Analytical Base Transit Time of a Bipolar Junction Transistor (BJT) Considering Kirk Effect

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by
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Declaration

It is hereby declared that this thesis or any part of it has not been submitted elsewhere for the award of any degree or diploma.

Signature of the candidate

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Dedication

To My Parents.

Acknowledgement

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Abstract

In this thesis analytical expressions of effective base width and base transit time for exponentially doped base of a BJT considering Kirk effect are developed. The present treatment also includes bandgap narrowing and finite velocity saturation effect in the calculation of base transit time. In modern bipolar junction transistors, the collector is more lightly doped than the base. Therefore, high values of current and collector-emitter voltage saturate the carrier velocity. With carrier-velocity saturation the positions of the space-charge region boundaries become function of collector current. We have studied the influence of velocity saturation in the base-collector depletion layer, and compared the base transit time with velocity saturation to that without velocity saturation. From the comparison we have observed that the base transit time is increased by velocity saturation. Considering bandgap narrowing we found that the base transit time will be underestimated if we do not consider it. The aiding electric field weakens if bandgap narrowing takes place, thereby reducing the drift-aided movement of minority carriers in the base which increases the base transit time. In our work we have incorporated Kirk effect for calculating effective base width and base transit time and found that incorporating Kirk effect the effective base width is increased significantly. Since carriers injected from the emitter have to spend more time in the effective base region, the base transit time becomes higher than that without considering base width widening condition or Kirk effect. So for proper modeling of bipolar junction transistors finite velocity saturation, bandgap narrowing and Kirk effect must be considered.

Chapter 1

Introduction

1.1 Introduction

The purpose of this thesis is to derive an analytical expression for the base transit time of bipolar junction transistor (BJT) considering Kirk effect. In this chapter general description of a BJT, base width stretching mechanism and base transit time are given. Recent literature on analytical base transit time and base width widening mechanism has been reviewed. Finally objective of the thesis and a summary has been included.

1.2 Bipolar Junction Transistor

The bipolar junction transistor (BJT), one of the most important semiconductor devices, was invented by a research team at Bell Laboratories in 1947 [1]. It has had an unprecedented impact on the electronic industry in general and on solid-state research in particular. After the invention of BJT the transistor theory has been extended to include high-frequency, high-power and switching behaviors.

Many developments and research works have been made in transistor technology. The basic bipolar junction transistors are of npn or pnp structure in which emitters and collectors are heavily doped. But this structure is not suitable for high-voltage power transistors. Because for a power transistor switch, the desired features are current handling capability in the on-state and blocking voltage in the off-state together with switching times and losses. These features can be successfully achieved by introducing an extra lightly doped layer between base and substrate.

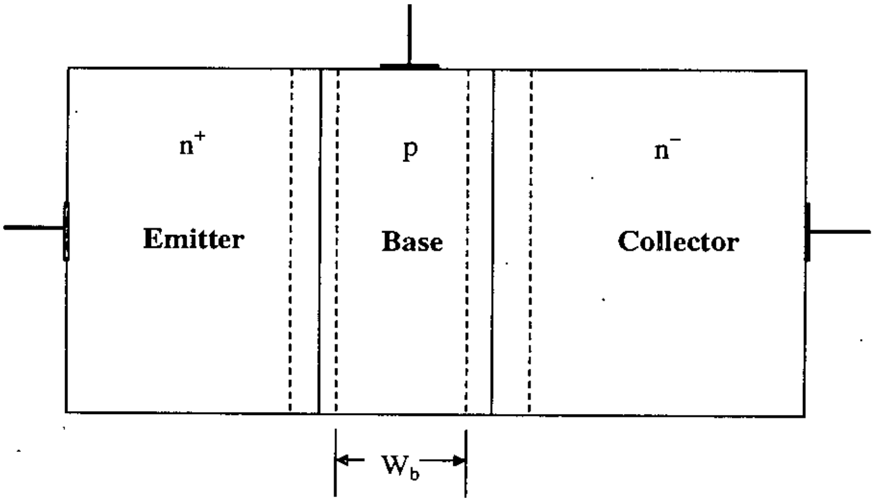

Fig. 1.1: n^+pn^- transistor structure.

1.3 Base Width Widening Mechanism

The structure of a BJT is shown in fig. 1.1. In many cases we can assume that the effective base width W_b is independent of the bias voltages applied to the collector and emitter junctions. This is a reasonable approximation when the collector current is small. In modern bipolar junction transistors the collector is more lightly doped than the base [2]. Therefore high values of current and collector emitter voltage effect predominantly in the collector region before such effects in the base region become noticeable. Even in the forward mode of operation, due to internal voltage drops in the lightly doped collector region, at high current density the base-collector junction may become internally forward biased. When the base-collector junction is forward biased there is charge storage in the collector. In the state of quasi-saturation this charge contributes to the total base charge. The effective base width then increases with the increase of collector current. This phenomenon is known as Kirk effect [3] . This effect plays an important role at high current level in the study of the behavior and the general characteristics of a BJT. As the base width is increased, the base Gummel number [4] (the number of impurities per unit area in the base) increases. Therefore, the current gain decreases at high collector currents. At the same time the increased base width reduces the cut-off frequency and the transistor becomes slower, which causes the increase in base transit time. The base width widening mechanism can be explained considering two different mechanisms - one is quasi-saturation and another is Kirk effect. In the next subsections these will be explained in details.

1.3.1 Quasi-Saturation

To explain the quasi-saturation assume that the base-emitter voltage V_{BE} is varied keeping the collector-emitter voltage V_{CE} fixed. The collector current density J_n has exponential dependence upon base-emitter junction voltage V_{jBE}. Since large change in collector current is caused by a small variation of V_{BE}, we can neglect the change in collector-base voltage V_{CB}. Now V_{jCB} can be expressed as

$$V_{jCB} = V_{CB} - r_C J_n - r_B J_B \tag{1.1}$$

Where V_{jCB} is the collector-base junction voltage, V_{CB} is the externally applied collector-base voltage, J_n and J_B are the collector and base current densities respectively and r_C and r_B are the collector and base resistances per unit cross sectional area (Ω/cm^2) respectively. Now if J_n is increased by increasing the base-emitter voltage V_{BE} keeping the collector-base voltage V_{CB} fixed, the ohmic drop in the collector will increase. So the collector-base junction voltage V_{jCB} will decrease. If this process is continued, at a certain high current level the collector-base junction will become forward biased. In this case the external collector-base terminal is reversed biased but the internal junction will be forward biased. This phenomenon is known as quasi-saturation condition [3]. At that position if we force more current to flow through the collector, the ohmic resistance in the collector region must be reduced to follow the drift-diffusion theory at collector-base junction. As a result, the effective collector-base junction shifts toward the collector to make the collector region smaller than that before. Again there is a negligible electric field or voltage drop in the base region compared to that in the collector region. So the effective base width will increase.

1.3.2 Kirk Effect

The Kirk effect comes into consideration when the base-collector voltage is high. To explain the Kirk effect let us consider a typical integrated transistor in which the base doping is significantly higher than that of the collector. The electric current crossing the collector junction is given by

$$J_n = qv(x)n(x) \tag{1.2}$$

where $v(x)$ is the carrier velocity and $n(x)$ is the electron density. Inside the depletion layer, the electric field is high so that the velocity may be assumed to reach the saturation velocity v_s. Therefore, $n(x) = J_n/qv_s$. This equation states that the electron concentration increases with the collector current. As the current is increased, the electron density will first approach the density of the lightly doped collector. The net space charge there, $N_C - n(x)$, will decrease, leading to a lower electric field gradient given by

$$\frac{d\xi}{dx} = \frac{q}{\epsilon}\left(N_C - \frac{J_n}{qv_s}\right). \tag{1.3}$$

Since the collector voltage is a constant, the integration of the electric field will remain the same with or without current. In other words, the two areas under the electric field curves are identical. So with the injection of collector current the gradient of electric field changes reducing the peak value of the electric field $\xi(0)$. This rise to an increase of the depletion region in the collector side and decrease of depletion layer in the base side, so the effective base width is increased. However, this stretching of the base is quite small. As the current is further increased, collector becomes fully depleted and $\xi(0)$ continues to decrease. Eventually, it becomes zero, which corresponds to the collapse of the original collector space charge layer. That is to say, the collector-base junction is forward-biased. The base spreads over the collector and its width becomes very large.

1.4 Base Transit Time

For an npn transistor, operating in active mode, electrons are injected into the base from emitter. Injected electrons travel towards the collector through the base. The time that the minority carriers take to traverse the quasi-neutral base or the average time that an excess electron spends in the quasi-neutral base is called the base transit time (τ_B).

The total charge of the injected carriers $n(x)$ in the base per unit area is given by

$$Q_b(x) = q\int_0^{W_b} n(x)dx \tag{1.4}$$

The base transit time (τ_B) for the transfer of minority carriers across the base is given by the ratio of minority carrier charge density to the collector current density [5] ,

$$\tau_B = \frac{Q_b(x)}{J_n} = \frac{q}{J_n}\int_0^{W_b} n(x)dx \tag{1.5}$$

where J_n is the collector current density and W_b is the effective base width.

The base transit time depends upon different factors such as (i) base width, (ii) base resistance, (iii) various concentration profile in the base region (i,e. uniform, linier, exponential, Gaussion, box), (iv) velocity saturation effect, (v) collector current density, (vi) quasi-saturation, (vii) built-in electric field in the base region, (viii) mobility, (ix) band gap narrowing etc.

When the base widening effect occurs, the base transit time increases. The different features of transistor parameters such as maximum frequency of operation, cut-off frequency, noise figure etc. are also changed also with the change of base transit time.

1.5 Literature Review

In 1986 J. J. H. Van Biesen [6] used a regional analysis to study the transit times of a BJT as a function of base-emitter bias. He subdivided the total transit time from emitter to collector contact into five components. But no closed form solution was obtained.

In 1994 J. S. Yuan [7] studied the effect of the base profile on the base transit time of the bipolar transistor for all levels of injection. He proposed equation for the minority carrier distribution within the base for different types of base doping. Using boundary conditions and the proposed equation he numerically evaluated the base transit time. The form of equations for J_n and τ_B in his work are not concise to express J_n and τ_B as the function defined by some existing models.

In 1996 P. Ma and L. Zhang [8] proposed an improved model of J_n and τ_B considering the electric field dependence on the minority carrier mobility. The method is based on iteration technique. So the equation of electron current density and base transit time are not concise and they are inconvenient for understanding device physics.

In 1991 K. Suzuki [9] showed that the base transit time strongly depends on the base concentration and base doping profile. He analyzed the base transit time for exponential, Gaussian and box doping profiles for minimizing the base transit time and proposed that the exponential base doping is the optimum profile for getting lower base transit time. But in his analysis he assumed constant base width. This is a reasonable approximation when the collector current is small. At high current and high collector-emitter voltage, the density of injected electron traversing the collector depletion region becomes comparable to the space-charge density. Thus, the collector junction and base width are modified.

In 1992 K. Suzuki and N. Nakayama [10] studied the influence of the velocity saturation in the base-collector depletion layer and compared the injected electron concentration profile, collector current density, and base transit time with velocity saturation to those without velocity saturation. They showed that the base transit time is increased by velocity saturation. They carried out their analysis before the onset of Kirk effect .

Szeto and Reif [11] used the bandgap narrowing in their analysis effect. They showed that to reduce the base transit time the use of nonuniform base doping introduces an aiding built-in electric field which causes subsequent bandgap narrowing effect. They suggested that the bandgap narrowing effect should be considered in order not to overestimate the total reduction of base transit time.

An analytical expression for base transit time which is equally applicable to both heterojunction and homojunction bipolar junction transistors with exponentially doped base was developed by M. M. Jahan and A. F. M. Anwar [12]. In their analysis they incorporated dopant dependent mobility, bandgap narrowing and finite velocity saturation effects. In this paper the analysis was done before the onset of Kirk effect.

In 1973 D. L. Bowler and F. A. Lindholm [13] showed that for a bipolar junction transistor where collector doping is substantially lower than base doping and if the collector-base voltage V_{CB} is low, then with the increase of collector current the depletion layer on the collector side will collapse. If the applied voltage (V_{CB}) is greater than some critical value, the collector depletion layer will expand with increasing collector current until it reaches the end of the collector. Any further increase in current will result in the onset of the mode associated with space-charge-limited flow.

S. G. Lee and R. M. Fox [14] explained that with carrier-velocity saturation the carrier concentration in the space-charge region is proportional to the collector current, and the position of the space-charge region boundaries become functions of the carrier concentration. For high values of V_{CB} with the increase in collector current the depletion region on the base side decreases whereas the depletion region on the collector side increases resulting the effective base width to increase.

In the present work, an analytical expression for the base transit time of an exponentially doped base is obtained considering Kirk effect i.e, the effect of effective base width widening. Nonuniform bandgap narrowing and velocity saturation at the base-collector junction is incorporated in this analysis. The dependence of base transit time on various parameters of BJT is also studied.

1.6 Objectives of the Thesis

The transit time through the neutral region of the base (referred to as the base transit time) often dominates the emitter-to-collector transit time and therefore determines the high frequency figures of merit of a transistor, such as the unity current gain frequency and the maximum frequency of oscillation. Analytical expression of base transit time is available for exponentially doped base without considering Kirk effect i.e, neglecting base width stretching. The objective of this thesis is to derive an analytical expression for base transit time of a bipolar junction transistor (BJT) considering Kirk effect. For this purpose, minority carrier distribution in the base is first obtained considering bandgap narrowing, finite velocity saturation in the base-collector depletion layer and exponential base doping. Then the Poisson's equation is solved to find out the expression of modulated base width. Finally, the expression for base transit time is obtained.

1.7 Thesis Layout

This thesis consists of four chapters. Chapter 1 gives an introduction followed by literature review and justification of carrying out the research on base transit time. In chapter 2 mathematical expressions for base transit time is derived. The expression for minority carrier concentration in the base is obtained by solving the continuity equations for electron and hole. The electric field distribution within the base is obtained by applying the condition $J_P = 0$ and considering nonuniform bandgap narrowing. Then one dimentional Poisson's equation is solved to obtain the expression of effective base width considering three operating conditions (a) part of the collector region is quasi-neutral, (b) collector is entirely space-charged and (c) nonohmic quasi-saturation. Finally, expression for base transit time is obtained. The results obtained are presented in chapter 3 and dependence of base transit time on different transistor parameters are studied. The thesis concludes by presenting an overall discussion on the work and pointing out some unsolved problems for future work in chapter 4.

Chapter 2

Mathematical Analysis

2.1 Introduction

Base transit time is an important parameter of modern bipolar transistors. It dominates the emitter-to-collector transit time and therefore, dominates the high frequency figures of merit of transistors [15]. In modern bipolar transistors, the collector is lightly doped than the base. So at high current and high collector-emitter voltage the effective base width increases with the increase of injection. The carriers injected from the emitter will spend more time in the effective base region and the new base transit time will be greater than that without considering base widening condition. To determine the actual base transit time, the effective base width needs to be calculated. In this chapter analytical expressions for effective base width and base transit time for exponential base doping is derived.

2.2 Effective Base Width

The one dimensional view of an n^+pn^- transistor operating in active mode is shown in fig. 2.1. Commonly used approximation is that the carrier concentration is zero at the base side of the collector-base depletion layer and effective base width is a constant quantity. But with carrier-velocity satu-

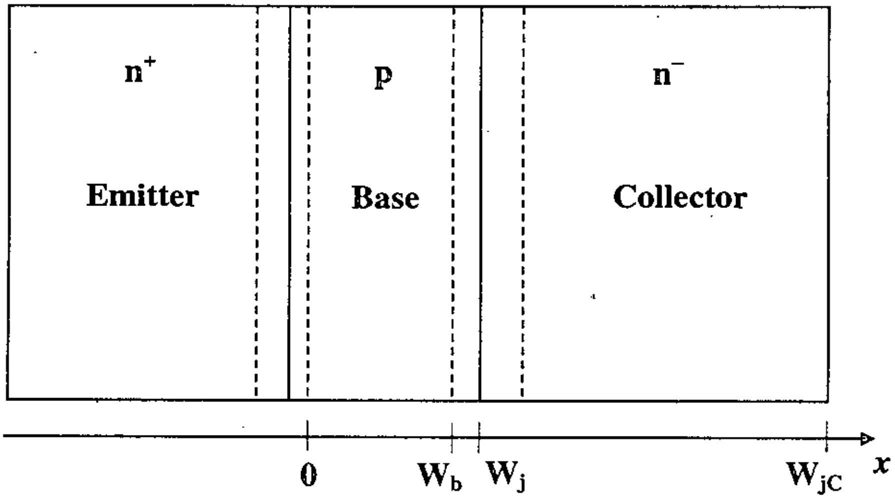

Fig. 2.1: One dimensional view of an n^+pn^- bipolar transistor.

ration the carrier concentration in the space-charge region is proportional to the collector current and the position of space-charge boundary becomes function of the carrier concentration [14]. So the collector junction and the base width are modified. For explaining this effect three operating conditions are considered:

Case 1: Part of the collector region is quasi-neutral.
Case 2: Collector is entirely space-charged.
Case 3: Nonohmic quasi-saturation.

The following assumptions are taken throughout the thesis,

(a) Low level of injection in the base.

(b) The electric field intensity outside the depletion regions is so small that the drift current of minority carriers is negligible.

(c) No recombination and generation in the depletion region.

(d) The widths of the emitter and collector regions are greater than the diffusion length of the minority carriers so that the minority carrier densities have their equilibrium values at the contacts.

(e) The collector area is much larger than the emitter area so as to collect all electrons crossing the collector junction.

(f) The active part of the base and two junctions are of uniform cross sectional area; current flow in the base is essentially one-directional from emitter to collector.

2.2.1 Effective Base Width for Case 1

The relationship between the charge distribution and the electric field is given by the Poisson's equation as

$$\frac{d\xi}{dx} = -\frac{q}{\epsilon_s}[(n-p)+(N_a-N_d)] \tag{2.1}$$

where n is the electron concentration, p is the hole concentration, N_a is the acceptor impurity concentration and N_d is the donor impurity concentration. For completely depleted region, free-carrier densities are zero ($n = p = 0$) and eqn.(2.1) becomes

$$\frac{d\xi}{dx} = -\frac{q}{\epsilon_s}(N_a - N_d) \tag{2.2}$$

For n-type material $N_a = 0$ and for p-type material $N_d = 0$.
The current crossing the collector junction is given by

$$J_n = qn_c v(x) \tag{2.3}$$

where J_n is the collector current density, $v(x)$ is the velocity of electron and n_c is the electron concentration in the space-charge region required to support the current. Inside the depletion layer, the electric field is high so that the velocity may be assumed as having reached the saturation velocity v_s. Therefore, from eqn. (2.3) we have

$$n_c = \frac{J_n}{qv_s} \tag{2.4}$$

The above equation states that the electron concentration increases with the collector current.

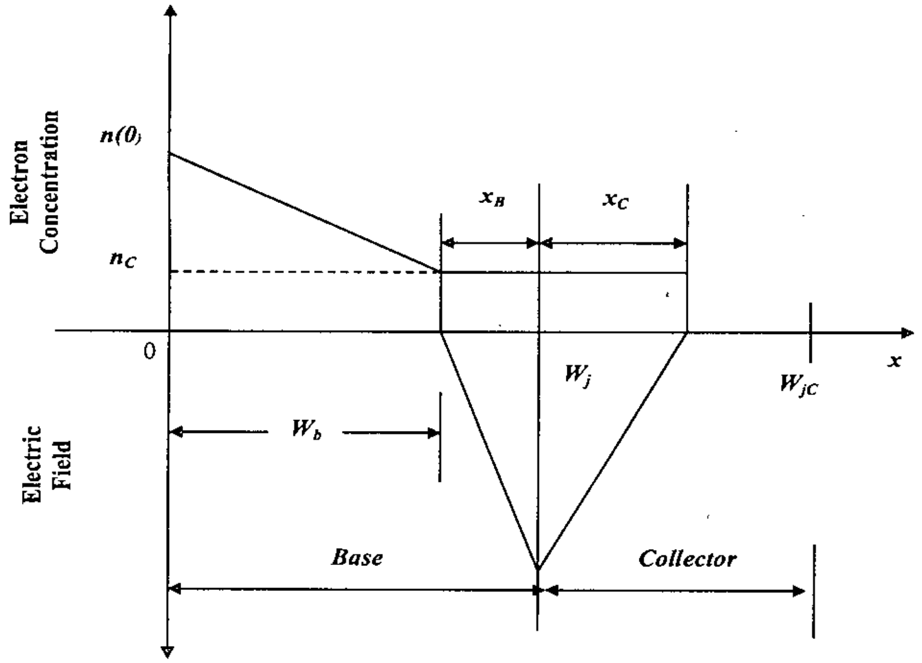

Fig. 2.2: Typical electron concentration and electric field distribution for case 1.

Poisson's equation in the base side of the space-charge region is

$$\frac{d\xi_b}{dx} = -\frac{q}{\epsilon_s}\left\{N_B(x) + \frac{J_n}{qv_s}\right\}; \qquad \text{for} - x_B < x < 0 \tag{2.5}$$

And in the collector side of space-charge region is

$$\frac{d\xi_c}{dx} = \frac{q}{\epsilon_s}\left(N_C - \frac{J_n}{qv_s}\right); \qquad \text{for } \; 0 < x < x_C \tag{2.6}$$

where x_B is the penetration of the space-charge region into the base, x_C is the penetration of the space-charge region into the collector, $N_B(x)$ is the base doping concentration and N_C is the collector doping concentration. Assuming exponential base doping profile $N_B(x)$ is given by [9]

$$N_B(x) = N_B(0)\exp\left[-\left(\frac{\eta x}{W_b}\right)\right] \tag{2.7}$$

where $N_B(0)$ is the base doping concentration at the edge of the emitter-base space-charge region and η ($= \ln\{N_B(0)/N_B(W_b)\}$) is the slope of base doping.

Fig. 2.2 shows the electron concentration and electric field distribution for the case of forward-active operation with part of the collector is quasi-neutral. Since the electric field is zero in the neutral regions and at the edges of the depletion layer, we have

$$\xi_b(-x_B) = \xi_c(x_C) = 0$$

Now, integrating eqns. (2.5) and (2.6) and using $\xi_b(0) = \xi_c(0)$ leads to the relation

$$A_3 x_C = A_2 \left\{ \exp\left(\frac{\eta x_B}{W_b} \right) - 1 \right\} - A_1 x_B \tag{2.8}$$

where $A_1 = -J_n / v_s \epsilon_s$, $A_2 = q N_B(0) W_b / \epsilon_s \eta$ and $A_3 = (q N_C v_s - J_n)/v_s \epsilon_s$.

Poisson's equation can be solved in the collector junction space-charge region to yield the following equation:

$$2(V_{CB} + \phi_c) = A_3 x_C^2 - A_1 x_B^2 - 2A_2 \left\{ \left(\frac{W_b}{\eta} \right) \exp\left(\frac{\eta x_B}{W_b} \right) - \frac{W_b}{\eta} \right\} \tag{2.9}$$

where V_{CB} is the collector junction reverse bias voltage and ϕ_c is the collector junction built-in potential.

From eqns.(2.8) and (2.9) x_B and x_C can be written as

$$x_B = \sqrt{\frac{2 V_0 A_3}{(A_4 - A_1)^2 + 2 A_4 A_3 - A_1 A_3}} \tag{2.10}$$

and

$$x_C = \frac{x_B (A_4 - A_1)}{A_3} \tag{2.11}$$

where $V_0 = V_{CB} + \phi_c$ and $A_4 = q N_B(0)/\epsilon_s$. So the effective base width W_b can be written as

$$W_b = W_j - x_B \tag{2.12}$$

where W_j is the metalargical base width. Equation (2.12) is valid only up to the value of J_n for which the base-collector space-charge region extends all the way to the collector for a given value of V_{CB}, i.e., when $x_C = W_{jC}$.

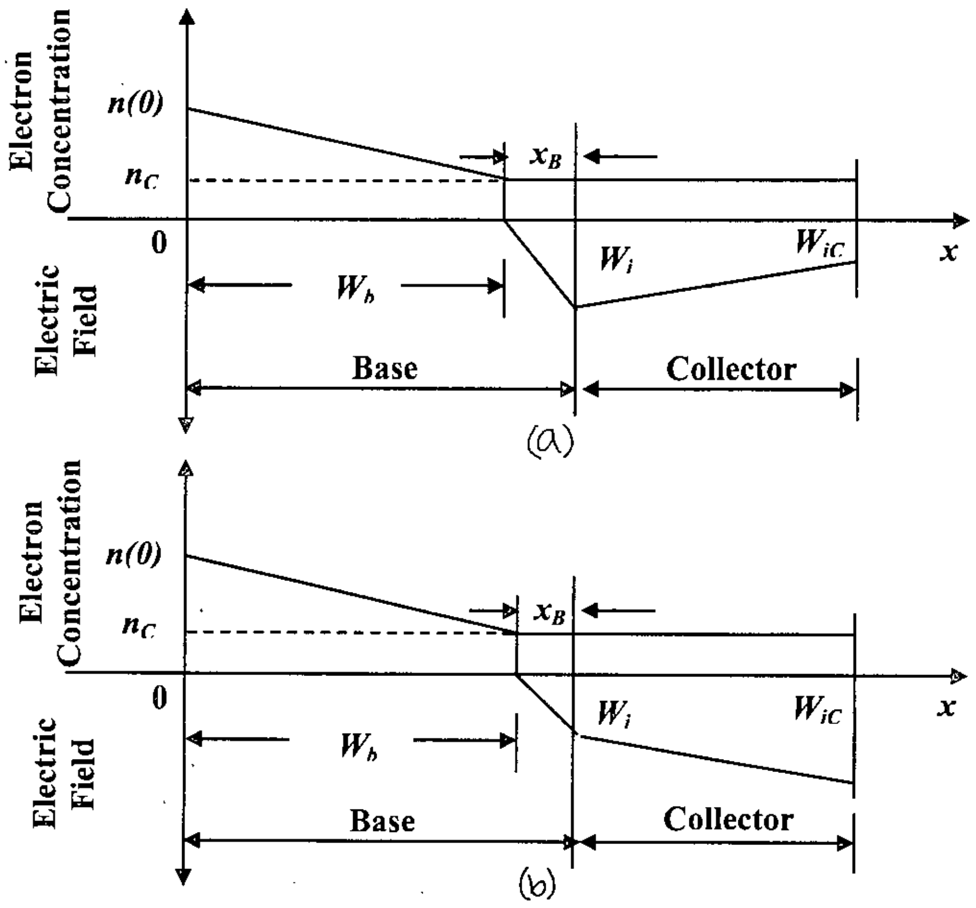

Fig. 2.3: Typical electron concentration and electric field distribution for case 2; (a) for $J_n < J_C$, (b) for $J_n > J_C$.

The upper limit of validity for eqn. (2.12) can be found from eqn. (2.11) with x_C replaced by W_{jC}

$$J_n|_{x_C=W_{jC}} \cong -v_s\epsilon_s \left\{ \frac{-b_1 - \sqrt{b_1^2 - 4a_1c_1}}{2a_1} \right\} \tag{2.13}$$

where $A_5 = qN_C/\epsilon_s$, $A_6 = 2V_0/W_{jC}^2$, $a_1 = A_5 + A_6$, $b_1 = A_5^2 - A_4^2 - 2A_4(A_5 + A_6)$ and $c_1 = A_4^2(A_6 - A_5) - 2A_4A_5^2$.

2.2.2 Effective Base Width for Case 2

As J_n increases above $J_n|_{x_C=W_{jC}}$ the collector-base space-charge region begins to uncover charge in the buried layer. The resulting electron concentration and electric field distribution are shown in fig. 2.3 (a). As the current increases above $J_C(= qN_C v_s)$, the electron concentration in the collector rises above N_C, leading to electric field and electron concentration distributions as shown in fig. 2.3 (b).

In this case the electric field distribution in the base and collector region will be

$$\xi_b(x) = A_1(x + x_B) + A_2\left\{\exp\left(-\frac{\eta x}{W_b}\right) - \exp\left(\frac{\eta x_B}{W_b}\right)\right\} \quad (2.14)$$

$$\text{and} \qquad \xi_c(x) = \xi_c(W_{jC}) - A_3(W_{jC} - x) \quad (2.15)$$

$$\text{where,} \qquad \xi_c(W_{jC}) = A_1 x_B + A_2\left\{1 - \exp\left(\frac{\eta x_B}{W_b}\right)\right\} + A_3 W_{jC}$$

Using charge neutrality and solving Poission's equation over the collector junction space-charge region, x_B for case 2 can be written as

$$x_B = \frac{-b + \sqrt{b^2 - 4ac}}{2a} \quad (2.16)$$

where, $a = A_4 - A_1/2$, $b = (A_4 - A_1)W_{jC}$ and $c = -(V_0 + A_3 W_{jC}^2/2)$.

The effective base width W_b for case 2 can be written as

$$W_b = W_j - x_B \quad (2.17)$$

2.2.3 Effective Base Width for Case 3

Further increment in J_n drives the BJT into nonohmic quasi-saturation. Fig. 2.4 shows a typical electron concentration and electric field distribution in nonohmic quasi-saturation condition. In this condition of operation, the collector space-charge region excess free-electron concentration $(n_c - N_C)$ is

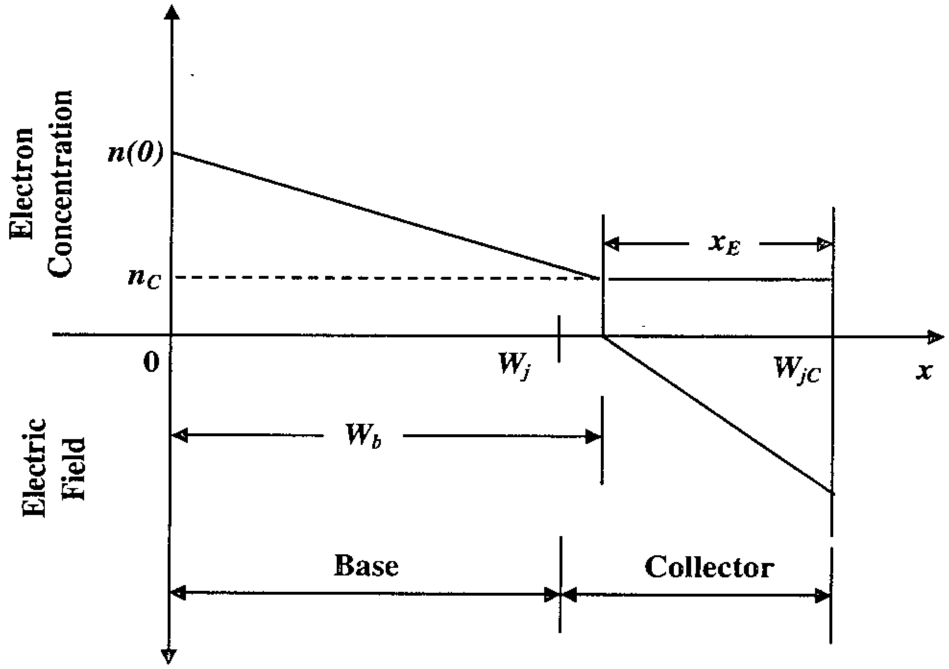

Fig. 2.4: Typical electron concentration and electric field distribution for case 3.

balanced by the heavily doped buried-layer charge. Solution of Poission's equation leads to

$$x_E = \sqrt{\frac{2(V_{CB} + \phi_c)v_s\epsilon_s}{J_n - qN_C v_s}} \tag{2.18}$$

where x_E is the collector space-charge width in nonohmic quasi-saturation operation. In this case W_b must be redefined as

$$W_b = W_j + W_{jC} - x_E \tag{2.19}$$

The upper limit of validity for eqn. (2.17) and lower limit for eqn. (2.19) can be found by replacing x_E with W_{jC} in eqn. (2.18) as

$$J_n|_{x_E=W_{jC}} = \frac{2\epsilon_s v_s(V_{CB} + \phi_c)}{W_{jC}^2} + qN_C v_s \tag{2.20}$$

2.3 Electric Field in the Base

For a bipolar junction transistor with a reasonable current gain the hole current density is negligible ($J_p \approx 0$) compared to electron current density. The electric field in the base is given by [16]

$$E(x) = \frac{kT}{q}\left\{\left[\frac{1}{p(x)}.\frac{dp(x)}{dx}\right] - \left[\frac{1}{n_{ie}^2(x)}.\frac{dn_{ie}^2(x)}{dx}\right]\right\} \tag{2.21}$$

where $p(x) = n(x) + N_B(x)$ and $n_{ie}(x)$ is the effective intrinsic carrier concentration in the base. The first term in eqn. (2.21) represents electric field due to concentration gradient and the second term represents the quasi-field due to nonuniform bandgap narrowing [17]. For low level of injection $N_B(x) \gg n(x)$ and hence $p(x) \approx N_B(x)$.

The intrinsic carrier concentration is given by [12]

$$n_{ie}^2(x) = n_{ie}^2(0)\left[\frac{N_B(x)}{10^{17}}\right]^{2V_g/V_T}, \tag{2.22}$$

where $n_{ie}(0)$ is the intrinsic carrier concentration without bandgap narrowing, V_g is the bandgap and $V_T = kT/q$ is the thermal voltage.
Taking $s = V_g/V_T$ eqn. (2.22) can be written as

$$n_{ie}^2(x) = n_{ie}^2(0)\left[\frac{N_B(x)}{10^{17}}\right]^{2s} \tag{2.23}$$

Substituting the values of $N_B(x)$ and $n_{ie}^2(x)$ in eqn. (2.21) expression of electric field reduces to

$$E(x) = \frac{\eta V_T}{W_b}(2s - 1) \tag{2.24}$$

2.4 Collector Current Density

Neglecting the recombination within the base the electron current density, J_n of a bipolar transistor can be written as

$$-J_n = qD_n \frac{dn(x)}{dx} + q\mu_n E(x) n(x) \tag{2.25}$$

where q is the electron charge, D_n is the electron diffusion coefficient, μ_n is the electron mobility, n is the electron concentration and E is the electric field. Inserting the value of $E(x)$ in eqn. (2.25) and integrating it over the effective base region we have

$$\exp(p).n(W_b) - n(0) = -\frac{J_n}{qD_n}.\frac{W_b}{p}\{\exp(p) - 1\} \tag{2.26}$$

where $p = \eta(2s-1)$, $n(0)$ is the electron concentration in the base at the edge of emitter-base space-charge region and $n(W_b)$ is the electron concentration in the base at the edge of collector-base space-charge region.

The boundary condition at the emitter-base junction is given by [18]

$$n(0) = \frac{n_{ie}^2(0)}{N_B(0)} \exp\left(\frac{qV_{BE}}{kT}\right) \tag{2.27}$$

and, assuming that the electron velocity in the base-collector depletion region saturates at v_s, $n(W_b)$ can be expressed as [19]

$$n(W_b) = \frac{J_n}{qv_s} \tag{2.28}$$

Putting the values of $n(0)$ and $n(W_b)$ in eqn.(2.26), the electron current density can be obtained as

$$J_n = \frac{\dfrac{n_{ie}^2(0)}{N_B(0)} \exp\left(\dfrac{V_{BE}}{V_T}\right)}{\dfrac{\exp(p)}{qv_s} + \dfrac{W_b}{qD_n p}\{\exp(p) - 1\}} \tag{2.29}$$

2.5 Base Transit Time

For a thin base bipolar junction transistor neglecting recombination in the base and considering an exponential base doping profile the electron concentration in the base can be represented as [7]

$$n(x) = C_1 + C_2 \exp\left(-\frac{qEx}{kT}\right) \tag{2.30}$$

where C_1 and C_2 are two arbitrary constants.

$$\text{at} \quad x = 0, \qquad n(0) = C_1 + C_2 \tag{2.31}$$

$$\text{and} \quad \text{at} \quad x = W_b, \quad n(W_b) = C_1 + C_2 \exp\left\{-\frac{qEW_b}{kT}\right\} \tag{2.32}$$

Substituting the values of $n(0)$, $n(W_b)$ and E, C_1 and C_2 can be solved from the eqns. (2.31) and (2.32) as

$$C_1 = \frac{J_n}{qv_s} + \frac{n_{ie}^2(0)\exp\{(V_{BE}/V_T) - \eta(2s-1)\}}{N_B(0)[\exp\{-\eta(2s-1)\} - 1]} \tag{2.33}$$

$$\text{and} \quad C_2 = \frac{n_{ie}^2(0)\exp(V_{BE}/V_T) - (J_n/qv_s)}{N_B(0)[1 - \exp\{-\eta(2s-1)\}]} \tag{2.34}$$

So, the injected minority carrier distribution within the base is

$$n(x) = C_1 + C_2 \exp\left\{-\frac{\eta(2s-1)x}{W_b}\right\} \tag{2.35}$$

Integrating the injected minority carrier over the effective base region and dividing it by J_n/q [5], base transit time τ_B can be solved as

$$\tau_B = \frac{qW_b[C_1 - \frac{C_2}{p}\{\exp(p) - 1\}]}{\dfrac{\dfrac{n_{ie}^2(0)}{N_B(0)}\exp\left(\dfrac{V_{BE}}{V_T}\right)}{\dfrac{\exp(p)}{qv_s} + \dfrac{W_b}{qD_n p}\{\exp(p) - 1\}}} \tag{2.36}$$

2.6 Conclusion

The analytical expressions for effective base width and base transit time of a bipolar junction transistor are obtained in this chapter. The transistor under our consideration has lightly doped collector and exponentially doped base. Kirk effect is considered for obtaining expressions of effective base width and base transit time. The transistor is assumed to be operated under quasi-saturation condition. Low injection in the base and collector region is taken into consideration and recombination in the base is neglected.

Chapter 3

Results and Discussion

3.1 Introduction

The mathematical equations related to the work have been derived in the previous chapter. Based on the derived equations simulation programs have been developed using MATLAB software. Required parameters have been taken from practical transistors. The data of various parameters generated by the programs are plotted in this chapter to study the effects of those parameters on base transit time. The variation of base transit time with different parameters are also discussed.

3.2 Results and Discussion

In the following sub-sections the dependence of effective base width, collector current density and base transit time upon different parameters of transistor will be discussed with the help of different plots.

3.2.1 Dependence of effective base width upon collector current density

In many cases it is assumed that the base width W_b is independent of the bias voltage applied to the collector and emitter junctions. In modern bipolar junction transistors the collector is more lightly doped than the base. Therefore high values of collector current and collector-emitter voltage can reasonably change the effective base width. From the eqns. (2.12), (2.17) and (2.19) it is clear that the effective base width is not constant - it is a function of collector current density J_n. The dependence of effective base width (W_b) with collector current density (J_n) for different values of peak base doping is shown in Fig. 3.1. From the figure it is clear that the effective base width is almost constant at the lower values of J_n. But at the higher values of J_n effective base width increases rapidly with the increase of collector current density J_n. This is because when the collector is fully depleted, increase in J_n decreases the depletion layer in the base side rapidly and further increase in J_n forces the base spreads over the lightly doped collector region and the effective base width becomes very high as shown in Fig. 3.2. Since in case 3 the expression of effective base width is independent of base doping concentration, we have got same line for different peak base doping. From the Fig. 3.1 we also see that with the increase of peak base doping effective base width increases. This is because, with the increase of the base doping the depletion region in the base side decreases causing the increase of effective base width.

3.2.2 Variation of collector current density with base-emitter voltage

The variation of collector current density with base-emitter voltage for different values of peak base doping is shown in Fig. 3.3 and for different values of slope of base doping η is shown in Fig. 3.4. It is clear from the figures that with the increase of V_{BE}, J_n increases. For the lower values of V_{BE} the increase in J_n is small but when V_{BE} crosses a certain value, J_n rises abruptly. From the Fig. 3.3 we also see that for a certain value of V_{BE}, J_n is high for low value of peak base doping. This is because if the base doping is low the excess holes in the base region is small. So electrons can enter into the base region spontaneously and crosses the base, and the

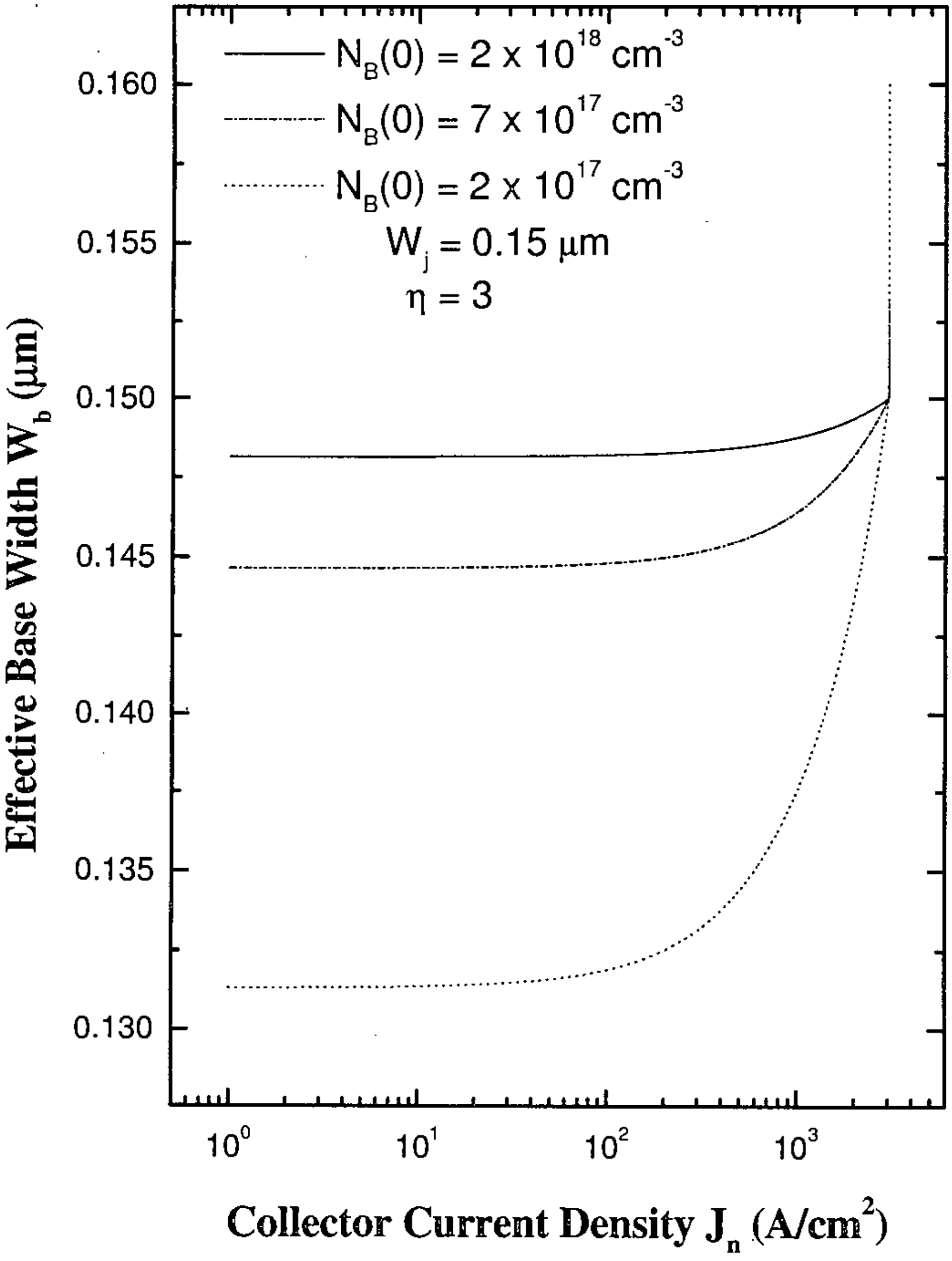

Fig. 3.1: Effective base width as a function of collector current density for different values of peak base doping $N_B(0)$. Here $\eta = 3$ and $W_j = 0.15\mu m$.

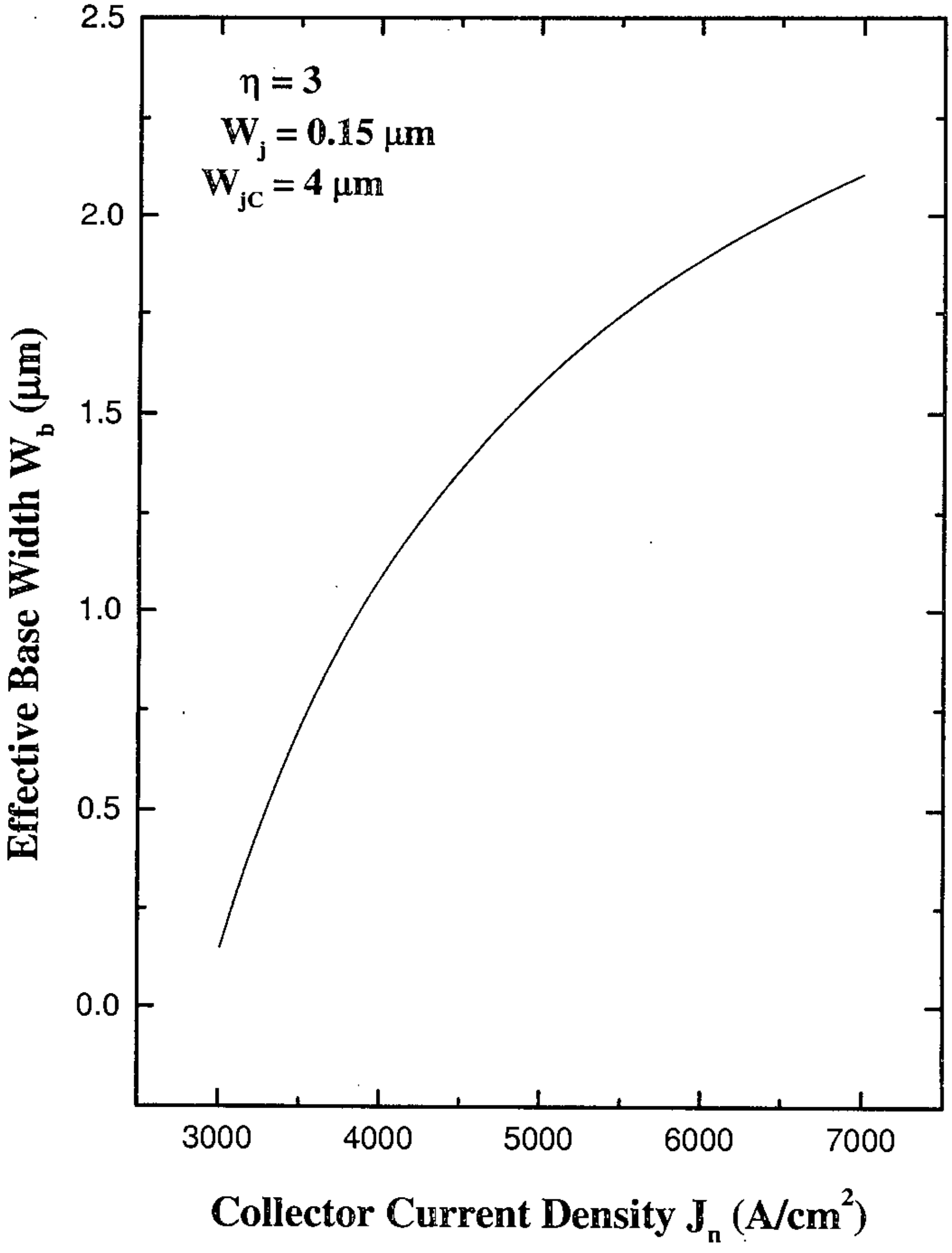

Fig. 3.2: Effective base width as a function of collector current density for case 3. Here $\eta = 3$, $W_j = 0.15\mu m$ and $W_{jC} = 4\mu m$.

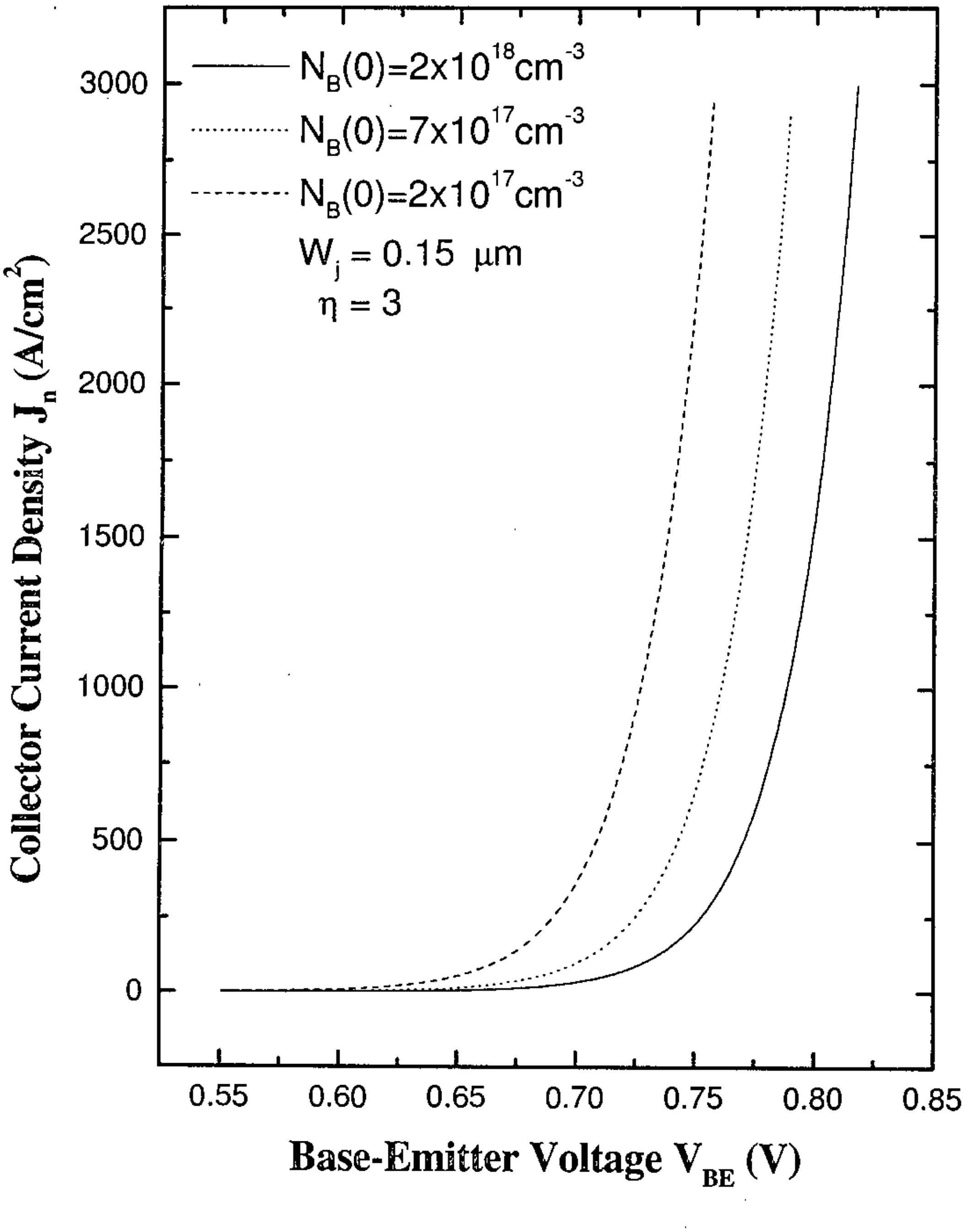

Fig. 3.3: Collector current density as a function of base-emitter voltage for different values of peak base doping $N_B(0)$. Here $\eta = 3$ and $W_j = 0.15\mu m$.

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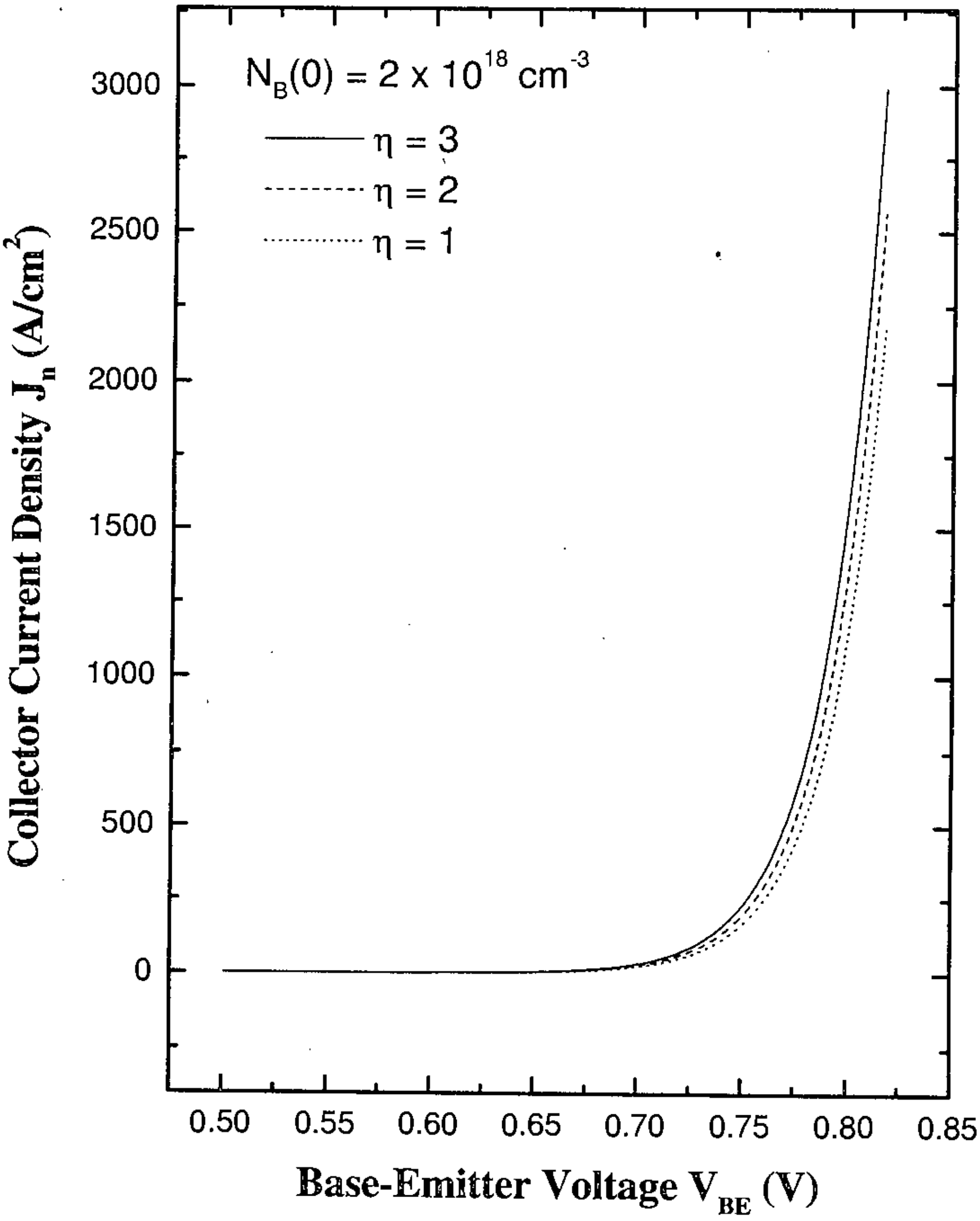

Fig. 3.4: Collector current density as a function of base-emitter voltage for different values of η. Here $N_B(0) = 2 \times 10^{18} cm^{-3}$ and $W_j = 0.15 \mu m$.

collector current density becomes high. From the Fig. 3.4 we see that for a fixed value of V_{BE}, J_n increases with the increase in slope of base doping.

3.2.3 Dependence of effective base width upon base-emitter voltage

The dependence of effective base width upon base-emitter voltage for different values of peak base doping is shown in Fig. 3.5. From the figure it is clear that with the increase in base-emitter voltage effective base width increases. This is because with the increase in base-emitter voltage carrier injection into the base increases which increases the effective base width. From the figure we also see that effective base width is large for high values of peak base doping. This is because with the increase in base doping depletion layers in the base side of the base-emitter junction and the base-collector junction are reduced which increases the effective base width.

3.2.4 Minority carrier distribution within the base

The distribution of minority carrier $n(x)$ within the base for different slopes of base doping is shown in Fig. 3.6. It can be seen from the figure that the minority carrier has the maximum value at the base-emitter junction (at $x = 0$) and the minimum value at the collector-base junction (at $x = W_j$). It decreases from its peak value with the increase of distance from the base-emitter junction. In this work, velocity saturation of electron at the collector-base junction is taken into consideration. So, at collector-base junction minority carrier concentration is not zero. From the figure we also see that as the slope of base doping decreases the minority carrier distribution within the base becomes more and more linear. This is because, as the slope of base doping decreases the nonuniformity of the doping profile decreases.

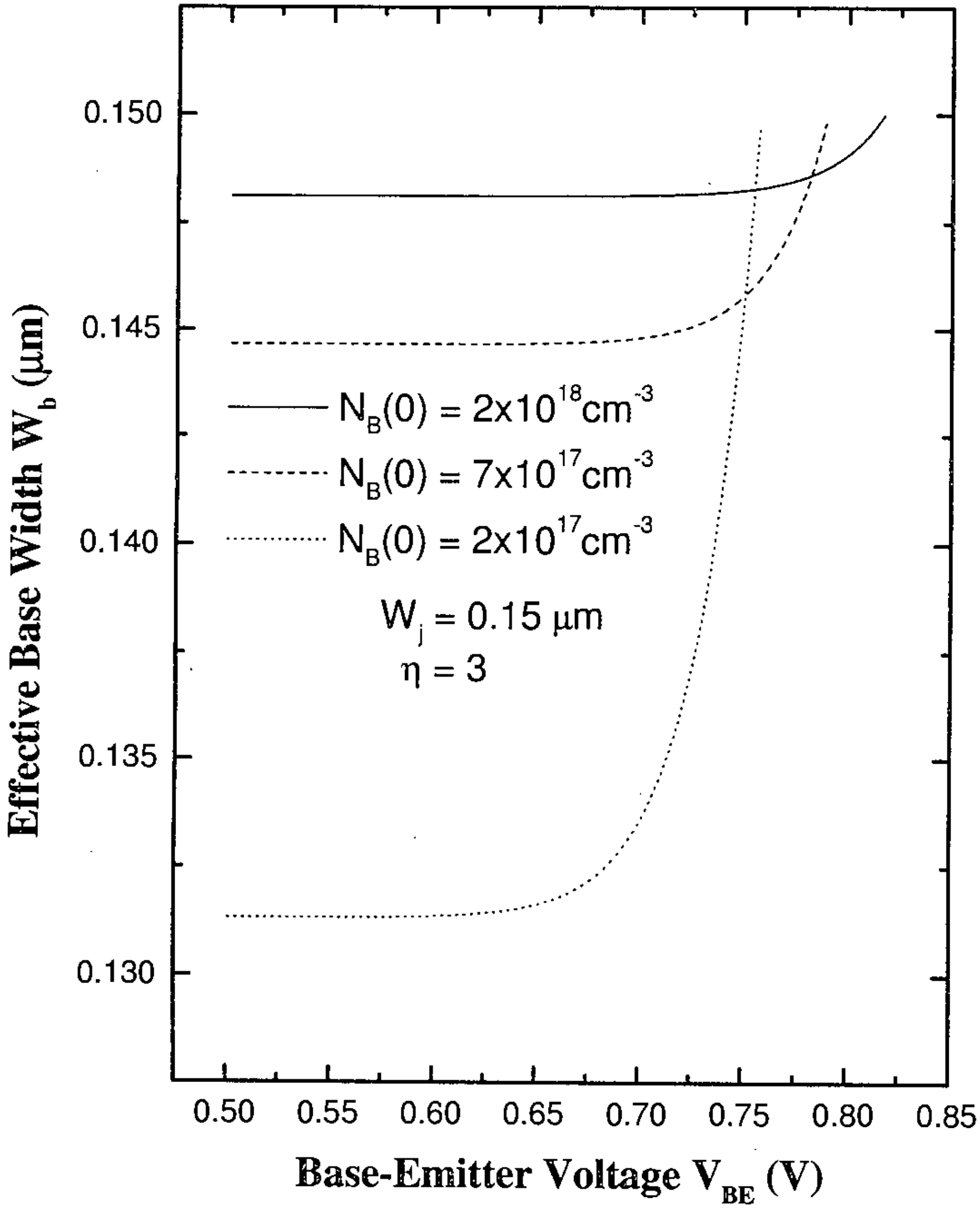

Fig. 3.5: Effective base width as a function of base-emitter voltage for different values of peak base doping $N_B(0)$. Here $\eta = 3$ and $W_j = 0.15\mu m$.

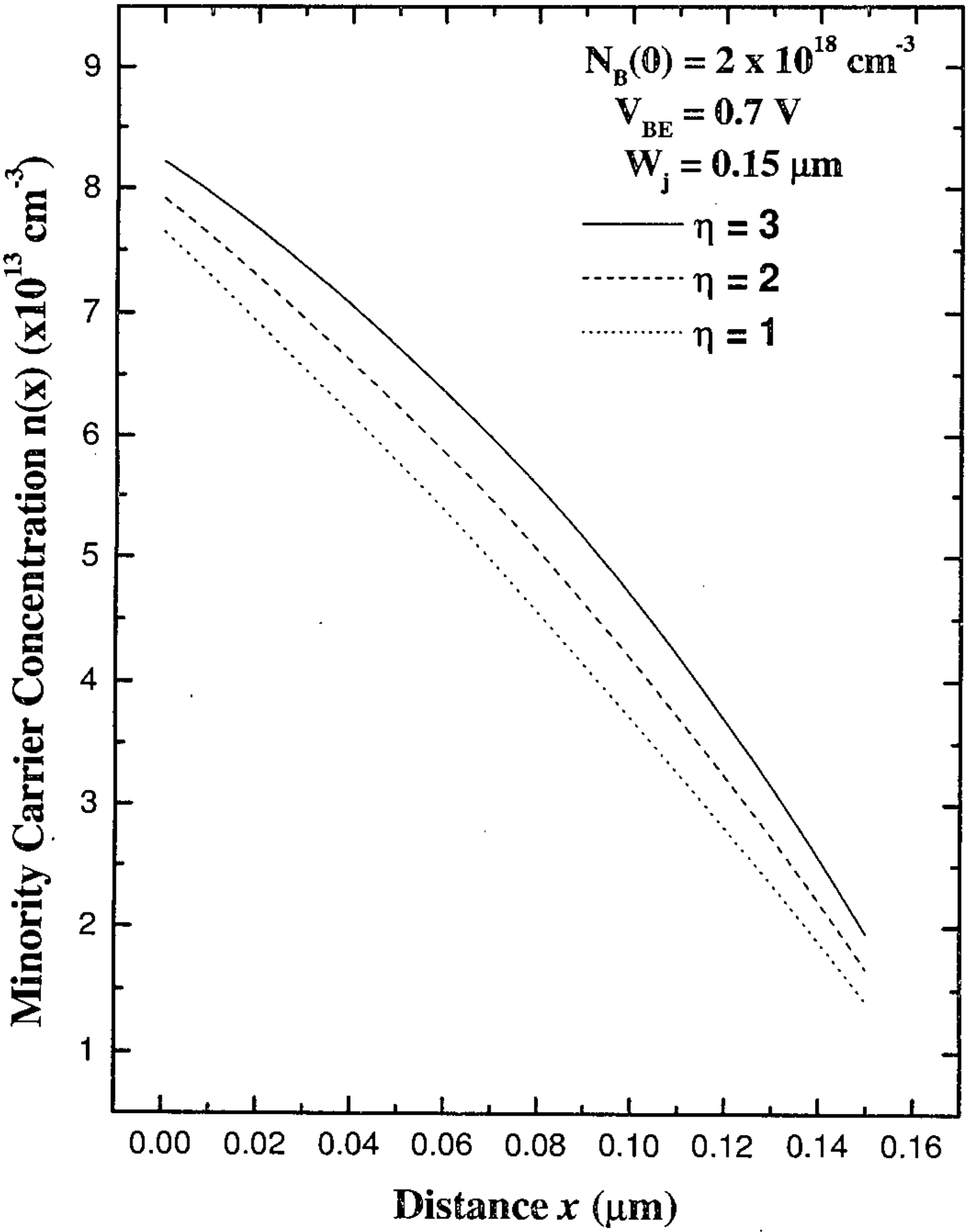

Fig. 3.6: Minority carrier distribution within the base for different values of slope of base doping η. Here $V_{BE} = 0.7V$ and $N_B(0) = 2 \times 10^{18}\, cm^{-3}$.

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3.2.5 Base transit time as a function of base-emitter voltage

The variation of base transit time with base-emitter voltage for different peak base concentrations is shown in Figs. 3.7 and 3.8. Fig. 3.7 is for case 1 and case 2, and Fig. 3.8 is for case 3. The variation of base transit time for different slopes of base doping is shown in Fig. 3.9. It is clear from the figures that with the increase of base-emitter voltage base transit time increases. With the increase of base-emitter voltage the collector current increases, as we see in Figs. 3.3 and 3.4, which can reduce the base transit time. But at the same time effective base width increases with the increase in base-emitter voltage, as shown in Fig. 3.5. Since with the increase of effective base width electrons injected from the emitter have to traverse more distance, the base transit time increases.

3.2.6 Dependence of base transit time upon peak base doping concentration

The dependence of base transit time upon peak base doping concentration for different slopes of base doping is shown in Fig. 3.10. From this figure we see that the base transit time increases with peak base doping concentration. This is because the mobility of electrons in the base decreases with the increment of peak base doping [7] and hence the base transit time increases.

3.2.7 Dependence of base transit time upon the slope of base doping

The variation of base transit time with the slope of base doping concentration for different peak base doping is shown in Fig. 3.11. From this figure we see that the base transit time is a decreasing function of slope of base doping. This is because the aiding electric field in the base increases with η. The aiding electric field increases the drift-aided movement of minority carrier in the base, so the base transit time decreases.

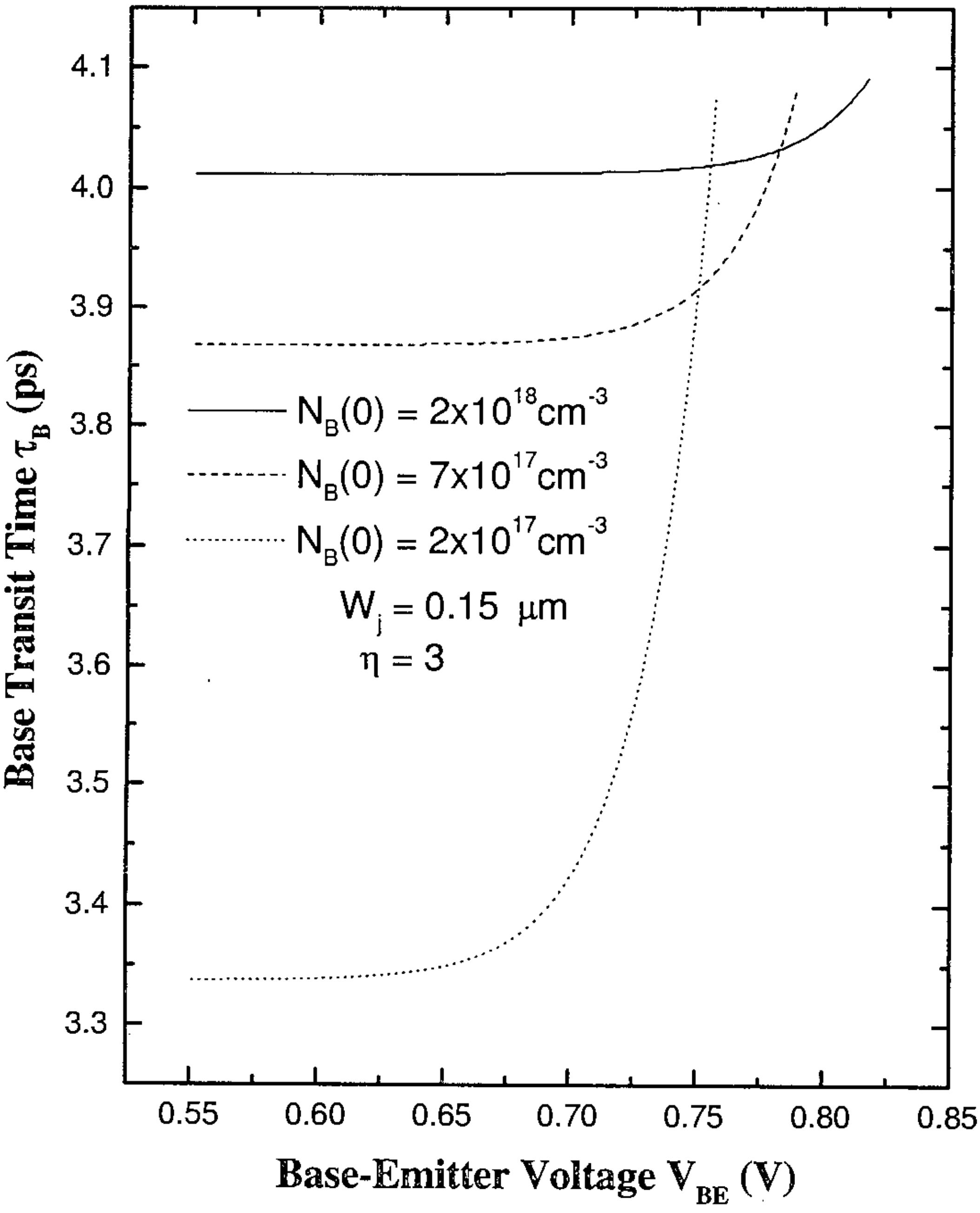

Fig. 3.7: Base transit time as a function of base-emitter voltage for different values of $N_B(0)$. Here $\eta = 3$ and $W_j = 0.15\mu m$ (for case 1 and case 2).

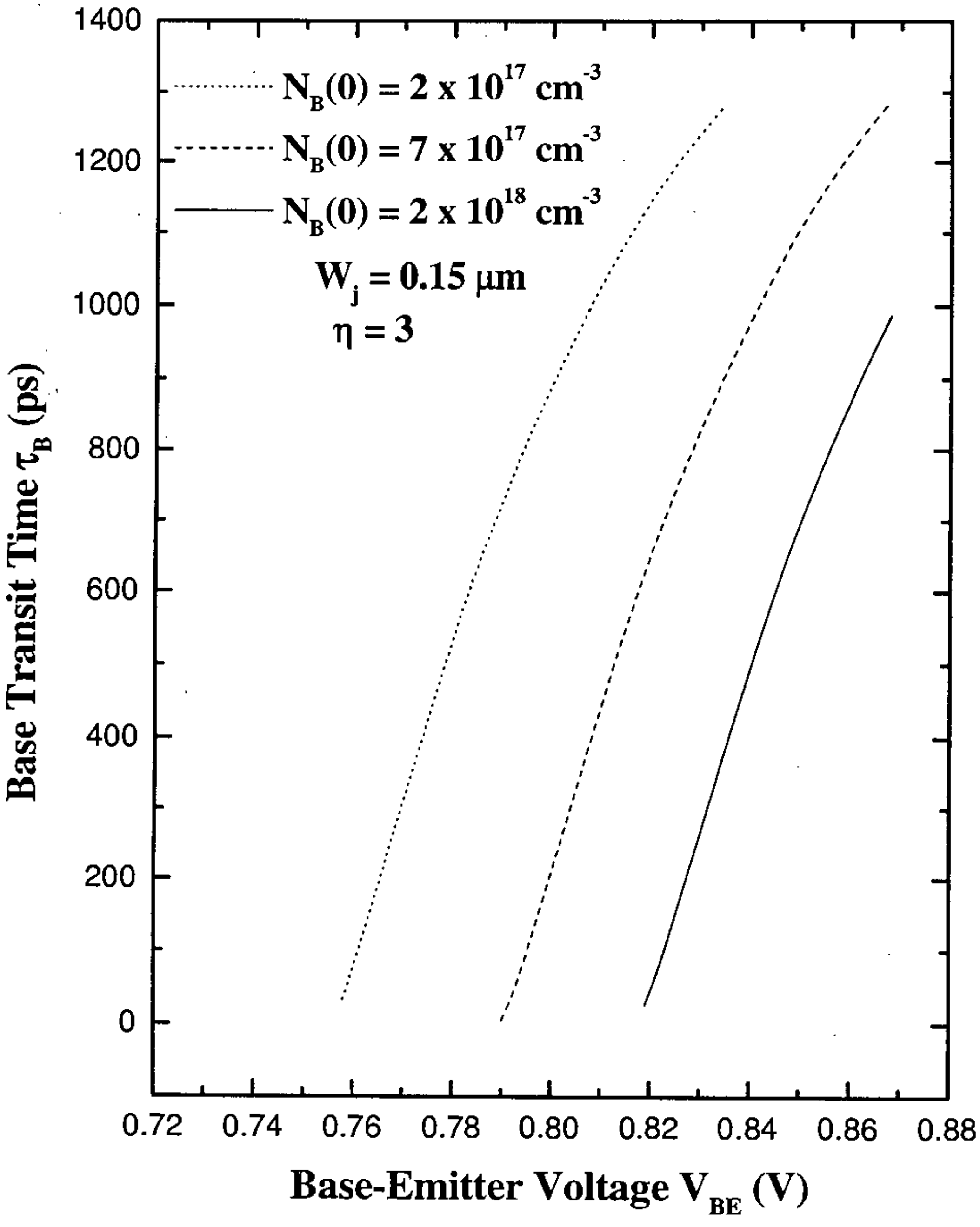

Fig. 3.8: Base transit time as a function of base-emitter voltage for different values of $N_B(0)$. Here $\eta = 3$ and $W_j = 0.15\mu m$ (for case 3).

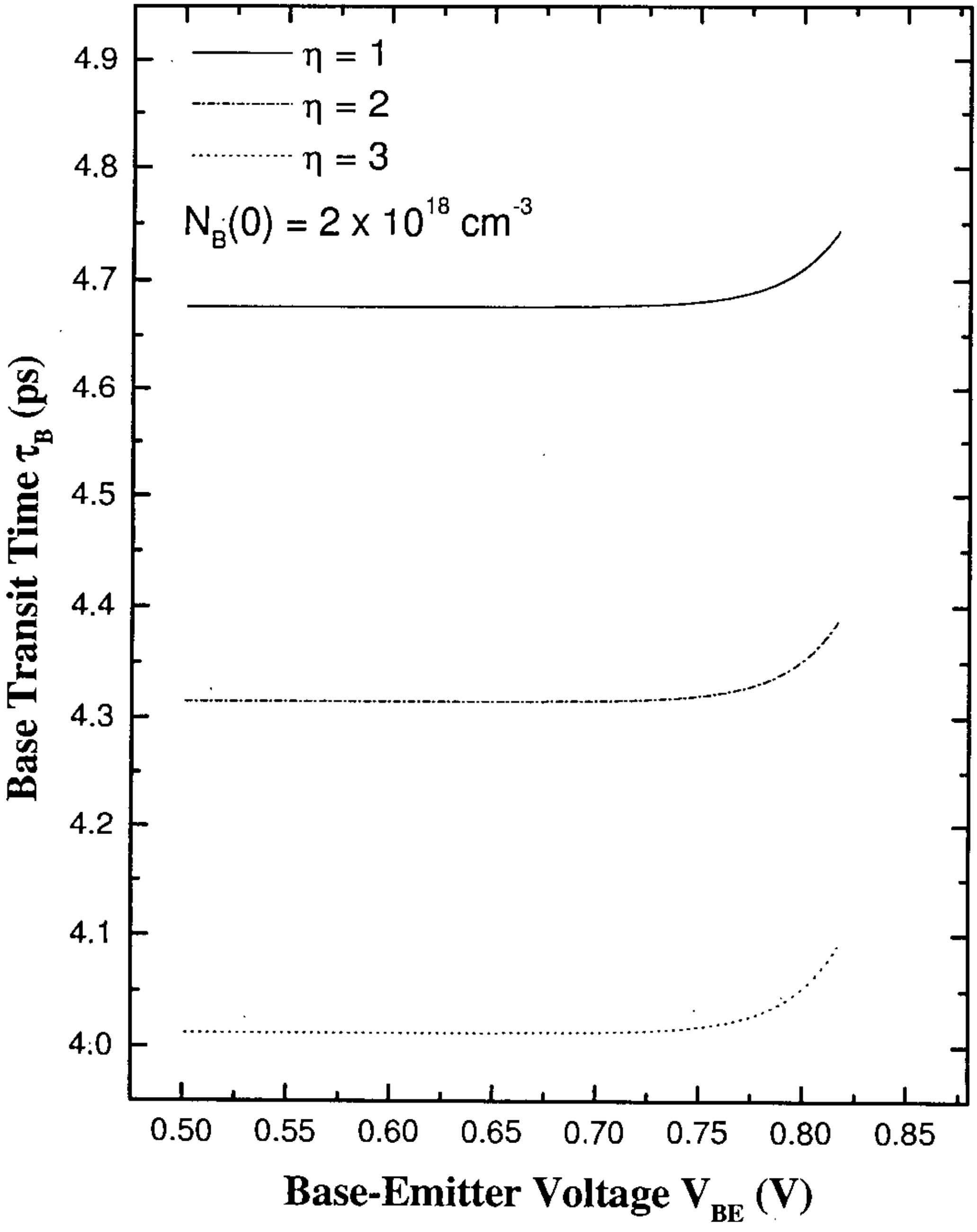

Fig. 3.9: Base transit time as a function of base-emitter voltage for different slopes of base doping η. Here $N_B(0) = 2 \times 10^{18}\, cm^{-3}$ and $W_j = 0.15\mu m$.

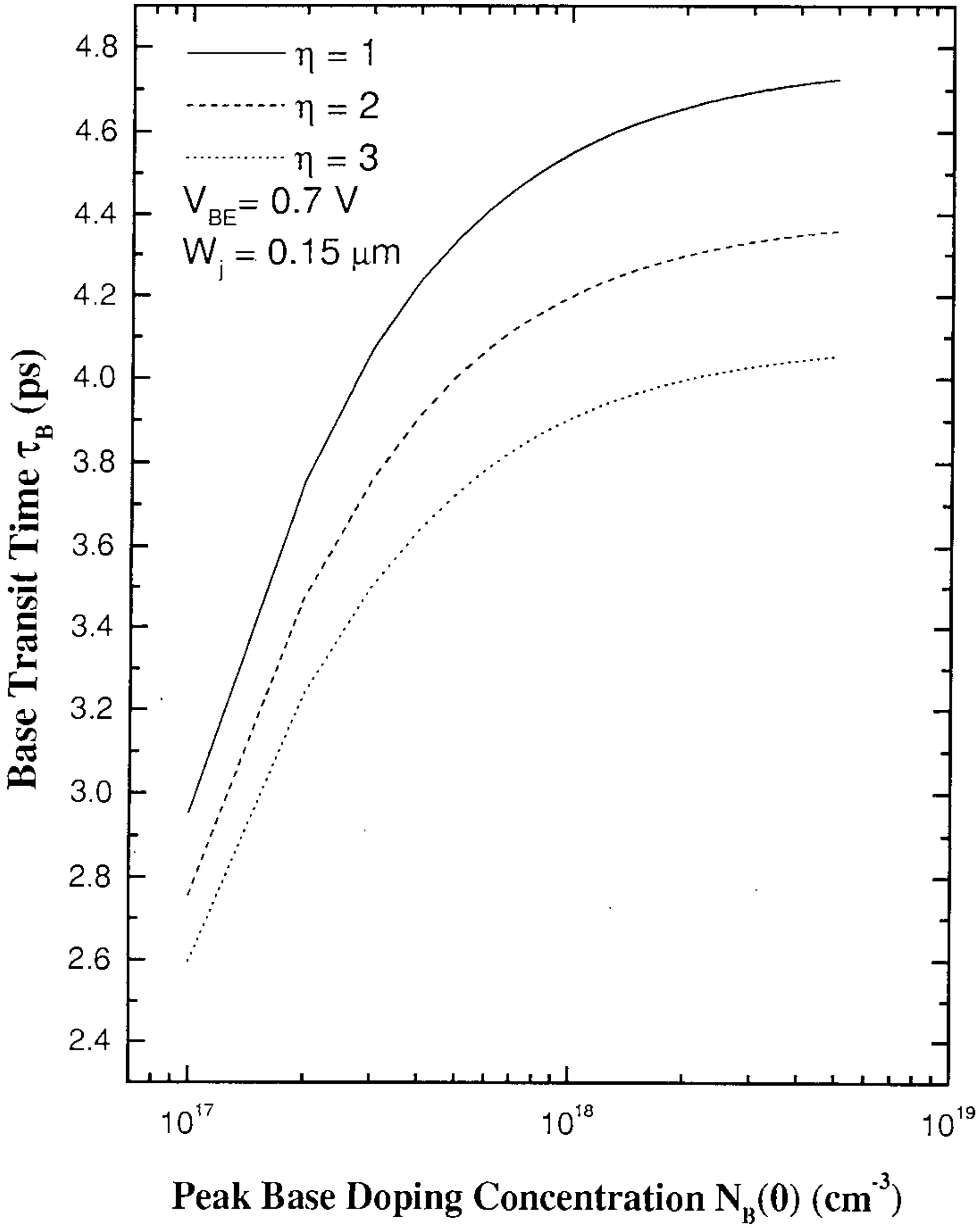

Fig. 3.10: Base transit time as a function of peak base doping concentration for different slopes of base doping η.

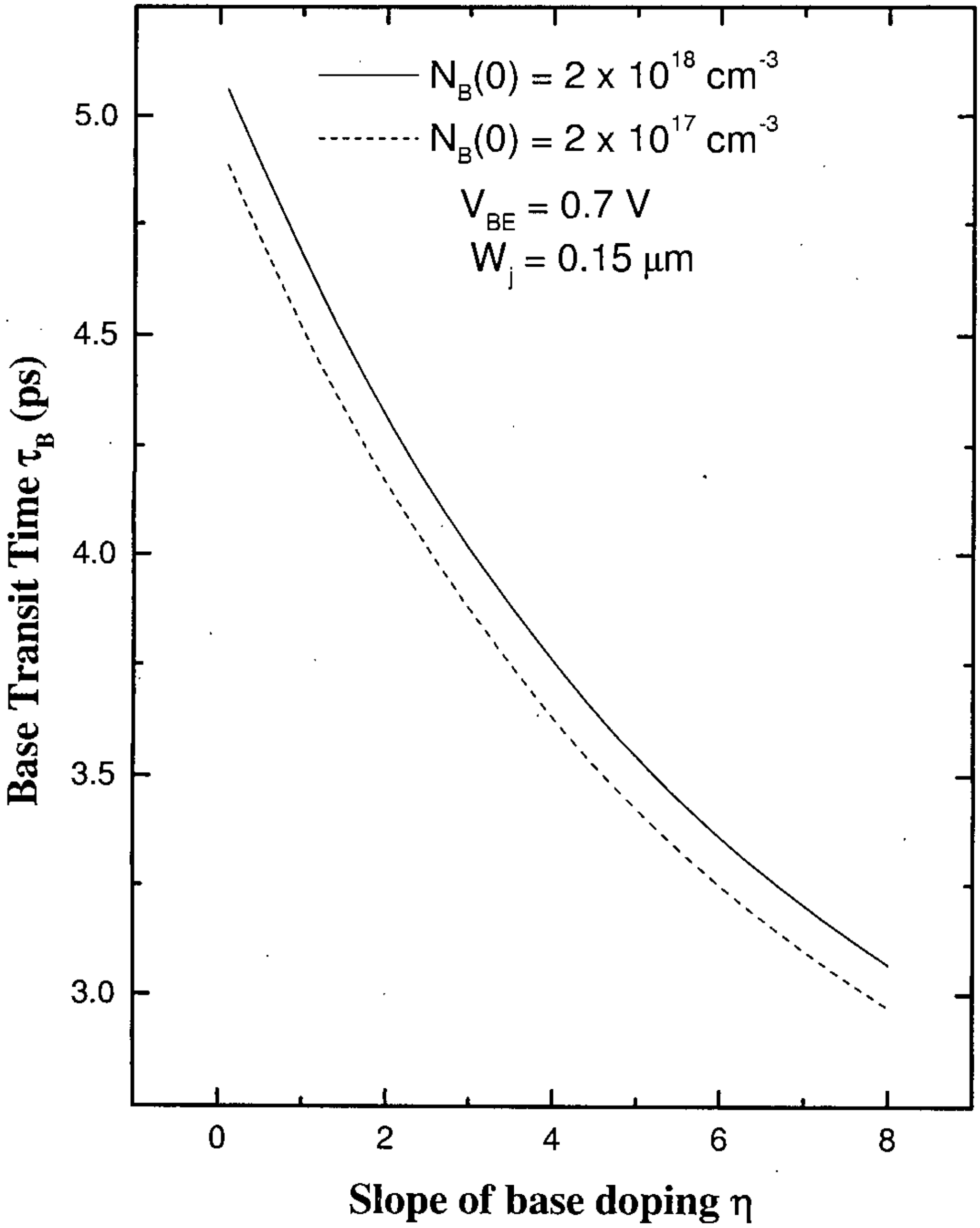

Fig. 3.11: Base transit time as a function of base doping gradient η for $N_B(0) = 2 \times 10^{17}$ and $N_B(0) = 2 \times 10^{18}\, cm^{-3}$.

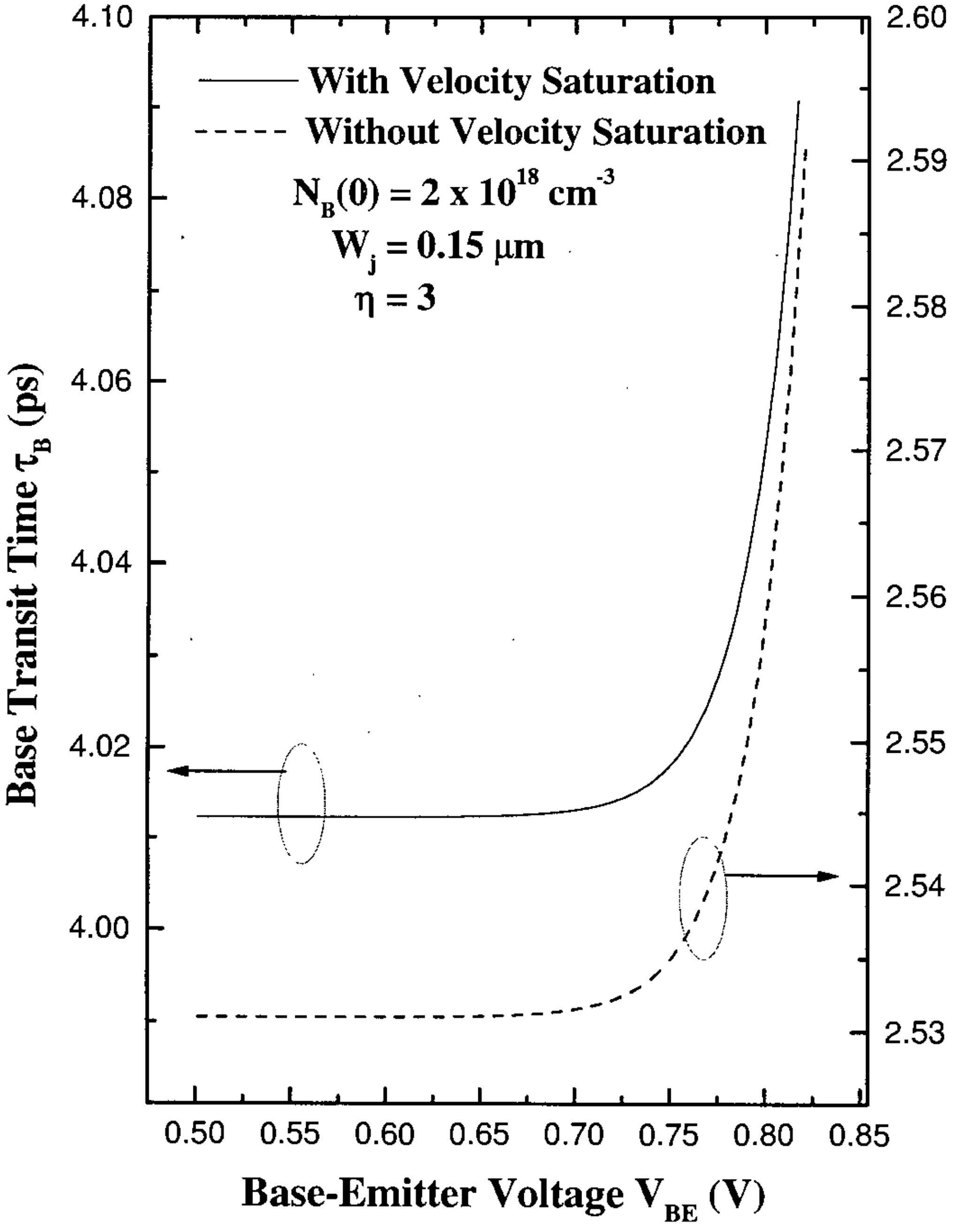

Fig. 3.12: Comparison of base transit time with and without considering velocity saturation for $\eta = 3$ and $N_B(0) = 2 \times 10^{18}\, cm^{-3}$.

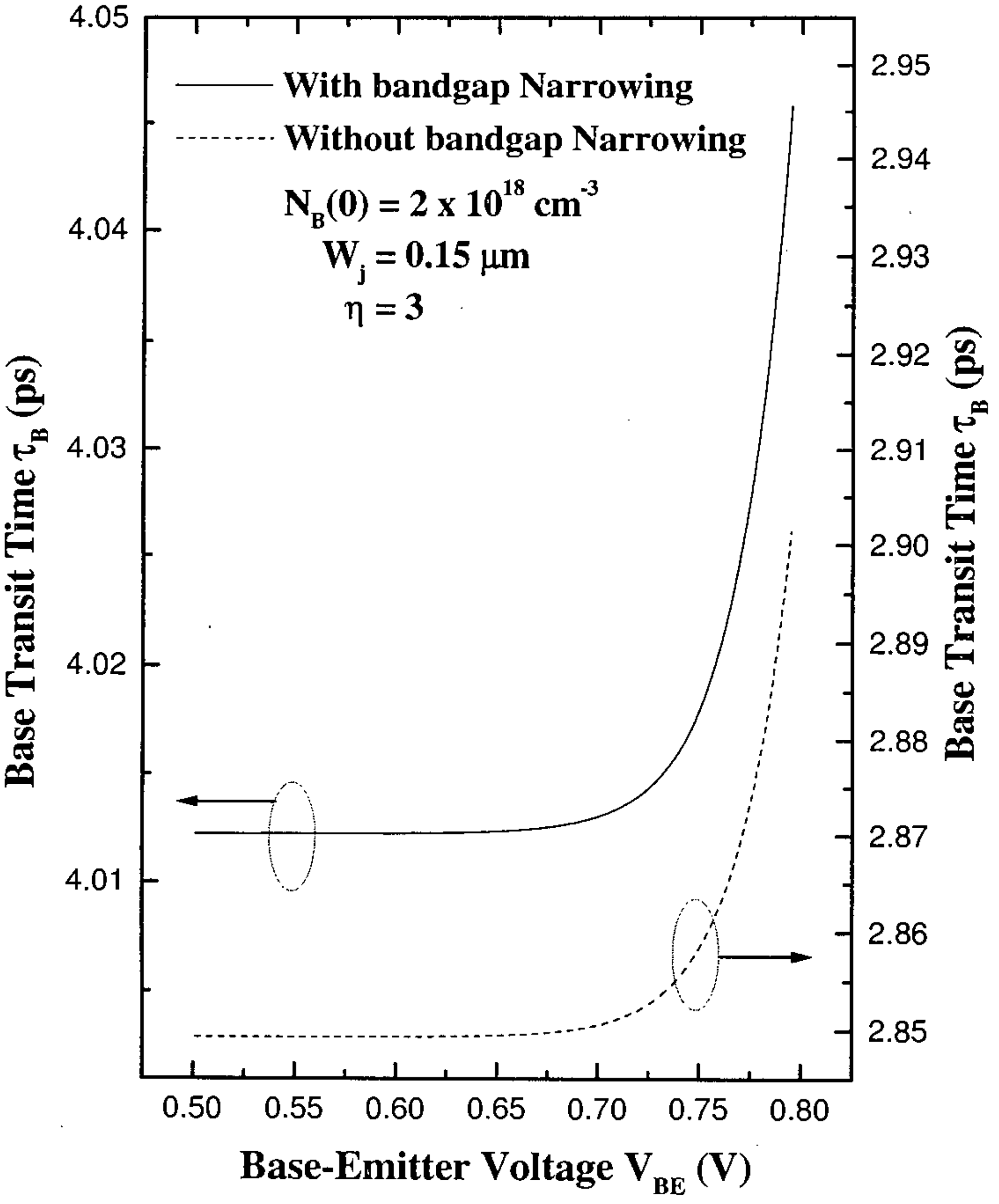

Fig. 3.13: Comparison of base transit time with and without considering bandgap narrowing for $\eta = 3$ and $N_B(0) = 2 \times 10^{18}\,cm^{-3}$.

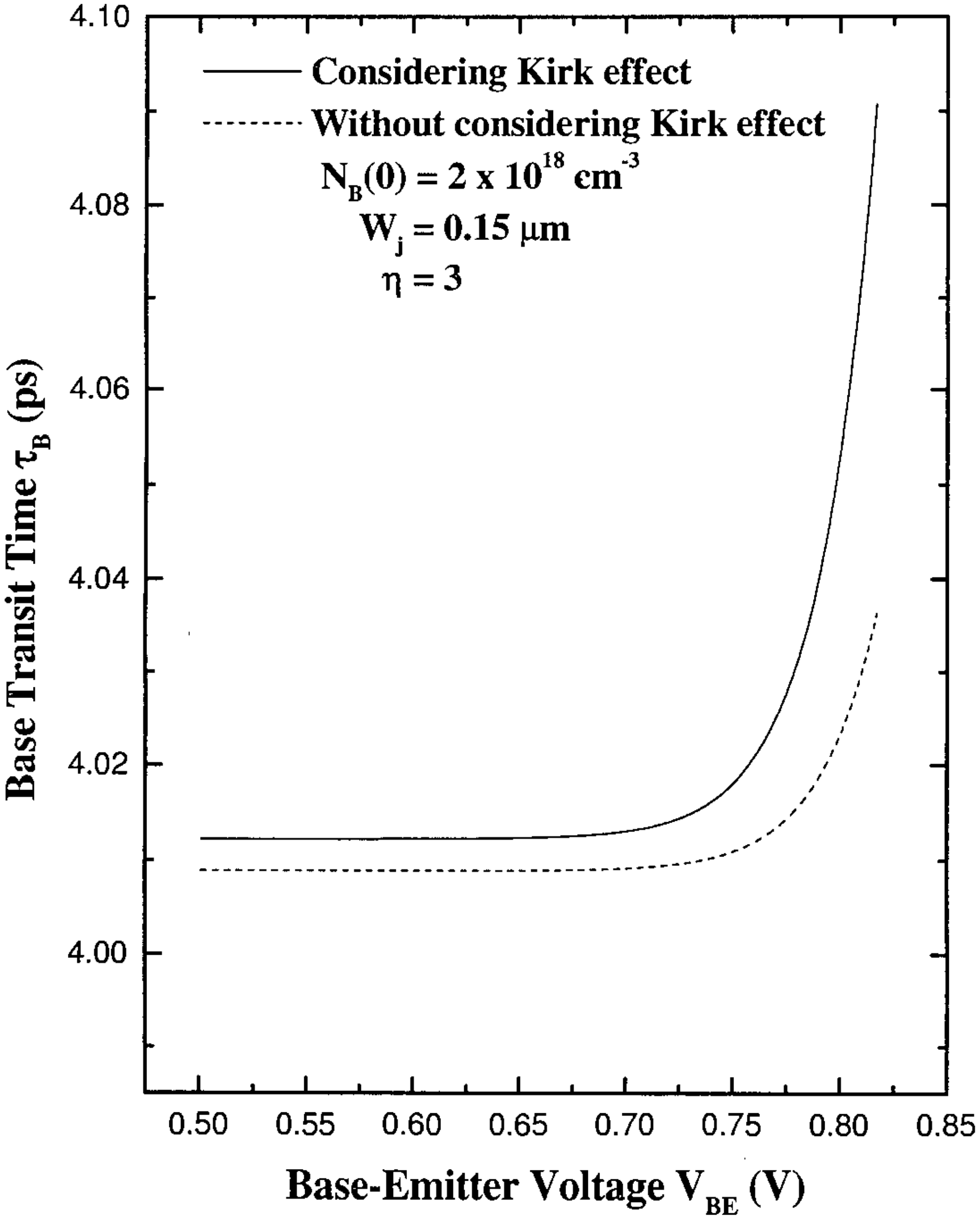

Fig. 3.14: Comparison of base transit time with and without considering Kirk effect for $\eta = 3$ and $N_B(0) = 2 \times 10^{18}\,cm^{-3}$(case 1 and case 2).

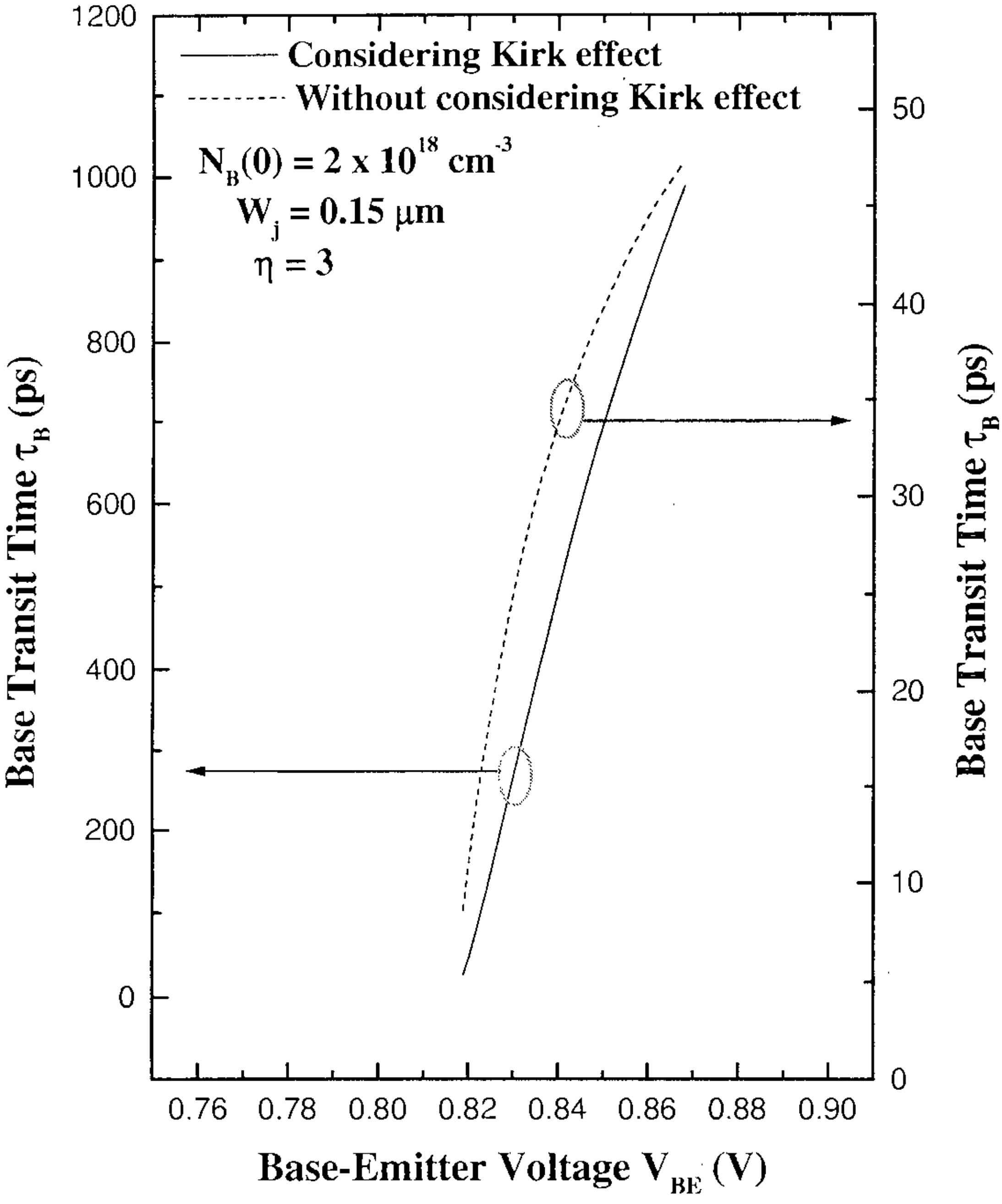

Fig. 3.15: Comparison of base transit time with and without considering Kirk effect for case 3.

3.2.8 Comparison between present and previous works

In the present work we have considered finite velocity saturation, bandgap narrowing and Kirk effect for calculating base transit time of a bipolar junction transistor. A comparison of base transit time with and without considering velocity saturation and bandgap narrowing are shown in Figs. 3.12 and 3.13. From these figures it is clear that the base transit time will be underestimated if we do not consider these effects. In Fig. 3.14 base transit time with and without considering Kirk effect for case 1 and case 2 are shown. From this figure we see that base transit time without considering Kirk effect is slightly lower than that with considering this effect. But for case 3 which is shown in Fig. 3.15 we see that base transit time considering Kirk effect is approximately twenty times that of without considering Kirk effect. So base transit time will be severely underestimated if Kirk effect is not considered.

3.3 Conclusion

The analytical expressions obtained in chapter 2 are used in this chapter to show the dependence of base transit time on different parameters. The results show that base transit time increases with base-emitter voltage and peak base doping concentration. We also see that the base transit time is a decreasing function of slope of base doping. The results obtained are compared with the results in which Kirk effect was neglected. The comparison shows that the neglegence of Kirk effect would severely underestimate the base transit time as is obvious from Fig. 3.15.

Chapter 4

Conclusions

4.1 Summary

In the present work analytical expressions for effective base width and base transit time of a bipolar junction transistor are studied. In this work the trend of modern BJTs having the collector more lightly doped than the base is taken into consideration. Exponentially doped base is also considered in this work because literature shows that exponential base doping is the optimum for obtaining minimum base transit time at any base width [9]. This is the best profile if there is no other base concentration limiting factor. The base of modern transistors is not uniformly doped. The influence of the velocity saturation in the base-collector depletion layer has been studied in this work. Since the collector is more lightly doped than the base, high values of current and collector-emitter voltage saturate the carrier velocity. With carrier-velocity saturation the carrier concentration in the space-charge region is proportional to the collector current, and the positions of the space-charge region boundaries become functions of the carrier concentration [14]. When the base doping becomes very high ($> 10^{17} cm^{-3}$), we have to consider the bandgap narrowing effect which causes reduction of current gain [20]. In our work, we have considered bandgap narrowing effect to obtain the expression of electric field in the base. In many cases for obtaining base transit time and finding other parameters of a bipolar junction transistor the base width is assumed to be constant. It is a reasonable approximation when the collector current is low. For lightly doped collector and heavily doped base if the collector current increases, the effective base width becomes function of collector current and it also increases. This phenomena is known as Kirk effect. As the base width increases, the carriers injected from the emitter have to traverse more distance in the base and the base transit increases. We have also considered this effect in our work.

The analytical expression of base transit time and the consequent figures show that the base transit time increases with base-emitter voltage and peak base doping, and decreases with the slope of the base doping. While comparing with previous works it is seen that base transit time considerably increases due to incorporation of velocity saturation, bandgap narrowing and Kirk effect. The results show that the neglegence of velocity saturation, bandgap narrowing and Kirk effect would severely underestimate the base transit time.

4.2 Suggestions for Future Work

In this work, we have developed an analytical expression for base transit time considering only low level of injection in the base. But a numerical method can be employed in determining the base transit time considering Kirk effect for high level of injection in the base. The boundary conditions has to be modified accordingly.

One dimensional analysis is carried out in this research. But under the flow of high current density, nonuniform distribution of current need to be considered. Further research is needed to include a two dimensional analysis.

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