A SIMPLIFIED EQUILIBRIUM APPROACH TO THE DESIGN OF FREE STANDING STAIRS BASED ON FINITE ELEMENT ANALYSIS

A THESIS BY

MD. ANWAR ZAHID

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Approved as to the style and content by:

Dr. Sohrabuddin Ahmad Professor Department of Civil Engineering BUET, Dhaka.	Chairman (Supervisor)
Dr. M. Shamim Z. Bosunia Professor Department of Civil Engineering BUET, Dhaka.	Member
Dr. Md. Humayun Kabir Professor and Head Department of Civil Engineering BUET, Dhaka.	Member
Dr. Khan Mahmud Amanat Assistant Professor Department of Civil Engineering BUET, Dhaka.	Member
Dr. Md. Wahhaj Uddin Professor Department of Mechanical Engineering BUET, Dhaka.	Member (External)

To My

Mother & Father

DECLARATION

Declared that, except where specified by references to other investigators, the work embodied in this thesis is the result of investigation carried out by the author under the supervision of Professor Sohrabuddin Ahmad of Civil Engineering Deapartment, BUET.

Neither the thesis nor any part thereof has been submitted or is being concurrently submitted in candidature for any degree at any other institution.

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Author

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CONTENTS

ABSTRACT

Because of the absence of any specific code, the process of design of a free standing stair is dependent on some approximate analytical methods. These methods fail to recognise the variation of stress resultants across any cross section of the stair slab and require a lot of computational works. Amanat[4] made an extensive study on free standing stairs using the idealisation of *Ahmad's[2]* thick shell element and proposed a simplified guideline for easy analysis of free standing stairs. His approach also recognises the stress variation across different sections of the free standing stair, based on finite element analysis, and developed seven empirical design equations.

If the loadings and the geometric conditions of the free standing stairs are symmetric, then, only two redundants, namely the lateral shear and the bending moment at the mid landing section are present. Hence, the number of empirical equations can be reduced to two for finding the values of these two redundants. The moments and forces at the other critical locations can be calculated by using equilibrium equations. With a view to developing a more rational and realistic approach by reducing the number of equilibrium equations, and also for increasing the range of applicability of Amanat's equations, this research was undertaken. Sensitivity analysis of the different geometric parameters is carried out over an extended range for two cases of loading. It is found that, in the higher range, the linear equations proposed by Amanat fail to represent the moments and forces as seen from the results of finite element analysis. Based on this study, two sets of empirical equations are developed for two cases of loadings. Other necessary design parameters are obtained from equilibrium equations which reduce the total number of empirical relations, developed by Amanat, from seven to four. In this research, detailed study is carried out to select the equations which nearly represent the actual situation. Ten prototype stairs are solved by using the proposed equations, Amanat's equations, and finite element analysis. These results establish the acceptability of the proposed equations.

The proposed method has made the calculations for forces and moments, required for design of free standing stairs, very easy, and its estimation is found to be in the acceptable limits for commonly used dimensional ranges of stairs. Recommendations regarding the reinforcement layout have also been made.

LIST OF SYMBOLS

A	Horizontal gap between flights
B	Width of landing
C	Width of flights
E_c	Modulus of elasticity of concrete
E_s	Modulus of elasticity of steel
$F_{Am1}, F_{Bm1}..$	Factors corresponding to A, B for mid landing section moment in load case one.
$F_{Am2}, F_{Bm2}..$	Factors corresponding to A, B for mid landing section moment in load case two .
F_{Ah1}, F_{Bh1}	Factors corresponding to A, B for mid landing lateral shear in load case one.
H	Floor to floor height
H_0	Lateral shear at mid landing section
K_{h1}	Numeric constant for lateral shear in load case one
K_{m1}	Numeric constant for moment in load case one
L	Horizontal length of flight
M_b	Bending moment at support
M_i	In-plane moment
M_0	Bending moment at mid section landing
M_t	Torsional moment
T_1	Waist thickness of flight slab
T_2	Thickness of landing slab
V_x	Axial force on a section
V_y	Lateral shear on a section
V_z	Transverse shear on a section
X, Y, Z	Global co-ordinate system
X', Y', Z'	Local co-ordinate system
f_c'	28 day cylinder strength of concrete

f_s	Allowable stress in steel
f_y	Yield strength of steel
p	Transverse shear stress
α	Angle of inclination of flights with horizontal
ϕ	Angular distance of a point on bow girder from centre line
σ	Normal stress
τ	Lateral shear stress

CHAPTER 1

INTRODUCTION

1.1 GENERAL

An important functional element in a building, whether tall or low rise, is the stair. It may be defined as a series of steps arranged for the purpose of connecting different floors of a building. At the time of any emergency evacuation, like an earthquake or a fire, stair is subjected to its maximum loading. At the peak hour in a commercial, business centre or market, a stair is used not only to facilitate transport of people between floors but also to provide an elegant look to the building. Architecturally, it must be fascinating and beautiful; structurally, it must be strong, stiff, and efficient; functionally it must allow smooth and free transit of people from floor to floor; and seismically, it must be flexible enough to withstand the shock load.

Depending on various architectural forms, stairs may be classified as (*Fig. 1.1*) :

- Free standing stair,
- Slabless stair,
- Helical stair, and
- Simple straight stair.

Compared with other structural components of a building, stairs have some unusual characteristics. Stairs are an assemblage of interconnected plates in a three-dimensional space and supported at the outer edges of these plates. Both in-plane and out-of-plane forces may be predominant in the stair depending on the arrangement of supports. In simple stairs, supports are provided at floor levels and at intermediate landing. Previous investigations on this type of stairs revealed that load is transferred primarily through bending of stairs slabs. In plane forces, although present, are of little significance. The situation changes totally when the support at landing is removed and the stair is allowed to stand freely between supports at floor levels only. The structural functioning of this free standing stair is completely different from that of the simple stairs.

While the functioning of simple stairways can be approximated as a two dimensional flexure problem, there is no way of similar treatment to the free standing stair which is truly a three dimensional problem. Consequently, the structural analysis of a free standing stair is much more complex in comparison to that of the ordinary stairs.

From architectural view-point, free standing stairs are more attractive than simple stairs. Another architecturally fascinating stair of similar structural action is the helicoidal stair.

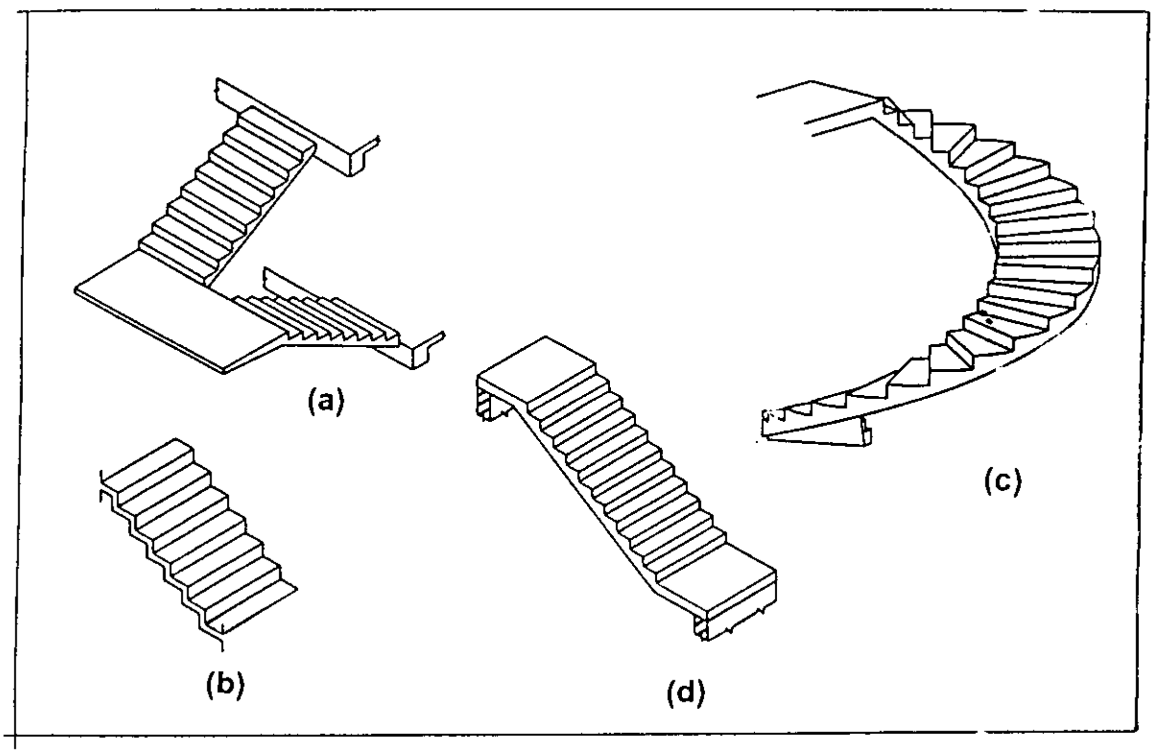


Fig. 1.1 Stairs of Different Types

a) Free standing stair

b) Slabless stair

c) Helical stair

d) Simple straight stair

But construction of a helicoidal stair is difficult because of its geometry. A considerable part of the total cost goes to the construction of the formwork of a helicoid. On the contrary, the formwork of a free standing stair is simple. Unlike simple stairs, a free standing stair does not need a separate stair hall for its construction. Despite this, construction of free standing stairs has been limited in the past. This is mainly due to the lack of adequate knowledge about its complex three dimensional behaviour and absence of a simple design procedure. Now, with the advent of finite element method along with the availability of high speed digital computers, it is possible to analyse the free standing stair more accurately and rationally.

1.2 BACKGROUND OF RESEARCH

Free standing stairs received the attention of a number of researchers in the past. The methods of analysis developed by various authors indicate that it is a structure carrying torsion and in-plane moments besides bending and shear. In their analysis they made several assumptions to simplify the actual structural behaviour of the stair. Sauter [10], Taleb [12] and Cusens et. al.[5] replaced the stair slab by an equivalent skeletal rigid frame and then they solved for the redundants, similar to those occurring in a space latticed structure. They did not account for any variation of stress resultants across the sections of slab. Liebenberg [7] retained the space plate configuration of the stair. His solution is based on assumption that primary load carrying system is produced by the membrane forces caused by the interaction of plates, support, etc. These 'points' or 'lines' of intersection of the slab elements become supports to the secondary load carrying system of bending forces in the slab elements. Siev [11] suggested a method which is similar to that applied in the analysis of folded plate. He introduced a support along the flight landing junction and called the resulting structure as primary system where loads are carried by bending and by reactions at support. Later the reaction at the kink is applied in the reverse order and secondary stresses are found. Thus, in Siev's approach, torsional stresses comes out as a secondary stress and it was Siev's conclusion that such secondary quantity can be neglected in design. He assumed that the bending, torsion, etc., are uncoupled. This is, in fact, not true.

While using the approximate methods, there is a general tendency of designing the stair with an unnecessarily high factor of safety with an eventual loss of slenderness and economy. An exact analysis of the problem by solving the differential equations of equilibrium and compatibility in accordance with the theory of elasticity is beyond the scope of existing rigorous mathematical methods. The only alternative is to employ a numerical technique. Among the available numerical methods, finite element approach is the most powerful one provided a high-speed digital computer is available. Rahman [8] and Adhikari [1] used finite element technique to analyse the problem. They used the thick-shell element developed by Ahmad [2]. They studied the same prototype used by Siev and Cusens et. al. to facilitate comparison. Their finite element study clearly established the characteristic three dimensional action of the stair. But their study was limited in scope, and no suggestions were made regarding the design of the stair. Adhikari did however show an example of reinforcement design using the stresses found from finite element analysis.

Amanat[4] carried out an extensive computational study on the free standing stair based on finite element modelling. He used Ahmad's general thick shell finite element to model the stair slabs. Based on a detailed parametric study, he proposed a few explicit equations for its design. These equations are in terms of some geometric parameters of a free standing stair. Once these geometric parameters are selected, various design forces and moments at different critical sections of the stair can be directly estimated from these equations. His study also revealed that bending moment at some critical locations are not uniformly distributed across the cross sections, rather moments are concentrated at one side of such critical sections in the lateral direction. These phenomena must be acknowledged in the reinforcement design according to his suggestions. Once this non-uniformity of stress distribution is taken into consideration, determination of the reinforcement becomes a routine matter.

1.3 OBJECTIVE OF THE STUDY

Amanat's[4] proposed approach include every possible geometric parameters of this stairs, each of which can be varied independently. However, in most practical cases it is observed that some of these parameters are interdependent and thus the dependent parameters could have been eliminated from the proposed equations, simplifying the process of analysis. Also, under symmetric loading and boundary conditions it may be argued that free standing stairs have only two redundants :

- Lateral Shear at Mid Landing Section, and
- Bending Moment at Mid Landing Section.

Once these two redundants are determined other forces and moments can be calculated using the simple equations of static equilibrium. This eliminates the use of other approximate equations of Amanat's approach. In most practical cases, stairways are symmetrical in nature. So, there is scope for further simplification of Amanat's proposed method for design of free standing stair which will be of great help to practising engineers and designers.

The objectives of this study are, therefore, as follows :

- To simplify the analysis and design procedure proposed by Amanat,
- To increase the applicable limit of Amanat's proposed design equations,
- To derive new emperical equations based on the sensitivity analysis of parameters, and
- To develop a design guide-line for free standing stairs, simpler than the present ones.

1.4 SCOPE AND METHODOLOGY OF RESEARCH

In an attempt to simplify the existing methods of analysis for free standing stairs the steps followed here are given below :

- The stair is modelled using Ahmad's[2] thick shell finite element. Validity and correctness of the model are established through comparison with Amanat's[4] results.
- A detailed parametric study is made under symmetric loading and boundary conditions. Based on this study, simplifications of Amanat's equations are proposed.
- Through the analysing of (a) Full live load over the entire stairway, and (b) Full live load only on the flights, over a larger applicable range, two sets of new equations are proposed for *Lateral Shear* and *Bending Moment at Mid Landing Section.*
- Other necessary equilibrium equations are derived in order to calculate the forces and moments at critical locations.
- Ten examples with varying range are solved and compared with both the Finite Element Method Model and Amanat's equations for establishing the validity of the proposed simplification, and
- A practical design example is presented to show the stepwise application of the proposed simplified equations.

CHAPTER 2

FREE STANDING STAIR PARAMETERS AND PROTOTYPE STAIRS

2.1 INTRODUCTION

The first part of this chapter deals with the definition of the geometry of the free standing stair, its loading, co-ordinate systems, sign convention, boundary conditions, redundants, and material properties. The last part of this chapter contains the geometric data of the ten prototype stairs, referred to in different chapters of this presentation.

2.2 GEOMETRY OF THE STAIR

Geometrically, a free standing stair is a structure composed of interconnected plates, each located in different planes in a three dimensional space, like the one in *Fig.1.1*(a). Usually it consists of three rectangular plates, namely,

- a lower flight,
- a landing, and
- a upper flight.

The upper end of the upper flight is held fixed at upper floor level. Similarly, the lower end of the lower flight is held fixed at lower floor level. The other ends of the flights are connected to a horizontal landing slab with no other support. Steps are provided on the flights.

According to the relative position of the flights, a free standing stair may be right handed or left handed, as illustrated in *Figs.2.1* and *2.2,* respectively. The front, back, right, left and inner sides of the stair are arbitrarily defined, as shown in *Fig.2.3.*

The geometry of a free standing stair may be symmetric or unsymmetric. In plan, if the flights are identical, the stair is symmetric. Otherwise, the stair is unsymmetric. In the present study, a right handed symmetric stair is considered.

The dimensions required to describe the geometry of the stair are shown in *Fig.2.4*. In this figure :

A = *Horizontal gap between flights*

B = *Width of landing slab*

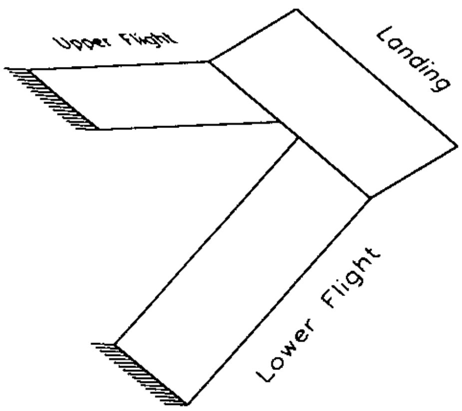


Fig 2.1 Right Handed Stair

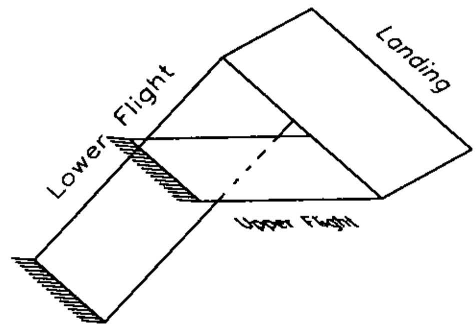


Fig 2.2 Left Handed Stair

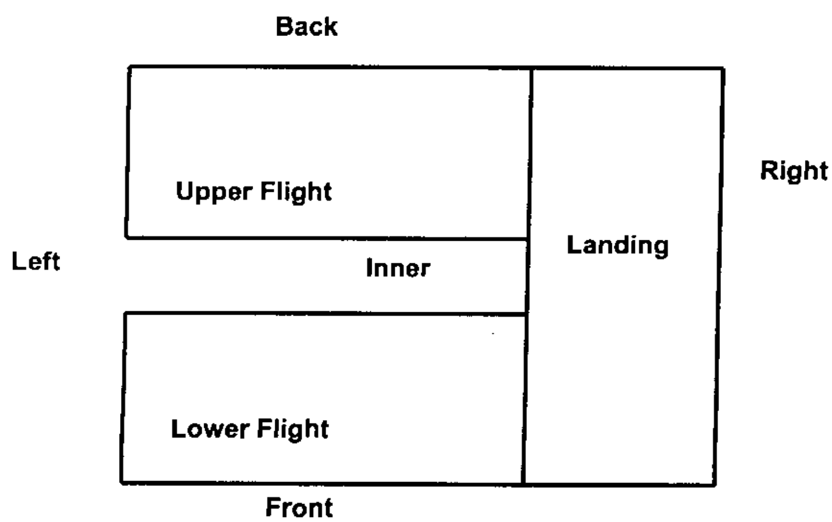


Fig. 2.3 Orientation of Different Location

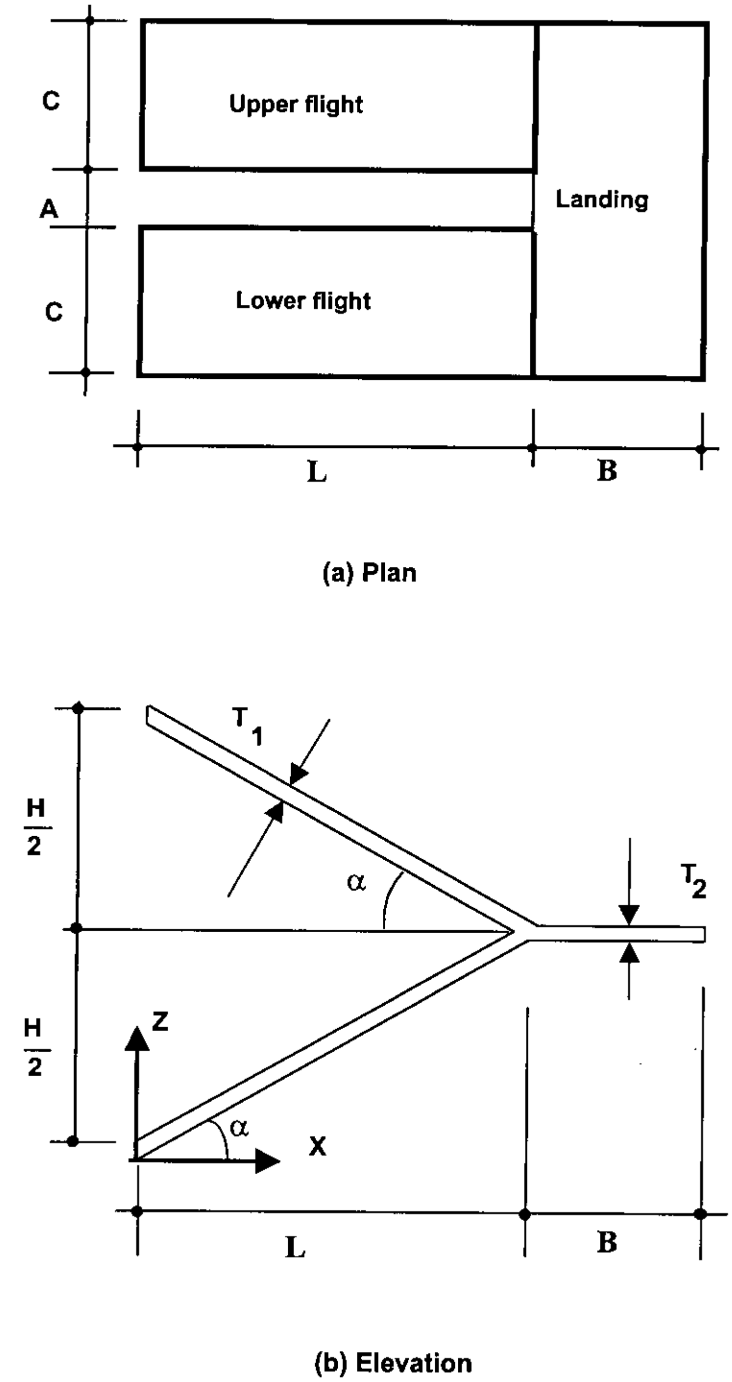


Fig. 2.4 Plan and Elevation of Free Standing Stair

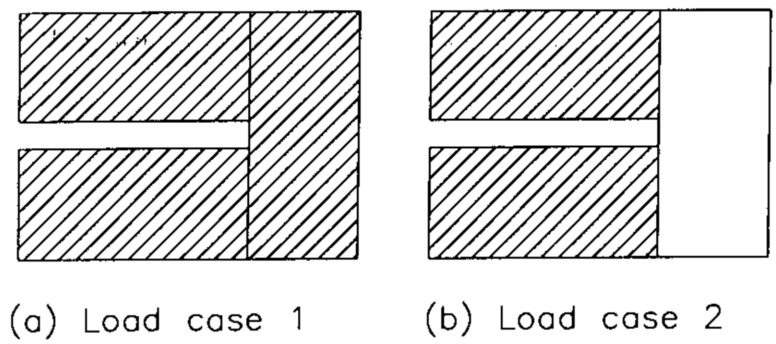


Fig. 2.5 Load Cases

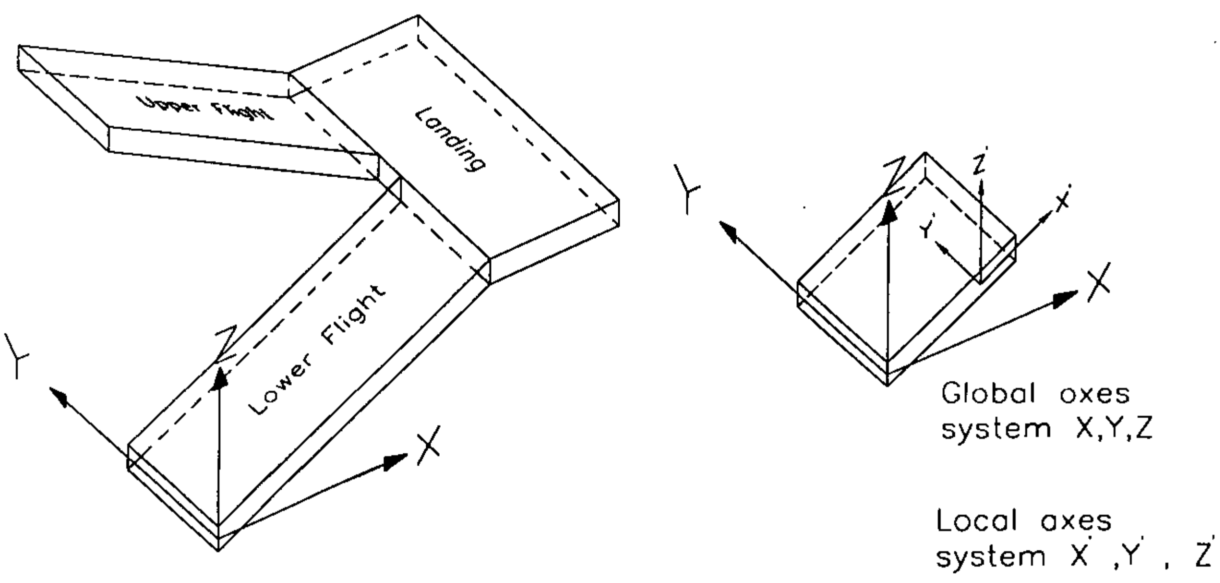


Fig. 2.6 Co-ordinate Systems

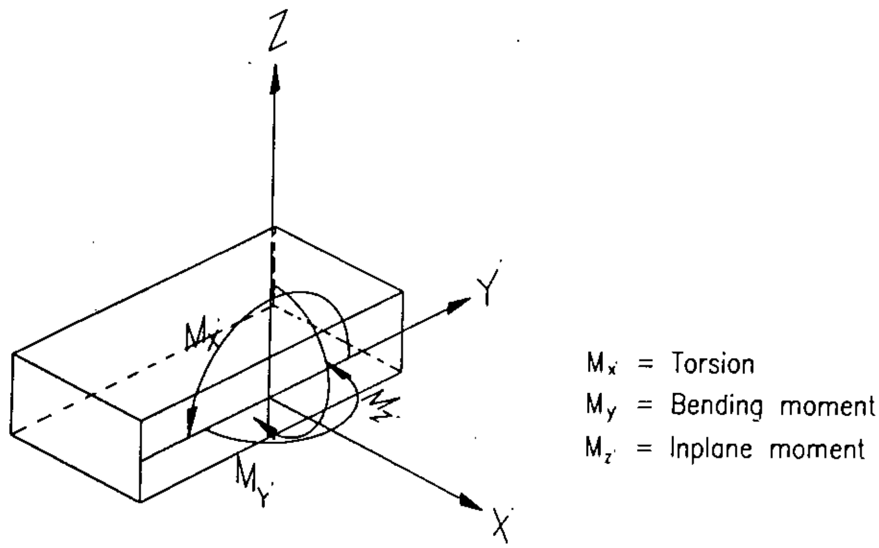


Fig. 2.7 Positive Direction of Moments with respect to Local Axes

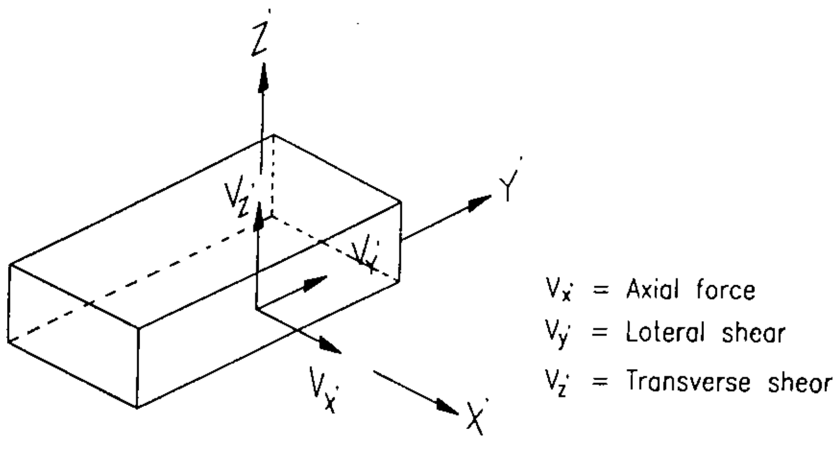


Fig. 2.8 Positive Direction of Forces with respect to Local Axes

C = *Width of flights*

L = *Horizontal span of flight*

H = *Floor to floor height*

T_1 = *Flight slab waist- thickness*

T_2 = *Landing-slab thickness*

2.3 LOADS ON THE STAIR SLABS:

The stair is primarily a functional unit in a building. As a consequence, live loads may act on it in several ways. But, for analysis and design, only those few possibilities are considered which produce maximum design moments and shears at some critical sections. In this study, only two possible load combinations are considered. These are,

a) Loading 1: Full live load over the entire stairway.

b) Loading 2: Full live load on the flights only

The above two loadings are schematically described in *Fig.2.5*, shaded portions indicating position of live load. Both the loadings are symmetric. There are other arrangements of live load such as full live load only on one of the flights. Previous findings of other investigators and of Amanat[4] revealed that such unsymmetric loadings produce unsymmetric stress in the stair. It was, however, observed that the magnituds of the stresses do not become critical for such unsymmetric loadings. Hence only the two loadings as described earlier, are considered in this study.

2.4 CO-ORDINATE SYSTEMS

Two types of co-ordinate systems are used for the free standing stair : (i) the global co-ordinate system and (ii) the local system, as shown in *Fig. 2.6*. The geometry of the stair is defined with respect to the global system. On the other hand, stresses, moments, etc., are calculated in local system. The global system for lower flight, landing and upper flight and the local system for lower flight are shown in *Fig. 2.6*.

2.5 SIGN CONVENTION

For consistency and for clarity of the analysis and of results presented in the subsequent chapters, a unique sign convention for stresses, moments, etc., is followed throughout.

Deflection is presented in global system and is positive in positive direction of axes. The positive directions of moments, shears, etc., at a section are defined with respect to local axes system. Stresses and forces are positive when they act in the positive direction of respective local axis. For moments, right hand screw rule is followed. These are defined in *Figs 2.7* and *2.8*.

2.6 BOUNDARY CONDITIONS AND REDUNDANTS

Although the free standing stair is a three dimensional plate structure involving 3-D interaction of plates, the current practice is to simplify it into a space frame structure consisting of linear bar elements. The boundary conditions are that the flight members are held completely fixed at their floor level supports. With such linear bar idealisation and boundary conditions, the stair slab becomes, in general, indeterminant of sixth degree. The three moments and tension and two shear forces can be taken as redundants or these actions at the mid landing section can be considered as redundants.

If both the stair and the loading are symmetric then, for the actual condition at mid landing section, the readers may refer to the two examples of *Irons and Ahmad [6]*.

In free standing stair, the landing is unsupported, or considered as supported by the upper and lower flights of stairs acting as beams in tension, torsion and bending. In *Fig. 2.9*, using mirror images, the deflection and the rotation in the YZ plane are zero in the middle of the landing, proving that the stair is symmetric about this line.

In *Fig. 2.10*, the interaction between the two half-landings is considered. Since it is monolithic, the bending and twisting moments and the tensile and shear stresses are considered. After the two mirror images the loads are acting at original direction but these four components are acting at opposite direction, summing these initial and final pictures, the values of the four components are zero as load is doubled, which prove that they are non existent. Thus, *Fig. 2.10* shows that the tensile and vertical shear forces, σ_{YY} and τ_{YZ} are zero. Also the bending and twisting moments in the XZ and XY planes are zero.

In *Fig. 2.11* the other two remaining components, lateral shear and bending moment at mid landing section, after the two mirror images are not opposite. This proves the existence of these two components at mid landing section.

From the above discussion it is understood that, if both the flights and the loading are symmetric, the degree of static indeterminacy becomes two. Here, the bending moment

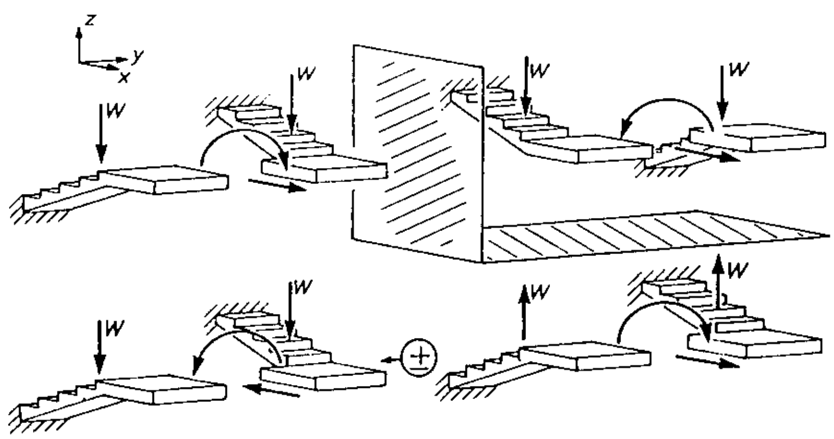


Fig. 2.9 Symmetry about Mid Landing Section

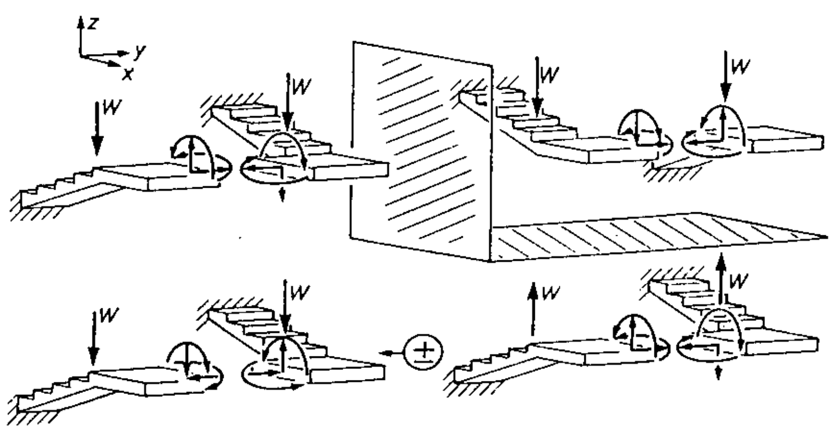
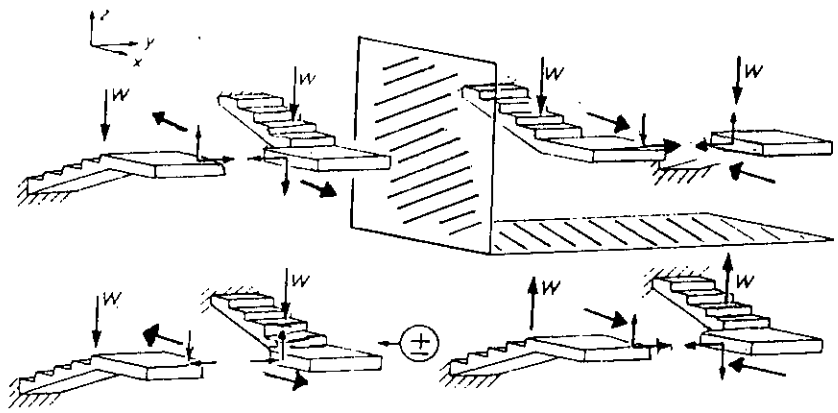


Fig. 2.10 Zero Forces and Moments at the Mid Landing Section

(a)



(b)

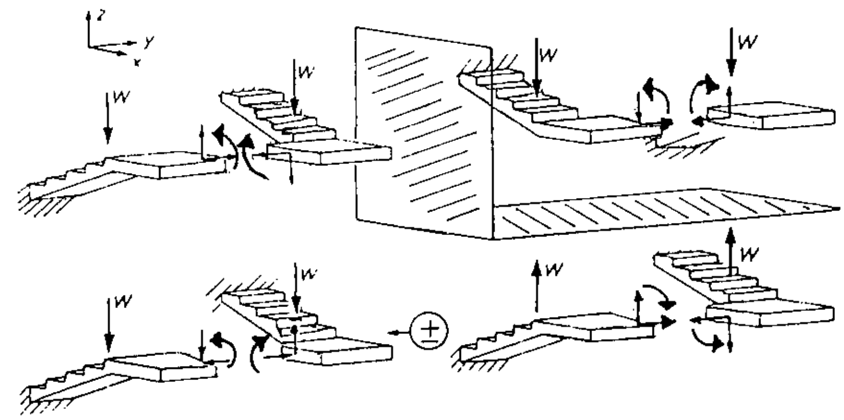


Fig. 2.11 Redundants at Mid Landing Section

(a) Lateral Shear

(b) Bending Moment

M_0 and in-plane shear V_0 at mid-landing section can be considered as the two redundants (*Fig.2.12*).

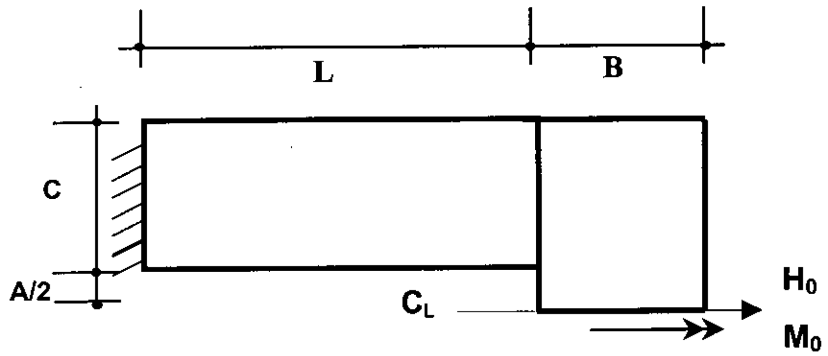


Fig. 2.12 Planar Presentation of Redundants at Mid Landing Section

2.7 MATERIAL PROPERTIES:

The stair under study is a monolithic reinforced concrete structure, built integrally with supports. Reinforced concrete is a non-homogeneous material. From past experience it is found that no serious error occurs if reinforced concrete is considered homogeneous, provided that the stresses are within allowable limits. Assumption of reinforced concrete as a homogeneous material greatly simplifies the analysis of structures. Keeping this in mind, in this study, it is assumed that the reinforced concrete used in the stair-slab is a homogeneous, isotropic material and it obeys Hooke's law. In the proposed equations a factor for f_c' (28 day cylinder strength of concrete) is introduced but for the prototype stairs $f_c' = 20.68\ Mpa$ and the Poisson's ratio of concrete, $\nu = 0.15$ is used all through.

2.8 THE PROTOTYPE STAIR SLABS:

Ten prototype stairs of different dimensions, but of common properties, are studied here. These studies of stairs are compared later with their finite element results and with the predictions of equations proposed by Amanat. The data used for the prototype stairs are given in Table 2.1.

Table 2.1 Geometric Data of the Prototype Stairs

Example	Horizontal gap between flights A(mm)	Width of landing B(mm)	Width of flight C(mm)	Horizontal projection of flight L(mm)	Floor height H(mm)	Slab thickness T(mm)
1	510	915	915	2290	2745	130
2	710	1525	1220	2540	3050	150
3	800	1175	1100	2800	3250	180
4	900	1700	1750	3250	3500	230
5	950	1475	1800	3450	4000	260
6	1100	2000	2000	3700	4500	290
7	1200	2200	2200	3800	4600	180
8	1300	2300	2300	3900	4700	300
9	1400	2400	2400	3950	4850	310
10	1500	2500	2500	4000	5000	330

Loads

Dead load = as calculated

live load = 4786.4 Pa on horizontal projection.

Material properties

Concrete strength, f_c' = 20.68 MPa

Yield strength of steel, f_y = 275.84 MPa

Poisson's ratio of concrete, ν = 0.15

Modulus of elasticity of concrete, E_C = $4700\sqrt{f_c'}$ MPa

CHAPTER 3

LITERATURE REVIEW

3.1 INTRODUCTION

This chapter reviews the earlier analyses of free standing stairs by different researchers with particular attention to the assumptions used in their analyses.

Shell or plate theories generally lead to considerable complexities in the analysis and design of stairs. It is possible to analyse the stair using an analytical method or a modern numerical technique such as finite element method. But practicing engineers eagerly look for a simple and straight forward design procedure which may be somewhat approximate but rational. Such demand resulted in approximation of the stair-slab as a space frame structure composed of bar elements or as a determinate slab structure. A number of papers on these approximate methods of solution of free standing stair are available in the literature. Amanat[4] made an useful attempt to propose a simple and direct way of finding the moments and forces required to design a free standing stair, based on finite element approach.

3.2 CODES AND PRACTICES

The structural behaviour of a free standing stair is quite different from other ordinary two flight stairs where the landings are supported by beams or side walls. In the ACI and British Code of Practices there are no specific guidelines for the design of a free standing stair. It is true for the Indian and the other codes of practices. Absence of guidelines in these codes discourages practicing engineers to design or construct a free standing stair. Reynold's[9] handbook includes a design method for a free standing stair based on the paper published by Cusens and Kuang [5]. But the procedure described there is not very straight forward and is somewhat abstract due to some simplification.

3.3 ANALYTICAL APPROACHES

The analytical approaches for the free standing stair slabs may be divided in to two main types. The first type idealises the stair structure as a space frame of some type. The methods of Sauter[10], Cusens and Kuang[5], Taleb[12] fall in this category. Idealisation of a plate as a straight frame element can only be justified when the width to length ratio of the plate is small. But this is not true for a free standing stair where width of plates is of comparable magnitude with respect to its length. In such cases, space frame idealisation

(a)

(b)

(c)

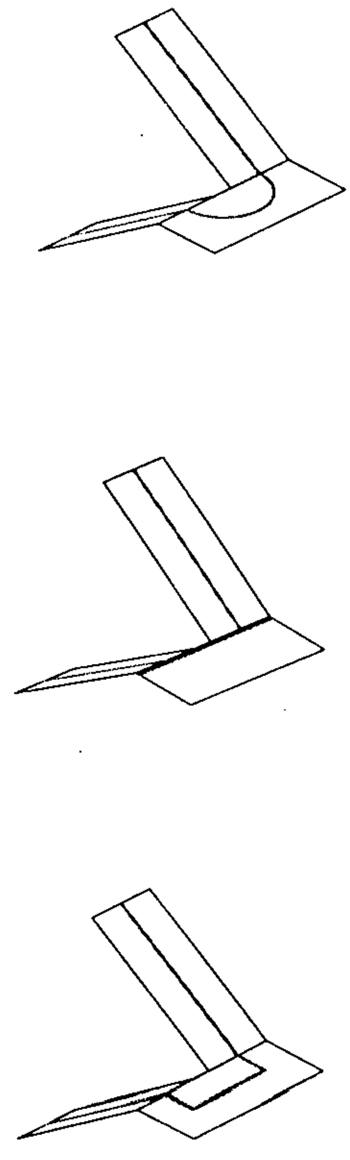


Fig 3.1 Idealisation in Different Analytical Approaches

a) Fuchssteiner, Sauter

b) Cusens & Kuang

c) Taleb

does not seem to be a good approximation. The second type retains the plate configuration of the stair but considers the structure to be a determinate one based on some assumptions. The overall structural rigidity resulting from the indeterminacy is lost when such assumptions are made. The methods of Siev[11] and Liebenberg[7] are of this type.

Of the various analytical approaches, those of Cusens and Kuang, and of Siev's are preferred in design because of their relatively simple structural idealisation and analyses. The analysis suggested by Siev is the most elaborate one. However, the analysis becomes simple, only when, the secondary stresses are ignored. In Sauter's approach, once the bar idealisation is made, the remaining treatment for analysis is rigorous which is basically the same as the flexibility method of structural frame analysis. Some of the useful approaches are discussed in the following articles.

3.3.1 CUSENS AND KUANG'S METHOD

Cusens and Kuang[5] analysed the free standing stair assuming that its structural behaviour could be simulated by the skeletal rigid frame shown in *Fig.3.2.* In their procedure a cut is introduced at the mid landing section and the horizontal restraining force V_{yo} and bending moment M_{yo} are applied at the two halves of the frame. These two quantities are taken as the redundants. Equations for bending moment, shear and axial forces, etc., in the flights are then readily expressed in terms of these two redundants .

Neglecting the effect of axial and shearing forces on deformation, the total strain energy in the structure is evaluated in terms of V_{yo} and M_{yo}. Following the concept of least work, the first partial derivative of the strain energy with respect to V_{yo} and M_{yo} are evaluated and then they are equated to zero. This results in a set of two simultaneous equations in terms of V_{yo} and M_{yo}. Solving these two simultaneous equations, one can determine the magnitudes of V_{yo} and M_{yo}. Once these redundants are known, the structure becomes statically determinate and the shear forces, bending moments and axial forces at any point of the structure can be calculated using equations of static equilibrium. Cusens and Kuang[5], in their paper, dealt only symmetrical stair cases under symmetrical loading.

3.3.2 SAUTER'S METHOD

Sauter's[10] method of analysis is based on considering the stair as a space frame composed of linear bar elements. The idealised stair frame consists of two cantilevered

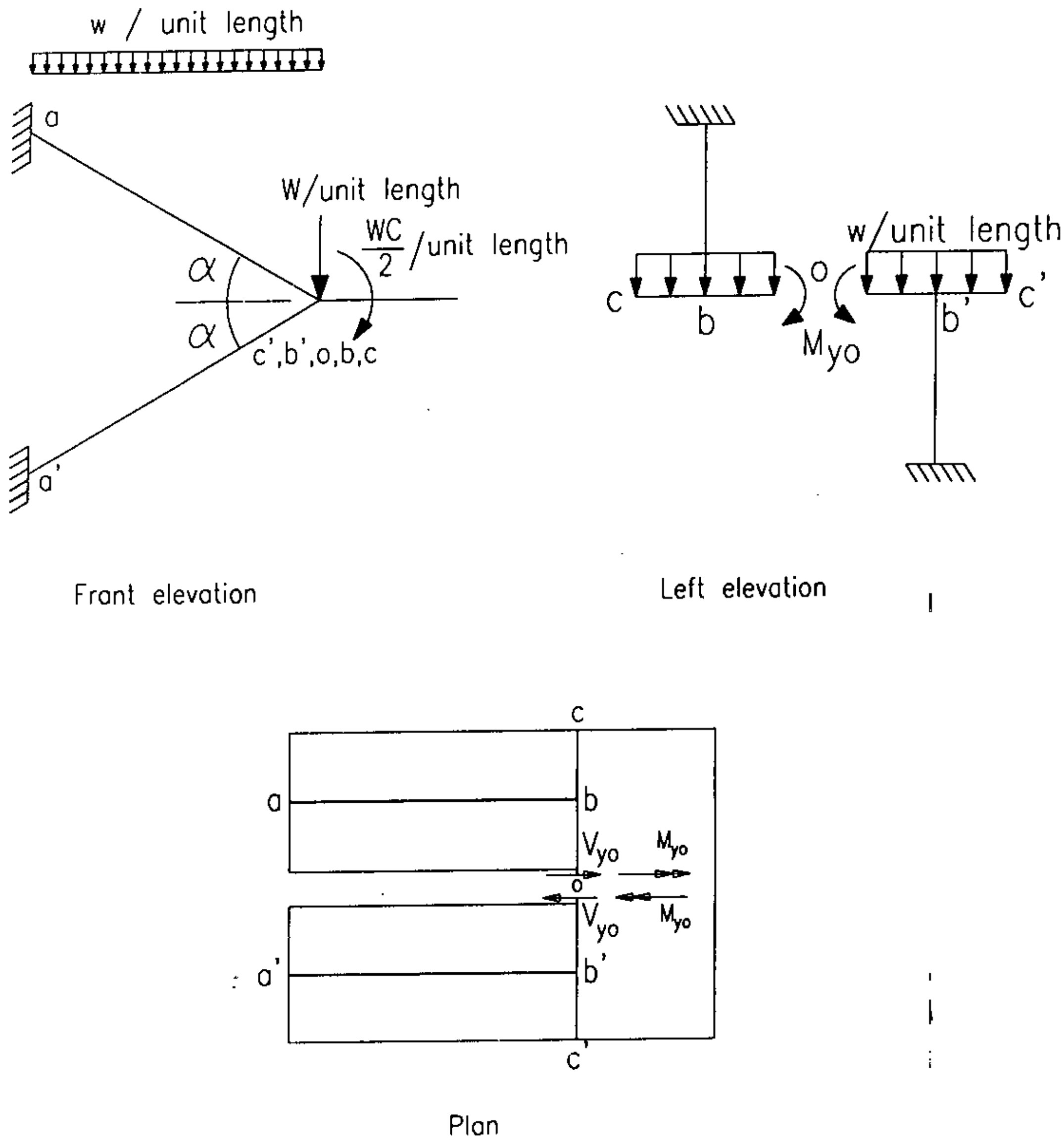
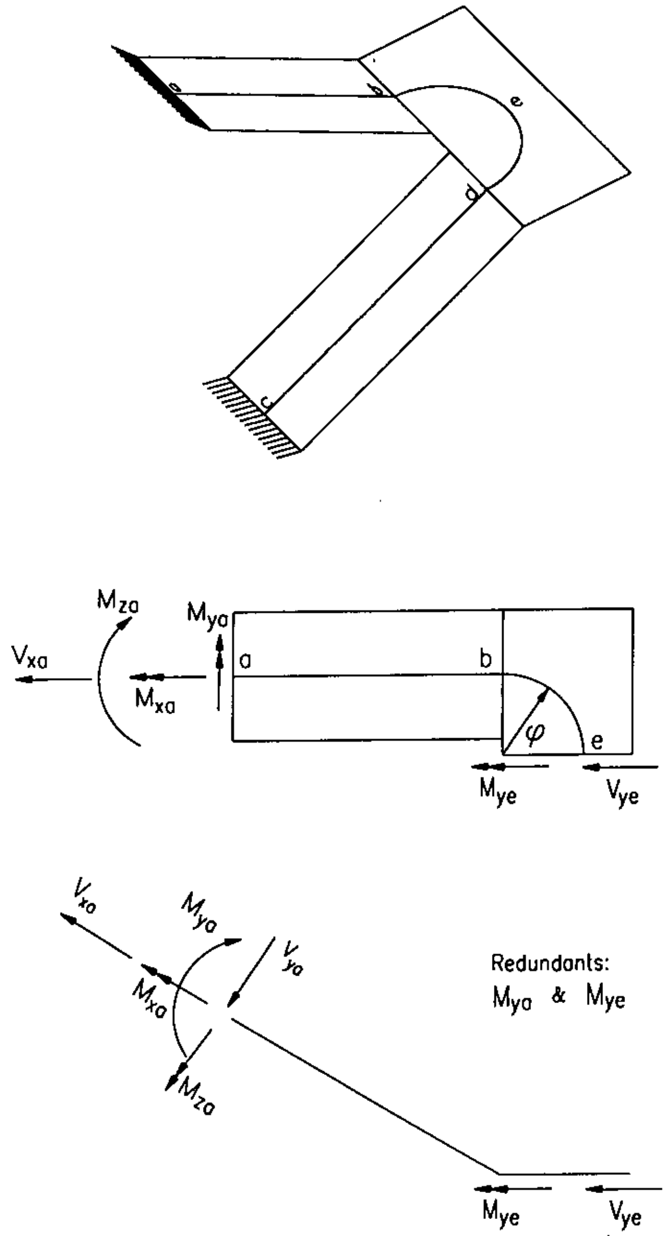


Fig. 3.2 Cusens and Kuang's Method



Moments and forces acting on the stair slab

Fig. 3.3 Sauter's Method of Analysis

straight members projecting out from the upper and lower floor supports and connected at their ends by a horizontal bow girder representing the landing. In his method the two redundants are the bending moments M_y at upper flight support and at mid-landing section.

First, the equations of moments and forces are written for unit values of the redundants in turn for the unloaded structure and corresponding deformations are calculated using work integral. Next, considering the redundants to be zero, the structure is analysed as a determinate structure for unit distributed load in flights and then on landing separately. Deformations are also evaluated for these loading in the usual manner. In evaluating the work integrals, effect of shear and axial forces are neglected. Once the deformations are determined for each separate case, the redundants can be calculated from compatibility conditions of deformations *(Fig.3.3)*. In his analysis, Sauter dealt only symmetric stair slab with symmetric loading.

3.3.3 SIEV'S METHOD

Siev [11] developed methods for solving stresses in free standing stair slabs under symmetrical and anti-symmetrical loading conditions based on the concept of statically determinate structures. In his procedure the overall stress analysis is accomplished in two stages. In the first stage, the line of intersection between flights and landing is considered simply supported *(Fig.3.4)*. The resulting structure is referred to as the primary slab structure. Bending moments and reactions at the supports are calculated under various possible positions of live load. In the second stage, the flights and landing are considered as interconnected plates. This is referred to as the secondary plate structure. The reaction at the assumed kink line support will act as the only load in opposite direction on this plate structure. This load is then resolved in the plane of flights, which develops in-plane moment and axial forces in flight slabs. Due to these forces, in-plane deflection of flights occur which is calculated using the ordinary flexure formulas. Assuming that the landing slab is very rigid in its plane, compatibility in deformation requires that the kink line will remain straight.

From this compatibility condition, the secondary stresses are determined. Torsional moments are calculated as secondary stresses. The final solution is the combination of primary and secondary stresses. Later it was Siev's conclusion that the secondary effects can be ignored in design.

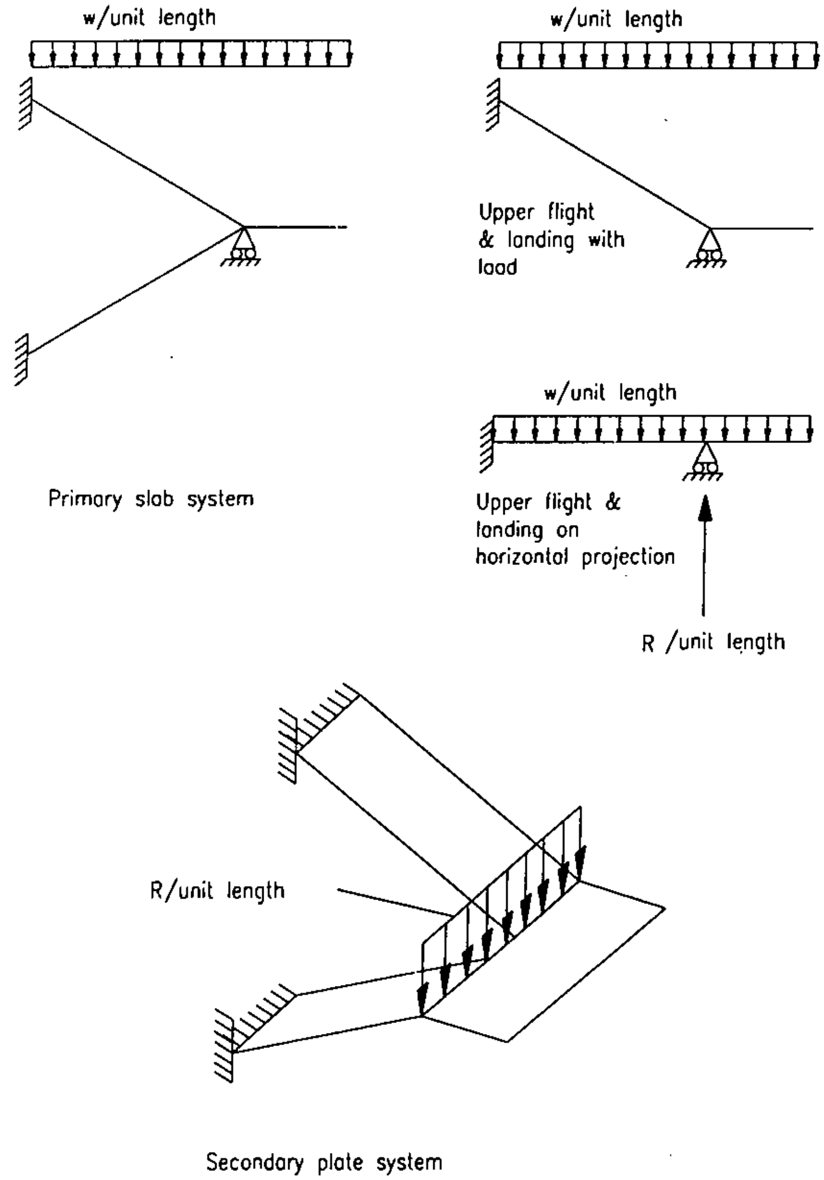


Fig. 3.4 Siev's Method of Analysis

3.4 FINITE ELEMENT APPROACH

The analytical approaches are not fully dependable for the analysis of free standing stair as long as economy and efficiency in design are concerned. These methods fail to simulate the actual interaction of plates in three dimension. Also, these approaches cannot demonstrate the variation of stress resultants along the width of the stair.

Amanat[4] found that a free standing stair is a three dimensional plate structure whose simplification to skeletal frame structure or to a determinate slab system is questionable. The structural behaviour of the stairway shows that load is transferred predominantly through in-plane bending and torsional moments, assisted by some axial forces. For a symmetric stair under symmetric loading, both flights are equally stressed. That is, stresses and deflections of one flight will be the mirror image of the same in other flight. This implies that the bending moment diagram of both flights will be the same in magnitude and in direction. Axial forces of equal magnitude but opposite sign will develop in flights. In-plane moment of the same magnitude and direction will be developed in flights while flights will be under equal and opposite torsion. The flights have usual transverse shear and zero lateral shear.

In landing, the predominant forces are the bending moments in both X and Y direction, lateral shear at mid-landing section and some torsion at quarter span. All the three components of displacements are equally dominant. Along with the deflection occurring vertically, the whole stairway experiences a horizontal sway towards upper flight.

Amanat demonstrated that the prediction of bending moments at different critical locations by approximate analytical methods might not always be acceptable. Support bending moment is overestimated in Cusens and Kuang's[5] approach and underestimated in Siev's[11] approach. In predicting the in-plane moment in landing, Siev's method seems to overestimate it. Sauter's[10] and Cusens and Kuang's approaches give reasonably good values of in-plane moment. In case of flight torsion all the analytical methods overestimate it substantially. For the mid-landing section, all the analytical methods underestimate the bending moments. Despite all such inaccuracies and approximations, some of these analytical methods have been used with success in the past. Conservative design approach combined with high factor of safety is attributed to be associated with such success.

With the advancement in the techniques of structural analysis it is now possible to analyse virtually all types of structures employing the finite element technique. But for the design of a small structure, like a stair, this technique seems to be too elaborate to be used in each and every case. A straightforward method of analysis that will enable to carry out the calculation more easily but with acceptable accuracy is preferable.

Amanat assumed that the stair is built of reinforced concrete. As a construction material, concrete is widely used. Now, if it is possible to develop explicit expressions for a conservative estimation of moments and forces at various critical locations within the limiting inaccuracy of 15% to 20%, it will greatly reduce the effort necessary in calculations and speed up the design process. Following the above discussion and the arguments forwarded therein, a set of expressions for moments and forces [Table 3.1] at the critical sections are suggested by Amanat. These expressions are valid within a certain range of variation of the geometric parameters of the stairs.

The equations are valid for fully fixed support condition. With the above limitations and assumptions, the equations are presented below. All of the following equations are of the form,

$$\textbf{Force or Moment} = K.F_A.F_B.F_C.F_L.F_H.F_T.F_f$$

Here K is a numeric constant. F_A is factor corresponding to the geometric parameter 'A' and so on. F_f is the factor corresponding to f_c'.

The uses of the above equations are straightforward. Values of forces and moments can readily be calculated once the design values of geometric parameters are prescribed.

Table 3.1 Equations Proposed by Amanat[4] (SI Unit)

		PARAMETRIC FUNCTIONS OF STAIR							
		F_A	F_B	F_C	F_L	F_H	F_T	F_f	
Design Parameters	Const.	150 < A(mm)< 1000	915 < B(mm) < 1875	915 < C(mm) < 1900	2030 <L(mm)< 3550	2440 < H(mm) < 4320	100 < T(mm) < 280	14 < f_c' (Mpa) < 40	Values of Design Parameters
Vertical deflection at landing corner, mm	2.03	$1+0.00545(A-125)^{0.94}$	$1+0.00114(B-914)^{1.1}$	$1+0.00165(C-914)^{0.93}$	$1-7.87x10^{-6}$ (L - 2030)	$1-19.68x10^{-6}$(H-2440)	$1-0.161\ (T-100)^{0.334}$	$1-1.074x10^{-6}(f_c'-14)^{.93}$	
Support negative moment, kN-m	-4.712	1.555+0.000787(A-50)	1.06-0.00022 (B-860)	1.2+0.00276(C-864)	1+0.000748(L-2030)	$1+5.9x10^{-6}$(H-2440)	0.39+0.00173(T-90)	1.0	
Flight midspan positive moment, kN-m	-1.526	$1.1-31.48x10^{-6}(A-150)^{1.52}$	$1-70.11x10^{-6}(B-915)^{1.365}$	1.0	$1+0.128x10^{-6}(L-2030)^{2.66}$	$1+0.899x10^{-9}(H-2440)^{2.77}$	$1-.00165(T-100)^{1.17}$	1	
Negative moment at kink, kN-m	-3.447	1.23+0.000512(A-125)	1.01+0.00323(B-915)	.85+0.000709(C-915)	1.0	1.0	$0.95+0.00447(T-100)^{1.03}$	1	
Negative moment at midsection of landing, kN-m	-6.14	1+0.000303(A-150)	1+0.00118(B-915)	1+0.00106(C-915)	1+0.000409(L-2030)	$1+26.37x10^{-6}$(H-2040)	1+0.00185(T-100)	1	
Axial force in flights, kN	34.69	1+0.000236(A-125)	1+0.000787(B-915)	1+0.000827(C-915)	1+0.000354(L-2030)	1-0.000157(H-2440)	1+0.00276(T-100)	1	
Torsion in flights, kN-m.	2.312	1+0.00177(A-125)	1+0.00063(B-915)	1+0.00268(C-915)	$1-8.0x10^{-6}(L-2030)^{.75}$	1.0	1+0.00358(T-100)	1	
Inplane moment in flights, kN-m	-14.35	1.1+0.000866(A-150)	1+0.000984(B-915)	1+0.00157(C-915)	1+0.00059(L-2030)	1-0.000197(H-2440)	1+0.0026(T-100)	1	
Lateral shear in mid-section of landing, kN	30.17	1-0.000276(A-150)	1+0.00138(B-915)	1+0.000709(C-915)	1+0.000669(L-2030)	1-0.00024(H-2440)	$1+0.000746(T-100)^{1.3}$	1	

CHAPTER 4

FINITE ELEMENT AND ANALYTICAL ANALYSES OF FREE STANDING STAIR

4.1 INTRODUCTION

Finite element technique is a powerful and versatile tool for the analysis of problems of structural and continuum mechanics. Analysis of the free standing stair using this technique is discussed here following that of simplified equilibrium approach.

The element-wise discretization procedure in finite element process reduces a continuum problem of infinite number of variables to one of a finite number of unknowns. The analysis of a continuum by finite element technique has three basic steps : structural idealisation which is the subdivision of the actual continuum into an assemblage of discrete structural elements, evaluation of element characteristics such as stiffness, stress and mass matrices and structural analysis of the element assemblage.

4.2 FINITE ELEMENT APPROACH

4.2.1 Choice of Element

In finite element technique, selection of a suitable element shape and type is important. There are various types of elements that is beam elements, frame elements, plate elements, shell elements, solid elements, etc. Each of these elements is suitable for some particular types of structures. For example, frame elements are suitable for ordinary beam-column latticed structures. Similarly, shell elements are suitable for shell structures. There are many variations of shell elements, each with their own characteristics. The free standing stair is a space structure composed of interconnected plates. Hence, shell elements are preferred for this structure.

4.2.2 Shell Elements

There are various types of shell elements developed so far. All of them fall in either of the two general categories, namely the thin shell elements and thick shell elements. In the thin shell elements, it is assumed that the normal to the middle surface before deformation remains normal after deformation. This means that the out of plane shear, that is, transverse shear stress is neglected in this type of elements. The thick shell elements include bending as well as shear deformations. In these elements, nodal lines are straight and 'normal' to the undeformed middle surface. They are inextensible and remain straight after deformation. However, they are in general not normal to the deformed middle surface after deformation, allowing the calculation of transverse shear stresses and strains. Consequently, this type of elements is suitable for modelling plates and shells where transverse shear is important. For the free standing stair under study, the 8 noded general thick shell element developed by Ahmad [3] is adopted.

4.2.3 Features of the Computer Program

Ahmad developed a computer program, which can analyse any shell or plate structure using the thick shell elements. The program was written in FORTRAN language and is quite general. It requires a considerable amount of data for input. This is the obvious consequence of a program, which is general in nature. The output of the program gives deflections and stresses at nodes with respect to global co-ordinate system. To simplify the input output process, separate computer programs were developed which can generate necessary data for the Ahmad's program from minimum input and can calculate the necessary design parameters such as bending moments, shear forces, etc., from the output of the general thick shell program.

4.2.4 Assumptions and Limitations

The thick shell elements developed by Ahmad are based on some assumptions such as the material within an element is isotropic and elastic and obeys Hooke's law. However such elastic properties are allowed to differ from element to element, which allows the program to handle structures made of composite materials. Besides these global assumptions, the additional assumptions, particular to the free standing stair are as follows:

- Waist-slab of flight and landing-slab the have same thickness.
- The additional stiffening effect provided by the steps to the waist-slab of flights is neglected. They only contribute to dead weight.
- The slab thickness is assumed to be sufficient to withstand the stresses developed and no account was given to slenderness.
- The supports at floor levels are assumed to be rigidly fixed.

4.2.5 Modification of the Thick Shell Program

Both the 8-noded and 12-noded thick shell elements of Ahmad[2] give very good results in predicting the normal stresses at nodes. But the out of plane shear stress at the nodal points is not very good. In fact, transverse shear is not very important in ordinary curved shell structures, as because such out of plane shear in doubly curved shell structures are usually low. But in free standing stair the transverse shear is important. So, in Amanat's work, some modifications were incorporated in the stress output routines. The program uses Gauss integration technique to evaluate the stiffness matrix of the elements and it was observed that Gauss points within the elements were the best sampling points for stress calculation with excellent accuracy. Details about the Gauss points of an element are given in the appendix. Shear and torsion are, therefore, calculated on the basis of the stresses at these Gauss points.

4.3 FINITE ELEMENT IDEALISATION OF THE FREE STANDING STAIR

4.3.1 The Thick Shell Element

The thick shell elements may be curved about either one or both of its planes and may have variable thickness along its axes. The edges of elements may thus be either curved or straight. The program allows the users to use any of the two types of elements, that is, either the 8-noded parabolic element or the 12-noded cubic element. In the present study the 8-noded elements are used.

Proper idealisation of a structure into an assemblage of finite number of elements is important in the analysis. The process involves division of the structure into elements in a suitable manner, numbering the elements and nodes in an efficient way, selection of local and global axes system, etc. Efficiency in these steps is necessary for optimal utilisation of the computer memory and computational speed. Some of these aspects of finite element idealisation of the stair slab are discussed in the following articles.

4.3.2 Element Mesh Configuration

A structure can be subdivided into elements in many possible ways. It depends on many factors such as the element characteristics, accuracy needed, available computer memory, etc. It is in general accepted that the finer the subdivision the better the idealisation. However, with proper choice of elements, the rate of gain in accuracy decreases after a certain degree of fineness in subdivision. Element mesh can be finer near points of stress concentration in comparison to the other parts of the structure. Considering all these factors the subdivision shown in *Figs.4.1* and *4.2* were adopted for the study. The floor level supports, flight-landing junction and the mid-landing section are the places of possible stress concentration and hence the element sizes there are taken to be smaller in these areas in comparison to the other parts of the stair.

4.3.3 Element and Node Numbering Scheme

The thick shell finite element program uses frontal solution technique to evaluate the displacement vectors of elements. Front width is dependent on the numbering sequence of elements. To keep the front width minimum the following two general rules are followed while numbering the elements:

- Numbering should preferably start at one end of the structure and terminate at the other end.
- The difference in element numbers between two adjacent elements should be kept minimum.

Following the above concept, the element numbering scheme shown in *Fig.4.1* is adopted here for the stair.

The original program requires the global co-ordinates of the nodes as input data. It is rather troublesome to enter the co-ordinates of all the nodes. However, the program offers one advantage in this regard. If only the corner nodes data are given, the program can automatically generate the co-ordinates of mid-side nodes assuming a straight line interpolation between the corner nodes. As the elements of the stair slab are perfectly rectangular, advantage of this feature of the program has been utilised in the study. First, the corner nodes are numbered from left to right starting from the front towards the back of the stair. There are 44 such corner nodes. Then the mid-side nodes are numbered in a systematic sequence as shown in *Fig.4.1*. Although nodes can be numbered in any arbitrary way, the regular pattern of *Fig.4.1* is adopted for ease in developing the data generation program.

4.3.4 Adjusted Unit Weight for Gravity Loading

The general thick shell program can accept loading data in many ways - depending upon the nature of loading. For the present study, live load due to transiting people is assumed uniformly distributed on horizontal plane. In this case the easiest way to incorporate the load is to increase the weight density of the material. The weights of the steps and uniformly distributed live load have been included in the stair slab by properly increasing

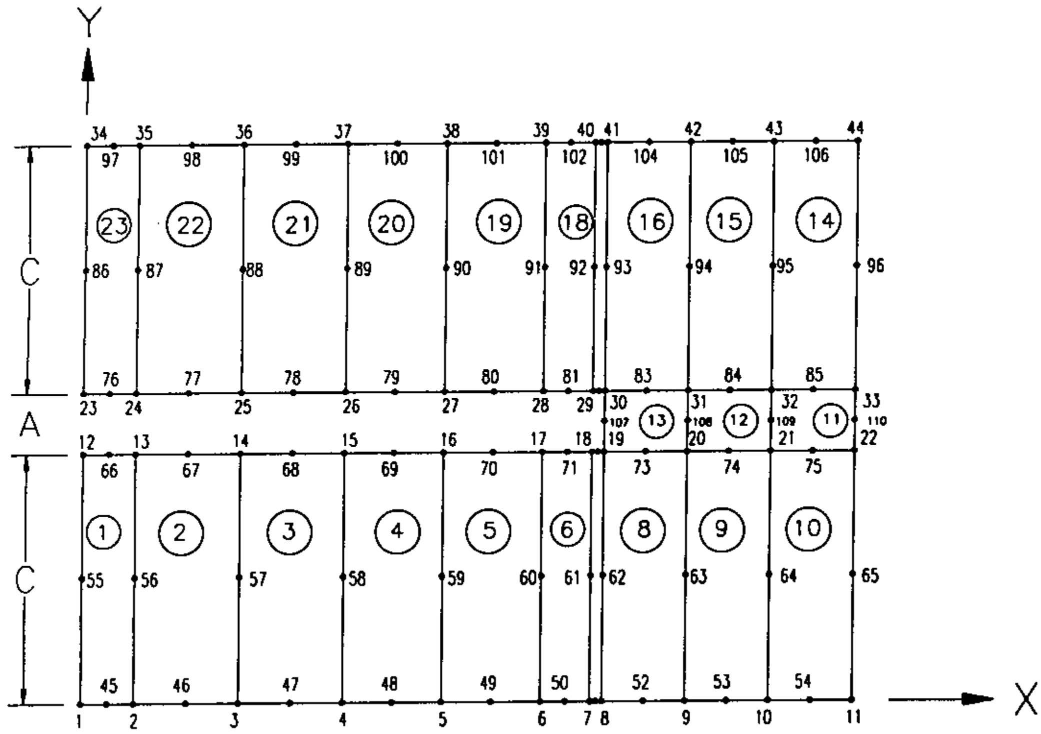


Fig. 4.1 Element and Nodal Numbering of the Stair(Horizontal Plane)

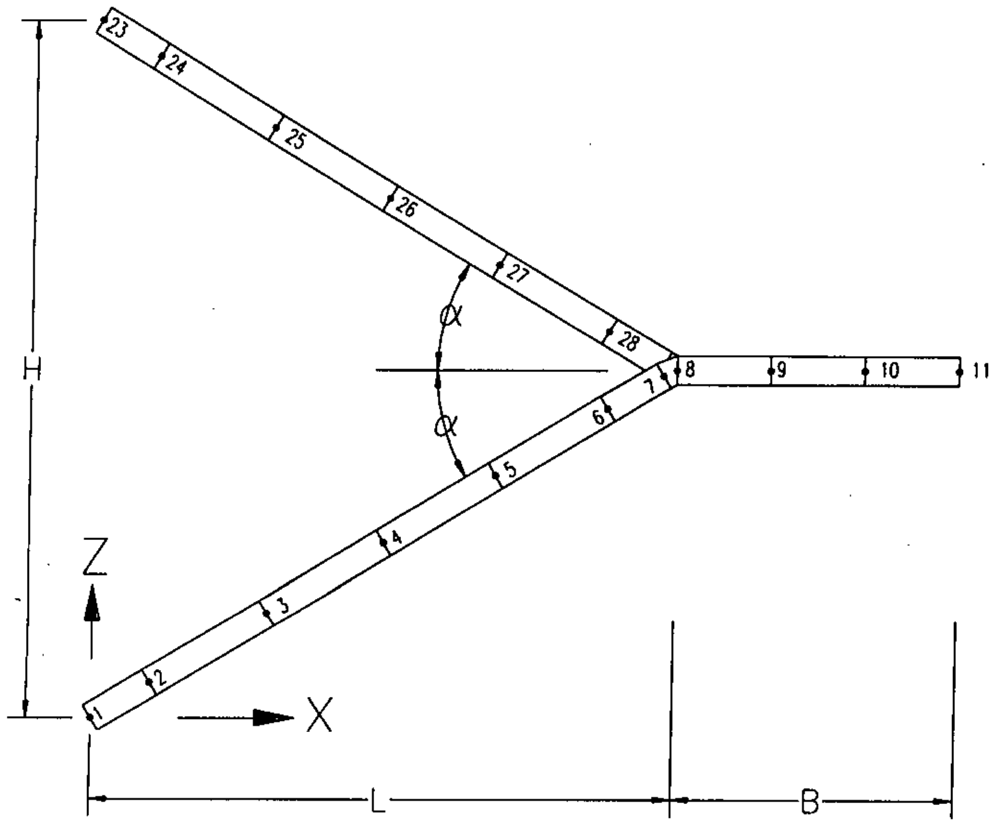


Fig. 4.2 Element and Nodal Numbering of the Stair(Vertical Plane)

the unit weight of slab material. It is observed that the flight slabs are inclined at an angle α whereas the landing slab is horizontal. For this reason the thickness measured vertically is different in flight slabs and in landing slab. Hence adjusted unit weight for flight slab and landing slab differ although the slab thickness is same everywhere.

4.3.5 Transformation of Stresses

The original finite element program gives stresses as output with respect to global co-ordinate system. The global stresses cannot be used directly for design. These are to be transformed into local stresses with respect to local axes system for determining forces and moments, which will subsequently be required for design.

The stress output is in tensor form with respect to global axes. The state of stress at any point can completely be defined by the stress tensor with respect to a set of three mutually perpendicular axes. If the stress tensor corresponding to X, Y, Z system is known, it can readily be transformed to any other X', Y', Z' axes system with the help of the well known stress transformation rule,

$$[\sigma'] = [A][\sigma][A]^T$$

where,

$[\sigma']$ = Stress tensor in X',Y',Z' system

$[\sigma]$ = Stress tensor in X,Y,Z system

[A] = Direction cosine matrix of X',Y',Z' system with respect to X,Y,Z system

$[A]^T$ = Transpose of [A]

This transformation rule can best be represented by the index notation as,

$$[\ i=1,3 \quad \text{and} \quad \sigma'_{ij} = \sum_{k=1}^{3} \sum_{l=1}^{3} a_{ik} \cdot \sigma_{kl} \cdot a_{lj} \quad j=1,3]$$

Here σ_{ij} and σ'_{ij} are the element at i-th row and j-th column of $[\sigma]$ and $[\sigma']$ respectively. The direction cosine matrix is :

$$[\mathbf{A}] = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix}$$

where ,

(a_{11}, a_{12}, a_{13}) = Direction cosines of X' axis w.r.t. X,Y, Z axes

(a_{21}, a_{22}, a_{23}) = Direction cosines of Y' axis w.r.t. X,Y,Z axes

(a_{31}, a_{32}, a_{33}) = Direction cosines of Z' axis w.r.t. X,Y,Z axes

4.3.6 Determination of Forces and Moments at a Section

The forces and moments that act on a transverse section are described in article 2.5. These quantities are required for design and calculated from the local nodal stresses. The procedure described below demonstrates how to calculate these forces from stresses. A transverse section with three nodes, numbered 1, 2 and 3, are shown in *Fig. 4.3.* where the stresses are:

σ = normal stress

τ = lateral shear stress

p= transverse shear stress

The stresses in *Fig. 4.3* have two subscripts. The first subscript indicates whether it is, at the top or bottom of the node, and the second subscript indicates the node. For example σ_{b1} means the normal stress at bottom of node 1. The calculation is carried out in two steps. First the nodal stress resultants are calculated as follows:

$$V_{x1} = \frac{(\sigma_{t1} + \sigma_{b1})}{2} t$$

$$V_{y1} = \frac{(\tau_{t1} + \tau_{b1})}{2} t$$

$$V_{z1} = \frac{(p_{t1} + p_{b1})}{2} t$$

$$M_{x1} = (\tau_{b1} - \tau_{t1}) . \frac{t^2}{12}$$

$$M_{y1} = (\sigma_{b1} - \sigma_{t1}) . \frac{t^2}{12}$$

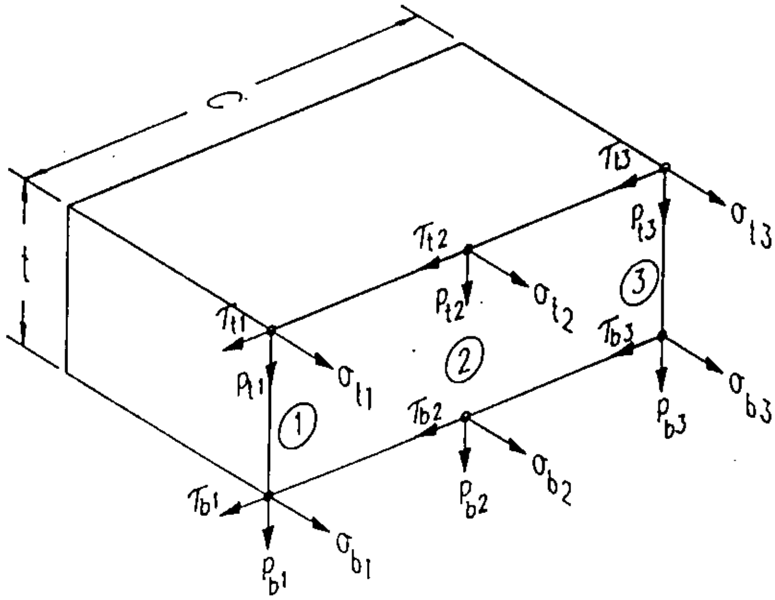


Fig 4.3 Calculation of Forces from Nodal Stresses

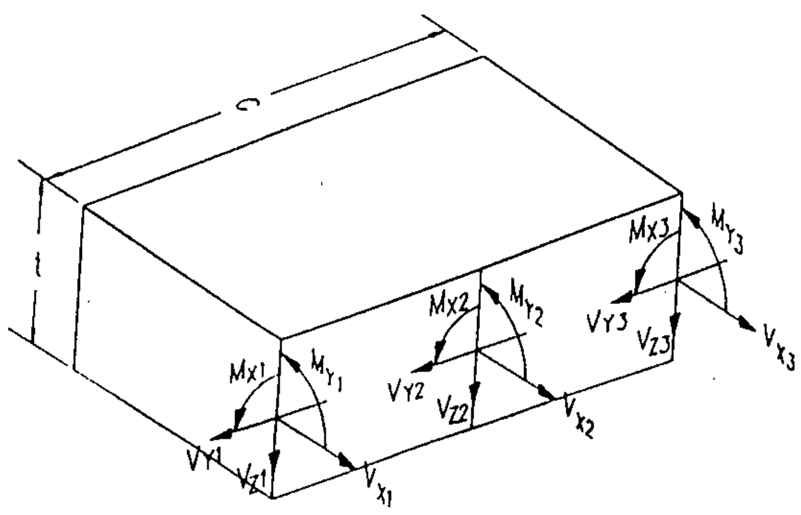


Fig 4.4 Calculation of Moments from Nodal Stresses

The above equations are for node 1 *(Fig.4.3).* Such quantities at other nodes can be calculated in a similar way. The forces and moments from stress resultants are calculated as follows. For a three noded section the variation of stress resultants across the width of the section can be assumed parabolic so that Simpson's integration rule can be applied. Thus:

$$V_x = (V_{x1} + 4V_{x2} + V_{x3}).\frac{C}{6}$$

$$V_y = (V_{y1} + 4V_{y2} + V_{y3}).\frac{C}{6}$$

$$V_z = (V_{z1} + 4V_{z2} + V_{z3}).\frac{C}{6}$$

$$M_x = (M_{x1} + 4M_{x2} + M_{x3}).\frac{C}{6} + (V_{z1} - V_{z3}).\frac{C^2}{12}$$

$$M_y = (M_{y1} + 4M_{y2} + M_{y3}).\frac{C}{6}$$

$$M_z = (V_{x3} - V_{x1}).\frac{C^2}{12}$$

If there are more than three nodes, a similar approach can be used with appropriate integration rule.

4.4 EQUILIBRIUM APPROACH

The critical sections for the analysis and design of free standing stair slab is shown in *Fig. 4.5.*

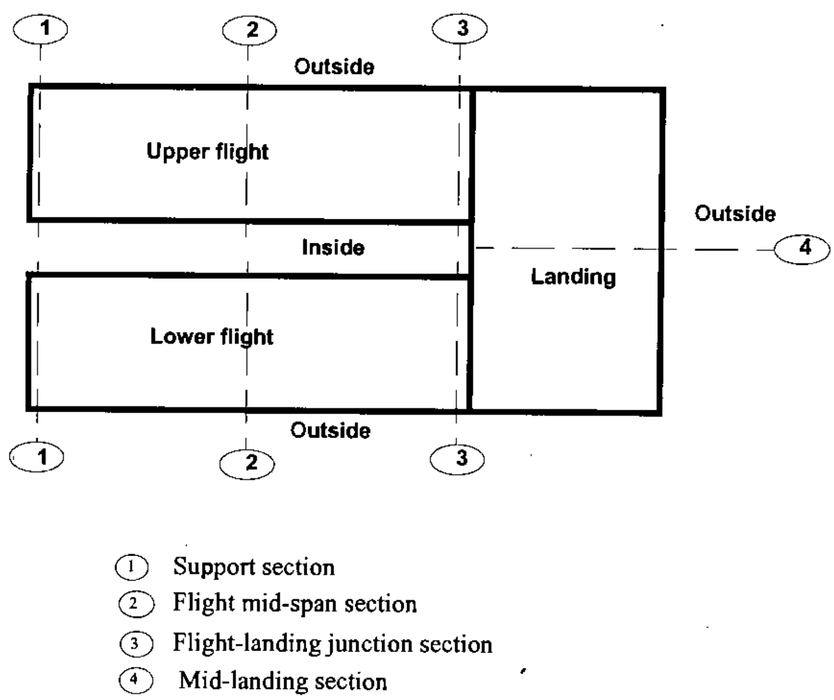


Fig.4.5 Critical Sections of the Stair

When both the stair slab and the loading are symmetric then the degree of static indeterminacy becomes two. The bending moment M_o and in-plane shear H_o at mid-landing section can only be considered as redundants(Section 4). The relevant quantities used in the equilibrium equations are shown below:

Live Load:

In the following process the Live Load is assumed as 4786.4 Pa which is equivalent to 100 psf, as used by Amanat[4].

Average Dead Load from Slab:

$$\frac{T(Thickness\ of\ slab\ \ \text{in mm})}{1000} \times 0.024 \times 10^6\ (Pa)$$

Average Dead Load from Steps:

$$\frac{1}{2} \times \frac{Rise \times Tread}{\sqrt{(Rise^2 + Tread^2)}} \times \frac{1}{1000} \times 0.024 \times 10^6\ (Pa)$$

W_1 (The load acting on mid point of flight):

$$\left[Live\ Load \times L \times C + (Dead\ Load\ from\ Slab + Dead\ Load\ from\ Steps) \times C \times \frac{L}{Cos\alpha}\right] \times \frac{1}{10^9}\ (kN)$$

W_2(The load acting on mid point of landing):

Loading 1:

$$(Live\ \ Load\ \ +\ Dead\ \ Load\ \ from\ \ Slab\ ) \times B \times \frac{\left(C + \frac{A}{2}\right)}{10^9}\ (kN\ )$$

Loading 2:

$$(Dead\ \ Load\ \ from\ \ Slab\ ) \times B \times \frac{\left(C + \frac{A}{2}\right)}{10^9}\ (kN\ )$$

After calculating the values of the bending moment M_o and in-plane shear H_o at mid-landing section by the empirical equations the equation of forces and moments at other

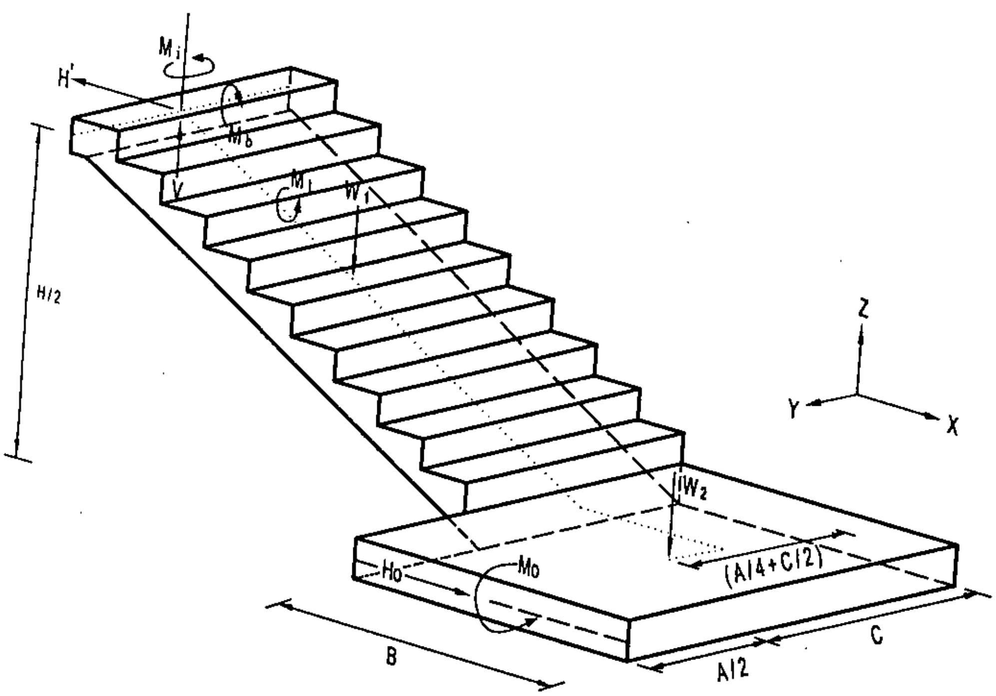


Fig. 4.6 Free Body of Free Standing Stair at Equilibrium State

critical location can be found from the equations of equilibrium*(Fig. 4.6)*. The required equations are stated below:

1) Support Negative Moment

$$= -H_0\left(\frac{H}{2}\right) + W_2\left(\frac{B}{2} + L\right) + W_1\left(\frac{L}{2}\right) \tag{4.1}$$

2) Flight Midspan Positive Moment

$$= H_0\left(\frac{H}{4}\right) - W_2\left(\frac{L}{2} + \frac{B}{2}\right) - \left(\frac{W_1}{2}\right)\frac{L}{4} \quad \text{(From right side)} \tag{4.2}$$

$$= -M_b - H_0 \times \frac{H}{4} + \frac{(W_1 + W_2)L}{2} - \frac{W_1}{2} \times \frac{L}{4} \, \text{(From left side)} \tag{4.3}$$

3) *Negative Moment at Kink*

$$= W_2 \times \frac{B}{2} \tag{4.4}$$

4) Axial Force in Flights

$= \text{H}'\text{(Horizontal Force) Cos}\alpha + \text{V(Vertical Force) Sin}\alpha$

$$= H_0 Cos\alpha + (W_1 + W_2)\, Sin\alpha \tag{4.5}$$

5)Torsion In Flights

$$= \left\{M_0 + W_2 \times \frac{A}{4}\right\} Cos\alpha - H_0 Sin\alpha\left(\frac{C}{2} + \frac{A}{2}\right) \tag{4.6}$$

6) In Plane Moment in Flights

$$= H_0\left(\frac{C}{2} + \frac{A}{2}\right) Cos\alpha + W_2 Sin\alpha \times \frac{A}{4} + M_0 Sin\alpha \tag{4.7}$$

CHAPTER 5

SENSITIVITY ANALYSIS

5.1 INTRODUCTION

The approximate analytical approaches and the finite element approach for the analysis of the free standing stairway have been discussed in chapters three and four, respectively. A stairway of most common dimensions is used throughout, as the model of analysis. Changing one dimension within the selected range and keeping the other dimensions constant the sensitivity analysis is done in order to analysed the effect of these parameters and find some new sets of equations for a new design rationale.

5.2 STAIR DESIGN PARAMETERS

To determine the forces and moments at every section it is possible to analyse the free standing stair using any suitable method, but in this equilibrium approach, because of symmetry, only two redundants required are :

- Lateral shear at mid-landing section, and
- Bending moment at mid-landing section.

The other necessary quantities at the critical locations can be derived from the equations of equilibrium. So extensive parametric study is done to verify the change in these two design parameters with the change of geometric parameters. The study is made for the two cases of loading. The general arrangements and boundary conditions defined in chapter two are maintained throughout this study.

5.3 GEOMETRIC PARAMETERS OF THE STAIR

The effects of geometric parameters studied here are :

- Horizontal gap between flight, A
- Width of landing, B
- Width of flights, C
- Horizontal span of flights, L
- Total height of stairway, H
- Slab thickness, T_1 and T_2

In this study the thickness of the flight waist-slab and landing-slab are kept the same ($T_1 = T_2$). Each of these parameters is varied independently keeping the other remaining parameters constant at their most usual values. The most usual values are taken as:

Horizontal gap between flight (A)	= 300mm
Width of landing (B)	= 1250mm
Width of flights(C)	= 1250mm
Horizontal span of flights (L)	= 2800mm
Total height of stairway (H)	= 3350mm
Slab thickness ($T= T_1,T_2$)	= 100mm
Rise	= 150 mm
Tread	= 150/Tanα = 250.75 mm

The range of the data, which is used in this sensitivity analysis, is presented in table 5.1.

Table 5.1 Range of Geometric Parameters Used in Sensitivity Analysis.

Geometric parameters	Range
A	150 to 1500 mm
B	915 to 2500 mm
C	915 to 2500 mm
L	2030 to 4000 mm
H	2440 to 5000 mm
T	100 to 350 mm
f_c'	14 to 40 MPa

5.4 RESULTS OF PARAMETRIC STUDY

In the following sections the findings of the parametric study are discussed.

a) The effect of variation of horizontal gap between flights (A), on lateral shear and bending moment at mid landing section, for both the loading cases, is presented in *Figs. 5.1* and *5.2*. In both the loading cases, the lateral shear at mid landing section decreased

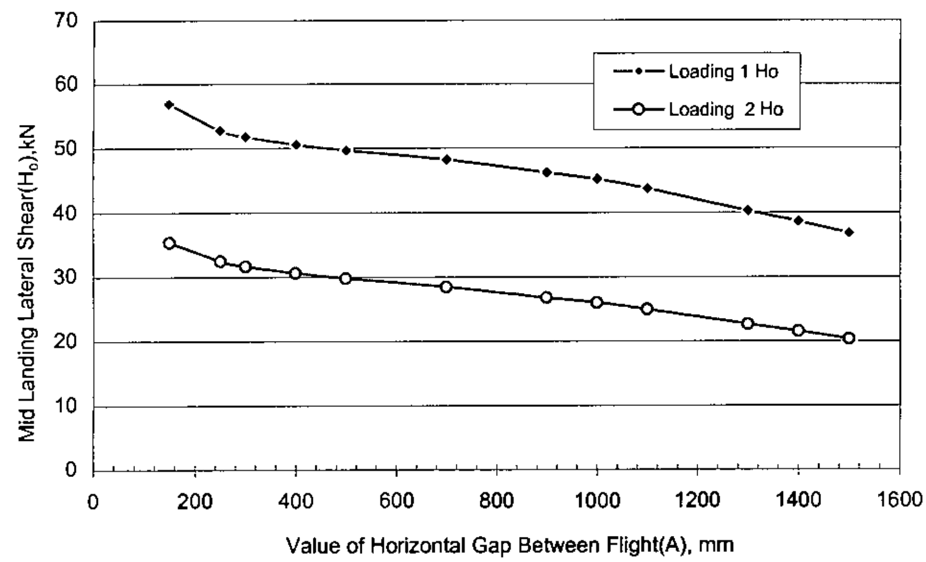


Fig. 5.1 Variation of Mid Landing Lateral Shear Due to Variation of Horizontal Gap Between flight(A)

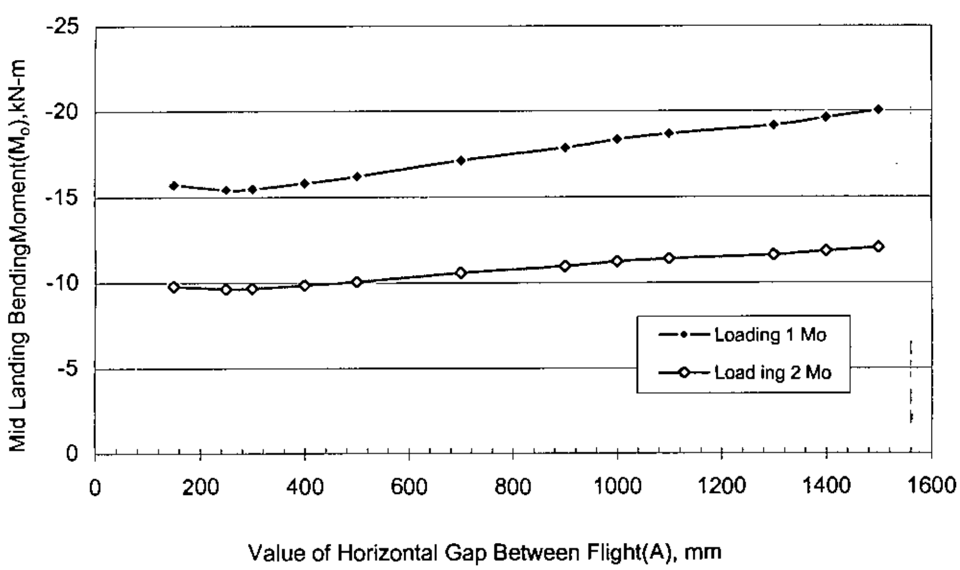


Fig. 5.2 Variation of Mid Landing Bending Moment Due to Variation of Horizontal Gap Between Flight(A)

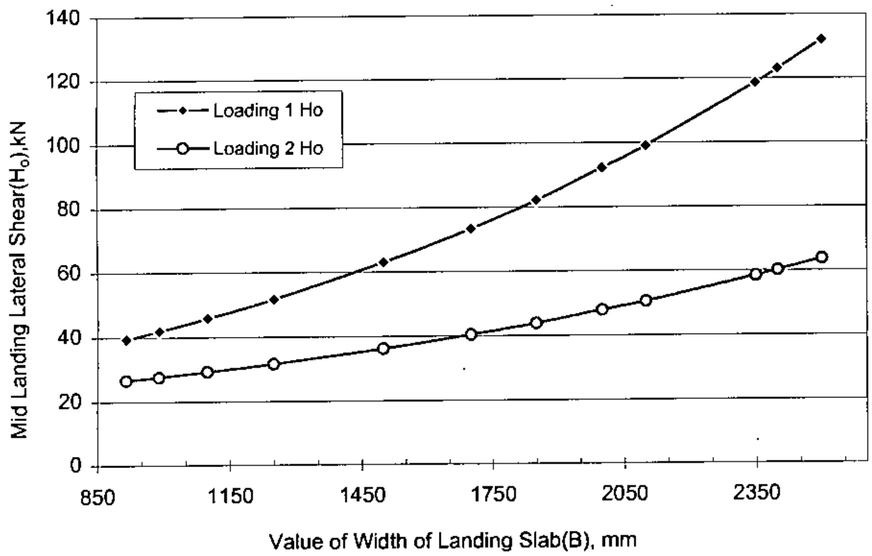


Fig. 5.3 Variation of Mid Landing Lateral Shear Due to Variation of Width of Landing Slab(B)

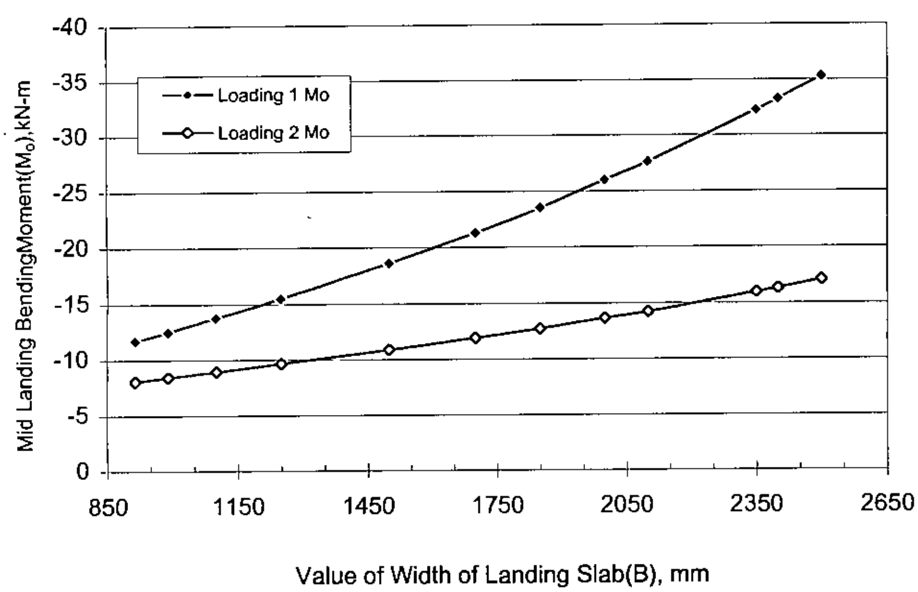


Fig. 5.4 Variation of Mid Landing Bending Moment Due to Variation of Width of Landing Slab(B)

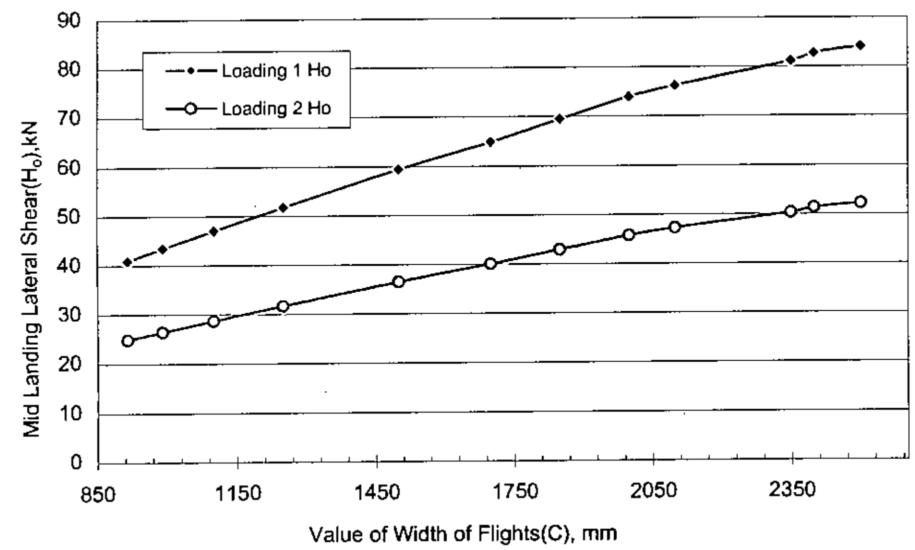


Fig. 5.5 Variation of Mid Landing Lateral Shear Due to Variation of Width of Flights(C)

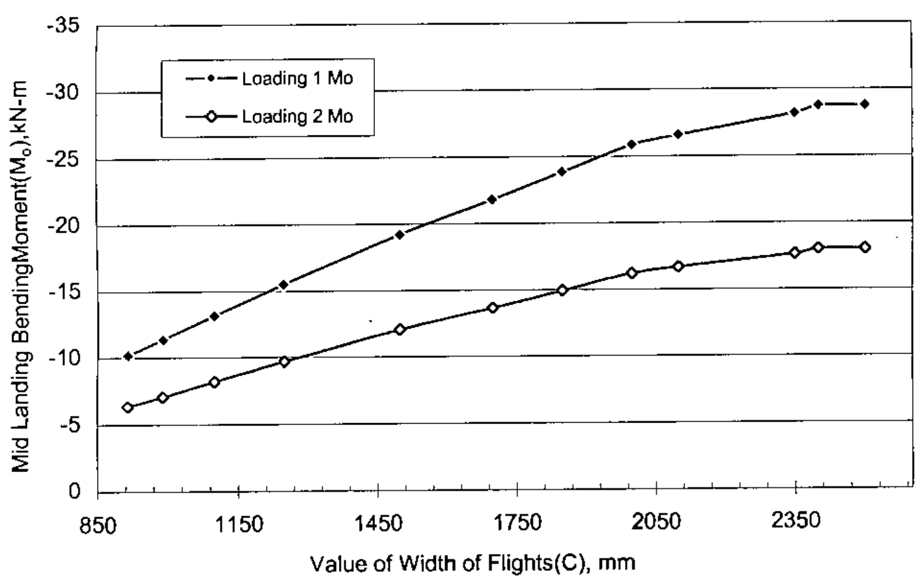


Fig. 5.6 Variation of Mid Landing Bending Moment Due to Variation of Width of Flights(C)

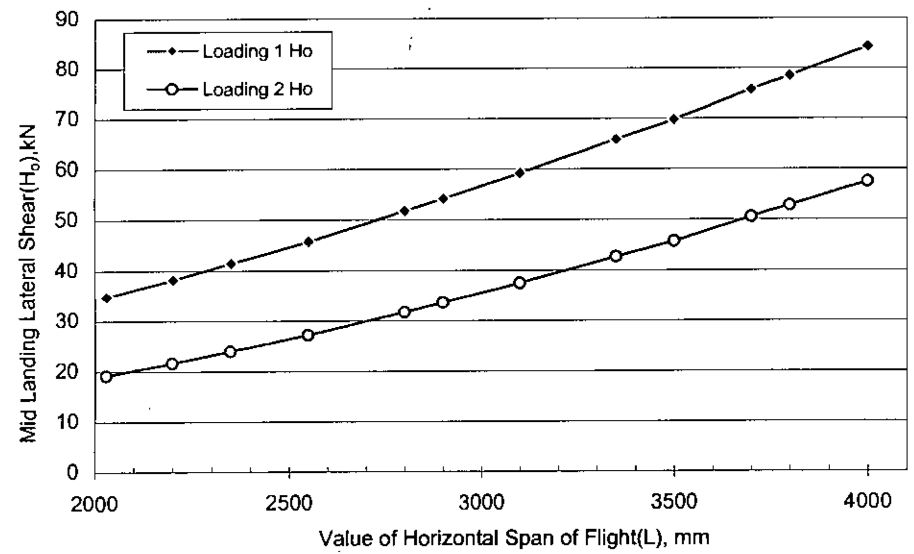


Fig. 5.7 Variation of Mid Landing Lateral Shear Due to Variation of Horizontal Span of Flight(L)

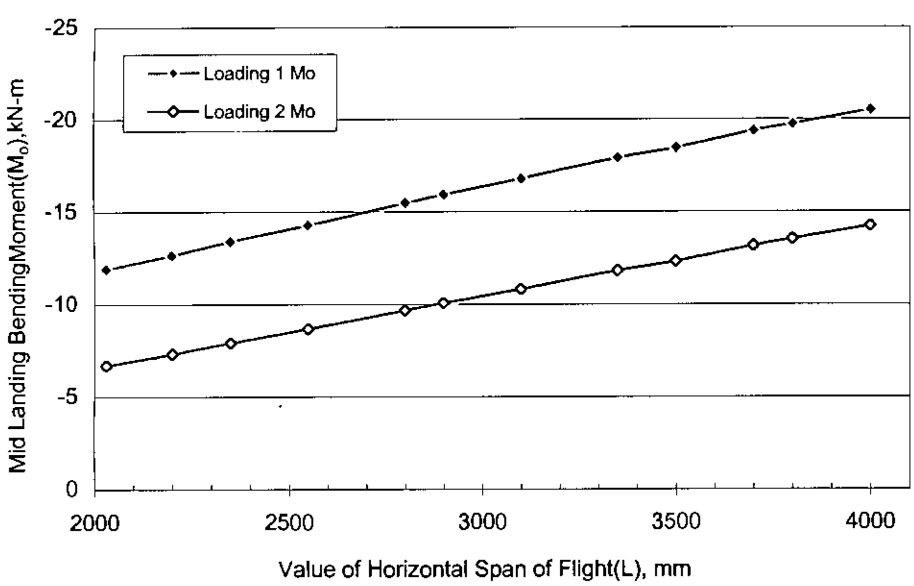


Fig. 5.8 Variation of Mid Landing Bending Moment Due to Variation of Horizontal Span of Flight(L)

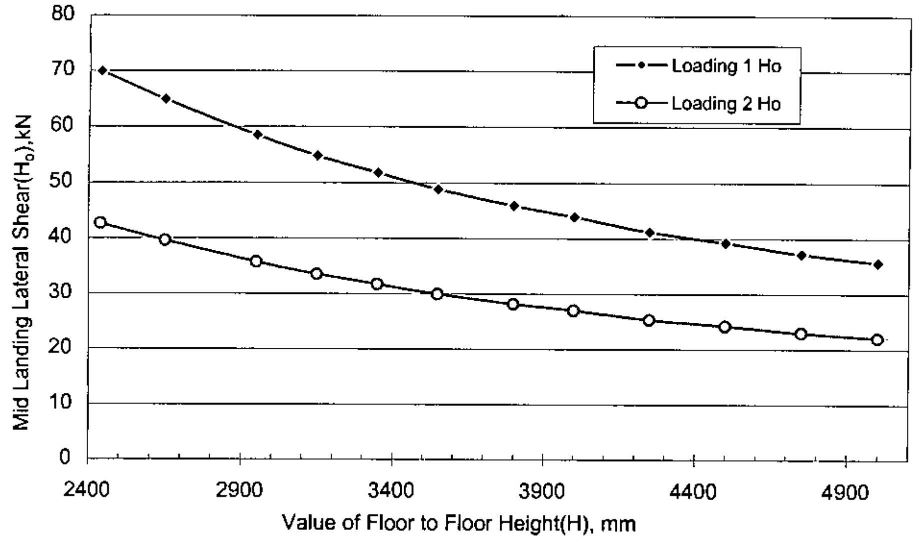


Fig. 5.9 Variation of Mid Landing Lateral Shear Due to Variation of Floor to Floor Height(H)

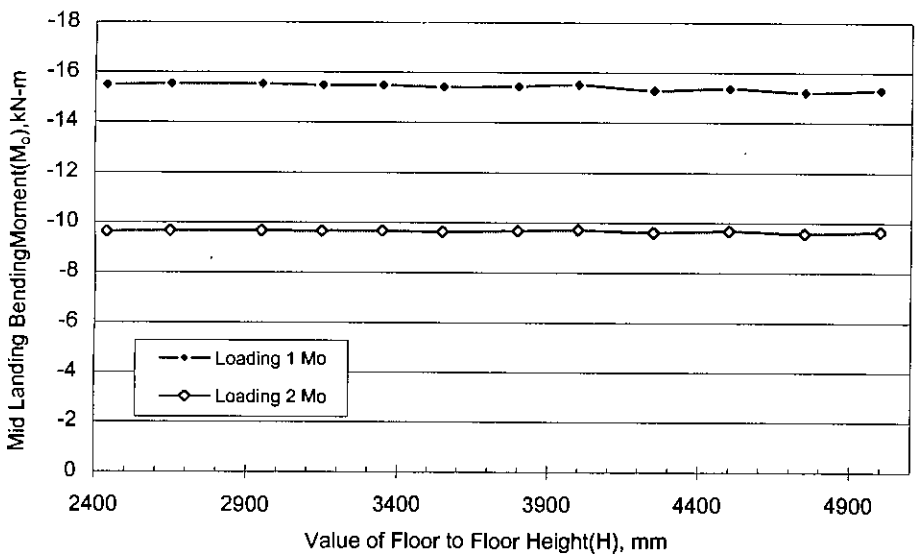


Fig. 5.10 Variation of Mid Landing Bending Moment Due to Variation of Floor to Floor Height(H)

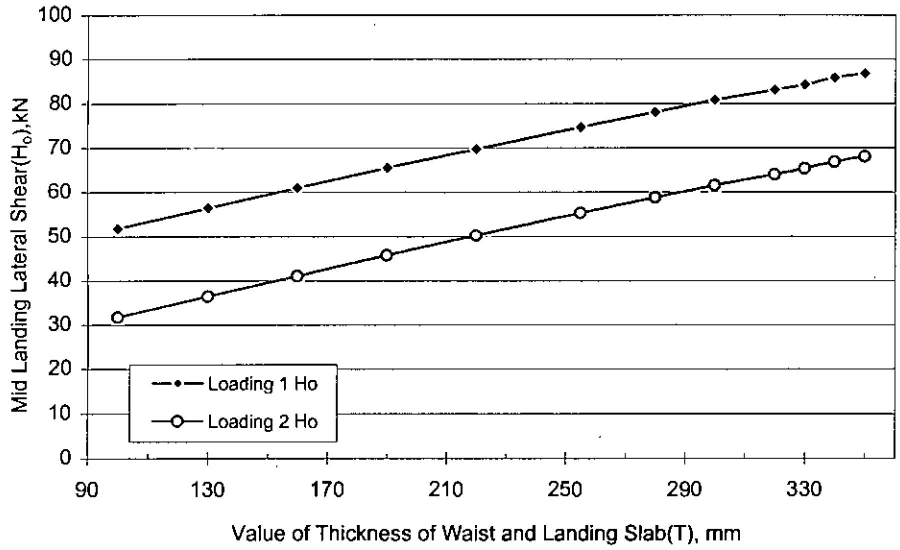


Fig. 5.11 Variation of Mid Landing Lateral Shear Due to Variation of Thickness of Waist and Landing Slab(T)

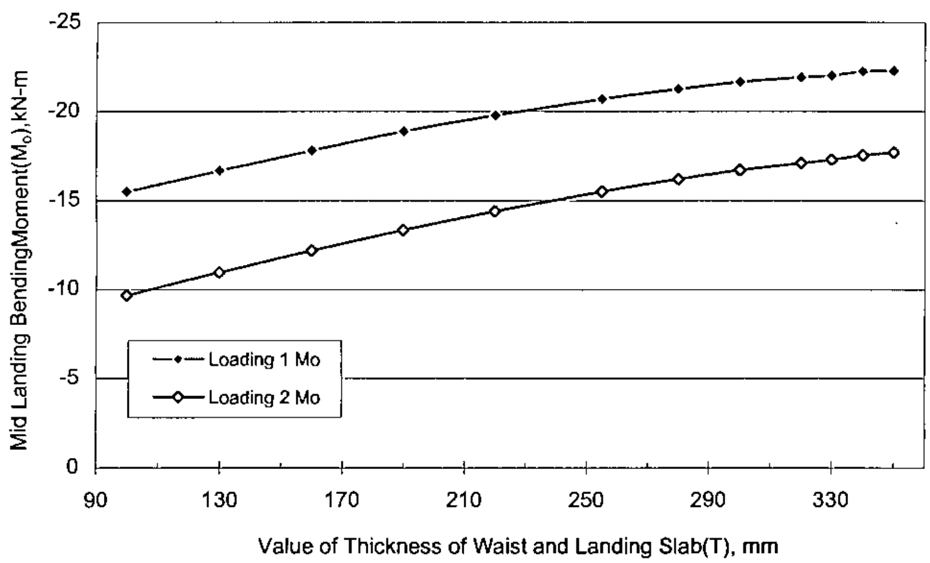


Fig. 5.12 Variation of Mid Landing Bending Moment Due to Variation of Thickness of Waist and Landing Slab(T)

with the increasing values of 'A'. Similarly, the bending moment at mid landing section also increased with the increasing values of 'A'.

b) The effect of variation of landing width (B), on lateral shear and bending moment at mid landing section, for both the loading cases, is presented in *Figs. 5.3* and *5.4.* For both the loading cases these two quantities increased with the increasing values of 'B'. The variation is not linear. In the higher range of values for 'B' the values of H_0 and M_0 increase at an increasing rate.

c) The influence of width of flights (C), on lateral shear and bending moment at mid landing section is presented in *Figs. 5.5* and *5.6*. Both the values increased with the increasing values of 'C'. For the lateral shear it shows almost a linear variation throughout the range but for bending moment, at the higher values of 'C', the moments increased in a decreasing rate.

d) Figs. 5.7 and *5.8* show the behaviour of lateral shear and bending moment at mid landing section in response to the variation of horizontal span of flights (L). In both the loading cases, the values of H_0 and M_0 increase with the increasing values of 'L'. The natures of the curves are nearly linear.

e) Figs. 5.9 and *5.10* show the behaviour of H_0 and M_0 in response to the variation of floor-to-floor height (H). When 'H' increases the lateral shear H_0 decreases under both the cases of loading. For bending moment M_0, the values are nearly constant through out the range of H.

f) The effect of slab thickness (T) on lateral shear and bending moment at mid landing section is presented in *Figs. 5.11* and *5.12*. The lateral shear values and moment values, in both the loading cases, show almost a linearly increasing trend.

Amanat[4] studied the influence of concrete strength f_c' on different design parameters. He found that concrete strength has no effect on stresses. That is, forces and moments at different sections remain unchanged for any variation of f_c'. At first instance, this may seem illogical but actually f_c' is a material property and only contributes to the evaluation of modulus of elasticity *E*. The differential equation of equilibrium at a point, as derived in the theory of elasticity, does not contain any term related to material properties. Hence stresses are independent of material properties as long as we are concerned with small deflection theory. However, f_c' does have an effect on deflection. Because, strains are related to stresses through material properties such as modulus of elasticity ***E*** and Poisson's ratio ν.

CHAPTER 6

DEVELOPMENT OF SIMPLIFIED DESIGN RATIONALE

6.1 GENERAL

Considering the characteristics of the free standing stairway and the equilibrium equations in chapter four it is required to find suitable empirical equations for finding the bending moment M_o and in-plane shear H_o at mid-landing section for both the loadings. In the following articles the method of adapting the empirical equations and comparison of results with the finite element methods and method by Amanat[4] is done. Finally a complete design of a prototype stair is presented.

6.2 DEVELOPMENT OF EMPIRICAL EQUATIONS:

Amanat presented a table *(table 3.1)* for finding the quantities required in the design of free standing stair. From the table, negative moment at support and positive moment at flight midspan are calculated directly for the maximum values at loading two, whereas the bending moment M_o and in-plane shear H_o at mid-landing section are calculated for loading one. In the present method, the bending moment M_o and in-plane shear H_o at mid-landing section are the two required quantities, which have to be determined from the empirical equations. To achieve this objective, two sets of empirical equations for the two cases of loading are required.

By analysing the results of sensitivity study it is easily found that, within the specified range of Amanat*(table 3.1)* these two quantities almost varied linearly but when the range is increased higher order functions must be introduced to incorporate this change in range. Amanat in his method presented some empirical equations but the basis of selecting these equations was not specified. In the present study a model equation was developed as:

$$Y = a + b\,(X\text{- Lower Range})^{c}$$

Y= stands for either H_0 or M_0, and

X= values of the concerned geometric parameters

The results of finite element analysis were fitted with this function and error was minimised to find the values of a, b and c. This finally lead to an optimum empirical equation for each governing factor. The results of this analysis are presented in *Figs. 6.1 to 6.6.* in order to find the equations for lateral shear in loading one.

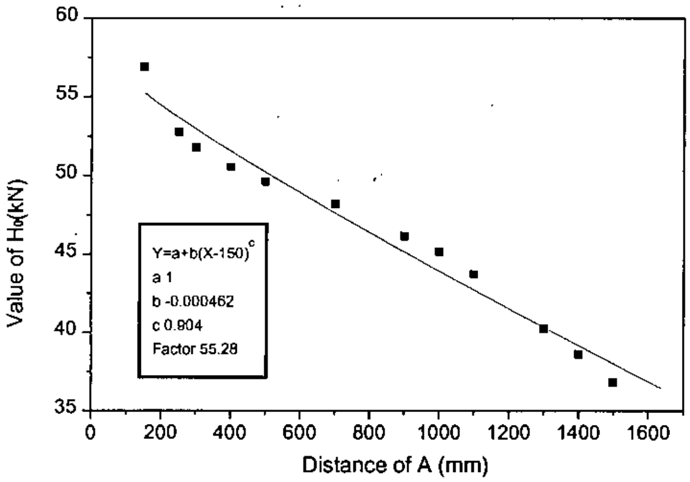


Fig 6.1 Equation of A for H_0(Load Case 1)

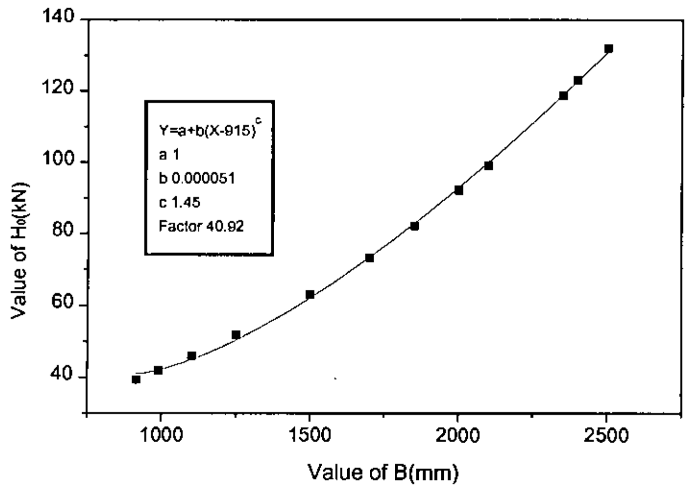


Fig 6.2 Equation of B for H_0(Load Case 1)

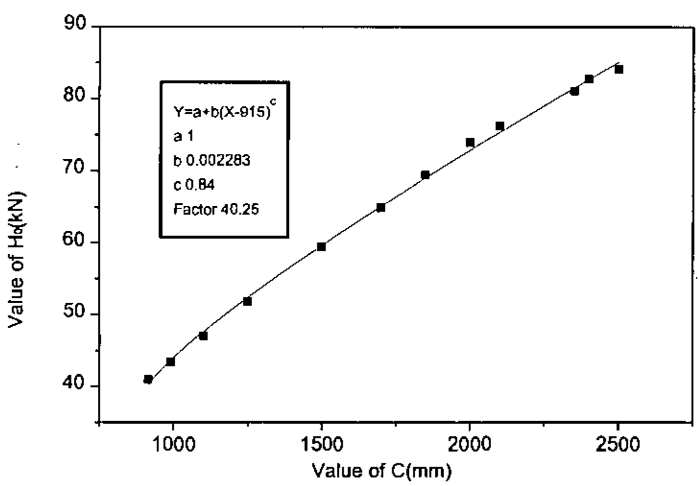


Fig 6.3 Equation of C for H_0(Load Case 1)

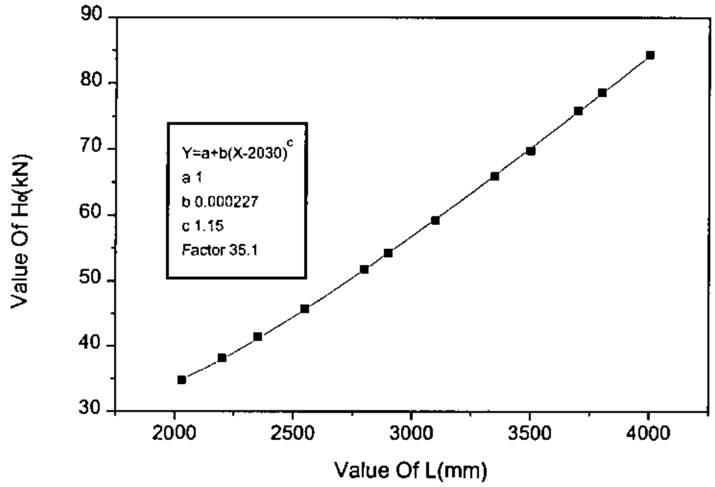


Fig 6.4 Equation of L for H_0(Load Case 1)

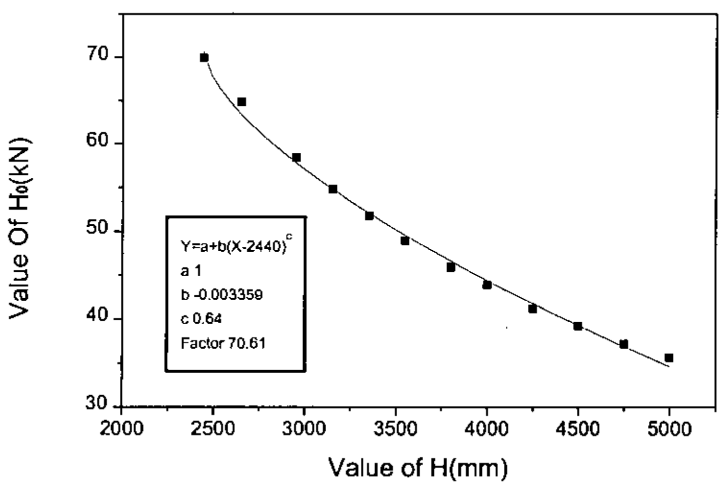


Fig 6.5 Equation of H for H_0(Load Case 1)

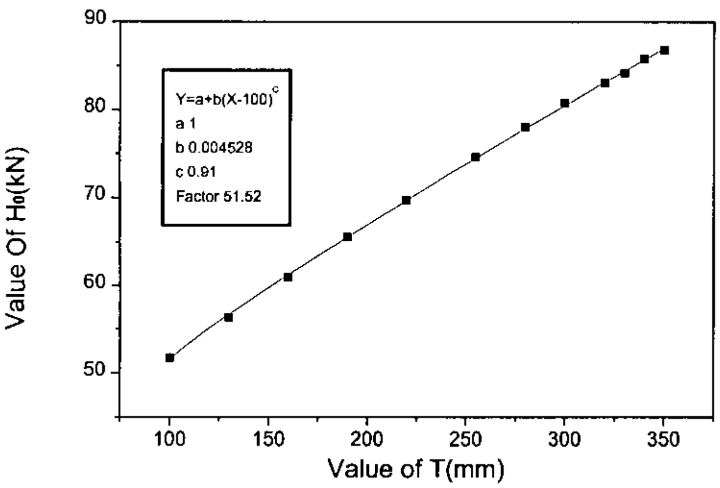


Fig 6.6 Equation of T for H_0(Load Case 1)

The equations obtained from fitting the finite element results for different geometric parameters give directly the corresponding values of H_0 and M_0 in each case. However, these equations are used only to visualise the contribution of different geometric parameters on the values of H_0 and M_0. The exact value from the equations are not needed, rather the nature of the graph is used to determine the contribution of different parameters. These equations, obtained from finite element results are, therefore, modified by dividing the values of 'a' and 'b' with the value of 'a' in each equation and thus converting the value of 'a' into one for all the equations. The modification proposed is explained through an example.

For the calculation of H_0, the effect of width of landing slab (B) in the case of loading one, the equation of the graph representing the finite element results *(Fig 6.2)* is actually:

$$40.92+0.00206(B-915)^{1.45} \qquad (6.1)$$

but the corresponding contributory equation (F_{Bh1} in *equation 6.4*)is written as:

$$1+0.000051(B-915)^{1.45} \qquad (6.2)$$

Equation 6.1 is divided by 40.92 to get *equation 6.2*, which is used in the subsequent analysis to find the contribution of Width of Landing Slab (B) on the value of H_0 under loading one. This term, which is used to divide the equations obtained from plotting finite element results, is termed 'factor' and presented in *Figs. 6.1 to 6.6.*

Based on this scheme, two tables (*Tables 6.1 and 6.2)* are presented for finding the values of the bending moment M_o and lateral shear H_o at mid-landing section. Similarly other modified equations are obtained and presented in the appendix B.

The expressions are explicit and of empirical nature. Hence, care must be taken to use proper units of measurements. In these equations, the unit of force is kilo Newton (kN) and that of length is mm. The unit of moment is 'kN-m' for convenience. The range of the different geometric parameters in the proposed equations is similar to that of *table 5.1*.

One limitation of the proposed guideline is that the thickness of the flight and landing slabs are assumed to be equal. The equations are derived for individual loading condition. The equations give values of moments and forces corresponding to Live Load of 4786.4 Pa and appropriate dead load of slab and steps. The Live Load is taken equivalent to 100 psf, as used by Amanat, to facilitate the comparison between the two methods. Since elastic analysis is made throughout, it is possible to calculate forces and moments for other values of live load by simple proportioning. The equations are valid for fully fixed support condition only.

Table 6.1 Equations for Calculating H_0 and M_0 under Loading-1

Loading-1		PARAMETRIC FUNCTIONS OF STAIR							
		F_A	F_B	F_C	F_L	F_H	F_T	F_f	
Design Parameters	Const.	150 < A(mm)< 1500	915 < B(mm) < 2500	915 < C(mm) < 2500	2030 <L(mm)< 4000	2440 < H(mm) < 5000	100 < T(mm) < 350	$14 < f_c'$ (Mpa) < 40	Values of Design Parameters
Negative moment at midsection of landing, (kN-m)	-5.75	$1+0.000078(A-50)^{1.15}$	$1+0.000142(B-915)^{1.29}$	$1+0.009069(C-915)^{0.74}$	$1+0.000589(L-2030)^{0.94}$	$1-0.0000071(H-2440)$	$1+0.009311(T-100)^{0.71}$	1	
Lateral shear in mid-section of landing, (kN)	30.97	$1-0.000462(A-150)^{0.904}$	$1+0.000051(B-915)^{1.45}$	$1+0.002283(C-915)^{0.84}$	$1+0.000227(L-2030)^{1.15}$	$1-0.003359(H2440)^{0.64}$	$1+0.004528(T-100)^{0.91}$	1	

Table 6.2 Equations for Calculating H_0 and M_0 under Loading-2

Loading-2		PARAMETRIC FUNCTIONS OF STAIR							
		F_A	F_B	F_C	F_L	F_H	F_T	F_f	
Design Parameters	Const.	150 < A(mm)< 1500	915 < B(mm) < 2500	915 < C(mm) < 2500	2030 <L(mm)< 4000	2440 < H(mm) < 5000	100 < T(mm) < 350	14 < f_c' (Mpa) < 40	Values of Design Parameters
Negative moment at midsection of landing, (kN-m)	-3.70	$1+0.000095(A-150)^{1.1}$	$1+0.000143(B-915)^{1.21}$	$1+0.009144(C-915)^{0.74}$	$1+0.000578(L-2030)$	$1-0.000001(H-2440)$	$1+0.013169(T-100)^{0.76}$	1	
Lateral shear in mid-section of landing, (kN)	22.77	$1-0.001144(A-150)^{0.81}$	$1+0.000025(B-915)^{1.475}$	$1+0.003693(C-915)^{0.785}$	$1+0.000154(L-2030)^{1.24}$	$1-0.01748(H-2440)^{0.44}$	$1+0.006851(T-100)^{0.93}$	1	

With the above mentioned limitations and assumptions, all the proposed equations are of the form :

$$\textbf{\textit{Force or Moment}} = K\, F_A F_B F_C F_L F_H F_T\, F_f$$

here K is a numeric constant. F_A is factor corresponding to the geometric parameter 'A' and so on. F_f is the factor corresponding to f_c'.

A) LOADING 1

i) Negative Moment at Mid-section of Landing

$$M_{01} = K_{m1}\, F_{Am1} F_{Bm1}\, F_{Cm1} F_{Lm1}\, F_{Hm1} F_{Tm1}\, F_{fm1} \quad \text{(kN-m)} \ldots (6.3)$$

K_{m1} = -5.75

F_{Am1} = $1.0 + 0.000078\,(A - 150)^{1.15}$

F_{Bm1} = $1.0 + 0.000142\,(B - 915)^{1.29}$

F_{Cm1} = $1.0 + 0.009069\,(C - 915)^{0.74}$

F_{Lm1} = $1.0 + 0.000589\,(L - 2030)^{0.94}$

F_{Hm1} = $1.0 - 0.0000071\,(H - 2440)$

F_{Tm1} = $1.0 + 0.009311\,(T - 100)^{0.71}$

F_{fm1} = 1.0

ii) Lateral Shear at Mid-section Landing

$$H_{01} = K_{h1}\, F_{Ah1} F_{Bh1}\, F_{Ch1} F_{Lh1}\, F_{Hh1} F_{Th1}\, F_{fh1} \quad \text{(kN)} \ \ldots \quad (6.4)$$

K_{h1} = 30.97

F_{Ah1} = $1.0 - 0.000462\,(A - 150)^{0.904}$

F_{Bh1} = $1.0 + 0.000051\,(B - 915)^{1.45}$

F_{Ch1} = $1.0 + 0.002283\,(C - 915)^{0.84}$

F_{Lh1} = $1.0 + 0.000227\,(L - 2030)^{1.15}$

F_{Hh1} = $1.0 - 0.003359\,(H - 2440)^{0.64}$

F_{Th1} = $1.0 + 0.004528\,(T - 100)^{0.91}$

F_{fh1} = 1.0

B) LOADING 2

i) Negative Moment at Mid-section of Landing

$$M_{02} = K_{m2} F_{Am2} F_{Bm2} F_{Cm2} F_{Lm2} F_{Hm2} F_{Tm2} F_{fm2} \quad \text{(kN-m)} \ldots (6.5)$$

K_{m2} = -3.70

F_{Am2} = $1.0 + 0.000095\,(A - 150)^{1.1}$

F_{Bm2} = $1.0 + 0.000143\,(B - 915)^{1.21}$

F_{Cm2} = $1.0 + 0.009144\,(C - 915)^{0.74}$

F_{Lm2} = $1.0 + 0.000578\,(L - 2030)$

F_{Hm2} = $1.0 - 0.000001\,(H - 2440)$

F_{Tm2} = $1.0 + 0.013169\,(T - 100)^{0.76}$

F_{fm2} = 1.0

ii) Lateral Shear at Mid-section Landing

$$H_{02} = K_{h2} F_{Ah2} F_{Bh2} F_{Ch2} F_{Lh2} F_{Hh2} F_{Th2} F_{fh2} \quad \text{(kN)} \quad \ldots. \quad (6.6)$$

K_{h2} = 22.77

F_{Ah2} = $1.0 - 0.001144\,(A - 150)^{0.81}$

F_{Bh2} = $1.0 + 0.000025\,(B - 915)^{1.475}$

F_{Ch2} = $1.0 + 0.003693\,(C - 915)^{0.785}$

F_{Lh2} = $1.0 + 0.000154\,(L - 2030)^{1.24}$

F_{Hh2} = $1.0 - 0.01748\,(H - 2440)^{0.44}$

F_{Th2} = $1.0 + 0.006851\,(T - 100)^{0.93}$

F_{fh2} = 1.0

The use of the above equations is straightforward. Values of forces and moments can readily be calculated once the design values of geometric parameters are known. Using these equations a computer program in '*Microsoft Excel*' is developed to carry out the calculation in a more convenient way on a worksheet, in tabular form, to get the values of forces or moments.

6.3 COMPARATIVE ASSESSMENT OF THE PROPOSED METHOD

The geometric data of the ten prototype stairs *(Table 2.1)* are used to verify the applicability of the present design equations. The values of the bending moment M_o and in-plane shear H_o at mid-landing section are calculated by using proposed *equations 6.3 to 6.6.*

From the analysis of Amanat it is found that the other required quantities for the analysis and design of the free standing stair slabs are:

- *Support Negative Moment*
- *Flight Midspan Positive Moment*
- *Negative Moment at Kink*
- *Axial Force in Flights*
- *Torsion In Flights*
- *In Plane Moment in Flights*

These quantities are calculated from the equilibrium *equations 4.1 to 4.7.* The ten prototype stairs are analysed by finite element method and also by the equations proposed by Amanat. All three sets of results are then compared in *Figs. 6.7 to 6.14.*

6.4 DISCUSSION ON THE RESULTS

In *Figs. 6.7 to 6.15*, the geometric parameters of the first five examples are within the range of Amanat's proposed equations. The geometric parameters of examples six to ten are outside the range of Amanat's equations. The values of moment and shears in these examples are obtained after linear extrapolation of his proposed equations.

From *Fig. 6.7* it is found that the proposed equations give comparable values with finite element results in all the ten examples for negative moment at support. In examples six to ten, the values obtained from Amanat's equations, after linear extrapolation are lower than those of the finite element.

Fig. 6.8 shows that flight mid span positive moments are well above the finite element results. For example three it is slightly low, but the difference is less than 1 kN-m, which can be balanced by the other reinforcements. *Equation 4.2* shows that the effect of Lateral shear at mid section landing (H_0) is prominent on flight mid span positive moment. In finding the equation for H_0 under loading two, emphasis was given to keep the results

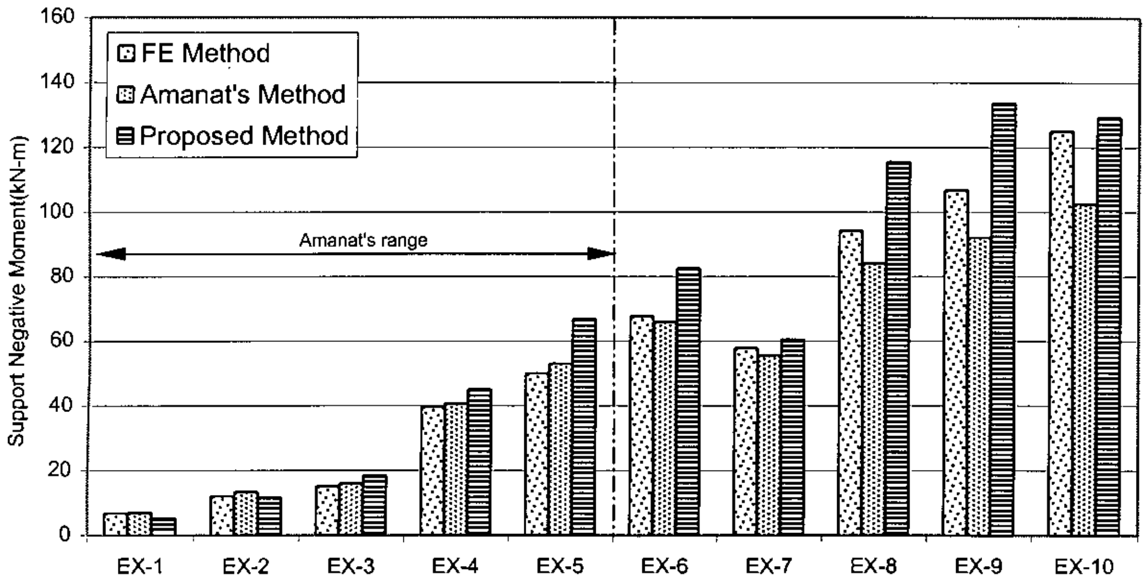


Fig. 6.7 Comparison of Support Negative Moment

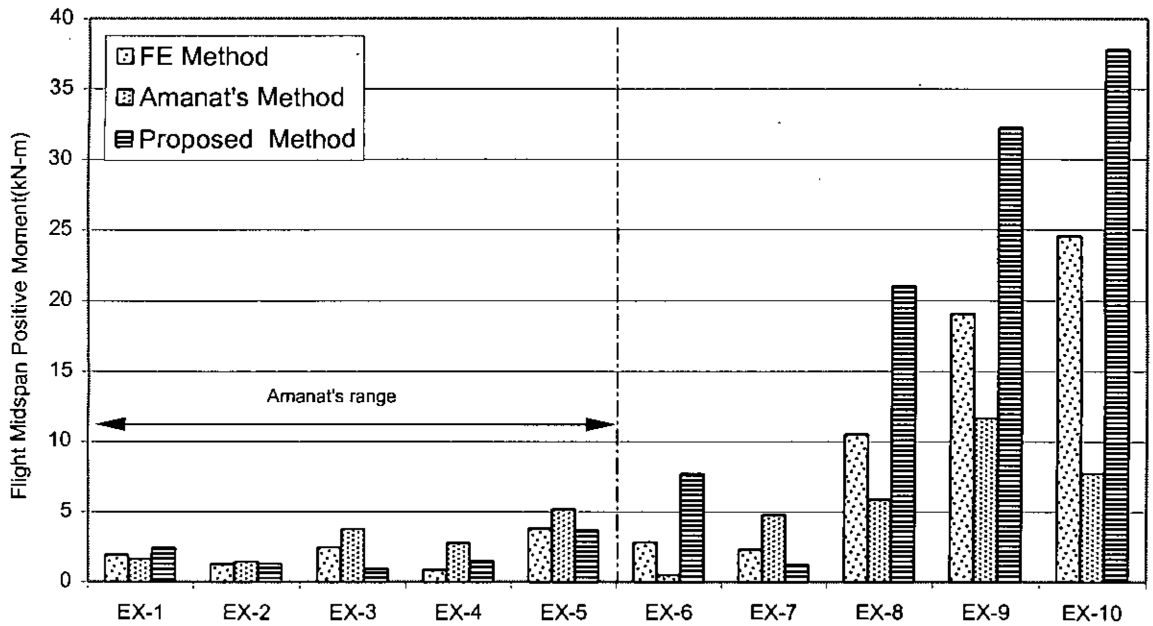


Fig. 6.8 Comparison of Flight Midspan Positive Moment

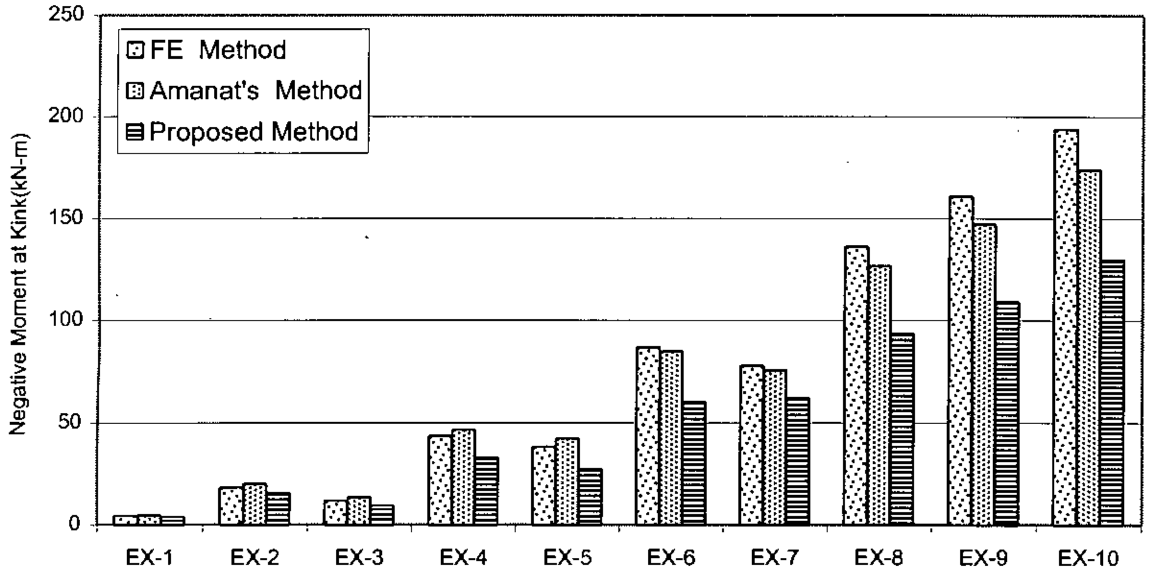


Fig 6.9 Comparison of Negative Moment at Kink

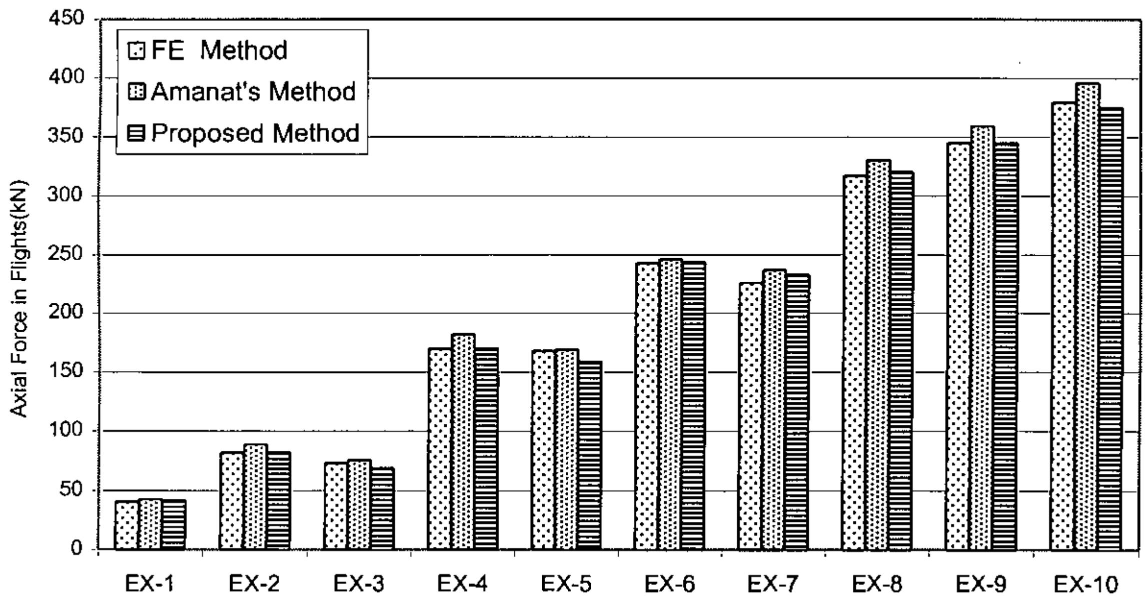


Fig 6.10 Comparison of Axial Force in Flights

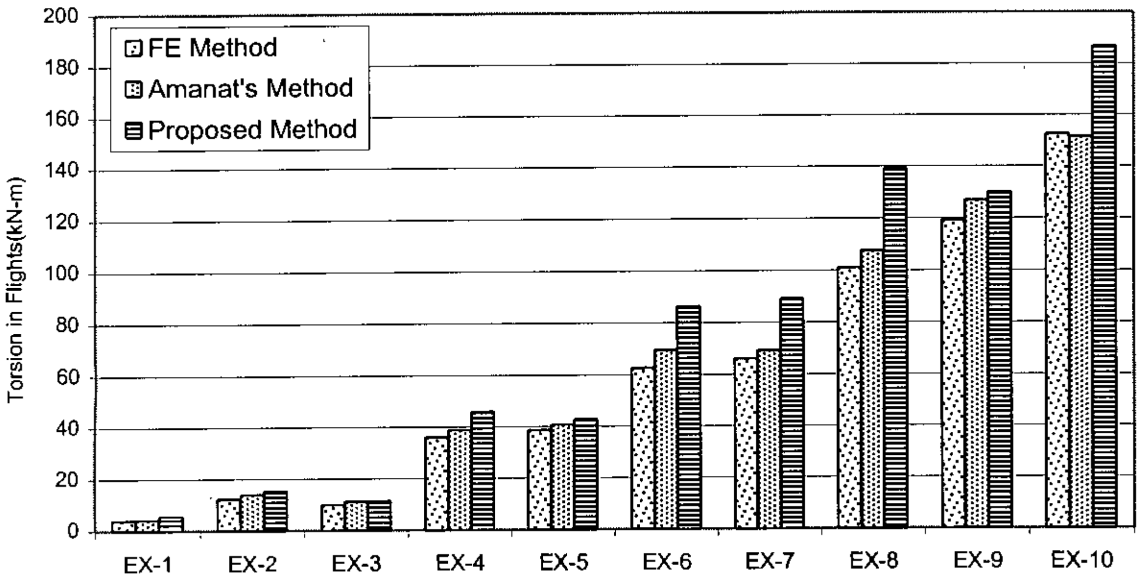


Fig 6.11 Comparison of Torsion in Flights

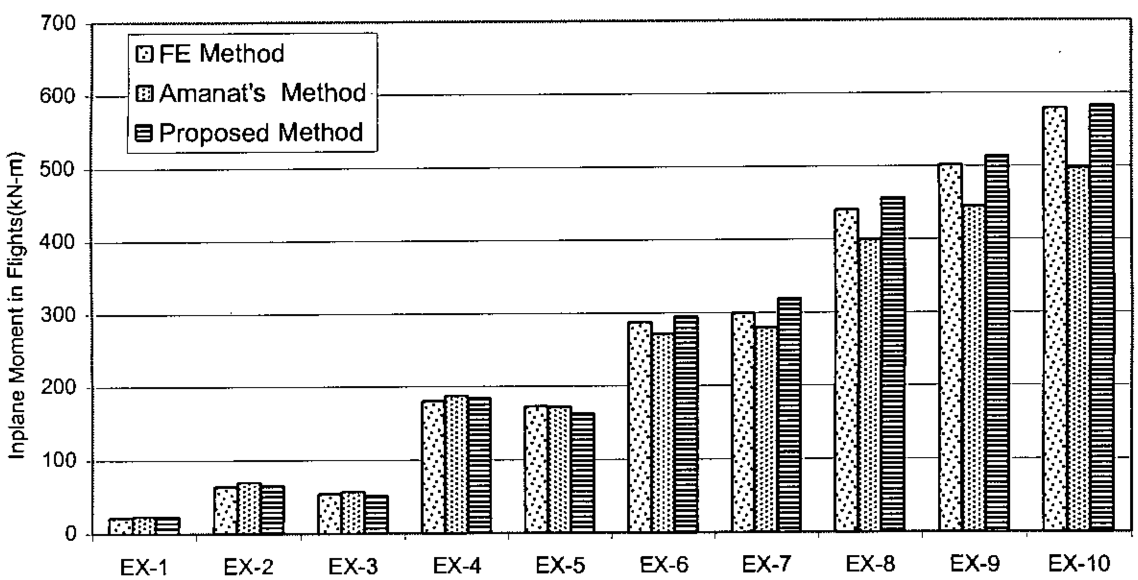


Fig 6.12 Comparison of Inplane Moment in Flights

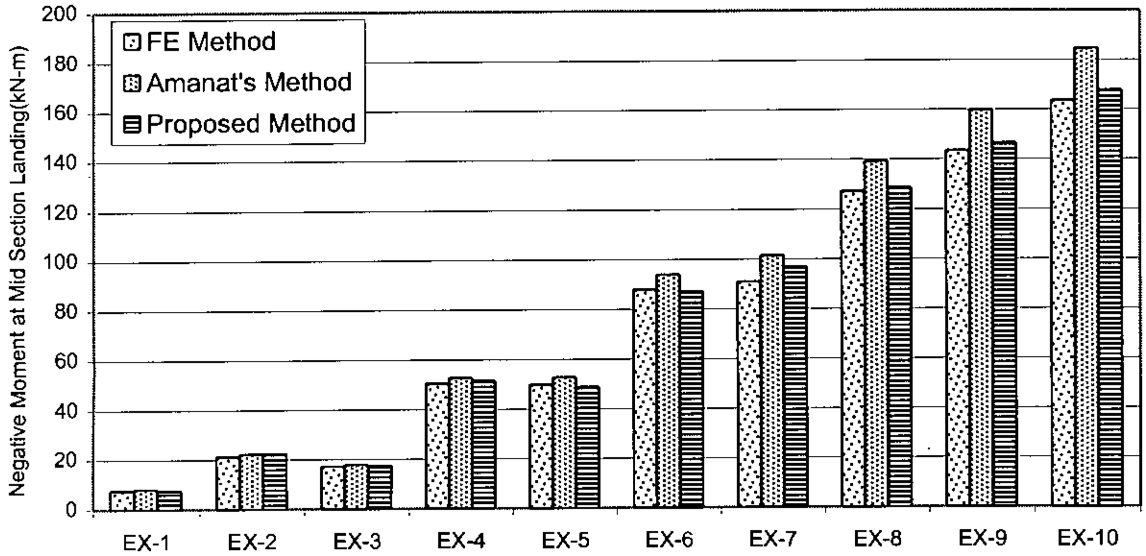


Fig 6.13 Comparison of Negative Moment at Mid Section Landing

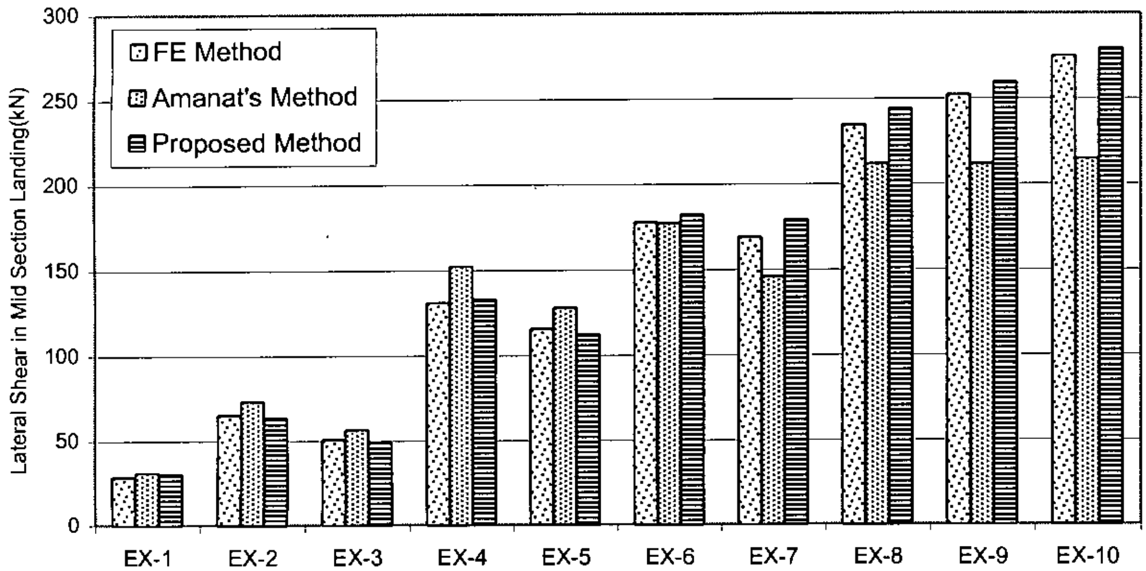


Fig 6.14 Comparison of Lateral Shear in Mid Section Landing

higher than finite element results. As H_0 was on higher side, positive moment at flight mid span is more conservative side for most of the examples. Considering this, a factor of 0.9 is used as a multiplier in *equation 4.2* to bring back the higher estimation of the results.

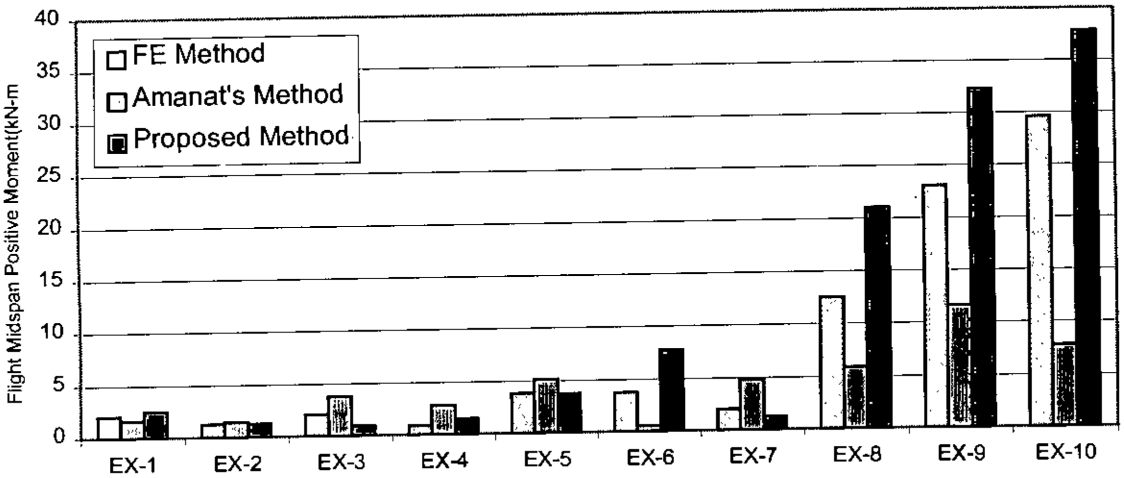


Fig 6.15 Comparison of Factored Flight Midspan Positive Moment

The proposed design flight mid span positive moment is then given by *equation 6.7* and the results are plotted in *Fig. 6.15.*

Design Flight Mid Span Positive Moment:

$$= 0.9 \times \{H_0\left(\frac{H}{4}\right) - W_2\left(\frac{L}{2}+\frac{B}{2}\right) - \left(\frac{W_1}{2}\right)\frac{L}{4}\} \quad ..(6.7)$$

Fig. B.1 in appendix B shows that, for 'A' between 250 to 450 mm, the values of horizontal shear at mid section landing (H_0) are overestimated in loading two. H_0 is negative for calculating the values of support negative moment from *equation 4.1.* This higher estimation of H_0, in this range, reduces the value of support negative moment by a considerable amount. In *Fig. 6.7* for example one, the similar phenomenon is observed. These values of support negative moments within the permitted range of 'A' are small in magnitude in comparison to the magnitude of moments for other examples. To increase the moment values in this range of horizontal gap between flights, a factor of 1.8 is used in *equation 4.1* to obtain the design moments in this range *(Equation 6.8).*

Design Support Negative Moment:

$$= 1.8 \times \{-H_0\left(\frac{H}{2}\right) + W_2\left(\frac{B}{2}+L\right) + W_1\left(\frac{L}{2}\right)\} \quad \ldots(6.8) \qquad \text{(For A= 250 to 450 mm)}$$

Fig. 6.9 shows that the negative moment at kink, calculated by the proposed method, is lower than the values of other two methods. Amanat calculated kink moment at node seven in place of node eight *(Fig. 4.2)*. The use of node seven increased the moment-arm, which resulted in higher estimation of moments. The present approach finds the kink moment at the junction between flight and landing, with reduced moment arm, which results in its lower values compared to the other two methods.

Fig. 6.10, 6.11, and 6.12 presented flight axial forces, torsion and flight in plane moments, respectively. The values presented are very much comparable with those of finite element methods.

Negative moments at mid section landing (M_0) are maximum in load case one. *Fig. 6.13* shows the values of negative moments at mid section landing (M_0) from different methods. Results of the proposed method are very much comparable with those of finite elements. Amanat's method overestimated M_0 in comparison to proposed and the finite element methods.

Lateral shear at mid landing section (H_0) is maximum under loading one. *Fig. 6.14* showed the values of lateral shear at mid landing section (H_0) from different methods. The results of the proposed method are close to finite element results. Amanat's equations over estimate H_0 in comparison to finite element results for examples one to five and underestimate it in examples six to ten.

6.5 PROPOSAL FOR A DESIGN GUIDE

The structural design procedure of a free standing stair may be divided into three stages, namely:

a) Analysis for stresses (forces and moments)

b) Checking of the thickness and calculation of the reinforcements

c) Detailing of the reinforcement layout.

These steps are described in the following sub-sections.

6.5.1 Analysis

The values of forces and moments at the critical locations are obtained for *equations 6.3 to 6.8 and 4.1 to 4.7*. These equations are valid within certain range of the geometric parameters and concrete strength. Effort has been made to increase their usefulness by increasing the range of the previous methods.

6.5.2 Calculation of Reinforcement

Four types of reinforcements are to be determined for the stair. These are:

- Longitudinal steel for bending moments.
- Longitudinal steel for axial force.
- Longitudinal steel for in-plane moment.
- Transverse stirrup for the torsion or for the combined effect of torsion and shear.

The theoretical details of the reinforcement calculation are not given here. They are found in any standard text book on reinforced concrete design. Since *WSD* method does not treat torsion adequately, *USD* method is preferred to *WSD* method in this study. The equations presented in the previous sections give working values of moments and forces. Since elastic analysis is followed throughout, these forces and moments are directly proportional to load. To convert these working values to ultimate design values they are to be multiplied by some factor. This factor can be calculated as the ratio of factored ultimate load to working load. Details of these design aspects are shown later through a design example.

6.5.3 Design Example

The application of the proposed analysis and design guide is shown here through an example. A prototype stair model is used for the design. The input data for design is shown below,

Geometry

Horizontal gap between flight (A)	=	305 mm
Width of landing (B)	=	1220 mm
Width of flights(C)	=	1220 mm
Horizontal span of flights (L)	=	2550 mm
Total height of stairway (H)	=	3050 mm
Slab thickness ($T = T_1, T_2$)	=	125 mm
Rise	=	150 mm
Tread	=	$150/\mathrm{Tan}\alpha = 250$ mm

It should be noted here that T is initially assumed.

Loads

Live load	= 4786.4 Pa (on horizontal projection)
Floor finish	= 718 Pa
Dead load	= as calculated.

Material properties

$f_c' = 20.68$ MPa

$f_y = 275.84$ MPa

Analysis

Forces and moments are calculated using *equation 6.1 to 6.4 and 4.1 to 4.7, as* given below. The calculations are also shown in the worksheet on the next page.

Support negative moment	= 3.51 kN-m.
Flight midspan positive moment	= 6.04 kN-m.
Flight landing junction negative moment	= 7.95 kN-m.
Axial force in flight	= 66.78 kN.
Torsion in flight	= 7.63 kN-m.
In-plane moment in flight	= 42.00 kN-m.
Negative moment at mid-section of landing	= 15.48 kN-m.
Lateral shear at mid-section of landing	= 51.25 kN.

Multiplying Factor for ultimate design values

The calculated values of forces and moments are working stress values, which do not include the effect of load due to floor finish. They should be multiplied by an appropriate factor so that ultimate design values can be obtained including the effect of floor finish. The procedure is straight forward and is shown below:

Inclination of flight slab:

$$\alpha = \tan^{-1}\frac{H/2}{L}$$

$$= \tan^{-1}\frac{3050/2}{2550}$$

Table 6.3 Tabular Presentation of Proposed Method

Input Line	--->								
A(mm) =	**305**	H(mm)=	**3050**	W_1(KN)=	31.365	Sinα =	0.513	DLSlab(Pa)=	3000
B(mm) =	**1220**	T(mm)=	**125**	L-1W_2(KN)=	13.038	Cosα =	0.858	DLStep(Pa)=	1544.82
C(mm) =	**1220**	f'C(Mpa)=	**20.68**	α(Radian) =	0.5390	Tread(mm)=	250.82	L-2 W_2(Kn=	5.02
L(mm) =	**2550**	ThicknessSlab(mm)=	**125**	Rise(mm)=	150	γconcrete(MPa/m)=	0.024	LL(Pa)=	4786.4

Load Case -2=>		(150<**A**<1500)	915<**B**<2500	915<**C**<2500	2030<**L**<4000	2440<**H**<5000	100<**T**<350	14<**f'c**<40	**Value**
	CONST.	305	1220	1220	2550	3050	125	20.68	
LC-2 (Mo)(KN-m)	*-3.70*	1.0244	1.1450	1.6303	1.3006	0.9990	1.1521	1.0000	-10.59
LC-2 (Ho)(KN)	*22.77*	0.9320	1.1154	1.3293	1.3592	0.7062	1.1367	1.0000	34.33

Load Case -1=>		(150<**A**<1500)	915<**B**<2500	915<**C**<2500	2030<**L**<4000	2440<**H**<5000	100<**T**<350	14<**f'c**<40	**Value**
	CONST.	305	1220	1220	2550	3050	125	20.68	
LC-1 (Mo)(KN-m)	*-5.75*	1.0258	1.2275	1.6251	1.2105	0.9957	1.0915	1.0000	-15.48
LC-1 (Ho)(KN)	*30.97*	0.9559	1.2041	1.2788	1.3016	0.7964	1.0847	1.0000	51.25

Load Case-1				***Load Case-2***
Ho=	*51.25*		Ho=	34.33
Mo=	*-15.48*	**Equilibrium Values**	Mo=	-10.59
SNM(Kn-m)	SNM	3.51	MAX LD CASE-2	
FMPM(Kn-m)	FMPM	6.04	MAX LD CASE-2	
NMAK(Kn-m)	NMAK	7.95		
AFF(KN)	AFF	66.78		
TiF(KN-m)	TIF	7.63		
IPMIF(KN-m)	IPMIF	42.00		
NMAMSL(KN-m)	Mo(LC-1)	15.48		
LSIMSL(KN)	Ho(LC-1)	51.25		

$\therefore\ \alpha\ =\ 30.88^{0}$

Dead load of slab on inclined surface of flights:

$$= \frac{T(Thickness\ of\ slab\ \ \text{in mm})}{1000} \times 0.024 \times 10^{6}\ (Pa)$$

$$= \frac{125}{1000} \times 0.024 \times 10^{6} = 3000\ \text{Pa}$$

The steps are 150 mm high. Hence the tread is:

$$= \frac{150}{\tan\alpha}\,(mm)$$

$$= 250.82mm$$

Average dead load from steps :

$$= \frac{1}{2} \times \frac{Rise \times Tread}{\sqrt{(Rise^{2} + Tread^{2})}} \times \frac{1}{1000} \times 0.024 \times 10^{6}\ (Pa)$$

$$= \frac{1}{2} \times \frac{150 \times 250.82}{\sqrt{(150^{2} + 250.82^{2})}} \times \frac{1}{1000} \times 0.024 \times 10^{6}\ (Pa)$$

= 1544.82 Pa.

Dead load without floor finish :

= 3000+1544.82 = 4544.82 Pa.

Dead load with floor finish:

= 4544.82+718 = 5262.82 Pa.

The above dead loads are on inclined surface of flights. On horizontal projection, these are,

without floor finish	= 4544.82/cosα	= 5297 Pa.
with floor finish	= 5262.82/cosα	= 6133.8 Pa.

Therefore, the correction factor for floor finish becomes,

$$\frac{\text{Total Load With FF}}{\text{Total Load Without FF}} = \frac{4786.4 + 6133.8}{4786.4 + 5297} = 1.083$$

For ultimate values:

factored live load = 1.7×4786.4 = 8136.88 Pa.

factored dead load = 1.4×6133.8 = 8587.32 Pa.

Total factored load = 8136.88+ 8587.32= 16724.2 Pa.

Total unfactored load = 4786.4 + 6133.8 = 10920.2 Pa.

Therefore multiplying factor for conversion to ultimate value is

$$\frac{16724.2}{10920.2} = 1.531$$

Hence final multiplying factor is = 1.531×1.083 = 1.658

Checking of thickness requirement

Adequacy of slab thickness should be checked using the maximum bending moment which is, for the present case, the moment at mid landing section. Amanat[4] mentioned that the inner one-third of the width of the section carries 50% of the total moment. Hence the thickness should be checked accordingly.

Now, 50% of the ultimate moment = $\frac{1}{2} \times 1.7 \times 15.48$ = 13.158 kN-m.

One third of the width = $\frac{1220}{3}$ =406.67 mm.

Balanced steel ratio, $\rho_b = .85\beta_1 \frac{f_c'}{f_y} \frac{600}{600 + f_y}$

For 20.68 MPa concrete, $\beta_1 = 0.85$, hence $\rho_b = 0.037$

Therefore, $\rho_{max} = 0.75\rho_b = 0.0277$

Now, $M_u = \varphi \rho f_y b d^2 (1 - 0.59 \frac{\rho f_y}{f_c'})$

$\therefore 13.158\times10^6 = 0.9\times0.0277\times275.84\times406.67\times d^2(1\text{-}0.59\dfrac{0.0277\times275.84}{20.68})$

Solving, $d_{required} = 77.57$ mm

$d_{provided}$ = Total thickness - clear cover - stirrup dia - $\frac{1}{2}$ main bar dia.

$= 125 - 25.4 - 9.525 - \frac{1}{2}\cdot 9.525$ mm [assumed main bar size = #3]

= 85.31 mm.

$\therefore d_{provided} > d_{required}$ OK.

Negative steel at support

$$A_s = \frac{M_u}{\varphi f_y(d-a/2)}$$

$$= \frac{1.7\times1.8\times3.51\times10^6}{.9\times275.84\times(85.31-12/2)}$$ [assuming a = 12mm]

$\therefore$ A_s = 545.50 mm^2.

check, $a = \dfrac{A_s f_y}{.85 f_c' b} = \dfrac{545.50\times275.84}{.85\times20.68\times1220} = 7.02$ mm.

$\therefore$ revised A_s = 528.91 mm^2

$\therefore \rho = \dfrac{A_s}{bd} = \dfrac{528.91}{1220\times85.31} = 0.00508 > \dfrac{1.4}{f_y}$ OK.

provide φ 12 mm bar 5 nos.

Through similar calculations as made above, the bending steel at other critical sections are,

Positive steel at midspan of flights

A_s = 503.73 mm^2, provide φ 12 mm 4 nos.

Negative steel at kink

$A_s = 671.43\ mm^2$, provide φ 12 mm 6 nos.

Negative steel at mid-section of landing

$A_s = 1271\ mm^2$, provide φ 12 mm 12 nos.

Steel for in-plane moment in flight

$$A_s = \frac{1.7 \times 42.0 \times 10^6}{.9 \times 275.84 \times (1120 - 5/2)} \qquad \text{[assumed } a = 5 \text{ mm]}$$

$$= 257.36\ mm^2$$

check, $a = \dfrac{257.36 \times 275.84}{.85 \times 20.68 \times 125} = 32.31$

revised $A_s = 260.6\ mm^2$, provide φ 12 mm 3 nos.

Steel for axial force (tensile) in upper flight

Axial force = 66.78 Nt.

$$\therefore\ A_s = \frac{1.7 \times 66.78 \times 10^3}{.275.84} = 411.56\ mm^2$$

provide φ 12 mm bar 4 nos.

Stirrup for torsion in flights

Dimension of flight cross section is 1220 mm ×125 mm

$\therefore\ x = 125\ mm$, $y = 1220\ mm$, $x_1 = 75$ mm, $y_1 = 1170$ mm.

$$\alpha_t = 0.66 + 0.33\frac{y_1}{x_1} \le 1.5$$

$$= 0.66 + 0.33\frac{1170}{75}$$

$$= 5.81 > 1.5$$

$$\therefore \qquad \alpha_t = 1.5$$

stirrup spacing, $s = \dfrac{\varphi\alpha_t A_t x_1 y_1 f_y}{T_u - 0.07\varphi\sqrt{f_c'}x^2 y}$

using φ 10 mm bar, A_t = 78.53 mm^2

∴ spacing, s $= \dfrac{.85\times 1.5\times 78.53\times 75\times 1170\times 275.84}{1.7\times 7.63\times 10^6 - 0.07\times .85\times\sqrt{20.68}\times 125^2\times 1220}$

= 309.7 mm c/c

maximum spacing permitted by code is $\dfrac{x_1+y_1}{4}$ (311.25)or 305 mm ok.

∴ use closed rectangular stirrups of φ 10 mm bar @ 305 mm c/c.

check minimum web reinforcement, $(2A_t + A_v) \geq 0.34\dfrac{b_w s}{f_y}$

now, $0.34\dfrac{b_w s}{f_y} = 0.34\times\dfrac{125\times 280}{275.84} = 43.14 < 129$ OK.

longitudinal steel for torsion, $A_l = 2A_t\dfrac{x_1+y_1}{s} = 2\times 113.09\times\dfrac{75+1170}{280}$

∴ A_l = 1005.69 mm^2. This steel will be added to other longitudinal steels.

Stirrups for landing

Ultimate lateral shear in landing, V_u = 1.7×51.25× 10^3 = 87125 Nt.

Taking half of the width of section as effective in shear, the shear capacity of the section is,

$$V_c = 0.17\sqrt{f_c'}bd = 0.17\sqrt{20.68}\times 125\times 560 = 54115.6 \text{ Nt.}$$

stirrup spacing, $s = \dfrac{\varphi A_v f_y d}{V_u - \varphi V_c}$

$= \dfrac{.85\times(2\times 78.53)\times 275.84\times 560}{87125 - .85\times 54115.6}$ [Using φ 10 mm]

= 501.5 mm.

but $s_{max} \leq {}^{d}/_{2}$. Hence stirrup spacing = 280 mm.

$\therefore$ use closed rectangular stirrups of φ 10 mm bar @ 280 mm c/c.

The layout of the reinforcement is shown in *Figs. 6.16* and *6.17.*

Reactions on supporting beam at floor level

The free body of upper flight plus landing cut at mid-section is shown in *Fig. 4.6*. With reference to this figure:

W_1 (The load acting on mid point of flight):

$$\left[Live\ Load \times L \times C + (Dead\ Load\ from\ Slab + Dead\ Load\ from\ Steps) \times C \times \frac{L}{Cos\,\alpha} \right] \times \frac{1}{10^9}\,(kN)$$

= 31.365 kN

W_2 (The load acting on mid point of landing):

Load Case 1:

$$(Live\ \ Load\ \ + Dead\ \ Load\ \ from\ \ Slab\ ) \times B \times \frac{\left(C + \frac{A}{2}\right)}{10^{9}}\,(kN\,)$$

W_2 =13.038 kN

Load Case 2:

$$(Dead\ Load\ from\ Slab) \times B \times \frac{\left(C + \frac{A}{2}\right)}{10^9}\,(kN)$$

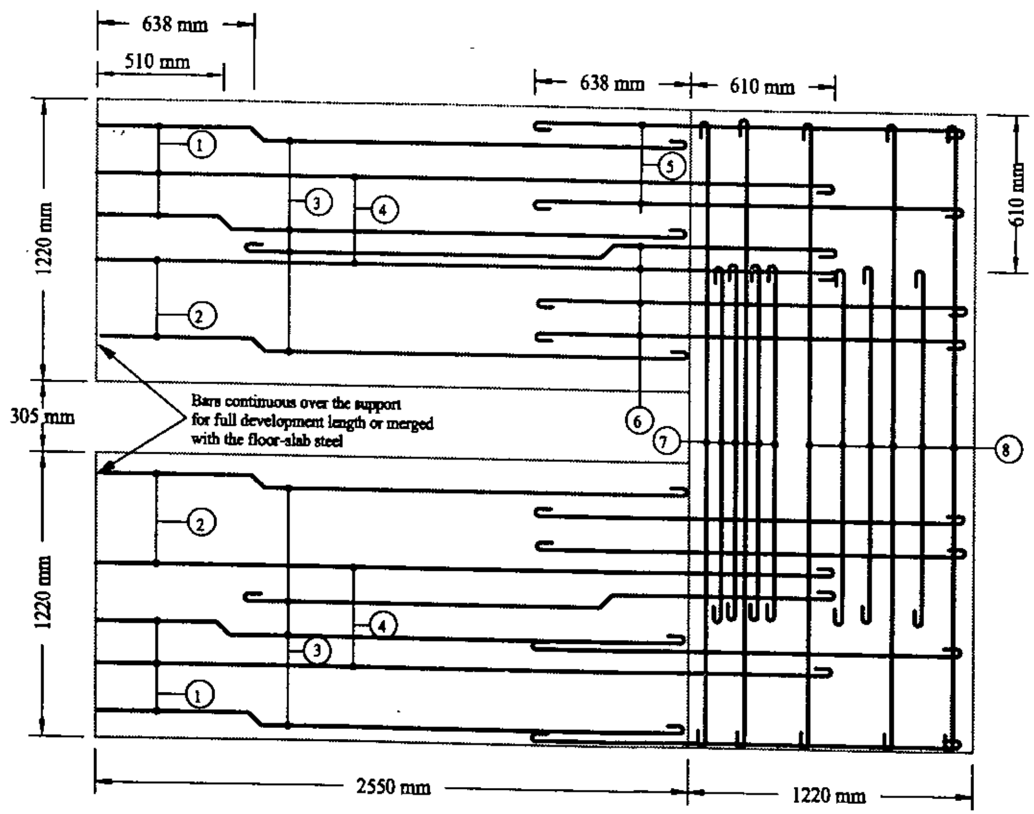
W_2= 5.02 kN.

Hence by ΣF_Z =0 ,

$V = (W_1 + W_2) = (31.365+13.038) = 44.403$ kN.

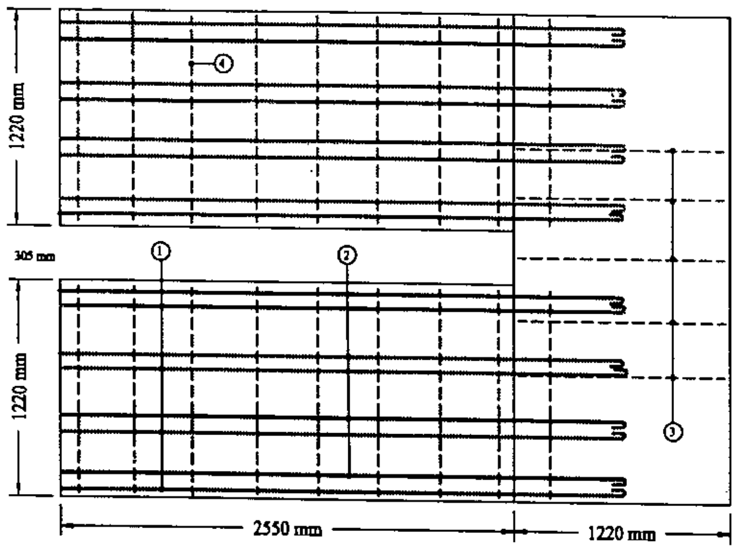
ΣF_X =0

$H^{I} = H_O = 51.25$ kN.



1. Three φ 12 mm bars, top, equally spaced within outer 610 mm
2. Two φ 12 mm bars, top, equally spaced within inner 610 mm
3. Four φ 12 mm bars at bottom.
4. Two φ 12 mm bars, top, continuous.
5. Two φ 12 mm bars, equally spaced within outer half of the width.
6. Four φ 12 mm bars, top, equally spaced within inner half of the width.
7. Six φ 12 mm bars, placed within inner one third of the width of landing.
8. Six φ 12 mm bars, placed within outer two third of the width of landing.

Fig. 6.16 Layout of Flexural Steel



1. Four φ 12 mm bars, bottom.
2. Four φ 12 mm bars, top.
3. φ 10 mm bar, closed stirrup @ 305 mm c/c. (Landing)
4. φ 10 mm bar, closed stirrup @ 280 mm c/c. (Flight)

Fig. 6.17 Layout of Stirrups and Longitudinal Steel

CHAPTER 7

CONCLUSIONS

7.1 GENERAL :

The findings of this thick-shell finite-elements study of the behaviour of free standing stairs and the brief reviewing of the earlier works on it are summarised in this chapter. The generalised computer program is capable of dealing with the arbitrary geometry, loadings and boundary conditions. Conclusions are drawn from the study of the prototypes, the sensitivity analysis, the results obtained from the finite element method, Amanat's[4] method and the proposed method. Unless otherwise specified, the conclusions listed here refer to the free standing stair slab held fixed at upper and lower floor levels and acted upon by uniformly distributed symmetrical loading.

Chapter one contains the previous works done on free standing stairs and the objectives, scope and methodology of the present research.

Chapter two deals with the geometry of the free standing stair slabs, required loading conditions, co-ordinate systems, sign convention, boundary conditions, redundants, and material properties.

Codes and practices, existing analytical methods and Amanat's method are described in chapter three.

Chapter four deals with the finite element model and the required modification done on the thick shell program. The proposed approach, load calculation scheme and other required equations for calculating moments and shears at the critical locations are presented at the end of this chapter.

Lateral shear and bending moment at the mid section of landing are the two redundants for the symmetrical condition of the free standing stair. Sensitivity analysis was carried out in chapter five with a view to understand the effect of geometric parameters on these two redundants. Changing one parameter at a time, while keeping others constant, the sensitivity analysis is done for all the geometric parameters.

The basis of adopting the proposed equations is stated in chapter six. Comparison of the results obtained from finite element analysis, Amanat's method and proposed method are presented here in graphical form. To show the necessary calculation details of the proposed approach, a sample calculation and reinforcement details of an example is presented at the end.

A computer program in *'Microsoft Excel'* is developed to facilitate the necessary calculation of this method in tabular form.

7.2 FINDINGS OF THE PRESENT STUDY

Amanat's equations give values of different design parameters, corresponding to that case of loading in which the parameter is maximised. For example, negative moment at support and positive moment at flight mid span are maximised in loading two but lateral shear and bending moment at mid section landing are maximised in loading one. His equations give directly the design values of negative moment at support and positive moment at flight mid span for loading two and lateral shear and bending moment at mid section landing for loading one. But in this present approach, to calculate negative moment at support and positive moment at flight mid span, values of lateral shear and bending moment of mid landing section are required in loading two rather than loading one. This fact initiated the need to propose two equations for lateral shear and bending moment at mid landing section in each case of loading. This leads to solving the stair as an equilibrium problem for individual loading case in a more realistic way.

From the results of sensitivity analysis it is found that within Amanat's range it varied almost linearly but when the range is increased certain parameters changed nonlinearly. To accommodate this behaviour higher order functions are introduced replacing the linear equations. Amanat, in his method, presented some empirical equations but the basis of selecting these equations was not specified. A model equation is introduced and modified later to find optimum empirical equations for each governing factor. In order to obtain more rational values for positive moment at flight mid span and negative moment at support, some multiplying factors are introduced in the equilibrium equations to find the design equations.

7.3 THE DESIGN RATIONALE :

Based on this study, a simple design example is presented in chapter six. The range of validity of the method covers the most frequently occurring cases. The main advantages of the proposed method are :

a) The number of empirical equations proposed by Amanat[4] for individual stair was seven. Here it is reduced to four. The effects of loading one and two are considered separately in these equations. The required forces and moments are obtained easily from the suggested equations. There is no need for an elaborate structural analysis. This relieves the designer from the rigorous calculation required even in the approximate analytical methods.

b) The basis for selecting the equations is defined and the range of geometric parameters is increased which make the approach more realistic and useful.

7.4 SCOPE FOR FUTURE INVESTIGATION

Consistent with the objectives of the present study, the overall behaviour of the free standing stair, held fixed at supports and acted upon by symmetric loading, has been analysed and specific design guidelines have been formulated. However, further works remain to be done, some indications of which are given below.

- For gaining confidence in the application of the proposed equations, a physical model may be constructed and tested.
- The stairs analysed are considered fixed at supports. Equations can also be derived for stairs partially fixed conditions. A detailed study may be made considering partially flexible supports at floor levels.
- This study was primarily limited to stair of symmetric geometry. It can be extended to cover non-symmetric geometry.
- Linear elastic analysis was made throughout. A finite element analysis with non-linear material properties can be attempted in future.
- Two possible symmetric loadings were used in the analysis. Investigation for unsymmetric loading, such as live load only on upper flight, may be carried out.

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APPENDIX A

APPENDIX

A.1 BRIEF DESCRIPTION OF THICK SHELL ELEMENT

The thick shell program used here is a FORTRAN code, a modified version of the general thick shell element code developed by Ahmad [3]. A brief description of the program is given in art.A.2.

Typical thick shell elements are shown in Fig.A.1.1. In thick shells, bending effect can be expected to be significant. The transverse shear deformation is also significant. From a three dimensional point of view the elements have two degeneracies. Firstly, the original normals to the middle surface are assumed to remain straight. Secondly, the distance of a point along the normal from the middle surface remains unaffected.

Geometric Definition of the Element:

The external faces of the element are curved, while the lateral faces are planes. Pairs of points i_{top} and i_{bottom}, each with given Cartesian co-ordinates, describe the shape of the element.

If ξ, η be the two curvilinear co-ordinates in the middle plane of the shell element (Fig.A.1.2) and ζ be the normal to the middle surface and further if it is assumed that ξ, η and ζ vary between +1 and −1 on the respective faces of the element then a relationship can be written between the Cartesian co-ordinates of any point of the shell and the curvilinear co-ordinates in the form,

$$\begin{Bmatrix} x \\ y \\ z \end{Bmatrix} = \Sigma N_i(\xi,\eta)\frac{1+\zeta}{2}\begin{Bmatrix} x_i \\ y_i \\ z_i \end{Bmatrix}_{top} + \Sigma N_i(\xi,\eta)\frac{1+\zeta}{2}\begin{Bmatrix} x_i \\ y_i \\ z_i \end{Bmatrix}_{bottom}$$

Here $N_i(\xi, \eta)$ is a shape function taking a value of unity at the nodes i and zero at all other nodes. If the basic functions N_i are derived as 'shape functions' of a 'parent' two dimensional element, square or even triangular in plan and are so designed that compatibility is achieved at interfaces, then the curved shape elements will fit into each other. Arbitrarily curved shapes of the element can be achieved by using shape functions of different orders. Only parabolic and cubic types are shown in Fig.A.1.1. For the purpose of present analysis a parabolic element has been used. By placing a larger

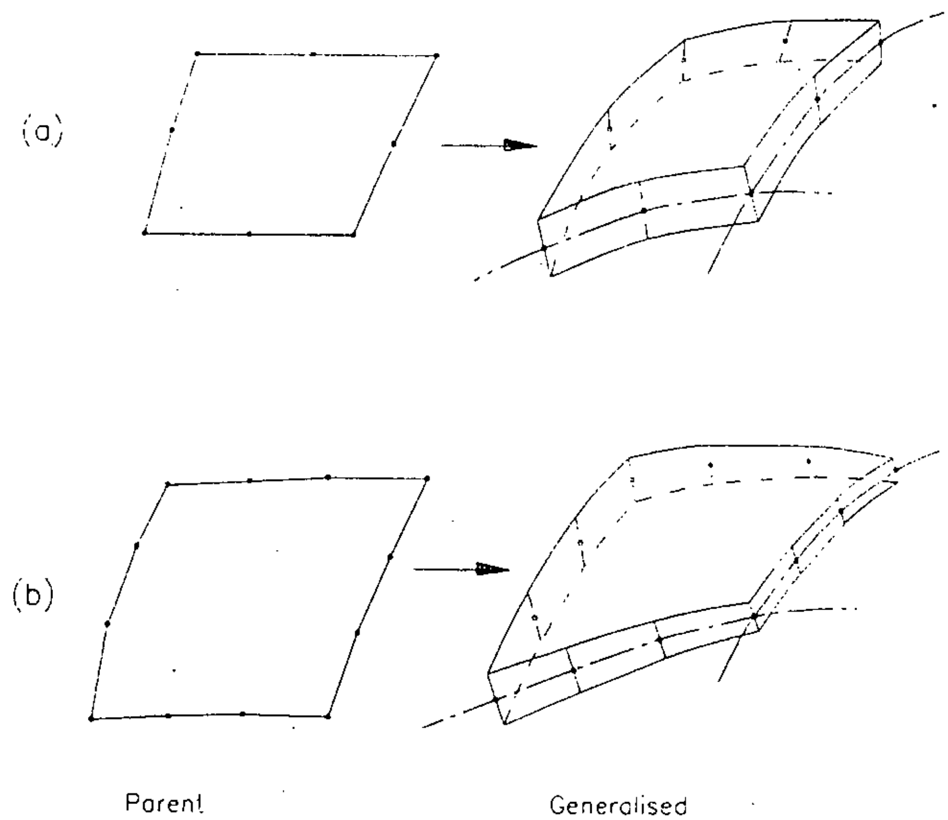


Fig. A.1.1 Thick Shell Elements

a) Parabolic

b) **Cubic**

number of nodes on the surfaces of the element more elaborate shapes can be achieved if so desired. It should be noted that the co-ordinate direction is only approximately normal to the middle surface. The relationship between the Cartesian co-ordinates and the curvilinear co-ordinates can be written conveniently in a form specified by the 'vector' connecting the upper and lower points (*i.e.* a vector of length equal to the shell thickness, t) and mid-surface co-ordinates (shown in Fig.A.1.3).

$$\begin{Bmatrix} x \\ y \\ z \end{Bmatrix} = \Sigma \mathrm{N}_i \begin{Bmatrix} x_i \\ y_i \\ z_i \end{Bmatrix}_{mid} + \quad \Sigma \mathrm{N}_i \frac{\zeta}{2} \, \overline{\mathbf{v}}_{3i}$$

and $$\overline{\mathbf{v}}_{3i} = \begin{Bmatrix} x_i \\ y_i \\ z_i \end{Bmatrix}_{top} - \begin{Bmatrix} x_i \\ y_i \\ z_i \end{Bmatrix}_{bottom}$$

here $\overline{\mathbf{v}}_{3i}$ is a vector whose length is the shell thickness.

Displacement Field

Since the strains in the direction normal to the mid-surface is assumed to be negligible, the displacement throughout the element will be taken to be uniquely defined by the three Cartesian components of the mid-surface node displacements and two rotations of the nodal vector $\overline{\mathbf{v}}_{3i}$ about orthogonal directions normal to it. If two such orthogonal directions are given by vector $\overline{\mathbf{v}}_{2i}$ and $\overline{\mathbf{v}}_{1i}$ of unit magnitude, with corresponding scalar rotation α_i and β_i, it can be written, similar to the previous equation, but now dropping the suffix *mid* for simplicity,

$$\begin{Bmatrix} u \\ v \\ w \end{Bmatrix} = \Sigma \mathrm{N}_i \begin{Bmatrix} u_i \\ v_i \\ w_i \end{Bmatrix} + \Sigma \mathrm{N}_i \zeta \frac{\mathrm{t}_i}{2} \left[\overline{\mathbf{v}}_{1i} - \overline{\mathbf{v}}_{2i} \right] \begin{Bmatrix} \alpha_i \\ \beta_i \end{Bmatrix}$$

A.2 GENERAL FEATURES OF THE PROGRAM

The general thick shell finite element program is a generalised program. The geometry of a structure is defined in a global system. The loading and boundary conditions must be

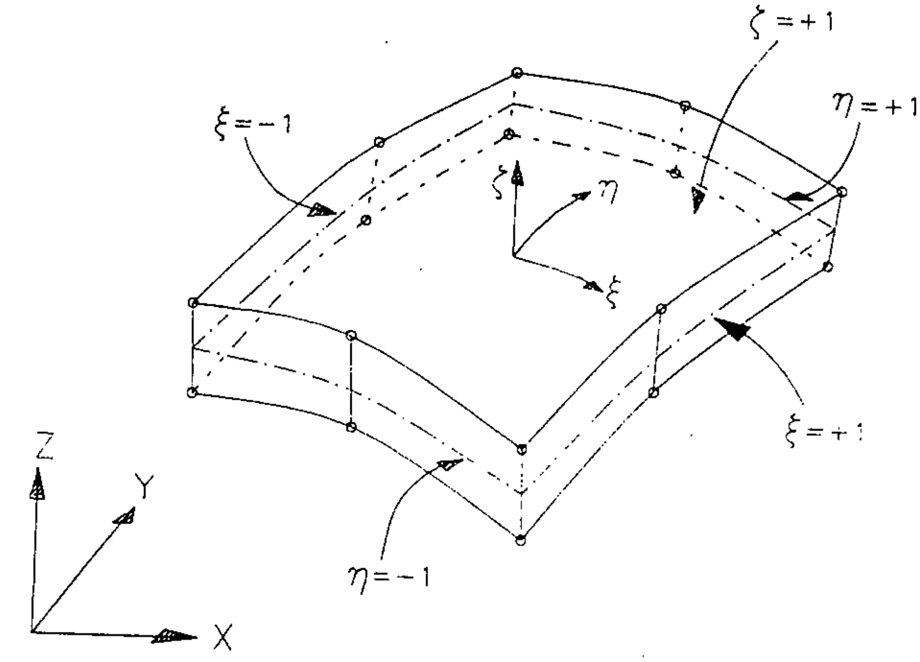


Fig. A.1.2 Geometry of the Element

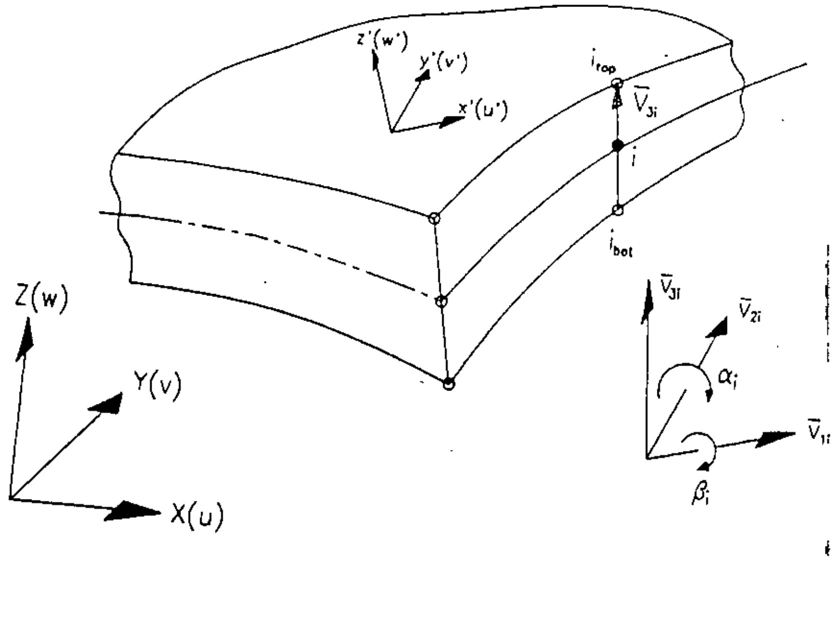


Fig. A.1.3 Local and Global Co-ordinate Systems and Nodal Degrees of Freedom

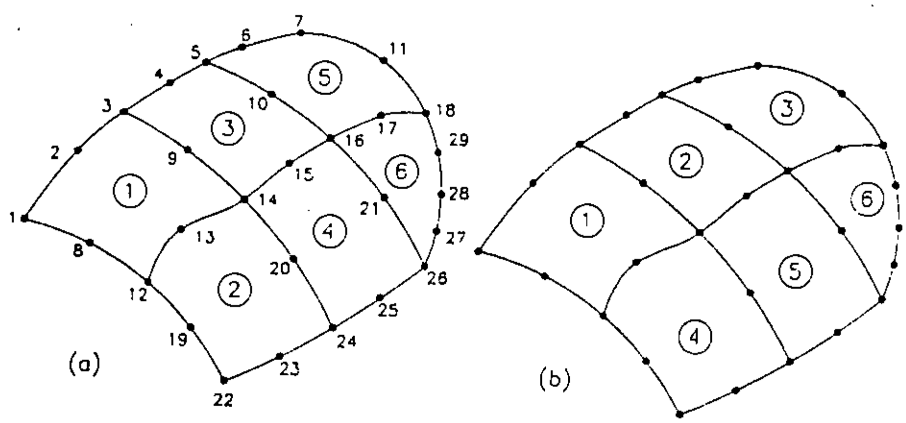
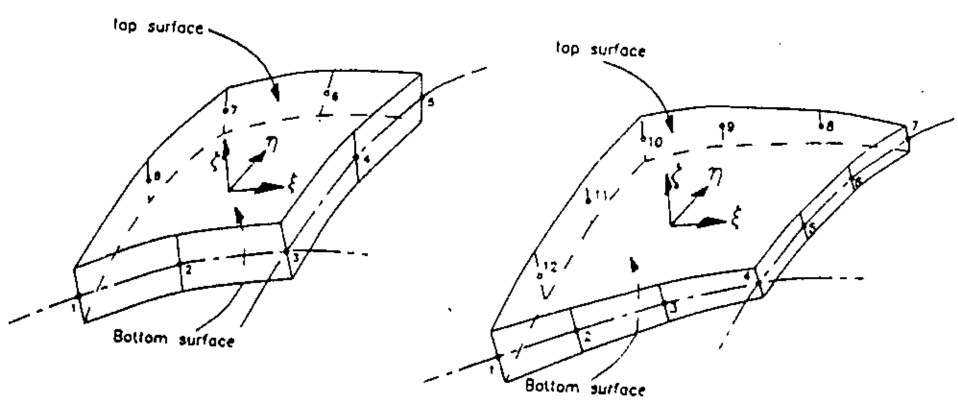


Fig. A.2.1 Division of Structure with Parabolic Elements



(a) Parabolic

(b) Cubic

Fig. A.2.2 Definition of Element Topology

given in the same unit as the nodal displacements of an element. The stresses are usually calculated at the nodal points in the global system.

The top and bottom co-ordinates of each node with respect to Cartesian co-ordinate are fed into the program. Co-ordinates for non-corner nodes lying on straight edges are not required to be given. If these co-ordinates of the nodes are fed into the program, then the shape of the element is automatically defined in the program. Therefore the thickness of the element can vary from node to node and the edges may be curved parabolically and cubically depending upon the type of element used. The program as at present can handle isotropic elastic material. The material properties are defined for every element separately, thus allowing the program to deal with materials varying from element to element. The temperature and pressure can be varied from node to node.

Output from the Program

The displacements are calculated and printed against each node in the ascending order for every loading case. Stresses are first calculated in the local orthogonal system and then transformed to the global Cartesian system. For every node the top surface stresses are followed by the bottom surface stresses .

The global stresses are also stored separately for top and bottom surfaces against nodal numbers and at the end a simple averaging is performed on them. The average stresses are then printed out in the ascending order of the nodal numbers. The top surface stresses for all the loading cases are followed by the bottom surface stresses.

Division of Structure into Elements

The structure is first of all divided into suitable elements and the nodes are numbered in any suitable way as shown for example in Fig.A.2.1. The elements are also suitably numbered in some sequence on which they are fed in the computer. Two probable sequences are shown in Figs.A.2.1a and A.2.1b. Each element is topologically defined by its nodal numbers in a consistent right hand screw system shown in Fig.A.2.2a and A.2.2b.

Front Width and Selection of Order of Elimination

To carryout the analysis of a structure using minimum possible computer storage, the elements selected in such a sequence that the maximum number variables to be handled at any particular time (the front width) is minimum. For example, the prescribed order of elements in Fig.A.2.1a gives the smallest front width. This is evident even from inspection in a simple structure.

The thick shell program uses the frontal solution technique. Here the assembly of an element stiffness and the corresponding right hand sides is immediately followed by the process of elimination of the variables corresponding to nodes which occur for the last time. This is indicated to the program by inserting a -ve sign before these nodes. This can easily be put in most shell structures once the element sequence has been selected.

A.3 EVALUATION OF STRESSES AT GAUSS POINTS

Formulation of the element stiffness matrix involves integration of complex polynomial shape functions of the elements and their derivatives. Exact integration of such functions are very troublesome and numerical integration becomes essential. Numerical integration of any function involves evaluation of the function at some representative points within the element. The number of representative points required for a certain degree of accuracy depends on the technique of integration employed. It is found that number of sampling points are minimum when Gauss' quadrature is applied. For this reason virtually all finite element programs uses Gauss' quadrature. The Gauss quadrature formula is,

$$I = \int_{-1}^{+1} f(\xi)\,d\xi = \sum_{i=1}^{n} H_i f(a_i)$$

In the above formula integration is performed over the range -1 to +1, a_i is the abscissa of the Gauss points, $f(a_i)$ is the ordinate, H_i is the weight coefficient and n is the number of sampling points. Value of a_i and H_i depends on the value of n. The following table lists these values for n upto 3.

	$\pm a$	H
$n = 1$	0.0	2.0
$n = 2$	0.577350269	1.0
$n = 3$	0.774596692 0.000000000	0.5555555555 0.8888888888

In Ahmad's program 2-point and 3-point Gauss' quadratures are applied for the 8-noded and 12-noded elements respectively. Integration is performed in the local element co-ordinate (ξ , η , ζ) system. The co-ordinates of the points within the element varies between +1 and -1. If 2-point (n = 2) integration is performed then, on the middle surface, there are four Gauss points because integration is carried out in both ξ *and* η direction. The co-ordinates of these four points with respect to ξ, η system will be (±0.577350269,± 0.577350269). These points are shown in Fig.A.3.1

In finite element analysis using displacement methods, the stresses are discontinuous between elements, because of the nature of the assumed displacement variation. For this reason stress at a node is calculated by averaging the stresses obtained at that node from the elements common to that node. Experience has shown that in the case of isoparametric elements the Gauss integration points are the best stress sampling points. The element nodes, which are the most useful points for output and interpretation of stresses, appear to be the worst sampling points. However, it has been observed that shape function derivatives (and hence stresses) evaluated at interior of the elements are more accurate than those calculated on the element boundary.

The thick shell elements developed by Ahmad [2] are good for analysing singly or doubly curved shell structures where load is carried by bending as well as inplane forces. In most shell structures transverse shear is not a very important quantity. It has been found from previous experience that out-of-plane shear in such ordinary shell structures are small in magnitude and they can safely be neglected in design. Consequently, while formulating element characteristics, less importance was given to the evaluation of transverse shear stresses. For this reason, the thick shell program of Ahmad gives somewhat erroneous nodal transverse shear stresses. However, stresses at Gauss points are predicted with good accuracy. In shell or plate structures, like a free standing stair, transverse shear is important. Total torsion on a given section can correctly be determined only if shear stresses are correctly known. For this reason determination of stresses at Gauss points becomes essential if one uses Ahmad's program to analyse any structure where shear and torsion are important.

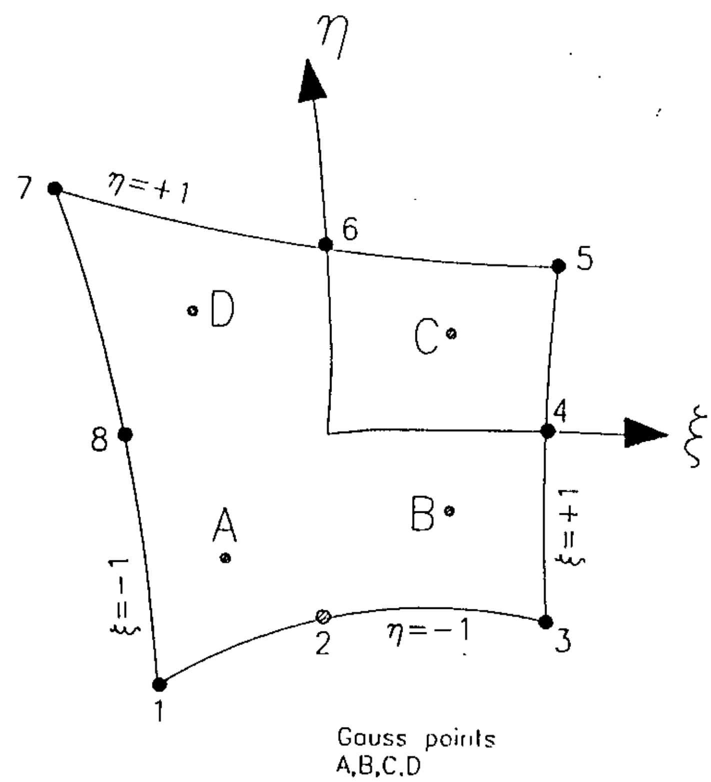


Fig. A.3.1 Gauss Points in a 8-Noded Element

APPENDIX B

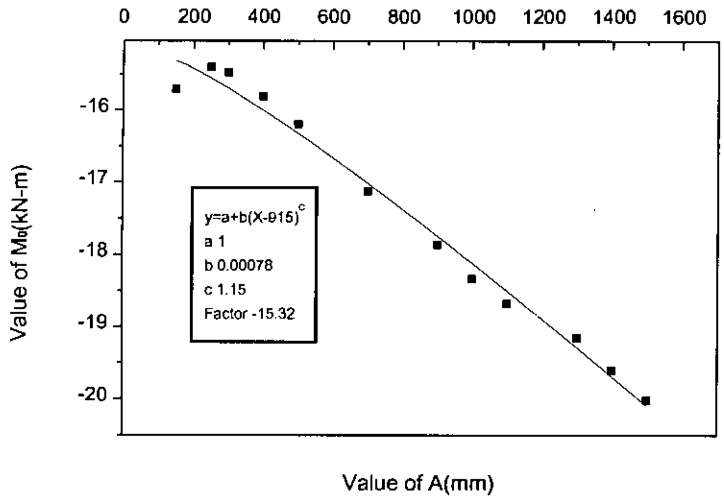


Fig B.1 Equation of A for Mo(Load Case 1)

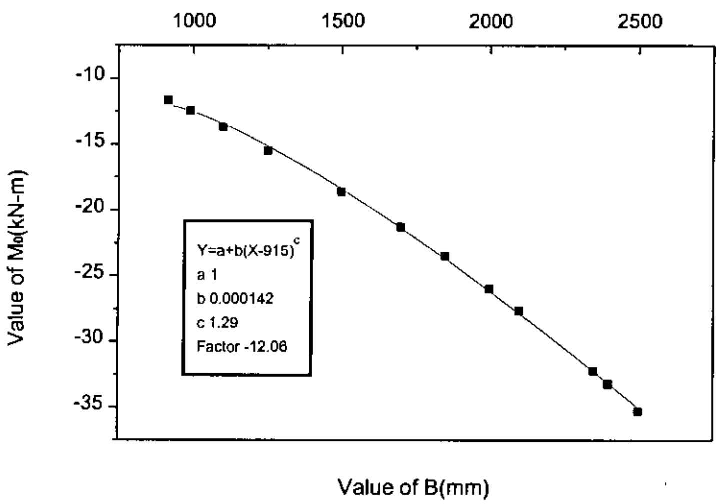


Fig B.2 Equation of B for Mo(Load Case 1)

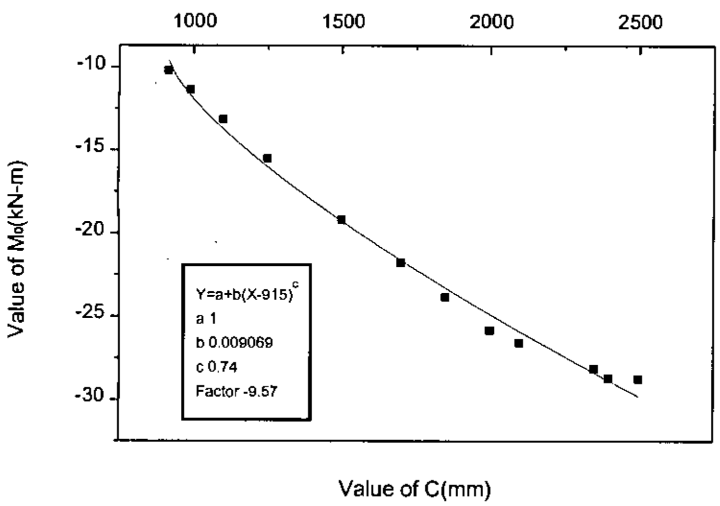


Fig B.3 Equation of C for Mo(Load Case 1)

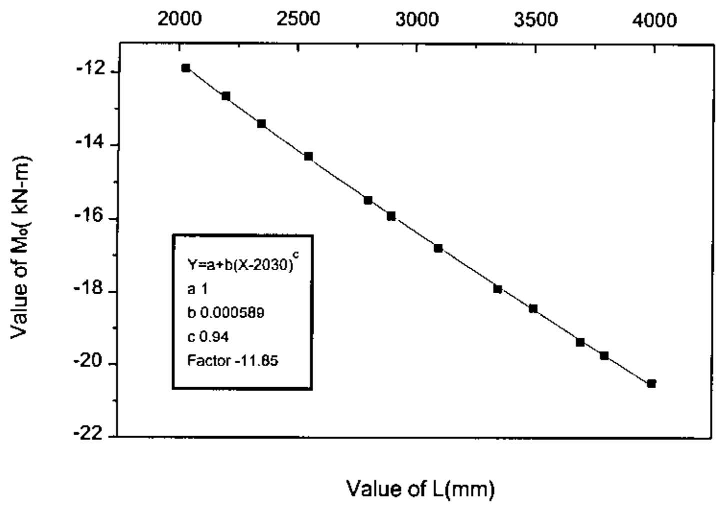


Fig B.4 Equation of L for Mo(Load Case 1)

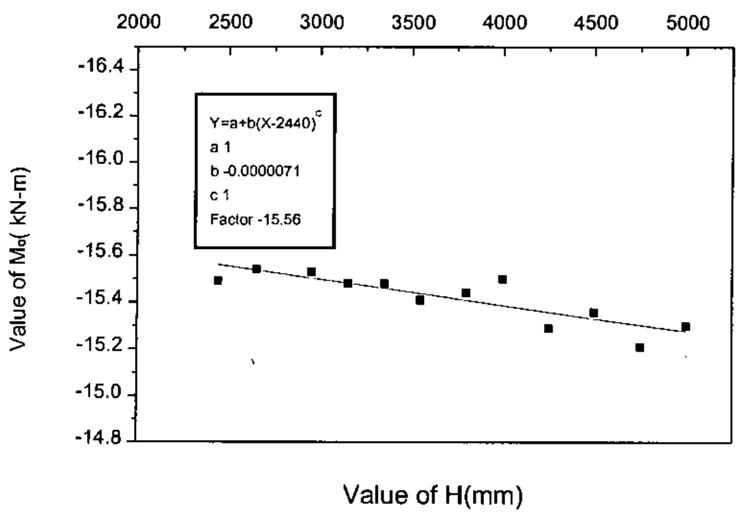


Fig B.5 Equation of H for Mo(Load Case 1)

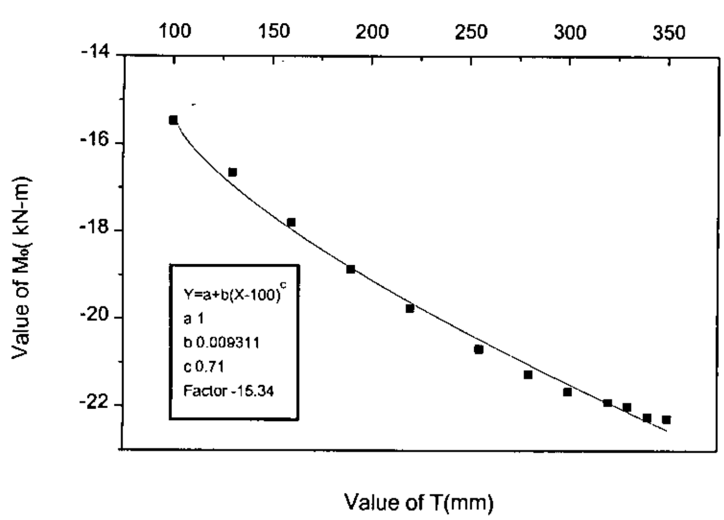


Fig B.6 Equation of T for Mo(Load Case 1)

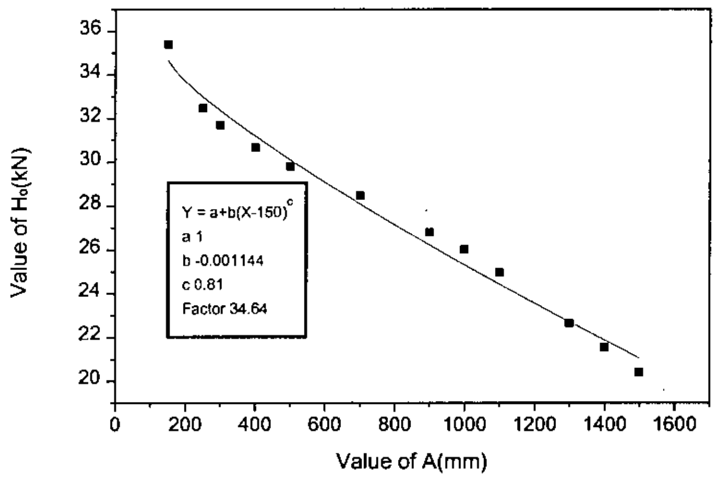


Fig B.7 Equation of A for H_o(Load Case 2)

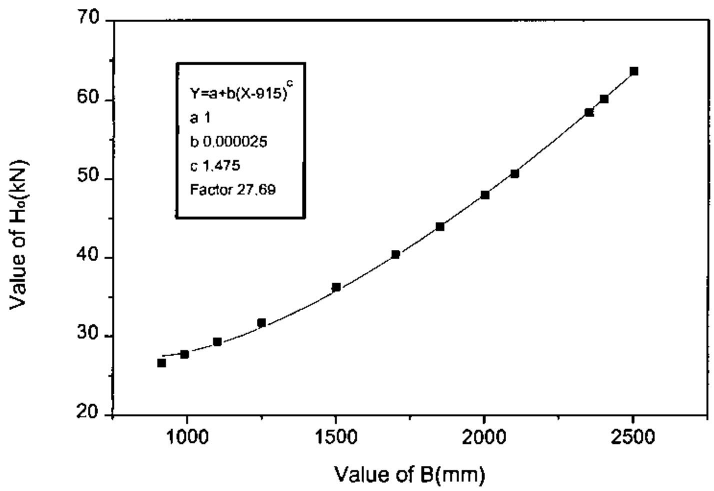


Fig B.8 Equation of B for H_o(Load Case 2)

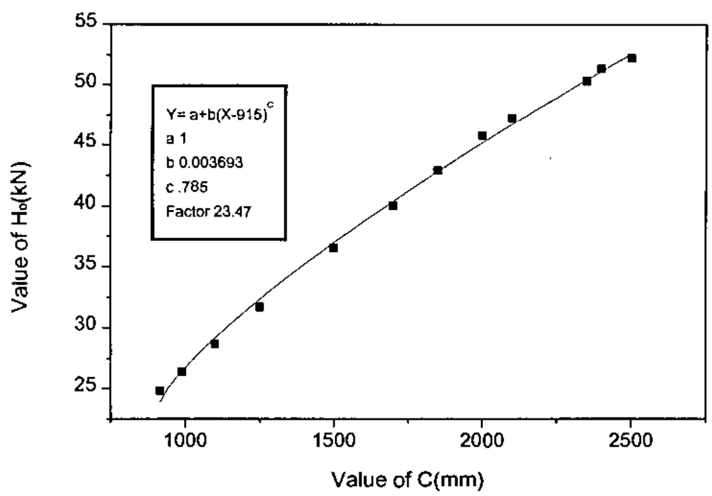


Fig B.9 Equation of C for H_o(Load Case 2)

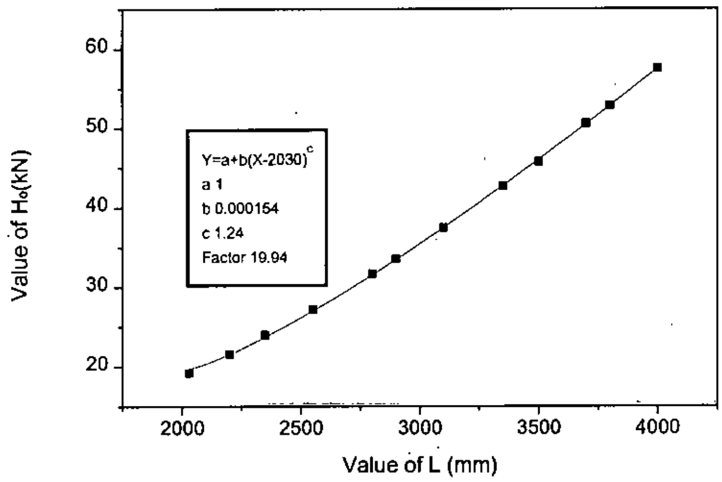


Fig B.10 Equation of L for H_o(Load Case2)

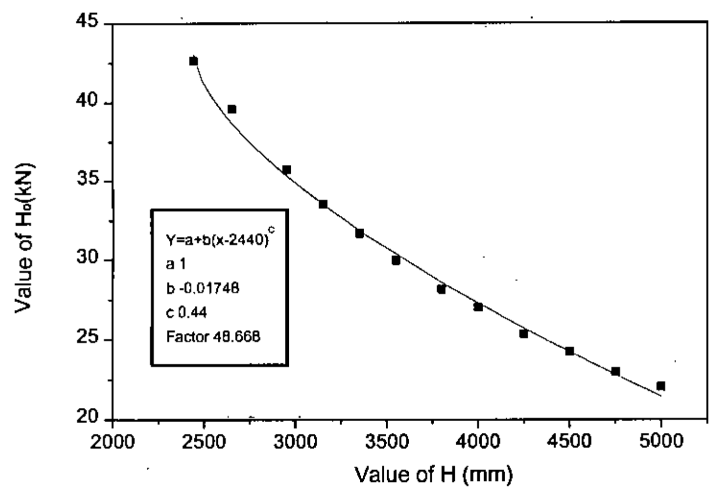


Fig B.11 Equation of H for H_o(Load Case2)

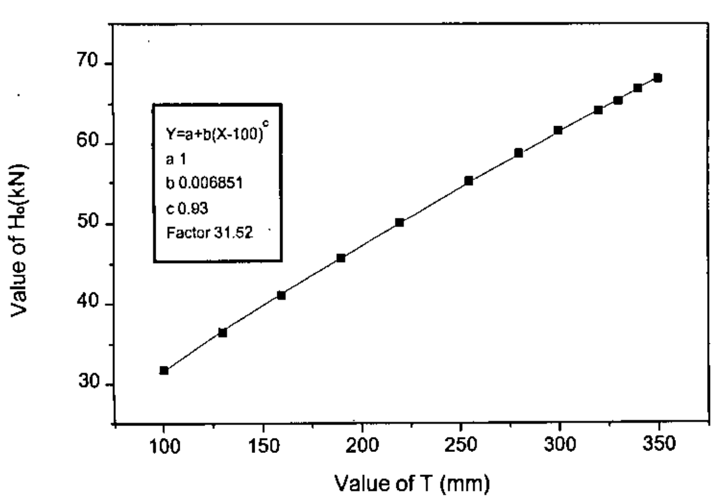


Fig B.12 Equation of T for H_o(Load Case2)

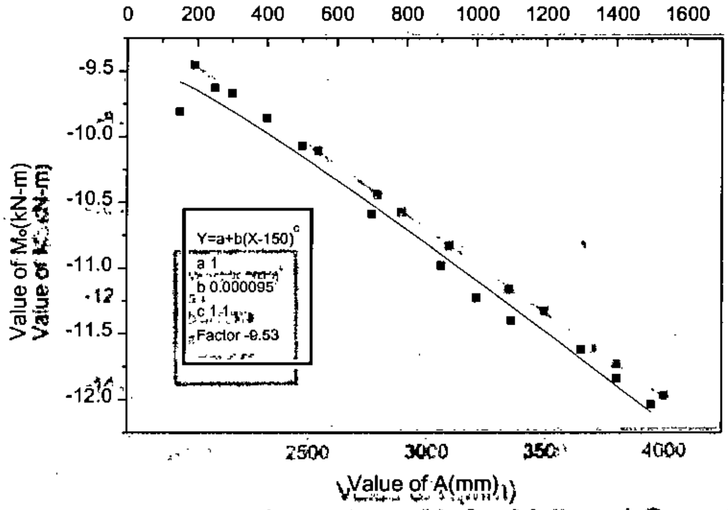


Fig B.13 Equation of A for M_o(Load Case2)

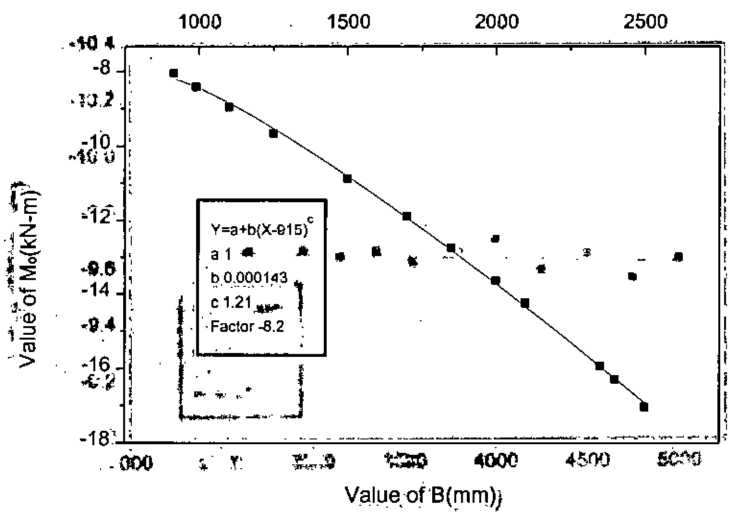


Fig B.14 Equation of B for M_o(Load Case2)

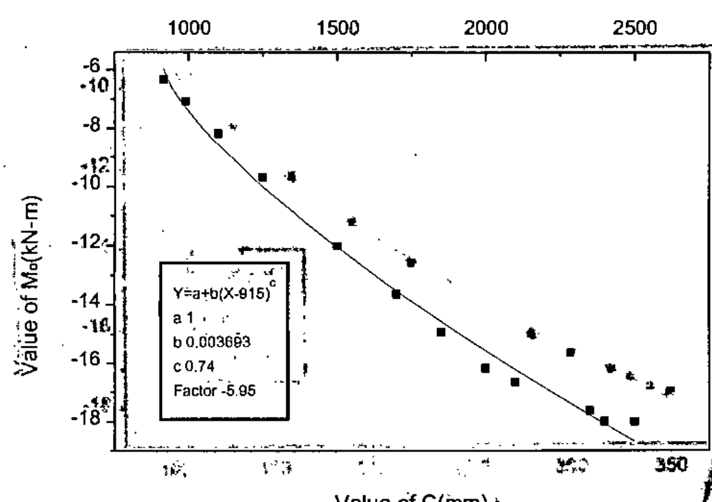


Fig B.15 Equation of C for M_o(Load Case2)

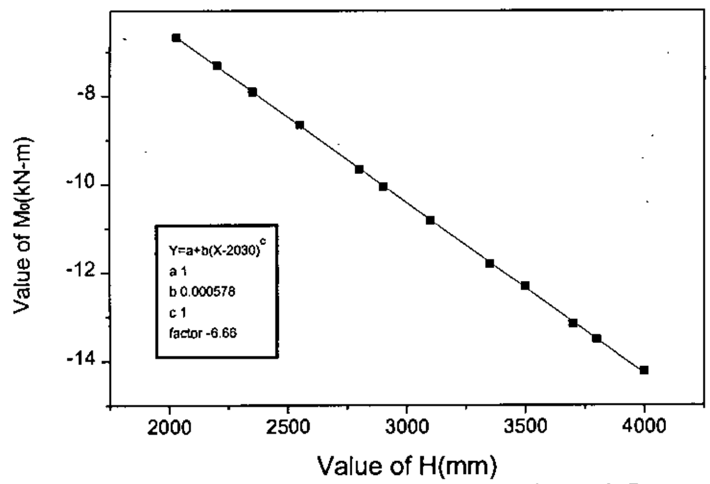


Fig. B.16 Equation of L for Mo(Load Case 2)

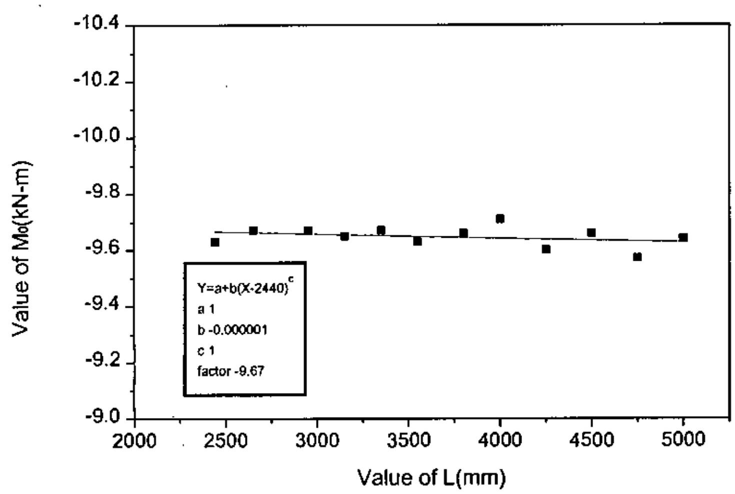


Fig. B.17 Equation of H for Mo(Load Case 2)

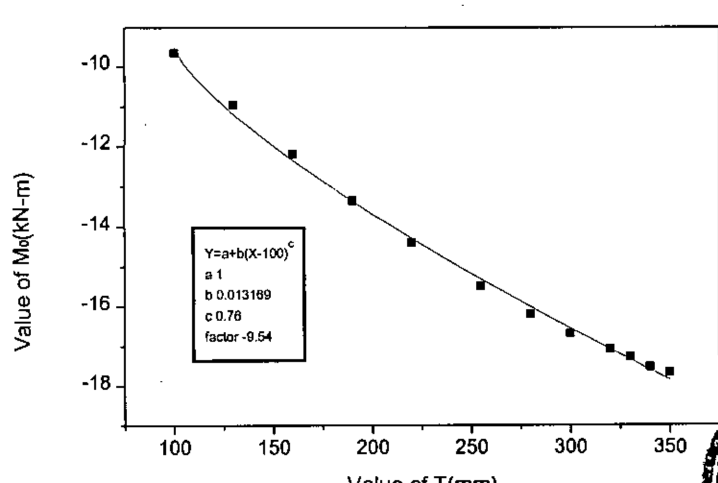


Fig. B.18 Equation of T for Mo(Load Case 2)