

**EFFECT OF RANDOMLY DISTRIBUTED INFILLS ON
THE VIBRATION CHARACTERISTICS OF REINFORCED
CONCRETE FRAMES**

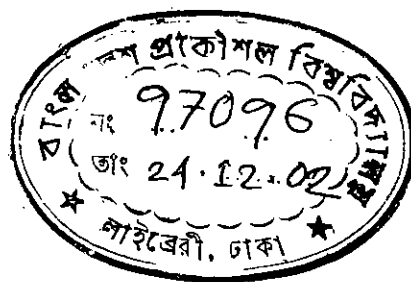
by

MD. EKRAMUL HAQUE

A thesis submitted to the Department of Civil Engineering of
Bangladesh University of Engineering and Technology, Dhaka in
partial fulfillment of the requirement for the degree

of

MASTER OF SCIENCE IN CIVIL ENGINEERING (STRUCTURAL)



OCTOBER 2002

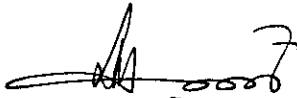


EFFECT OF RANDOMLY DISTRIBUTED INFILLS ON THE
VIBRATION CHARACTERISTICS OF REINFORCED
CONCRETE FRAMES

by

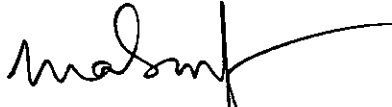
MD. EKRAMUL HAQUE

A thesis approved as to style and content for the degree of M.Sc.
Engineering (Civil and Structural) on 20th of October, 2002.



Dr. K. M. Amanat
Associate Professor,
Department of Civil Engineering
BUET, Dhaka-1000.

:Chairman
(Supervisor)



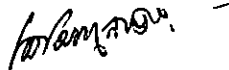
Dr. Md. Abdur Rouf
Professor and Head
Department of Civil Engineering
BUET, Dhaka-1000.

:Member



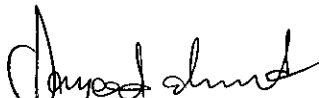
Dr. Sk. Sekender Ali
Professor
Department of Civil Engineering
BUET, Dhaka-1000.

:Member



Dr. A. M. M Taufiqul Anwar
Professor
Department of Civil Engineering
BUET, Dhaka-1000.

:Member

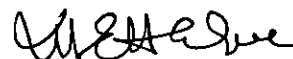


Dr. Sayeed Ahmed
Executive Director,
SMEC (Bangladesh) Ltd.
House-200, Lane-2,
DOHS, Baridhara, Dhaka

:Member (External)

DECLARATION

I hereby certify that the research work reported in this thesis has been performed by the author and that this work has not been submitted elsewhere for any other purpose (except for publication).



October, 2002

(Md. Ekramul Haque)

ACKNOWLEDGMENT

The author wishes to express his deepest gratitude to Dr. K. M. Amanat, Associate Professor, Department of Civil Engineering, BUET, Dhaka for his continuous guidance, invaluable suggestions and affectionate encouragement at every stage of this study.

The author wishes to express his gratitude to his beloved wife for her continuous encouragement and cooperation during this study.

The author is grateful to Bangladesh Open University (BOU) Authority for permission of getting admission and granting a study leave of one year for this study.

ABSTRACT

Conventional modeling of reinforced concrete structures by FE method results in too long and unrealistic value of period of buildings. A reason of this phenomenon is that a structure usually contains many secondary elements like infills, which are generally ignored to have any structural contribution. As such the numerical models become too flexible resulting in longer time period. For this reason, code provisions usually impose a limit on the period value obtained from numerical analysis with conventional FE modeling. In this thesis, an extensive computational study has been conducted to determine the fundamental period of buildings considering the structural effect of infills.

A numerical investigation has been performed to calculate the fundamental period of a series of regular building frames under various conditions. Variation of number of floors, floor height, number of span, number of bays, floor panel size, percentage of infills etc. are considered during the investigation. The infills are modeled as equivalent diagonal strut. Beams and columns are modeled using three-dimensional frame element and floor slab is modeled using shell element. The period of the same buildings are also evaluated using empirical code equations and a comparison with the results of FE analysis are made.

On the basis of the investigation it has been found that the period found by modal analysis for models without infill are much higher than the same found by code equations and from modal analysis with infill. It has been found that the code equations generally produce shorter values of periods when compared to the periods obtained from modal analysis with infill, although the difference is not as high as with the models without infill.

Three parameters namely (i) number of stories or total height of the building, (ii) floor panel size and (iii) amount of infilled panels present in the building have been identified to have significant influence the building period. Incorporating the effects of these parameters, a few correction factors have been proposed which may be used to further refine the period value predicted by code equations.

The period of a few arbitrary examples are determined on the basis of the proposal and are compared with modal analysis. It has been found that the periods determined by applying the suggested correction factors match better with modal analysis than existing code equations.

CONTENTS

	<u>Page No.</u>
DECLARATION	III
ACKNOWLEDGMENT	IV
ABSTRACT	V
CONTENTS	VI
LIST OF FIGURES	X
LIST OF TABLES	XIII
Chapter - 1 : INTRODUCTION	
1.1 General	1
1.2 Static approach : Equivalent Static Force Method	2
1.3 Structure period and Earthquake forces	3
1.4 Role of infill on period of vibration	4
1.5 Objectives and scope	5
1.6 Methodology and Outline	6
Chapter - 2 : REVIEW OF CODES AND THEORIES FOR ESTIMATING TIME PERIOD.	
2.1 Historical review of works	8
2.2 Codes of practice	17
2.2.1 Uniform Building code	18
2.2.2 Bangladesh National Building code (BNBC 1993)	21
2.2.3 Standard Association of Australian Earthquake code (SAA)	21
2.2.4 Indian Standard criteria for earthquake resistant design of structures (IS)	22
2.2.5 The National building code of Canada (NBC), 1995.	23
2.2.6 The Building Standard Law of Japan (BSLJ), 1987.	24

2.3	Comparison of methods for determination of period by codes	25
2.4	Free Vibration Analysis of Frame Structures	26
2.4.1	Analysis of Vibration frequencies and period	26
2.4.2	Analysis of Vibration mode shapes	28
2.4.3	Structure parameters that influence period	31

Chapter - 3 : ANALYSIS OF INFILLED FRAME STRUCTURES

3.1	Introduction	32
3.2	Characteristics of infilled frames	32
3.3	Overview of the analysis of infilled frames	35
3.3.1	Approximate Method	37
3.3.2	Equivalent Strut Method	39
3.3.3	Plasticity Model	42
3.3.4	Coupled Boundary Element Method	43
3.4	Choice of the Model	44
3.5	Equivalent Strut Modeling	45
3.6	Beam and Column Moment Capacity	46
3.6.1	Example for calculation of Beam and column Moment capacity.	47
3.7	Determination of Equivalent Strut Stiffness (K_0)	48
3.7.1	Example for determination of K_0	54

Chapter - 4 : FINITE ELEMENT MODELING FOR TIME PERIOD ANALYSIS.

4.1	Introduction	56
4.2	Finite Element Analysis by Ansys	56
4.2.1	Modal analysis	57
4.2.2	Assumptions & Restrictions	57
4.3	Modal analysis procedure	57
4.4	Description of elements:	
4.4.1	Beam Element	59
4.4.2	Mass Element:	59
4.4.3	Shell Element	62

4.4.4	COMBIN14 Spring -Damper	63
4.5	Modeling of T beam :	63
4.5.1	Different Amount of Offsets of Beam 44	64
4.5.2	Model Examples	64
4.5.3	Remarks	72
4.6	Prototype Model for study	72
4.7	Results of Reference model	73
4.8	Remarks	77

Chapter - 5: INFLUENCE OF STRUCTURAL PARAMETERS ON TIME PERIOD

5.1	Introduction :	78
5.2	Structural parameters and Different Models	79
5.2.1	Description of Models of Group A	79
5.2.2	Description of Models of Group B	81
5.2.3	Description of Models of Group C	84
5.2.4	Description of Models of Group D	86
5.2.5	Description of Models of Group E	89
5.2.6	Description of Models of Group F	91
5.2.7	Description of Models of Group G	94
5.2.8	Description of Models of Group H	95
5.3	Effect of structure parameters on Natural time Period	97
5.3.1	Effect of Variation of Column Stiffness on period of vibration	98
5.3.2	Effect of Variation of Number of Story on period of vibration	100
5.3.3	Effect of Variation of Panel Size on period of vibration	101
5.3.4	Effect of Variation of Floor Height on period of vibration	103
5.3.5	Effect of Variation of Beam Stiffness on period of vibration	106

5.3.6	Effect of Variation of Number of Bay on period of vibration	106
5.3.7	Effect of Variation of Number of Span on period of vibration	108
5.3.8	Effect of Variation of Infill Density on period of vibration	110

**Chapter - 6 : ANALYSIS OF MODELS WITH RANDOMLY
DISTRIBUTED INFILLS**

6.1	Introduction	114
6.2	Random Distribution Of Infill	114
6.3	Description of Models	115
6.4	Effect of Random Distribution of Infills on Time Period	120
6.5	Remarks	123

**Chapter - 7 : ESTIMATING NATURAL PERIOD OF VIBRATION OF
INFILLED FRAMES**

7.1	Introduction	124
7.2	Parameters Controlling Time Period	124
7.3	Assessing the Influence of Controlling Parameters on Time Period	126
7.4	Rationale for Estimating Period of Vibration	128
7.5	Verification of the Proposal	129
7.6	Effect on Base Shear	133

Chapter - 8 : CONCLUSIONS

8.1	General	134
8.2	Findings in Brief	134
8.3	A Rationale for Estimating Period of Vibration	135
8.4	Recommendations.	136

REFERENCES	137
-------------------	-----

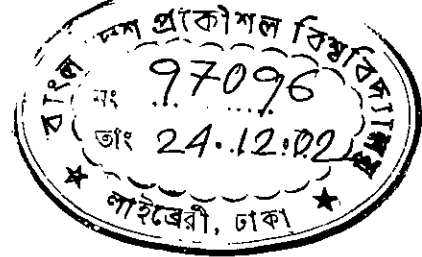
LIST OF FIGURES

	Page
Fig. 3.1 Interactive behavior of frame and infill	33
Fig. 3.2 Analogous braced frames	33
Fig. 3.3 Modes of in fill failure	34
Fig. 3.4 Modes of frame failure	34
Fig. 3.5 Masonry in fill frame sub assemblage in masonry in fill panel frame structures.	40
Fig. 3.6 Masonry in fill panel in frame structures.	40
Fig. 3.7 Constitutive Model for masonry infill panel	41
Fig. 3.8 Strength envelope for masonry infill panel	41
Fig. 3.9 Composite yield criterion with iso-shear stress lines.	43
Fig. 3.10 Beam and column sections showing reinforcement	48
Fig. 3.11 Frame forces equilibrium.	49
Fig. 4.1 3-Dimensional elastic beam element (Beam 4)	60
Fig. 4.2 3-Dimensional elastic beam with offsets (Beam 44)	61
Fig. 4.3 Structural Mass element	61
Fig. 4.4 Elastic shell element	62
Fig. 4.5 Combin 14 Spring Damper element	63
Fig. 4.6 T-beam showing different offsets of beam 44	65
Fig. 4.7 T-Beam of example 1 showing different offsets of beam 4 and beam 44	66
Fig. 4.8 T-Beam section of example 2	68
Fig. 4.9 T-Beam section of example 3	69
Fig. 4.10 Comparison of deflections for different solutions	71-72

Fig. 4.11	3-D view, Plan and Elevation of prototype model	74
Fig. 4.12	Different mode shapes of prototype model	75-77
Fig. 5.1	Plan configuration of different models of group A	82
Fig. 5.2	Elevation of different models of group B	85
Fig. 5.3	Plan configuration of different model of group C	87
Fig. 5.4	Elevation of different models of group D	89
Fig. 5.5	Elevation of different models of group E	92
Fig. 5.6	Plan configuration of different models of group F	93
Fig. 5.7	Plan configuration of different models of group G	95
Fig. 5.8	Typical plan of models of group H	96
Fig. 5.9	Natural Time Period vs. column stiffness	99
Fig. 5.10	Natural Time Period vs. number of story	102
Fig. 5.11	Natural Time Period vs. Panel size	104
Fig. 5.12	Natural Time Period vs. floor height	105
Fig. 5.13	Natural Time Period vs. beam stiffness	107
Fig. 5.14	Natural Time Period vs. number of bay	109
Fig. 5.15	Natural Time Period vs. number of span	111
Fig. 5.16	Natural Time Period vs. percentage of infill	112
Fig. 6.1	X-direction frames of model R-20 showing the random distribution of infills	116
Fig. 6.2	X-direction frames of model R-40 showing the random distribution of infills	117
Fig. 6.3	X-direction frames of model R-60 showing the random distribution of infills	118
Fig. 6.4	X-direction frames of model R-80 showing the random distribution of infills	119
Fig. 6.5	Time period for 20% infill placed in different modes	121

Fig. 6.6	Time period for 40% infill placed in different modes	121
Fig. 6.7	Time period for 60% infill placed in different modes	122
Fig. 6.8	Time period for 80% infill placed in different modes	122
Fig. 6.9	Natural time period vs. percentage of infill.	123
Fig. 7.1	Ratio $T_{\text{Modal}}/T_{\text{BNBC}}$ for varying number of story	127
Fig. 7.2	Ratio $T_{\text{Modal}}/T_{\text{BNBC}}$ for varying panel size	127
Fig. 7.3	Ratio $T_{\text{Modal}}/T_{\text{BNBC}}$ for varying percentage of infill	128
Fig. 7.4(a)	Comparison of time period obtained by various methods of example 1	131
Fig. 7.4(b)	Comparison of time period obtained by various methods of example 2	131
Fig. 7.4(c)	Comparison of time period obtained by various methods of example 3	132
Fig. 7.4(d)	Comparison of time period obtained by various methods of example 4	132

	LIST OF TABLE	Page
Table 2.1	Fundamental period and Co-efficient depending on fundamental period for moment resisting frames, deforming freely	25
Table 4.1	List of element used in Models for Finite element analysis by ANSYS.	58
Table 4.2	Comparison of results of T-Beams	70
Table 4.3	Values of different structure parameters of prototype model	73
Table 5.1	Values of different structure parameters used in different models.	79
Table 7.1	Examples for verification of the proposal	130



CHAPTER 1

INTRODUCTION

1.1 GENERAL

In recent years, structural engineers are giving increasing attention to the design of buildings capable of withstanding earthquakes. A great deal of time and effort are going into the development of better methods of design. This has led to a better understanding of earthquakes and the forces they exert on buildings. New concepts are developed concerning the earthquake resistance of buildings as determined by their ability to absorb the energy input from the earth vibration. The inertia of the structure tending to stay at rest as the ground starts to move and then tending to move in the direction of the initial ground motion as the ground reverses its motions, imposes lateral forces on the structural framing. Seismic forces, since they are due to inertia, are directly related to the weight of the structure.

Seismic codes for designing earthquake resistant structures have been developed bearing in mind that the phenomena that is being dealt with is dynamic in nature. But for most structures which are built, a rigorous dynamic analysis is not always practicable even with the present high speed computer capabilities. The framers of the codes therefore sought provisions, which would approximate the results of the more rigorous approach in an easily usable procedure applicable to a large range of types of structures. The code levels of equivalent ground accelerations are lower than the recorded accelerations during moderate to strong earthquakes. The inelastic response mentioned above is one of the reasons why code forces have been lower than that obtained using

recorded motions and an assumed elastic structural response. Most structures, and especially buildings which house people, have many secondary strengths and energy absorbing features which are not taken in to account in the structural seismic force resisting system, and thus [justify lower code values without jeopardizing safety]

The earthquake resistant design of structures can be done following the accepted approaches and methods, namely:

- 1) Static approach
 - (a) The equivalent static force or the building code method.
- 2) Dynamic Approach
 - (a) Response spectrum method.
 - (b) Time history method.

According to the specifications given by different earthquake resistant codes, the response spectrum and time history method of analysis are not essential for earthquake resistant design of low and medium rise regular frame structures. For such structures, equivalent static force method is usually recommended. [As the present study will focus on low and medium rise regular frame structures only, equivalent static force method will be discussed further as appropriate design method for such structures]

1.2 STATIC APPROACH: EQUIVALENT STATIC FORCE METHOD

In equivalent static force method, static analysis is used with equivalent static loading to represent the dynamic action of the earthquake on the structure. Recent editions of widely used building codes like UBC 1997 still recommends equivalent static force method. The vertical seismic forces are not generally considered for the design of structures except for the effect of uplift forces and for very important structures such as nuclear reactor building. In typical building design only the horizontal components of earthquake forces are considered and are assumed to act non-concurrently along the two major structural axes. Earthquake

resistance calls for energy absorption rather than strength resistance only. [For convenience in design, an earthquake is translated into an equivalent static load acting horizontally on the building and is intended to represent the inertia force, which is mass times acceleration occurring at the critical instant of maximum deflection and zero velocity during the largest cycle of vibration as the structure responds to the earthquake motion.] These design loading, however, incorporate modifying factors to cover local variations in seismicity, type of construction, soil conditions, usage of building, etc.

The total force is distributed over the height of the structure by considering the response of the structure during an earthquake. The seismic coefficient is modified by factors, such as dynamic properties of the structures, seismicity of the region, importance of the structure, subsoil conditions, allowable stresses and load factors.

[The dynamic properties of the structure are dependent on natural period of vibration, modal shapes and damping characteristics of the structures. The code formulae may, therefore, underestimate the response of longer period buildings because it does not properly recognize the contributions of higher modes of vibrations.] (Chopra and Yim, 1987).

1.3 STRUCTURE PERIOD AND EARTHQUAKE FORCES

Natural frequency of the structure is a very important design parameter in earthquake resistant design by equivalent static force method. [The natural frequency can be defined as the frequency with which the whole system oscillates in the absence of external forces] or for a system with more than one degree of freedom, the frequency of one of the normal modes of vibration. Mathematically, the natural frequency is inversely proportional to the natural period of a system.

For earthquake resistant design of regular frame structures design codes generally recommend simplified formulae for the determination of natural period. These formulae are either function of story height or number of stories. But actually the [period of vibration of an elastic system is a function of its mass and stiffness properties.] Hence [the base shear of the

earthquake forces are also function of mass and stiffness distribution of the structure. [The factors affecting the stiffness distributions include beam and column size, story height, number of story, number of bay and bay width, amount of infills etc. Therefore, all these factors are to be considered in determination of period of structures.]

1.4 ROLE OF INFILL ON PERIOD OF VIBRATION

It is said earlier that the natural period of vibration is an important design parameter for earthquake resistant design. It is well recognized that if any rational method such as eigenvalues extraction or Releigh's method is applied to determine the time period based on conventional modeling, the resulting time periods are too large and do not agree well with the observed values of time period of real structures during ground motion. For this reason, code provisions usually impose a limiting value of time period if it is obtained from modal analysis. This discrepancy between the observed frequency and frequency obtained from modal analysis (Hossain², 1997) is probably due to the fact that a real building structure contains some reserve strength and stiffness contributed from components which are conventionally regarded as non-structural such as infills and are not included in analysis.

The infilled frame consists of a steel or reinforced concrete column-and-girder frame with infills of brickwork or concrete block work. In addition to functioning as partitions, the infills may also serve structurally to brace the frame against horizontal loading. The wall braces the frame partly by its in-plane shear resistance and partly by its behavior as a diagonal bracing strut in the frame. When the frame is subjected to horizontal loading, it deforms with double-curvature bending of the columns and girders. The translation of the upper part of the column in each story and the shortening of the leading diagonal of the frame cause the column to lean against the wall as well as to compress the wall along its diagonal. It is roughly analogous to a diagonally braced frame.

Infills influence the behavior of structure in many ways. Their presence in the structure reduces the sway against lateral load. Presence of infill also modifies the vibrational characteristics of a building frame. Infills influence

the natural period in two ways: (a) their mass contributes to the total weight of the building which tends to increase the period and (b) their in-plane rigidity increases the stiffness of the structure causing some reduction in the period of vibration. In conventional structural analysis of framed structures by equivalent static force method, these effects of infills are generally not considered.

A number of researches in the past e.g., (Holmes, 1961), (Stafford Smith, 1962,1966,1967), (Dhanesekar and Page, 1986), (Mander et al., 1993) etc. investigated the various characteristics of infill and infilled frame. It appears that the dynamic characteristics of infilled frame structures is one of the least understood area. Since infilled frame structures are very common and engineers are becoming more and more concerned with the earthquake resistant design, it is essential that the vibration characteristics of this type of building structure be adequately understood. Research in this area will eventually lead to a better understanding of the characteristics of infilled frame resulting in safer and more economic structures.

1.5 OBJECTIVES AND SCOPE

Equivalent static force method of structural analysis uses a number of dynamic properties of soil and structure. Natural period of vibration is the only dynamic parameter related to the structure that is used in the code equations (BNBC 1993, UBC, 1997 etc.). As such, it is important that this parameter be properly estimated. The principal objective of the present study is to investigate the natural time period of common reinforced concrete framed structures considering the structural contribution of the infill. Specific objectives and research interests are as follows:

- _ To review the existing literature and code equations to identify their applicability and limitations and area of further improvements.
- _ To carry out a systematic investigation of multistoried building frames to identify the effect of various parameters like number of stories, number of bays etc. on the natural period of vibration incorporating the effect of infills.

- To investigate the effect of various amount of infilled panels ranging from twenty percent to eighty percent of the total frame panels filled by infills.
- investigate the effect of randomly distributed infills to reflect the realistic nature of occurrence of infills in framed structures.

1.6 METHODOLOGY AND OUTLINE

The study presented in this thesis is about the natural period of vibration of ordinary reinforced concrete framed structures having infills. To carry out a systematic computational investigation, the important structural parameters which have influence on period would be first identified based on available literature (Hossain, 1997; Noor 1997).

Then an appropriate finite element modeling technique to adequately represent the infilled frame structure would be described. Choice of elements and technique of appropriate meshing would have also been enumerated. [The finite element modeling and modal analysis (eigenvalue and eigenvector extraction) based on three-dimensional modeling of some typical frames using frame and plate elements would be carried out using ANSYS software.

A systematic parametric study would be carried out to determine period of vibration for different variations of structural parameters like number of stories, story height, span length, column stiffness etc. for frames with and without infill. Afterwards the results would be compared graphically with the period values given by different codes to establish the relative importance of different parameters as well as the influence of infill. In all cases of investigation, a parallel analysis shall be made without modeling the infill so that a comparison can be made with conventional analysis. The effect of different infill density over the whole structure would also be studied for regular and random distribution of infill panels. Afterwards, based on the findings of the study, conclusion and recommendations are drawn.

The organization of the thesis is as follows:

The current chapter is Chapter 1, which introduces the reader with the thesis work. Review of codes and theories for estimating the time period are described in Chapter 2. The effect of infill to the stiffness and parameters of infill are described in Chapter 3. Chapter 4 describes the methodology for determination of period of frame structure by modal analysis by ANSYS. Chapter 5 is dedicated to a detail parametric study on influence of different structure parameters on period. Investigation conducted in this chapter leads to a recommendation on choice of structure parameters to reduce earthquake forces. In chapter 6, the analysis for the effect of randomly distributed infills on period of vibration has been described. Chapter 7 is produced to estimate natural period of vibration of infilled frames taking BNBC code equations as reference. The conclusions made from the study are presented in Chapter 8. This chapter also recommends future work or extension of this work.

CHAPTER 2

REVIEW OF CODES AND THEORIES FOR ESTIMATING TIME PERIOD

2.1 HISTORICAL REVIEW OF WORKS

The fundamental period plays a key role in dictating the magnitude of design earthquake forces of frame structures. For making frame structures earthquake resistant, it is important to determine the design earthquake forces as precisely as possible so that they can reflect the actual conditions. It is essential, therefore, to determine the period of the structures with high accuracy. Advancements have been made to develop simple and approximate formulae for determining the period of a structure. But still these formulae have got ample rooms for further improvement. Even now these formulae suffer from shortcomings and limitations, which warrant further investigations and study for necessary improvement.

Building codes specify design and construction requirements, which are intended to protect buildings from major structural damages and the public from loss of life and injury. These requirements are based to a large extent on past earthquake experience and judgment. Because of differences in the magnitude of earthquakes, geological formations, types of construction, and other factors, the philosophy of seismic design among different countries of engineers varied in different aspects. The first edition of the Uniform Building Code was published in 1927. It was prepared by the Pacific Coast Building Officials Conference after the Santa Barbare Earthquake of June 29, 1925. As experience and usage increased, provisions were modified and revised editions have appeared

time to time. The Uniform Building Code, 1927 edition, required that all buildings more than 20 ft in height except exposed steel-frame and wood-frame buildings, be designed for earthquake forces using 7.5% of the specified vertical loads as the seismic coefficient when allowable bearing capacity of soil was 2 or more tsf and 10% for soils of lesser bearing capacity. An increase of 33.2% in the allowable unit stresses was permitted for all materials, except for structural steel, which was allowed a 50% increase.

Andrus (1952) summarized the chronological historical development of Uniform Building Codes published in different times. The Uniform Building Code, 1937 edition, adopted a formula for calculating the lateral force F as

$$F = CW \quad (2.1)$$

Where,

C = seismic coefficient

W = total dead load plus half live load

Here the seismic coefficients were established on zone basis.

Los Angeles Code, 1940 edition, proposed a formula for calculating the base shear of a structure whose height is not more than 150 feet as (Housner, 1959)

$$V = \left(\frac{.6}{n + 3.5} \right) W \quad (2.2)$$

Where,

n = number of stories

W = weight of the structure

Uniform Building Code, 1949 edition, modified the procedure for determining the seismic coefficient (Biggs et al., 1959) by introducing a formula

$$C = \frac{.15}{N + 4.5} \quad (2.3)$$

Where, N is number of stories above the story under consideration.

Joint Committee of San Francisco, California Section (1951) prepared the Lateral Force Code, which represents a work of outstanding significance and value in pointing out a rational approach to the dynamic problem of earthquake resistant design. For the first time, the determination of the seismic coefficient C took into consideration the dynamic behavior of building.

Ulrich and Carder (1952) listed some of the results of the work done by the Coast and Geodetic Survey. The Coast and Geodetic Survey measured the period of vibration of hundreds of buildings, water-tank towers, and ground vibrations. Basing on this data, the Joint Committee of San Francisco proposed an empirical relation for the period of vibration of buildings taking into consideration the height and width of buildings as follows

$$T_n = .06 \frac{H}{\sqrt{b}} \quad (2.4)$$

Where,

H = height, ft

b = width in ft, in the direction of motion considered

The Joint Committee however, recommended the use of a more conservative coefficient (.05) in the above equation in the computation of the seismic coefficient $C = .015/T$. The total seismic force V is determined from the relationship $V = CW$. Where, W represents the weight of the building considered in seismic computation.

The Seismology Committee of Structural Engineers Association, California (1958) proposed a formula for calculating base shear of frame structures for earthquake resistant design as,

$$V = KCW \quad (2.5)$$

Where,

$$C = \frac{.05}{3\sqrt{T}} \quad (2.6)$$

$$T = \frac{.05H}{\sqrt{D}} \quad (2.7)$$

Above, T is considered to be the fundamental period of vibration; H and D are the height and width of the structure, respectively.

Apart from developing empirical formulae for determining base shear and period of structure by different codes, simultaneous research works by the individuals were carried out for relating theory of structural dynamics for earthquake resistant design of structures.

Blume (1958) analyzed the frame structure applying the theory of dynamics and concluded that the fundamental period of vibration is a logical index of base shear for general code purposes. However, the height and width of the building considered either individually or together are not adequate indicators of period of modern, flexible structures without walls. He also suggested that, these periods should be specifically computed or improved approximate method should be utilized. He showed that the present-day seismic codes are reasonably adequate for moderate earthquakes but, because of the oversimplification of a very complex subject, they cannot produce consistent results. The real earthquake values (i.e. earthquake magnitudes) of various types of buildings and structures are not necessarily proportionate to the lateral force factors used in design. Stated in another way, code requirements are not producing consistent safety factors because of too much reliance on coefficients without adequate regard to the structural-dynamics

phenomenon, ultimate resistances involved in various types and rigidities of construction.

Clough (1960) summarized the principal factors controlling the dynamic response of structure to the earthquake. He also related these principal factors controlling the dynamic response of structures to the earthquakes with the lateral force provisions recommended for inclusion in the Uniform Building Code by Structural Engineers Association of California. He showed that the base shear, V , could be determined with the following formula:

$$V = W \frac{\ddot{u}}{g} \frac{1}{1 - \left[\frac{T}{T_p} \right]^2} \quad (2.8)$$

Where

$$T = 2 \pi \sqrt{\frac{W}{gk}} \quad (2.9)$$

T = fundamental period of vibration

W = weight of the structure

k = stiffness of structure

g = acceleration due to gravity

T_p = period of structure due to harmonic motion

\ddot{u} = ground acceleration

It is seen that the response of a structure depends on the natural period of the structure, which depends, in turn, on its stiffness and weight.

Blume (1967) studied how the natural period of vibration of structure can be determined applying the theories of structural dynamics. He showed that an idealized model of a building can be analyzed and its

dynamic properties can be obtained according to the established theories. He stressed the need for calculating natural period of vibration of structures for undamped condition with the formula

$$T_i = \frac{2\pi}{P_i} \quad (2.10)$$

Where,

T_i = natural period of vibration in sec

P_i = natural angular frequency of the I -th mode in radian per sec

Uniform Building Code, 1971 edition developed by the International Conference of Building Officials (1971) modified the earlier empirical formula for the lateral force. According to this code the base shear of moment resisting space frame can be determined with the following formula

$$V = Z K C W \quad (2.11)$$

Where,

Z = numerical coefficient dependent upon zone.

$$C = \frac{.05}{\sqrt[3]{T}} \quad (2.12)$$

$$T = 0.1 N \quad (2.13)$$

C = numerical coefficient for base shear

T = fundamental period of vibration

N = number of stories

K = horizontal force factor

W = weight of the structure

Here, it is seen that the fundamental period of vibration of structure is considered in computing seismic coefficient. The fundamental period of vibration was taken as the function of the number of stories of the structure.

Uniform Building Code, 1991 edition, of International Conference of Building Officials (1991) modified the earlier formula for calculating the design earthquake forces of structure. Here the fundamental period of vibration of structure, T is considered to be the function of height of the structure.

Selvam and Jayasree (1993) simplified manual procedure for estimating the period and amplitudes of concrete shear buildings. From the analysis of nearly one hundred and fifty buildings; the following formula was evolved for determining the fundamental period of structures.

$$T^2 = \frac{M}{K} [16n^2 + 16n + 7.3] \quad (2.14)$$

Where,

T = natural period in sec

M = total mass of all floors in a frame

K = total stiffness of all columns in a frame

n = number of stories

Bangladesh National Building Code (1993) and Uniform Building Code, 1994 edition formulated identical approximate formula for calculating period of structure. The empirical relationship for base shear calculation is

$$V = \frac{ZIC}{R} W \quad (2.15)$$

Where,

Z = seismic zone coefficient

I = structure importance coefficient

R = response modification coefficient for structural system

W = total seismic load

C = numerical coefficient given by the relation

$$C = \frac{1.25S}{T^{\frac{2}{3}}} \quad (2.16)$$

T = fundamental period of vibration in sec

S = site coefficient for soil characteristics

For regular concrete frames, period T may be approximated as (UBC))

$$T = .03 (h_n)^{3/4} \quad (2.17)$$

Where,

h_n = height of structure in feet above base

Here, it is seen that according to the above formula the period of regular frame structure is function of height of structure only.

Li et al. (1994) measured the first natural periods of many tall buildings in China using ambient, initial displacement-pull and release, initial velocity-rockets, and resonance methods in the case of small amplitude. A lot of measurement results show that the measured natural period of frame buildings are greater than the true values, but the measured natural frequencies of shear wall buildings are barely greater than the true values. The formulae proposed for estimating the first natural frequencies of tall buildings are as follows (Li et al., 1994):

$$T_i = 0.085N \text{ for frame buildings} \quad (2.18a)$$

$$T_i = 0.045N \text{ for shear-wall buildings} \quad (2.18b)$$

$$T_t = 0.060N \text{ for frame-shear-wall buildings} \quad (2.18c)$$

and

$$T_t = 0.070N \text{ for frame buildings} \quad (2.19a)$$

$$T_t = 0.040N \text{ for shear-wall buildings} \quad (2.19b)$$

$$T_t = 0.055N \text{ for frame-shear-wall buildings} \quad (2.19c)$$

In which, T_t = transverse period; T_l = longitudinal period; N = story number of the building considered.

Recently, Goel and Chopra (1997) expressed the time period in a form convenient for shear wall buildings,

$$\text{Time period, } T = 40 \sqrt{\frac{\rho}{kG} \frac{1}{\sqrt{\bar{A}_c}}} H \quad (2.20)$$

Where, ρ = average mass density, defined as the total building mass (mH) divided by the total building volume ($A_B H$; A_B = building plan area),

i.e., $\rho = m/A_B$;

\bar{A}_c = the equivalent shear expressed as a percentage of A_B , i.e., $\bar{A}_c = \frac{A_c}{A_B}$,

G = Shear modulus = $E/2(1 + \mu)$

μ = Poisson's ratio = 0.2 for concrete

Eq.(2.21) applies only for those buildings in which lateral load resistance is provided by uncoupled shear walls. Theoretical formulas for the fundamental period of buildings with coupled shear walls are available in Rutenberg (1975), and for buildings with a combination of shear walls and moment-resisting frames in Heidebrecht and Stafford-Smith (1973) and Stafford-Smith and Crowe (1986). It seems that these formulas cannot be simplified to the form of equation 2.20.

A lot of research works have been carried out to formulate simple empirical formulae for calculating base shear and period of frame structures for earthquake resistant design. The base shear depends on several coefficients and the accuracy of design earthquake forces certainly depends on the precise evaluation of these coefficients. Of all the coefficients, the coefficient related with fundamental period of vibration needs special attention.

The empirical formulae for calculating period of frame structures are simple and handy but approximate. All approximate formulae except very few developed so far, for determining the period of structure are based on a single structure parameter. According to the theory of dynamics, however, the period of a structure cannot be a function of single structure parameter; rather it is dependent on total mass and stiffness of the structure. It would be appropriate; therefore, to investigate the effect of different structure parameters on the period of regular frame structures by finite element based computer softwares. Frequency evaluated through modal analysis procedure incorporating the effect of infill could be used for calculating corresponding period. This study would further lead to evaluate the limitations of the approximate code formulae, now being used for determining the period of a structure.

2.2 CODES OF PRACTICE

The design of buildings considering earthquakes usually employs static loads that are determined in accordance with provisions in the applicable building code. The appropriate earthquake-resistant regulations for one country are not necessarily the same as for other countries. Emphasis has been given on the equivalent static force procedure, as has been described in the present day codes. Some of the codes in use in Bangladesh are,

- (i) The Uniform Building Code (UBC)
- (ii) Bangladesh National Building Code (BNBC)

- (ii) Standards Association of Australian Earthquake Code (SAA)
- (iv) Indian Standard Criteria for Earthquake Resistant Design of Structures (IS)
- (v) The National Building Code of Canada (NBC)
- (vi) The Building Standard Law of Japan (BSLJ).

Equivalent static force method of design for regular structures suggested by the above codes is discussed below for the comparative study of seismic design and evaluation of time period of structures.

2.2.1 Uniform Building Code

The procedures and limitations for the design of structures by Uniform Building Code (UBC) are determined considering zoning, site characteristics, occupancy, configuration of structural system and height. Two of the major parameters in the selection of design criteria are occupancy and structural configuration.

The total lateral force or base shear, according to Uniform Building Code 1985, 1991, 1994 and 1997 are enumerated below.

UBC – 85

The base shear, V is given by (Uniform Building Code, 1985)

$$V = ZIKCSW \quad (2.21)$$

Where,

Z = seismic zone factor

I = occupancy importance factor

K = horizontal force factor

S = site coefficient for soil characteristics

W = the total seismic dead load

C = vibration characteristic factor expressed as a coefficient related to the flexibility of a structure.

$$C = \frac{1}{15\sqrt{T}} \leq 0.12 \quad (2.22)$$

Where, T is the elastic undamped fundamental period of vibration of the building (in seconds) in the direction of the motion considered. The value of T may be determined from one of the following methods:

Method A: The fundamental period of vibration, T , is established using the structural properties and deformation characteristics of the resisting elements in a properly substantiated analysis. In absence of precise determination of T , the fundamental period of a building, in which lateral-load resisting system consists of moment-resisting space frames capable of resisting 100% of the required lateral forces and the frames are not enclosed or adjoined by more rigid elements tending to prevent them from resisting the lateral forces,

$$T = 0.1N$$

Where,

N = total number of stories between the base and the upper most level in the main portion of the structure.

Method B

The fundamental period T may be calculated using the structural properties and deformational characteristics of the resisting elements in a properly substantiated analysis. The formula for determining the period is

$$T = 2\pi \sqrt{\left(\sum_{i=1}^n w_i \delta_i^2 \right) \div \left(g \sum_{i=1}^n f_i \delta_i \right)} \quad (2.23)$$

The values of f_i represent any lateral force distributed approximately in accordance with rational distribution. The elastic deflections, δ_i , shall be

calculated using the applied lateral forces, f_i . The value of C shall be not less than 80 percent of the value obtained by using T from Method A.

UBC - 91,94,97

UBC-91 and 97 are identical in defining base shear, which, is given by

$$V = \frac{ZIC}{R_w} W \quad (2.24)$$

Where,

Z = seismic zone factor

I = occupancy importance factor

R_w = numerical coefficient

W = total seismic weight

$$C = \frac{1.25S}{T^{2.3}} \geq .075R_w \leq 2.75$$

S = site coefficient for soil characteristics

T = fundamental period of vibration in seconds.

The value of the fundamental period, T of the structure shall be determined from one of the following methods:

Method A: For all buildings the value of T may be approximated by the following formula:

$$T = C_t h_n^{3/4} \quad (2.25)$$

Where,

C_t = numerical coefficient

h_n = building height in ft above base

For moment resisting concrete frame structures

$$C_i = .03$$

Method B: This is same as the method B described previously for UBC-85.

2.2.2 Bangladesh National Building Code (BNBC 1993)

Bangladesh National Building Code was published in 1993. The empirical formula to determine the total design base shear in this code has been described in Article 2.1 through the equations 2.15 to 2.17 of this thesis.

The value of C (equation 2.16) need not exceed 2.75 and this value may be used for any structure without regard to soil type or structure period. Except for those requirements where code prescribed forces is scaled up by $.375R$, the minimum value of the ratio C/R shall be $.075$.

The value of the fundamental period, T of the structure shall be determined from one of the following methods:

Method A: The value of the fundamental period, T for reinforced concrete moment resisting frames can be determined approximately from the following empirical formula:

$$T = 0.073 h_n^{3/4} \quad (2.26)$$

Where,

h_n = height in meters above the base to level n .

Method B: This is same as described for UBC-85 (Article-2.2.1)

2.2.3 Standard Association of Australian Earthquake Code (SAA)

According to Standards Association of Australian Earthquake Code (1979), the minimum total horizontal earthquake force to be resisted by a building shall be

$$H = ZIKCSW \quad (2.27)$$

Where,

H = the equivalent static horizontal force for which the structure may be designed instead of the actual inertia forces generated in the structure by the movement of the ground. The minimum value of $H = 0.02W$, for non-zero values of Z . For the purpose of design, this force shall be taken to act non-concurrently in the direction of each of the main axes of the building.

Z = seismic zone coefficient

I = occupancy importance factor

K = horizontal force factor

S = site structure resource factor

W = total seismic dead load

C = the seismic response factor

$$= \frac{I}{15\sqrt{T}} > .12 \quad (2.28)$$

T = fundamental period of vibration

$$= 0.1n$$

Where,

n = total number of levels above the base up to and including level n .

2.2.4 Indian Standard Criteria for Earthquake Resistant Design of Structures (IS)

In Indian Standard Criteria for Earthquake Resistant design of Structures of 1970, the base shear V_B is given by the following formula:

$$V_B = C \alpha_n \beta W \quad (2.29)$$

Where,

α_n = seismic coefficient

β = a coefficient depending upon the soil foundation system

W = total amount of seismic load

C = a coefficient defining the flexibility of structure with the increase in number of stories = $0.5/T^{1/3}$

T = fundamental time period of the building in seconds. For moment resisting frames without bracing or shear walls, the fundamental period is taken to be $.1n$.

n = number of stories including the basement floors.

2.2.5 The National building code of Canada (NBC), 1995.

In NBC, the base shear V_B is given by the following formula.

$$V_B = \nu S I F W \left(\frac{U}{R} \right) \quad (2.30)$$

Where,

ν = Zonal Velocity ratio

I = Importance factor

R = Force modification factor

S = Seismic response factor function of fundamental natural period.

F = Foundation factor depends on soil type and depth.

W = Total dead load and applicable portion of live load.

NBC (1995) includes an extra factor in its code provisions. This factor called calibration factor U , is applied to maintain the design base shears at the same level of proportion of buildings with good to excellent capability of resisting seismic load consistent with the force modification factor R , as reported by Uzumeri (1995).

2.2.6 The Building Standard Law of Japan (BSLJ), 1987.

According to BSLJ (1987) the base shear formula is given by

$$V_B = Z R_t A_i C_o W \quad (2.31)$$

Where,

Z = Seismic hazard zoning Co-efficient.

R_t = design spectral co-efficient which depends on soil profile and fundamental natural period of the buildings.

W = sum of dead load and applicable portion of live load.

C_o and A_i are two extra factors BSLJ (1987) includes in its codes. C_o is the standard shear co-efficient which should not be less than 0.2 and 1.0 for moderate earthquake motions and for severe earthquake motions respectively. A_i is the lateral shear distribution factor, which should be determined by the fundamental natural period and weight distributions of the buildings.

$$A_i = 1 + \left(\frac{1}{\sqrt{\alpha_i}} - \alpha_i \right) \frac{2T}{1+3T} \quad (2.32)$$

$$\alpha_i = W/W_n$$

W = weight above i-th story W_n = weight above ground level

T = Fundamental natural period (Sec.)

W_n = weight above ground level

T = Fundamental natural period (Sec.)

= $h(0.02+0.01\alpha)$; where, h = height of the building in meters, α =the ratio of the total height of stories of steel construction to the height of the building.

2.3 COMPARISON OF METHODS FOR DETERMINATION OF PERIOD BY CODES

The analytical formulae for the fundamental period and the coefficient depending on fundamental period of the structure are suggested to be computed by UBC-85, UBC-91, 94, 97, Bangladesh National Building Code (BNBC, following Method A), Standard Association of Australian Earthquake Code(SAA), Indian Standard Criteria for Earthquake Resistant Design of Structures (IS), National building code of Canada (NBC), and Building Standard Law of Japan (BSLJ) are summarized in Table 2.1.

Table 2.1 Fundamental period and coefficient depending on fundamental period for moment resisting frames, deforming freely:

	UBC-85	UBC-91, 94, 97	BNBC	SAA	IS	NBC	BSLJ
T	$0.1N$	$0.03h_n^{3/4}$	$0.073h_n^{3/4}$	$0.1n$	$0.1n$	$0.1N$	$h(0.02+0.01\alpha)$
C	$\frac{1}{15\sqrt{T}}$	$\frac{1.25S}{T^{2/3}}$	$\frac{1.25S}{T^{2/3}}$	$\frac{1}{15\sqrt{T}}$	$0.5/T^{1/3}$		
	≤ 0.12	$\geq 0.75 R_w$ ≤ 2.75	$\geq 0.75 R_w$ ≤ 2.75	> 0.12			

According to UBC-85, SAA and IS, the fundamental period of vibration is a function of number of stories. According to UBC-91, 94, 97, BNBC, and BSLJ however, it is a function of height of the structure. As per UBC-85 and SAA, the co-efficient of fundamental period of vibration is proportional to $1/\sqrt{T}$. According to UBC-91, 94, 97 and BNBC it is proportional to $1/T^{2/3}$ and according to IS, $1/T^{1/3}$.

2.4 FREE VIBRATION ANALYSIS OF FRAME STRUCTURES

Determining the behaviour of a structure during an earthquake is basically a dynamic problem. The seismic motions of the ground cause the structure to vibrate and the amplitude of vibration of its dynamic deformation and its duration are of concern to the engineer. If the vibration of an elastic structure takes place in the absence of any externally imposed force but in the presence of external and internal frictional forces, the motion is termed as damped free vibration (Chandrapatla and Belegmdu, 1991). In the hypothetical case where it is assumed that the frictional forces are also absent, the motion is called an undamped free vibration. The undamped motion is defined simply by the elastic resistance and the inertial forces of the system, by the initial condition of the state of motion, and by the boundary conditions of the structure. The response for the hypothetical undamped vibration is an approximation of the actual damped vibration experienced by real structures.

Damping has much less importance in controlling the maximum response of a structure to short duration loads (e.g., earthquake) than for long periodic or harmonic loads because the maximum response to a particular impulsive load will be reached in a very short time, before the damping forces can absorb much energy from the structure. For this reason the hypothetical undamped vibration is considered in the analysis of time period of structures. A brief outline of the theory of undamped free vibration of framed structures shown by Clough and Penzien (1993) is presented in the following sub-sections.

2.4.1 Analysis Of Vibration Frequencies And Period

The equations of motion for a freely vibrating undamped (Clough and Penzien, 1993) system can be written as

$$\mathbf{m} \ddot{\mathbf{v}} + \mathbf{k} \mathbf{v} = \mathbf{0} \quad (2.33)$$

Where, \mathbf{m} = mass matrix of the structure, $\ddot{\mathbf{v}}$ = acceleration vector, \mathbf{k} = stiffness matrix and, \mathbf{v} = displacement vector and $\mathbf{0}$ = zero vector. The problem of vibration analysis consists of determining the condition under

which the equilibrium condition expressed by Eq. 2.34 will be satisfied. By analogy with the behavior of SDOF systems, it will be assumed that the free-vibration motion is simple harmonic, which may be expressed as

$$\mathbf{v}(t) = \hat{\mathbf{v}} \sin(\omega t + \theta) \quad (2.34)$$

In this expression $\hat{\mathbf{v}}$ represents the shape of the system (which does not change with time; only the amplitude varies) and θ is a phase angle. When the second time derivative of Eq. 2.35 is taken, the accelerations in free vibration are

$$\ddot{\mathbf{v}} = \omega^2 \hat{\mathbf{v}} \sin(\omega t + \theta) = -\omega^2 \mathbf{v} \quad (2.35)$$

Substituting Eq. 2.34 and 2.35 into Eq. 2.33 gives

$$-\omega^2 \mathbf{m} \hat{\mathbf{v}} \sin(\omega t + \theta) + \mathbf{k} \hat{\mathbf{v}} \sin(\omega t + \theta) = 0$$

which (since the sine term is arbitrary and may be omitted) may be written as

$$[\mathbf{k} - \omega^2 \mathbf{m}] \hat{\mathbf{v}} = 0 \quad (2.36)$$

Eq. 2.36 constitutes an eigen value or characteristic value problem. The quantities ω^2 are the eigen values or characteristic values indicating the square of the free vibration frequencies, while the corresponding displacement vectors $\hat{\mathbf{v}}$ express the corresponding shapes of the vibrating system, known as the eigenvectors or mode shapes. Now it can be shown by Cramer's rule that the solution of this set of simultaneous equations is of the form:

$$\hat{\mathbf{v}} = \frac{\mathbf{0}}{\|\mathbf{k} - \omega^2 \mathbf{m}\|} \quad (2.37)$$

Hence, a nontrivial solution is possible only when the denominator determinant vanishes. In other words, finite-amplitude free vibrations are possible only when

$$\|\mathbf{k} - \omega^2 \mathbf{m}\| = 0 \quad (2.38)$$

Eq. 2.38 is called the frequency equation of the system. Expanding the determinant will give an algebraic equation of the i th degree in the frequency parameter ω^2 for a system having n degrees of freedom. The n roots of this equation $(\omega_1^2, \omega_2^2, \omega_3^2, \dots, \omega_n^2)$ represent the square of frequencies of the n modes of vibration, which are possible in the system. The mode having the lowest frequency is the first mode; the next higher frequency is the second mode, etc. The vector made up of the entire set of modal frequencies, arranged in sequence, will be the frequency vector ω .

$$\omega = \begin{Bmatrix} \omega_1 \\ \omega_2 \\ \omega_3 \\ \vdots \\ \omega_n \end{Bmatrix} \quad (2.39)$$

For the real, symmetric, positive definite mass and stiffness matrices which pertain to stable structural systems, all roots of the frequency equation will be real and positive.

The i th period T_i may be found from i th frequency, as

$$T_i = \frac{2\pi}{\omega_i} \quad (2.40)$$

Therefore, the fundamental period is

$$T = \frac{2\pi}{\omega_1} \quad (2.41)$$

2.4.2 Analysis Of Vibration Mode Shapes

When the frequencies of vibration have been determined from Eq. 2.38, the equations of motion, Eq. 2.36, may be expressed as

$$\tilde{\mathbf{E}}^{(n)} \hat{\mathbf{V}}_n = \mathbf{0} \quad (2.42)$$

in which,

$$\tilde{\mathbf{E}}^{(n)} = \mathbf{k} - \omega_n^2 \mathbf{m} \quad (2.43)$$

Thus, $\tilde{\mathbf{E}}^{(n)}$ represents the matrix obtained by subtracting $\omega_n^2 \mathbf{m}$ from the stiffness matrix; since it depends on the frequency, it is different for each mode. Eq. 2.42 is satisfied identically because the frequencies were evaluated from this condition; therefore the amplitude of the vibrations is indeterminate. However, the shape of the vibrating system can be determined by solving for all the displacements in terms of any one coordinate.

For this purpose it may be assumed that the first element of the displacement vector has a unit amplitude; that is

$$\begin{Bmatrix} \hat{v}_{1n} \\ \hat{v}_{2n} \\ \hat{v}_{3n} \\ \vdots \\ \hat{v}_{Nn} \end{Bmatrix} = \begin{Bmatrix} 1 \\ \hat{v}_{2n} \\ \hat{v}_{3n} \\ \vdots \\ \hat{v}_{Nn} \end{Bmatrix} \quad (2.44)$$

In expanded form, Eq. 2.42 may then be written

$$\begin{bmatrix} e_{11}^{(n)} & e_{12}^{(n)} & e_{13}^{(n)} & \dots & e_{1N}^{(n)} \\ \dots & \dots & \dots & \dots & \dots \\ e_{21}^{(n)} & e_{22}^{(n)} & e_{23}^{(n)} & \dots & e_{2N}^{(n)} \\ e_{31}^{(n)} & e_{32}^{(n)} & e_{33}^{(n)} & \dots & e_{3N}^{(n)} \\ \dots & \dots & \dots & \dots & \dots \\ e_{N1}^{(n)} & e_{N2}^{(n)} & e_{N3}^{(n)} & \dots & e_{NN}^{(n)} \end{bmatrix} \begin{Bmatrix} 1 \\ \dots \\ \hat{v}_{2n} \\ \hat{v}_{3n} \\ \dots \\ \hat{v}_{Nn} \end{Bmatrix} = \begin{Bmatrix} 0 \\ \dots \\ 0 \\ 0 \\ \dots \\ 0 \end{Bmatrix} \quad (2.45)$$

in which partitioning is indicated to correspond with the as yet unknown displacement amplitudes. For convenience, Eq. 2.45 is expressed in matrix notations as

$$\begin{bmatrix} e_{11}^{(n)} & \tilde{E}_{00}^{(n)} \\ \tilde{E}_{01}^{(n)} & \tilde{E}_{00}^{(n)} \end{bmatrix} \begin{Bmatrix} 1 \\ \hat{v}_{0n} \end{Bmatrix} = \begin{Bmatrix} 0 \\ 0 \end{Bmatrix} \quad (2.46)$$

from which,

$$\tilde{\mathbf{E}}_{01}^{(n)} + \tilde{\mathbf{E}}_{00}^{(n)} \hat{\mathbf{v}}_{0n} = \mathbf{0} \quad (2.47)$$

and

$$e_{01}^{(n)} + \tilde{\mathbf{E}}_{10}^{(n)} \hat{\mathbf{v}}_{0n} = \mathbf{0} \quad (2.48)$$

Eq. 2.48 can be solved simultaneously for the displacement amplitudes

$$\hat{\mathbf{v}}_{0n} = -(\tilde{\mathbf{E}}_{00}^{(n)})^{-1} \tilde{\mathbf{E}}_{01}^{(n)} \quad (2.49)$$

and Eq. 2.48 is redundant; the redundancy corresponds to the fact that it is satisfied identically. The displacement vector obtained in Eq. 2.49 must satisfy Eq. 2.48 however, and this condition provides a useful check on the accuracy of the solution.

Here it may be mentioned that it is not always wise to let the first element of the displacement vector unity; numerical accuracy will be improved if the unit element is associated with one of the larger displacement amplitudes. The same solution process can be employed in any case, however, by merely rearranging the order of the rows and columns of $\tilde{\mathbf{E}}^{(n)}$ appropriately.

The displacement amplitudes obtained from Eq. 2.49 together with the unit amplitude of the first component constitute the displacement vector associated with the i th mode of vibration. For convenience the vector is usually expressed in dimensionless form by dividing all the components by one reference component (usually the largest).

The resulting vector is called the n th mode shape ϕ_n ; thus

$$\phi_n = \begin{Bmatrix} \phi_{1n} \\ \phi_{2n} \\ \dots \\ \phi_{Nn} \end{Bmatrix} \equiv \frac{1}{\hat{v}_{kn}} \begin{Bmatrix} 1 \\ \dots \\ \hat{v}_{2n} \\ \hat{v}_{3n} \\ \dots \\ \hat{v}_{Nn} \end{Bmatrix} \quad (2.50)$$

in which,

\hat{v}_{1n} is the reference component, taken as the first component here.

The shape of each of the N modes of vibration can be found by the same process; the square matrix made up of the N mode shapes will be represented by ϕ as

$$\phi = [\phi_1 \ \phi_2 \ \phi_3 \ \dots \ \phi_N] = \begin{bmatrix} \phi_{11} & \phi_{12} & \dots & \phi_{1n} \\ \phi_{21} & \phi_{22} & \dots & \phi_{2n} \\ \dots & \dots & \dots & \dots \\ \phi_{N1} & \phi_{N2} & \dots & \phi_{NN} \end{bmatrix} \quad (2.51)$$

2.4.3 Structure parameters that influence period

The Eq. 2.38 clearly states that the frequency of a structure is directly proportional to the stiffness properties and inversely proportional to the mass properties of the structure. The stiffness properties are directly proportional to the modulus of elasticity of the materials and moment of inertia of the structural elements. The stiffness properties are also inversely proportional to height of vertical members and span of horizontal members. Therefore the stiffness properties and hence the frequency will be influenced by modulus of elasticity as well as by sectional dimensions, width and height of a structure. Again the effect of shearing deformation could be significant on the periods corresponding to the higher mode (Basu et al., 1982).

Mass properties depend on the density of the material. Thus the material density will certainly affect the frequency. Since the period of the structure is inversely proportional to the frequency, the period of the structure will similarly be a function of the stiffness and mass properties of the structure. Furthermore, the material properties will also influence the magnitude of the period of a structure. Hence, the period of a structure will be a function of its parameters.

CHAPTER 3

ANALYSIS OF INFILLED FRAME STRUCTURES

3.1 INTRODUCTION

An infilled frame consists of a steel or reinforced concrete column-and-girder frame with infills of brickwork or concrete block work. In addition to functioning as partitions, the infills may also serve structurally to brace the frame against horizontal loading. A large number of buildings are constructed with masonry infills for architectural needs or aesthetic reasons. In non-earthquake regions where the wind forces are not severe, the masonry infilled frame is one of the most common structural forms for high-rise construction. The frame is designed for gravity loading only and, in the absence of accepted design method, the infills are presumed as to contribute sufficiently to the lateral strength of the structure for it to withstand the horizontal loading. The simplicity of construction, and the highly developed expertise in building that type of structure have made the infilled frame one of the most rapid and economical structural forms for tall buildings.

3.2 CHARACTERISTICS OF INFILLED FRAMES

The behavior of masonry infilled frames has been extensively studied in the last decades (Smith and Coull, 1991; Saneinejad and Hobbs, 1995 etc.) in attempts to develop a rational approach for design of such frames. The use of a masonry infill to brace a frame combines some of the desirable structural characteristics of each, while overcoming some of their deficiencies. The high in-plane rigidity of the masonry wall significantly

stiffens the relatively flexible frame. The result is, therefore a relatively stiff and tough bracing system.

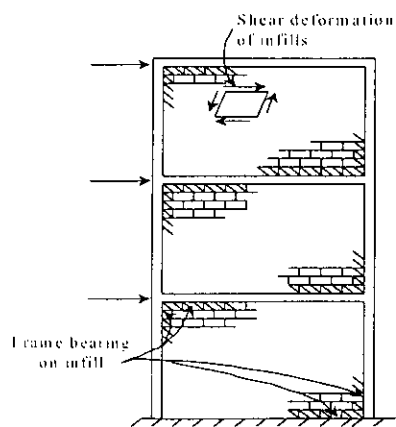


Fig. 3.1 Interactive behavior of frame and infill
(Smith and Coull, 1995)

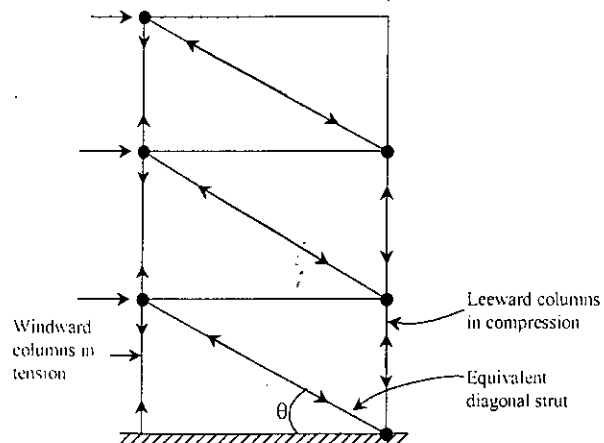


Fig. 3.2 Analogous braced frame
(Smith and Coull, 1995)

The wall braces the frame partly by its in-plane shear resistance and partly by its behavior as a diagonal bracing strut in the frame. Figure 3.1 shows such modes of behavior. When the frame is subjected to horizontal loading, it deforms with double-curvature bending of the columns and beams. The translation of the upper part of the column in each story and the shortening of the leading diagonal of the frame cause the column to

lean against the wall as well as to compress the wall along its diagonal. It is roughly analogous to a diagonally braced frame (fig.3.2)

The potential modes of failure of the wall arises as a result of its interaction with the frame are given below:

1. Tension failure of the tension column due to overturning moments.
2. Flexure or shear failure of the columns.
3. Compression failure of the diagonal strut.
4. Diagonal tension cracking of the panel and
5. Sliding shear failure of the masonry along horizontal mortar beds.

The above failure modes are shown in fig. 3.3 and 3.4

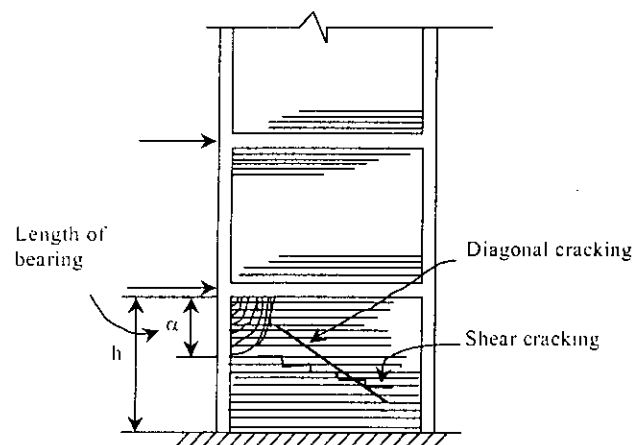


Fig.3.3 Modes of infill failure

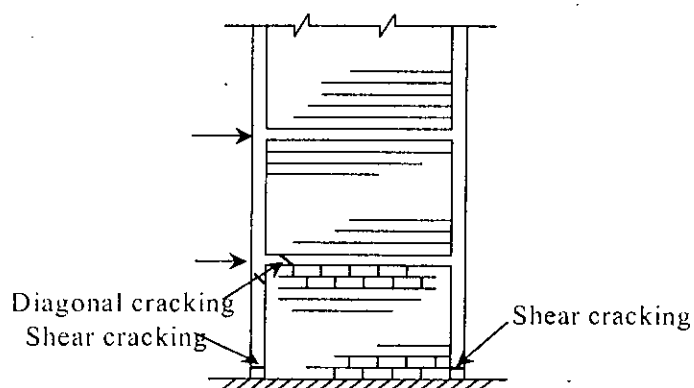


Fig.3.4 Modes of frame failure

The 'perpendicular' tensile stresses are caused by the divergence of the compressive stress trajectories on opposite sides of the leading diagonal as they approach the middle region of the infill. The diagonal cracking is initiated at and spreads from the middle of the infill, where the tensile stresses are a maximum, tending to stop near the compression corners, where the tension is suppressed.

The nature of the forces in the frame can be understood by referring to the analogous braced frame shown in fig. 3.2. The windward column is in tension and the leeward column is in compression. Since the infill bears on the frame not as a concentrated force exactly at the corners, but over short lengths of the beam and column adjacent to each compression corner, the frame members are subjected also to transverse shear and a small amount of bending. Consequently, the frame members or their connections are liable to fail by axial force or shear, and especially by tension at the base of the windward column shown in fig. 3.4.

3.3 OVERVIEW OF THE ANALYSIS OF INFILLED FRAMES.

The behavior of infilled frames under lateral loads has been investigated by a number of researchers. Holmes (1961), Stafford Smith (1962, 1966, 1967), and Mainstone and Weeks (1970) conducted experimental and analytical investigations on the lateral stiffness and strength of steel frames infilled with mortar and concrete panels. Dawe and Seah (1989), Flanagan et al. (1992), and Mander et al. (1993) have studied the behavior of masonry-infilled steel frames under in-plane and out-of-plane loads. Dhanasekar and Page (1986) have developed finite-element models for analyzing masonry-infilled steel frames. Wood (1978) and Liauw and Kwan (1985) have developed plastic analysis methods to predict the in-plane limit loads of steel infilled frames. The behavior of masonry-infilled RC frames is generally more complicated than that of steel infilled frames and has been examined by Fiorato et al. (1970), Klingner and Bertero (1976), Kanh and Hanson (1979), Bertero and Brokken (1983), Zarnic and Tomazevic (1990), and others. However, most of these studies have been conducted with either

small-scale frame specimens or frame designs that might not truly reflect existing structures. In spite of this, these studies have identified a number of complicated failure mechanisms that can be possibly caused by the frame-panel interaction.

More recently, a study was conducted by Angel et al. (1994) to investigate the behavior of RC frames infilled with masonry panels. In that study, the frame specimens were designed in accordance with current code provisions, and the main emphasis was to examine the out-of-plane resistance of the infill panels. To investigate the performance of masonry-infilled RC frames under in-plane lateral loads, a comprehensive study was carried out at the University of Colorado in conjunction with Atkinson-Noland & Associates. This study focused on RC frames that were designed in accordance with current code provisions, with and without the consideration of strong earth quake loadings. The influence of the frame aspect ratio and vertical load distribution on the lateral resistance of infilled frames was also examined. Furthermore, extensive material and component tests were carried out to obtain the properties of the concrete and masonry materials as well as the shear strength of masonry mortar joints. These tests allow a proper calibration of different refined analysis methods. Both simple analytical models and inelastic finite-element analysis methods have been developed and validated with the experimental results.

Recently, Saneinejad and Hobbs (1995) developed a method based on the equivalent diagonal strut approach for the analysis and design of steel or concrete frames with concrete or masonry infill walls subjected to in-plane forces. The method takes into account the elastoplastic behavior of infilled frames considering the limited ductility of infill materials. Various governing factors such as the infill aspect ratio, the shear stresses at the infill-frame interface, and relative beam and column strengths are accounted for in this development. However, the formulation furnishes only extreme or boundary values for design purposes.

Some of the above methods of analysis of infilled frame is described in the following sections.

3.3.1 Approximate Method

A concept of the behavior of infilled frames has been developed from approximate analyses (Smith and Coull, 1991). The method presented here draws from a combination of test observations and the results of analyses. It may be classified as an elastic approach except for the criterion used to predict the infill crushing, for which a plastic type of failure of the masonry infill is assumed.

Stresses in the infill

Relating to shear failure : Shear failure of the infill is related to the combination of shear and normal stresses induced at points in the infill when the frame bears on it as the structure is subjected to the external lateral shear. An extensive series of plane-stress membrane finite-element analyses has shown that the critical value of this combination of stresses occur at the center of the infill and that they can be empirically determined by

$$\text{Shear stress, } \tau_{xy} = \frac{1.43}{L_t} Q \quad (3.1)$$

$$\text{Vertical compressive stress, } \sigma_y = \frac{(0.8h/l - 0.2)}{Lt} Q \quad (3.2)$$

Where Q is the horizontal shear load applied by the frame to the infill of length L , height h , and thickness t .

Relating to diagonal tensile failure: Similarly, diagonal cracking of the infill is related to the maximum value of diagonal tensile stress in the infill. This also occurs at the center of the infill and, based on the results of the analyses, maybe expressed empirically as

$$\text{Diagonal tensile stress, } \sigma_d = \frac{0.58}{Lt} \quad (3.3)$$

These stresses are governed mainly by the proportions of the infill. They are little influenced by the stiffness properties of the frame because they occur at the center of the infill, away from the region of the frame.

Relating to compressive failure of the corners : Tests on model infilled frames have shown that the length of bearing of each story-height column against its adjacent infill is governed by the flexural stiffness of the column relative to the inplane bearing stiffness of the infill. The stiffer the column, the longer the length of bearing and the lower the compressive stresses at the interface. Tests to failure have borne out the deduction that the stiffer the column, the higher the strength of the infill against compressive failure. They have also shown that crushing failure of the infill occurs over a length approximately equal to the length of bearing of the column against the infill shown in fig.3.3.

As a crude approximation, an analogy may be drawn with the theory for a beam on an elastic foundation, from which it has been proposed that the length of column bearing α may be estimated by

$$\alpha = \frac{\pi}{2\lambda} \quad (3.4)$$

$$\text{where, } \lambda = \sqrt[4]{\frac{E_m t}{4EIh}} \quad (3.5)$$

in which E_m is the elastic modulus of the masonry and EI is the flexural rigidity of the column. The parameter λ expresses the bearing stiffness of the infill relative to the flexural rigidity of the column: the stiffer the column, the smaller the value of λ and the longer the length of bearing.

If it is assumed that when the corner of the infill crushes, the masonry bearing against the column within the length α is at the masonry ultimate compressive stress f'_m , then the corresponding ultimate horizontal shear Q'_c on the infill is given by

$$Q'_c = f'_m \alpha t \quad (3.6)$$

$$\text{or } Q'_c = f'_m t \cdot \frac{\pi}{2} \sqrt{\frac{4EIh}{E_m t}} \quad (3.7)$$

Considering now the allowable horizontal shear Q_c on the infill, and assuming a value for E/E_m of 30 in the case of a steel frame and 3 in the

case of a reinforced concrete frame, the allowable horizontal shear on a steel framed infill corresponding to a compressive failure is given by

$$Q_c = 5.2 f_m \sqrt[4]{Iht^3} \quad (3.8)$$

and for a reinforced concrete framed infill

$$Q_c = 2.9 f_m \sqrt[4]{Iht^3} \quad (3.9)$$

in which f_m is the masonry allowable compressive stress.

These semi empirical formulas indicate the significant parameters that influence the horizontal shear strength of an infill when it is governed by a compressive failure of one of its corners. The masonry compressive strength and the wall thickness have the most direct influence on the infill strength. While the column inertia and infill height exert control in proportion to their fourth roots. The infill strengths indicated by Eq. (3.8) and (3.9) are very approximate. Experimental evidence has shown them to overestimate the real values; therefore, they need to be modified before being used in the design procedure.

3.3.2 Equivalent Strut Method

Saneinejad and Hobbs (1995) developed a method based on the equivalent diagonal strut approach for the analysis and design of steel and concrete frames with concrete or masonry infill walls subjected to in-plane forces. The proposed analytical development assumes that the contribution of the masonry infill panel shown in fig. 3.5 to the response of the infilled frame can be modeled by "replacing the panel" by a system of two diagonal masonry compression struts shown in fig. 3.6. The stress-strain relationship for masonry in compression shown in fig 3.7 is used to determine the strength envelope of the equivalent strut, can be idealized by a polynomial function. Since the tensile strength of masonry is negligible, the individual masonry struts are considered to be ineffective in tension. However, the combination of both diagonal struts provides a lateral load resisting mechanism for the opposite lateral directions of loading.

The lateral force-deformation relationship for the structural masonry infill panel is assumed to be a smooth curve bounded by a bilinear strength envelope with an initial elastic stiffness until the yield force V_y and there on a post yield degraded stiffness until the maximum force V_m is reached shown in fig.3.8. The corresponding lateral displacement values are denoted as u_y and u_m respectively. The analytical formulations for the strength envelope parameters were developed on the basis of the available "equivalent strut model" for infilled frames.

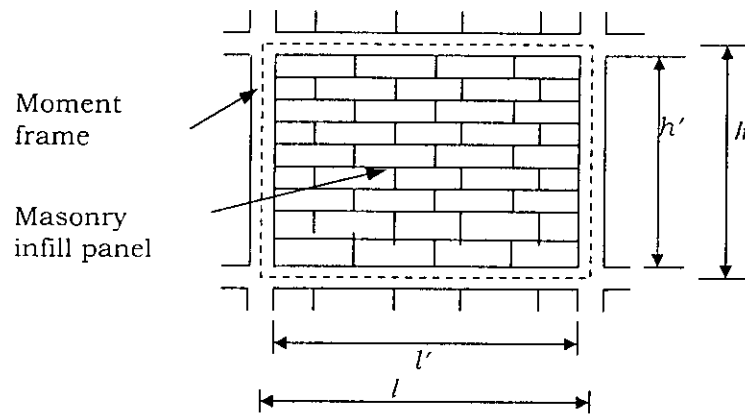


Fig 3.5 Masonry infill frame sub-assembly in masonry infill panel frame structures.

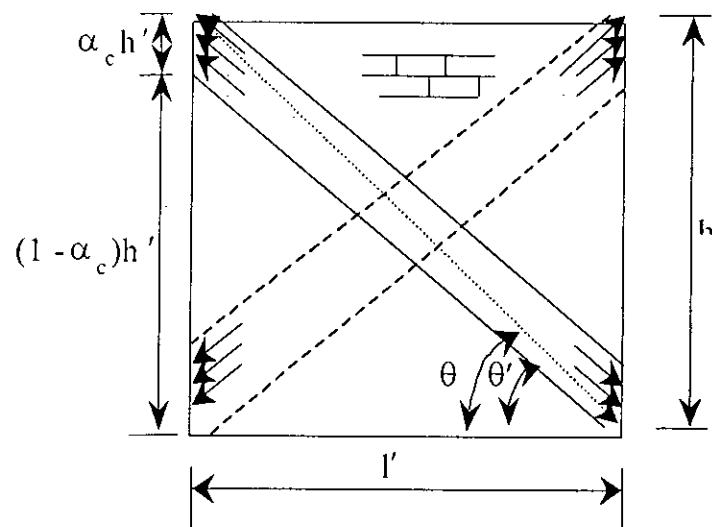


Fig. 3.6 Masonry infill panel in frame structures.

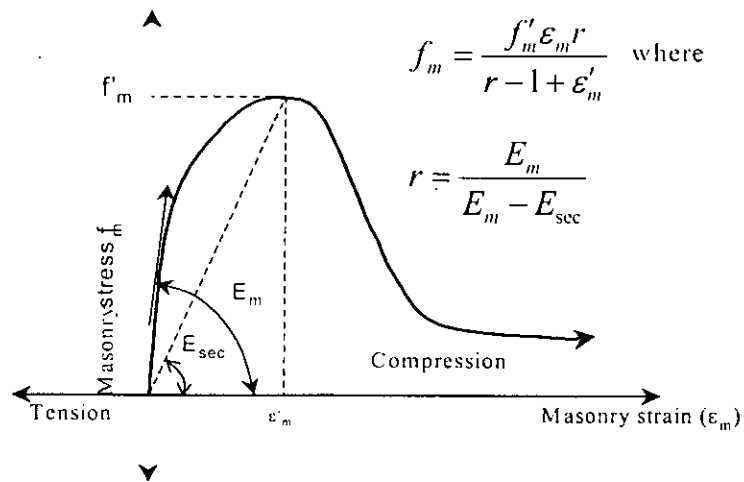


Fig.3.7 Constitutive model for masonry infill panel (Saneinejad and Hobbs, 1995)

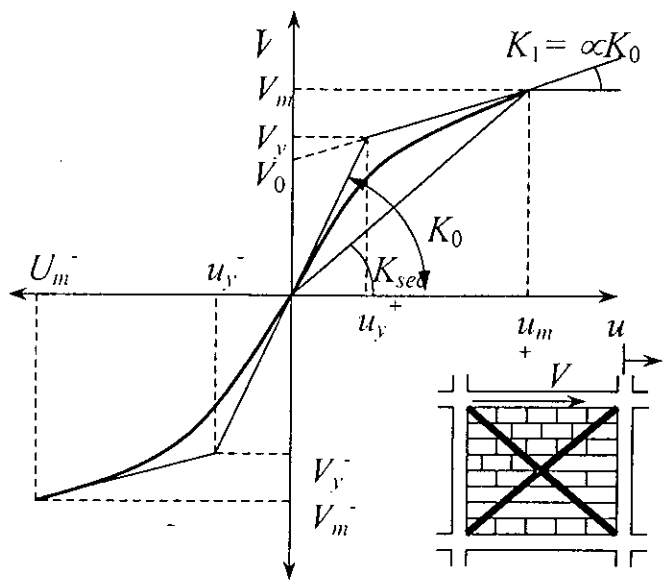


Fig. 3.8 Strength envelope for masonry infill panel (Saneinejad and Hobbs, 1995).

3.3.3 Plasticity Model

A model is developed by Lorenzo et al.(1997) based on the theory of plasticity, which is adopted to describe the inelastic behavior, utilizes modern algorithmic concepts, including an implicit Euler background return mapping scheme, a local Newton-Raphson method and a consistent tangential stiffness matrix. The model is capable of predicting independent response along the material axes. It features a tensile fracture energy and a compressive fracture energy, which are different for each material axis.

A large number of anisotropic materials exist in engineering such as masonry, plastics, wood and most composites. The frame work of plasticity theory is general enough to apply to both isotropic and an-isotropic behavior. Indeed, the past decade has witnessed numerous publications on second numerical implementations of isotropic plasticity models. Nevertheless, it appears that, while some anisotropic plasticity models have been proposed from purely theoretical and experimental standpoints, only a few numerical implementations and calculations have actually been carried out. Examples include the work of de Borst and Feenstra (1990) and Schellekens and de Borst (1990) who fully treated the implementation of elastic-perfectly-plastic Hill (1948) and Hoffman (1967) criteria, respectively. In these publications hardening behavior has been simulated with the fracture model of Besseling (1958). More recently, linear tensorial hardening has been incorporated in the Hill criterion. It is not surprising that only a few anisotropic models have been implemented and tested successfully. An accurate analysis of anisotropic materials requires a description for all stress states. The yield criterion proposed combines the advantages of modern plasticity concepts with a powerful representation of an-isotropic material behavior, which includes different hardening/softening behavior along each material axis.

In order to model orthotropic material behavior, a Hill-type criterion (1948) for compression and a Rankine-type criterion for tension are proposed (fig.3.9). The internal damage due to these failure mechanisms is represented with two internal parameters, one for damage in tension and one for damage in compression. The model is formulated in such a way that

each internal parameter is related to two independent fracture energies along each material axis.

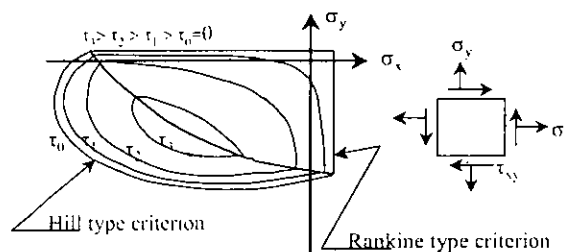


Fig 3.9 Composite yield criterion with iso-shear stress lines
(Proposed by Lorenzo et al.1997)

3.3.4 Coupled Boundary Element Method

The behavior of infilled frames subjected to horizontal loads is analyzed by an iterative numerical procedure (Papia, M.1998). The stiffness of the structural system is determined with variations in geometrical and mechanical characteristics.

The stiffness contribution by brickwork or concrete panels in reinforced concrete or steel frames can provide to be decisive in relation to structure safety. Neglecting the presence of such systems in the calculation of structures subjected to horizontal loads leads to an evaluation of stresses in the frames which is often far from the real situation and may compromise safety. In fact, on account of the high degree of stiffness, panels not placed symmetrically in the plan produce very dangerous unforeseen torsional effects. Such effects may also occur when the infills are distributed symmetrically, if there are openings for doors and windows in some of them, which cause a loss of stiffness.

The analysis is carried out utilizing the boundary element method (BEM) for the infill and opportunely dividing the frame into finite elements, so as to transform the mutual interactions of the two subsystems into stresses distributed along the boundary for the infill and into nodal actions for the frame. This makes it possible to take into account the separation arising between the two substructures when mutual tensile stresses are involved.

At first, infills without openings are considered, using BEM with constant elements for two-dimensional problems in elasticity. Then the results are compared with those obtained using the simplified equivalent pin-jointed strut model, which is very common in literature.

Subsequently, using an analogous procedure, panels with openings are considered. For these systems, for which no satisfactory simplified models exist, the loss of stiffness in relation to the size of the opening is evaluated.

3.4 CHOICE OF THE MODEL

In the previous articles several computational models are described which can be used to model and analyze infills. Of these models the first one described in section 3.3.1 is an approximate method primarily intended for preliminary design purpose through manual calculation. The last two models are based on continuum plasticity approach in which the infill is modeled as an assemblage of several plane stress elements interacting with frame elements via special interface element. The material properties for the plane stress elements are plasticity or damage model approach. Such modeling is suitable for a detailed and micro level study of the infill panels where stress, strain, damage, cracks and failure etc. at various locations of the infill are of primary importance. Such model requires a considerable amount of computational effort due to their highly nonlinear iterative solution procedure. Such modeling is not suitable for investigating overall structural behavior of building where infill is only a structural component. In such a situation the equivalent strut model is more suitable. The equivalent strut model proposed by Saneinejad and Hobbs (1995) is a relatively recent model capable of representing the behavior of infill satisfactorily. The model is based on an equivalent diagonal strut and uses a time rate-independent constitutive model which can be used for a static nonlinear analysis as well as time-history analysis. The same model with hysteretic formulation has been successfully used by Madan et al.(1997) for static monotonic analysis, quasi-static cyclic analysis. They have successfully verified the model by simulating experimental behavior of tested masonry infill frame sub-assemblage. The equivalent diagonal strut

model considers the entire infill panel as a single unit and takes into account only the equivalent global behavior. As a result the approach does not permit study of local effects such as frame-infill interaction within the individual infilled frame sub-assembly. More detailed micro modeling approaches such as the plasticity approach and the boundary element approach discussed earlier need to be used to capture the spatial and temporal variations of local conditions within the infill. However, the equivalent strut model allows for adequate evaluation of the nonlinear force deformation response of the structure and individual components under lateral load. The computed force-deformation response may be used to asses the overall structure damage and its distribution to a sufficient degree of accuracy. Thus, the equivalent strut model is better suited for representing the behavior of infills in nonlinear time-history analysis of large or complex structures with multiple components particularly in cases where the focus is on evaluating the global structural response. In thesis, the equivalent strut modeling, therefore, is chosen for modeling and studying the behavior of plane frames.

3.5 EQUIVALENT STRUT MODELLING

Some details of the equivalent strut model of Saneinjad and Hobbs (1995) as implemented by Madan et al.(1997) are described here. Considering the infilled masonry frame shown in Fig.3.5, the maximum lateral force V_m and corresponding displacement u_m in the infill masonry panel (Madan, 1997) are

$$V_m^+ (V_m^-) \leq A_d f_m' \cos \theta \leq \frac{vtl'}{(1-0.45 \tan \theta) \cos \theta} \leq \frac{0.83tl'}{\cos \theta} \quad (3.10)$$

$$u_m^+ (u_m^-) = \frac{\varepsilon_m' L_d}{\cos \theta} \quad (3.11)$$

in which,

t = thickness of the infill panel;

l' = lateral dimension of the infill panel;

f_m' = masonry prism strength;

ε' = corresponding strain;

θ = inclination of the diagonal strut;

v = basic shear strength of masonry;

A_d = area of the equivalent diagonal strut and

L_d = length of the equivalent diagonal struts. A_d and L_d can be calculated as

$$A_d = \frac{(1 - \alpha_c) \alpha_c t h \frac{\sigma_c}{f_c} + \alpha_b t l \frac{\tau_b}{f_c}}{\cos \theta} \leq 0.5 \frac{t h' \frac{f_u}{f_c}}{\cos \theta} \quad (3.12)$$

$$L_d = \sqrt{(1 - \alpha_c)^2 h'^2 + l'^2} \quad (3.13)$$

The quantities α_c , α_b , σ_c , τ_b , f_u and f_c depends on the geometric and material properties of the frame and infill panel. These can be estimated using the formulations of the "equivalent strut model". The lateral yield force V_y and displacement u_y of the infill panel may be calculated from geometry.

$$V_v^+ (V_v^-) = \frac{V_m - \alpha K_0 u_m}{(1 - \alpha)} \quad (3.14)$$

$$u_v^+ (u_v^-) = \frac{V_m - \alpha K_0 u_m}{K_0 (1 - \alpha)} \quad (3.15)$$

The initial stiffness K_0 of the infill masonry panel may be estimated using the following formula (Madan et al. 1997):

$$K_0 = 2(V_m / u_m) \quad (3.16)$$

The parameters V_m , V_y , u_m , u_y , K_0 , K_1 etc. are clearly shown in Fig.3.8. The degradation of strut stiffness from K_0 to K_1 was assumed to be a bilinear curve by Madan et al.(1997).

3.6 BEAM AND COLUMN MOMENT CAPACITY.

To find out the stiffness of equivalent strut (K_0), it required to determine the following properties of beam, column and joint,

M_{pc} = Plastic resisting moment of column.

M_{pb} = Plastic resisting moment of beam.

M_{pj} = Plastic resisting moment of beam and column joint. M_{pj} is the sum of all M_{pb} and M_{pc} at the joint.

To determine the M_{pb} , M_{pc} ; it requires to provide reinforcement in beam and column. These moments can be calculated on the basis of USD formulae,

$$M_n = A_s f_y (d - a/2) \quad (3.16)$$

$$\text{where, } a = \frac{A_s f_y}{0.85 f'_c b} \quad (3.17)$$

4.6.1 Example for Calculation of Beam and Column Moment Capacity:

For a 10-storied building, considering the beam of 250mm×600mm size and column of 600mm×600mm size, the beam and column moment capacity (M_{pb} , M_{pc} , M_{pj}) are calculated as below:

Beam Moment Capacity (M_{pb})

Given, $f_y = 60 \text{ Ksi} = 400 \text{ MPa}$, $f_c = 4 \text{ ksi} = 28 \text{ MPa}$

$$\rho_{min} = \frac{200}{f_y} = .0033$$

$$\rho_{max} = 0.75 \times 85 \times k_1 \frac{f_c}{f_y} \cdot \frac{87}{87 + f_y} = 0.75 \times (.85)^2 \times \frac{4}{60} \times \frac{87}{47 + 60} = .0214$$

Assumed steel ratio, $\rho = 1/2 (\rho_{max} + \rho_{min}) = 0.01235$

$$A_s = \rho b d = 0.01235 \times 250 \times 550 = 1698 \text{ mm}^2$$

$$= 4 \times 491 = 1964 \text{ mm}^2 \text{ (using 4 -25 mm diameter steel)}$$

$$a = \frac{A_s f_y}{.85 f'_c b} = \frac{1964 \times 400}{.85 \times 28 \times 250} = 132$$

$$M_u = \phi f_y A_s (d - a/2) = .9 \times 400 \times 1964 \times (550 - \frac{132}{2}) = 3.42 \times 10^7$$

i.e., $M_{pb} = 3.42 \times 10^7 \text{ N-mm}$

Column Moment Capacity (M_{pc}):

Assuming 2% reinforcement used (i.e., $\rho = 0.02$), required area of steel = $0.02 \times 600 \times 550 = 6600 \text{ mm}^2$. Providing 4 - 40mm and 4-25 mm diameter steel, total steel area = 6988 mm²

From fig. 4.10 (b), we get, $A_s = \text{Effective steel area} = 3003\text{mm}^2$

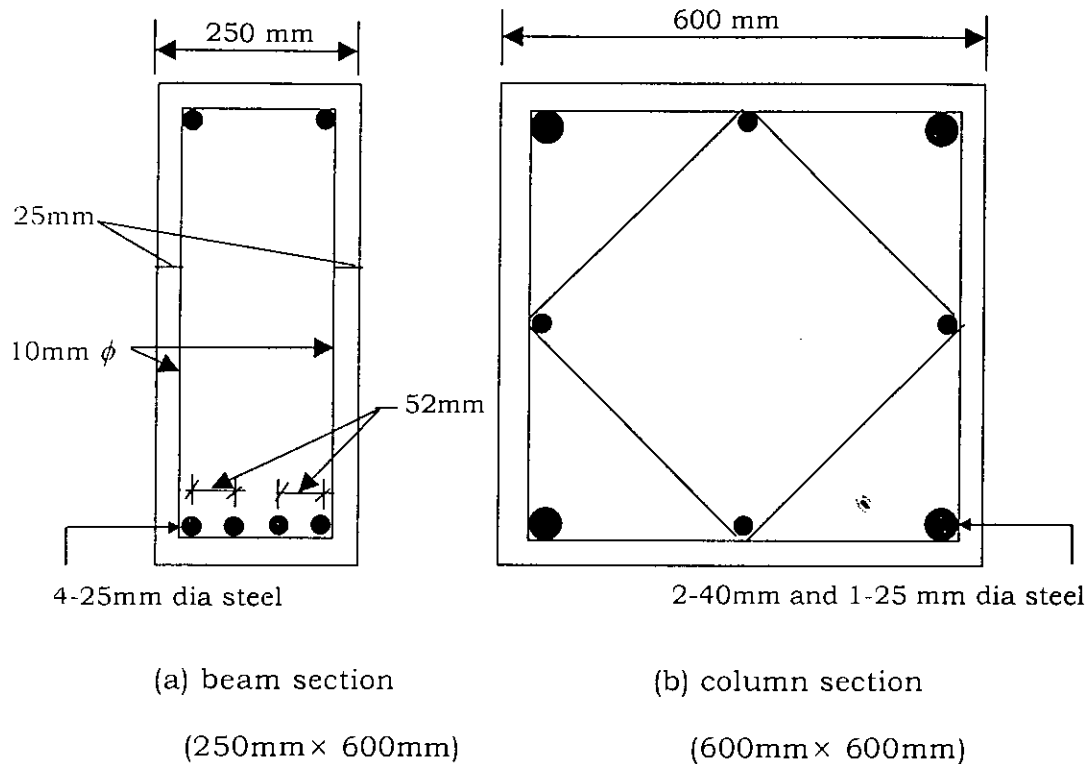


Fig. 4.10 Beam and column sections showing reinforcement

$$a = \frac{A_s f_y}{85 f_c b} = 84$$

$$M_u = \phi f_y A_s (d - a/2)$$

$$= 0.9 \times 400 \times 3003 \times (550 - 40/2) = 5.49 \times 10^8$$

$$M_{pc} = 5.49 \times 10^8 \text{ N-mm.}$$

$$M_{pj} = 2(M_{pc} + M_{pb}) = 2 \times (3.42 \times 10^7 + 5.49 \times 10^8)$$

$$= 1.17 \times 10^9 \text{ N-mm.}$$

3.7 DETERMINATION OF EQUIVALENT STRUT STIFFNESS (K_0).

The equivalent strut model proposed by Saneinejad and Hobbs (1995) and later enhanced by Madan et al. (1997) is discussed in details here. The mathematical derivation of the equivalent strut model begins with an

idealized free body diagram of an infill panel and the surrounding frame as shown in Fig.3.11.

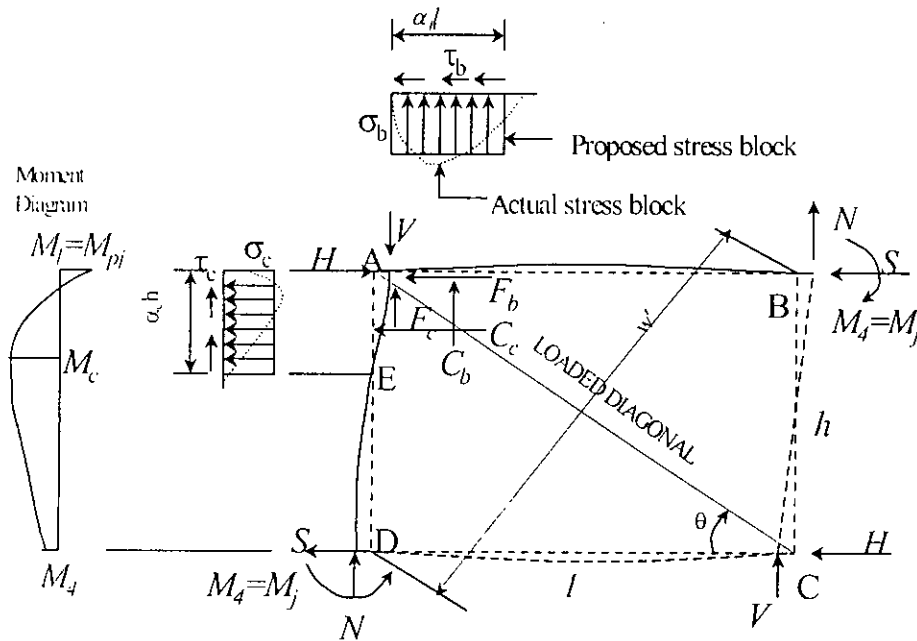


Fig 3.11 Frame forces equilibrium (Saneinjad and Hobbs, 1995)

From fig 3.5 and 3.11,

$$r = \frac{h}{l} < 1 \quad (3.19)$$

where, r = aspect ratio of the frame.

h = center to center height of beam.

l = center to center length of column.

$$r' = \frac{h'}{l'} \quad (3.20)$$

where h' = height of infill.

l' = length of infill.

$$\theta = \tan^{-1}\left(\frac{h}{l}\right) \quad (3.21)$$

$$\theta' = \tan^{-1}\left(\frac{h'}{l'}\right) \quad (3.22)$$

where, θ = inclination of diagonal strut.

The effective compressive strength of infill, f_c can be calculated by

$$f_c = 0.6\phi f'_m \quad (3.23)$$

where, ϕ is a constant and its value is 0.65 and

f'_m = compressive strength of masonry.

The length of the proposed stress blocks (Fig 4.11) may not exceed 0.4 times the corresponding infill dimensions

$$\alpha_c h \leq 0.4h' \quad (3.24)$$

$$\alpha_b l \leq 0.4l' \quad (3.25)$$

where α = normalized length of contact; and subscripts c and b designate column and beam, respectively.

Frame/infill interaction is associated with shear forces that may be evaluated closely by the following:

$$F_c = \mu r^2 C_c \quad (3.26)$$

$$F_b = \mu C_b \quad (3.27)$$

where C and F = frame/infill contact normal and shear forces (Fig.3.11) and μ = coefficient of friction of the frame/infill interface. At the peak load

$$M_A = M_C = M_{pj} \quad (3.28)$$

where M_A and M_C = bending moments at loaded corners (points A and C in Fig.3.11); and M_{pj} = the joint plastic resisting moment.

$$M_D = M_B = M_j < M_{pj} \quad (3.29)$$

$$M_c = \beta_c M_{pc} \quad (3.30)$$

$$M_b = \beta_b M_{pb} \quad (3.31)$$

where M_B and M_D = bending moments at the unloaded corners (Fig.3.11) and M_j represents either of these values. Also M_c and M_b are maximum intermediate elastic moment of column and beam.

$$\beta_c \leq \beta_0 = 0.2 \quad (3.32)$$

$$\beta_b \leq \beta_0 = 0.2 \quad (3.33)$$

where β_0 = nominal, or rather upper-bound value of the reduction factors.

Let $h' = h$ and $l' = l$

Frame forces equilibrium requires the following:

$$V = H \tan \theta \quad (3.34)$$

$$H = C_c + F_b + 2S \quad (3.35)$$

$$V = C_b + F_c + 2N \quad (3.36)$$

Rotational equilibrium of the infill requires the following:

$$C_c \left(\frac{h}{2} - a_c \frac{h}{2} \right) - F_c \frac{l}{2} - C_b \left(\frac{l}{2} - a_b \frac{l}{2} \right) + F_b \frac{h}{2} = 0 \quad (3.37)$$

$$\text{where } C_c = \sigma_c t a_c h,$$

$$C_b = \sigma_b t a_b l,$$

$$F_c = \tau_c t a_c h \text{ and}$$

$$F_b = \tau_b t a_b l.$$

H and V = horizontal and vertical components of the external forces; S and N = shear and axial forces, respectively, over the uncontacted length of the column; σ and τ = proposed uniform frame/infill contact normal and shear forces, θ = sloping angle of the infill diagonal. Taking the static moment of the forces acting on the column and beam about point A:

$$S = 0.5 \sigma_c t a_c^2 h + \frac{(M_{pj} + M_j)}{h} \quad (3.38)$$

$$N = 0.5 \sigma_b t a_b^2 l + \frac{(M_{pj} + M_j)}{l} \quad (3.39)$$

Substituting for contact forces, C_c and F_b , and also column shear force, S into Eq. 3.35 leads to the collapse load, as follows:

$$H = \sigma_c t (1 - a_c) a_c h + \tau_b t a_b l + 2 \frac{(M_{pj} + M_j)}{h} \quad (3.40)$$

At peak load, the infill is subjected to failure resulting from combined normal and shear stresses acting on the contacted surfaces in the loaded corners. The well known Tresca hexagonal yield criterion, described by

Chen (1982), is mathematically convenient for this combination, and is given by

$$\sigma^2 + 3\tau^2 = f_c^2 \quad (3.41)$$

where, f_c = effective compressive strength of the infill.

Assuming rectangular stress blocks, as shown in Fig. 3.13, can be written also in terms of the contact stresses, as follows:

$$\tau_c = \mu r^2 \sigma_c \quad (3.42)$$

$$\tau_b = \mu \sigma_b \quad (3.43)$$

Combining Eqs.3.41, 3.42 and 3.43 and solving for the contact stresses leads to the nominal (upper-bound) values of the contact normal stresses

$$\sigma_{c0} = \frac{f_c}{\sqrt{1+3\mu^2 r^4}}; \quad (3.44)$$

$$\sigma_{b0} = \frac{f_c}{\sqrt{1+3\mu^2}} \quad (3.45)$$

Taking the static moment of the forces acting on the column along EA gives the following:

$$M_{pj} + M_c - 0.5(a_c h)^2 \sigma_c t = 0 \quad (3.46)$$

A similar relation can be written for the beam as follows:

$$M_{pj} + M_b - 0.5(a_b l)^2 \sigma_b t = 0 \quad (3.47)$$

Substituting for M_c and M_b from equations 3.30 and 3.31 into Eqs. 3.46 and 3.47 and solving for the contact lengths, leads to the following:

$$a_c h = \sqrt{\frac{2M_{pj} + 2\beta_c M_{pc}}{\sigma_c t}}; \quad (3.48)$$

$$a_b l = \sqrt{\frac{2M_{pj} + 2\beta_b M_{pb}}{\sigma_b t}}; \quad (3.49)$$

Either β_c or β_b would approach their upper-bound value, $\beta_0 = 0.2$, when the contact surface in question develops the corresponding nominal normal stress. Substituting for these nominal values and combining with Eqs. 3.24 and 3.25 leads to the following:

$$a_c h = \sqrt{\frac{2M_{pj} + 2\beta_0 M_{pc}}{\sigma_{c0} t}} \quad (0.4h) \quad (3.50)$$

$$a_b l = \sqrt{\frac{2M_{pj} + 2\beta_0 M_{pb}}{\sigma_{b0} t}} \quad (0.4l) \quad (3.51)$$

Substituting for the contact forces into Eqs. 3.40 gives the following:

$$\sigma_b \alpha_b (1 - \alpha_b - \mu r) = r^2 \sigma_c \alpha_c (1 - \alpha_c - \mu r) \quad (3.52)$$

This relation would be satisfied only with the real contact stresses, generated from the nominal contact stresses, (Eqs. 3.44 and 3.45) are as follows:

$$\text{If } A_c > A_b, \text{ then } \sigma_b = \sigma_{b0} \text{ and } \sigma_c = \sigma_{c0} (A_b / A_c) \quad (3.53)$$

$$\text{If } A_b > A_c, \text{ then } \sigma_c = \sigma_{c0} \text{ and } \sigma_b = \sigma_{b0} (A_c / A_b) \quad (3.54)$$

Where,

$$A_c = r^2 \sigma_{c0} \alpha_c (1 - \alpha_c - \mu r) \quad (3.55)$$

$$A_b = \sigma_{b0} \alpha_b (1 - \alpha_b - \mu r) \quad (3.56)$$

The actual compressive strength of masonry depends on the direction of stresses and it can be found by following

$$f_u = f_c \left[1 - \left(\frac{L_d}{40t} \right)^2 \right] \quad (3.57)$$

The effective length of diagonal strut, L_d and the cross sectional area of the diagonal strut, A_d for effective compressive strength can be calculated from equations 3.12 and 3.13. Here L_d is not greater than $40t$ and f_c is effective compressive strength of masonry. The maximum lateral force V_m and corresponding displacement u_m in the infill masonry panel can be calculated from equations 3.10 and 3.11. Finally, the initial stiffness K_0 of the infill masonry panel may be estimated using the equation 3.16.

3.7.1 Example of Determination of Equivalent Strut Stiffness

For a 10- storied frame structure with 4×4 bay, span of 4500mm, floor height of 3000mm, beam size of 250mm×600mm and column size of 600mm×600mm, the equivalent strut stiffness (K_o) are calculated as follows:

Given values are:

$$h = 3000 \text{ mm} \quad h' = 3000 - 600 = 2400 \text{ mm}$$

$$l = 4500 \text{ mm} \quad l' = 4500 - 600 = 3900 \text{ mm}$$

$$\phi = 0.65 \quad \epsilon_m = 0.002 \quad \mu = 0.6$$

$$f_m' = 12 \text{ MPa} \quad \nu = 6 \text{ MPa} \quad \beta_o = .02$$

For same structure the beam and column moment capacity (M_{pb} , M_{pc} , M_{pj}) are calculated in article 3.6.1 as follows:

$$M_{pb} = 3.42 \times 10^7$$

$$M_{pc} = 5.49 \times 10^8$$

$$M_{pj} = 1.17 \times 10^9$$

Using the above values in equations 3.19 to 3.57 and 3.10 to 3.14, the value of K_o are determined. The results obtained from those equations are written in tabular form as follows:

t	h	h'	l	l'	ϕ
250	3000	2400	4500	3900	0.65

μ	f_m'	β_o	M_{pj}	M_{pc}	M_{pb}
0.6	12 MPa	0.2	3.42×10^7	5.49×10^8	1.17×10^9

r	r'	θ	θ'	f_c	σ_{c0}
.667	.615	.588	.552	4.68 MPa	4.248 N/mm ²

σ_{b0}	α_c	α_b	A_c	A_b	σ_c
3.245	0.224	.378	.159	.272	4.248 N/mm ²

σ_b	τ_b	τ_c	L_d	f_a	A_d
1.90	1.14	1.13	4321.9	2.25	94211

μ_m	V_m	K_0
10.39	583558	112331 N/mm

CHAPTER 4

FINITE ELEMENT MODELLING FOR TIME PERIOD ANALYSIS

4.1 INTRODUCTION

The number of displacement components, which must be considered in order to represent the effect of all significant inertia forces of a structure, may be termed as the number of dynamic degrees of freedom. In reality, every point of structure has certain numbers of degrees of freedom (DOF). Thus a structure has infinite numbers of DOF. But in most practical cases, this can be replaced by a finite number of DOF. Such type of structures is represented as MDOF system. The dynamic response of a structure cannot be described adequately by single degree of freedom (SDOF) model. Such behavior can be expressed only by MDOF system. The detailed theory may be found in any standard text book (e.g., "Dynamics of Structures" by Clough and Penzien). For determining the period of three dimensional frame structures of high degree of indeterminacy, the use of computer program, specially finite element based type of analysis, is essential (Rankaj and Gambhir 1992). ANSYS is a powerful general-purpose finite element package, which may be used for the purpose.

4.2 FINITE ELEMENT MODELLING BY ANSYS

The ANSYS program, was introduced in 1970, by Swanson Analysis System (1995). Since that time ANSYS Supports Distributors have grown as part of a commitment to provide latest finite element analysis and design technology to engineers, worldwide. ANSYS capabilities can be utilized in computers that range from PCs to super main frames. ANSYS is a general-purpose program constantly updated with new features,

enhancements of existing features, and error corrections. The version of the program followed in this research work, is Revision 5.4.

4.2.1 Modal Analysis

Modal analysis helps the determination of the vibration characteristics of structure. It is used to determine the natural frequencies and mode shapes of a structure or its components. The natural frequencies and mode shapes of a structure are important parameters in the design of a structure for dynamic loading conditions. They are also required for spectrum analysis or mode superposition harmonic or transient analysis. Modal analysis in the ANSYS family of products is a linear type analysis. Any nonlinearity such as plasticity and contact (gap) elements are ignored even if they are defined. There are several mode extraction methods: Subspace, Block Lanczos, Power dynamics, Reduced, Unsymmetric and Damped. In this analysis Block Lanczos method is used. (Detailed procedures and steps can be seen in ANSYS Guide Manual)

4.2.2 Assumptions and Restrictions

The assumptions and restrictions in ANSYS are enumerated below :

- i) Valid for structural and fluid degrees of freedom
- ii) The structure has constant stiffness and mass effects
- iii) There is no damping unless the damped eigensolver is selected
- iv) The structure has no time varying forces, displacements, pressures, or temperature applied (that is, free vibration).

4.3 MODAL ANALYSIS PROCEDURE

The procedure of modal analysis consists of four main steps:

1. Building the model
2. Load application and solution
3. Expansion of modes
4. Reviewing the results

Building the Model: In this step the model geometry is defined. The structure, to be analyzed, has to be formulated as a model following the frame work of modal analysis.

Models developed must adequately characterize the actual response. The elements with which the model is made of, represent the actual components. ANSYS program has a large library of different types of element. Out of these large numbers of different elements, suitable elements for building up the required models are chosen. For modeling of three- dimensional frame structure the following elements are used in the present study:

Table 4.1: List of elements used in models for finite element analysis by ANSYS

Element Designation	Type of Element	Name of member of structure
Beam 4	3-D elastic beam	Columns
Beam 44	3-D tapered, unsymmetrical, elastic beam	Beams
Shell 63	Elastic shell	Slabs
COMBIN14	Spring-Damper	Infills (partition walls)
Mass 21	Structural mass	Non-structural dead load (Loads of infills and floor finish)

Load Application and Solution: In this step, the type of analysis, analysis option, load step options etc., is defined to initiate the finite element solution for natural frequencies. In this work modal analysis has been used as analysis type and reduced methods are used as analysis options.

Expansion of Modes: In the modal analysis, the term “expansion” means writing the mode shapes to the result file. This step is required to review the mode shapes in result file.

Reviewing the Results: In this step, the required parts of results are brought out for easy reference.

4.4 DESCRIPTION OF ELEMENTS:

4.4.1 Beam Element: Two types of beam element have been used. For modeling of column, beam-4 and for modeling of beams, beam-44 has been used.

Beam-4: Beam-4 element is an elastic, uni-axial, 3-dimensional element which can withstand tension, compressions, torsion and bending. The element has two nodes with six degrees of freedom at each node; translations in the nodal x, y, and z axes and rotations about the nodal x, y, z axes. The geometry, node locations, and the coordinate system for this element are shown in fig. 4.1. The element is defined by two or three nodes, the cross-sectional area, two area moments of inertia (IZZ and IYY), an angle of orientation (θ or ν) about the element x-axis, and the material properties.

Beam 44: Beam-44 is an elastic, uniaxial, 3-dimensional element, which can withstand tension, compressions, torsion and bending. The element has two nodes with six degrees of freedom at each node; translations in the nodal x, y, and z-axes and rotations about the nodal x, y, z axes. This element allows a different unsymmetrical geometry at each end and permits the end nodes to be offset from the centroidal axis of the beam. The geometry, node locations, and the coordinate system for this element are shown in fig. 4.2. The element is located by a reference coordinate system (x', y', z') and offsets. The reference system is defined by nodes I, J, K, or an orientation angle as shown in figure 4.2. The principal axes of the beam are in the element coordinate system (x, y, z) with x along the cross-section centroid (C.G).

4.4.2 Mass Element:

Mass element is a point element used as a structural mass on each nodal points of the model formulated. The node of mass element has got six

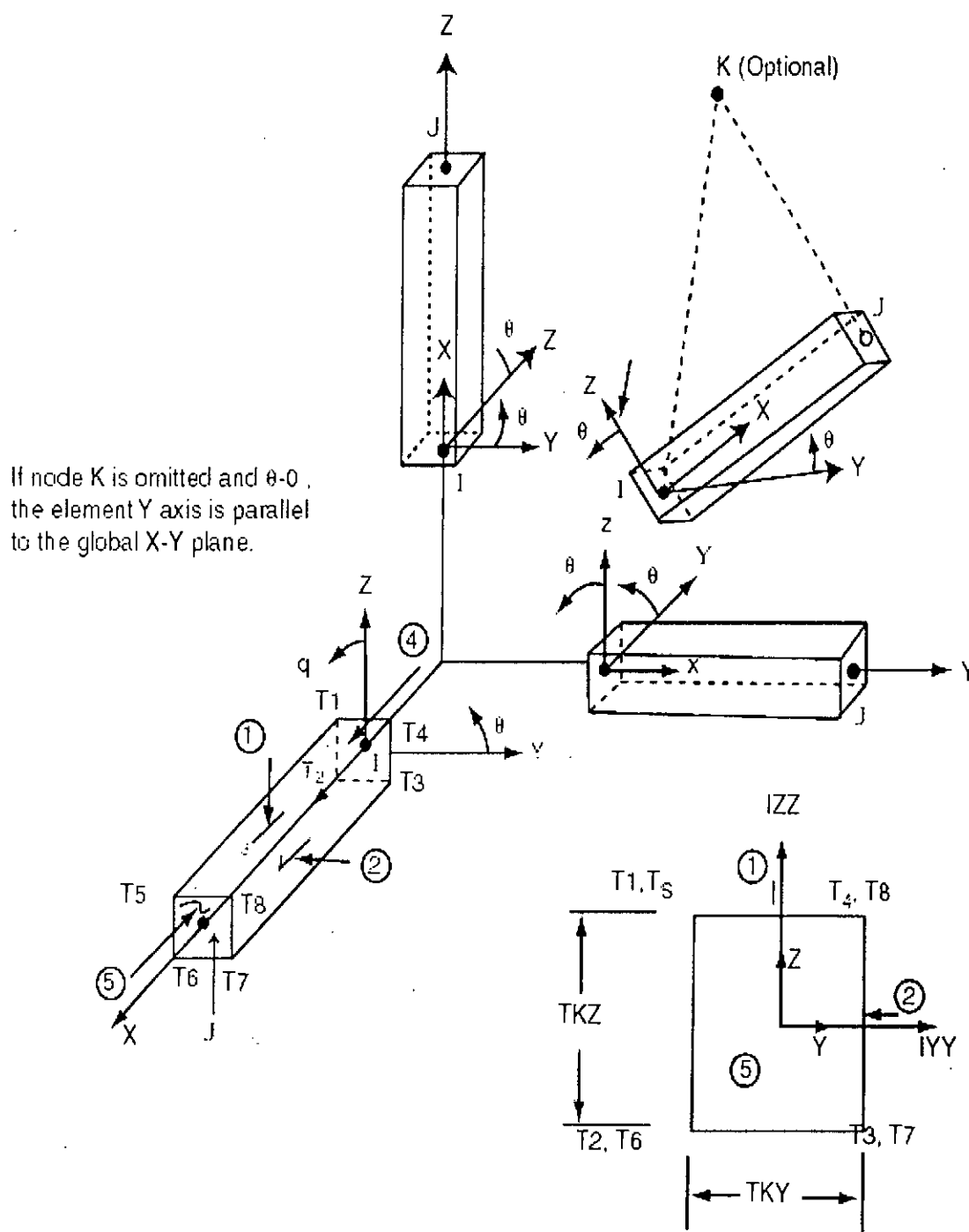


Fig. 4.1: 3-D elastic beam element (beam-4)

degrees of freedom: translations in the nodal x , y and z directions and rotations about nodal, x , y and z axes (Zienkiewicz, 1979). The geometry, node locations, and coordinate system, for typical mass element are given in fig.4.3. The vertical walls, widely used as partition walls in between

rooms of frame structure, are considered as mass elements. So mass elements actually represent the vertical wall loads at each nodal point of the frame structure. Also the loads of Floor Finish (F.F) are included as mass at nodal points.

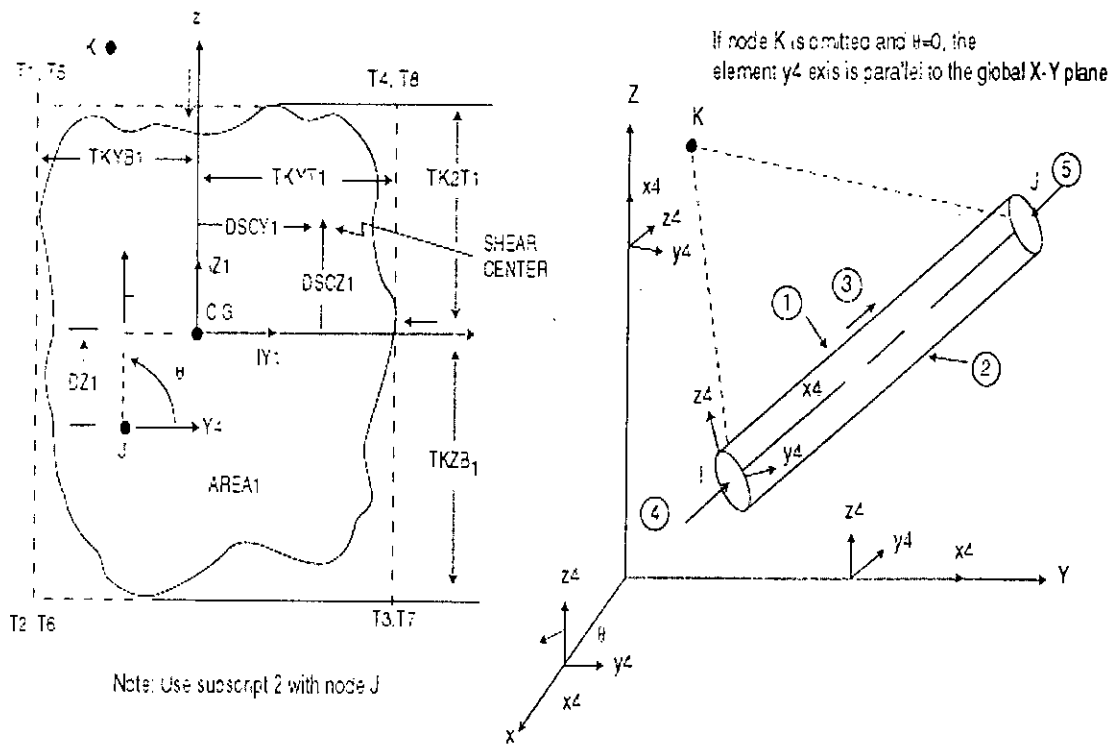


Fig. 4.2: 3-D tapered un-symmetric beam element (beam-44)

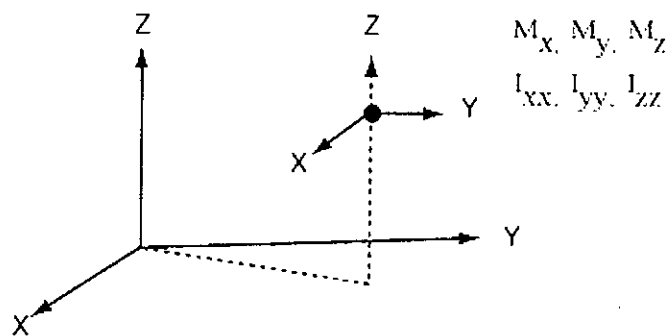
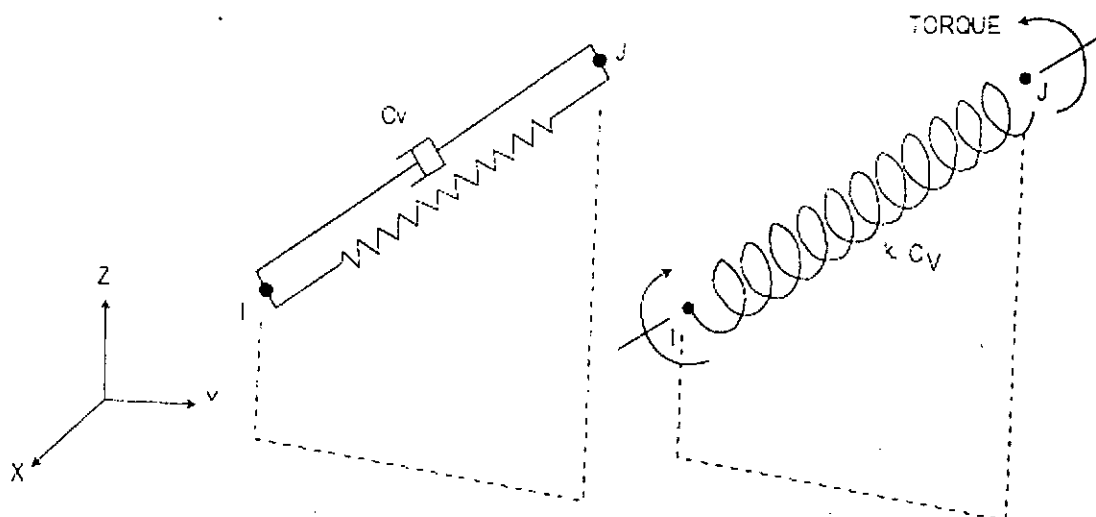


Fig. 4.3: 3-D structural mass element

4.4.4 COMBIN14 Spring -Damper:

The element has longitudinal or torsional capability in one, two, or three-dimensional applications. The longitudinal spring-damper option is a uniaxial tension-compression element with up to three degrees of freedom at each node; translations in the nodal x, y, and z directions. No bending or torsion is considered. The torsional spring-damper option is a purely rotational element with three degrees of freedom at each node; rotations about the nodal x, y, and z-axes. No bending or axial loads are considered. The longitudinal spring-damper option is adopted for modeling the infills as diagonal struts. The geometry, node locations, and the coordinate system for this element are shown in fig.4.5.



Note- Two-dimensional elements must lie in the X-Y plane

Fig. 4.5: COMBIN 14 Spring Damper

4.5 MODELING OF T BEAM:

All reinforced concrete floors are always monolithic. Forms are built for beam soffits and sides and for the underside of the slabs, and the entire construction is poured at once, from the bottom of the deepest beams to the top of the slab. The resulting beam cross section is T-shaped. When flat slab structure is modeled using plate element and ordinary beam element for the column line beams, plate elements are connected to the nodes of beams. These beam nodes are along the center line of the

beams. This type of modeling is not capable of representing the T beam action. In order to represent T beam behaviour properly it is necessary to have the beam element offset downward by appropriate amount. In ANSYS Beam 44 provides such facilities.

In order to determine the proper amount of offset, a study has been performed with several examples of T beam. Here, deflection is chosen as the criterion of comparison. Solution for maximum deflection is determined by finite element modeling using ANSYS. The results obtained from the above analysis are compared with the results obtained from analytical solution. The T beam modeling with different amount of offsets of beam 44 are described in the following sections.

4.5.1 Different amount of offsets of beam 44

Consider a T beam with thickness of slab t and the depth of beam (web portion) d . In the FE model, the slab portion is modeled using 4-noded plate/shell elements. The node point is considered at mid depth of the slab. Now the T beam may be constructed with different amount of offsets of beam-44. Three possible combination of slab and beam location are considered as described below.

Type 1: In this model, the top of beam is placed at bottom of the slab i.e., at $t/2$ distance below the node point. Here, the total offset of beam 44 is $t/2 + d/2$. This set is represented as T_1 .

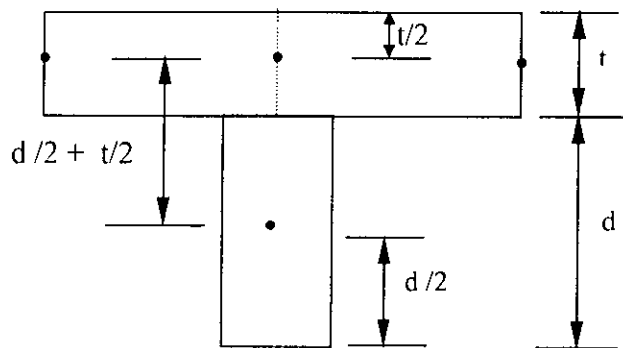
Type 2: The top of beam is placed at the middle of the slab i.e., at node point. The offset of beam 44 is $d/2$. This set is represented as T_2 .

Type 3: The top of beam is placed at the top of the slab i.e., at $t/2$ distance above the node point. Here, the offset of beam 44 is $d/2 - t/2$. This set is represented as T_3 .

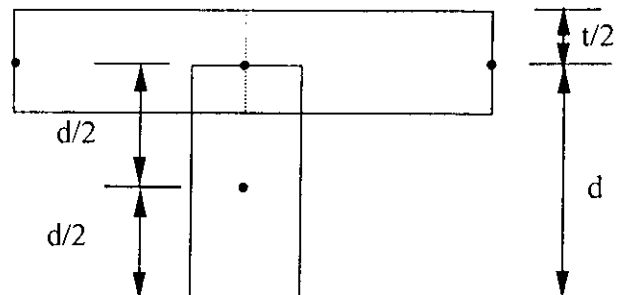
T beam sections with different amount of offsets are shown in figure 5.6.

4.5.2 MODEL EXAMPLES

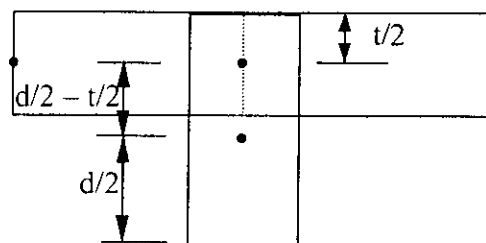
Now, we shall find the deflections of several model T beams using both analytical method and finite element analysis (with beam-4 and beam-44).



(a) Type 1: total offset is ' $d/2 + t/2$ '

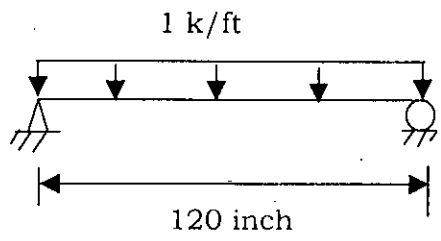


(b) Type 2: total offset is ' $d/2$ '

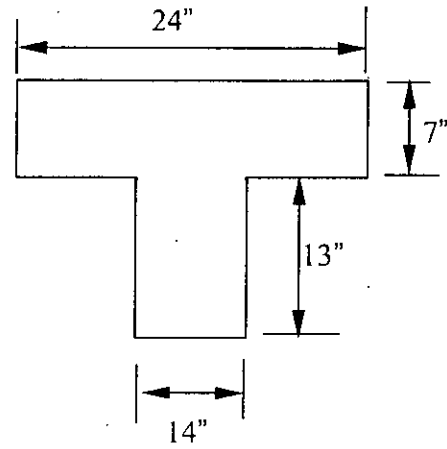


(c) Type 3: total offset is ' $d/2 - t/2$ '

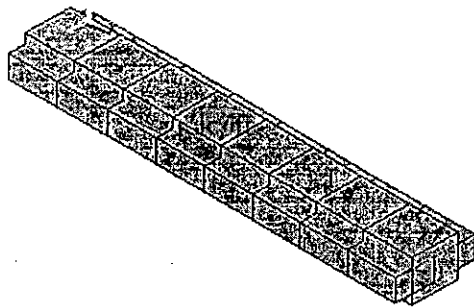
Fig 4.6 (a-c): T beam sections showing different amount of offsets of beam 44



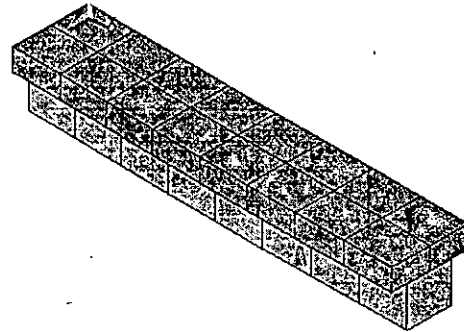
(a) T-beam example 1



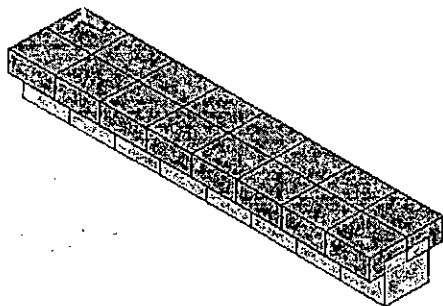
(b): T beam section of example 1



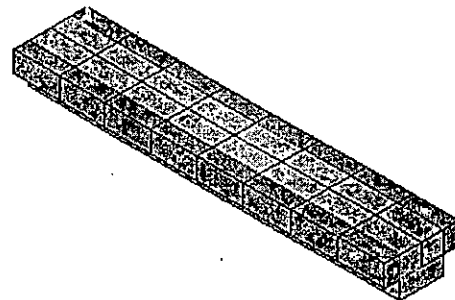
(c) Isometric view with beam 4



(d) Isometric view with beam 44 type 1



(e) Isometric view with beam 44 type 2



(f) Isometric view with beam 44 type 3

Fig 4.7(a-f): T beam of example 1 showing different offsets in beam 4 and beam 44 elements

Example 1: A simply supported T beam of 120 inch span with flange width 24 inch and thickness 7 inch, web width 14 inch, beam depth 20 inch. Given uniformly distributed load is 1000 #/ft and $E= 3 \times 10^6$ psi. Compute the vertical deflection.

Analytical Method: For simply supported beam with uniformly distributed load, the maximum deflection, Δ_{\max} may be calculated by the formula:

$$\Delta_{\max} = \frac{5}{384} \times \frac{\omega L^4}{EI}. \text{ The calculation for deflection is shown below.}$$

Flange width	Flange depth	Flange Area	Web width	Web depth	Web Area
24 inch	7 inch	168 inch ²	14 inch	13 inch	182 inch ²

Depth of neutral axis	Moment of inertia	Deflection
11.3 inch	11985 inch ⁴	0.0906 inch

Finite Element analysis: Deflection of example 1 by Finite Element analysis is calculated as follows:

Deflection using beam 4			
Area (total)	I_{zz}	I_{yy}	Deflection
350 inch ²	2563 inch ⁴	2972 inch ⁴	0.331 inch

Deflection using beam 44					
Set 1		Set 2		Set 3	
offset	ΔT_1	offset	ΔT_2	Offset	ΔT_3
10 inch	0.096 inch	6.5 inch	0.161 inch	3 inch	0.271 inch

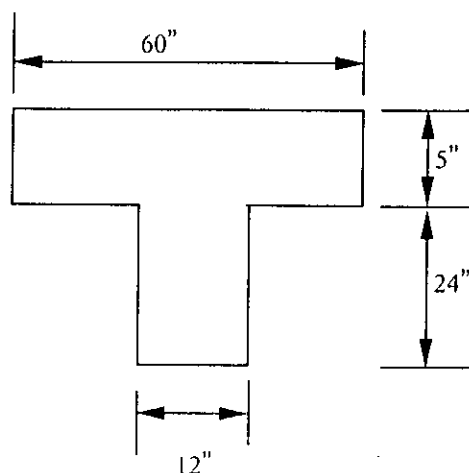


Fig 4.8: T beam section of example 2

Example 2: A simply supported T beam of 192 inch span with flange width 60 inch and thickness 5 inch, web width 12 inch, web depth 24 inch. Given UDL is 1000 #/ft and $E_c = 3 \times 10^6$ #/inch². Compute the vertical deflection. The T beam section is shown in fig.4.8.

The maximum deflection, Δ_{\max} is calculated following the same process as described in example 1.

1. Analytical Method:

Flange width	Flange depth	Flange Area	Web width	Web depth	Web Area
60 inch	5 inch	300 inch ²	12 inch	24 inch	288 inch ²
Depth of neutral axis		Moment of inertia		Deflection	
19.4 inch		45343 inch ⁴		0.1308 inch	

2. Finite Element Solution: Deflection of example 2 by Finite Element analysis is calculated as follows.

Deflection using beam 4			
Area (total)	I_{zz}	I_{yy}	Deflection
588 Inch ²	13824 Inch ⁴	3456 Inch ⁴	0.408 inch

Deflection using beam 44					
Set 1		Set 2		Set 3	
offset	ΔT_1	offset	ΔT_2	offset	ΔT_3
14.5 inch	0.142 inch	12 inch	0.178 inch	9.5 inch	0.225 inch

Example 3: A simply supported T beam of 240 inch span with flange width 60 inch and thickness 6 inch, web width 15 inch, web depth 30 inch. Given UDL is 1000 #/ft and $E_c = 3 \times 10^6$ #/inch². Compute the vertical deflection.

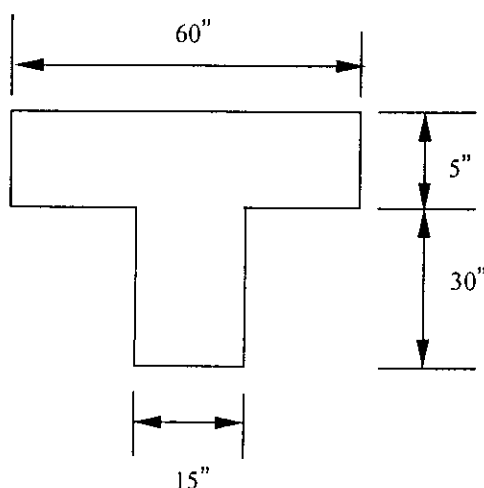


Fig 4.9: T beam section of example 3

The maximum deflection, Δ_{\max} is calculated following the same process as described in example 1 and 2.

1. Analytical Method:

Flange width	Flange depth	Flange Area	Web width	Web depth	Web Area
60 inch	6 inch	360 inch ²	15 inch	30 inch	450 inch ²

Depth of neutral axis	Moment of inertia	Deflection
23 inch	99630 inch ⁴	0.145 inch

2. Finite Element Solution: Deflection of example 2 by Finite Element analysis is as follows.

Deflection using beam 4					
Area (total)		I_{zz}	I_{yy}	Deflection	
750 inch ²		33750 inch ⁴	8438 inch ⁴	0.421 inch	
Deflection using beam 44					
Set 1		Set 2		Set 3	
Offset	ΔT_1	offset	ΔT_2	offset	ΔT_3
18 inch	0.153 inch	15 inch	0.189 inch	12 inch	0.235 inch

The results of T beam examples calculated by different methods are compared in following table (table 5.2) and in figure 5.10.

Table 4.2: Comparison of results of T beams

Example	Analytical Solution.	Deflection using beam 4	Deflection using beam 44		
			Set 1	Set 2	Set 3
1	0.0906	0.331 (265% higher)	0.096 (6.0% higher)	0.161 (78% higher)	0.271 (199% higher)
2	0.1308	0.408 (212% higher)	0.142 (6.5% higher)	0.178 (36% higher)	0.225 (72% higher)
3	0.145	0.421 (190% higher)	0.153 (5.5% higher)	0.189 (30% higher)	0.235 (62% higher)

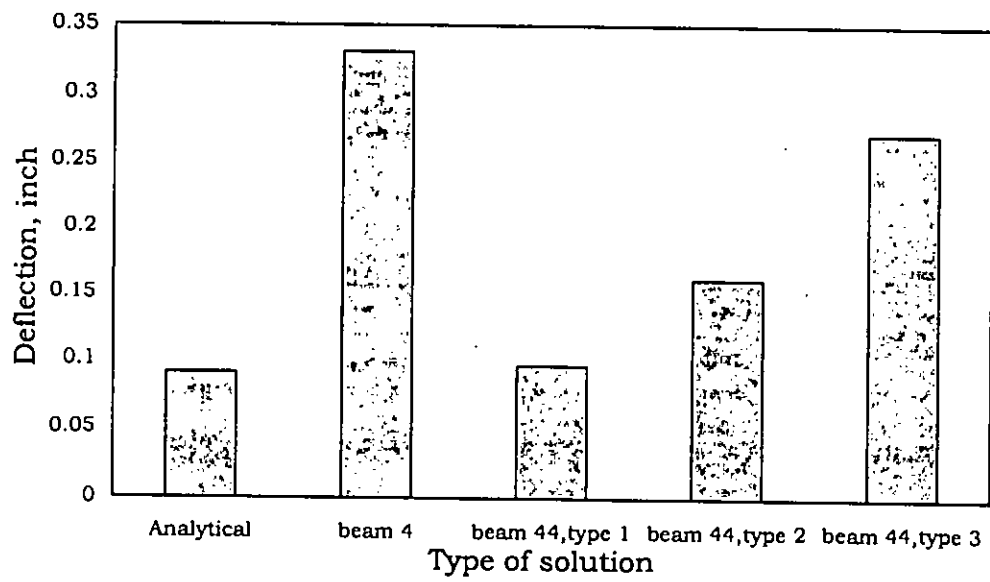


Fig.4.10(a) Comparison of deflections of different types for example 1

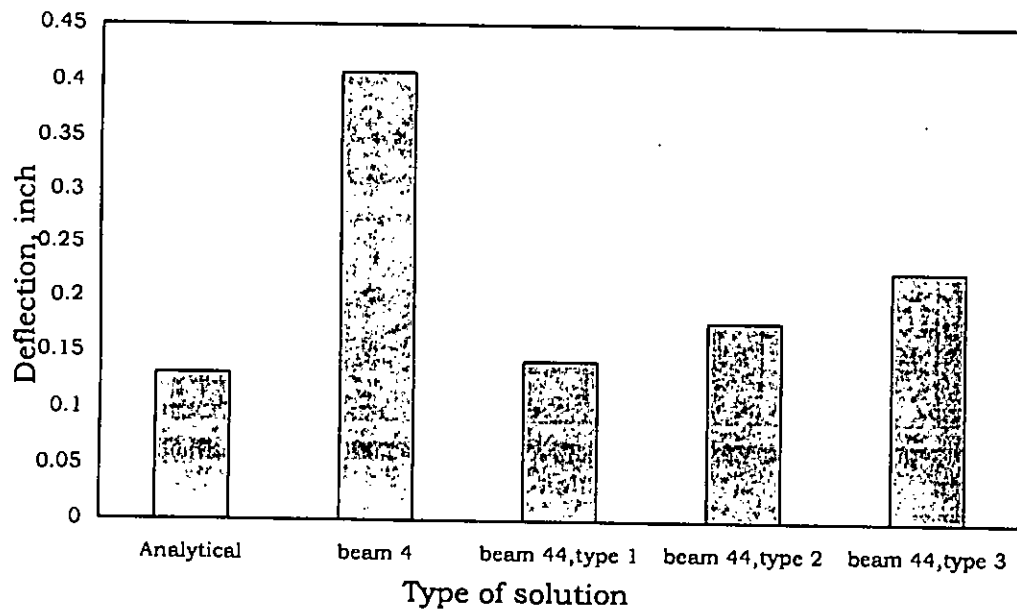


Fig.4.10.b Comparison of deflections of different types for example 2

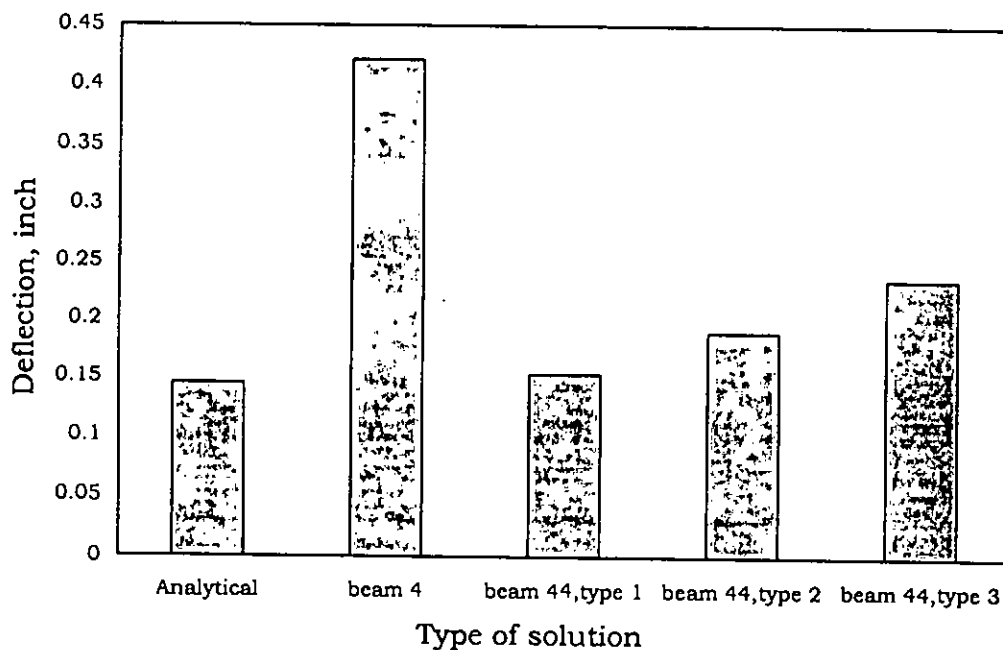


Fig.4.10.c Comparison of deflections of different types for example 3

4.5.2 Remarks:

From the table 4.2 and figure 4.10, following observations can be made.

- 1) The result of finite element modeling for T beam using beam 4 element significantly differs with the analytical solution.
- 2) The deflection using Beam-44 element with offset equal to the half of slab thickness plus web depth, that is, $(d/2 + t/2)$ is closest to the analytical solution. So in the modeling of frame structures in this thesis, Beam-44 element is used with offset equal to $(d/2 + t/2)$ to obtain the maximum accuracy in results.

4.6 REFERENCE MODEL FOR STUDY:

For a parametric study, a reference model of a 10-storied building of 4 x 4 bay was analyzed by ANSYS for determination of time period. The values of important parameters of the reference model are listed in Table 5.3 as shown below:

Table 4.3: Values of parameters of the generalized reference model

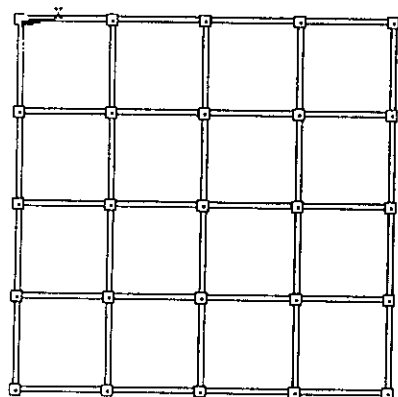
Sl. No.	Parameters	Values
1	Modulus of elasticity of concrete	2×10^4 N/mm ²
2	Density of concrete	2.4×10^{-9} ton/mm ³
3	Size of column	500 mm × 500 mm
4	Size of beam	350 mm × 500 mm
5	Number of story	10
6	Height of each story	3500 mm
7	Number of span and bays (along the direction of motion and its transverse direction)	4 × 4
8	Width of each bay	6000 mm
9	Thickness of slab	150 mm
10	Amount of infill (percentage)	40% of the panels
11	Thickness of infill	250 mm
12	Structural mass (Mass 21) at each node	9.6 ton
13	Acceleration due to gravity	9810 mm/sec ²
14	Equivalent strut stiffness, K_0	150255 N/mm

The 3-D view, plan and elevation of the prototype model with and without considering the presence of infill are shown in figure 4.11.

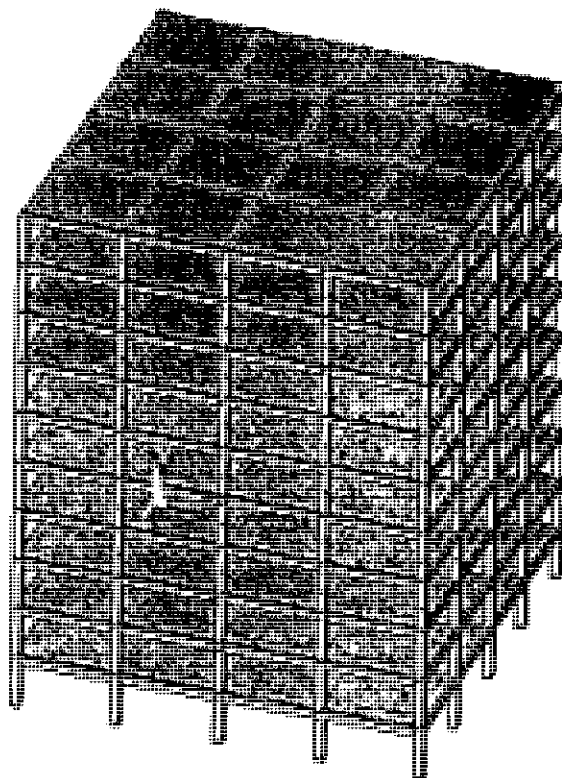
4.7 RESULTS OF REFERENCE MODEL:

The analysis of the reference model is performed by ANSYS using the procedure and steps mentioned in article 4.3. The results including the frequencies and mode shapes of the reference model are recorded.

Actually the number of mode shape is equal to number of (D.O.F). In mode shape lateral sway is considered and the time period corresponding to that mode shape is taken. The first few mode shapes for models considering and without considering the presence of infills are given below. The theory of frequency and mode shape analysis has been described in chapter 2 of this thesis.



(a) Plan of prototype model



(b) 3-d view of prototype model

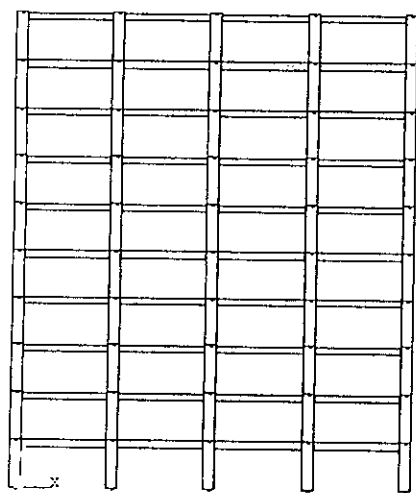
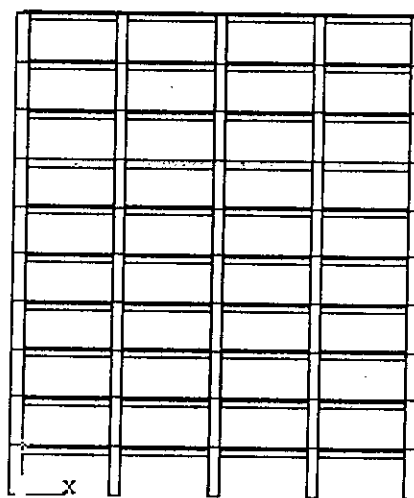
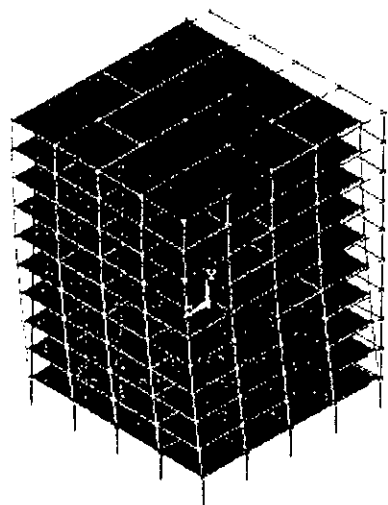
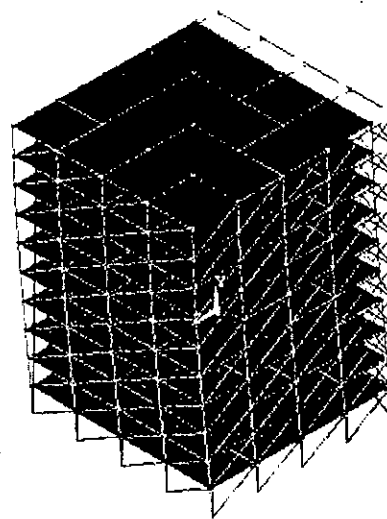
(c) Elevation of model Frame
(without infill))(d) Elevation of Frame (with infill)
Infills are shown as diagonal
COMBIN14 Spring Damper element

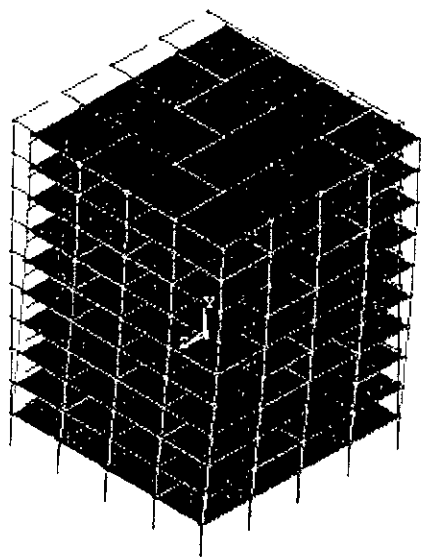
Fig 4.11: 3-d view, plan and elevation of prototype model.



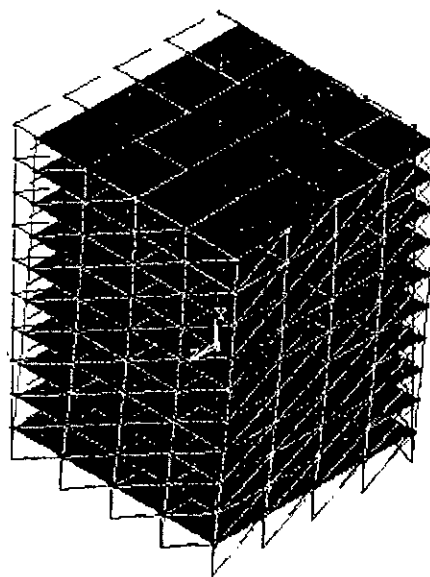
(a) 1st mode without infill
frequency=0.4719 Hz



(b) 1st mode with infill
frequency=0.9437 Hz

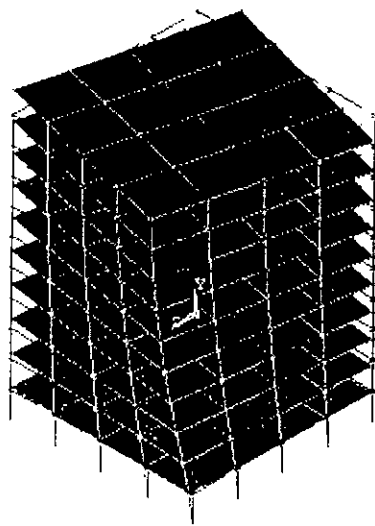


(c) 2nd mode without infill
frequency=0.4719 Hz

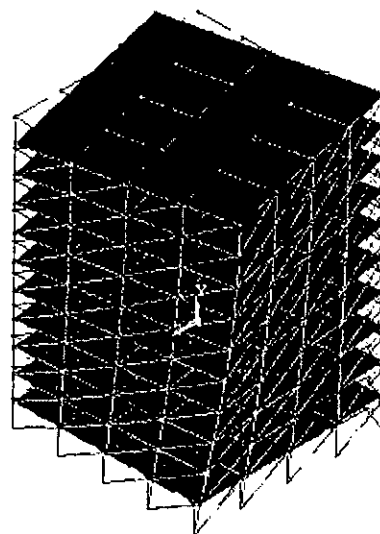


(d) 2nd mode with infill
frequency=0.9437 Hz

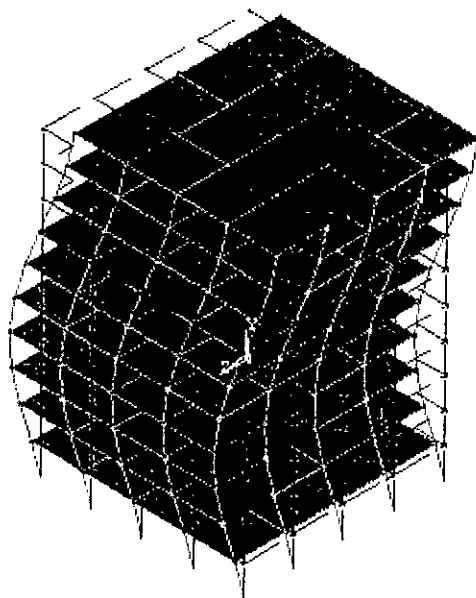
Fig 4.12 (a-d): Different mode shapes (1st and 2nd mode) of reference model



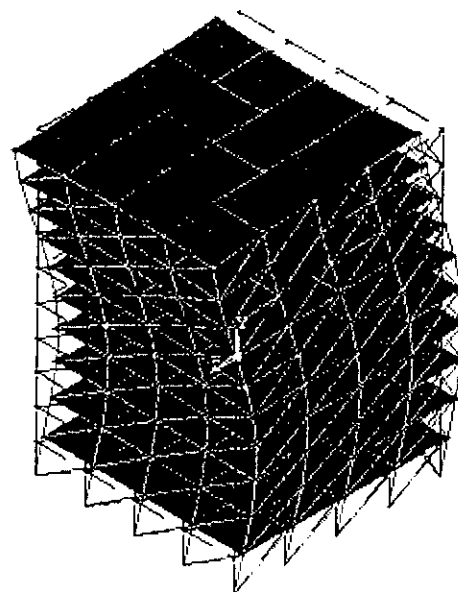
(e) 3rd mode without infill
frequency = .5018 Hz



(f) 3rd mode with infill
frequency=1.443 Hz

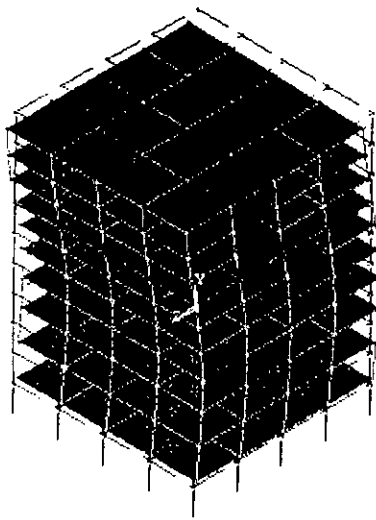


(g) 4th mode without infill
frequency =1.444 Hz

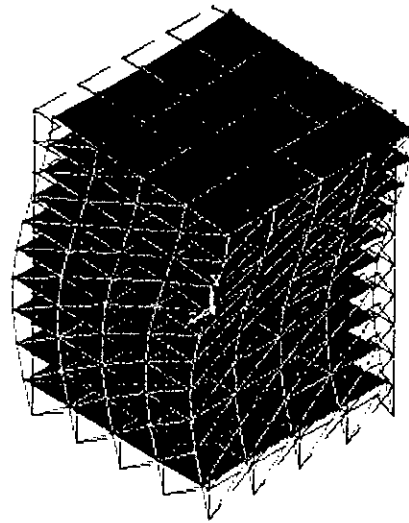


(h) 4th mode with infill
frequency=2.845 Hz

Fig 4.12 (e-h): Different mode shapes (3rd and 4th mode) of reference model



(i) 5th mode without infill
frequency = 1.444 Hz



(j) 5th mode with infill
frequency=2.845 Hz

Fig 4.12 (i-j): Different mode shapes (5th mode) of reference model

4.8 REMARKS :

In this chapter, it has been discussed how to use ANSYS and how to extract frequencies or time periods and mode shapes from modal analysis. The detailed parametric study will be discussed in next chapter.

CHAPTER 5

INFLUENCE OF STRUCTURAL PARAMETERS ON TIME PERIOD

5.1 INTRODUCTION:

The procedure and methodology for calculating the period of frame structures are described in the previous chapter. The empirical formulae proposed by different codes for calculating the period of regular frame structures are discussed in chapter 2. In order to ascertain the degree of influence of the structure parameters on period, a set of model frame structure are first selected. In selecting such models, care has been taken so that they can adequately reflect influence of different parameters. Series of models are designed so that influence of different parameters may be clearly established. In this chapter, the selected models are first described. The period of vibration of the models are then evaluated following the approximate code formulae and modal analysis technique. A detail parametric study is conducted further to identify influence of different parameters. The period of same models are calculated by different code formulae viz. (1) BNBC and UBC (1994), (2) NBC (1995), (3) IS (1984), (4) BSLJ (1987). Results of modal analysis are compared with those obtained from using approximate code formulae. Limitations and applicability of code formulae are established. It may be noted that the investigation carried out in this work applies to regular three-dimensional concrete frame structures only.

The models are also analyzed considering the structural effects of infill. Generally, the infills have been placed in outer panels, which cover 40% of total panels. The effect of infill is distinguished by comparing the results with the same for structures without infill.

5.2 STRUCTURAL PARAMETERS AND DIFFERENT MODELS :

Various numbers of regular three-dimensional model frame structures are selected. The values of parameters of reference model has been described in the article 4.6. The parameters are varied within a certain range which are normally found in frame structures. The variation of the parameters is listed in table 5.1.

Table 5.1: Values of structural parameters used in different models

Sl. No	Parameters	Values	Reference Value
1	Size of column	300×300, 400×400, 500×500, 600×600, 700×700 mm ²	500 ×500 mm ²
2	Number of story	4,7,10,13,16	10
3	Floor Panel size	4000,5000,6000,7000,8000 mm	6000mm
4	Story height	2500, 3000, 3500, 4000, 4500 mm	3500 mm
5	Size of beam	350×300, 350×400, 350×500, 350×600, 350*700 mm ²	350×500 mm ²
6	Number of bay	2, 3, 4, 5, 6	4
7	Number of span	2, 3, 4, 5, 6	4
8	Amount of Infill	20%, 40%, 60%, 80%	40%

On the basis of different structural parameters mentioned above, the models are divided in eight major groups. All the models are analyzed separately. The time period of models for different values of parameters are recorded. For each model five mode shapes are observed. The different groups on the basis of variation of structure parameters are as follows:

5.2.1 Description of different models of Group A:

Group A represents the effect on time period with variation of column stiffness, that is column sizes. In this group, five models of varying column sizes have been analyzed without considering the effect of infill and again

those five models were analyzed considering the effect of infill. In the analysis of model of this group the other parameters are kept at their reference value (table 5.1). The different column sizes are 300mm×300mm, 400mm×400mm, 500mm×500mm, 600mm×600mm, 700mm×700mm and the corresponding models are represented symbolically as A1, A2, A3, A4, A5 respectively. The models considering the effect of infill of this group are represented by A1k, A2k, A3k, A4k, A5k accordingly.

The models of this group are divided in 5(five) subgroups. The subgroups and different models are described below.

Subgroup A1: Consists of two models, namely, model A1 and A1k.

Model A1 is composed of frame structures with 4×4 bay. The column size in this model is 300mm×300mm, providing cross-sectional area = 90000mm² and moment of inertia $I_{yy} = I_{zz} = 6.75 \times 10^8 \text{ mm}^4$. Mass of the infill is considered but, the structural effect of infill is not considered in this model. The other parameters of the model (Reference model) described in article 4.6 are remained unchanged. Model A1k is same as Model A1 except the structural effect of infill is considered. To represent the structural effect of infill the "COMBIN 14 Spring Damper" element is used. The value of "Equivalent Strut Stiffness", K_0 is calculated for each model. For model A1k, the value of K_0 is 149823 N/mm. The example for calculation of K_0 is shown in article 4.6. The plan configuration of model A1 and A1k is shown in figure 5.1(a).

Subgroup A2: Consists of two models, namely, model A2 and A2k.

Model A2 is same as model A1 except the column size in this model is 400mm×400mm, providing cross-sectional area = 160000mm² and moment of inertia $I_{yy} = I_{zz} = 2.133 \times 10^9 \text{ mm}^4$. Model A2k is same as Model A2 except the structural effect of infill is considered. For model A2k, the value of K_0 is 150464 N/mm. The plan configuration of model A2 and A2k is shown in figure 5.1(b).

Subgroup A3: Consists of two models, namely, model A3 and A3k

Model A3 is same as model A1 except the column size in this model is 500mm×500mm, providing cross-sectional area = 250000mm² and moment of inertia $I_{yy} = I_{zz} = 5.21 \times 10^9 \text{ mm}^4$. Model A3k is same as Model A3 except the structural effect of infill is considered. For model A3k, the value of K_0 is

150255 N/mm. The plan configuration of model A3 and A3k is shown in figure 5.1(c).

Subgroup A4: Consists of two models, namely, model A4 and A4k

Model A4 is same as model A1 except the column size in this model is 600mm×600mm, providing cross-sectional area = 360000mm² and moment of inertia $I_{yy} = I_{zz} = 1.08 \times 10^{10}$ mm⁴. Model A4k is same as Model A4 except the structural effect of infill is considered. For model A4k, the value of K_0 is 149936 N/mm. The plan configuration of model A4 and A4k is shown in figure 5.1(d).

Subgroup A5: Consists of two models, namely, model A5 and A5k

Model A5 is same as model A1 except the column size in this model is 700mm×700mm, providing cross-sectional area = 490000mm² and moment of inertia $I_{yy} = I_{zz} = 2.01 \times 10^{10}$ mm⁴. Model A5k is same as Model A5 except the structural effect of infill is considered. For model A5k, the value of K_0 is 149601 N/mm. The plan configuration of model A5 and A5k is shown in figure 5.1(e).

5.2.2 Description of different models of Group B:

This group represents the effect on time period with variation in number of story. Five models have been analyzed with variation of number of story without considering the effect of infill and the same models were analyzed considering the effect of infill. Also calculations have been made to calculate the time periods of those five models by the empirical formulae described in chapter 2. The numbers of story considered are 4, 7, 10, 13, and 16. The different models of this group are symbolically represented by B1, B2, B3, B4, and B5 respectively. The models considering the effect of infill of this group are represented by B1k, B2k, B3k, B4k, and B5k accordingly.

Subgroup B1: Consists of two models, namely, model B1 and B1k

Model B1 is composed of frame structures with 4×4 bay. The number of story in this model is 4. Mass of the infill is considered, but the structural

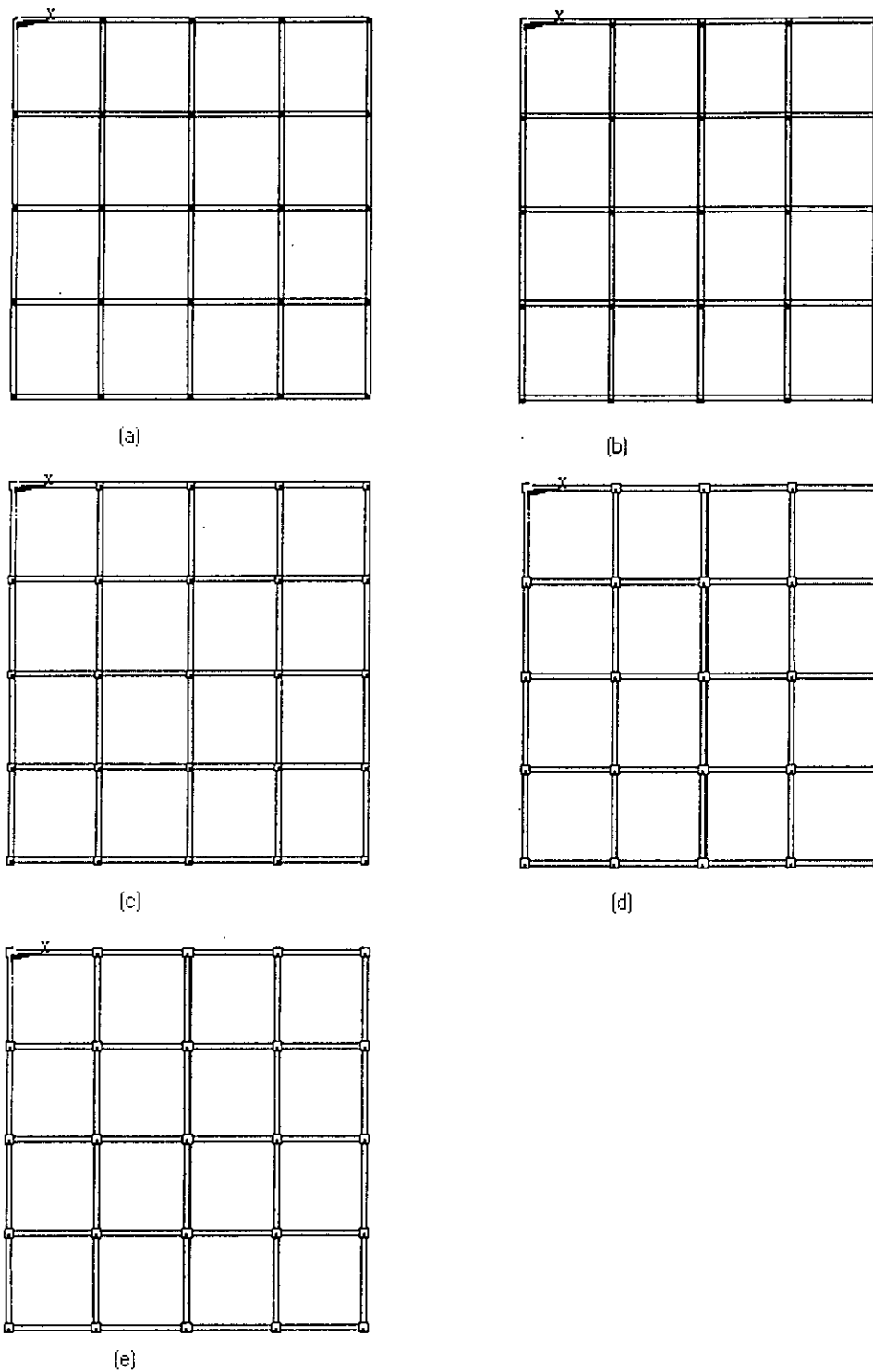


Fig. 5.1 Plan of different models of group A showing variation in column stiffness. (a) Model A1, (b) Model A2, (c) Model A3, (d) Model A4, (e) Model A5.

effect of infill is not considered in this model. The other parameters have been kept at their reference value. Model B1k is same with Model B1 except the structural effect of infill is considered. The "Equivalent Strut Stiffness", K_0 is calculated for this model. The value of K_0 is found as 150255 N/mm. To compare with the number of story of other subgroup models the elevation of the models B1 and B1k shown in figure 5.2(a).

Subgroup B2: Consists of two models, namely, model B2 and B2k.

Model B2 is composed of frame structures with 4×4 bay. The number of story in this model is 7. Mass of the infill is considered, but the structural effect of infill is not considered in this model. The other parameters have been kept at their reference value. Model B2k is same with Model B2 except the structural effect of infill is considered. The value of K_0 for this model is 150255 N/mm. The elevation of models of this subgroup is shown in figure 5.2(b).

Subgroup B3: Consists of two models, namely, model B3 and B3k

Model B3 is composed of frame structures with 4×4 bay. The number of story in this model is 10. Mass of the infill is considered, but the structural effect of infill is not considered in this model. The other parameters have been kept at their reference value. Model B3k is same with Model B3 except the structural effect of infill is considered. The value of K_0 for this model is 150255 N/mm. The elevation of models of this subgroup is shown in figure 5.2(c).

Subgroup B4: Consists of two models, namely, model B4 and B4k

Model B4 is composed of frame structures with 4×4 bay. The number of story in this model is 13. Mass of the infill is considered, but the structural effect of infill is not considered in this model. The other parameters have been kept at their reference value. Model B4k is same with Model B4 except the structural effect of infill is considered. The value of K_0 for this model is 150255 N/mm. The elevation of models of this subgroup is shown in figure 5.2(d).

Subgroup B5: Consists of two models , namely, model B5 and B5k

Model B5 is composed of frame structures with 4×4 bay. The number of story in this model is 16. Mass of the infill is considered, but the structural effect of infill is not considered in this model. The other parameters have been kept at their reference value. Model B5k is same with Model B5 except the structural effect of infill is considered. The value of K_0 for this model is 150255 N/mm. The elevation of models of this subgroup is shown in figure 5.2(e).

5.2.3 Description of different models of Group C:

This group represents the effect on time period with variation in panel size. Five models have been analyzed with variation in bay width without considering the effect of infill and the same models have been analyzed considering the effect of infill. Also calculations have been made to calculate the time periods of those five models by the empirical formulae described in chapter 2. The panel size of different models of this group are considered as 4000mm, 5000mm, 6000mm, 7000mm, 8000mm. The different models of this group are symbolically represented by C1, C2, C3, C4, C5 respectively. The models considering the effect of infill of this group are represented by C1k, C2k, C3k, C4k, C5k accordingly.

Subgroup C1: Consists of two models, namely, model C1 and C1k

Model C1 is composed of frame structures with 4×4 bay. The panel size of this model is 4000mm. Mass of the infill is considered but the structural effect of infill is not considered in this model. The other parameters have been kept at their reference value. Model C1k is same with Model C1 except the structural effect of infill is considered. The “Equivalent Strut Stiffness”, K_0 is calculated for this model. The value of K_0 for this model is found as 139071 N/mm. The plan configuration of the model of this subgroup is shown in figure 5.3(a).

Subgroup C2: Consists of two models, namely, model C2 and C2k

Model C2 is composed of frame structures with 4×4 bay. The panel size of this model is 5000mm. Mass of the infill is considered but the structural effect of infill is not considered in this model. The other parameters have been kept at their reference value. Model C2k is same with Model C2 except

the structural effect of infill is considered. The value of K_0 for this model is found as 146201 N/mm. The plan configuration of the model of this subgroup is shown in figure 5.3(b).

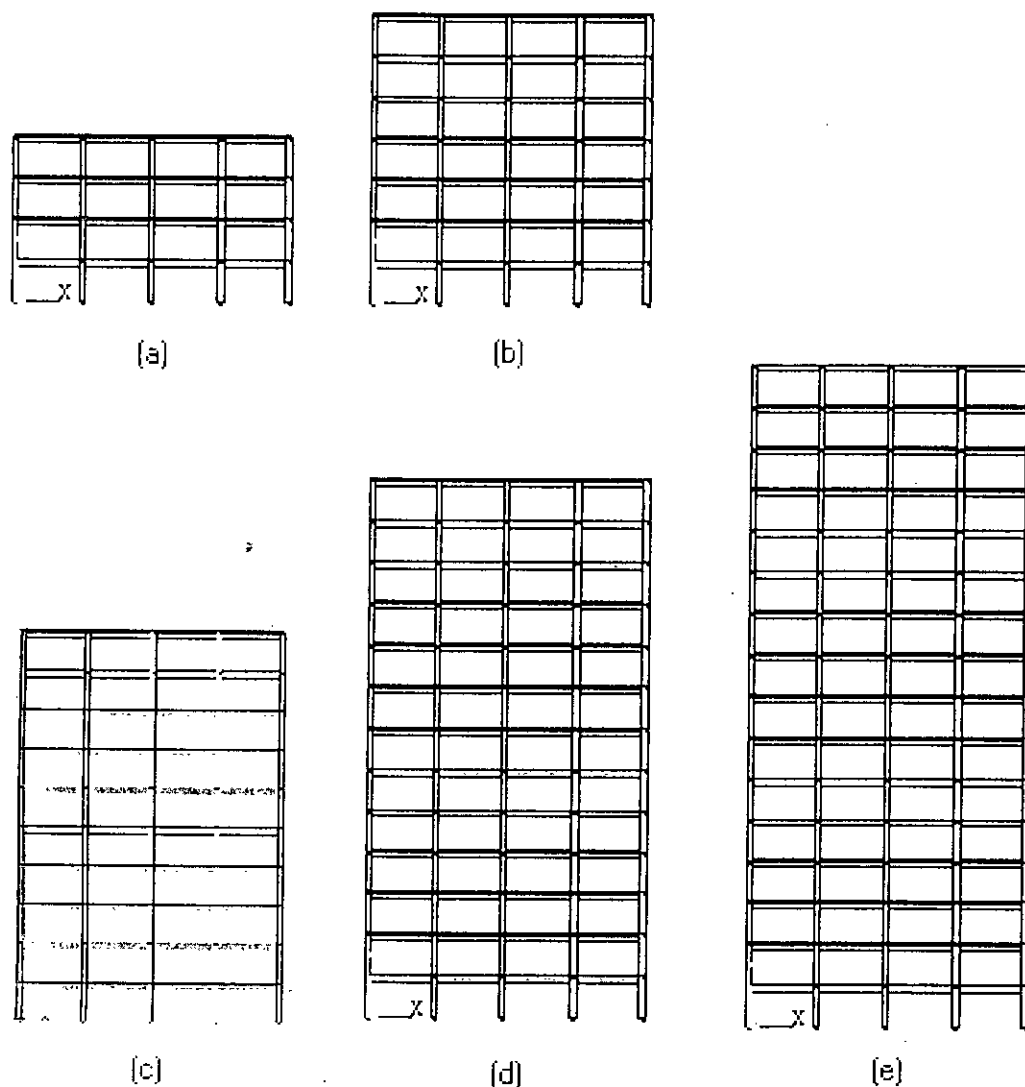


Fig.5.2 Elevation of different models of group B showing variation in number of story. (a) Model B1, (b) Model B2, (c) Model B3, (d) Model B4, (e) Model B5.

Subgroup C3: Consists of two models, namely, model C3 and C3k

Model C3 is composed of frame structures with 4×4 bay. The panel size of this model is 6000mm. Mass of the infill is considered but the structural

effect of infill is not considered in this model. The other parameters have been kept at their reference value. Model C3k is same with Model C3 except the structural effect of infill is considered. The value of K_0 for this model is found as 150255 N/mm. The plan configuration of the model of this subgroup is shown in figure 5.3(c).

Subgroup C4: Consists of two models, namely, model C4 and C4k

Model C4 is composed of frame structures with 4×4 bay. The panel size of this model is 7000mm. Mass of the infill is considered but the structural effect of infill is not considered in this model. The other parameters have been kept at their reference value. Model C4k is same with Model C4 except the structural effect of infill is considered. The value of K_0 for this model is found as 152745 N/mm. The plan configuration of the model of this subgroup is shown in figure 5.3(d).

Subgroup C5: Consists of two models, namely, model C5 and C5k

Model C5 is composed of frame structures with 4×4 bay. The panel size of this model is 8000mm. Mass of the infill is considered but the structural effect of infill is not considered in this model. The other parameters have been kept at their reference value. Model C5k is same with Model C5 except the structural effect of infill is considered. The value of K_0 for this model is found as 154371 N/mm. The plan configuration of the model of this subgroup is shown in figure 5.3(e).

5.2.4 Description of different models of Group - D

This group represents the effect on time period with variation in floor height. Five models have been analyzed with variation in floor height without considering the effect of infill and the same models have been analyzed with considering the effect of infill. Also calculations have been made to calculate the time periods of those five models by the empirical formulae described in chapter 2. The floor height of different models of this group are considered as 2500mm, 3000mm, 3500mm, 4000mm, 4500mm. The different models of this group are symbolically represented by

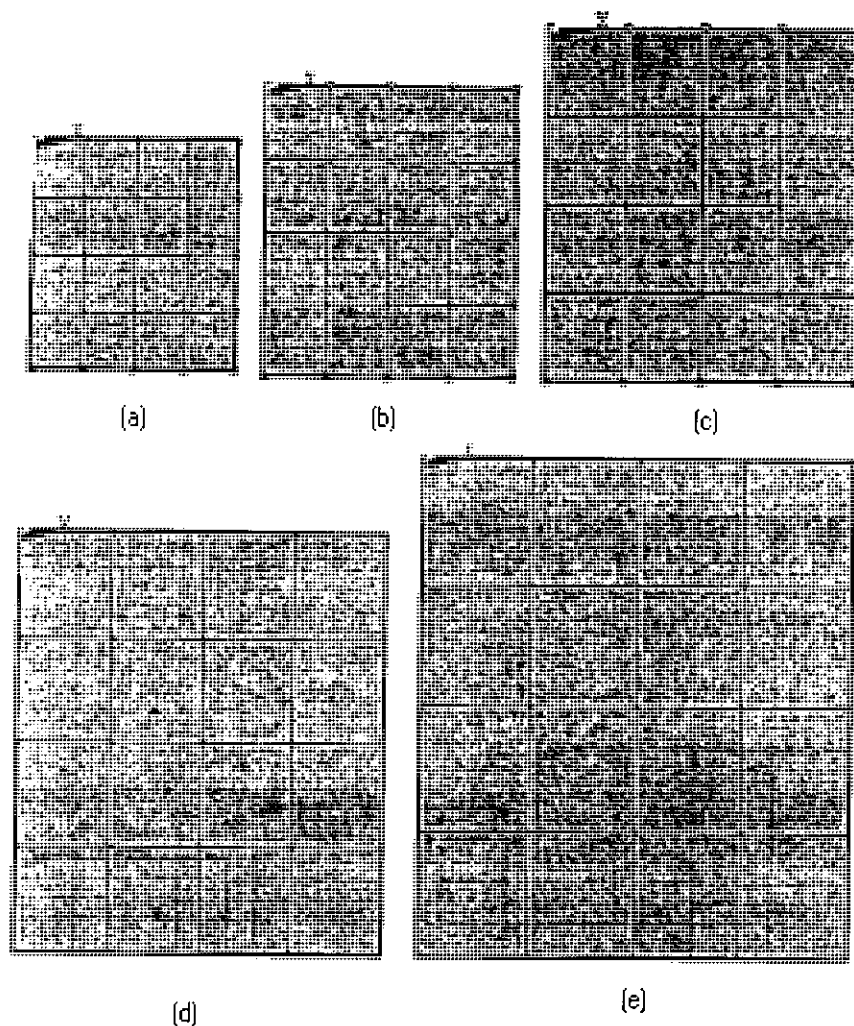


Fig. 5.3: Plan of different models of group C showing variation of panel size (a) Model C1, (b) Model C2, (c) Model C3, (d) Model C4, (e) Model C5.

D1, D2, D3, D4, D5 respectively. The models considering the effect of infill of this group are represented by D1k, D2k, D3k, D4k, and D5k accordingly.

Subgroup D1: Consists of two models, namely, model D1 and D1k.

Model D1 is composed of frame structures with 4x4 bay. The floor height of the structure in this model is 2500mm. Mass of the infill is considered but the structural effect of infill is not considered in this model. The other parameters have been kept at their reference value. Model D1k is same with Model D1 except the structural effect of infill is considered. The value of K_0

is for this model is calculated as 154949 N/mm. The elevation of models of this subgroup is shown in figure 5.4(a).

Subgroup D2: Consists of two models, namely, model D2 and D2k.

Model D2 is composed of frame structures with 4×4 bay. The floor height of the structure in this model is 3000mm. Mass of the infill is considered but the structural effect of infill is not considered in this model. The other parameters have been kept at their reference value. Model D2k is same with Model D2 except the structural effect of infill is considered. The value of K_0 is for this model is calculated as 152756 N/mm. The elevation of models of this subgroup is shown in figure 5.4(b).

Subgroup D3: Consists of two models; namely, model D3 and D3k.

Model D3 is composed of frame structures with 4×4 bay. The floor height of the structure in this model is 3500mm. Mass of the infill is considered but the structural effect of infill is not considered in this model. The other parameters have been kept at their reference value. Model D3k is same with model D3 except the structural effect of infill is considered. The value of K_0 is for this model is calculated as 150255 N/mm. The elevation of models of this subgroup is shown in figure 5.4(c).

Subgroup D4: Consists of two models; namely, model D4 and D4k.

Model D4 is composed of frame structures with 4×4 bay. The floor height of the structure in this model is 4000mm. Mass of the infill is considered but the structural effect of infill is not considered in this model. The other parameters have been kept at their reference value. Model D4k is same with Model D4 except the structural effect of infill is considered. The value of K_0 is for this model is calculated as 147496 N/mm. The elevation of models of this subgroup is shown in figure 5.4(d).

Subgroup D5: Consists of two models; namely, model D5 and D5k

Model D5 is composed of frame structures with 4×4 bay. The floor height of the structure in this model is 4500mm. Mass of the infill is considered but the structural effect of infill is not considered in this model. The other parameters have been kept at their reference value. Model D5k is same with Model D5 except the structural effect of infill is considered. The value of K_0

is for this model is calculated as 143521 N/mm. The elevation of models of this subgroup is shown in figure 5.4(e).

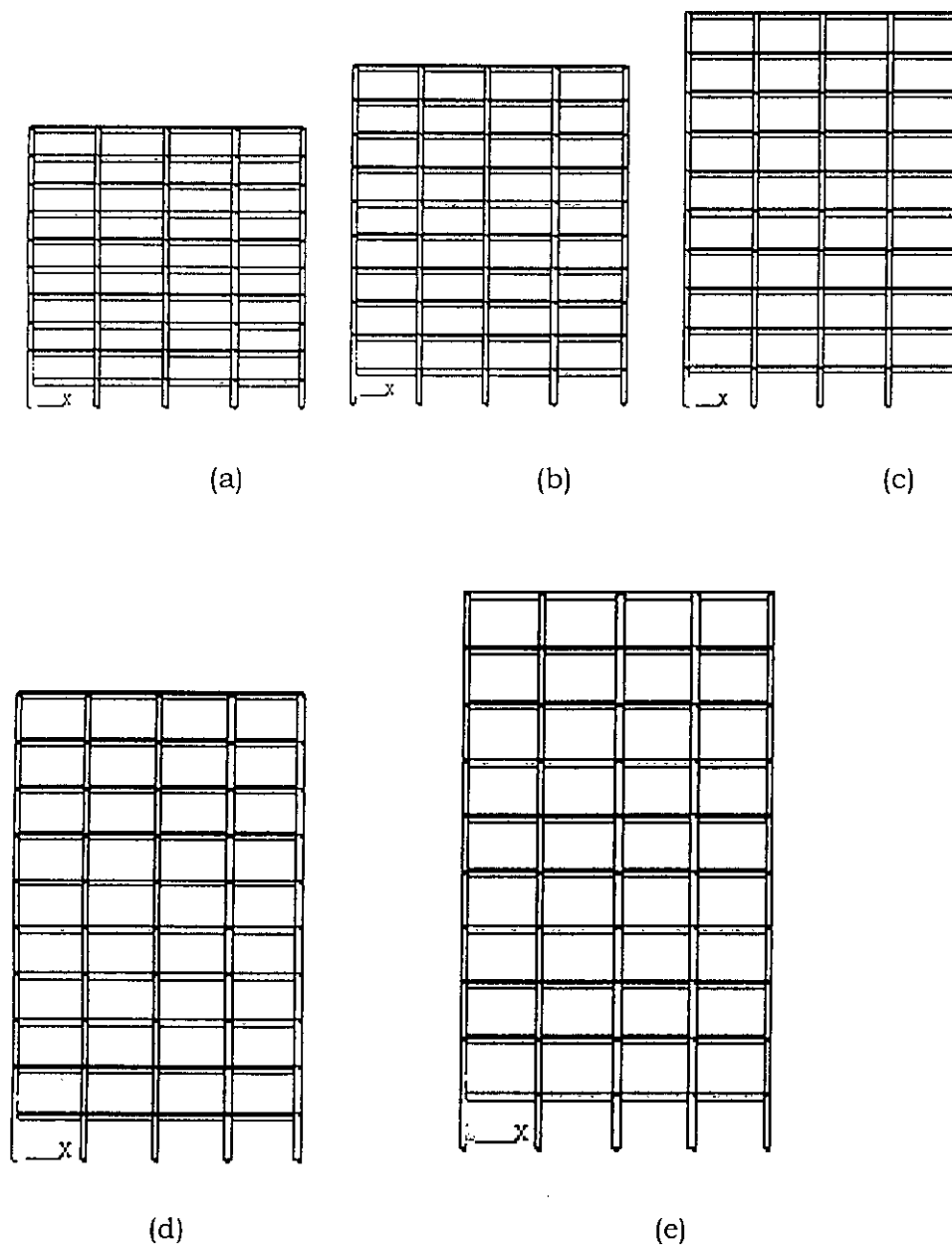


Fig.5.4: Elevation of different models of group D showing variation of floor height. (a) Model D1, (b) Model D2, (c) Model D3, (d) Model D4, (e) Model D5.

5.2.5 Description of different models of Group E

Group E represents the effect on time period with variation of beam stiffness, that is beam sizes. In this group, five models of varying column

sizes have been analyzed without considering the effect of infill and again those five models were analyzed with considering the effect of infill. In the analysis of model of this group the other parameters are kept at their reference value. The different beam sizes are 300mm×350mm, 350mm×400mm, 350mm×500mm, 350mm×600mm, 350mm×700mm and the corresponding models are represented symbolically as E1, E2, E3, E4, E5 respectively. The models considering the effect of infill of this group are represented by E1k, E2k, E3k, E4k, and E5k accordingly. The models of this group are divided in 5(five) subgroups. The subgroups and different models are described below.

Subgroup E1: Consists of two models, namely, model E1 and E1k.

Model E1 is composed of frame structures with 4×4 bay. The beam size in this model is 350mm×300mm, providing cross-sectional area=105000mm² and moment of inertia, $I_{yy} = 1.429 \times 10^9 \text{ mm}^4$, $I_{zz} = 7.875 \times 10^8 \text{ mm}^4$. Mass of the infill is considered but the structural effect of infill is not considered in this model. Model E1k is same with Model E1 except the structural effect of infill is considered. The "Equivalent Strut Stiffness", K_0 is calculated for this model. The value of K_0 is 150224 N/mm. The elevation of models of this subgroup is shown in figure 5.5(a).

Subgroup E2: Consists of two models; namely, model E2 and E2k.

Model E2 is composed of frame structures with 4×4 bay. The beam size in this model is 350mm×400mm, providing cross-sectional area=140000mm² and moment of inertia $I_{yy} = 1.429 \times 10^9 \text{ mm}^4$, $I_{zz} = 1.867 \times 10^9 \text{ mm}^4$. Mass of the infill is considered but, the structural effect of infill is not considered in this model. Model E2k is same with model E2 except the structural effect of infill is considered. The elevation of models of this subgroup is shown in figure 5.5(b).

Subgroup E3: Consists of two models; namely, model E3 and E3k.

Model E3 is composed of frame structures with 4×4 bay. The beam size in this model is 350mm×500mm., providing cross-sectional area=175000mm² and moment of inertia $I_{yy} = 1.786 \times 10^9 \text{ mm}^4$, $I_{zz} = 3.646 \times 10^9 \text{ mm}^4$. Mass of the infill is considered but, the structural effect of infill is not considered in this model. Model E3k is same with model E3 except the structural effect

of infill is considered. The value of K_0 is calculated as 150255 N/mm for this model. The elevation of models of this subgroup is shown in figure 5.5(c).

Subgroup E4: Consists of two models; namely, model E4 and E4k.

Model E4 is composed of frame structures with 4×4 bay. The beam size in this model is 350mm×600mm, providing cross-sectional area=210000mm² and moment of inertia, $I_{yy} = 2.144 \times 10^9 \text{ mm}^4$ and $I_{zz} = 6.3 \times 10^9 \text{ mm}^4$. Mass of the infill is considered but the structural effect of infill is not considered in this model. Model E4k is same with model E4 except the structural effect of infill is considered. The value of K_0 is 150255 N/mm. The elevation of models of this subgroup is shown in figure 5.5(d).

Subgroup E5: Consists of two models; namely, model E5 and E5k.

Model E5 is composed of frame structures with 4×4 bay. The beam size in this model is 350mm×700mm., providing cross-sectional area=245000mm² and moment of inertia, $I_{yy} = 2.50 \times 10^9 \text{ mm}^4$ and $I_{zz} = 1.0 \times 10^{10} \text{ mm}^4$. Mass of the infill is considered but the structural effect of infill is not considered in this model. Model E5k is same with Model E5 except the structural effect of infill is considered. The value of K_0 is 150255 N/mm. The elevation of models of this subgroup is shown in figure 5.5(e).

5.2.6 Description of different models of Group F

This group represents the effect on time period with variation in number of bay (along the direction transverse to motion). Five models have been analyzed with variation of number of bay without considering the effect of infill and the same models were analyzed considering the effect of infill. Also calculations have been made to calculate the time periods of those five models by the empirical formulae described in chapter 2. The numbers of bay considered are 2, 3, 4, 5, 6. The different models of this group are symbolically represented by F1, F2, F3, F4, F5 respectively. The models considering the effect of infill of this group are represented by F1k, F2k, F3k, F4k, and F5k accordingly. The models of this group are divided in 5(five) subgroups as described below.

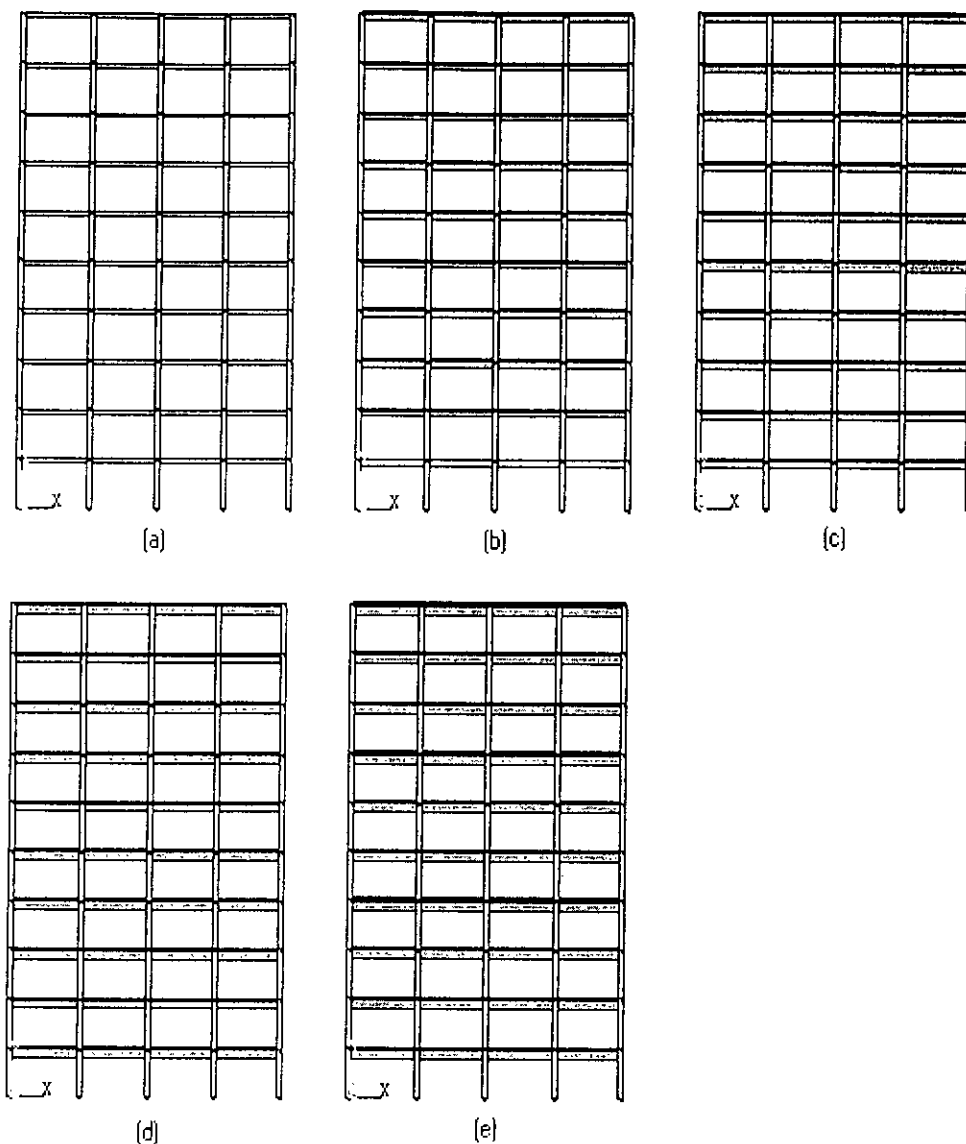


Fig.5.5 Elevation of different models of group E showing variation in beam stiffness (a)Model E1, (b)Model E2, (c)Model E3, (d)Model E4, (e)Model E5.

Subgroup F1: Consists of two models, namely, model F1 and F1k.

Model F1 is composed of frame structures with 2×4 bay. In this model, the number of bay along the transverse direction of motion is 2. Mass of the infill is considered but, the structural effect of infill is not considered in this model. Model F1k is same with model F1 except the structural effect of infill is considered. The value of K_0 is calculated as 150255 N/mm. The plan configuration of the models of this subgroup is shown in figure 5.6(a).

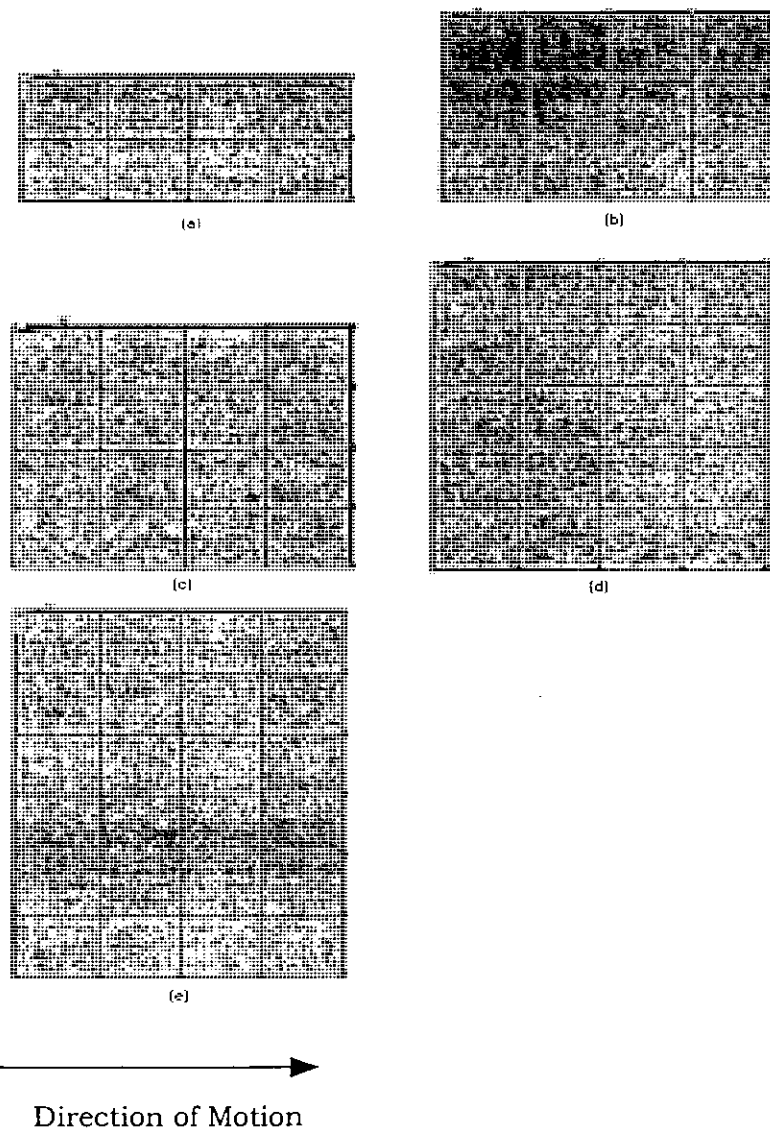


Fig.5.6 Plan of different models of group F showing variation in number of bay along the transverse direction of motion. (a) Model F1, (b) Model F2, (c) Model F3, (d) Model F4, (e) Model F5.

Subgroup F2: Consists of two models, namely, model F2 and F2k.

Model F2 is composed of frame structures with 3x4 bay. In this model, the number of bay along the transverse direction of motion is 3. Mass of the infill is considered but the structural effect of infill is not considered in this model. Model F2k is same with Model F2 except the structural effect of infill

is considered. The value of K_0 is 150255 N/mm. The plan configuration of the models of this subgroup is shown in figure 5.6(b).

Subgroup F3: Consists of two models, namely, model F3 and F3k

Model F3 is composed of frame structures with 4×4 bay. In this model, the number of bay along the transverse direction of motion is 4. Mass of the infill is considered but the structural effect of infill is not considered in this model. Model F3k is same with model F3 except the structural effect of infill is considered. The value of K_0 is 150255 N/mm. The plan configuration of the models of this subgroup is shown in figure 5.6(c).

Subgroup F4: Consists of two models, namely, model F4 and F4k.

Model F4 is composed of frame structures with 5×4 bay. In this model, the number of bay along the transverse direction of motion is 5. Mass of the infill is considered but. the structural effect of infill is not considered in this model. Model F4k is same with model F4 except the structural effect of infill is considered. The value of K_0 is 150255 N/mm. The plan configuration of the models of this subgroup is shown in figure 5.6(d).

Subgroup F5: Consists of two models, namely, model F5 and F5k.

Model F5 is composed of frame structures with 6×4 bay. In this model, the number of bay along the transverse direction of motion is 6. Mass of the infill is considered but. the structural effect of infill is not considered in this model. Model F5k is same with Model F5 except the structural effect of infill is considered. For this model, the value of K_0 is 150255 N/mm. The plan configuration of the models of this subgroup is shown in figure 5.6(e).

5.2.7 Description of different models of Group G

This group represents the effect on time period with variation in number of bay along the direction of motion. The same models of Group F are used in this group but the direction of motion is considered to be perpendicular (transverse) to the direction of those models. The period of vibration is found considering the vibration in changed direction. Five models have been analyzed with variation of number of bay without considering the effect of infill and the same models were analyzed with considering the effect of infill.

Also calculations have been made to calculate the time periods of those five models by the empirical formulae described in chapter 2. The numbers of bay are 2, 3, 4, 5, 6. The different models of this group are symbolically represented by G1, G2, G3, G4, G5 respectively. The models considering the effect of infill of this group are represented by G1k, G2k, G3k, G4k, and G5k accordingly. The plan configuration of the model of this group is shown in figure 5.7.

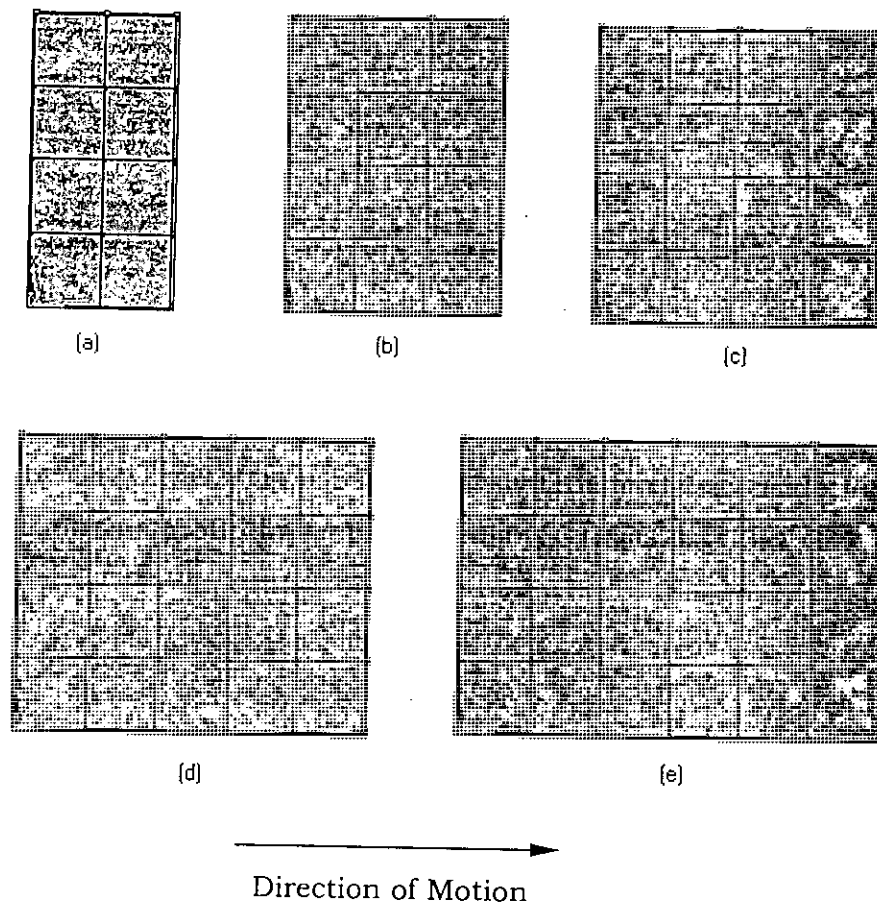


Fig.5.7: Plan of different models of group G showing variation in number of bay along the direction of motion. (a) Model G1, (b) Model G2, (c) Model G3, (d) Model G4, (e) Model G5.

6.2.8 Description of different models of Group H

This group represents the effect on time period in variation of infill density. The amount of infills are varied in different model from 20% to 80% .

Depending on the amount of infill this group is subdivided in four subgroups.

Subgroup H1: Consists of two models, namely, model H1 and H1k

Model H1 is composed of frame structures with 4×4 bay. In this model the infill density is 20%. The infills are provided in the innermost eight panels that is, on panel 3-3 and c-c of fig. 5.8 to cover 20% infill. Mass of the infill is considered but, the structural effect of infill is not considered in this model. Model H1k is same with model H1 except the structural effect of infill is also considered. The value of K_0 is 150255 N/mm.

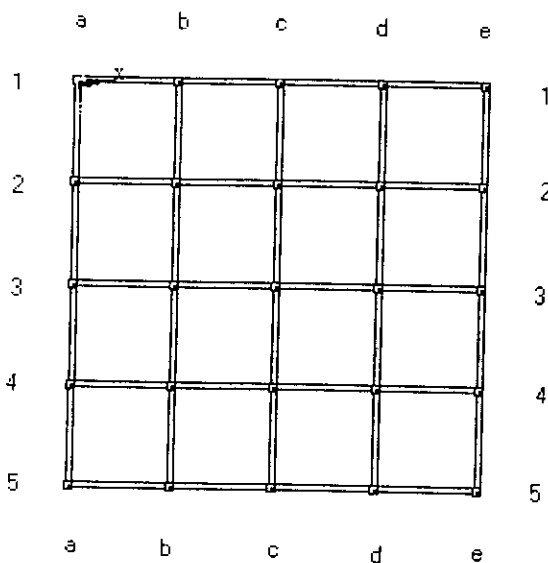


Fig 5.8: Typical Plan of models of group H. (The x- and y- direction vertical frames are numbered as 1-1 through 5-5 and a-a through e-e to distribute the different amount of infills in a regular manner.)

Subgroup H2: Consists of two models; namely, model H2 and H2k .

Model H2 is composed of frame structures with 4×4 bay. In this model the infill density is 40%. The infills are provided in the outer panels i.e, on panel 1-1, 5-5, a-a, and e-e to cover 40% infill. Mass of the infill is considered but the structural effect of infill is not considered in this model. Model H2k is same with model H2 except the structural effect of infill is considered. The value of K_0 is 150255 N/mm. The infills of this model is shown in figure 5.8(b).

Subgroup H3: Consists of two models , namely, model H3 and H3k

Model H3 is composed of frame structures with 4×4 bay. In this model the infill density is 60%. For 4×4 bay frame structure infills in the outer and innermost panels is provided to cover 60% infill. Mass of the infill is considered, but the structural effect of infill is not considered in this model. Model H3k is same with model H3 except the structural effect of infill is considered. The value of K_0 is 150255 N/mm. The infills of the models are shown in figure 5.8(c).

Subgroup H4: Consists of two models; namely, model H4 and H4k

Model H4 is composed of frame structures with 4×4 bay. In this model the infill density is 80%. For 4×4 bay frame structure infills in the sixteen outer panels and sixteen innermost panels are provided to cover 80% infill. Mass of the infill is considered but, the structural effect of infill is not considered in this model. Model H4k is same with model H4 except the structural effect of infill is considered. The value of K_0 is 150255 N/mm. The infills of the models are shown in figure 5.8(d).

5.3 EFFECT OF STRUCTURE PARAMETERS ON NATURAL TIME PERIOD:

The natural time period of vibration of the models described in Art-5.2 are found by modal analysis method. The time periods of vibration of the same models are also calculated using the empirical formulae given by different codes as described in chapter 2. The periods of vibration of the models obtained by modal analysis are compared with the periods of vibration calculated from empirical formulae. Necessary graphs are drawn to study the effect of the structure parameters on the magnitude of period of vibration as well as to study the variations of the results of modal analysis (finite element analysis) with the results of empirical formulae. Discussions on effect of individual structure parameters on time periods of vibration and comparison of modal analysis method with empirical formulae are presented in the next sections.

5.3.1 Effect of variation of column stiffness on period of vibration

For studying the effect of column stiffness on period of vibration, models of Group A are considered. These models represent 10-storied 4×4 frames with different column sizes varying from 300mm × 300mm through 700mm × 700mm. The time periods of vibration of the models are found by modal analysis method. For each model 5(five) different mode shapes are observed. It is found that in all the models of this group, first mode shows the lateral sway. So, the time period corresponding to the first mode is considered. The period of vibration of the same models are also calculated by code (approximate) formulae described in chapter 2. The results of both modal analysis method and code formulae are plotted against column stiffness as shown in figure 5.9. From fig. 5.9, the following observations have been made about the variation of time period with the variation of column stiffness. Also a comparison between the results of modal analysis and empirical formulae has been made.

- 1) The magnitude of natural period of structure obtained by modal analysis decreases with increase of column stiffness. The rate of decreasing diminishes with the increasing column stiffness. But empirical formulae do not show any change in period with change in column stiffness.
- 2) The magnitudes of time period found by modal analysis for models without considering the effect of infills (models A1 through A5) are much higher than those found by empirical formulae or from analysis with infill.
- 3) The time period considering the effect of infill is much less (35% to 55%) than that without considering the effect of infill.
- 4) For medium or higher column stiffness, the magnitude obtained by both modal analysis (considering the effect of infill) and by empirical formulae e.g., BNBC, UBC, IS, BSLJ etc. are almost same but the magnitude obtained by modal analysis is slightly higher for low column stiffness.

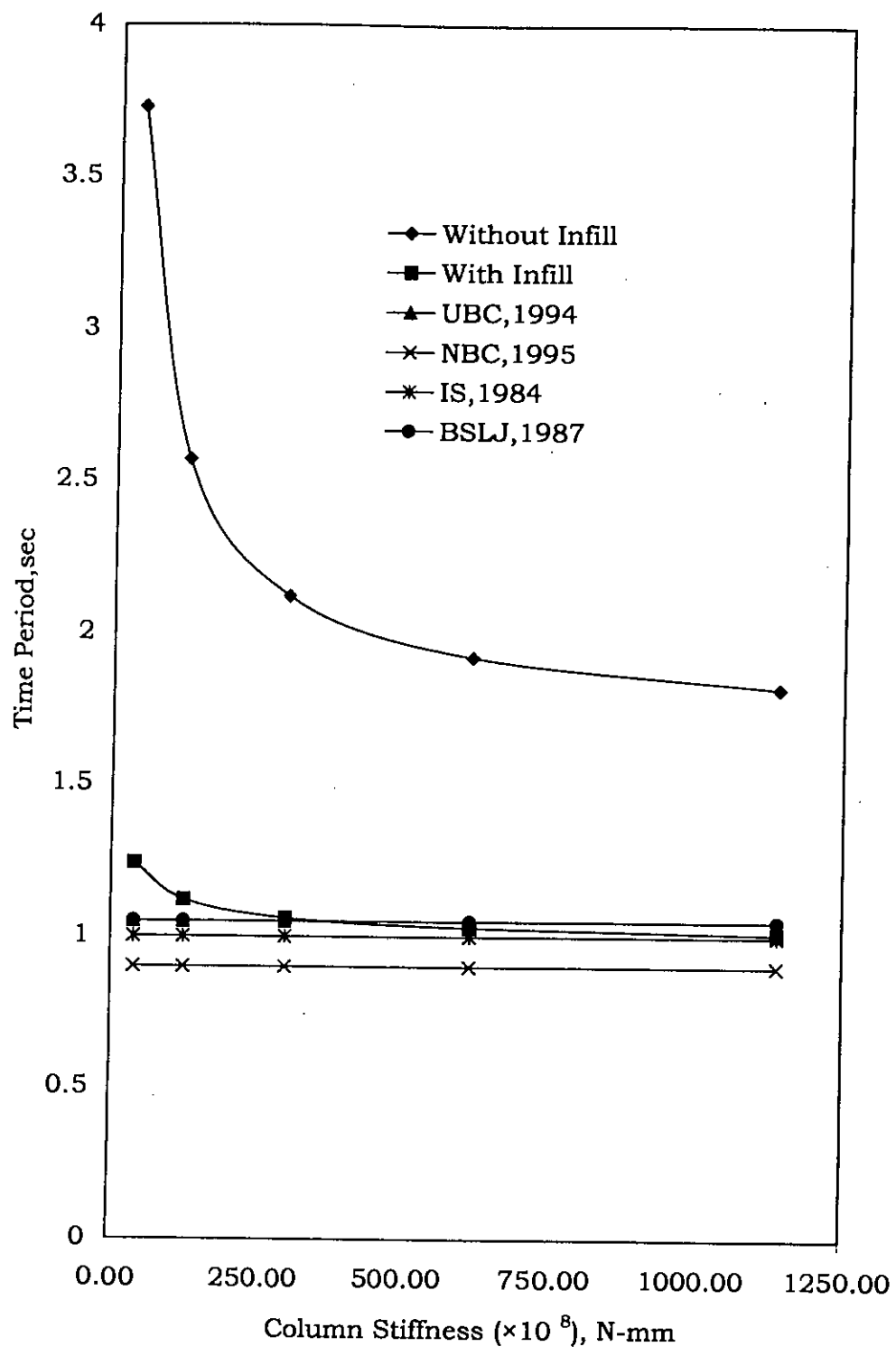


Fig. 5.9 -Natural Time Periods vs. Column Stiffness

- 5) The time period obtained from the empirical formulae are very close to the results found from the modal analysis considering the effect of infill. Hence, it can be decided that the empirical formulae are reasonable to calculate the time period of structures.

5.3.2 Effect of variation of number of story on period of vibration

For studying the effect of number of story on period of vibration, models of Group B are considered. These models represent 10-storied 4 x 4 frames with number of story varying from 4 to 16. The time periods of vibration of the models are found by modal analysis method. For each model 5(five) different mode shapes are observed. It is found that in all the models of this group, first mode shows the lateral sway. So, the time period corresponding to the first mode is considered. The period of vibration of the same models are also calculated by empirical (approximate) formulae described in chapter 2. The results of both modal analysis method and empirical formulae are plotted against number of story as shown in figure 5.10. From fig. 5.10, the following observations have been made about the variation of time period with the variation of number of story.

- 1) The results obtained from modal analysis as well as from empirical formulae shows that time period increases with increase in number of story i.e., increase of building height. This is due to the fact that frequency is proportional to stiffness and increase of number of story causes increase of building height and decrease in overall stiffness against lateral sway.
- 2) The period of vibration obtained by modal analysis considering the effect of infill is significantly smaller than the period of vibration without considering the infill.
- 3) The difference, though not very high, between results obtained from approximate codes and the results of model analysis increases with increasing number of story.
- 4) The time periods obtained from modal analysis without considering the effect of infill differ to a great extent from the results of modal

analysis considering the effect of infill. So, the modal analysis of bare frames should not be used in time period calculation.

- 5) Time periods obtained for infilled frames are much closer to the code values as compared to bare frames. So it may be concluded that the approximate code results give satisfactory estimate of time period of frame structures and can be used in base shear calculation.

5.3.3 Effect of Variation of Panel Size on Period of Vibration

For studying the effect of bay width on period of vibration, models of Group C are considered. These models represent 10-storied 4×4 frames with different bay width as 4000mm, 5000mm, 6000mm, 7000mm and 8000mm. Floor panel size. The periods of vibration of the models are found by modal analysis method. For each model 5(five) different mode shapes are observed. It is found that in all the models of this group, first mode shows the lateral sway. So, the time period corresponding to the first mode is considered. The period of vibration of the same models are also calculated by empirical (approximate) formulae described in chapter 2. The results of both modal analysis method and empirical formulae are plotted against panel size as shown in figure 5.11. From this figure, the following observations have been made about the variation of time period with the variation of panel size. Also a comparison between the results of modal analysis and empirical formulae has been made.

- 1) Increase in floor panel size causes decrease in stiffness and increases mass. The time period is inversely proportional to stiffness. So, the time period increases with increase in floor panel size.
- 2) At medium range of bay width (5500mm to 6500mm), empirical formulae give results those are very close to the result obtained by modal analysis considering the effect of infill. At higher or lower range of span, the codes' result vary to some extent.

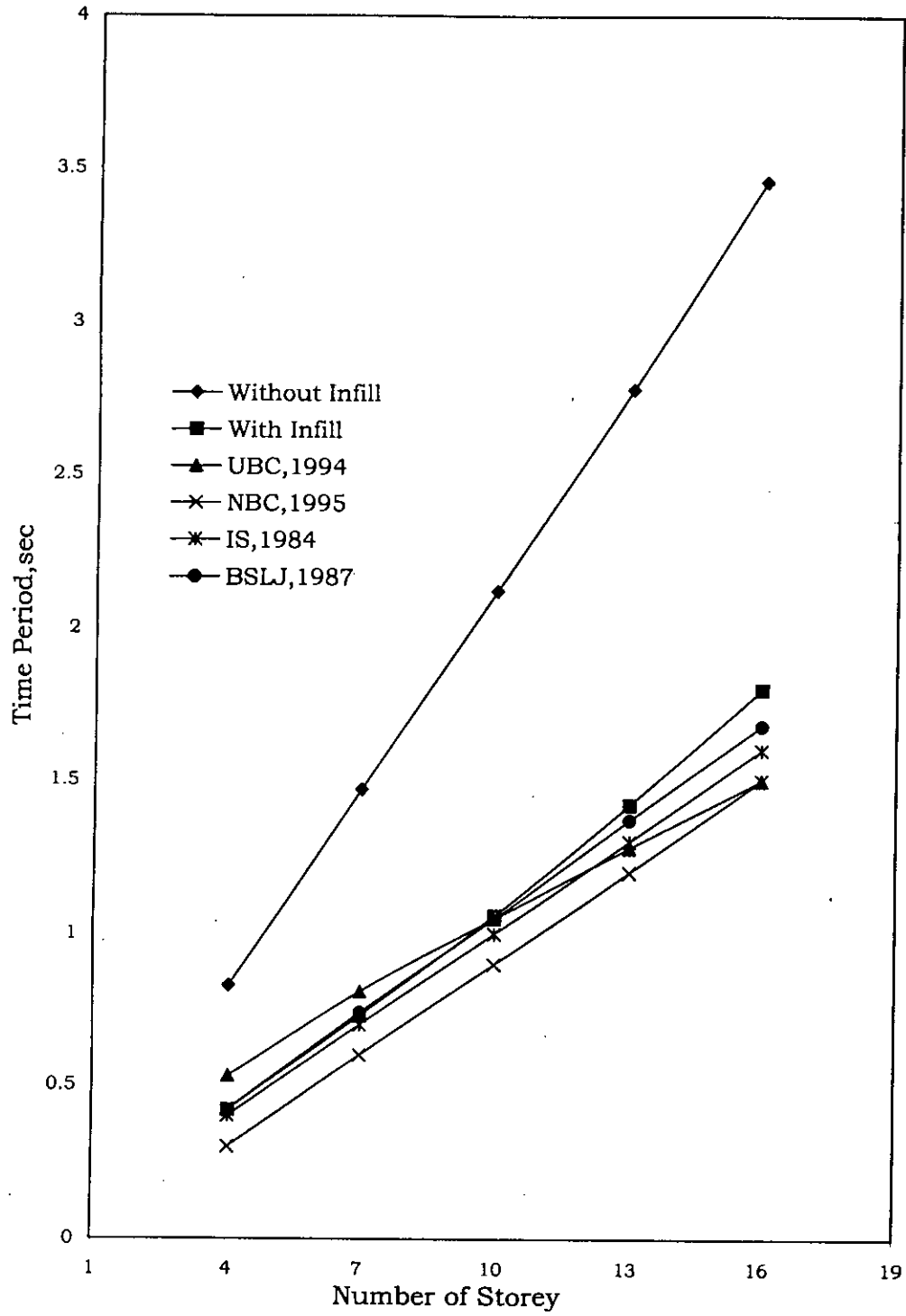


Figure 5.10 : Natural Time Period vs. Number of Storey

- 3) The time period obtained by modal analysis without considering the effect of infill is almost 100% higher than that considering the effect of infill. So, the model analysis of frames without considering the effect of infills should not be used in time period calculation.

At medium range of panel sizes (5500mm to 6500mm), the time period obtained from all the codes are close to the results of infilled frames. So all the codes can satisfactorily be used at such ranges of panel size. At larger panel sizes, the codes show conservative results.

5.3.4 Effect of variation of floor height on period of vibration

For studying the effect of floor height on period of vibration, models of Group D are considered. These models represent 10-storied 4×4 frames with different floor height e.g., 2500mm, 3000mm, 3500mm, 4000mm, 4500mm. The time periods of vibration of the models are found by modal analysis method. For each model 5(five) different mode shapes are observed. It is found that in all the models of this group, first mode shows the lateral sway. So, the time period corresponding to the first mode is considered. The period of vibration of the same models are also calculated by empirical (approximate) formulae described in chapter 2. The results of both modal analysis method and empirical formulae are plotted against floor height column stiffness as shown in figure 5.12. From fig. 5.12, the following observations have been made about the variation of time period with the variation of floor height. Also a comparison between the results of modal analysis and empirical formulae has been made.

- 1) The period increases with increase in floor height because increase in floor height increases column lengths thereby decreasing stiffness against lateral sway.
- 2) The modal analysis without considering the effect of infill shows higher time periods than the periods calculated from codes.
- 3) IS (1984) and NBC (1995) codes do not show any variation of time period with change in floor height.
- 4) The time periods obtained from modal analysis considering the effect of infill are very close to the code results

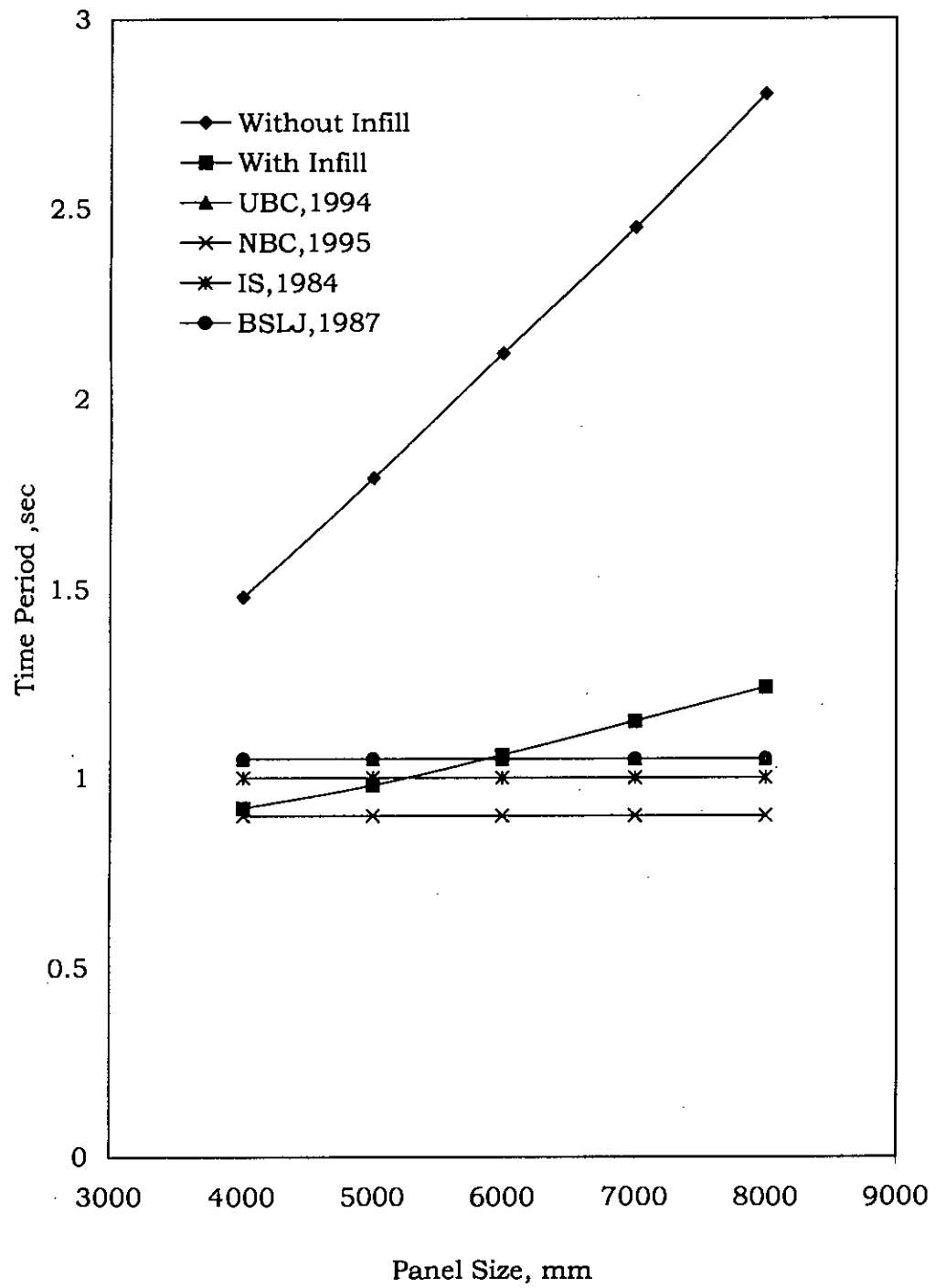


Fig. 5.11-Natural Time Period vs.Panel Size

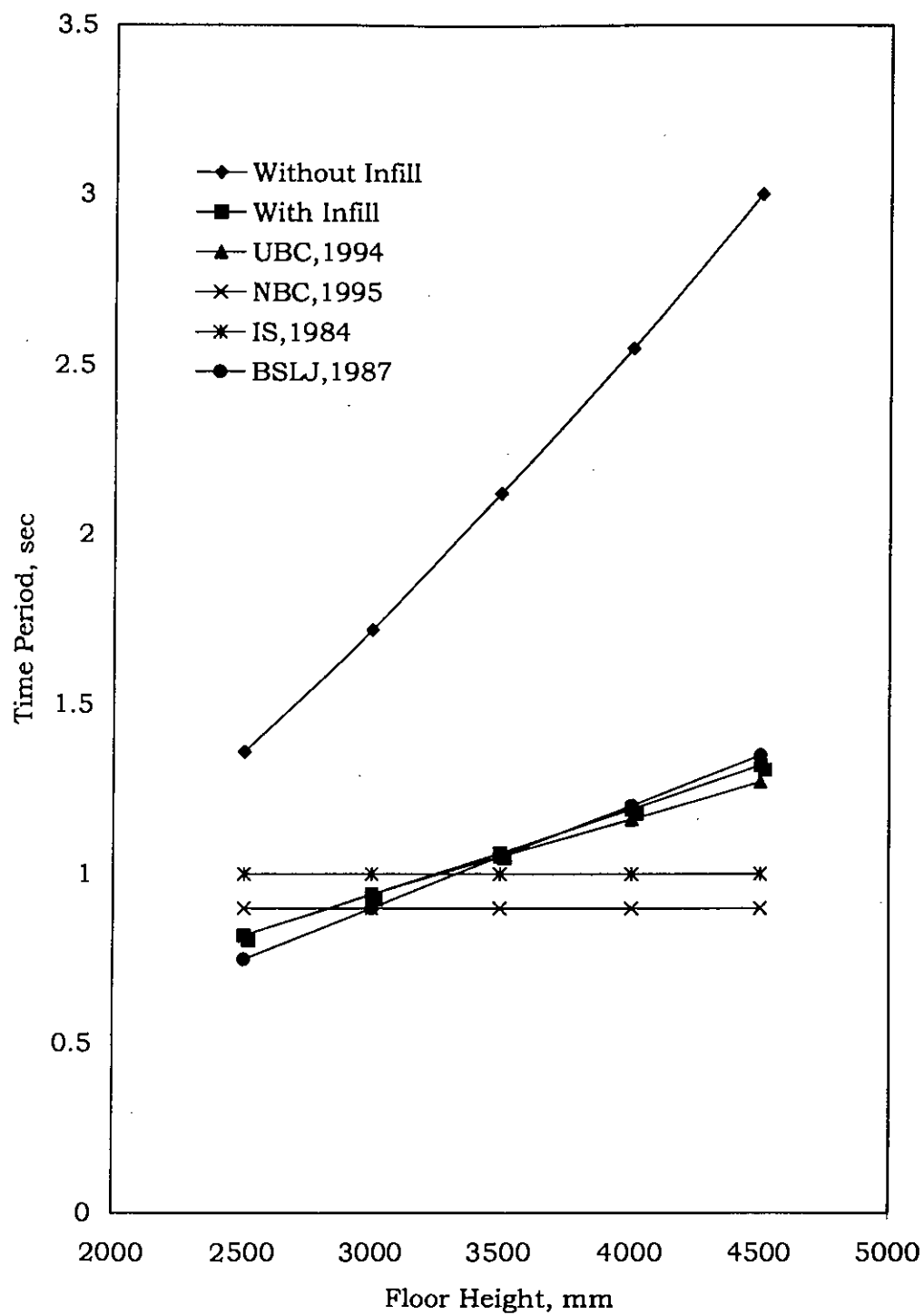


Fig. 5.12 Natural Time Period vs. Floor Height

5.3.5 Effect of variation of beam stiffness on period of vibration

For studying the effect of beam stiffness on period of vibration, models of Group E are considered. These models represent 10-storied 4 x 4 bay frames with different beam sizes e.g., 350mm x 300mm through 350mm x 700mm. The time periods of vibration of the models are found by modal analysis method. For each model 5(five) different mode shapes are observed. It is found that in all the models of this group, first mode shows the lateral sway. So, the time period corresponding to the first mode is considered. The period of vibration of the same models are also calculated by empirical (approximate) formulae described in chapter 2. The results of both modal analysis method and empirical formulae are plotted against column stiffness as shown in figure 5.13. From fig. 5.13, the following observations have been made about the variation of time period with the variation of beam stiffness. Also a comparison between the results of modal analysis and empirical formulae has been made.

- 1) The period of structure decreases with the increase of stiffness for bare frames. As the height and lateral dimension remains unchanged, the code formulae do not show any change in time period with change in beam stiffness.
- 2) The time period without considering the effect of infills give very much higher value as compared with the code results.
- 3) The time period obtained from modal analysis with considering the effect of infills are very close to the time period calculated from codes.
- 4) Effect of beam stiffness is negligible when infills are considered in the analysis.

5.3.6 Effect of variation of number of bay on period of vibration

For studying the effect of number of bay (along the transverse direction of motion) on period of vibration, models of Group F are considered. These models represent 10-storied 4 x 4 bay frames with different number of bay along the transverse direction of motion e.g., 2, 3, 4, 5, 6.

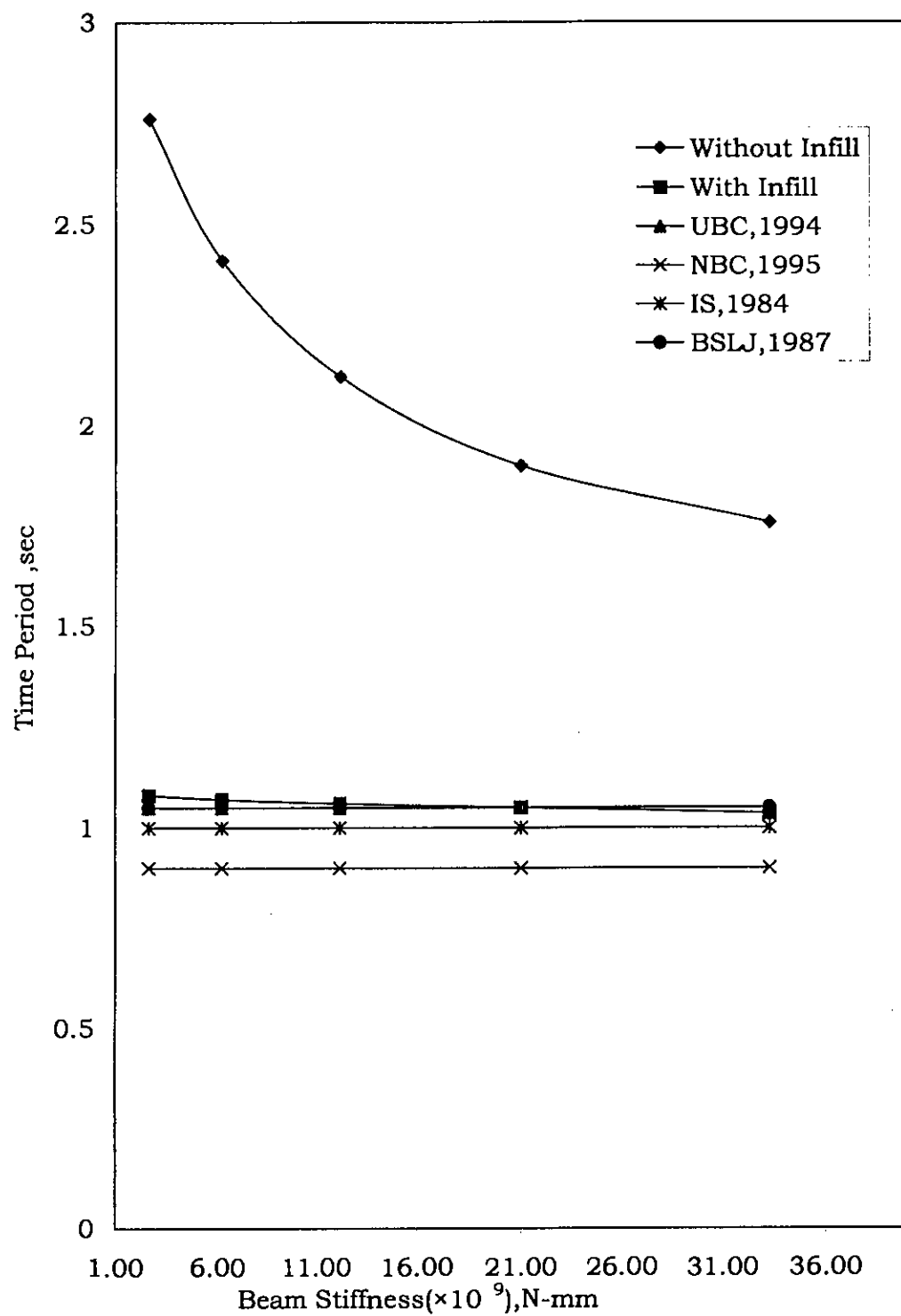


Fig.-5.13 : Natural Time Period vs. Beam Stiffness

The time periods of vibration of the models are found by modal analysis method. For each model 5(five) different mode shapes are observed. It is found that in all the models of this group, first mode shows the lateral sway. So, the time period corresponding to the first mode is considered. The period of vibration of the same models are also calculated by empirical (approximate) formulae described in chapter 2. The results of both modal analysis method and empirical formulae are plotted against column stiffness as shown in figure 5.14. From fig. 5.14, the following observations have been made about the variation of time period with the variation of column stiffness. Also a comparison between the results of modal analysis and empirical formulae has been made.

- 1) The codes do not show any change with change in number of bay along the transverse direction of motion.
- 3) The time periods obtained from modal analysis are varied with change in number of bay along the transverse direction of motion, but this variation is insignificant and may be ignored.
- 3) The modal analysis without considering the effect of infill shows higher time periods than the time periods calculated from codes.
- 4) The time period obtained from modal analysis considering the effect of infills are close to the time period calculated from codes.

5.3.7 Effect of Variation of Number of Span on period of vibration

For studying the effect of number of span (along the direction of motion) on period of vibration, models of Group G are considered. These models represent 10-storied 4 x 4 bay frames with different number of bay along the direction of motion e.g., 2, 3, 4, 5, 6. The time periods of vibration of the models are found by modal analysis method. For each model 5(five) different mode shapes are observed. It is found that in all the models of this group, first mode shows the lateral sway. So, the time period corresponding to the first mode is considered. The period of vibration of the same models are also calculated by empirical (approximate) formulae described in chapter 2. The

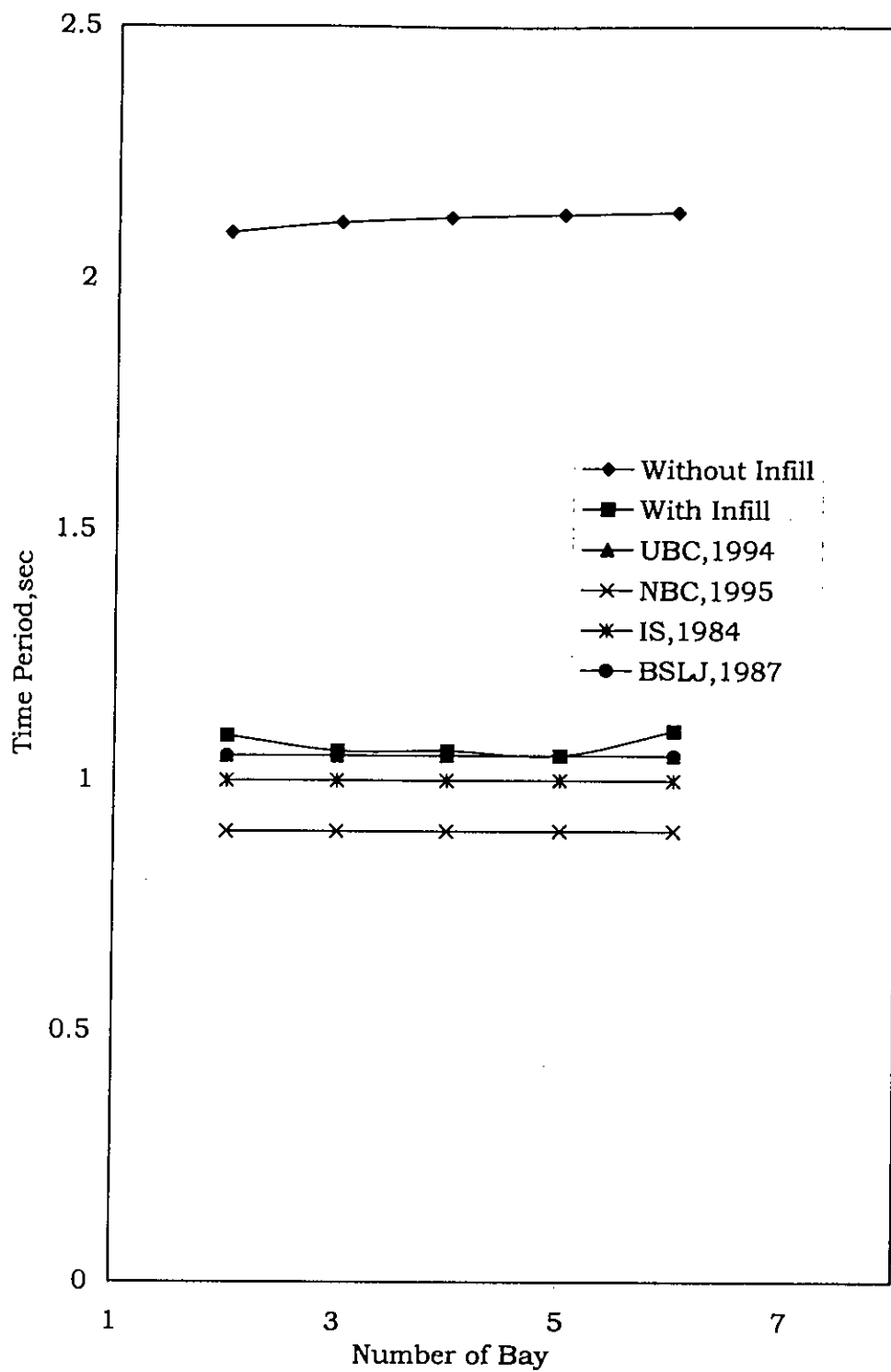


Fig. 5.14: Natural Time Period vs. Number of Bay

results of both modal analysis method and empirical formulae are plotted against column stiffness as shown in figure 5.15. From fig. 5.15, the following observations have been made about the variation of time period with the variation of column stiffness. Also a comparison between the results of modal analysis and empirical formulae has been made.

- 1) The Period of vibration decreases as the number of span increases along the direction of motion. This is due to the reason that the frequency is directly proportional to stiffness and increase in number along the direction of motion of bay increases, stiffness of the structure increases and as a result, period of vibration decreases. The rate of decrease diminishes with increasing number of bays.
- 2) The time period obtained by modal analysis considering the effect of infill is close to the time period calculated from code formulae but the time period obtained by modal analysis without considering the effect of infill is much higher than code formulae.
- 4) The code formulae give good approximation for estimating time period.

5.3.8 Effect of variation of infill density on period of vibration

For studying the effect of infill density on period of vibration, models of Group H are considered. These models represent 10-storied 4 x 4 bay frames with different amount of infills e.g., 20%, 40%, 60%, 80%. In the models of this group infills are placed in regular manner. The time periods of vibration of the models are found by modal analysis method. For each model 5 (five) different mode shapes are observed. It is found that in all the models of this group, first mode shows the lateral sway. So, the time period corresponding to the first mode is considered. The period of vibration of the same models are also calculated by empirical (approximate) formulae described in chapter 2. The results of both modal analysis method and empirical formulae are plotted against infill density as shown in figure 5.16.

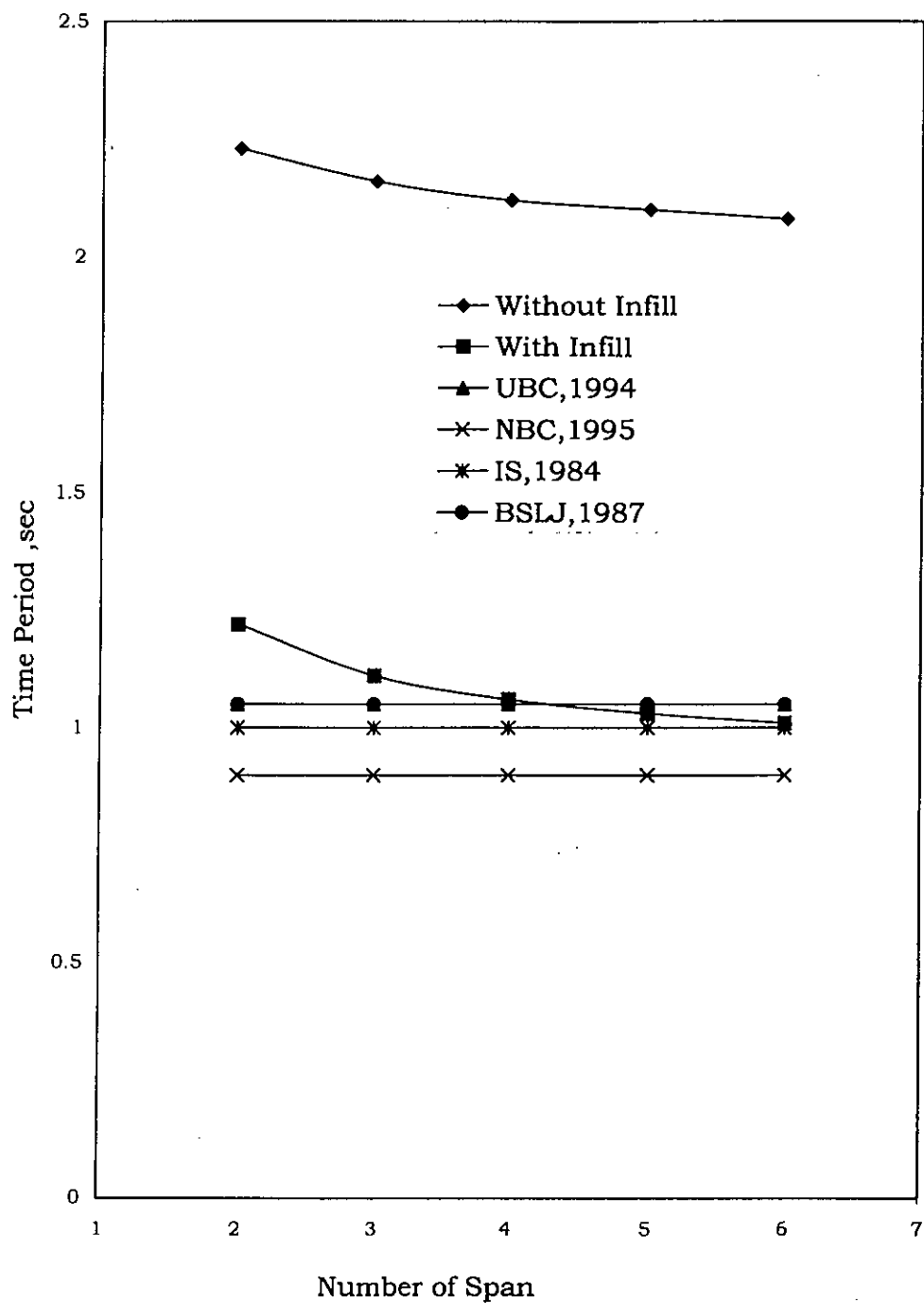


Fig. 6.15: Natural Time Period vs. Number of Span

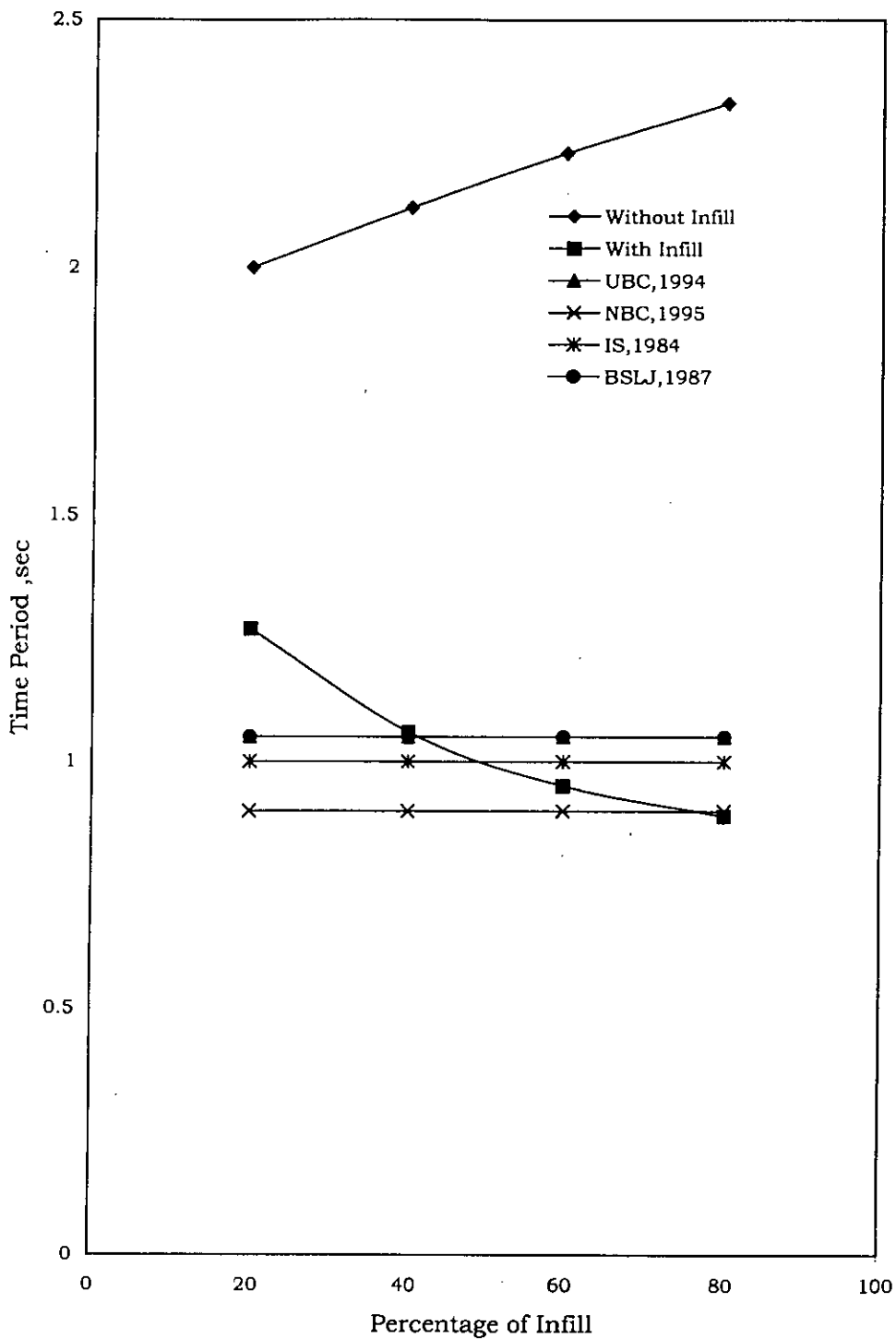


Fig.5.16-Natural Time Period vs.Percentage of Infill

From fig. 5.16, the following observations have been made about the variation of time period with the variation of infill density. Also a comparison between the results of modal analysis and empirical formulae has been made.

- 1) The time period is increasing with increase of infill density when the effect of infill is not considered (only mass of infill is considered).
- 2) The time period is decreasing with increase of infill density when the effect of infill is considered. This is due to the fact that additional stiffness due to the presence of infill, the overall stiffness of the structure increases and as a result, the time period decreases.
- 3) The codes do not show any change with change in percentage of infill.
- 4) It is observed that modal analysis considering the presence of 40% infills gives time period very much close to the empirical codes. So, for calculation of base shear for frame structures with such amount of infills, the use of codes is satisfactory.

CHAPTER 6

ANALYSIS OF MODELS WITH RANDOMLY DISTRIBUTED INFILLS

6.1 INTRODUCTION:

In the previous chapter the models have been analyzed considering the infill placed in regular manner i.e., distributed in a regular pattern. In the models of group A through group G, infill placed in the outermost panels, which cover 40% of the total panels. In group H, models have been analyzed with different amount of infill (20%, 40%, 60% and 80%) and all the infill were distributed in a regular pattern. But in practical cases, the infill are not uniformly distributed as well as the total amount of infill is not same for all structures. Rather, the infills are randomly distributed in different panels of different floors and their amount vary from structure to structure. So it is necessary to study the models considering the effect of random distribution of infill.

In this chapter, the infill of models of group H described in previous chapter are further analyzed considering the random distribution of infill. The amounts of infill are varied from 20% to 80%. The results of the models with randomly distributed infill are compared with the results of models with uniformly distributed infill as well as with the results of empirical codes.

6.2 RANDOM DISTRIBUTION OF INFILL

Random distribution of infill for different models is performed manually as follows:

A 10-storied 4 x 4 bay structure consists of 400 numbers of frame panels (200 in x- direction and 200 in z direction). In each direction (either x or z), there are 5(five) vertical frames numbered as 1 through 5. Each vertical frame has 10(ten) floors (stories) numbered as 1 through 10 and frame

panels of each floor has 4(four) numbered as 1 through 4. For random distribution, 200 pieces of 'tickets' (papers) are numbered according to their geometric locations. For example, a ticket numbered as "4 10 3" is meant for the panel at location of 3rd bay, 10th floor of 4th frame. Then a manual draw is performed. To provide 20% infill in x -direction, 40 (forty) numbers of tickets are drawn out of 200 tickets and to provide 20% infill in z -direction, a similar draw is performed. For 20% infill, five separate draw is performed to have five models with different random distribution of infill.

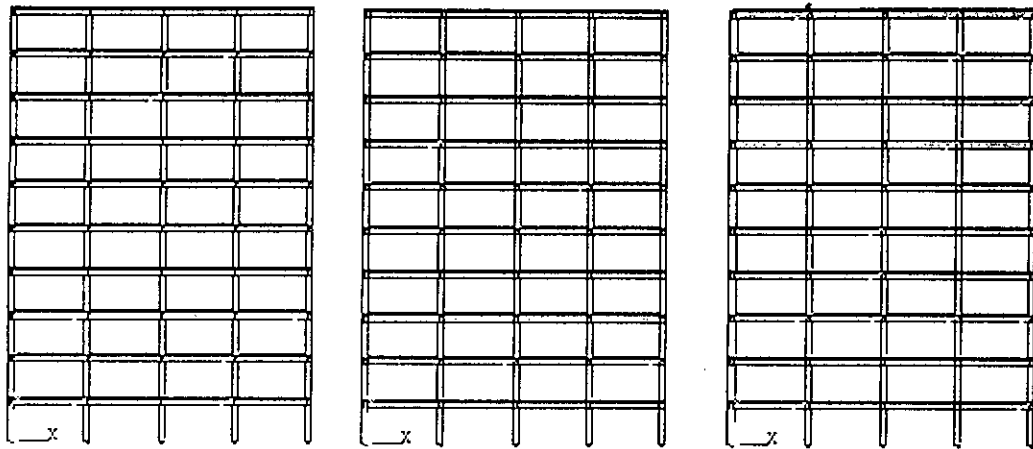
Similarly, to provide 40% infill in x-direction, 80 (eighty) numbers of tickets are drawn out of 200 tickets and to provide 40% infill in z -direction, another 80 (eighty) numbers of tickets are drawn out of remaining 120 tickets and the process is repeated to obtain five sets of models for 40%. Similar process is followed to obtain five sets of models for 60% and five sets of models for 80% infill.

6.3 DESCRIPTION OF MODELS

The different models with randomly distributed infill are symbolically represented as R20, R40, R60 and R80 as described below.

Model R20: Composed of frame structures with 4×4 bay. The amount of infill is 20% distributed randomly as described in article 6.2. Five sets of draw are performed for 5 (five) separate models as represented by R20-1, R20-2, R20-3, R20-4, R20-5. All other parameters of these models are same with the parameters of model H1k (models with regular distribution of infill described in article 6.2.8 of previous chapter). The infill of different frames in x-direction of model R20-1 are shown in figure 6.1.

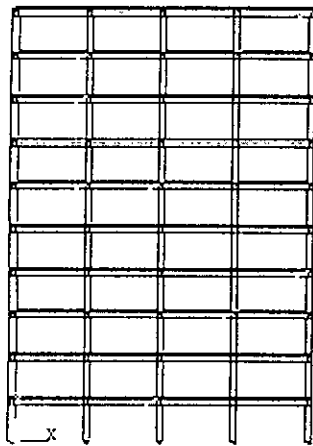
Model R40: Composed of frame structures with 4×4 bay. The amount of infill is 40% distributed randomly as described in article 6.2. Five sets of draw are performed for 5 (five) separate models as represented by R40-1, R40-2, R40-3, R40-4, R40-5. All other parameters of these models are same with the parameters of model H2k (models with regular distribution of infill). The infill of different frames in x-direction of model R40-1 are shown in figure 6.2.



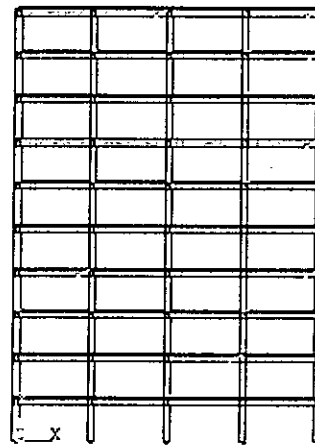
(a)

(b)

(c)

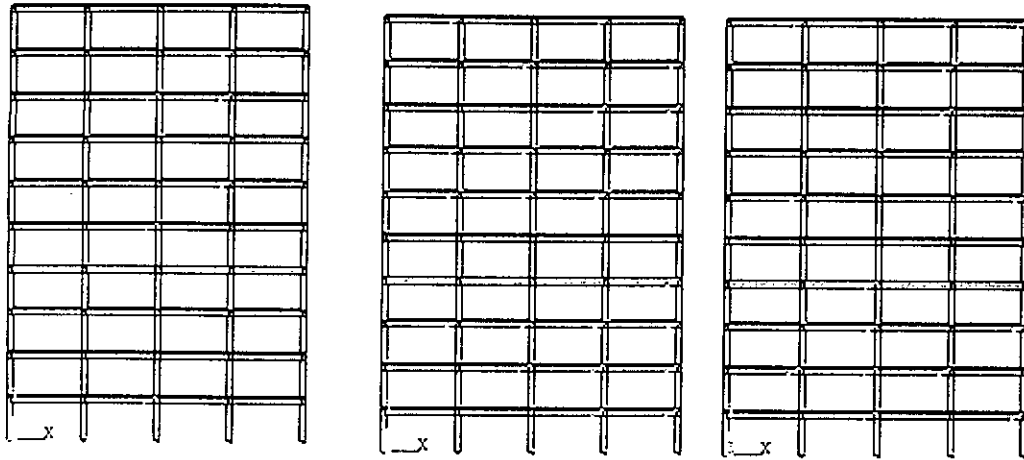


(d)



(e)

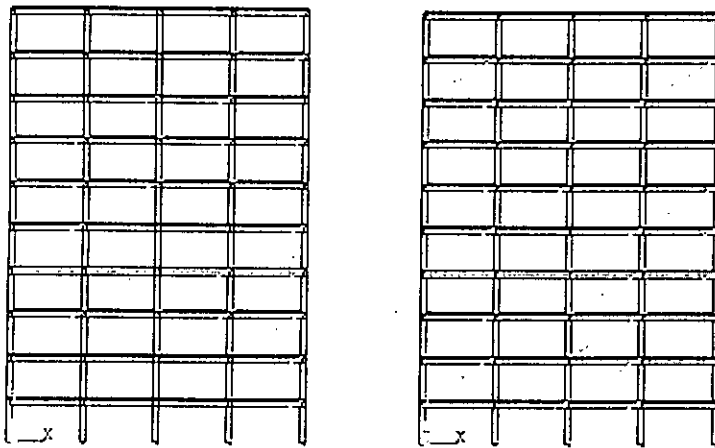
Fig.6.1: x-direction frames of model R-20-1 showing the random distribution of infills



(a)

(b)

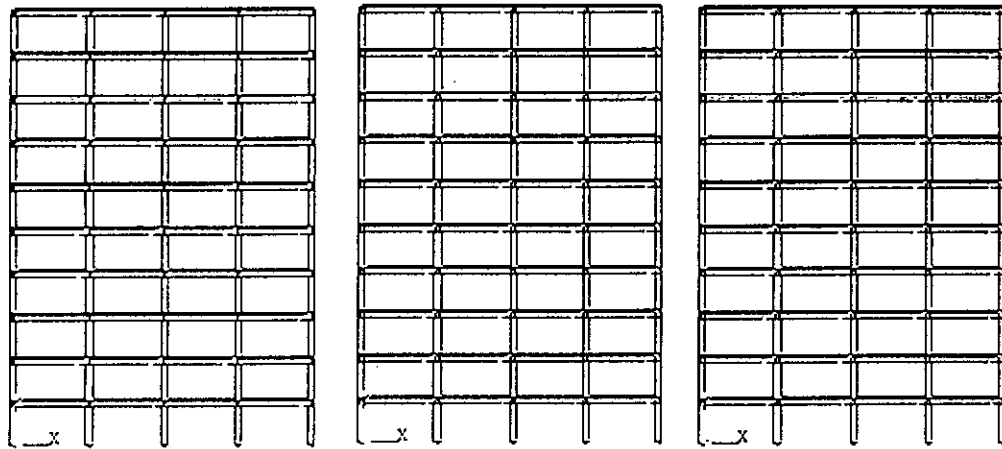
(c)



(d)

(e)

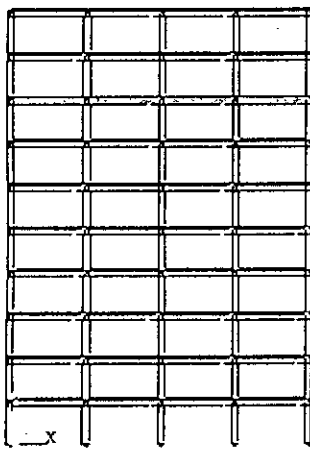
Fig.6.2: x-direction frames of model R-40-1 showing the random distribution of infills



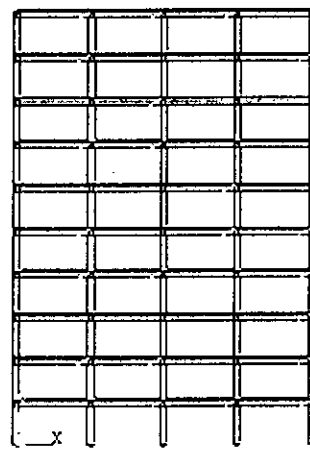
(a)

(b)

(c)

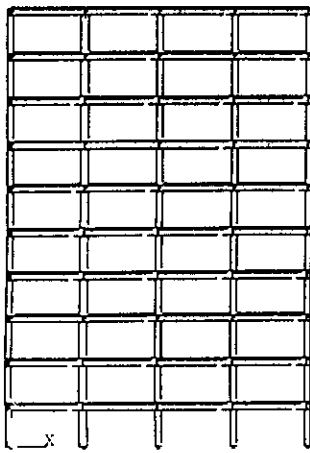


(d)

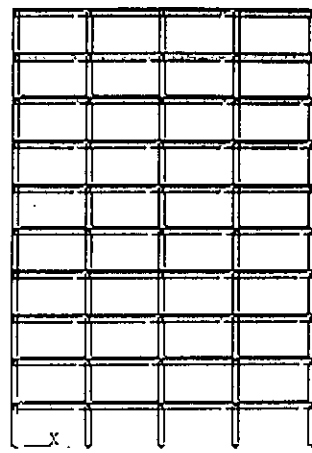


(e)

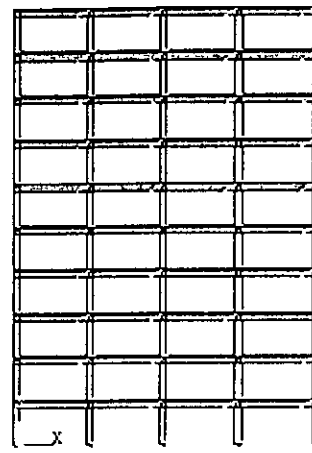
Fig.6.3: x-direction frames of model R-60-1 showing the distribution of infills



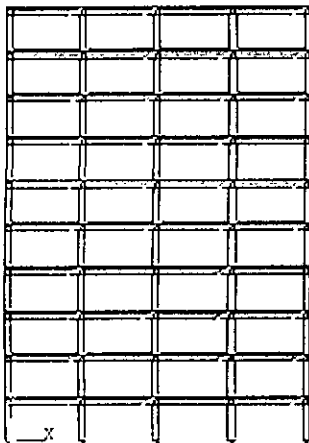
a)



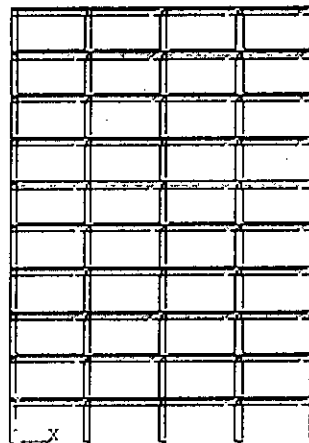
(b)



(c)



(d)



(e)

Fig.6.4: x-direction frames of model R-80-1 showing the random distribution of infills

Model R60: Composed of frame structures with 4×4 bay. The amount of infill is 60% distributed randomly as described in article 6.2. Five sets of draw are performed for 5 (five) separate models as represented by R60-1, R60-2, R60-3, R60-4, R60-5. All other parameters of these models are same with the parameters of model H3k (models with regular distribution of infills). The infills of different frames of model R60-1 in x-direction are shown in figure 6.3.

Model R80: Composed of frame structures with 4×4 bay. The amount of infill is 80% distributed randomly as described in article 7.2. Five sets of draw are performed for 5 (five) separate models as represented by R80-1, R80-2, R80-3, R80-4, R80-5. All other parameters of these models are same with the parameters of model H4k (models with regular distribution of infills). The infills of different frames in x-direction of model R80-1 are shown in figure 6.4.

6.4 Effect of random distribution of infills on time period

The models with random distribution infill are analyzed (by modal analysis) to obtain the time period. The results of models R20-1 through R20-5 are recorded. Average of these results are considered to represent the time period of model R20. Similarly, the time period of model R40, R60 and R80 are obtained. The results of each model have been shown in figure 6.5 through 6.8. Curves are drawn to compare the time period for regular and random distribution of infill with the time period obtained by codes and modal analysis (figure 6.9). From the results obtained by above analysis and the figures 6.5 through 6.9, the following observations have been made.

- 1) The time period obtained by random analysis is about 3 - 4% higher than the time period obtained by regular analysis.
- 2) The time period of modal analysis decreases with increasing amount of infills.
- 3) The results of random analysis are very much close to the code results.

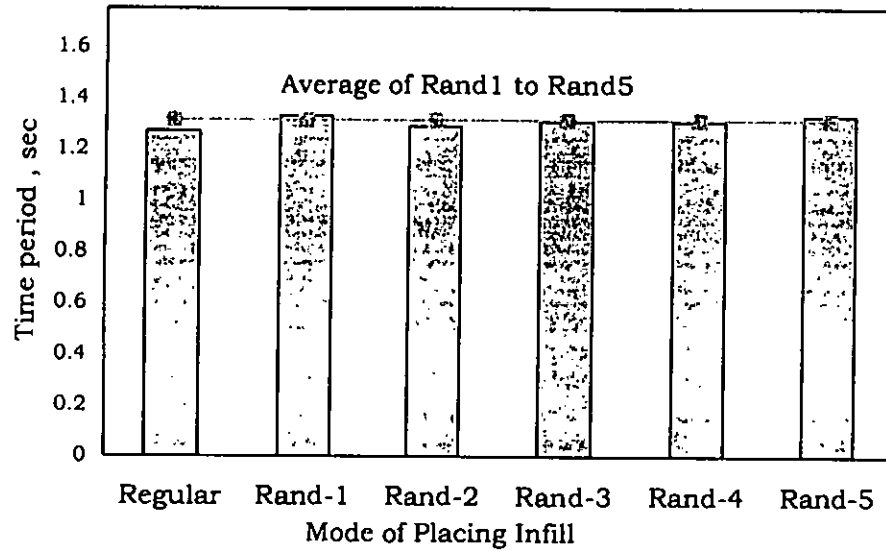


Fig. 6.5 Time Period for Structure with 20% Infills placed in different modes

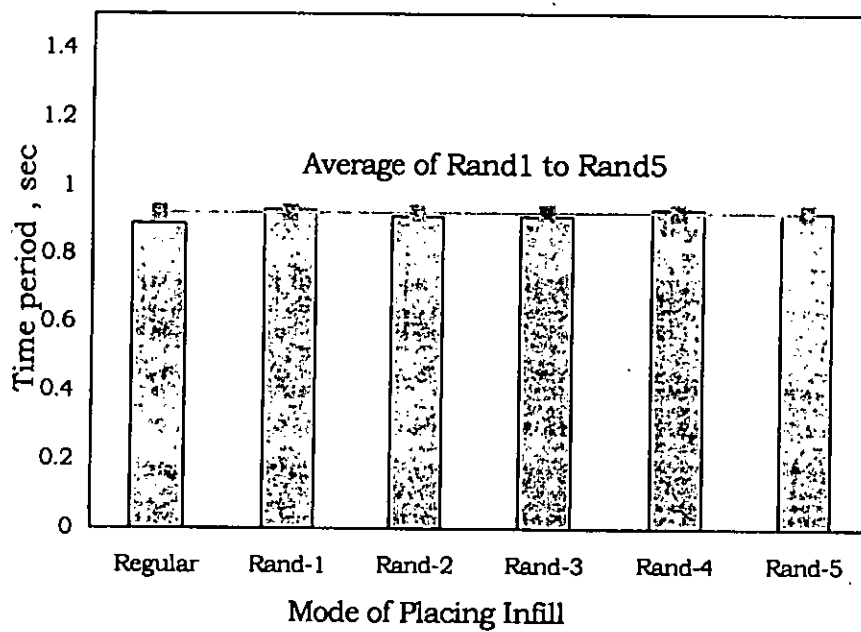


Fig. 6.8 Time Period for Structure with 80% Infills placed in different modes

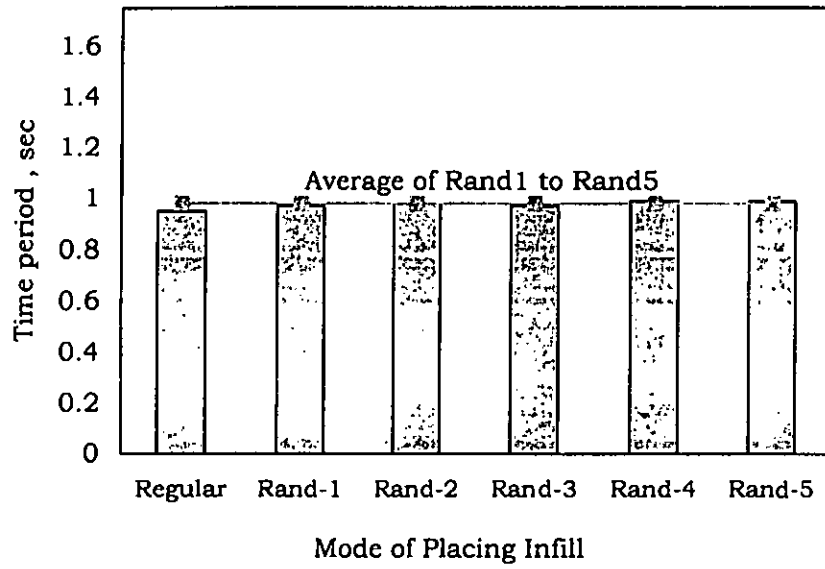


Fig. 6.7 Time Period for Structure with 60% Infills placed in different modes

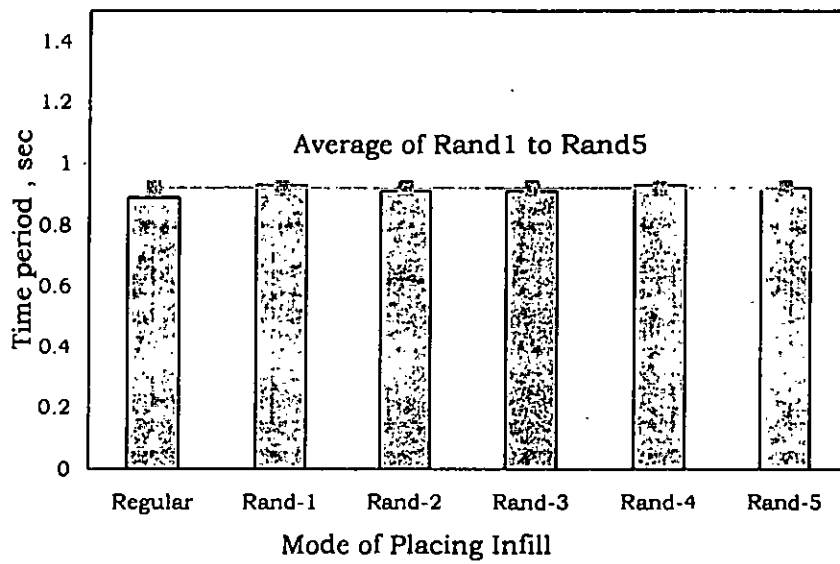


Fig. 6.8 Time Period for Structure with 80% Infills placed in different modes

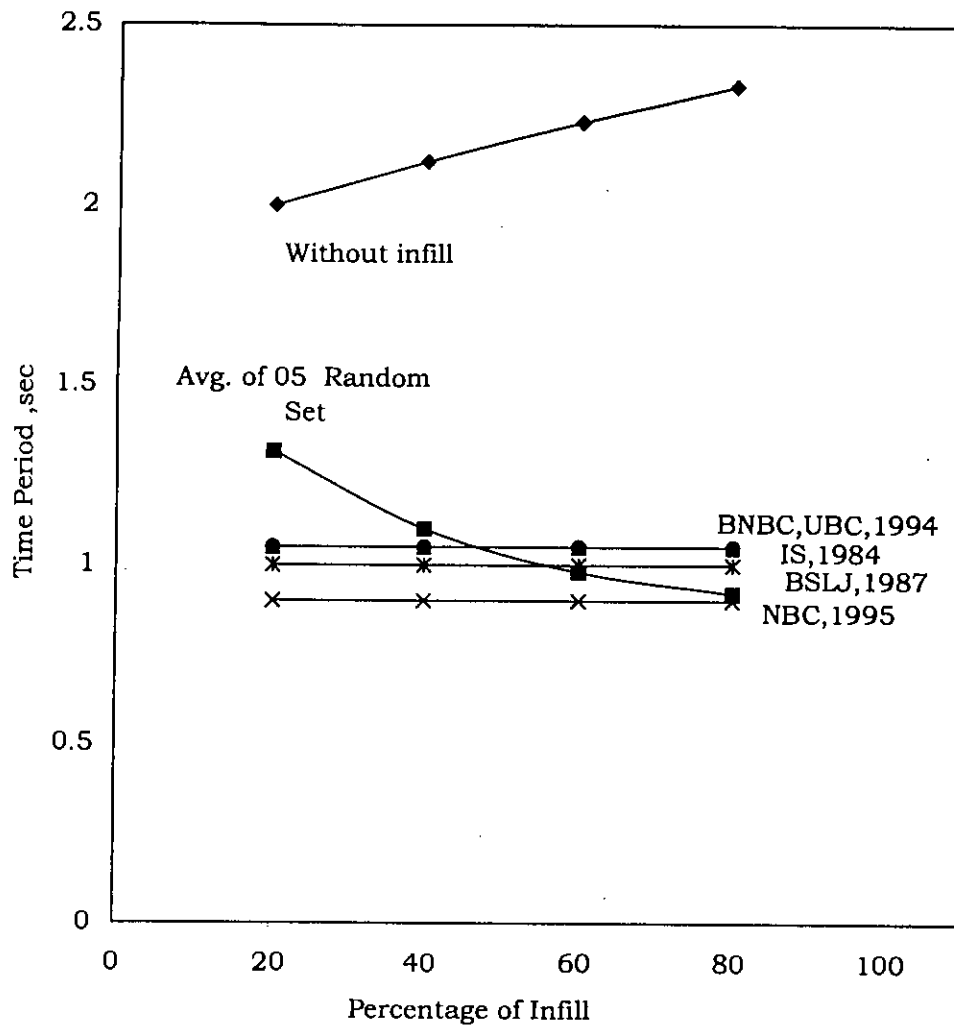


Fig.6.9-Natural Time Period vs.Percentage of Infill

6.4 REMARKS

The time period obtained with random distribution is very much close to the time period with regular distribution of infill. So, the distribution of infill (either regular or random) has very little influence on the time period. For a certain amount of infill, analysis with regular distribution of infill is sufficient to determine the time period for earthquake calculation, although, in practical cases, infill are placed in random manner. It may be noted that the time period decreases with increase in amount of infill.

CHAPTER 7

ESTIMATING NATURAL PERIOD OF VIBRATION OF

INFILLED FRAMES

7.1 INTRODUCTION

In the previous three chapters, a rigorous computation and investigation has been made on natural period of vibration of reinforced concrete frames having infills. Effect of various parameters such as story height, column stiffness, beam stiffness, panel size, number of bay, number of span, number of story, amount of infill are studied in detailed and the results are discussed in chapter 5. Recognizing the fact that in real structures, infills may appear in a random manner, a study with random distribution of infills has also been carried out as described in Chapter 6. From these studies, the relative importance of various parameters can easily be established.

7.2 PARAMETERS CONTROLLING TIME PERIOD

From the study presented in Chapter 5, specially from fig. 5.9 through fig. 5.16, the relative importance of different parameters can easily be established as follows:

* For frames without infill, column stiffness influences the period significantly but, for infilled frames, column stiffness does not play any significant role. Since most structures do have infill within the panels, it is therefore imperative that for real structures, column stiffness need not to be considered in determining the time period.

01

- * Time period increases as the number of story increases or in other words, the total height of the building increases and it is found that the code equations can estimate the time period of infilled frame reasonably well. But some modification in the code equations is necessary to have better prediction.
- * Panel sizes (Floor panel) influences the time period as compared to code equations. In fact, code equations cannot reflect any influence of floor panels. Increasing size of floor panel, actually increases the mass of the structure without much increasing the lateral stiffness. This ultimately increases time period.
- * Code equations can satisfactorily predict the time period with respect to floor height as seen from fig. 5.12 for 40% infill.
- * Beam stiffness has no significant influence on time period when infill is present. Therefore, effect of beam stiffness can be neglected for all practical purposes in approximate determination of time period.
- * Number of bay in the direction transverse to motion has no significant influence on time period as seen from fig. 5.14.
- * Varying the number of span in the direction of motion does not appreciably change the period of vibration for infilled frame structure as seen from fig. 5.15. Also the period predicted by modal analysis for different spans, match closely with code equation, for four spans or higher. For lower number of spans the period is slightly higher than the code equations. Therefore, number of span is not an important parameter in determining period of vibration.
- * The amount of infills present in a structure has significant influence on period of vibration as seen from fig 5.16. Structure period decreases with increasing amount of infill.
- * Modal analysis of reinforced concrete frames with random distribution of infills has been performed in chapter 6 and it was found that time period is almost same with regular distribution of infill. Thus randomness of distribution does not have any appreciable effect.

From the above discussion, the following three parameters are identified to have significant influence on period of vibration.

- 1) Number of story or height of the building
- 2) Panel size
- 3) Amount of infilled panels.

7.3 ASSESSING THE INFLUENCE OF CONTROLLING PARAMETERS ON TIME PERIOD.

Of the various code equations it is observed that BNBC, UBC and BSLJ do comparatively well in approximating the period of vibration. Of these codes, BNBC is chosen as the reference code based on which suggestion on improving the estimations of time period shall be made. BNBC is chosen as the reference code because, this code is applicable to the structures of Bangladesh and it is also consistent with widely followed UBC and BSLJ. Now, the time period predicted by codes are compared with the same found from modal analysis.

Fig. 8.1 shows the ratio of period obtained from modal analysis and the same from BNBC equations ratio for varying number of story. From this figure, it is observed that the ratio T_{Modal} / T_{BNBC} varies between 0.79 to 1.2 for number of story varying between 4 to 16 and the variation is almost linear. This variation can be analytically expressed as,

$$T_{Modal}/T_{BNBC} = 0.0343N_s + 0.6587 \quad (7.1)$$

Where, N_s is the number of story.

The above equation may also be expressed as,

$$T_{Modal}/T_{BNBC} = 0.0098H + 0.6587 \quad (7.1.a)$$

Where, H=height of the building.

Fig. 8.2 shows the ratio of period obtained from modal analysis and the same from BNBC equations for varying panel sizes. From this figure, it is observed that the ratio T_{Modal} / T_{BNBC} varies between 0.85 to 1.2 for panel sizes between 4

m to 8 m and the variation is almost linear. This variation can be analytically expressed as,

$$T_{\text{Modal}}/T_{\text{BNBC}} = 0.0774L + 0.557 \quad (7.2)$$

Where, L is the panel size in meter.

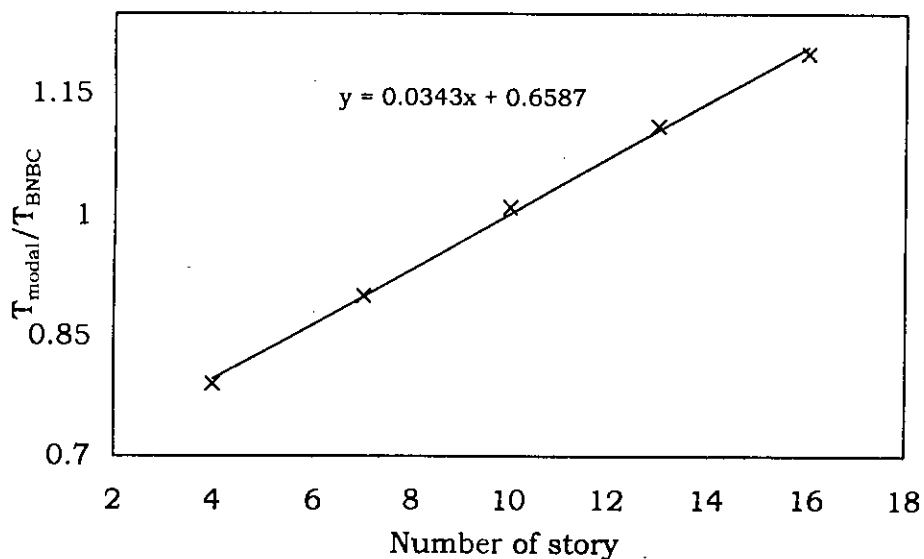


Fig. 7.1 Ratio $T_{\text{modal}}/T_{\text{BNBC}}$ for varying number of Story

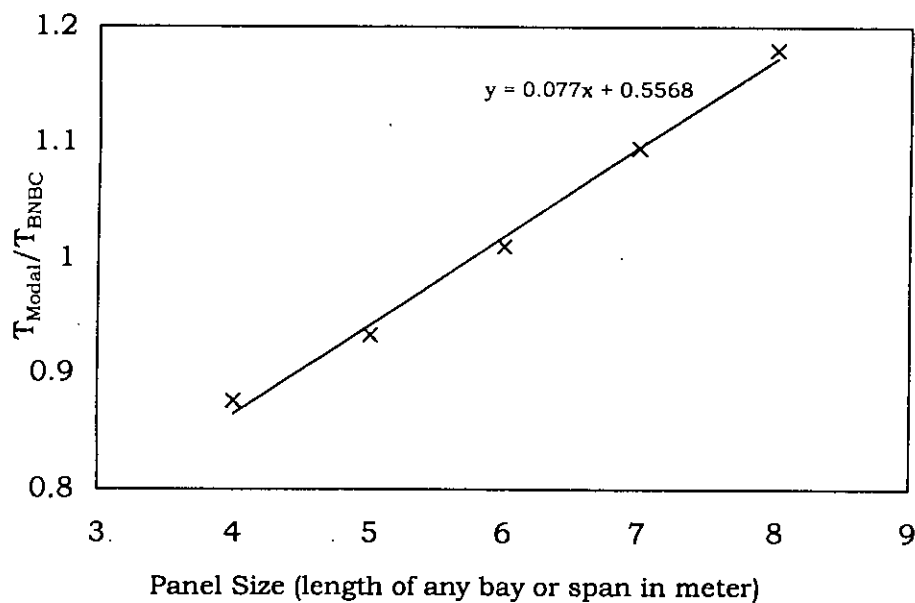


Fig.7.2: Ratio $T_{\text{Modal}}/T_{\text{BNBC}}$ for varying Panel Size

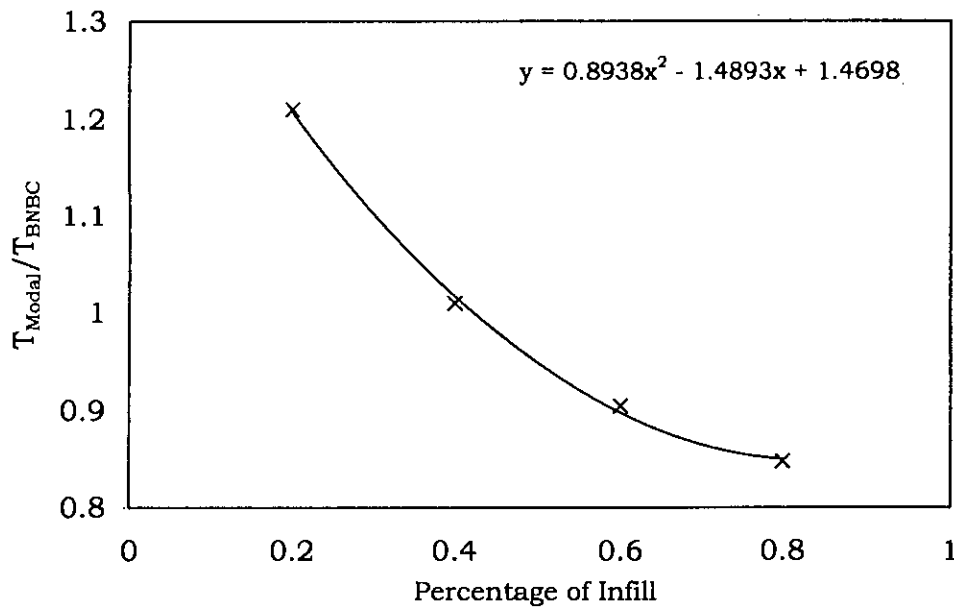


Fig 7.3. Ratio $T_{\text{Modal}}/T_{\text{BNBC}}$ for varying Percentage of Infill

Similarly the ratio of time period for varying amount of infill present in the structure is graphically represented in fig. 7.3, it is seen that the ratio varies between 1.2 to 0.85, for different amount of infills. This variation can analytically be expressed as

$$T_{\text{Modal}}/T_{\text{BNBC}} = 0.894(P_i)^2 - 1.49 P_i + 1.47 \quad (7.3)$$

Where, P_i indicates the amount of infilled panel in the structure with respect to the total number of panels.

7.4 RATIONALE FOR ESTIMATING PERIOD OF VIBRATION

In the previous article, it is shown how the time period of vibration of infilled frames is influenced by different parameters. Analytical representation of these influences is also shown by equations 8.1 through 8.3. These

equations can be used as the basis for improving the accuracy of time period predication by the BNBC equation. Thus it can be suggested that, after determining time period from BNBC equation, correction factors or multiplying factors given by Eq. 7.1 through 7.3 may be applied to finally calculate the time period. Thus we can write,

$$\text{Period, } T = T_{BNBC} \times m_1 \times m_2 \times m_3 \quad (7.4)$$

where, m_1 is the T_{Modal}/T_{BNBC} obtained from equation 7.1 and so on.

7.5 VERIFICATION OF THE PROPOSAL

The proposed modification factor described in the previous article needs to be verified to establish the acceptability. In order to do this, a few arbitrarily chosen infilled framed building are considered. These are shown in table 7.1. For each of the structure the time period is calculated using BNBC equation as well as using the modal analysis. The period predicted by BNBC is corrected using multiplying factor, as given equation 7.4. These results are graphically shown in figs. 7.4(a) through 7.4(d). From these figures, we see that the proposed multiplying factors can effectively improve the period predicted by BNBC. Also the amount reduction of base shear using the value of time period according to the predicted formula has been shown in Table 7.1. From the BNBC code equation it can be obtained that percentage change in base shear

$$\text{is proportional to } \left\{ \left(\frac{T}{T_{BNBC}} \right)^{2/3} - 1 \right\} \times 100$$

It is seen that the base shear calculated by predicted formula differ to the base shear calculated by BNBC code is almost 0.5-16.5% and this prediction matches the result of modal analysis. Thus the validity of the proposed multiplying factor equation (7.1 through to 7.3) is established.

Table 7.1: Examples for verification of the proposal

Sl. No	Parameters	Examples			
		1	2	3	4
1	Size of column	500 ×500 mm ²	500 ×500 mm ²	500 ×500 mm ²	500 ×500 mm ²
2	Num. of Story	10	13	10	7
	Value of m_1	1.002	1.1046	1.002	0.8988
3	Panel Size	6 m	5 m	8 m	6 m
	Value of m_2	1.019	0.9418	1.1728	1.019
4	Floor height	3500 mm	3500 mm	3500 mm	3500 mm
5	Size of beam	350×500 mm ²	350×500 mm ²	350×500 mm ²	350×500 mm ²
6	Number of bay	4	4	4	4
7	Number of Span	4	4	4	4
7	Amount of Infiles (percentage of total panels)	80%	40%	40%	80%
	Value of m_3	0.85	1.017	1.017	0.85
T_{modal}		.89	1.32	1.24	0.55
T_{BNBC}		1.05	1.28	1.05	0.81
T		0.91	1.35	1.25	0.62
% Reduction in base shear		-14.0%	+3.61%	+12.33%	-16.3%

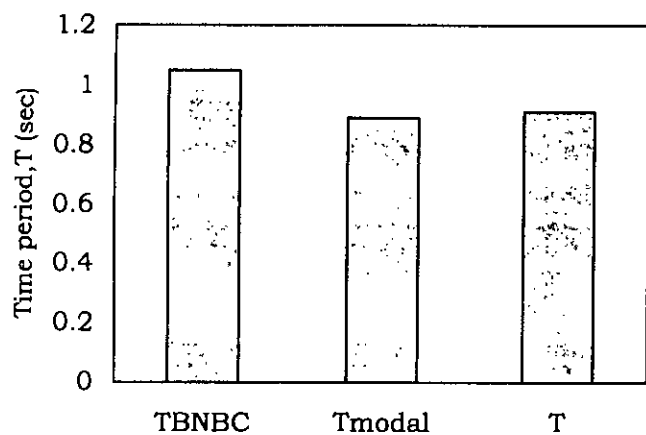


Fig.7.4(a): Comparison of time period obtained by various methods of example 1

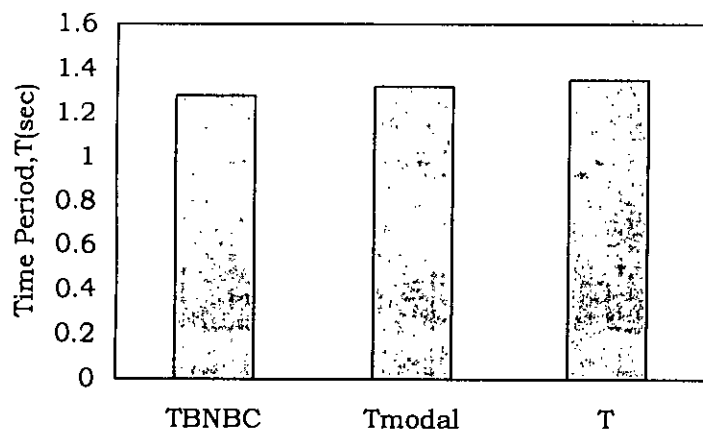


Fig.7.4(b): Comparison of time period obtained by various methods of example 2

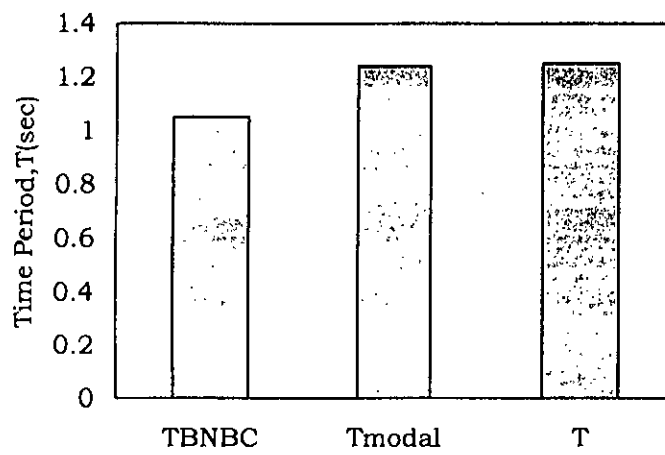


Fig.7.4(c): Comparison of time period obtained by various methods of example 3

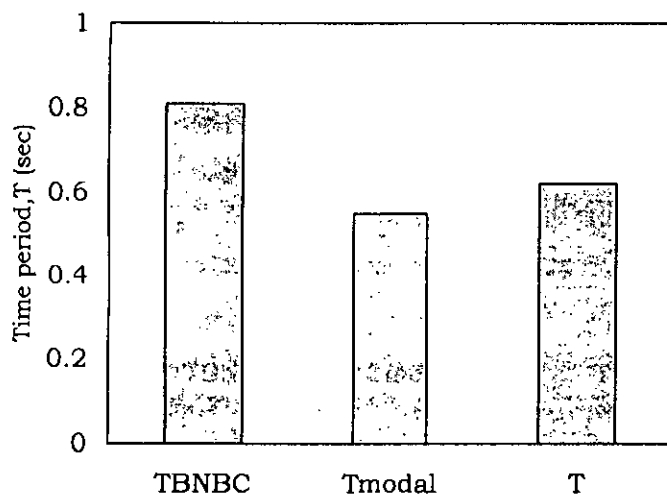


Fig.7.4(d): Comparison of time period obtained by various methods of example 4

7.6 EFFECT ON BASE SHEAR

As discussed in Art. 2.2.1; the formula for calculating the base shear by UBC-94 and BNBC - 1993 is

$$V = \frac{ZIC}{R_w} W \quad \text{where, } C = \frac{1.25S}{T^{2/3}}$$

(the parameters have been described in article 2.2.1)

It is seen that the base shear increases as the C value increases. Again, the C value is inversely proportional to the period of the structure. So, as the period of the structure increases, the C value of the structure decreases and thereby the magnitude of the base shear decreases. With the decrease of the value of the base shear the design earthquake forces also decreases. The variation of the structure parameters that influence the magnitude of the period and thereby influence the C-value of the structure leading to change of earthquake forces are discussed in art. 7.2.

From the discussion of art.7.2, the variation of the structure parameters those effectively can reduce design earthquake forces on a structure are discussed below.

- By increasing the number of story or by increasing total building height, the time period can be increased. Hence, the C-value can be reduced and as a result, base shear will be decreased.
- By increasing panel size, the time period can be increased and hence the C-value can be reduced. Hence, increasing the panel size may effectively reduce the earthquake forces on structure.
- The time period is inversely proportional to the amount of infill present in the panels. So reducing the amount of infill can effectively increase the time period and hence the C-value will be decreased and thus the base shear will be decreased.

CHAPTER 8

CONCLUSIONS AND RECOMMENDATIONS

8.1 GENERAL

An extensive computational investigation has been performed on reinforced concrete framed buildings having masonry infilled panels to identify the relative importance of different structure parameters as required in earthquake resistant design. Focus is primarily given on reasonable estimation of the natural period of vibration of structures, which is an important parameter of earthquake resistant design by equivalent static force method. Based on the investigation a few parameters have been identified to have significant influence on the natural period of vibration. These parameters are as follows:

- 1) Number of story or height of the building
- 2) Panel size
- 3) Percentage of infill.

It has been found that the code equations can predict period of vibration adequately. However, there are scopes of further improvement of code equations incorporating the effect of these three parameters. Based on the findings of the study, proposal is made to apply correction factors to the time period obtained by code equations (BNBC) to make the estimation more reasonable.

8.2 FINDINGS IN BRIEF

The findings of the study presented in chapter 4,5,6 and 7 on the characteristics of infilled frame are summarized below:

- 1) The period is found to increase with increasing story height and number of stories. The code formulae recognize this fact. The results of codes are approximate and may be modified to be more consistent with the results of the modal analysis considering the effect of infill.

- 2) In presence of infill, column stiffness or beam stiffness does not have any appreciable effect in influencing the period. Hence the idea of making flexible floor (by reducing the column or beam size) to increase period and consequent reduction in base shear may not be effective, if infills are present in a floor.
- 3) Modal analysis with infill results in period close to the same predicted by code equations while analysis without infill results in significantly longer period. This phenomenon has been observed for all parameters studied.
- 4) The code equations cannot recognize the effect of panel size on period of vibration.
- 5) For different amount of floor height, the period predicted by code equations (except IS and NBC) agrees reasonably well with the period given by modal analysis with infilled frame.
- 6) When the number of span varied along the direction of motion the time period also varied to some extent, which the code equations do not take into account.
- 7) When the amount of infill varies in the structure, the resulting period from modal analysis shows some variation. Code equations are not capable of representing this variation.

Based on the above findings it is felt that the code equations may be refined for a more reasonable prediction of time period.

8.3 A RATIONALE FOR ESTIMATING PERIOD OF VIBRATION

On the basis of the investigation presented in this thesis, three parameters namely (i) Number of stories or total height of the building, (ii) Panel size and (iii) Amount of infill have been identified as the most important parameters influencing the natural period of vibration. As such, any modification scheme to the period proposed by code equation must include these three parameters. Based on this notion, three multiplying factors corresponding to these three quantities have been proposed. Choosing BNBC as the reference code, a reasonable estimation of the period can be made by multiplying the time period predicted by BNBC equation with these three factors. The details of this proposal can be found in chapter 7. The validity

of the proposal has also been established by calculating and comparing time period of a few arbitrary examples.

8.4 RECOMMENDATIONS

The study presented in the investigation has been carried out for framed structures having regular geometry. The number of story was limited from 4 to 16. In plan, the number of bays and spans were also limited. The study was conducted on the assumption that infill may occur on any floor including the ground floor.

The findings of the present investigation should be interpreted with the frame work of these limitations. It can therefore be said that more work can be done in this line to overcome such limitations. Following is a list of recommendations that may be carried out for further advancement of research.

- 1) The study may be carried out for structures having different floor plans at different height.
- 2) The scope of the present investigation can be expanded by including more stories, bays and spans under investigation.
- 3) Equivalent strut model is used in this study to represent the infill. The behavior of structure using other models of infills can be studied.
- 4) Recognizing the fact that, ground floors are sometimes used as parking and hence contain no infill, similar study can be carried out with infills only on upper floors except on ground floor.

REFERENCES

- Andrus, F.M. (1952). Earthquake Design Requirement of Uniform Building Code. Proceedings of Symposium on Earthquake and Blast Effects on Structures, Los Angeles, June.
- Angel, R., Abrams, D., Uzarski, J. and Webster, M.(1994). Behavior of reinforced concrete frames with masonry infills. Rep.UILU-94-2005, Dept. of Civ. Engrg., Univ. of Illinois, Urbana- Champaign,III.
- Basu, A.K. Nagpal, A.K. and Nagar, A.K. (1982). Dynamic Characteristics of Frame-Wall System. J. Struct. Engrg., ASCE, Vol. 94, ST2, February.
- Bertero, V.V. and Brokken, S. (1983). Infills in seismic resistant building. J. Struct. Engrg., ASCE, 109(6), 1337-1361.
- Besseling, J.F.(1958). A theory of elastic, plastic and creep deformations of an initially isotropic material showing anistropic strain-hardening, creep recovery and secondary creep. J.Appl.Mech.,22,529-536.
- Blume, J.A. (1958). Structural Dynamics in Earthquake-Resistant Design, Transactions, ASCE, V.125, Paper No. 3054.
- Blume, J.A. (1967). Dynamic Characteristics of Multistory Building ASCE, Vol, 94, ST 2, February.
- Borst, R. de and. Feenstra, P.H (1990). Studies in anistropic plasticity with reference to the Hill criterion, Int.J.Numer.Meth.Engng.,29,315-336.
- Chandrapatla, T.R. and Belegmdu, A.D. (1991), Introduction to Finite Elements in Engineering, Prentice-Hall of India, New Delhi, 110001.
- Chopra, A.K. and Yim, S.C.S. (1987). Dynamics of Structures Uplift, ASCE, Vol. 624, No. 1 April, 906-930.

Clough, R.W. (1960). Dynamic Effects of Earthquakes, Transactions, ASCE, V. 126, Part II, Paper No. 3252.

Clough, R.W. and Penzien, J. (1993). Dynamics of Structures, Mc Graw-Hill Book Company, New York.

Council of the Standards Association of Australia (1979). Standard Association of Australia Earthquake Code, Australia.

Dawe, J.L., and Seah, C.K. (1989). Behavior of masonry infilled steel frames. Can. J. Civ. Engrg., Ottawa, Canada 16(6), 856-876.

Dhanasekar, M., and Page, A. W. (1986). The influence of brick masonry infill properties on the behavior of infilled frames. Proc., Inst. of Civ. Engrs., London, U.K., Part 2, 81(Dec.), 593-605.

Fintel, M. (1974). Hand Book of Concrete Engineering, CBS Publishers and Distributors, 485, Jain Bhawa Bholanath Nagar, Shahdra, Delhi, 110032.

Fiorato, A. E., Sozen, M.A. and Gamble, W.L. (1970). An investigation of the interaction reinforced concrete frames with masonry filler walls. Rep. UILU-ENG 70-100, Univ. of Illinois, Urbana -Champaign, III.

Flanagan, R.D., Bennett, R.M. and Barclay, G.A. (1992). Experimental testing of hollow clay tile infilled frames. Proc., 6th Can. Masonry Symp., Univ. of Saskatchewan, Canada, 633-644.

Ghosh, S.K. and Domel, J.R. (1992). Design of Concrete Buildings for Earthquake and Wind Forces. Portland Cement Association and International Conference of Building Officials, 5420, Old Orchard Road, Skokie, Illinois, 60077-1083.

Goel R.K. and Chopra A. K. (1998). Period Formulas For Concrete Shear Wall Buildings. J. Struct. Engrg., ASCE, Vol. 124, No. 4. April. 426-433.

Heidebrecht, A.C., and Stafford-Smith, B. (1973). Approximate Analysis of Tall Wall -frame Structures. J. Struct. Div., ASCE, 99(2), 199-221.

Hill, R. (1948). A theory of the yielding and plastic flow of anisotropic metals. Proc. Roy. Soc., (London) A193, 281-288.

Hoffman, O., (1967). The brittle strength of orthotropic materials. J. Composit Mat., 1, 200-206.

Holmes, M. (1961). Steel frames with brick work and concrete infilling. Proc., Inst. Civ. Engrs., 19(6501), 473-478.

Hossain, M. (1997). Non linear finite element analysis of Wall-Beam structures. Ph.D. thesis, Department of Civil Engineering, BUET.

Hossain, Zahid. M. (1997) . Influence of structure parameters on period of frame structures for earthquake resistant design. M. Sc. Thesis Department of Civil Engineering, BUET.

Housing and Building Research Institute and Bangladesh Standards and Testing Institution (1993). Bangladesh National Building Code (BNBC).

Housner, G.W. (1959). Behavior of Structures During Earthquakes, Proceedings, ASCE, Vol. 85, EM 4, October.

Indian Standard Institutions (1970). Indian Standard Criteria for Earthquake Resistant Design of Structures. Manak Bhavan, 9 Bahadur Shah Zafar Mang, New Delhi, India.

International Association for Earthquake Engineering (1973). Earthquake Resistant Regulations. A World List, Gakujuthu Bunken Iukyu-Kai, Oh-Okayama, Meguroku, Tokyo, 152.

International Conference of Building Officials. Uniform Building Code (1971, 1985, 1991, 1994, 1997), Willier, California.

Joint Committee of San Francisco, California Section (1951). Lateral Forces of Earthquakes and Wind, Proc, ASCE, Vol. 77, Separate No. 66, April.

Kahn, L.F., and Hanson, R.D., (1979). Infilled walls for earthquake strengthening. J. Struct. Div., ASCE, 105(2), 283-296.

Klingner, R. E. and Bertero, V.V. (1976). Infilled frames in earthquake-resistant construction. Rep. EERC 76-82, Univ. of California, Berkeley, Calif.

Liau, T.C. and Kwan, K.H. (1985). Unified plastic analysis for infilled frames. *J. Struct. Engrg.*, ASCE, 111(7), 1427-1449.

Li Q., Cao H. and Li Kwan, G.(1994). Analysis of Free Vibrations of Tall Buildings. *J. Engrg. Mech.*, ASCE, Vol. 120, No. 9, 1861-1876.

Lourenco, P. B., Brost, R. D. and Rots, J. G. (1997). "A Plane Stress Softening Plasticity Model For Orthotropic Materials", *International Journal For Numerical Methods In Engineering*, Vol. 40, 4033-4057.

Madan, A. Reinhorn, A. M., Mander, J. B., and Valles, R. E.(1997). Modeling of Masonry Infill Panels For Structural Analysis. *J. Struct. Engrg.*, ASCE Vol. 123, No. 10, October, 1295-1297.

Mainstone, R.J. and Weeks, G. A. (1970). The influence of bounding frame on the racking stiffness and strength of brick walls. *Proc.*, 2nd Int. Brick Masonry Conf., Stoke-on-Trent, England, 165-171.

Mander, J.B., Nair, B., Wojtkowski, K., and Ma, J. (1993). An experimental study on the seismic performance of brick-infilled steel frames with and without retrofit. Rep. NCEER-93-0001, State Univ. of New York at Buffalo, N.Y.

Ministry of Construction and other Relevant Order, Japan (1987). The Building Standard Law of Japan (BSLJ). Earthquake-Resistant Design Method for Buildings, Part 2.

Noor, Munaz A. (1997). Reappraisal of Code Provisions for Structures Subjected to Seismic Loading. M.Sc. Thesis, Department of Civil Engineering, BUET, Dhaka.

Papia, M.(1998). Analysis of Infilled Frames Using a Coupled Finite Element And Boundary Element Solution Scheme. *International Journal For Numerical Methods In Engineering*, Vol. 26, 731-742.

- Rankaj and Gambhir, S. (1992). On Critical Earthquake Direction and Treatment of Accidental Torsional Response in 3 Dimensional Frame Analysis. Bull, Ind. Soc. Earth, Tech, Paper No. 326, Vol. 29, No 4, Dec. PP. 37-52.
- Rutenberg, A. (1975). Approximate Natural Frequencies for Coupled Shear Walls. J. Earthquake Engrg. and Struct. Dyn., 4(1), 95-100.
- Selvam, V.K. and Jayasree, K. (1993). A Note On Modal Analysis. Bull. Ind. Soc. Earth, Tech, Paper No. 330, Vol. 30. No. 1. Mar. PP. 27-33.
- Saneinejad, A. and Hobbs, B. (1995). Inelastic Design of Infilled Frames. J. Struct. Engrg., ASCE, Vol. 121, No. 4. April. 634-643.
- Schellekens, J.C.J. and Borst, R.de. (1990). The use of the Hoffman yield criterion in finite element analysis of anistropic composites. Comput.Struct.,37,1087-1096.
- Schueller, W. (1977). High-Rise Building Structures. John Willer and Sons, Inc. Canada.
- Seismology Committee (1958). Recommended Lateral Force Requirements. Structural Engineers Association of California, Yosemite California, October.
- Shen, M.S. and Wang, P.C. (1987). A Review of Horizontal Force Factors For Moment-Resisting R.C. Frames. US-ASIA Conference On Engineering for Mitigating Natural Hazards Damage, Bangkok, Thailand.
- Smith, B. S. and Coull (1991). A.Infilled-Frame Structures, Tall Building Structures Analysis And Design, John Wiley & Sons, inc. 168-174.
- Stafford Smith, B. (1962). Lateral stiffness of infilled frames. J. Struct. Div., ASCE, 88(6), 183-199.
- Stafford Smith, B. (1966). Behavior of the square infilled frames. J. Struct. Div., ASCE, 92(1), 381-403.

Stafford Smith, B. (1967). Methods for predicting the lateral stiffness and strength of multi-story infilled frames. *Build. Sci.*, Vol. 2, Pergamon Press, Oxford, U.K., 247-257.

Stafford-Smith, B., and Crowe, E. (1986). Estimating Periods of Vibration of Tall Buildings. *J. Struct. Engrg.*, ASCE, 112(5), 1005-1019.

Standard Association of Australian Earthquake Code (SAA, 1979).

Swanson Analysis System (1995), ANSYS Users Manual, Vol I, Vol II, Vol III, Vol IV, Po. Box. 65, Johnson Road, Houston.

Taranath, B.S. (1976). *Structural Analysis and Design of Tall Buildings*. McGraw-Hill Book Company, New York.

The National Research Council of Canada (1995). National building code of Canada (NBC).

Ulrich, F.P. and Carder, D.S. (1952). Vibrations of Structure, Proceedings of Symposium on Earthquake and Blast Effect on Structures. EERI and University of California, Los Angeles, June.

Uzumeri, S.M. (1995). Development of Canadian Seismic Resistant Design Code for Reinforcement Concrete Buildings. Thomas Poly Symposium. Recent Development in lateral force transfer in buildings, SP157, ACI, Detroit, PP.301-330.

Wood, R.H. (1978). Plasticity, composite action and collapse design of reinforced shear wall panel in frames. *Proc., Inst. of Civ. Engrg.*, London, U.K., Part 2, 65, 381-411.

Zarnic, R. and Tomazevic, M. (1990). Behavior of repaired masonry infilled R/C frames. Rep. To U.S.-Yugoslav Joint Board on Scientific and Tech. Cooperation, Inst. For Testing and Res. In Mat. And Struct., Ljubljana, Yugoslavia.

Zienkiewicz, O.C. (1979). *The Finite Element Method*. TMII edition, McGraw-Hill Publishing Company Limited, New Delhi.

