# RATIONAL DESIGN OF DEEP BEAMS WITH OPENINGS

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## RATIONAL DESIGN OF DEEP BEAMS WITH OPENINGS

# A Project Report

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## DECLARATION

It is to declare that the work embodied in this thesis is the result of the investigation carried out by the author under the supervision of Dr. Salek M. Seraj, Professor, Department of Civil Engineering, Bangladesh University of Engineering and Technology, Dhaka, Bangladesh, except where specific references are made to the other investigators. Neither this thesis nor any part of it has been submitted or is being concurrently submitted elsewhere for any other purpose (except for publication).

December, 2002

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#### **ABSTRACT**

Deep beam is a very important structural element in various types of concrete structures such as transfer girder, panel beams, foundation walls of rectangular tanks, bins, shear walls, etc. Beams whose span to depth ratio is about 5 or less are categorized by the ACI Building Code as deep beams. The current ACI Building Code has provisions for the design of deep beams subjected to flexure and shear only. Several new lines of thinking have developed in the recent past in order to unify the design of various structural members. In the design of reinforced concrete deep beams, it is sometimes necessary to provide opening for service or access. Deep beam with web opening is not yet covered by the current code of practice. A very limited study on deep beams with openings and some studies on beams with openings are available in the literature.

In the present study a laboratory investigation has been carried out on seven model deep beam specimens having shear-span-to-depth ratio equal to 1.0. Whereas only one of the deep beams was solid in nature, the other six deep beams had openings. All the seven model deep beams were of 1600 mm x 400 mm x 130 mm size. Three types of web openings having sizes 150 mm x 150 mm, 300 mm x 160 mm and 450 mm x 160 mm were kept in the deep beam model specimens. The solid deep beam was designed in compliance to the concept of compressive force path (CFP). The other six deep beams were designed following the tenets of the CFP method coupled with the recommendations of Mansur and Tan in their works related to the analysis and design of concrete beams with web openings. Out of the two groups of deep beams having openings, the top and bottom chord above and below the openings of the first group were provided with special confinement reinforcement at a spacing half the effective chord depth. The other set of deep beams with openings were very much similar to the first group, but had additional horizontal confinement steel and diagonal steel around the openings.

The results of the study revealed that deep beams with or without openings may be designed following the method adopted in the study as all the deep beams attained similar initial cracking load as well as final failure load. The beneficial effect of confining steel in arresting early crack propagation and attainment of higher failure load could also be ascertained from the study. For all the deep beams tested, the compressive force path has played a significant role in carrying the applied loads from the loading points to the supports. A limited numerical investigation has displayed that numerical tools may essentially play a vital role in determing the CFP in a reinforced concrete member.

# CONTENTS

DEC	LARATION	iv
		**
	NOWLEDGEMENT	v
	TRACT	vi
	TENTS	vii
LIST	OF TABLES	x
LIST	OF FIGURES	xi
NOT	ATION	xiii
CHA	APTER ONE GEN	ERAL CONSIDERATION
1.1	Introduction	. 1
1.2	Statement and the problem	1
1.3	Objective and Methodology of the	research 1
1.4	Scope of the experimental study	2
1.5	Outline of research	2
CHA	APTER TWO REVI	EW OF THE LITERATURE
2.1	General	4
2.2	Study on the previous research pa	per 5
2.2.1	Analytical methods	5
2.2.2	Experimental methods	7
2.2.3	Truss analogy/truss (strut and tie)	
2.2.4	Deep beams with web opening	15
2.2.5		17

# CHAPTER THREE METHODS FOR THE ANALYSIS AND DESIGN OF DEEP BEAMS

3.1	Introduction	25
3.2	ACI concept for shear and diagonal tension	27
3.2.1	Reinforced concrete beam without shear reinforcement	27
3.2.2	Reinforced concrete beams with web reinforcement	30
3.2.3	ACI concept on the shear strength of deep beams	33
3.3	BS 8110 concept for shear and diagonal tension	36
3.3.1	Shear behaviour of beams without shear reinforcement	36
3.3.2	Shear resistance of web reinforcement	37
3.3.3	British Standards Institution code for the design of shear strength	42
3.4	Kotsovos design method for deep reinforced concrete beams	45
3.4.1	Modeling	45
3.4.2	Design method	48
3.5	Deep beam model and design with opening	53
3.6	Solid element	55
CHA	APTER FOUR LABORATORY INVESTIGATION	N
4.1	General	57
4.2	Design of test beams	57
4.3	Determination of the properties of the constituents of	
	reinforced concrete	65
4.4	Reinforcement	65
1.5	Design of concrete mix	67
1.6	Preparation of moulds	67
1.7	Curing of test beams and control cylinder	68
1.8	Preparation of arrangement of load testing machine	68
1.9	Testing procedure	68
1.10	Compressive strength of associated cylinder	70

# CHAPTER FIVE RESULTS AND DISCUSSION ON PHYSICAL AND NUMERICAL EXPERIMENTATION

5.1	General		72
5.2	Behaviou	r of test beam under load	72
5.2.1	Crack pa	ttern and load carrying capacity	72
5.2.2	Mode of	failure of test beams	74
5.2.3	Deflection	n pattern	82
5.3	Numerica	al experimental	85
CHA	APTER :	SIX CONCLUSIONS AND RECOMMEND	ATIONS
6.1	Conclusio	on .	100
6.2	Recomme	endations for future study	101
REF	ERENC	ES	
Appe	endix - A	Design of beams B1, B2, B3, B4, B5, B6 and B7	106
Appe	endix - B	Calibration of loading jack	111
Anne	ndiv - C	Mid - Span deflection of all test enecimens	113

# LIST OF TABLES

Table no.	Description	Page no.
Table 3.1	Design concrete shear stress v <sub>c</sub> for fcu≥ 40 N/mm <sup>2</sup>	43
Table 4.1	Grading of aggregates	66
Table 4.2	Physical properties of aggregates	66
Table 4.3	Physical properties of reinforcements	71
Table 4.4	Compressive strength of associated cylinder's	71
Table 5.1	Observed cracking and ultimate loads of test Beams	84

# LIST OF FIGURES

Figure no.	Description	Page no
Figure 2.1	Schematic representation of the path of compressive force and corresponding out line of compressive stress trajectories for RC deep beams with various a/d ratio	24
Figure 2.2	Typical inclined compression failures of RC deep beams under (a) Uniform and (b)Two-point loading	24
Figure 3.1	Stresses in homogeneous rectangular beams	26
Figure 3.2	Forces at diagonal crack in beams with vertical stirrups	32
Figure 3.3	Forces at diagonal crack in beams with inclined web reinforcement	32
Figure 3.4	Crack pattern for different shear span-to-depth (a <sub>v</sub> /d) ratios	38
Figure 3.5	Typical model of Truss Analogy	38
Figure 3.6	Proposed model for simply supported reinforced concrete beam	46
Figure 3.7	Proposed model for deep beams under (a) Single point (b) Two-Point and / or for uniform loading	46
Figure 3.8	Proposed model for continuos reinforced concrete deep beams	47
Figure 3.9	Typical types of behavior exhibited by RC beams without shear reinforcement	47
Figure 3.10	Design procedures of Types iv behavior	49
Figure 3.11	Schematic representation of members of proposed model design as column	49
Figure 3.12	Additional internal actions developing due to presence of transverse reinforcement in region where path changes direction for Type II behaviour	50
Figure 4.1a	Reinforcement arrangement of Beam B1	58
Figure 4.1b	Reinforcement arrangement of Beam B2	59
Figure 4.1c	Reinforcement arrangement of Beam B3	60
Figure 4.1d	Reinforcement arrangement of Beam B4	61
Figure 4.1e	Reinforcement arrangement of Beam B5	62
Figure 4.1f	Reinforcement arrangement of Beam B6	63
Figure 4.1g	Reinforcement arrangement of Beam B7	64
Figure 4.2	Loading of model test specimen	69
Figure 5.1a	Crack pattern and individual jack load intensities (P3=P4) in tonnes of beam B1	75
Figure 5.1b	Crack pattern and individual jack load intensities (P3=P4) in tonnes of beam B2	76
Figure 5.1c	Crack pattern and individual jack load intensities (P3=P4) in tonnes of beam B3	77
		78
Figure 5.1d	Crack pattern and individual jack load intensities (P3=P4) in tonnes of beam B4	
Figure 5.1e	Crack pattern and individual jack load intensities (P3=P4) in	79

tonnes of beam B5	
Crack pattern and individual jack load intensities (P3=P4) in tonnes of beam B6	80
Crack pattern and individual jack load intensities (P3=P4) in tonnes of beam B7	81
Load deflection curves of the deep beams tested	83
Von Mises stresses of beam B1	86
Von Mises stresses of beam B2	87
Von Mises stresses of beam B3	87
Von Mises stresses of beam B4	89
Von Mises stresses of beam B5	90
Von Mises stresses of beam B6	91
Von Mises stresses of beam B7	92
Crack pattern of beam B1	93
Crack pattern of beam B2	94
Crack pattern of beam B3	95
Crack pattern of beam B4	96
Crack pattern of beam B5	97
Crack pattern of beam B6	98
Crack pattern of beam B7	99
	Crack pattern and individual jack load intensities (P3=P4) in tonnes of beam B6 Crack pattern and individual jack load intensities (P3=P4) in tonnes of beam B7 Load deflection curves of the deep beams tested Von Mises stresses of beam B1 Von Mises stresses of beam B2 Von Mises stresses of beam B3 Von Mises stresses of beam B4 Von Mises stresses of beam B5 Von Mises stresses of beam B6 Von Mises stresses of beam B7 Crack pattern of beam B1 Crack pattern of beam B2 Crack pattern of beam B3 Crack pattern of beam B4 Crack pattern of beam B4 Crack pattern of beam B5 Crack pattern of beam B5 Crack pattern of beam B6

#### **NOTATIONS**

 $\theta$  = Inclination of shear crack

 $\alpha$  = Inclination of shear reinforcement

 $\rho_h$  = Horizontal web steel ratio

 $\rho_{\rm v}$  = Vertical web steel ratio

 $\rho_w$  = Main tension steel ratio

 $a_v$  = Shear span

 $A_s$  = Area of main tension steel

 $A_{sh}$  = Area of horizontal web steel ratio

 $A_{sv}$  = Area of vertical web steel ratio

A<sub>v</sub> = Distance between center of support to center of load

b = Width of beam

d' = Distance of the centroid of compression steel from top surface

d = Effective depth

f = Bending stress

fc avg. = Average compression strength of 3 sample cylinders =  $f_c$ 

 $f_c$  = Concrete cylinder strength

 $f_h$  = Yield strength of horizontal web steel

 $f_{\rm v}$  = Yield strength of vertical web steel

 $f_y$  = Yield strength of main tension steel

h = Total height of the beam

l = Effective span

L = Span; center to center of supports

 $L_n/I_0$  = Clear span; distance between faces of supports

M = Moment at critical section

 $M_a$  = Applied moment

M<sub>c</sub> = Moment resisted by concrete

M<sub>cr</sub> = Maximum moment at initial diagonal cracking load

M<sub>f</sub> = Flexural moment capacity

M<sub>u</sub> = Maximum moment at ultimate failure load

704

p = Horizontal distance of shear crack

p<sub>cr</sub> = Single jack load at initial diagonal crack

p<sub>f</sub> = Single jack load at flexural crack

p<sub>u</sub> = Single jack load at ultimate failure

s = Distance of the critical cross section from support

s<sub>h</sub> = Spacing of horizontal web steel

 $s_v$  = Spacing of vertical web steel

t = Principal stress

v = Average shear stress

V = Shear force at critical section

 $V_c$  = Shear resisted by concrete

v<sub>c</sub> = Shear stress taken by concrete

x = Clear shear span

 $x_e$  = Effective clear shear span

z = Lever arm of flexural moment

# CHAPTER ONE GENERAL CONSIDERATION

#### 1.1 INTRODUCTION

In the construction of modern buildings, many pipes and ducts are necessary to accommodate essential services like water supply, sewage, air-conditioning, electricity, telephone and computer network. Usually these pipes and ducts are placed under sofit of the beam and, for aesthetic reasons, are covered by a suspended ceiling, thus creating a dead space. Beams whose span to depth ratio is about 5 or less and that are loaded at the top or compression faces are used occasionally as structural members. Such members are categorized by the ACI building code as deep beams. The present study deals with the design of such structural members with openings.

#### 1.2 STATEMENT AND THE PROBLEM

The investigation of the stresses in the deep beam is not new. But stress investigation in deep beam with web opening is rare. The behaviour of deep beam with web opening is of complex nature. Due to an abrupt change in the cross-sectional dimensions of the deep beam with stress concentration that may lead to wide cracking that is unacceptable from aesthetic and durability view point. The reduced stiffness of the beam may also give rise to excessive deflection under service load and result is considerable redistribution of internal forces and moments unless special reinforcement is presided in sufficient quantity, the strength of such a deep beams may be reduced to a critical degree.

#### 1.3 OBJECTIVE AND METHODOLOGY OF THE RESEACH

There are many structural elements the behaviour of which is significantly different from the beam's with normal proportions. It is the purpose of this study to try to understand the load carrying capacity of deep beams with openings and to arrive at a rational design methodology. In order to achieve this objective, a number of deep

beams has been designed, manufactured and tested at the concrete structures laboratory of Bangladesh University of Engineering and Technology (BUET). While the first beam was designed in accordance to the Compressive Force Path (CFP) concept, Kotsovos(1988b), six other deep beams with various sized openings were designed by following the basic tenets of the CFP concept. During the course of designing the deep beams with openings, reinforcement arrangement around the opening was varied in order to assess performance of the beams. Apart from physically testing the deep beams in the laboratory, limited numerical experimentation was also conducted by using STAAD. Pro as a numerical tool.

#### 1.4 SCOPE OF THE EXPERIMENTAL STUDY

A limited number of number of tests were conducted to verify the application of compressive force path in a deep beams with or without openings. One solid deep beam and six deep beams with openings were designed and tested in the laboratory. Load carrying capacity, crack pattern, mode of failure and deflection pattern of all these deep beam have been studied and compared.

For all the beams, a span to depth (a/d) ratio of one has been chosen. The size of the opening has been varied. Two concentrated loads were applied on the top of the beams maintaining the a/d ratio. Loads were placed at the top and supports were paced at the bottom. Size of the bearing plates was such that they distribute the loads within the safe limit of the bearing stress of the concrete. Deflectometers were suitably placed and test conditions like support mechanism, loading condition and rate of loading were kept practically the same for all the beams tested.

#### 1.5 OUTLINE OF THE RESEARCH

The present study encompasses experimental and numerical study on deep beams with various sized openings. Chapter 2 of the thesis presents a general review of various available on the general area of research on deep beams. Chapter 3 also describes the Compressive Force Path Method as applied to the design of deep beams. Chapter 4 narrates the details of the experimental work conducted during the present study. Test results and interpretation of test results have been discussed in

chapter 5. This chapter also briefly addresses the numerical experimentation conducted in this study. Finally conclusions have been drawn and recommendations have been made in Chapter 6.

## **CHAPTER TWO**

#### REVIEW OF THE LITERATURE

#### 2.1 GENERAL

A review of the literature reveals that the investigation on the behaviour of deep beams is not a new subject. It started at the end of 19th century. Deep beams have been subjected to many papers (as referred) covering theoretical and experimental aspects that have been included in the design documents. Analysis of deep beams under shear is quite complex because large numbers of parameters involve in this problem. Because of the complexities involved, exact theoretical basis for the analysis of deep beams in shear is not still developed. On the other hand, deep beams are frequently continuous. Due to lack of understanding the exact mechanism of shear failure, the design concept, design methods are very different throughout the different nations of the world. Most of them are empirical or semi-empirical which have come out depending on some test results. So the codes do not usually provide an accurate prediction of shear strength of deep beams, rather they usually predict very conservative strength. There are two basic approaches used to analyse the shear and torsion problem; one is mechanism method and the other is truss model theory. Mechanism method is the basis for ACI code. In this method it is assumed that the major contribution to shear resistance in beams without shear reinforcement is provided by the region of the beam below the neutral axis through the mechanism of aggregate interlock and dowel action of the longitudinal reinforcement. But recent test results, however, clearly raise doubts over these generally accepted views. By fitting the mechanism method to the test results, the ACI method becomes empirical or at best semi-empirical. From the theoretical point of view this method cannot satisfy the compatibility condition and in certain cases, even equilibrium condition. Although new concept, e.g., compression field theory, modified compression field theory and the strut and tie model have recently evolved in the general field of structural concrete design, most of these deviate very little from the basis on which present-day design is founded; and, thus, they carry implicit assumptions, which in many cases, are incompatible with the fundamental properties of concrete. So, due to lack of exact mechanism, when one's method is applied to tested data of others, it

predicts unsatisfactory results. So it is obvious that further close investigation and analysis are required in this field. However, over the few years it has been argued that, in contrast to these widely held views, the shear resistance of RC beams is in fact provided by the region of the path along which the compressive force is transmitted to the supports, rather than the region of the beam below the neutral axis. In the absence of an exact and simplified method, this new thinking (Compressive Force Path) seems to give an adequate explanation of the shear transfer mechanism of deep beams. The CFP, which departs radically from the established design concept, seems to give an adequate and rational explanation of the behaviour of various structural concrete members.

## 2.2 STUDY ON THE PREVIOUS RESEARCH PAPERS

## 2.2.1 Analytical Methods

Dischinger (1932) started the investigation of stresses in deep beams. He solved the problem of a continuous deep beam subjected to periodic loading by representing the load in terms of a Fourier series. He constructed a stress function satisfying the boundary conditions and governing differential equation in terms of hyperbolic functions. To handle a deep beam of finite span, Dischinger suggested that the results of the continuous deep beam be used. His solution for a finite beam did not satisfy the free end boundary conditions because the finite beam was being supported by shearing stresses and the finite span did not satisfy the condition of zero bending stress at its free ends.

Chow et al. (1951) undertook the investigation of a single span deep beam. He used the solution of a continuous deep beam, which satisfied all but one of the boundary conditions. He then constructed another stress function via the principle of least work, which negated the residual boundary stresses. Chow checked his results by solving the governing differential equation by the method of finite differences. Although, Chow claimed that the principle of least work yielded more accurate results than the method of finite differences, he reverted to the method of finite differences in his next study of a single span beam subjected to five loading cases

and three depth-to-span ratios (1953). Although Chow was able to predict the general behaviour of finite deep beams, he could not produce accurate results at all cross sections of the beam.

Kitchen and Archer (1956) utilised the method of least work to construct a stress function, but it was different from Chow's work in that it did not depend on the solution of the continuous beam. Their bending stress values agreed fairly well with the results of Chow, but their shear stress results were not in good agreement.

Geer (1960) studied the effects of load length vs. clear span. He used the method of finite differences but with a much finer computational grid than Chow. His results had shown that the distribution of the load could significantly affect the normal stress characteristics and the maximum values. Furthermore, maximum compressive bending stresses were found to exist below the mid-height of the beam. No data for the distribution of the shear stresses were given.

Holmes and Mason (1972) also used the method of least work to solve the problem of a single span deep beam supported by a parabolic shearing force applied at the vertical edges. This loading condition did not take into account the high bearing pressures normally associated with the reactions. Their results did not differ from the results obtained by shallow beam theory by as much as might be expected for beams of deep proportions.

Barry and Heino (1983) used the multiple Fourier technique to compare the stress fields in single span deep beams due to uniform loading at the top edge and at the bottom edge. The method involved the superposition of stress functions. The first stress function was used to satisfy the boundary conditions on the upper and lower edges of the beam. The second and third stress functions were used to satisfy the boundary conditions on the vertical edges of the beam. This approach allows satisfying all the required boundary conditions. Contour maps of the stress field revealed the existence of the regions of pure tension and pure compression. These regions indicated proneness to spalling, bursting or crushing.

#### 2.2.2 Experimental Methods

Pavia and Siess (1965) tested 19 simply supported beams to describe and investigation of the shear strength behaviour of some moderately deep reinforced concrete beams. The major variables involved in his study were the amount of tension reinforcement, the concrete strength, and the amount of web reinforcement and span depth ratios. The beams were loaded at third points. Current shear and flexural strength concepts, as applied to shallow members, were considered in relation to the strength and behaviour of the test specimen. From the test results the authors suggested that the shallow beam theory is not applicable to deep beams because of the effect of load concentrations on the distribution of internal stresses. There seems to be, however, a gradual transition from shallow beam behaviour to deep beam behaviour. The transition range appeared to be between span-depth ratios, L/d, of 2 and 6. Such beams are defined as moderately deep beams. From the load deflection curves, two major stages of behaviour were observed. First, the elastic behaviour of the beams up to yielding of the tension reinforcement, and, second, the "inelastic" behaviour after yielding and up to the ultimate when deformation becomes large with respect to the load. Flexural cracks were observed at sections of maximum moment, and inclined cracks originated near the support and propagate upward and toward mid-span. Once the inclined cracks have formed, the strain in the tension reinforcement is nearly uniform along the beam, and the concrete strains tend to concentrate near the mid-span over the ends of the inclined cracks. Arch behaviour in deep beams causes high stresses in the tension reinforcement at the supports. The failure was similar in appearance to the shear compression failure of conventional beams but was usually accompanied by extensive inelastic deformation. For the test beams that failed in shear, inclined cracks formed prior to failure. A second inclined crack, which extended from the load point to the support outside the first inclined crack, gave the beam a "strut like" appearance. The failure of this strut in compression was accompanied by the shearing off of the unloaded part of the beam outside the load blocks and the unbonding of the tension steel over the supports.

Kong, Robins and Cole (1970) tested 35 rectangular deep beams of 30 in. simple span, 36 in. over-al length, 3 in. width and over-all span/depth ratios (L/h) ranging from 1 to 3. The test beams were divided into seven series, according to the type and amount of web reinforcement. Each of the beams was tested under two point loading. Each time one type of web steel was used to identify the individual effect of web reinforcement. He found that relatively widely spaced horizontal and vertical web reinforcements were not effective in controlling deflections but reduced significantly even by a moderate amount of horizontal bars placed near the bottom. Deflection did not significantly affected by the cylinder strength of concrete. As L/h and x/h (clear shear span/total depth) ratios increased, the relative effectiveness of horizontal web reinforcement decreased. Average crack width was smaller of closely spaced horizontal bars near the bottom. This type of bars was largely effective for low L/h and x/h ratios. Vertical stirrups became effective with increasing L/h and x/h ratios. The effectiveness of horizontal and vertical stirrups was insignificant for moderately web reinforced and moderately deep beams. The crack widths were largest for beams with no web reinforcement. Crack widths were increased with increasing L/h and x/h ratios. With a few exceptions, the crack pattern and mode of failure did not affect by the web reinforcement and in L/h and x/h ratios. The typical behaviour was first flexural cracks formed and propagate rapidly and then diagonal cracks formed at higher loads. When the load reached about 70 to 90 percent of the ultimate, a new type of diagonal crack suddenly appeared within the shear span. These diagonal cracks were the dangerous cracks and they had the common property of initiating not at the soffit but at points about h/3 up from the soffit. On further loading, failures were occurred in one of the three ways. (a) one or more of these existing diagonal cracks might penetrate so deeply into the compression zone at the loading point or at the support that immediate failure occurred by crushing of the concrete there. (b) The propagation of these cracks might split the beam approximately along the line joining the load blocks at the support and loading point; or a new diagonal crack might form outside of the existing one, in which case failure could occur by either one of the above methods. (c) By crushing of the strut-like portion of concrete between two diagonal cracks. The size of the bearing block might have significantly influenced the general cracking behaviour and the final mode of failure.

Manuel, Slight and Suter (1971) tested 12 reinforced concrete deep beams in which the variables a/d and L/d were systematically varied i.e., a/d has been separated from L/d and other major variables were kept constant. All the beams were tested with two points loading. This investigation was to establish the relative importance of a/d and L/d on deep beam behaviour. The term M/Vd has been related to shear capacity of a reinforced concrete beam. For simple beam under symmetrical two-point loading or single point loading, the term M/Vd is equivalent to the term a/d at the load point sections. After analyzing the test results the authors concluded that the ultimate strength of reinforced concrete deep beams appeared to be influenced significantly by a/d and insignificantly by L/d. The influence of a/d relates to the mode of failure. While the diagonal cracking capacity of deep beams was not influenced significantly by L/d, there was an overall tendency for the diagonal cracking capacity to increase with an increase in a/d from 0.30 to 1.00.

Subedi et al. (1986) tested 13 simply supported deep beams with span depth ratio, (L/h) 1, 2 and 3. He varied span, overall depth, main longitudinal steel, vertical web steel and horizontal web steel. He also used two types of support condition, one is resemblance of practical condition and the other is more or less identical with the theoretical roller. Electric strain gauges were incorporated to some steel. The beams were divided into three series on the basis of loading types and support conditions. Different modes of failure, and a comparative study with CIRIA GUIDE-2 have been presented in this paper. He claimed that the model predict not only the shear strength but also the mode of failure of a deep beam. He observed both flexural and shear failures in the test beams. Flexural cracks at mid-span dominated the behaviour in the long span beams. However, the flexural cracks at mid-span were accompanied by inclined shear cracks emanating from the edge of the loading blocks for short span beams. In case of beams with relatively low amounts of main reinforcement failed in a ductile manner. Diagonal splitting failures were observed in which the beams contain higher amounts of main reinforcement. Most of these cases failure appeared from the diagonal splitting; bond failure occurs along the main steel. When the diagonal split first occurred, the events were accompanied by distinctive cracking

sound, sometimes quite loud. As the cracks extended, they appeared to do so relatively quietly, but splitting noises were heard frequently, probably as a result of the formation of new cracks.

Subedi (1988) proposed a method of analysis for deep beams, based on the equilibrium of forces at failure. The method was tested against the test results on 19 deep beams over a wide range of parameters. The proposed method was also used to analyze 87 deep beams tested by other researchers. In case of Beams tested by Kong et al., the author claimed that the observed mode of failure was same as predicted and the typical behaviour was diagonal splitting. The beams with higher depth showed good results but inconsistency in the failure modes. For moderate depth it was not so good and for lower depth it shown bad results. In case of Beams tested by Smith and Vantsiotis, the mode of failure had been correctly predicted for the beams that failed in the diagonal splitting mode. The predicated ultimate loads were considerably lower than the experimental values for the beams with either a small or a large amount of web reinforcement in one direction only, shown greater difference with the beams with a large amount of web reinforcement in both directions.

Ali and Habib (1992a) presented a comprehensive experimental study on the deflection and stress distribution characteristic in deep beam under uniformly distributed load. They investigated the effect of L/d ratio, variation of web reinforcement on stress distribution and deflection of deep beams. The author stated that the available theories for stress computation do not predict the stress in concrete, in flexural steel or in web steel of a deep beam. The mid span deflection could not be predicted accurately using the deflection computation methods that generally used for ordinary shallow beams. The smaller the span/depth ratio the more pronounced is the deviation of stress patterns from that of Bernoulli and Navier. Design of deep beams based on the linear distribution of bending stresses as used in shallow beams may be seriously in error, since the simple theory of flexure does not take into account the effect of normal pressure on the top and bottom edges of the beam caused by the load and reactions. Mid-span deflections were measured and compared with the theoretical values calculated from deflection formulas for ordinary shallow beam theory with both cracked and untracked sections. Efforts were made in the test

program in order to observe the effects of the variation of horizontal and vertical web reinforcement on the deflection and the stress distribution in the concrete masses, in the flexural steel and in the horizontal and vertical web reinforcements. Brick aggregates were also used in the test program because it is widely used in developing countries like Bangladesh. After investigation the authors concluded that, before the initiation of cracks, deflection of beams with L/D=2 was fairly accurately predicated by the ordinary shallow beam formula using uncracked sections. However, for beams with L/D=1, this approach grossly underestimated the actual deflection of beams. On the other hand, after initiation of cracks, ordinary shallow beam theory with cracked sections, predicts the deflections of beams fairly accurately in both the test series (L/D=1 and L/D=2).

Ali and Habib (1992b) presented another research paper depending on the test results as previously discussed. They found that the amount of web reinforcements influenced only the diagonal cracking load not the ultimate load capacity. Diagonal cracks developed first in relatively deeper beams (L/D=1) and flexural cracks developed first in the shallower deep beams (L/D=2). ACI Code underestimates the diagonal cracking shear stress for deep beams when subjected to UDL. However, the upper limit of the shear stress causing diagonal cracks set by ACI Code is in conformity with the test results, although somewhat conservative. On the other hand, the Cosio and Siess (1960) equation is not conservative for calculating diagonal shear stress for deep beams having lower L/D ratios. Vertical web reinforcement is more effective in resisting diagonal cracking in deep beams than is horizontal web reinforcement. The upper limit of  $8\sqrt{f'_c}$  for the ultimate shear stress suggested by the ACI Code was fairly conservative estimate for RC deep beams when adequate web reinforcement was provided. Moreover, the upper limit of  $6\sqrt{f'_c}$  for the contribution of concrete in ultimate shear was also conservative for RC deep beams when subjected to UDL. Shear compression failure (by crushing of concrete) was observed in relatively shallow beams (L/D=2) whereas in the deeper beams (L/D=1) diagonal tension failure was predominant.

Sultan (2000) tested five simply supported and five two span continuos deep beams and found that shear failure is common in all the cases of deep beams. Whereas, clear span to effective depth ratio has demonstrated insignificant effect, shear span to effective depth ratio has been found to be a governing parameter in controlling the shear strength. Vertical web reinforcement, strength of the concrete and amount of flexural steel have shown some influence on the shear strength.

# 2.2.3 Truss analogy / Truss (strut and tie) Model Method

Rogowsky et al. (1986b) reported the results of 7 simply supported and 17 two-span deep beams with varying shear span to depth ratios and varying combinations of vertical and horizontal web reinforcement. The design implications (strut and tie model) of these tests were discussed in a companion paper (1986a). Analysing the test results, he found that ACI design procedures for deep beams are not applicable for continuous deep beams. In the simple span beams with light stirrups or no stirrups, major inclined cracks developed almost instantaneously in both the shear span at about 50 to 60 percent of failure load. The cracks appeared to be very sever even though the load was only about 25 percent of the eventual failure load. After inclined cracking, the behaviour was essentially that of a truss or tied-arch. The stirrups crossing the major inclined cracks were at or near yield at failure. Some of these reached yield at 50 to 60 percent of the failure load. Ultimate failure was due to crushing of the compression strut. In the continuous deep beams, even though there were a large number of stirrups present, inclined cracking still occurred with a loud thud. There were crack fans over the interior support and under each load. Compression struts formed between the cracks making up these fans but tended to be less well defined than in other continuous beams with fewer stirrups. Failure was due to crushing near the top of the compression strut in the interior shear span. There was little difference in behaviour for beams with no web reinforcement, minimum stirrups, minimum horizontal web reinforcement, or maximum horizontal web reinforcement. He also found that the increase in shear strength with decrease in a/d is due to a direct compression strut from the load to the support. This has two important consequences. The slope of the compression strut is the important

parameter and is better defined by a/d than by M/Vd. Both is essentially the same in simple beams, but in continuous beams M/Vd varies greatly depending on how close the critical section is to the point of inflection. The effective concrete strength was not found to be very significant. The specimens were under-reinforced so that the stirrups and much of the flexural steel yielded before the concrete crushed. In such a beam, the actual failure strength of the concrete was not nearly as significant as was the correct determination of which steel yields. In other words, selecting an appropriate truss model is of great importance in design. An appropriate truss model is one, which correctly identifies the reinforcement, which is at yield at failure of the beam and discounts the remaining reinforcement. If the model requires a redistribution of internal forces, such redistribution must not exceed the deformation capacities of the various concrete elements involved. This has been illustrated for three cases. (a) The capacity of the equilibrium truss model for simple beams was reached when the stirrups yield and the longitudinal steel yield at mid-span. For simply supported beams, very little redistribution was required to reach that state. It was noted that the longitudinal steel carried significant force at the faces of the supports. (b) There are two trusses for a beam with one layer of horizontal web reinforcement. For an ideal plastic material, the shear capacity of the beam would be the sum of the shear capacities of the two trusses. For reinforced concrete this is not necessarily the case. The lower chord would reach yield first. The additional deformation would require for the upper layer of steel to yield so that the upper truss can reach its capacity will generally be large enough to destroy the bottom truss. So the authors suggested that, horizontal web reinforcement is ineffective in increasing the shear strength of deep beam. (c) The two-span beams with heavy stirrups were ductile and developed yielding of the top and bottom reinforcement producing full plastic mechanisms before failure.

Mau and Hsu (1987) presented a theoretical treatment of the shear behaviour of deep beam. The theoretical model consists of the application of soften truss model with an assumed effective transverse compressive stress in the beam web. He stated that, on the basis of some test results, empirical formulas have been adopted for the design of deep beams in the current ACI Code. Theoretical treatment of the subject,

however, has been scarce and unsuccessful. He identified several possible modes of failure but the predominant one is that of shear failure. The shear action in the beam web leads to compression in the diagonal direction and tension in a direction perpendicular to that. The observation of the web cracks before failure leads to a theoretical model that assumes the dependency of shear strength on concrete splitting strength. Comparing the test result of T-beams to the plasticity truss model, he indicated that an empirical factor must be included to reduce the compressive strength of concrete. He also found that the compatibility truss model also tends to overestimate the shear strength considerably if the compression strength of concrete is not reduced empirically. The extent of reduction can be related to the ratio of the two principal strains in the concrete. This phenomenon of the strength reduction has been called the softening of concrete. The softened truss model theory, which has been successfully developed for low rise shear walls and torsion, is extended to deep beams. He found that the soften truss model theory agrees well with the test results of the beams with shear span to effective depth ratio, a/d between 2.5 and 6. However, for a/d below 2.5, i.e., for deep beams, the theory underestimates the shear strength considerably because some other factors are involved in the shear strength of deep beams. For the prediction of shear strength of a deep beam, he used soften truss model theory by introducing an effective transverse compressive stress in the beam web.

Hsu (1988) summarised the soften truss model theory, which has been developed for shear and torsion of reinforced concrete member, in a systemic and unified manner. The softened truss model theory for shear and torsion consists of eleven equations involving fourteen variables has been derived from equilibrium, compatibility and materials conditions to solve the shear problem. The author has claimed that the soften Truss Model theory has been successfully applied to structures where shear behaviour predominates, such as low-rise shear walls, framed wall panels, deep beams, and shear transfer strengths. It also worked very well for members subjected to torsion. The theory holds that a concrete element reinforced with orthogonal steel bars and subjected to shear stresses would develop diagonal cracks at an angle inclined to the main steel bars. These cracks could separate the concrete into a series

of diagonal concrete struts, which are assumed to resists axial compression. Together with the steel bars, which are assumed to take only axial tension, they form a truss action to resist the applied shear stresses. For simplicity, the concrete struts were assumed to be inclined at 450 to the steel bars. The equations, derived from the preliminary truss concept, overestimated the test results. To improve the prediction of the truss model, the theory had undergone three major developments. The first development was the generalisation of the angle of inclination of concrete struts. They assumed that the angle of inclination might deviate from 45°. On this basis three equilibrium equations had been derived, which could explain why longitudinal and transverse steel with different percentages can both yield at failure. The theory is known as variable angle truss model. Since plasticity is assumed at failure, it could also be called the plasticity truss model. The second development was the deviation of the compatibility equation to determine the angle of inclination of the concrete strut. Since this angle was assumed to coincide with the angle of inclination of the principle compression stress and strain, this theory was known as the compression field theory.

## 2.2.4 Deep Beams with Web Opening

Since in most building utility ducts and pipes, water line, swear line etc. are passed through beams. However, provision of these openings may cause serviceability problems of excessive cracking and deflection and may severely affect the strength of a beam. Several investigation has been carried out by the researchers like Nasser et al., 1967; Ragan and Warwarcik 1967; Hanson 1969; Barney et al., 1977; Mansur et al., 1984; 1985; and Mansur 1988; and treated the strength problem with recommendations for the design and detailing of reinforcement near the opening to achieve adequate crack control. However, these researches were held on the normal beams. Researches on the deep beams with web opening are very scarce. So adequate research must be needed in this field. Moreover, the available methods for ultimate shear strength design can be applied only to simply supported beams where bending moment and shear force distributions are uniquely defined. In the case of continuos beams, the reduction of internal stress and a mark deviation of internal

stresses from elastic analysis, a reduction in stiffness due to presence of openings causes again a redistribution internal forces and moments, the amount of which needs to be properly evaluated to achieve a satisfactory design.

Kong et al. (1978) proposed a simple structural idealization for predicting the ultimate shear strength of deep beams with web openings. The structural idealization was based on simplifying assumptions derived from ultimate-load test on 56 smallscale deep beams; all made of lightweight concrete. Further experimental evidence was produced in 1978 to support the simple structural idealization for predicting the shear strength of RC deep beams with web openings. The shear behavior of deep beams is complex and the presence of web opening makes it cumbersome. The author mentioned that the experiments have confirmed that the strength and serviceability of such beams are governed mainly by a small number of identifiable variables and that for design purposes, the proposed structural idealization could prove a useful new tool. Deep beams with web openings are not yet covered by the current codes of practice such as CP 110, ACI 318 and the CEB-FIP recommendations and also because the scarcity of the experimental evidence, the provisions of CIRIA Guide are necessarily rather caution and restrictive. The author observed that there was more scatter value among the measured ultimate load for beams without web reinforcement than among those for beams with web reinforcement. The scatter was mainly due to unavoidable initial imperfections in the beams, and that the presence of web reinforcement reduces the effect of such imperfections. From the different detailing of web reinforcement, the author observed that the detailing of web reinforcement is more important than to provide an extra amount of web reinforcement. Web reinforcement should be so arranged as to protect both diagonal cracking regions, above and below the web opening.

Mansur et al. (1991) experimentally investigated on eight reinforced concrete continuous beams, each containing a large transverse opening. The beams are rectangular in cross section and all contain the same amount and arrangement of longitudinal reinforcement. The number of spans, the size of opening, and its location along the span were considered as major variables. Final failure of the beam

occurred by the formation of a mechanism i.e. the formation of hinge and the two opening ends represents the most vulnerable locations for the development of plastic hinges. Besides early cracking, the strength and stiffness of the beam decrease with an increase either in the length or depth of opening. Similarly, openings located in the high moment region produce larger deflections and result in early collapse of the beam. The author observed that the location of opening had very little influence on cracking load but affects the load deflection response. Similarly the opening location had no influence on the mode of failure.

# 2.2.5 Compressive Force Path Method

From the discussion of previous paper it can be concluded that the shear behaviour of reinforced concrete beam is so complicated that though there are a huge number of research papers, that behaviour is still ambiguous because a large number of parameters are involved with shear behaviour. Because of this ambiguity different researchers have thought in different angles and presented different solution procedures. Each of them claimed that their procedures have shown good accuracy to predict the shear behaviour of deep beams. But if one's procedure is applied to other's set of test data, the results have shown a very poor accuracy. Since shear failure occurs in many types of building element, so it is important to find out the shear behaviour of a deep beam with exact failure mechanism. The present day design models are explicitly based on the truss-analogy concept and thus dependent very much upon the residual strength of cracked concrete (Seraj et al., 1993). The new theories or models, like the compressive field theory, the modified compressive field theory, or the strut-and-tie model, are no exception, because Truss analogy forms the core of these concepts as well (Seraj et al., 1993). However, in the recent past, the widely held view that concrete exhibits strain softening material characteristic under any state of stress has been challenged by Kotsovos (1983) with experimental evidence, which has indicated that concrete, as a materials, is brittle in nature. He showed that conventional strain-softening response is not a material characteristic, as extensively considered, but merely a "descriptor" of secondary testing effects. The shallowness of present-day design concepts becomes evident by

looking at the extensive number of design equations put forward by different researchers or by the same researchers on different occasions. Incidentally most of the investigators seldom deviate from truss-analogy, yet failed to put forward a generalised design solution. The design equations were calibrated to set of experiments and, thus, are severely limited in their application (Kotsovos, 1983)

Kotsovos (1982) assessed the average longitudinal stress in the compressive zone, which was 75% higher than the uniaxial compressive strength of concrete (f'c). This large stress was due to the triaxial compressive stress conditions in the compressive zone. Kotsovos (1983) argued that, in the absence of stirrups, a triaxial compressive state of stress could be developed due to the occurrence of volume dilation in localised regions within the compressive zone. His views have been supported by the experimentally established shape of the transverse deformation profile (Kotsovos, 1982) of the top face of the beam. The characteristic feature of this profile is the large transverse expansion (indicative of volume dilation) that occurs in the region of cross-sections coinciding with a deep flexural crack when the load-carrying capacity of the beam is approached. This localised transverse expansion is restrained by concrete in the adjacent regions and such a restraint may be considered to be equivalent to the application of a confining pressure that has been assessed to be at least 10% of f'c. Kotsovos and Newman (1981) has found that small confining stress is sufficient to increase the load-carrying capacity by more than 50% and this should be the cause of large longitudinal compressive stress developing in the compressive zone. Concurrently, the expanding concrete induces tensile stresses in adjacent regions and this rise to a compression/tension state of stress. Such a state of stress reduces the strength of concrete in the longitudinal dimension. In fact Kotsovos and Newman (1981) have found that a tensile stress of about 5% of fc is sufficient to reduce the compressive strength by about 50%. Kotsovos (1984) has been shown that collapse occurs due to horizontal splitting of the compressive zone in the region between deep flexural cracks.

Kotsovos et al. (1987) investigated the behaviour of a number of RC beams, with various arrangements of shear reinforcement, subjected to two-point loading with

various shear spans to depth ratio. The test results were in conflicts with the "truss analogy" concept. A large portion of the shear span of beams could not behave as truss because the absence of shear reinforcement did not allow the formation of ties and the presence of a wide inclined crack did not permit the development of suitable struts. And yet, the beams sustained loads significantly larger than those widely expected. These test results clearly rise doubts over widely held views and, in contrast, indicate that "Truss" behaviour is not a necessary condition for the beams to attain their flexural capacity once their "shear capacity" is exceeded. Their results support that the portion of the beam below the neutral axis is not the main contributor to shear resistance of a concrete structure but the region of the beam above the neutral axis. The inclined cracks, which eventually caused failure of a beam, locally reduced the neutral axis to less than 5% of the beam depth. In view of such a small depth of compressive zone it may be argued that concrete is unlikely to be able to sustain such high tensile stresses caused by the presence of shear force. Such an argument is usually based on the erroneous assumption that the concrete behaviour within the compressive zone is realistically described by uniaxial stress strain characteristics. As for the case of compressive zone in the region of the section coinciding with a flexural crack, concrete in the region of a section through the tip of a deep inclined crack is also subjected to a wholly compressive state of stress. It appeared that a part of vertical and horizontal components of this compressive state of stress counteracts the tensile stresses developing in the presence of shear force. Hence, in spite of the presence of such a force, the state of stress remains compressive and this causes a significant enhancement of the local strength. However, eventually the shear force increases beyond a critical level and results in the development of tensile stresses that are sufficient to eliminate the restraining effect of the adjacent concrete, thus creating a compression/tension stress filed which reduces abruptly the local strength and causes failure.

Kotsovos (1988a and 1988b) presented two papers introducing the concept "Compressive Force Path (CFP)" as a basis for reinforced concrete ultimate limit state design. It was shown that the causes of beam failure are associated with the development of tensile stresses in the region of the path along which the compressive

force is transmitted to the support and not, as is widely considered by the region of the beam below the neutral axis through aggregate interlock. The proposed concept in identifies the mechanisms that may give rise to such stresses within the path. The author suggested that, for a RC beam at its ultimate limit state, the compressive force at the middle cross section is transmitted to the supports by following a path that is essentially bilinear (Figure. 2.1 (b)). For a beam subjected to two-point loading with a/d greater than a value of approximately 2.0, the change in the path direction occurs at a distance of approximately twice the beam depth d; for smaller values of a/d, the change in path direction is considered to occur at the cross section including the load point. The path of the compressive force may be visualized as the "flow" of compressive stresses with varying sections perpendicular to the path direction and with the compressive force representing the stress resultant at each section (Figure. 2.1 (a)). It has been proposed that shear failure is associated with the presence of tensile stresses developing in the region of the compressive force path. Tensile stresses may develop perpendicular to the path due to a number of causes such as (a) A tensile stress resultant (T in figure 2.1) develops for equilibrium purposes at locations where the path changes direction. (b) The compressive stress will reach a critical level at the smallest section of the path, where the stress intensity is the highest, before this level is reached in adjacent sections. This level marks the start of an abrupt and large material dilation that induces tensile stress (t1 in figure 2.1 (a)) in the surrounding concrete. (c) Large tensile stress (t<sub>1</sub> in figure 2.1 (a)) develops perpendicular to the direction of the maximum principal compressive stress in the region of the crack tip. (d) Bond failure at the level of the tension reinforcement between two consecutive flexural cracks changes the stress conditions in the compressive zone of the beam element between these cracks. This change in the stress intensity should give rise to tensile stresses in the compressive zone. The author indicated that the force sustained by the tensile reinforcement within the length of the horizontal projection of the inclined portion of the path is constant and this implies that the compressive force acting along the inclined portion of the path is also constant. The causes for such behaviour appeared to be associated with the occurrence of inclined cracking; as the beam deflects under increased load. The compressive stress trajectories along the path are such that, for equilibrium purposes,

they give rise to tensile stresses in the orthogonal direction. This combined compression tension state of stress leads to the formation of an inclined crack at a location where the concrete strength is exceeded. It should be noted, however, that, for a deep beam, the occurrence of such a crack within the inclined portion of the path does not interrupt the flow of the compressive stresses and thus the beam is capable of sustaining a significantly higher load. Figures 2.1and 2.2 indicate that the pat of the compressive force may be visualised as a 'flow' of compressive stress with varying section perpendicular to the path direction and with the compressive force representing the stress resultant at each section. The shape of the stress 'flow' and the intensity of the stress field are very much dependent on the beam boundary conditions. For a simply supported deep beam subjected to a load uniformly distributed on its top face, the stress flow may have a shape similar to that indicated in figure 2.2 (a). It is realistic to consider that the difference in shape between the above 'flow' and that caused by an equivalent load concentrated at the two third points affects only the location of failure initiation within a particular portion (inclined or horizontal) of the path and not the magnitude of the force that can be carried along this portion (Figure. 2.2). Based on this reasoning, it is considered realistic for design purposes to replace the actual stress 'flow' with a uniform stress 'flow' of intensity equal to the uniaxial cylinder compressive strength (f'c). The cross section of the 'flow' should be chosen such that the actual maximum compressive force carried along the path remains unchanged.

Kotsovos (1988a) outlined the experimental evidence of RC beams, with various arrangement of shear reinforcement, subjected to two-point loading and with various shear span-to-depth ratios, a/d. The inclined crack that formed within each shear span of the beams, with a/d=1.5. reinforced with stirrups within the middle span only, did not penetrate into the region of the load point and caused crushing of this region, as widely considered. Instead, it bypassed this region, and extended toward the middle zone. Apparently, the presence of stirrups within the middle zone prevented further extension and failure eventually occurred due to yielding of the tension steel. It has been indicated that the absence of stirrups in the portion of the shear span adjacent to the support (a/d=3.3) led to the formation of a deep inclined

crack that reduced the neutral axis depth. Further extension of this crack toward the load point was prevented apparently by the presence of stirrups in the portion of the shear span adjacent to the load point, and the beam eventually failed in flexure. It is interesting to note that a flexural model of failure was also exhibited by Beam with a/d = 3.3. The inclined cracks that formed within the shear span apparently did not reduce the depth of the neutral axis sufficiently to overcome the shear resistance of the compressive zone. In contrast with this behaviour, the beam with a/d= 4.4 failed in shear. However, failure occurred under a load significantly higher than that predicted by current code provisions for shear design. The beam with a/d = 3.3 (which exhibited a flexural mode of failure within the middle span), beam with a/d = 4.4 was characterized by a flexural mode of failure within the shear span. The lack of stirrups in the region extending to a distance of 2d from the support led to the formation of a deep inclined crack that reduced the depth of the neutral axis to a very small value approximately equal to 0.05d. Such a small neutral axis depth combined with the large width of the inclined crack must have led to yielding of the tension reinforcement, and failure eventually occurred due to failure of the compressive zone in the region between the horizontal branch of the crack and the top face of the beam. Shear failure can be prevented only by placing shear reinforcement throughout the shear span. Yet the results indicated that placing of stirrups in the middle, and not the shear span of the beams with a/d = 1.5, leads to a load-carrying capacity significantly larger than that of beams without shear reinforcement. Similarly the absence of stirrups in a large portion of the shear span of beams with a/d = 3.3, did not prevent the beams from reaching their flexural capacity and similar to the beam which was reinforced with stirrups throughout the shear span. In fact, the load-carrying capacity of these beams was almost 100 percent higher than that of the beam without stirrups, as expected, failed in shear. The results for beams with a/d = 4.4, sustained a load significantly larger than that of beam without shear reinforcement, but it did not reach its flexural capacity. However, this beam sustained a load similar to that corresponding to flexural capacity but failed within the shear span. These results indicate clearly that such behaviour cannot be explained in terms of the concept of the shear capacity of a critical section. A prerequisite for the application of the shear capacity of critical sections concept is the widely accepted view that the main

contributor to the shear resistance of an RC beams without shear reinforcement is aggregate interlock. In view of this, it is realistic to suggest that, in the absence of shear reinforcement, the main contributor to the shear resistance of a RC beam at its ultimate limit state is the compressive zone, with the region of the beam below the neutral axis. For a beam under two-point loading with a/d <2.0, critical tensile stress were considered to develop in the region adjacent to the load point within the middle span. Reinforcing this region with stirrups, therefore, should prevent local failure before the flexural capacity of the beam is reached. It has been suggested also that, for a beam under two-point loading with a/d> 2.0, the critical stresses develop within the region of the path beyond a distance of approximately 2d from the support. Reinforcing this region with stirrups, therefore, is essential if the beam is to fail in flexure. On the other hand, placing stirrups in regions where inclined cracks were expected to form did not appeared to safeguard against shear failure (a/d = 4.0). Such behaviour that was also in agreement with the compressive force path concept.

Kotsovos and Lefas (1990) verified the method developed in compliance with the concept of the "Compressive force path" for the design of reinforced concrete beams. The method had been used to design a number of beams whose behaviour is subsequently investigated by experiment. The beams chosen were such that the design details specified by the proposed method were significantly different than those specified by current code provisions. In fact, on the basis of current code concept, the beams are deemed incapable of sustaining the design load; yet the beams not only sustained safely the specified design load, but also did so with an about of transverse reinforcement 70% less, in some cases, than specified by current methods. It has been demonstrated that the proposed model and the ensuring design method can be extended easily to apply to any type of skeletal structural concrete configuration where the model represents the structural elements between consecutive points of inflection with the interaction between the elements in the region of such points being modified as an "internal hinged support affected by the provision of transverse reinforcement.

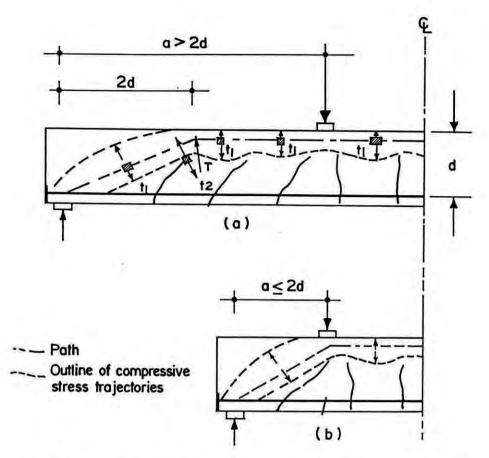


Figure 2.1 Schematic representation of the path of compressive force and corresponding out line of compressive stress trajectories for RC deep beams with various a/d ratio

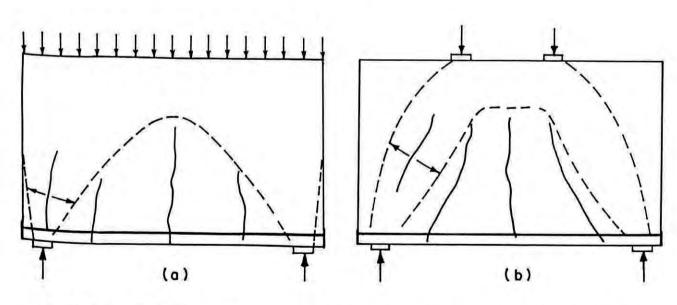


Figure 2.2 Typical inclined compression failures of RC deep beams under (a) Uniform and (b)Two-point loading

#### CHAPTER THREE

# METHODS FOR THE ANALYSIS AND DESIGN OFDEEP BEAMS

#### 3.1 INTRODUCTION

Shear an important but controversial topic in structural concrete (Hsu 1982, Kotsovos 1983, Kong 1987). In design it is generally desirable to ensure that ultimate strengths are governed by flexure rather than by shear. Shear failures, which in reality are failures under combined shear forces and ending moments, are characterized by small deflections and lack of ductility. There is sometimes little warning before failure occurs, and this makes shear failures particularly undesirable. When shear force act on a beam and goes into shear deformation, the shear stress prevent the in place sliding. The shear stress of a slender beam varies parabolic ally through the beam height with maximum at the neutral axis and zero at the top and bottom surface. If a small square element located at the neutral axis of such a beam is isolated, figure 3.1 (a), the stresses in the element are as in figure 3.1 (b), which is equivalent to the element as in figure 3.1 (c). That is, the action of two pairs of shear stresses on the vertical and horizontal faces is the same as that of two pairs of normal stresses, one tension and one compression, acting on the 450 faces and of numerical value equal to that of the shear stresses. If an element of the beam is not considered which is located neither at the neutral axis nor at the outer edges, its vertical faces are subjected not only to the shear stresses (v = VQ/Ib) but also to the familiar bending stresses f = My/I). The six stresses, which now act on the element, can again be combined into a pair of inclined compression stresses and a pair of inclined tension stresses, which act at right angles to each other. They are known as principal stresses. Their value is given by,  $t = f/2 \pm \sqrt{(f^2/4 + v^2)}$  and their inclination  $\alpha$  by  $\tan 2\alpha = 2v/f$ . Since the magnitude of the shear stresses v and the bending stresses f changes both along the beam and vertically with distance from neutral axis, the inclination as well as the magnitudes of the resulting principal stresses "t" also very from one place to another. Tensile stresses are of particular concern in view of the low tensile strength of concrete. Tension stresses of various inclinations and magnitudes, resulting from

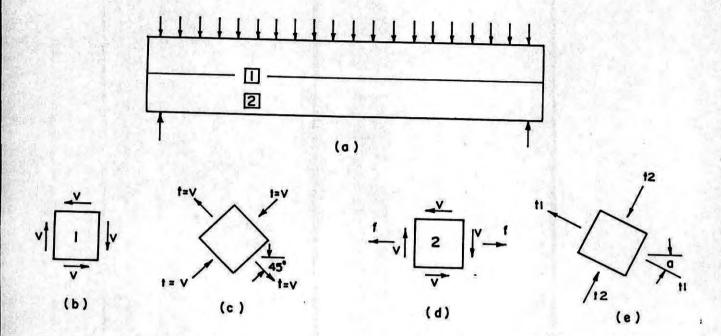


Figure 3.1 Stresses in homogeneous rectangular beams

shear alone (at neutral axis) or from the combined action of shear and bending, exist in all parts of a beam. It is for this reason that inclined tension stresses, known as diagonal tension, must be carefully considered in reinforced concrete design.

# 3.2 ACI CONCEPT FOR SHEAR AND DIAGONAL TENSION

## 3.2.1 Reinforced Concrete Beam without shear reinforcement

The ACI-ASCE Committee (1962), MacGregor and Peter (1977) and joint ASCE-ACI Task Committee (1973) explained the shear and diagonal tension behaviour. As load is increased on a homogeneous elastic beam without reinforcement, tension cracks will form where the tension stresses are largest and will immediately cause the beam to fail. The outer fibre at the section of maximum bending moment experience maximum tension. However when tension reinforcement is provided, the required flexural tension strength is furnished by the steel and much higher loads can be carried. Shear stresses increase proportionally to the loads. In consequence, diagonal tension stresses of significant intensity are created in regions of high shear forces, chiefly close to the supports. Longitudinal tension steel does not reinforce the tensionally weak concrete against those diagonal tension stresses which occur elsewhere, caused by shear alone or by combined effect of shear and flexure. Eventually, these stresses attain magnitudes sufficient to open additional tension cracks in a direction perpendicular to the local tension stress. These are known as diagonal cracks. Flexural cracks occur in regions of high moments and diagonal cracks in regions of high shear forces.

Formation of diagonal cracks: The diagonal tension stress t represent the combined effect of the shear stresses v and bending stresses f which are proportional to shear force V and the bending moment M, respectively. Eventually, the relative values of M and V will affect the magnitude as well as the direction of the diagonal tension stresses. At location of large shear force V and small bending moment M there will be little flexural cracking, if any, prior to the development of a diagonal tension crack. Consequently, the average intensity of shear stresses in the section

prior to crack formation is, v = V/bd. The exact distribution of these shear stresses over the depth of the cross section is not known. It can not also be computed from equation, v = VQ/Ib, because this equation does not account for the influence of the reinforcement and because concrete is not an elastic homogeneous material. If flexural stresses are negligibly small at the particular location, the diagonal tension stresses are numerically equal to the shear stresses, with a maximum at the neutral axis. Consequently, diagonal cracks from mostly at or near the neutral axis and propagate from that location. These so-called web-shear cracks can be expected to form when diagonal tension stress in the vicinity of the neutral axis becomes equal to the tension strength of concrete. It was found that the tension strength of the concrete varies from about  $3\sqrt{f}_c$  to about  $5\sqrt{f}_c$  (Winter and Nilson, 1991). It was also found that in regions with large shear and small moment, the diagonal tension cracks format an average or nominal shear stress  $v_{cr}$  of about  $3.5\sqrt{f}_c$ , that is

$$v_{cr} = V_{cr}/bd = 3.5\sqrt{f_{c}}$$
 3.1a

where  $V_{cr}$  = shear force at which diagonal tension crack is formed.

The locations at which the shear force and bending moment have large values, flexural tension cracks first form. Their width and length are well controlled by longitudinal tension steel. However, when the diagonal tension stress at the upper end of one or more of these cracks exceeds the tensile strength of the concrete, the crack bends in a diagonal direction and continues to grow in length and width. These cracks are known as flexural shear cracks. As the diagonal tension cracks develops, the average shear stress is larger than that given by the equation, v = V/bd, because the cracks reduce the area of uncracked concrete which is available to resist shear. The amount of this reduction will vary, depending on the unpredictable length of the pre-existing flexural tension crack. A large number beam test (ACI-ASCE Committee 326, 1962) has been evaluated for this purpose. They show that in the presence of large moments (for which adequate longitudinal reinforcement has been provided) the nominal shear stress at which diagonal tension cracks form is

$$v_{cr} = V_{cr}/bd = 1.9\sqrt{f'_{c}}$$
 3.1b

It has been found that the shear at which diagonal cracks develop depends on the ratio of shear force to bending moment or more precisely, on the ratio of shear stress v to bending stress f at the top of the flexural crack. Therefore  $v = k_1(V/bd)$  and  $f = k_2(M/bd^2)$  where  $k_1$  depends on the depth of penetration of flexural crack and  $k_2$  depends on the crack configuration. Therefore, the ratio,  $v/f = k_1 Vd/k_2M$ , must be expected to affect that load at which flexural cracks develop into flexure-shear cracks, the unknown quantity  $k_1/k_2$  to be explored by test. Equation (3.1a) gives the cracking shear for very large values of Vd/M, and Equation (3.1b) for very small values. Moderate values of Vd/M,  $v_{cr}$  will be intermediate between these extremes. From the evaluation of large number of tests (ACI-ASCE Committee 326, 1962) it has been found that the nominal shear stress at which diagonal flexure-shear cracking develops is conservatively predicted from

$$v_{cr} = V_{cr}/bd = 1.9\sqrt{f_c} + 2500 p_w Vd/M \le 3.5\sqrt{f_c}$$
 3.1c

Where,  $p_w = A_s/bd = longitudinal tension steel$ 

To summarise in reference (MacGregor and Peter, 1977) indicate that the stress at which flexural shear cracks form in the presence of significant moments depend on the reinforcement ratio  $p_w$  in addition to the primary variable  $\sqrt{f'_c}$ . The empirical equation for flexure-shear cracking, proposed (MacGregor and Peter, 1977) to replace  $v_{cr} = 1.9\sqrt{f'_c}$  in equation (3.1b) is

$$v_{cr} = \sqrt{f'_c} \le (0.8 + 120p_w)\sqrt{f'_c} \le 2.3 \sqrt{f'_c}$$
 3.1d

For steel ratio p<sub>w</sub> smaller than about 0.009, Eqn. (3.1d) gives values progressively smaller than Eqn. (3.1b), while for p<sub>w</sub> larger than about 0.012, the reverse is true. Changes of code have been proposed on this basis (MacGregor and Peter, 1977) that

would eliminate (3.1c). They would take account of the effect of Vd/M only for unusually deep beams or when sizable concentrated loads are located close to supports. Two types of behaviour have been observed in diagonally cracked beams: (1) The diagonal crack, once formed, spreads either immediately or at only slightly higher load, traversing the entire beam from the tension reinforcement to the compression face, splitting it in two and failing the beam. This process is sudden and without warning chiefly in the shallower beams, i.e. beams with span depth ratio of about 8 or more. (2) Alternatively, the diagonal crack, once formed, spreads toward and partially into the compression zone but stops short of penetrating to the compression face. In this case sudden collapse occurs, and the failure load may be significantly higher than that at which the diagonal crack first formed. This behaviour is chiefly observed in the deep beams with smaller span-depth ratios.

## 3.2.2 Reinforced Concrete Beams with Web Reinforcement

Economical design of a flexural member should be such that it develops its full moment capacity without premature shear failure. Shear failure is sudden and explosive, shown inadequate ductility and fails without warning. When the concrete has the shear strength less than the applied shear, special shear reinforcement, known as web reinforcement, is used to increase this strength. Shear reinforcement is placed either vertically or inclindly with the main tension bars. Web reinforcement has no noticeable effect prior to the formation of diagonal cracks. After diagonal cracks, web reinforcement augments shear resistance in four separate ways: (a) Part of the shear force is resisted by the bars, which traverse a particular crack, (b) These bars restricts the growth of diagonal cracks and reduce their penetration into the compression zone. This leaves more uncracked concrete for resisting the combined action of shear and compression. (c) The stirrups also counteract the widening of the cracks. This makes a significant and reliable interface force. (d) The stirrups are so arranged that they tie the longitudinal reinforcement into the main bulk of the concrete. These resist the splitting of concrete along the longitudinal reinforcement and increase the shear resisted in dowel action.

Behaviour with Vertical Reinforcement: Most frequently, web reinforcement consist of vertical stirrup. The forces acting on a cracked plane of a beam of vertical reinforcement are shown in figure 3.2. Each stirrup traversing the crack exerts a force  $A_v f_v$ .  $A_v$  is the cross-section area of each leg of stirrup. For equilibrium in the vertical direction,

$$V_{ext} = V_c + V_s + V_d + V_{iy}$$
 3.2a

Where,  $V_s = \sum_n A_v$   $f_v$  is the vertical force in the stirrup, n being the number of stirrups traversing the crack = p/s, p= horizontal projection of the crack; s = spacing of the stirrup. It is seen that after inclined cracking the portion of the shear,  $V_s = \sum_n A_v$   $f_v$  carried by the stirrups increases linearly, while the sum of the three other components,  $V_c + V_d + V_{iy}$  stays nearly constant (Winter and Nilson, 1991). When the stirrups yield  $\sum_n A_v$   $f_v$  remains constant but  $V_{iy}$  and  $V_d$  falls off rapidly. It is conservatively assume that just prior to failure of a web reinforced beam

$$V_{\alpha} = V_c + V_d + V_{iy}$$
 3.2b

It is assumed that the inclined cracks are normally developed at  $45^{\circ}$ , then the horizontal projection p will be equal to effective depth d of the beam. At failure when  $V_{\text{ext}} = V_n$  then

$$V_n = V_{cr} + \frac{A_v F_y d}{s}$$
3.3

This equation predicts very conservative strength than observed value. The observed value is about 45% larger than the predicted value (Winter and Nilson, 1991)

Beams with inclined web reinforcements: The functions of inclined web reinforcements are very similar to vertical web reinforcements. Figure 3.3 indicates the forces acting on one part of the inclined crack where the inclined crack length  $I=p/\cos\theta$  is crossed by inclined bars horizontally spaced a distance s apart. Therefore

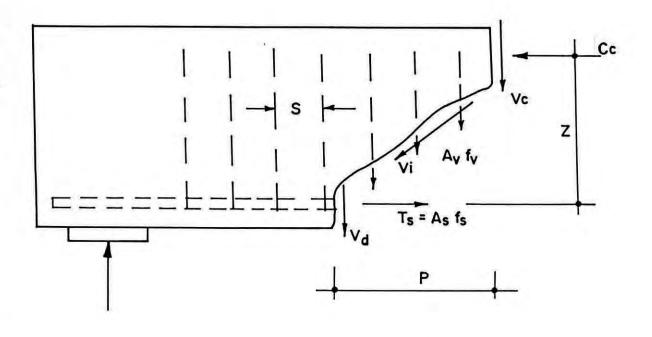


Figure 3.2 Forces at diagonal crack in beams with vertical stirrups

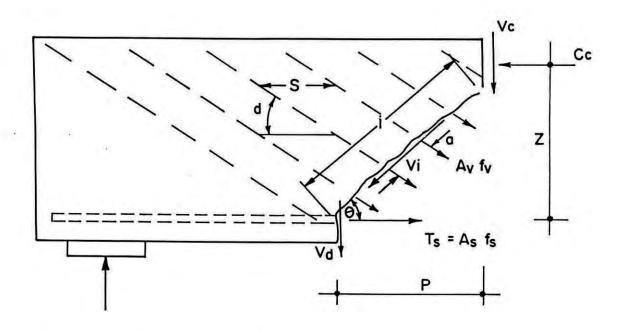


Figure 3.3 Forces at diagonal crack in beams with inclined web reinforcement

$$a = \frac{s}{\sin\theta (\cot\theta + \cot\alpha)}$$
 3.4

Where, a = distance between the bars measured parallel to the direction of cracks

S= horizontal distance of the inclined bars

 $\theta$  = inclination of the crack

 $\alpha$  = inclination of the bars

The number of bars crossing the cracks

$$n = i/a$$
, after some transformation  
 $n = p/s (1+\cot \alpha \tan \theta)$ 

The total vertical component of the forces in all bars which cross the crack is,  $V_s = nA_v f_v \sin \alpha = A_v f_v p/s$  (sin  $\alpha + \cos \alpha \tan \theta$ ). It is assumed that at shear failure, the web reinforcement reaches the yield point, the horizontal projection of the crack is the effective depth d and  $V_c + V_d + V_{iy}$  is equal to  $V_{cr}$ . Therefore the ultimate strength when failure caused by shear is obtained as

$$V_n = V_{cr} + \frac{A_r f_r d(\sin \alpha + \cos \alpha)}{s}$$
 3.5

Equation (3.3) and (3.5) apply only if web reinforcement is so spaced that any conceivable diagonal crack is traversed by at least one stirrup or inclined bar.

# 3.2.3 ACI Concept on the Shear Strength of Deep Beams

Behaviour of deep beams: According to ACI code for deep beams, flexural strength can be predicted with sufficient accuracy using the same methods employed for beams of normal proportions. The shear strength of deep beam may be as much as 2 or 3 times greater than that predicted using expressions developed for normal

members because a significant part of the shear force is transferred directly from the load to supports by tied-arch action. The ultimate strength of deep beams depends upon tied-arch action, in which the main steel is fully stressed over nearly its entire length rather than only at the maximum moment section. So special attention must be paid to the anchorage of such steel. When diagonal cracking occurs, it will be at a slope steeper than 45° in most cases, consequently, while it is important to include vertical stirrup, they are apt to be less effective than horizontal web steel (Winter and Nilson, 1991). The horizontal bars are effective because they act more in the direction perpendicular to the diagonal crack, thus improving the shear transfer by aggregate interlock and dowel action

## ACI Code for deep beam

Special provision for shear in deep beams is provided for 1<sub>n</sub>/d≤5

$$V_u < \phi V_n$$
 3.6

Where  $\phi = 0.85$  for shear, and

$$V_n = V_c + V_s 3.7$$

Regardless of the amount of reinforcement provided, the normal shear strength  $V_n$  is not to be taken greater than  $8\sqrt{f'_c}$  bd when  $l_n/d$  is less than 2 and is not to be greater than

$$V_{n} = \frac{2}{3} (10 + \frac{I_{n}}{d}) \sqrt{f'_{c}} bd$$
 3.8

When  $l_n/d$  is between 2 and 5.

The critical section for shear is to be taken a distance  $0.15\ l_n$  from the face of supports for uniformly distributed loads and 0.5a for beams with concentrated loads, but not to exceed a distance d from the support face in either case. Shear reinforcement required by calculation or other code provisions at the critical section

is to be used throughout the span. The concrete contribution to the shear strength can be computed from

$$V_c = (3.5 - 2.5 \frac{M_u}{V_u d})(1.9 \sqrt{f'_c} + 2500 \ \rho_w \frac{V_u d}{M_u}) \ bd$$
 3.9

With the restrictions that the multiplier  $(3.5-2.5~M_u/V_ud)$  must not exceed 2.5 and that  $V_c$  must not be taken greater than  $6\sqrt{f'_c}bd$ .  $M_u$  and  $V_u$  are the moment and shear force, at factored loads, occurring simultaneously at the critical section. When the shear force  $V_u$  at factored loads exceeds the design shear strength of the concrete  $\phi V_c$ , shear reinforcement must be provided to carry the excess shear. The contribution of the web steel  $V_s$  is to be calculated from

$$V_{s} = \left[\frac{A_{v}}{S} \left\{ \frac{1 + \frac{l_{n}}{d}}{12} \right\} + \frac{A_{vh}}{s_{2}} \left\{ \frac{11 - \frac{l_{n}}{d}}{12} \right\} \right] f_{v}d$$
3.10

Where  $A_v$  = Area of the shear reinforcement perpendicular to the main flexural steel within a distance s.

 $A_{vh}$  = Area of the shear reinforcement parallel to the main flexural steel within a distance  $s_2$ .

Combining equations (3.6), (3.7) and (3.10) and rearranging terms, one obtained

$$\left[\frac{A_{v}}{S}\left\{\frac{1+\frac{l_{n}}{d}}{12}\right\} + \frac{A_{vh}}{s_{2}}\left\{\frac{11-\frac{l_{n}}{d}}{12}\right\}\right] = \frac{V_{u}-\phi V_{c}}{\phi f_{yv}d}$$
3.11

The relative amounts of horizontal and vertical web steel that are used, based on equation (3.11), may vary within the following restrictions: the are  $A_v$  must not be less than 0.0015  $b_w$ s, and s must not exceed d/5 or 18 inch. The area  $A_{vh}$  must not be less than 0.0025  $b_w$ s<sub>2</sub>, and s<sub>2</sub> must not exceed d/3 or 18 inch. For design purposes, it is useful to note that coefficients in parenthesis in equation (3.11) are weighting factors for the relative effectiveness of the vertical and horizontal web steel. It is seen that, for very deep beams with small  $l_n$ /d, the horizontal steel  $A_{vh}$  is dominantly effective, and the addition of vertical web steel  $A_v$  will have little effect in increasing strength (Winter and Nilson, 1991).

## 3.3 BS 8110 CONCEPT FOR SHEAR AND DIAGONAL TENSION

## 3.3.1 Shear Behavior of Beams without Shear Reinforcement

British Standards Institution Code (BS 8110, 1985) identified that the failure made is strongly depended on the shear span to depth ratio, a/d as follows:

- (a) a/d>6: Beams with such a high a/d ratio usually fail in bending.
- (b) 6>a/d>2.5: Beams with a/d lower than about 6 tend to fail in shear. With reference to figure 3.4a, as the force V is increased, the flexural crack a-b nearest the support would propagate toward the loading point, gradually becoming an inclined crack, which is known as a flexure-shear crack but which is offer referred to simply as a diagonal crack (Figure. 3.4a: crack a-b-c). With further increase of V, failure usually occurs in one of two modes, If the a/d ratio is relatively high, the diagonal crack would rapidly spread to e, resulting in collapse by splitting the beam into two pieces. This mode of failure is often called diagonal tension failure; for such a failure mode, the ultimate load is sensibly the same as that at the formation of the diagonal crack. If the a/d ratio is relatively low, the diagonal crack tends to stop somewhere at j (Figure. 3.4a); a number of random cracks may develop in the concrete around the longitudinal tension reinforcement. As V is further increased, the diagonal crack widens and propagates along the level of the tension reinforcement (Figure. 3.4a: crack g-h). The increased shear force presses down the longitudinal steel and causes

the destruction of the bond between the concrete and the steel, usually leading to the splitting of the concrete along g-h

- (c) 2.5> a/d> 1: For a/d lower than about 2.5 but greater than 1, the diagonal crack often forms independently and not as a development of a flexural crack (Figure. 3.4b) (Taylor, 1973). The beam usually remains stable after such cracking. Further increase in the force V will cause the diagonal crack to penetrate into the concrete compression zone at the loading point, until eventually crushing failure of the concrete occurs there, sometimes explosively. This failure mode is usually called shear-compression failure; for this mode, the ultimate load is sometimes more than twice that at diagonal cracking.
- (d) a/d <1: the behaviour of beams with such low a/d ratio approaches that of deep beams. The diagonal crack forms approximately along a line joining the loading and support points (Figure. 3.4c). It forms mainly as a result of the splitting action of the compression force that is transmitted directly from the loading point to the support; it initiates frequently at about d/3 above the bottom face of the beam. As the force V is increased, the diagonal crack would propagate simultaneously towards the loading and support points. When the crack has penetrated sufficiently deeply into the concrete zone at the loading point, or, more frequently, at the support point, crushing failure of the concrete occurs. For a deep beam failure mode, the ultimate load is often several times that at diagonal cracking.

### 3.3.2 Shear Resistance of Web Reinforcement

The stresses in the shear reinforcement have been analyzed by the truss analogy, illustrated in figure 3.5, in which the web bars are assumed to form the tension members of an imaginary truss, while the thrusts in the concrete constitute the compression members. The figure shows a general case of links at a longitudinal spacing  $s_{\nu}$ . The links and the concrete 'struts' are shown inclined at the general angles  $\alpha$  and  $\beta$ , respectively, to the beam axis. To derive design equations using the truss analysis, draw a line A-A in figure 3.5, parallel to the concrete 'struts'.

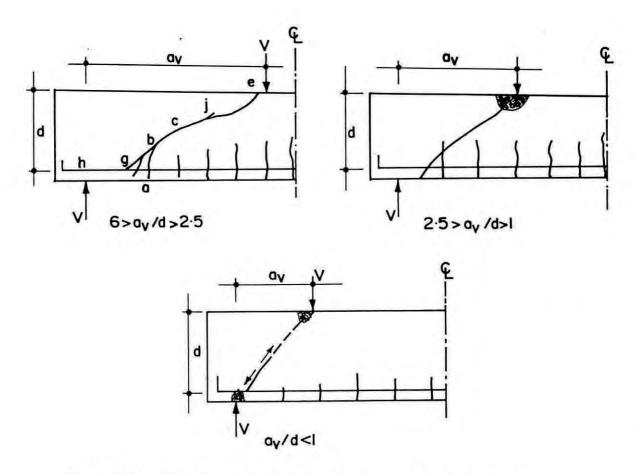


Figure 3.4 Crack pattern for different shear span-to-depth (a<sub>v</sub>/d) ratios

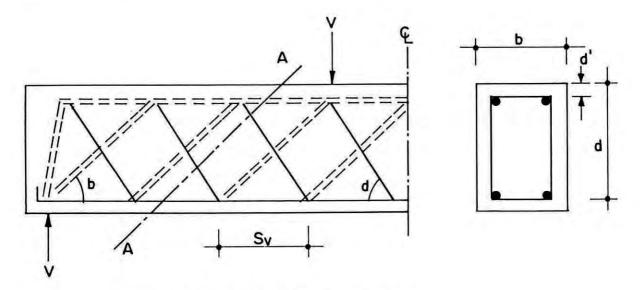


Figure 3.5 Typical model of Truss Analogy

Consider the vertical equilibrium of the free body to the left of the line A-A. The web resistance  $V_s$  is contributed by the vertical components of the tension  $A_v f_{yv}$  in the individual links that are crossed by A-A:

$$Vs = A_v f_{yv} \sin \alpha$$
 (Number of links crossed by A-A)

$$V_s = A_v f_{yv} (\cos \alpha + \cot \beta) \left( \frac{d - d'}{s_v} \right)$$
 3.12a

Where  $A_v$  is the area of both legs of each link and  $f_{yv}$  is the characteristic strength of the links.

In the particular case of vertical links,  $\alpha = 90^{\circ}$  and Eqn. (3.12a) because

$$V_{s} = A_{v} f_{yv} \left( \frac{d - d'}{s_{v}} \right) \cot \beta$$

$$V_s \approx A_v f_{vy} \left(\frac{d}{s_v}\right) \cot \beta$$
 3.12b

Tests (ACI-ASCE Committee 426, 1973) have led to the recommendation that  $\beta$  in Eqn. (3.12b) could be taken as 45°, so that

$$Vs = A_v f_{yv} \left( \frac{d}{s_v} \right)$$
 3.12c

It is well known that,  $V_s = V - V_c$ ; and if we write V = vbd and  $V_c = v_c$  bd, the above equation for vertical links becomes

$$\mathbf{v} - \mathbf{v_c} = \left(\frac{A_{\mathbf{y}}f_{\mathbf{y}\mathbf{v}}}{bs_{\mathbf{v}}}\right)$$
 3.12d

when the web reinforcement consists of a system of bent-up bars, it would be reasonable to use Eqn. (3.12a).

It should be noted once again that the truss analogy is no more than a design tool; though conceptually convenient, it presents an over simplified model of the reinforced concrete beam in shear (ACI-ASCE Committee 426, 1973). The truss analogy model completely ignores the favorable interaction between the web reinforcement and the aggregate interlock capacity and the dowel force capacity; to this extent it tends to give conservative results, though the conservatism reduces as the amount of web steel increases. The truss analogy also assumes that the failure of the beam is initiated by the yielding or excessive deformation of the web reinforcement, but in very thin webbed reinforced or prestresses concrete beams (e.g. T-beams), failure may in fact be initiated by web crushing; in such a case the truss analogy would give unsafe results. The above account of shear behaviour, together with that of beams without web reinforcement, may be amplified by the following summary statements:

- (a) For a beam without web reinforcement, for a given concrete strength  $f_{cu}$  and the longitudinal steel ratio  $p_w$ , a safe lower bound value can be assigned to the nominal shear stress at collapse. Designating this nominal shear stress as  $v_c$ , a safe estimate of the ultimate shear strength of a beam without web reinforcement would be  $V_c = v_c bd$ .
- (b) Where web reinforcement is used, it remains practically unstressed until diagonal cracking occurs, at which instant those web bars that intercept the diagonal crack will receive a sudden increase in stress (Evans and Kong, 1967). If the amount of web steel is too small, the sudden stress increase may cause the instant yielding of the web bars. In that case the author suggested that  $p_v f_{yv}$  should not be less than 0.38 N/mm<sup>2</sup>. So rounding up

 $p_v f_{yv}$  (minimum)  $\geq 0.4 \text{ N/mm}^2$ 

(c) Web reinforcement may confidently be assumed to be effective, only if every potential diagonal crack is intercepted by at least one web bar. This is to be possible if the spacing of vertical links must not exceed.

$$S_{v,max}$$
 (vertical links) = d

More precisely taking 75% of this value for design

$$S_{v,max}$$
 (vertical links) = 0.75d

For bent-up bars (or inclined links),

$$S_{v,max}$$
 (bent-up bars) =  $(\cot \alpha + \cot \beta)d$ 

But both  $\alpha$  and  $\beta$  should not be less than  $45^{\circ}$ , so that

$$S_{v,max}$$
 (bent-up bars) =2d

Again, taking 75% of this value for design,

$$S_{v,max}$$
 (bent-up bars) = 1.5d

- (d) When  $p_v$  and  $s_v$  satisfy the requirements in (b) and (c) above, the capacity of the web reinforcement may for design purposes be estimated by the truss analogy. The truss analogy assumes that the web steel can reach its yield stress before other failure modes occur, e.g. web crushing or compression zone failure.
- (e) For a beam with web reinforcement, the shear resistance may be regarded as  $V = V_c + V_s$ , where  $V_c$  may conservatively be obtained from test results for beams without web reinforcement.

(f) The truss analogy does not differentiate between links and bent-up bars. When they are used in combination, the analogy gives their shear capacity as the sum of their capacities when used separately. The effects of links and bent-up bars used in combination are actually more than additive (Evans and Kong, 1967). Bent-up bars are more effective than links in restricting the widening of the diagonal crack; but links can perform the important function of preventing the pressing down of the longitudinal reinforcement and hence maintain the dowel capacity.

## 3.3.3 British Standards Institution Code for the Design of Shear Strength

BS 8110's design procedure is based on the principles explained in Sections 3.3.1 and 3.3.2. The procedure is summarized below:

Step 1: The design shear tress, v = V/bd, where V = the ultimate shear force; b = the beam width; and d = the effective depth of the beam.

Step 2:  $v > 0.8 \sqrt{f'_{cu}}$  or 5 N/mm<sup>2</sup>

if v of step 1  $0.8\sqrt{f'_{cu}}$  or 5 N/mm<sup>2</sup> exceeds whichever is less, the product bd must be increased to reduce v to the limit.

Step 3: v<0.5 v<sub>c</sub>

Compare the design shear stress v with the design concrete shear stress  $v_c$  in Table 3.1, If  $v < 0.5 \ v_c$ , then

- (a) No shear reinforcement is required for members of minor structural importance, such as lintels.
- (b) For all other structural members, provide minimum links, which

Table 3.1 Design concrete shear stress  $v_c$  for  $f_{cu}\!\ge\!40~\text{N/mm}^2$ 

100A <sub>s</sub> /bd	Effective depth d (mm)						
	150	175	200	225	250	300	≥400
≤0.15	0.50	0.48	0.47	0.45	0.44	0.42	0.40
0.25	0.60	0.57	0.55	0.54	0.53	0.50	0.47
0.50	0.75	0.73	0.70	0.68	0.65	0.63	0.59
0.75	0.85	0.83	0.80	0.77	0.76	0.72	0.67
1.00	0.95	0.91	0.88	0.85	0.83	0.80	0.74
1.50	1.08	1.04	1.01	0.97	0.95	0.91	0.84
2.00	1.19	1.15	1.11	1.08	1.04	1.01	0.94
≥3.00	1.36	1.31	1.27	1.23	1.19	1.15	1.07

Are defined as shear links that will provide a shear resistance of 0.4 N/mm<sup>2</sup>, i.e.

$$A_{v} \text{ (min, links)} \ge \frac{0.4bs_{v}}{0.87h_{v}}$$
 3.13

And in the direction of the span, the link spacing s<sub>v</sub> should not exceed 0.75d.

Step 4: 
$$0.5v_c \le v \le (v_c + 0.4)$$

If v is between  $0.5v_c$  and  $(v_c + 0.4)$ , provide minimum links as defined by Eqn. (3.13) for the whole length of the beam.

Step 5: 
$$(v_c + 0.4) < v$$

If v exceeds  $v_c + 0.4$  provide links as follows:

$$A_{v} \ge \frac{(v - v_c)bs_{v}}{0.87f_{yv}}$$

$$3.14$$

## Step 6: Anchorage of links

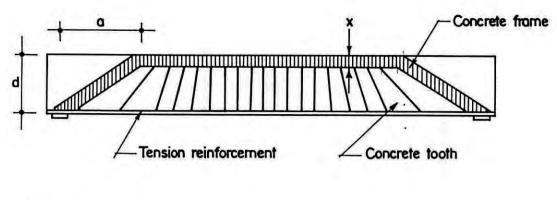
- (a) A link should pass round another bar of at least its own size, through an angle of 90°, and continue for a length of at least eight times its own size: or
- (b) It should pass round another bar of at least its own size, through an angle of 180°, and continue for a length of at least four times its own size.

# 3.4 KOTSOVOS DESIGN METHOD FOR DEEP REINFORCED CONCRETE BEAMS

## 3.4.1 Modelling

The recently developed concept, "Compressive Force Path (CFP)" may be introduced in structural concrete design by developing physical models of structural concrete members, at their ultimate limit state, compatible with the concept. For the case of a simply supported RC beam without web reinforcement, such a model may take the form of a "comb-like" structure tied by the tension reinforcement, as indicated in figure 3.6 (Kotsovos and Borowski, 1993). The authors shown that the "comb" compresses a frame with inclined legs, providing a simplified yet realistic representation of the compressive force path, and a number of "teeth" representing the concrete cantilevers that form between consecutive flexural or inclined cracks. The horizontal distance 'a' of the joint of the horizontal and inclined members of the frame from the support is dependent on the beam slenderness and type of loading, as indicated also in figure 3.6, while the cross section of the model is that of the actual beam.

Figure 3.7 shows two such simplified compressive force paths for the case of a deep beam subjected to single and two-point loading, respectively. The CFP for a two-point loading may also be valid for the case of a uniform loading if the equivalent two-point load is applied at the third points. The stress 'flow' is considered to have a rectangular cross-section with a width equal to the beam width. The depth of the horizontal portion of the stress 'flow' of the path may be assessed such that the compressive force equals the force sustained by the tensile reinforcement. The inclined stress "flow" of the path is symmetrical with respect to the line connecting the intersection of the directions of the applied load and the horizontal path of the compressive force, with the intersection of the directions of the reaction and the tensile reinforcement. Figure 3.8 shows the simplified compressive force path for two span beam subjected to two point loading.



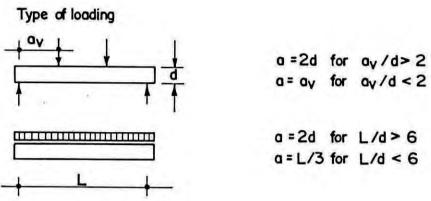


Figure 3.6 Proposed model for simply supported reinforced concrete beam

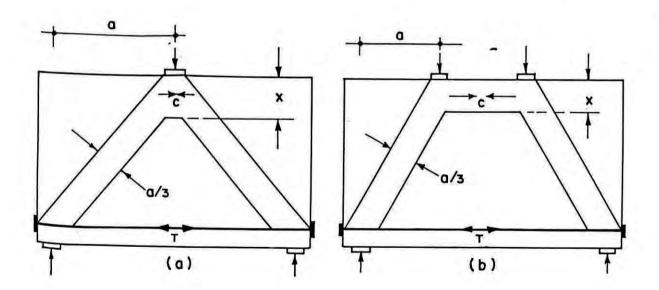


Figure 3.7 Proposed model for deep beams under (a) Single point (b) Two-Point and / or for uniform loading

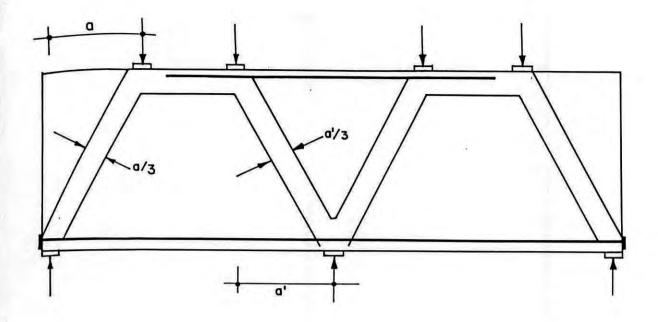


Figure 3.8 Proposed model for continuos reinforced concrete deep beams

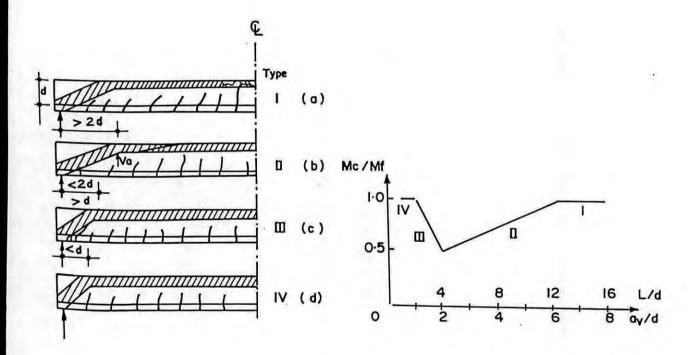


Figure 3.9 Typical types of behavior exhibited by RC beams without shear reinforcement

## 3.4.2 Design Method

To implement the preceding model in design, it is essential to complement it with a failure criterion, capable of yielding close predications of the load-carrying capacity of a RC beam. Such a criterion may take the form of the well-established relationship between the strength of an RC beam (expressed as the moment corresponding to the failure load) and the shear span av or span-to-depth (L/d) ratio, as depicted by the curved of figure 3.9. The figure indicates that the curve is divided essential into four portions each one corresponding to a particular type of beam behaviour characterized by a particular mode of failure. It may also be interesting to note that, while the horizontal portions of the curve describe the ultimate moment of resistance of a section in pure bending, the inclined portions essentially reflect the effect of the shear force on the maximum moment that can be sustained by the section. It should be noted that, although the curve in figure 3.9 implies that for  $a/d \le 1$  (L/d  $\le 2$ ), the beam exhibits a flexural mode of failure, this, in fact, may not always be true. Failure may occur within the shear span and be brittle in nature under a failure load corresponding to a maximum bending moment which is not usually significantly different from flexural capacity M<sub>f</sub>. To prevent such failure, the legs of the frame may be designed as follows (Figure. 3.10):

- (1) Moment equilibrium Cz = Pa, which yields x
- (2) Horizontal force equilibrium T = C, which yield  $A_s$  where,  $\sigma_c = 0.88 f_{cyl} = 0.64 f_{cu}$
- (3) Check whether a/3 satisfies vertical force equilibrium  $C_{\emptyset}$  sin $\emptyset$  =P; If not, adjust b and repeat.

$$M_f = T*z = C*z$$
, Where  $C = bx\sigma_c$ , and  $T = A_s f_y$   
 $f_{cyl} = Cylinder$  concrete strength  
 $f_{cu} = Cube$  concrete strength  
 $f_y = Steel$  characteristic strength

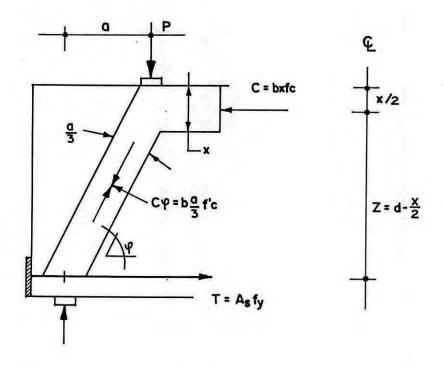


Figure 3.10 Design procedures of Types iv behavior

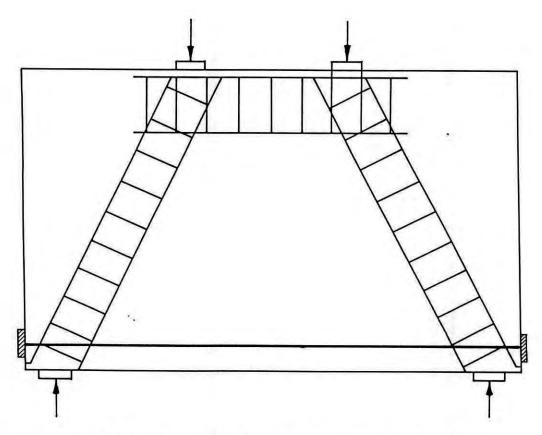


Figure 3.11 Schematic representation of members of proposed model design as column

The author suggested that if the inclined concrete strut can not carry the total compressive force and the width of the beam could not be increase to improve the strut capacity, then the strut may be design as a column (Figure. 3.11) to sustain this loads.

An analytical expression of the inclined portion of the curve (Figure. 3.9) for  $a_v/d \ge 2(L/d \ge 4.0)$  has been proposed by Kotsovos and Bobrowski (1993) as follows:

$$M_c = 0.875 \text{sd} \left( 0.342b + 0.3 \frac{M_f}{d^2} \sqrt{\frac{z}{s}} \right) \sqrt[4]{\frac{16.66}{\rho_s f_r}}$$
 3.15

Where s is the distance of the critical cross-section from the support (mm) shear span for two-point loading; s = 2d for uniformly distributed loading (all parameters refer to the critical cross section)

 $M_c$  = the moment corresponding to failure load (N-mm)

M<sub>f</sub> = the flexural capacity (N-mm)

d = the effective depth (mm)

z =The lever arm of the horizontal internal actions (mm)

 $\rho_w$  = tension steel ratio

 $f_y$  = the characteristic strength of the tension steel (N/mm2)

b = the effective width (mm)

The portion of the curve of  $1 < a_v/d < 2$  (2 < L/d < 4) can be defined by linear interpolation between the values, of  $M_c = M_f$  for  $a_v/d = 1$  (L/d = 2) and  $M_c$  as obtained form Eqn. (3.15) for  $a_v/d = 2$  (L/d = 4). In this case the critical section will coincide with the section through the joint of the horizontal and inclined members of the frame.

The use of the preceding failure criterion in design will involve (a) designing the horizontal member of the frame to be capable of sustaining a compressive force C

such that C = T = M/z (Figure. 3.10); and (b) checking whether the applied moment  $(M_a)$  applied at the critical cross section is larger or smaller than  $M_c$ . If  $M_a < M_c$ , only nominal web reinforcement would be required similar to that specified by current codes of practice, whereas if  $M_a > M_c$ , web reinforcement should be designed in accordance with the requirements (Kotsovos and Bobrowski, 1993) described as follows:

#### Web reinforcement

In compliance with the concept of the compressive force path, failure of the model should occur due to failure of either the horizontal or inclined members of the frame or the joint of these members. When such failure occurs under a load level lower than that corresponding to the flexural capacity of an under-reinforced concrete beam, it will be brittle in nature and, therefore, should be prevented. However, there cannot be a single method for preventing brittle failure since as figure 3.9 indicates, there are a number of different types of failure characterizing beam behaviour.

For a ≤ d the model essentially represents a deep beam, and brittle failure is associated with failure of the inclined leg of the frame. It has been suggested that the most effective way to prevent such failure is by adjusting the cross-sectional area of the beam to satisfy condition (3) of figure 3.10 and providing nominal web reinforcement such as that specified in current practice for deep beam design. Designing web reinforcement in compliance with current design methods should not be replied upon to safeguard against brittle failure since there has been experimental evidence (Kotsovos and Bobrowski, 1993) that shown that the presence of such reinforcement has an insignificant effect on load-carrying capacity. On the other hand, the provision of nominal reinforcement is essential, since it practically eliminates the possibility of instability caused by unforeseen out of plane actions.

For d < a < 2d, brittle failure is associated with failure of the horizontal member of the frame in the region of the joint. The penetration of the inclined crack deeply into the compressive zone reduces the zone cross-sectional area and thus leads to a reduction of both the load carrying capacity of the zone as well as the moment that can be sustained by the model section. Failure can be prevented by increasing the section moment of resistance to the level of the applied bending moment through the provision of web reinforcement in the form of stirrups uniformly distributed throughout the portion of the model between the support and the joint of the frame member. Such reinforcement should be designed such that at yield, it would be capable of sustaining a total tensile force  $T_{sv} = A_v f_{yv}$ , applying halfway between the support and the joint, the contribution of which to the moment capacity of the critical section s is  $M_{sv} = T_{sv}a/2 = M_a - M_c$ . Hence, the total amount of stirrups required to provide the critical section with a moment of resistance larger than, or equal to, the applied moment will be

$$Av \ge \frac{T_{sv}}{f_{vv}} = \frac{2(M_a - M_c)}{af_{vv}}$$
3.16

For  $a \ge 2d$ , the most likely cause of brittle failure of the model is the tensile force  $V_a$  that develops in the region of the joint to balance the action of the resultant of the compressive force acting on the joint in the direction of the frame members (Figure. 3.9b). A measure of the tensile force that can be resisted by concrete alone in this region is given by

$$V_c = \frac{M_c}{S}$$
 3.17

Where  $M_c$  and s are defined by Eqn. (3.15).

To prevent brittle failure of the joint, stirrups should be provided to sustain the force  $(V_a - V_c)$ . The presence of such reinforcement will slightly modify the "comb-like" model as indicated in figure 3.12. The figure provides a schematic representation of the additional internal actions developing in the region of the joint and indicates that the stirrups not only sustain the action of the vertical component  $(V = V_a - V_c)$  of the compressive force resultant, but also subject the shaded concrete block, where it is anchored, to a compressive force D. This force balance the shear force V acting at

the right hand side of the preceding block. It should be noted that the stirrups will be activated only when the capacity of concrete to sustain alone the action of the internal tensile force is attained. When this occurs, the excess tensile force will be sustained by the stirrups with a cross-sectional area equal to

$$A_{v} = \frac{V_{a} - V_{c}}{f_{vv}}$$
 3.18

Placed over the length d of the shaded portion of the block in figure 3.12. It has been found that the inclined and horizontal members of the concrete frame are capable of sustaining the internal actions which develop for equilibrium purposes as indicated in figure 3.12, and therefore, it will be sufficient to provide nominal web reinforcement outside the shaded block to comply with current codes of practice. It should be noted that, if the beam is subjected to point loads, stirrups may also be required within the horizontal member of the frame of the model in the region of the point load. The need for such reinforcement arise due to the development of tensile stresses within this region, which occurs when the tension reinforcement is depended form the concrete in the vicinity of the section including the load point.

### 3.5 DEEP BEAM MODEL AND DESIGN WITH OPENING

Mansur and Tan (1999) describe the behavior of beams under bending, shear, and torsion. Design methods based on plastic hinge mechanism, plasticity truss and strutand-tie models, and skew-bending theory. Suitable guidelines on the detailing of beams with web openings are also included.

In the design of concrete deep beam with web opening, the usual acceptance criteria for structural members should apply. There are basically the Strength and serviceability requirements. When the depth of the opening is less that 40% of the over all depth then it in called small opening. As in the case of small openings, the

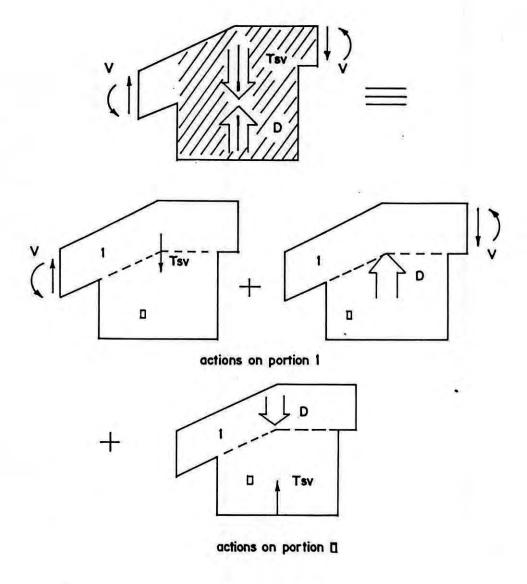


Figure 3.12 Additional internal actions developing due to presence of transverse reinforcement in region where path changes direction for Type II behaviour

flexural strength of a beam depends on the area of concrete in compression that is available above the opening to provide the moment of resistance. The analysis therefore applicable to beams with large openings under pure bending. However, noting that the chord members above and below the opening behave as compression strut and tension tie, respectively, it is essential that the slenderest effect in the compression strut be accounted for. As the length of the opening and, hence, the length of the compressive strut becomes longer, premature instability failure of the beam may occur. According to the ACI Code (1995), for compression members in a non-sway frame, the effects of slenderness may, in general, be neglected when

$$\frac{kl_u}{r}$$
 < 22

in which k is the effective length factor which may be taken as 1,  $l_u$  is the unsupported length of the compression chord, and r is the radius of gyration which may be taken as

$$r = 0.3d_c$$

where dc can be taken as the depth of the compression chord.

Mansur stated a simplified equation to find out the axial force in bottom chord and top chord as per

$$(N_u)_b = \frac{M_m}{z}$$
 3.19

Where  $M_m$  is the applied moment at center of opening above gives the magnitude of axial force in the chord members directly irrespective of the amount of reinforcement

#### 3.6 SOLID ELEMENT

Solid elements enable the solution of structural problems involving general three dimensional stresses. There is a class of problems such as stress distribution in concrete dams, soil and rock strata where finite element analysis using solid elements provides a powerful tool. The solid element used in STAAD Pro is of eight nodded

isoparametric type. These elements have three translational degrees-of-freedom per node.

The stiffness matrix of the solid element is evaluated by numerical integration with eight Gauss-Legendry points. To facilitate the numerical integration, the geometry of the element is expressed by interpolating functions using natural coordinate system, (r,s,t) of the element with its origin at the center of gravity. The interpolating functions are shown below:

$$X = \sum_{i=1}^{8} hixi, y = \sum_{i=1}^{8} hiyi,, z = \sum_{i=1}^{8} hizi$$
 3.20

Where x, and z are the coordinates of any point in the element and  $x_i$ ,  $y_i$ ,  $z_i$  i=1,...,8 are the coordinates of nodes defined in the global coordinate system. The interpolation functions,  $h_i$  are defined in the natural coordinate system, (r, s, t). Each of r, s and t varies between -1 and +1. The fundamental property of the unknown interpolation functions  $h_i$  is that their values in natural coordinate system is unity at done, I, and zero at all other nodes of the element. The element displacements are also interpreted the same way as the geometry. For completeness, the functions are given below:

$$u = \sum_{i=1}^{8} hiui, v = \sum_{i=1}^{8} hivi,, w = \sum_{i=1}^{8} hiwi$$
 3.21

where u, v and w are displacements at any point in the element and  $u_i$ ,  $v_i$ ,  $w_i$ , i=1, 8 are corresponding nodal displacements in the coordinate system used to describe the geometry.

# CHAPTER FOUR LABORATORY INVESTIGATION

#### 4.1 INTRODUCTION

The experimental phase of the study has been distinctly divided into a sequence of works:

- Design of test beams
- Determination of the properties of the constituents of reinforced concrete.
- Fabrication of Reinforcement
- Design of concrete mix
- Preparation of test beams and control cylinders
- Curing of test beams and control cylinder
- Preparation of arrangement of load testing machine
- Testing procedure

## 4.2 DESIGN OF TEST BEAMS

One solid deep beam was designed according to CFP method (detail calculations are shown in the Appendix –A). Another six deep beams were designed as per simple beam equation of (Mansur and Tan) considering the openings only (detail calculations are shown in the Appendix –A). Figure 4.1a to 4.1g show the reinforcement details of the deep beams designed, fabricated and tested during the course of study. It is clear from the figure that the reinforcement detailing of beams B2 and B5, B3 and B6, and B3 and B7 are very much similar, except the fact that B5, B6 and B7 had additional diagonal reinforcement around the opening, in comparison to B2, B3, and B4, respectively.

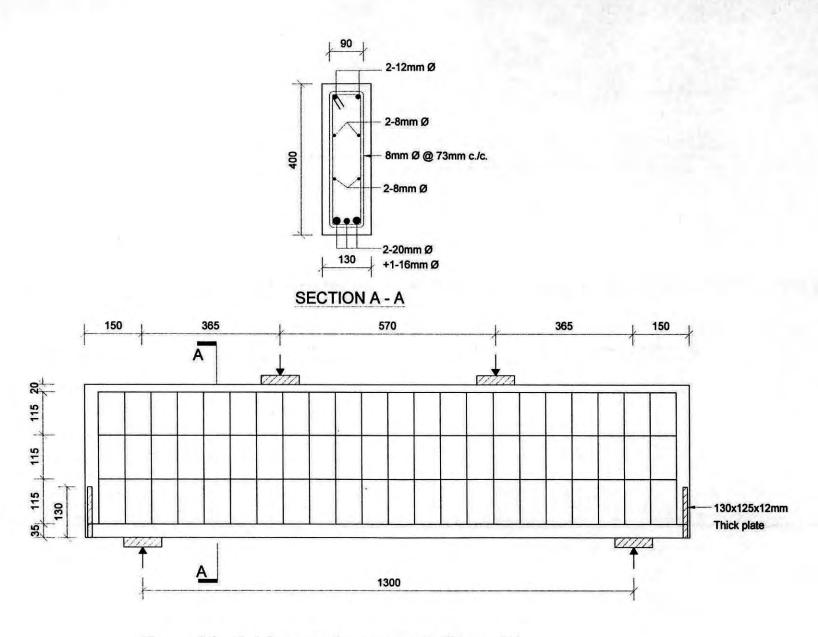


Figure 4.1a Reinforcement arrangement of Beam B1

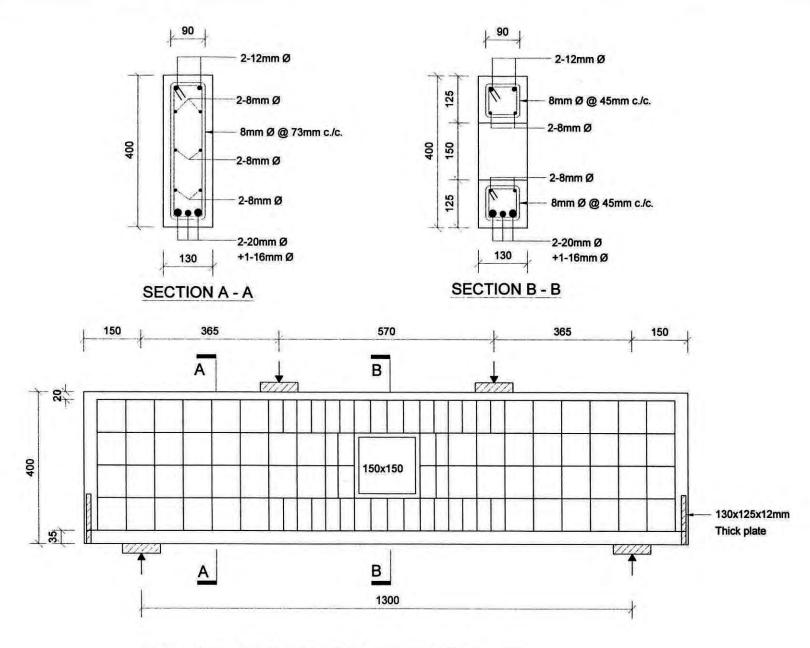


Figure 4.1b Reinforcement arrangement of Beam B2

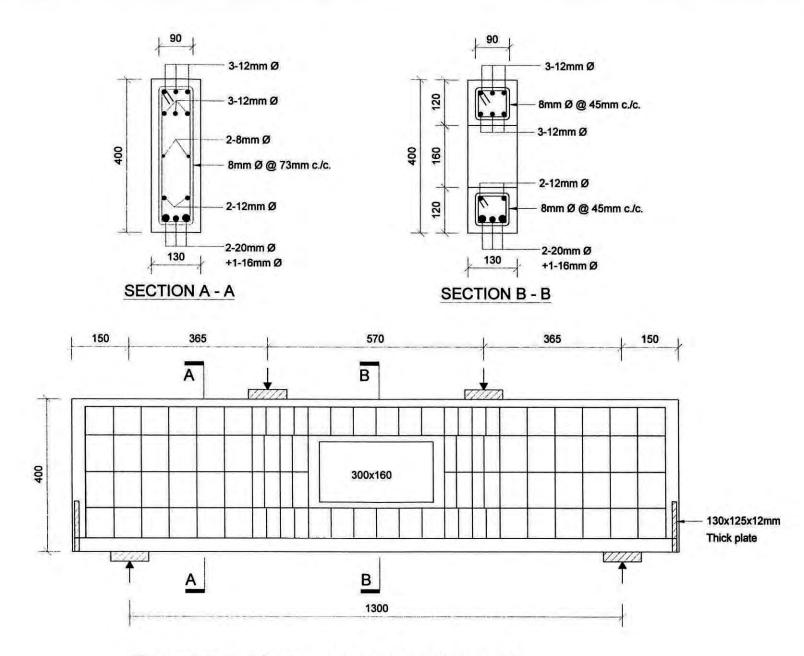


Figure 4.1c Reinforcement arrangement of Beam B3

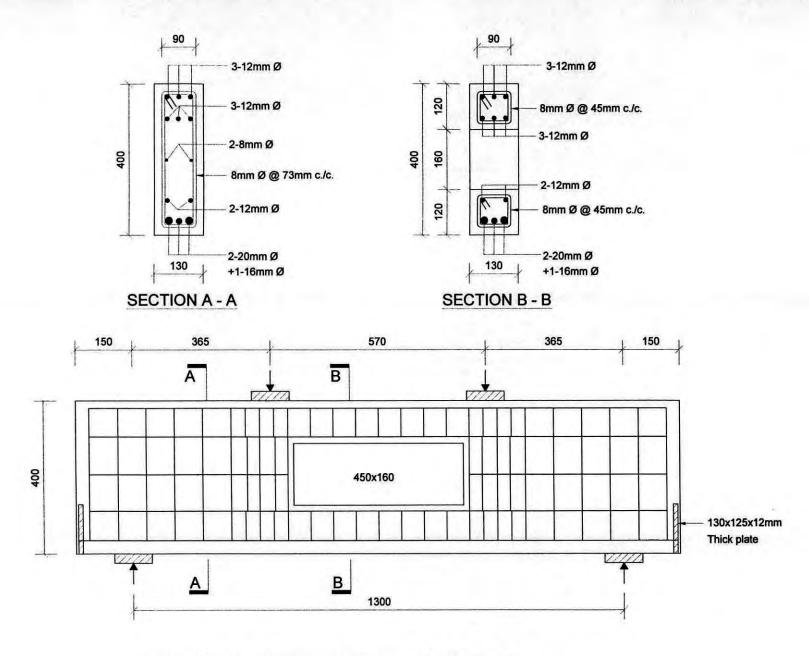


Figure 4.1d Reinforcement arrangement of Beam B4

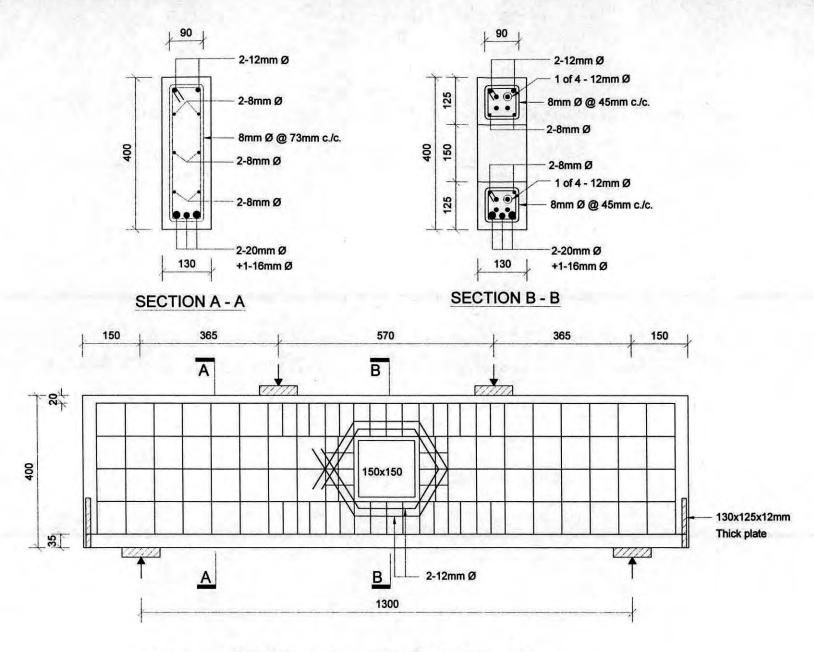


Figure 4.1e Reinforcement arrangement of Beam B5

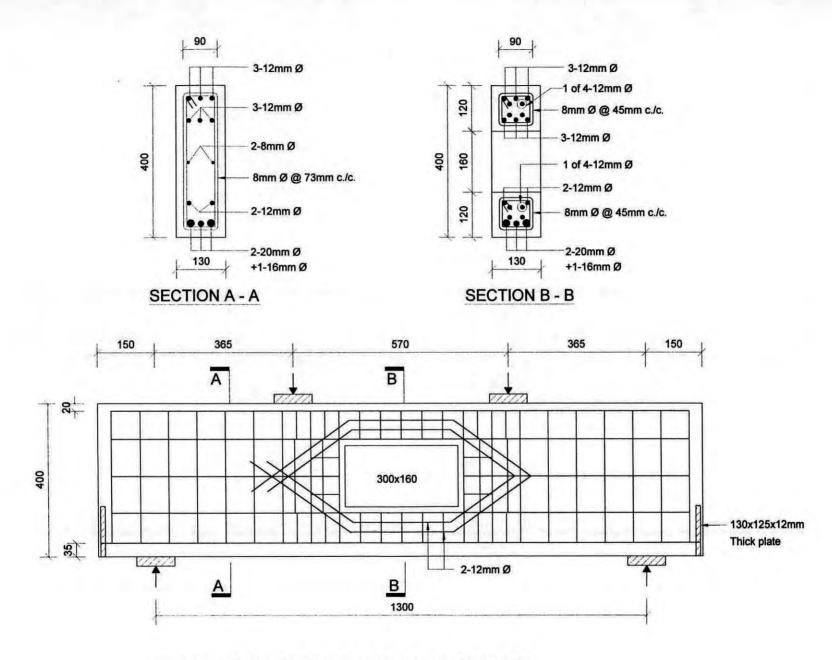


Figure 4.1f Reinforcement arrangement of Beam B6

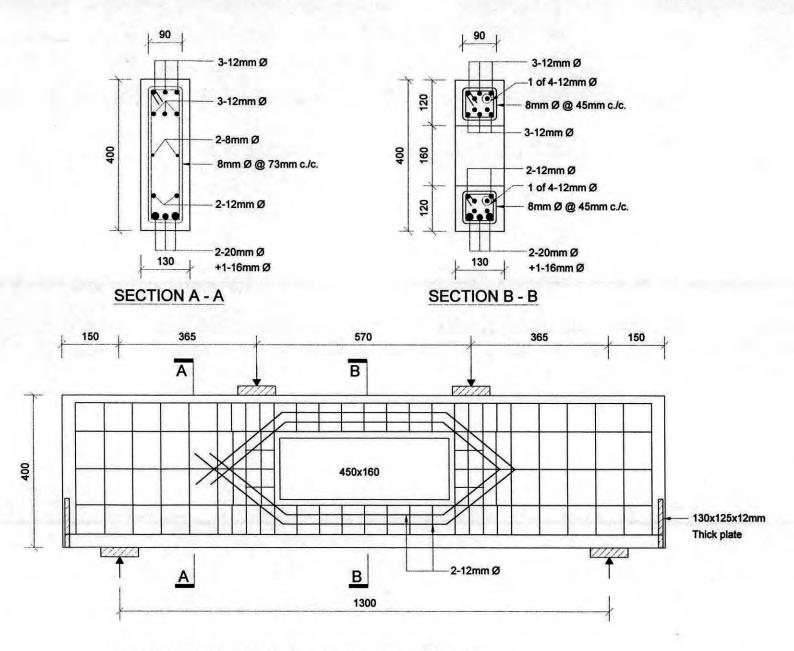


Figure 4.1g Reinforcement arrangement of Beam B7

### 4.3 DETERMINATION OF THE PROPERTIES OF THE CONSTITUENTS OF REINFORCED CONCRETE

Stone aggregates from boulder were used as coarse aggregate. The boulders were crushed to 12 mm down size because of small size of model of the actual deep beam. Sylhet sand passing No. 4 was used as fine aggregate. The absorption test, specific gravity, unit weight and fineness modulus of the aggregates were determining as per ASTM recommendation.

The grading and other physical properties of the coarse and fine aggregates are shown in Table 4.1 and Table 4.2 respectively.

#### 4.4 REINFORCEMENT

Mild steel deformed bars of 12,16 and 20 mm nominal diameter were used as main flexural reinforcement and 8 mm deformed bars were used as web reinforcement. These specimens of all sizes were tested as per ASTM A 370 - 77. The results are shown in Table 4.3.

Table 4.1 Grading of Aggregates

Sieve Size	Cumulative Percentage Retained			
Sieve Size	Coarse Aggregate	Fine Aggregate		
1 ½				
1	V =			
3/4				
1/2	14.1			
3/8	42			
1/4	98.5			
#4	98.8	0		
#8	100	2.3		
#16	100	13.4		
#30	100	43.4		
#10	100	8.3		
#100	100	97.6		
Fineness modulus	6.54	2.4		

Table 4.2 Physical Properties of Aggregates

Property	Stone chips	Sylhet Sand	
Fineness modulus	6.54	2.4	
Unit weight (loose)	1553.3 Kg/m <sup>3</sup>	1580.1 Kg/m <sup>3</sup>	
Bulk sp gravity	2.71	2 .21	
Bulk sp gravity (ssd)	2.73	2.30	

#### 4.5 DESIGN OF CONCRETE MIX

There are some standard methods for concrete mix design. Most of the time the ingredients remain in filed condition and to bring it at required condition as per recommendation is a difficult job. Here a trial mix method for proportioning required cylinder strength has been followed on the basis of the test results of the trial mix, the ratio of cement, sand and aggregate were finally selected. Ordinary Portland cement was used through out the test programme. The mix was designed to attain a nominal cylinder crushing strength of 30 N/mm<sup>2</sup> at 28 days of curing. Mix details are given below.

Design Strength 30 N/mm<sup>2</sup>

Mix proportion (by volume)

Cement: sand: stone Aggregate = 1:1.5:3.

Water cement ratio =0.45

The slump of the fresh concrete = 50 mm

#### 4.6 PREPARATION OF MOULDS

Moulds for casting of concrete deep beams with opening were made in the laboratory using timber. For mould one inch wooden plank was used. Care was taken to keep the moulds nearly water tight during casting of the test beams. In all the beams, reinforcements were fabricated according to the design. The flexural reinforcements were welded at both the ends with the steel plates and the web reinforcements were brought in position by binding them with each other and with the flexural reinforcement by mean of wire. (Figures 4.1a to 4.1g)

Concrete mixes were prepared in the laboratory with mixing machine. In one day all seven beams were cast and twenty eight control cylinders were prepared. Beams were cast in three layers and after each layer, compaction of concrete was achieved by vibrating the mould filled with fresh concrete on a machine vibrator. Care was taken such that web reinforcement assembly was not changed under the action of the

vibrator. Two extended stirrups were provided near the ends of each beam and these were used as lifting hooks.

### 4.7 CURING OF TEST BEAMS AND CONTROL CYLINDER

Curing of test beams were performed for 28 days by warping these with moist gunny bags. Those gunny bags were moistened thrice daily. The control cylinders were kept under water for curing 28 days.

## 4.8 PREPARATION OF ARRANGEMENT OF LOAD TESTING MACHINE

The specimens were white washed on either sides to facilitate visual observation of the preparation of cracks on the beam surface. A 50 mm square grid mesh was drawn on the white washed face between the supports only to establish the relative locations of cracks. The specimens were tested using Universal Testing machine of 100 tones capacity. Each of the loading jacks were calibrated by proving column of 1600 Kg. (ID 1052-9-3015) The calibration results are shown in Appendix - B. The simply supported beams were tested under two point loading (Figure 4.2). The loads were applied on the top surface through 50 mm thick, 130 mm by 100 mm steel plate and the reactions were supported by the same size of plates at the bottom of the beam. The reaction plates were supported on 16 mm dia steel rollers to give an ideal support condition. The center lines of the loading and support region of each beam were clearly marked on both sides of all the beams.

#### 4.9 TESTING PROCEDURE.

One deflectometer graduated in 0.001 in./division was employed to measure the central deflection of each span. Defectometer readings were recorded at each load increment. Before loading, deflectometer readings were set to zero. The beam was then loaded and deflections were recorded at regular intervals of load increment. An

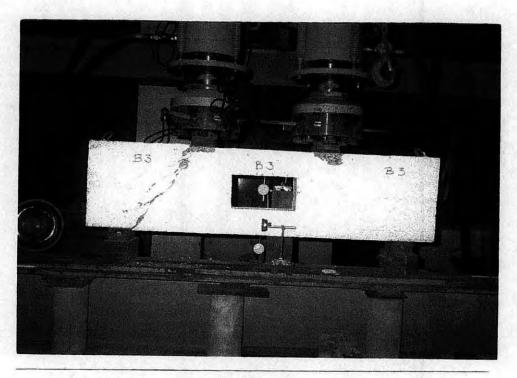


Figure 4.2 Loading of model test specimen

increment of 1 Ton of load in each of the jacks was applied within one minute and then the load was kept constant for another minute. Cracks were searched by a magnifying glass and were deeply marked with a soft pencil upon their formation and propagation on the beam surface and the load intensity at which it was formed was noted by the side of the crack. The results will be reported in Chapter 6

### 4.10 COMPRESSIVE STRENGTH OF ASSOCIATED CYLINDERS

Three cylinders, which were cast along with each of the beams, were tested under axial compression to determine the ultimate compressive strength  $(f_c)$  of concrete. The associated cylinders were tested at the time of testing the beam. The test results are shown in Table 4.4

Table 4.3 Physical Properties of Reinforcement

Nominal diameter	yield/proof strength (MPa)	Ultimate strength (MPa)	elongation (%)  23  19	
20 mm	482.75	728.44		
16 mm	444.93	671.85		
12 mm	433.49	662.06	20	
8 mm	531.10	699.28	18	

Table 4.4 Compressive strength of associated cylinder's

Beam	f' <sub>c</sub> N/mm <sup>2</sup>	f'c N/mm²	
	At 28 days	At days of testing	
B1	32.9	38.0	
B2	34.0	36.8	
В3	27.0	37.5	
B4	34.0	36.5	
B5	32.4	36.2	
В6	28.6	34.7	
В7	31.3	35.8	

#### CHAPTER FIVE

# RESULTS AND DISCUSSION ON PHYSICAL AND NUMERICAL EXPERIMENTATION

#### 5.1 GENERAL

A total of seven reinforced concrete deep beams were tested under concentrated loading as previously discussed. Opening of the beam, length, breath, over all depth, steel arrangement, and loading condition were discussed in the previous chapter. The investigation is conducted to find out the role of opening in the deep beam. The special interest of this investigation was to ascertain the validity of the concept of compressive force path method for deep beams with openings. The results of the tests have been presented in this chapter. The results have been critically analyzed and discussed in order to further investigate and understand the behaviour of deep beams with openings.

#### 5.2 BEHAVIOUR OF TEST BEAMS UNDER LOAD.

#### 5.2.1 Crack Pattern and Load Carrying capacity

**Beam B1:** This beam was a solid beam. The first flexural crack was observed almost at the mid span at a load of 148.5 kN. The crack gradually extended upwards. The first diagonal crack was observed at the load of 189.2 kN, which then propagated upwards. The failure is the split of concrete near the loading point at the load of 856.3 kN. Just before failure a wide crack line is formed between edge of the loading point and the support.

Beam B2:- This beam had a 150 mm x 150 mm opening at the center. First flexural crack was at corner of the opening and at mid span of the beam at load of 111.5 kN. The first diagonal crack was found at the load of 189.2 kN. The beam is failed by splitting of concrete at the left side of loading point. Just before failure, splaying of concrete occurred near the loading point of load 838.7 kN.

Beam B3:- This beam had a 300 mm x 160 mm opening at the center. First flexural crack was at the corner and at the mid span near end of the opening at the load of 130.3 kN. The first diagonal crack was formed at the load of 209.3 kN. After further loading the crack widened and several cracks were found at different loads. The beam failed at a load of 847.5 kN by spalling of concrete. This spalling of concrete occurred on the line of loading point to support.

Beam B4: This beam had a 450 mm x 160 mm opening at the center. The first flexural crack was found at the load of 148.5 kN. The first diagonal crack at the load of 189.2 kN. Several flexural crack is formed in the lower chord. No crack is formed in the upper chord. When load is increased the diagonal crack is formed and wider to wider. The beam is failed at the load of 838.7 kN.

Beam B5:- This beam had a 150 x 150 mm opening. Additional steel in comparison to B2 was provided on the sides of the opening. The first crack was formed at the load of 189.2 kN and was flexural in nature and the first diagonal crack was formed at the load of 209.3 kN. During the increase of load the concrete is splaying in the loading point at the both side. The beam failed at a load of 838.7 kN.

**Beam B6**:- This beam had a 300 mm x 160 mm opening. Additional steel in comparison to B3 was provided on the sides of the opening. The first flexural crack was formed at a load of 111.5 kN and the first diagonal crack was formed at 229.3 kN. During the increase of the load, several diagonal cracks were formed. The beam failed at a load of 838.7 kN.

**Beam B**<sub>7</sub>:- This beam had a 450 mm x 160 mm opening. Additional steel in comparison to B4 was provided on the sides of the opening. The first flexural crack was formed at a load of 93.64 kN. Due to increase of load this crack propagated upward to the edge of the opening. First diagonal crack was formed at a load of 189.2 kN. This beam failed at the end of 820.3 kN load.

A summary of the observed cracking and ultimate loads of the beams tested is given in Table 5.1. It is clear from Table 5.1 that all the deep beams sustained nearly similar amount of load, despite the presence of opening of varying sizes. The cracking load of all the beams was almost the same, although beams B5 and B6 developed initial cracking at a load of about 10% higher than their counterparts i.e. beams B2 and B3. The average of the failure loads of the six deep beams with opening was 837.1 kN, only 2% below the failure load of the deep beam without opening. It appears that for the deep beams with opening, a significant part of the load was transferred directly from the point of application to the supports by diagonal compression struts, i.e. load is transferred by tied—arch mechanism, following compressive path concept.

#### 5.2.2 Mode of failure of test beams

Figures 5.1 a to 5.1g show the crack patterns at the failure of the test beams together with loads at which each crack was first observed, and the extent of the crack at that load. Figure 5.4 a to 5.4 g are the photographs showing the general crack pattern as well as the enlarged portion showing the cracks that caused failure of the beams B1, B2, B3, B4, B5, B6 and B7, respectively.

The crack pattern and the mode of failure of all the test beams were similar despite the web opening. On loading, first flexural cracks formed nearly at center of the beam. Due to increase of the load this crack is propagated upwards. At some higher loads crack were formed near the supports.

As the applied load was increased further, the diagonal cracks formed between the loading point and the support point. There were several cracks due to increase of load, between loading point and support. As the load approached nearly 90% of the failure load, the process of further crack formation was stopped and the system stabilized. However the already visible cracks got widened in size. No horizontal crack was visible at this stage.

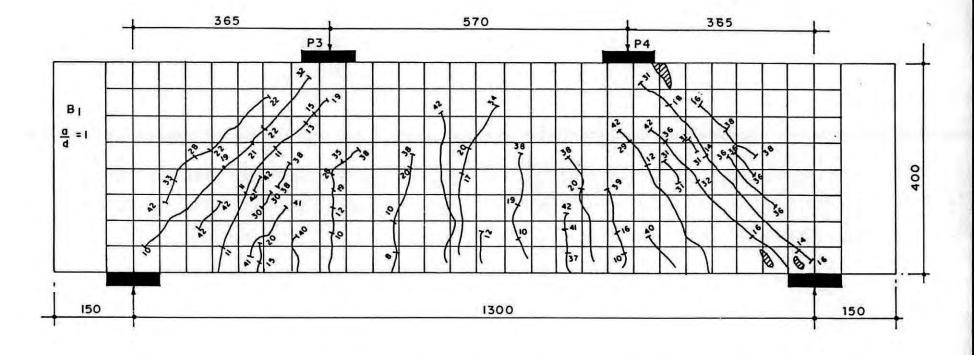


Figure 5.1a Crack pattern and individual jack load intensities (P3=P4) in tonnes of beam B1

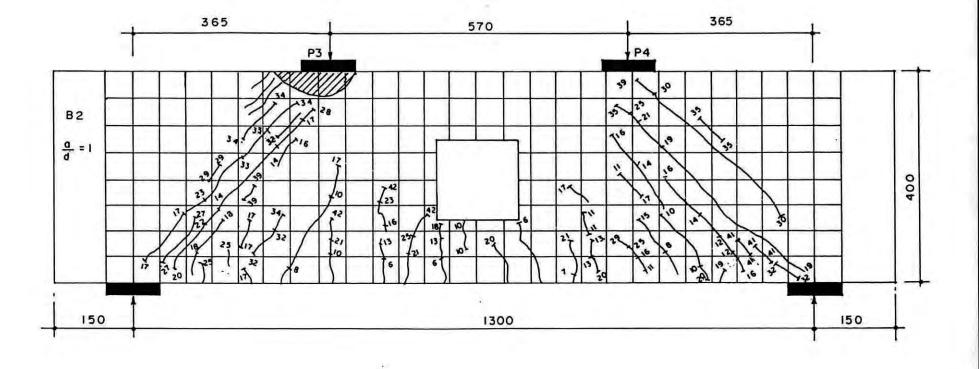


Figure 5.1b Crack pattern and individual jack load intensities (P3=P4) in tonnes of beam B2

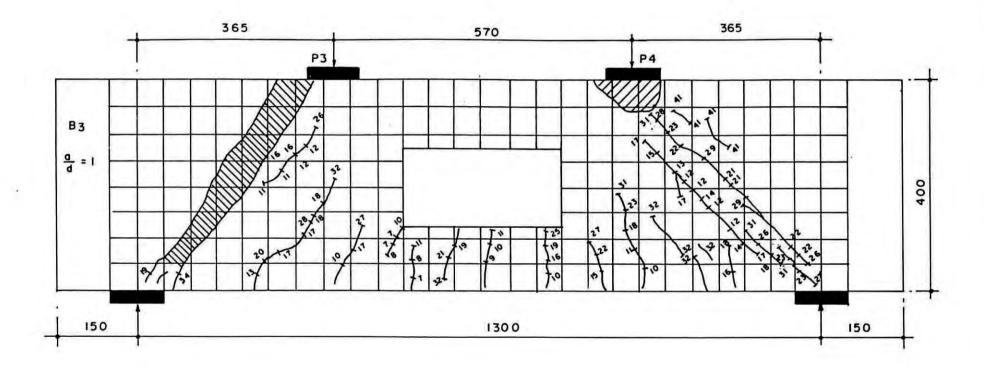


Figure 5.1c Crack pattern and individual jack load intensities (P3=P4) in tonnes of beam B3

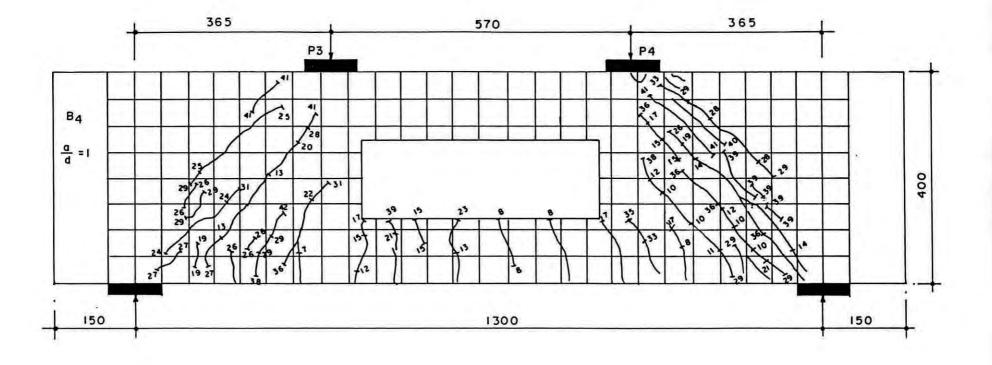


Figure 5.1d Crack pattern and individual jack load intensities (P3=P4) in tonnes of beam B4

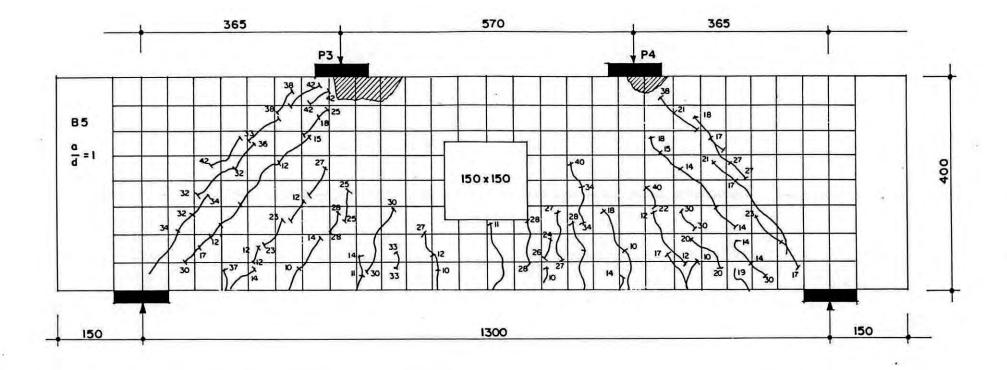


Figure 5.1e Crack pattern and individual jack load intensities (P3=P4) in tonnes of beam B5

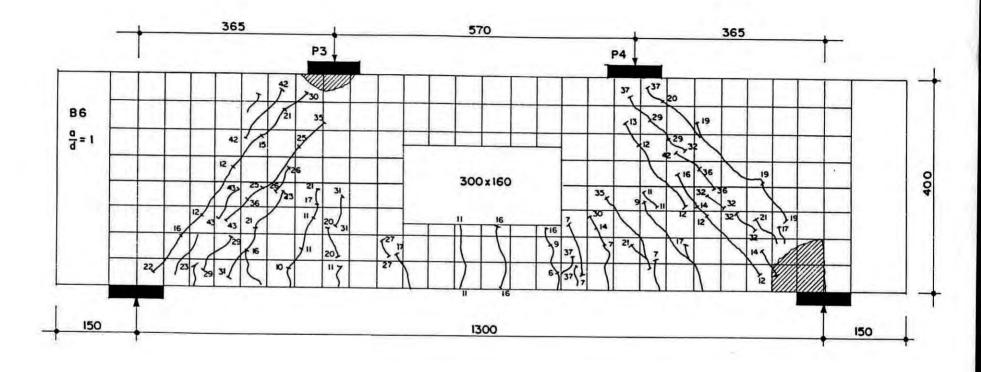


Figure 5.1f Crack pattern and individual jack load intensities (P3=P4) in tonnes of beam B6

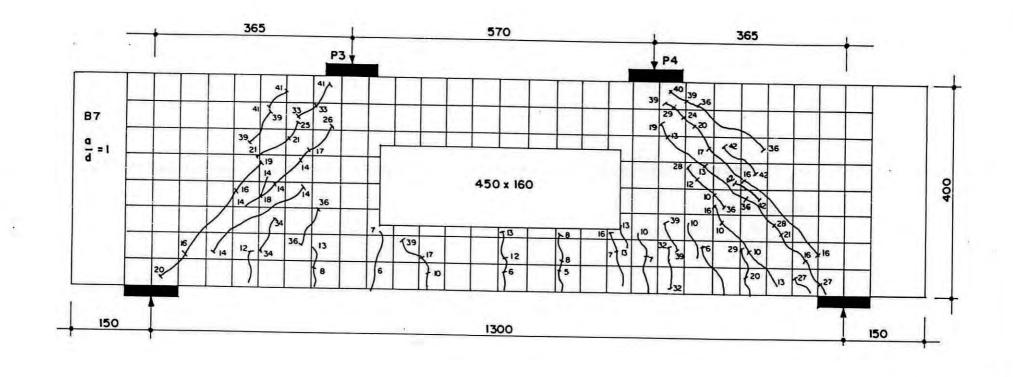


Figure 5.1g Crack pattern and individual jack load intensities (P3=P4) in tonnes of beam B7

No crack is formed at the top portion of the concrete as well as in the top chord of the opening between the loading points.

#### 5.2.3 Deflection Pattern

The deflection pattern of all the beams B1 to B7 is given in Figure 5.2. It is clear from the figure that the solid beam B1 underwent the least amount of deformation. The overall deformation of beams B5, B6 and B7 (having additional diagonal reinforcements around the openings) were lower than their B2, B3, and B4 counterparts. The crack pattern and failure mode of these beams (B5, B6 and B7) as shown earlier in Figures 5.1 and Figures 5.4 also substantiates the fact that the additional diagonal reinforcements around openings arrested early propagation of cracks to the compressive zone, thus reducing the amount of deformation at failure. All the beams, however, reached similar failure loads at varying amount of deformations. Appendix- C shows the mid span deflections of different test specimens at every one tonnes increment.

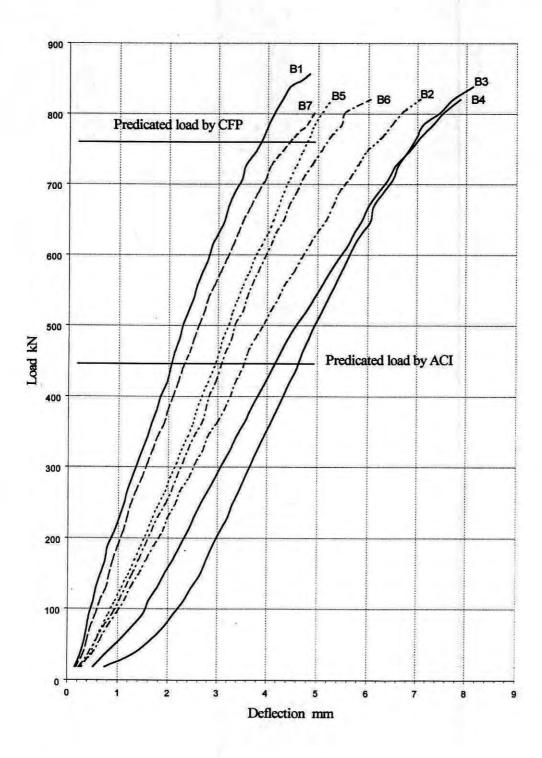


Figure 5.2 Load Deflection Curves of the Deep Beams Tested

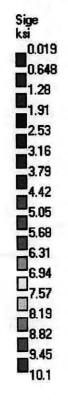
Table 5.1 Observed cracking and ultimate Loads of Test Beams

Beam No	Size of Opening	Flexural Cracking Load	Diagonal Cracking Load	Ultimate Failure Load	Design Load Assuming fc'=30MPa	Predicated Load as per ACI (at actual f <sub>c</sub> 'on the day of testing)	Expected failure of compressive force path (at actual fc'on the day of testing)	Mode of failure
	mm x mm	kN	kN	kN	kN	kN	kN	
B1	Solid	148.5	189.2	856.3	600	446	760	Inclined compression failure (ICF
B2	150 x 150	111.5	189.2	838.7	600	438	736	ICF
В3	300 x 160	130.3	209.3	847.5	600	442	750	ICF
B4	450 x 160	148.5	189.2	838.7	600	436	730	ICF
B5	150 x 150	189.2	209.3	838.7	600	434	724	ICF
В6	300 x 160	111.5	229.3	838.7	600	425	694	ICF
В7	450 x 160	93.64	189.2	820.3	600	432	716	ICF



#### 5.3 NUMERICAL EXPERIMENTATION

Physical tests of reinforced concrete members are time consuming and expensive. Numerical experimentation, on the other hand, relatively less expensive, but needs skills of using commercially available software. In the present study, STAAD Pro (2002) has been use to analyze the deep beams tested in this research. Here deep beams have been analyzed using by modeling the beams with solid elements of 30x43x40 mm size. Linear analysis have been conducted. The objective of the analyses was to ascertain the areas of the deep beams which should need special detailing during design and fabrication of the members. Figures 5.3a to 5.3h show the Von Mises stresses of the beams tested. Compressive Force Path is clearly visible. It appears that from these and other force envelopes, design of reinforced concrete members can be performed.



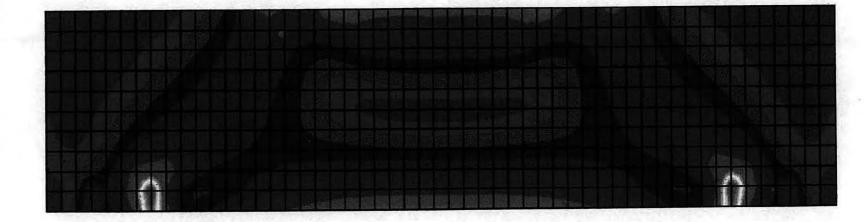


Figure 5.3a Von Mises stresses of Beam B1

Sige ksi 0.018

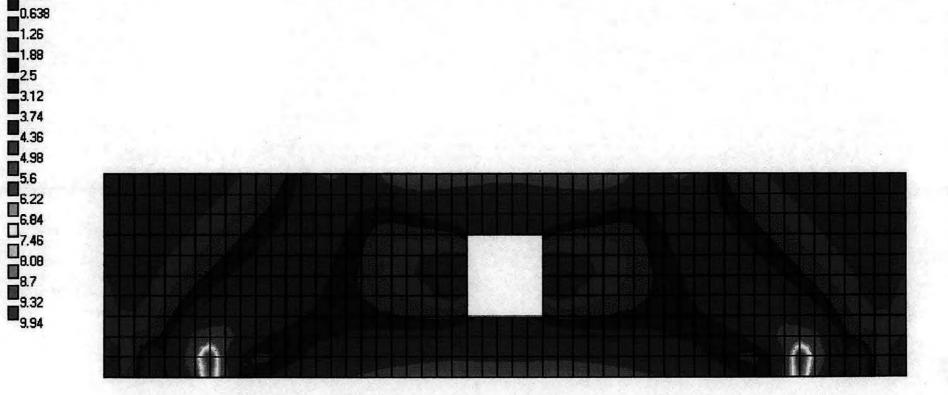


Figure 5.3b Von Mises stresses of Beam B2



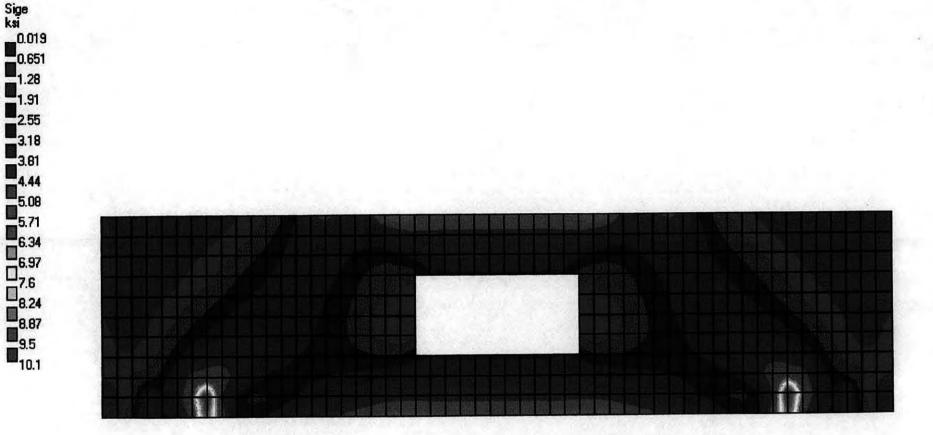
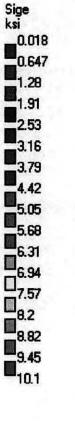
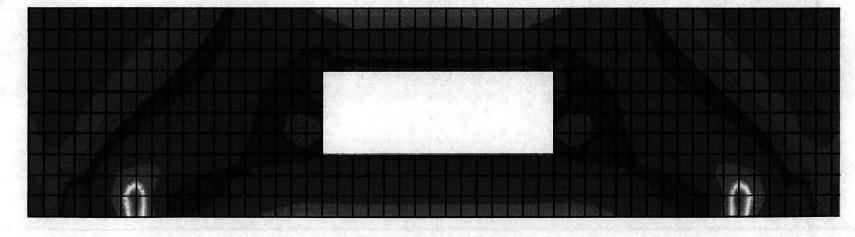


Figure 5.3c Von Mises stresses of Beam B3





 $1 \text{ kip/in}^2 = 6.894 \text{ MPa}$ 

Figure 5.3d Von Mises stresses of Beam B4

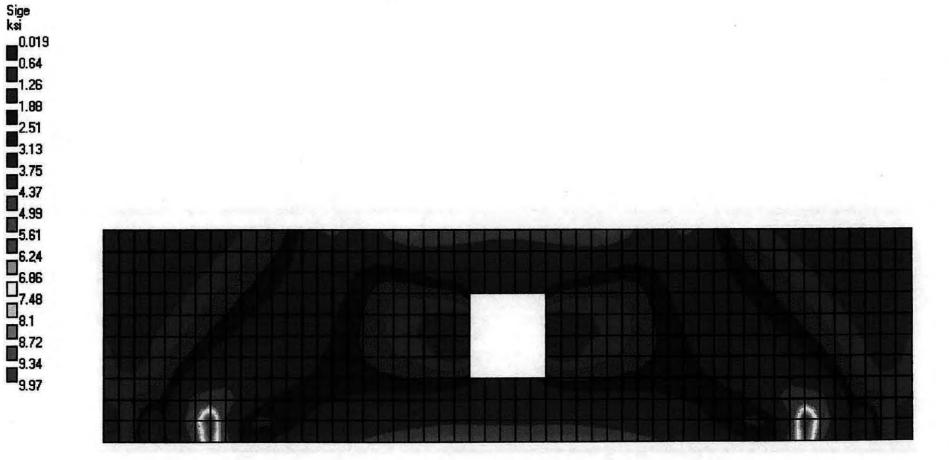
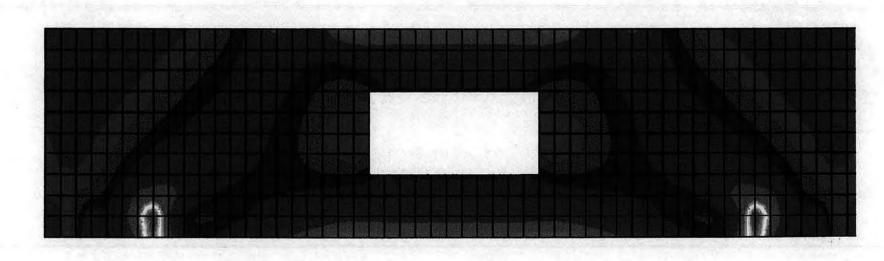


Figure 5.3e Von Mises stresses of Beam B5

Sige ksi

0.018 0.644 1.27 1.89 2.52 3.15 3.77 4.4 5.02 5.65 6.27 6.9 7.52 8.15 8.77 9.4 10



 $1 \text{ kip/in}^2 = 6.894 \text{ MPa}$ 

Figure 5.3f Von Mises stresses of Beam B6

Sige ksi 0.018

0.633 1.25 1.86 2.48 3.09 3.71 4.33 4.94 5.56 6.17 6.79 7.4 8.02 8.63 9.25 9.87

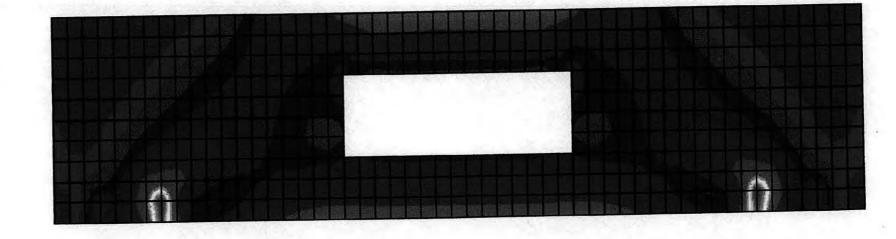


Figure 5.3g Von Mises stresses of Beam B7



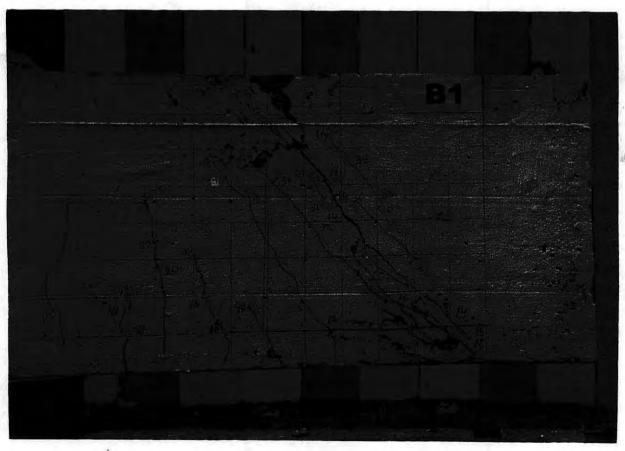
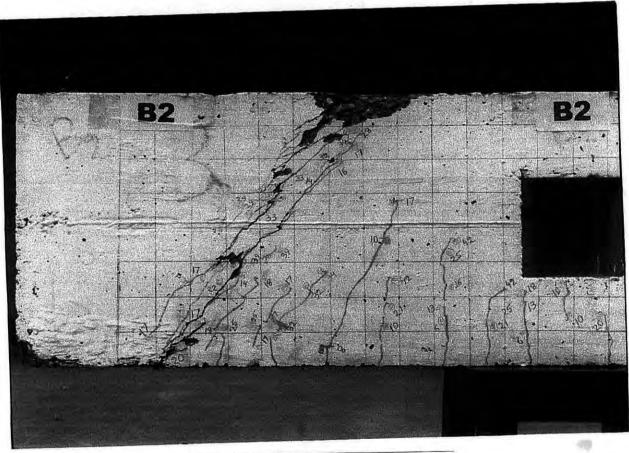


Figure 5.4a Crack pattern of beam B1





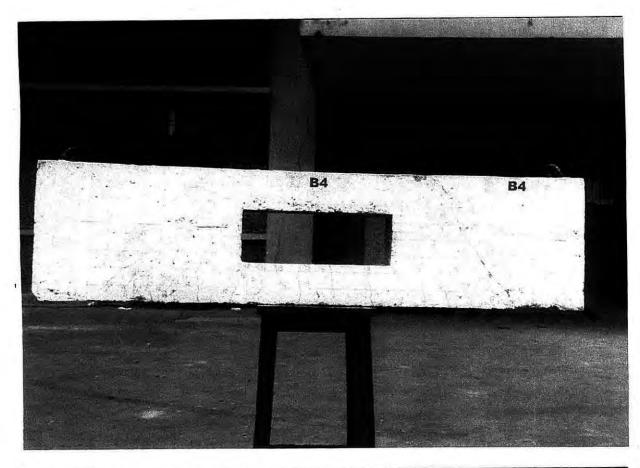
da

Figure 5.4b Crack pattern of beam B2





Figure 5.4c Crack pattern of beam B3



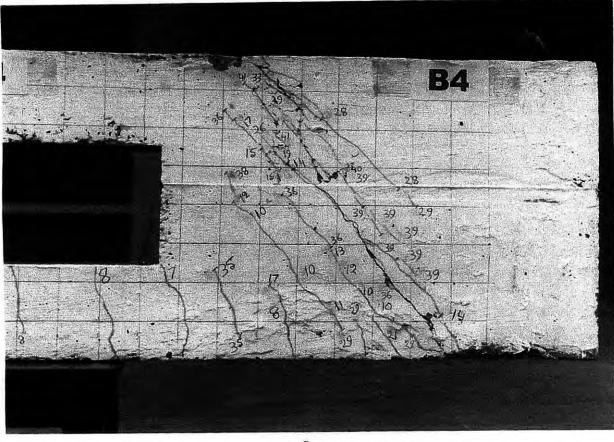
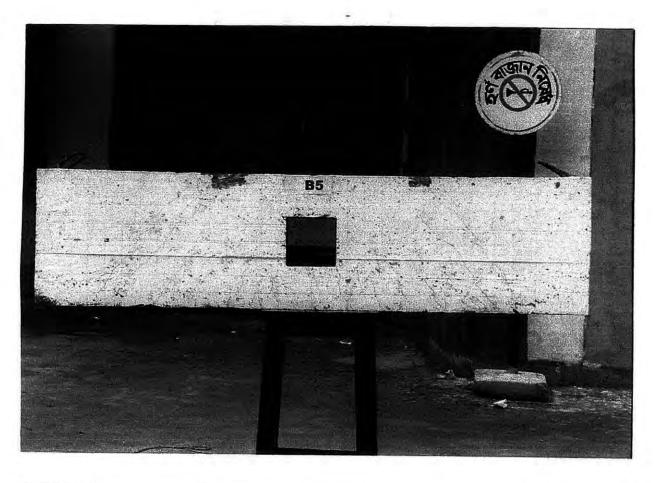


Figure 5.4d Crack pattern of beam B4



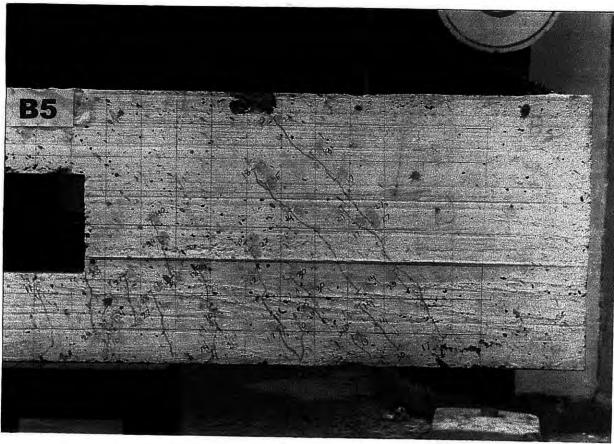


Figure 5.4e Crack pattern of beam B5





Figure 5.4f Crack pattern of beam B6





Figure 5.4g Crack pattern of beam B7

### CHAPTER SIX

### CONCLUSIONS AND RECOMMENDATIONS

#### 6.1 CONCLUSIONS

The present study presents the results of a laboratory investigation carried out on seven deep beams having shear span to depth ratio equal to 1.0. Out of these seven specimens one (Beam B1) was solid deep beam, whereas the other six had openings of varying sizes. The deep beams with openings were of two groups: In the first group (Beams B2, B3 and B4), top and bottom chords above and below the openings were provided with special confinement reinforcement at a spacing half of the effective chord depth following the recommendations of Mansur and Tan (1999). The confining stirrups were placed in between the loading points. In the other group (Beams B5, B6 and B7), in addition to strengthening the chords with confining steel as in the case of first group, diagonal reinforcement as well as horizontal confinement around the openings were provided. The following are the conclusions that could be derived, albeit tentatively, on the basis of the limited experimentation conducted during the present study:

- 1. The ultimate failure load of all the deep beams, with or without openings, tested in this study were very much similar to each other pointing at the fact that the design methodology adopted in designing the deep beams was appropriate.
- The first cracking load of all the deep beams tested was quite close to one another. However provision of additional diagonal reinforcement and horizontal confinement around the web opening have resulted in slight delay in the formation of the initial crack.
- 3. Among all the deep beams tested, the deep beam without opening underwent the least amount of deformation and was also the stiffest. Whereas the deformation behaviour of all the deep beams tested was brittle in nature, the deep beams with openings deformed slightly more than the solid beam. Additional diagonal reinforcement and horizontal confinement around the web openings have resulted

- in slightly stiffer load-deformation behaviour of deep beams with opening in comparison to similar beams which did not have such reinforcement.
- 4. For all the deep beams tested, the compressive force path from the loading points to the supports have played a significant role in carrying the loads.
- Diagonal splitting caused by failure of the inclined compressive strut has been found to be the principle cause behind the failure of deep beams with or without openings.
- A very limited numerical study on the deep beams tested have revealed the
  potentiality of possible use of numerical tools in identifying the compressive
  force path of a reinforced concrete member so that design can be done
  accordingly.

#### 6.2 RECOMMENDATIONS FOR FUTURE STUDY

The behaviour of deep beams in general and deep beams with openings in particular is a complex problem. Although out of the present study a number of points could be recognized, further studies in the following areas may be under taken to further consolidate the understanding:

- The effect of the shear-span-to-depth ratio on the strength and deformation behaviour of deep beams with openings may be studied.
- 2. Study of the behaviour of deep beams subjected to loading above the web opening may be studied.
- Deep beams with eccentric opening or spanning over a number of spans may also be studied.

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### Design calculation of Beam B1

Width of beam b: 130 mm

Height of beam h: 400 mm

Depth of beam d: 365 mm

Length of beam 1: 1600 mm

Span length s: 1300 mm

Self weight w: 1.96 kN

Support Reaction R =  $\frac{600 + 1.96}{2}$  = 300.98*kN* 

As per figure 3.11

$$C*Z = P*a$$

$$b*x*f_{cyl}*(d-x/2) = P*a$$

$$d*x - x^2/2 = \frac{P*a}{b*fcyl} = \frac{300.98*365}{130x30} = 28168$$

$$x^2 - 365*x*2 + 56337 = 0$$

$$x = (730 \pm \sqrt{(730/2 - 4*56337)})/2$$

$$= 87.71$$
mm

$$z = d - x/2 = 365 - \frac{87.71}{2} = 321.14 \text{ mm}$$

$$C \otimes \sin \Theta = \frac{b * a}{3} * f_{cyl} * \frac{z}{\sqrt{a^2 + z^2}} = \frac{130 * 365}{3} * 30 * \frac{321.14}{\sqrt{365^2 + 321.14^2}} = 313 \text{ kN}$$

$$C = b*x* f_{cyl} = 130* 87.71* 30$$

$$T = C = A_s f_y$$

As = 
$$\frac{T}{fy} = \frac{342.069*1000}{415} = 824 \text{ mm}^2$$

Provide 2 – 20 mm ø + 1-16 mm ø

Vertical stirrup 
$$=\frac{d}{5} = \frac{365}{5} = 73$$

Horizontal Steel = 
$$\frac{d}{3} = \frac{365}{3} = 121mm$$

Provide 3 No 8 mm ø

### Design Calculation of Beam B2 150 X 150 (Opening)

Compressive stress resultant

$$C = 0.85 \text{ f}'_C \text{ b* t} = 0.85 * 30* 130* 125$$
  
= 414.375 kN

For Equilibrium T=C

$$T = A_s f_S = 414.375 \text{ kN}$$

$$f_S = \frac{414.375X1000}{930} = 45 \text{ N/mm}^2 (2-20 \text{ mm } \varnothing +1-16 \text{ mm } \varnothing +2-8 \text{ mm } \varnothing = 930 \text{ mm}^2)$$

Nominal flexural strength

$$M_n = C*(d - h_C/2)$$
= 414 (365 -  $\frac{125}{2}$ )
= 125 kN - m. > 109.5 kN - m

Depth of the Compression stress block

$$h_{C} = \frac{Asfy}{0.85 fcb} = \frac{929 * 415}{0.85 * 30 * 130} = 116 mm$$

As the depth of h<sub>c</sub> is less than the compression chord so beam will not fail through opening.

Stirrups spacing 
$$\frac{125-35}{2} = 45 \,\text{mm}$$

Provide 8 mm ø @ 45 mm c/c

### Design calculation of beam B3 (Opening 300 x 160)

The axial load in the Bottom Chord (Tension)

$$N_{ub} = \frac{109.5}{0.4 - 0.12} = 391kN$$

Steel Area 
$$A_S = \frac{391*1000}{415*0.9} = 1047 \text{ mm}^2$$

Provide 2 - 20 mm ø + 1-16mm ø + 2 -12 mm ø

Top chord compression member slenderness effected neglected when

$$\frac{KL_u}{r} < 22$$

$$= \frac{1*300}{0.3*0.12}$$

$$= 8.33$$

The member design as column

$$391*1000 = 0.85*0.7 [0.85*30*(130*120 - A_{st}) + 415*A_{st}]$$

$$A_{st} = 665 \text{ mm}^2$$

Provide 6 no. 12 mm ø

Stirrups spacing 
$$\frac{120-35}{2} = 42.5 \text{ mm}$$

Provide 8 mm ø @ 45 mm c/c

### Design calculation of beam B4 (Opening 450 x 160)

The axial load in the Bottom Chord (Tension)

$$N_{ub} = \frac{109.5}{0.4 - 0.12} = 391kN$$

Steel Area 
$$A_S = \frac{391*1000}{415*0.9} = 1047 \text{ mm}^2$$

Provide 2 - 20 mm ø + 1-16mm ø + 2 -12 mm ø

Top chord compression member slenderness effected neglected when

$$\frac{KL_u}{r} < 22$$

$$= \frac{1*450}{0.3*0.12}$$

$$= 12.5$$

The member design as column

$$391*1000 = 0.85*0.7 [0.85*30*(130*120 - A_{st}) + 415*A_{st}]$$

$$A_{st} = 665 \text{ mm}^2$$

Provide 6 no. 12 mm ø

Stirrups spacing 
$$\frac{120-35}{2} = 42.5 \text{ mm}$$

Provide 8 mm ø @ 45 mm c/c

The Design calculations of beam B5,B6 and B7 are similar to B2, B3, and B4, respectively. B5,B6 and B7, however, had extra diagonal reinforcement.

Design calculation of beam B1 (By ACI)

$$V_c = (3.5 - 2.5 \frac{M_u}{V_u d})(1.9 \sqrt{f'_c} + 2500 \ \rho_w \frac{V_u d}{M_w}) \ bd$$

Multiplier 
$$(3.5-2.5\frac{80.75}{67.44X14.36}) = 3.29$$

This multiplier should not be greater than 2.5

$$V_c = 2.5 (1.9 \sqrt{f'_c} + 2500 \ \rho_w \frac{V_u d}{M_u}) \ bd$$

$$V_c = 2.5 (1.9\sqrt{5292} + 2500 \times 0.0174 \times \frac{67.44X14.37}{80.75}) \times 14.37 \times 5.11$$

$$V_c = 121 \text{ kip}$$

$$V_{s} = \left[ \frac{A_{v}}{S} \left\{ \frac{1 + \frac{l_{n}}{d}}{12} \right\} + \frac{A_{vh}}{s_{2}} \left\{ \frac{11 - \frac{l_{n}}{d}}{12} \right\} \right] f_{vh} d$$

$$V_{s} = \left[ \frac{0.077X2}{2.87} \left\{ \frac{1 + \frac{54.31}{14.37}}{12} \right\} + \frac{0.077X2}{4.72} \left\{ \frac{11 - \frac{54.31}{14.37}}{12} \right\} \right] X60000X14.37$$

$$V_s = 35.26 \text{ kip}$$

$$V_n = 121 + 35.26 = 156.54 \text{ kip}$$

The nominal shear strength V<sub>n</sub> is not to be greater than

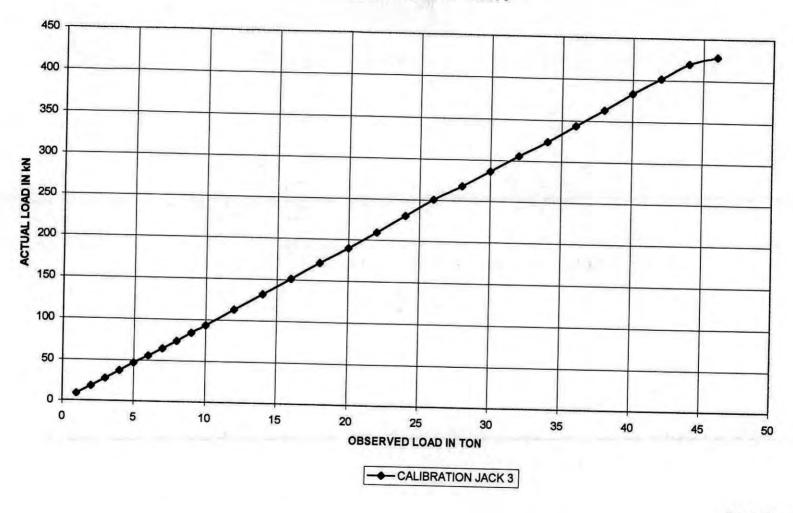
$$V_n = \frac{2}{3}(10 + \frac{I_n}{d})\sqrt{f'_c} bd$$

When  $l_n/d$  is between 2 and 5.

$$V_n = \frac{2}{3}(10 + \frac{54.31}{14.37})\sqrt{5292} \times 5.11 \times 14.37$$

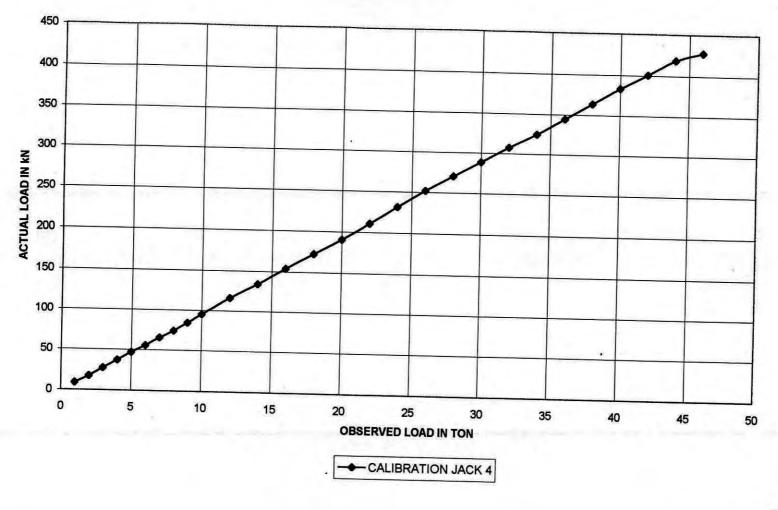
$$V_n = 49.36 \text{ kip}$$

## **CALIBRATION OF JACK 3**



Appendix -B

### **CALIBRATION OF JACK4**



# Mid span deflection of all test specimens

served	Jack	Jack /	Average	Total	Beam(		Beam		Beam(	
	LICONOVI 1	Load			Soli	id	150X150	opening	300X160	opening
load tonne	3 kN.	4 kN.	load kN.	load kN.	Ctrain	Deflection mm	Strain	Deflection mm	Strain gauge reading	Deflection mm
Approx.	4000			17.0	reading 6	0.1524	9	0.2286	29	0.7366
1	8.89	8.89	8.89	17.8	10		19	0.4826	50	1.27
2	18.14	17.52	17.83	35.66 54.78	10 13	2242	25	0.635	64	1.6256
3	27.39	27.39	27.39 37.26	54.78 74.52	13		31	0.7874	75	1.905
4		37.26	37.26 46.82	74.52 93.64	15		38	0.9652	83	2.1082
5		46.82			20		44	1.1176	91	2.3114
6		55.76 65.32	65.165	111.5	20		49	1.2446	97	2.4638
7					25		55	1.397	104	2.6416
8				168.9	29		61	1.5494	109	2.7686
9					31		67	1.7018	114	
10 11				1. 17.00	36	0.9144	74	1.8796	120	3.048
11 12					40	1.016	78	1.9812	126	
12 13		100			43.5	1,1049	84	2.1336	130	
13				266	46	1.1684	88	2.2352	135	
14					50	1.27	95	2.413	140	
15 16	The second second				54		100	2.54	145	
			4		58		106	2.6924	150	
17 18			The state of the s		62	2 1.5748	110	2.794	155	3.93
18	The second second second				65		117	2.9718	160	4.06
19					69	All the same of th	123	3.1242	165	4.19
20			110000		72		128	3.2512	170	4.31
21					77		132	3,3528	175	4.44
22					80	The state of the s	138	3,5052	181	4.597
23				and the second	83		142	3.6068	185	4.69
24					87		148	3.7592	190	4.82
25 26	No. of the last of				90			3.937	195	5 4.95
26 27	2							4.064	200	5.0
				_			166	4.2164	205	5.20
28								1 4.3434	210	5.33
29 30		_			1000			8 4.5212	215	5 5.46
30				The second second				4 4.6736	220	0 5.58
31						A. C.		0 4.826	225	5 5,71
32		C. P. C. C. C. C. C. C. C.	Control of the Control		The second secon			6 4.9784	231	1 5.867
33						777		3 5.1562	2 238	8 6.045
34		2.1						8 5.2832	2 240	0 6.09
35								3 5.4102	2 248	8 6.299
36								1 5.6134	256	6 6.502
37				Company of the Compan					2 260	0 6.60
38			The state of the s						9 268	8 6.80
39									8 275	5 6.98
40									7 280	7.1
41		_	_					201203 90		7.46
42				200			72000			
43						42.90	5 FAILURE		320	
44	The state of the s								FAILURE	-
4.	1000			_	25.00	19			- LUCKLI	1111
40	6 428.28	8 428	28 428,125	5 856.3	19	90 4.826 113	الا	1	4	

# Mid span deflection of all test specimens

bserved	21.2	1 2 2 2 3 2 5 6 7	Average	Total		Beam(B4)		m(B5)	Beam(B6)	
1000	Load	T. T. S. C. S.		1 2 -2		opening	150X15	0 opening	300X16	0 opening
load	kN.	kN.	load kN.	load kN.	Strain gauge reading	Deflectio mm	gauge reading	Deflection mm	Ctuain	Deflection mm
1	8.89		8.89	The second secon		0.50	8 11	0.279	4 10	0.25
2			17.83			0.736		0.457		0.406
3	27.39		27.39			1.01		0.558	8 22	0.558
5	37.26 46.82		37.26		50	1.2		0.685		0.711
6	55.76	3-25-97-124-9	46.82			1.498		0.812		0.88
7	65.01		55.76 65.165		63	1.600		0.939		1.01
8	74.2		74.23		70	1.77		1.066		1.14
9	84.44	0.45	84.44	148.46	75	1.90		1.193		1.2
10	93.69		94.615	168,88	82	2.082		1.320	2.01	1.39
11	103.4		104.635	189.23	88	2.235		1.447		1.52
12	113.12	12000000	114.66	209.27 229.32	94	2.3876	- 100	1.574		1.65
13	122.52	The second second second	123.83		99	2.5146		1.676	10.7	1.77
14	131.93		133.01	247.66 266.02	104	2.6416		1.803		1.930
15	141.49	143.95	142.72		110	2.794	6-27	1.9304		2.057
16	151.05	153.82	152.435	285.44	116	2.9464		2.032	A Party of the Par	2.159
17	160.92	163.07	161.995	304.87 323.99	122	3.0988		2.159		2.286
18	170.78	172.33	171.555	343.11	128	3.2512	200.1.	2.2606		2.3876
19	180.19	181.42	180.805	361.61	133	3.3782		2.3876		2.5146
20	189.59	190.52	190.055		139	3.5306		2,4892	2.00	2.667
21	199.46	200.54	200	380.11 400	144	3.6576	2010	2.5654		2.7432
22	209.33	210.56	209.945	419.89	150	3,81	105	2.667	Carrier Co.	2.8194
23	219.81	221.05	220.43	440.86	156	3.9624		2.794	2.50	2.9464
24	230.3	231.53	230.915	461.83	162	4.1148		2.921	The state of the s	3.048
		241.71	241.095	482.19	168	4.2672		3.0226		3.1496
	250.65	251.89	251.27	502.54	174	4.4196		3,0988	100 100 100	3.2766
	259.29	261.14	260.215	520.43	181	4.5974	127	3,2258		3,3528
1	267.92	270.39	269.155	538.31	188 194	4.7752	130	3.302		3,5052
	277.33	279.49	278.41	556.82	200	4.9276	134	3.4036		3.5814
	286.73	288.58	287.655	575.31	206	5.08	138	3.5052		3.683
	296.29	298.14	297.215	594.43	213	5.2324		3.6322		3.81
	305.85	307.7	306.775	613.55	220	5.4102	147	3.7338		3.937
	314.79		315.485	630.97	225	5.588	153	3.8862	160	4.064
	323.74		324.2	648.4	231	5.715	158	4.0132	164	4.1656
	333.76		334.145	668.29	236	5.8674	162	4.1148	170	4.318
	343.78	344.4	344.09	688.18	244	5.9944 6.1976	166	4.2164	174	4.4196
37	353.49	354.11	353.8	707.6	252	6.4008	170	4.318	180	4.572
		363.83	363.52	727,04	258	6.5532	175	4.445	185	4.699
	373.23		373,385	746.77	269	6.8326	180	4.572	192	4.8768
		383.25	383.25	766.5	278	7.0612	185	4.699	199	5.0546
		391.89	392.12	784.24	288	7.3152	190	4.826	206	5.2324
		400.52	400.985	801.97	297	7.5438	194	4.9276	215	5.461
43 4				820.31	310	7.874	200	5.08	218	5,5372
	19.64			838.66 F.			207	5.2578	238	6.0452
	23.96	423.5		847.46	LLOILE		FAILURE		FAILURE	
				856.25		/				

served	Jack	Jack	Average	Total	Beam(B7)		
	Load	Load			450X160 opening		
load	3	4	load	load	Strain	Deflection mm	
tonne	kN.	kN.	kN.	kN.	gauge reading		
1	8.89	8.89	8.89	17.78	7	0.1778	
2	18.14	17.52	17.83	35,66	12	0.3048	
3	27.39	27.39	27.39	54.78	15	0.381	
4	37.26	37.26	37.26	74.52	18	0.4572	
5	46.82	46.82	46.82	93.64	22	0.5588	
6	55.76	55.76	55.76	111.52	25	0.63:	
7	65.01	65,32	65,165	130.33	30	0.76	
8	74.2	74.26	74.23	148.46	32	0.812	
9	84.44	84.44	84.44	168.88	35	0.88	
10	93.69	95.54	94,615	189.23	39	0.990	
11	103.4	105.87	104.635	209.27	43	1.092	
12	113.12	116.2	114.66	229.32	46	1.168	
13	122.52	125.14	123.83	247.66	49	1.244	
14	131.93	134.09	133.01	266.02	53	1.346	
	141.49	143.95	142.72	285.44	58		
15	151.05	153.82	152,435	304.87	62	1.574	
16	160.92	163.07	161.995	323.99	66		
17	170.78	172.33	171.555	343.11	70		
18	180.19	181.42	180.805	361.61	75		
19	189.59	190.52	190.055	380.11	78		
20			200	400	82		
21	199.46			419.89	86		
22		210.56		440.86	90		
23		221.05			95		
24				461.83	99		
25			241.095	482.19			
26	The second second			502.54	103		
27				520.43	107		
28				538.31	110		
29				556.82	115		
30				575.31	120		
31				594.43	125		
32				613.55	130		
33				630.97	134		
34				648.4	138		
35				668.29	144		
36					150		
37					150		
38					160		
39					169		
4(	383.25	383.25			178		
41	392.35	391.89	392.12	784.24	183		
42	401.45	400.52	400,985			4.92	
42	410.54	409.77	410.155	820.31	FAILURE		
44			419.33	838.66			
4:				847.46			
40		The second second					

