DEVELOPMENT OF A PRODUCTION INVENTORY MODEL WITH FUZZY RANDOM DEMAND AND INVENTORY COST WITH RELIABILITY CONSIDERATIONS

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BY

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CERTIFICATE OF APPROVAL

The thesis titled "**Development of a Production Inventory Model with Fuzzy Random Demand and Inventory Cost with Reliability Considerations**" submitted by **Sanjoy Kumar Paul**, Roll no.: **0409082001P**, has been accepted as satisfactory in partial fulfillment of the requirement of the degree of **Master of Science in Industrial and Production Engineering** on March 30, 2011.

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It is hereby declared that this thesis or any part of it has not been submitted elsewhere for the award of any degree or diploma.

Sanjoy Kumar Paul

To the Almighty

To my family

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ABSTRACT

This thesis addresses a production inventory model to maximize the expected average profit considering fuzziness of random demand and inventory holding cost along with reliability of the production process. The classical inventory control models assume that products are produced by perfectly reliable production process with a fixed set-up cost. While the reliability of the production process cannot be increased without a price, its set-up cost can be reduced with investment in flexibility and reliability improvement. In this thesis, a production inventory model with reliability of production process consideration is developed in an imprecise and uncertain mixed fuzzy environment. The goal of this thesis is to introduce demand and inventory holding cost as a fuzzy random variable in an imperfect production process. Here, set-up cost and reliability of the production process along with production period are decision variables and expected average profit is the objective function which is to be maximized. Expected average profit of the model is a fuzzy quantity due to fuzzy-randomness of the demand and inventory holding cost and its graded mean integration value (GMIV) is optimized using unconstraint signomial geometric programming to determine optimal decision for the decision maker. A numerical example has been presented to explain the model.

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NOMENCLATURE

K = production rate per day (constant).

$\widetilde{D}$= demand per day (fuzzy random in nature).

P = production cost per unit time.

$\tilde{h}$= holding cost per unit quantity per day (fuzzy random in nature).

C_0 = set-up cost per cycle (a decision variable).

S_1 = selling price of fresh units.

S_2 = selling price of defective units.

r = the reliability of the production process (a decision variable).

$\tilde{T}$ = duration of each cycle which is fuzzy random in nature due to fuzzy-randomness of $\widetilde{D}$.

T_1 = production period (a decision variable).

$Y(C_0, r)$= total cost of interest and depreciation for a production process per production cycle.

$F(C_0, r, \mathrm{T}_1)$= total profit function of C_0, r, T_1.

$q(t)$= inventory level at time t.

$\tilde{Z}$= profit function which is fuzzy random in nature due to fuzzy-randomness of $\widetilde{D}$.

ABBREVIATIONS

EOQ	Economic order quantity
EPL	Economic production lot
EPQ	Economic production quantity
JIT	Just-in-time
DCF	Discounted cash flow
FEPQ	Fuzzy economic production quantity
FTPIC	Fuzzy total production inventory cost
GMIV	Graded mean integration value
GA	Genetic algorithm
BP	Basic period
FSGA	Fuzzy simulation based genetic algorithm
WIP	Work-in-process
FAGP	Fuzzy additive goal programming
MGP	Modified geometric programming
GP	Geometric programming
FGA	Fuzzy genetic algorithm
ROII	Return on inventory investment
IWFNLP	Inverse weight fuzzy non-linear programming
DE	Differential evolution
NPV	Net present value
GCM	Global criteria method
FRI	Fuzzy riemann integral
GRG	Generalized reduced gradient
EMQ	Economic manufacturing quantity
TFN	Triangular fuzzy number
FRV	Fuzzy random variable
DD	Degrees of difficulty
FIFO	First in first out

CHAPTER I
INTRODUCTION

Inventory control is the process of managing the timing and the quantities of goods to be ordered and stocked, so that demands can be met satisfactorily and economically. Inventory control policies are decision rules that focus on the trade-off between the costs and benefits of alternative solutions to questions of when and how much to order for each different type of item. Success of inventory control depends on some important issues i.e. uncertainty about the size of future demands, uncertainty of inventory cost, uncertainty of lead time, reliability of the production process etc. Inventory is the costliest operating expense for a manufacturer and properly managing that function is crucial to optimizing productivity and profitability. As the profit of an organization depends on production inventory, it is important to optimize the production inventory problem.

1.1 Rationale of the study

In the global competitive market, manufacturers must maintain optimum quantity of inventory to maximize profit and efficiency of supply chain. Companies aim to maintain the required amount of inventory at right place and at right time with right cost. If material is not available precisely, the supply chain may incur expensive repercussions and production downtime. In this regard, optimization of production inventory model and finding optimum amount of inventory level has paid extensive attention among the researchers. Considering some realistic factors such as fuzzy random demand, fuzzy inventory holding cost and reliability of the production system a production inventory model is developed to maximize the expected average profit.

1.2 Objectives of the study

The objectives of this thesis work are:

i. To incorporate uncertainty in production inventory model by integrating fuzzy random variables.

ii. To develop a mathematical model of production inventory considering fuzzy random demand and inventory cost with reliability of the production process.

iii. To optimize the production inventory model by maximizing total profit and set-up cost. reliability of the production process along with the production period are as the decision variables.

This thesis, however, presents possible clues in the development of production inventory model by providing mathematical results to help on understanding, formulation and analysis of such mathematical inventory model.

1.3 Outline of methodology

The research work is theoretical in nature. A mathematical inventory model is developed considering some practical situations such as fuzzy random demand, fuzzy inventory holding cost and reliability of the production process. The production inventory model is composed of some mathematical equations which are used to determine the numerical values of different decision variables (set-up cost per cycle, reliability of the production process, duration of time until the production is being held and maximum profit). The methodology is as follows:

i. A profit function is developed considering selling price of fresh units, selling price of defective units, production cost. holding cost, set-up cost and cost of interest and depreciation cost

ii. Expected average profit and graded mean integrated value (GMIV) of expected average profit are determined and optimized.

iii. Mathematical equations are obtained from optimal feasible solutions considering some important factors (fuzzy random demand. fuzzy inventory holding cost and reliability of the production process) which are present in real life production inventory problem.

iv. Equations for set-up cost per cycle, reliability of the production process and duration of time until production is being held are developed for this production inventory problem and graded mean integration value (GMIV) of total profit is optimized using unconstraint signomial geometric programming.

v. A numerical example is considered to illustrate and explain the production inventory model.

CHAPTER II
LITERATURE REVIEW

In the production inventory mathematical modeling, the problems are complex and multi objectives and theoretical in nature. That is, there exist several criteria that must be taken into consideration when formulating and solving the model. It is common that some or all of these criteria are conflicting, perhaps incommensurable and set by more than one decision maker. Among these criteria there are for example: demand, inventory holding costs, set-up costs, reliability of the production process, imperfection and impreciseness of the production process etc. Traditionally, these problems have been tackled as single objective optimization problems after combining the multiple criteria into single scalar value. On the other hand, inherent complexity and uncertainty of some variables make the inventory problem complicated. This imposes a pressure upon the researcher to implement an appropriate and realistic mathematical modeling of the production inventory problem.

2.1 Literature review

Some research papers have been studied to understand the background of the study. Many factors have been considered to develop the previous production inventory models. Different methodologies have been followed to develop and solve models. In this section, some research papers have been studied to understand the factors and methodologies considered by researchers.

Andersson and Melchiors (2001) developed a one warehouse several retailers' inventory system assuming lost sales at the retailers. Using the well-known METRIC-approximation as a framework, they presented a heuristic for finding cost effective base-stock policies. Almost all multi-echelon inventory models assume that demand not satisfied immediately can be backordered. In some situations this assumption may not be realistic. For example, it may be more representative to model stock outs as lost sales when the retailers are in a competitive market and customers can easily turn to another firm when purchasing the good. The research dealing with multi-echelon inventory models has focused mainly on the backorder case, and the number of models dealing with lost sales is rather limited. The main reason for this is the added complexity of the lost sales case. The researchers considered a two-echelon inventory system with one central warehouse and an arbitrary number of retailers. The retailers face customer demand and replenish their stocks from the central warehouse. The warehouse, in turn, replenishes its stock from an outside supplier. In the paper the researchers analyzed a model for a one warehouse, multiple retailers' inventory system. Demand occurs only at the retailers and follows independent Poisson processes. All lead times are assumed to be constant. All installations use (S-1, S)-policies with continuous review. It is assumed that backlogging of customer demand is not allowed. The analysis departs in one of the most widely known multi-echelon inventory models, the METRIC-model. They demonstrated how the METRIC-model can be modified to handle the lost sales case. Their approach gives an approximate model which is quite simple and efficient from a computational point of view. The inventory system under consideration consists of one central warehouse and an arbitrary number of retailers. The retailers face Poisson customer demand. No backlogging is allowed at the retailers. Consequently, the customers that arrive to a retailer that is out of stock will become lost sales for the retailer. When stock outs occur at the warehouse, all demands from the retailers are fully backlogged and the backorders are filled according to a FIFO-policy. The transportation time between the warehouse and a given retailer is assumed to be constant as well as the transportation time from the external supplier to the warehouse. The cost of replenishment is assumed to be zero or negligible compared to the holding and stock out costs. The external supplier is assumed to have infinite capacity, which means that the replenishment lead time for the central warehouse is constant. A heuristic method for evaluation and optimization of (S-1, S)-policies for one warehouse, multiple retailers inventory system is presented. The evaluation technique uses the well-known METRIC-approximation as a framework. From a computational point of view the presented technique is very efficient and simple. The original backorder METRIC-model is one of the most widely used multi-echelon inventory models. The lost sales generalization of this model makes the policy evaluation a bit more complex, since an iterative procedure is needed to obtain the cost. Still, the model is rather simple and easy to implement. Moreover, in many practical situations lost sales is a reasonable way to model stock outs.

Hsieh (2002) proposed two fuzzy production inventory models with fuzzy parameters for crisp production quantity, or for fuzzy production quantity. The fuzzy total production inventory costs of these models under the fuzzy arithmetical operations of function principle are proposed. The final purpose is to find optimal solutions of these models by using graded mean integration representation method for defuzzifing fuzzy total production inventory cost, and by using extension of the Lagrangean method for solving inequality constrain problem. Furthermore, he founds that the optimal solutions are all crisp real numbers. In addition, when the fuzzy parameters (fuzzy inventory cost, fuzzy demand, fuzzy setup cost, fuzzy demand rate, and fuzzy production rate) are all crisp real numbers, the optimal solutions of proposed models can be specified to meet classical production inventory models. In the real world, the parameters and variables in inventory model may be almost uncertain datum. Graded mean integration representation method is used to defuzzify the trapezoidal fuzzy total production inventory cost. In the fuzzy production inventory model for crisp production quantity, the first derivative of fuzzy total production inventory cost is used to solve the optimal production quantity. Furthermore, the algorithm of extension of the Lagrangean method is used to solve inequality constrains in fuzzy production inventory model for fuzzy production quantity.

Lin et al. (2003) deals with inventory models that unify the inventory problems of raw materials and finished products for a single product imperfect manufacturing system. The products are manufactured in batches, and raw materials are jointly replenished from outside suppliers. The system is assumed to deteriorate during the production process. As a result, some proportion of defective items is produced. The defective items are reworked at some cost either before or after a sale. Periodic inspections at equally spaced times and restorations of the production process are used to operate the system. The objective is to minimize the expected total cost for the system. A solution procedure is developed to find a near optimal solution for the basic model. The analysis is extended to various cases where the defect rate is a function of the setup cost, the proportion of defective items is not constant, or the inventory system has a limited capacity for raw materials. Production of a single product produced in batches on a single facility or production process is considered in this study. Authors assumed that at the beginning of each production run the production process is in an "in-control" state and producing items that conform to the specification. After a period of time, the production process deteriorates. As a result, the process shifts from the in-control state to the out-of-control state in which a certain percentage of items produced are non-conforming. The imperfect production process and the raw materials are considered which are required for each production run. After the process is shifted from the in-control state to the out-of-control state, the state change will be discovered by inspection and followed by some restorative work. The inspection and restoration times are assumed to be negligible. Although this assumption can be relaxed, it is quite plausible, given the current advances in the technologies used for inspection. Moreover, the use of modular designs in complex production systems enables speedy repair of failed machines and restorations. The models are generalized to consider the case where the shift of the production process can be detected and corrected during a production run. The elapsed time until the process shifts is a random variable and assumed to have an exponential distribution with mean 1/p. It is assumed that the percentage of detectives, p, is a constant throughout the duration when the production process is in an out-of-control state. Furthermore, the defect rate, p, is assumed to be independent of the setup cost. In the development of the basic model, it is assumed that once the production process is out of control, the percentage of defective items produced (p) remains constant throughout the entire cycle. However, this assumption may not be realistic. It is conceivable that as the production system deteriorates over time, the percentage of defective items produced may increase accordingly. In this section, the basic model is extended to incorporate two dynamic cases: the production system deteriorates linearly after the system is in the out-of-control state, and the production system deteriorates exponentially after the process is in the out-of-control state. The integrated EPQ models deal with the joint effects of maintenance policy by inspection and the production-inventory system, including raw materials on the cost of operating a single facility. Periodic inspections and equally spaced inspection times are adopted to develop the basic model and to determine the relationship of reworking cost and warranty cost. A solution procedure is developed to obtain a near optimal production run time for the finished product, economic ordering quantity for each raw material, and an inspection schedule. Such a production run time is found to be shorter than that of the perfect production system. The time decreases as the number of raw materials or the warranty cost increases. The model is extended for the situation where both the mean elapsed time of the shift and the percentage of defective items produced is a function of the production setup cost. The analysis is further extended to incorporate cases where the process will deteriorate linearly or exponentially after a certain time. Finally, the model is directed toward a case that has a limited capacity for raw materials.

Cal et al. (2004) studied a single-period two-product inventory model with stochastic demands and downward substitution. Optimal order quantities provided some properties are represented in Cal et al. (2004) study. They considered that there are two products and two demand classes. The downward substitution means that, demands from class 2 can be satisfied using stocks of product 1, but stocks of product 2 cannot be used to satisfy demands from class 1. This substitution structure widely exists in real life, such as the product with higher capabilities or more functions can satisfy the demands for the product with lower capabilities or less functions. The researchers assumed that the demands for each product are stochastic and the retailer just places orders before the demands are realized for once. The order, holding, penalty, and salvage costs are proportional to the quantity, and the revenue earned is also linear in the quantity sold. The objective is to decide the order quantity for each product. They developed a general profit maximization model for the single-period two-product substitution problem, obtained the optimal condition for the order quantities, and some properties with respect to the optimal order quantities. In the mathematical model, the researchers studied a single-period two-product inventory model with stochastic demands, proportional revenues and costs, downward substitution. They developed a general profit maximization model and show that it is concave and sub modular, and then obtained the optimal condition for the order quantities. For the optimal quantities, they studied the impact of the parameters, and get some properties.

Generally, production quality is not always perfect. It is usually depend on the operating state of a production process which may shift from an in-control state to an out-of-control state due to occurrence of some assignable cause(s). When a signal for an assignable cause is triggered, a search is initiated and is terminated upon finding the cause within a pre-specified target time. The process is then brought back to an in-control state by repair. However, if the assignable cause is not discovered within the pre-specified time, production is allowed to continue until the next sampling or warning, whichever occurs first. In this case, either the alarm is considered to be false with a probability of Type I error, or the assignable cause has not been eliminated with a probability of Type II error. In the latter case, the process produces products in an out-of-control state until the next sampling or warning, whichever occurs first. However, this state does not indicate any severe damage to the system. Traditionally, $\overline{X}$ and/or R control charts have been used to monitor the stability of a production process. An $\overline{X}$ -chart is used to control process mean and an R chart to control process variance. It is possible both the process mean and process variance to vary simultaneously during a production cycle. Under these conditions, a generalized economic model for the joint determination of production quantity, an inspection schedule, and the design of the $\overline{X}$ and R control charts are developed by Rahim (2004). A direct search optimization method is used to determine the optimal decision variables of the economic model. This research makes an assumption that there is a pre-specified time to find an assignable cause when there is an out-of-control signal. If the source of the assignable cause cannot be detected within a pre-specified time, the process is allowed to continue until the next warning signal is triggered. This leads to Type I or Type II errors depending on the actual existence or non-existence of an error. The time for searching an assignable cause is considered to be an additional decision variable. This assumption may invite some criticism and deserves some explanation and/or justification. Some researchers might interpret it as contradicting the modern view in quality management that quality is the number one priority or as supporting the traditional concept of quantity over quality. In other words, time is considered to be more important than quality; hence, time is not to be wasted in fixing a problem. The implication of this assumption is that, in the long run, resources may be wasted in producing inferior goods. Customers may end up receiving poor quality products that should have been detected in the process. Loss of the customer's goodwill may be incurred, and the manufacturer will eventually lose competitiveness. From the numerical studies of this model, the optimal value of Type II error lies, approximately, between 0.01 and 0.05. That is, the probability of catching the shift if there is one, lies between 0.95 and 0.99. This indicates that if the assignable cause occurs and is not detected immediately, it will be detected, on average, after one sampling interval.

Marketing researchers and practitioners have long recognized the demand of many retail items is proportional to the amount of inventory displayed. Recently, two distinct types of inventory control models reflecting this relationship have appeared in the literature, models in which the demand rate of an item is a function of the initial inventory level and those in which it is dependent on the instantaneous inventory level. Urban (2005) presented a comprehensive overview of this literature and demonstrate the equivalence of the two types of models through the use of a simple, periodic-review model. An alternative approach to sensitivity analysis for inventory models with inventory-level-dependent demand is also presented. The operations management/operations research literature has recently focused considerable attention on inventory models for products exhibiting an inventory-level-dependent demand rate. In this paper, comprehensive review of this literature is conducted, distinguishing between Type I models in which the demand rate of an item is a function of the initial inventory level and Type II models in which it is dependent on the instantaneous inventory level. A periodic-review model is then developed, first solving the general Type I problem, then illustrating how the more complex, Type II model can be solved. Although the literature has evolved into these two distinct streams of research, the authors show that in situations in which the costs are not dependent on the inventory level throughout the period, the Type II model can be represented by an equivalent Type I model. The two types of models are not identical in all situations; for example, an equivalent Type I model (as presented) would not be suitable for a Type II model using average inventory levels for holding costs or average backorder levels for shortage costs.

Chang et al. (2006) considered a mixed inventory model involving variable lead time with backorders and lost sales. They first fuzzify the random lead-time demand to be a fuzzy random variable and obtain the total cost in the fuzzy sense. Then. they further fuzzify the total demand to be the triangular fuzzy number and derive the fuzzy total cost. By the centroid method of defuzzification, they derive the estimate of total cost in the fuzzy sense. Also, they find the optimal solution for order quantity and lead time in the fuzzy sense such that the total cost has a minimum value. The issue of lead-time reduction has received a great deal of attention in the field of production/inventory management. Lead time usually consists of the following components: order preparation, order transit, supplier lead time, delivery time and setup time. Although most of the literature dealing with inventory problems viewed lead time as an uncontrollable variable, however, in some practical situations, lead time can be reduced by controlling some or all of its components. The benefits associated with efforts to reduce lead time, such as lower the safety stock, reduce the loss caused by stock out, increase the service level to the customer, and gain the competitive advantages in business, can be clearly perceived through the Japanese successful experiences of using Just-in-time (JIT) production. The previous lead-time reduction models are based on the continuous review inventory systems in which the uncertainty of demand during lead time is tackled from the traditional probability theory and the annual average demand is assessed by a crisp value. However, various types of uncertainties and imprecision including randomness and fuzziness are inherent in real inventory environments, in the article; the authors address the issue of lead-time reduction under such circumstances. Specifically, they attempt to develop a fuzzy inventory model by considering the fuzziness and randomness for lead time demand, which is represented by a fuzzy random variable. Also, for the annual average demand, due to the fact that it may fluctuate a little in an unstable environment and is difficult to assess by a crisp value, they consider it as the fuzzy number. The purpose of this article is to recast Chang et al.'s mixture inventory model involving variable lead time with backorders and lost sales by further considering the fuzziness of lead-time demand and annual average demand. The authors aim at providing an alternative approach of modeling uncertainty that may appear in real situations; whereas they do not attempt to establish the superiority of proposing a new model to reduce more inventory cost than previous one. Authors also consider a mixture inventory model and address the issue of lead-time reduction in the fuzzy environments. Building upon Chang et al.'s (2006) model in which the annual average demand is a crisp value and the random lead-time demand is normally distributed, the authors first fuzzify lead time demand to be a fuzzy random variable and derive the total expected annual cost in the fuzzy sense. Then, they further fuzzify demand to be the triangular fuzzy number and obtain the fuzzy total cost. After defuzzification, they derive the estimate of total expected annual cost in the fuzzy sense and obtain the corresponding optimal order quantity and lead time.

An inventory model for deteriorating items with stock-dependent consumption rate and shortages under inflation and time discounting over a finite planning horizon is derived by Hou (2006). The total cost function is convex. With the convexity, a simple solution algorithm is presented to determine the optimal order quantity and the optimal interval of the total cost function. A sensitivity analysis of the optimal solution with respect to the parameters of the system is carried out. This model incorporates some realistic features that are likely to be associated with some kinds of inventory. First, deterioration over time is a natural feature for goods. Secondly, occurrence of shortages in inventory is a natural phenomenon in real situations. Thirdly, it has been observed in supermarkets that the demand is usually influenced by the amount of stock displayed on the shelves, i.e., the demand rate may go up or down if the on-hand inventory level increases or decreases. Fourthly, the DCF approach permits a proper recognition of the financial implication of the opportunity cost in inventory analysis. Next, since the inventory systems always need to invest large capital to purchase inventories, which it is highly correlated to the return of investment. Hence, it is important to consider the effects of inflation and the time value of money in formulating inventory replenishment policy. In keeping with this reality, these factors are incorporated into the present model. The model is very useful in the retail business. It can be used for electronic components, fashionable clothes, domestic goods and other products which are more likely with the characteristics above. Here, it has been given an analytic formulation of the problem on the framework described above and have presented an optimal solution procedure to find optimal replenishment policy. From this research results, it is also verified that the effects of inflation and the time value of money in formulating replenishment policy result in smaller discounted total cost than a policy which does ignore the effects of these factors.

Finally, the sensitivity of the solution to changes in the values of different parameters has been discussed. It is seen that changes in the consumption rate, the ordering cost and shortage cost lead to significant effects on the order quantity. The total cost is sensitive to changes in the consumption rate parameter, the unit cost and net discount rate of inflation (R). Moreover, special cases that influence the optimal total cost are also discussed. It is obviously observed that the total cost increases considerably with the case of the inflation and time value of money are not considered, that is, net discount rate of inflation is zero.

Islam and Roy (2006) developed an economic production quantity (EPQ) model with flexibility and reliability consideration of production process and demand dependent unit production cost. The model has involved one storage space constraint. It is formulated in fuzzy environment introducing fuzziness in objective and constraint goals, coefficient and indexes of objective function and constraint. The model is proposed to by fuzzy geometric programming technique. The determination of the most cost-effective production quantity is commonly known as classical economic production quantity model. Over the last three decades tremendous amount of research effort has been expanded on this topic. The inventory problem is controlled by some constraints. Here one constraint is capacity constraint. But, in real life problems, it is almost impossible to predict the restricted resource amount precisely. Decision maker may change it within some limits as per the demand of the situation. Hence it may be assumed uncertain in non-stochastic sense but fuzzy in nature. The researchers have formulated a fuzzy economic production quantity (FEPQ) model with flexibility and reliability consideration of production process and demand dependent unit production cost under the storage space constraint. Shortages are not permitted in this model.

Law and Wee (2006) developed an inventory model of an integrated production-inventory model from the perspectives of both the manufacturer and the retailer. They considered both ameliorating and deteriorating effects taking account of multiple deliveries, partial backordering and time discounting. The amelioration and deterioration are assumed to follow the Weibull distribution. The discounted cash flow and optimization technique are used to derive the optimal solution. This model is particularly useful for items that ameliorate and deteriorate at the same time. Amelioration occurs when the value or utility of a product increases over time. Weibull ameliorating rates with decreasing function with time and Weibull deteriorating rates with increasing function with time are used in this model. Weibull distribution is used to represent the product in stock as it deteriorates with time. The deterioration rate is assumed to increase with age; that is, the longer the items remained unused, and the higher the deteriorating rate. Partial backordering is considered for a fair competitive market because some customers are impatient, and may find an alternative source when there is a shortage. The main contribution of the model is to incorporate manufacturer–retail cooperation and consider the factors of amelioration, deterioration, Weibull distribution, multiple deliveries, partial backordering and time discounting simultaneously. They applied the discounted cash flow (DCF) approach and optimization technique to determine the optimal production and replenishment policy. The model considered deterioration, amelioration, multiple deliveries, partial backordering and time discounting. The model is particularly useful for items that ameliorate and deteriorate at the same time. The discounted cash flow and optimization technique are used to derive the optimal solution.

Mohebbi (2006) considered that a production-inventory system with limited storage capacity in which demand for a single item arises according to a compound Poisson stream whose occurrence rate and random batch sizes at any point in time depend on the state of a random (external) environment, and production occurs continuously and uniformly over time at a rate determined by the state of another independent and randomly changing (internal) environment. It is assumed that each of the two randomly changing environments is represented by a continuous-time homogeneous Markov chain with a discrete state space. The facility starts production as soon as the inventory level drops below the storage limit and the production continues until the inventory level reaches the storage full capacity. Assuming that the shortages are lost, the author has derived the steady-state distribution of the inventory level which is used to formulate some measures of common interest regarding the performance of such systems. The author has also formulated an expected-net-revenue maximization problem and provides some numerical results. In the production-inventory model, in which, the production and demand rates are modulated by two independent continuous-time homogeneous Markov chains. For stochastic piecewise linear production and compound Poisson demands, the author has applied level-crossing theory to derive the limiting distribution of the inventory level in a limited capacity production-storage system with lost sales. A number of important performance measures including an expected revenue accumulation rate function were formulated and some numerical results in terms of sensitivity analysis and optimality of the system parameters were presented. These results clearly demonstrate the critical impact of the variability induced by randomly changing environments on the system performance.

Boute et al. (2007) considered a two-echelon supply chain: a single retailer holds a finished goods inventory to meet customer demand, and a single manufacturer produces the retailer's replenishment orders on a make-to-order basis. In the setting the retailer's order decision has a direct impact on the manufacturer's production. It is a well known phenomenon that inventory control policies at the retailer level often propagate customer demand variability towards the manufacturer, sometimes even in an amplified form (known as the bullwhip effect). The manufacturer, however, prefers to smooth production, and thus he prefers a smooth order pattern from the retailer. At first sight a decrease in order variability comes at the cost of an increased variance of the retailer's inventory levels, inflating the retailer's safety stock requirements. However, integrating the impact of the retailer's order decision on the manufacturer's production leads to new insights. A smooth order pattern generates shorter and less variable (production/replenishment) lead times, introducing a compensating effect on the retailer's safety stock. They show that by including the impact of the order decision on lead times, the order pattern can be smoothed to a considerable extent without increasing stock levels.

Dutta et al. (2007) developed a continuous review inventory system where the annual average demand is treated as a fuzzy random variable. Also, the lead-time demand is assessed by a triangular fuzzy number. Since the annual demand is a fuzzy random variable, the associated total cost function is also a fuzzy random variable. As a result, the total expected cost becomes a fuzzy quantity. They find the optimal order quantity along with the reorder point so that the associated total cost is minimum. Using possibilistic mean value of a fuzzy number the fuzzy expected cost has been minimized here.

Atici and Uysal (2008) proposed to investigate the optimal production and inventory paths of HMMS type models (proposed by Holt, Modigliani, Muth and Simon) on complex time domains. Time scale calculus which is a rapidly growing theory is a main tool for solving and for analyzing the model. In the case, one can use the results of the qualitative analysis in time scales, whose development is in progress. A flexible and capable modeling technique is considered here.

Chen and Chang (2008) introduced a fuzzy economic production quantity (FEPQ) model with defective productions that cannot be repaired. Authors consider a fuzzy opportunity cost and trapezoidal fuzzy costs under crisp production quantity or fuzzy production quantity in order to extend the traditional production inventory model to the fuzzy environment. Authors use Function Principle as arithmetical operations of fuzzy total production inventory cost (FTPIC), and use the Graded Mean Integration Representation method to defuzzify the fuzzy total production and inventory cost. Then they use the Kuhn–Tucker method to find the optimal economic production quantity of the fuzzy production inventory model.

Lin (2008) developed periodic review inventory model with variable lead time by considering the fuzziness of expected demand shortage and backorder rate. Author fuzzified the expected shortage quantity at the end of cycle and the backorder (or lost sales) rate, and then obtained the fuzzy total expected annual cost. Using the signed distance method to defuzzify, the estimate of total expected annual cost in the fuzzy sense is derived. For the proposed model, author obtained the optimal review period and optimal lead time in the fuzzy sense so that the total expected annual cost in the fuzzy sense has a minimum value. For a periodic review inventory control system with variable lead time, Ouyang and Chuang (2001) have treated the review period and lead time as decision variables in crisp set. This paper explores a similar inventory model in which not only the expected demand shortage is fuzzified as a fuzzy random variable in the fuzzy sense, but the lost sales rate is also considered to be a fuzzy number. The researcher then obtained the total expected annual cost in the fuzzy sense. Here, author used signed distance method to defuzzify the fuzzy total expected annual cost and obtain an estimate of the total expected annual cost in the fuzzy sense.

Mohebbi and Hao (2008) assumed that an unreliable supplier in a single-item stochastic inventory system alternates randomly between two possible states (i.e., available and unavailable), following a two-state continuous-time homogeneous Markov chain. For a compound Poisson stream of demands and Erlang lead times, their model considers the scenario where the processing of the outstanding order (if any) is interrupted at every supplier's transition epoch from the available to the unavailable state, and is restarted from the outset upon the supplier's regaining its available state. They derived the stationary distribution of the on-hand inventory under a continuous-review policy. The model takes an explicit new account of the impact of supplier's availability on the replenishment lead time and therefore, expands the analytical treatment of supply interruption to a broader extent. The primary objective in modeling the system is to derive the stationary distribution of the inventory level as an enabler for establishing various performance measures. The authors utilize a level-crossing approach to accomplish this objective and use the resulting distribution to obtain the exact functional form of the long-run expected total cost (i.e. the sum of ordering, holding and shortage costs) per unit time of operating this inventory system.

Maiti (2008) developed multi-item inventory model with stock-dependent demand and two-storage facilities is developed in fuzzy environment (purchase cost, investment amount and storehouse capacity are imprecise) under inflation and time value of money. Joint replenishment and simultaneous transfer of items from one warehouse to another is proposed using basic period (BP) policy. As some parameters are fuzzy in nature, objective (average profit) functions as well as some constraints are imprecise in nature. Model is formulated as to optimize the possibility/necessity measure of the fuzzy goal of the objective function and constraints are satisfied with some pre-defined necessity. A genetic algorithm (GA) is developed with roulette wheel selection, binary crossover and mutation and is used to solve the model when the equivalent crisp form of the model is available. In other cases fuzzy simulation process is proposed to measure possibility/necessity of the fuzzy goal as well as to check the constraints of the problem and finally the model is solved using fuzzy simulation based genetic algorithm (FSGA). Also a simulation approach is proposed to determine this possibility/ necessity measure in complex fuzzy environment.

Panda et al. (2008) developed a mathematical model for a single period multi-product manufacturing system of stochastically imperfect items with continuous stochastic demand under budget and shortage constraints. After calculating expected profit in general form in terms of density functions of the demand and percentage of imperfectness, particular expressions for those density functions are considered. Here the constraints are of three types: both are stochastic, one stochastic and other one imprecise (fuzzy) and both imprecise. The stochastic constraints have been represented by chance constraints and fuzzy constraints in the form of possibility/necessity constraints. Stochastic and fuzzy constraints are transformed to equivalent deterministic ones using 'here and now' approach and fuzzy relations respectively. The deterministic problems are solved using a non-linear optimization technique-generalized reduced gradient method. The authors propose an extension to economic production lot size model for imperfect items in which the production rate is assumed to be finite and demand rate is stochastic (continuous) under uncertain budget and shortage constraints.

Rezaei and Davoodi(2008) developed a deterministic inventory model considering the scenario of supply chain with multiple products and multiple suppliers, all of which have limited capacity. They also considered the situation of imperfect quality. The researchers introduced imperfect items and storage capacity in the lot sizing with supplier selection problem and formulate the problem as a mixed integer programming model. Then the model is solved with a genetic algorithm. Using classical optimization methods the single optimal solution can be obtained but it is true when the problem is small. Some higher level information (such as social and cultural consideration) could not be included in the model using classical optimization methods. Although these solutions (obtained via GA) are not necessarily optimal and are almost near-optimal, it can be possible for decision maker to select one of them that matches with the real world condition.

Maiti et al. (2008) provided two defuzzification techniques for two fuzzy inventory models using extension principle and duality theory of non-linear programming and interval arithmetic. An EOQ model for deteriorating items is considered in fuzzy environment. Here, the inventory cost coefficients, storage space and budgetary cost are fuzzy and represented by fuzzy numbers. Demands and rates of deterioration of the items are constant. The fuzzy total average cost for the inventory control system is derived and minimized. Firstly, using the Zadeh's extension principle a pair of mathematical models are formulated to calculate the lower and upper bounds of the α -cuts of total average fuzzy inventory cost at possibility level α. The first model is solved easily using a gradient based optimization technique (Generalized Reduced Gradient method), but to solve the second model the authors proposed the duality theorem of non-linear programming such that the primal and dual models have the same objective value.

Tarim and Smith (2008) proposed a constraint programming model for computing the finite horizon single-item inventory problem with stochastic demands in discrete time periods with service-level constraints under the non-stationary version of the "periodic review, order-up-to-level" policy (i.e., non-stationary (R,S) or, simply (R_n, S_n)). The computational tests show that the approach is more tractable than the conventional MIP formulation. Two different domain reduction methods are proposed to improve the computational performance of solution algorithms.

Vijayan and Kumaran (2008) introduced fuzziness by allowing the cost components imprecise and vague to certain extent. Continuous review and periodic review inventory models in which a fraction of demand is backordered and the remaining fraction is lost during the stock out period are considered under fuzzy environment. Authors used trapezoidal fuzzy numbers to represent these characteristics. The optimum policies of these models under fuzzy costs are also derived by them. Inventory models which involve both backorders and lost sales are known as models with a mixture of backorders and lost sales. Stochastic inventory models with a mixture of backorders and lost sales are described by introducing fuzziness in the cost parameters. The inventory model also implies that the impact of fuzziness is nullified when each of the cost components are allowed fuzziness in equal percentages.

Chung et al. (2009) proposed an inventory model which showed that the total cost function per unit time is convex by a rigorous proof, derived the closed forms for the upper and lower bounds on the optimal cycle time of the total cost function per unit time, thereby enabling straightforward application of the standard bisection algorithm to numerically compute the optimal cycle time and compared optimal solutions obtained by using this approach (the bisection algorithm) and Park's approach.

Chung et al. (2009) established a new inventory model with two warehouses for items with imperfect quality. They developed a mathematical model by maximizing the annual total profit. They developed an inventory model to incorporate concepts of two warehouses and imperfect quality to relax assumptions (All units produced are of perfect quality and. The inventories are stored by a single warehouse with unlimited capacity) to establish a new economic production quantity model. In practice, the above two assumptions are unrealistic and product quality is not always perfect but directly affected by the reliability of the production process used to produce the products. On the other hand, as we all know, the capacity of any warehouse is limited. The researchers have incorporated concepts of the basic two warehouses and imperfect quality in their mathematical model. They concluded that expected total profit per unit time ETPU(y) is piecewise concave.

Banerjee et al. (2009) recast an inventory model to be more relevant to current situations, where the penalty cost for a shortage occurrence at a downstream stage in a supply chain is continually transmitted to the upstream stages. The supplier, in this case, at the upstream of the supply chain is responsible for all the downstream shortages due to the chain reaction of its backlog. Authors developed a model in which the backorder cost per unit time is a linearly increasing function of shortage time, and it claims that the optimal policy for the supplier is setting the optimal shortage time per inventory cycle to minimize its total relevant cost in a JIT environment.

Liu and Lian (2009) considered the cost-effective inventory control of work-in-process (WIP) and finished products in a two-stage distributed manufacturing system. The first stage produces a common WIP, and the second stage consists of several production sites that produce differentiated products with different capacity and service level requirements. The unit inventory holding cost is higher at the second stage. Authors used a network of inventory-queue model to evaluate the inventory cost and service level achievable for given inventory control policy, and then derives a very simple algorithm to find the optimal inventory control policy that minimizes the overall inventory holding cost and satisfies the given service level requirements. The researchers consider a distributed manufacturing system that consists of two production stages. The first production stage produces a common important component with limited capacity, and the second stage is a differentiation stage that consists of several stations which produce different finished products. taking the output of first stage and other possible components as their raw materials. It develops a simple "frontier curve" optimization method to find the optimal inventory control policies at all sites that provide the required service levels for all finished products simultaneously. The optimization algorithm is iterative and allows for easy "what-if" analysis, and considers explicitly the joint cost and performance effect of inventory control policy at all sites to the system.

Mondal et al. (2009) considered an economic production lot (EPL) model considering, the unit production cost depends upon the cost of raw materials, labor charges advertisement cost, produced units, etc.. demand rate is a deterministic function of selling price and the advertisement cost and Selling price is determined by a mark-up over the unit production cost. The authors have made an extension to the economic production lot size model incorporating both marketing decision regarding the defective items and variable unit price depending on the rate of production. It is a common belief that more production gives more profit. Another popular myth is that profit increases with the increase of selling price per unit item.

Panda and Maiti (2009) considered, multi-item economic production quantity (EPQ) models with selling price dependent demand, infinite production rate, stock dependent unit production and holding costs in this paper. Flexibility and reliability consideration are introduced in the production process. The models are developed under two fuzzy environments - one with fuzzy goal and fuzzy restrictions on storage area and the other with unit cost as fuzzy and possibility - necessity restrictions on storage space. The objective goal and constraint goal are defined by membership functions and the presence of fuzzy parameters in the objective function is dealt with fuzzy possibility/necessity measures. The models are formed as maximization problems. The first one-the fuzzy goal programming problem is solved using fuzzy additive goal programming (FAGP) and modified geometric programming (MGP) methods. The second model with fuzzy possibility/necessity measures is solved by geometric programming (GP) method. The authors have considered demand as power function of selling price, unit costs dependent on inventory level, holding costs again as functions of unit costs. Here they have formulated multi-item profit maximization production inventory models with limited storage area in fuzzy/fuzzy possibility and necessity sense under process reliability and flexibility. They have considered fuzzy possibility and necessity measures of the objective function when its some parameters are fuzzy.

A production inventory model for a newly launched product is developed by Roy et al. (2009) incorporating inflation and time value of money. It is assumed that demand of the item is displayed stock dependent and lifetime of the product is random in nature and follows exponential distribution with a known mean. Here learning effect on production and setup cost is incorporated. Model is formulated to maximize the expected profit from the whole planning horizon. A fuzzy-based lifetime extension of genetic algorithms is considered. A genetic algorithm (GA) with varying population size is used to solve the model where crossover probability is a function of parent's age type (young, middle-aged, old, etc.) and is obtained using a fuzzy rule base and possibility theory. In this GA a subset of better children is included with the parent population for next generation and size of this subset is a percentage of the size of its parent set. This GA is named fuzzy genetic algorithm (FGA) and is used to make decision for above production inventory model in different cases. The model is illustrated with some numerical data. Sensitivity analysis on expected profit function is also presented. Performance of this GA with respect to some other GAs is compared. An EPQ model has been considered under inflation and time discounting over a stochastic time horizon incorporating the learning effect on both the production and setup cost.

Maiti et al. (2009) developed an inventory model for a deteriorating item (seasonal product) with linearly displayed stock dependent demand in imprecise environment (involving both fuzzy and random parameters) under inflation and time value of money. Authors assumed that time horizon, i.e., period of business are random and follow exponential distribution with a known mean. The resultant effect of inflation and time value of money is assumed as fuzzy in nature. The particular case, when resultant effect of inflation and time value is crisp in nature, is also analyzed. For crisp inflation effect, the total expected profit for the planning horizon is maximized using the GA (Genetic Algorithm) to derive optimal inventory decision. On the other hand when inflationary effect is fuzzy then the expected profit is fuzzy in nature too. For crisp model expected profit is proposed to maximize using a GA with roulette wheel selection, arithmetic crossover and random mutation. In the case of fuzzy model, a fuzzy simulation process is proposed to maximize the optimistic/pessimistic return of the objective function and a fuzzy simulation based genetic algorithm with GA operators is developed to solve the model. To solve the stochastic model (model-1) GA is used. The basic technique to deal problems of the fuzzy stochastic model (model-2) is to convert the possibility/necessity constraint to its deterministic equivalent.

Expected inventory order crossovers occur if at the moment of ordering it is expected that orders will not arrive in the sequence they are ordered. Recent research has shown that, expected inventory order crossovers will be encountered more frequently in future. and that use of a myopic order-up-to policy based on a stochastic dynamic programming approach leads to improved performance compared to the classical approach. Riezebos and Gaalman (2009) showed that the improved policy is still heuristic in nature, as it neglects several control options that are available on the various ordering moments and makes some restrictive assumptions with respect to the separability (i.e., decomposability) of the stochastic dynâmic programming problem.

Sakaguchi (2009) discussed a method to get an economic order quantity in inventory systems with discrete demand to determine whether it increases or decreases from period to period. The researchers assumed h and p are the holding cost and the shortage costs per unit per period, respectively, and c is the purchasing cost per unit, z is the amount on hand in initial period after a regular order is received, t length of period, N number of periods and they also assumed that demand in a single period be a discrete random variable. The decision criterion of single period is the minimization of the expected cost which includes the purchasing, holding and shortage costs.

Sharma (2009) developed a composite for a generalized environment of a manufacturing situation, where the procurement of multiple input items has been considered along with the production of a finished item and fractional backordering is included. The model can be used for a variety of situations. They provided the output parameters for possible combinations of input parameters, which can be used directly and are also suitable for further analysis. According to the researcher a 100% inspection of manufactured product is carried out and therefore the finished product inventory includes non-defective items only. However the manufacturing facility will produce certain defective items also in general and therefore these are incorporated in the relevant cost formulation and the generalized procedure. The composite model can be used for a wide variety of cases. For example, the formulation can be adjusted for the situation when a particular facility is generating the acceptable items only. The researcher developed this composite model incorporating the manufacturing and procurement of input items. Author included fractional/partial backordering along with the finite replenishment rate of input items. The model is suitable for adapting to the variety of situations available in industrial/business environment. In the context of continuous manufacturing, the demands are usually more than the production rate and these demands are looked at from a different perspective in any manufacturing system. But there are cases in the batch production where the production rate is usually greater than the demand rate. Therefore it becomes relevant to formulate the total cost and to obtain the output parameters after optimizing the costs and this model serves this purpose properly.

Hsu et al. (2009) developed a fuzzy multi-objective joint replenishment inventory model of deteriorating items. The model maximizes the profit and return on inventory investment (ROII) under fuzzy demand and shortage cost constraint. They propose a novel inverse weight fuzzy non-linear programming (IWFNLP) to formulate the fuzzy model. A soft computing, differential evolution (DE) with/without migration operation, is proposed to solve the problem. The performances of the proposed fuzzy method and the conventional fuzzy additive goal programming (FAGP) are compared. They show that the solution derived from the IWFNLP method satisfies the decision maker's desirable achievement level of the profit objective, ROII objective and shortage cost constraint goal under the desirable possible level of fuzzy demand. It is an effective decision tool since it can really reflect the relative importance of each fuzzy component. A multi-objective joint replenishment deteriorating items inventory model with stock-dependent demand and total average shortage cost constraint is formulated in a fuzzy environment. Under the basic period (BP) approach, the objective of this model is to maximize the profit and ROII objectives. ROII is defined as the ratio of profit to the average inventory investment. Two fuzzy scenarios are considered in this study. A multi-objective inventory model with fuzzy shortage cost constraint is first developed. Then they extend the model by assuming the demands are also fuzzy in nature. The fuzzy multi-objective models are formulated using FAGP method. A novel method IWFNLP is also proposed. This method embeds the idea of inverse weights into the Max–Min fuzzy model. The results of the FAGP and IWFNLP methods solved by DE with/without migration operation are illustrated numerically. In the numerical examples, different patterns of weights are assigned to each fuzzy component including the possible level of fuzzy demand, profit objective, ROII objective and total shortage cost constraint goal. A multi-objective joint replenishment deteriorating items inventory model with fuzzy demand and shortage cost constraint is developed. The model with fuzzy demand and shortage cost constraint that simultaneously maximizes the profit and return on inventory investment objectives is non-existence. The researchers also initiate the formulation of a multi-objective inventory model by the inverse weight fuzzy non-linear programming (IWFNLP) and fuzzy additive goal programming (FAGP) methods simultaneously. An efficient constraint handling method is embedded into the differential evolution (DE) with/without migration technique to solve the complex inventory model.

In order to understand the potential benefits of the consignment stock policy, an analytical model is offered by Zavanella and Zanoni (2009) with reference to the interesting industrial case of a single-vendor and multiple buyer productive situation, thus obtaining the optimal replenishment decisions for both the vendor and buyers in such a situation. The results show how the CS policy works better than the uncoordinated optimization. The study aimed at proposing a model for a single-vendor multi-buyer system, integrated in a shared management of the buyers' inventory, so as to pursue a reduction or the stability of the holding costs while descending the chain. The inventory management is carried out according to the CS practice and, consequently, the model appears to be simpler. The results show that the joint management of the inventory gives rise to economic benefits, which, however, may be modest or relevant according to the structure of the chain. The results themselves suggested the development of a sensitivity analysis, which allowed drawing some interesting remarks on the influence of the parameters relevant to the economic performance of the supply chain.

Benjaafar et al. (2010) considered the control of a production-inventory system with impatient customers. The optimal policy can be described using two thresholds: a production base-stock level that determines when production takes place and an admission threshold that determines when orders should be accepted. Here, it is described that an algorithm for computing the performance of the system for any choice of base-stock level and admission threshold. In a numerical study, the authors compare the performance of the optimal policy against several other policies. A system is considered where a single product is produced at a single facility to fulfill demand from customers who place orders continuously over time according to a Poisson process with rate λ. Items are produced one unit at a time with exponentially distributed production times with mean $1/\mu$. The production facility can produce ahead of demand in a make-to-stock fashion. However, items in inventory incur a holding cost h per unit per unit time. Upon arrival, an order is either fulfilled from inventory, if any is available, backordered, or rejected. If an order is rejected, the system incurs a rejection cost r. If an order is backordered, the system incurs no immediate cost. However, customers are impatient and may decide to cancel their orders if their waiting time in backlog exceeds a patience time.

Chen and Kang (2010) developed the integrated inventory models with permissible delay in payment, in which customers' demand is sensitive to the buyer's price. The models consider the two-level trade policy in the vendor–buyer and buyer–customer relationships in supply chain management. A simple recursive solution procedure is proposed for the integrated models to determine the buyer's optimal pricing and production/order strategy. Although the total profit from the buyer and vendor increases together, the buyer's share lessens. To compensate the buyer's loss due to the cooperative relationship, a negotiation system is presented in order to allocate the profit increase to the vendor and buyer to determine the pricing and production/order strategy.

Haji and Darabi (2010) investigated the effect of information update on a specific class of problems specifically the effect on the decisions related to a single-period inventory model .The goal is to find the best set of decisions to minimize the total cost of the inventory systems. The paper helps to find the optimal policy for a single-period inventory system with information update choice. The introduction of update decision generated a total cost function that is not necessarily convex.

Liao and Huang (2010) considered the impact of a replenishment policy on the timing of the cash flows associated with payments to suppliers and revenue streams from customers. That is, the same cash amount will possess different money value at different future time. The more appropriate net present value (NPV) object instead of the average cost objective is adopted in this paper. In addition, the deteriorating effects will be incorporated in this inventory model, and the time to deterioration of each item follows an exponential distribution. The discounted cash flow (DCF) approach is used to derive the optimal solution in this study. Furthermore, is has been first found that the optimal solution not only exists bus also is unique. Then, the authors provided a theorem to locate the optimal ordering policy of two levels trade credit, deterioration and time discounting simultaneously. The discounted cash flow (DCF) approach to determine the optimal ordering policy is applied. The paper incorporates some realistic features that are likely to be associated with the inventory of some kinds of items. One is that deterioration over time is a natural feature for items; the other is that the DCF approach permits a proper recognition of the financial implication of the opportunity cost in inventory analysis with two levels of trade credit.

He et al. (2010) developed a production-inventory model for deteriorating items with multiple-market demand, where each market has a different selling season and a different constant demand rate. To foster additional managerial insights, the authors performed extensive sensitivity analyses and illustrated the results with a simulation study. Global markets offer selling opportunities and pose production management challenges for manufacturers of deteriorating items. Exploiting the difference in timing of the selling season of the deteriorating items at different markets is a unique opportunity to improve the profitability of a deteriorating items' manufacturer. In this paper, it has suggested a method for finding the optimal production and inventory schedule for manufacturers of deteriorating items. Here, it is assumed that the manufacturer produces in one location and sells in different markets that have different selling seasons. It has showed that our method helps minimize costs. The model considered the demand rate in each market as constant.

Kevork (2010) considers the classical single-period inventory model, also known as the Newsboy Problem, with the demand normally distributed and fully observed in successive inventory cycles. The extent of applicability of such a model to inventory management depends upon demand estimation. Appropriate estimators for the optimal order quantity and the maximum expected profit are developed. The statistical properties of the two estimators are explored for both small and large samples, analytically and through Monte-Carlo simulations. For small samples, both estimators are biased. The form of distribution of the optimal order quantity estimator depends upon the critical fractile, while the distribution of the maximum expected profit estimator is always left-skewed. Small samples properties of the estimators indicate that, when the critical fractile is set over a half, the optimal order quantity is underestimated and the maximum expected profit is overestimated with probability over 50%, whereas the probability of overestimating both quantities exceeds again 50% when the critical fractile is below a half. For large samples, based on the asymptotic properties of the two estimators, confidence intervals are derived for the corresponding true population values. The validity of confidence intervals using small samples is tested by developing appropriate Monte-Carlo simulations.

Leung (2010) first generalized a number of integrated models with/without lot streaming and with/ without complete backorders under the integer–multiplier coordination mechanism, and then individually derived the optimal solution to the three- and four-stage model, using algebraic methods of complete squares and perfect squares. It is subsequently deduced optimal expressions for some well-known models. For this model, Leung checked that the optimal solution, which is algebraically derived, is a global one. Two ready extensions of this model that warrant future research endeavors in this field are: First, following the evolution of three- and four-stage multi-firm supply chains, can the integrated model of a five- or higher-stage multi-firm supply chain is formulated and algebraically analyzed. Secondly, using complete and perfect squares, we can solve the integrated model of a n-stage multi-firm supply chain either for an equal cycle time, or an integer multiplier at each stage with a fixed ratio partial backordering allowed for some/all downstream firms (or retailers), with or without lot streaming can be solved.

An optimal production inventory model with fuzzy time period and fuzzy inventory costs for defective items is formulated and solved by Mandal et al. (2010) under fuzzy space constraint. Here, the rate of production is assumed to be a function of time and considered as a control variable. Also the demand is linearly stock dependent. The defective rate is taken as random, the inventory holding cost and production cost are imprecise. The fuzzy parameters are converted to crisp ones using credibility measure theory. The different items have the different imprecise time periods and the minimization of cost for each item leads to a multi-objective optimization problem. The model is under the single management house and desired inventory level and product cost for each item is prescribed. The multi-objective problem is reduced to a single objective problem using global criteria method (GCM) and solved with the help of fuzzy riemann integral (FRI) method, Kuhn–Tucker condition and generalized reduced gradient (GRG) technique. In optimum results including production functions and corresponding optimum costs for the different models are obtained. A multi-objective and multi-item defective dynamic system with a resource constraint with different fuzzy time periods has been solved for the first time via GCM, FRI, Kuhn–Tucker condition and GRG methods.

For the first time, a dynamic production inventory model with imprecise time periods under space constraint has been formulated and solved.

Rieksts and Ventura (2010) discusses inventory models over an infinite planning horizon with constant demand rate and two modes of transportation. These transportation options include truckloads and a less than truck- load carrier. An optimal algorithm is derived for a one-warehouse one-retailer system. A power-of-two heuristic algorithm is also proposed for a one-warehouse multi-retailer system. Computational results are provided to show that, on the average, the heuristic algorithm is at least 94% effective. An optimal policy is derived for a one-warehouse one-retailer inventory system that includes the option of transporting freight with two different modes. Although the optimal policy may have non-stationary order intervals, a tractable optimal algorithm is developed.

Sana (2010) developed a model to determine the optimal product reliability and production rate that achieves the biggest total integrated profit for an imperfect manufacturing process. The basic assumption of the classical economic manufacturing quantity (EMQ) model is that all manufacturing items are of perfect quality. The assumption is not true in practice. Most of the production system produces perfect and imperfect quality items. In some cases the imperfect quality (nonconforming) items are reworked at a cost to restore its quality to the original one. Rework cost may be reduced by improvements in product reliability (i.e., decreasing in product reliability parameter). Lower value of product reliability parameter results in increase development cost of production and also smaller quantity of nonconforming products. The unit production cost is a function of product reliability parameter and production rate. As a result, higher development cost increases unit production cost. The problem of optimal planning work and rework processes belongs to the broad field of production–inventory model which deals with all kinds of reuse processes in supply chains. These processes aim to recover defective product items in such a way that they meet the quality level of 'good item'. The benefits from imperfect quality items are: regaining the material and value added on defective items and improving the environment protection. In this point of view, a model is introduced here to guide a firm/industry in addressing variable product reliability factor, variable unit production cost and dynamic production rate for time-varying demand. The paper provides an optimal control formulation of the problem and develops necessary and sufficient conditions for optimality of the dynamic variables. In this purpose, the Euler–Lagrange method is used to obtain optimal solutions for product reliability parameter and dynamic production rate.

Production, remanufacture, and waste disposal EPQ type models are developed and analyzed by Saadany and Jaber (2010), where a manufacturer serves a stationary demand by producing new items of a product as well as by remanufacturing collected used/returned items. In these developed models, the return rate of used items is modeled as a demand-like function of purchasing price and acceptance quality level of returns. The model developed herein is a decision tool that helps managers in determining the optimum acceptable acquisition quality level and its corresponding price for used items that are collected for recovery purposes and that minimizes the total system cost. The model developed by assuming the return rate of used items follows a demand-like function dependent on two decision variables which are the purchasing price, P, and the acceptance quality level, q, for these returned items. In addition, this paper accounts for the cost of raw materials required to produce a single new unit of the product, C_n, where the monetary value of the purchasing price for a returned item is $PM = P - C_n$. Two models are developed in this paper. The first model assumes a single production cycle and a single remanufacturing cycle per interval T. The second model, a generalization of the first, assumes m remanufacturing cycles and n production cycles per interval T.

Yang (2010) developed an economic order quantity (EOQ) model, in which, shortages are partial back- logging to reflect the fact that longer the waiting time; the smaller the backlogging rate, the effects of inflation and time value of money are relevant or vital, and the replenishment cycles and the shortage intervals are time- varying. As a result, the proposed model is in a general framework that includes numerous previous models. The optimal replenishment schedule uniquely exists and the total profit associated with the inventory system is a concave function of the number of replenishments is proved.

The above mentioned papers did not consider the inventory model with fuzzy random demand and inventory cost with reliability of the production process which are most realistic factors for a production process. This thesis paper develops an inventory model to incorporate concept of reliability with fuzzy random demand and inventory cost consideration to establish a new economic production quantity model. Consequently, the inventory model in this paper is more practical than the traditional EPQ model.

CHAPTER III
PROBLEM IDENTIFICATION

A retailer may estimate that the demand of a commodity may follow a particular distribution. Again, inventory holding cost of the product may follow a particular distribution. However, it is very difficult to estimate the exact value of the parameters of the distribution. In this case, these parameters are considered as fuzzy numbers. Consequently, the distribution of these parameters are fuzzy random distribution and it can be said that the demand and inventory holding cost are fuzzy random. A basic assumption in the inventory management system is that set-up cost for production is fixed. In addition, the models also implicitly assume that items produced are of perfect quality. However, in reality, products are not always perfect but are directly affected by the reliability of the production process employed to manufacture the product.

3.1 Problem Definition

In this thesis work, an economic production quantity (EPQ) model is considered, where demand and inventory holding cost of the product are fuzzy random in nature with known probability distribution and the production process is assumed to be not 100% perfect, i.e. a fraction of the produced items are defective. Moreover, it is assumed that the defective items are sold at a reduced price and the selling price of fresh units is taken as a mark-up over the unit production cost. The model is formulated to maximize the expected average profit. Since demand and inventory holding cost are fuzzy random in nature, expected profit is a fuzzy number. So α-cut of the expected profit is obtained and by using this graded mean integration value (GMIV) of expected profit is obtained and this is then optimized using unconstraint signomial geometric programming to determine optimal decision of decision maker. Mathematical equations are obtained from optimal feasible solutions considering fuzzy random demand and inventory holding cost along with reliability of the production process which are very important in real life production inventory problem. Equations for set-up cost per cycle, reliability of the production process, duration of time until production is being held and maximum total profit are developed for this production inventory problem.

3.2 Assumption of the study

Some assumptions are considered in this thesis works. Assumptions are as follows:

i. Preparation time is negligible.

ii. Production starts immediately after receiving the order.

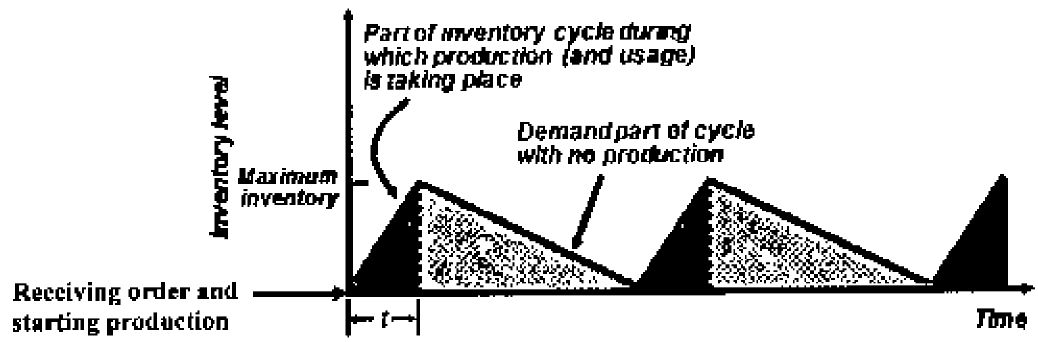

Fig. 3.1: Instantaneous production in EPQ model.

iii. Here, production period (T_1), production process reliability (r) and set-up cost (C_0) are decision variables.

iv. Defective items are sold immediately with a lower price than fresh items.

v. Selling price (S_1) of fresh units is mark-up (m) of production cost (P) such that, $S_1 = mP; m > 1$.

vi. Selling price (S) of defective units is mark up (m_1) of production cost (P) such that, $S_2 = m_1 P; 0 < m_1 \leq 1$.

vii. The total fresh units are greater than the demand, i.e. $rK > \tilde{D}$.

Where K is production rate per day and $\tilde{D}$ is demand per day which in fuzzy random in nature.

viii. Total cost of interest and depreciation per production cycle $Y(C_0, r)$ is inversely related to set-up cost co and directly related to process reliability (r) according to following general power function (Cheng, 1989):

$$Y(C_0, r) = aC_0^{-b} r^c$$

Where a, b and c are positive constants chosen to provide best fit of the estimated cost function. This assumption is based on the fact that to reduce the costs of production set-up and scrap and rework on shoddy products, substantial investment in improving the flexibility and reliability of the production process is necessary. Consequently, the total cost of interest and depreciation per production cycle of the modern flexible production process is much higher than that of the conventional inflexible process. In reality, this relationship should be discrete but a continuous function is used here as an approximation which is needed to simplify the subsequent mathematical analysis. A similar equation to model the relationship between set-up cost and interest and depreciation cost has been suggested by Beek and Puttin (1987).

CHAPTER IV
MODEL FORMULATION

4.1 Preliminary concepts

Let X be a classical set of objects, called the universe, whose generic elements are denoted by x. Membership in a classical subset A of X is often viewed as a characteristic function μ_A from X to $\{0,1\}$ such that

$$\mu_A = \begin{cases} 1 & for\ x \in A \\ 0 & otherwise \end{cases} \tag{4.1}$$

Here {0, 1} is called a valuation set. If the valuation set is allowed to be the real interval [0, 1], A is called a fuzzy set and to distinguish from classical set, it is denoted by $\tilde{A}$. In this case characteristic function μ_A is called membership function of $\tilde{A}$ and is denoted by $\mu_{\tilde{A}}$. The closer the value $\mu_{\tilde{A}}(x)$ of to 1, the more x belongs to $\tilde{A}$. So a fuzzy set $\tilde{A}$, in the universe of discourse X is completely characterized by the set of pairs as below

$$\tilde{A} = \{(x, \mu_A(x)), \quad x \in X\}$$

Clearly $\tilde{A}$ is a subset of X that has no sharp boundary and in this case it is normally written as $\tilde{A} \subseteq X$. A fuzzy set $\tilde{A} \subseteq X$ is said to be normal if $\exists$ at least one $x_0 \in X$ such that $\mu_{\tilde{A}}(x_0) = 1$. A fuzzy set $\tilde{A} \subseteq X$ is said to be convex if $\forall x_1 \in X,\ \forall x_2 \in X$, and $\forall \lambda \in [0,1]$

$$\mu_{\tilde{A}}(\lambda x_1 + (1-\lambda)x_2) \geq \min(\mu_{\tilde{A}}(x_1), \mu_{\tilde{A}}(x_2))$$

Any convex normalized fuzzy subset $\tilde{a}$ of R (where R represents set of real numbers) with membership function $\mu_{\tilde{a}}: R \to [0,1]$, is called a fuzzy number (Dubois and Prade (1980).

4.2 L–R representation of fuzzy numbers

L-R representation of fuzzy numbers is presented by Dubois and Prade (1980). A fuzzy number $\tilde{A} \subseteq R$ is said to be an L–R type fuzzy number if its membership function $\mu_{\tilde{A}}$ is given by

$$\mu_{\tilde{A}} = \begin{cases} L\left(\frac{m-x}{\alpha}\right) & for \quad x \leq m, \ \alpha > 0 \\ R\left(\frac{x-m}{\beta}\right) & for \quad x \geq m, \ \beta > 0 \end{cases} \tag{4.2}$$

Where, L is for left and R is for right reference, m is the mean value of $\tilde{A}$, α and β are called left and right spreads, respectively.

4.3 α - Level set

α-Level set (or interval of confidence at level α) of a fuzzy set $\tilde{A}$ in X is a crisp subset of X denoted by A (α) and is defined by

$$A(\alpha) = \{x \in X / \mu_{\tilde{A}}(x) \geq \alpha, \qquad \forall \alpha \in [0,1]\}$$

Let F be the set of all fuzzy numbers. Then for any $\tilde{A}$, $\tilde{B} \in$ F and for any $\lambda \in R$, $(\tilde{A}*\tilde{B})(\alpha)$=A($\alpha$)*B($\alpha$), $(\lambda\tilde{A})(\alpha) = (\lambda A)(\alpha)$, where $* \in \{+ , - , . , /\}$and for $* = /$, $0 \notin A_\alpha$ (Bector and Chandra, 2005)

4.4 Triangular fuzzy number (TFN)

A TFN $\tilde{A}$ is specified by the triplet (a_1,a_2,a_3) and is defined by its continuous membership function $\mu_{\tilde{A}}(x)$: X→[0, 1] as follows:

$$\mu_{\tilde{A}}(x) = \begin{cases} L(x) = \left(\frac{x-a_1}{a_2-a_1}\right) & if a_1 \leq x \leq a_2 \\ R(x) = \left(\frac{a_3-x}{a_3-a_2}\right) & if a_2 \leq x \leq a_3 \\ 0 & otherwise \end{cases} \tag{4.3}$$

So an α-cut of $\tilde{A}$ can be expressed by the following interval

A $(\alpha) = [a_1 + (a_2 - a_1)\alpha, a_3 - (a_3 - a_2)\alpha]$, $\alpha \in [0,1]$.

4.5 Graded mean integration value (GMIV) of fuzzy number

Chen and Hsieh (1999) introduced graded mean integration representation method based on the integral value of graded mean α-level of LR-fuzzy number for defuzzifing LR-fuzzy numbers. Suppose Ã is a LR-fuzzy number. Then according to Chen and Hsieh (1999), GMIV of Ã is denoted by P (Ã) and is defined as

$$P(\tilde{A}) = \int_0^1 \left(\frac{x}{2}\right)\{L^{-1}(x) + R^{-1}(x)\}\ dx \Big/ \int_0^1 x\,dx$$

$$= \int_0^1 x\{L^{-1}(x) + R^{-1}(x)\}\, dx \qquad (4.4)$$

4.6 Interval arithmetic

Let $* \in \{+, -, ., /\}$ be a binary operation on the set of positive real numbers. If A and B are closed intervals then $A * B = \{a * b : a \in A, b \in B\}$ defines a binary operation on the set of closed intervals (Moore, 1966). In the case of division, it is assumed that $0 \notin B$. The operations on intervals used here may be explicitly calculated from the above definition as

$$A + B = [a_L, a_R] + [b_L, b_R] = [a_L + b_L, a_R + b_R] \qquad (4.5)$$

$$A - B = [a_L, a_R] - [b_L, b_R] = [a_L - b_L, a_R - b_R] \qquad (4.6)$$

$$A.B = [a_L, a_R].[b_L, b_R] =$$
$$[\min\{a_L b_L, a_L b_R, a_R b_L, a_R b_R\}, \max\{a_L b_L, a_L b_R, a_R b_L, a_R b_R\}] \qquad (4.7)$$

$$\frac{A}{B} = \frac{[a_L, a_R]}{[b_L, b_R]} = [a_L, a_R].\left[\frac{1}{b_R}, \frac{1}{b_L}\right] \quad where\ 0 \notin B \qquad (4.8)$$

$$kA = \begin{cases} [ka_L, ka_R], & for\ k \geq 0 \\ [ka_R, ka_L], & for\ k < 0 \end{cases} \qquad (4.9)$$

Where, k is a real number.

4.7 Fuzzy random variable (FRV) and its expectation

Let F be the set of all fuzzy numbers. A metric d on F is defined by

$$d(\tilde{A},\tilde{B}) = \sqrt{\int_0^1 \{A_L(\alpha) - B_L(\alpha)\}^2\, d\alpha + \int_0^1 \{A_R(\alpha) - B_R(\alpha)\}^2\, d\alpha} \qquad (4.10)$$

Where, $A_L(\alpha), A_R(\alpha)$ are lower and upper end points of $\tilde{A}_\alpha$; $B_L(\alpha), B_R(\alpha)$are lower and upper end points of $\tilde{B}_\alpha$, then (F,d) is a complete metric space.

Let (Ω, A, P) be a complete probability space. A FRV is a Borel measurable function $\tilde{X}: (\Omega, A, P) \to (F, d)$.

If $\tilde{X}$ is a FRV, then $[X]_\alpha = [X_\alpha^-, X_\alpha^+]$, $\alpha \in [0,1]$ is a random closed interval set, and X_α^-, X_α^+ are real valued random variable.

The expectation of a FRV $\tilde{X}$ is defined as a unique fuzzy number $\tilde{u} \in F$, whose $\alpha - cut$, u_α is given by $u_\alpha = [EX]_\alpha = E[X_\alpha] = [E(X_\alpha^-), E(X_\alpha^+)]$, $\alpha \in [0,1]$.

For a FRV, $\tilde{X} = \{(X_\alpha^-, X_\alpha^+): 0 \le \alpha \le 1\}$, the expectation of $\tilde{X}$ is defined by,

$$\widetilde{EX} = \int \tilde{X} dP = \{\left(\int X_\alpha^-\, dP, \int X_\alpha^+\, dP\right): 0 \le \alpha \le 1\}$$

If $\tilde{X}$ is discrete FRV, such that $\left(\tilde{X} = \tilde{x}_i\right) = \tilde{p}_i$; $i = 1, 2, 3, \dots\dots\dots$ then its fuzzy expectation number is given by $\widetilde{EX} = \sum_{i=1}^{\infty} \tilde{x}_i \tilde{p}_i$.

It is also proved (Lopez-Diaz and Gil, 1998) that $\widetilde{EX} \in F$ and $[\widetilde{EX}]_0 = \int_\Omega X_0 dP = [EX_0^-, EX_0^+]$, for $\alpha = 0$. Hence it can be said that, the fuzzy expected value is a summarizing fuzzy value of the central tendency of FRV.

4.8 Mathematical modeling

According to assumptions, inventory level at time t, $q(t)$ is given by

$$\frac{dq(t)}{dt} = \begin{cases} rK - \tilde{D} & \text{for } 0 \leq t \leq T_1 \\ -\tilde{D} & \text{for } T_1 < t \leq \tilde{T} \end{cases} \tag{4.11}$$

Where, $q(0) = 0, q(T_1) = q_1, \; q(\tilde{T}) = 0$

According to Dubois and Prade (1980) fuzzy integration on $\frac{dq(t)}{dt} = rK - \tilde{D}$, for $0 \leq t \leq T_1$ is done. Now integrating from 0 to T_1, $\tilde{q_1}$ is obtained which is given by,

$$\tilde{q_1} = (rK - \tilde{D})T_1$$

And $\frac{dq(t)}{dt} = -\tilde{D}$, for $T_1 < t \leq \tilde{T}$

Now integrating from T_1 to $\tilde{T}$, $\tilde{q_1}$ is obtained which is given by,

$$\tilde{q_1} = \tilde{D}(\tilde{T} - T_1)$$

So at $t = T_1$

$$(rK - \tilde{D})T_1 = \tilde{D}(\tilde{T} - T_1)$$

This gives,

$$\tilde{T} = \frac{rKT_1}{\tilde{D}} \tag{4.12}$$

Total production holding cost per cycle is equal to,

$$\tilde{h}\int_0^{\tilde{T}} q(t)dt = \tilde{h}[\int_0^{T_1} q(t)dt + \int_{T_1}^{\tilde{T}} q(t)dt]$$

$$= \tilde{h}[\int_0^{T_1}(rK - \tilde{D})tdt + \int_{T_1}^{\tilde{T}} \tilde{D}\,(\tilde{T} - T_1)dt]$$

(4.13)

After solving equation (4.13), total production holding cost per cycle is obtained and given by

$$\frac{\tilde{h}\tilde{D}\tilde{T}^2}{2}[1-\frac{\tilde{D}}{rK}]$$

A profit function is developed considering selling price of fresh units, selling price of defective units, production cost, holding cost, set-up cost and cost of interest and depreciation cost.

Total profit incurred per production cycle

= (selling price of fresh units) + (selling price of defective units)

− (production cost) − (setup cost) − (holding cost)

− (cost of interest and depreciation cost)

Selling price for defective units and production cost are depended to the reliability of the production process. From Bag et al. (2009),

Selling price for defective units $= m_1 P\ (\frac{1-r}{r})\tilde{D}$

Production cost $= \frac{P\tilde{D}}{r}$

So, $Total\ profit\ per\ cycle, Z$

$$= mP\tilde{D} + m_1 P\left(\frac{1-r}{r}\right)\tilde{D} - \frac{P\tilde{D}}{r} - \frac{C_0}{\tilde{T}} - \frac{\tilde{h}\tilde{D}\tilde{T}^2}{2}\left[1-\frac{\tilde{D}}{rK}\right]/\tilde{T} - aC_0{}^{-b}r^c/\tilde{T}$$

Putting the value of $\tilde{T} = \frac{rKT_1}{\tilde{D}}$ and after simplifying,

$Total\ profit\ per\ cycle, Z$

$$= \left[mP + \left(\frac{1-r}{r}\right)m_1 P + \frac{\tilde{h}T_1}{2}\right]\tilde{D} - \left[\frac{P}{r} + \frac{C_0}{rKT_1} + \frac{aC_0{}^{-b}r^c}{rKT_1}\right]\tilde{D} - \frac{\tilde{h}rkT_1}{2} \quad (4.14)$$

Here as fuzziness and randomness appear simultaneously into the optimization setting, it is essential to interpret the problem and to transform it into the deterministic equivalent one. GMIV of $\widetilde{EZ}$ is employed to defuzzify. For this, it is only needed to know the several α − level set of $\widetilde{EZ}$ for all $\alpha \in [0,1]$.

Now, considering the fuzzy random demand $\tilde{D}$ and holding cost $\tilde{h}$ with the given set of data

$(\tilde{d}_1, \tilde{p}_1), (\tilde{d}_2, \tilde{p}_2), (\tilde{d}_3, \tilde{p}_3), \ldots\ldots, (\tilde{d}_n, \tilde{p}_n)$ and $(\tilde{h}_1, \tilde{p}_{h1}), (\tilde{h}_2, \tilde{p}_{h2}), (\tilde{h}_3, \tilde{p}_{h3}), \ldots\ldots, (\tilde{h}_j, \tilde{p}_{hj})$ respectively, the profit $\tilde{Z}$ is FRV and its expectation is a unique fuzzy number which is,

$$\widetilde{EZ} = \sum_{i=1}^{n}\left[mP + \left(\frac{1-r}{r}\right)m_1P + \sum_{j=1}^{u}\frac{\tilde{h}_j\tilde{p}_{h_j}T_1}{2}\right]\tilde{d}_i\tilde{p}_i - \sum_{i=1}^{n}\left[\frac{P}{r} + \frac{C_0}{rKT_1} + \frac{aC_0{}^{-b}r^{c}}{rKT_1}\right]\tilde{d}_i\tilde{p}_i - \sum_{j=1}^{u}\frac{\tilde{h}_j\tilde{p}_{h_j}rKT_1}{2} \tag{4.15}$$

As the demand and holding cost data are imprecise with fuzzy probability, so for the sake of simplicity, all the data set and its corresponding probabilities are considered as TFN and shown in Table 4.1 and Table 4.2.

Table 4.1: TFN for demand and associated probabilities

Demand	Probability
$(\underline{d}_i, d_i, \overline{d}_i)$	$(\underline{p}_i, p_i, \overline{p}_i)$

Table 4.2: TFN for holding cost and associated probabilities

Holding cost	Probability
$(\underline{h}_j, h_j, \overline{h}_j)$	$(\underline{p}_{hj}, p_{hj}, \overline{p}_{hj})$

Where, $i = 1,2, \ldots\ldots, n$ and $j = 1,2, \ldots\ldots, u$

Then the fuzzy expected profit function will also be a TFN, $\widetilde{EZ} = (\underline{EZ}, EZ, \overline{EZ})$
Where,

$$EZ = E[Z(\alpha = 1)]$$

$$= \sum_{i=1}^{n}\left[mP + \left(\frac{1-r}{r}\right)m_1P + \sum_{j=1}^{u}\frac{h_j p_{hj} T_1}{2}\right] d_i p_i$$

$$- \sum_{i=1}^{n}\left[\frac{P}{r} + \frac{C_0}{rKT_1} + \frac{aC_0{}^{-b}r^c}{rKT_1}\right] d_i p_i - \sum_{j=1}^{u}\frac{h_j p_{hj} rKT_1}{2}$$

$$\underline{EZ} = E[Z_L(\alpha = 0)]$$

$$= \sum_{i=1}^{n}\left[mP + \left(\frac{1-r}{r}\right)m_1P + \sum_{j=1}^{u}\frac{\underline{h_j}\,\underline{p_{hj}} T_1}{2}\right] \underline{d_i p_i}$$

$$- \sum_{i=1}^{n}\left[\frac{P}{r} + \frac{C_0}{rKT_1} + \frac{aC_0{}^{-b}r^c}{rKT_1}\right] \underline{d_i}\,\underline{p_i} - \sum_{j=1}^{u}\frac{\underline{h_j}\,\underline{p_{hj}} rKT_1}{2}$$

$$\overline{EZ} = E[Z_R(\alpha = 0)]$$

$$= \sum_{i=1}^{n}\left[mP + \left(\frac{1-r}{r}\right)m_1P + \sum_{j=1}^{u}\frac{\overline{h_j}\,\overline{p_{hj}} T_1}{2}\right] \overline{d_i}\,\overline{p_i}$$

$$- \sum_{i=1}^{n}\left[\frac{P}{r} + \frac{C_0}{rKT_1} + \frac{aC_0{}^{-b}r^c}{rKT_1}\right] \overline{d_i}\,\overline{p_i} - \sum_{j=1}^{u}\frac{\overline{h_j}\,\overline{p_{hj}} rKT_1}{2}$$

Here the α-level set of the fuzzy number $\widetilde{EZ}$ are considered as follows

$$EZ(\alpha) = E[Z(\alpha)] = \left[E\left(Z_L(\alpha)\right), E\left(Z_R(\alpha)\right)\right]; \quad 0 \le \alpha \le 1$$

And different α -cut intervals for the fuzzy number $\widetilde{EZ}$ are obtained for different a between 0 and 1.Taking, α-cut on both sides of equation (4.15)

$$\widetilde{EZ}_{\alpha} = \sum_{i=1}^{n}\left[mP + \left(\frac{1-r}{r}\right)m_1P + \sum_{j=1}^{u}\frac{\tilde{h}_{j\alpha}\tilde{p}_{hj\alpha}T_1}{2}\right]\tilde{d}_{i\alpha}\tilde{p}_{i\alpha} - \sum_{i=1}^{n}\left[\frac{P}{r} + \frac{C_0}{rKT_1} + \frac{aC_0{}^{-b}r^{c}}{rKT_1}\right]\tilde{d}_{i\alpha}\,\tilde{p}_{i\alpha} - \sum_{j=1}^{u}\frac{\tilde{h}_{j\alpha}\tilde{p}_{hj\alpha}rKT_1}{2}$$

(4.16)

Now, α-cut value for demand, holding cost and their associated probabilities

$$\tilde{d}_{i\alpha} = [\underline{d_i} + \alpha\left(d_i - \underline{d_i}\right), \quad \overline{d_i} - \alpha(\overline{d_i} - d_i)]$$

$$\tilde{p}_{i\alpha} = [\underline{p_i} + \alpha\left(p_i - \underline{p_i}\right), \quad \overline{p_i} - \alpha(\overline{p_i} - p_i)]$$

$$\tilde{h}_{j\alpha} = [\underline{h_j} + \alpha\left(h_j - \underline{h_j}\right), \quad \overline{h_j} - \alpha(\overline{h_j} - h_j)]$$

$$\tilde{p}_{hj\alpha} = [\underline{p_{hj}} + \alpha\left(p_{hj} - \underline{p_{hj}}\right), \quad \overline{p_{hj}} - \alpha(\overline{p_{hj}} - p_{hj})]$$

Using these and arithmetic interval, from equation (4.16) it is obtained

$$
\begin{aligned}
\widetilde{EZ}_a = \Bigg[\Bigg[\sum_{i=1}^{n} \Bigg[& mP + \left(\frac{1-r}{r}\right) m_1 P \\
& + \sum_{j=1}^{u} \frac{\{\underline{h_j} + \alpha\left(h_j - \underline{h_j}\right)\}\{\underline{p_{hj}} + \alpha\left(p_{hj} - \underline{p_{hj}}\right)\} T_1}{2} \Bigg] \{\underline{d_i} \\
& + \alpha\left(d_i - \underline{d_i}\right)\}\{\underline{p_i} + \alpha\left(p_i - \underline{p_i}\right)\} \\
& - \sum_{i=1}^{n} \left[\frac{P}{r} + \frac{C_0}{rKT_1} + \frac{aC_0^{-b} r^c}{rKT_1}\right] \{\overline{d_i} - \alpha(\overline{d_i} - d_i)\}\{\overline{p_i} - \alpha(\overline{p_i} \\
& - p_i)]\} - \sum_{j=1}^{u} \frac{\{\overline{h_j} - \alpha(\overline{h_j} - h_j)\}\{\overline{p_{hj}} - \alpha(\overline{p_{hj}} - p_{hj})\} rKT_1}{2} \Bigg], \\
& \Bigg[\sum_{i=1}^{n} \Bigg[mP + \left(\frac{1-r}{r}\right) m_1 P \\
& + \sum_{j=1}^{u} \frac{\{\overline{h_j} - \alpha(\overline{h_j} - h_j)\}\{\overline{p_{hj}} - \alpha(\overline{p_{hj}} - p_{hj})\} T_1}{2} \Bigg] \{\overline{d_i} - \alpha(\overline{d_i} - d_i)\}\{\overline{p_i} \\
& - \alpha(\overline{p_i} - p_i)\} \\
& - \sum_{i=1}^{n} \left[\frac{P}{r} + \frac{C_0}{rKT_1} + \frac{aC_0^{-b} r^c}{rKT_1}\right] \{\underline{d_i} + \alpha\left(d_i - \underline{d_i}\right)\}\{\underline{p_i} + \alpha\left(p_i - \underline{p_i}\right)]\} \\
& - \sum_{j=1}^{u} \frac{\{\underline{h_j} + \alpha\left(h_j - \underline{h_j}\right)\}\{\underline{p_{hj}} + \alpha\left(p_{hj} - \underline{p_{hj}}\right)\} rKT_1}{2} \Bigg] \Bigg]
\end{aligned}
\tag{4.17}
$$

From equation (4.17), $L^{-1}(\alpha)$ and $R^{-1}(\alpha)$ are obtained,

$$\mathrm{L}^{-1}(\alpha) = \left[\sum_{i=1}^{n}\left[mP + \left(\frac{1-r}{r}\right)m_1 P + \sum_{j=1}^{u}\frac{\{\underline{h_j} + \alpha(h_j - \underline{h_j})\}\{\underline{p_{hj}} + \alpha(p_{hj} - \underline{p_{hj}})\}T_1}{2}\right]\{\underline{d_i} + \alpha(d_i - \underline{d_i})\}\{\underline{p_i} + \alpha(p_i - \underline{p_i})\} - \sum_{i=1}^{n}\left[\frac{P}{r} + \frac{C_0}{rKT_1} + \frac{aC_0{}^{-b}r^c}{rKT_1}\right]\{\overline{d_i} - \alpha(\overline{d_i} - d_i)\}\{\overline{p_i} - \alpha(\overline{p_i} - p_i)\}]\} - \sum_{j=1}^{u}\frac{\{\overline{h_j} - \alpha(\overline{h_j} - h_j)\}\{\overline{p_{hj}} - \alpha(\overline{p_{hj}} - p_{hj})\}rKT_1}{2}\right] \tag{4.18}$$

And

$$\mathrm{R}^{-1}(\alpha) = \left[\sum_{i=1}^{n}\left[mP + \left(\frac{1-r}{r}\right)m_1 P + \sum_{j=1}^{u}\frac{\{\overline{h_j} - \alpha(\overline{h_j} - h_j)\}\{\overline{p_{hj}} - \alpha(\overline{p_{hj}} - p_{hj})\}T_1}{2}\right]\{\overline{d_i} - \alpha(\overline{d_i} - d_i)\}\{\overline{p_i} - \alpha(\overline{p_i} - p_i)\} - \sum_{i=1}^{n}\left[\frac{P}{r} + \frac{C_0}{rKT_1} + \frac{aC_0{}^{-b}r^c}{rKT_1}\right]\{\underline{d_i} + \alpha(d_i - \underline{d_i})\}\{\underline{p_i} + \alpha(p_i - \underline{p_i})\}]\} - \sum_{j=1}^{u}\frac{\{\underline{h_j} + \alpha(h_j - \underline{h_j})\}\{\underline{p_{hj}} + \alpha(p_{hj} - \underline{p_{hj}})\}rKT_1}{2}\right] \tag{4.19}$$

Now, using the method of representation of generalized fuzzy number based on the integral values of graded mean-α -level, a defuzzified representation of the unique fuzzy number is found,

$$G(\widetilde{EZ}) = \frac{\int_0^1 \left(\frac{\alpha}{2}\right)\{\mathrm{L}^{-1}(\alpha) + \mathrm{R}^{-1}(\alpha)\}\,d\alpha}{\int_0^1 \alpha\,d\alpha}$$

$$= \int_0^1 \alpha\{\mathrm{L}^{-1}(\alpha) + \mathrm{R}^{-1}(\alpha)\}\,d\alpha \tag{4.20}$$

From equations (4.18) and (4.19) substituting value of $L^{-1}(\alpha)$ and $R^{-1}(\alpha)$ in equation (4.20) and after simplifying $G(\widetilde{EZ})$ is obtained,

$$G(\widetilde{EZ}) = a_1 - a_2 r^{-1} + a_3 T_1 - a_4 C_0 r^{-1} T_1^{-1} - a_5 C_0^{-b} r^{c-1} T_1^{-1} - a_6 r T_1 = F(C_0, r, T_1) \tag{4.21}$$

Where,

$$a_1 = P(m - m_1)(E_1 + E_5)$$

$$a_2 = P(E_1 - m_1 E_1 + E_5 - m_1 E_5)$$

$$a_3 = \frac{1}{2}(E_2 - E_3 + E_4 + E_7 - E_8 + E_9)$$

$$a_4 = \frac{1}{K}(E_1 + E_5)$$

$$a_5 = \frac{a}{K}(E_1 + E_5)$$

$$a_6 = \frac{K}{2}(E_6 + E_{10})$$

And,

$$E_1 = \sum_{i=1}^{n} \frac{1}{2}\underline{d_i}\,\underline{p_i} + \frac{1}{3}\left(\underline{d_i}p_i - 2\underline{d_i}\,\underline{p_i} + d_i\underline{p_i}\right) + \frac{1}{4}\left(d_i - \underline{d_i}\right)(p_i - \underline{p_i})$$

$$E_2 = \left[\sum_{j=1}^{u}\left\{\underline{h_j}\,\underline{p_{hj}} + \left(\underline{h_j}p_{hj} - 2\underline{h_j}\,\underline{p_{hj}} + h_j\underline{p_{hj}}\right) + \left(h_j - \underline{h_j}\right)(p_{hj} - \underline{p_{hj}})\right\}\right]$$
$$\times\left[\sum_{i=1}^{n}\left\{\frac{1}{2}\underline{d_i}\,\underline{p_i} + \frac{1}{3}\left(\underline{d_i}p_i - 2\underline{d_i}\,\underline{p_i} + d_i\underline{p_i}\right) + \frac{1}{4}\left(d_i - \underline{d_i}\right)(p_i - \underline{p_i})\right\}\right]$$

$$E_3 = \left[\sum_{j=1}^{u}\left\{\left(\underline{h_j}p_{hj} - 2\underline{h_j}\,\underline{p_{hj}} + h_j\underline{p_{hj}}\right) + 2\left(h_j - \underline{h_j}\right)(p_{hj} - \underline{p_{hj}})\right\}\right]$$

$$\times\left[\sum_{i=1}^{n}\left\{\frac{1}{6}\underline{d_i}\,\underline{p_i} + \frac{1}{12}\left(\underline{d_i}p_i - 2\underline{d_i}\,\underline{p_i} + d_i\underline{p_i}\right) + \frac{1}{20}\left(d_i - \underline{d_i}\right)(p_i - \underline{p_i})\right\}\right]$$

$$E_4 = \left[\sum_{j=1}^{u}\left\{2\left(h_j - \underline{h_j}\right)(p_{hj} - \underline{p_{hj}})\right\}\right]$$

$$\times\left[\sum_{i=1}^{n}\left\{\frac{1}{120}\,\underline{d_i}\,\underline{p_i} + \frac{1}{360}\left(\underline{d_i}p_i - 2\underline{d_i}\,\underline{p_i} + d_i\underline{p_i}\right) + \frac{1}{840}\left(d_i - \underline{d_i}\right)(p_i\right.$$

$$\left. - \underline{p_i})\right\}\right]$$

$$E_5 = \sum_{i=1}^{n}\left\{\frac{1}{2}\,\overline{d_i}\,\overline{p_i} + \frac{1}{3}\left(\overline{d_i}\,p_i - 2\overline{d_i}\,\overline{p_i} + d_i\overline{p_i}\right) + \frac{1}{4}\,(\overline{d_i} - d_i)(\overline{p_i} - p_i)\right\}$$

$$E_6 = \sum_{j=1}^{u}\left[\frac{1}{2}\,\overline{h_j}\,\overline{p_{hj}} + \frac{1}{3}\,\left(\overline{h_j}\,p_{hj} - 2\overline{h_j}\,\overline{p_{hj}} + h_j\overline{p_{hj}}\right) + \frac{1}{4}\,(\overline{h_j} - h_j)(\overline{p_{hj}} - p_{hj})\right]$$

$$E_7 = \sum_{j=1}^{u}\left[\overline{h_j}\,\overline{p_{hj}} + \left(\overline{h_j}\,p_{hj} - 2\overline{h_j}\,\overline{p_{hj}} + h_j\overline{p_{hj}}\right) + (\overline{h_j} - h_j)(\overline{p_{hj}} - p_{hj})\right]$$

$$\times\sum_{i=1}^{n}\left[\frac{1}{2}\,\overline{d_i}\,\overline{p_i} + \frac{1}{3}\left(\overline{d_i}\,p_i - 2\overline{d_i}\,\overline{p_i} + d_i\overline{p_i}\right) + \frac{1}{4}\,(\overline{d_i} - d_i)(\overline{p_i} - p_i)\right]$$

$$E_8 = \sum_{j=1}^{u}\left[\left(\overline{h_j}\, p_{hj} - 2\overline{h_j}\, \overline{p_{hj}} + h_j\overline{p_{hj}}\right) + 2(\overline{h_j} - h_j)(\overline{p_{hj}} - p_{hj})\right]$$

$$\times \sum_{i=1}^{n}\left[\frac{1}{6}\overline{d_i}\overline{p_i} + \frac{1}{12}\left(\overline{d_i}\, p_i - 2\overline{d_i}\, \overline{p_i} + d_i\overline{p_i}\right) + \frac{1}{20}\, (\overline{d_i} - d_i)(\overline{p_i} - p_i)\right]$$

$$E_9 = \sum_{j=1}^{u}\left[2(\overline{h_j} - h_j)(\overline{p_{hj}} - p_{hj})\right]$$

$$\times \sum_{i=1}^{n}\left[\frac{1}{120}\overline{d_i}\overline{p_i} + \frac{1}{360}\left(\overline{d_i}\, p_i - 2\overline{d_i}\, \overline{p_i} + d_i\overline{p_i}\right) + \frac{1}{840}\, (\overline{d_i} - d_i)(\overline{p_i} - p_i)\right]$$

$$E_{10} = \sum_{j=1}^{u}\left\{\frac{1}{2}\underline{h_j}\, \underline{p_{hj}} + \frac{1}{3}\left(h_j p_{hj} - 2\underline{h_j}\, \underline{p_{hj}} + h_j\underline{p_{hj}}\right) + \frac{1}{4}\left(h_j - \underline{h_j}\right)(p_{hj} - \underline{p_{hj}})\right\}$$

GMIV of the expected average profit is determined and optimized. According to this, the problem is to determine(C_0, r, T_1), so that GMIV of $\widetilde{EZ}$, $F(C_0, r, T_1)$ is maximum. Here the function $F(C_0, r, T_1)$ is optimized by unconstraint signomial geometric programming. The problem is to,

$$Maximize\ F(C_0, r, T_1) = a_1 - a_2 r^{-1} + a_3 T_1 - a_4 C_0 r^{-1} T_1^{-1} - a_5 C_0^{-b} r^{c-1} T_1^{-1} - a_6 r T_1 \quad (4.22)$$

This is equivalent to,

$$Minimize\ F^*(C_0, r, T_1) = -a_1 + a_2 r^{-1} - a_3 T_1 + a_4 C_0 r^{-1} T_1^{-1} + a_5 C_0^{-b} r^{c-1} T_1^{-1} + a_6 r T_1$$

As a_1 is independent of the decision variables C_0, r, T_1, so it can be neglected to derive the optimal value of the objective function. Then the problem reduces to

$$Minimize\ F^{*}(C_0, r, T_1) = a_2 r^{-1} - a_3 T_1 + a_4 C_0 r^{-1} T_1^{-1} + a_5 C_0^{-b} r^{c-1} T_1^{-1} + a_6 r T_1$$

This is an unconstraint signomial geometric programming problem with degrees of difficulty (DD) = number of terms - (number of variables +1) = 5 - (3 + 1) = 1

The dual problem of the above problem is

$$d(w) = \left(\frac{a_2}{w_1}\right)^{w_1} \left(\frac{a_3}{w_2}\right)^{-w_2} \left(\frac{a_4}{w_3}\right)^{w_3} \left(\frac{a_5}{w_4}\right)^{w_4} \left(\frac{a_6}{w_5}\right)^{w_5} \tag{4.23}$$

Subject to the normality and orthogonality conditions,

$$w_1 - w_2 + w_3 + w_4 + w_5 = 1$$

$$w_3 - bw_4 = 0$$

$$-w_1 - w_3 + (c-1)w_4 + w_5 = 0$$

$$-w_2 - w_3 - w_4 + w_5 = 0$$

Solving the normality and orthogonality conditions,

$$w_1 = 1 - 2(b+1)w_4$$

$$w_2 = 1 - (2b + c + 2)w_4$$

$$w_3 = bw_4$$

$$w_5 = 1 - (b + c + 1)w_4$$

Substituting the above dual weights into the dual function equation (4.23), $d(w_4)$ is obtained,

$$d(w_4) = \left[\frac{a_2}{1-2(b+1)w_4}\right]^{1-2(b+1)w_4} \times \left[\frac{a_3}{1-(2b+c+2)w_4}\right]^{-1+(2b+c+2)w_4}$$
$$\times \left[\frac{a_4}{bw_4}\right]^{bw_4} \times \left[\frac{a_5}{w_4}\right]^{w_4} \times \left[\frac{a_6}{1-(b+c+1)w_4}\right]^{1-(b+c+1)w_4}$$

Taking logarithm and $d(w_4) = y(w_4)$ then equating $\frac{dy}{dw_4} = 0$, it is obtained

$$\log\left[\left[\frac{a_2}{1-2(b+1)w_4}\right]^{-2(b+1)} \times \left[\frac{a_3}{1-(2b+c+2)w_4}\right]^{(2b+c+2)} \times \left[\frac{a_4}{bw_4}\right]^{b} \times \left[\frac{a_5}{w_4}\right]^{1}\right.$$
$$\left.\times \left[\frac{a_6}{1-(b+c+1)w_4}\right]^{-(b+c+1)}\right] = 0$$

This is equivalent to,

$$a_2^{-2(b+1)} \times a_3^{(2b+c+2)} \times a_4^{b} \times a_5 \times a_6^{-(b+c+1)}$$
$$\times \left[\left[\frac{1}{1-2(b+1)w_4}\right]^{-2(b+1)} \times \left[\frac{1}{1-(2b+c+2)w_4}\right]^{(2b+c+2)} \times \left[\frac{1}{bw_4}\right]^{b}\right.$$
$$\left.\times \left[\frac{1}{w_4}\right]^{1} \times \left[\frac{1}{1-(b+c+1)w_4}\right]^{-(b+c+1)}\right] = 1$$

(4.24)

From equation (4.24) optimal value of w_4 is obtained, which is written as $w_4{}^*$

The optimal feasible solution of dual problem of equation (4.23)

$$w_1{}^* = 1 - 2(b+1)w_4{}^*$$

$$w_2{}^* = 1 - (2b+c+2)w_4{}^*$$

$$w_3{}^* = bw_4{}^*$$

$$w_5{}^* = \{1 - (b+c+1)w_4{}^*\}$$

Putting these values into the objective function in equation (4.23) optimum value of $d(w)$ is obtained as $d(w^*)$ which is written as F_0^*

$$F_0^* = d(w^*) = \left[\frac{a_2}{1-2(b+1)w_4^*}\right]^{1-2(b+1)w_4^*} \times \left[\frac{a_3}{1-(2b+c+2)w_4^*}\right]^{-1+(2b+c+2)w_4^*}$$
$$\times \left[\frac{a_4}{bw_4^*}\right]^{bw_4^*} \times \left[\frac{a_5}{w_4^*}\right]^{w_4^*} \times \left[\frac{a_6}{1-(b+c+1)w_4^*}\right]^{1-(b+c+1)w_4^*}$$

Putting values of a_2, a_3, a_4, a_5 *and* a_6 , the equation becomes,

$$F_0^* = d(w^*) = \left[\frac{P(E_1 - m_1E_1 + E_5 - m_1E_5)}{1-2(b+1)w_4^*}\right]^{1-2(b+1)w_4^*}$$
$$\times \left[\frac{\frac{1}{2}(E_2 - E_3 + E_4 + E_7 - E_8 + E_9)}{1-(2b+c+2)w_4^*}\right]^{-1+(2b+c+2)w_4^*} \times \left[\frac{\frac{1}{K}(E_1 + E_5)}{bw_4^*}\right]^{bw_4^*}$$
$$\times \left[\frac{\frac{a}{K}(E_1 + E_5)}{w_4^*}\right]^{w_4^*} \times \left[\frac{\frac{K}{2}(E_6 + E_{10})}{1-(b+c+1)w_4^*}\right]^{1-(b+c+1)w_4^*}$$

(4.25)

Again from the relationship between primal-dual variables (Beightler and Phillips, 1976; Duffin et al. 1967), following relations can be determined:

$$P(E_1 - m_1E_1 + E_5 - m_1E_5)r^{-1} = w_1^*d(w^*) \tag{4.26}$$

$$-\frac{1}{2}(E_2 - E_3 + E_4 + E_7 - E_8 + E_9)T_1 = w_2^*d(w^*) \tag{4.27}$$

$$\frac{1}{K}(E_1 + E_5)C_0r^{-1}T_1^{-1} = w_3^*d(w^*) \tag{4.28}$$

$$\frac{a}{K}(E_1 + E_5)C_0^{-b}r^{c-1}T_1^{-1} = w_4^*d(w^*) \tag{4.29}$$

$$\frac{K}{2}(E_6 + E_{10})rT_1 = w_5^*d(w^*) \tag{4.30}$$

Now, to solve the above simultaneous equations to obtain the optimal solution of the primal problem, equations (4.28) and (4.30) are multiplied and optimal value of set-up cost per cycle (C_0) is obtained which is as follows:

$$C_0^* = \left[\frac{2bw_4^*\{1-(b+c+1)w_4^*\}\{d(w^*)\}^2}{(E_1+E_5)(E_6+E_{10})}\right] \tag{4.31}$$

Now equations (4.29) and (4.30) are multiplied to obtain optimal value of reliability (r) of the production process which is as follows:

$$r^* = \left[\frac{2^{b+1}(bw_4^*)^b w_4^*[1-(b+c+1)w_4^*]^{b+1}[d(w^*)]^{2b+2}}{a[(E_1+E_5)(E_6+E_{10})]^{b+1}}\right]^{\frac{1}{c}} \tag{4.32}$$

To obtain optimal value of production period (T_1) equations (4.30) and (4.32) are multiplied and optimal value of T_1 is as follows:

$$T_1^* = \left[\frac{a^{\frac{1}{c}}(E_1+E_5)^{\frac{b+1}{c}}(E_6+E_{10})^{\frac{b+1}{c}-1}}{2^{\frac{b+1}{c}-1}K(bw_4^*)^{\frac{b}{c}}(w_4^*)^{\frac{1}{c}}\{1-(b+c+1)w_4^*\}^{\frac{b+1}{c}-1}\{d(w^*)\}^{\frac{2b+2}{c}-1}}\right] \tag{4.33}$$

Maximum value of profit is determined as,

$$F_{max} = a_1 - F_0^*$$

$$F_{max} = P(m - m_1)(E_1 + E_5) - F_0^* \tag{4.34}$$

CHAPTER V
RESULTS AND DISCUSSIONS

The thesis work is theoretical in nature. A mathematical production inventory model is developed considering some practical situations such as fuzzy random demand, fuzzy inventory cost and reliability of the production process. The production inventory model is composed of some mathematical equations which can be used to determine the optimal values of different decision variables i.e. set-up cost per cycle, reliability of the production process and duration of time until the production is being held. The equation for maximum total profit is also developed which can be used to get the maximum profit of production inventory model. The model is discussed by illustrating a numerical example.

5.1 Numerical illustration

To illustrate the model, a particular EPQ problem is considered. Suppose for a particular EPQ problem, following data are considered:

$$K = 60 \; units$$

$$m = 2.0$$

$$m_1 = 0.80$$

$$P = 21.7 \; units$$

$$a = 1600$$

$$b = 0.50$$

$$c = 0.75$$

Demand and holding cost data are associated with the probability. Demand and holding cost data are given in Table 5.1 and Table 5.2 respectively. Triangular fuzzy number is considered for demand and holding cost

Table 5.1: Demand data and associated probabilities

Demand	Probability
(18, 20, 22)	(0.045, 0.05, 0.055)
(23, 25, 27)	(0.143, 0.15, 0.157)
(28, 30, 32)	(0.292, 0.30, 0.308)
(33, 35, 37)	(0.192, 0.20, 0.208)
(38, 40, 42)	(0.092, 0.10, 0.108)
(43, 45, 47)	(0.093, 0.10, 0.107)
(48, 50, 52)	(0.094, 0.10, 0.106)

Table 5.2: Holding cost data and associated probabilities

Holding Cost	Probability
(0.5, 1.0, 1.5)	(0.054, 0.060, 0.065)
(2.0, 2.5, 3.0)	(0.216, 0.220, 0.227)
(3.5, 4.0, 4.5)	(0.383, 0.390, 0.398)
(5.0, 5.5, 6.0)	(0.227, 0.230, 0.236)
(6.5, 7.0, 7.5)	(0.090, 0.100, 0.108)

From given data, value of E_1, E_2, E_3, E_4, E_5, E_6, E_7, E_8, E_9, E_{10} are determined, then value of $a_1, a_2, a_3, a_4, a_5, a_6$ are also evaluated using following equations:

$$E_1 = \sum_{i=1}^{n} \frac{1}{2}\underline{d_i}\,\underline{p_i} + \frac{1}{3}\left(\underline{d_i}p_i - 2\underline{d_i}\,\underline{p_i} + d_i\underline{p_i}\right) + \frac{1}{4}\left(d_i - \underline{d_i}\right)\left(p_i - \underline{p_i}\right)$$

$$= 16.512$$

$$E_2 = \left[\sum_{j=1}^{u}\left\{\underline{h_j}\,\underline{p_{hj}} + \left(\underline{h_j}p_{hj} - 2\underline{h_j}\,\underline{p_{hj}} + h_j\underline{p_{hj}}\right) + \left(h_j - \underline{h_j}\right)\left(p_{hj} - \underline{p_{hj}}\right)\right\}\right]$$

$$\times\left[\sum_{i=1}^{n}\left\{\frac{1}{2}\underline{d_i}\,\underline{p_i} + \frac{1}{3}\left(\underline{d_i}p_i - 2\underline{d_i}\,\underline{p_i} + d_i\underline{p_i}\right) + \frac{1}{4}\left(d_i - \underline{d_i}\right)\left(p_i - \underline{p_i}\right)\right\}\right]$$

$$= 57.558$$

$$E_3 = \left[\sum_{j=1}^{u}\left\{\left(\underline{h_j}p_{hj} - 2\underline{h_j}\,\underline{p_{hj}} + h_j\underline{p_{hj}}\right) + 2\left(h_j - \underline{h_j}\right)\left(p_{hj} - \underline{p_{hj}}\right)\right\}\right]$$

$$\times\left[\sum_{i=1}^{n}\left\{\frac{1}{6}\underline{d_i}\,\underline{p_i} + \frac{1}{12}\left(\underline{d_i}p_i - 2\underline{d_i}\,\underline{p_i} + d_i\underline{p_i}\right) + \frac{1}{20}\left(d_i - \underline{d_i}\right)\left(p_i - \underline{p_i}\right)\right\}\right]$$

$$= 3.3790$$

$$E_4 = \left[\sum_{j=1}^{u}\left\{2\left(h_j - \underline{h_j}\right)\left(p_{hj} - \underline{p_{hj}}\right)\right\}\right]$$

$$\times\left[\sum_{i=1}^{n}\left\{\frac{1}{120}\,\underline{d_i}\,\underline{p_i} + \frac{1}{360}\left(\underline{d_i}p_i - 2\underline{d_i}\,\underline{p_i} + d_i\underline{p_i}\right) + \frac{1}{840}\left(d_i - \underline{d_i}\right)\left(p_i - \underline{p_i}\right)\right\}\right] = 0.0080$$

$$E_5 = \sum_{i=1}^{n}\left\{\frac{1}{2}\overline{d_i}\,\overline{p_i} + \frac{1}{3}\left(\overline{d_i}p_i - 2\overline{d_i}\,\overline{p_i} + d_i\overline{p_i}\right) + \frac{1}{4}\left(\overline{d_i} - d_i\right)\left(\overline{p_i} - p_i\right)\right\}$$

$$= 17.755$$

$$E_6 = \sum_{j=1}^{u}\left[\frac{1}{2}\,\overline{h_j}\,\overline{p_{hj}} + \frac{1}{3}\left(\overline{h_j}\,p_{hj} - 2\overline{h_j}\,\overline{p_{hj}} + h_j\overline{p_{hj}}\right) + \frac{1}{4}(\overline{h_j} - h_j)(\overline{p_{hj}} - p_{hj})\right]$$

$$= 2.1505$$

$$E_7 = \sum_{j=1}^{u}\left[\overline{h_j}\,\overline{p_{hj}} + \left(\overline{h_j}\,p_{hj} - 2\overline{h_j}\,\overline{p_{hj}} + h_j\overline{p_{hj}}\right) + (\overline{h_j} - h_j)(\overline{p_{hj}} - p_{hj})\right]$$

$$\times \sum_{i=1}^{n}\left[\frac{1}{2}\,\overline{d_i}\overline{p_i} + \frac{1}{3}\left(\overline{d_i}\,p_i - 2\overline{d_i}\,\overline{p_i} + d_i\overline{p_i}\right) + \frac{1}{4}\,(\overline{d_i} - d_i)(\overline{p_i} - p_i)\right]$$

$$= 72.528$$

$$E_8 = \sum_{j=1}^{u}\left[\left(\overline{h_j}\,p_{hj} - 2\overline{h_j}\,\overline{p_{hj}} + h_j\overline{p_{hj}}\right) + 2(\overline{h_j} - h_j)(\overline{p_{hj}} - p_{hj})\right]$$

$$\times \sum_{i=1}^{n}\left[\frac{1}{6}\overline{d_i}\overline{p_i} + \frac{1}{12}\left(\overline{d_i}\,p_i - 2\overline{d_i}\,\overline{p_i} + d_i\overline{p_i}\right) + \frac{1}{20}\,(\overline{d_i} - d_i)(\overline{p_i} - p_i)\right]$$

$$= -3.8520$$

$$E_9 = \sum_{j=1}^{u}\left[2(\overline{h_j} - h_j)(\overline{p_{hj}} - p_{hj})\right]$$

$$\times \sum_{i=1}^{n}\left[\frac{1}{120}\overline{d_i}\overline{p_i} + \frac{1}{360}\left(\overline{d_i}\,p_i - 2\overline{d_i}\,\overline{p_i} + d_i\overline{p_i}\right) + \frac{1}{840}\,(\overline{d_i} - d_i)(\overline{p_i} - p_i)\right]$$

$$= 0.0104$$

$$E_{10} = \sum_{j=1}^{u}\left\{\frac{1}{2}\underline{h_j}\,\underline{p_{hj}} + \frac{1}{3}\left(\underline{h_j}p_{hj} - 2\underline{h_j}\,\underline{p_{hj}} + h_j\underline{p_{hj}}\right) + \frac{1}{4}\left(h_j - \underline{h_j}\right)(p_{hj} - \underline{p_{hj}})\right\}$$

$$= 1.9395$$

And

$$a_1 = P(m - m_1)(E_1 + E_5) = 892.295$$

$$a_2 = P(E_1 - m_1E_1 + E_5 - m_1E_5) = 148.716$$

$$a_3 = \frac{1}{2}(E_2 - E_3 + E_4 + E_7 - E_8 + E_9) = 65.289$$

$$a_4 = \frac{1}{K}(E_1 + E_5) = 0.5711$$

$$a_5 = \frac{a}{K}(E_1 + E_5) = 913.77$$

$$a_6 = \frac{K}{2}(E_6 + E_{10}) = 122.70$$

Now optimal value of w_4, which is written as w_4^*, is calculated using equation (4.24),

$$a_2^{-2(b+1)} \times a_3^{(2b+c+2)} \times a_4^{b} \times a_5 \times a_6^{-(b+c+1)} \times \left[\left[\frac{1}{1-2(b+1)w_4}\right]^{-2(b+1)} \times \left[\frac{1}{1-(2b+c+2)w_4}\right]^{(2b+c+2)} \times \left[\frac{1}{bw_4}\right]^{b} \times \left[\frac{1}{w_4}\right]^{1} \times \left[\frac{1}{1-(b+c+1)w_4}\right]^{-(b+c+1)}\right] = 1$$

$$\text{or,} \qquad w_4^* = 0.1775338$$

Using the value of w_4^*, values of w_1^*, w_2^*, w_3^*, and w_5^* are also calculated using the following equations.

$$w_1^* = 1 - 2(b+1)w_4^* = 0.4674$$

$$w_2^* = 1 - (2b + c + 2)w_4^* = 0.33425$$

$$w_3^* = bw_4^* = 0.08877$$

$$w_5^* = 1 - (b + c + 1)w_4^* = 0.6005$$

Now value of F_0^* or $d(w^*)$ is calculated using equation (4.25)

$$F_0^* = d(w^*) = \left[\frac{P(E_1 - m_1E_1 + E_5 - m_1E_5)}{1-2(b+1)w_4^*}\right]^{1-2(b+1)w_4^*}$$

$$\times\left[\frac{\frac{1}{2}(E_2 - E_3 + E_4 + E_7 - E_8 + E_9)}{1-(2b+c+2)w_4^*}\right]^{-1+(2b+c+2)w_4^*} \times\left[\frac{\frac{1}{K}(E_1 + E_5)}{bw_4^*}\right]^{bw_4^*}$$

$$\times\left[\frac{\frac{a}{K}(E_1 + E_5)}{w_4^*}\right]^{w_4^*} \times\left[\frac{\frac{K}{2}(E_6 + E_{10})}{1-(b+c+1)w_4^*}\right]^{1-(b+c+1)w_4^*}$$

$$\text{or, } F_0^* = 332.81 \text{ units}$$

C_0^*, r^* and T_1^* are evaluated using equation (4.31), (4.32) and (4.33) respectively,

$$C_0^* = \left[\frac{2bw_4^*\{1-(b+c+1)w_4^*\}\{d(w^*)\}^2}{(E_1+E_5)(E_6+E_{10})}\right] = 84.30$$

$$r^* = \left[\frac{2^{b+1}(bw_4^*)^b w_4^*\{1-(b+c+1)w_4^*\}^{b+1}\{d(w^*)\}^{2b+2}}{a[(E_1+E_5)(E_6+E_{10})]^{b+1}}\right]^{\frac{1}{c}} = 0.9560$$

$$T_1^* = \left[\frac{a^{\frac{1}{c}}(E_1+E_5)^{\frac{b+1}{c}}(E_6+E_{10})^{\frac{b+1}{c}-1}}{2^{\frac{b+1}{c}-1}K(bw_4^*)^{\frac{b}{c}}(w_4^*)^{\frac{1}{c}}\{1-(b+c+1)w_4^*\}^{\frac{b+1}{c}-1}\{d(w^*)\}^{\frac{2b+2}{c}-1}}\right] = 1.704$$

Now maximum total profit is determined from equation (4.34) as following,

$= a_1 - F_0^*$

$= P(m - m_1)(E_1 + E_5) - F_0^*$

$= 559.50$ units

Three decision variables are related to each other. Figure 5.1 shows the relationship between production period and reliability. With improvement of reliability of the system, production period is decreased. From equation (4.30), it is shown that production period and reliability are inversely related.

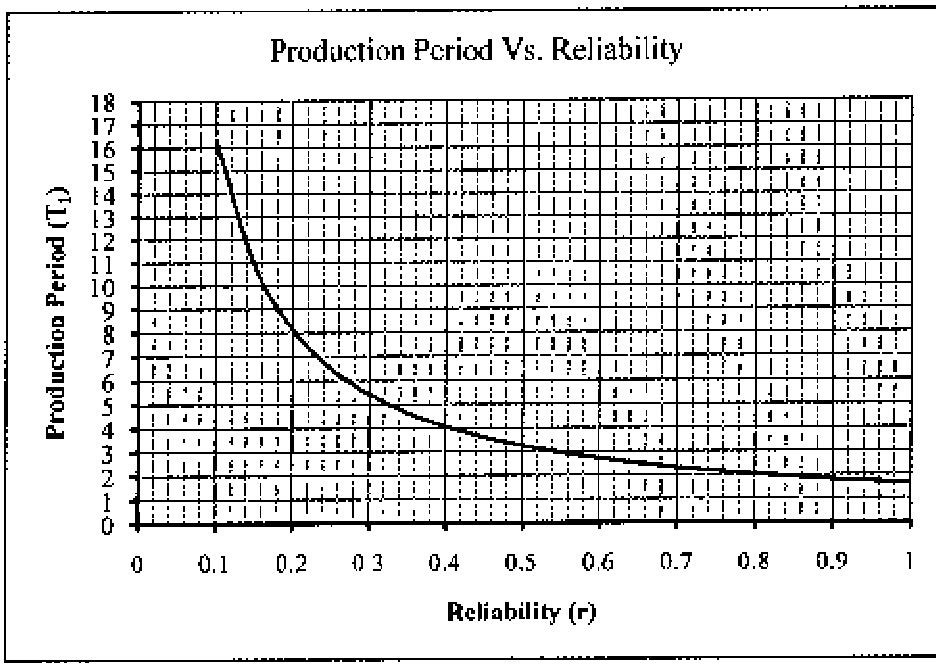

Fig. 5.1: Relationship between production period and reliability

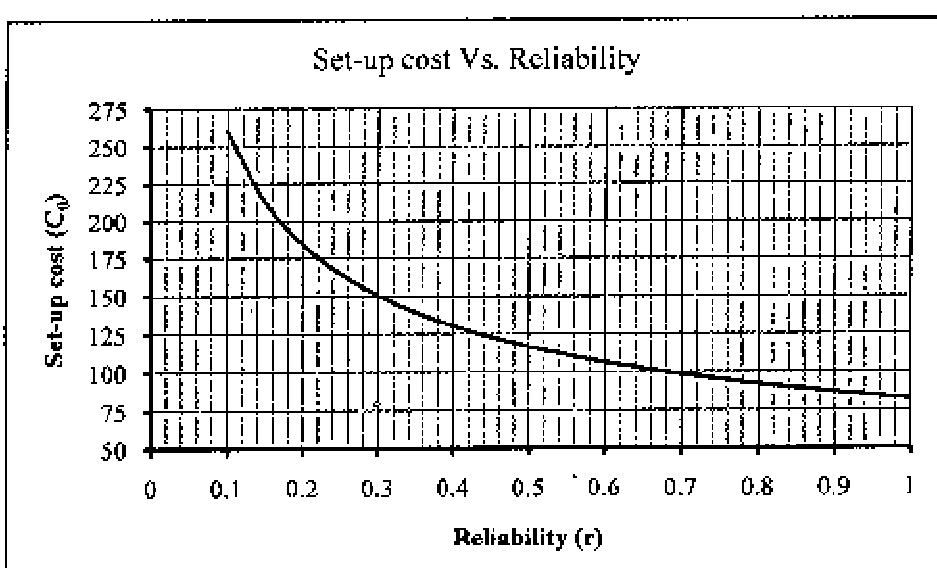

Fig. 5.2: Relationship between set-up cost and reliability

Set-up cost and reliability are also related to each other. Set-up cost is decreased with improvement of reliability of the production process. The relationship between set-up cost and reliability is shown in Figure 5.2.

Both production period and set-up cost decrease with improving reliability. But set-up cost and production period are proportionally related. With increasing production period, set-up cost increases. Figure 5.3 shows the relationship between set-up cost and production period.

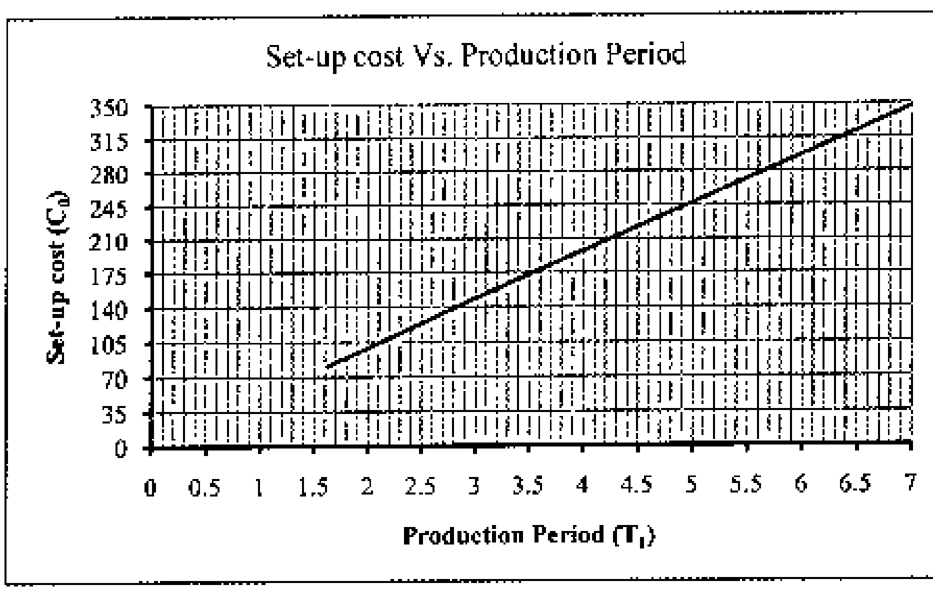

Fig. 5.3: Relationship between set-up cost and production period

Total profit is effected by three decision variables namely reliability of the production process, production period and set-up cost. The effect of reliability on total profit is shown in Figure 5.4. Total profit increases with improvement of reliability up to 0.9560. Total profit is negative when reliability is less than 0.20245 and total profit goes downward when reliability is greater than 0.9560. Set-up cost and production period are assumed constant values of 84.3 units and 1.704 units respectively to obtain the relationship between profit and reliability.

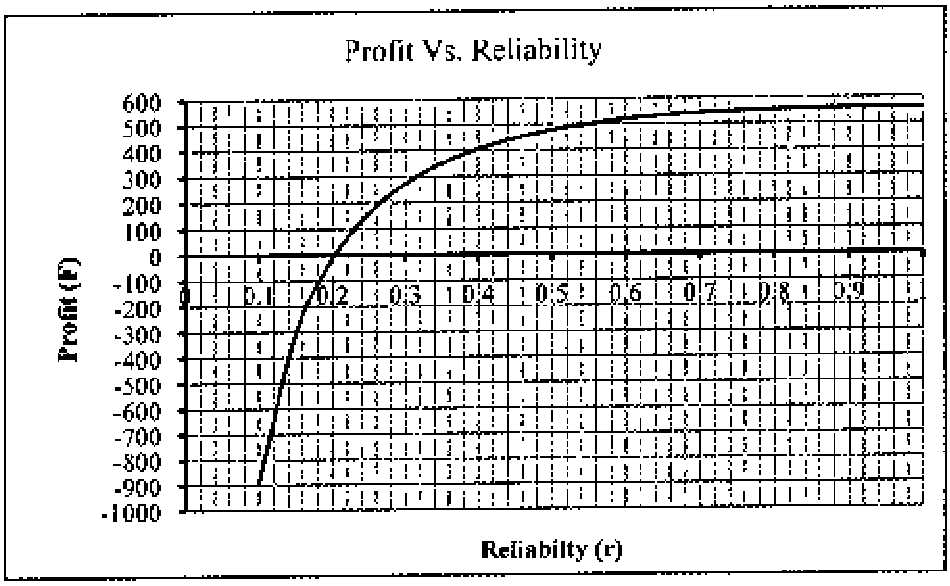

Fig. 5.4: Variation of total profit with reliability of the production system

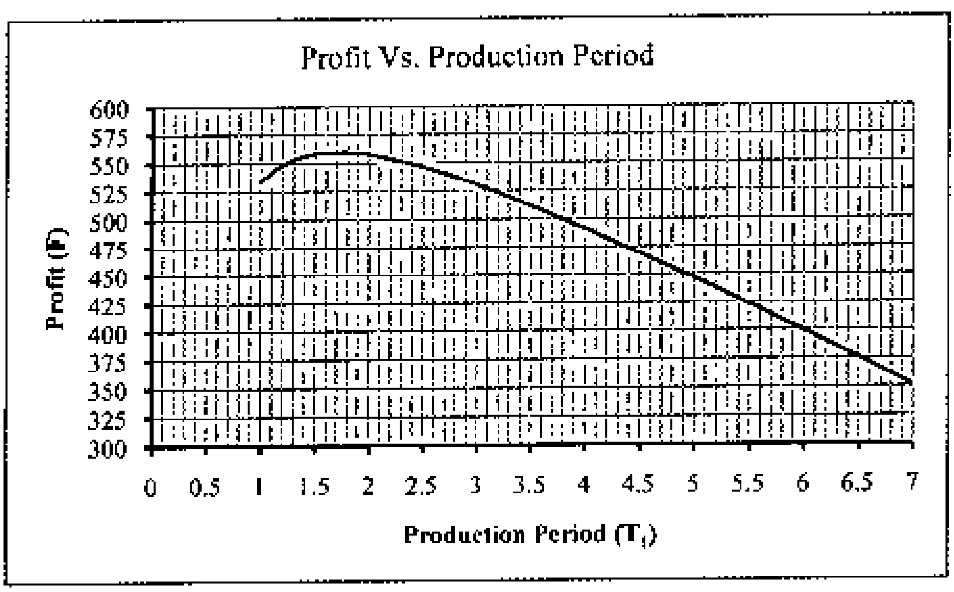

Fig. 5.5: Variation of total profit with production period

It is observed that total profit varies with varying production period. Figure 5.5 presents the relationship between profit and production period. Reliability and set-up cost are kept constant to obtain the relationship between profit and production period . For production period less than 1.704 units, profit is increased and after that profit decreases.

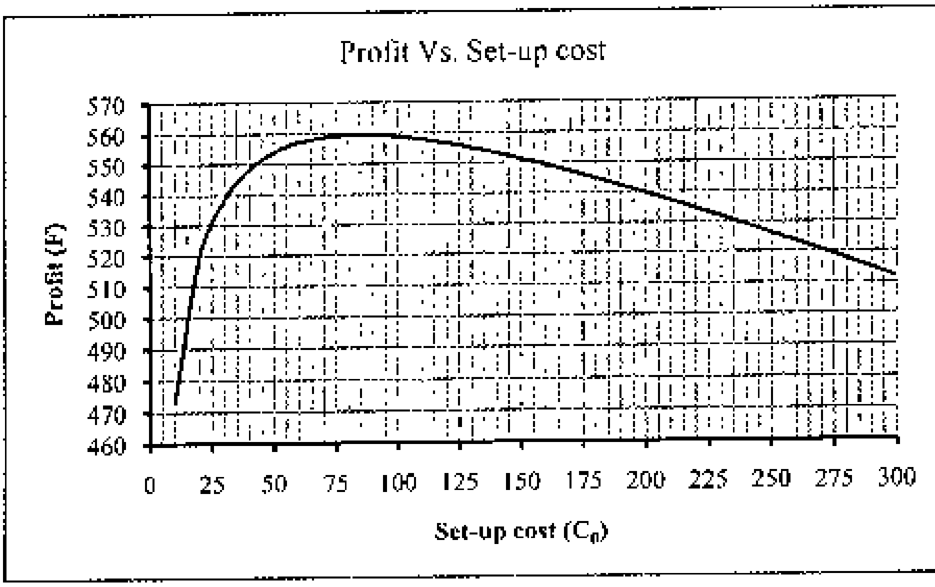

Fig. 5.6: Variation of total profit with set-up cost

It is also observed that profit depends on set-up cost. The relationship between profit and set-up cost is illustrated in Figure 5.6. Profit is maximum at set-up cost 84.3 units. Reliability and production period are assumed as constant to explain the relationship between profit and set-up cost.

The optimal value of set-up cost 84.30 units, reliability of the production process 0.9560 and production period 1.704 units are obtained for the above mentioned production inventory problem. Maximum total profit is also determined for the model which is 559.50 units.

CHAPTER VI
CONCLUSIONS AND RECOMMENDATIONS

6.1 Conclusions

As the profit of an organization largely depends on production inventory, optimization of production inventory model is very important due to complex relationship exists between decision variables and objective function. Uncertainty, imprecision and reliability of the production process have significant impact on the production system. It is also important to incorporate uncertainty, imperfection and reliability of the system to optimize the production inventory model.

The objective of this thesis work is to develop a mathematical model of production inventory by maximizing the total profit. Set-up cost per cycle, reliability of the production process and production period are decision variables. To maximize the total profit, equations for optimal value of decision variables are also obtained. Equation for maximum profit is evaluated using the optimal equations of decision variables. Finally, the model is solved by providing a numeral example.

The implementation of fuzzy random variable as demand and inventory holding cost give more realistic information where the variable values are imprecise. So the model is more realistic and applicable than traditional production inventory models.

Imprecision and uncertainty in imperfect production process are incorporated in the production inventory problem. Reliability is an important factor for a production process which is incorporated in this model. The model is applicable in an imperfect production process where reliability is an important factor.

6.2 Recommendations

Geometric programming is used to find optimal decision for decision maker is very complex situations. This approach may be followed to deal with different optimization problems involving imprecision and uncertainty in science and technology. This model can be extended considering lead time, shortage etc. as fuzzy random variables. Unconstraint signomial geometric programming technique is used to optimize the model. Some other optimization techniques such as non linear geometric programming etc. may be used to optimize the model. Triangular fuzzy number is considered to develop this inventory model. Trapezoidal and gaussian fuzzy number can also be considered to incorporate uncertainty in this model

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