# INFLUENCE OF STRUCTURE PARAMETERS ON PERIOD OF FRAME STRUCTURES FOR EARTHQUAKE RESISTANT DESIGN 



## BY

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A thesis submitted to the Department of Civil Engineering of Bangladesh University of Engineering and Technology, Dhaka in partial fulfilment of the requirement for the degree of

## MASTER OF SCIENCE IN ENGINEERING (CIVIL)

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## DECLARATION

I hereby certify that the research work reported in this thesis has been performed by me and that this work has not been submitted elsewhere for any other purpose (except for publication).

January, 1997

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## ACKNOWLEDGEMENT

The author wishes to express his deepest gratitude to Dr. A.M.M. Taufiqul Anwar, Professor, Department of Civil Engineering, BUET for his continuous guidance, invaluable suggestions and affectionate encouragement at every stage of this study.

The author wishes to express his profound gratitude to Professor M. Azadur Rahman Professor and Head, Department of Civil Engineering, BUET for the interest taken by him in this research and for his constant encouragement and guidance.

The author is grateful to Dr. Salek M. Seraj, Associate Professor, Department of Civil Engineering, BUET for the interest taken by him in this research and for his constant encouragement and guidance.

The author is grateful to Dr. Md. Shamsul Hoque, Associate Professor, Department of Civil Engineering, BUET, for rendering his helping hands during the course of this study.

The author expresses his thanks to Dr. A.B.M. Badruzzaman for his co-operation in using the computer facility. The author is thankful to Mr. M. A. Malek for the care he has taken in typing the manuscript.


#### Abstract

Codes of practice suggest equivalent static force method for earthquake resistant design of frame structures of moderate height. For regular frames, the codes propose approximate formulae for fundamental period required for the evaluation of base shear. These formulae describe period either as a function of height of the structure or number of stories. Lack of reflection of other structure parameters makes them grossly approximate. An investigation is conducted in this work to assess the influence of different structure parameters on the period of regular frame structures. Model frames, divided into six major groups are subjected to modal analysis to evaluate the fundamental frequency and corresponding period. General purpose finite element package ANSYS, Revision 5.2, has been employed to conduct the analyses. The analysis tool assumes constant stiffness and mass effects and neglects damping. Appropriate elements are selected from ANSYS element library to model the frames. ANSYS option of lumped mass approach has been used for mesh elements. Masses of secondary structures are added as point mass elements at nodal points. Modal analysis has been used as analysis type and reduced method as analysis option. The extracted eigenvaleus and the corresponding eigenvectors represent the frequencies and mode shapes respectively. The fundamental period is obtained from the lowest frequency.

An extensive parametric study is conducted to identify influence of parameters like numbers and width of bays, numbers and height of stories, stiffness of columns and beams and strength and density of concrete on fundamental period. Effect of inclusion of floor slabs and masses of secondary structural elements in the analysis is also studied. The period has been found to increase with decreasing number of bays along the direction of motion or increasing number of bays transverse to the direction of motion. An increase of bay width, story height or number of stories leads to an increase of period. Approximate code formulae are found to become increasingly conservative with increasing structure height in evaluation of earthquake forces.

Use of stiffer columns and beams or concrete of higher strength has a reducing effect on period. It has been revealed that application of approximate code formulae in cases of low rise structures with stocky columns and beams or with concrete of higher strength may lead to an underestimation of earthquake forces. A reduction in beam height or column dimension in the direction of earthquake forces or choice of low strength concrete are found to reduce design earthquake forces. In most practical cases, the approximate code formulae lead to a conservative estimation of earthquake forces.


## CONTENTS

Page
Declaration ..... iii
Acknowledgment ..... iv
Abstract ..... v
Contents ..... vi
List of Figures ..... ix
List of Plates ..... xii
List of Tables ..... xiv
List of Symbols ..... xV
Chapter 1: INTRODUCTION
1.1 General ..... 1
1.2 Predominance of Frame as Structural System ..... 2
1.3 Earthquake Resistant Design ..... 5
1.3.1 General Principles ..... 5
1.3.2 Equivalent Static Force Method ..... 9
1.4 Structure Period and Earthquake Forces ..... 11
1.5 Historical Review of Works ..... 12
1.6 Objectives of the Research ..... 22
1.7 Scope of the Work ..... 22
Chapter 2: STRUCTURAL PROPERTY FORMULATION FOR MDOF SYSTEMS
2.1 Introduction ..... 25
2.2 Structural Discretization ..... 25
2.3 Derivation of Equations of Motion for Multiple Degrees of Freedom Systems ..... 28
2.4 Formulation of Structural Property Matrices ..... 34
2.4.1 Elastic Stiffness Matrix ..... 34
2.4.2 Mass Properties ..... 38
2.4.3 Lumped Mass Matrix ..... 38
2.4.4 Consistent Mass Matrix ..... 41
2.4.5 Damping Matrix ..... 44
2.4.6 External Loading ..... 44
Chapter 3: FREE VIBRATION ANALYSIS AND PERIOD OF FRAME STRUCTURES
3.1 Introduction ..... 45
3.2 Analysis of Vibration Frequencies and Period ..... 45
3.3 Analysis of Vibration Mode Shapes ..... 48
3.4 Structure Parameters that Influence Period ..... 51
Chapter 4: REVIEW OF SOME CODES OF PRACTICE FOR EARTHQUAKE RESISTANT DESIGN
4.1 Introduction ..... 52
4.2 Uniform Building Code ..... 52
4.3 Bangladesh National Building Code (BNBC 1993) ..... 56
4.4 Standard Association of Australian Earthquake Code (SAA) ..... 57
4.5 Indian Standard Criteria for Earthquake Resistant Design of Structures (IS) ..... 58
4.6 Comparision of Methods for Determination of Period by Codes ..... 59
Chapter 5: DETERMINATION OF PERIOD BY MODAL ANALYSIS USING ANSYS
5.1 Introduction ..... 62
5.2 General Discussions on ANSYS ..... 62
5.3 Modal Analysis by ANSYS ..... 63
5.3.1 Introduction ..... 63
5.3.2 Assumptions and Restrictions ..... 63
5.3.3 Analysis of Undamped Structures ..... 63
5.3.4 Eigenvalue and Eigenvector Extraction ..... 64
5.3.5 Modal Analysis Procedure ..... 68
Chapter 6: INFLUENCE OF STRUCTURAL PARAMETERS ON PERIOD OF FRAME STRUCTURES
6.1 Introduction ..... 73
6.2 Description of Models ..... 74
6.3 Determination of period of Models Following Approximate Code Formulae ..... 85
6.4 Effect of Structure Parameters on Period ..... 93
6.4.1 Determination of Period of Models by Modal Analysis ..... 93
6.4.2 Effect of Number of Bays ..... 94
6.4.3 Effect of Bay Width ..... 96
6.4.4 Effect of Story Height ..... 98
6.4.5 Effect of Stiffness of Columns ..... 100
6.4.6 Effect of Stiffness of Beams ..... 103
6.4.7 Effect of Strength of Concrete ..... 105
6.4.8 Effect of Density of Concrete ..... 107
6.4.9 Effect of Number of Stories ..... 109
6,4.10 Effect of Inclusion of Floor Slabs and Mass of Secondary structural Elements ..... 111
6.5 Comparison of Results of Modal Analysis and Code Formulae ..... 127
6.6 Investigating the Limitations of Approximate Code Formulae ..... 127
6.7 Choosing Structure Parameters to reduce Earthquake Forces ..... 131
Chapter 7: CONCLUSIONS AND RECOMMENDATIONS
7.1 Conclusions ..... 134
7.2 Recommendations for Future Study ..... 135
REFERENCES ..... 136
APPENDIX ANSYS Inputs and outputs

## LIST OF FIGURES

Fig. 1.1 Structural systems (a) Frame (b) Shear wall (c) Frame-shear wall (d) Framed-tube4

Fig. 1.2 Structural concepts versus number of stories 6
Fig. 2.1 Discretization of a general beam-type structure
Fig. 2.2 (a) Non-uniform straight beam segment (b) Beam deflection due to unit nodal translation at left end (c) Beam deflection due to unit nodal rotation at left end (d) Real rotation and virtual translation of node37

Fig. 2.3 Lumping of mass at beam nodes
40

Fig. 2.4 Node subjected to real angular acceleration and virtual translation

Fig. 4.1 Variation of fundamental period with story number
Fig. 4.2 Variation of proportionality factor, C with period, T (value of S is considered to be 1 )61

Fig. 5.1 3-Dimensional elastic beam element71

Fig. 5.2 Elastic shell element 72
Fig. 5.3 Structural mass element 72
Fig. 6.1 Plan configurations of models included in group GpA representing 6 -story and 12 -story 3-D frame structures a) Model A1 b) Model A2 c) Model A3 d) Model A4

Fig. 6.2 Plan configurations of models included in group GpB representing 6 -story and 12 -story 3 -D frame structures (a) Model B1 (b) Model B2 (c) Model B3 (d) Model B4

Fig. 6.3 Plan configurations of models included in group GpC representing 6 -story and 12 -story 3-D frame structures (a) Model C1 (b) Model C2 (c) Model C3 (d)ModelC4

Fig. 6.4 Plan configurations of models included in group GpD representing 6 -story and 12 -story 3 -D frame structures (a) $3 \times 3$ bays (b) $4 \times 4$ bays (c) $5 \times 5$ bays

Fig. 6.5 Plans of models included in group GpE representing 6 -story, 8 -story, 10 -story, 12 -story and 14-story 3-D frame structures (a) $3 \times 3$ bays (b) $4 \times 4$ bays c) $5 \times 5$ bays
Fig. 6.6 Plan configurations of models included in group GpF representing 6 -story and 12 -story 3-D frame structures (a) $2 \times 2$ bays (b) $3 \times 3$ bays (c) $4 \times 4$ bays (d) $5 \times 5$ bays

Fig. 6.7 Periods of 6-story and 12-story models following approximate code formulae89

Fig. 6.8 Effect of story height on period of frame structures 91
$\begin{array}{ll}\text { Fig. 6.9 } & \begin{array}{l}\text { Effect of number of stories on period of structures } \\ \text { following approximate code formulae }\end{array} \\ 92\end{array}$
Fig. 6.10 $\begin{aligned} & \text { Effect of numbers of bays on period of 6-story } \\ & \text { frame structures }\end{aligned}$
Fig. 6.11 Effect numbers of bays on period of 12-story frame structures 95
$\begin{array}{ll}\text { Fig. 6.12 } & \begin{array}{l}\text { Effect of bay width on period of 6-story frame } \\ \text { structures }\end{array}\end{array}$
Fig. 6.13 $\begin{aligned} & \text { Effect of bay width on period of 12-story frame } \\ & \text { structures }\end{aligned}$
Fig. 6.14 $\begin{aligned} & \text { Effect of story height on period of 6-story frame } \\ & \text { structures }\end{aligned}$
Fig. 6.15 $\begin{aligned} & \text { Effect of story height on period of 12-story frame } \\ & \text { structures }\end{aligned}$

Fig. 6.16 Effect of column stiffness on period of 6-story frame
$\begin{array}{ll}\text { Fig. 6.17 } & \begin{array}{l}\text { Effect of column stiffness on period of 12-story } \\ \text { frame structures }\end{array}\end{array}$
Fig. 6.18 Effect of beam stiffness on period of 6-story frame structures104
$\begin{array}{ll}\text { Fig. 6.19 } & \begin{array}{l}\text { Effect of beam stiffness on period of 12-story } \\ \text { frame structures }\end{array}\end{array}$
$\begin{array}{ll}\text { Fig. 6.20 } & \begin{array}{l}\text { Effect of concrete strength on period of 6-story } \\ \text { frame structures }\end{array}\end{array}$
Fig. 6.21 $\begin{aligned} & \text { Effect of concrete strength on period of 12-story } \\ & \text { frame structures }\end{aligned}$
$\begin{array}{ll}\text { Fig. 6.22 } & \begin{array}{l}\text { Effect of density of concrete on period of 6-story } \\ \text { frame structures }\end{array}\end{array}$
Fig. 6.23 Effect of density of concrete on period of 12-story frame structures

108

Fig. 6.24 Effect of number of stories on period of frame structures

Fig. 6.25 Effect of inclusion of floor slabs \& mass of secondary elements on period of 6-story frame structures

Fig. 6.26 Effect of inclusion of floor slabs \& mass of secondary elements on period of 12 -story frame structures
$\begin{array}{lll}\text { Fig. 6.27 } & \begin{array}{l}\text { Comparison of periods determined by modal } \\ \text { analysis and approximate code formulae }\end{array} & 128\end{array}$

## LIST OF PLATES

## Page

Plate 6.1 3-Dimensional ANSYS display of $5 \times 5$ bay 6 story frame structure without floor slab ..... 86
Plate 6.2 3-Dimensional ANSYS display of $5 \times 5$ bay 12 story frame structure without floor slab ..... 86
Plate 6.3 3-Dimensional ANSYS display of $5 \times 5$ bay 6 -story frame structure with floor slab ..... 87
Plate 6.4 3-Dimensional ANSYS display of $5 \times 5$ bay 12-story frame structure with floor slab ..... 87
Plate 6.5 Isometric view of $5 \times 5$ bay 6-story frame structure without floor slab showing symbolic indication of degrees of freedom and direction of excitation ..... 115
Plate 6.6 Modal shape of $5 \times 5$ bay 6 -story frame structure without floor slab for 1st mode ..... 115
Plate 6.7 Modal shape of $5 \times 5$ bay 6-story frame structure without floor slab for 2nd mode ..... 116
Plate 6.8 Modal shape of $5 \times 5$ bay 6-story frame structure without floor slab for 3rd mode ..... 116
Plate 6.9 Modal shape of $5 \times 5$ bay 6-story frame structure without floor slab for 4 th mode ..... 117
Plate 6.10 Modal shape of $5 \times 5$ bay 6-story frame structure without floor slab for 5th mode ..... 117
Plate 6.11 Isometric view of $5 \times 5$ bay 12-story frame structure without floor slab showing symbolic indication of degrees of freedom and direction of excitation ..... 118
Plate 6.12 Modal shape of $5 \times 5$ bay 12-story framestructure without floor slab for 1st mode ..... 118
Plate 6.13 Modal shape of $5 \times 5$ bay 12-story frame structure without floor slab for 2nd mode ..... 119
Plate 6.14 Modal shape of $5 \times 5$ bay 12-story frame structure without floor slab for 3rd mode ..... 119
Plate 6.15 Modal shape of $5 \times 5$ bay 12 -story frame structure without floor slab for 4th mode ..... 120
Plate 6.16 Modal shape of $5 \times 5$ bay 12 -story frame structure without floor slab for 5th mode ..... 120
Plate 6.17 Isometric view of $5 \times 5$ bay 6 -story frame structure with floor slab showing symbolic indication of degreesof freedom and direction of excitation ..... 121
Plate 6.18 Modal shape of $5 \times 5$ bay 6-story frame structure with floor slab for 1st mode ..... 121
Plate 6.19 Modal shape of $5 \times 5$ bay 6-story frame structure with floor slab for 2 nd mode ..... 122
Plate 6.20 Modal shape of $5 \times 5$ bay 6-story frame structure with floor slab for 3rd mode ..... 122
Plate 6.21 Modal shape of $5 \times 5$ bay 6-story frame structure with floor slab for 4th mode ..... 123
Plate 6.22 Modal shape of $5 \times 5$ bay 6-story frame structure with floor slab for 5th mode ..... 123
Plate 6.23 Isometric view of $5 \times 5$ bay 12-story frame structure with floor slab showing symbolic indication of degrees of freedom and direction of excitation ..... 124
Plate 6.24 Modal shape of $5 \times 5$ bay 12-story frame structure with floor slab for 1st mode ..... 124
Plate 6.25 Modal shape of $5 \times 5$ bay 12-story frame structure with floor slab for 2 nd mode ..... 125
Plate 6.26 Modal shape of $5 \times 5$ bay 12-story frame structure with floor slab for 3rd mode ..... 125
Plate 6.27 Modal shape of $5 \times 5$ bay 12-story frame structure with floor slab for 4th mode ..... 126
Plate 6.28 Modal shape of $5 \times 5$ bay 12-story frame structure with floor slab for 5 th mode ..... 126

## LIST OF TABLES

## Page

Table 4.1 Fundamental period and coefficient depending on fundamental period for moment resisting frames, deforming freely59

Table 6.1 Structure parameters applicable, in general to
selected models

## LIST OF SYMBOLS

a Coefficient, distance, maximum acceleration of building
[A] Symmetric matrix
[B] Tridiagonalized form of [A]
C Seismic coefficient
$\mathrm{c}_{\mathrm{ij}} \quad$ Damping influence coefficient
c Damping matrix
E Modulus of elasticity
$\left\{\mathrm{F}_{\mathrm{s}}\right\} \quad$ Elastic force matrix
$F_{D} \quad$ Force due to damping
$\mathrm{F}_{\mathrm{z}} \quad$ Force due to inertia
$\mathrm{F}_{\mathrm{s}} \quad$ Force due to spring
F(t) Externally applied load
$f_{c}^{\prime} \quad$ Cylinder strength of concrete
$\mathrm{f}_{\mathrm{i}} \quad$ ith natural frequency (cycles per unit time)
g Acceleration due to gravity
H Horizontal earthquake force
$h_{n} \quad$ Height of building
I Moment of inertia, occupancy importance factor
K Horizontal force factor
k Stiffness
k Structural stiffness matrix
$\mathrm{k}_{\mathrm{ij}} \quad$ Stiffness influence coefficient
[L] Lower triangular matrix
L Length of beam
M Mass of building
$\mathrm{M}(\mathrm{x})$ Internal moment
m mass matrix
$\mathrm{m}_{\mathrm{ij}} \quad$ Mass influence coefficient
N, n Number of stories
$P_{a} \quad$ External nodal force
R Response modification factor
S Site coefficient for soil characteristics
T Fundamental period of vibration
[T] Matrix constructed to tridiagonalize [A]
$\{\dot{u}\}$ Nodal velocity vector

| $\{i i\}$ | Nodal acceleration vector |
| :---: | :---: |
| $\{u\}$ | Nodal displacement vector |
| V | Total horizontal seismic force |
| $v$ | Displacement |
| \{v\} | Displacement vector |
| $\{\dot{v}\}$ | Velocity vector |
| $\{\ddot{v}\}$ | Acceleration vector |
| $\hat{v}$ | Displacement vector |
| $\mathrm{v}_{\mathrm{a}}$ | Virtual displacement |
| W | Building weight, total seismic load |
| w | Unit weight of concrete |
| $X_{u}$ | Displacement |
| \{x\} | Displacement vector |
| $\mathrm{W}_{\mathrm{E}}$ | External work |
| $\mathrm{W}_{\text {I }}$ | Internal work |
| $\delta \mathrm{W}_{\mathrm{E}}$ | External virtual work |
| $\delta \mathrm{W}_{\text {I }}$ | Internal virtual work |
| t | Time |
| Z | Seismic zone factor |
| $\psi$ | Generalized displacement function |
| $\theta$ | Rotation, phase angle |
| $\omega$ | Angular velocity |
| $\{\omega\}$ | Frequency vector |
| $\omega_{1}$ | Lowest frequency |
| $\alpha_{n}$ | Seismic coefficient |
| $\beta$ | A coefficient depending upon soil foundation system |
| $\phi$ | Modal displacement |
| $\{\phi\}_{i}$ | mode shape of ith natural frequecy |
| $\omega_{i} \quad$ i | ith natural circular frequency (radians per unit time) |
| $\left\{\phi_{i}\right\}$ | mode shape of mode i |
| $\left\{\psi_{0}\right\}$ E | Eigen vector |
| $\lambda \quad \mathrm{E}$ | Eigen values |
| $\left\{\hat{\phi}_{i}\right\}$ E | Eigen vector (unknown) mode shape |
| $\phi_{n} \quad \mathrm{n}$ | $n$ nh mode shape |

## CHAPTER 1

INTRODUCTION

### 1.1 GENERAL

Reinforced concrete frames are the most widely used structural system in building construction practice. Codes recommend equivalent static force method for earthquake resistant design of such structures upto certain height. Equivalent static force method suggests calculation of base shear and distribution of this quantity as earthquake forces over the height of the structure. Major earthquake resistant design codes give different formulae to calculate the base shear. Irrespective of the formula, the base shear is always expressed as a function of the period of the structure.

Precise determination of the structure period for using it in base shear calculation often constitutes certain difficulties. In the preliminary design stage when the structure proportions are still not known, period may only be determined approximately. Even when a structure is finalized in geometry and proportion, time consuming precise period calculation may not be wise from the economy point of view or may be prohibitive for a ordinary designer without access to a high level software. Codes of practice take these factors in account and propose approximate formulae for the determination of structure period.

The formulae proposed by codes for the determination of period are usually very simple. Most of them describe period as a sole function of a single
structure parameter such as height or number of stories. It is anticipated that the period should depend on all the stiffness and mass characteristics of the structure. It is, therefore, intended in this work to study the influence of such characteristics on the period of frame structures. Such a study might help in finding boundaries of application of simple code formulae for the determination of period.

### 1.2 PREDOMINANCE OF FRAME AS STRUCTURAL SYSTEM

In general, the structural system of a building is a three-dimensional complex assemblage of various combinations of interconnected structural elements. These may be discrete members or continuous assemblages. The primary function of the structural system is to carry effectively and safely all the loads acting on the building, and eventually to transmit them to the foundation. The different types of structural systems those are being used commonly in reinforced concrete buildings for resisting lateral loads, like earthquake, winds, etc. are enumerated below.

Frame: The term 'Frame' denotes a structure that derives its resistance to lateral forces from the rigidity of the connections between beams, columns, and slabs. Moment resistant frames consist of linear, horizontal members (beams) in plane and connected to linear, vertical members (columns) with rigid or semirigid joints (Fig. 1.1(a)). A moment resistant frame is identified by the prominence of its flexibility due to the flexure of the individual beams and columns and the rotation at their joints. The strength and stiffness of the frame are proportional to the column and beam size, and inversely proportional to the story height and column spacing.

Shear Wall: Shear wall can be defined as the structural system which provides strength, stiffness, and stability to a building subjected to lateral load, deriving their strength and stiffness from the inherent shape (Fig. 1.1(b)). Shear walls are supposed to resist predominantly bending produced by lateral loads.

Frame-Shear Wall: The term frame-shear wall denotes any combination of frames and shear wall (Fig. 1.1(c)). Considerations of shear wall-frame interaction leads to a more economical design.

Framed-Tube: The framed tube consists of a closely spaced grid of external columns, connected with beams (Fig. 1.1(d)). The framed tube represents a logical evolution of the conventional frame structure possessing the necessary lateral stiffness with excellent qualities while retaining the planning flexibility of interior columns.

As the height of building increases, the structural system that has to be adopted for neutralizing the effect of lateral forces economically, will be different. The likely structural system that has to be applied for various heights of the buildings (Schueller, 1977 ) are given in Figure 1.2.

From Fig. 1.2 it appears that frame structural systems are suitable for construction of low \& medium rise buildings. Due to economic reasons and easy availability of reinforced concrete, construction of R.C. frame structures has gained enormous popularity in our country. The simplicity in design \& construction is another major factor for choosing frame structural systems to a large extent. Furthermore, with growing tendencytowards high rise construction, 15 to 20 storied buildings are being extensively constructed in a growing city like Dhaka. Thus the


Fig. 1.1 Structural systems (a) Frame (b) Shear wall (c) Frame-shear wall (d) Framed-tube
quantum of frame structure construction has become more than any other type of structural systems.

Since the construction of frame structures has become so extensive in our country, it is appropriate to study the basic approximations laid in their design. It is, therefore, intended here to investigate the influence of structure parameters on period of frame structures, in particular and also to find limitations of approximate formula for calculation of period of such structures.

### 1.3 EARTHQUAKE RESISTANT DESIGN

### 1.3.1 General Principles

In earthquake-resistant design, it is not sufficient to make a member `strong'. It must also have a reserve of ductility. The seismic loads on the structure during an earthquake result from internal inertia force, which are created by ground accelerations. Forces are imposed on structures by earthquake ground motions in response to the basic law of physics that bodies at rest tend to stay at rest and bodies in motion tend to stay in motion. The inertia of the structure tending to stay at rest as the ground starts to move and then tending to move in the direction of the initial ground motion as the ground reverses, imposes lateral forces on the structural framing. Seismic forces, since they are due to inertia, are directly related to the height of the structure.

The magnitude of the lateral forces which a structure will experience is dependent on a number of factors. The intensity of ground motion and the duration of shaking are of primary importance. Many structures little recognizable to seismic forces may survive small intensity shaking and may not be adversely affected by a fairly intense shaking of very short duration.


Fig.1.2 Structural concepts versus number of stories

The buildings which are to survive in an area of prolonged strong shaking must be competently designed and well constructed.

The response of the structure to the ground motion is also a primary factor ( Fintel, 1974 ). The fundamental period and higher modes of the structure and their relationship to the frequency content of the ground motion affect the magnitude of the forces in the structures. Resonance between the structure and the ground although noted in past earthquakes is not usually important. The motions in the area close to the epicenter are highly erratic and are in the short period range, so that resonance effects in this area are not so pronounced. As the distance from the source of the ground motion increases, the shorter period motions are dissipated and the long period motions predominate. At these greater distances the longer period, taller structures may respond resonantly (Council of the Standards Association of Australia, 1979 ).

As mentioned above, the forces are directly related to the weight of the structure so that a reduction in the weight means a reduction in force. There is, therefore, an advantage to be gained in reducing the weight of the structure for seismic design. Heavy weights placed high in the structure will increase the fundamental period of the structure.

In speaking of response, the normal thinking by the designer is that the structure is remaining in an elastic state, that is, the structure will be displaced by the lateral forces, and return to its original configuration without having experienced any permanent deformation in any of its members. This assumption is valid for most structures subjected to minor seismic forces. This assumption is not valid for the strong ground motion associated with a major earthquake. It would be economically unfeasible to attempt to design a structure that would behave completely elastically in
such an event, and might even be impossible since increased stiffness attracts increased forces.

Seismic codes for designing earthquake resistant structures have been developed bearing in mind that the phenomena that is being dealt with, is dynamic in nature, but that for most structures which are built, a rigorous dynamic analysis is not practicable even with the present high speed computer capabilities. The framers of the codes, therefore sought provisions which would approximate the results of the more rigorous approach in an easily usable procedure applicable to a large range of types of structures. The code levels of equivalent ground accelerations are lower than the recorded accelerations during moderate to strong earthquakes. The inelastic response described above is one of the reasons why code forces have been lower than that obtained using recorded motions and an assumed elastic structural response. Most structures, and especially buildings which house people, have many secondary strengths and energy absorbing features which are not calculated in the structural seismic force resisting system, and thus justify lower code values without jeopardizing safety.

Girder and column stiffnesses appear to be the important factors in determining the magnitude and distribution of deformations and member forces which will result from a ground excitation. For low-rise building the base shear decreases with the increase of girder stiffness (Ahmad, 1983).

The earthquake resistant design of structures can be done ( Taranath, 1976) following the three accepted methods, namely,
i) The equivalent static force or the building code method.
ii) The response spectrum method
iii) The time history method

According to the specifications given by different earthquake resistant codes, the response spectrum and time history method of analysis are not essential for earthquake resistant design of low and medium rise regular frame structures. For such structures, equivalent static force method is usually recommended. As the present study will focus on low and medium rise regular frame structures only equivalent static force method will be described further as appropriate design method for such structures.

### 1.3.2 Equivalent Static Force Method

In Equivalent Static Force Method, static analysis is used with equivalent static loadings to represent the dynamic action of the earthquake on the structure. The continued use of the static approach in the design of buildings is based on the premise that buildings so designed have performed well in the past earthquakes. These good performances may have been made due to other factors that were present in the older buildings but ignored in the analysis, such as contribution to strength and stiffness by constructional elements and ductility of the structure. The vertical seismic forces are not generally considered for the design of structures except for the effect of uplift forces and for very important structures such as nuclear reactor building. In typical building design only the horizontal components of earthquake forces are considered and are assumed to act nonconcurrently along the two major structural axes. Earthquake-resistant design involves engineering judgment and experience as well as application of scientific principles. The design involves economic considerations and probabilities. Earthquake resistance calls for energy absorption rather than strength resistance only. For convenience in design, an earthquake is translated into an equivalent static load acting horizontally on the building and is intended
to represent the inertia force, which is mass times acceleration occuring at the critical instant of maximum deflection and zero velocity during the largest cycle of vibration as the structure responds to the earthquake motion. These design loadings, however, incorporate modifying factors to cover local variations in seismicity, type of construction, soil conditions, usage of building, etc.

The concept of an assumed constant lateral acceleration permits the determination of lateral force as simply the product of weight of the element considered and the ratio of the selected lateral acceleration to the acceleration due to gravity. This is called the seismic coefficient and is the basis of most codes.

Using Newton's second law of motion, the total lateral seismic force, also called the base shear, is determined by the relation,

$$
V=M a
$$

where,

> V = total horizontal seismic force over the height of the building $\quad$ (also called the base shear)
> M = mass of the building
> a = the maximum acceleration of the building

Since, $M=W / g$
where,

$$
\begin{aligned}
& \mathrm{W}=\text { building weight } \\
& \mathrm{g}=\text { acceleration due to gravity } \\
& \mathrm{V}=\mathrm{Wa} / \mathrm{g}=\mathrm{Wc}
\end{aligned}
$$

where, c is called the seismic coefficient, which represents the ratio of maximum earthquake acceleration to the acceleration due to gravity.The base shear is distributed over the height of the structure by considering the response of the structure during an earthquake.The seismic coefficient is modified by factors which take into account the following:

- Dynamic properties of the structures
- Seismicity of the region
- Importance of the structure
- Subsoil conditions
- Allowable stresses and load factors

Here, again, the dynamic properties of the structure are dependent on natural period of vibration, modal shapes and damping characteristics of the structures. The code formula may therefore underestimate the response of longer period buildings because it does not properly recongize the contributions of higher modes of vibrations (Chopra and Yim, 1987)

### 1.4 STRUCTURE PERIOD AND EARTHQUAKE FORCES

The natural frequency can be defined as the frequency with which the whole system oscillates in the absence of external forces, or for a system with more than one degree of freedom, the frequency of one of the normal modes of vibration. The natural period of vibration can be defined as the period of the free oscillation of a body or system, when the period varies with amplitude. The natural period is the period when the amplitude approaches to zero. Mathematically, the natural frequency is inversely proportional to the natural period of a system.

For earthquake resistant design of regular frame structures design codes generally recommend simplified formulae for the determination of natural period. These formulae are either function of story height or number of stories. But actually the period of vibration of an elastic system is a function of its mass and stiffness properties. Hence the base shear or the earthquake forces are also function of mass and stiffness distribution of the structure.

Again the period of vibration of a structure is proportional to the stiffness, mass and dynamic properties of materials of structures. So, the base shear is dependent on the stiffness, mass and dynamic properties of materials of structures.

### 1.5 HISTORICAL REVIEW OF WORKS

The fundamental period plays a key role in dictating the magnitude of design earthquake forces of frame structures. For making frame structures earthquake resistant, it is important to determine the design earthquake forces as precisely as possible so that they can reflect the actual conditions. It is essential, therefore, to determine the period of the structure with high accuracy. Many advances have been made to develop simple approximate formulae for determining the period of a structure. But still these formulae have got ample rooms for further improvement. Even now these formulae suffer from serious shortcomings and limitations which warrant further investigations and study for necessary improvement.

Tangachi of Tokyo Institute of Technology took an initiative for determining the periods of structures by actual observations of the vibrations of the buildings. Based on his numerous observations, he suggested an empirical relation for the fundamental period of buildings as

$$
\mathrm{T}_{\mathrm{N}}=(.07 \text { to } .09) \mathrm{N} \mathrm{Sec}
$$

where,
N is the number of stories

Again, it was seen that, in addition to the proportions of buildings, relative rigidity influences the period. The effect of rigidity on the period was illustrated by the observations on the Marunonchi Building taking Tokyo Station. When the building was completed, its period was .94 sec . After semi destructive earthquake of Apr. 26, 1922, when the building suffered slight damage, the period increased to 1.01 sec . Repairing and strengthening of the building reduced the period to .71 sec . The building suffered serious damage in the 1923 earthquake and in this condition it had a period of 1.18 sec . After rehabilitation, the period of free vibration became .48 sec , which was indicative of its high rigidity.

Building codes specify design and construction requirements which are intended to protect buildings from major structural damage and the public from loss of life and injury. These requirements are based to a large extent on past earthquake experience and judgment. Because of differences in the magnitude of earthquakes, geological formations, types of construction, and other factors, the philosophy of seismic design among different countries of engineers varied in different aspects. The first edition of the Uniform Building Code was published in 1927. It was prepared by the Pacific Coast Building Officials Conference after the Santa Barbare earthquake of June 29, 1925. As experience and usage increased, provisions were modified and revised editions have appeared time to time. The Uniform Building Code, 1927 edition, required that all building more than 20 ft in height, except exposed steel-frame and wood-frame buildings, be designed for earthquake forces using $7.5 \%$ of the specified vertical loads as the seismic coefficient when allowable bearing capacity of soil was 2 or more tons per square foot
and $10 \%$ for soils of lesser bearing capacity. An increase of $33.2 \%$ in the allowable unit stresses was permitted for all materials, except for structural steel, which was allowed a $50 \%$ increase.

Andrus (1952) summarised the chronological historical development of Uniform Building Codes published in different times. The Uniform Building Code, 1937 edition, adopted a formula for calculating the lateral force F as

$$
\mathrm{F}=\mathrm{CW}
$$

where,

$$
\begin{aligned}
& \mathrm{C}=\text { seismic coefficient } \\
& \mathrm{W}=\text { total dead load plus half live load }
\end{aligned}
$$

Here the seismic coefficients were established on zone basis.

Los Angles Code, 1940 edition, proposed formula for calculating the base shear of a structure whose height is not more than 150 feet as (Housner, 1959)

$$
V=\left(\frac{.6}{n+3.5}\right) W
$$

where,

$$
\begin{aligned}
& \mathrm{n}=\text { number of stories } \\
& \mathrm{W}=\text { weight of the structure }
\end{aligned}
$$

Uniform Building Code, 1949 edition, modified the procedure for determining the seismic coefficient (Biggs et al., 1959) by introducing a formula

$$
C=\frac{.15}{N+4.5}
$$

where,
N is number of stories above the story under consideration.

Joint Committee of San Francisco, California Section (1951) prepared the Lateral Force Code which represents a work of outstanding significance and value in pointing out a rational approach to the dynamic problem of earthquake resistant design. For the first time, the determination of the seismic coefficient $C$ took into consideration the dynamic behavior of buildings.

Ulrich and Carder (1952) listed some of the results of the work done by the Coast and Geodetic Survey. The Coast and Geodetic Survey measured the period of vibration of hundreds of buildings, water-tank towers, and ground vibrations. Basing on this data, the Joint Committee of San Francisco proposed an empirical relation for the period of vibration of buildings taking into consideration the height and width of buildings as follows

$$
T_{n}=.06 \frac{H}{\sqrt{b}}
$$

where,
$\mathrm{H}=$ height, ft
$\mathrm{b}=$ width in ft , in the direction of motion considered

The Joint Committee however, recommended the use of a more conservative coefficient (.05) in the above equation in the computation of the seismic coefficient $\mathrm{C}=.015 / \mathrm{T}$. The total seismic force V is determined from the relationship $\mathrm{V}=\mathrm{CW}$. Where, W represents the weight of the building considered in seismic computation.

The Seismology Committee of Structural Engineers Association, California (1958) proposed a formula for calculating base shear of frame structures for earthquake resistant design as,

$$
\mathrm{V}=\mathrm{KCW}
$$

where,

$$
\begin{aligned}
& C=\frac{.05}{3 \sqrt{T}} \\
& T=\frac{.05 H}{\sqrt{D}}
\end{aligned}
$$

Above, $T$ is considered to be the fundamental period of vibration; $H \& D$ are the height and width of the structure, respectively.

Apart from developing empirical formulae for determining base shear and period of structure by different codes, simultaneous research works by the individuals were carried out for relating theory of structural dynamics for earthquake resistant design of structures.

Blume (1958) analyzed the frame structure applying the theory of dynamics and concluded that the fundamental period of vibration is a logical index of base shear for general code purposes. However, the height and width of the building considered either individually or together are not adequate indicators of period of modern, flexible structures without walls. He also suggested that, these periods should be specifically computed or improved approximate method should be utilized. He narrated that the present-day seismic codes are reasonably adequate for moderate earthquakes but, because of the oversimplification of a very complex subject, they can not produce consistent results. The real earthquake values of various types of buildings and structures are not necessarily proportionate to the lateral force
factors used in design. Stated in another way, code requirements are not producing constant safety factors because of too much reliance on coefficients without adequate regard to the structural-dynamics phenomenaand ultimate resistances involved in various types and rigidities of construction.

Clough (1960) summarized the principal factors controlling the dynamic response of structures to the earthquake. He also related these principal factors controlling the dynamic response of structures to the earthquakes with the lateral force provisions recommended for inclusion in the Uniform Building Code by Structural Engineers Association of California. He showed that the base shear, V , can be determined with the following formula:

$$
V=W \frac{\ddot{u}}{g}\left[\frac{1}{1-\left[\frac{T}{T_{P}}\right]^{2}}\right]
$$

where,

$$
\begin{aligned}
& T=2 \pi \sqrt{\frac{W}{g k}} \\
& \mathrm{~T}=\text { fundamental period of vibration } \\
& \mathrm{W}=\text { weight of the structure } \\
& \mathrm{k}=\text { stiffness of structure } \\
& \mathrm{g}=\text { acceleration due to gravity } \\
& \mathrm{T}_{\mathrm{p}}=\text { period of structure due to harmonic motion } \\
& \ddot{u}=\text { ground acceleration }
\end{aligned}
$$

It is seen that the response of a structure depends, in a very direct fashion, on the natural period of the structure, which depends, in turn, on its stiffness and weight.

Blume (1967) studied how the natural period of vibration of structure can be determined applying the theory of structural dynamics. He has shown that an idealized model of a building can be analyzed and its dynamic properties can be obtained according to the established theory. He stressed the need for calculating natural period of vibration of structure for undamped condition with the formula

$$
T_{i}=\frac{2 \pi}{p_{i}}
$$

where,
$\mathrm{T}_{\mathrm{i}}=$ natural period of vibration in sec
$p_{i}=$ natural angular frequency of the ith mode in radian per sec

Uniform Building Code, 1971 edition developed by the International Conference of Building Officials (1971) modified the earlier empirical formula for the lateral force. According to this code the base shear of moment resisting space frame can be determined with the following formula

$$
V=Z K C W
$$

where,
$\mathrm{Z}=$ numerical coefficient dependent upon zone.
$C=\frac{.05}{\sqrt[3]{T}}$
$\mathrm{T}=0.1 \mathrm{~N}$
$\mathrm{C}=$ numerical coefficient for base shear
$\mathrm{T}=$ fundamental period of vibration
$\mathrm{N}=$ number of stories
$\mathrm{K}=$ horizontal force factor
$\mathrm{W}=$ weight of the structure

Here, it is seen that the fundamental period of vibration of structure is considered in computing seismic coefficient. The fundamental period of vibration was taken as the function of the number of stories of the structure. Uniform Building Code, 1991 edition, of International Conference of Building Officials (1991) modified the earlier formula for calculating the design earthquake forces of structure. Here the fundamental period of vibration of structure, $T$ is considered to be the function of height of the structure.

Selvam and Jayasree (1993) presented a paper on the simplified manual procedure for estimating the period and amplitudes of concrete shear buildings. From the analysis of nearly one hundred and fifty buildings, the following formula was evolved for determining the fundamental period of structures

$$
T^{2}=\frac{M}{K}\left[16 n^{2}+16 n+7.3\right]
$$

where,
$\mathrm{T}=$ natural period in sec
$\mathrm{M}=$ total mass of all floors in a frame
$\mathrm{K}=$ total stiffness of all columns in a frame
$\mathrm{n}=$ number of stories

Bangladesh National Building Code (1993) and Uniform Building Code, 1994 edition, developed by International Conference of Building Officials (1994) formulated identical approximate formula for calculating period of structure. They formulated empirical relationship for base shear calculation as

$$
V=\frac{Z I C}{R} W
$$

where,
$Z=$ seismic zone coefficient
I = structure importance coefficient
$\mathrm{R}=$ response modification coefficient for structural system
$\mathrm{W}=$ total seismic load
$\mathrm{C}=$ numerical coefficient given by the relation

$$
C=\frac{1.25 S}{T^{\frac{2}{3}}}
$$

$\mathrm{T}=$ fundamental period of vibration in sec
$S=$ site coefficient for soil characteristics

For regular concrete frames, period T may be approximated as

$$
T=.03\left(h_{n}\right)^{3 / 4}
$$

where,

$$
h_{n}=\text { height of structure }
$$

Here, it is seen that according to the above formula the period of regular frame structure is function of height of structure only.

A lot of research works is being carried out to formulate simple empirical formula for calculating base shear and period of frame structure for earthquake resistant design. The base shear depends on several coefficients and the accuracy of design earthquake forces certainly depends on the precise evaluation of these coefficients. It is also apparent that all
coefficients related with base shear calculation doesn't suffer with much of limitations except fundamental period of vibration, T, and horizontal force factor or response modification factor, K. Shen and Wang (1987) have shown that the horizontal force factor, K , specified by UBC, does not ensure the ultimate lateral resistant capacity equal to design lateral load, 1.403 V . They have also shown with the help of equal energy concept, that K has got upper limit as well as lower limit. Ultimately, they have suggested rational K value for pure plane frames which decreases with increasing degrees of redundancy .

It appears from the above review that formulae suggested by design codes for base shear of structures largely depend on the coefficients. Of all the coefficients, the coefficient related with fundamental period of vibration needs special attention.

This reviewing shows that a lot of empirical formulae have been developed so far for determining the period of frame structures. These formulae are simple and handy but approximate. All approximate formulae except very few developed so far, for determining the period of structure are based on a single structure parameter. According to the theory of dynamics, however, the period of a structure can not be a function of single structure parameter, rather it is dependent on total mass and stiffness of the structure. It would be appropriate, therefore, to investigate the effect of different structure parameters on the period of regular frame structures by finite element based computer softwares. Frequency evaluated through modal analysis procedure could be used for calculating corresponding period. This study would further lead to evaluate the limitations of the approximate code formulae, now being used for determining the period of a structure.

### 1.6 OBJECTIVES OF THE RESEARCH

The objectives of the present research are:
i) To review the major codes for earthquake resistant design and approximate evaluation of period of regular frame structures.
ii) To investigate the limitations of approximate formulae suggested by codes for the determination of period of regular frames.
iii) To evaluate period of selected model frames through modal analysis procedure of ANSYS and to study the influence of stiffness and mass parameters of frame structures on their period.
iv) To study the effect of bay widths, story heights, number of bays, column and beam proportions and other parameters on the period.
v) To investigate the effect of inclusion of slabs and secondary structural elements on the determination of period of frame structures.
vi) To identify factors that increase period and reduce earthquake forces and thus to provide designers with a guidance towards a rational design.

### 1.7 SCOPE OF THE WORK

The work reported here represents an investigation of the influence of structure parameters on the period of frame structures for earthquake resistant design. A preliminary review of major codes including the Bangladesh National Building Code for earthquake resistant design was
conducted to identify the code recommendations for determining the period of frame structures. A study was conducted for finding the limitations of the approximate code formulae for the determination of the period of frame structures. An extensive parametric study was undertaken to clearly identify the influence of stiffness and mass parameters of frame structures on their period.

The study was conducted following modal analysis method. Numerical determination of structure period was carried out by using the powerful software, ANSYS. Effect of varying the stiffness, story heights and other parameters, including the dynamic properties of the materials, on the period of structures were studied.

Periods obtained by modal analysis were compared with those obtained from approximate formulae of method A suggested by codes. Influence of rigid slabs, and secondary structural elements on the period of structures were also studied.

The problem is introduced and the objectives are stated in Chapter 1. Chapter 2 and 3 deal with formulation of structural property matrices for MDOF structures and describe the theoretical basis of vibration analysis and determination of period of frame structures. Structure parameters that may influence period are also identified in Chapter 3. Major Codes for earthquake resistant design are reviewed in Chapter 4. A comparison of approximate methods for the determination of period of regular frame structures by design codes is also done in this chapter.

Chapter 5 describes the methodology of determination of period of frame structures by modal analysis by ANSYS. Chapter 6 is dedicated to a detail parametric study on influence of different structure parameters on period.

Investigation conducted in this chapter leads to a recommendation on choice of structure parameters to reduce earthquake forces.

The conclusions made from the study are presented in Chapter 7. The chapter also recommends future work in this and related fields. Finally, ANSYS inputs and outputs for selected sample models are given in the Appendix.

## CHAPTER 2

## STRUCTURAL PROPERTY FORMULATION FOR MDOF SYSTEMS

### 2.1 INTRODUCTION

The response (i.e the distribution of internal forces and displacements) of a frame structure resulting from its base motion is influenced by the properties of both the structure and the foundation, as well as the character of exciting motion. The character of the exciting motion can be considered to be dynamic in nature. It is, therefore, necessary to formulate the problem in terms of differential equations by relating the inertia forces to the second derivative of structural displacements. The resulting equations are called equations of motion which express the dynamic equilibrium of all forces, including the inertia forces acting on the structure.

### 2.2 STRUCTURAL DISCRETIZATION

In the dynamic system, the analysis obviously is greatly complicated by the fact that the inertia forces result from structural displacements which in turn are influenced by the magnitudes of inertia forces. This close cycle of cause and effect can be attached directly only by formulating the problem in terms of differential equations. Furthermore, because the mass of a system may be distributed continuously along its length, the displacements and accelerations must be defined for each point along the axis if the inertia forces are to be completely defined. In this case the analysis must be formulated in terms of partial differential equations because the position along the span as well as the time must be taken as independent variables. There are several methods of transforming a continuous system to a discrete one in order to reduce the dimensions of the problem and simplify its solution. Some of these methods of discretization are described below.

Lumped-mass Procedure: If the mass of a system were concentrated in a series of discrete points or lumps, the analytical problem would be greatly
simplified because inertia forces would be developed only at these mass points. In this case it is necessary to define the displacements and accelerations only at these discrete points.

The lumped-mass idealization described above provides a simple means of limiting the number of degrees of freedom that must be considered in the analysis of arbitrary problems in structural dynamics. The lumping procedure is most effective in treating systems in which a large proportion of the total mass actually is concentrated in a few discrete points. Then it may be assumed that the mass of the structure which supports those point concentration are included in the lumps and the structure itself is weightless.

Generalized Displacements: If the mass of the system is quite uniformly distributed throughout, an alternative approach to limiting the degrees of freedom may be preferable. This procedure is based on the assumption that the deflected shape of the structure can be expressed as the sum of a series of specified displacement patterns; these patterns then become the displacement coordinates of the structure. A simple example of this approach of expressing deflections in structures is the trigonometric series representation of the deflection of a simple beam. In this case, the deflection may be expressed as the sum of independent sine-wave contributions.

In general, any arbitrary shape compatible with the prescribed support conditions can be represented by an infinite series of such sine-wave components. The amplitudes of the sine-wave shapes may be considered to be the coordinates of the system, and the infinite number of degrees of freedom of the actual beam are represented by the infinite number of terms included in the series. The advantage of this approach is that a good approximation to the actual shape can be achieved by a truncated series of sine-wave components.

This concept can be further generalized by recognizing that the sine-wave shapes used as the assumed displacement patterns were an arbitrary choice in this example. In general, any shape which is compatible with the prescribed geometric support conditions and which maintain the necessary continuity of internal displacement may be assumed. For any assumed set of displacements functions, the resulting shape of the structure depends upon
the amplitude terms which may be referred to as generalized coordinates. The number of assumed shape patterns represents the number of degrees of freedom considered in this form of idealization. In general, better accuracy can be achieved in a dynamic analysis for a given number of degrees of freedom by using the shape-function method of idealization than by the lumped-mass approach. However, it also should be recognized that greater computational effort is required for each degree of freedom when such generalized coordinates are employed.

Finite-Element Concept: The displacements of any given structure can be expressed in terms of finite number of discrete displacement coordinates, which combines certain features of both the lumped-mass and the generalized coordinate procedures. This approach, which is the basis of the finite-element method of analysis of structural continua, provides a convenient and reliable idealization of the system and is particularly effective in computer analysis.

The finite-element type of idealization is applicable to structures of all types: frame structures, which comprise assemblages of one-dimensional members (beams, columns, etc.); plane-stress or plate or shell-type structures, which are made up of two-dimensional components; and general three dimensional solids.

The first step in the finite-element idealization of any structure, involves dividing it into an appropriate number of segments or elements. Their size is arbitrary, they may be all of the same size or all different. The ends of the segments, at which they are interconnected, are called nodal points. The displacements of these nodal points then become the generalized coordinates of the structure.

The deflection of the complete structure can now be expressed in terms of these generalized coordinates by means of an appropriate set of assumed displacement functions. Sometimes the displacement functions are called interpolation functions because they define the shape between the specified nodal displacements. These interpolation functions could be any curve which is internally continuous and which satisfies the geometric displacement conditions imposed by nodal displacements. For example, in case of one
dimensional elements it is convenient to use the shapes which would be produced by the nodal displacements in a uniform beam. From the above discussion, it is apparent that coordinates used in the finite element method are just a special form of generalized coordinates.

The advantages of this special procedure are as follows:

- Any desired number of generalized coordinates can be introduced merely by dividing the structure into an appropriate number of segments.
- Since the displacement functions chosen for each segment may be identical, computations are simplified.
- The equations which are developed by this approach are largely uncoupled because each nodal displacement affects only the neighbouring elements; thus the solution process is greatly simplified.

In general, the finite-element approach provides the most efficient procedure for expressing the displacements of arbitrary structural configurations by means of a discrete set of coordinates.

### 2.3 DERIVATION OF EQUATIONS OF MOTION FOR MULTIPLE DEGREES OF FREEDOM SYSTEMS

Mass, elastic properties (flexibility or stiffness), energy-loss mechanism, or damping, and the external source of excitation or loading are the essential properties of any linearly elastic structural system subjected to dynamic loads. The dynamic response of a single degree of freedom system can be evaluated by the solution of a single differential equation of motion. If the physical properties of the system are such that its motion can be described by a single co-ordinate and no other motion is possible, then it actually is a single degree of freedom system and the solution of the equation provides the exact dynamic response. On the other hand, if the structure actually has more than one possible mode of displacement and it is reduced mathematically to a single degree of freedom approximation by assuming its deformed shape, the solution of the equation of motion is only an approximation of the true dynamic behaviour. The quality of the result
obtained with a single degree of freedom approximation depends on many factors, principally the spatial distribution and time variation of the loading and the stiffness and mass properties of the structure. If the physical properties of the system, constrain it to move most easily with the assumed shape, and if the loading is such as to excite a significant response in this shape, the single degree of freedom solution will probably be a good approximation; otherwise, the true behaviour may bear little resemblance to the computed response. One of the major disadvantages of the single degree of freedom approximation is that it is difficult to asses the reliability of the results obtained from it.

In general, the dynamic response of a structure can not be described adequately by a single degree of freedom model; usually the response includes time variation of the displacement shape as well as its amplitude. Such behaviour can be described only in terms of more than one displacement coordinate; that is, the motion must be represented by more than one degree of freedom. As noted earlier, the degrees of freedom in a discrete parameter system may be taken as the displacement amplitudes of certain selected points in the structure, or they may be generalized coordinates representing the amplitudes of a specified set of displacement patterns.

1 In deriving the equations of motion of a general multiple degrees of freedom system, it will be convenient to refer to the general simple beam shown in Fig. 2.1 as a typical example. The discussion applies equally to any type of structure, but the visualization of the physical factors involved in evaluating all the forces acting is simplified for this type of structure. The motion of this structure will be assumed to be defined by the displacement of a set of discrete points on the beam: $v_{1}(t), v_{2}(t) \ldots \ldots v_{i}(t) \ldots \ldots v_{N}(t)$

In principle, these points may be located arbitrarily on the structure; in practice, they should be associated with any specific features of the physical properties which may be significant and should be distributed so as to provide a good definition of the deflected shape. The number of degrees of freedom (displacement components) to be considered may vary. However, greater numbers provide better approximations of the true dynamic behavior.

In the beam of Fig. 2.1 only one displacement component has been associated with each nodal point on the beam. It should be noted, however, that several displacement components could be identified with each point; e.g the rotation and longitudinal motions might be used as additional degrees of freedom at each point. The equation of motion of the system of Fig. 2.1 can be formulated by expressing the equilibrium of the effective forces associated with each of its degrees of freedom. In general, four types of forces will be involved at any point i ; the externally applied load $p_{i}(t)$ and the forces resulting from the motion, that is, inertia $f_{I i}$ damping $f_{D i}$ and elastic $f_{s i}$ Thus for each of the several degrees of freedom the dynamic equilibrium may be expressed as

$$
\begin{align*}
& \mathrm{f}_{\mathrm{I} 1}+\mathrm{f}_{\mathrm{D} 1}+\mathrm{f}_{\mathrm{S} 1}=\mathrm{p}_{1}(\mathrm{t}) \\
& \mathrm{f}_{\mathrm{I} 2}+\mathrm{f}_{\mathrm{D} 2}+\mathrm{f}_{\mathrm{S} 2}=\mathrm{p}_{2}(\mathrm{t})  \tag{2.1}\\
& \mathrm{f}_{\mathrm{I} 3}+\mathrm{f}_{\mathrm{D} 3}+\mathrm{f}_{\mathrm{S} 3}=\mathrm{p}_{3}(\mathrm{t}) \\
& \\
& \\
& \mathrm{f}_{\mathrm{In}}+\ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots
\end{align*}
$$

or when the force vectors are represented in matrix form,

$$
\begin{equation*}
\mathbf{f}_{t}+\mathbf{f}_{D}+\mathbf{f}_{s}=\mathbf{p}(t) \tag{2.2}
\end{equation*}
$$

Each of the resisting forces is expressed most conveniently by means of an appropriate set of influence coefficients. Let, for example, the elastic force component developed at point 1 be considered ; this depends, in general, upon the displacement components developed at all points of the structure.

$$
\begin{equation*}
f_{S 1}=k_{11} v_{1}+k_{12} v_{2}+k_{13} v_{3}+\ldots .+k_{1 N} v_{N} \tag{2.3a}
\end{equation*}
$$

and, in general,

$$
\begin{equation*}
f_{S i}=k_{i 1} v_{1}+k_{i 2} v_{2}+k_{i 3} v_{3}+k_{i N} v_{N} \tag{2.3b}
\end{equation*}
$$



Fig. 2.1 Discretization of a general beam-type structure

In these expressions it has been assumed that the structural behaviour is linear, so that the principle of superposition applies. The coefficients $k_{i j}$ are called stiffness influence coefficients, defined as follows:

$$
\begin{align*}
k_{i j}= & \text { force corresponding to coordinate } \mathrm{i} \text { due to } \\
& \mathrm{a} \text { unit displacement of coordinate } \mathrm{j} \tag{2.4}
\end{align*}
$$

In matrix form, the complete set of elastic force relationship may be written as:

$$
\left\{\begin{array}{c}
f_{S 1}  \tag{2.5}\\
f_{S 2} \\
. \\
f_{S i} \\
.
\end{array}\right\}=\left[\begin{array}{ccccccc}
k_{11} & k_{12} & k_{13} & \ldots & k_{1 i} & \ldots & k_{1 N} \\
k_{21} & k_{22} & k_{23} & \ldots & k_{2 i} & \ldots & k_{2 N} \\
\ldots \ldots & \ldots \ldots & \ldots \ldots & \ldots \ldots & \ldots \ldots . . \\
k_{i 1} & k_{i 2} & k_{i 3} & \ldots & k_{i i} & \ldots & k_{i N} \\
\ldots \ldots & \ldots \ldots & \ldots \ldots & \ldots & \ldots \ldots & \ldots
\end{array}\right]\left\{\begin{array}{c}
v_{1} \\
v_{2} \\
.
\end{array}\right\}
$$

or,

$$
\begin{equation*}
\mathbf{f}_{s}=\mathbf{k v} \tag{2.6}
\end{equation*}
$$

in which the matrix of stiffness coefficients $\mathbf{k}$ is called the stiffness matrix of the structure (or the specified set of displacement coordinates) and $\mathbf{v}$ is the displacement vector representing the displaced shape of the structure. If it is assumed that the damping depends on the velocity, that is, viscous type, the damping forces corresponding to the selected degrees of freedom may be expressed by means of damping influence coefficients in similar fashion. By analogy with Eq. 2.5, the complete set of damping forces is given by

$$
\left\{\begin{array}{c}
f_{D 1}  \tag{2.7}\\
f_{D 2} \\
\vdots \\
f_{D i} \\
\cdots
\end{array}\right\}=\left[\begin{array}{ccccccc}
c_{11} & c_{12} & c_{13} & \cdots & c_{1 i} & \cdots & c_{1 N} \\
c_{11} & c_{11} & c_{11} & \cdots & c_{11} & \cdots & c_{1 N} \\
\cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots \\
c_{11} & c_{11} & c_{11} & \cdots & c_{11} & \cdots & c_{1 N} \\
\cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots
\end{array}\right]\left\{\begin{array}{c}
\dot{v}_{1} \\
\dot{v}_{2} \\
\cdots \\
\dot{v}_{i} \\
\cdots
\end{array}\right\}
$$

in which $\dot{v}_{i}$ represents velocity of the ith displacement coordinate and the coefficients $c_{i j}$ are called the damping influence coefficients. The definition of these coefficients is identical to that in Eq. 2.4:

$$
\begin{align*}
c_{i j}= & \text { force at coordinate } \mathrm{i} \text { due to unit velocity at } \\
& \text { coordinate } \mathrm{j} \tag{2.8}
\end{align*}
$$

Using matrix notations, Eq. 2.7 may be written as

$$
\begin{equation*}
\mathbf{f}_{D}=\mathbf{c} \dot{\mathrm{v}} \tag{2.9}
\end{equation*}
$$

in which the matrix of damping coefficients $\mathbf{c}$ is called the damping matrix of the structure (for the specified degrees of freedom) and $\dot{v}$ is called the velocity vector.

The inertia forces may also be expressed by a set of influence coefficients called the mass coefficients. These represent the relationship between the accelerations of the degrees of freedom and the resulting inertia forces; by analogy with Eq. 2.5, the inertia forces may be expressed as

$$
\left\{\begin{array}{c}
f_{I 1}  \tag{2.10}\\
f_{I 2} \\
\vdots \\
f_{I i} \\
\cdots
\end{array}\right\}=\left[\begin{array}{ccccccc}
m_{11} & m_{12} & m_{13} & \cdots & m_{1 i} & \cdots & m_{1 N} \\
m_{21} & m_{22} & m_{23} & \cdots & m_{2 i} & & m_{2 N} \\
\cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots \\
m_{i 1} & m_{i 2} & m_{i 3} & \cdots & m_{i 1} & \cdots & m_{i N} \\
\cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots
\end{array}\right]\left\{\begin{array}{c}
\ddot{v}_{1} \\
\ddot{v}_{2} \\
\cdots \\
\ddot{v}_{i} \\
\cdots
\end{array}\right\}
$$

where $\ddot{v}_{i}$ is the acceleration of the $i$ th displacement coordinate and the coefficients $\mathrm{m}_{\mathrm{ij}}$ are the mass influence coefficients, defined as follows:

$$
\begin{align*}
& \mathrm{m}_{\mathrm{ij}}=\text { force at coordinate } \mathrm{i} \text { due to unit acceleration } \\
& \text { at coordinate } \mathrm{j} \tag{2.11}
\end{align*}
$$

In matrix notations, Eq . 2.10 may be written as

$$
\begin{equation*}
f_{\mathrm{I}}=\mathbf{m} \ddot{\mathrm{v}} \tag{2.12}
\end{equation*}
$$

in which $\mathbf{m}$ is the mass matrix of the structure, $\ddot{\mathbf{v}}$ acceleration vector and $\mathbf{f}_{I}$ inertia force vector. Substitution of Eqs. 2.6, 2.9 and 2.12 into Eq. 2.2 gives the complete dynamic equilibrium of the structure, considering all degrees of freedom. Thus,

$$
\begin{equation*}
\mathbf{m} \ddot{\mathbf{v}}+\mathbf{c} \dot{\mathbf{v}}+\mathbf{k} \mathbf{v}=\mathbf{p}(t) \tag{2.13}
\end{equation*}
$$

The equation expressed above is the equation of multiple degrees of freedom systems. Matrix Eq. 2.13 represents N algebraic equations of motion which serve to define the response of the multiple degrees of freedom system.

### 2.4 FORMULATION OF STRUCTURAL PROPERTY MATRICES

### 2.4.1 Elastic Stiffness Matrix

Several methods have been developed for the elements of a structure stiffness matrix. In this work the finite-element method has been used for it.

In principle, the stiffness coefficient associated with any prescribed set of nodal displacements can be obtained by direct application of their definitions. In practice, however, the finite element concept frequently provides the most convenient means for evaluating the elastic properties. By this approach the structure is assumed to be divided into a system of discrete elements which are interconnected only at a finite number of nodal points. The properties of the complete structure are then found by evaluating the properties of the individual finite elements and superposing them appropriately.

The problem of defining the stiffness properties of any structure is thus reduced basically to the evaluation of the stiffness of a typical element. Stiffness coefficients of the element represent the nodal forces due to unit nodal displacements. The nodal forces associated with any nodal displacement component can be determined by the principle of virtual displacements. Stiffness coefficients are developed here for a nonuniform straight beam segment shown in Fig. 2.2a. Let $v_{1}$, and $v_{2}$ be the displacements of ends $a$ and $b$ respectively and $v_{3}$ and $v_{4}$ are corresponding rotations. The deflected shapes resulting from applying a unit displacement
of each type at the left end of the element are shown in Fig.2b \& c .These displacement functions are selected as shapes which satisfy nodal and internal continuity requirements. Generally, cubic hermitian polynomials satisfy these conditions. Hence corresponding shapes may be expressed as

$$
\begin{align*}
& \psi_{1}(x)=1-3\left(\frac{x}{L}\right)^{2}+2\left(\frac{x}{L}\right)^{3}  \tag{2,14}\\
& \psi_{3}(x)=x\left(1-\frac{x}{L}\right)^{2} \tag{2.15}
\end{align*}
$$

The equivalent shape functions for displacements applied at the right end are

$$
\begin{align*}
& \psi_{2}(x)=3\left(\frac{x}{L}\right)^{2}-2\left(\frac{x}{L}\right)^{3}  \tag{2.16}\\
& \psi_{4}(x)=\frac{x^{2}}{L}\left(\frac{x}{L}-1\right) \tag{2.17}
\end{align*}
$$

With these four interpolation functions, the deflected shapes of the element can now be expressed in terms of its nodal displacements as

$$
\begin{equation*}
v(x)=\psi_{1}(x) v_{1}+\psi_{2}(x) v_{2}+\psi_{3}(x) v_{3}+\psi_{4}(x) v_{4} \tag{2.18}
\end{equation*}
$$

For example, the vertical force developed at end 'a' due to a unit rotation applied at that point can be termed as stiffness coefficients $\mathrm{k}_{13}$. This force component can be evaluated by introducing a virtual vertical displacement at end a, as shown in Fig. 2.2d, while the unit rotation is applied as shown, and equating the work done by the external forces to the work done by the internal forces: $W_{E}=W_{I}$. In this case, the external work is done only by the vertical force component at "a" because the initial displacements of all other nodal components vanish; so

$$
\begin{equation*}
W_{E}=\delta v_{a} p_{a}=\delta v_{1} k_{13} \tag{2.19}
\end{equation*}
$$

The internal virtual work is done by the internal moments associated with $\theta_{\mathrm{a}}=1$ acting on the virtual curvatures,

$$
\frac{\delta^{2}}{\delta x^{2}}[\delta v(x)]=\psi_{1}^{\prime}(x) \delta v_{1} \text { (neglecting the effects of shear distortion) }
$$

However, the internal moment due to $\theta_{a}=\mathrm{v}_{3}=1$ may be expressed as

$$
\begin{equation*}
\mathrm{M}(\mathrm{x})=\mathrm{EI}(\mathrm{x}) \psi_{3}^{\prime \prime}(\mathrm{x}) \tag{2.20}
\end{equation*}
$$

Thus the internal work is given by

$$
\begin{equation*}
W_{I}=\delta v_{1} \int_{0}^{L} E I(x) \psi_{1}^{\prime \prime}(x) \psi_{3}^{\prime}(x) d x \tag{2.21}
\end{equation*}
$$

When the work expressions of Eq. 2.19 and 2.21 are equated, the expression for this stiffness coefficient is

$$
\begin{equation*}
k_{13}=\int_{0}^{L} E I(x) \psi_{1}^{\prime}(x) \psi_{3}^{\prime \prime}(x) d x \tag{2.22}
\end{equation*}
$$

Any stiffness coefficient associated with beam flexure, therefore, may be written equivalently as

$$
\begin{equation*}
k_{i j}=\int_{0}^{L} E I(x) \psi_{i}^{*}(x) d x \tag{2.23}
\end{equation*}
$$

When the stiffness coefficients of all the finite elements in a structure have been evaluated, the stiffness of the complete structure is obtained by merely adding the element stiffness coefficients appropriately. The method is known as direct stiffness method. In effect, any stiffness coefficient $\mathrm{k}_{\mathrm{ij}}$ of the complete structure can be obtained by adding together the corresponding stiffness coefficients of the elements associated with those nodal points. Thus if the elements $m$, $a$, and $p$ were all attached to nodal point $i$ of the complete structure, the structure stiffness coefficient for this point would be

$$
\begin{equation*}
\hat{\hat{k}}_{i i}=\hat{\hat{k}}_{i i}^{(m)}+\hat{\hat{k}}_{i i}^{(n)}+\hat{\hat{k}}_{i i}^{(p)} \tag{2.24}
\end{equation*}
$$


(a)

(b)


Fig. 2.2 (a) Non uniform straight beam segment (b) Beam deflection due to unit nodal translation at left end (c) Beam deflection due to unit nodal rotation at left end (d) Real rotation and virtual translation of node
in which the superscripts identify the individual elements. Before the element stiffness can be superposed in this fashion, they are expressed in a common global coordinate system which is applied to the entire structure. The double hats are placed over each element stiffness symbol in Eq. 2.24 to indicate that they have been transformed from their local coordinate to the global coordinates.

### 2.4.2 Mass Properties

The mass of an actual structure is continuously distributed over the spatial extent of the structure and, as a result, actual structures have an infinite number of degrees of freedom. Often, however, the important features of the dynamic response of an actual structure may be adequately approximated with far less tedious computations by idealizing the structure. In such an idealization, the mass of the structure is considered to be lumped or concentrated at certain finite number of mass points and the resistance of the structure to deflection is then represented by members of elements which are considered to be weightless but have structural strength and stiffness. These idealized structures are said to be concentrated mass structures.

In general six co-ordinates may be required to specify the position of each mass of a concentrated-mass system, three linear displacements and three angular displacements. Thus, such a system would have six degrees of freedom per mass.

It is also possible to consider the mass distributed over the length of the member. In this case, a consistent mass matrix may be generated and the approach is known as consistent mass approach.The techniques of generation of lumped mass matrix and consistent mass matrix are described below.

### 2.4.3 Lumped Mass Matrix

The simplest procedure for defining the mass properties of any structure is to assume that the entire mass is concentrated at the points at which the translational displacements are defined. The usual procedure for defining the
point mass to be located at each node is to assume that the structure is divided into elements, the nodes serving as connection points.

Fig. 2.3 illustrates the procedure for a beam-type structure. The mass of each segment is assumed to be concentrated in point masses at each of its nodes, the distribution of the segment mass to these points being determined by statics.

The total mass concentrated at any node of the complete structure then is the sum of the nodal contributions form all the elements attached to that node. In the beam system of Fig. 2.3, there are two segments contributing to each node; for example,

$$
\mathrm{m}_{1}=\mathrm{m}_{1 \mathrm{a}}+\mathrm{m}_{1 \mathrm{~b}}
$$

For a system in which only translational degrees of freedom are defined, the lumped-mass matrix has a diagonal form; for the system of Fig. 2.3 it would be written as

$$
\mathbf{m}=\left[\begin{array}{ccccccc}
m_{1} & 0 & 0 & \cdots & 0 & \cdots & 0  \tag{2.25}\\
0 & m_{2} & 0 & \cdots & 0 & \cdots & 0 \\
0 & 0 & m_{3} & \cdots & 0 & \cdots & 0 \\
\cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots \\
0 & 0 & 0 & \cdots & m_{i} & \cdots & 0 \\
\cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots \\
0 & 0 & 0 & \cdots & 0 & \cdots & m_{N}
\end{array}\right]
$$

in which there are as many terms as there are degrees of freedom. The offdiagonal terms $\mathrm{m}_{\mathrm{ij}}$ of this matrix vanish because an acceleration of any mass point produces an inertia force at that point only. The inertia force at $i$ due to a unit acceleration of point $i$ is obviously equal to the mass concentrated at that point; thus, the mass influence coefficient $\mathrm{m}_{\mathrm{ij}}=\mathrm{m}_{\mathrm{i}}$ in a lumped mass system.





Fig. 2.3 Lumping of mass at beam nodes

If more than one transitional degree of freedom is specified at any nodal point, the same point mass will be associated with each degree of freedom. On the other hand, the mass associated with any rotational degree of freedom will be zero because of the assumption that the mass is lumped in points which have no rotational inertia. Of course, if a rigid mass having a finite rotational inertia is associated with a rotational degree of freedom, the diagonal mass coefficient for that degree of freedom would be the rotational inertia of the mass. Thus the lumped mass matrix is a diagonal matrix which includes zero diagonal elements for the rotational degrees of freedom, in general.

### 2.4.4 Consistent Mass Matrix

Making use of the finite element concept, it is possible to evaluate mass influence coefficients for each element of a structure by a procedure similar to the analysis of element stiffness coefficients. A non uniform beam segment is considered for the purpose as shown in Fig. 2.4 The degrees of freedom of the segment are the translation and rotation at each end, and it will be assumed that the displacements within the span are defined by the same interpolation functions $\psi_{i}(x)$ used in deriving the element stiffness.

If the beam were subjected to a unit angular acceleration of the left end, $\ddot{v}_{3}=\ddot{\theta}_{a}=1$, accelerations would be developed along its length, as follows

$$
\begin{equation*}
\ddot{\mathrm{v}}(\mathrm{x})=\psi_{3}(\mathrm{x}) \ddot{\mathrm{v}}_{3} \tag{2.26}
\end{equation*}
$$

By d'Alembert's principle, the inertia force resisting this acceleration is

$$
\begin{equation*}
f_{1}(x)=m(x) \ddot{v}(x)=m(x) \psi_{3}(x) \ddot{v}_{3} \tag{2.27}
\end{equation*}
$$

Now the mass influence coefficients associated with this acceleration are defined as the nodal inertia forces produced by it; these can be evaluated from the distributed inertia force of Eq. 2.27 by the principle of virtual displacements. The vertical force at the left end can be evaluated by introducing a vertical virtual displacement and equating the work done by the external nodal force $p_{a}$ to the work done on the distributed inertia forces $f_{i}(x)$. Thus

$$
\begin{equation*}
p_{a} \delta v_{a}=\int_{0}^{f_{1}} f_{1}(x) \delta v(x) d x \tag{2.28}
\end{equation*}
$$

Expressing the vertical virtual displacement in terms of the interpolation function and substitution of Eq. 2.27 leads finally to

$$
\begin{equation*}
\mathrm{m}_{13}=\int_{0}^{\mathrm{L}} \mathrm{~m}(\mathrm{x}) \psi_{1}(\mathrm{x}) \psi_{3}(\mathrm{x}) \mathrm{dx} \tag{2.29}
\end{equation*}
$$

It should be noted in Fig. 2.4 that the mass influence coefficient represents the inertia force opposing the acceleration, but it is also numerically equal to the external force producing the acceleration.

From Eq. 2.29 it is evident that any mass influence coefficient $\mathrm{m}_{\mathrm{ij}}$ can be evaluated from the expression.

$$
\begin{equation*}
\mathrm{m}_{\mathrm{ij}}=\int_{o}^{L} m(x) \psi_{i}(x) \psi_{j}(x) d x \tag{2.30}
\end{equation*}
$$

The symmetric form of this equation shows that the mass matrix is symmetric; that is $m_{i j}=m_{\mathrm{j} i}$. When the mass coefficients are computed, using the interpolation functions, the result is called the consistent mass matrix.

When the mass coefficients of the elements of a structure have been evaluated, the mass matrix of the complete element assemblage can be developed by adding the element mass coefficients appropriately. In effect, any mass coefficient $\mathrm{m}_{\mathrm{ij}}$ of the complete structure can be obtained by adding together the corresponding mass coefficients of the elements associated with those nodal points.

The dynamic analysis of a consistent mass system generally requires considerably more computational effort than a lumped-mass system does, for two reasons:
(i) The lumped mass matrix is diagonal, while the consistent-mass matrix has many off-diagonal terms.


Fig 2.4 Node subjected to real angular acceleration and virtual translation
(ii) The rotational degrees of freedom can be eliminated from a lumped-mass analysis, whereas all rotational and transitional degrees of freedom must be included in a consistent-mass analysis.

### 2.4.5 Damping Matrix

If the various damping forces acting on a structure could be determined quantitatively, the finite element concept could be used to define the damping coefficients of the system. After determining the element damping influence coefficients, the damping matrix of the complete structure could be determined by a superposition process equivalent to the direct stiffness method. In practice, however, evaluation of the damping property by this method is very cumbersome. For this reason, the damping is generally expressed in terms of a percentage of mass or in terms of damping ratios established from experiments on similar structures. When damping is assumed as a percentage of mass, mathematically, it is expressed as

$$
\mathbf{c}=\lambda \mathbf{m}
$$

where,
$\lambda$ is a constant.

### 2.4.6 External Loading

If the dynamic loading acting on a structure consists of concentrated forces corresponding with the displacement coordinates, the load vector of Eq. 2.2 can be written directly. In general, however, the load is applied at other points as well as at the nodes and may include distributed landings. In this case, the load terms in Eq. 2.2 are generalized forces associated with the corresponding displacement components. The most direct means of determining the effective nodal forces generated by loads distributed between the nodes is by application of the principles of simple statics. In other words, the nodal forces are defined as a set of concentrated loads which are statically equivalent to the distributed loading.

## CHAPTER 3

## FREE VIBRATION ANALYSIS AND PERIOD OF FRAME STRUCTURES

### 3.1 INTRODUCTION

Determining the behaviour of a structure during an earthquake is basically a vibration problem. The seismic motions of the ground cause the structure to vibrate and the amplitude of vibration of its dynamic deformation and its duration are of concern to the engineer. If the motion of a body is reciprocating or oscillating in character, it is called vibration. If the vibration of an elastic structure takes place in the absence of any externaly imposed force but in the presence of external and internal frictional forces, the motion is termed as damped free vibration (Chandrapatla and Belegmdu, 1991). In the hypothetical case where it is assumed that the frictional forces are also absent, the motion is called an undamped free vibration. The undamped motion is defined simply by the elastic resistance and the inertial forces of the system, by the initial condition of the state of motion, and by the boundary conditions of the structure. The response for the hypothetical undamped vibration is an approximation of the actual damped vibration experienced by the real strictures.

### 3.2 ANALYSIS OF VIBRATION FREQUENCIES AND PERIOD

The equations of motion for a freely vibrating undamped ( Clough and Penzien, 1993 ) system can be obtained by omitting the damping matrix and appliedload vector from Eq. 2.13. Thus,

$$
\begin{equation*}
m \ddot{v}+k v=0 \tag{3.1}
\end{equation*}
$$

in which $\mathbf{o}$ is a zero vector. The problem of vibration analysis consists of determining the condition under which the equilibrium condition expressed by

Eq. 3.1 will be satisfied. By analogy with the behavior of MDOF systems, it will be assumed that the free-vibration motion is simple harmonic, which may be expressed as

$$
\begin{equation*}
\mathbf{v}(\mathrm{t})=\hat{\mathbf{v}} \sin (\omega t+\theta) \tag{3.2}
\end{equation*}
$$

In this expression $\hat{\mathbf{v}}$ represents the shape of the system (which does not change with time; only the amplitude varies) and $\theta$ is a phase angle. When the second time derivative of Eq. 3.2 is taken, the accelerations in free vibration are

$$
\begin{equation*}
\ddot{\mathbf{v}}=\omega^{2} \hat{\mathbf{v}} \operatorname{Sin}(\omega t+\theta)=-\omega^{2} \mathbf{v} \tag{3.3}
\end{equation*}
$$

Substituting Eq. 3.2 and 3.3 into Eq. 3.1 gives

$$
-\omega^{2} \mathbf{m} \hat{\mathbf{v}} \sin (\omega t+\theta)+\mathbf{k} \hat{\mathbf{v}} \sin (\omega t+\theta)=\mathbf{0}
$$

which (since the sine term is arbitrary and may be omitted) may be written as

$$
\begin{equation*}
\left[\mathbf{k}-\omega^{2} \mathbf{m}\right] \hat{\mathbf{v}}=\mathbf{0} \tag{3.4}
\end{equation*}
$$

Eq. 3.4 constitutes an eigenvalue or characteristic value problem. The quantities $\omega^{2}$ are the eigenvalues or characteristic values indicating the square of the freevibration frequencies, while the corresponding displacement vectors $\hat{\mathbf{v}}$ express the corresponding shapes of the vibrating system, known as the eigenvectors or mode shapes. Now it can be shown by Cramer's rule that the solution of this set of simultaneous equations is of the form

$$
\begin{equation*}
\hat{\mathbf{v}}=\frac{0}{\left\|\mathbf{k}-\omega^{2} \mathbf{m}\right\|} \tag{3.5}
\end{equation*}
$$

Hence, a nontrivial solution is possible only when the denominator determinant vanishes. In other words, finite-amplitude free vibrations are possible only when

$$
\begin{equation*}
\left\|\mathbf{k}-\omega^{2} \mathbf{m}\right\|=0 \tag{3.6}
\end{equation*}
$$

Eq. 3.6 is called the frequency equation of the system. Expanding the determinant will give an algebraic equation of the Nth degree in the frequency parameter $\omega^{2}$ for a system having N degrees of freedom. The N roots of this equation $\left(\omega_{1}^{2}, \omega_{2}^{2}, \omega_{3}^{2}, \ldots \ldots . . \omega_{N}^{2}\right)$ represent the frequencies of the $N$ modes of vibration which are possible in the system. The mode having the lowest frequency is the first mode, the next higher frequency is the second mode, etc. The vector made up of the entire set of modal frequencies, arranged in sequence, will be the frequency vector $\omega$ :

$$
\omega=\left\{\begin{array}{c}
\omega_{1}  \tag{3.7}\\
\omega_{2} \\
\omega_{3} \\
\vdots \\
\omega_{N}
\end{array}\right\}
$$

For the real, symmetric, positive definite mass and stiffness matrices which pertain to stable structural systems, all roots of the frequency equation will be real and positive.

The ith period $T_{i}$ may be found from ith frequency, as

$$
\begin{equation*}
T_{i}=\frac{2 \pi}{\omega_{i}} \tag{3.8}
\end{equation*}
$$

Therefore, the fundamental period is

$$
\begin{equation*}
T=\frac{2 \pi}{\omega_{1}} \tag{3.9}
\end{equation*}
$$

### 3.3 ANALYSIS OF VIBRATION MODE SHAPES

When the frequencies of vibration have been determined from Eq. 3.6, the equations of motion, Eq. 3.4, may be expressed as

$$
\begin{equation*}
\widetilde{\mathbf{E}}^{(n)} \hat{\mathbf{v}}_{n}=\mathbf{0} \tag{3.10}
\end{equation*}
$$

in which,

$$
\begin{equation*}
\widetilde{\mathbf{E}}^{(n)}=\mathbf{k}-\omega_{n}^{2} \mathbf{m} \tag{3.11}
\end{equation*}
$$

Thus, $\widetilde{\mathbf{E}}^{(n)}$ represents the matrix obtained by subtracting $\omega_{n}^{2} \mathbf{m}$ from the stiffness matrix; since it depends on the frequency, it is different for each mode. Eq. 3.10 is satisfied identically because the frequencies were evaluated from this condition; therefore the amplitude of the vibrations is indeterminate. However, the shape of the vibrating system can be determined by solving for all the displacements in terms of any one coordinate.

For this purpose it may be assumed that the first element of the displacement vector has a unit amplitude; that is

$$
\left\{\begin{array}{c}
\hat{v}_{1 \mathrm{n}}  \tag{3.12}\\
\hat{\mathrm{v}}_{2 \mathrm{n}} \\
\hat{\mathrm{v}}_{3 \mathrm{n}} \\
\vdots \\
\hat{\mathrm{v}}_{\mathrm{Nn}}
\end{array}\right\}=\left\{\begin{array}{c}
1 \\
\hat{v}_{2 \mathrm{n}} \\
\hat{v}_{3 \mathrm{n}} \\
\vdots \\
\hat{\mathrm{v}}_{\mathrm{Nn}}
\end{array}\right\}
$$

In expanded form, Eq. 3.10 may then be written

$$
\left[\begin{array}{ccccc}
e_{11}{ }^{(n)} & e_{12}{ }^{(n)} & e_{13}{ }^{(n)} & & e_{1 N}{ }^{(n)}  \tag{3.13}\\
\cdots & \cdots & \cdots & \ldots & \cdots \\
e_{21}{ }^{(n)} & e_{22}{ }^{(n)} & e_{23}{ }^{(n)} & & e_{2 N}{ }^{(n)} \\
e_{31}^{(n)} & e_{32}{ }^{(n)} & e_{33}{ }^{(n)} & & e_{3 N}{ }^{(n)} \\
\cdots & \cdots & \cdots & \cdots & \cdots \\
e_{N 1}{ }^{(n)} & e_{N 2}{ }^{(n)} & e_{N 3}{ }^{(n)} & & e_{N N}{ }^{(n)}
\end{array}\right]\left\{\begin{array}{c}
1 \\
\cdots \\
\hat{v}_{2 n} \\
\hat{v}_{3 n} \\
\cdots \\
\hat{v}_{n n}
\end{array}\right\}=\left\{\begin{array}{c}
0 \\
\cdots \\
0 \\
0 \\
\cdots \\
0
\end{array}\right\}
$$

in which partitioning is indicated to correspond with the as yet unknown displacement amplitudes. For convenience, Eq. 3.13 is expressed in matrix notations as

$$
\left[\begin{array}{ll}
e_{11}^{(n)} & \tilde{\mathbf{E}}_{00}^{(n)}  \tag{3.14}\\
\widetilde{\mathbf{E}}_{01}^{(n)} & \widetilde{\mathbf{E}}_{00}^{(n)}
\end{array}\right]\left\{\begin{array}{c}
1 \\
\hat{\mathbf{v}}_{0 n}
\end{array}\right\}=\left\{\begin{array}{l}
0 \\
\mathbf{0}
\end{array}\right\}
$$

from which,

$$
\begin{equation*}
\widetilde{\mathbf{E}}_{01}^{(n)}+\widetilde{\mathbf{E}}_{o o}{ }^{(n)} \hat{\mathbf{V}}_{o n}=\mathbf{0} \tag{3.15}
\end{equation*}
$$

and

$$
\begin{equation*}
e_{11^{(m)}}+\widetilde{\mathbf{E}}_{10}{ }^{(n)} \hat{\mathbf{v}}_{0 n}=0 \tag{3.16}
\end{equation*}
$$

Eq.3.16 can be solved simultaneously for the displacement amplitudes

$$
\begin{equation*}
\hat{\mathbf{v}}_{o n}=-\left(\widetilde{\mathbf{E}}_{00}^{(n)}\right)^{-1} \widetilde{\mathbf{E}}_{01}^{(n)} \tag{3.17}
\end{equation*}
$$

and Eq. 3.16 is redundant; the redundancy corresponds to the fact that it is satisfied identically. The displacement vector obtained in Eq. 3.17 must satisfy

Eq. 3.16, however, and this condition provides a useful check on the accuracy of the solution.

Here it may be mentioned that it is not always wise to let the first element of the displacement vector unity; numerical accuracy will be improved if the unit element is associated with one of the larger displacement amplitudes. The same solution process can be employed in any case, however, by merely rearranging the order of the rows and columns of $\widetilde{\mathbf{E}}^{(n)}$ appropriately.

The displacement amplitudes obtained from Eq. 3.17 together with the unit amplitude of the first component constitute the displacement vector associated with the nth mode of vibration. For convenience the vector is usually expressed in dimensionless form by dividing all the components by one reference component (usually the largest).

The resulting vector is called the nth mode shape $\phi_{n}$; thus

$$
\phi_{n}=\left\{\begin{array}{c}
\phi_{1 n}  \tag{3.18}\\
\phi_{2 n} \\
\cdots \\
\phi_{N n}
\end{array}\right\} \equiv \frac{1}{\hat{v}_{k n}}\left\{\begin{array}{c}
1 \\
\hat{v}_{2 n} \\
\hat{v}_{3 n} \\
\cdots \\
\hat{v}_{N n}
\end{array}\right\}
$$

in which,
$\hat{v}_{k n}$ is the reference component, taken as the first component here.

The shape of each of the N modes of vibration can be found by the same process; the square matrix made up of the N mode shapes will be represented by $\phi$ as

$$
\phi=\left[\begin{array}{llll}
\phi_{1} & \phi_{2} & \phi_{3} & \cdots \phi_{N}
\end{array}\right]=\left[\begin{array}{cccc}
\phi_{11} & \phi_{12} & \cdots & \phi_{1 N}  \tag{3.19}\\
\phi_{21} & \phi_{22} & \cdots & \phi_{2 N} \\
\cdots & \cdots & \cdots & \cdots \\
\phi_{N 1} & \phi_{N 2} & \cdots & \phi_{N N}
\end{array}\right]
$$

### 3.4 STRUCTURE PARAMETERS THAT INFLUENCE PERIOD

The Eq. 3.6 clearly states that the frequency of a structure is directly proportional to the stiffness properties and inversely proportional to the mass properties of the structure. The stiffness properties are directly proportional to the modulus of elasticity of the materials and moment of inertia of the structural elements. The stiffness properties are also inversely proportional to height of vertical members and span of horizontal members. Therefore the stiffness properties and hence the frequency will be influenced by modulus of elasticity as well as by sectional dimensions, width and height of a structure. Again the effect of shearing deformation could be significant on the periods corresponding to the higher mode ( Basu et al., 1982).

Mass properties depend on the density of the material. Thus the material density will certainly affect the frequency. Since the period of the structure is inversely proportional to the frequency, the period of the structure will similarly be a function of the stiffness and mass properties of the structure. Furthermore, the material properties will also influence the magnitude of the period of a structure. Hence, the period of a structure will be a function of its parameters.

## CHAPTER 4

## REVIEW OF SOME CODES OF PRACTICE FOR EARTHQUAKE RESISTANT DESIGN

### 4.1 INTRODUCTION

The design of buildings considering earthquakes usually employs static loads that are determined in accordance with provisions in the applicable building code. The appropriate earthquake-resistant regulations for one country are not necessarily the same as for other countries. But the prime objectives and aims of formulating empirical formulae and code for earthquake resistant structures ( International Association for Earthquake Engineering, 1973 ) are mainly

- to keep the number of deaths from earthquakes to an acceptably small number, and
- to design and construct buildings so that the long term cost of repairing damage does not exceed the extra cost that would have been required to prevent damage.

The determination of earthquake forces by the Uniform Building Code, Bangladesh National Building Code, Standards Association of Australian Earthquake Code and Indian Standard Criteria for Earthquake Resistant Design of Structures are mainly based on constant acceleration concept. Equivalent static force method of design for regular structures of height less than 240 ft suggested by the above codes are discussed below for the comparative study of seismic design and evaluation of period.

### 4.2 UNIFORM BUILDING CODE

The procedures and limitations for the design of structures by Uniform Building Code (UBC) are determined considering zoning, site characteristics, occupancy, configuration, structural system and height. Two
of the major parameters in the selection of design criteria are occupancy and structural configuration.

The total lateral force or base shear, according to Uniform Building Code 1985, 1991, and 1994 are enumerated below.

## UBC-85

The base shear, V is given by ( Uniform Building Code, 1985 )

$$
\mathrm{V}=\mathrm{ZIKCSW}
$$

where,
$Z=$ seismic zone factor
I = occupancy importance factor
$\mathrm{K}=$ horizontal force factor
$\mathrm{S}=$ site coefficient for soil characteristics
$\mathrm{W}=$ the total seismic dead load
$\mathrm{C}=$ vibration characteristic factor expressed as a coefficient related to the flexibility of a structure,

$$
\mathrm{C}=\frac{1}{15 \sqrt{T}} \leq 0.12
$$

where $T$ is the elastic undamped fundamental period of vibration of the building (in secs) in the direction of the motion considered. The value of $T$ may be determined from one of the following methods:

## Method A

The fundamental period of vibration, $T$, is established using the structural properties and deformation characteristics of the resisting elements in a properly substantiated analysis. In absence of precise determination of T, the fundamental period of a building, in which lateral-load resisting system consists of moment-resisting space frames capable of resisting $100 \%$ of the required lateral forces and the frames are not enclosed or adjoined by more rigid elements tending to prevent them from resisting the lateral forces,
where,
$\mathrm{N}=$ total number of stories between the base and the upper most level in the main portion of the structure.

## Method B

The fundamental period T may be calculated using the strcutural properties and deformational characteristics of the resisting elements in a properly substantiated analysis. The formula for determining the period is

$$
\mathrm{T}=2 \pi \sqrt{\left(\sum_{\mathrm{i}=1}^{\mathrm{n}} \mathrm{w}_{\mathrm{i}} \delta_{\mathrm{i}}^{2}\right) \div\left(\mathrm{g} \sum_{\mathrm{i}=1}^{\mathrm{n}} \mathrm{f}_{\mathrm{i}} \delta_{\mathrm{i}}\right)}
$$

The values of $f_{i}$ represent any lateral force disributed approximately in accordance with rational distribution. The elastic deflections, $\delta_{i}$, shall be calculated using the applied lateral forces, $f_{i}$. The value of $C$ shall be not less than 80 percent of the value obtained by using T from Method A ..

## UBC-91 and 94

UBC-91 and 94 are identical in defining base shear, which, according to them, is given by

$$
\mathrm{V}=\frac{Z I C}{R_{w}} W
$$

where,
$Z=$ seismic zone factor
$I=$ occupancy importance factor
$\mathrm{R}_{\mathrm{W}}=$ numerical coefficient
$\mathrm{W}=$ total seismic load

$$
\mathrm{C}=\frac{1.25 S}{T^{2 / 3}} \geq .075 R_{w} \leq 2.75
$$

$S=$ site coefficient for soil characteristics

## $T=$ fundamental period of vibration in secs

The value of the fundamental period, T of the structure shall be determined from one of the following methods:

## Method A

For all buildings the value of T may be approximated by the following formula:

$$
\mathrm{T}=C_{t} h_{n}^{3 / 4}
$$

where,
$\mathrm{C}_{\mathrm{t}}=$ numerical coefficient
$\mathrm{h}_{\mathrm{n}}=$ building height in ft above base .

For moment resisting concrete frame structures

$$
C_{t}=.03
$$

## Method B

The fundamental period T may be calculated using the strcutural properties and deformational characteristics of the resisting elements in a properly substantiated analysis. The requirement may be satisfied by using the formula :

$$
T=2 \pi \sqrt{\left(\sum_{i=1}^{n} w_{i} \delta_{i}^{2}\right) \div\left(g \sum_{i=1}^{n} f_{i} \delta_{i}\right)}
$$

The values of $f_{i}$ represent any lateral force disributed approximately in accordance with rational distribution. The elastic deflections, $\delta_{\mathrm{i}}$, shall be calculated using the applied lateral forces, $\mathrm{f}_{\mathrm{i}}$. The value of T from Method B shall not be over 30 percent than the value of T obtained from Method A.

### 4.3 BANGLADESH NATIONAL BUILDING CODE (BNBC 1993)

Bangladesh National Building Code was published in 1993. The code suggests to determine the total design base shear in a given direction from the following relationship:

$$
\mathrm{V}=\frac{Z I C}{R} W
$$

where,
$\mathrm{Z}=$ seismic zone coefficient
$\mathrm{I}=$ structure importance coefficient
$R=$ response modification coefficient for structural system
$\mathrm{W}=$ the total seismic dead load
$\mathrm{C}=$ numerical coefficient given by the relation
$\mathrm{C}=\frac{1.25 \mathrm{~S}}{\mathrm{~T}^{2 / 3}}$
where,
$\mathrm{S}=$ site coefficient for soil characteristics
$\mathrm{T}=$ fundamental period of vibration in seconds, of the structure for the direction under consideration.

The value of $C$ need not exceed 2.75 and this value may be used for any structure without regard to soil type or structure period. Except for those requirements where code prescribed forces are scaled up by .375 R , the minimum value of the ratio $\mathrm{C} / \mathrm{R}$ shall be .075 .

The value of the fundamental period, T of the structure shall be determined from one of the following methods:

## Method A

The value of the fundamental period, T for reinforced concrete moment resisting frames can be determined approximately from the following empirical formula:

$$
\mathrm{T}=.073 \mathrm{~h}_{\mathrm{n}}^{3 / 4}
$$

where,
$h_{n}=$ height in meters above the base to level $n$.

## Method B

The fundamental period T may be calculated using the strcutural properties and deformational characteristics of the resisting elements in a properly substantiated analysis. This requirement may be satisfied by using the following formula:

$$
\mathrm{T}=2 \pi \sqrt{\left(\sum_{\mathrm{i}=1}^{\mathrm{n}} \mathrm{w}_{\mathrm{i}} \delta_{\mathrm{i}}^{2}\right) \div\left(\mathrm{g} \sum_{\mathrm{i}=1}^{\mathrm{n}} \mathrm{f}_{\mathrm{i}} \delta_{\mathrm{i}}\right)}
$$

The values of $f_{i}$ represent any lateral force disributed approximately in accordance with rational distribution. The elastic deflections, $\delta_{i}$, shall be calculated using the applied lateral forces, $f_{i}$. The value of $T$ determined from the above formula shall not exceed that calculated from Method A by more than $40 \%$.

### 4.4 STANDARD ASSOCIATION OF AUSTRALIAN EARTHQUAKE CODE (SAA)

According to Standards Association of Australian Earthquake Code (1979), the minimum total horizontal earthquake force to be resisted by a building shall be

$$
\mathrm{H}=\mathrm{ZIKCSW}
$$

with a minimum value of $\mathrm{H}=.02 \mathrm{~W}$, for non-zero values of Z . For the purpose of design, this force shall be taken to act non-concurrently in the direction of each of the main axes of the building.

Above,
$H=$ the equivalent static horizontal force for which the structure may be designed instead of the actual inertia forces generated in the structure by the movement of the ground.
$Z=$ seismic zone coefficient
$I=$ occupancy importance factor
$\mathrm{K}=$ horizontal force factor
$\mathrm{S}=$ site structure resource factor
$W=$ total seismic dead load
C $=$ the seismic response factor

$$
=\frac{1}{15 \sqrt{T}}>.12
$$

$\mathrm{T}=$ fundamental period of vibration

$$
=0.1 \mathrm{n}
$$

where,
$\mathrm{n}=$ total number of levels above the base upto and including level n .

### 4.5 INDIAN STANDARD CRITERIA FOR EARTHQUAKE RESISTANT DESIGN OF STRUCTURES (IS)

In Indian Standard Criteria for Earthquake Resistant design of Structures of 1970, the base shear $V_{B}$ is given by the following formula:

$$
V_{B}=C \alpha_{n} \beta W
$$

where, $\alpha_{\mathrm{n}}=$ seismic coefficient
$\beta=$ a coefficient depending upon the soil foundation system $\mathrm{W}=$ total amount of seismic load
$C=a$ coefficient defining the flexibility of structure with the increase in number of stories $=5 / \mathrm{T}^{1 / 3}$
$\mathrm{T}=$ fundamental time period of the building in seconds. For moment resisting frames without bracing or shear walls, the fundamental period is taken to be . In.
$\mathrm{n}=$ number of stories including the basement floors.

### 4.6 COMPARISION OF METHODS FOR DETERMINATION OF PERIOD BY CODES

The analytical formulae for the fundamental period and the coefficient depending on fundamental period of the structure are suggested to be computed by UBC-85, UBC-91, UBC-94, Bangladesh National Building Code (following Method A), Standard Association of Australian Earthquake Code and Indian Standard Criteria for Earthquake Resistant Design of Structures, are summarized in Table 4.1

Table 4.1 Fundamental period and coefficient depending on fundamental period for moment resisting frames, deforming freely

| UBC-85 |  <br> UBC-94 | BNBC | SAA | IS |
| :--- | :--- | :--- | :---: | :---: |
| $\mathrm{T}=0.1 \mathrm{~N}$ | $\mathrm{~T}=.03 \mathrm{~h}_{\mathrm{n}}^{3 / 4}$ | $\mathrm{~T}=.03 \mathrm{~h}_{n}^{3 / 4}$ | $\mathrm{~T}=0.1 \mathrm{n}$ | $\mathrm{T}=0.1 \mathrm{n}$ |
| $C=\frac{1}{15 \sqrt{T}}$ | $C=\frac{1.25 S}{T^{2 / 3}}$ | $C=\frac{1.25 S}{T^{2 / 3}}$ | $\mathrm{C}=\frac{1}{15 \sqrt{\mathrm{~T}}}$ | $\mathrm{C}=.5 / \mathrm{T}^{1 / 3}$ |
| $\leq .12$ | $\geq 0.75 \mathrm{R}_{\mathrm{w}}$ <br> $\leq 2.75$ | $\geq 0.75 \mathrm{R}_{\mathrm{w}}$ <br> $\leq 2.75$ | $>2.75$ |  |

According to UBC-85, SAA \& IS, the fundamental period of vibration is a function of number of stories. According to UBC-94, and BNBC, however, it is a function of height of the structure .

In Fig. 4.1, its variation with story number or height are plotted following different codes. It has been assumed that the height of each story is 10 ft . As
per UBC-85, and SAA the coefficient of fundamental period of vibration is proportional to $1 / \sqrt{T}$. According to UBC-91, UBC-94 and BNBC, it is proportional to $1 / \mathrm{T}^{2 / 3}$ and according to IS $1 / \mathrm{T}^{1 / 3}$. The variation of, C , with the fundamental period T , following above codes, are shown in Fig. 4.2.


Fig. 4.1 Variation of fundamental period with story number


Fig. 4.2 Variation of proportionality factor, C with period, $T$ ( value of $S$ is considered to be 1)

## CHAPTER 5

## DETERMINATION OF PERIOD BY MODAL ANALYSIS USING ANSYS

### 5.1 INTRODUCTION

The period of a frame structure can be determined by applying the theory of structural dynamics. The application of the theory involves a great deal of calculation and requires solution of differential equations, leading to evalution of eigenvalues. It is cumbersome except for very small, simple structures. For determining the period of three dimensional frame structures of high degree of indeterminancy, the use of computer programme, specially, finite element based type of analysis, is essential (Rankaj and Gambhir, 1992). ANSYS is the most powerful general purpose finite element package, which may be used for the purpose. The effects of variation of different structure parameters on the period of three dimensional frame structures is studied here using ANSYS.

### 5.2 GENERAL DISCUSSION ON ANSYS

The ANSYS program, was introduced in 1970, by Swanson Analysis System (1995). Since that time ANSYS Supports Distributors have grown as part of a commitment to provide latest finite element analysis and design technology to engineers, world wide. ANSYS capabilities can be utilized in computers that range from PCs to super main frames. ANSYS is a general-purpose program constantly updated with new features, enhancements of existing features, and error corrections. The current release of the program followed in this research work, is Revision 5.2.

### 5.3 MODAL ANALYSIS BY ANSYS

### 5.3.1 Introduction

Modal analysis helps the determination of the vibration characteristics of structure. It is used to determine the natural frequencies and mode of a structure or its component. Modal analysis in the ANSYS is a linear analysis. Any nonlinearities such as plasticity and contact elements, are ignored even if they are defined.

### 5.3.2 Assumptions \& Restrictions

The assumptions and restrictions in ANSYS are enumerated below:
i) Valid for structural and fluid degrees of freedom
ii) The structure has constant stiffness and mass effects
iii) There is no damping unless the damped eigensolver is selected
iv) The structure has no time varying forces, displacements, pressures, or temperature applied ( that is, free vibration).

### 5.3.3 Analysis of Undamped Structures

This analysis is used for natural frequency and mode shape. The governing equation in matrix notation is

$$
\begin{equation*}
[\mathrm{M}]\{\ddot{\mathrm{u}}\}+[\mathrm{k}]\{\mathrm{u}\}=\{0\} \tag{5.1}
\end{equation*}
$$

where,
$[\mathrm{M}]=$ structural mass matrix,
[k] = structural stiffness matrix
$\{\ddot{u}\}=$ nodal acceleration vector
$\{u\}=$ nodal displacement vector

For a linear system,

$$
\begin{equation*}
\{u\}=\{\phi\}_{i} \operatorname{Cos} \omega_{i} t \tag{5.2}
\end{equation*}
$$

where,
$\{\phi\}_{\mathrm{i}}=$ eigenvector representing the mode shape of the ith natural frequency.
$\omega_{\mathrm{i}}=$ angular frequency corresponding to i th mode
t = time

Substituting of Eq. 5.2 in Eq. 5.1 leads to

$$
\begin{equation*}
\left(-\omega_{i}^{2}[M]+[k]\right)\{\phi\}_{i}=\{0\} \tag{5.3}
\end{equation*}
$$

This equlity is satisfied if either $\{\phi\}_{i}=\{0\}$ or if the determinant $\left([k]-\omega^{2}[M]\right)$ is zero. The first option is trivial and, therefore, is not of interest. Hence,

$$
\begin{equation*}
\left|[\mathrm{k}]-\omega^{2}[\mathrm{M}]\right|=0 \tag{5.4}
\end{equation*}
$$

This is an eigenvalue problem which may be solved for n values of $\omega^{2}$ and n eigenvectors $\{\phi\}_{i}$. Here, n is the number of degrees of freedom.

### 5.3.4 Eigenvalue and Eigenvector Extraction

The eigenvalue and eigenvector problem needs to be solved for modefrequency analysis. It has the form of:

$$
\begin{equation*}
[\mathrm{k}]\left\{\phi_{\mathrm{i}}\right\}=\lambda_{\mathrm{i}}[\mathrm{M}]\left\{\phi_{\mathrm{i}}\right\} \tag{5.5}
\end{equation*}
$$

where,
$[\mathrm{k}]=$ structural stiffness matrix
$\left\{\phi_{i}\right\}=$ eigenvector

$$
\begin{aligned}
& \lambda_{\mathrm{i}}=\text { eigenvalue } \\
& {[\mathrm{M}]=\text { structure mass matrix }}
\end{aligned}
$$

The Eq. 5.3 can be rewritten as

$$
\begin{equation*}
[k]\left\{\phi_{i}\right\}=\omega_{i}^{2}[M]\left\{\phi_{i}\right\} \tag{5.6}
\end{equation*}
$$

The Eq. 5.5 and Eq. 5.6 are of same nature, where $\omega_{\mathrm{i}}$ is the natural circular frequency of mode i ( $\omega_{\mathrm{i}}^{2}$ is the eigenvalue). The natural frequency $\mathrm{f}_{\mathrm{i}}$ and period $T_{i}$ are given by

$$
\begin{align*}
& \mathrm{f}_{\mathrm{i}}=\omega_{\mathrm{i}} / 2 \pi  \tag{5.7}\\
& \mathrm{~T}_{\mathrm{i}}=2 \pi / \omega_{\mathrm{i}} \tag{5.8}
\end{align*}
$$

The eigenvalues and eigenvectors may be extracted following four different methods. These are reduced method, subspace method, unsymmetric method and damped method. Eigenvalues and eigenvectors, extracted by ANSYS following reduced method, which has been used in the present work, is elaborated below.

## Reduced Method

For the reduced procedure, the system of equations is first condensed down to those degrees of freedoms (DOFs)associated with the master DOFs by Guyan Reduction. Master DOFs actually specify which existing DOFs are to be retained. The set of n master DOFs characterize the natural frequencies of interest in the system. The number of master DOFs selected should usually be at least equal to twice the number of frequencies of interest. The reduced form may be expressed as

$$
[\hat{k}]\left\{\hat{\phi}_{i}\right\}=x_{i}\left[\begin{array}{lll}
{[\hat{M}]}
\end{array}\right]\left\{\begin{array}{ll}
\hat{\phi}_{i}
\end{array}\right\}
$$

where,

$$
\begin{aligned}
& {[\hat{k}]=\text { reduced stiffness matrix (known) }} \\
& \left\{\hat{\phi}_{\mathrm{i}}\right\}=\text { eigenvector (unknown) }
\end{aligned}
$$

$$
\lambda_{\mathrm{i}} \quad=\text { eigen value (unknown) }
$$

$$
[\hat{\mathrm{M}}]=\text { reduced mass matrix }(\text { known })
$$

The actual eigenvalue extraction is performed. The extraction employed is the HBI (Householder Bisection Inverse iteration) extraction technique and consists of the following five steps.

Step 1: Transformation of the Generalized Eigen Problem to a Standard Eigen Problem. The Eq. 5.9 must be transformed to the desired form of standard eigen problem. This form may be of following type

$$
[\mathrm{A}]\{\psi\}=\lambda\{\psi\}
$$

where,
[A] is symmetric.

Transformation to above form may be as follows:
Premultiplying both sides of Eq. 5.9 by $[\hat{\mathrm{M}}]^{-1}$.

$$
\begin{equation*}
\left.[\hat{\hat{1}}]^{-1}[\hat{k}] \mid \hat{\phi}\right\}=\lambda\{\hat{\phi}\} \tag{5.10}
\end{equation*}
$$

Decomposing $[\hat{\mathrm{M}}]$ into $[\mathrm{L}][\mathrm{L}]^{\mathrm{T}}$ by Cholesky decomposition, where [L] is a lower triangular matrix Eq. 5.10 , becomes

$$
\begin{equation*}
[L]^{-T}\left[L L^{-}[\hat{k}]\{\hat{\phi}\}=\lambda\{\hat{\phi}\}\right. \tag{5.11}
\end{equation*}
$$

It is convenient to define

$$
\begin{equation*}
\{\hat{\phi}\}=[\mathrm{L}]^{-\mathrm{T}}\{\psi\} \tag{5.12}
\end{equation*}
$$

Substitution of Eq. 5.11 into Eq. 5.12 yields

$$
\begin{equation*}
[L]^{-1}[\hat{k}][L]^{-1}\{\psi\}=\lambda\{\psi\} \tag{5.13}
\end{equation*}
$$

or ,

$$
\begin{equation*}
[\mathrm{A}]\{\psi\}=\lambda\{\psi\} \tag{5.14}
\end{equation*}
$$

where,

$$
[A]=[L]^{-1}[\hat{k}][L]^{-T}
$$

Hence the symmetry of [A] has been ensured by this procedure.

Step 2: Reducing [A] to Tridiagonal Form

This step is performed by Householder's method through a series of a similarity transformations, yielding

$$
[\mathrm{B}]=[\mathrm{T}]^{\mathrm{T}}[\mathrm{~A}][\mathrm{T}]
$$

where,
$[\mathrm{B}]=$ tridiagonalized form of $[\mathrm{A}]$
[T] = matrix constructed to tridiagonalize [A], solved for iteratively

The eigenproblem is reduced to

$$
[\mathrm{B}]\{\psi\}=\lambda\{\psi\}
$$

Here it can be mentioned that the eigenvalues ( $\lambda$ ) have not changed through these transformations, but the eigenvectors are related by

$$
\begin{equation*}
\{\hat{\phi}\}=[L]^{-T}[T]\left\{\psi_{i}\right\} \tag{5.15}
\end{equation*}
$$

## Step 3: Eigenvalue Calculation

Strum sequence checks are used with the bisection method to determine the eigenvalues. The strum sequence check computes the number of negative main diagonals ecountered during the traingularization of the shifted matrix.

Step 4: Eigenvector Calculation

The eigenvectors are evaluated using increase iteration with shifting. The eigenvectors associated with multiple eigenvalues are evaluated using initial vector deflation by Grain-Schmidt orthogonalization in the inverse iteration procedure.

## Step 5: Eigenvector Transformation

After the eigenvectors $\left\{\varphi_{i}\right\}$ are evaluated, $\left\{\hat{\phi}_{i}\right\}$ mode shapes are recovered through Eq. 5.15. In the expansion pass, the eigenvectors are expanded from the master DOFs to the total DOFs.

### 5.3.5 Modal Analysis Procedure

The procedure for a modal analysis consists of four main steps:

- Building the model
- Load application and solution
- Expansion of modes
- Reviewing the results

Building the Model: In this step the model geometry is defined. The structure, to be analysed, has to be formulated as a model following the frame work of modal analysis.

Models developed must adequately characterize the actual response. The elements with which the model is made of, represent the actual components. ANSYS program has a large library of different types of element. Out of these large numbers of different elements, elements which are suitable for building up the required models are choosen. For modeling 3 dimensional frame structures, beam elements, shell elements, and mass elements were choosen. The individual elements are described below.

Beam Element: Beam element is an elastic, uniaxial, 3 dimensional element which can withstand tension, compression, torsion and bending. The element
has two nodes with six degrees of freedom at each node; translations in the nodal $x, y$, and $z$ directions and rotations about the nodal $x, y$ and $z$ axes. The geometry, node locations and coordinate systems for this element are shown in Fig 5.1. The beams and columns of 3 dimensional frame structures are represented by beam elements while formulating the model of the frame structure.

Shell Element: Elastic shell element has been used for slab. It withstands both bending and membrane forces. The element is capable of taking both the inplane and normal loads. The element has six degrees of freedom at each node; translations in the nodal $x, y$ and $z$ directions and rotations about the nodal $x, y$ and $z$ axes (Ahmed et al. 1968 ). The geometry, node locations, and the coordinate system for this element are shown in Fig. 5.2. The element is defined by four nodes, thickness and orthrotropic material properties. For building the model, the shell element is used as thick plate element. The curvature of the shell element is considered as zero. The effect of bending stiffness is considered only and the effect of membrane stiffness is not considered. So the shell element will response like thick plate element which will represent the slab of the frame structure.

Mass Element: Mass element is a point element used as a structural mass on each nodal points of the model formulated. The node of mass element has got six degrees of freedom: translations in the nodal $x, y$ and $z$ directions and rotations about nodal, $\mathrm{x}, \mathrm{y}$ and z axes (Zienkiewicz, 1979 ). The geometry, node locations, and coordinate systems for typical mass element are given in Fig: 5.3. The vertical walls, widely used as partition walls in between rooms of frame structure, are considered as mass elements. So mass elements actually represent the vertical wall loads at each nodal point of the frame structure.

Load Application and Solution: In this step, the type of analysis, analysis option, load step options, etc., are defined to initiate the finite element solution for the natural frequencies. In this work modal analysis has been used as analysis type and reduced methods were used as analysis options.

Expansion of Modes: In the modal analysis, the term "expansion" means writing the mode shapes to the result file. This step is required to review the mode shapes in result file.

Reviewing the Results: In this step the required parts of results written in the result file are brought out for easy reference.

If node $K$ is omitted and $\theta=0^{\circ}$, the element $y$ axis is parallel


Fig. 5.1 3-Dimensional elastic beam element


Fig. 5.2 Elastic shell element


Fig. 5.3 Structural mass element

## CHAPTER 6

## INFLUENCE OF STRUCTURAL PARAMETERS ON PERIOD OF FRAME STRUCTURES

### 6.1 INTRODUCTION

In the preceding chapters theories for calculating the period of frame structures are described. The procedure and methodology following which ANSYS works for calculating period are described in Chapter 5. The empirical formulae proposed by different codes for calculating the period of regular frame structures are discussed in Chapter 4.

Discussions in Chapter 4 revealed that the approximate formulae proposed by codes for evaluation of period describe period as a sole function of structure height or number of stories. From the theories of evaluation of period, presented in Chapter $3 \& 5$, it is clear that all the parameters contributing to stiffness and mass properties should have influence on period. $\mathrm{In}_{\beta}$ order to ascertain the degree of influence of such structure parameters on period, a set of model frame structure are first selected in this chapter. In selecting such models, care has been taken so that they can adequately reflect influence of different parameters after proper analysis is conducted for period. While designing the models, emphasis is given on clear identification of parameters, which may significantly affect period. Series of models are designed so that influence of desired parameters may be clearly established.

In this chapter, the selected models are first described. The period of the models are then evaluated following both approximate code formulae (following Method A ) and modal analysis technique. In evaluating period by modal analysis, ANSYS has been extensively used. A detail parametric study is conducted further to identify influence of different parameters. Results of modal analysis are compared with those obtained from using approximate code formulae. Limitations of code formulae of Method A are
established. Finally, a guideline is provided to choose structure parameters for reducing earthquake forces on the structure in its design.

It should be noted that the investigation carried out in this work applies to regular 3-D concrete frame structures only.

### 6.2 DESCRIPTION OF MODELS

Various numbers of regular 3-dimensional model frame structures are selected following the framework given by finite element based computer software ANSYS. For convenience of studying the influence of various structure parameters on period, such models are grouped. The values of parameters, in general, applicable for all groups of models are given in Table 6.1.

Table 6.1 Structure parameters applicable, in general, to selected models

| Sl. No. | Parameters | Values |
| :---: | :---: | :---: |
| 1 | Modulus of Elasticity | $2.07 \times 10^{7} \mathrm{kN} / \mathrm{m}^{2}$ |
| 2 | Density of Concrete | $23.56 \mathrm{kN} / \mathrm{m}^{3}$ |
| 3 | Acceleration due to Gravity | $9.81 \mathrm{~m} / \mathrm{sec}^{2}$ |
| 4 | Width of the Bay | 5.0 m |
| 5 | Story Height | 3.2 m |
| 6 | Size of Columns | $0.5 \mathrm{~m} \times 0.5 \mathrm{~m}$ |
| 7 | Size of Beams | $0.35 \mathrm{~m} \times 0.5 \mathrm{~m}$ |
| 8 | Thickness of Slab | 0.15 m |

The models are divided into six major groups. These are $\mathrm{GpA}, \mathrm{GpB}, \mathrm{GpC}$, $\mathrm{GpD}, \mathrm{GpE}, \& \mathrm{GpF}$ respectively. Several submodels are developed within a group by addition of different structural members and elements and or variation of their parameters. The models developed are described below.


(c)

(d)

Fig. 6. 1 Plan configurations of models included in group GpA representing 6 -story and 12 -story 3-D frame structures (a) Model A1 (b) Model A2 (c) Model A3 (d) Model A4

Models of GpA, GpB \& GpC: These are models of 6 -and 12 -story 3 dimensional frame structures with slabs. Values of parameters cited in Table 6.1 are applicable to all of them. Plan configuration of models included in groups $\mathrm{GpA}, \mathrm{GpB}$ and GpC are shown in Fig.6.1, Fig.6.2 and Fig. 6.3 respectively. The models of groups $\mathrm{GpA}, \mathrm{GpB} \& \mathrm{GpC}$ account for investigating the variation of the period of the structure for increase or decrease of the number of bays along and transverse to the direction of motion.

Models of GPA: All models of GpA have two bays in the direction perpendicular to the direction of motion. They differ among themselves by the number of bays along the direction of motion (Fig.6.1). Models included in this group are described below.

Model A1: Composed of frame structures with 2 bays along the direction of motion and 2 bays in the direction perpendicular to the direction of motion (Fig. 6.1(a)).

Model A2: Composed of frame structures with 3 bays along the direction of motion and 2 bays along the direction perpendicular to the direction of motion (Fig.6.1 (b)).

Model A3: Composed of frame structures with 4 bays along the direction of motion and 2 bays along the direction perpendicular to the direction of motion (Fig. 6.1(c)).

Model A4: Composed of frame structures with 5 bays along the direction of motion and 2 bays along the direction perpendicular to the direction of motion (Fig.6.1(d)).

Models of GpB : All models of this group have three bays in the direction perpendicular to the direction of motion. They differ among themselves by the number of bays along the direction of motion (Fig. 6.2). Models included in GpB are described below.

(a)

(b)
$\longrightarrow$ Direction of motion

( 61

(d)

Fig. 6. 2 Plan configurations of models included in group GPB representing 6 -story and 12 -story $3-D$ frame structures (a) Model B1
(b) Model B2
(c) Model B3 (d) Model B4

Model B1: Composed of frame structures with 2 bays along the direction of motion and 3 bays along the direction perpendicular to the direction of motion (Fig. 62 (a)).

Model B2: Composed of frame structures with 3 bays along the direction of motion and 3 bays along the direction perpendicular to the direction of motion (Fig.6.2 (b)).

Model B3: Composed by frame structures with 4 bays along the direction of motion and 3 bays along the direction perpendicular to the direction of motion (Fig. 6.2 (c)).

Model B4: Composed of frame structure with 5 bays along the direction of motion and 3 bays along the direction perpendicular to the direction of motion ( Fig. 6.2 (d)).

Models of GpC: All models of this group have four bays in the direction perpendicular to the direction of motion. They differ among themselves by the number of bays along the direction of motion (Fig. 6.3). Models of GpC are described below.

Model C1: Composed of frame structure with 2 bays along the direction of motion and 4 bays along the direction perpendicular to the direction of motion (Fig. 6.3 (a)).

Model C2: Composed of frame structures with 3 bays along the direction of motion and 4 bays along the direction perpendicular to the direction of motion (Fig. 6.3 (b)).

Model C3: Composed of frame structures with 4 bays along the direction of motion and 4 bays along the direction perpendicular to the direction of motion (Fig. 6.3 (c)).

Model C4: Composed of frame structures with 5 bays along the direction of motion and 4 bays along the direction perpendicular to the direction of motion (Fig. 6.3 (d)).

(a)
$\longrightarrow$ Direction of notion

(c)

(b)

(d)

Fig. 6. 3 Plan configurations of models included in group Gp C representing 6 -story and 12 -story 3-D frame structures (a) Model C1 (b) Model C2 $(c)$ Model C3 (d) Model C4

Models of GpD: Models of GpD are selected to investigate the effect of variation of bay width, story height, sizes of columns, sizes of beams, cylinder strength of concrete and density of concrete on period of frame structures. Models of GpD comprise of 6 -story and 12 -story $3 \times 3$ bays, $4 \times 4$ bays and $5 \times 5$ bays in plan 3 -dimensional frame structures. In GpD , a total of 6 numbers of models are designed following the numerical data of Table.6.1. Plans of models of GpD are shown in Fig.6. 4. Models included in group GpD are described below.

Model Db: Models Db are generated to investigate the effect of variation of bay width on the period of the structures. The bay widths of all the models of GpD, shown in Fig. 6.4 are varied from 5.0 m to 7.0 m at a rate of 0.5 m . For each of the above bay widths the periods are calculated. In this case, all numerical data given in Table. 6.1 remains unaltered except bay width.

Model Dh: Models Dh are designed to study the effect of variation of story height on the period of the structure. Models Dh are generated from models of GpD . The plan configurations of the models of Dh correspond to those shown in Fig. 6.4. All model parameters except story height remain same as those given in Table 6.1. The story height for the models is varied from 3 m to 4 m at a step of 0.25 m .

Model Dc: Models Dc are used to study the effect of variation of column sizes on the period of the structures. Models Dc are developed from models of GpD . All model parameters except column sizes remain same as those shown in Table.6.1.Column size are chosen as $.4 \mathrm{~m} \times .4 \mathrm{~m}, 0.45 \mathrm{~m} \times 0.45 \mathrm{~m}$, $0.5 \mathrm{~m} \times 0.5 \mathrm{~m}$ and $0.55 \mathrm{~m} \times 0.55 \mathrm{~m}$. For each size of columns, the periods of the 6 -story and 12 -story models are calculated. The plan configurations of models Dc correspond to those shown in Fig. 6.4.

Model Dbm: Models Dbm are used for investigating the effect of sizes of beams on the period of the structure. These models are developed from the models of GpD . All the numerical data given in Table 6.1 are used except the size of the beams. The periods of 6 -story and 12 -story models, the plan configurations of which are shown in Fig. 6.4, are calculated for beam sizes $0.25 \mathrm{mxx} 0.4 \mathrm{~m}, 0.3 \mathrm{~m} \times 0.45 \mathrm{~m}, 0.35 \mathrm{~m} \times 0.5 \mathrm{~m}, 0.4 \mathrm{~m} \times 0.55 \mathrm{~m}$ and 0.45 mx 0.6 m .

(a)

(b)

(c)

Fig. 6. 4 Plan configurations of models included in group GpD representing 6 -story and 12 -story 3-D frame structures (a) $3 \times 3$ bays (b) $4 \times 4$ bays
(c) $5 \times 5$ bays

Model Dcs: Models Dcs are used for investigating the effect of cylinder strength of concrete on period of the structure. These models are developed from models of GpD. All the numerical data given in Table 6.1 are used except the cylinder strength of the concrete. The periods of 6 -story and 12story models, the plan configurations of which are shown in Fig. 6.4 are calculated for cylinder strength $17244 \mathrm{kN} / \mathrm{m}^{2}, 20692.8 \mathrm{kN} / \mathrm{m}^{2}, 27590.4$ $\mathrm{kN} / \mathrm{m}^{2}, 34488 \mathrm{kN} / \mathrm{m}^{2}$ and $41385.6 \mathrm{kN} / \mathrm{m}^{2}$.

Model Dd: Models Dd are used for investigating the effect of density of concrete on the period of the structures. These models are developed from models of GpD . All the numerical data given in Table.6.1 are used except the density of the concrete. The periods of the 6 -story and 12 -story models, the plan configurations of which are shown in Fig. 6.4, are calculated for density of concrete $22 \mathrm{kN} / \mathrm{m}^{3}, 22.78 \mathrm{kN} / \mathrm{m}^{3}, 23.565 \mathrm{kN} / \mathrm{m}^{3}, 24.3505 \mathrm{kN} / \mathrm{m}^{3}$, and $25.136 \mathrm{kN} / \mathrm{m}^{3}$.

Model of GpE: Models of GpE are utilized to study the effect of numbers of stories on the period of the structures. Models of GpE are $3 \times 3$ bays, $4 \times 4$ bays and $5 \times 5$ bays, in plan 3 -dimensional frame structures which are shown in Fig.6.5. The frames shown in Fig.6.5 are 6 -story, 8 -story, 10 -story, 12story and 14 -story high. In GpE a total of 15 numbers of models are included and the numerical data given in Table.6.1 apply for these models.

Models of GpF: Models of GpF are 6 -story and 12 -story, $2 \times 2$ bays, $3 \times 3$ bays, $4 \times 4$ bays, and $5 \times 5$ bays frame structures, the plan configurations of which are shown in Fig.6.6. These models are developed to study the effect of inclusion of slabs and mass distribution of secondary structural elements on the period of 3 dimensional frame structures. In GpF a total of 8 numbers of models are included. The numerical data given in Table.6.1 are applicable for the models. Models of GpF are utilized to generate three subgroups of models which are, models FF, models FS and models FMS. These models are described below.

(a)

(b)

(c)

Fig. 6.5 Plans of models included in group GPE representing 6 -story, 8 -story, 10 -story, 12 -story and 14 -story 3-D frame structures (a) $3 \times 3$ bays (b) $4 \times 4$ bays (c) $5 \times 5$ bays

(a)

(b)

(c)

(d)

Fig. 6. 6 Plan configurations of models included in group GpF representing 6 -story and 12 -story 3-D frame structures (a) $2 \times 2$ bays (b) $3 \times 3$ bays (c) $4 \times 4$ bays (d) $5 \times 5$ bays

Model FF: Models FF are 3-dimensional frame structures without slab. Models FF are used to calculate the periods of $2 \times 2$ bays, $3 \times 3$ bays, $4 \times 4$ bays and $5 \times 5$ bays 6 -and 12 -storied frame structures. Model parameters correspond to the numerical data given in Table 6.1. The plans of models are shown in Fig. 6.6.

The 3-Dimensional ANSYS display of $5 \times 5$ bay 6 -story and 12 -story frame structure without floorslabs are shown in plate 6.1 and plate 6.2 respectively.

Model FS: Models FS are similar to models FF, except that they include slabs at each story level. Here the periods are calculated with the inclusion of slabs at each story. Models FS are used to investigate the effect of inclusion of slabs on period of frame structures.

The 3-Dimensional ANSYS display of $5 \times 5$ bay 6 -story and 12-story frame structure with floorslabs are shown in plate 6.3 and plate 6.4 respectively.

Models FMS: These models are developed from models FS by adding mass of secondary structural elements to the respective nodal points. Here the periods of the models are calculated to investigate the effect of inclusion slabs and masses of secondary structural elements on period of frame structures.

### 6.3 DETERMINATION OF PERIOD OF MODELS FOLLOWING APPROXIMATE CODE FORMULAE

In this article the periods of the models, described in Art. 6.2, are calculated following approximate empirical formulae given by design codes. The design codes state that the period of a regular unbraced concrete frame structure is either function of height or function of number of stories of the structure.

According to UBC-85, SAA and IS codes, the period may be calculated by the formula,


Plate 6.1
3-Dimensional ANSYS display of $5 \times 5$ bay 6-story frame structure without floor slab


Plate 6.2 3-Dimensional ANSYS display of $5 \times 5$ bay 12-story frame structure without floor slab


Plate 6.3 3-Dimensional ANSYS display of $5 \times 5$ bay 6-story frame structure with floor slab


Plate 6.4 3-Dimensional ANSYS display of $5 \times 5$ bay 12-story frame structure with floor slab

$$
\begin{equation*}
\mathrm{T}=0.1 \mathrm{~N} \tag{6.1}
\end{equation*}
$$

where,

$$
\mathrm{N}=\text { number of stories }
$$

Again, according to UBC-91, UBC-94 and BNBC the may be evaluated by the formula,

$$
\begin{equation*}
\mathrm{T}=.03\left(\mathrm{~h}_{\mathrm{n}}\right)^{3 / 4} \tag{6.2}
\end{equation*}
$$

where,

$$
\mathrm{h}_{\mathrm{n}}=\text { height of the building in } \mathrm{ft} .
$$

The periods of the models are calculated by the above formulae and described below.

Periods of models $\mathbf{G p A}, \mathbf{G p B}, \mathbf{G p C}, \mathbf{G p D}$ and $\mathbf{G p F}$ : All the models of these groups are composed of 6 -story and 12 -story frame structures. The total height of the 6 -story and 12 -story models (except model Dh) are taken as 19.2 m and 38.4 m respectively. According to Eq. 6.1, the periods of the 6 -story and 12 -story models mentioned above are 0.6 and 1.2 seconds respectively.

Again, according to Eq. 6.2, the period of the same models are 0.67 and 1.14 seconds for 6 -and 12 -storied structures respectively.

The periods of the models calculated by approximate code formulae are plotted in Fig. 6.7. Periods of 6-story models are plotted in Fig. 6.7 (a) and periods of 12 -story models are shown in Fig. 6.7 (b).

From Fig. 6.7 the following conclusions may be drawn.
i) The period of 6-story models calculated by Eq. 6.2 is higher than the period calculated by Eq.6.1. In other words, period of 6 -story building


Fig. 6.7 Periods of 6-story and 12-story models following approximate code formulae
calculated by UBC-85, SAA and IS design Code is lower in magnitude than the period obtained by following UBC-91, UBC-94 and BNBC formulae.
ii) The period of 12 -story models calculated by Eq. 6.2 is smaller than the period calculated by Eq. 6.1. The period of 12 -story building calculated by UBC-85, SAA and IS formulae is higher in magnitude than the period calculated by following UBC-91, UBC-94 and BNBC.
iii) The period of the frame structure increases as its height or number of stories increases.
iv) Eq. 6.1 will lead to more conservative evaluation of base shear for low rise structures compared to Eq. 6.2. Whereas, for high rise structures Eq. 6.2 becomes more conservative than Eq. 6.1.

Periods of models Dh: Models Dh are of $3 \times 3,4 \times 4$ and $5 \times 5$ bays of 6 -story and 12 -story frame structures. The story heights of 6 -story and 12 -story models are varied for calculating their period. The periods determined by using Eqs. $6.1 \& 6.2$ for 6 -story and 12 -story models are plotted against story height in Fig .6.8.

From Fig. 6.8, the following conclusions are drawn
i) Period increases as the story height increases when the UBC-91, UBC-94 and BNBC design codes are used. This is because Eq. 6.2 depends on structure height.
ii) Period remains constant as the story height increases if the formulae given by UBC-85, SAA and IS are used. This is because Eq. 6.1 depends on number of stories only.

Periods of models GpE: Group GpE comprises of models which are 6-, 8 -, 10-, 12-, and 14 -story high. The period of the models are calculated following Eq. 6.1 and Eq. 6.2 respectively. The results are plotted in Fig. 6.9 Eq. 6.1 is invariant of story height; whereas Eq. 6.2 depends on height of stories. That is why two curves for story heights 2.8 m and 3.2 m are plotted using Eq. 6.2.


Fig. 6.8 Effect of story height on period of structures


Fig. 6.9 Effect of number of stories on period of structures following approximate code formulae

The following observations are made from Fig. 6.9.
i) The period of a structure increases with the increase of both number of stories and height of the structure. Constant positive slope for the curves corresponding to both Eq. 6.1 and Eq. 6.2 confirms this observation.
ii) For story height of 3.2 m , Eq. 6.1 yields lower period than Eq. 6.2 when a structure is less than 10 storied in height. If the story height would be less than 3.2 m , this observation would be, in general, true, but for structures of lower than 10 storied. To visualize this, a curve following Eq. 6.2 and with story height 2.8 m is added in Fig. 6.9. This curve intersects the curve for Eq. 6.1 near 6 story level which is less than 10 story obtained for 3.2 m story height.
iii) For structures of more than 10 storied, Eq. 6.1 gives higher periods than Eq. 6.2, when story height is 3.2 m . However, for story height of 2.8 m , Eq. 6.1 yields higher period than Eq. 6.2 for structures above 6 -story.

### 6.4 EFFECT OF STRUCTURE PARAMETERS ON PERIOD

### 6.4.1 Determination of Period of Models by Modal Analysis

The periods of the models developed in Art. 6.2 are calculated below by modal analysis method using ANSYS. The analyses for period of these models are already done in the preceding article using empirical formulae given by design codes. A numerical investigation is carried out here to compare periods of the models obtained by modal analysis and by the empirical formula of UBC-94. Necessary curves are drawn to study the effect of the structure parameters on the magnitude of period of the structures. Discussions on effect of individual structure parameters on period and comparison of period determined by modal analysis with that by UBC94 follows.

### 6.4.2 Effect of Number of Bays

For studying the effect of increase or decrease of number of bays on the period of a structure, models of $\mathrm{GpA}, \mathrm{GpB}$ and GpC are analyzed by modal analysis. Periods of 6 -story models given by ANSYS are plotted against number of bays along the direction of motion in Fig.6.10. The corresponding curves for 12 -story models are plotted in Fig.6.11. The period-number of bay relationships of same models obtained by using approximate formula of UBC-94 are also plotted in Fig.6.10 and Fig.6.11. In each of Fig. 6.10 \& Fig. 6.11, top three curves represent period-number of bay relationships as obtained by modal analysis for 2,3 and 4 numbers of transverse bays respectively. While the bottom most curve represents the same relationship corresponding to approximate formula of UBC-94. The code formula gives a curve which is invariant of number of bays along any direction and which is a sole function of total structure height only.

The following observations are made from Fig. 6.10 and Fig. 6.11.
i) The period of a structure decreases as the number of bays increases along the direction of motion. This may be explained by the fact that as frequency is directly proportional to stiffness and as increase of bays along the direction of motion makes the structure stiffer, this leads to increase in frequency and corresponding decrease in period.
ii) On the contrary, the period of a structure increases as the number of bays increases transverse to the direction of motion. This may be explained by the fact that increase of bays transverse to direction of motion adds mass, while the stiffness along the direction of motion remains virtually unaffected. As frequency of a structure is inversely proportional to mass, addition of mass leads to decrease in frequency and corresponding increase in period.
iii) The increase of period due to increase of number of bays transverse to the direction of motion is more significant than the decrease of period due to increase of number of bays along the direction of motion. This becomes more prominent with increase in number of stories.


Fig. 6.10 Effect of numbers of bays on period of 6-story frame structures


Fig. 6.11 Effect of numbers of bays on period of 12 -story frame structures
iv) The period of these models according to UBC-94 formula remains constant with the variation of number of bays both transverse and along the direction of motion. However, for higher storied frames the period increases. This is because the corresponding formula is a direct function of structure height only.
v) UBC-94 formula yields lower periods than those obtained by modal analysis. This difference increases with increase in number of transverse bays and decreases with increase in number of bays along the direction of motion. Again, the difference becomes more significant for higher storied structures. Hence, approximate UBC-94 formula is supposed to lead to more conservative base shear computation. For higher storied structures, this conservativeness would be more prominent.

### 6.4.3 Effect of Bay Width

For studying the effect of bay width on the period of a structure, models Db are subjected to modal analysis by ANSYS. The variation of period with bay width for six storied models of group Db are plotted in Fig.6.12. The corresponding variation for twelve storied models of Db are plotted in Fig. 6.13. In both the figures, bay widths are varied from 5 m to 7 m at an increment of 0.5 m . The top three curves in these figures represent periodbay width relationships for structures with $3 \times 3,4 \times 4$, and $5 \times 5$ bays in plan respectively. The bottom most curve in both these figures corresponds to empirical formula of UBC-94 for period evaluation. The curves obtained by using empirical formula of UBC-94 are invariant of both numbers and widths of bays. This is because the formula represents period as a sole and direct function of structure height.

The following observations are made from Fig. 6.12 and Fig. 6.13.
i) Curves drawn from modal analysis show that the period of a structure increases as its bay width increases. With increase in bay width, the stiffness of a structure decreases and mass increases. This leads to a decrease of structure frequency which is directly proportional to stiffness and inversely proportional to mass. Decrease in frequency leads to increase in period. Therefore, the results obtained are consistent.


Fig. 6.12 Effect of bay width on period of 6-story frame structures


Fig. 6.13 Effect of bay width on period of 12-story frame structures
ii) The period of a structure increases with the increase of number of bays. This is so because with increase in number of bays, both the stiffness and mass of a structure increase. As frequency is directly proportional to stiffness, and inversely proportional to mass, increase in stiffness leads to increase in frequency while increase in mass leads to decrease in frequency. However, influence of mass dominates over influence of stiffness which leads to a resultant decrease in frequency with corresponding increase in period.
iii) The periods of the models calculated by the approximate formula of UBC-94 remain constant for any bay width or number of bays. However, for higher storied building the period is higher because the approximate formula is a direct function of structure height.
iv) The magnitude of the period obtained through modal analysis is always higher than the magnitude of the period computed with the approximate formula of UBC-94. However, this difference is more significant with increasing bay width and increasing number of stories. For example, for $5 \times 5$ bay 6 -storied structures this difference for 7 m bay width is $38.65 \%$ higher than that for 5 m bay width. The corresponding difference for 12-storied structures in case of 7 m bay width is $37.48 \%$ higher than that of 5 m bay width. Again, with 7 m bay width, the difference in frequency for $5 \times 5$ bay 12 -storied structure is $267.28 \%$ higher than that of 6 -storied structure. Thus, the approximate code formula is supposed to lead to more conservative evaluation of base shear. This conservativeness increases with increasing bay width and increasing number of stories.

### 6.4.4 Effect of Story Height

For studying the effect of story height on the period of a structure, models Dh are considered. Models Dh are 6 -and 12 -storied structures of $3 \times 3,4 \times 4$ and $5 \times 5$ bays in plan with story heights varying from 3 m to 4 m at an interval of 0.25 m . The period of the models are calculated by modal analysis method using ANSYS. The period of the same models are also calculated using empirical formula of UBC-94. Periods of 6 storied models are plotted against story height in Fig.6.14. In Fig. 6.15, the corresponding


Fig. 6.14 Effect of story height on period of 6-story frame structures


Fig. 6.15 Effect of story height on period of 12-story frame structures
relationships for 12-storied models are plotted. In both of these figures, the top three curves represent period-story height relationships for $3 \times 3,4 \times 4$ and $5 \times 5$ bay structures respectively. The bottom curve in both of them represents the period-story height relationship as defined by approximate code formula of UBC-94. The UBC-94 curve is invariant of number of bays and therefore, is represented by a single curve for all three cases of $3 \times 3,4 \times 4$ and $5 \times 5$ bays. However, the period defined by UBC-94 curve increases with increasing story height, as the UBC-94 formula expresses period as a direct function of building height.

The following observations are made from Fig. 6.14 and Fig. 6.15.
i) The period of a structure, in general, increases as the story height of the structure increases.
ii) The increase in period for increase in number of bays is more for 6story buildings than for 12 -story buildings. However, this increase is not very significant.
iii) Modal analysis of structures yields higher periods than that given by approximate code formula of UBC-94. The difference increases with increasing story height. It signifies that the code formula becomes more conservative with increase in story height. Again the difference between periods obtained through modal analysis and defined by approximate code formula increases with increasing number of stories. For example, this difference for $5 \times 5$ bays 12 -storied structure is $202.69 \%$ higher than that of 6 -storied structure when story height is taken 3 m .

### 6.4.5 Effect of Stiffness of Columns

For studying the effect of stiffness of columns on the period of a structure, models Dc are considered. These models represent 6 -story and 12 -story frames $3 \times 3,4 \times 4$ and $5 \times 5$ bays in plan with column sizes varying from $0.45 \times 0.45 \mathrm{~m}$ to $0.6 \times 0.6 \mathrm{~m}$. The periods of the models are calculated by modal analysis method using ANSYS. The periods of the same models are also calculated by approximate formula of UBC-94. The periods of the 6storied models determined by modal analysis method and by approximate

UBC-94 formula are plotted against column stiffness in Fig. 6.16. Identical relationships for 12 -storied models are plotted in Fig 6.17. In all the analyses, the column height, h and modulus of elasticity, E of the column material do not change. The stiffness of columns is represented by EI / h of the respective models.

In Fig 6.16 and Fig 6.17, the top three curves represent relationships established through modal analysis for $3 \times 3,4 \times 4$, and $5 \times 5$ bays structures. The bottom curve represents the relationship obtained through approximate code formula for the same structures. The code formula, however, remains invariant of number of bays and column stiffness. As already mentioned, the approximate formula expresses period as a sole function of structure height only.

The following salient features may be identified from Fig 6.16 and Fig 6.17
i) The period of a structure decreases with increase of the stiffness of the columns. However, the rate of decrease of period diminishes with increasing column stiffness. This fact, however, remains unidentified by the approximate code formula.
ii) The period of a structure, in general, increases as the number of bays of the structure increases. The increase of period due to increase of number of bays is more for 6 -story buildings than 12 -story buildings.
iii) The magnitude of the period of 12 -story frames calculated by ANSYS is higher than the period obtained from the empirical formula of UBC-94.
iv) The magnitude of the period of 6-story frames calculated by ANSYS is higher than the period obtained from UBC-94 as long as the stiffness of the model column is less than 0.0485 GNm . For higher column stiffnesses, the approximate code formula will no longer remain conservative in base shear calculation. This leads to a very important conclusion. Designers must be very careful in the application of approximate code formula, because in every individual case there may be a boundary of column stiffness beyond which the approximate code formula no longer remains conservative. In other words, application of approximate code formula must be limited in


Fig. 6.16 Effect of column stiffness on period of 6-story frame structures


Fig. 6.17 Effect of column stiffness on period of 12-story frame structures
cases of stocky columns. For the range of column stiffnesses studied for the 12 -story frames, however, the code formula yields lower period than the modal analysis.

### 6.4.6 Effect of Stiffness of Beams

For studying the effect of stiffness of beams on the period of a structure, models Dbm are considered. These models represent 6- and 12- story frames $3 \times 3,4 \times 4$ and $5 \times 5$ bays in plan, with beam sizes varying from $0.25 \times 0.4 \mathrm{~m}$ to $0.45 \times 0.6 \mathrm{~m}$. The periods of the models are calculated by modal analysis technique with ANSYS. The periods of the same models are also calculated by the empirical formula of UBC-94. The periods of 6 -story frames are plotted against beam stiffness in Fig. 6.18. The same relationships for 12story frames are plotted in Fig. 6.19. Here the beam span, L and modulus of elasticity, E of beam material do not change for the models. The beam stiffness is represented by EI/L of the respective model.

The top three curves in these figures represent period-beam stiffness relationships for $3 \times 3,4 \times 4$, and $5 \times 5$ bay structures respectively as obtained through modal analysis. The bottom curve represents the same relationship for the same models but obtained by using approximate UBC-94 formula. The approximate code formula is invariant of number of bays in plan or beam stiffness. As mentioned earlier the formula is a sole function of structure height only.

The following observations are made from Figs. 6.18 and 6.19.
i) The period of a structure decreases with the increase of the stiffness of beams. However, the rate of decrease of period diminishes with increasing stiffness. The approximate code formula can not identify this fact.
ii) The period of a structure increases as the number of bays of the structure increases. This increase in period due to increase of number of bays is more for 6 -storied structures than for 12 -storied structures.
iii) The period remains constant with the increase of the stiffness of the beams, according to empirical formula of UBC-94.


Fig. 6.18 Effect of beam stiffness on period of 6-story frame structures


Fig. 6.19 Effect of beam stiffness on period of 12-story frame structures
iv) The periods of 6-story frames obtained through modal analysis are higher than those calculated by approximate formula of UBC-94, as long as the stiffnesses of the beams of the models are less than 0.0228 GNm . For higher beam stiffnesses, the approximate code formula will no longer remain conservative in base shear evaluation. Designers must, therefore, be very careful in application of approximate code formula, because in every individual case, there may be a boundary of beam stiffness beyond which the formula no longer remains conservative. In other words, application of approximate formula must be limited for stocky beams like that for stocky columns. For the range of beam stiffnesses considered for 12-story frames, however, the code formula yields lower period than modal analysis.

### 6.4.7 Effect of Strength of Concrete

For studying the effect of cylinder strength of concrete on period, models Dcs are considered. These models represent 6 -story and 12 -story frame structures, $3 \times 3,4 \times 4$, and $5 \times 5$ bays in plan with cylinder strength of concrete varying from $17244 \mathrm{kN} / \mathrm{m}^{2}$ to $41386 \mathrm{kN} / \mathrm{m}^{2}$. The value of cylinder strength of concrete is related with the modulus of elasticity by

$$
E=33 w^{1.5} \sqrt{f_{c}^{\prime}}
$$

Where, $E$ is modulus of elasticity, $w$ is unit weight, and $f_{c}^{\prime}$ is cylinder strength of concrete respectively. While the cylinder strength varies between the above range, the modulus of elasticity will vary between $2.09 \times 10^{7} \mathrm{kN} / \mathrm{m}^{2}$ and $3.24 \times 10^{7} \mathrm{kN} / \mathrm{m}^{2}$. The periods of these models are first determined by modal analysis using ANSYS. The periods of the same models are also calculated with the empirical formula of UBC-94. The periods of 6-story models are plotted against concrete strength in Fig. 6.20. The identical relationships for 12 -story models are plotted in Fig. 6.21. The three inclined curves in each of these figures correspond to periods of $3 \times 3,4 \times 4$ and $5 \times 5$ bay structures. The horizontal curves in these figures represent periodcylinder strength relationships for the same models but defined by approximate UBC-94 formula. The later ones are invariant of number of bays or concrete strength. This is because the code formula defines period as a sole function of structure height.

The following observations are made from the figures drawn.


Fig. 6.20 Effect of concrete strength on period of 6-story frame structures


Fig. 6.21 Effect of concrete strength on period of 12-story frame structures
i) The period of a structure decreases with the increase of cylinder strength of concrete. The rate of decreases of period slightly diminishes with increase in concrete strength. This fact, however, remains unrecognised by approximate formula of UBC-94.
ii) The period of a structure increases as the number of bays of the structure increases. This increase of period due to increase of number of bays are more for 6 -story buildings than for 12 -story buildings.
iii) The period remains constant with the increase of cylinder strength of concrete when determined from the empirical formula of UBC-94.
iv) The magnitude of the period of 6-story frame structures obtained through modal analysis is higher than the magnitude of the period calculated by approximate UBC-94 formula as long as the cylinder strength of concrete remains less than $20693 \mathrm{kN} / \mathrm{m}^{2}$. For higher strengths, UBC-94 formula gives higher periods making base shear evaluation by approximate formula no longer conservative. It may be concluded, therefore, that there is a limit in material strength in each individual case beyond which base shear evaluation based on approximate code formula will no longer remain conservative.
v) There must be a limit in application of approximate UBC-94 formula when higher strength concretes are used. This will be specially vital when low-rise structures are built using high strength concretes. For the range of concrete strength investigated, such a limit could not be attained for 12story structures. It appears from Fig. 6.21 that such a limit lies beyond the concrete strength of $41386 \mathrm{kN} / \mathrm{m}^{2}$.

### 6.4.8 Effect of Density of Concrete

For studying the effect of density of concrete on period, models Dd are considered. These models represent 6 -story and 12 -story frame structures, $3 \times 3,4 \times 4$ and $5 \times 5$ bays in plan with density of concrete varying from 22.0 to $25.2 \mathrm{kN} / \mathrm{m}^{3}$. The periods of these models are first determined by modal analysis using ANSYS. The periods of the same models are also calculated with empirical formula of UBC-94. The periods of the 6 -tory models are


Fig. 6.22 Effect of density of concrete on period of 6-story frame structures


Fig. 6.23 Effect of density of concrete on period of 12-story frame structures
plotted against density of concrete in Fig. 6.22. The identical relationships for 12 -story models are plotted in Fig. 6.23. The three top lines in each of these figures correspond to periods of $3 \times 3,4 \times 4$ and $5 \times 5$ bay structures, as determined by modal analysis technique. The horizontal curves in these figures represent period-concrete density relationships for the same models defined by approximate UBC-94 formula. The code formula is invariant of number of bays or density of concrete. Because it defines period as a sole function of structure height.

The following observations are made from Fig. 6.22 and Fig. 6.23
i) The period of a structure increases with the increase in density of the concrete.
ii) The period remains constant with the increase in density of concrete, when calculated with empirical formula of UBC-94.
iii) The magnitude of the period determined by modal analysis method is found to be higher than the magnitude of the period found by empirical formula of the code.
iv) The difference between the magnitude of period of a structure determined by modal analysis and empirical formula of UBC-94 increases as number of stories increases.

### 6.4.9 Effect of Number of Stories:

For studying the effect of number of stories on the period, models of group GpE are considered. These models are analyzed by modal analysis technique using ANSYS. Models of GpE consist of 6, 8, 10, 12 and 14 storied structures, $3 \times 3,4 \times 4$ and $5 \times 5$ bays in plan each. The periods obtained through modal analysis of these models are plotted in Fig. 6.24. The top three curves represent period-number of story relationships for $3 \times 3,4 \times 4$ and $5 \times 5$ bays respectively. The periods of the same models are calculated by the empirical formula proposed by UBC-94. The corresponding period-number of story relationship is represented by the bottom curve in Fig. 6.24. It may be noted that the curve is invariant of number of bays in plan and, therefore, is


Fig. 6.24 Effect of number of stories on period of frame structures
represented by a single curve for all three case of $3 \times 3,4 \times 4$ and $5 \times 5$ bays. However, the period varies linearly with the number of stories as the formula expresses period as a direct function of structure height. Fig. 6.24 leads to following observations:
i) The period of a structure increases as the number of stories increases. In other words, the period of a structure increases with the increase of the height of the structure.
ii) The magnitude of the period of a model obtained through modal analysis is greater than the magnitude of the period of the same model calculated by approximate code formula of UBC-94. This difference of periods increases as number of stories or height of a building increases. For example, for a $5 \times 5$ bay structure, the corresponding difference in case of 14 storied structure is $237.63 \%$ higher than that for 6 -storied structure.
iii) Modal analysis of structures show that the difference in periods for $3 \times 3,4 \times 4$, and $5 \times 5$ bays structures are insignificant for any number of stories and this difference reduces with increasing number of stories. As both mass and stiffness of a structure increases with increasing number of bays in plan, perhaps effect of mass neutralizes the effect of stiffness on period and thereby leads to insignificant difference.
iv) As slopes of the modal analysis curves are greater than the slope of the UBC-94 curve, the approximate UBC-94 formula becomes more and more conservative in base shear evaluation with increasing number of stories.

### 6.4.10 Effect of Inclusion of Floor Slabs and Mass of Secondary Structural Elements

Models included in GpF are analyzed to study the effect of inclusion of floor slabs and mass of secondary structural elements. Models FF, FS and FMS constitute models GpF . These are 6 -story and 12 -story structures $2 \times 2,3 \times 3$, $4 \times 4$ and $5 \times 5$ bays in plan. Models FF represent bare frame structures, models FS represent frames with floor slabs and models FMS represent


Fig. 6.25 Effect of inclusion of floor slabs \& mass of secondary elements on period of 6 -story frame structures


Fig. 6.26 Effect of inclusion of floor slabs \& mass of secondary elements on period of 12-story frame structures
frames including both floor slabs and masses of secondary structural elements.

The periods of the above mentioned models are first calculated by modal analysis with ANSYS. The periods of these models are also calculated by empirical formula of UBC-94. The periods of the models determined by modal analysis and by approximate formula are plotted against number of bays in plan. The curves for 6 -story models are plotted in Fig. 6.25 and those for 12 -story models in Fig. 6.26. Three curves represent results of modal analysis for bare frames, for frames with floor slabs and for frames with floor slabs and masses of secondary structural elements respectively. The fourth curve in each of these figures represents the period-number of bays relationship if approximate UBC-94 formula is used.

The following observations are made from Fig. 6.25 and Fig. 6.26.
i) The period of a frame structure without floor slab decreases with the increase of number of bays. However, this decrease is not very significant and the rate of decrease diminishes with increasing number of bays.
ii) The period of a frame structure with floor slabs increase with the increase of number of bays. Again, this increase is not very significant. However, inclusion of floor slabs at story levels increases the magnitude of period significantly when compared to cases without floor slabs.
iii) The period of a frame structure with floor slabs and mass of secondary structural elements included, decreases with the increase of number of bays of the structures. The rate of decrease diminishes with increasing number of bays. The magnitude of the period of a frame structure with floor slabs and masses of secondary elements included is much higher than cases of bare frames or frames with floor slabs only.
iv) The period of a frame structure remains invariant of number of bays or inclusion of floor slabs and masses of secondary structural elements according to UBC-94 formula.
v) The periods calculated by approximate formula of UBC-94 are always lower than the periods determined by including effect of floor slabs or masses of secondary elements. However, periods determined by UBC-94 formula are higher then those for bare frames. UBC-94 formula will,
therefore, lead to conservative base shear evaluation for frames with floor slabs or with masses of secondary elements included. Ignorance of floor slabs and secondary elements may lead to underestimation of the earthquake forces.

Plate 6.5 presents the isometric view of $5 \times 5$ bays 6 -story frame structure with floor slabs in each floor and ANSYS symbolic indication of degrees of freedom at the base support of the structure and direction of excitation at each node of the structure. Plate $6.6,6.7,6.8,6.9 \& 6.10$ present the mode shapes of the structure for Ist, 2nd, 3rd, 4th, \& 5th mode respectively. The dot lines in the plot indicate the original shape of the structure and the firm lines indicate the modal shape of the structure

Plate 6.11 presents the isometric view of $5 \times 5$ bays 12 -story frame structure with floor slabs in each floor and ANSYS symbolic indication of degrees of freedom at the base support of the structure and direction of excitation at each node of the structure. Plate $6.12,6.13,6.14,6.15 \& 6.16$ present the mode shapes of the structure for Ist, 2nd, 3rd, 4th, \& 5th mode respectively. The dot lines in the plot indicate the original shape of the structure and the firm lines indicate the modal shape of the structure

Plate 6.17 presents the isometric view of $5 \times 5$ bays 6 -story frame structure without floor slabs in each floor and ANSYS symbolic indication of degrees of freedom at the base support of the structure and direction of excitation at each node of the structure. Plate $6.18,6.19,6.20,6.21 \& 6.22$ present the mode shapes of the structure for Ist, 2nd, 3rd, 4th , \& 5th mode respectively. The dot lines in the plot indicate the original shape of the structure and the firm lines indicate the modal shape of the structure.

Plate 6.23 presents the isometric view of $5 \times 5$ bays 12 -story frame structure without floor slabs in each floor and ANSYS symbolic indication of degrees of freedom at the base support of the structure and direction of excitation at each node of the structure. Plate $6.24,6.25,6.26,6.27 \& 6.28$ present the mode shapes of the structure for Ist, 2nd, 3rd, 4th , \& 5th mode respectively. The dot lines in the plot indicate the original shape of the structure and the firm lines indicate the modal shape of the structure.


Plate 6.5 Isometric view of $5 \times 5$ bay 6-story frame structure without floor slab showing symbolic indication of degrees of freedom and direction of excitation


Plate 6.6 Modal shape of $5 \times 5$ bay 6-story frame structure without floor slab for 1st mode


Plate 6.7 Modal shape of $5 \times 5$ bay 6-story frame structure without floor slab for 2nd mode


Plate 6.8 Modal shape of $5 \times 5$ bay 6-story frame structure without floor slab for 3rd mode


Plate 6.9 Modal shape of $5 \times 5$ bay 6-story frame structure without floor slab for 4th mode


Plate 6.10 Modal shape of $5 \times 5$ bay 6-story frame structure without floor slab for 5th mode


Plate 6.11 Isometric view of $5 \times 5$ bay 12-story frame structure without floor slab showing symbolic indication of degrees of freedom and direction of excitation


Plate 6.12 Modal shape of $5 \times 5$ bay 12-story frame structure without floor slab for 1st mode


Plate 6.13 Modal shape of $5 \times 5$ bay 12-story frame structure without floor slab for 2nd mode


Plate 6.14 Modal shape of $5 \times 5$ bay 12-story frame structure without floor slab for 3rd mode


Plate 6.15 Modal shape of $5 \times 5$ bay 12-story frame structure without floor slab for 4th mode


Plate 6.16 Modal shape of $5 \times 5$ bay 12-story frame structure without floor slab for 5th mode


Plate 6.17 Isometric view of $5 \times 5$ bay 6-story frame structure with floor slab showing symbolic indication of degrees of freedom and direction of excitation


Plate 6.18 Modal shape of $5 \times 5$ bay 6 -story frame structure with floor slab for 1st mode


Plate 6.19 Modal shape of $5 \times 5$ bay 6-story frame structure with floor slab for 2nd mode


Plate 6.20
Modal shape of $5 \times 5$ bay 6-story frame structure with floor slab for 3rd mode


Plate 6.21 Modal shape of $5 \times 5$ bay 6-story frame structure with floor slab for 4th mode


Plate 6.22 Modal shape of $5 \times 5$ bay 6-story frame structure with floor slab for 5th mode


Plate 6.23 Isometric view of $5 \times 5$ bay 12-story frame structure with floor slab showing symbolic indication of degrees of freedom and direction of excitation


Plate 6.24 Modal shape of $5 \times 5$ bay 12-story frame structure with floor slab for 1st mode


Plate 6.25 Modal shape of $5 \times 5$ bay 12-story frame structure with floor slab for 2 nd mode


Plate 6.26 Modal shape of $5 \times 5$ bay 12-story frame structure with floor slab for 3rd mode


Plate 6.27 Modal shape of $5 \times 5$ bay 12-story frame structure with floor slab for 4th mode


Plate 6.28 Modal shape of $5 \times 5$ bay 12-story frame structure with floor slab for 5th mode

### 6.5 COMPARISON OF RESULTS OF MODAL ANALYSIS AND CODE FORMULAE

From a comparative review of design codes, it has been observed in Art. 4.6 that the codes propose approximate evaluation of structure period by two different formulae. These are given by Eqs. 6.1 and 6.2 respectively. Eq. 6.1 assumes period as a sole function of number of stories, while Eq. 6.2 expresses period as a sole function of structure height. UBC-94 and BNBC suggest identical approximate formula given by Eq. 6.2.

In the preceding article, the periods of model frames found by modal analysis and by approximate UBC-94 formula are compared. It has been observed that the UBC-94 formula is invariant of any structural parameter other than structure height.

For a comparison of period evaluation by code formulae (Eq. 6.1 and Eq. 6.2 ) with that by modal analysis, models of GpE are considered. These are 6 , $8,10,12$ and 14 story frames $3 \times 3,4 \times 4,5 \times 5$ bays in plan. Periods of these models determined by Eq. 6.1 and Eq. 6.2 and by modal analysis are plotted against structure height (number of stories) in Fig.6.27. It may be observed that both Eqs. 6.1 and 6.2 yield lower periods than modal analysis. All the curves indicate that period of a structure increases with increasing structure height. However, the rate of increase of period is not same for all of them. For number of stories less than 10, Eq. 6.2 yields higher periods while for number of stories greater than 10, Eq. 6.1 yields higher period. Thus Eq. 6.1 is more conservative compared to Eq. 6.2 for structures less than 10 -story high. For structures more than 10 -storied, the reverse is true. Eq. 6.1 shows higher rate of increase of period with height compared to Eq. 6.2.

### 6.6 INVESTIGATING THE LIMITATIONS OF APPROXIMATE CODE FORMULAE

According to the empirical formula, given by UBC-85, SAA and IS, the period of a structure is a function of the number of stories of the structure. Again, according to UBC-91, UBC-94 and BNBC the period of a structure is


Fig. 6.27 Comparison of periods determined by modal analysis and approximate code formulae
a function of the height of the structure only. While deriving the formula from the principles and theory of structural dynamics for calculating the period of a structure in Art. 3.2, it was observed that it is a function of stiffness and mass properties of the structure.

While analyzing for periods of the models by modal analysis in Art.6.4, it was observed that the period of a structure is not only dependent on the height of the structure, but it is also dependent on the number of bays, width of bays, height of the stories, stiffness of the beams and columns, strength of the materials, density of concrete and masses of structural and nonstructural elements. However the approximate empirical formulae proposed by codes do not include all these factors. The approximate formulae reviewed in this work expresses period either as a function of structure height or as a function of number of stories. In other words, the approximate code formulae for period evaluation in some way or other are functions of structure height only. Ignorance of other factors makes the code formulae grossly approximate. Although, in most cases they have been found to be conservative in evaluation of earthquake forces.

Though the height of the structure is a major factor in determining the period of a structure, other factors may influence the period significantly. Hence, empirical formulae developed without considering these effects suffer from large approximations. Some evidences in support of this observation are given in the following paragraphs.

For example, it is observed from Fig. 6.13 that, for increase of bay width of a structure by $40 \%$, the period of the structure increases by about $37 \%$. From Fig. 6.17 it is seen that for increase of stiffness of columns by $267 \%$ the period of the structure reduces by $18 \%$. From Fig. 6.19 it is observed that for increase of stiffness of beams by $567 \%$, the period of the structure reduces by $21 \%$. From Fig. 6.21 it is seen that for increase of cylinder strength of concrete by $240 \%$ the period of a structure decreases by $20 \%$. From Figs. 6.25 and 6.26, it can be observed that the inclusion of the floor slabs and masses of secondary structural elements increases the magnitude of the period of a structure.

The period of a structure determined by UBC-94 formula is generally lower than the period of the structure calculated by modal analysis. For the ranges of values of structural parameters considered for different models, it has been observed that the UBC-94 formula may yield $56 \%$ lower period than the modal analysis for not considering other structural parameters.

Inclusion of certain structure parameters in analysis may lead to lower period of a structure by modal analysis than by UBC-94 formula. These are column and beam sizes and cylinder strength of concrete. For example, designers must be very careful in application of UBC-94 formula in cases of stocky columns and beams and in cases of use of higher strength concrete in low- rise structures.

From the above discussions it may be concluded that the empirical formulae given by codes suffer from limitations by not considering the effect of structural parameters like bay widths, number of bays, stiffness of beams and columns, cylinder strength of concrete, density of concrete and masses of secondary structural elements. These limitations could be overcomed only by appropriate modification of the formulae. However, the code formulae in their present form have to be used carefully in cases of stocky columns and beams and in cases of use of higher strength concrete.

### 6.7 CHOOSING STRUCTURE PARAMETERS TO REDUCE EARTHQUAKE FORCES

As discussed in Art. 4.2, the formula for calculating the base shear by UBC 94 is

$$
\begin{equation*}
V=\frac{Z I C}{R_{W}} W \tag{6.3}
\end{equation*}
$$

It is seen that the base shear increases as the C value increases. Again, the C value is inversely proportional to the period of the structure. So, as the period of the structure increases, the $C$ value of the structure decreases and thereby the magnitude of the base shear decreases. With the decrease of the value of the base shear the design earthquake forces also decrease. The variation of the structure parameters that increase the magnitude of the period and thereby decrease the C -value of the structure leading to reduction of earthquake forces are enumerated below.

- Increasing the number of bays of the structure in the direction transverse to the direction of motion.
- Increasing the bay width
- Reducing the column size
- Reducing the beam size
- Increasing the masses of secondary structural elements.
- Reducing cylinder strength of concrete.
- Increasing density of concrete

The above variation of the parameters will increase the magnitude of the period of a structure. But due to these variations the weight/mass of the structure may also increase or decrease. Eq. 6.3 shows that the earthquake
forces are directly proportional to structure weight as well as to C . Therefore, any variation of weight while trying to influence C -value would additionally affect the earthquake forces on the structure. As mentioned earlier, the increase of the period of the structure will decrease the $C$ value of the structure. So the variation of the structure parameters that would reduce both the C value and the weight of the structure would positively reduce earthquake forces.

The variation of the structure parameters those effectively can reduce design earthquake forces on a structure are discussed below.

- By increasing the number of bays of the structure in the direction perpendicular to the direction of motion, the magnitude of the period of the structure will increase and thereby the C -value of the structure will decrease. But due to the increase of the number of bays the weight of the structure will also increase. If the reduction of earthquake forces caused by reduction of C -value exceeds the increment caused by the increase of the weight of the structure, the magnitude of the earthquake forces will be effectively reduced.
- By increasing bay width, the C-value may be reduced. Hence, the earthquake forces on the structure may be effectively reduced by increasing bay width, only if the weight of the structure is contained accordingly.
- By selecting column size as small as possible, both the C-value and the weight of the structure will be reduced. As the C -value and the weight of the structure reduce simultaneously, the magnitude of the design earthquake forces will also be effectively reduced.
- By decreasing the beam size as much as possible, simultaneous reduction in values of C and weight of the structure can be achieved. Hence, the design earthquake forces may be effectively reduced by decreasing beam size.
- Use of lower cylinder strengths of concrete leads to higher values of period and lower values of C . As the weight of the structure remains unaffected, therefore use of concrete of lower cylinder strengths will effectively reduce design earthquake forces on a structure.
- Inclusion of additional masses increases the period of the structure. Increase of period of the structure means decrease of $C$ value and decrease of design earthquake forces. However, weights of these masses will increase the weight the structure itself which will increase the design earthquake forces of the structure. Hence, secondary masses can only lead to reduction of earthquake forces, if their influence on C -values can supersede their effect on total structure weight.
- Increase in density of concrete leads to simultaneous increase of C -value and structure weight. Hence increase in density of concrete may only reduce earthquake forces provided its effect on period becomes more prominent than its effect on weight.


## CHAPTER 7

## CONCLUSIONS AND RECOMMENDATIONS

### 7.1 CONCLUSIONS

The effects of different structure parameters on period of regular frame structures for earthquake resistant design are investigated in this work. The structure parameters those play key role in reducing the design earthquake forces are also identified. Major earthquake resistant design codes and the empirical formulae proposed by them for determining the periods of structures are studied. Based on the investigations conducted in this work, the following conclusions are drawn regarding period of regular frame structures:

1) The period increases with decreasing number of bays along the direction of motion and increasing number of bays transverse to the direction of motion. Also, an increase in bay width leads to an increase in period. Approximate code formulae can not recognise these facts.
2) The period is found to increase with increasing story height and number of stories. The code formulae recognize this fact. Although, the codes become increasingly conservative with taller structures in calculating period.
3) Inclusion of floor slabs and masses of secondary structural elements in analyzing frame structures gives higher period than that for bare frames.
4) Use of stiffer columns and beams leads to a reduction in the period of a frame. Beyond certain limit of stiffness, the approximate code formulae may no longer remain conservative in calculation of period for base shear evaluation. Designers thus should be cautious in applying approximate code formulae in cases where stocky columns and beams are selected. Because these formulae may lead in these cases to an underestimation of earthquake forces. Special care has to be taken in cases of low-rise structures with stocky columns and beams.
5) Use of concrete of higher strengths reduce period. Especially for lowrise structures with high strength concrete code formulae may evaluate higher period leading to an underestimation of earthquake forces. Hence, for a certain height of a structure, there is a limit of concrete strength, beyond which application of approximate code formulae for period must be restricted.
6) Design earthquake forces on a structure may be reduced by choosing smaller dimensions for beams and columns. Especially, reduction of dimension of a column cross section in the direction of earthquake forces and the height of the beam cross section can effectively minimize these forces. Another means of reducing design earthquake forces is to use lower strength concrete.
7) Approximate code formulae for evaluation of period are invariant of structure parameters other than height. However, in most practical cases these formulae are found to lead to conservative estimation of earthquake forces.

### 7.2 RECOMMENDATIONS FOR FUTURE STUDY

The following recommendations are made for future study:

1) Investigations may be carried out for reviewing and reforming the present code formulae for period, so that influence of structure parameters other than height may be incorporated in the formula. Such study would facilitate the determination of period and the design earthquake forces more accurately.
2) Influence of structure parameters on period of structural systems other than frames may be studied.

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## APPENDIX

## ANSYS INPUTS AND OUTPUTS

## INPUT DATA FOR MODELS

## MODEL A1 ( $6 \& 12$ story building )

1 Modulus of elasticity $=2.07 \times 10^{6} \mathrm{kN} / \mathrm{m}^{2}$
2 Density of concrete $=23.56 \mathrm{kN} / \mathrm{m}^{2}$
3 Acceleration due to gravity $=9.81 \mathrm{~m} / \mathrm{sec}^{2}$
4 Bay width $=5.0 \mathrm{~m}$
5 Story height $=3.2 \mathrm{~m}$
6 Cylinder strength of concrete $=17244 \mathrm{kN} / \mathrm{m}^{2}$
7 Column size $=.5 \mathrm{~m} \times .5 \mathrm{~m}$
$8 \quad$ Beam size $=.35 \mathrm{~m} \times .5 \mathrm{~m}$
9 Slab thickness $=.15 \mathrm{~m}$
10 Number of bays along the direction of motion $=2$
11 Number of bays along the direction perpendicular to the direction of motion $=2$

## MODEL A2 (6 \& 12 story building )

1 Modulus of elasticity $=2.07 \times 10^{6} \mathrm{kN} / \mathrm{m}^{2}$
2 Density of concrete $=23.56 \mathrm{kN} / \mathrm{m}^{2}$
3 Acceleration due to gravity $=9.81 \mathrm{~m} / \mathrm{sec}^{2}$
$4 \quad$ Bay width $=5.0 \mathrm{~m}$
5 Story height $=3.2 \mathrm{~m}$
6 Cylinder strength of concrete $=17244 \mathrm{kN} / \mathrm{m}^{2}$
$7 \quad$ Column size $=.5 \mathrm{~m} \times .5 \mathrm{~m}$
$8 \quad$ Beam size $=.35 \mathrm{~m} \times .5 \mathrm{~m}$
9 Slab thickness $=.15 \mathrm{~m}$
10 Number of bays along the direction of motion $=3$
11 Number of bays along the direction perpendicular to the direction of motion $=2$

## MODEL A3 (6 \& 12 story building)

1 Modulus of elasticity $=2.07 \times 10^{6} \mathrm{kN} / \mathrm{m}^{2}$
2 Density of concrete $=23.56 \mathrm{kN} / \mathrm{m}^{2}$
3 Acceleration due to gravity $=9.81 \mathrm{~m} / \mathrm{sec}^{2}$
$4 \quad$ Bay width $=5.0 \mathrm{~m}$
5 Story height $=3.2 \mathrm{~m}$
6 Cylinder strength of concrete $=17244 \mathrm{kN} / \mathrm{m}^{2}$
$7 \quad$ Column size $=.5 \mathrm{~m} \times .5 \mathrm{~m}$
$8 \quad$ Beam size $=.35 \mathrm{~m} \times .5 \mathrm{~m}$
9 Slab thickness $=.15 \mathrm{~m}$
10 Number of bays along the direction of motion $=4$
11 Number of bays along the direction perpendicular to the direction of motion $=2$

## MODEL A4 (6 \& 12 story building)

1 Modulus of elasticity $=2.07 \times 10^{6} \mathrm{kN} / \mathrm{m}^{2}$
2 Density of concrete $=23.56 \mathrm{kN} / \mathrm{m}^{2}$
3 Acceleration due to gravity $=9.81 \mathrm{~m} / \mathrm{sec}^{2}$
$4 \quad$ Bay width $=5.0 \mathrm{~m}$
5 Story height $=3.2 \mathrm{~m}$
6 Cylinder strength of concrete $=17244 \mathrm{kN} / \mathrm{m}^{2}$
$7 \quad$ Column size $=.5 \mathrm{~m} \times .5 \mathrm{~m}$
$8 \quad$ Beam size $=.35 \mathrm{~m} \times .5 \mathrm{~m}$
$9 \quad$ Slab thickness $=.15 \mathrm{~m}$
10 Number of bays along the direction of motion $=5$
11 Number of bays along the direction perpendicular to the direction of motion $=2$

## MODEL B1 (6 \& 12 story building)

1 Modulus of elasticity $=2.07 \times 10^{6} \mathrm{kN} / \mathrm{m}^{2}$
2 Density of concrete $=23.56 \mathrm{kN} / \mathrm{m}^{2}$
3 Acceleration due to gravity $=9.81 \mathrm{~m} / \mathrm{sec}^{2}$
$4 \quad$ Bay width $=5.0 \mathrm{~m}$
5 Story height $=3.2 \mathrm{~m}$
6 Cylinder strength of concrete $=17244 \mathrm{kN} / \mathrm{m}^{2}$
7 Column size $=.5 \mathrm{~m} \times .5 \mathrm{~m}$
$8 \quad$ Beam size $=.35 \mathrm{~m} \times .5 \mathrm{~m}$
$9 \quad$ Slab thickness $=.15 \mathrm{~m}$
10 Number of bays along the direction of motion $=2$
11 Number of bays along the direction perpendicular to the direction of motion $=3$

## MODEL B2 (6 \& 12 story building)

1 Modulus of elasticity $=2.07 \times 10^{6} \mathrm{kN} / \mathrm{m}^{2}$
2 Density of concrete $=23.56 \mathrm{kN} / \mathrm{m}^{2}$
3 Acceleration due to gravity $=9.81 \mathrm{~m} / \mathrm{sec}^{2}$
$4 \quad$ Bay width $=5.0 \mathrm{~m}$
5 Story height $=3.2 \mathrm{~m}$
6 Cylinder strength of concrete $=17244 \mathrm{kN} / \mathrm{m}^{2}$
$7 \quad$ Column size $=.5 \mathrm{~m} \times .5 \mathrm{~m}$
$8 \quad$ Beam size $=.35 \mathrm{~m} \times .5 \mathrm{~m}$
9 Slab thickness $=.15 \mathrm{~m}$
10 Number of bays along the direction of motion $=3$
11 Number of bays along the direction perpendicular to the direction of motion $=3$

## MODEL B3 (6 \& 12 story building)

1 Modulus of elasticity $=2.07 \times 10^{6} \mathrm{kN} / \mathrm{m}^{2}$
2 Density of concrete $=23.56 \mathrm{kN} / \mathrm{m}^{2}$
3 Acceleration due to gravity $=9.81 \mathrm{~m} / \mathrm{sec}^{2}$
$4 \quad$ Bay width $=5.0 \mathrm{~m}$
5 Story height $=3.2 \mathrm{~m}$
6 Cylinder strength of concrete $=17244 \mathrm{kN} / \mathrm{m}^{2}$
$7 \quad$ Column size $=.5 \mathrm{~m} \times .5 \mathrm{~m}$
$8 \quad$ Beam size $=.35 \mathrm{~m} \times .5 \mathrm{~m}$
9 Slab thickness $=.15 \mathrm{~m}$
10 Number of bays along the direction of motion $=4$
11 Number of bays along the direction perpendicular to the motion $=3$

## MODEL B4 (6 \& 12 story building)

1 Modulus of elasticity $=2.07 \times 10^{6} \mathrm{kN} / \mathrm{m}^{2}$
2 Density of concrete $=23.56 \mathrm{kN} / \mathrm{m}^{2}$
3 Acceleration due to gravity $=9.81 \mathrm{~m} / \mathrm{sec}^{2}$
$4 \quad$ Bay width $=5.0 \mathrm{~m}$
5 Story height $=3.2 \mathrm{~m}$
6 Cylinder strength of concrete $=17244 \mathrm{kN} / \mathrm{m}^{2}$
$7 \quad$ Column size $=.5 \mathrm{~m} \times .5 \mathrm{~m}$
$8 \quad$ Beam size $=.35 \mathrm{~m} \times .5 \mathrm{~m}$
9 Slab thickness $=.15 \mathrm{~m}$
10 Number of bays along the direction of motion $=4$
11 Number of bays along the direction perpendicular to the direction of motion $=3$

## MODEL C1 ( 6 \& 12 story building)

1 Modulus of elasticity $=2.07 \times 10^{6} \mathrm{kN} / \mathrm{m}^{2}$
2 Density of concrete $=23.56 \mathrm{kN} / \mathrm{m}^{2}$
3 Acceleration due to gravity $=9.81 \mathrm{~m} / \mathrm{sec}^{2}$
$4 \quad$ Bay width $=5.0 \mathrm{~m}$
5 Story height $=3.2 \mathrm{~m}$
6 Cylinder strength of concrete $=17244 \mathrm{kN} / \mathrm{m}^{2}$
7 Column size $=.5 \mathrm{mx} .5 \mathrm{~m}$
$8 \quad$ Beam size $=.35 \mathrm{~m} \times .5 \mathrm{~m}$
9 Slab thickness $=.15 \mathrm{~m}$
10 Number of bays along the direction of motion $=2$
11 Number of bays along the direction perpendicular to the direction of motion $=4$

MODEL C2 (6 \& 12 story building)
1 Modulus of elasticity $=2.07 \times 10^{6} \mathrm{kN} / \mathrm{m}^{2}$
2 Density of concrete $=23.56 \mathrm{kN} / \mathrm{m}^{2}$
3 Acceleration due to gravity $=9.81 \mathrm{~m} / \mathrm{sec}^{2}$
$4 \quad$ Bay width $=5.0 \mathrm{~m}$
5 Story height $=3.2 \mathrm{~m}$
6 Cylinder strength of concrete $=17244 \mathrm{kN} / \mathrm{m}^{2}$
$7 \quad$ Column size $=.5 \mathrm{~m} \times .5 \mathrm{~m}$
$8 \quad$ Beam size $=.35 \mathrm{~m} \times .5 \mathrm{~m}$
9 Slab thickness $=.15 \mathrm{~m}$
10 Number of bays along the direction of motion $=5$
11 Number of bays along the direction perpendicular to the direction of motion $=3$

MODEL C3 ( 6 \& 12 story building )

$$
\begin{array}{ll}
1 & \text { Modulus of elasticity }=2.07 \mathrm{X} 10^{6} \mathrm{kN} / \mathrm{m}^{2} \\
2 & \text { Density of concrete }=23.56 \mathrm{kN} / \mathrm{m}^{2} \\
3 & \text { Acceleration due to gravity }=9.81 \mathrm{~m} / \mathrm{sec}^{2} \\
4 & \text { Bay width }=5.0 \mathrm{~m} \\
5 & \text { Story height }=3.2 \mathrm{~m} \\
6 & \text { Cylinder strength of concrete }=17244 \mathrm{kN} / \mathrm{m}^{2} \\
7 & \text { Column size }=.5 \mathrm{~m} \mathrm{x} .5 \mathrm{~m} \\
8 & \text { Beam size }=.35 \mathrm{~m} \mathrm{x.5m} \\
9 & \text { Slab thickness }=.15 \mathrm{~m} \\
10 & \begin{array}{l}
\text { Number of bays along the direction } \\
\text { of motion }=4
\end{array}
\end{array}
$$

11 Number of bays along the direction perpendicular to the direction of motion $=4$

MODEL C4 (6 \& 12 story building )

$$
\begin{array}{ll}
1 & \text { Modulus of elasticity }=2.07 \times 10^{6} \mathrm{kN} / \mathrm{m}^{2} \\
2 & \text { Density of concrete }=23.56 \mathrm{kN} / \mathrm{m}^{2} \\
3 & \text { Acceleration due to gravity }=9.81 \mathrm{~m} / \mathrm{sec}^{2} \\
4 & \text { Bay width }=5.0 \mathrm{~m} \\
5 & \text { Story height }=3.2 \mathrm{~m} \\
6 & \text { Cylinder strength of concrete }=17244 \mathrm{kN} / \mathrm{m}^{2} \\
7 & \text { Column size }=.5 \mathrm{~m} \times .5 \mathrm{~m} \\
8 & \text { Beam size }=.35 \mathrm{~m} \times .5 \mathrm{~m} \\
9 & \text { Slab thickness }=.15 \mathrm{~m} \\
10 & \text { Number of bays along the direction } \\
& \text { of motion }=5
\end{array}
$$

11 Number of bays along the direction perpendicular to the direction of motion $=4$

## MODEL Db1 ( $6 \& 12$ story $3 \times 3,4 \times 4,5 \times 5$ bays structures)

1 Modulus of elasticity $=2.07 \times 10^{6} \mathrm{kN} / \mathrm{m}^{2}$
2 Density of concrete $=23.56 \mathrm{kN} / \mathrm{m}^{2}$
3 Acceleration due to gravity $=9.81 \mathrm{~m} / \mathrm{sec}^{2}$
$4 \quad$ Bay width $=5.0 \mathrm{~m}$
5 Story height $=3.2 \mathrm{~m}$
6 Cylinder strength of concrete $=17244 \mathrm{kN} / \mathrm{m}^{2}$
$7 \quad$ Column size $=.5 \mathrm{~m} \times .5 \mathrm{~m}$
8 Beam size $=.35 \mathrm{~m} \times .5 \mathrm{~m}$
$9 \quad$ Slab thickness $=.15 \mathrm{~m}$
10 Number of bays along the direction of motion $=3$
11 Number of bays along the direction perpendicular to the direction of motion $=3$

## MODEL Db2 (6\& 12 story, 3x3, 4x4, 5x5 bays structures)

1 Modulus of elasticity $=2.07 \times 10^{6} \mathrm{kN} / \mathrm{m}^{2}$
2 Density of concrete $=23.56 \mathrm{kN} / \mathrm{m}^{2}$
3 Acceleration due to gravity $=9.81 \mathrm{~m} / \mathrm{sec}^{2}$
$4 \quad$ Bay width $=5.5 \mathrm{~m}$
5 Story height $=3.2 \mathrm{~m}$
6 Cylinder strength of concrete $=17244 \mathrm{kN} / \mathrm{m}^{2}$
$7 \quad$ Column size $=.5 \mathrm{~m} \times .5 \mathrm{~m}$
$8 \quad$ Beam size $=.35 \mathrm{~m} \times .5 \mathrm{~m}$
$9 \quad$ Slab thickness $=.15 \mathrm{~m}$
10 Number of bays along the direction of motion $=3,4,5$
11 Number of bays along the direction perpendicular to the direction of motion $=3,4,5$

MODEL Db3 ( $6 \& 12$ story, $3 \times 3,4 \times 4,5 \times 5$ bays structures )
1 Modulus of elasticity $=2.07 \times 10^{6} \mathrm{kN} / \mathrm{m}^{2}$
2 Density of concrete $=23.56 \mathrm{kN} / \mathrm{m}^{2}$
3 Acceleration due to gravity $=9.81 \mathrm{~m} / \mathrm{sec}^{2}$
$4 \quad$ Bay width $=6.0 \mathrm{~m}$
5 Story height $=3.2 \mathrm{~m}$
6 Cylinder strength of concrete $=17244 \mathrm{kN} / \mathrm{m}^{2}$
$7 \quad$ Column size $=.5 \mathrm{mx} .5 \mathrm{~m}$
$8 \quad$ Beam size $=.35 \mathrm{~m} \times .5 \mathrm{~m}$
9 Slab thickness $=.15 \mathrm{~m}$
10 Number of bays along the direction of motion $=3,4,5$
11 Number of bays along the direction perpendicular to the direction of motion. $=3,4,5$

MODEL Db4 ( 6 \& 12 story, $3 \times 3,4 \times 4,5 \times 5$ bays structures)

1 Modulus of elasticity $=2.07 \times 10^{6} \mathrm{kN} / \mathrm{m}^{2}$
2 Density of concrete $=23.56 \mathrm{kN} / \mathrm{m}^{2}$
3 Acceleration due to gravity $=9.81 \mathrm{~m} / \mathrm{sec}^{2}$
$4 \quad$ Bay width $=6.5 \mathrm{~m}$
5 Story height $=3.2 \mathrm{~m}$
6 Cylinder strength of concrete $=17244 \mathrm{kN} / \mathrm{m}^{2}$
$7 \quad$ Column size $=.5 \mathrm{~m} \times .5 \mathrm{~m}$
$8 \quad$ Beam size $=.35 \mathrm{~m} \times .5 \mathrm{~m}$
9 Slab thickness $=.15 \mathrm{~m}$
10 Number of bays along the direction of motion $=3,4,5$
11 Number of bays along the direction perpendicular to the direction of motion $=3,4,5$

## MODEL Db5 ( 6 \& 12 story, $3 \times 3,4 \times 4,5 \times 5$ bays structures)

1 Modulus of elasticity $=2.07 \times 10^{6} \mathrm{kN} / \mathrm{m}^{2}$
2 Density of concrete $=23.56 \mathrm{kN} / \mathrm{m}^{2}$
3 Acceleration due to gravity $=9.81 \mathrm{~m} / \mathrm{sec}^{2}$
$4 \quad$ Bay width $=7.0 \mathrm{~m}$
5 Story height $=3.2 \mathrm{~m}$
6 Cylinder strength of concrete $=17244 \mathrm{kN} / \mathrm{m}^{2}$
7 Column size $=.5 \mathrm{~m} \times .5 \mathrm{~m}$
$8 \quad$ Beam size $=.35 \mathrm{~m} \times .5 \mathrm{~m}$
9 Slab thickness $=.15 \mathrm{~m}$
10 Number of bays along the direction
of motion $=3,4,5$
11 Number of bays along the direction perpendicular to the diction of motion $=3,4,5$

MODEL Dh1 ( $6 \& 12$ story, $3 \times 3,4 \times 4,5 \times 5$ bays structures)

1 Modulus of elasticity $=2.07 \times 10^{6} \mathrm{kN} / \mathrm{m}^{2}$
2 Density of concrete $=23.56 \mathrm{kN} / \mathrm{m}^{2}$
3 Acceleration due to gravity $=9.81 \mathrm{~m} / \mathrm{sec}^{2}$
$4 \quad$ Bay width $=5.0 \mathrm{~m}$
5 Story height $=3.0 \mathrm{~m}$
6 Cylinder strength of concrete $=17244 \mathrm{kN} / \mathrm{m}^{2}$
$7 \quad$ Column size $=.5 \mathrm{~m} \times .5 \mathrm{~m}$
$8 \quad$ Beam size $=.35 \mathrm{~m} \times .5 \mathrm{~m}$
9 Slab thickness $=.15 \mathrm{~m}$
10 Number of bays along the direction of motion $=3,4,5$
11 Number of bays along the direction perpendicular to the direction of motion $=3,4,5$

1 Modulus of elasticity $=2.07 \times 10^{6} \mathrm{kN} / \mathrm{m}^{2}$
2 Density of concrete $=23.56 \mathrm{kN} / \mathrm{m}^{2}$
3 Acceleration due to gravity $=9.81 \mathrm{~m} / \mathrm{sec}^{2}$
$4 \quad$ Bay width $=5.0 \mathrm{~m}$
5 Story height $=3.25 \mathrm{~m}$
6 Cylinder strength of concrete $=17244 \mathrm{kN} / \mathrm{m}^{2}$
$7 \quad$ Column size $=.5 \mathrm{~m} \times .5 \mathrm{~m}$
$8 \quad$ Beam size $=.35 \mathrm{~m} \times .5 \mathrm{~m}$
9 Slab thickness $=.15 \mathrm{~m}$
10 Number of bays along the direction of motion $=3,4,5$
11 Number of bays along the direction perpendicular to the direction of motion $=3,4,5$

## MODEL Dh3 ( 6 \& 12 story, $3 \times 3,4 \times 4,5 \times 5$ bays structures)

1 Modulus of elasticity $=2.07 \times 10^{6} \mathrm{kN} / \mathrm{m}^{2}$
2 Density of concrete $=23.56 \mathrm{kN} / \mathrm{m}^{2}$
3 Acceleration due to gravity $=9.81 \mathrm{~m} / \mathrm{sec}^{2}$
$4 \quad$ Bay width $=5.0 \mathrm{~m}$
5 Story height $=3.5 \mathrm{~m}$
6 Cylinder strength of concrete $=17244 \mathrm{kN} / \mathrm{m}^{2}$
$7 \quad$ Column size $=.5 \mathrm{~m} \times .5 \mathrm{~m}$
$8 \quad$ Beam size $=.35 \mathrm{~m} \times .5 \mathrm{~m}$
9 Slab thickness $=.15 \mathrm{~m}$
10 Number of bays along the direction of motion $=3,4,5$
11 Number of bays along the direction perpendicular to the direction of motion $=3,4,5$

1 Modulus of elasticity $=2.07 \times 10^{6} \mathrm{kN} / \mathrm{m}^{2}$
2 Density of concrete $=23.56 \mathrm{kN} / \mathrm{m}^{2}$
3 Acceleration due to gravity $=9.81 \mathrm{~m} / \mathrm{sec}^{2}$
$4 \quad$ Bay width $=5.0 \mathrm{~m}$
5 Story height $=3.75 \mathrm{~m}$
6 Cylinder strength of concrete $=17244 \mathrm{kN} / \mathrm{m}^{2}$
$7 \quad$ Column size $=.5 \mathrm{~m} \times .5 \mathrm{~m}$
$8 \quad$ Beam size $=.35 \mathrm{~m} \times .5 \mathrm{~m}$
$9 \quad$ Slab thickness $=.15 \mathrm{~m}$
10 Number of bays along the direction of motion $=3,4,5$
11 Number of bays along the direction perpendicular to the direction of motion. $=3,4,5$

## MODEL Dh5 ( $6 \& 12$ story, $3 \times 3,4 \times 4,5 \times 5$ bays structures)

1 Modulus of elasticity $=2.07 \times 10^{6} \mathrm{kN} / \mathrm{m}^{2}$
2 Density of concrete $=23.56 \mathrm{kN} / \mathrm{m}^{2}$
3 Acceleration due to gravity $=9.81 \mathrm{~m} / \mathrm{sec}^{2}$
$4 \quad$ Bay width $=5.0 \mathrm{~m}$
5 Story height $=4.0 \mathrm{~m}$
6 Cylinder strength of concrete $=17244 \mathrm{kN} / \mathrm{m}^{2}$
$7 \quad$ Column size $=.5 \mathrm{~m} \times .5 \mathrm{~m}$
$8 \quad$ Beam size $=.35 \mathrm{~m} \times .5 \mathrm{~m}$
$9 \quad$ Slab thickness $=.15 \mathrm{~m}$
10 Number of bays along the direction of motion $=3,4,5$
11 Number of bays along the direction perpendicular to the direction of motion. $=3,4,5$

MODEL Dc1 ( $6 \& 12$ story, $3 \times 3,4 \times 4,5 \times 5$ bays structures)

1 Modulus of elasticity $=2.07 \times 10^{6} \mathrm{kN} / \mathrm{m}^{2}$
2 Density of concrete $=23.56 \mathrm{kN} / \mathrm{m}^{2}$
3 Acceleration due to gravity $=9.81 \mathrm{~m} / \mathrm{sec}^{2}$
$4 \quad$ Bay width $=5.0 \mathrm{~m}$
5 Story height $=3.2 \mathrm{~m}$
$6 \quad$ Cylinder strength of concrete $=17244 \mathrm{kN} / \mathrm{m}^{2}$
7 Column size $=.4 \mathrm{~m} \times .4 \mathrm{~m}$
$8 \quad$ Beam size $=.35 \mathrm{~m} \times .5 \mathrm{~m}$
9 Slab thickness $=.15 \mathrm{~m}$
10 Number of bays along the direction of motion $=3,4,5$
11 Number of bays along the direction perpendicular to the direction of motion. $=3,4,5$

MODEL Dc2 ( $6 \& 12$ story, $3 \times 3,4 \times 4,5 \times 5$ bays structures)

1 Modulus of elasticity $=2.07 \times 10^{6} \mathrm{kN} / \mathrm{m}^{2}$
2 Density of concrete $=23.56 \mathrm{kN} / \mathrm{m}^{2}$
3 Acceleration due to gravity $=9.81 \mathrm{~m} / \mathrm{sec}^{2}$
$4 \quad$ Bay width $=5.0 \mathrm{~m}$
5 Story height $=3.2 \mathrm{~m}$
6 Cylinder strength of concrete $=17244 \mathrm{kN} / \mathrm{m}^{2}$
7 Column size $=.45 \mathrm{~m} \times .45 \mathrm{~m}$
$8 \quad$ Beam size $=.35 \mathrm{~m} \times .5 \mathrm{~m}$
9 Slab thickness $=.15 \mathrm{~m}$
10 Number of bays along the direction
of motion $=3,4,5$
11 Number of bays along the direction perpendicular to the direction of motion $=3,4,5$

MODEL Dc3 (6 \& 12 story, 3x3, 4x4, 5x5 bays structures)
1 Modulus of elasticity $=2.07 \times 10^{6} \mathrm{kN} / \mathrm{m}^{2}$
2 Density of concrete $=23.56 \mathrm{kN} / \mathrm{m}^{2}$
3 Acceleration due to gravity $=9.81 \mathrm{~m} / \mathrm{sec}^{2}$
$4 \quad$ Bay width $=5.0 \mathrm{~m}$
5 Story height $=3.2 \mathrm{~m}$
6 Cylinder strength of concrete $=17244 \mathrm{kN} / \mathrm{m}^{2}$
$7 \quad$ Column size $=.5 \mathrm{~m} \times .5 \mathrm{~m}$
$8 \quad$ Beam size $=.35 \mathrm{mx} .5 \mathrm{~m}$
9 Slab thickness $=.15 \mathrm{~m}$
10 Number of bays along the direction of motion $=3,4,5$
11 Number of bays along the direction perpendicular to the direction of motion $=3,4,5$

MODEL Dc 4 ( 6 \& 12 story, $3 \times 3,4 \times 4,5 \times 5$ bays structures)
1 Modulus of elasticity $=2.07 \times 10^{6} \mathrm{kN} / \mathrm{m}^{2}$
2 Density of concrete $=23.56 \mathrm{kN} / \mathrm{m}^{2}$
3 Acceleration due to gravity $=9.81 \mathrm{~m} / \mathrm{sec}^{2}$
$4 \quad$ Bay width $=5.0 \mathrm{~m}$
5 Story height $=3.2 \mathrm{~m}$
6 Cylinder strength of concrete $=17244 \mathrm{kN} / \mathrm{m}^{2}$
7 Column size $=.55 \mathrm{~m} \times .55 \mathrm{~m}$
$8 \quad$ Beam size $=.35 \mathrm{mx} .5 \mathrm{~m}$
9 Slab thickness $=.15 \mathrm{~m}$
10 Number of bays along the direction of motion $=3,4,5$
11 Number of bays along the direction perpendicular to the direction of motion $=3,4,5$

1 Modulus of elasticity $=2.07 \times 10^{6} \mathrm{kN} / \mathrm{m}^{2}$
2 Density of concrete $=23.56 \mathrm{kN} / \mathrm{m}^{2}$
3 Acceleration due to gravity $=9.81 \mathrm{~m} / \mathrm{sec}^{2}$
$4 \quad$ Bay width $=5.0 \mathrm{~m}$
5 Story height $=3.2 \mathrm{~m}$
6 Cylinder strength of concrete $=17244 \mathrm{kN} / \mathrm{m}^{2}$
7 Column size $=.5 \mathrm{~m} \times .5 \mathrm{~m}$
$8 \quad$ Beam size $=.25 \mathrm{mx} .4 \mathrm{~m}$
9 Slab thickness $=.15 \mathrm{~m}$
10 Number of bays along the direction
of motion $=3,4,5$
11 Number of bays along the direction perpendicular to the direction of motion $=3,4,5$

## MODEL Dbm2 ( 6 \& 12 story, $3 \times 3,4 \times 4,5 \times 5$ bays structures)

1 Modulus of elasticity $=2.07 \times 10^{6} \mathrm{kN} / \mathrm{m}^{2}$
2 Density of concrete $=23.56 \mathrm{kN} / \mathrm{m}^{2}$
3 Acceleration due to gravity $=9.81 \mathrm{~m} / \mathrm{sec}^{2}$
$4 \quad$ Bay width $=5.0 \mathrm{~m}$
5 Story height $=3.2 \mathrm{~m}$
6 Cylinder strength of concrete $=17244 \mathrm{kN} / \mathrm{m}^{2}$
$7 \quad$ Column size $=.5 \mathrm{~m} \times .5 \mathrm{~m}$
$8 \quad$ Beam size $=.3 \mathrm{~m} \times .45 \mathrm{~m}$
9 Slab thickness $=.15 \mathrm{~m}$
10 Number of bays along the direction of motion $=3,4,5$
11 Number of bays along the direction perpendicular to the direction of motion $=3,4,5$

MODEL Dbm3 ( 6 \& 12 story, $3 \times 3,4 \times 4,5 \times 5$ bays structures)

1 Modulus of elasticity $=2.07 \times 10^{6} \mathrm{kN} / \mathrm{m}^{2}$
2 Density of concrete $=23.56 \mathrm{kN} / \mathrm{m}^{2}$
3 Acceleration due to gravity $=9.81 \mathrm{~m} / \mathrm{sec}^{2}$
$4 \quad$ Bay width $=5.0 \mathrm{~m}$
5 Story height $=3.2 \mathrm{~m}$
6 Cylinder strength of concrete $=17244 \mathrm{kN} / \mathrm{m}^{2}$
$7 \quad$ Column size $=.5 \mathrm{~m} \times .5 \mathrm{~m}$
$8 \quad$ Beam size $=.35 \mathrm{~m} \times .5 \mathrm{~m}$
$9 \quad$ Slab thickness $=.15 \mathrm{~m}$
10 Number of bays along the direction of motion $=3,4,5$
11 Number of bays along the direction perpendicular to the direction of motion $=3,4,5$

MODEL Dbm4 ( 6 \& 12 story, $3 \times 3,4 \times 4,5 \times 5$ bays structures)
1 Modulus of elasticity $=2.07 \times 10^{6} \mathrm{kN} / \mathrm{m}^{2}$
2 Density of concrete $=23.56 \mathrm{kN} / \mathrm{m}^{2}$
3 Acceleration due to gravity $=9.81 \mathrm{~m} / \mathrm{sec}^{2}$
$4 \quad$ Bay width $=5.0 \mathrm{~m}$
5 Story height $=3.2 \mathrm{~m}$
6 Cylinder strength of concrete $=17244 \mathrm{kN} / \mathrm{m}^{2}$
$7 \quad$ Column size $=.5 \mathrm{~m} \times .5 \mathrm{~m}$
$8 \quad$ Beam size $=.4 \mathrm{~m} \times .55 \mathrm{~m}$
9 Slab thickness $=.15 \mathrm{~m}$
10 Number of bays along the direction of motion $=3,4,5$
11 Number of bays along the direction perpendicular to the direction of motion $=3,4,5$

MODEL Dbm5 ( 6 \& 12 story, $3 \times 3,4 \times 4,5 \times 5$ bays structures)
1 Modulus of elasticity $=2.07 \times 10^{6} \mathrm{kN} / \mathrm{m}^{2}$
2 Density of concrete $=23.56 \mathrm{kN} / \mathrm{m}^{2}$
3 Acceleration due to gravity $=9.81 \mathrm{~m} / \mathrm{sec}^{2}$
$4 \quad$ Bay width $=5.0 \mathrm{~m}$
5 Story height $=3.2 \mathrm{~m}$
6 Cylinder strength of concrete $=17244 \mathrm{kN} / \mathrm{m}^{2}$
7 Column size $=.5 \mathrm{~m} \times .5 \mathrm{~m}$
$8 \quad$ Beam size $=.45 \mathrm{~m} \times .6 \mathrm{~m}$
$9 \quad$ Slab thickness $=.15 \mathrm{~m}$
10 Number of bays along the direction of motion $=3,4,5$
11 Number of bays along the direction perpendicular to the direction of motion $=3,4,5$

MODEL Dcs1 ( $6 \& 12$ story, $3 \times 3,4 \times 4,5 \times 5$ bays structures)
1 Modulus of elasticity $=2.07 \times 10^{6} \mathrm{kN} / \mathrm{m}^{2}$
2 Density of concrete $=23.56 \mathrm{kN} / \mathrm{m}^{2}$
3 Acceleration due to gravity $=9.81 \mathrm{~m} / \mathrm{sec}^{2}$
$4 \quad$ Bay width $=5.0 \mathrm{~m}$
5 Story height $=3.2 \mathrm{~m}$
6 Cylinder strength of concrete $=17244 \mathrm{kN} / \mathrm{m}^{2}$
7 Column size $=.5 \mathrm{~m} \times .5 \mathrm{~m}$
$8 \quad$ Beam size $=.35 \mathrm{~m} \times .5 \mathrm{~m}$
9 Slab thickness $=.15 \mathrm{~m}$
10 Number of bays along the direction of motion $=3,4,5$
11 Number of bays along the direction perpendicular to the direction of motion $=3,4,5$

MODEL Dcs2 ( 6 \& 12 story, $3 \times 3,4 \times 4,5 \times 5$ bays structures)

1 Modulus of elasticity $=2.07 \times 10^{6} \mathrm{kN} / \mathrm{m}^{2}$
2 Density of concrete $=23.56 \mathrm{kN} / \mathrm{m}^{2}$
3 Acceleration due to gravity $=9.81 \mathrm{~m} / \mathrm{sec}^{2}$
$4 \quad$ Bay width $=5.0 \mathrm{~m}$
5 Story height $=3.2 \mathrm{~m}$
6 Cylinder strength of concrete $=20692.8 \mathrm{kN} / \mathrm{m}^{2}$
7 Column size $=.5 \mathrm{~m} \times .5 \mathrm{~m}$
$8 \quad$ Beam size $=.35 \mathrm{~m} \times .5 \mathrm{~m}$
$9 \quad$ Slab thickness $=.15 \mathrm{~m}$
10 Number of bays along the direction of motion $=3,4,5$
11 Number of bays along the direction perpendicular to the direction of motion $=3,4,5$

MODEL Dcs3 ( 6 \& 12 story, $3 \times 3,4 \times 4,5 \times 5$ bays structures)

1 Modulus of elasticity $=2.07 \times 10^{6} \mathrm{kN} / \mathrm{m}^{2}$
2 Density of concrete $=23.56 \mathrm{kN} / \mathrm{m}^{2}$
3 Acceleration due to gravity $=9.81 \mathrm{~m} / \mathrm{sec}^{2}$
$4 \quad$ Bay width $=5.0 \mathrm{~m}$
5 Story height $=3.2 \mathrm{~m}$
6 Cylinder strength of concrete $=27590.4 \mathrm{kN} / \mathrm{m}^{2}$
$7 \quad$ Column size $=.5 \mathrm{~m} \times .5 \mathrm{~m}$
$8 \quad$ Beam size $=.35 \mathrm{~m} \times .5 \mathrm{~m}$
$9 \quad$ Slab thickness $=.15 \mathrm{~m}$
10 Number of bays along the direction of motion $=3,4,5$
11 Number of bays along the direction perpendicular to the direction of motion $=3,4,5$

MODEL Dcs 4 ( 6 \& 12 story, $3 \times 3,4 \times 4,5 \times 5$ bays structures)

1 Modulus of elasticity $=2.07 \times 10^{6} \mathrm{kN} / \mathrm{m}^{2}$
2 Density of concrete $=23.56 \mathrm{kN} / \mathrm{m}^{2}$
3 Acceleration due to gravity $=9.81 \mathrm{~m} / \mathrm{sec}^{2}$
$4 \quad$ Bay width $=5.0 \mathrm{~m}$
5 Story height $=3.2 \mathrm{~m}$
6 Cylinder strength of concrete $=34488 \mathrm{kN} / \mathrm{m}^{2}$
$7 \quad$ Column size $=.5 \mathrm{~m} \times .5 \mathrm{~m}$
$8 \quad$ Beam size $=.35 \mathrm{~m} \times .5 \mathrm{~m}$
$9 \quad$ Slab thickness $=.15 \mathrm{~m}$
10 Number of bays along the direction of motion $=3,4,5$
11 Number of bays along the direction perpendicular to the direction of motion $=3,4,5$

MODEL Dcs5 (6 \& 12 story, $3 \times 3,4 \times 4,5 \times 5$ bays structures)

1 Modulus of elasticity $=2.07 \times 10^{6} \mathrm{kN} / \mathrm{m}^{2}$
2 Density of concrete $=23.56 \mathrm{kN} / \mathrm{m}^{2}$
3 Acceleration due to gravity $=9.81 \mathrm{~m} / \mathrm{sec}^{2}$
$4 \quad$ Bay width $=5.0 \mathrm{~m}$
5 Story height $=3.2 \mathrm{~m}$
6 Cylinder strength of concrete $=41385.65 \mathrm{kN} / \mathrm{m}^{2}$
7 Column size $=.5 \mathrm{~m} \times .5 \mathrm{~m}$
$8 \quad$ Beam size $=.35 \mathrm{~m} \times .5 \mathrm{~m}$
9 Slab thickness $=.15 \mathrm{~m}$
10 Number of bays along the direction of motion $=3,4,5$
11 Number of bays along the direction perpendicular to the direction of motion $=3,4,5$

MODEL Dd1 ( $6 \& 12$ story, $3 \times 3,4 \times 4,5 \times 5$ bays structures)
1 Modulus of elasticity $=2.07 \times 10^{6} \mathrm{kN} / \mathrm{m}^{2}$
2 Density of concrete $=22.0 \mathrm{kN} / \mathrm{m}^{2}$
3 Acceleration due to gravity $=9.81 \mathrm{~m} / \mathrm{sec}^{2}$
$4 \quad$ Bay width $=5.0 \mathrm{~m}$
5 Story height $=3.2 \mathrm{~m}$
6 Cylinder strength of concrete $=41385.65 \mathrm{kN} / \mathrm{m}^{2}$
$7 \quad$ Column size $=.5 \mathrm{~m} \times .5 \mathrm{~m}$
$8 \quad$ Beam size $=.35 \mathrm{~m} \times .5 \mathrm{~m}$
9 Slab thickness $=.15 \mathrm{~m}$
10 Number of bays along the direction of motion $=3,4,5$
11 Number of bays along the direction perpendicular to the direction of motion. $=3,4,5$

## MODEL Dd2 ( $6 \& 12$ story, $3 \times 3,4 \times 4,5 \times 5$ bays structures)

1 Modulus of elasticity $=2.07 \times 10^{6} \mathrm{kN} / \mathrm{m}^{2}$
2 Density of concrete $=22.78 \mathrm{kN} / \mathrm{m}^{2}$
3 Acceleration due to gravity $=9.81 \mathrm{~m} / \mathrm{sec}^{2}$
$4 \quad$ Bay width $=5.0 \mathrm{~m}$
5 Story height $=3.2 \mathrm{~m}$
6 Cylinder strength of concrete $=41385.65 \mathrm{kN} / \mathrm{m}^{2}$
$7 \quad$ Column size $=.5 \mathrm{~m} \times .5 \mathrm{~m}$
$8 \quad$ Beam size $=.35 \mathrm{~m} \times .5 \mathrm{~m}$
$9 \quad$ Slab thickness $=.15 \mathrm{~m}$
10 Number of bays along the direction of motion $=3,4,5$
11 Number of bays along the direction perpendicular to the direction of motion $=3,4,5$

## MODEL Dd3 (6 \& 12 story, 3x3, 4x4, 5x5 bays structures)

1 Modulus of elasticity $=2.07 \times 10^{6} \mathrm{kN} / \mathrm{m}^{2}$
2 Density of concrete $=23.565 \mathrm{kN} / \mathrm{m}^{3}$
3 Acceleration due to gravity $=9.81 \mathrm{~m} / \mathrm{sec}^{2}$
$4 \quad$ Bay width $=5.0 \mathrm{~m}$
5 Story height $=3.2 \mathrm{~m}$
6 Cylinder strength of concrete $=41385.65 \mathrm{kN} / \mathrm{m}^{2}$
$7 \quad$ Column size $=.5 \mathrm{~m} \times .5 \mathrm{~m}$
$8 \quad$ Beam size $=.35 \mathrm{~m} \times .5 \mathrm{~m}$
9 Slab thickness $=.15 \mathrm{~m}$
10 Number of bays along the direction of motion $=3,4,5$
11 Number of bays along the direction perpendicular to the direction of motion $=3,4,5$

MODEL Dd4 ( 6 story \&12 story ,3x3,4x4,5x5 bays structures)
1 Modulus of elasticity $=2.07 \times 10^{6} \mathrm{kN} / \mathrm{m}^{2}$
2 Density of concrete $=24.3505 \mathrm{kN} / \mathrm{m}^{3}$
3 Acceleration due to gravity $=9.81 \mathrm{~m} / \mathrm{sec}^{2}$
$4 \quad$ Bay width $=5.0 \mathrm{~m}$
5 Story height $=3.2 \mathrm{~m}$
6 Cylinder strength of concrete $=41385.65 \mathrm{kN} / \mathrm{m}^{2}$
$7 \quad$ Column size $=.5 \mathrm{~m} \times .5 \mathrm{~m}$
$8 \quad$ Beam size $=.35 \mathrm{~m} \times .5 \mathrm{~m}$
$9 \quad$ Slab thickness $=.15 \mathrm{~m}$
10 Number of bays along the direction of motion $=3,4,5$
11 Number of bays along the direction perpendicular to the direction of motion $=3,4,5$

## MODEL Dd5 (6 story \& 12 story, $3 \times 3,4 \times 4,5 \times 5$ bays structures)

1 Modulus of elasticity $=2.07 \times 10^{6} \mathrm{kN} / \mathrm{m}^{2}$
2 Density of concrete $=25.136 \mathrm{kN} / \mathrm{m}^{3}$
3 Acceleration due to gravity $=9.81 \mathrm{~m} / \mathrm{sec}^{2}$
$4 \quad$ Bay width $=5.0 \mathrm{~m}$
5 Story height $=3.2 \mathrm{~m}$
6 Cylinder strength of concrete $=41385.65 \mathrm{kN} / \mathrm{m}^{2}$
$7 \quad$ Column size $=.5 \mathrm{mx} .5 \mathrm{~m}$
$8 \quad$ Beam size $=.35 \mathrm{~m} \times .5 \mathrm{~m}$
9 Slab thickness $=.15 \mathrm{~m}$
10 Number of bays along the direction of motion $=3,4,5$
11 Number of bays along the direction perpendicular to the direction of motion $=3,4,5$

MODEL E1 (4 story, 3x3, 4x4, 5x5 bays structures)

1 Modulus of elasticity $=2.07 \times 10^{6} \mathrm{kN} / \mathrm{m}^{2}$
2 Density of concrete $=23.56 \mathrm{kN} / \mathrm{m}^{3}$
3 Acceleration due to gravity $=9.81 \mathrm{~m} / \mathrm{sec}^{2}$
$4 \quad$ Bay width $=5.0 \mathrm{~m}$
5 Story height $=3.2 \mathrm{~m}$
6 Cylinder strength of concrete $=41385.65 \mathrm{kN} / \mathrm{m}^{2}$
$7 \quad$ Column size $=.5 \mathrm{~m} \times .5 \mathrm{~m}$
$8 \quad$ Beam size $=.35 \mathrm{~m} \times .5 \mathrm{~m}$
$9 \quad$ Slab thickness $=.15 \mathrm{~m}$
10 Number of bays along the direction of motion $=3,4,5$
11 Number of bays along the direction perpendicular to the direction of motion $=3,4,5$

1 Modulus of elasticity $=2.07 \times 10^{6} \mathrm{kN} / \mathrm{m}^{2}$
2 Density of concrete $=23.56 \mathrm{kN} / \mathrm{m}^{3}$
3 Acceleration due to gravity $=9.81 \mathrm{~m} / \mathrm{sec}^{2}$
$4 \quad$ Bay width $=5.0 \mathrm{~m}$
5 Story height $=3.2 \mathrm{~m}$
6 Cylinder strength of concrete $=41385.65 \mathrm{kN} / \mathrm{m}^{2}$
7 Column size $=.5 \mathrm{~m} \times .5 \mathrm{~m}$
$8 \quad$ Beam size $=.35 \mathrm{~m} \times .5 \mathrm{~m}$
$9 \quad$ Slab thickness $=.15 \mathrm{~m}$
10 Number of bays along the direction of motion $=3,4,5$
11 Number of bays along the direction perpendicular to the direction of motion $=3,4,5$

## MODEL E3 ( 8 story, $3 \times 3,4 \times 4,5 \times 5$ bays structures)

1 Modulus of elasticity $=2.07 \times 10^{6} \mathrm{kN} / \mathrm{m}^{2}$
2 Density of concrete $=23.56 \mathrm{kN} / \mathrm{m}^{3}$
3 Acceleration due to gravity $=9.81 \mathrm{~m} / \mathrm{sec}^{2}$
$4 \quad$ Bay width $=5.0 \mathrm{~m}$
5 Story height $=3.2 \mathrm{~m}$
6 Cylinder strength of concrete $=41385.65 \mathrm{kN} / \mathrm{m}^{2}$
$7 \quad$ Column size $=.5 \mathrm{~m} \times .5 \mathrm{~m}$
8 Beam size $=.35 \mathrm{~m} \times .5 \mathrm{~m}$
9 Slab thickness $=.15 \mathrm{~m}$
10 Number of bays along the direction of motion $=3,4,5$
11 Number of bays along the direction perpendicular to the direction of motion $=3,4,5$

MODEL E4 ( 10 story, $3 \times 3,4 \times 4,5 \times 5$ bays structures)

1 Modulus of elasticity $=2.07 \times 10^{6} \mathrm{kN} / \mathrm{m}^{2}$
2 Density of concrete $=23.56 \mathrm{kN} / \mathrm{m}^{3}$
3 Acceleration due to gravity $=9.81 \mathrm{~m} / \mathrm{sec}^{2}$
$4 \quad$ Bay width $=5.0 \mathrm{~m}$
5 Story height $=3.2 \mathrm{~m}$
6 Cylinder strength of concrete $=41385.65 \mathrm{kN} / \mathrm{m}^{2}$
7 Column size $=.5 \mathrm{~m} \times .5 \mathrm{~m}$
$8 \quad$ Beam size $=.35 \mathrm{~m} \times .5 \mathrm{~m}$
9 Slab thickness $=.15 \mathrm{~m}$
10 Number of bays along the direction of motion $=3,4,5$
11 Number of bays along the direction perpendicular to the direction of motion $=3,4,5$

## MODEL E5 ( 12 story, $3 \times 3,4 \times 4$, $5 \times 5$ bays structures)

1 Modulus of elasticity $=2.07 \mathrm{X} 10^{6} \mathrm{kN} / \mathrm{m}^{2}$
2 Density of concrete $=23.56 \mathrm{kN} / \mathrm{m}^{3}$
3 Acceleration due to gravity $=9.81 \mathrm{~m} / \mathrm{sec}^{2}$
$4 \quad$ Bay width $=5.0 \mathrm{~m}$
5 Story height $=3.2 \mathrm{~m}$
$6 \quad$ Cylinder strength of concrete $=41385.65 \mathrm{kN} / \mathrm{m}^{2}$
$7 \quad$ Column size $=.5 \mathrm{~m} \times .5 \mathrm{~m}$
$8 \quad$ Beam size $=.35 \mathrm{~m} \times .5 \mathrm{~m}$
9 Slab thickness $=.15 \mathrm{~m}$
10 Number of bays along the direction of motion $=3,4,5$
11 Number of bays along the direction perpendicular to the direction of motion $=3,4,5$

## MODEL E6 (14 story, 3x3, 4x4, 5x5 bays structures)

1 Modulus of elasticity $=2.07 \times 10^{6} \mathrm{kN} / \mathrm{m}^{2}$
2 Density of concrete $=23.56 \mathrm{kN} / \mathrm{m}^{3}$
3 Acceleration due to gravity $=9.81 \mathrm{~m} / \mathrm{sec}^{2}$
$4 \quad$ Bay width $=5.0 \mathrm{~m}$
5 Story height $=3.2 \mathrm{~m}$
6 Cylinder strength of concrete $=41385.65 \mathrm{kN} / \mathrm{m}^{2}$
$7 \quad$ Column size $=.5 \mathrm{~m} \times .5 \mathrm{~m}$
$8 \quad$ Beam size $=.35 \mathrm{~m} \times .5 \mathrm{~m}$
9 Slab thickness $=.15 \mathrm{~m}$
10 Number of bays along the direction
of motion $=3,4,5$
11 Number of bays along the direction perpendicular to the direction of motion $=3,4,5$

MODEL FF ( 6 \& 12 story, $2 \times 2,3 \times 3,4 \times 4,5 \times 5$ bays structures)

1 Modulus of elasticity $=2.07 \times 10^{6} \mathrm{kN} / \mathrm{m}^{2}$
2 Density of concrete $=23.56 \mathrm{kN} / \mathrm{m}^{3}$
3 Acceleration due to gravity $=9.81 \mathrm{~m} / \mathrm{sec}^{2}$
$4 \quad$ Bay width $=5.0 \mathrm{~m}$
5 Story height $=3.2 \mathrm{~m}$
6 Cylinder strength of concrete $=41385.65 \mathrm{kN} / \mathrm{m}^{2}$
7 Column size $=.5 \mathrm{mx} .5 \mathrm{~m}$
$8 \quad$ Beam size $=.35 \mathrm{~m} \times .5 \mathrm{~m}$
9 Slab thickness $=.15 \mathrm{~m}$

1 Modulus of elasticity $=2.07 \times 10^{6} \mathrm{kN} / \mathrm{m}^{2}$
2 Density of concrete $=23.56 \mathrm{kN} / \mathrm{m}^{3}$
3 Acceleration due to gravity $=9.81 \mathrm{~m} / \mathrm{sec}^{2}$
$4 \quad$ Bay width $=5.0 \mathrm{~m}$
5 Story height $=3.2 \mathrm{~m}$
$6 \quad$ Cylinder strength of concrete $=41385.65 \mathrm{kN} / \mathrm{m}^{2}$
$7 \quad$ Column size $=.5 \mathrm{~m} \times .5 \mathrm{~m}$
$8 \quad$ Beam size $=.35 \mathrm{mx} .5 \mathrm{~m}$
$9 \quad$ Slab thickness $=.15 \mathrm{~m}$

## MODEL FMS ( 6 \& 12 story, $2 \times 2,3 \times 3,4 \times 4,5 \times 5$ bays structures)

1 Modulus of elasticity $=2.07 \times 10^{6} \mathrm{kN} / \mathrm{m}^{2}$
2 Density of concrete $=23.56 \mathrm{kN} / \mathrm{m}^{3}$
3 Mass $=8.0299 \mathrm{kN}-\mathrm{sec}^{2} / \mathrm{m}$
3 Acceleration due to gravity $=9.81 \mathrm{~m} / \mathrm{sec}^{2}$
$4 \quad$ Bay width $=5.0 \mathrm{~m}$
5 Story height $=3.2 \mathrm{~m}$
6 Cylinder strength of concrete $=41385.65 \mathrm{kN} / \mathrm{m}^{2}$
7 Column size $=.5 \mathrm{~m} \times .5 \mathrm{~m}$
$8 \quad$ Beam size $=.35 \mathrm{~m} \times .5 \mathrm{~m}$
9 Slab thickness $=.15 \mathrm{~m}$

## OUTPUT FOR MODELS

| $\begin{aligned} & \mathrm{SL} / \\ & \mathrm{NO} \end{aligned}$ | TYPEOF MODEL | FREQUENCY | PERIOD | REMARKS |
| :---: | :---: | :---: | :---: | :---: |
| 1 | A1 | 1.4475 | . 690846 | for |
| 2 | A2 | 1.4535 | . 687994 | 6story |
| 3 | A3 | 1.4564 | . 6866245 | structures |
| 4 | A4 | 1.46 | . 6849315 |  |
| 5 | A1 | . 68581 | 1.458129 | for |
| 6 | A2 | .70136 | 1.425801 | 12 story |
| 7 | A3 | . 70909 | 1.410258 | structures |
| 8 | A4 | . 712 | 1.404494 |  |
| 9 | B1 | 1.4009 | . 7138268 | for |
| 10 | B2 | 1.4059 | . 7112881 | 6 story |
| 11 | B3 | 1.4083 | . 7100759 | structures |
| 12 | B4 | 1.4096 | . 7094211 |  |
| 13 | B1 | . 66474 | 1.504347 | for |
| 14 | B2 | . 67955 | 1.471562 | 12 story |
| 15 | B3 | .. 6869 | 1.455815 | structures |
| 16 | B4 | . 69127 | 1.446612 |  |
| 17 | C1 | 1.3754 | . 7270612 | for |
| 18 | C2 | 1.3799 | . 72469 | 6 story |
| 19 | C3 | 1.382 | . 723589 | structures |
| 20 | C4 | 1.3832 | . 722961 |  |
| 21 | C1 | . 65322 | 1.53087 | for |
| 22 | C2 | . 66763 | 1.497835 | 12 story |
| 23 | C3 | . 67479 | 1.48194 | structures |
| 24 | C4 | . 7904 | 1.472667 |  |


| 25 | Db for 5.0 m | 1.4059 | . 711288 | for |
| :---: | :---: | :---: | :---: | :---: |
| 26 | Db for 5.5 m | 1.2894 | . 77555 | $3 \times 3$ bays |
| 27 | Db for 6.0 m | 1.1883 | . 841538 | 6 story |
| 28 | Db for 6.5 m | 1.0999 | . 90917 | frames |
| 29 | Db for 7.0 m | 1.0222 | . 97828 |  |
| 30 | Db for 5.0 m | 1.382 | . 723589 | for |
| 31 | Db for 5.5 m | 1.2653 | . 790326 | $4 \times 4$ bays |
| 32 | Db for 6.0 m | 1.1644 | . 85881 | 6 story |
| 33 | Db for 6.5 m | 1.0766 | . 92885 | frames |
| 34 | Db for 7.0 m | . 99956 | 1.0 |  |
| 35 | Db for 5.0 m | 1.3665 | . 731796 | for |
| 36 | Db for 5.5 m | 1.2499 | . 8 | $5 \times 5$ bays |
| 37 | Db for 6.0 m | 1.1493 | . 870095 | 6 story |
| 38 | Db for 6.5 m | 1.0619 | . 941708 | frames |
| 39 | Db for 7.0 m | . 98549 | 1.0147 |  |
| 40 | Db for 5.0 m | . 67955 | 1.471562 | for |
| 41 | Db for 5.5 m | . 62649 | 1.59619 | $3 \times 3$ bays |
| 42 | Db for 6.0 m | . 57963 | 1.72523 | 12 story |
| 43 | Db for 6.5 m | . 53821 | 1.85801 | frames |
| 44 | Db for 7.0 m | . 50148 | 1.9941 |  |
| 45 | Db for 5.0 m | 1.4059 | . 711288 | for |
| 46 | Db for 5.5 m | 1.2894 | . 77555 | $4 \times 4$ bays |
| 47 | Db for 6.0 m | 1.1883 | . 841538 | 12 story |
| 48 | Db for 6.5 m | 1.0999 | . 90917 | frames |
| 49 | Db for 7.0 m | 1.0222 | . 97828 |  |
| 50 | Db for 5.0 m | 1.382 | . 723589 | for |
| 51 | Db for 5.5 m | 1.2653 | . 790326 | 5 x 5 bays |
| 52 | Db for 6.0 m | 1.1644 | . 85881 | 12 story |
| 53 | Db for 6.5 m | 1.0766 | . 92885 | frames |


| 55 | Dh for 3.0 m | 1.5269 | . 65492 | for |
| :---: | :---: | :---: | :---: | :---: |
| 56 | Dh for 3.25 m | 1.3782 | . 72558 | $3 \times 3$ bays |
| 57 | Dh for 3.5 m | 1.2522 | . 798594 | 6 story |
| 58 | Dh for 3.75 m | 1.442 | . 87397 | frames |
| 59 | Dh for 4.00 m | 1.0509 | . 95156 |  |
| 60 | Dh for 3.0 m | 1.5004 | . 666488 | for |
| 61 | Dh for 3.25 m | 1.3548 | . 738116 | $4 \times 4$ bays |
| 62 | Dh for 3.5 m | 1.2314 | . 81208 | 6 story |
| 63 | Dh for 3.75 m | 1.1256 | . 88841 | frames |
| 64 | Dh for 4.00 m | 1.034 | . 96711 |  |
| 65 | Dh for 3.0 m | 1.4834 | . 67412 | for |
| 66 | Dh for 3.25m | 1.3397 | . 7464 | $5 \times 5$ bays |
| 67 | Dh for 3.5 m | 1.2179 | . 821085 | 6 story |
| 68 | Dh for 3.75 m | 1.1135 | . 89806 | frames |
| 69 | Dh for 4.00 m | 1.0231 | . 97742 |  |
| 70 | Dh for 3.0 m | . 73825 | 1.35455 | for |
| 71 | Dh for 3.25 m | . 66609 | 1.5012 | $3 \times 3$ bays |
| 72 | Dh for 3.5 m | . 60496 | 1.653 | 12 story |
| 73 | Dh for 3.75 m | . 55259 | 1.80965 | frames |
| 74 | Dh for 4.00 m | . 50731 | 1.97118 |  |
| 75 | Dh for 3.0 m | . 73272 | 1.364778 | for |
| 76 | Dh for 3.25 m | . 66149 | 1.5117 | 4 x 4 bays |
| 77 | Dh for 3.5 m | . 60112 | 1.66356 | 12 story |
| 78 | Dh for 3.75 m | . 54938 | 1.820233 | frames |
| 79 | Dh for 4.00 m | . 50461 | 1.98173 |  |
| 80 | Dh for 3.0 m | . 73180 | 1.36649 | for |
| 81 | Dh for 3.25 m | . 66087 | 1.5132 | $5 \times 5$ bays |
| 82 | Dh for 3.5 m | . 60073 | 1.6646 | 12 story |
| 83 | Dh for 3.75 m | . 54918 | 1.82089 | frames |


| 85 | Dc for .4 mx .4 m | 1.1637 | . 85932 | for |
| :---: | :---: | :---: | :---: | :---: |
| 86 | Dc for $.45 \mathrm{~m} \times .45 \mathrm{~m}$ | 1.3 | . 76923 | $3 \times 3$ bays |
| 87 | Dc for $.5 \mathrm{~m} \times .5 \mathrm{~m}$ | 1.4059 | . 711288 | 6 story |
| 88 | Dc for $.55 \mathrm{~m} \times .5 \mathrm{~m}$ | 1.4864 | . 672766 | frames |
| 89 | Dc for .4 mx .4 m | 1.1339 | . 8819 | for |
| 90 | Dc for .45 mx .45 m | 1.2729 | . 7856 | $4 \times 4$ bays |
| 91 | Dc for $.5 \mathrm{~m} \times .5 \mathrm{~m}$ | 1.382 | . 723589 | 6 story |
| 92 | Dc for $.55 \mathrm{~m} \times .5 \mathrm{~m}$ | 1.4657 | . 682267 | frames |
| 93 | Dc for .4 mx .4 m | 1.1152 | . 8967 | for |
| 94 | Dc for $.45 \mathrm{~m} \times .45 \mathrm{~m}$ | 1.2556 | . 79643 | 5 x 5 bays |
| 95 | Dc for $.5 \mathrm{~m} \times .5 \mathrm{~m}$ | 1.3665 | . 731797 | 6 story |
| 96 | Dc for .55 mx .5 m | 1.4521 | . 68865 | frames |
| 97 | Dc for .4 mx .4 m | . 57237 | 1.74712 | for |
| 98 | Dc for .45 mx .45 m | . 63387 | 1.57761 | 3x3 bays |
| 99 | Dc for .5 mx .5 m | . 67955 | 1.47156 | 12 story |
| 100 | Dc for $.55 \mathrm{~m} \times .5 \mathrm{~m}$ | . 71192 | 1.4046 | frames |
| 101 | Dc for .4 mx .4 m | . 56318 | 1.77563 | for |
| 102 | Dc for $.45 \mathrm{~m} \times .45 \mathrm{~m}$ | . 62691 | 1.5951 | 4 x 4 bays |
| 103 | Dc for .5 mx .5 m | . 67479 | 1.48194 | 12 story |
| 104 | Dc for .55 mx .5 m | . 7091 | 1.4102 | frames |
| 105 | Dc for .4 mx .4 m | . 56006 | 1.78553 | for |
| 106 | Dc for .45 mx .45 m | . 62505 | 1.59987 | $5 \times 5$ bays |
| 107 | Dc for .5 mx .5 m | . 67411 | 1.48343 | 12 story |
| 108 | Dc for .55 mx .5 m | . 70939 | 1.40966 | frames |


| 109 | Dbm for .25 mx .4 m | 1.2161 | . 8223 | for |
| :---: | :---: | :---: | :---: | :---: |
| 110 | Dbm for .3 mx .45 m | 1.3127 | . 761788 | $3 \times 3$ bays |
| 111 | Dbm for $.35 \mathrm{~m} \times .5 \mathrm{~m}$ | 1.4059 | . 71128 | 6 story |
| 112 | Dbm for $.4 \mathrm{~m} \times .55 \mathrm{~m}$ | 1.4852 | . 67331 | frames |
| 113 | Dbm for $.45 \mathrm{~m} \times .6 \mathrm{~m}$ | 1.5445 | . 647458 |  |
| 114 | Dbm for $.25 \mathrm{~m} \times .4 \mathrm{~m}$ | 1.201 | . 832639 | for |
| 115 | Dbm for .3 mx .45 m | 1.2929 | . 77345 | $4 \times 4$ bays |
| 116 | Dbm for $.35 \mathrm{~m} \times .5 \mathrm{~m}$ | 1.382 | . 723589 | 6 story |
| 117 | Dbm for .4 mx .55 m | 1.4576 | . 686059 | frames |
| 118 | Dbm for .45 mx .6 m | 1.5139 | . 660545 |  |
| 119 | Dbm for .25 mx .4 m | 1.1912 | . 839489 | for |
| 120 | Dbm for .3 mx .45 m | 1.2802 | . 781127 | $5 \times 5$ bays |
| 121 | Dbm for .35 mx .5 m | 1.3665 | . 731796 | 6 story |
| 122 | Dbm for .4 mx .55 m | 1.4399 | . 69449 | frames |
| 123 | Dbm for .45 mx .6 m | 1.4944 | . 6691648 |  |
| 124 | Dbm for $.25 \mathrm{~m} \times .4 \mathrm{~m}$ | . 58211 | 1.71788 | for |
| 125 | Dbm for .3 mx .45 m | . 63211 | 1.582 | $3 \times 3$ bays |
| 126 | Dbm for .35 mx .5 m | . 67955 | 1.47156 | 12 story |
| 127 | Dbm for $.4 \mathrm{~m} \times .55 \mathrm{~m}$ | . 71916 | 1.3905 | frames |
| 128 | Dbm for .45 mx .6 m | . 74818 | 1.3365 |  |
| 129 | Dbm for $.25 \mathrm{~m} \times .4 \mathrm{~m}$ | . 5799 | 1.72443 | for |
| 130 | Dbm for .3 mx .45 m | . 62838 | 1.59139 | $4 \times 4$ bays |
| 131 | Dbm for .35 mx .5 m | . 67479 | 1.48194 | 12 story |
| 132 | Dbm for .4 mx .55 m | . 71385 | 1.4008 | frames |
| 133 | Dbm for .45 mx .6 m | . 7427 | 1.34643 |  |
| 134 | Dbm for .25 mx .4 m | . 5799 | 1.72443 | for |
| 135 | Dbm for .3 mx .45 m | . 62838 | 1.59139 | 5 x 5 bays |
| 136 | Dbm for .35 mx .5 m | . 67479 | 1.48194 | 12 story |
| 137 | Dbm for .4 mx .55 m | . 71385 | 1.4008 | frames |


| 139 | Dcs for $17244 \mathrm{kN} / \mathrm{m}^{2}$ | 1.413 | . 70771 | for |
| :---: | :---: | :---: | :---: | :---: |
| 140 | Dcs for $20692.8 \mathrm{kN} / \mathrm{m}^{2}$ | 1.4788 | . 676223 | $3 \times 3$ bays |
| 141 | Dcs for $27590.4 \mathrm{kN} / \mathrm{m}^{2}$ | 1.5892 | . 629247 | 6 story |
| 142 | Dcs for $34488 \mathrm{kN} / \mathrm{m}^{2}$ | 1.6803 | . 59513 | frames |
| 143 | Dcs for $41385.6 \mathrm{kN} / \mathrm{m}^{2}$ | 1.7587 | . 5686 |  |
| 144 | Dcs for $17244 \mathrm{kN} / \mathrm{m}^{2}$ | 1.389 | . 71994 | for |
| 145 | Dcs for $20692.8 \mathrm{kN} / \mathrm{m}^{2}$ | 1.4537 | . 687899 | $4 \times 4$ bays |
| 146 | Dcs for $27590.4 \mathrm{kN} / \mathrm{m}^{2}$ | 1.5622 | . 640123 | 6 story |
| 147 | Dcs for $34488 \mathrm{kN} / \mathrm{m}^{2}$ | 1.6517 | . 605436 | frames |
| 148 | Dcs for $41385.6 \mathrm{kN} / \mathrm{m}^{2}$ | 1.7287 | . 578469 |  |
| 149 | Dcs for $17244 \mathrm{kN} / \mathrm{m}^{2}$ | 1.3735 | . 728066 | for |
| 150 | Dcs for $20692.8 \mathrm{kN} / \mathrm{m}^{2}$ | 1.4374 | . 6957 | 5x5 bays |
| 151 | Dcs for $27590.4 \mathrm{kN} / \mathrm{m}^{2}$ | 1.5447 | . 647374 | 6 story |
| 152 | Dcs for $34488 \mathrm{kN} / \mathrm{m}^{2}$ | 1.6333 | . 61225 | frames |
| 153 | Dcs for $41385.6 \mathrm{kN} / \mathrm{m}^{2}$ | 1.7094 | . 585 |  |
| 154 | Dcs for $17244 \mathrm{kN} / \mathrm{m}^{2}$ | . 68299 | 1.46415 | for |
| 155 | Dcs for $20692.8 \mathrm{kN} / \mathrm{m}^{2}$ | . 71480 | 1.39899 | $3 \times 3$ bays |
| 156 | Dcs for $27590.4 \mathrm{kN} / \mathrm{m}^{2}$ | . 76816 | 1.3018 | 12 story |
| 157 | Dcs for $34488 \mathrm{kN} / \mathrm{m}^{2}$ | . 81218 | 1.231254 | frames |
| 158 | Dcs for $41385.6 \mathrm{kN} / \mathrm{m}^{2}$ | . 85005 | 1.1764 |  |
| 159 | Dcs for $17244 \mathrm{kN} / \mathrm{m}^{2}$ | . 6782 | 1.4745 | for |
| 160 | Dcs for $20692.8 \mathrm{kN} / \mathrm{m}^{2}$ | . 70979 | 1.408867 | $4 \times 4$ bays |
| 161 | Dcs for $27590.4 \mathrm{kN} / \mathrm{m}^{2}$ | . 76277 | 1.31101 | 12 story |
| 162 | Dcs for $34488 \mathrm{kN} / \mathrm{m}^{2}$ | . 80649 | 1.23994 | frames |
| 163 | Dcs for $41385.6 \mathrm{kN} / \mathrm{m}^{2}$ | . 84409 | 1.1847 |  |
| 164 | Dcs for $17244 \mathrm{kN} / \mathrm{m}^{2}$ | . 67752 | 1.47597 | for |
| 165 | Dcs for $20692.8 \mathrm{kN} / \mathrm{m}^{2}$ | . 70907 | 1.41029 | $5 \times 5$ bays |
| 166 | Dcs for $27590.4 \mathrm{kN} / \mathrm{m}^{2}$ | . 762 | 1.3123 | 12 story |


| 167 | Dcs for $34488 \mathrm{kN} / \mathrm{m}^{2}$ | . 86568 | 1.241187 | frames |
| :---: | :---: | :---: | :---: | :---: |
| 168 | Dcs for $41385.6 \mathrm{kN} / \mathrm{m}^{2}$ | . 84324 | 1.1859 |  |
| 169 | Dd for $22 \mathrm{kN} / \mathrm{m}^{3}$ | . 70371 | 1.421 | for |
| 170 | Dd for $22.78 \mathrm{kN} / \mathrm{m}^{3}$ | . 69117 | 1.4468 | $3 \times 3$ bays |
| 171 | Dd for $23.565 \mathrm{kN} / \mathrm{m}^{3}$ | . 67955 | 1,4716 | 6 story |
| 172 | Dd for $24.3505 \mathrm{kN} / \mathrm{m}^{3}$ | . 6685 | 1.49588 | frames |
| 173 | Dd for $25.136 \mathrm{kN} / \mathrm{m}^{3}$ | . 65826 | 1.5192 |  |
| 174 | Dd for $22 \mathrm{kN} / \mathrm{m}^{3}$ | . 69878 | 1.4311 | for |
| 175 | Dd for $22.78 \mathrm{kN} / \mathrm{m}^{3}$ | . 68633 | 1.457 | $4 \times 4$ bays |
| 176 | Dd for $23.565 \mathrm{kN} / \mathrm{m}^{3}$ | . 67479 | 1.4819 | 6 story |
| 177 | Dd for $24.3505 \mathrm{kN} / \mathrm{m}^{3}$ | . 66381 | 1.5065 | frames |
| 178 | Dd for $25.136 \mathrm{kN} / \mathrm{m}^{3}$ | . 65364 | 1.53 |  |
| 179 | Dd for $22 \mathrm{kN} / \mathrm{m}^{3}$ | . 69807 | 1.43252 | for |
| 180 | Dd for $22.78 \mathrm{kN} / \mathrm{m}^{3}$ | . 68564 | 1.45849 | $5 \times 5$ bays |
| 181 | Dd for $23.565 \mathrm{kN} / \mathrm{m}^{3}$ | . 67411 | 1.48343 | 6 story |
| 182 | Dd for $24.3505 \mathrm{kN} / \mathrm{m}^{3}$ | . 66314 | 1.5079 | frames |
| 183 | Dd for $25.136 \mathrm{kN} / \mathrm{m}^{3}$ | . 65299 | 1.5314 |  |
| 184 | Dd for $22 \mathrm{kN} / \mathrm{m}^{3}$ | 1.4559 | . 6868 | for |
| 185 | Dd for $22.78 \mathrm{kN} / \mathrm{m}^{3}$ | 1.43 | . 6993 | $3 \times 3$ bays |
| 186 | Dd for $23.565 \mathrm{kN} / \mathrm{m}^{3}$ | 1.4059 | . 71128 | 12 story |
| 187 | Dd for $24.3505 \mathrm{kN} / \mathrm{m}^{3}$ | 1.3831 | . 723 | frames |
| 188 | Dd for $25.136 \mathrm{kN} / \mathrm{m}^{3}$ | 1.3619 | . 73426 |  |
| 189 | Dd for $22 \mathrm{kN} / \mathrm{m}^{3}$ | 1.4311 | . 69876 | for |
| 190 | Dd for $22.77 \mathrm{kN} / \mathrm{m}^{3}$ | 1.4056 | . 71143 | 4 x 4 bays |
| 191 | Dd for $23.565 \mathrm{kN} / \mathrm{m}^{3}$ | 1.382 | . 72358 | 12 story |
| 192 | Dd for $24.3505 \mathrm{kN} / \mathrm{m}^{3}$ | 1.3595 | . 73556 | frames |
| 193 | Dd for $25.136 \mathrm{kN} / \mathrm{m}^{3}$ | 1.3387 | . 74699 |  |
| 194 | Dd for $22 \mathrm{kN} / \mathrm{m}^{3}$ | 1.4151 | . 70666 | for |
| 195 | Dd for $22.78 \mathrm{kN} / \mathrm{m}^{3}$ | 1.3899 | . 71947 | $5 \times 5$ bays |
| 196 | Dd for $23.565 \mathrm{kN} / \mathrm{m}^{3}$ | 1.3665 | . 73179 | 12 story |
| 197 | Dd for $24.3505 \mathrm{kN} / \mathrm{m}^{3}$ | 1.3445 | . 74388 | frames |
| 198 | Dd for $25.136 \mathrm{kN} / \mathrm{m}^{3}$ | 1.3237 | . 75545 |  |


| 199 | E for 6 story | 1.4059 | . 711288 | All models |
| :---: | :---: | :---: | :---: | :---: |
| 200 | E for 8 story | 1.043 | . 95877 | are composed of |
| 201 | E for 10 story | . 82518 | 1.211856 | $3 \times 3$ bays |
| 202 | E for 12 story | . 67955 | 1.47156 | frame structures |
| 203 | E for 14 story | . 57509 | 1.738858 |  |
| 204 | E for 6 story | 1.3820 | . 72358 | All models |
| 205 | E for 8 story | 1.0289 | . 971911 | are composed of |
| 206 | E for 10 story | . 82969 | 1.205269 | $4 \times 4$ bays |
| 207 | E for 12 story | . 67479 | 1.4819 | frame structures |
| 208 | E for 14 story | . 58494 | 1.7389 |  |
| 209 | E for 6 story | 1.3665 | . 73179 | All models |
| 210 | E for 8 story | 1.0196 | . 98077 | are composed of |
| 211 | E for 10 story | . 81085 | 1.23327 | $5 \times 5$ bays |
| 212 | E for 12 story | . 67411 | 1.4834 | frame structures |
| 213 | E for 14 story | . 575043 | 1.739 |  |
| 214 | FF for $2 \times 2$ bays | 1.8739 | . 53364 | All models |
| 215 | FF for $3 \times 3$ bays | 1.8835 | . 5309 | are 6 storied |
| 216 | FF for $4 \times 4$ bays | 1.881 | . 52963 | without floor slabs |
| 217 | FF for $5 \times 5$ bays | 1.8909 | . 528848 | in each story |
| 218 | FF for 2x2 bays | . 87987 | 1.13653 | All models |
| 219 | FF for $3 \times 3$ bays | . 89977 | 1.1114 | are 12 storied |
| 220 | FF for $4 \times 4$ bays | . 90979 | 1.1 | without floor slabs |
| 221 | FF for $5 \times 5$ bays | . 91582 | 1.092 | in each story |
| 222 | FS for $2 \times 2$ bays | 1.4475 | . 69084 | All models |
| 223 | FS for $3 \times 3$ bays | 1.4059 | . 711288 | are 6 storied |
| 224 | FS for $4 \times 4$ bays | 1.382 | . 723589 | with floor slabs |
| 225 | FS for 5 x 5 bays | 1.3665 | . 73179 | in each story |
| 226 | FS for $2 \times 2$ bays | . 68581 | 1.4581 | All models |
| 227 | FS for $3 \times 3$ bays | . 67955 | 1.4716 | are 12 storied |


| 228 | FS for $4 \times 4$ bays | .67479 | 1.482 | with floor slabs |
| :--- | :--- | :--- | :--- | :--- |
| 229 | FS for $5 \times 5$ bays | .67411 | 1.48343 | in each story |
| 230 | Fms for $2 \times 2$ bays | 1.0365 | .96478 | All models |
| 231 | Fms for $3 \times 3$ bays | 1.0453 | .95666 | are 6 storied |
| 232 | Fms for $4 \times 4$ bays | 1.0484 | .95383 |  |
| 233 | Fms for $5 \times 5$ bays | 1.0496 | .952743 | masses in each stor |
|  |  |  |  |  |
| 234 | Fms for $2 \times 2$ bays | .49268 | 2.0297 | All models |
| 235 | Fms for $3 \times 3$ bays | .50605 | 1.9761 | are 12 storied |
| 236 | Fms for $4 \times 4$ bays | .51224 | 1.9522 |  |
| 237 | Fms for $5 \times 5$ bays | .51648 | 1.9361 | masses in each stor |

## ANSYS INPUT FILE AND OUTPUT FILE FOR 5X5 BAYS 6 STORY FRAME STRUCTURE WITHOUT FLOOR SLABS IN EACH STORY

/BATCH
/PREP7
/TITLE,F666
KAN,2
KAY,2,5
ET,1,4
ET,2,4
R,1,.175,.0036458,.00178645,.35,. 5
R,2,.25,.005208,.005208,.5,. 5
MP,EX, 1,20700000
DENS,1,2.4021
MP,NUXY,1,. 15
$\mathrm{N}, 1$
$\mathrm{N}, 6,0,0,25$
FILL, 1,6
NGEN, 7,6,1,6,1,0,3.2,0
NGEN,6,42,1,42,1,5,0,0
REAL,2
E,1,7
EGEN,5,1,73
EGEN,6,6,67,77,1
REAL, 2
E,85,91
EGEN,6,1,133
REAL, 1
E,91,92
EGEN,5,1,139
EGEN,6,6,133,143,1
REAL,2
E,127,133
EGEN,6,1,199
REAL, 1
E,133,134
EGEN,5,1,205
EGEN,6,6,199,209,1
REAL,2
E,169,175
EGEN,6,1,265
REAL,1
E,175,176
EGEN,5,1,271
EGEN,6,6,265,275,1
REAL,2
E,211,217
EGEN,6,1,331
REAL, 1
E,217,218
EGEN,5,1,337
EGEN,6,6,331,341,1
E,7,49
EGEN,5,42,397
EGEN,6,6,397,401,1
E,8,50
EGEN,5,42,427
EGEN,6,6,427,431,1
E,9,51
EGEN,5,42,457

EGEN,6,6,457,461,1
E,10,52
EGEN,5,42,487
EGEN,6,6,487,491,1
E,11,53
EGEN,5,42,517
EGEN,6,6,517,521,1
E, 12,54
EGEN,5,42,547
EGEN,6,6,547,551,1
/VIEW,1,1,1,1
/EDGE,1,1
/ESHAPE, 1
ENUM,1
NNUM, 1
EPLOT
FINISH
/SOLU
D,1,ALL,0,,6,1
D,43,ALL, $0,48,1$
D,85,ALL,0,,90,1
D,127,ALL, $0,132,1$
D,169,ALL,0,,174,1
D,211,ALL,0,,216,1
M,7,UX,42,1
M,49,UX,84,1
M,91,UX,126,1
M,133,UX,168,1
M,175,UX,210,1
M,217,UX,252,1
TOTAL, 10
LUMPM,ON
SAVE
SOLVE
FINISH
/POST1
NUMBER,1
EPLOT
SET,1,1
PLDISP,1
/wait,10
SET,1,2
PLDISP,1
/WAIT, 10
SET,1,3
PLDISP, 1
/WAIT, 10
SET,1,4
PLDISP, 1
/WAIT, 10
SET,1,5
PLDISP, 1
/WAIT, 10
FINISH
/EOF
/ansys52/bin/ansysunv.e52


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1 -*


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[^0][^1]GENERATE 6 TOTAL SETS OF ELEMENTS WITH NODE INCREMENT OF

## REAL CONSTANT NUMBER= 2

$\begin{array}{lllll}\text { ELEMENT } & 265 & 169 & 175 & 0\end{array}$
GENERATE 6 TOTAL SETS OF ELEMENTS WITH NODE INCREMENT OF SET IS SELECTED ELEMENTS IN RANGE 265 TO 265 IN STEPS OF 1 MAXIMUM ELEMENT NUMBER $=270$

REAL CONSTANT NUMBER $=1$
ELEMENT $271 \begin{array}{llll}271 & 175 & 176 & 0\end{array}$
GENERATE 5 TOTAL SETS OF ELEMENTS WITH NODE INCREMENT OF 1 SET IS SELECTED ELEMENTS IN RANGE 271 TO 271 IN STEPS OF 1 MAXIMUM ELEMENT NUMBER $=275$

GENERATE 6 TOTAL SETS OF ELEMENTS WITH NODE INCREMENT OF SET IS SELECTED ELEMENTS IN RANGE 265 TO 275 IN STEPS OF 1 MAXIMUM ELEMENT NUMBER $=330$

REAL CONSTANT NUMBER= 2

ELEMENT $\begin{array}{lllll}331 & 211 & 217 & 0\end{array}$
GENERATE 6 TOTAL SETS OF ELEMENTS WITH NODE INCREMENT OF 1 SET IS SELECTED ELEMENTS IN RANGE 331 TO 331 IN STEPS OF 1 MAXIMUM ELEMENT NUMBER $=336$

REAL CONSTANT NUMBER $=1$
ELEMENT $\begin{array}{lllll}337 & 217 & 218 & 0\end{array}$
GENERATE 5 TOTAL SETS OF ELEMENTS WITH NODE INCREMENT OF 1 SET IS SELECTED ELEMENTS IN RANGE 337 TO 337 IN STEPS OF 1 MAXIMUM ELEMENT NUMBER $=341$

GENERATE 6 TOTAL SETS OF ELEMENTS WITH NODE INCREMENT OF SET IS SELECTED ELEMENTS IN RANGE 331 TO 341 IN STEPS OF 1
MAXIMUM ELEMENT NUMBER $=396$ MAXIMUM ELEMENT NUMBER $=396$
$\begin{array}{lllll}\text { ELEMENT } & 397 & 7 & 49 & 0\end{array}$
GENERATE 5 TOTAL SETS OF ELEMENTS WITH NODE INCREMENT OF 42 SET IS SELECTED ELEMENTS IN RANGE 397 TO 397 IN STEPS OF 1 MAXIMUM ELEMENT NUMBER $=401$

GENERATE 6 TOTAL SETS OF ELEMENTS WITH NODE INCREMENT OF 6 SET IS SELECTED ELEMENTS IN RANGE 397 TO 401 IN STEPS OF 1 MAXIMUM ELEMENT NUMBER $=426$

ELEMENT $427 \quad 8 \quad 50$
GENERATE 5 TOTAL SETS OF ELEMENTS WITH NODE INCREMENT OF
SET IS SELECTED ELEMENTS IN RANGE 427 TO 427 IN STEPS OF
MAXIMUM ELEMENT NUMBER $=431$
GENERATE 6 TOTAL SETS OF ELEMENTS WITH NODE INCREMENT OF SET IS SELECTED ELEMENTS IN RANGE 427 TO 431 IN STEPS OF 1

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```
                            ANSYS COMMAND LINE ARGUMENTS
INITIAL JOBNAME = file
MEMORY REQUESTED (MB) = 64.0
BATCH MODE REQUESTED = NOLIST
START-UP FILE MODE = READ
DATABASE SIZE REQUESTED (MB) = 16
*** NOTE *** CP= 1.040 TIME= 14:37:20
There are no parameters and no abbreviations defined.
```

```
00011-52 VERSION=RS6000 REVISION =5.2
```

00011-52 VERSION=RS6000 REVISION =5.2
FOR SUPPORT CALL SUPPORT PERSON PHONE SUPPORT PHONE FAX SUPPORT FAX
FOR SUPPORT CALL SUPPORT PERSON PHONE SUPPORT PHONE FAX SUPPORT FAX
CURRENT JOBNAME=file 14:37:20 OCT 13,1996 CP= 1.060

```
CURRENT JOBNAME=file 14:37:20 OCT 13,1996 CP= 1.060
```

RUN SETUP PROCEDURE FROM FILE=/ansys52/docu/start.ans
/INPUT FILE=/ansys52/docu/start.ans LINE $=0$
1
***** ANSYS -ENGINEERING ANALYSIS SYSTEM REVISION 5.2 ***** 00011-52 VERSION=RS6000 14:37:21 OCT 13, 1996 CP= 1.570 FOR SUPPORT CALL SUPPORT PERSON PHONE SUPPORT PHONE FAX SUPPORT FAX

[^2]ANALYSIS TYPE $=2$ (MODE-FREQUENCY ANALYSIS)
EXPAND FIRST 5 MODE SHAPES (MODEXP= 5 )


```
        CURRENT NODAL DOF SET IS UX UY UZ ROTX ROTY ROTZ
        THREE-DIMENSIONAL MODEL
    ELEMENT TYPE 2 IS BEAM4 3-D ELASTIC BEAM
    KEYOPT(1-12)= 00000 0000}000000000
    CURRENT NODAL DOF SET IS UX UY UZ ROTX ROTY ROTZ
    THREE-DIMENSIONAL MODEL
    ELEMENT TYPE 
    CURRENT NODAL DOF SET IS UX UY UZ ROTX ROTY ROTZ
    THREE-DIMENSIONAL MODEL
    REAL CONSTANT SET }1\mathrm{ ITEMS 1 TO 6
    .17500 .36458E-02 .17864E-02 .35000 .50000 .00000E+00
    REAL CONSTANT SET 2 ITEMS 1 TO 6
    .25000 .52080\textrm{E}-02 .52080\textrm{E}-02 .50000 .50000 .00000\textrm{E}+00
    REAL CONSTANT SET 3 ITEMS 1 TO 6
    .15000 .00000E+00 .00000\textrm{E}+00}..00000\textrm{E}+00 .00000\textrm{E}+00\quad.00000\textrm{E}+0
MATERIAL 1 EX = .2070000E+08
MATERIAL 1 DENS = 2.402100
MATERIAL 1 NUXY = .1500000
ELEMENT TYPE 3 IS SHELL63 ELASTIC SHELL
KEYOPT(1-12)= 20000000 0)00000
CURRENT NODAL DOF SET IS UX UY UZ ROTX ROTY ROTZ
THREE-DIMENSIONAL MODEL
NODE 1 KCS = 0 X,Y,Z= .00000E+00 .00000E+00 .00000E+00
NODE 6 KCS = 0 X,Y,Z= .00000E+00 .00000E+00 25.000
FILL 4 POINTS BETWEEN NODE 1 AND NODE }
START WITH NODE 2 AND INCREMENT BY 1
GENERATE 7 TOTAL SETS OF NODES WITH INCREMENT 6 SET IS SELECTED NODES IN RANGE \(\quad 1\) TO 6 IN STEPS OF \(\quad 1\) GEOMETRY INCREMENTS ARE \(.00000 \mathrm{E}+00 \quad 3.2000 \quad .00000 \mathrm{E}+00 \mathrm{SPACING}=1.0000\)
GENERATE 6 TOTAL SETS OF NODES WITH INCREMENT 42 SET IS SELECTED NODES IN RANGE 1 TO 42 IN STEPS OF 1 GEOMETRY INCREMENTS ARE \(5.0000 \quad .00000 \mathrm{E}+00.00000 \mathrm{E}+00 \mathrm{SPACING}=1.0000\)
REAL CONSTANT NUMBER=
ELEMENT \(1 \begin{array}{llll} & 1 & 7 & 0\end{array}\)
GENERATE 6 TOTAL SETS OF ELEMENTS WITH NODE INCREMENT OF 1 SET IS SELECTED ELEMENTS IN RANGE 1 TO 1 IN STEPS OF 1 MAXIMUM ELEMENT NUMBER \(=6\)
REAL CONSTANT NUMBER \(=1\)
ELEMENT \(\begin{array}{lllll}7 & 7 & 8 & 0\end{array}\)
```


# GENERATE 5 TOTAL SETS OF ELEMENTS WITH NODE INCREMENT OF SET IS SELECTED ELEMENTS IN RANGE 7 TO 7 IN STEPS OF 1 MAXIMUM ELEMENT NUMBER= 11 

# GENERATE 6 TOTAL SETS OF ELEMENTS WITH NODE INCREMENT OF 6 SET IS SELECTED ELEMENTS IN RANGE 1 TO 11 IN STEPS OF MAXIMUM ELEMENT NUMBER $=66$ 

## REAL CONSTANT NUMBER= <br> 2

ELEMENT $67 \quad 43 \quad 49 \quad 0$
GENERATE 6 TOTAL SETS OF ELEMENTS WITH NODE INCREMENT OF 1 SET IS SELECTED ELEMENTS IN RANGE 67 TO 67 IN STEPS OF 1 MAXIMUM ELEMENT NUMBER= 72

REAL CONSTANT NUMBER $=1$
$\begin{array}{lllll}\text { ELEMENT } & 73 & 49 & 50 & 0\end{array}$
GENERATE 5 TOTAL SETS OF ELEMENTS WITH NODE INCREMENT OF 1 SET IS SELECTED ELEMENTS IN RANGE 73 TO 73 IN STEPS OF 1 MAXIMUM ELEMENT NUMBER= 77

> GENERATE 6 TOTAL SETS OF ELEMENTS WITH NODE INCREMENT OF SET IS SELECTED ELEMENTS IN RANGE 67 TO 77 IN STEPS OF MAXIMUM ELEMENT NUMBER $=132$

REAL CONSTANT NUMBER $=2$
ELEMENT $133 \quad 85 \quad 91 \quad 0$

> GENERATE 6 TOTAL SETS OF ELEMENTS WITH NODE INCREMENT OF SET IS SELECTED ELEMENTS IN RANGE 133 TO 133 IN STEPS OF 1 MAXIMUM ELEMENT NUMBER $=138$

REAL CONSTANT NUMBER $=1$
ELEMENT $139 \quad 91 \quad 92 \quad 0$
GENERATE 5 TOTAL SETS OF ELEMENTS WITH NODE INCREMENT OF SET IS SELECTED ELEMENTS IN RANGE 139 TO 139 IN STEPS OF 1 MAXIMUM ELEMENT NUMBER $=143$

GENERATE 6 TOTAL SETS OF ELEMENTS WITH NODE INCREMENT OF SET IS SELECTED ELEMENTS IN RANGE 133 TO 143 IN STEPS OF 1 MAXIMUM ELEMENT NUMBER $=198$

REAL CONSTANT NUMBER $=2$
ELEMENT $199 \quad 127 \quad 133 \quad 0$
GENERATE 6 TOTAL SETS OF ELEMENTS WITH NODE INCREMENT OF 1 SET IS SELECTED ELEMENTS IN RANGE 199 TO 199 IN STEPS OF 1 MAXIMUM ELEMENT NUMBER= 204

REAL CONSTANT NUMBER $=1$
ELEMENT $205133134 \quad 0$
GENERATE 5 TOTAL SETS OF ELEMENTS WITH NODE INCREMENT OF 1 SET IS SELECTED ELEMENTS IN RANGE 205 TO 205 IN STEPS OF 1 MAXIMUM ELEMENT NUMBER= 209

# GENERATE 5 TOTAL SETS OF ELEMENTS WITH NODE INCREMENT OFSET IS SELECTED ELEMENTS IN RANGE 457 TO 457 IN STEPS OF 1MAXIMUM ELEMENT NUMBER $=461$ 

GENERATE 6 TOTAL SETS OF ELEMENTS WITH NODE INCREMENT OF SET IS SELECTED ELEMENTS IN RANGE 457 TO 461 IN STEPS OF 1 MAXIMUM ELEMENT NUMBER= 486

ELEMENT $487 \quad 10 \quad 52 \quad 0$
GENERATE 5 TOTAL SETS OF ELEMENTS WITH NODE INCREMENT OF SET IS SELECTED ELEMENTS IN RANGE 487 TO 487 IN STEPS OF 1 MAXIMUM ELEMENT NUMBER= 491

GENERATE 6 TOTAL SETS OF ELEMENTS WITH NODE INCREMENT OF SET IS SELECTED ELEMENTS IN RANGE 487 TO 491 IN STEPS OF 1 MAXIMUM ELEMENT NUMBER= 516

ELEMENT 517 11 530
GENERATE 5 TOTAL SETS OF ELEMENTS WITH NODE INCREMENT OF 42 SET IS SELECTED ELEMENTS IN RANGE 517 TO 517 IN STEPS OF 1 MAXIMUM ELEMENT NUMBER $=521$

GENERATE 6 TOTAL SETS OF ELEMENTS WITH NODE INCREMENT OF SET IS SELECTED ELEMENTS IN RANGE 517 TO 521 IN STEPS OF 1 MAXIMUM ELEMENT NUMBER= 546

ELEMENT $547 \quad 12 \quad 54 \quad 0$
GENERATE 5 TOTAL SETS OF ELEMENTS WITH NODE INCREMENT OF 42 SET IS SELECTED ELEMENTS IN RANGE 547 TO 547 IN STEPS OF 1 MAXIMUM ELEMENT NUMBER= 551

GENERATE 6 TOTAL SETS OF ELEMENTS WITH NODE INCREMENT OF 6 SET IS SELECTED ELEMENTS IN RANGE 547 TO 551 IN STEPS OF 1 MAXIMUM ELEMENT NUMBER $=576$
$\begin{array}{llll}\text { view point for window } & 1 & 1.0000 & 1.0000 \\ 1.0000\end{array}$
EDGE KEY FOR WINDOW 1 IS 1 WITH TOLERANCE ANGLE OF 45.0 DEGREES. ELEMENT DISPLAYS USING REAL CONSTANT DATA WITH FACTOR 1.00 ELEMENT NUMBER KEY=1

NODE NUMBER KEY=1

[^3]
## ***** ANSYS SOLUTION ROUTINE

```
        SPECIFIED CONSTRAINT UX FOR SELECTED NODES 43 TO 48 BY 1
        REAL}=0.000000000\textrm{E}+00 IMAG=0.000000000\textrm{E}+0
        ADDITIONAL DOFS= UY UZ ROTX ROTY ROTZ
        SPECIFIED CONSTRAINT UX FOR SELECTED NODES 85 TO 90 BY I
        REAL}=0.000000000\textrm{E}+00 IMAG=0.000000000\textrm{E}+0
        ADDITIONAL DOFS= UY UZ ROTX ROTY ROTZ
        SPECIFIED CONSTRAINT UX FOR SELECTED NODES 127 TO 132 BY 1
        REAL}=0.000000000E+00 IMAG=0.000000000E+0
        ADDITIONAL DOFS= UY UZ ROTX ROTY ROTZ
        SPECIFIED CONSTRAINT UX FOR SELECTED NODES 169 TO 174 BY 1
        REAL}=0.000000000\textrm{E}+00 IMAG=0.000000000\textrm{E}+0
        ADDITIONAL DOFS= UY UZ ROTX ROTY ROTZ
        SPECIFIED CONSTRAINT UX FOR SELECTED NODES 211 TO 216 BY 1
        REAL=0.000000000E +00 IMAG=0.000000000E+00
        ADDITIONAL DOFS= UY UZ ROTX ROTY ROTZ
        MASTER DOF UX FOR SELECTED NODES IN RANGE 7TO 42 IN STEPS OF 1
ADDITIONAL DOFS=
NUMBER OF MASTER DOF = 36
MASTER DOF UX FOR SELECTED NODES IN RANGE 49 TO 84 IN STEPS OF I
ADDITIONAL DOFS=
NUMBER OF MASTER DOF= }7
MASTER DOF UX FOR SELECTED NODES IN RANGE 91 TO 126 IN STEPS OF 1
ADDITIONAL DOFS=
NUMBER OF MASTER DOF= }10
MASTER DOF UX FOR SELECTED NODES IN RANGE 133 TO 168 IN STEPS OF 1
ADDITIONAL DOFS=
NUMBER OF MASTER DOF= }14
MASTER DOF UX FOR SELECTED NODES IN RANGE 175 TO 210 IN STEPS OF 1
ADDITIONAL DOFS=
NUMBER OF MASTER DOF = }18
MASTER DOF UX FOR SELECTED NODES IN RANGE 217 TO 252 IN STEPS OF 1
ADDITIONAL DOFS=
NUMBER OF MASTER DOF= }21
TOTAL MASTER D.O.F.= }1
USE LUMPED MASS MATRIX APPROXIMATION
ALL CURRENT ANSYS DATA WRITTEN TO FILE NAME= file.db
FOR POSSIBLE RESUME FROM THIS POINT
***** ANSYS SOLVE COMMAND *****
1
***** ANSYS - ENGINEERING ANALYSIS SYSTEM REVISION 5.2 *****
00011-52 VERSION=RS6000 14:37:26 OCT 13,1996 CP= 4.370
FOR SUPPORT CALL SUPPORT PERSON PHONE SUPPORT PHONE FAX SUPPORT FAX
F666
** ANSYS VERSION FOR UNIVERSITY TEACHING PURPOSES ONLY **
SOLUTION OPTIONS
```

PROBLEM DIMENSIONALITY. . . . . . . . . . . . 3-D

```
DEGREES OF FREEDOM. . . . . UX UY UZ ROTX ROTY ROTZ
ANALYSIS TYPE . . . . . . . . . . . . . . . MODAL
    EXTRACTION METHOD . . . . . . . . . . . . RED
LUMPED MASS MATRICES. . . . . . . . . . . . .ON
NUMBER OF MODES TO EXTRACT . . . . . . . . . .ALL MDOF
NUMBER OF REDUCED MODES TO PRINT. . . . . . . . 0
NUMBER OF MASTER DOF. . . . . . . . . . . . . . . 216
NUMBER OF MODES TO EXPAND . . . . . . . . . . . 5
ELEMENT RESULTS CALCULATION . . . . . . . . . OFF
```

LOAD STEP OPTIONS
LOAD STEP NUMBER. 1
PRINT OUTPUT CONTROLS NO PRINTOUT
DATABASE OUTPUT CONTROLS. . . . . . . . . . . . ALL DATA WRITTEN
***** CENTROID, MASS, AND MASS MOMENTS OF INERTIA
CALCULATIONS ASSUME ELEMENT MASS AT ELEMENT CENTROID
TOTAL MASS $=1171.7$
$\begin{array}{lcc} & \text { MOM. OF INERTIA } & \text { MOM. OF INERTIA } \\ \text { CENTROID } & \text { ABOUT ORIGIN } & \text { ABOUT CENTROID }\end{array}$

```
XC=12.500 IXX =.4280E+06 IXX = .1125E+06
YC=10.633 IYY = .5197E+06 IYY = .1535E+06
ZC=12.500 IZZ =.4280E+06 IZZ = .1125E+06
IXY= -.1557E+06 IXY = -.9030E-09
IYZ = -.1557E+06 IYZ = -.9502E-09
IZX= -.1831E+06 IZX = -.2752E-08
```


## *** MASS SUMMARY BY ELEMENT TYPE ***

TYPE MASS
$1 \quad 1171.74$
Range of element maximum matrix coefficients in global coordinates
Maximum $=1617187.5$ at element 34.
Minimum=724500 at element 397 .
*** ELEMENT MATRIX FORMULATION TIMES
TYPE NUMBER ENAME TOTAL CP AVE CP
1576 BEAM4 $3.210 \quad .006$
Time at end of element matrix formulation $\mathrm{CP}=9.9$.
Estimated number of active DOF $=1296$.
Maximum wavefront $=327$.
Number of Master DOF $=216$.
Time at end of matrix triangularization $\mathrm{CP}=23$,
Equation solver maximum pivot $=1752818.38$ at node 136 UY.
Equation solver minimum pivot= 83677.941 at node 37 ROTY.

MODE FREQUENCY (CYCLES/TIME)

| 1 | 1.89085822 |
| :--- | ---: |
| 2 | 2.19200504 |
| 3 | 3.69862780 |
| 4 | 4.84532886 |
| 5 | 5.88161417 |
| 6.29448826 |  |
| 6.65807752 |  |
| 7.01577388 |  |
| 7.75472347 |  |
| 7.92774534 |  |
| 8.97149070 |  |
| 9.81841653 |  |
| 10.4450425 |  |
| 10.8949625 |  |
| 11.5574802 |  |
| 12.2705526 |  |
| 12.8981308 |  |
| 13.3295935 |  |
| 15.5761046 |  |
| 16.0023790 |  |
| 16.6367197 |  |
| 17.3313403 |  |
| 17.9674056 |  |
| 18.0489677 |  |
| 20.7805292 |  |
| 21.1301417 |  |
| 21.6461572 |  |
| 22.1801971 |  |
| 23.4024682 |  |
| 23.4081209 |  |
| 24.8380416 |  |
| 25.1045471 |  |
| 25.4989224 |  |
| 25.8937560 |  |
| 27.7167408 |  |
| 27.7178246 |  |
| 30.2424717 |  |
| 30.3597303 |  |
| 30.7341580 |  |
| 31.0451580 |  |
| 31.0884229 |  |
| 31.2488289 |  |
| 31.4236270 |  |
| 31.6307351 |  |
| 32.0124321 |  |
| 32.6503298 |  |
| 32.8779742 |  |
| 33.2307299 |  |
| 33.6116758 |  |
| 33.7918057 |  |
| 34.4921729 |  |
| 34.5107956 |  |
| 34.7741622 |  |
| 34.7926911 |  |
| 35.1562024 |  |
| 35.5443768 |  |
| 36.4578653 |  |
| 36.6942458 |  |
| 36.8390243 |  |
| 37.0849484 |  |


| 61 | 37.4407743 |
| :---: | :---: |
| 62 | 37.7860914 |
| 63 | 38.8282432 |
| 64 | 39.0575574 |
| 65 | 39.3098725 |
| 66 | 39.5003032 |
| 67 | 39.7797456 |
| 68 | 40.0513559 |
| 69. | 41.3317445 |
| 70 | 41.4234427 |
| 71 | 43.8964752 |
| 72 | 43.9054417 |
| 73 | 57.4977494 |
| 74 | 57.5291428 |
| 75 | 57.7613061 |
| 76 | 57.9234491 |
| 77 | 57.9832916 |
| 78 | 58.0910017 |
| 79 | 58.2884185 |
| 80 | 58.4859961 |
| 81 | 59.0257408 |
| 82 | 59.1561117 |
| 83 | 59.3682902 |
| 84 | 59.5709327 |
| 85 | 60.6931301 |
| 86 | 60.8185650 |
| 87 | 61.0279798 |
| 88 | 61.2063119 |
| 89 | 61.4144087 |
| 90 | 62.4811409 |
| 91 | 62.5998982 |
| 92 | 62.7716466 |
| 93 | 62.9427763 |
| 94 | 63.9234410 |
| 95 | 63.9869459 |
| 96 | 64.1103681 |
| 97 | 64.2197629 |
| 98 | 64.3901763 |
| 99 | 64.4897137 |
| 100 | 64.6372300 |
| 101 | 65.5537155 |
| 102 | 65.6724172 |
| 103 | 67.4766998 |
| 104 | 67.5529898 |
| 105 | 69.4980453 |
| 106 | 69.5461626 |
| 107 | 72.0398253 |
| 108 | 72.8611952 |
| 109 | 80.1083251 |
| 110 | 80.1407225 |
| 111 | 80.2923195 |
| 112 | 80.4158061 |
| 113 | 80.4373272 |
| 114 | 80.5150352 |
| 115 | 80.6559055 |
| 116 | 80.8001269 |
| 117 | 81.1667609 |
| 118 | 81.2668145 |
| 119 | 81.4216461 |
| 120 | 81.5729633 |
| 121 | 82.3977687 |
| 122 | 82.4997858 |
| 123 | 82.6501118 |



| 187 | 107.936301 |
| :--- | :--- |
| 188 | 108.012757 |
| 189 | 108.123614 |
| 190 | 108.230588 |
| 191 | 108.692905 |
| 192 | 108.704473 |
| 193 | 108.922325 |
| 194 | 108.985143 |
| 195 | 109.078229 |
| 196 | 109.168944 |
| 197 | 109.920265 |
| 198 | 109.923853 |
| 199 | 116.203768 |
| 200 | 116.272221 |
| 201 | 116.386317 |
| 202 | 116.510613 |
| 203 | 116.949729 |
| 204 | 117.029565 |
| 205 | 117.251899 |
| 206 | 117.271388 |
| 207 | 117.835455 |
| 208 | 117.841903 |
| 209 | 118.818602 |
| 210 | 118.820682 |
| 211 | 119.372198 |
| 212 | 119.620490 |
| 213 | 119.898022 |
| 214 | 119.898637 |
| 215 | 130.542021 |
| 216 | 130.578608 |

```
*** PROBLEM STATISTICS
ACTUAL NO. OF ACTIVE DEGREES OF FREEDOM \(=1080\)
R.M.S. WAVEFRONT SIZE \(=260.3\)
NUMBER OF MASTER DEGREES OF FREEDOM \(=216\)
```

*** ANSYS BINARY FILE STATISTICS
BUFFER SIZE USED= 16384
. 625 MB WRITTEN ON ELEMENT MATRIX FILE: file.emat . 375 MB WRITTEN ON ELEMENT SAVED DATA FILE: file.esav 2.188 MB WRITTEN ON TRIANGULARIZED MATRIX FILE: file.tri 1.125 MB WRITTEN ON MODAL MATRIX FILE: file.mode . 250 MB WRITTEN ON RESULTS FILE: file.rst

FINISH SOLUTION PROCESSING

```
***** ROUTINE COMPLETED ***** CP = 37.830
```

1
***** ANSYS -ENGINEERING ANALYSIS SYSTEM REVISION 5.2 ***** 00011-52 VERSION=RS6000 14:38:13 OCT 13, 1996 CP= 37.850 FOR SUPPORT CALL SUPPORT PERSON PHONE SUPPORT PHONE FAX SUPPORT FAX F666

[^4]NUMBER KEY SET TO $1-1=$ NONE $0=$ BOTH $1=$ COLOR $2=$ NUMBER

USE LOAD STEP 1 SUBSTEP 1 FOR LOAD CASE 0
SET COMMAND GOT LOAD STEP $=1$ SUBSTEP $=1$ CUMULATIVE ITERATION $=1$ TIME/FREQUENCY $=1.8909$
TITLE $=\mathrm{F} 666$
TASK SUSPENDED FOR 10.000 SECONDS

USE LOAD STEP 1 SUBSTEP 2 FOR LOAD CASE 0
SET COMMAND GOT LOAD STEP $=1$ SUBSTEP $=2$ CUMULATIVE ITERATION $=2$ TTME/FREQUENCY= 2.1920
TITLE $=$ F666
TASK SUSPENDED FOR 10.000 SECONDS

USE LOAD STEP 1 SUBSTEP 3 FOR LOAD CASE 0
SET COMMAND GOT LOAD STEP $=1$ SUBSTEP $=3$ CUMULATIVE ITERATION $=3$ TIME/FREQUENCY= 3.6986
TTTLE= F666
TASK SUSPENDED FOR 10.000 SECONDS

USE LOAD STEP 1 SUBSTEP 4 FOR LOAD CASE 0
SET COMMAND GOT LOAD STEP $=1$ SUBSTEP $=4$ CUMULATIVE ITERATION $=4$ TIME/FREQUENCY $=4.8453$
TITLE=F666
TASK SUSPENDED FOR 10.000 SECONDS

USE LOAD STEP 1 SUBSTEP 5 FOR LOAD CASE 0
SET COMMAND GOT LOAD STEP $=1$ SUBSTEP $=5$ CUMULATIVE ITERATION $=5$ TIME/FREQUENCY $=5.8816$
TTILE $=\mathrm{F} 666$
TASK SUSPENDED FOR 10.000 SECONDS
EXIT THE ANSYS POST1 DATABASE PROCESSOR
***** ROUTINE COMPLETED ***** $\mathrm{CP}=38.690$
***** END OF INPUT ENCOUNTERED *****
PURGE ALL SOLUTION AND POST DATA
SAVE ALL MODEL DATA
ALL CURRENT ANSYS DATA WRITTEN TO FILE NAME= file.db FOR POSSIBLE RESUME FROM THIS POINT

NUMBER OF WARNING MESSAGES ENCOUNTERED=



[^0]:    Examples, solutions, theories, results, methods, and references to other documents are provided in the Help System and User's Manual for guidance and explanation only. THEY ARE NOT GUARANTEED TO PROVIDE THE USER WITH THE MOST ACCURATE, USEFUL, OR CORRECT ANALYSIS FOR ANY PARTICULAR PROBLEM.

[^1]:    The Program, Help System, and Documentation are protected by United States copyright law. You may print pages from the Help

[^2]:    ** ANSYS VERSION FOR UNIVERSITY TEACHING PURPOSES ONLY ** ***** ANSYS ANALYSIS DEFINTTION (PREP7) *****

    F666

[^3]:    ***** ROUTINE COMPLETED ${ }^{* * * * *} \mathrm{CP}=$ 2.710

[^4]:    ** ANSYS VERSION FOR UNIVERSITY TEACHING PURPOSES ONLY **
    ANSYS RESULTS INTERPRETATION (POST1)

