Development of an optimal production allocation model for multistage multi location multi period system

By
MD.MAHABUBUR RAHAMAN

A THESIS SUBMITTED TO THE DEPARTMENT OF INDUSTRIAL AND PRODUCTION ENGINEERING, BANGLADESH UNIVERSITY OF ENGINEERING AND TECHNOLOGY, DHAKA IN PARTIAL FULFILMENT OF THE REQUIREMENT FOR THE DEGREE OF

MASTER OF ENGINEERING
(INDUSTRIAL & PRODUCTION)

August, 1999

Department of Industrial and Production Engineering Bangladesh University of Engineering and Technology
Dhaka-1000

Development of an optimal production allocation model for multistage multi location multi period system

The thesis titled "**Development of an optimal production allocation model for multistage multi location multi period system**" submitted by Md. Mahabubur Rahaman, Roll No. 9408002F, session 1993-94-95 to the Department of Industrial and Production Engineering (IPE) of Bangladesh University of Engineering and Technology (BUET), has been accepted as satisfactory for the partial fulfillment of the requirements for the degree of MASTEROF ENGINEERING (Industrial & Production).

Board of Examiners

1. Anwarul Haque

Dr. A.F.M. Anwarul Haque
Professor
Dept. of IPE, BUET, Dhaka.

Chairman
(Supervisor)

2. Dr. Golam Mohiuddin
Professor & Head
Dept. of IPE, BUET, Dhaka.

Member

3. Mustafizul Karim

Dr. A. N. Mustafizul Karim
Associate professor
Dept. of IPE, BUET, Dhaka.

Member

Department of Industrial and production Engineering Bangladesh University of Engineering and Technology
Dhaka-1000

DECLARATION

I do hereby declare that this work has been done by me and neither this thesis nor any part of it has been submitted elsewhere for the award of any degree or diploma except for publication

Countersigned

Dr. A. F. M. Anwarul Haque
Supervisor

Md. Mahabubur Rahaman

ACKNOWLEDGEMENT

The author expresses his immense gratitude and indebtedness to his supervisor **Dr. A. F. M. Anwarul Haque**, Professor, Department of Industrial and Production Engineering (IPE), Bangladesh University of Engineering and Technology (BUET) for his constant guidance, supervision, valuable suggestions and kindest support throughout the preparation of this thesis work.

The author owes his heartfelt gratefulness to Dr. Md. Golam Mohiuddin, Professor & Head, IPE Department, BUET, for his all-out co-operation during his graduate studies, specially for the software support (LINDO) without which it would have been almost impossible to complete this project work smoothly.

The author would like to express his profound thanks and gratitude to Dr. A.N. Mustafizul Karim, Associate Professor, IPE Department, BUET for his effective criticism & constructive suggestions.

The author acknowledges the immense help Mr. Mijanur Rahman, Chief Sales Manager, Mr. Golam Kabir, MD (HRD), M.A. Dewan, Chief Manager (Factory), Rahim Afrooz, Bangladesh Ltd., and other personnel of Rahim Afrooz, extended for providing data during various stages of the researeh work.

A very special thank is due to Mr. Md. Shahidullah, Computer Operator, Department of Mechanical Engineering, BUET for the long hours of work he painstakingly devoted to editing, formatting and composing the thesis paper in a very short period.

The author would like to express his thanks and gratitude to all of his friends specially Mahbubul Haque and Bellal Hossain for their inspiration during different stages of his graduate program.

Last but not the least the author owes his deep sense of gratitude to his parents and sister for their moral support during the entire period of thesis work.

ABSTRACT

Multistage multilocation multiperiod production systems are very common in manufacturing and business arena. Production allocation for multistage multilocation mltiperiod system is of great importance to the academics, researchers and managers. The task of production allocation for multistage system is a highly complex job because of the requirement of incorporating various aspects of overall manufacturing and business scenario and inventory oriented factors. This task becomes uphill especially, for assembling oriented enterprises. To get a realistic solution of production allocation, the search for effective quantitative techniques is an essential one. Mathematical modelling is an efficient approach to these cases.

In this research work attempt has been made to develop mathematical models relating to various situations, -starting from very simple cases to the complex assembling oriented systems. All the developed models were tested with data and found to provide optimum solutions. The linear programming methodology was employed for solving the problems assuming the associated costs linear.

The output analysis of the model was found to be a helpful guideline for the management in terms of decision upon inventory policy making, resource allocation, marketing strategy, possible plant expansion and capital budgeting.

Beside the analytical ability and conceptual maturity of the designer, the success of this modelling approach stems from its nature of incorporating updated and correct data. Models relied highly on modern information technology. The presence of management information system (MIS) in the enterprise was felt to be very essential.

TABLE OF CONTENTS

Chapter-3: Mathematical modelling

LIST OF TABLES

LIST OF CHARTS

LIST OF FIGURES

ACRONYM

DP	:	Dynamic programming
MRP	:	Material requirement planning
ERP	:	Enterprise resources planning
MIS	:	Management information system

Chapter-1
Introduction

1.1 General Introduction :

Production allocation and inventory management are interwoven inextricably. The objectives of production-allocation and inventory management are to contribute to the overall corporate net profit and return on investment by establishing and achieving customer service, inventory investment and plant utilization objectives consistent with corporate objectives. To reach the desired point, it requires an organized set of processes that provides information to managers to support the operations and decision making within the organization. In quest for an organized set of processes, management looks for comprehensive managerial tools and efficient database management system. In any manufacturing organization, organizational growth revolves around sound production planning and control. Converting plans into reality requires systems and procedures for execution. Control closes the loop by measuring actual results, comparing them with planned results and deciding if objectives, decision processes, or methods of execution should be revised.

While it is easier to exercise planning and controlling in any simple production-inventory system, in case of a complex production-inventory system such as a multistage one, it becomes quite an uphill task. Complexity mounts enormously should it become desirable to model multifacility features of a system-a multistage production inventory system with assembling operations – when facilities are to be operated on different schedules and at different locations. This type of modeling involves a series of separate stages of decision making, all linked together by some coupling relationships such as the inventory balance one.

The success of the design & implementation of a complex production – inventory system calls for sufficient attention and knowledge vis-a-vis mainly two major aspects-the overall manufacturing and business scenario and inventory management. These aspects will now be addressed briefly in the following two sections before presenting the problem description and objectives of the present research work.

1.2 Overall Scenario of Manufacturing & Business :

A. Strategic aspects:

Due to free market economy, there exists a fierce competition of survival for the manufacturer and the marketer alike. Business both domestic as well as international has changed tremendously over the last few decades. This has culminated to the recent globalization of production of goods and services. In these times of intense global competition, the beginning of any business planning exercise should start from defining the company's vision & mission. This should then be followed by on-going strategic planning. Strategic plan is all about plans to bridge the gap from the current scenario to where the organization intends to be in future. The current times have seen big companies merge together and try to get the birds' eye view from global perspective. Multinational companies collect and collate information regarding market opportunity, cheap and skilled labor, raw material cost, suitable infrastructure & government regulation, communication system, environmental concern etc. Apart from being concerned about its global market share and full employment, the corporate management eyes a long-term growth of the corporation by carrying an intense customer focus. To ensure this, besides other factors, one important factor is optimum production allocation to satisfy the customer's demand keeping inventory at a minimum possible level.

B. **Production planning aspects**:

Now-a-days it is very rare that a firm that starts with raw materials and manufactures a finished consumer product at a single location has its sales outlets at the factory gate. As an example, a plant may be considered in which an automobile is assembled rather than fabricated. Hundreds of vendors supply various components. Sometimes semi-finished products are procured or produced by purchasing raw materials. This calls for an efficient process planning, implementing and controlling cost-effective flow and storage of raw materials, in-process inventory & finished goods.

C. **Diversity aspects**:

Not only many organizations are large, they are also diverse in respect of the variety of products they offer, the number of subsidiary plants they maintain and the number of firms they deal with. In many cases, company operations are scattered geographically. It is not unlikely that parts needed in one plant of a company may be stored unused on shelves of another plant, just increasing its inventory cost. Most manufacturing organization has a multitude of finished products & hundreds of raw material items to make the finished products. It takes time to manufacture these products and also to order the raw materials from the suppliers. If the raw materials are received early or in more quantity than ordered for, there would be excess inventory. Conversely, if they are received late, the quantity is short, or the materials are not of the right specification or the right quality, there would be shortage of raw materials & customer's order would remain unfulfilled or get delayed. As the organization gets more diverse & confronted with stiff competition, so does the challenge in synchronizing the timing of manufacture of finished products with the receipt of the raw materials. And here lies the importance of an effective database management, which would be capable of managing huge data relating to inventory of products/parts among the plants.

D. **Information aspects:**

The availability of information about "in-house" inventory among the company personnel is of paramount importance. The information on the specifications, price, availability, shipping time and reliability of the components becomes sometimes more critical and more difficult to maintain than that of on "in-house" inventories. Information regarding market opportunity, growth, potential rivals, technological updating, alternative resources is sine qua non for successful planning and controlling of the organization. In this business world of present times, every organization is faced with the stark reality of making & marketing products of right quality, at the right time, at the right price, at the right place with right communication & promotion. This warrants the necessity of coordinating the multifarious activities of the system which, if supported by information technology (IT), would go a long way in meeting the above. As Oliver Wight has noted that 'when the computer came along, the production & inventory manager was suddenly in a position to do some of the things he had never been able to do before'.[1] But for the development of management information system (MIS) in an organization, all the patient work of its planning would go awry & the targets amiss. Hence the all important need of cost effective information flow among various tiers of the organization.

1.3 Inventory Management:

1.3.1 Inventory as a concept:

Inventories are materials that a company carries on hand and that usually represent a sizable portion of the company's total assets. The sales department frequently sees inventory as an unlimited resource, and feels that the production control department has failed if any item is not available in case of necessity. The financial people look inventory as a necessary evil that ties up capital useable elsewhere. Financial people usually want to be bogged down with the question of tied up capital. Factory people have difficulty in understanding the cost associated with carrying inventories and they frequently look upon the effects of inventory control with displeasure because of the apparent inefficiency forced on the plant by increased setups. From overall corporate viewpoint, inventories in a business serve much as the suspension system of an automobile. Obviously the problem lies in the differences in perspective.

A frequently heard cliche in business is "You can't sell from an empty wagon". In business, people want to sell to make profit. There is every possibility that someone biased for high profit could make " the wagon too heavy to pull". Yes! In business there is a need to have the capacity to pull the wagon having something inside to sell for making profit. And here comes the question of reaching rational inventory decisions. As sales volume may fluctuate from time to time, these fluctuating sales need to be absorbed by the resilient inventory system to surmount the problem of shortages. Sometimes manufacturing operations may be halted due to shortages of components on which final assembly is dependent. Lot size inventories make it possible fewer machine setups and higher machine utilization.

1.3.2 Types of inventories:

Inventories comprise the following five general categories of materials in industries-

A. Production inventories: Items that go into the final products, raw materials, and bought out components;

B. Maintenance repair and operating inventories: Items that do not form a part of final product but are consumed in the production process, spare parts, consumable items etc;

C. In process inventories: Semi finished products at various stages of production;

D. Finished goods inventories: Completed products ready for dispatch;

E. Miscellaneous inventories: Which arise out of the above four types of inventories, scrap, surplus and obsolete items which are not supposed to be disposed off.

1.3.3 Costs of inventory:

The costs that are affected by each specific decision must be determined when deciding how many inventories to carry. The problem before the management is to balance the following opposing inventory related costs:

Sl no	Cost to have inventory	Cost not to have inventory
1.	Return on investment	Stock out cost
2.	Handling costs	Loss of customers
3.	(Labor & industry) Handling equipment	Down time cost
4.	Obsolescence	Idle time &idle production
5.	Spoilage & shelf life	Capacity &other cost.

Every business enterprise must have faced the problem of either being out of stock or a large amount of money tied up in the form of inventories. When the company operations are scattered geographically, due to lack of efficient database management system the tied up capital may rise further.

1.4 Problem description and objectives:

Multistage multilocation production system is a reality in today's manufacturing & business environment. The current business scenario & the inventory management, as discussed above, call for closer interdependence among stages of a multistage system. This interdependence is especially true when the facilities can be operated on different schedules, with coordination required only because some facilities obtain parts or semifinished products from one or more other facilities. By allowing flexibility in scheduling, reduced production costs may be realized; however, this usually is at the expense of additional inventory holding costs resulting from increased inventories between stages. The between-stage inventories act as a buffer to absorb the effect of imbalances between the production rates of successive stages. The larger the inventory, the more the independence between the stages. But it is always desirable to keep the inventory low in production systems, which in turn asks for more interdependence and close coordination. Ultimately the mathematical models for multistage system requires close representation of these interdependence among the stages.

The keys to modelling a multistage system are the decisions about what groupings of production operations constitute the stages in the system, whether or not each stage is to have multiple facilities operating in parallel, and how many inventory points between stages are to be defined.

When a stage consists of parallel sequences of production centers, where each sequence produces the product of that stage, it requires to model the stage as parallel facilities and define separate decision variables for each facility. This would nearly always be true in cases where a stage involves multiple plants. Due to the perplex situation it is difficult to determine production schedule for multistage system.

In multistage production system, usually the demand of finished product occurs at the last stage. Each finished product requires single or a set of sub assemblies. Thus in the assembling plant the demand of the components or sub assemblies occurs in multiple of demanded finished product. Same component of same identification/specification may be required for assembling of multiple finished products. So it is very likely for the management to search for quantitative technique that enables to ascertain the necessity of the number of components to satisfy the entire manufacturing demand. The development of realistic model depends on appropriate attention to various costs. These costs are production/procurement costs at various stages of production/procurement, shipping costs due to shipment among the plants & holding costs at various warehouses. Besides costs, the necessity to address various restrictions also demands equal attention. Resource constraints or capacity constraints, production capacities, assembling capacities, storage limitations and transportation limitations are among the prominent restrictions to be taken care of by the developed model.

This project work is directed to the mathematical modeling of the system under various situations & restrictions. The main thrust of this project work was to model an assembling oriented multistage multilocation multiple product manufacturing system. With this view in mind, the objectives of this project work have been set as follows:

A. To formulate a mathematical model for multistage multilocation multiperiod production allocation system;

B. To test and verify the model through application in a real-life setting with a case problem;

C. To study the output and analyze the results;

D. To perform a sensitivity analysis on the different parameters of the model.

Chapter-2

Background study

2.1 Introduction:

As mentioned in the proceeding chapter, the main thrust of the present research work is to formulate mathematical models related to multistage multilocation multiperiod production system under different environments. This chapter first addresses extensively the various aspects of the modelling of the system followed by some relevant conceptual thoughts on operational planning, procurement & production allocation. Finally it illustrates various solution methodologies that are in use in industries & found in the literature.

2.2 Modelling aspects of multistage, multiperiod production system:

Multistage multi location production system is a reality in to-day's manufacturing & business environment. Due to this reality, mathematical models for multistage system is of great importance to the academics and researchers. Inventory situation exists at various stages of production allocation system. The raw materials are found awaiting processing. The semi finished items initiate queue in between production stages. Finished goods are stored at the factory dispatch sections, in transit, in warehouse distributing points and in sales outlets. There are many industries where it is notable that raw material cost fluctuates over periods. Government import regulations could sometimes play vital roles in this regard. Labor availability, power availability etc. are also important factors to be considered. Under this type of situation, it becomes a challenge to the management to provide a unique set of decisions. Mathematical modelling allows the complexities and uncertainties associated with decision problem to be put into a logical framework amenable to analytic manipulation and solution. Today most consumer and industrial finished goods are distributed through multi stage inventory system of one sort or another. Computers, automobiles, spare parts, aircraft manufacturing (Boeing) industry and military hardware are commonly provided through multi stage system. Any enterprise with geographically dispersed demand, economies of scale in production and/or transportation, and market driven service requirements typically relies on multistage inventory system to remain competitive. Multi stage inventory systems are also common in production contexts, particularly in multi plant operations where the inventories act to decouple one facility from another. Over the past decades there has been much progress in developing an inventory theory for these multi stage systems. For deterministic demand, there are very effective procedures for setting reorder intervals for a wide range of systems [2, 3]. For serial systems with stochastic demand, there exist approaches for finding optimal order policies for both the periodic review case [4, 5] and continuous review case [6]. Different types of situations may arise during mathematical modelling of multi stage models. Different authors highlighted many cases. When modelling for multi stage system, case that usually arises - multi stage serial production – inventory system deciding upon regular time or over time production quantities at each stage. Hence management should decide whether it would go for overtime production in addition to regular time production capacity or not. As an alternative to this case, models can be formulated considering different sources of productiou at each stage. i.e. choosing the best source of product at each stage, assuming separate production variables for each stage. Alternative production routings for products within a plant in any stage is also a prominent case to be analyzed. In practice it is sometimes recommended that a multi stage system can be scheduled effectively by modelling the most important stages as a single stages system, sequentially scheduling operatious at the other stages, often using more elementary planning models. Whether a large multi stage model, which simultaneously considers decisions at each stage, or a set of smaller single-stage models, which plan stages in sequence, should be used is a matter of relevant economics. Modelling of production systems may or may not incorporate backlogging case depending on demand pattern and capacity scenario. In case of multi stage multi product inventory system, management may be encountered by product mix decisions. The sales potential of different products is supposed to be different. Taking into account the maximum and minimum possible sales of a particular product, in particular periods and possible revenue earnings against a particular product for those periods, models can be developed to find the most profitable production program over the planning horizon.

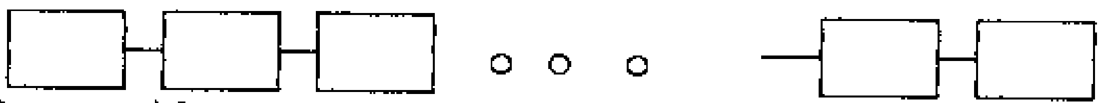
Again multi stage multi location production distribution models can be developed for single semi finished product and single finished product considering no backlogging. This model can be modified for additional considerations; as for example, sale or purchase of semi finished product at any stage. Overtime and work force level decision problems at each plant, multiple routings within the plant, production rate change costs, limitation of the size of shipments between plants. Inventory maximum and minimums at each stage and backlogging of finished goods demand could also add new features to the model.

Since the above discussion was made for single semi finished and finished product, this model however can be implemented for multiple semi finished product and multiple finished product. In this case, each of the finished products uses one or more of the multiple semi finished product produced by plants at previous stage. All the cases as discussed above were based on linear cost components. But there are exceptions those in which, models can be formulated with nonlinear cost components if appropriate; however, such models are sometimes difficult to solve especially when the problem tends to be large.

There are different approaches to this type of problems such as network structure of multi stage models. For network model, a common approach is first considering a network-based algorithm for solving a single product planning problem for a serial system with concave costs, and subsequently move general network structures with linear costs.

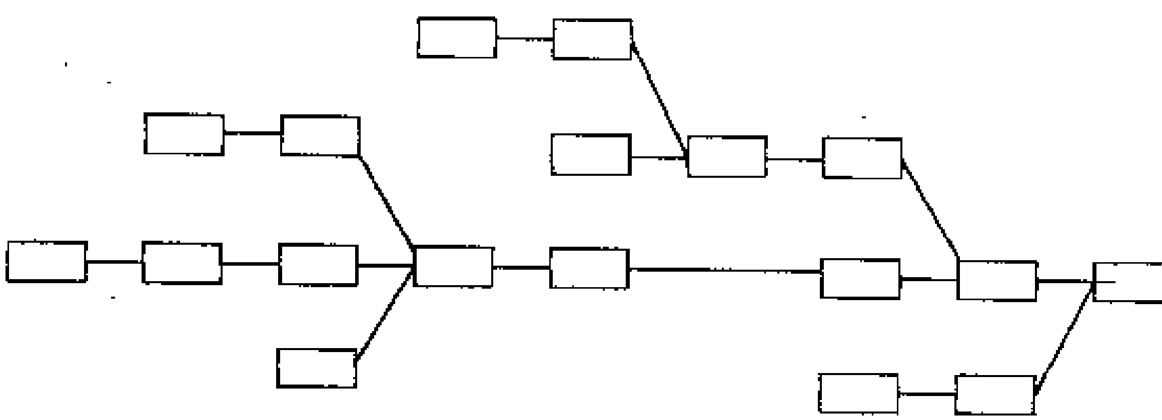
Multistage production system has been analyzed by hundreds of authors. Fig-1 (a) illustrates a single product produced in a series of steps, where no capacity constraints exist. Zangwill[7] and Love[8] used a concave cost structure (which allows e. g for setup cost and linear holding cost) and present relatively efficient solution techniques. Fig- 1(b) represents a product made by a complex assembly process. Lot-sizing decision in such a system has been studied by many authors, including Wagner[9] and Crownston, Wagner and Williams[10]. The complexity added by multiple products has been adequately analyzed. Multistage multi location production allocation has also been studied by Lynwood A. Johnson and Doughlas C.

Montgomery[11] considering single semi finished product and single finished product using linear programming techniques.



Raw material Finished product

Figure-1(a): Multistage serial production system for single product.



Raw material Finished product

Figure-1(b): Multistage system for assembling finished product.

Model formulation is largely an art, and requires considerable knowledge of the system under study. Successful modelling requires sound understanding regarding the interaction between the model and the system. The road to reach a correct decision becomes smooth by considerable enrichment of knowledge regarding operational planning, procurement concepts, MRP, MRP II, and production allocation concepts as described in the following pages. Searching of appropriate tools for appropriate cases is also important to defuse the complexity.

2.3 **Operational planning**: The starting point in any organizational plan is to determine how much the organization can sell from period to period. For any manufacturing firm, sales forecasting is a critical process that is often neglected and often done in isolation as ritual by the sales department and there is no accountability for accurate sales forecasting. Actual sales are often way out compared to forecast. As a result, other department like production, procurement and finance often do not rely on this forecasts but make their own one. The net result is that there is a high level of inventory or obsolescence. In case of operational planning, management should evaluate decisions concerning workforce size and stability, overtime, inventory fluctuations, and cash flow over given operating ranges of demand and capacity. Material management function should properly focus on priority of item wise demand for the finished product at different operating warehouses. Some times, there are conflicting arguments on safety stock versus safety time strategy. Buzacott. J.A. and Shanthikumer J.G. [12] have shown that safety time is usually only preferable to safety stock, when it is possible to make accurate forecasts of future required shipments over the lead time, otherwise safety stock is more robust in coping with changes in customer requirements in the lead time or with fluctuations in forecasts of lead time demand. During operational planning, multitude of problems may arise before management, and no single technique will necessarily meet all the needs of any one company. Many organizations, consequently, use a combination of different operational planning techniques.

2.4 New concept of procurement: The role of procurement department begins at the end of materials planning exercise. The role of purchasing department in any world class company is crucial since savings can be brought about through efficient purchasing practices. Latest purchasing is all about developing partnership concepts with suppliers thereby ensuring lean or just-in-time inventories, vendors managed inventories, quality at the source, vendors scheduling and such other techniques.

2.5 Production allocation concepts:

Production allocation is the activity of reching a future goal regarding production decision for future time period. The objective of production allocation management is to have inventory in the right place, at right time, at reasonable cost, so that the corporate management can achieve a desired level of customer service at or below a specified cost meeting anticipated production requirements through optimal use of resources. The various factor that act as inputs in production allocation includes, among others, current inventory level & backlog position, current work in process inventory, current work force levels, capacities of each production center, material availability, production standards, cost standards & selling price etc. Since allocation decisions affect manifold activities, eg. facilities, inventory management, stock-out frequency, manufacturing, transportation, communication & data processing, allocation strategies and policies should be part of an overall integrated organizational strategy encompassing all these functional areas. Decision made by marketing, finance, manufacturing shipping & logistics and engineering are to be linked systematically as well.

2.6 MRP, MRP II and Advent of new technology: To determine production schedule/allocation in multi stage manufacturing system, material requirement planning (MRP) is used throughout the industry. The basic idea (Orlicky[13] or McClain and Thomas[14]) is that a production schedule of a finished item translates into known quantity and timing needs.

MRP is a system that uses bill of materials, inventory and open order data, and master production schedule information to calculate requirements for materials. Fortunately today there is computer to help make our task easier.

Over the years computer packages have been developed that can aid material planning. There are other terms to be mentioned such as ERP (Enterprise resources planning).

MRP II is a set of integrated business processes or methodologies supported by computer systems to plan and control the resources of an organization most effectively. These integrated businesses processes have already been told earlier such as strategic planning, demand management, sales and operations planning, materials requirement planning an supplier scheduling. All these are key processes from an MRP II point of view that need to be fully integrated to derive maximum business benefits. The strength of MRP II philosophy lies in integrating business with computer systems and in its emphasis on sound people management and total quality and continuous improvement process.

2.7 Solution algorithms employed to solve inventory and production planning problems-

Dynamic programming model

Network model

Heuristic (silver meal) model.

Linear programming model.

2.7.1 Dynamic programming model:

Dynamic programming (DP) is a mathematical technique dealing with the optimization of multistage decision problems. The technique was originated in 1952 by Richard Bellman and G.B Dantzig, and was initially referred to as the stochastic linear programming. The name dynamic programming was evolved because of its use with time. Though the originator of the technique, Richard Bellman, himself, has said, 'we have coined the term dynamic programming' to emphasize that there are problems in which time plays an essential role", yet, in many dynamic programming problems time is not a relevant variable.

Dynamic programming may also be called multistage programming since the procedure typically determines the solution stages. The main unifying theory in DP is the principle of optimality. It basically dictates how a properly decomposed problem may be solved in stages (rather than as one entity) through the use of recursive computations. The computations are carried out in stages by breaking down the problem into subproblems (stages). To reduce the volume and complexity of computation each sub-problem is then considered separately. Since there exists an interdependence between the sub-problems, a procedure is required to link the computations that guarantees a feasible solution for each stage. This procedure is based on understanding of stage, state and the life function "recursive equation". The state of the system is perhaps most important concept in a dynamic programming model. It represents "link" between (successive) stages. In DP, the computations are actually recursive. Computation at each stage is based on computations of previous stage. This dynamic programming approach however becomes tedious and tiresome as the number of state variable increases. Due to this limitation, it is very difficult to use this approach. While dealing with large number of variables, formulation of recursive equation also becomes a perplex job as the number of variable increases.

2.7.2 Network model:

Production planning problems can be conceptualized as network models. Network models perform efficiently in case of production planning problems employing single product. Several network algorithms for the cases of linear, convex, concave and piecewise concave costs has been studied. This particular modelling approach was pioneered mainly by Zangwill. Zangwill gave a theorem of backward algorithm for solving the backward case, which states "In a minimum cost network flow problem where the objective function is a concave function of the arc flows and the network has a single source, there is an optimal solution having the properly that each node has an inward flow from at most one arc". In case of no backlogging case, Zangwill's backward algorithm can be used with minor modification/assumption.

Florian and Robillard have constructed a branch-and-bound algorithm for solving the concave cost network flow problem with capacity constraints.

Multistage production planning problems can also be handled by network concept by considering first a network - based – algorithm for solving a single product planning problem for a serial system with concave costs, and then more general network structures with linear costs. Multistage multiperiod network flow model has been analyzed by Zangwill with the objective function of minimizing total production and inventory cost over the planning horizon.

2.7.3 Heuristic (silver meal) model:

The heuristic model is an approximation of the dynamic programming model provided that the unit production/purchasing costs are identical and constant for all periods. This represents a slight restriction over the DP model where any concave cost function is allowable. In the heuristic, however it is permissible to have different setup and holding costs for the different periods.

With this restriction, the variable production is ignored and the heuristic is designed to balance the setup and inventory holding costs only. The decision variable in the heuristic is defined as the number of successive periods whose demand can be "lumped" in to a single production and or purchasing lot for the purpose of balancing the set up cost against the inventory holding cost.

2.7.4 Linear programming:

Since the introduction of OR technique in the late 1940s, linear programming has proven to be one of the most efficient management science tools. Its success stems from its flexibility in describing multitude of real life situation in varions areas.

The general form of a linear programming problem is a set of linear relationships defining the trade-off for each resource which is to be allocated and a single objective function which gives the contribution of each decision variable.

Linear programming model has a simple form that can be easily understood and, more importantly, is often realistic description of the real world production planning problems can be solved easily by employing linear programming technique. If the relevant costs can be assumed to be linear functions of the variables defining the production problem, the planning problem can be formulated as a linear programming, provided any constraints are also linear.

Linear programming models can easily incorporate resource constraints at each stages and translate required quantities of production in a rather. Spontaneous fashion. The planning for a multistage multilocation multiperiod system using multiple plants in any stage for multiple products relies more on linear programming technique than other techniques discussed in this section.

The availability of powerful algorithms for solving the general LP problem means that the large-scale production planning problems then can be analyzed mathematically.

In this project linear programming was employed as the solution technique. The obvious reason for concentrating on linear programming models is the availability of methods for solving very large linear programming problems. Dynamic multistage models tends to be large scale, since generally, number of constraints and variables is proportional to number of stages, as well as to the number of time periods. Dynamic programming and heuristic models could proved to be tedious, tiresome and perplex approach for solving problems this magnitude.

Chapter –3

Mathematical Modelling

3.1 Introduction: The essence of production planning lies in the use of mathematical model to describe a decision problem and to determine an optimal decision or policy by analysis or solution of that model. The development of multistage models will be addressed in the following sections of this chapter. In this research work, attempts were taken to characterize various multistage multi location and multi period production allocation systems. The nature and feature of the model gradually shifted to incorporate various production and manufacturing scenario. Application of the model has also been changed from very simple cases to complex manufacturing systems. The feature and application of all the models are shown in the chart 3.1.

Chart 3.1: Feature and application of all the models to be discussed.

	FEATURE	APPLICATION
Model-1	This model was constructed for single semifinished and single finished product where the output of the first stage was treated as necessary input for the second stage. The demand of finished product at the finished product ware house (Distribution Point) was considered known with certainty.	This model can be applied to the case of milk pasteurization and distribution enterprises

3.1.1 Subscript Meanings:

i: Location (identity) of semifinished product producing/purchasing plant at first stage; i ranges from 1 to n.

j: Location (identity) of finished product producing plant at second stage; j ranges from 1 to m.

k: Location (identity) of finished product warehouses at third stage; κ ranges from 1 to l.

t: Time period; t ranges from 1 to T.

a: Identity of semifinished product 'a' ranges from a to b

p: Identity of semifinished product; 'p' ranges from 1 to r.

- Subscript 'j' in model-5 refers to the identity / type of finished product

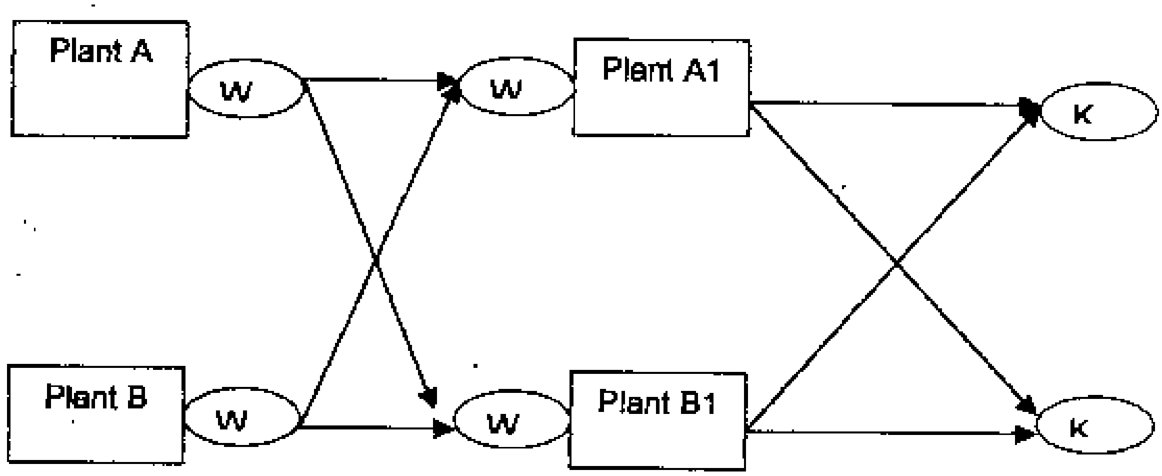
3.2 Assumptions made as in the models:

- There is no wastage, or spillage in different production stages.
- Holding cost includes holding cost, handling cost and storage cost.
- Shipping cost includes handling cost, shipping cost and transportation cost.
- Plants were assumed to be geographically separated.

Model-1

3.3.1.1 Introduction:

In order to get a glimpse of this model, let us first consider the case of fig-2 as illustrated, where the demand of finished product at the finished product ware house and the other factors may change with time.



Stage-1: *(Production & storage of Semi finished product)*

Stage-2: *(Storage of semi finished product Coming from 1st stage & production Of finished product.)*

Stage-3: *(Finished product warehouse)*

Fig-2: Material flow for algorithm developed in model–1.

Formulation of model I is applicable to processing industry such as pasteurization of milk, where raw milk is collected from different collecting points and then brought to the pasteurization plant. After pasteurizing the milk, packed liquid milks are then transported to different sales warehouses. This model is also applicable to foundry and casting industry. For this model 'i' ranges from 1 to 2, 'j' ranges from 1 to 2, 'k' ranges from 1 to 2, 't' ranges from 1 to 6.

Considering the case in fig-2 , it is found that a single semi finished product is produced at plant A and plant B. Each of this semi finished product producing plant maintains its own inventory of semi finished product at storage facilities associated with the plants (w).

At the second stage, there are finished product producing plants namely plant A1 and plant B1. Prior to finishing operation, each plant maintains its own inventory of semi finished product coming from stage 1.

Finally, there are finished product warehouses to offer storage & distribution facility. A single finished product produced by two plants. plant A1 and plant B1, are immediately distributed to warehouses of third stage, where it is stored.

All customer demand is satisfied from these locations. Each of the four plants and two regional warehouses is located at different geographical locations. Production cost varies among plants. Shipping costs are function of source and destination and function of mode of transportation.

3.3.1.2 Cost coefficients of objective function

The costs associated in the model are the following:

1. Production or procurement cost of semifinished product, at first stage.
2. Holding cost of semifinished product at first stage.
3. Shipping cost of semifinished product between first stage and second stage.
4. Holding cost of semifinished product at second stage.
5. Production cost of finished product and shipping cost to the third stage
6. Holding cost of finished product at third stage.

Each of these terms are discussed in the following paragraph.

1: Production or procurement cost of semifinished product, at first stage: Semifinished products can be produced/procured at different plants of first stage located at different locations. This cost term in objective function for each period can be written as

$$\sum_{i=1}^{2} (C_{1it} X_{it})$$

i = 1, 2
t = 1, 2, 3, 4, 5, 6

where,

C_{1it} = Cost to produce/procure a unit of semi finished product at plant i of stage 1 in period t.

X_{it}= Number of units of semi finished product produced at plant i of stage 1 in period t.

2. *Holding cost of semifinished product at first stage:*

After produciltion/purchasing of semifinished product it can be stored in the associated warehouses. This cost term in objective function for each period can be written as

$$\sum_{i=1}^{2} (h_{1it} I_{1it})$$

i = 1, 2
t = 1, 2, 3, 4, 5, 6

where,

I_{1it}= Semi finished product inventory at plant i of stage 1 at the end of Period t.

h_{1it}, = Unit inventory carrying costs at stage-1 in period t.

3. *Shipping cost of semifinished product between first stage and second stage:*

Costs incurred due shipping of semifinished product from warehouse of first stage to warehouse of second stage. This cost term in objective function for each period can be

$$\sum_{i=1}^{2} \sum_{j=1}^{2} g_{ijt} V_{ijt}$$

i = 1, 2
j = 1, 2
t = 1, 2, 3, 4, 5, 6

where,

g_{ijt}= Cost to ship a unit of semi finished product from plant i of stage 1 to storage facility at plant j at stage 2 in period t.

V_{ijt}= Number of units of semi finished product shipped from plant i of stage 1 to plant j at stage 2 in period t.

4. *Holding cost of semifinished product at second stage:*

Cost incurred due to inventory holding at second stage. Before finishing operation there is a provision of material storage in the second stage, if it requires. This term in objective function for each period can be written as

$$\sum_{j=1}^{2} (h_{2jt} I_{2jt})$$

j = 1, 2
t = 1, 2, 3, 4, 5, 6

where,

I_{2jt}= Semi finished product inventory at plant j, stage 2 at the end of period t.

h_{2jt} =Unit inventory carrying costs at stage-2 in period t.

5. *Production cost of finished product and shipping cost to the third stage:*

Cost incurred due to finishing operation in the second stage and the transportation cost of finished product between second stage to third stage. Here provision has been made such that the finished products can be shipped directly to third stage after finishing operation. This cost term in objective function for each period can be written as

$$\sum_{j=1}^{2} \sum_{k=1}^{2} (C_{2jt} + g'_{jkt}) Y_{jkt}$$

j = 1, 2
k = 1, 2
t = 1, 2, 3, 4, 5, 6

where,

C_{2jt}=Cost to produce a unit of finished product at plant j of stage 2 in period t.

g'_{jkt}= Cost to ship a unit of finished product from plant j of stage 2 to regional warehouse k in period t.

Y_{jkt}= Number of units of finished product produced at plant j of stage 2, and shipped to warehouse k of final stage.

6. *Holding cost of finished product at 3rd stage:*

Cost to hold finished product at the warehouse or distribution centers of third stage. This cost term in objective function for each period can be written as

$$\sum_{k=1}^{2} (h_{3kt} I_{3kt})$$

k = 1, 2
t = 1, 2, 3, 4, 5, 6

where,

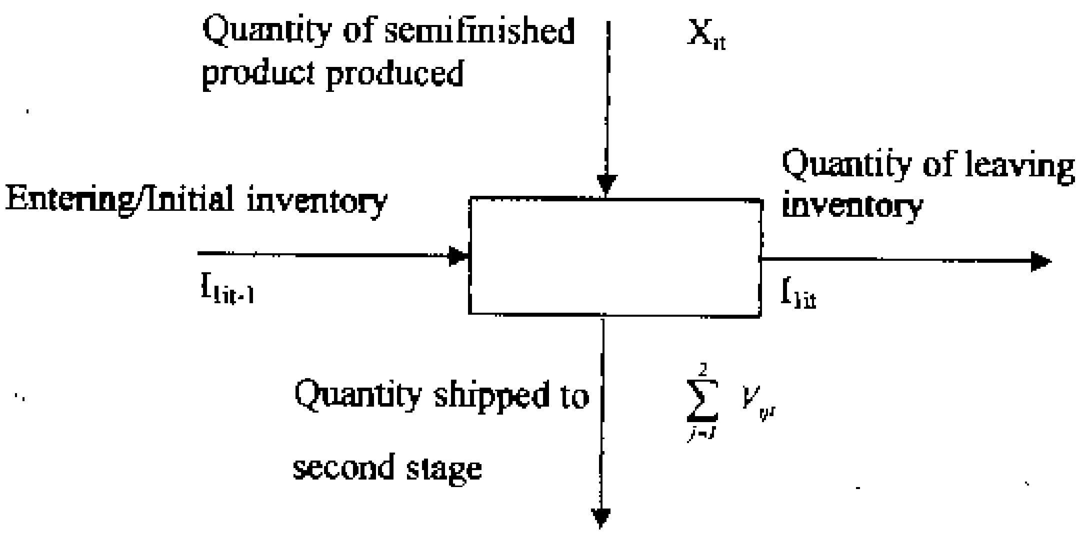
I_{3kt} = Finished product inventory at warehouse k at the end of period t.

h_{3kt} =Unit inventory carrying costs at stage-3 in period t.

3.3.1.3 Constraints:

1. *Inventory balance at first stage:*

At any period, there should be a balance of semifinished product inventory at the warehouses of first stage.



This may be expressed mathematically

$$I_{1it} = I_{1it} - 1 + X_{it} - \sum_{j=1}^{2} V_{ijt} \qquad \begin{vmatrix} i = 1,2 \\ t = 1,2,3,4,5,6 \end{vmatrix}$$

where,

I_{1it}= Semi finished product inventory at plant i of stage 1 at the end of Period t.

X_{it}= Number of units of semi finished product produced at plant i of stage 1 in period t.

V_{ijt}= Number of units of semi finished product shipped from plant i of stage 1 to plant j at stage 2 in period t.

2. *Material balance:*

Total quantity of products shipped from first stage to second stage should be equal to the quantity of finished products shipped from second stage to third stage. This may be stated as:

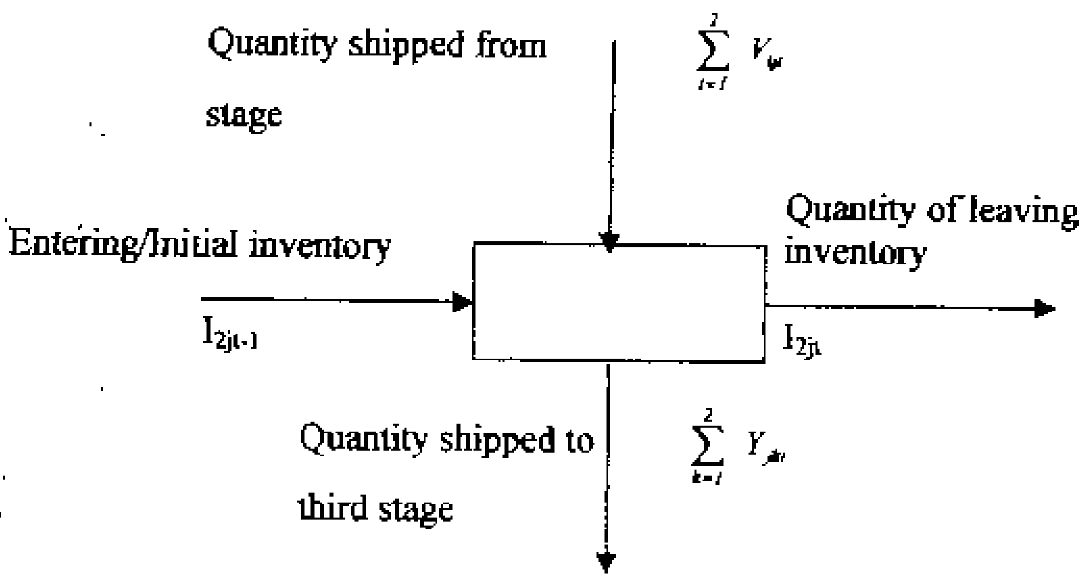
$$\sum_{i=1}^{2} Y_{ijl} = \sum_{k=1}^{2} Y_{jkl} \qquad \begin{vmatrix} j = 1,2 \\ l = 1,2,3,4,5,6 \end{vmatrix}$$

where,

Y_{jkl}= Number of units of finished product produced at plant j of stage 2, and shipped to warehouse k of final stage.

3. Inventory balance at second stage (semifinished):

At any period there should be a balance of semifinished product inventory at the warehouses of second stage.



This may be expressed mathematically

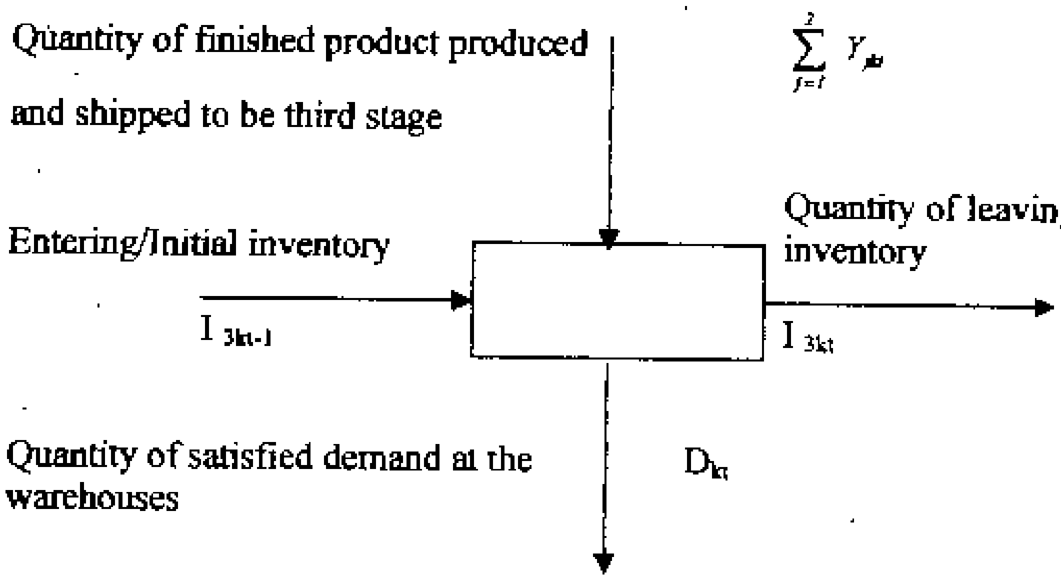
$$I_{2jt} = I_{2j,t-1} + \sum_{i=1}^{2} V_{ijt} - \sum_{k=1}^{2} Y_{jkt} \qquad \begin{vmatrix} j = 1,2 \\ t = 1,2,3,4,5,6 \end{vmatrix}$$

where,

I_{2jt}= Semi finished product inventory at plant j, stage 2 at the end of period t.

4. *Inventory balance at third stage:*

At any period, there should be a balance of finished product inventory at each warehouse of distribution center of third stage.



This may be expressed mathematically

$$I_{3kt} = I_{3kt-1} + \sum_{j=1}^{2} Y_{jkt} - D_{kt} \qquad \begin{vmatrix} j = 1,2 \\ t = 1,2,3,4,5,6 \end{vmatrix}$$

where,

I_{3kt} = Finished product inventory at warehouse k at the end of period t.

D_{kt} = Given demand of finished product at warehouse k in period t.

5. *Capacity constraint (samifinished product):*

production of number of units of semifinished product should be always less than the production capacity of each plant at each period. This can be stated as:

$$X_{it} <= P_{1it} \qquad \Big| t = 1,2$$

where,

P_{1it} = Maximum production capacity at plant i of stage-1

6. *Capacity constraint (finished product):*

Production of number of units of finished product should always be less than the production capacity of each plant at each period. This can be stated as.

$$\sum_{k=1}^{2} Y_{jkt} <= P_{2jt} \qquad \Big| \begin{array}{l} j = 1,2 \\ t = 1,2,3,4,5,6 \end{array}$$

where,

P_{2jt} = Maximum production capacity at plant j of stage-2

Non negativity of variables:

$I_{1it} >= 0$
$I_{2jt} >= 0$
$X_{it} >= 0$
$V_{ijt} >= 0$
$Y_{jkt} >= 0$
$I_{3kt} >= 0$

3.3.1.4 Complete model:

The model in its complete form can now be defined as:

Minimize,

$$Z=\sum_{t=1}^{T}\left[\begin{array}{l}\sum_{i=1}^{2}(C_{1it}X_{it}+h_{1it}I_{1it})+\sum_{i=1}^{2}\sum_{j=1}^{2}g_{ijt}V_{ijt}+\sum_{j=1}^{2}h_{2jt}I_{2jt}\\+\sum_{j=1}^{2}\sum_{k=1}^{2}(C_{2jt}+g'_{jkt})Y_{jkt}+\sum_{k=1}^{2}h_{3kt}I_{3kt}\end{array}\right]$$

Subject to

Inventory balance at first stage:

$$I_{1it}=I_{1it-1}+X_{it}-\sum_{j=1}^{2}V_{ijt}\qquad \left|\begin{array}{l}i=1,2\\t=1,2,3,4,5,6\end{array}\right.$$

material balance:

$$\sum_{i=1}^{2}V_{ijt}=\sum_{k=1}^{2}Y_{jkt}\qquad \left|\begin{array}{l}j=1,2\\t=1,2,3,4,5,6\end{array}\right.$$

Inventory balance at second stage (semifinished):

$$I_{2jt}=I_{2jt-1}+\sum_{i=1}^{2}V_{ijt}-\sum_{k=1}^{2}Y_{jkt}\qquad \left|\begin{array}{l}j=1,2\\t=1,2,3,4,5,6\end{array}\right.$$

Inventory balance at third stage:

$$I_{3kt}=I_{3kt-1}+\sum_{j=1}^{2}Y_{jkt}-D_{kt}\qquad \left|\begin{array}{l}j=1,2\\t=1,2,3,4,5,6\end{array}\right.$$

Capacity canstraint (samifinished product):

$$X_{it}<=P_{kt}\qquad \left|\,i=1,2\right.$$

Capacity constraint (finished product):

$$\sum_{k=1}^{2} Y_{jkt} <= P_{2jt} \qquad \begin{vmatrix} j = 1,2 \\ t = 1,2,3,4,5,6 \end{vmatrix}$$

Non negativity of variables:

$I_{hit} >= 0$

$I_{2jt} >= 0$

$X_{it} >= 0$

$V_{ijt} >= 0$

$Y_{jkt} >= 0$

$I_{3kt} >= 0$

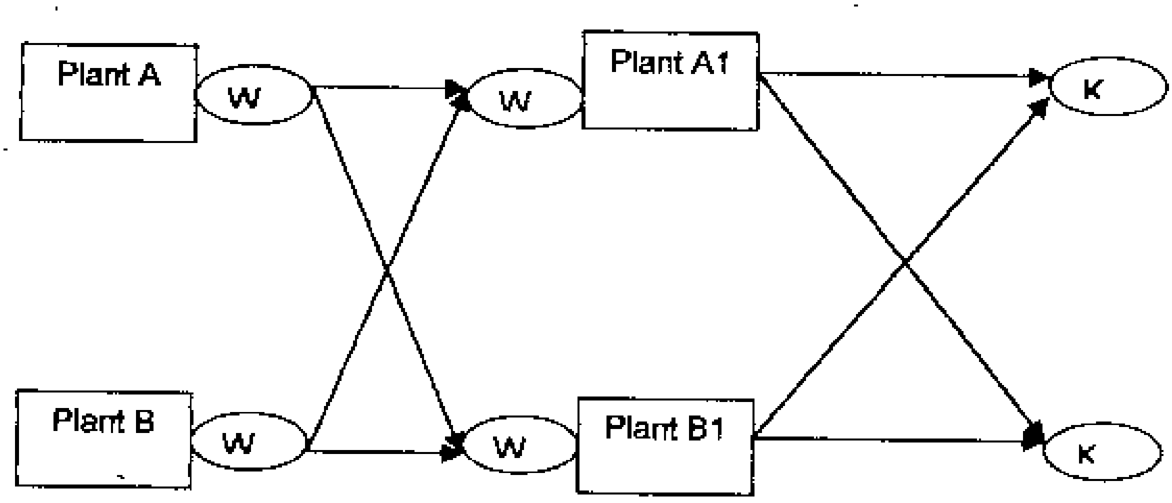
Z= Total production and shipping cost during the entire planning horizon

Model-2

3.3.2.1 Introduction:

This model is applicable to foundry & casting industry for manufacturing spare parts, machine tools etc. A hypothetical case has been considered here as illustrated in fig-2. For this model 'i' ranges from 1 to 2, 'j' ranges from 1 to 2, 'k' ranges from 1 to 2, 'a' ranges from 1 to 2 and 't' ranges from 1 to 6.

In this model multiple semi finished products were considered both in first and second stages of production. It was assumed that each semi finished product produced/procured in first stage would act as an essential component for the finished product produced in second stage, keeping the other assumptions intact. The material flow was also assumed the same as that assumed in model 1.



Stage-1:
(Production & storage of semifinished products)

Stage-2:
(Storage of semifinished product coming from stage – 1 & production of finished products)

Stage-3:
(Finished product warehouses)

Fig-3: Material flow for algorithm developed in model – 2.

3.3.2.2 Cost coefficients of objective function & constraints:

The notation and subscripts of this model is similar to the previous model. In this model only change was an introduction of subscript 'a' which is used to indicate individual semifinished/finished product, since this model considers multiple semifinished/finished product.

3.3.2.3 Complete model:

The model in its complete form can be defined as:

Minimize,

$$Z = \sum_{t=1}^{T} \left[\begin{array}{l} \sum_{a}^{b} \sum_{i=1}^{2} (C_{1ait} X_{ait} + h_{1ait} I_{1ait}) + \sum_{a}^{b} \sum_{i=1}^{2} \sum_{j=1}^{2} (g_{aijt} V_{aijt}) + \sum_{a}^{b} \sum_{j=1}^{2} h_{2ajt} I_{2ajt} + \\ \sum_{a}^{b} \sum_{j=1}^{2} \sum_{k=1}^{2} (C_{2ajt} + g'_{ajkt}) Y_{ajkt} + \sum_{a}^{b} \sum_{k=1}^{2} h_{3kt} I_{3kt} \end{array} \right]$$

Subject to

Inventory balance:

$$I_{1ait} = I_{1ait-1} + X_{it} - \sum_{j=1}^{2} V_{aijt} \qquad \left| \begin{array}{l} i = 1,2 \\ t = 1,2,3,4,5,6 \\ a = a,b \end{array} \right.$$

Material balance:

$$\sum_{i=1}^{2} V_{aijt} = \sum_{k=1}^{2} Y_{ajkt} \qquad \begin{vmatrix} j = 1,2 \\ t = 1,2,3,4,5,6 \\ a = a,b \end{vmatrix}$$

Inventory balance:

$$I_{2ajt} = I_{2ajt-1} + \sum_{i=1}^{2} V_{aijt} - \sum_{k=1}^{2} Y_{ajkt} \qquad \begin{vmatrix} j = 1,2 \\ t = 1,2,3,4,5,6 \\ a = a,b \end{vmatrix}$$

Inventory balance:

$$I_{3akt} = I_{3akt-1} + \sum_{j=1}^{2} Y_{ajkt} - D_{akt} \qquad \begin{vmatrix} j = 1,2 \\ t = 1,2,3,4,5,6 \\ a = a,b \end{vmatrix}$$

Capacity limitation (semifinished):

$$X_{ait} <= P_{1ait} \qquad \begin{vmatrix} i = 1,2 \\ a = a,b \\ t = 1,2,3,4,5,6 \end{vmatrix}$$

Capacity limitation (finished):

$$\sum_{k=1}^{2} Y_{ajkt} <= P_{2ajt} \qquad \begin{vmatrix} j = 1,2 \\ t = 1,2,3,4,5,6 \\ a = a,b \end{vmatrix}$$

Non negativity of variables:

$I_{1ait} >= 0$

$I_{2ajt} >= 0$

$X_{ait} >= 0$

$V_{aijt} >= 0$

$Y_{ajkt} >= 0$

$I_{3akt} >= 0$

Z= Total production and shipping cost during the entire planning horizon.

Model-3

3.3.3.1 Introduction:

This model was developed with a different approach as compared to previous models. The concept of assembling was incorporated in this model. A hypothetical manufacturing system has been considered as illustrated in fig-3 for the formulation of this model. For this model 'i' is equals to 1, 'j' is equals to 1, 'k' is equals to 1 and 't' ranges from 1 to 6.

As an initial effort, two differently located manufacturing plants were considered, along with a regional warehouse. Material flow was studied between these plants and the warehouse. At first stage, it produces semi finished product or components needed to assemble finished product at stage 2. At second stage it assembles the components. The finished products are directly shipped to the warehouse on demand at the third stage.

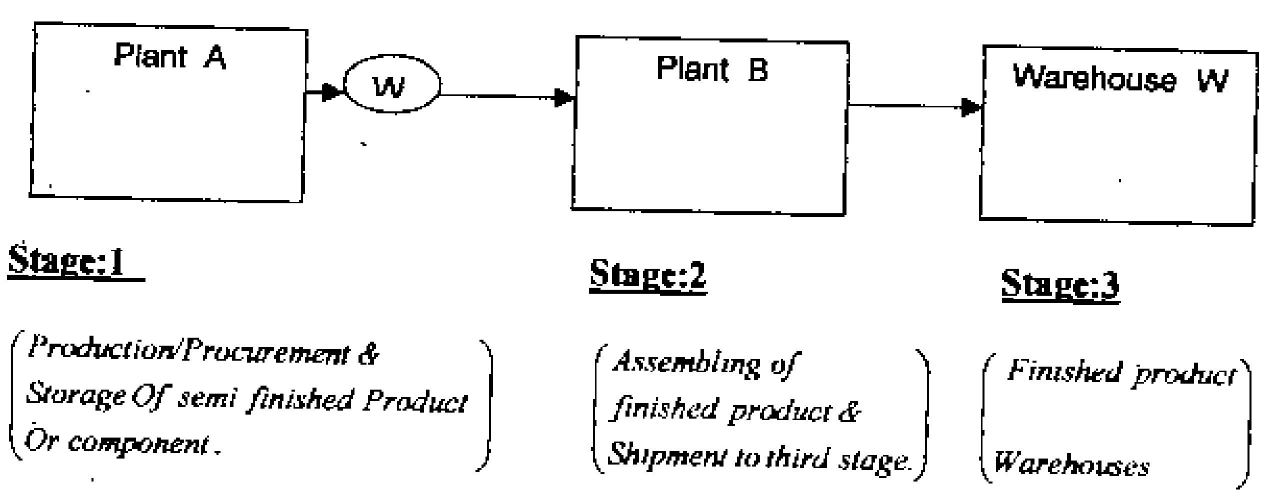


Fig-4: Material flow diagram for algorithm developed in model -3.

Formulation of model III is applicable to simple assembling and distribution system where the enterprise relies on manufacturing and marketing of single finished product.

3.3.3.2 Cost coefficients of objective function:

The costs associated in the model are:

1. *Production/procurement cost of semifinished products or components at first stage;*

Cost incurred due to production or procurement of components in the plants of first stage. This cost term in objective function for each period can be written as

$$\sum_{a=1}^{3} (C_{at} X_{at})$$

$a = 1, 2, 3$
t = 1, 2, 3, 4, 5, 6

where,

C_{at} = Cost to produce/procure a unit of semi finished product "a" at plant A of stage 1 in period t.

X_{at}= Number of units of semi finished product of identity "a" produced /procured at plant A of stage 1 in period t.

2. *Holding cost of components at first stage:*

After production of components it can be stored at the associated warehouses cost incurred due to holding of these components are holding costs. This term in objective function can be writteu as

$$\sum_{a=1}^{3} (C_{1at} I_{1at})$$

$a = 1, 2, 3$
$t = 1, 2, 3, 4, 5, 6$

where,

C_{1at} =Unit inventory carrying costs at stage-1 in period t.

I_{1at}= Semi finished product inventory of product "*a*" at plant A of stage 1 at the end of period t.

3. *Shipping cost of semifinished product:*

Cost incurred due to transportation of components from first stage to the assembling plant of 3rd stage. This cost term in objective functiou for each period can now be written as

$$\sum_{a=1}^{3} (V_{at}\, g_{at})$$

$a = 1, 2, 3$
$t = 1, 2, 3, 4, 5, 6$

where,

V_{at}= Number of units of semi finished product/component of identity "*a*"shipped from plant A of stage 1 to plant B at stage 2 in period t.

g_{at}= Cost to ship a unit of semi finished product "*a*" from plant A of stage 1 to plant B at stage 2 in period t.

4. Assembling & shipping cost:

Cost incurred due to assembling of finished product and shipment to the third stage. This cost term in objective function for each period can now be written as

$$(C_t + S_t)\, Y_t \qquad t = 1, 2, 3, 4, 5, 6$$

where,

C_t= Cost to produce/assemble a unit of finished product at plant B of stage 2 in period t.

S_t= Cost to ship a unit of finished product from plant B of stage 2 to regional warehouse W in period t.

Y_t= Number of units of finished product assembled at plant B of stage 2.

5. Holding cost of finished product:

Cost incurred due to holding of assembled product at the third stage warehouse. This cost term in the objective function can be written as

$$C_{3Yt}\, I_{3Yt}$$

where,

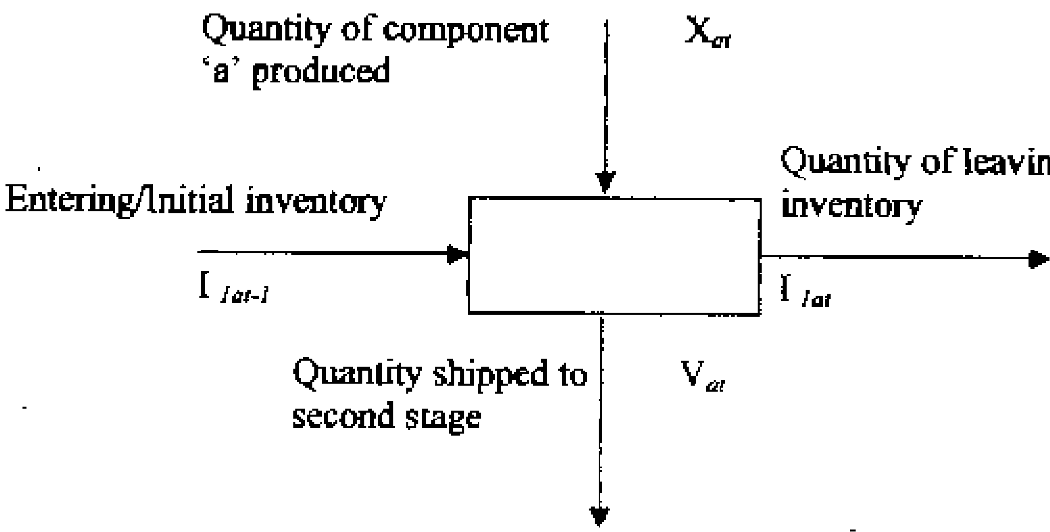
C_{3yt} = Unit inventory carrying cost at stage-3 in period t.

I_{3yt}= Finished product inventory at warehouse W at the end of period t.

3.3.3.3 Constraints:

1. *Inventory balance at first stage:*

At any period there should be a balance of semifinished product inventory at the warehouse of first stage.



This may be expressed mathematically

$$I_{1at} = I_{1at-1} + X_{at} - V_{at} \qquad \begin{vmatrix} a = 1,2,3 \\ t = 1,2,3,4,5,6 \end{vmatrix}$$

where,

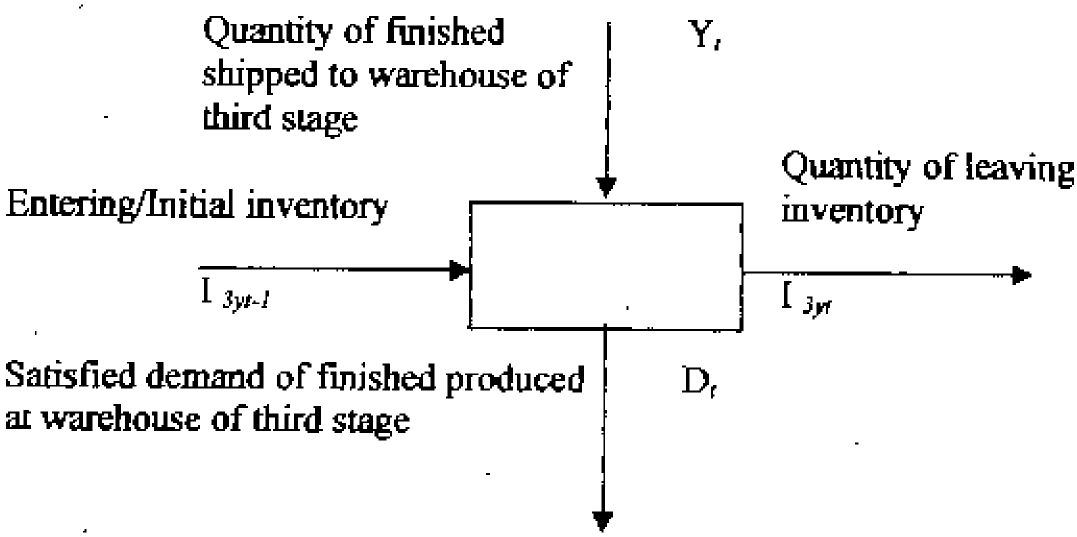
I_{1at}= Semi finished product inventory of product "*a*" at plant A of stage 1 at the end of period t.

X_{at}= Number of units of semi finished product of identity "*a*" produced /procured at plant A of stage 1 in period t.

V_{at}= Number of units of semi finished product/component of identity "*a*" shipped from plant A of stage 1 to plant B at stage 2 in period t.

2. *Inventory balance at third stage:*

At any period, there should be a balance of finished (assembled) product inventory at the warehouse/distribution center of the third stage. This can be stated as:



This may be expressed mathematically

$$I_{3yt} = I_{3yt-1} + Y_t - D_t \qquad \forall t = 1,2,3,4,5,6$$

where,

I_{3yt}= Finished product inventory at warehouse W at the end of period t.

Y_t= Number of units of finished product assembled at plant B of stage 2.

D_t= Given demand of assembled product at warehouse W in period t.

3. Capacity limitation (components):

Production of number of semifinished products/components should be always less than the production capacity of the plant at each period. This can be stated as:

$$X_{at} <= P_{at}$$

$t = 1, 2, 3, 4, 5, 6$

where,

P_{at} = Production capacities at stage-1 in period t.

4. Material balance:

Number of production of units of each component should be greater than the minimum numbers of components necessary to assemble the finished product in any period (according to product explosion). This can be stated as:

$$X_a/m_a => Y_t$$

$t = 1, 2, 3, 4, 5, 6$

where,

m_a = Minimum number of component 'a' required to assemble a finished product 'Y'.

5. *Capacity limitation:*

Number of units of assembled product should always be less than the production capacity of the plant at any period. This can be stated as:

$$Y_t <= P_t \qquad t = 1, 2, 3, 4, 5, 6$$

where

P_t= Assembling capacity of stage-2 in period t.

3.3.3.4 Complete model:

The model in its complete form can be defined as:

Minimize,

$$Z = \sum_{t=1}^{6}\left[\sum_{a=1}^{3}(C_{at}X_{at} + C_{1at}I_{1at}) + \sum_{a=1}^{3}V_{at}g_{at} + (C_t + S_t)Y_t + C_{3yt}I_{3yt}\right]$$

Subject to

Inventory balance at first stage:

$$I_{1at} = I_{1at-1} + X_{at} - V_{at} \qquad \begin{vmatrix} a = 1,2,3 \\ t = 1,2,3,4,5,6 \end{vmatrix}$$

Inventory balance at third stage:

$$I_{3yt} = I_{3yt-1} + Y_t - D_t \qquad |t = 1,2,3,4,5,6$$

Capacity limitation (components):

$$X_{at} <= P_{at} \qquad \begin{cases} a = 1,2,3 \\ t = 1,2,3,4,5,6 \end{cases}$$

Material balance:

$$X_{at} / m_a >= Y_t \qquad \begin{cases} a = 1,2,3 \\ t = 1,2,3,4,5,6 \\ ma = \text{zero or any positive integer value.} \end{cases}$$

Capacity limitation:

$$Y_t <= P_t \qquad \{ t = 1,2,3,4,5,6$$

Non negativity of variables:

$$I_{1at} >= 0 \qquad \begin{cases} a = 1,2,3 \\ t = 1,2,3,4,5,6 \end{cases}$$

$$I_{3yt} >= 0$$

$$X_{at} >= 0$$

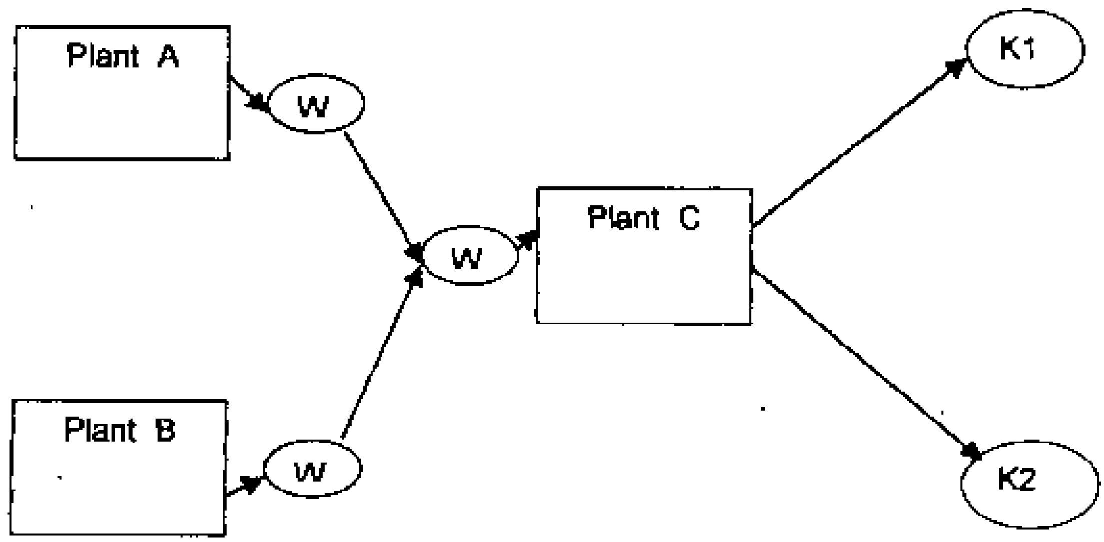
$$Y_t >= 0$$

Z= Total production and shipping cost during the entire planning horizon.

Model-4

3.3.4.1 Introduction:

In this model, relatively complex situation was considered, as compared to the previous model. It is very likely that the subassemblies can be brought to assembling plant from different plants located at a diverse location, using multiple plants at the first stage. For this model 'i' ranges from 1 to 2, 'k' ranges from 1 to 2, 'a' ranges from 1 to 2 and 't' ranges from 1 to 6. At the first stage there are two plants namely, plant A and plant B, which produces same products/components (semi finished) then holds inventory at the storage facilities. The components were then shipped to plant C at second stage for assembling. The components can be stored at the storage facility prior to the assembling in case of necessity. All the components were used to assemble the finished product. Finished products were immediately shipped to the finished product warehouse after the assembling operation . The finished product warehouses are $\kappa 1$ and $\kappa 2$.



Stage:1

(Production/Procurement & storage of semi finished Product.(Components))

Stage:2

(Storage of components & assembling of finished Product.)

Stage:3

(Storage of finished product at regional Warehouses.)

Fig-5: Material flow diagram for algorithm developed in model –4.

Formulation of model IV is suitable for assembling finished products of any type. As for example the case of automobile assembling can be considered. Where subassemblies and components can be manufactured or procured at different locations, and the final shape takes place to an assembling plant.

where,

I_{1ait} = Inventory of semifinished product of identity 'a' at plant i, of stage-1 at the end of period t.

h_{1ait} = Cost to hold a unit of semifinished product of identity 'a' at plant i in period t.

3. *Holding cost of components at warehouse of second stage:*

Cost incurred due to holding of components at any period in the warehouses of second stage. Prior to assembling, components coming from stage-1 can be stored at the stage-2 warehouse. This can be stated as:

$$\sum_{a=1}^{2} (h_{2at} I_{2at}) \qquad \Big| t = 1,2,3,4,5,6$$

where,

h_{2at} = cost to hold a unit of semi finished product of identity "a" at second stage in period t.

I_{2at} = Inventory of semi finished product of identity "a" at stage 2, at the end of period t.

4. *Shipping cost of semifinished product:*

Cost incurred due to transportation of components from stage-1 to the assembling plant of stage-2. This can be stated as:

$$\sum_{a=1}^{2} \sum_{i=1}^{2} \left(V_{ait}\, g_{ait}\right) \qquad \forall t = 1,2,3,4,5,6$$

where,

V_{ait} = Number of units of semi finished product or component of identity "a" shipped from plant i, of stage 1 to storage facility of plant C, at stage 2 .

g_{ait} = Cost to ship a unit of semi finished product of identity "a" from ith plant of first stage to the assembling plant at second stage, in period t.

5. *Assembling and shipping cost to stage-3:*

Cost incurred due to assembling of finished product & shipment to stage-3. This can be stated as:

$$\sum_{k=1}^{2} \left(I_{3kt}\, h_{3kt} + C_{kt}\, Y_{kt}\right) \qquad \forall t = 1,2,3,4,5,6$$

where,

I_{3kt} = Inventory of assembled (finished) product at the warehouse k, at the end of period t.

h_{3kt} = Cost to hold a unit of finished product in warehouse k at period t.

C_{kt} = Cost to assemble a unit of finished product at the assembling plant and ship to warehouse k in period t.

Y_{kt} = Number of units of finished(assembled) product assembled at assembling plant and shipped to the warehouse k, in period t.

3.3.4.3 Constraints of model:

Constraint of this model acts similar to the previous model.

3.3.4.4 Complete model:

The model in its complete form can be defined as:

Minimize,

$$Z = \sum_{t=1}^{6}\Big[\sum_{a=1}^{2}\sum_{i=1}^{2}(C_{ait}^{1}X_{ait} + I_{1ait}h_{1ait}) + \sum_{a=1}^{2}h_{2at}I_{2at} + \sum_{a=1}^{2}\sum_{i=1}^{2}V_{ait}g_{ait} + \sum_{k=1}^{2}(I_{3kt}h_{3kt} + C_{kt}Y_{kt}) \Big]$$

Subject to

Inventory balance:

$$I_{1ait} = I_{1ait-1} + X_{ait} - V_{ait} \qquad \begin{cases} a = 1,2 \\ i = 1,2 \\ t = 1,2,3,4,5,6 \end{cases}$$

Inventory balance:

$$I_{2at} = I_{2at-1} + \sum_{i=1}^{2} V_{ait} - m_a \sum_{k=1}^{2} Y_{kt} \qquad \begin{cases} t = 1,2,3,4,5,6 \\ a = 1,2 \end{cases}$$

Inventory balance:

$$I_{3kt} = I_{3kt-1} + Y_{kt} - D_{kt} \qquad \begin{cases} k = 1,2 \\ t = 1,2,3,4,5,6 \end{cases}$$

Material balance:

$$\sum_{k=1}^{2} Y_{kt} <= \sum_{i=1}^{2} X_{ait} / m_a \qquad \begin{cases} a = 1,2 \\ t = 1,2,3,4,5,6 \\ ma = \text{zero or any positive integer value} \end{cases}$$

Capacity limitation:

$$X_{ait} <= P_{1ait} \qquad \begin{cases} i = 1,2 \\ a = 1,2 \\ t = 1,2,3,4,5,6 \end{cases}$$

Capacity limitation:

$$Y_{kt} <= P_{2kt} \qquad \begin{cases} k = 1,2 \\ t = 1,2,3,4,5,6 \end{cases}$$

Non negativity of variables:

$I_{1ait} >= 0$

$I_{2at} >= 0$

$X_{ait} >= 0$

$V_{ait} >= 0$

$I_{3kt} >= 0$

$Y_{kt} >= 0$

Z = Total production and distribution cost for the entire planning horizon.

Model-5

3.3.5.1 Introduction:

In the previous model, a single finished (assembled) product was assumed. In this case multiple finished (assembled) product was considered in the second stage in addition to the multiple semi finished product at stage 1. For this model 'i' ranges from 1 to 2, 'j' ranges from 1 to 4, 'p' ranges from 1 to 16, 'k' ranges from 1 to 3 and 't' ranges from 1 to 4.

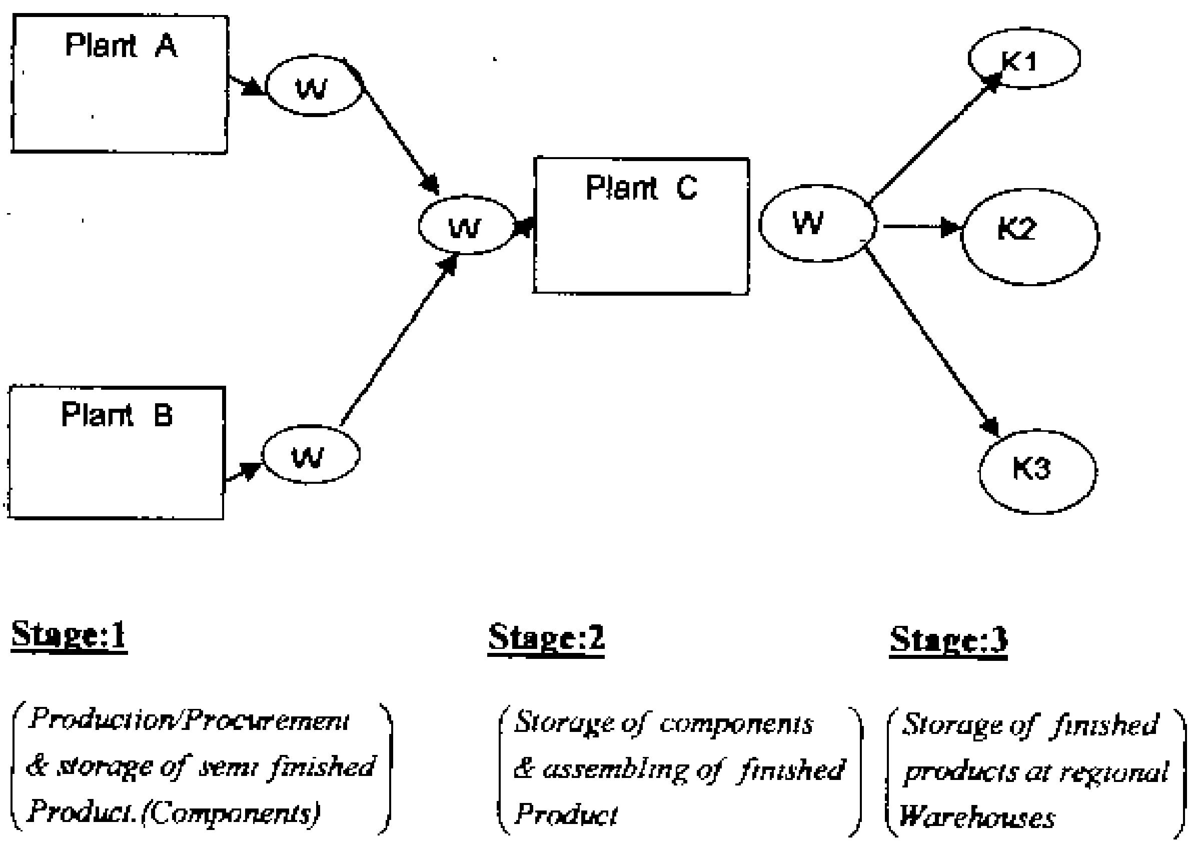


Fig-6: Material flow diagram for algorithm developed in model –5.

The developed model was tested with real life data availed from a local battery manufacturing company Rahim Afrooz (Bangladesh ltd.) This model was developed for enterprises that, relies on manufacturing and marketing of multiple finished item. The case of automobile, computer hardware and electronics are the common examples of this model.

3.3.5.2 Cost coefficients of objective functioo:

The costs associated in the model are the following:

1. Production/procurement and inventory holding cost of semifinished products/components at first stage.
2. Holding cost of semifinished products at second stage.
3. Shipping cost of semifinished products/components between first and second stage.
4. Assembling cost of the finished products at the second stage.
5. Holding cost of the finished products at second stage.
6. Shipping cost of finished product between second stage and last stage
7. Holding cost of finished products at the warehouses of third stage.

Each of these terms are discussed in the following paragraph:

1. ***Production/procurement and inventory holding cost of semifinished products/components at first stage.***

Cost incurred due to production/procurement of semifinished products/components at first stage plants located at different locations. This cost term in objective function for each period can be written as:

$$\sum_{i=1}^{2} \sum_{p=1}^{16} \left(C_{pit} X_{pit} + I_{1pit} h_{1pit}\right)$$

p = 1, 2, 3, 4, ... 16
i = 1, 2
t = 1, 2, 3, 4

where,

C_{pit} = cost to produce/procure a unit of semi finished product of identity "p" at plant i, in stage-1 at period t.

X_{pit} = Number of units of semi finished product/compouent of identity "p" produced/procured at plant i, of stage-1 in period t.

I_{1pit} = Inventory of semi finished product of identity "p" at plant i of stage 1 at the end of period t.

h_{1pit} = Cost to hold a unit of semi finished product of identity "p" , in storage facility of plant i at period t.

2. ***Holding cost of semifinished products at second stage:***

Cost incurred due to inventory holding of components coming from first stage at the second stage. This cost term in objective function for each period can be written as:

$$\sum_{p=1}^{16} h_{2pt} I_{2pt}$$

p = 1, 2, 3, 4, ... 16
t = 1, 2, 3, 4

where,

h_{2pt} = Cost to hold a unit of semi finished product of identity "p" in the storage facility of plant C at period t.

I_{2pt} = Inventory of semi finished product or components of identity "p" in the storage facility of plant C of stage 2, at the end of period t.

3. ***Shipping cost of semifinished products/components between first and second stage.***

Cost incurred due to transportation of components from first stage to second stage. This cost term of objective function for each period can be written as:

$$\sum_{i=1}^{2} \sum_{p=1}^{16} V_{pit} g_{pit}$$

p = 1, 2, 3, 4, ... 16
i = 1, 2
t = 1, 2, 3, 4

where,

V_{pit} = Number of units of semi finished product or component of identity "p" shipped from plant i of stage 1 to the storage facility of plant C at stage2 in period t.

g_{pit} = cost to ship a unit of semi finished product of identity "p" from first stage of plant i, to the storage facility of second stage in period t.

4. *Assembling cost of the finished products at the second stage:*

Cost incurred due to assembling of semifinished products at the assembling plant of second stage. This cost term of objective function for each period can be written as:

$$\sum_{j=1}^{4} C_{jt} Y_{jt}$$

j = 1, 2, 3, 4
t = 1, 2, 3, 4

where,

C_{jt} = Cost to assemble a unit of finished product of identity "j" in the assembling plant at period t.

Y_{jt} = Number of units of finished product of identity 'j' assembled at plant C of stage 2, in period t.

5. *Holding cost of the finished products at second stage:*

Cost incurred due to inventory holding of finished (assembled) products at the warehouse of second stage. This cost term of objective function for each period can be written as:

$$\sum_{j=1}^{4} h_{3jt} I_{3jt}$$

j = 1, 2, 3, 4
t = 1, 2, 3, 4

where,

h_{3jt} = Cost to hold a unit of finished product (assembled) of identity "j" in the storage facility of plant C.

I_{3jt} = Inventory of finished (assembled) product of identity " j" in the storage facility of plant C (after assembling), of stage 2, at the end of period t.

6. *Shipping cost of finished product between second stage and last stage*

Cost incurred due to transportation of finished products (assembled) between warehouses of second and third stage. This cost term of objective function for each period can be written as:

$$\sum_{j=1}^{4} \sum_{k=1}^{3} C_{jkt} W_{jkt}$$

j = 1, 2, 3, 4
k = 1, 2, 3
t = 1, 2, 3, 4

where,

C_{jkt} = Cost to ship a unit of finished product of identity "j" from storage facility of second stage to warehouse k of final stage in period t.

W_{jkt} = Number of units of finished product of identity "j" shipped to warehouse k in period t.

7. *Holding cost of finished products at the warehouses of third stage*

Cost incurred due to inventory holding of finished (assembled) products at the warehouse of third stage. This cost term of objective function for each period can be written as:

$$\sum_{j=1}^{4} \sum_{k=1}^{3} h_{4jkt} I_{4jkt}$$

$$j = 1, 2, 3, 4$$
$$k = 1, 2, 3$$
$$t = 1, 2, 3, 4$$

where,

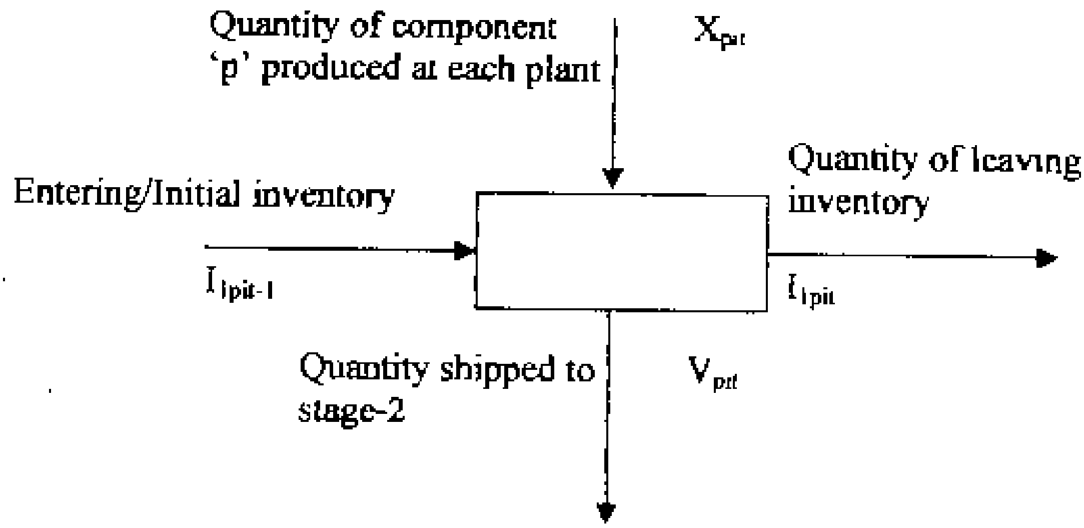
h_{4jkt} = Cost to hold a unit of finished product of identity "j" in warehouse k of final stage, in period t.

I_{4jkt} = Inventory of finished (assembled) product of identity "j" in the warehouse k of stage 3 at the end of period t.

3.3.5.3 Constraints:

1. *Inventory balance at stage-1:*

At any period, there should be a balance of semifinished product inventory of each type, at the warehouses of the stage-1.

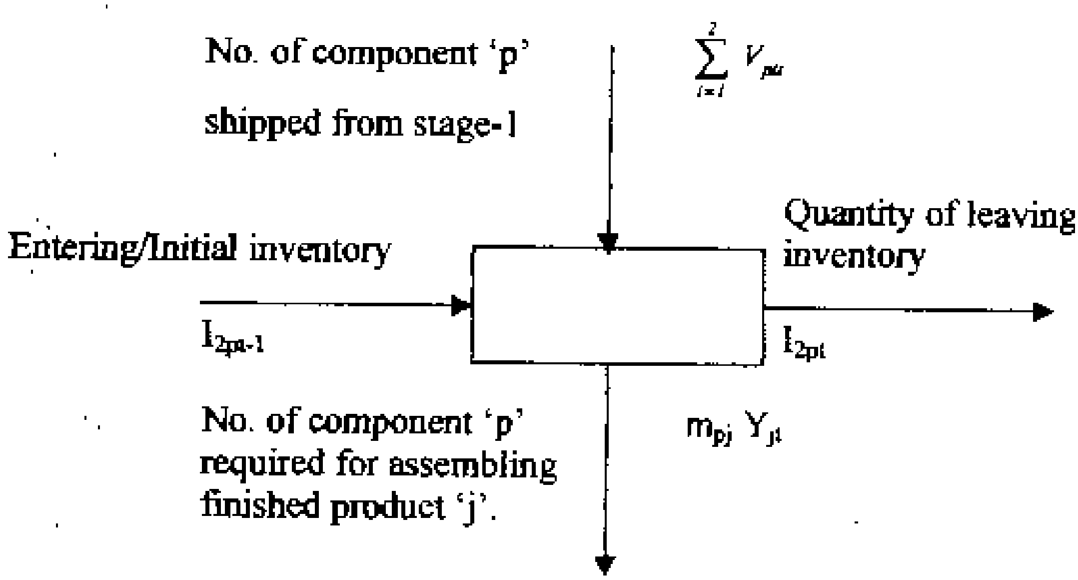


This may be expressed mathematically

$$I_{1pit} = I_{1pit-1} + X_{pit} - V_{pit} \qquad \begin{cases} p = 1,2,3,4 \ldots\ldots\ldots\ldots 16 \\ i = 1,2 \\ t = 1,2,3,4 \end{cases}$$

2. *Inventory balance at stage-2 (component):*

At any period, there should be a balance of semifinished product inventory of the type 'p', at the warehouse of the stage-2.



This may be expressed mathematically

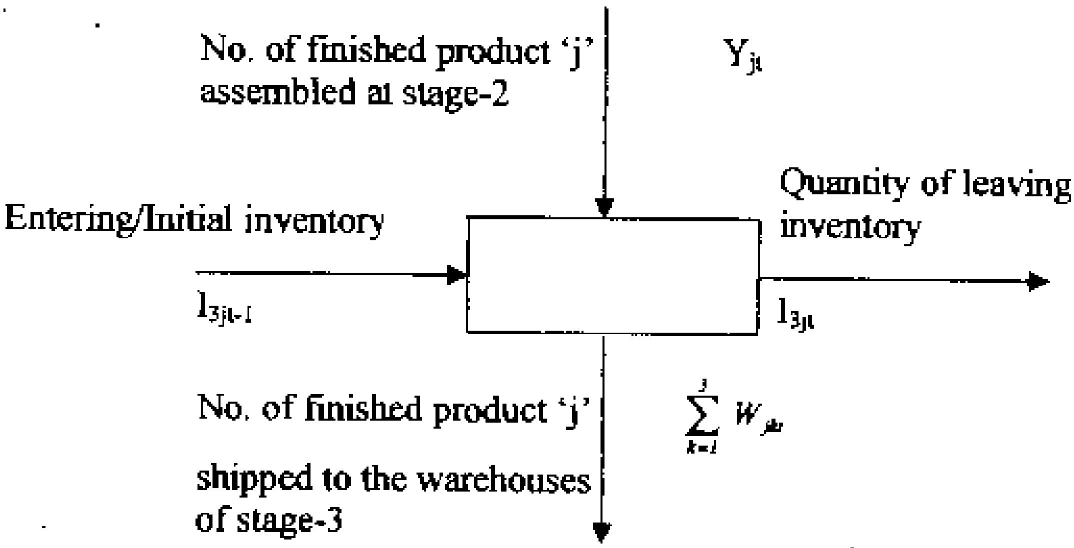
$$I_{2pt} = I_{2pt-1} + \sum_{i=1}^{2} V_{pit} - \sum_{j=1}^{4} m_{pj} Y_{jt} \qquad \begin{array}{l} t = 1,2,3,4 \\ j = 1,2,3,4 \\ mpj = 0 \text{ or any positive integer.} \end{array}$$

where,

m_{pj} = No. of component 'p' needed for assembling each unit of finished product 'j'.

3. *Inventory balance at stage-2 (assembled product):*

At any period, there should be a balance of assembled product inventory of the type 'j', at the warehouse of the stage-2.



This may be expressed mathematically

$$I_{3jt} = I_{3jt-1} + Y_{jt} - \sum_{k=1}^{3} W_{jkt} \qquad \begin{vmatrix} j = 1,2,3,4 \\ t = 1,2,3,4 \end{vmatrix}$$

4. *Component supply:*

The production quantity of component 'p' in any period at stage-1 must be greater than the number of components needed for assembling finished product 'j' at stage-2. This can be stated as:

$$\sum_{j=1}^{4} m_{pj} Y_{jt} <= \sum_{i=1}^{2} X_{pit} \qquad \begin{vmatrix} p = 1,2,3,4..............16 \\ t = 1,2,3,4 \end{vmatrix}$$

5. *Capacity limitation:*

The number of units of component 'p' produced, should always be less than the plant capacity at any period. This can be stated as:

$$X_{pit} <= q_{1pit} \qquad \left| \begin{array}{l} i = 1,2 \\ p = 1,2,3,4..............16 \\ t = 1,2,3,4 \end{array} \right.$$

where,

q_{1pit} = Maximum production capacity of semifinished product 'p' at plant i, in period t.

6. *Capacity limitation:*

The number of units of finished product 'j' assembled, should always be less than the plant capacity at any period. This can be stated as:

$$Y_{jt} <= q_{2jt} \qquad \left| \begin{array}{l} j = 1,2,3,4 \\ t = 1,2,3,4 \end{array} \right.$$

where,

q_{2jt} = Maximum assembling capacity of finished product 'j' in plant C, in period t.

7. *Storage limitation:*

Due to the space limitation in the warehouses, it is not possible to store the finished product, more than a specified quantity, at any period. This can be stated as:

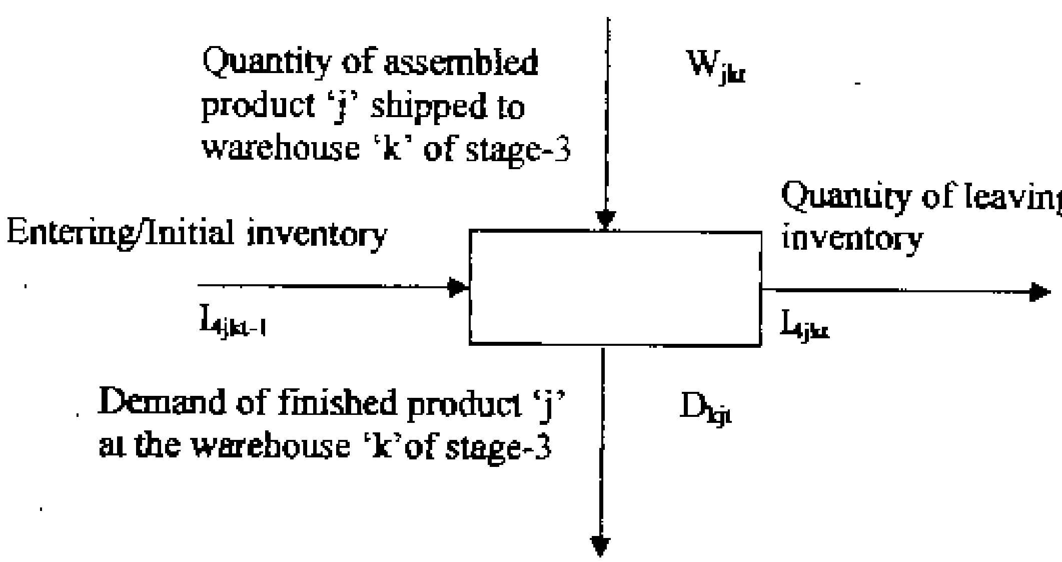
$$\sum_{j=1}^{4} Y_{jt} <= Q_t \qquad \begin{vmatrix} t = 1,2,3,4 \\ j = 1,2,3,4 \end{vmatrix}$$

where,

Q_t = Maximum quantity of finished product that can avail the storage facility at the storage facility of assembling plant in period t.

8. Inventory balance:

The warehouses of stage-3 should have the capacity to satisfy the market demand of any finished product 'j'. This can be stated as:



This may be expressed mathematically

$$I_{4jkt} >= I_{4jkt-1} + W_{jkt} - D_{jkt} \qquad \begin{vmatrix} j = 1,2,3,4 \\ k = 1,2,3 \\ t = 1,2,3,4 \end{vmatrix}$$

where,

D_{jkt} = Demand of finished product "j" in warehouse k at period t.

3.3.5.4 Complete model:

The model in its complete form can be define as:

Minimize,

$$Z = \sum_{t=1}^{4} \left[\begin{array}{l} \sum_{i=1}^{2}\sum_{p=1}^{16}(C_{pit}X_{pit} + I_{1pit}h_{1pit}) + \sum_{p=1}^{16} h_{2pt}I_{2pt} + \sum_{i=1}^{2}\sum_{p=1}^{16} V_{pit}g_{pit} \\ + \sum_{j=1}^{4}(C_{jt}Y_{jt} + h_{3jt}I_{3jt}) + \sum_{j=1}^{4}\sum_{k=1}^{3} C_{jkt}W_{jkt} + \sum_{j=1}^{4}\sum_{k=1}^{3} h_{4jkt}I_{4jkt} \end{array} \right]$$

Subject to

Inventory balance at stage-1:

$$I_{1pit} = I_{1pit-1} + X_{pit} - V_{pit} \qquad \left| \begin{array}{l} p = 1,2,3,4 \ldots\ldots\ldots\ldots 16 \\ i = 1,2 \\ t = 1,2,3,4 \end{array} \right.$$

Inventory balance at stage-2 (component):

$$I_{2pt} = I_{2pt-1} + \sum_{i=1}^{2} V_{pit} - \sum_{j=1}^{4} m_{pj}Y_{jt} \qquad \left| \begin{array}{l} t = 1,2,3,4 \\ j = 1,2,3,4 \\ mpj = 0 \text{ or any positive integer.} \end{array} \right.$$

Inventory balance at stage-2 (assembled product):

$$I_{3jt} = I_{3jt-1} + Y_{jt} - \sum_{k=1}^{3} W_{jkt} \qquad \left| \begin{array}{l} j = 1,2,3,4 \\ t = 1,2,3,4 \end{array} \right.$$

Component supply:

$$\sum_{j=1}^{4} m_{pj} Y_{jt} <= \sum_{i=1}^{2} X_{pit} \qquad \begin{vmatrix} p = 1,2,3,4 \ldots\ldots\ldots\ldots 16 \\ t = 1,2,3,4 \end{vmatrix}$$

Capocity limitation:

$$X_{pit} <= q_{1pit} \qquad \begin{vmatrix} i = 1,2 \\ p = 1,2,3,4 \ldots \ldots\ldots\ldots 16 \\ t = 1,2,3,4 \end{vmatrix}$$

Capacity limitation:

$$Y_{jt} <= q_{2jt} \qquad \begin{vmatrix} j = 1,2,3,4 \\ t = 1,2,3,4 \end{vmatrix}$$

Storage limitation:

$$\sum_{j=1}^{4} Y_{jt} <= Q_t \qquad \begin{vmatrix} t = 1,2,3,4 \\ j = 1,2,3,4 \end{vmatrix}$$

Inventory balance:

$$I_{4jkt} >= I_{4jkt-1} + W_{jkt} - D_{jkt} \qquad \begin{vmatrix} j = 1,2,3,4 \\ k = 1,2,3 \\ t = 1,2,3,4 \end{vmatrix}$$

Non negativity of variables:

$X_{pit} >= 0$

$I_{1pit} >= 0$

$I_{2pt} >= 0$

$V_{pkt} >= 0$

$Y_{jt} >= 0$

$I_{3jt} >= 0$

$I_{4jkt} >= 0$

$W_{jkt} >= 0$

Z = Total production and distribution cost for the entire planning horizon.

Chapter-4

Input of the Model

4.1 Introduction: The mathematical models developed in the preceding chapter have been tested by the realistic data. The hypothetical data were used for the first four models. The last model was tested with real life data collected from a local battery manufacturing company named Rahim Afrooz (BD) Ltd. This chapter presents first the flow process chart and the various input parameters of each model. The parameters were either assumed or collected from the real life situation.

4.2 Input of Model-1:

A hypothetical Milk pasteurization and distribution system was considered for this model. This system consists of two milk production or collection centers, two pasteurization plants and two regional warehouses or distribution centers. In first stage, besides production centers, there are storage facilities. In second stage also, there are storage facilities prior to pasteurization. The raw milk collection at the first stage can either be made by producing in the firm house or purchasing from local producers. The flow chart is shown in figure 4.1. The production cost for raw milk, inventory holding cost, cost of pasteurization, cost of shipping, production capacities, pasteurization capacities are shown respectively in table-4.1, 4.2, 4.3, 4.4, 4.5 and 4.6.

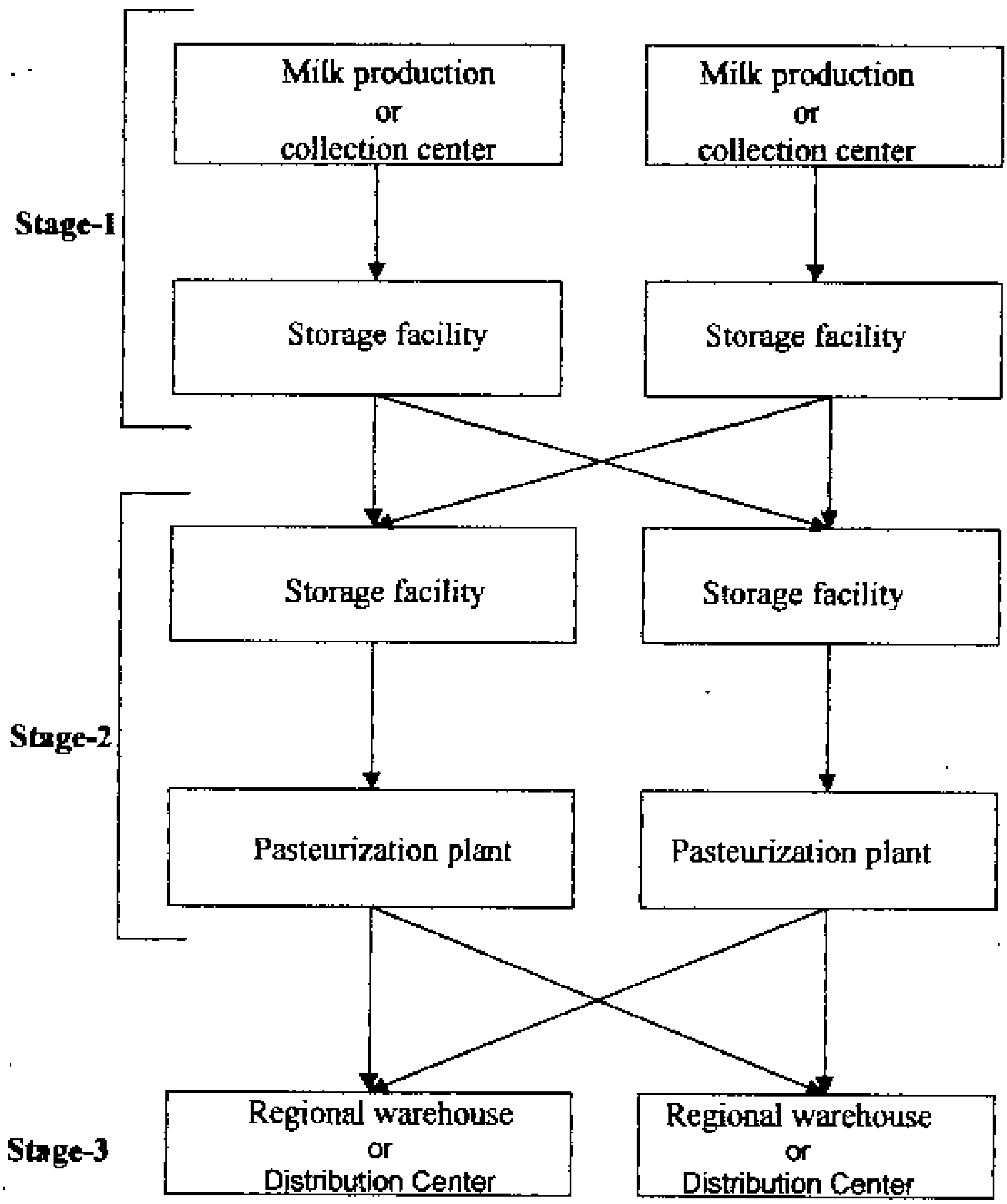


Fig. 4.1 Flow chart for model-1

Table 4.1: The production cost for raw milk per unit at first stage for each period:

Period	Collection center-1 in Tk.	Collection Center-2 in Tk.
1	12	11
2	12	11
3	12	11
4	13	12
5	13	12
6	12	11

Inventory holding costs at all the stages were assumed constant over period and same cost was assumed in all the locations within a stage.

Table 4.2: The inventory holding cost per unit in Tk. for all the stages:

Stage	Storage facility-1	Storage facility-2
1	1	1
2	2	2
3	1	1

Table 4.3: Cost of pasteurization at second stage and shipping to 3rd stage per unit pack of milk in Tk.:

Period	Pasteurization in plant-1		Pasteurization in plant-2	
	Shipped to warehouse		Shipped to warehouse	
	1	2	1	2
1	7	9	8	8
2	7	9	8	8
3	7	9	8	8
4	7	9	8	8
5	7	9	8	8
6	7	9	8	8

Table 4.4: Cost of shipping per unit raw milk from storage facilities of stage-1 to storage facilities of stage-2 for all periods in Tk.

Period	From storage facility-1 of first stage to storage facility-1 of 2nd stage	From storage facility-2 of first stage to storage facility-1 of 2nd stage	From storage facility-1 of first stage to storage facility-2 of 2nd stage	From storage facility-2 of first stage to storage facility-2 of 2nd stage
1	2	3	3	2
2	2	3	3	2
3	2	3	3	2
4	2	3	3	2
5	2	3	3	2
6	2	3	3	2

Table 4.5: Production capacities of raw milk at stage-1 in units.

Production Center-1	Production Center-2
54000	60000

Capacities of raw milk production were assumed constant over time at the collection centers of first stage.

Table 4.6: Maximum pasteurization capacities at stage-2 in units.

Plant-1	Plant-2
56000	62000

These capacities were also assumed constant for the entire planning horizon.

4.3 Input of Model-2

The concept of multiple products was introduced in this model, assuming the manufacturing and distribution scenario similar to the first model. The application of this model fits well to the case of manufacturing small parts by employing foundry and machining processes. This formulation was tested by assumed data. Due to simplification this model was tested assuming two products only. Two different types of products product A and product B are first produced by casting method at the plants of first stage. These products can be stored if necessary at the storage facilities associated with the foundry plants. At the second stage there are storage facilities to hold products coming from first stage. The plants at this stage provide machining operation. The finished products are then shipped to the warehouse (distribution center) of the final stage. The flow chart is shown in figure 4.2 and the relevant input parameters are presented in table 4.7 to 4.13.

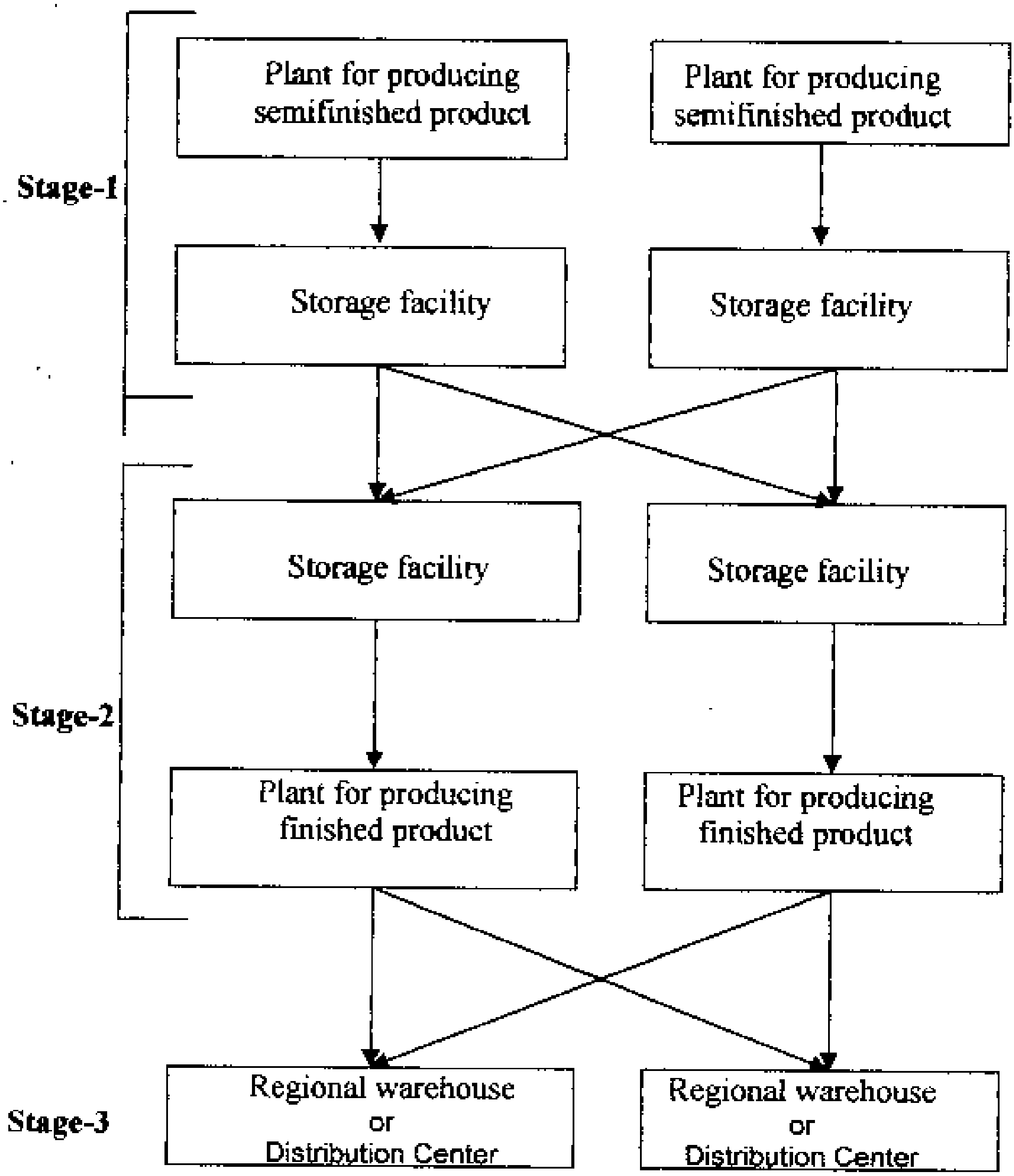


Fig. 4.2 Flow chart for model-2

Table 4.7: Cost to produce each unit of semifinished product at plants of stage-1 (These costs were assumed constant over period.) in Tk.:

Costs at Plant-1		Costs at Plant-2	
Product A	Product B	Product A	Product B
12	14	11	15

Table 4.8: Inventory carrying cost per item at the warehouses of stage-1. (These costs were assumed constant over time):

Costs associated with the warehouse of plant-1		Costs associated with the warehouse of plant-2	
Product A	Product B	Product A	Product B
2	2	2	2

Table 4.9: Shipping costs of the semifinished product (A) from first stage to second stage per item in Tk. (These costs were assumed constant over time):

Between warehouse-1 of stage-1 to warehouse-1 of second stage-2	Between warehouse-2 of stage-1 to warehouse-1 of stage-2	Between warehouse-1 of stage-1 to warehouse-2 of stage-2	Between warehouse-2 of stage-1 to warehouse-2 of stage-2
3	4	4	4

Table 4.10: Shipping costs of the semifinished product (B) from first stage to second stage per item in Tk. (These costs were assumed constant over time):

Between warehouse-1 of stage-1 to warehouse-1 of second stage-2	Between warehouse-2 of stage-1 to warehouse-1 of stage-2	Between warehouse-1 of stage-1 to warehouse-2 of stage-2	Between warehouse-2 of stage-1 to warehouse-2 of stage-2
4	5	3	6

Table 4.11: Cost for finishing operation of a unit of product A and ship to the final stage in Tk. (These costs were assumed constant over time):

Cost of producing at plant-1 of stage-2 and shipping to warehouse-1 of stage-3	Cost of producing at plant-2 of stage-2 and shipping to warehouse-1 of stage-3	Cost of producing at plant-1 of stage-2 and shipping to warehouse-2 of stage-3	Cost of producing at plant-2 of stage-2 and shipping to warehouse-2 of stage-3
10	11	10	10

Table 4.12: Cost for finishing operation of a unit of product B, and ship to the final stage in Tk. (These costs were assumed constant over time):

Cost of producing at plant-1 of stage-2 and shipping to warehouse-1 of stage-3	Cost of producing at plant-2 of stage-2 and shipping to warehouse-1 of stage-3	Cost of producing at plant-1 of stage-2 and shipping to warehouse-2 of stage-3	Cost of producing at plant-2 of stage-2 and shipping to warehouse-2 of stage-3
11	12	12	12

Table 4.13: Inventory carrying cost for each unit of finished product in the final stage at each warehouse in Tk. (These costs were assumed constant for the entire planning horizon):

Warehouse-1		Warehouse-2	
Product-A	Product-B	Product-A	Product-B
3	3	3	3

4.4 Input for Model-3

This model was developed to consider the case of assembling small parts or components. The three components were produced at the first stage and then transported to the second stage for assembling into a single finished product. It was assumed that after assembling the finished products are shipped to the warehouse of the final stage directly. The flow process chart with respect to this model is shown in figure 4.3 and the relevant input parameters are presented in tables 4.14 to 4.16.

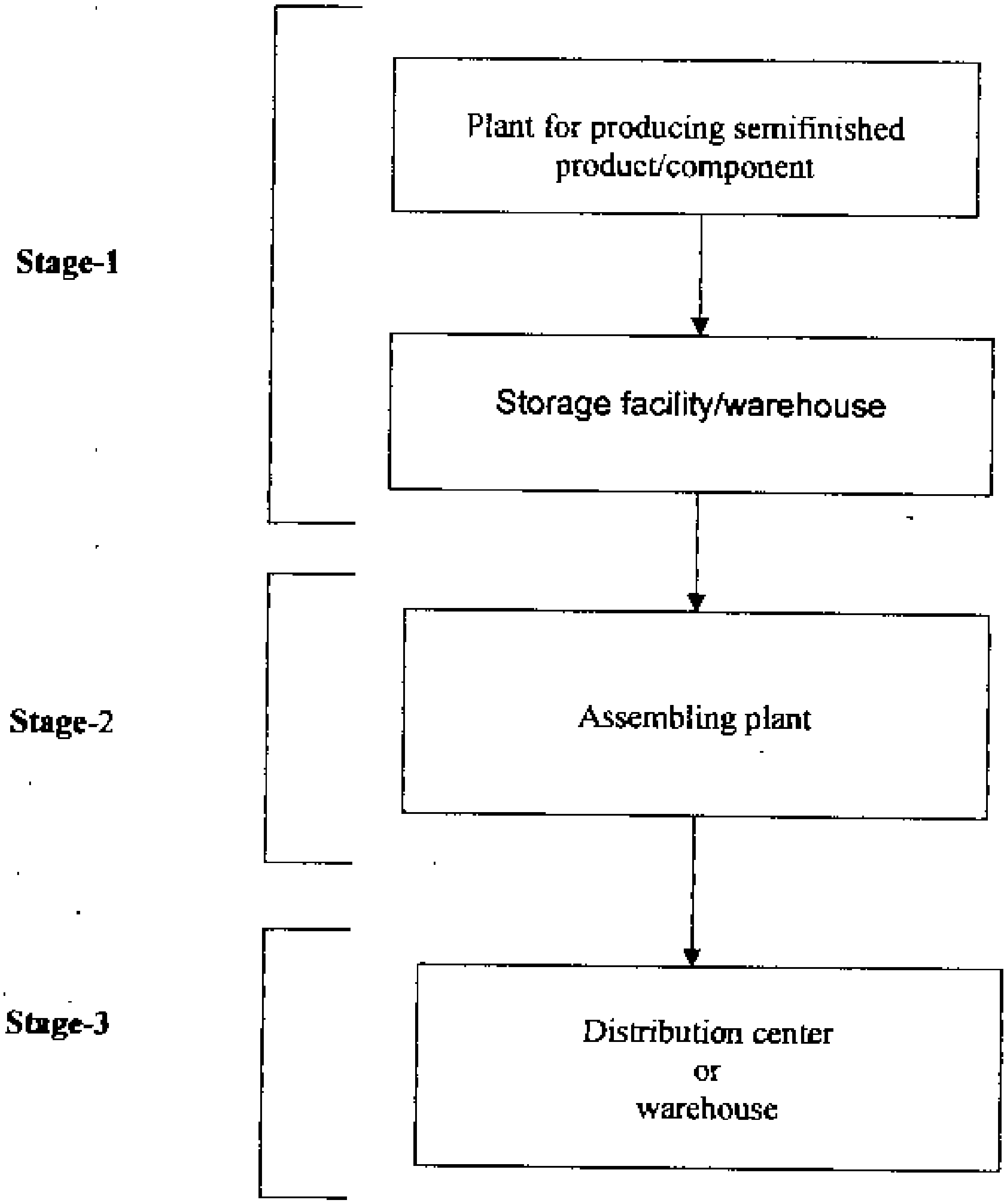


Fig. 4.3: Flow chart for model-3

Table 4.14: Costs to produce each unit of semifinished product at stage-1 in Tk. (These costs were assumed constant over time):

Production cost for components-A	Production cost for components-B	Production cost for components-B
40	50	60

Table 4.15: Unit inventory holding cost of each component at stage-1. (These costs were assumed constant over period):

Inventory holding cost for component-A in Tk.	Inventory holding cost for component-B in Tk.	Inventory holding cost for component-C in Tk.
4	6	8

Table 4.16: Unit shipping cost for each component from first stage to second stage. (These costs were assumed constant over period):

Shipping cost for component-A in Tk.	Shipping cost for component-B in Tk.	Shipping cost for component-C in Tk.
2	3	4

The cost to assemble a unit of finished product and ship to a warehouse of final stage was assumed Tk. 27. The unit inventory holding cost of finished product at final stage was assumed to be equal to Tk. 10.

Both of these costs were assumed constant for the entire planning horizon.

4.5 Input of model-4

This model is an extension of the previous model. In this model multiple plants were considered in the first stage and multiple distribution centers in the last stage. To simplify the situation, only two semifinished products and single finished product were considered. The figure 4.4 presents the flow process chart and the relevant input parameters are in entered in tables 4.17 to 4.20.

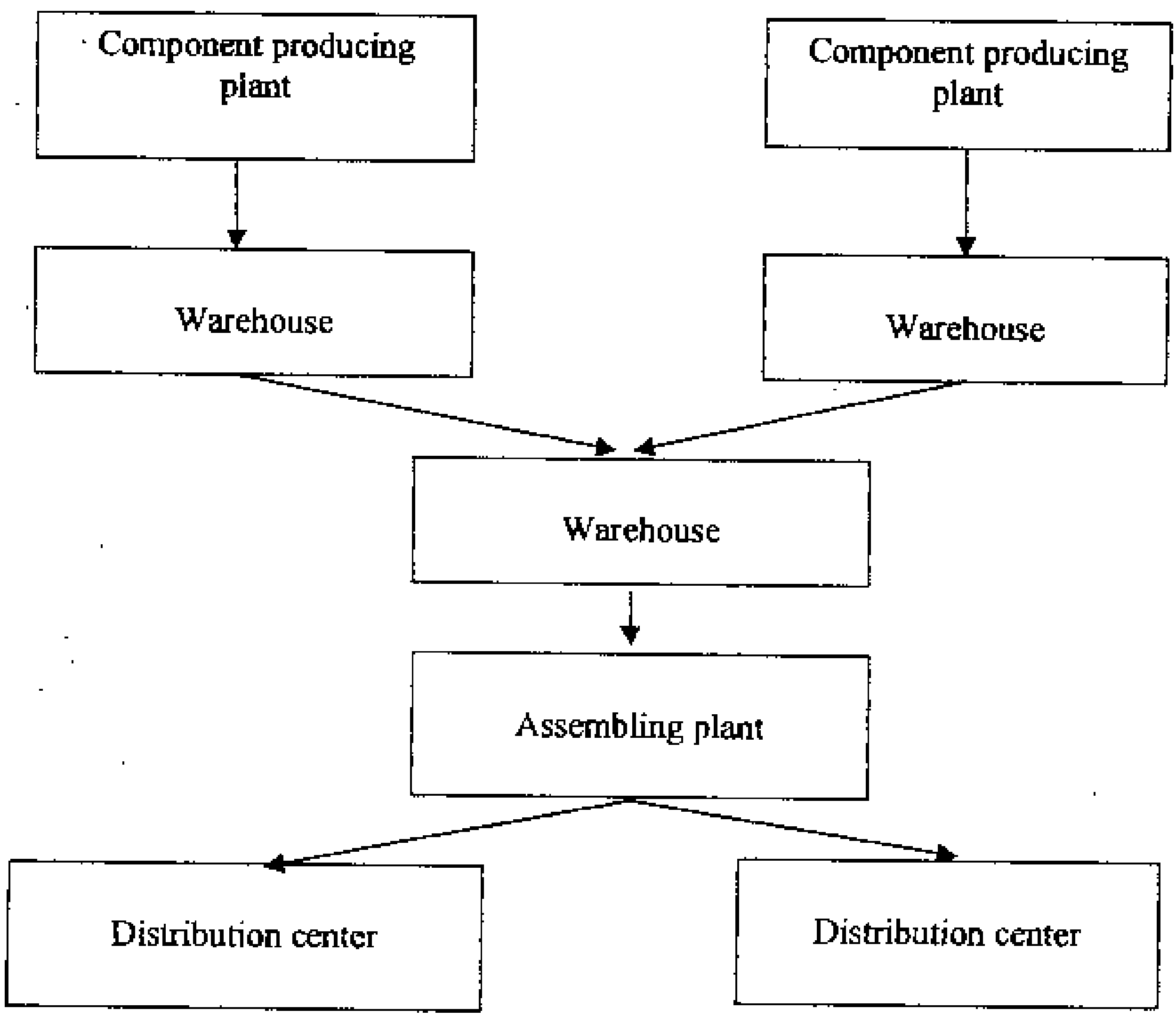


Fig. 4.4: Flow chart for model-4

Table 4.17: Cost to produce each unit of component at stage-1. (These costs were assumed constant over periods):

First plant-1		Second plant-2	
Cost for component A in Tk.	Cost for component B in Tk.	Cost for component A in Tk.	Cost for component B in Tk.
40	69	42	70

Table 4.18: Inventory holding cost for each unit of component at warehouses of stage-1. (These costs were assumed constant over periods):

Costs at warehouse associated with plant-1		Costs at warehouse associated with plant-2	
Holding cost for component A in Tk.	Holding cost for component B in Tk.	Holding cost for component A in Tk.	Holding cost for component B in Tk.
6	9	6	9

Table 4.19: Cost to assemble a unit of finished product and ship to warehouse of last stage. (These costs were assumed constant over periods):

Shipping cost due to shipment to distribution center-1 in Tk.	Shipping cost due to shipment to distribution center-2 in Tk.
38	37

Table 4.20: Unit inventory holding costs at the distribution center of last stage. (These costs were assumed constant over periods):

Distribution center-1 in Tk.	Distribution center-2 in Tk.
10	11

4.6 Input of the model-5

This model was tested with real life data as stated earlier from Rahim Afrooz (BD) Ltd. It manufactures several models of automotive batteries. In reality Rahim Afrooz operates two manufacturing plants one at Savar and another in Tejgaon, Dhaka. In addition to manufacturing, it also maintains several distribution centers at different locations of Bangladesh. For justifying the underlying concept of the model, it is assumed that, the company manufactures subassemblies at Savar and Tejgaon and assembling process is carried out in another plant before making shipment to distribution centers. In actual practice production/purchase of the subassemblies and assembling operation are all carried out in a single plant at Savar. However, through personal contact and interview with the relevant personnel, it is learnt that the company would go for massive expansion of its existing facilities. In fact the oecessary assumptions made in the model have been incorporated taking their views into account. The values of costs and other input parameters are also assumed according to their suggestion. This has been due to fit the model in line with the proposed future expansion plan of the company.

The hypothetical flow chart is shown in figure 4.5.

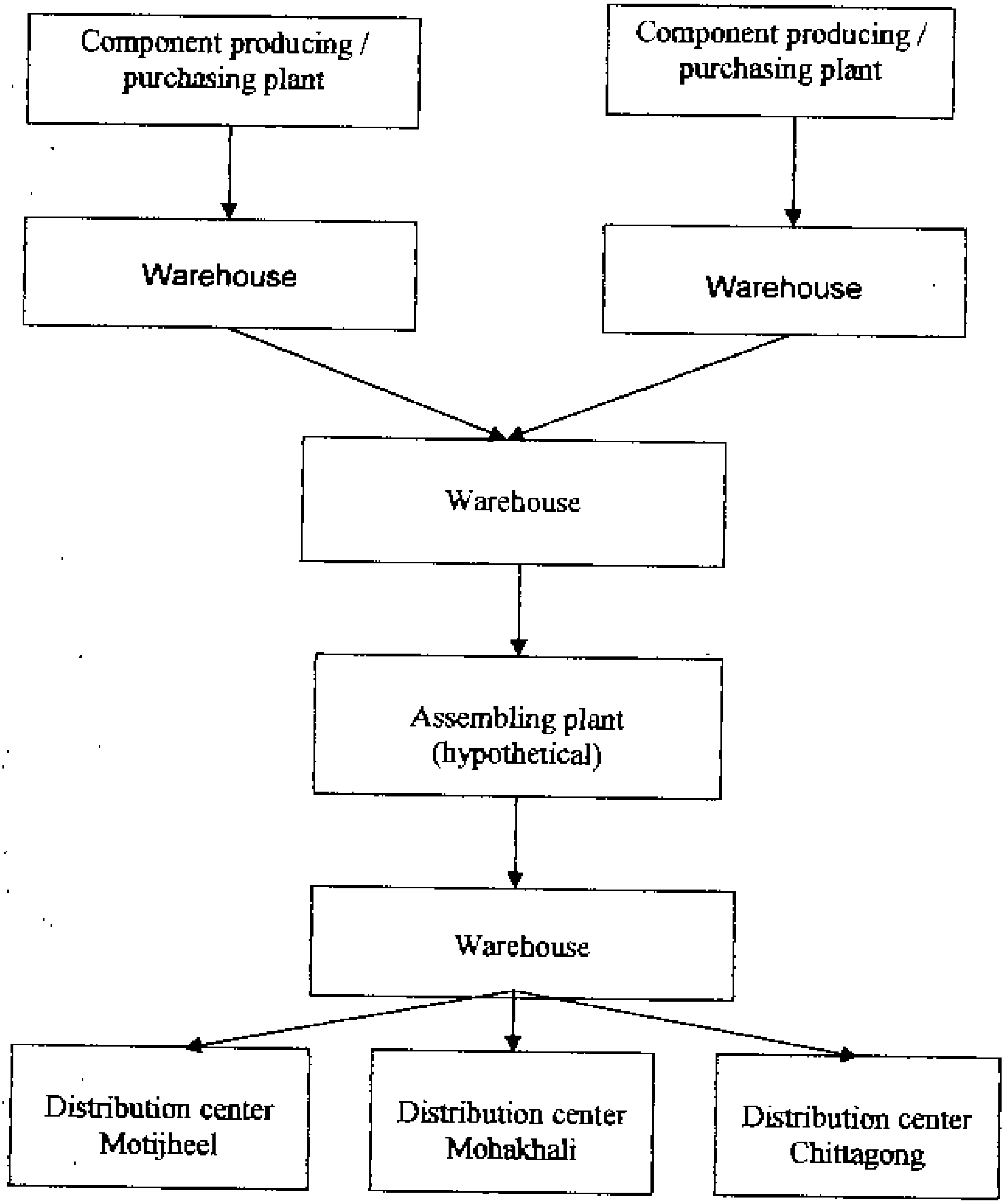


Fig. 4.5: Flow chart for model-5

Among the fifty models manufactured by Rahim Afrooz (Bangladesh ltd.) four models of automotive batteries were chosen for this case study . The selected automotive battery models are: NS40Z, NS60, N50, N70. For the assembling of these four models, 6 types of components of 16 different identities (specifications) are needed. Each item manufactured in two plants of the first stage. The components are:

A) Container.

B) Positive plate.

C). Negative plate.

D) Separator.

E) Small parts.

F) Carton.

The relevant input parameters are entered in tables 4.21 to 4.32.

Table 4.21: Relevant cost for the subassemblies of NS40Z at plant –1:

Sl No.	Name of the component.	Production/Procurement cost. In Tk	Inventory holding cost at 1st stage in Tk	Transportation cost from 1st stage to 2nd stage. In Tk.	Inventory holding cost at 2nd stage in Tk	Contribution to the finished product. In Nos.
1	Container	110	10	1.00	7	1
2	Positive plate	12	1.2	0.10	1	30
3	Negative plate	10	1.2	0.10	1	36
4	Separator	0.80	0.08	0.01	0.1	60
5	Small parts	18.2	1.82	0.05	1	1
6	Carton	23	0.50	0.20	0.5	1

Table 4.22: Relevant costs for subassemblies of NS60 at plant-1:

Sl.No	Name of the component.	Production/Procurement cost. In Tk	Inventory holding cost at 1st stage in Tk.	Transportation cost from 1st stage to 2nd stage. InTk	Inventory holding cost at 2nd stage in Tk.	Contribution to the finished product. In Nos
1	Container	122	11	1.2	8	1
2	Positive plate	12	1.2	0.10	1	36
3	Negative plate	10	1.2	0.10	1	42
4	Separator	0.80	0.08	0.01	0.10	72
5	Small parts	18.2	1.82	0.05	1	1
6	Carton	24	0.50	0.20	0.50	1

Table 4.23: Relevant costs for subassemblies of N50 at plant-1:

Sl No.	Name of the component.	Production/ Procurement cost. In Tk	Inventory holding cost at 1st stage in Tk.	Transportation cost from 1st stage to 2nd stage InTk.	Inventory holding cost at 2nd stage in Tk	Contribution to the finished product. In Nos.
1	Container	179	15.5	1.5	10	1
2	Positive plate	15	1.5	0.15	1	24
3	Negative plate	14	1.5	0.15	1	30
4	Separator	1.0	0.10	0.01	0.1	48
5	Small parts	24.2	1.80	0.15	1	1
6	Carton	26	0.50	0.20	0.5	1

Table 4.24: Relevant costs for subassemblies of N70 at plant-1:

Sl No.	Name of the component.	Production/ Procurement cost. In Tk	Inventory holding cost at 1st stage in Tk	Transportation cost from 1st stage to 2nd stage. InTk	Inventory holding cost at 2nd stage in Tk.	Contribution to the finished product. In Nos.
1	Container	185	20	1.5	15	1
2	Positive plate	15	1.5	0.15	1	36
3	Negative plate	14	1.5	0.15	1	42
4	Separator	1.0	0.10	0.01	0.1	72
5	Small parts	24.2	1.80	0.15	1	1
6	Carton	30	0.50	0.20	0.5	1

Table 4.25: Relevant costs for subassemblies of NS40Z at plant-2:

Sl.No.	Name of the component	Production/ Procurement cost In Tk	Inventory holding cost at 1st stage in Tk	Transportation cost from 1st stage to 2nd stage InTk	Inventory holding cost at 2nd stage in Tk.	Contribution to the finished product In Nos
1	Container	108	10	1.5	7	1
2	Positive plate	13	1.5	0.30	1	30
3	Negative plate	11	1.4	0.20	1	36
4	Separator	1	0.1	0.01	0.10	60
5	Small parts	18.5	1.8	0.06	1	1
6	Carton	22	0.60	0.30	0.50	1

Table 4.26: Relevant costs for subassemblies of NS60 at plant-2:

Sl.No.	Name of the component.	Production/ Procurement cost. In Tk	Inventory holding cost at 1st stage in Tk.	Transportation cost from 1st stage to 2nd stage InTk.	Inventory holding cost at 2nd stage in Tk.	Contribution to the finished product. In Nos
1	Container	125	10	2	8	1
2	Positive plate	13	1.5	0.30	1	36
3	Negative plate	11	1.4	0.20	1	42
4	Separator	1	0.1	0.01	0.10	72
5	Small parts	18.5	1.8	0.06	1	1
6	Carton	25	0.60	0.30	0.50	1

Table 4.27: Relevant costs for subassemblies of N50 at plant-2:

Sl.No.	Name of the component.	Production/ Procurement cost. In Tk	Inventory holding cost at 1st stage in Tk.	Transportation cost from 1st stage to 2nd stage InTk	Inventory holding cost at 2nd stage in Tk.	Contribution to the finished product In Nos
1	Container	180	16	2	10	1
2	Positive plate	14	1.4	0.20	1	24
3	Negative plate	13	1.5	0.30	1	30
4	Separator	0.90	0.10	0.01	0.10	48
5	Small parts	24	1.7	0.15	1	1
6	Carton	27	0.60	0.30	0.50	1

Table 4.28: Relevant costs for subassemblies of N70 at plant-2:

Sl.No	Name of the component.	Production/ Procurement cost In Tk	Inventory holding cost at 1st stage in Tk	Transportation cost from 1st stage to 2nd stage In Tk.	Inventory holding cost at 2nd stage in Tk.	Contribution to the finished product. In Nos
1	Container	184	21	1.5	15	1
2	Positive plate	14	1.4	0.20	1	36
3	Negative plate	13	1.5	0.30	1	42
4	Separator	0.90	0.10	0.01	0.10	72
5	Small parts	24	1.7	0.15	1	1
6	Carton	28	0.60	0.30	0.50	1

Table 4.29: Relevant costs for the assembled product NS40Z:

Name of the Assembled Product	Assembling cost (In Tk.)	Inventory Holding cost at Stage 2. (In Tk.)	Transportation cost From stage 2 to the Warehouse in stage 3. (In Tk.)			Holding cost at the Warehouse of the last Stage (In Tk)		
			1	2	3	1	2	3
NZ40Z	10	1	4	4	10	3	3	2

Table 4.30: Relevant costs for the assembled product NS60:

Name of the Assembled Product	Assembling cost (In Tk.)	Inventory Holding cost at Stage 2. (In Tk.)	Transportation cost From stage 2 to the Warehouse in stage 3. (In Tk.)			Holding cost at the Warehouse of the last Stage.(in Tk.)		
			1	2	3	1	2	3
NS60	10	1	4	4	10	3	3	2

Table 4.31: Relevant costs for the assembled product N50:

Name of the Assembled Product	Assembling cost (In Tk)	Inventory Holding cost at Stage 2 (In Tk)	Transportation cost From stage 2 to the Warehouse in stage 3 (In Tk)			Holding cost at the Warehouse of the last Stage.(in Tk)		
			1	2	3	1	2	3
N50	15	1	4	4	10	4	4	3

Table 4.32: Relevant costs for the assembled product N70:

Name of the Assembled Product	Assembling cost (In Tk.)	Inventory Holding cost at Stage 2. (In Tk)	Transportation cost From stage 2 to the Warehouse in stage 3. (In Tk.)			Holding cost at the Warehouse of the last Stage (in Tk.)		
			1	2	3	1	2	3
N70	15	1	4	4	10	4	4	3

Chapter-5

Results and Discussion

5.1 Introduction: The models formulated in chapter-3 for multistage multilocation multiperiod production system were run on an IBM computer using LINDO program developed by Linns Schrage Graduate School of Business, University of Chicago. All the models gave feasible solutions. This chapter presents the output of all the models – in which the values of the decision variables and the objective functious are mentioned. However, for the model in which real-life data were used from Rahim Afrooz (BD) Ltd., the output was followed by the discussion and sensitivity analysis.

5.2 The Output of Model-1

Number of variables	102
Number of rows	77
Number of iterations	49

The objective value is Tk. 13927900.00

Table 5.2.1: Number of units of raw milk to be procured/produced at stage-1

Period	Xit	
	At collection center-1	At collection center-2
1	X11 = 47600	X21 = 60000
2	X12 = 48000	X22 = 54000
3	X13 = 51000	X23 = 60000
4	X14 = 54000	X24 = 60000
5	X15 = 46000	X25 = 60000
6	X16 = 54000	X26 = 58000

Table 5.2.2: Number of units of raw milk to be shipped to second stage from first stage.

Period	Vijt			
	From warehouse-1 of first stage to warehouse-1 of stage-2	From warehouse-2 of first stage to warehouse-1 of stage-2	From warehouse-1 of first stage to warehouse-2 of stage-2	From warehouse-2 of first stage to warehouse-2 of stage-2
1	V111 = 47700	V211 = 2200	V121 = 0	V221 = 57900
2	V112 = 48000	V212 = 0	V122 = 0	V222 = 54000
3	V113 = 51000	V213 = 0	V123 = 0	V223 = 60000
4	V114 = 54000	V21 4 = 2000	V124 = 0	V224 = 58000
5	V115 = 46000	V215 = 0	V125 = 0	V225 = 60000
6	V116 = 54000	V211 6 = 2000	V126 = 0	V226 = 56000

Table 5.2.3: Number of units of milk pasteurized and shipped to warehouses of third stage are:

Period	Yjkt			
	From plant-1 of stage-2 to warehouse-1 of stage-3	From plant-2 of stage-2 to warehouse-1 of stage-3	From plant-1 of stage-2 to warehouse-2 of stage-3	From plant-2 of stage-2 to warehouse-2 of stage-3
1	Y111= 49900	Y211= 0	Y121= 0	Y221= 57900
2	Y112= 48000	Y212= 0	Y122= 0	Y222= 54000
3	Y113= 51000	Y213= 0	Y123= 0	Y223= 60000
4	Y114= 56000	Y214= 0	Y124= 0	Y224= 58000
5	Y115= 46000	Y215= 8000	Y125= 0	Y225= 52000
6	Y116= 56000	Y216= 0	Y126= 0	Y226= 56000

5.3 Output of Model-2

Number of variables 140

Number of Rows 105

Number of iterations 76

The value of the objective function Tk. 268035.00

Table 5.3.1: Number of units of semifinished product to be produced at first stage are

Period	*Xait*			
	At plant-1		At plant-2	
	Product-A	Product-B	Product-A	Product-B
1	XA11 = 564	XB11 = 850	XA21 = 650	XB21 = 505
2	XA12 = 530	XB12 = 850	XA22 = 770	XB22 = 530
3	XA13 = 800	XB13 = 850	XA23 = 380	XB23 = 250
4	XA14 = 700	XB14 = 850	XA24 = 550	XB24 = 150

Table 5.3.2: Number of units of semifinished product to be shipped from stage-2 to stage-3.

Period	Vaijt			
	From warehouse-1 of stage-1 to warehouse-1 of stage-2	From warehouse-2 of stage-1 to warehouse-1 of stage-2	From warehouse-1 of stage –1 to warehouse-2 of stage-2	From warehouse-2 of stage-1 to warehouse-2 of stage-2
	Product-A			
1	VA111 = 574	VA211 = 0	VA121 = 0	VA221 = 670
2	VA112 = 530	VA212 = 120	VA122 = 0	VA222 = 650
3	VA113 = 800	VA213 = 100	VA123 = 0	VA223 = 280
4	VA114 = 700	VA214 = 0	VA124 = 0	VA224 = 550
Period	Product-B			
1	VB111 = 140	VB211 = 545	VB121 = 730	VB221 = 0
2	VB112 = 150	VB212 = 530	VB122 = 700	VB222 = 0
3	VB113 = 250	VB213 = 250	VB123 = 600	VB223 = 0
4	VB114 = 0	VB214 = 150	VB124 = 850	VB224 = 0

Table 5.3.3: Number of units of finished product to be shipped to the warehouses of third stage after finishing operation are:

Period	Yajkt			
	From plant-1 of stage-2 to warehouse-1 of stage-3	From plant-2 of stage-2 to warehouse-1 of stage-3	From plant-1 of stage-2 to warehouse-2 of stage-3	From plant-2 of stage-2 to warehouse-2 of stage-3
	For Product-A			
1	YA111 = 574	YA211 = 0	YA 121 = 0	YA221 = 670
2	YA112 = 650	YA212 = 0	YA 122 = 0	YA222 = 650
3	YA113 = 580	YA213 = 0	YA 1213= 320	YA223 = 280
4	YA114 = 700	YA214 = 0	YA 124 = 0	YA224 = 550
Period	For Product-B			
1.	YB111 = 685	YB211 = 0	YB 121 = 0	YB221 = 730
2	YB112 = 680	YB212 = 0	YB 122 = 0	YB222 = 700
3	YB113 = 500	YB213 = 0	YB 123 = 0	YB223 = 600
4	YB114 = 150	YB2114= 450	YB 124 = 0	YB224 = 400

5.4 Output of Model-3

Number of variables	70
Number of Rows	62
Number of iterations	47

The value of the objective function Tk. 694351.00

Table 5.4.1: Number of units of components to be produced at stage-1 are:

Period	Xat		
	Product-A	Product-B	Product-C
1	XA1 = 900	XB1 = 600	XC1 = 600
2	XA2 = 900	XB2 = 600	XC2 = 600
3	XA3 = 900	XB3 = 600	XC3 = 600
4	XA4 = 879	XB4 = 586	XC4 = 586
5	XA5 = 900	XB5 = 600	XC5 = 600
6	XA6 = 900	XB6 = 600	XC6 = 600

Table 5.4.2: Number of units of products to be shipped to second stage from first stage are:

Period	Vat		
	Product-A	Product-B	Product-C
1	VA1 = 900	VB1 = 600	VC1 = 600
2	VA2 = 900	VB2 = 600	VC2 = 600
3	VA3 = 900	VB3 = 600	VC3 = 600
4	VA4 = 879	VB4 = 586	VC4 = 586
5	VA5 = 900	VB5 = 600	VC5 = 600
6	VA6 = 900	VB6 = 600	VC6 = 600

Table 5.4.3: Number of finished product to be assembled and shipped to the third stage are:

Period	Yt
	Number of assembled & shipped units
1	Y1 = 300
2	Y2 = 300
3	Y3 = 300
4	Y4 = 293
5	Y5 = 300
6	Y6 = 300

5.5 The output of Model-4

Number of variables 116

Number of Rows 99

Number of iterations 83

The value of the objective function Tk. 1662107.00

Table 5.5.1: Number of units of semifinished products to be produced at the first stage are:

Period	Xait			
	Plant-1		Plant-2	
	Product-A	Product-B	Product-A	Product-B
1	XA11 = 1500	XB11 = 640	XA21 = 810	XB21 = 900
2	XA12 = 1500	XB12 = 680	XA22 = 870	XB22 = 900
3	XA13 = 1500	XB13 = 720	XA23 = 930	XB23 = 900
4	XA14 = 1500	XB14 = 700	XA24 = 900	XB24 = 900
5	XA15 = 1500	XB15 = 684	XA25 = 876	XB25 = 900
6	XA16 = 1500	XB16 = 684	XA26 = 876	XB26 = 900

Table 5.5.2: Number of units of finished product to be produced and shipped to warehouses are:

Period	Ykt	
	Warehouse-1	Warehouse-2
1	Y11 = 360	Y21 = 410
2	Y12 = 390	Y22 = 400
3	Y13 = 400	Y23 = 410
4	Y14 = 370	Y24 = 430
5	Y15 = 390	Y25 = 430
6	Y16 = 387	Y26 = 405

Table 5.5.3: Number of units of semifinished product to be shipped to second stage from first stage are:

Period	Vait			
	From Plant-1 of first stage		From Plant-2 of first stage	
	Product-A	Product-B	Product-A	Product-B
1	VA11 = 1500	VB11 = 640	VA21 = 810	VB21 = 900
2	VA12 = 1500	VB12 = 680	VA22 = 870	VB22 = 900
3	VA13 = 1500	VB13 = 720	VA23 = 930	VB23 = 900
4	VA14 = 1500	VB14 = 700	VA24 = 900	VB24 = 900
5	VA15 = 1500	VB15 = 740	VA25 = 960	VB25 = 900
6	VA16 = 1500	VB16 = 684	VA26 = 876	VB26 = 900

5.6 The Output of Model-5

The model contains the following number of rows and variables.

Rows: 485

Variables: 541

Iterations: 519

The output of the model includes:

Decision variables	Interpretation
Xpit Where, p = 1, 2, 3, 4 ....., 16 i = 1, 2 t = 1, 2, 3, 4 Total, 128 variables	Number of units of semi finished products to be produced/procured in plants of stage –1 at different periods.
Yjt Where, j = 1, 2, 3, 4 t = 1, 2, 3, 4 Total, 16 variables	Number of units of finished products to be assembled in assembling plant of stage-2 at different periods.
Wjkt Where, j = 1, 2, 3, 4 k = 1, 2, 3 t = 1, 2, 3, 4 Total, 48 variables	• Number of units of finished products to be shipped to the warehouses of the final stage at different periods.

- Total manufacturing and distribution cost for future planning horizon.

5.6.1 Number of units of semi finished product needed to be produced/procured in plants of stage-1:

The assembling operation of finished products at the assembling plant is dependent on the supply of semi finished products from preceding stage. In this model it was assumed that, the semi finished products can either be produced or procured depending on the requirements. There were total 16 components manufactured/procured at two different plants. The quantities of semi finished products needed to be produced/ procured to keep the production of finished product unhindered at the assembling plant are shown bellow in the charts.

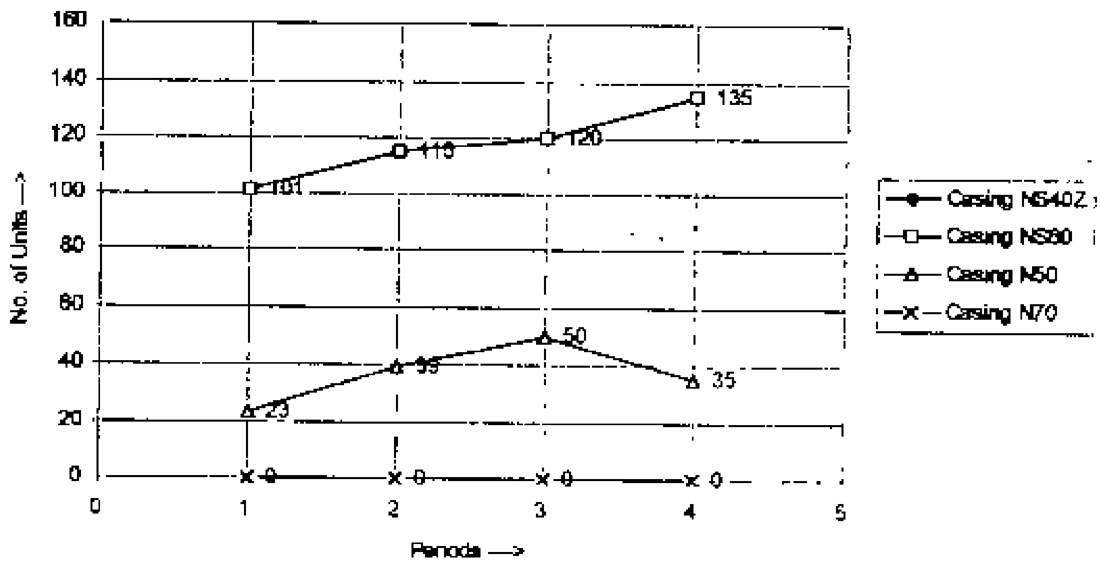


Chart-1: Number of units of Casings produced/procured at plant 1 of stage-1

In chart-1, it shows that two types of casings are produced at the plant-1 of stage-1 at different periods. These are Ns60 & N50 casings. The quantities of Ns60 casings are 101,115,120 and 135 in consecutive four periods. The quantities of N50 casings are 23,39,50 and 35 in consecutive four periods.

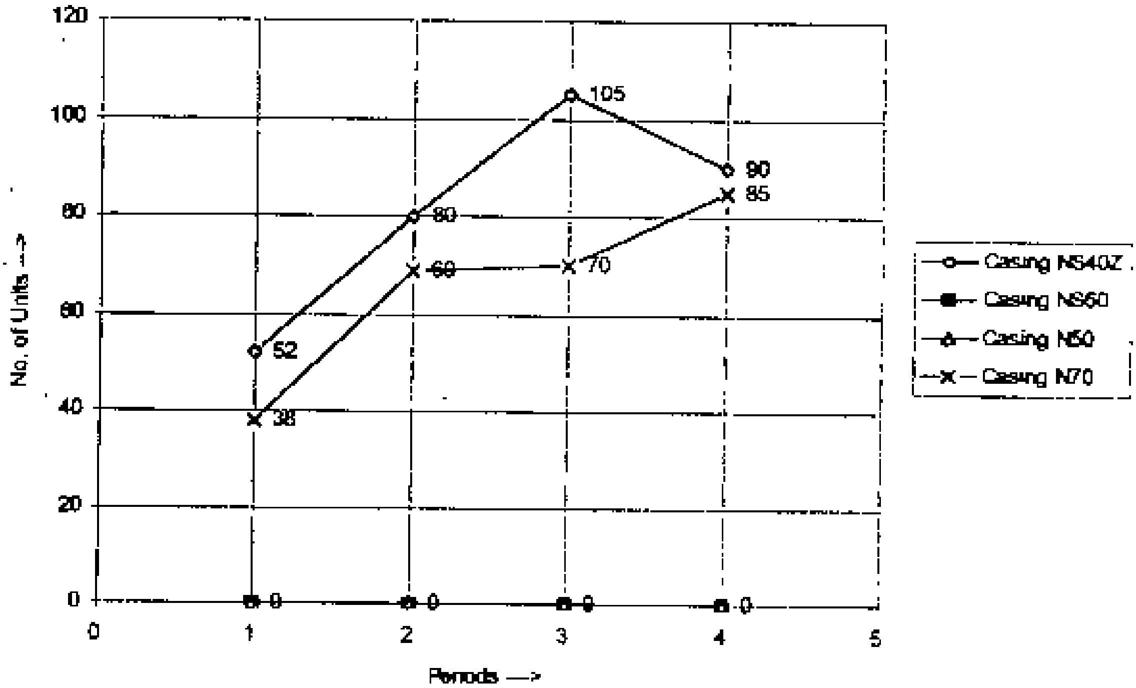


Chart-2: Number of units of casings produced at plant 2 of stage –1.

In chart-2, it shows that, two types of casings are produced at the plant-2 of stage-1 at different periods. These are Ns40Z & N70 casings. The quantities of Ns40Z casings are 52,80,105 and 90 in consecutive four periods. The quantities of N70 casings are 38,69,70 and 85 in consecutive four periods.

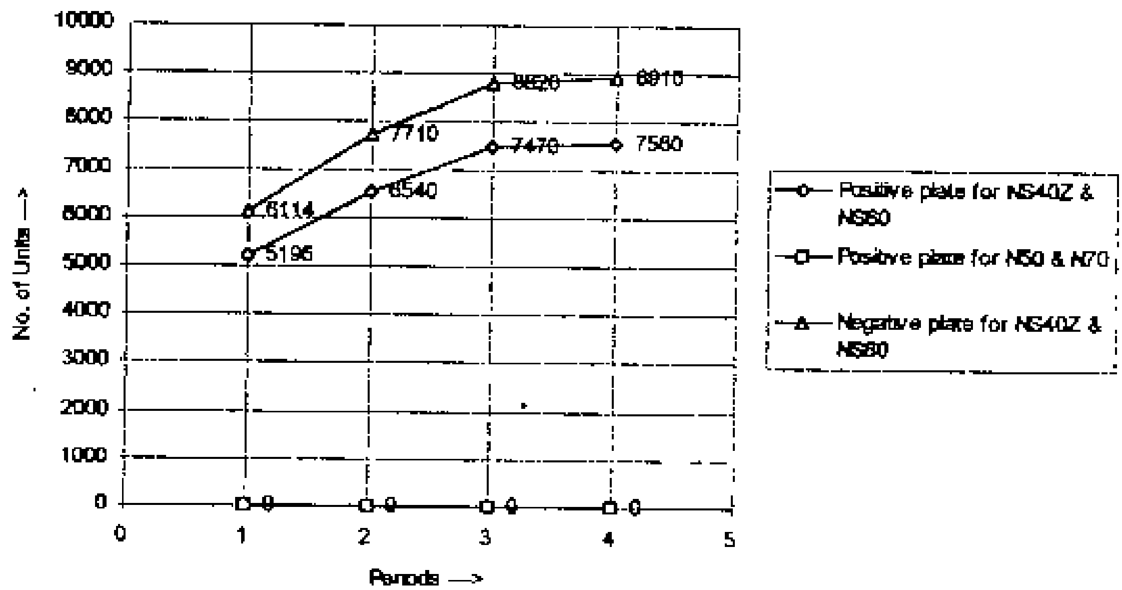


Chart-3: Number of units of positive plates & Negative plates produced at plant 1 of stage 1.

In chart-3, it shows the quantities of positive plate and negative plates produced at the plant-1 of stage-1 at different periods. These are positive plate for Ns40Z & Ns60 models and negative plates for Ns40Z & Ns60 models. The quantities of Ns40Z & Ns60 positive plates are 5196,6540,7470 and 7560 in consecutive four periods. The quantities of Ns40Z & Ns60 negative plates are 6114,7710,8820 and 8910 in consecutive four periods.

In chart-4, it shows the quantities of positive plate and negative plates produced at the plant-2 of stage-1 at different periods. These are positive plate for N50 & N70 models and negative plates for N50 & N70 models. The quantities of N50 & N70 positive plates are 1920,3420,3720 and 3900 in consecutive four periods. The quantities of N50 & N70 negative plates are 2286,3420,3720 and 3900 in consecutive four periods.

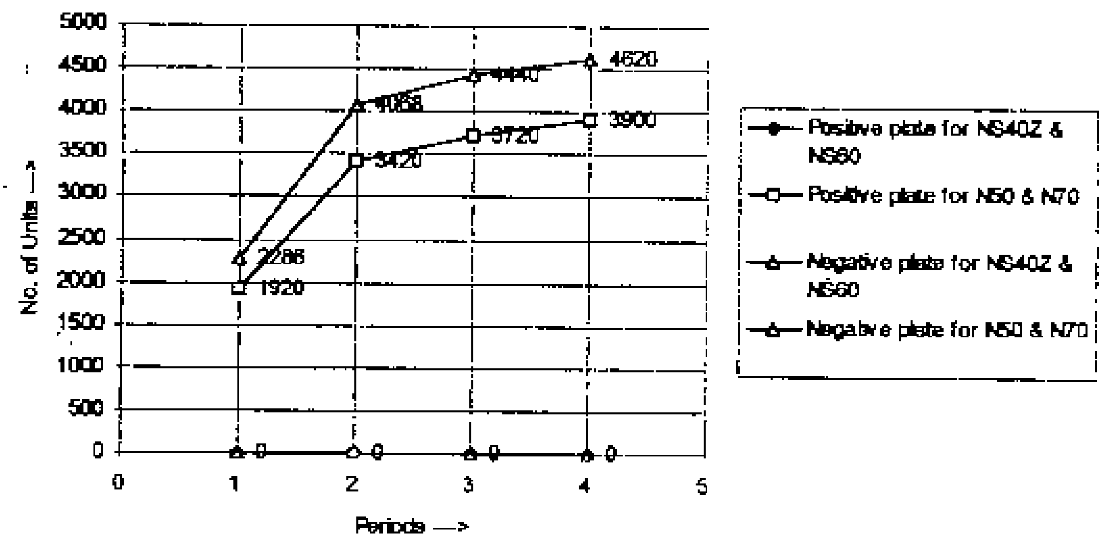


Chart-4: Number of units of Positive plates and Negative plates produced/procured at plant 2 of stage-1.

In chart-5, it shows the quantities of separators produced at the plant-1 of stage-1 at different periods. These are separators used for Ns40Z and Ns60 models. The quantities of separators are 10392,13080,14940 and 15120 in consecutive four periods.

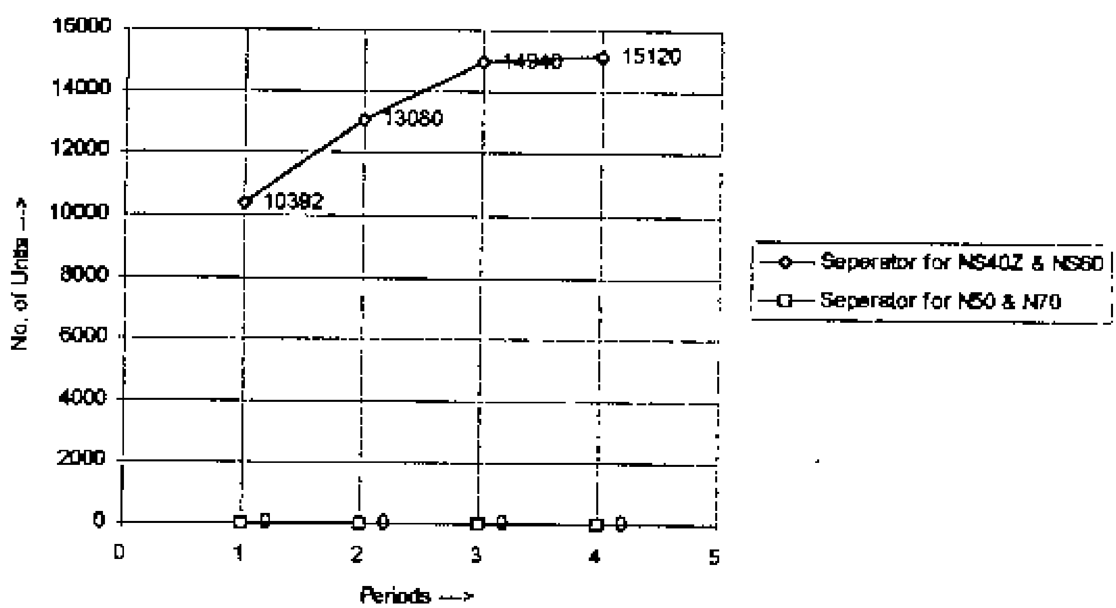


Chart-5: Number of units of Separators produced/procured at plant 1 of stage-1.

In chart-6, it shows the quantities of separators produced at the plant-2 of stage-1 at different periods. These are separators used for N50 and N70 models. The quantities of separators are 3840,6840,7440 and 7800 in consecutive four periods.

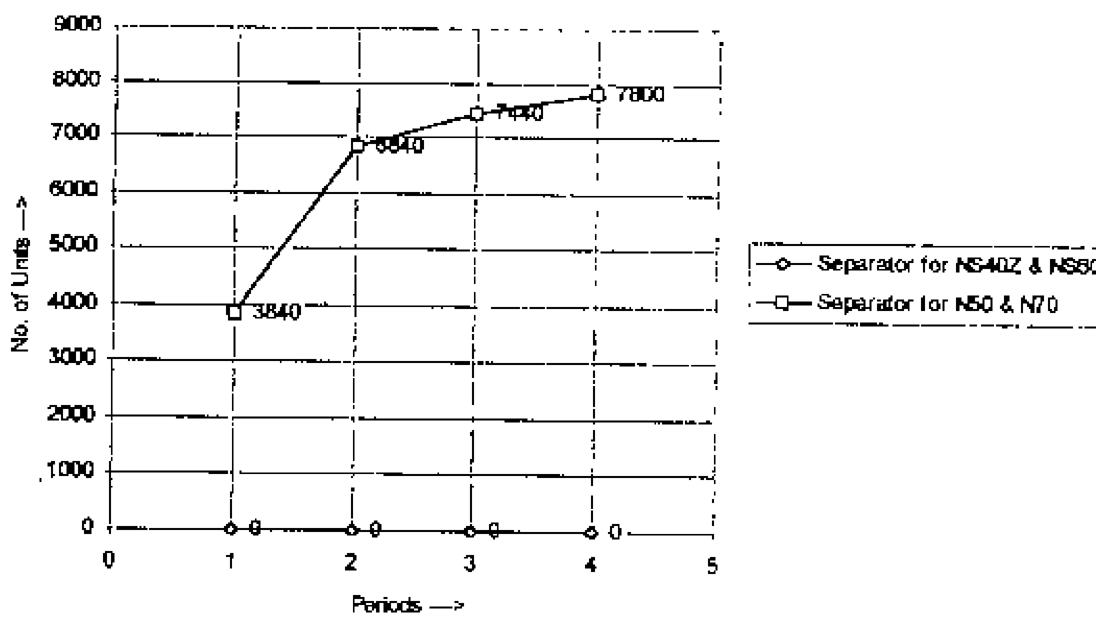


Chart-6: Number of units of Separators produced/procured at plant 2 of stage-1.

In chart-7, it shows the quantities of small parts pack for models of Ns40Z & Ns60, carton for model Ns60 and carton for model N50 procured at the plant-1 of stage-1 at different periods. The quantities of small parts pack for Ns40Z & Ns60 procured are 153,195,225 and 225 in consecutive four periods. The quantities of carton for model Ns60 are 101,115,120 and 135 and the quantities of carton for N50 are 23,39,50 and 35 in consecutive four periods.

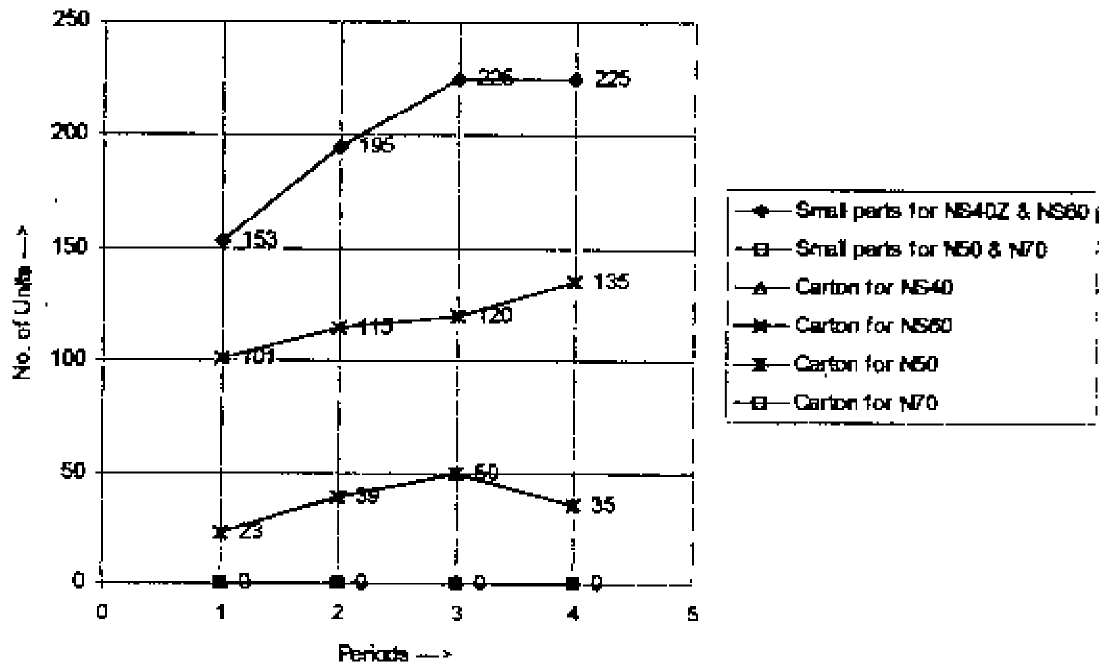


Chart-7: Small parts and Cartons procured at plant 1 of stag1.

In chart-8, it shows the quantities of small parts pack for models of N50 & N70, carton for model Ns40Z and carton for model N70 procured at the plant-2 of stage-1 at different periods. The quantities of small parts pack for N50 & N70 procured are 61,108,120 and 120 in consecutive four periods. The quantities of carton for model Ns40Z are 52,80,105 and 90 and the quantities of carton for N70 are 38,69,70 and 85 in consecutive four periods.

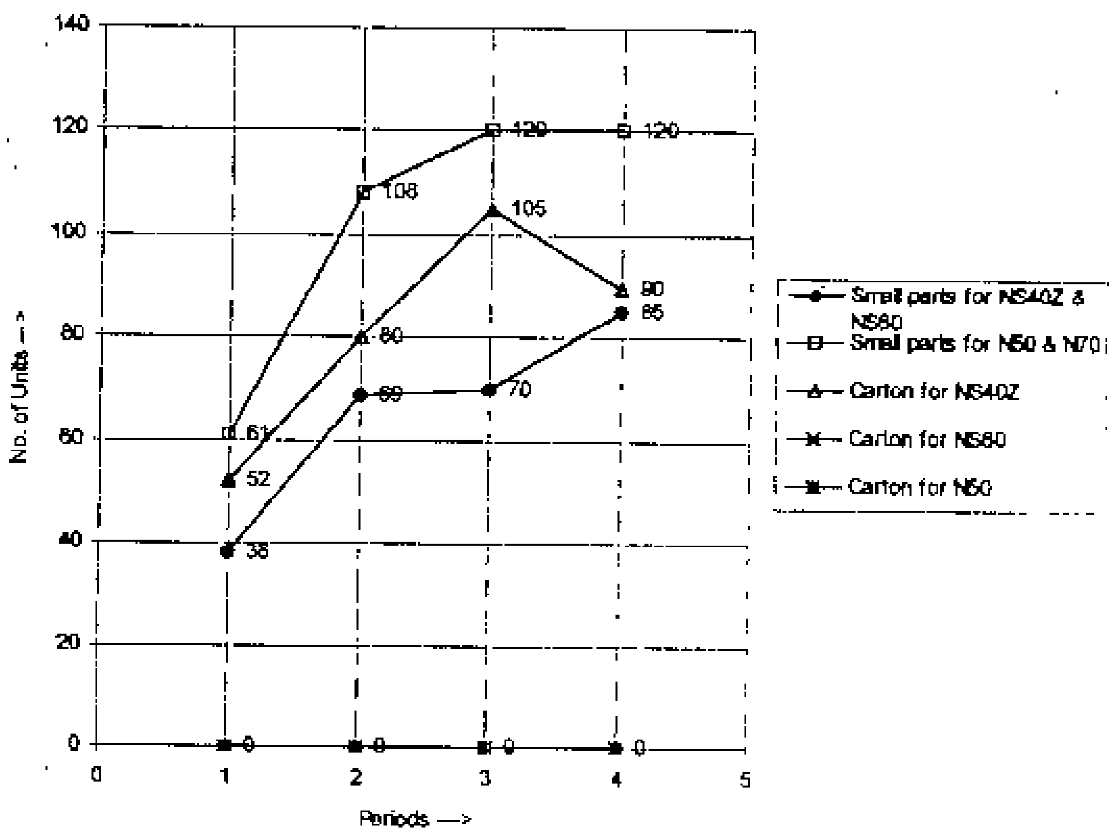


Chart-8: Small parts and Cartons produced/procured at plant 2 of stage-1.

5.6.2 Number of units of finished product produced to be assembled at assembling plant of stage –2:

Four models of automotive batteries were assembled at the assembling plant at each period. Total planning horizon was divided into four periods. The quantities of the batteries of different models assembled in different periods for satisfying the demand has shown in the chart bellow:

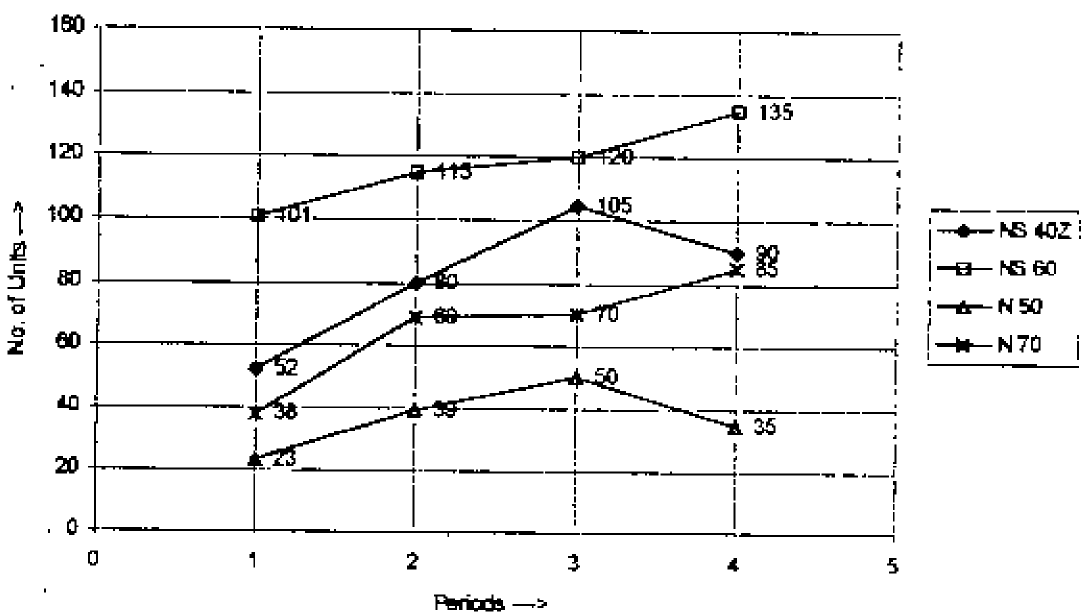


Chart-9: Number of units finished products assembled In the Assembling plant (Considering initial inventory).

Chart-9 shows the number of units of finished products of different types assembled at different period The quantities of assembled Ns40Z batteries are 52,80,105 and 90 in consecutive four periods. The quantities of assembled Ns60 batteries are 101,115,120 and 135. The quantities of assembled N50 batteries are 23,39,50 and 35. The quantities of assembled N70 batteries are 38,69,70 and 85.

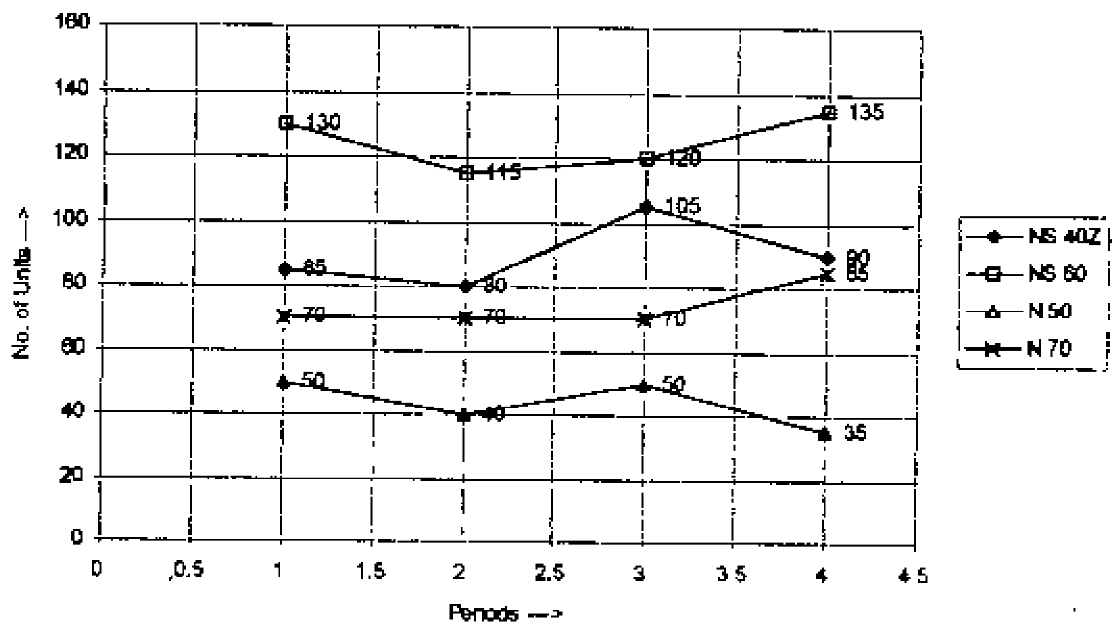


Chart-10: Number of units finished products Assembled in the Assembling plant (Considering initial inventory equal to zero).

Chart-10 shows the number of units of finished products of different types assembled at different periods. The quantities of assembled Ns40Z batteries are 85,80,105 and 90 in consecutive four periods. The quantities of assembled Ns60 batteries are 130,115,120 and 135. The quantities of assembled N50 batteries are 50,40,50 and 35. The quantities of assembled N70 batteries are 70,70,70 and 85. In this case initial inventory of the entire finished product at different storage facilities were considered zero.

5.6.3 Number of units of finished products to be shipped from assembling plant to the finished product warehouses:

Since the ultimate demand of finished product occurs at the warehouses, the assembled finished products were shipped to the warehouses at Motijheel, Mohakhali and Chittagong. On the basis of demand, management ships automotive batteries of different type to these sales outlets. This shipment quantity provides necessary information to develop requisite logistic planning. The shipment quantities were shown in the chart bellow

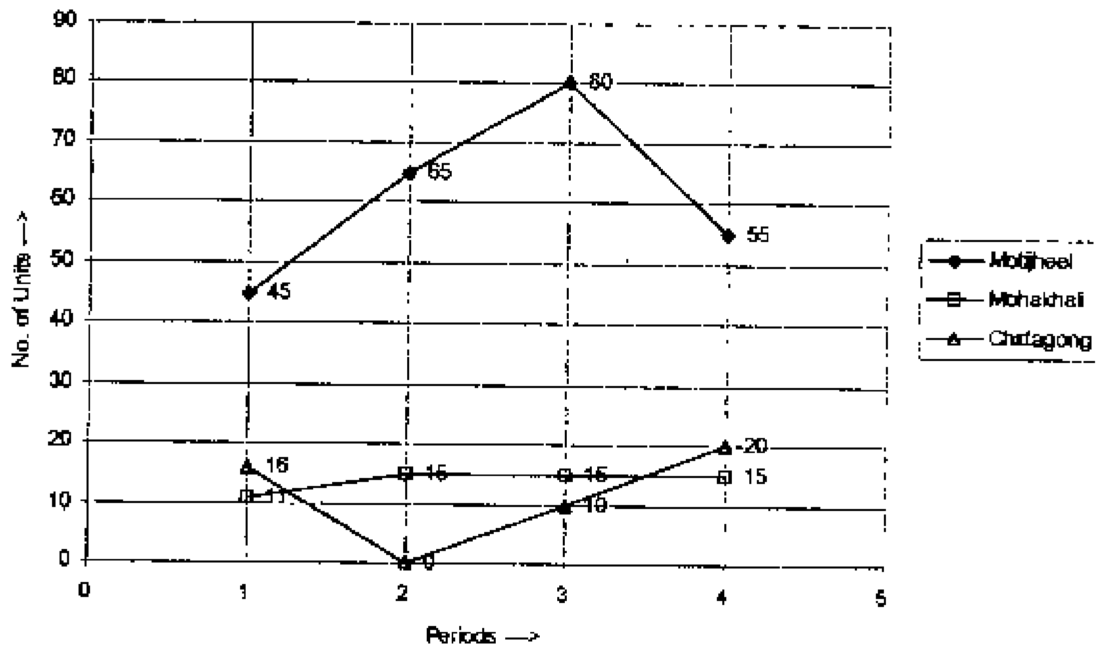


Chart-11: Number of units of finished product (Ns40Z) shipped to different warehouses.

Chart-11 shows the quantities of shipment of finished product Ns40Z at different warehouses at different periods. The quantities of shipment of Ns40Z battery to Motijheel warehouse are 45,65,80 and 55 in consecutive four periods. The quantities of shipment of Ns40Z battery to Mohakhali warehouse are 11,15,15 and 15 in consecutive four periods. The quantities of shipment of Ns40Z battery to Chittagong warehouse are 16,0,10 and 20 in consecutive four periods.

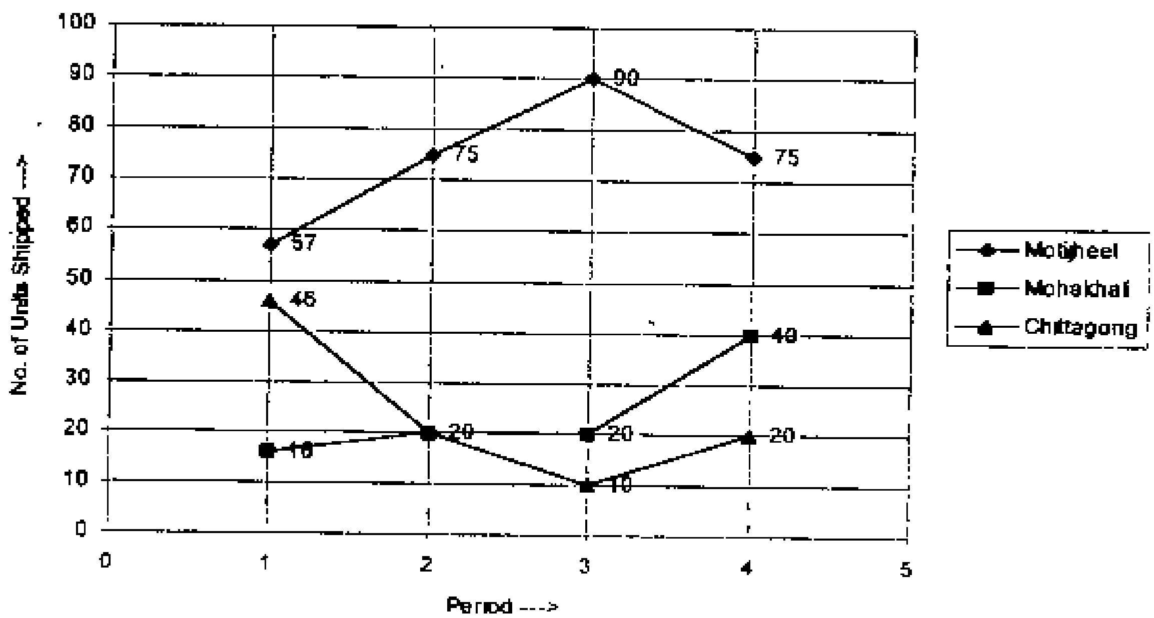


Chart-12: Number of units of finished product (Ns60) shipped to Different warehouses.

Chart-12 shows the quantities of shipment of finished product Ns60 at different warehouses at different periods. The quantities of shipment of Ns60 battery to Motijheel warehouse are 57,75,90 and 75 in consecutive four periods. The quantities of shipment of Ns60 battery to Mohakhali warehouse are 16,20,20 and 40in consecutive four periods. The quantities of shipment of Ns60 battery to Chittagong warehouse are 46,20,10 and 20 in consecutive four periods.

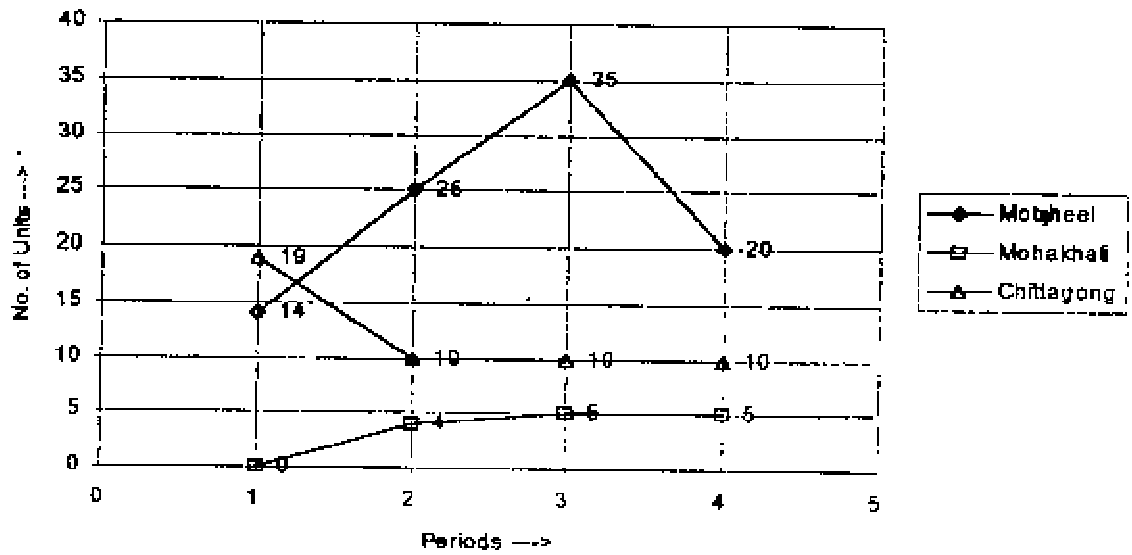


Chart-13: Number of units of finished product (N50) shipped to Different warehouses.

Chart-13 shows the quantities of shipment of finished product N50 at different warehouses at different periods. The quantities of shipment of N50 battery to Motijheel warehouse are 14,25,35 and 20 in consecutive four periods. The quantities of shipment of N50 battery to Mohakhali warehouse are 0,4,5 and 5 in consecutive four periods. The quantities of shipment of N50 battery to Chittagong warehouse are 19,10,10 and 10 in consecutive four periods.

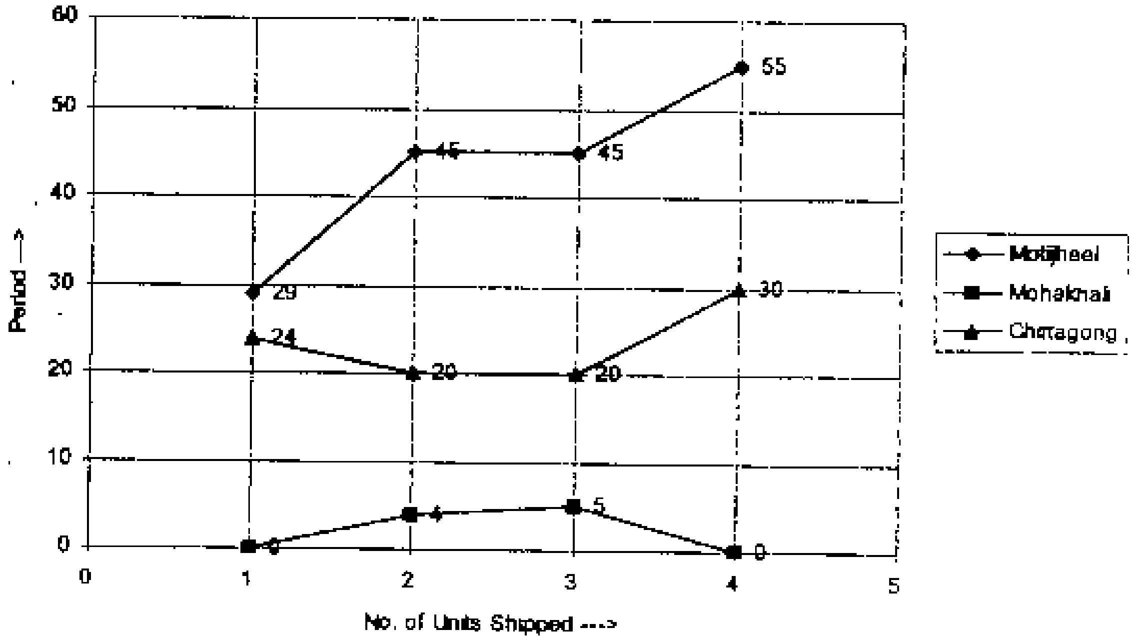


Chart-14: Number of units of finished product (N70) shipped to Different warehouses

Chart-14 shows the quantities of shipment of finished product N70 at different warehouses at different periods. The quantities of shipment of N70 battery to Motijheel warehouse are 29,45,45 and 55 in consecutive four periods. The quantities of shipment of N70 battery to Mohakhali warehouse are 0,4,5 and 0 in consecutive four periods. The quantities of shipment of N70 battery to Chittagong warehouse are 24,20,20 and 30 in consecutive four periods.

5.6.4 Total manufacturing and distribution cost for future planning horizon:

Total cost for the entire planning horizon was found Tk. 1343088.00. This figure will invariably help the management to decide upon future budget and investment planning. This figure is of great importance to the financial people of the company.

5.7 Discussion: Analysis of the result could be a helpful guideline for management. This model gives the optimum values of variables and in addition it provides the sensitivity analysis of the report. The sensitivity analysis may be of great importance to the management. It gives the model a dynamic characteristic that allows the analyst to study the behavior of the optimal solution as a result of possible changes in the values of the model parameters.

To analyze the sensitivity of production and shipping cost this model was tested with an assumed gross increase (gross increase) in production and shipping costs. The cost curves has been shown in chart 15, 16.

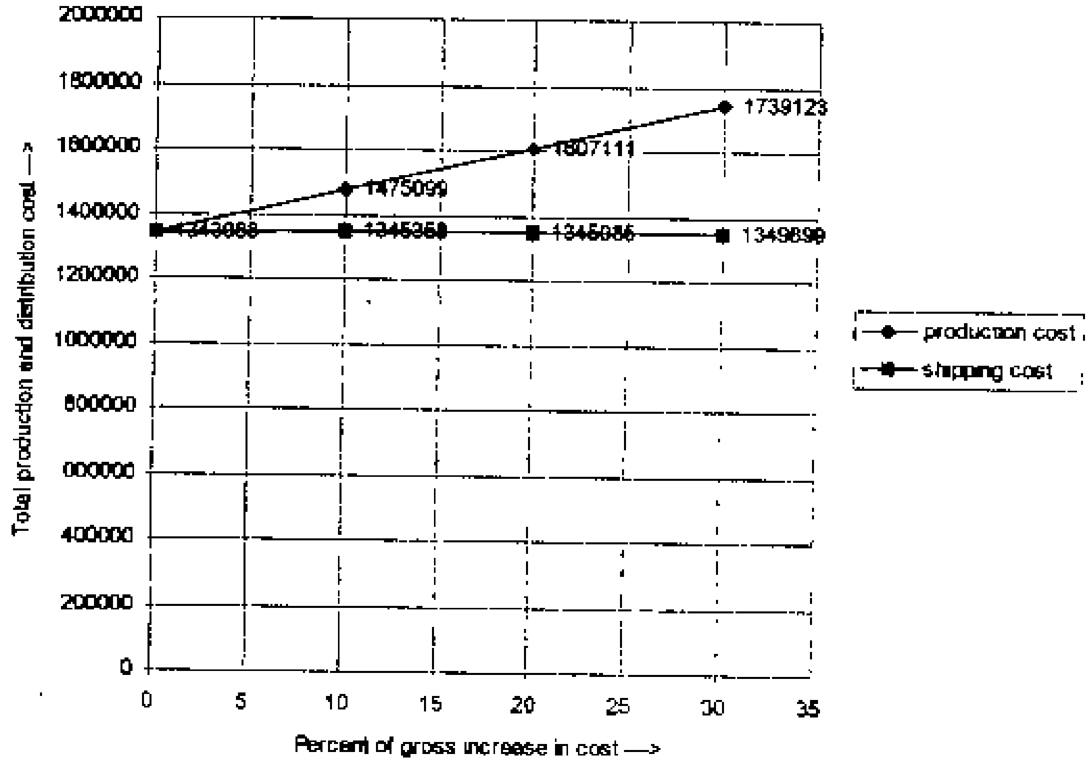


Chart-15: Total cost Vs rate of increase in production and shipping cost.

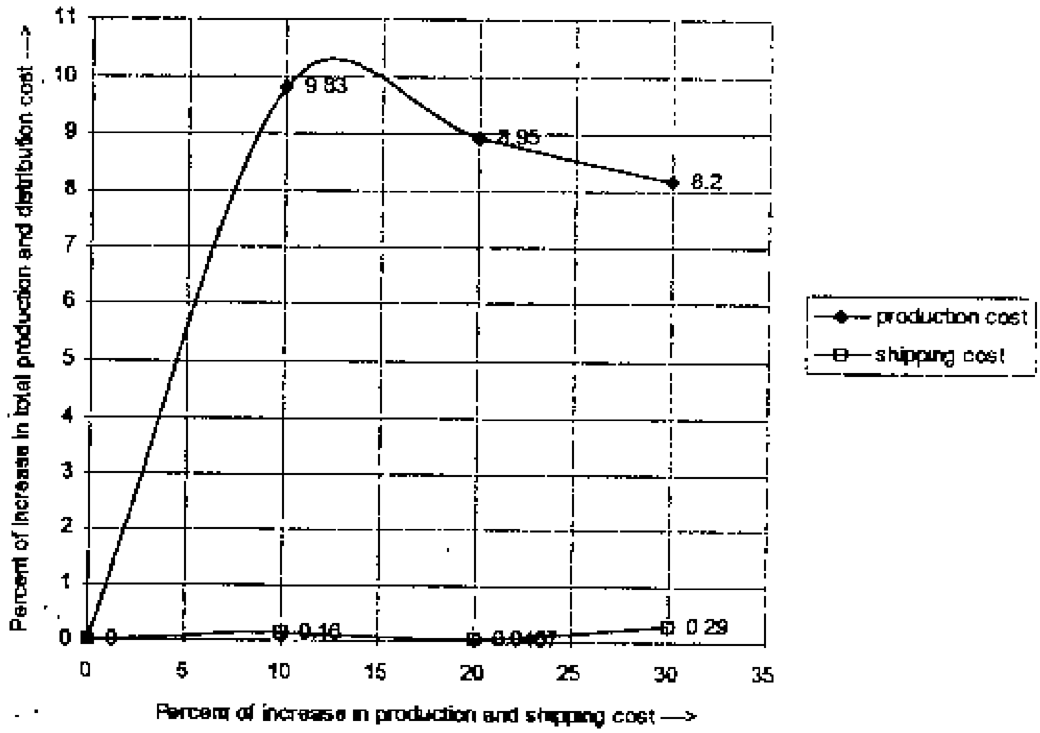


Chart-16: Rate of increase in total cost vs rate of gross increase in production cost & shipping cost.

The curves show that the total production and distribution cost for the entire planning horizon increases as the production cost and shipping increases. From the percent of increase in total cost it is found that the production costs are moderately sensitive over the shipping cost. The curve in chart 16 shows that at the beginning stage the percent of increase in total cost rises as the percent of increase in production/shipping cost rises. At the later stage the curve declines. Since only production & shipping costs are subjected to increase, this declining nature of the curve seems natural as other components of the total costs are not affected by the percent increase.

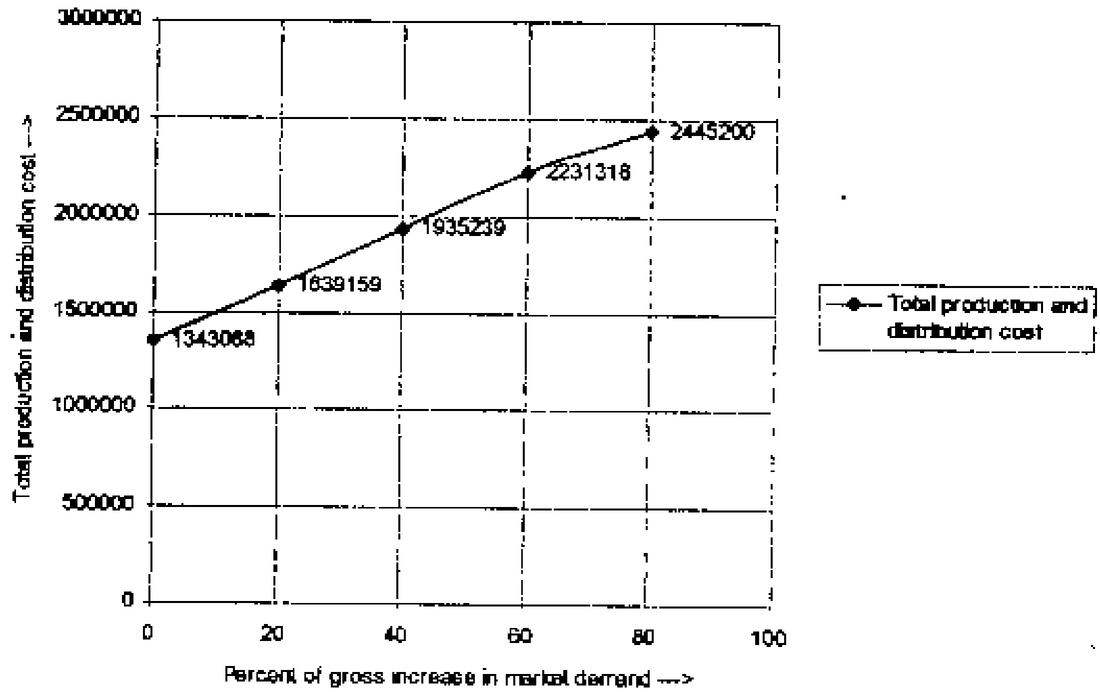


Chart-17: Total cost Vs rate of increase in market demand.

The model was also studied assuming increased market demand. It was tested with an assumed increase (gross) in market demand. The curve in chart 17 shows that the total cost for the entire planning horizon will increase if the enterprise wants to satisfy higher market demand.

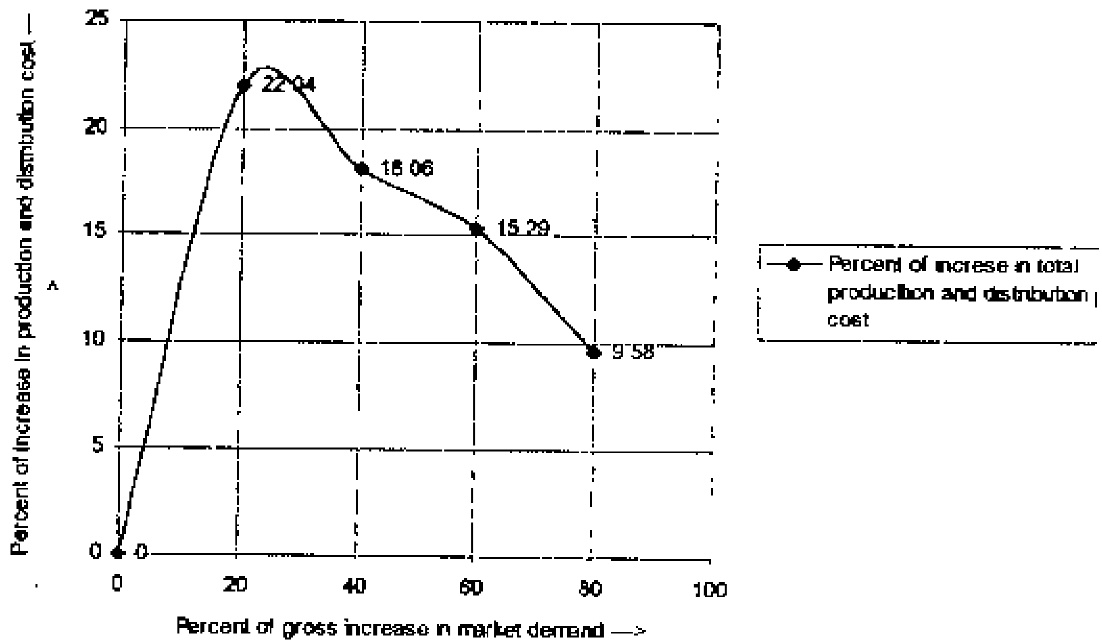


Chart-18: Total cost Vs percent of increase in market demand.

The curve in chart 18 show that at the beginning stage the percent of increase in total cost rises as the percent of demand rises. In later stage the percent of growth in total cost declines with the rise in market demand. This phenomena has already been discussed in case of chart 16.

From the analysis it is found that the company has abundant production capacities. It can withstand the pressure of larger growth rate of market demand and gives feasible solutions. The model is also flexible enough to cope with a sudden demand at any period in any distribution center.

Table 5.5.4: Range Report for semifinished product (partial)

Ranges in which the basis is unchanged.

Variable	Current coefficient	Allowable increase	Allowable decrease
XA11	110	Infinity	1.000
XB11	122	3.8	1.000
XC11	179	1.5	.9998
XD11	185	Infinity	1.000
XA21	108	1.5	1.000
XB21	125	0.199	3.8
XC21	180	Infinity	1.5
XD11	184	1.000	1.000

Table 5.5.5: Ranges Report for finished product (partial)

Variable	Current coefficient	Allowable increase	Allowable decrease
Y11	10	Infinity	1.000
Y21	10	Infinity	1.000
Y31	15	Infinity	.9998
Y41	15	Infinity	1.000
Y12	10	1.000	1.000
Y22	10	1.000	1.000
Y32	15	0.998	1.000
Y42	15	1.000	1.000

From the range report it is found that there are scopes to reduce cost of production of semifinished product slightly for any individual item. Also there are scopes to reduce inventory holding costs of particular items at any particular period.

During modeling it was found that the information regarding the initial inventory of finished product at the sales outlets or at any inventory holding stations, is very important. If the initial inventory exists in abundance, the model will try to allocate demanded finished product as much as possible from the initial inventory with out going for production. The phenomena exhibited by the model of allocating demanded product from unrestricted initial inventory without going for production seems logical. However the model, is flexible enough to cope with situation of dealing with zero or nonzero inventory. In this project both the above mentioned cases were tested and the result was found satisfactory. The results has been shown in chart 9 and chart 10.

Chapter-6

Conclusions & Recommendations

In this project work, attempts have been made to Characterize various multistage multi location production planning systems- starting from single finished product to multiple finished products. Necessary features were considered from simple processing line to critical assembling line of production systems.

The mathematical models developed were all tested with data and found to provide optimum results. It amply demonstrates that it is possible, through mathematical modelling, to examine various production planning situations of multistage multilocation multiperiod assembliug operations, incorporate different constraints and arrive at the optimum (or at least feasible) solutions. These solutions may not always meet all practical limitations. but these would definitely help the practitioner make more objective analysis of the system and thereby take various realistic management decisions. As for example, from the output of the case study of Rahim Afrooz(BD), management may consider few alternatives which are:

(I) to increase the market demand of existing set of automotive battery models by aggressive advertising, focusing on other sales promotion activities and explore new market demand abroad;

(II) to expand the existing product line by introducing automotive batteries of new model;

(III) to use the underutilized capacity by new product development, innovating the new product and adding it to the product mix.

In all the models developed in this study, associated costs were considered linear. In many situations this may not be a realistic assumption. In future work, this assumption may be relaxed and non linear functions of the cost may be incorporated into the models. The other aspects that may be included in the future work are the consideration of multiple plants at the assembling stage, multiple routing within the plant and backlogging of finished goods demand etc. The output of the models indicates that the values of the decision variables are integers. But in cases where the production capacities are critical, the linear programming models may generate non integer values for the decision variables. In such cases, the approach of integer linear programming may be called for.

Lastly to conclude, management must be exposed to modern information technology. In order to make a realistic production plan, especially in a complex situation like multistage multilocation multiperiod production system, proper recording and timely updating of various data relating to in-house stock of materials, procurement procedure for raw materials, components or subassemblies, production capacity, costs demand forecast etc must be ensured. This calls for an efficient computerized database management system, without which the real benefit of modelling can hardly be expected.

Data Input of the Model

(Only for Model-5)

APPENDIX

Interpretation of the variables as used during computation

P	X_{pjt}	V_{pjt}	I_{1pjt}	I_{2pt}
	$j = 1, 2$ $t = 1, 2, 3, 4$	$j = 1, 2$ $t = 1, 2, 3, 4$	$j = 1, 2$ $t = 1, 2, 3, 4$	$t = 1, 2, 3, 4$
P = 1	X_{Ajt}	V_{Ajt}	I_{1Ajt}	I_{2At}
P = 2	X_{Bjt}	V_{Bjt}	I_{1Bjt}	I_{2Bt}
P = 3	X_{Cjt}	V_{Cjt}	I_{1Cjt}	I_{2Ct}
P = 4	X_{Djt}	V_{Djt}	I_{1Djt}	I_{2Dt}
P = 5	X_{Ejt}	V_{Ejt}	I_{1Ejt}	I_{2Et}
P = 6	X_{Fjt}	V_{Fjt}	I_{1Fjt}	I_{2Ft}
P = 7	X_{Gjt}	V_{Gjt}	I_{1Gjt}	I_{2Gt}
P = 8	X_{Hjt}	V_{Hjt}	I_{1Hjt}	I_{2Ht}
P = 9	X_{Jjt}	V_{Jjt}	I_{1Jjt}	I_{2Jt}
P = 10	X_{Kjt}	V_{Kjt}	I_{1Kjt}	I_{2Kt}
P = 11	X_{Ljt}	V_{Ljt}	I_{1Ljt}	I_{2Lt}
P = 12	X_{Mjt}	V_{Mjt}	I_{1Mjt}	I_{2Mt}
P = 13	X_{Njt}	V_{Njt}	I_{1Njt}	I_{2Nt}
P = 14	X_{Ojt}	V_{Ojt}	I_{1Ojt}	I_{2Ot}
P = 15	X_{Pjt}	V_{Pjt}	I_{1Pjt}	I_{2Pt}
P = 16	X_{Qjt}	V_{Qjt}	I_{1Qjt}	I_{2Qt}

MIN 110 XA11 + 122 XB11 + 179 XC11 + 185 XD11 + 108 XA21 + 125 XB21
+ 180 XC21 + 184 XD21 + 12 XE11 + 15 XF11 + 13 XE21 + 14 XF21
+ 10 XG11 + 14 XH11 + 11 XG21 + 13 XH21 + .8 XJ11 + XK11 + XJ21
+ .9 XK21 + 18.2 XL11 + 24.2 XM11 + 18.5 XL21 + 24 XM21 + 23 XN11
+ 24 XO11 + 26 XP11 + 30 XQ11 + 22 XN21 + 25 XO21 + 27 XP21 + 28 XQ21
+ 110 XA12 + 122 XB12 + 179 XC12 + 185 XD12 + 108 XA22 + 125 XB22
+ 180 XC22 + 184 XD22 + 12 XE12 + 15 XF12 + 13 XE22 + 14 XF22
+ 10 XG12 + 14 XH12 + 11 XG22 + 13 XH22 + .8 XJ12 + XK12 + XJ22
+ .9 XK22 + 18.2 XL12 + 24.2 XM12 + 18.5 XL22 + 24 XM22 + 23 XN12
+ 24 XO12 + 26 XP12 + 30 XQ12 + 22 XN22 + 25 XO22 + 27 XP22 + 28 XQ22
+ 110 XA13 + 122 XB13 + 179 XC13 + 185 XD13 + 108 XA23 + 125 XB23
+ 180 XC23 + 184 XD23 + 12 XE13 + 15 XF13 + 13 XE23 + 14 XF23
+ 10 XG13 + 14 XH13 + 11 XG23 + 13 XH23 + .8 XJ13 + XK13 + XJ23
+ .9 XK23 + 18.2 XL13 + 24.2 XM13 + 18.5 XL23 + 24 XM23 + 23 XN13
+ 24 XO13 + 26 XP13 + 30 XQ13 + 22 XN23 + 25 XO23 + 27 XP23 + 28 XQ23
+ 110 XA14 + 122 XB14 + 179 XC14 + 185 XD14 + 108 XA24 + 125 XB24
+ 180 XC24 + 184 XD24 + 12 XE14 + 15 XF14 + 13 XE24 + 14 XF24
+ 10 XG14 + 14 XH14 + 11 XG24 + 13 XH24 + .8 XJ14 + XK14 + XJ24
+ .9 XK24 + 18.2 XL14 + 24.2 XM14 + 18.5 XL24 + 24 XM24 + 23 XN14
+ 24 XO14 + 26 XP14 + 30 XQ14 + 22 XN24 + 25 XO24 + 27 XP24 + 28 XQ24
+ 10 I1A10 + 11 I1B10 + 15.5 I1C10 + 20 I1D10 + 10 I1A20 + 10 I1B20
+ 16 I1C20 + 21 I1D20 + 1.2 I1E10 + 1.5 I1F10 + 1.5 I1E20 + 1.4 I1F20
+ 1.2 I1G10 + 1.5 I1H10 + 1.4 I1G20 + 1.5 I1H20 + .08 I1J10 + .1 I1K10
+ .1 I1J20 + .1 I1K20 + 1.82 I1L10 + 1.8 I1M10 + 1.8 I1L20 + 1.7 I1M20
--More--

+ .5 I1N10 + .5 I1O10 + .5 I1P10 + .5 I1Q10 + .6 I1N20 + .6 I1O20
+ .6 I1P20 + .6 I1Q20 + 10 I1A11 + 11 I1B11 + 15.5 I1C11 + 20 I1D11
+ 10 I1A21 + 10 I1B21 + 16 I1C21 + 21 I1D21 + 1.2 I1E11 + 1.5 I1F11
+ 1.5 I1E21 + 1.4 I1F21 + 1.2 I1G11 + 1.5 I1H11 + 1.4 I1G21
+ 1.5 I1H21 + .08 I1J11 + .1 I1K11 + .1 I1J21 + .1 I1K21 + 1.82 I1L11
+ 1.8 I1M11 + 1.8 I1L21 + 1.7 I1M21 + .5 I1N11 + .5 I1O11 + .5 I1P11
+ .5 I1Q11 + .6 I1N21 + .6 I1O21 + .6 I1P21 + .6 I1Q21 + 10 I1A12
+ 11 I1B12 + 15.5 I1C12 + 20 I1D12 + 10 I1A22 + 10 I1B22 + 16 I1C22
+ 21 I1D22 + 1.2 I1E12 + 1.5 I1F12 + 1.5 I1E22 + 1.4 I1F22 + 1.2 I1G12
+ 1.5 I1H12 + 1.4 I1G22 + 1.5 I1H22 + .08 I1J12 + .1 I1K12 + .1 I1J22
+ .1 I1K22 + 1.82 I1L12 + 1.8 I1M12 + 1.8 I1L22 + 1.7 I1M22 + .5 I1N12
+ .5 I1O12 + .5 I1P12 + .5 I1Q12 + .6 I1N22 + .6 I1O22 + .6 I1P22
+ .6 I1Q22 + 10 I1A13 + 11 I1B13 + 15.5 I1C13 + 20 I1D13 + 10 I1A23
+ 10 I1B23 + 16 I1C23 + 21 I1D23 + 1.2 I1E13 + 1.5 I1F13 + 1.5 I1E23
+ 1.4 I1F23 + 1.2 I1G13 + 1.5 I1H13 + 1.4 I1G23 + 1.5 I1H23
+ .08 I1J13 + .1 I1K13 + .1 I1J23 + .1 I1K23 + 1.82 I1L13 + 1.8 I1M13
+ 1.8 I1L23 + 1.7 I1M23 + .5 I1N13 + .5 I1O13 + .5 I1P13 + .5 I1Q13
+ .6 I1N23 + .6 I1O23 + .6 I1P23 + .6 I1Q23 + 10 I1A14 + 11 I1B14
+ 15.5 I1C14 + 20 I1D14 + 10 I1A24 + 10 I1B24 + 16 I1C24 + 21 I1D24
+ 1.2 I1E14 + 1.5 I1F14 + 1.5 I1E24 + 1.4 I1F24 + 1.2 I1G14
+ 1.5 I1H14 + 1.4 I1G24 + 1.5 I1H24 + .08 I1J14 + .1 I1K14 + .1 I1J24
+ .1 I1K24 + 1.82 I1L14 + 1.8 I1M14 + 1.8 I1L24 + 1.7 I1M24 + .5 I1N14
+ .5 I1O14 + .5 I1P14 + .5 I1Q14 + .6 I1N24 + .6 I1O24 + .6 I1P24
+ .6 I1Q24 + 7 I2A0 + 8 I2B0 + 10 I2C0 + 15 I2D0 + I2E0 + I2F0 + I2G0
--More--

+ I2H0 + .1 I2J0 + .1 I2K0 + I2L0 + I2M0 + .5 I2N0 + .5 I2O0 + .5 I2P0
+ .5 I2Q0 + 7 I2A1 + 8 I2B1 + 10 I2C1 + 15 I2D1 + I2E1 + I2F1 + I2G1
+ I2H1 + .1 I2J1 + .1 I2K1 + I2L1 + I2M1 + .5 I2N1 + .5 I2O1 + .5 I2P1
+ .5 I2Q1 + 7 I2A2 + 8 I2B2 + 10 I2C2 + 15 I2D2 + I2E2 + I2F2 + I2G2
+ I2H2 + .1 I2J2 + .1 I2K2 + I2L2 + I2M2 + .5 I2N2 + .5 I2O2 + .5 I2P2
+ .5 I2Q2 + 7 I2A3 + 8 I2B3 + 10 I2C3 + 15 I2D3 + I2E3 + I2F3 + I2G3
+ I2H3 + .1 I2J3 + .1 I2K3 + I2L3 + I2M3 + .5 I2N3 + .5 I2O3 + .5 I2P3
+ .5 I2Q3 + 7 I2A4 + 8 I2B4 + 10 I2C4 + 15 I2D4 + I2E4 + I2F4 + I2G4
+ I2H4 + .1 I2J4 + .1 I2K4 + I2L4 + I2M4 + .5 I2N4 + .5 I2O4 + .5 I2P4
+ .5 I2Q4 + VA11 + 1.2 VB11 + 1.5 VC11 + 1.5 VD11 + 1.5 VA21 + 2 VB21
+ 2 VC21 + 1.5 VD21 + .1 VE11 + .15 VF11 + .3 VE21 + .2 VF21 + .1 VG11
+ .15 VH11 + .2 VG21 + .3 VH21 + .01 VJ11 + .01 VK11 + .01 VJ21
+ .01 VK21 + .05 VL11 + .15 VM11 + .06 VL21 + .15 VM21 + .2 VN11
+ .2 VO11 + .2 VP11 + .2 VQ11 + .3 VN21 + .3 VO21 + .3 VP21 + .3 VQ21
+ VA12 + 1.2 VB12 + 1.5 VC12 + 1.5 VD12 + 1.5 VA22 + 2 VB22 + 2 VC22
+ 1.5 VD22 + .1 VE12 + .15 VF12 + .3 VE22 + .2 VF22 + .1 VG12
+ .15 VH12 + .2 VG22 + .3 VH22 + .01 VJ12 + .01 VK12 + .01 VJ22
+ .01 VK22 + .05 VL12 + .15 VM12 + .06 VL22 + .15 VM22 + .2 VN12
+ .2 VO12 + .2 VP12 + .2 VQ12 + .3 VN22 + .3 VO22 + .3 VP22 + .3 VQ22
+ VA13 + 1.2 VB13 + 1.5 VC13 + 1.5 VD13 + 1.5 VA23 + 2 VB23 + 2 VC23
+ 1.5 VD23 + .1 VE13 + .15 VF13 + .3 VE23 + .2 VF23 + .1 VG13
+ .15 VH13 + .2 VG23 + .3 VH23 + .01 VJ13 + .01 VK13 + .01 VJ23
+ .01 VK23 + .05 VL13 + .15 VM13 + .06 VL23 + .15 VM23 + .2 VN13
+ .2 VO13 + .2 VP13 + .2 VQ13 + .3 VN23 + .3 VO23 + .3 VP23 + .3 VQ23

--More--

```
   + VA14 + 1.2 VB14 + 1.5 VC14 + 1.5 VD14 + 1.5 VA24 + 2 VB24 + 2 VC24
   + 1.5 VD24 + .1 VE14 + .15 VF14 + .3 VE24 + .2 VF24 + .1 VG14
   + .15 VH14 + .2 VG24 + .3 VH24 + .01 VJ14 + .01 VK14 +  01 VJ24
   + .01 VK24 + .05 VL14 + .15 VM14 + .06 VL24 + .15 VM24 + .2 VN14
   + .2 VO14 + .2 VP14 + .2 VQ14 + .3 VN24 + .3 VO24 +  3 VP24 + .3 VQ24
   + 10 Y11 + 10 Y21 + 15 Y31 + 15 Y41 + 10 Y12 + 10 Y22 + 15 Y32
   + 15 Y42 + 10 Y13 + 10 Y23 + 15 Y33 + 15 Y43 + 10 Y14 + 10 Y24
   + 15 Y34 + 15 Y44 + I310 + I311 + I312 + I313 + I314 + I320 + I321
   + I322 + I323 + I324 + I330 + I331 + I332 + I333 + I334 + I340 + I341
   + I342 + I343 + I344 + 4 W111 + 4 W211 + 4 W311 + 4 W411 + 4 W121
   + 4 W221 + 4 W321 + 4 W421 + 10 W131 + 10 W231 + 10 W331 + 10 W431
   + 4 W112 + 4 W212 + 4 W312 + 4 W412 + 4 W122 + 4 W222 + 4 W322
   + 4 W422 + 10 W132 + 10 W232 + 10 W332 + 10 W432 + 4 W113 + 4 W213
   + 4 W313 + 4 W413 + 4 W123 + 4 W223 + 4 W323 + 4 W423 + 10 W133
   + 10 W233 + 10 W333 + 10 W433 + 4 W114 + 4 W214 + 4 W314 + 4 W414
   + 4 W124 + 4 W224 + 4 W324 + 4 W424 + 10 W134 + 10 W234 + 10 W334
   + 10 W434 + 3 I4110 + 3 I4210 + 4 I4310 + 4 I4410 + 3 I4120 + 3 I4220
   + 4 I4320 + 4 I4420 + 2 I4130 + 2 I4230 + 3 I4330 + 3 I4430 + 3 I4111
   + 3 I4211 + 4 I4311 + 4 I4411 + 3 I4121 + 3 I4221 + 4 I4321 + 4 I4421
   + 2 I4131 + 2 I4231 + 3 I4331 + 3 I4431 + 3 I4112 + 3 I4212 + 4 I4312
   + 4 I4412 + 3 I4122 + 3 I4222 + 4 I4322 + 4 I4422 + 2 I4132 + 2 I4232
   + 3 I4332 + 3 I4432 + 3 I4113 + 3 I4213 + 4 I4313 + 4 I4413 + 3 I4123
   + 3 I4223 + 4 I4323 + 4 I4423 + 2 I4133 + 2 I4233 + 3 I4333 + 3 I4433
   + 3 I4114 + 3 I4214 + 4 I4314 + 4 I4414 + 3 I4124 + 3 I4224 + 4 I4324
--More--
```

SUBJECT TO
2) - XA11 - I1A10 + I1A11 + VA11 = 0
3) - XC11 - I1C10 + I1C11 + VC11 = 0
4) - XB11 - I1B10 + I1B11 + VB11 = 0
5) - XD11 - I1D10 + I1D11 + VD11 = 0
6) - XA21 - I1A20 + I1A21 + VA21 = 0
7) - XB21 - I1B20 + I1B21 + VB21 = 0
8) - XC21 - I1C20 + I1C21 + VC21 = 0
9) - XD21 - I1D20 + I1D21 + VD21 = 0
10) - XE11 - I1E10 + I1E11 + VE11 = 0
11) - XF11 - I1F10 + I1F11 + VF11 = 0
12) - XE21 - I1E20 + I1E21 + VE21 = 0
13) - XF21 - I1F20 + I1F21 + VF21 = 0
14) - XG11 - I1G10 + I1G11 + VG11 = 0
15) - XH11 - I1H10 + I1H11 + VH11 = 0
16) - XG21 - I1G20 + I1G21 + VG21 = 0
17) - XH21 - I1H20 + I1H21 + VH21 = 0
18) - XJ11 - I1J10 + I1J11 + VJ11 = 0
19) - XK11 - I1K10 + I1K11 + VK11 = 0
20) - XJ21 - I1J20 + I1J21 + VJ21 = 0
21) - XK21 - I1K20 + I1K21 + VK21 = 0
22) - XL11 - I1L10 + I1L11 + VL11 = 0
23) - XM11 - I1M10 + I1M11 + VM11 = 0
--More--

```
24) - XL21 - I1L20 + I1L21 + VL21 =    0
25) - XM21 - I1M20 + I1M21 + VM21 =    0
26) - XN11 - I1N10 + I1N11 + VN11 =    0
27) - XO11 - I1O10 + I1O11 + VO11 =    0
28) - XP11 - I1P10 + I1P11 + VP11 =    0
29) - XQ11 - I1Q10 + I1Q11 + VQ11 =    0
30) - XN21 - I1N20 + I1N21 + VN21 =    0
31) - XO21 - I1O20 + I1O21 + VO21 =    0
32) - XP21 - I1P20 + I1P21 + VP21 =    0
33) - XQ21 - I1Q20 + I1Q21 + VQ21 =    0
34) - XA12 - I1A11 + I1A12 + VA12 =    0
35) - XB12 - I1B11 + I1B12 + VB12 =    0
36) - XC12 - I1C11 + I1C12 + VC12 =    0
37) - XD12 - I1D11 + I1D12 + VD12 =    0
38) - XA22 - I1A21 + I1A22 + VA22 =    0
39) - XB22 - I1B21 + I1B22 + VB22 =    0
40) - XC22 - I1C21 + I1C22 + VC22 =    0
41) - XD22 - I1D21 + I1D22 + VD22 =    0
42) - XE12 - I1E11 + I1E12 + VE12 =    0
43) - XF12 - I1F11 + I1F12 + VF12 =    0
44) - XE22 - I1E21 + I1E22 + VE22 =    0
45) - XF22 - I1F21 + I1F22 + VF22 =    0
46) - XG12 - I1G11 + I1G12 + VG12 =    0
47) - XH12 - I1H11 + I1H12 + VH12 =    0
--More--
```

48) - XG22 - I1G21 + I1G22 + VG22 = 0
49) - XH22 - I1H21 + I1H22 + VH22 = 0
50) - XJ12 - I1J11 + I1J12 + VJ12 = 0
51) - XK12 - I1K11 + I1K12 + VK12 = 0
52) - XJ22 - I1J21 + I1J22 + VJ22 = 0
53) - XK22 - I1K21 + I1K22 + VK22 = 0
54) - XL12 - I1L11 + I1L12 + VL12 = 0
55) - XM12 - I1M11 + I1M12 + VM12 = 0
56) - XL22 - I1L21 + I1L22 + VL22 = 0
57) - XM22 - I1M21 + I1M22 + VM22 = 0
58) - XN12 - I1N11 + I1N12 + VN12 = 0
59) - XO12 - I1O11 + I1O12 + VO12 = 0
60) - XP12 - I1P11 + I1P12 + VP12 = 0
61) - XQ12 - I1Q11 + I1Q12 + VQ12 = 0
62) - XN22 - I1N21 + I1N22 + VN22 = 0
63) - XO22 - I1O21 + I1O22 + VO22 = 0
64) - XP22 - I1P21 + I1P22 + VP22 = 0
65) - XQ22 - I1Q21 + I1Q22 + VQ22 = 0
66) - XA13 - I1A12 + I1A13 + VA13 = 0
67) - XB13 - I1B12 + I1B13 + VB13 = 0
68) - XC13 - I1C12 + I1C13 + VC13 = 0
69) - XD13 - I1D12 + I1D13 + VD13 = 0
70) - XA23 - I1A22 + I1A23 + VA23 = 0
71) - XB23 - I1B22 + I1B23 + VB23 = 0

--More--

72) - XC23 - I1C22 + I1C23 + VC23 = 0
73) - XD23 - I1D22 + I1D23 + VD23 = 0
74) - XE13 - I1E12 + I1E13 + VE13 = 0
75) - XF13 - I1F12 + I1F13 + VF13 = 0
76) - XE23 - I1E22 + I1E23 + VE23 = 0
77) - XF23 - I1F22 + I1F23 + VF23 = 0
78) - XG13 - I1G12 + I1G13 + VG13 = 0
79) - XH13 - I1H12 + I1H13 + VH13 = 0
80) - XG23 - I1G22 + I1G23 + VG23 = 0
81) - XH23 - I1H22 + I1H23 + VH23 = 0
82) - XJ13 - I1J12 + I1J13 + VJ13 = 0
83) - XK13 - I1K12 + I1K13 + VK13 = 0
84) - XJ23 - I1J22 + I1J23 + VJ23 = 0
85) - XK23 - I1K22 + I1K23 + VK23 = 0
86) - XL13 - I1L12 + I1L13 + VL13 = 0
87) - XM13 - I1M12 + I1M13 + VM13 = 0
88) - XL23 - I1L22 + I1L23 + VL23 = 0
89) - XM23 - I1M22 + I1M23 + VM23 = 0
90) - XN13 - I1N12 + I1N13 + VN13 = 0
91) - XO13 - I1O12 + I1O13 + VO13 = 0
92) - XP13 - I1P12 + I1P13 + VP13 = 0
93) - XQ13 - I1Q12 + I1Q13 + VQ13 = 0
94) - XN23 - I1N22 + I1N23 + VN23 = 0
95) - XO23 - I1O22 + I1O23 + VO23 = 0

--More--

96) - XP23 - I1P22 + I1P23 + VP23 = 0
97) - XQ23 - I1Q22 + I1Q23 + VQ23 = 0
98) - XA14 - I1A13 + I1A14 + VA14 = 0
99) - XB14 - I1B13 + I1B14 + VB14 = 0
100) - XC14 - I1C13 + I1C14 + VC14 = 0
101) - XD14 - I1D13 + I1D14 + VD14 = 0
102) - XA24 - I1A23 + I1A24 + VA24 = 0
103) - XB24 - I1B23 + I1B24 + VB24 = 0
104) - XC24 - I1C23 + I1C24 + VC24 = 0
105) - XD24 - I1D23 + I1D24 + VD24 = 0
106) - XE14 - I1E13 + I1E14 + VE14 = 0
107) - XF14 - I1F13 + I1F14 + VF14 = 0
108) - XE24 - I1E23 + I1E24 + VE24 = 0
109) - XF24 - I1F23 + I1F24 + VF24 = 0
110) - XG14 - I1G13 + I1G14 + VG14 = 0
111) - XH14 - I1H13 + I1H14 + VH14 = 0
112) - XG24 - I1G23 + I1G24 + VG24 = 0
113) - XH24 - I1H23 + I1H24 + VH24 = 0
114) - XJ14 - I1J13 + I1J14 + VJ14 = 0
115) - XK14 - I1K13 + I1K14 + VK14 = 0
116) - XJ24 - I1J23 + I1J24 + VJ24 = 0
117) - XK24 - I1K23 + I1K24 + VK24 = 0
118) - XL14 - I1L13 + I1L14 + VL14 = 0
119) - XM14 - I1M13 + I1M14 + VM14 = 0
--More--

120) - XL24 - I1L23 + I1L24 + VL24 = 0
121) - XM24 - I1M23 + I1M24 + VM24 = 0
122) - XN14 - I1N13 + I1N14 + VN14 = 0
123) - XO14 - I1O13 + I1O14 + VO14 = 0
124) - XP14 - I1P13 + I1P14 + VP14 = 0
125) - XQ14 - I1Q13 + I1Q14 + VQ14 = 0
126) - XN24 - I1N23 + I1N24 + VN24 = 0
127) - XO24 - I1O23 + I1O24 + VO24 = 0
128) - XP24 - I1P23 + I1P24 + VP24 = 0
129) - XQ24 - I1Q23 + I1Q24 + VQ24 = 0
130) - I2A0 + I2A1 - VA11 - VA21 + Y11 = 0
131) - I2A1 + I2A2 - VA12 - VA22 + Y12 = 0
132) - I2A2 + I2A3 - VA13 - VA23 + Y13 = 0
133) - I2A3 + I2A4 - VA14 - VA24 + Y14 = 0
134) - I2B0 + I2B1 - VB11 - VB21 + Y21 = 0
135) - I2B1 + I2B2 - VB12 - VB22 + Y22 = 0
136) - I2B2 + I2B3 - VB13 - VB23 + Y23 = 0
137) - I2B3 + I2B4 - VB14 - VB24 + Y24 = 0
138) - I2C0 + I2C1 - VC11 - VC21 + Y31 = 0
139) - I2C1 + I2C2 - VC12 - VC22 + Y32 = 0
140) - I2C2 + I2C3 - VC13 - VC23 + Y33 = 0
141) - I2C3 + I2C4 - VC14 - VC24 + Y34 = 0
142) - I2D0 + I2D1 - VD11 - VD21 + Y41 = 0
143) - I2D1 + I2D2 - VD12 - VD22 + Y42 = 0

--More--

144) - I2D2 + I2D3 - VD13 - VD23 + Y43 = 0
145) - I2D3 + I2D4 - VD14 - VD24 + Y44 = 0
146) - I2E0 + I2E1 - VE11 - VE21 + 30 Y11 + 36 Y21 = 0
147) - I2E1 + I2E2 - VE12 - VE22 + 30 Y12 + 36 Y22 = 0
148) - I2E2 + I2E3 - VE13 - VE23 + 30 Y13 + 36 Y23 = 0
149) - I2E3 + I2E4 - VE14 - VE24 + 30 Y14 + 36 Y24 = 0
150) - I2F0 + I2F1 - VF11 - VF21 + 24 Y31 + 36 Y41 = 0
151) - I2F1 + I2F2 - VF12 - VF22 + 24 Y32 + 36 Y42 = 0
152) - I2F2 + I2F3 - VF13 - VF23 + 24 Y33 + 36 Y43 = 0
153) - I2F3 + I2F4 - VF14 - VF24 + 24 Y34 + 36 Y44 = 0
154) - I2G0 + I2G1 - VG11 - VG21 + 36 Y11 + 42 Y21 = 0
155) - I2G1 + I2G2 - VG12 - VG22 + 36 Y12 + 42 Y22 = 0
156) - I2G2 + I2G3 - VG13 - VG23 + 36 Y13 + 42 Y23 = 0
157) - I2G3 + I2G4 - VG14 - VG24 + 36 Y14 + 42 Y24 = 0
158) - I2H0 + I2H1 - VH11 - VH21 + 30 Y31 + 42 Y41 = 0
159) - I2H1 + I2H2 - VH12 - VH22 + 30 Y32 + 42 Y42 = 0
160) - I2H2 + I2H3 - VH13 - VH23 + 30 Y33 + 42 Y43 = 0
161) - I2H3 + I2H4 - VH14 - VH24 + 30 Y34 + 42 Y44 = 0
162) - I2J0 + I2J1 - VJ11 - VJ21 + 60 Y11 + 72 Y21 = 0
163) - I2J1 + I2J2 - VJ12 - VJ22 + 60 Y12 + 72 Y22 = 0
164) - I2J2 + I2J3 - VJ13 - VJ23 + 60 Y13 + 72 Y23 = 0
165) - I2J3 + I2J4 - VJ14 - VJ24 + 60 Y14 + 72 Y24 = 0
166) - I2K0 + I2K1 - VK11 - VK21 + 48 Y31 + 72 Y41 = 0
167) - I2K1 + I2K2 - VK12 - VK22 + 48 Y32 + 72 Y42 = 0
--More--

168) - I2K2 + I2K3 - VK13 - VK23 + 48 Y33 + 72 Y43 = 0
169) - I2K3 + I2K4 - VK14 - VK24 + 48 Y34 + 72 Y44 = 0
170) - I2L0 + I2L1 - VL11 - VL21 + Y11 + Y21 = 0
171) - I2L1 + I2L2 - VL12 - VL22 + Y12 + Y22 = 0
172) - I2L2 + I2L3 - VL13 - VL23 + Y13 + Y23 = 0
173) - I2L3 + I2L4 - VL14 - VL24 + Y14 + Y24 = 0
174) - I2M0 + I2M1 - VM11 - VM21 + Y31 + Y41 = 0
175) - I2M1 + I2M2 - VM12 - VM22 + Y32 + Y42 = 0
176) - I2M2 + I2M3 - VM13 - VM23 + Y33 + Y43 = 0
177) - I2M3 + I2M4 - VM14 - VM24 + Y34 + Y44 = 0
178) - I2N0 + I2N1 - VN11 - VN21 + Y11 = 0
179) - I2N1 + I2N2 - VN12 - VN22 + Y12 = 0
180) - I2N2 + I2N3 - VN13 - VN23 + Y13 = 0
181) - I2N3 + I2N4 - VN14 - VN24 + Y14 = 0
182) - I2O0 + I2O1 - VO11 - VO21 + Y21 = 0
183) - I2O1 + I2O2 - VO12 - VO22 + Y22 = 0
184) - I2O2 + I2O3 - VO13 - VO23 + Y23 = 0
185) - I2O3 + I2O4 - VO14 - VO24 + Y24 = 0
186) - I2P0 + I2P1 - VP11 - VP21 + Y31 = 0
187) - I2P1 + I2P2 - VP12 - VP22 + Y32 = 0
188) - I2P2 + I2P3 - VP13 - VP23 + Y33 = 0
189) - I2P3 + I2P4 - VP14 - VP24 + Y34 = 0
190) - I2Q0 + I2Q1 - VQ11 - VQ21 + Y41 = 0
191) - I2Q1 + I2Q2 - VQ12 - VQ22 + Y42 = 0

--More--

```
192) - I2Q2 + I2Q3 - VQ13 - VQ23 + Y43 =    0
193) - I2Q3 + I2Q4 - VQ14 - VQ24 + Y44 =    0
194) - Y11 - I310 + I311 + W111 + W121 + W131 =    0
195) - Y12 - I311 + I312 + W112 + W122 + W132 =    0
196) - Y13 - I312 + I313 + W113 + W123 + W133 =    0
197) - Y14 - I313 + I314 + W114 + W124 + W134 =    0
198) - Y21 - I320 + I321 + W211 + W221 + W231 =    0
199) - Y22 - I321 + I322 + W212 + W222 + W232 =    0
200) - Y23 - I322 + I323 + W213 + W223 + W233 =    0
201) - Y24 - I323 + I324 + W214 + W224 + W234 =    0
202) - Y31 - I330 + I331 + W311 + W321 + W331 =    0
203) - Y32 - I331 + I332 + W312 + W322 + W332 =    0
204) - Y33 - I332 + I333 + W313 + W323 + W333 =    0
205) - Y34 - I333 + I334 + W314 + W324 + W334 =    0
206) - Y41 - I340 + I341 + W411 + W421 + W431 =    0
207) - Y42 - I341 + I342 + W412 + W422 + W432 =    0
208) - Y43 - I342 + I343 + W413 + W423 + W433 =    0
209) - Y44 - I343 + I344 + W414 + W424 + W434 =    0
210) - XA11 - XA21 + Y11 <=   0
211) - XB11 - XB21 + Y21 <=   0
212) - XC11 - XC21 + Y31 <=   0
213) - XD11 - XD21 + Y41 <=   0
214) - XA12 - XA22 + Y12 <=   0
215) - XB12 - XB22 + Y22 <=   0
--More--
```

```
216) - XC12 - XC22 + Y32 <=   0
217) - XD12 - XD22 + Y42 <=   0
218) - XA13 - XA23 + Y13 <=   0
219) - XB13 - XB23 + Y23 <=   0
220) - XC13 - XC23 + Y33 <=   0
221) - XD13 - XD23 + Y43 <=   0
222) - XA14 - XA24 + Y14 <=   0
223) - XB14 - XB24 + Y24 <=   0
224) - XC14 - XC24 + Y34 <=   0
225) - XD14 - XD24 + Y44 <=   0
226) - XE11 - XE21 + 30 Y11 + 36 Y21 <=   0
227) - XE12 - XE22 + 30 Y12 + 36 Y22 <=   0
228) - XE13 - XE23 + 30 Y13 + 36 Y23 <=   0
229) - XE14 - XE24 + 30 Y14 + 36 Y24 <=   0
230) - XF11 - XF21 + 24 Y31 + 36 Y41 <=   0
231) - XF12 - XF22 + 24 Y32 + 36 Y42 <=   0
232) - XF13 - XF23 + 24 Y33 + 36 Y43 <=   0
233) - XF14 - XF24 + 24 Y34 + 36 Y44 <=   0
234) - XG11 - XG21 + 36 Y11 + 42 Y21 <=   0
235) - XG12 - XG22 + 36 Y12 + 42 Y22 <=   0
236) - XG13 - XG23 + 36 Y13 + 42 Y23 <=   0
237) - XG14 - XG24 + 36 Y14 + 42 Y24 <=   0
238) - XH11 - XH21 + 30 Y31 + 42 Y41 <=   0
239) - XH12 - XH22 + 30 Y32 + 42 Y42 <=   0
--More--
```

```
240) - XH13 - XH23 + 30 Y33 + 42 Y43 <=  0
241) - XH14 - XH24 + 30 Y34 + 42 Y44 <=  0
242) - XJ11 - XJ21 + 60 Y11 + 72 Y21 <=  0
243) - XJ12 - XJ22 + 60 Y12 + 72 Y22 <=  0
244) - XJ13 - XJ23 + 60 Y13 + 72 Y23 <=  0
245) - XJ14 - XJ24 + 60 Y14 + 72 Y24 <=  0
246) - XK11 - XK21 + 48 Y31 + 72 Y41 <=  0
247) - XK12 - XK22 + 48 Y32 + 72 Y42 <=  0
248) - XK13 - XK23 + 48 Y33 + 72 Y43 <=  0
249) - XK14 - XK24 + 48 Y34 + 72 Y44 <=  0
250) - XL11 - XL21 + Y11 + Y21 <=  0
251) - XL12 - XL22 + Y12 + Y22 <=  0
252) - XL13 - XL23 + Y13 + Y23 <=  0
253) - XL14 - XL24 + Y14 + Y24 <=  0
254) - XM11 - XM21 + Y31 + Y41 <=  0
255) - XM12 - XM22 + Y32 + Y42 <=  0
256) - XM13 - XM23 + Y33 + Y43 <=  0
257) - XM14 - XM24 + Y34 + Y44 <=  0
258) - XN11 - XN21 + Y11 <=  0
259) - XN12 - XN22 + Y12 <=  0
260) - XN13 - XN23 + Y13 <=  0
261) - XN14 - XN24 + Y14 <=  0
262) - XO11 - XO21 + Y21 <=  0
263) - XO12 - XO22 + Y22 <=  0
--More--
```

```
264) - XO13 - XO23 + Y23 <=   0
265) - XO14 - XO24 + Y24 <=   0
266) - XP11 - XP21 + Y31 <=   0
267) - XP12 - XP22 + Y32 <=   0
268) - XP13 - XP23 + Y33 <=   0
269) - XP14 - XP24 + Y34 <=   0
270) - XQ11 - XQ21 + Y41 <=   0
271) - XQ12 - XQ22 + Y42 <=   0
272) - XQ13 - XQ23 + Y43 <=   0
273) - XQ14 - XQ24 + Y44 <=   0
274)   XA11 <=   100000
275)   XA12 <=   100000
276)   XA13 <=   100000
277)   XA14 <=   100000
278)   XA21 <=   100000
279)   XA22 <=   100000
280)   XA23 <=   100000
281)   XA24 <=   100000
282)   XB11 <=   100000
283)   XB12 <=   100000
284)   XB13 <=   100000
285)   XB14.<=   100000
286)   XB21 <=   100000
287)   XB22 <=   100000
--More--
```

```
288)  XB23 <=   100000
289)  XB24 <=   100000
290)  XC11 <=   100000
291)  XC12 <=   100000
292)  XC13 <=   100000
293)  XC14 <=   100000
294)  XC21 <=   100000
295)  XC22 <=   100000
296)  XC23 <=   100000
297)  XC24 <=   100000
298)  XD11 <=   100000
299)  XD12 <=   100000
300)  XD13 <=   100000
301)  XD14 <=   100000
302)  XD21 <=   100000
303)  XD22 <=   100000
304)  XD23 <=   100000
305)  XD24 <=   100000
306)  XE11 <=   30000
307)  XE12 <=   30000
308)  XE13 <=   30000
309)  XE14 <=   30000
310)  XE21 <=   27600
311)  XE22 <=   27600
--More--
```

```
312)   XE23 <=   27600
313)   XE24 <=   27600
314)   XF11 <=   24800
315)   XF12 <=   24800
316)   XF13 <=   24800
317)   XF14 <=   24800
318)   XF21 <=   40000
319)   XF22 <=   40000
320)   XF23 <=   40000
321)   XF24 <=   40000
322)   XG11 <=   30000
323)   XG12 <=   30000
324)   XG13 <=   30000
325)   XG14 <=   30000
326)   XG21 <=   32400
327)   XG22 <=   32400
328)   XG23 <=   32400
329)   XG24 <=   32400
330)   XH11 <=   33000
331)   XH12 <=   33000
332)   XH13 <=   33000
333)   XH14 <=   33000
334)   XH21 <=   36120
335)   XH22 <=   36120
--More--
```

336) XH23 <= 36120
337) XH24 <= 36120
338) XJ11 <= 100000
339) XJ12 <= 100000
340) XJ13 <= 100000
341) XJ14 <= 100000
342) XJ21 <= 100000
343) XJ22 <= 100000
344) XJ23 <= 100000
345) XJ24 <= 100000
346) XK11 <= 100000
347) XK12 <= 100000
348) XK13 <= 100000
349) XK14 <= 100000
350) XK21 <= 100000
351) XK22 <= 100000
352) XK23 <= 100000
353) XK24 <= 100000
354) XL11 <= 1800
355) XL12 <= 1800
356) XL13 <= 1800
357) XL14 <= 1800
358) XL21 <= 1800
359) XL22 <= 1800
--More--

```
360)  XL23 <=   1800
361)  XL24 <=   1800
362)  XM11 <=   1800
363)  XM12 <=   1800
364)  XM13 <=   1800
365)  XM14 <=   1800
366)  XM21 <=   1800
367)  XM22 <=   1800
368)  XM23 <=   1800
369)  XM24 <=   1800
370)  XN11 <=   100000
371)  XN12 <=   100000
372)  XN13 <=   100000
373)  XN14 <=   100000
374)  XN21 <=   100000
375)  XN22 <=   100000
376)  XN23 <=   100000
377)  XN24 <=   100000
378)  XO11 <=   100000
379)  XO12 <=   100000
380)  XO13 <=   100000
381)  XO14 <=   100000
382)  XO21 <=   100000
383)  XO22 <=   100000
--More--
```

```
384)  XO23 <=   100000
385)  XO24 <=   100000
386)  XP11 <=   100000
387)  XP12 <=   100000
388)  XP13 <=   100000
389)  XP14 <=   100000
390)  XP21 <=   100000
391)  XP22 <=   100000
392)  XP23 <=   100000
393)  XP24 <=   100000
394)  XQ11 <=   100000
395)  XQ12 <=   100000
396)  XQ13 <=   100000
397)  XQ14 <=   100000
398)  XQ21 <=   100000
399)  XQ22 <=   100000
400)  XQ23 <=   100000
401)  XQ24 <=   100000
402)  Y11 <=   324
403)  Y12 <=   324
404)  Y13 <=   324
405)  Y14 <=   324
406)  Y21 <=   450
407)  Y22 <=   450
--More--
```

```
 408)  Y23 <=  450
 409)  Y24 <=  450
 410)  Y31 <=  150
 411)  Y32 <=  150
 412)  Y33 <=  150
 413)  Y34 <=  150
 414)  Y41 <=  300
 415)  Y42 <=  300
 416)  Y43 <=  300
 417)  Y44 <=  300
 418)  Y11 + Y21 + Y31 + Y41 <=  576
 419)  Y12 + Y22 + Y32 + Y42 <=  576
 420)  Y13 + Y23 + Y33 + Y43 <=  576
 421)  Y14 + Y24 + Y34 + Y44 <=  576
 422)  I310 <=  20
 423)  I320 <=  18
 424)  I340 <=  15
 425)  I4110 <=  5
 426)  I4210 <=  3
 427)  I4310 <=  6
 428)  I4410 <=  6
 429)  I4120 <=  4
 430)  I4220 <=  4
 431)  I4320 <=  6
--More--
```

```
432)  I4420 <=   6
433)  I4130 <=   4
434)  I4230 <=   4
435)  I4330 <=   6
436)  I4430 <=   6
437)  I330 <=   10
438)  W111 + I4110 - I4111 >=   50
439)  W121 + I4120 - I4121 >=   15
440)  W131 + I4130 - I4131 >=   20
441)  W112 + I4111 - I4112 >=   65
442)  W122 + I4121 - I4122 >=   15
443)  W132 + I4131 - I4132 >=   0
444)  W113 + I4112 - I4113 >=   80
445)  W123 + I4122 - I4123 >=   15
446)  W133 + I4132 - I4133 >=   10
447)  W114 + I4113 - I4114 >=   55
448)  W124 + I4123 - I4124 >=   15
449)  W134 + I4133 - I4134 >=   20
450)  W211 + I4210 - I4211 >=   60
451)  W221 + I4220 - I4221 >=   20
452)  W231 + I4230 - I4231 >=   50
453)  W212 + I4211 - I4212 >=   75
454)  W222 + I4221 - I4222 >=   20
455)  W232 + I4231 - I4232 >=   20
--More--
```

- - - - -

456) W213 + I4212 - I4213 >= 90
457) W223 + I4222 - I4223 >= 20
458) W233 + I4232 - I4233 >= 10
459) W214 + I4213 - I4214 >= 75
460) W224 + I4223 - I4224 >= 40
461) W234 + I4233 - I4234 >= 20
462) W311 + I4310 - I4311 >= 20
463) W321 + I4320 - I4321 >= 5
464) W331 + I4330 - I4331 >= 25
465) W312 + I4311 - I4312 >= 25
466) W322 + I4321 - I4322 >= 5
467) W332 + I4331 - I4332 >= 10
468) W313 + I4312 - I4313 >= 35
469) W323 + I4322 - I4323 >= 5
470) W333 + I4332 - I4333 >= 10
471) W314 + I4313 - I4314 >= 20
472) W324 + I4323 - I4324 >= 5
473) W334 + I4333 - I4334 >= 10
474) W411 + I4410 - I4411 >= 35
475) W421 + I4420 - I4421 >= 5
476) W431 + I4430 - I4431 >= 30
477) W412 + I4411 - I4412 >= 45
478) W422 + I4421 - I4422 >= 5
479) W432 + I4431 - I4432 >= 20

--More--

```
    465)  W312 + I4311 - I4312 >=  25
    466)  W322 + I4321 - I4322 >=  5
    467)  W332 + I4331 - I4332 >=  10
    468)  W313 + I4312 - I4313 >=  35
    469)  W323 + I4322 - I4323 >=  5
    470)  W333 + I4332 - I4333 >=  10
    471)  W314 + I4313 - I4314 >=  20
    472)  W324 + I4323 - I4324 >=  5
    473)  W334 + I4333 - I4334 >=  10
    474)  W411 + I4410 - I4411 >=  35
    475)  W421 + I4420 - I4421 >=  5
    476)  W431 + I4430 - I4431 >=  30
    477)  W412 + I4411 - I4412 >=  45
    478)  W422 + I4421 - I4422 >=  5
    479)  W432 + I4431 - I4432 >=  20
--More--
    480)  W413 + I4412 - I4413 >=  45
    481)  W423 + I4422 - I4423 >=  5
    482)  W433 + I4432 - I4433 >=  20
    483)  W414 + I4413 - I4414 >=  55
    484)  W424 + I4423 - I4424 >=  0
    485)  W434 + I4433 - I4434 >=  30
END
```

OBJECTIVE FUNCTION VALUE

1) 1343088.00

VARIABLE	VALUE	REDUCED COST
XA11	.000000	1.500000
XB11	101.000000	.000000
XC11	23.000000	.000000
XD11	.000000	1.000000
XA21	52.000000	.000000
XB21	.000000	.000000
XC21	.000000	1.500000
XD21	38.000000	.000000
XE11	5196.000000	.000000
XF11	.000000	.300000
XE21	.000000	.400000
XF21	1920.000000	.000000
XG11	6114.000000	.000000
XH11	.000000	.500000
XG21	.000000	.500000
XH21	2286.000000	.000000
XJ11	10392.000000	.000000
XK11	.000000	.000000
XJ21	.000000	.200000

--More--

XK21	3840.000000	.000000
XL11	153.000000	.000000
XM11	.000000	.200001
XL21	.000000	.000000
XM21	61.000000	.000000
XN11	.000000	.700001
XO11	101.000000	.000000
XP11	23.000000	.000000
XQ11	.000000	.000000
XN21	52.000000	.000000
XO21	.000000	1.099998
XP21	.000000	.699999
XQ21	38.000000	.000000
XA12	.000000	1.500000
XB12	115.000000	.000000
XC12	39.000000	000000
XD12	.000000	1.000000
XA22	80.000000	.000000
XB22	.000000	2.000000
XC22	.000000	1.500000
XD22	69.000000	.000000
XE12	6540.000000	.000000
XF12	.000000	.950000
XE22	.000000	1.200000

--More--

XF22	3420 000000	.000000
XG12	7710.000000	.000000
XH12	.000000	.000000
XG22	.000000	.800000
XH22	4068.000000	.000000
XJ12	13080.000000	.000000
XK12	.000000	.000000
XJ22	.000000	000000
XK22	6840.000000	.000000
XL12	195.000000	.000000
XM12	.000000	.000000
XL22	.000000	.309999
XM22	108.000000	.000000
XN12	.000000	700001
XO12	115.000000	.000000
XP12	39 000000	000000
XQ12	.000000	1.900000
XN22	80.000000	.000000
XO22	.000000	1.099998
XP22	.000000	1.099998
XQ22	69.000000	.000000
XA13	.000000	.000000
XB13	120.000000	.000000
XC13	50.000000	.000000

--More--

XD13	.000000	1.000000
XA23	105.000000	.000000
XB23	.000000	.000000
XC23	.000000	1.500000
XD23	70.000000	.000000
XE13	7470.000000	.000000
XF13	.000000	.950000
XE23	.000000	1.200000
XF23	3720.000000	.000000
XG13	8820.000000	.000000
XH13	.000000	.849999
XG23	.000000	.400000
XH23	4440.000000	.000000
XJ13	14940.000000	.000000
XK13	.000000	.000000
XJ23	.000000	.200000
XK23	7440.000000	.000000
XL13	225.000000	.000000
XM13	.000000	.200001
XL23	.000000	.309999
XM23	120.000000	.000000
XN13	.000000	.000000
XO13	120.000000	.000000
XP13	50.000000	.000000

--More--

XQ13	.000000	1.900000
XN23	105.000000	.000000
XO23	.000000	1.099998
XP23	.000000	1.099998
XQ23	70.000000	.000000
XA14	.000000	.000000
XB14	135.000000	.000000
XC14	-35.000000	.000000
XD14	.000000	1.000000
XA24	90.000000	.000000
XB24	.000000	.000000
XC24	.000000	1.500000
XD24	85.000000	.000000
XE14	7560.000000	.000000
XF14	000000	.950000
XE24	.000000	1.200000
XF24	3900.000000	.000000
XG14	8910.000000	.000000
XH14	.000000	.849999
XG24	.000000	.000000
XH24	4620.000000	.000000
XJ14	15120.000000	.000000
XK14	.000000	.000000
XJ24	.000000	.200000

--More--

I1F20	.000000	.600000
I1G10	.000000	.300000
I1H10	.000000	.300000
I1G20	.000000	.000000
I1H20	.000000	.800000
I1J10	.000000	.000000
I1K10	.000000	.000000
I1J20	.000000	.020000
I1K20	-.000000	.100000
I1L10	.000000	.870000
I1M10	.000000	.950000
I1L20	..000000	.550000
I1M20	.000000	.850000
I1N10	.000000	.000000
I1O10	.000000	.200000
I1P10	.000000	.200000
I1Q10	.000000	.000000
I1N20	.000000	.400000
I1O20	.000000	.400000
I1P20	.000000	.000000
I1Q20	000000	2.100000
I1A11	.000000	3.000000
I1B11	.000000	2 999999
I1C11	.000000	5.500000

--More--

I1D11	.000000	5.000000
I1A21	.000000	3.000000
I1B21	.000000	4 000000
I1C21	.000000	6.000000
I1D21	.000000	6.000000
I1E11	.000000	.200000
I1F11	.000000	1.150000
I1E21	.000000	1.300000
I1F21	.000000	.400000
I1G11	.000000	5.200000
I1H11	.000000	.000000
I1G21	.000000	5.700000
I1H21	.000000	500000
I1J11	.000000	.180000
I1K11	.000000	.000000
I1J21	.000000	.000000
I1K21	.000000	.000000
I1L11	.000000	.820000
I1M11	000000	.599999
I1L21	.000000	1 110000
I1M21	.000000	.700000
I1N11	.000000	.000000
I1O11	.000000	000000
I1P11	.000000	.000000

--More--

VA11	.000000	.000000
VB11	101.000000	.000000
VC11	23.000000	.000000
VD11	.000000	.000000
VA21	52 000000	.000000
VB21	.000000	3.800003
VC21	.000000	.000000
VD21	38 000000	.000000
VE11	5196.000000	.000000
VF11	.000000	.650000
VE21	.000000	.800000

--More--

VF21	1920.000000	000000
VG11	6114.000000	.000000
VH11	.000000	.350000
VG21	.000000	.600000
VH21	2286.000000	.000000
VJ11	10392.000000	.000000
VK11	.000000	.100000
VJ21	.000000	.000000
VK21	3840.000000	.000000
VL11	153.000000	.000000
VM11	.000000	.000000
VL21	.000000	.310000
VM21	61.000000	.000000
VN11	.000000	.200000
VO11	101.000000	.000000
VP11	23.000000	.000000
VQ11	.000000	1.900000
VN21	52.000000	000000
VO21	.000000	.000000
VP21	.000000	.400000
VQ21	38.000000	.000000
VA12	.000000	.000000
VB12	115.000000	.000000
VC12	39.000000	.000000

--More--

VD12	.000000	.000000
VA22	80.000000	.000000
VB22	.000000	1.800003
VC22	.000000	.000000
VD22	69.000000	.000000
VE12	6540.000000	.000000
VF12	000000	.000000
VE22	.000000	.000000
VF22	3420.000000	.000000
VG12	7710.000000	.000000
VH12	.000000	.850000
VG22	.000000	.300000
VH22	4068.000000	.000000
VJ12	13080.000000	.000000
VK12	000000	.100000
VJ22	.000000	.200000
VK22	6840.000000	.000000
VL12	195.000000	.000000
VM12	.000000	.200001
VL22	.000000	.000000
VM22	108.000000	.000000
VN12	.000000	200000
VO12	115.000000	.000000
VP12	39.000000	.000000

--More--

VQ12	.000000	.000000
VN22	80.000000	.000000
VO22	.000000	.000000
VP22	.000000	.000000
VQ22	69.000000	.000000
VA13	.000000	1.500000
VB13	120.000000	.000000
VC13	50.000000	.000000
VD13	.000000	.000000
VA23	105.000000	.000000
VB23	.000000	3.800003
VC23	.000000	.000000
VD23	70.000000	.000000
VE13	7470.000000	.000000
VF13	.000000	.000000
VE23	.000000	.000000
VF23	3720 000000	000000
VG13	8820.000000	000000
VH13	.000000	.000000
VG23	.000000	.700000
VH23	4440.000000	.000000
VJ13	14940.000000	.000000
VK13	.000000	.100000
VJ23	.000000	.000000

VK23	7440.000000	000000
VL13	225.000000	.000000
VM13	.000000	.000000
VL23	.000000	.000000
VM23	120.000000	000000
VN13	.000000	.900000
VO13	120.000000	.000000
VP13	50.000000	.000000
VQ13	.000000	000000
VN23	105.000000	.000000
VO23	.000000	.000000
VP23	-.000000	.000000
VQ23	70.000000	.000000
VA14	.000000	1.500000
VB14	135.000000	.000000
VC14.	35.000000	.000000
VD14	.000000	.000000
VA24	90.000000	.000000
VB24	.000000	3.800003
VC24	.000000	.000000
VD24	85.000000	.000000
VE14	7560.000000	000000
VF14	.000000	.000000
VE24	.000000	.000000

--More--

VF24	3900.000000	000000
VG14	8910.000000	.000000
VH14	.000000	.000000
VG24	.000000	1.100000
VH24	4620.000000	.000000
VJ14	15120.000000	.000000
VK14	.000000	.100000
VJ24	.000000	.000000
VK24	7800.000000	.000000
VL14	225.000000	.000000
VM14	.000000	.000000
VL24	.000000	.309999
VM24	120.000000	.000000
VN14	.000000	.900000
VO14	135.000000	.000000
VP14	35.000000	.000000
VQ14	.000000	.000000
VN24	90.000000	.000000
VO24	.000000	000000
VP24	.000000	.000000
VQ24	85.000000	.000000
Y11	52.000000	.000000
Y21	101.000000	.000000
Y31	23.000000	.000000

--More--

Y41	38.000000	.000000
Y12	80.000000	.000000
Y22	115.000000	.000000
Y32	39.000000	.000000
Y42	69.000000	.000000
Y13	105.000000	.000000
Y23	120.000000	.000000
Y33	50.000000	.000000
Y43	70.000000	.000000
Y14	90.000000	.000000
Y24	135.000000	.000000
Y34	35.000000	.000000
Y44	85.000000	.000000

[illegible]	[illegible]	[illegible]
W134	20.000000	.000000
W234	20.000000	.000000
W334	10.000000	.000000
W434	30.000000	.000000
I4110	5.000000	.000000
I4210	3.000000	.000000
I4310	6.000000	000000
I4410	6.000000	.000000
I4120	4.000000	.000000
I4220	4.000000	.000000
I4320	6.000000	.000000
I4420	6.000000	.000000
I4130	4.000000	.000000
I4230	4.000000	.000000
I4330	6.000000	.000000
I4430	6.000000	.000000
I4111	000000	3.000000
I4211	.000000	3.000000
I4311	.000000	3.999878

--More--

I4413	.000000	4 000000
I4123	.000000	3 000000
I4223	.000000	3.000000
I4323	.000000	4.000000
I4423	.000000	1396.270000
I4133	.000000	2.000000
I4233	000000	2.000000
I4333	.000000	3 000000
I4433	.000000	3.000000
I4114	.000000	942.250000
I4214	.000000	1100.770000
I4314	.000000	1037.330000
I4414	.000000	1396.270000
I4124	.000000	942.250000
I4224	.000000	1100.770000
I4324	.000000	1037.330000
I4424	.000000	4.000000
I4134	.000000	947.250000
I4234	.000000	1105.770000
I4334	.000000	1042.330000
I4434	.000000	1401.270000

2)	.000000	6.000000
3)	.000000	8.500000
4)	.000000	6.800000
5)	.000000	13.500000
6)	.000000	5.500000
7)	.000000	9.800003
8)	.000000	8.000000
9)	.000000	13.500000
10)	.000000	.900000
11)	.000000	1.500000
12)	.000000	1 500000
13)	.000000	.800000
14)	.000000	.900000
15)	.000000	1.200000
16)	.000000	1.400000
17)	.000000	.700000
18)	.000000	.080000
19)	.000000	.100000
20)	.000000	.080000
21)	.000000	.000000
22)	.000000	.950000
23)	.000000	.850000
24)	.000000	1.250000
25)	.000000	.850000

---More---

ROW	SLACK OR SURPLUS	DUAL PRICES

--More--

2)	.000000	6.000000
3)	.000000	8.500000
4)	.000000	6.800000
5)	.000000	13.500000
6)	.000000	5.500000
7)	.000000	9.800003
8)	.000000	8.000000
9)	.000000	13.500000
10)	.000000	.900000
11)	.000000	1.500000
12)	.000000	1.500000
13)	000000	.800000
14)	.000000	.900000
15)	.000000	1.200000
16)	.000000	1.400000
17)	.000000	.700000
18)	.000000	.080000
19)	.000000	100000
20)	.000000	.080000
21)	.000000	.000000
22)	.000000	.950000
23)	.000000	.850000
24)	.000000	1.250000
25)	.000000	.850000

--More--

26)	.000000	500000
27)	.000000	.300000
28)	.000000	.300000
29)	.000000	.500000
30)	.000000	.200000
31)	.000000	.200000
32)	.000000	.600000
33)	.000000	-1.500000
34)	.000000	13.000000
35)	.000000	14.800000
36)	.000000	18.500000
37)	.000000	28.500000
38)	000000	12 500000
39)	.000000	15.800000
40)	.000000	18.000000
41)	.000000	28.500000
42)	.000000	1.900000
43)	.000000	1.850000
44)	.000000	1.700000
45)	.000000	1.800000
46)	.000000	-3.100000
47)	.000000	2.700000
48)	.000000	-2.900000
49)	.000000	1.700000

--More--

50)	.000000	-.020000
51)	.000000	.200000
52)	.000000	.180000
53)	.000000	.100000
54)	.000000	1.950000
55)	.000000	2.050001
56)	.000000	1.940000
57)	.000000	1.850000
58)	.000000	1.000000
59)	.000000	.800000
60)	.000000	.800000
61)	.000000	-.900000
62)	.000000	.700000
63)	.000000	700000

468)	.000000	-1033.330000
469)	.000000	-1033.330000
470)	.000000	-1039.330000
471)	000000	-1033.330000
472)	.000000	-1033.330000
473)	.000000	-1039.330000
474)	.000000	-1392.270000
475)	.000000	-1388.270000
476)	.000000	-1398.270000
477)	.000000	-1392.270000
478)	.000000	-1392.270000
479)	.000000	-1398.270000
480)	.000000	-1392.270000
481)	.000000	-1392.270000

--More--

482)	.000000	-1398.270000
483)	.000000	-1392.270000
484)	.000000	.000000
485)	.000000	-1398.270000

NO. ITERATIONS= 519

DO RANGE(SENSITIVITY) ANALYSIS?
?

RANGES IN WHICH THE BASIS IS UNCHANGED:

OBJ COEFFICIENT RANGES

VARIABLE	CURRENT COEF	ALLOWABLE INCREASE	ALLOWABLE DECREASE
XA11	110.000000	INFINITY	1.500000
XB11	122.000000	3.800003	.199997
XC11	179.000000	1.500000	.999878
XD11	185.000000	INFINITY	1.000000
XA21	108.000000	1.500000	1 000000
XB21	125.000000	.199997	3.800003
XC21	180.000000	INFINITY	1.500000
XD21	184.000000	1.000000	1.000000
XE11	12.000000	.400000	027778
XF11	15.000000	INFINITY	.300000
XE21	13.000000	INFINITY	.400000
XF21	14.000000	.300000	.027778
XG11	10.000000	.500000	.023810
XH11	14.000000	INFINITY	.500000
XG21	11.000000	INFINITY	.500000
XH21	13.000000	.500000	.023810
XJ11	.800000	.200000	.013889
XK11	1.000000	INFINITY	000000
XJ21	1.000000	INFINITY	.200000

--More--

VARIABLE	CURRENT COEF	ALLOWABLE INCREASE	ALLOWABLE DECREASE
XK21	.900000	.000000	.013889
XL11	18.200000	.310000	.550000
XM11	24.200000	INFINITY	.200001
XL21	18.500000	.550000	.310000
XM21	24.000000	.200001	.999878
XN11	23.000000	INFINITY	.700001
XO11	24.000000	1.099998	1.000000
XP11	26.000000	.699999	.999878
XQ11	30.000000	.600000	1.700000
XN21	22.000000	.700001	1.000000
XO21	25.000000	INFINITY	1.099998
XP21	27.000000	INFINITY	.699999
XQ21	28.000000	1.700000	.600000
XA12	110.000000	INFINITY	1 500000
XB12	122.000000	1.000000	1.000000
XC12	179.000000	.999878	1.000122
XD12	185.000000	INFINITY	1.000000

XD22	184.000000	1.000000	1.000000
XE12	12.000000	.027778	.027778
XF12	15.000000	INFINITY	.950000
XE22	13.000000	INFINITY	1.200000

--More--

XF22	14.000000	.027778	.027778
XG12	10.000000	.023810	.023810
XH12	14.000000	.300000	.350000
XG22	11.000000	INFINITY	.800000
XH22	13.000000	.023810	.023810
XJ12	.800000	.013889	.013889
XK12	1.000000	INFINITY	.000000
XJ22	1.000000	220000	.180000
XK22	.900000	.000000	.013889
XL12	18.200000	.309999	1.000000
XM12	24.200000	.599999	200001
XL22	18.500000	INFINITY	.309999
XM22	24.000000	.200001	.599999
XN12	23.000000	INFINITY	.700001
XO12	24.000000	1.000000	1.000000
XP12	26.000000	.999878	1.000122
XQ12	30.000000	INFINITY	1.900000
XN22	22.000000	.700001	1.000061
XO22	25.000000	INFINITY	1.099998
XP22	27.000000	INFINITY	1.099998
XQ22	28.000000	1.000000	1.000000
XA13	110.000000	1.500000	1.500000
XB13	122.000000	1.000000	1.000000
XC13	179.000000	1.000122	1.000000

--More--

XD13	185.000000	INFINITY	1.000000
XA23	108.000000	1.000061	1.000000
XB23	125.000000	2.000000	1.800003
XC23	180.000000	INFINITY	1.500000
XD23	184.000000	1.000000	1.000000
XE13	12.000000	.027778	.027778
XF13	15.000000	INFINITY	.950000
XE23	13.000000	INFINITY	1.200000
XF23	14.000000	.027778	.027778
XG13	10.000000	.023810	.023810
XH13	14.000000	INFINITY	.849999
XG23	11.000000	INFINITY	.400000
XH23	13.000000	.023810	.023810
XJ13	.800000	.013889	.013889
XK13	1.000000	INFINITY	.000000
XJ23	1.000000	INFINITY	.200000
XK23	.900000	.000000	.013889
XL13	18.200000	.309999	1.000000
XM13	24.200000	INFINITY	.200001
XL23	18.500000	INFINITY	.309999
XM23	24.000000	.200001	1.000000
XN13	23.000000	INFINITY	.000000
XO13	24.000000	1.000000	1.000000
XP13	26.000000	1.000122	1.000000

--More--

XQ13	30.000000	INFINITY	1.900000
XN23	22.000000	.000000	1.000000
XO23	25.000000	INFINITY	1.099998
XP23	27.000000	INFINITY	1.099998
XQ23	28.000000	1.000000	1.000000
XA14	110.000000	3.000000	1.500000
XB14	122.000000	1.000000	2.000000
XC14	179.000000	1.000000	140.500000
XD14	185.000000	INFINITY	1.000000
XA24	108.000000	1.000000	3.000000

[illegible]	[illegible]	[illegible]	[illegible]
XE14	12.000000	.027778	8.100000
XF14	15.000000	INFINITY	.950000
XE24	13.000000	INFINITY	1.200000
XF24	14.000000	.027778	10.200000
XG14	10.000000	.023810	400000
XH14	14.000000	INFINITY	.849999
XG24	11.000000	.400000	.300000
XH24	13.000000	.023810	9.300000
XJ14	.800000	.013889	.880000
XK14	1.000000	.000000	.090000
XJ24	1.000000	INFINITY	.200000

--More--

XK24	.900000	.013889	.000000
XL14	18 200000	.309999	.490001
XM14	24.200000	INFINITY	.200001
XL24	18.500000	.490001	.309999
XM24	24.000000	.200001	20.150000
XN14	23.000000	.000000	.700000
XO14	24.000000	1.000000	22.200000
XP14	26.000000	1.000000	24.200000
XQ14	30.000000	INFINITY	1.900000
XN24	22.000000	.700000	.000000
XO24	25.000000	INFINITY	1.099998
XP24	27.000000	INFINITY	1.099998
XQ24	28.000000	1.000000	28.000000
I1A10	10.000000	INFINITY	4.000000
I1B10	11.000000	INFINITY	4.200000
I1C10	15 500000	INFINITY	7 000000
I1D10	20.000000	INFINITY	6 500000
I1A20	10.000000	INFINITY	4.500000
I1B20	10 000000	INFINITY	.199997
I1C20	16.000000	INFINITY	8.000000
I1D20	21.000000	INFINITY	7.500000
I1E10	1.200000	INFINITY	.300000
I1F10	1.500000	.300000	.650000
I1E20	1.500000	.400000	.800000

--More--

I1F20	1.400000	INFINITY	.600000
I1G10	1.200000	INFINITY	.300000
I1H10	1.500000	INFINITY	.300000
I1G20	1.400000	.500000	.600000
I1H20	1.500000	INFINITY	.800000
I1J10	.080000	.010000	.220000
I1K10	.100000	.090000	.400000
I1J20	.100000	INFINITY	.020000
I1K20	.100000	INFINITY	.100000
I1L10	1.820000	INFINITY	.870000
I1M10	1.800000	INFINITY	.950000
I1L20	1.800000	INFINITY	.550000
I1M20	1.700000	INFINITY	.850000
I1N10	.500000	700000	.200000
I1O10	.500000	INFINITY	.200000
I1P10	.500000	INFINITY	200000
I1Q10	.500000	1.700000	600000
I1N20	.600000	INFINITY	.400000
I1O20	.600000	INFINITY	.400000
I1P20	.600000	.699999	.400000
I1Q20	.600000	INFINITY	2.100000
I1A11	10.000000	INFINITY	3.000000
I1B11	11.000000	INFINITY	2.999999
I1C11	15.500000	INFINITY	5.500000

--More--

I1D11	20.000000	INFINITY	5 000000
I1A21	10.000000	INFINITY	3.000000
I1B21	10.000000	INFINITY	4.000000

I1H21	1.500000	INFINITY	.500000
I1J11	.080000	INFINITY	.180000
I1K11	.100000	.090000	.000000
I1J21	.100000	.180000	.220000
I1K21	.100000	.000000	.090000
I1L11	1.820000	INFINITY	.820000
I1M11	1.800000	INFINITY	.599999
I1L21	1.800000	INFINITY	1.110000
I1M21	1.700000	INFINITY	.700000
I1N11	.500000	.700000	.200000
I1O11	.500000	INFINITY	.000000
I1P11	.500000	.000000	2.300000

--More--

I1Q11	.500000	INFINITY	1.900000
I1N21	.600000	INFINITY	.100000
I1O21	.600000	INFINITY	.100000
I1P21	.600000	INFINITY	.500000
I1Q21	.600000	INFINITY	.100000
I1A12	10.000000	INFINITY	1.500000
I1B12	11.000000	INFINITY	3.000000
I1C12	15.500000	INFINITY	5.500000
I1D12	20.000000	INFINITY	5.000000
I1A22	10.000000	INFINITY	3.000000
I1B22	10.000000	1.800003	2.000000
I1C22	16.000000	INFINITY	6.000000
I1D22	21.000000	INFINITY	6.000000
I1E12	1.200000	INFINITY	.200000
I1F12	1.500000	INFINITY	.500000
I1E22	1.500000	INFINITY	.500000
I1F22	1.400000	INFINITY	.400000
I1G12	1.200000	INFINITY	.200000
I1H12	1.500000	INFINITY	1.350000
I1G22	1.400000	.300000	800000
I1H22	1.500000	INFINITY	.500000
I1J12	.080000	INFINITY	.220000
I1K12	.100000	.090000	.000000
I1J22	.100000	INFINITY	.440000

--More--

I1K22	.100000	.000000	.090000
I1L12	1.820000	INFINITY	.820000
I1M12	1.800000	INFINITY	1.000001
I1L22	1.800000	INFINITY	.800000
I1M22	1.700000	INFINITY	.700000
I1N12	.500000	.700000	1.600000
I1O12	.500000	.000000	2.300000
I1P12	.500000	.000000	2.300000
I1Q12	.500000	.000000	1.900000
I1N22	.600000	INFINITY	.800000
I1O22	.600000	INFINITY	.100000
I1P22	.600000	INFINITY	.100000
I1Q22	.600000	INFINITY	.100000
I1A13	10.000000	INFINITY	3.000000
I1B13	11.000000	INFINITY	3.000000
I1C13	15.500000	INFINITY	5.500000
I1D13	20.000000	INFINITY	5.000000
I1A23	10.000000	INFINITY	3.000000
I1B23	10.000000	INFINITY	2.000000
I1C23	16.000000	INFINITY	6.000000
I1D23	21.000000	INFINITY	6.000000
I1E13	1.200000	INFINITY	.200000
I1F13	1.500000	INFINITY	.500000
I1E23	1.500000	INFINITY	.500000

--More--

I1F23	1.400000	INFINITY	.400000
I1G13	1.200000	INFINITY	.200000
I1H13	1.500000	INFINITY	.500000

References

1. Oliver W. Wight, Production and Inventory Management in the Computer Age, Cahners Books, 1974.

2. Roundy, R., " 98% Effective Integer- Ratio Lot sizing for One warehouse Multi-Retailer Systems." Management science, 31(1985).

3. Maxwell, W. L and J. A. Muckstadt, "Establishing Consistent and Realistic Reorder Intervals in Production-Distribution Systems." Operations res. 33(1985)

4. Clark, A. J and H. Scarf, "Optimal policies for a multi-echelon Inventory Problem." Management science, 6(1960).

5. Federgruen, A. and P. Zipkin, " Approximations of Dynamic Multilocation Production and Inventory Problems." Management Science 30 (1984)

6. De Bodt, M. A. and S. C. Graves, " Continuous Review Policies for a Multi-Echelon Inventory Problem with stochastic demand" Management Science 31(1985).

7. Zangwill. W., "A Backlogging model and Multi-Echelon model of a Dynamic Economic lot-size Production system- A Network Approach" Management science (May 1969).

8. Love. S., "A facilities in series Inventory model with Nested Schedules" Management science (January 1972, part-1)

9. Wagner.M., and Crowsten. W., "Dynamic lot-size models for multi- stage assembly systems" Management science(September 1973)

10. Wagner. M. and Williams. J., "Economic lot-size Determination in Multi-stage Assembly systems." Management science January, 1973.

11 Johnson. L. A. and Montgomery. D.C., Operations Research in Production Planning, Scheduling, and inventory control. John Wiley & Sons, Inc. 1974.

12. Buzacott, J.A. and Shanthikumar J.G. "Safety stock versus safety time in MRP controlled production systems" Management science, December 1994.

13. Orlicky. J., Material Requirements Planning, Mc Graw-hill, New York, 1975.

14. Mc Clain. J. and Thomas. J., Operations Management: Production of goods and serevices, Prentice-Hall, Englewood cliffs, NJ.,1980.

15. Taha. H. A. Operations Research. Prentice Hall 1996.

16. W. B. Lee and C.P. Mclaughlin, " Corporate simulation model for Aggregate Materials Management". Production and Inventory management vol. 15 no 1, First quarter 1974.

17. R.E. Markland "Improving fuel utilization in steel meal operations using linear programming" Journal of Operations Management, vol. 1. No 2, November 1980.