LOAD DEFLECTION PREDICTION FOR LATERALLY LOADED PILES IN LAYERED SOILS.

A Thesis

By

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ABSTRACT

The present investigation has been conducted to study the load-displacement response and horizontal load capacity of vertical unrestrained piles subjected to lateral load using a finite difference method. Generally uniform soil deposits are not found in nature and the piles are embedded in layered soils. The effect of two-layer cohesive soil system in the engineering behaviour of a laterally loaded pile is investigated analytically. Pile deflection often governs the design of laterally loaded piles, as such the possibility of reducing the pile head deflection using a stiff layer near the ground level has been investigated.

For piles subjected to a given horizontal load, the deflection at pile head, is smaller for piles embedded in stiff soils than piles embedded in soft soils. Therefore the allowable capacity [the allowable horizontal load capacity according to BNBC (1993) is half of the horizontal load which produces 1 inch deflection at pile head] of a pile is greater if the pile is embedded in stiff soil. However the maximum moment corresponding to allowable load increases with the increase of soil modulus. The reason behind this is the increase in allowable horizontal capacity in stiff soil. Higher horizontal load produces higher maximum moment. To attain higher allowable capacity of a pile embedded in a better soil the reinforcement of the pile should be increased according to the value of maximum moment.

The horizontal capacity and corresponding moment of a pile embedded in soft soil and tip resting on a stiff soil is similar to a pile embedded in a uniform soil having properties of the top soft layer.
When a pile is subjected to a given horizontal load, the presence of a thin stiff layer at ground surface reduces the deflections and moments to a great extent. That is the horizontal capacity of a vertical pile can be increased significantly by replacing the upper soil with a thin stiff layer. The horizontal capacity depends on the modulus of subgrade reaction and thickness of the top stiff layer.

When a pile is subjected to a given horizontal load the deflection of piles decreases with the increase of thickness of the top stiff layer. However effect of thickness on the values of maximum moment is negligible.
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$A_m$ dimensionless moment co-efficient
$A_x$ dimensionless moment co-efficient
$A'_p$ dimensionless moment co-efficient
$A_v$ dimensionless moment co-efficient
$B$ width of pile
$B_m$ dimensionless moment co-efficient
$B_x$ dimensionless moment co-efficient
$B'_p$ dimensionless moment co-efficient
$B_v$ dimensionless moment co-efficient
$C_u$ undrained shear strength
$D$ pile diameter
$D_x$ distance above the ground for application of loading in inches
$E_p$ Young's modulus of the pile material
$E_{30}$ Secand modulus
$f$ location of maximum moment
$H_u$ ultimate horizontal load
$I_p$ Moment of inertia of the pile section
$k$ modulus of subgrade reaction
$k_s$ soil modulus of subgrade reaction
$L$ length of pile
$M_c$ characteristic moment
$M_g$ moment
$M_{max}$ maximum lateral moment
$M_i$ ground line moment
$M_y$ yield moment
<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>( M_z )</td>
<td>moment of pile at any depth</td>
</tr>
<tr>
<td>( P )</td>
<td>pile load</td>
</tr>
<tr>
<td>( P' )</td>
<td>pressure on soil</td>
</tr>
<tr>
<td>( P_c )</td>
<td>characteristic load</td>
</tr>
<tr>
<td>( P_t )</td>
<td>ground line load</td>
</tr>
<tr>
<td>( P'_z )</td>
<td>Soil reaction at any depth</td>
</tr>
<tr>
<td>( Q_g )</td>
<td>lateral force</td>
</tr>
<tr>
<td>( q )</td>
<td>intensity of soil pressure</td>
</tr>
<tr>
<td>( R )</td>
<td>characteristic length</td>
</tr>
<tr>
<td>( R_1 )</td>
<td>moment of inertia ratio</td>
</tr>
<tr>
<td>( S_u )</td>
<td>undrained shear strength of clay</td>
</tr>
<tr>
<td>( T )</td>
<td>characteristic length</td>
</tr>
<tr>
<td>( V_z )</td>
<td>shear force on pile at any depth</td>
</tr>
<tr>
<td>( x )</td>
<td>deflection</td>
</tr>
<tr>
<td>( x_z )</td>
<td>pile deflection at any depth</td>
</tr>
<tr>
<td>( y_{\text{Combined}} )</td>
<td>estimated ground line deflection due to both load and moment</td>
</tr>
<tr>
<td>( y_t )</td>
<td>ground line deflection</td>
</tr>
<tr>
<td>( z )</td>
<td>distance from ground surface</td>
</tr>
<tr>
<td>( Z_{\text{max}} )</td>
<td>maximum distance from ground surface</td>
</tr>
<tr>
<td>( \Delta )</td>
<td>settlement</td>
</tr>
</tbody>
</table>
CHAPTER-1
INTRODUCTION

1.1 GENERAL

The uses of piles is man’s oldest method of overcoming the difficulties of founding on soft soils. Although it dates back to prehistoric lake villages, until the late nineteenth century, the design of pile foundations was based entirely on experience, or even divine providence. Modern literature on piles can be said to date from the publication of piles and pile driving, edited by Wellington of the Engineering News (Later to become the Engineering News-record) in 1893, in which the widely known Engineering News pile-driving formula was proposed. Since this first attempt at a theoretical assessment of the capacity of a pile, a great volume of field experience and empirical data on the performance of pile foundations has been published. In recent years, the increasing demand on the foundation engineer to predict reliably the behaviour of his pile designs has stimulated more-sophisticated theoretical research into the interaction between a pile or piles and the embedding soil, so that a large volume of empirical knowledge is now balanced by a comparable theoretical understanding.

Piles are frequently subjected to lateral forces, and moments, for example, in quarry and harbor structures, where horizontal forces are caused by the impact of ships during berthing and wave action, in offshore structures subjected to wind and wave action, in pile-supported earth-retaining structures, in lock structures, in transmission-tower foundations, where high wind forces may act, and in structures constructed in
earthquake areas where some building codes specify that piles supporting such structures should have the lateral load.

In designing pile foundations to resist lateral loads, the criterion for design in the majority of cases is not the ultimate lateral capacity of the piles, but the maximum deflection of the piles. The allowable deflection may be relatively large for temporary structures or tied retaining walls, but only small movements can be tolerated in such structures as tied abutments to bridges, or in the foundations of tall structures.

Design practice in the past has frequently made use of empirical information for pile design, derived from full-scale lateral-load tests. In recent years, however, theoretical approaches for predicting lateral movements have been developed extensively. Two approaches have generally been employed. i) The subgrade reaction approach, in which the continuous nature of the soil medium is ignored and the pile reaction at a point is simply related to the deflection at that point. ii) The elastic approach, which assumes the soil to be an ideal elastic continuum.

The effect of layered soil system on the engineering behaviour of a laterally loaded pile can be investigated analytically. A modulus of subgrade reaction is used to define the soil stiffness, stiffness of other layer can be defined in terms of relative stiffness of the original layer.

1.2 SCOPE AND OBJECTIVE OF THE RESEARCH

In an attempt to investigate the load-deflection prediction and load carrying capacity of laterally loaded pile, a review of related literature is made. A finite difference
procedure is used to idealize the soil-pile system. Results obtained from this research are expected to be useful in the design of pile foundation embedded in layered soils and subjected to lateral loads caused by wind, earth-quake etc.

Pile deflection often governs the design of laterally loaded piles, as such the possibility of reducing the pile head deflection using an improved stiff layer near the ground level is an important issue.

With the above in view, the research is carried out with the following objectives.

i) To study the effect of variation of modulus of subgrade reaction on the behaviour of laterally loaded piles.

ii) To study the effect of pile dimension on the behaviour of laterally loaded piles.

iii) To carry out analysis of laterally loaded piles embedded in layered soil and to study the effect of a thin stiff layer near ground surface.
CHAPTER-2

LITERATURE REVIEW

2.1 GENERAL

The behaviour of a laterally loaded pile is a classical example of soil-structure interaction; the properties of the soil control the behaviour of the embedded structure. Recent experimental investigations and advances in analytical techniques have added greatly to the general understanding of the problem. In the analysis of a laterally loaded pile, it is now possible to include the soil properties that are more realistic than those used previously. Therefore, more confidence may now be placed in the computed behaviour of a pile-supported structures subjected to lateral loads.

It is known that a soft soil deposit provides less resistance to a laterally loaded pile. Engineers have long recognized the beneficial effect that the stiffening of the surficial soil has in reducing the lateral deflection of a laterally loaded pile.
2.3 DIFFERENT METHODS FOR SOLUTION OF LATERALLY LOADED PILES IN COHESIVE SOIL

There are different methods suggested by various authors for solving the problem of laterally loaded vertical piles in clay. Each has its own merits and demerits. Important methods in use are:

1. Empirical method
2. Davisson and Gill method (1963)
3. Broms’s method (1964)

2.3.1 Empirical method

McNulty (1956) [referred by Poulos & Davis (1980)] studied a number of piles and arrived at the allowable lateral-pile loads presented in table 2.1. It was also concluded that lateral-pile movements should be limited to no more than \( \frac{1}{4} \) in. for buildings. Other structures might tolerate a somewhat larger movement.
Table 2.1 Suggested safe allowable lateral forces on vertical piles, Kips (McNulty 1956)*

<table>
<thead>
<tr>
<th>Pile type</th>
<th>Medium Sand</th>
<th>Fine Clay</th>
<th>Medium Clay</th>
</tr>
</thead>
<tbody>
<tr>
<td>Free-head timber, 12-in. dia.</td>
<td>1.5</td>
<td>1.5</td>
<td>1.5</td>
</tr>
<tr>
<td>Fixed-head timber, 12-in. dia.</td>
<td>5.0</td>
<td>4.5</td>
<td>4.0</td>
</tr>
<tr>
<td>Free-head concrete, 16-in. dia.</td>
<td>7.0</td>
<td>5.5</td>
<td>5.0</td>
</tr>
<tr>
<td>Fixed-head concrete, 16-in. dia.</td>
<td>7.0</td>
<td>5.5</td>
<td>5.0</td>
</tr>
</tbody>
</table>

Based on a safety factor of 3 applied to the load required for 0.25-in. deflection

*Cited by [Poulos & Davis (1980)]
2.3.2 Davisson and Gill method (1963)

A general solution for determination of moments and displacements of a vertical pile subjected to lateral load and moment at the ground surface has been given by Matlock and Reese (1960). Consider a pile of length L subjected to a lateral force $Q_s$ and a moment $M_s$ at the ground surface (that is, at $z=0$), as shown in Fig. 2.3a. Fig. 2.3b shows the general nature of the deflected shape of the pile and the soil resistance caused by the applied load and the moment.

According to a simpler Winkler's model, an elastic medium (which is soil in this case) can be replaced by a series of finite closely independent elastic springs. With this assumption, one can write that

$$k = \frac{p'}{x}$$

where $k =$ modulus of subgrade reaction

$$p' = $$ pressure on soil

$$x = $$ deflection

Referring to Fig 2.3b and using the theory of beams on an elastic foundation, one can write

$$E_p I_p \frac{d^4 x}{dz^4} = p'$$

where $E_p =$ Young's modulus of the pile material

$I_p =$ Moment of inertia of the pile section

Based on Winkler's model

$$p' = -kx$$
Fig. 2.3  a) Laterally loaded pile

   b) Soil resistance on pile caused by lateral load
The sign in the preceding equation is negative because the soil reaction is in the opposite direction to the pile deflection.

Combining equation 2.2 and 2.3

\[ \frac{d^4 x}{dz^4} + kx = 0 \quad (2.4) \]

Solution of the preceding equation results in the following expressions

Pile deflection at any depth \([x_d(z)]\)

\[ x_d(z) = A_\alpha Q_s T^3 \frac{T^3}{E_p I_p} + B_\alpha M_g \frac{T^2}{E_p I_p} \quad (2.5) \]

Moment of pile at any depth \([M_d(z)]\)

\[ M_d(z) = A_m Q_s T + B_m M_g \quad (2.6) \]

Shear force on pile at any depth \([V_d(z)]\)

\[ V_d(z) = A_v Q_s + B_v \frac{M_g}{T} \quad (2.7) \]

Soil reaction at any depth \([P'_d(z)]\)

\[ P'_d(z) = A'_v \frac{Q_s}{T} + B'_v \frac{M_g}{T^2} \quad (2.8) \]

where \(A_\alpha, B_\alpha, A_m, B_m, A_v, B_v, A'_v, B'_v\) are co-efficients

Cohesive soils:

Solutions similar to those given in Eqs (2.5) to (2.8) have been given by Davisson and Gill (1963) for the case of piles embedded in clay. According to these solutions
To see equations (2.9) and (2.10), one must know the magnitude of the characteristic Length $R$. This can be calculated from equation (2.11) provided the co-efficient of the

$$x_x(z) = A_x \frac{Q_x R^3}{E_p I_p} + B_x \frac{M_x R^2}{E_p I_p}$$  \hspace{1cm} (2.9)$$

and $$M_x(z) = A_m Q_m R + B_m M_g$$  \hspace{1cm} (2.10)$$

where $A_x$, $B_x$, $A_m$ and $B_m$ are co-efficients, and

$$R = 4 \sqrt[4]{\frac{E_p I_p}{k}}$$  \hspace{1cm} (2.11)$$

The values of the $A$ and $B$ co-efficients are given in Fig.2.4.

Note that, in this Fig.

$$Z = \frac{z}{R}$$  \hspace{1cm} (2.12)$$

and $$Z_{\text{max}} = \frac{L}{R}$$  \hspace{1cm} (2.13)$$

To use equation (2.9) and (2.10), one must know the magnitude of the characteristic Length $R$. This can be calculated from equation 2.11 provided the co-efficient of the subgrade reaction is known.
Fig. 2.4 Variation of $A'_x$, $B'_x$, $A'_m$ and $B'_m$ with $z$ [after Davisson & Gill, (1963)]
2.3.3 Broms’s method (1964)

Broms (1964)* assumed, a simplified distribution of soil resistance as being zero from the ground surface to a depth of 1.5d and a constant value of 9C_u below this depth.

Possible failure mechanisms for unrestrained piles according to Broms (1964) are shown for "short" and "Long" piles in Fig. 2.5 together with the soil-reaction distributions.

Short pile :- Short piles (termed rigid piles) are those in which the lateral capacity is dependent wholly on the soil resistance.

Long pile :- Long piles are those whose lateral capacity is primarily dependent on the yield moment of the pile itself.

In Fig. 2.5 f defines the location of the maximum moment, and since the shear there is zero

\[ f = \frac{H_u}{9C_u d} \]  

(2.14)

Also, taking moments about the maximum moment location

\[ M_{\text{max}} = H_u (e + 1.5d + 0.5f) \]  

(2.15a)

*Cited by [Poulos and Davis (1980)]
Fig. 2.5 Failure mechanisms for piles, in cohesive soil (Broms 1964) *

*Cited by [Poulos & Davis (1980)]
also \[ M_{\text{max}} = 2.25dg^2C_u \] (2.15b)

since \[ L = 1.5d + f + g \], Equation 2.14 and 2.15 can be solved for the ultimate lateral load \( H_u \).

The solution is plotted in Fig. 2.6 in terms of dimensionless parameter \( \frac{L}{d} \) and \( \frac{H_u}{C_u d^3} \) and applies for short piles in which the yield moment \( M_y > M_{\text{max}} \), the inequality being checked by using Equation 2.14 and 2.15a.

For long piles, Equation 2.15b no longer holds, and \( H_u \) is obtained from equation 2.14 and 2.15a by setting \( M_{\text{max}} \) equal to the known value of yield moment, \( M_y \).

This solution is plotted in Fig. 2.6b in terms of dimensionless parameter \( \frac{H_u}{C_u d^3} \) and \( \frac{M_y}{C_u d^3} \). It should be noted that Broms’s solution for short piles can easily be recovered from the simple statical solution for uniform soil by using an equivalent length of pile equal to \( L - 1.5d \) and an equivalent eccentricity of loading equal to \( c + 1.5d \).
Fig. 2.6 Ultimate lateral resistance in cohesive soil

a) Short piles

b) Long piles (Broms, 1964)*

*Cited by [Poulos & Davis (1980)]
2.3.4 Bowles Method (1977)

Bowles (1977) suggested a method for determining the deflection, moment, shear and point force of a laterally loaded pile. He used finite difference technique. Where modulus of subgrade reaction is used and develop a computer program. The detailed of the method are discussed in the next chapter.

2.3.5 Duncan et al's method (1982)

The characteristic load method (CLM) was developed by Duncan, Evans and Phillip (1982). The method uses dimensional analysis to characterize the nonlinear behaviour of laterally loaded piles and drilled shafts by means of relationships among dimensionless variables. The new method is simple enough for use by manual calculation, and it can also be adapted for computer use. Lateral deflections and maximum bending moments calculated using CLM have been found to be in good agreement with values measured in field load tests.
The characteristic load method [Duncan et al (1982)] closely approximates the results of nonlinear p-y analysis. It was developed by performing nonlinear p-y analysis for a wide range of free-head and fixed-head piles and drilled shafts in clay and in sand, and representing the results in the form of relationships among dimensionless variables. The method can be used to determine

1. Ground-line deflections due to lateral load for free-head conditions, fixed head conditions, and the flagpole condition
2. Ground-line deflections due to moments applied at the ground line
3. Maximum moments for free head conditions, fixed head conditions and the flagpole conditions and
4. The location of the maximum moment in the pile or drilled shaft.

The use of dimensionless variables makes it possible to represent a wide range of real conditions by means of a single relationship. To form these dimensionless relationships, loads are divided by a characteristic load $P_c$, moments are divided by a characteristic moment $M_c$, and deflections are divided by the pile width $D$.

The characteristic load and moment that form the basis for the dimensionless relationships are given by the following expressions;

For clay

$$P_c = 7.34D^2(E_pR) \left( \frac{S_u}{E_pR} \right)^{0.60} \quad (2.16)$$
\[ M_c = 3.86D^3 \left( \frac{E_p}{E_pR_t} \right) \left( \frac{S_u}{E_pR_t} \right)^{0.46} \]  

(2.17)

in which  
- \( P_c \) = Characteristic load (F);  
- \( M_c \) = Characteristic moment (F.L);  
- \( D \) = Pile or drilled shaft width or diameter (L)  
- \( E_p \) = Pile or drilled shaft modulus of elasticity \( \left( \frac{F}{L^2} \right) \)  
- \( R_t \) = Moment of inertia ratio = ratio of moment of inertia of the pile or drilled shaft to the moment of inertia of a solid circular cross section (dimensionless)  
- \( = \frac{I_p}{I_{circular}} \)  
- \( S_u \) = Undrained shear strength of clay \( \left( \frac{F}{L^2} \right) \)

Values of \( R_t \) are calculated by dividing the moment of inertia of the pile or drilled shaft by the moment of inertia of a solid circular cross section with diameter equal to \( D \).

Values of undrained shear strength for use in developing p-y curves for clays have been based on either vane shear tests or triaxial tests on undisturbed samples (Matlock 1970; Welch and Reese 1972; Reese et al. 1975; Reese and Welch 1975) and these tests are therefore the most appropriate means of evaluating values of \( S_u \) for use in p-y analysis and the CLM.

Deflection due to loads applied at ground line

Dimensionless relationships between load and deflection for piles and drilled shafts subjected to lateral loads at the ground line are shown in Fig. 2.7. The vertical axis shows values of load divided by characteristic load \( \frac{P_t}{P_c} \) and the horizontal axis shows
Fig. 2.7 Load deflection curves for clay [after Duncan et al. (1982)]
ground line deflection divided by the pile or drilled shaft width or diameter \( \frac{y_t}{D} \). The numerical values used in plotting these curves are tabulated in Table 2.2. The tabulated values are convenient for setting up spreadsheet calculations, and for performing manual calculations.

To estimate the ground line deflection using Fig. 2.7. Calculate the value of \( P_e \). Divided the ground line load by \( P_t \) to determine the value of \( \frac{P_t}{P_e} \). Using the appropriate curve in Fig. 2.7 determine the value of \( \frac{y_t}{D} \) and multiply the value by \( D \) to determine the ground line deflection \( y_t \).

Maximum moments due to loads at or above ground line

In free-head piles and drilled shafts loaded at or above the ground line, the maximum moment occurs at some depth below ground. The magnitude and location of the maximum moment can be estimated using the theory for soil modulus increasing linearly with depth.

The first step is to determine the “Characteristic length T for the pile and soil conditions being analysed. After the deflection \( y_{\text{combined}} \) has been determined as described
Table 2.2 Load deflection co-efficients [after Duncan et al (1982)]

<table>
<thead>
<tr>
<th>y/b (1)</th>
<th>Clay Free Head ( P_r/P_c ) (2)</th>
<th>Clay Fixed Head ( P_r/P_c ) (3)</th>
<th>Sand Free Head ( P_r/P_c ) (4)</th>
<th>Sand Fixed Head ( P_r/P_c ) (5)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.0000</td>
<td>0.0000</td>
<td>0.0000</td>
<td>0.0000</td>
<td>0.0000</td>
</tr>
<tr>
<td>0.0025</td>
<td>0.0040</td>
<td>0.0088</td>
<td>0.0008</td>
<td>0.0016</td>
</tr>
<tr>
<td>0.0050</td>
<td>0.0065</td>
<td>0.0133</td>
<td>0.0013</td>
<td>0.0028</td>
</tr>
<tr>
<td>0.0075</td>
<td>0.0078</td>
<td>0.0168</td>
<td>0.0017</td>
<td>0.0039</td>
</tr>
<tr>
<td>0.0100</td>
<td>0.0091</td>
<td>0.0197</td>
<td>0.0021</td>
<td>0.0049</td>
</tr>
<tr>
<td>0.0150</td>
<td>0.0113</td>
<td>0.0247</td>
<td>0.0027</td>
<td>0.0065</td>
</tr>
<tr>
<td>0.0200</td>
<td>0.0135</td>
<td>0.0289</td>
<td>0.0033</td>
<td>0.0079</td>
</tr>
<tr>
<td>0.0300</td>
<td>0.0171</td>
<td>0.0359</td>
<td>0.0043</td>
<td>0.0104</td>
</tr>
<tr>
<td>0.0400</td>
<td>0.0200</td>
<td>0.0419</td>
<td>0.0052</td>
<td>0.0125</td>
</tr>
<tr>
<td>0.0500</td>
<td>0.0226</td>
<td>0.0471</td>
<td>0.0060</td>
<td>0.0144</td>
</tr>
<tr>
<td>0.0600</td>
<td>0.0250</td>
<td>—</td>
<td>0.0068</td>
<td>—</td>
</tr>
<tr>
<td>0.0800</td>
<td>0.0292</td>
<td>—</td>
<td>0.0083</td>
<td>—</td>
</tr>
<tr>
<td>0.1000</td>
<td>0.0332</td>
<td>—</td>
<td>0.0097</td>
<td>—</td>
</tr>
<tr>
<td>0.1500</td>
<td>0.0412</td>
<td>—</td>
<td>0.0124</td>
<td>—</td>
</tr>
</tbody>
</table>
previously, all values in (2.18) are kept except T. The value of T is determined by solving (2.18) for T by repeated trial.

\[ y_{\text{combined}} = \frac{2.43P_t}{E_p I_p} T^3 + \frac{1.62M_t}{E_p I_p} T^2 \]  

(2.18)

in which T=Characteristic length (L); y_{\text{combined}} = Estimated ground line deflection due to both load and moment (L); y_{\text{combined}} = y_t if p_t=0 or M_t=0, p_t=Ground line load (F), M_t=Ground line moment (F.L), E_p=Pile or drilled shaft modulus of elasticity (\( \frac{F}{L^2} \)) and I_p=Pile or drilled shaft moment of inertia (L^4)

When the value of T has been determined, the bending moments in the upper part of the pile or drilled shaft can be calculated using the following equation;

\[ M_z = A_m P_t T + B_m M_t \]  

(2.19)

in which M_z=Moment at depth Z (F.L); z=Depth below ground line (L); A_m=Dimensionless moment co-efficient and B_m=Dimensionless moment co-efficient. Values of A_m and B_m are given in Table 2.3.

The component of moment resulting from ground line load reaches a maximum at a depth equal to \( z=1.3T \), Where A_m is a maximum. The component of moment that results from the ground line moment is largest at the ground line (\( z=0 \)). Where the value of B_m is a maximum. When both moment and load act at the ground line, the maximum moment occurs between \( z=0 \) and \( z=1.3T \). The value of the maximum moment and the depth at which it occurs can be estimated by calculating values of M_z for a number of values of z, using the co-efficients in Table 2.3
Table 2.3  Moment co-efficients $A_m$ and $B_m$ [after Matlock and Reese (1961)]

<table>
<thead>
<tr>
<th>$z/T$ (1)</th>
<th>$A_m$ (2)</th>
<th>$B_m$ (3)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.0</td>
<td>0.00</td>
<td>1.00</td>
</tr>
<tr>
<td>0.5</td>
<td>0.46</td>
<td>0.98</td>
</tr>
<tr>
<td>1.0</td>
<td>0.73</td>
<td>0.85</td>
</tr>
<tr>
<td>1.3</td>
<td>0.77</td>
<td>0.73</td>
</tr>
<tr>
<td>1.5</td>
<td>0.76</td>
<td>0.64</td>
</tr>
<tr>
<td>2.0</td>
<td>0.63</td>
<td>0.40</td>
</tr>
</tbody>
</table>
2.4 DETERMINATION OF MODULUS OF SUBGRADE REACTION

In the analysis of laterally loaded piles the modulus of subgrade reaction of the surrounding soil is required. Determination of the modulus of subgrade reaction is generally carried out by one of the following methods [Bowles (1977)]

1. Full-scale lateral-loading test on a pile
2. Plate-loading tests
3. Empirical correlations using other soil properties

Full-scale lateral-loading test on a pile

The most direct means of using pile loading tests is to instrument the pile so that the soil pressures and pile deflections along the pile can be measured directly. This method has been used for a number of piles (e.g. Matlock and Ripperberger 1958) but is time-consuming, requires care, and is relatively expensive. A more convenient and or rotation and to backfigure the value of $k_s$, assuming an appropriate distribution with depth. Reese and Cox (1969) describe the interpretation of the tests in which both deflection and rotation is measured.
Plate-loading tests

If a foundation of width B as shown Fig. 2.8 is subjected to a load per unit area of q, it will undergo a settlement Δ. The co-efficient of subgrade modulus k can be defined as

\[ k = \frac{q}{\Delta} \]  \hspace{1cm} (2.20)

The unit of k is in kN/m^3 (or lb/in.\(^3\)). The value of the co-efficient of subgrade reaction is not a constant for a given soil. It depends on several factors, such as the length (L) and width (B) of the foundation and also the depth of embedment of the foundation. A comprehensive study of the parameters affecting the co-efficient of subgrade reaction has been given by Terzaghi (1955). According to this study, the value of the co-efficient of subgrade reaction decreases with the width of the foundation. In the field, load tests can be carried out by means of square plates measuring 0.3 m x 0.3 m (1 ft x 1 ft), and value of k can be calculated. The value of k can be related to large foundations measuring BxL as follow as suggested by B. M. Das (1987)

Foundations on Clays

\[ k \text{ (kN/m}^3\text{)} = k_{0.3}\text{(kN/m}^3\text{)[0.3 (m)/B (m)]} \]  \hspace{1cm} (2.21)

For rectangular foundations having dimensions of BxL (for similar soil and q)

\[ k = \frac{k_{(BxL)}(1 + \frac{B}{L})}{1.5} \]  \hspace{1cm} (2.22)
Fig. 2.8 Definition of co-efficient of subgrade reaction k
Where $k$ = co-efficient of subgrade modulus of the rectangular foundation (BxL)

$$k_{(B \times B)} = \text{co-efficient of subgrade modulus of a square foundation having dimension of } B \times B$$

The preceding equation indicates that the value of $k$ of a very long foundation with a width $B$ is approximately equal to $0.67k_{(B \times B)}$

Terzaghi (1955) recommended the value of co-efficient of subgrade reaction $k_s$ (to be used for pile) as equal to $k$ the value of subgrade reaction determined for vertically loaded plate test. Some typical values of co-efficient of subgrade reaction as shown in Table 2.4
# Table 2.4 Some typical values of co-efficient of subgrade reaction

<table>
<thead>
<tr>
<th>Material</th>
<th>Condition</th>
<th>Values</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Sand (dry or moist)</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Loose</td>
<td></td>
<td>8-25 MN/m³ (29-92 lb/in.³)</td>
</tr>
<tr>
<td>Medium</td>
<td></td>
<td>25-125 MN/m³ (91-460 lb/in.³)</td>
</tr>
<tr>
<td>Dense</td>
<td></td>
<td>125-375 MN/m³ (460-1380 lb/in.³)</td>
</tr>
<tr>
<td><strong>Sand (saturated)</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Loose</td>
<td></td>
<td>10-15 MN/m³ (38-55 lb/in.³)</td>
</tr>
<tr>
<td>Medium</td>
<td></td>
<td>35-40 MN/m³ (128-147 lb/in.³)</td>
</tr>
<tr>
<td>Dense</td>
<td></td>
<td>130-150 MN/m³ (478-552 lb/in.³)</td>
</tr>
<tr>
<td><strong>Clay</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Stiff (q_u=100-200 kN/m²)</td>
<td></td>
<td>12-25 MN/m³ (44-92 lb/in.³)</td>
</tr>
<tr>
<td>Very stiff (q_u=200-400 kN/m²)</td>
<td></td>
<td>25-50 MN/m³ (92-184 lb/in.³)</td>
</tr>
<tr>
<td>Hard (q_u&gt;400 kN/m²)</td>
<td></td>
<td>&gt;50 MN/m³ (&gt;184 lb/in.³)</td>
</tr>
</tbody>
</table>

(Note: q_u=unconfined compression strength)
**Empirical method**

A number of empirical methods for determination of modulus of subgrade reaction are available.

**Skempton method (1951)**

Skempton (1951) have suggested the following relation to evaluate $k_s$

$$k_s = (80 - 320) \frac{C_u}{d}$$  \hspace{1cm} (2.23)

Where $C_u =$ undrained shear strength of soil

$d =$ diameter of piles

**Vesic’s Method (1961)**

Vesic (1961) analyzed an infinite horizontal beam on an elastic foundation and by comparing the results with those obtained by the use of subgrade-reaction theory, related the modulus of subgrade reaction $k_s$ to the elastic parameters $E_s$ and $\eta_s$ of the mass, as follows

$$k_s = \left( \frac{0.65}{d} \right) 12 \left[ \frac{E_s d^4}{E_p I_p} \left( \frac{E_s}{1 - \eta_s^2} \right) \right]$$  \hspace{1cm} (2.24)

Where $E_p I_p =$ Pile stiffness

$d =$ pile diameter
Broms method (1964)

For clays, Broms (1964) has related $k_s$ to the secant modulus $E_{50}$ at half the ultimate stress in an undrained test as

$$k_s = 1.67 \frac{E_{50}}{d}$$  \hspace{1cm} (2.25)

Davisson method (1970)

Davisson (1970) suggests a more conservative value of

$$k_s = 67 \frac{C_u}{d}$$  \hspace{1cm} (2.26)
CHAPTER-3

PROGRAMME METHODOLOGY

3.1 GENERAL

A computer programme using finite difference techniques was developed by Bowles(1977) to determine the deflection, moment and shear of a laterally loaded piles embedded in uniform soil. The programme was extended for two layer soil system. Details are presented in the following paragraph.

3.2 ANALYSIS BY FINITE DIFFERENCES

The principal advantages of the finite-differences method are that the modulus of subgrade reaction or the cross section of the footing (beam, pile) can vary in any reasonable fashion along the member. The method requires writing and solving a large number of simultaneous equations, however, the technique can be programmed on the computer to both write and solve the set of equations, and once this is done, the problem is as simple.

The assumption is

The soil pressure at any point is proportional to the deflection, and the deflection of a point is independent of the deflections of adjacent points; in other words, at each point the foundation rests on a spring.
From Fig. 3.1 the average slope of the elastic curve (first forward difference), and approximating the derivative with the finite element $\Delta x = h$, is

$$\left( \frac{dy}{dx} \right)_2 \Rightarrow \left( \frac{\Delta y}{\Delta x} \right)_2 = \left( \frac{y_3 - y_2}{h} \right)_2$$

where $\left( \frac{\Delta y}{\Delta x} \right)_2$ indicates the slope at station 2

If we take backward differences of station 2 and 3 in this expression, we obtain

$$\left( \frac{dy}{dx} \right)_3 \Rightarrow \left( \frac{\Delta y}{\Delta x} \right)_3 = \left( \frac{y_2 - y_1}{h} \right)_3$$

$$\left( \frac{dy}{dx} \right)_2 \Rightarrow \left( \frac{\Delta y}{\Delta x} \right)_2 = \left( \frac{y_2 - y_1}{h} \right)_2$$

Now approximating the second derivative with finite elements

$$\frac{d^2 y}{dx^2} \Rightarrow \frac{\Delta^2 y}{\Delta x^2} = \Delta \left( \frac{\Delta y}{\Delta x} \right) = \Delta \left( \frac{y_3 - y_2 - y_2 - y_1}{h} \right) = 1 \left( \frac{y_3 - 2y_2 + y_1}{h} \right)$$

$$\left( \frac{\Delta^2 y}{\Delta x^2} \right)_2 = \frac{1}{h^2} (y_3 - 2y_2 + y_1)$$

This expression is termed a central difference equation for the second derivative.

From mechanics of solids

$$\frac{d^2 y}{dx^2} = \frac{M}{EI}$$

and using finite lengths and replacing $h$ by $\Delta x$ and subscripting the deflections for matrix notation, we obtain the desired finite-difference equation for bending moment in terms of the deflections

$$\frac{d^2 y}{dx^2} = \frac{M}{EI}$$

$$M = EI \frac{d^2 y}{dx^2}$$
Note that $\frac{dy}{dx} \neq \frac{y_3 - y_2}{h}$ unless $h$ is small or the elastic curve is linear.

Fig. 3.1 Finite-difference definition of slope
1. Replace the system of loads on the pile with a horizontal force and a moment on the free end of the pile as shown in Fig. 3.2

2. Divide the pile into increments of $x$, which are preferably (but not necessarily) of equal length. The use of $x=\text{constant}$ simplifies the computational work considerably.

3. Assume distribution of pressure intensity $q$ along the pile, as shown in Fig. 3.3 and Fig. 3.4. The distribution may be assumed to be stepped if sufficient divisions ($x$) are used. When $k_b$ (horizontal modulus of subgrade reaction) is not constant, the stepped solution is the simplest, and is not seriously in error with 10 or more divisions.

4. Compute equivalent concentrated loads (reactions) from the pressure distribution at points along the pile 1, 2, 3, ..............$n$.

5. Sum moments at each point except point 1 as

$$M = EI \frac{1}{(\Delta x)^2} (y_3 - 2y_2 + y_1)$$

$$M_n = EI \frac{1}{(\Delta x)^2} (y_{n+1} - 2y_n + y_{n-1})$$

The procedure to use to solve a problem in this manner is as follows:

1. Replace the system of loads on the pile with a horizontal force and a moment on the free end of the pile as shown in Fig. 3.2

2. Divide the pile into increments of $x$, which are preferably (but not necessarily) of equal length. The use of $x=\text{constant}$ simplifies the computational work considerably.

3. Assume distribution of pressure intensity $q$ along the pile, as shown in Fig. 3.3 and Fig. 3.4. The distribution may be assumed to be stepped if sufficient divisions ($x$) are used. When $k_b$ (horizontal modulus of subgrade reaction) is not constant, the stepped solution is the simplest, and is not seriously in error with 10 or more divisions.

4. Compute equivalent concentrated loads (reactions) from the pressure distribution at points along the pile 1, 2, 3, ..............$n$.

5. Sum moments at each point except point 1 as

$$M = EI \frac{1}{(\Delta x)^2} (y_3 - 2y_2 + y_1)$$

$$M_n = EI \frac{1}{(\Delta x)^2} (y_{n+1} - 2y_n + y_{n-1})$$

This step yields $n$ equations.

6. Write an additional equation as $\sum F_h = 0$
Fig. 3.3 Computation of Equivalent Reactions for Various Assumptions for Pressure Distribution

\[
R_1 = \frac{1}{2} hky_1, \quad R_2 = hky_2; \quad R_3 = hky_3, \quad R_n = \frac{1}{2} hky_n
\]
Fig. 3.4 Pile with Modulus of Subgrade Reaction Constant with Depth
The simultaneous solution of this n+1 set of equations yields the deflections at each point. From the deflections, the concentrated loads at the points can be computed. The moments at each points are evaluated by substitution of the calculated deflections into Equation (3.3)
Calculation of Reaction

From Fig. 3.4

\[ R_1 = \frac{1}{2} x q \]  \hspace{1cm} (3.4)

where \( R_1 \) = Point force in kips
\( x \) = Length of each division in inch or ft
\( q \) = Intensity of soil pressure (Units of force x L^{-1})

\[ R_1 = \frac{1}{2} x_k B y_1 \]  \hspace{1cm} (3.5)

where \( k_s \) = Lateral modulus of subgrade reaction
(Units of force x length^{-2})

\( B \) = width of pile in inch or ft
\( y \) = deflection at point 1 in inch or ft

\[ R_2 = R_3 = R_4 = R_5 = x q = x_k B y_2 \]  \hspace{1cm} (3.6)

\[ R_n = \frac{1}{2} x q = \frac{1}{2} x_k B y_n \]  \hspace{1cm} (3.7)

Calculation of Moment

From Fig. 3.4 shown above

At point 2 to the left, the moment equation is

\[ \sum M_{2, x} = 0 \Rightarrow P.(D_x + x) - R_1 x - M_2 = 0 \]

\[ \Rightarrow M_2 = P.(D_x + x) - R_1 x \]

\[ \Rightarrow M_2 = - R_1 x + P.(D_x + x) \]

\[ \Rightarrow \frac{EI}{x^2} (y_1 - 2y_2 + y_3) + R_1 x = P.(D_x + x) \]
\[ EI \frac{x^2}{x^2} (y_1 - 2y_2 + y_3) + \frac{1}{2} x k_s D y_1, x = P.(D_x + x) \]
\[ \Rightarrow EI \frac{x^2}{x^2} (y_1 - 2y_2 + y_3) + \frac{1}{2} x^2 k_s D y_1 = P.(D_x + x) \]  

At point 3 to the left, the moment equation is

\[ \sum M_3 = 0, \Rightarrow P.(D_x + 2x) - R_1.2x - R_2.x - M_3 = 0 \]
\[ \Rightarrow M_3 = -R_1.2x - R_2.x + P.(D_x + 2x) \]
\[ \Rightarrow EI \frac{x^2}{x^2} (y_2 - 2y_3 + y_4) + R_1.2x + R_2.x = P.(D_x + 2x) \]
\[ \Rightarrow EI \frac{x^2}{x^2} (y_2 - 2y_3 + y_4) + x^2 k_s D y_1 + x^2 k_s D y_2 = P.(D_x + 2x) \]  

(3.8)

Similarly other equations are found out

The programme code is presented in the Appendix.
CHAPTER-4
RESULTS AND DISCUSSION

4.1 GENERAL

In this study, different aspects those affect load-displacement response and load carrying capacity of laterally loaded unrestrained piles were investigated. These include the study of the effect of soil modulus of subgrade reaction, pile length and diameter. Special emphasis have been given on the study of the effect of (i) thin stiff layer at bottom which is a common case for end bearing pile (ii) thin stiff layer at top which improves the capacity of the pile. An attempt has been made to develop design curves. To study these effects a computer programme was used.

In this chapter the analytical results obtained from the investigation have been presented and discussed.

4.2 PILES IN UNIFORM SOIL

The behaviour of laterally loaded piles embedded in uniform cohesive soil was studied. Soils having different soil modulus of subgrade reaction, different pile length and different pile diameter were used for analysis.
Fig. 4.2 Deflected Shape of Piles Embedded in Soils Having Different Values of Soil Modulus of subgrade reaction

Pile Length = 40 ft
Pile Dia. = 24 inch
Horizontal Load = 10 kip

\( k_s = 5 \text{ kip/ft} \) when Max. Deflection = 1.3784 inch
\( k_s = 10 \text{ kip/ft} \) when Max. Deflection = 0.7668 inch
\( k_s = 15 \text{ kip/ft} \) when Max. Deflection = 0.5537 inch
Table 4.1 The maximum deflection and the maximum moment of a pile embedded in soils having different values of soil modulus of subgrade reaction due to various horizontal load

<table>
<thead>
<tr>
<th>Pile length in ft</th>
<th>Pile diameter in inch</th>
<th>Soil modulus $k_s$ in kip/ft$^3$</th>
<th>Horizontal load</th>
<th>Maximum deflection in inch</th>
<th>$\Delta_{\text{max}}/H$</th>
<th>Max. moment in kip-ft</th>
<th>Location of Max. moment from ground surface in ft</th>
</tr>
</thead>
<tbody>
<tr>
<td>40</td>
<td>24</td>
<td>5</td>
<td>5</td>
<td>0.6892</td>
<td>0.012</td>
<td>26.85</td>
<td>13</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>10</td>
<td>1.3784</td>
<td>0.035</td>
<td>53.70</td>
<td>13</td>
</tr>
<tr>
<td>40</td>
<td>24</td>
<td>10</td>
<td>5</td>
<td>0.3834</td>
<td>0.009</td>
<td>24.59</td>
<td>11</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>10</td>
<td>0.7668</td>
<td>0.019</td>
<td>49.18</td>
<td>11</td>
</tr>
<tr>
<td>40</td>
<td>24</td>
<td>15</td>
<td>5</td>
<td>0.2769</td>
<td>0.007</td>
<td>22.79</td>
<td>10</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>10</td>
<td>0.5537</td>
<td>0.014</td>
<td>45.59</td>
<td>10</td>
</tr>
</tbody>
</table>
For 10 kip horizontal load
For 5 kip horizontal load

Pile Length = 40 ft
Pile Dia. = 24 inch
Horizontal Load = 5 kip and 10 kip

$k_s = 5 \text{ kip/ft}^3, 10 \text{ kip/ft}^3 \text{ and } 15 \text{ kip/ft}^3$

Fig. 4.3 Relationship Between the Maximum Deflection and Soil Modulus of Subgrade Reaction for a Pile
Fig. 4.4 Distribution of Moment Along the Pile Length

Moment in kip-ft

Pile Length = 40 ft
Pile Dia. = 24 inch
Horizontal Load = 10 kip

- $k_p = 5$ kip/ft
- $k_p = 10$ kip/ft
- $k_p = 15$ kip/ft
moments occur at a depth between 10 to 13 ft. For softer soil the maximum moment occur at a deeper depth. However the location of maximum moment does not shift significantly with the increase of soil modulus. The values of the maximum moment due to 5 kip and 10 kip horizontal load for a pile embedded in soils having modulus of subgrade reaction 5 kip/ft³, 10 kip/ft³ and 15 kip/ft³ are tabulated in Table 4.1. The values of maximum moment and corresponding kₘ are plotted in Fig. 4.5. It can be observed that for a particular lateral load the values of maximum moment decreases when a pile is embedded in soil having higher soil modulus of subgrade reaction.

The distribution of point force along the pile length for pile embedded in clay having different modulus of subgrade reaction are plotted in Fig. 4.6. The maximum point force occur at a depth between 2 to 4 ft. For softer soil the maximum point force occur at a deeper depth. However the location of maximum point force does not shift significantly with the increase of soil modulus. The values of the maximum point force due to 5 kip and 10 kip horizontal load for a pile embedded in soils having modulus of subgrade reaction 5 kip/ft³, 10 kip/ft³ and 15 kip/ft³ are 2.06 kip, 2.25 kip and 2.41 kip respectively.

It is interesting to note that the deflection due to a given horizontal load for a pile embedded in soil having 10 kip/ft³ modulus of subgrade reaction is 1.37/0.76=1.8 times less than the deflection in a soil having kₘ=5 kip/ft³. Whereas the ratio between the maximum moments is 54/50=1.08 for pile embedded in similar soils. Therefore it could be concluded that the deflection of the pile head is very sensitive to modulus of subgrade reaction however the value of the maximum moment is less sensitive.
Fig. 4.5 Relationship Between the Maximum Moment and Soil Modulus of Subgrade Reaction
Pile Length = 40 ft
Pile Dia. = 24 inch
Horizontal Load = 10 kip

$k_s = 5$ kip/ft when Max. Point force = 2.06 kip
$k_s = 10$ kip/ft when Max. Point force = 2.25 kip
$k_s = 15$ kip/ft when Max. Point force = 2.41 kip

Fig. 4.6 Distribution of Point Force Along the Pile Length
The maximum deflections and moments due to horizontal load 5 kip and 10 kip for a 24 inch diameter pile having length 40 ft embedded in soil with $k_s=5$ kip/ft$^3$, 10 kip/ft$^3$ and 15 kip/ft$^3$ are tabulated in Table 4.1. It can be observed from the table that the maximum deflections and moments produced are directly proportional to the applied horizontal load.
4.2.2 Effect of length

To study the effect of length on deflection and lateral capacity, a 24 inch diameter pile having lengths 30 ft, 40 ft, 50 ft, 60 ft, 80 ft and 100 ft were analyzed. A lateral load of 10 kip was applied at the pile head as shown in Fig 4.1. The soil modulus of subgrade reaction (the value of $k_s$) 10 kip/ft² was used. The maximum deflections for piles of different lengths subjected to a given 10 kip horizontal load are plotted in Fig. 4.7 and tabulated in Table 4.2. It can be seen from Fig. 4.7. that the deflection of the pile decreases with the increase of pile length up to a certain length after that the deflection remains constant. The deflections of 30 ft and 40 ft long piles for 10 kip horizontal load are 0.8747 inch and 0.7668 inch respectively whereas the deflections of 70 ft and 80 ft long piles are 0.7298 inch and 0.7249 inch respectively. The difference in deflection for 30 ft and 40 ft long pile is 0.1 inch whereas difference in deflection is insignificant (0.005 inch) for 70 ft and 80 ft long piles. It may be concluded that the difference between the maximum deflection for shorter piles is greater than the difference between the maximum deflection for longer piles.

The values of the maximum moments due to a given horizontal load (10 kip) on piles having lengths 30 ft, 40 ft, 50 ft, 60 ft, 80 ft, and 100 ft are tabulated in Table 4.2. The maximum moments and corresponding pile lengths are plotted in Fig. 4.8. It can be observed that for a given horizontal load the maximum moment increases with the increase of pile lengths, for pile lengths between 30 ft and 50 ft. However the increase of the maximum moment is insignificant for pile lengths between 50 ft and 100 ft.
Fig. 4.7 Relationship Between the Maximum Deflection and Pile Length for a Given Horizontal Load

Pile Dia = 24 inch
Horizontal Load = 10 kip
$k_c = 10$ kip/ft$^1$
Table 4.2  The maximum deflection and the maximum moment due to (10 kip) horizontal load for different lengths of pile

<table>
<thead>
<tr>
<th>Pile length in ft</th>
<th>Pile diameter in inch</th>
<th>Soil modulus k₃ in kip/ft³</th>
<th>Maximum deflection in inch</th>
<th>Maximum moment in kip-ft</th>
</tr>
</thead>
<tbody>
<tr>
<td>30</td>
<td>24</td>
<td>10</td>
<td>0.8747</td>
<td>41.72</td>
</tr>
<tr>
<td>40</td>
<td>24</td>
<td>10</td>
<td>0.7668</td>
<td>49.18</td>
</tr>
<tr>
<td>50</td>
<td>24</td>
<td>10</td>
<td>0.7408</td>
<td>51.66</td>
</tr>
<tr>
<td>60</td>
<td>24</td>
<td>10</td>
<td>0.7351</td>
<td>51.97</td>
</tr>
<tr>
<td>80</td>
<td>24</td>
<td>10</td>
<td>0.7249</td>
<td>52.21</td>
</tr>
<tr>
<td>100</td>
<td>24</td>
<td>10</td>
<td>0.7101</td>
<td>51.78</td>
</tr>
</tbody>
</table>
Fig. 4.8 Relationship Between the Maximum Moment and Pile Length for a Given Horizontal Load
It is interesting to note that the deflection for a 30 ft long pile is greater than the deflection of 40 ft long pile for a given horizontal load. Whereas the maximum moments for a 30 ft long pile is smaller than a 40 ft long pile under a given load. This can be explained by the fact that the soil reaction developed is higher of shorter pile (as the deflection is more) than the longer pile.

For a given horizontal load (10 kip) the maximum deflections for piles of different lengths embedded in different soils ($k_s = 5$ kip/ft$^2$, 10 kip/ft$^2$ and 15 kip/ft$^2$) are plotted in Fig. 4.9. It can be observed from the figure that the maximum deflection due to a given horizontal load does not increases after a certain length of pile. This critical lengths are 30 ft, 40 ft and 50 ft for $k_s=5$, 10, and 15 kip/ft$^3$ respectively. This critical length at which the deflection becomes constant is smaller for stronger soils.
Fig. 4.9  Relationship Between the Maximum Deflection and Pile Length for Pile Embedded in Soils Having Different Soil Modulus of Subgrade Reaction.
4.2.3 Effect of diameter

To study the effect of diameter on deflection and lateral capacity of vertical piles, a 40 ft pile having 20 inch, 24 inch and 30 inch diameters were analyzed. A lateral load of 10 kip was applied at the pile head. The soil modulus of subgrade reaction (the value of $k_s$) 5 kip/ft$^3$, 10 kip/ft$^3$ and 15 kip/ft$^3$ were used. The maximum deflections and corresponding pile diameters for a 40 ft long pile embedded in soil having $k_s=10$ kip/ft$^3$ are plotted in Fig. 4.10 and tabulated in Table 4.3. It can be observed that the deflection decreases with the increase of diameter.

The maximum moments corresponding to pile diameters are plotted in Fig. 4.11 and tabulated in Table 4.3. For piles of a given length subjected to a given horizontal load, the maximum moment increases gradually with the increase of pile diameter. The maximum deflection under the given horizontal load is smaller for larger diameter piles, whereas the maximum moment is higher for larger diameter pile. This is due to higher moment of inertia of larger diameter pile. The ratios of maximum deflection for pile diameter 30 inch and 20 inch is $0.55/1.03=0.53$ whereas the ratio between the maximum moment of respective pile diameters is $53.57/44.40=1.2$. The maximum deflection is very sensitive of pile diameter but the maximum moment is not that sensitive.

The maximum deflections and corresponding pile diameter due to a given horizontal load on piles embedded in different soils ($k_s=5$, 10, and 15 kip/ft$^3$) are plotted in Fig. 4.12. It can be observed from the figure that the maximum deflection due to a given horizontal load decreases with the increases of soil modulus for different diameter of piles.
Fig. 4.10 Relationship Between the Maximum Deflection and Pile Diameter for a Given Horizontal Load
Table 4.3  The maximum deflection and the maximum moment due to 10 kip horizontal load for piles having different diameter

<table>
<thead>
<tr>
<th>Pile length in ft</th>
<th>Pile diameter in inch</th>
<th>Soil modulus k_s in kip/ft^2</th>
<th>Maximum deflection in inch</th>
<th>Maximum moment in kip-ft</th>
</tr>
</thead>
<tbody>
<tr>
<td>40</td>
<td>20</td>
<td>10</td>
<td>1.0275</td>
<td>44.40</td>
</tr>
<tr>
<td>40</td>
<td>24</td>
<td>10</td>
<td>0.7668</td>
<td>49.18</td>
</tr>
<tr>
<td>40</td>
<td>30</td>
<td>10</td>
<td>0.5530</td>
<td>53.57</td>
</tr>
</tbody>
</table>
Fig. 4.11 Relationship Between the Maximum Moment and Pile Diameter for a Given Horizontal Load

Pile Length = 40 ft
Horizontal Load = 10 kip
$k_c = 10 \text{ kip/ft}^3$
Fig. 4.12 Relationship Between the Maximum Deflection and Pile Diameter for a Given Horizontal Load for Different Values of Soil Modulus of Subgrade Reaction
4.3 DESIGN CURVES AND DESIGN TABLES

The criteria for determining the allowable horizontal load on a pile is the deflection of the pile at the pile head. According to BNBC [Bangladesh National Building Code (1993)] the allowable horizontal load on the pile is \( \frac{1}{2} \) the load that produces 1 inch deflection at pile head. Therefore design curves and tables are developed on the basis of 1 inch deflection of pile head. The load corresponding to 1 inch horizontal deflection for a 40 ft long pile with different diameters embedded in a soil having \( k_s=10 \) kip/ft\(^3\) are plotted in Fig. 4.13. The corresponding maximum moments for 1 inch horizontal deflection are also plotted in this figure. From this figure it is possible to determine maximum allowable horizontal capacity of the pile and the corresponding maximum moment.

Similar curves for 30 ft, 40 ft, and 50 ft long pile having different diameters and embedded in soils having different \( k_s \) (5 kip/ft\(^3\), 10 kip/ft\(^3\) and 15 kip/ft\(^3\)) are plotted in Fig. 4.14(a), (b), (c). The figure can be used to determine the allowable horizontal load and corresponding moments for design purpose.

The allowable horizontal loads according to BNBC [Bangladesh National Building Code (1993)] and corresponding maximum moments on 24 inch diameter pile embedded in soils having modulus of subgrade reaction 5 kip/ft\(^3\), 10 kip/ft\(^3\) and 15 kip/ft\(^3\) are presented in Table 4.4(a). It can be noted that for 24 inch diameter pile embedded in soil having \( k_s=5 \) kip/ft\(^3\) the allowable horizontal loads are 3 kip, 3.25 kip and 3.9 kip for 30 ft, 40 ft, and 50 ft long piles, whereas for 24 inch diameter pile embedded in soil having
Fig. 4.13 Relationship Between the Horizontal Load and the Corresponding Maximum Moment to the Pile Diameter for Producing 1 inch Deflection for 40 ft Pile

For 1 inch Deflection
Pile Length = 40 ft
\( k_s = 10 \text{ kip/ft}^3 \)
Fig. 4.14(a) Relationship Between the Horizontal Load and the Corresponding Maximum Moment for 1 inch Deflection [for 40 ft long pile embedded in soil having different soil modulus ($k_s$)]
For 1 inch Deflection
Pile = 30 ft
ks1 = 5 kip/ft³
ks2 = 10 kip/ft³
ks3 = 15 kip/ft³

Fig. 4.14(b) Relationship Between the Horizontal Load and the Corresponding Maximum Moment for 1 inch Deflection [for 30 ft long pile embedded in soil having different soil modulus (ks)]
Fig. 4.14(c) Relationship Between the Horizontal Load and the Corresponding Maximum Moment for 1 inch Deflection [for 50 ft long pile embedded in soil having different soil modulus (k_s)]
Table 4.4(a). Allowable horizontal loads according to BNBC (1993) for 24 inch diameter pile and the corresponding maximum moments

$k_s = 5 \text{kip/ft}^3$

<table>
<thead>
<tr>
<th>Pile length in ft</th>
<th>Pile dia. in inch</th>
<th>Horizontal load in kip for 1 inch deflection at the pile head</th>
<th>Allowable horizontal load in kip</th>
<th>Maximum moment in kip-ft for 1 inch deflection</th>
<th>Max. moment corresponding to allowable horizontal load in kip-ft</th>
<th>Location of max. moment in ft from ground</th>
</tr>
</thead>
<tbody>
<tr>
<td>30</td>
<td>24</td>
<td>6.00</td>
<td>3.00</td>
<td>25.966</td>
<td>12.98</td>
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<td>7.50</td>
<td>3.75</td>
<td>40.273</td>
<td>20.13</td>
<td>12.0</td>
</tr>
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<td>50</td>
<td>24</td>
<td>7.87</td>
<td>3.94</td>
<td>46.730</td>
<td>23.36</td>
<td>15.0</td>
</tr>
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</table>

$k_s = 10 \text{kip/ft}^3$

<table>
<thead>
<tr>
<th>Pile length in ft</th>
<th>Pile dia. in inch</th>
<th>Horizontal load in kip for 1 inch deflection at the pile head</th>
<th>Allowable horizontal load in kip</th>
<th>Maximum moment in kip-ft for 1 inch deflection</th>
<th>Max. moment corresponding to allowable horizontal load in kip-ft</th>
<th>Location of max. moment in ft from ground</th>
</tr>
</thead>
<tbody>
<tr>
<td>30</td>
<td>24</td>
<td>11.50</td>
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<td>47.97</td>
<td>23.99</td>
<td>9.0</td>
</tr>
<tr>
<td>40</td>
<td>24</td>
<td>13.25</td>
<td>6.62</td>
<td>65.17</td>
<td>32.58</td>
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</tr>
<tr>
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<td>24</td>
<td>13.51</td>
<td>6.75</td>
<td>69.80</td>
<td>34.90</td>
<td>12.5</td>
</tr>
</tbody>
</table>
### Horizontal Load Allowable Maximum Max. moment Location

<table>
<thead>
<tr>
<th>Pile length in ft</th>
<th>Pile dia. in inch</th>
<th>Horizontal load in kip for 1 inch deflection at the pile head</th>
<th>Allowable horizontal load in kip</th>
<th>Maximum moment in kip-ft for 1 inch deflection</th>
<th>Max. moment corresponding to allowable horizontal load in kip-ft</th>
<th>Location of max. moment in ft from ground</th>
</tr>
</thead>
<tbody>
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<td>16.75</td>
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<td>9.0</td>
</tr>
<tr>
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<td>24</td>
<td>18.50</td>
<td>9.25</td>
<td>84.34</td>
<td>42.17</td>
<td>12.0</td>
</tr>
<tr>
<td>50</td>
<td>24</td>
<td>18.52</td>
<td>9.26</td>
<td>86.51</td>
<td>43.25</td>
<td>12.5</td>
</tr>
</tbody>
</table>

$k_s = 15 \text{ kip/ft}^3$
For a 40 ft long and 24 inch diameter pile embedded in soils having $k_s = 5$, 10, and 15 kip/ft$^3$ the allowable horizontal loads are 3.75 kip, 6.62 kip, and 9.25 kip respectively and corresponding moments are 20.13 kip-ft, 32.58 kip-ft and 42.17 kip-ft respectively. It can be concluded that the horizontal capacity is increased if the pile is embedded in better soil. However the corresponding maximum moments are higher in better soil. To get higher horizontal pile capacity in better soil the pile should reinforced sufficiently to take care of the bending moments.

Piles having diameter 20 inch and 30 inch, embedded in soils having modulus of subgrade reaction 5 kip/ft$^3$, 10 kip/ft$^3$ and 15 kip/ft$^3$ are presented in Table 4.4(b) and 4.4(c). These Tables can also be used for determining horizontal load capacity and corresponding moments. It can be noted from the table that larger diameter piles have greater horizontal load capacity.
Table 4.4(b) Allowable horizontal loads according to BNBC (1993) for 20 inch diameter pile and the corresponding maximum moments

\( k_s = 5 \, \text{kip/ft}^3 \)

<table>
<thead>
<tr>
<th>Pile length in ft</th>
<th>Pile dia. in inch</th>
<th>Horizontal load in kip for 1 inch deflection at the pile head</th>
<th>Allowable horizontal load in kip</th>
<th>Maximum moment in kip-ft for 1 inch deflection</th>
<th>Max. moment corresponding to allowable horizontal load in kip-ft</th>
<th>Location of max. moment in ft from ground</th>
</tr>
</thead>
<tbody>
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<td>20.62</td>
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</tr>
<tr>
<td>40</td>
<td>20</td>
<td>5.60</td>
<td>2.80</td>
<td>28.17</td>
<td>14.08</td>
<td>12.0</td>
</tr>
<tr>
<td>50</td>
<td>20</td>
<td>5.85</td>
<td>2.92</td>
<td>31.20</td>
<td>15.60</td>
<td>12.5</td>
</tr>
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</table>

\( k_s = 10 \, \text{kip/ft}^3 \)

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<tr>
<th>Pile length in ft</th>
<th>Pile dia. in inch</th>
<th>Horizontal load in kip for 1 inch deflection at the pile head</th>
<th>Allowable horizontal load in kip</th>
<th>Maximum moment in kip-ft for 1 inch deflection</th>
<th>Max. moment corresponding to allowable horizontal load in kip-ft</th>
<th>Location of max. moment in ft from ground</th>
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<tbody>
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<td>9.0</td>
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<td>20</td>
<td>9.81</td>
<td>4.91</td>
<td>43.56</td>
<td>21.78</td>
<td>10.0</td>
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<td>20</td>
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<td>5.00</td>
<td>45.11</td>
<td>22.56</td>
<td>10.0</td>
</tr>
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</table>
\( k_s = 15 \text{ kip/ft}^3 \)

<table>
<thead>
<tr>
<th>Pile length in ft</th>
<th>Pile dia. in inch</th>
<th>Horizontal load in kip for 1 inch deflection at the pile head</th>
<th>Allowable horizontal load in kip</th>
<th>Maximum moment in kip-ft for 1 inch deflection</th>
<th>Max. moment corresponding to allowable horizontal load in kip-ft</th>
<th>Location of max. moment in ft from ground</th>
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</thead>
<tbody>
<tr>
<td>30</td>
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<td>12.70</td>
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<td>48.28</td>
<td>24.14</td>
<td>9.0</td>
</tr>
<tr>
<td>40</td>
<td>20</td>
<td>13.34</td>
<td>6.67</td>
<td>54.38</td>
<td>27.19</td>
<td>10.0</td>
</tr>
<tr>
<td>50</td>
<td>20</td>
<td>13.51</td>
<td>6.75</td>
<td>56.56</td>
<td>28.28</td>
<td>12.5</td>
</tr>
</tbody>
</table>
Table 4.4 (c) Allowable horizontal loads according to BNBC (1993) for 30 inch diameter pile and the corresponding maximum moments

$k_s=5 \text{ kip/ft}^2$

<table>
<thead>
<tr>
<th>Pile length in ft</th>
<th>Pile dia. in inch</th>
<th>Horizontal load in kip for 1 inch deflection at the pile head</th>
<th>Allowable horizontal load in kip</th>
<th>Maximum moment in kip-ft for 1 inch deflection</th>
<th>Max. moment corresponding to allowable horizontal load in kip-ft</th>
<th>Location of max. moment in ft from ground</th>
</tr>
</thead>
<tbody>
<tr>
<td>30</td>
<td>30</td>
<td>7.70</td>
<td>3.85</td>
<td>33.97</td>
<td>16.98</td>
<td>10.5</td>
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<td>30</td>
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<td>4.85</td>
<td>54.65</td>
<td>27.32</td>
<td>12.0</td>
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<tr>
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<td>30</td>
<td>11.12</td>
<td>5.56</td>
<td>72.90</td>
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<td>15.0</td>
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</table>

$k_s=10 \text{ kip/ft}^2$

<table>
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<tr>
<th>Pile length in ft</th>
<th>Pile dia. in inch</th>
<th>Horizontal load in kip for 1 inch deflection at the pile head</th>
<th>Allowable horizontal load in kip</th>
<th>Maximum moment in kip-ft for 1 inch deflection</th>
<th>Max. moment corresponding to allowable horizontal load in kip-ft</th>
<th>Location of max. moment in ft from ground</th>
</tr>
</thead>
<tbody>
<tr>
<td>30</td>
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<td>7.50</td>
<td>64.86</td>
<td>32.43</td>
<td>10.5</td>
</tr>
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<td>30</td>
<td>18.18</td>
<td>9.09</td>
<td>97.37</td>
<td>48.68</td>
<td>12.0</td>
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<td>19.60</td>
<td>9.80</td>
<td>115.88</td>
<td>57.94</td>
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</tbody>
</table>
$k_s = 15 \text{ kip/ft}^2$

<table>
<thead>
<tr>
<th>Pile length in ft</th>
<th>Pile dia. in inch</th>
<th>Horizontal load in kip for 1 inch deflection at the pile head</th>
<th>Allowable horizontal load in kip</th>
<th>Maximum moment in kip-ft for 1 inch deflection</th>
<th>Max. moment corresponding to allowable horizontal load in kip-ft</th>
<th>Location of max. moment in ft from ground</th>
</tr>
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<tbody>
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<td>22.23</td>
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<td>12.0</td>
</tr>
<tr>
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<td>30</td>
<td>27.03</td>
<td>13.52</td>
<td>147.67</td>
<td>73.83</td>
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</table>
4.4 PILES IN LAYERED SOIL

Generally uniform soil deposits are not found in nature and the piles are embedded in layered soils. The effect of two-layer soil system in engineering behaviour of a laterally loaded vertical pile is investigated analytically. Two modulus of subgrade reaction were used to define the stiffness of two soil layer. Two cases of layered soil system were considered one with a thin stiff layer at the bottom, and the other with a thin stiff layer on the top. The first case is very common. Generally pile tip rests on good soil layer and the top soil is poor. If lateral capacity of a pile is increased by providing a thin artificial layer near the top, the second case is applicable.

4.4.1 Effect of a Thin Stiff Layer at Bottom

This is the general case, normally piles are design in soils where the top soil is poor. The pile tip rests on a hard soil. The vertical capacity of such piles are normally high.

The pile embedded in soil conditions shown in Fig. 4.15 was analyzed. The top 34 ft soil is a poor having soil modulus of subgrade reaction, \( k_s = 10 \text{ kip/ft}^3 \) and the bottom 6 ft (3D) soil is hard having soil modulus of subgrade reaction, \( k_s = 50 \text{ kip/ft}^3 \). Deflection and moment of piles (as shown in Fig. 4.15) having different diameters 20 inch, 24 inch, and 30 inch subjected to a lateral load of 10 kip were determined. The results of the maximum deflections (at pile head) and the maximum moments are plotted.
Fig. 4.15 Pile Subjected to Lateral Load Embedded in Layered Soil

**Diagram Description:**
- **P = 10 kip**
- **Pile Length = 40 ft**
- **Pile Dia. = 24 inch**
- **Horizontal Load = 10 kip**
- **k_s = 10 kip/ft^3**
- **k_s = 50 kip/ft^3**

**Legend:**
- **Xxxxxx** represents the pile segment.
in Fig. 4.16(a) and 4.16(b) respectively. These results are compared with similar piles (length and diameter same) embedded in uniform soil of $k_8=10$ kip/ft$^3$. It can be observed from the figure that the maximum deflections and the maximum moments of piles embedded in such soils are almost equal. For piles subjected to a given horizontal load, it is of interest to note from Fig. 4.16(a) and 4.16(b) that the deflection of piles in such layered soils are slightly less than the deflections of piles embedded in uniform soils, but the values of maximum moments of piles embedded in such layered soils are slightly greater than the values of maximum moment of piles embedded in uniform soil. Smaller deflection causes smaller soil reaction, which shifts the location of maximum moment deeper. Thus the moment is higher. However, for all practical cases the horizontal capacity of such piles (tip resting on stiff soil) may be determined by using the design charts of piles embedded in uniform soil having $k_8$ of top soft layer.
Fig. 4.16(a) Maximum deflection of piles embedded in uniform soil and in layered soils [pile tips resting on hard soil]
Fig. 4.16(b) Maximum Moments in piles embedded in uniform soil and in layered soils [pile tips resting on hard soil]
4.4.2 Effect of a Thin Stiff Layer at Top

To study the effect of a thin stiff layer at top on the lateral capacity of vertical piles, a 40 ft pile having 20 inch, 24 inch, and 30 inch diameter embedded in soil profile as shown in Fig. 4.17 was analyzed. The upper layer and lower layer having soil modulus of subgrade reaction 50 kip/ft$^3$ and 10 kip/ft$^3$ respectively. The thickness of the upper layer is 2 ft. Replacing the top soil layer with $[k_s=50 \text{ kip/ft}^3]$ by a soil layer with $[k_s=100 \text{ kip/ft}^3]$ similar piles were analyzed by applying 10 kip horizontal load. The results of maximum deflections are plotted in Fig. 4.18(a). The results of similar piles embedded in a uniform soil having $k_s=10 \text{ kip/ft}^3$ are also plotted in this figure. It can be observed from the figure that the influence of a 2 ft thick layer at top is immense the deflection of the pile reduces to a great extent. For example, for a 24 inch diameter pile embedded in a soft soil having $k_s=10 \text{ kip/ft}^3$, the maximum deflection at the top is 0.7668 inch when a 10 kip load is applied. But when the top 2 ft soft soil ($k_s=10 \text{ kip/ft}^3$) is replaced by a stiff soil having $k_s=50 \text{ kip/ft}^3$ under the similar conditions the deflection is only 0.2502 inch. If the top 2 ft soil is replaced by a very stiff soil $k_s=100 \text{ kip/ft}^3$ under the same load the deflection is 0.1422 inch. Similar results were obtained for a pile having diameter 20 inch and 30 inch.

For piles embedded in soil layer as shown in Fig. 4.17 the maximum moments due to 10 kip horizontal load are shown in Fig 4.18(b). The results of similar piles embedded in uniform soil having $k_s=10 \text{ kip/ft}^3$ are also plotted in this figure. For a given horizontal load (10 kip) the value of maximum moment reduces to a great extent. For example, for a 24 inch diameter pile embedded in a soft soil having $k_s=10 \text{ kip/ft}^3$, the maximum moment is 49.10 kip-ft when a 10 kip load is applied. But when the top 2 ft soft soil ($k_s=10 \text{ kip/ft}^3$) is replaced by a stiff soil having $k_s=50 \text{ kip/ft}^3$, under similar conditions
Fig. 4.17 Pile Subjected to Lateral Load Embedded in Layered Soil

- Pile Length = 40 ft
- Pile Dia. = 24 inch
- Horizontal Load = 10 kip

- 38 ft, $k_s = 10 \, \text{kip/ft}^3$

P = 10 kip
Fig. 4.18(a) Maximum deflection of piles embedded in uniform soil and in layered soils [with a thin stiff soil layer at top]
Fig. 4.18(b) Maximum Moments in piles embedded in uniform soil and in layered soils (with a thin stiff soil layer at top)
the moment is only 21.05 kip-ft. If the top 2 ft soil is replaced by a very stiff soil $k_s=100$ kip/ft$^2$, under the same load the moment is 16.22 kip-ft. Similar results were obtained for a pile having diameter 20 inch and 30 inch.

A thin stiff layer at the ground surface reduces the deflections and moments to a great extent when a pile is subjected to a given horizontal load. That is the horizontal capacity of a vertical pile can be increased significantly by replacing soil of the top layer with a thin stiff layer. Stiff soil at top causes greater soil reaction at the top then the moment is smaller.
4.4.3 Effect of the thickness of top stiff layer

To study the effect of layer thickness of the top stiff layer, two layer soil system was used. The top layer having \( k_s = 50 \text{ kip/ft}^2 \) and bottom layer \( k_s = 10 \text{ kip/ft}^2 \), is shown in Fig. 4.17. The thickness of the top layer was 2 ft. The pile was subjected to a horizontal load of 10 kip. By changing the thickness of the upper layer by 4 ft and 6 ft and using a 10 kip horizontal load at the pile head, the deflections, moments and point forces were determined. The distribution of deflections, moments and point forces along the pile length are plotted in Fig. 4.19(a), 4.19(b) and 4.19(c). The results of the maximum deflections are plotted in Fig. 4.20(a). It can be observed from the figure that for a given horizontal load the deflection is less when the thickness of the upper stiff layer is thicker. However the mere presence of a thin stiff layer reduces the deflection to a great extend. But due to the variation of thickness of top layer the change in maximum deflection is not that significant.

The thickness of the top layer varying from 2 ft to 6 ft the maximum moments due to 10 kip horizontal load for piles embedded in soil layers as shown in Fig. 4.17 are plotted in Fig. 4.20(b). The results of similar piles embedded in uniform soil having \( k_s = 10 \text{ kip/ft}^2 \) are also plotted in this figure. It is interesting to note that the maximum deflection of the pile reduces with the increases of top layer thickness from 2 ft to 6 ft but the maximum moments increases with increase of layer thickness from 2 ft to 6 ft. However the increase in moment is not that significant. The reason behind this is smaller deflection caused smaller soil reaction which shifts the location of maximum moments deeper with higher maximum values of moment.
Fig. 4.19(a) Distribution of Deflections Along the Pile Length of Layered Soils Having Different Layer Thickness

- Pile Length = 40 ft
- Pile Dia. = 24 inch
- Horizontal Load = 10 kip
- Uniform Soil $k_s = 10$ kip/ft$^3$
- Top Layer $k_{st} = 50$ kip/ft$^3$ and Bottom Layer $k_b = 10$ kip/ft$^3$
- Thickness of the Layers are 2 ft, 4 ft and 6 ft
Fig. 4.19(b) Distribution of Moments Along the Pile Length of Layered Soils Having Different Layer Thickness

Pile Length = 40 ft
Pile Dia. = 24 inch
Horizontal Load = 10 kip
Uniform Soil $k_s = 10$ kip/ft$^3$
Top Layer $k_s = 50$ kip/ft$^3$ and Bottom Layer $k_s = 10$ kip/ft$^3$
Thickness of the Layers are 2 ft, 4 ft and 6 ft
Fig. 4.19(c) Distribution of Point Forces Along the Pile Length of Layered Soils Having Different Layer Thickness
Fig. 4.20(a) Maximum deflection of piles embedded in uniform soil and in layered soils [with a thin stiff soil layer at top]
Figure 4.20(b)  Maximum Moments in piles embedded in uniform soil and in layered soils [with a thin stiff soil layer at top]
Piles embedded in uniform soil of having $k_s = 5$ kip/ft$^3$ and $k_s = 15$ kip/ft$^3$ and similar piles embedded in layered soil (top layer thickness 2 ft and $k_s = 50$ kip/ft$^3$) as shown in Fig. 4.17. were analyzed. The maximum deflections and the maximum moments are plotted in Fig. 4.20(c), 4.20(d), 4.20(e), 4.20(f) and tabulated in Table 4.5. The results are as similar, that is with the increase of thickness of the top layer the maximum deflection reduces. The maximum moment increases with the increase of layer thickness. However the increase in moment is not that significant.
Pile Length = 40 ft
Horizontal Load = 10 kip
Top layer \(k_{s1} = 50 \text{ kip/ft}^3\) and bottom layer \(k_{s6} = 15 \text{ kip/ft}^3\)
Thickness of the layers are 2 ft, 4 ft and 6 ft

Fig. 4.20 (c) Maximum deflection of piles embedded in uniform soil and in layered soils [with a thin stiff soil layer at top]
Fig. 4.20(d)  Maximum deflection of piles embedded in uniform soil and in layered soils [with a thin stiff soil layer at top]
Fig. 4.20(e) Maximum Moments in piles embedded in uniform soil and in layered soils [with a thin stiff soil layer at top]
Fig. 4.20(f) Maximum Moments in piles embedded in uniform soil and in layered soils [with a thin stiff soil layer at top]

Pile Length = 40 ft
Horizontal Load = 10 kip
Top layer $k_u = 50$ kip/ft$^3$ and bottom layer $k_u = 15$ kip/ft
Thickness of the layers are 2 ft, 4 ft and 6 ft
Table 4.5  The maximum deflection and the maximum moment due to 10 kip horizontal load on a pile embedded in layer soils

$k_{su}=5 \text{kip/ft}^3$ and $k_{sl}=50 \text{kip/ft}^3$

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<th>Pile length in ft</th>
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<th>Soil modulus $k_s$ in kip/ft$^3$</th>
<th>Thickness of the upper layer in ft</th>
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\( k_{3b} = 10 \text{ kip/ft}^3 \) and \( k_{3f} = 50 \text{ kip/ft}^3 \)

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4.5 DESIGN CURVES FOR PILES EMBEDDED IN LAYERED SOILS

According to BNBC [Bangladesh National Building Code (1993)] the allowable horizontal load on the pile is 1/2 of the load that produces 1 inch deflection at the pile head. To develop a design curve a 40 ft long pile having 20 inch, 24 inch and 30 inch diameters were analyzed. The piles are embedded in two layer soil system as shown in Fig. 4.17. The upper and lower layers having soil modulus of subgrade reaction 50 kip/ft³ and 10 kip/ft³. The thickness of the upper layers are 2 ft, 4 ft, and 6 ft. The design curves and tables are developed on the basis of 1 inch deflection at pile head. Horizontal loads corresponding to 1 inch deflection for different sizes of pile diameters are plotted in Fig 4.21(a) and tabulated in Table 4.6. The capacity of the pile increase when thickness of the top layer increase. The corresponding moments are also plotted in Fig 4.21(b).

The load which produces 1 inch deflection at head for piles having length 40 ft with various diameter 20 inch, 24 inch and 30 inch embedded in soils having different values of ks are plotted in Fig. 4.21(c), and 4.21(d). This figure can be used for design purposes to calculate the allowable horizontal load on the pile. The allowable horizontal load shall be 1/2 the load determined from the figure. The moment corresponding to the allowable horizontal load will be 1/2 of moment shown in figure.
Fig. 4.21(a) Horizontal Load that Produces the 1 inch Deflection for piles embedded in layered soils [with stiff soil layer at top]
Table 4.6  Allowable horizontal loads according to BNBC (1993) on 24 inch diameter pile and corresponding maximum moments for layered soil

$k_{sl}=5$ kip/ft$^3$ and $k_{st}=50$ kip/ft$^3$

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Fig. 4.21(b)  Maximum Moment Results from the Load That Produces 1 inch Deflection for piles embedded in layered soils [with stiff soil layer at top]
Fig. 4.21(c) Horizontal Load that Produces the 1 inch Deflection for piles embedded in layered soils [with stiff soil layer at top]
Fig. 4.21(d) Maximum Moment Results from the Load that Produces 1 inch Deflection for piles embedded in layered soils [with stiff soil layer at top]
CHAPTER-5

CONCLUSIONS & RECOMMENDATIONS FOR FURTHER STUDIES

5.1 GENERAL

The present investigation has been conducted to study the load-displacement response and horizontal load capacity of vertical piles subjected to lateral load using a finite difference method. Generally uniform soil deposits are not found in nature and the piles are embedded in layered soils. The effect of two-layer cohesive soil system in the engineering behaviour of a laterally loaded pile with unrestrained head is investigated using a finite difference computer programme. Pile deflection often governs the design of laterally loaded piles, as such the possibility of reducing the pile head deflection using a stiff layer near the ground level has been investigated with particular interest.

5.2 CONCLUSIONS

Laterally loaded piles embedded in uniform soil and in layered soils were analyzed using a finite difference computer programme. From the present study following conclusions can be drawn
Piles embedded in uniform soil

1. Under a given horizontal load the deflection of the pile head depends on the modulus of subgrade reaction of the soil. The deflection decreases with the increase of subgrade reaction. The value of maximum moment decreases with the increase of soil modulus but the reduction of the value of maximum moment is insignificant.

2. The maximum deflection due to given horizontal load decreases if the length of pile increases. But the value of the maximum deflection does not increase after a certain length of pile. The critical length at which the deflection becomes constant is smaller for piles embedded in stronger soil.

3. The allowable horizontal load capacity according to BNBC [Bangladesh National Building code (1993)] is half of the horizontal load which produces 1 inch deflection at pile head. The deflection at pile head, subjected to a given horizontal load is smaller for piles embedded in stiff soils than piles embedded in soft soils. Therefore the allowable capacity of a pile is greater if the pile is embedded in stiff soil (when soil modulus is higher). However the maximum moment corresponding to allowable load capacity increases with the increase of soil modulus. The reason behind this is the increase in allowable horizontal capacity. Higher horizontal load produces higher maximum moment. To attain higher allowable capacity of a pile embedded in a better soil the reinforcement of the pile should be increased according to the value of maximum moment.
Piles embedded in Layered soils

1. The horizontal capacity and corresponding moment of a pile embedded in soft soil and
   tip resting on a stiff soil is similar to a pile embedded in a uniform soil having properties
   of the top soft layer.

2. When a pile is subjected to a given horizontal load, the presence of a thin stiff layer at
   ground surface reduces the deflections and moments to a great extent. That is the
   horizontal capacity of a vertical pile can be increased significantly by replacing the upper
   soil with a thin stiff layer. The horizontal capacity depends on the modulus of subgrade
   reaction and thickness of the top stiff layer.

3. When a pile is subjected to a given horizontal load the deflection of piles decreases
   with the increase of thickness of the top stiff layer. However the effect of thickness of the
   top layer on the values of maximum moment is insignificant.

5.3 RECOMMENDATIONS FOR FURTHER STUDY

The present research has covered the engineering behaviour of a laterally
loaded vertical piles embedded in a two layer cohesive soil system. It is recommended to
extend this research in the following fields to establish a better picture covering the
behaviour of laterally loaded piles.
(i) To perform field test on laterally loaded piles embedded in uniform and layered soil to compare the analytical results.

(ii) Three or more than three layer soil system can be investigated.

(iii) Piles subjected to inclined loads can be investigated.

(iv) Investigation may be carried out on restrained piles.
References


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APPENDIX

COMPUTER PROGRAMME
LATERAL-PILE SOLUTION BY FINITE DIFFERENCES TECHNIQUE
CO-EFFICIENT MATRIX FOR LATERALLY LOADED PILE
VALID FOR VERTICAL PILE ONLY
MUST MODIFY X FOR BATTER PILE
SOIL MODULUS OF SUBGRADE REACTION CONSTANT THAT MEANS THE
SOIL IS CLAY
THIS IS THE 20 DIVISION PROGRAM

PPILE LOADING
E-PILE MODULUS OF ELASTICITY
PI-PILE MOMENT OF INERTIA
ESM-SOIL MODULUS
PL-PILE LENGTH
H-SLOPE OF PILE
T-NO. OF PILE DIVISIONS
SKO-SUBGRADE MODULUS AT GROUND SURFACE
DX-DISTANCE ABOVE THE GROUND FOR APPLICATION OF LOADING
LAST EQUATION HAS BEEN DIVIDED BY A CONSTANT FACTOR OF 1000

DIMENSION SK(21),A(21),CM(21,21),EM(21,21),PF(21),FM(21)
DIMENSION PM(21,1),DEF(21,1)
INTEGER T,SKO
OPEN(UNIT=5,FILE='DATA',STATUS='OLD')
OPEN(UNIT=6,FILE='SSS30UT',STATUS='NEW')
READ(5,10)T,SKO,E,PI,PL,ESM
10 FORMAT(213,4F10.4)
READ(5,15)DX,P,D
15 FORMAT(3F10.3)
READ(5,17)ESM2
17 FORMAT(F10.4)

FORMATION OF THE SK(21) AND A(21) MATRIX

WRITE(6,18)
18 FORMAT('THE SK(21) MATRIX ARE AS follows',THE A(21)
1 MATRIX ARE AS FOLLOWS'///)
   X=PL/T
   B=(E*PI)/(X**2)
   N=T+1
   M=N-1
   L=N-2
   LL=N-3
   NN=N-17
   MM=I-3
   DO 90 I=1,N
      IF(I.LE.NN)THEN
         ESM=ESM1
         D=1.5*D1
         D=D1
         ELSE
         ESM=ESM2
         D=D1
         ENDIF
         SK(I)=(D*ESM)+SKO
         A(I)=(SK(I)*(X**2))/(6.0*B)
         WRITE(6,20)SK(I),A(I)
      CONTINUE
   DO 12 I=1,NN
      ESM=ESM1
      D=1.5*D1
      D=D1
      SK(I)=(D*ESM)+SKO
      A(I)=(SK(I)*(X**2))/(6.0*B)
      WRITE(6,13)SK(I),A(I)
   CONTINUE
   DO 14 I=NN+1,LL
      ESM=ESM2
      D=D1
      D=1.5*D1
      SK(I)=(D*ESM)+SKO
      A(I)=(SK(I)*(X**2))/(6.0*B)
      WRITE(6,16)SK(I),A(I)
   CONTINUE
   DO 21 I=LL+1,N
      ESM=ESM3
      D=D1
      D=1.5*D1
      SK(I)=(D*ESM)+SKO
      A(I)=(SK(I)*(X**2))/(6.0*B)
      WRITE(6,22)SK(I),A(I)
   CONTINUE
IF (I.GT.0.0.AND.I.LT.NN) GOTO 57
PI=PI1
D=D1
DO 90 I=1,N
IF (I.GT.NN) THEN
PI=PI2
D=D2
ELSE
PI=PI1
D=D1
ENDIF
B=(E*PI)/(X**2)
SK(I)=(D*ESM)+SKO
A(I)=(SK(I)*(X**2))/(6.0*B)
WRITE(6,20)SK(I),A(I)
20 FORMAT(10X,F12.7,10X,F12.7,10X,F12.7)
90 CONTINUE

DO 90 I=1,N
B=(E*PI)/(X**2)
SK(I)=(D*ESM)+SKO
A(I)=(SK(I)*(X**2))/(6.0*B)
WRITE(6,20)SK(I),A(I),B
20 FORMAT(10X,F12.7,10X,F12.7,10X,F12.7,10X,F12.7)
90 CONTINUE

GENERATE THE CM(21,21) MATRIX

WRITE(6,23)

THE CM(21,21) MATRIX ARE AS FOLLOWS:

CM(1,1)=1.0+(2.0*A(1))
DO 200 I=2,L
J=I
CM(I,J)=1.0+6.0*A(I)
200 CONTINUE
CM(M,M)=-6.0*A(M)
CM(2,1)=5.0*A(1)
DO 260 I=3,L
J=1
K=I-1
CM(I,J)=CM(K,J)+3.0*A(J)
260 CONTINUE
DO 270 J=2,L
CMC=12.0
R=0.0
K=J+1
DO 270 I=K,M

CM(I,J) = (CMC+R)*A(J)
R = R + 6.0

270 CONTINUE
CM(M,L) = 12.0*A(L)
CM(N,1) = (3.0*SK(1)*X)/1000
CM(N,N) = (3.0*SK(N)*X)/1000
DO 280 J = 2, M
I = N
CM(I,J) = (6.0*SK(J)*X)/1000
280 CONTINUE
DO 400 I = 1, L
J = I + 1
CM(I,J) = -2.0 + A(J)
K = I + 2
CM(I,K) = 1.0
400 CONTINUE
CM(M,N) = A(N)
DO 600 I = 1, LL
MM = I + 3
DO 600 J = MM, N
CM(I,J) = 0.0
600 CONTINUE
WRITE(6,25)((CM(I,J), J = 1, N), I = 1, N)
25 FORMAT(1X, F11.5, 2X, F11.5, 2X, F11.5, 2X, F11.5)
IF (DX - 0.0) 605, 610, 605
605 CONTINUE

GENERATE THE EM(21,21) MATRIX

WRITE(6,28)
28 FORMAT(/ / / / / /)
THE EM(21,21) MATRIX ARE AS FOLLOWS'
1 / / / / / /
EM(1,1) = 2.0
EM(1,2) = -5.0
EM(1,3) = 4.0
EM(1,4) = -1.0
EM(2,1) = -5.0
EM(2,2) = 18.0
EM(2,3) = -24.0
EM(2,4) = 14.0
EM(2,5) = -3.0
DO 635 J = 5, N
EM(1,J) = 0.0
635 CONTINUE
DO 636 J = 6, N
EM(2,J) = 0.0
636 CONTINUE
DO 650 I = 3, N
DO 650 J = 1, N
EM(I,J)=CM(I,J)
CONTINUE
WRITE(6,30)((EM(I,J),J=1,N),I=1,N)
FORMAT(10X,F11.7,4X,F11.7,4X,F11.7,4X,F11.7)

GENERATE THE PM(21,1) MATRIX

WRITE(6,33)
FORMAT(///' THE PM(21,1) MATRIX ARE AS FOLLOWS'///)
PM(1,1)=(P*DX)/B
PM(2,1)=(2.0*X*P)/B
PM(1,1)=(P*DX+P*X)/B
PM(2,1)=(P*DX+2.0*X*P)/B
DO 700 I=3,M
Z=I
PM(I,1)=P*((Z*X+DX)/B)
CONTINUE
PM(N,1)=(6.0*P)/1000

GENERATE THE PM(21,1) MATRIX

WRITE(6,34)
FORMAT(///' THE PM(21,1) MATRIX ARE AS FOLLOWS'///)
WRITE(6,35)(PM(I,1),I=1,N)
FORMAT(20X,F12.8)

INVERT THE CO-EFFICIENT MATRIX

CALC MATINV(EM,EM,N)
GO TO 715
IF(N-11)655,657,655
IF(N-11)656,657,656
WRITE(6,40)((EM(I,J),J=1,N),I=1,N)
FORMAT(10X,F11.7,4X,F11.7,4X,F11.7,4X,F11.7)
CALC MATINV(CM,CM,N)
DO 712 I=1,N
DO 712 J=1,N
EM(I,J)=CM(I,J)
CONTINUE
DO 750 K=1,N
  DEF(K,1)=0.0
  DO 750 J=1,N
    DEF(K,1)=DEF(K,1)+(EM(K,J)*PM(J,1))
  CONTINUE
PF(N)=X*(2.0*SK(N)*DEF(N,1)+SK(M)*DEF(M,1))*(0.1667)
PF(1)=X*(2.0*SK(1)*DEF(1,1)+SK(2)*DEF(2,1))*(0.1667)
DO 775 I=2,M
  J=I-1
  K=I+1
  PF(I)=(X/6.0)*(SK(J)*DEF(J,1)+4.0*SK(I)*DEF(I,1)+SK(K)*DEF(K,1))
CONTINUE
FM(1)=P*DX
FM(2)=(P*(DX+X)-PF(1)*X)
FM(3)=(P*(DX+2.0*X)-PF(1)*2.0*X-PF(2)*X)
FM(N)=0.0
DO 774 I=4,M
  J=I-1
  K=I+1
  FM(I)=B*(DEF(J,1)-2.0*DEF(I,1)+DEF(K,1))
CONTINUE
DO 755 I=1,N
  FM(I)=FM(I)/12.0
CONTINUE
WRITE(6,50)(FM(I),I=1,N)
FORMAT(6X,4(F10.2,3X))
WRITE(6,44)
FORMAT(///1X,'THE DEFLECTIONS ARE :- ',2X,'THE POINT FORCES ARE '
THE MOMENTS ARE :-'///)
DO 900 I=1,N
  WRITE(6,60)DEF(I,1),PF(I),FM(I)
FORMAT(6X,F10.4,12X,F12.5,12X,F12.5)
CONTINUE
STOP
END

SUBROUTINE MATINV(A1,AINV1,N)
REAL A1(21,21),AINV1(21,21),R(21,42)
DO 101 I=1,N
  DO 101 J=1,N
    R(I,J)=A1(I,J)
  J1=N+1
  J2=2*N
  DO 102 I=1,N
    DO 102 J=J1,J2
      R(I,J)=0.0
DO 103 I=1,N
   J=I+N
103 R(I,J)=1.0
DO 104 K=1,N
   KP1=K+1
   IF(K.EQ.N) GO TO 105
   L=K
   DO 106 I=KP1,N
   C=ABS(R(I,K))
   D=ABS(R(L,K))
106 IF(C.GT.D) L=I
   IF(L.EQ.K) GO TO 105
   DO 107 J=K,J2
   TEMP=R(K,J)
   R(K,J)=R(L,J)
107 R(L,J)=TEMP
105 DO 108 J=KP1,J2
108 R(K,J)=R(K,J)/R(K,K)
   IF(K.EQ.1) GO TO 109
   KM1=K-1
   DO 111 I=1,KM1
   DO 111 J=KP1,J2
111 R(I,J)=R(I,J)-R(I,K)*R(K,J)
   IF(K.EQ.N) GO TO 112
109 DO 104 I=KP1,N
   DO 104 J=KP1,J2
104 R(I,J)=R(I,J)-R(I,K)*R(K,J)
112 DO 113 I=1,N
   DO 113 J=1,N
   K=J+N
113 AINV1(I,J)=R(I,K)
RETURN
END