63168	1.08921	0.65592	0.34468
	4.26883	4.03687	3.74892	3.39743	2.97788	2.49251	1.95615	1.40288	0.88836	0.47296
0.70	6.92659	6.04792	5.19189	4.36414	3.57203	2.82516	2.13501	1.52037	0.99706	0.58499
	4.43578	4.22885	3.97395	3.66379	3.29224	2.85657	2.36116	1.82321	1.27897	0.77157
0.80	7.54023	6.65185	5.78290	4.93795	4.12294	3.34557	2.61595	1.94720	1.35593	0.86163
	4.53938	4.35211	4.12321	3.84600	3.51413	3.12289	2.67151	2.16714	1.63060	1.07192
0.90	8.09000	7.19403	6.31512	5.45708	4.62484	3.82474	3.06512	2.35689	1.71426	1.15516
	4.59649	4.42504	4.21698	3.96631	3.66700	3.31365	2.90309	2.43692	1.92584	1.34402
1.00	8.58529	7.68323	6.79640	5.92808	5.08242	4.26480	3.48221	2.74384	2.06182	1.45195
	4.61928	4.46080	4.26977	4.04079	3.76829	3.44688	3.07226	2.64307	2.16387	1.57761
1.10	9.03378	8.12670	7.23343	6.35680	5.50042	4.66890	3.86824	3.10627	2.39342	1.74346
	4.61663	4.46898	4.29209	4.08107	3.83091	3.53638	3.19281	2.79732	2.35080	1.77327
1.20	9.44173	8.53043	7.63182	6.74833	5.38318	5.04040	4.22517	3.44432	2.70685	2.02482
	4.59505	4.45655	4.29159	4.09568	3.86429	3.59251	3.27568	2.91012	2.49458	1.93536
1.30	9.81437	8.89948	7.99635	7.10711	6.23466	5.38256	4.55536	3.75907	3.00160	2.29352
	4.55947	4.42879	4.27403	4.09093	3.87551	3.62317	3.32936	2.99011	2.60295	2.06865
1.40	10.15611	9.23810	8.33112	7.43696	6.55834	5.69838	4.86119	4.05208	3.27806	2.54848
	4.51355	4.38963	4.24370	4.07160	3.86991	3.63427	3.36040	3.04422	2.68261	2.17723

NUMBER OF SOURCES Q=20

ALPHA	1	2	3	4	5	6	7	8	9	10
0.10	1.16170	0.64833	0.30264	0.11375	0.03373	0.00789	0.00147	0.00022	0.00003	0.00000
	1.24057	0.78049	0.38878	0.14771	0.04269	0.00960	0.00173	0.00025	0.00003	0.00000
0.20	2.55235	1.84389	1.23189	0.74153	0.38998	0.17380	0.06413	0.01936	0.00477	0.00096
	2.44029	2.00500	1.49669	0.97856	0.53356	0.24146	0.08585	0.02513	0.00592	0.00114
0.30	3.78238	2.99450	2.26622	1.61583	1.06448	0.63202	0.32867	0.14558	0.05374	0.01632
	3.27466	2.91572	2.46985	1.95043	1.39749	0.87878	0.46829	0.20557	0.07332	0.02124
0.40	4.85273	4.02318	3.23499	2.59040	1.33497	1.25731	0.78678	0.43762	0.21012	0.08480
	3.84905	3.54954	3.17405	2.71964	2.19590	1.63304	1.08595	0.62398	0.29979	0.11770
0.50	5.78705	4.93206	4.10827	3.32427	2.59127	1.92349	1.33821	0.85418	0.48690	0.24037
	4.24265	3.98638	3.66651	3.27587	2.81258	2.28543	1.72070	1.16691	0.68872	0.34141
0.60	6.60793	5.73585	4.88869	4.07285	3.29667	2.57092	1.90936	1.32878	0.84768	0.48185
	4.50861	4.28437	4.00668	3.66783	3.26221	2.78932	2.25859	1.69560	1.14671	0.67398
0.70	7.33407	6.44972	5.58612	4.74825	3.94258	3.17746	2.46377	1.81533	1.24908	0.78364
	4.68303	4.48321	4.23783	3.93971	3.58233	3.16155	2.67855	2.14444	1.58625	1.05108
0.80	7.98069	7.08710	6.21131	5.35734	4.53341	3.73720	2.98541	2.28933	1.66040	1.11719
	4.79100	4.61031	4.39020	4.12423	3.80601	3.43016	2.99410	2.50103	1.96476	1.41551
0.90	8.55993	7.65912	6.77394	5.90773	5.06482	4.25068	3.47248	2.73950	2.06387	1.46103
	4.85020	4.68487	4.48495	4.24472	3.95827	3.61999	3.22568	2.77449	2.27226	1.73656
1.00	9.08173	8.17512	7.28247	6.40666	5.55130	4.72105	3.92189	3.16160	2.45041	1.80162
	4.87343	4.72066	4.53718	4.31790	4.05745	3.75043	3.39200	2.97936	2.51379	2.00452
1.10	9.55418	8.64281	7.74409	6.85051	5.99320	5.15218	4.33654	3.55490	2.81599	2.13127
	4.86999	4.72769	4.55782	4.35585	4.11694	3.83599	3.50814	3.12962	2.69916	2.22082
1.20	9.98391	9.06854	8.16478	7.27479	6.40136	5.54797	4.71904	3.92034	3.15936	2.44605
	4.84672	4.71327	4.55488	4.36740	4.14654	3.88756	3.58574	3.23695	2.83855	2.39141
1.30	10.37648	9.45769	8.54967	7.65430	6.77408	5.91213	5.07228	4.25964	3.48073	2.74426
	4.80879	4.68290	4.53430	4.35912	4.15356	3.91324	3.63370	3.31071	2.94093	2.52338
1.40	10.73643	9.81470	8.90303	8.00307	7.11711	6.24794	5.39897	4.57474	3.78103	3.02549
	4.76002	4.64057	4.50052	4.33589	4.14345	3.91913	3.65878	3.35820	3.01381	2.62342

NUMBER OF SOURCES Q=21

ALPHA	1	2	3	4	5	6	7	8	9	10
0.10	1.24195	0.71195	0.34523	0.13641	0.04296	0.01075	0.00216	0.00035	0.00005	0.00001
	1.32459	0.85764	0.44596	0.17912	0.05518	0.01329	0.00257	0.00041	0.00005	0.00001
0.20	2.71114	1.99037	1.36020	0.84511	0.46402	0.21866	0.08534	0.02818	0.00757	0.00168
	2.58486	2.15519	1.64695	1.11557	0.64515	0.30817	0.11934	0.03745	0.00962	0.00204
0.30	4.00682	3.20958	2.46764	1.79786	1.22013	0.75445	0.41407	0.19638	0.07869	0.02626
	3.45838	3.10780	2.67062	2.15573	1.59578	1.05124	0.59624	0.28278	0.11023	0.03519
0.40	5.13308	4.29589	3.49691	2.74720	2.06084	1.45502	0.94850	0.55736	0.28736	0.12664
	4.05907	3.76801	3.40308	2.95958	2.44307	1.87681	1.30746	0.80066	0.41744	0.18065
0.50	6.11567	5.25417	4.42141	3.62511	2.87525	2.18456	1.56876	1.04567	0.63204	0.33700
	4.47040	4.22205	3.91258	3.53431	3.08354	2.56508	1.99837	1.42327	0.89901	0.48702
0.60	6.97865	6.10083	5.24594	4.41971	3.62950	2.88483	2.19786	1.58370	1.05972	0.64290
	4.74810	4.53114	4.26317	3.93646	3.54470	3.08538	2.56374	1.99829	1.42631	0.90380
0.70	7.74194	6.85241	5.98196	5.13503	4.31732	3.53616	2.80103	2.12405	1.52030	1.00725
	4.93001	4.73687	4.50045	4.21373	3.87003	3.46427	2.99530	2.46961	1.90655	1.34365
0.80	8.42145	7.52312	6.64114	5.77912	4.94161	4.13447	3.36524	2.64368	1.98230	1.39665
	5.04233	4.86781	4.65593	4.40054	4.09529	3.73445	3.31420	2.83493	2.30508	1.74635
0.90	9.03016	8.12494	7.23408	6.35059	5.50819	4.68172	3.88734	3.13302	2.42912	1.78892
	5.10365	4.94403	4.75171	4.52125	4.24692	3.92306	3.54482	3.10973	2.62013	2.08720
1.00	9.57850	8.65776	7.76984	6.88731	6.02333	5.18197	4.36338	3.58919	2.85309	2.17133
	5.12735	4.97991	4.80346	4.59323	4.34405	4.05062	3.70786	3.31203	2.86216	2.36305
1.10	10.07487	9.15959	8.25593	7.35612	6.49288	5.63973	4.81102	4.01244	3.25133	2.53729
	5.12311	4.98580	4.82249	4.62890	4.40048	4.13229	3.81940	3.45756	3.04412	2.58009
1.20	10.52638	9.60729	8.69886	7.80304	6.92222	6.05950	5.21863	4.40453	3.62354	2.88387
	5.09815	4.96942	4.81714	4.63749	4.42640	4.17934	3.89170	3.55908	3.17800	2.74733
1.30	10.93882	10.01649	9.10402	8.20316	7.31603	6.44539	5.59449	4.76774	3.97057	3.21021
	5.05787	4.93645	4.79357	4.62572	4.42929	4.20014	3.93399	3.62647	3.27381	2.87366
1.40	11.31698	10.39184	9.47593	8.57077	7.67831	6.80098	5.94172	5.10443	4.29391	3.51653
	5.00626	4.89116	4.75639	4.59867	4.41479	4.20097	3.95319	3.66736	3.33957	2.96681

NUMBER OF SOURCES Q=22

ALPHA	1	2	3	4	5	6	7	8	9	10
0.10	1.32286	0.77717	0.39022	0.16144	0.05375	0.01433	0.00308	0.00054	0.00008	0.00001
	1.40897	0.93644	0.50646	0.21417	0.07002	0.01799	0.00372	0.00063	0.00009	0.00001
0.20	2.87048	2.13836	1.49142	0.95329	0.54402	0.26956	0.11321	0.03969	0.01155	0.00279
	2.72935	2.30566	1.79900	1.25748	0.76022	0.38462	0.15943	0.05392	0.01500	0.00347
0.30	4.23175	3.42590	2.67151	1.98408	1.38225	0.88571	0.50972	0.25685	0.11078	0.04025
	3.64188	3.29951	2.87133	2.36250	1.79919	1.23436	0.73970	0.37603	0.15894	0.05549
0.40	5.41383	4.56965	3.76088	2.99750	2.29232	1.66111	1.12175	0.69117	0.37909	0.18049
	4.26882	3.98586	3.63125	3.19897	2.69135	2.12574	1.54095	0.99691	0.55835	0.26361
0.50	6.44453	5.57717	4.73629	3.92891	3.16397	2.45290	1.80990	1.25162	0.79507	0.45270
	4.69787	4.45702	4.15752	3.79134	3.35347	2.84560	2.28155	1.69319	1.13261	0.66218
0.60	7.34970	6.45652	5.60474	4.76915	3.96642	3.20497	2.49560	1.85180	1.28961	0.82597
	4.98731	4.77722	4.51846	4.20337	3.82521	3.38001	2.86982	2.30743	1.72189	1.16137
0.70	8.15007	7.25580	6.37914	5.52407	4.69563	3.90028	3.14632	2.44441	1.80787	1.25268
	5.17569	4.98985	4.76186	4.48593	4.15536	3.76445	3.31041	2.79621	2.23512	1.65594
0.80	8.86252	7.95983	7.07225	6.20297	5.35503	4.53656	3.75116	3.00828	2.31879	1.69635
	5.29341	5.12467	4.92051	4.67505	4.38209	4.03574	3.63133	3.16728	2.64806	2.08854
0.90	9.50070	8.59144	7.69543	6.81535	5.95449	5.11709	4.30852	3.53565	2.80738	2.13513
	5.35688	5.20261	5.01738	4.79605	4.53312	4.22295	3.86036	3.44180	2.96711	2.44274
1.00	10.07547	9.15094	8.25824	7.35967	6.49798	5.64681	4.82053	4.02493	3.26742	2.55752
	5.38101	5.23856	5.06867	4.86686	4.62823	4.34760	4.01984	3.64062	3.20748	2.72209
1.10	10.59580	9.67695	8.76880	7.87337	6.99305	6.13091	5.29073	4.47742	3.69721	2.95819
	5.37500	5.24337	5.08615	4.90038	4.68172	4.42546	4.12674	3.78104	3.38482	2.93710
1.20	11.06910	10.14661	9.23394	8.33284	7.44344	6.57444	5.72303	4.89557	4.09731	3.33524
	5.34939	5.22505	5.07845	4.90608	4.70407	4.46815	4.19382	3.87661	3.51257	3.09929
1.30	11.50136	10.57578	9.65930	8.75345	7.86018	6.98178	6.12117	5.28212	4.46923	3.68854
	5.30674	5.18948	5.05194	4.89090	4.70295	4.48419	4.23050	3.93760	3.60147	3.21892
1.40	11.89774	10.96948	10.04973	9.13986	8.24158	7.35697	6.48871	5.64001	4.81497	4.01900
	5.25231	5.14115	5.01140	4.86009	4.68416	4.48004	4.24401	3.97197	3.66002	3.30463

NUMBER OF SOURCES Q=23

ALPHA	1	2	3	4	5	6	7	8	9	10
0.10	1.40436	0.84386	0.43749	0.18884	0.06621	0.01872	0.00429	0.00081	0.00013	0.00002
	1.49366	1.01670	0.57007	0.25286	0.08742	0.02385	0.00526	0.00095	0.00014	0.00002
0.20	3.03032	2.28769	1.62524	1.06565	0.62961	0.32645	0.14501	0.05428	0.01700	0.00445
	2.87376	2.45630	1.95246	1.40356	0.88300	0.47068	0.20772	0.07532	0.02258	0.00565
0.30	4.45703	3.64331	2.87753	2.17400	1.55014	1.02500	0.61512	0.32713	0.15073	0.05915
	3.82515	3.49080	3.07178	2.57020	2.00652	1.42644	0.89739	0.48548	0.22094	0.08373
0.40	5.69493	4.84434	4.02663	3.25088	2.52871	1.87459	1.30533	0.83802	0.48507	0.24718
	4.47831	4.20311	3.85854	3.43759	2.94002	2.37825	1.78388	1.21011	0.72163	0.36822
0.50	6.77392	5.90098	5.05270	4.23530	3.45682	2.72751	2.06023	1.47032	0.97442	0.58688
	4.92507	4.69136	4.40136	4.04692	3.62205	3.12607	2.56819	1.97323	1.38562	0.86484
0.60	7.72105	6.83315	5.96488	5.12083	4.30687	3.53049	2.80127	2.13125	1.53505	1.02895
	5.22626	5.02266	4.77261	4.46861	4.10366	3.67277	3.17560	2.62035	2.02898	1.44142
0.70	8.55847	7.65935	6.77755	5.91510	5.07704	4.26905	3.49848	2.77465	2.10936	1.51702
	5.42314	5.24221	5.02215	4.75642	4.43837	4.06197	3.62323	3.12255	2.56839	1.98211
0.80	9.30384	8.39716	7.50446	6.62862	5.77320	4.94277	4.14310	3.38155	2.66758	2.01317
	5.54426	5.38095	5.18404	4.94790	4.66655	4.33405	3.94524	3.49715	2.99128	2.43726
0.90	9.97143	9.05846	8.15777	7.27172	6.40326	5.55616	4.73505	3.94595	3.19653	2.49663
	5.60986	5.46061	5.28202	5.06925	4.81701	4.51980	4.17229	3.77020	3.31166	2.79969
1.00	10.57267	9.65466	8.74761	7.85355	6.97496	6.11500	5.27754	4.46758	3.69148	2.95740
	5.63446	5.49668	5.33293	5.13898	4.91020	4.64160	4.32808	3.96501	3.54893	3.07933
1.10	11.11699	10.19483	9.28260	8.38207	7.49534	6.62523	5.77488	4.94868	4.15189	3.39143
	5.62871	5.50045	5.34892	5.17044	4.96088	4.71578	4.43040	4.10011	3.72090	3.29041
1.20	11.61199	10.68636	9.76983	8.86397	7.97065	7.09224	6.23154	5.39232	4.57909	3.79778
	5.60041	5.48017	5.33890	5.17330	4.97974	4.75418	4.49231	4.18967	3.84212	3.44640
1.30	12.06412	11.13554	10.21539	9.30504	8.40619	7.52091	5.65171	5.80188	4.97528	4.17715
	5.55543	5.44205	5.30949	5.15479	4.97471	4.76560	4.52357	4.24440	3.92400	3.55877
1.40	12.47873	11.54760	10.62432	9.71016	8.80660	7.91554	7.03926	6.18052	5.34291	4.53096
	5.49819	5.39073	5.26567	5.12028	4.95173	4.75664	4.53147	4.27236	3.97537	3.63673

NUMBER OF SOURCES Q=24

ALPHA	1	2	3	4	5	6	7	8	9	10
0.10	1.48641	0.91192	0.48690	0.21857	0.08042	0.02402	0.00585	0.00117	0.00020	0.00003
	1.57862	1.09824	0.63657	0.29515	0.10753	0.03105	0.00727	0.00141	0.00023	0.00003
0.20	3.19060	2.43822	1.76141	1.18185	0.72044	0.38920	0.18196	0.07234	0.02424	0.00684
	3.01807	2.60701	2.10700	1.55313	1.01271	0.56606	0.26467	0.10242	0.03291	0.00887
0.30	4.68279	3.85170	3.08546	2.36722	1.72318	1.17162	0.72970	0.40713	0.19907	0.08378
	4.00822	3.68166	3.27185	2.77840	2.21583	1.62596	1.06785	0.61084	0.29734	0.12158
0.40	5.97637	5.11985	4.29396	3.50695	2.76941	2.09456	1.49811	0.99683	0.60478	0.32719
	4.68755	4.41979	4.08493	3.67528	3.18856	2.63310	2.03408	1.43763	0.90566	0.49523
0.50	7.10349	6.22551	5.37045	4.54395	3.75327	3.00760	2.31857	1.70024	1.16849	0.73851
	5.15203	4.92509	4.64416	4.30104	3.88911	3.40583	2.85570	2.26055	1.65422	1.09201
0.60	8.09269	7.20034	6.32623	5.47451	4.65043	3.86066	3.11377	2.42049	1.79404	1.24968
	5.46501	5.26754	5.02572	4.73227	4.38007	3.96343	3.48023	2.93504	2.34390	1.73893
0.70	8.96711	8.06448	7.17701	6.30789	5.46114	4.64184	3.85550	3.11334	2.42278	1.79769
	5.66936	5.49401	5.28142	5.02532	4.71918	4.35681	3.93341	3.44745	2.90372	2.31746
0.80	9.74541	8.83505	7.93768	7.05587	6.19281	5.35257	4.54021	3.76220	3.02681	2.34450
	5.79491	5.63672	5.44660	5.21921	4.94882	4.62953	4.25587	3.82394	3.33312	2.78898
0.90	10.44233	9.52599	8.62101	7.72955	6.85425	5.99843	5.16619	4.36278	3.59487	2.87097
	5.86263	5.71811	5.54575	5.34098	5.09876	4.81379	4.48067	4.09473	3.65291	3.15572
1.00	11.07008	10.14887	9.23781	8.33877	7.45393	6.58613	5.73872	4.91611	4.12376	3.36871
	5.88772	5.75432	5.59630	5.40972	5.19014	4.93284	4.63280	4.28521	3.88607	3.43336
1.10	11.63831	10.71312	9.79715	8.89198	7.99955	7.12219	6.26285	5.42524	4.61397	3.83492
	5.88122	5.75705	5.61085	5.43916	5.23810	5.00341	4.73059	4.41493	4.05220	3.63918
1.20	12.15507	11.22655	10.30649	9.39628	8.49761	7.61251	6.74355	5.89398	5.06763	4.26966
	5.85126	5.73487	5.59857	5.43928	5.25360	5.03769	4.78745	4.49857	4.16677	3.78834
1.30	12.62708	11.69573	10.77219	9.85776	8.95387	8.06234	7.18555	6.32613	5.48757	4.67431
	5.80397	5.69422	5.56632	5.41751	5.24475	5.04463	4.81343	4.54711	4.24167	3.89315
1.40	13.05985	12.12606	11.19957	10.28149	9.37318	8.47626	7.59285	6.72528	5.87665	5.05080
	5.74389	5.63988	5.51921	5.37935	5.21764	5.03091	4.81590	4.56884	4.28590	3.96327

NUMBER OF SOURCES Q=25

ALPHA	1	2	3	4	5	6	7	8	9	10
0.10	1.56894	0.98124	0.53835	0.25060	0.09644	0.03034	0.00782	0.00167	0.00030	0.00004
	1.66379	1.18091	0.70573	0.34100	0.13051	0.03977	0.00985	0.00202	0.00035	0.00005
0.20	3.35129	2.58984	1.89970	1.30153	0.81516	0.45763	0.22419	0.09422	0.03362	0.01016
	3.16227	2.75772	2.26234	1.70562	1.14859	0.67034	0.33058	0.13593	0.04663	0.01346
0.30	4.90882	4.08096	3.29510	2.55336	1.90082	1.32487	0.85282	0.49665	0.25617	0.11490
	4.19108	3.87208	3.47146	2.98675	2.42928	1.83153	1.24955	0.75140	0.38889	0.17056
0.40	6.25812	5.39612	4.56271	3.76543	3.01392	2.32028	1.69909	1.16651	0.73749	0.42060
	4.89661	4.63595	4.31045	3.91195	3.43659	2.88931	2.28969	1.67694	1.10836	0.64449
0.50	7.43332	6.55067	5.68939	4.85461	4.05287	3.29246	2.58390	1.94002	1.37573	0.90633
	5.37876	5.15825	4.88595	4.55374	4.15453	3.68440	3.14593	2.55282	1.93496	1.34023
0.60	8.46452	7.56806	6.68860	5.82988	4.99664	4.19485	3.43217	2.71818	2.06480	1.48611
	5.70350	5.51184	5.27782	4.99438	4.65443	4.25183	3.78308	3.25000	2.66374	2.04944
0.70	9.37598	8.45954	7.57742	6.70220	5.84760	5.01814	4.21962	3.45930	2.74640	2.09240
	5.91537	5.74530	5.53974	5.29271	4.99787	4.64900	4.24072	3.77015	3.23923	2.65832
0.80	10.18715	9.27339	8.37172	7.48447	6.61451	5.76544	4.94175	4.14915	3.39492	2.68818
	6.04533	5.89196	5.70823	5.48909	5.22901	4.92225	4.56318	4.14732	3.67246	3.14118
0.90	10.91352	9.99398	9.08503	8.18865	7.30710	6.44347	5.60128	4.78521	4.00106	3.25617
	6.11522	5.97514	5.80862	5.61136	5.37856	5.10509	4.78567	4.41537	3.99033	3.50931
1.00	11.56771	10.64352	9.72881	8.82522	7.93471	7.05982	6.20349	5.36966	4.56303	3.78968
	6.14082	6.01154	5.85891	5.67920	5.46825	5.22153	4.93418	4.60139	4.21878	3.78334
1.10	12.15981	11.23182	10.31240	9.40299	8.50534	7.62152	6.75408	5.90639	5.08234	4.28710
	6.13355	6.01323	5.87203	5.70669	5.51356	5.28862	5.02753	4.72573	4.37883	3.98309
1.20	12.69835	11.76717	10.84384	9.92964	9.02610	8.13498	7.25368	6.39984	5.56194	4.74946
	6.10196	5.98920	5.85755	5.70414	5.52580	5.31888	5.07957	4.80358	4.48672	4.12501
1.30	13.19017	12.25626	11.32963	10.41142	9.50297	8.60583	7.72217	6.85425	6.00516	5.17866
	6.05233	5.94600	5.82246	5.67915	5.51323	5.32143	5.10036	4.84609	4.55466	4.22213
1.40	13.64112	12.70487	11.77542	10.85381	9.94116	9.03899	8.14916	7.27364	6.41528	5.57727
	5.98944	5.88867	5.77211	5.63742	5.48207	5.30315	5.09755	4.86172	4.59196	4.28450

NUMBER OF SOURCES Q=26

ALPHA	1	2	3	4	5	6	7	8	9	10
0.10	1.65194	1.05171	0.59172	0.28488	0.11433	0.03775	0.01026	0.00232	0.00044	0.00007
	1.74914	1.26458	0.77735	0.39031	0.15548	0.05018	0.01311	0.00285	0.00052	0.00008
0.20	3.51236	2.74244	2.03991	1.42440	0.91643	0.53153	0.27177	0.12022	0.04547	0.01465
	3.30637	2.90836	2.41828	1.86051	1.28991	0.78298	0.40557	0.17649	0.06438	0.01983
0.30	5.13516	4.30101	3.50626	2.75212	2.08259	1.48413	0.98386	0.59533	0.32222	0.15313
	4.37373	4.06208	3.67054	3.19502	2.64324	2.04196	1.44099	0.90619	0.49587	0.23204
0.40	6.54011	5.67302	4.83269	4.02601	3.26177	2.55106	1.90736	1.34604	0.88234	0.52719
	5.10542	4.85155	4.53508	4.14752	3.68380	3.14609	2.54917	1.92573	1.32743	0.81511
0.50	7.76333	6.87641	6.00941	5.16704	4.35524	3.58152	2.85535	2.18847	1.59469	1.08899
	5.60528	5.39088	5.12681	4.80505	4.41826	3.96147	3.43495	2.84820	2.22484	1.60595
0.60	8.83657	7.93631	7.05191	6.18679	5.34523	4.53258	3.75572	3.02323	2.34581	1.73635
	5.94179	5.75563	5.52899	5.25506	4.92685	4.53791	4.08380	3.56416	2.98627	2.36918
0.70	9.78503	8.87526	7.97867	7.09787	6.23611	5.39750	4.58716	3.81153	3.07884	2.39925
	6.16119	5.99609	5.79717	5.55873	5.27460	4.93862	4.54509	4.09015	3.57360	3.00190
0.80	10.62907	9.71215	8.80653	7.91430	7.03804	6.18101	5.34716	4.54158	3.77066	3.04241
	6.29558	6.14676	5.96904	5.75764	5.50728	5.21239	4.86732	4.46718	4.00868	3.49215
0.90	11.38487	10.45239	9.54981	8.64889	7.76170	6.89097	6.03987	5.21250	4.41393	3.65065
	6.36766	6.23177	6.07073	5.88051	5.65654	5.39394	5.08749	4.73219	4.32369	3.85950
1.00	12.06544	11.13850	10.22040	9.31264	8.41699	7.53569	6.67139	5.82753	5.00824	4.21883
	6.39372	6.26834	6.12075	5.94748	5.74461	5.50782	5.23242	4.91368	4.54703	4.12879
1.10	12.68148	11.75088	10.82828	9.91501	9.01258	8.12286	7.24822	6.39146	5.55611	4.74666
	6.38573	6.26904	6.13252	5.97311	5.78741	5.57157	5.32150	5.03276	4.70094	4.32197
1.20	13.24175	12.30808	11.38173	10.45391	9.55591	8.65935	7.77540	6.90931	6.06121	5.23589
	6.35249	6.24313	6.11584	5.96796	5.79648	5.59794	5.36880	5.10493	4.80222	4.45653
1.30	13.75340	12.81712	11.88764	10.95598	10.05334	9.15112	8.26128	7.38578	6.52733	5.68915
	6.30055	6.19743	6.07798	5.93982	5.78026	5.59627	5.38460	5.14158	4.86334	4.54600
1.40	14.22257	13.28404	12.35186	11.42692	10.51039	9.60341	8.70780	7.82518	6.95819	6.10947
	6.23487	6.13715	6.02444	5.89456	5.74518	5.57350	5.37569	5.15129	4.89384	4.60067

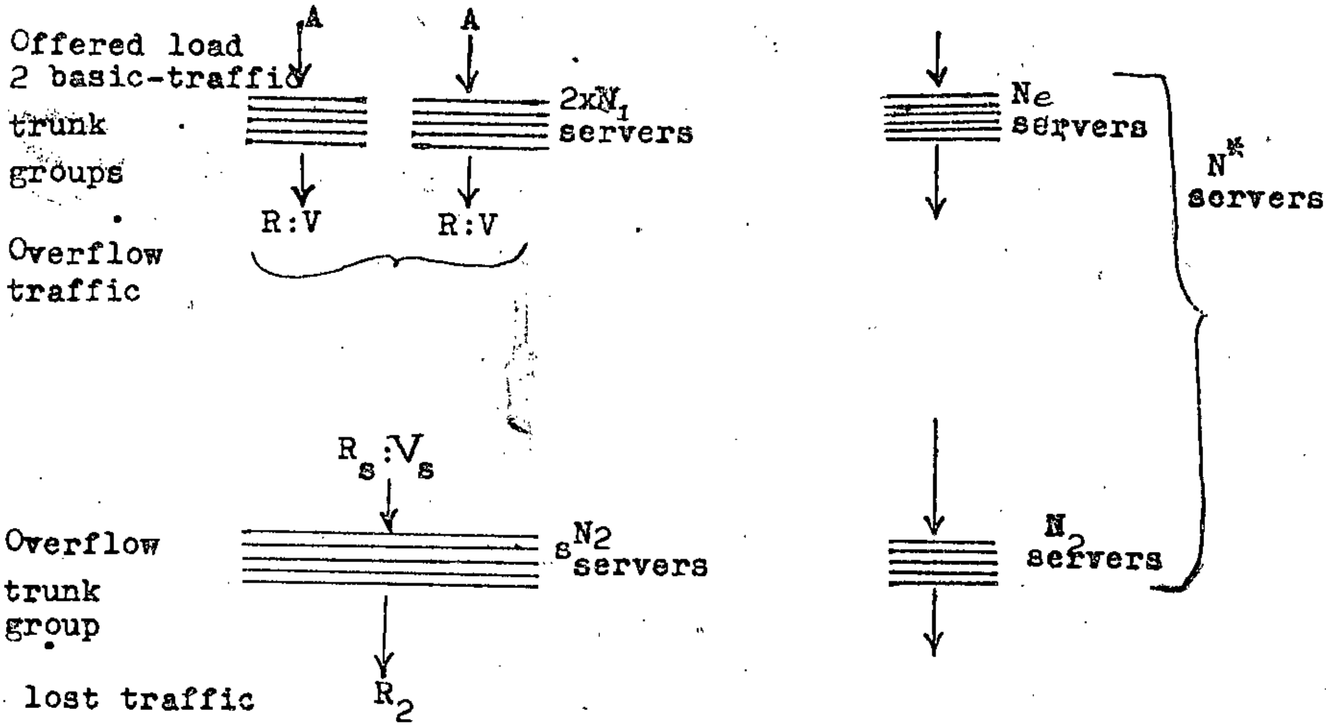
NUMBER OF SOURCES Q=27

ALPHA	1	2	3	4	5	6	7	8	9	10
0.10	1.73535	1.12327	0.64691	0.32136	0.13412	0.04636	0.01326	0.00316	0.00063	0.00011
	1.83464	1.34912	0.85124	0.44298	0.18552	0.06244	0.01716	0.00393	0.00076	0.00013
0.20	3.67375	2.89593	2.18186	1.55017	1.02092	0.61067	0.32470	0.15058	0.06013	0.02056
	3.45034	3.05899	2.57461	2.01737	1.43598	0.90342	0.48962	0.22465	0.08684	0.02843
0.30	5.36179	4.52178	3.71879	2.96322	2.26807	1.64883	1.12221	0.70276	0.39724	0.19899
	4.55621	4.25165	3.86906	3.40300	2.85817	2.25619	1.64073	1.07401	0.61821	0.30710
0.40	6.82232	5.95051	5.10379	4.28847	3.51260	2.78634	2.12213	1.53446	1.03843	0.64653
	5.31402	5.06656	4.75886	4.38198	3.92999	3.40286	2.81126	2.18194	1.56046	1.00559
0.50	8.09365	7.20265	6.33037	5.43104	4.66007	3.87426	3.13220	2.44455	1.82409	1.28512
	5.83160	5.62301	5.36677	5.05502	4.68028	4.23685	3.72314	3.14522	2.52129	1.88575
0.50	9.20881	8.30501	7.41605	6.54506	5.69589	4.87342	4.08378	3.33469	2.63577	1.99874
	6.17990	5.99895	5.77930	5.51439	5.19737	4.82168	4.38214	3.87681	3.30978	2.69508
0.70	10.19429	9.28133	8.38069	7.49477	6.62649	5.77957	4.95357	4.16925	3.41891	2.71661
	6.40684	6.24646	6.05380	5.82344	5.54947	5.22579	4.84655	4.40719	3.90592	3.34618
0.90	11.07115	10.15131	9.24202	8.34522	7.46321	6.59895	5.75594	4.93873	4.15298	3.40570
	6.54566	6.40114	6.22908	6.02497	5.78376	5.50009	5.16836	4.78348	4.34137	3.84071
0.90	11.85633	10.93113	10.01519	9.11009	8.21777	7.34060	6.48139	5.64394	4.83259	4.05302
	6.61991	6.48797	6.33211	6.14850	5.93283	5.68043	5.38626	5.04528	4.65290	4.20574
1.00	12.56333	11.63383	10.71259	9.80100	8.90067	8.01349	7.14200	6.28916	5.45859	4.65503
	6.64648	6.52476	6.38193	6.21471	6.01940	5.79190	5.52775	5.22230	4.87093	4.46959
1.10	13.20333	12.27030	11.34475	10.42788	9.52106	8.62606	7.74184	6.88003	6.03456	5.21252
	6.63779	6.52452	6.39240	6.23856	6.05978	5.85247	5.61274	5.33628	5.01873	4.65593
1.20	13.78527	12.84929	11.92019	10.99897	10.08693	9.18540	8.29644	7.42191	6.56473	5.72806
	6.60289	6.49674	6.37356	6.23083	6.06575	5.87506	5.65541	5.40289	5.11344	4.78309
1.30	14.31680	13.37831	12.44519	11.52132	10.60485	9.69803	8.80263	7.92016	7.05340	6.20493
	6.54865	6.44857	6.33298	6.19961	6.04599	5.86929	5.66546	5.43383	5.16797	4.86500
1.40	14.80410	13.86345	12.92875	12.00076	11.08059	10.16930	9.26343	8.37939	7.50472	6.64645
	6.48016	6.38531	6.27621	6.15085	6.00701	5.84215	5.65348	5.43776	5.19193	4.91214

7.3 A CASE STUDY

Given :

• Two basic traffic trunk group each is loaded with traffic $A=3.5$ Erl from finite number of sources $q=21$. On the basis of economic considerations the number of servers $N_1 = 7$ is provided in each basic traffic trunk group. A loss of B=.75% for example, is permitted for the traffic in each direction. Traffic overflow from basic trunk groups is fed to a overflow trunk group. All trunk groups are operated with full availability.



a) Given arrangement

b) Equivalent arrangement

Fig.7.1 Application of the table

To find :

The number N_2 servers in the overflow trunk group(Fig.7.1)

Solution :

In accordance with the permitted loss B=.75% of the total offered load of 2A = 7Erl, altogether $R_2 = B.2A = .0525$ Erl may be lost. As per the formulae derived in section 5.3.6 let it be considered that $R_{21} = R_{22} = R_{2/2} = .02625$.

Now $\quad 3.5 = \frac{\alpha}{1+\alpha}(21 + .02625)$ according to eqn. 5.52.

Or $\quad .1664 + .1664\alpha = \alpha$

Or $\quad \alpha = \frac{.1664}{.833} \simeq .2$

For q=21 , α(ALPHA) = .2 and $N_1=7$ the overflow tables in Sec.7.2 give for each traffic direction an overflow traffic with the characteristics R= .08 Erl (top number) and V= .11 (bottom number).

Hence a composite traffic with the characteristics $R_s = 2.08$ Erl = .16 Erl and $V_s = 2.11 = .22$ is offered to the overflow trunk group. A reading from the table shows that the same characteristics result as for the composite traffic offered to the overflow trunk group can be obtained if $q_e = 22$, $\alpha_e = .1$ and $N_e = 4$ servers. The number N_2 of servers of the overflow trunk group must now be calculated so that for $q_e=22$, $\alpha_e = .1$ and $N^* = N_e + N_2$ servers there is an overflow traffic with the traffic intensity $R_2 = .0525$.

For the values $R_2 = .0525$ Erl, $q_e = 22$, $\alpha_e = .1$ the working table gives $N^* = 5$ servers. The number N_2 of servers is now $N_2 = N^* - N_e = 5 - 4 = 1$ must be provided.

CHAPTER 8

DISCUSSION AND CONCLUSION

8.1 GENERAL REMARKS

In this thesis exact and approximate methods have been used in the case of single primary group to find overflow traffic values. The two way of finding overflow parameters agree to each other to a considerable extent giving almost identical results. Approximate method (ERT method) for several primary groups has been employed to evaluate overflow traffic values. Simulation results with selected dimension also supports the results obtained.

8.2 OBSERVATIONS

In chapter 5 exact method for calculating single group overflow traffic has been discussed. The equations derived are complex and require much computing effort for solution. Overflow parameters in that case could be found only by means of digital computers; they are shown graphically in Sec. 7.1.

In figures 7.1 and 7.2 the variation of variance (V_1) to mean (R_1) ratio with mean has been shown. Variance to mean ratio (V_1/R_1) is maximum in the range of relatively small mean values. For larger R_1 values V_1/R_1 is decreasing.

Fig. 7.3 indicates the change of mean values with the change of the calling rate α. R_1 increases almost linearly in the lower value of α, while it does not change appreciably in the wider range of .

Fig. 7.4 shows mean values which have increased with the decrease of primary trunks (n_1).

Fig. 7.5 shows several plot of R_1 vs q (number of sources). For a particular calling rate and primary trunks the mean value increases almost linearly with the increase of sources. The dotted lines in this figure indicates the same plot with approximate method.

Fig. 7.6 shows the variation of V_1 with the change of the calling rate α. For a set of sources and primary trunks V_1 rises to a peak value at a particular value of α and then it starts decreasing with the increase of α. The variance is zero at the lower range of α. The dotted lines in this plot also show the same curves with approximate method.

Each of the figures 7.7 and 7.8 shows two curves . One of these in each case has been obtained from simulation run while the other curves are from exact calculatioh methods.

Further, a table has been prepared in Sec. 7.2 using exact formulae. With the help of this table overflow traffic parameters for several groups can be evaluated.

8.3 EFFECT OF INCREASE NUMBER OF SOURCES

In the case of poisson traffic the increase of V_1/R_1 with the increase of R_1 is not considerable. The variance is more in this case. From curves (a) and (b) of figure 7.2 it is observed that as the number of sources are increased for the same number of primary trunks the curve has a shift upward which agrees with the characteristics of pure chance traffic from infinite number of sources.

8.4 EFFECT OF INCREASING PRIMARY TRUNKS

As the number of primary trunks is increased variance increases, but R_1 decreases. So V_1/R_1 increases. This can be checked with the help of figures 7.1 to 7.4.

When $q=50$, $\alpha = 1$, $n_1 = 10$ $R_1=21$ and $V_1/R_1 = 0.55$

and when $q=50$, $\alpha=1$, $n_1 = 25$, $R_1=3$ and $V_1/R_1 = 1.58$

So, V_1/R_1 increases as n_1 increases.

8.5 EFFECT OF CALLING RATE ON VARIANCE

Fig. 7.6 shows that the approximate formula yields values of high accuracy in finding variance for calling rates upto about $\alpha = 1$. This range of calling rate comprises practically all cases occuring in telephone traffic. Zero V_1 for small α also indicates less traffic offered to the system.

8.6 APPROXIMATE METHOD

Approximate method (ERT method) for several primary groups (Sec. 5.3) is different from Wilkinson's ERT method (Sec.4.4) in some respect. Unlike Wilkinson's method the sources are considered finite and the calling rate is used instead of offered traffic. The table prepared in Sec. 7.2 is simple in representation and handy for design purpose. The use of this table has been discussed in the example in Sec. 7.3.

8.7 SIMULATION RESULTS

Simulation gives practically useful results but it costs immense programming efforts and computer time. Inspite of this, simulation technique is very oftem used to assess the performance of a definite network structure before the actual design. In this work simulation results support the accuracy of the approximate and exact calculation figures. However, this simulation results should be accompanied by confidence interval calculations. Even in the absence of confidence interval the results are considerably in agreement with the results calculated. Further work on simulation run will be useful.

8.8 CONCLUSION

Overflow tables are available for the case of infinite sources. For infinite sources these tables yield overflow traffic values which differ a lot from the tables those have been prepared in the case of finite sources (Sec.7.2). Both the approximate methods — one for single group and the other for several groups can be used widely for design purpose. Exact method for single group though lengthy is easily solveable by digital computers. Simulation technique on different network structures with limited number of sources can be performed to find better and simple approximate formulae.

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