UNSTEADY FREE CONVECTION BOUNDARY LAYER FLOW OVER A HEATED PLATE WITH DIFFERENT INCLINATIONS

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Abstract

Unsteady free convection boundary layer flow over a heated plate with different inclinations have been studied numerically. The governing equations are then made dimensionless by using usual similarity transformations. The dimensionless equations are solved numerically by applying Nachtsheim-Swigert shooting iteration technique along with Runge-Kutta sixth order integration method.

Two cases of the motion of the flow have been considered; one is the impulsive motion and the other is the uniformly accelerated motion . The plate temperature is assumed to be a function of time. The dimensionless velocity, temperature and pressure profiles are shown graphically for different values of the parameters entering into the problem .
The numerical values of the local skin-friction and local Nusselt number are also presented in tabular form.

Finite difference analysis of natural convection flow over an inclined heated plate have been studied numerically. We have studied the natural convection flow over an inclined flat plate by employing implicit finite-difference method of Crank-Nicolson method. The transient velocity and temperature profiles are displayed graphically and discussed thereafter. The stability and convergence of the implicit finite-difference scheme are established as well. The software FORTRAN 90 is used to perform computational job and the post processing software MS Excel has been used to display the numerical results graphically.

Author's Declaration

I hereby declare that this thesis work submitted to the Department of Mathematics, Bangladesh University of Engineering and Technology (BUET) in partial fulfillment of the requirements for the degree of Master of Philosophy in Mathematics has not been submitted elsewhere (Universities or Institutions) for the any other degree.

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Table of Content

List of Contents **Page No.**

Nomenclature

c_p	: Specific heat at constant pressure
U_0	: Constant or mean velocity
T_∞	: Free stream temperature
T	: Temperature at any point of the flow field
T_w	: Wall temperature
x, y	: Cartesian coordinates
u, v	: Velocity components in the x and y directions,
	: respectively.
g_0	: Gravity acceleration
N_u	: Nusselt Number
Gr	: Grashof Number
Pr	: Prandtl Number
f	: Dimensionless stream function
v_0	: Suction parameter
t	: Time
$\overline{P}$	: Dimensionless pressure of the fluid
p	: Fluid pressure
$\overline{q}$	: Velocity field
k	: Thermal conductivity
m	: Any integer
X, Y	: Dimensionless co-ordinates
U,V	: Components of the dimensionless velocity field.
L	: Characteristic length of the plate.

Greek Symbols

α	: Arbitrary angle
β	: Thermal expansion coefficients
θ	: Dimensionless temperature
η	: Coordinate transformation in terms of x and y
υ	: Fluid kinematics viscosity
ρ	: Fluid density
σ	: Time dependent length scale
τ_w	: Wall skin friction
q_w	: Local surface heat flux

LIST OF FIGURES

LIST OF TABLES

Chapter one

1.1 Introduction

Natural convection is a mechanism of transport in which fluid motion is not generated by any external source like pump, fan, suction device, and moving solid object in contact with fluid but only by density differences in fluid occurring due to temperature gradients. The heat transfer coefficient in natural convection is less than that of in forced convection. The movements of the fluid in free convection whether it is a gas or liquid result from the buoyancy forces imposed on the fluid when its density in the proximity of the heat transfer surface is decreased as a result of the heating process. The presence of buoyancy forces like gravitation or an equivalent force arising from the equivalence principle is essential for natural convection. It is important to note that body forces other than gravity can act to move the fluid in a natural convection problem. For example centripetal acceleration is a body force that exits in rotating machinery. The Coriolis force is a body force that is significant in oceanic and atmospheric motions. Although a number of variations can exist, the common situation is working with natural-convection problems where the body force is gravitational and the density gradient is due to a temperature gradient.

The temperature distribution in natural convection depends on the intensity of fluid currents that depends on the temperature potential. So the quantitative and qualitative analysis of natural convection heat transfer is difficult. Experimental and numerical investigations take place for solving heat transfer and fluid flow problems. Also an overall comparison of experimental, numerical method with theoretical analysis established here to show the reliability of methods. The aim of the present work is to study the free convection boundary layer flow over a heated plate with different inclinations considering time dependent plate temperature.

Natural convection attracted a great deal of attention of researchers because of its presence in nature like the rising plume of hot air from fire, oceanic currents, sea-wind formation, soil-flow dynamics, fog formation and also in engineering applications like formation of microstructures during the cooling of molten metals, shrouded fins, solar ponds, nuclear reactor operation. The most common analysis of heat transfer is on flat plate. Recently heat transfer for non-plane surface plates are in concern. The natural convection procedures are governed essentially by three features namely the body force, the temperature difference in the flow field and the fluid density discrepancy with temperature. Natural convection is the most important style of heat transfer form pipes, transmission lines, refrigeration coils, burning radiators and various realistic situations. The manipulation of natural convection of the heat transfer can be deserted in the case of large Reynolds number and very small Grashof number. Alternatively, the natural convection should be the governing aspect for large Grashof number and Reynolds number.

The main objectives of the project are as follows-

- To observe the effect of inclination on natural convection heat transfer for different surface geometry.
- To evaluate wall shear stress in terms of local skin friction and rate of heat transfer in terms of local Nusselt number.
- To displayed the velocity, temperature and pressure profiles for different values of the controlling parameters.
- To solve numerically by using shooting method, the basic equations are transformed to non dimensional boundary layer equations.

Natural convection is observed as a result of the motion of the fluid due to the density changes arising form the heating process. This case occurs at very small velocities of motion in the process of large temperature differences. A hot radiator used for heating a room is one example of a practical device which transfers heat by natural convection. The movement of the fluid in natural convection, whether it is gas or liquid, results from the buoyancy forces imposed on the fluid when its density in the proximity of the heat transfer surface is decreased as a result of the heating process. The buoyancy forces would not be present if the fluid were not acted upon by same external force field such as gravity, although gravity is not the only type of force field which can produce the natural convection currents ; a fluid enclosed in a rotating machine is acted upon by a centrifugal force field, and thus could experience natural convection currents if one or more of the surface in contact with fluid were heated. The buoyancy forces which give rise to the natural convection currents are called body forces.

For natural convective flows to be dynamically similar, the Grashof number ($G_r = \dfrac{g_0 \beta \Delta T L^3}{\upsilon^2}$, where g_0 is the gravitational acceleration, β is the thermal expansion coeffient, ΔT is the temperature difference, L is the length scale and υ is the kinematic viscosity) and the Prandtle number ($P_r = \dfrac{\upsilon \rho c_p}{\kappa}$, where κ is the termal conductivity, ρ is the fluid density and c_p is the specific heat at constant pressure) must be the same in both the systems.

The governing equations for convection flow are coupled elliptic partial differential equations and, therefore, of considerable complexity. The major problems in obtaining a solutions to these equations lie in the inevitable variation of density with temperature, or concentration, and in their partial, elliptic nature. Several approximations are generally made to considerably simplify these equations. Among them Boussinesq approximation attributed to Boussinesq (1903) is considered here. In flows accompanied by heat transfer, the fluid properties are normally functions of temperature. The variations may be small and yet be the cause of the fluid motion. If the density variation is not large, one may treat the density as constant in the unsteady and convection terms, and treat it as variable only in the gravitational term. This is called Boussinesq approximation.

1.2 Literature Survey

Natural convection flows under the influence of gravitational force have been investigated most extensively because they occur frequently in nature as well as in science and engineering applications. When a heated surface is in contact with the fluid , temperature difference causes buoyancy force, which induces the natural convection. Natural convection flow has been studied in various types of surface such as flat plate, sphere, cylinder, vertical cone etc. The study of temperature and heat transfer is of great importance to the engineers because of its almost universal occurrence in many branches of science and engineering. Although heat transfer analysis is most important for the proper sizing of fuel elements in the nuclear reactors cores to prevent burnout. The performance of aircraft also depends upon the case with which the structure and engines can be cooled. Heat transfer is commonly associated with fluid dynamics. The knowledge of temperature distribution is essential in heat transfer studies because of the fact that the heat flow takes place only whenever there is a temperature gradient in a system. The three different manners of heat transfer namely; conduction, convection and radiation must be considered.

The theoretical analysis and experiments carried out by Schmidt and Bechmann (1930) of the free convection boundary layer flow of air subject to the gravitational force about an isothermal, vertical flat plate constitute one of the earliest comprehensive studies of natural convection flow. Eckert and Soehngen (1948) verified and extended the experimental work of Schmidt. Schuh (1948) obtained the numerical solutions by computing velocity and temperature distributions for several Prandtl numbers. Ostrach (1953) studied aspects of natural convection heat transfer. He also analyzed laminar free convection flow and heat transfer about a flat plate parallel to the direction of generating body force. Yang (1960) studied the unsteady laminar boundary layer equations for free convection on vertical plates to establish necessary and sufficient conditions under which similarity solutions were possible. Soundalgekar (1972) analyzed viscous dissipation effects on unsteady free convection flow past an infinite, vertical porous plate with constant suction. Ganapathy (1997) studied time dependent free convection motion and heat transfer in an infinite porous medium by a heated sphere. Elliot (1970) analysed the problem of unsteady free convection boundary layer flow over two-dimensional and axisymetric bodies for a step input in the surface temperature. Williams et al. (1987) studied the unsteady free convection flow over a vertical flat plate under the assumption of variations of the wall temperature with time and distance. They found possible semi-similar solutions for a variety of classes of wall temperature distributions. Sattar and Alam (1994) investigated the unsteady free convection flow of a viscous, incompressible and electrically conducting fluid past a moving infinite vertical porous plate taking into account the thermal diffusion effect. The unsteadiness in the flow field was introduced by time-dependent velocity of the moving plate.

Natural convection boundary layer flow over horizontal and slightly inclined surfaces was studied by Pera and Gebhart (1972); the effects of a small surface inclination were analyzed by perturbing flow over a horizontal surface. These studies concern two-dimensional flows whose principal direction is normal to the single leading edge of horizontal and slightly inclined surfaces. Umemura and Law(1990) investigated natural-convection boundary-layer flow over a heated plate with arbitrary inclination by identifying a set of combined boundary-layer variables and then casting the governing equations into a universal form. This concerns that the flow characteristics depend not only on the extent of inclination but also on the distance from the leading edge. Ekambavanan and Ganesan(1994) have considered the transient free convection over a semi-infinite flat plate which is inclined with a positive angle to the horizontal and in which both the wall temperature and spices concentration vary together with the power of the coordinate along the plate .Recently the steady laminar free convection boundary layer over the upper surface of a semi-infinite flat plate which is inclined at a small angle to the horizontal under the combined buoyancy effects of thermal and mass (concentration effects) is theoretically studied by Angel et al.(2001). Both positive and negative inclinations of the plate are concerned. The Keller-box scheme has been used to complete the solution. Results for the skin friction coefficient, the local Nusselt and Shearwood number are presented on graphs. Chamkha et al.(2001) have considered steady, laminar, hydromagnetic coupled heat and mass buoyancy induced natural convection boundary later flow of an electrically- conducting and heat generating and absorbing fluid along a semi infinite permeable inclined flat plate. The governing equations were developed and transformed using appropriate similarity transformations.

The transformed equations were then solved numerically by an implicit, iterative finite-difference scheme and the results were also shown in graphs. Very recently Hasan et al.(2003) have studied the mathematical modeling for free convection flow of a particulate suspension over an infinite, inclined, permeable and isothermal plate in the presence of both magnetic field and fluid heat absorbing effects. The general effects of the magnetic field and the heat absorbing fluid were found to damp the flow and the thermal profiles. Ganesan and Palani(2003) investigated the natural convection effects on impulsively started inclined plate with heat and mass transfer. Crank- Nicolson implicit finite difference method was used to solve the unsteady, non-linear and coupled governing equations. Numerical results were obtained for various parameters. The steady state velocity , temperature and concentration profiles, local and average skin friction, Nusselt number and Sherwood number are shown graphically. It was observed that the local wall shear stress decreases as an angle of inclination decreases. Other important contributions on heat and mass transfer flow over inclined flat plate are due Jones (1973), Chen and Yeh(1980), Moutsoglou and Chen(1980), Ganesan and Ekambavannan(1992), Hossain et al. (1996), Ramadan and Chamkha(2003), and Yu and Lin(1998).

Unsteady free convection flow past a vertical porous plate was investigated by Helmy(1998), Acharya et al(2000) have studied free convection and mass transfer flow through a porous medium bounded by vertical infinite surface with constant suction and heat flux. But In this study they considered the flow to be steady. Coming back to unsteady case, Kim(2000) investigated unsteady MHD convection heat transfer past a semi-infinite vertical porous moving plate with variable suction. Little extension to this problem has been done by Chamkha(2004).

Free convective steady hydro magnetic flow about a heated vertical flat plate was considered by Gupta(1961), Poots(1961), Osterle and Yound(1961). The problem of the free convection boundary layer on a vertical plate with prescribed surface heat flux was studied by Merkin and Mahmood(1990).A transformation of the boundary layer equations for free convection past a vertical plate with arbitrary blowing and wall temperature variations was studied by Vedhnyagam et al.(1980). The case of a heated isothermal horizontal surface with transpiration was discussed in some detail first by Clarke and Riley(1975,1976) and then recently by Lin and Yu(1988).

The unsteady mixed convection flow past an infinite vertical isothermal plate of an incompressible fluid is a physical situation, which is often experienced, in the industrial application. Some important contributions in this aspect have been given by Schneider(1979), Jha(1991) and Sattar et al.(1997) investigated the MHD unsteady mixed convection flow through a porous medium. He, however, obtained the solutions by employing Laplace transform technique and taking the value of the Prandtl number to be equal to one. Latter Satter et al.(1997) obtained an analytical solution of an unsteady flow through a porous medium taking a constant heat source and a variable suction velocity. Unlike the work of Jha(1991), the solutions have been obtained for different Prandtl number.

In the present work, Unsteady free convection boundary layer flow over a heated plate with different inclinations have been investigated. The results have been obtained for different values of relevant physical parameters. The governing equations are then made dimensionless by using usual similarity transformations. The dimensionless equations are solved numerically by applying Nachtsheim-Swigert shooting iteration technique along with Runge-Kutta sixth order integration method. Numerical results of the velocity, temperature and skin friction coefficient for different values of Prandtl number Pr , Grashoff number Gr, and angle α are presented graphically.

In chapter two, Time dependent free convection analysis over an inclined heated plate have been analyzed. The non- dimensional boundary layer equations have been solved by using Nachtsheim-Swigert shooting iteration technique along with Runge-Kutta sixth order integration method. The results in terms of local skin friction coefficient and Nusselt number have been shown in tabular forms. Velocity profiles, Temperature profiles and pressure have been presented graphically for various values of Prandtl number Pr , Grashoff number Gr, and angle α .

In chapter three, Finite difference analysis of natural convection flow over an inclined heated plate has been described. The non- dimensional boundary layer equations have been solved by using implicit finite difference technique. Numerically results are presented by velocity, temperature and Prandtl number Pr.

In chapter four, we have presented overall conclusions of the models studied. Finally all references quoted in the text can be found at the end of the thesis.

Chapter Two

Time Dependent Free Convection Analysis Over An Inclined Heated Plate .

Free convection flow adjacent to inclined surface bounded by an extensive body of fluid is of considerable importance in micrometeorological and industrial applications. Some of the earlier workers in this respect are Ostrach (1953), Stewartson (1958), Gill et al. (1965). Most of the existing analyses have used the similarity solutions for the vertical case with the buoyant force being the component of the body force along the plate. Many researchers such as Merkin (1989,1996) and Harvet and Blay (1999) have investigated the problem of free convection over a vertical plate. Karkus (1968) applied perturbation technique to study the natural convection flow adjacent to inclined isothermal and finite-length surfaces. Free convection boundary layer flow over a horizontal and slightly inclined surface has been studied by Pera and Gebhart (1972). In the recent past, Umemura and Law (1990) developed a generalized formulation for the natural convection boundary layer flow over a flat plate of arbitrary inclination. Recently Hossian et al. (1995) studied the free convection flow from an isothermal plate inclined at a small angle to the horizontal . Angel et al.(2001) presented numerical solution of free convection flow past an inclined surface. He studied the flow characteristics depended not only on the extent of inclination but also on the distance from the leading edge. The above works were on steady flows. The aim of the present works is to study the free convection boundary layer flow over a heated plate with different inclinations considering time dependent plate temperature.

Free convection boundary layer flow over a semi-infinite heated flat plate with inclined arbitrary angle $\alpha(0 \le \alpha \le \frac{\pi}{2})$ to the horizontal is considered. For this purpose let us consider the unsteady motion of an incompressible fluid. The flow is assumed to be in the x-direction and y-axis is normal to it. At time t>0, the plate temperature is instantly raised to $T(>T'_\infty)$, where T_∞ be the temperature of the uniform flow and the plate starts with a velocity $U(t)$ in its own plane. Consider u and v be the velocity components in the rectangular co-ordinate system, U_0 be the mean velocity of the plate in the x-direction and ρ be the fluid density and C_p is the specific heat at constant pressure. The physical flow configuration is shown in the following Figure 2.1

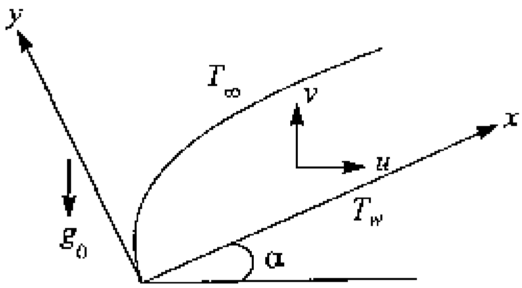

Figure 2.1 : Configuration of the problem

The flow model considered is of unsteady free convection boundary layer flow over a flat plate with arbitrary inclination under the influence of applied field. We have considered a time dependent suction and plate temperature. It is assumed that the flow is one-dimensional, unsteady state, laminar and the fluid is incompressible. The solutions of the governing equations have been done taking suitable similarity transformations . The nonlinear similarity equations, which are locally similar, are solved numerically by Nachtscheim-Swigert(1995) iteration technique.Two cases are considered 1) Impulsively started plate moving in its own plane and 2) Uniformly accelerated plate . The suction velocity is taken to be inversely proportional to the length-scale .

2.1 Governing equations of the flow

The continuity, momentum and energy equations for unsteady, viscous and incompressible flows are respectively given by

$$\nabla.\bar{q}=0 \tag{2.1}$$

$$\frac{\partial\bar{q}}{\partial t}+\rho(\bar{q}.\nabla)\bar{q}=-\nabla p+\upsilon\nabla^{2}\bar{q}+\bar{F} \tag{2.2}$$

$$\frac{\partial T}{\partial t}+(\bar{q}.\nabla)T=\frac{k}{\rho C_{p}}\nabla^{2}T \tag{2.3}$$

Here $\bar{q}=\bar{q}(u,v)$ is the velocity vector , $\bar{F}$ is the body force per unit volume which defined as - ρg , p is the pressure force, ρ is the density of the fluid, υ is the kinemetic viscosity , β is the thermal expansion coefficients, g_0 is the gravitational acceleration, T is the temperature inside the boundary layer, α is the inclination angle from horizontal direction, k is the thermal conductivity, c_p is the specific heat at a constant pressure and ∇ is the vector differential operator which is defined by

$$\nabla=\hat{I}_{x}\frac{\partial}{\partial x}+\hat{I}_{y}\frac{\partial}{\partial y}$$

where $\hat{I}_x$ and $\hat{I}_y$ are the unit vectors along x and y axes respectively.

The above mentioned equations (2.1)-(2.2) would serve as the governing equations of the problems under the Boussinesq's approximation

$$\frac{\partial v}{\partial y}=0 \tag{2.4}$$

$$\frac{\partial u}{\partial t}+v\frac{\partial u}{\partial y}=\upsilon\frac{\partial^{2}u}{\partial y^{2}}+g_{0}\beta(T-T_{\infty})\sin\alpha \tag{2.5}$$

$$0=-\frac{1}{\rho}\frac{\partial p}{\partial y}+g_{0}\beta(T-T_{\infty})\cos\alpha \tag{2.6}$$

$$\rho c_{p}\left(\frac{\partial T}{\partial t}+v\frac{\partial T}{\partial y}\right)=k\frac{\partial^{2}T}{\partial y^{2}} \tag{2.7}$$

Where the variables and related quantities are defined in the Nomenclature.

The appropriate boundary conditions for the above problem are as follows:

$$u = U(t),\ v = v(t),\ T = T(t) \quad at\ y = 0 \tag{2.8a}$$

$$u = 0,\ v = 0,\ T = T_\infty, P = 0 \quad as\ y \to \infty \tag{2.8b}$$

Two cases have been considered for the problem . They are:

Case I: Impulsively started plate (ISP), i.e., when the plate is impulsively started and moves in its own plane.

Case II: Uniformly accelerated plate (UAP), i.e., when the plate moves with a velocity taken to be a function of time.

2.2 Similarity analysis

Case I: Impulsive motion

We introduce a similarity parameter σ as

$$\sigma = \sigma(t) \tag{2.9}$$

where σ is the time dependent length scale . In terms of σ, a convenient solution of the equation (2.4) is considered to be

$$v = -v_0 \frac{\upsilon}{\sigma} \tag{2.10}$$

here the constant v_0 represents a dimensionless normal velocity at the plate which is positive for suction and negative for blowing.

Now we introduce the following dimensionless variables

$$u = U(t) = U_0, \eta = \frac{y}{\sigma}, f(\eta) = \frac{u}{U_0}$$
$$\theta(\eta) = \frac{T - T_\infty}{T_0 - T_\infty}, \overline{P}(\eta) = \frac{p\sigma}{U_0 \upsilon \rho} \tag{2.11}$$

where U_0 is the mean velocity , T_0 is the mean temperature and $\overline{p}$ is the dimensionless pressure , all being constant.

Using equations (2.9),(2.10) and (2.11) in equations (2.5)-(2.7), we obtain

$$-\frac{\sigma}{\upsilon}\frac{\partial\sigma}{\partial t}\eta f' = f'' + G_r\theta\sin\alpha \tag{2.12}$$

$$0 = -\overline{P'} + G_r\theta\cos\alpha \tag{2.13}$$

$$-\frac{\sigma}{\upsilon}\frac{\partial\sigma}{\partial t}\eta\theta' - v_0\theta' = \frac{1}{P_r}\theta'' \tag{2.14}$$

where

$G_r(=\dfrac{g_0\beta(T_0-T_\infty)\sigma^2}{U_0\upsilon})$, is the Grashoff number,

$P_r(=\dfrac{\upsilon}{k/\rho c_p})$, is the Prandtl number

all are the dimensionless local parameters .

The boundary conditions (2.8a) and (2.8b) then becomes

$$f = 0, \quad \theta = 1 \qquad at \quad \eta = 0 \tag{2.15}$$

$$f = 0, \; \theta = 0 \; \overline{P} = 0 \quad as \quad \eta \to \infty$$

So following the works of Sattar and Hossain (1992), assuming that

$$\frac{\sigma}{\upsilon}\frac{\partial\sigma}{\partial t} = c \text{ (a constant)} \tag{2.16}$$

The equations (2.12) and (2.14) are similar except for the term $\frac{\sigma}{\upsilon}\frac{\partial\sigma}{\partial t}$, where time t appears explicitly.

Integrating (2.16) we obtain

$$\sigma = \sqrt{2c\upsilon t} \tag{2.17}$$

where the constant of integration is determined through the condition that $\sigma = 0$ when t=0. It thus appears from (2.17) that by making a realistic choice of c to be equal to 2, then in (2.16) $\sigma = 2\sqrt{\upsilon t}$ which exactly corresponds to the usual scaling factor considered for various non steady boundary layer flow Schlichting (1968). Since σ is a scaling factor as well as a similarity parameter, one other value of c in (2.16) would not change the nature of the solution except that the scale would be different. Lastly, introducing (2.16) with c=2 in equations (2.12) and (2.14) respectively we have the following dimensionless ordinary differential equations :

$$f'' = -2\zeta f' - G_r\theta\sin\alpha \tag{2.18}$$

$$\overline{P'} = G_r\theta\cos\alpha \tag{2.19}$$

$$\theta'' = -2\zeta P_r\theta' \tag{2.20}$$

where $\zeta = \eta + \frac{v_0}{2}$

Now it is required to solve the above equations subject to the boundary conditions (2.15).

Case II: Uniformly Accelerated Motion

In this case $U(t)$ is the free stream velocity and $T(t)$ is the plate temperature are assumed to have the following forms :

$$U(t) = U_0\sigma_*^{2m+2} \tag{2.21}$$

$$T(t) = T_\infty + (T_0 - T_\infty)\sigma_*^{2m} \tag{2.22}$$

where m is an integer and $\sigma_* = \frac{\sigma}{\sigma_0}$

Now introducing

$$u = U(t)f(\eta) = U_0\sigma_*^{2m+2}f(\eta) \tag{2.23}$$

$$T = T_\infty + (T - T_\infty)\sigma_*^{2m}\theta(\eta) \tag{2.24}$$

$$p = \frac{\rho U(t)\upsilon}{\sigma}\overline{P_a}(\eta) = \frac{\rho U_0\sigma_*^{2m+2}\upsilon}{\sigma}\overline{P_a}(\eta) \tag{2.25}$$

where $\vec{P}_a$ is the dimensionless pressure for accelerated motion.

Introducing the relations (2.9),(2.10) and (2.16)-(2.20) in equations (2.5),(2.6) and (2.7) and also introducing the following dimensionless parameters

$G_r(=\frac{g_0\beta(T_0 - T_\infty)\sigma^2}{U_0\upsilon})$, is the Grashoff number,

and $P_r(=\frac{\upsilon}{k/\rho c_p})$, is the Prandtl number

We obtain the following dimensionless differential equations :

$$\frac{\upsilon}{\sigma}\frac{\partial\sigma}{\partial t}\left[(2m+2)f-\eta f'\right]-v_0 f'=f''+G_r\theta\sin\alpha \tag{2.26}$$

$$\overline{P_a}' = G_r\theta\cos\alpha \tag{2.27}$$

$$\frac{\upsilon}{\sigma}\frac{\partial\sigma}{\partial t}\left[2m\theta-\eta\theta'\right]-v_0\theta'=\frac{1}{P_r}\theta'' \tag{2.28}$$

The boundary conditions (2.8a),(2.8b) now reduce to

$$f=1,\ \theta=1 \qquad at \quad \eta=0 \tag{2.29a}$$

$$f=0,\ \theta=0,\ \overline{P_a}=0 \quad as \quad \eta\to\infty \tag{2.29b}$$

Now following the arguments in case I, equations (2.21) and (2.22) become respectively

$$f''+2\zeta f'-4(m+1)f+G_r\theta\sin\alpha=0 \tag{2.30}$$

$$\overline{P_a}' = G_r\theta\cos\alpha \tag{2.31}$$

$$\theta''+2P_r\zeta\theta'-4mP_r\theta=0 \tag{2.32}$$

where $\qquad \zeta=\eta+\frac{v_0}{2}$

The solution of the equation (2.18)-(2.20) for the case I and the equation (2.30)-(2.32) for the case II are solved numerically under the respective boundary conditions using Nachtsheim - Swigert iteration technique.

2.3 Skin- friction coefficieut and Nusselt number :

The parameters of engineering interest for the present problem are the local skin-friction coefficient and local Nussel number which indicate physically wall shear stress and rate of heat transfer respectively .

The equation defining the wall skin-friction is given by

$$\tau_w=\mu\left(\frac{\partial u}{\partial y}\right)_{y=0} \tag{2.33}$$

and the local surface heat flux is defined

$$q_w=-k\left(\frac{\partial t}{\partial y}\right)_{y=0} \tag{2.34}$$

The dimensionless local wall shear stress and local surface heat flux for impulsively stared plate respectively are obtained as

$$\frac{\tau_w \sigma}{\mu U_0} = f'(0) \tag{2.35}$$

and $$\frac{q_w \sigma}{k(T - T\infty)} = -\theta'(0) \tag{2.36}$$

Hence for impulsively started plate the dimensionless skin- friction coefficient and the Nusselt number are given by

$$\tau_i = \frac{\tau_w \sigma}{\mu U_0} = f'(0) \tag{2.37}$$

and $$N_{ui} = \frac{q_w \sigma}{k(T - T\infty)} = -\theta'(0) \tag{2.38}$$

for accelerated plate they are obtained as

$$\tau_a = \frac{\tau_w \sigma_0}{\mu U_0 \sigma_*^{2m+1}} = f'(0) \tag{2.39}$$

and $$N_{ua} = \frac{q_w \sigma_0}{k(T - T\infty)\sigma_*^{2m+1}} \tag{2.40}$$

Thus the dimensionless values of the local skin-friction and the Nusselt number for impulsive as well as accelerated plate are obtained numerically.

2.4 Numerical analysis

The systems of equations (2.18-2.20) for case I and (2.30-2.32) for case II with the boundary conditions are non linear and coupled. It is difficult to solve them analytically Hence we adopt a procedure to obtain the solution numerical . Here we use the sixth order Runge - Kutta method namely Nachtsheim-Swigert iteration technique (guessing the missing value) and Runge-Kutta Merson method , in collaboration with Runge-Kutta shooting method .

In a shooting method , the missing (unspecified) initial condition at the initial point of the interval is assumed , and the differential equation is then integrated numerically as an initial value problem to the terminal point . The accuracy of the assumed missing initial condition is then checked by comparing the calculated value of independent variable at the terminal point with its given value there . If a difference exists, another value of the missing initial condition must be assumed and the process is repeated .This process is repeated until the agreement between the calculated and the given condition at the terminal point is within the specified degree of accuracy .

The Nachtsheim-Swigert iteration technique thus needs to be discussed elaborately. The boundary conditions (2.15)-(2.29) associated with nonlinear ordinary differential equations (2.18)-(2.20) and (2.30-2.32) are the two points asymptotic class . Two point boundary conditions have values of the dependent variable specified at two different values of independent variable. Specification of an asymptotic boundary condition implies that the velocity f and temperature θ tend to unity as the independent variable tends to outer specified value .

The method of numerically integrating a two point asymptotic boundary value problem of the boundary layer type , the initial value method is recast as an initial value problem . Thus it is necessary to estimate as many boundary conditions at the surface as were given at infinity . The governing differential equations are integrating with these assumed surface boundary conditions . The solution has been achieved , if the required outer boundary condition is satisfied . But this is not so easy , because selecting a value may result in this divergence of the trial integration or in slow convergence of surface boundary conditions required satisfying the asymptotic outer boundary condition .

Selecting too large a value of the independent variable is expensive in terms of computer time .

Nachtsheim-Swigert (1965) developed an iteration method to these difficulties. In equation there are three asymptotic boundary conditions and hence three unknown surface conditions

$$f'(0) = g_1, \theta'(0) = g_2, \text{ and } \overline{P}(0) = g_3 \tag{2.41}$$

Within the contest of the initial value method and Nachtsheim-Swigert iteration technique the outer boundary conditions may be functionally represented as

$$f(\eta_{max}) = f(f'(0), \theta'(0), \overline{P}(0)) = \delta_1 \tag{2.42}$$

$$\theta(\eta_{max}) = \theta(f'(0), \theta'(0), \overline{P}(0)) = \delta_2 \tag{2.43}$$

with the asymptotic convergence criteria given by

$$f'(\eta_{max}) = f'(f'(0), \theta'(0), \overline{P}(0)) = \delta_3 \tag{2.44}$$

$$\theta'(\eta_{max}) = \theta'(f'(0), \theta'(0), \overline{P}(0)) = \delta_4 \tag{2.45}$$

$$\overline{P}(\eta_{max}) = \overline{P}(f'(0), \theta'(0), \overline{P}(0)) = \delta_5 \tag{2.46}$$

Choosing $f'(0) = g_1, \theta'(0) = g_2$ and $\overline{P}(0) = g_3$ and expanding in a first order Taylor's series after using equations

$$f(\eta_{max}) = F_c + \frac{\partial f}{\partial g_1}\Delta g_1 + \frac{\partial f}{\partial g_2}\Delta g_2 + \frac{\partial f}{\partial g_3}\Delta g_3 = \delta_1 \tag{2.47}$$

$$\theta(\eta_{max}) = \theta_c + \frac{\partial \theta}{\partial g_1}\Delta g_1 + \frac{\partial \theta}{\partial g_2}\Delta g_2 + \frac{\partial \theta}{\partial g_3}\Delta g_3 = \delta_2 \tag{2.48}$$

$$\overline{P}(\eta_{max}) = \overline{P_c} + \frac{\partial \overline{P}}{\partial g_1}\Delta g_1 + \frac{\partial \overline{P}}{\partial g_2}\Delta g_2 + \frac{\partial \overline{P}}{\partial g_3}\Delta g_3 = \delta_3 \tag{2.49}$$

$$f'(\eta_{max}) = f_c' + \frac{\partial f'}{\partial g_1}\Delta g_1 + \frac{\partial f'}{\partial g_2}\Delta g_2 + \frac{\partial f'}{\partial g_3}\Delta g_3 = \delta_4 \tag{2.50}$$

$$\theta'(\eta_{max}) = \theta_c' + \frac{\partial \theta'}{\partial g_1}\Delta g_1 + \frac{\partial \theta'}{\partial g_2}\Delta g_2 + \frac{\partial \theta'}{\partial g_3}\Delta g_3 = \delta_5 \tag{2.51}$$

yields where subscript 'c' indicates the value of the function at η_{max} determined from the trial integration .

Solution of these equations in a least – squares sense requires determining the minimum value of

$$E = \delta_1^2 + \delta_2^2 + \delta_3^2 + \delta_4^2 + \delta_5^2 \tag{2.52}$$

with respect to g_1, g_2 and g_3

Now differentiating E with respect to g_1 yields

$$2\delta_1 \frac{\partial \delta_1}{\partial g_1} + 2\delta_2 \frac{\partial \delta_2}{\partial g_1} + 2\delta_3 \frac{\partial \delta_3}{\partial g_1} + 2\delta_4 \frac{\partial \delta_4}{\partial g_1} + 2\delta_5 \frac{\partial \delta_5}{\partial g_1} = 0$$

$$or, \left(f_c + \frac{\partial f}{\partial g_1}\Delta g_1 + \frac{\partial f}{\partial g_2}\Delta g_2 + \frac{\partial f}{\partial g_3}\Delta g_3 \right)\frac{\partial f}{\partial g_1} + \left(\theta_c + \frac{\partial \theta}{\partial g_1}\Delta g_1 + \frac{\partial \theta}{\partial g_2}\Delta g_2 + \frac{\partial \theta}{\partial g_3}\Delta g_3 \right)\frac{\partial \theta}{\partial g_1}$$
$$+ \left(\overline{P}_c + \frac{\partial \overline{P}}{\partial g_1}\Delta g_1 + \frac{\partial \overline{P}}{\partial g_2}\Delta g_2 + \frac{\partial \overline{P}}{\partial g_3}\Delta g_3 \right)\frac{\partial \overline{P}}{\partial g_1} + \left(f_c' + \frac{\partial f'}{\partial g_1}\Delta g_1 + \frac{\partial f'}{\partial g_2}\Delta g_2 + \frac{\partial f'}{\partial g_3}\Delta g_3 \right)\frac{\partial f'}{\partial g_1}$$
$$+ \left(\theta_c' + \frac{\partial \theta'}{\partial g_1}\Delta g_1 + \frac{\partial \theta'}{\partial g_2}\Delta g_2 + \frac{\partial \theta'}{\partial g_3}\Delta g_3 \right)\frac{\partial \theta'}{\partial g_1} = 0$$

$$or, \left[\left(\frac{\partial f}{\partial g_1}\right)^2 + \left(\frac{\partial \theta}{\partial g_1}\right)^2 + \left(\frac{\partial \overline{P}}{\partial g_1}\right)^2 + \left(\frac{\partial f'}{\partial g_1}\right)^2 + \left(\frac{\partial \theta'}{\partial g_1}\right)^2 \right]\Delta g_1$$
$$+ \left[\frac{\partial f}{\partial g_2}.\frac{\partial f}{\partial g_1} + \frac{\partial \theta}{\partial g_2}.\frac{\partial \theta}{\partial g_1} + \frac{\partial \overline{P}}{\partial g_2}.\frac{\partial \overline{P}}{\partial g_1} + \frac{\partial f'}{\partial g_2}.\frac{\partial f'}{\partial g_1} + \frac{\partial \theta'}{\partial g_2}.\frac{\partial \theta'}{\partial g_1} \right]\Delta g_2 \tag{2.53}$$
$$+ \left[\frac{\partial f}{\partial g_3}.\frac{\partial f}{\partial g_1} + \frac{\partial \theta}{\partial g_3}.\frac{\partial \theta}{\partial g_1} + \frac{\partial \overline{P}}{\partial g_3}.\frac{\partial \overline{P}}{\partial g_1} + \frac{\partial f'}{\partial g_3}.\frac{\partial f'}{\partial g_1} + \frac{\partial \theta'}{\partial g_3}.\frac{\partial \theta'}{\partial g_1} \right]\Delta g_3$$
$$= -\left[f_c \frac{\partial f}{\partial g_1} + \theta_c \frac{\partial \theta}{\partial g_1} + \overline{P}_c \frac{\partial \overline{P}}{\partial g_1} + f_c' \frac{\partial f'}{\partial g_1} + \theta_c' \frac{\partial \theta'}{\partial g_1} \right]$$

Similarly differentiating E with respect to g_2 and g_3 , we obtain respectively

$$2\delta_1 \frac{\partial \delta_1}{\partial g_2} + 2\delta_2 \frac{\partial \delta_2}{\partial g_2} + 2\delta_3 \frac{\partial \delta_3}{\partial g_2} + 2\delta_4 \frac{\partial \delta_4}{\partial g_2} + 2\delta_5 \frac{\partial \delta_5}{\partial g_2} = 0$$

$$
\begin{aligned}
or, &\left(\frac{\partial f}{\partial g_1}\cdot\frac{\partial f}{\partial g_2}+\frac{\partial \theta}{\partial g_1}\cdot\frac{\partial \theta}{\partial g_2}+\frac{\partial \overline{P}}{\partial g_1}\cdot\frac{\partial \overline{P}}{\partial g_2}+\frac{\partial f'}{\partial g_1}\cdot\frac{\partial f'}{\partial g_2}+\frac{\partial \theta'}{\partial g_1}\cdot\frac{\partial \theta'}{\partial g_2}\right)\Delta g_1 \\
&+\left[\left(\frac{\partial f}{\partial g_2}\right)^2+\left(\frac{\partial \theta}{\partial g_2}\right)^2+\left(\frac{\partial \overline{P}}{\partial g_2}\right)^2+\left(\frac{\partial f'}{\partial g_2}\right)^2+\left(\frac{\partial \theta'}{\partial g_2}\right)^2\right]\Delta g_2 \\
&+\left(\frac{\partial f}{\partial g_3}\cdot\frac{\partial f}{\partial g_2}+\frac{\partial \theta}{\partial g_3}\cdot\frac{\partial \theta}{\partial g_2}+\frac{\partial \overline{P}}{\partial g_3}\cdot\frac{\partial \overline{P}}{\partial g_2}+\frac{\partial f'}{\partial g_3}\cdot\frac{\partial f'}{\partial g_2}+\frac{\partial \theta'}{\partial g_3}\cdot\frac{\partial \theta'}{\partial g_2}\right)\Delta g_3 \\
&=-\left[f_c\frac{\partial f}{\partial g_2}+\theta_c\frac{\partial \theta}{\partial g_2}+\overline{P}_c\frac{\partial \overline{P}}{\partial g_2}+f_c'\frac{\partial f'}{\partial g_2}+\theta_c'\frac{\partial \theta'}{\partial g_2}\right]
\end{aligned}
\tag{2.54}
$$

and

$$
2\delta_1\frac{\partial \delta_1}{\partial g_3}+2\delta_2\frac{\partial \delta_2}{\partial g_3}+2\delta_3\frac{\partial \delta_3}{\partial g_3}+2\delta_4\frac{\partial \delta_4}{\partial g_3}+2\delta_5\frac{\partial \delta_5}{\partial g_3}=0
$$

$$
\begin{aligned}
or, &\left(\frac{\partial f}{\partial g_1}\cdot\frac{\partial f}{\partial g_3}+\frac{\partial \theta}{\partial g_1}\cdot\frac{\partial \theta}{\partial g_3}+\frac{\partial \overline{P}}{\partial g_1}\cdot\frac{\partial \overline{P}}{\partial g_3}+\frac{\partial f'}{\partial g_1}\cdot\frac{\partial f'}{\partial g_3}+\frac{\partial \theta'}{\partial g_1}\cdot\frac{\partial \theta'}{\partial g_3}\right)\Delta g_1 \\
&+\left(\frac{\partial f}{\partial g_2}\cdot\frac{\partial f}{\partial g_3}+\frac{\partial \theta}{\partial g_2}\cdot\frac{\partial \theta}{\partial g_3}+\frac{\partial \overline{P}}{\partial g_2}\cdot\frac{\partial \overline{P}}{\partial g_3}+\frac{\partial f'}{\partial g_2}\cdot\frac{\partial f'}{\partial g_3}+\frac{\partial \theta'}{\partial g_2}\cdot\frac{\partial \theta'}{\partial g_3}\right)\Delta g_2 \\
&+\left[\left(\frac{\partial f}{\partial g_3}\right)^2+\left(\frac{\partial \theta}{\partial g_3}\right)^2+\left(\frac{\partial \overline{P}}{\partial g_3}\right)^2+\left(\frac{\partial f'}{\partial g_3}\right)^2+\left(\frac{\partial \theta'}{\partial g_3}\right)^2\right]\Delta g_3 \\
&=-\left[f_c\frac{\partial f}{\partial g_3}+\theta_c\frac{\partial \theta}{\partial g_3}+\overline{P}_c\frac{\partial \overline{P}}{\partial g_3}+f_c'\frac{\partial f'}{\partial g_3}+\theta_c'\frac{\partial \theta'}{\partial g_3}\right]
\end{aligned}
\tag{2.55}
$$

We can write equations (2.53)-(2.55) in a system of linear equations as follows:

$$a_{11}\Delta g_1 + a_{12}\Delta g_2 + a_{13}\Delta g_3 = b_1 \tag{2.56}$$

$$a_{21}\Delta g_1 + a_{22}\Delta g_2 + a_{23}\Delta g_3 = b_2 \tag{2.57}$$

$$a_{31}\Delta g_1 + a_{32}\Delta g_2 + a_{33}\Delta g_3 = b_3 \tag{2.58}$$

Here

$$a_{11}=\left[\left(\frac{\partial f}{\partial g_1}\right)^2+\left(\frac{\partial \theta}{\partial g_1}\right)^2+\left(\frac{\partial \overline{P}}{\partial g_1}\right)^2+\left(\frac{\partial f'}{\partial g_1}\right)^2+\left(\frac{\partial \theta'}{\partial g_1}\right)^2\right]$$

$$a_{12}=\frac{\partial f}{\partial g_1}\cdot\frac{\partial f}{\partial g_2}+\frac{\partial \theta}{\partial g_1}\cdot\frac{\partial \theta}{\partial g_2}+\frac{\partial \overline{P}}{\partial g_1}\cdot\frac{\partial \overline{P}}{\partial g_2}+\frac{\partial f'}{\partial g_1}\cdot\frac{\partial f'}{\partial g_2}+\frac{\partial \theta'}{\partial g_1}\cdot\frac{\partial \theta'}{\partial g_2}$$

$$a_{13}=\frac{\partial f}{\partial g_1}.\frac{\partial f}{\partial g_3}+\frac{\partial \theta}{\partial g_1}.\frac{\partial \theta}{\partial g_3}+\frac{\partial \overline{P}}{\partial g_1}.\frac{\partial \overline{P}}{\partial g_3}+\frac{\partial f'}{\partial g_1}.\frac{\partial f'}{\partial g_3}+\frac{\partial \theta'}{\partial g_1}.\frac{\partial \theta'}{\partial g_3}$$

$$a_{21}=\frac{\partial f}{\partial g_2}.\frac{\partial f}{\partial g_1}+\frac{\partial \theta}{\partial g_2}.\frac{\partial \theta}{\partial g_1}+\frac{\partial \overline{P}}{\partial g_2}.\frac{\partial \overline{P}}{\partial g_1}+\frac{\partial f'}{\partial g_2}.\frac{\partial f'}{\partial g_1}+\frac{\partial \theta'}{\partial g_2}.\frac{\partial \theta'}{\partial g_1}$$

$$a_{22}=\left[\left(\frac{\partial f}{\partial g_2}\right)^2+\left(\frac{\partial \theta}{\partial g_2}\right)^2+\left(\frac{\partial \overline{P}}{\partial g_2}\right)^2+\left(\frac{\partial f'}{\partial g_2}\right)^2+\left(\frac{\partial \theta'}{\partial g_2}\right)^2\right]$$

$$a_{23}=\frac{\partial f}{\partial g_2}.\frac{\partial f}{\partial g_3}+\frac{\partial \theta}{\partial g_2}.\frac{\partial \theta}{\partial g_3}+\frac{\partial \overline{P}}{\partial g_2}.\frac{\partial \overline{P}}{\partial g_3}+\frac{\partial f'}{\partial g_2}.\frac{\partial f'}{\partial g_3}+\frac{\partial \theta'}{\partial g_2}.\frac{\partial \theta'}{\partial g_3}$$

$$a_{31}=\frac{\partial f}{\partial g_3}.\frac{\partial f}{\partial g_1}+\frac{\partial \theta}{\partial g_3}.\frac{\partial \theta}{\partial g_1}+\frac{\partial \overline{P}}{\partial g_3}.\frac{\partial \overline{P}}{\partial g_1}+\frac{\partial f'}{\partial g_3}.\frac{\partial f'}{\partial g_1}+\frac{\partial \theta'}{\partial g_3}.\frac{\partial \theta'}{\partial g_1}$$

$$a_{32}=\frac{\partial f}{\partial g_3}.\frac{\partial f}{\partial g_2}+\frac{\partial \theta}{\partial g_3}.\frac{\partial \theta}{\partial g_2}+\frac{\partial \overline{P}}{\partial g_3}.\frac{\partial \overline{P}}{\partial g_2}+\frac{\partial f'}{\partial g_3}.\frac{\partial f'}{\partial g_2}+\frac{\partial \theta'}{\partial g_3}.\frac{\partial \theta'}{\partial g_2}$$

$$a_{33}=\left[\left(\frac{\partial f}{\partial g_3}\right)^2+\left(\frac{\partial \theta}{\partial g_3}\right)^2+\left(\frac{\partial \overline{P}}{\partial g_3}\right)^2+\left(\frac{\partial f'}{\partial g_3}\right)^2+\left(\frac{\partial \theta'}{\partial g_3}\right)^2\right]$$

$$b_1=-\left[f_c\frac{\partial f}{\partial g_1}+\theta_c\frac{\partial \theta}{\partial g_1}+\overline{P}_c\frac{\partial \overline{P}}{\partial g_1}+f'_c\frac{\partial f'}{\partial g_1}+\theta'_c\frac{\partial \theta'}{\partial g_1}\right]$$

$$b_2=-\left[f_c\frac{\partial f}{\partial g_2}+\theta_c\frac{\partial \theta}{\partial g_2}+\overline{P}_c\frac{\partial \overline{P}}{\partial g_2}+f'_c\frac{\partial f'}{\partial g_2}+\theta'_c\frac{\partial \theta'}{\partial g_2}\right]$$

and

$$b_3=-\left[f_c\frac{\partial f}{\partial g_3}+\theta_c\frac{\partial \theta}{\partial g_3}+\overline{P}_c\frac{\partial \overline{P}}{\partial g_3}+f'_c\frac{\partial f'}{\partial g_3}+\theta'_c\frac{\partial \theta'}{\partial g_3}\right]$$

Now solving the equations (2.56-2.58) using Cramer's rule , we have

$$\Delta g_1=\frac{\det A_1}{\det A},\quad \Delta g_2=\frac{\det A_2}{\det A}\quad\text{and}\quad \Delta g_3=\frac{\det A_3}{\det A}$$

where

$$\det A_1 = \begin{vmatrix} b_1 & a_{12} & a_{13} \\ b_2 & a_{22} & a_{23} \\ b_3 & a_{32} & a_{33} \end{vmatrix} = b_1(a_{22}a_{33} - a_{32}a_{23}) + b_2(a_{32}a_{13} - a_{12}a_{33}) + b_3(a_{12}a_{23} - a_{22}a_{13})$$

$$\det A_2 = \begin{vmatrix} a_{11} & b_1 & a_{13} \\ a_{21} & b_2 & a_{23} \\ a_{31} & b_3 & a_{33} \end{vmatrix} = b_1(a_{31}a_{23} - a_{21}a_{33}) + b_2(a_{11}a_{33} - a_{31}a_{13}) + b_3(a_{21}a_{13} - a_{11}a_{23})$$

$$\det A_3 = \begin{vmatrix} a_{11} & a_{12} & b_1 \\ a_{21} & a_{22} & b_2 \\ a_{31} & a_{32} & b_3 \end{vmatrix} = b_1(a_{21}a_{32} - a_{31}a_{22}) + b_2(a_{31}a_{12} - a_{11}a_{32}) + b_3(a_{11}a_{22} - a_{21}a_{12})$$

$$\det A = \begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix} = a_{11}(a_{22}a_{33} - a_{23}a_{32}) + a_{21}(a_{32}a_{13} - a_{12}a_{33}) + a_{31}(a_{12}a_{23} - a_{22}a_{13})$$

Then we obtain the missing values g_1, g_2 and g_3 as

$$g_1 = g_1 + \Delta g_1 \,,$$

$$g_2 = g_2 + \Delta g_2 \,,$$

$$g_3 = g_3 + \Delta g_3 \,.$$

Thus adopting the numerical technique aforementioned ,the solution of the equations (2.18)-(2.20) with boundary conditions (2.15) for case I and the equations (2.30)-(2.32) with boundary conditions (2.29) for case II are obtained together with sixth-order implicit Runge-Kutta initial value solver and determine the velocity, temperature and concentration as a function of the coordinate η. In the process of integration the skin-friction coefficient $f''(0)$ and heat transfer rate $-\theta'(0)$ are also calculated .

2.5 Results and discussion

The system of nonlinear ordinary differential equations (2.18)–(2.20) together with the boundary conditions (2.15) for the case I and (2.30)-(2.32) together with the boundary condition (2.29) for the case II have been solved numerically by using sixth-order Runge-Kutta shooting method. Various groups of the parameters $\alpha, Gr, \Pr, v_0, m$ were considered in different phases. In all the computations the step size $\Delta\eta = 0.005$ was selected that satisfied a convergence criterion of 10^{-6} in almost all of different phases mentioned above. However, different step sizes such as $\Delta\eta = 0.01$ to $\Delta\eta = 0.001$ were also tried and the obtained solutions have been found to be independent of the step sizes as observed in figure 2.2.

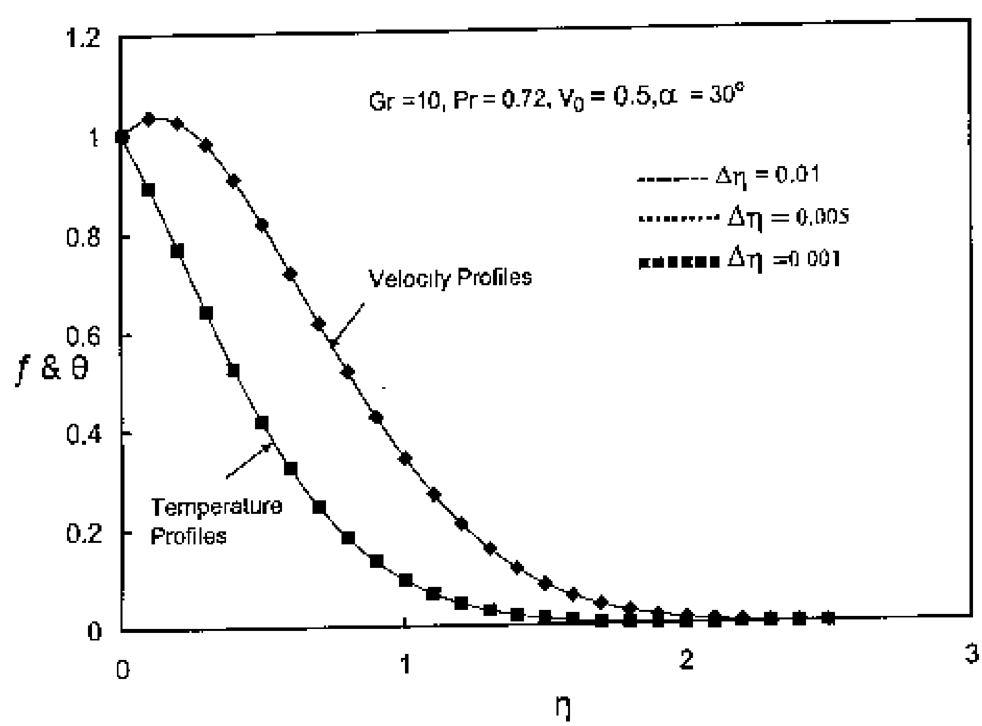

Fig. 2.2: Velocity & temperature profiles for different step sizes.

The results for the two cases considered above are displayed graphically in Figures (2.2.1)-(2.2.12) respectively for dimensionless forms of velocity, temperature and pressure . Numerical computations have been carried out for the study of the effects of various parameters on the velocity, temperature and pressure distribution for both the case . For this purpose the effects of different parameters Gr, v_0, Pr , m and α on the fluid flow have been investigated . The value of Prandtl number Pr is taken equal to 0.72, 1.0 and 7.0 that corresponds physically 0.72 is suitable for air at 20°c , 1.0 correspond to electrolyte solution as salt water and 7.0 corresponds to water. The value Grashoff number Gr is taken to be large 10.0 , where larger values of Gr correspond to a cooling problem that is generally encountered in nuclear engineering in connection with the cooling of reactors . The positive or negatives values of Gr respectively represent cooling and heating of the plate . The suction velocity v_0 is taken to be equal to 0.5 , 1.0 and 2.0 which are appropriate for the liquid metals . The values of α and m are chosen arbitrarily .

Case I : The mentioned parameters the velocity , temperature and pressure distribution profiles are represented graphically in Figures (2.2.1)-(2.2.6) for both the heating and cooling of impulsively started plate . From Figures (2.2.1) and (2.2.2) for the case when $Gr > 0$ (in the presence of cooling of the plate by natural convection currents) we observe that i) there is a rise in the velocity profiles due to an increase in α , ii) an increase in the suction parameter v_0 causes a fall in the velocity fields. From Figures (2.2.3) for the case when Gr positive and negative (in the presence of heating and cooling of the plate by natural convection currents) it is seen that there is fall in the velocity profiles due to an increase. Figures (2.2.4) for the case when $Gr>0$ in the presence of cooling of the plate by natural convection currents.

We also plotted temperature profiles in Figure (2.2.5) in case of $Gr > 0$ for a comparison in different Prandtl number Pr.We see that there is a decrease in temperature due to increases which is very large in case of water (Pr=7.0). Figure (2.2.6) shows the pressure distribution profiles for two cases $Gr < 0$ and $Gr > 0$ for fixed values Pr , v_0 and α. We observe that there is a rise in pressure due to increase for the case $Gr > 0$, on the other hand , a reverse phenomenon occurs in case $Gr < 0$.

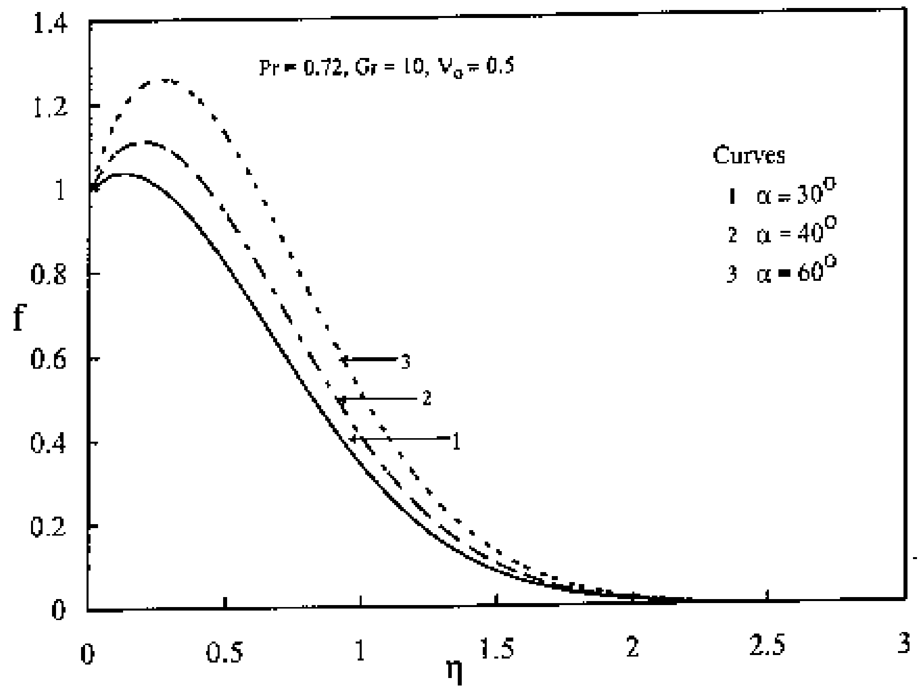

Fig:2.2.1: Velocity profiles due to cooling of impulsively started plate for different values of α.

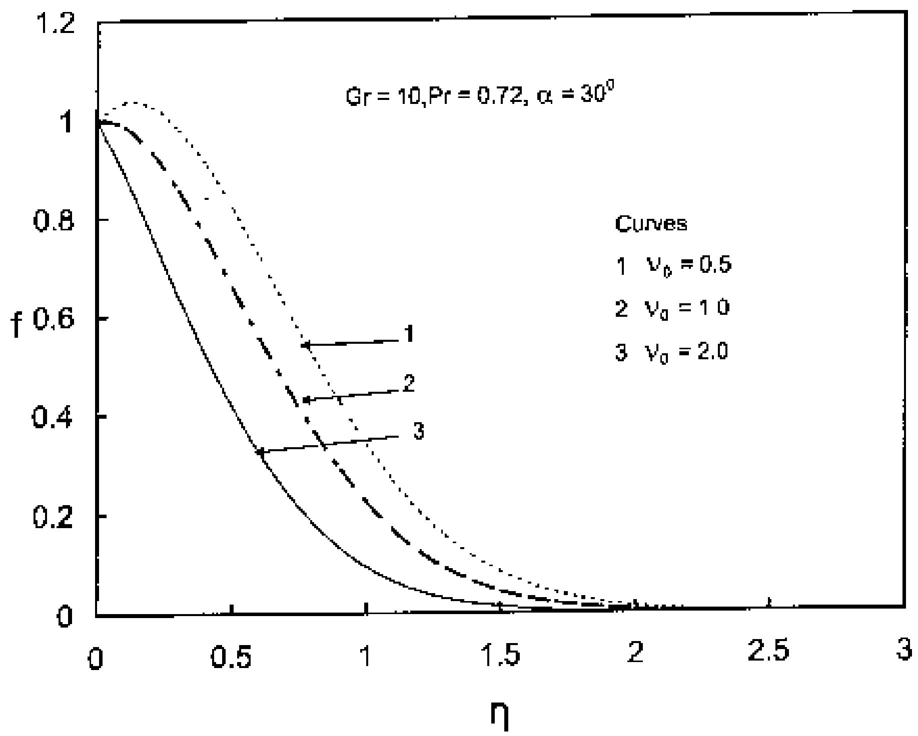

Fig:2.2.2: Velocity profiles due to cooling of impulsively started plate for different values of ν_0.

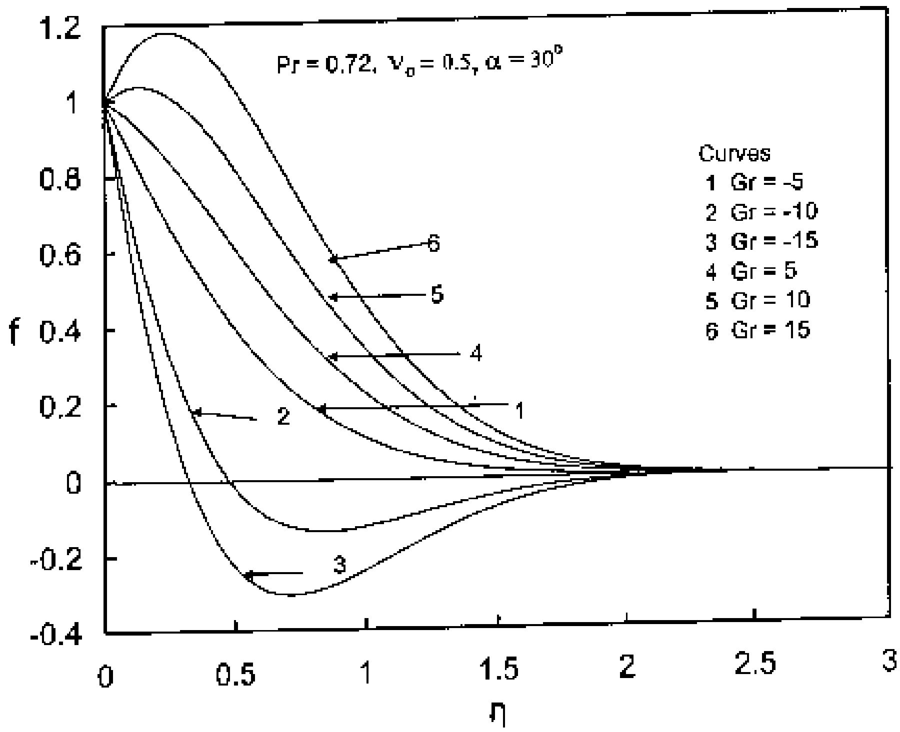

Fig:2.2.3: Velocity profiles due to heating and cooling of impulsively started plate for different values of Gr.

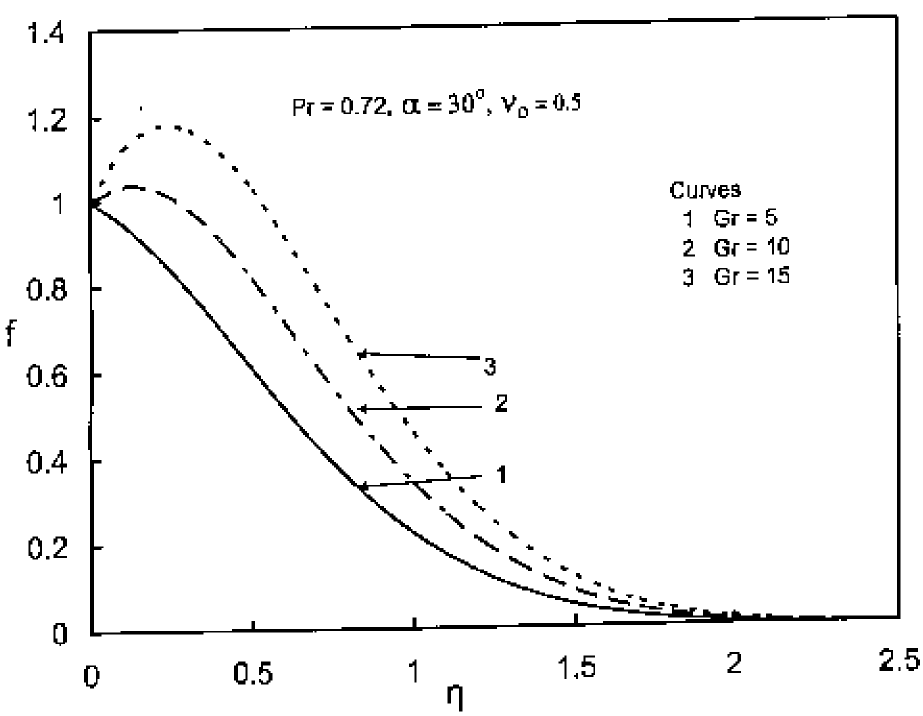

Fig:2.2.4: Velocity profiles due to cooling of impulsively started plate for different values of Gr.

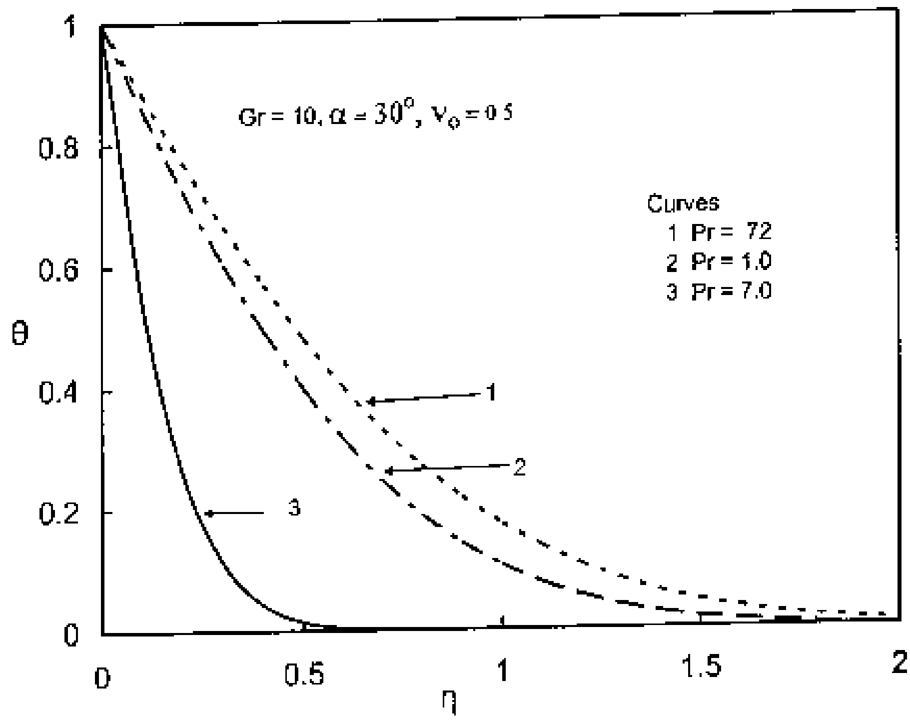

Fig:2.2.5: Temperature profiles for impulsively started plate for different values of *Pr*.

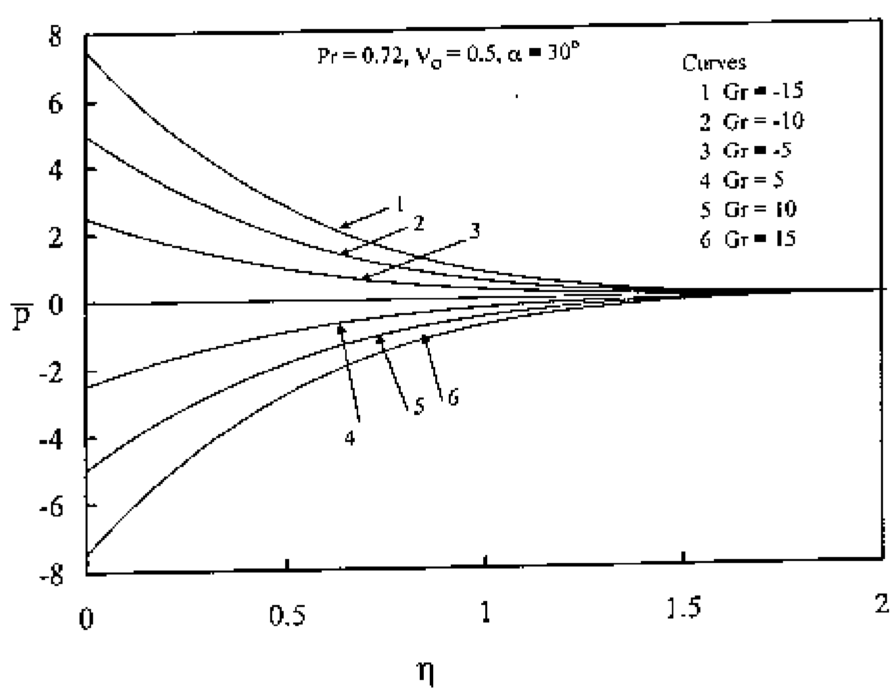

Fig:2.2.6: Pressure distribution profiles for impulsively started plate for different values of *Gr*.

Case II: Figures (2.2.7-2.2.10) show the variation of the velocity profiles due to accelerated motion of the plate when it is cooled and heated by natural convection currents respectively. Figure (2.2.11) shows the temperature profiles and Figure (2.2.12) shows the pressure distribution profiles. We observe from the figures that the effects of various parameters on velocity and temperature are similar to those of the impulsively started plate. Thus the discussion of the results in this case is not produced for brevity.

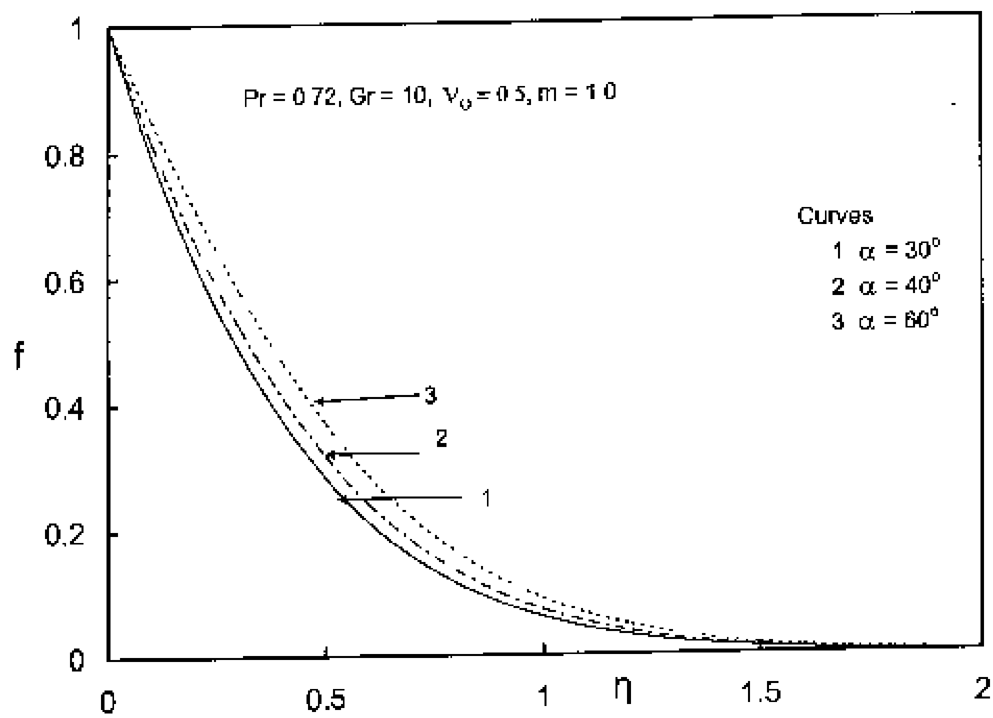

Fig:2.2.7: Velocity profiles due to cooling of uniformly accelerated plate for different values of α.

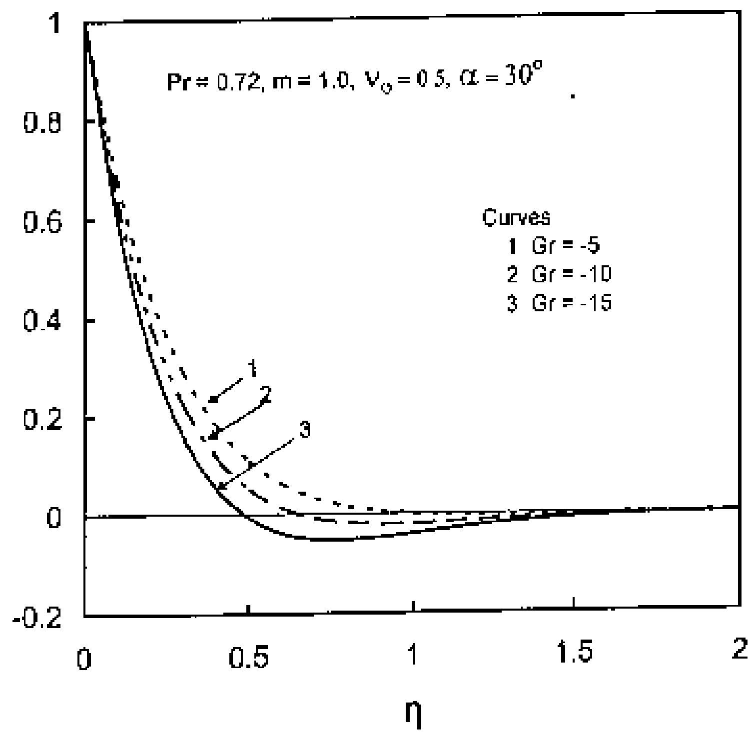

Fig:2.2.8: Velocity profiles due to heating of uniformly accelerated plate for different values of *Gr*.

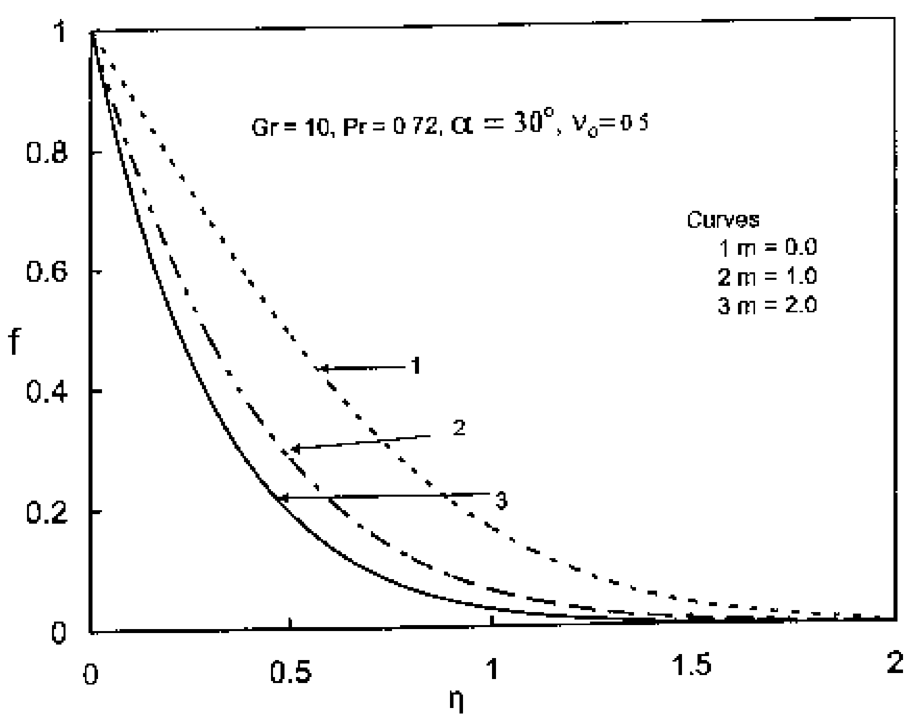

Fig:2.2.9: Velocity profiles due to cooling of uniformly accelerated plate for different values of *m*.

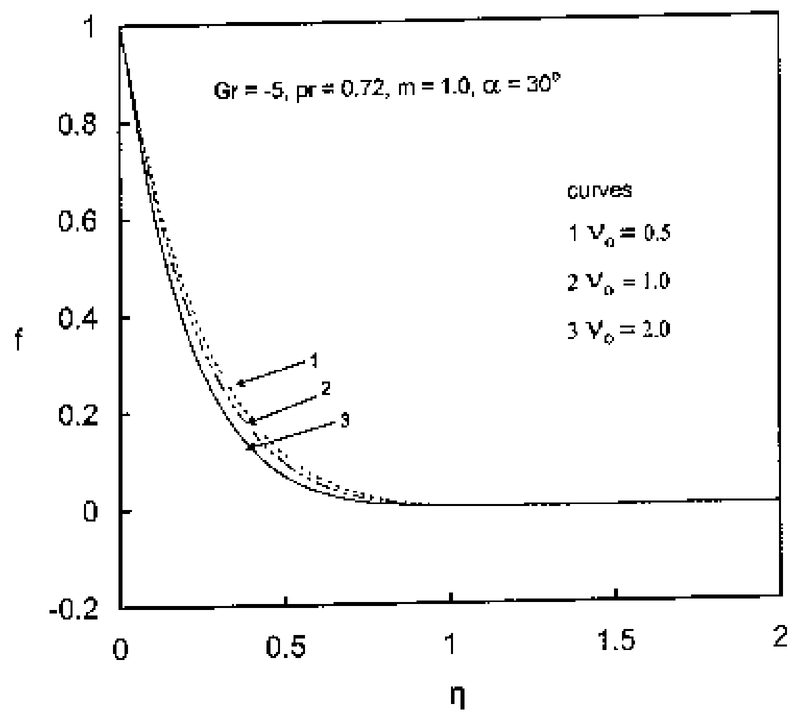

Fig:2.2.10: Velocity profiles due to heating of uniformly accelerated plate for different values of v_0.

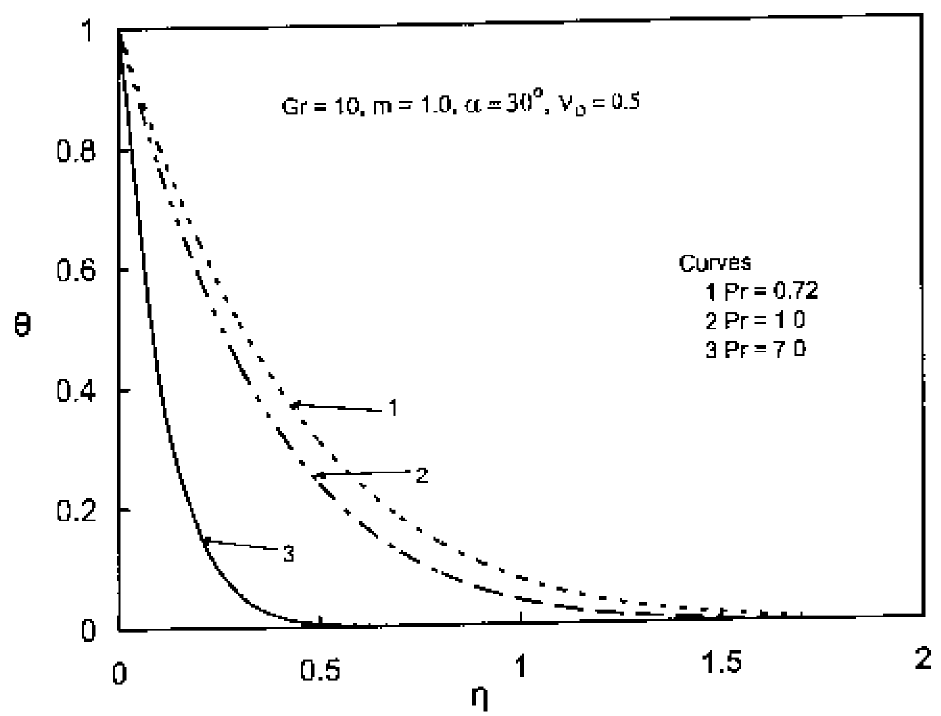

Fig:2.2.11: Temperature profiles for uniformly accelerated plate for different values of Pr.

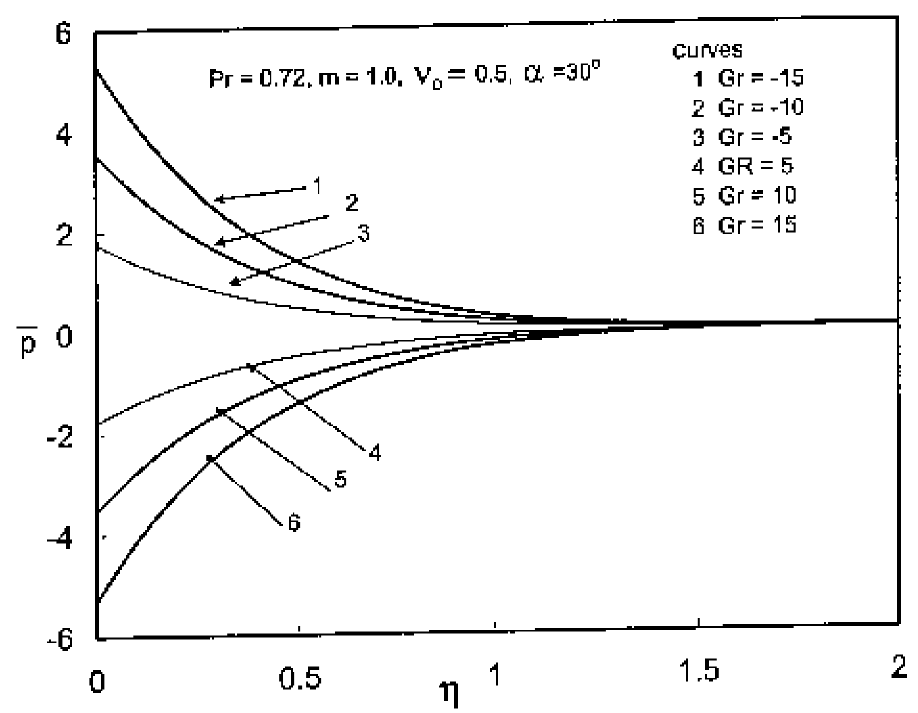

Fig:2.2.12: Pressure distribution profiles for uniformly accelerated plate for different values of α.

Finally in Tables 2.1 & 2.2 numerical values of the skin friction and Nusselt Number respectively proportional to $f'(0)$ and $-\theta'(0)$ are given for impulsive as well as uniformly accelerated motion of the plate. In Table 2.1, it appears that the skin friction coefficients increase with the increase of α and Gr but decreases with the increase of ν_0 and Pr. On the other hand, Nusselt number decreases with the increase of ν_0 and Pr. The Table 2.2 Indicates that the skin friction coefficients increase with the increase of α, Gr, m but decreases with the increase of ν_0 and $\Pr$. The coefficient of Nusselt number increases with the increase of m, ν_0 and $\Pr$. We see from both the table that the wall shear stress has a larger effect in case of impulsively started plate as compared to the uniformly accelerated plate.

Table 2.1

Numerical values of skin friction coefficient, τ_i and nusselt number N_{ui} for impulsively started plate

Gr	Pr	α	v_0	τ_i	N_{ui}
10	0.72	30^0	0.5	0.6058	1.2005
10	7.0	30^0	0.5	- 0.8207	5.0515
10	0.72	40^0	0.5	1.1960	1.2008
-10	0.72	30^0	0.5	-3.5365	1.2003
5	0.72	30^0	1.0	-0.8609	1.4594
-5	0.72	30^0	0.5	-2.5021	1.1993
5	0.72	60^0	0.5	0.3282	1.2005

Table 2.2

Numerical values of skin friction coefficient, τ_a and nusselt number N_{ua} for uniformly accelerated plate

Gr	Pr	m	α	v_0	τ_a	N_{ua}
10	7.0	1.0	30^0	0.5	-2.8525	8.1123
10	0.72	1.0	30^0	0.5	-3.0716	2.3389
10	0.72	0.0	30^0	0.5	-1.7969	1.1997
10	0.72	1.0	40^0	0.5	-1.9762	2.1195
10	0.72	1.0	30^0	1.0	-2.5674	2.3391
-10	0.72	1.0	30^0	0.5	-4.2988	2.1196

Chapter Three

Finite Difference Analysis of Natural Convection Flow Over an Inclined Heated Plate

Two dimensional natural convection heat and mass transfer flow past a semi-infinite flat plate have been receiving the attention of many researchers because of its wide application in industry and technological fields. Natural convection along an inclined plate has received less attention than the case of vertical and horizontal plates. Finite-difference technique has been used in natural convective flow analysis by many researchers. Callahan and Marner (1976) have presented a paper on transient free convection with mass transfer effects and to solve the problem by explicit finite difference technique. Soundalgekar and Ganesan (1985) solved the same problem using implicit finite difference technique and compared the result with those of Callahan and Marner (1976) and both the results agree well. Recently Chamkha et al. (2001) have presented a paper on similarity solutions for hydromagnetic simultaneous heat and mass transfer by natural convection from inclined plate with thermal heat generation or absorption and to solved by implicit finite difference technique. Very recently Ganesan and Palani (2003) have studied free convection effects on the flow of water at 4°C past a semi-infinite inclined flat plate to solved the problem using implicit finite difference technique. The explicit finite difference technique is conditionally stable but implicit finite difference technique is unconditionally stable and hence during computation large time-steps as compared to explicit technique are permitted which leads to saving computer time which is a distinct advantage. For this convenience we have solved our problem using implicit finite difference technique. Numerically results are presented by velocity, temperature and Prandtl number *Pr*. In the following section detailed derivations of the governing equations for the flow and the method of solutions along with the results and discussions are presented.

For this purpose let us consider a two-dimensional unsteady flow of a viscous incompressible fluid past an inclined flat plate. Initially it is assumed that the flow is to be in the X-direction which is permeable inclined at an angle α to the horizontal and the Y-axis is perpendicular to the plate. The physical flow configuration is shown in the following Figure 3.1

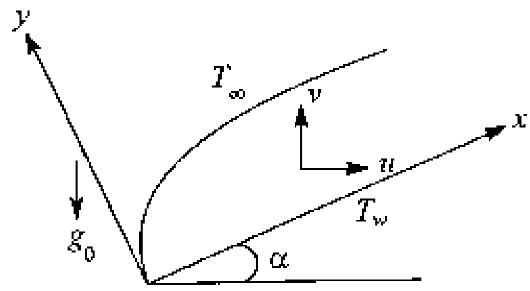

Figure 3.1 : Configuration of the problem

3.1 Governing equations of the flow:

The mathematical statement of the basic conservation laws of mass, momentum and energy for the unsteady viscous incompressible flow are given by

$$\nabla.\vec{q} = 0 \tag{3.1}$$

$$\frac{\partial \vec{q}}{\partial t} + \rho(\vec{q}.\nabla)\vec{q} = -\nabla p + \mu\nabla^2\vec{q} + \vec{F} \tag{3.2}$$

$$\frac{\partial T}{\partial t} + (\vec{q}.\nabla)T = \frac{\kappa}{\rho c_p}\nabla^2 T \tag{3.3}$$

where $\vec{q} = \vec{q}\,(u, v\,)$, u and v are the velocity components along the x and y axes respectively, $\vec{F}$ is the body force per unit volume which is defined as $-\rho g$, T is the temperature of the fluid in the boundary layer, g_0 is the acceleration due to gravity, k is the thermal conductivity, c_p is the specific heat at constant pressure and μ is the viscosity of the fluid. ∇ is the vector differential operator and is defined by

$$\nabla = \hat{i}_x \frac{\partial}{\partial x} + \hat{i}_y \frac{\partial}{\partial y}$$

where $\hat{i}_x$ and the $\hat{i}_y$ are the un it vector along x and y axes respectively.

The above mentioned equations (3.1) to (3.3) would serve as the governing equations of the problems under the Boussinesq's approximation

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \tag{3.4}$$

$$\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = \nu\frac{\partial^2 u}{\partial y^2} + g_0\beta(T - T_\infty)\sin\alpha \tag{3.5}$$

$$\frac{\partial T}{\partial t} + u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y} = \frac{\kappa}{\rho c_p}\frac{\partial^2 T}{\partial y^2} \tag{3.6}$$

The appropriate boundary conditions to be satisfied by the above equations are

$$t \le 0, u = 0, v = 0, T = T_\infty \tag{3.7a}$$

$t > 0, u = 0, T = T_\infty$ at x=o

u=0,v=0, T=T_w at y=o (3.7b)

u=0, T=T_w as $y \to \infty$

3.2 Transformation of the governing equations

Equations (3.4) to (3.6) may now be nondimensionalized by using the following dimensionless variables:

$$X = \frac{x}{L},\ Y = \frac{y}{L}Gr^{1/4},\ U = \frac{uL}{\upsilon}Gr^{-1/2},\ V = \frac{vL}{\upsilon}Gr^{-1/4},\ \theta = \frac{T - T_\infty}{T_w - T_\infty},\ \tau = \frac{tGr^{1/2}\upsilon}{L^2} \tag{3.8}$$

Where L is the characteristic length of the plate, Gr is the Grashof number, θ is the non dimensional temperature.

Substituting the relations (3.8) into the equations (3.4) to (3.6) then the following non-dimensional equations

$$\frac{\partial U}{\partial X}+\frac{\partial V}{\partial Y}=0 \tag{3.9}$$

$$\frac{\partial U}{\partial \tau}+U\frac{\partial U}{\partial X}+V\frac{\partial U}{\partial Y}=\frac{\partial^2 U}{\partial Y^2}+\theta\,\mathrm{Sin}\alpha \tag{3.10}$$

$$\frac{\partial \theta}{\partial \tau}+U\frac{\partial \theta}{\partial X}+V\frac{\partial \theta}{\partial Y}=\frac{1}{\Pr}\frac{\partial^2 \theta}{\partial Y^2} \tag{3.11}$$

Where $Gr = \dfrac{g_0\beta L^3(T_w-T_\infty)}{\upsilon^2}$, is the Grashof number and $P_r = \dfrac{\upsilon\rho c_p}{\kappa}$, is the Prandtl number.

The corresponding boundary conditions (3.7a) to (3.7b) take the following form:

$U=0,\ V=0,\ \theta=0$ for every $\tau \le 0$ (3.12a)

for $\tau > 0$

$U=0,\ V=0,\ \theta=0$ at $X=0$

$U=0,\ V=0,\ \theta=1$ at $Y=0$ (3.12b)

$U=0,\ \theta=0$ as $Y\rightarrow\infty$

3.3 Numerical analysis

The non-linear equations (3.9)-(3.11) subject to the initial and boundary conditions (3.12a) and (3.12b) are solved numerically for the velocity and temperature using the implicit finite differences scheme of Crank-Nicholson technique. For this purpose, we consider a plate of height $X_{max}(=1.0)$ and regarded $Y=Y_{max}(=4.0)$ which corresponds to $Y=\infty$. The value of Y is assumed to lie well outside both the momentum and energy boundary layers. The computer time, variable mesh sizes are taken both in X and Y directions, where X direction is taken along the plate inclined at an angle α to the horizontal and Y direction is taken normal to the plate. In figure 3.2, there are $\overline{M}$ and $\overline{N}$ grid spacing in the X and Y direction respectively.

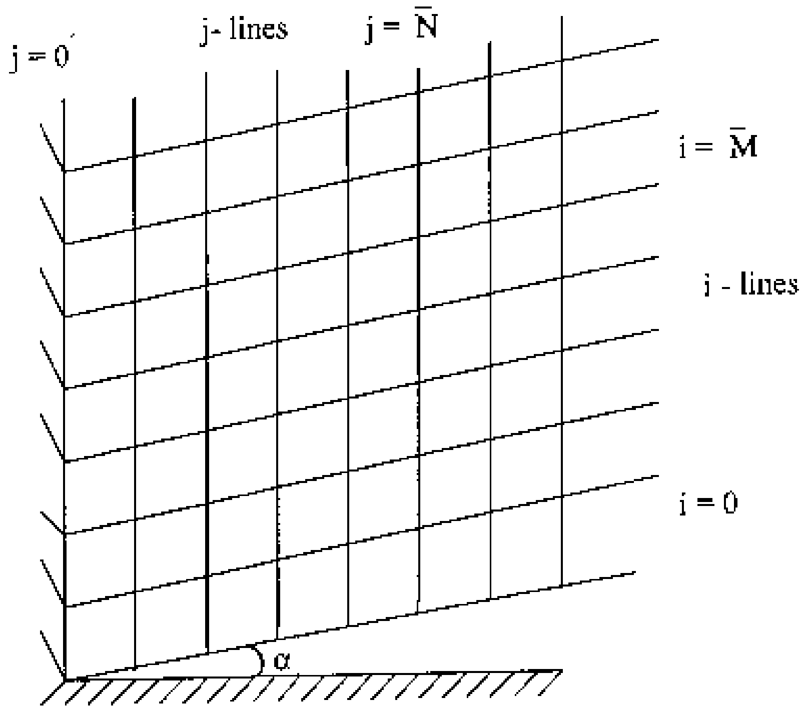

Figure 3.2: Grid system

The mesh sizes are given by

$\Delta X = 0.10\ (0 \le X \le 1.0)$

$\Delta Y = 0.20\ (0 \le Y \le 4.0)$

$\Delta \tau = 0.10$

The partial differential equations (3.9)-(3.11) then can be written in the following finite-difference approximations

$$\frac{U_{i,j}^{n+1} - U_{i-1,j}^{n+1} + U_{i,j}^{n} - U_{i-1,j}^{n} + U_{i,j-1}^{n+1} - U_{i-1,j-1}^{n+1} + U_{i,j-1}^{n} - U_{i-1,j-1}^{n}}{2\Delta X} \tag{3.13}$$

$$+\frac{V_{i,j}^{n+1} - V_{i,j-1}^{n+1} + V_{i,j}^{n} - V_{i,j-1}^{n}}{\Delta Y} = 0$$

$$\frac{U_{i,j}^{n+1}-U_{i,j}^{n}}{\Delta\tau}+U_{i,j}^{n}\frac{U_{i,j}^{n+1}-U_{i-1,j}^{n+1}+U_{i,j}^{n}-U_{i-1,j}^{n}}{2\Delta X}+V_{i,j}^{n}\frac{U_{i,j+1}^{n+1}-U_{i,j-1}^{n+1}+U_{i,j+1}^{n}-U_{i,j-1}^{n}}{4\Delta Y}$$
$$=\frac{U_{i,j-1}^{n+1}-2U_{i,j}^{n+1}+U_{i,j+1}^{n+1}+U_{i,j-1}^{n}-2U_{i,j}^{n}+U_{i,j+1}^{n}}{2(\Delta Y)^2}+\frac{\theta_{i,j}^{n+1}+\theta_{i,j}^{n}}{2}\sin\alpha \tag{3.14}$$

$$\frac{\theta_{i,j}^{n+1}-\theta_{i,j}^{n}}{\Delta\tau}+U_{i,j}^{n}\frac{\theta_{i,j}^{n+1}-\theta_{i-1,j}^{n+1}+\theta_{i,j}^{n}-\theta_{i-1,j}^{n}}{2\Delta X}+V_{i,j}^{n}\frac{\theta_{i,j+1}^{n}-\theta_{i,j-1}^{n+1}+\theta_{i,j+1}^{n}-\theta_{i,j-1}^{n}}{4\Delta Y}$$
$$=\frac{1}{\Pr}\frac{\theta_{i,j-1}^{n+1}-2\theta_{i,j}^{n+1}+\theta_{i,j+1}^{n+1}+\theta_{i,j-1}^{n}-2\theta_{i,j}^{n}+\theta_{i,j+1}^{n}}{2(\Delta Y)^2} \tag{3.15}$$

Here the subscripts i designates the grid points with x- coordinates and j designates the grid points with y-coordinates and n designates the value at a time $\tau = n\Delta\tau$, where $n=1,2,3......$etc.

The initial and boundary conditions

$$U_{i,j}^{0}=0, V_{i,j}^{0}=0, \theta_{i,j}^{0}=0 \tag{3.16}$$

$$\begin{aligned} &U_{0,j}^{n}=0, V_{0,j}^{n}=0, \theta_{0,j}^{n}=0\\ &U_{i,0}^{n}=0, V_{i,0}^{n}=0, \theta_{i,0}^{n}=1\\ &U_{i,h}^{n}=0, \theta_{i,h}^{n}=0 \end{aligned} \tag{3.17}$$

where $h\to\infty$

Equations (3.13)-(3.15) are written in the following form

$$A_1U_{i,j-1}^{n+1}+B_1U_{i,j}^{n+1}+C_1U_{i,j+1}^{n+1}=E_1 \tag{3.18}$$

$$A_2\theta_{i,j-1}^{n+1}+B_2\theta_{i,j}^{n+1}+C_2\theta_{i,j+1}^{n+1}=E_2 \tag{3.19}$$

$$V_{i,j}^{n+1}=E_3 \tag{3.20}$$

where

$$A_1 = -\frac{V_{i,j}^n}{h\Delta Y} - \frac{1}{2(\Delta Y)^2}$$

$$A_2 = -\frac{V_{i,j}^n}{4\Delta Y} - \frac{1}{2\Pr(\Delta Y)^2}$$

$$B_1 = \frac{1}{\Delta\tau} + \frac{U_{i,j}^n}{2\Delta X} + \frac{1}{(\Delta Y)^2}$$

$$B_2 = \frac{1}{\Delta\tau} + \frac{U_{i,j}^n}{2\Delta X} + \frac{1}{\Pr(\Delta Y)^2}$$

$$C_1 = \frac{V_{i,j}^n}{4\Delta Y} - \frac{1}{2(\Delta Y)^2}$$

$$C_2 = \frac{V_{i,j}^n}{4\Delta Y} - \frac{1}{2\Pr(\Delta Y)^2}$$

$$E_1 = \frac{U_{i,j}^n}{2\Delta X}\left(U_{i-1,j}^{n+1} + U_{i-1,j}^n - U_{i,j}^n\right) + \frac{V_{i,j}^n}{4\Delta Y}\left(U_{i,j-1}^n - U_{i,j+1}^n\right)$$

$$+\frac{U_{i,j}^n}{\Delta\tau} + \frac{1}{2(\Delta Y)^2}\left(U_{i,j-1}^n - 2U_{i,j}^n + U_{i,j+1}^n\right) + \frac{1}{2}\left(\theta_{i,j}^{n+1} + \theta_{i,j}^n\right)\sin\alpha$$

$$E_2 = \frac{\theta_{i,j}^n}{\Delta\tau} + \frac{U_{i,j}^n}{2\Delta X}\left(\theta_{i-1,j}^{n+1} + \theta_{i-1,j}^n - \theta_{i,j}^n\right) + \frac{V_{i,j}^n}{4\Delta Y}\left(\theta_{i,j-1}^n - \theta_{i,j+1}^n\right)$$

$$+\frac{1}{2\Pr(\Delta Y)^2}\left(\theta_{i,j-1}^n - 2\theta_{i,j}^n + \theta_{i,j+1}^n\right)$$

$$E_3 = V_{i,j-1}^{n+1} - V_{i,j}^n + V_{i,j-1}^n - \frac{\Delta\tau}{2\Delta X}$$

$$\left(U_{i,j}^{n+1} - U_{i-1,j}^{n+1} + U_{i,j}^n - U_{i-1,j}^n + U_{i,j-1}^{n+1} - U_{i-1,j-1}^{n+1} + U_{i,j-1}^n - U_{i-1,j-1}^n\right)$$

At a time τ calculating the values of θ, U, V at all the nodal points, their values at a time $\tau+\Delta\tau$ are calculated in the following manner

Knowing the values of θ, U, V at a time $\tau = n\Delta\tau$, the line i=1 is considered. The values of θ are found from equation (3.19) at every nodal point on $i=1$ at $(n+1)$th time level. Finally the values of U are found from equation (3.18) in the same procedure. In all the process the values of V are derived from the equation (3.20). Processing on this way and get the values of θ, U and V for all $i=2,3,\ldots\ldots\overline{M}$. This process is repeated for $n=2,3,$.until the steady state is reached.

3.4 Stability of the finite difference equation

It is necessary to discuss the stability and convergence of the finite difference scheme for convenience of analysis. For constant mesh sizes, the general term of Fourier expension for U and θ at a time arbitrarily called $\tau = 0$ are assumed to be the form $e^{i\phi X} e^{i\phi Y}$.

These terms can be written as follows at a later time τ

$$
\begin{aligned}
U &= F(\tau) e^{i\phi X} e^{i\phi Y} \\
\theta &= H(\tau) e^{i\phi X} e^{i\phi Y}
\end{aligned}
\tag{3.21}
$$

The values of F and H after one time step are denoted by F' and H' respectively. Substituting (3.21) in the finite difference equations (3.13)-(3.15) and the coefficients U and V treated as constant after any one time step. We obtain

$$
\begin{aligned}
&\frac{F'-F}{\Delta\tau} + \frac{U}{2\Delta X}(F'+F)\left(1-e^{-i\phi\Delta X}\right) + \frac{V}{2\Delta Y}(F'+F)i\sin\phi\Delta Y \\
&= \frac{1}{(\Delta Y)^2}(F'+F)(\cos\phi\Delta Y - 1) + \frac{1}{2}(H'+H)\sin\alpha
\end{aligned}
\tag{3.22}
$$

$$
\begin{aligned}
&\frac{H'-H}{\Delta\tau} + \frac{U}{2\Delta X}(H'+H)\left(1-e^{-i\phi\Delta X}\right) + \frac{V}{2\Delta Y}(H'+H)i\sin\phi\Delta Y \\
&= \frac{1}{\Pr(\Delta Y)^2}(H'+H)x(\cos\phi\Delta Y - 1)
\end{aligned}
\tag{3.23}
$$

Let us define

$$
\begin{aligned}
A &= \frac{\Delta\tau U}{2\Delta X}\left(1-e^{-i\phi\Delta X}\right) + \frac{\Delta\tau V}{2\Delta Y} i\sin\phi\Delta Y - \frac{\Delta\tau}{(\Delta Y)^2}(\cos\phi\Delta Y - 1) \\
B &= \frac{\Delta\tau U}{2\Delta X}\left(1-e^{-i\alpha\Delta X}\right) + \frac{\Delta\tau V}{2\Delta Y} i\sin\phi\Delta Y - \frac{\Delta\tau}{\Pr(\Delta Y)^2}(\cos\phi\Delta Y - 1)
\end{aligned}
\tag{3.24}
$$

Then the equations (3.22) and (3.23) can be written as

$$
(1+A)F' = (1-A)F + \frac{\Delta\tau}{2}(H'+H)Sin\alpha \tag{3.25}
$$

$$
(1+B)H' = (1-B)H \tag{3.26}
$$

where

$$
\begin{aligned}
F' &= \left(\frac{1+A}{1-A}\right)F + \frac{\Delta\tau}{(1+A)(1+B)} \\
H' &= \left(\frac{1-B}{1+B}\right)H
\end{aligned}
\tag{3.27}
$$

which can be expressed in the following matrix

$$\begin{bmatrix} F' \\ H' \end{bmatrix} = \begin{bmatrix} a_{11} & a_{12} \\ 0 & a_{22} \end{bmatrix} \begin{bmatrix} F \\ H \end{bmatrix}$$

i.e. $\bar{\eta}' = \bar{E}\bar{\eta}$

$$a_{11} = \frac{1-A}{1+A}, a_{12} = \frac{\Delta\tau}{(1+A)(!+B)}, a_{22} = \frac{1-B}{1+B}$$

The eigen values of the application matrix $\bar{E}$ are obtained from the characteristics equation

$$\left| I\lambda - \bar{E} \right| = 0 \text{ i.e.}$$

$$\Rightarrow \begin{vmatrix} \lambda - a_{11} & -a_{12} \\ 0 & \lambda - a_{22} \end{vmatrix} = 0$$

Thus eigen values are

$$\lambda_1 = \frac{1-A}{!+A}, \lambda_2 = \frac{1-B}{1+B}$$

Let, a $= \frac{\Delta\tau U}{2\Delta X}$, b $= \frac{\Delta\tau}{(\Delta Y)^2}$

We can write

$$A = 2a\sin^2\left(\frac{\phi\Delta X}{2}\right) + 2b\sin^2\left(\frac{\phi\Delta Y}{2}\right) + ia\sin\phi\Delta X$$

$$= Z_1 + iZ_2$$

$$B = 2a\sin^2\left(\frac{\phi\Delta X}{2}\right) + \frac{2}{\Pr}b\sin^2\left(\frac{\phi\Delta Y}{2}\right) + ia\sin\phi\Delta X$$

$$= Z_3 + iZ_4$$

Since the real part of A or B are greater than or equal to zero.

Therefore the scheme is unconditionally stable.

3.5 Results and discussion

The implicit finite-difference method of Crank-Nicolson technique is employed in the present work for numerical computation. The transient primary velocities and temperature profiles are shown graphically in Figures 3.5.1- 3.5.5 and discussed thereafter. The solutions are obtained at the upper edge of the plate, namely $X = 1.0$ for different values of the given dimensionless parameters. The values 0.72, 1.00 and 7.00 are considered for the Prandtl number Pr which represent specific conditions of the flow.

At first we have plotted the transient primary velocity profiles for different values of inclined angle α respectively in Figure 3.5.1, it is seen that the primary velocity increases with the increasing values of α. To show the effect of Prandtl number, Pr, in the velocity field, we depict Figure 3.5.2. We have seen from this Figure, the increase of the Prandtl number reflects to the primary velocity profiles. However, the velocity profiles for lower time values are also calculated and displayed. Figure 3.5.3 represent respectively of the primary velocity for different values of τ.

We have also plotted the transient temperature profiles considering effects of various values of non-dimensional parameters Figure 3.5.4 and 3.5.5 show the effect of Prandtl number Pr and τ in the temperature profiles.

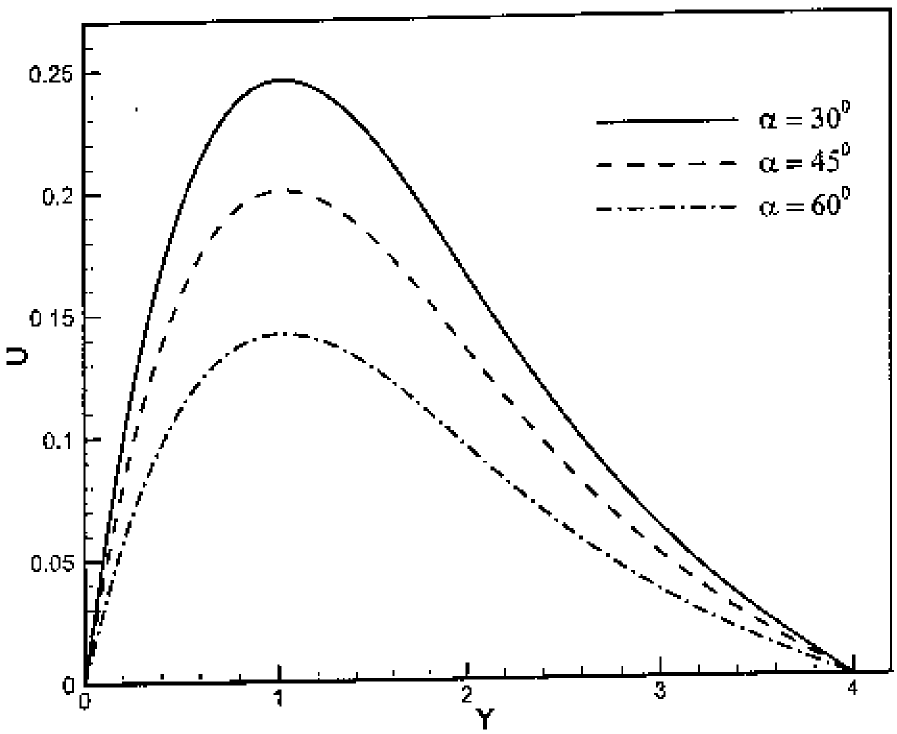

Fig: 3.5.1: Primary velocity profiles for Pr = 0.72.

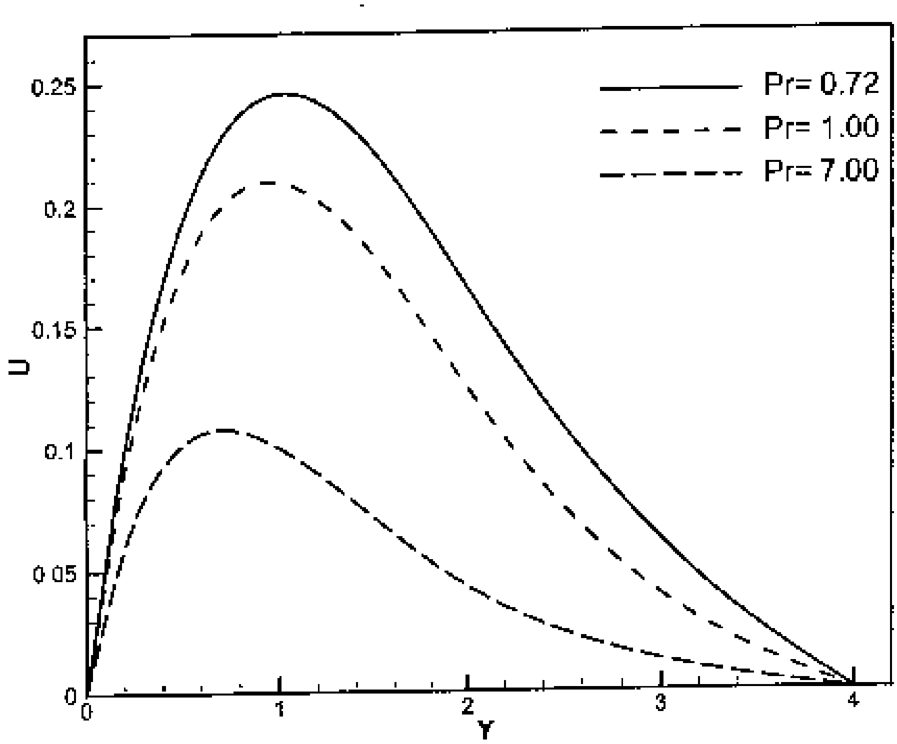

Fig: 3.5.2: Primary velocity profiles for $\alpha = 30°$.

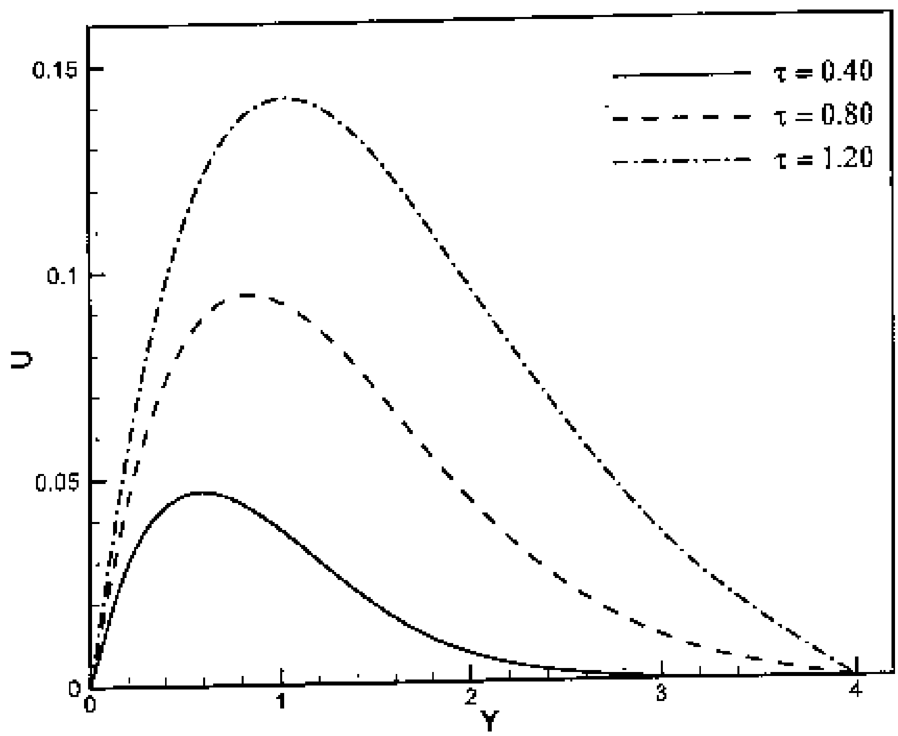

Fig: 3.5.3: Primary velocity profiles for Pr = 0.72 and $\alpha = 60°$.

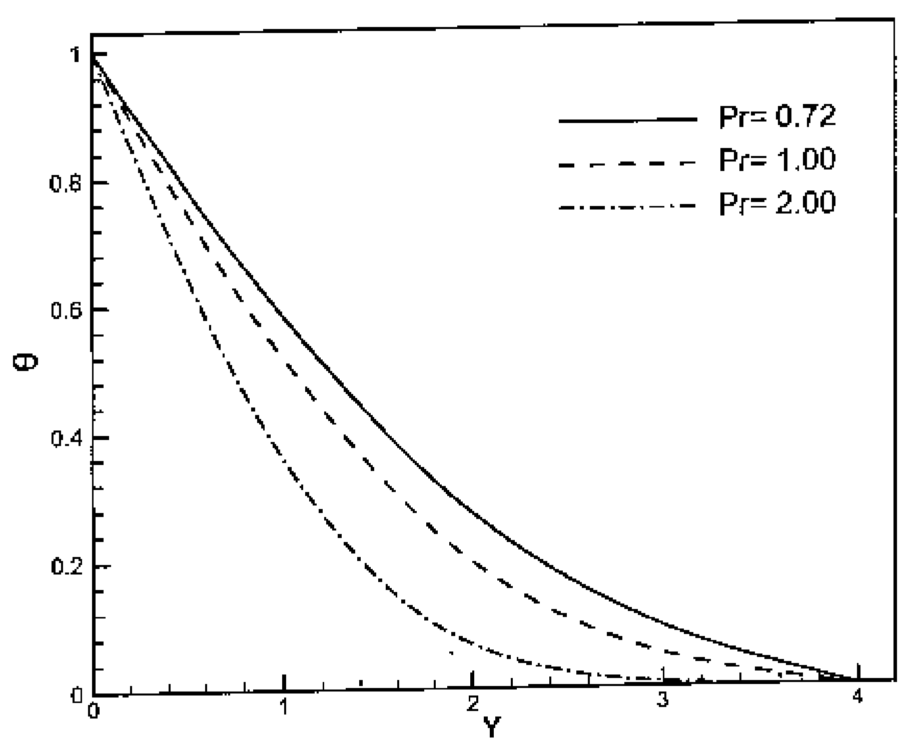

Fig: 3.5.4: Temperature profiles for $\alpha = 30°$.

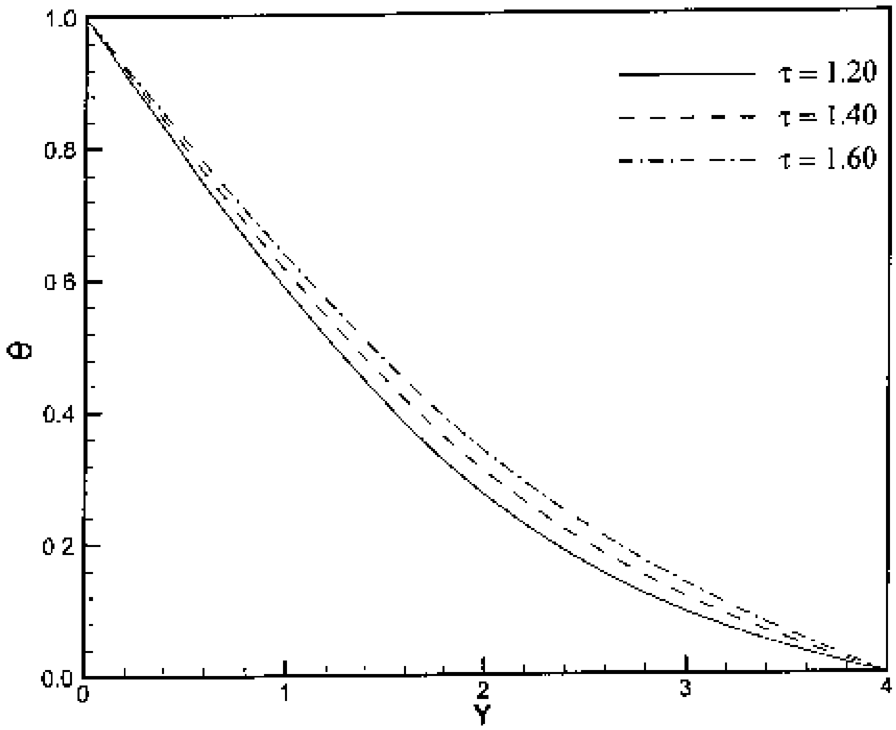

Fig: 3.5.5: Temperature profiles for Pr = 0.72 and $\alpha = 60°$.

Chapter Four

4.1 Conclusion

Unsteady free convection boundary layer flow over a heated plate with different inclinations has been studied. The present work is Time dependent free convection analysis over an inclined heated plate. And another work is Finite difference analysis of natural convection flow over an inclined heated plate. From the investigation the flowing conclusions may be drawn

The mentioned parameters the velocity profiles are represented graphically for both the cooling and heating of impulsively started plate. When Gr >0, there is a rise in the velocity profiles due to an increase in α and an increase in the suction parameter v_0 causes a fall in the velocity fields. When Gr positive and negative the velocity profiles due to an increase. On the other hand the variation of the velocity profiles due to accelerated motion of the plate when it is cooled and heated by natural convection currents respectively.

The temperature profiles in case of Gr >0 for a comparison in different Prandtl number Pr. There is a decrease in temperature due to increase which is large in case of water (Pr =7.0)

The pressure distribution profiles due to increase for the case Gr >0, on the other hand, a reverse phenomenon occurs in case Gr <0.

The skin friction coefficients increase with the increase of α and Gr but decreases with the increase of v_0 and Pr and Nusselt number decreases with the increase of v_0 and Pr.

The primary velocity increases with the increasing values of α.

The increase of the Prandtl number reflects to the primary velocity profiles.

4.2 Extension of this work

The present work can be extended in different ways. Some of those are:

Temperature dependent thermal conductivity has been considered in the present study.

The problem can be extended considering the concentration effects.

Inclusion of Joule heating effects may be another extension.

Considering the Radiation heat transfer effects.

Critical behavior of the flow may be studied.

References

Ali. J. Chamkha,(2004) Unsteady MHD convective Heat and Mass transfer past a semi-infinite vertical permeable moving plate with heat absorption, Int.J.engg. Sci, Vol 42, pp 217-230.

Angel, M., Hossain, M. A., Zeb, S. and Pop, I.(2001), Int. J. Appl. Mech & Eng., 2, pp 473.

Callahan, G. D. and Marner, W. J. (1976), Int. J. Heat and Mass Trans., pp 19,165.

Chamkha, A. J., Rahim, A. and Khalid, A.(2001), Heat and Mass trans., 37 pp 117.

Chamkha, A. J., Rahim, A.A. and Khalid, A., (2001) Similarity solutions for hydromagnetic simultaneous heat and mass transfer by natural convection from an inclined plate with integral heat generation or absorption., Heat and Mass Transfer, 37 pp 117-123.

Chen, T.S. and Yuh, C.F.(1980), Int. J. Heat mass Trans., 23 pp 527.

Clarke J.F. and Riley N.,(1975) Natural convection induced in a gas by the presence of a hot porous horizontal surface, Q.J.Mech. Appl. Math., vol. 28, pp. 373-396,

Clarke J.F. and Riley N.,(1976) Free convection and the burning of a horizontal fuel surface, J. Fluid Mech., vol. 74, pp. 415-431

Eckert and Soehngen (1948) Studies on heat transfer in laminar free convection with Zehnden-Mech interferometer. Tech. Rept 5747, ATI 44580, Air Materiel Command (Dayton, Ohio).

Ekambvannan, K and Ganesan, P.(1994), Warme Stoffubert.,30, pp 63.

Elliot, L. (1970) Free convection on a two-dimensional or axisymmetric body. Quarterly Journal of Mechanics and Applied Mathematics 23: pp 153-62.

Ganapathy, R (1997) Time dependent free convection motion and heat transfer in an infinite porous medium induced by a heated sphere. Int. J. Heat Mass Transfer. Vol. 40, No. 7, pp 1551-1557.

Ganesan, P. and Ekambvannan, K.(1992), Indian J. Techn., 30 pp 471.

Ganesan, P. and Palani, G.(2003), Heat and Mass transfer, 39 pp 277.

Ganesan, P.and Palani, G., (2003) Free convection effects on the flow of water at 4^{0}C past a semi-infinite inclined plate., Heat and mass Transfer, 39 pp 785-789.

Gill, W.N.,Zeh,D.W.and Del-Casele, E (1965), Free convection on a horizontal plate., Z.angew. Math. Phys, 16 pp. 532

Gupta A.S.,(1961) Steady and transient free convection of an electrically conducting fluid form a vertical plate in the presence of magnetic field. Applied Scientific Research., Vol. 9A, pp. 319-333

Harvet and Blay (1999), Natural convection over a non-isothermal vertical plate, Int. J. heat and mass Transfer , 42 pp. 3103

Hasan, M., Ramadan, Chamkha, A.J.(2003), Heat and mass transfer, 39 pp 367.

Hossain, M. A., Pop, I. and ahmad, M.(1996), J. Theoritical Appi. Fluid Mech., 1, pp 194.

Jha, B.K.(1991) MHD unsteady mixed convection flow through a porous medium. Astrophys. Space Sci., 175, 101.

Jones, D.R.(1973), Q.J. Mech. Appl. Math., 26 pp 77.

K.A.Helmy, MHD(1998) unsteady free convection flow past a vertical porous plate, ZAMM Vol 78(4), pp 225-270.

Karkus, W.T. (1968), Natural convection flow adjacent to inclined isothermal and finite-length surfaces, Int. J. Mass Transfer, 11 pp. 241

Lin H.T. and Yu W.S.(1988) Free convection on a horizontal plate with blowing and suction, J. Heat Trans., ASME, Vol.110, pp. 793-796

M.Acharya, G.C.Dash, and L.P.Singh,(2000) Magnetic field effects on the free convection and mass transfer flow through porous medium with constant suction and constant heat flux, Indian J. pure appl.Math.,31(1), pp 1-18

Merkin (1989), Free convection on a heated vertical plate: the solution of small Prandtl number, J. Eng. Math. 23, pp. 273

Merkin (1996), Conjugate free convection on a vertical wavy flux. Int. J. heat mass transfer, 39,7, pp. 1527

Merkin J.H. and Mahmood T.,(1990) On the free convection boundary layer on a vertical plate with prescribed surface heat flux, J. Engg. Math, Vol. 24, pp. 95-107

Nachtsheim, P.R. and Swigert, P.(1965) Satisfaction of the asymptotic boundary conditions in numerical solution of the system of non-linear equations of boundary layer type. NASA TN-D3004.

Osterle J.F. and Yound F.J.,(1961) Natural convection between heated vertical plates in horizontal magnetic fields. J., Fluid Mech., Vol 11, No.4, pp. 512

Ostrach, S. (1953) An analysis of laminar free-convection and heat transfer about a flat plate parallel to direction of the generating body force., NACA Tech.

Ostrach, S. (1953) New aspects of natural convection heat transfer. Trans. ASME 75, 1287.

Pera, L. and Gebhart, B.(1972), Natural convection boundary layer flow over horizontal and slightly inclined. Int. J. Mass Transfer, 16 pp. 1131.

Poots G.,(1961) Laminar natural convection flow in magneto hydrodynamics, International Journal of Heat and Mass Transfer, Vol.3, No.1, pp. 1-25

Ramadan, H. M. and Chamkha, A. J.(2003), Int. J. Heat Mass Transfer, 39 pp 367.

Sattar, M.A., and Alam, M.M. (1994) Thermal diffusion as well as transpiration effects on MHD free convection and mass transfer flow past an accelerated vertical porous plate Indian Journal of Pure and Applied Mathematics 25, pp 679-688.

Sattar, M.A., Samad, M. A. and Kalim, M.H.(1997) Unsteady MHD forced and free convection flow through a porous with constant heat source and variable suction. Dhaka Univ. J. Sci., 45(2), pp 205.

Satter, M.A. and Maleque, M.A.(2000) Unsteady MHD natural convection flow along an accelerated porous plate with hall current and mass transfer in a rotating porous medium, J. Energy, Heat and Mass Transfer, 22, pp 67-72.

Schmidt, E., and Bechmann, W. (1930) Das Temperature-und Geschwidigkeitsfeld vor einer Warme abgebenden senkrechter platte bei naturlicher convection. Tech. Mech. U. Termodynamik, Bd. I, (10), pp 341-349.

Schneider, W. (1979) A similarity solution for combined forced and free convection flow over a horizontal plate. Int. J. Heat and Mass Transfer, 22, pp 1401.

Schuh, H. (1948) Boundary Layer of temperature. Gt. Brit. Ministry of Air Production Volkenrode Repts. And Transls, pp 1007.

Soundalgekar, V. M., and Ganesan, P., (1985) Finite difference analysis of transient free convection with mass transfer on an isothermal vertical flat plate., Int. J. Engg Sci. 19, pp 757-770.

Soundalgekar, V.M. (1972) Viscous dissipation effects on unsteady free convective flow past an infinite, vertical porous plate with constant suction. Int. J. Heat Mass Transfer, Vol. 15, pp 1253-1261.

Stewartson, K.(1958), on free convection from a horizontal plate. Z. angew. Math. Phys. 9a, pp. 276.

Umemura, a. and Law, C.K.(1990), Natural convection boundary flow over a heated plate with arbitary inclination. J. Fluid Mecha, 219 pp. 571.

Vedhanayagam M., Altenkirch R.A. and Eichhorn R., (1980) A transformation of the boundary layer equations for free convection past a vertical flat plate with arbitrary blowing and wall temperature variations, International Journal of Heat and Mass Transfer, Vol. 23, pp. 1286-1288,

Williams, J.C., Mulligan, J.C., and Rhyne, T.B. (1987) Semi-similar solutions for unsteady free-convection boundary layer flow on a vertical that plate. Journal Fluid Mechanics 175, pp 309-332.

Yang, K.T. (1960) Possible similarity solutions for laminar free convection on vertical plates and cylinders. J. Appl. Mech. pp 27, 230.

Youn. J. Kim,(2000) Unsteady MHD convective heat transfer past a semi-infinite vertical porous moving plate with variable suction, Int. J. Eng. Sci, Vol 38, pp 833-845.

Yu, W.S. and Lin H. T.(1998), Warme-und Stoffubertr., 23 pp 203.