Analytical Investigation of Steady Flow Around a Circular Wellbore in an Elliptical Domain

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I hereby declare that this thesis or any part of it has not been submitted elsewhere for the award of any degree or diploma.

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This work is dedicated to

My Uncle

Md. Abdul Hakim Sarker

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ABSTRACT

Flow behavior through a porous medium depends on fluid properties as well as the characteristics of the medium. In a homogeneous and isotropic porous medium, flow around a wellbore is generally radial or linear. In some cases, flow around a wellbore may also become elliptical instead of radial or linear. Many authors discussed the elliptical flow behavior assuming a vertical fracture at the center of the reservoir. Elliptical flow may also be observed around a circular wellbore at the center of an anisotropic reservoir even if there is no vertical fracture. Elliptical flow situation around a circular wellbore also occurs during the production phase of cyclic steam stimulation (CSS) when a shear zone of enhanced permeability is created around the wellbore during the injection phase of this process. No discussion is found in petroleum literature on elliptical flow behavior with circular wellbore at the center. This study modified the analytical solutions available in literature to investigate steady flow around a circular wellbore in an elliptical domain.

The new model is used to analyze the effect of eccentricity of the elliptical domain on flow rate. It was found that radial flow model will produce as much as nine percent error in flow calculation if it is used in elliptical system. Using "Continuous Succession of Steady States" (CSSS) method, the model is also applied to estimate the depth of investigation for a system with elliptical flow domain. The depth of investigation in radial and elliptical systems are also compared in this study. The elliptical model was found in accordance with the radial model at eccentricity equal to zero. The area of drainage was also found to be independent of the flow rate, pressure level and shape of the elliptical domain.

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Greek symbols

θ	=	angular coordinate
ρ	=	fluid density
ϕ	=	porosity
μ	=	viscosity

Symbols

a	=	half of the major axis of the elliptical domain
A_{XO}	=	geometric factor of the elliptical domain
b	=	half of the minor axis of the elliptical domain
c_f	=	fluid compressibility
c_m	=	rock/pore compressibility
c_t	=	total compressibility
dV	=	differential volume
dG	=	differential mass
e	=	eccentricity of the elliptical domain
g	=	acceleration due to gravity
h	=	reservoir pay thickness
K	=	hydraulic conductivity
k_{max}	=	maximum permeability of the anisotropic medium
k_{min}	=	minimum permeability of the anisotropic medium
p	=	pressure
q	=	well flow rate
r	=	radial coordinate
R	=	distance of the outer boundary of the elliptical domain
r_w	=	wellbore radius
t	=	time
t_D	=	dimensionless time

u_m	=	geometric coefficient
z	=	vertical distance

Suffix

D	=	dimensionless
i	=	at outer boundary or at the boundary of investigated region
w	=	wellbore

CHAPTER 1

INTRODUCTION

Study of fluid flow through porous media has a great importance in Petroleum Engineering. Without knowing the characteristics of fluid flow through a porous medium, it is impossible to design and optimize the production facilities of petroleum substances from a hydrocarbon reservoir.

Flow through porous media depends on the characteristics of the media such as homogeneity, permeability distribution, shape of the reservoir outer boundary, shape of the aquifer inner boundary, existence of fractures etc. Generally, in a homogeneous and isotropic medium, when two-dimensional flow is considered, a radial or linear flow situation is found. On the other hand, in the area surrounding a vertical fracture, in an anisotropic formation or in a reservoir with elliptical outer boundary, flow will become elliptical (Kucuk and Brigham, 1979). Similar flow situation also occurs in Cyclic Steam Stimulation (CSS) when a vertical fracture is created during injection period creating an elliptical flow domain (Tamim, 1995).

The concepts of radial and linear flows were developed many years ago in fluid mechanics as well as in reservoir engineering. However, elliptical flow in the porous media is a relatively new concept. The earliest discussion on elliptical flow behavior is attributed to Muskat in 1937. After that many authors tried to model their reservoir incorporating elliptical flow.

In case of a reservoir having a vertical fracture (Fig 1.1) at the center of the reservoir, flow along the fracture axis will be higher than that of other directions due to the infinite conductivity of the fracture. This variation of fluid velocity along the different directions creates an elliptical drainage area around the fracture.

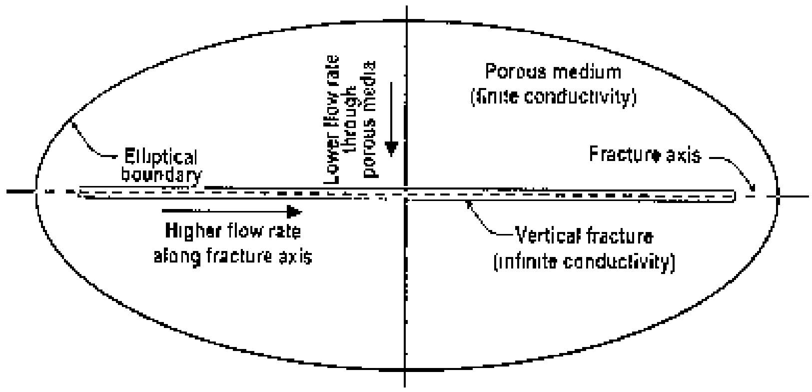

Fig 1.1: Elliptical Flow with Fracture at the Middle of the reservoir

In petroleum engineering study, there are several situations where the flow is around a circular wellbore in an elliptical domain. Flow in anisotropic reservoir is an example of such case. If the reservoir is homogeneous but anisotropic i.e. if directional permeability is present then a circular wellbore will not produce drainage area in a circular fashion. In this case the circular drainage area will be distorted and an elliptical domain around the wellbore will be created. Fluid velocity along the direction of maximum permeability (say k_{max}) becomes the highest and that along the lowest permeability (say k_{min}) becomes the lowest. As a result, fluid will be drained to a higher distance along the direction of k_{max} than the direction of k_{min}. Eventually, it will create an elliptical drainage area around the wellbore (Fig 1.2). It is possible to determine the directions of maximum and minimum permeability of the anisotropic reservoir. The techniques of measuring anisotropy are available in many references (Arnold *et al.*, 1962, Head *et al.*, 1993, Wannell *et al.*, 1993, Iverson and Ajdari, 1996).

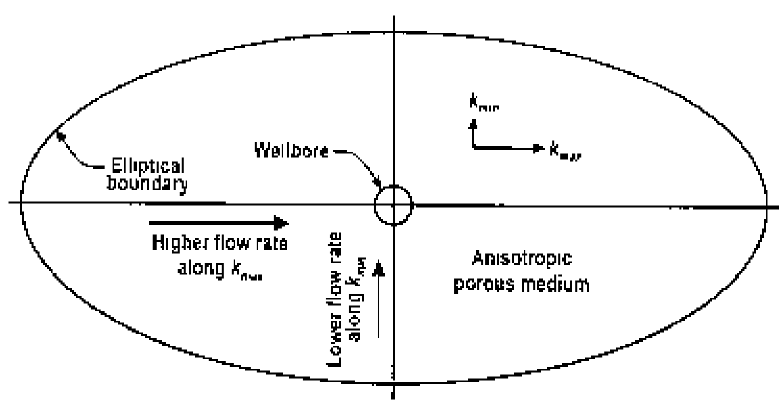

Fig 1.2: Elliptical Flow in Anisotropic Media (without Vertical Fracture)

During the production phase of CSS process, the fractures which were created at the injection phase can be closed These closed fractures create a "sheer zone" around the wellbore into the reservoir. This sheer zone exhibits directional permeability and creates elliptical flow around the wellbore.

In this study, the steady state flow equation for a reservoir with incompressible fluid and elliptical flow boundary is considered. The circular wellbore is considered at the center of the reservoir. The depth of investigation is also determined from the model and is compared with other models in literature.

1.1 Statement of the Problem

Radial and linear flow situations are observed very commonly in reservoir engineering. Elliptical flow situation, on the other hand, does not occur as frequently as radial or linear flows. In case of a reservoir having vertical fracture, anisotropy or elliptical shaped outer boundary, elliptical flow is found in the reservoir around the wellbore. Analytical flow equations for an elliptical flow domain with vertical fractures are available in petroleum literature (Muskat, 1937; Coats *et al.*, 1959; Prats, 1961; Prats *et al.*, 1962; Kucuk and Brigham, 1979, Obut and Ertekin, 1987). In absence of vertical fracture, the anisotropic formation produces elliptical flow domain around a circular wellbore. In Cyclic Steam Stimulation (CSS) process, the production period also encounters elliptical flow situation around a circular wellbore upon the creation of shear zone of enhanced permeability due to the injection of high-pressure steam during the injection period (Settari and Raisbeck, 1981). No reference is found in petroleum literature discussing flow around a circular wellbore in an elliptical domain in absence of a vertical fracture.

The objective of this study is to adopt the steady state water flow equation of van der Ploeg *et al.* (1971) modeling the flow around a circular wellbore at the center of the elliptical domain that will be appropriate in Reservoir Engineering application. The effect of the shape and size of the elliptical domain on flow rates will also be investigated. The new model will be applied for determining 'the depth of investigation' in an elliptical domain.

1.2 Solution Methodology

In developing the steady flow equation in elliptical domain with circular wellbore at the center, steady state well flow theory for a confined elliptical aquifer developed by van der Ploeg *et al.*, (1971) was used

The following features highlighted the methodology that has been used for this study:

1) Modification of equation of van der Ploeg *et al.* (1971) which is related to water flow in porous media, and use it in reservoir engineering scenario.

2) Comparison of the results with radial model to check the validity using same set of data.

3) Derivation of the equation for 'depth of investigation' using the method of "Continuous Succession of Steady States" (CSSS).

4) Validation of the proposed equation by comparing results with the radial model.

5) Determination of depth of investigation for elliptical domain for different eccentricities.

1.3 Organization of the Thesis

This study intends to establish a new approach to understand the elliptical flow through porous media with a circular wellbore at the center of the flow domain.

The literature review is presented in Chapter 2. The whole review is presented under two sections – 'elliptical flow' section and 'radius of drainage' section.

In Chapter 3, development of flow equation and its analysis are discussed. The analysis is done on the basis of eccentricity of the elliptical domain. The difference in radial and elliptical flow model is also discussed in the analysis.

Chapter 4 contains discussions of application of this new approach in determining depth of investigation in elliptical domain. The expression of radius of drainage has been derived by the method of "Continuous Succession of Steady States".

Chapter 5 presents general discussions, conclusions, recommendations etc. from this study.

The data tables for this study are presented in Appendix A. Appendix B presents the derivation of flow equation by Van der Ploeg, Kirkham and Boast (1971). Computer programs for generating of data from the proposed equation, written in Turbo C++, version 3.0 are presented in Appendix C and Appendix D.

CHAPTER 2

LITERATURE REVIEW

A lot of work is found in the literature on elliptical flow and radius of investigation. The works are reviewed here in the following two sections.

2.1 Elliptical Flow

Elliptical flow in porous media is well established in petroleum literature. A series of authors considered elliptical flow in their reservoir modeling, but it is mostly discussed in modeling of the vertical fracture of the reservoir.

Muskat (1937) first presented an analytical solution for the steady-state flow from a finite line source in an infinite reservoir. The author considered a canal or river as a line source and the theory of conjugate functions to derive the model. It was shown that equipressure curves are confocal ellipses and streamlines are confocal hyperbolas.

Coats *et al.* (1959) considered unsteady-state liquid flow through porous media having elliptical boundaries. Their flow model was based on two confocal ellipses, called inner and outer boundaries. They started with diffusivity equation governing unsteady-state liquid flow through porous media. They also assumed uniform porosity and permeability throughout the flow model. They transformed the Cartesian coordinate system to elliptical coordinate system in their solution. The final form of the flow equation was solved numerically and compared with other flow models. They compared their elliptical model with radial model on the basis of equal area encompassed by the exterior ellipse and the exterior circle and equal areas included within the interior ellipse and the interior circle. They showed that the water influx calculation was about 7% in error when an elliptical boundary was approximated by an equal area circle for short dimensionless time.

Prats (1961) considered vertical fracture at the center of a cylindrical reservoir. His incompressible fluid model showed elliptical pressure distribution near the wellbore. His model characterizes the effect of vertical fracture by defining a parameter which is the ratio of the flow abilities of the formation and the fracture. He showed that the flow around the fracture becomes circular from elliptical as the parameter approaches infinity from zero. Prats, like other authors, also used the transformation of axes to elliptical coordinate to solve his model. Prats, Hazebroek and Strickler (1962) also studied with compressible fluids and found that the effect of fractures on compressible fluid could also be ignored.

Arnold and Gonzalez (1962) presented a graphical method of estimating reservoir anisotropy from production data. Their study was based on the transient pressure distribution in a porous and permeable medium. The method considered elliptical isopotential lines for anisotropic reservoirs. They showed that the ratio of major and minor axes is related to the maximum and minimum permeability ratio.

A closed-form solution for saturated flow into a fully penetrating well in elliptical flow geometry was proposed by van der Ploeg, Kirkham and Boast (1971). Their work was related to water flow in a confined elliptical aquifer. A free surface was assumed at the water head. Steady-state flow equations were developed for various well locations using gravity flow. They started with Laplace's equation in polar coordinate and solve it using the Gram-Schimdt method as modified by Powers *et al.* (1967). Results and flow nets were presented for several cases.

Kucuk and Brigham (1979) worked on the transient flow model in elliptical systems. They used two-dimensional diffusivity equation and transform the equation to elliptical coordinates. Their model is applicable to infinite conductivity vertical fractured wells, elliptically shaped reservoirs, and anisotropic reservoirs producing at a constant rate or pressure. They generated type curves for both constant rate and pressure. In their following work (1981), they applied the elliptic flow solution to water influx problems in elliptical and anisotropic aquifer. They presented solutions for the dimensionless cumulative water influx and influx rate as a function of the dimensionless time in tabular and graphical forms.

Hale and Evers (1981) developed elliptical flow equation for vertically fractured gas wells in low permeability or tight reservoir. The equation was developed for use with short-term flow tests at either constant rate or constant pressure test conditions. Their generalized elliptical equation combines linear properties of early time behavior with the radial properties of late time flow in a single equation. They used the conformal mapping technique to solve the problem for steady state elliptical flow regime and modified the solution for unsteady state problems by defining radius of investigation which changes as a function of dimensionless time

A composite system in elliptical flow geometry is considered by Obut and Ertekin (1984). Their study was based on transient data on injection well having infinite conductivity vertical fracture. They idealized the swept volume as an elliptical region with focii located at the tips of the fracture wings. Assuming equal pressure and flux at the interface they gave solutions for both constant pressure and constant rate at the wellbore using the technique of separation of variables. Their solution is a good approximation of the stated problem.

While studying the effect of thermoelastic stresses on injection well fracturing, Perkins and Gonzales (1985) found elliptical flooded zone around the fracture. They stated that along the growth of the fracture, the flow system evolved from an essentially circular geometry in plan view to one characterized more nearly as elliptical. Their study was related to change of stress due to the temperature difference of the injected fluid and the reservoir fluid. They derived an expression to find the major and minor axes of the ellipse.

Okoye *et al.* (1988) presented an analytical solution to unsteady state and pseudo-steady state flow in naturally fractured reservoirs with elliptical boundaries. They started from diffusivity equation and applied Laplace Transformation to develop their model. Finally they inverted the Laplace space solution using numerical Laplace inversion algorithm to calculate the dimensionless pressure and time values.

Several authors (Dietrich (1986), Arthur *et al.* (1991), Tamim *et al.* (1995)) considered elliptical flow geometry in Cyclic Steam Stimulation (CSS) process of enhanced oil recovery. Dietrich (1986) considered CSS model of tar sands with hydraulically induced fracture. He assumed an elliptical heating pattern as hot injection fluid moved off a vertical fracture face. He found that the major and minor axes of the heated ellipsoid which is produced out of steam injection process had great importance in the choice of well spacing and the time of inter-well heat communication

Arthur *et al.* (1991) in their analytical model for CSS also considered elliptical flow geometry. They assumed that the length of the fracture obtained during the steam injection for a particular cycle does not change at the end of the production period for that cycle. They divided the entire elliptical flow geometry produced from the fracture during the steam injection into a hot and warm zone to evaluate the performance of CSS for the purpose of optimization and process control of commercial operations.

Tamim and Farouq Ali (1995) presented another analytical CSS model including formation parting. Their model was based on a fracture heating computation coupled with fluid flow. They also considered elliptical flow geometry in their CSS model.

2.2 Radius of Drainage

Radius of drainage is the distance that a pressure transient has moved into a formation following a rate change in a well. Radius of drainage is characterized by the two criteria. One is pressure change and the other is flow rate. Many authors considered these two criteria as a basis in defining the radius of drainage.

Tek *et al.* (1957) developed a mathematical formulation of the radius of drainage based on a developed field where gas production approached steady state condition. They defined the radius of drainage as the radius at which the flow rate is one per cent of that at the wellbore at any instant.

Hurst *et al.* (1961) compared the Kelvin solution (line source approximation), which applies to an infinite reservoir, to that for a bounded reservoir, and found that initially the two solutions were essentially identical. They correlated the time till which this match continues and introduced a correlation for radius of drainage.

van Poolen (1964) presented radius of drainage equation by combining the solution of diffusivity equation for both finite and infinite acting reservoir. He based his derivation on the *Y*- function (pressure derivative) of Jones (1962).

Ishteiwy and van Poolen (1969) derived radius of drainage equation for pressure buildup analysis for a single well. They considered two boundary conditions. The first boundary condition considered a single well near an extensive linear barrier in an otherwise homogeneous and infinite reservoir. In the second case, a single well is assumed near a linear and extensive pressure source in an otherwise homogeneous and infinite reservoir.

Kutasov and Hejri (1984) used the material balance condition to determine well drainage radius. Their radius of drainage was based on a well at constant bottom-hole pressure in an infinite acting reservoir They found the relationship between dimensionless drainage radius and dimensionless time.

Johnson (1988) defined radius of drainage as the radius enclosed a volume in the reservoir that accounts for a specified faction (ε) of the cumulative production. He considered constant production rate and constant formation volume factor with the line-source solution of diffusivity equation. He used volumetric material balance by assuming constant compressibility and porosity of a homogeneous reservoir.

Liao and Lee (1994) determined depth of investigation for elliptical flow problems for hydraulically fractured wells. They assumed an elliptical shaped wellbore formed due to fracture at the center of the reservoir. They also used their model to elliptically composite reservoirs. They used the method of "continuous succession of steady states" in their derivation of depth of investigation from steady-state flow equation.

CHAPTER 3

RESERVOIR MODELING

The flow theory of van der Ploeg, Kirkham and Boast (1971) used in this study was related to water flow in a confined elliptical aquifer. They developed a closed-formed solution for steady saturated flow into a fully penetrating well in elliptical flow geometry. A free surface was assumed at the water head. Steady-state solutions were developed for various well locations using gravity flow. Results and flow nets were presented for several cases. Fig 3.1 is the geometric representation of their model. The figure is redrawn showing the well at the center. They considered different well locations for their study. The essence of their approach was to derive orthonormal functions for the problem using the methods of Powers *et al.* (1967). Although van der Ploeg *et al.* presented solutions for different well locations, only the solution for a well at the center is considered in this study.

3.1 Flow Equation

It is important to design an accurate model to extract any meaningful result from its analysis. For the ease of calculation it is also critical to make the appropriate assumptions. In the present work, a reservoir with elliptical drainage system is considered. The following assumptions are to be considered in the development of the model –

1. Flow is elliptical into a common source or sink
2. Reservoir has a constant thickness
3. Reservoir is considered homogeneous in all rock properties
4. Formation is completely saturated with single fluid
5. Fluid saturation is constant
6. Steady-state, single phase flow

7. Compressibility of the system is constant and small
8. Viscosity of the fluid is constant throughout the reservoir
9. Well is completed across the entire formation thickness

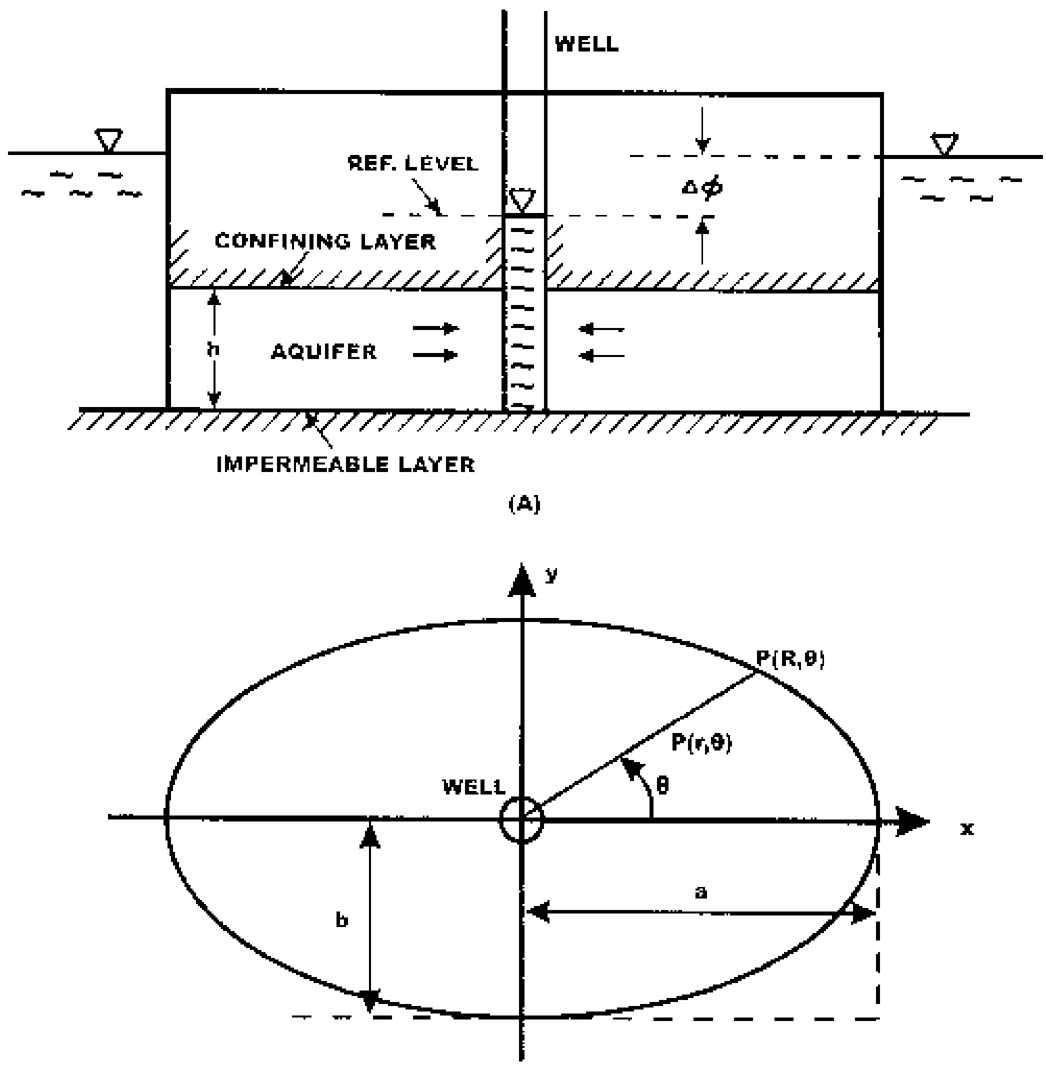

Fig 3.1: Geometric Representation of van der Ploeg et al. Model

(A) Cross-sectional View, (B) Plan View

The flow equation from a circular wellbore of an elliptical drainage boundary as developed by van der Ploeg *et al.* (Appendix B) is given by –

$$q = -\frac{2\pi K h A_{\lambda,w} \Delta\phi}{\ln \frac{a}{r_w}} \qquad 3.1$$

where,

$\Delta\phi$ is the difference in hydraulic head,

K is the hydraulic conductivity of the reservoir,

a is the major axes of the reservoir,

r_w is the wellbore radius and

h is the reservoir pay thickness.

$A_{N\theta}$ is a dimensionless coefficient that depends on the geometry of the reservoir (eccentricity) and the location of the well. A detailed description of the development and solution procedure of this equation is given in Appendix A.

The hydraulic conductivity K used in groundwater flow is related to permeability, density and viscosity used in petroleum engineering. Hydraulic conductivity can be written as –

$$K = \frac{k\rho g}{\mu} \quad 3.2$$

where,

k is the reservoir permeability,

ρ is the fluid density,

g is the acceleration due to gravity and

μ is the fluid viscosity.

When density is assumed to be constant, the hydraulic head ϕ can be expressed as

$$\phi = \frac{p}{\rho g} + z \quad 3.3$$

Therefore,

$$\Delta\phi = \frac{\Delta(p + \rho g z)}{\rho g} \quad 3.4$$

So, Eqn 3.1 becomes –

$$q = -\frac{2\pi k h A_{N0}\Delta(p+\rho g z)}{\mu \ln\frac{a}{r_w}} \qquad 3.5$$

For horizontal flow Δz is zero and hence Eqn 3.5 becomes –

$$q = -\frac{2\pi k h A_{N0}\Delta p}{\mu \ln\frac{a}{r_w}} \qquad 3.6$$

Ignoring the minus sign and rearranging the Eqn 3.5, a dimensionless flow equation may be found to be –

$$Q_D = \frac{q\mu}{kh\Delta(p+\rho g z)} = \frac{2\pi A_{N0}}{\ln(a/r_w)} \qquad 3.7$$

3.1.1 Permeability

In Eqn 3.5, the permeability term k, appears in the numerator, has different values for isotropic and anisotropic reservoirs. For isotropic reservoir it is simply k, an average permeability of the reservoir. Typically, this permeability is determined either from core-analysis data or from well testing. On the other hand, in anisotropic reservoir the average permeability value has to deal with at least two directional permeabilities, k_{max} and k_{min}. Kuchuk and Brigham (1979) reported average permeability for anisotropic porous media as –

$$k = \bar{k} = \sqrt{k_{max} k_{min}} \qquad 3.8$$

3.1.2 Axes of the elliptical domain

a and b are the axes of the elliptical domain. These values depend on the characteristics of reservoir.

3.1.2.1 Elliptical reservoir

For elliptical isotropic reservoir, major axis 'a' and minor axis 'b' will be as good as the geometric axes of the reservoir.

3.1.2.2 Anisotropic reservoir

For anisotropic reservoir the permeability will be considered according to Eqn 3.8. The major and minor axes of the elliptical drainage system formed due to anisotropy of the reservoir are related to the directional permeability of the reservoir. Arnold and Gonzalez (1962) expressed the relationship of major and minor axes with permeabilities. They showed that the ratio of major and minor axes becomes –

$$a/b = \sqrt{k_{max}/k_{min}} \qquad 3.9$$

3.2 Characteristics of $A_{N\theta}$

In radial system the flow equation is given by –

$$q = \frac{2\pi kh\Delta(p + \rho gz)}{\mu \ln \frac{r_e}{r_w}} \qquad 3.10$$

The flow equation in elliptical drainage system (Eqn 3.5) is very similar to radial system (Eqn 3.10). The only difference between these two equations is the presence of an additional term $A_{N\theta}$ in the elliptical one. Also the outer boundary radius r_e is replaced by the major axis a of elliptical domain. Development of sequential formulas for elliptical equation by Powers *et al.* (Appendix B) reveals that the coefficient $A_{N\theta}$ depends on $u_m(\theta)$ only. The term $u_m(\theta)$ is given by –

$$u_m(\theta) = \frac{\left(\frac{R^2}{a^2}\right)^m - \left[\frac{r_w^2}{a^2}\frac{r_w^2}{R^2}\right]^m}{1 - \left(\frac{r_w^2}{a^2}\right)^{2m}} \cos 2m\theta \qquad 3.11$$

It is clear from Eqn 3.11 that $u_m(\theta)$ is a dimensionless quantity and depends not on the values of wellbore radius, major and minor axes of the elliptic domain but on their ratios. So the coefficient in elliptic equation A_{N0} is a dimensionless quantity and depends on the shape of the elliptic domain with respect to the wellbore radius. In other words, A_{N0} is a geometric factor of the reservoir. The suffix N of the constant A_{N0} tends to infinity but the value of A_{N0} converges to a number with only few values of N. Fig 3.2 depicts the convergence of A_{N0}. Fig 3.2 is plotted for major axes = 100, minor axes = 50 and radius of wellbore = 1 unit. The figure shows that only at N = 5 the value of A_{N0} converges. Table A. 1 and Table A. 2 are constructed to realize how A_{N0} varies with different sets of major axes, minor axes and wellbore radius ratios. The data are plotted in Fig 3.3. The figure indicates that A_{N0} increases with eccentricity but decreases when the ratio of major axes and wellbore radius increases.

The values of eccentricities for an elliptical domain vary from zero to unity. The eccentricity values, which are close to zero, indicate the circular shape or closely circular shape of the domain. The values of A_{N0} in these cases are unity or close to unity. So the constant A_{N0} is a kind of deviation factor which corresponds to the effect of eccentricity or the deviation of the domain from the circle. When the wellbore dimension is small compared to other dimensions such as major or minor axes of the elliptical domain, the variation of A_{N0} values becomes small as the eccentricity increases.

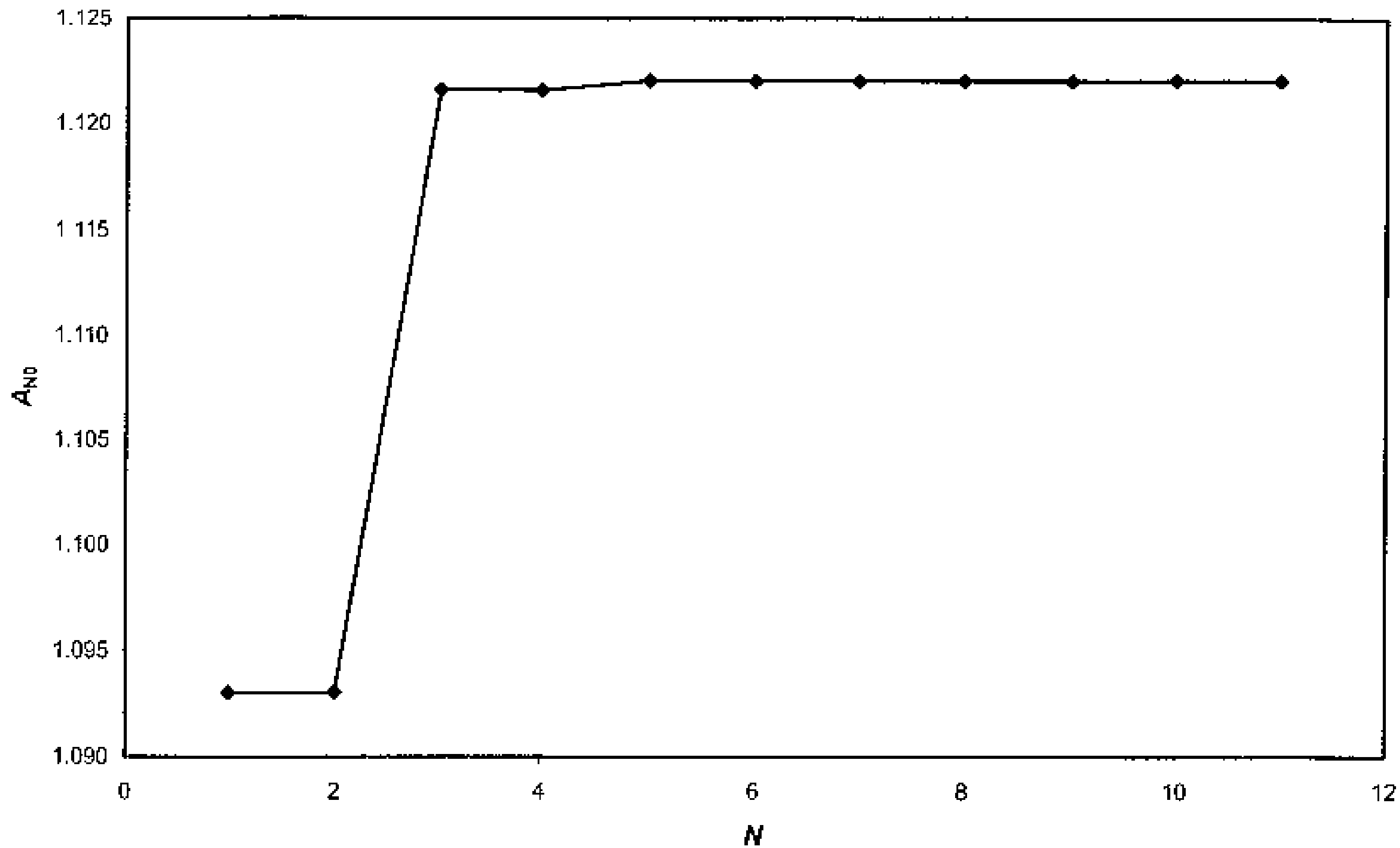

Fig 3.2: Converging Behavior of A_{NO}

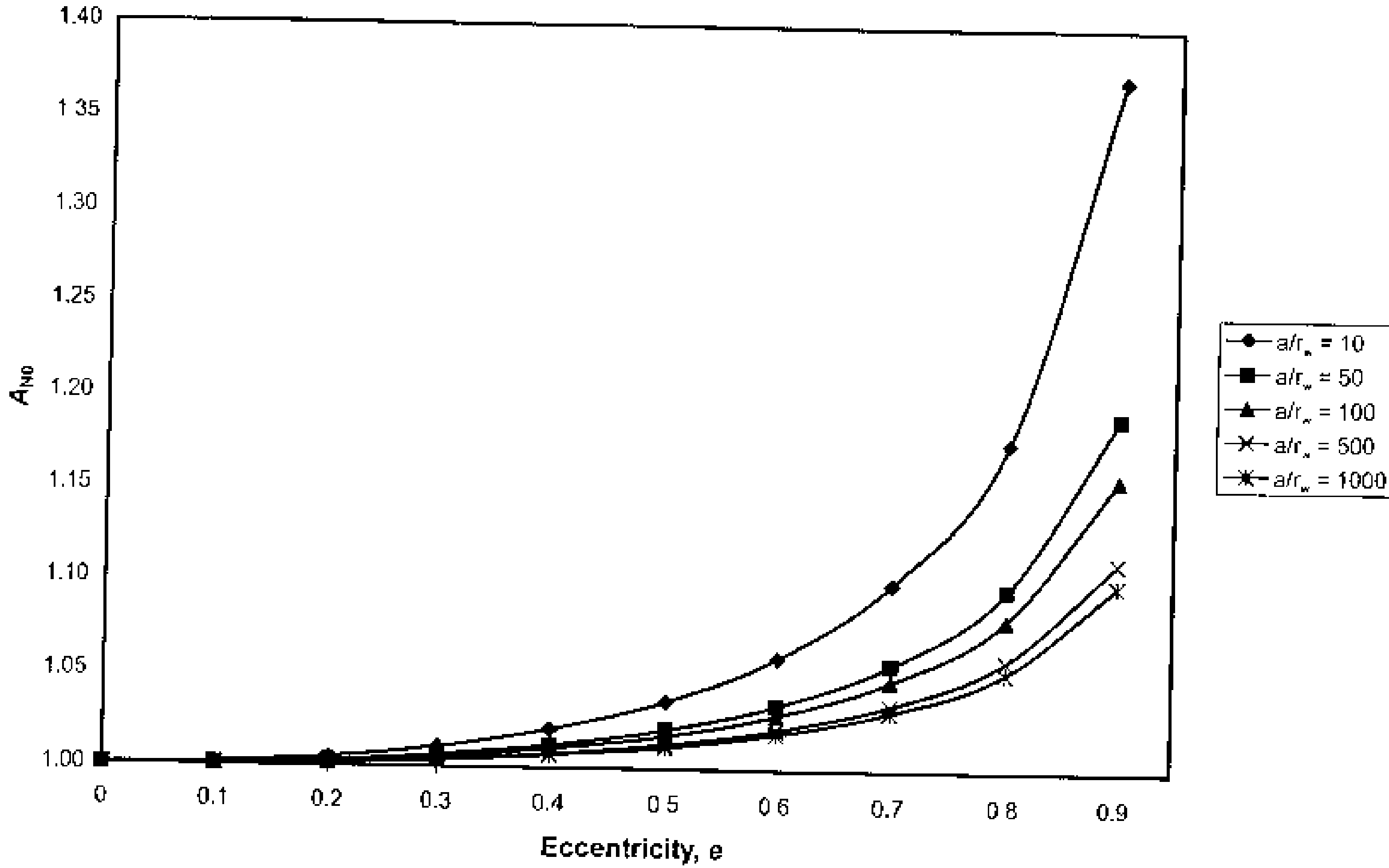

Fig 3.3: Variation of A_{N0} with Eccentricty

3.3 Comparison of the flow equations

In this section the elliptical flow equation is compared with the radial flow equation. The comparison is shown in tabular form as well as graphically. To compare radial flow equation and elliptical flow equation, primarily a circular reservoir is considered. Keeping the area of the reservoir constant, equivalent elliptical domains are constructed (Fig 3.4) for different eccentricity to apply the elliptical equation. The radius of the wellbore is kept constant for both the systems.

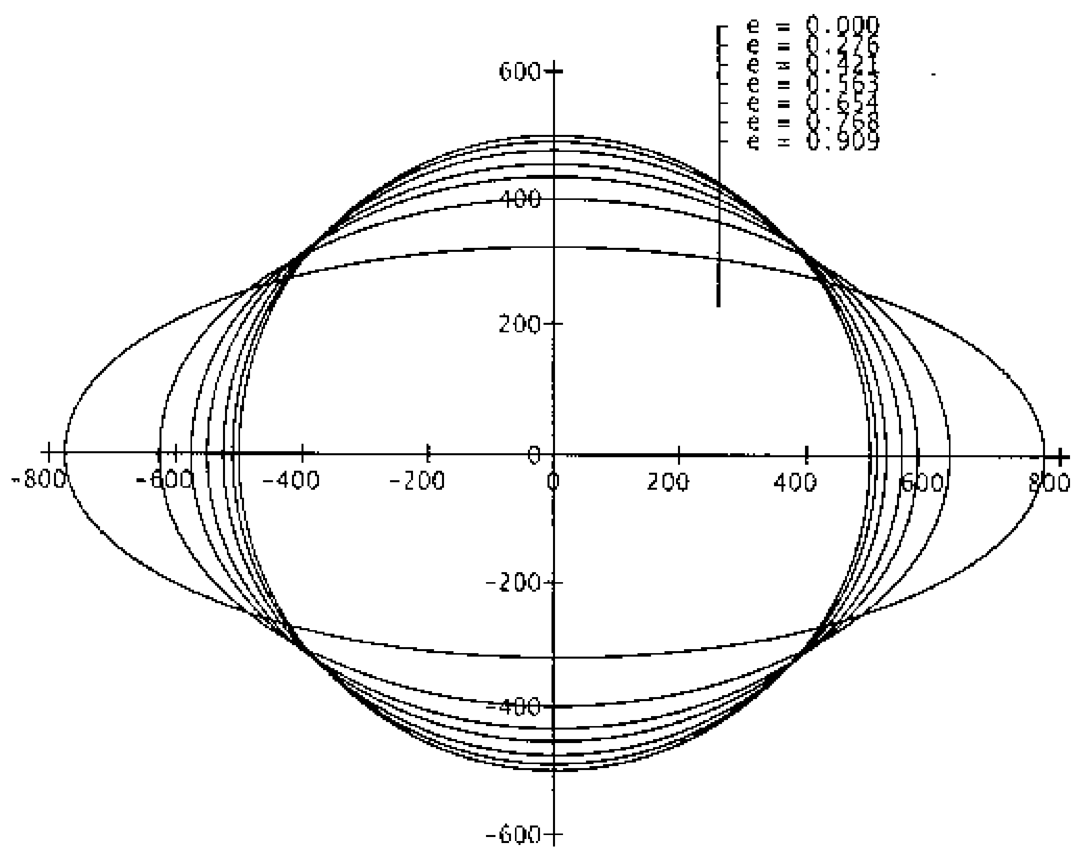

Fig 3.4: Circular and Equivalent Elliptical Reservoirs (equal area)

In Table A. 3 dimensionless flow rates are shown where axes of the elliptical domain are expressed by the ratio of r_w.

From the Fig 3.5 it is clear that at eccentricity below 0.5, the dimensionless flow rates in both radial and elliptical drainage systems do not vary significantly. With the increase in eccentricity, as the value of A_{Nq} increases, the dimensionless flow rate in elliptical drainage system also increases. It can be noted that the value of $Q_{D(elliptical)}$ does not change up to three decimal places when eccentricity changes from zero to around 0.5. For an eccentricity value above 0.5, deviation of $Q_{D(elliptical)}$ becomes significant. Percentage of error and corresponding to eccentricity of the elliptical drainage area is shown in Fig 3.6. An eccentricity above 0.9 may produce as high as nine-percent error in flow calculation. A similar error in flow calculation was reported by Coats *et al.* (1959).

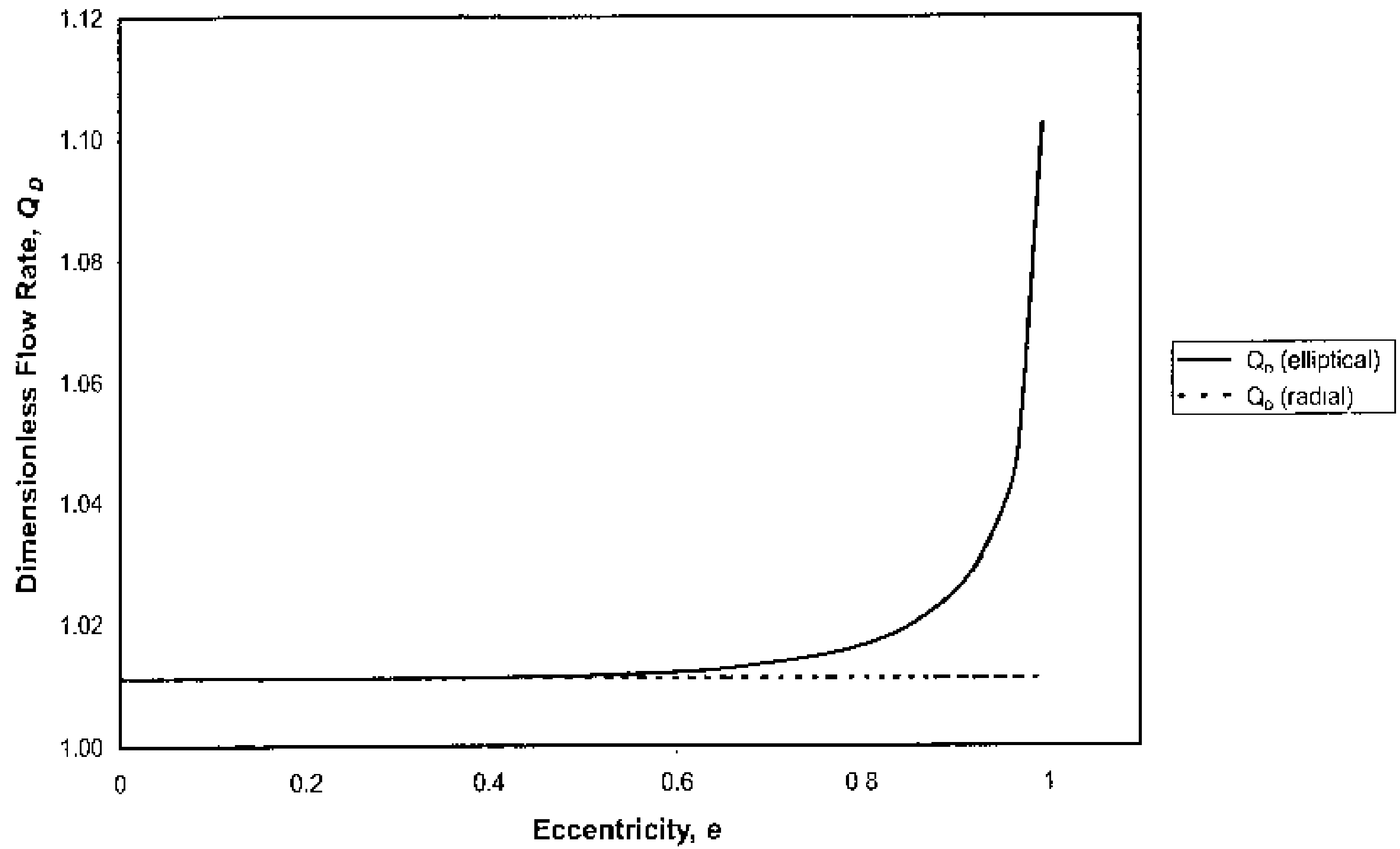

Fig 3.5: Dimensionless Flow Rate at Different Eccentricity

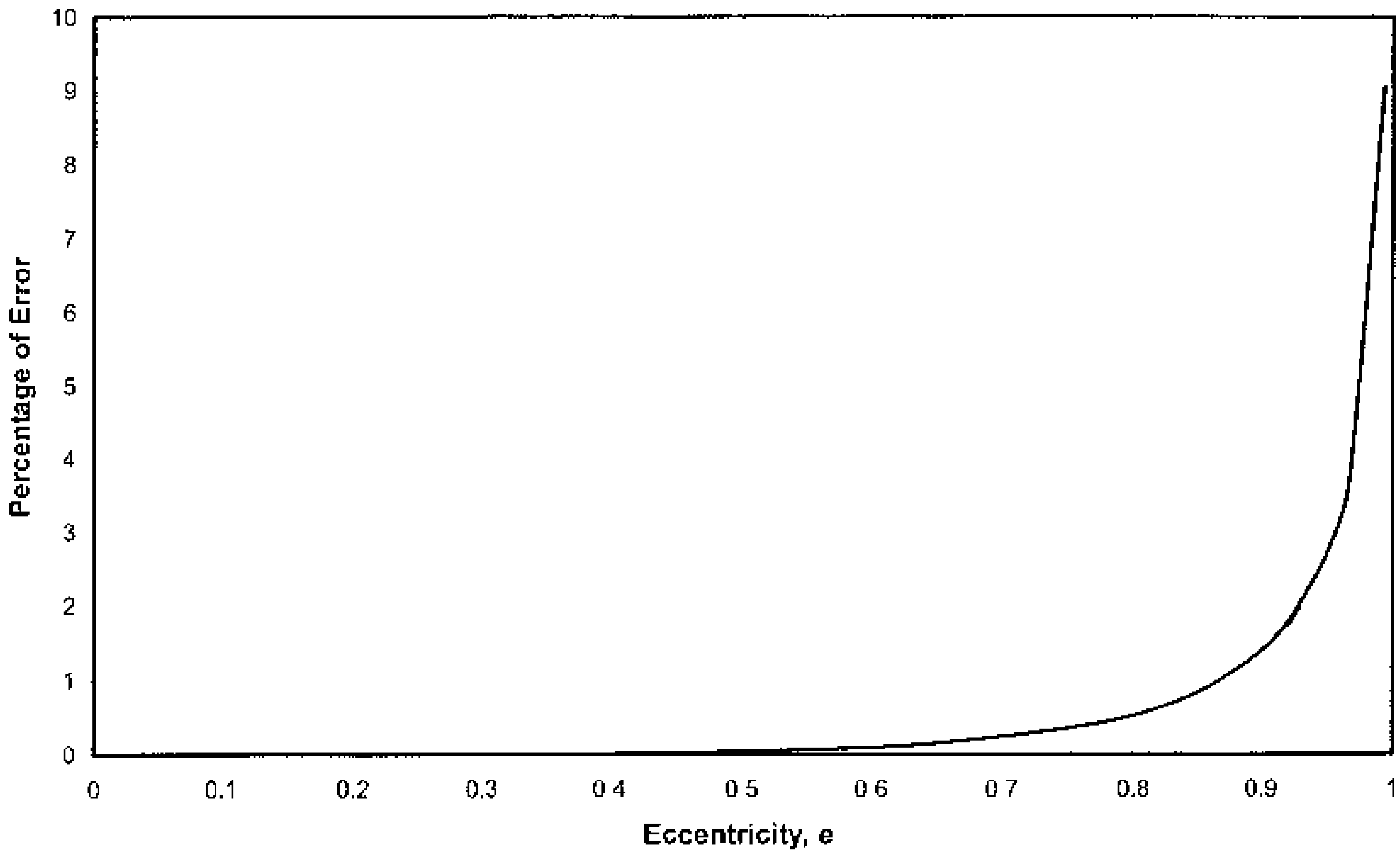

Fig 3.6: Percentage of Error in Flow Calculation in Elliptical Drainage System When Flow is Calculated Using Radial Model

CHAPTER 4

DERIVATION OF "DEPTH OF INVESTIGATION"

In this section, an equation of depth of investigation for elliptical flow systems, such as for anisotropic homogeneous reservoir is going to be developed. Depth of investigation is a very important parameter as it is used in well test design and analysis. The depth of investigation for elliptical flow systems can be used in a way similar to radius of investigation for radial flow problems.

4.1 Depth of Investigation

For radial flow problems, the concept of "radius of investigation" has been established and widely used in well test design and analysis. When a change in production rate of a well in a reservoir takes place, it can be considered as a disturbance or a pulse and it will propagate throughout the reservoir to be adjusted with time. The distance over which the pressure transient has moved into a formation following a rate change in a well can be defined as the radius of investigation.

For radial flow systems, the disturbance propagates radially and the investigated area is circular. A circle should have a radius and so the term "radius of investigation" is suitable for radial systems. In case of linear or elliptical flow problems, the scenario is different. For linear flow cases investigated area is rectangular and for elliptical cases it is elliptical. So the term "radius of investigation" is not suitable for the latter systems. It is better to use the term "depth of investigation" instead of "radius of investigation" for these cases.

If the pressure transient of a reservoir can be expressed as a function of distance from the well, r and time, t, i.e.

$$p = p(r,t) \quad 4.0a$$

the depth of investigation at time t can be defined implicitly by Eqn 4.0b as –

$$\frac{\partial}{\partial r} p(r,t) = 0 \quad 4.0b$$

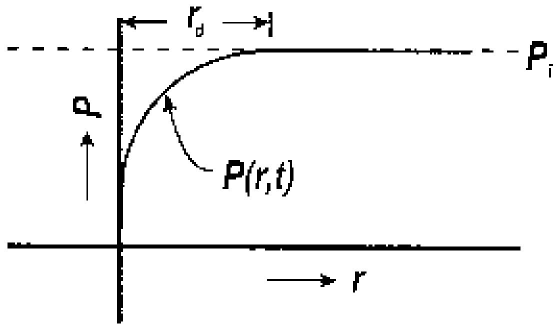

Fig 4.1: Depth of Investigation

4.2 Derivation of Depth of Investigation

The method of "Continuous Succession of Steady States (CSSS)" is used to derive the depth of investigation for elliptical flow system using steady state flow equation around a circular wellbore in an elliptical domain.

The concept of CSSS was first introduced by Muskat (1937). This method considers a transient unsteady-state flow as continuous succession of steady state flow pattern. Each steady state flow pattern is different from the others.

Determination of depth of investigation deals with transient flow problem, or time dependent problem. In the method of CSSS, the time variable plays the role of a parameter rather than an independent variable, and does not occur in the governing equation. Thus while the analytical details are carried through with the explicit assumption of time dependent boundary conditions, this is done with the understanding that if the boundary conditions do not vary with time, the pressure distribution would be a steady-state distribution appropriate to the corresponding instantaneous values of boundary conditions.

The steady-state flow in an elliptical domain with circular wellbore located at the center as expressed earlier by the Eqn 3.5 can be written with slight modification as –

$$q = \frac{2\pi kh A_{NO}(p - p_w)}{\mu \ln(a/r_w)} \quad 4.1$$

For a porous medium, Liao and Lee (1994) found the expression of porosity as –

$$\varphi = \varphi_o e^{c_m(p-p_o)} \quad 4.2$$

and for a fluid, the density is –

$$\rho = \rho_o e^{c_f(p-p_o)} \quad 4.3$$

where,

ϕ = porosity at pressure p

ϕ_o = porosity at pressure p_o

c_m = pore compressibility and

ρ = fluid density at pressure p

ρ_o = fluid density at pressure p_o

c_f = fluid compressibility

If the compressibilities c_m and c_f are small then using the approximation of exponential series (*e*-series) one can write –

$$\phi\rho = (\phi\rho)_o + (\phi\rho)_o c_t (p - p_o) \quad 4.4$$

Where, $c_t = c_m + c_f$, and c_t is called total compressibility.

Thus it can be written –

$$(\phi\rho)_i - (\phi\rho) = (\phi\rho)_i c_t (p_i - p) \quad 4.5$$

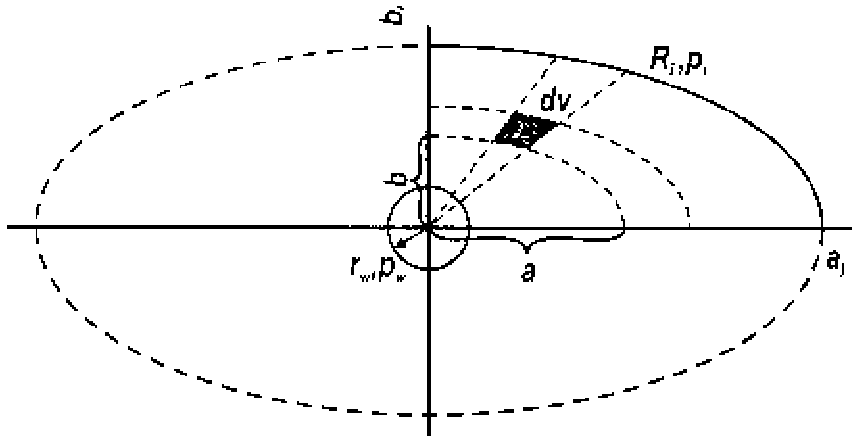

Fig 4.2: Elliptical Domain with Circular Wellbore at the Center

For a given time t, if it is assumed that the disturbance (pressure variation) has influenced an area from r_w to r_i where the pressures are p_w and p_i respectively, then the instantaneous pressure distribution at this time obeys the steady state distribution of Eqn 4.1. The equation can be rewritten as –

$$p - p_w = \left(\frac{q\mu}{2\pi kh}\right)\frac{\ln(a/r_w)}{A_{so}} \qquad 4.6$$

$$\text{or, } \ln(a/r_w) = \frac{2\pi kh}{q\mu} A_{so}(p - p_w) \qquad 4.7$$

and,

$$p_i - p_w = \left(\frac{q\mu}{2\pi kh}\right)\frac{\ln(a_i/r_w)}{A_{so}} \qquad 4.8$$

$$\text{or, } \ln(a_i/r_w) = \frac{2\pi kh}{q\mu} A_{so}(p_i - p_w) \qquad 4.9$$

Now,

$$\ln\frac{a_i}{a} = \ln\left(\frac{a_i}{r_w}\right) - \ln\left(\frac{a}{r_w}\right) \qquad 4.10$$

$$\text{or, } \ln\frac{a_i}{a} = \frac{2\pi kh}{q\mu}\left[A_{NOi}(p_i - p_w) - A_{NO}(p - p_w)\right] \quad 4.11$$

$$\text{or, } A_{NOi}(p_i - p_w) - A_{NO}(p - p_w) = \frac{q\mu}{2\pi kh}\ln\frac{a_i}{a} \quad 4.12$$

Putting the value of $\frac{q\mu}{2\pi kh}$ in Eqn 4.12 from Eqn 4.8 one can write,

$$A_{NOi}(p_i - p_w) - A_{NO}(p - p_w) = \frac{p_i - p_w}{\ln(a_i/r_w)} A_{NOi}\ln\frac{a_i}{a} \quad 4.13$$

$$\text{or, } \frac{A_{NOi}}{A_{NO}}(p_i - p_w) - (p - p_w) = \frac{p_i - p_w}{\ln(a_i/r_w)}\frac{A_{NOi}}{A_{NO}}\ln\frac{a_i}{a} \quad 4.14$$

$$\text{or, } \frac{A_{NOi}}{A_{NO}}(p_i - p_w) - (p - p_w) = \frac{A_{NOi}}{A_{NO}}\frac{p_i - p_w}{\ln(a_i/r_w)}\ln\frac{a_i}{a} \quad 4.15$$

$$\text{or, } p - p_w = \frac{A_{NOi}}{A_{NO}}\left[(p_i - p_w) - \frac{p_i - p_w}{\ln(a_i/r_w)}\ln\frac{a_i}{a}\right] \quad 4.16$$

$$\text{or, } p = p_w + \frac{A_{NOi}}{A_{NO}}\left[(p_i - p_w) - \frac{p_i - p_w}{\ln(a_i/r_w)}\ln\frac{a_i}{a}\right] \quad 4.17$$

Subtracting p_i from both sides –

$$p - p_i = -p_i + p_w + \frac{A_{NOi}}{A_{NO}}\left[(p_i - p_w) - \frac{p_i - p_w}{\ln(a_i/r_w)}\ln\frac{a_i}{a}\right] \quad 4.18$$

$$\text{or, } p - p_i = -(p_i - p_w) + \frac{A_{NOi}}{A_{NO}}(p_i - p_w) - \frac{A_{NOi}}{A_{NO}}\frac{p_i - p_w}{\ln(a_i/r_w)}\ln\frac{a_i}{a} \quad 4.19$$

$$\text{or, } p - p_i = \left(\frac{A_{NOi}}{A_{NO}} - 1\right)(p_i - p_w) - \frac{A_{NOi}}{A_{NO}}\frac{p_i - p_w}{\ln(a_i/r_w)}\ln\frac{a_i}{a} \quad 4.20$$

$$\text{or, } p = p_i - \left[\left(1 - \frac{A_{NOi}}{A_{NO}}\right)(p_i - p_w) + \frac{A_{NOi}}{A_{NO}}\frac{p_i - p_w}{\ln(a_i/r_w)}\ln\frac{a_i}{a}\right] \quad 4.21$$

The instantaneous production rate is given by Eqn 4.1 and it is rewritten as –

$$q(t) = \frac{2\pi k h A_{NO}(p - p_w)}{\mu \ln(a/r_w)} \qquad 4.22$$

An incremental volume within the drainage volume is dV.

$$dV = h(r\,dr\,d\theta) \qquad 4.23$$

Up to this time t, this small volume has produced an incremental liquid mass dG.

$$dG = [(\phi\rho)_i - (\phi\rho)]dV \qquad 4.24$$

$$dG = [(\varphi\rho)_i - (\phi\rho)]h(r\,dr\,d\theta) \qquad 4.25$$

Thus up to this time t, the total liquid mass from this volume is –

$$G(t) = 4\int_0^{\pi/2}\int_{r_w}^{R}[(\phi\rho)_i - (\phi\rho)]h\,r\,dr\,d\theta \qquad 4.26$$

where, $R = \dfrac{ab}{\sqrt{a^2\sin^2\theta - b^2\cos^2\theta}}$ 4.27

From Eqns 4.26 and 4.5 one can write

$$G(t) = 4\int_0^{\pi/2}\int_{r_w}^{R}(\phi\rho)_o c_t(p_i - p)h\,r\,dr\,d\theta \qquad 4.28$$

$$= 4(\phi\rho)_o c_t h\int_0^{\pi/2}\int_{r_w}^{R}(p_i - p)r\,dr\,d\theta \qquad 4.29$$

$$= 4(\varphi\rho)_o c_t h\int_0^{\pi/2}\int_{r_w}^{R}\left[\left(1 - \frac{A_{NOi}}{A_{NO}}\right)(p_i - p_w) + \frac{A_{NOi}}{A_{NO}}\frac{p_i - p_w}{\ln(a_i/r_w)}\ln\frac{a_i}{a}\right]r\,dr\,d\theta \qquad 4.30$$

$$= 4(\phi\rho)_o c_t h(p_i - p_w)\int_0^{\pi/2}\int_{r_w}^{R}\left[\left(1 - \frac{A_{NOi}}{A_{NO}}\right) + \frac{A_{NOi}}{\ln(a_i/r_w)}\frac{\ln(a_i/a)}{A_{NO}}\right]r\,dr\,d\theta \qquad 4.31$$

$$= 4(\phi\rho)_o c_t h(p_i - p_w)\int_0^{\pi/2}\int_{r_w}^{R}\left[1 - \frac{A_{NOi}}{A_{NO}} + \frac{A_{NOi}}{A_{NO}}\frac{\ln(a_i/a)}{\ln(a_i/r_w)}\right]r\,dr\,d\theta \qquad 4.32$$

$$= 4(\phi\rho)_o c_t h(p_i - p_w) \int_0^{\pi/2} \int_{r_w}^{R} \left[1 - \frac{A_{NOt}}{A_{NO}} \left\{1 - \frac{\ln(a_i/a)}{\ln(a_i/r_w)}\right\}\right] r\,dr\,d\theta \qquad 4.33$$

$$= 4(\phi\rho)_o c_t h(p_i - p_w) \int_0^{\pi/2} \int_{r_w}^{R} \left[1 - \frac{A_{NOt}}{A_{NO}} \left\{\frac{\ln(a_i/r_w) - \ln(a_i/a)}{\ln(a_i/r_w)}\right\}\right] r\,dr\,d\theta \qquad 4.34$$

$$= 4(\phi\rho)_o c_t h(p_i - p_w) \int_0^{\pi/2} \int_{r_w}^{R} \left[1 - \frac{A_{NOt}}{A_{NO}} \frac{\ln(a/r_w)}{\ln(a_i/r_w)}\right] r\,dr\,d\theta \qquad 4.35$$

$$= \frac{4(\phi\rho)_o c_t h(p_i - p_w)}{\ln(a_i/r_w)} \int_0^{\pi/2} \int_{r_w}^{R} \left[\ln(a_i/r_w) - \frac{A_{NOt}}{A_{NO}} \ln(a/r_w)\right] r\,dr\,d\theta \qquad 4.36$$

On the other hand, for constant rate production,

$$G(t) = qt\rho_o = \frac{2\pi khA_{NOt}(p_i - p_w)}{\mu \ln(a_i/r_w)} \rho_o t \qquad 4.37$$

Comparison of Eqns 4.36 and 4.37 yields

$$\frac{2\pi khA_{NOt}(p_i - p_w)}{\mu \ln(a_i/r_w)} \gamma_o t = \frac{4(\phi\gamma)_o c_t h(p_i - p_w)}{\ln(a_i/r_w)} \int_0^{\pi/2} \int_{r_w}^{R} \left[\ln(a_i/r_w) - \frac{A_{NOt}}{A_{NO}} \ln(a/r_w)\right] r\,dr\,d\theta$$

4.38

$$\text{or, } \frac{kt}{\phi_o c_t \mu} \frac{\pi A_{NOt}}{2} = \int_0^{\pi/2} \int_{r_w}^{R} \left[\ln(a_i/r_w) - \frac{A_{NOt}}{A_{NO}} \ln(a/r_w)\right] r\,dr\,d\theta \qquad 4.39$$

Now, the following dimensionless groups are defined

$$r_D = \frac{r}{r_w}, \qquad 4.40$$

$$t_D = \frac{kt}{\phi\mu c_t r_w^2}, \qquad 4.41$$

$$R_D = \frac{R}{r_w} \qquad 4.42$$

and $a_{De} = \frac{a_e}{r_w}$ 4.43

Therefore,

$$r^2 = r_w^2 r_D^2 \qquad 4.44$$

or, $2rdr = 2r_w^2 r_D dr_D$ 4.45

or, $rdr = r_w^2 r_D dr_D$ 4.46

Introducing dimensionless groups in Eqn 4.39 it is found

$$\frac{\pi A_{NDe}}{2} t_D = \int_0^{\pi/2} \int_1^{R_D} \left[\ln(a_{De}) - \frac{A_{NDe}}{A_{ND}} \ln(a_D) \right] r_D \, dr_D \, d\theta \qquad 4.47$$

or, $\frac{\pi A_{NDe}}{2} t_D = \int_0^{\pi/2} \int_1^{R_D} \ln(a_{De}) r_D \, dr_D \, d\theta - A_{NDe} \int_0^{\pi/2} \int_1^{R_D} \frac{\ln(a_D)}{A_{ND}} r_D \, dr_D \, d\theta$ 4.48

Now, from equation of an ellipse –

$$\frac{r^2 \cos^2\theta}{a^2} + \frac{r^2 \sin^2\theta}{b^2} = 1 \qquad 4.49$$

and, $b^2 = a^2(1 - e^2)$ 4.50

where, e is the eccentricity of the ellipse

From Eqns 4.49 and 4.50 it is deduced that

$$\frac{r^2 \cos^2\theta}{a^2} + \frac{r^2 \sin^2\theta}{a^2(1 - e^2)} = 1 \qquad 4.51$$

or, $a^2 = r^2 \cos^2\theta + \frac{r^2 \sin^2\theta}{(1 - e^2)}$ 4.52

or, $a = r\sqrt{\cos^2\theta + \frac{\sin^2\theta}{(1 - e^2)}}$ 4.53

$$\text{or, } r = a\frac{\sqrt{(1-e^2)}}{\sqrt{\sin^2\theta + (1-e^2)\cos^2\theta}} \qquad 4.54$$

$$\text{if, } f(\theta) = \frac{\sqrt{(1-e^2)}}{\sqrt{\sin^2\theta + (1-e^2)\cos^2\theta}}, \qquad 4.55$$

then

$$r = a\,f(\theta) \qquad 4.56$$

$$\text{and } a = r/f(\theta) \qquad 4.57$$

So, Eqn 4.48 becomes

$$\frac{\pi A_{NDr}}{2} t_D = \ln(a_{Dr})\int_0^{\pi/2}\int_1^{a_{Dr} f(\theta)} r_D\,dr_D\,d\theta - A_{NDr}\int_0^{\pi/2}\int_1^{a_{Dr} f(\theta)} \frac{1}{A_{ND}}\ln\left(\frac{r_D}{f(\theta)}\right) r_D\,dr_D\,d\theta \qquad 4.58$$

In the left hand side of Eqn 4.58 the there are two parts. These are –

$$I_1 = \ln(a_{Dr})\int_0^{\pi/2}\int_1^{a_{Dr} f(\theta)} r_D\,dr_D\,d\theta \qquad 4.59$$

And

$$I_2 = \int_0^{\pi/2}\int_1^{a_{Dr} f(\theta)} \frac{1}{A_{ND}}\ln\left(\frac{r_D}{f(\theta)}\right) r_D\,dr_D\,d\theta \qquad 4.60$$

I_1 can be evaluated by integration and it can be written as –

$$I_1 = \frac{\pi\ln(a_{Dr})}{4}\left[a_{Dr}^2\sqrt{1-e^2} - 1\right] \qquad 4.61$$

On the other hand, I_2 is evaluated numerically. So the final expression becomes –

$$t_D = \frac{2}{\pi A_{NDr}}\left[\frac{\pi}{4}\ln(a_{Dr})\left(a_{Dr}^2\sqrt{1-e^2} - 1\right) - A_{NDr}\times I_2\right] \qquad 4.62$$

Eqn 4.62 is the equation of the depth of investigation, a_{Dr}. It is an implicit equation of a_{Dr}. One can readily find the DOI from this equation if I_2 can be evaluated. The I_2

contains r_D or a_D term So for a given r_D or a_D, t_D can be calculated from this equation very easily.

4.3 Validation of the Model

The implicit expression of depth of investigation, Eqn 4.62 is derived for elliptical flow with circular wellbore at the center. Different eccentricity will represent different shape of the elliptical flow. Circle is a special case of ellipse whose eccentricity is zero. In literature an expression of depth of investigation for radial flow system whose drainage area is circular is available. Liao and Lee (1994) discussed the depth of investigation in details for radial flow systems as well as elliptical flow systems with vertical fracture at the middle. For radial flow systems, they found the depth of investigation as –

$$t_D = \frac{1}{4}\left(r_D^2 - 1 - 2\ln r_D\right) \qquad 4.63$$

It is also an implicit expression for depth of investigation r_D. Eqn 4.62 and the Eqn 4.63 both are in dimensionless form. Eqn 4.62 is solved with computer program which is given in Appendix D. The generated data are given in Table A. 4. As Eqn 4.62 is not an explicit expression for dimensionless depth of investigation a_{Di}, it was not suitable to calculate a_{Di} for different dimensionless time t_{Di}. For the convenience, t_{Di} values were generated for different a_{Di}'s taking the eccentricity of the elliptical domain as the parameter.

When the eccentricity of the elliptical domain is zero it becomes circular. In this case i.e. at e = 0, Eqn 4.62 should produce the depth of investigation similar to the radius of investigation in circular domain as given in Eqn 4.63. Both the equations are plotted in Fig 4.3. First ten data for Fig 4.3 are shown in Table A. 5.

In Fig 4.3, it is found that the curves are overlapping each other giving merely the same values of depth of investigation or radius of investigation. This is a good support for the validity of the elliptical model of this study.

4.4 Results

The depth of investigation for elliptical system is expressed in terms of the length of major axes. The elliptical drainage boundary can be obtained from major axes and eccentricity of the elliptical domain. In Fig 4.4, dimensionless depth of investigation is plotted against the dimensionless time at different eccentricities. The values for this figure are given in Table A. 6.

The depth of investigation in terms of minor axis can easily be calculated from the data of Table A. 4. The calculated data of minor axis are given in Table A. 6 and Table A. 7. The data were plotted in Fig 4.5.

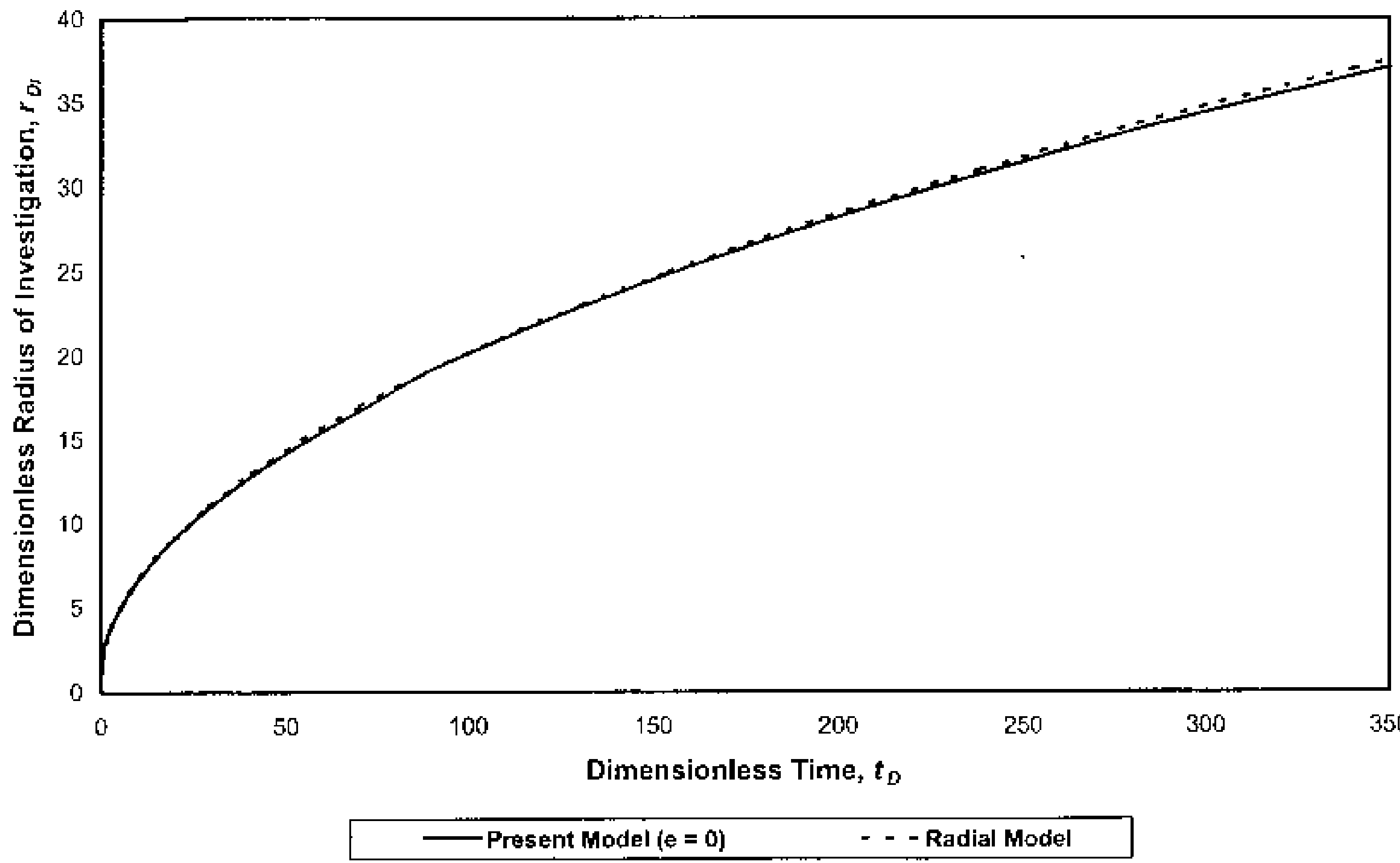

Fig 4.3: Comparison of Depth of Investigation from Elliptical and Radial Models

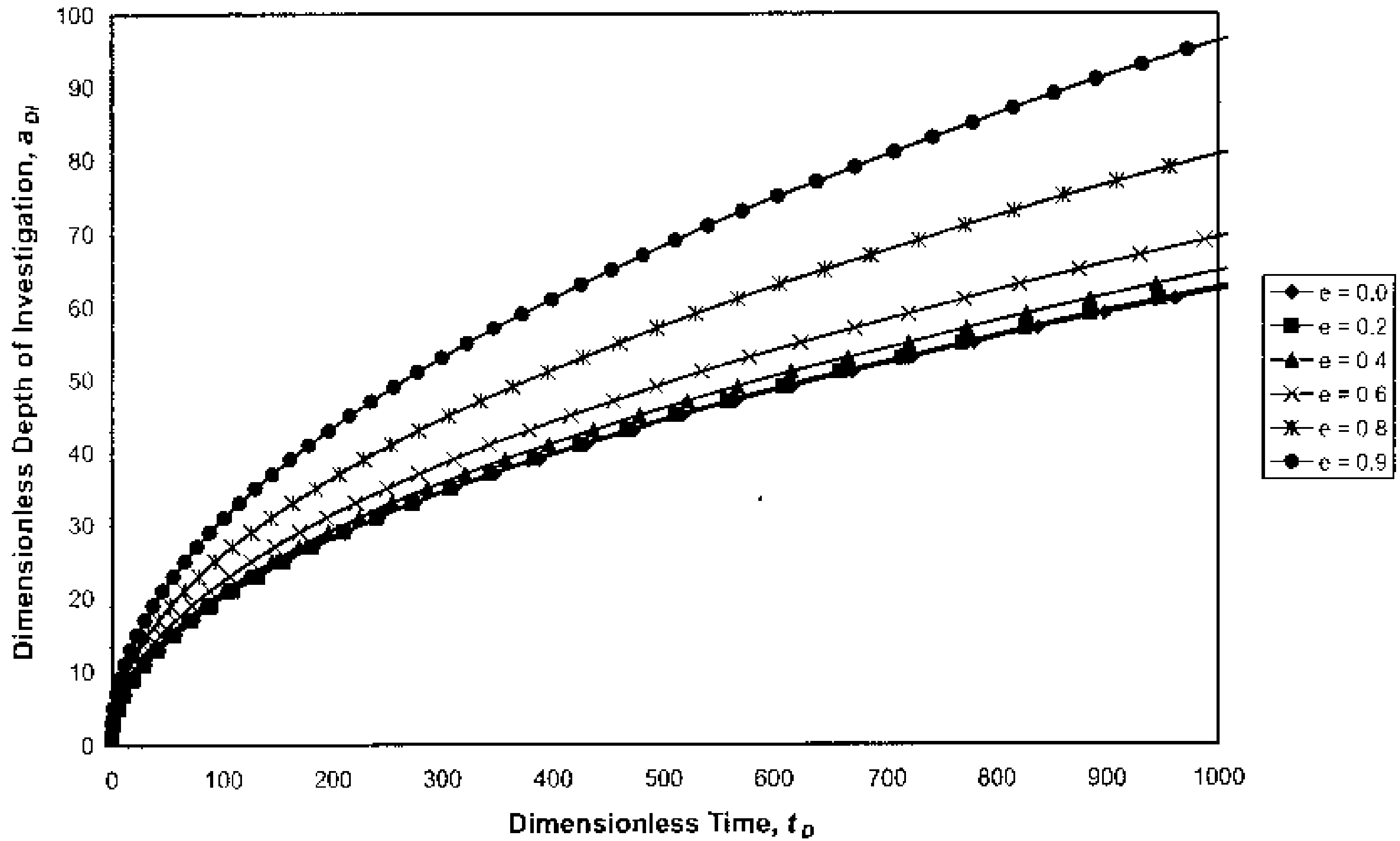

Fig 4.4: Depth of Investigation of Elliptical Domain (in Terms of Major Axes) with Different Eccentricity

From Fig 4.4 and Fig 4.5, it is found that the depths of investigation or the lengths of the major and minor axes of the elliptical propagation increases with increase in dimensionless time. With the increase in eccentricities, major axes of the elliptical domain increases but the minor axes decreases which can easily be understood from the properties of ellipse.

After a definite dimensionless time period, different elliptical domains for corresponding eccentricities produce different depths of investigations. Depths of investigation for different eccentricities are tabulated in Table A. 8 for different values of t_D. These data of Table A. 8 are read from Fig 4.4. The values are plotted in Fig 4.6 and it shows the increasing trends of depth of investigation with eccentricities. Cullender (1955) found that the extent of drainage area is a function of time and is independent of the rate of flow and pressure level. The drainage areas are calculated in Table A. 9 and plotted in Fig 4.7. It shows that the drainage areas are almost constant and do not change with the shape of the domain as well. Fig 4.8 is another representation of the data of Table A. 9. It is plotted to check the variation of drainage area with dimensionless time for different eccentricities. The little deviation of the points for different e but particular t_D as observed in the Fig 4.8 is due to the error of numerical integration of the computer. This error can be minimized with the refinement of the mesh points with improved algorithm. Equation of the depth of investigation, Eqn 3.62 is independent of flow rate and pressure terms. So the depth of investigation and hence the drainage area do not depend on flow rate or pressure level, but it depends on time. Therefore, it can be concluded that the elliptical model is consistent with Cullender's statement. In addition to this, it is also understood from the elliptical model that the drainage area is independent of the shape of the elliptical domain.

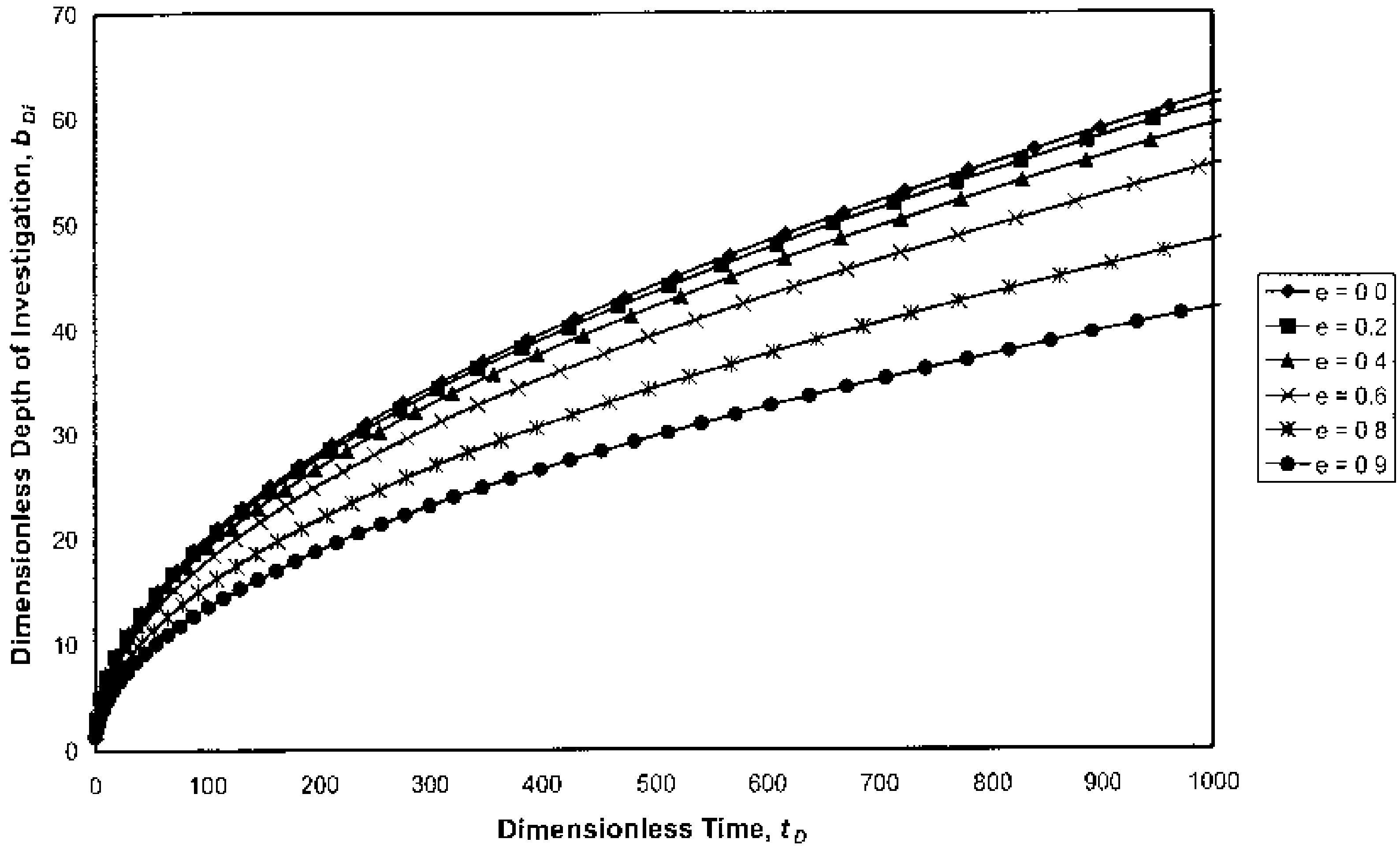

Fig 4.5: Depth of Investigation of Elliptical Domain (in Terms of Minor Axes) with Different Eccentricity

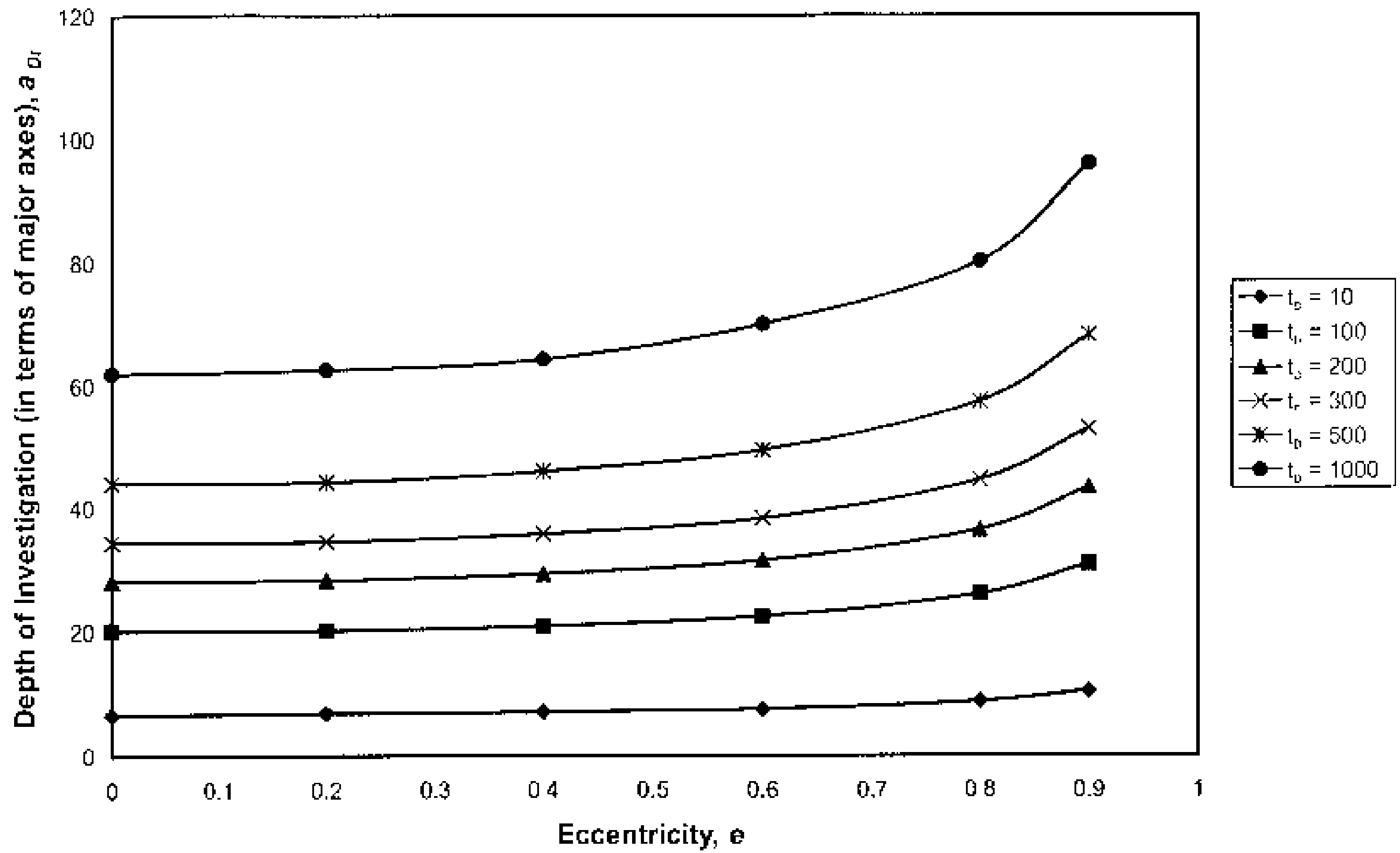

Fig 4.6: Variation of Depth of Investigation with Eccentricities for Different t_D

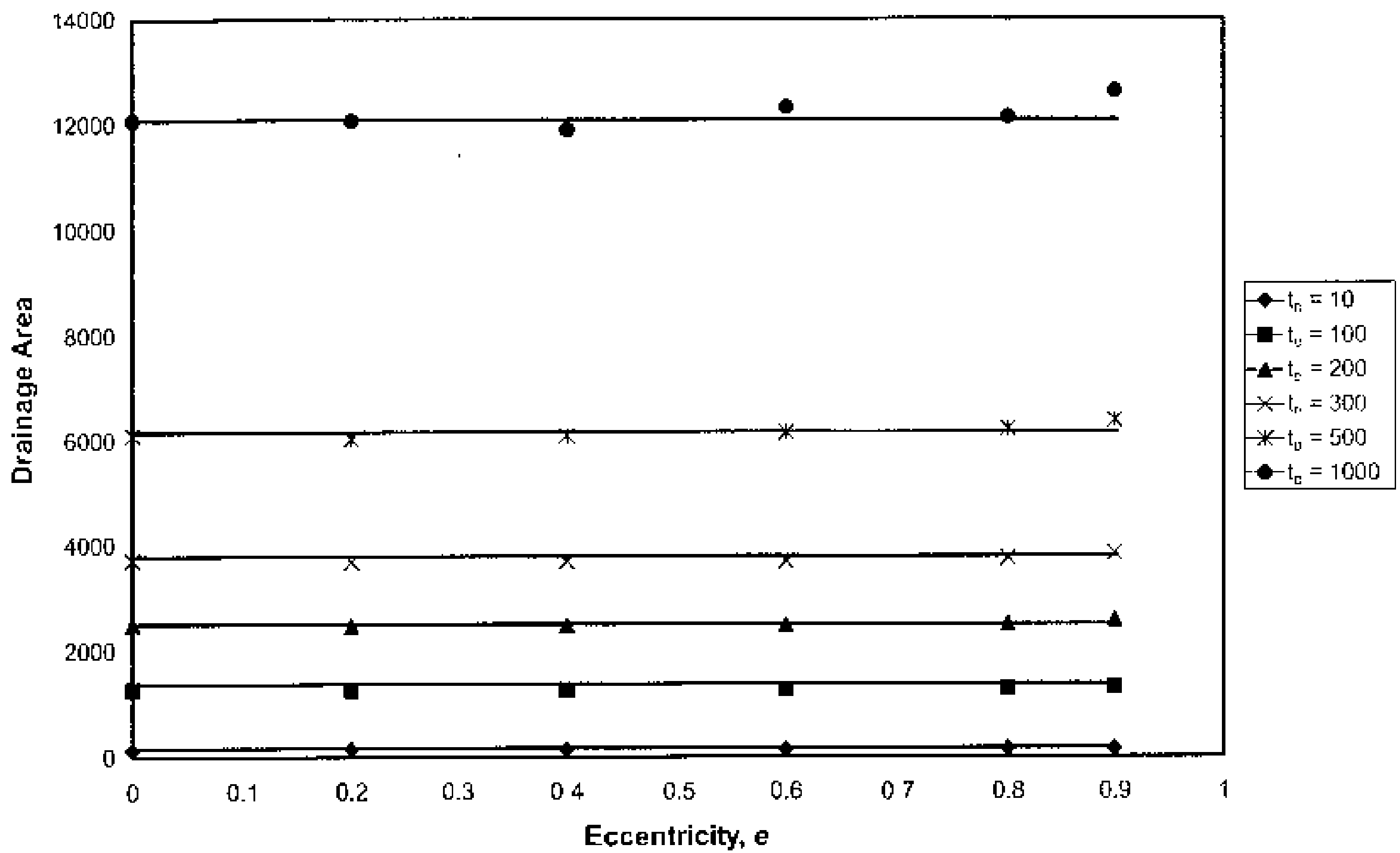

Fig 4.7: Variation od Drainage Area for Different Shape of the Elliptical Domain

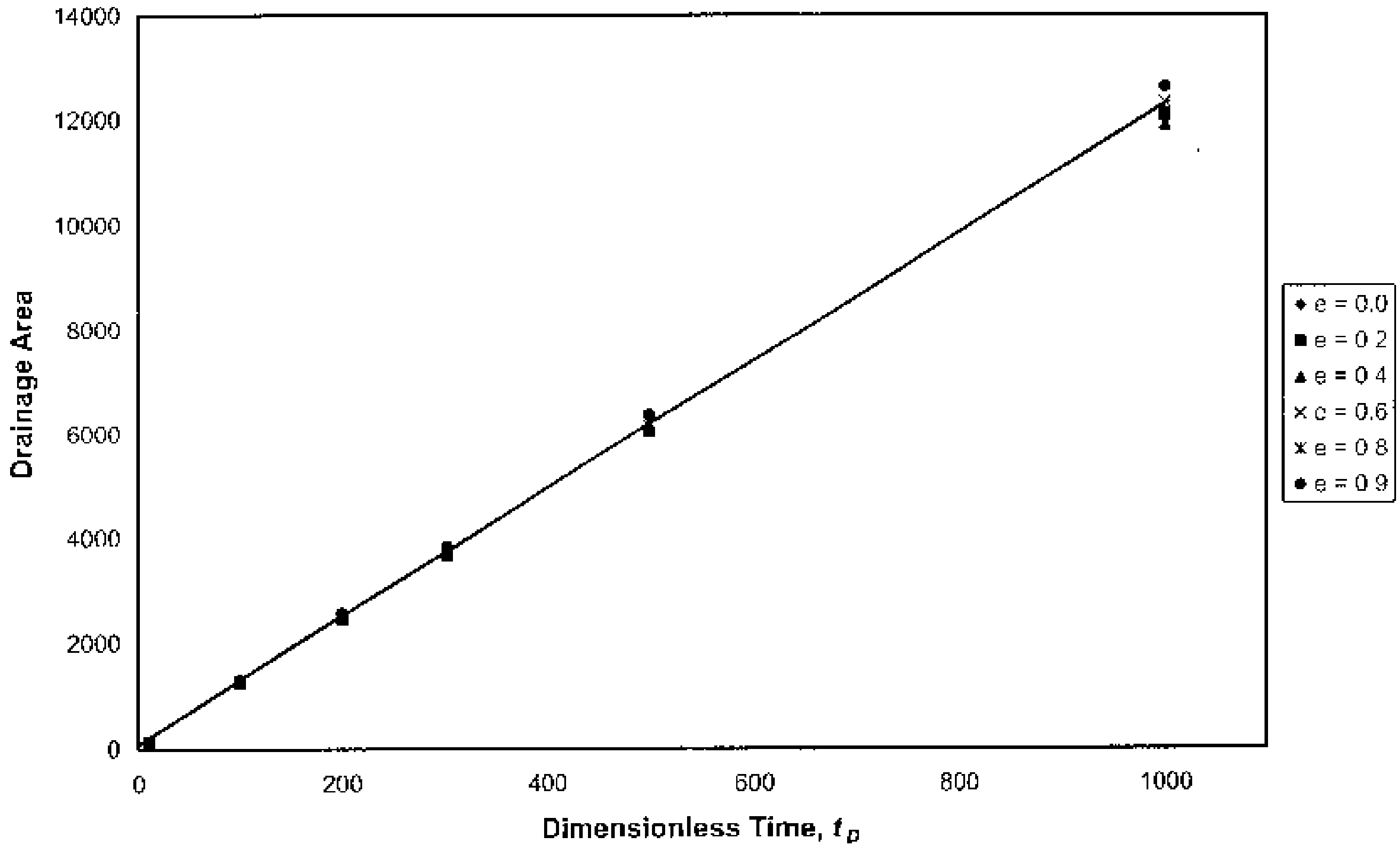

Fig 4.8: Variation od Drainage Area with Dimensionless Time

CHAPTER 5

CONCLUSIONS

In this work the closed form solution for flow into circular wellbore in an elliptical domain proposed by van der Ploeg *et al.* is modified and adopted for petroleum engineering application. The flow equation derived from their proposition is for steady-state condition. Using the method of continuous succession of steady states, the equation is applied to calculate the depth of investigation in elliptical flow in a petroleum reservoir system. The concept of depth of investigation is related to the transient flow behavior of a reservoir. When the pressure transient propagates from the wellbore, the drainage area and hence the depth of investigation increases and ultimately the pressure transient touches the outer boundary of the reservoir. The pseudo-steady state or the steady state condition begins from this point. Estimating the depth of investigation is very important on many counts. Depth of investigation concept in elliptical flow systems is as good as radius of investigation of radial systems. A well test analysis provides important reservoir information on an average basis. This information is good for the region within the depth of investigation i.e. within the drainage area. So it is very important to know the extent of the reservoir that is being tested to provide the parameters like permeability and storage capacity of the reservoir from a well-test analysis. Another important aspect of knowing the depth of investigation is to optimize the locations of new wells to be drilled in a field. It is very difficult to identify the well test run time without an estimation of radius of drainage.

5.1 Conclusions

- Flow around a circular wellbore in elliptical domain should be treated according to this new model to incorporate the elliptical flow behavior of the system.

- Radial flow model will produce as much as nine percent error in flow calculation if it is used in elliptical flow system.
- A new equation for the calculation of depth of investigation in an elliptical domain has been developed that can be successfully used in well test modeling for such system.
- The depth of investigation for elliptical flow model produces the same result as of a radial model when eccentricity of the domain is zero.
- Depth of investigation in terms of major axis increases with the increase in eccentricity of the elliptical domain.
- An equal dimensionless time produces the equal drainage area irrespective of the eccentricity of the domain.

5.2 General Recommendation

- The wellbore is considered at the center of the elliptical domain in the flow equations. Further study can be made considering wellbore at locations other than the center of the domain.
- The study considered single-phase fluid flow in the reservoir. Further study can be made for multiple-phase fluid flow in the reservoir.

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APPENDIX A

TABLES

Table A. 1: Variation of A_{N0} (a/r_w = 10, 50, 100)

a/r_w	10		50		100	
e	b/r_w	A_{N0}	b/r_w	A_{N0}	b/r_w	A_{N0}
0	10.000	1.000000	50.000	1.000000	100.000	1.000000
0.1	9.950	1.001094	49.749	1.000646	99.499	1.000548
0.2	9.798	1.004519	48.990	1.002655	97.980	1.002255
0.3	9.539	1.010725	47.697	1.006279	95.394	1.005329
0.4	9.165	1.020584	45.826	1.012010	91.652	1.010184
0.5	8.660	1.035826	43.301	1.020776	86.603	1.017593
0.6	8.000	1.059792	40.000	1.034347	80.000	1.029027
0.7	7.141	1.099930	35.707	1.056480	71.414	1.047575
0.8	6.000	1.176170	30.000	1.096674	60.000	1.080945
0.9	4.359	1.372535	21.794	1.190080	43.589	1.156972

Table A. 2: Variation of A_{N0} (a/r_w = 500, 1000)

a/r_w	500		1000	
e	b/r_w	A_{N0}	b/r_w	A_{N0}
0	500.000	1.000000	1000.000	1.000000
0.1	497.494	1.000406	994.987	1.000365
0.2	489.898	1.001670	979.796	1.001502
0.3	476.970	1.003943	953.939	1.003546
0.4	458.258	1.007527	916.515	1.006767
0.5	433.013	1.012978	866.025	1.011661
0.6	400.000	1.021349	800.000	1.019166
0.7	357.071	1.034825	714.143	1.031221

a/r_w	500		1000	
e	b/r_w	A_{N0}	b/r_w	A_{N0}
0.8	300.000	1.058751	600.000	1.052546
0.9	217.945	1.111776	435.890	1.099445

Table A. 3: Percentage of Error due to Eccentricity ($Q_{D\,(radial)}$ = 1.011037)

a/r_w	b/r_w	e	A_{N0}	$Q_{D\,(elliptical)}$	% error
500.00	500.00	0.000	1.000000	1.011037	0.000000
510.00	490.20	0.276	1.003217	1.011068	0.003044
525.00	476.19	0.421	1.008044	1.011231	0.019161
550.00	454.55	0.563	1.016078	1.011776	0.073032
575.00	434.78	0.654	1.024095	1.012625	0.157042
625.00	400.00	0.768	1.040043	1.015075	0.399332
675.00	370.37	0.836	1.055838	1.018317	0.720012
725.00	344.83	0.880	1.071726	1.022425	1.126382
775.00	322.58	0.909	1.086828	1.026439	1.523360
825.00	303.03	0.930	1.101980	1.031060	1.980385
800.00	312.50	0.921	1.094434	1.028713	1.748294
900.00	277.78	0.951	1.124244	1.038436	2.709944
1000.00	250.00	0.968	1.153047	1.048796	3.734642
1500.00	166.67	0.994	1.283042	1.102334	9.029987

Table A. 4: Different Values of Dimensionless Time and Corresponding Dimensionless Depth of Investigations for Different Eccentricities

a_{Di}	t_{Di}					
	e = 0.0	e = 0.2	e = 0.4	e = 0.6	e = 0.8	e = 0.9
1	0.000	0.000	0.000	0.000	0.000	0.000
3	1.312	1.381	1.231	0.999	0.574	0.091
5	5.311	5.006	4.628	3.875	2.660	1.532
7	11.293	10.746	9.948	8.529	6.081	3.879
9	19.399	18.587	17.236	14.869	10.728	7.206
11	29.617	28.470	26.460	22.797	16.588	11.339
13	41.944	40.416	37.371	32.506	23.796	16.444
15	56.382	54.299	50.660	43.700	32.079	22.336

a_{pi}	t_{pi}					
	e = 0.0	e = 0.2	e = 0.4	e = 0.6	e = 0.8	e = 0.9
17	72.930	70.203	65.604	56.760	41.757	29.219
19	88.741	88.385	82.406	71.600	52.496	36.673
21	109.119	108.664	100.987	87.730	64.849	45.261
23	131.603	130.926	121.991	105.869	78.060	54.736
25	156.196	155.010	144.509	125.414	92.400	65.128
27	182.901	181.400	169.461	147.162	108.179	76.333
29	211.719	209.712	196.268	169.662	125.567	88.043
31	242.654	240.243	224.731	194.393	143.640	100.828
33	275.706	273.037	254.619	221.324	162.990	114.560
35	310.878	307.552	286.912	249.830	184.152	129.347
37	348.171	343.917	321.133	279.596	206.127	144.888
39	387.587	383.203	357.492	310.974	228.950	161.323
41	429.128	423.921	396.507	343.395	253.932	178.368
43	472.796	467.460	436.135	379.217	279.219	196.609
45	518.591	512.074	478.373	415.893	305.982	215.559
47	566.515	559.004	522.329	454.660	334.873	235.308
49	616.569	608.893	567.950	493.780	364.273	256.345
51	668.755	659.045	616.057	535.583	395.338	277.832
53	723.074	713.818	665.819	578.946	427.152	300.184
55	779.526	768.736	719.240	624.154	459.901	322.531
57	838.114	826.772	772.272	671.243	494.420	347.478
59	898.838	885.804	827.533	718.863	530.157	372.477
61	961.699	947.268	885.515	770.062	567.839	398.864
63	1026.698	1012.849	944.640	821.330	605.848	424.926
65	1093.836	1078.541	1006.086	875.387	644.476	452.731
67	1163.113	1146.728	1070.300	930.031	685.430	481.538
69	1234.529	1216.560	1136.620	987.538	728.229	511.263
71	1308.085	1288.465	1204.426	1045.272	770.610	541.060
73	1383.782	1362.925	1273.276	1107.304	815.796	572.131
75	1461.621	1438.987	1345.008	1169.131	861.179	603.531

a_{Di}	t_{Di}					
	e = 0.0	e = 0.2	e = 0.4	e = 0.6	e = 0.8	e = 0.9
77	1558.637	1518.705	1417.745	1232.028	909.125	638.032
79	1641.294	1598.424	1493.923	1298.629	956.459	671.383
81	1726.098	1682.413	1572.687	1366.904	1006.922	706.231
83	1813.049	1768.002	1650.205	1434.492	1058.269	740.895
85	1902.148	1854.550	1731.752	1505.234	1107.965	778.089
87	1993.395	1943.772	1815.904	1577.183	1162.819	815.377
89	2086.792	2034.481	1900.897	1652.961	1216.114	852.950
91	2182.338	2128.547	1986.547	1728.922	1272.517	890.758
93	2280.035	2222.784	2077.939	1806.543	1331.110	932.371
95	2379.883	2321.093	2168.257	1885.729	1388.335	972.369
97	2481.882	2421.539	2261.088	1964.535	1448.939	1014.146
99	2586 033	2520.581	2356.458	2048.409	1509 424	1056.150

Table A. 5: Dimensionless Time Values for Elliptical Model (e = 0.0) and Radial Model

r_{Di}	t_{Di}	
	Elliptical Model (Eqn 3.58)	Radial Model (Eqn 3.59)
1	0	0
3	1.312	1.451
5	5.311	5.195
7	11.293	11.027
9	19.399	18.901
11	29.617	28.801
13	41.944	40.718
15	56.382	54.646
17	72.93	70.583
19	88.741	88.528

Table A. 6: Different values of dimensionless time and corresponding dimensionless depth of investigations (in terms of minor axes) for eccentricity 0 to 0.4

e = 0.0	e = 0.2	e = 0.4

t_{Di}	b_{Di}	t_{Di}	b_{Di}	t_{Di}	b_{Di}
1.312	3.000	1.381	2.939	1.231	2.750
5.311	5.000	5.006	4.899	4.628	4.583
11.293	7.000	10.746	6.859	9.948	6.416
19.399	9.000	18.587	8.818	17.236	8.249
29.617	11.000	28.470	10.778	26.460	10.082
41.944	13.000	40.416	12.737	37.371	11.915
56.382	15.000	54.299	14.697	50.660	13.748
72.930	17.000	70.203	16.657	65.604	15.581
88.741	19.000	88.385	18.616	82.406	17.414
109.119	21.000	108.664	20.576	100.987	19.247
131.603	23.000	130.926	22.535	121.991	21.080
156.196	25.000	155.010	24.495	144.509	22.913
182.901	27.000	181.400	26.454	169.461	24.746
211.719	29.000	209.712	28.414	196.268	26.579
242.654	31.000	240.243	30.374	224.731	28.412
275.706	33.000	273.037	32.333	254.619	30.245
310.878	35.000	307.552	34.293	286.912	32.078
348.171	37.000	343.917	36.252	321.133	33.911
387.587	39.000	383.203	38.212	357.492	35.744
429.128	41.000	423.921	40.172	396.507	37.577
472.796	43.000	467.460	42.131	436.135	39.410
518.591	45.000	512.074	44.091	478.373	41.243
566.515	47.000	559.004	46.050	522.329	43.076
616.569	49.000	608.893	48.010	567.950	44.909
668.755	51.000	659.045	49.970	616.057	46.742
723.074	53.000	713.818	51.929	665.819	48.575
779.526	55.000	768.736	53.889	719.240	50.408
838.114	57.000	826.772	55.848	772.272	52.241
898.838	59.000	885.804	57.808	827.533	54.074
961.699	61.000	947.268	59.768	885.515	55.907
1026.698	63.000	1012.849	61.727	944.640	57.740

e = 0.0		e = 0.2		e = 0.4	
t_{Di}	b_{Di}	t_{Di}	b_{Di}	t_{Di}	b_{Di}
1093.836	65.000	1078.541	63.687	1006.086	59.573
1163.113	67.000	1146 728	65.646	1070 300	61.407
1234 529	69.000	1216.560	67.606	1136.620	63.240
1308.085	71.000	1288 465	69 566	1204.426	65.073
1383.782	73.000	1362.925	71.525	1273.276	66.906
1461.621	75.000	1438.987	73 485	1345.008	68.739
1558.637	77.000	1518.705	75.444	1417.745	70.572
1641.294	79 000	1598.424	77.404	1493.923	72.405
1726.098	81 000	1682.413	79 363	1572.687	74.238
1813.049	83.000	1768.002	81.323	1650.205	76.071
1902.148	85.000	1854.550	83.283	1731.752	77.904
1993.395	87.000	1943.772	85.242	1815.904	79.737
2086.792	89.000	2034 481	87.202	1900.897	81.570
2182.338	91.000	2128.547	89.161	1986.547	83.403
2280.035	93.000	2222.784	91.121	2077.939	85.236
2379.883	95.000	2321.093	93.081	2168.257	87.069
2481.882	97.000	2421.539	95.040	2261.088	88.902
2586.033	99.000	2520.581	97.000	2356.458	90.735

Table A. 7: Different values of dimensionless time and corresponding dimensionless depth of investigations (in terms of minor axes) for eccentricity 0.6 to 0.9

e = 0.6		e = 0.8		e = 0.9	
t_{Di}	b_{Di}	t_{Di}	b_{Di}	t_{Di}	b_{Di}
0.999	2.400	0.574	1.800	0.091	1.308
3.875	4.000	2.660	3.000	1.532	2.179
8.529	5.600	6.081	4.200	3.879	3.051
14.869	7.200	10.728	5.400	7.206	3.923
22.797	8 800	16.588	6 600	11.339	4 795
32.506	10.400	23.796	7.800	16.444	5.667
43.700	12.000	32.079	9.000	22.336	6.538
56.760	13.600	41.757	10.200	29.219	7.410
71.600	15.200	52.496	11.400	36 673	8.282

e = 0.6		e = 0.8		e = 0.9	
t_{Di}	b_{Di}	t_{Di}	b_{Di}	t_{Di}	b_{Di}
87.730	16.800	64.849	12.600	45.261	9.154
105.869	18.400	78.060	13.800	54.736	10.025
125.414	20.000	92.400	15.000	65.128	10.897
147.162	21.600	108.179	16.200	76.333	11.769
169.662	23.200	125.567	17.400	88.043	12.641
194.393	24.800	143.640	18.600	100.828	13.513
221.324	26.400	162.990	19.800	114.560	14.384
249.830	28.000	184.152	21.000	129.347	15.256
279.596	29.600	206.127	22.200	144.888	16.128
310.974	31.200	228.950	23.400	161.323	17.000
343.395	32.800	253.932	24.600	178.368	17.871
379.217	34.400	279.219	25.800	196.609	18.743
415.893	36.000	305.982	27.000	215.559	19.615
454.660	37.600	334.873	28.200	235.308	20.487
493.780	39.200	364.273	29.400	256.345	21.359
535.583	40.800	395.338	30.600	277.832	22.230
578.946	42.400	427.152	31.800	300.184	23.102
624.154	44.000	459.901	33.000	322.531	23.974
671.243	45.600	494.420	34.200	347.478	24.846
718.863	47.200	530.157	35.400	372.477	25.718
770.062	48.800	567.839	36.600	398.864	26.589
821.330	50.400	605.848	37.800	424.926	27.461
875.387	52.000	644.476	39.000	452.731	28.333
930.031	53.600	685.430	40.200	481.538	29.205
987.538	55.200	728.229	41.400	511.263	30.076
1045.272	56.800	770.610	42.600	541.060	30.948
1107.304	58.400	815.796	43.800	572.131	31.820
1169.131	60.000	861.179	45.000	603.531	32.692
1232.028	61.600	909.125	46.200	638.032	33.564
1298.629	63.200	956.459	47.400	671.383	34.435

e = 0.6		e = 0.8		e = 0.9	
t_{Di}	b_{Di}	t_{Di}	b_{Di}	t_{Di}	b_{Di}
1366.904	64.800	1006.922	48.600	706.231	35.307
1434.492	66.400	1058.269	49.800	740.895	36.179
1505.234	68 000	1107.965	51.000	778.089	37.051
1577.183	69.600	1162.819	52.200	815.377	37.922
1652.961	71 200	1216.114	53.400	852.950	38.794
1728.922	72 800	1272.517	54.600	890 758	39.666
1806.543	74 400	1331.110	55.800	932.371	40.538
1885.729	76.000	1388.335	57.000	972.369	41.410
1964.535	77.600	1448.939	58.200	1014.146	42.281
2048.409	79.200	1509.424	59.400	1056.150	43.153

Table A. 8: Depths of Investigation a_{Di} for Different Eccentricities

t_D	e					
	0	0.2	0.4	0.6	0.8	0.9
10	6.60	6 80	7.00	7 50	8.70	10.40
100	20.15	20.17	20.90	22 37	25.99	30.84
200	28.21	28.33	29.27	31.43	36.45	43.36
300	34.39	34 57	35.78	38.31	44.57	52.95
500	44.07	44 27	46.00	49.46	57.42	68.23
1000	62.00	62 60	64 27	70.00	80.21	96.00

Table A. 9: Drainage Area for Different Eccentricities

t_D	e					
	0	0.2	0.4	0.6	0.8	0.9
10	136.8481	142.3326	141.0869	141.372	142.6726	148.1134
100	1274.927	1252.271	1257.717	1257.688	1273.253	1302.436
200	2499.212	2470.47	2466 812	2482.731	2504.363	2574.579
300	3715.483	3678.623	3686.135	3688.631	3743.773	3838.642
500	6101.505	6032.614	6092.65	6148.216	6214.819	6374.974
1000	12076.31	12062.44	11893.43	12315 07	12127.16	12620.31

APPENDIX B

FLOW INTO A WELL LOCATED AT THE CENTRE OF AN ELLIPTICAL BOUNDARY

Van der Ploeg, Kirkham and Boast (1971) presented a closed-form solution for steady state saturated flow into a finite circular wellbore in an elliptical confined aquifer. They solved the flow problem for different well location in an isotropic and homogeneous aquifer.

B.1 Well at the center of an ellipse

The equation of ellipse in a rectangular coordinate is –

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1 \qquad \text{(B.1)}$$

Using polar coordinates (r, θ), x and y may be written as

$$x = r\cos\theta \qquad \text{(B.2)}$$

$$y = r\sin\theta, \qquad \text{(B.3)}$$

and the equation of ellipse for a point $P(r, \theta)$ at the boundary is

$$r^2\left(\frac{\cos^2\theta}{a^2} + \frac{\sin^2\theta}{b^2}\right) = 1 \qquad \text{(B.4)}$$

B.2 Boundary conditions

a. $\varphi = 0 \qquad for\ r = r_o \qquad 0 \le \theta \le \pi/2$ (B.5)

b. $\phi = 1$ $\quad$ *for* $r = R$ $\quad$ $0 \leq \theta \leq \pi/2$ (B.6)

c. $\dfrac{\partial \phi}{\partial \theta} = 0$ $\quad$ *for* $\theta = 0$ $\quad$ $r_w < r < a$ (B.7)

d. $\dfrac{\partial \phi}{\partial \theta} = 0$ $\quad$ *for* $\theta = \pi/2$ $\quad$ $r_w < r < b$ (B.8)

Laplace's equation in polar coordinates is –

$$\frac{\partial^2 \phi}{\partial r^2} + \frac{1}{r}\frac{\partial \varphi}{\partial r} + \frac{1}{r^2}\frac{\partial^2 \phi}{\partial \theta^2} = 0 \tag{B.9}$$

The solution of this problem should give an expression of ϕ which should satisfy the boundary conditions and Laplace's equation.

B.3 Solution

The authors used the Gram-Schimdt method as modified by Powers, Krikham and Snowden (1967) to determine the solution as

$$\phi = \sum_{m=0}^{N} A_{Nm} u_m(r,\theta) \tag{B.10}$$

where

m = 0, 1, 2, … , N;

N = 0, 1, 2, …, ∞;

$$u_m(r,\theta) = \frac{\left(\dfrac{r}{a}\right)^{2m} - \left[\dfrac{r_w^2}{(ar)}\right]^{2m}}{1 - \left(\dfrac{r_w^2}{a^2}\right)^{2m}} \cos 2m\theta \tag{B.11}$$

Replacing r with R at the boundary and using Eqn A.4 to express R in term of θ, $u(r,\theta)$ may be written as

$$u_m(\theta)=\frac{\left(\frac{R^2}{a^2}\right)^m-\left[\frac{r_w^2}{a^2}\frac{r_w^2}{R^2}\right]^m}{1-\left(\frac{r_w^2}{a^2}\right)^{2m}}\cos 2m\theta \tag{B.12}$$

where,

$$R^2=\left(\frac{\cos^2\theta}{a^2}+\frac{\sin^2\theta}{b^2}\right)^{-1} \tag{B.13}$$

The hydraulic head may be defined as

$$\phi_{r=R}=\sum_{m=0}^{N}A_{Nm}u_m(\theta) \qquad 0\leq\theta\leq\pi/2 \tag{B.14}$$

Applying boundary condition (b), the hydraulic head may be written as

$$\varphi_{r=R}=1=\sum_{m=0}^{N}A_{Nm}u_m(\theta) \qquad 0\leq\theta\leq\pi/2 \tag{B.15}$$

Powers *et al.* (1967) derived a table of orthogonal functions to solve potential flow problems like seepage of steady rain through soil bedding. In accordance with Powers *et al.*, the two constants to determine A_{Nm} in this problem are

$$w_m=\int_0^{\frac{\pi}{2}}(1)u_m(\theta)d\theta, \qquad m=0,1,2,\ldots,N \tag{B.16}$$

and

$$u_{mn}=\int_0^{\frac{\pi}{2}}u_m(\theta)u_n(\theta)d\theta, \qquad m=0,1,2,\ldots,N \quad n\leq m \tag{B.17}$$

The parameters $u_m(\theta)$ and $u_n(\theta)$ may be determined from Eqn A.12. When $N\rightarrow\infty$, boundary condition (b) is satisfied exactly. It can be seen that the term with zero subscripts produce indeterminate forms. Using L'Hôpital's rule u_0 would be

$$u_0(\theta) = \frac{\ln \dfrac{r}{r_w}}{\ln \dfrac{a}{r_w}}. \tag{B.18}$$

Therefore, the hydraulic head may be written as

$$\phi = A_{N0}\frac{\ln \dfrac{r}{r_w}}{\ln \dfrac{a}{r_w}} + \sum_{m=1}^{N} A_{Nm}\frac{\left(\dfrac{r}{a}\right)^{2m} - \left[\dfrac{r_w^2}{(ar)}\right]^{2m}}{1-\left(\dfrac{r_w^2}{a^2}\right)^{2m}}\cos 2m\theta \tag{B.19}$$

After the values of w_m and u_{mn} are determined, all values can be calculated by using Table 2 of Powers *et al.* (1967).

B.4 Sequential Formulas presented by Powers *et al.*

$$A_{Nm} = E_m - \sum_{p=m+1}^{N} E_p J_{pm} \tag{B.20}$$

$$E_m = G_m / D_m \qquad m = 0, 1, 2, \ldots, N \tag{B.21}$$

$$G_m = w_m - \sum_{n=0}^{m-1} c_{mn} G_n \qquad m = 0, 1, 2, \ldots, N \tag{B.22}$$

$$J_{m0} = c_{m0} - \sum_{n=1}^{m-1} c_{mn} J_{n0} \qquad m = 0, 1, 2, \ldots \tag{B.23}$$

$$J_{mn} = c_{mn} - \sum_{r=n+1}^{m-1} c_{mr} J_{rn} \qquad m = 2, 3, 4, \ldots \tag{B.24}$$

$$n = 1, 2, 3, \ldots, m-1$$

$$c_{mn} = \frac{(u_m u_n) - \sum_{r=0}^{n-1} J_{nr}(u_m u_r)}{D_n} \qquad m = 1, 2, 3, \ldots \tag{B.25}$$

$$n = 0, 1, 2, \ldots, m-1$$

$$D_m = \left(u_m u_m\right) - \sum_{n=0}^{m-1} c_{mn}^2 D_n \qquad m = 1, 2, 3, \ldots \tag{B.26}$$

$$n = 0, 1, 2, \ldots$$

B.5 Well Discharge

Using Darcy's equation for potential flow for unit thickness, the flow rate q can be written as

$$q_{n-1} = -K\Delta\phi \int_0^{2\pi} \left(\frac{d\varphi}{dr}\right)_{r=r_w} r d\theta \quad r_w \rightarrow 0, \tag{B.27}$$

where, K is the hydraulic conductivity. Multiplying the above equation with the thickness h, the total flow is

$$q = -Kh\Delta\phi \int_0^{2\pi} \left(\frac{d\phi}{dr}\right)_{r=r_w} r d\theta \quad r_w \rightarrow 0, \tag{B.28}$$

The flow rate for a well at the center of an elliptical drainage system is

$$q = -\frac{2\pi Kh A_{n0} \Delta\phi}{\ln \frac{a}{r_w}} \tag{B.29}$$

APPENDIX C

COMPUTER PROGRAM TO CALCULATE $A_{N\theta}$-VALUES

To calculate the values of A_{N0} for reservoirs with circular wellbore at the centre of the elliptical drainage area of different eccentricity, the following programming codes are written in C language.

```
/*
Programming Language: Turbo C++
Version: 3.0
The position of the well is at centre of the reservoir.
*/

#include<stdio.h>
#include<conio.h>
#include<math.h>

#define  pie 3.14159265359
#define  err 0.001

/* Global Variable Decleration */
        double K, h, delphi, a, b, rw;

/* Function Decleration */
        double find_um(int m, int theta);
        double find_u_mn(int m,int n);
        double find_w(int m);
        double R(int theta);

/* Function Definitions */
```

```
double find_um(int m, int theta)
{
 double factor= pie/180.0,
 double result=0,

 if(m==0) result=(log(R(theta)/rw))/(log(a/rw));
 else {
 result=cos(2*m*theta*factor)*((pow((R(theta*factor)/a),(2*m))-
pow((rw*rw/(a*R(theta*factor))),(2*m)))/(1-pow((rw/a),(4*m))));
 }
 return result;
}

double find_u_mn(int m,int n)  /* Intigration is done using TRAPIZOIDAL Method */
[
 int x=0;
 double result=0,
 for(x=0;x<=89,++x) {
 result+=(find_um(m,x)*find_um(n,x)+find_um(m,(x+1))*find_um(n,(x+1)))*0.5;
 }
 return result;
}

double find_w(int m)
[
 int x=0;
 double result=0;
 for(x=0;x<=89,++x) [
 result+=(find_um(m,x)+find_um(m,(x+1)))*0.5,
 }
 return result;
}
double R(int theta)
{
```

```
 double factor= pie/180.0;
 return
(sqrt(1/(cos(theta*factor)*cos(theta*factor)/(a*a)+sin(theta*factor)*sin(theta*factor)/(b
*b))));
}

void main (void)
{
 clrscr();
 printf("\n\nData input Part:\n\n\n");
 printf("Properties of the Elliptical Acquifer:\n");
 printf("Major axes (a) =\t");
 scanf("%lf",&a);
 printf("Minor axes (b) =\t");
 scanf("%lf",&b);
 printf("\n\n***************\n");
 printf("Welbore radius (rw) =\t");
 scanf("%lf",&rw);
 printf("\n\n\nData input completed......");
 printf("\n\n\nDesired value of N :\t");
 int N;
 scanf("%d",&N);
 printf("\n\nPress any key to continue . ..  ");
 getch();
 clrscr();
 textcolor(WHITE+BLINK);
 cprintf("\n\n\n\n\n\n\n\n\n                    Please wait....");

// Local Variable decleration

 double D[50];
 double C[50][50];
 double J[50][50]
 double G[50],F[50];
```

```
int i,j,k,l;

for(i=0;i<=N;++i)
 {
   D[i]=find_u_mn(i,i);
   for(j=0;j<i;++j)
              {
              D[i]=D[i]-C[i][j]*C[i][j]*D[j];
              }

   for(j=0;j<=i;++j)
              {
              C[i+1][j]=find_u_mn(i+1,j);
              for(k=0;k<=j-1;++k)
                     {
                     C[i+1][j]=C[i+1][j]-J[j][k]*find_u_mn(i+1,k);
                     }
              C[i+1][j]=C[i+1][j]/D[j];
              }

   for(j=0;j<=i;++j)
              {
              J[i+1][j]=C[i+1][j];
              for(k=j+1;k<=i;++k)
                     {
                     J[i+1][j]=J[i+1][j]-C[i+1][k]*J[k][j];
                     }
              }

   G[i]=find_w(i);
   for(j=0;j<i;++j)
              {
              G[i]=G[i]-C[i][j]*G[j];
              }
```

```
    E[i]=G[i]/D[i];
  }

// Calculation of constants A(Nm)

  double A[50][50];
  for(i=0;i<=N;++i)
        {
        for(j=0;j<=i;++j)
                {
                A[i][j]=E[j];
                for(k=j+1;k<=i;++k)
                        {
                        A[i][j]=A[i][j]-E[k]*J[k][j];
                        }
                }
        }

  textcolor(LIGHTGRAY);
  clrscr();

  for(i=0;i<=N;++i)
        {
        for(j=0;j<=i;++j)
                {
                printf("%lf   ",A[i][j]);
                }
        printf("\n");
        }
  printf("\n\n\n\n");
  textcolor(LIGHTGRAY);
  cprintf("Press any key to return...");
  getch();}
```

APPENDIX D

COMPUTER PROGRAM TO CALCULATE t_D AND a_{Di}-VALUES

```
#include<stdio.h>
#include<conio.h>
#include<math.h>
#include<stdlib.h>

#define pi 3.14159265359
#define err 0.001

//Global variables

float aDi,e;   // aDi = dimensionless major axes length, (ai/rw)
               // e   = eccentricity of the elliptical propagation

//Global variable as used in ANO

double a_ANO, b_ANO,rw_ANO;

//Functions declaration

double f(float theta);
double ANO(float a);
double find_um(int m, int theta);
double find_u_mn(int m,int n);
double find_w(int m);
```

```
double R(int theta);

//Function definition

double f(float theta)

 {
  double lob = 1-e*e;
  double s = sin(theta);
  double c = cos(theta);
  double result =sqrt(lob)/sqrt(s*s+lob*c*c);
  return result;
 }

double ANO(float a)
 {
  rw_ANO = 1.0;
  a_ANO = a;
  b_ANO = a*sqrt(1-e*e);

  if(b_ANO<=rw_ANO) return 1.0;

  int N = 15;

// Local Variable decleration

  double D[20];
  double C[20][20];
  double J[20][20];
  double G[20],E[20];
  int i,j,k,l;

  for(i=0;i<=N;++i)
   {
```

```
        D[i]=find_u_mn(i,i);
        for(j=0;j<i;++j)
                {
                D[i]=D[i]-C[i][j]*C[i][j]*D[j];
                }

        for(j=0;j<=i;++j)
                {
                C[i+1][j]=find_u_mn(i+1,j);
                for(k=0;k<=j-1;++k)
                        {
                        C[i+1][j]=C[i+1][j]-J[j][k]*find_u_mn(i+1,k);
                        }
                C[i+1][j]=C[i+1][j]/D[j];
                }

        for(j=0;j<=i;++j)
                {
                J[i+1][j]=C[i+1][j];
                for(k=j+1;k<=i;++k)
                        {
                        J[i+1][j]=J[i+1][j]-C[i+1][k]*J[k][j];
                        }
                }

        G[i]=find_w(i);
        for(j=0;j<i;++j)
                {
                G[i]=G[i]-C[i][j]*G[j];
                }

        E[i]=G[i]/D[i];
    }
// Calculation of constants A(Nm)
```

```
 double A[20][20];
 for(i =0;i<=N;++i)
        {
        for(j=0;j<=i;++j)
                {
                A[i][j]=E[j];
                for(k=j+1;k<=i;++k)
                        {
                        A[i][j]=A[i][j]-E[k]*J[k][j];
                        }
                }
        }
 printf("\nANO = %lf",A[15][0]);
 return A[15][0];

}

double find_um(int m, int theta)
{
 double factor= pi/180.0;
 double result=0;

 if(m==0)
        result=(log(R(theta)/rw_ANO))/(log(a_ANO/rw_ANO));
 else
        {
        result=cos(2*m*theta*factor)*((pow((R(theta*factor)/a_ANO),(2*m))-
pow((rw_ANO*rw_ANO/(a_ANO*R(theta*factor))),(2*m)))/(1-
pow((rw_ANO/a_ANO),(4*m))));
        }
 return result;
}
double find_u_mn(int m,int n)  /* Intigration is done using TRAPIZOIDAL Method */
```

```
{
 int x=0;
 double result=0;
 for(x=0;x<=89;++x)
         {
         result+=(find_um(m,x)*find_um(n,x)+find_um(m,(x+1))*find_um(n,(x+1)))*0.
5;
         }
 return result;
}

double find_w(int m)
{
 int x=0;
 double result=0;
 for(x=0;x<=89;++x)
         {
         result+=(find_um(m,x)+find_um(m,(x+1)))*0.5;
         }
 return result;
}

double R(int theta)
{
 double factor= pi/180.0;
 return
(sqrt(1/(cos(theta*factor)*cos(theta*factor)/(a_ANO*a_ANO)+sin(theta*factor)*sin(the
ta*factor)/(b_ANO*b_ANO))));
}


void main()
{
```

```
FILE *fp;

double tD, integral = 0.0;
clrscr();

printf("\nEccentricity = ");
scanf("%g",&e);


if((fp = fopen("ANO.txt", "a+"))==NULL)
        {
        printf("Cannot open file");
        exit(1);
        }
fprintf(fp,"\nEccentricity : %g\n",e);
fclose(fp);


double uptheta = pi/2;  //uptheta is the upper limit of theta i.e. 90 degree or pi/2
double h1 = 0.05;       //h1 is the increment in theta direction
double h2 = 0.05;       //h2 is the increment in r direction

int noflines = uptheta/h1+1;


double *line_endpoint;
line_endpoint = (double *) malloc(sizeof(double)*noflines);
if(line_endpoint==NULL)
        {
         printf("\nNot enough memory to run the program...\nPress any key to exit...");
         getch();
         exit(1);
        }
/* Looping */
```

```
for(int j=0;j<noflines;j++)
 {
   line_endpoint[j] = 1.0
 }



for(aDi=1;aDi<=100;aDi+=2)
      {
      double tD = 0.0;
      int i=0;

      for(double theta = 0; theta<uptheta; theta+=h1)
         {
           printf("\nTheta = %lf radian",theta);
           double ftheta = f(theta);  //theta is in radian
           double RD = aDi*ftheta;
           for(float rD=line_endpoint[i];rD<RD;rD+=h2)
              {
                printf("\nRD => %g of %lf",rD,RD);

                double r2a = rD/ftheta;    //r2a is the conversion of a from r (a = r/f(theta))
                integral = integral + rD*h1*h2*log(r2a)/ANO(r2a);
              }
           line_endpoint[i] = rD+h2;
           i++;
         }

      double part1 = (pi/4)*log(aDi)*(aDi*aDi*sqrt(1-e*e)-1);
      double ANOi=ANO(aDi);

      tD = (part1-ANOi*integral)/(uptheta*ANOi);  //here uptheta = pi/2 which is defined earlier
```

```
        printf("\n\taDi = %lf\t\ttD = %lf",aDi,tD);

        if((fp = fopen("ANO.txt", "a+"))==NULL)
                {
                printf("Cannot open file");
                exit(1);
                }

        fprintf(fp,"\n\taDi = %lf\t\ttD = %lf",aDi,tD);
        fclose(fp);
        }
    getch();
}
```