RESONATING GROUP FORMALISM AND EQUIVALENT POTENTIAL FOR ALPHA-ALPHA INTERACTION

BY

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CERTIFICATE

This is to certify that the author is solely responsible for the work reported in this thesis and that this work has not been submitted to any University or elsewhere for the award of a degree/diploma.

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ABSTRACT

The resonating group formalism has been applied to the alpha-alpha cluster model of ^{8}Be. On the way of solving the Schrodinger wave equation using this method, an integro-differential equation appears which contains a non-local and non-separable kernel expression. The detailed calculation of the kernel involves a huge and complicated task. For higher nuclei the calculation of the kernel gets more and more complicated and laborious. That is why the systematic calculation of resonating group formalism hardly passes the nuclei of mass number 15-20.

In order to break this barrier we have, in our present study, introduced a new simple non-local and separable kernel having same properties and same number of parameters as the original kernel. We have replaced the original kernel by this new kernel keeping the other terms namely direct potential part and Coulomb part as obtained from resonating group method into account. The integro-differential equation with this new kernel has been solved numerically to calculate the phase shifts for different energies. The results we obtain agree well with the experimental phase shifts. This shows that resonating group formalism can be applied at least partially to nuclei of any higher mass number.

CONTENTS

CHAPTER 1

GENERAL INTRODUCTION

1.1 DIFFERENT NUCLEAR MODELS

Exact form of gravitational force and of electromagnetic force is known but in the case of nuclear force this is not the case. So one of the important studies of quantum mechanice is to determine the form of the nuclear force. Two basic properties of nuclear force are known, the range and the strength of the force. The range of the nuclear force is 1 Fermi i.e. 10^{-13} cm. and the force is very strong in comparison with the gravitational force or electromagnetic force. To study nuclear force one is to take account of the ways in which the nucleons i.e. neutrons and protons are situated inside the nucleus. Several nuclear models have been proposed, each is based on a set of simplified assumptions and is useful in a limited way. Each model serves to correlate a portion of our experimental knowledge about nuclei, usually within more or less narrow range of phenomena, but fails when applied to data outside this range. A short description of some of the models is given here.

Alpha-particles are ejected by some nuclei on disintegration in both natural and artificial ways. Based on this fact Wheeler and Fano have proposed the α particle model according to which the nucleus of multiple of four nucleons contains α particles, at least as substructures, although they cannot maintain their identity for a very long time inside condensed nuclear matter but will dissolve into more elementary particles. The α particle model can approximately explain several low lying levels of nuclei of multiple of four nucleons. For example, the low energy levels of ^{8}Be are explained as rotational levels produced by the two α particles which rotate round each other. This model can be applied to the nuclei which are composed of a whole number of α particles i.e. $^{8}Be, ^{12}C, ^{16}O$ etc.

The liquid drop model was first proposed by Bohr and Kalckar and then elaborated by Herzenberg, Majorana, Wheeler and others. This model compares the nucleus to a liquid drop, the nucleons corresponding to the molecules of the liquid, due to several points of similarity such as large interaction between constituent particles, nearly constant density, surface tension etc. This model has been utilized with a certain amount of success in the interpretation of intra nuclear forces and of nuclear transformations and in particular nuclear fission.

The nuclear shell model is one of the most important and useful models of nuclear structure. The experimental facts indicate that especially stable nuclei result when either of the number of protons Z or the number of neutrons N=A-Z is equal to one of the numbers 2,8,20,50,82,126. These numbers are commonly referred to as magic numbers. The magic numbers of neutrons and protons have been interpreted as forming closed shells of neutrons or protons in analogy with the filling of electron shells in atoms. This model haa been successfully used to explain certain nuclear phenomena such as stability, spin, magnetic moment.

It has long been known that there are certain properties of nuclei which suggest that nucleons tend to cluster in groups within a nucleus. Evidence that nucleons tend to form α particle clusters is common. Unstable heavy nuclei can decay by emitting α particles as if they existed as essentially independent entities in nuclear matter. ^{6}Li can be considered as composed of two cluster one is alpha particle containing four nucleons and the other is deuteron particle containing two nucleons. Similarly ^{8}Be consists of two alpha clusters.

1.2 RESONATING GROUP STRUCTURE

For a quantitative description of the compound nucleus behavior J.A. Wheeler [1,2] proposed the Resonating group structure or Resonating group method. In this formalism the main idea is that there exists in nuclei relatively long range correlation which manifest themselves through the formation of nucleon clusters. This structure regards the neutrons and protons in the nucleus as being divided into various groups (e.g. α particle) which do not maintain their identity for ever but undergo continual changes redistributing themselves into new groups. The intricate phenomena exhibited by nuclear systems are therefore considered to be a consequence of the dynamical interplay between various cluster structures. This method was extensively used to study problems of nuclear scattering and reactions.

Following Wheeler's method one obtains for the relative motion of the two groups an integral equation in which appears an interaction generated from two nucleon forces. This consists of two parts; a direct part which involves no particle exchange between the two groups and another part appearing in the form of a non-local, non-separable kernel interaction containing terms corresponding to the exchange of one, two or more nucleons between the groups. This method has some advantages over other methods for the following reasons:

(i) It is a microscopic formulation which explicitly takes cluster correlations into consideration.

(ii) It employs totally antisymmetric wave functions and therefore the Pauli's exclusion principle is fully accounted for.

(iii) It utilizes a nucleon-nucleon potential which explains the two-nucleon low-energy scattering data.

(iv) It treats correctly the motion of the total center of mass.

(v) It considers nuclear bound state, scattering and reaction problems in a unified manner.

(vi) It can be used to study cases where the particles involved in the incoming and outgoing channels are both arbitrary composite nuclei.

(vii) It is based on a variational principle; consequently, the accuracy of the result can be tested and improved by systematically expanding the basic-function space employed in the calculation.

As is unavoidable with a microscopic description possessing these features, the main difficulty is that practical calculations become frequently rather involved. In spite of this there already exist many investigations [3,4] which convincingly demonstrate the flexibility and the power of this unified theory. In our works we shall consider the case of ^{8}Be.

1.3 α-α INTERACTION AND THE PHASE SHIFT

The entire subject of α-α interaction occupies an important role in the nuclear structural problem. The purposes of the α-α scattering experiments have been first to get information about the nature of the α-α interaction and second to investigate the energy levels of ^{8}Be. The first α-α scattering experiment was performed by Rutherford and Chadwick. They observed that at large distances of collisions, force between the particles was given by Coulomb's law. But there was an indication of a departure from Coulomb's law at closer distances. They tentatively explained it by invoking some strong additional forces which increased much more rapidly with distance than an ordinary inverse square Coulomb field. An explanation was also sought in terms of the effects of nuclear forces, a more definite nomenclature for the so called additional forces. By this time the idea of nucleus being composed of neutrons and protons was gaining ground and the concept of nuclear force (responsible for keeping the neutrons and protons together in the nucleus) was introduced. It was reasonable therefore to assume that this nuclear force would provide a nuclear potential between the α particles and thus α-α scattering should be governed not only by the Coulomb forces but also by the nuclear forces. One would then expect that the effect of nuclear forces would be to cause a change in the phase of the initial wave describing the

incident α particles. This phase change, usually called the phase shift denoted by δ was seen to contain all the essential information about the nuclear potential.

1.4 OBJECTIVES OF THE PRESENT WORK

To find the phase shift of the α-α interaction we have to solve the Schrodinger equation for 8 nucleons. Resonating group method employs totally antisymmetric wave function and two body fundamental nucleon-nucleon interaction. Because of this a huge calculation is involved in the process of solving Schrodinger wave equation. A non-local and non-separable kernel expression appears from the antisymmetrization operator. Further the terms are lengthy and complicated. Moreover for higher nuclei the calculation of this kernel is troublesome. Our aim is to avoid this situation. We shall replace the non-local and non-separable kernel expreasion by a simple non-local but separable kernel. But we shall fully take into account of other effects of resonating group formalism e.g. direct part nuclear potential and Coulomb potential. Integro-differential equation with this new kernel will be solved numerically by computer. From the numerical solution of the equation we shall calculate the phaae shifts for different bombarding energies and a comparative study of theoretical and experimental result will be made.

CHAPTER 2

REVIEW OF THE EARLIER WORKS

The α-α interaction plays an important role in the solution of the nuclear structural problem. We give here a brief account of the historical background.

The α particle was discovered with the discovery of radioactivity by Becquerel and Curies in 1896-1898. Subsequently some of its properties were studied by Rutherford and his collaborators who found that α rays are positively charged. Each α particle carries two units of electronic charge and is nothing but ^4He nuclei. Some other important properties of α particles are

(1) It has a radius of 1.44 fm.

(2) It is a tightly bound system with a binding energy 28 Mev.

(3) It has an intrinsic spin and parity of 0^+ and thus obeys Bose-Einstein statistics. The wave function describing two α particles must remain symmetric with respect to exchange of particles.

The first α-α scattering was performed when Rutherford

and Chadwick (1927) [5] investigated the scattering of α particles from ^{4}He nuclei. Up to 1939 the only sources available for α scattering were natural α emitters e.g. radium, thorium and polonium. The succeeding development of the cyclotron, Van der graff and other high energy accelerators has provided α particles with higher, more controlled bombarding energies. One can broadly divide the whole range of α-α scattering experiments into two periods.

(1) 1927-1939, when the natural α emitters were used as sources.

(2) 1940 onwards, when accelerators were used as source beam.

The α-α scattering experiments performed during the period 1927-1939 were all based on the "annular ring method". A small chamber with an annular ring scattering volume was used. The energies of the α particles emitted from the radioactive nuclei at the scattering volume were determined by range measurements. The scattering of α particles from ^{4}He nuclei was investigated as a function of energy by slowing down natural α particles with absorbers. They observed that at large distances of collision the force between the particles was given by Coulomb's law, but there was an indication of a departure from Coulomb's law at closer distances. Rutherford and Chadwick (1927) [5] explained it by

invoking some strong additional forces which increased much more rapidly with distance than an ordinary inverse square Coulomb's field. Blackett and Champion (1931) [6] performed an α scattering experiment in a Wilson cloud chamber. Other experiments were carried out later on by Wright (1932) [7], Mohr and Pringle (1937) [8] and Devon (1939) [9] to study this deviation. By this time the concept of nuclear force that keeps the neutrons and protons together was introduced. It was reasonable, therefore, to assume that α-α scattering should be governed not only by the Coulomb's forces but also by the nuclear forces and the effect of nuclear forces would be to cause a change in the phase of the initial wave describing the incident α particles. Phase shift measurements thus became the main concern of the latter experiments.

The first post-war experiment was done by Cowie, Heydenburg, Temmer and Little (1952) [10] at the Department of Terrestrial magnetism, Carnegie Institution of Washington using Van der graff accelerator. They measured the differential scattering cross section for α scattering in helium in the energy range of 400 to 500 Kev(lab) over the angular range of 10^0 to 45^0(lab). Russel, Phillips and Reich (1956) [11] performed α-α scattering experiment in the Rice institute for laboratory bombarding energies of 3 to 6 Mev. After this, many experiments [12] were carried out with more and more bombarding energies using high energy accelerators.

Darriulat, Igo, Pugh and Holmgren (1965) [13] used the cyclotron as accelerator and covered the energy range 53-120 Mev.

Side by side the theoretical investigation also continued. The earliest of such theoretical studies began with the celebrated "Resonating Group Formalism" of Wheeler (1937) [1,2]. By theoretical studies we mean those studies which begin with an eight nucleon (four protons and four neutrons) system and try to develop an interaction between two α clusters; starting from first principles and basic two nucleon forces.

Margenau (1941) [14], Herzenberg (1957) [15] attempted to study the two body α-α interaction using the resonating group approach. Van der spuy (1959) [16] simplied the wave equation in great detail and calculated the non-local kernel function. The scattering phase shifts were calculated by Butcher and McNamee (1959) [17] within the framework of resonating group theory. As pointed out by Schmid and Wildermuth (1961) [18], their force mixture ($\frac{2}{3}$ Serber plus $\frac{1}{3}$ Rosenfeld) seemed to contradict the results of other analyses. Nevertheless, theirs were the first calculations which took complete account of the exchange part of the Coulomb interaction. Okai and Park (1966) [19] calculated the phase shifts for different partial waves

with proper choice of the parameters and compared the result with the experimental phase shift. The agreement of the calculated phase shift with the experimental phase shift was fairly good. Reichstein and Tang (1969) [20] studied the α-α scattering problem with the resonating group method in one channel approximation, using an improved nucleon-nucleon potential. The results were significantly better than those obtained in previous α-α calculations using this method.

Resonating group calculation had been quite successful in describing the α-α system below the reaction threshold of 17.36 Mev (c.m) [20,21]. In order to extend these calculations to energies above this threshold Brown and Tang (1971) [22] introduced a local, phenomenological, imaginary potential into the usual resonating group formulation; the purpose being to account approximately for the influence of reactions on the elastic scattering. They found that a simple, surface peaked, imaginary potential with a strength which varies smoothly with energy was quite adequate to reproduce the imaginary parta of the experimentally determined phase shifts from 25 to 60 Mev (c.m). The introduction of such a potential was found to cause only a small change in the calculated real parts of the α-α phase shifts. The agreement of these calculated real phases with those determined from experiment was better below the reaction threshold than above it.

In the resonating group model calculation of the α-α interaction [21] α particle wave function parameter which enters into the calculation of kernel was fixed from the experimental r.m.s. radius of the α particle. Afzal and Ali (1976) [23] used a consistent variational procedure for the α particle energy to fix the α particle wave function parameter. In fact the r.m.s radius of the α particle determined by the variational procedure was found to be only 10-15% smaller than the experimentally observed value. The calculated phase shifts showed a good agreement with the experimental phase shifts.

The α-α scattering with specific distortion effect was studied by Thompson, LeMere and Tang (1977) [24]. The calculation was performed with sufficient number of distortion function to ensure proper convergence of the phase shift values. It was shown that there was a good overall agreement between the calculated result with distortion and the empirical result. In all the angular momentum states, the effect of including apecific distortion was not too large in this case which is related to the fact that the α clusters have a rather low compressibility.

To find the position and width of the resonances Kruppa, Lovas and Gyarmati (1988) [25] introduced a complex scaling on

the inter cluster relative co-ordinate into the resonating group model. In the generator coordinate technique used to solve the resonating group equation, the complex scaling required minor changes in the formulae and code. The finding of resonances did not need any preliminary guess or explicit reference to any asymptotic presumption. The procedure was applied to the resonances in the relative motion of the two ground state α clusters in ^{8}Be.

CHAPTER 3

ALPHA-ALPHA RESONATING GROUP MODEL OF THE ^{8}Be NUCLEUS

3.1 THE WAVE FUNCTION

In resonating group formalism the neutrons and protons in the nucleus are regarded as being divided into various groups (e.g. α particles) which do not maintain their identity for ever but undergo continual changes redistributing themselves into new groups. Here we shall consider the resonating group model of the ^{8}Be nucleus. The nucleus of ^{8}Be consists of eight nucleons of which four are protons and four neutrons. Although this is unstable yet this provides us with a typical and an interesting example of two α clusters.

Let us suppose that the nucleons 1, 2, 3, 4 form the structure of one α particle a and 5, 6, 7, 8 that of another α particle b where 1, 2, 5, 6 are neutrons and 3, 4, 7, 8 are protons. Starting from this pair of α particles, other groups of two α's may then be generated by allowing the exchange of nucleons between the pair. These nucleon exchanges must however be governed by Pauli exclusion principle. The wave

function of the composite nucleus is written as totally antisymmetrized combination of the wave function for the various possible groups in the nucleons. Thus following resonating group formalism we assume that the wave function of the eight nucleons system is given by

$$\Psi = A[\psi_a(12,34)\ \psi_b(56,78)R(\vec{r}_{a,b})] \qquad (3.1.1)$$

where the antisymmetrized operator A is written as

$$A = [1-H(25)-H(26)-H(15)-H(16)+H(15)H(26)]\times [1-H(47)-H(48)-H(37)-H(38)+H(37)H(48)]$$

Here H(ij) is the Heisenberg exchange operator exchanging both space and spin co-ordinates of particles i and j i.e.

$$H(ij) = P_x(ij)P_\sigma(ij)$$

ψ_a and ψ_b are the internal wave functions of the α particles a and b respectively. $R(\vec{r}_{a,b})$ is the wave function for the relative motion of the two α particles with

$$\vec{r}_{a,b}=\vec{r}=\frac{1}{4}(\vec{r}_1+\vec{r}_2+\vec{r}_3+\vec{r}_4)-\frac{1}{4}(\vec{r}_5+\vec{r}_6+\vec{r}_7+\vec{r}_8),$$

denoting the vector between the centre of mass of the α particles. $\vec{r}_1,\vec{r}_2,\vec{r}_3,\vec{r}_4,\vec{r}_5,\vec{r}_6,\vec{r}_7,\vec{r}_8$ are the position vectors of the eight nucleons.

Since 1,2 and 5,6 are neutrons the exchange operators

H(15), H(25), H(16), H(26) effectively represent the same operator. Hence one operator can be replaced by other. We write H(15) for these operators. Similarly we can write H(37) for the operators H(37), H(47), H(38), H(48). Hence the operator A becomes

$$\begin{aligned} A &= [1-4H(15)+H(15)H(26)][1-4H(37)+H(37)H(48)] \\ &= 1-4H(37)+H(37)H(48)-4H(15)+16H(15)H(37) \\ &\quad -4H(15)H(37)H(48)+H(15)H(26)-4H(15)H(26)H(37) \\ &\quad +H(15)H(26)H(37)H(48) \\ &= 1+a \end{aligned}$$

where

$$\begin{aligned} a &= -4H(37)+H(37)H(48)-4H(15)+16H(15)H(37) \\ &\quad -4H(15)H(37)H(48)+H(15)H(26) \\ &\quad -4H(15)H(26)H(37)+H(15)H(26)H(37)H(48) \end{aligned} \qquad (3.1.2)$$

Therefore the wave function can be written as

$$\begin{aligned} \psi &= (1+a)[\phi_a\phi_b R(\vec{r})] \\ &= \psi_a\psi_b R(\vec{r})+a[\phi_a\phi_b R(\vec{r})] \end{aligned} \qquad (3.1.3)$$

The α particle wave function ψ_a has the form

$$\psi_a(12,34) = \chi_a(12,34)\ \phi_a(12,34) \qquad (3.1.4)$$

where $\chi_a(12,34)=\frac{1}{2}\{\alpha(1)\beta(2))-\beta(1)\alpha(2)\}\{\alpha(3)\beta(4)-\beta(3)\alpha(4)\}$ is the antisymmetric epin wave function with α,β corresponding

to + and - spin respectively and $\phi_a(12,34)$ is the symmetric spatial part of α wave function. For simplicity $\phi_a(12,34)$ [12] is assumed to be of the form

$$\phi_a(12,34) = N \exp\{-\alpha(r_{12}^2+r_{13}^2+r_{14}^2+r_{23}^2+r_{24}^2+r_{34}^2)\}$$

with $N^2 = 2^{\frac{21}{2}}(\frac{\alpha}{\pi})^{\frac{9}{2}}$ by normalization, $r_{12}=|\bar{r}_1-\bar{r}_2|$ and α being a parameter. Similarly

$$\psi_b(56,78) = \chi_b(56,78)\phi_b(56,78) \qquad (3.1.5)$$

3.2 THE WAVE EQUATION

The eight body schrodinger wave equation is

$$H\psi = E\psi \qquad (3.2.1)$$

where ψ is the total wave function for the α-α system. The Hamiltonian H of the system is given by

$$H = T+V$$

where the kinetic energy operator T and the potential energy V are

$T=-\frac{\hbar^2}{2M}\sum_{i=1}^{8}\nabla_i^2$, M being the nucleon mass

$$V = \sum_{i<j=1}^{8} V(ij)$$

V(ij) [12] is assumed to be of the form

$$V(ij) = (w+mP_x(ij)+bP_\sigma(ij)+hH(ij))v(ij) \qquad (3.2.2)$$

w, m, b, h are the coefficients of Wigner, Majornana, Bartlett and Heisenberg exchange operators with normalization w+m+b+h=1; the operators $P_x(ij)$ and $P_\sigma(ij)$ being the space and spin exchange operators for the particles i,j and v(ij) the common radial dependence of the central exchange type nucleon-nucleon force law. For simplicity the Gaussian form of v(ij) is assumed

$$v(ij) = V_0 \exp(-\beta r_{ij}^2)$$

Now let us define the following set of internal co-ordinates

$$\vec{\rho}_1 = \vec{r}_1 - \vec{r}_2$$

$$\vec{\rho}_2 = \vec{r}_3 - \frac{\vec{r}_1+\vec{r}_2}{2}$$

$$\vec{\rho}_3 = \vec{r}_4 - \frac{\vec{r}_1+\vec{r}_2+\vec{r}_3}{3}$$

$$\vec{R}_a = \frac{\vec{r}_1+\vec{r}_2+\vec{r}_3+\vec{r}_4}{4}$$

$$\vec{\rho}_4 = \vec{r}_5 - \vec{r}_6$$

$$\vec{\rho}_5 = \vec{r}_7 - \frac{\vec{r}_5+\vec{r}_6}{2}$$

$$\vec{\rho}_6 = \vec{r}_8 - \frac{\vec{r}_5+\vec{r}_6+\vec{r}_7}{3}$$

$$\vec{R}_b = \frac{\vec{r}_5+\vec{r}_6+\vec{r}_7+\vec{r}_8}{4}$$

$$\vec{r}_{a,b} = \vec{r} = \frac{\vec{r}_1+\vec{r}_2+\vec{r}_3+\vec{r}_4}{4} - \frac{\vec{r}_5+\vec{r}_6+\vec{r}_7+\vec{r}_8}{4}$$

$$\vec{R}_c = \frac{\vec{r}_1+\vec{r}_2+\vec{r}_3+\vec{r}_4+\vec{r}_5+\vec{r}_6+\vec{r}_7+\vec{r}_8}{8}$$

For this new co-ordinate system the kinetic energy operator T becomes [16]

$$T = -\frac{\hbar^2}{2M}\sum_{i=1}^{8}\nabla_i^2 = -\frac{\hbar^2}{2M}[2\nabla_{\rho_1}^2+\frac{3}{2}\nabla_{\rho_2}^2+\frac{4}{3}\nabla_{\rho_3}^2+2\nabla_{\rho_4}^2+\frac{3}{2}\nabla_{\rho_5}^2$$
$$+\frac{4}{3}\nabla_{\rho_6}^2+\frac{1}{2}\nabla_r^2+\frac{1}{8}\nabla_{R_c}^2]$$

$$= -\frac{\hbar^2}{2M}[2\nabla_{\rho_1}^2+\frac{3}{2}\nabla_{\rho_2}^2+\frac{4}{3}\nabla_{\rho_3}^2]-\frac{\hbar^2}{2M}[2\nabla_{\rho_4}^2+\frac{3}{2}\nabla_{\rho_5}^2+\frac{4}{3}\nabla_{\rho_6}^2]$$
$$-\frac{\hbar^2}{4M}\nabla_r^2-\frac{\hbar^2}{16M}\nabla_{R_c}^2 \qquad (3.2.3)$$

The last term in the expression is the kinetic energy of the c.m. motion. This term is omitted because we are not interested with the c.m. motion. We write T_3 (the kinetic

energy of the α particle a) for the first three terms and T_b for the second three terms. Then the expression for T becomes

$$T = T_a + T_b - \frac{\hbar^2}{4M}\nabla_r^2$$

The wave equation (3.2.1) can be written as

$$(T + \sum_{i<j=1}^{8} V(ij) - E)\psi = 0 \qquad (3.2.4)$$

To obtain the equation for the relative motion of the two α particles i.e.wave equation for $R(\vec{r})$ the nucleon co-ordinates must be integrated out. In this connection use is made of the variational equations. The variational equations for the two α particles a and b are respectively

$$\int \chi_a(12,34)\phi_a^*(12,34)\{T_a - E_\alpha + \sum_{i<j=1}^{4} V(ij)\}$$
$$\chi_a(12,34)\phi_a(12,34)\,d\tau_a d\sigma_a = 0 \qquad (3.2.5)$$

$$\int \chi_b(56,78)\phi_b^*(56,78)\{T_b - E_\alpha + \sum_{i<j=5}^{8} V(ij)\}$$
$$\chi_b(56,78)\phi_b(56,78)\,d\tau_b d\sigma_b = 0 \qquad (3.2.6)$$

where $\int$ denotes the integration over the space and the spin co-ordinates, E_α is the binding energy of α particle.

Multiplying the equation (3.2.4) by $\psi_a^* \psi_b^*$ and integrating over the space and the spin variables of α particles a and b we obtain

$$\int\int \psi_a^* \psi_b^* \{T + \sum_{i<j=1}^{8} V(ij) - E\} \psi \, d\tau_a d\sigma_a d\tau_b d\sigma_b = 0$$

Using (3.1.3) in this equation we get

$$\int\int \psi_a^* \psi_b^* \{T + \sum_{i<j=1}^{8} V(ij) - E\} (\psi_a \psi_b R(\vec{r}) + a\psi_a \psi_b R(\vec{r})) \, d\tau_a d\sigma_a d\tau_b d\sigma_b = 0$$

$$\text{or, } \int\int \psi_a^* \psi_b^* \{T + \sum_{i<j=1}^{8} V(ij) - E\} (\psi_a \psi_b R(\vec{r}) \, d\tau_a d\sigma_a d\tau_b d\sigma_b$$

$$+ \int\int \psi_a^* \psi_b^* \{T + \sum_{i<j=1}^{8} V(ij) - E\} a\psi_a \psi_b R(\vec{r}) \, d\tau_a d\sigma_a d\tau_b d\sigma_b = 0$$

$$\text{or, } \int\int \psi_a^* \psi_b^* \{T_a + T_b - \frac{\hbar^2}{4M} \nabla_r^2 - E_\alpha - E_\alpha - (E - 2E_\alpha) + \sum_{i<j=1}^{4} V(ij)$$

$$+ \sum_{i<j=5}^{8} V(ij) + \sum_{1234,5678} V(ij) \} \psi_a \psi_b R(\vec{r}) \, d\tau_a d\sigma_a d\tau_b d\sigma_b$$

$$+ \int\int \psi_a^* \psi_b^* \{T + \sum_{i<j=1}^{8} V(ij) - E\} a\psi_a \psi_b R(\vec{r}) \, d\tau_a d\sigma_a d\tau_b d\sigma_b = 0$$

$$or,\ \iint \psi_a^* \psi_b^* \{T_a - E_a + \sum_{i<j=1}^{4} V(ij)\} \psi_a \psi_b R(\vec{r})\, d\tau_a d\sigma_a d\tau_b d\sigma_b$$

$$+ \iint \Phi_a^* \Phi_b^* \{T_b - E_a + \sum_{i<j=5}^{8} V(ij)\} \psi_a \psi_b R(\vec{r})\, d\tau_a d\sigma_a d\tau_b d\sigma_b$$

$$+ \iint \psi_a^* \Phi_b^* \{-\frac{\hbar^2}{4M}\nabla_r^2 - (E - 2E_a)\} \psi_a \psi_b R(\vec{r})\, d\tau_a d\sigma_a d\tau_b d\sigma_b$$

$$+ \iint \psi_a^* \Phi_b^* \{\sum_{1234,5678} V(ij)\} \psi_a \psi_b R(\vec{r})\, d\tau_a d\sigma_a d\tau_b d\sigma_b$$

$$+ \iint \psi_a^* \psi_b^* [T - E + \sum_{i<j=1}^{8} V(ij)]\, a\psi_a \psi_b R(\vec{r})\, d\tau_a d\sigma_a d\tau_b d\sigma_b = 0$$

Using the variational equations (3.2.5) and (3.2.6) in this equation we obtain

$$\{\frac{\hbar^2}{4M}\nabla_r^2 + (E - 2E_a)\} R(\vec{r}) - \iint \psi_a^* \psi_b^* \{\sum_{1234,5678} V(ij)\}\, \psi_a \psi_b R(\vec{r})\, d\tau_a d\sigma_a d\tau_b d\sigma_b$$

$$= \iint \psi_a^* \Phi_b^* (T + \sum_{i<j=1}^{8} V(ij) - E)\, a\psi_a \psi_b R(\vec{r})\, d\tau_a d\sigma_a d\tau_b d\sigma_b$$

Using the expressions for a, ψ_a, ψ_b, V(ij) and T from the equations (3.1.2), (3.1.4), (3.1.5), (3.2.2) and (3.2.3) in this equation and carrying out many many multidimensional integrations the following equation [12] is obtained

$$\{\frac{\hbar^2}{4M}\nabla_r^2 + (E - 2E_a) - V_D(r)\} R(\vec{r})$$

$$= \int K(\vec{r}, \vec{r}')\, R(\vec{r}')\, d\vec{r}' \qquad (3.2.7)$$

where $\vec{r}'$ is the co-ordinate obtained from $\vec{r}$ by exchanging the particles, The direct interaction term $V_D(r)$ [12] is

$$V_D(r) = 4(4w+2b-2h-m)V_0\left(\frac{16\alpha}{16\alpha+3\beta}\right)^{\frac{3}{2}}\exp\left(-\frac{16\alpha\beta}{16\alpha+3\beta}r^2\right)$$

and the kernel [16] is

$$K(\vec{r},\vec{r}') = 2^{11}\left(\frac{\alpha}{3\pi}\right)^{\frac{3}{2}}\exp\left(-\frac{8}{3}\alpha(5r^2+5r'^2-8\vec{r}.\vec{r}')\right.$$

$$\times\left[\frac{\alpha\hbar^2}{M}\left(\frac{256}{9}\alpha(7r^2+7r'^2-13\vec{r}.\vec{r}')-62\right)+E-6V_0(w+m)\left(\frac{4\alpha}{4\alpha+\beta}\right)^{\frac{3}{2}}\right.$$

$$\left.+(4m+2h-2b-w)V_0\exp\left(-4\beta(r^2+r'^2-2\vec{r}.\vec{r}')\right)\right\}$$

$$-3(2b+3w-2m-2h)V_0\left(\frac{6\alpha}{6\alpha+\beta}\right)^{\frac{3}{2}}\exp\left(-\frac{8\alpha\beta}{18\alpha+3\beta}(r^2+r'^2+2\vec{r}.\vec{r}')\right)$$

$$-6(w+m)V_0\left(\frac{6\alpha}{6\alpha+\beta}\right)^{\frac{3}{2}}\exp\left(-\frac{8\alpha\beta}{18\alpha+3\beta}(r^2+r'^2-4\vec{r}.\vec{r}')\right)$$

$$\left.\times\left\{\exp\left(-\frac{8\alpha\beta}{6\alpha+\beta}r^2\right)+\exp\left(-\frac{8\alpha\beta}{6\alpha+\beta}r'^2\right)\right\}\right]$$

$$+3\times2^9\left(\frac{\alpha}{4\pi}\right)^{\frac{3}{2}}\exp\left(-8\alpha(r^2+r'^2)\right)\times\left\{-\left(\frac{\alpha\hbar^2}{M}[64\alpha(r^2+r'^2)-54]\right.\right.$$

$$\left.+E-4(w+m)V_0\left(\frac{4\alpha}{4\alpha+\beta}\right)^{\frac{3}{2}}\right)+4(2b+2w-3m-2h)V_0$$

$$\left(\frac{8\alpha}{8\alpha+\beta}\right)^{\frac{3}{2}}\exp\left(-\frac{8\alpha\beta}{8\alpha+\beta}(r^2+r'^2-2\vec{r}.\vec{r}')\right)+8(w+m)V_0\left(\frac{16\alpha}{16\alpha+3\beta}\right)^{\frac{3}{2}}$$

$$\left.\exp\left(-\frac{16\alpha\beta}{16\alpha+3\beta}r'^2\right)+\exp\left(-\frac{16\alpha\beta}{16\alpha+3\beta}r^2\right)\right]$$

The self adjoint kernel $K(\vec{r},\vec{r}')$ describes a very

complicated function and represents all the effects of antisymmetrization and the exchange nuclear interaction.

$V_0(r)$ is the direct interaction (i.e. in the absence of nucleon exchange) which originates from the identity element of the antisymmetrization.

The direct part $V_c(r)$ [12] of the Coulomb interaction $4\frac{e^2}{r}erf(4r\sqrt{\frac{1}{3}\alpha})$ is also added to $V_0(r)$ to give a total direct part $V_0'(r) = V_0(r) + V_c(r)$.

Equation (3.2.7) shows that in order to know $R(\vec{r})$ at the point $\vec{r}$ in space, one needs to know $R(\vec{r}')$ at all other points $\vec{r}'$ in space. Thus equation (3.2.7) describes a non-local process. The kernel of the Integro-differential equation $K(\vec{r},\vec{r}')$ involving both $\vec{r}$ and $\vec{r}'$ represents a non-local interaction and is symmetric. Mathematically speaking, the origin of the kernel interaction lies in the antisymmetrization of the wave function. But since antisymmetrization means physically that particles having same spin and charge should not come too close to one another, it is expected that $K(\vec{r},\vec{r}')$ should incorporate the character of repulsion.

We can separate out the partial waves by making the

following expansion in terms of Legendre polynomials

$$R(\vec{r}) = r^{-1} \sum_{l=even} f_l(r) P_l(\cos\theta) \tag{3.2.8}$$

where the odd partial waves are missing on account of the identity of the two colliding clusters.

$$and \quad K_l(\vec{r},\vec{r}') = (4\pi r r')^{-1} \sum_{l=0}^{\infty} (2l+1) k_l(r,r') P_l(\cos\theta) \tag{3.2.9}$$

$$Thus \quad k_l(r,r') = 2\pi r r' \int_{-1}^{1} K_l(\vec{r},\vec{r}') P_l(\cos\theta)\, d(\cos\theta) \tag{3.2.9a}$$

where θ is the angle between $\vec{r}$ and $\vec{r}'$

With (3.2.8) and (3.2.9) the equation (3.2.7) reduces to [12]

$$\left[\frac{\hbar^2}{4M}\left(\frac{d^2}{dr^2} - \frac{l(l+1)}{r^2}\right) + E' - V_D'(r)\right] f_l(r) = \int_0^{\infty} k_l(r,r') f_l(r')\, dr' \tag{3.2.10}$$

where $E' = E - 2E_\alpha$ is the relative energy of the two α particles.

The price for the antisymmetrization of the wave function in order not to violate the Pauli principle is to solve the

integro-differential equation (3.2.10) with a complicated kernel $k_1(r,r')$. The detailed calculation of the kernel has not been shown here. But it is a huge task. However these calculation has been done by different authors [16,26] and the phase shift was calculated. But our main point here is that although the resonating group formalism is very consistent and systematic and the result obtained are highly satisfactory, yet it has got certain limitations: for higher nuclei the calculation of this kernel gets more and more complicated and laborious. That is why the systematic calculation of resonating group formalism hardly passes the nucleus of mass number 20. In order to break this barrier we like to propose that non-local, non separable kernel should be replaced by a non-local separable simple kernel and at the same time the direct part of the potential (both nuclear and Coulomb) obtained from resonating group formalism is fully taken into account. In that case resonating group formalism at least partially can be applied to nuclei of any mass number. So here we like to test this method with nucleus of 8 particles namely ^{8}Be case.

CHAPTER 4

PHASE SHIFT OF ALPHA-ALPHA INTERACTION USING A NEW KERNEL

4.1 INTRODUCTION

In resonating group calculation the major task is to compute the kernel function. To compute the kernel function one needs to carry out a substantial number of multidimensional spatial integrations. Because of the complicated nature of the resonating group wave function caused by the requirement of total antisymmetrization these integrations are generally tedious to perform and almost impossible, especially when the number of nucleons involved in the system is large. Indeed this was the main reason which led a number of people to comment in the past that resonating group method is practicable only for very light nucleon system which involve two s shell nuclei in both the incident and outgoing channels.

The integration technique which is generally used is the cluster co-ordinate technique. In this technique one

introduces as spatial variables of integrations the internal and the relative co-ordinates of the clusters. This seems to a natural set of integration variables to use, because the resonating group trial wave function is explicitly expressed in terms of these co-ordinates. Then by choosing functions of Gaussian dependence for both the spatial part of the wave function and the spatial part of the nucleon-nucleon potential, multidimensional integrals can be evaluated analytically.

The evaluation of matrix elements by the cluster co-ordinate technique is however, usually a quite tedious procedure. To compute the multidimensional integrals which involve integrands of Gaussian functions multiplied by polynomials of spatial co-ordinates, one must first diagonalize the quadratic terms in the exponents by applying linear co-ordinate transformation. For systems containing relatively small number of nucleons this is not too difficult. However when one deals with systems which contain a rather large number of nucleons, this diagonalization procedure can become very tedious due to the antisymmetrization of the wave function, because for every permutation of nucleons in different clusters one has to introduce a different co-ordinate transformation.

Saito [28] proposed the orthogonality condition model

(OCM) as an approximation to the RGM equation of motion. It has been extensively applied to treat especially multi cluster structure problem where a straightforward application of the RGM approach is quite difficult. This semi microscopic model was specially proposed for the explicit purpose of avoiding the derivation of the complicated kernel function occurring in RGM by taking the Pauli principle only approximately into account.

In this work we propose a new procedure to avoid the difficulties mentioned above. We shall replace the original complicated kernel by an arbitrary simple kernel and calculate the phase shift with this new kernel.

4.2 THE NEW KERNEL

Now we shall develop a new kernel which will replace the original kernel. The original kernel $k_1(r,r')$ given by (3.2.9a) contains the parameters α, β, V_0, w, m, b, h with w+m+b+h = 1.

Tables below shows the values of $K_0(r,r')$fm and $K_2(r,r')$fm in Mev for the parameters

α = .066875 f^{-2} β = .46 f^{-2} V_0 = -72.98

w = m = .4075 b = h = .0925

with E = 2 and $\frac{\hbar^2}{M}$ = 41.47 X 10^{-26} cm^2.

Table 1

r	0	1	2	3	4	5	6
$k_0(r,1)f$	0	45.64	24.16	-1.90	-.43	-.01	.00
$k_0(r,2)f$	0	24.16	12.02	14.70	1.42	.01	.00
$k_0(r,3)f$	0	-1.90	14.70	-.25	2.04	0.15	.00
$k_0(r,4)f$	0	-.43	1.42	2.04	.00	.14	.01
$k_0(r,5)f$	0	-.01	.01	.15	.14	.02	.01
$k_0(r,6)f$	0	.00	.00	.00	.01	.01	.00

Table 2

r	0	1	2	3	4	5	6
$k_2(r,1)f$	0	-15.74	20.52	3.42	.07	.00	.00
$k_2(r,2)f$	0	20.52	1.55	10.56	1.18	.01	.00
$k_2(r,3)f$	0	3.42	10.56	-2.02	1.55	-.13	.00
$k_2(r,4)f$	0	.07	1.18	1.55	-.13	.12	.01
$k_2(r,5)f$	0	.00	.01	.13	.12	.01	.01
$k_2(r,6)f$	0	.00	.00	.00	.01	.01	.00

It is clear from the above tables that both the kernel $k_0(r,r')$ and $k_1(r,r')$ decreases sharply as a Gaussian function with increasing r and r'. They are effective over a finite domain.

The original kernel contains six parameters and it has the form

$$k_1(r,r') = k_1^1(r,r') + Ek_1^2(r,r')$$

So we introduce a new kernel having the same form and the same number of parameters

We write

$$k_1^1(r,r') = Ae^{-Pr}e^{-Qr'}$$

$$k_1^2(r,r') = Be^{-Sr}e^{-Tr'}$$

Thus the new kernel is

$$k_1(r,r') = Ae^{-Pr}e^{-Qr'} + E\,Be^{-Sr}e^{-Tr'}$$

where A, B, P, Q, S, T are parameters.

We shall replace the original kernel in the equation (3.2.10) by this new kernel and solve the equation numerically.

4.3 NUMERICAL CALCULATION

The equation (3.2.10) can be written as

$$\frac{d^2 f_l(r)}{dr^2} + \frac{1}{D}\left(E' - \frac{D\, l(l+1)}{r^2} - V_D'(r)\right) f_l(r) = \frac{1}{D}\int_0^\infty k_l(r, r')\, f_l(r')\, dr' \qquad (4.3.1)$$

where $D = \frac{\hbar^2}{4M}$

There are six parameters in the equation (4.3.1). They are to be adjusted by fitting to the two nucleon data and matter distribution of an α particle.

Five of these are the parameters of the two-nucleon interaction, well depth V_0, force range parameter β, and the parameters of the force mixture w, m, b, h which are reduced to three in number owing to the normalization w+m+b+h=1. The parameters of the two nucleon interaction are chosen to be

V_0=-72.98 Mev $\beta = .46 \times 10^{-26} cm^2$

w+m-b-h=0.63, (w+m+b+h=1)

Two parameters of the force mixture still remain free. We consider here pure Serber force in which w=m and b=h.

Therefore w=m=.4075 and b=h=.0925.

One of the six parameters is the range parameter α connected with the matter probability distribution of an α particle. We assume that α cluster has approximately the same radius as the experimental one of a free α particle. The choice α = .066875 f^{-2} corresponds to the cluster radius 1.45f. The calculated binding energy of the α cluster is 28.4 Mev using the values of the parameter chosen here. This is in good agreement with the experimental binding energy 28.2 Mev. of an α particle.

Table 1 and Table 2 show that the kernel function $k_l(r,r')$ decreases sufficiently rapidly as r and r' increases. This enables us to replace the infinite upper limit in the integral by some finite value r_c=5.6.

Robertson [26] has described the solution of the integro-differential equation of the general form

$$\frac{d^2 f_l(r)}{dr^2} + \frac{1}{D}\left(E' - \frac{Dl(l+1)}{r^2} - V_D'(r)\right) f_l(r) = \frac{1}{D}\int_0^{r_c} k_l(r,r') f_l(r')\, dr' \qquad (4.3.2)$$

by a simultaneous equation method.

The range of integration can now be divided into many intervals of equal distances h(=.2) with pivotal points $r_0(=0), r_1, r_2, r_3 \ldots r_c (=Nh)$. The integral involving the kernel function is then evaluated by use of Simpson formula for numerical integration.

We have

$$r_c = 5.6$$

or $$Nh = 5.6$$

$$\therefore \quad N = 28$$

Equation (4.3.2) can be written as

$$\frac{d^2 f_l(r)}{dr^2} - \frac{1}{D} U_l(r) f_l(r) = \frac{1}{D} \int_0^{r_c} k_l(r, r') f_l(r') dr'$$

$$\text{where } U_l(r) = V_D'(r) + \frac{D\, l(l+1)}{r^2} - E' \qquad (4.3.3)$$

$$\text{Now } \int_0^{r_c} k_l(r, r') f_l(r') dr' = \sum_{m=0}^{N} T(m)\, k(r_n, r_m')\, f(r_m')$$

$$= \sum_{m=0}^{28} T(m)\, k_{n,m} f_m$$

where the coefficients T(m) are the integrating factors appropriate to the method of integration.

Equation (4.3.3) now becomes

$$f''(r_n) - \frac{1}{D}U(r_n)\,f(r_n) = \frac{1}{D}\sum_{m=0}^{28} T(m)\,k_{n,m} f_m \quad at \quad r = r_n$$

$$or, \quad f''_n - \frac{1}{D}U_n f_n = \frac{1}{D}\sum_{m=0}^{28} T(m)\,k_{n,m} f_m \qquad with \quad f(r_n) = f_n \tag{4.3.4}$$

We know that

$$\delta^2 f_n = h^2\left(1 + \frac{\delta^2}{12} - \frac{\delta^4}{240} + \ \ldots\ldots\ldots\ \right) f''_n$$

where δ is a central difference operator.

Now $\delta^2 f_n = h^2\left(1 + \frac{\delta^2}{12}\right) f''_n$ *neglecting higher orders of* δ

$$= h^2\left(1 + \frac{\delta^2}{12}\right)\left[\frac{1}{D}\sum_{m=0}^{28} T(m)\,k_{n,m} f_m + \frac{1}{D}U_n f_n\right] \qquad (using\ 4.3.4)$$

$$or, \quad D\delta^2 f_n = h^2 U_n f_n + h^2 \sum_{m=0}^{28} T(m)\,k_{n,m} f_m + \frac{h^2}{12}\delta^2 U_n f_n + \frac{h^2}{12}\delta^2 \sum_{m=0}^{28} T(m)\,k_{n,m} f_m$$

$$or, \quad D(f_{n+1} - 2f_n + f_{n-1}) = h^2 U_n f_n + h^2 \sum_{m=0}^{28} T(m)\,k_{n,m} f_m + \frac{h^2}{12}\left[U_{n+1} f_{n+1}\right.$$

$$\left. - 2U_n f_n + U_{n-1} f_{n-1}\right] + \frac{h^2}{12}\sum_{m=0}^{28}\left[T(m)\,k_{n+1,m} f_m\right.$$

$$\left. - 2T(m)\,k_{n,m} f_m + T(m)\,k_{n-1,m} f_m\right]$$

$$or, \quad \left(D - \frac{h^2}{12}U_{n+1}\right) f_{n+1} - \left(2D + \frac{5h^2}{6}U_n\right) f_n + \left(D - \frac{h^2}{12}U_{n-1}\right) f_{n-1}$$

$$= \frac{h^2}{12}\sum_{m=0}^{28}\left(k_{n+1,m} + 10k_{n,m} + k_{n-1,m}\right) T(m)\,f_m$$

$$\left(\frac{12D}{h^2}-U_{n+1}\right) f_{n+1}-\left(2\frac{12D}{h^2}+10U_n\right) f_n+\left(\frac{12D}{h^2}-U_{n-1}\right) f_{n-1}$$

$$= \sum_{m=0}^{28} (k_{n+1,m}+10k_{n,m}+k_{n-1,m})\, T(m)\, f_m$$

The kernel has the form $k(r,r')=k'(r,r')+Ek^2(r,r')$. So the above equation with $\frac{12D}{h^2}=DH$ takes the form

$$(DH-U_{n+1})\, f_{n+1}-(2DH+10U_n)\, f_n+(DH-U_{n-1})\, f_{n-1}$$

$$= \sum_{m=0}^{28} (k^1_{n+1,m}+10k^1_{n,m}+k^1_{n-1,m})\, T(m)\, f_m$$

$$+ \quad (E'+2E_a) \sum_{m=0}^{28} (k^2_{n+1,m}+10k^2_{n,m}+k^2_{n-1,m})\, T(m)\, f_m$$

Putting n= 1,2,3 ..... 27 in this equation we get 27 simultaneous equations.

For n = 1

$$(DH-U(r_2))\, f(r_2)-(2DH+10U(r_1))\, f(r_1)+(DH-U(r_0))\, f(r_0)$$

$$- T(0)\,[\{k^1(r_2,r_0')+10k^1(r_1,r_0')+k^1(r_0,r_0')\}+(E'+2E_a)$$

$$\{k^2(r_2,r_0')+10k^2(r_1,r_0')+k^2(r_0,r_0')\}]\, f(r_0)$$

$$-T(1)\,[\{k^1(r_2,r_1')+10k^1(r_1,r_1')+k^1(r_0,r_1')\}+(E'+2E_a)$$

$$\{k^2(r_2,r_1')+10k^2(r_1,r_1')+k^2(r_0,r_1')\}]\, f(r_1)$$

$$- T(2)\,[\{k^1(x_2,x_2')+10k^1(x_1,x_2')+k^1(x_0,x_2')\}+(E'+2E_a)\{k^2(x_2,x_2') +10k^2(x_1,x_2')+k^2(x_0,x_2')\}]\,f(x_2)$$

... ...

... ...

$$- T(28)\,[\{k^1(x_2,x_{28}')+10k^1(x_1,x_{28}')+k^1(x_0,x_{28}')\}+(E'+2E_a)\{k^2(x_2,x_{28}') + 10k^2(x_1,x_{28}')+k^2(x_0,x_{28}')\}]\,f(x_{28}) = 0$$

We assume that

$$f(x_0) = f(0) = 0 \qquad (4.3.5)$$

$$f(x_1) = f(.2) = .2 \qquad (4.3.6)$$

Also we have

$$k^1(x_0,x_n')=0 \quad \textit{and} \quad k^2(x_0,x_n')=0, \ \textit{for } n=0,1,2,..28 \qquad (4.3.7)$$

Using (4.3.7) and (4.3.5) in the above equation we get

$$[-T(1)\{(k^1(x_2,x_1')+10k^1(x_1,x_1')+(E'+2E_a)\,(k^2(x_2,x_1')+10k^2(x_1,x_1'))\} - \{2DH+10U(x_1)\}]\,f(x_1$$

$$+[-T(2)\{(k^1(x_2,x_2')+10k^1(x_1,x_2'))+(E'+2E_a)\,(k^2(x_2,x_2')+10k^2(x_1,x_2') - \{DH-U(x_2)\}]\,f(x_2)$$

$$-T(3)\{(k^1(r_2,r_3')+10k^1(r_1,r_3'))+(E'+2E_a)(k^2(r_2,r_3')+10k^2(r_1,r_3'))\}f(r_3)$$

... ...

... ...

$$-T(28)\{(k^1(r_2,r_{28}')+10k^1(r_1,r_{28}))+(E'+2E_a)(k^2(r_2,r_{28}')+10k^2(r_g,r_{28}'))\}f(r_{28})=0$$

or, $W(1,1)f(r_1)+W(1,2)f(r_2)+W(1,3)f(r_3)+\ldots+W(1,28)f(r_{28})=0$

Now using (4.3.6) we get

$$W(1,2)f(r_2)+W(1,3)f(r_3)+\ldots\ldots+W(1,28)f(r_{28})=-W(1,1)f(r_1)$$
$$=-W(1,1)x.2$$
$$=B(1)$$

or, $A(1,1)f(r_2)+A(1,2)f(r_3)+\ldots+A(1,27)f(r_{28})=B(1)$

Similarly for n=2, 3, 4, ... 27 we get

$$A(2,1)f(r_2)+A(2,2)f(r_3)+A(2,3)f(r_4)+\ldots\ldots+A(2,27)f(r_{28})=B(2)$$

$$A(3,1)f(r_2)+A(3,2)f(r_3)+A(3,3)f(r_4)+\ldots\ldots+A(3,27)f(r_{28})=B(3)$$

... ...

... ...

$$A(27,1)f(r_2)+A(27,2)f(r_3)+\ldots\ldots\ldots\ldots+A(27,27)f(r_{28})=B(27)$$

This set of 27 simultaneous linear Algebraic equations are

solved by computer to find the values of $f(r_2), f(r_3), \ldots . f(r_{28})$ i.e. f(.4), f(.6), ...f(5.6).

Since the kernel function is negligibly small beyond the point r_c(=5.6), the contribution to the integro-differential equation from this kernel can be neglected for values larger than r_c(=5.6).

The equation (3.2.10) then reduces to

$$[\frac{\hbar^2}{4M}(\frac{d^2}{dr^2}-\frac{l(l+1)}{r^2})+E'-V_D'(r)]f_l(r)=0$$

or, $f_n''+U_nf_n=0$ where $U_n=\frac{E'-V_D'(r)}{D}-\frac{l(l+1)}{r^2}$

The Fox-Goodwin [27] method will be applied to obtain $f_l(r)$ at large distance r=R.

We have

$$\delta^2 f_n = h^2(1+\frac{\delta^2}{12})f_n''$$

$$or,\ f_{n+1}-2f_n+f_{n-1} = h^2(1+\frac{\delta^2}{12})(-U_nf_n)$$

$$= -h^2U_nf_n-\frac{h^2}{12}\delta^2(U_nf_n)$$

$$= -h^2U_nf_n-\frac{h^2}{12}(U_{n+1}f_{n+1}-2U_nf_n+U_{n-1}f_{n-1})$$

$$or,\ (1+\frac{h^2}{12}U_{n+1})f_{n+1}-(2-\frac{5h^2}{6}U_n)f_n+(1+\frac{h^2}{12}U_{n-1})f_{n-1}=0$$

This recursion formula is then used repeatedly to obtain $f_l(r)$ through the point r_m=9.2 fm at which the direct nuclear potential $V_n(r)$ in $V_0(r)$ becomes negligibly small in comparison with the Coulomb potential in $V_0(r)$. At this point the logarithm derivative of $f_l(r)$ is computed by the use of central difference formula

$$L=\frac{1}{f_l(r)}f_l'(r)=\frac{1}{f_l(9.2)}f_l'(9.2)$$

$$=\frac{f_l(9.6)-9f_l(9.4)+45f_l(9.2)-45f_l(9.0)+9f_l(8.8)-f_l(8.6)}{60h.f_l(9.2)}$$

This result is matched with the logarithm derivative of the exterior solution which consists of a linear combination of the regular and the irregular Coulomb wave function.

The procedure for determination of nuclear phase shift is given below.

The radial part of the Schrodinger wave equation for two α particles may be written as

$$\frac{d^2f_l(r)}{dr^2}+[k^2-U(r)-\frac{l(l+1)}{r^2}]f_l(r)=0 \qquad (4.3.8)$$

where $k=\sqrt{\frac{2\mu E'}{\hbar^2}}$, μ being the reduced mass of the two α

particles, E' their relative energy and $U(r)=\frac{2\mu}{\hbar^2}V_{\alpha\alpha}(r)$. Beyond some large enough distance R, the nuclear part of the complete α-α potential $V_{\alpha\alpha}(r)$ becomes negligible and the Coulomb part dominates. The required solutions are those which vanish at the origin and behave asymptotically as an incoming Coulomb distorted plane wave plus outgoing Coulomb plus nuclear distorted spherical wave. The solution of (4.3.8) in the region $r \geq R$ are then [12]

$$[f_l(kr)]_{r\geq R}=\frac{\exp[i(\delta_l+\sigma_l)]}{k}\times[\cos\delta_l F_l(kr)+\sin\delta_l G_l(kr)]$$

where δ_l is the phase shift for lth partial wave and σ_l is the Coulomb phase shift of rank l. $F_l(kr)$ and $G_l(kr)$ are the regular and the irregular Coulomb wave functions respectively.

The nuclear phase shift δ_l can be obtained from the condition of continuity of $f_l(kr)$ and $f'_l(kr)$ at r=R.

Now

$$\frac{df_l(kr)}{dr}=\frac{\exp\{i(\delta_l+\sigma_l)\}}{k}\times[\cos\delta_l\frac{dF_l(kr)}{dr}+\sin\delta_l\frac{dG_l(kr)}{dr}]$$

$$or\quad f'_l(kr)=\frac{\exp\{i(\delta_l+\sigma_l)\}}{k}\times[\cos\delta_l\frac{dF_l(kr)}{d(kr)}k+\sin\delta_l\frac{dG_l(kr)}{d(kr)}k]$$

$$=\frac{\exp\{i(\delta_l+\sigma_l)\}}{k}\times[\cos\delta_l F'_l(kr).k+\sin\delta_l G'_l(kr).k]$$

The prime on the left side denotes differentiation w.r.t. r

and the primes on the right side denote differentiation w.r.t. $\rho(=kr)$.

Now

$$\frac{f_l(kr)}{f_l'(kr)} = \frac{\cos\delta_l F_l(kr)+\sin\delta_l G_l(kr)}{\cos\delta_l F_l'(kr).k+\sin\delta_l G_l'(kr).k}$$

$$or,\ 1/L = \frac{\cos\delta_l F_l+\sin\delta_l G_l}{\cos\delta_l F_l' k+\sin\delta_l G_l' k}$$

$$or,\ \tan\delta_l = \frac{kF_l'-F_l L}{G_l L-kG_l'} \quad at\ r=R$$

For given values of the parameter ρ (at r=R=9.2), the Coulomb wave functions and their derivatives F_l, G_l, F_l' and G_l' can be calculated numerically using the series expansion method of Froberg [29].

The whole system of equations is solved by computer using new kernel and the phase shifts are calculated for the partial waves s,d,g and i for the bombarding energies .5 Mev to 40 Mev in the c.m. system. There are six parameters A, B, P, Q, S and T in the new kernel to be adjusted by iterating procedure until the phase shifts are in good agreement with the experimental phase shifts.

The phase shifts obtained by Okai and Park [19] using original kernel is also shown in the graph for comparison.

CHAPTER 5

RESULTS, DISCUSSION AND CONCLUSION

The present study is concerned with the analysis of α-α scattering with the energy range of the bombarding energies upto 40 Mev. Within this bombarding energy range there are several inelastic and reaction channels. Since the phase shifts calculated here are real quantities, we compare them with the real parts of the experimental phase shifts. This is a somewhat arbitrary procedure, since the opening of inelalstic and reaction channels not only introduces the imaginary parts of the phase shifts but also affects the real parts of the phase shifts for the entrance channel. It is aasumed that the effect is negligible.

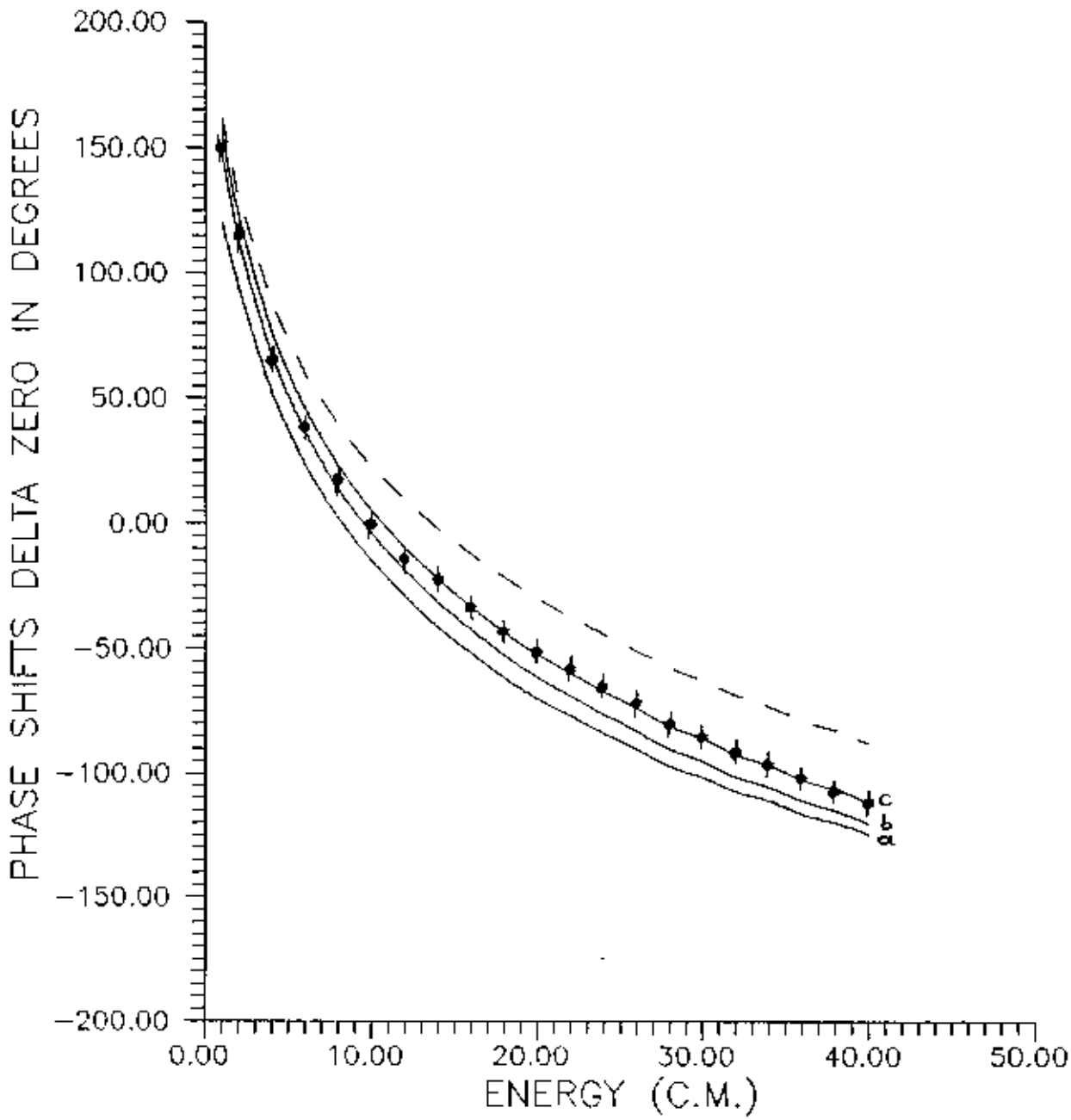
Fig. 1, 2, 3 and 4 show the energy dependence of the phase shifts for L = 0, 2, 4 and 6 respectively. The three solid curves a, b and c represent phase shifts obtained for the new kernel for different sets of parameter values. The dashed curves represent phase shifts calculated by Okai and Park [19] using original kernel. Marks ϕ show the experimental phase shifts.

Fig.1 shows the energy dependence of the phase shifts δ_0. Table 3 shows the values of the six parameters in the new kernel for the curves a, b and c in this figure.

Table 3

Curve	A	B	P	Q	S	T
a	1	1	1	1	1	1
b	5	5	1	1	1	1
c	20	20	1	1	1	1

It is clear from the figure that the curve b showe a better agreement with the experimental values in the lower energy region less than 10 Mev. and the curve c shows a better agreement in the higher energy region greater than 12 Mev.



Fig—1 Calculated phase shifts of alpha—alpha scattering are shown by three solid curves a, b and c for three different sets of parameters as shown in the table 3, phase shifts calculated by Okai and Park [19] from original kernel are shown by dashed curve. Marks ϕ show the experimental values.

Fig. 2 shows the energy dependence of the phase shift δ_2. Table 4 shows the values of the parameters in the new kernel for the curves a, b and c in this figure.

Table 4

Curve	A	B	P	Q	S	T
a	1	1	1	1	1	1
b	3.75	3.75	1	1	1	1
c	6	6	1	1	1	1

In this case the curve a is lower than the experimental values. The curve b shows the best agreement with the experimental phase shifts and the curve c is higher than the experiments values.

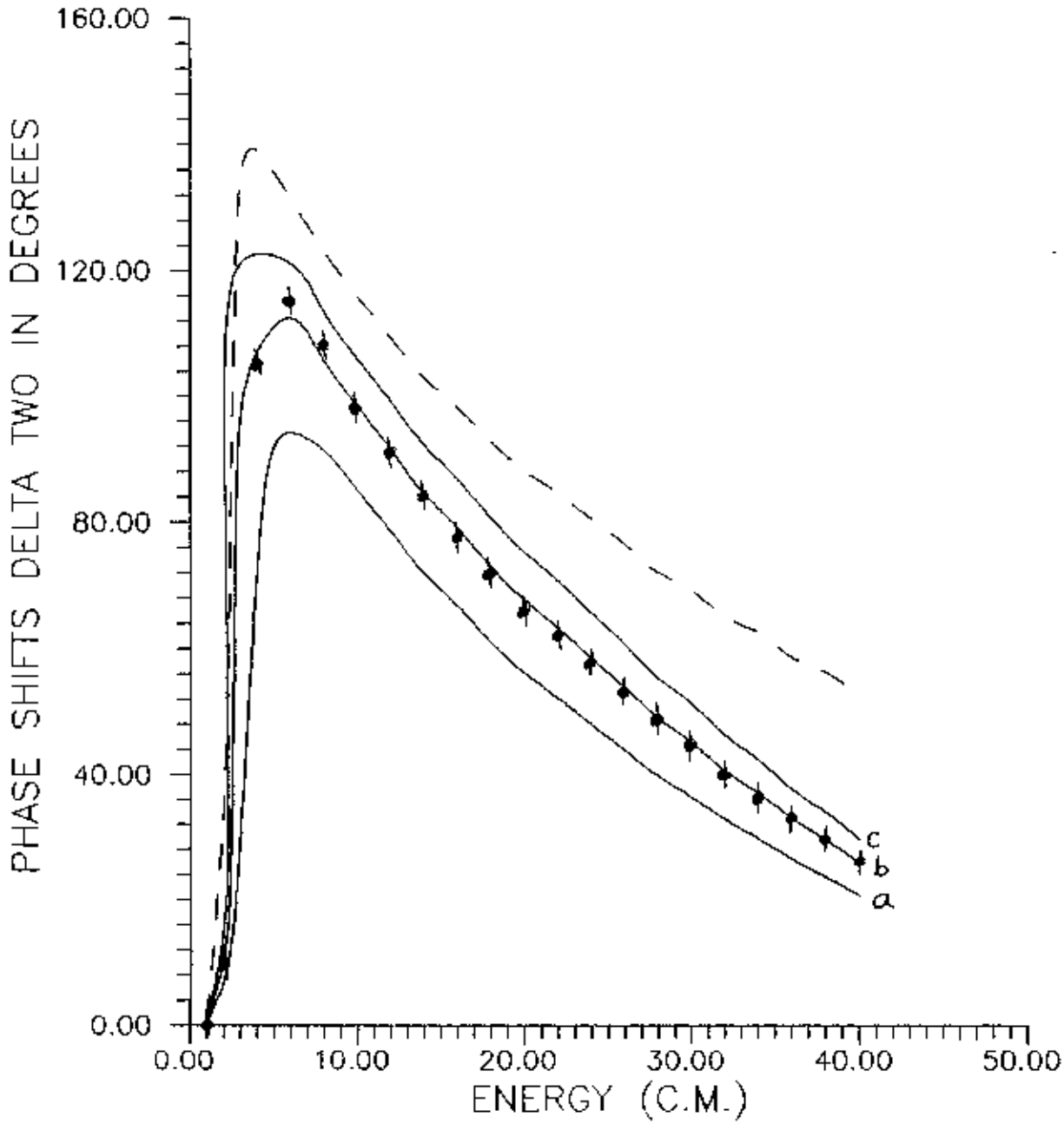


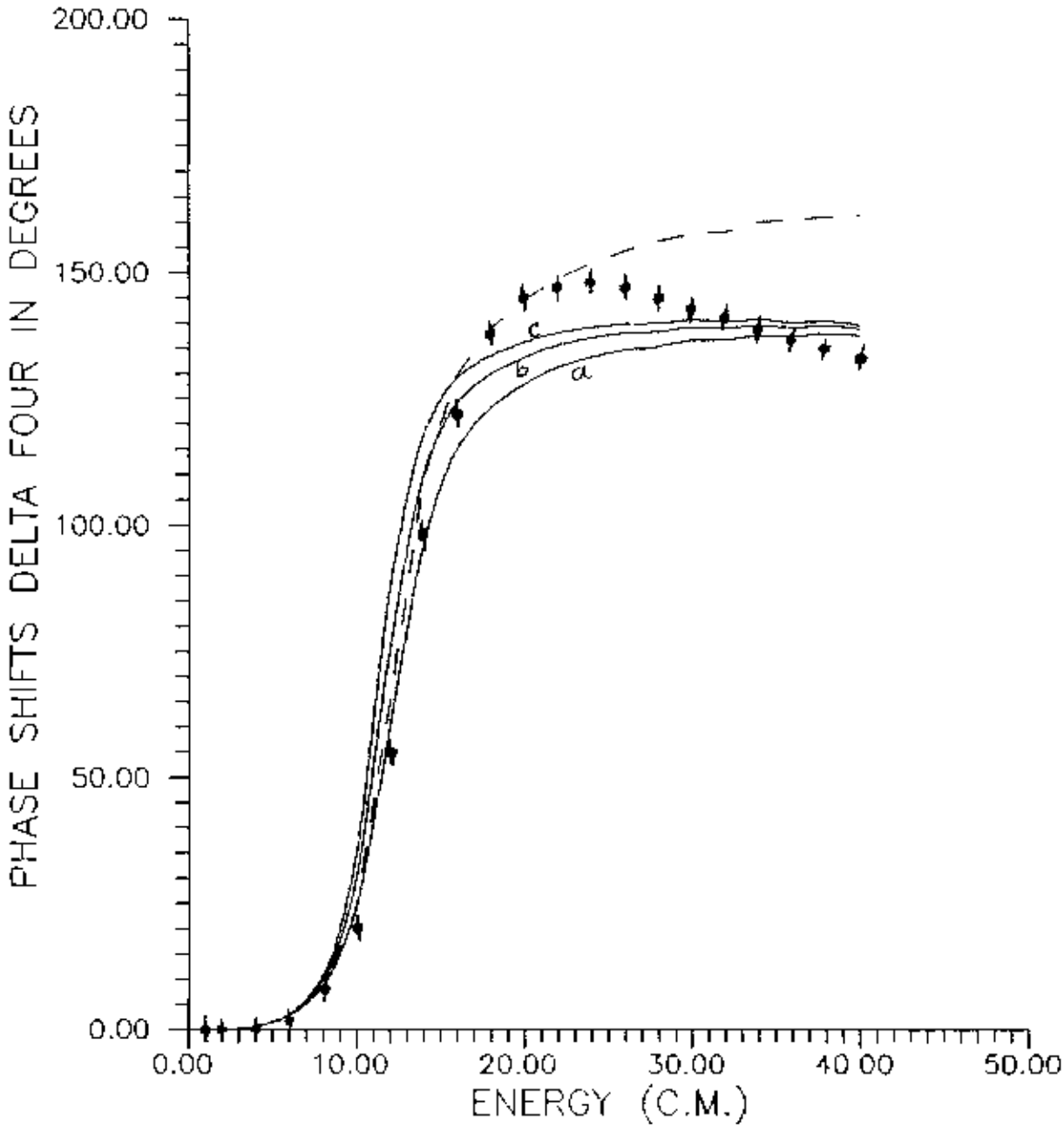
Fig-2 Calculated phase shifts of alpha-alpha scattering are shown by three solid curves a, b and c for three different sets of parameters as shown in the table 4, the phase shifts calculated by Okai and Park [19] from original kernel are shown by dashed curve. Marks ϕ show the experimental values.

Fig. 3 shows the energy dependence of the phase shift δ_4. Table 5 shows the values of the parameters used in the curves a, b and c in this figure.

Table 5

Curve	A	B	P	Q	S	T
a	.1	.1	1	1	1	1
b	1	1	1	1	1	1
c	1.5	1.5	1	1	1	1

In this figure in the energy region less than 16 Mev the curve b is higher than the experimental phase shifts and in the region $16<E<32$ it is lower than the experimental values and again in the energy region greater than 32 Mev it is higher than the experimental values. We have drawn other two curves a and c just below and above the curve b by changing the parameter values. We take the curve b as the better one.



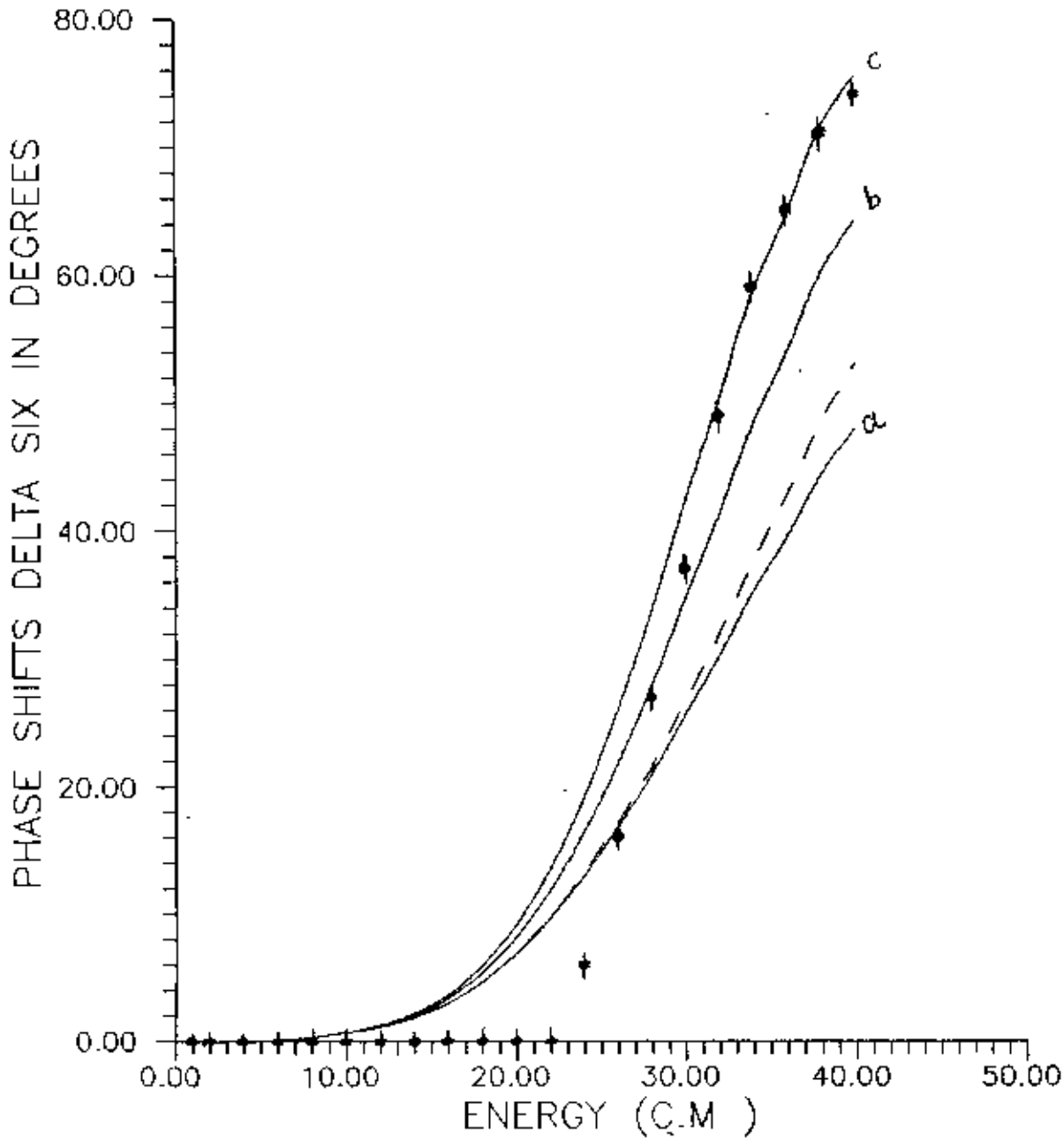
Fig−3 Calculated phase shifts of alpha−alpha scattering are shown by three solid curves a, b and c for three different sets of parameters as shown in the table 5, the phase shifts calculated by Okai and Park [19] from original kernel are shown by dashed curve. Marks ↓ show the experimental values.

Fig. 4 shows the energy dependence of the phase shift δ_s. Table 6 shows the values of the parameters used in the curves a, b and c in this figure.

Table 6

Curve	A	B	P	Q	S	T
a	1	1	1	1	1	1
b	-300	1	1	1	1	1
c	-425	.1	1	1	1	1

In the figure 4 the curve a is obtained for the parameters values unity. This curve is lower than the experimental values. In this case if we increase the values of the parameters A and B we see that the curve does not rise up. But if we take the negative values of the parameter A the curve rises up. Actually the values of the original kernel in this case is negative. We have drawn two other curves b and c. We take the curve c as the better one although the discrepancy in the lower energy region is considerable. Here we see that the discrepancy between the experimental values and the curve obtained from the original kernel is considerable.



Fig−4 Calculated phase shifts of alpha−alpha scattering are shown by three solid curves a, b and c for three different sets of parameters as shown in the table 6, the phase shifts calculated by Okai and Park [19] from original kernel are shown by dashed curve. Marks ♦ show the experimental values.

It is clear from figures 1, 2, 3 and 4 that as far as the general trend of the energy dependence of the phase shifts is concerned the calculation based on the simplest cluster model assumption, with the replacement of original kernel by a new kernel reproduces the experimental results fairly well. For the s and d partial waves, according to figure 1 and 2 the calculated phase shifts given by the curves c and b respectively have almost the same energy dependence as the experimental ones. For the g and i partial waves we obtain a good agreement with the experimental values with some systematic discrepancies.

Finally we like to make some observations that we have fully taken into account of the direct potential (nuclear and Coulomb) as obtained from resonating group formalism and at the same time we have replaced the complicated non-local non-separable kernel by a simple non-local separable kernel. And the results, we obtain, agree well with the experimental phase shifts. This clearly shows that such a simple non-local separable kernel can do the needful for the complicated non-local non-separable kernel. In that case we can extend this type of work to any nuclei of higher mass number and this will open the way to apply resonating group formalism at least partially to higher nuclei which is not possible by earlier method.

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