COMBINED OPTIMIZATION OF AGE BASED REPLACEMENT AND CONTINUOUS REVIEW SPARE PROVISIONING

A Thesis

By

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Department of Industrial & Production Engineering
Bangladesh University of Engineering & Technology

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CERTIFICATE OF RESEARCH

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DECLARATION

I do hereby declare that this work has not been submitted elsewhere for the award of any degree or diploma.

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The 20 th of December, 1995

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To Md. Mizanul Haque...

A rare brother, friend, and comrade

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ABSTRACT

Block replacement is a preventive maintenance policy where a large number of identical, low cost operating components are replaced group-wise after certain time intervals. To ensure availability of spare items when operating components are to be replaced after breakdown or block replacement interval, there is always a tendency to overstock the components. Excess inventory of the components involves substantial capital. In a maintenance organization, normally, maintenance and spare parts provisioning are treated as separate problems, with special attention to maintenance policy. Inventory systems matching stochastic demand pattern could also be designed and therefore combined optimization of maintenance and inventory functions can be anticipated to prove more cost-effective than considering these policies separately. This general problem forms the basis of the present research.

In the present research a manufacturing system has been considered to solve the general problem of optimizing joint maintenance and inventory policies. This system comprises of several work centers and the focus is on the stochastic failures, replacements and order lead times of a single type of statistically identical items, common to each of the work centers. The system is highly complex to be described by a mathematical model. A simulation model has, therefore been developed for the system operating under block replacement maintenance policy. The system response in terms of total cost, system downtime and work in process inventory accumulation have been studied for continuous review inventory policy. Various cost and system parameters have found to bear direct and significant effect on the optimal values of the decision variables. The results obtained clearly show that the joint optimized policy produces better results than that from the combinations of separately or sequentially optimized policies.

CHAPTER ONE

INTRODUCTION

- Introduction
- Scope of the Present Work
- Objectives of the Present Work
- Importance of this Topic
- Applications
- Organization of this Thesis

1.1 INTRODUCTION

It has been time immemorial since human being practised maintenance function knowingly or not knowingly. With the development of civilizations and consequent technological advancements, man has seen multitudes of tools and appliances that are, as parts of nature, inclined to deterioration. He has been striving to extend the useful lives of these objects and thus fight their deteriorating through aging. Maintenance, the best tool to solve this problem, is often defined as the combination of any actions carried out to retain an item in, or restore it to, an acceptable condition. Nowadays it is granted to be one of the most important function in any industrial enterprise. This position seems justified after a quick browse over the principal objectives of a well-organized engineering maintenance management system. These are,

a. To ensure operational readiness of all equipment and thus assure the optimum availability of installed equipment for production (or service) and obtain the maximum possible return on investment.

b. To extend the useful life of assets (i.e., every part of a production plant: site, building and contents).

c. To conserve energy, spare parts and maintenance materials usage.

d. To ensure the safety of personnel using the facilities.

In view of the present trend of manually controlled to automatically controlled production where equipment is operated at higher speeds and products are turned out with closer tolerances, the maintenance function is growing, not only in numbers of people employed, but also in capital value and quality of measures. The increasing importance of maintenance function has thus found a new and better definition in the name of *Terotechnology* [Corder, 1976] which is the combination of management, financial, engineering and other practices applied to physical assets in pursuit of economical life cycle costs. It is concerned with the specification and design for reliability and maintainability of plant, machinery, equipment, buildings and structures, with their installation, commissioning, maintenance, modification and replacement, and with the feedback of information on design, performance and costs.

This new definition of maintenance function stresses on the cost and service optimality of such organization and the integration of maintenance activities with other major department activities. The major contribution to the total maintenance cost of an industry typically comes from the maintenance materials and activities. The focus is then naturally on them. Irrespective of the nature of the industry, inventory control of maintenance materials is indispensable, as maintenance itself with the industry. Inventory and maintenance management are then two important terms in industrial jargon, each complementing the other and both serving towards a cost and service optimal industrial environment. The present work is intended to investigate the influences impressed on the total cost by both maintenance and inventory related costs. The underlying concept is that the problem of unavailability of maintenance materials in time of need, and that of inventorying them in excess amount for safety are unavoidable in real time situations. The frequent complaint of maintenance supervision is the spare parts shortage. Often there is a tendency to stock them in excess amount to remove this conflict and thereby incur substantial capital cost. These problems can be solved by joint, rather than sequential optimization of maintenance and inventory policies. Moreover, this would be more cost effective when a trade off between these two types of costs are deduced logically. Researches on relevant fields have yielded several maintenance and inventory policies which produce optimal results separately or sequentially, but a little effort has been exerted to joint optimization. The ever increasing importance of this issue therefore calls for further research in this field.

1.2 SCOPE OF THE PRESENT WORK

Literature on maintenance and inventory control reveal that most of the past studies consider these two functions either separately or sequentially for optimization. This is reflected by the individually established maintenance and inventory models. Extensive researches have been carried out on these fields. Though maintenance management accentuates the integration of these two functions, efforts exerted for this purpose remain strikingly little.

The general problem to be considered in this research is that of merging maintenance and inventory models. This is required to drive away the conflict between demand structure and supply of the maintenance materials imposed by the maintenance policy undertaken. Solution to this problem forms the basis of this research. It involves the determination of a cost optimal maintenance scheme and precluding possible failure of this scheme by proper inventory control. Separately or sequentially optimized policies produce solutions for separate problems and simple consolidation of these solutions cannot suggest the truly optimal solution, especially in case of preventive maintenance. Failure and preventive maintenance policies are the most important and practised ones. Operating items may be replaced preventively either individually or in groups. The maintenance model to be studied is block or group replacement, but failure maintenance policy is also included to compare their individual performances while inventory of maintenance materials are controlled by continuous review approach. Block replacement is a preventive maintenance scheme where identical items are replaced after certain time interval. This interval is designed to lower the cost than that in case of failure or individual maintenance. The theme is that [Sarker, 1995] it is expensive to replace any machine or any of its components after failure than replacing them before failure. This policy is widely acclaimed to be the most cost and service effective policy for large number of low/medium cost items. This suits very much with a major portion of common maintenance materials.

Continuous review of inventory approach is also widely practised as block replacement policy by its virtue of simplicity in operation. This method determines the combination of a maximum allowable stock level (S) and a reorder level (s). Demand for the material depletes the stock and brings the stock level from maximum allowable stock level to reorder level. At this point the inventory repletes so as to reach the maximum allowable stock level again. It is also known as two bin inventory control.

The present work aims at the joint optimization of these two well established maintenance and inventory policies. A manufacturing system has been taken as the point of interest. This system consists of several work centers in three production lines and is capable of producing three products. These work centers employ a specific type of machine component which fails stochastically. The items are replaced upon breakdown and after each block replacement interval, which comes first. The problem is then to *maintain* this system while considering continuous review inventory control. This is to ensure availability of spares without excess stocking of them. Replacement times are also stochastic in nature and imbalance the production lines causing system down time and accumulation of work in process inventory. Simulation has been used as the analyzing tool. This is because the development of a mathematical model becomes extremely difficult due to the stochastic nature of item failure, replacement and procurement events. Besides the determination of optimal values of decision variables for a set of operating conditions, results have been obtained for the separately optimized policies also. The influences of cost and system parameters on system behavior have been analyzed. System performance has been adjudged by cost optimality and counting system downtime, service level, accumulation of work in process inventory etc.

1.3 OBJECTIVES OF THE PRESENT WORK

The objectives of this work are as follows;

a. To study both the separate and the joint optimization of maintenance and inventory policies.

b. To develop a simulation model to minimize the maintenance and inventory related costs for various system and cost parameters under block replacement maintenance policy with continuous review inventory control system.

c. To determine the optimal operating policies for a given set of parameters and conditions.

d. To analyze the obtained results and compare them with those obtained from sequential optimization of maintenance and inventory policies.

e. To study the effects of the decision variables on other performance characteristics, such as system down time and work-in-process-inventory.

f. To draw conclusion on the light of obtained results and recommend for further extensions of this research.

1.4 IMPORTANCE OF THIS TOPIC

The considerable amount of capital and effort involved with the maintenance organization require for quite the same level of attention that are to be given to any other departments by the management authority. The Working Party on Maintenance Engineering set up by Mintech in England, 1967/68, estimated that over 3000 million pounds per year [Corder, 1976] be spent on maintenance engineering in manufacturing industry alone, excluding nationalized industries and services. To put in to perspective, this figure exceeded the annual expenditure for the whole National Health Service. Also implicitly remains the other values of environmental, human safety, national economy growth etc. factors. The workmanship required is also of same importance. Historically, the typical size of a plant maintenance group in a manufacturing organization ranged 5-10% of the operating force (1 to 17 in 1969 and 1 to 12 in 1981) [Niebel, 1985]. This value is logically anticipated to increase due to tendency for any industry to increase mechanization and the complete automation of many processes, including the use of robots. Inclusion of maintenance material management is the essence of this topic since roughly 40-50% of the total maintenance cost is from maintenance materials.

The afore-mentioned importance of maintenance function and inventory control of maintenance materials highlight the significance of this topic. By joint optimization of these two factors, total maintenance cost can be dramatically reduced, yet maintaining the same product quality. High system reliability with low cost ensures advantageous position in a competitive market. The study involves most widely used maintenance and inventory policies of the current time and the findings on this topic can easily be applied to all types of industrial enterprises. This widespread applicability renders this topic very useful and important both in practice and academic research.

1.5 APPLICATIONS

Perhaps there is no production or service organization that does not require maintenance and consequently, inventorying of maintenance materials. The present research deals with block replacement policy combined with continuous review inventory policy. Therefore the findings of this research are anticipated to be of high value where these policies are practised. Any plant which has a considerably large demand for low or medium cost spare items, commonly known as *Class B or C* items and suffering from the following problems, is to be benefited the most.

a. Low machine and equipment utilization because of unscheduled work stoppages due to breakdowns.

b. Frequent and high idle time of machines and operators who have to wait for the facilities. This happens when the maintenance scheme is not an effective one, or spare shortage paralyzes the maintenance activities.

c. High scrap and rejects due to unreliability of equipment. This also happens due to improper maintenance. Over aged or faulty setup has always to be detected and repaired/replaced promptly by the maintenance department.

For cost optimality, medium to large sized industries are supposed to be the main application areas where capital requirement for maintenance program and spare inventory are considerably large. Real time data, when incorporated to the developed model will provide the industries with cost optimal and feasible solutions to maintenance problems. However, the concept and the findings can be always extended to almost every real time maintenance organization practices.

1.6 ORGANIZATION OF THIS THESIS

A detailed literature survey illustrating the various established maintenance and inventory models has been given in Chapter Two. The separate maintenance and inventory models have been described here along with the combined models. Chapter Three provides the development of the simulation model. Here the system is described thoroughly with cost, down time and work in process formulation. Logical relationships have been provided along with a brief touch on the coding of the simulation program. Based on the previous researches and the complexity of the problem, the use of simulation as the analyzing tool has been justified in this chapter. Design of the experimentation with the developed model has been described in Chapter Four. This chapter deals with input data analysis and details on designed system characteristic parameters. Statistical techniques employed for output data analysis are provided here. Chapter Five contains the results of the present work. Numerical outputs and explanations on the experimentation have been discussed in the view of the research objectives. Conclusion and recommendations for further study are given in Chapter Six. At the end of this thesis, the necessary references and appendices have been given.

CHAPTER TWO

LITERATURE REVIEW

- Introduction
- Maintenance Management
- Inventory Management for Maintenance
- Joint Maintenance and Inventory Policy
- Summary of Literature Review.

2.1 INTRODUCTION

The widely acknowledged importance of maintenance and inventory functions has been demanding extensive research in these fields. With the development of various maintenance and inventory models and their ever continuing extensions, they have flourished in to separate research fields. Previous studies considered these problems separately but they provide a rich background toward the conceptualization of the general problem and also objective of the present work. The present chapter is intended to survey the existing research literature keeping in mind the scope and objective of the present study. The gist of previous studies on maintenance and inventory management have been discussed separately in the next two sections. The section after that describes the research activities in detail, that have been carried out in effort to merge these functions. Appropriate previous findings are selected in order to serve the purpose of creating a comprehensive background.

2.2 MAINTENANCE MANAGEMENT

Intense competition together with a rapidly advancing technology has wrought many changes in the pattern and outlook of both the production and maintenance side of industry. Modern plant and equipment require heavy capital expenditure, making *downtime* extremely costly. To ensure maximum plant availability and reliability, maintenance management must prosper accordingly. Maintenance is expected to be carefully planned in conjunction with production requirements and schedules so that it causes the minimum disturbances and loss of production. Inadequate or improper maintenance can lead to damage which is extremely costly not only in repairs but also in lost productions. New products are being developed continuously and techniques, processes, systems and methods are being updated from time to time. Therefore the maintenance management science has been continuously updated to keep pace with the advancement of production technology. The following terms are most frequently encountered in maintenance management.

Maintenance: A combination of any actions carried out to retain an item in, or to restore it to, an acceptable condition.

Emergency Maintenance: Maintenance which it is necessary to put in hand immediately to avoid serious consequences.

Planned Maintenance: Maintenance organized and carried out with forethought, control and records to a predetermined plan.

Breakdown: Failure resulting in the terminated life time of an item.

Corrective Maintenance: Maintenance carried out to restore (including adjustment and repair) an item which has ceased to meet an acceptable condition.

Preventive Maintenance: Maintenance carried out at predetermined intervals, or to other prescribed criteria, and intended to reduce the likelihood of an item not meeting an acceptable condition.

Running Maintenance: Maintenance which can be carried out while the item is in service.

Shut-down Maintenance: Maintenance which can only be carried out while the item is out of service.

Overhaul: A comprehensive examination and restoration of an item, or major part thereof, to an acceptable condition.

Downtime: The period of time during which an item is not in a condition to perform its intended function.

Work-in-process-inventory: The accumulation of finished sub-assembly parts before a work center due to its unavailability.

2.2.1 Statistical Principles Governing Item Failure

Understanding the nature of the failure mechanisms of operating equipment is the first step required for designing proper maintenance and age replacement strategies. Preventive maintenance can decrease the likelihood of unplanned failures of equipment. By gaining an understanding of probability laws governing failure, we can more effectively design economically sound maintenance strategies. It is evident that all components of a given type, construction, manufacture and operating condition will not fail after the same operating time but at different times in the future [Billinton & Allan, 1983]. Consequently, these times-to failure obey a probability distribution which may, or may not, be known and which describes the probability that a given fails within a certain specified time or survives beyond a certain specified time. This probability value is a function of the time that is specified or considered. If the construction, or operating condition changes, or if the components are obtained from a different manufacturer, the distribution describing the times-to-failure is also likely to be changed causing different values of probability of failure within a given specified time. Similarly, a system that is failed and being repaired or replaced is unlikely to have a constant repair or replacement time and these time-to-repair or time-to-replace are distributed according to a probability distribution which again may or may not be known. In all practical cases, the appropriate probability distribution must be deduced from sample testing or from data collection scheme associated with the operation of the component.

For statistical consideration, the most important random variable in component failure study is the life time of the component. If at $t = 0$, the component or the system is known to be operating then its probability of failure at $t = 0$ is zero. At $t = \infty$, however, the probability of failure tends to unity as it is a certainty that the component or the system will fail given that the exposure time to failure is long enough. This characteristic is equivalent to *cumulative distribution function* which increases from zero to unity as the random variable increases from it smallest to its largest value.

Let the random variable representing the lifetime of any component be T and its cumulative distribution function $F(t)$ be given by

$$F(t) = P\{T \leq t\}$$

The function $F(t)$ is treated as a differentiable function of t, so that the probability density function $f(t)$ given by the equation below exists.

$$f(t) = \frac{dF(t)}{dt}$$

In addition to the distribution and density functions of the random variable T, the interest is projected towards related functions. One is the *reliability function* also known as the survival function. The reliability function of the component, denoted by $R(t)$, is given by

$$R(t) = P(T > t) = 1 - F(t)$$

In words, $R(t)$ is the probability that a new component will survive past time t. It is to be noted that $F(t)$ represents the probability of failure of the component to survive past time t.

Considering the following conditional probability

$$P\{t < T \le t + s | T > t\}$$

which means that a new component will fail between t and $t+s$ given that it lasts beyond t. This conditional probability may be interpreted as; think t as the *present time* and s as an increment of time in the future. The event$\{T > t\}$ means that the component has survived until the present, or in other words, it is still working. The conditional event $\{t < T \le t + s | T > t\}$ means that the component is working now but will fail before an additional s units of time have passed. From elementary probability theory for any events A and B it can be written,

$$P\{A|B\} = \frac{P\{A \cap B\}}{P\{B\}}$$

In the special case where $A \subset B$, $A \cap B = A$, so that

$$P\{A|B\} = \frac{P\{A\}}{P\{B\}} \text{ when } A \subset B.$$

After identifying the event $A = \{t < T \le t + s\}$ and $B = \{T > t\}$, a little reflection shows that $A \subset B$ in this particular case, so that

$$P\{t < T \le t + s | T > t\} = \frac{P\{t < T \le t + s\}}{P\{T > t\}} = \frac{F(t+s) - F(t)}{R(t)}$$

Dividing the above expression by s and letting s approach zero [Sarker, 1995],

$$\lim_{s \to 0} \frac{1}{s} \frac{\{F(t+s) - F(t)\}}{R(t)} = \frac{f(t)}{R(t)} = \lambda(t)$$

which may be defined as the failure rate function, hazard rate function, age specific failure rate etc. In terms of failure the hazard rate $\lambda(t)$ is a measure of the rate at which failures occur. However this is not simply the number of failures that occur in a given time period of time because this is dependent upon the size of the sample being considered. In order to evaluate the hazard rate, the number of failures must be related per unit to the number of components that are exposed to failure, giving the following definition of $\lambda(t)$ [Billinton & Allan , 1983]

$$\lambda(t) = \frac{\text{Number of failures per unit time}}{\text{Number of components exposed to failure}}$$

The next step is then data collection for any component or system, identification of the appropriate failure probability distribution and analysis for the selection of a maintenance policy. Depending upon the failure probability distribution, the failure rate may be decreasing, constant, or increasing. These failure patterns affect maintenance strategies immensely. Before surveying the maintenance models to be employed in the present research, a review of the above mentioned failure laws seems to be extremely necessary.

2.2.2 Some Selected Failure Laws

Constant failure rate of components implies that the failure characteristics can be best described by *Exponential* distribution. Here λ, the rate of occurrence, is the single parameter. If lifetime T is exponentially distributed with parameter λ, then T corresponds to the lifetime of a component that exhibits no aging over time; that is, a component that has survived up until time t_1 is equally likely to fail in the next instant of time as one that has survived up until time t_2 for any times t_1 and t_2. The expected failure time is $1/\lambda$. The standard deviation of the failure time is also $1/\lambda$.

Although the constant failure rate function that leads to the exponential law is significant in real time reliability and maintenance theory, there are other important failure laws as well. Researchers are familiar with items that possess increasing failure rate functions. Decreasing failure rate functions also occur frequently. New products often have a high failure rate because of the *burn in* phase, in which the defective items in the population are weeded out.

An important class of failure rate functions that includes both increasing and decreasing failure rate functions is of the form [Sarker, 1995]

$\lambda(t) = \alpha\beta t^{\beta-1}$ where α and $\beta > 0$

Here $\lambda(t)$ is a polynomial function in the variable t that depends on the two parameters α and β. When $\beta > 1$, $\lambda(t)$ is increasing, and when $0 < \beta \leq 1$, $\lambda(t)$ is decreasing. This form of failure rate is best described by the Weibull distribution which is also known as a family of distributions. Since it is often true that empirical failure rate functions (i.e., those obtained from real test data) are closely approximated by polynomials, accurate

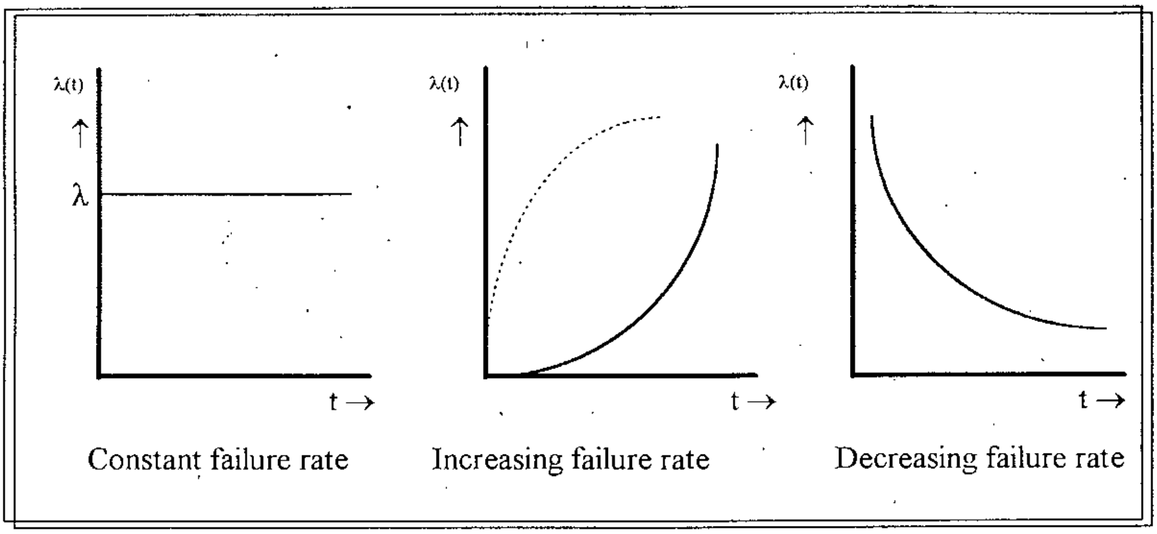


Figure 2.1. Various failure rate functions [Sarker, 1995].

description of the failure law of many types of operating equipment can be given by the Weibull distribution. Figure 2.1 provides a graphical presentation of these typical failure rate functions. Further details on this distribution have been in the Appendix A with its various forms.

Sometimes neither increasing nor decreasing failure rate functions accurately describe the failure characteristics of particular equipment. A typical case in point is pictured in figure 2.2 and is often referred as *bath-tub* curve [Billinton and Allan, 1983] for self-evident reasons. Three distinct regions can be identified in this curve. In the region I, which is also known as infant mortality or de-bugging period, the failure rate is decreasing . This is due to manufacturing errors and improper design. Region II represents the normal operating or useful life. Since all defective items are weeded out the failure rate is more or less constant until aging begins. After that region III starts where items enter the *wear-out* phases and failure rate starts to increase.

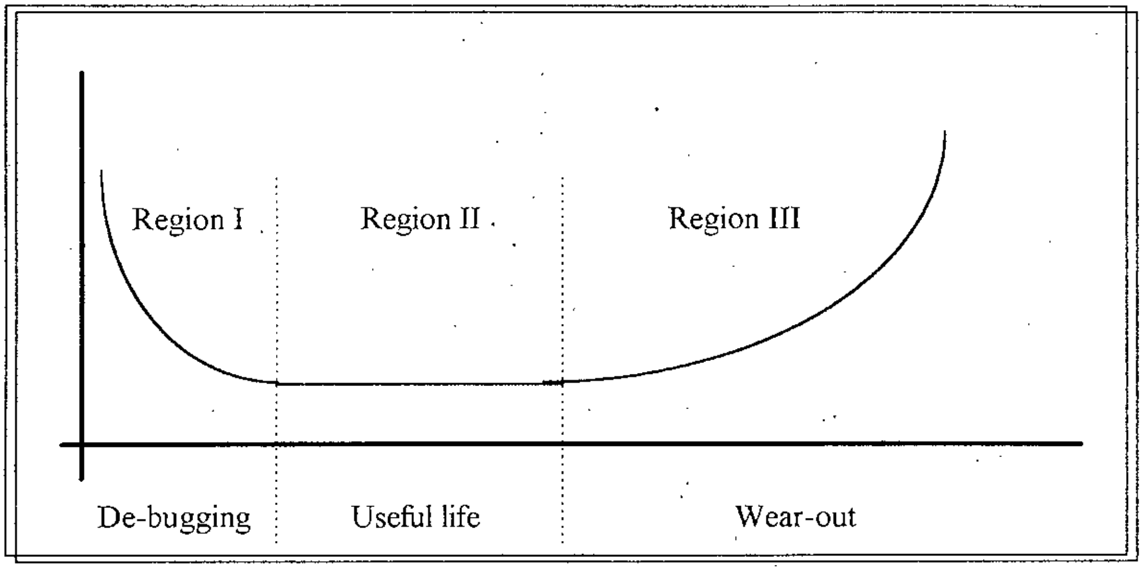


Figure 2.2. A typical component failure rate curve as a function of age.

From the above literature it can be concluded that high system reliability can be achieved, in practice, for extended periods of operation provided proper debugging procedures are adopted to eliminate early or premature failures and strict component replacement schedules are followed during preventive maintenance so that components do not enter their wear-out region. This component replacement is essential if reliable system operation is required beyond the component's normal wear-out time. Such replacement restores the system to an operational condition of low failure probability. Thus, when good quality preventive maintenance is performed, reliable system operation becomes possible for very long periods. Bad preventive maintenance on the other hand can degrade the system reliability and therefore the skill and quality of the maintenance personnel is a big factor in such situations.

2.2.3 Preventive Maintenance

After a thorough investigation on failure laws of operating equipment, various preventive maintenance models are now left to be considered. The purpose of preventive maintenance is to decrease the likelihood that an item will require replacement due to failure. At the heart of such a policy is the assumption that it costs

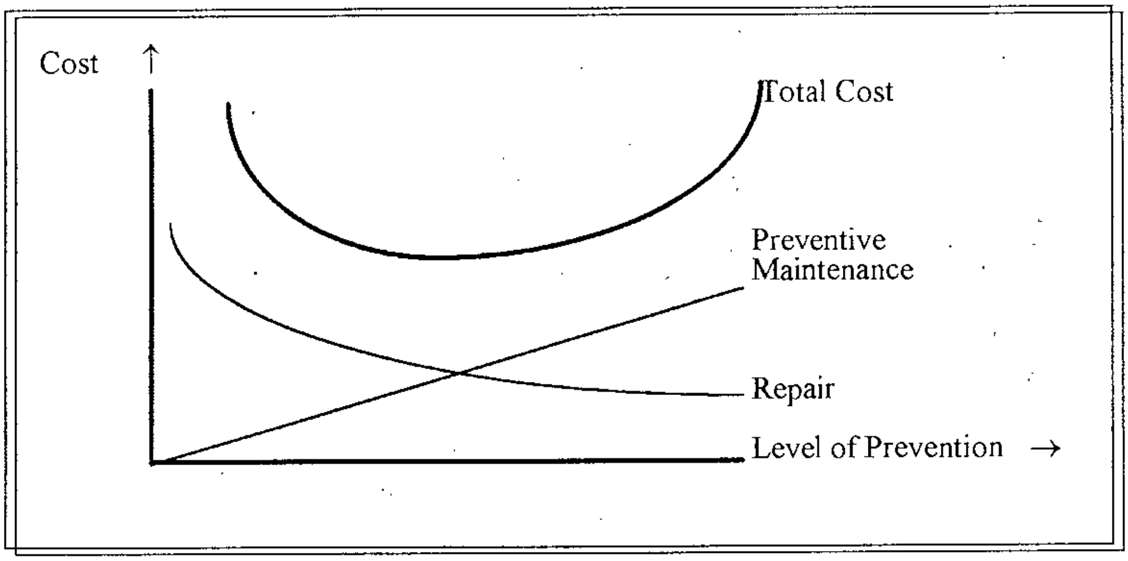


Figure 2.3. Cost of preventive maintenance and cost of repair.

more to make a repair or replacement at the time of failure than at some predetermined time. It is obvious that failure of an operating item means a production line must be stopped to determine the cause of the failure and repair the problem. Since preventive maintenance can be accomplished at a convenient time when the system is not operating, the cost of planned replacement is less than the cost of unplanned replacements. Figure 2.3 [Dilworth, 1989] shows the cost curves for both preventive maintenance and remedial repair. From the curves it is evident that preventive maintenance policy has a convex total cost function and the cost optimality of planned replacement over unplanned replacement depends upon the level of prevention. Both individual and block preventive maintenance, then, have optimal operating points which depend upon the operating conditions.

2.2.4 Individual Replacement Preventive Maintenance

Individual preventive maintenance i.e., planned replacement of items considering each unit individually and naturally, is best suited for very small number and expensive equipment. Consider a single piece of continuously operating equipment whose lifetime is a continuous random variable T with known cumulative distribution function $F(t)$. It is assumed that it costs c_1 to replace the item when it fails and c_2, ($c_2 < c_1$) to replace it prior to failure and planned replacements are made exactly t units of time after the last replacement. The goal is then to find the optimal value of t to minimize the average cost per unit time of both planned and unplanned replacements if any. Keeping in mind that a cycle is the time between successive replacements, the following expression can be used to determine the expected cost per unit time. Successive replacement cycles are shown in Figure 2.4.

$$\textit{Expected cost per unit time} = \frac{\textit{Expected cost per cycle}}{\textit{Expected length of a cycle}}$$

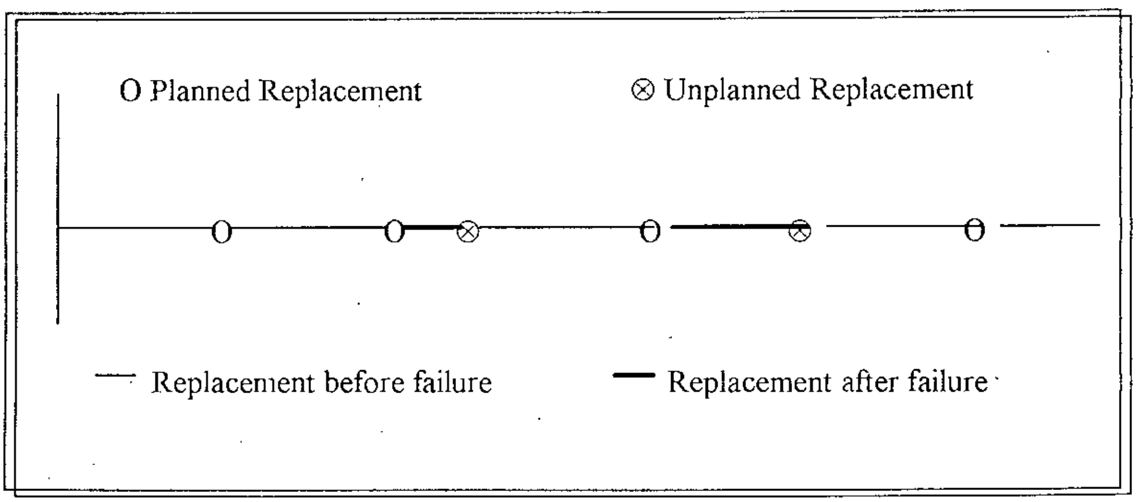


Figure 2.4. Successive cycles for planned and unplanned replacement of a single item.

It now is evident that

$$E(\textit{cost per cycle}) = c_1 \, P\{\textit{replacement due to failure}\} + c_2 \, P\{\textit{replacement is planned}\}.$$

Noticing that P{replacement due to failure} $= P\{T \leq t\} = F(t)$, and *P{replacement is planned}* $= P\{T > t\} = 1 - F(t)$, where T is the lifetime of the item placed into service at the end of the previous cycle. It follows that,

$$E(cost\ per\ cycle) = c_1\ F(t) + c_2\ \{1\text{-}F(t)\},$$

and clearly, the next replacement will occur at the minimum (T, t) [Sarker, 1995]. Hence if the life of the item be expressed by x and the probability of surviving upto this time by $f(x)$,

$$E(length\ of\ cycle) = E[min.(T,\ t)] = \int_0^{\infty} \min(x,t) f(x) dx$$

$$= \int_0^{t} x f(x) dx + t \int_t^{\infty} f(x) dx$$

$$= \int_0^{t} x f(x) dx + t[1 - F(t)]$$

It follows that the expected cost per unit time, denoted by $G(t)$, is given by

$$G(t) = \frac{c_1\ F(t) + c_2\ [1\text{-}F(t)]}{\int_0^{t} x f(x) dx + t[1 - F(t)]}$$

The rest is then the search for t to minimize $G(t)$. The optimization may be cumbersome depending upon the form of the life time distribution $F(t)$. For component life to be exponentially distributed, it is quite simple to evaluate the functional values of $G(t)$ for different values of t. Unfortunately it is extremely difficult to find the solution for other important distributions like the Weibull distribution and ironically the function do not show any convex shape which implies that there is no economy in replacing an item prior to the time it fails for both decreasing and constant failure rates.

The difficulty with calculating the optimal solution for this problem with distributions other than exponential distribution is in determining an expression for the term

$\int_0^t xf(x)dx$ that appears in the denominator of $G(t)$. While it seems a difficult task to obtain an analytical expression for this integral, a discrete approximation approach can be utilized. This means the lifetime distribution of operating components is taken to be discrete, rather than continuous one. Probabilities of failure for discrete lifetimes are calculated and these discrete probabilities are used directly to compute $G(t)$. This yields [Sarker, 1995],

$$\int_0^t xf(x)dx \approx \sum_{k=1}^{t} k\, P_k$$ where k is discrete lifetime and P_k the probability.

This value when utilized facilitates the approximation of the cost function $G(t)$.

2.2.5 Block Replacement Preventive Maintenance

In certain circumstances, it is more economical to replace groups of items at the same time than one by one. This holds true particularly in case of large number and low cost items. As with individual replacement, the problem here is to obtain an analytical expression for the integral given in the previous section. However, the same discrete approximation method can be employed here also.

The total cost for preventive maintenance is the aggregate of the cost of group replacement incurred after each replacement interval and the expected cost for replacing those units that break down in spite of the preventive maintenance. Let us assume that N items are placed into service at time 0 and P_n is the probability of a breakdown in period n. Then the expected number of breakdowns in n periods with preventive maintenance, performed every n time periods is the number of items expected to break down during the time period plus the number expected to breakdown more than once [Tersine, 1985]. If B_n is the expected number of such breakdowns, then

$$B_n = N \sum P_n + B_1 P_{n-1} + B_2 P_{n-2} + \cdots\cdots + B_{n-1}\ P_1.$$

The total cost can then be calculated as

$$Expected\ total\ cost = C_p + B_n\ c_b$$

where C_p and c_b are the costs for block replacement and cost for a single replacement respectively. The total cost per unit time period then be expressed by the following function.

$$G(n) = \frac{C_p + B_n\ c_b}{n}$$

The optimal number of periods to replace all N items is the value of n that minimizes $G(n)$. The minimum value of $G(n)$ should be compared to expected cost per period assuming items are replaced as they fail only. To calculate this, the first step is to determine the expected lifetime of the single item by the following equation.

$$E(T) = \sum_{n=1}^{\infty} n P_n$$

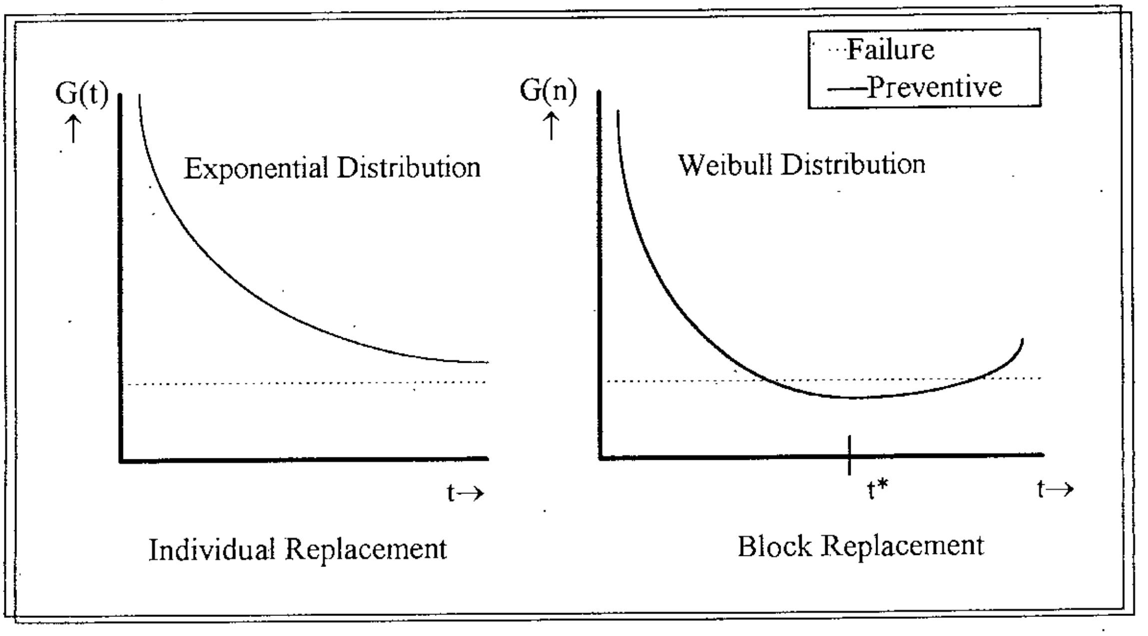


Figure 2.5. Values of G(t) and G(n) functions with time. [Sarker, 1995]

Then the failure rate for a single item λ can be can be calculated by $\lambda = 1/T$ and the cost of making replacement to items on one-at-a-time basis by $c_b\ \lambda$ per item or $Nc_b\lambda$ for the entire block of items. This should be compared to the optimal value of $G(n)$ to determine if a block replacement strategy is economical. Figure 2.5 shows typical cost function curves for Exponential and Weibull distributions.

2.3 INVENTORY MANAGEMENT FOR MAINTENANCE

Total maintenance costs generally consist of (1) the cost associated with maintenance labor, (2) the cost of required material and spare parts and (3) the cost of production downtime when breakdowns occur [Higgins & Morrow, 1977]. The cost of production downtime can be interpreted as a *shortage cost* or the penalty for the unavailability of required material and spares. Perhaps the major complaint of maintenance supervision is the unavailability of spare parts and materials at the time they are needed. With the increasing complexity of modern manufacturing equipment, the cost of stocking spare parts is high. Maintaining an adequate inventory of parts and supplies to ensure the availability of these materials when needed is essential in an modern maintenance management program. Thus spare stocking is dependent on the spare requirement in the maintenance operation. The integration of inventory management with maintenance management is therefore indispensable for solving the continuing conflict of reducing inventory capital while avoiding stockouts. The objectives can be enumerated as follows.

a. To relate stock and stores quantities to demand, thus avoiding both over stocking and under stocking.

b. To determine an inventory policy that is best suited to an appropriate maintenance policy.

2.3.1 Classification of Maintenance Materials

The first step to organize inventory system is the classification of the inventory materials. This is very much important in determining the nature of inventory control. Classification of various maintenance materials is usually done by *ABC analysis* which is described below in detail. ABC classification, or *distribution by value*, is used to provide an initial sorting of items into groups according to the annual expenditures they cause. The items with highest annual value use are called A items. A items should be controlled closely. They often justify perpetual inventory records in a fixed quantity system or frequent review in a fixed interval system. B class inventory items such as bearing, bracket, bushing, coil, disk, fuse, lever, plug, screw, socket, spring, valve etc., have the second highest annual value use. These items would be reviewed less frequently than A items and minimum-maximum version of continuous review system is recommended [Dilworth, 1989]. C items represent a large percentage of the items, often half of them but less than one-fourth of the annual value volume. Very simple approximate inventory control may be used to control C items. A glance at this analysis then suggests that for a maintenance organization where relatively large numbers of medium/low cost spare (B class) items are required, the choice of block replacement as the operating policy comes naturally. The remaining job is then simply the selection of appropriate inventory policy.

2.3.2 Continuous Review Inventory Control System

The fixed interval system can sometimes result in the placing of very small orders. A continuous review or maximum-minimum inventory system, denoted by (S, s) system eliminates the handling of quantities that are considered too small to be economical. Here at first the inventory level is set to a maximum level S. The variable demand rate depletes the stock and an order for $(S - s)$ items is placed whenever the inventory level reaches or goes below the reorder level s. The inventory is reviewed after each demand. The placement of orders is therefore triggered by the inventory level status rather than a fixed period of time as with the periodic review system. Figure 2.6 [Dilworth, 1989] shows the behavior of such system. Here the cycle is expressed in terms of inventory items. Under this system, the reorder point and the order quantity may be fixed or variable depending upon the status of the inventory level with respect to the reorder level s. Demand rate, and the lead time are variable.

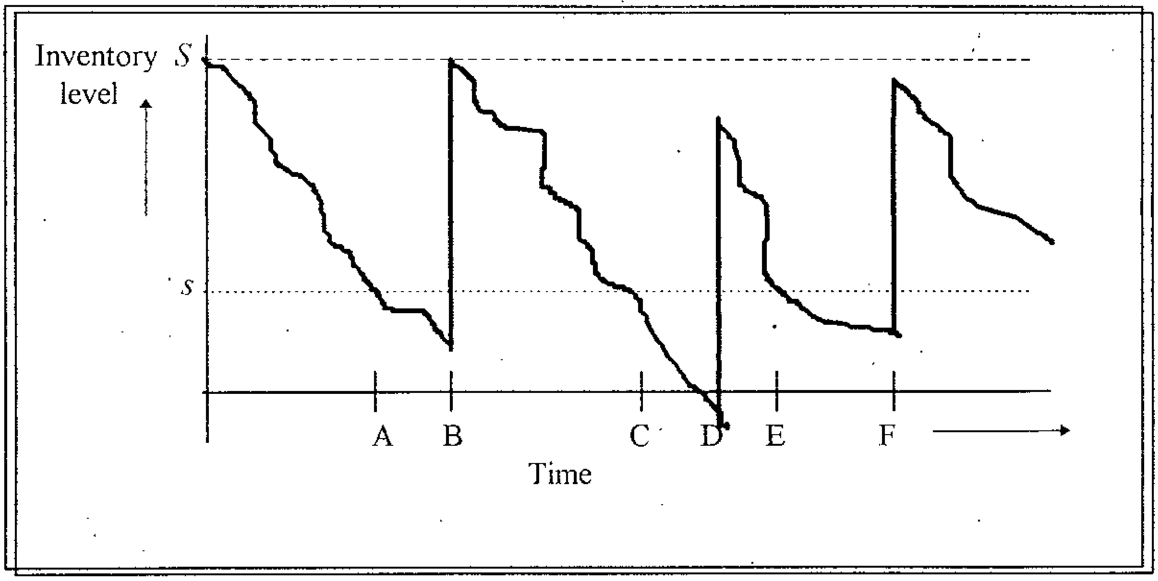


Figure 2.6. Behavior of continuous review inventory control system.

2.4 JOINT MAINTENANCE AND INVENTORY POLICY

Before the conception of joint optimization of maintenance and inventory functions, it was commonly assumed that an unlimited number of spare units are immediately available when a replacement is needed [Thomas and Osaki, 1978]. The first age replacement policy was mathematically formulated by Barlow and Proschan [Kabir & Olayan, 1995a] which held this assumption and ignored spare provisioning and repair/replace times also. For real time situations this is not a judicious assumption and the lead time for delivering a spare and an appropriate ordering policy have to be incorporated to the original model. Thomas and Osaki developed an ordering policy for a single operating item considering perfect and instantaneous switch over of spares, constant lead time for spare delivery and the provision of expedited ordering. The essence of the model can be described as,

> An item is considered which starts working at time 0. If that item does not fail up to a pre-specified ordering time t_0, we order at t_0 a spare for replacement (regular ordering) and the delivered spare is installed immediately when it

arrives, irrespective of the original item. On the other hand, if the item fails before the regular ordering time t_0 ; we order a spare immediately (expedited ordering) and the delivered spare is installed immediately when it arrives. Once the delivered spare starts working, the same behavior repeats itself for an infinite time span. A cycle can thus be defined as the period from the time at which the delivered spare starts working to the time at which the next delivered spare starts working. The optimal ordering policy can then be determined by minimizing the cost function for each cycle.

Osaki et. al. [1981] produced a generalized form of the model considering no repair and minimal repair of the operating item. In general setting, the ordering policy can be stated as,

> An order for a spare unit is placed at a specified time t_0, and on delivery after a constant lead time L, the spare is kept in stock up to another specified time t_l . ($\geq t_0 + L$). When the operating unit fails before t_0, a spare unit is ordered immediately. When the operating unit fails between $t_0 + L$ and t_l, it is replaced by the spare unit in stock. The jointly optimal ordering time t_0^* $(0 \leq t_0^* \leq \infty)$ and replacement time t_l $(t_0^* + L$ or $\infty)$ are determined by minimizing the expected total inventory costs (i.e., ordering, holding and shortage costs) per unit time.

The proposed model above considered constant lead time for spare delivery which, again, is not a reasonable assumption. This shortcoming was eliminated by Park and Park [1986] when they introduced the lead time variability to their model.

All the models mentioned above aimed at the determination of an optimal ordering policy considering inventory functions only. They take the failure pattern of the operating item in account only to determine the order placement period and no consideration was given to the maintenance related costs of the operating item. It is evident that these policies suit class A items only. The maintenance related costs of these items are also reasonably high and thus can not be ignored in real time situations.

Kabir and Olayan [Kabir and Olayan, 1995b] indicated the extreme complexity in developing a mathematical model that incorporate the maintenance related costs and also extend towards the case of multiple unit situation. They avoided this problem by using simulation as the tool. This allowed the application of established inventory and maintenance policies in the study. They developed a (S, s, T) simulation model, where S is the maximum inventory level, s is the reorder level and T the preventive replacement period. This model considered continuous review inventory policy with individual preventive maintenance of the operating item. The operating principle of the policy can be described as follows,

> An order for $(S - s)$ spare units will be placed whenever the inventory level falls to s. The operating unit will be preventively replaced after interval T provided a spare is available. Otherwise, the unit will be replaced as soon a the stock is replenished. If the operating unit fails before T, it will be replaced as soon as a spare is available. The order lead time is considered to be randomly distributed. Optimal values of the decision variables (S, s, T) are determined by minimizing the expected total cost per unit time, where the cost components include both replacement and inventory related costs. The cost formulation of the model is as follows,
>
> i. Ordering cost occurs at a unit replacement if the inventory level is equal to or below the reorder level.
> ii. Shortage cost for a unit is incurred over the length of time for which a failure replacement is delayed due to late arrival of stocks.
> iii. Holding cost is computed for each spare unit individually and over the time it is held. This is the time between its arrival time and the start of its operation at a replacement point.
> iv. Replacement cost for a unit is according to the type of replacement.

The results obtained by Kabir and Olayan are very much impressive in terms of cost effectiveness, when compared to Barlow and Proschan age replacement policy supported by optimal (S, s) policy. This clearly shows that separate optimizations of replacement and spare inventory policies do not ensure global optimality when total system cost has to be minimized. However, this model did not consider repair/replacement times and also the total system consisted of single or few items. Individual preventive maintenance is very much practicable for this type of system. But in real time situations, many operating items are also to be preventively maintained and if these items could be classified as *B* or *C* class items, block replacement policy might prove more cost effective [Tersine, 1985]. This keeps yet another provision of testing block replacement policy with continuous review inventory policies.

Acharya et al [1986] developed a mathematical model that considered block replacement of operating items for the first time, with periodic review of spares inventory. They assumed that the ordering for replenishment of replacement equipment stock is done at preventive replacement instants only. This facilitated the optimization procedure as the review period and replacement interval coincided. Unfortunately simplification of this type could not be done for continuous review inventory policy since the cycle here is in terms of inventory level, not in time units.

The model considers a system comprising *n* identical units which are replaced on failure and preventively at times *T, 2T,* ...., etc. The demand for replacement units is met through ordering new units at each replacement point. The inventory policy is of base-stock where the stock level is replenished up to a certain point after each review period. The total cost rate could be calculated from the following expression,

Total cost rate = Replacement cost rate + Inventory holding cost rate + Backorder cost rate.

The replacement related cost rate for *n* identical items with the same block replacement interval *T* is,

$$\frac{n\,[C_p + C_f H(T)]}{T}$$

Where C_p and C_f are costs of preventive and corrective replacement of one unit respectively. H(T) is the mean number of replacements in the interval $(0,\ T)$ of a unit.

The inventory holding cost rate is the sum of the cycle stock holding cost rate plus the safety stock holding cost rate. The variation of cycle stock with time is shown in figure 2.7.

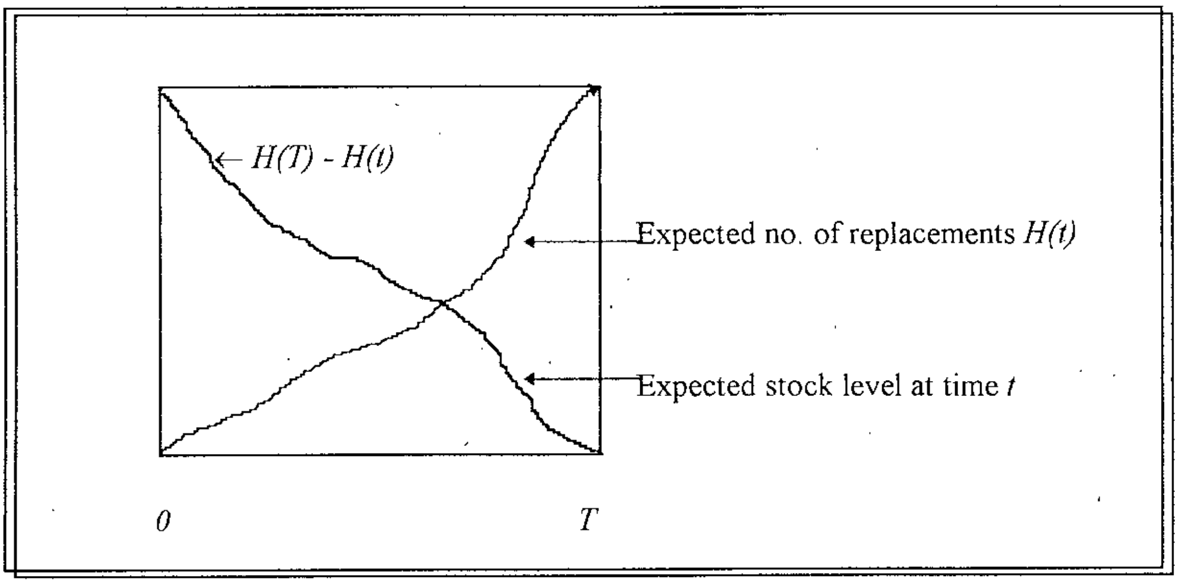


Figure 2.7. Variation of cycle stock of replacement units.

The cycle stock holding rate is expressed by,

$$\frac{n}{T}\int_0^T [H(T) - H(t)]dt$$

The model keeps provision of safety stock in order to prevent backordering. The average number of backorder incurred per year is the product of the average number of backorders incurred per cycle and the average number of cycles per year. If the demand during $(0, T)$ is x, the number of backorders during a cycle is,

$\eta(x, s) = 0$, if $x<s$

$= (x-s)$, otherwise, where s is the safety stock.

The average number of back orders per period [Hadley & Whitin, 1963] is,

$\bar{\eta}(s) = \int_0^\infty \eta(x,s) f(x)dx$, where $f(x)$ is the pdf. of the spares demand for all n units.

$$= \int_{s}^{\infty} (x - s) f(x) dx$$

The total average cost rate of backorders is then simply,

$C_b \int_{s}^{\infty} (x - s) f(x) dx$, where C_b is backorder cost rate per unit.

Continuing with a similar argument, the total average holding cost rate due to the provision of safety stock is,

$C_h \int_{s}^{\infty} (x - s) f(x) dx$, where C_h is the holding cost rate per unit.

Assuming A to be the ordering cost and recognizing that an order is placed after every T time units, the ordering cost rate becomes A/T. The total cost rate is then,

$$TC = \frac{n}{T}[C_p + C_f H(T)] + C_h \frac{n}{T}\int_{0}^{T} [H(T) - H(t)]dt + A/T + C_b \int_{s}^{\infty} (x - s) f(x) dx +$$

$$C_h \int_{s}^{\infty} (x - s) f(x) dx$$

Acharya et. al. have presented an algorithm to solve this problem with a numerical example. Their results show clearly that mere combination of separately optimal maintenance and inventory policies do not produce global optimality and hence joint optimization of these two function proves to be the best choice.

2.5 SUMMARY OF LITERATURE REVIEW

A browse on the established theories and the findings of current researches makes it clear that maintenance and inventory control are two important functions and too critical to be malpractised. In the early periods intense competition, quality concern, tendency towards mechanization and complete automation were not so prominent. Therefore there was no need to think about the joint optimization of these two functions for yet better cost effectiveness. Accordingly all researches were devoted towards maintenance and inventory in a separate manner. This is reflected by the development of the maintenance models which did not consider the inventory related problems. In the present world of high technology, and more complex market structure, these two functions should not be treated separately. This notion has been pioneered by Barlow and Proschan and later on nurtured by Osaki et al, Acharya et al, and Kabir et al and other researchers. However, it is clear that much more effort has to be exerted on this relatively new field. This necessitates further research on this field and thereby encourages the present research very much.

Based on the extent of the previous studies, the present research has been designed so as to contribute further to this field. It includes block replacement of a single type of component with continuous review spare provisioning and stochastic replacement times and also takes into account the system down time and work-in-process-inventory which did not appear in the existing literature. All these inclusions serve to update the present art of the state of the general problem.

CHAPTER THREE

SIMULATION MODEL DEVELOPMENT

- Introduction
- The Art of Simulation
- The Conceptual Model
- Simulation Software and Coding

3.1 INTRODUCTION

This chapter is devoted to the concept of simulation and the development of simulation models, especially in context of the present research problem. At first the nature of simulation is discussed along with the steps to be followed to develop a valid and credible simulation model. The role and applicability of simulation as an analyzing tool is then described. The next section describes the general problem stated in chapter one in the context of a specific maintenance organization system, where the importance of inventory functions can not be ignored. This system and the components within it are mathematically and logically related to develop a simulation model. The final section of this chapter introduces the computer program and the SIMSCRIPT II.5 simulation language used to code the program.

3.2 THE ART OF SIMULATION

Simulation can be defined as follows [Russell, 1989],

> A simulation of as system is the operation of a model that is a representation of the system. The model is amenable to manipulation that would be impossible, too expensive, or impractical to perform on the system it portrays. The operation of the model can be studied and from it properties concerning the behavior of the actual system can be inferred.

Simulation or the imitation of real world situations, stems from three origins [Tocher, 1963]. These are the immense difficulties in problem solving with the theory of mathematical statistics, the demands of applied mathematics to solve real world problems, and the development of the new science of operations research.

In simulation, a system is the focus of interest. Real world facilities or processes are the system elements. A set of assumptions, usually in the form of mathematical or logical relationships is employed to construct the model. Instead of experimenting with the real world object, experimentation is carried out to study the responses from the model over time [Kleijnen, 1974]. If the relationships which compose the model are simple enough, it may be possible to use mathematical methods to obtain exact or analytic solutions. However, most real world systems are too complex to allow realistic models to be evaluated numerically, and these models must be studied by means of simulation. The art of simulation is then to focus the problems in a system to a model, evaluate the model numerically over a time period and analyze the outputs to estimate the true characteristics of the model [Law & Larmey, 1983] The power of this tool is self-evident from its emergence, but often is underestimated as *the method of last resort*. This is due to the fact that working with a complex simulation model is also time consuming, costly and arduous task [Law & Kelton, 1991]. Even the results may simply be valueless, since it is nothing but a statistical sampling experiment with the model of a system [Kleijnen, 1974].

It is the nature of the system elements that defines the type of the models, which may be of three dimensions; a. static or dynamic, b. deterministic or stochastic and c. continuous or discrete event. A model simulating maintenance and inventory functions is expected to contain discrete, probabilistic events since the system experiences stochastic changes of its states only at countable numbers of points in time. Accordingly the simulation model developed in this chapter has to be dynamic, stochastic and a discrete event in type.

3.2.1 Steps in Simulation Modeling

Detailed simulation modeling and coding are in fact just part of an overall simulation effort to understand or design a complex system. For a comprehensive study, attention must be paid to a variety of other concerns, ranging from statistical experimental design to budget and personnel management. Figure 3.1 [Law & Kelton, 1991] shows the necessary steps that will compose a typical, sound simulation study, and the relationship among them. Not all studies are supposed to contain all these steps, and in the order stated.

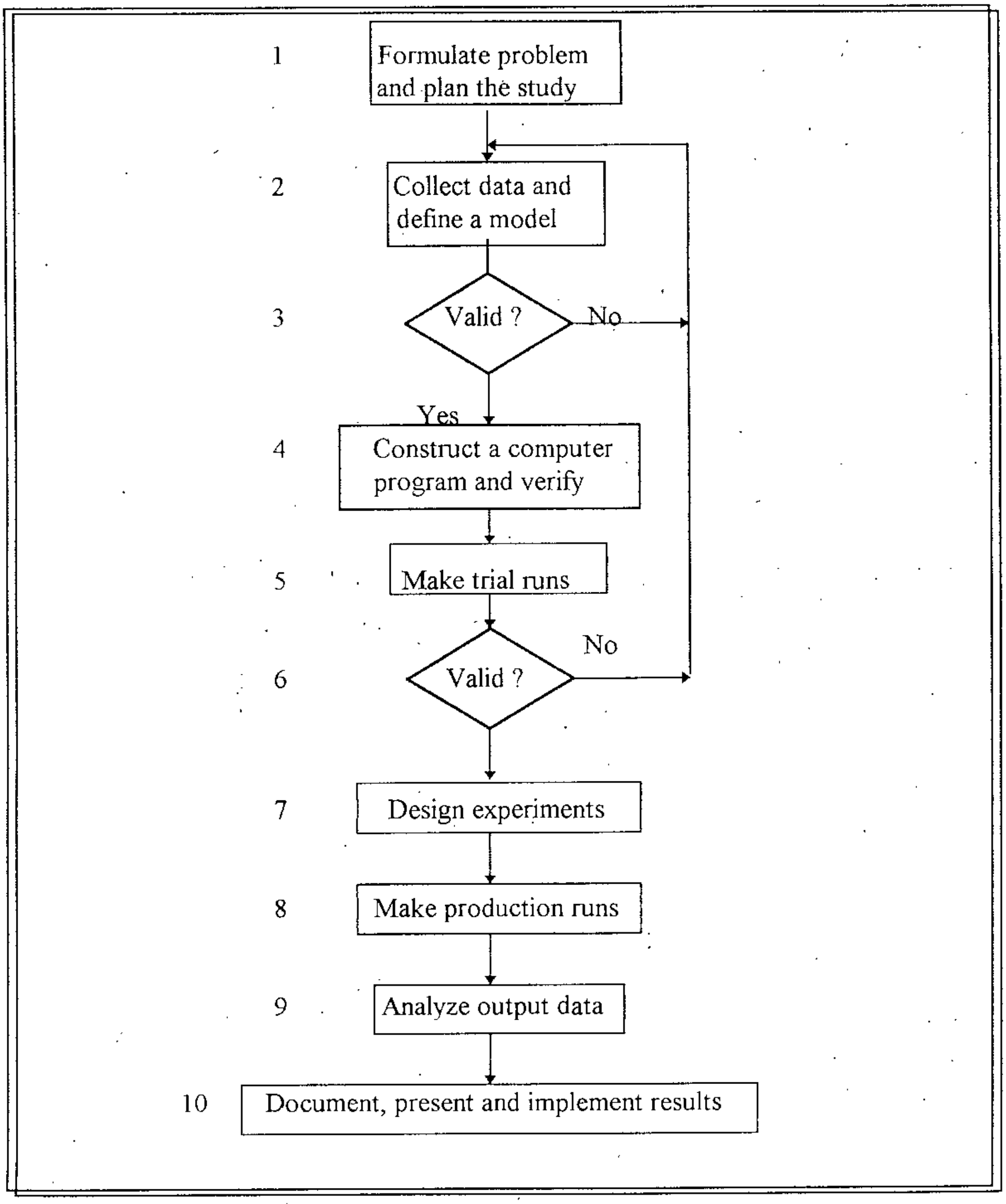


Figure 3.1. Steps in a simulation study.

Some studies may contain steps that do not fit neatly into the sequence shown in figure 3.1. Moreover, simulation studies do not have a hard and fast sequential process. Therefore it often may be a good decision to reshuffle the steps, in case new insights are obtained after any one of them.

ruitful simulation models could only be developed after the verification (checking of whether program is performing as intended) , and validation (checking of whether the conceptual model is an accurate representation of the system under study). Credibility of the model is marked with successful implementation of the simulation results. Figure 3.2 [Law & Kelton, 1991] shows the timing and relationships of validation, verification and establishing credibility.

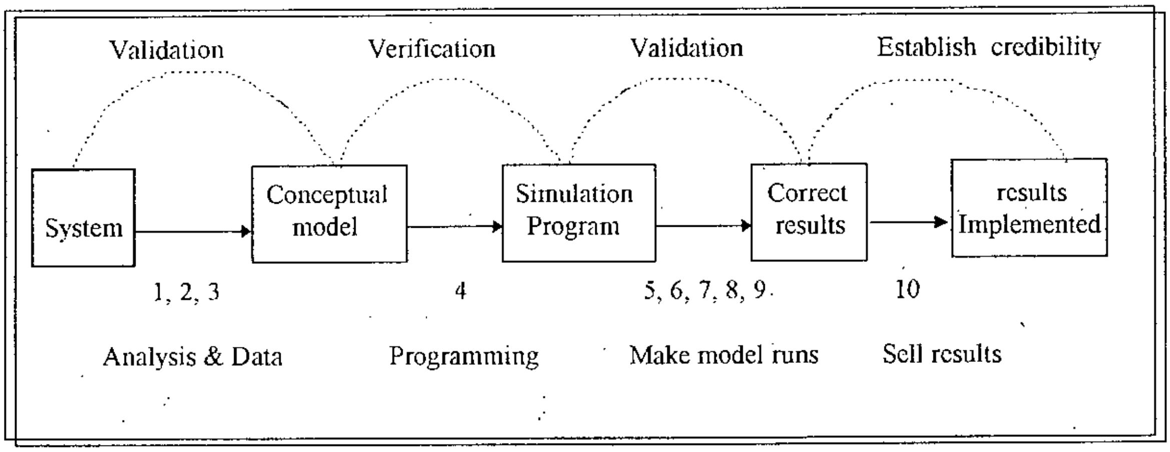


Figure 3.2. Timing and relationships of validation, verification, and credibility.

3.3 THE CONCEPTUAL MODEL

The general problem considered in the present research is to jointly, rather than separately or sequentially, optimize maintenance and inventory policies in term of total operating cost per unit time. Block replacement and continuous review have been chosen to be the operating maintenance and inventory policies respectively. The system is assumed to have a certain type operating component which is large in number and of low/medium cost. The focus, then, is naturally on the stochastic failure and replacement of this specific item. The model is then further extended by considering stochastic replacement times which allow the computation two other measures of performance: system down time and work-in-process-inventory.

In the present research work, a manufacturing system with its maintenance and inventory related functions has been simulated. The system, as shown in figure 3.3, comprises of three production lines. The production lines are shown in straight lines, and the system can be adapted to any extension to its configuration. Each of the production line consists of several work centers. The machines in these work centers could be identical or non-identical, but all of them have one common type operating components. The number of operating units in each work center (shown as the rectangular blocks in figure) has been shown on the blocks in parentheses. For example, there are 20 identical units of interest in operation in the work center 'A'. The probability of a given failed item being located in a specific work center, and also the production line is given in table 3.1. As an example, for a failed item, the probability of its being located in production line 1 is 0.33. In such case it is located

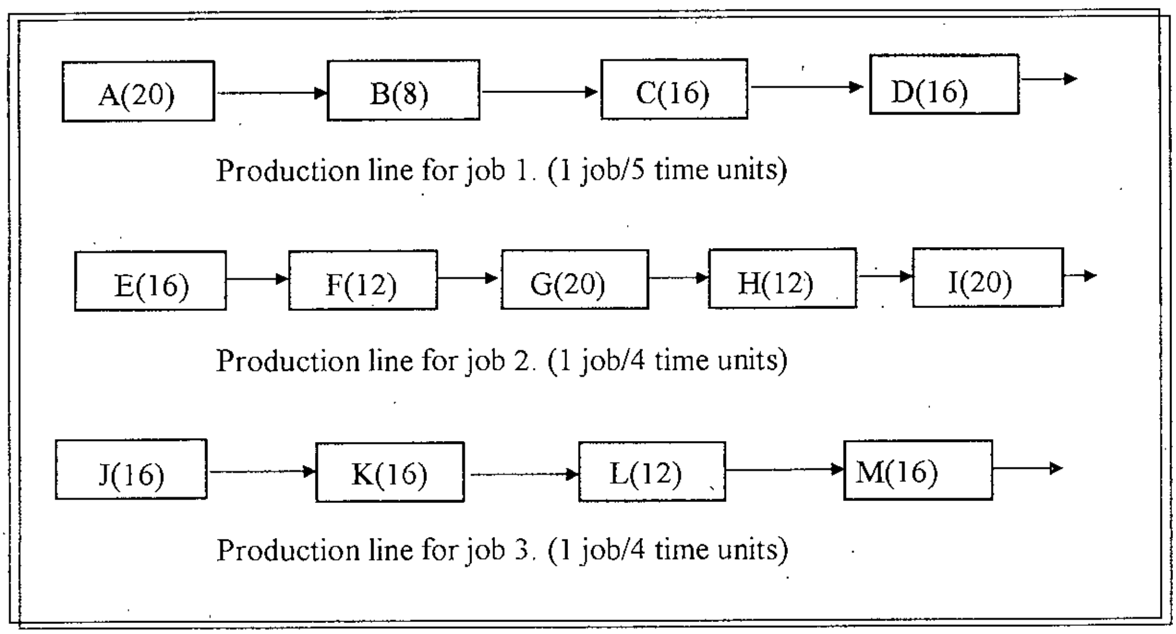


Figure 3.3. Manufacturing system configuration.

in the work center 'A' with probability 0.2. The following assumptions regarding the governing principles of the manufacturing system have been held to develop the conceptual model.

1. A single type of operating component is considered only. The item can be present in any machine. All the spares for the item, along with the operating one are identical in terms of their parameters and characteristics such as life time, design etc., so that they are identical from statistical point of view.

Production line	Probability	Production line	Probability	Production line	Probability
1	0.33	2	0.33	3	0.33
Work Center A	0.20	Work Center E	0.20	Work Center J	0.25
Work Center B	0.30	Work Center F	0.30	Work Center K	0.30
Work Center C	0.20	Work Center G	0.15	Work Center L	0.15
Work Center D	0.30	Work Center H	0.15	Work Center M	0.30
*	*	Work Center I	0.20	*	*

Table 3.1. Item failure probabilities within the manufacturing system.

2. It is more cost effective to replace rather than repair any of the failed items. This assumption is quite reasonable since the chosen component have earlier been taken to fall in B or C class inventory stores.

3. All items are replaced preventively after certain time intervals called the block replacement period. Any item failing between these group replacement instants are replaced instantaneously if spares are available.

4. Failure of any operating component is revealed instantaneously.

5. The spares in inventory do not deteriorate with the passage of time.

6. The switch over of spare items to operation is perfect.

7. The failure rate for each item increases with time and each replacement introduces a new item with new life.

8. The unit cost of the item is a constant independent of time or order quantity and thus its effect can be ignored.

9. The inventory policy to be investigated is continuous review (S, s) type.

10. An emergency order is placed only if the inventory level falls abruptly from a value greater than s to a value less than zero, while no other order is pending. For periodic review no such provision has been kept.

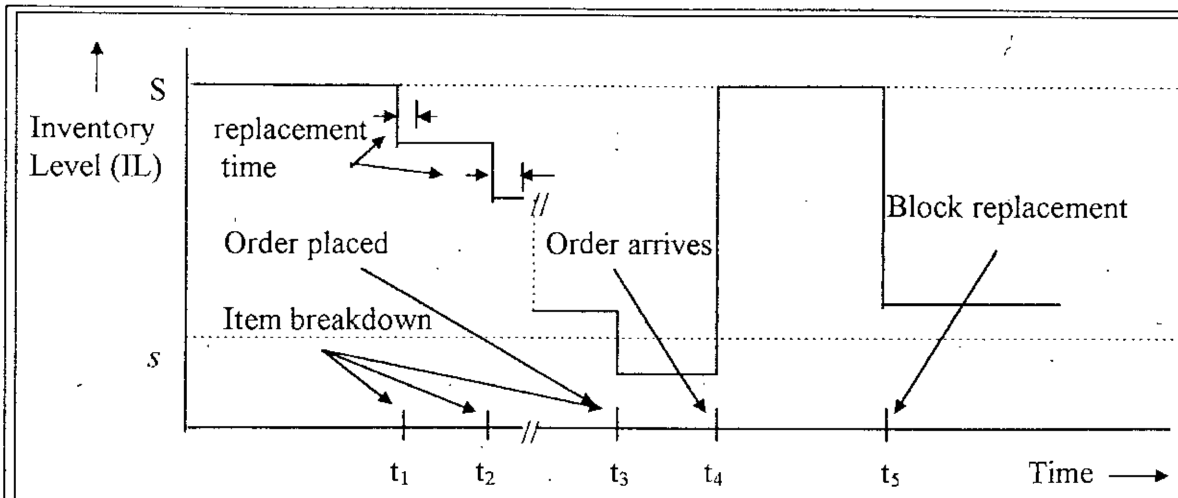
11. The inventory shortage cost includes the direct loss of production due to idling of the system per item per unit of time and also indirect losses like organization bad reputation.

12. The production lines are initially perfectly balanced. Whenever a failure takes place, the corresponding work station, with other ones continue to work until the respective jobs are done. After that they wait for the job of the failed work station to be completed and then the production line starts performing with balance again.

These assumptions and the theories on block preventive replacement and continuous review inventory (*S*, *s*) policy are applied to the manufacturing system considered, configure the conceptual model of the system. The operating rule can then be described briefly as follows,

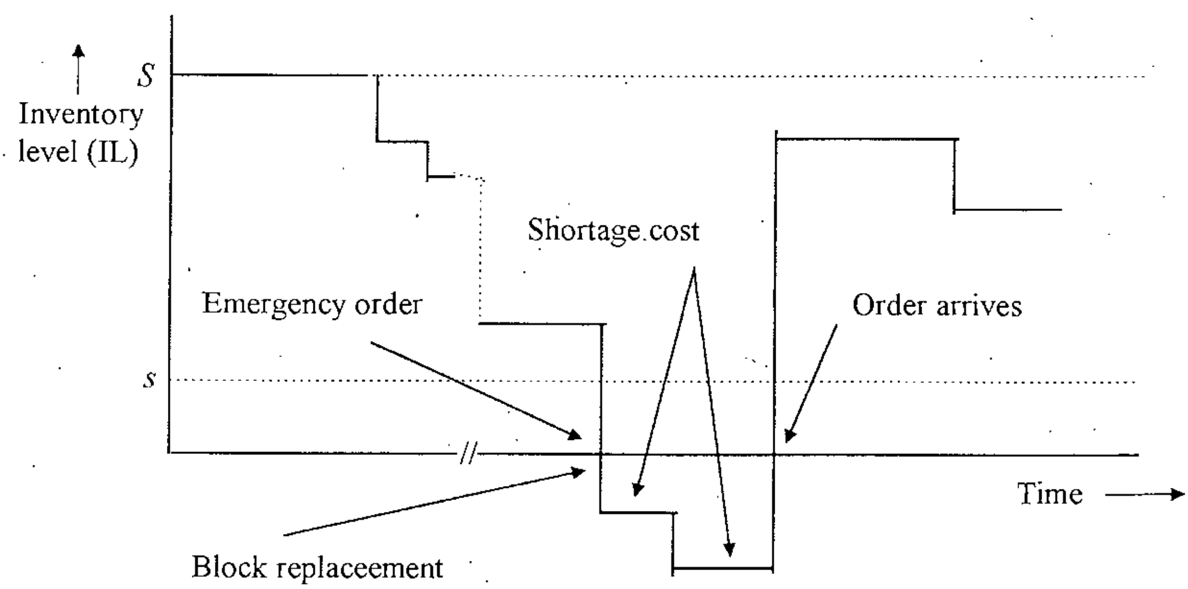
> At time *0*, the inventory level is *S* and the system components starts to work from the beginning of their life time. The next event is the individual breakdown of operating items before they could be preventively replaced, or the block or group-wise replacement of all the components after the replacement period *T*, which ever comes first. Replacement time for the items is considered to be stochastic, depending upon location, detection of the failed item and also spare availability. A regular order for (*S* - *s*) items is placed whenever the inventory level goes equals or below the reorder level s. Order lead times are also stochastic, depending upon the factors external to the system. If a spare is not available when needed, shortage cost is charged for each time unit until this item becomes available. Inventory level and system status are checked after every replacement, breakdown or preventive one. The optimal operating principle is that combination of *S*, *s*, *T* which minimizes the total cost per unit time.

The events that take place during the operation of the system described above are shown in figure 3.4.a and figure 3.4.b below.



Inventory holding cost for S items for time t_1
Failure replacement cost at t_1, t_2 and t_3, stochastic replacement times
Inventory holding cost for S-1 items for time t_2-t_1 and thus on.
Ordering cost at t_3 and order arrives at t_4. Lead time is t_4-t_3
Block replacement cost at t_5

Figure 3.4.a. Events in the simulation model.



Block replacement caused inventory level fall below zero.
Shortage cost for each unavailable item for each time period.
Emergency order placed and order arrives after lead time.

Figure 3.4.b. Events in the simulation model.

3.3.1 Cost Formulations

Maintenance and inventory related costs and assumptions on their nature have been considered in this model that conform with the research objectives. The maintenance costs are the individual breakdown replacement cost per unit item (C_f)and the block preventive replacement cost per unit item (C_b). These two costs are consolidated by the assumption that they include all the direct, standby and degradation costs associated with maintenance work [Niebel, 1985]. This makes it possible to realize a real world system without making the model entangled with complex cost structure.

Among the inventory costs, the most important ones are the costs for carrying and that for shortage of the inventory. The cost of carrying inventory is not simply the storage cost but it includes the cost of the money tied up with the inventory and also other considerations such as cost of insurance. These costs have been pulled under the name of inventory holding cost (C_h) for each spare item hold for each time unit. The cost of not not having the required inventory or out-of-stock cost is accrued to the total cost when a spare part is not available at hand has to be backordered. This type of cost has direct effects on machine idle time and indirect costs such as reputation of the organization etc.. Inventory shortage cost (C_s) is considered to be calculated for each unavailable spare item for each time unit as long as this unavailability remains. The ordering cost includes all the component encountered throughout the inventory item procurement process. It may be regular (C_o) or emergency (C_e) type, depending upon the inventory status at the time of the order placement.

Computation of the total cost for the system over time requires the consideration of the following principles.

1. Ordering cost (C_o or C_e) is incurred after every instant the inventory level (*IL*) goes below the reorder level for the (*S, s, T*) system, and is charged instantaneously.

$$TC_u = TC_p + C_o, \quad \text{for } s \geq IL > 0$$

$$TC_u = TC_p + C_e, \quad \text{for } 0 \geq IL$$

where TC_u is the latest of most recently updated total cost and TC_p is the total cost previous to the placement of the order. The ordering quantity for (*S, s, T*) system is,

$$S - Current\ inventory\ level, \quad \text{for } s \geq IL$$

2. Inventory holding cost is computed for the inventory level between any two events taking place in the system that generate demand for one or more spare parts. If t is the present simulated time and t_{pe} the time of the previous demand or inventory replenishment event then the holding cost is added to the total cost according to the following manner.

$$TC_u = TC_p + C_h(t-t_{pe})IL \quad \text{for } IL > 0$$

3. The inventory shortage cost is accrued for each spare part remaining unavailable for each time unit. When the inventory level falls below zero, no further demands could be satisfied and these are backordered, while the work centers remain idle until the shipment arrives and the inventory level is updated. This cost is calculated in a similar fashion as the holding cost.

$$TC_u = TC_p + C_s(t-t_{pe})IL \quad \text{for } IL < 0$$

4. The individual breakdown cost is calculated in the event of failure of N_i items (where $i = 1, 2, \ldots, 200$) and if required number of spares is available. In case of insufficient inventory, the available spares are exhausted to the work centers. The rest (N_i -IL) items are then back ordered and the disrupted work centers are kept idle until the inventory is replenished. These unsatisfied demands are then charged with inventory shortage cost only.

Taking t as the present simulated time, t_{ne} the time of the next event,

$$TC_u = TC_p + N_i\ C_f, \quad \text{if all demands could be satisfied}$$
$$= Tc_p + IL\ C_f + (N_i - IL).C_s.(t_{ne} - t), \quad \text{otherwise}$$

and

$$TTF_i = t + 1$$

where TTF_i is the next time to failure of i-th item which cannot be replaced due to unavailability of a spare

5. Replacement cost is included in the total cost after the fixed block replacement periods (T) in the following manner.

$$TC_u = TC_p + N_0.C_b \quad \text{if } t = mT \text{ (} m \text{ is any non zero integer)}$$

Where N_0 is the total number of operating item in the system (200 for the present problem). If the inventory level does not permit all the N_0 items to be preventively replaced, block replacement is carried out until the inventory is depleted. After that the rest (N_0 - IL) items are assigned a special mode which enables their replacement, when spares become available. Under such conditions the inventory level falls below zero and shortage cost is invoked accordingly. For symbols carrying meanings as above,

$$TC_u = TC_p + IL.C_b + (N_0 - IL).C_s.(t_{ne} - t)$$

The total cost is zero for $t = 0$ and increases with time as all these various costs are added to it. This continues until a specified simulation period $ST_{max.}$ is reached. After that the total cost may be divided by the simulation period to get the total cost per unit time. The formulation for the manufacturing system downtime, service level of inventory system and accumulation of work-in-process-inventory (WIPI) have been described in the next section.

3.3.2 System Downtime & WIPI Formulations

System Downtime: The system remains down or idle when any replacement is on progress, or no spare is available to satisfy a demand. In this model the replacement time is assumed to be stochastic and therefore the total downtime has to be simulated also with the total cost. The three cases discussed below are to be considered.

1. A failure replacement is taking place. In this case the model generates a replacement time t_r with certain probability distribution. At first the amount of job left on that machine is calculated by the following equation,

$$Job\ Left = Modulus\left[\frac{Present\ simulation\ time}{Time\ units\ required\ to\ complete\ the\ job}\right]$$

The down time DT_n for each production line n is then calculated as,

$$DT_n = Replacement\ time$$

To count in the effect of the number of work centers, a composite term *down time - work center*, has been devised. This is denoted by $DTWC_n$ and is the total accumulation of the product of down time and the number of suffering work centers in a certain production line *n*.

$$DTWC_n = (Number\ of\ work\ center).(DT_n) \qquad \text{if job left} = 0$$

$$= (Number\ of\ work\ center - 1).(DT_n) \qquad \text{if job left} \neq 0$$

2. A block replacement is taking place. Here the calculation logic is the same as the previous case with the exception of the replacement time. In this case the time for block replacement is taken to be the maximum value of all the individual replacement time values for the operating components.

$$Replacement\ time = max.\left|\ replacement\ time\ for\ i\text{-}th\ component\ \right|_{i=1,2,\ldots,200.}$$

$$\text{DTWC}_n = (Replacement\ time).(Number\ of\ work\ centers\ in\ line\ 'n')$$

3. No spare available for replacement. This is the simplest case where the down time is 1 time unit for each event of inventory status checking, since a failed item that could not be replaced due to spare shortage is advanced to the next time unit in an *idle* mode. The down time-work center is then simply 1 unit of time multiplied by the number of work centers

$$DT_n = 1 + Replacement\ time \quad \text{and}$$

$$DTWC_n = (1 + Replacement\ time).\ (Number\ of\ work\ centers\ in\ line\ 'n')$$

Work-in-process-inventory : It is the accumulation of semi-processed products between two work centers. A system with high work-in-process-inventory accumulation indicates poor management and technological efficiency. The work-in-process-inventory ($WIPI_n$), in this study has been computed by multiplying the number of such items with the time for which this inventory exists. To calculate this for each of the '*n*' production lines, amount of job left is at first determined using the expressions given in the previous section. To calculate the accumulated work in process inventory through out a simulation period, the following cases have been considered.

1. The work-in-process-inventory is due to the unavailability of a spare item when any failed item demands replacement and the product(s) must wait until the next time unit.

$$WIPI_n = 1.[(\textit{number of work centers in line } n) - 1] \quad \text{if job left} = 0$$

2. The disrupted work center is the last one in the production line.

$$WIPI_n = (\textit{replacement}\ \text{time}).[(\textit{number of work centers in line } n) - 1]$$

3. The disrupted work center is not the last one in the production line.

$$WIPI_n = (\textit{replacement}\ \text{time}).[(\textit{number of work centers in line } n) - 2]$$

4. A block preventive replacement is taking place in the instant when all the work centers have finished their job and waiting for the next assignment.

$$WIPI_n = (\text{block } \textit{replacement}\ \text{time}).(\textit{number of work centers in line } n - 1)$$
$$\text{if job left} = 0$$

3.4 SIMULATION SOFTWARE AND CODING

SIMSCRIPT II.5 is a general programming language containing the capabilities for building discrete-event, continuous, or combined simulation models. It has the programming features of FORTRAN, ALGOL AND PL/I [Law & Kelton, 1991]. Furthermore its English-like and freeform syntax make SIMSCRIPT II.5 simulation programs almost self-documenting. Because of its general process approach, its sophisticated data structures, and its powerful control statements, this language is often used for large, complex simulation models.

The conceptual simulation model described in the previous sections falls in to the discrete event category because it is evident that the state of the proposed system described in the previous section changes with discrete points in time. All discrete event models share a number of common components and there is a logical organization for these components that promotes the coding, debugging and future changing of the computer program. The following components are expected to be found in most discrete event models and this was no exception to the computer program coded for the present study [Law & Kelton, 1991].

System state: The collection of state variables necessary to describe the system at a particular time.

Simulation clock: A variable giving the current value of the *simulated* time.

Event list: A list containing the next time when each type of event will occur.

Statistical counters: Variable used for storing statistical information about system performance.

Initialization routine: A subprogram to initialize the simulation model at time zero.

Timing routine: A subprogram that determines the next event from the event list and then advances the simulation clock to the time when that event is to occur.

Event routine: A subprogram that updates the system state when a particular type of event occurs.

Library routines: A set of subprograms used to generate random observations from probability distributions that were determined as part of the simulation model.

Report generator: A subprogram that computes estimates (from the statistical counters) of the desired measures of performance and produces a report when the simulation ends.

Main program: A subprogram that invokes the timing routine to determine the next event and then transfers the control to the corresponding event routine to update the system state appropriately. The main program may also check for termination and invoke the report generator when the simulation is over. The logical relationships among these components is shown in figure 3.5 [Law & Kelton, 1991].

The simulation begins at time *0* with the main program invoking the initialization routine, where the simulation clock is set to zero, the system state and the statistical counters are initialized, and the event list is initialized. After control is returned to the main program, it invokes the timing routine to determine which type of routine is the most imminent. If an event of type *i* is the next to occur, the simulation clock is advanced to the time that event *i* will occur and control is returned to the main program. Then the main program invokes routine *i*, where typically three types of activities occur: (1) the system state is updated to account for the fact that an event of type i has occurred; (2) information about system performance is gathered by updating the statistical counters; and (3) the times of occurrence of future events are generated and this information is added to the event list. After all processing has been completed, either in event routine i or in the main program.. a check is typically made to determine if the simulation should now be terminated. If it the time to terminate, the report generator is invoked from the main program to compute estimate (from the statistical counters) of the desired measures of performance and to produce a report. If not, control is passed back to the main program and the main program-timing routine-main program-event routine-termination check cycle is repeated until the stopping condition is eventually satisfied.

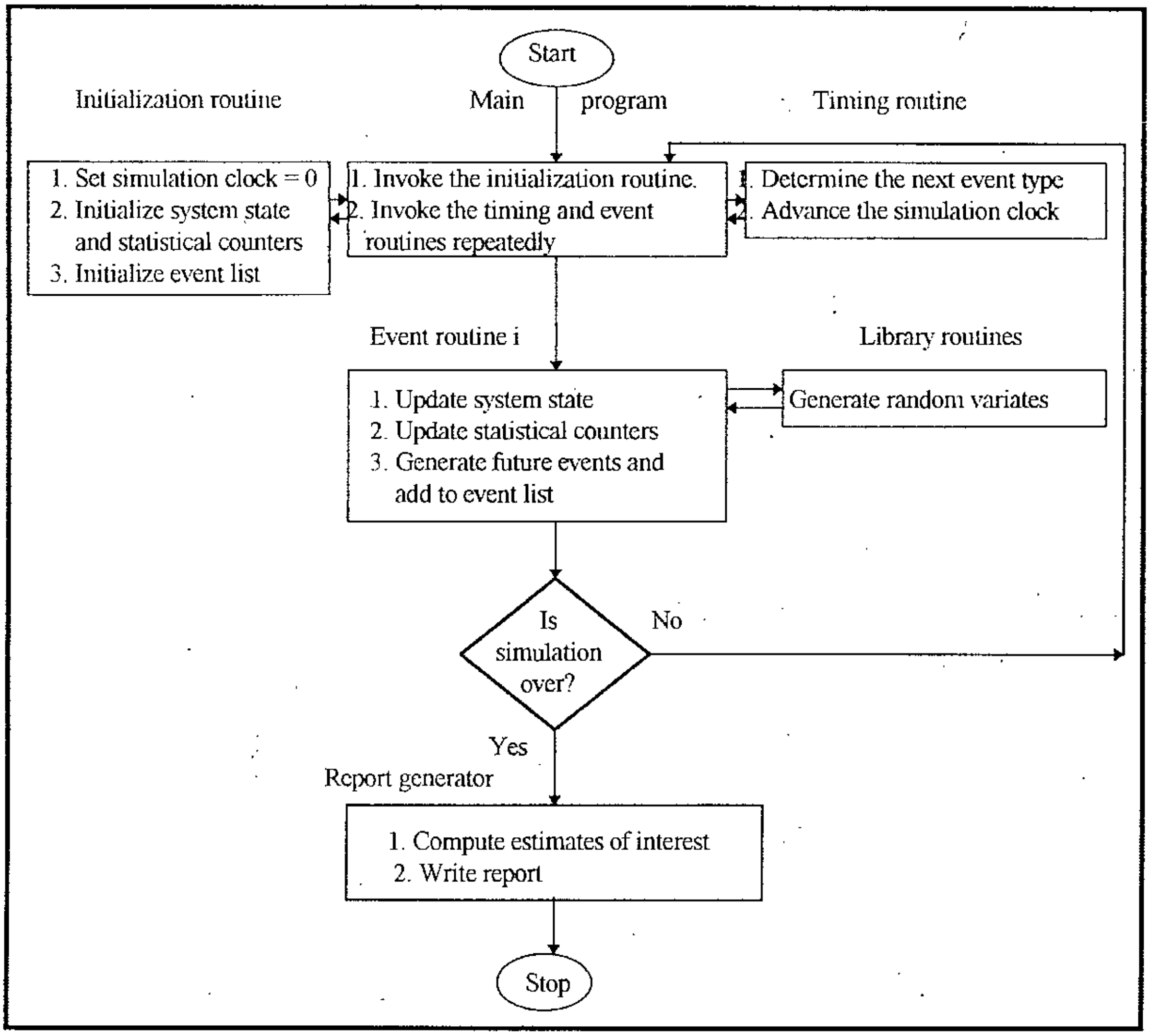


Figure 3.5 Basic structure of a discrete event simulation program

3.4.1 Organization of the Program

A computer program for the conceptual model developed in the previous section has been coded with the PC version of SIMSCRIPT II.5. Features on programming with SIMSCRIPT II.5 can be obtained from [Russell, 1983, and Law & Larmey, 1984]. The developed computer simulation program consists of the following modules. The interdisciplinary structure of these modules for a single data set and a single replication have been given in figure 3.6.

PREAMBLE: It is the introductory module whose statements are declarative and non-executable. This module is used to define all the building blocks, the global variables, the basic time unit of time for the simulation clock, and the desired measure of system performance.

MAIN: This is the module where the execution of a SIMSCRIPT program begins. In the present program this routine is used to read input cost and statistical parameters for the simulation by calling the COST and STATISTICS routines, to initialize the system by calling the INITIALIZE routine, and to place the *initial* event records in to the event list using the ACTIVATE statement. The simulation actually begins after the execution of the START SIMULATION statement, which is nothing but just a call to the in-built timing routine. The control then is transferred between the three basic events, arrival of a regular or expedited order, block preventive and breakdown replacement. These events are described in the CHECK, FAILURE and REPLACEMENT routines respectively. Whichever event takes place, the system operating cost is updated by the routine EVALUATION. Execution of the END statement in the MAIN module will occur only when the event list no longer contains any process notices. The simulation will also terminate if a STOP statement is executed in some routine.

COSTS: This routine is called from the MAIN module to read the input cost parameters. Costs are assigned for both regular and emergency order, inventory holding and shortage.. and block preventive replacement and breakdown replacement.

STATISTICS: This routine called from the MAIN module to assign the values for the statistical parameters for the system. These are the statistical parameters of the component life time and order lead time distributions, variance of replacement time distribution and the number of the presently employed random number generating stream.

INITIALIZE: This routine, when called from the MAIN, assigns the initial state to the system parameters. The operating components are assigned their lifetimes, the performance parameters such as cost, system downtime etc. are set to their initial value of zero. The simulation clock remains still at time equal to zero.

CHECK: This is one of the three event routines that are invoked in the MAIN, demanding the transfer of control to any of these. If the present simulated time indicates an arrival of regular or expedited order, this routine is given the control from the MAIN. The inventory level is updated and the order status is assigned an *off* value.

REPLACEMENT: Control is transferred from the MAIN to this routine at the instants of block preventive replacement. The following actions are taken through this event routine when invoked.

1. A demand for the replacement of all the operating components is created.

2. The items that are due to fail at the present simulated time are identified and replacement is carried out as long as spares are available.

3. All other items are then replaced as long as the inventory level remains above zero.

4. System downtime and work-in-process-inventory accumulation are computed taking the replacement time in account. This is done by calling the routines DOWNR and WIPIR.

5. If there are some items that are failed but still could not be replaced due to spare shortage are assigned to remain failed until the next time unit and the routines DOWNR and WIPIR are called again.

6. If there are some items that are still operating, then these are assigned a special mode which enables the program to mark them and carry out replacements whenever spares are available.

7. The block preventive replacement costs are calculated according to the cost formulations and the total cost is updated.

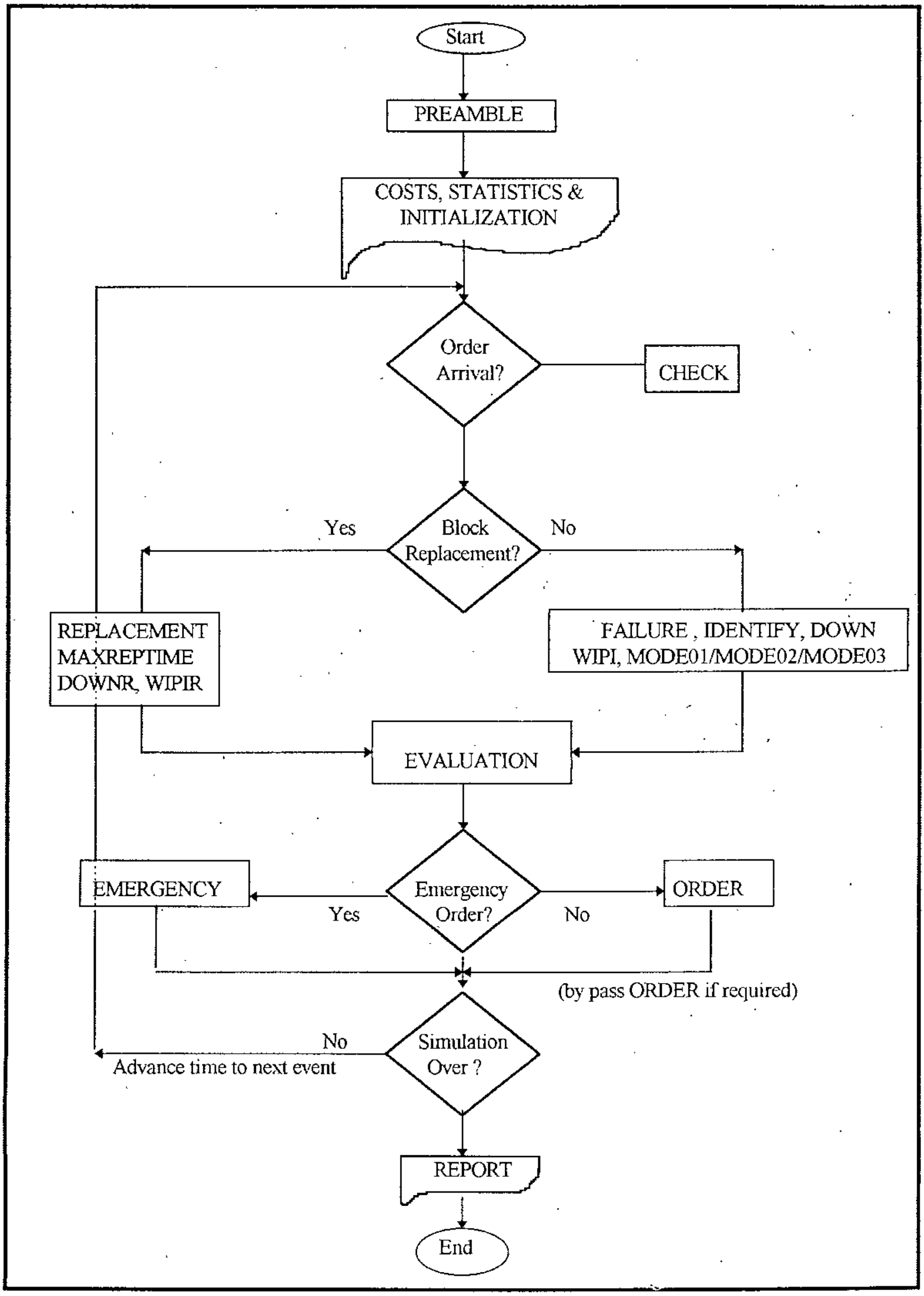


Figure 3.6. Various modules and transfer of control among them.

FAILURE: This routine is also invoked from the MAIN when the present simulation time coincides with the breakdown time of one or more of the operating components. The following actions are taken accordingly.

1. The total number of the components failed on this instant is determined after identifying the disrupted work centers by calling the routine IDENTIFY. The demand for spares is created for these items. Failed items are replaced with new ones if spares are available.

2. In case of spare unavailability, the work centers are kept inoperable until the next time unit.

3. A check is carried out if there are such items to be replaced as stated in the action number six in the previous routine. These items are replaced if spares are available.

4. Routines DOWN and WIPI are then called to update the system downtime and work-in-process accumulation.

IDENTIFY: This routine is called from FAILURE and is intended to locate the work center and production line whenever an item breaks down. It also generates the replacement times for the item. This is done according to the probabilities considered in table 3.1 and the mean replacement times given in table 4.1 of Chapter Four.

DOWN: This routine is called from FAILURE and is intended to calculate the system downtime accumulation throughout the simulation period. It calls the routines MODE01, MODE02 and MODE03 in case of any unsatisfied demand in the work centers in the production lines 1, 2, and 3. For all satisfied demands, it calculates the system downtime according to the formulations given in section 3.3.2 of this chapter.

WIPI: This routine is called from FAILURE to calculate the work-in-process-inventory accumulation throughout the simulation period according to the formulations given in section 3.3.2 of this chapter.

MODE01/MODE02/MODE03: These routines are also called from FAILURE. They calculate the system downtime when any failed item located in the production lines 1 or 2 or 3 could not be replaced due to spare shortage.

MAXREPTIME: This routine is called from REPLACEMENT to determine the block preventive replacement time for all the operating components.

DOWNR: This routine is called from REPLACEMENT to calculate the system downtime during the block preventive replacement time for all the operating components.

WIPIR: This routine also is called from REPLACEMENT to calculate the work-in-process-inventory (if any) during the block preventive replacement time for all the operating components.

EVALUATION: This routine is called from MAIN to calculate the total operating cost for the simulation period. It also checks the inventory level and prevailing order status to create a regular or emergency order.

ORDER: This routine is called from EVALUATION to dispatch a regular order spare parts. An order for (*S - present inventory level*) items for the (*S, s, T*) system is placed if the order status is in *off* state. The delivery time is then determined.

EMERGENCY: This routine is activated from EVALUATION whenever the inventory level falls below zero. The actions taken are similar to those described in the previous routine.

REPORT: This routine is activated from MAIN after the simulation is over. It prints the system and cost parameter inputs and also the output values of total cost per unit time, system downtime and work-in-process-inventory accumulation for the simulation period.

CHAPTER FOUR

DESIGN OF EXPERIMENTS

- Introduction
- Input Data Structure
- Experimental Design
- Simulation Procedure
- Output Data Analysis

4.1 INTRODUCTION

This chapter deals with the selection of input probability distributions, design of experiments and output data analysis, one by one. For a valid and credible model, the selection of input data distributions is extremely important. A valid stochastic simulation model has the primary requirement of the selection of appropriate probability distributions, so that the conceptual model is an accurate representation of the system under study.

After the development of a valid and credible simulation model, the next task is to design the experiments or the production runs of the program. This is the determination of initial conditions for the simulation runs, the length of warm-up period, the length of simulation runs and the number of independent simulation runs [Law and Kelton, 1991]. Equally important is the output data analysis. This is due to the fact that simulation is nothing but computer based statistical sampling experiment. Simulation results, therefore, must be passed through standard statistical analysis techniques before drawing inferences that are supposed to be representative enough to be acceptable to the physical system.

4.2 INPUT DATA STRUCTURE

To develop a valid simulation model, the step next to problem formulation is the collection of information and data on the system of interest. Sometimes they are applied directly in the simulation as in *(trace-driven)* simulation, or an empirical distribution is defined by the data values. Standard techniques of statistical inference are frequently used to `fit' a theoretical distribution form [Law and Kelton, 1991]. The model developed in the present study is rather complex one. Both maintenance and inventory related data are difficult to be apportioned. This makes the collection of such real time data extremely difficult. The same is true for data on life time of operating equipment, the replacement times for single failure and block replacements.

The problems in data collection, thus undermine the validity of the model. Law and Kelton suggests two heuristic procedures to cope with such problem [Law and Kelton, 1991]. One is to identify an interval [a, b] in which the 'experts' feel that the random variable will lie with probability alone to 1, and then use this uniform distribution for that random variable. Another procedure is to set up a triangular density function (a, b, c) by incorporating the model (c) of the distribution with the intervals (a, b). Both of these procedures require the subjective estimates of field experts and apparently, they could not be employed in this research.

The probability distributions of the various random variables, therefore had to be assumed as theoretical ones. Choice for input probability distribution were further validated by investigating the previous researchers as described in the literature review.

4.2.1 Selection of Input Distributions

Component Life Time and Order Lead Time Distributions : For all the case problems discussed in the next section, Weibull distribution has been used to represent both unit failure time and order lead time distributions. This continuous distribution, widely known as a *family of distribution*, [Kabir & Olayan, 1995a] is versatile and play a very important role in the statistical analysis. In SIMSCRIPT II.5, pseudo-random data, matching Weibull distribution can be generated by calling the in-built routine Weibull.f (α, β, n), where n is the number of random number generating stream. The selection of the values of the shape and scale parameters (α_t, β_t for item life time and α_o, β_o for order lead time) have been given in the next section which describes the experimental setup. The statistical information on the Weibull distribution is given in Appendix A along with the typical shapes that can be produced by the Weibull distribution are shown for the failure density function, cumulative failure distribution and hazard rate.

Item Replacement Time Distribution : The model has provisions for both individual failure and block preventive replacements. Law and Kelton [Law & Kelton, 1991] suggests that the distribution of time to complete some task (item replacement, in this case) suits Gamma distribution the most. Accordingly the replacement times have been chosen to follow this distribution. Like the Weibull distribution, Gamma distribution also is a two parameter (α, β) continuous probability distribution. In SIMSCRIPT II.5, the data matching gamma distribution are generated by calling the in-built routine Gamma.f (μ, σ, n), where μ is the mean, σ the variance and n, the number of random number generating stream. The table 4.1 shows the data regarding the mean of the replacement times for the various work centers in the proposed system. The variance has arbitrarily been taken to be 4.0 for all the cases. In case of block replacement, the maximum amount of replacement time required for any of the component has been taken as the replacement time for the whole system. Statistical information along with the various reliability functions of the Gamma distribution has been given in Appendix A.

Production line 1	Mean Time	Production line 2	Mean Time	Production line 3	Mean Time
Work Center A	0.4	Work Center E	0.4	Work Center J	0.3
Work Center B	0.5	Work Center F	0.3	Work Center K	0.4
Work Center C	0.3	Work Center G	0.4	Work Center L	0.3
Work Center D	0.4	Work Center H	0.4	Work Center M	0.3
*	*	Work Center I	0.5	*	*

Table 4.1. Mean replacement times for the items located in various work centers.

4.2.2 Selection of The Cost Values

Rigorous trial runs have been conducted to test the models output to small changes in the input parameters. This was done for estimating the various cost parameters in the model. The cost data have been selected close to those employed by Kabir and Olayan [Kabir & Olayan, 1995a] after test for variance and sensitivity to output.

Inventory Related Costs: The inventory related costs are regular and emergency ordering costs, inventory holding and shortage costs. The regular ordering cost (C_o) has been taken from *5* Taka to *40* Taka with increments of *5* Taka. Emergency ordering cost (C_e) has been taken to be three times that of the regular ordering cost, other parameters remaining constant. The inventory holding cost per spare part per unit time (C_h) has been varied from *0.25* Taka to *2.50* Taka with increments of *0.25* Taka per spare part per time unit. Similarly, the inventory shortage cost has been varied from *6* Taka to *24* Taka with increments of *2* Taka per spare part per unit time.

Maintenance Related Costs: Maintenance related costs are the costs incurred during either, i. failure replacement for individual item or, ii. preventive block replacement for a group of items before failure. The cost of failure replacement per item (C_f) has been taken from *20* Taka to *100* Taka with increments of *10* Taka. Block replacement cost (C_b) has been varied from *10* Taka to *50* Taka with increments of *4* Taka per replacement.

4.3 EXPERIMENTAL DESIGN

From the chosen values of the various input parameters, a number of data groups have been designed to study the effect of these parameters on system response. The simulation program has been executed with these data sets with necessary replications. This is due to the fact that randomization of experiments is essential to smooth out the effects of dependency or auto-correlation, if the generated variable data are supposed to be truly random or independent [Cochran & Cox, 1957]. Otherwise the simulation results seem to be worthless. SIMSCRIPT II.5 provides 10 random number generator streams with which the replications have been done to randomize the experiments and get representative output. The experiment consisted of testing the system with the following data set.

Study On Life Time Shape Parameter: The employed data sets given below are designed to test the effect of the shape parameter of the item life time distribution α_t.

β_t	α_o	β_o	C_o	C_f	C_b	C_h	C_s
50	3.0	5	10.00	60.00	10.00	0.50	12.00

Data Set	1	2	3	4	5	6	7	8	9	10
α_t	2.00	2.25	2.50	2.75	3.00	3.25	3.50	3.75	4.00	4.25

Table 4.2.a & 4.2.b. Data sets for life time distribution parameters.

<u>Study on Order Lead Time Shape Parameter:</u> The employed data sets to study the effect of order lead time distribution shape parameter are,

α_t	β_t	β_o	C_o	C_f	C_b	C_h	C_s
4.00	50	5	10.00	60.00	10.00	0.25	12.00

Data Set	1	2	3	4	5	6	7	8	9	10
α_o	2.00	2.25	2.50	2.75	3.00	3.25	3.50	3.75	4.00	4.25

Table 4.3.a & 4.3.b Data sets to study the effect of order lead time distribution shape parameter.

<u>Study on Ordering Cost:</u> The data sets employed to study the effect of regular ordering cost are,

α_t	β_t	α_o	β_o	C_f	C_b	C_h	C_s
4.00	50	3.00	5	60.00	10.00	0.50	12.00

Data Set	1	2	3	4	5	6	7	8	9	10
C_o	5.0	10.0	15.0	20.0	25.0	30.0	35.0	40.0	45.0	50.0

Table 4.4.a & 4.4.b. Data sets to study the effect of regular ordering cost.

<u>Study on Inventory Holding Cost:</u> The following data sets have been taken to test the effect of inventory holding cost on the total cost optimality.

α_t	β_t	α_o	β_o	C_o	C_f	C_b	C_s
4.0	50	3.0	5	10.00	60.00	10.00	12.00

Data Set	1	2	3	4	5	6	7	8	9	10
C_h	0.25	0.50	0.75	1.00	1.25	1.50	1.75	2.00	2.25	2.50

Table 4.5.a & 4.5.b. Data sets to study the effect of inventory holding cost.

Study on Inventory Shortage Cost: The following data sets have been utilized to study the effect of inventory shortage cost on total cost optimality.

α_t	β_t	α_o	β_o	C_o	C_f	C_b	C_h
4.00	50	3.00	5	10.00	60.00	10.00	.050

Data Set	1	2	3	4	5	6	7	8	9	10
C_s	6.0	8.0	10.0	12.0	14.0	16.0	18.0	20.0	22.0	24.0

Table 4.6.a & 4.6.b. Data sets to study the effect of inventory shortage cost.

Study on Block Replacement Cost: The data sets utilized to study the effect of block replacement of the operating items have been given below.

α_t	β_t	α_o	β_o	C_o	C_f	C_h	C_s
4.00	50	3.00	5	10.00	60.00	0.50	12.00

Data Set	1	2	3	4	5	6	7	8	9	10
C_b	10.0	14.0	18.0	22.0	26.0	30.0	34.0	38.0	42.0	46.0

Table 4.7.a & 4.7.b. Data sets to study on the effect of block replacement cost.

Study on Failure Replacement Cost: The effect of the failure replacement cost per item on system total cost optimality has been tested with the following set of data.

α_t	β_t	α_o	β_o	C_o	C_b	C_h	C_s
4.00	50	3.00	5	10.00	10.00	0.50	12.00

Data Set	1	2	3	4	5	6	7	8	9	10
C_f	20.0	30.0	40.0	50.0	60.0	70.0	80.0	90.0	100.0	110.0

Table 4.8.a & 4.8.b. Data set to study the effect of individual item failure replacement cost.

4.4 SIMULATION PROCEDURE

After setting the experimental setup, the job left remains determination of the procedure to search for the optimum values of the decision variables. These are the parameters of continuous inventory review technique, the optimal values of maximum allowable stock level (S), reorder level (s) and the parameter of block preventive maintenance, the block replacement interval (T). The search procedure is described below with a logic diagram in figure 4.1.

a. Selection of the ranges of the values of the decision variables has been made at first. Priority has been given to the choice for suitable values for the system and at the same time avoid heavy computational time. The maximum allowable stock level S and the stock reorder level s have been varied from 200 to 250 units of spare items and 0 to 50 units of spare items respectively with increments of 10 units. The jointly optimal block replacement interval T has been searched from 50 to 350 time units with increments of 25 time units. In the cases the optimal values of T (separately or sequentially optimized) exceed above the search range, it is assumed that failure replacement is the better maintenance policy in those cases. Results falling to this category could be found in Chapter Five with '*' (asterisk) signs in places of optimal values of T.

b. Determination of the warm-up period for the model. This is the period after which the output data shows steady state characteristics.

c. Execution of the simulation program with the chosen values of the system parameters and decision variables. Jointly optimal results were searched from the various combinations of the decision variables together rather than considering them separately or sequentially. Later on the system has been optimized for both maintenance and inventory functions sequentially.

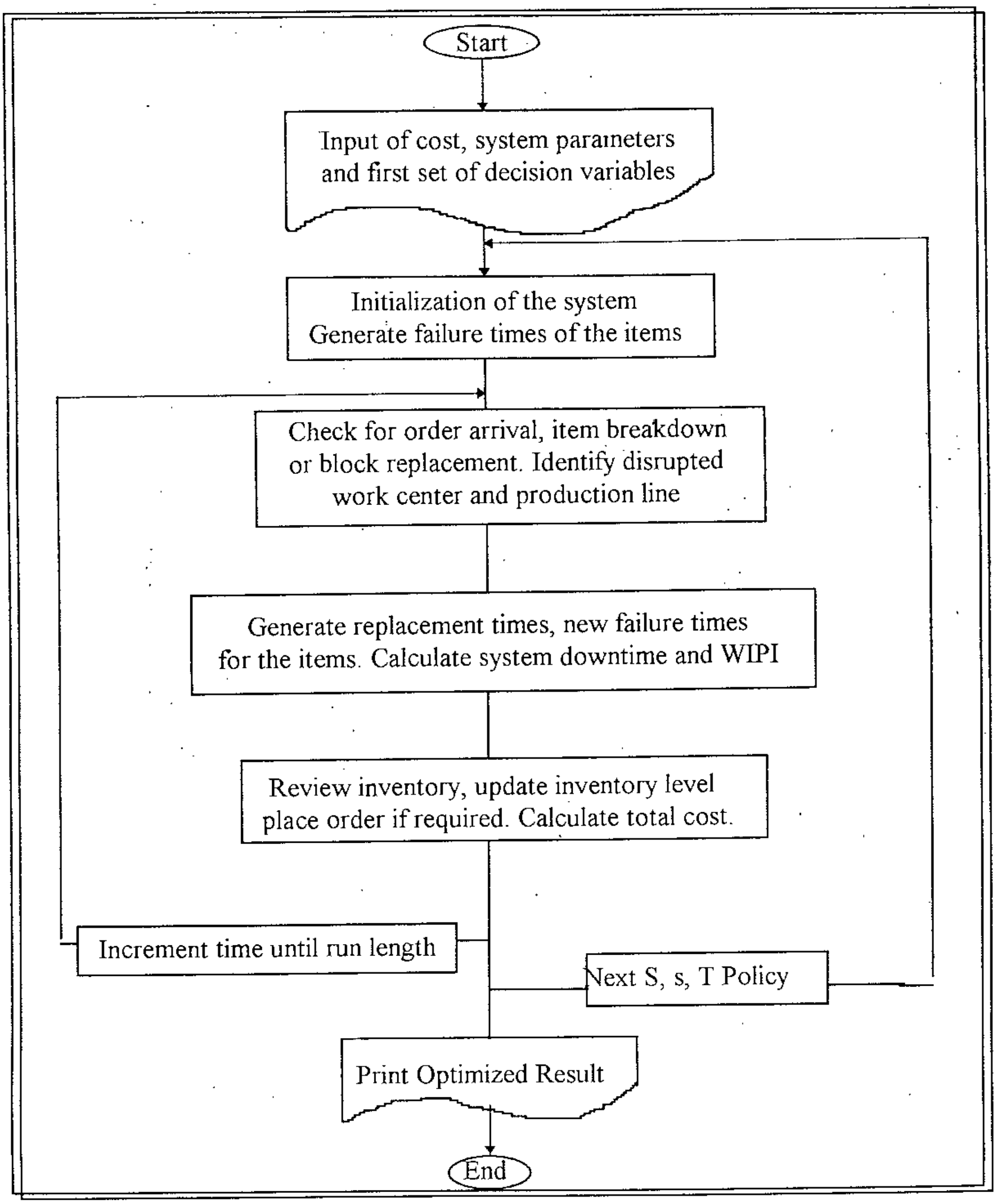


Figure 4.1. Logic diagram of the simulation procedure in terms of system events.

d. Repetition of (c) for 10 random number streams. The final results have been taken to be the mean of these replications after the construction of 90% confidence intervals for these estimated values.

e. Determination of the sequentially optimized values of S, s, T. The optimal inventory related decision variables are computed by ignoring the maintenance related costs. The same technique has been followed to determine those for maintenance function also.

f. Investigation on the cost optimality of the jointly optimized policy by comparing the cost values of jointly optimized and the aggregate of the sequentially optimized maintenance and inventory policies.

g. Repetition of (d), (e) and (f) for all designed data points in a data set.

h. Repetition of above steps for all data sets.

4.5 OUTPUT DATA ANALYSIS

The developed simulation model has been found to be a non-terminating type. Such type of simulation does not have any specified condition to set the length of a simulation run. [Law & Kelton, 1991]. A measure of performance for such a simulation is said to be a steady state parameter if that parameter possesses the characteristics of the steady state distribution of any stochastic process output. Non-terminating and steady state type simulation results do not depend on the initial conditions. The figure 4.2 shows the typical response curve for such output. If the random variable Y in the figure has the steady state distribution, the most important estimate would be the steady state mean value $\nu = E(Y)$.

The problem is then to warm up the model or deletion of initial data. The simplest and most general technique for determining the warm-up period is the graphical procedure devised by Welch [Law & Kelton, 1991]. Its specific goal is to determine a time index such that $E(Y_i) \approx \nu$ for $i > l$, where l is the warm-up period. The inherent variability of the process Y_1, Y_2, ... makes it difficult to determine l from a single replication. Welch's procedure is based on n independent replications and the following steps.

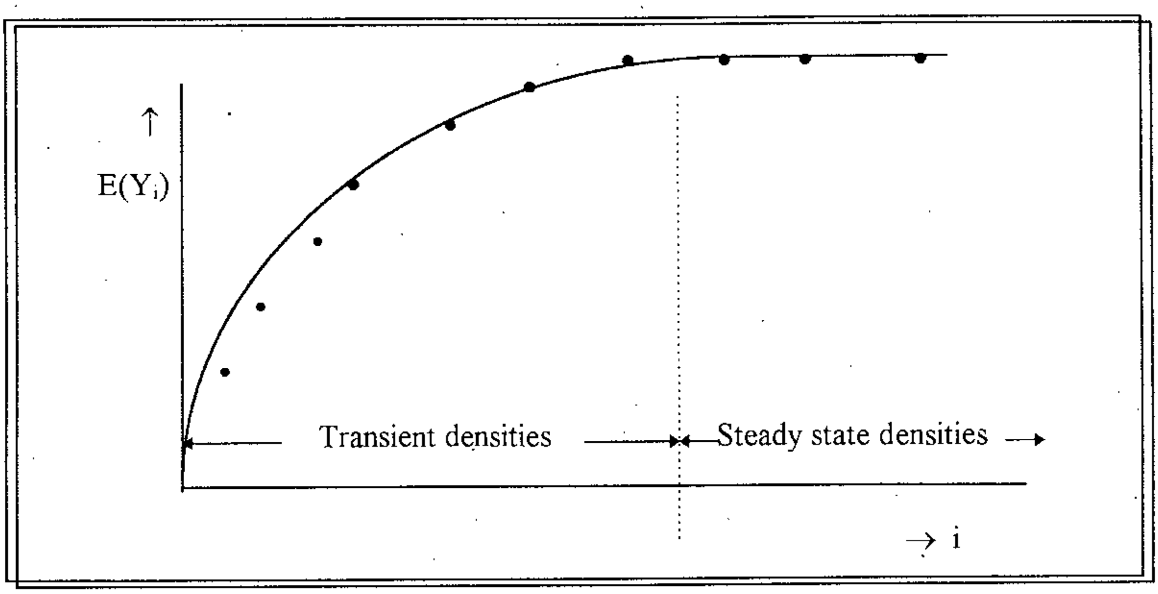


Figure 4.2. Transient and steady state density functions for a particular stochastic process Y_1, Y_2, ..., and initial condition i.

1. Make n replications ($n \geq 5$), each of length m (where m is large). Let Y_{ji} be the i-th observation from j-th replication ($j = 1, 2, ..., n$; $i = 1, 2, ..., m$).

2. Let $\bar{Y}_i = \sum_{j=1}^{n} Y_{ji}/n$ for $i = 1, 2, ...m$.

3. To smooth out high frequency oscillations, a moving average has to be defined by $\bar{Y}_i(w)$ (where w is the *window* and is a positive integer such that $[m/2] \geq w$).

$$\bar{Y}_i(w) = \sum_{s=-w}^{w} \bar{Y}_{i+s}, \quad \text{if } i = w + 1, \ldots\ldots, m - w$$

$$= \sum_{s=-(i-1)}^{i-1} \bar{Y}_{i+s}, \quad \text{if } i = 1, 2, \ldots\ldots, w$$

4. Plot $\bar{Y}_i(w)$ for $I = 1, 2, ..., m - w$ and choose l to be that value of i beyond which $\bar{Y}_1(w)$, $\bar{Y}_2(w)$, ..., appear to be converged.

The determination of mean and variance of the statistical samples from simulation have been carried out by employing the replication/deletion approach. This approach obtains a point estimate and construct confidence interval for this estimate of ν [Law & Kelton, 1991]. It is assumed that n' replications of the simulation are made, each of length m', where m' is much larger than the warm-up period l. The output system response X_j from the warm-up period to run length, for j-th replication is then,

$$X_j = \sum_{i=l+1}^{m} Y_{ji}, \text{ for } j = 1, 2, \ldots\ldots n'$$

Thus it can be said that X_j 's are random variables with $E(X_j) \approx \nu$. $\overline{X}(n')$ is then said to be an approximately unbiased point estimate of ν, and an approximately 100(1 - α) percent confidence interval for ν is given by,

$$\overline{X}(n') \pm t_{n'-1,1-\alpha/2}\sqrt{\frac{S^2(n')}{n'}}$$

where $\overline{X}(n')$, and $S^2(n')$ are computed by the following two equations [Law & Kelton, 1991].

$$\overline{X}(n') = \frac{\sum_{i=1}^{n'} X_i}{n'}, \text{ and}$$

$$S^2(n') = \frac{\sum_{i=1}^{n'} [Xi - \overline{X}(n')]^2}{n' - 1}.$$

CHAPTER FIVE

RESULTS AND DISCUSSION

- Introduction
- Computational Experience
- Results on Cost Effectiveness
- Results on Downtime & WIPI

5.1 INTRODUCTION

This chapter presents the numerical results obtained after the previously designed experimentation. The next section contains the results on the warm up period determination. The pattern of output total cost and determination of the optimal point is then elaborated with the output for a sample data set. The results for all the data sets have been presented in both tabular and graphical forms. Effects of the various system and cost parameter on system performance in terms of total cost, downtime and work-in-process accumulation have been analyzed for the (*S, s, T*) model.

5.2 COMPUTATIONAL EXPERIENCE

The execution of the simulation program with the experimental design described in the previous chapter yielded the output data of the present research. The following procedures and a numerical example below demonstrate the computational effort required in collection and processing of the output data and their conversion into representative results of the study.

The following data set has been utilized to produce the output of the numerical example: (α_t: 4.0, β_t: 50, α_o:3.0, β_o: 5, C_o: 10.0, C_f: 60, C_b:10.0, C_h: 0.50, C_s: 12.0). All the graphs and consequent analyses in this section have been drawn after experimentation with this data set. The table 5.1 provides the sample computation of mean and confidence interval of the output obtained after 10 replications of the experiment. The output values have been obtained by deleting the transient initial outputs produced before the attainment of warm-up period.

At first the mean and variance values of the outputs of the 10 replications have been calculated using formulae given in the previous chapter. The 90% confidence interval (or expectation that 90% of the collected output data will fall within this interval) was constructed by getting the value of t distribution for $\alpha = 0.05$ and *9* degrees of freedom. This value was consecutively added and subtracted with the mean value to determine the confidence level. The values in the above sample calculation were for the jointly optimized maintenance and inventory policy. The same procedure has been applied for the output of the sequential optimization of the policies.

Replication No.	Total cost/unit time	Mean	Variance	90% confidence interval
1	175.37			
2	178.92			
3	185.73			
4	180.16			
5	172.44	181.63	29.72	184.79 – 178.47
6	187.69			
7	181.33			
8	183.47			
9	179.72			
10	191.80			

Table 5.1. Sample calculation of output data mean and construction of confidence interval.

5.2.1 Determination of the Warm-up Period

Experimentation with the simulation model developed in Chapter Three has been carried out with a sample data set. Output cost data have been obtained for this data set for *50* run lengths from *0* to *10000* time units. For each run length, the program was executed with each of the ten random number generating stream and the means of these values were taken. The graphical method devised by Welch as described in section 4.4 of Chapter Four has been employed to find out the warm-up period. The figure 5.1 shows the graphical presentation of the raw mean total cost data for the run lengths and the refined ones using a window value of *5*. From the figure it is clear that a warm-up period of *2000* time units is sufficient to filter the initial transient outputs.

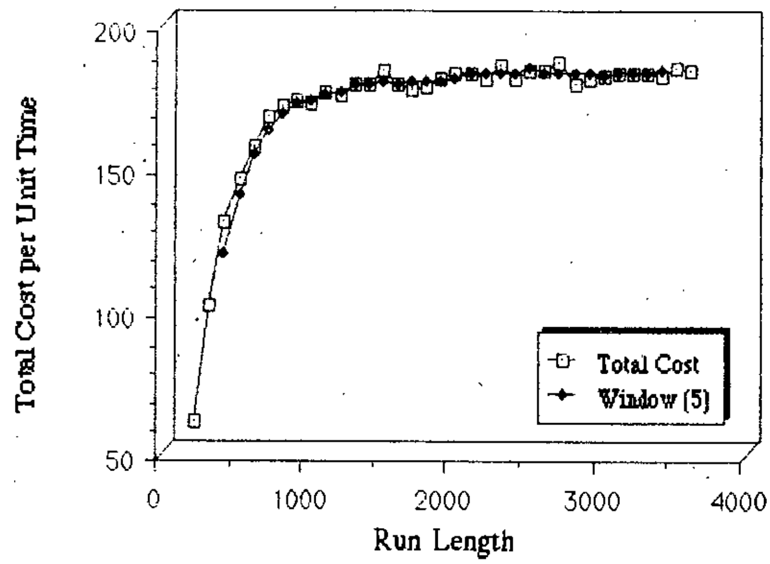


Figure 5.1. Graphical determination of the warm-up period.

5.2.2 Effect of Maximum Inventory Level on Total Cost

Experiment has been conducted to test the effect of the decision variable maximum inventory level (*S*) on the total operating cost per unit time when other parameters aree fiiixed. The experiment has been conducted for ten replications and each time the data for the first *2000* time units have been deleted. The cost values have then been obtained by calculating the mean of these replications. For each set of replications, variances have also been calculated to construct *90%* confidence intervals for the output data according to the procedure described in section 4.4 of the previous chapter. In fact this procedure has been employed for all the experiments described hereafter. The obtained cost function as shown in figure 5.2 has been found to be a convex one. This is because lower values of (*S*) implies lesser inventory on hand and initiates more frequent orders. It may even induce significant inventory shortage cost. On the other hand, keeping larger maximum inventory level is penalized by substantial load of inventory holding cost.

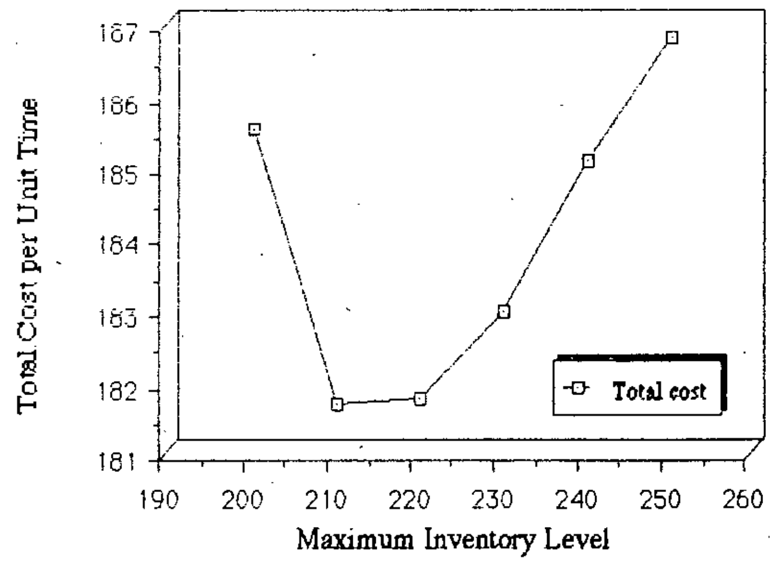


Figure 5.2 Effect of maximum inventory level on total operating cost per unit time.

5.2.3 Effect of Reorder Level on Total Cost

The figure 5.3 shows the effect of the reorder level (s) on the total operating cost per unit time. The cost values have been obtained for various values of the reorder level while keeping the other parameters constant. The cost function in this case has also been found to be convex. This can be explained due to the fact that smaller reorder levels cannot guard against sudden large demands (those in cases of block replacements) and thus cause it to incur higher inventory shortage cost. At the same time, larger reorder level means a considerable inventory has to be carried throughout, which again, increases the total cost.

5.2.4 Effect of Block Replacement Period on Total Cost

Experimentation on the developed simulation model have been carried out to investigate the effect of block replacement period on total cost. The computer program was executed for a single set of (*S*, *s*) values and different values of block replacement periods while keeping the other parameters constant. The cost function, as shown in figure 5.4 has not been found to be strictly convex. It rather shows several minimum points with an overall

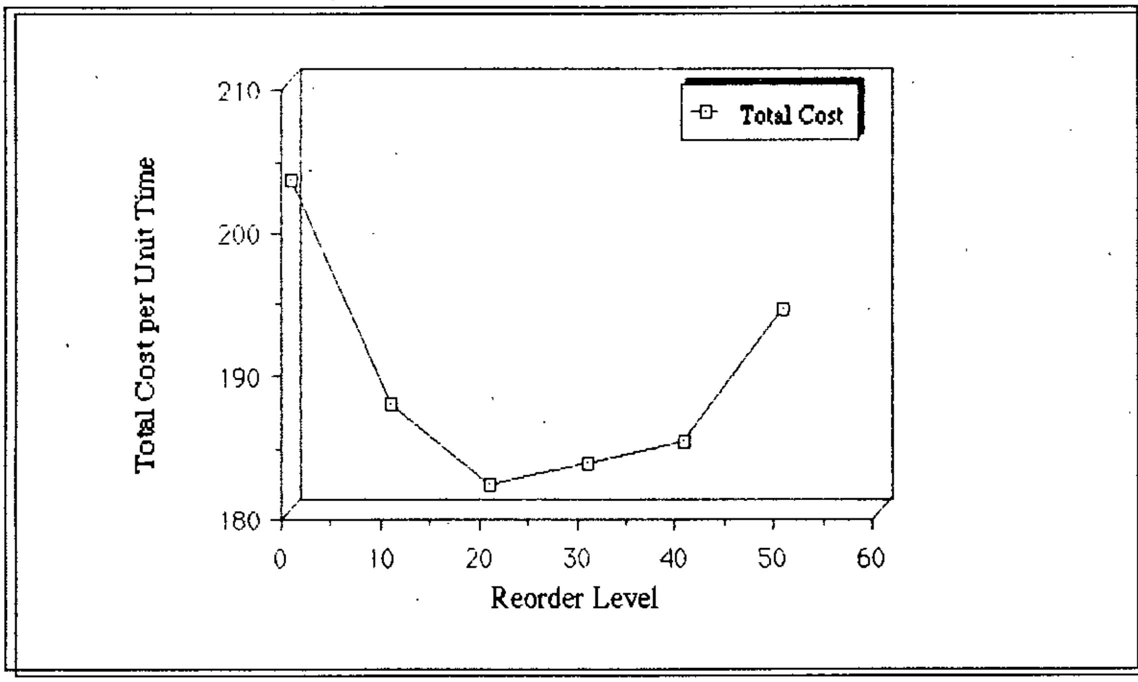


Figure 5.3. Effect of reorder level on the total operating cost per unit time.

downward trend. The corresponding cost values considering the same data and individual replacement instead of block replacement have also been shown. Riggs [Riggs, 1981] suggests that the first minimum point of the cost function is taken to be the optimum point if this value is lesser than the cost obtained considering individual breakdown costs only. All the optimum total costs per unit time given in the next sections have been determined in the same manner.

5.3 RESULTS ON COST EFFECTIVENESS

After the determination of the warm-up period of the simulation model.. numerical results and analysis on the investigation of the various statistical and cost parameters on the behavior of the proposed (*S*, *s*, *T*) system have been presented along with necessary tables and figures. Corresponding values considering separate optimization of the (*S*, *s*) and (*T*) policies have also been juxtaposed to compare their relative cost effectiveness. The values tabulated are the means of the ten replications. The confidence intervals for the experiments have been given in Appendix B. The obtained output indicates that the combined optimization of maintenance and inventory functions is more cost-effective than that of the

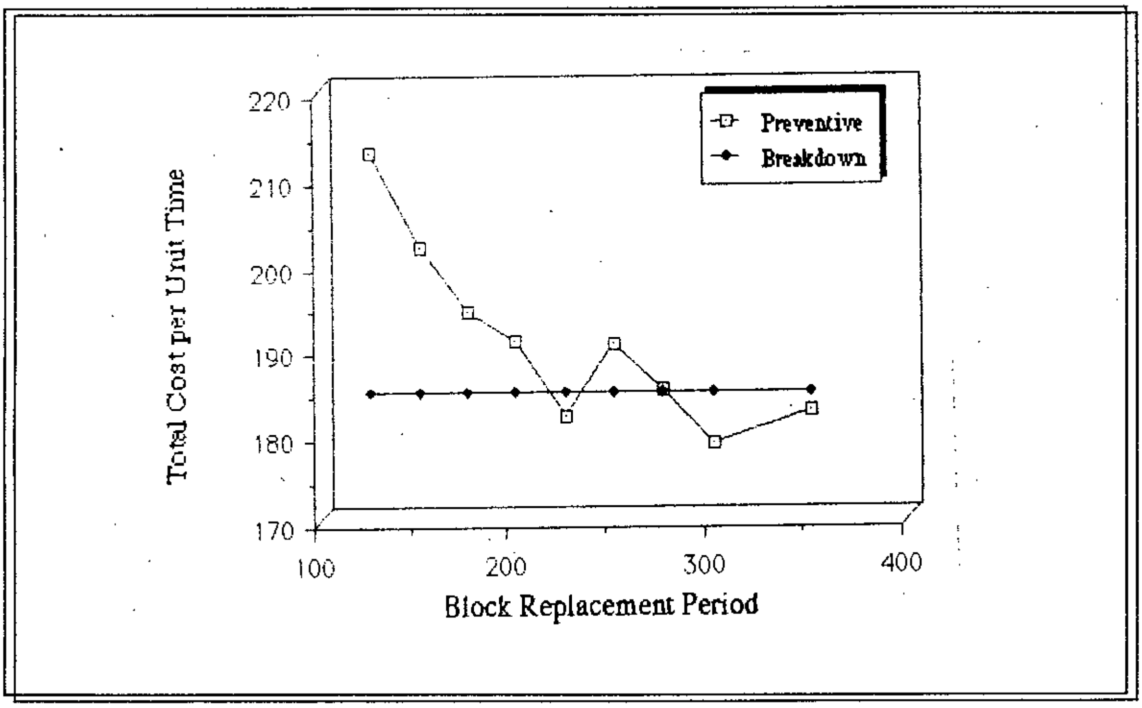


Figure 5.4. Effect of block replacement period on the total operating cost per unit time.

combination of their separate optimizations. This is because all the three decision variables get chances to adjust themselves in order to produce optimal solutions. This is not possible with their separate optimizations. For an example, any variation in an inventory related variable (S or s) will not affect the maintenance policy so that an appropriate policy can never be devised.

5.3.1 Effects of Component Life Time Shape Parameter

Table 5.2 shows the optimal values of the decision variables of the combined optimized maintenance and inventory policy.

Data Set	Jointly Optimized Decision Variables				Separately Optimized Decision Variables			
*	S	s	T	Total Cost	S	s	T	Total Cost
1	200	10	225	176.82	230	10	225	183.40
2	210	10	225	180.19	230	10	225	183.40
3	210	20	225	181.63	230	20	225	197.59
4	210	20	225	181.63	230	20	225	196.32
5	210	20	225	181.63	240	30	225	197.69
6	220	30	250	185.56	240	30	225	197.69
7	230	30	250	185.79	240	30	225	197.69
8	230	40	250	186.90	240	40	225	203.23
9	240	40	275	190.37	240	40	225	203.23
10	250	40	275	192.11	240	40	225	203.23

Table 5.2. Results of the study on the item life time shape parameter on system behavior.

The results indicate that the component life time shape parameter has significant effects on the decision variables whether they are optimized combined or separately. The resultant of these effects are manifested in the values of total operating cost per unit time as shown in figure 5.5. The most significant bearing of the shape parameter is on the block replacement interval (*T*). This is explained due to the fact that the smaller values of the life time shape parameter implies smaller hazard rates. This causes more and more early failures of the operating components. In fact, the components do not undergo the actual aging process. As a result, large numbers of the components are to be replaced between two consecutive block replacement instants which render the notion of block replacement futile. No optimal solution for separate block replacement maintenance policy has been found for the first two data sets which implies that failure maintenance proved to be the best policy in these cases. The jointly optimized system reacted to this phenomenon by decreasing the block replacement period and increasing the inventory level to satisfy all the demands for the spares. Naturally it could not yield the cost effective solutions.

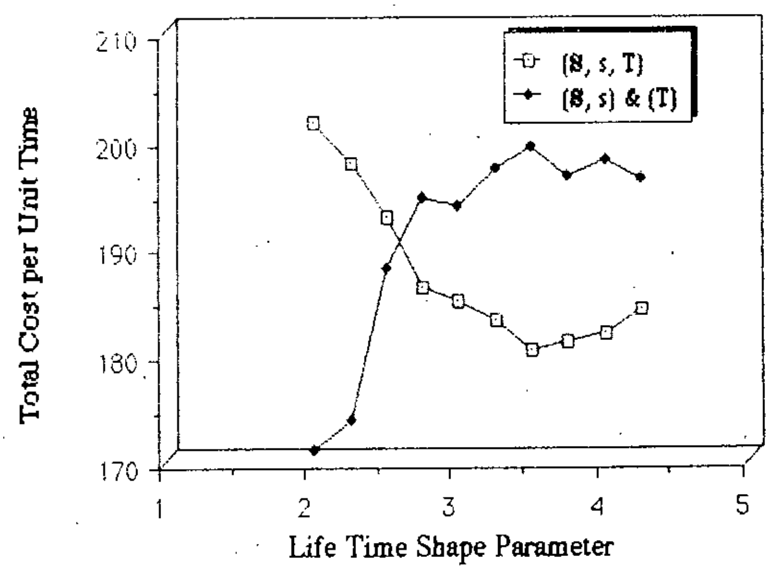


Figure 5.5. Effect of component life time shape parameter on total cost per unit time.

With the increase in values of the shape parameter the operating component show age based failure and optimal solutions could be found for both joint and separate optimization. However separate optimization can not count for the effects of inventory and maintenance related costs at the same time. This causes the extra charge due to the relatively high inventory level required and for these cases, the joint optimization prove to be more cost effective. The aging of the components induce a defined failure pattern rather than a sporadic one. Consequently the inventory related variables (S, s) could be optimized along with the maintenance related variable (T). These effects have been illustrated in figure 5.6 next page.

5.3.2 Effects of Order Lead Time Shape Parameter

The results of the investigation on the effects of order lead time shape parameter have been given in table 5.3.

Data Set	Jointly Optimized Decision Variables				Separately Optimized Decision Variables			
*	S	s	T	Total Cost	S	s	T	Total Cost
1	200	10	225	176.82	230	10	225	183.40
2	210	10	225	180.19	230	10	225	183.40
3	210	20	225	181.63	230	20	225	197.59
4	210	20	225	181.63	230	20	225	196.32
5	210	20	225	181.63	240	30	225	197.69
6	220	30	250	185.56	240	30	225	197.69
7	230	30	250	185.79	240	30	225	197.69
8	230	40	250	186.90	240	40	225	203.23
9	240	40	275	190.37	240	40	225	203.23
10	250	40	275	192.11	240	40	225	203.23

Table 5.3. Results illustrating the effects of order lead time shape parameter on system behavior.

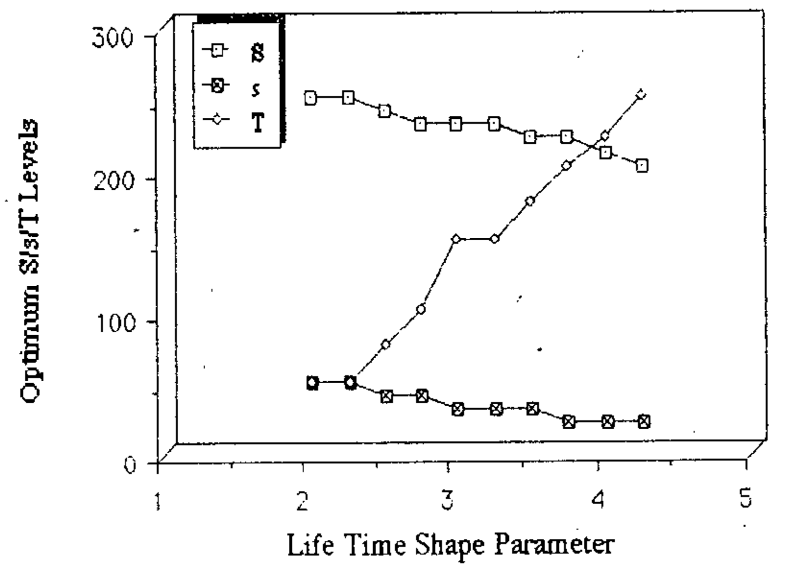


Figure 5.6. Effect of life time shape parameter on the jointly optimized decision variables.

As before, the smaller values of this parameter indicate the early delivery of orders for spares. This enables the system to operate with smaller inventory levels. The separately optimized value of block replacement period remained constant since in this case the inventory related events do not influence the maintenance policy. However, to cope with larger order lead time separately optimized inventory related variables take higher values and thus increasing the total cost per unit time. On the other hand taking the inventory and maintenance related costs at the same time in the joint optimization yields larger replacement intervals and this guards against sudden large demands (at the replacement instants) when the lead times are large. In other words, inventory shortage cost is not

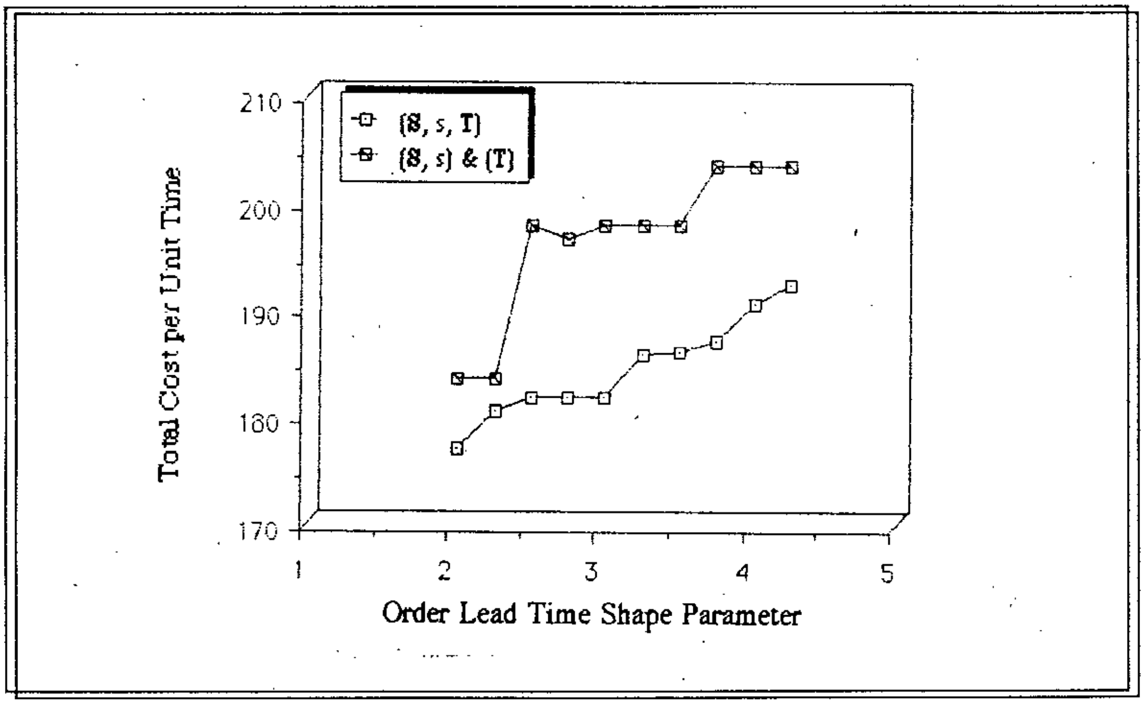


Figure 5.7. Effect of order lead time shape parameter on total cost per unit time.

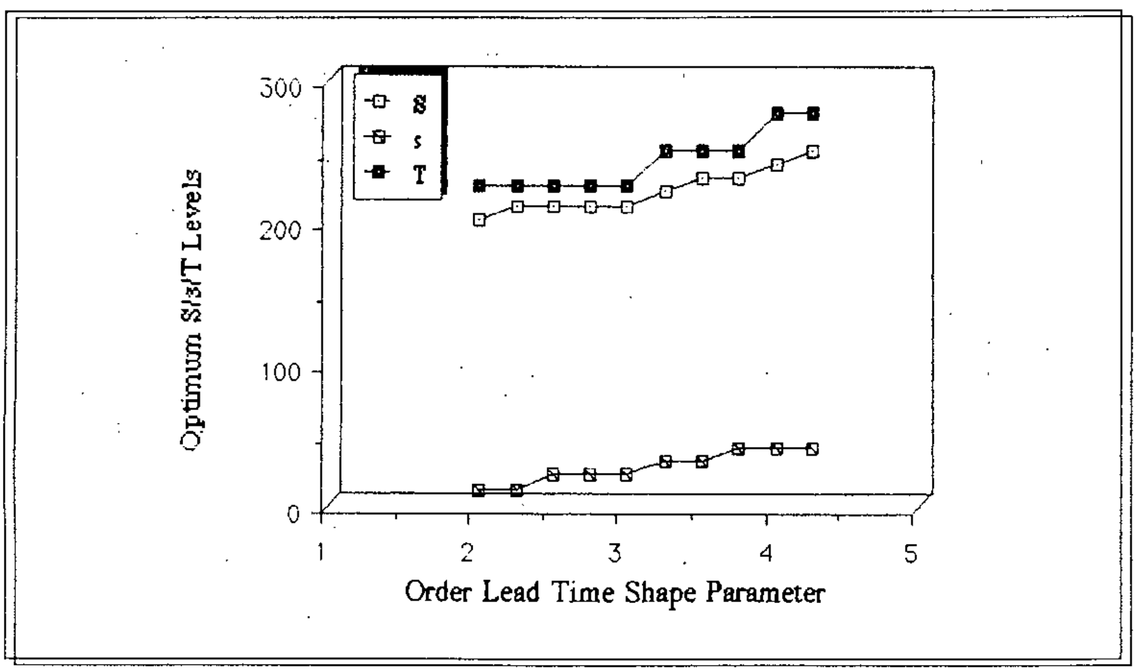


Figure 5.8. Effect of order lead time shape parameter on the jointly optimized decision variables.

allowed to be in action. Consequently the jointly optimized policy acts more cost effectively as shown in figure 5.7. The probability of larger lead time increases with higher values of the shape parameter so that larger maximum inventory level and reorder level have to be maintained after adjustments with the maintenance policy to minimize the spare shortage. This is evident from the figure 5.8.

5.3.3 Effects of Ordering Cost

Ordering cost directly influences the optimization of inventory related variables and thus have significant effect on both the joint and separate optimization of maintenance and inventory policies. This is reflected in the numerical results presented in table 5.4.

Data Set	Jointly Optimized Decision Variables				Separately Optimized Decision Variables			
	S	s	T	Total Cost	S	s	T	Total Cost
1	210	10	200	183.50	220	20	225	185.77
2	210	20	225	181.63	240	30	225	197.69
3	210	20	225	181.63	240	30	225	197.69
4	220	20	225	183.21	240	30	225	197.69
5	220	20	225	183.21	250	30	225	204.20
6	220	30	250	185.56	250	30	225	203.43
7	230	30	275	187.65	250	40	225	208.56
8	240	30	275	189.69	250	40	225	208.56
9	240	40	300	194.88	250	50	225	210.13
10	250	40	300	204.20	250	50	225	210.13

Table 5.4. Results illustrating the effects of ordering cost on system behavior.

Increase in ordering cost causes the system to operate with higher inventory levels and block replacement intervals as shown in figure 5.9 in order to preclude the events of more frequent ordering. For the separately optimized policies, ordering cost affects the inventory policy only. The separately optimized maintenance policy therefore cannot produce the actual optimal solution when combined with the inventory policy. The effect of any variation in the ordering cost on the decision variables is distributed in case of joint optimization so that maintenance policy is adjusted according to inventory levels. This explains the changes in the maintenance policies along with inventory levels in the results for the combined optimization of these policies. Table 5.3 reflects that these effects are similar to those observed for the order lead time shape parameter. This is verified by the total cost curves in figure 5.10.

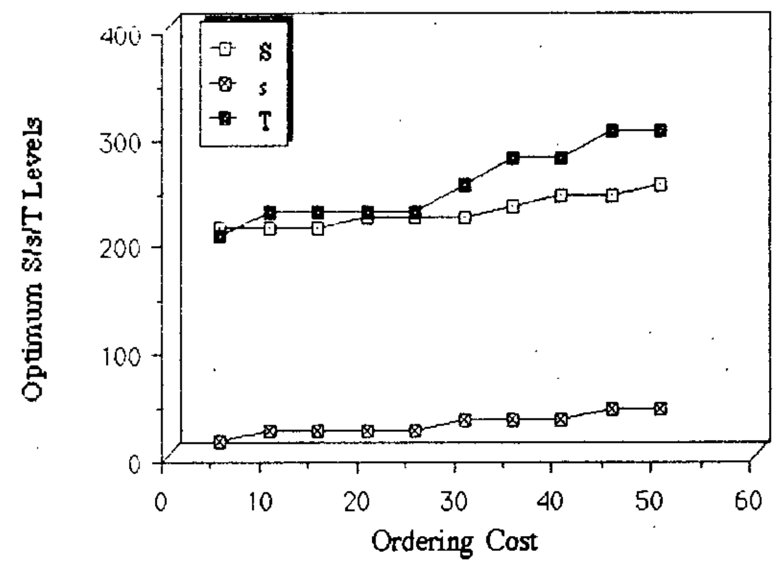


Figure 5.9. Effect of ordering cost on the jointly optimized decision variables.

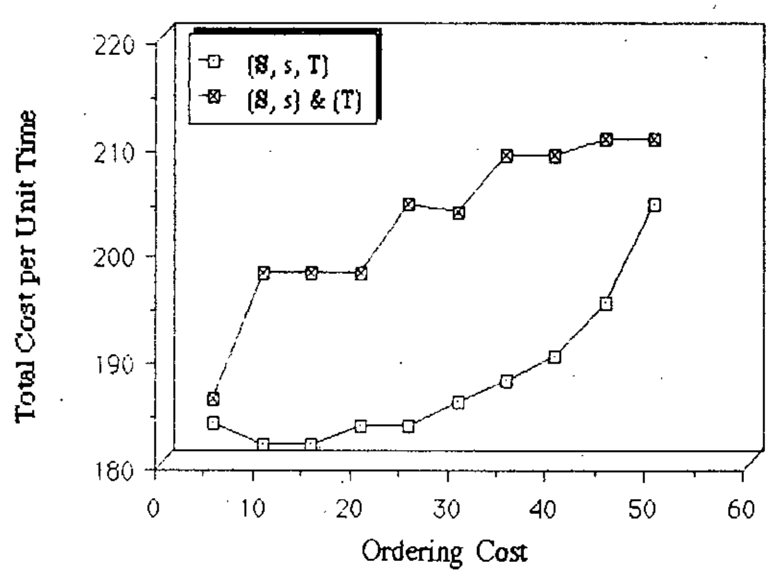


Figure 5.10. Effect of ordering cost on total cost per unit time.

5.3.4 Effects of Inventory Holding Cost

Investigation on the effects of inventory holding cost on the system behavior yielded the numerical results presented in table 5.4.

Data Set	Jointly Optimized Decision Variables				Separately Optimized Decision Variables			
*	S	s	T	Total Cost	S	s	T	Total Cost
1	230	30	200	192.06	240	30	225	203.67
2	210	20	225	181.63	240	30	225	203.67
3	210	20	225	181.63	240	20	225	203.67
4	210	10	225	183.50	230	20	225	207.11
5	210	10	200	183.50	230	20	225	204.90
6	210	10	200	184.25	220	30	225	205.70
7	210	10	200	185.79	210	20	225	208.87
8	200	0	200	184.87	200	10	225	208.70
9	200	0	200	188.70	200	10	225	210.07
10	200	0	175	194.41	200	0	225	213.81

Table 5.5. Results illustrating the effects of inventory holding cost on system behavior.

The concept of joint optimization of maintenance and inventory policies stems partly from the idea of this investigation. The inventory holding cost is supposed to be traded-off with the maintenance costs in their combined optimization. Once again this inventory related parameter could not affect the maintenance policy in case of their separate optimization. With the increase of inventory cost, naturally the maximum inventory level falls down even with the jointly optimized policy as shown in figure 5.11. The fall in reorder level is quite notable because this amount of inventory is usually intended to be carried throughout. This reduction in inventory levels can not be matched with the maintenance policy in case of their separate optimization as happens with the combined optimization. The jointly optimized policy produces more cost effective solutions. This is achieved due to the fact that the maintenance related decision variable is adjusted so that their combined effect produces global minimums for the total cost function. This is reflected in figure 5.12.

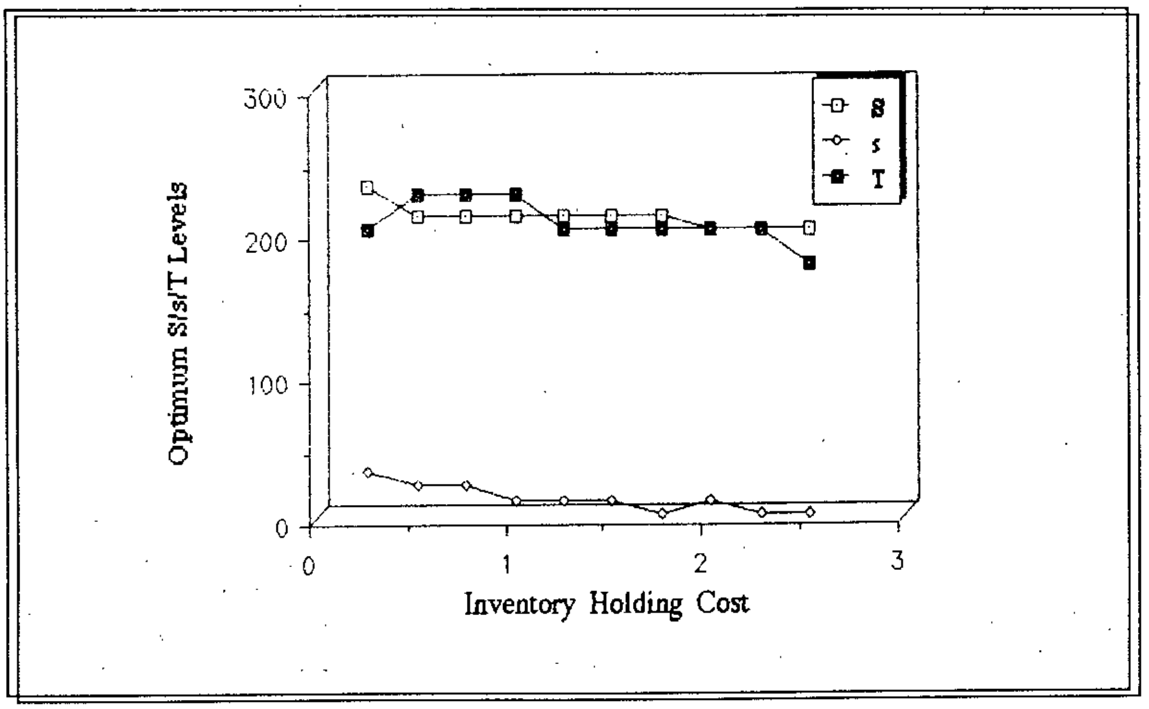


Figure 5.11. Effect of inventory holding cost on the jointly optimized decision variables.

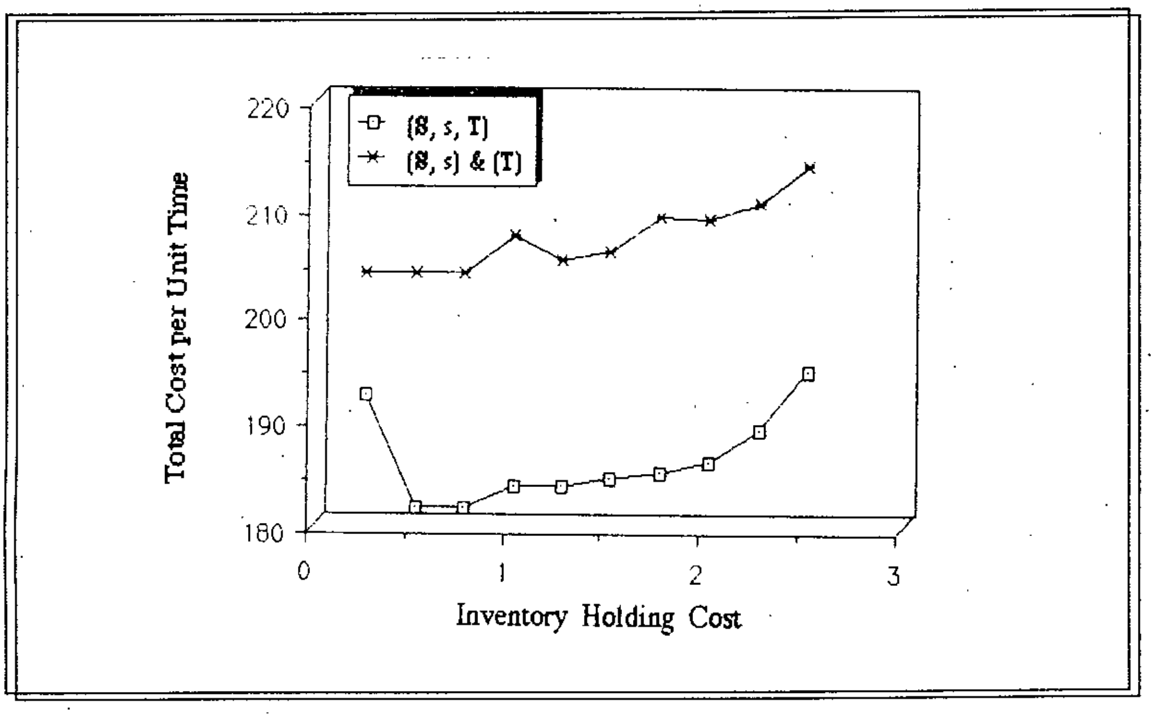


Figure 5.12. Effect of inventory holding cost on total cost per unit time.

5.3.5 Effect of Inventory Shortage Cost

Results for the investigation on the effects of inventory shortage cost on system behavior is given in table 5.6.

Data Set	Jointly Optimized Decision Variables				Separately Optimized Decision Variables			
*	S	s	T.	Total Cost	S	s	T	Total Cost
1	230	10	250	178.91	220	10	225	184.54
2	230	10	250	178.94	220	20	225	186.29
3	220	10	250	180.20	230	30	225	194.30
4	210	20	225	181.63	240	30	225	203.67
5	220	20	225	183.98	240	30	225	203.67
6	220	30	200	185.21	240	40	225	207.53
7	230	30	200	186.20	240	40	225	207.53
8	230	40	200	187.71	250	50	225	214.19
9	220	50	200	188.10	250	50	225	214.19
10	220	50	200	193.77.	250	50	225	214.19

Table 5.6. Results illustrating the effects of inventory shortage cost on system behavior.

The inventory shortage cost is another factor that initiated the joint optimization of maintenance and inventory policies. While the effect of inventory holding cost prevents the possibility of stocking excess spares, inventory shortage cost routes out the possibility for under stocking them with adjustment to a maintenance policy. Therefore the effect of inventory shortage cost is expected to increase the reorder level and consequently the inventory holding cost. This simulation results for the proposed policy conforms with this as shown in figure 5.13. The jointly optimized policy adjusts the maximum inventory level and block replacement interval accordingly but this is never attained by the separate optimization of these policies. Consequently the proposed jointly optimized policy has been observed to produce more cost effective results. The comparative cost figures have been graphically presented in figure 5.14.

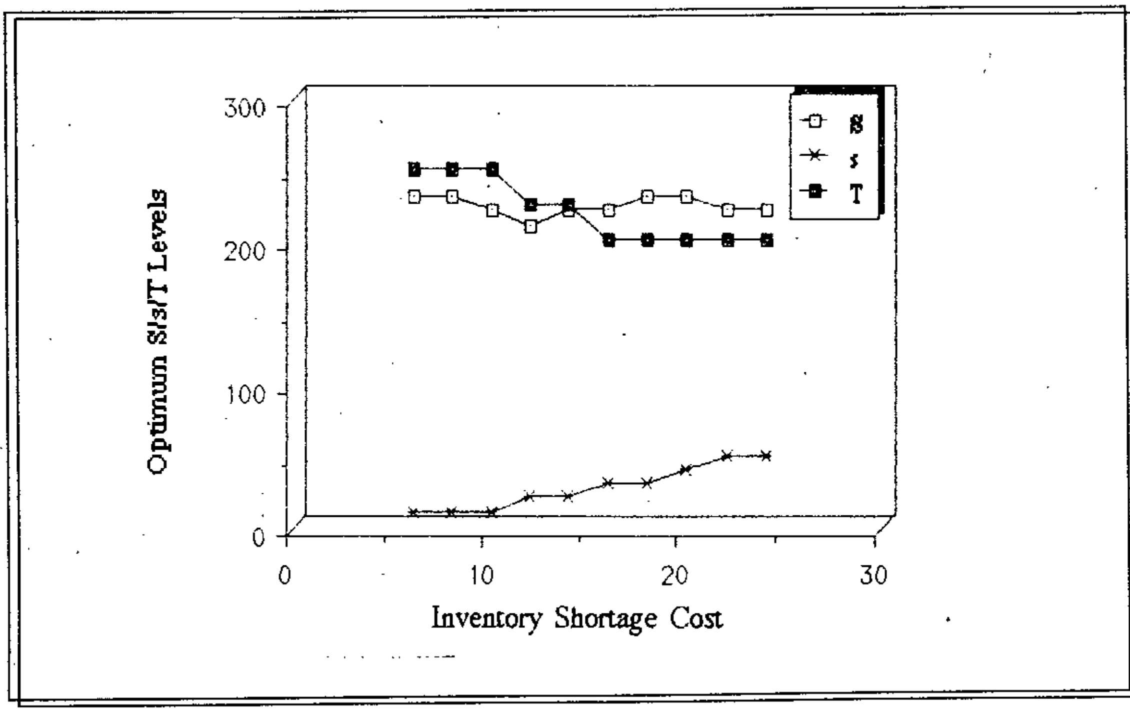


Figure 5.13. Effect of inventory shortage cost on the jointly optimized decision variables.

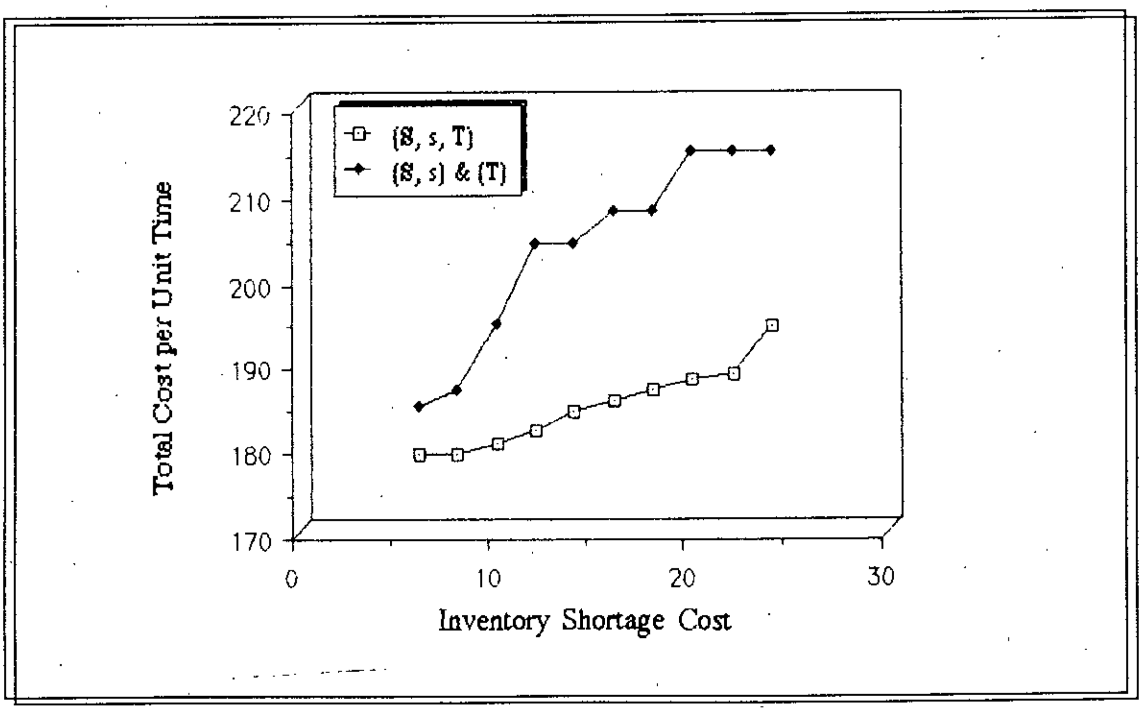


Figure 5.14. Effect of inventory shortage cost on total cost per unit time.

5.3.6 Effect of Block Preventive Replacement Cost

Table 5.7 shows the results of the investigation on the effect of the block replacement cost on the system behavior. The effect of the block replacement cost directly affects the combined optimization of maintenance and inventory policies by dominating the maintenance strategy. With the increase of this cost the block replacement also increases in order to reduce huge preventive maintenance cost and the tendency is towards adopting a breakdown maintenance strategy. The inventory level in such cases need not be too high since there are no sudden large demands (as happens in block replacement instants) and it is unlikely that the inventory shortage cost is incurred. This is shown in figure 5.15. For separate optimization of these policies, only the maintenance strategy is affected.

Data Set	Jointly Optimized Decision Variables				Separately Optimized Decision Variables			
*	S	s	T	Total Cost	S	s	T	Total Cost
1	210	20	225	181.63	240	30	275	203.67
2	210	20	225	181.63	240	30	275	203.67
3	210	20	225	181.63	240	30	300	205.79
4	220	10	250	185.39	240	30	325	205.81
5	220	10	250	186.47	240	30	325	205.81
6	210	10	275	193.33	240	30	*	201.33
7	210	10	300	191.94	240	30	*	201.33
8	200	10	*	188.31	240	30	*	201.33
9	200	10	*	190.09	240	30	*	201.33
10	200	10	*	189.78	240	30	*	201.33

Table 5.7. Results illustrating the effects of block replacement cost on system behavior.

Here the inventory level is never optimized in relation to the maintenance policy and this is reflected in the table 5.7. Therefore the more the block replacement cost, the more the tendency towards breakdown replacement and the insensitivity towards inventory policy causes the separately optimized policy to cost more when compared to their jointly optimized counterpart. This has been clearly illustrated in figure 5.16.

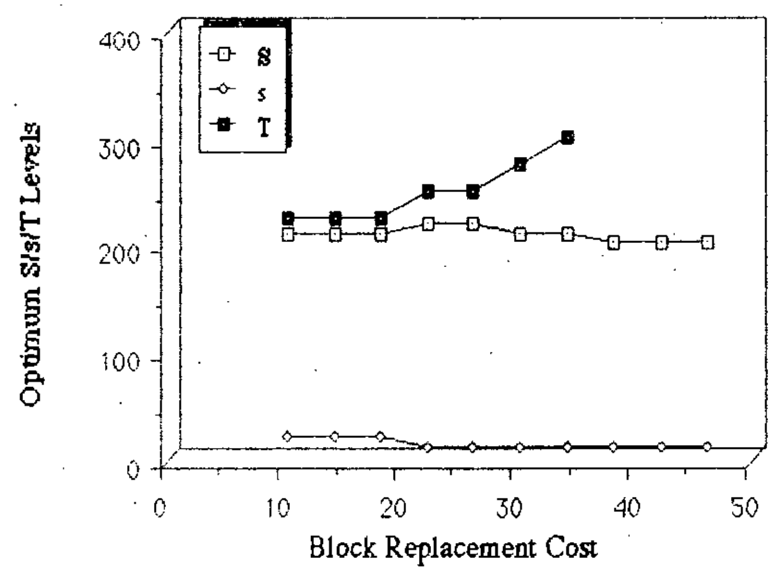


Figure 5.15. Effect of block replacement cost on the jointly optimized decision variables.

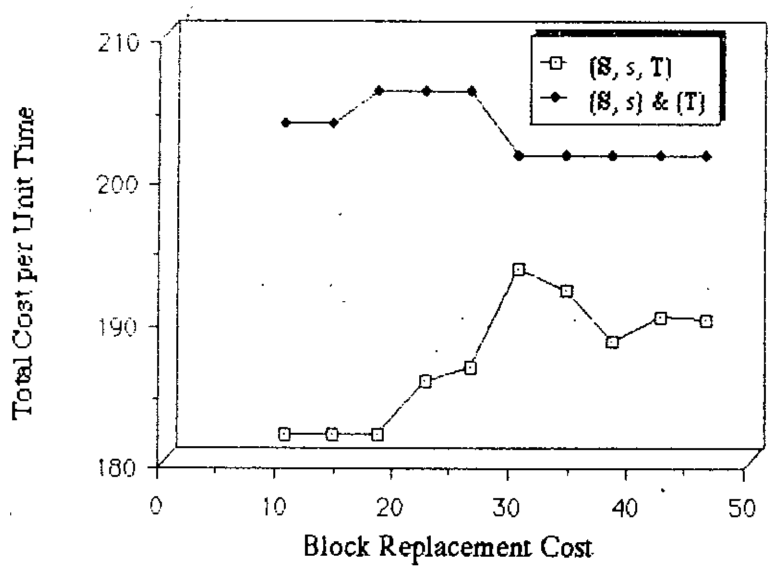


Figure 5.16. Effect of block replacement cost on total cost per unit time.

5.3.7 Effect of Failure Replacement Cost

This cost parameter also bears significant influence on the performance of the combined optimization of maintenance and inventory policies. Table 5.8 presents the numerical values obtained after the experimentation.

Data Set	Jointly Optimized Decision Variables				Separately Optimized Decision Variables			
*	S	s	T	Total Cost	S	s	T	Total Cost
1	230	10	*	205.54	240	30	*	189.90
2	230	10	275	194.30	240	30	275	197.37
3	220	20	250	185.63	240	30	275	197.37
4	220	30	250	185.63	240	30	250	196.57
5	220	20	225	183.98	240	30	225	203.67
6	210	20	225	180.31	240	30	225	203.67
7	220	30	225	178.66	240	30	225	203.67
8	210	20	200	184.97	240	30	200	197.22
9	210	30	225	186.69	240	30	200	197.22
10	220	30	200	182.37	240	30	175	198.09

Table 5.8. Results illustrating the effects of failure replacement cost on system behavior.

The effects of individual failure or breakdown cost on the decision variables of the proposed (*S*, *s*, *T*) system is exactly opposite to those of the block replacement cost. This is quite anticipated because smaller values of breakdown cost will encourage failure, rather than preventive maintenance. Therefore the increase of this cost will induce a switch over from failure to preventive maintenance. This is the same with the combination of separately optimized (*S*, *s*) and (*T*) policies except that the inventory related variables remain unchanged in value. The penalty in terms of high inventory holding cost for this insensitivity towards the inventory policy makes the separately optimized policies cost more when compared to their jointly optimized counterpart. This is reflected in the figure 5.17.

For higher breakdown cost values the inventory levels also become high as shown in figure 5.18. This can be explained due to the fact that adoption of a preventive maintenance policy requires sufficient inventory to meet the large demands at the block replacement instants and yet prevent the inventory shortage cost to accrue to the total cost.

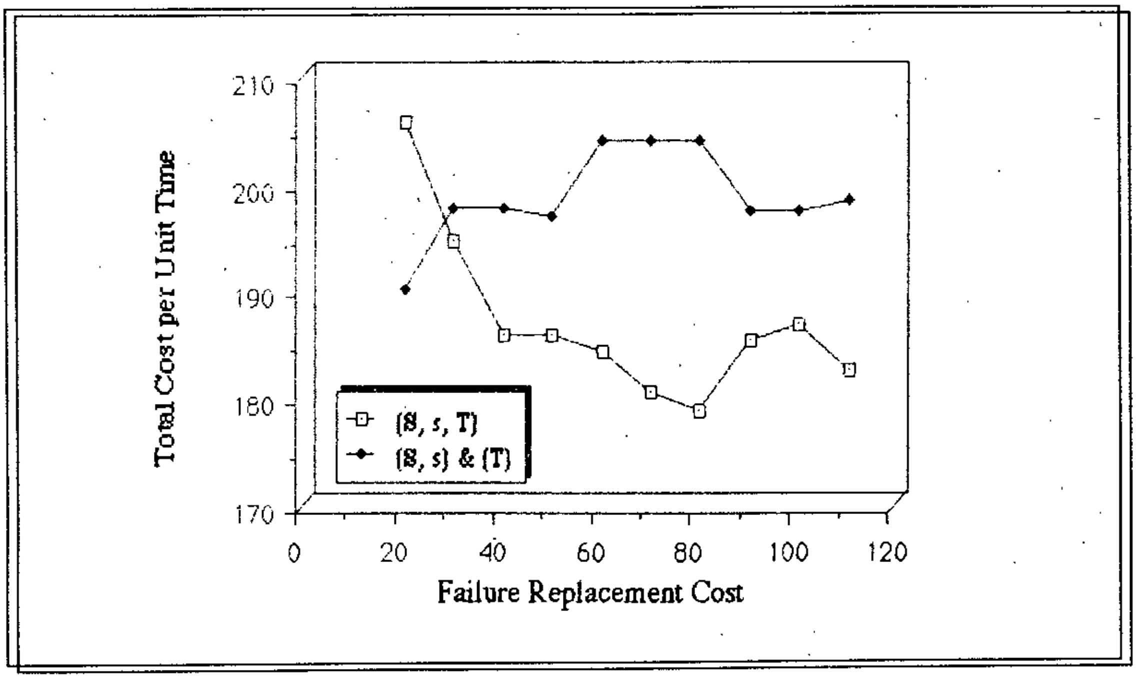


Figure 5.17. Effect of failure replacement cost on total cost per unit time.

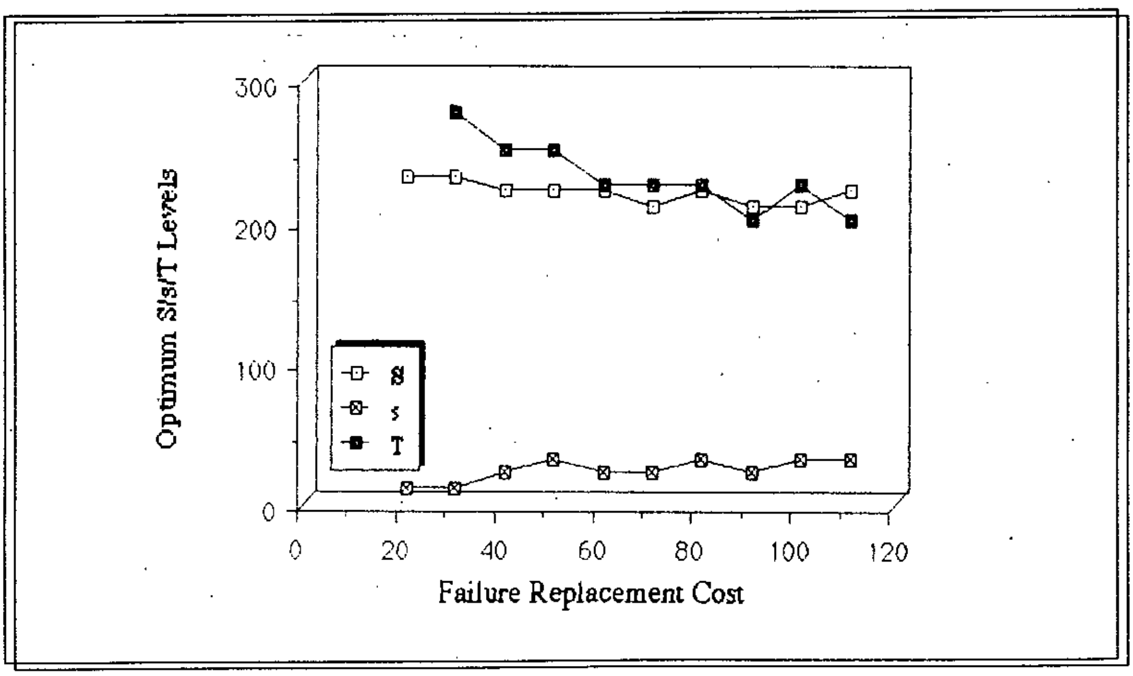


Figure 5.18. Effect of failure replacement cost on the jointly optimized decision variables.

5.4 RESULTS ON DOWNTIME AND WIPI

Experimentation carried out for each of the data sets given in Chapter Four produced results on system downtime and work-in-process-inventory accumulations as well as the total cost per unit time. Numerical results of these experiments have been presented in tabular form in Appendix C. Though the system has been optimized for the total cost per unit time, the effects of the various system and cost parameters can be analyzed from the obtained results.

System downtime and work-in-process-inventory are two measures of performance which depend on the values of the decision variables of maintenance and inventory policies. A cost effective policy may not prove to be optimal in terms of these two measures since in many cases it may be more profitable to keep lower inventory and allow the work centers to keep *idle* (as in cases of lower inventory shortage costs). The proposed jointly optimized (*S*,, *s*, *T*) system searches the combination of the decision variables which minimize the total cost per unit time. The results given in the previous section reveal that various system and cost parameters affect system downtime and work-in-process accumulation significantly. These effects are indirect since they do not show any correlation with the various parameters such as inventory holding or block replacement costs. This is quite anticipated because the system has not been optimized for downtime or work-in-process-inventory. Rather, they are best interpreted in terms of the decision variables.

A thorough study on the results regarding downtime and work-in-process-inventory show that the values of maximum inventory and reorder level influence these two quantities the most, especially the reorder level. The system experienced low downtime and work-in-process-inventory whenever it is operated with a policy that have ample supply of spares. This is verified by the results obtained from the separate optimization of the maintenance and inventory policies. These policies appear to be optimal in terms of downtime and work-in-process-inventory because the separate optimization of maintenance and inventory policies ensures that the spare inventory is kept at a level that guards against system unavailability. These policies are not cost effective because the inventory policy is not jointly optimized but availability of spares when needed cause them to produce more impressive results on downtime and work-in-process-inventory. The best results are obtained with large values of reorder level. This can be explained due to the fact that this decision variable plays the predominant role in spare availability.

Block replacement period also influence these two performance measures. This effect is most prominent in cases of smaller block replacement period. High block replacement frequency of the operating components make the system suffer from frequent plant shutdown for maintenance purpose and consequently both downtime and work-in-process-inventory increase. This effect is not negligible since the number of work centers down or experiencing work-in-process-inventory is also taken into account by the composite terms (work center multiplied by downtime or work-in-process-inventory multiplied by time spent in this state) designed in Chapter Three. For large block replacement periods this effect is negligible with respect to those for the inventory levels.

CHAPTER SIX

CONCLUSIONS AND RECOMMENDATIONS

- Introduction
- Conclusions
- Recommendations for Future Work

6.1 INTRODUCTION

This chapter is intended to draw conclusions on the present research and to put forth some recommended actions that are supposed to complement and consolidate the study. Conclusions are drawn on the basis of the findings of the study in view of the scope and objectives of the present research. The next section contains the concluding remarks on the performance and significance of the proposed jointly optimized block replacement maintenance and continuous review inventory control (*S*, *s*, *T*) policy. The last section is devoted to furnish some recommendations for future work.

6.2 CONCLUSIONS

The general problem considered in the present research is to optimize the maintenance and inventory policies in terms of total operating cost. These two policies are historically found to be optimized either separately or sequentially and the obtained results are then applied in practical cases. Since maintenance policy does not account for inventory related considerations (such as inventory holding or shortage costs), their separate or even sequential optimization, naturally cannot yield the cost optimal solutions to the problem. In the present work a manufacturing system has been considered which operates with block replacement (*T*) maintenance, and continuous review (S, s) spare provisioning policy. The problem of the combined optimization of these two policies are very complex to be solved analytically due to the stochastic nature of the system events. Therefore simulation has been used as the analyzing tool. A (*S*, *s*, *T*) model has been constructed which merges these separate policies. Experimentation has been carried out with this model for its performance under different operating situations. The outputs for both combined and sequential optimization of the two policies clearly show that the proposed model performs more cost optimally. Considering the performances of the proposed (*S*, *s*, *T*) simulation model under different operating situations, the following conclusions can be drawn.

1. The combined optimization of maintenance and inventory policies is a better way to reduce the total system operating cost than by the combination of their separate

optimization. This is particularly true when the contributions of both the maintenance and inventory related costs are significant and the components become *aged* before they fail. Global optimality for this problem is ensured because the effects of all the three decision variables (*S, s, T*) on the total cost are taken into account simultaneously.

2. Block or group-wise replacement of operating components is the best maintenance policy when they are low in cost and quite large in number. Almost all the data that have been collected in the experimentation prove this since in these cases the cost figures for block replacement were lower than those considering failure replacement as described in section 5.3 of Chapter Five. Block replacement policy appeared to be a poor choice in the cases of lower values of the ratio of individual breakdown replacement cost to block replacement cost and lower values of component lifetime shape parameter. This can be explained by the facts that lower values of the replacement cost ratio mentioned above renders the advantage of block replacement useless as do lower values of the shape parameter which implies that the components do not show aging over the passage of time.

3. The continuous review inventory control policy is a good choice to work in conjunction with the block replacement maintenance policy because of its simplicity in basic mechanism. It is easy to understand and implement, even for floor level workers. This policy can track the inventory status directly in terms of the number of items on hand after each demand. Therefore it is sensitive to significant variability that might exist in demand patterns.

4. The shape parameter of the component lifetime probability distribution plays an important role in the combined optimization of maintenance and inventory policies. For smaller values of this parameter the hazard rates are also lower and operating components actually do not show aging. This is supported by the analytical solutions for component lifetime following exponential distribution where no optimum preventive maintenance policy could be determined. Higher values of the shape parameter initiate aging process of the components and the advantage of block replacement is successfully implemented.

5. The effects of ordering cost and the shape parameter of order lead time on the combined optimization cannot be neglected. Ordering costs directly influences the (*S, s, T*) policies,

being incurred after each order for spare items. Higher values of the shape parameter indicates the larger probability of larger lead time and cause the system to react by increasing the reorder level to preclude the introduction of inventory shortage cost. This in turn tends to increase in the inventory holding cost.

6. The combined optimization technique proves to be more cost effective for higher inventory related cost figures since trade-off between maintenance and inventory costs is not feasible if they are not evenly poised. The inventory holding cost is more important to consider than the inventory shortage cost. This is because considerable amount of spare inventory must be kept throughout the maintenance cycle in order to ensure availability of spares in time of need. Higher values of inventory holding cost thus significantly affects the jointly optimized policy by lowering the inventory related decision variables (S, s) and increasing of the maintenance cycle time (T). Inventory shortage cost on the other hand has the tendency to increase the values of the reorder level (s) for the (S, s, T) policy.

7. The maintenance related costs are the most predominant contributors to the total cost and therefore affect the combined optimization most significantly. The block replacement cost plays the vital role since it is the key factor which determines the optimality of a block maintenance policy. Individual breakdown replacement cost is also important and the effects of these costs are best described by the relative values of these two costs or the ratio of individual breakdown replacement cost to block replacement cost. The higher values of this ratio implies better cost effectiveness of the block replacement policy. For lower values, individual replacements are encouraged and naturally the margin of cost savings made by the combined optimization are lowered.

8. A cost effective policy may not prove to be optimal in terms of system downtime and work-in-process-inventory. The proposed jointly optimized (S,, s, T) system searches the combination of the decision variables which minimize the total cost per unit time and not the downtime or work-in-process-inventory. Therefore the effects of various system and cost parameters on these quantities are best interpreted in terms of the decision variables.

The values of maximum inventory and reorder level influence these two quantities the most, especially the reorder level. The best results are obtained with large values of reorder level. This can be explained due to the fact that this decision variable plays the predominant role in spare availability. Block replacement period also influence these two performance measures. This effect is most prominent in cases of smaller block replacement period. High block replacement frequency of the operating components make the system suffer from frequent plant shutdown for maintenance purpose and consequently both downtime and work-in-process-inventory increase. For large block replacement periods this effect is negligible with respect to those for the inventory levels.

9. Finally, it remains to conclude on the simulation method itself as an analyzing tool in the present study. Obtaining analytical results for the system considered could be extremely difficult. This is because the total cost estimation for the combined maintenance and inventory cycle in terms of either inventory level or time units seems to be an insurmountable problem. The stochasticity in component failures and order delivery adds further complexity and thus simulation comes to play the role of savior. However, simulation itself is statistical sampling only and therefore the results can be accepted only after rigorous statistical analysis technique, in which this study lacks. The process is very much time consuming and selection of the searching values of the decision variable affects the findings significantly. Nevertheless, simulation has proved to be a powerful tool in analyzing the behavior of complex systems as reflected from the study.

6.3 RECOMMENDATIONS FOR FUTURE WORK

In perspective of the problem and findings of the present research the following recommendations can be put forth which would complement and find further extensions on this research.

1. The findings of the present study encourages more rigorous design of experiment and statistical analysis in order to improve the credibility of the results. Based on the

significance of the various decision factors and their levels on system behavior, the most critical factors and levels can be sorted out to make a more powerful design of experiments. This has to be followed by statistical analysis which perform tests for interaction and variances of the factors and their levels.

2. Further experimentation can be conducted in order to optimize the system considering system downtime, work-in-process-inventory and service level as measures of performance.

3. The model can be tested for investigation on the effects of replacement time distribution parameters on the system downtime and work-in-process-inventory accumulation.

4. The research can be extended to the case of investigation on the component repair policy rather than replacing them or on both repair and replacement of failed components. Inventory related performance measures like the service level can also be included in the model.

5. Experiments can be carried out to study the effect of the total number of operating components in the system in order to identify the operating situations that create the demarcations between the choices of individual breakdown and block or group-wise replacement of the operating components.

6. Further researches can be carried out considering non-identical components that may or may not have the same failure patterns and other cost parameters.

7. All the recommendations above can be implemented towards the joint optimization of block replacement maintenance and periodic review spare provisioning policies. This will develop a comprehensive overview on the present topic.

REFERENCES AND APPENDICES

- References
- Appendix A
- Appendiix B
- Appendix C

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APPENDIX A

Weibull	Weibull (α, β)
Density	$f(x) = \alpha\beta^{\alpha} x^{\alpha-1} e^{-(x/\beta)\alpha}$ if $x>0$ $= 0$ otherwise
Distribution	$F(x) = 1 - e^{-(x/\beta)\alpha}$ if $x>0$ $=0$ otherwise
Parameters	Shape parameter $\alpha>0$, Scale parameter $\beta>0$
Range	$[0\text{-}\infty]$
Mean	$[\beta/\alpha].\Gamma(1/\alpha)$
Variance	$[\beta^2/\alpha]\cdot\{2\Gamma(2/\alpha) - 1/\alpha[\Gamma(1/\alpha)]^2\}$
Comments	The exponential (α) and Weibull $(1, \beta)$ are the same.
	$\beta < 1$ represents a decreasing hazard rate or the debugging period. $\beta = 1$ represents a constant hazard rate or the normal life period. $\beta > 1$ represents an increasing hazard rate or the wearout period. Possible Applications: Time to complete some task, time to failure of a piece of equipment etc.

Table A. 1 Information regarding the Weibull distribution.

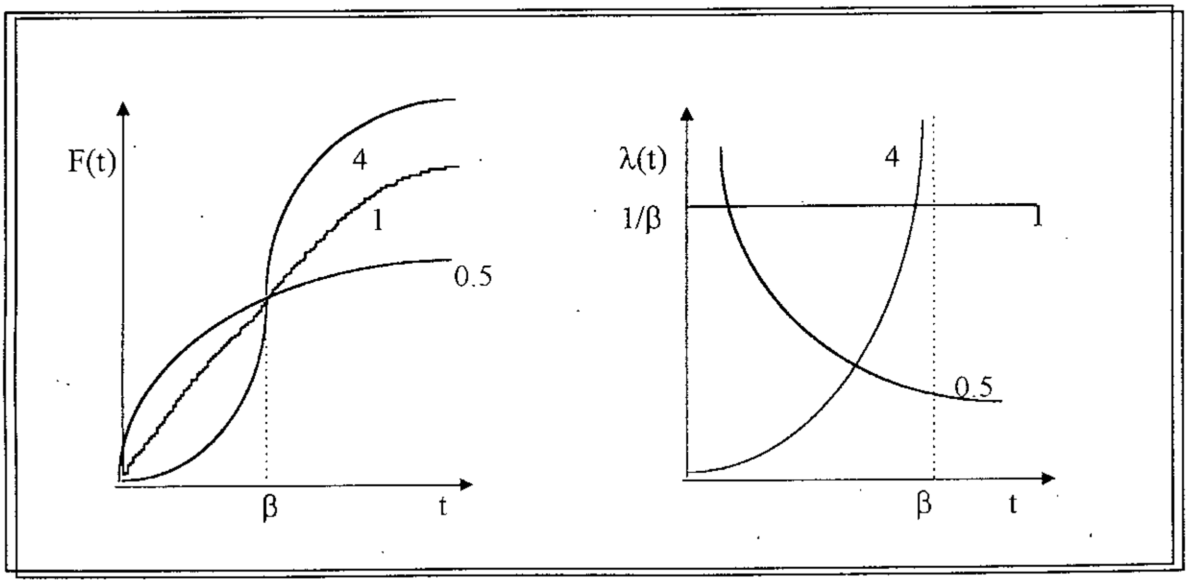

Figure A.1 : Weibull reliability functions. Parameters are the values of α .

Gamma	Gamma (α, β)
Density	$f(x) = \beta^{-\alpha} x^{\alpha-1} e^{-x/\beta}$ if $x > 0$ $= 0$ otherwise
Distribution	If α is not an integer, there is no closed form. If α is a positive integer then $F(x) = 1 - e^{-x/\beta} \sum_{j=0}^{\alpha-1} \frac{(x/\beta)^j}{j!} (x/\beta)^j$ if $x > 0$ $= 0$ otherwise
Parameters	Shape parameter $\alpha > 0$, Scale parameter $\beta > 0$
Range	[0-∞]
Mean	$\alpha\beta$
Variance	$\alpha\beta^2$
Comments	Possible Applications: Time to complete some task, e.g., customer service or machine repair.

Table A. 2. Information regarding the Gamma distribution.

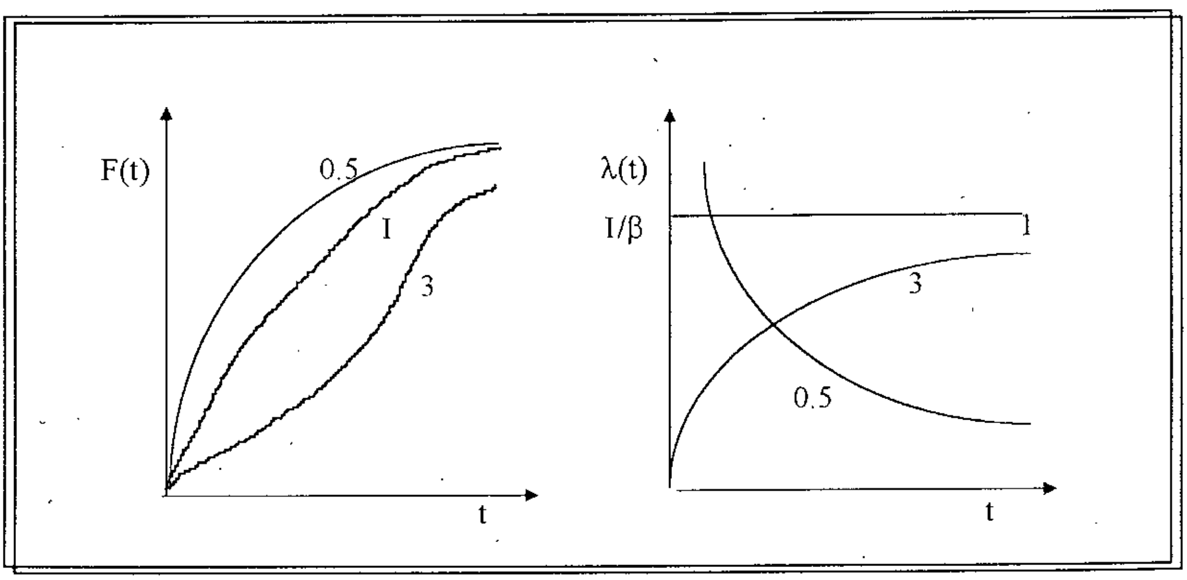

Figure A.1 : Gamma reliability functions. Parameters are the values of α .

APPENDIX B

Data Set	1	2	3	4	5	6	7	8	9	10
Confidence Interval	± 4.21	± 3.97	± 3.23	± 5.66	± 3.31	± 6.21	± 5.19	± 4.98	± 3.16	± 5.11

Table B.1. Confidence intervals for output data in table 5.2

Data Set	1	2	3	4	5	6	7	8	9	10
Confidence Interval	± 2.29	± 6.81	± 3.16	± 3.16	± 3.16	± 4.25	± 5.19	± 3.69	± 4.87	± 3.33

Table B.2. Confidence intervals for output data in table 5.3

Data Set	1	2	3	4	5	6	7	8	9	10
Confidence Interval	± 4.55	± 3.16	± 3.16	± 4.88	± 5.31	± 2.29	± 3.78	± 4.63	± 3.71	± 2.96

Table B.3. Confidence intervals for output data in table 5.4

Data Set	1	2	3	4	5	6	7	8	9	10
Confidence Interval	± 5..37	± 3.16	± 3.16	± 2.38	± 3.98	± 6.98	± 5.64	± 4.19	± 2.77	± 4.27

Table B.4. Confidence intervals for output data in table 5.5

Data Set	1	2	3	4	5	6	7	8	9	10
Confidence Interval	± 5.11	± 4.87	± 7.08	± 3.16	± 6.88	± 3.56	± 3.49	± 5.08	± 4.91	± 3.43

Table B.5. Confidence intervals for output data in table 5.6

Data Set	1	2	3	4	5	6	7	8	9	10
Confidence Interval	± 4.14	± 2.09	± 3.94	± 7.04	± 6.48	± 5.31	± 4.44	± 2.97	± 3.43	± 5.83

Table B.6. Confidence intervals for output data in table 5.7

Data Set	1	2	3	4	5	6	7	8	9	10
Confidence Interval	± 3.33	± 5.41	± 4.28	± 3.68	± 4.70	± 8.03	± 4.96	± 6.33	± 4.21	± 2.99

Table B.7. Confidence intervals for output data in table 5.8

Data Set	Down time (Work center-time units)			WIPI (No. of jobs-time units)		
*	Line 1	Line 2	Line 3	Line 1	Line 2	Line 3
1	5615	5240	5432	255	261	243
2	5695	5430	5337	257	249	264
3	5123	4973	5877	231	229	237
4	6291	6773	6348	203	218	225
5	6539	6449	6261	205	219	208
6	6597	6492	6304	197	204	193
7	6709	6779	6531	196	190	181
8	6752	6607	6608	182	177	179
9	6558	6339	6435	178	189	188
10	6073	6295	6371	174	186	173

Table C.1. Output in terms of down time and WIPI for experiment with component life time shape factor variation.

Data Set	Down time (Work center-time units)			WIPI (No. of jobs-time units)		
*	Line 1	Line 2	Line 3	Line 1	Line 2	Line 3
1	7342	7295	7463	283	271	279
2	7034	7106	7395	288	279	266
3	6904	6873	6852	276	274	269
4	6910	6988	6879	271	269	264
5	6753	6710	6817	265	265	255
6	5907	6163	6211	266	266	257
7	5496	6008	5879	259	259	248
8	5612	5379	5603	254	254	241
9	4988	4852	4796	247	246	239
10	4081	4513	4491	244	239	236

Table C.2. Output in terms of down time and WIPI for experiment with order lead time shape factor variation.

Data Set	Down time (Work center-time units)			WIPI (No. of jobs-time units)		
*	Line 1	Line 2	Line 3	Line 1	Line 2	Line 3
1	7192	7045	7068	266	258	269
2	6993	7079	7195	251	246	252
3	6752	6883	7081	248	253	248
4	6613	6705	6673	239	245	241
5	5897	6011	5927	233	231	238
6	5809	5921	5887	237	229	236
7	5531	5691	5719	221	222	232
8	5304	4963	5997	227	211	217
9	4891	4956	4903	219	227	220
10	4677	4830	4751	225	203	218

Table C.3. Output in terms of down time and WIPI for experiment with ordering cost variation.

Data Set	Down time (Work center-time units)			WIPI (No. of jobs-time units)		
*	Line 1	Line 2	Line 3	Line 1	Line 2	Line 3
1	5476	5403	4989	166	176	149
2	5529	5428	5622	178	170	168
3	5853	5329	5695	183	184	187
4	6047	5947	5988	181	196	182
5	6433	5991	6234	195	185	199
6	6441	6238	6476	208	211	205
7	7019	7155	7392	226	219	237
8	7811	7739	7904	244	237	249
9	8069	8113	8058	258	261	263
10	8879	9045	9091	267	262	255

Table C.4. Output in terms of down time and WIPI for experiment with inventory holding cost variation.

Data Set	Down time (Work center-time units)			WIPI (No. of jobs-time units)		
*	Line 1	Line 2	Line 3	Line 1	Line 2	Line 3
1	6397	6218	6358	281	273	287
2	6121	6139	6173	277	269	285
3	5953	5981	5927	265	259	282
4	5879	5802	5965	262	247	274
5	5477	5481	5396	264	254	261
6	5119	5127	5195	253	239	246
7	4976	4935	4987	241	232	242
8	4761	4707	4801	235	221	234
9	4283	4351	4297	220	218	209
10	4077	4122	4153	229	215	202

Table C.5. Output in terms of down time and WIPI for experiment with inventory shortage cost variation.

Data Set	Down time (Work center-time units)			WIPI (No. of jobs-time units)		
*	Line 1	Line 2	Line 3	Line 1	Line 2	Line 3
1	5833	5796	5781	223	215	217
2	5711	5709	5689	220	211	222
3	5704	5673	5681	218	205	214
4	5681	5706	5935	208	201	213
5	5336	4985	5127	192	196	199
6	5253	4991	4907	186	184	187
7	4879	4878	4891	174	188	179
8	4765	4987	4918	178	170	177
9	4288	5113	5037	172	167	171
10	4077	4976	4993	169	158	166

Table C.6. Output in terms of down time and WIPI for experiment with block replacement cost variation.

Data Set	Down time (Work center-time units)			WIPI (No. of jobs-time units)		
*	Line 1	Line 2	Line 3	Line 1	Line 2	Line 3
1	4712	4769	4783	215	227	232
2	4824	4831	4875	221	219	222
3	4859	4866	4829	210	203	217
4	4988	5033	5019	198	191	212
5	5217	4967	5012	187	184	204
6	5563	5511	5496	183	177	183
7	5741	5690	5681	176	175	180
8	5822	5871	6019	179	169	179
9	5953	6003	5874	162	166	161
10	5992	5913	6108	170	154	156

Table C.7. Output in terms of down time and WIPI for experiment with individual breakdown replacement cost variation.