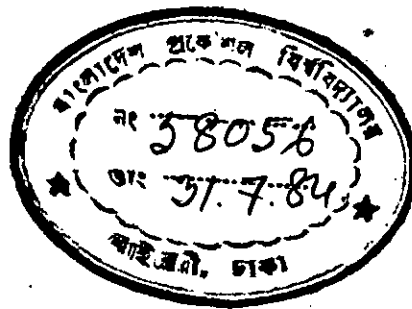


EFFECT OF PARTICLE SHAPE AND ANGULARITY ON ENGINEERING  
PROPERTIES OF NATURAL GRANULAR SOIL DEPOSITS OF BANGLADESH

A Thesis  
by  
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Submitted to the Department of Civil Engineering,  
Bangladesh University of Engineering & Technology, Dhaka  
in partial fulfilment of the requirements for the degree  
of  
MASTER OF SCIENCE IN CIVIL ENGINEERING



July, 1984



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ABSTRACT

Granular soils were collected from the river beds of five different locations of Bangladesh. The locations are Dhaka, Mymensingh, Jamalpur, Rangpur and Rajshahi and the rivers include Meghna, Brahmaputra, Teesta and Padma.

The particle characteristics designated by sphericity, elongation and flakiness were determined for each soil. These were taken as measures of particle shape and angularity. Shear strength and permeability were also determined for each of these soils at three different void ratios.

From the analysis of the results it was found that the shear strength decreases with increasing particle sphericity and increases with increasing elongation and flakiness. But the permeability increases with sphericity and decreases with elongation and flakiness. From the test results it was possible to formulate a model for the determination of peak angle of shearing resistance from the known values of void ratio and particle characteristics.

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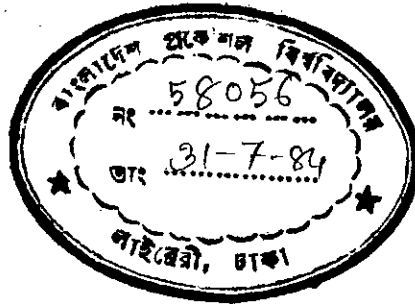
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Chapter 1  
INTRODUCTION



1.1 General

Every structure, whether it be a building, a bridge or a dam derives its support from the soil. Texturally soils fall into two categories - granular and cohesive. A granular media may be considered to consist of more or less rounded particles of sand or gravel which interact in a simple way by their contact forces only. This soil usually maintains single-grained structure and unlike the second group does not possess any cohesion.

The lower size range of the granular media is sand. According to MIT classification system the grain size of sands ranges from 0.06 mm to 2.00 mm. Very often it is required to place foundation on natural sand deposits, that is deposits of sand formed by the natural processes.

Among the engineering properties, the one that plays the most important role in fixing the design criteria for foundations on sand is its shear strength. The shear strength of sand in particular and granular soil in general, is mainly determined by the parameter  $\phi$ , the angle of shearing resistance.

Porosity is frequently used as an index property of granular soils. This may suggest a priori that two samples having the same void ratio should have the same mechanical behaviour. No doubt, porosity of granular materials is an

important variable which characterizes the over all deformation behaviour but it alone is not sufficient. The arrangement of grains, that is fabric, should also have an influence. The fabric of granular media should include a measure of geometrical arrangement of the individual particles and an adequate description to reflect mutual positions of the grains. Even if the stress history and the history of deposition are disregarded the fabric will greatly be influenced by the particle size, shape and angularity.

The grain size and shape of a granular media will depend on the degree of weathering to which they have been subjected during their formation from firm bed rock and on the wear and tear to which they have been subjected during the process of erosion, transportation and deposition. The weathering and erosional processes are largely dependent on the climatic conditions. Since it is unlikely that climatic conditions of two geographical area will be same, therefore the grain size, shape and angularity of soils from two different climates are likely to be different even if the bed rock from which they are produced is similar.

Some works have been done in various parts of the world to establish the effect of grain size, shape and angularity on engineering properties of granular soils. It is now recognised that the above mentioned grain characteristics have important bearing on their engineering behaviour. Unfortunately no study to relate above behaviour is known to the

author for soils of Bangladesh. For this reason this research makes an attempt to relate the effect of grain shape and angularity on some of the engineering properties such as shear strength and permeability of sands from recent alluvium of Bangladesh. For the above purpose soil samples were collected from the river beds of five selected locations.

### 1.2 Grain Size, Shape and Angularity

The most direct method of determining the grain size distribution of granular soils is by sieving. A representative sample of the material is placed at the top of a nest of sieves of successively smaller openings and the system is shaken so that the particles may pass through the openings large enough to permit their passage. A particle diameter, called the equivalent diameter, is generally taken to be equivalent to the minimum square aperture through which the particle will pass. The purpose of particle size determination is to determine the frequency distribution of the various constituent sizes within a sample of material.

In the previous paragraph, particle shape was tacitly assumed as a sphere the diameter of which is equal to the sieve opening through which it passed. Implicit in this concept was a correspondence between the true shape of a particle and that of an equivalent but imaginary sphere, with its diameter taken as the aperture of the minimum sieve

through which the particle will pass. This definition of the shape of a particle is adequate if measured distances between extreme limits on the surface of the particles, along lines that pass through the particles' center of gravity, do not vary greatly. The variation in the distances from the imaginary spherical diameter is usually reckoned as angularity.

It is a common practice to indicate particle shape and angularity by a variety of terms like sphericity, flakiness, elongation and by the roundness of the particle. A well known fact is that a particle of a given volume attains the minimum volume when it is spherical, and sphericity is taken as the ratio of the surface area of a sphere with the same volume as the particle to the surface area of the particle. Flakiness is the ratio of width to the thickness of the particle and elongation is the ratio of the length with that of the width. Roundness, on the contrary, is the ratio of the radius of the angular projections of the particle to that with the radius of the inscribed circle on the projected plane when the particle is in its most stable position. For this research work particle sphericity, elongation and flakiness will be considered to be a measure of shape and angularity.

### 1.3 The Recent Alluvium of Bangladesh

Since the soil samples for this research have been collected from the river beds of the recent alluvium of

Bangladesh a description of the recent alluvium is being presented below.

Bangladesh consists primarily of a large alluvium basin floored with Quaternary sediments deposited by the Ganges and Brahmaputra rivers and their numerous associated streams and distributaries. The Rajmahal Hills which border the country on the northwest and west are composed of trap and are considered to be lower Jurassic. The hills and the trap plateaus range from 500 to 800 feet above sea level, although some individual hills exceed 1500 feet in elevation.

On the northeast it is bounded by the Shillong or Assam plateau. The plateau measures about 60 miles from north to south, about 150 miles from east to west, and is surrounded on the north, west, and south by low river basins. The basins eastern limits are the Tripura Hills to the north and the Chittagong Hills to the south. These hills are composed of sediments of Paleocene through Pliocene age.

The land-building process of Bangladesh must have been due to the sediments washed down from the high lands on the three sides of it. In broad terms, there are three major geological formations in Bangladesh. These formations are important in relation to soils. With Tertiary hill sediments in the northern and eastern hills, Madhupur Clay of the Madhupur and Barind tracts in the center and west, the remainder of the country areas are occupied by the recent alluvium in the floodplains and estuaries.

The tertiary hill sediments comprise mainly of unconsolidated and little consolidated beds of sandstones, siltsstones, shales and some conglomerates. They have been folded into a succession of pitching anticlines and synclines. These are aligned approximately north-northwest to south-southwest in Chittagong and the south of Sylhet district, swinging round to almost east-west in the north of Sylhet and Mymensingh districts.

The formation of the Madhupur Clay underlies the Madhupur and Barind tracts. It possibly occurs also on the Akhaura terrace and on the summit of the Lalmai Hills in the east of Comilla. The formation is remarkably homogeneous in appearance, both vertically and laterally. It comprises of a layer of unconsolidated clay, about 25 feet thick near Dhaka but progressively thinner to the east and possibly much thicker in the west of Rajshahi district. The formation is generally almost horizontal but has broken into a number of fault blocks.

The unconsolidated floodplain sediments of the recent alluvium occupy the greater part of the country. These sediments are far from homogeneous in age, texture and mineralogy. They have been deposited under peidmont, meander floodplain, estuarine and tidal conditions in different areas. New alluvium is still being deposited near active river channels, but most floodplain land has apparently received little or no new alluvium for several hundred years or more. Rivers have changed

their courses from time to time in the past, abandoning and re-occupying various parts of their floodplains, and thus providing sediments of different ages in different areas. Some floodplain areas have also been uplifted or downwarped by earth movements, specially in Sylhet and Mymensingh districts and there are numerous sand filled earthquake fissures in parts of these districts.

Most floodplain sediments have a high silt contents. This is particularly so in the case of Brahmaputra/Jamuna and Meghna sediments. Sandy sediments occur extensively in the substratum of soils in the north of Teesta floodplain and in the west of Ganges floodplain. Clay deposits occur on the surface over most of the Ganges floodplain. And the beds of flowing rivers usually accommodate sand during seasons of high currents and silt or even clay when the current recedes. It is the sand deposits collected from five river beds that have been used in the present study.

#### 1.4 Area of the Research

The misleading concept that granular soils interact in a simple way by their contact forces only has led to the use of reconstituted samples. The logic behind the use of reconstituted samples is that void ratio is the only factor that influences the engineering properties of granular materials. It is undoubtedly an important factor but the factors like stress history, history of deposition, and



above all fabric should not be ignored. The basic objective of this research is to study the influence of particle shape and angularity on engineering properties of sands of Bangladesh. As particle shape and angularity may affect a number of engineering characteristics like strength, deformation and permeability. This research is limited to determination of particle characteristics and their effect on shear strength and permeability at various void ratio conditions. The result thus obtained is expected to provide a better understanding of the fundamentals of deformation behaviour of sands and their permeability characteristics.

## Chapter 2

### LITERATURE REVIEW

#### 2.1 Introduction

Because of its particulate nature the mechanical behaviour of a soil mass is influenced by a number of factors. It is difficult to take into account all the factors. This research aims at evaluating the effect of particle shape and angularity on two engineering properties - the shear strength and the permeability of some selected local granular soils. A brief review of literature on various aspects of this research is presented in the subsequent articles.

#### 2.2 Particle Shape and Angularity

Any method of quantifying particle shape and angularity is complicated due to its complex three-dimensional morphology and can only be approximated. Various methods are in use to quantify particle shape and angularity but unfortunately such measures are not sufficiently broad or general to include all aspects of the said properties.

E. Frossard (1982) quoted the reference of Whalley (1972) who pointed out that if the overall three-dimensional morphology of a particle is considered as its form  $F$ , it can be functionally be defined in terms of several attributes as follows:

$$F = f(S_h, S_p, A_f, R_f, T_f) \quad \dots \quad (2.1)$$

where  $S_h$  is the shape,  $S_p$  the sphericity,  $A_f$  the angularity,  $R_f$  the roundness and  $T_f$  the surface texture. Size is usually taken to be independent of form. The topological relationships between these components are complex and rather arbitrary and such a division in the past has been largely one of geological pragmatism. Shape and sphericity are related as are roundness and angularity.

Perhaps Wadell (1932) was the first researcher to define particle shape and angularity in numerical terms. He proposed that the definition of shape should be the ratio of the surface area of a sphere equal in volume to the particle considered to the actual surface area of the particle. The ratio has a maximum value of 1.0 which is the value when the particle is a sphere. All other shapes have value less than 1.0. This ratio is termed as sphericity and has been found to be suitable for expressing shapes of sedimentary particles. The term roundness was also used by him to indicate the angularity of the particle's corners. Degree of roundness of a particle in one plane may be expressed as

$$\frac{\sum\left(\frac{r}{R}\right)}{N}$$

where  $r$  is the radius of curvature of the corners,  $R$  is the radius of the maximum inscribed circle in the plane of measurement and  $N$  is the number of corners. Here a corner is defined as every such part of the outline of an area (projected area) which has a radius of curvature equal to or less

than the radius of curvature of the maximum inscribed circle. Finally Wadell (1932) concluded that sphericity expresses the shape and the roundness a summarized expression for certain detail characteristic of the solid. Sphericity and roundness together expresses the image of the solid and for convenience may be expressed in the form shown below:

$$\frac{\text{Roundness}}{\text{Sphericity}} = \text{Image of Solid.}$$

The double line separating the values is used to indicate that the image expression is not a ratio.

Krumbein (1941) described a rapid but rigorous method for the measurement of shape and roundness of sedimentary particles. His method is based on the measurement of long, intermediate and short diameters of the particles and on the use of the formula:

$$\left(\frac{b}{a}\right)^2 = \frac{\psi^3}{(c/b)} \quad \dots \quad (2.2)$$

where  $\psi$  is the sphericity;  $a$ ,  $b$  and  $c$  are long, intermediate and short diameters respectively. This method lends itself to graphical presentation. Krumbein actually presented a series of curves for sphericity with  $b/a$  as ordinate and  $c/b$  as abscissa. The curves are shown in Fig. 2.1. He also made the determination of roundness even simpler by providing a chart as shown in Fig. 2.2 for visual comparison.

Flatness ratio, the ratio of short diameter to intermediate diameter, was used by Rittenhouse (1943) to define

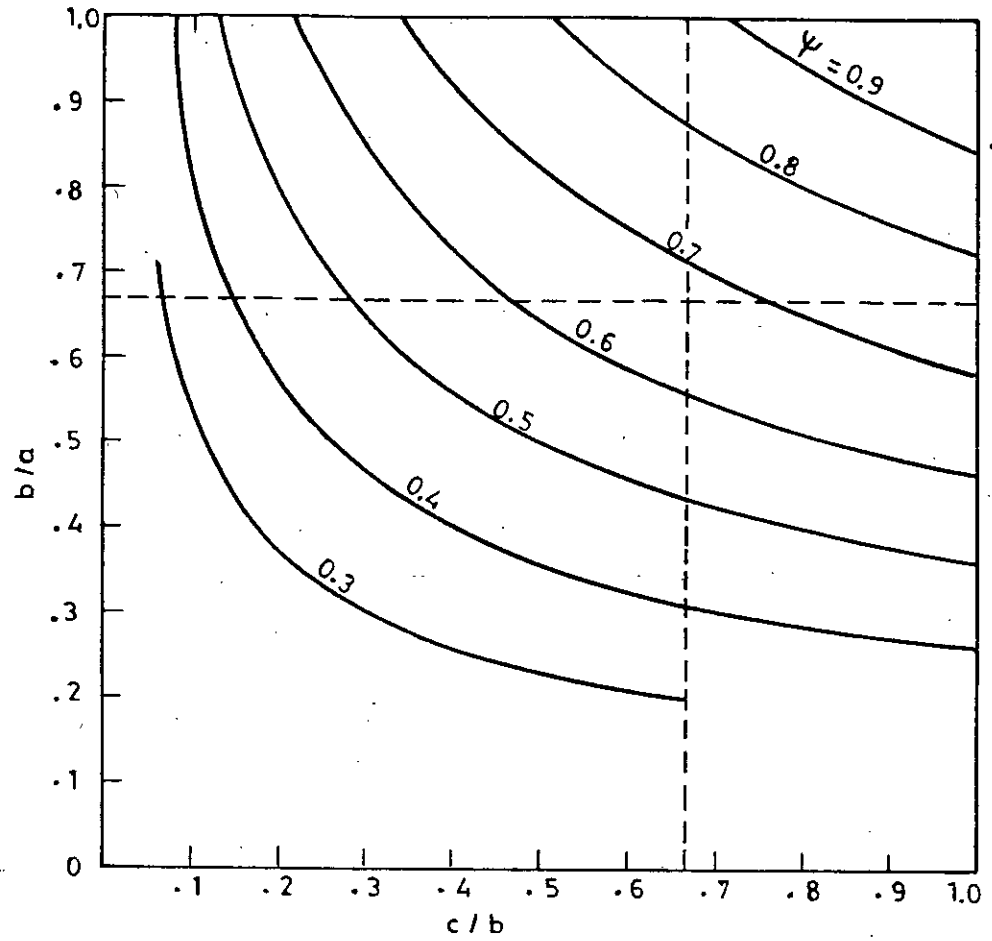


Fig.2.1: Krumbein's chart for determining sphericity  
(After Krumbein, 1941).

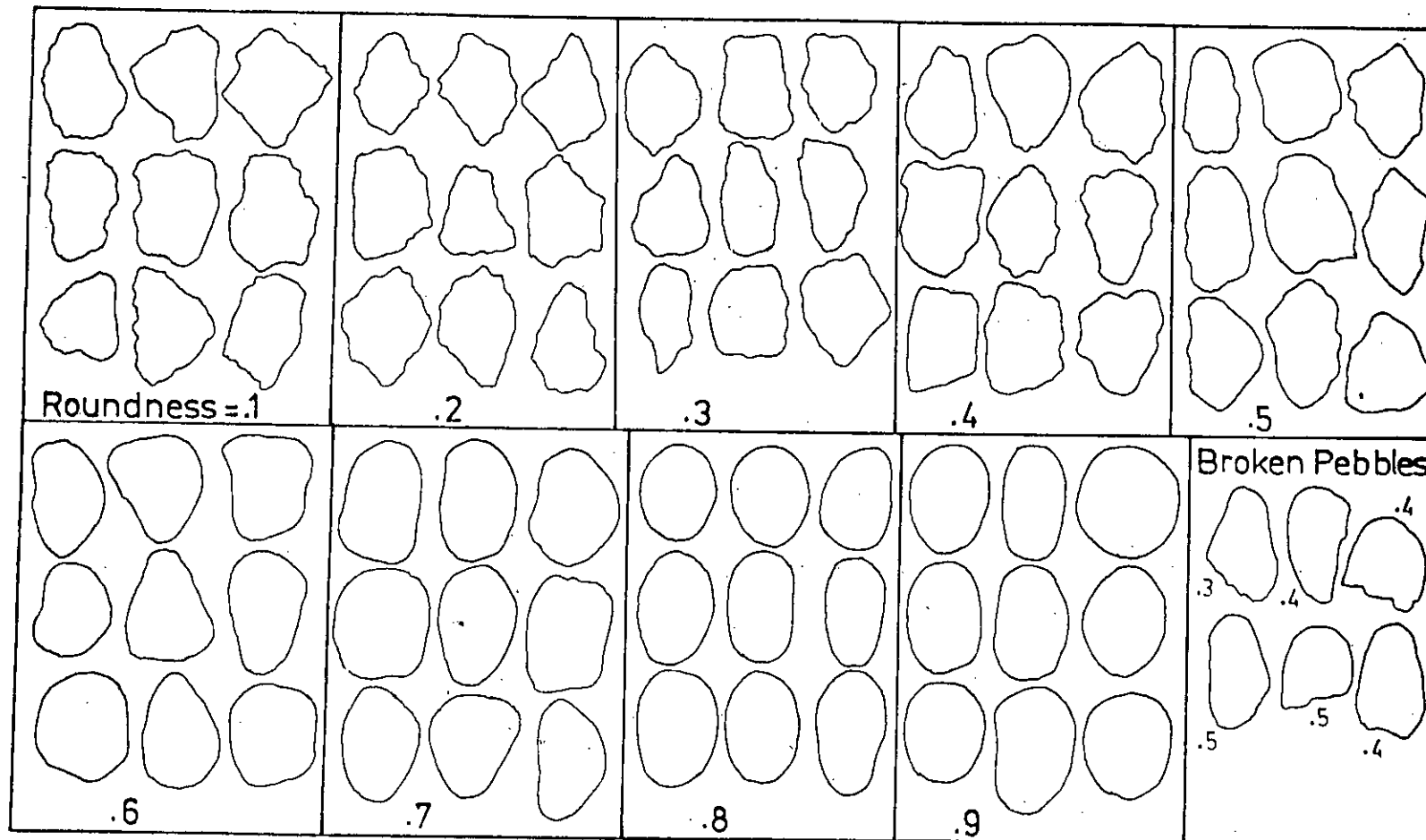


Fig.2.2: Pebble Image for Visual Roundness  
for 16-32 mm. Pebbles. (After Krumbein, 1941)

the shape of soil particles. He took the average of the opening of two sieves which defines the boundary of the particle size as intermediate diameter and correlated it with flatness ratio as in Fig. 2.3. The figure provides the variation of flatness ratio with average intermediate diameter for four size grades. The size grades are 0.351 to 0.246 mm for A, 0.246 to 0.175 mm for B, 0.175 to 0.124 mm for C and 0.124 to 0.088 mm for D.

Due to inadequacy of the existing methods for describing shape Lees (1964) developed a new non-empirical method for measuring the angularity of particles. The method takes into account three main characteristics of angularity, namely the degree of acuteness of the angles of the corners, the number of angular corners and the degree of projection of the corners from the main mass of the particles. He also criticised Wadell's sphericity as being inadequate to depict particle shape without the use of the ratio of flatness to elongation.

Lambe (1969) considered particles in the silt size range or coarser to be nearly equidimensional as spheres and based on this reasoning he suggested to specify particles of this size range by a single parameter called equivalent diameter.

Ehrlich and Weinberg (1970) developed method for characterization of grain shape. This method yields a mathematical model of the grain that will regenerate the

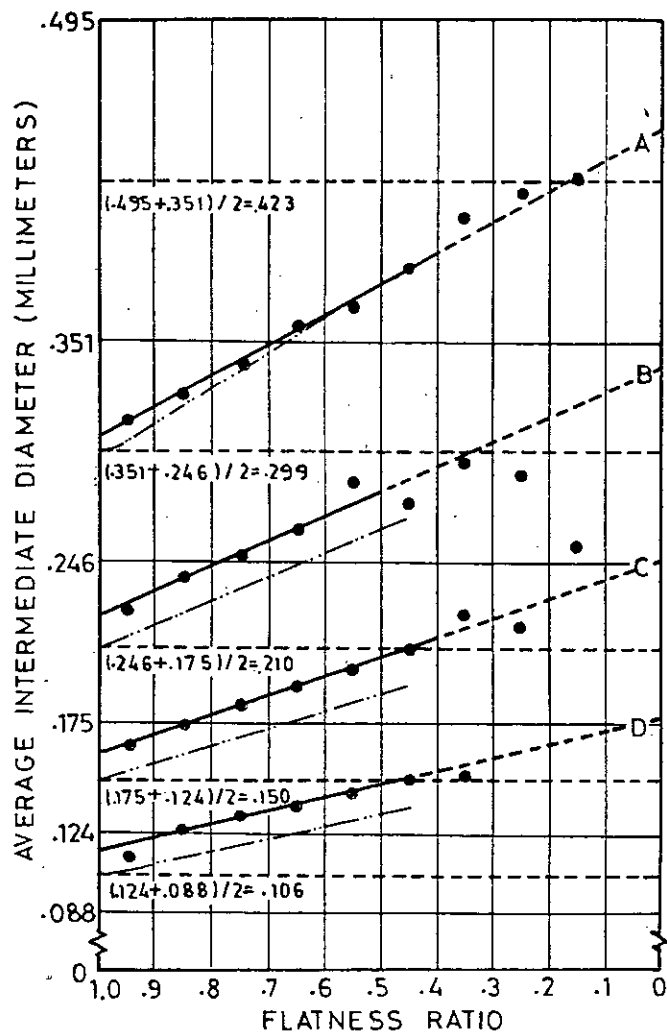


Fig. 2.3 : Relation of flatness ratio, measured intermediate diameter, and arithmetic mean intermediate diameter.

A, B, C and D are for 0.351-to 0.246-mm., 0.246-to 0.175-mm., 0.175-to 0.124-mm. and 0.124-to 0.088 mm. size grades, respectively.

( After Rittenhouse, 1943 )



grain shape as precisely as required. In addition, by representing grain shape as a linear equation with an indefinite number of terms, with each term representing the contribution of a known shape component, shape and changes in shape can be readily analysed and monitored. In this method grain shape is estimated by an expansion of the periphery radius as a function of angle about the grains center of gravity by the fourier series:

$$R(\theta) = R_0 + \sum_{n=1}^{\infty} R_n \cos(n\theta - \phi_n) \quad \dots \quad (2.3)$$

where  $\theta$  is the polar angle measured from an arbitrary reference line. The first term in the series,  $R_0$ , is equivalent to the average radius of the grain in the plane of interest. For the remainder of the terms,  $n$  is the harmonic order,  $R_n$  is the harmonic amplitude and  $\phi_n$  is the phase angle.

Heywood (1947) used the word shape to indicate two distinct characteristics of particles (Allen, 1975). These two characteristics would be the degree to which a particle approaches a definite form such as cube, tetrahedron or sphere and relative proportions of the particle to which group each of these particles belong spheriod from another of the same class. With these two points in mind he forwarded a listing of shape factors to be used in defining the shape of a particle. He also proposed the use of the following ratios in cases where three mutually perpendicular

dimensions of a particle can be determined:

$$\text{elongation ratio, } \epsilon = \frac{L}{B} \quad \dots \quad (2.4)$$

$$\text{flakiness ratio, } f = \frac{B}{T}$$

where T is the thickness, B is the breadth and L is the length of the particle.

### 2.3 The Strength of Soil

The crushing strength of a soil particle is generally very high compared with the normal stress in the soil mass. Changes in normal stress therefore cause only limited volume change due to rearrangements of particles. However, very large deformations may result from sliding at the particle surfaces as a result of applied shear stresses which exceed the shearing resistance on these surfaces. The shear strength of a soil is the resistance to such shear deformations on a potential rupture surface within the soil mass.

The shear strength of any soil material depends on the way in which the failure is defined. The usual form of the shear-stress-deformation curve for a soil is shown in Fig. 2.4. In the case of loosely packed sands and some normally consolidated clays, the maximum and residual values of the shear strength are generally the same and the only question in practice is whether one can tolerate the large deformations which may occur before the maximum shearing resistance is developed.

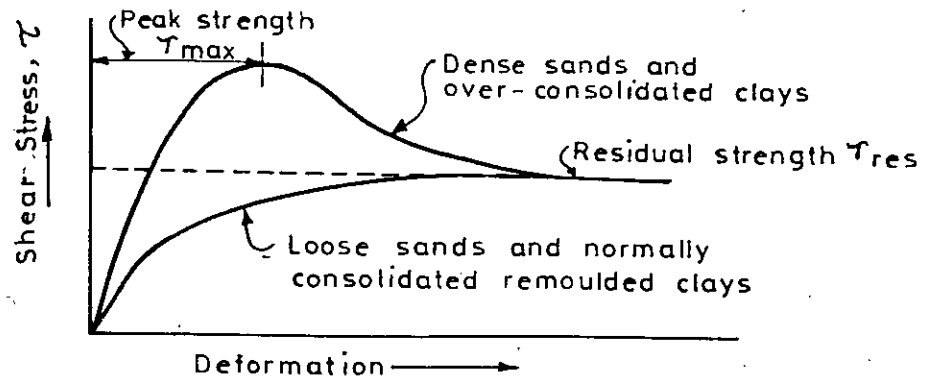


Fig. 2.4: Typical shear stress/deformation curves for soils

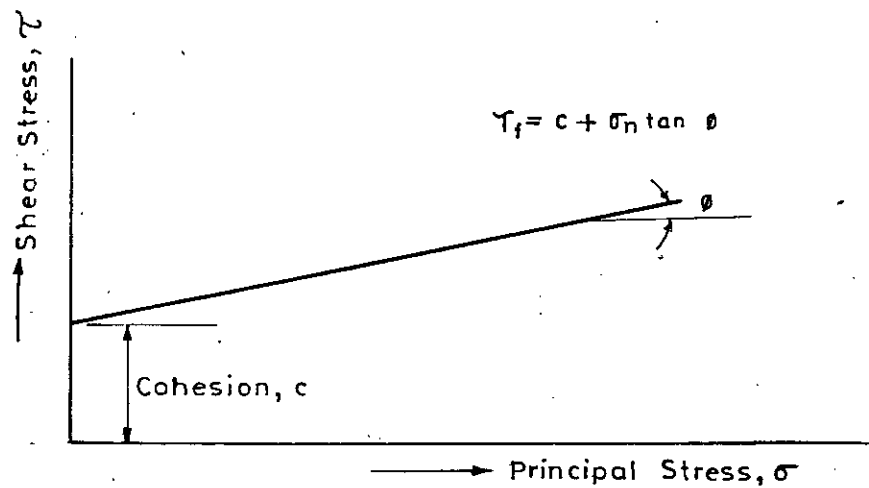


Fig. 2.5: Coulomb's failure theory

In densely packed sands and over consolidated clays the peak shear strength may be considerably greater than the residual value which develops after a large deformation. The peak shear strength however develops at small deformation. In considering the stability of an earth structure therefore it may not be reasonable to assume that the maximum shear strength is developed everywhere simultaneously. The question whether to base the calculations on the peak or on the residual strength or on some intermediate value is an important one. The shear strength is conventionally designated either by the peak or by the residual value.

For stress-strain curves obtained from triaxial tests Henkel (1959) defined 'failure' as the condition existing when the deviator stress reaches a maximum. This is not a general failure criterion since the deviator stress is a function of pore pressure as well as soil properties. A more fundamental definition of failure should be the stress state when the ratio of principal stresses is a maximum

Scott, R.F. (1963) defined failure from a simplified model of cohesionless soil consisting of rigid spheres in hexagonal packing as the condition when the obliquity of the resultant force on the surface of sliding becomes equal to a constant value. Here obliquity is the angle that the resultant force makes with the normal on the shearing plane.

For soils in general the resistance to sliding at the particle surfaces is partly frictional and for granular soils it is mostly frictional in nature.

In the derivation of Coulomb's failure theory, for the frictional component of resistance, it is assumed that the laws of friction apply. Two laws of friction have been recognised. These were first stated by Leonardo da Vinci in about 1500 and then were restated by Amantons in 1699 and are frequently referred to as Amanton's Laws (Rosenak, 1963; Mitchell, 1976). They are:

- i) the frictional force is directly proportional to normal force and
- ii) the frictional resistance between two bodies is independent of the size of the two bodies.

Coulomb first suggested that the shear strength of a soil might be expressed in the form

$$\tau_f = C + \sigma_n \tan \phi \quad \dots \quad (2.6)$$

where  $\tau_f$  is the shear strength of the soil,  
 $\sigma_n$  is the normal stress on the failure surface,  
 $C, \phi$  are parameters which are approximately constant  
 for a particular soil.

A graphical representation of equation (2.6) is shown in Fig. 2.5.

Mohr's theory (Scott, C.R., 1969) predicts that failure will take place when the major and minor principal stresses are related by some function of the form:

$$(\sigma_1 - \sigma_3) = f(\sigma_1 + \sigma_3)$$

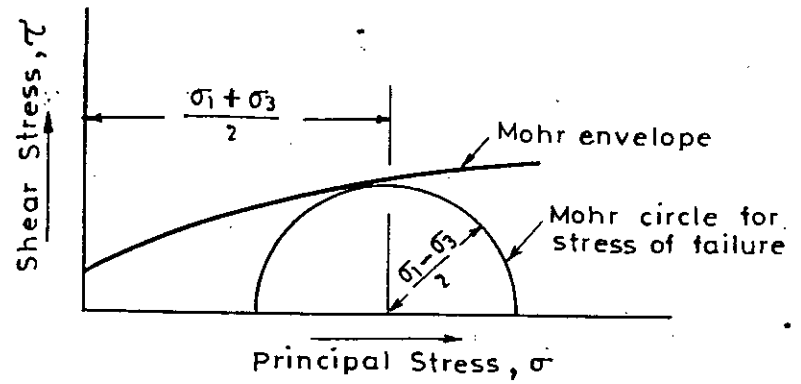


Fig. 2.6: Mohr's failure theory.

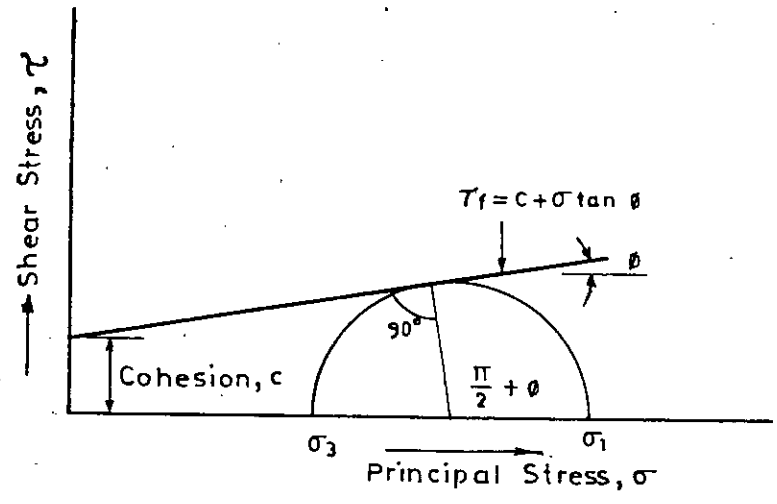


Fig. 2.7: The Mohr - Coulomb failure theory.

with  $\sigma_1$  as the major principal stress and  $\sigma_3$  as the minor principal stress.

At failure the radius of the Mohr's stress circle is some function of the mean principal stress  $(\sigma_1 + \sigma_3)/2$ . A series of such circles at various failure condition may be connected by a line tangent to all the circles. Such a line is known as Mohr's envelope. Fig. 2.6 shows such an envelope. In practice the Mohr's envelope of a soil is commonly found to be approximately straight over a considerable range of normal stress and may be expressed in the same form as Coulomb's equation for shear strength. A close approximation of Coulomb's equation may be taken to be identical with the Mohr envelope and the combination of these two failure conditions is known as the Mohr-Coulomb failure theory. This is depicted in Fig. 2.7.

Using Thurston and Deresiewicz's (1959) analysis Scott, R.F. (1963) mathematically showed that for an ideal packing of equal spheres under triaxial compression the failure can uniquely be defined by Mohr-Coulomb theory. Mitchell (1964) from his study of shearing resistance as a rate process derived an equation for failure which closely resembles the Mohr-Coulomb equation.

#### 2.4 Angle of Shearing Resistance for Granular Soils

The non-existence of cohesion in granular soils does not necessarily mean that the strength of such soils can be accounted

for entirely in terms of intergrain sliding friction. The peak angle of shearing resistance  $\phi_m$  can be represented as the sum of three contributions (Rowe, 1962): the frictional resistance at contacts, the particle rearrangements and the dilation. In this connection dilation may be defined as the volume change of the soil mass during shearing. Breakdown of various frictional components of  $\phi$  is shown in Fig. 2.8. In this figure  $\phi_\mu$  is the component due to true friction or sliding between grains,  $\phi_f$  is the component corrected for dilation,  $\phi_m$  is the peak value of  $\phi$  and  $\phi_{cv}$  is the angle of shearing resistance at constant volume deformation.

At the lowest porosities, the peak strength is reached before significant interparticle movement can develop and thus for this porosity the work for rearrangement is small. Failure requires volume expansion against the confining stress giving the large dilation contribution as shown. If the confining stress is very high there will be less dilation but more grain crushing to accommodate the shear deformations. At higher porosities some rearrangements develop prior to failure as particles roll and slide along planes inclined at various angles.

The critical void ratio shown in Fig. 2.8 represents a condition where failure occurs at constant volume. In this case no work is required to produce dilation, and  $\phi_m$  is composed only of  $\phi_\mu$  and particle rearrangements. Fig. 2.8 provides a graphic illustration of the mechanism of  $\phi_m$  mobilization and shows clearly the component of strength at different porosities.



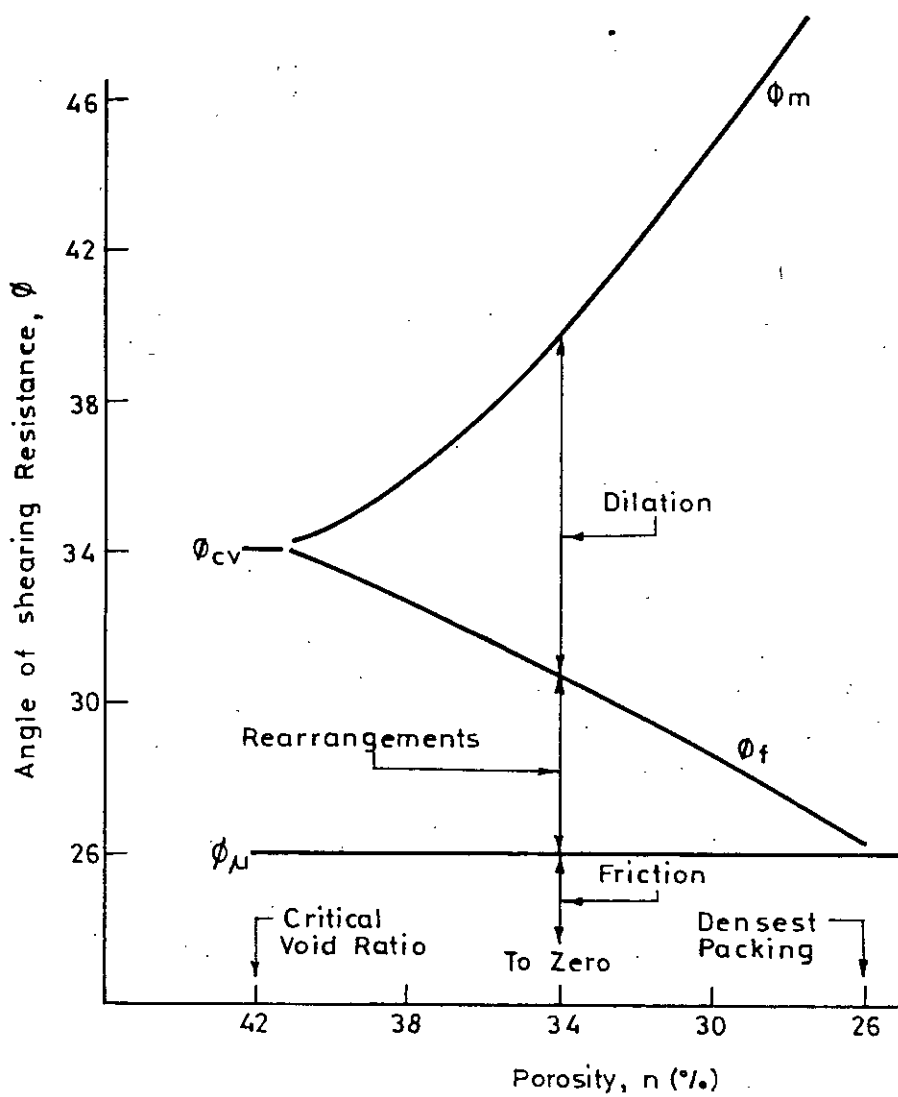


Fig. 2.8: Components of shear strength in granular soils  
( After Rowe, 1962 )

## 2.5 Particle Shape and Angularity as Contributing Factors to the Shear Strength of Granular Soil

Although in soil mechanics very little efforts have been made to express particle shape and angularity in numerical terms, the effect of these two factors is well recognised qualitatively. Terzaghi and Peck (1967) pointed out that for granular soils the grain size distribution and shape of the grains have an influence on the angle of shearing resistance,  $\phi$ . Representative values for  $\phi$  as quoted by them are shown in Table 2.1.

Lambe and Whitman (1969) discussed critically the factors affecting the strength of a cohesionless soil. As regards angularity they suggested that angular particles would interlock more thoroughly than rounded particles and hence sands composed of angular particles would have larger angle of shearing resistance. The data of peak angle of shearing resistance presented in Table 2.2 confirm this prediction. Even when a sand is strained to its ultimate condition that is when no further volume change takes place and that the sand is in a loose condition, the sand with angular particles has the greater angle of shearing resistance.

Koerner (1970) reported that with increase in particle angularity and decrease in sphericity,  $\phi_m$  and  $\phi_f$  increases. He defined sphericity as the ratio of projected particle area to area of smallest circumscribing sphere and used it as a measure of particle shape.

Table 2.1: Representative values of  $\phi$  for sands and silts  
(After Terzaghi and Peck, 1967)

Material	Angle of shearing resistance	
	Loose	Dense
1	2	3
Sand, round grains, uniform	27.5°	34°
Sand, angular grains, well graded	33.0°	45°
Sandy gravels	35.0°	50°
Silty sand	27.0°-33.0°	30°-34°
Inorganic silt	27.0°-30.0°	30°-35°

Table 2.2: Effect of angularity and grading on angle of shearing resistance (After Sowers and Sowers, 1961).

Shape and Grading	Angle of shearing resistance	
	Loose	Dense
1	2	3
Rounded, uniform	30.0°	37°
Rounded, well graded	34.0°	40°
Angular, uniform	35.0°	43°
Angular, well graded	39.0°	45°

Determining sphericity according to Rittenhouse classification method (1943) and roundness according to Krumbein's method (1941) and using these values in the analysis of test results Bros (1979) found that in general the angle of shearing resistance decreases with increasing value of sphericity and roundness.

Although not dealing with soils Eerola and Ylosjoki's (1970) working on the effect of particle shape on the friction angle of coarse-grained aggregates projects an idea on the effect of particle shape and angularity on shear strength of granular soils. Their test results indicate that particle shape has a distinct effect upon the value of the angle of shearing resistance. They found that at a porosity of 26% the angle  $\phi$  of elongated Macadam and rounded gravel differed from each other by approximately 13 degrees. Fig. 2.9 shows their experimental results showing the effect of angularity on friction angle.

For the low strength of graded aggregate Gur, Shklarsky and Livneh (1967) reasoned that comparatively large surface area of the flaky coarse fraction of the aggregate and their tendency to horizontal alignment favour higher resistance to mutual vertical displacement that is reduced packability. As a result excessive flakiness causes low density and therefore reduction in strength.

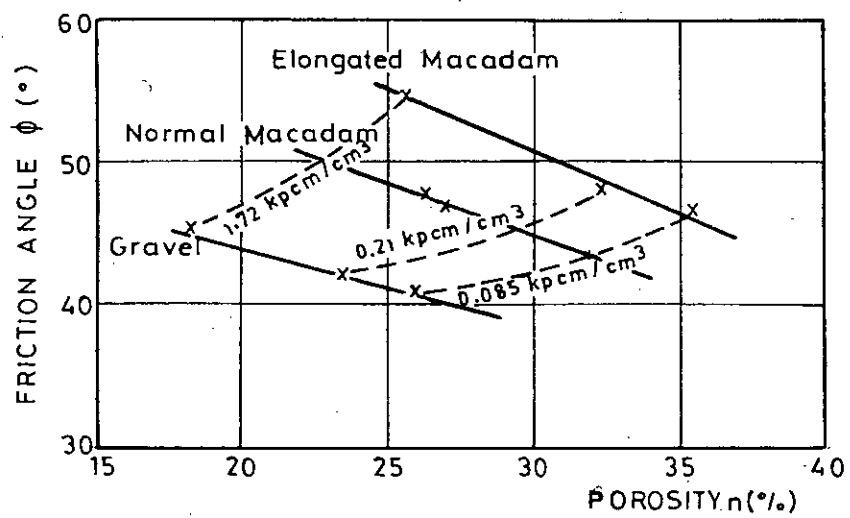


Fig. 2.9: Effect of particle shape and porosity on friction angle. Dotted curves are from same compacting work.

( After Eerola, 1970 )

## 2.6 Permeability

Permeability is the capacity of a soil to conduct or discharge water under a given hydraulic gradient. The coarse grained soils are considered highly pervious and therefore this type of soil has high permeability. With fine grained soils the situation is reversed.

About one hundred and twenty five years ago Darcy (1856) becoming interested in the problem of flow through porous media devised an experiment which gave a very simple result. The experiment illustrated in Fig. 2.10 showed that volume of flow  $Q$  through the filter bed in time  $t$  was directly proportional to the cross-sectional area  $A$  and to the difference in piezometric levels  $\Delta h$  and inversely proportional to the length  $l$  between the piezometers. Expressed mathematically,

$$\frac{Q}{t} \propto \frac{A\Delta h}{l}$$

$$\text{or } q = kA \frac{\Delta h}{l}$$

$$\text{or } \frac{q}{A} = k \frac{\Delta h}{l}$$

$$v = ki \quad \dots \quad (2.7)$$

where  $v$  = total volume flow rate per unit of total cross-sectional area perpendicular to the direction of macroscopic flow, commonly called discharge or approach velocity,

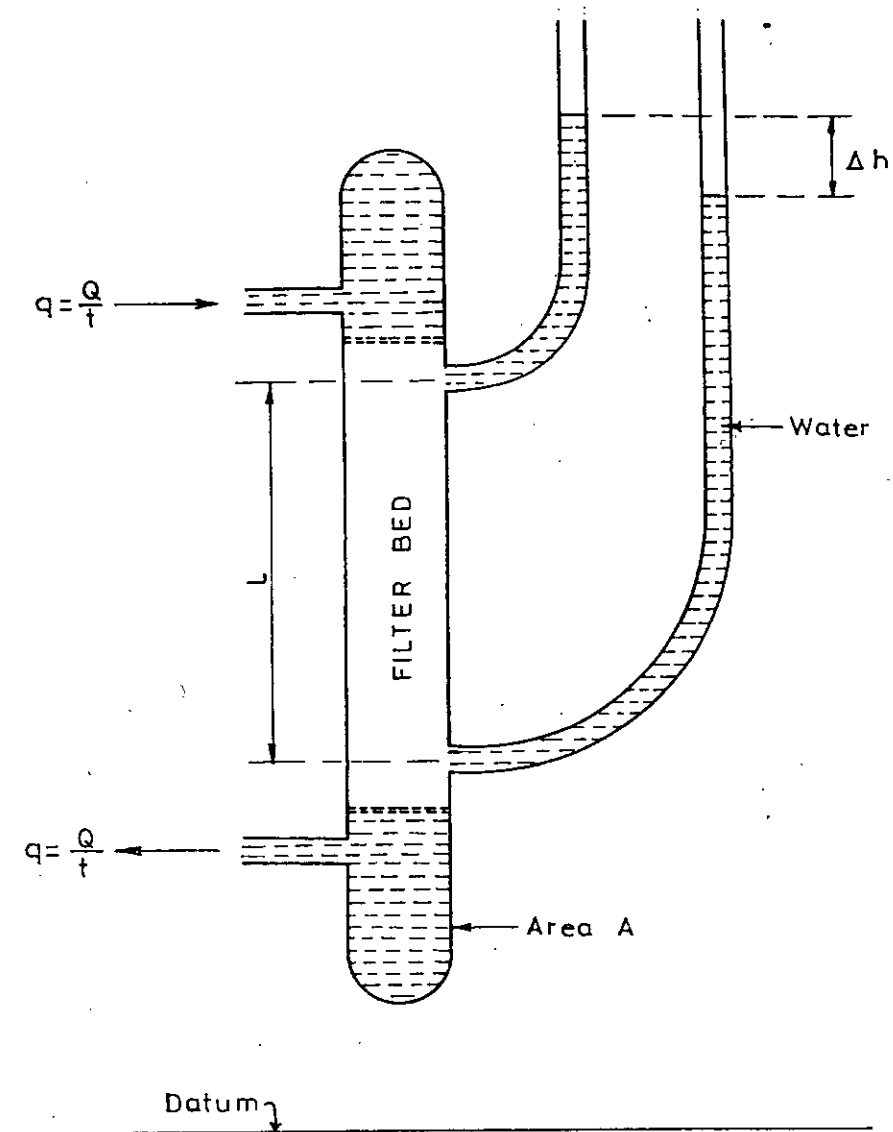


Fig. 2.10: Schematic diagram of Darcy's sand filtration experiment (after Leonards, 1962)

$i$  = total head lost per unit length of macroscopic flow path, called hydraulic gradient, and

$k$  = Darcy's constant of permeability.

This constant  $k$  is a coefficient and considered as a measure of permeability of a porous media.

## 2.7 Permeability Prediction

Darcy's law was the result of simple experimental observations. Many attempts have been made to achieve a theoretical relation between permeability and grain characteristics of the soil mass and its porosity. Many of these relations have been derived from the Hagen-Poiseuille (Hagen, G. 1839; Poiseuille, J. 1841) equation for viscous flow through a small capillary tube of radius  $r$  given by the expression:

$$q = \frac{\pi \gamma s}{8\eta} r^4 \quad \dots \quad (2.8)$$

where  $q$  = flow rate

$\gamma$  = unit weight of the fluid

$\eta$  = viscosity of the fluid, and

$s$  = hydraulic gradient under which flow takes place.

A model may then be considered to consist of a bundle of capillary tubes arranged parallel to each other with a ratio of pore cross-section to total area equal to porosity of the soil  $n$ . On the basis of this parallel-tube capillary model permeability may be expressed as



$$k = C_s r^2 \frac{\gamma}{\eta} n \quad \dots \quad (2.9)$$

where  $k$  = Darcy's permeability constant

$C_s$  = shape factor which takes into account the variation  
in shape of the capillary

$r$  = radius of the capillary tube

$\eta$  = viscosity of flowing fluid, and

$\gamma$  = unit weight of the fluid.

Although the pore size may be related to the dimensions of the grains composing the soil and that a collection of grains can be arranged in various assemblages containing different pore sizes there can be no unique relationship between grain size and pore size in a soil mass. Therefore, in order to correlate between model and actual soil condition an average pore diameter should be used. This average diameter concept as used by Kozeny is a function of hydraulic radius, that is the ratio between flow area in a capillary and its wetted perimeter. When mathematically represented the permeability equation on the basis of the above concept becomes:

$$k = C_s \frac{\gamma}{\eta} \frac{n^3}{S_v^2} \quad \dots \quad (2.10)$$

where  $k$  = Darcy's permeability constant

$C_s$  = shape factor that takes into account the  
variability in pore shape

$\gamma$  = unit weight of fluid

$\eta$  = viscosity of fluid

$n$  = porosity, and

$S_v$  = surface area per unit of total volume.

Apart from these theoretical models empirical formulae are also available which use simple grain parameters to determine permeability. Hazen (1892) gave such a formula for permeability as

$$k = C D_{10}^2 \quad \dots \quad (2.11)$$

Here  $D_{10}$  is defined as that grain diameter at which 10 percent of the material is of smaller diameter. The constant  $C$  is approximately equal to 100 for filter sands of uniform grain sizes. For other types of soils  $C$  is reported to vary between 41 and 146 (Taylor, 1948). A particular value to be used for any soil should be determined experimentally.

Slichter (1899) derived an equation involving uniform spheres of diameter  $d$  and the empirical constant  $c$  as

$$k = \frac{771d^2}{c} \quad \dots \quad (2.12)$$

Terzaghi (1925) used an empirical relation between porosity  $n$ , viscosity of water  $\eta$ , the constant  $C$ , effective grain size  $D_{10}$  and permeability as

$$k = \frac{C}{\eta} \left[ \frac{n-0.13}{\sqrt[3]{1-n}} \right]^2 D_{10}^2 \quad \dots \quad (2.13)$$

where the parameter  $C/\eta$  varies from 800 for rounded sands to 460 for angular sands.

## 2.8 Factors Affecting Permeability of Soils

The flow of water in soils is laminar in nature. Poiseuille's equation for the laminar flow through a round capillary tube may be written as

$$q = \frac{\pi r^4 \gamma_w}{8\eta} i \quad \dots \quad (2.14)$$

where  $q$  is the rate of flow,  $r$  is the radius of the tube,  $\gamma_w$  is the unit weight of water,  $i$  is the hydraulic gradient under which flow takes place and  $\eta$  is the viscosity of water. If the area of the tube  $\pi r^2$  is represented by 'a' and the hydraulic mean radius of the tube, defined as the ratio of area to wetted perimeter, by  $m$ , that is

$$\frac{\pi r^2}{2\pi r} = \frac{r}{2} = m, \text{ then the equation (2.14)}$$

may be reduced to

$$q = \frac{1}{2} m^2 a \frac{\gamma_w}{\eta} i \quad \dots \quad (2.15A)$$

If the cross-section of the tube through which the water flows is not circular, it can be inferred that the rate of flow will be similar to the equation (2.15A), the only variation will occur in the numerical coefficient on the right handside. For tube cross-section of any type the rate of flow may be given by,

$$q = C_s m^2 a \frac{\gamma_w}{\eta} i \quad \dots \quad (2.15B)$$

where  $C_s$  is the coefficient depending upon the type or shape of the cross-section and hence may be called the shape factor.

Flow through soils is much more complicated than it is through regular tubes of any cross-sectional shape. In the first place the path of flow in soils is zigzag depending upon the configuration of particles in a soil mass, compared to much smoother path in tubes or pipes and secondly the concept of mean hydraulic radius for soils is based upon some hypothetical grain diameter. However the preceding equation can be used to indicate factors on which the coefficient of permeability depends.

In equation 2.15B 'a' represents the opening of the pipe. In the case of soils, the water flows through the pores and if  $A$  is the total cross-sectional area of a soil section as used in the Darcy's equation, the area of voids  $A_v$  is given by  $nA$  with  $n$  being the porosity of the soil mass. The term  $nA$  is the same as the area 'a' of the pipe. The porosity  $n$  can be replaced by  $\frac{e}{1+e}A$  that is 'a'. Also the hydraulic mean radius  $m$  for pipes is the ratio of area to the wetted perimeter  $p$  that is  $\frac{a}{p}$  where 'a' denotes the cross-sectional area of the pipe and  $p$  its perimeter. If the numerator and denominator in the ratio  $\frac{a}{p}$  are multiplied by the length  $l$  of the pipes, the hydraulic mean radius may be defined as the ratio of the volume of the flow channel to the surface area. This definition can be used to derive an expression for hydraulic mean radius.

The volume of flow channel in soils may be taken as the pore volume or the volume of voids which is equal to  $eV_s$  where  $e$  is the void ratio and  $V_s$  is the volume of solids. The surface area  $A_s$  for the soil grains may be worked out on the basis of a hypothetical spherical grain diameter  $d$  having the same volume to surface area ratio of the soil grains collectively.

$$\text{So for soils } m = \frac{eV_s}{A_s} = e \frac{\frac{\pi d^3}{6}}{\pi d^2} = \frac{1}{6} de \quad \dots \quad (2.16)$$

Further the coefficient  $C_s$  may be replaced by a more general coefficient  $C_1$  for soils to take into account the factor  $C_s$  and any other shape, grain size and configuration effects of the soil particles.

Substituting the values of 'a' and  $m$  as found for soils and replacing  $C_s$  by  $C_1$  the following may be obtained:

$$\begin{aligned} q &= C_1 \left(\frac{1}{6} de\right)^2 \left(\frac{e}{1+e} A\right) \frac{\gamma_w}{\eta} i \\ &= \left(\frac{C_1}{6^2} d^2 \frac{e^3}{1+e} A\right) \frac{\gamma_w}{\eta} i \\ &= \left(C_2 d^2 \frac{e^3}{1+e} \frac{\gamma_w}{\eta}\right) i A \quad \dots \quad (2.17) \end{aligned}$$

where  $C_2$  absorbs the fraction  $\frac{C_1}{6^2}$ . Comparing this equation with Darcy's law

$$k = C_2 d^2 \frac{e^3}{1+e} \frac{\gamma_w}{\eta} \quad \dots \quad (2.18)$$

This equation gives a clear indication of the factors on which the permeability of a soil mass depends.

The permeability of a soil is a function of the shape of the soil pores which depends on the shape and size of the soil particles and their arrangements in the mass. In other words, permeability depends upon the structure of the soil mass represented by the factor  $C_2$ . For example results reported by Bros (1979) are shown in Table 2.3 for coefficient of permeability at maximum dry density obtained by Standard Proctor compaction for different types of sand grains. It is evident from the data that permeability in general increases with particle roundness and sphericity.

In equation (2.18) the value  $d$ , though a hypothetical grain diameter, is a measure of the grain size of the soil. It can be concluded therefore that the value of  $k$  depends upon the second power of some measures of the grain size of the soil. Fig. 2.11 collected from Loudon (1952) shows relationship between permeability and Hazen's effective grain diameter.

The factor  $C_2$  in equation (2.18) depends upon the shape of void space in the soil and consequently may be considered to depend to some extent on the void ratio. Fig. 2.12 and Fig. 2.13 collected from the work of Loudon (1952) depict the dependence of permeability on the porosity for various types of granular material.

The term  $\frac{\gamma_w}{\eta}$  in the equation (2.18) represent the properties of pore water. Both of these quantities change with

Table 2.3: Effect of particle shape on engineering properties of sands (After Bros, 1979).

Type of sand	Roundness	Sphericity	Permeability		Shear strength	
			Test void ratio	k in cm/s	Initial void ratio	Angle of shearing resistance
1	2	3	4	5	6	7
Sand 1	0.1	0.67-0.69	0.643	$5.00 \times 10^{-5}$	0.728	43.0°
Sand 2	0.1-0.2	0.69	0.588	$2.99 \times 10^{-6}$	0.642	40.0°
Sand 3	0.1-0.2	0.75	0.451	$5.19 \times 10^{-6}$	0.463	41.5°
Sand 4	0.5-0.6	0.83-0.85	0.595	$9.41 \times 10^{-4}$	0.648	38.5°
Sand 5	0.6	0.81	0.610	$1.06 \times 10^{-2}$	0.656	37.0°
Sand 6	0.6-0.7	0.87	0.608	$5.44 \times 10^{-3}$	0.653	37.5°
Sand 7	0.5-0.6	0.77	0.652	$2.06 \times 10^{-3}$	0.656	37.5°
Sand 8	0.8-0.95	0.87	0.720	$1.83 \times 10^{-3}$	0.758	36.0°
Sand 9	1.0	1.00	0.691	$6.45 \times 10^{-3}$	0.757	29.5°

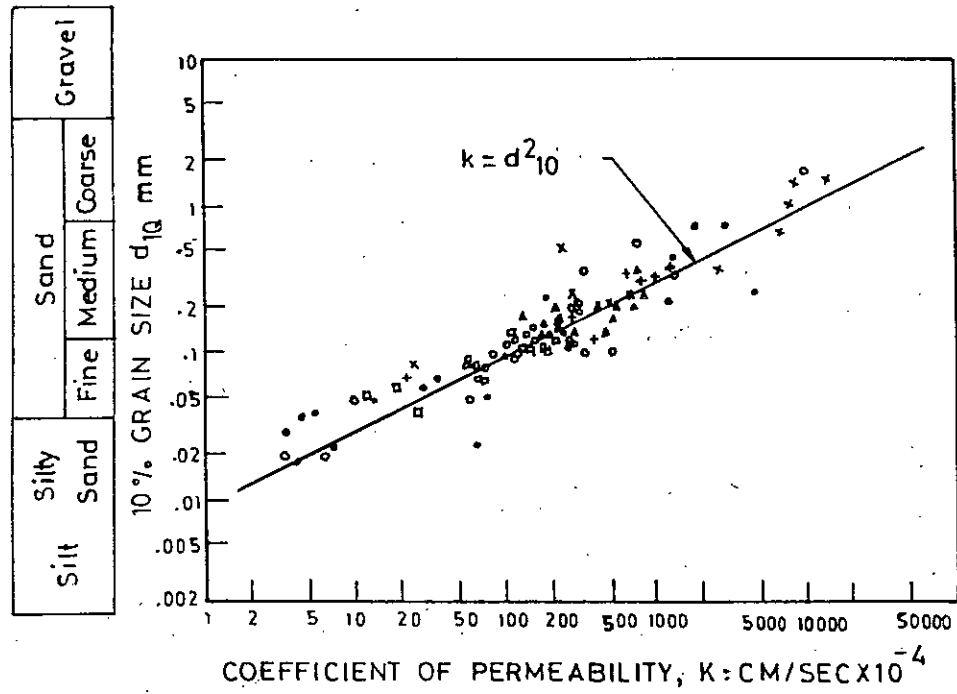


Fig. 2.11: Data from various published papers showing relation between permeability and Hazen's effective size ( after Loudon, 1952 )



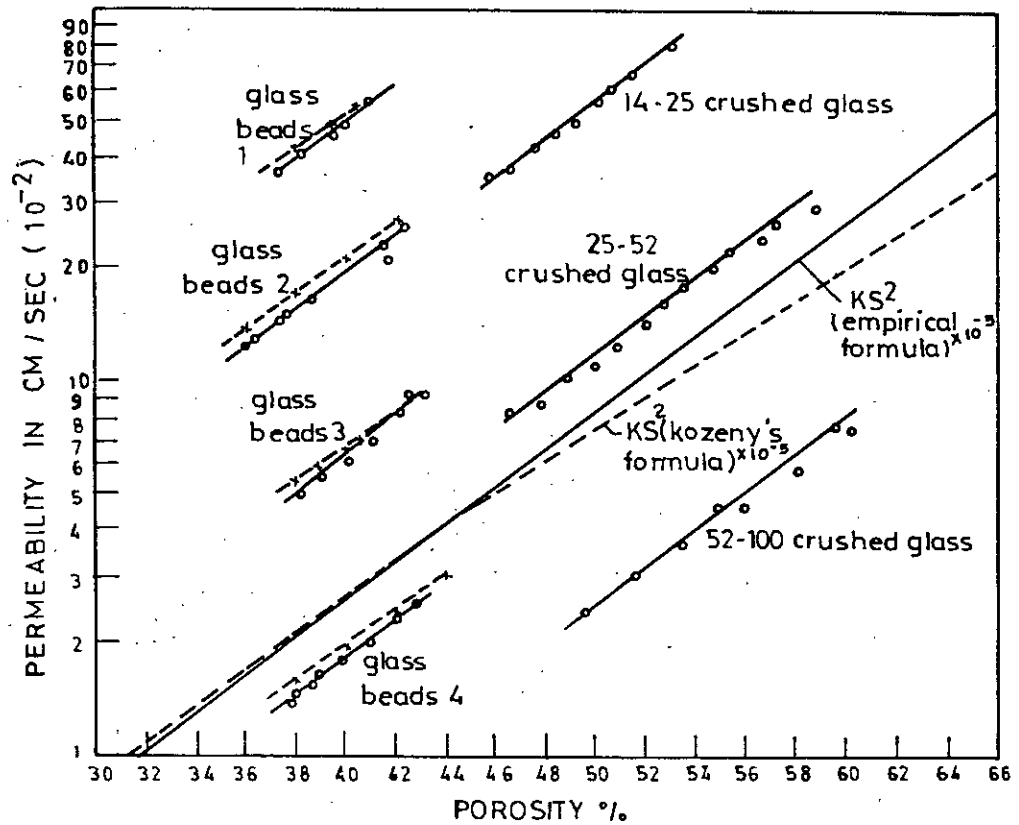


Fig. 2.12: Permeability of glass beads and crushed pyrex plotted against porosity. Broken lines calculated from Kozeny's formula. (after Loudon, 1952)

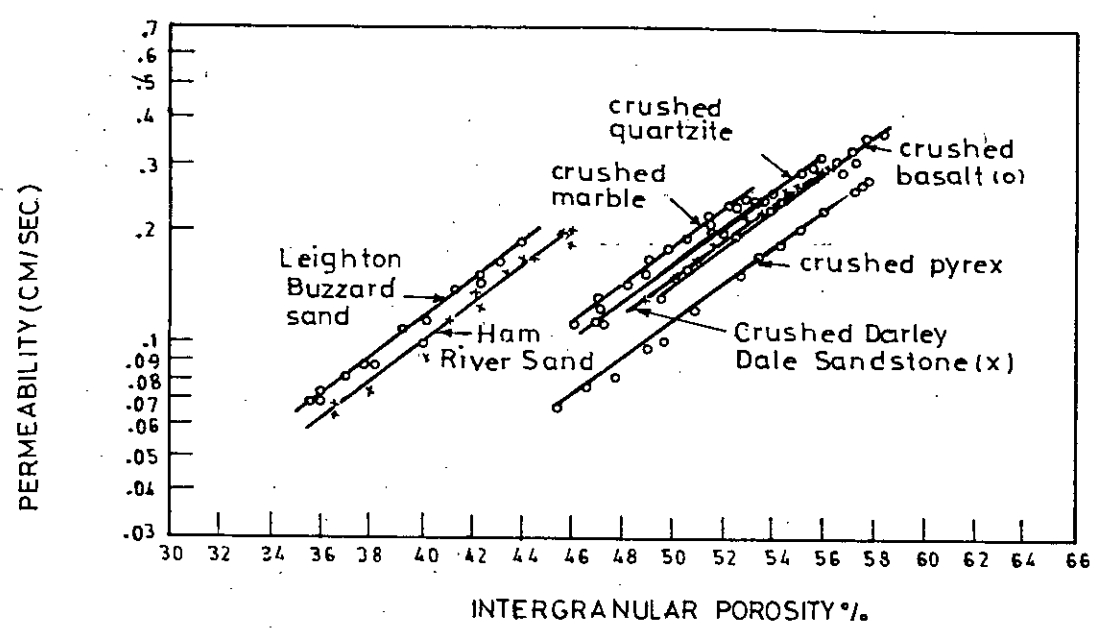


Fig. 2:13: Permeability of sands of grading 25-52 plotted against intergranular porosity (after Loudon, 1952)

temperature. However, it may be noted that the unit weight of water does not change with temperature as much as does the viscosity.

Finally air trapped in the voids can obstruct the free flow of water in the soil mass. As such the presence of air or degree of unsaturation affects the value of permeability.

## 2.9 Salient Features of Literature Review

The review of literature reveals that any method of quantifying particle shape and angularity is at best an approximation of the complex three dimensional morphology of sand grains. Although various methods are available to express grain shape and angularity in numerical terms none seem to be completely adequate in all respect.

Among many factors void ratio is considered to be an important one that influences shear strength. From microscopic point of view it is the fabric that is most important. For a particular soil with the same void ratio the strength may vary depending upon the particle arrangement or fabric. The fabric depends upon the particle size, shape and angularity. It is found that with the increase of angularity the shear strength increases.

Permeability is influenced by the shape of pores in the soil mass, viscosity of the fluid which is flowing through the soil and by the temperature. Here the shape of the pores

and the void ratio is determined by the grain size, shape and angularity. It has been found that larger the grain size the greater the permeability. It has also been established by experiments that a soil mass with spherical particles shows higher permeability than soils with angular grains.

An examination of the published work for granular soils of Bangladesh revealed that no information is available to relate the effect of particle shape and angularity on engineering properties of these local soils.

## Chapter 3

### THE RESEARCH SCHEME

#### 3.1 Introduction

It has been referred earlier that a number of definitions of grain shape and angularity exists. This chapter specifies the definitions adopted for this research and also describes the objectives of the research.

#### 3.2 Definitions Adopted for Particle Shape and Angularity

Although particle shape and angularity can be described by a variety of terms, it is more common to use terms like sphericity, elongation and flakiness. Sphericity is usually taken as a measure of particle shape. It can also be used to describe angularity. Allen (1975) defined the inverse of sphericity as angularity.

In this thesis sphericity has been used as a ratio of the surface area of a sphere equivalent in volume to that of the particle to its actual surface area. This definition of sphericity was suggested by Wadell (1932). Flakiness has been used as the ratio of width to the thickness of the particle and elongation as the ratio of the length of the particle to its width. In this connection length is taken as the largest dimension of the particle while width as the largest dimension in any plane perpendicular to the plane containing the length. Finally thickness is defined as the volume of the particle divided by its length and width.

### 3.3 Objective of the Research

From the literature review it is apparent that little published informations are available to account for the effect of particle shape and angularity on engineering properties of sands. This research work has been undertaken to achieve the following objectives:

- i) To determine the shape and angularity characteristics as defined in the preceeding article of a number of selected local soils.
- ii) To evaluate the effect of particle characteristics on shear strength of the soils at various void ratios.
- iii) To determine the effect of particle shape and angularity on permeability of the soils at three different porosities.

### 3.4 Collection of Soil Samples

A number of soil samples were collected from the river beds at different locations in Bangladesh. From a preliminary study of shape and angularity characteristics of these samples five samples were selected for further study. The reason for selecting these five soil samples were that they varied markedly in their individual grain characteristics. These selected soils were from the river beds of Meghna at Dhaka, Padma at Rajshahi, Teesta at Ranpur, Old Brahmaputra at Jamalpur and Mymensingh.

### 3.5 The Test Program

Grain size analysis and specific gravity determinations were made on representative fractions of all the soil samples. From the soils analysed for size distribution a representative portion obtained by successive fractioning was examined under microscope for the determination of particle length and width. Average weight of a particle was computed from the collective weight of a known number of particles. These quantities formed the basis for the determination of particle sphericity, elongation and flakiness.

Subsequent to the determination of grain characteristics direct shear and permeability tests were accomplished on representative samples of each of the collected soils. Both the shear and permeability tests were carried out at three void ratios. For direct shear test three normal loads were applied at each void ratio.

## Chapter 4

### DETERMINATION OF GRAIN CHARACTERISTICS

and

### ENGINEERING PROPERTIES OF THE SOILS

#### 4.1 Introduction

Determination of surface area is quite complicated although equipments are available for the purpose. Due to the nonavailability of any such equipment direct measurement of surface area of soil particles could not be made. Consequently a simple idealization of the actual shape of the particles was assumed. It was assumed that the soil particles are rectangular parallelepipeds having the same volume as the particle's actual volume. The following articles describe methods for determining the shape characteristics based on the above idealization and also the methods for determining their engineering properties.

#### 4.2 Measurement and Computation of Grain Characteristics

The grain size analysis was performed by following the ASTM standard (D 422, 1972). After the grain size analysis the fractions of dry soils retained on each sieve were preserved separately. The specific gravity of the soil solids were determined by the method suggested in ASTM D 854, 1972.

A maximum of four hundred oven dried soil particles were counted for each preserved fractions and then weighed by a sensitive balance capable of reading upto four decimal



positions of a gram. Dividing this weight by the number of particles and their specific gravity, the average volume of a particle was obtained. If  $W$  be the weight of  $N$  particles and  $G_s$  be their specific gravity, then  $V$ , the average volume of particle was obtained as:

$$V = (W/N)/G_s \quad \dots \quad (4.1)$$

From the above expression it is apparent that the radius  $R_1$  of a sphere having the same volume as  $V$  can be expressed as:

$$R_1 = (3V/4\pi)^{1/3} \quad \dots \quad (4.2)$$

Following the computation of the equivalent spherical radius ten grains were picked randomly from each preserved soil fraction. The length  $L$  and width  $B$  of these ten particles were measured directly by a microscope fitted with a sliding micrometer by a procedure described below:

- i) Sand particles were placed on a glass slide. It was observed that sand grains rested on its most stable position.
- ii) The length of the particle was measured from the longest dimension of the image of the particle when examined under the microscope and when it was free to rest on the glass slide.
- iii) The width was taken as the distance between two extreme points on the boundary of the image perpendicular to the direction of length measured.

iv) Thickness  $T$  was then computed by the following equation:

$$T = \frac{V}{LB} \quad \dots \quad (4.3)$$

where  $V$  is the volume of the particle determined by equation 4.1,  $L$  and  $B$  are its measured length and width respectively.

v)  $L$ ,  $B$  and  $T$  were then averaged for the ten particles.

From the measured values of  $L$ ,  $B$  and computed values of  $T$  and  $R_1$  the particle shape and angularity in terms of sphericity, elongation and flakiness were computed by using the following expressions:

$$\text{Sphericity, } \psi = \frac{4\pi R_1^2}{2(LB+BT+TL)} \quad \dots \quad (4.4)$$

$$\text{Elongation, } \epsilon = \frac{L}{B} \quad \dots \quad (4.5)$$

$$\text{Flakiness, } f = \frac{B}{T} \quad \dots \quad (4.6)$$

The values thus obtained represented a particular sieve fraction only. To make them representative of the whole soil sample the weighted average of all the soil fractions were calculated by the expressions given below:

$$\psi_{\text{ave}} = \frac{w_1 \psi_1 + w_2 \psi_2 + w_3 \psi_3 + \dots + w_n \psi_n}{w_1 + w_2 + w_3 + \dots + w_n} \quad (4.7)$$

$$\epsilon_{ave} = \frac{w_1 \epsilon_1 + w_2 \epsilon_2 + w_3 \epsilon_3 + \dots + w_n \epsilon_n}{w_1 + w_2 + w_3 + \dots + w_n} \quad (4.8)$$

$$f_{ave} = \frac{w_1 f_1 + w_2 f_2 + w_3 f_3 + \dots + w_n f_n}{w_1 + w_2 + w_3 + \dots + w_n} \quad (4.9)$$

where  $w$  subscripted by 1,2,3... n indicate the weight of fractions of the soil retained on each sieve. The  $\psi$ ,  $\epsilon$ , and  $f$  are the corresponding values of sphericity, elongation and flakiness respectively. Here  $\psi_{ave}$ ,  $\epsilon_{ave}$ , and  $f_{ave}$  are the weighted average sphericity, elongation and flakiness respectively for the particular soil sample.

#### 4.3 Determination of Shear Strength

The engineering properties determined in this research include shear strength and permeability. Direct shear tests were performed to determine the shear strength. Fig. 4.1 presents the schematic diagram of direct shear test apparatus. The shear box used was circular in shape with a diameter of 6.35 cm. The initial thickness of the soil specimen in the shear box was always maintained at 2 cm. The shearing speed was 0.12 cm/minute. Soil used in the test was air dried. The test void ratios were 0.63, 0.76 and 0.81 respectively. For each void ratio three normal loads of 0.25 kg/cm<sup>2</sup>, 0.50 kg/cm<sup>2</sup> and 1.00 kg/cm<sup>2</sup> were used. The test procedures described below were followed:

**LEGEND**

- (a) - Dial gauge to measure vertical movement and to observe consolidation for "consolidated" test
- (b) - Lateral deformation measuring gauge
- (c) - Set screws to fix load head into position
- (d) - Serrated edges to hold sample
- (e) - Set screws to separate shear box
- (f) - Loading bar
- (g) - Alignment pins
- (h) - Load head
- (i) - Soil sample
- ( $P_h$ ) - Shear load

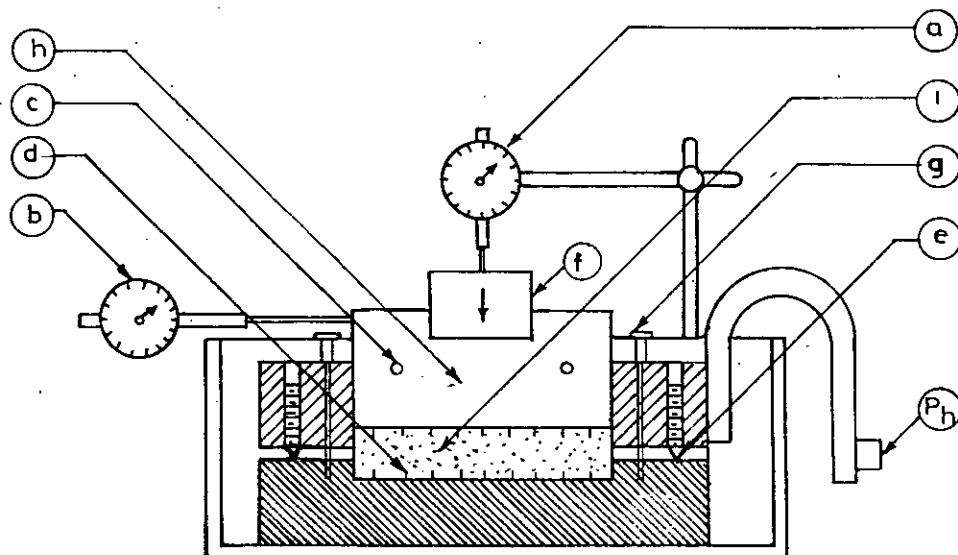


Fig. 4.1 Schematic Diagram of Direct Shear Test Apparatus

- i) A dish capable of containing approximately 500 gms of air dried sand was weighed.
- ii) The separation screws (e) on the top half of the shear box was lowered and then the two parts of the box was locked together.
- iii) Predetermined amount of sand to produce the required void ratio was placed on the shear box from the dish.
- iv) The loading block (f) was placed on the top of the serrated porous stone placed on the soil sample.
- v) Normal load of desired magnitude, which include  $0.25 \text{ kg/cm}^2$ ,  $0.50 \text{ kg/cm}^2$  and  $1.00 \text{ kg/cm}^2$ , was applied to the soil through the loading block.
- vi) The two parts of the shear box were separated by advancing the spacing screws in the upper half of the mold. The space was slightly larger than the largest grains of soil used for the sample.
- vii) Two dial gauges (a) and (b) were attached to measure the vertical and shear displacements.
- viii) The shearing load  $P_h$  was applied through a calibrated proving ring fitted with a load dial. The corresponding vertical and shear displacements readings by dials (a) and (b) were maintained.
- ix) The data thus obtained were used to plot the stress-deformation and volume-change-deformation curves.

#### 4.4 · Determination of Permeability

For the determination of permeability the falling head method was adopted. Fig. 4.2 presents the schematic diagram of the falling head permeability setup. The diameter of the permeameter was 3.556 cm. and its height was 7.112 cm. The diameter of the stand pipe was 0.720 cm. For each soil permeability was determined at three porosities. The porosities are 40%, 44% and 49% respectively. A brief account of the test procedure is given below:

- i) Soil was placed in the permeameter after placing a Watman 100 filter paper at its bottom. The weight of the soil placed was such that it completely filled the permeameter. This weight of the soil required was computed from the known value of specific gravity of soil particles, intended void ratio of the soil specimen and volume of the permeameter.
- ii) The soil in the permeameter was compacted to the desired void ratio in three layers by using a 1" dia rod as a compactor.
- iii) Another filter paper was placed at the top of the soil sample and then the block containing the stand-pipe was placed and properly screwed to ensure tightness.
- iv) Closing both the valves, suction was applied through the top of stand pipe and it was continued for twenty minutes.

**LEGEND**

- ① - Control Valve
- ② - Porous Stone
- ③ - Permeameter
- ④ - Filter Paper
- ⑤ - Standpipe
- ⑥ - Soil Specimen

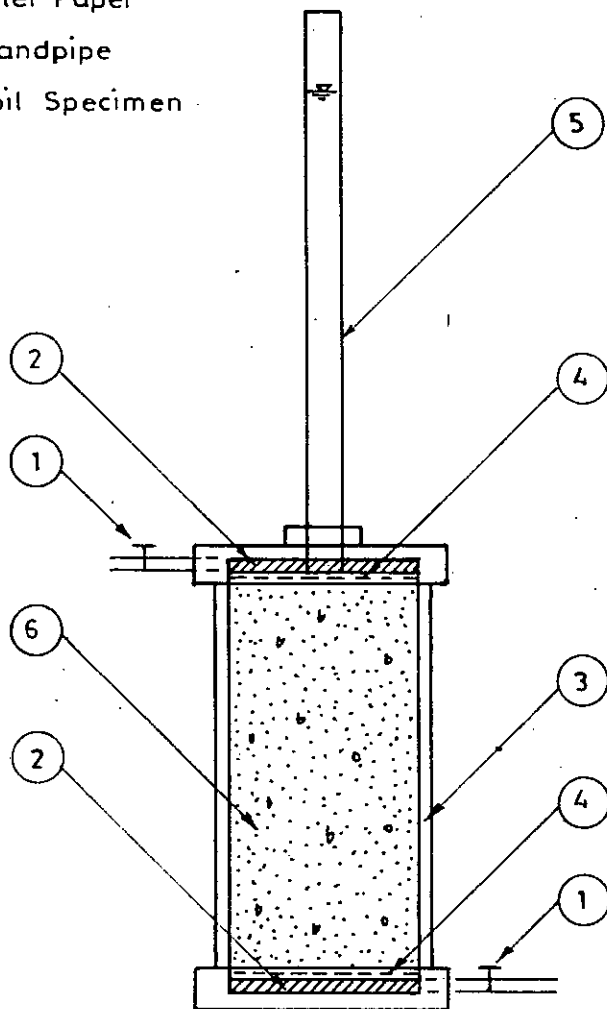


Fig. 4.2 Schematic Diagram of the Falling Head Permeability Test Apparatus

- v) Through the bottom valve the soil sample was allowed to suck boiled distilled water for about 15 minutes.
- vi) Keeping the upper valve closed the lower valve was opened and a back pressure of magnitude  $0.05 \text{ kg/cm}^2$  was applied through it to ensure saturation. Boiled distilled water was used in the application of back pressure.
- vii) The back pressure was maintained for 10 minutes. Later the top valve was opened and boiled distilled water was circulated through the bottom for another 5 minutes to remove the entrapped air bubbles in the soil of the permeameter.
- viii) With top valve closed and bottom valve open time required for the water column in the standpipe to fall a certain height was recorded by a stop watch. Three such measurements were made for the calculation of permeability.
- ix) Finally the permeability was computed by the following formula:

$$k = 2.3 \frac{a_s l_p}{A_p t} \log_{10} \frac{h_1}{h_2}$$

where  $k$  = Darcy's coefficient of permeability

$a_s$  = Cross sectional area of the standpipe

$A_p$  = Cross-sectional area of the permeameter



$l_p$  = Height of the permeameter .

$t$  = Time required by the water surface in the standpipe in falling from a head of  $h_1$  to  $h_2$ .

All through the test boiled distilled water was used. The water flowing out of the soil in the permeameter through the bottom valve was collected. Also the volume of water flowing through the soil from the standpipe for a particular height of fall was computed from the height of fall and the diameter of the standpipe. The volume of water flowing through and coming out of the soil should be equal. These two quantities were checked during all the permeability measurements.

EXPERIMENTAL RESULTS AND DISCUSSIONS

5.1 Introduction

The soil samples used in this research are designated by the symbols listed in Table 5.1. The table also shows their location, particle characteristics and other grain properties.

5.2 Grain Size Distribution and Particle Characteristics

The typical grain size distributions of the soils are presented in Fig. 5.1. None of the soils contain any clay particles. The figure shows that the amount of fines, that is particles less than 0.074 mm, in the soil samples is very insignificant. The soil SS-5 collected from the bed of Old Brahmaputra at Mymensingh contains the maximum fines of 4% and the soil SS-3 from Padma at Rajshahi contains less than 1%. The uniformity coefficient of the soils vary from 1.92 for SS-4 to 2.67 for SS-5. The intermediate values are 2.16 for SS-1, 2.44 for SS-4 and 2.58 for SS-2. The grain size distribution features and the particle characteristics are summarized in Table 5.1. The contents of the table are arranged in order of increasing value of sphericity. That is the soil sample SS-1 has the least sphericity while the soil SS-5 has the maximum sphericity.

5.3 Shear Test Results

Nine direct shear tests were performed on each of the five selected soils at different void ratios and loading condition to evaluate the shearing characteristics of the soils.

Table 5.1: Grain properties of the soils investigated

Sample designation	Location of sample	Specific gravity	Effective grain dia., $D_{10}$ :mm	Uniformity coefficient, U	Sphericity $\psi_{ave}$ $mm^2/mm^2$	Elongation $\epsilon_{ave}$ :mm/mm	Flakiness $f_{ave}$ :mm/mm
1	2	3	4	5	6	7	8
SS-1	Jamalpur (Brahmaputra) Old	2.61	0.155	2.16	0.25	1.50	11.00
SS-2	Rangpur (Teesta)	2.68	0.120	2.58	0.37	1.37	9.12
SS-3	Rajshahi (Padma)	2.65	0.145	2.44	0.45	1.18	7.40
SS-4	Dhaka (Meghna)	2.66	0.135	1.92	0.57	1.27	5.48
SS-5	Mymensingh (Brahmaputra)	2.70	0.150	2.67	0.73	1.10	3.60

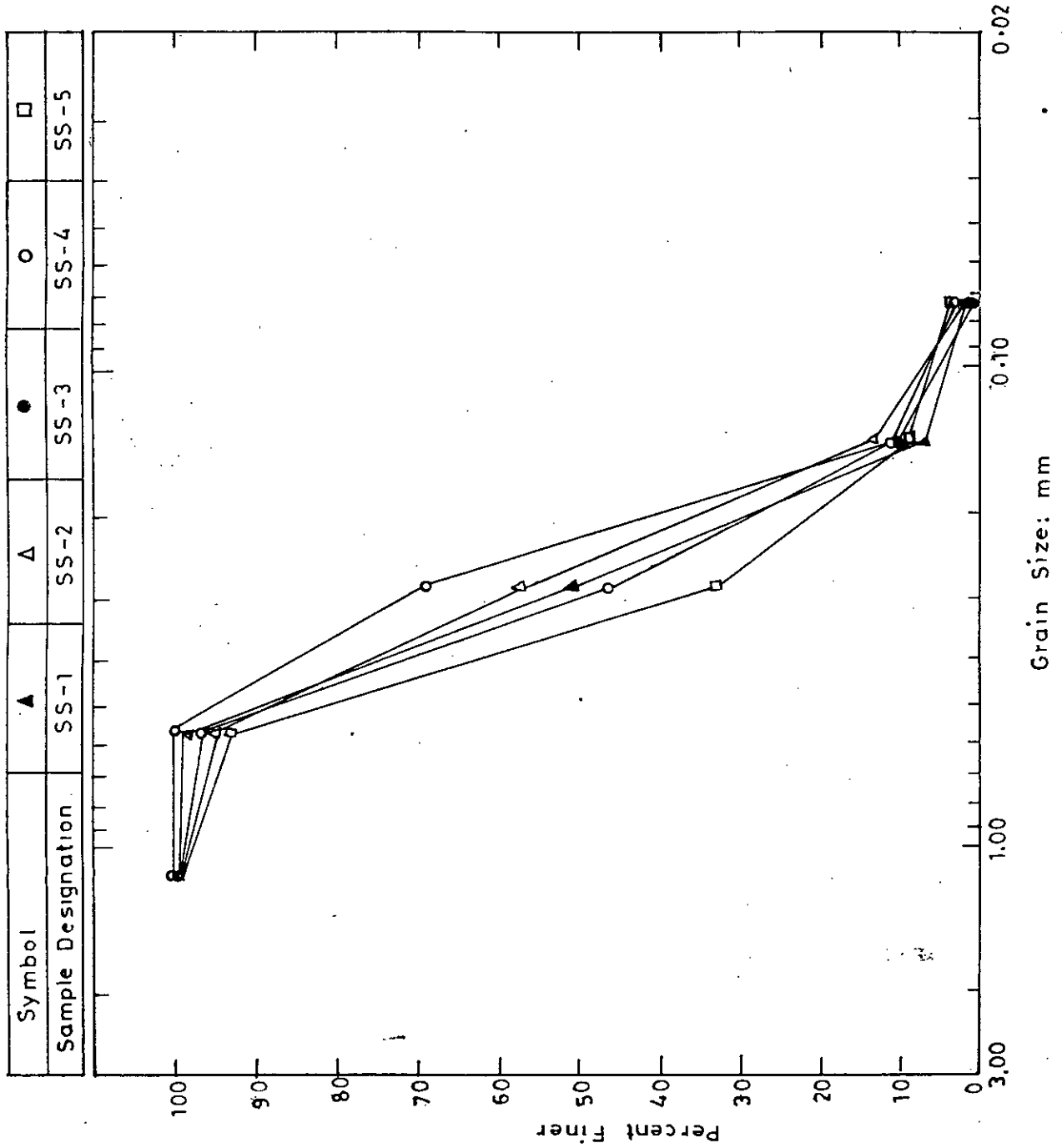


Fig. 5.1: Grain Size Distribution of the Soils Used.

Since a large number of stress-deformation - volume change curves were obtained, the results are summarized in Table 5.2 and in Fig. 5.2 some of the typical curves are presented. As mentioned earlier the direct shear test was performed at three different loading condition for each void ratio. The values of the shearing resistance at peak and ultimate conditions are presented in the fourth and fifth column in Table 5.2 while the 6th and 7th column contain the values of the angle of shearing resistance at peak and ultimate conditions respectively obtained from plotting of the failure envelope for peak and ultimate condition from the direct shear test results.

The curves presented in the Fig. 5.2 describe the general nature of the soil behaviour when subjected to direct shearing. From the curves it is evident that the shearing resistance increases with increasing deformation and on reaching a peak value the resistance gradually decreases and finally assumes a constant residual value at large deformations. The volume change curves, on the other hand, show that there is always decrease in volume during initial load application and shearing. This decrease in volume with increase in shear deformation fades away giving rise to an increase. This phenomena of increase or decrease in volume during shearing is termed as dilation. At large deformation the shearing resistance assumes a constant residual value, the corresponding volume change curves at this stage of deformation are observed to remain horizontal. The

Table 5.2: Peak and ultimate values of shear strength and angle of shearing resistance for various initial void ratios and normal loads.

Soil Type	Location	Initial void ratio	Normal load <sub>2</sub> (kg/cm <sup>2</sup> )	Shearing Resistance (kg/cm <sup>2</sup> )		Angle of shearing resistance (degrees)	
				Peak	Ultimate	Peak	Ultimate
1	2	3	4	5	6	7	8
SS-1	Jamalpur (Brahmaputra) Old	0.63	1.00	.98	.62	45.0	33.0
			0.50	.53	.32		
			0.25	.29	.18		
SS-1	Jamalpur (Brahmaputra) Old	0.76	1.00	.83	.62	41.0	33.0
			0.50	.44	.32		
			0.25	.24	.18		
SS-1	Jamalpur (Brahmaputra) Old	0.81	1.00	.70	.62	36.0	33.0
			0.50	.36	.32		
			0.25	.21	.18		
SS-2	Rangpur (Teesta)	0.63	1.00	.96	.63	44.0	33.0
			0.50	.50	.31		
			0.25	.27	.17		
SS-2	Rangpur (Teesta)	0.76	1.00	.76	.63	38.0	33.0
			0.50	.40	.31		
			0.25	.20	.17		
SS-2	Rangpur (Teesta)	0.81	1.00	.64	.63	33.0	33.0
			0.50	.32	.31		
			0.25	.17	.17		
SS-3	Rajshahi (Padma)	0.63	1.00	.89	.65	42.5	33.5
			0.50	.48	.36		
			0.25	.26	.19		
SS-3	Rajshahi (Padma)	0.76	1.00	.78	.65	39.0	33.5
			0.50	.39	.36		
			0.25	.23	.19		
SS-3	Rajshahi (Padma)	0.81	1.00	.70	.65	34.5	33.5
			0.50	.36	.36		
			0.25	.19	.19		

Contd.....

Table 5.2 (Contd.....)

1	2	3	4	5	6	7	8
		0.63	1.00 0.50 0.25	.84 .49 .26	.64 .34 .17	42.0	33.0
SS-4	Dhaka (Meghna)	0.76	1.00 0.50 0.25	.73 .39 .21	.64 .34 .17	37.0	33.0
		0.81	1.00 0.50 0.25	.67 .34 .19	.64 .34 .17	33.0	33.0
		0.63	1.00 0.50 0.25	.72 .44 .24	.62 .28 .16	39.0	32.5
SS-5	Mymensingh (Brahmaputra) Old	0.76	1.00 0.50 0.25	.68 .32 .19	.62 .28 .16	35.0	32.5
		0.81	1.00 0.50 0.25	.62 .36 .17	.62 .28 .16	33.0	32.5

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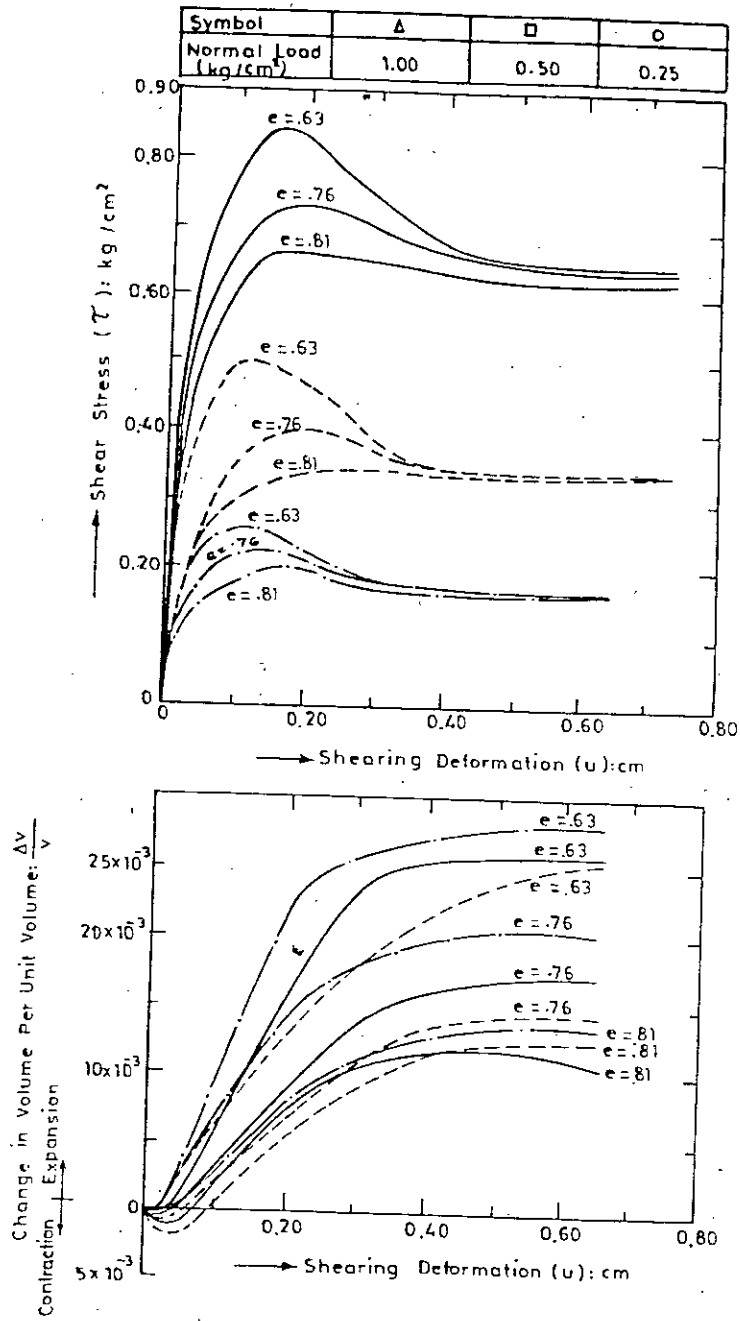


Fig. 5.2: Typical Direct Shear Test Result.



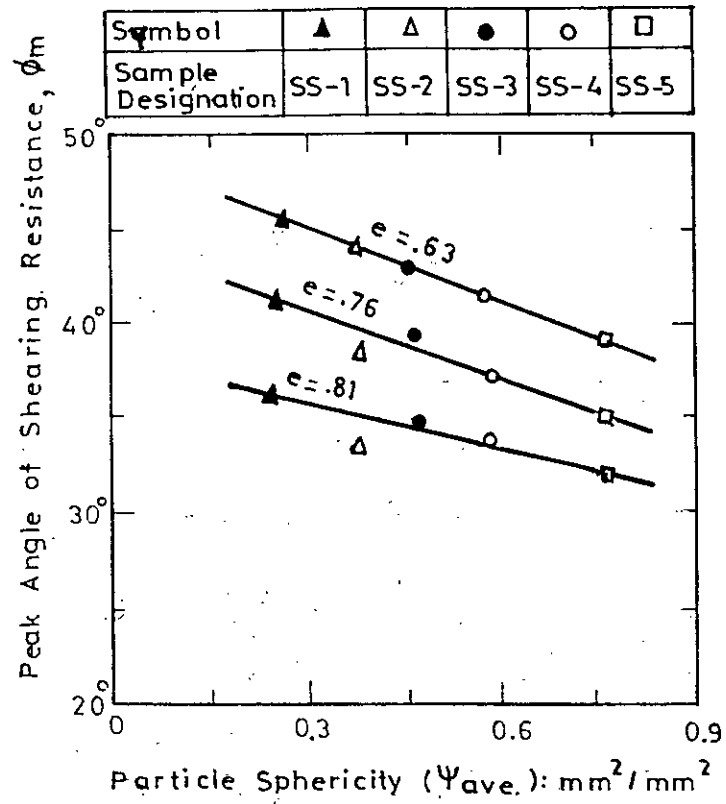


Fig. 5.3: Effect of Particle Sphericity on Peak Angle of Shearing Resistance

stress and the corresponding normal pressure at each void ratio. The variation of the angle of shearing resistance has been approximated by straight lines. From the curves it is obvious that the angle of shearing resistance decreases with increasing sphericity and it has higher magnitude for lower initial void ratio. This finding conforms with the general reasoning. This establishes the fact that lower the initial void ratio the greater will be the interlocking hence the greater the value of the angle of shearing resistance. On the contrary when sphericity increases the interlocking is reduced and hence the strength decreases. The curves also disclose that for lower initial void ratio the rate of change of angle of shearing resistance is higher than for that for higher void ratio.

The straight lines of Fig. 5.4 and Fig. 5.5 indicate the variation of the peak angle of shearing resistance with average values of particle elongation and flakiness respectively at three void ratios as in the case of sphericity. In these figures the variation has been approximated by straight lines. It is found that the strength increases with increasing elongation and flakiness. It is likely because with elongation and flakiness the interlocking becomes higher for a particular void ratio. Moreover with increasing value of sphericity the flakiness and elongation decreases because of the more spherical shape of the soil particles. The angle of shearing resistance should increase with increasing values of these two quantities representing particle shape characteristics. Also it is observed that

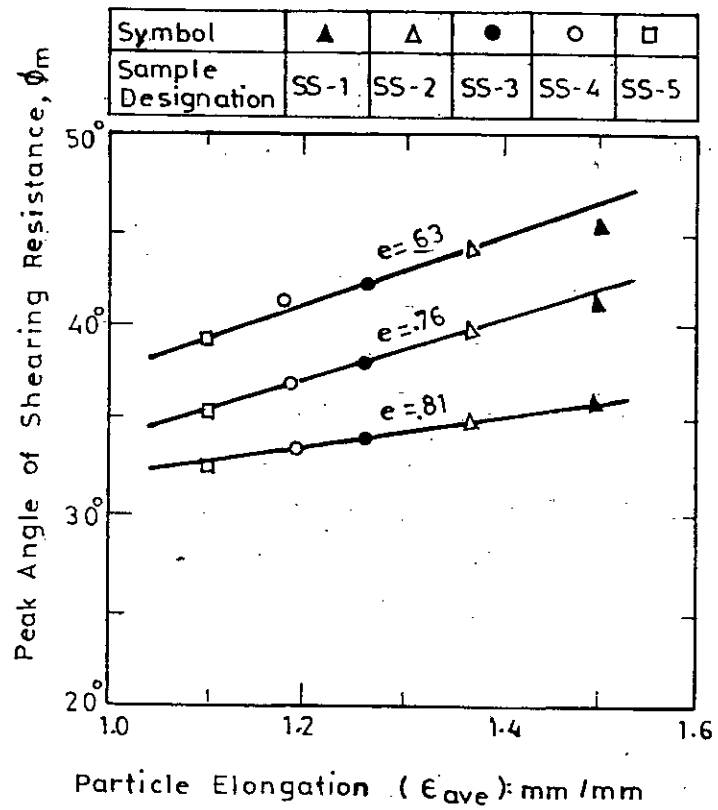


Fig. 5.4: Effect of Particle Elongation on Peak Angle at Shearing Resistance

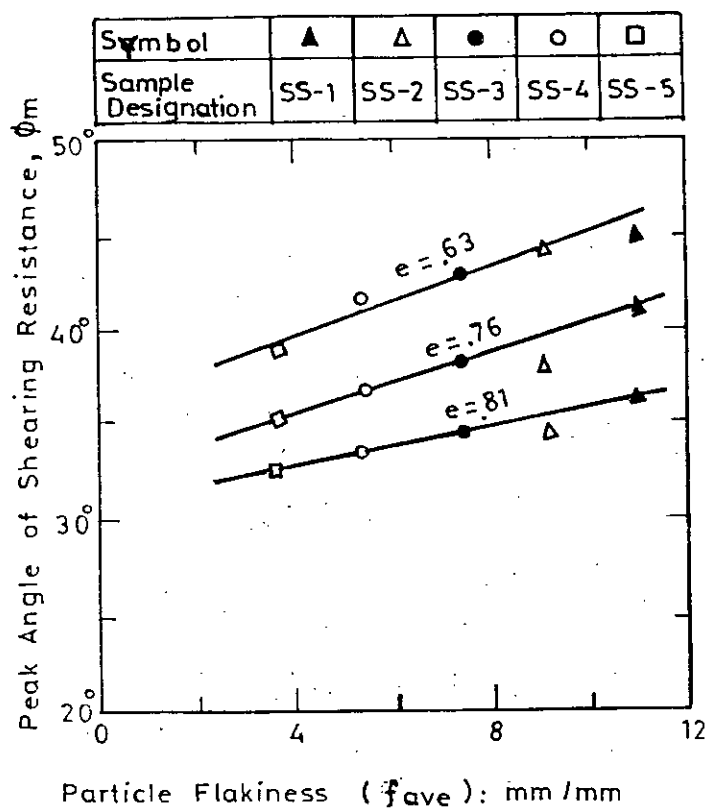


Fig. 5.5: Effect of Particle Flakiness on Peak Angle of Shearing Resistance.

the rate of change of angle of shearing resistance with elongation and flakiness is higher for lower initial void ratio.

Fig. 5.6 presents the ultimate angle of shearing resistance as a function of particle sphericity, elongation and flakiness in the same plot. It is observed that the ultimate angle of shearing resistance vary neither with sphericity nor with particle elongation and flakiness. Here the ultimate angle of shearing resistance has been obtained from a failure envelope plotted from the ultimate value of the shearing resistance and the corresponding normal stress. This happens because at the ultimate stage the soil mass shears without any volume change and hence no interlocking.

The ultimate angle of shearing resistance is independent of particle sphericity, elongation and flakiness. Also it has been established from Fig. 5.3, Fig. 5.4 and Fig. 5.5 that peak angle of shearing resistance decreases with sphericity and increases with elongation and flakiness. The variation is found to be almost linear. Depending on this linearity a mathematical expression is being proposed to determine the peak angle of shearing resistance:

$$\phi_m = \phi_{cv} + (\alpha + \beta x) \dots \quad (5.1)$$

where  $\phi_m$  is the peak angle of shearing resistance,  $\phi_{cv}$  is the angle of shearing resistance at constant volume deformation,  $\alpha$  and  $\beta$  are two constants, and  $x$  is a measure of particle characteristics. In this context particle characteristics include sphericity, elongation and flakiness.

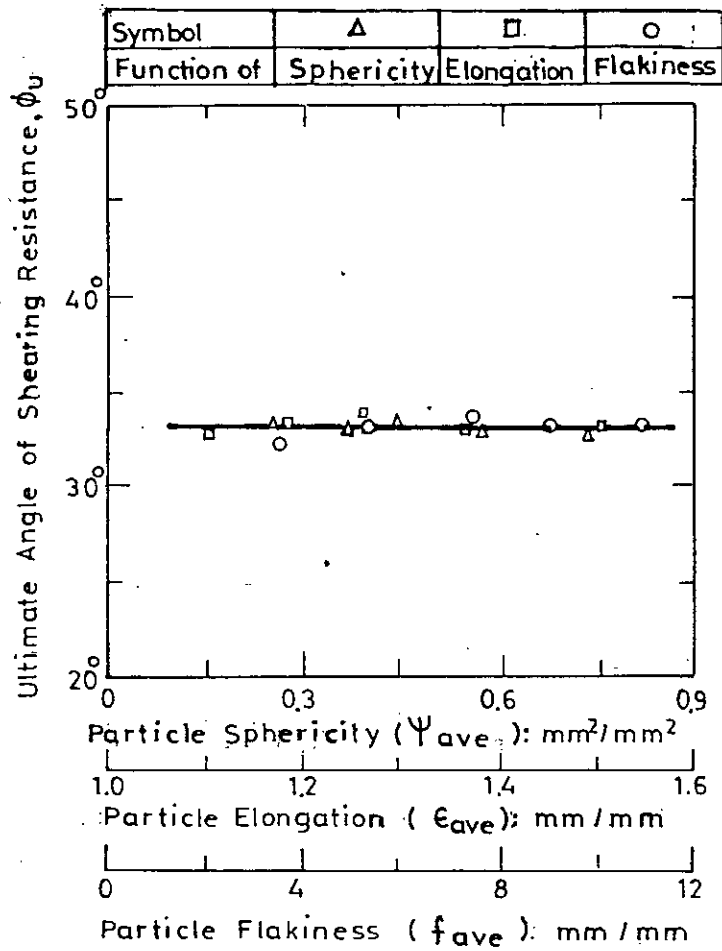


Fig. 5.6 Effect of Particle Sphericity, Elongation and Flakiness on Ultimate Angle of Shearing Resistance.

The constants  $\alpha$  and  $\beta$  depend on the initial void ratio of the soil mass and on the particle characteristics. The following equation is proposed for determining the peak angle of shearing resistance when sphericity is taken as a measure of particle characteristics:

$$\phi_m = 33 + (\alpha_1 + \beta_1 x) \quad \dots \quad (5.1A)$$

Equation 5.1A represents the straight lines of Fig. 5.3. The values of  $\alpha_1$  and  $\beta_1$  for various void ratios are presented in Table 5.3.

Similarly from the values of peak angle of shearing resistance from Fig. 5.4 and Fig. 5.5 the following equations are also proposed to determine the peak angle of shearing resistance when elongation and flakiness are considered as measures of particle characteristics respectively:

$$\phi_m = 33 + (\alpha_2 + \beta_2 x) \quad \dots \quad (5.1B)$$

$$\phi_m = 33 + (\alpha_3 + \beta_3 x) \quad \dots \quad (5.1C)$$

The values of  $\alpha_2$ ,  $\beta_2$  and  $\alpha_3$ ,  $\beta_3$  are provided in Table 5.4 and Table 5.5 respectively. Fig. 5.6 shows the value of  $\phi_{cv}$  for the above equations. From this figure the value of  $\phi_{cv}$  is read as  $33^\circ$ .

The magnitude of  $\phi_m$  at other void ratios intermediate to those presented in Tables 5.3, 5.4 and 5.5 can be approximated by linear interpolation. It is also to be noted that the above equations will apply only when the soil is clean sand and when

Table 5.3: Values of  $\alpha_1$  and  $\beta_1$  when sphericity is taken as a measure of particle characteristics.

Void Ratio	$\alpha_1$	$\beta_1$
1	2	3
0.63	15.89	-13.54
0.76	11.89	-13.54
0.81	5.08	-8.33

Table 5.4: Values of  $\alpha_2$  and  $\beta_2$  when elongation is taken as a measure of particle characteristics.

Void Ratio	$\alpha_2$	$\beta_2$
1	2	3
0.63	-14.63	18.75
0.76	-16.25	17.50
0.81	- 8.25	7.50

Table 5.5: Values of  $\alpha_3$  and  $\beta_3$  when flakiness is taken as a measure of particle characteristics

Void Ratio	$\alpha_3$	$\beta_3$
1	2	3
0.63	2.58	0.95
0.76	-0.92	0.81
0.81	-2.20	0.47



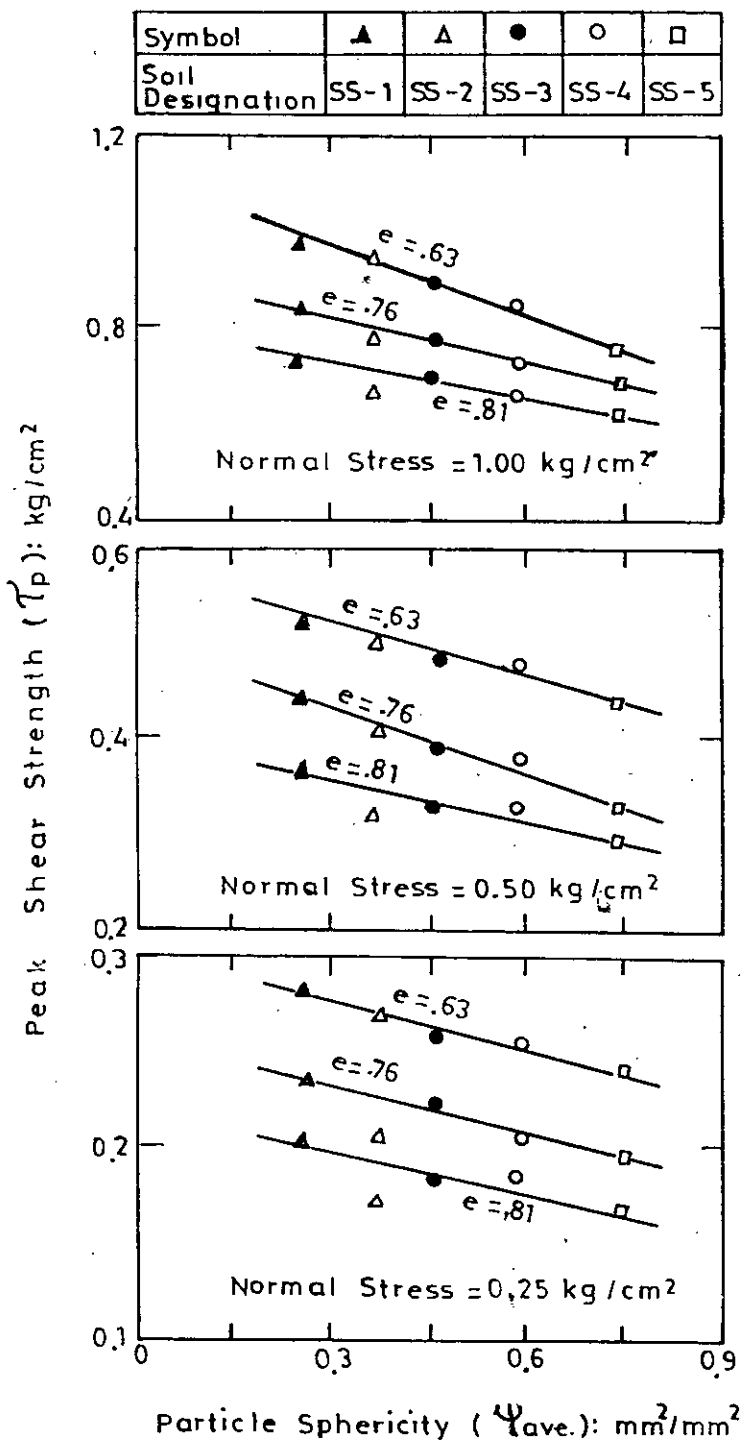


Fig. 5.7: Effect of Particle Sphericity on Peak Shear Strength.

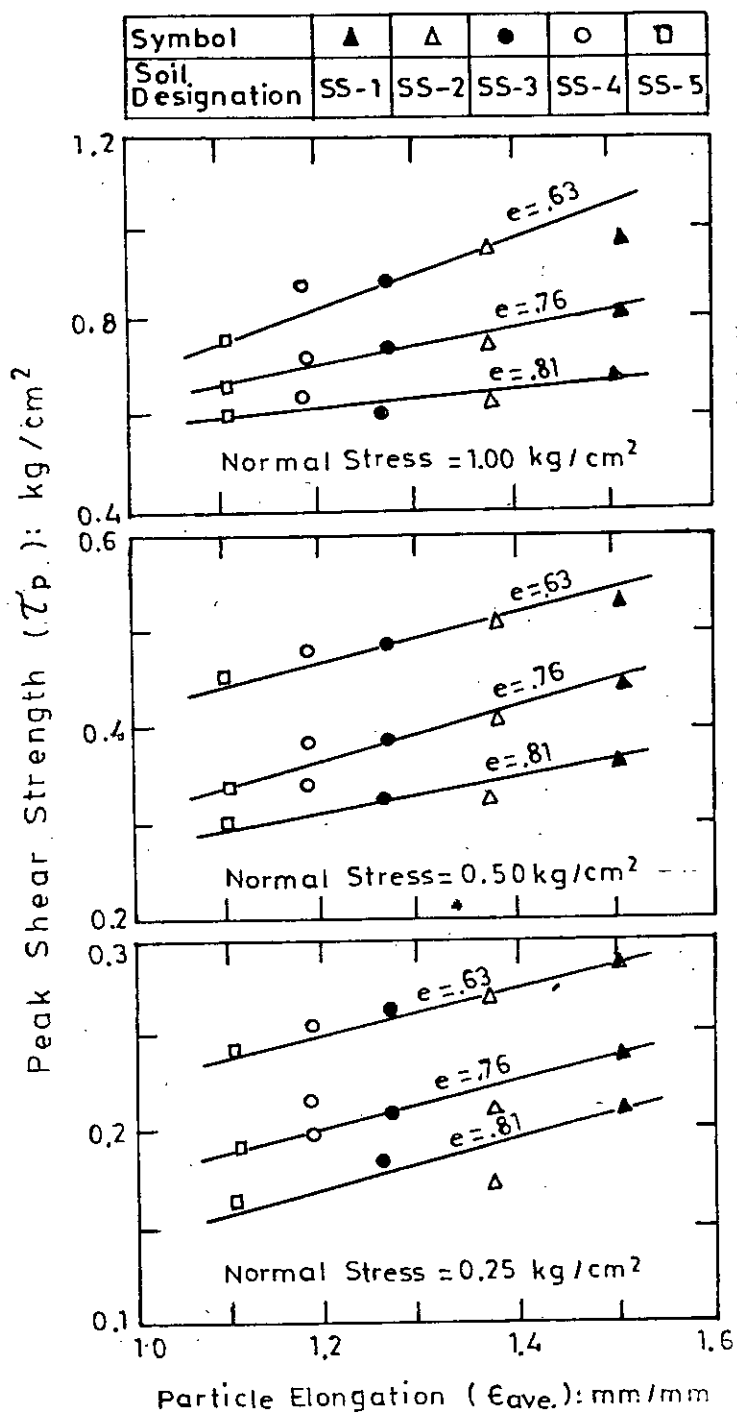


Fig. 5.8: Effect of Particle Elongation on Peak Shear Strength.

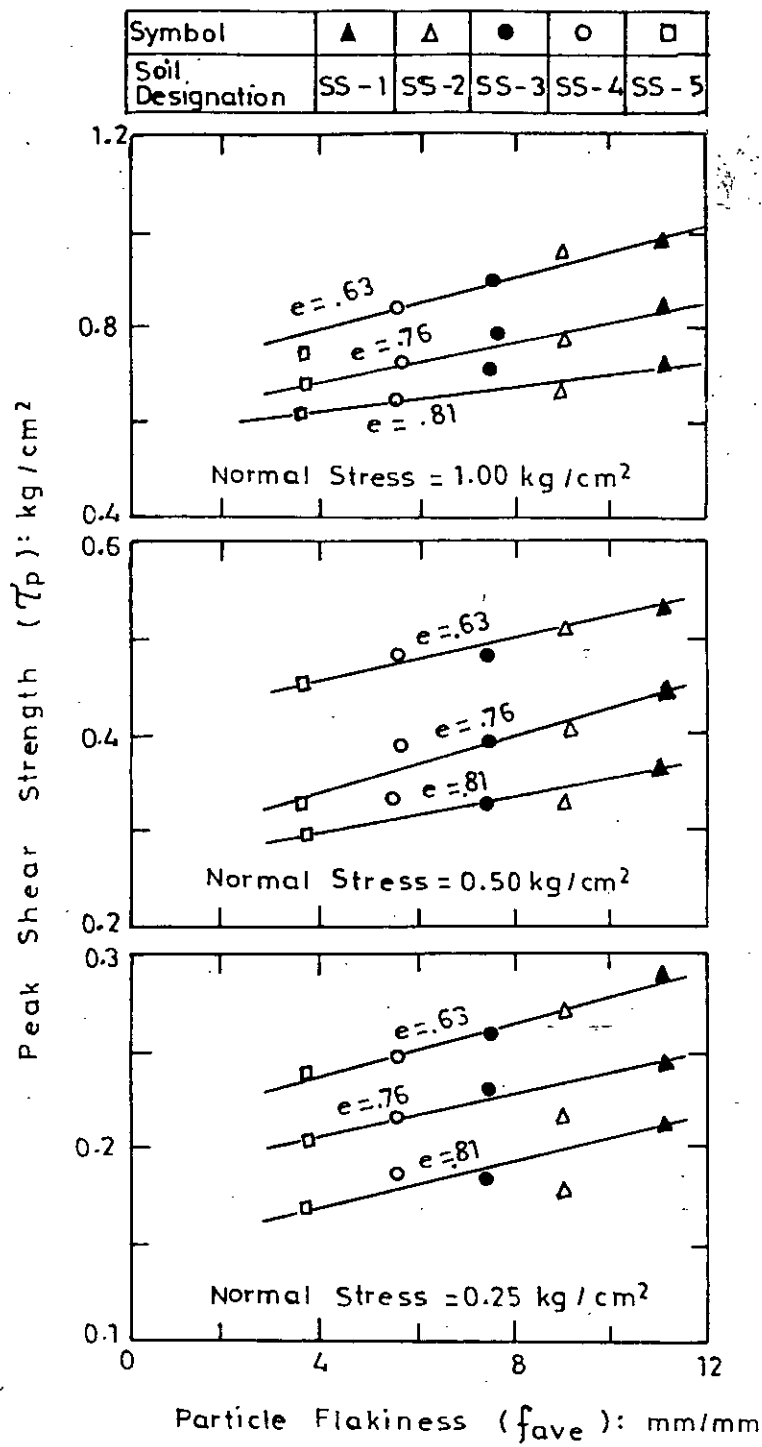


Fig. 5.9: Effect of Particle Flakiness on Peak Shear Strength.

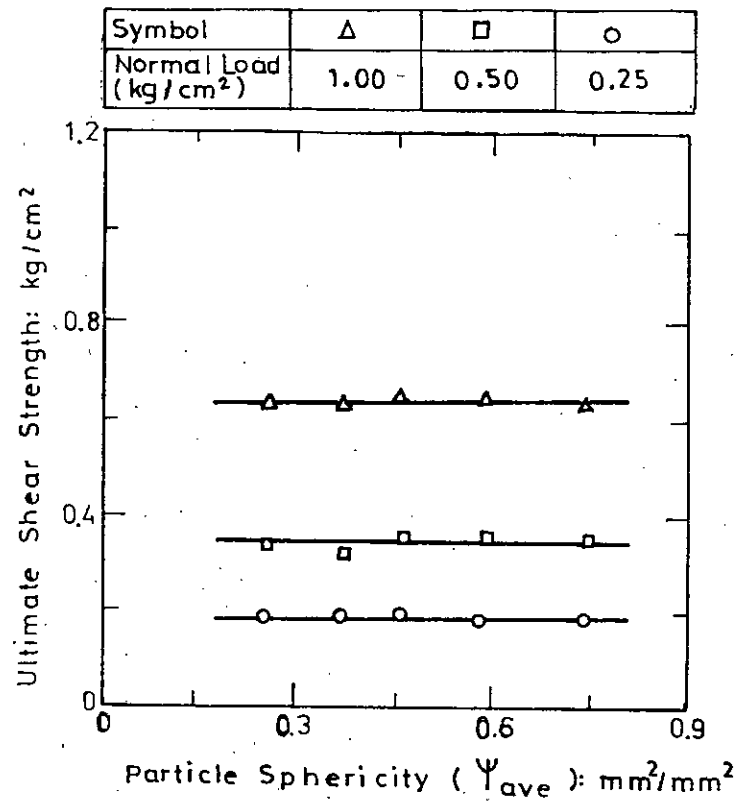


Fig. 5.10: Effect of Particle Sphericity on Ultimate Shear Strength.

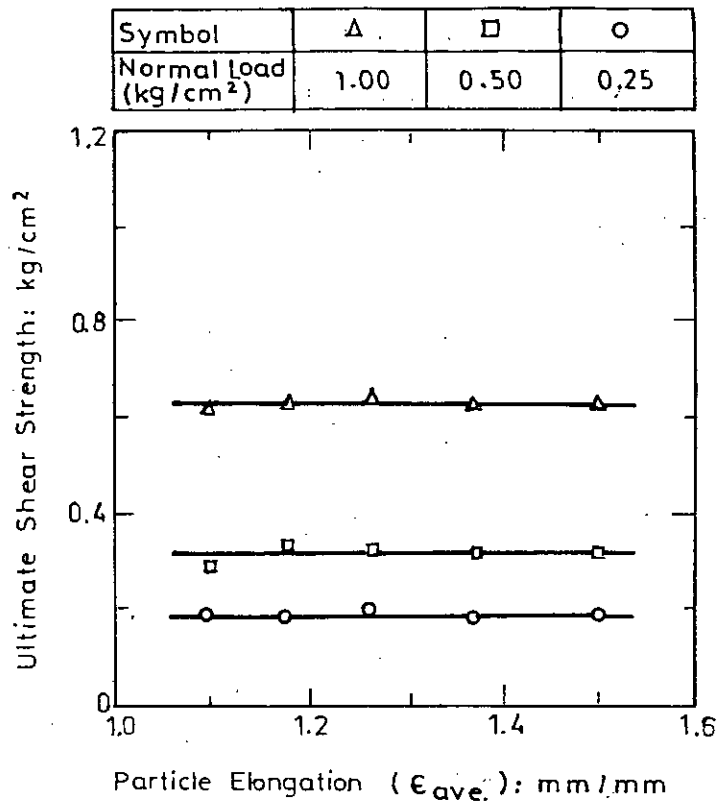


Fig. 5.11: Effect of Particle Elongation on Ultimate Shear Strength.

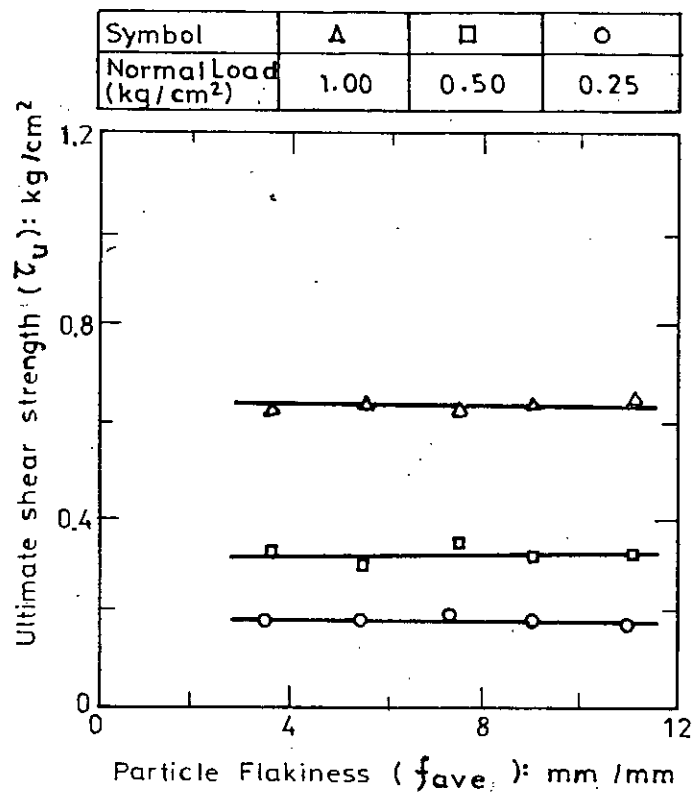


Fig. 5J2; Effect of Particle Flakiness on Ultimate Shear Strength

the uniformity coefficient lies between 1.92 and 2.67 and the effective grain diameter ranges from 0.12 mm to 0.155 mm.

Fig. 5.7 presents peak shear strength as a function of average sphericity for various void ratios and loading conditions. It is observed that the shear strength decreases with increase in sphericity. Similarly Fig. 5.8 and Fig. 5.9 depicts the variation of the peak shear strength with elongation and flakiness respectively. As in the case of angle of shearing resistance here also it is observed that the strength increases with increasing elongation and flakiness. Fig. 5.10, Fig. 5.11 and Fig. 5.12, on the contrary, presents the ultimate value of shear strength as a function of particle sphericity, elongation and flakiness respectively. These figures disclose that the ultimate strength is independent of these particle characteristics. No attempt has been made to develop equation for the determination of shearing strength because for granular soils it is the angle of shearing resistance that describes the strength completely.

### 5.5 Permeability of the Soils

From the test data obtained from the falling head permeability test the coefficient of permeability was computed at the test temperature. This was then converted to correspond to a test for a temperature of 20°C. The permeability measurements were made at three different porosities for each soil. The permeabilities at 20°C in log scale have been plotted

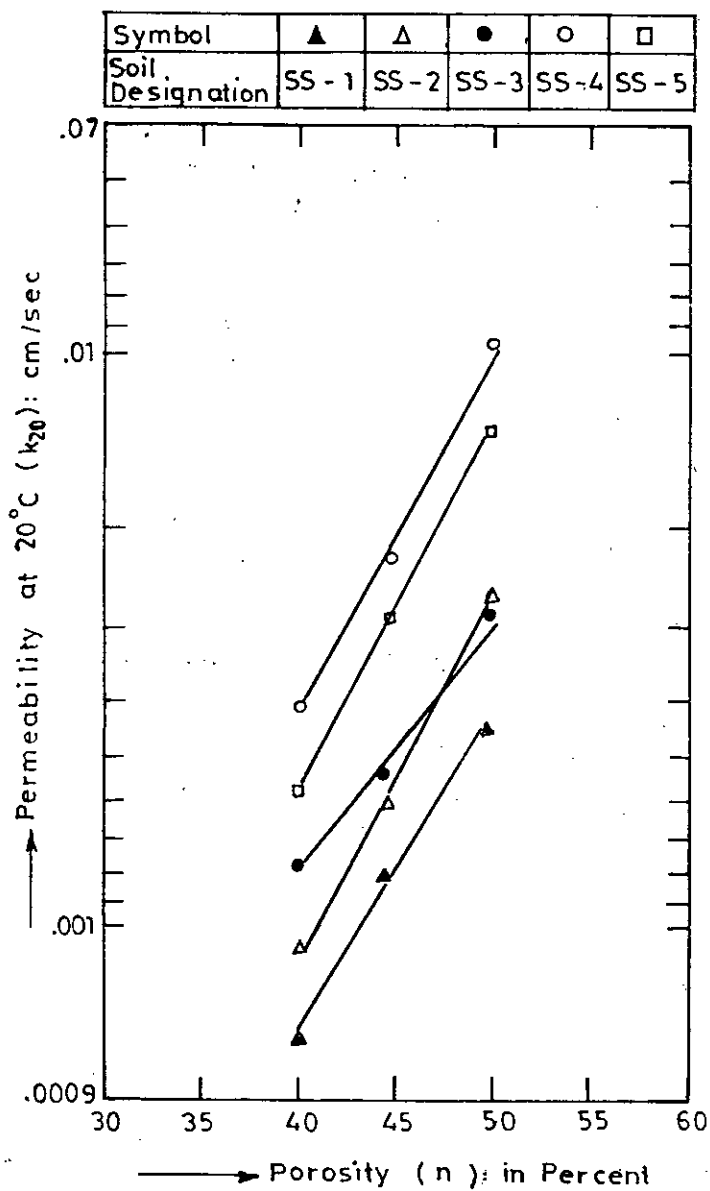


Fig. 5.13: Permeability in Log Scale  
as a Function of Porosity



against porosity in Fig. 5.13. For each soil the plots closely approximates a straight line.

5.6 Effect of Particle Sphericity, Elongation and Flakiness on Permeability

Fig. 5.14 through Fig. 5.16 show permeability at 20°C as a function of sphericity, elongation and flakiness respectively. In these figures log of permeability has been plotted against the above mentioned particle characteristics in ordinary scale. In Fig. 4.14 it was possible to draw an average curve for all the void ratios. Fig. 4.15 in which log of permeability has been plotted against particle elongation, the points for  $n = 49.0\%$  are so scattered that the variation has been indicated by a narrow strip bounded by two curves. The same has been done for Fig. 4.16 at the lowest porosity.

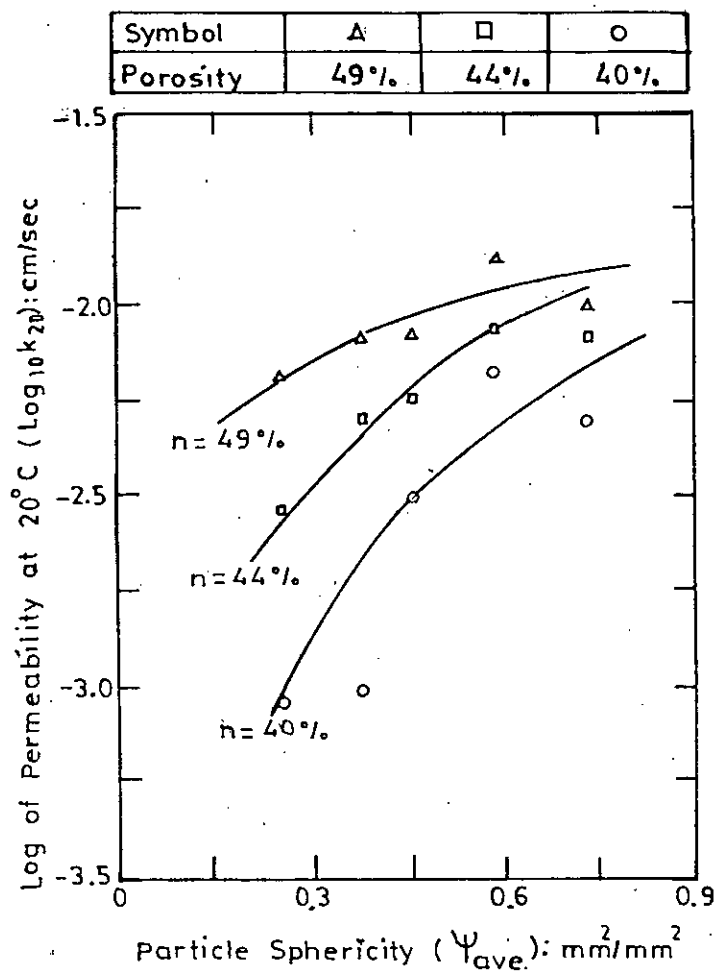


Fig. 75J4: Effect of Particle Sphericity on Permeability at 20°C

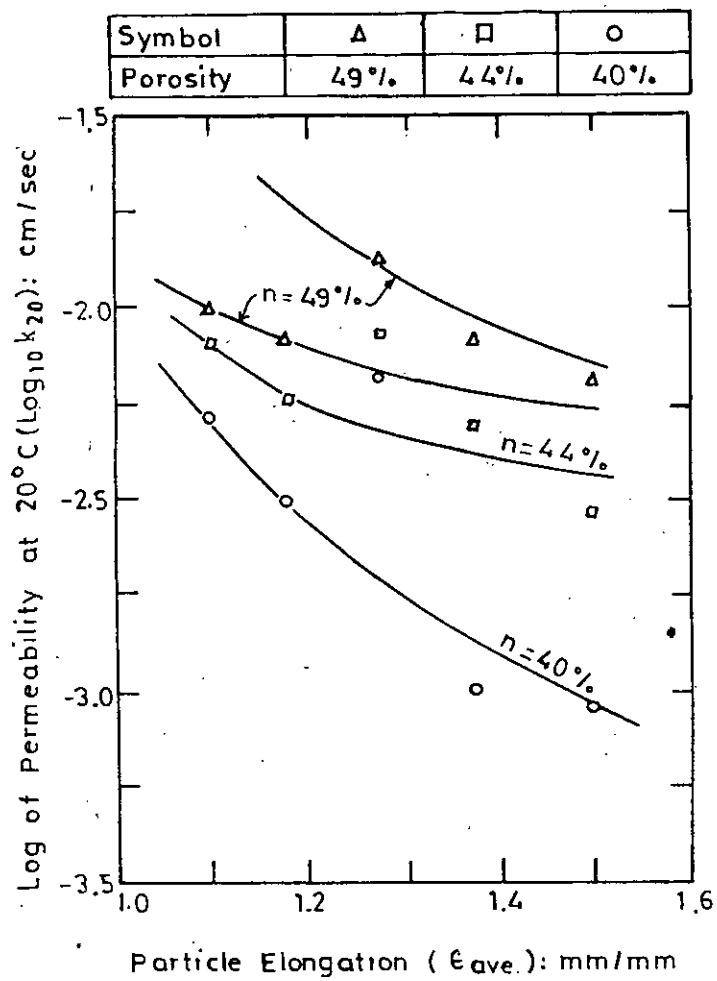


Fig. 5.15: Effect of Particle Elongation on Permeability at 20°C

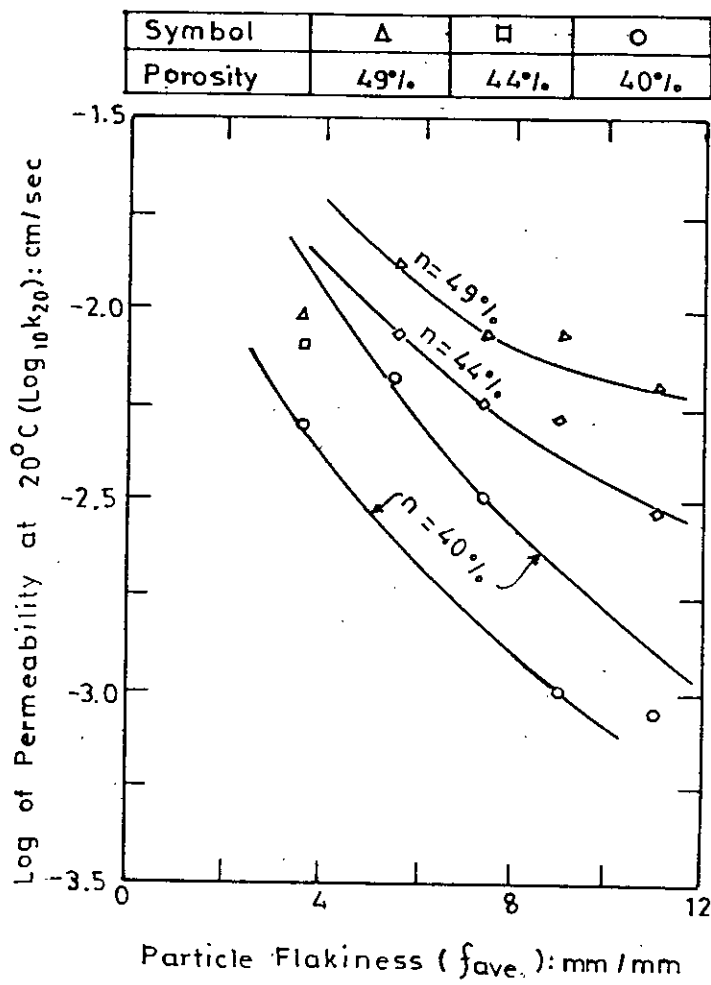


Fig. 5.16: Effect of Particle Flakiness on Permeability at 20°C

## Chapter 6

### CONCLUSION AND RECOMMENDATION

for

### FUTURE RESEARCH

#### 6.1 Conclusions

The major conclusions stemming from this study are presented below. Among them the first, second and the last one confirm the findings of the earlier researchers.

- i) The peak angle of shearing resistance decreases with increasing sphericity and increases with increasing elongation and flakiness. Consequently the peak shear strength decreases with increasing sphericity and increases with increasing elongation and flakiness..
- ii) The effect of particle sphericity, elongation and flakiness is higher at lower void ratio than that at higher void ratios.
- iii) The ultimate angle of shearing resistance is independent not only of particle sphericity but also of elongation and flakiness. As a result the ultimate value of the shearing resistance is independent of these three particle characteristics.
- iv) For granular soils, collected from the river beds of Bangladesh at some selected locations, with uniformity coefficient between 1.92 and 2.67 and effective grain size between 0.120 mm and 0.155 mm

the peak angle of shearing resistance can be approximated by the equation

$$\phi_m = 33 + (\alpha + \beta x)$$

In which  $\alpha$  and  $\beta$  are constants depending upon the grain characteristics  $x$ . Table 5.3, 5.4 and 5.5 provide values of  $\alpha$  and  $\beta$  for different grain characteristics and void ratio conditions.

- v) With particle sphericity increasing Darcy's coefficient of permeability increases and it has a decreasing trend with increasing elongation and flakiness.

## 6.2 Recommendation for Future Research

This research work was restricted to the evaluation of the effect of particle shape and angularity on shear strength and permeability of granular soils. No attempt has been made to separate the strength component.

The evaluation of shear strength parameter  $\phi_f$ , the angle of shearing resistance corrected for dilation, is of practical importance. The reason for such a statement requires visualizing a shear failure in any plastic equilibrium problem involving a soil mass. If the strain at every point along the failure path could be measured it would vary from near zero at the most recently mobilized section to a relatively high value at the

the initially mobilized section. But the maximum value of the angle of shearing resistance occurs at only one value of strain. Therefore it can not be counted upon along the entire failure path. In fact, as shear progresses the value of the angle of shearing resistance falls to a limiting value of  $\phi_f$ . In other words the dilational component of the shearing resistance can not be relied upon.

Need often arises for an approximate value of the angle of shearing resistance without going to the expense of undisturbed samples and laboratory tests. In addition, cohesionless soils are usually difficult to sample in the undisturbed state. On the other hand, properties like particle shape, size and gradation can be readily and economically obtained. It is, therefore, suggested that a formula be developed for  $\phi_f$  as a function of particle shape, size, distribution and relative density of the soil mass.

As regards permeability it is recommended that all the existing formulae to determine permeability from simple grain property be studied and their validity be determined for various particle shape, size distributions and relative density ranges.

*Appendix*

TYPICAL PHOTOGRAPHS OF THE SOILS STUDIED

A.1 Introduction

For the visualization of the particle shape and angularity of the soils investigated photographs of representative fractions are provided in the following article.

A.2 Typical Photographs

The following are the photographs of representative fractions of the soils used in this research. The identification of each soil is attached at its bottom. Except the soil SS-1 all the soils have been magnified 60 times. The magnification of SS-1 is 120.

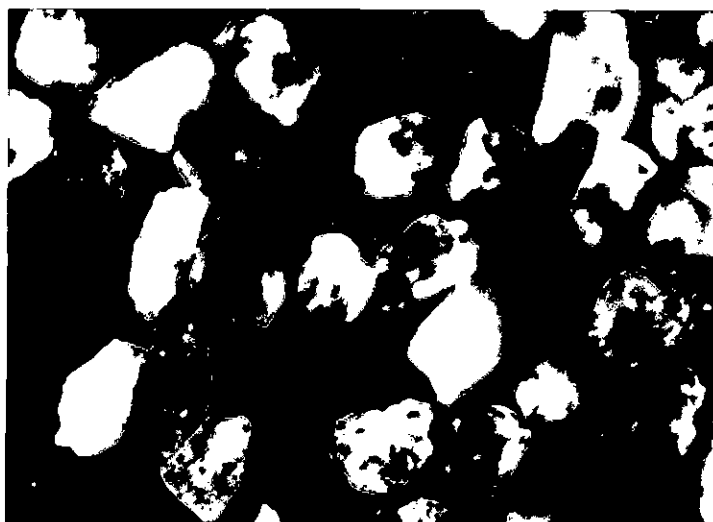


Fig. A.1: SS-1, collected from the bed of Old Brahmaputra at Jamalpur.



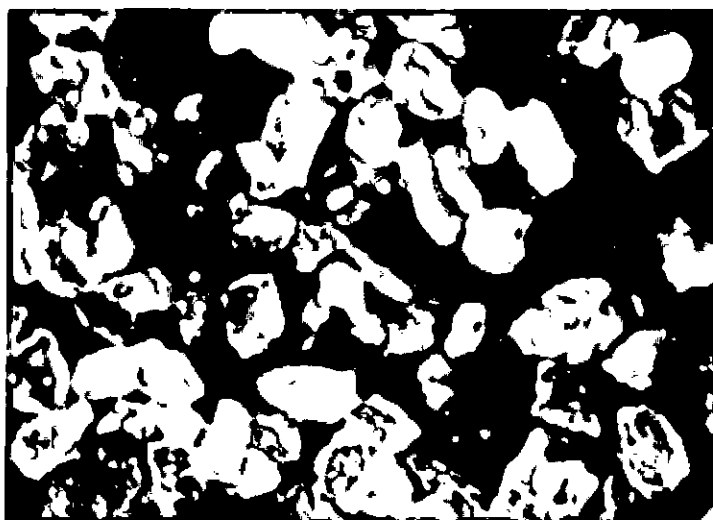


Fig. A.2: SS-2, collected from the bed of Teesta at Rangpur.

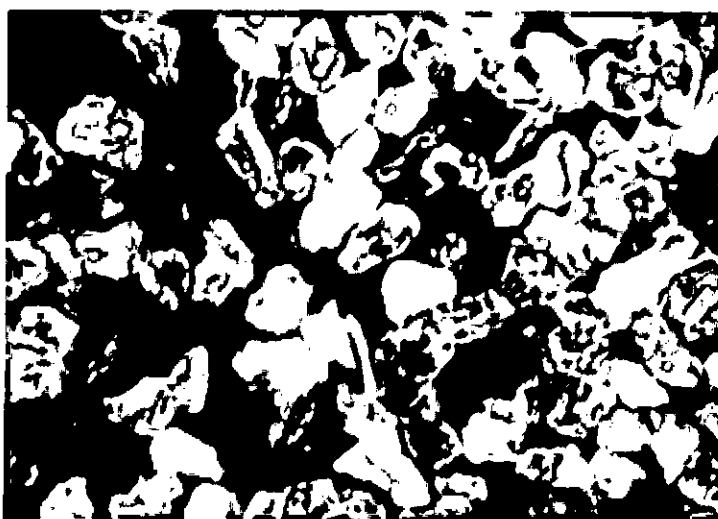


Fig. A.3: SS-3, collected from the bed of Padma at Rajshahi.

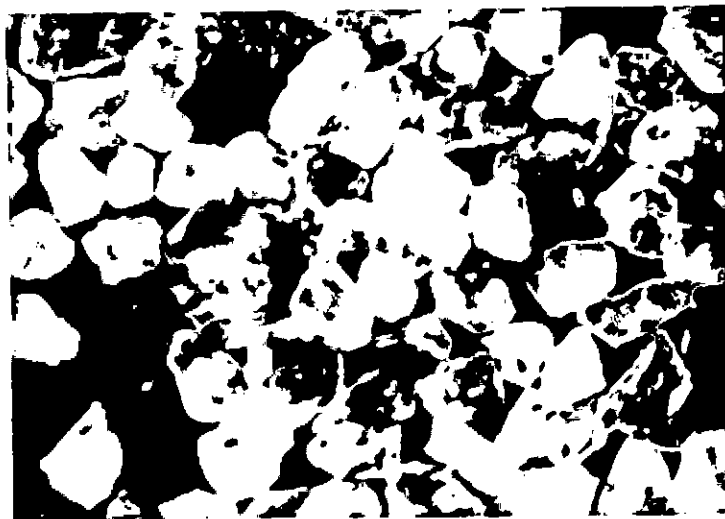


Fig. A.4: SS-4, collected from the bed of Meghna at Dhaka.

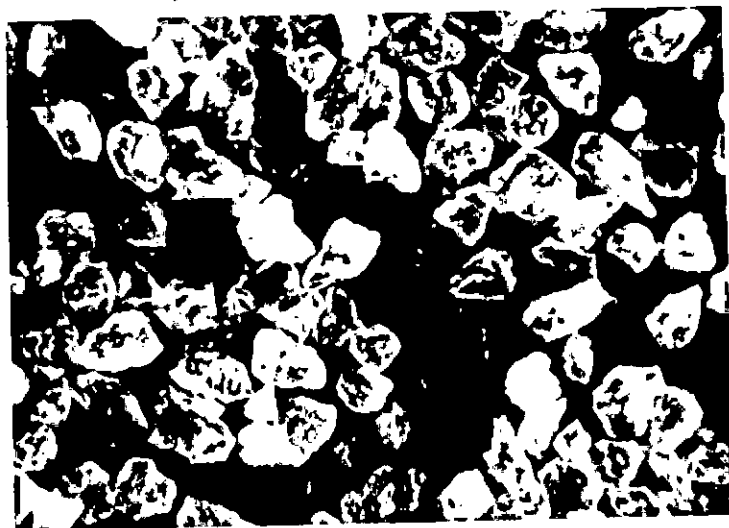


Fig. A.5: SS-5, collected from the bed of Old Brahmaputra at Mymensingh.

## REFERENCES

- ✓ 1. Allen, T. (1975). Particle size measurement, Chap. 4. London: Chapman and Hall.
- ✓ 2. Bros, B. & Orzeszyna, H. (1979). The influence of particles' angularity and surface roughness on engineering properties of sands. Proc. 7th European conference on soil mechanics, Brighton, Vol.2, 11-14.
3. Coulomb, C.A. (1776). Essai sur une application des regles des maximis et minimis a quelques problemes de statique relatif a L'architecture, Mem. Acad. Roy. Press. a Div. Sav. Etr. Reported by C.R. Scott (1969).
4. Darcy, H. (1856). Les fontaines publiques de la ville de Dijon, Dalmon, Paris. Reported by Leonard (1962).
- ✓ 5. Eerola, M. & Ylosjoki, M. (1970). The effect of particle shape on the friction angle of coarse-grained aggregates. proc. 1st Int. Congr. Int. Ass. Engng Geol. 1, 444-456.
6. Ehrlich, R. & Weinberg, B. (1970). An exact method for characterization of grain shape. J. Sedim. Petrol. 40, No. 1, 205-212.
7. Frossard, E. (1979). Effect of sand grain shape on inter-particle friction; indirect measurements by Rowe's stress dilatancy theory. Geotechnique 29, No. 3, 341-350.
8. Gur, Y., Shklarsky, E., & Livneh, H. (1967). Effect of coarse-fraction flakiness on the strength of graded materials. Proc. 3rd Asian Reg. Conf. on Soil Mech. and Fdns Engng. Haifa. 276-280.
9. Hagen, G. (1839). Uber die Bewegung des Wassers in engen zylindrischen Rohren, Annalen der physik und Chemie, Leipzig, Vol. 46, p423. Reported by Leonards (1962).
10. Hazen, A. (1892). Some physical properties of sands and gravels with special reference to their use in filtration. 24th Annual Report. Massachusetts State Board of Health. Reported by Loudon (1952).
11. Henkel, D.J. (1959). The relationship between the strength, pore water pressure, and volume change characteristics of saturated clays, Geotechnique 9, 119-127.
12. Heywood, H. (1947). The scope of particle size analysis and standardization. Institution of Chemical Engineers. Vol. 25, Loudon. Reported by Allen, T. (1975).

- ✓ 13. Koerner, R.M. (1970). Effect of particle characteristics on soil strength. J. Soil Mech. Fdns. Div. Am. Soc. Civ. Engrs 96, SM4, 1221-1234.
14. Kozeny, J. (1927). Uber kapillare leitung des wassers in Boden. Ber. Wein. Akad. 136-271. Reported by Loudon (1952).
- ✓ 15. Krumbein, W.C. (1941). Measurement and geological significance of shape and roundness of sedimentary particles. J. Sedim. Petrol. 11. No.2, 64-72.
16. Lambe, T.W. & Whitman, R.V. (1969). Soil Mechanics, Chap.4 and Chap. 11. New York: Wiley.
- ✓ 17. Lees, G. (1964). A new method for determining the angularity of particles, Sedimentol. 3,2-21.
18. Loudon, A.G. (1952). The computation of permeability from simple soil tests. Geotechnique, Vol.3, No.2, 165-183.
19. Mitchell, J.K. (1964). Shearing resistance of soils as a rate process. J. Soil Mech. Fdns Div. Am Soc. Civ. Engrs 90, SM1, 29-61.
- ✓ 20. Mitchell, J.K. (1976). Fundamentals of soil behaviour, Chap. 4. New York: Wiley.
21. Mohr, O. (1914). Abhandlungen aus den Gegiete der Technischer Mechanik, W. Ernst (Berlin), 2nd Ed. Reported by C.R. Scott (1969).
22. Poiseuille, J. (1841). Recherches experimentales sur le mouvement des liquides dans les tubes de tres petits diametres, Comptes rendus, Paris, Vol.2, 961-1041. Reported by Leonards (1962).
23. Rittenhouse, G. (1943). Relation of shape to the passage of grains through sieves. Industrial Engg. Chemistry. Vol. 15. No.2. 153-155.
24. Rosenak, S. (1963). Soil mechanics, Chap. 4. London: B.T. Batsford Ltd.
25. Rowe, P.W. (1962). Energy components during the triaxial cell and direct shear tests, Geotechnique 14. 247-261.
26. Scott, C.R. (1969). Soil mechanics and foundations, Chap. 6. London: Maclaren and Sons.
27. Scott. R.F.. (1963). Principles of soil mechanics, Chap. 7. London: Addison-Wesley Publishing Company, Inc.
28. Slichter, C.S. (1899). Theoretical investigation of the motion of ground water. 19th Annual Report. U.S. Geological Survey. 2:305. Reported by Loudon (1952).

29. Sowers, G.B. and Sowers, B.F. (1951). Introductory Soil Mechanics and Foundations. Macmillan, New York.
30. Taylor, D.W. (1948). Fundamentals of soil mechanics, Chap. 6. New York: Wiley.
31. Terzaghi, K. (1925). Erdbaumechanik; P. 120. Deuticke, Vienna. Reported by Loudon (1952).
32. Terzaghi, K. & Peck, R.B. (1967). Soil mechanics in engineering practice, Chap. 2. New York: Wiley.
33. Thurston, C.W. & Deresiewicz, H. (1959). Analysis of a compression test of a model of granular medium. J. App. Mech. 26, Trans. ASME 81, 251-267.
34. Wadell, H. (1932). Volume, shape and roundness of rock particles. Journal of geology. Vol. 40, 443-451.
35. Whalley, W.B. (1972). The description and measurement of sedimentary particles and the concept of form. J. Sedim. Petrol. 42, No.4, 961-965.

