IMPROVED DESIGN PROCEDURE FOR STAIR SLABS

A Thesis

by

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I do hereby declare that neither this thesis nor any part there of has been submitted or is being concurrently submitted in candidature for any degree at any other university.

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SYNOPSIS

The design of stair slabs, based on different codes and practices, has been very much conservative, Stair slabs, due to its folded shape and support restraints, are capable of taking far greater loads than it is usually thought to be. Based on this awareness, a recent study has been carried out in this department using finite element method of analysis and an improved design procedure has been indicated. The author, in the present study, has further investigated the actual behaviour of and stress conditions in various types of stair slabs, in order to substantiate and further improve the indidated design procedure.

Theoretical investigations have been carried out, involving computer analysis by general thick shell finite element program, on two flight and two flight open well stair slabs with different boundary conditions by using suitable finite element subdivision. Variations of deflections, axial forces and moments along different sections have been presented. Equilibrium of different segments of stair slabs and accuracy of results have been checked for the calculated stresses.

Results of these investigations indicates that stair slab does not act like a simple oneway slab. Appreciable

amount of axial forces occur near the junction of landing and waist slab. These forces are less pronounced in the open well stair slab. Distance between the points of contraflexure (effective simply supported span) lies within a range of 65% to 67% of horizontal distance of the waist slab. Effectively this reduces the magnitude of positive bending moment to less than 25% of that obtained from following the British Code. Most of the load in the landing slab, both in two flight and two flgiht open well stair slabs, are carried in the longitudinal direction of the flight and load carried in the transverse direction are negligible.

An improved design procedure has been suggested for two flight and two flight-open well stair slabs. It has been shown that cost of stair slab construction can be reduced by 40% to 60% depending on whether one follows the British Code or American practice.

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CHAPTER 1

INTRODUCTION

1.1 General

The modern trend in designing office buildings, apartment buildings and other structures are to save the construction cost by optimum utilization of material. This can be done only by proper investigation into the behaviour of the structures. Though stair-case is an important and essential part of a building, proper attention was not given by the researchers until very recently[10,1]. The leading code of practices does not provide definite guidelines for the design of stair slabs. In a recent research Irtishad Uddin Ahmad[1] suggested a design procedure with the limited findings corresponding to one specific boundary condition. Some points are, however, left unanswered in Ahmad's suggestions. For further insight and closer look, more investigations are required. The design procedure for two flight stair-case with different boundary conditions and two flight stair-case with open well remains uninvestigated. Further investigations on the behaviour of stair slabs are therefore required with a view to formulating definite guidelines and developing proper code provisions regarding design of stair slabs.

1.2 Background of the Research

Stair-case, due to its typical support configuration, has a significant amount of restraining effect while carrying the applied load. This fact should be taken into consideration in developing a rational and economic design procedure. Out of the two leading code of practices, British[12] and ACI[14] only the former recognizes this fact to some extent. According to British Code the effective span of a stair slab can safely be taken as a distance equal to the 'going' of the waist slab plus at each end either half the width of the landing or 1 meter whichever is smaller. The ACI code mentions nothing about the restraining effect of stair slab owing to its shape and support condition. Consequently, the followers of this code usually design a stair slab as a simple one way slab with a span equal to the horizontal distance between the supports. Reduction in the effective span as suggested by the British code results into a considerable saving in the design. Findings of a recent study by Saquib and Ahmad[10] on stair slab carried out in the Department of Civil Engineering,, of this University, indicated that the stair slab does not respond in the same manner as do the simple slab. The study further indicated that the effective span of stair slab may even be less than that prescribed by British code. It has been found that a typical stair slab of same thickness designed in accordance with the British code requires nearly half of the reinforcement required by the conventional American design practice. A nearly full size model of stair slab, designed according to British code, was tested and it sustained a load which was 33 percent in excess of the load corresponding to the ultimate moment capacity without showing any sign of failure. This indicated that even though the British code of practice is liberal than American design practice yet there was scope for making further saving in the design of stair slab. Although it is recognised fact that landing slab, owing to their typical support configuration, has some stiffening effect on the stair slab, no definite finding is available to ascertain its extent. Distinctive features of a stair slab and a simple slab should therefore be taken into consideration for suitable and economic design. With these objectives Ahmad(1) investigated numerically and experimentally the actual behaviour of stair slab in order to determine the extent of restraining effect of landing slabs. This resulted into a suggestion for improved design procedure for two flight stair indicating a substantial savings of cost of materials (35% to 53%). However, Ahmad observed that an axial force of considerable magnitude persisted near the kink zone, requiring some additional reinforcement. Also no research has yet been carried out on the behaviour of other stair slabs especially with open well. Therefore, there was enough scope for further study into the behaviour of stair slabs.

1.3 Scope of the Research

As a part of the literature survey, a brief outline and comparison of the conventional methods of design of stair slabs is presented. In this context design of stair slab understudy by different methods are presented as comparative case studies to give a clear idea about the usual design practices of stair slab.

With the advent of modern digital computer, finite element method emerged as a powerful tool doing real wonders in solving highly complex problems in structural engineering and other fields with enormous speed and accuracy. The author used this modern sophosticated technique to solve the problems. Numerical investigations using thick shell finite element program(3) were carried out on two flight stair-case with or without open well and having various support conditions. Particular attention was given on the axial force near the kink zone found by Ahmad(1), effect of boundary condition and open well stair-case.

1.4 Objective of Research

The objective of the study was

i) To investigate the behaviour of stair slabs with various support conditions, using thick shell finite elements(2).

ii) To carryout analysis with finer mesh subdivision at boundaries and at kink zone to eleminate undesirable effects.

iii) To investigate the extent of axial force in the stair slabs.

iv) To check the equilibrium of segments of the stair slabs.

v) To investigate the behaviour of stair slabs with open well.

vi) To develop rational design procedure i.e. to formulate definite guidelines regarding design of stair slabs.

CHAPTER 2

LITERATURE SURVEY

2.1 Terminology of Stair-case

The commonly used technical terms[4,6,8,9,11] of stair-case construction and design are shown in Fig. 2.1 and briefly defined below:

i) Steps: A portion of stairway comprising the tread and riser which permits ascent or descent from one floor to another.

ii) Tread: The horizontal upper part of a step on which foot is placed in ascending or descending a stairway.

iii) Riser: The vertical portion of a step providing a support to the tread.

iv) Flight: A series of steps without any platform, break or landing in their direction.

v) Landing: A platform or resting place provided between two flights. A landing extending right across stair-case is termed as half space landing and the one extending only half across a stair-case is called a quarter-space landing.

vi) Pitch: The angle of inclination of the stair with the floor is known as a pitch.

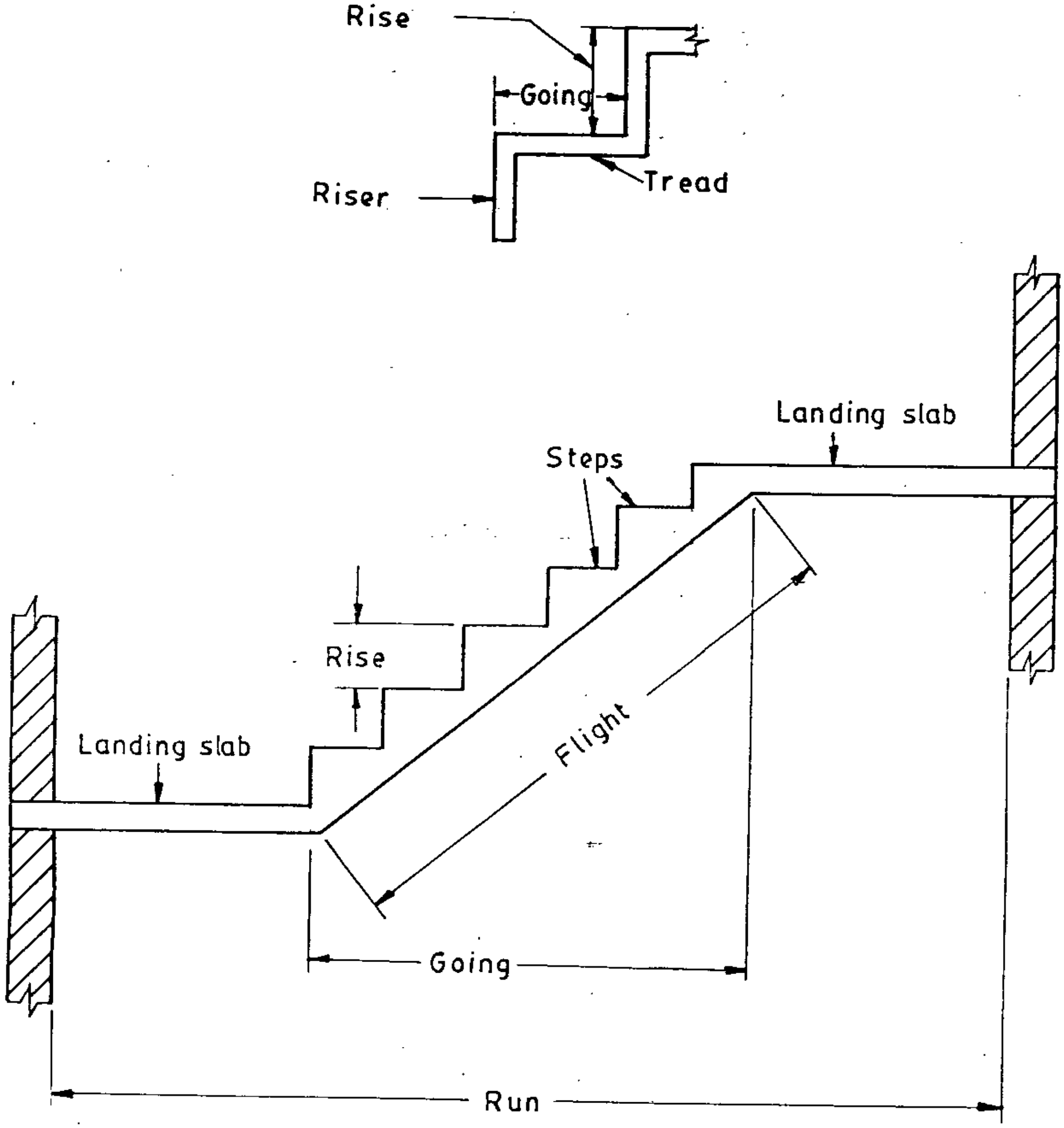


Fig. 2.1 Components of stair slab.

vii) Rise: This is the vertical distance between two successive treads.

viii) Run: The total length of a stair in a horizontal plane is known as a run and it includes the length of landing also.

ix) Waist: The thickness of structural slab in case of a R.C.C. stair is known as a waist.

x) Stringer beam: Beam running in the direction of flight to support the steps in a stair.

2.2 Classification of Stairs

The defferent form of stairs(4,6,13) may be classified under the following main heads:

i) Straight stairs: These are the stairs along which there is no change in direction on any flight between two successive floors (Fig. 2.2a).

ii) Dog-legged stairs: These consist of two straight flights of step with abrupt turn between them. Usually a level landing is placed across the two flights at the change of direction (Fig. 2.2b).

iii) Open-newel or open-well stairs: These consist of two or more straight-flights changed in such a manner that a clear space, called a 'well' occurs between the backward and forward flights (Fig.2.2c).

iv) Geometrical stairs: These are similar to the open-newel stair with the difference that the open well between the forward and the backward flight are curved (Fig. 2.2d).

v) Circular stairs: These stairs when viewed from above appear to follow with a single centre of curvature and large radius (Fig. 2.2e).

vi) Bifurcated stairs: These stairs are so arranged that there is a wide flight at the start which is subdivided into narrow flights at the mid-landing. The two narrow flights start from either side of the mid-landing (Fig. 2.2f).

2.3 Review of Codes and Practices

2.3.1 American design practice

ACI code provides no guideline for designing stair slab. A tendency is apparent among followers of this code to design stairslabs as a simple one way slab, without regard to its support condition.

The simplest form of reinforced concrete stairway consists of an inclined slab supported at the ends upon beams, with steps formed upon its upper surface. Such a stair slab is usually designed as a simple slab with a span

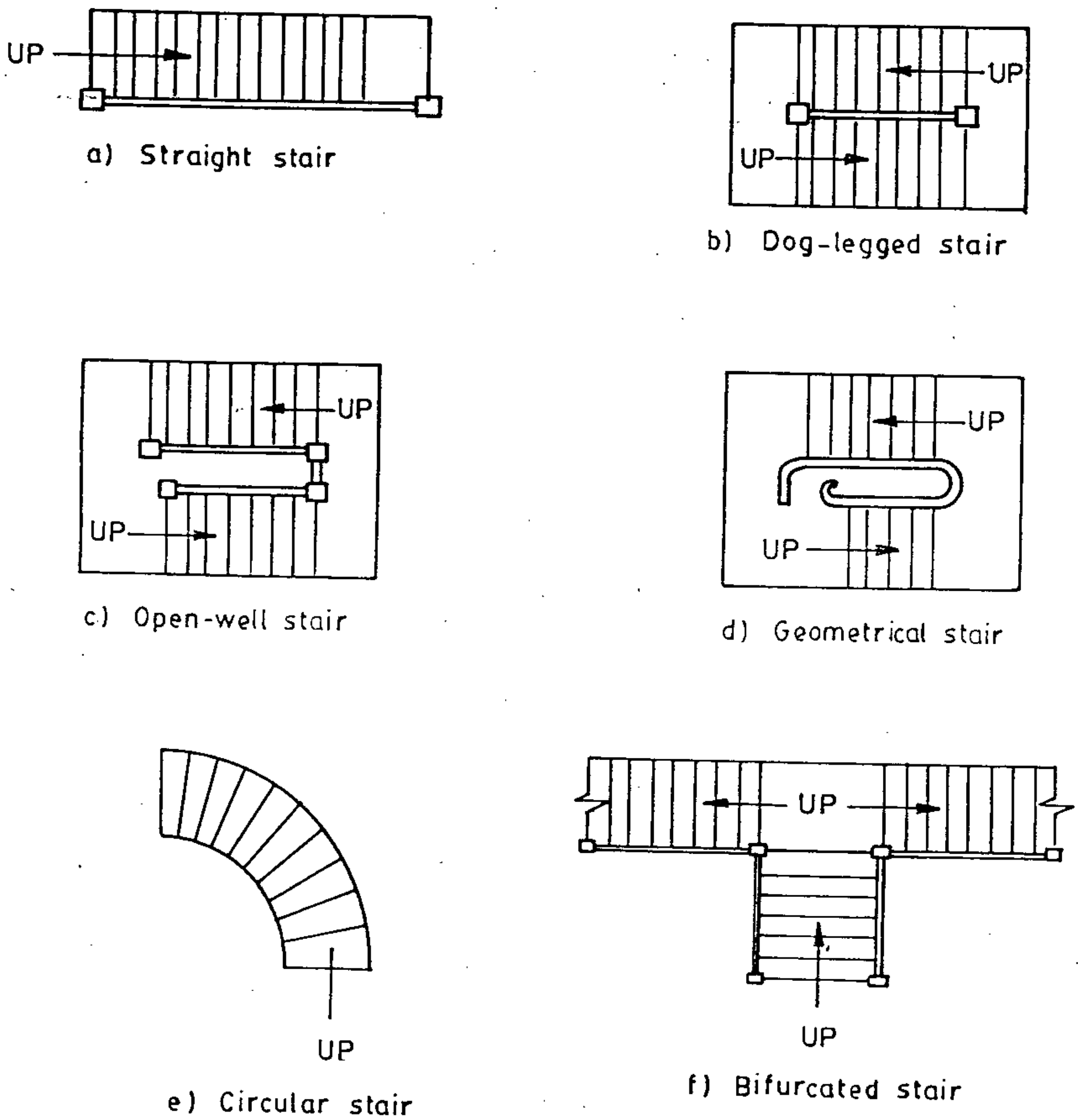


Fig. 2.2 Different types of stairs.

equal to the horizontal distance between supports. This method of design require steel to be placed only in the direction of the length of the slab. Transverse steel, usually one bar to each tread, is used only to assist in distribution of the load and to provide temperature reinforcement. The stair-cases with landing slab are designed as a simple slab with span equal to the horizontal distance between supports, the effect of the angle that occurs in a stair slab of this type is disregarded.

2.3.2 British code of practice

British code of practice(12) suggests effective span of stairs as given below:

The effective span of stairs without stringer beams should be taken as the following distances:

i) Where supported at top and bottom risers by beams spanning parallel with the risers, the distance centre to centre of beams.

ii) Where spanning on the edge of a landing slab which spans parallel to the risers. A distance equal to the going of the stairs plus at each end either half the width of the landing or one metre whichever is smaller.

iii) Where the landing slab spans in the same direction as the stairs, they should be considered as acting together to form a single slab and the span determined as the distance centre to centre of the supporting-beams or walls, the going being measured horizontally.

British code also gives specifications regarding distribution of loads in open well stair case.

Distribution of loading on stairs: In case of stairs with open wells, where spans partly crossing at right angles occur, the load on area common to any two such spans may be taken as one half in each direction as shown in Fig. 2.3.

Where flights or landings are built into walls at a distance of not less than 110 mm and are designed to span in the direction of the flight, a 150 mm strip may be deducted from the loaded area and the effective breadth of the section may be increased by two-thirds of the thickness that embedded, up to a maximum of 80 mm as shown in Fig. 2.4.

2.3.3 Indian code of practice

As per indian practice[5,6,7,13] stair slabs are divided into two categories, depending upon the direction in which the stair slab spans.

i) Stair slab spanning horizontally.

ii) Stair slab spanning longitudinally.

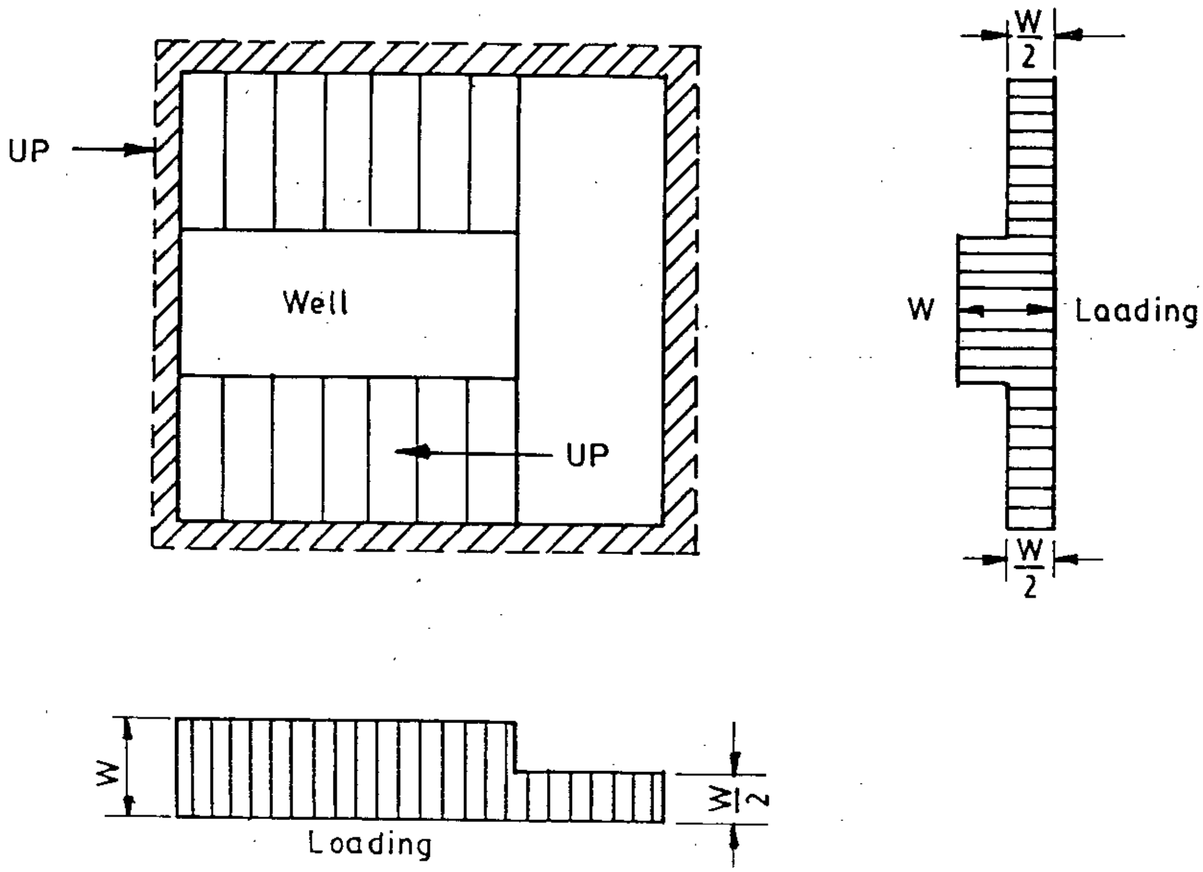


Fig. 2.3 Loading on stairs with open well.

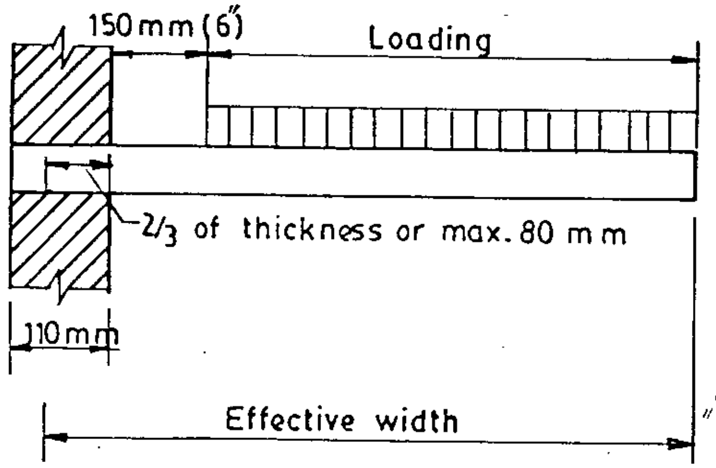
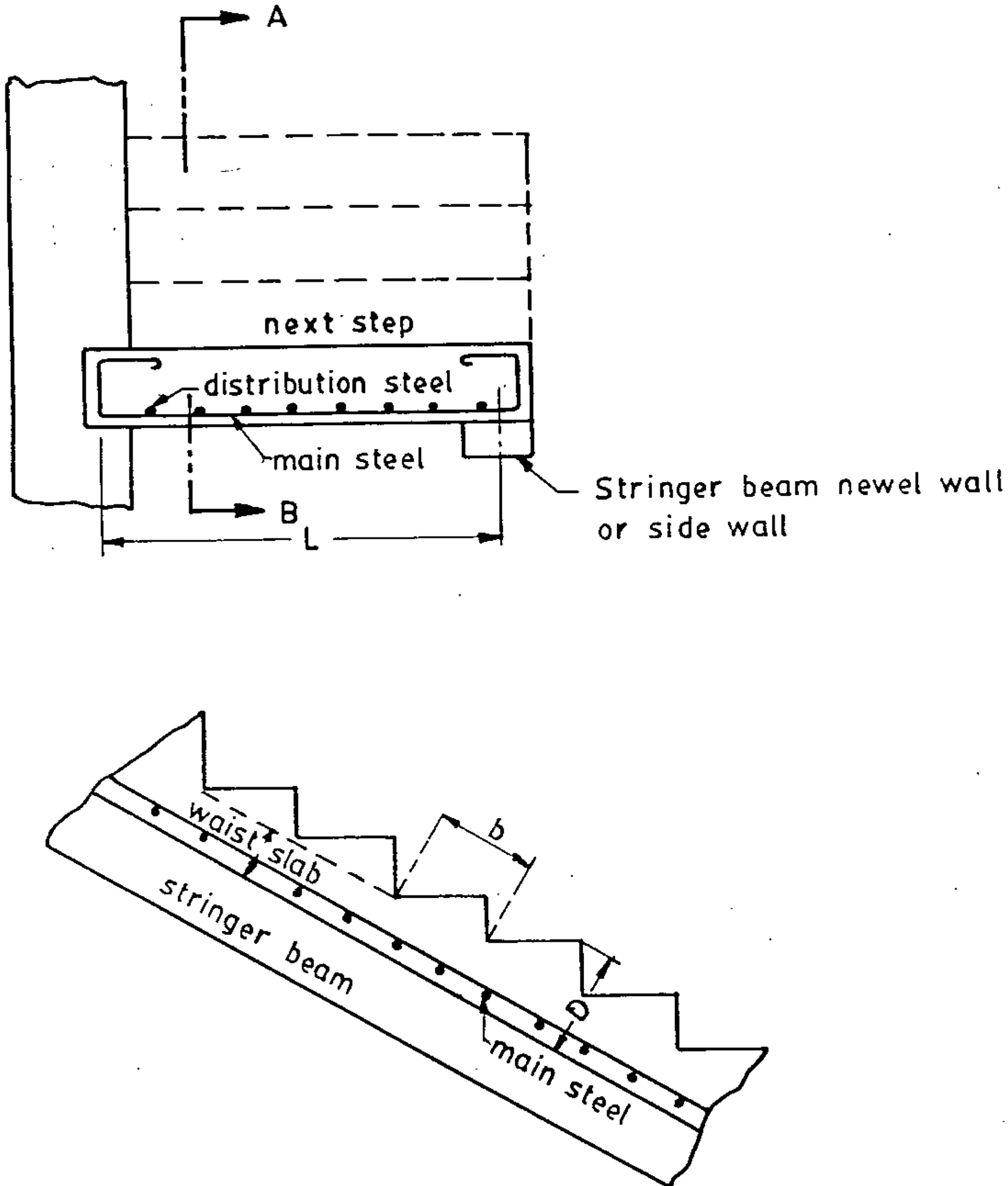


Fig. 2.4 Loading on stairs built into wall.

i) Stair slab spanning horizontally: In this category, the slab is supported on each side by side wall or stringer beam on one side and beam on the other side. Sometimes, as in the case of straight stair, the slab may also be supported on both the sides by the two side walls. The slab may also be supported horizontally by side wall on one side of each flight and the common newel on the other side between the backward and forward flights. In such a case the effective span L is the horizontal distance between centre to centre of supports. Each step is designed as spanning horizontally with a bending moment equal to $wL^2/8$. Each step is considered equivalent to a rectangular beam of width b (measured parallel to the slope of the stair) and an effective depth equal to D/2, as shown in Fig. 2.5.

ii) Stairs spanning longitudinally: In this type, the waist slab is supported at bottom and top of the flight and unsupported at the sides. The flight of the stair is continuous having beams both at top and bottom. The waist slab may be designed with a horizontal span equal to the centre to centre distance of the supporting beams. The waist slab is designed for a bending moment of $wL^2/10$. Where the flights or landing are built into wall, the landing slab is assumed to span in the same direction as the stairs. They should be considered as acting together to form a single slab. The



Section-AB

Fig. 2.5 Stair slab spanning horizontally.

effective span for each flight shall be taken as clear distance covered by the flight including the whole of landing plus 8 cm at either end for end bearing. The formula for bending moment is given by $wL^2/8$.

Sometimes the landing slab may span at right angles to the direction of stairs. In such a case the landing slab may be supposed to be acting as a beam supporting the flights. The effective span in such a case should be taken as a distance equal to the going of the stairs plus at each end either half the width of the landing or one metre whichever is smaller (similar to the british code of practice). Regarding the distribution of loading on stairs with open wells Indian code of practice follows the British code.

2.3.4 Discussion on the Code of Practices

It is clear from the above articles that ACI code does not provide any guideline for the design of stair slabs. For this reason followers of this code go for the conservative design considering the stair slab as simply supported slab regardless of the support condition. But it is likely that behaviour of stair slab depends significantly on the type of support.

On the other hand, British code has shed some light on the reduction of span considering a few type of support conditions and on the reduction of intensity of load partly on the common space where spans at right angle occur.

Indian code of practice classifies stair slabs into two types, one is spanning horizontally another is spanning longitudinally, depending upon the support provided. In other respects this code is almost similar to that of British code of practice, except in one cases, when waist slab is supported by beams at the top and bottom. In such a case continuity of the slab and a bending moment of $wL^2/10$ at the middle of the going of stairs is suggested by the Indian practice. But no suggestion is made regarding the bending moment at the beam supports. British code also does not provide any such suggestion.

Though there is a sharp change in angle at the kink, no code consider the effect of this angle changed. There is a common practice to provide negative reinforcement near the junction of waist slab and is extended upto supports. But nothing is found in any of the code provisions about the placement of such negative reinforcement.

Stair slabs commonly rest upon brick walls and more frequently landing slabs are built into the walls. Support conditions of such type can neither be termed as 'fixed' nor 'simply supported'. Actual condition of support is likely to be somewhat in between these two. There is no mention of the extent of 'fixity', imparted by the wall supports, in code provisions.

2.4 Comparative Case Studies

In this investigation, two types of stair-case have been considered, two flight type and two flight open well type. Each type has been studied for only one size and dimension (Fig. 2.6 and Fig. 2.7) with various support conditions. The chosen dimensions are most common in this field.

Design of the two flight stair slabs by American practice, British code and the method developed by Ahmad[1], and the design of two flight open well stair slabs by American practice and British code are presented below: WSD method of design has been followed with f'_c = 2500 psi and f_s = 20000 psi. Live load is taken as 100 lb/ft^2 and unit wt. of concrete is assumed to be 150 lb/cft.

a) American design practice

i) Two flight stair slab (Fig. 2.6):

Span = 17.08'

Slab thickness = 8"

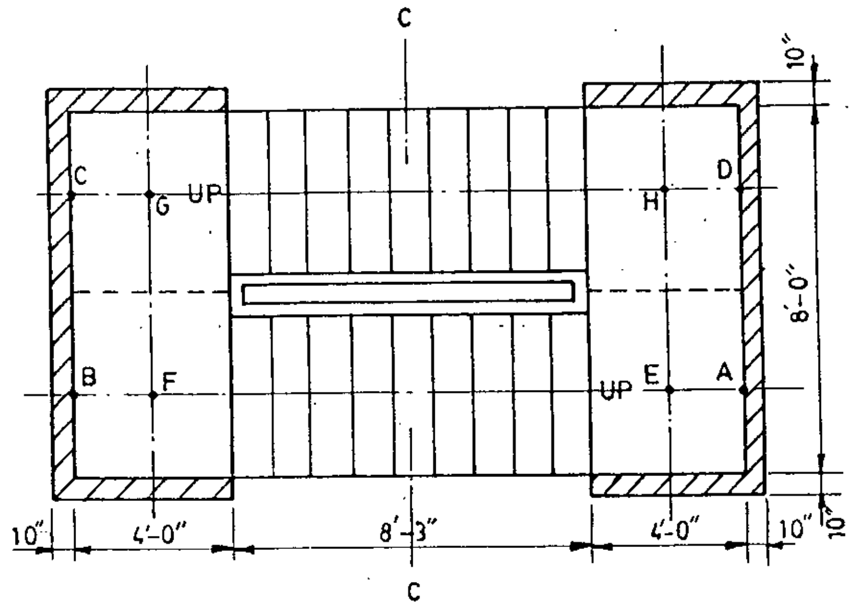


Fig. 2.6 Plan view of two flight stair slabs under study.

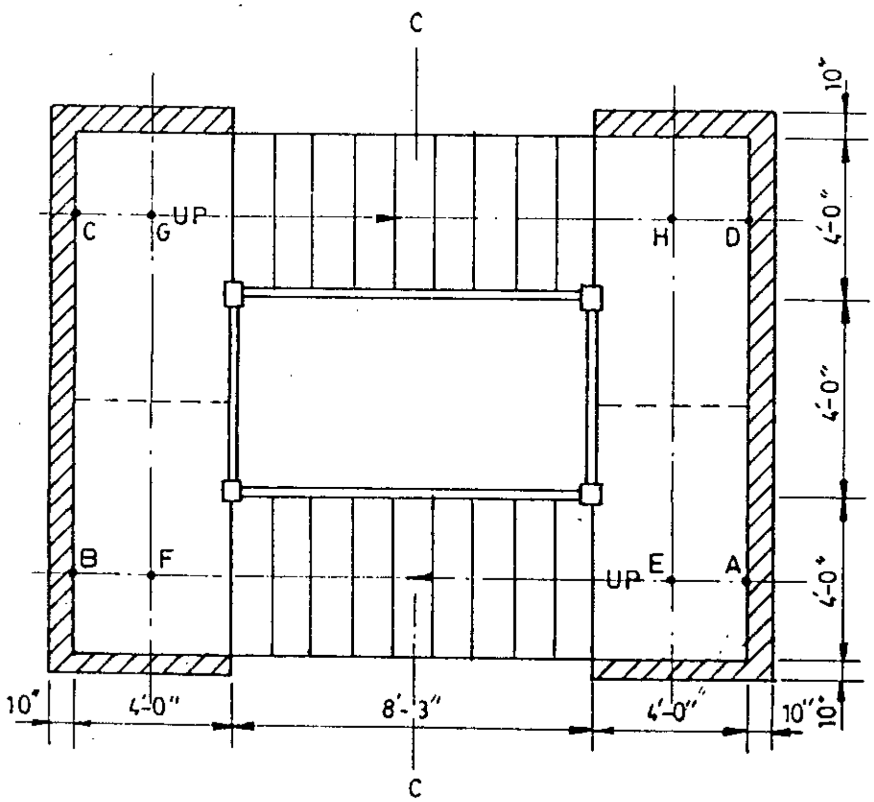


Fig. 2.7 Plan view of two flight open well stair slab.

Landing slab:

Dead load = 100 lb/ft^1

Live load = 100 lb/ft

Total : 200 lb/ft^2

Waist slab:

Dead load = 116.93 lb/ft^2

Wt. due to steps = 37.5 lb/ft^2

Live load = 100 lb/ft^2

Total: 254.43 lb/ft^2

Maximum bending moment at midspan = 8738.87 ft-lb/ft
Therefore slab thickness of 8" with steel area 0.86 in^2/ft are found to be required.

(ii) Two flight span well stair slab (Fig. 2.7):

Design for the span AB:

Span = 17.08'

Slab thickness = 8"

The design of this span is same as that of the previous problem.

Therefore slab thickness 8" with steel area 0.86 in^2/ft are required.

Design for the span BC:

Span = 12.83'

Dead load = 100 lb/ft^2

Live load = 100 lb/ft^2

Total = 200 lb/ft^2

Maximum bending moment at midspan = 3854.62 ft-lb/ft.

The overall thickness of flight AB is 8" with steel area 0.86 in^2/ft and the overall depth of landing slab BC is 8" with steel area 0.377 in^2/ft.

b) British code of practice

(i) Two flight stair slab (Fig. 2.6):

Span = 12.25'

Slab thickness = 6"

Total load on landing slab = 175 lb/ft^2

Total load on waist slab = 225.2 lb/ft^2

Max. bending moment of mid span = 3948.86 $ft\text{-}lb/ft^2$

Area of steel required = 0.54 in^2/ft

(ii) Two flight open well stair-slab (Fig. 2.7)

Design of the span AB:

Span = 12.25'

Slab thickness = 6"

Landing slab:

Total load on landing slab=175 lb/ft^2

Total load on waist slab = 225.2 lb/ft^2

Maximum bending moment at the midspan = 3948.86 lb-ft/ft

Area of steel A_s = 0.54 in^2/ft

Design of the span BC :

Maximum bending moment at midspan = 3450.55 ft-lb/ft

Area of steel A_s = 0.47 in^2/ft

The thickness of span AB is 5½" with steel area 0.49 in^2/ft and the thickness of span BC is the same as that of AB with steel area 0.18 in^2/ft.

c) <u>Design by the method developed by Ahmad</u>

Two flight stair slab (Fig.2.6)

Thickness of the slab = 4 in.

Effective span = 0.75x8.25 = 6.2 ft

Total load = 195.97 lb/ft

Maximum positive moment = $1/8 \times 195.97 \times (6.2)^2$ = 941.62 ft-lb/ft

Maximum negative moment = 1.25×941.62 = 1177.02 ft-lb/ft

Required steel, $A_s(+) = 0.22\ in^2/ft$

$A_s(-) = 0.27\ in^2/ft$

Additional reinforcement, at upper kink (wall side)

$= 2 \times 0.27 = 0.54\ in^2/ft$

and at lower kink (inner side) = $0.27\ in^2/ft$.

Estimate of material requirements for a single flight of the stair slab designed by the different methods are summarized in the following Table 2.1.

Table 2.1 Amount of concrete and steel for a single flight of the stair slab required by various design procedures

Method of design	Thickness of slab, in.	Volume of concrete, cft	Weight of steel, lb.
Ahmad	4	25.75	125
British code of practice	6	38.62	224
American practice	8	51.52	370

From the above Table 2.1 the stand of various methods can be visualized. Now comparing the amount of concrete and steel required by the Ahmad's method with those by British code of practice it is observed that the reduction in the amount of concrete and steel are of the order of 33% and 44% respectively and that with the American practice are of the order of 50% and 66% respectively.

Estimate of material requirements for a single flight open-well stair-case by the different method are summarized in the following Table 2.2.

Table 2.2 Amount of concrete and steel for a single flight of the open well stair slab required by various design procedures-

Method of design	Thickness of slab, in.	Volume of concrete, cft	Weight of steel, lb
British code of practice	6	48.29	276
American practice	8	64.38	464

Comparing the amount of concrete and steel required by the British code of practice with those by American design practice it is observed that the reduction in the amount of concrete and steel are of the order of 25% and 41% respectively.

CHAPTER 3

FINITE ELEMENT ANALYSIS OF STAIR SLABS

3.1 General

A finite element program[3] for general thick shell finite element analysis, developed by Ahmad[2] has been used for the analysis of stair slabs under study. A brief description and characteristic features of thick shell element and program are given in the appendix. The assumed data used in the investigation is the same as that of Ahmad[1]. It has been done in order to compare the two results. The overall slab thickness of 4 in. has been used. Although steps on the waist slab of a stair-case are generally constructed by monolithic casting, the effect has not been considered. The modulus of elasticity of the concrete has been considered to be 3×10^6 psi and unit weight is that of 150 lb/cft. Live load has been taken to be 100 psf. The weight of the steps has not been considered and the live load has been applied as a gravity load.

3.2 The Finite Element Program

Ahmad[3] developed a general thick shell finite element computer program for analysis of shells and plates. The author has used the same program with a minor modifications. The modifications were made for the following reasons:

i) In the original program the output for stresses were in global co-ordinates, which could not be directly used for design purposes. As such modifications were made to obtain the stress resultants in local co-ordinates.

ii) For the purpose of design, bending moment and axial forces were required. For this reason a simple routine was added to obtain the bending moments and axial forces.

3.3 Method of Computation of Normal Stresses

To obtain the normal stresses, the technique of transformation of stresses were used. In Fig. 3.1, x and z are the reference axes in which the stresses are obtained from the original program while x' and z' are local axes.

Let ABC be a differential element with an unit length perpendicular to the paper.

Now summing all the forces in x' direction

$$\sigma_{x'} dA - \sigma_x dA \cos\theta \times \cos\theta - \sigma_z dA \times \sin\theta \times \sin\theta$$
$$- \tau_{xz} dA \cos\theta \times \sin\theta - \tau_{zx} dA \sin\theta \times \cos\theta = 0$$

where dA is the differential area of the surface AC and since we know $\tau_{xz} = \tau_{zx}$

$$\therefore \sigma_x = \sigma_x \cos^2\theta + \sigma_z \sin^2\theta + \tau_{zx} \sin 2\theta \tag{3.1}$$

Summing all the force in z' direction

$$\tau_{x'z'}dA - \sigma_z dA \sin\theta \times \cos\theta + \sigma_{x'} dA \cos\theta \times \sin\theta$$

$$+\tau_{xz} dA \sin\theta \times \sin\theta - \sigma_{xz} \cos\theta \times \cos\theta = 0$$

$$\tau_{x'z'} = -\frac{\sigma_x - \sigma_z}{2} \sin 2\theta + \tau_{xz} \cos 2\theta \tag{3.2}$$

3.4 Method of Computation of Moment and Axial Forces

Moment Computation

After obtaining the top and bottom normal stresses for every node, the moments can be calculated from the formula

$$M = \frac{S_n I}{C} \tag{3.3}$$

where, S_n = bottom fibre bending stresses

I = moment of inertia of the section considered.

C = distance of bottom fibre from centroid of the section.

Now, from Fig. 3.2 bottom fibre bending stress $S_n = (S_b - S_t)/2$

where, S_b = bottom normal stress

S_t = top normal stress

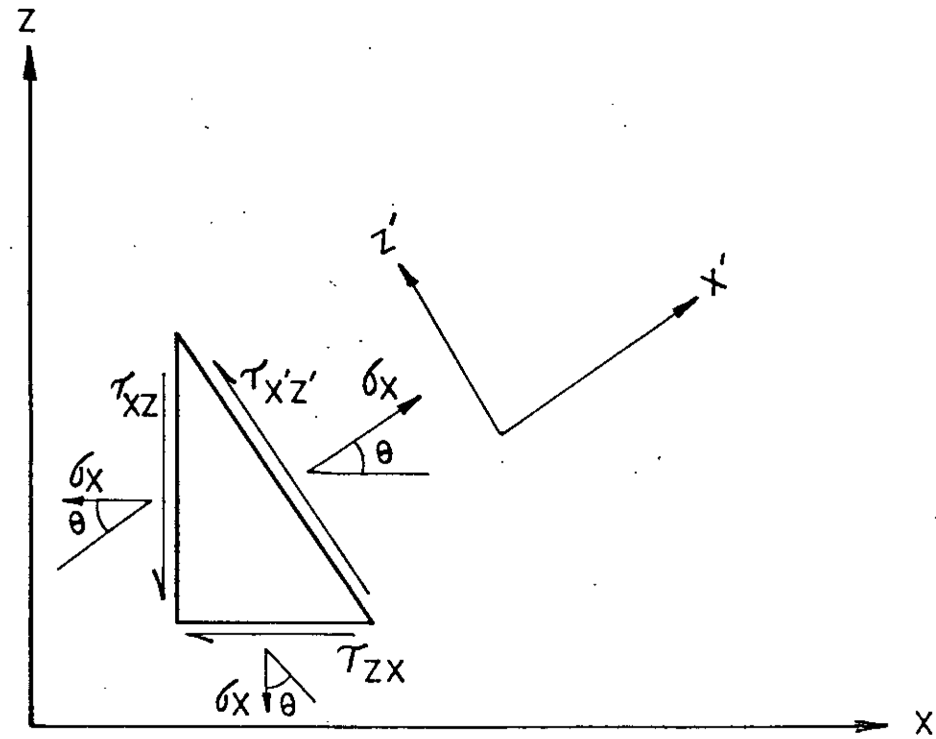


Fig. 3.1 Differential element with global and local axes.

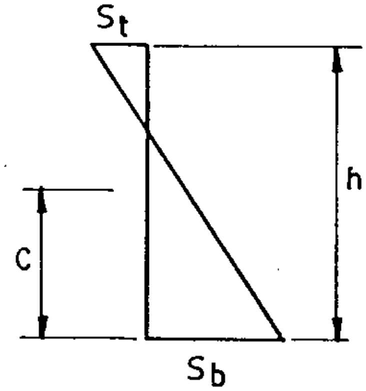


Fig. 3.2 Assumed stress diagram.

In the problem concerned the section is uniform and rectangular. Now putting the value of $I = h^3/12$ and $C = h/2$ in Eqn. (3.3), the expression for moment is

$$M = \frac{S_n h^2}{6} \qquad 3.4$$

Axial force computation

The axial force at different nodes can be calculated from

$$N = S \times h$$

where S = average of top and bottom normal stress

3.5 Idealization of Support Condition

Brick wall supports cannot be idealized either as 'fixed' or 'simple support'. But it is expected that the rotation of the boundary node would be somewhat in between the two. Hence stair slabs under study have been analysed for both types of boundary conditions: one considering fixed supported another one considering simply supported. It has been further assumed that behaviour of the stair slabs would be symmetrical about the dotted places shown in Fig. 2.6 and Fig. 2.7. Nodal lines on the plane of symmetry were kept restrained against displacement in the perpendicular direction of the plane of symmetry and against rotation out of the plane of symmetry. Thus the condition of symmetry were realized. The boundary conditions which would be considered in the analysis are shown in Fig. 3.3.

3.6 Element Subdivision

To tackle uncertain stress condition at kink zone a number of element subdivision were analysed. The subdivision first tried was the same as that used by Ahmad,(1), shown in Fig. 3.4. The performance of this subdivision at kink was not satisfactory. Then a finer mesh of element subdivision at kink shown in Fig. 3.5 was used. This subdivision gave satisfactory result upto element 4 from the landing side. But the performance of elements 5 and 6 were not satisfactory.

It was seen in the analysis that further subdivision of elements 5 and 6 (Fig. 3.5) did not improve the results either. For this reason, the elements, whose performance was good, were extended towards the kink. And finally an element subdivision was found as shown in Fig. 3.6 which gave satisfactory results at the kink zone as well. Subsequently this element subdivision (Fig. 3.6) was used for further analysis.

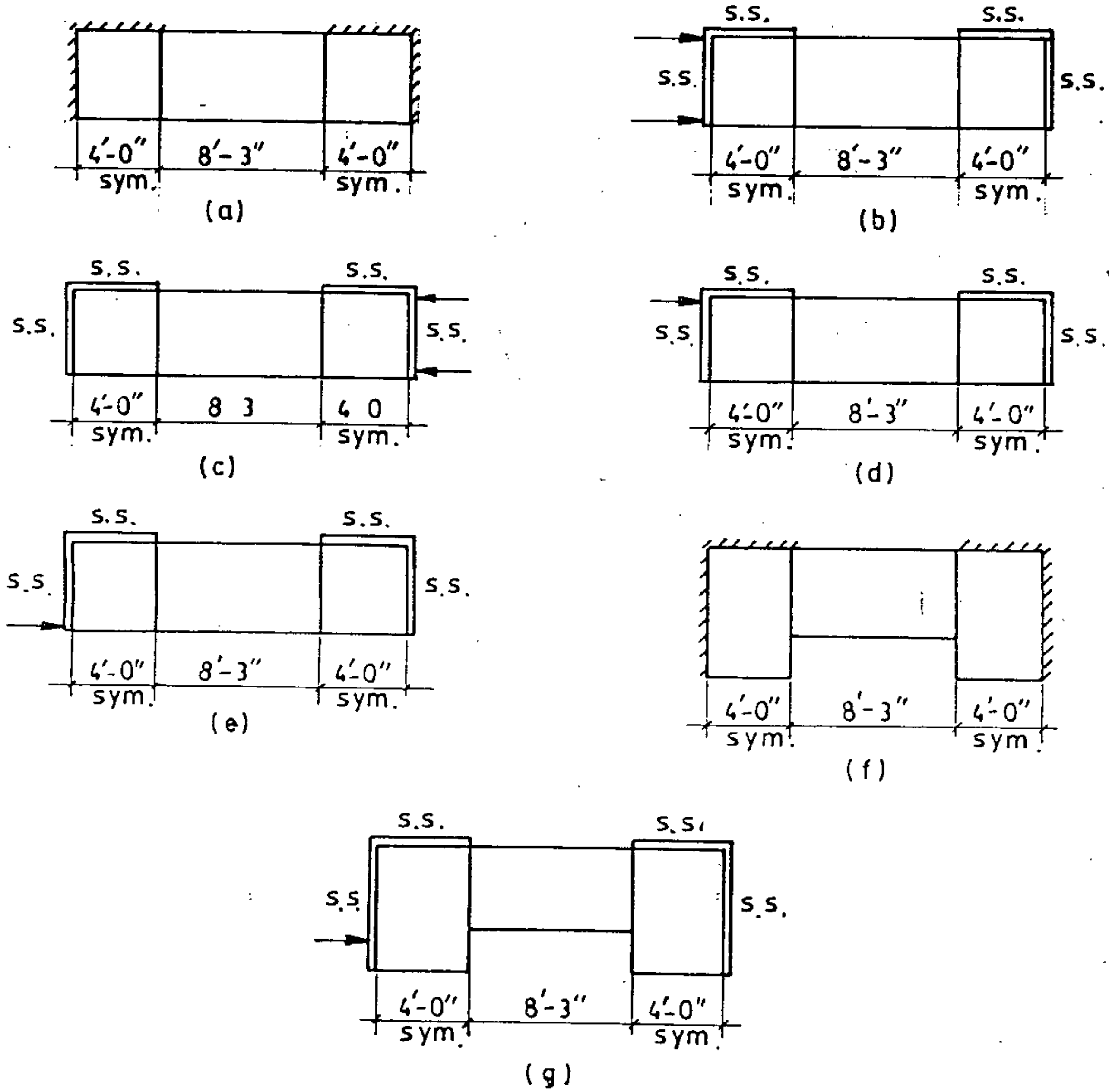


Fig. 3.3 The boundary conditions, considered for analysis.

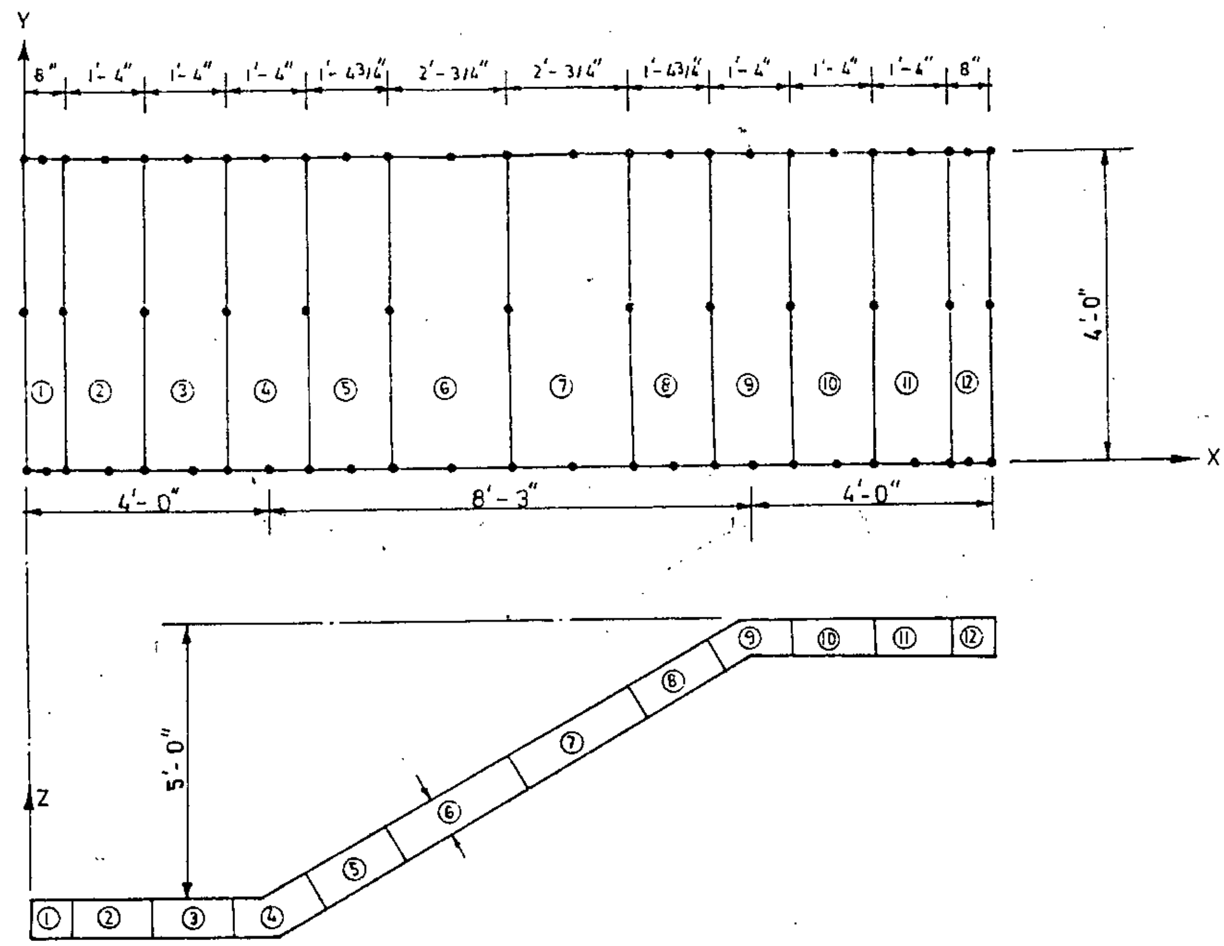


Fig. 3.4 Plan of first element subdivision.

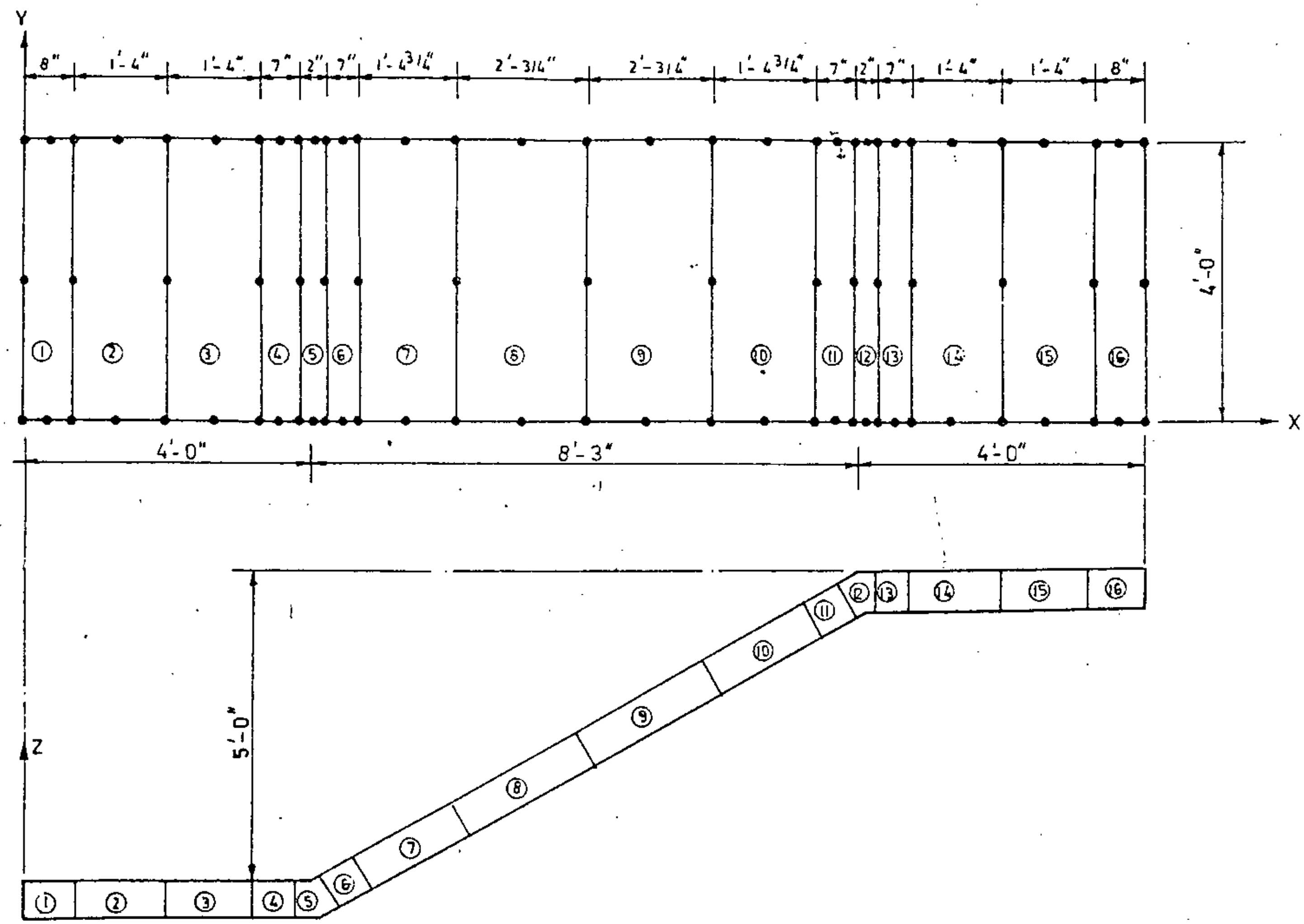


Fig. 3.5 Plan of second element subdivision.

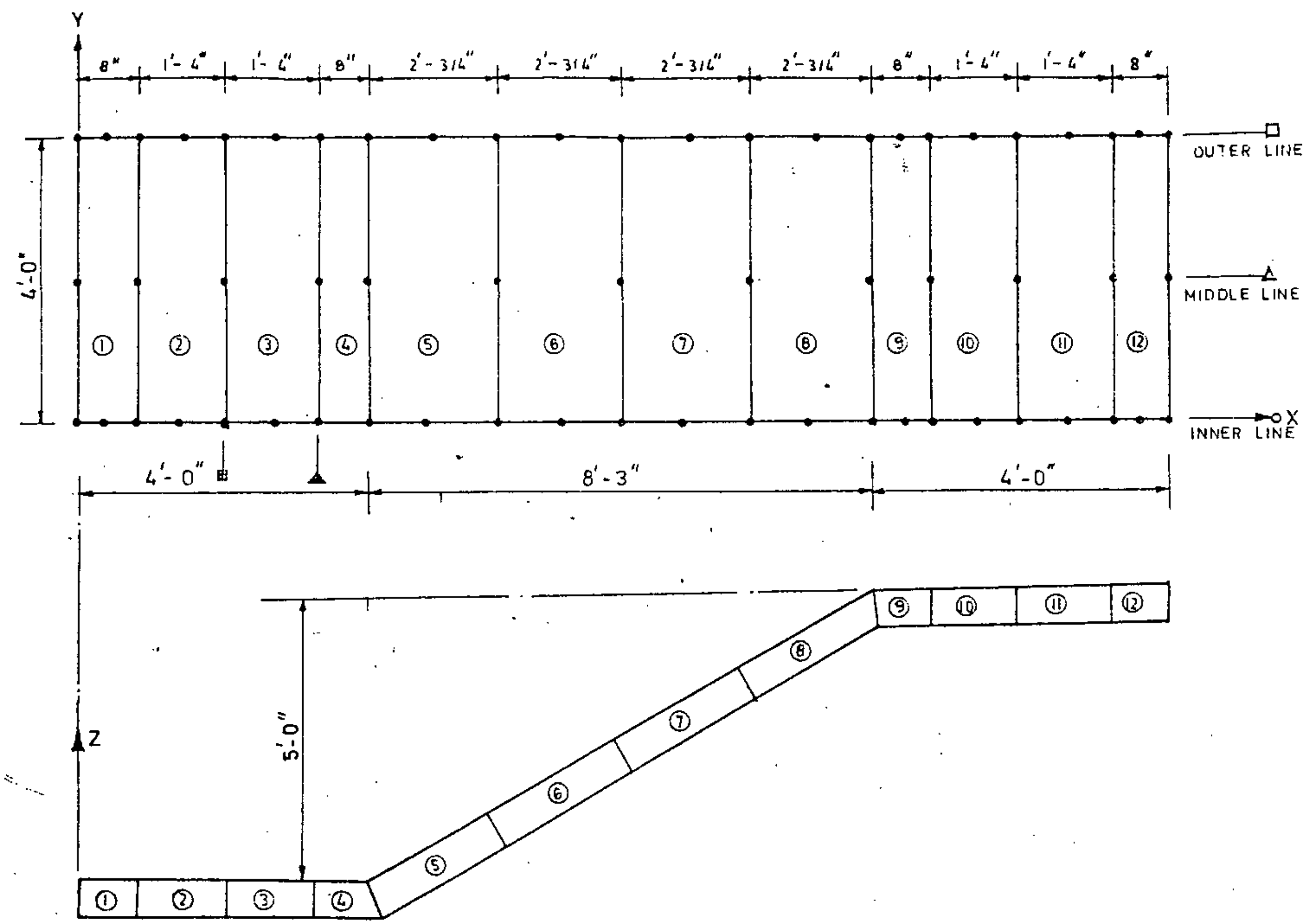


Fig. 3.6 Plan of third element subdivision.

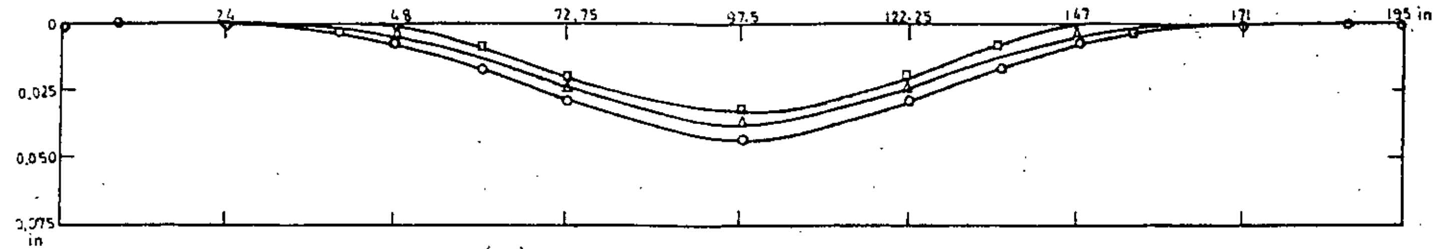
3.7 Analysis of two Flight Stair Slab with Different Boundary Conditions

Computer output from the program gives three displacements, two rotations (described in appendix), top and bottom stresses, and bending moment and axial force at each node. To investigate the effect of boundary various support conditions were used. These results are summarized under the following sub-articles.

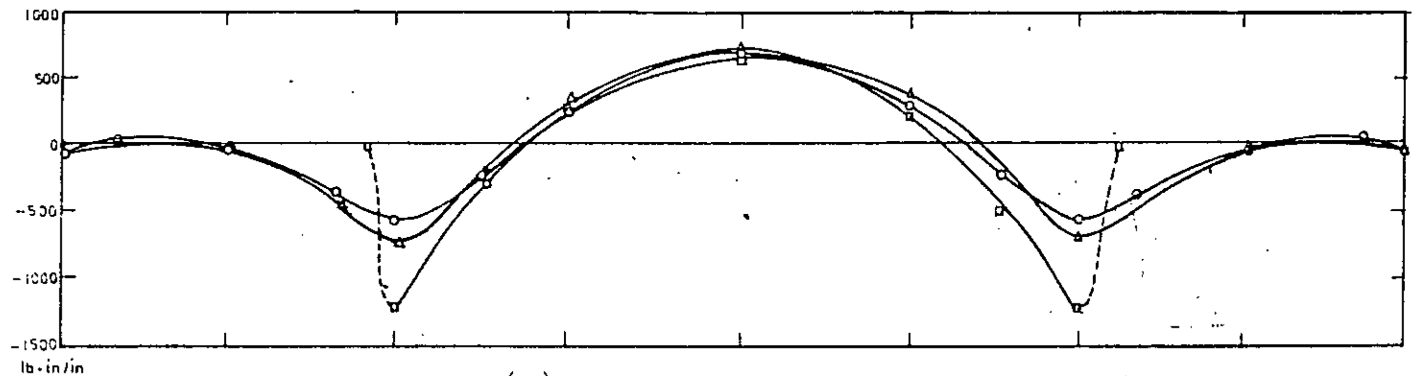
3.7.1 Fixed Boundary Condition

Waist slab of the type of Fig. 2.6 is supported on the landing slabs at two ends. Landing slabs are supported by walls on three sides. In this boundary condition, the boundary nodes were considered fixed. The boundary condition is shown in Fig. 3.3(a). Deflection, bending moment and axial force diagrams in the longitudinal direction are shown in Fig. 3.7.

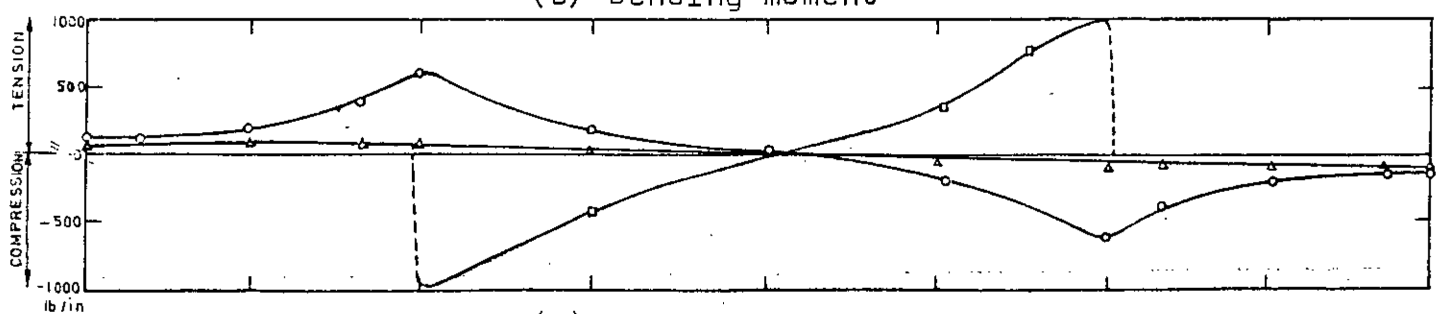
From the deflection diagram of Fig. 3.7 it is seen that the points of contraflexure lie within the waist slab and the distance between the points of contraflexure is about 65% of going of the waist slab. Thissargument can be confirmed from the bending moment diagrams also. The maximum positive bending moment occurs at the middle of the waist slab and his moment is more or less uniform in the transverse direction



(a) Deflection



(b) Bending moment



(c) Axial force

3.7 Results of finite element analysis of fixedly supported stair slab.

the average value of this moment is about 638 ft-lbs/ft. The maximum negative bending moment occurs at the support of the kink and it gradually reduces to about 46% at the inner point. The average value of this moment is about 772 ft-lb/ft. From the axial force diagram, it is seen that at the lower kink the inner point remains under maximum tension, the middle point has negligible stress and support point has maximum compression. Conversely at upper kink, the maximum compression occurs at the inner point, the stress is quite small at the middle point and the outer point remains under maximum tension.

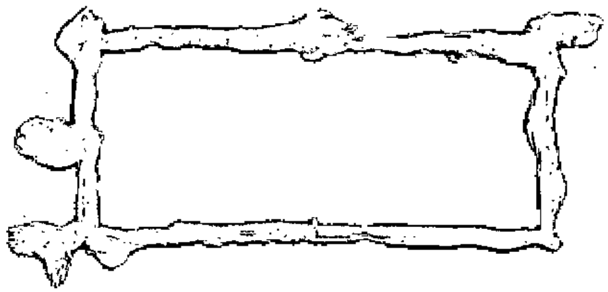
The deflection, bending moment and axial force diagram of the stair slab in the transverse direction along two transverse lines (Fig. 3.6) are shown in Fig. 3.8. These figures show that bending moment and axial force is negligible in the transverse direction.

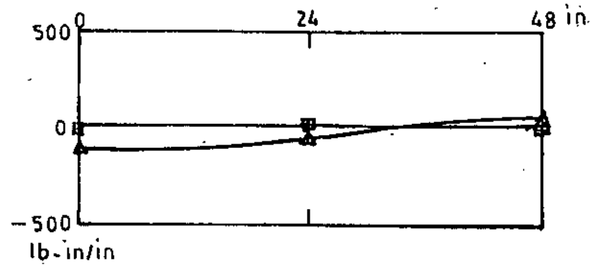
3.7.2 Simply supported boundary conditions

Four types of simply supported boundary conditions have been investigated, which are presented in what follows:

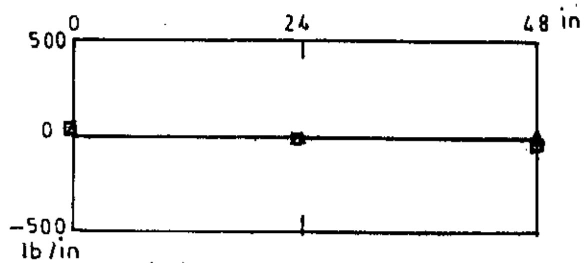
(a) Horizontal restraint at two points of the lower landing:

The configuration and dimension of the stairslab of this problem is the same as that of previous one but only





(a) Bending moment



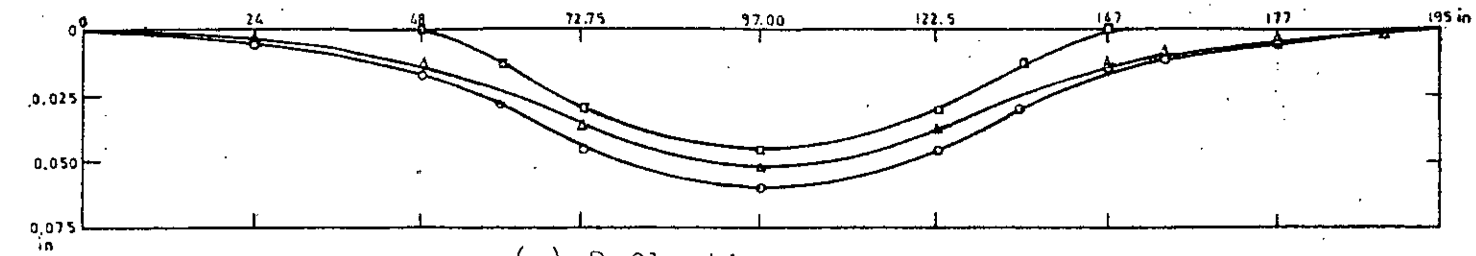
(b) Axial force

Fig. 3.8 Results of finiteeelement analysis of fixed boundary stair slab in the transverse direction.

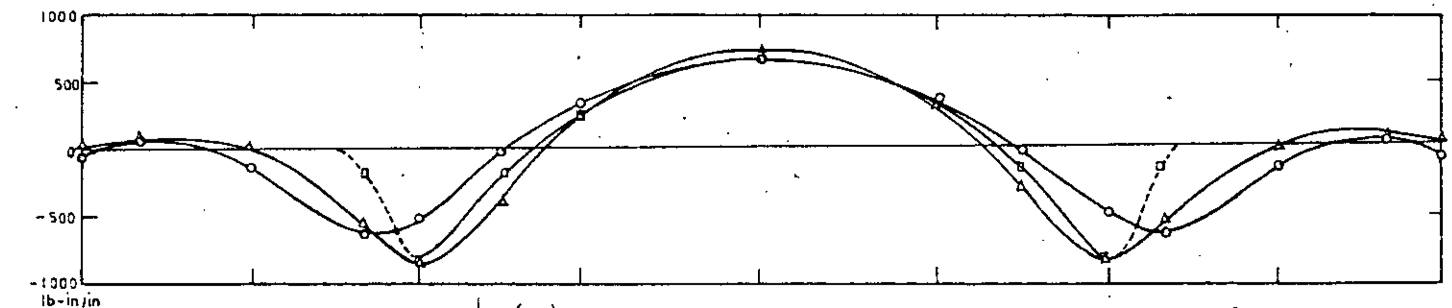
difference is that the boundary nodes were considered simply supported and two of the lower landing nodes were restrained against movement in the horizontal direction as shown in Fig. 3.3(b). Deflection, bending moment and axial force diagrams in thelongitudinal direction are shown in Fig. 3.9.

From the deflection diagram of Fig. 3.9 it is seen that the points of contraflexure is about 67% of the going of the waist slab. This fact is confirmed from the bending moment diagram also. The maximum positive bending moment occurs at the middle of the waist slab and this moment is more or less uniform in the transverse direction. The magnitude of this moment is about 685 ft-lb/ft. The maximum negative bending moment occurs at the kink zone. But in this problem this moment is more or less uniform and the magnitude of this moment is about 785 lb-ft/ft. From the axial force diagram it is seen that at the bottom kink the inner point remains under maximum tension, at the middle point the tension is quite small and at the outer point the maximum compression occurs. Conversely at the top kink, the maximum compression occurs at the inner point, the compression is quite small at the middle point and the outer point remains under maximum tension.

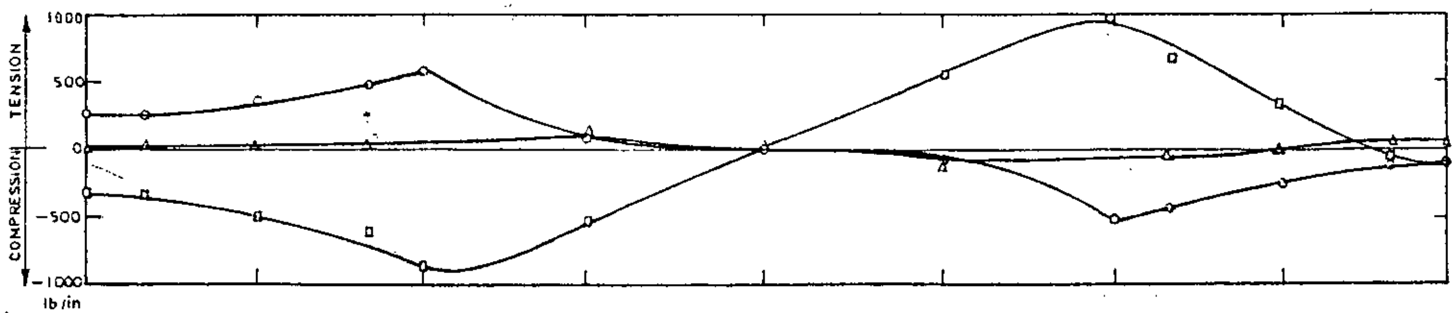
The bending moment and axial force of this problem in the transverse direction along the two transverse lines (Fig. 3.6) are once again negligible.



(a) Deflection



(b) Bending moment



(c) Axial force

Fig. 3.9 Results of finite element analysis of simply supported stair slab of type (a).

b) Horizontal restraint at two points of the upper landing:

The boundary condition of this problem is shown in Fig. 3.3(c). Deflection, bending moment and axial force diagramsin the longitudinal direction are shown in Fig. 3.10.

From the deflection diagram of Fig. 3.10 it is seen that the points of contraflexure is about 66% of the going of the waist slab. This fact is confirmed from the bending moment diagram also. The maximum positive bending moment occurs at the middle of the waist slab and this moment is more or less uniform in the transverse direction and the average value of this moment is about 767 ft-lb/ft. From the axial force diagram it is seen that at the bottom kink the inner point remains under maximum tension, at the middle point the tension is quite small and at the outer point maximum compression occurs. On the other hand at the top kink, the maximum compression occurs at the inner point, the compression is quite small at the middle point and the outer point remains under maximum tension. All these results are very similar to that of type (a).

c) Horizontal restraint at the corner node of the lower landing:

The boundary condition of this problem is shown in Fig. 3.3(d). This type of boundary condition was chosen in order to relax zone of the horizontal restraints on the edge

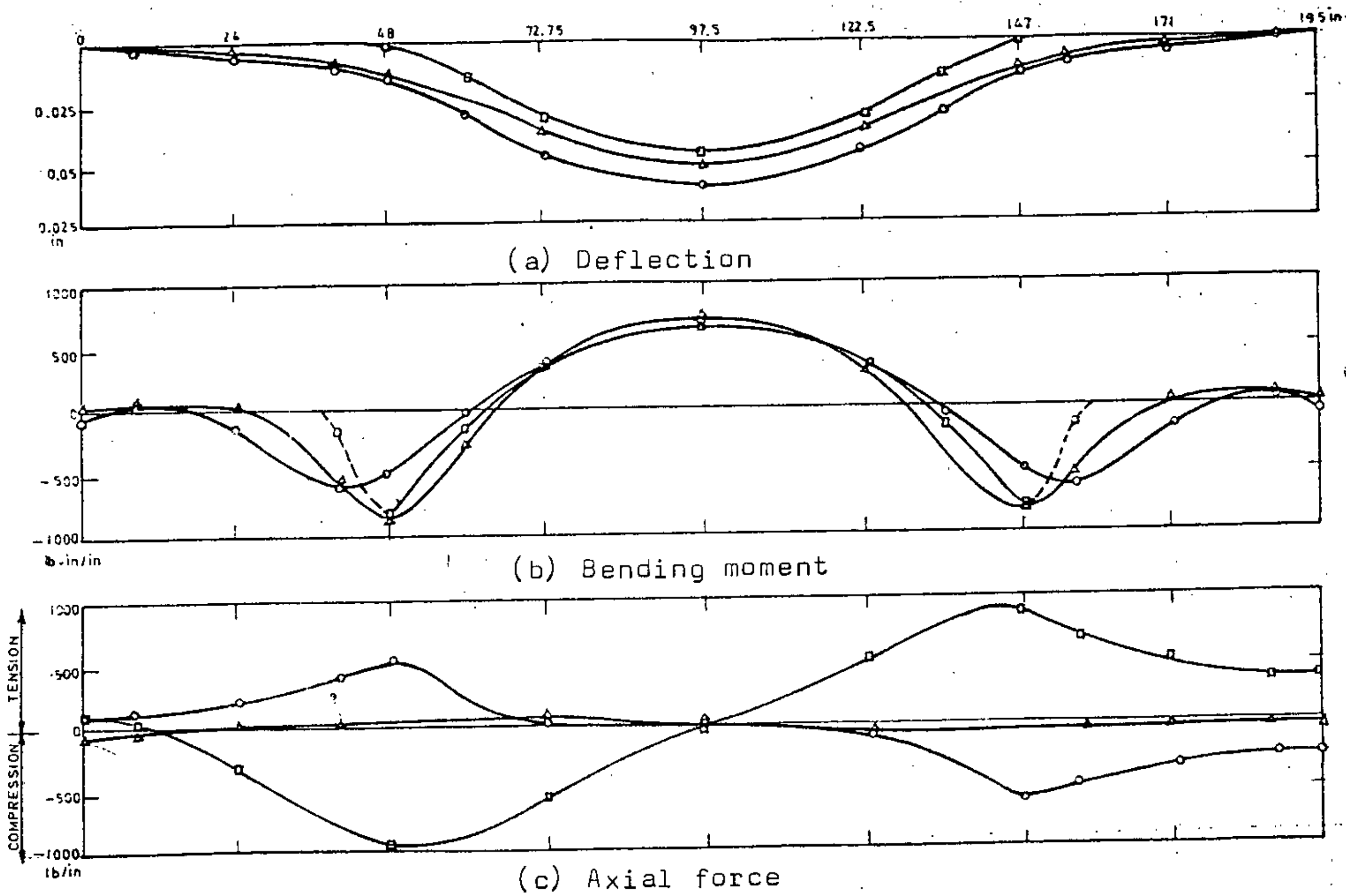


Fig.3.10 Results of finite element analysis of simply supported stair slab of type (b).

boundary. Deflection, bending moment and axial force diagrams in the longitudinal direction are shown in Fig. 3.11.

From the deflection diagram of Fig. 3.11 it is seen that the points of contraflexure is about 66% of the going of the waist slab. This value is verified from the bending moment diagram also. The maximum positive bending moments occurs at the middle of the waist slab and this moment is more or less uniform in the transverse direction and the average value of these moments is about 680 ft-lb/ft. The maximum negative bending moments occur near the kink zone. These are more or less uniform in the transverse direction. The average value of these moments is about 775 ft-lb/ft. The nature of the curves are more or less the same as that of type (a) and (b).

d) Horizontal restraint at the inner node of the lower landing:

The boundary condition of this problem is shown in Fig. 3.3(a). This boundary condition was used to find the effect of the position of the restraint against horizontal displacement. The deflection, bending moment and axial force diagrams in the longitudinal direction are practically identical to those obtained from Type (c).

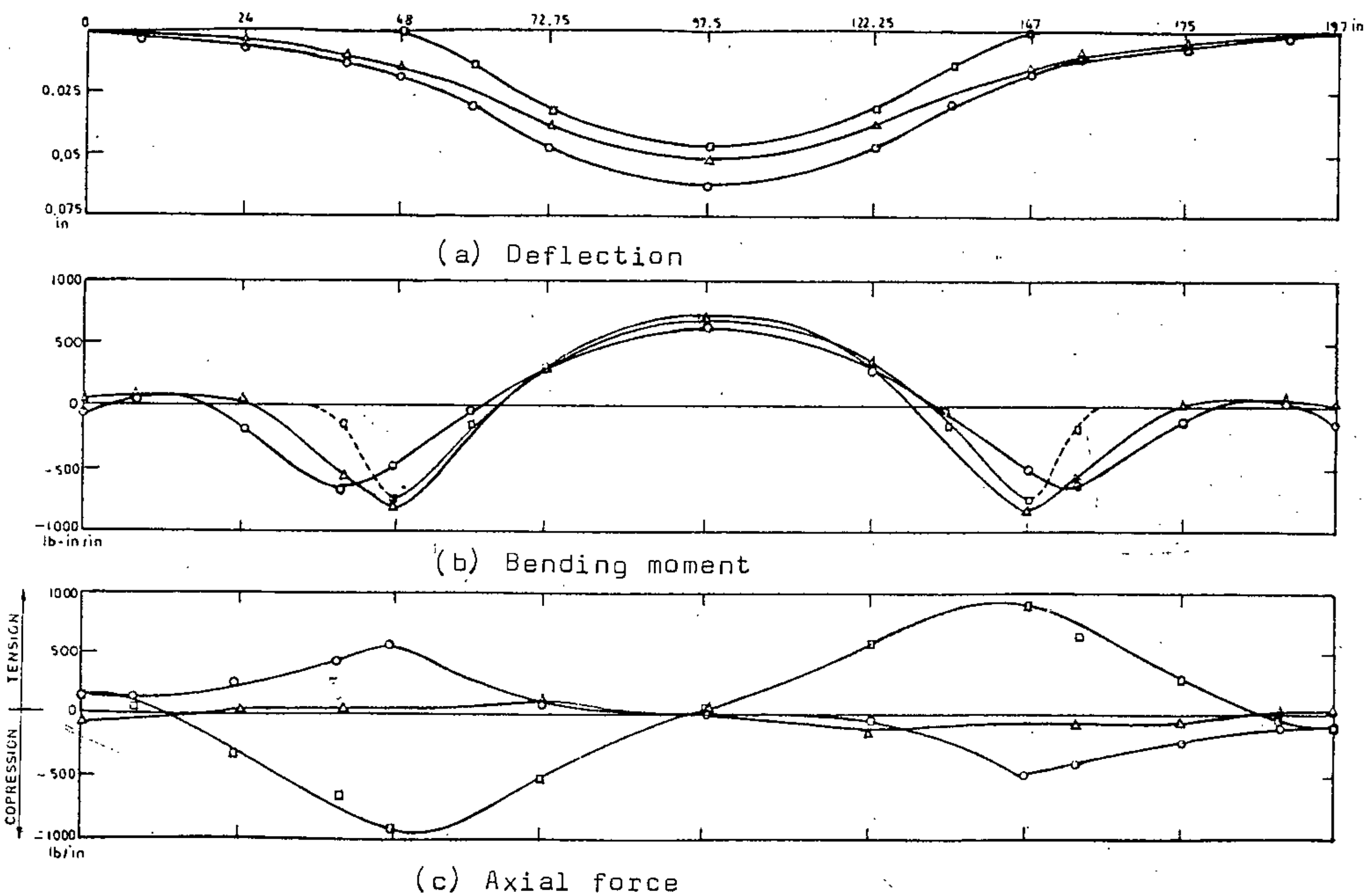


Fig. 3.11 Results of finite element analysis of simply supported stair slab of type (c).

3.8 Two Flight Open Well Stair Slab with Different Boundary Conditions

Dimensions of the stair slab were similar to those of the previous problems except the open well. Once again investigations were carried out with fixed and simply supported boundary conditions. The results are presented in what follows:

3.8.1 Fixed Boundary condition

Waist slab of the type of Fig. 2.7 is supported on landing slabs at two ends. Landing slabs are supported by walls on three sides. In this boundary condition, the boundary nodes were considered fixed, as shown in Fig. 3.3(f). The finite element subdivision is shown in Fig. 3.12. Deflection, bending moment and axial force diagrams in the longitudinal direction are shown in Fig. 3.13. Deflection, bending moment and axial force diagrams of open well landing slab in the x-direction are shown in Fig. 3.14.

From the deflection diagram of Fig. 3.13 it is seen that the points of contraflexure are within the waist slab and the distance between the points of contraflexure is about 65% of the going of the waist slab. This observation can be confirmed from the bending moment diagram. The maximum positive bending moment occurs at the middle of the waist

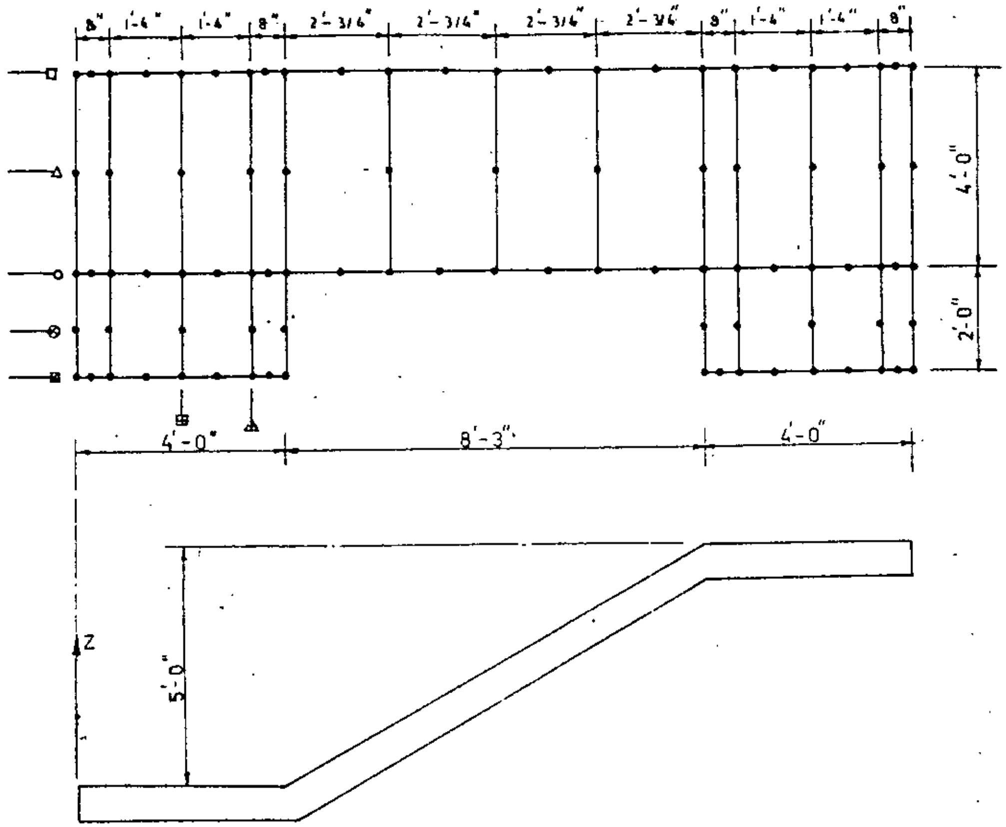


Fig. 3.12 Finite element subdivision of open well stair slab.

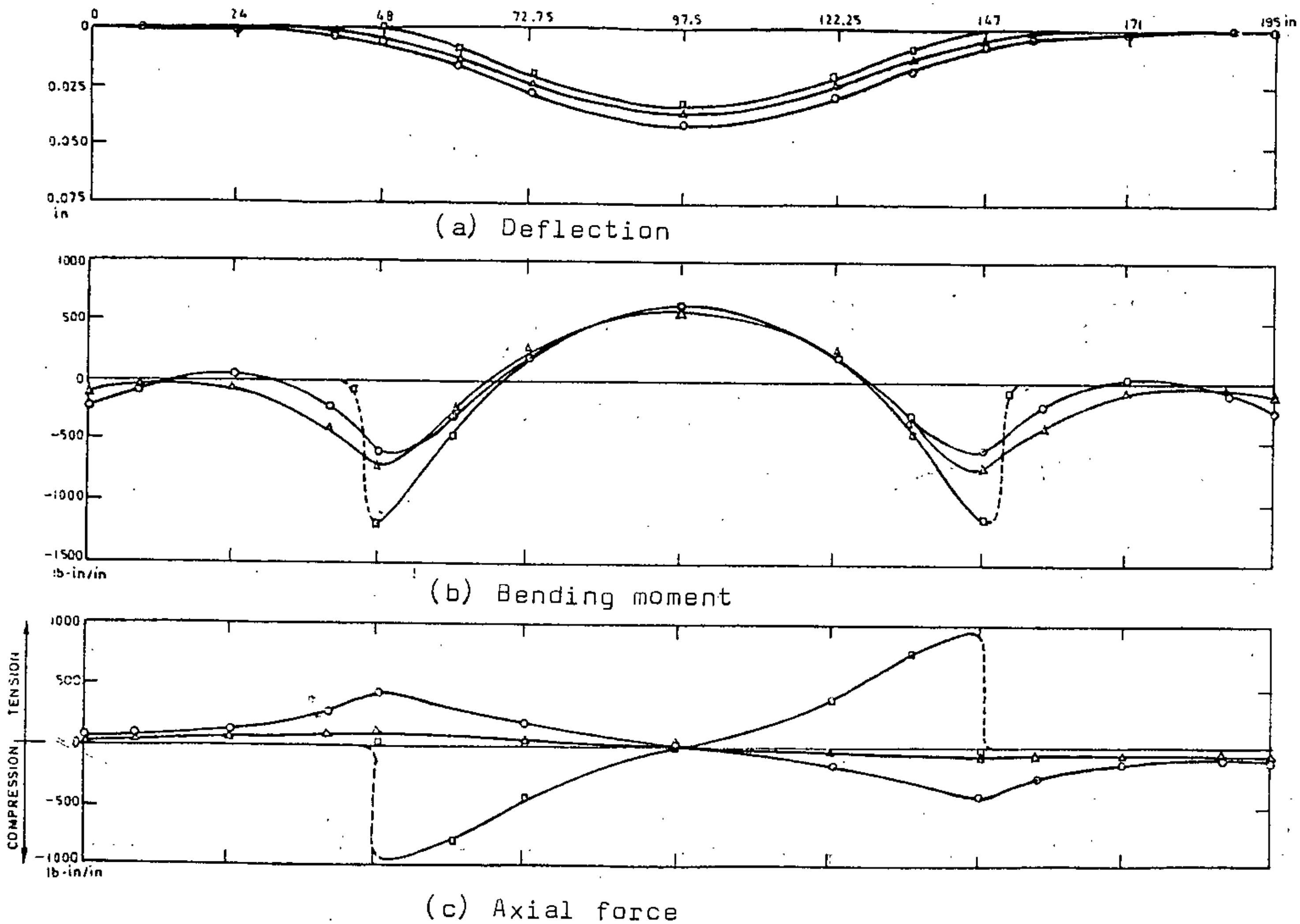


Fig. 3.13 Results of finite element analysis of fixed boundary open well stair slab.

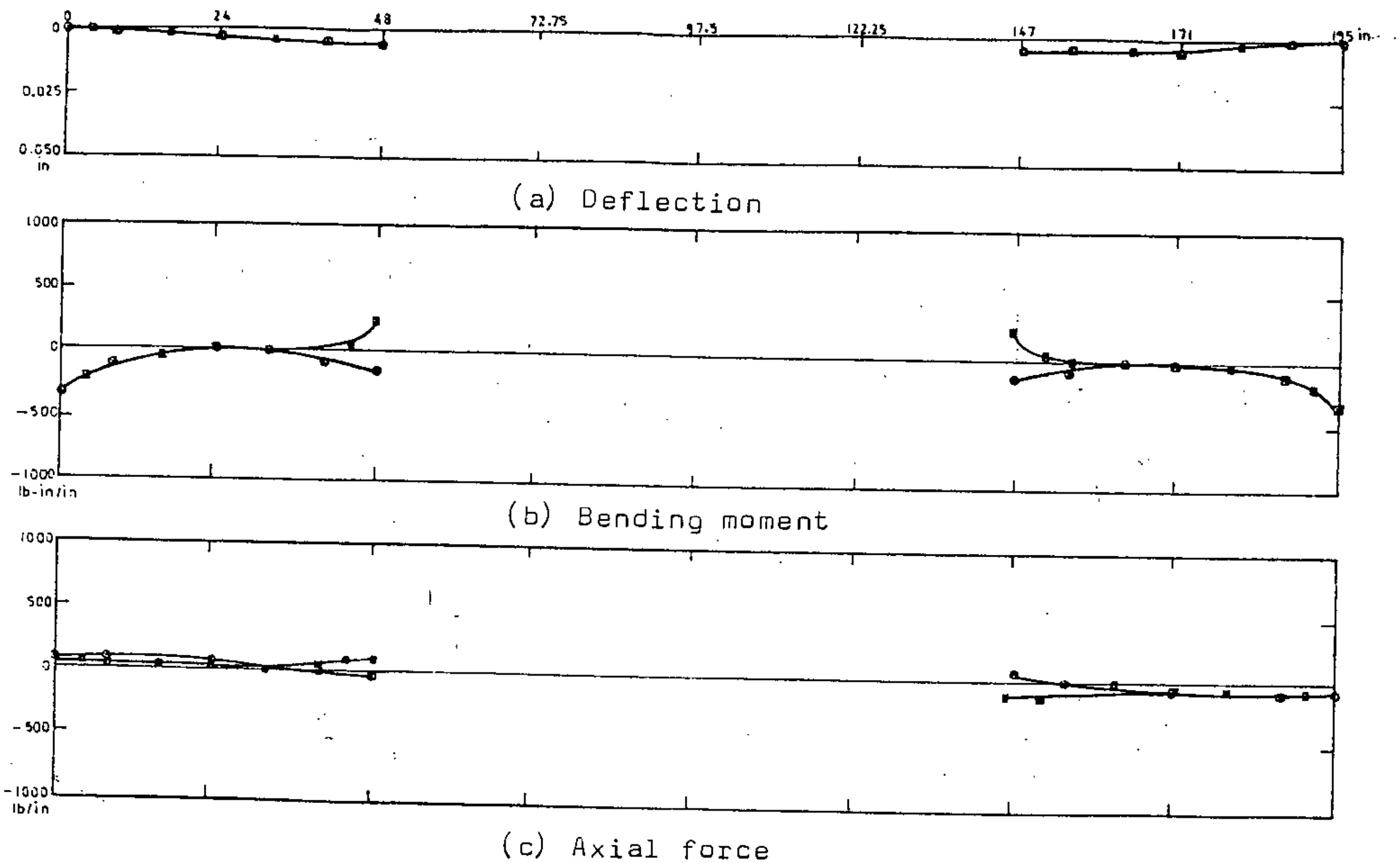


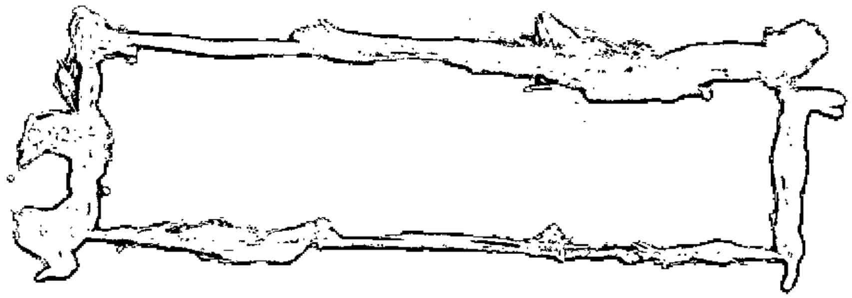
Fig. 3.14 Results of finite element analysis at landing slab of fixed boundary open well stair slab in the x-direction.

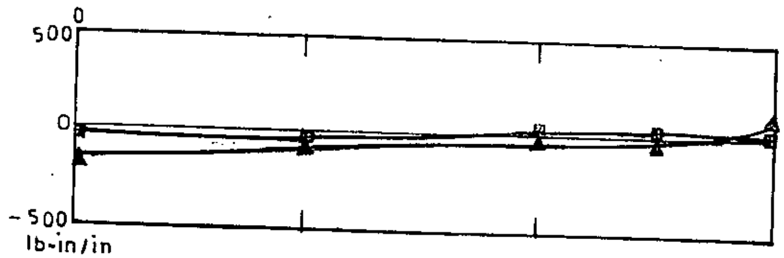
slab and this moment is more or uniform in the transverse direction, the average value of this moment is about 620 ft-lb/ft. The maximum negative bending moment occurs at the support of the kinks and it gradually reduces to about 52% at the inner point. The average value of this moment is about 758 lb-ft/ft. From the axial force diagram it is seen that at the lower kink the inner point remains under maximum tension, at the middle point the tension is quite small, and at the outer point maximum compression occurs. Conversely at the upper kink, the maximum compression occurs at the inner point, the compression is quite small at the middle point and the outer point experiences maximum tension.

The deflection, bending moment and axial force diagrams of this problem in the transverse direction along the two transverse line (Fig. 3.12) are shown in Fig. 3.15. These figures show that bending moments and axial forces, are negligible in the transverse direction.

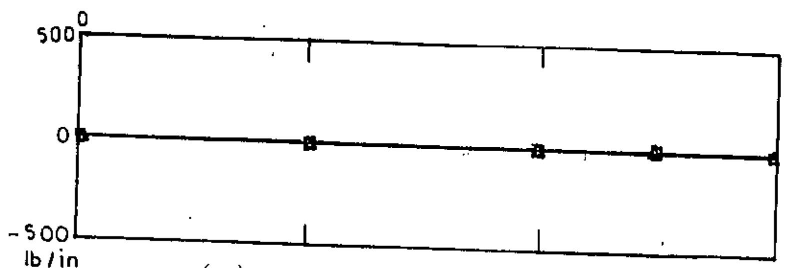
3.8.2 Simply supported boundary condition

Here the boundary nodes were considered simply supported and a node of the lower landing was restrained against movement in the horizontal direction as shown in Fig. 3.3(g). The deflection, bending moment and axial force diagrams in the longitudinal direction are shown in Fig. 3.16. Deflection,



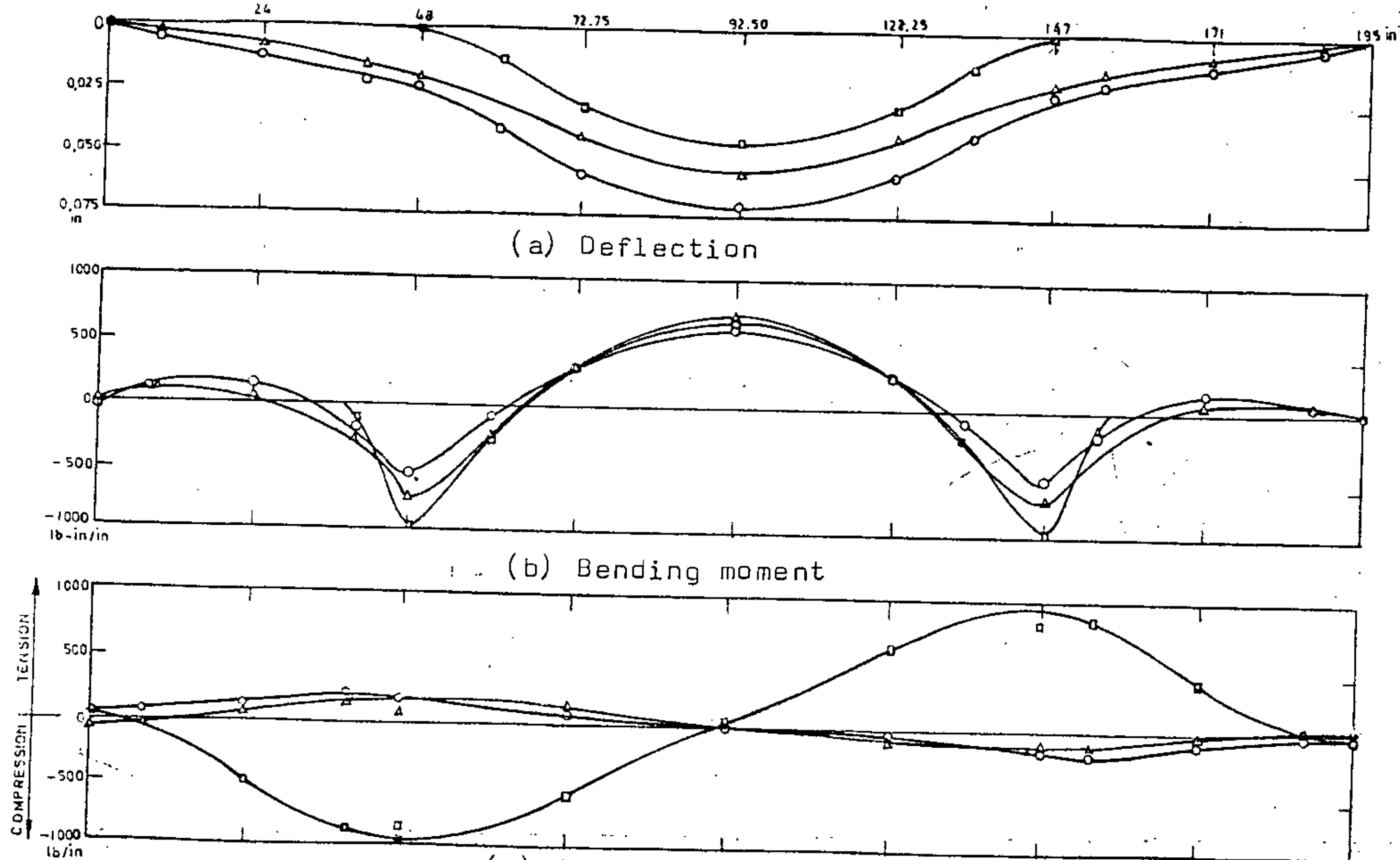


(a) Bending moment



(b) Axial force

Fig. 3.15 Results of finiteelement analysis of fixed boundary open well stair slab in the transverse direction.



(a) Deflection

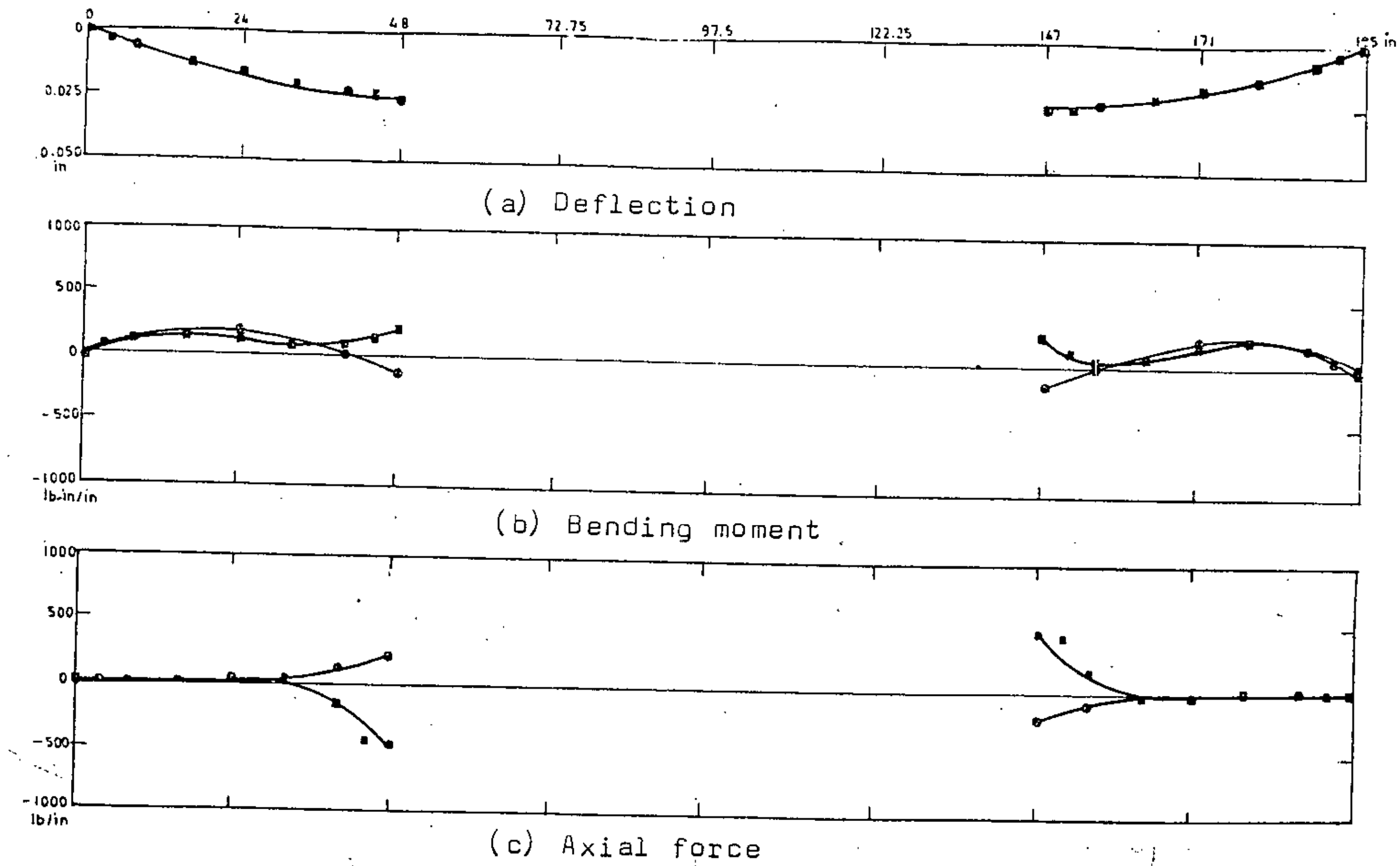
(b) Bending moment

(c) Axial force

Fig. 3.16 Results of finite element analysis of simply supported open well stair slab in the longitudinal direction.

bending moment and axial force diagrams of open well landing slab in the x-direction are shown in Fig. 3.17.

From the deflection diagram of Fig. 3.16 it is seen that the points of contraflexure is about 66% of going of the waist slab. This figure also corresponds to the bending moment diagram. As in the case of fixed boundary condition, the maximum positive bending moment occurs at the middle of the waist slab and this moment is more or less uniform in the transverse direction and the average value of this moment is about 690 ft-lb/ft. The maximum negative bending moment occurs at the kink zones and this moment is more or less uniform and the average value of this moment is about 740 ft-lb/ft.



(a) Deflection

(b) Bending moment

(c) Axial force

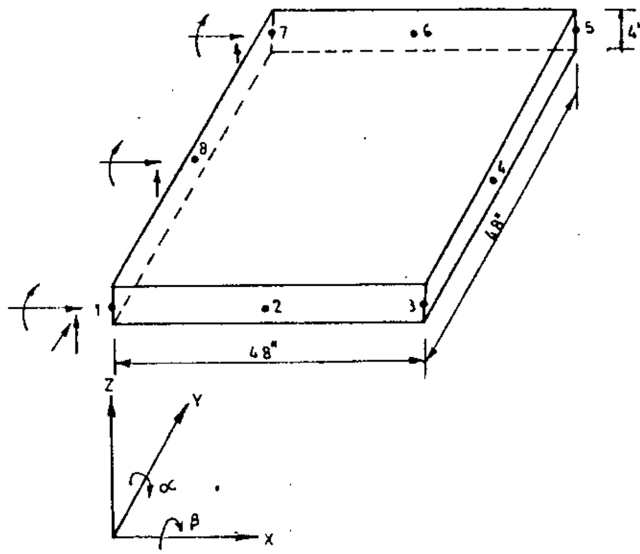
Fig. 3.17 Results of finite element analysis at landing of simply supported open well stair slab in the x-direction.

CHAPTER 4

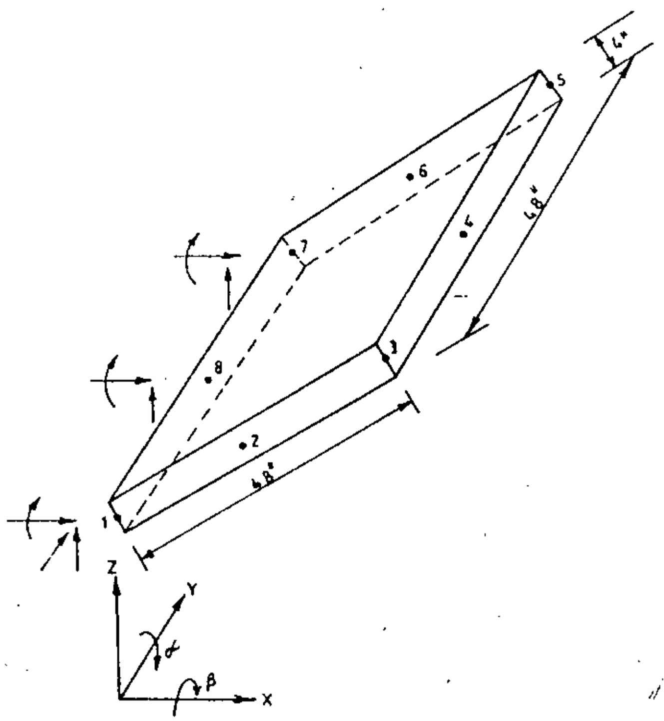
ANALYSIS AND DISCUSSIONS OF RESULTS

4.1 General

The first step to assess the results obtained from the general thick shell computer program was to check the equilibrium of segments of the structure. In doing that it was observed that the out of plane shear stress was oscillatory in nature and unusually high in magnitude. To examine this further, two simple slabs were analysed. In Fig. 4.1(a) a horizontal cantilever slab is shown. The directions which were restrained are also shown in the figure. It was found from the computer results that the stresses in the nodal plane comprising the nodes 1,2 and 3 and the stresses in the nodal plane comprising 7,6 and 5 were the same respectively. As such the stresses in the line of nodes, 1,2, and 3 only were considered. In Fig. 4.2 the bending moment and shear force diagrams from the strength of material theories are drawn and the results from the finite element analysis are shown in the corresponding diagram. It is seen from the diagram that the variation in the bending moment is reasonable considering that only one element was used in the analysis. In the case of out of plane shear stresses the variation is unacceptable, although the resultant maximum value is quite reasonable. For further

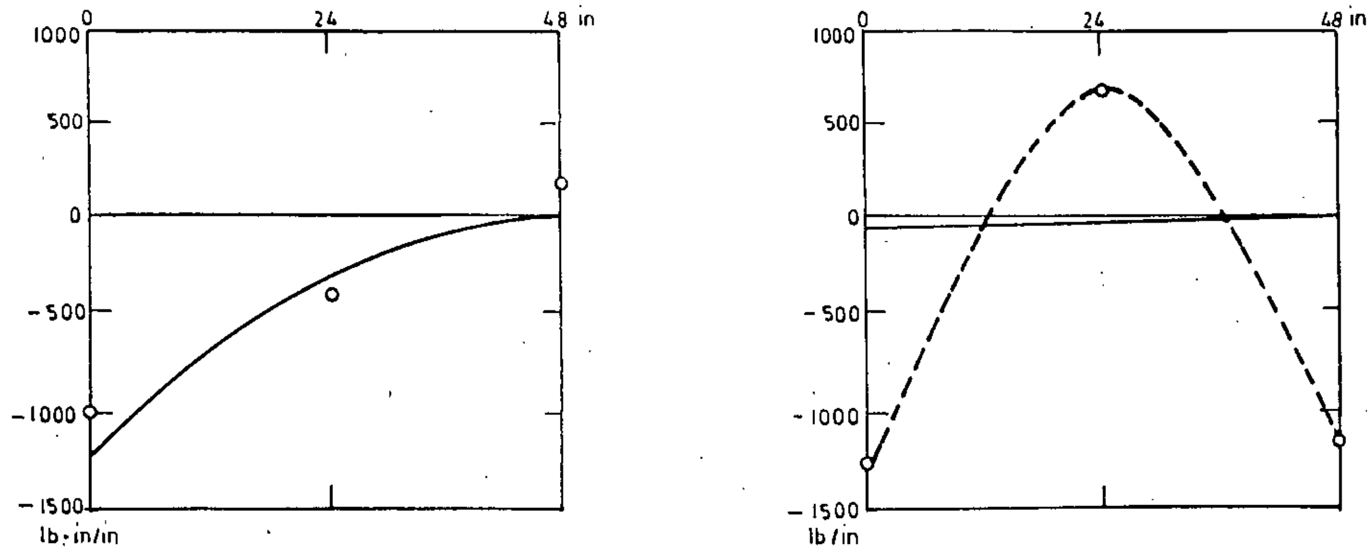


a) Horizontal cantilever slab.



b) Inclined cantilever slab at an angle 30°.

Fig. 4.1 Two simple slabs.



a) Conventional bending moment diagram, the points represent the result from finite element analysis.

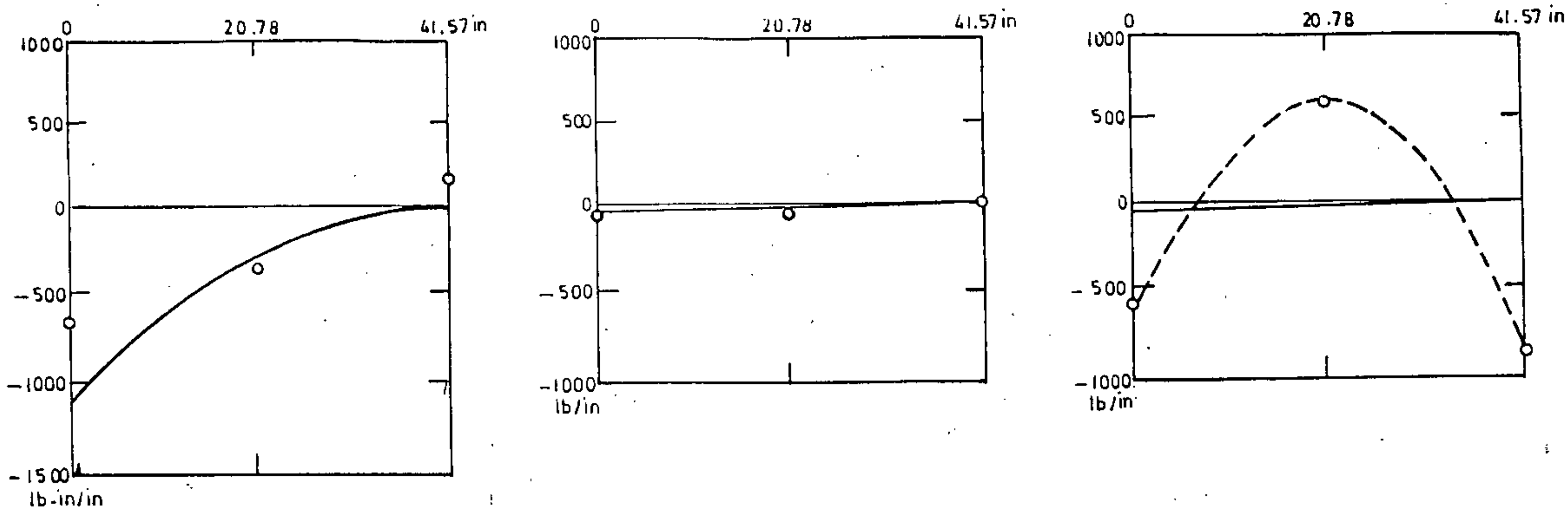
b) Conventional shear force diagram, the points represent the result from finite element analysis

Fig. 4.2 Bending moment and shear force diagram per in. of horizontal cantilever slab.

insight, another simple slab, Fig. 4.1(b) was analysed. In Fig. 4.3 the bending moment, axial force and shear force diagrams from the strength of material theories are drawn and the results from the finite element analysis are shown in the corresponding diagram. In this example the variations for bending moment and axial force are reasonable, But the out of plane shear stress variation is once again unacceptable. It was concluded from these investigations that the spurious behaviour of the out of plane shear stress was limited to this particular stress component and it was not disturbing the bending or inplane stress components. The out of plane shear stresses calculated from finite element analysis was therefore conveniently ignored and values easily calculated from shear consiserations were taken in checking the internal equilibrium of segments of the structures.

4.2 Check for Equilibrium of the Stair Slab with Fixed Boundary Condition

The two segments selected for checking equilibrium are shown in Fig. 4.4. The segments cover the most critical zones having maximum bending moments and maximum axial forces. The total bending moment and total axial force along the ends of the segments were calculated graphically.



a) Conventional bending moment diagram and the points represent the result from finite element analysis.

b) Conventional axial force diagram and the points represent the result from finite element analysis.

c) Conventional shears force diagram and the points represent the result from finite element analysis.

Fig. 4.3 Bending moment, axial force and shear force diagram per in. of inclined cantilever slab.

a) Equilibrium check on the first segment:

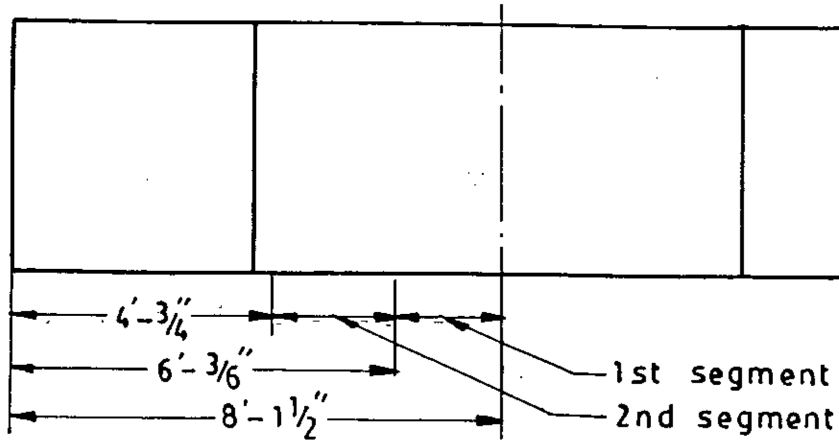
The free body of the first segment is shown in Fig. 4.5. The bending moments and axial forces shown in this figure are taken from the Fig. 3.7. The computed shear force is zero along the transverse centre line of the structure (line c.c. of Fig. 2.6). This is evident from symmetry considerations. The shear force at the other end can therefore be found out easily. Bending moment is plotted in Fig. 4.6. The total bending moment M_{AB} and M_{CD} are found to be 30800 lb-in and 12800 lb-in respectively. Axial forces are plotted in Fig. 4.7, from which the total axial force at CD is - 750 lb, (compressive). The resultant free body is then drawn as shown in Fig. 4.8, from which the following equilibrium checks are performed.

i) Check for equilibrium of moment

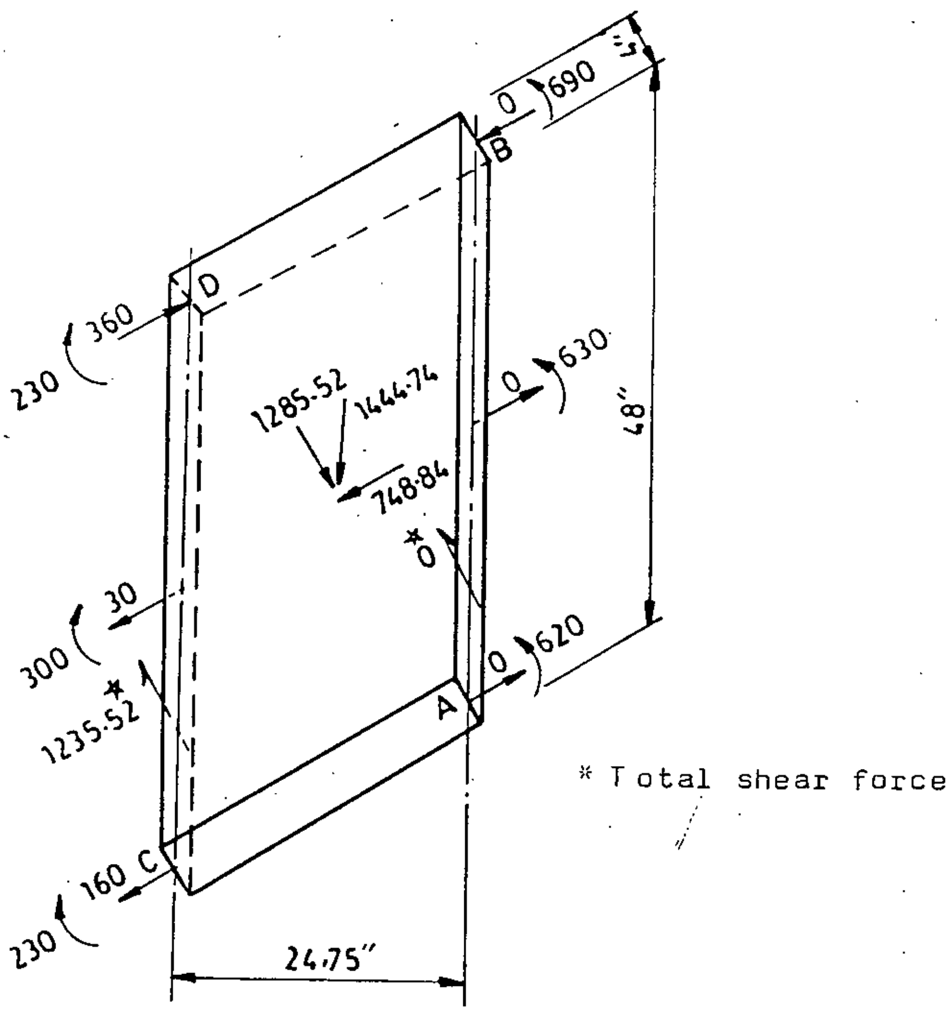
For equilibrium of moment the summation of moment about AB is considered.

$$\Sigma M_{AB} = 30800 - 12800 + 1235.5 \times \frac{28.94}{2} - 1235.5 \times 28.94$$
$$= 122 \text{ lb-in.}$$

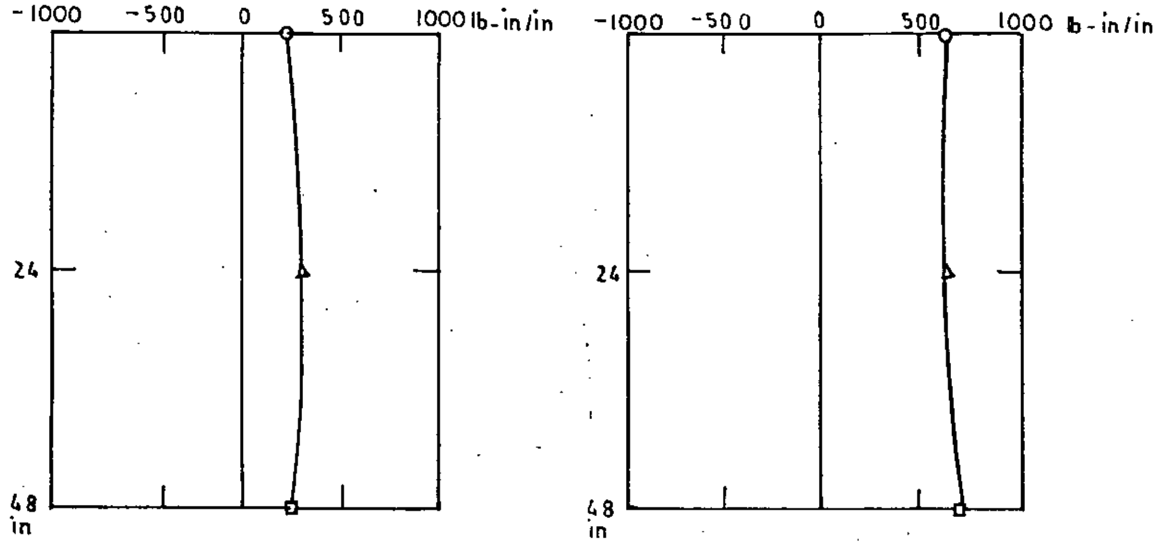
The percent error is of the order of 0.25% which is negligible. Hence equilibrium is satisfied.



F ig. 4.4 T he segements of fixed support stair slab.



F ig. 4.5 F ree body diagram of first segement.



(a) Along the edge CD   b) Along the edge AB

Fig. 4.6 Bending moment (about y-axis) diagrams.

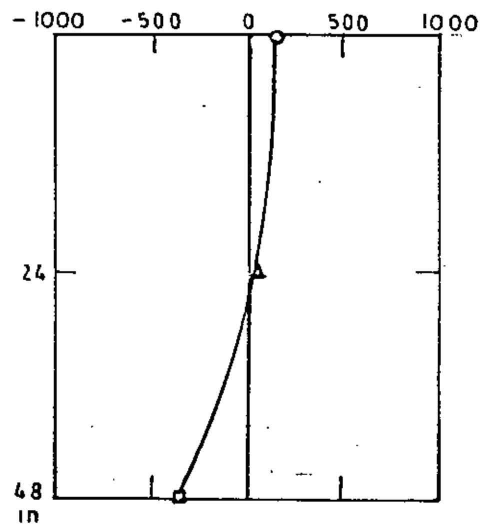


Fig. 4.7 Axial force diagram along the edge CD.

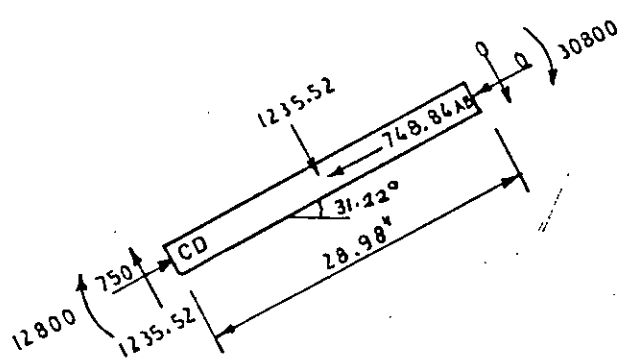


Fig. 4.8 Resultant free body diagram of first segement.

ii) Check for equilibrium of axial force

For checking equilibrium of axial forces

$$\Sigma F = 749 - 750 = -1$$

The percent error is negligible.

b) Equilibrium check on second segment

The free body of the second segment is shown in Fig. 4.9. The bending moments and axial forces shown in this figure are taken from the Fig. 3.7. The shear force is computed from statics and symmetry consideration. Bending moment is plotted in Fig. 4.10. The total bending moments M_{AB} and M_{CD} are found to be 13,600 lb-in and -37,800 lb-in respectively, here negative sign indicates top fibre is in tension. Axial forces are plotted in Fig. 4.11. The total axial forces F_{AB} and F_{CD} are found to be -750 lb and -1500 lb respectively. The resultant free body is then drawn as shown in Fig. 4.12, from which the following equilibrium checks are performed.

i) Check for equilibrium of moment

For equilibrium of moment the summation of moment about AB is considered.

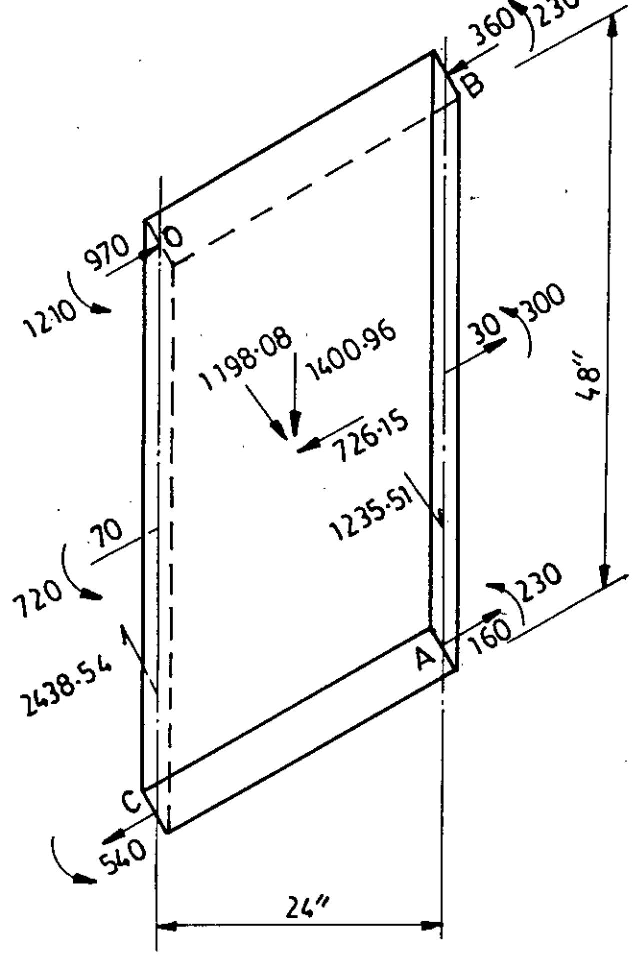
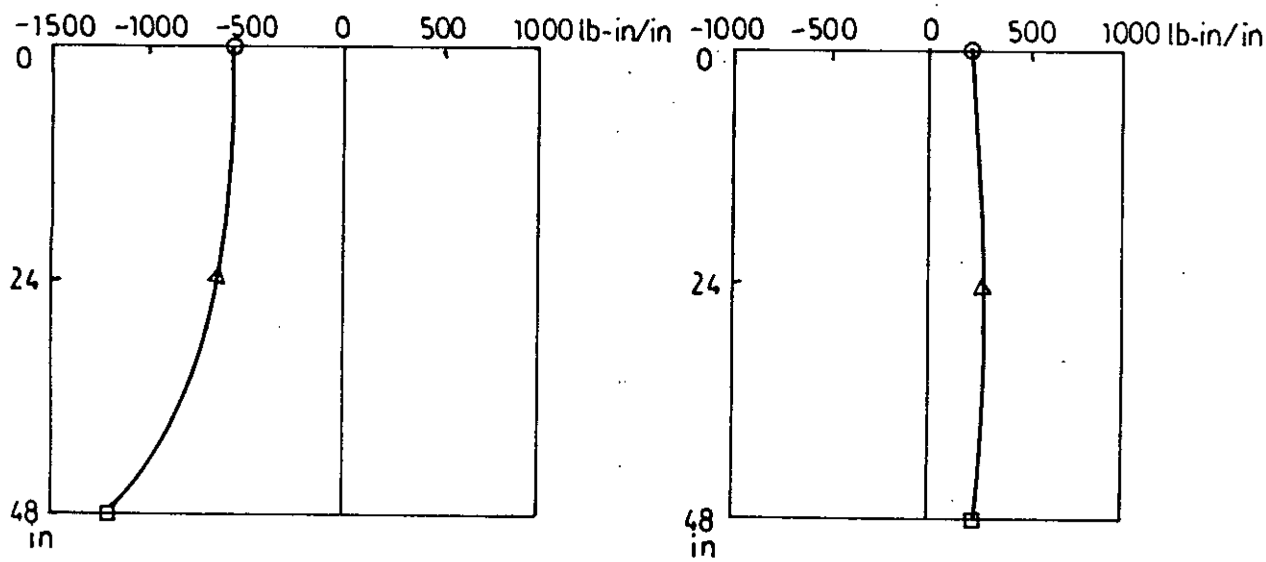


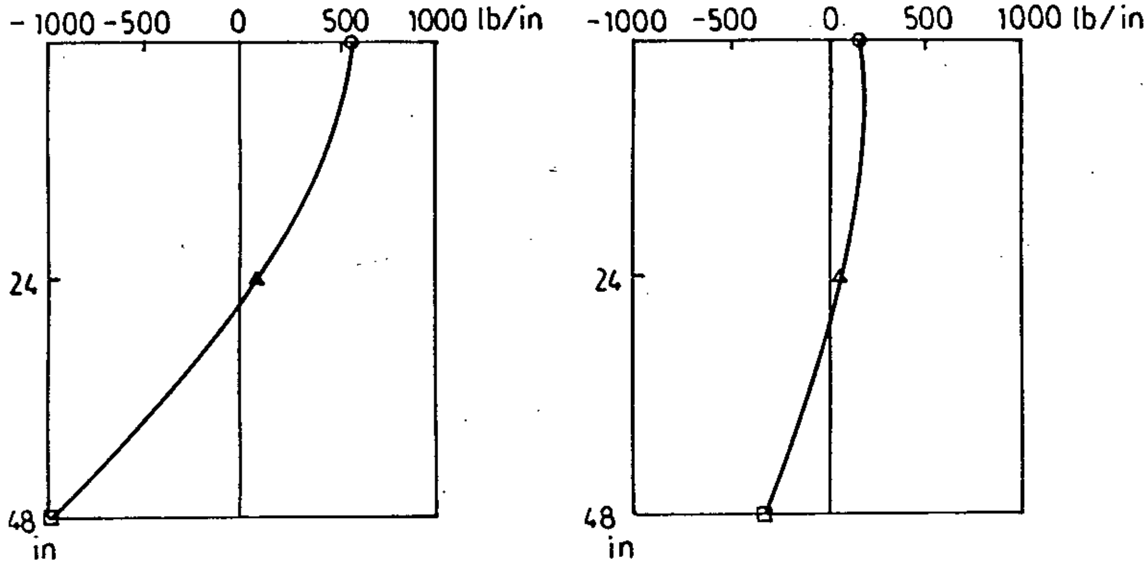
Fig. 4.9 Free body diagram of second segement.



a) Along the edge C D

b) Along the edge AB

Fig. 4.10 Bending moment (about y-axis) diagrams.



a) Along the edge C D b) Along the edge AB

Fig. 4.11 Axial force diagrams.

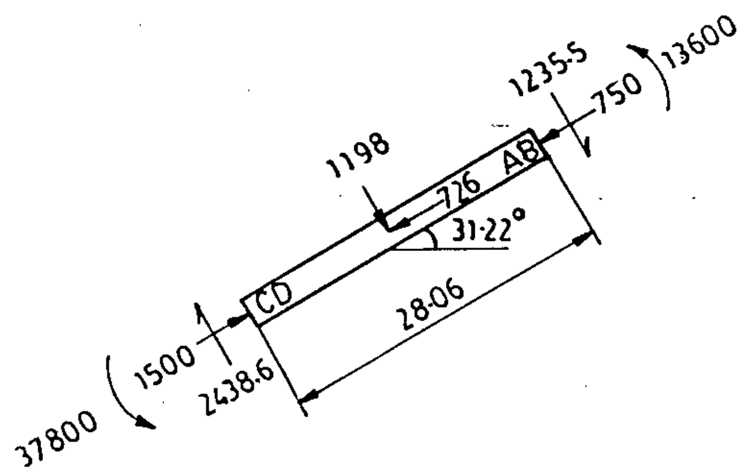


Fig. 4.12 Resultant free body diagram of second segment.

$$\Sigma M_{AB} = 113 \text{ lb-in}$$

The percent error is of the order of 0.15% which is negligible. Hence equilibrium is satisfied.

ii) Check for equilibrium of axial forces

$$\Sigma F = 24 \text{ lb}$$

The percent error is of the order of 1.5% which is small enough to be considered as negligible.

4.3 Check for Equilibrium of Stair Slab with Simply Supported Boundary Condition

The three segments selected for checking equilibrium are shown in Fig. 4.13.

The segments cover the most critical zones having maximum bending moments and maximum axial forces. For the segment selected at the landing only horizontal equilibrium was checked. The total bending moment and total axial force along the ends of the segments were once again calculated graphically.

a) Equilibrium check on first segment

The free body of the first segment is shown in Fig. 4.14. The bending moments, axial forces shown in this figure are

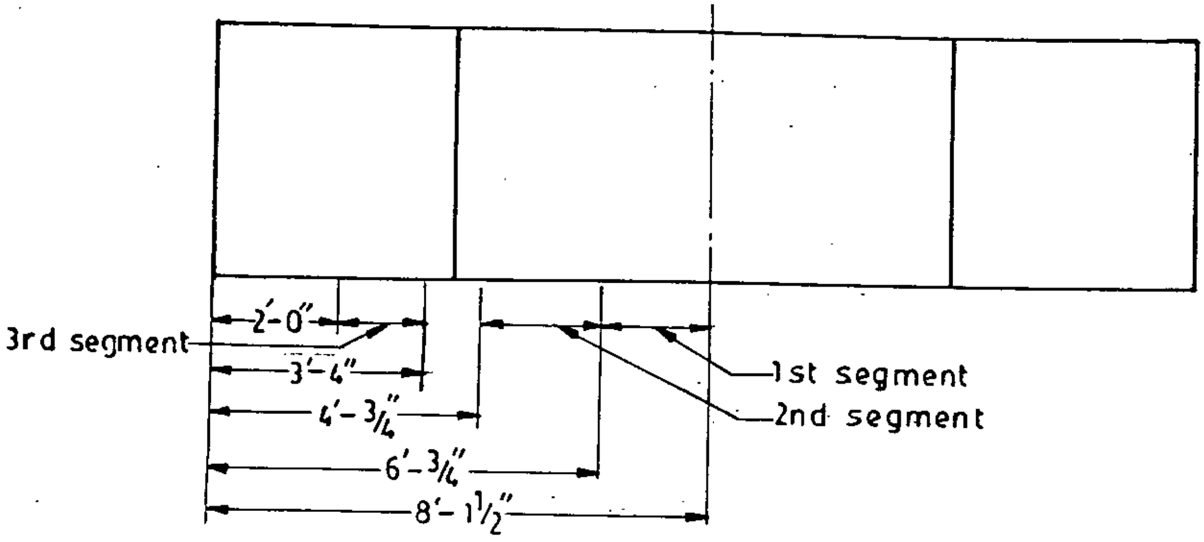


Fig. 4.13 The segments of simple support stair slab.

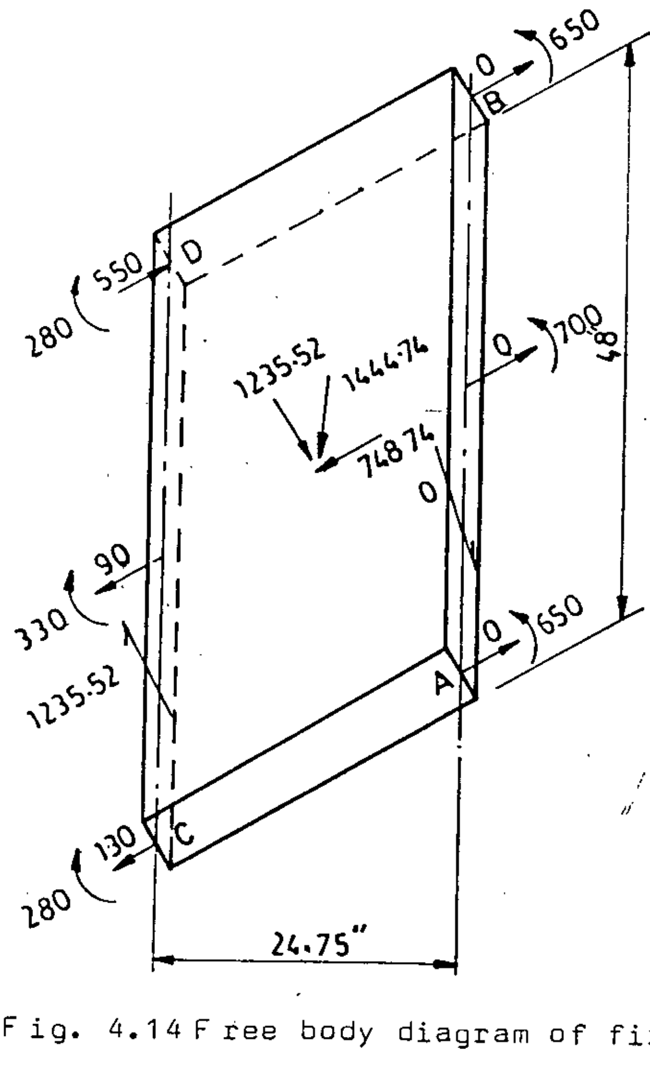


Fig. 4.14 Free body diagram of first segment.

taken from Fig. 3.9. As before computed shear force is zero along the transverse centreline of the structure and the shear force at the other end can be calculated easily. Bending moment is plotted in Fig. 4.15. The total bending moment M_{AB} and M_{CD} are found to be 32700 lb-in and 14800 lb-in respectively. Axial forces are plotted in Fig. 4.16, from which the total axial force at CD is -750. The resultant free body is then drawn as shown in Fig. 4.17, from which the following equilibrium checks are performed.

i) Check for equilibrium of moment

For equilibrium of moment the summation of moment about AB is considered.

$$\Sigma M_{AB} = 22.0 \text{ lb-in}$$

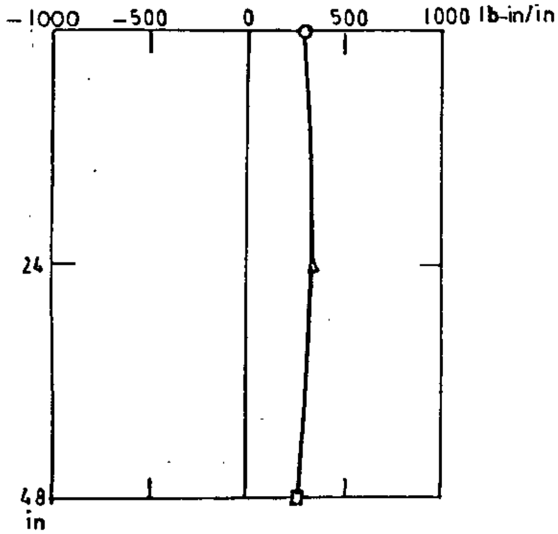
This is negligible compared to the total positive or total negative bending moment.

ii) Check for equilibrium of axial force

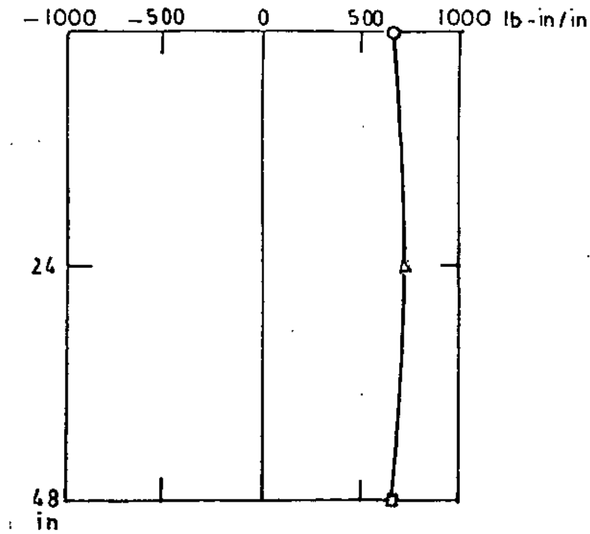
For checking equilibrium of axial forces

$$\Sigma F = -1$$

This is negligible compared to the total compressive axial force.



a) Along the edge C D



b) Along the edge AB

Fig. 4.15 Bending moment (about y-axis) diagram.

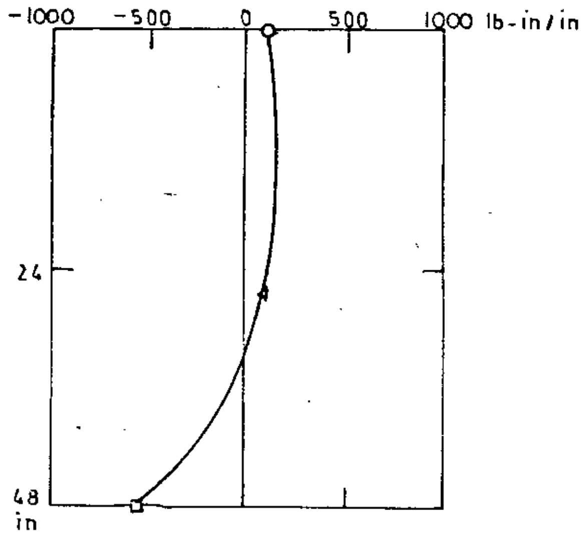


Fig. 4.16 Axial force diagram along the edge C D.

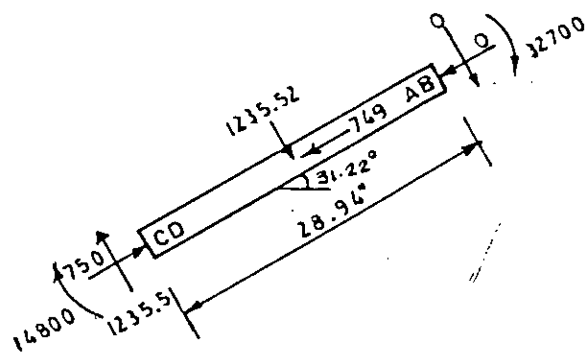


Fig. 4.17 Resultant free body diagram of first segement.

b) Equilibrium check on Second Segment

The free body of the second element is shown in Fig. 4.18. The bending moments, axial forces shown in this figure are taken from Fig. 3.9. The shear force is computed from statics and symmetry considerations. Bending moment is plotted in Fig. 4.12. The total bending moments M_{AB} and M_{CD} are found to be 14700 lb-in and 36800 respectively. Axial forces are plotted in Fig. 4.20. The total axial forces F_{AB} and F_{CD} are found to be - 750 lb and -1500 lb- respectively. The resultant free body is then drawn as shown in Fig. 4.21, from which the following equilibrium checks are performed.

i) Check for equilibrium of moment

For equilibrium of moment the summation of moment about AB is considered.

$$\Sigma M_{AB} = - 22 \text{ lb.}$$

which is negligible compared to the total positive or total negative moment i.e., equilibrium is satisfied.

ii) Check for equilibrium of axial force

For checking equilibrium of axial forces

$$\Sigma F = 24 \text{ lb}$$

The percent error is of the order of 1.5% which is negligible, hence equilibrium is satisfied.

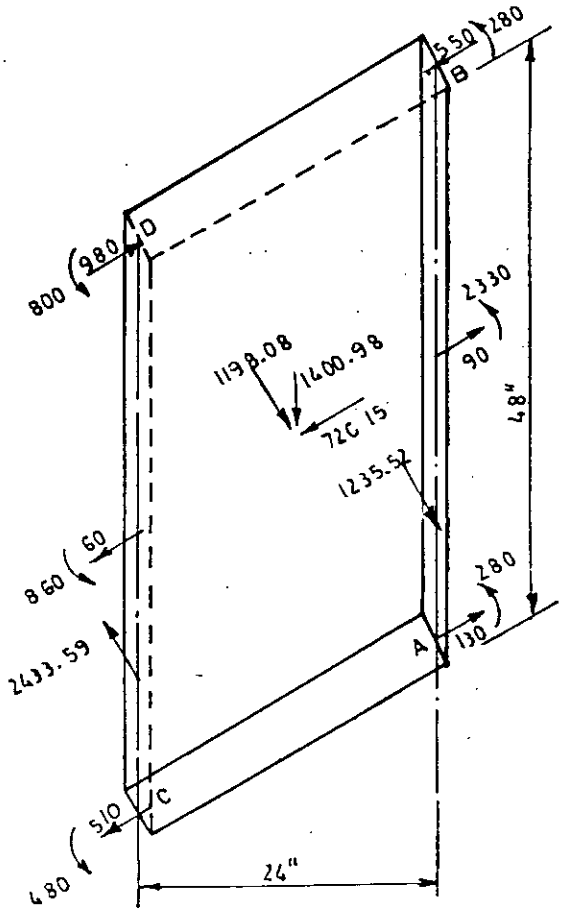


Fig. 4.18 Free body diagram of second segment.

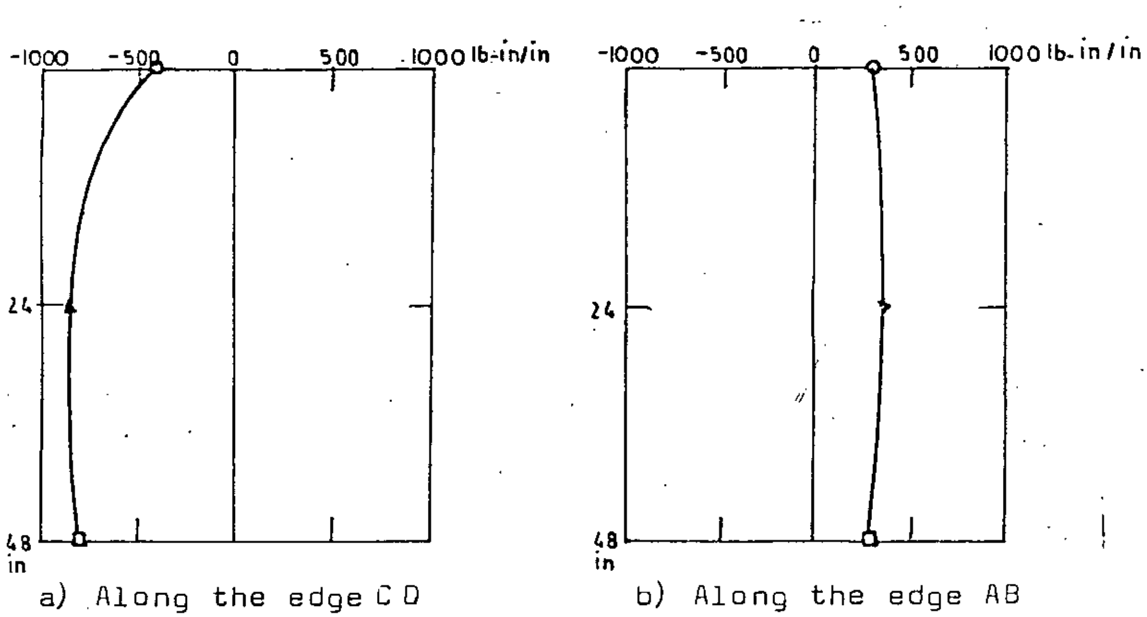
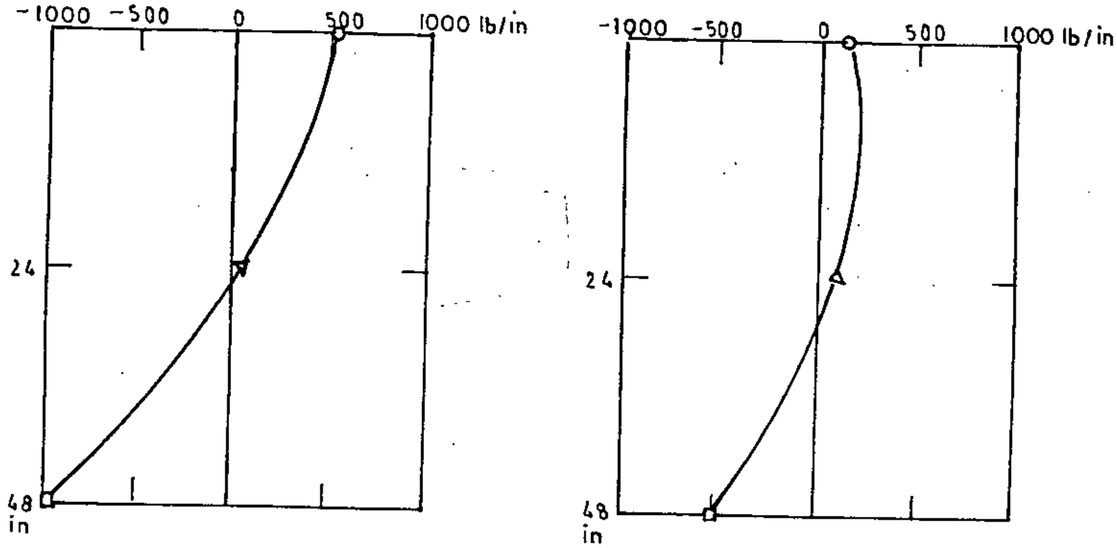


Fig. 4.19 Bending moment (about y-axis) diagrams.



a) Along the edge CD

b) Along the edge AB

Fig. 4.20 Axial force diagrams.

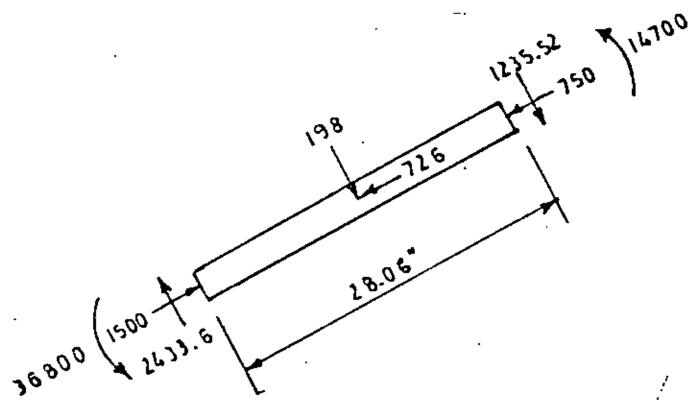


Fig. 4.21 Resultant free body diagram of second segement.

c) Equilibrium check on third segment

The free body of the third element is shown in Fig. 4.22. In the case of this segment only the equilibrium of axial force is checked. Axial forces are plotted in Fig. 4.23. The total axial forces F_{AB} and F_{CD} are found to be -312 lb and -300 lb respectively. The resultant free body is then drawn as shown in Fig. 4.24, from which the equilibrium check for axial forces is performed as shown below:

$$\Sigma F = 12 \text{ lb}$$

The percent error is of the order of 4%. It is noted that the axial forces themselves are very small, and as such the error is not of much significance.

4.4 Check for Equilibrium of Openwell Stair Slab with Fixed Boundary Concition

The two segment selected for checking equilibrium are shown in Fig. 4.25. The segment cover the most critical zones having maximum bending moments and axial forces. The total bending moment and axial force along the ends of the elements were once again calculated graphically. The out of plane shear forces has been calculated from the elementary theories and symmetry considerations.

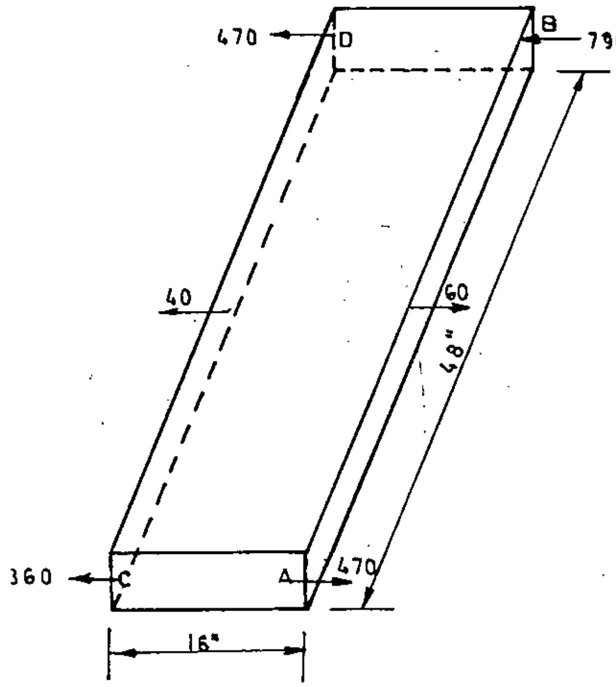
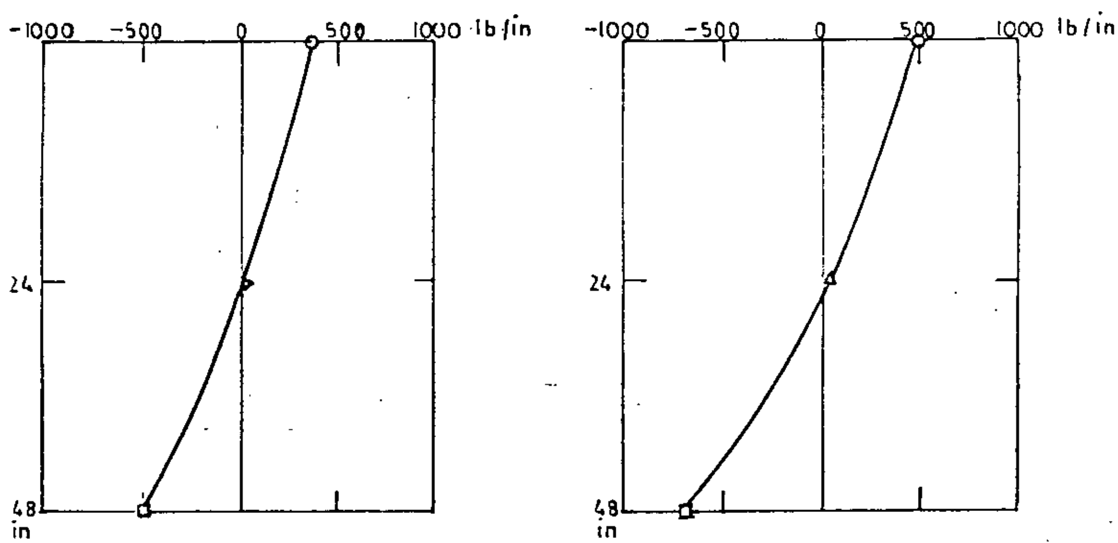


Fig. 4.22 Free body diagram of third segment.



a) Along the edge CD b) Along the edge AB

Fig. 4.23 Axial force diagrams.

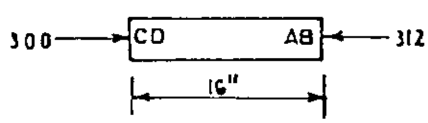


Fig. 4.24 Resultant free body diagram of third segment.

a) Equilibrium Check on First Segment

The free body of the first segment is shown in Fig. 4.26. The bending moments and axial forces shown in this figure are taken from Fig. 3.13. Bending moment is plotted in the Fig. 4.27. The total bending moment M_{AB} and M_{CD} are found to be 30400 lb-in and 12500 lb-in. respectively. Axial forces are plotted in Fig. 4.28, from which the total axial force at CD is -750 lb. The resultant free body is then drawn as shown in Fig. 4.29, from which following equilibrium checks are performed.

i) Check for equilibrium of moment

For equilibrium of moment the summation of moment about AB is considered.

$$\Sigma M_{AB} = 22 \text{ lb-in.}$$

This is negligible with compared to the total positive or negative moments.

ii) Check for equilibrium of axial force

For checking equilibrium of axial forces

$$\Sigma F = -1 \text{ lb}$$

This is negligible compared to the total compressive force.

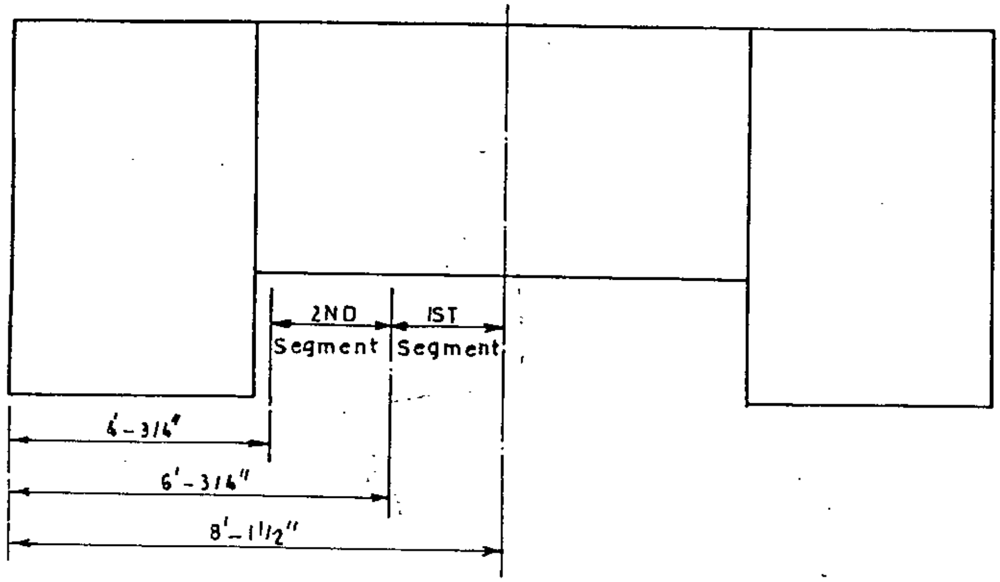


Fig. 4.25 The segments of fixed support open well stair slab.

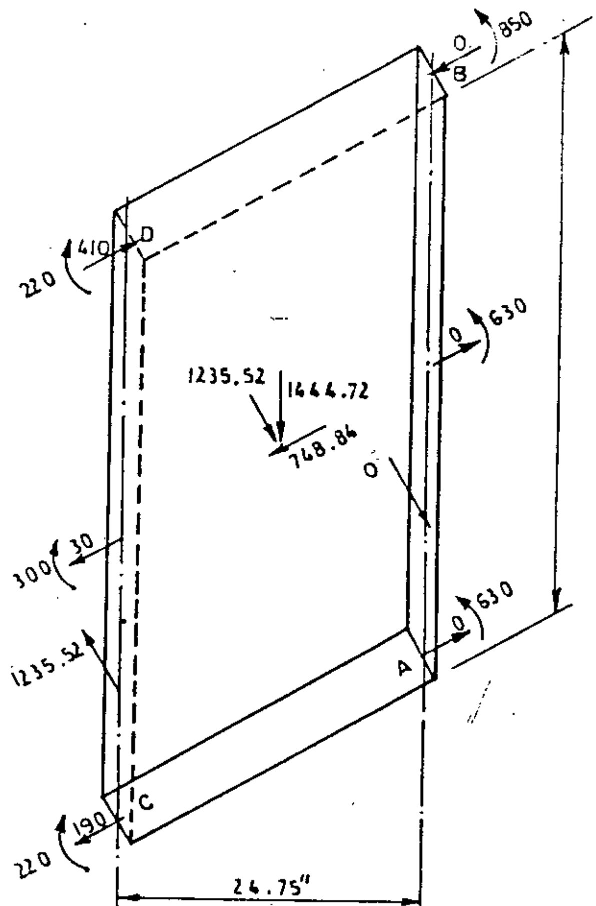
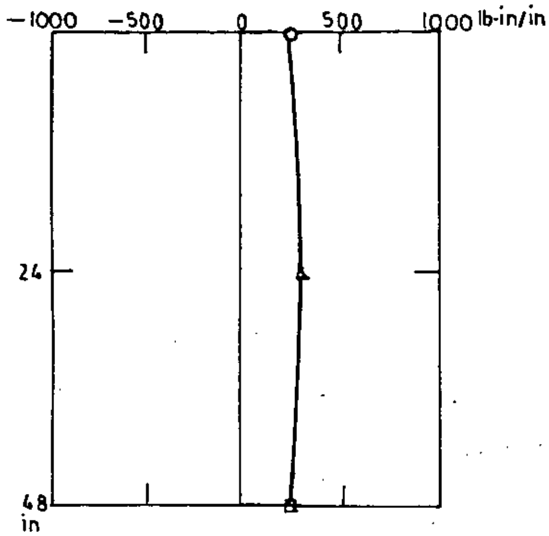
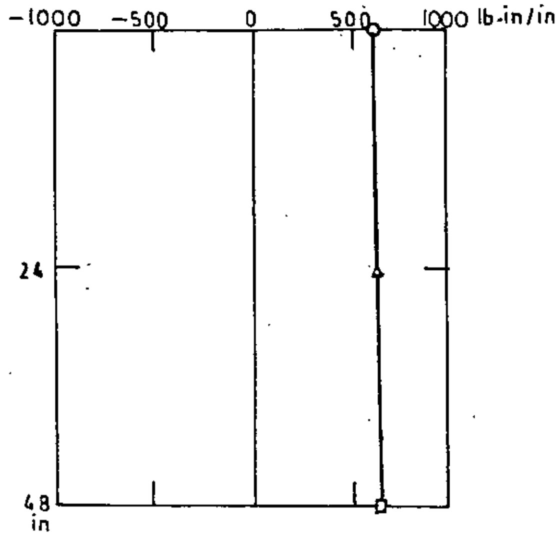


Fig. 4.26 Free body diagram of first segment.



a) Along the edge C D



b) Along the edge AB

Fig. 4.27 Bending moment (about y-axis) diagrams.

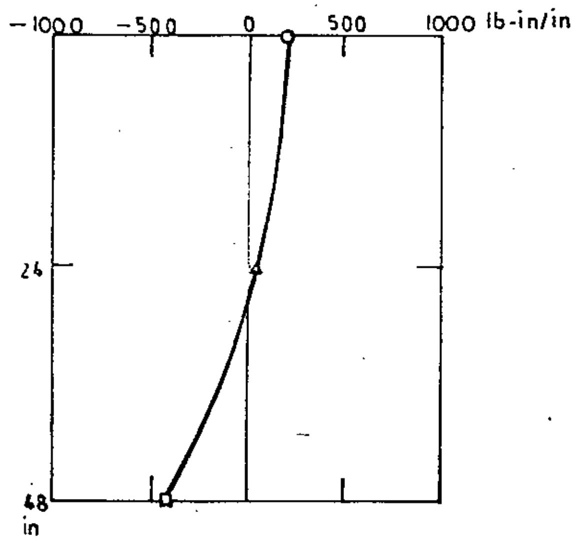


Fig. 4.28 Axial force diagram along the edge C D.

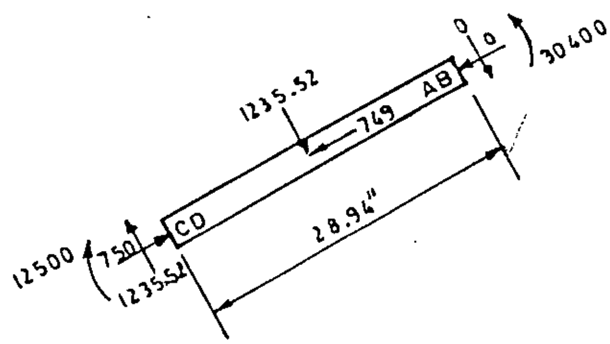


Fig. 4.29 Resultant free body diagram of first segment.

b) Equilibrium check on second segment

The free body of the second segment is shown in Fig. 4.30. The bending moments and axial forces shown in this figure are taken from Fig. 3.13. Bending moment is plotted in Fig. 4.31. The total bending moments M_{AB} and M_{CD} are found to be 13500 lb-in and -37900 lb-in respectively. Axial force are plotted in Fig. 4.32. The total axial forces F_{AB} and F_{CD} are found to be -750 lb and -1500 lb respectively. The resultant free body is then drawn as shown in Fig. 4.33, from which the following checks are performed.

i) Check for equilibrium of moment

For equilibrium of moment the summation of moment about AB is considered.

$$\Sigma M_{AB} = 78 \text{ lb-in.}$$

which is very small compared to the total positive or total negative moment.

ii) Check for equilibrium of axial force

For checking equilibrium of axial forces

$$\Sigma F = 24 \text{ lbs}$$

The percent error is of the order of 1.5% compared to the total compressive forces.

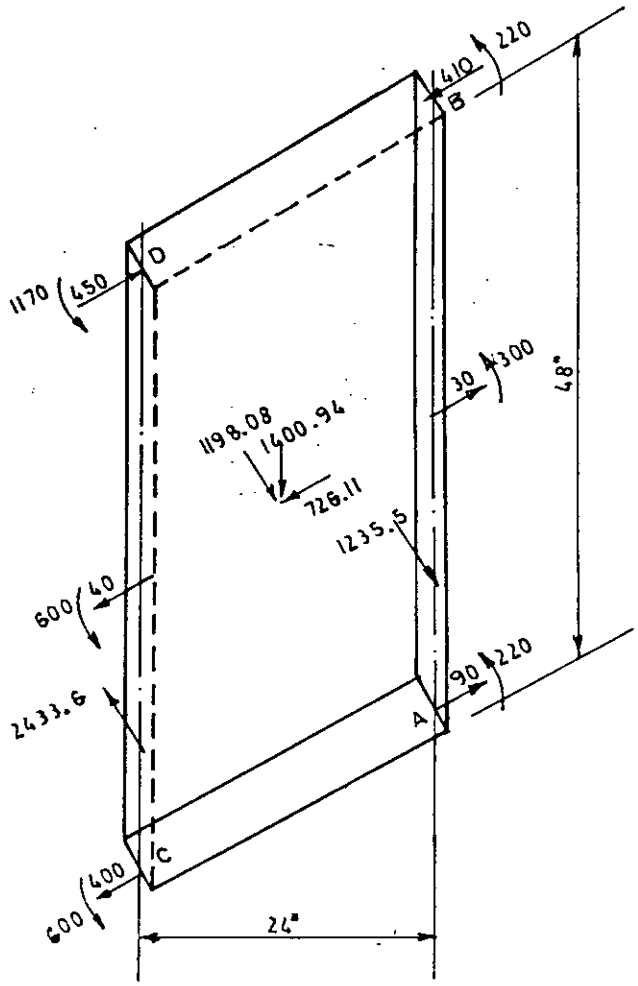
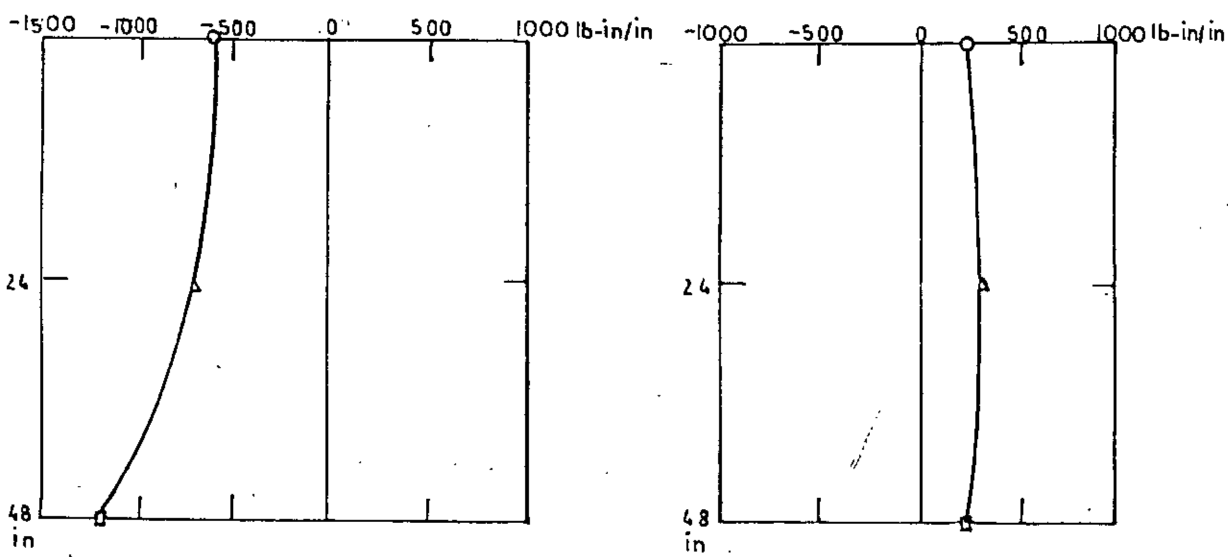


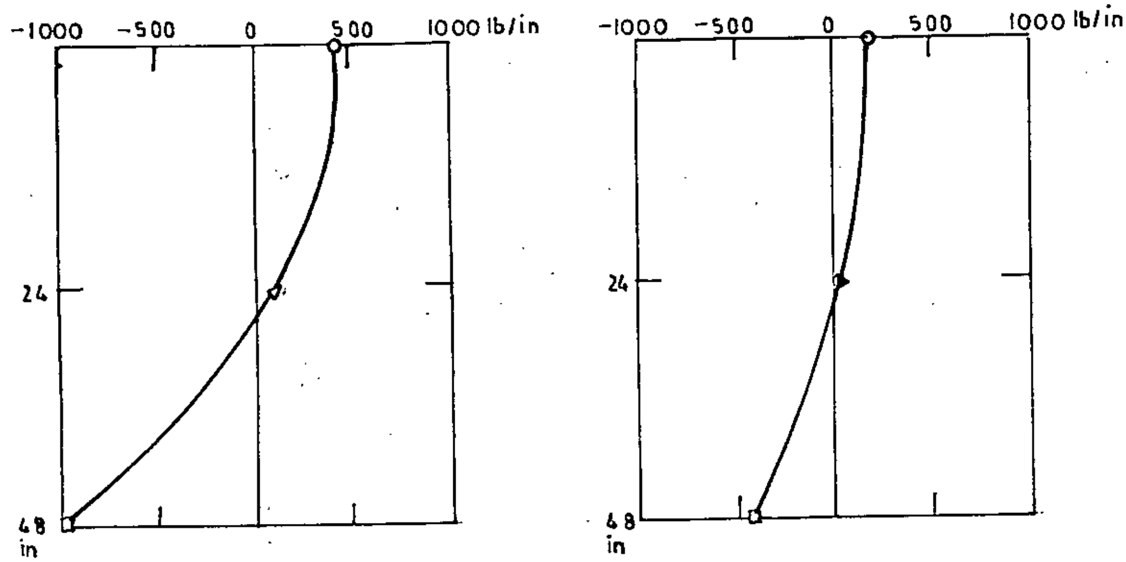
Fig. 4.30 Free body diagram of second segment.



a) Along the edge CD

b) Along the edge AB

Fig. 4.31 Bending moment (about y-axis) diagram.



a) Along the edge CD

b) Along the edge AB

Fig. 4.32 Axial force diagrams.

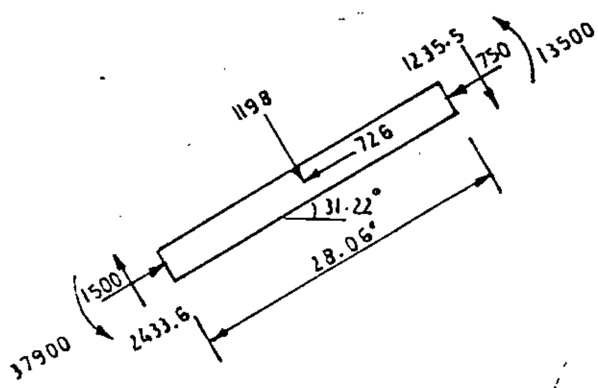


Fig. 4.33 Resultant free body diagram of second segment.

4.5 Check for Equilibrium of Open Well Stair Slab with Simply Supported Boundary Condition

The three segments selected for checking equilibrium are shown in Fig. 4.34. The segment cover the most critical zones having maximum bending moment and maximum axial forces. For the segment selected at the landing only the horizontal equilibrium was checked.

a) Equilibrium check on first segment

The free body of the first segment is shown in Fig. 4.35. The bending moments and axial forces shown in this figure are taken from Fig. 3.16. The shear forces has again been calculated from elementary theories and symmetry considerations. Bending moment is plotted in Fig. 4.36. The total bending moment M_{AB} and M_{CD} are found to be 33000 lb-in and 15100 lb-in respectively. Axial forces are plotted in Fig. 4.37, from which the total axial force at CD is -750. The resultant free body is then drawn as shown in Fig. 4.38, from which the following equilibrium checks are performed.

i) Check for equilibrium of moment

For equilibrium of moment the summation of moment about AB is considered.

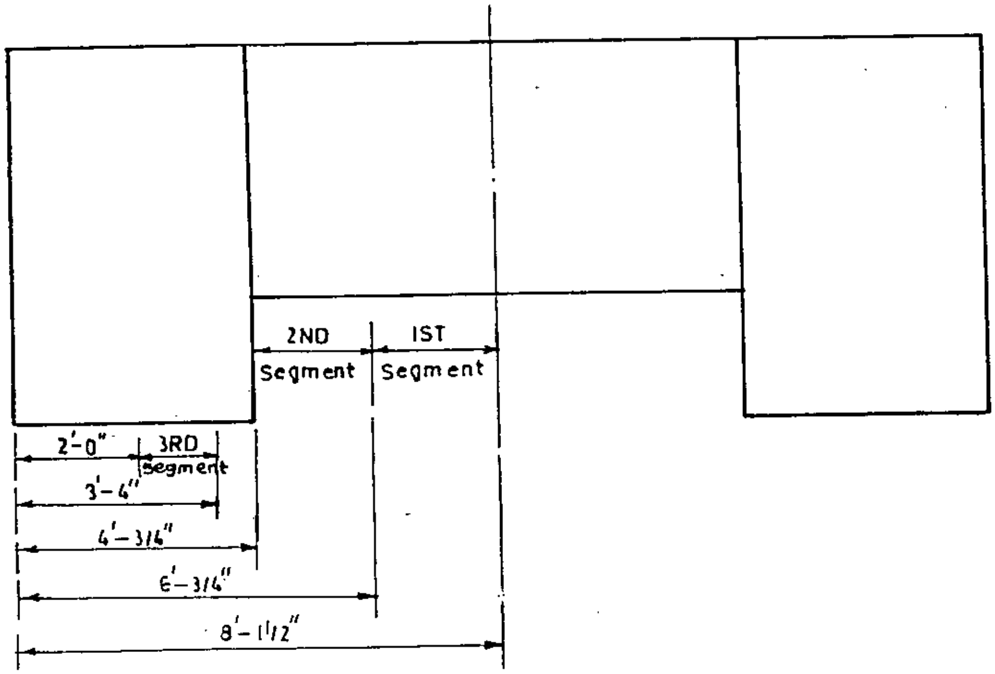


Fig. 4.34 The segements of simple support stair slab.

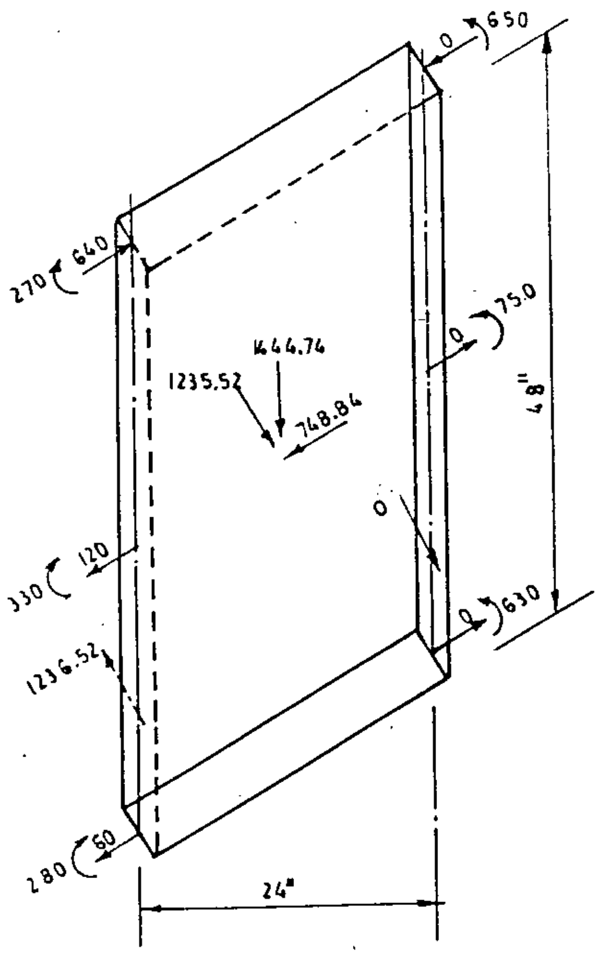
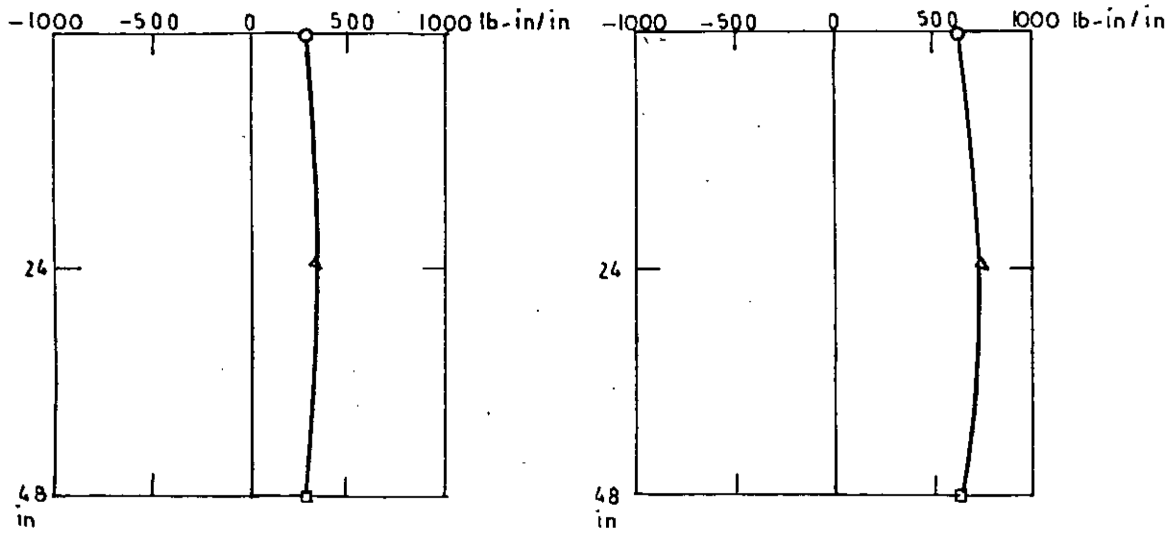


Fig. 4.35 Free body diagram of first segment.



a) Along the eedge C D

b) Along the edge AB

Fig. 4.36 Bending moment (about y-axis) diagrams.

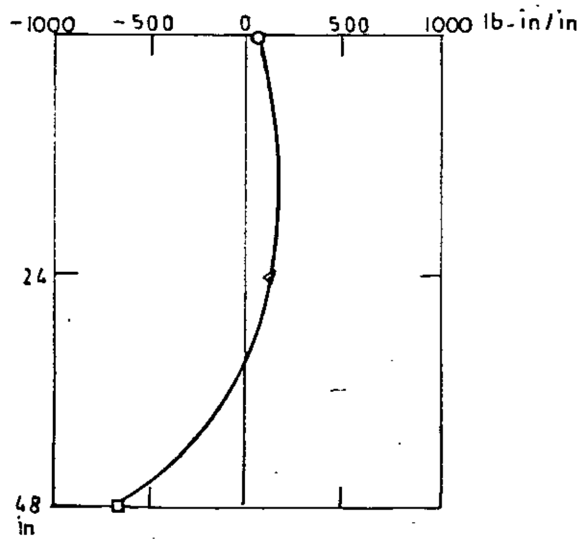


Fig. 4.37 Axial force diagram along the edge C D of first segment.

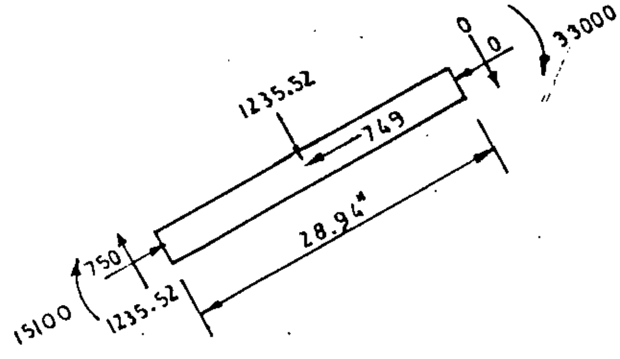


Fig. 4.38 Resultant free body diagram of first segment.

$$\Sigma M_{AB} = 22 \text{ lb-in.}$$

This is negligible compared to the total positive or total negative bending moment.

ii) Check for equilibrium of axial force

For checking equilibrium of axial forces

$$\Sigma F = -1 \text{ lb}$$

This is negligible compared to the total compressive axial force.

b) Equilibrium check on Second segment

The free body of the second element is shown in Fig. 4.39. The bending-moments and axial forces shown in this figure are taken from Fig. 3.16. The shear force has been calculated from elementary theories and symmetry considerations. Bending moment is plotted in Fig. 4.40. The total bending moment M_{AB} and M_{CD} are found to be 37100 lb-in and 14400 lb-in respectively. Axial forces are plotted in the Fig. 4.41. The total axial forces F_{AB} and F_{CD} are found to be - 750 lb and 1500 lb respectively. The resultant free body is then drawn as shown in Fig. 4.42, from which following equilibrium checks are performed.

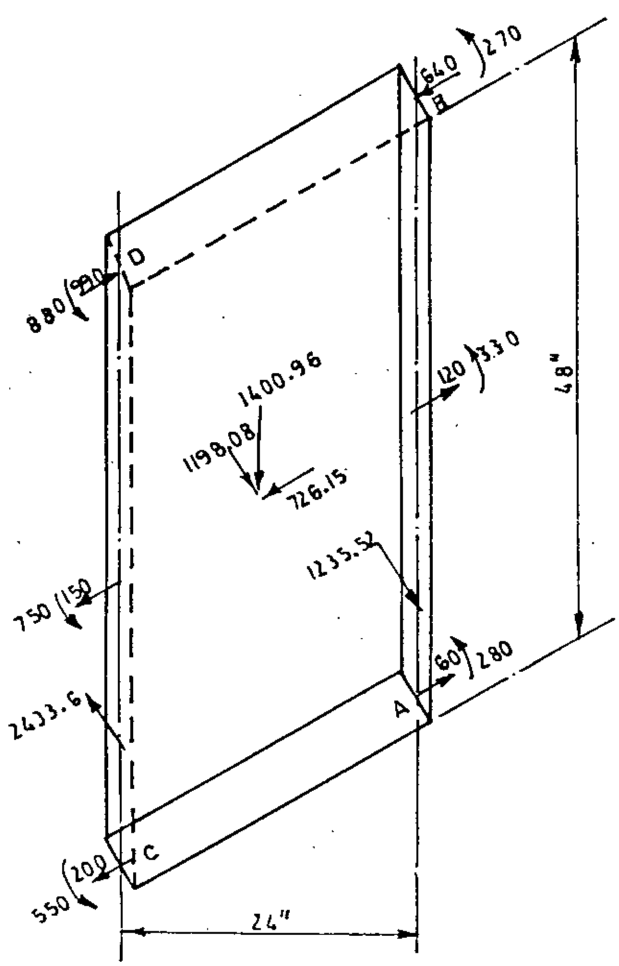


Fig. 4.39 Free body diagram of second segment.

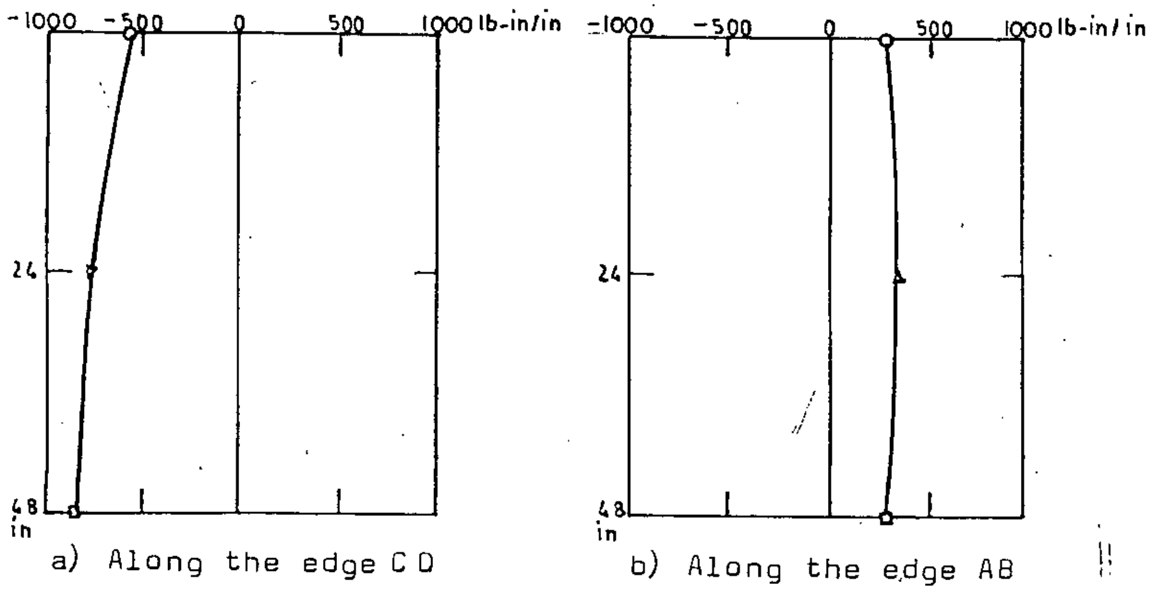


Fig. 4.40 Bending moment (about y-axis) diagrams.

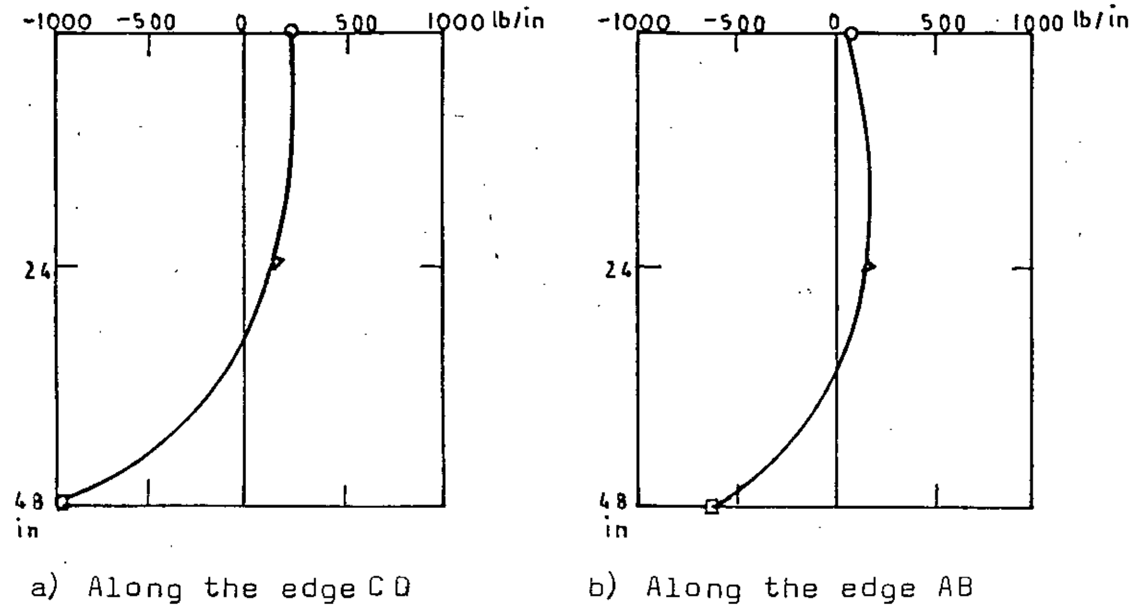


Fig. 4.41 Axial force diagrams.

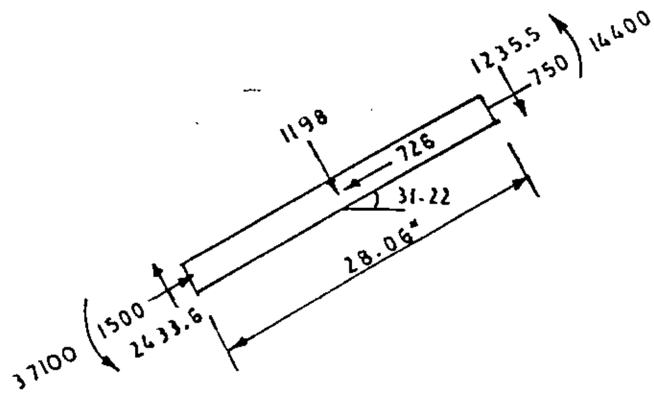


Fig. 4.42 Resultant free body diagram of second segment.

i) Check for equilibrium of moment

For equilibrium of moment the summation of moment about AB is considered

$$\Sigma M_{AB} = 22 \text{ lb-in}$$

This is negligible compared to the total positive or negative bending moment.

ii) Check for equilibrium of axial force

For checking equilibrium of axial forces

$$\Sigma F = 24 \text{ lb}$$

The percent error is of the order of 1.5% which is negligible compared to total compressive force.

c) Equilibrium check on third segment

The free body of the third segment is shown in Fig. 4.43. In the case of this segment only the equilibrium of axial force is checked. Axial forces are plotted in Fig. 4.44. The total axial forces F_{AB} and F_{CD} are found to be -612 lb and -600 lb respectively. The resultant free body is then drawn as shown in Fig. 4.45, from which equilibrium check for axial forces is performed as follows:

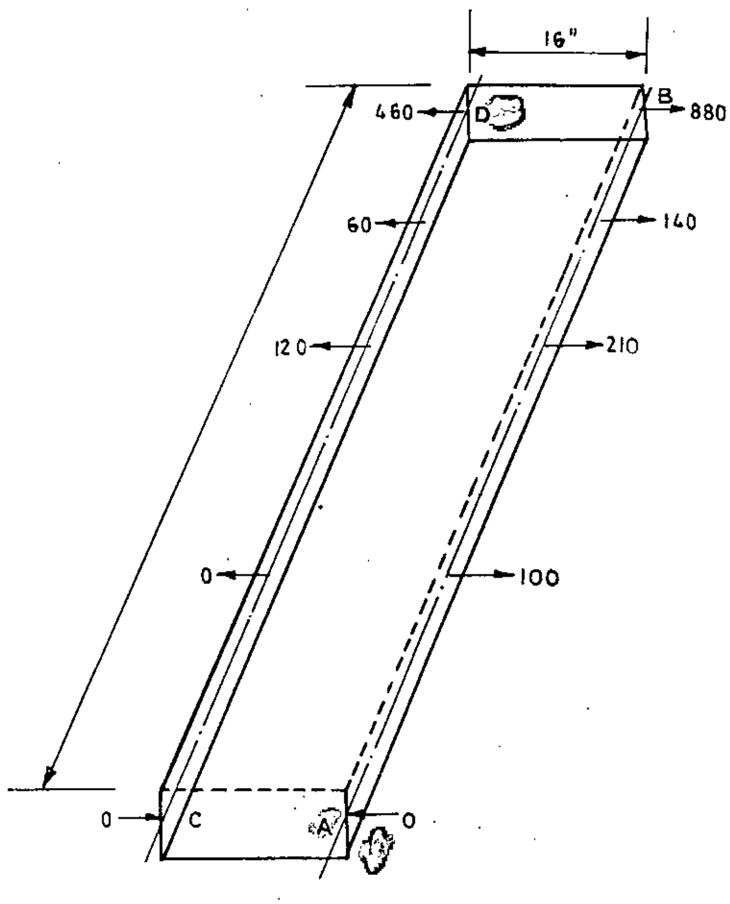
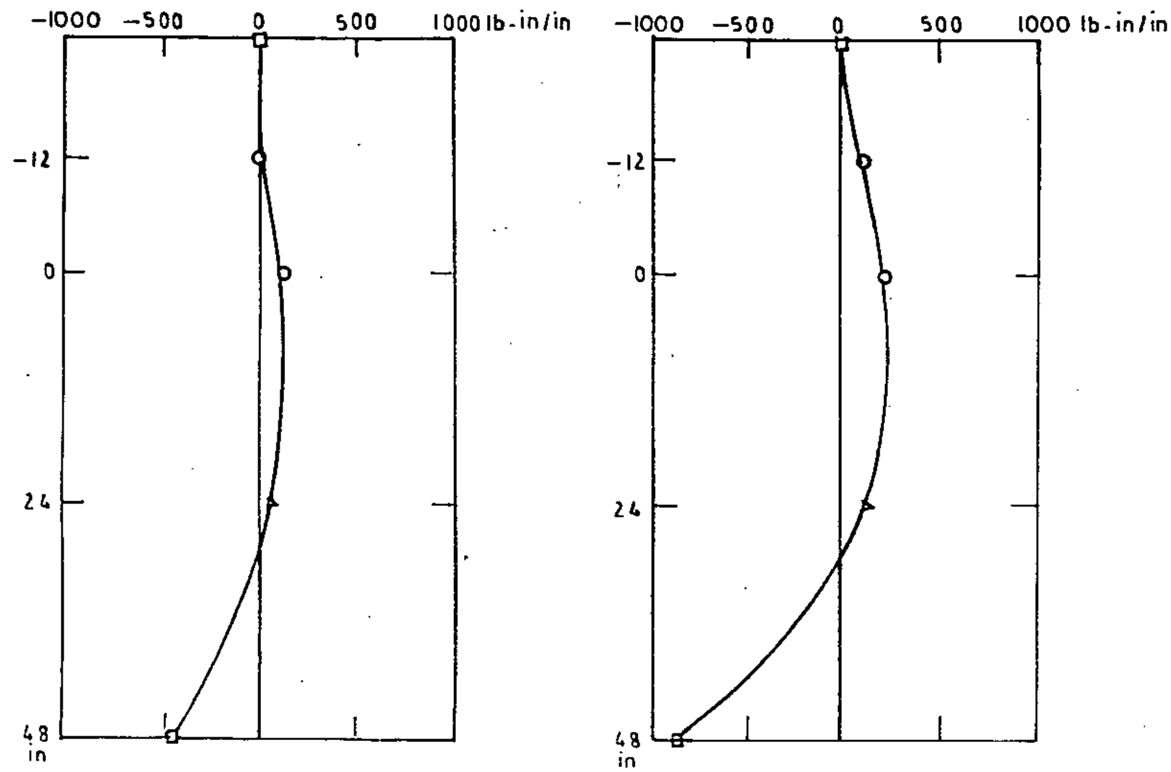


Fig. 4.43 Free body diagram of third segment.



a) Along the edge CD b) Along the edge AB

Fig. 4.44 Axial force diagrams.

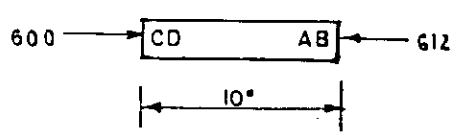


Fig. 4.45 Resultant free body diagram of third segment.

$\Sigma F = 12$ lb.

The percent error is of the order of 2% . The axial stresses themselves being very small, this error can be neglected.

4.6 Accuracy of Results

From the discussion of the above articles it is seen that the maximum level of error is of the order of 2% , but in the maximum cases this error level is below 1% , which is practically negligible. From this it is clear that equilibrium of structure is satisfied. Hence now it can confidently be concluded that the stresses obtained from finite element analysis are sufficiently accurate. The percent error of the results are summarized in Table 4.1.

Table 4.1 Percent error of the stresses obtained by analysis

Type of stairs	Type of support	No. of segments	Percent error	
			Moment	Axial force
Two flight stair	Fixed support	1st	0.25	0.0
		2nd	0.15	1.5
	Simple support	1st	0.0	0.0
		2nd	0.0	1.5
		3rd	-	4
Two flight open well stair	Fixed support	1st	0.0	0.0
		2nd	0.0	1.5
	Simple support	1st	0.0	0.0
		2nd	0.0	1.5
		3rd	-	2.0

CHAPTER 5

IMPROVEMENT IN DESIGN PROCEDURE

5.1 General

From finite element analysis it was observed that the stress distribution in the stair slab is not as simple as in the simply supported one way slab. Various equilibrium checks have demonstrated that the stresses are of acceptable accuracy. This is essential for the development of a rational design procedure.

5.2 Behaviour of Two Flight Stair Slab

It was seen from the Art. 3.7 that the points of contraflexure of all the two flight stairs with five boundary conditions are within the waist slab. In case of simple support it was seen that the effect of boundary on the position of the points of contraflexure, the maximum positive and negative bending moment and the maximum axial forces is very little. The main characteristic features of fixed supported and simply supported stair slab are summarized below.

a) Fixed supported stair slab

i) The distance between the points of contraflexure = 66% of horizontal distance of the waist slab.

ii) The average maximum positive moment
= 638 ft-lb/ft

iii) The average maximum negative moment
= 772 ft-lb/ft

iv) The average tension over the half width at the inner side of the lower kink = 340 lb/in.

v) The average compression over the one-third width at the outer side of the lower kink = 600 lb/in.

vi) The average tension over the one-third width at the outer side of the upper kink = 600 lb/in.

vii) The average compression over the half width at the inner side of the upper kink = 340 lb/in.

viii) The average tension over the half width at the inner side of 0.25 l from the lower kink = 100 lb/in, where l is the length of the waist slab.

ix) The average tension over the one-third width at the outer side of 0.25 l from the upper kink = 200 lb/in.

x) The average positive moment at a distance 0.25 l from the lower or the upper kink = 283 lb-in/in.

b) Simply supported stair slab

i) The distance between the points of contraflexure = 67% of horizontal distance of the waist slab.

ii) The average maximum positive moment = 680 ft-lb/ft.

iii) The average maximum negative moment = 775 ft-lb/ft

iv) The average tension over the half width at the inner side of the lower kink = 335 lb/in.

v) The average compression over the one-third width at the outerside of the lower kink = 600 lb/in.

vi) The average tension over the one-third width at the outer side of the upper kink = 600 lb/in.

vii) The average compression over the half width at the inner side of the upper kind = 335 lb/in.

viii) The average tension over the half width at the inner side of 0.25 l from the lower kink = 150 lb/in.

ix) The average tension over the one-third width at the outer side of 0.25 l from the upper kink = 269 lb/in.

x) The average positive moment at 0.25 l from the lower or the upper kink = 300 lb-in/in.

From the above description it is evident that the fixed support or simple support has no significant effect on the stress condition in the stair slab. Therefore for the design purpose, the average value of the above two boundary conditions can be used. This is also logical since actual condition is somewhere in between. The average values are given below:

i) Effective span = 66% of the going of the waist slab.

ii) The average maximum positive moment = 659 ft-lb/ft.

iii) The average maximum negative moment = 773 ft-lb/ft.

iv) The average tension over the half width at the inner side of the lower kink = 337 lb/in.

v) The average compression over the one-third width at the outer side of the lower kink = 600 lb/in.

vi) The average tension over the one-third width at the outer side of the upper kink = 600 lb/in.

vii) The average compression over the half width at the inner side of the upper kink = 337 lb/in.

viii) The average tension over the half width at the inner side of waist slab, 0.25 l from the lower kink = 125 lb/in.

ix) The average tension over the one-third width at the outer side of 0.25 l from the upper kink = 234 lb/in.

x) The average positive moment at 0.25 l from the lower or the upper kink = 292 lb-in/in.

These results, along with similar results of two flight open well stair slabs, are summarized in the Table 5.1.

5.3 Stress Conditions of Two Flight Stair Slab

The stress conditions at various sections are studied below:

a) Lower kink

i) Compressive stress from the axial force at the outerside = $\frac{600}{4}$ = 150 psi.

ii) Tensile stress from the axial force at the inner side = $\frac{337}{4}$ = 84 psi

iii) Maximum compressive stress in concrete from bending moment (considering cracked section)

$$f_c = \frac{2M}{kjbd^2} = 650 \text{ psi}$$

Therefore, the maximum compressive stress at the outerside = 150 + 650 = 800 psi which is much lower than allowable stress of concrete.

Combined stress diagram at lower kink (inner side) is shown in Fig. 5.1.

Table 5.1 Stress condition of two flight and two flight open well stair-case

		Two flight stair		Two flight open well stair	
		Fixed support	Simple support	Fixed support	Simple support
The distance between the points of contraflexure (% going)		66	67	65	677
The average maximum positive moment		638 lb-in/in	680 lb-in/in	620 lb-in/in	690 lb-in/in
The average maximum negative moment		772 lb-in/in	775 lb-in/in	758 lb-in/in	740 lb-in/in
The average tension over the half width	At the inner side of lower kink	340 lb/in	335 lb/in	238 lb/in	200 lb/in
	At the inner side of 0.25 l from lower kink	100 lb/in	150 lb/in	-	-
The average tension over the one-third width	At the outer side of the upper kink	600 lb/in	600 lb/in	519 lb/in	394 lb/in
	At the outer side of 0.25 l from the upper kink	200 lb/in	269 lb/in	-	-
The average compression over the one-third width at the outerside of the lower kink		600 lb/in	600 lb/in	519 lb/in	394 lb/in
The average compression over the half width of the inner side of the upper kink.		340 lb/in	335 lb/in	283 lb/in	200 lb/in
The average positive moment at 0.25 l from the kinks.		283 lb-in/in	300 lb-in/in	-	-

b) Upper kink

i) Compressive stress from the axial force at inner side = 84 psi.

ii) Tensile stress from the axial force at the outerside = 150 psi.

iii) Maximum compressive stress in concrete from bending moment (considering cracked section) f_c = 650 psi.

Therefore, the maximum compressive stress at the innerside = 84 + 650 = 734 psi. which is much less than allowable stress of concrete.

Combined stress diagram at upper kink (outer side) is shown in Fig. 5.2.

c) Waist slab quarter distance from the lower kink

i) Tensile stress from the axial force at the inner side = 31 psi.

ii) Maximum compressive stress in concrete from bending moment (considering cracked section) = 372 psi.

Combined stress diagram at a distance 0.25 l from the lower kink is shown in Fig. 5.3.

d) Waist slab quarter distance from the upper kink

i) Tensile stress from the axial force at the outerside = 58 psi.

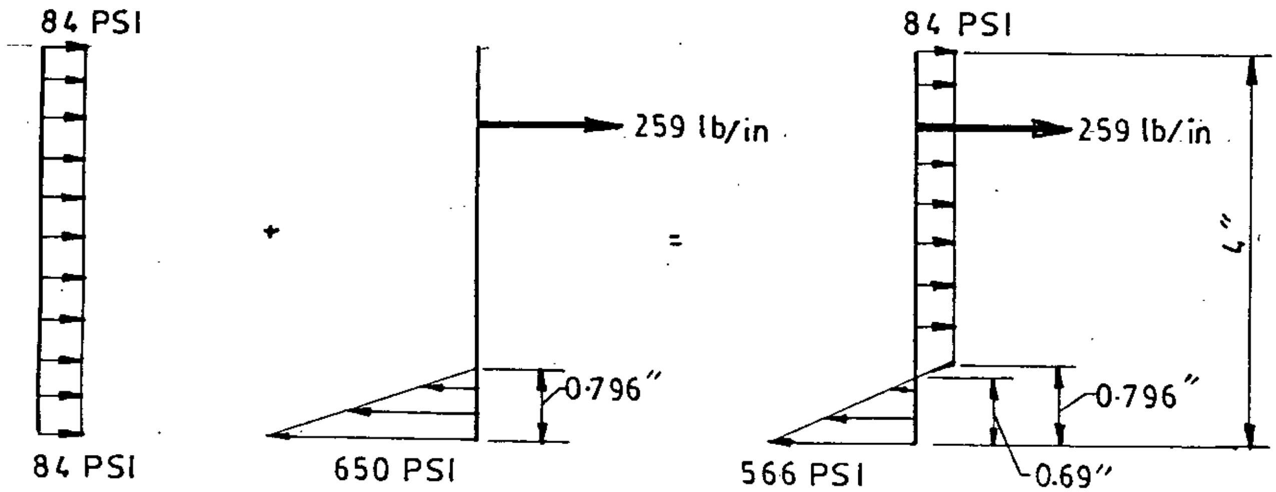


Fig. 5.1 Combined stress diagram at lower kink (inner side).

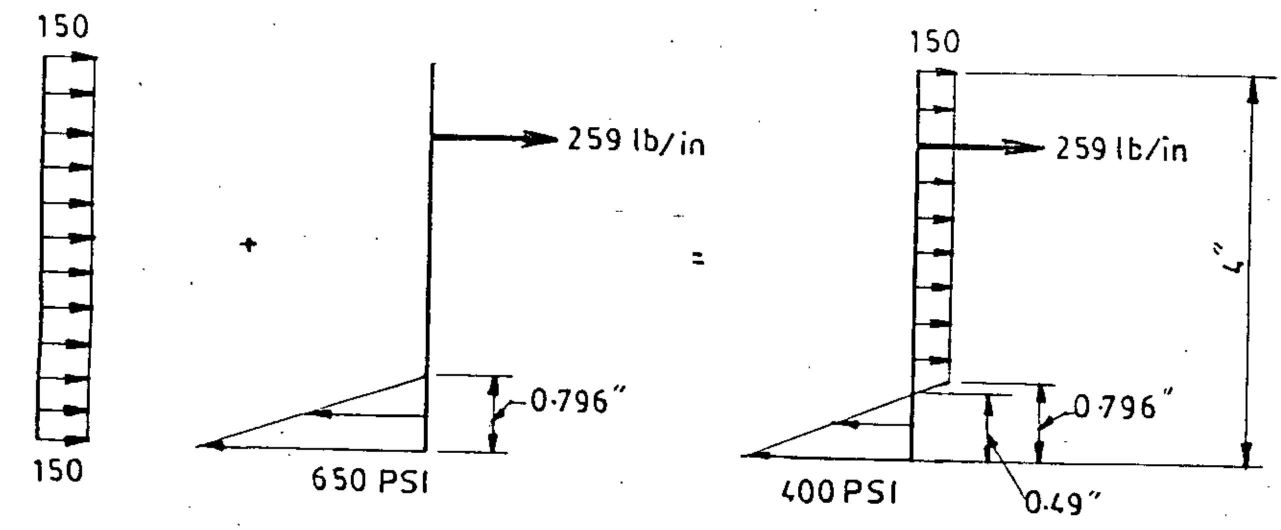


Fig. 5.2 Combined stress diagram at upper kink (outer side).

ii) Maximum compressive stress in concrete from bending moment (considering cracked section)

$f_c = 372$ psi.

Combined stress diagram at a distance 0.25 l from the upper kink are shown in Fig. 5.4.

e) Middle of the waist slab

At this point the axial force is zero and positive bending moment is maximum, which is more or less uniform in the transverse direction.

If the points of contraflexure are thought to be hinge point then the maximum positive bending moment can be found out by the formula $wl^2/8$ where l is the horizontal distance between the points of contraflexure or effective span. Therefore the moment calculated by this formula is $= \frac{175.4 \times (5.45)^2}{8} = 650$ ft-lb/ft which is very much close to the moment 659 ft-lb/ft obtained by analysis.

f) Transverse direction

The moments and axial forces in the transverse direction are insignificant.

5.4 <u>Stress Conditions of Two Flight Open Well Stair Slab</u>

The stress conditions at various sections are studied below:

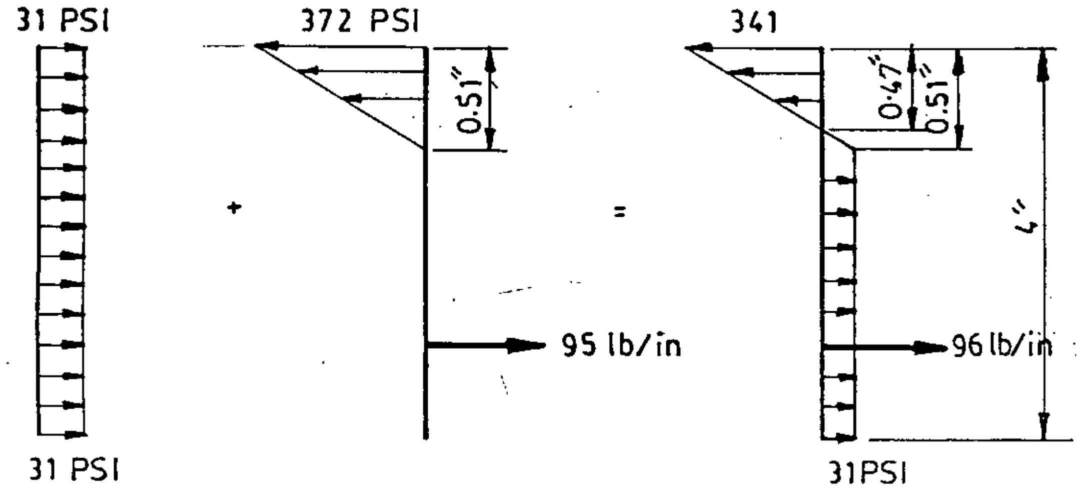


Fig. 5.3 Combined stress diagram at a distance 0.25 l from the lower kink (inner side).

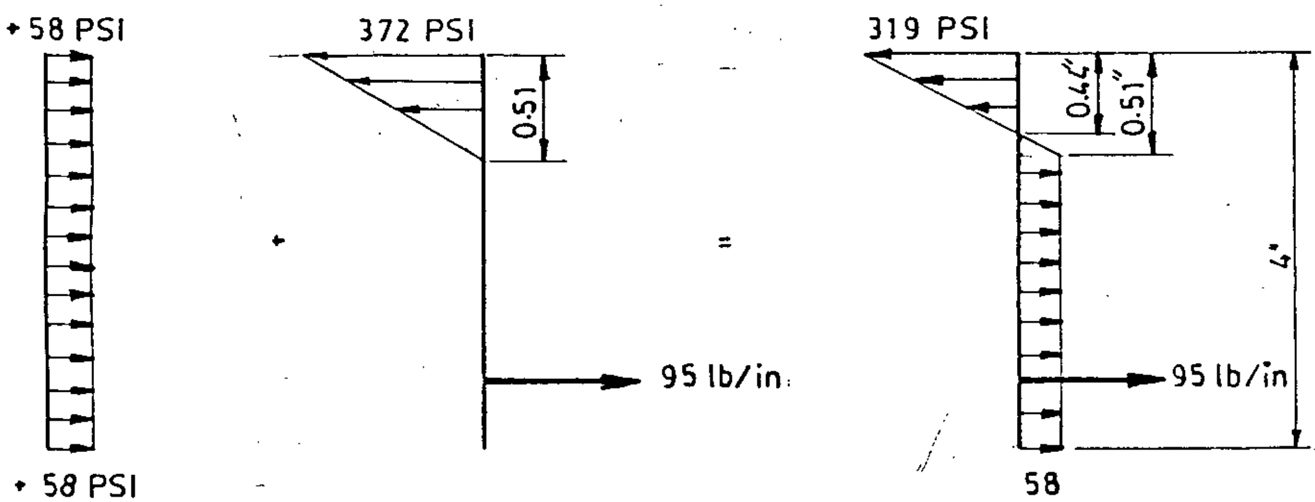


Fig. 5.4 Combined stress diagram at a distance 0.25 l from the upper kink (outer side).

a) Lower kink

i) Compressive stress from the axial force at the outer side = 114 psi.

ii) Tensile stress from the axial force at the inner side = 60 psi.

iii) Maximum compressive stress in concrete from bending moment (considering cracked section) = 638 psi.

Therefore, the maximum compressive stress at the outerside = 114+638 = 752 psi which is much lower than allowable stress of concrete.

Combined stress diagram at lower kink (inner side) is shown in Fig. 5.5.

b) Upper kink

i) Compressive stress from axial force at the inner side = 60 psi.

ii) Tensile stress from the axial force at the outer side = 114 psi

iii) Maximum compressive stress in concrete from bending moment (considering cracked section) = 638 psi.

Therefore, the maximum compressive stress at the inner side = 60 + 638 = 698 psi. which is much lower than the allowable stress of concrete.

Combined stress diagram at lower kink (inner side) is shown in Fig. 5.5.

b) Upper kink

i) Compressive stress from axial force at the inner side = 60 psi

ii) Tensile stress from the axial force at the outer side = 114 psi

iii) Maximum compressive stress in concrete from bending moment (considering cracked section) = 638 psi.

Therefore, the maximum compressive stress at the inner side = 60 + 638 = 698 psi. which is much lower than the allowable stress of concrete.

Combined stress diagram at upper kink (outer side) is shown in Fig. 5.6.

c) Middle of the waist slab

As in the case of two flight stair slab, in this case also, the axial force is zero and maximum positive bending moment occurs at this point. This moment is more or less uniform in the transverse direction.

If the points of contraflexure are thought to be hinge point then the maximum positive bending moment can be found out by the formula $wl^2/8$ where l is the horizontal distance between the points of contraflexure or effective span.

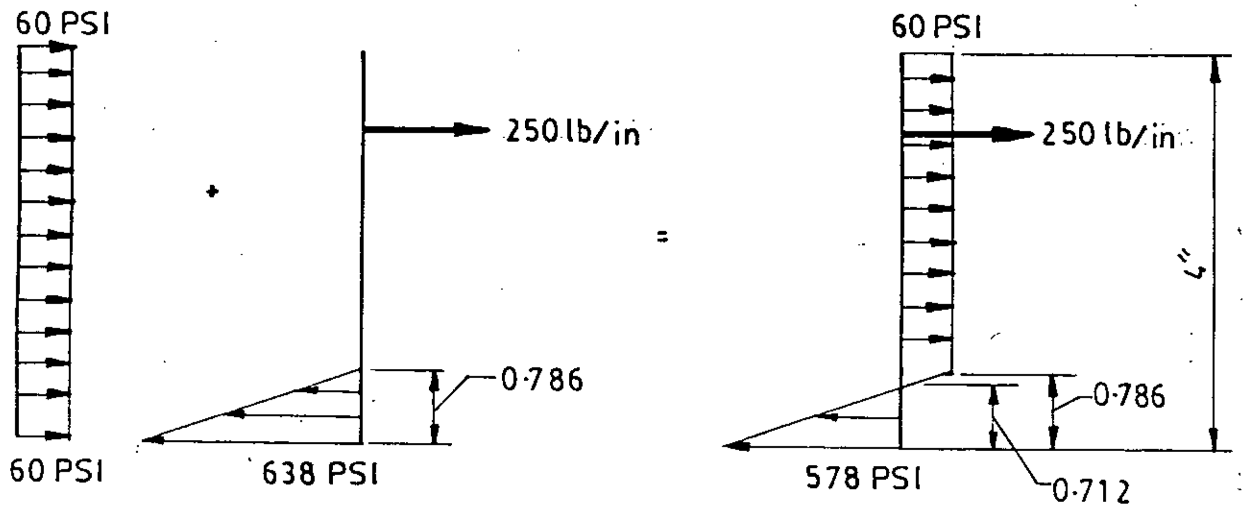


Fig. 5.5 Combined stress diagram at lower kink (inner side).

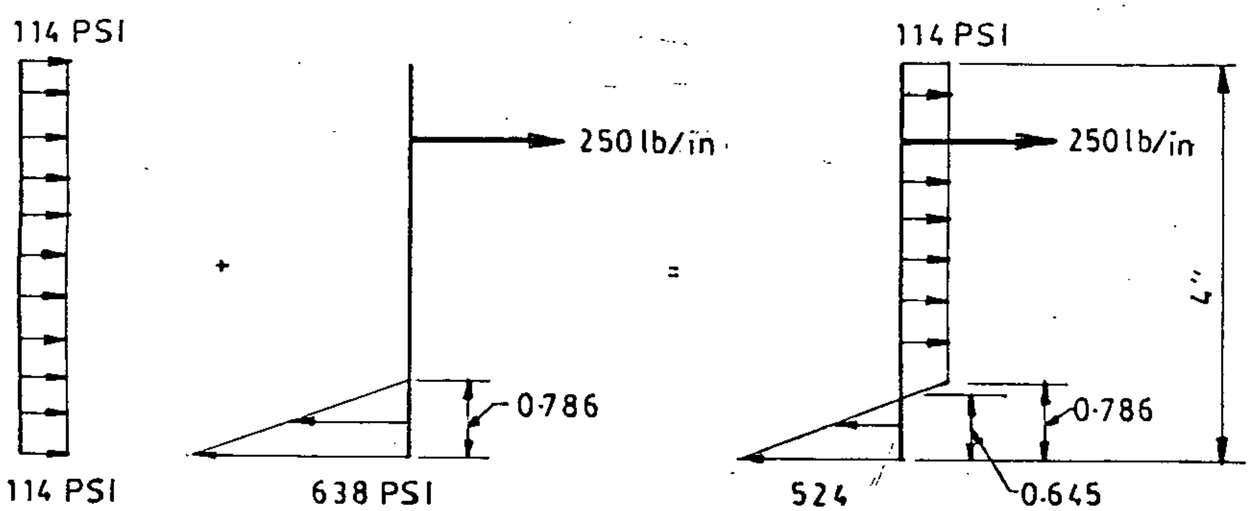


Fig. 5.6 Combined stress diagram at upper kink (outer side).

Therefore the moment calculated by this formula is

$$= \frac{175.4x(5.45)^2}{8} = 650 \text{ ft-lb/ft}$$ which is very much close to the moment 655 ft-lb/ft obtained by analysis.

d) Transverse direction

The moments and axial forces in the transverse direction are once again insignificant.

5.5 Improvement in Design Procedure of Stair Slab

On the basis of finite element analysis of stair slabs, an improved design procedure is presented here.

Considering some practical constraints such as non-uniformity in the behaviour of concrete, lack of quality control measures, imperfection in workmanship etc. somewhat conservative suggestions are being made.

a) Effective span can be taken as a distance equal to 70% of going of the waist slab. Maximum positive moment can be calculated by using the relationship, $M = wl^2/8$, where l is the effective span (Fig. 5.7). Location of the maximum positive moment is at the middle of the waist slab. Reinforcement required by the maximum positive moment shall be placed throughout the waist slab.

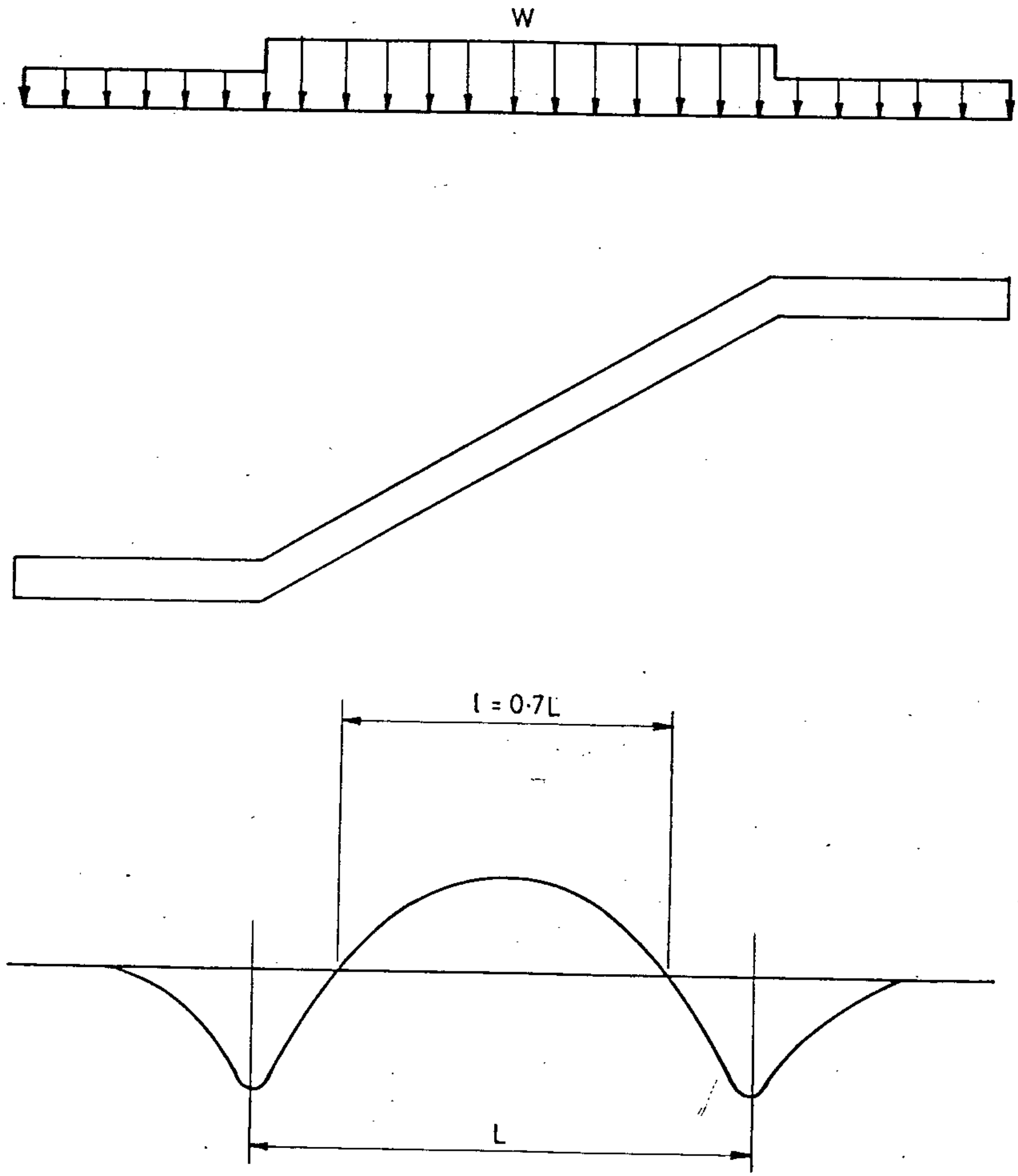


Fig. 5.7 Loading condition and idealized bending moment diagram of stair slab.

b) Magnitude of maximum negative moment can be taken as 1.2 times maximum positive moment. It is seen from the finite element analysis that the maximum negative moment for the two flight stair slab and that of 14% more for two flight open well stair slab. A somewhat conservative factor is suggested. Location of maximum negative moment are practically at the junction of waist and landing slab. Reinforcement required by the maximum negative moment should be provided at top in the waist slab and extended on both sides as required.

c) At top of the upper kink, due to the presence of axial tension, additional reinforcement, equal to two thirds the reinforcement required for negative moment, shall be provided. This additional reinforcement will be placed in the longitudinal direction spread over a zone of width equal to one-third of the width of the waist slab. Additional reinforcement, equal to one-third the reinforcement required for negative moment, shall be provided at top near inner boundary of the lower kink in the longitudinal direction and shall be spread over a zone of width equal to half of the width of the waist slab. Length of these reinforcements should be equal to the length of the reinforcements provided for negative moments. In the case of open well these additional reinforcement can be reduced by 25%.

d) Transverse reinforcement in the landing slab shall be provided at the bottom. Bending moment along this direction was found to be too insignificant to govern the design and reinforcements can be determined by minimum code requirements. Half of these reinforcements shall be bent upto provide for any eventual negative moment developed at the supports.

5.6 Design of Stair Slab by the Suggested Procedure

A design of the stair slab under investigation following WSD method is presented here on the basis of suggestions in the previous article.

Thickness of the slab = 4 in

Effective span = 0.7x8.25 = 5.77 ft

Total load = 195.97 lb/ft^2

f'_c = 2500 psi

f_s = 20000 psi

Maximum positive moment = 1/8x195.97x(5.77)^2

= 815.55 ft-lb/ft

Maximum negative moment = 1.2x815.55

= 978.66 ft-lb/ft

Required effective depth = 2.34 in

d provided = 3 in

$$\text{Required, } A_s(+) = 0.19 \text{ in}^2/\text{ft}$$

$$A_s(-) = 0.22 \text{ in}^2/\text{ft}$$

Additional reinforcement, at upper kink (for one-third width at the wall side) = 0.44 in^2/ft.

Additional reinforcement at the lower kink (for half width at the inner side) = 0.22 in^2/ft.

Details of reinforcements are shown in Fig. 5.8.

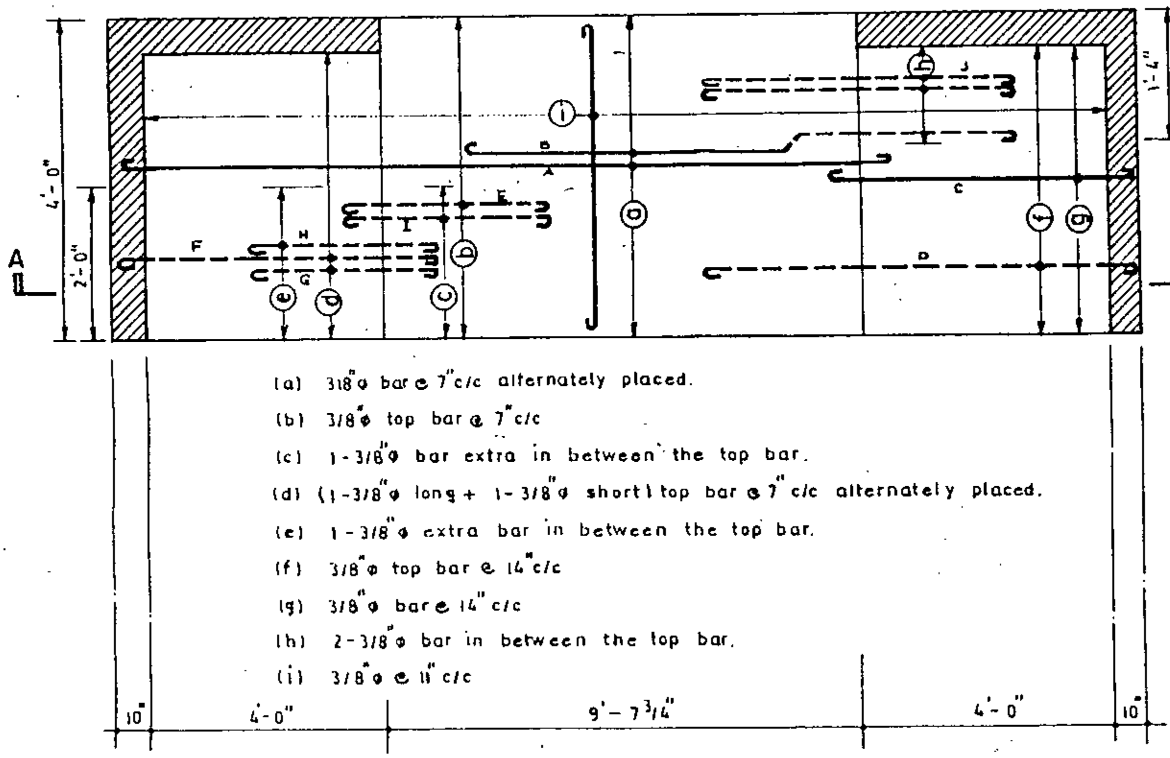
5.7 Cost Comparison of Material Required by Different Procedures

The comprison of material requirements for a single flight stair slab by different method is discussed in Chapter-2. A summary of the cost involvement in various methods, including the presently suggested one is presented below:

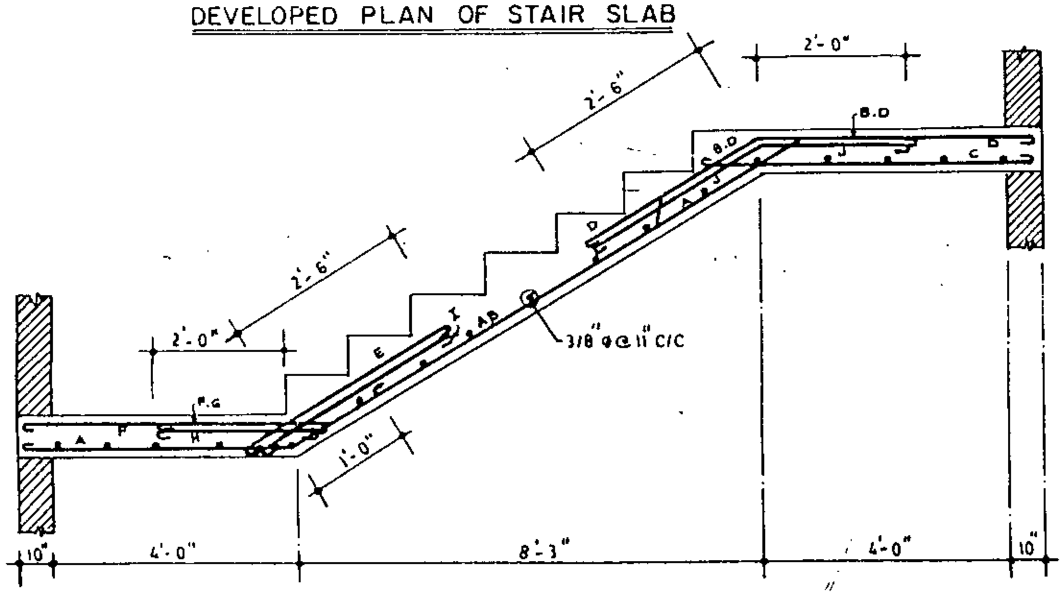
Table 4.1 Cost of construction corresponding to the various method

Method of design	Thickness of slab in	Volume of concrete, cft	Weight-of steel, lb	Cost of construction* Taka
Suggested procedure	4	25.75	106	1950
Ahmad	4	25.75	125	2070
British code of practice	6	38.62	224	3331
American practice	8	51.5	370	4890

*Cost of concrete = 50 Tk/cft (including labour charge)
Cost of steel = 700 Tk/cwt (including labour charge)



DEVELOPED PLAN OF STAIR SLAB



SECTION A-A

Fig. 5.8 Reinforcement detail of the stair slab designed by improved design procedure.

Comparing the cost of construction required by the suggested design procedure with those by the Ahmad's method, the British code of practice and American design practice- it was observed that the reduction in the cost of construction was of the order of 5%, 40% and 60% respectively.

CHAPTER 6

CONCLUSION

6.1 General

The main objective of this research was to carryout an extensive study of the stress conditions prevailing in the stair slab and to substantiate and further rationalize the improved design procedure suggested by Ahmad[1]. On the basis of the investigation , the following conclusion can, in general, be drawn.

a) Stair slab cannot be treated as simple oneway slab.

b) Load carrying capacity of stair slabs is much higher than that anticipated in usual design practices.

c) Simple flexural analysis for stair slabs is inadequate. Design of such slabs can be made economic as well as rational by taking into consideration their actual stress distributions as revealed by more sophisticated analysis.

d) The effects of support condition, i.e., whether the stair slab is fixed supported or simply supported, are negligible.

e) The effect of the presence open well on bending moment is insignificant but it has a positive effect on the distribution of axial forces.

f) The stresses in the transverse direction of stair slab are negligible.

6.2 Stress Conditions in Stair Slab

The observed features regarding the stress conditions of the stair slab are listed below:

a) Both for open well and without open well staircases the points of contraflexure lie within the waist slab portion, effective span for positive moment being within the range of 65% to 67% of horizontal distance of the waist slab.

b) Maximum positive moment is located at the middle of the waist slab.

c) Maximum negative moments for the stair slab occur near the kink zone both in simple support and fixed support conditions.

d) Appreciable axial forces develop at the kinks of stair slab. In the case of open well stair slab these forces are less pronounced.

e) Bending moments, along the transverse direction (in landing slab) is negligible in comparison to those along the longitudinal direction, indicating that most of the loads are carried along the later direction.

6.3 Suggestion for Future Research

Stair slabs, small but important elements of a building, have not received sufficient attention from researchers previously. Findings and subsequent conclusions of this study are largely based on the results obtained for a few types of stair slabs by using the thick shell finite element program. Hence the conclusions drawn earlier in this chapter are limited by the scope of research. Therefore further investigations on the behaviour of stairslabs should be continued with a view to formulating definite guide lines and developing proper code provisions regarding the design of stair slabs. After evaluating the findings of this thesis there remains scope for further studies in the following fields.

a) To create confidence in the suggested design procedure model tests may be carried out to the point of destruction.

b) Intensive finite element investigations can be carried out on other types of stair cases.

c) Effects of variations in going of the waist slab, width of the landing slab, and width of the open well against the width of waist slab should be studied.

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APPENDIX

A.1 Element Characteristics

A.1.1 General

Typical thick shell elements are shown in Fig. A.1. In thick shells bending effect can be expected to be significant. In addition, the transverse shear deformation may also be significant. From a three dimensional point of view the elements have two degeneracies. Firstly, the original normals to the middle surface are assumed to remain straight. Secondly, the distance of a point along the normal from the middle surface remains unaffected.

A.1.2 Geometric Definition of the Element

The external faces of the element are curved, while the sections across the thickness are generated by straight lines, pairs of points i_{top} and i_{bottom}, each with given cartesian co-ordinates, prescribe the shape of the element.

If ξ , η be the two curvilinear coordinates in the middle plane of the shell (Fig. A.2) and ζ a linear co-ordinate in the thickness direction. If further it is assumed that ξ , η , ζ vary between 1 and -1 on the respective faces of the element then it can be written a relationship between the cartesian co-ordinates of any point of the shell and the

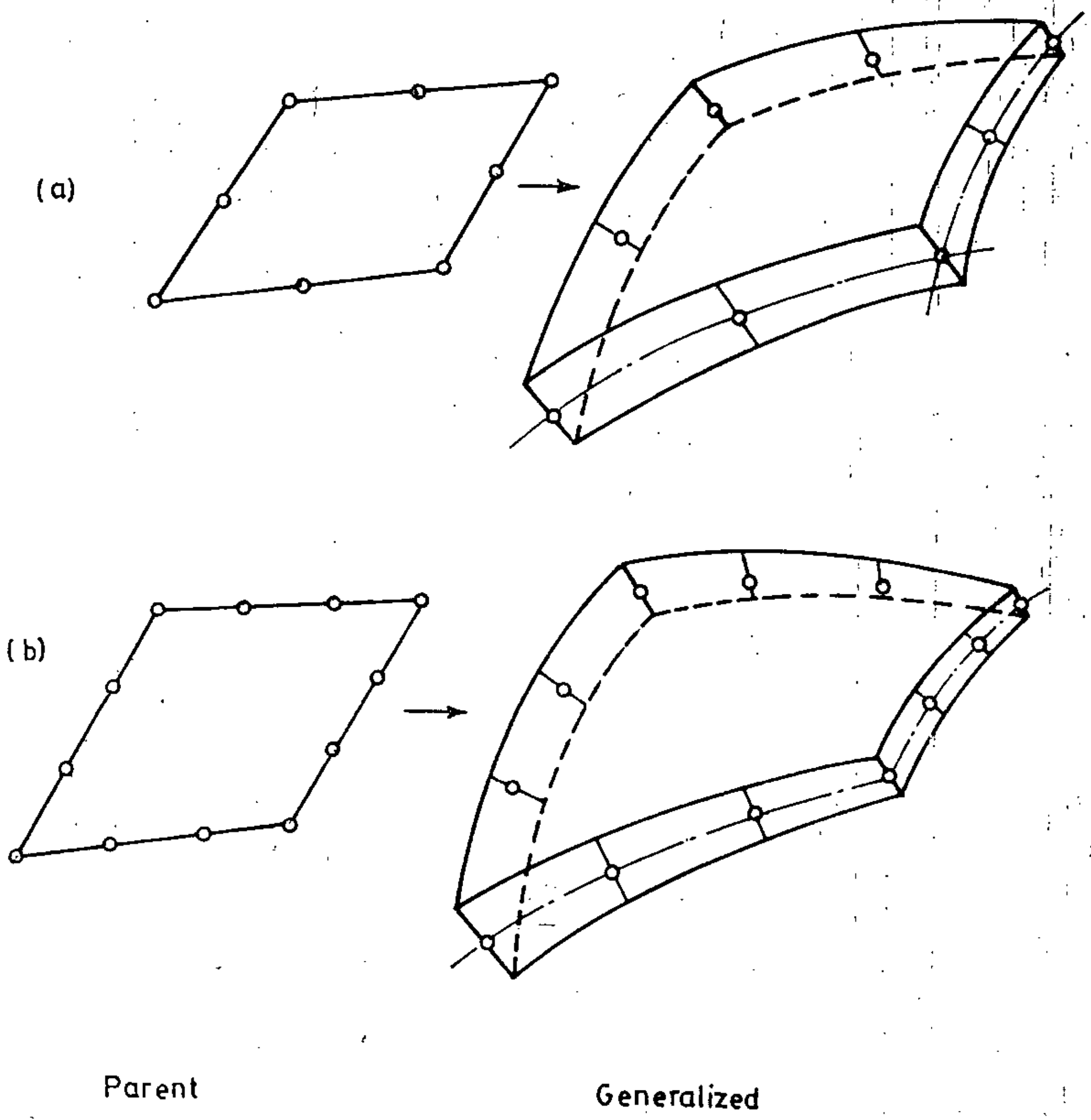
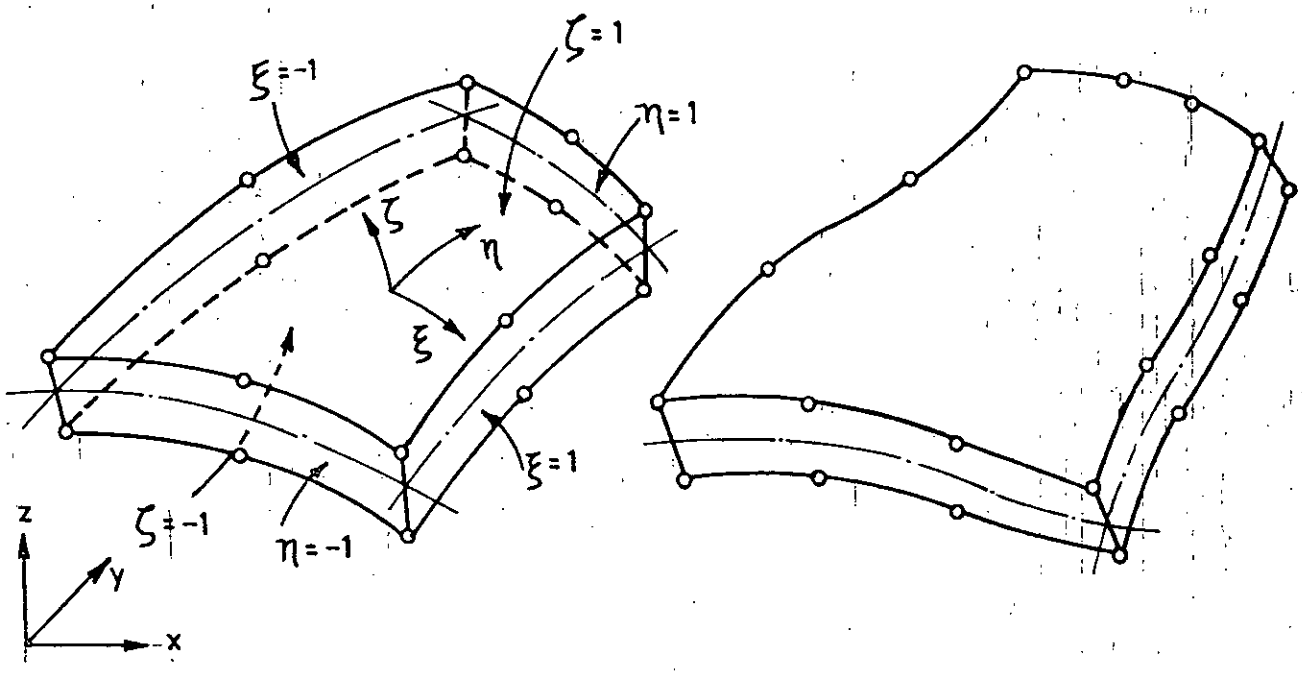


Fig. A.1 Thick Shell Elements
(a) Parabolic (b) Cubic

curvilinear co-ordinates in the form

$$\begin{Bmatrix} x \\ y \\ z \end{Bmatrix} = \Sigma\, N_i(\xi,\eta)\,\frac{(1+\zeta)}{2}\begin{Bmatrix} x_i \\ y_i \\ z_i \end{Bmatrix}_{top} + \Sigma N_i(\xi,\eta)\,\frac{(1-\zeta)}{2}\begin{Bmatrix} x_i \\ y_i \\ z_i \end{Bmatrix}_{bottom}$$

Here $N_i(\xi,\eta)$ is a shape function taking a value of unity at the nodes i and zero of all other nodes. If the basic functions N_i are derived as 'shape functions' of a 'parent', two dimensional element, square or even triangular in plan and are so designed that compatibility is achieved at interfaces, then the curved shape elements will fit into each other. Arbitrary curved shapes of the element can be achieved by using shape functions of different orders. Only parabolic and cubic types are shown in Fig. A.1. By placing a larger number of nodes on the surfaces of the element more elaborate shapes can be achieved if so designed. It should be noted that the co-ordinate direction ζ is only approximately normal to the middle surface. The relationship between the cartesian co-ordinates and curvilinear co-ordinates can be written conveniently in a form specified by the 'vector' connecting the upper and lower points (i.e. a vector of length equal to the shell thickness t) and mid-surface co-ordinates (shown in Fig. A.3).



(a) Parabolic (b) Cubic

Fig. A.2 Geometry of Element

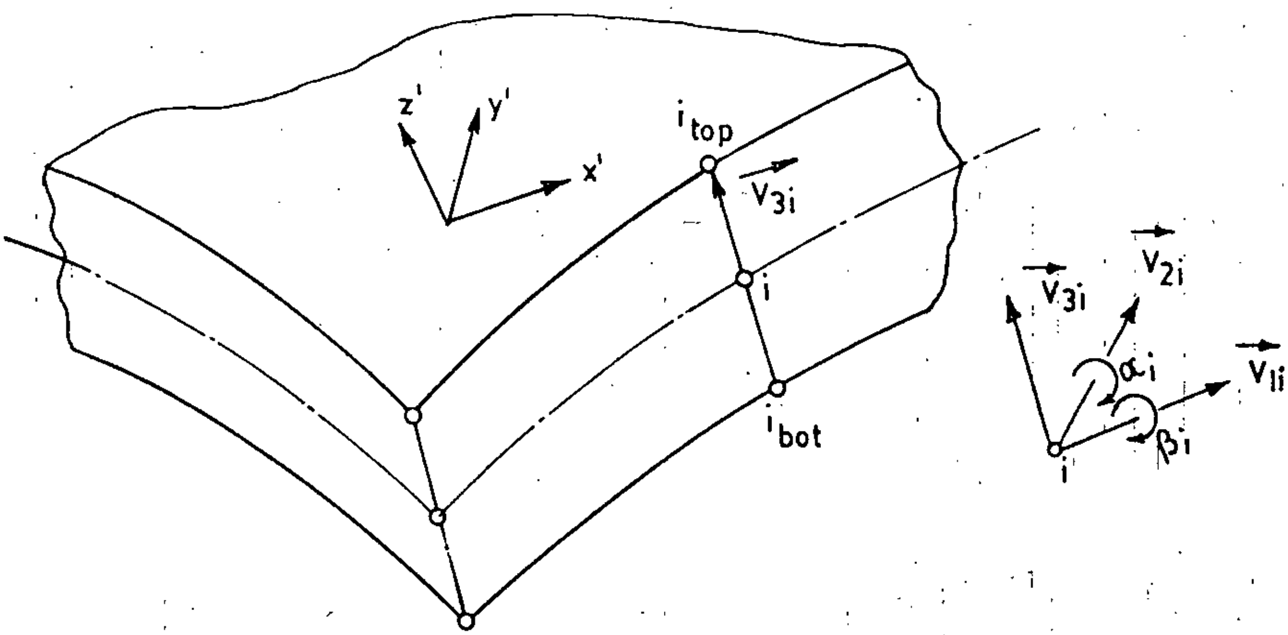


Fig. A.3 Local Coordinates and Nodal Degrees of Freedom

$$\begin{Bmatrix} x \\ y \\ z \end{Bmatrix} = \Sigma N_i \begin{Bmatrix} x_i \\ y_i \\ z_i \end{Bmatrix}_{mid} + \Sigma N_i \frac{\zeta}{2} \bar{V}_{3i}$$

with

$$\bar{V}_{3i} = \begin{Bmatrix} x_i \\ y_i \\ z_i \end{Bmatrix}_{top} - \begin{Bmatrix} x_i \\ y_i \\ z_i \end{Bmatrix}_{bottom}$$

defining a vector whose length is the shell thickness.

A.1.3 Displacement field

Since the strains in the direction normal to the mid-surface is assumed to be negligible, the displacement throughout the element will be taken to be uniquely defined by the three cartesian components of the mid-surface node displacement and two rotations of the nodal vector $\bar{V}_{3i}$ about orthogonal directions normal to it. If two such orthogonal directions are given by vectors $\bar{V}_{2i}$ and $\bar{V}_{1i}$ of unit magnitude, with corresponding (scalar) rotation α_i and β_i it can be written, similar to the previous equation, but now dropping the suffix mid for simplicity.

$$\begin{Bmatrix} u \\ v \\ w \end{Bmatrix} = \Sigma N_i \begin{Bmatrix} u_i \\ v_i \\ w_i \end{Bmatrix} + \Sigma N_i \zeta \frac{t_i}{2} \left[V_{1i} - V_{2i} \right] \begin{Bmatrix} \alpha_i \\ \beta_i \end{Bmatrix}$$

A.2 General Features of the Program

The general thick shell finite element program is a generalized program. The geometry of a structure is defined in a global system i.e. rectangular cartesian. The loading and boundary conditions must be given in the same unit as the nodal displacements of an element. The stresses are usually calculated at the nodal points in the global system.

The top and bottom co-ordinates of each node with respect to cartesian co-ordinate are fed into the program. Co-ordinates for non-corner nodes lying on straight edges are not required to be given. If these co-ordinates of the nodes are fed into the program, then the shape of the element is automatically defined in the program. Therefore the thickness of the element can vary from node to node and the edges may be curved parabolically and cubically depending upon the type of element used. The program as at present can handle isotropic elastic material. The material properties are defined for every element, thus allowing the program to deal with materials varying from element to element. The temperature and pressure can be varied from node to node.

A.3 Output from the Program

The displacements are calculated and printed against each node in the ascending order for every loading case.

Stresses are first calculated in the local orthogonal system and then transformed to the global cartesial system. For every node the top surface stresses are followed by the bottom surface stresses.

The global stresses are also stored separately for top and bottom surfaces against nodal numbers and at the end a simple averaging is performed on them. The average stresses are then printed out in the ascending order of the nodal numbers. The top surface stresses for all the loading cases are followed by the bottom surface stresses.

A.4 Division of Structure into Elements

The structure is first of all divided into suitable elements and the order numbered in any suitable way as shown for example in Fig. A.4(a). The elements are also suitably numbered in some sequence on which they are fed in the computer. Two probable sequence are shown in Figs. A.4(a) and (b).

Each element is topologically defined by its nodal numbers in a consistent right hand screw system shown in Fig. A.5(a) and (b).

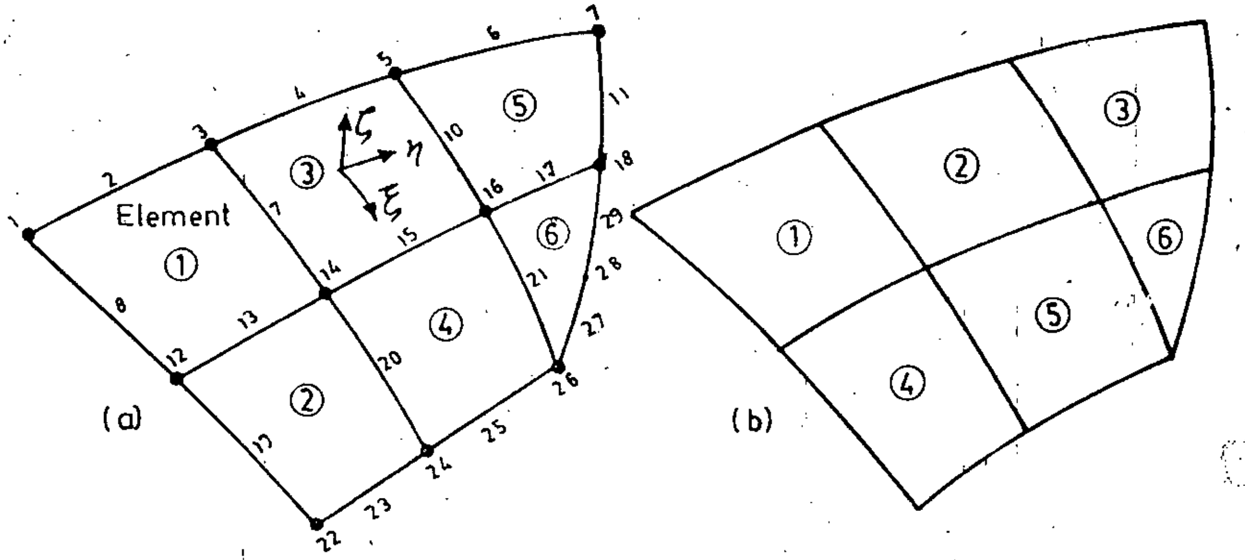
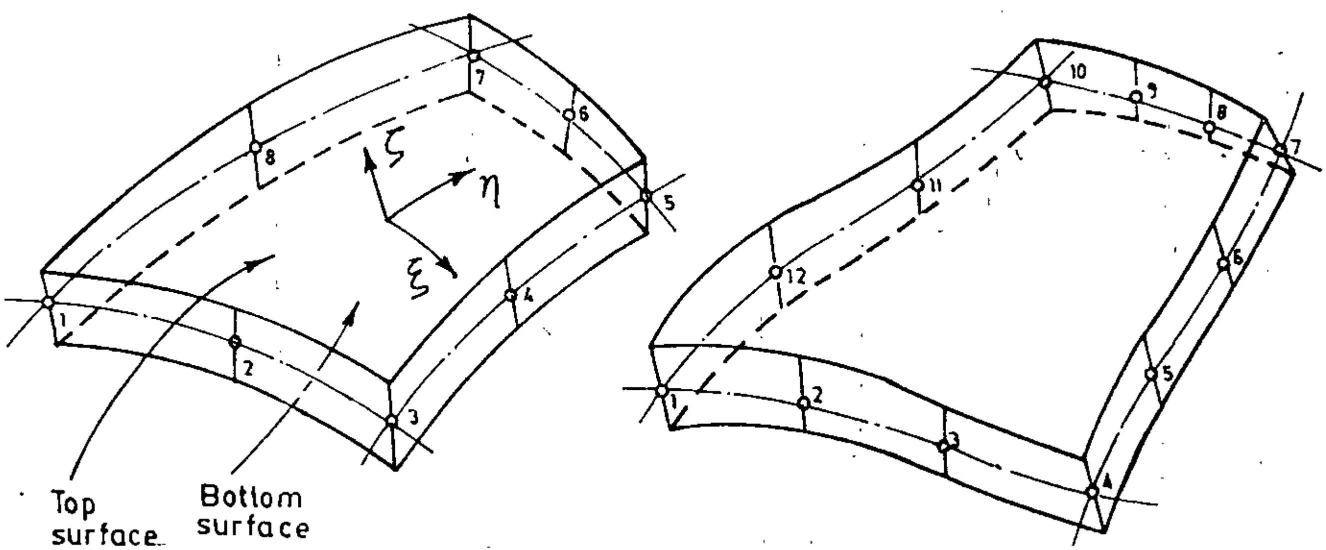


Fig. A.4 Division of a Structure with Parabolic Elements



(a) Parabolic (b) Cubic

Fig. A.5 Definition of Element Topology

A.5 Front Width and Selection of Order of Elemination

To carry out the analysis of a structure using the minimum possible computer storage, the elements are selected in such a sequence that the maximum number of variables to be handled at any particular time (the front width) is minimum. For example, the prescribed order of elements in Fig. A.4(a) will give the smallest front width. This is evident even from inspection in a simple structure.

The thick shell program uses the Frontal Solution technique. Here the assembly of an element stiffness and the corresponding right hand sides is immediately followed by the process of elemination of the variables corresponding to nodes which occur for the last time. This is indicated to the program by inserting a -ve sign before these nodes. This can easily be put in most shell structures once the element sequence has been selected.