A SIMPLIFIED ANALYSIS AND DESIGN OF FREE STANDING STAIRS RESTING ON PARTIALLY RIGID SUPPORT

A THESIS BY

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CONTENTS

DECLARATION

Declared that, except where specified by references to other investigators, the work embodied in this thesis is the results of investigation carried out by the author under the supervision of Dr. Khan Mahmud Amanat, Associate professor of Civil Engineering Department, BUET.

Neither the thesis nor any part thereof has been submitted or is being concurrently submitted in candidature for any degree at any other institution.

Author

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ABSTRUCT

Conventional design of a free standing stair is dependent on some approximate analytical methods. The conventional methods fail to recognize the variation of stress resultants across any cross section of the stair slab. Amanat [1993] made an extensive finite element study on free standing stairs and proposed a simplified but rational guideline for easy analysis of free standing stairs based on seven semi-empirical equations for direct evaluation of forces and moments at critical locations. His approach recognizes the stress variation across different sections of the free standing stair..

Later, in the study of Zahid [1999], two equations were developed for two redundants, namely the lateral shear and the bending moment at the mid-landing section. The moments and forces at the other critical locations can be calculated using equilibrium equations. In all these studies, supports were assumed completely fixed at floor levels. In the present study, two sets of empirical equations are developed for three cases of loading for stairs resting directly on the floor slab which can provide only partial rigidity as support. Other necessary design parameters are obtained from equilibrium equations. Ten example stairs are solved by using the proposed equations and finite element analysis. Comparison of these results establish the acceptability of the proposed equations.

The proposed method has rendered the calculations of forces and moments very easy, and the estimation is found to be within the acceptable limits for commonly used proportions of stairs.

LIST OF SYMBLES

A	Extended portion of upper landing
B	Width of flights
C	Horizontal gap between flights
D	Width of flights
E	extended portion of upper landing
E_C	Modulus of elasticity of concrete
E_S	Modulus of elasticity of steel
F	Width of upper landing
F_{B1}, F_{C1}	Factors corresponding to B,C for load case one
G	Projected length of flights
H	Width of lower landing
H_o	Lateral shear at mid-landing section
h	Floor to floor height
$K_{1,2}$	Numerical constant for load case one and two
M_o	Bending moment at mid-landing section
M_s	Bending moment at slab-flight junction
M_t	Torsional moment
T_1	Slab thickness of upper landing
T_2	Waist slabs & lower landing thickness
V_x	Axial force on a section
V_Y	Lateral shear on a section
X, Y, Z	Global co-ordinate system
X', Y', Z'	Local co-ordinate system
$f'c$	28 day cylinder strength of concrete
f_s	Allowable stress in steel
f_y	Yield strength of steel
p	Transverse shear stress
σ	Normal stress
α	Angle of inclination of flights with horizontal
τ	Lateral shear stress

CHAPTER 1
INTRODUCTION

1.1 GENERAL

Stair is an important functional element in a building, whether tall or low rise. It may be defined as a series of steps arranged for the purpose of connecting different floors of a building. At the time of any emergency evacuation, like an earthquake or a fire, stair is subjected to its maximum loading. At the peak hour in a commercial, center or market, a stair is used not only to facilitate transport of people between floors but also to provide an elegant look to the building. Architecturally, it must be fascinating and beautiful; structurally, it must be strong, stiff, efficient; functionally it must allow smooth and free transit of people from floor to floor; and seismically, it must be flexible enough to withstand the shock load.

Depending on various architectural forms, stairs may be classified as (Fig. 1.1) :

- Free standing stair,
- Slabless stair,
- Helical stair, and
- Simple straight stair.

Compared with other structural components of a building, stairs have some unusual characteristics. Stairs are an assemblage of interconnected plates in a three-dimensional space and supported at the outer edges of these plates. Both in-plane and out-of-plane forces may be predominant in the stair depending on the arrangement of supports. In simple stairs, supports are provided at floor levels and at intermediate landing. Previous investigations on this type of stairs revealed that load is transferred primarily through bending of stairs slabs. In plane forces, although present, are of little significance. The situation changes totally when the supports at landing is removed and the stair is allowed to stand freely between supports at floor levels only. The structural functioning of this free standing stair is completely different from that of the simple stairs. While the functioning of simple stairways can be approximated as a two dimensional flexure

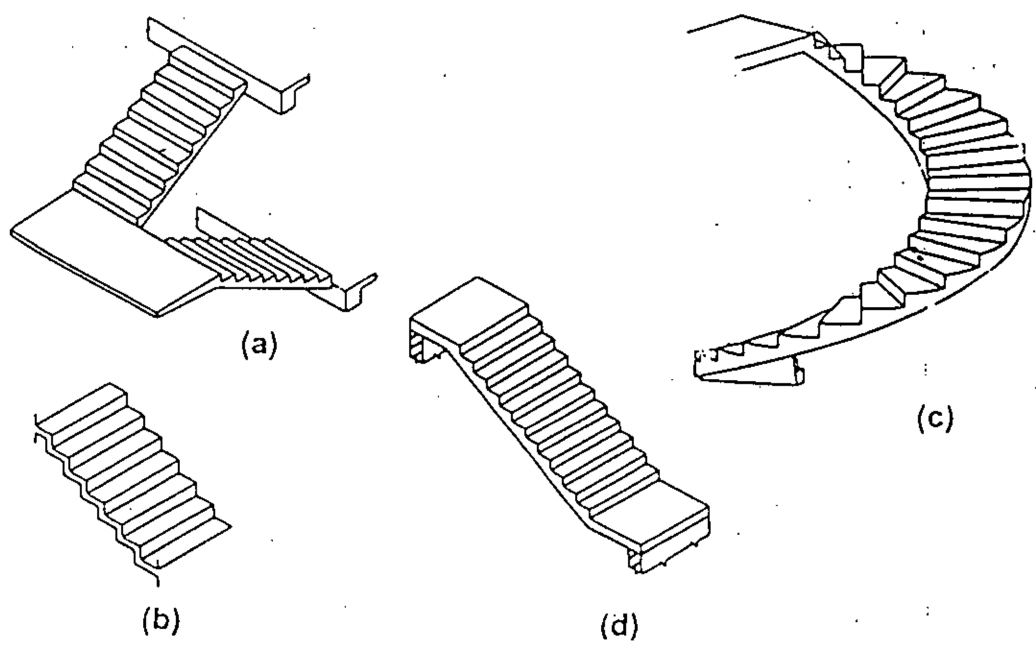


Fig. 1.1 Stairs of Different Types

a) Free standing stair

b) Slabless stair

c) Helical stair

d) Simple straight stair

problem, there is no way of similar treatment to the free standing stair which is truly a three dimensional problem. Consequently, the structural analysis of a free standing stair is much more complex in comparison to that of the ordinary stairs. From architectural viewpoint, free standing stairs are more attractive than simple stairs. Another architecturally fascinating stair of similar structural action is the helicoidal stair. But construction of a helicoidal stair is difficult because of its geometry. A considerable part of the total cost goes to the construction of the formwork of a helicoid. On the contrary, the formwork of a free standing stair is simple. Unlike simple stairs, a free standing stair does not need a separate stair hall for its construction. Despite this , construction of free standing stairs has been limited in the past. This is mainly due to the lack of adequate knowledge about its complex three-dimensional behavior and absence of a simple design procedure. Now, with the advent of finite element method along with the availability of high speed digital computers, it is possible to analyze the free standing stair more accurately and rationally.

1.2 BACKGROUND OF RESEARCH

Stair is the most essential feature of all types of buildings. Stairs, especially free-standing stair received attention from researchers in the past. The subsequent methods of analysis developed by various authors indicate it to be a structure carrying torsion and in-plane moments besides bending and shear. In their analysis they made several assumptions to simplify the actual structural behavior of the stair. Sauter (1964), Taleb (1964), Cusens and Kuang (1966) replaced the slab structure by an equivalent skeletal rigid frame. Liebenberg (1956) and Sieve (1962) retained the space plate configuration of the stair. Amanat (1993) proposed a set of explicit empirical equations to determine the various forces and moments at different critical locations of the stair which do not necessitate any formal analysis of the stair. This simplified the analysis and design of the stair to a great extent. Zahid (1999) simplified it further by developing the equations on the basis of equilibrium of the stair. In all these studies, the stair is assumed to be held completely fixed at floor levels. This assumption is reasonable if the stair is supported by a sufficiently rigid beam at floor levels. In present days, with the increase in quality of concrete materials, there is a general tendency of eliminating the supporting beam to give the stair a better architectural look. In this case, the stair rests directly on the floor slab, which can provide only partial rigidity as support. In such a case the concept of rigid support no longer holds valid and the stair analysis methods discussed so far becomes too approximate to be followed for design. It is thus necessary to develop some guideline to analyze and design such a free-standing stair resting directly on floor slabs

1.3 OBJECTIVES AND ASSUMPTIONS

The objective of the present investigation is to develop semi-empirical design equations for free standing stairs resting directly on floor slabs. For a free standing stair having symmetric loading and fixed boundary conditions the redundants are the bending moment and lateral shear at mid-landing section. It is assumed that this also holds reasonably true for stairs resting on floor slabs. Thus, if these two redundants are known, the other forces and moments can easily be determined from the equilibrium of the stair. The present investigation is aimed at developing some explicit equations to determine these redundants in terms of the basic design parameters of the stair. The proposed design equations will simplify the analysis and design procedure. It is expected that this study will lead to a simpler and time saving design guideline for free standing stairs resting on floor slabs.

1.4 OUTLINE OF MEHTODOLOGY

A typical stair slab will be modeled with standard dimensions including a portion of floor slab. Ahmad's (1969) thick shell finite element will be used. A detailed parametric study will be carried out. The effect of different design parameters on the bending moment and lateral shear at the middle of landing will be studied. Based on the study a set of equations will be developed to determine these redundants. Other design forces and moments can be calculated considering the static equilibrium of the whole stair once these redundants are known. A few comparisons will be made with the results from the finite element analysis to establish the validity of the proposed equation. Finally, a practical design example will be solved to show the application of the proposed simplified equations.

CHAPTER 2

FREE-STANDING STAIR PARAMETERS

2.1 GEOMETRY OF STAIR SLAB

A free-standing stair is a structure composed of interconnected plates, each located in different planes in three-dimensional space. In its simplest form, it consists of three rectangular plates, namely, lower flight, landing and upper flight. The upper end of upper flight is held at upper floor level. Similarly, the lower end of the lower flight is held at lower floor level. The other ends of flights are connected to a horizontal-landing slab. Steps are provided on the flights.

According to the relative position of the flights, a free-standing stair may be right handed or left handed, as shown in Figs 2.1 and 2.2 respectively. The dimensions required to describe the geometry of the stair slab are shown in Figs 2.3 and 2.4. In these figures:

A =E = Extended portion of upper landing

B =D = Width of flights

C =Gap between flights

F =Width of upper landing

G = Projected length of flights

H = Width of lower landing =B=D

T_1 = Thickness of floor slab

T_2 = Thickness of flights and lower landing

2.2 LOADS ON THE STAIR SLABS

A stair slab is primarily a functional unit in a building system. As a consequence, live loads may act on it in several ways. But in analysis and design, only those few possibilities are considered which produce maximum design moments, shear etc. at some critical points. In this study three possible load combinations are considered. These are:

a) Load caes-1 : Full live load throughout the stairway.

b) Load case-2: Full live load in both flights only.

c) Load case-3: Full live load in landing slab only.

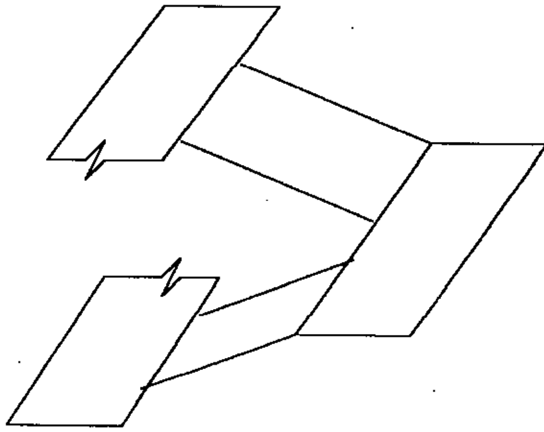


Fig. 2.1 Right Handed Stair

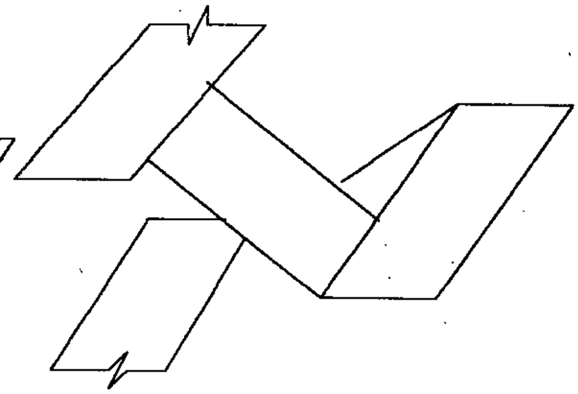


Fig. 2.2. Left Handed stair

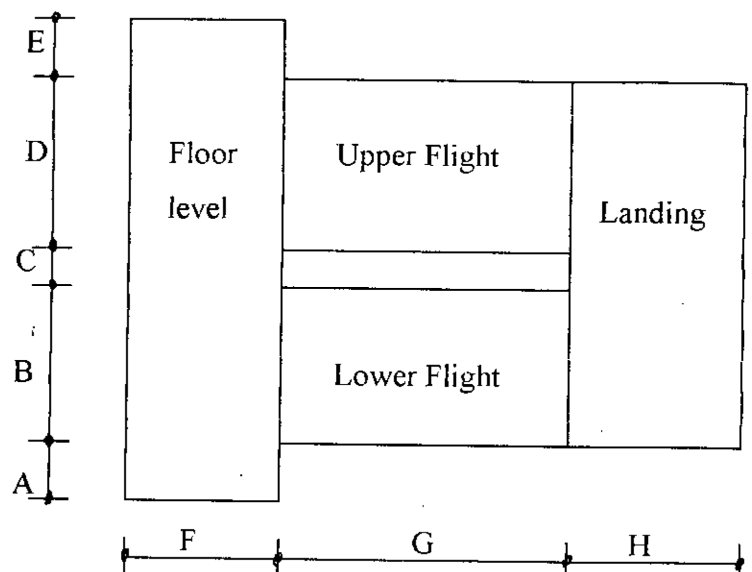


Fig. 2.3 Plan of a free standing Stair

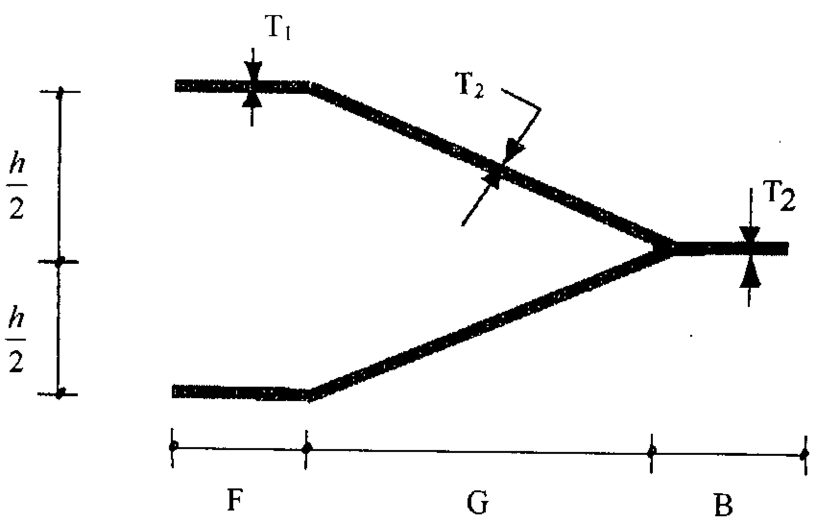
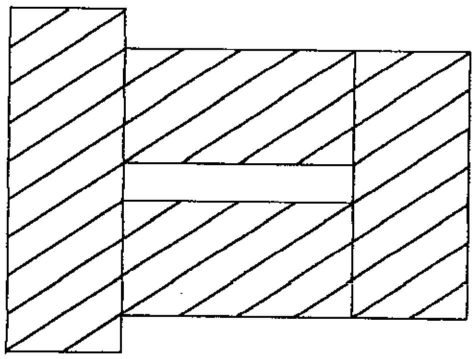
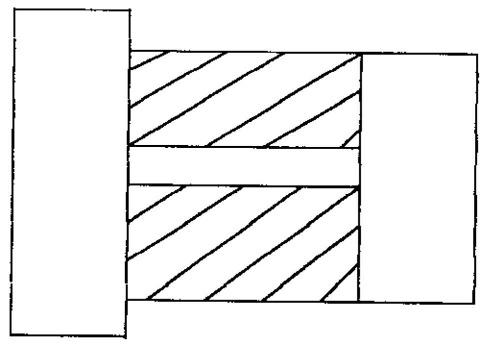


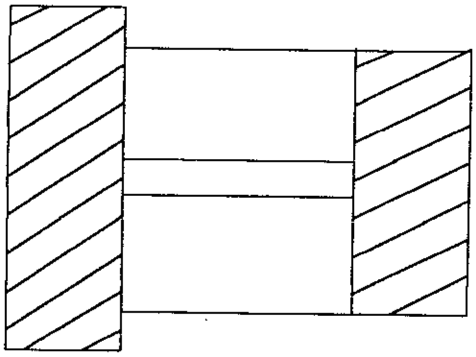
Fig. 2.4 Stair slab geometry (Elevation)



Load Case-1



Load Case-2



Load Case -3

Note : Shaded portion indicates occupation of live load

Fig. 2.5 Load Cases

The above three load cases are schematically described in Fig. 2.5 where shaded portions indicate occupation of live load. All of the above three load cases are symmetric.

There are other arrangements of live load such as full live load on one of the flights only. Such unsymmetrical loading produce unsymmetrical stress in the stair. It was, however, observed that the magnitude of the stresses does not become critical for such unsymmetric loading. Hence only the three loading cases described above are adopted for analysis.

2.3 SIGN CONVENTION

In order to keep consistency and clarity in the analysis and results presented in the subsequent chapters, a unique sign convention for stresses, moments, deflections, etc.is followed throughout. The deflection is presented in global system and is positive when it occurs in positive direction of axes. The positive direction of moments, shears, etc. at a section are set with respect to local axes system. Stresses and forces are positive when they act in the positive direction of respective local axes. For moments, right hand screw rule is followed. These are described in Figs. 2.6 and 2.7.

2.4 BOUNDARY CONDITIONS AND REDUNDANTS

Although the free-standing stair slab is a three dimensional plate structure involving 3-D interaction of plates , the current practice to analyze it involves simplification of it into a space frame structure consisting of linear bar elements. With such linear bar idealization, the stair slab becomes, in general, indeterminate to sixth degrees. The moments M_X, M_Y, and M_Z, shear forces V_Z, V_Y and axial force V_X at the upper flight or lower flight support can be taken as redundants or these actions at the mid-landing section can be considered as redundants. If both the stair slab and the loading are symmetric then the degree of static indeterminacy becomes two. The bending moment (M_o) and in-plane shear (H_o) at mid-landing section can be considered as redundants [Zahid, 1999]. The planner representation of redundants are shown in Fig. 2.8.

The degree of indeterminacy cannot be described in such a simple way if we consider the slab system as a 3-D plate structure. In a plate structure moments, shears, etc. are

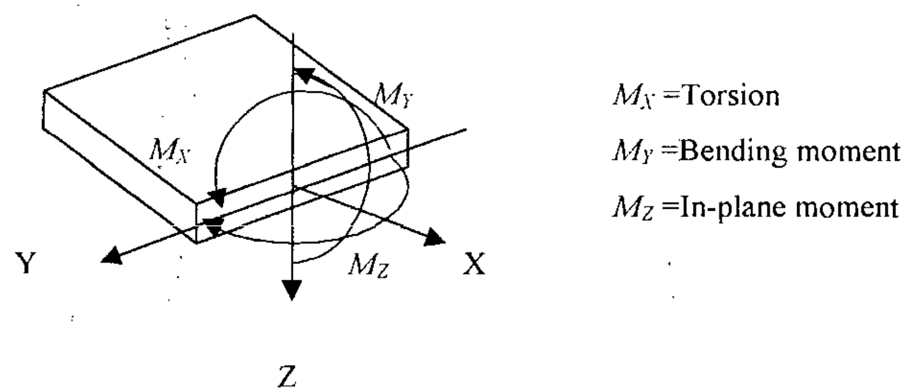


Fig. 2.6 Positive direction of moments.

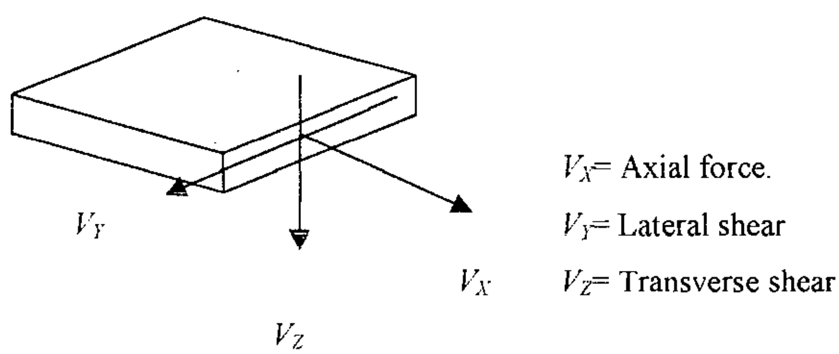


Fig. 2.7 Positive direction of forces.

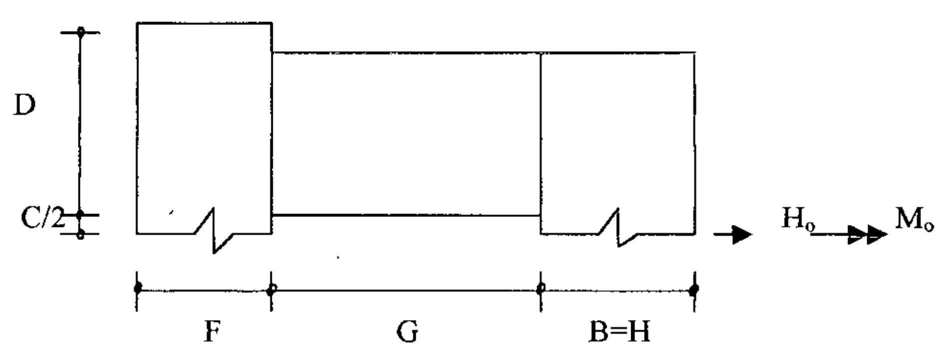


Fig. 2.8 Planer Presentation of redundants at Mid-Landing Section

expressed in terms of stress-resultants at a point. Solution of such a plate structure involves determination of all the stress resultants at all points of the structure.

2.5 MATERIAL PROPERTIES

In this study, it is assumed that the reinforced cement concrete used in the stair-slab is a monolithic structure, homogeneous, isotropic materiel and it obeys Hook's law.

2.6 STAIR SLAB UNDER STUDY

Ten stairs of different dimensions, are studied here. The dimensions and other proportions of these stairs are chosen arbitrarily within practical range. The data for the reference stair is given below.

Extended portion of upper landing (A=E)	= 500mm (Fixed)
Width of lower landing and flights (B=D=H)	= 800-2000mm
Horizontal gap between flights (C)	= 100-1000mm
Width of upper landing (F)	= 1500mm (Fixed)
Horizontal span of flights (G)	= 2250-3500mm
Floor to floor height (h)	= 3000mm (Fixed)
Waist slab and lower landing thickness (T_2)	= 125-250mm
Loads	
Dead load	= as calculated
Live load	= 4.79 kN/m^2 on horizontal projection
Material properties	
Concrete strength, f_c'	= 20680 kN/m^2
Yield strength of steel, f_y	= 275760 kN/m^2
Poisson's ratio of concrete, μ	= 0.15
Modulus of elasticity of concrete, E_c	= 21525 $\sqrt{f_c'}$ kN/m^2

Examples of ten arbitrary stairs selected for study are shown in table 2.1

Table 2.1 Geometric Data of the arbitrary ten stairs

Examples	Width of lower Landing and Flights	Gap between Flights	Horizontal projection of flights	Thickness of lower landing and flights
	B(mm)	C(mm)	G(mm)	T2 (mm)
1	1000	200	2500	125
2	1500	300	2800	150
3	1800	500	3200	175
4	2000	200	3500	225
5	1200	800	2700	150
6	1000	250	2500	125
7	1200	250	2600	150
8	1500	200	2800	150
9	1800	200	3000	175
10	2000	225	3200	200

A =500mm (fixed), B=D=H, and F=1500mm (fixed)

CHAPTER 3
LITERATURE REVIEW

3.1 INTRODUCTION

Shell or plate theories generally lead to considerable complexities in the analysis and design of stairs. Analytical method or a modern numerical technique such as finite element method can be used to analyse the stairs. But practising engineers eagerly look for a simple and straightforward design procedure, which may be somewhat approximate but rational. Such demand resulted in approximation of the stair-slab as a space frame structure composed of bar elements or as a determinate slab structure. A number of papers on these approximate methods of solution of free-standing stair are available in the literature. Amanat [1993] made an useful attempt to propose a simple and direct way of finding the moments and forces required to design a free-standing stair, based on finite element approach. The seven empirical equations proposed by Amanat[1993] give reasonable values of design moments and forces within the specified limit. He also proposed a reinforcement layout scheme for the stair recognising the variation of stress resultants across sections. Zahid [1999] further simplified the analysis by reducing the number of empirical equations from seven to four. However, these proposals are valid only for stairs with completely rigid supports at floor levels.

3.2 CODES AND PRACTICES

The structural behaviour of a free- standing stair is quite different from other ordinary two flight stairs where the landings are supported by beams or side walls. In the ACI and British Code of Practices there are no specific guidelines for the design of a free-standing stair. It is true for the Indian and the other codes of practices as well. Absence of guidelines in these codes discourages practising engineers to design or construct a free-standing stair. Reinforced Concrete Designer's Handbook [Reynolds,1988] includes a design method for a free-standing stair based on the paper published by Cusens and Kuang [1965]. But the procedure described there is not very straightforward and is somewhat abstract due to some simplification.

3.3 ANALYTICAL APPROACHES

The analytical approaches for the free-standing stair slabs may be divided into two types. The first type idealises the stair structure as a space frame of some type. The methods of Sauter [1964], Cusens and Kuang [1965], fall in this category. Idealisation of a plate as a straight frame element can only be justified when the width to length ratio of the plate is small. But this is not true for a free-standing stair where width of plates is of comparable magnitude with respect to its length. In such cases, space frame idealisation does not seem to be a good approximation. The second type retains the plate configuration of the stair but considers the structure to be a determinate one based on some assumptions. The overall structural rigidity resulting from the indeterminacy is lost when such assumption is made. The methods of Siev [1962] and Liebenberg [1956] are of this type.

Of the various analytical approaches, those of Cusens and Kuang[1965], and of Siev's [1962] are preferred in design because of their relatively simple structural idealization and analysis. The analysis suggested by Siev[1962] is the most elaborate one. However, the analysis becomes simple only when the secondary stresses are ignored. In Sauter's approach, once the bar idealization is made, the remaining treatment for analysis is basically the same as discussed in the following articles.

3.3.1 CUSENS AND KUANG'S METHOD

Cusens and Kuang [1965] analysed the free standing stair assuming that its structural behaviour could be simulated by the skeletal rigid frame shown in Fig. 3.2. In their procedure a cut is introduced at the mid landing section and the horizontal restraining force V_{yo} and bending moment M_{yo} are applied at the two halves of the frame. These two quantities are taken as the redundants. Equations for bending moment, shear and axial forces, etc. in the flights are then readily expressed in terms of these two redundants.
Neglecting the effect of axial and shearing forces on deformation, the total strain energy in the structure is evaluated in terms of V_{yo} and M_{yo}. Following the concept of least work, first partial derivative of the strain energy with respect to V_{yo} and M_o are evaluated and then these are equated to zero. This results in a set of two simultaneous equations in terms of V_{yo} and M_{yo}. Once these redundants are known, the structure becomes statically

determinate and the shear forces, bending moments and axial forces at any point of the structure can be calculated using equations of static equilibrium. Cusens and Kuang [1965], in their paper, dealt only symmetrical staircases under symmetrical loading.

3.3.2. SAUTER'S METHOD

Sauter's [1964] method of analysis is based on considering the stair as a space frame composed of linear bar elements. The idealised stair frame consists of two cantilevered straight members projecting out from the upper and lower floor supports and connected at their ends by a horizontal bow girder representing the landing. In his method the two redundants are the bending moments M_y at upper flight support and at mid-landing section. First, the equations of moments and forces are written for unit values of the redundants in turn for the unloaded structure and corresponding deformations are calculated using work integral. Next, considering the redundants to be zero, the structure is analysed and a determinate structure for unit distributed load in flights and then on landing separately. Deformations are also evaluated for these loading in the usual manner. In evaluating the work integrals, effect of shear and axial forces are neglected. Once the deformations are determined for each separate case, the redundants can be calculated from compatibility conditions of deformations (Fig.3.3). In his analysis, Sauter [1964] dealt only symmetric stair slab with symmetric loading.

3.3.3 SIEV'S METHOD

Siev [1962] developed methods for solving stresses in free standing stair slabs under symmetrical and anti-symmetrical loading conditions based on the concept of statically determinate structures. In his procedure the overall stress analysis is accomplished in two stages. In the first stage, the line of intersection between flights and landing is considered simply supported (Fig.3.4). The resulting structure is referred to as the primary slab structure. Bending moments and reactions at the supports are calculated under various possible positions of live load. In the second stage, the flights and landing are considered as interconnected plates. This is referred to as the secondary plate structure. The reaction at the assumed kink line support will act as the only load in opposite direction on this

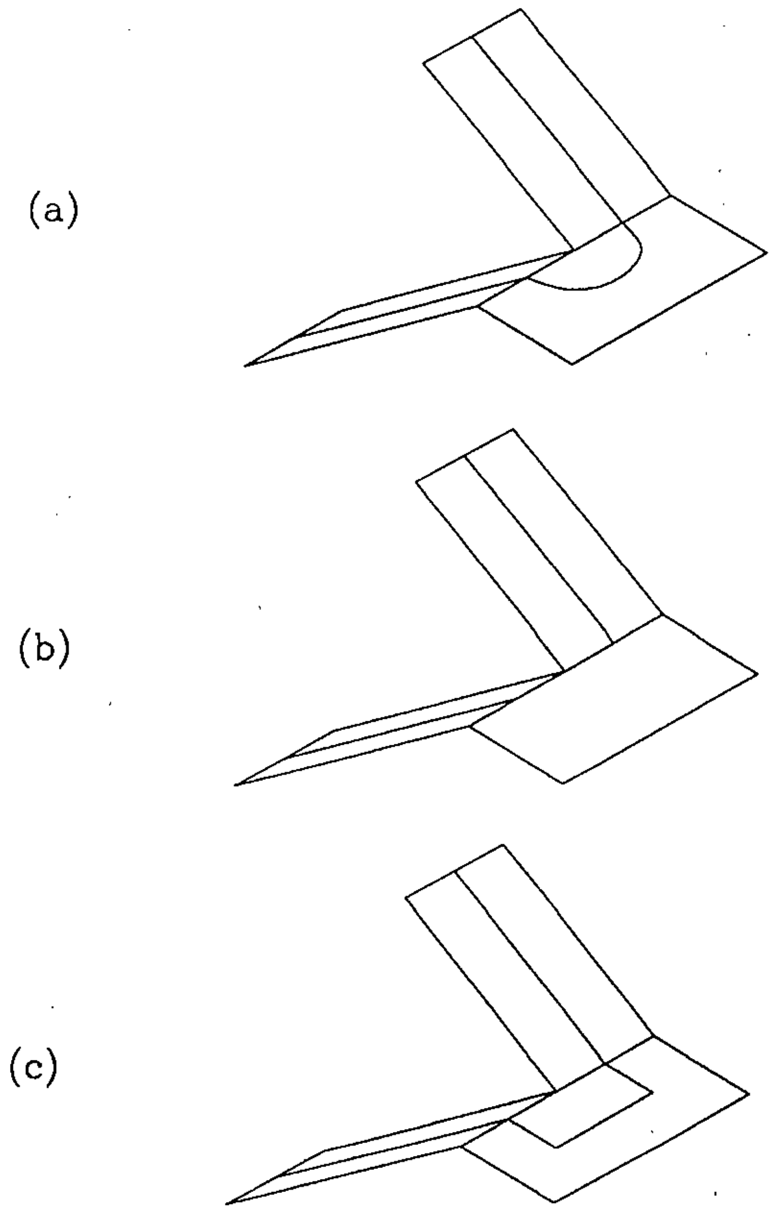


Fig. 3.1 Idealisation in Different Analytical Approaches

a) Fuchsteiner, Sauter [1964]

b) Cusens & Kuang [1965]

c) Taleb [1964]

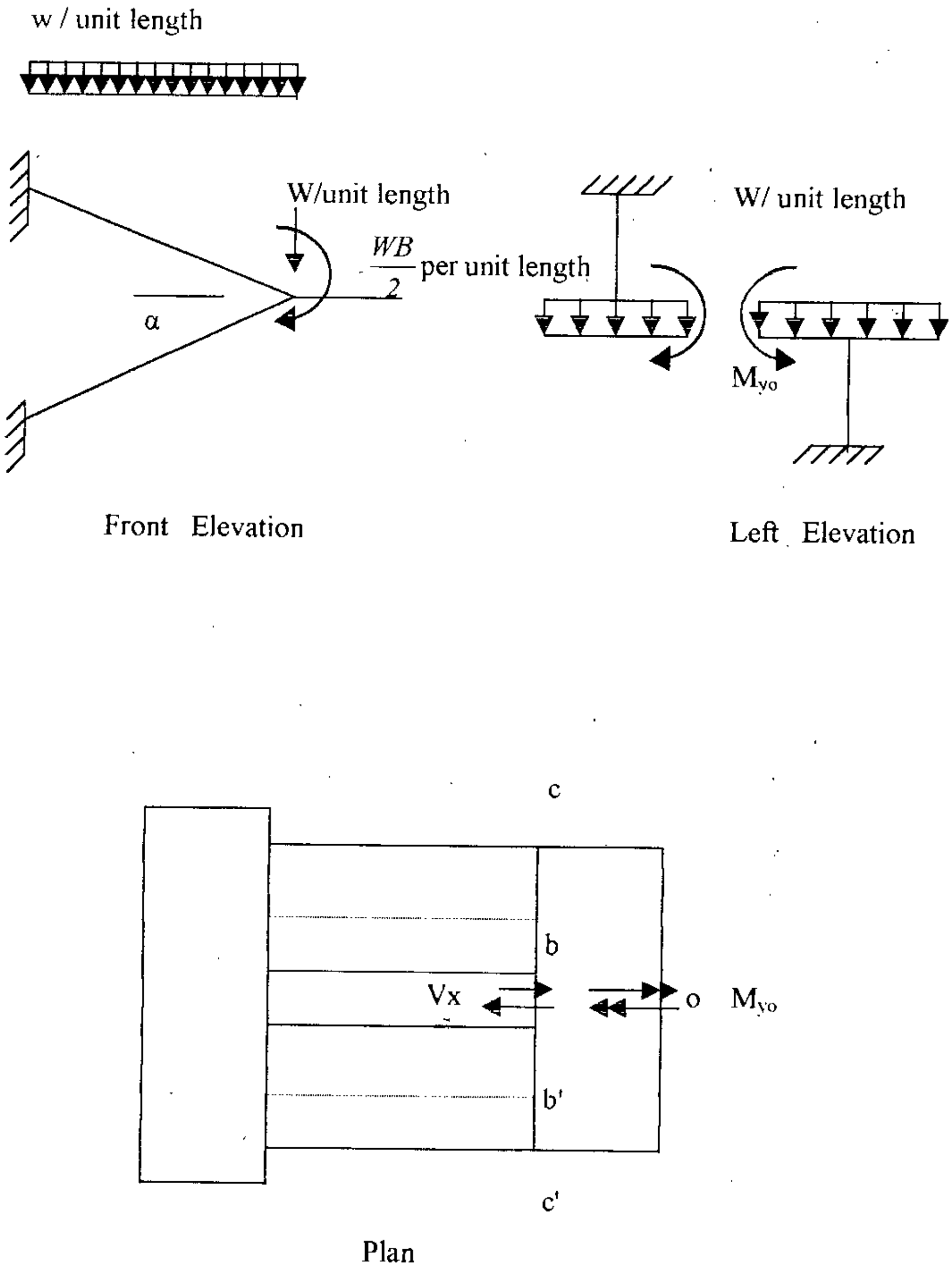
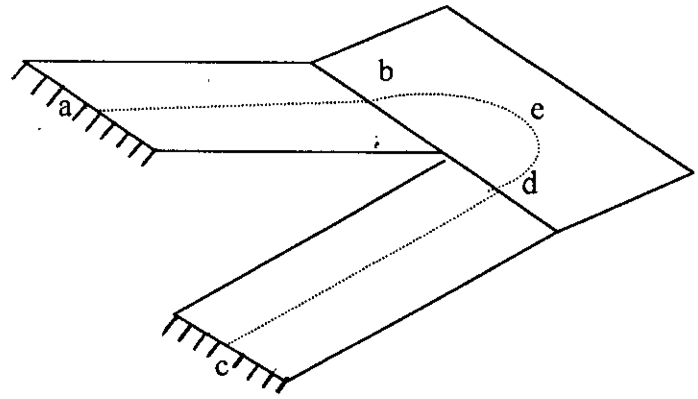
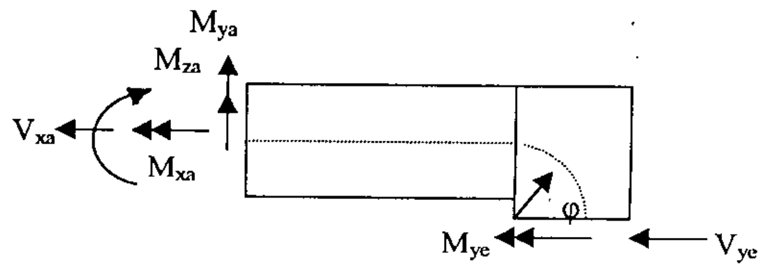


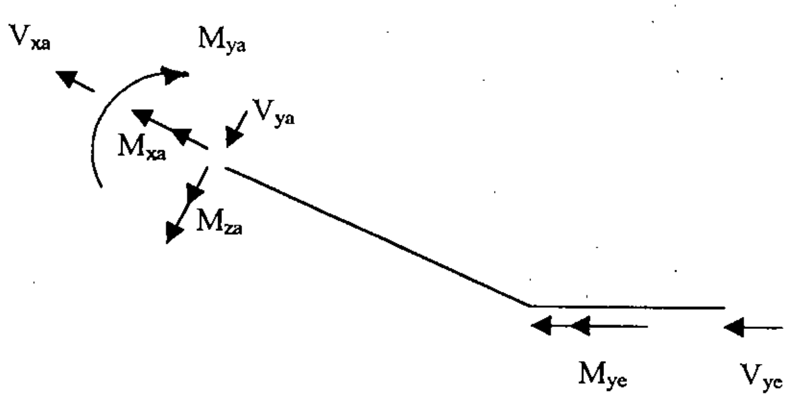
Fig. 3.2 Cusens and Kuang's [1965] Method



Isometric of a free standing stair (Part)



Redundant:
M_{ya} and M_{Ye}



Moments and forces acting on the stair slab

Fig. 3.3 Sauter's Method of Analysis

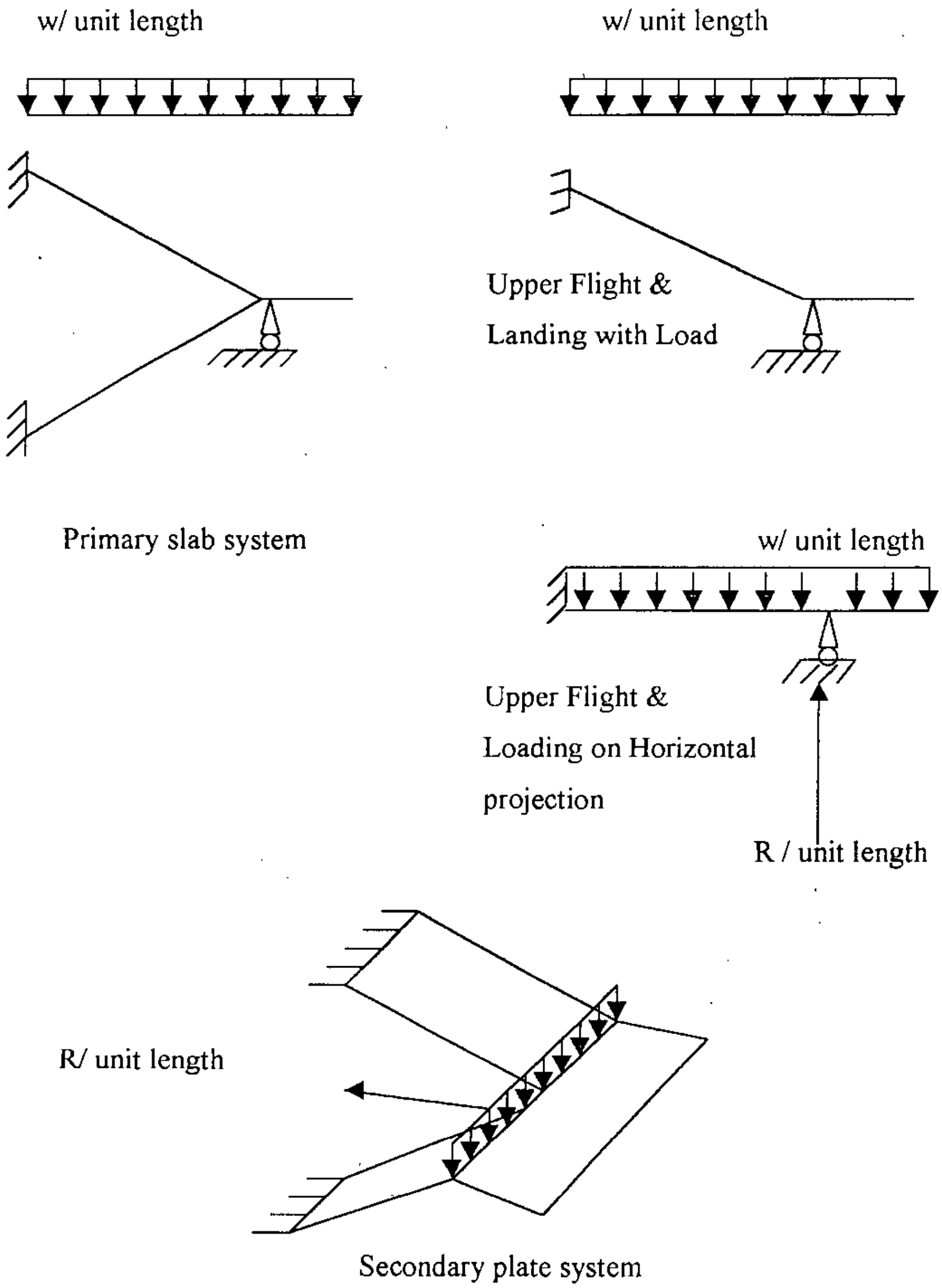


Fig. 3.4 Siev's [1962] Method of Analysis

plate structure. This load is then resolved in the plane of flights, which develops in plane moment and axial forces in flight slabs. Due to these forces, in-plane deflection of flights occur which is calculated using the ordinary flexure formulas. Assuming that the landing slab is very rigid in its plane, compatibility in deformation requires that the kink line will remain straight.

From this compatibility condition, the secondary stresses are determined. Torsional moments are calculated as secondary stresses. The final solution is the combination of primary and secondary stresses. Later it was Siev's conclusion that the secondary effects can be ignored in design.

3.4 SEMI-EMPIRICAL APPROACHES BASED ON FE ANALYSIS

The analytical approaches may not be dependable for the analysis of free-standing stair as long as economy and efficiency in design are concerned. These methods fail to simulate the actual interaction of plates in three dimensions. Also, these approaches cannot demonstrate the variation of stress resultants across the width of the stair. Amanat [1993] made a comprehensive study of the free standing stair based on rigorous finite element analysis. Ahmad's [1969] thick shell elements were used in the analysis. It was assumed that the flights are held completely fixed at floor levels, i.e. the supports are fully rigid. The structural behaviour of the stairway shows that load is transferred predominantly through in-plane bending and torsional moments, assisted by some axial forces. For a symmetric stair under symmetric loading, both flights are equally stressed. That is, stresses and deflections of one flight will be the mirror image of the same in other flight. This implies that the bending moment diagram of both flights will be the same in magnitude and in direction. Axial forces of equal magnitude but opposite sign will develop in flights. In-plane moment of the same magnitude and direction will be developed in flights while flights will be under equal and opposite torsion. The flights have usual transverse shear and zero lateral shears.

In landing, the predominant forces are the bending moments in both X and Y direction, lateral shear at mid-landing section and some torsion at quarter span. All the three components of displacements are equally dominant. Along with the deflection occurring vertically, the whole stairway experiences a horizontal sway towards upper flight.

Amanat [1993] demonstrated that the prediction of bending moments at different critical locations by approximate analytical methods might not always be acceptable. Support bending moment is overestimated in Cusens and Kuang's [1965] approach and underestimated in Siev's [1962] approach. In predicting the in-plane moment in landing, Siev's method seems to overestimate it. Sauter's [1964] and Cusens and Kuang's approaches give reasonably good values of in-plane moment. In case of flight torsion all the analytical methods underestimate the bending moments. Based on the study, Amanat [1993] proposed a set of empirical equations to explicitly determine the required design forces and moments in terms of basic design parameters. These equations are presented in table 3.1. It has been shown that these equations give reasonably accurate values within their limit of applicability. The equations are valid for fully fixed support condition.. All of the equations are of the form,

$$\textbf{Force or Moment} = K.F_A.F_B.F_C.F_L.F_H.F_T.F_f \qquad (3.1)$$

Here K is a numeric constant. F_A is factor corresponding to the geometric parameter 'A' and so on. F_f is the factor corresponding to f_c'. The use of the above equations are straightforward. Values of forces and moments can readily be calculated once the design values of geometric parameters are prescribed.

Zahid [1999] proposed two sets of equations for the lateral shear (V_o) and bending moment (M_o) at the middle of landing for two load cases. The moments and forces at other sections can be determined from equilibrium of the stair. In his study a reference equation was adopted as:

$$Y = a + b\,(X - lower\ rage)^c \qquad (3.2)$$

Where, Y stands for either H_0 , M_0 ; X are the value of the concerned geometric parameters and a , b and c are numerical constants. Using the results of finite element

Table 3.1 Equations Proposed by Amanat[1993] (SI Unit)

		PARAMETRIC FUNCTIONS OF STAIR							
		F_A	F_B	F_C	F_L	F_H	F_T	F_f	
Design Parameters	Const.	$150 < A(mm) < 1000$	$915 < B(mm) < 1875$	$915 < C(mm) < 1900$	$2030 < L(mm) < 3550$	$2440 < H(mm) < 4320$	$100 < T(mm) < 280$	$14 < f_c'$ (Mpa) < 40	Values of Design Parameters
Vertical deflection at landing corner, mm	2.03	$1+0.00545(A-125)^{0.94}$	$1-0.00114(B-914)^{1.1}$	$1+0.00165(C-914)^{0.93}$	$1-7.87\times10^{4}(L-2030)$	$1-19.68\times10^{4}(H-2440)$	$1-0.161(T-100)^{0.334}$	$1-1.074\times10^{4}(f_c'-14)^{.93}$	
Support negative moment, kN-m	-4.712	$1.555+0.000787(A-50)$	$1.06-0.00022(B-860)$	$1.2+0.00276(C-864)$	$1+0.000748(L-2030)$	$1+5.9\times10^{4}(H-2440)$	$0.39-0.00173(T-90)$	1.0	
Flight midspan positive moment, kN-m	-1.526	$1.1-31.48\times10^{4}(A-150)^{1.52}$	$1-70.11\times10^{4}(B-915)^{1.365}$	1.0	$1+0.128\times10^{4}(L-2030)^{2.66}$	$1+0.899\times10^{4}(H-2440)^{2.77}$	$1-.00165(T-100)^{1.17}$	1	
Negative moment at kink, kN-m	-3.447	$1.23+0.000512(A-125)$	$1.01+0.00323(B-915)$	$.85+0.000709(C-915)$	1.0	1.0	$0.95+0.00447(T-100)^{1.03}$	1	
Negative moment at midsection of landing, kN-m	-6.14	$1+0.000303(A-150)$	$1+0.00118(B-915)$	$1+0.00106(C-915)$	$1+0.000409(L-2030)$	$1+26.37\times10^{4}(H-2040)$	$1-0.00185(T-100)$	1	
Axial force in flights, kN	34.69	$1+0.000236(A-125)$	$1+0.000787(B-915)$	$1+0.000827(C-915)$	$1+0.000354(L-2030)$	$1-0.000157(H-2440)$	$1+0.00276(T-100)$	1	
Torsion in flights, kN-m.	2.312	$1+0.00177(A-125)$	$1+0.00063(B-915)$	$1+0.00268(C-915)$	$1-8.0\times10^{4}(L-2030)^{.75}$	1.0	$1+0.00358(T-100)$	1	
Inplane moment in flights, kN-m	-14.35	$1.1-0.000866(A-150)$	$1+0.000984(B-915)$	$1+0.00157(C-915)$	$1+0.00059(L-2030)$	$1-0.000197(H-2440)$	$1+0.0026(T-100)$	1	
Lateral shear in mid-section of landing, kN	30.17	$1-0.000276(A-150)$	$1+0.00138(B-915)$	$1-0.000709(C-915)$	$1+0.000669(L-2030)$	$1-0.00024(H-2440)$	$1-0.000746(T-100)^{1.3}$	1	

Table 3.2 Equations for Calculating Ho and Mo under Loading-1 [Zahid,1999]

Loading-1		PARAMETRIC FUNCTIONS OF STAIR							
		F_A	F_B	F_C	F_L	F_H	F_T	F_f	
Design Parameters	Const.	150 < A(mm)< 1500	915 < B(mm) < 2500	915 < C(mm) < 2500	2030 <L(mm)< 4000	2440 < H(mm) < 5000	100 < T(mm) < 350	$14 < f_c'$ (Mpa) < 40	Values of Design Parameters
Negative moment at midsection of landing. (kN-m)	-5.75	$1+0.000078(A-50)^{1.15}$	$1+0.000142(B-915)^{1.29}$	$1+0.009069(C-915)^{0.74}$	$1+0.000589(L-2030)^{0.94}$	$1-0.0000071(H-2440)$	$1+0.009311(T-100)^{0.71}$	1	
Lateral shear in mid-section of landing. (kN)	30.97	$1-0.000462(A-150)^{0.904}$	$1+0.000051(B-915)^{1.45}$	$1+0.002283(C-915)^{0.84}$	$1+0.000227(L-2030)^{1.15}$	$1-0.003359(H2440)^{0.64}$	$1+0.004528(T-100)^{0.91}$	1	

Table 3.3 Equations for Calculating Ho and Mo under Loading-2 [Zahid,1999]

Loading-2		PARAMETRIC FUNCTIONS OF STAIR							
		F_A	F_B	F_C	F_L	F_H	F_T	F_f	
Design Parameters	Const.	$150 < A(mm) < 1500$	$915 < B(mm) < 2500$	$915 < C(mm) < 2500$	$2030 < L(mm) < 4000$	$2440 < H(mm) < 5000$	$100 < T(mm) < 350$	$14 < f_c'$ (Mpa) < 40	Values of Design Parameters
Negative moment at midsection of landing. (kN-m)	-3.70	$1+0.000095(A-150)^{1.1}$	$1+0.000143(B-915)^{1.21}$	$1+0.009144(C-915)^{0.74}$	$1+0.000578(L-2030)$	$1-0.000001(H-2440)$	$1+0.013169(T-100)^{0.76}$	1	
Lateral shear in mid-section of landing. (kN)	22.77	$1-0.001144(A-150)^{0.81}$	$1+0.000025(B-915)^{1.475}$	$1+0.003693(C-915)^{0.785}$	$1+0.000154(L-2030)^{1.24}$	$1-0.01748(H-2440)^{0.44}$	$1+0.006851(T-100)^{0.93}$	1	

analysis and minimising the error we found the values of constants. This finally led to an optimum empirical equation for each governing factor. The equations give directly the corresponding values of H_0 and M_0 in each case. Based on this Scheme, two tables (Table 3.2 and 3.3) were presented for finding the values of H_0 and M_0 at mid-landing section.

3.5 REMARKS

All the previous study of free standing stair were made for fully fixed support condition. In the present study, effort is given to study the behaviour of free standing stair resting directly on floor slabs without the presence of any beams or other stiffening member. Based on the investigation an attempt shall be made to propose semi-empirical equations for lateral shear (V_o) and bending moment (M_o) at the middle of landing for this kind of free standing stair.

CHAPTER 4

FINITE ELEMENT MODELING AND ANALYSIS OF FREE STANDING STAIR

4.1 INTRODUCTION

Finite element technique is a powerful and versatile tool for the analysis of problems of structural and continuum mechanics. Analysis of the free standing stair using this technique is discussed here. The element-wise discretisation procedure in finite element process reduces a continuum problem of infinite number of variables to one of a finite number of unknowns. The analysis of a continuum by finite element technique has three basic steps : structural idealization which is the sub-division of the actual continuum into an assemblage of discrete structural elements, evaluation of element characteristics such as stiffness, stress & mass matrices and structural analysis of the element assemblage.

4.2 FINITE ELEMENT APPROACH

4.2.1 Choice of Element

In finite element technique, selection of a suitable element type is important. There are various types of elements, that is, beam elements, frame elements, solid elements, plate elements, shell elements, etc. Each of these elements is suitable for some particular types of structures. For example, frame elements are suitable for ordinary beam-column latticed structures. Similarly, shell elements are suitable for shell structures. There are many variations of shell elements, each with their own characteristics. The free standing stair is a space structure composed of interconnected plates. Hence, shell elements are preferred for this structure.

4.2.2 Shell Element

There are various types of shell elements developed so far. All of them fall in either of the two general categories, namely the thin shell elements and thick shell elements. In the thin shell elements, it is assumed that the normal to the middle surface before deformation remains normal after deformation. This means that the out of plane shear, that is, transverse shear stress is neglected in this type of elements. The thick shell elements include bending as well as shear deformations. In these elements, nodal lines are straight and 'normal' to the un-deformed middle surface. They are in-extensible and remain straight after deformation. However , they are in general not normal to the deformed middle surface after deformation, allowing the calculation of transverse shear stresses and strains. Consequently, this type of elements is suitable for modeling plates and shells where transverse shear is important. For the free standing stair under study, the 8-noded general thick shell element developed by Ahmad [1969] is adopted.

4.2.3 Features of the Computer program

Ahmad [1969] developed a computer program, which can analyze any shell or plate structure using the thick shell elements. The program was written in FORTRAN language and is quite general. It requires a considerable amount of data for input. The output of the program gives deflections and stresses at nodes with respect to global co-ordinate system. To simplify the input output process, separate computer programs were developed which can generate necessary data for the Ahmad's program from minimum input and can calculate the necessary design parameters such as bending moments, shear forces, etc., from the output of the general thick shell program.

4.2.4 Assumptions and Limitations

The thick shell elements developed by Ahmad [1969] are based on some assumptions such as the material within an element is isotropic and elastic and obeys Hooke's law. However such elastic properties are allowed to differ from element to element, which allows the program to handle structures made of composite materials. Besides these global assumptions, the additional assumptions, particular to the free standing stair are as follows:

- The additional stiffening effect provided by the steps to the waist-slab of flights is neglected. They only contribute to dead weight.
- The slab thickness is assumed to be sufficient to withstand the stresses developed and no account was given to slenderness.

4.3 FINITE ELEMENT IDEALISATION OF THE FREE STANDING STAIR

4.3.1 The Thick Shell Element

The thick shell elements may be curved about either one or both of its planes and may have variable thickness along its axes. The edges of elements may thus be either curved or straight. The program allows the users to use any of the two types of elements , that is , either the 8-noded parabolic element or the 12-noded cubic element. In the present study the 8-noded elements are used.

Proper idealization of a structure into an assemblage of finite number of elements is important in the analysis. The process involves division of the structure into elements in a suitable manner, numbering the elements and nodes in an efficient way, selection of local and global axes system, etc. Efficiency in these steps is necessary for optimal utilization of the computer memory and computational speed. Some of these aspects of finite element idealization of the stair slab are discussed in the following articles.

4.3.2 Element Mesh Configuration

A structure can be subdivided into elements in many possible ways. It depends on many factors such as the element characteristics, accuracy needed, available computer memory, etc. It is in general accepted that the finer the subdivision the better the idealization. However, with proper choice of elements, the rate of gain in accuracy decreases after a certain degree of fineness in subdivision. Element mesh can be finer near points of stress concentration in comparison to the other parts of the structure. Considering all these factors the subdivision shown in Figs. 4.1(a, b, c) and 4.2 were adopted for the study. The floor level supports, flight-landing junction and the mid-landing section are the places of possible stress concentration and hence the element sizes there are taken to be smaller in these areas in comparison to the other parts of the stair.

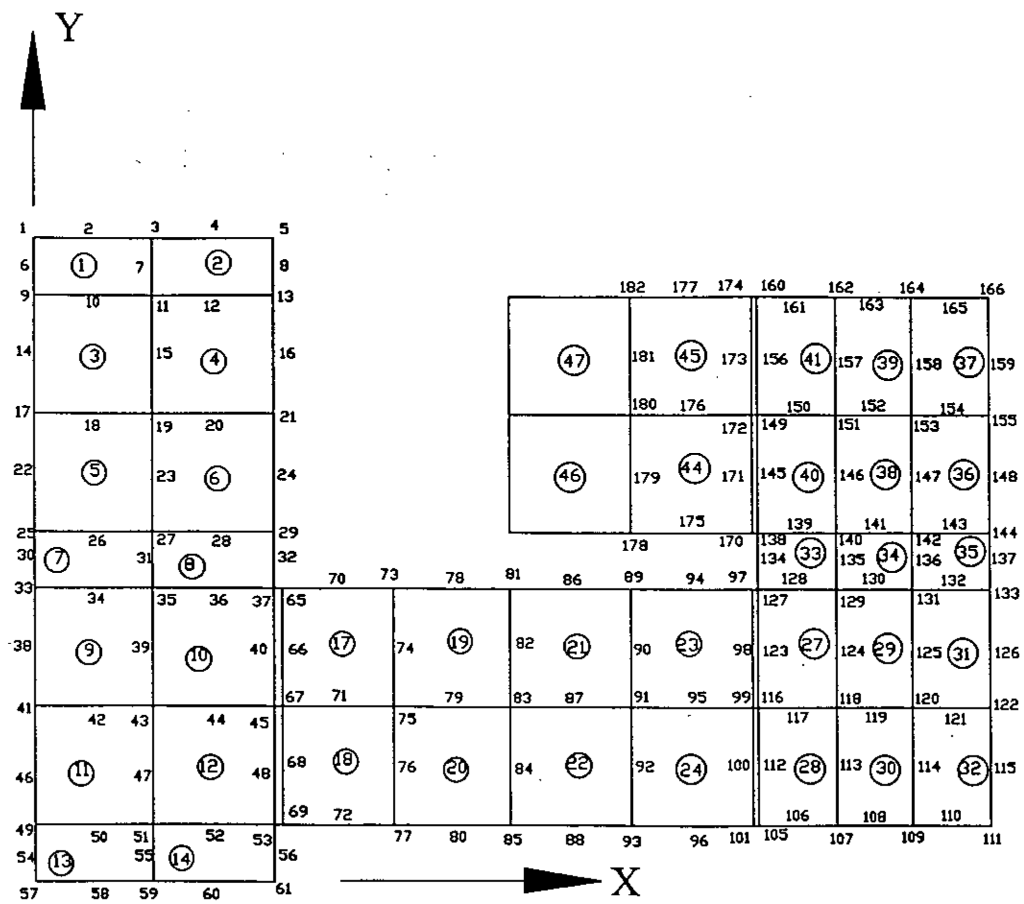


Fig 4.1a Element and node configuration

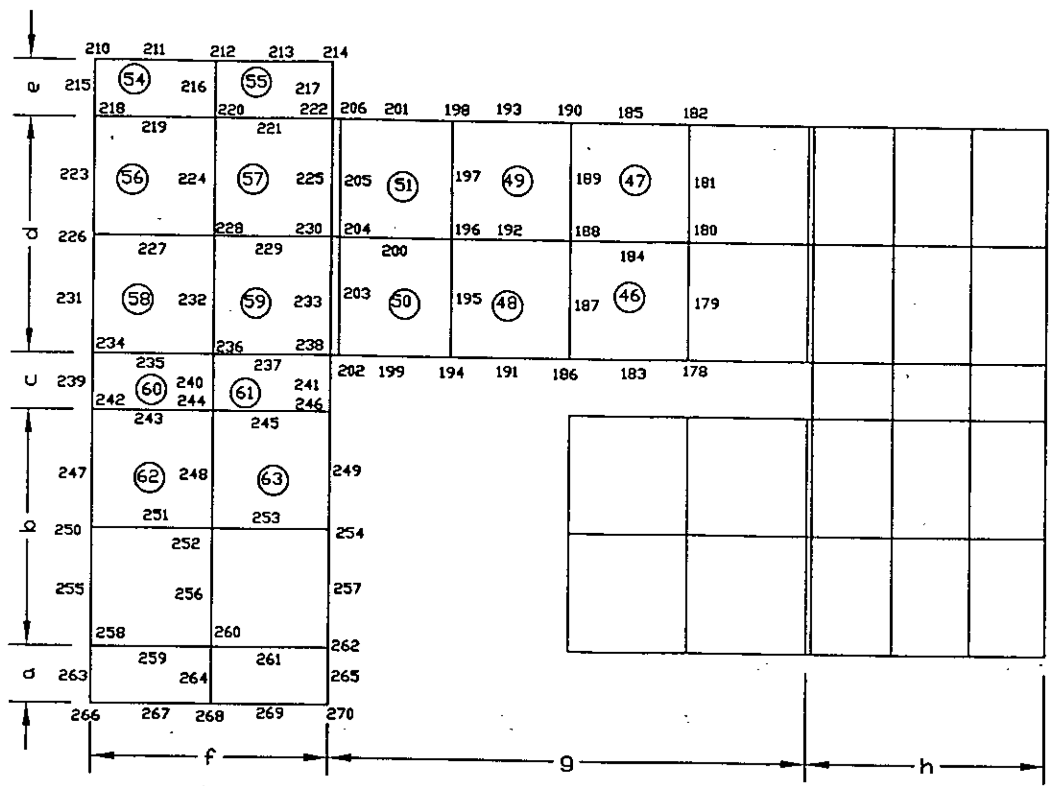


Fig 4.1b Element and node configuration

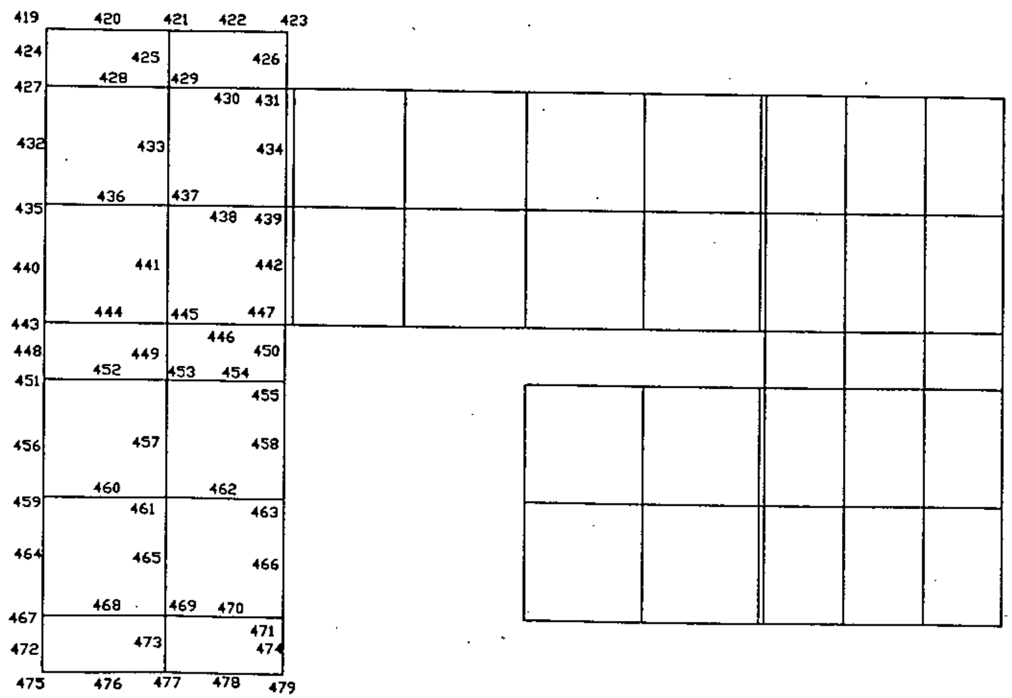


Fig 4.1c Element and node configuration

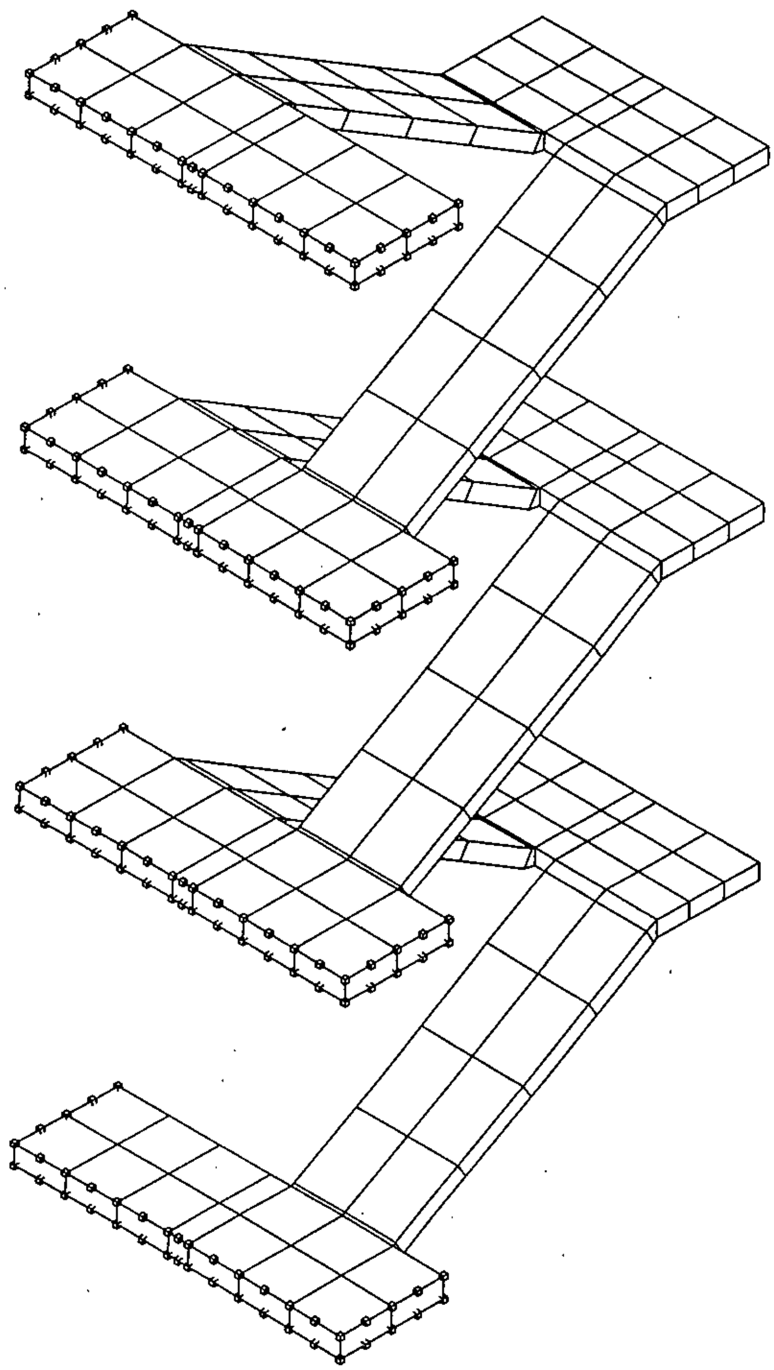


Fig. 4.2 Isometric view of the FE Model

4.3.3 Element and Node Numbering Scheme

The thick shell finite element program uses frontal solution technique to evaluate the displacement vectors of elements. Front width is dependent on the numbering sequence of elements. To keep the front width minimum the following two general rules are followed while numbering the elements:

- Numbering should preferably start at one end of the structure and terminate at the other end.
- The difference in element numbers between two adjacent elements should be kept minimum.

Following the above concept, the element numbering scheme shown in Figs. 4.1a, 4.1b, and 4.1c is adopted here for the stair. The original program requires the global co-ordinates of the nodes as input data. It is rather troublesome to enter the co-ordinates of all the nodes. However , the program offers one advantage in this regard. If only the corner nodes data are given, the program can automatically generate the co-ordinates of mid-side nodes assuming a straight line interpolation between the corner nodes. As the elements of the stair slab are perfectly rectangular, advantage of this feature of the program has been utilized in the study. First, the corner nodes are numbered from left to right starting from the front towards the back of the stair. Then the mid-side nodes are numbered in a systematic sequence as shown in Figs. 4.1(a, b, c). Although nodes can be numbered in any arbitrary way, the regular pattern of Figs. 4.1(a, b, c) is adopted for ease in developing the data generation program.

4.3.4 Adjusted Unit Weight for Gravity Loading

The general thick shell program can accept loading data in many ways, depending upon the nature of loading. For the present study, live load due to transiting people is assumed uniformly distributed on horizontal plane. In this case the easiest way to incorporate the load is to increase the weight density of the material. The weights of the steps and uniformly distributed live load have been included in the stair slab by properly increasing the unit weight of material. It is observed that the flight slabs are inclined at an angle α whereas the landing slab is horizontal. For this reason the thickness measured vertically is different in flight slabs and in landing slab. Hence adjusted unit weight for flight slab and landing slab differ although the slab thickness is same everywhere.

4.3.5 Transformation of Stresses

The original finite element program gives stresses as output with respect to global co-ordinate system. The global stresses cannot be used directly for design. These are to be transformed into local stresses with respect to local axes system for determining forces and moments, which will subsequently be required for design.

The stress output is in tensor form with respect to global axes. The state of stress at any point can completely be defined by the stress tensor with respect to a set of three mutually perpendicular axes. If the stress tensor corresponding to X, Y, Z system is known, it can readily be transformed to any other X', Y', Z' axes system with the help of the well known stress transformation rule,

$$[\sigma'] = [A][\sigma][A]^T \qquad (4.1)$$

where,

$[\sigma']$ = Stress tensor in X', Y', Z' system

$[\sigma]$ = Stress tensor in X, Y, Z system

$[A]$ = Direction cosine matrix of X', Y', Z' system with respect to X, Y, Z system

$[A]^T$ = Transpose of $[A]$

This transformation rule can best be represented by the index notation as:

$$\sigma'_{ij} = \sum_{\kappa=1}^{3} \sum_{l=1}^{3} a_{ik}\, a_{kl}\, a_{lj} \qquad (4.2)$$

Where i=1,3 and j=1,3

Here σ_{ij} and σ'_{ij} are the element at i-th row and j-th column of $[\sigma]$ and $[\sigma']$ respectively.

The direction cosine matrix is:

$$[A] = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix} \qquad (4.3)$$

where, (a_{11}, a_{12}, a_{13}) = Direction cosines of X' axis w.r.t. X, Y, Z axes

(a_{21}, a_{22}, a_{23}) = Direction cosines of Y' axis w.r.t. X, Y, Z axes

(a_{31}, a_{32}, a_{33}) = Direction cosines of Z' axis w.r.t X, Y, Z axes

4.3.6 Determination of Forces and Moments at a section

The procedure described below demonstrates how to calculate these forces from stresses. A transverse section with three nodes, numbered 1,2 and 3 are shown in (Fig.4.3) where the stresses are :

σ = normal stress

τ = lateral shear stress and

ρ = transverse shear stress

The stresses in Fig. 4.3 have two subscripts. The first subscript indicates whether it is , at the top or at the bottom of the node, and the second subscript indicates the node. For example σ_{b1} means the normal stress at bottom of node 1. The calculation is carried out in

$$V_{x1} = \left[\frac{\sigma_{t1} + \sigma_{b1}}{2}\right] t \tag{4.4}$$

$$V_{Y1} = \left[\frac{\tau_{t1} + \tau_{b1}}{2}\right] t \tag{4.5}$$

$$V_{z1} = \left[\frac{p_{t1} + p_{b1}}{2}\right] t \tag{4.6}$$

$$M_{x1} = \left[\frac{\tau_{b1} - \tau_{t1}}{12}\right] t^2 \tag{4.7}$$

$$M_{Y1} = \left[\frac{\sigma_{b1} - \sigma_{t1}}{12}\right] t^2 \tag{4.8}$$

two steps. First the nodal stress resultants are calculated with the above mentioned equations. The above equations are for node 1 (Fig.4.3). Such quantities at other nodes can be calculated in a similar way. The forces and moments from stress resultants (shown in Fig. 4.4) are calculated as follows. For a three nodded section the variation of

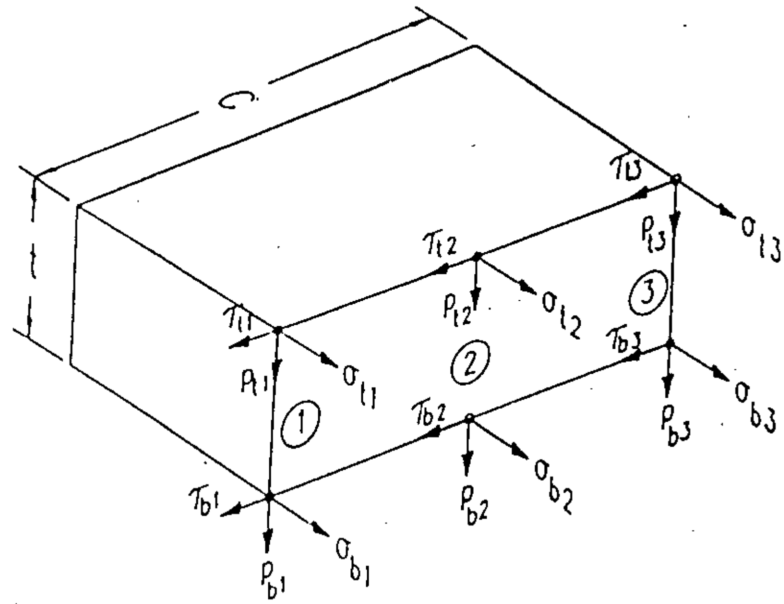


Fig 4.3 Calculation of Forces from Nodal Stresses

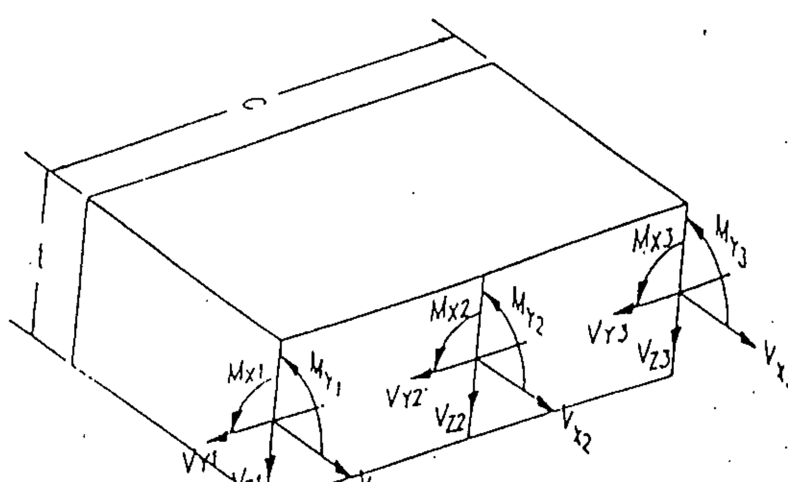


Fig 4.4 Calculation of Moments from Nodal Stresses

stress resultants across the width of the section can be assumed parabolic so that Simpson's integration rule can be applied. Thus:

$$V_x = (V_{x1} + 4V_{x2} + V_{x3}) \cdot \frac{C}{6} \tag{4.9}$$

$$V_y = (V_{y1} + 4V_{y2} + V_{y3}) \cdot \frac{C}{6} \tag{4.10}$$

$$V_z = (V_{z1} + 4V_{z2} + V_{z3}) \frac{C}{6} \tag{4.11}$$

$$M_x = (M_{x1} + 4M_{x2} + M_{x3}) \frac{C}{6} + (V_{z1} - V_{z3}) \cdot \frac{C^2}{12} \tag{4.12}$$

$$M_y = (M_{y1} + 4M_{y2} + M_{y3}) \cdot \frac{C}{6} \tag{4.13}$$

$$M_z = (V_{x3} - V_{x1}) \cdot \frac{C^2}{12} \tag{4.14}$$

If there are more than three nodes, a similar approach can be used with appropriate integration rule.

4.4 EQUILIBRIUM APPROACH

The critical sections for the analysis and design of free standing stair slab is shown in Fig. 4.5.

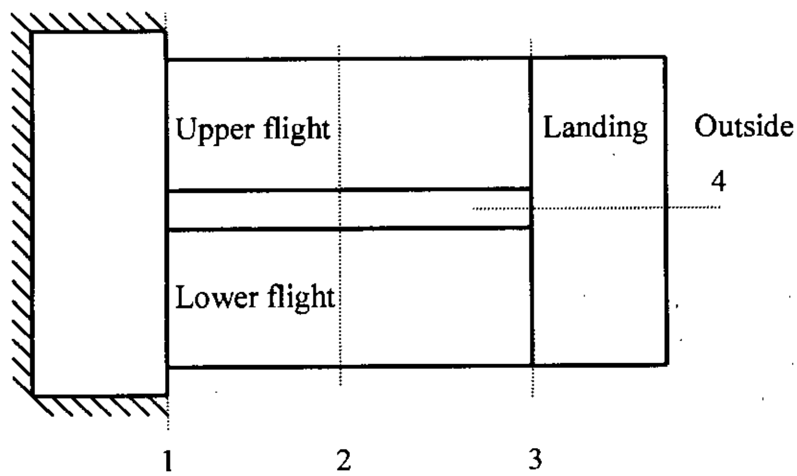


Fig. 4.5 Critical Sections of the Stair

1. Support section
2. Flight mid-span section
3. Flight-landing junction section
4. Mid-landing section

When both the stair slab and the loading are symmetric then the degree of static indeterminacy becomes two. The bending moment M_o and in-plane shear H_o at mid-landing section can be considered as redundant. The relevant quantities used in the equilibrium equations are shown below:

Live load

In the following process the live load is assumed as 4786 N/m^2 which is equivalent to 100 psf, as used by Amanat [1993] and Zahid [1999].

Average dead load from slab

$$\frac{T\left(Thickness\ \ of\ \ slab\ \ in\ \ mm\right)}{1000} \times 23576\ \left(N/m^{2}\right)$$

Average Dead Load from steps

$$\frac{1}{2}\times\frac{Rise\times Tread}{\sqrt{\left(Rise^{2}+Tread^{2}\right)}}\times\frac{1}{1000}\times 23576 \quad \left(N/m^{2}\right)$$

W_1 = Resultant load acting on mid point of flight

W_2 =Resultant load acting on mid point of landing

After calculating the values of the bending moment M_o and in-plane shear H_o at mid-landing section by the empirical equations, the equation of forces and moments at other critical location can be found from the equations of equilibrium (Fig. 4.5). Taking moments about the critical locations the following equations are developed as stated below:

1) Support Negative Moment

$$=-H_O\left[\frac{h}{2}\right]+\frac{W_2}{2}\left[\frac{B}{2}+G\right]+W_1\left[\frac{G}{2}\right] \tag{4.15}$$

2) Flight Mid span Positive Moment

$$=H_O\left[\frac{h}{4}\right]-\frac{W_2}{2}\left[\frac{G}{2}+\frac{B}{2}\right]-\left[\frac{W_1}{2}\right]\left[\frac{G}{4}\right] \qquad (From\ right\ side) \tag{4.16}$$

$$=-M_s-H_O\frac{h}{4}+\left[W_1+\frac{W_2}{2}\right]\frac{G}{2}-\left[\frac{W_1}{2}\right]\left[\frac{G}{4}\right] \qquad (\ From\ left\ side) \tag{4.17}$$

3) Negative moment at kink

$$=W_2\times\left[\frac{B}{2}\right] \tag{4.18}$$

4) Axial force in flights

= (Horizontal force)*Cosa* + (Vertical force) *Sina*

$$H_OCos\alpha+\left(W_1+\frac{W_2}{2}\right)Sin\alpha \tag{4.19}$$

5) Torsion in Flights

$$\left[M_O + \frac{W_2}{2} \times \frac{C}{4}\right] Cos\alpha \; H_O \; Sin\alpha \left[\frac{B}{2} + \frac{C}{2}\right] \tag{4.20}$$

6) In plane Moment in Flights

$$H_O \left[\frac{B}{2} + \frac{C}{2}\right] Cos\alpha + \frac{W_2}{2} Sin\alpha \frac{C}{4} + M_O \; Sin\alpha \tag{4.21}$$

CHAPTER 5
SENSITIVITY ANALYSIS

5.1 INTRODUCTION

As a reference model of analysis, a stairway of most common dimensions is used throughout. Changing one dimension within the selected range and keeping the other dimensions constant the sensitivity analysis is done in order to analyze the effect of these parameters.

5.2 STAIR DESIGN PARAMETER

To determine the forces and moments at every section, in this equilibrium approach, only two redundants are required because of symmetry which are :

- Lateral shear at mid-landing section, and
- Bending moment at mid-landing section.

Other necessary quantities at the critical locations can be derived from the equations of equilibrium. Extensive parametric study is done to investigate the change in these two design parameters with the variation of geometric parameters. The study is made for the three cases of loading. The general arrangements and boundary conditions defined in chapter two are maintained throughout this study.

5.3 GEOMETRIC PARAMETERS OF THE STAIR

The geometric parameters studied here are shown in Fig.5.1.

The thickness of upper landing is T_1, which is equal to $1.25T_2$. Each of these parameters is varied independently keeping the other remaining parameters constant at their initial values. The initial values are taken as:

Extended portion of upper landing (A)	= 500mm (Fixed)
Width of landing and Flights (B)	= 1200mm
Horizontal gap between Flights (C)	= 250mm

Width of upper landing (F) = 1500mm (Fixed)

Horizontal span of flights (G) = 2250mm

Waist slab and lower landing thickness (T_2) = 150mm

Slop of waist slab (α) = 0.54 radian

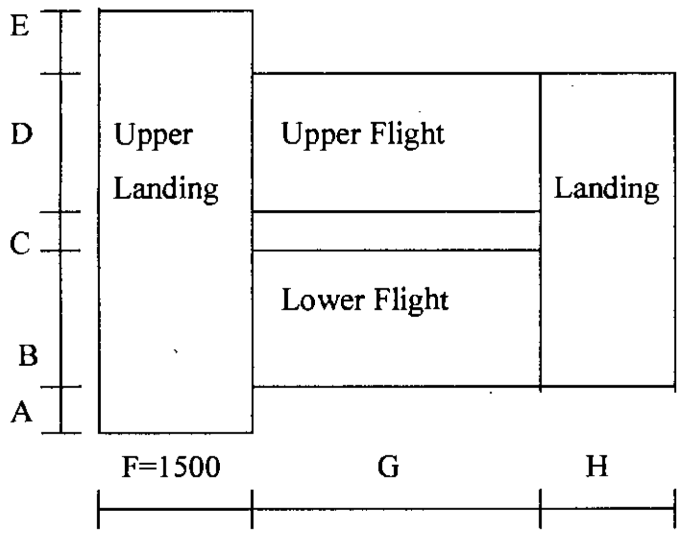


Fig. 5.1 Plan of a free standing stair

The range of variation of different parameters which are used in this analysis, is presented in table 5.1

Range of Geometric Parameters used in Sensitivity analysis.

Geometric parameters	Range
A	500mm(Fixed)
B	800 - 2000mm
C	100 -1000mm
F	1500mm (Fixed)
G	2250 - 3500mm
T_2	125 - 250mm

The variation of Moment (M_o) and Lateral shear (H_o) at mid-landing section for different values of design parameters are Tabulated in Table 5.2 and 5.3 for three different load cases. The graphical representation of the variations of M_o and H_o are shown in Fig. 5.2 to 5.9. The mentioned equations will more or less satisfy the curve plotted in Fig.'s 5.2 to 5.9 with the results found from the Finite Element Analysis, for variation of one parameter but keeping the other parameters constant.

5.4 RESULTS OF PARAMETRIC STUDY

The findings of the parametric study are discussed in the following sections:

a) The effect of variation of landing width (B), on bending moment and lateral shear at the mid-landing section, for three loading cases, is presented in Fig. 5.2 and 5.3. For all the three cases M_o and H_o increased with the increasing values of 'B'. The variation is not linear. In the higher range of values for 'B' the values of M_o and H_o increase at an increasing rate.

b) The effect of variation of horizontal gap between flights (C), on bending moment and lateral shear at mid-landing section, is represented in Figs 5.4 and 5.5. In all the three cases, the lateral shear at mid-landing section decreased with the increasing values of 'C'. But the bending moment at mid-landing section (M_o) initially decreases and then increases with the increasing values of 'C'.

c) Figs 5.6 and 5.7 show the behavior of bending moment and lateral shear at mid-landing section in response to the variation of horizontal span of flights (G). In all the cases, the values of M_o and H_o increase with the increasing value of 'G'. The nature of the curves are nearly linear.

d) The effect of slab thickness(T_2) on bending moment and lateral shear at mid-landing section is presented in Figs 5.8 and 5.9. The lateral shear for all the cases, shows almost a linearly increasing trend, and the moment increases with the increasing values of T_2, but the nature of the curves are nearly linear.

TABLE 5.2 VARIATION OF MOMENT (M_0) AT MID-LANDING SECTION FOR DEFFERENT VALUES OF GEOMETRIC PARAMETERS OF STAIR (From finite element analysis)

Geometric parameters of stair (mm) Moment , M_o (N-mm)

B	C	G	T_2	Load Case-1	Load Case-2	Load Case3
800	100	2250	125	4750895	7362483	6564932
1000	"	"	"	8563712	14136350	12434530
1200	"	"	"	14013130	24326670	21177020
1500	"	"	"	25930450	47697940	41050380
1800	"	"	"	43132020	82843570	70716090
2000	"	"	"	57865350	113749700	96683240
800	100	2250	125	4750895	7362483	6564932
"	150	"	"	4235059	6597435	5875991
"	250	"	"	4043944	6330887	5632479
"	375	"	"	4155710	6526703	5802628
"	600	"	"	4542160	7170005	6367489
"	1000	"	"	5233045	8358619	7404103
800	100	2250	125	4750895	7362483	6564932
"	"	2500	"	5291127	7936850	7128875
"	"	2800	"	5930126	8605402	7788402
"	"	3000	"	6352388	9042643	8221069
"	"	3200	"	6772581	9475113	8649789
"	"	3500	"	7400140	10117330	9287527
800	100	2250	125	4750895	7362483	6564932
"	"	"	150	5151118	7735185	6947052
"	"	"	175	5498749	8036149	7262592
"	"	"	200	5793896	8268912	7514483
"	"	"	225	6036365	8434850	7703694
"	"	"	250	6227167	8539525	7834373

TABLE 5.3 VARIATION OF LATERAL SHEAR (H_o) AT MID-LANDING SECTION FOR DEFFERENT VALUES OF GEOMETRIC PARAMETERS OF STAIR (From finite element analysis)

Geometric parameters of stair (mm) Lateral shear , H_o (N)

B	C	G	T2	Load Case-1	Load Case-2	Load Case3
800	100	2250	125	-27905	-41239	-37167
1000	"	"	"	-41976	-65602	-58387
1200	"	"	"	-59647	-97711	-86086
1500	"	"	"	-93737	-162681	-141626
1800	"	"	"	-138057	-250914	-216449
2000	"	"	"	-173986	-324510	-278541
800	100	2250	125	-27905	-41239	-37167
"	150	"	"	-25848	-38443	-34597
"	250	"	"	-24076	-36256	-32536
"	375	"	"	-22760	-34788	-31114
"	600	"	"	-20357	-31902	-28376
"	1000	"	"	-14011	-22845	-20147
800	100	2250	125	-27905	-41239	-37167
"	"	2500	"	-29601	-42674	-38682
"	"	2800	"	-31676	-44492	-40578
"	"	3000	"	-33079	-45749	-41880
"	"	3200	"	-34494	-47036	-43206
"	"	3500	"	-36638	-49012	-45233
800	100	2250	125	-27905	-41239	-37167
"	"	"	150	-30637	-43921	-39869
"	"	"	175	-33319	-46548	-42515
"	"	"	200	-35950	-49116	-45103
"	"	"	225	-38525	-51612	-47622
"	"	"	250	-41046	-54044	-50080

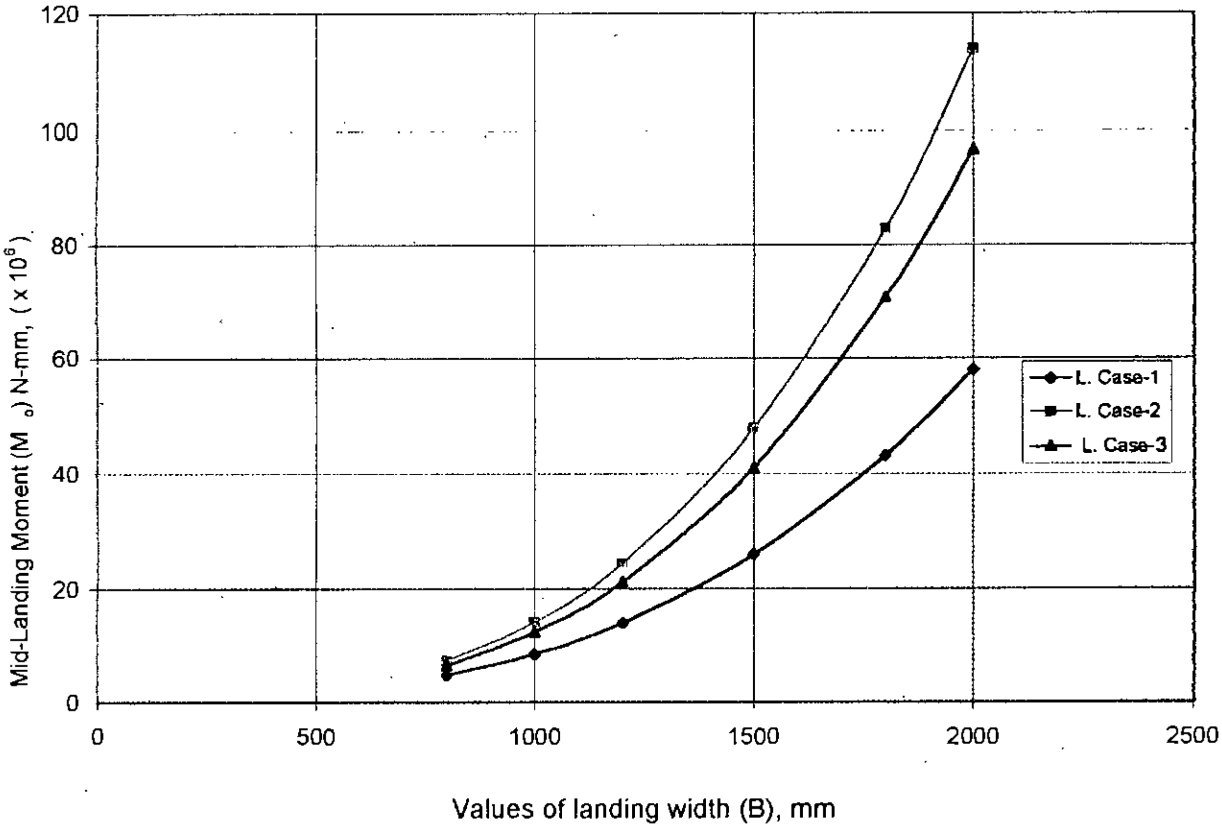


Fig. 5.2 Variation of Mid-landing Moment Due to Variation of Width of Landing (B)

RELATION BETWEEN M_o WITH LANDING WIDTH (B)

Load Case	M_o (N-mm)
1	$4750895\,[1+1.8 \times 10^{-4}(B-800)^{1.55}]$
2	$7362483\,[1+1.4 \times 10^{-4}(B-800)^{1.63}]$
3	$6564932\,[1+1.4 \times 10^{-4}(B-800)^{1.62}]$

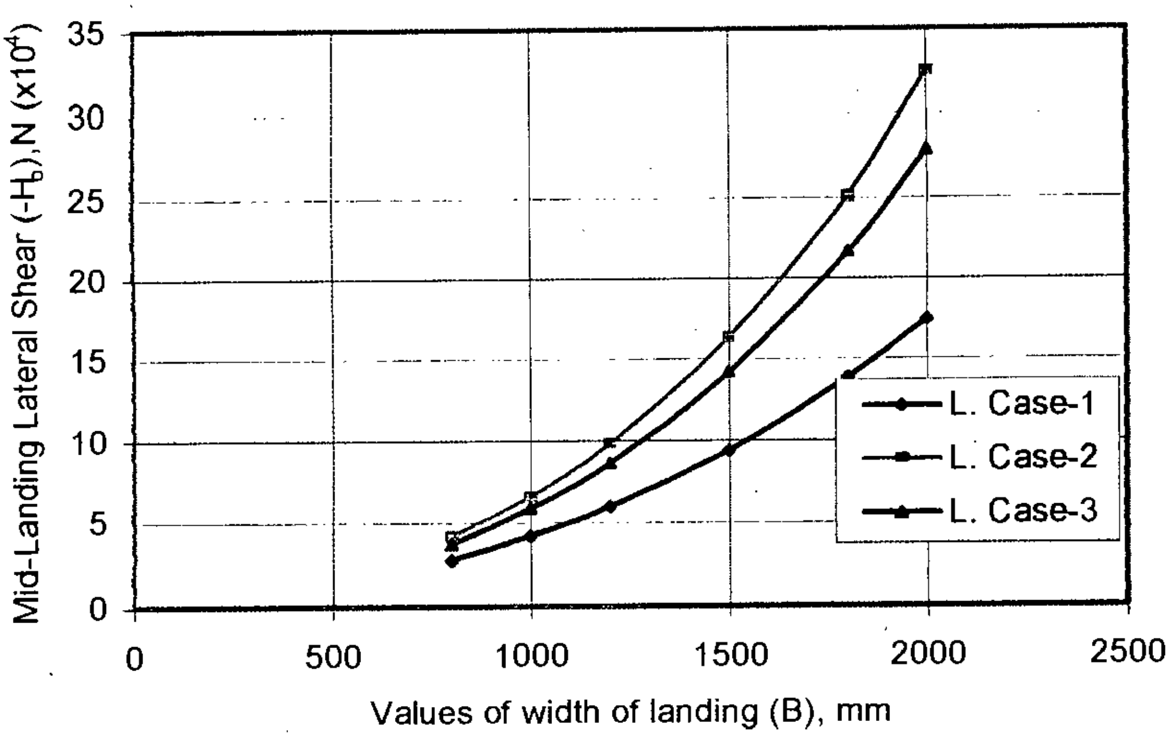


Fig. 5.3 Variation of Lateral shear (H_o) at Mid-landing due to variation of Width of Landing (B)

RELATION BETWEEN H_o WITH LANDING WIDTH(B)

Load Case	H_o (N)
1	$-27905\,[(1+3.33 \times 10^{-4}(B-800)^{1.358})]$
2	$-41239\,[1+2.57 \times 10^{-4}(B-800)^{1.43})]$
3	$-37167\,[1+2.688 \times 10^{-4}(B-800)^{1.42})]$

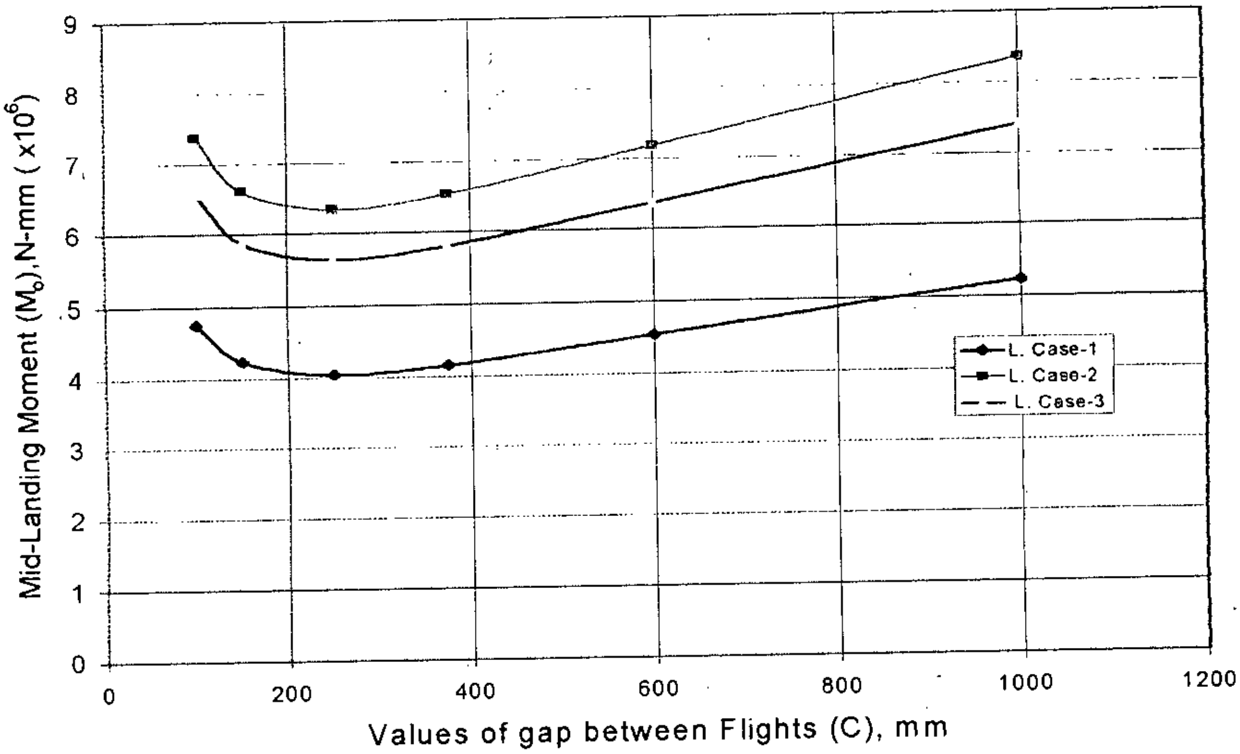


Fig. 5.4 Variation of Mid-landing Moment (M_o) Due to Variation of Horizontal gap between Flights (C)

RELATION BETWEEN M_o WITH GAP BETWEEN FLIGHTS(C)

Load Case	M_o (N-mm)
1	$5162284\,[1-0.0514(C-100)^{0.287}+1.897\times10^{-4}(C-375)]$
2	$7999275\,[1-0.053(C-100)^{0.272}+5.3\times10^{-4}(C-250)]$
3	$7133668\,[1-0.052(C-100)^{0.276}+5.31\times10^{-4}(C-250)]$

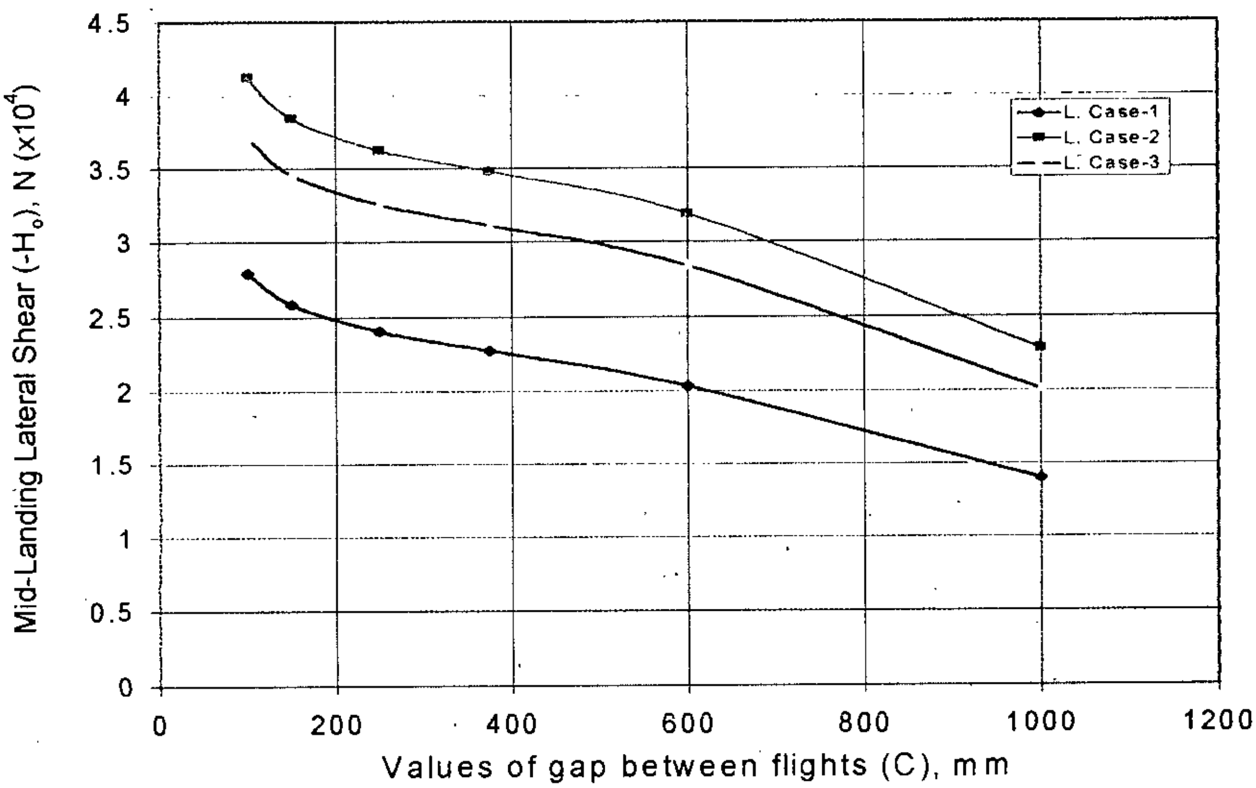


Fig.5.5 Variation of Mid-landing shear due to Variation of Horizontal gap between Flights (C)

RELATION BETWEEN H_o WITH GAP BETWEEN FLIGHTS(C)

Load Case	H_o, (N)
1	$-26522\,[1-9.2\times10^{-3}(C-100)^{0.487}-1.897\times10^{-4}(C-375)]$
2	$-39322\,[1-1.06\times10^{-2}(C-100)^{0.425}-1.78\times10^{-4}(C-375)]$
3	$-35401\,[1-1.02\times10^{-2}(C-100)^{0.441}-1.814\times10^{-4}(C-375)]$

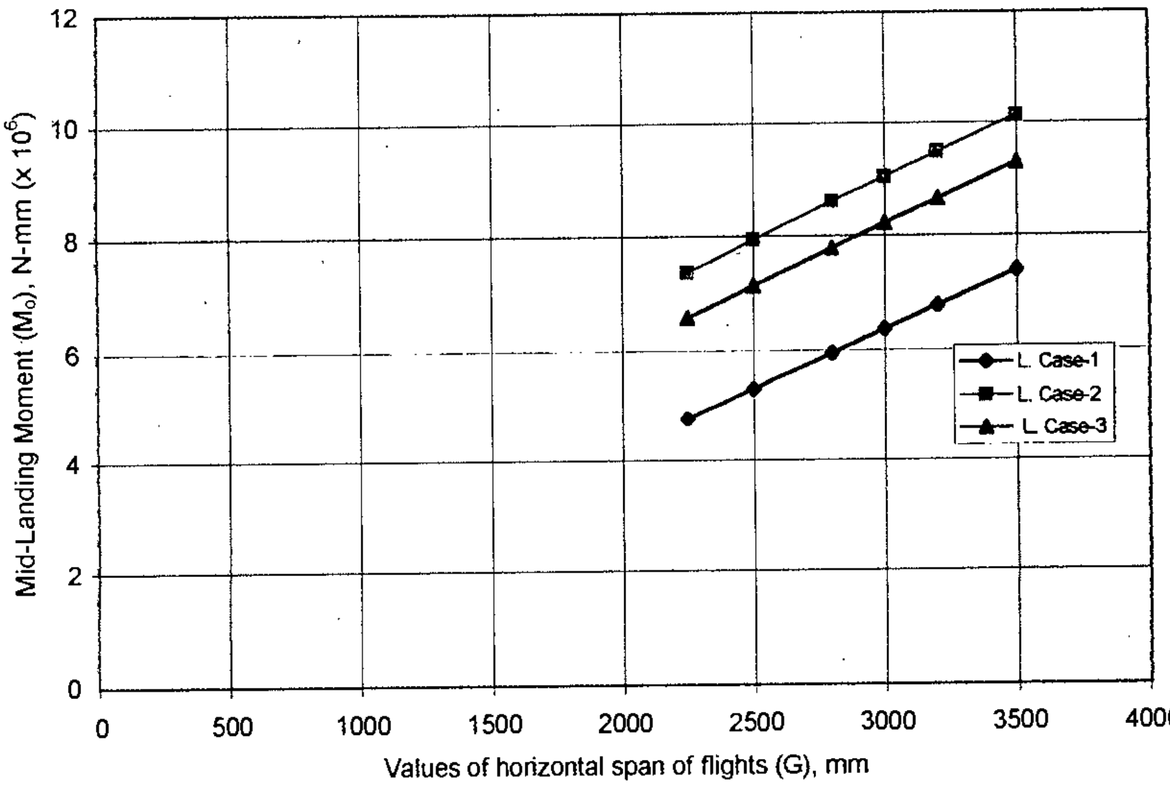


Fig. 5,6 Variation of Mid-landing Moment (M_o) due to Variation of Horizontal Span of Flights (G)

RELATION BETWEEN M_o WITH PROJECTED FLIGHT LENTH(G)

Load Case	M_o (N-mm)
1	$4750895\ [1+ 4.55 \times 10^{-4}(G-2250)]$
2	$7362483\ [1+ 3.12 \times 10^{-4}(G-2250)]$
3	$6564932\ [1+ 3.43 \times 10^{-4}(G-2250)]$

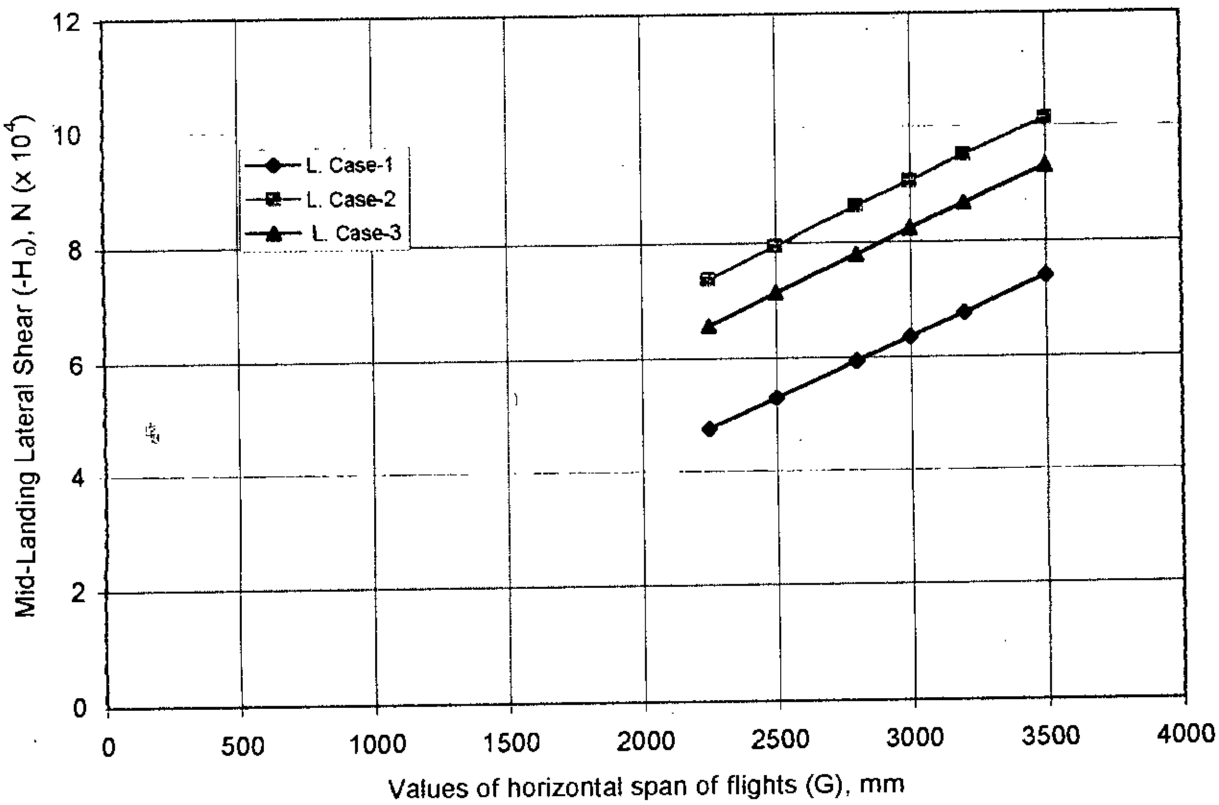


Fig. 5.7 Variation of Mid-landing Lateral Shear (H_o) due to Variation of Horizontal Span of Flights (G)

RELATION BETWEEN H_o WITH PROJECTED FLIGHT LENTH(G)

Load Case	H_o (N)
1	$-27905\,[1+2.43 \times 10^{-4}(G-2250)]$
2	$-41239\,[1+1.39 \times 10^{-4}(G-2250)]$
3	$-37167\,[1+1.39 \times 10^{-4}(G-2250)]$

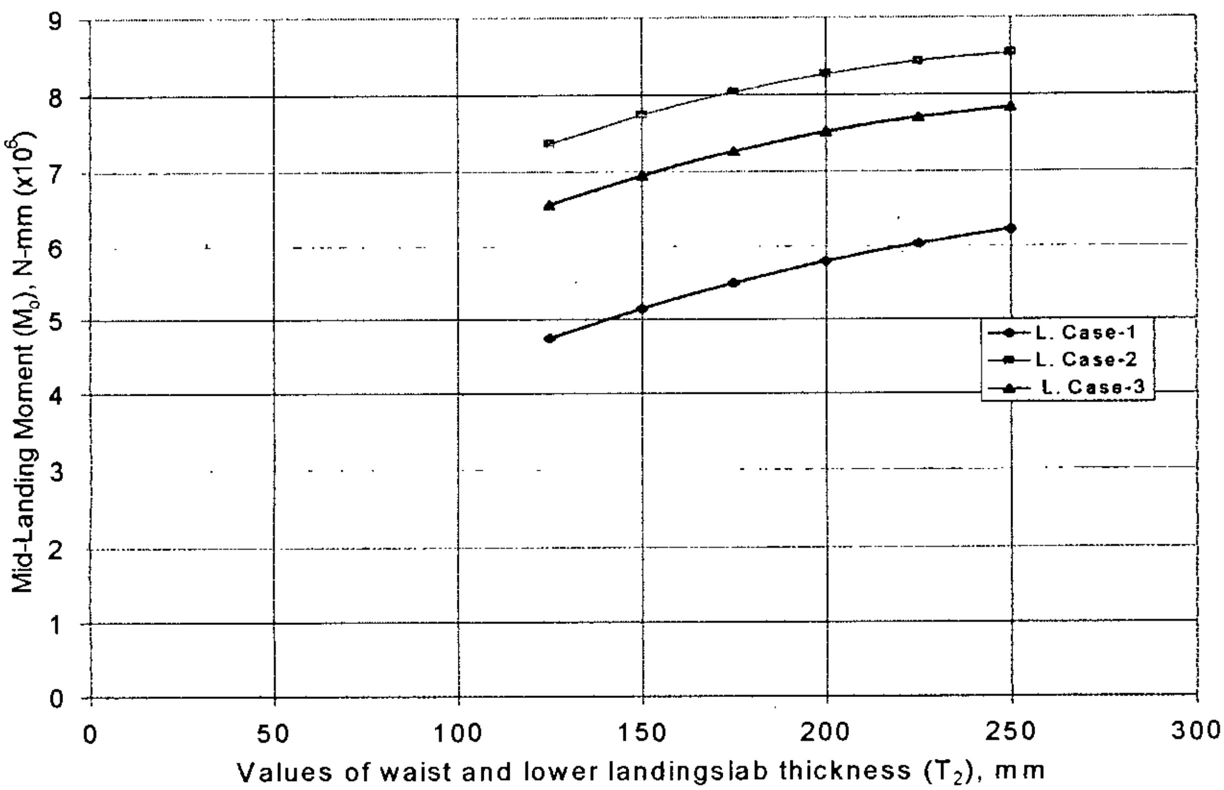


Fig. 5.8 Variation of Mid-landing Moment due to Variation of Waist & Lower Landing Slab Thickness (T_2)

RELATION BETWEEN Mo WITH LANDING THICKNESS(T_2)

Load Case	M_o (N-mm)
1	$4750895 [1+ 4.62 \times 10^{-3}(T_2-125)^{0.902})]$
2	$7362483 [1+ 3.24 \times 10^{-3}(T_2-125)^{0.854})]$
3	$6564932 [1+ 3.55 \times 10^{-3}(T_2-125)^{0.868})]$

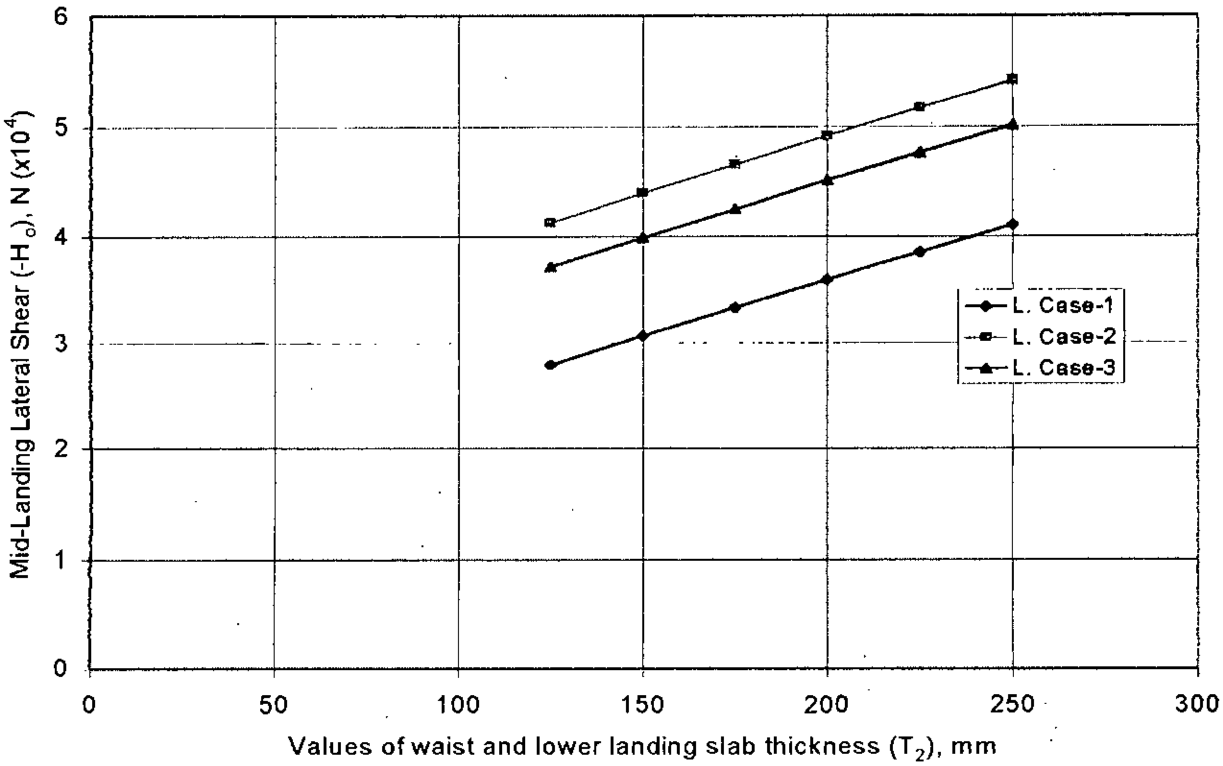


Fig. 5.9 Variation of Mid-landing Lateral Shear due to Variation of Waist &Lower landing Slab Thickness (T_2)

RELATION BETWEEN H_o WITH LANDING THICKNESS (T_2)

Load Case	H_o (N)
1	$-27905[1+3.88 \times 10^{-3}(T_2-125)]$
2	$-41239[1+2.95 \times 10^{-3}(T_2-125)^{0.965}]$
3	$-37167[1+3.27 \times 10^{-3}(T_2-125)^{0.967}]$

CHAPTER 6

DEVELOPMENT OF SIMPLIFIED DESIGN RATIONALE

6.1 GENERAL

In this chapter an attempt is made to quantify the effects of design parameters on the bending moment and lateral shear at mid-landing section. Semi-empirical equations for M_o and H_o are proposed. In these equations M_o and H_o are expressed as functions of design parameters such as B, C, G, etc. Once the values of M_o and H_o are known from these equations, the moments and forces at other sections can be approximated by using the equilibrium equations.

6.2 DEVELOPMENT OF EMPIRICAL EQUATIONS

The variation of M_o and H_o with respect to a single design parameter is assumed to be of the following form

$$Y = a + b\,(X - d)^{c} \qquad (6.1)$$

where Y is either M_o or H_o.

X is the value of the parameter concerned .

d is the lowest values of X.

a, b and c are constants.

Each of the curves of Fig. 5.2 to 5.9 are least-square fitted with the above form of equations. For each curve, the valued of a, b and c are thus determined from the least-square fit.

The expressions are explicit and of empirical nature. Hence care must be taken to use proper units of measurements. In these equations, the unit of force is Newton (N) and that of length is mm. The unit of moment is N-mm. The range of the different geometric parameters in the proposed equations is similar to that of table 5.1.

Since elastic analysis is made throughout, it is possible to calculate forces and moments for other values of live load by simple proportioning. The equations thus developed are shown with their corresponding figures (Fig. 5.2 to 5.9). Each of these equations represents the effect of a single parameter on M_o and H_o. In order to formulate general expressions for M_o and H_o, these equations are combined in the following form,

$$\boldsymbol{M_o \text{ or } H_o = K.F_B.\ F_C.\ F_G.\ F_{T2}} \tag{6.2}$$

Here, K is a numeric constant. The factors F_B, F_C represent the effect of a single parameter. For example F_B represents the effect of B in the form of equation (6.1). After combining in the form of equation (6.2), numeric constant a, b and c for each of the parameters are further adjusted. The adjustment is made by comparing the values of M_o and H_o given by the proposed equations with those obtained from finite element modeling of several stairs whose geometry were arbitrarily selected within the practical range.

The Equation of Moment (M_o) are as follows:

For load Case-1

$$M_{01} = K_1.F_{B1}.F_{C1}.F_{G1}.FT_{21} \text{ (N-mm)} \tag{6.3}$$

$K_1 = 4750895$

$F_{B1} = [1+ 1.9 \times 10^{-4} (B-800)^{1.60}]$ $800<B<2000$mm

$F_{C1} = [1- 0.060 (C-100)^{0.315} +2.5 \times 10^{-4} (C-250)]$ $100<C<1000$mm

$F_{G1} = [1+ 5\times 10^{-5} (G-2250)]$ $2250<G<3500$mm

$FT_{21} = [1+ 4.5 \times 10^{-3} (T_2 -120)^{0.9}]$ $125<T_2<250$mm

For load Case-2

$$M_{02} = K_2.F_{B2}.F_{C2}.F_{G2}.FT_{22} \text{ (N-mm)} \tag{6.4}$$

$K_2 = 7362483$

$F_{B2} = [1+1.5 \times 10^{-4} (B-800)^{1.60}]$ $800<B<2000$mm

$F_{C2} = [1- 0.050 (C-100)^{0.25} +5 \times 10^{-5} (C-100)]$ $100<C<1000$mm

$F_{G2} = [1+2.5 \times 10^{-4} (G-2250)]$ $2250<G<3500$mm

$FT_{22} = [1+ 0.003(T_2-120)^{0.85}]$ $125<T_2<250$mm

For load Case-3

$$M_{03} = K_3.F_{B3}.F_{C3}.F_{G3}.FT_{23} \text{ (N-mm)} \tag{6.5}$$

$K_3 = 6564932$

$F_{B3} = [1+ 1.3 \times 10^{-4} (B-800)^{1.60}]$ $800<B<2000$mm

$F_{C3} = [1- 0.040 (C-100)^{0.30} +5 \times 10^{-5} (C-250)]$ $100<C<1000$mm

$F_{G3} = [1+ 5 \times 10^{-4} (G-2250)]$ $2250<G<3500$mm

$FT_{23} = [1+ 3 \times 10^{-3}(T_2 -125)^{0.85}]$ $125<T_2<250$mm

The Equation of force (H_o) are as follows:

For load Case-1

$$H_0 = K_1 . F_{B1} . F_{C1} . F_{G1} . FT_{21} \quad (N) \tag{6.6}$$

$K_1 = -27905$

$F_{B1} = [1 + 2.5 \times 10^{-4} (B-800)^{1.36}]$ $\quad$ $800<B<2000mm$

$F_{C1} = [1 - 9 \times 10^{-3} (C-100)^{0.48} - 1.8* \times 10^{-4} (C-375)]$ $\quad$ $100<C<1000mm$

$F_{G1} = [1 + 2.5 \times 10^{-4} (G-2250)]$ $\quad$ $2250<G<3500mm$

$FT_{21} = [1 + 4 \times 10^{-3} (T_2 - 125)^{0.9}]$ $\quad$ $125<T_2<250mm$

For load Case-2

$$H_0 = K_2 . F_{B2} . F_{C2} . F_{G2} . FT_{22} \quad (N) \tag{6.7}$$

$K_2 = -41239$

$F_{B2} = [1 + 2.5 \times 10^{-4} (B-800)^{1.37}]$ $\quad$ $800<B<2000mm$

$F_{C2} = [1 - 1 \times 10^{-2} (C-100)^{0.45} - 1.8 \times 10^{-4} (C-375)]$ $\quad$ $100<C<1000mm$

$F_{G2} = [1 + 2.5 \times 10^{-4} (G-2250)]$ $\quad$ $2250<G<3500mm$

$FT_{22} = [1 + 3.5 \times 10^{-3} (T_2 - 125)^{0.9}]$ $\quad$ $125<T_2<250mm$

For load Case-3

$$H_0 = K_3 . F_{B3} . F_{C3} . F_{G3} . FT_{23} \quad (N) \tag{6.8}$$

$K_3 = -37167$

$F_{B3} = [1 + 2.6 \times 10^{-4} (B-800)^{1.36}]$

$F_{C3} = [1 - 1 \times 10^{-2} (C-100)^{0.42} - 1.8 \times 10^{-4} (C-375)]$

$F_{G3} = [1 + 2.5 \times 10^{-4} (G-2250)]$

$FT_{23} = [1 + 3.2 \times 10^{-3} (T_2 - 125)^{0.92}]$

The use of the above equations is straightforward. Values of forces and moments can readily be calculated once the design values of geometric parameters are known.

6.3 EFFICIENCY OF THE PROPOSED EQUATIONS

In order to justify the applicability of the proposed equations, the results given by the equations are compared with the corresponding results obtained from finite element modeling. Ten examples of stairs are chosen with arbitrarily selected values of parameters within practical range. A description of these stairs is already given in chapter-2.

Comparison is made for the mid-landing lateral shear H_o and bending moment M_o for different load cases. A summarized comparison of moment M_o is given in table 6.1 and the same for shear H_o is given in table 6.2. The same comparison is graphically shown in figs 6.1 through 6.6 by means of bar graph. A visual inspection of the bar graphs reveals that in most cases the proposed equations are able to predict the values with very good accuracy as compared to analysis. For H_o the variation is below 5% in most cases while only for two cases the variation exceeds 5%. But in all cases of H_o the variation is less than 7%. For M_o the variation is somewhat higher but still within acceptable range. The highest variation is around 17%, but in majority of the cases the variation is around 10% to 12%.

It is thus established that the proposed semi-empirical equations are capable of predicting the values of M_o and H_o with reasonable accuracy.

Table 6.1 For ten arbitrary selected stairs, following are the comparison (in %) between the negative moments (M_o) found from finite element method and that (M_o) calculated from empirical formula for three different cases

Variable geometric parameters				Moment at mid-landing section (M_0) N-mm								
				Load Case-1			Load Case-2			Load Case-3		
B	C	G	T_2	F.E.M	Emp. m	Variation (%)	F.E.M	Emp. m	Variation (%)	F.E.M	Emp. m	Variation (%)
1000	200	2500	125	8075042	6774067	-16%	12889570	11533584	-10.5	11419270	10057785	-11.92
1500	300	2800	150	24312970	28874851	16%	40485770	46065023	13.8	35559110	39801523	11.95
1800	500	3200	175	44640680	48995290	9.75	72232800	82797502	14.6	63821000	74447434	16.65
2000	200	3500	225	91354710	79612665	-12.85	138867600	129404905	-6.81	124383700	123247585	-9.1
1200	800	2700	150	14747090	13082219	-11.29	23600370	21385598	-9.38	20900140	18064671	-13.57
1000	250	2500	125	7897937	6566929	-16.85	12629210	11337910	-10.2	11184330	9840735	-12.01
1200	250	2600	150	13612680	13882170	1.98	21831640	22375240	2.49	19324880	19152252	-0.9
1500	200	2800	150	26466780	29735174	12.35	43980370	47461386	7.91	38638770	41371381	7.07
1800	200	3000	175	50027620	53271632	6.48	82075050	84489572	2.94	72304990	75441169	4.36
2000	225	3200	200	73891780	73327078	-0.76	118010800	117503470	-0.43	104562500	107431414	2.74

Table 6.2 .For ten arbitrary selected stairs, followings are the comparison (in%) between the Lateral Shear (H_0) at mid-landing, found from finite element method analysis and (H_o) calculated from empirical formula for three different cases

Variable geometric parameters				Lateral shear at mid-landing section (H_0) N								
				Load Case-1			Load Case-2			Load Case-3		
B	C	G	T_2	F.E.M	Emp.m	Variation (%)	F.E.M	Emp.m	Variation (%)	F.E.M	Emp.m	Variation (%)
1000	200	2500	125	-36731	-37629	2.45	-57118	-56230	-1.03	-50890	-51312	0.82
1500	300	2800	150	-81604	-87237	6.9	-131940	-134338	1.82	-116587	-120904	3.7
1800	500	3200	175	-121519	-128426	5.68	-193207	-199757	3.4	-171352	-180950	5.6
2000	200	3500	225	-216645	-211276	-2.48	-321628	-323018	0.43	-289625	-286997	-0.91
1200	800	2700	150	-46099	-44361	-3.77	-70479	-68568	-2.71	-65545	-63968	-2.45
1000	250	2500	125	-35539	-36573	2.91	-55548	-55051	-0.9	-49437	-50147	1.44
1200	250	2600	150	-54636	-55995	2.49	-85155	-84803	-0.41	-75847	-76652	1.06
1500	200	2800	150	-88212	-92128	4.44	-141736	-141326	-2.9	-125411	-126337	0.74
1800	200	3000	175	-141088	-143067	1.4	-224314	-220070	-1.89	-198942	-195741	-1.61
2000	225	3200	200	-187729	-187287	-0.24	-291773	-288039	-1.28	-260059	-256149	-1.52

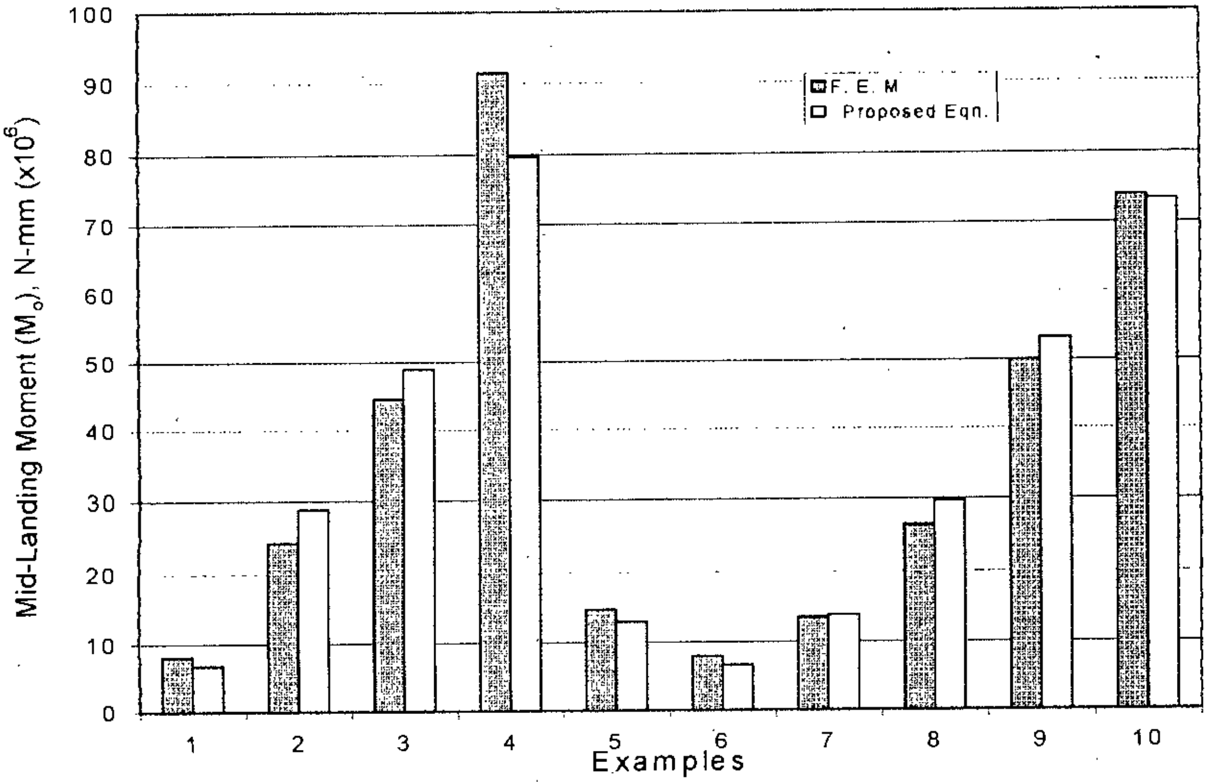


Fig. 6.1 Comparison of Moments (M_o) found from finite element analysis and that from proposed equations for case-1

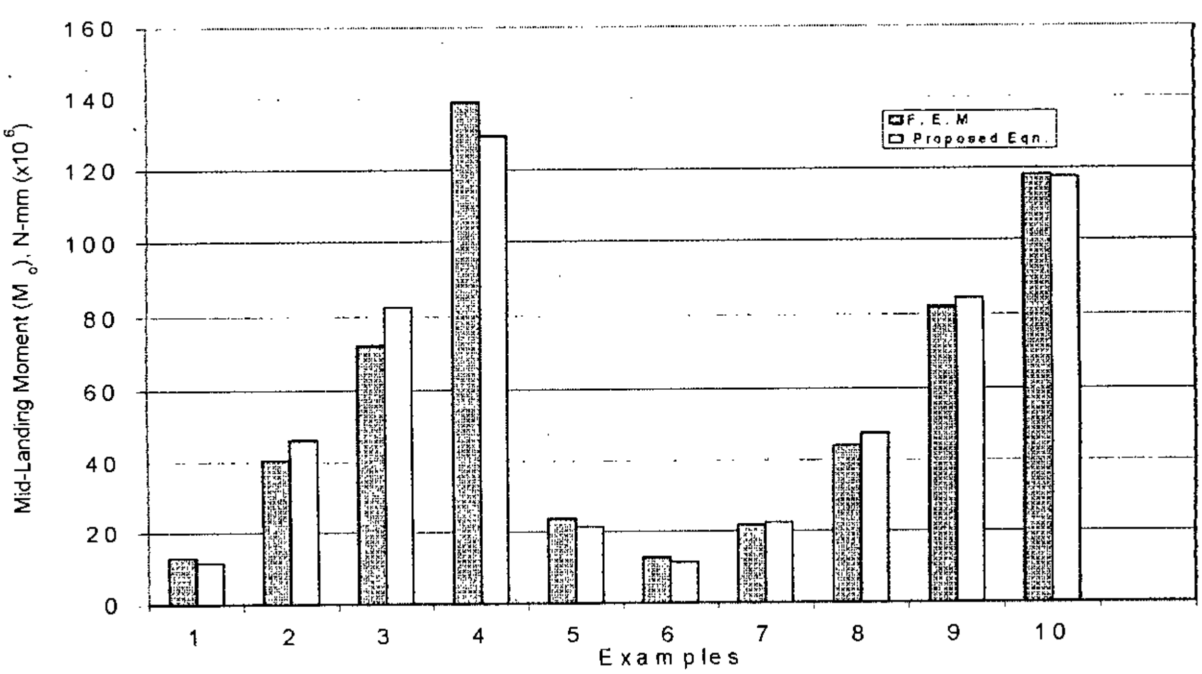


Fig. 6.2 Comparison of Moments (M_o) found from finite element analysis and that from proposed equations for case-2

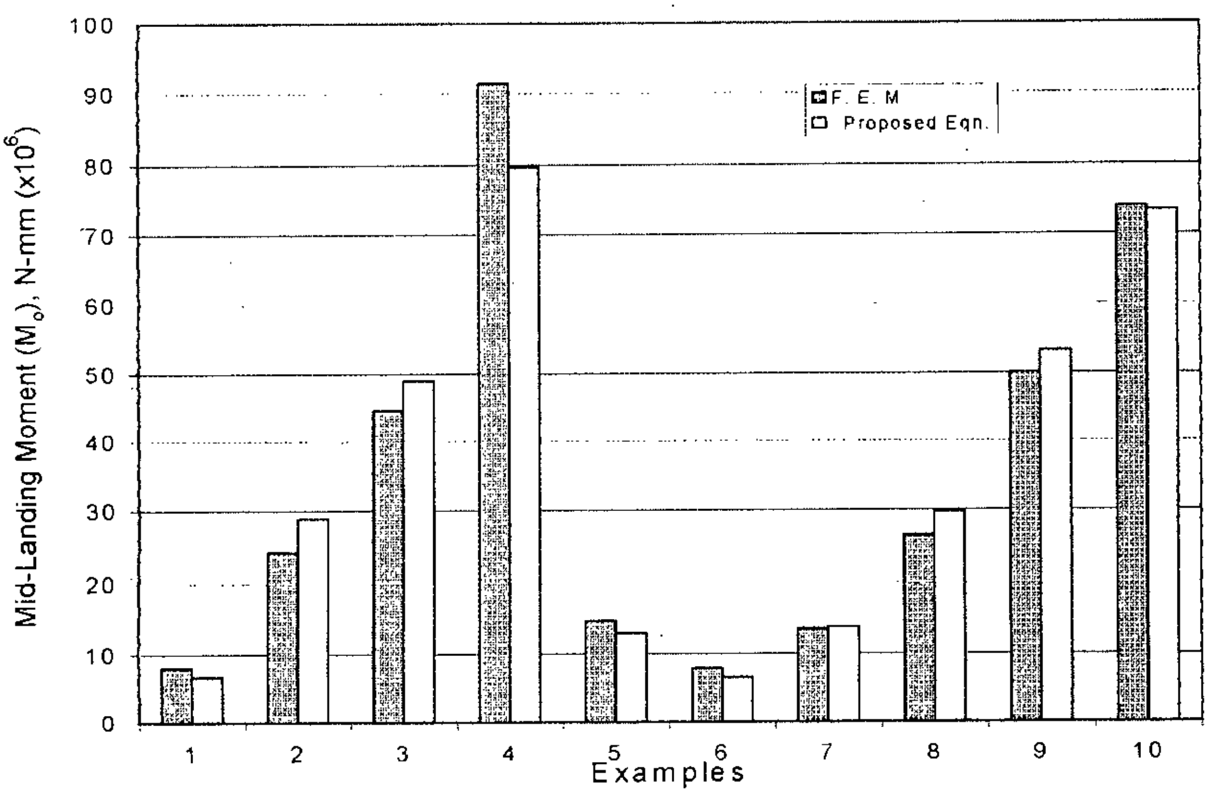


Fig. 6.1 Comparison of Moments (M_o) found from finite element analysis and that from proposed equations for case-1

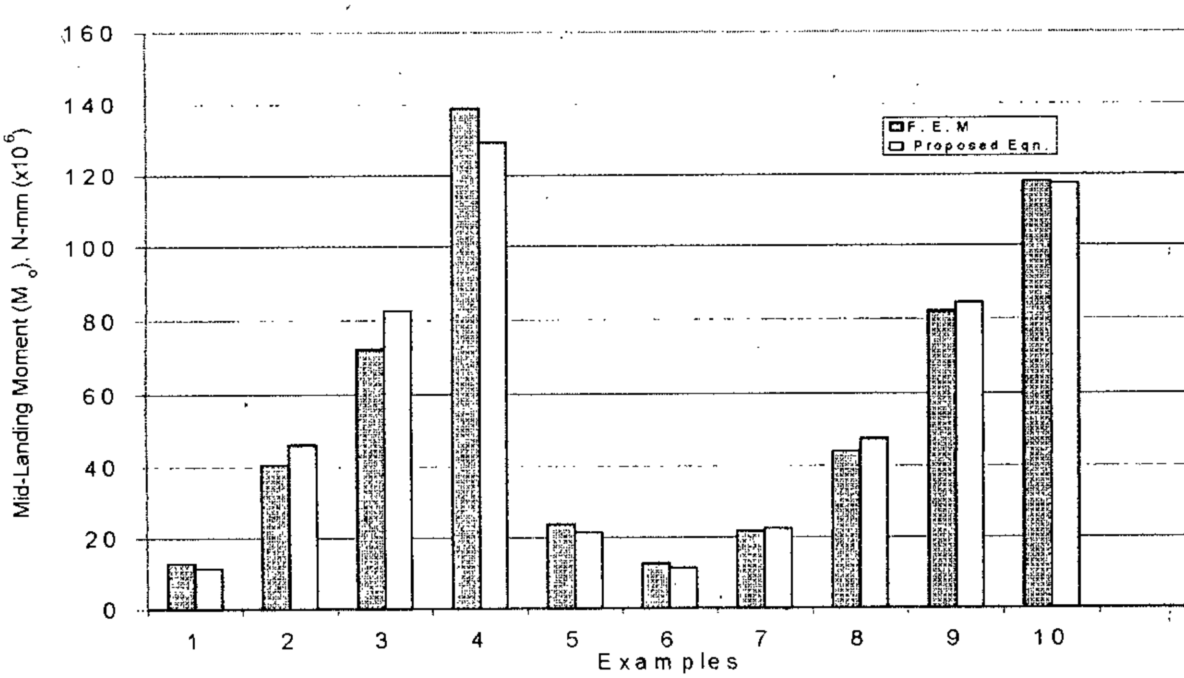


Fig. 6.2 Comparison of Moments (M_o) found from finite element analysis and that from proposed equations for case-2

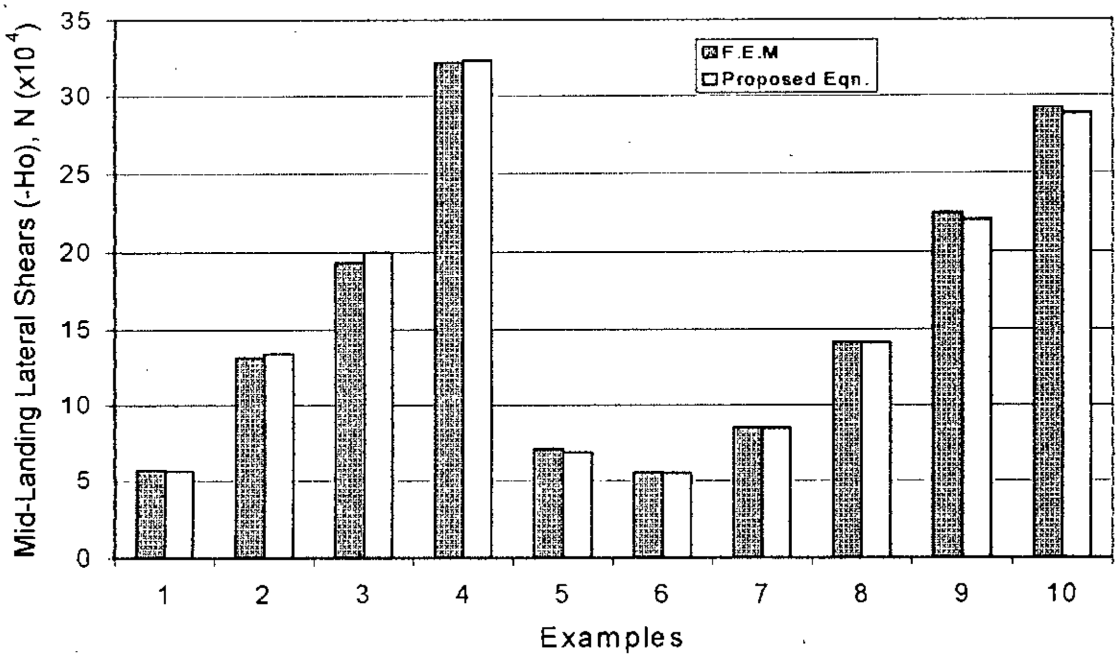


Fig. 6.5 Comparison of Lateral Shear (H_o) found from finite element analysis and that from proposed equations for case-2

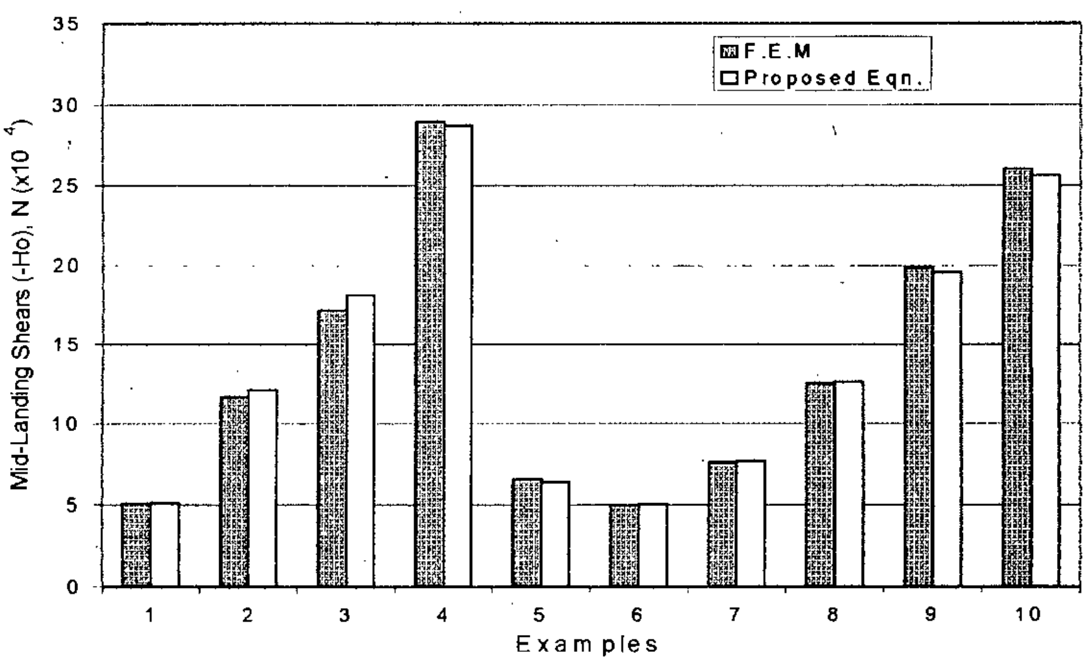


Fig. 6.6 Comparison of Lateral Shear (H_o) found from finite element analysis and that from proposed equations for case-3

6.4 PROPOSAL FOR A DESIGN GUIDE

The structural design procedure of a free standing stair may be divided into three stages, namely:

a) Analysis for stresses (forces and moments)

b) Checking of the thickness and calculation of the reinforcements

c) Detailing of the reinforcement layout.

These steps are described in the following sub-sections.

6.4.1 Analysis

The values of forces and moments at the critical locations are obtained from equations 6.3 to 6.8 and equations of equilibrium.

6.4.2 Calculation of reinforcement

- Longitudinal steel for bending moments.
- Longitudinal steel for axial force.
- Longitudinal steel for in-plane moment.
- Transverse stirrup for the for the combined effect of torsion and shear.

The theoretical details of the reinforcement calculation are not given here. They are found in any standard text book on reinforced concrete design. The equations presented in the previous sections give working values of moments and forces. Since elastic analysis is followed throughout, these forces and moments are directly proportional to load. To convert these working values to ultimate design values these are to be multiplied by some factor. This factor can be calculated as the ratio of factored ultimate load to working load. Details of these design aspects are shown through a design example.

6.4.3 Design Example

The application of the proposed analysis and design guide is shown here through an example. A reference stair model is used for the design. The data for design is shown below.

Geometry

Extended-portions of upper landing (A)	= 500mm (Fixed)
Width of flights and lower landing (B)	= 1200mm
Horizontal gap between flights (C)	= 250mm
Horizontal span of flights (G)	= 2250 mm
Width of upper landing (F)	= 1500 mm (Fixed)
Waist and lower landing slab thickness (T_2)	= 150mm
Slop of flights (α)	= 6:10 or 0.54 Radian (Fixed)

Load

Live load = 4786 N/m^2, Dead load = As calculated.

Material

f'_c = 20.68 N/mm^2

f_y = 275.76 N/mm^2

Forces and moments are calculated using equation of equilibrium.(Eqns.4.15 to 4.21).

The geometric parameters of a design example are shown in Fig. 6.7.

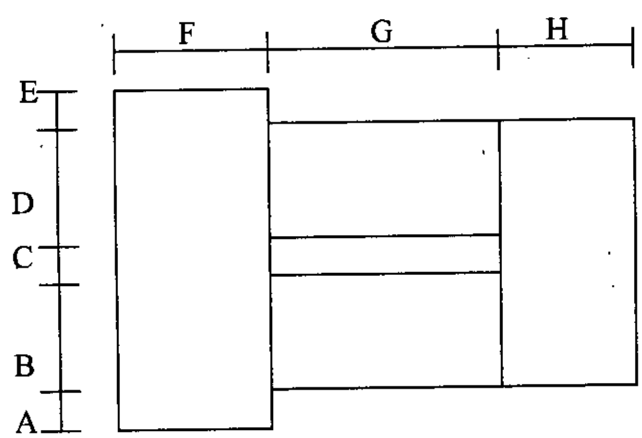


Fig. 6.7 Plan of free standing stair for design example

CALCULATION

Using the equation 6.3 for Case1

Negative moment (M_0) at mid section of landing for the above mentioned dimensions is

$$M_0 = 4750895\,[1+1.9\times10^{-4}(B-800)^{1.6}][1-.06(C-100)^{.315}+2.5\times10^{-4}(C-250)]$$
$$[1+5\times10^{-5}(G-2250)][1+4.5\times10^{-3}(T2-120)^{.9}]\ \text{N-mm}$$

$= 13911.69$ kN-mm

About 45% [Amanat-1993] of the M_o will be distributed among the inner one-third of the section.

So max. moment per meter width of mid-landing

$$= 13911.69 \times .45 \times \frac{1}{0.4}\ \text{kN - mm per meter width}$$

$= 15650.65$ kN-mm per meter width

Factored live load	=1.7 X4786 N/m^2	= 8137 N/m^2
Factored Dead load	=1.4 X 4712 N/m^2	=6597 N/m^2
Total Factored Load	=14734 N/m^2	
Total unfactored load	=9498 N/m^2	

Therefore multiplying factor for conversion to ultimate value is

$$\frac{14734}{9498} = 1.55$$

Negative steel at Mid-Landing section

$$A_s = \frac{M_u}{\varphi f_y\left(d - \frac{a}{2}\right)} \tag{6.9}$$

$$= \frac{1.55 \times 15650650}{.0.85 \times 275.76 \times \left(119 - \frac{15}{2}\right)} \qquad \left[\text{assuming a} = 15\text{mm}\right]$$

$= 928.19$ mm^2

check, $\quad a = \dfrac{A_s\, f_y}{.85\, f_c' b} = \dfrac{928.19 \times 275.76}{.85 \times 20.68 \times 1000} = 14.56\,\text{mm}$ OK

A_s =928.19 mm^2

Provide 12mm dia @ 120mm c/c

Using the equation 6.6 for load case-1

Lateral shear (H_o) at mid-landing section

$$H_o = -27905\,[1+2.5 \times 10^{-4}(B-800)^{1.36}][1-9 \times 10^{-3}(C-100)^{0.48} - 1.8 \times 10^{-4}(C-375)]$$
$$[1+2.5 \times 10^{-4}(G-2250)][1+4 \times 10^{-3}(T2-125)^{0.9}]$$

$= -51489.31$ N

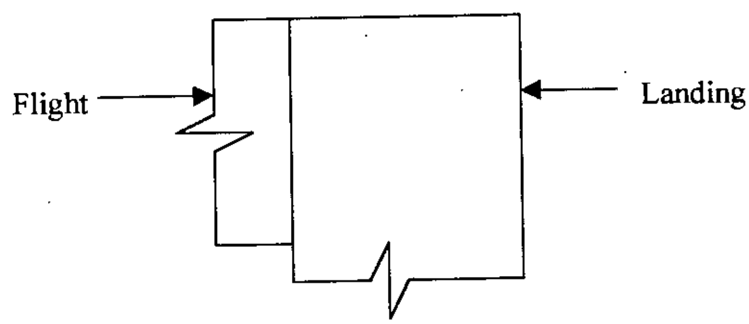
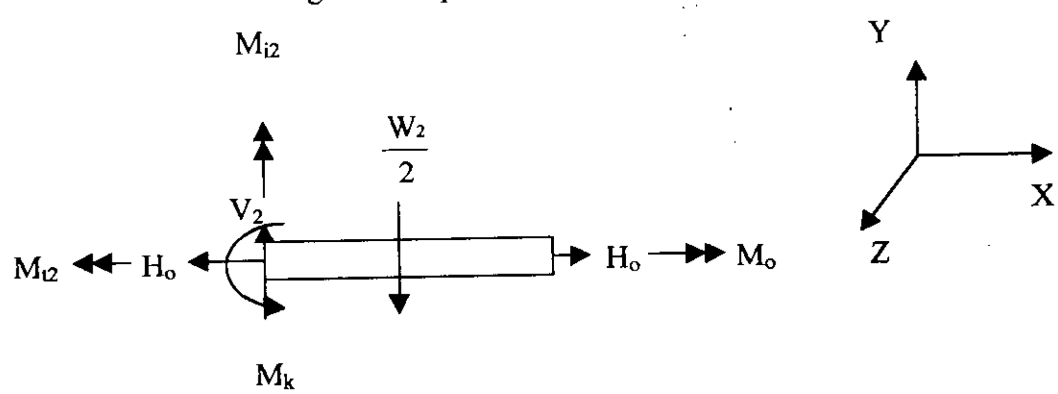


Fig.6.8 Part plan



Free body diagram of Fig. 6.8

Torsion in Flights (M_{t2})

$$= \left[M_O + \frac{W_2}{2} \times \frac{C}{4}\right] Cos\alpha - H_O\ Sin\alpha \left[\frac{B}{2} + \frac{C}{2}\right] \qquad (6.10)$$

$= -6467241.3$ N-mm per width of flight

Ultimate value of M_{t2u} $= 1.55 \times M_{t2}$

$= -10024224$ N-mm

Vertical Shear at flight - landing junction (V_2) = Total load on half landing

$= 1200 \times 1325 \times .009501$

$= 15107$ N

Ultimate value of $V_2 = V_{2u} = 1.55 \text{ X} 15107$ N $= 23416$ N

Stirrup for torsion in flights

Dimension of flight cross section is 1200mm X 150mm

$x = 150$ mm , $y = 1200$ mm , $x_1 = 100$ mm , $y_1 = 1150$ mm.

$$\alpha_t = 0.66 + 0.33\frac{y_1}{x_1} \le 1.5 \tag{6.11}$$

$$= 4.45 > 1.5$$

Therefore $\alpha_t = 1.5$

$$C_t = \frac{bd}{x^2 y} = \frac{1200 \times 119}{150 \times 150 \times 1200} = 0.0053 \tag{6.12}$$

Torsion resisted by concrete(T_c)

$$T_c = \frac{0.2\sqrt{f'_c}\, x^2 y}{\sqrt{1 + \left(0.4\frac{V_u}{C_t T_u}\right)^2}} \tag{6.13}$$

$=$ 50967372.2 N-mm which is greater than Torsional moment

Shear resisted by concrete (V_c) = 31812 N

Maximum spacing permitted by code is

$$\frac{x_1 + y_1}{4} = 312.5 \text{ mm} \tag{6.14}$$

Use closed rectangular stirrups of dia 10mm bar @ 300mm c/c

$$\text{check minimum web reinforcement, } (2A_t + A_v) \ge 0.34\frac{b_w s}{f_y} \tag{6.15}$$

$$\text{now, } 0.34\frac{b_w s}{f_y} = 0.34 \times \frac{150 \times 300}{275.76} < 157 \qquad \text{OK}$$

Longitudinal steel for torsion ,(A_l)

$$= 2A_t \frac{x_1 + y_1}{s} = 2 \times 113.09 \times \frac{100 + 1150}{300} \tag{6.16}$$

$= 942.42$ mm^2 per 1.2 meter width. [Using 12mm dia bar]

$= 785.34$ mm^2 per meter width, which will be added to other longitudinal steels.

$$\text{Negative Moment at kink} = \frac{W_2}{2} \times \frac{B}{2} \tag{6.17}$$

$$= \left[\frac{1}{2} \times 1200 \times 2650 \times .009501\right]\left[\frac{1200}{2}\right] \text{N - mm per width of flight}$$

= 9064200 N-mm per width of Flight

Negative moment at slab-flight junction is

$M_s = 10155400$ N-mm. [From equation of equilibrium]

Maximum moment occurs within the outer half of the flight , which is about 65%.[Amanat 1993]

$$\text{Therefore Moment at junction} = \left[\frac{0.65 \times 10155400}{0.6}\right] \text{N - mm per meter width}$$

= 11001683.33 N-mm per meter width

Ultimate value of Moment at junction (M_s)

$$= 1.55 \times 11001683.33 = 17052609.17 \text{ N - mm per meter width}$$

$$\text{Negative steel, } A_S = \frac{M_u}{\varphi f_y\left(d - \frac{a}{2}\right)}$$

$$= \frac{17052609.17}{0.85 \times 275.76 \times \left(119 - \frac{12}{2}\right)} \qquad \text{[assuming a = 12mm]}$$

$= 643.82$ mm^2

check ,

$$a = \frac{A_S f_y}{0.85 f_c b} = \frac{643.82 \times 275.76}{0.85 \times 20.68 \times 1000} = 10.10 \text{ mm}$$

So revised steel = 586.47 mm^2

Total steel including torsion effect 586.47 + 50 % of 785.34 =979.14 mm^2 per meter width. i.e. 12mm dia @ 115.50 mm c/c . Say 12 dia @ 115 mm c/c

Stirrup for Landing

Ultimate Lateral Shear in Landing , (V_u) =1.55 X 51489.31 N = 79808.43 N

Taking half of the width of section as effective in shear (Zahid-1999), the shear capacity of the section is,

$$V_c = 0.17\sqrt{f_c}\,bd = 0.17 \times \sqrt{20.68} \times 150 \times 570 = 66098\,\text{N} \tag{6.18}$$

Stirrup spacing, s

$$= \frac{\varphi A_v f_y d}{V_u - \varphi V_c} = \frac{0.85 \times (2 \times 78.53) \times 275.76 \times 570}{79808.43 - 0.85 \times 66098} \quad [\text{Using } \varphi\,10\text{mm}] \tag{6.19}$$

=1045mm

But $s_{max} \le \frac{d}{2}$

Hence stirrup spacing =285mm

Use closed rectangular stirrup of 10mm dia bar @ 285mm c/c

Minimum steel area in the bottom layer of the landing is 0.25% of the concrete area [ACI code], which is =0.0025 x 1000 x 150

= 375mm^2

=φ10mm bar @ 209.41mm c/c

Say φ10mm bar @ 200 mm c/c

W_1 (The load acting on mid-point of flight) =

$$[\text{Liveload} \times G \times B + (\text{Dead load from slab} + \text{Dead load from steps}) \times B \times \frac{G}{\text{Cos}\alpha}\ ]\ \ (\text{N})$$

=32543.16 N

W_2 (The load acting on mid-point of landing) = 30214 N

Flight Mid-span Moment

$$= H_o\left[\frac{h}{4}\right] - \frac{W_2}{2}\left[\frac{B}{2} + \frac{G}{2}\right] - \left[\frac{W_1}{2}\right]\left[\frac{G}{4}\right] \tag{6.20}$$

= -457054.50 N-mm

Ultimate Moment = -1.55 X 457054.50N-mm = -708434.48 N-mm

Maximum Moment within one-third , which is about 45%(Amanat,1993)

$$= 0.45 \times 708434.48 \times \frac{1}{0.4} \quad \text{(N - mm)} \quad \text{per meter width}$$

$= -796988.78$ N-mm per meter width

Steel area (A_s) = 30.09 mm^2 [assuming a=12mm]

Check, a = 0.472mm. Revised A_s = 28.52 mm^2 per meter width

Total steel (A_s) including torsion effect =28.52 + 50% of 785.34 = 392.67mm^2

= φ 10 mm bar @ 180.73 mm c/c

Say φ 10 mm bar @ 180 mm c/c

In plane moment in flights (M_{i2})

$$= H_0\left[\frac{B}{2}+\frac{C}{2}\right]\text{Cos}\alpha + \frac{W_2}{2}\text{Sin}\alpha\frac{C}{4} + M_0\text{Sin}\alpha \tag{6.21}$$

$$= 51489.31\left[\frac{1200}{2}+\frac{250}{2}\right]\text{Cos}\alpha + 30214\text{Sin}\alpha\frac{250}{4} + 13911690\text{Sin}\alpha$$

= 33211463.60 N-mm

Ultimate value of moment, (M_{i2u}) = 1.55 x 33211463.60 = 51477768.57 N-mm

A_s =201.49 mm^2 assuming a =120mm

Check, a = 3.16mm

Revised A_s = 191.23 mm^2

Provide φ 16mm bar 2 nos.

The moment Diagram along the length of the stair and sections of stair showing the reinforcements are shown in Fig. 6.9 and 6.10 respectively.

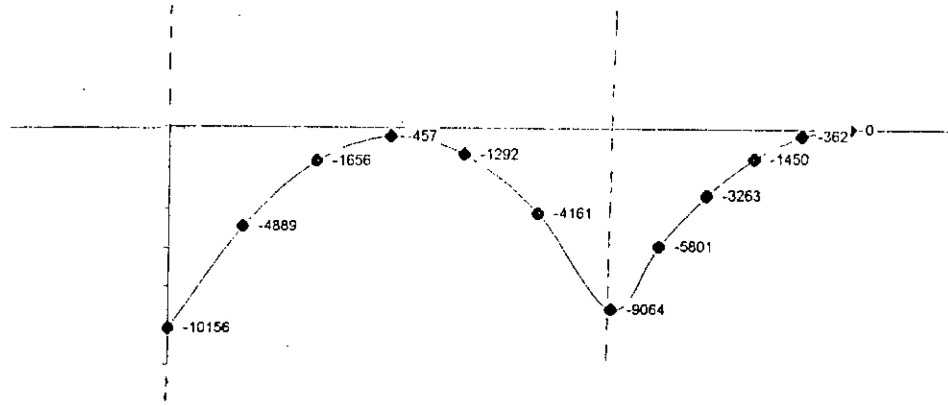


Fig. 6.9 Moment diagram along the stair projected length, kN-mm

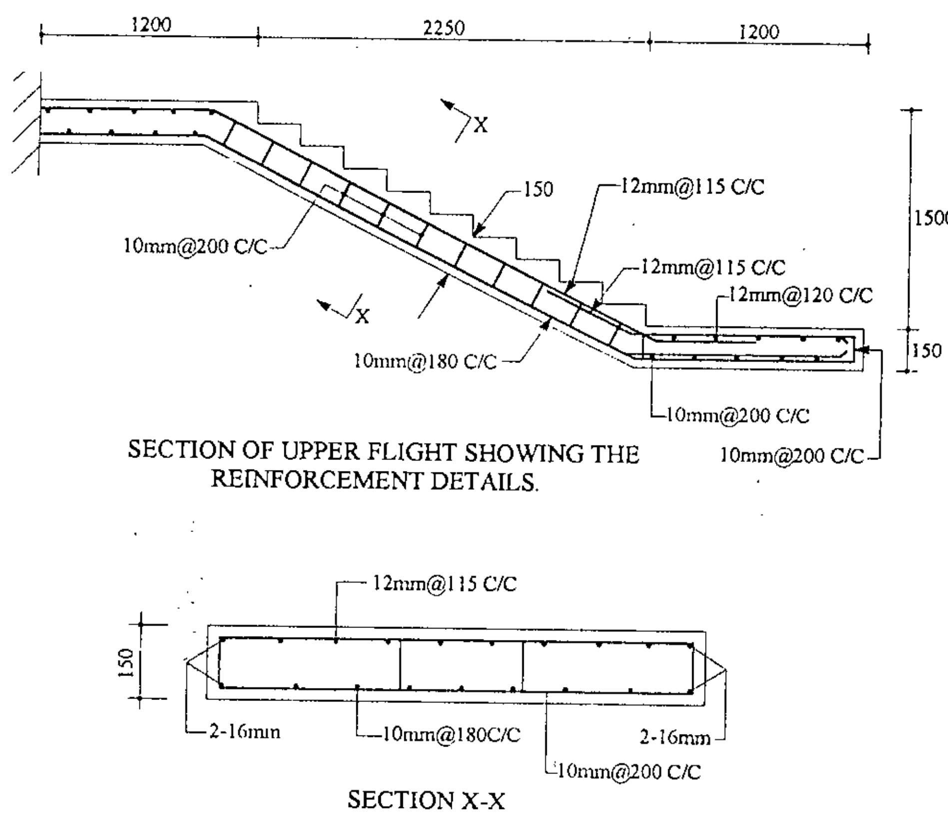


FIG: 6.10

CHAPTER 7
CONCLUSIONS

7.1 GENERAL

Conclusions are drawn from the study of the reference stair, the sensitivity analysis, the results obtained from the finite element method and the proposed method. Unless otherwise specified, the conclusions listed here refer to the free standing stair slab held partially fixed at upper and lower floor levels and acted upon by uniformly distributed symmetric loading.

7.2 FINDINGS OF THE PRESENT STUDY

Amanat's [1993] developed equations for forces and moments at different locations of the stair in terms of parameters. Zahid [1999] proposed the equations on the basis of equilibrium of the stair. In all these studies, the stair is assumed to be held completely fixed at floor levels. But in present days, there is a tendency of eliminating the supporting beams to give the stair better architectural looks. In this case, the stair rests directly on the floor slab, which can provide only partial rigidity as support. In this study, an attempt was made to develop some guideline to analyze and design such a free standing stair resting directly on the floor slab.

- A free standing stair resting on floor slab was modeled and analyzed in detail using Ahmed's thick shell finite element . The investigation reveals typical characteristics of free standing stair resting directly on floor slab.
- A detail sensitivity analysis is performed under different conditions of design parameters. The study established the relative importance of the design parameters.
- Based on the sensitivity analysis semi-empirical equations are suggested for the bending moment and lateral shear at mid-landing section for three different loading conditions.
- The performance of the proposed equations are investigated by comparing the values given by three equations with the corresponding values obtained from finite element method analysis for ten arbitrary chosen examples of free standing stair.

- It is found that the suggested semi-empirical equations are capable of predicting bending moments and lateral shears with acceptable accuracy. It is now possible to make an acceptable estimate of the bending moment and lateral shear at mid-landing section directly using these equations, without making any formal structural analysis of the stair.
- Once the moment and shear at mid-landing section are estimated with acceptable accuracy, moments and shears at other locations can be easily obtained using equilibrium equations. Thus the analysis of free standing stair resting directly on floor slab is greatly simplified.

7.3 THE DESIGN RATIONALE

Based on this study, a simple design example is presented in chapter six. The range of validity of the method covers the most frequently occurring cases. The main advantages of the proposed method are :

a) The number of empirical equations are only two for each load case. The other required forces and moments can be obtained by the equation of equilibrium directly. This relieves the designer from the rigorous calculation required even in the approximate analytical methods.

b) The empirical equations were made on the basis of the stair directly resting on the floor slab, which is a more realistic case.

c) The proposed equations can be applied for stair slabs of commonly occurring proportions.

7.4 SCOPE FOR FURTURE INVESTIGATION

Consistent with the objectives of the present study, the overall behavior of the free standing stair, held partially fixed at supports and acted upon by symmetric loading, has been analyzed and specific design guidelines have been formulated. Further works may be carried out, some indications of which are given below.

- To gain confidence in the application of the proposed equations, a physical model may be constructed and tested.
- The analysis of stair was made, considering partially fixed supports. The equations

can also be derived for stairs with simply supported conditions. A detail study may be made considering simple supports at floor levels.

- Linear elastic analysis was made throughout. A finite element analysis with non-linear material properties can be attempted in future.
- Two possible symmetric loading were used in the analysis. Investigation for unsymmetrical loading, such as live load only on upper flight, may be carried out.

REFERENCES

1) **Ahmad, S.**,"Curved Finite Elements in the Analysis of Solid, Shell and Plate Structure," Ph.D. Thesis, University College of Swansea,1969.

2) **Amanat, K.M.**, "A Design Rationale for Free Standing Stair Slabs Based on Finite Element Analysis", M.Sc. Engg.(Civil) thesis,Dept. of Civil Engg., BUET, 1993

3) **Cusens, A. R., and Kuang, J.G.**, "Analysis of Free-standing Stairs under symmetrical Loading," COCRETE AND CONSTRUCTIONAL ENGINEERING, Vol.60, No.5 May,1965, pp. 167-172.

4) **Liebenberg, A.C.**, "The Design of Slab Type Reinforced Concrete Stairways," THE STRUCTURAL ENGINEER, Vol. 38. 5, May 1960, pp. 156-164.

5) **Sauter, F.**, "Free Standing Stairs," AMERICAN CONCRETE INSTITUTE JOURNAL, PROCEEDINGS Vol. 61, No.7, July, 1964, pp. 847-870.

6) **Siev, A.**, "Analysis of free straight Multiflight Staircases," JOURNAL OF THE STRUCTURAL DIVISION, ASCE, Vol. 88, ST3, Proc. Paper 3168, june, 1962, pp.207-232.

7) **Taleb, N.J.**, " The Analysis of Stairs with Unsupported Intermediate landings," CONCRETE AND CONSTRUCTIONAL ENGINEERING, Vol. 59. No. 9 September, 1964, pp. 315-320.

7) **Zahid, A.**,'A simplified equilibrium approach to the design of free standing stairs based on Finite Element Analysis', M. Sc. Engg.(Civil) thesis, Dept. of Civil Engg., BUET, 1999.