Magnetohydrodynamic laminar free convective flow across a horizontal cylinder with adiabatic surface

By

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Magnetohydrodynamic laminar free convective flow across a horizontal cylinder with adiabatic surface

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Abstract

The present dissertation deals with the problem of two-dimensional steady Magnetohydrodynamic laminar free convective flow across a horizontal cylinder with adiabatic surface. The governing equations are transformed to non-linear ordinary differential equations by using the suitable substitution. These equations are then solved numerically using implicit finite difference method known as Keller box scheme.

The results are presented in terms of local skin friction, surface temperature distribution, the velocity profile and temperature profile. Effects of the physical quantities for fluid having different Prandtl numbers have been studied for the cases of isothermal and adiabatic part of the surface. The dimensionless skin friction co-efficient, the surface temperature distribution, the velocity distribution and the temperature profile over the whole boundary layer are shown graphically by using the software **TECHPLOT** for different values of the magnetic parameter M and the Prandtl number Pr. Conduction, convection and Joule heating effects on Magnetohydrodynamic flow from a vertical flat plate have also been studied.

Nomenclature

c_p	:	Specific heat at constant pressure
c_f	:	Local skin friction
f	:	Dimensionless stream function
h	:	Dimensionless temperature
g	:	Acceleration due to gravity
k	:	Thermal conductivity of the fluid
p	:	Fluid pressure
q_w	:	Surface heat flux
D	:	Diameter of the cylinder
T	:	Temperature in the boundary layer
T_b or T_w	:	Temperature at the surface
T_∞	:	Temperature of the ambient fluid
T_{so}	:	Plate temperature
b	:	Plate thickness
d	:	$(T_b - T_\infty)/ T_\infty$
l	:	Length of the plate
L	:	Reference length, $\nu^{2/3}/g^{1/3}$
p	:	Coupling parameter
u	:	Velocity component in the x-direction
v	:	Velocity component in the y-direction
x	:	Distance along the surface
y	:	Distance normal to the surface
B_0 or H_0	:	Applied magnetic field
Re	:	Reynolds number
Pr	:	Prandtl number
Gr	:	Grashof number
M	:	Magnetic parameter
J	:	Joule heating parameter

Greek symbols

β : Co-efficient of thermal expansion

ψ : Stream function

η : Dimensionless similarity variable

ρ : Density of the fluid inside the boundary layer

μ : Viscosity of the fluid

ν : Kinematic viscosity

κ : Thermal conductivity

κ_s : Thermal conductivity of the ambient solid

κ_f : Thermal conductivity of the ambient fluid

θ : Dimensionless temperature

σ : Electrical conductivity

τ : Skin friction

2φ : Angle formed by the adiabatic surface at the axis of the cylinder.

Contents

List of Figures

List of Tables

Chapter 1

Introduction

1.1 Overview

Fluid dynamics is one of the oldest branches of applied mathematics. It is also the branch in which some of the most significant advances have been made during the last fifty years. These advances have been motivated by exciting development in science and technology and facilitated by growth of computer capabilities and developments of sophisticated mathematical techniques.

An important contribution to the fluid dynamics is the concept of boundary-layer introduced by L. Prandtl(1904). The concept of boundary layer is the consequence of the fact that flows at high Reynolds numbers can be divided into two unequally spaced regions. A very thin layer (called boundary-layer) in the vicinity (of the object) in which the viscous effects dominate, must be taken into account, and for the bulk of the flow region, the viscosity can be neglected and the flow corresponds to the inviscid outer flow.

Although the boundary layer is very thin, it plays a very important role in the fluid dynamics. Boundary-layer theory has become an essential study now a days in analyzing the complex behaviors of real fluids. This concept can be utilized to simplify the Navier-Stokes' equations to such an extent that the viscous effects of flow parameters are evaluated, and these are usuable in many practical problems viz. the drag on ships and missiles, the efficiency of compressors and turbines in jet engines, the effectiveness of air intakes for ram and turbojets and so on.

The three distinct modes of heat transfer, namely conduction, convection and radiation must be considered. In reality, the combined effects of these three modes of heat transfer control temperature distribution in a medium.

Conduction occurs if energy exchange takes place from the region of high temperature to that of low temperature by the kinetic motion or direct impact of molecules, as in the case of fluid at rest, and by the drift of electrons, as in the case of metals. The radiation energy emitted by a body is transmitted in the space in the form of electromagnetic waves. Energy is emitted from a material due to its temperature level, being larger for a larger temperature, and is then transmitted to another surface, which may be vacuum or a medium, which may absorb, reflect or transmit the radiation depending on the nature and extent of the medium. Considerable effort has been directed at the convective mode of heat transfer. In this mode, relative motion of the fluid provides an additional mechanism for energy transfer. A study of convective heat transfer involves the mechanism of conduction and, sometimes, those of radiation processes as well. This makes the study of convective mode a very complicated one.

The convective mode of heat transfer is divided into two basic processes. If the motion of the fluid arises due to an external agent such as the externally imposed flow of a fluid over a heated object, the process is termed as forced convection. The fluid flow may be the result of a fan, a blower, the wind or the motion of the heated object itself. If the heat transfer to or from a body occurs due to an imposed flow of a fluid at a temperature different from that of the body, problems of forced convection encounters in technology.

If the externally induced flow is provided and flows arising naturally solely due to the effect of the differences in density, caused by temperature or concentration differences in the body force field(such as gravitational field) then these types of flow are called 'free convection' or 'natural convection' flows. The density difference causes buoyancy effects and these effects act as 'driving forces' due to which the flow is generated. Hence free convection is the process of heat transfer, which occurs due to movement of the fluid particles by density differences associated with temperature difference in a fluid.

The motion of an electrically conducting fluid, like mercury, under a magnetic field, in general, gives rise to induced electric currents on which mechanical forces are exerted by the magnetic field. On the other hand, the induced electric currents also produce induce magnetic field. Thus there is a two-way interaction between the flow field and the magnetic field, the magnetic field exerts force on the fluid by producing induced currents, and the induced currents change the original magnetic field. Therefore, the hydromagnetic flows (the flows of electrically conducting fluids in the presence of a magnetic field) are more complex than the ordinary hydrodynamic flows. Mathematically also, the hydromagnetic equations have three non-linear terms while in hydrodynamics we have only one. The number of governing equations are also increased The study of hydromagnetic flows is called magnetohydrodynamics (MHD).

Two developed branches of physics, namely electromagnetic theory and fluid dynamics interact to produce hydromagnetics and therefore the field of hydromagnetics is much richer than both the parent branches.

Magnetohydrodynamics (MHD) is that branch of continuum mechanics. which deals with the flow of electrically conducting fluids in electric and magnetic fields, Probably the largest advance towards understanding of such phenomena comes from the field of Astrophysics. Originally, MHD included only the study of strictly incompressible fluid, but today the terminology is applied to study ionized gases as well. Other names have been suggested, such as magnetofluid-mechanics or magneto-aerodynamics, but original nomenclature has persisted.

Many natural phenomena and engineering problems are susceptible to MHD analysis. It is useful in astrophysics. Geophysicists encounter MHD phenomena in the interactions of conducting fluids magnetic fields that are presented in and around heavenly bodies. Engineers employ MHD principles in the design of heat exchangers, pumps and flowmeters, in space vehicle propulsion, control and re-entry, in creating novel power generating systems, and in developing confinement schemes for controlled fusion.

1.2 Literature Review

Many researchers investigated the effects of MHD free and forced convection flow both experimentally and theoretically but few works were devoted to adiabatic surfaces. Luciano[19] in 1983 investigated the laminar free convection around horizontal cylinder The author considered the cylinder surface partly isothermal and partly adiabatic. Kuehn and Goldstein [17] determined the numerical solution of the Navier-Stokes's equations on laminar natural convective flow about a horizontal isothermal circular cylinder. Farouk and Guceri [18] investigated the natural convection from a horizontal cylinder. Merkin [14] studied free convection boundary layer flow on an isothermal horizontal circular cylinder. In all the aforementioned cases analyses established on free convection horizontal cylinder based on different conditions.

Sparrow and Cess [4] Investigated the effect of a magnetic field on free convection heat transfer Kuiken [10] studied MHD free convection in a strong cross-field. Hossain et. al. [22, 23] Investigated MHD forced and free convection boundary layer flow near the leading edge. They also Investigated MHD forced and free convection boundary layer flow along a vertical porous plate. Wilks [13] studied MHD free convection about a semi-infinite vertical plate in a strong cross-flow. The authors analysed the MHD natural convection flow also in various fields.

Free convection around horizontal cylinders has been extensively investigated, both analytically and numerically Little bit attention has so far been paid to other thermal conditions of noticeable practical interest. as are those relative to partly isothermal and partly adiabatic (it takes no heat from the fluid or heat is not being transformed to this part) cylinders.

Free convection heat transfer due to the simultaneous action of buoyancy and induced magnetic forces is very important in some practical problems. The analysis is carried out for laminar free convective flow across a horizontal cylinder with adiabatic surface It is found that the free convection heat transfer to liquid metals may be significantly affected by the presence of a magnetic field, but that other fields experience very small effects.

The literature on convection heat transfer is over helming and ever growing. In recent years, with the availability of high-speed and large-capacity digital computers, great advances have been made in the analysis of very complicated heat transfer problems.

The specific problem selected for study is the flow and heat transfer in an electrically conducting fluid adjacent to a horizontal cylinder which is isothermal with an adiabatic part.

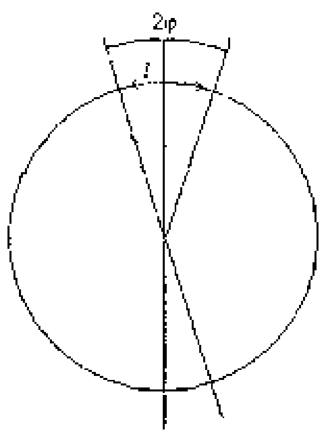

Figure-1: Geometry of the problem

The configuration is pictured in figure-1, where in particular, the adiabatic sector corresponding to an angle 2φ is symmetric with respect to the vertical diameter, whereas, the length of the relative arc is l, which could be reduced to zero in so restoring the fully isothermal situation

We shall investigate the magneto-hydrodynamic boundary layer flow resulting from the natural convection of a horizontal cylinder with adiabatic surface in an electrically conducting fluid in the presence of the transverse magnetic field.

The transformed non similar boundary layer equations governing the flow together with the boundary conditions based on free convection were solved numerically using the Keller box (implicit finite difference) along with Newton's linearization approximation method in the entire region starting from the upstream of the cylinder to the down stream for some values of the magnetic parameter M and the Prandtl number Pr. We have studied the effect of the parameters M, Pr and φ on the velocity and temperature fields as well as on the skin friction coefficient and the surface rate of heat transfer

Detailed derivations of the governing equations for the flow and the method of solutions along with the results and discussions are presented in the next chapters

1.3 Objectives

The present work has the following objectives:

1. To study the effects of the physical quantities for fluid having different Prandtl number for the cases of isothermal and adiabatic part of the surface. We also study the effect of the magnetic field on these physical quantities.
2. To obtain the solution of the governing equations. There are different types of numerical technique available depending on the types of problem to be solved. Finite difference method is one of the important methods both for programming and rapid convergence. Here we use finite difference method modified by Hossain et. al [22, 23]
3. To find the local skin friction, the surface rate of heat transfer and the velocity profile and temperature profile for the above mentioned problem.
4. The results in terms of local skin friction, local rate of heat transfer, the velocity and temperature profiles are shown graphically by using the software TECHPLOT.

1.4 Applications

The most important application of MHD is in the generation of electrical power with the flow of an electrical conducting fluid through a transverse magnetic field. Recently, experiments with ionized gases have been performed with the hope of producing power on a large scale in stationary plants with large magnetic fields. Generation of MHD power on a smaller scale is of interest for space applications.

The study of heat transfer is of great interest in many branches of science and engineering. In designing heat exchangers such as boilers, condensers and radiators etc. heat transfer analysis is essential for sizing such equipment. For example in the design of nuclear-reactor cores a thorough heat transfer analysis is important for proper sizing of fuel element to prevent burnout. In aerospace technology, heat transfer problems are crucial because of weight limitations and safety considerations. In heating and air conditioning applications for buildings a proper heat transfer analysis is necessary to estimate the amount of insulation needed to prevent excessive heat loses or gains.

In engineering, situations related to adiabatic surface describe about practical problem; for example, in heat transfer around metallic tubes partially covered by snow or ice, or around pipes where an internal layer of deposited salt greatly decreases in some areas of cylinder.

Chapter 2

Magnetohydrodynamic laminar free convective flow across a horizontal cylinder with adiabatic surface.

In the following sections detailed derivations of the governing equations for the flow and the method of solutions along with the results and discussions are presented.

2.1 Governing equations of the flow:

The starting point of the analysis is the basic conservation laws of mass, momentum and energy. To obtain the mathematical statement of these laws, we utilize the well known governing equations for free convection which are added terms appropriate to the magnetic effects. Characterizing the induced magnetic force by F_x we write the boundary layer equations governing the flow are:

Mass: $$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0 \tag{2.1}$$

Momentum: $$u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}=\nu\frac{\partial^2 u}{\partial y^2}\pm g\beta(T-T_\infty)\sin\left(\frac{x}{a}\right)+\frac{F_x}{\rho} \tag{2.2}$$

Energy: $$u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}=\alpha\frac{\partial^2 T}{\partial^2 y} \tag{2.3}$$

with boundary conditions:

$$u=v=0,\ T=T_w \text{ for isothermal part and } \frac{\partial T}{\partial y}=0 \text{ for adiabatic part at } y=0 \tag{2.4a}$$

$$u\to 0,\ T\to T_\infty \text{ as } y\to\infty \tag{2.4b}$$

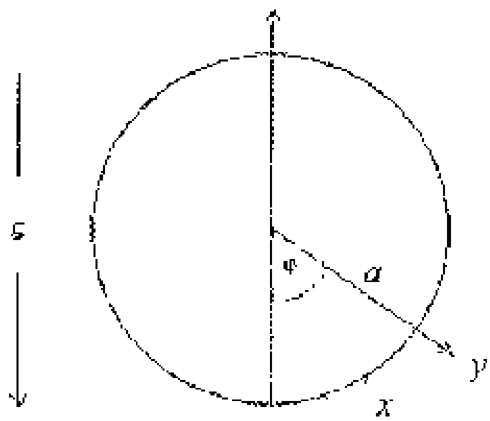

Figure 2: The co-ordinate system

Here u and v are the velocity components along the x and y direction respectively. T is the temperature of the fluid in the boundary layer, g is the acceleration due to gravity, ν is the kinematic viscosity of the fluid and β is the thermal expansion coefficient. The plus and minus signs attached to the buoyancy force $g\beta(T-T_\infty)$ in equation (2.2) apply to flow due to heating and cooling respectively. Fluid property variations have been considered only to the extent of a density variation, which provides a buoyancy force. For the energy equation we neglected the viscous dissipation, and the Joule heating term

The magnetic force $\boldsymbol{F}$ may, in the absence of excess charges, be written as

$$\boldsymbol{F}=\boldsymbol{J}\times\boldsymbol{B} \tag{2.5}$$

where J and B are respectively the current density and magnetic induction vectors. Further, when the external electric field is zero and the induced electric field negligible, the current density is related by Ohm's law as follows.

$$\boldsymbol{J}=\sigma(\boldsymbol{V}\times\boldsymbol{B}) \tag{2.6}$$

where σ denotes the electrical conductivity of the fluid. Next, under the condition that the magnetic Reynolds number is small, the induced magnetic field is negligible compared with the applied field. This condition is usually well satisfied in terrestrial applications, especially so in (low- velocity) free convection flows. So, we can write

$$\boldsymbol{B}=\boldsymbol{i}_z B_0 \tag{2.7}$$

Using equations (2.4), (2.5) and (2.6) we get the force component,

$$F_x = -\sigma u B_0^2 \tag{2.8}$$

This term is introduced into the momentum equation (2.2).

The specification of the boundary conditions is necessary to complete the statement of the problem. At the surface, the velocities are zero to satisfy the no slip conditions. In addition, temperature continuity requires that fluid and solid share the same temperature for isothermal part of the cylinder and temperature gradient is zero for adiabatic part of the cylinder. Far from the surface, the velocity approaches zero and the temperature approaches that of surroundings. Formally, these conditions were stated in equation (2.4). Using the equations (2.5) to (2.8) with respect to our considerations into the basic equations (2.1) and (2.3) we have the following form:

Mass $$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0 \tag{2.9}$$

Momentum: $$u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}=\pm g\beta(T-T_\infty)\sin\left(\frac{x}{a}\right)-\frac{\sigma u B_0^2}{\rho}+\nu\frac{\partial^2 u}{\partial y^2} \tag{2.10}$$

Energy: $$u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}=\alpha\frac{\partial^2 T}{\partial^2 y} \tag{2.11}$$

The appropriate boundary conditions to be satisfied by the above equations are:

$$u=v=0,\ T=T_w \text{ for isothermal part and } \frac{\partial T}{\partial y}=0 \text{ for adiabatic part at } y=0 \tag{2.12a}$$

$$u\to 0,\ T\to T_\infty \text{ as } y\to\infty \tag{2.12b}$$

2.2 Transformation of the governing equations:

Now we introduce the following group of transformations of co-ordinates from *(x, y)* to *(ξ, η)* coordinates system as

$$\eta=\frac{y}{a}Gr^{\frac{1}{4}},\quad \xi=\frac{x}{a} \tag{2.13}$$

Here the variable η is the free convection similarity variable. On the other hand, ξ, which is essentially a stretched x coordinate, is an index to the relative importance of the magnetic forces. Next, new dependent variables f and θ are defined as:

$$\psi(x,y) = \nu\xi Gr^{\frac{1}{4}} f(\xi,\eta) \tag{2.14}$$

$$\theta(\xi,\eta) = \frac{T(x,y) - T_\infty}{T_w - T_\infty} \tag{2.15}$$

where, the stream function $\psi(x,y)$ satisfies the mass conservation equation

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0, \text{ u and v defined as } u = \frac{\partial \psi}{\partial y},\ v = -\frac{\partial \psi}{\partial x} \tag{2.16}$$

$Gr = \dfrac{g\beta(T_w - T_\infty)}{\nu^2} a^3$ is the Grashof number.

Using the above transformation in equations (2.10) and (2.11) we get the non-dimensional equations of momentum and energy equations as

$$f''' + ff'' - f'^2 + \frac{\sin\xi}{\xi}\theta - \frac{\sigma a^2 B_0^2}{\nu Gr^{\frac{1}{2}}\rho} f' = \xi\left(f'\frac{\partial f'}{\partial \xi} - f''\frac{\partial f}{\partial \xi}\right) \tag{2.17}$$

$$\frac{1}{\Pr}\theta'' + f\theta' = \xi\left(f'\frac{\partial \theta}{\partial \xi} - \theta'\frac{\partial f}{\partial \xi}\right) \tag{2.18}$$

where, Prandtl number $\Pr = \dfrac{\mu C_p}{k} = \dfrac{\nu}{\alpha}$

And the boundary conditions in (2.12) become:

$$\left.\begin{aligned} &f' = \frac{\partial f}{\partial \eta} = 0 \\ &\theta = 1\ (for\ isothermal\ part)\ and \\ &\frac{\partial \theta}{\partial \eta} = 0\ (for\ adiabatic\ part) \end{aligned}\right\}\ at\ \eta = 0 \tag{2.19a}$$

$$\left.\begin{aligned} &\frac{\partial f}{\partial \eta} = 0\ and \\ &\theta \to 0 \end{aligned}\right\}\ as\ \eta \to \infty \tag{2.19b}$$

2.3 The important dimensionless parameters related to the problem:

The governing equations of the fluid flow are discussed at the previous sections in this chapter. These equations contain a number of variables It is difficult to study the effect of each variable on the process. Moreover these equations are nonlinear. There is no general method to find the solution of these nonlinear equations. In order to bring out the essential features of flow, it is necessary to find important dimensionless parameters, which characterize the flow. These parameters are very useful in the analysis of experimental results. Some non-dimensional parameters related to our problems are discussed below:

Reynolds number R_e

Reynolds number is the most important non-dimensional parameter of the fluid dynamics of a viscous fluid. It represents the ratio of the inertia force and the viscous force. It is denoted by R_e.

$$R_e = \frac{\text{Inertia force}}{\text{Viscous force}}$$

$$= \frac{\text{Mass} \times \text{Acceleration}}{\text{Shear stress} \times \text{Cross sectional area}}$$

$$= \frac{\text{Volume} \times \text{Density} \times (\text{Velocity/time})}{\text{Shear stress} \times \text{Cross sectional area}}$$

$$= \frac{\text{Cross sectional area} \times \text{Linear dimension} \times \text{Density} \times (\text{Velocity/time})}{\text{Shear stress} \times \text{Cross sectional area}}$$

$$= \frac{\rho V^2 L^2}{\mu V L} = \frac{V L}{\nu}$$

where, V, L, ρ and μ denote the characteristic velocity, the characteristic length, the density and the coefficient of viscosity of the fluid flow respectively. Here $\nu = \frac{\mu}{\rho}$ is the kinematics viscosity. This result implies that viscous forces are dominant for small Reynolds numbers and inertia forces are dominant for large Reynolds numbers. The Reynolds number was used as the criterion to determine the change from laminar to turbulent flow.

Prandtl number P_r

Prandtl gave an important number known as Prandtl number. The Prandtl number is a dimensionless parameter of a convective system that characterizes the regime of convection. It is the ratio of viscous force to the thermal force and is defined as follows:

$$P_r = \frac{\text{Viscous force}}{\text{Thermal force}}$$
$$= \frac{\text{Kinematic viscosity}}{\text{Thermal diffusivity}}$$
$$= \frac{\mu / \rho}{\kappa / \rho C_p} = \frac{\nu}{\alpha}$$

The Prandtl number is large when thermal conductivity is small and viscosity is large. and small when viscosity is small and thermal conductivity is large. For small value of ν, a thin region will be affected by viscosity, which is known as boundary layer region. For the small value of $\kappa / \rho C_p$, a thin region will be effected by heat conduction which is known as the thermal boundary layer. Prandtl number also gives the relative importance of viscous dissipation to the thermal dissipation. Thus it represents the relative importance of momentum and energy transport by the diffusion process. Usually for gases $\Pr \cong 1$, the transfer of momentum and energy by the diffusion process is comparable. For oils, $\Pr \gg 1$, hence the momentum diffusion is much greater than the energy diffusion; but for liquid metals, $\Pr \ll 1$ and the solution is reversed.

Grashof number G_r

The Grashof number gives the relative importance of buoyancy force to the viscous forces and is defined as

$$Gr = \frac{g \beta L^3 \left(T - T_0\right)}{\nu^2}$$

where g is the acceleration due to gravity. L the characteristic length of the problem, β the coefficient of volume expansion, and $T - T_0$ is the excess temperature of the fluid over the reference temperature T_0. This number is of great importance and plays a similar role in free convection as does the Reynolds number in forced convection. A critical value of the Grashof number is used to indicate transition from laminar to turbulent flow in free convection.

Magnetic Parameter M

The magnetic parameter is obtained from the ratio of the magnetic force to the inertia force and is defined as

$$M = \frac{\sigma B_0^2 L}{\rho \Omega L}$$

If this is of the order one then the magnetic force is important and the flow is to be considered as hydromagnetic flow. If it is very much less than one, them the flow can be taken as hydrodynamic. For small value of M, the motion is hardly affected by the magnetic field and for large value of M, the motion is largely controlled by the magnetic field.

Chapter 3

Method of Solution

3.1 Finite difference method

To get the solutions of the differential equations (2.17) and (2.18) along with the boundary condition (2.19a) and (2.19b), we shall employ a most practical, an efficient and accurate solution technique, known as implicit finite difference method together with Keller-box elimination technique which is well documented and widely used by Keller (1978) and Cebeci (1984) and recently by Hossain(1992).

To apply the aforementioned method, we first convert the equations (2.17) and (2.18) into the following system of first order differential equations with dependent variables $u(\xi,\eta)$, $v(\xi,\eta)$ and $p(\xi,\eta)$ along with the boundary condition (2.19a) and (2.19b) as

$$f' = u \tag{3.1}$$

$$u' = v \tag{3.2}$$

$$\theta' = p \tag{3.3}$$

Equations (2.17) and (2.18) transform to

$$v' + p_1 fv - p_2 u^2 + p_3\theta - p_4 u = \xi\left(u\frac{\partial u}{\partial \xi} - v\frac{\partial f}{\partial \xi}\right) \tag{3.4}$$

$$\frac{1}{\Pr}p' + p_1 fp = \xi\left(u\frac{\partial \theta}{\partial \xi} - p\frac{\partial f}{\partial \xi}\right) \tag{3.5}$$

Where

$$p_1 = 1,\ p_2 = 1,\ p_3 = \frac{\sin\xi}{\xi}\ and\ p_4 = \frac{\sigma a^2 B_0^2}{\nu Gr^{\frac{1}{2}}\rho} \tag{3.6}$$

The boundary conditions are:

$$\left.\begin{array}{l} f' = u = 0 \\ \theta = 1 \ (\text{for isothermal part}) \text{ and} \\ p = \dfrac{\partial \theta}{\partial \eta} = 0 \ (\text{for adiabatic part}) \end{array}\right\} at\ \eta = 0 \tag{3.7a}$$

$$\left.\begin{array}{l} u = \dfrac{\partial f}{\partial \eta} = 0 \text{ and} \\ \theta \to 0 \end{array}\right\} as\ \eta \to \infty \tag{3.7b}$$

We now consider the net rectangle on the (ξ, η) plane shown in the figure (3.1) and denote the net points by

$$\left.\begin{array}{l} \xi^0 = 0, \quad \xi^n = \xi^{n-1} + k_n \quad where \quad n = 1,2,\ldots N \\ \eta_0 = 0, \quad \eta_j = \eta_{j-1} + h_j \quad where \quad j = 1,2,\ldots J \end{array}\right\} \tag{3.8}$$

Here 'n' and 'j' are just sequence of numbers on the (ξ, η) plane, k_n and h_j are the variable mesh widths.

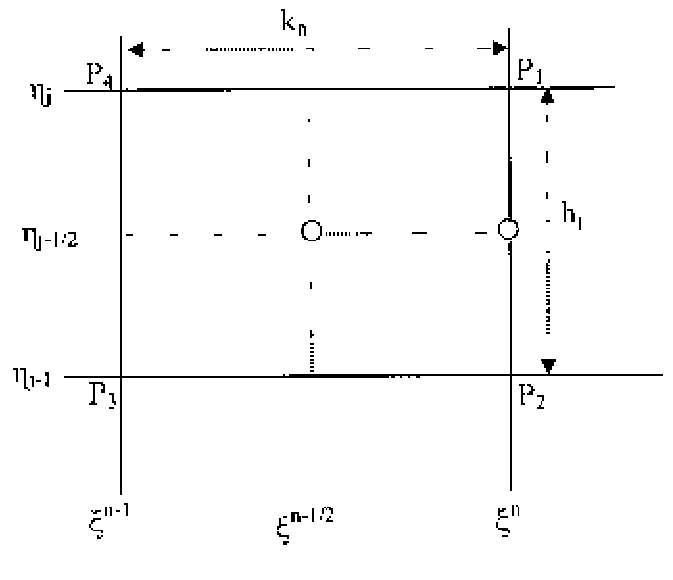

Figure 3.1: Net rectangle of the difference approximation for the Box scheme.

We approximate the quantities (f. u. v, p) at the points (ξ^n, η_j) of the net by$(f_j^n, u_j^n, v_j^n, p_j^n)$ which we call net function . We also employ the notation g_j^n for the quantities midway between net points shown in figure (3.1) and for any net function as

$$\xi^{n-1/2} = \frac{1}{2}(\xi^n + \xi^{n-1}) \tag{3.9a}$$

$$\eta_{j-1/2} = \frac{1}{2}(\eta_j - \eta_{j-1}) \tag{3.9b}$$

$$\theta_j^{n-1/2} = \frac{1}{2}(\theta_j^n + \theta_j^{n-1}) \tag{3.9c}$$

$$\theta_{j-1/2}^n = \frac{1}{2}(\theta_j^n + \theta_{j-1}^n) \tag{3.9d}$$

Now we write the difference equations that are to approximate the three first order ordinary differential equations (3.1)-(3.3) according to Box method by considering one mesh rectangle. We start by writing the finite difference approximation of the above three equations using central difference quotients and average about the mid-point $(\xi^n, \eta_{j-1/2})$ of the segment P_1P_2 shown in the figure (3.1) and the finite difference approximations to the two first order differential equations (3.4)-(3.5) are written for the mid point $(\xi^{n-1/2}, \eta_{j-1/2})$ of the rectangle $P_1P_2P_3P_4$. This procedure yields.

$$h_j^{-1}(f_j^n - f_{j-1}^n) = u_{j-1/2}^n = \frac{u_{j-1}^n + u_j^n}{2} \tag{3.10}$$

$$h_j^{-1}(u_j^n - u_{j-1}^n) = v_{j-1/2}^n = \frac{v_{j-1}^n + v_j^n}{2} \tag{3.11}$$

$$h_j^{-1}(\theta_j^n - \theta_{j-1}^n) = p_{j-1/2}^n = \frac{p_{j-1}^n + p_j^n}{2} \tag{3.12}$$

$$\frac{1}{2}\left[\frac{v_j^n - v_{j-1}^n}{h_j} + \frac{v_j^{n-1} - v_{j-1}^{n-1}}{h_j}\right] + (p_1 fv)_{j-1/2}^{n-1/2} - (p_2 u^2)_{j-1/2}^{n-1/2} + (p_3\theta)_{j-1/2}^{n-1/2}$$
$$-(p_4 u)_{j-1/2}^{n-1/2} = \xi_{j-1/2}^{n-1/2}\left[u_{j-1/2}^{n-1/2}\left\{\frac{u_{j-1/2}^n - u_{j-1/2}^{n-1}}{k_n}\right\} - v_{j-1/2}^{n-1/2}\left\{\frac{f_{j-1/2}^n - f_{j-1/2}^{n-1}}{k_n}\right\}\right] \tag{3.13}$$

$$\frac{1}{2P_r}\left[\frac{p_j^n - p_{j-1}^n}{h_j} + \frac{p_j^{n-1} - p_{j-1}^{n-1}}{h_j}\right] + (p_1 fp)_{j-1/2}^{n-1/2}$$
$$= \xi_{j-1/2}^{n-1/2}\left[u_{j-1/2}^{n-1/2}\left\{\frac{\theta_{j-1/2}^n - \theta_{j-1/2}^{n-1}}{k_n}\right\} - p_{j-1/2}^{n-1/2}\left\{\frac{f_{j-1/2}^n - f_{j-1/2}^{n-1}}{k_n}\right\}\right] \tag{3.14}$$

Now from the equation (3.13) we get

$$\frac{1}{2}\left(\frac{v_j^n - v_{j-1}^n}{h_j}\right)+\frac{1}{2}\left(\frac{v_j^{n-1} - v_{j-1}^{n-1}}{h_j}\right)+\frac{1}{2}\left\{(p_1 fv)_{j-1/2}^n + (p_1 fv)_{j-1/2}^{n-1}\right\}$$
$$-\frac{1}{2}\left\{(p_2 u^2)_{j-1/2}^n + (p_2 u^2)_{j-1/2}^{n-1}\right\}+\frac{1}{2}\left\{(p_3\theta)_{j-1/2}^n + (p_3\theta)_{j-1/2}^{n-1}\right\}$$
$$-\frac{1}{2}\left\{(p_4 u)_{j-1/2}^n + (p_4 u)_{j-1/2}^{n-1}\right\}=\frac{1}{2k_n}\xi_{j-1/2}^{n-1/2}\left(u_{j-1/2}^n + u_{j-1/2}^{n-1}\right)\left(u_{j-1/2}^n - u_{j-1/2}^{n-1}\right)$$
$$-\frac{1}{2k_n}\xi_{j-1/2}^{n-1/2}\left(v_{j-1/2}^n + v_{j-1/2}^{n-1}\right)\left(f_{j-1/2}^n - f_{j-1/2}^{n-1}\right)$$
$$\Rightarrow h_j^{-1}(v_j^n - v_{j-1}^n)+h_j^{-1}(v_j^{n-1} - v_{j-1}^{n-1})+(p_1)_{j-1/2}^n (fv)_{j-1/2}^n + (p_1)_{j-1/2}^{n-1}(fv)_{j-1/2}^{n-1}$$
$$-(p_2)_{j-1/2}^n (u^2)_{j-1/2}^n - (p_2)_{j-1/2}^{n-1}(u^2)_{j-1/2}^{n-1} + (p_3)_{j-1/2}^n(\theta)_{j-1/2}^n + (p_3)_{j-1/2}^{n-1}\theta_{j-1/2}^{n-1}$$
$$-(p_4)_{j-1/2}^n (u)_{j-1/2}^n - (p_4)_{j-1/2}^{n-1}(u)_{j-1/2}^{n-1} = \alpha_n\{(u^2)_{j-1/2}^n - (u^2)_{j-1/2}^{n-1}$$
$$-(fv)_{j-1/2}^n + v_{j-1/2}^n f_{j-1/2}^{n-1} - v_{j-1/2}^{n-1} f_{j-1/2}^n + (fv)_{j-1/2}^{n-1}\},\ \textit{Where}\ \alpha_n = \frac{\xi_{j-1/2}^{n-1/2}}{k_n}$$
$$\Rightarrow h_j^{-1}(v_j^n - v_{j-1}^n)+\left\{(p_1)_{j-1/2}^n + \alpha_n\right\}(fv)_{j-1/2}^n - \left\{(p_2)_{j-1/2}^n + \alpha_n\right\}(u^2)_{j-1/2}^n$$
$$+(p_3)_{j-1/2}^n(\theta)_{j-1/2}^n - (p_4)_{j-1/2}^n u_{j-1/2}^n = \alpha_n[-(u^2)_{j-1/2}^{n-1} + v_{j-1/2}^n f_{j-1/2}^{n-1} - v_{j-1/2}^{n-1} f_{j-1/2}^n$$
$$+(fv)_{j-1/2}^{n-1}] - (p_1)_{j-1/2}^{n-1}(fv)_{j-1/2}^{n-1} + (p_2)_{j-1/2}^{n-1}(u^2)_{j-1/2}^{n-1} - h_j^{-1}(v_j^{n-1} - v_{j-1}^{n-1})$$
$$-(p_3)_{j-1/2}^{n-1}\theta_{j-1/2}^{n-1} + (p_4)_{j-1/2}^{n-1}u_{j-1/2}^{n-1}$$

$$\Rightarrow h_j^{-1}(v_j^n - v_{j-1}^n)+\left\{(p_1)_{j-1/2}^n + \alpha_n\right\}(fv)_{j-1/2}^n - \left\{(p_2)_{j-1/2}^n + \alpha_n\right\}(u^2)_{j-1/2}^n$$
$$+(p_3)_{j-1/2}^n(\theta)_{j-1/2}^n - (p_4)_{j-1/2}^n u_{j-1/2}^n + \alpha_n(v_{j-1/2}^{n-1} f_{j-1/2}^n - v_{j-1/2}^n f_{j-1/2}^{n-1})$$
$$=\alpha_n\left\{(fv)_{j-1/2}^{n-1} - (u^2)_{j-1/2}^{n-1}\right\} - (p_1)_{j-1/2}^{n-1}(fv)_{j-1/2}^{n-1} + (p_2)_{j-1/2}^{n-1}(u^2)_{j-1/2}^{n-1}$$
$$-h_j^{-1}(v_j^{n-1} - v_{j-1}^{n-1}) - (p_3)_{j-1/2}^{n-1}(\theta)_{j-1/2}^{n-1} + (p_4)_{j-1/2}^{n-1}u_{j-1/2}^{n-1}$$
$$\Rightarrow h_j^{-1}(v_j^n - v_{j-1}^n)+\left\{(p_1)_{j-1/2}^n + \alpha_n\right\}(fv)_{j-1/2}^n - \left\{(p_2)_{j-1/2}^n + \alpha_n\right\}(u^2)_{j-1/2}^n$$
$$+(p_3)_{j-1/2}^n(\theta)_{j-1/2}^n - (p_4)_{j-1/2}^n u_{j-1/2}^n + \alpha_n(v_{j-1/2}^{n-1} f_{j-1/2}^n - v_{j-1/2}^n f_{j-1/2}^{n-1})$$
$$= -L_{j-1/2}^{n-1} + \alpha_n\left\{(fv)_{j-1/2}^{n-1} - (u^2)_{j-1/2}^{n-1}\right\}$$
$$L_{j-1/2}^{n-1} = (p_1)_{j-1/2}^{n-1}(fv)_{j-1/2}^{n-1} - (p_2)_{j-1/2}^{n-1}(u^2)_{j-1/2}^{n-1} + h_j^{-1}(v_j^{n-1} - v_{j-1}^{n-1})$$
$$+(p_3)_{j-1/2}^{n-1}(\theta)_{j-1/2}^{n-1} - (p_4)_{j-1/2}^{n-1}u_{j-1/2}^{n-1}$$

$$\Rightarrow h_j^{-1}(v_j^n - v_{j-1}^n)+\left\{(p_1)_{j-1/2}^n + \alpha_n\right\}(fv)_{j-1/2}^n - \left\{(p_2)_{j-1/2}^n + \alpha_n\right\}(u^2)_{j-1/2}^n$$
$$+(p_3)_{j-1/2}^n(\theta)_{j-1/2}^n - (p_4)_{j-1/2}^n \theta_{j-1/2}^n + \alpha_n(v_{j-1/2}^{n-1} f_{j-1/2}^n - v_{j-1/2}^n f_{j-1/2}^{n-1}) = R_{j-1/2}^{n-1} \qquad (3.15)$$
$$R_{j-1/2}^{n-1} = -L_{j-1/2}^n + \alpha_n\left\{(fv)_{j-1/2}^{n-1} - (u^2)_{j-1/2}^{n-1}\right\}$$

Since $p_i = p_i(\xi)$, so we may write the above equation as

$$h_j^{-1}(v_j^n - v_{j-1}^n) + \{(p_1)^n + \alpha_n\}(fv)_{j-1/2}^n - \{(p_2)^n + \alpha_n\}(u^2)_{j-1/2}^n$$
$$+ (p_3)^n(\theta)_{j-1/2}^n - (p_4)^n u_{j-1/2}^n + \alpha_n(v_{j-1/2}^{n-1} f_{j-1/2}^n - v_{j-1/2}^n f_{j-1/2}^{n-1}) = R_{j-1/2}^{n-1} \quad (3.16)$$

Again from the equation (3.14) we get

$$\frac{1}{2P_r}\left[\frac{p_j^n - p_{j-1}^n}{h_j} + \frac{p_j^{n-1} - p_{j-1}^{n-1}}{h_j}\right] + (p_1 fp)_{j-1/2}^{n-1/2}$$
$$= \xi_{j-1/2}^{n-1/2}\left[u_{j-1/2}^{n-1/2}\left\{\frac{\theta_{j-1/2}^n - \theta_{j-1/2}^{n-1}}{k_n}\right\} - p_{j-1/2}^{n-1/2}\left\{\frac{f_{j-1/2}^n - f_{j-1/2}^{n-1}}{k_n}\right\}\right]$$

$$\Rightarrow \frac{1}{P_r}h_j^{-1}(p_j^n - p_{j-1}^n) + \frac{1}{P_r}h_j^{-1}(p_j^{n-1} - p_{j-1}^{n-1}) + (p_1)_{j-1/2}^{n-1/2}(fp)_{j-1/2}^n + (p_1)_{j-1/2}^{n-1/2}(fp)_{j-1/2}^{n-1}$$
$$= \alpha_n\{(u_{j-1/2}^n + u_{j-1/2}^{n-1})(\theta_{j-1/2}^n - \theta_{j-1/2}^{n-1}) - (p_{j-1/2}^n + p_{j-1/2}^{n-1})(f_{j-1/2}^n - f_{j-1/2}^{n-1})\}$$
$$\Rightarrow \frac{1}{P_r}h_j^{-1}(p_j^n - p_{j-1}^n) + (p_1)_{j-1/2}^{n-1/2}(fp)_{j-1/2}^n = -M_{j-1/2}^{n-1} + \alpha_n\{(u\theta)_{j-1/2}^n - u_{j-1/2}^n\theta_{j-1/2}^{n-1}$$
$$+ u_{j-1/2}^{n-1}\theta_{j-1/2}^n - (pf)_{j-1/2}^n + p_{j-1/2}^n f_{j-1/2}^{n-1} - p_{j-1/2}^{n-1} f_{j-1/2}^n\} + \alpha_n\{(fp)_{j-1/2}^{n-1} - (u\theta)_{j-1/2}^{n-1}\}$$

Where, $M_{j-1/2}^{n-1} = \frac{1}{P_r}h_j^{-1}(p_j^{n-1} - p_{j-1}^{n-1}) + (p_1)_{j-1/2}^{n-1/2}(fp)_{j-1/2}^{n-1}$

$$\Rightarrow \frac{1}{P_r}h_j^{-1}(p_j^n - p_{j-1}^n) + \{(p_1)_{j-1/2}^{n-1/2} + \alpha_n\}(fp)_{j-1/2}^n - \alpha_n(u\theta)_{j-1/2}^n$$
$$- \alpha_n(u_{j-1/2}^{n-1}\theta_{j-1/2}^n - u_{j-1/2}^n\theta_{j-1/2}^{n-1} + p_{j-1/2}^n f_{j-1/2}^{n-1} - p_{j-1/2}^{n-1} f_{j-1/2}^n) = T_{j-1/2}^{n-1} \quad (3.17)$$
$$T_{j-1/2}^{n-1} = -M_{j-1/2}^{n-1} + \alpha_n\{(fp)_{j-1/2}^{n-1} - (u\theta)_{j-1/2}^{n-1}\}$$

The boundary conditions become

$$f_0^n = 0 \quad u_0^n = 0 \quad \theta_0^n = 1\,(\textit{for isothermal part}) \quad p_0^n = 0\,(\textit{for adiabatic part}) \quad (3.18)$$
$$u_J^n = 0 \quad \theta_J^n = 0$$

If we assume $f_j^{n-1}, u_j^{n-1}, v_j^{n-1}, \theta_j^{n-1}, p_j^{n-1}$ to be known for $0 \le j \le J$, equations (3.10) to (3.12) and (3.15) – (3.18) form a system of 5J + 5 non linear equations for the solutions of the 5J + 5 unknowns $(f_j^n, u_j^n, v_j^n, \theta_j^n, p_j^n)$, j = 0,1,2 ...J These non linear system of algebraic equations are to be linearized by Newton's Quassy linearization method . We define the iterates $[f_j^n, u_j^n, v_j^n, \theta_j^n, p_j^n]$, i = 0,1,2 ...N with initial values equal those at the previous x-station, which are usually the best initial guess availables. For the higher iterates we set:

$$f_j^{(i+1)} = f_j^{(i)} + \delta f_j^{(i)} \tag{3.19}$$

$$u_j^{(i+1)} = u_j^{(i)} + \delta u_j^{(i)} \tag{3.20}$$

$$v_j^{(i+1)} = v_j^{(i)} + \delta v_j^{(i)} \tag{3.21}$$

$$\theta_j^{(i+1)} = \theta_j^{(i)} + \delta \theta_j^{(i)} \tag{3.22}$$

$$p_j^{(i+1)} = p_j^{(i)} + \delta p_j^{(i)} \tag{3.23}$$

Now we substitute the right hand sides of the above equations in place of f_j^n, u_j^n, v_j^n, θ_j^n and p_j^n in equations (3.10) to (3.17) and (3.18) and omitting the terms that are quadratic in δf_j^i, δu_j^i, δv_j^i, $\delta \theta_j^i$ and δp_j^i we get the equations (3.10) to (3.12) in the following form:

$$\delta f_j^{(i)} - \delta f_{j-1}^{(i)} - \frac{h_j}{2} (\delta u_j^{(i)} + \delta u_{j-1}^{(i)}) = (r_1)_j \tag{3.24}$$

$$\text{Where } (r_1)_j = f_{j-1}^{(i)} - f_j^{(i)} + h_j u_{j-1/2}^{(i)} \tag{3.25}$$

$$\delta u_j^{(i)} - \delta u_{j-1}^{(i)} - \frac{h_j}{2} (\delta v_j^{(i)} + \delta v_{j-1}^{(i)}) = (r_4)_j \tag{3.26}$$

$$(r_4)_j = u_{j-1}^{(i)} - u_j^{(i)} + h_j v_{j-1/2}^{(i)} \tag{3.27}$$

$$\delta \theta_j^{(i)} - \delta \theta_{j-1}^{(i)} - \frac{h_j}{2} (\delta p_j^{(i)} + \delta p_{j-1}^{(i)}) = (r_5)_j \tag{3.28}$$

$$\text{Where } (r_5)_j = \theta_{j-1}^{(i)} - \theta_j^{(i)} + h_j p_{j-1/2}^{(i)} \tag{3.29}$$

$$\begin{aligned}
& h_j^{-1} (v_j^{(i)} + \delta v_j^{(i)} - v_{j-1}^{(i)} - \delta v_{j-1}^{(i)}) + \{(p_1)_{j-1/2}^n + \alpha_n\}\{(fv)_{j-1/2}^{(i)} + \delta (fv)_{j-1/2}^{(i)}\} \\
& - \{(p_2)_{j-1/2}^n + \alpha_n\}\{(u^2)_{j-1/2}^{(i)} + \delta (u^2)_{j-1/2}^{(i)}\} + (p_3)_{j-1/2}^n \{(\theta)_{j-1/2}^{(i)} + \delta (\theta)_{j-1/2}^{(i)}\} \\
& - (p_4)^n \{u_{j-1/2}^{(i)} + \delta u_{j-1/2}^{(i)}\} + \alpha_n (f_{j-1/2}^{(i)} + \delta f_{j-1/2}^{(i)}) v_{j-1/2}^{n-1} - \alpha_n (v_{j-1/2}^{(i)} + \delta v_{j-1/2}^{(i)}) f_{j-1/2}^{n-1} \\
& = R_{j-1/2}^{n-1}
\end{aligned}$$

$$\Rightarrow h_j^{-1}(v_j^{(i)}+\delta v_j^{(i)}-v_{j-1}^{(i)}-\delta v_{j-1}^{(i)})+\{(p_1)_{j-1/2}^n+\alpha_n\}$$

$$\left\{(fv)_{j-1/2}^{(i)}+\frac{1}{2}(f_j^{(i)}\delta(v)_j^{(i)}+v_j^{(i)}\delta(f)_j^{(i)}+f_{j-1}^{(i)}\delta(v)_{j-1}^{(i)}+v_{j-1}^{(i)}\delta(f)_{j-1}^{(i)}\right\}$$

$$-\{(p_2)_{j-1/2}^n+\alpha_n\}\left\{(u^2)_{j-1/2}^{(i)}+\frac{1}{2}\{\delta(u^2)_j^{(i)}+\delta(u^2)_{j-1}^{(i)}\right\}$$

$$+(p_3)_{j-1/2}^n\left\{(\theta)_{j-1/2}^{(i)}+\frac{1}{2}(\delta(\theta)_j^{(i)}+\delta(\theta)_{j-1}^{(i)})\right\}-(p_4)^n\{u_{j-1/2}^{(i)}+\frac{1}{2}(\delta u_j^{(i)}+\delta u_{j-1}^{(i)})$$

$$+\alpha_n\left[\left\{v_{j-1/2}^{n-1}(f_{j-1/2}^{(i)}+\frac{1}{2}(\delta f_j^{(i)}+\delta f_{j-1}^{(i)})\right\}-\left\{f_{j-1/2}^{n-1}(v_{j-1/2}^{(i)}+\frac{1}{2}(\delta v_j^{(i)}+\delta_{j-1}^{\prime}))\right\}\right]=R_{j-1/2}^{n-1}$$

$$\Rightarrow(s_1)_j\,\delta v_j^{(i)}+(s_2)_j\,\delta v_{j-1}^{(i)}+(s_3)_j\,\delta f_j^{(i)}+(s_4)_j\,\delta f_{j-1}^{(i)}+(s_5)_j\,\delta u_j^{(i)}$$
$$+(s_6)_j\,\delta u_{j-1}^{(i)}+(s_7)_j\,\delta\theta_j^{(i)}+(s_8)_j\,\delta\theta_{j-1}^{(i)}+(s_9)_j\,\delta p_j^{\prime}+(s_{10})_j\,\delta p_{j-1}^{\prime} \tag{3.30}$$
$$=(r_2)_j$$

Where $$(s_1)_j=(h_j^{-1}+\frac{(p_1)_{j-1/2}^n+\alpha_n}{2}f_j^{(i)}-\frac{1}{2}\alpha_n f_{j-1/2}^{n-1}) \tag{3.31}$$

$$(s_2)_j=(-h_j^{-1}+\frac{(p_1)_{j-1/2}^n+\alpha_n}{2}f_{j-1}^{(i)}-\frac{1}{2}\alpha_n f_{j-1/2}^{n-1}) \tag{3.32}$$

$$(s_3)_j=(\frac{(p_1)_{j-1/2}^n+\alpha_n}{2}v_j^{(i)}+\frac{1}{2}\alpha_n v_{j-1/2}^{n-1} \tag{3.33}$$

$$(s_4)_j=(\frac{(p_1)_{j-1/2}^n+\alpha_n}{2}v_{j-1}^{(i)}+\frac{1}{2}\alpha_n v_{j-1/2}^{n-1}) \tag{3.34}$$

$$(s_5)_j=-\{(p_2)_{j-1/2}^n+\alpha_n\}\,u_j^{(i)}-\frac{(p_4)_{j-1/2}^n}{2} \tag{3.35}$$

$$(s_6)_j=-\{(p_2)_{j-1/2}^n+\alpha_n\}\,u_{j-1}^{(i)}-\frac{(p_4)_{j-1/2}^n}{2}) \tag{3.36}$$

$$(s_7)_j=(p_3)^n/2 \tag{3.37}$$

$$(s_8)_j=(p_3)^n/2 \tag{3.38}$$

$$(s_9)_j=0 \tag{3.39}$$

$$(s_{10})_j=0 \tag{3.40}$$

$$(r_2)_j=R_{j-1/2}^{n-1}-\{h_j^{-1}(v_j^{(i)}-v_{j-1}^{(i)})+((p_1)_{j-1/2}^n+\alpha_n)(fv)_{j-1/2}^{(i)}\}$$
$$+((p_2)_{j-1/2}^n+\alpha_n)(u^2)_{j-1/2}^{(i)}-\alpha_n\{f_{j-1/2}^{(i)}v_{j-1/2}^{n-1}-f_{j-1/2}^{n-1}v_{j-1/2}^{(i)}\} \tag{3.41}$$
$$-\{(p_3)_{j-1/2}^n\,\theta_{j-1/2}^{(i)}-(p_4)^n u_{j-1/2}^{(i)}\}$$

Here the coefficients $(s_9)_j$ and $(s_{10})_j$, which are zero in this case, are included here for the generality.

Similarly by using the equations (3.19) to (3.23) in the equation (3.17) we get the following form:

$$(t_1)_j\,\delta p_j^{(i)}+(t_2)_j\,\delta p_{j-1}^{(i)}+(t_3)_j\,\delta f_j^{(i)}+(t_4)_j\,\delta f_{j-1}^{(i)}+(t_5)_j\,\delta u_j^{(i)}$$
$$+(t_6)_j\,\delta u_{j-1}^{(i)}+(t_7)_j\,\delta\theta_j^{(i)}+(t_8)_j\,\delta\theta_{j-1}^{(i)}+(t_9)_j\,\delta v_j^{(i)}+(t_{10})_j\,\delta v_{j-1}^{(i)}=(r_3)_j \tag{3.42}$$

Where $(t_1)_j=\frac{1}{P_r}h_j^{-1}+\frac{(p_1)_{j-1/2}^{n-1/2}+\alpha_n}{2}f_j^{(i)}-\frac{1}{2}\alpha_n f_{j-1/2}^{n-1}$ (3.43)

$$(t_2)_j=-\frac{1}{P_r}h_j^{-1}+\frac{(p_1)_{j-1/2}^{n-1/2}+\alpha_n}{2}f_{j-1}^{(i)}-\frac{1}{2}\alpha_n f_{j-1/2}^{n-1} \tag{3.44}$$

$$(t_3)_j=\frac{(p_1)_{j-1/2}^{n-1/2}+\alpha_n}{2}p_j^{(i)}+\frac{1}{2}\alpha_n p_{j-1/2}^{n-1} \tag{3.45}$$

$$(t_4)_j=\frac{(p_1)_{j-1/2}^{n-1/2}+\alpha_n}{2}p_{j-1}^{(i)}+\frac{1}{2}\alpha_n p_{j-1/2}^{n-1} \tag{3.46}$$

$$(t_5)_j=-\frac{\alpha_n}{2}\theta_j^{(i)}+\frac{1}{2}\alpha_n\theta_{j-1/2}^{n-1} \tag{3.47}$$

$$(t_6)_j=-\frac{\alpha_n}{2}\theta_{j-1}^{(i)}+\frac{1}{2}\alpha_n\theta_{j-1/2}^{n-1} \tag{3.48}$$

$$(t_7)_j=-\frac{\alpha_n}{2}u_j^{(i)}-\frac{1}{2}\alpha_n u_{j-1/2}^{n-1} \tag{3.49}$$

$$(t_8)_j=-\frac{\alpha_n}{2}u_{j-1}^{(i)}-\frac{1}{2}\alpha_n u_{j-1/2}^{n-1} \tag{3.50}$$

$$(t_9)_j=0 \tag{3.51}$$

$$(t_{10})_j=0 \tag{3.52}$$

$$(r_3)_j=T_{j-1/2}^{n-1}-\frac{1}{P_r}h_j^{-1}(p_j^{(i)}-p_{j-1}^{(i)})-\frac{(p_1)_{j-1/2}^{n-1/2}+\alpha_n}{2}(fp)_{j-1/2}^{(i)}$$
$$+\alpha_n\{(u\theta)_{j-1/2}^{(i)}+u_{j-1/2}^{n-1}\theta_{j-1/2}^{(i)}-\theta_{j-1/2}^{n-1}u_{j-1/2}^{i}\}$$
$$+\alpha_n\{f_{j-1/2}^{n-1}p_{j-1/2}^{(i)}-p_{j-1/2}^{n-1}f_{j-1/2}^{(i)}\} \tag{3.53}$$

The boundary conditions (3.18) become

$$\delta f_0^n=0,\ \delta u_0^n=0,\ \delta\theta_0^n=0,\ \delta p_0^n=0$$
$$\delta u_J^n=0,\ \delta\theta_J^n=0 \tag{3.54}$$

Which just express the requirement for the boundary conditions to remain during the iteration process.

Now the system of linear equations (3.24) - (3.30), (3.41), (3.42) and (3.53) together with the boundary conditions (3.54) can be written in a block matrix from a coefficient matrix, which are solved by modified 'Keller Box' methods especially introduced by Keller (1978). Later, this method has been used most efficiently by Cebeci and Bradshow (1984) and recently by Hossain (1992) . Hossain et all. (1994), taking the initial iteration to be given by convergent solution at $\xi = \xi_{i-1}$. Results are shown in graphical form by using the numerical values obtained from the above technique.

The solutions of the above equations (2.16) and (2.17) together with the boundary conditions (2.18) enable us to calculate the skin friction τ and the rate of heat transfer Q at the surface in the boundary layer from the following relations:

$$\tau = \mu\left(\frac{\partial u}{\partial y}\right)_{y=0} = \frac{\rho \nu^2}{a^2}\xi Gr^{\frac{3}{4}} f''(\xi,0) \qquad (3.55)$$

$$Q = -k\left(\frac{\partial T}{\partial y}\right)_{y=0} = -k\frac{T_w - T_\infty}{a} Gr^{\frac{1}{4}} \theta'(\xi,0) \qquad (3.56)$$

3.2 Results and discussion

In the present problem we have investigated the solutions of the nonsimilar boundary layer equations governing the laminar free convective flow across a horizontal cylinder with adiabatic surface in the presence of a transverse magnetic field.

Numerical values of the local skin friction coefficient and the rate of heat transfer for selected values of magnetic parameter M (=0.0, 0.2, 0.5, 0.8, 1.0) for the fluids having Prandtl number Pr (=1.0, 0.73, 0.2) with 2φ as adiabatic surface of the cylinder are obtained. Here $\varphi=0^0$ indicates that the total surface of the cylinder is isothermal. The values of the Prandtl number Pr are taken to 0.73 that corresponds physically the air and 1.0 corresponding to electrolyte solutions such as salt water and 0.2 have been used theoretically.

Here we have determined the values of the functions $f(\xi,\eta)$ and $\theta(\xi,\eta)$ and their derivatives for different values of the Prandtl number Pr and the magnetic parameter M. So, we may calculate the numerical values of the rate of heat transfer $\theta'(\xi,0)$ and the velocity gradient $f''(\xi,0)$ at the surface that are important from the physical point of view.

The local skin friction coefficient $f''(\xi, 0)$ and the rate of heat transfer $\theta'(\xi, 0)$ respectively are shown graphically for different values of the Prandtl number Pr (= 1.0, 0.73, 0.2) in fig.3.2 and fig.3.3 respectively when the value of the magnetic parameter M =0.2. From fig.3.2 it is clear that fluid having large Prandtl number has a lower local skin friction coefficient $f''(\xi, 0)$. Again from the graphs in fig.3.3 we observe that the fluid having large Prandtl number has higher rate of heat transfer $\theta'(\xi, 0)$.

Figs.3.4 and fig.3.5 deal with the effect of the magnetic parameter M(= 0.0, 0.2, 0.5, 0.8, 1.0) for different values of the controlling parameter and Pr=0.73. on the local skin friction coefficient $f''(\xi, 0)$ and the surface rate of heat transfer $\theta'(\xi, 0)$ respectively. From fig3.4, it can be seen that if the magnetic parameter increases the local skin friction coefficient $f''(\xi, 0)$ of the fluid decreases. On the other hand from fig.3.5 we observe that the rate of heat transfer $\theta'(\xi,0)$ decreases for the fluid having large magnetic parameter M.

We have also shown velocity profile $f'(\xi, \eta)$ and temperature profile $\theta(\xi, \eta)$ graphically for different values of M, Pr and φ as stated above. Both figs.3.6 and 3.7 are pictured for the fluid having Prandtl numbers Pr(=1.0, 0.73, 0.2), where magnetic parameter M=0.2 and $\varphi=30^0$. We may decide from fig.3.6 and 3.7 that the velocity profile and temperature profile are getting lower for higher values Prandtl numbers.

From fig.3.8, it is observed that increase in the value of the magnetic parameter M(=0.0, 0.2, 0.5, 0.8, 1.0) when other controlling parameter Pr=0.73 leads to decrease the value of the velocity profile $f'(\xi, \eta)$, which indicates that the magnetic parameter M retards the fluid motion. Again fig.3.9 shows that the increase in the magnetic parameter M leads to increase of the temperature profile $\theta(\xi, \eta)$.

An interesting change is seen in velocity profile $f'(\xi, \eta)$ in fig.3.10. Here we see velocity profile $f'(\xi, \eta)$ decreases for the increased value of φ (= 0^0, 30^0, 60^0, 90^0, 135^0 and 180^0) and it is true for a certain value of η. After that value of η, velocity profile $f'(\xi, \eta)$ is seen to increase for increasing value of φ. From fig.3.11 we observe that, the temperature profile $\theta(\xi,\eta)$ increases as φ increases.

The values of the skin friction and the rate of heat transfer have been presented in Table 3.1 and Table 3.2. From Table 3.1, it can be seen that the values of the skin friction coefficient and the rate of heat transfer decrease for increasing values of magnetic parameter M.

Again from Table 3.2, it can be observed that increasing values of Prandtl number *Pr* leads to decrease the values of the skin friction coefficients and to increase the rate of heat transfer.

Numerical values of velocity profile and temperature profile are also presented in Table 3.3, Table 3.4 and Table 3.5. It can be seen from Table 3.3 that increasing values of Magnetic parameter *M* retards velocity profile and accelerate temperature profile. Same results obtained for increasing values of φ. Again from Table 3.5 it can be observed that both the velocity profile and temperature profile decrease for the increasing values of Prandtl number *Pr*.

3.3 Conclusion

Magnetohydrodynamic (MHD) laminar free convective flow across a horizontal cylinder with adiabatic surface has been investigated introducing suitable transformations. Numerical solutions of the equations governing the flow are obtained by using the implicit finite difference method together with the Keller Box scheme. From the present investigation, the following conclusions may be drawn:

1. The skin friction coefficient, the rate of heat transfer and the velocity profile decrease for increasing value of the magnetic parameter.
2. It is also seen that increased value of the magnetic parameter *M* leads to increase the temperature profile.

3 It has been observed that the skin friction coefficient, the temperature profile and the velocity profile decrease for the fluid having large Prandtl number Pr but the rate of heat transfer increase for the fluid having large Prandtl number Pr.

4 Velocity profile decreases if we increase the angle of the adiabatic surface of the cylinder and it is true for a certain value of η. After that value of η, velocity profile may be increased for increasing angle of adiabatic surface of the cylinder. The temperature profile increases as the angle of the adiabatic surface increases.

TABLE 3.1

Numerical values of the local skin friction and the rate of heat transfer for different values of magnetic parameter M with Pr=0.73.

Values of ξ	M=0.2		M=0.5		M=1.0	
	$f''(\xi,0)$	$\theta'(\xi,0)$	$f''(\xi,0)$	$\theta'(\xi,0)$	$f''(\xi,0)$	$\theta'(\xi,0)$
10.0000	0.13992	0.36124	0.12923	0.34180	0.11542	0.31452
30.0000	0.40781	0.35692	0 37601	0 33722	0.33505	0.30963
50.0000	0.64116	0.34835	0.58899	0.32810	0.52233	0 29988
70.0000	0.81966	0.33538	0.74834	0.31429	0.65848	0 28508
90.0000	0.92638	0.31781	0.83766	0.29547	0.72822	0 26487
110.000	0.94856	0.29517	0.84475	0.27108	0 72072	0.23857
130.000	0 87759	0.26658	0.76167	0.23994	0.62994	0.20484
150.000	0.70737	0.22996	0.58301	0.19935	0.45362	0.16054
160.000	0.58213	0.20708	0.45504	0.17332	0 33257	0.13188
170.000	0.42537	0.17883	0.29589	0.13987	0.18676	0.09441

TABLE 3.2

Numerical values of the local skin friction and the rate of heat transfer for various values of Prandtl number Pr with M=0 2.

Values of ξ	Pr=0.2		Pr=0.73		Pr=1 0	
	$f''(\xi,0)$	$\theta'(\xi,0)$	$f''(\xi,0)$	$\theta'(\xi,0)$	$f''(\xi,0)$	$\theta'(\xi,0)$
10 0000	0.16124	0.21713	0.13992	0.36124	0.13407	0.40512
30.0000	0.47029	0.21466	0.40781	0.35692	0 39070	0.40022
50.0000	0.74045	0.20975	0.64116	0 34835	0.61407	0.39049
70 0000	0.94890	0.20236	0.81966	0.33538	0.78460	0 37577
90.0000	1.07662	0.19239	0.92638	0.31781	0.88598	0.35579
110.000	1.10936	0.17964	0 94856	0 29517	0.81120	0.30399
130.000	1.03781	0.16375	0.87759	0 26658	0.73053	0.26868
150 000	0.85605	0.14388	0.70737	0.22996	0.55750	0 22246
160.000	0.72149	0 13184	0 58213	0.20708	0.43349	0.19266
170.000	0.55410	0.11758	0 42537	0.17883	0.27898	0.15405

TABLE 3.3

Numerical values of the velocity profile and the temperature profile for different values of magnetic parameter M with Pr=0.73 and $\varphi=30^0$.

Values of η	M=0.2		M=0.5		M=1.0	
	$f'(\xi,\eta)$	$\theta(\xi,\eta)$	$f'(\xi,\eta)$	$\theta(\xi,\eta)$	$F'(\xi,\eta)$	$\theta(\xi,\eta)$
0.16068	0.11312	0.94270	0.10349	0.94586	0 09108	0.95029
0.49865	0.28035	0.82288	0.25306	0 83262	0.21806	0.84628
0.88811	0.37822	0 68825	0.34397	0.70512	0.28620	0.72885
1.43822	0.40129	0.51257	0.35600	0 53746	0.29821	0.57292
2.48059	0.28592	0 25870	0.25615	0.28835	0.21744	0 33279
3.47923	0.15862	0.11946	0.13353	0.14337	0.13061	0.18197
4.83720	0.05596	0.03661	0.05602	0.04951	0.05557	0.07306
5.03870	0.04695	0.03029	0.04775	0.04182	0.06329	0 04836
5.92876	0.01992	0 01216	0.02220	0.01885	0.02518	0 03252
6.97092	0.00547	0.00267	0.00747	0.00588	0.01047	0.01335

TABLE 3.4

Numerical values of the velocity profile and the temperature profile for different values of adiabatic surface 2φ with Pr=0.73 and M=0.2.

Values of η	φ=30⁰		φ=60⁰		φ=90⁰	
	$f'(\xi,\eta)$	$\theta(\xi,\eta)$	$f'(\xi,\eta)$	$\theta(\xi,\eta)$	$f'(\xi,\eta)$	$\theta(\xi,\eta)$
0.16068	0.11312	0.94270	0.10289	0.94502	0.08675	0.94897
0.49865	0.28035	0.82288	0.25785	0.83003	0.22158	0.84217
0.88811	0.37822	0.68825	0.35279	0.70048	0.31058	0.72137
1.43822	0.40129	0.51257	0.38226	0.53009	0.34888	0.56048
2.48059	0.28592	0.25870	0.28378	0.27802	0.27786	0.31322
3.47923	0.15862	0.11946	0.16386	0.13389	0.17169	0.16180
4.83720	0.05596	0.03661	0.06115	0.04360	0.07040	0.05831
5.03870	0.04695	0.03029	0.05175	0.03642	0.06044	0.04948
5.92876	0.01992	0.01216	0.02286	0.01532	0.02850	0.02245
6.97092	0.00547	0.00267	0.00660	0.00377	0.00891	0.00641

TABLE 3.5

Numerical values of the velocity profile and the temperature profile for various values of Prandtl number Pr while M=0.2 and $\varphi=30^0$.

Values of η	Pr=0.2		Pr=0.73		Pr=1.0	
	$f'(\xi,\eta)$	$\theta(\xi,\eta)$	$f'(\xi,\eta)$	$\theta(\xi,\eta)$	$F'(\xi,\eta)$	$\theta(\xi,\eta)$
0.16068	0.13220	0.96553	0.11312	0.94270	0.10789	0.93575
0.49865	0.33785	0.89330	0.28035	0.82288	0.26470	0.80155
0.88811	0.47478	0.81087	0.37822	0.68825	0.35236	0.65178
1.43822	0.54083	0.69766	0.40129	0.51257	0.36554	0.46059
2.48059	0.46615	0.50311	0.28592	0.25870	0.24661	0.20300
3.47923	0.33567	0.35205	0.15862	0.11946	0.12792	0.07975
4.83720	0.19225	0.20577	0.05596	0.03661	0.03986	0.01849
5.03870	0.17585	0.18924	0.04695	0.03029	0.03264	0.01448
5.92876	0.11667	0.12931	0.01992	0.01216	0.01198	0.00388
6.97092	0.06961	0.08097	0.00547	0.00267	0.00213	-0.00086

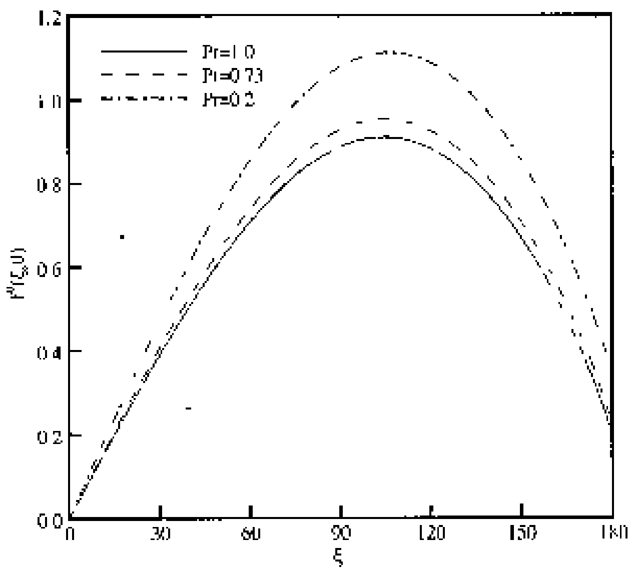

Fig.3.2: Skin friction for different values of Prandtl number Pr, where M=0.2.

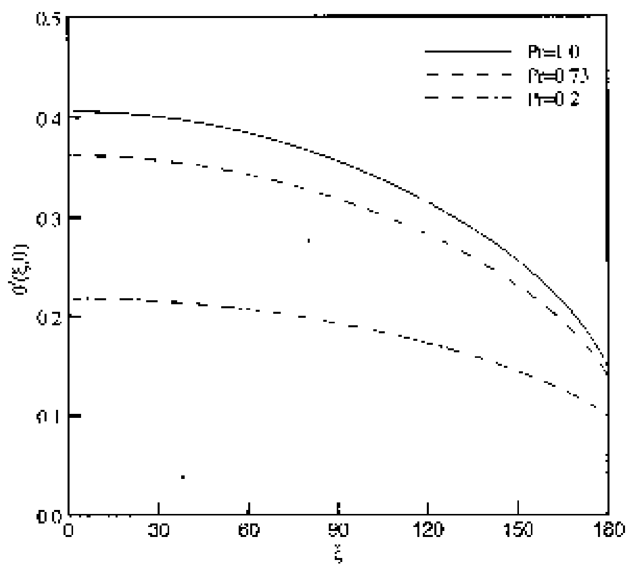

Fig.3.3: Rate of heat transfer for different values of Prandtl number, where M=0.2.

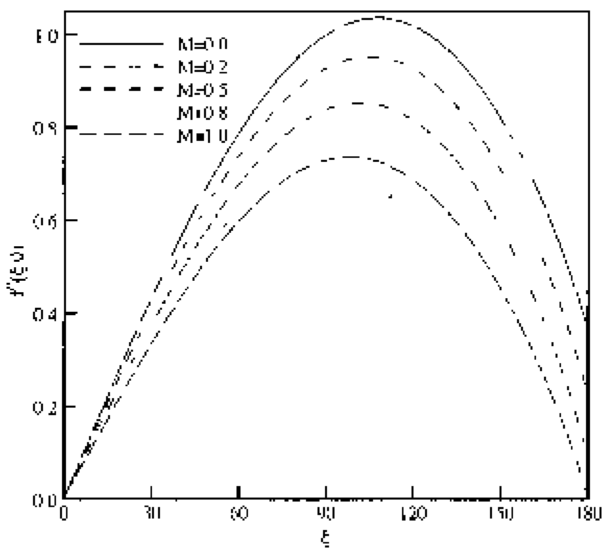

Fig.3.4: Skin frictions for different values of *M* where *Pr*=0.73

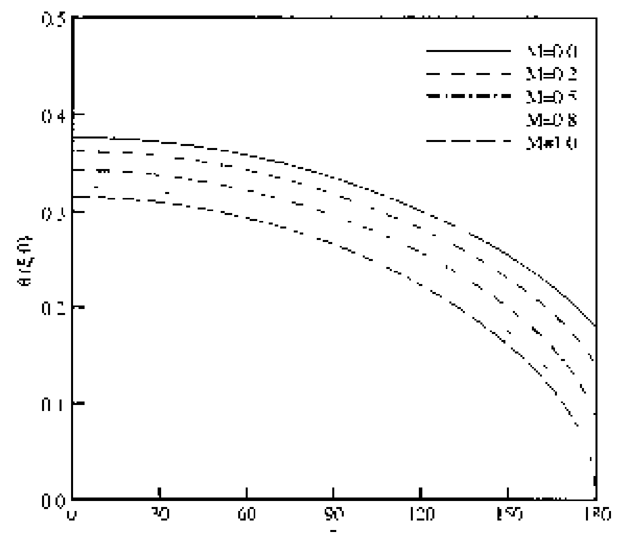

Fig.3.5: Rate of heat transfer for different values of *M* where *Pr*=0.73.

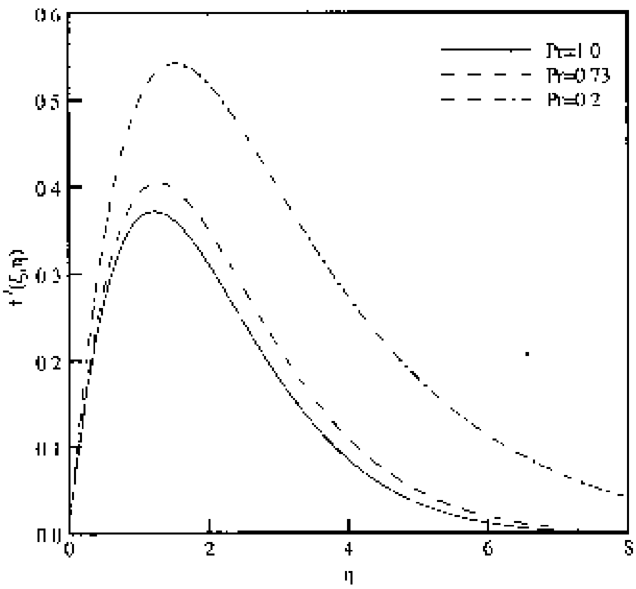

Fig.3.6: Velocity profile for different values of Prandtl numbers. where $\varphi=30^0$ and M=0.2.

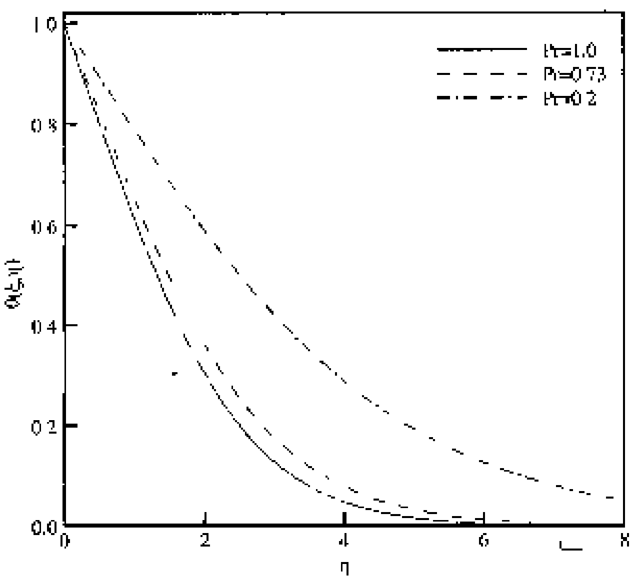

Fig.3.7: Temperature profile for different values of Prandtl numbers. where $\varphi=30^0$ and M=0.2

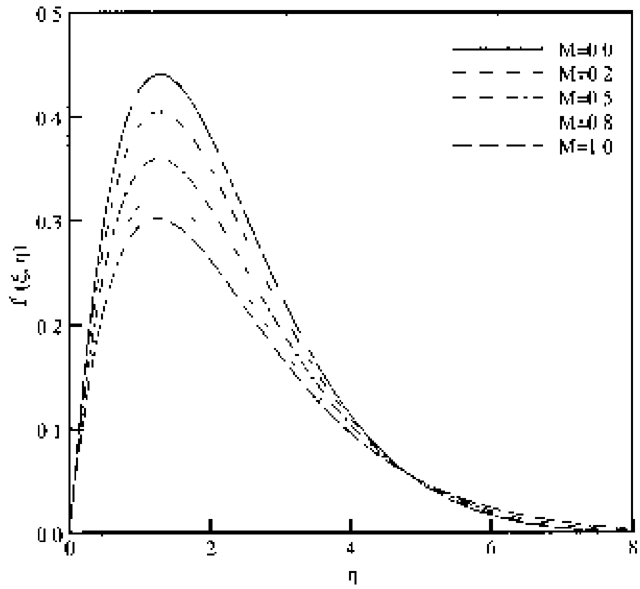

Fig.3.8: Velocity profile for different values of M, when Pr=0.73 and φ=30^0

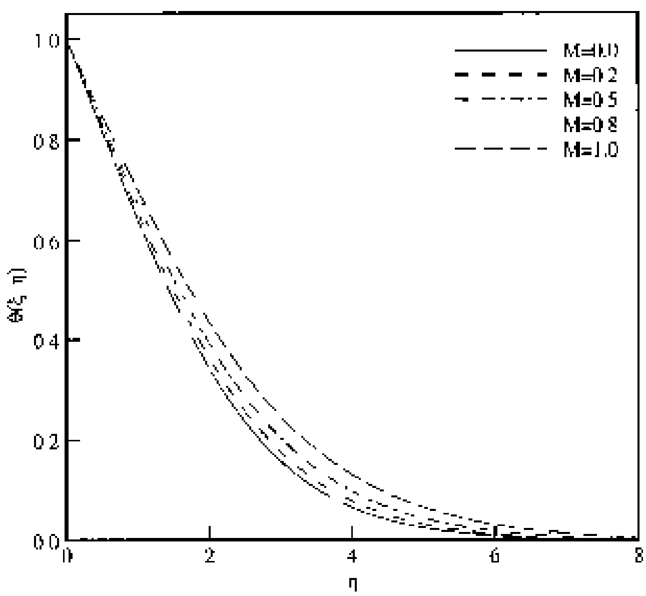

Fig.3.9: Temperature profile for different values of M, when Pr=0.73 and φ=30^0

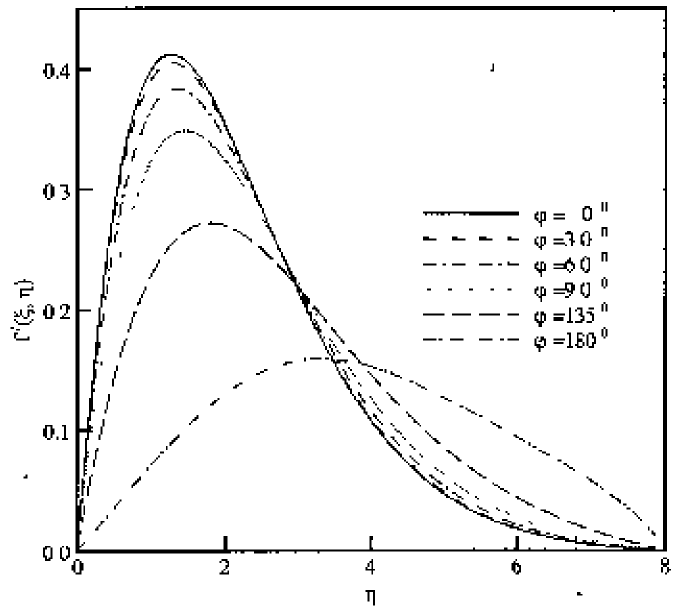

Fig.3.10: Velocity profile for different adiabatic surfaces when M=0.2 and Pr=0.73.

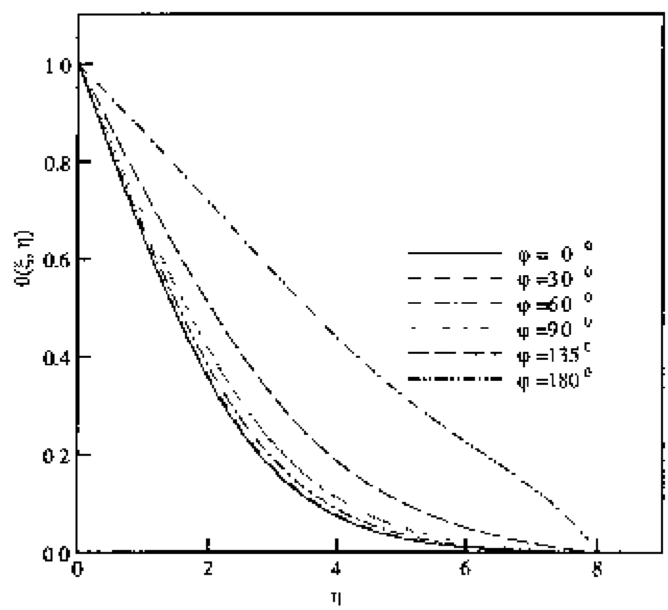

Fig.3.11: Temperature profile for different adiabatic surfaces when M=0.2 and Pr=0.73.

Chapter 4

Conduction, convection and Joule heating effects on Magnetohydrodynamic flow from a vertical flat plate

4.1 Introduction

In the traditional area of the convective heat transfer between a solid wall and a fluid flow the wall conduction resistance is usually neglected, i.e. the wall is assumed to be very thin. In this case it is usual to prescribe either the wall temperature or the wall heat flux and a considerable amount of research has done in order to understand the heat transfer characteristics over a wide range of flow configurations and fluid properties. However in many real engineering systems the wall conduction resistance cannot be neglected since conduction in the wall is able to significantly affect the fluid flow and the heat transfer characteristics of the fluid in the vicinity of the wall. In order to take account of physical reality, there has been a tendency to move away from considering mathematical problems in which the bounding wall is considered to be infinitely thin. Thus the conduction in solid wall and the convection in the fluid should be determined simultaneously. This type of convective heat transfer is referred to a conjugate heat transfer process and it arises due to the finite thickness of the wall. The phenomenon depends on several parameters. Therefore in many cases this strong dependence does exist.

Heat transfer from a heated vertical plate provides probably one of the most basic scenarios for natural convection problems. Variations of the problem occur frequently in the literature. Free convective steady hydromagnetic flow about a heated vertical flat plate has been considered by Gupta [1], Poots [2], Osterle and Yound [3], Sparrow and Cess [4], Lykoudis [6], Cramer [8] and Riley [9]. The similarity solutions were studied by Gupta [1] and Lykoudis [6] considering that the magnetic field differs inversely as the fourth root of the height above the bottom edge of the plate. Afterwards Nanda and Mohanty [11] made use of the similar technique to solve the hydromagnetic free convection of high and low Prandtl numbers because of realistic applications, as for liquid metals, the Prandtl number is low. Riley[9] considered a uniform magnetic field and integrated the boundary layer equations over a single boundary layer thickness. Effects of transversely applied magnetic field on free convection of an electrically conducting fluid past a semi-infinite plate were studied by Wilks [13]. Miyamoto et al. [18] have given an analysis of the relative importance of the parameters of the problem in particular with reference to coaxial heat conduction. Hossain and Ahmed [23] studied the MHD forced and free convection boundary layer flow near the leading edge. They also investigated the combined forced and free convection of an electrically conducting fluid past a vertical flat plate at which the surface heat flux was uniform and magnetic field was applied parallel to the direction normal to plate. The natural convection boundary layer flow of an electrically conducting fluid up a hot vertical wall in the presence of a strong magnetic field has been studied by several authors because of its application in nuclear engineering in connection with the cooling of reactors

In all the above studies, the effects of the viscous and joule heating were neglected because they are of the same order as well as negligibly small. But Gebhart [5] has shown that the viscous dissipation effect plays an important role in natural convection in various devices which are subjected to large deceleration or which operate at high rotative speeds and also in strong gravitational field processes on large scales (on the planets) and in geological processes. With this understanding Takhar and Soundalgekar [16] have studied the effects of viscous and Joule heating on the problem posed by Sparrow and Cess [4], using the series expansion method of Gebhart[5]. In the present study the Joule heating effects on Magnetohydrodynamic boundary layer flow and heat transfer resulting from the coupling of natural convection along and conduction inside a vertical flat plate will be investigated.

The transformed non similar boundary layer equations governing the flow together with the boundary conditions based on conduction and convection were solved numerically using the Keller box (implicit finite difference) along with Newton's linearization approximation method in the entire region starting from the lower part of the plate to the down stream for some values of the magnetic parameter M, the joule heating parameter J and the Prandtl number Pr. The effect of the parameters M, J and Pr on the velocity and temperature fields as well as on the skin friction coefficient and surface temperature distribution have been studied

In the following sections detailed derivations of the governing equations for the flow and heat transfer and the method of solutions along with the results and discussions are presented. All the investigations for the fluid with low Prandtl number appropriates for the liquid metals are carried out.

4.2 Governing equations of the flow:

We consider the steady two dimensional laminar free convection boundary layer flow of a viscous incompressible and electrically conducting fluid along a side of a vertical flat plate of thickness 'b' insulated on the edges and with temperature T_b maintained on the other side in the presence of a uniformly distributed transverse magnetic field. The flow configuration and the coordinates system are shown in Fig 4.1 The equations governing the flow are as follows:

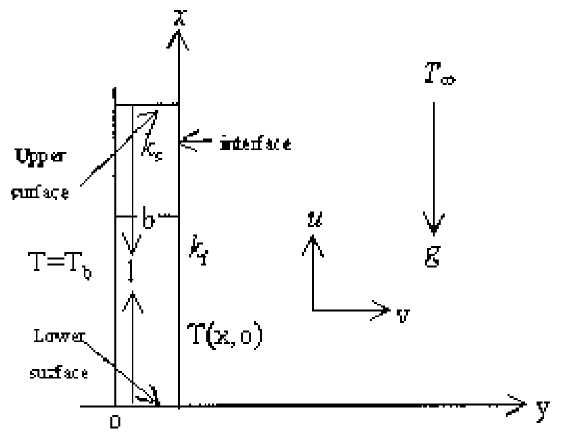

Fig.4 : Physical configuration and coordinates system.

$$\frac{\partial \bar{u}}{\partial \bar{x}}+\frac{\partial \bar{v}}{\partial \bar{y}}=0 \tag{4.1}$$

$$\bar{u}\frac{\partial \bar{u}}{\partial \bar{x}}+\bar{v}\frac{\partial \bar{u}}{\partial \bar{y}}=\nu\frac{\partial^2 \bar{u}}{\partial \bar{y}^2}+g\beta(T-T_\infty)-\frac{\sigma H_0^{\ 2}\bar{u}}{\rho} \tag{4.2}$$

$$\bar{u}\frac{\partial T}{\partial \bar{x}}+\bar{v}\frac{\partial T}{\partial \bar{y}}=\frac{\kappa_f}{\rho c_p}\frac{\partial^2 T}{\partial \bar{y}^2}+\frac{\sigma_0 H_0^2}{\rho c_p}\bar{u}^2 \tag{4.3}$$

The appropriate boundary conditions to be satisfied by the above equations are

$$\bar{u}=0, \bar{v}=0 \ \ at \ \ \bar{y}=0 \tag{4.4}$$

$$\bar{u}\to 0, T\to T_\infty \ as \ \bar{y}\to\infty$$

The coupling conditions require that the temperature and the heat flux be continuous at the solid-fluid interface and at the interface following Miyamoto [1] one can assume that

$$\frac{k_s}{k_f}\frac{\partial T_{so}}{\partial \bar{y}} = \left(\frac{\partial T}{\partial \bar{y}}\right)_{\bar{y}=0} \tag{4.5}$$

where k_s and k_f are the thermal conductivity of the solid and the fluid respectively. The temperature T_{so} in the solid as given by A. Pozzi and M. Lupo [21] is

$$T_s = T(\bar{x},0) - \{T_b - T(\bar{x},0)\}\bar{y}/b \tag{4.6}$$

where $T(\bar{x},0)$ is the unknown temperature at the interface to be determined from the solutions of the equations.

We observe that the equations (4.1) - (4.3) together with the boundary conditions (4.4) - (4.5) are non-linear partial differential equations. In the following sections the solution methods of these equations are discussed details.

4.3 Transformation of the governing equations

Equations (4.1) – (4.3) may now be nondimensionalized by using the following dimensionless dependent and independent variables:

$$x = \frac{\bar{x}}{L}, y = \frac{\bar{y}}{L}d^{\frac{1}{4}}, \bar{u} = \frac{\nu}{L}d^{\frac{1}{2}}u, \bar{v} = \frac{\nu}{L}d^{\frac{1}{4}}v, \frac{T - T_\infty}{T_b - T_\infty} = \theta$$
$$L = \frac{\nu^{2/3}}{g^{1/3}}, d = \beta(T_b - T_\infty) \tag{4.7}$$

As the problem of natural convection, its parabolic character, has no characteristic length, L has been defined in terms of ν and g, which are the intrinsic properties of the system. The reference length along the 'y' direction has been modified by a factor $d^{-1/4}$ in order to eliminate this quantity from the dimensionless equations and the boundary conditions.

The magneto hydrodynamic field in the fluid is governed by the boundary layer equations, which in the non-dimensional form obtained by introducing the dimensionless variables described in (4.7) may be written as

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0 \tag{4.8}$$

$$u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}+Mu=\frac{\partial^2 u}{\partial y^2}+\theta \tag{4.9}$$

$$u\frac{\partial \theta}{\partial x}+v\frac{\partial \theta}{\partial y}=\frac{1}{\Pr}\frac{\partial^2 \theta}{\partial y^2}+Ju^2 \tag{4.10}$$

Where $M=\frac{\sigma H_0{}^2 L^2}{\mu d^{1/2}}$, the dimensionless magnetic parameter, $P_r=\frac{\mu C_p}{\kappa}$, the Prandtl number and $J=\frac{\sigma H_0^2 \nu d^{\frac{1}{2}}}{\rho c_p (T_b-T_\infty)}$, the dimensionless joule heating parameter.

The corresponding boundary conditions (4.4) - (4.6) take the following form:

$$u=v=0\,,\ \theta - 1 = p\frac{\partial \theta}{\partial y}\ \text{at}\ y=0 \tag{4.11a}$$

$$u\rightarrow 0,\ v\rightarrow 0\quad \text{as}\ y\rightarrow\infty \tag{4.11b}$$

where p is the conjugate conduction parameter given by $p=\left(\frac{\kappa_f}{\kappa_s}\right)\left(\frac{b}{L}\right)d^{1/4}$

Here the coupling parameter 'p' governs the described problem The order of magnitude of 'p' depends actually on $\frac{b}{L}$ and $\frac{\kappa_f}{\kappa_s}$, $d^{1/4}$ being the order of unity The term $\frac{b}{L}$ attains values much greater than one because of L being small. In case of air, $\frac{\kappa_f}{\kappa_s}$ becomes very small when the vertical plate is highly conductive i.e $\kappa_s >> 1$ and for materials, $o\left(\frac{\kappa_f}{\kappa_s}\right) = 0.1$ such as glass. Therefore in different cases 'p' is different but not always a small number. In the present investigation we have considered $p = 1$ which is accepted for $\frac{b}{L}$ of $o\left(\frac{\kappa_f}{\kappa_s}\right)$.

To solve the equations (4.8) – (4.10) subject to the boundary conditions (4.11), the following transformations were introduced for the flow region starting from up stream to down stream.

$$\psi = x^{4/5}(1+x)^{-1/20} f(\eta,x),\ \eta = yx^{-1/5}(1+x)^{-1/20},\ \theta = x^{1/5}(1+x)^{-1/5} h(\eta,x) \qquad (4.12)$$

Here η is the dimensionless similarity variable and ψ is the stream function which satisfies the equation of continuity where $u = \frac{\partial\psi}{\partial y}$, $v = -\frac{\partial\psi}{\partial x}$ and $h(\eta,x)$ is the dimensionless temperature.

Substituting (4.12) into equations (4.9) and (4.10) we get, after some algebra the following transformed non-dimensional equations.

$$f''' + \frac{16+15x}{20(1+x)} ff'' - \frac{6+5x}{10(1+x)} f'^2 - Mx^{2/5}(1+x)^{1/10} f' + h = x\left(f'\frac{\partial f'}{\partial x} - f''\frac{\partial f}{\partial x}\right) \qquad (4.13)$$

$$\frac{1}{P_r} h'' + \frac{16+15x}{20(1+x)} fh' - \frac{1}{5(1+x)} f'h + Jx^{7/5}(1+x)^{1/10} f'^2 = x\left(f'\frac{\partial h}{\partial x} - h'\frac{\partial f}{\partial x}\right) \qquad (4.14)$$

In the above equations the primes denote differentiation with respect to η.

The boundary conditions (3.11) then takes the following form

$$\begin{aligned} &f(x,0) = f'(x,0) = 0,\ h'(x,0) = -(1+x)^{1/4} + x^{1/5}(1+x)^{1/20} h(x,0) \\ &f'(x,\infty) = 0,\ h'(x,\infty) = 0 \end{aligned} \qquad (4.15)$$

4.4 Joule heating parameter

In electronics, and in physics more broadly, **Joule heating** or **ohmic heating** refers to the increase in temperature of a conductor as a result of resistance to an electrical current flowing through it

At an atomic level, Joule heating is the result of moving electrons colliding with atoms in a conductor, where upon momentum is transferred to the atom, increasing its kinetic energy . Joule heating is named for James Prescott Joule, the first to articulate what is now Joule's law, relating the amount of heat released from an electrical resistor to its resistance and the charge passed through it. In our problem we got a dimensionless parameter $J = \dfrac{\sigma H_0^2 \nu d^{\frac{1}{2}}}{\rho c_p (T_h - T_\infty)}$, which is Joule heating parameter.

4.4 Method of Solution:

To get the solutions of the parabolic differential equations (4.13) and (4.14) along with the boundary condition (4.15), we shall employ implicit finite difference method together with Keller- box elimination technique which is well documented and widely used by Keller and Cebeci (1971) and recently by Hossain (1992). Since a good description of this method has been discussed in details in Chapter-3. further discussion is disregarded here. The numerical results obtained are presented in the following section.

4.5 Results and discussion

In this chapter we have investigated the problem of the steady two dimensional laminar free convection boundary layer flow of a viscous incompressible and electrically conducting fluid along a side of a vertical flat plate of thickness 'b' insulated on the edges with temperature T_b maintained on the other side in the presence of a uniformly distributed transverse magnetic field .

Skin frictions and surface temperature distributions are obtained for the fluid having Prandtl number Pr = 0.73. 2.55, 4.40 the joule heating parameter J = 0.01, 0.05. 0.1. 0.2 and the magnetic parameter M =0.01, 0.20, 0.50. 0.80.

If we know the values of the functions $f(\eta, x)$, $h(\eta, x)$ and their derivatives for different values of the Prandtl number Pr and the magnetic parameter M, we may calculate the numerical values of the surface temperature $\theta(0, x)$ and the velocity gradient $f''(0, x)$ at the surface that are important from the physical point of view.

Numerical values of the velocity gradient $f''(0, x)$ and the surface temperature $\theta(0, x)$ are depicted graphically in Fig.4.1 to 4.6 respectively against the axial distance x in the interval [0,20]. In Fig.4.1 and Fig.4.2, the shear stress coefficient $f''(0, x)$ and the surface temperature $\theta(0,x)$ are shown graphically for different values of the Prandtl number Pr (=0.73, 2.55, 4.40) when value of the magnetic parameter M is 0.8 and the joule heating parameter J = 0.2. In Fig.4.3 and Fig.4.4, the shear stress coefficient $f''(0, x)$ and the surface temperature $\theta(0,x)$ are shown graphically for different values of the magnetic parameter M(=0.01, 0.20, 0.50, 0.80) when value of the Pr is 4.40 and the joule heating parameter , J=0.02. Similarly in Fig.4.5 and Fig.4.6, the shear stress coefficient $f''(0, x)$ and the surface temperature $\theta(0,x)$ are shown graphically for different values of the joule heating parameter J(=0.01, 0.05, 0.1, 0.2) when value of the Pr is 4.40 and the magnetic parameter , M=0.2.

From Fig. 4.1, it is shown that the shear stress coefficient $f''(0, x)$ decreases monotonically with the increase of the Prandtl number Pr (=0.73, 2.55, 4.40) and from the Fig.4.2, the same result is observed on the surface temperature distribution due to increase of the value of the Prandtl number when the value of the magnetic parameter M is 0.8 and the value of the joule heating parameter $J = 0.2$.

From Fig. 4.3, it is shown that the skin friction coefficient $f''(0,x)$ decreases with the increase of the magnetic parameter M (=0.01, 0.2, 0.5, 0.8) and from the Fig.4.4, opposite result is observed on the surface temperature distribution due to increase of the value of the magnetic parameter M when the value of the Prandtl number is 4.40 and the value of the joule heating parameter $J = 0.02$.

From Fig. 4.5, it is observed that increase in the value of the joule heating parameter J leads to increase the value of the skin friction coefficient $f''(0,x)$ which is usually expected. Again Fig.4.6 shows that the increase of the joule heating parameter J leads to increase the surface temperature distribution $\theta(0,x)$.

Numerical values of the velocity profile $f'(\eta,x)$ and the temperature profile $\theta(\eta, x)$ are depicted graphically in Fig.4.7 to 4.12 respectively against the axial distance η. Fig. 4.7 depicts the velocity profile for different values of the Prandtl number Pr (= 0.05, 0.73, 1.0) while the magnetic parameter M=0.8, and the joule heating parameter J=0.2. Corresponding distribution of the temperature profile $\theta(\eta, x)$ in the fluids is shown in Fig. 4.8. From Fig. 4.7, it is seen that if the Prandtl number increases, the velocity profile of the fluid decreases. We also observe from Fig.4.8 that the temperature profile decreases within the boundary layer due to increase of the Prandtl number Pr.

In Fig 4.9 and Fig.4.10, the velocity profile $f'(\eta, x)$ and the temperature profile $\theta(\eta, x)$ are shown graphically for different values of the magnetic parameter M (=0.2, 0.5, 0.8, 1.0) when value of the Pr is 0.73 and the joule heating parameter, J=0.5. From Fig. 4.9, it is seen that if the magnetic parameter increases, the velocity profile of the fluid decreases. We also observe from Fig.4.10 that the temperature profile increases within the boundary layer due to increase of the magnetic parameter M.

Fig 4.11 and Fig.4.12 deal with the effect of the joule heating parameter J (= 0.2, 0.5, 0.8, 1.0) for Prandtl number Pr = 0.73 and for the magnetic parameter M = 0.8 on the velocity profile $f'(\eta, x)$ and the temperature profile $\theta(\eta, x)$. From Fig. 4.11, it is revealed that the velocity profile $f'(\eta, x)$ increases very small with the increase of the joule heating parameter J which indicates that joule heating increases the fluid motion. Small increment is shown from Fig 4.12 on the temperature profile $\theta(\eta, x)$ for increasing values of J.

Numerical values of the skin friction and the surface temperature distribution have been presented in Table 4.1 for different values of joule heating parameter J. It is observed that both the skin friction and the surface temperature distribution increase for the increasing values of joule heating parameter. Again the values of the velocity profile and the temperature profile for different values of joule heating parameter J are also presented in Table 4.2. It is seen that both the velocity profile and the temperature profile increase for the increasing values of Joule heating parameter.

4.6 Conclusion

We have studied the effects of joule heating parameter J and magnetic parameter M for different Prandtl number Pr on the magneto-hydrodynamic (MHD) natural convection boundary layer flow from a vertical flat plate by introducing a new class of transformations. The transformed non-similar boundary layer equations governing the flow together with the boundary conditions based on conduction and convection were solved numerically using implicit finite difference method together with Keller box scheme. The coupled effect of natural convection and conduction required that the temperature and the heat flux be continuous at the interface. From the present investigation, the following conclusions may be drawn:

1. The skin friction coefficient and the velocity distribution increase for increasing value of the joule heating parameter J.

2. Increased value of the joule heating parameter J leads to increase the surface temperature distribution as well as the temperature distribution.

3. It has been observed that the skin friction coefficient, the surface temperature distribution, the temperature distribution over the whole boundary layer and the velocity distribution decrease with the increase of the Prandtl number Pr.

4. Finally it follows that both the skin friction coefficient and the velocity distribution decrease with the increase of the magnetic parameter M. The surface temperature distribution and the temperature distribution over the whole boundary layer increase with the increase of the magnetic parameter M.

TABLE 4.1: Numerical values of the local skin friction and the surface temperature distribution for different values of Joule heating parameter J while Pr =0.73 and M=0.2.

Values of x	J=0.01		J=0.10		J=0.20	
	$f''(0,x)$	$\theta(0,x)$	$f''(0,x)$	$\theta(0,x)$	$f''(0,x)$	$\theta(0,x)$
1.0265	0.7020	0.7525	0.7091	0.7573	0.7171	0.7628
2.0369	0.8404	0.7859	0.8617	0.7977	0.8864	0.8116
3.0049	0.9266	0.8044	0.9662	0.8239	1.0132	0.8480
4.0219	0.9949	0.8182	1.0577	0.8467	1.1350	0.8838
5.1425	1.0549	0.8297	1.1480	0.8693	1.2668	0.9239
6.0502	1.0958	0.8374	1.2168	0.8869	1.3760	0.9583
7.1132	1.1377	0.8450	1.2951	0.9071	1.5098	1.0017
8.0285	1.1698	0.8508	1.3618	0.9246	1.6321	1.0425
9.0596	1.2024	0.8566	1.4374	0.9448	1.7791	1.0930
10.0179	1.2302	0.8615	1.5087	0.9642	1.9260	1.1449

TABLE 4.2: Numerical values of the velocity profile and the temperature profile for different values of Joule heating parameter J while Pr =0.73 and M=0.2.

Values of η	J=0.2		J=0.5		J=1.0	
	$f'(\eta,x)$	$\theta(\eta,x)$	$f'(\eta,x)$	$\theta(\eta,x)$	$f'(\eta,x)$	$\theta(\eta,x)$
0.5438	0.3113	0.7199	0.3234	0.7517	0.3445	0.8102
1.0554	0.4338	0.5815	0.4509	0.6203	0.4804	0.6916
1.5831	0.4480	0.4495	0.4650	0.4888	0.4936	0.5601
2.0827	0.4025	0.3400	0.4162	0.3743	0.4384	0.4349
2.5896	0.3314	0.2480	0.3405	0.2743	0.3544	0.3194
3.0689	0.2603	0.1791	0.2654	0.1978	0.2721	0.2285
3.4792	0.2044	0.1331	0.2066	0.1461	0.2087	0.1666
4.1056	0.1340	0.0820	0.1334	0.0887	0.1311	0.0984
5.0387	0.0635	0.0368	0.0615	0.0387	0.0577	0.0407
6.1741	0.0194	0.0107	0.0181	0.0108	0.0159	0.0106

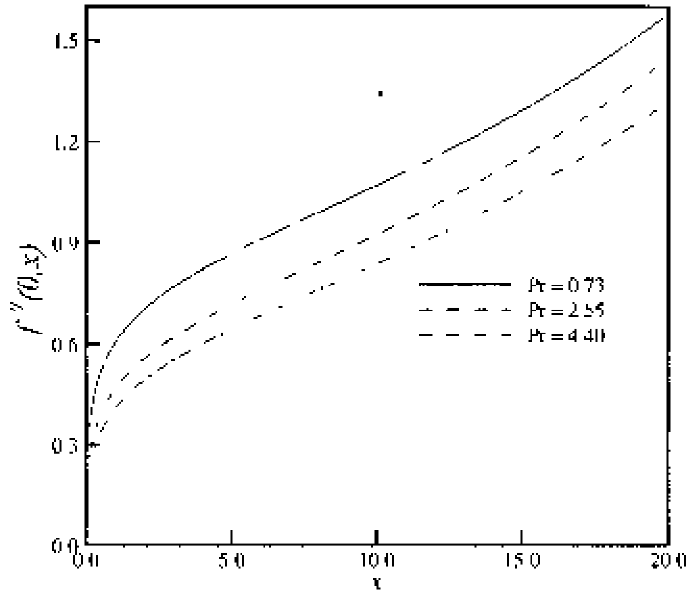

Fig.4.1: Skin frictions for different values of *Pr* when M = 0.8, J = 0.2

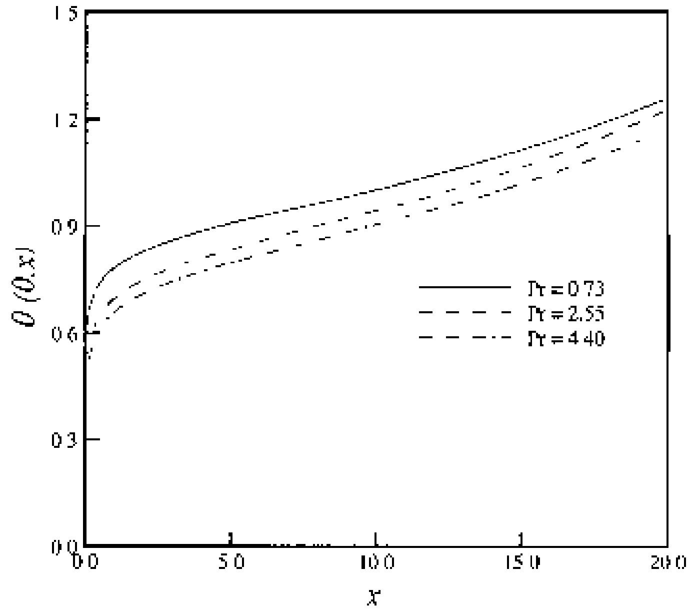

Fig.4.2: Surface temperature distribution for different values of *Pr* when M=0.8, J=0.2.

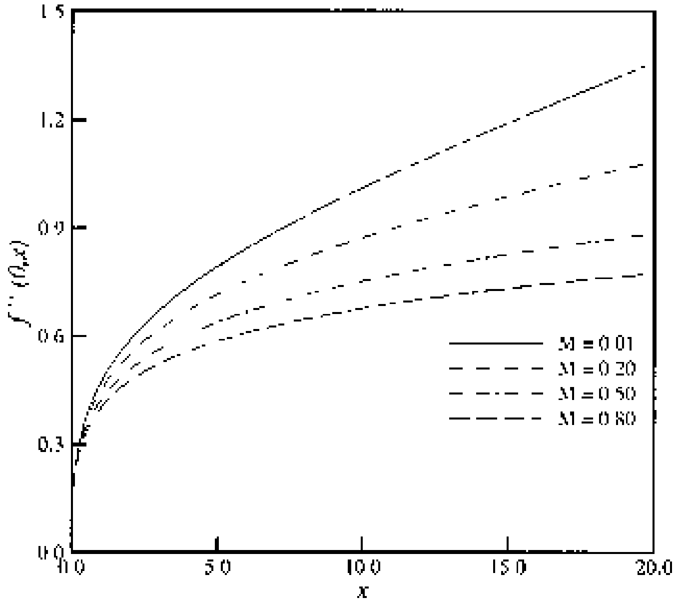

Fig.4.3: Skin frictions for different values of M when Pr=4.40, J= 0.02.

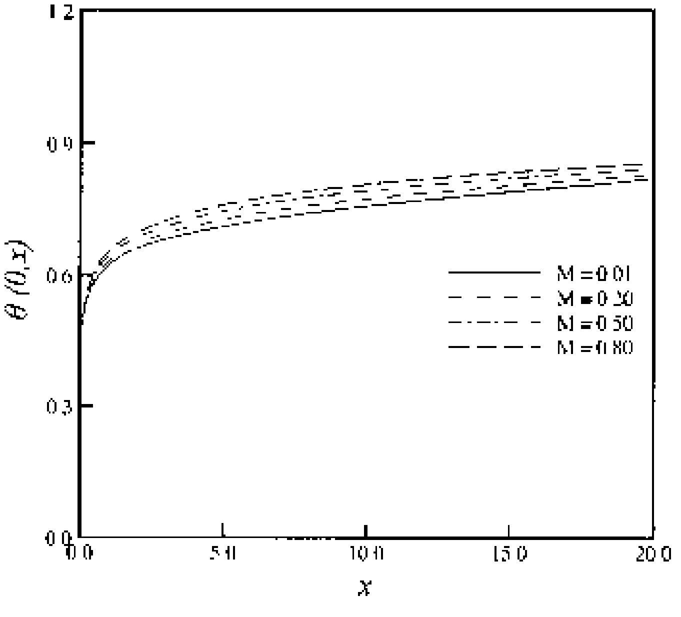

Fig.4.4: Surface temperature distribution for different values of M when Pr=4.40, J=0.02.

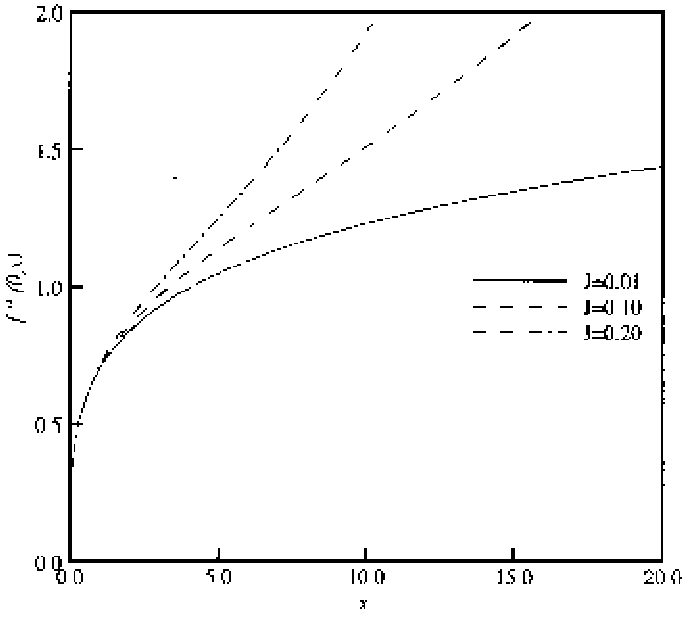

Fig.4.5: Skin frictions for different values of J when Pr =0 73 and M=0 2

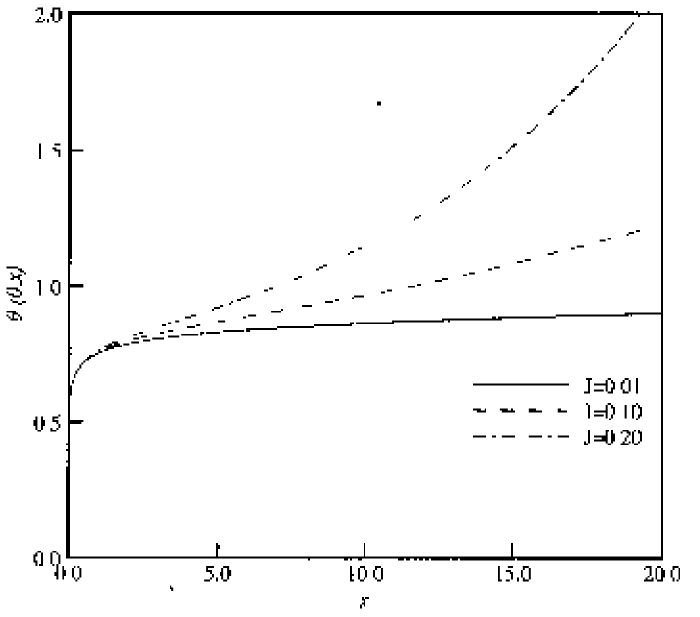

Fig.4.6: Surface temperature distribution for different values of J when Pr =0.73 and M=0.2

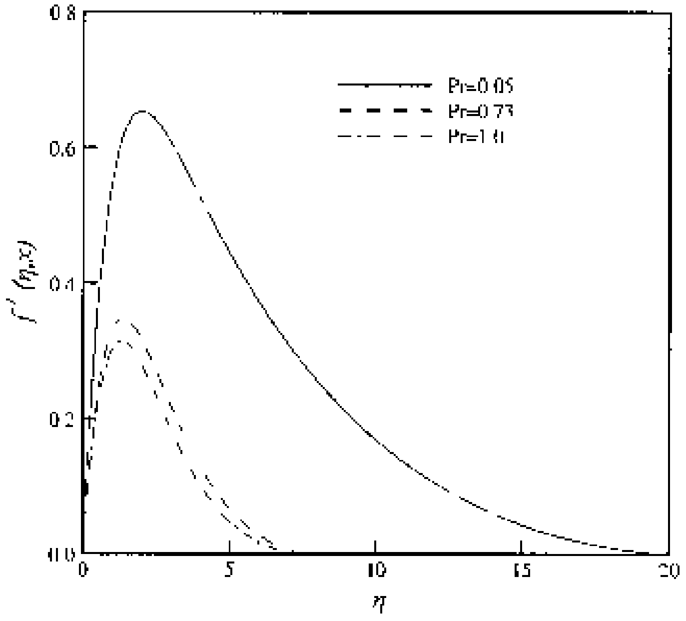

Fig.: 4.7: Velocity profile for different values of *Pr* when M=0.8 and J=0.2.

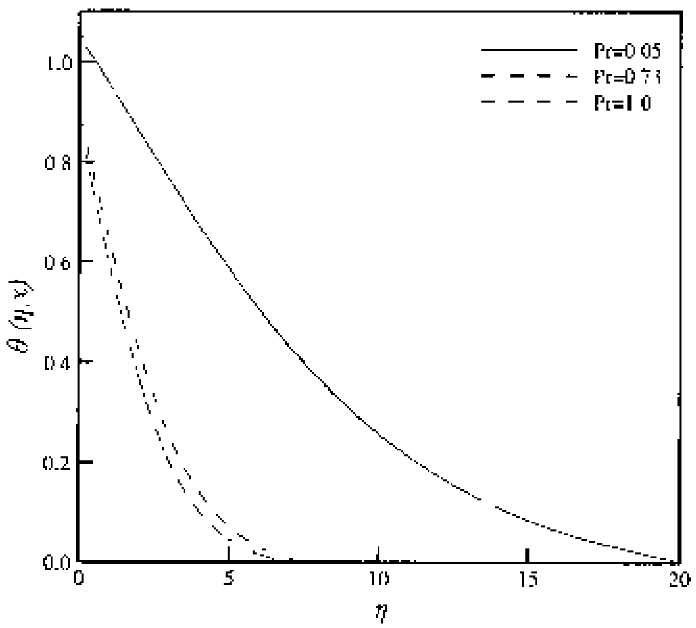

Fig.:4.8: Temperature profile for different values of *Pr* when M=0.8 and J=0.2.

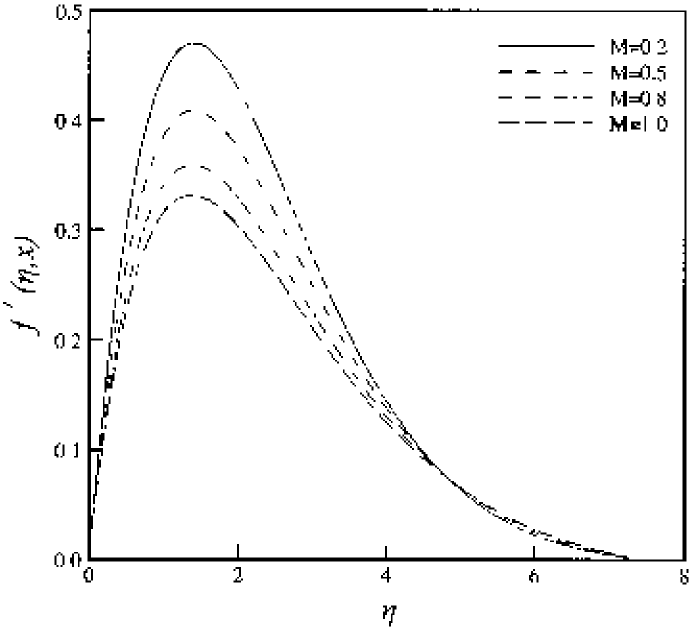

Fig.: 4.9: Velocity profile for different values of *M* when *Pr*=0.73 and *J*=0.5.

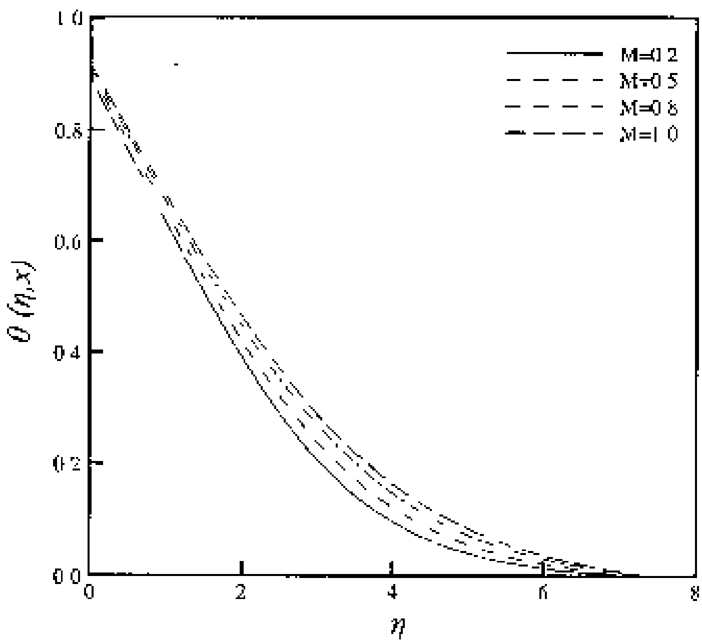

Fig.:4.10: Temperature profile for different values of *M* when *Pr*=0.73 and *J*=0.5.

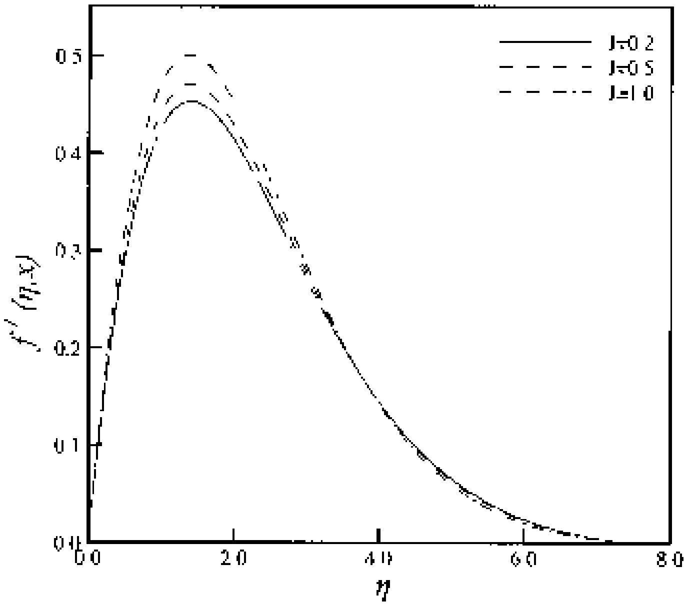

Fig.: 4.11: Velocity profile for different values of J when Pr=0.73 and M=0.2.

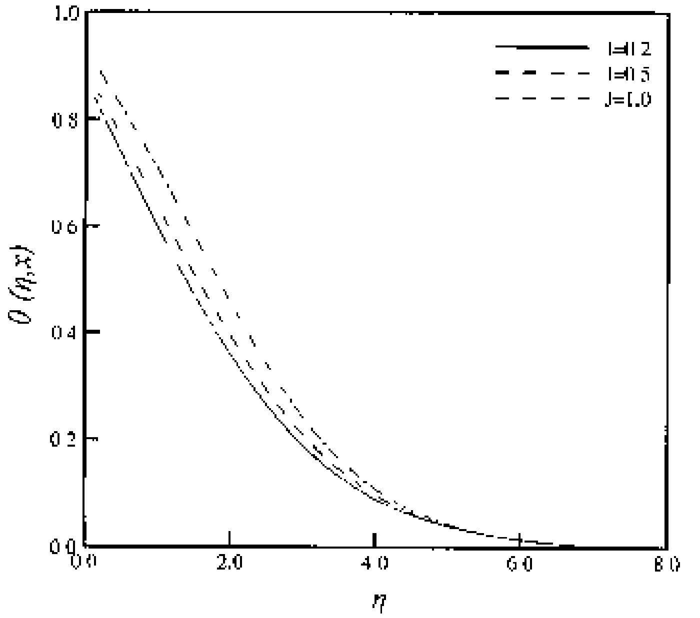

Fig.: 4.12: Temperature profile for different values of J when Pr=0.73 and M=0.2.

Chapter 5

Further Recommendation

The present studies may be extended further.

1. By using perturbation method or finite volume method to solve the governing equations.

2. By showing the effect of viscous dissipation in energy equation.

3. By considering the temperature dependent thermal conductivity and the temperature dependent viscosity, instead of taking constant thermal conductivity and viscosity.

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