OPTIMIZATION OF PORT FACILITIES

by

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B.Sc. Engg. (Naval
Architecture & Marine)

BANGLADESH UNIVERSITY OF ENGINEERING AND TECHNOLOGY, DHAKA
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A Thesis

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C O N T E N T

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ABSTRACT

The Chittagong Port, being the principal Port of Bangladesh, is the main link of the country with the outside world for its sea-borne trade and commerce. The export-import activities at the port have increased substantially during the last decade and it is expected that these will increase manifold in future with the development of the country. It is, therefore, of utmost necessity for management to make proper planning so that adequate service facilities may be provided at the port for different service requirements in future. Otherwise the marine congestion problem due to anticipated traffic demand may seriously affect the revenue earnings of the port.

In the present work, five years' data (January 1980 to December 1984) obtained from Chittagong Port are analyzed. To find their behaviours, different probability distributions are assumed and appropriate tests for goodness of fit are applied. For Weibull distribution, Kolmogorov-Smirnove test and for other distributions, Chi-square tests are applied.

The queueing models such as Mathematical model, Diffusion Approximation model and Simulation models are developed for the system. System parameters are estimated using these models and their values are compared with the observed ones. All three models are found applicable to the present system of operation at the port. However, each of the models has its own merits and limitations. Aspiration level models are also developed for the system for different conflicting measures such as mean waiting time in queue and the server's idle time which the decision maker wishes to

balance. These models make direct use of the operating characteristics of the system in deciding the optimum values of the design parameters.

Average revenues earned per ship and the costs of the port facilities are estimated. Using these estimates, a cost model is developed. The results obtained for this model are presented in a tabular form. These results, it is expected, will help management to take decisions more objectively in ascertaining optimum service facilities at the port to ensure maximum net incremental revenue at varying environments and service conditions.

GLOSSARY OF SYMBOLS

The following symbols have been used in this thesis.

M = Exponential distribution

E_K = Erlangian distribution of K phases

L = Mean number of customers in the system

Lq = Mean number of customers in the queue

W = Mean waiting time in the system

Wq = Mean waiting time in the queue

X = Server's idle time

λ = Mean arrival rate of customers

μ = Mean service rate per service channel

σ_u^2 = Variance of inter arrival time

σ_s^2 = Variance of service time

S = Number of parallel servers

ρ = $\frac{\lambda}{S\mu}$ = Utilization factor

$\frac{\lambda}{\mu}$ = Traffic intensity

x = Random variable for the number of units present in the system

f(x) = Probability density function (p.d.f) of x

$\beta(x)$ = Mean of the instantaneous change rate of x

$\alpha(x)$ = Variance of the instantaneous change rate of x

H_1, H_2 = Constants of integration

$P_n(t)$ = the probability that the number of customers in system at time t is n

P_n = The steady state probability of exactly n customers in the system

X(t) = Continuous valued variable for the number of customers in the system

$W(\tau)$ = Probability density function (p.d.f) for the waiting time in the system

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CHAPTER ONE

INTRODUCTION

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1.0 INTRODUCTION

1.1 General Introduction

The role of a port in the prosperity and economic growth of a country needs not to be over emphasized. Port is the only link for sea-borne trade and commerce with the outside world. Chittagong port,being the principal port of Bangladesh with shore based facilities, plays a vital role in the economic development of the country. Most of the country's export-import activities are carried out through this port. The big amount of revenue earned through services to the various arriving ships at the port, enriches the national economy and accelerates the country's development. The revenues earned during the year 1982-83 and 1983-84 are respectively Tk.49,36,04,000 and Tk.50,25,17,000. Number of ships and cargoes for imported and exported good handled during the year 1983-84 are respectively 858; 56,81,C37 and 3,93,057 [24]. At present the port is working with 15 jetties having ancilliary facilities for cargo handling,cargo storing etc.

It is expected that with the development of the country the export import activities at the port will increase substantially. To meet the increasing traffic demand it is an utmost necessity to take appropriate steps to provide adequate amount of facilities so that optimum service level can be maintained at the port. Otherwise it will cause a marine congestion problem which in turn will adversely affect the revenue earnings of the port.

1.2 System of Operation at the Port

The Chittagong port started functioning in 1888 with only four jetties constructed by the then Assam Bengal Railway. The port is situated on the right bank of the river KARNAFULI at a distance of about 9 nautical miles from the shore line of the Bay of Bengal. River KARNAFULI rising from the LUSHAI HILLS falls in the Bay of Bengal after taking a winding course of about 120 nautical miles through Chittagong Hill Tracts and Chittagong.

The Chittagong Port handles different types of cargo ships such as general cargo ships, oil tankers, bulk carriers (cement clinkers, food grains, sulpher), Container ships, 'RO-RO' ships etc. The different handling facilities are used to handle these ships. Shore jetty cranes are required for handling general cargo ships. Tanker terminals with adequate pipe line facilities are required for handling oil tankers. Bulk carriers need bucket, vacuum or grab unloaders depending on the nature of bulks.

The ships coming to the port cast anchor first at the outer anchorage. Then according to the berthing schedule, which is generally prepared weekly or fortnightly after getting necessary information from the shipping agents, ships are placed in the berths. It is important to mention here that for Chittagong port, the movement of ships depends on the tide and the draft. Ships move only at high tide and within a maximum draft of 9.14 m. For ships with higher draft like mother tankers, mother cargoes, lighterage facilities are arranged. After placing the ships in the berth cargo loading or unloading starts. The port has 15 jetties with jetty cranes, rail-road links and other shore based facilities. Sometimes after arrival at the port, the ship has to wait at the outer anchorage due to preoccupation of all jetties or terminals.

1.3 Problem Formulation

The system of operation at the port can be thought of as a typical queueing process. The basic process assumed by most queueing models is the following. "Customers" requiring service are generated over time by an "input source". These customers enter the queueing system and join a queue. At certain times a member of the queue is selected for service by some rule known as the service discipline. The required service is then performed for the customer by the service mechanism, after which the customer leaves the queueing system. This process is depicted in fig. 1.1.

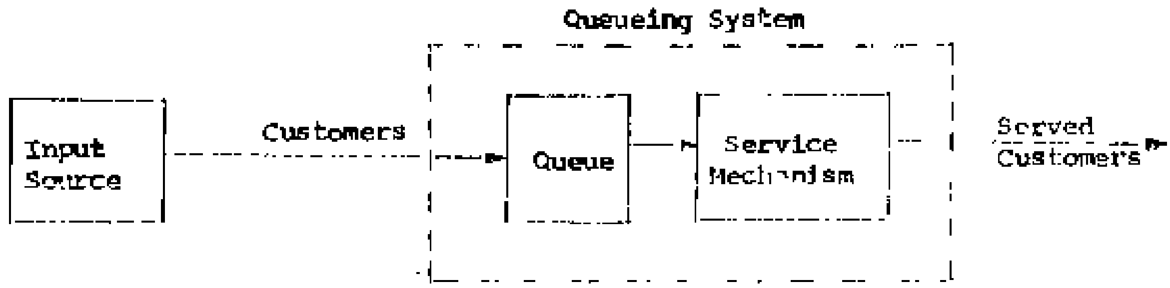

Fig. 1.1 : The basic queueing process

At the port ships come to the port as customers to get services and the facilities of the port render services to ships as servers. Here the services refer to handling of cargoes (loading or unloading) and facilities to jetties for berthing of ships or jetty cranes, mobile cranes, fork-lifters etc. for cargo handling, warehouse for cargo storing.

A large proportion of waiting-line problems that arise in practice involve making decisions on one or a combination of the followings:

1. Number of servers at a service facility
2. Efficiency of servers
3. Number of service facilities.

When such problems are formulated in terms of a queueing model, the corresponding decision variables usually would be number of servers at each facility, mean service rate per busy server and arrival rate at each facility. All the specific decisions mentioned above involve the general question of the appropriate level of service to provide in a queueing system. Decisions regarding the amount of service capacity to provide usually are based primarily on two considerations: (1) the cost incurred by providing the service and (2) the amount of waiting for that service. It is, however, readily apparent that these two considerations create conflicting pressures on the decision maker. The objective of reducing service costs recommends a minimal level of service. On the other hand, long waiting times are undesirable, which recommends a high level of service. Therefore it is necessary to strive for some type of compromise.

If adequate service facilities are not provided at the port for the incoming ships then the formation of waiting line or queue will occur. This may incur penalty costs for some items. Moreover if this situation prevails for a long time then in the long run the port has to lose 'customers' because of poor service condition. All these will cause loss of revenue. On the other hand, with respect to the frequency of the arriving ships, if the facilities having too big capacity are installed then there will be no queue but the service facilities will remain partially or fully idle which means excessive capital expenditure. So selection of optimal service level at the port is of utmost importance.

1.4 Objective of the Study

In view of the future possible marine congestion problem at the Chittagong port, management is looking for some methodology to ascertain the optimum capacity level of the service facilities which will be suitable at varying situations and service conditions. In this respect the present study is an attempt to provide an appropriate guideline to management. To keep this in mind, the objectives of the study have been identified as follows:

a. To study the current system of operation at the port and determine the behaviours (distribution) of the different system characteristics (parameters)

b. To select the appropriate queueing models for the system

c. To develop a suitable cost model to determine the optimal number of jetties with appropriate service level to ensure maximum net incremental revenue at different mean arrival rates of the ships;

CHAPTER TWO

LITERATURE SURVEY & BACKGROUND STUDY

2.0 LITERATURE SURVEY & BACKGROUND STUDY

2.1 Introduction

It is hardly possible to avoid a queue in day to day life activities. Waiting in a queue to receive any form of service from a given facility is a common phenomenon. Probably one of the major problems in many systems in this scientifically advanced world is the congestion problem. In the solutions of such problems designers try to minimize server idleness and customer waiting time or in other words they try to find out a compromise. Customers may get impatient if waiting is too long and thus sometimes leave the system even after waiting for sometime in the queue or may not join a long queue if they are yet to join. Thus organizations loose customers which in turn affects the profit margin if proper service is not available. But again to satisfy the customers' demand, management needs to invest more money for providing extra facilities. Therefore proper solutions of these are needed.

Since the last few decades, queueing problems have attracted the attention of operations analysts and industrial engineers. The available literature in congestion problems reveals that many mathematical models have been developed and applied quite successfully. However mathematical solutions of many complex queueing model can be formulated, it is sometimes very difficult to get the solution, sometimes the solutions are infact indeterminant. The literature shows that the researchers are forced to search for different approximation method to give answers to their problem within some resonable error zone when exact solutions are not available.

2.2 Characteristics of a Queueing Process

A queueing process in general has the following four basic characteristics:

i. Arrival process, ii. departure process, iii. queueing discipline and iv. system structure which are described in short below.

i. Arrival Process

Customers arrive to a service facility to receive service either from an infinite source or from a finite source population. Arrival may be independent or in group (bulk) and may have peaks at different times in a day e.g. rush hours. The arrival process is a stochastic process with known or unknown probability distribution for interarrival time. Sometimes, newly arriving customer decides not to join the system after seeing a long queue and thus balks. In addition, designers need to know if all customers are identical or some customers are significantly different from others with respect to arrival time distribution.

ii. Service or Departure Process

In some systems like cyclic or tandem systems service process of one stage becomes the arrival process to the next stage. Departure process has got all the similar characteristics of arrival process. Like arrival process, the departure process is also a stochastic process which may follow the different types of probability distributions depending on the type of service needed by the customers. Bulk service may sometimes take place like single service. Customers generally leave the system after receiving service but in some cases customer may get impatient due to long waiting time and leaves the queue without receiving services.

iii. Queue Discipline

Queue discipline refers to the manner in which the customers are selected for the service when a queue has formed. Generally customers are served in a service facility on first come first served basis. But there are cases of last come first serve or service at random order. Priorities in services are also seen in practical problems which may have different classes and types such as preemptive or nonpreemptive type. In preemptive situation, the customer with the highest priority is allowed to enter service immediately even if a customer with lower priority is already in service when the higher priority customer enters the system. In nonpreemptive situation the highest priority customer goes to the head of the queue but can not get into service until the customer presently in service is completed, even though this customer has a lower priority.

iv. System Structure

Series or parallel arrangements of service facilities are commonly found but series parallel combination or network of service stations are also seen sometimes. Multi-stage cyclic queueing system or line assembly process are the examples of series arrangement. There may be single or multiple servers at each stage in a series system. The system capacity or the number of waiting rooms may be infinite or finite. Blocking of service facility in a series arrangement or refusal of entry to a customer may take place due to finite number of waiting rooms.

2.3 Brief History of Work on Queueing Theory

The queueing theory was developed in 1903 when the Danish mathematician A.K. Erlang worked with the problem of congestion of telephone traffic[8]. Since then the development of the theory is continuing. Erlang published his works as "The Theory of Probabilities and Telephone Conversations"[7]. Molina worked on trunking problems and published his works as "Application of the Theory of Probability to Telephone Trunking Problems"[7]. Thornton C-Fry published his works as "Probability and its Engineering Uses" Which expanded much of Erlang's earlier work[8]. Felix Pollaczek did some further pioneering work on Poisson input, arbitrary output and single and multiple channel problems. Additional work was done at that time in Russia by Kolmogorov and Khintchine, in France by Crommelin and in Sweeden by Palm. Some representatives of the more recent contributions are works by D.R. Lindley on integral equations. N.T.J.Bailey and W. Lederman and G.E.Reuter on time dependent solutions, L. Takacs on waiting time, D.R.Cox on supplimentary variables, D.G.Kendall on imbedded Chains, D.G.Champernowne on the use of random walks and S.Karlin and J.L. McGregor on birth-death processes. Some additional researchers who have been very productive and have provided significant works to the literature are V.E. Benes, U.N.Bhat, R.W. Couway, D.P.Gaver, J.D.C.Little, W.L. Maxwell, P.M. Morse, M.F.Neuts, N.U. Prabhu, E.Reich, T.L. Saaty and R.Syski[7].

Biswas[2], Sunaga[21] worked with diffusion approximation method for different queueing model with Erlangian inter arrival and service time distribution. Halachmi[9],(10) worked with the diffusion approximation method with the knowledge of mean and variance of inter arrival and service time.

2.4 Mathematical Models and Limitations

Practical queueing problems vary widely both with respect to problem formulation and the type of solutions sought. It is sometimes very difficult to define the exact mathematical model to represent a practical queueing problem. As a result researchers tried to know the solutions of different queueing problems by making some hypothetical assumptions about the arrival and departure processes and the system structure. Researches on Markovian model with single or multiple servers have been started since the early days of research in this field. Solutions of many markovian Models are available now alongwith those of some models with semi-Markovian process assumption and some simple models with general distributions. But solutions of many complex queueing models are yet to be known. Formulation of mathematical models for complicated queueing problems are themselves difficult and even if the model can be formulated, it is sometimes very difficult to get the solution. Many known results of queueing problems are available in complicated transform form which is very difficult to invert and some times numerical approximation is used to invert them. Thus the exact solution of queueing problems are limited to systems with known probability distributions for arrival and departure process or to some systems with simple system structure. Moreover, solutions of such problems can not be applied to practical cases always as the practical problems do not possess the ideal assumptions which are generally made during the formulation of such models. Theoretical models are not always built to fit the practical problem rather the designers try to fit the problem to a known model. Exact solutions of queueing problems with general arrival and service time distributions, multi-stage series queueing systems are not known yet. However, considering Erlangian distribution for inter-arrival time and service time a comprehensive work has been done in this field for multi-servers with balking and two stage cyclic systems[2],[21].

Problems may be static or dynamic i.e. deterministic and probabilistics. Practical problems are generally dynamic i.e. system behaviour varies with time to a greater extent. But most of the solutions available for queueing problems are for static or equilibrium conditions.

2.5 Approximation Techniques

The approximations techniques are generally used to solve the practical problems where analytical solutions are difficult to find out. A considerable number of research papers have already been published on different approximation techniques, a detailed discussion on which is available in a survey paper by Bhat et al.[1]. Bhat et al. classified different approximation techniques into three major categories e.g. (i) system approximation (ii) process approximation and (iii) numerical approximation which are discussed below in short.

1. System Approximation

In this technique the system is observed first and then an approximate guess about the behaviour of the original system is made by comparison with some other known system with known behaviour. In other words, a simplified system is defined so as to resemble in behaviour as closely as possible to the original system to get the solution of the original one. The use of Erlangian distribution to represent a wide variety of practical input and output distributions is a type of system approximation. Page[19a] compared the results of models of multiple servers with different inter-arrival time and service time distribution and suggested an approximate method based on linear interpolation to get the value of model for multiple servers with Erlangian interarrival time and service time distribution (E/E/S). Maaloe[17] compared the results of single and multiple servers with exponential interarrival and service time (M/M/1 and M/M/S) and got a simple relation and suggested an approximation technique for multiple server with exponential inter-arrival time and Erlangian service time distribution (M/E/S). Cosmatatos[4] used the similarity of waiting line curves against co-efficient of variation for two systems to get an approximation for multiple servers with General Independent interarrival time and exponential service time distribution (GI/M/S).

ii. Process Approximation

In this approximation technique the process is simplified to get some approximation results. Thus in fluid or diffusion approximation the discrete nature of customer flow through the system is replaced with continuous flow to get the solution. This approximation technique will be discussed in detail in the next section.

iii. Numerical Approximation

It is the technique of manipulating the actual arithmatical expressions leading to an evaluation of certain measure.

2.5.1 Diffusion Approximation Method and Diffusion Equation

Real life queueing problems are stochastic in nature. If measurements of certain parameters of both arrival and departure processes are considered on successive days, the measurements will differ. That is the parameters will fluctuate around their mean values and with time these fluctuations will vary. The fluid flow approximation of queueing problems considers only means of arrival and departure processes. This approximation of queueing problems can be improved if the instantaneous variations in arrival per unit time and departure per unit time can be included in the mathematical model. Diffusion approximation method is such a technique where such variations are included along with the means. A considerable number of research papers have so far been devoted on this topic. Iglehart[13], Gaver[5], Newell[18],[19] & others did the pioneering research while the method is further developed by other researchers[2],[6],[10],[9],[11],[14],[16],[21], etc. A complete discussion on both fluid and diffusion approximation technique is available in [19] and a short but comprehensive description about both the methods is given in [15]. The diffusion equation is derived basing on the assumptions and discussions made in [2],[7],[9].

In diffusion approximation method, the discrete queueing process is replaced by a continuous process in such a way that the characteristics of the original process are not lost. Let $P_n(t)$ denote the time dependent probability that the number of customers in the system, $N(t)$, at time t is n. The diffusion approximation methodology suggests to replace the discrete variable $N(t)$ by a continuous variable $X(t)$, in a manner which allows recovery of estimates of $P_n(t)$ from the knowledge of the density function, $f(x,t).dx = P_r \left\{ x \leq X(t) \leq x + dx \right\}$ of $X(t)$. Since, by assumption made in (2), $X(t)$ behaves as a diffusion process,

$f(x,t)$ satisfies the chapman - Kolmogorov forward equation or Fokker-Plank equation and the following equation is obtained,

$$\frac{1}{2} \cdot \frac{\partial^2}{\partial x^2} \left\{ \alpha(x)\, f(x,t) \right\} - \frac{\partial}{\partial x} \left\{ \beta(x) \cdot f(x,t) \right\} = \frac{\partial}{\partial t} \left\{ f(x,t) \right\} \quad \ldots (2.1)$$

where, $\beta(x)$ and $\alpha(x)$, respectively, the mean and the variance of the instantaneous change in $X(t)$ given that $X(t) = x$. Mathematically $\beta(x)$ and $\alpha(x)$ are as follows,

$$\beta(x) = \underset{\Delta t \to 0}{Lt} \frac{E\left\{ X(t+\Delta t) - X(t) \mid X(t) = x \right\}}{\Delta t} \quad \ldots (2.2)$$

$$\alpha(x) = \underset{\Delta t \to 0}{Lt} \frac{Var\left\{ X(t+\Delta t) - X(t) \mid X(t) = x \right\}}{\Delta t} \quad \ldots (2.3)$$

Under steady-state condition, it is obvious from the process that $P_n(t)$ and $f(x,t)$ do not vary with t. Therefore mathematically,

$$\underset{t \to \infty}{Lt} P_n(t) = P_n \quad \ldots (2.4)$$

$$\underset{t \to \infty}{Lt} f(x,t) = f(x), \text{ and so} \quad \ldots (2.5)$$

$$\frac{\partial}{\partial t} \left\{ f(x,t) \right\} = 0 \quad \ldots (2.6)$$

Using the equations 2.5 & 2.6 in the equation 2.1 the following equation is obtained for steady state condition,

$$\frac{1}{2} \cdot \frac{d^2}{dx^2} \left\{ \alpha(x) \cdot f(x) \right\} - \frac{d}{dx} \left\{ \beta(x) \cdot f(x) \right\} = 0 \quad \ldots (2.7)$$

Applying 'Central Limit Theorem' to the arrival process and the departure process, it can be shown (2),(9) that for $0 \leq x \leq S$, $\beta(x) = \lambda - x\mu$ ... (2.8)

$$\alpha(x) = \lambda^3 \sigma_a^2 + x\mu^3 \sigma_s^2 \quad \text{... (2.9)}$$

for $x \geq S$, $\beta(x) = \lambda - S\mu$ ... (2.10)

$$\alpha(x) = \lambda^3 \sigma_a^2 + S\mu^3 \sigma_s^2 \quad \text{... (2.11)}$$

Where,

S = the number of servers

λ = the mean arrival rate

μ = the mean service rate per busy server.

σ_a^2 = variance of inter-arrival time

σ_s^2 = variance of service time.

The use of equations (2.7 to 2.11), after appropriate integrations, gives the following results[9]

$$f(x) = H_1 \left[\lambda^3 \sigma_a^2 + x\mu^3 \sigma_s^2 \right]^{u-1} \cdot \operatorname{Exp}\left\{ -\frac{2x}{\mu^2 \sigma_s^2} \right\} \quad \text{... (2.12)}$$

for $0 \leq x \leq S$

$$f(x) = H_2 \operatorname{Exp}\left\{ -2\left(\frac{S\mu - \lambda}{S\mu^2 \sigma_s^2 + \lambda^3 \sigma_a^2} \cdot x \right) \right\} . \quad \text{... (2.13)}$$

for $x \geq S$

where,

$$u = \frac{2\lambda}{\mu^3 \sigma_s^2} \left\{ \left(\frac{\lambda \sigma_a}{\mu \sigma_s} \right)^2 + 1 \right\}$$

H_1, H_2 are the constantsof integration

The procedure of calculating H_1 and H_2 is given in (2),(9),(21) considering continuous criteria and discretization criteria.

Biswas[2] and Sunaga[21] discussed about the system of multiple servers with Erlangian interarrival time and service time distribution (E/E/S). When Erlangian distribution is considered the mathematical expression becomes more simple and the system can be studied for different utilization factors.

2.5.2 Simulation

Simulation is generally used to solve practical queueing problems when exact results are not available. Simulation, a type of experimentation, needs many repetations to get the results within reasonable error limits and hence consumes enough computer time. Moreover, the results obtained are approximate ones and different types of tests are needs to be made to find out the reliability of the results and the model.

Simulation, which is called the management laboratory, is the technique to imitate the reality. It determines the effect of a number of alternative policies without disturbing the real system and helps to select the best policy. Many important managerial decision problems are too intricate to be solved by mathematical programming and experimentation with the actual system. Even if it is possible, it is too costly and risky. Simulation offers the solution by allowing experimentation with a model of the system without interfering with the real system. Simulation is thus often a by pass for complex mathematical analysis. Simulation models are comparatively flexible and can be modified to accommodate the changing environment of the real situation.

A simulation model can be considered as consisting of two basic phases; data generation and bookkeeping. Data generation involves the production of representative arrival of customers and their service times where needed throughout the queueing system. Generally this involves producing representative observations from prespecified probability distributions and this aspect is termed as Monte Carlo. Thus a Monte Carlo simulation is one in which it is necessary to generate at least one stream of random observations from some specified probability distribution.

Monte Carlo method of simulation is a very important tool to simulate the system characteristics. A detailed description and methodology of applying Monte Carlo simulation technique are given in [(7),(8),(12),(22)]. (10a, 10b).

The approximation techniques are not mathematically sound always but if experimentation proves them usefull, these methods can practically be used. In fact, engineers and system designers remain satisfied with approximate techniques if those techniques give some answers to their problems within some reasonable error zone when exact results are not available. However, when simulation method is used as an approximation method, the validity of the simulation is to be tested. Important works on validation are included in ref. 4a, 17a, 19c. 19d, 22a.

Sometimes historical data are used to build a model and then model output data are compared with the corresponding historical output data. If the agreement is not good, the parameters or the structure of the model are manipulated and the resulting output data are again compared with the historical data. This procedure , which is termed as callibration of a model, is continued until the two data sets agree closely.

CHAPTER THREE

DATA COLLECTION & ANALYSIS

3.1 Introduction

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3.3 Data Analysis

3.3.1 Analysis of Arrival of Ship at the Port

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3.0 DATA COLLECTION AND ANALYSIS

3.1 Introduction

In order to develop any kind of model for a system, the respective parameters must be known. In this chapter the relevant data collected from the traffic department of Chittagong Port Authority regarding the history of ship handling at the port have been analyzed to determine behaviour of the following parameters. These parameters will be the basis for the development of the appropriate queueing models for the system. Parameters are

i) Arrival of ships at the port

ii) Departure of ships from the port

iii) Waiting time of ships in queue

iv) Service time to serve a ship at the port

v) Number of ships in queue (queue length)

vi) Number of ships in service (in the berth)

vii) Number of ships in system

viii) Waiting time in system

ix) Cargo handling at the port.

3.2 Data Collection

The data of the ship handling history which includes the arrival date, the date of starting services, the date of departure from the port, the type of cargo handled, the amount of cargo handled (loaded-unloaded) was collected for a period of 5 years (January 1980 to December 1984). The following format was used to collect data:

SL NO	DATE OF ARRIVAL	DATE OF STARTING SERVICES	DATE OF DEPARTURE	TYPE OF CARGO HANDLED	AMOUNT OF CARGO HANDLED, LOADED-UNLOADED

During data collection only general cargo ships were considered as 81% of ships coming to the port are of this type (Table 3.1). The present study is restricted to the facilities of general cargo port only i.e. it excludes the facilities of tanker terminals, cement clinker jetties, silo jetties and Tripple Super Phosphate jetties.

TYPE OF SHIP \ YEAR	1978-79	79-80	80-81	81-82	82-83	83-84	Grand Total	%
General cargo ship	694	767	735	760	737	727	4420	81.25
Oil Tanker	43	57	64	57	51	44	316	5.81
Others	109	119	87	184	118	87	704	12.94
Total	846	943	886	1001	906	858	5440	100

Tab. 3.1 : Year-wise break up of types of ships

3.3 Data Analysis

The data of different system parameters are analyzed to determine the mean, variance and standard deviation. Tests are applied to check the goodness of fit of the probability distribution to the behaviour of system parameters.

3.3.1 Analysis of Arrival of Ship at the Port

The year -wise arrival history is given in Appendix A-1. Calculated values of Poisson distribution of probability & Chi-Square test are given below:

ARRIVAL OF SHIP PER DAY	OBSERVED FREQUENCY	PRODUCT FOR MEAN	PRODUCT FOR VARIANCE	EXPECTED PROBABILITY	EXPECTED FREQUENCY	CHI-SQUARE VALUE
(X)	(O)	(X).(O)	$(X^2).(O)$	$P(X)=\frac{e^{-\lambda}\lambda^X}{X!}$	(E)	$\chi^2=\frac{(O-E)^2}{E}$
0	246	-	-	0.131322	240	0.1500
1	502	502	502	0.266597	487	0.4620
2	476	952	1904	0.270610	494	0.6559
3	324	972	2916	0.183122	335	0.3612
4	167	668	2672	0.092939	170	0.0529
5	72	360	1800	0.037735	69	0.1304
6	29	174	1044	0.012768	23	1.5652
7	7	49	343	0.003703	7	0.0000
8	4	32	256	0.000940	2	2.0000
	1827	3709	11437		1827	5.3777

Mean arrival rate, $\lambda = \frac{3709}{1827} = 2.0301$ Ships per day.

Variance, $= \frac{1827 \times 11437 - (3709)^2}{1827 \times 1826} = 2.1398$

Standard deviation, $= 1.4628$ Ships per day.

Degrees of freedom, $\nu = 9 - 1 - 1 = 7.$

The critical value of Chi-square for 7 degrees of freedom at a significance level of 5% is

$$\chi^2_{0.05}(7) = 14.067 \text{ (from statistical table)}$$

Since the calculated value of Chi-square is less than the critical value, so the poisson distribution of probability provides a good fit to the distribution of arrival of ship to the port at 5% significance level. The observed and expected frequencies are shown in figure 3.1.

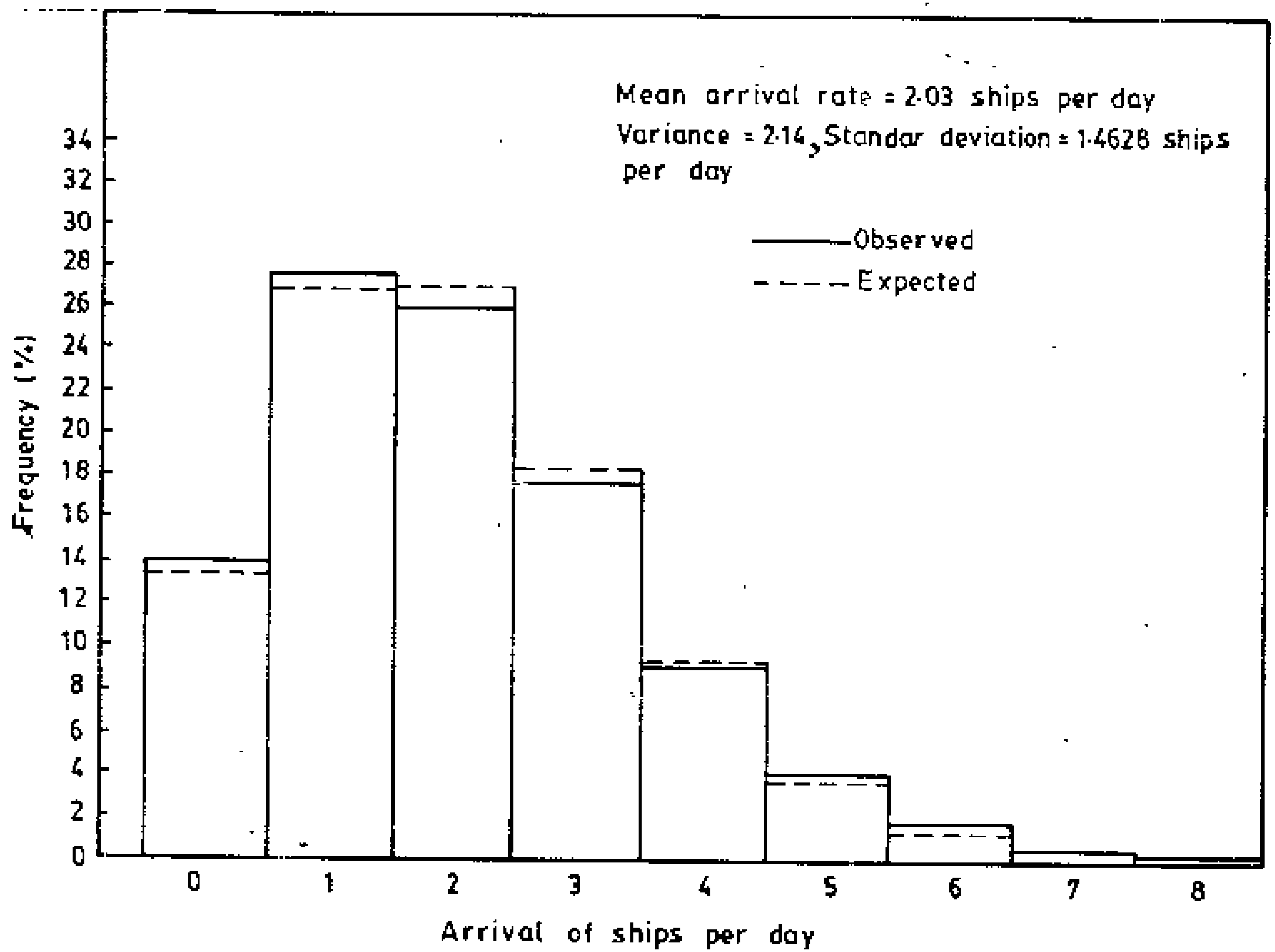


Fig. 3.1 Distribution of arrival of ships per day.

3.3.2 Analysis of Departure of Ships from the Port

The year-wise departure history is given in Appendix A-2. The following values are obtained for the departure process using the same procedure as used in arrival process.

Mean departure rate = 2.018 Ships per day

Variance = 2.135

Standard deviation = 1.461 Ships per day.

The Chi-square test shows that the poisson distribution of probability provides a good fit to the distribution of departure of ship from the port at 5% significance level. The observed and expected frequencies are shown in figure 3.2

3.3.3 Analysis of Service Time of a Ship

Year-wise history of service time of ship is given in Appendix A-3. Exponential distribution and Weibull distribution of probabilities are assumed for service time distribution per ship. Calculated values of exponential probability distribution & Chi-square test are given in table 3.2.

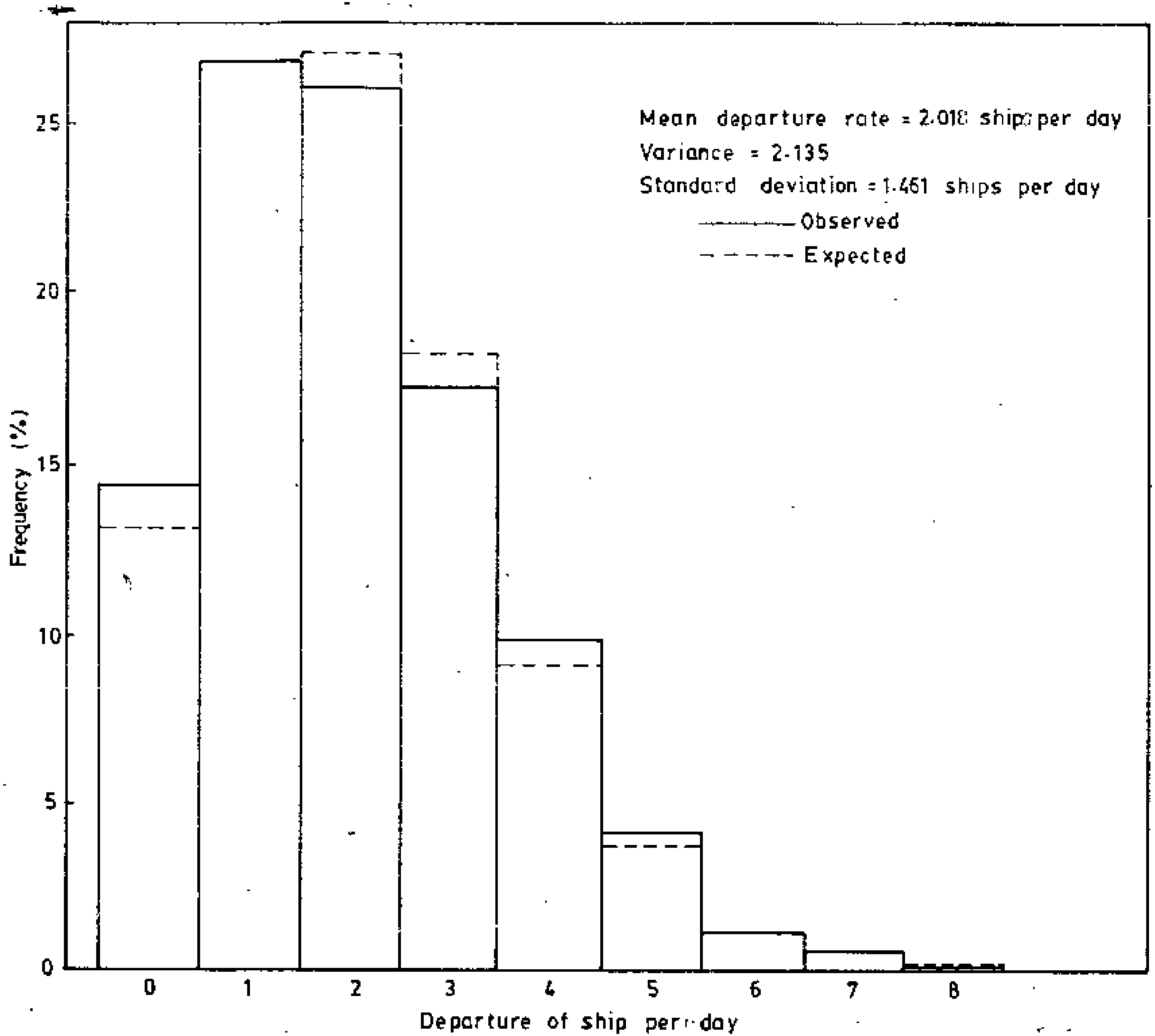


Fig. 3.2. Distribution of departure of ships per day

SERVICE TIME (t) Day	OBSERVED FREQUENCY (O)	t x (O)	t^2 x (O)	$F(t)=1-e^{-\alpha t}$	EXPECTED FREQUENCY (E)	$\chi^2 = \frac{(O-E)^2}{E}$
1	2	3	4	5	6	7
0 – 1	354	177.00	88.50	0.1434	525	55.69
1 – 2	455	682.50	1023.75	0.2662	450	0.06
2 – 3	487	1217.50	3043.75	0.3714	385	27.02
3 – 4	363	1270.50	4446.75	0.4616	330	3.30
4 – 5	311	1399.50	6297.75	0.5388	283	2.77
5 – 6	284	1562.00	8599.00	0.6049	242	7.28
6 – 7	225	1462.50	9506.25	0.6616	207	1.56
7 – 8	195	1462.50	10968.75	0.7101	178	1.62
8 – 9	171	1453.50	12354.75	0.7516	152	2.37
9 – 10	140	1330.00	12635.00	0.7873	130	0.76
10 – 11	86	903.00	9481.50	0.8178	112	6.03
11 – 12	79	908.50	10447.75	0.8439	96	3.01
12 – 13	63	787.50	9843.75	0.8663	82	4.40
13 – 14	59	796.50	10752.75	0.8854	70	1.72
14 – 15	56	812.00	11774.00	0.9019	60	0.27
15 – 16	42	651.00	10090.50	0.9159	52	1.92
16 – 17	40	660.00	10890.00	0.9280	44	0.35
17 – 18	29	507.50	8881.25	0.9383	38	2.13
18 – 19	27	499.50	9240.75	0.9467	32	0.78
19 – 20	28	546.00	10647.00	0.9543	28	0.00

Contd.

Table 3.2 : Calculated value of exponential probability distribution & Chi-square tests

1	2	3	4	5	6	7
20 - 21	17	348.50	7144.25	0.9609	24	2.04
21 - 22	16	344.00	7396.00	0.9665	20	0.80
22 - 23	14	315.00	7087.50	0.9713	17	0.52
23 - 24	17	399.50	9388.25	0.9754	15	0.27
24 - 25	11	269.50	6602.75	0.9789	13	0.31
25 - 26	13	321.50	8453.25	0.9819	11	0.36
26 - 27	6	159.00	4213.50	0.9845	9	1.00
27 - 28	15	412.50	11341.75	0.9867	8	6.13
28 - 29	5	142.50	4061.25	0.9886	7	0.57
29 - 30	11	324.50	9572.75	0.9902	6	4.17
30 - 31	9	274.50	8372.25	0.9916	5	3.20
31 - 32	4	126.00	3969.00	0.9928	4	0.00
32 - 33	6	195.00	6337.50	0.9939	4	1.00
33 - 34	5	167.50	5611.25	0.9947	3	1.33
34.-35	4	138.00	4761.00	0.9955	3	0.33
35 - 36	3	106.50	3780.75	0.9961	2	0.50
36 - 37	1	36.50	1332.25	0.9967	2	0.50
37 - 38	1	37.50	1406.25	0.9972	2	0.50
38 - 39	1	38.50	1482.25	0.9976	1	0.00
39 - 40	0	0.00	0.00	0.9979	1	1.00
40 - 41	1	40.50	1640.25	0.9982	1	0.00
41 - 42	1	41.50	1722.25	0.9985	1	0.00
42 - 43	1	42.50	1806.25	0.9987	1	0.00
43 - 44	2	87.00	3784.50	0.9989	1	1.00

Table 3.2 : Continued

1	2	3	4	5	6	7
44 – 45	0	0.00	0.00	0.9990	1	1.00
45 – 46	0	0.00	0.00	0.9992	1	1.00
46 – 47	2	93.00	4324.50	0.9993	–	–
47 – 48	0	0.00	0.00	0.9994	–	–
48 – 49	1	48.50	2352.25	0.9995	–	–
49 – 50	1	49.50	2450.25	0.9996	–	–
	3662	23658.00	301401.50		3662	149.65

Mean service time, $\frac{1}{\mu} = \frac{23658.00}{3662} = 6.46$ days per ship

$\alpha = \frac{1}{\text{Mean value}} = \frac{1}{6.46} = 0.1548$ ship per day

Variance $= \frac{3662 \times 301410.50 - (23658)^2}{3662 \times 3661} = 40.58$

Standard deviation $= 6.37$ days per ship.

Degrees of freedom = 50 - 1 - 1 = 48.

The critical value of Chi-square for 48 degrees of freedom at 5% level of significance is

$$\chi^2_{0.05}(48) = 66.13 \text{ (from the table by interpolation)}$$

Chi-square test shows that the exponential distribution of probability does not provide a good fit to the service time. The observed and expected frequencies are shown in the figures 3.3, 3.4 on the basis of exponential distribution. Fig. 3.3 is for discrete values and fig. 3.4 is for probability density function.

Weibull distribution covers a wide range of probability distribution. Also Kolmogorov - Smirnov (K-S) test is more powerful test than Chi-square test as (K-S) test uses expected and observed probabilities where as Chi-square test uses expected and observed frequencies. Hence Weibull distribution and (K-S) test are applied to check the goodness of fit of the distribution of the service time of a ship in the port. Calculated values of Weibull probability distribution and K-S test are given in Table 3.3.

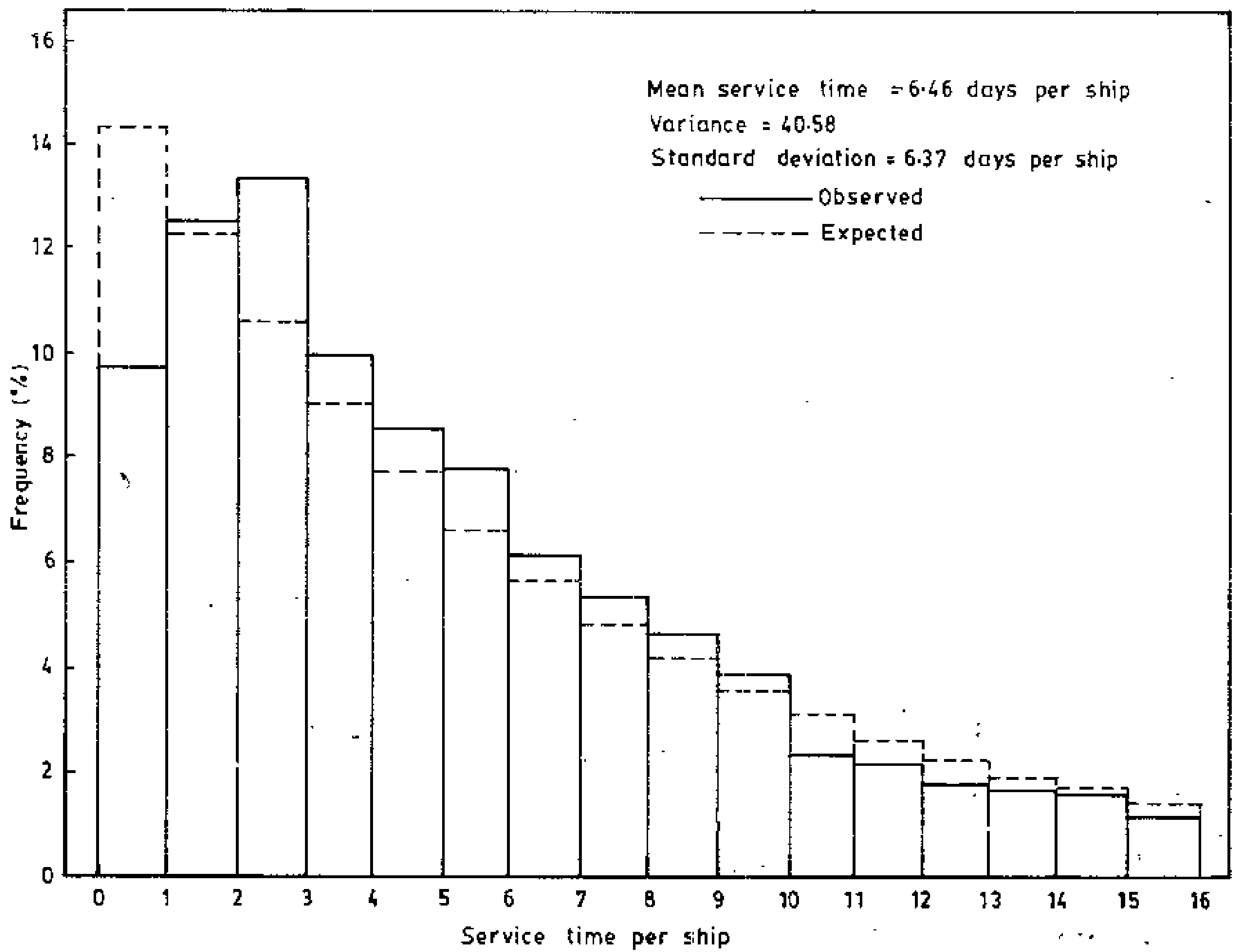


Fig-3·3 Distribution of service time – observed and expected (exponential)

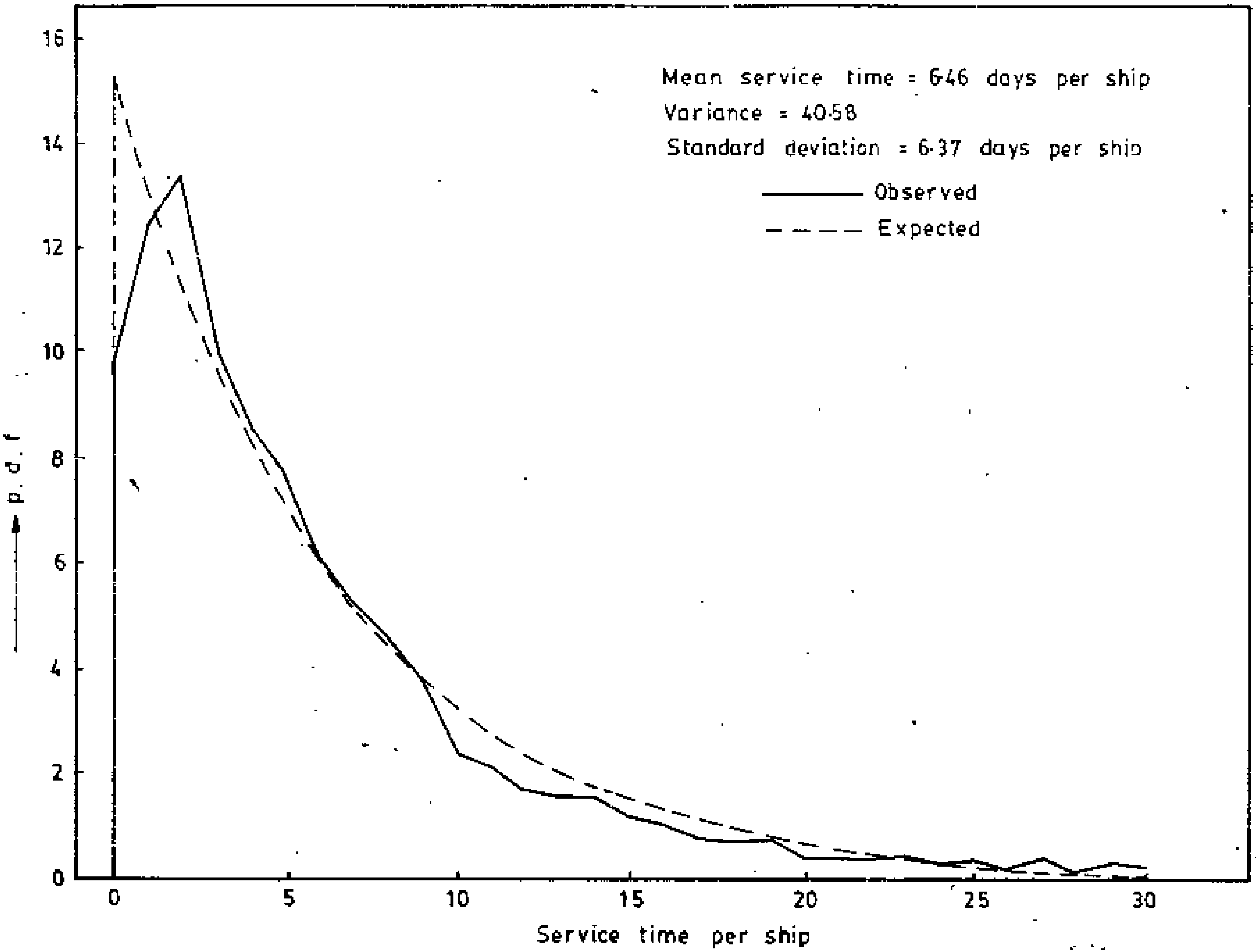


Fig. 3.4 p.d.f. for service time (exponential)

Class Time (day) t	Frequency	$S_t + \delta t$	$\frac{S_{t+\delta t} + S_{t-\delta t}}{2}$	r(t)	$\int r(t).dt$	$F = 1-e^{-\int r(t) dt}$	$F = 1-e^{-(\frac{t}{\eta})^\beta}$	(K-S) Value= $\|X_i - E_i\|$
Col-1	Col-2	Col-3	Col-4	Col-5 = $\frac{Col-2}{Col-4}$	Col-6	Col-7	Col-8	Col-9
0 - 1	354	3662	3485.0	0.10157	0.10157	0.0965	0.1367	0.0401
1 - 2	455	3308	3080.5	0.14770	0.24928	0.2206	0.2548	0.0341
2 - 3	487	2853	2609.5	0.18662	0.43590	0.3533	0.3567	0.0033
3 - 4	363	2366	2184.5	0.16616	0.60207	0.4523	0.4446	0.0076
4 - 5	311	2003	1847.5	0.16833	0.77041	0.5371	0.5206	0.0165
5 - 6	284	1692	1550.0	0.18322	0.95363	0.6146	0.5861	0.0284
6 - 7	225	1408	1295.5	0.17367	1.12731	0.6761	0.6427	0.0333
7 - 8	195	1183	1085.5	0.17964	1.30695	0.7293	0.6916	0.0377
8 - 9	171	988	902.5	0.18947	1.49642	0.7760	0.7338	0.0422
9 - 10	140	817	747.0	0.18741	1.68384	0.8143	0.7702	0.0441
10 - 11	86	677	634.0	0.13564	1.81949	0.8378	0.8016	0.0362
11 - 12	79	591	551.5	0.14324	1.96273	0.8595	0.8287	0.0307
12 - 13	63	512	480.5	0.13111	2.0938	0.8767	0.8521	0.0246
13 - 14	59	449	419.5	0.14064	2.23449	0.8929	0.8723	0.0205
14 - 15	56	390	362.0	0.15469	2.38918	0.9082	0.8898	0.0184
15 - 16	42	334	313.0	0.13418	2.52337	0.9198	0.9049	0.0149
16 - 17	40	292	272.0	0.14705	2.67043	0.9307	0.9179	0.0128
17 - 18	29	252	237.5	0.12210	2.79253	0.9387	0.9291	0.0095

Contd.

Table 3.3 : Calculated values of Weibull probability distribution & Chi-square test

Col-1	Col-2	Col-3	Col-4	Col-5	Col-6	Col-7	Col-8	Col-9
18 - 19	27	223	209.5	0.12887	2.92141	0.9461	0.9388	0.0073
19 - 20	28	196	182.0	0.15384	3.07525	0.9538	0.9471	0.0066
20 - 21	17	168	159.5	0.10658	3.18184	0.9584	0.9544	0.0040
21 - 22	10	151	143.0	0.11198	3.29373	0.9628	0.9606	0.0022
22 - 23	14	135	128.0	0.10937	3.40310	0.9667	0.9660	0.0006
23 - 24	17	121	112.5	0.15111	3.55421	0.9713	0.9706	0.0007
24 - 25	11	104	98.5	0.11167	3.66589	0.9744	0.9746	0.0002
25 - 26	13	93	86.5	0.15028	3.81617	0.9779	0.9781	0.0001
26 - 27	6	80	77.0	0.07792	3.89410	0.9796	0.9811	0.0014
27 - 28	15	74	66.5	0.22556	4.11966	0.9837	0.9837	0.0000
28 - 29	5	59	56.5	0.08849	4.20815	0.9851	0.9859	0.0008
29 - 30	11	54	48.5	0.22680	4.43496	0.9881	0.9878	0.0002
30 - 31	9	43	48.5	0.23376	4.66872	0.9906	0.9895	0.0010
31 - 32	4	34	32.0	0.12500	4.79372	0.9917	0.9909	0.0007
32 - 33	6	30	27.0	0.22222	5.01594	0.9933	0.9921	0.0011
33 - 34	5	24	21.5	0.23255	5.24850	0.9947	0.9932	0.0014
34 - 35	4	19	17.0	0.23529	5.48380	0.9958	0.9941	0.0016
35 - 36	3	15	13.5	0.22222	5.70602	0.9966	0.9949	0.0016
36 - 37	1	12	11.5	0.08695	5.79297	0.9969	0.9956	0.0012
37 - 38	1	11	10.5	0.09523	5.88821	0.9972	0.9962	0.0009
38 - 39	1	10	9.5	0.10526	5.99347	0.9975	0.9967	0.0007
39 - 40	-	9	9.0	0.00000	5.99347	0.9975	0.9972	0.0003

Table 3.3 : Continued

Col-1	Col-2	Col-3	Col-4	Col-5	Col-6	Col-7	Col-8	Col-9
40 - 41	1	9	8.5	0.11764	6.11112	0.9977	0.9975	0.0002
41 - 42	1	8	7.5	0.13333	6.24445	0.9980	0.9979	0.0001
42 - 43	1	7	6.5	0.15384	6.39830	0.9983	0.9982	0.0001
43 - 44	2	6	5.0	0.40000	6.79830	0.9988	0.9984	0.0004
44 - 45	-	4	4.0	-	6.79830	0.9988	0.9986	0.0002
45 - 46	-	4	4.0	-	6.79830	0.9988	0.9988	0.0000
46 - 47	2	4	3.0	0.66666	7.46496	0.9994	0.9990	0.0004
47 - 48	-	2	2.0	-	7.46496	0.9994	0.9991	0.0002
48 - 49	1	2	1.5	0.66666	8.13163	0.9997	0.9992	0.0004
49 - 50	1	1	0.5	2.00000	10.13163	0.9999	0.9993	0.0006

Results from Weibull probability paper shown in fig. 3.5, are as follows:

β = 0.98 $\approx$ 1.0

η = 6.8 days per ship

γ = 0.0

μ = 6.8 days per ship

where, β - Shape parameter or Weibull Slop

η - Scale parameter or Characteristic life

γ - Location parameter or Minimum life

μ - Mean value

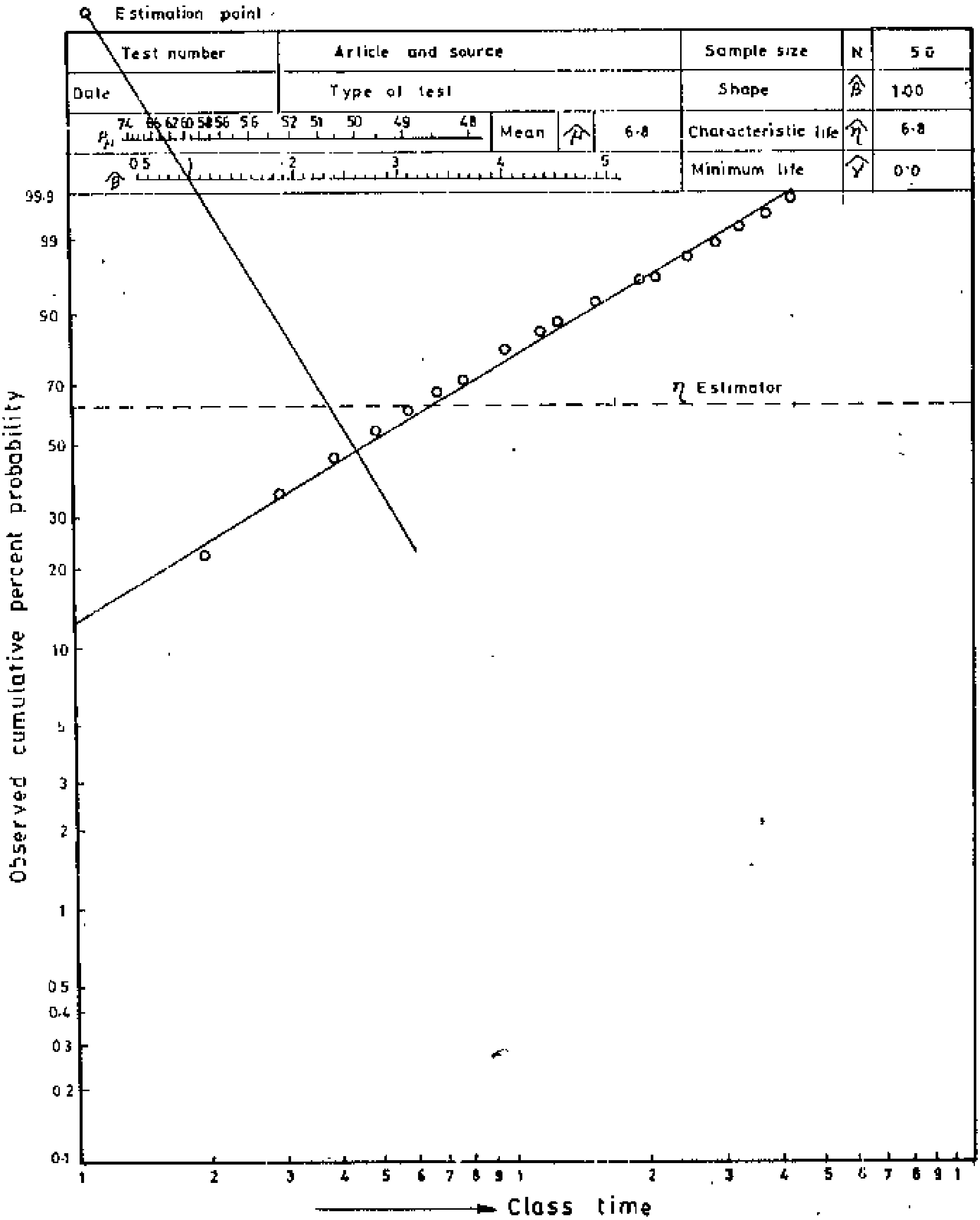


Fig. 3.5 Weibull paper plotting for service time

The calculated largest (K-S) value is 0.0441. The critical (K-S) value for n = 50 at 5% level of significance is 0.076933 (from (K-S) table), which shows that the service time distribution follows the Weibull distribution of probability with β = 1.0 & η = 6.8. Therefore it is concluded that the service time follows the exponential distribution of probabity with 5% level of significance. The observed and expected frequencies are shown in figure 3.6.

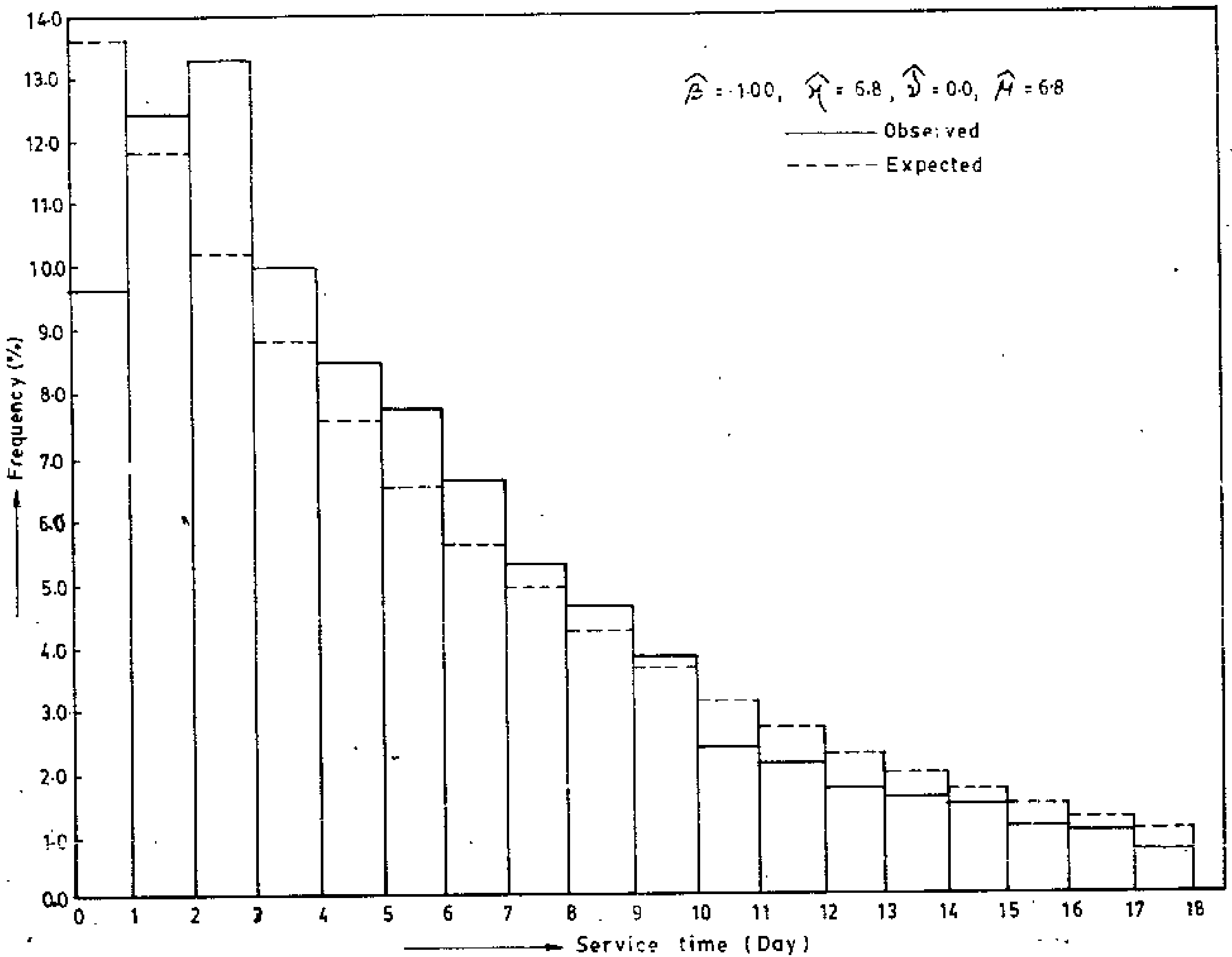


Fig. 3.6 Distribution of service time - observed and expected (weibull)

3.3.4 Analysis of Waiting Time in Queue

The exponential distribution of probability and the Weibull distribution of probability are assumed for waiting time in queue. -wise history of waiting time of ship in queue is given in Appendix A-4 The following values are determined for the waiting time of ship in the queue, following the procedure used in 3.3.3.

Mean waiting in queue = 2.869 days per ship

Parameter, $\alpha = \frac{1}{\text{Mean}} = \frac{1}{2.869} \approx 0.3485$ ship per day

Variance, = 14.115

Standard deviation = 3,757 days per ship.

The Chi-square test shows that the exponential distribution of probability does not provide a good fit to the waiting time of ship in queue at 5% significance level. The observed and the expected frequencies are shown in the figures 3.7, 3.8 on the basis of exponential distribution. Fig. 3.7 is for discrete value of probability and fig. 3.8 is for probability density function.

Weibull parameters, are estimated graphically by Weibull Probability Plotting Method, shown in figure 3.9 and they were,

β = 0.74

η = 2.35 days per ship

γ = 0.0

μ = 2.8 days per ship.

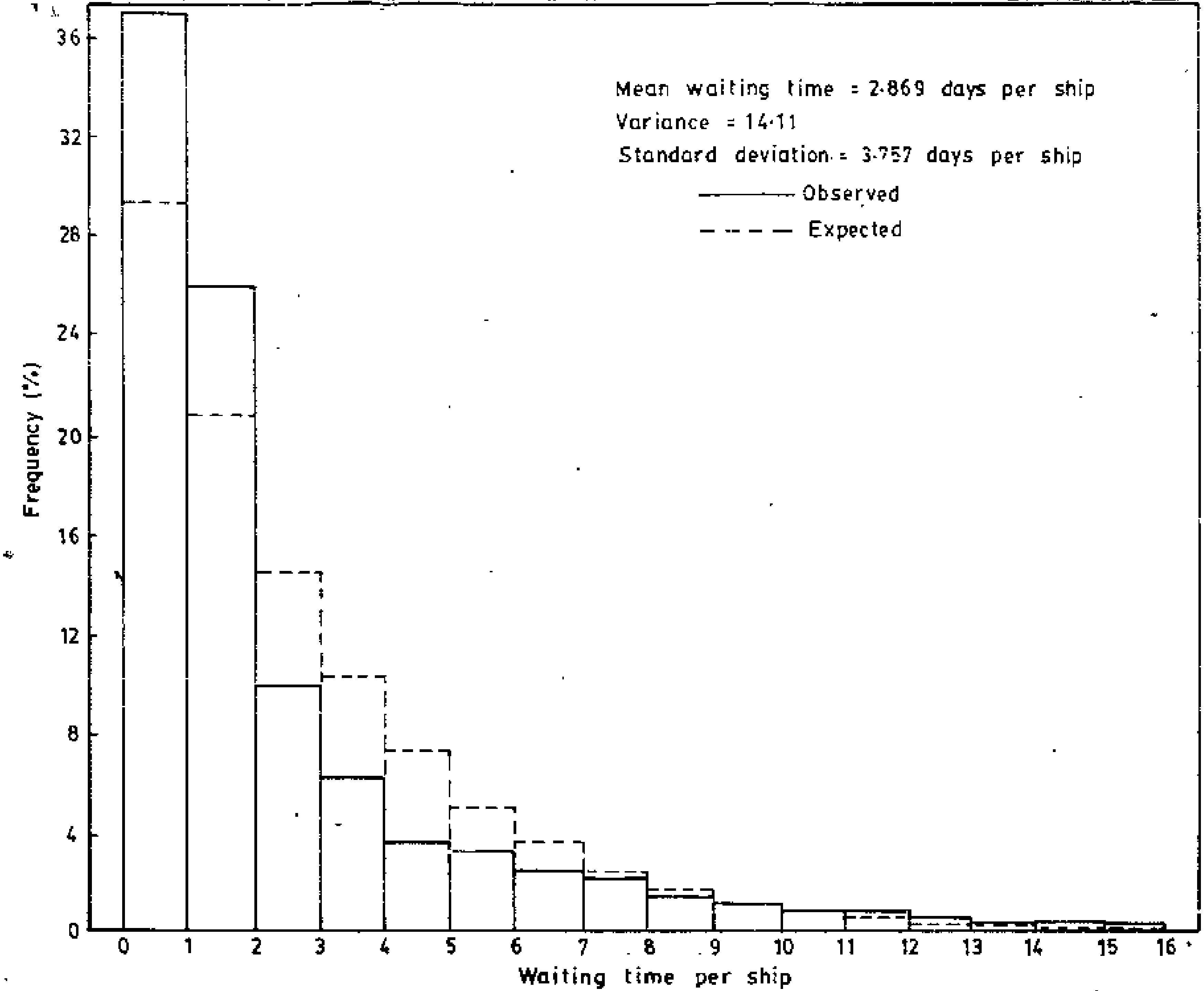


Fig. 3.7 Distribution of waiting time in queue--observed and expected (exponential)

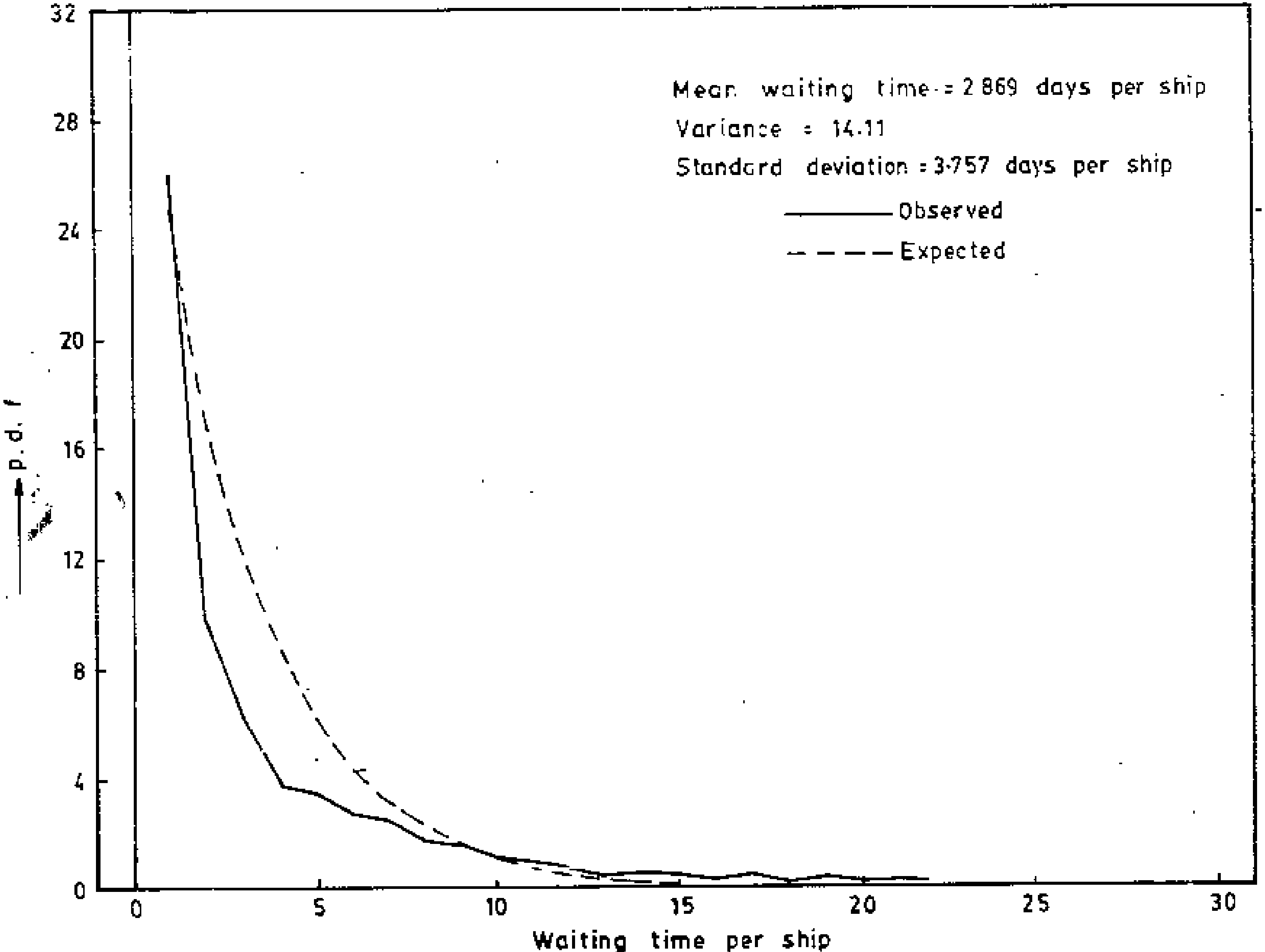


Fig.3.8 p.d.f for waiting time in queue (exponential)

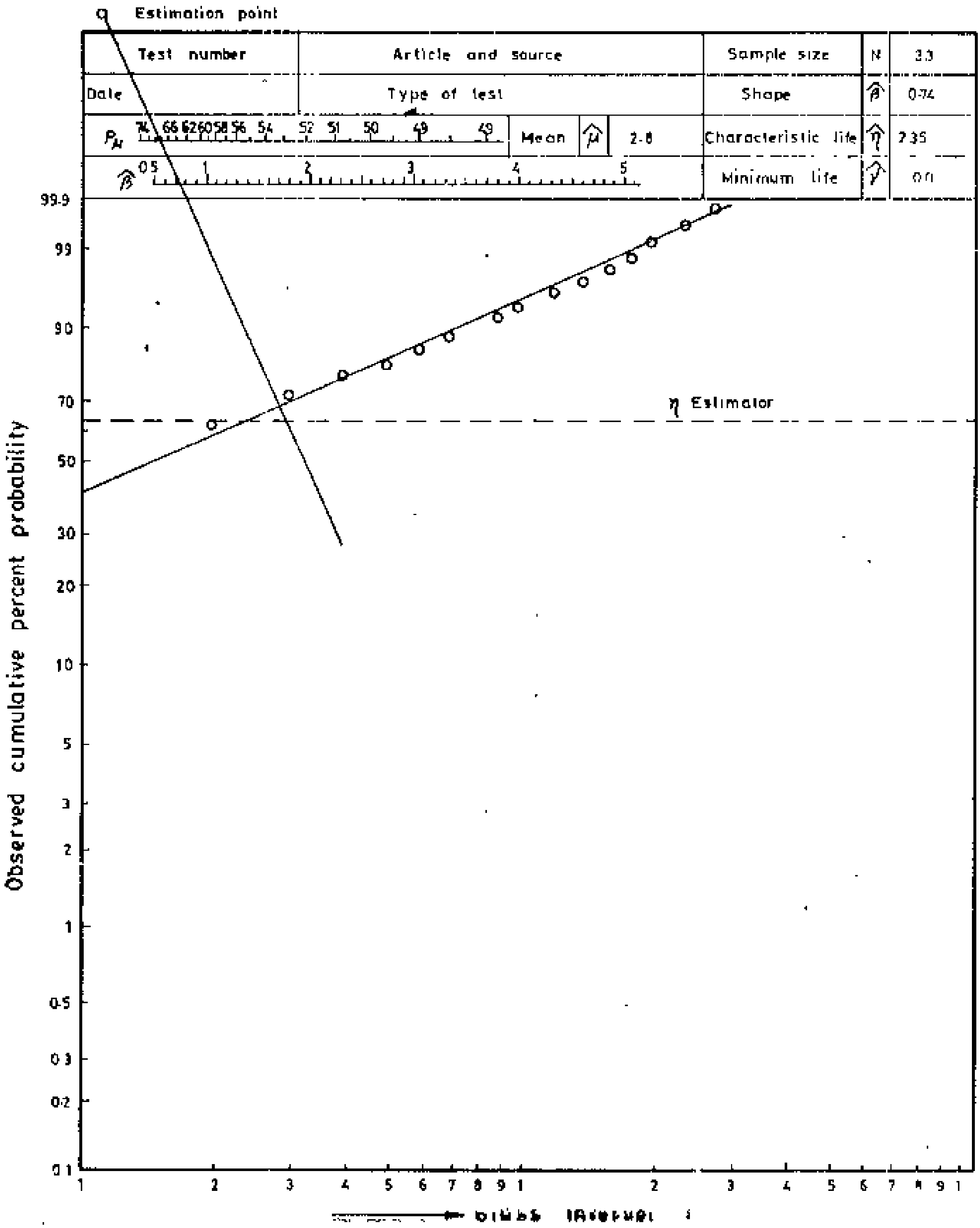


Fig. 3.9 Weibull paper plotting for waiting time in queue

The (K-S) test shows that the waiting time of ship in queue follows the Weibull distribution of probability with $\beta = 0.74$, $\eta = 2.35$, $\gamma = 0.0$, $\mu = 2.8$ at 5% significance level. The observed and expected probability are shown in figure 3.10.

3.3.5 Analysis of Number of Ships in Queue (queue length)

The Weibull distribution of probability is assumed for the number of ships in queue. Year-wise history of queue length is given in Appendix A-5. The following values are obtained for the number of ships in queue using the procedure as followed in 3.3.1.

Mean queue length (Lq)	=	4.825 ships
Variance	=	15.99
Standard deviation	=	3.999 ships

Results from Weibull probability paper, shown in figure 3.11 as follows:

$$\beta = 1.5, \eta = 5.75 \text{ ships}, \gamma = 0, \mu = 5.15 \text{ ships}.$$

The (K-S) test shows that the number of ships in queue (queue length) follows the Weibull distribution of probability with $\beta = 1.5$, $\eta = 5.75$, $\gamma = 0.0$ & $\mu = 5.15$ at 5% significance level. The observed and expected probability are shown in figure 3.12.

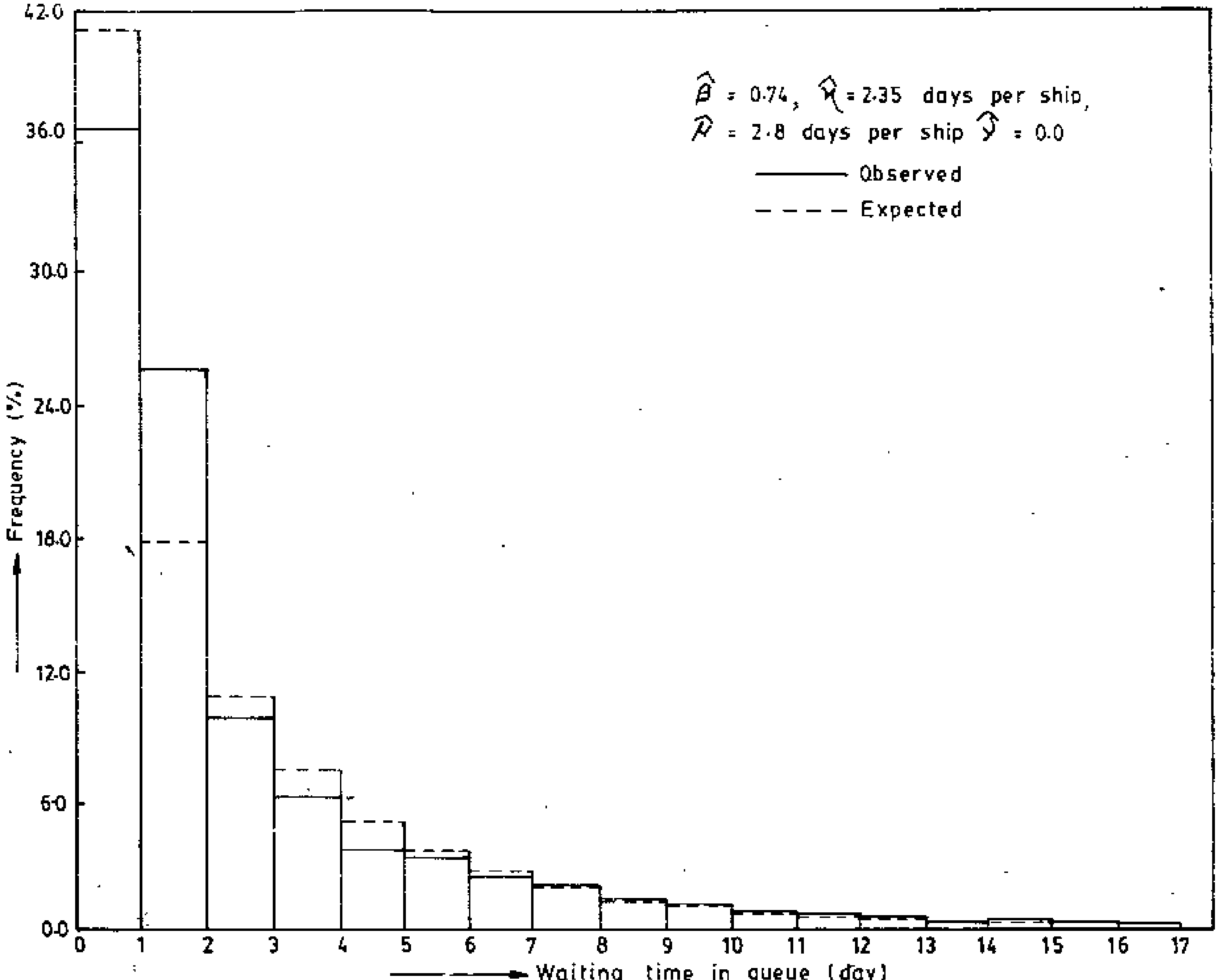


Fig. 3.10 Distribution of waiting time in queue - observed & expected (weibull)

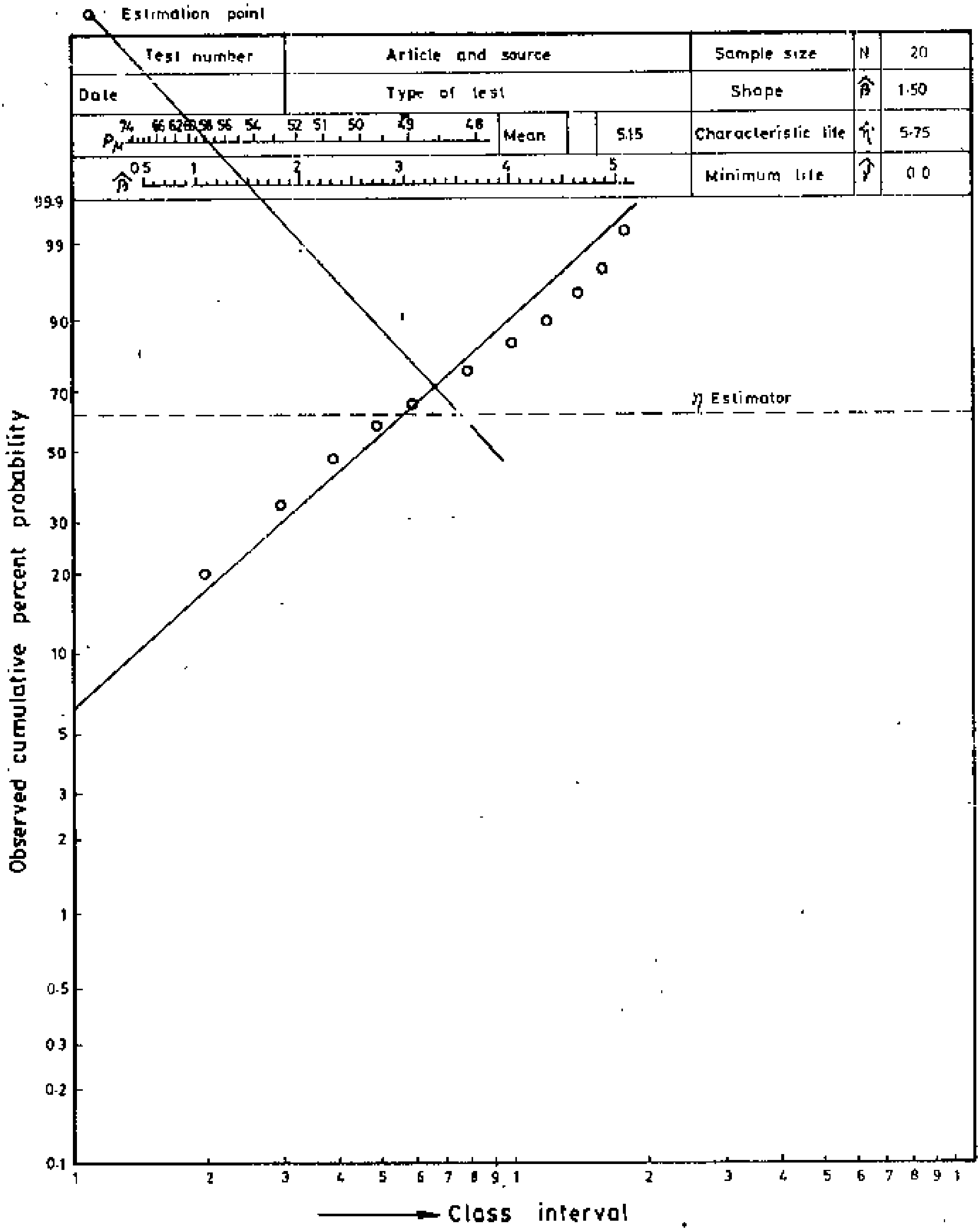


Fig.3.11 Weibull paper plotting for queue length

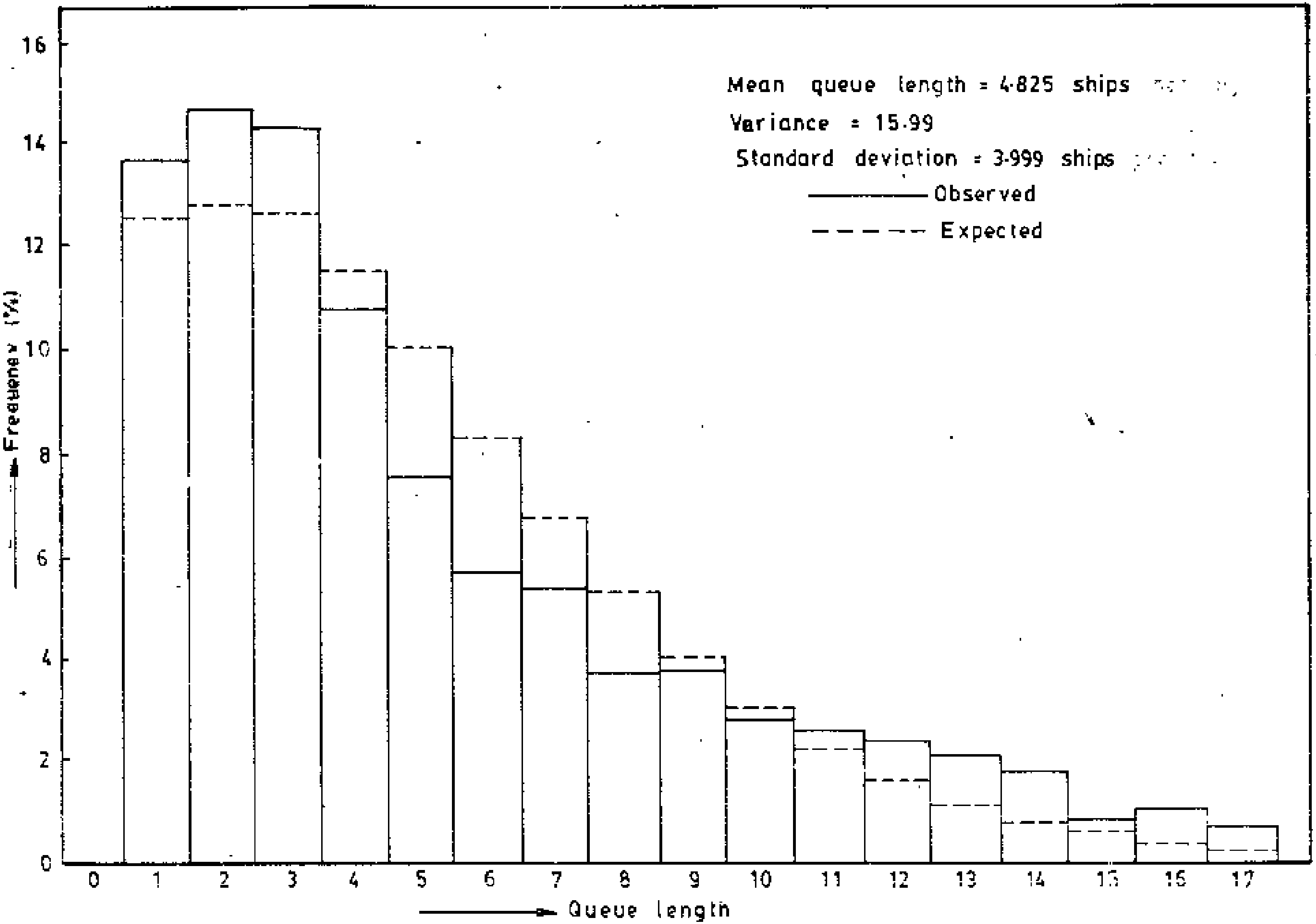


Fig.3.12 Distribution of queue length - observed and expected (weibull)

3.3.6 Analysis of Number of Ships in Service

The normal distribution and Weibull distribution of probabilities are assumed for the number of ships in service. Year -wise history of no. of ship in service is given in Appendix A-6. The following values are obtained for the number of ships in service using the procedure as followed in 3.3.1.

Mean no. of ships in service = 13.414 ships

Variance = 13.643

Standard deviation = 3.694 ships

The Chi-square test shows that the normal distribution of probability does not provides a good fit to the number of ships in service. The observed and expected frequencies are shown in figure 3.13.

Results from Weibull Probability Paper, shown in figure 3.14 are as follows:

β = 4.1

η = 15.8 ships

γ = 0.0

μ = 14.5 ships

The (K-S) test shows that the number of ships in service follows the Weibull distribution of probability with β = 4.1, η = 15.8 at 5% level of significance. The observed and expected frequencies are shown in figure 3.15

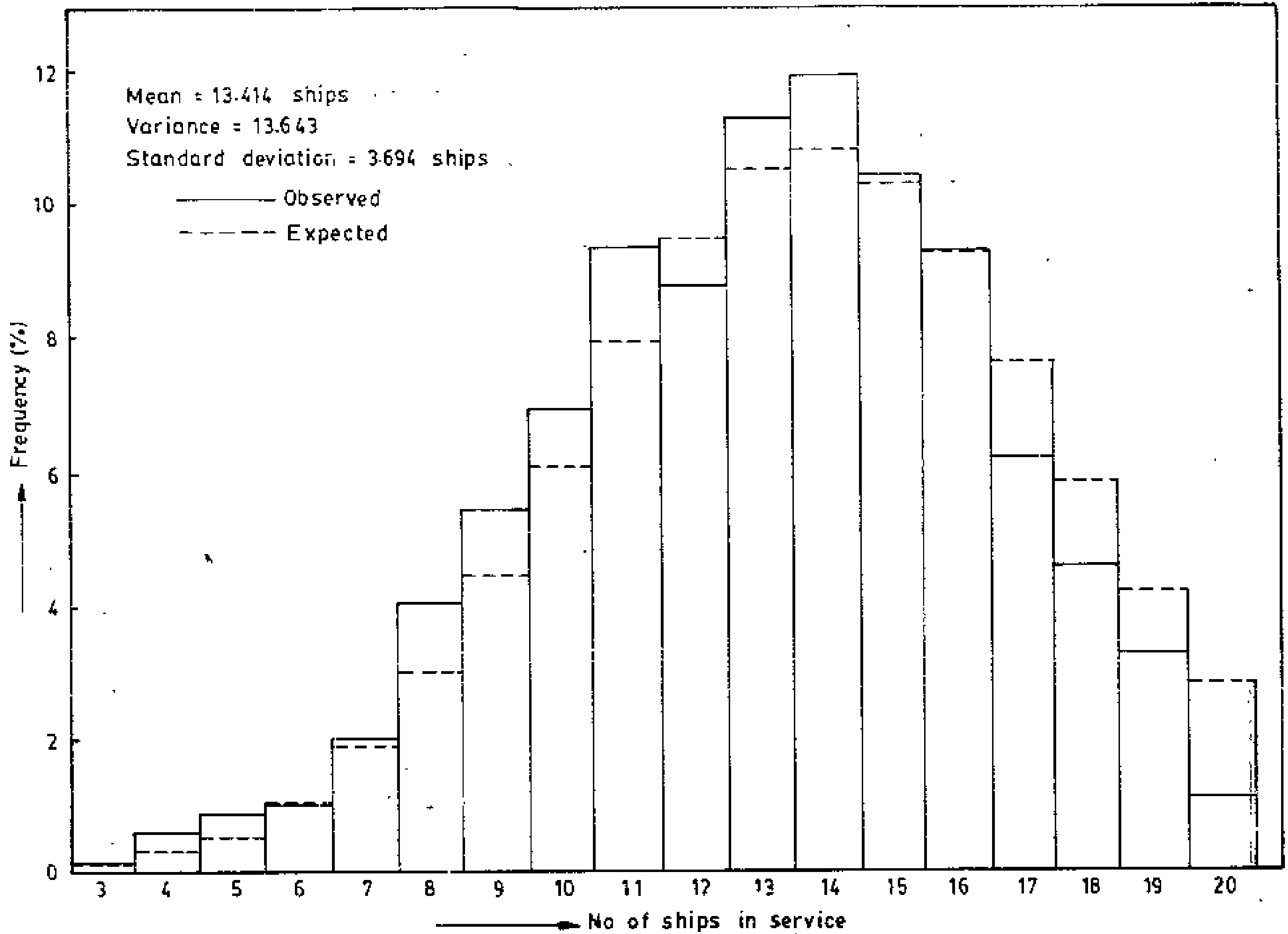


Fig.3.13 Distribution of No. of ships in service - observed and expected(normal)

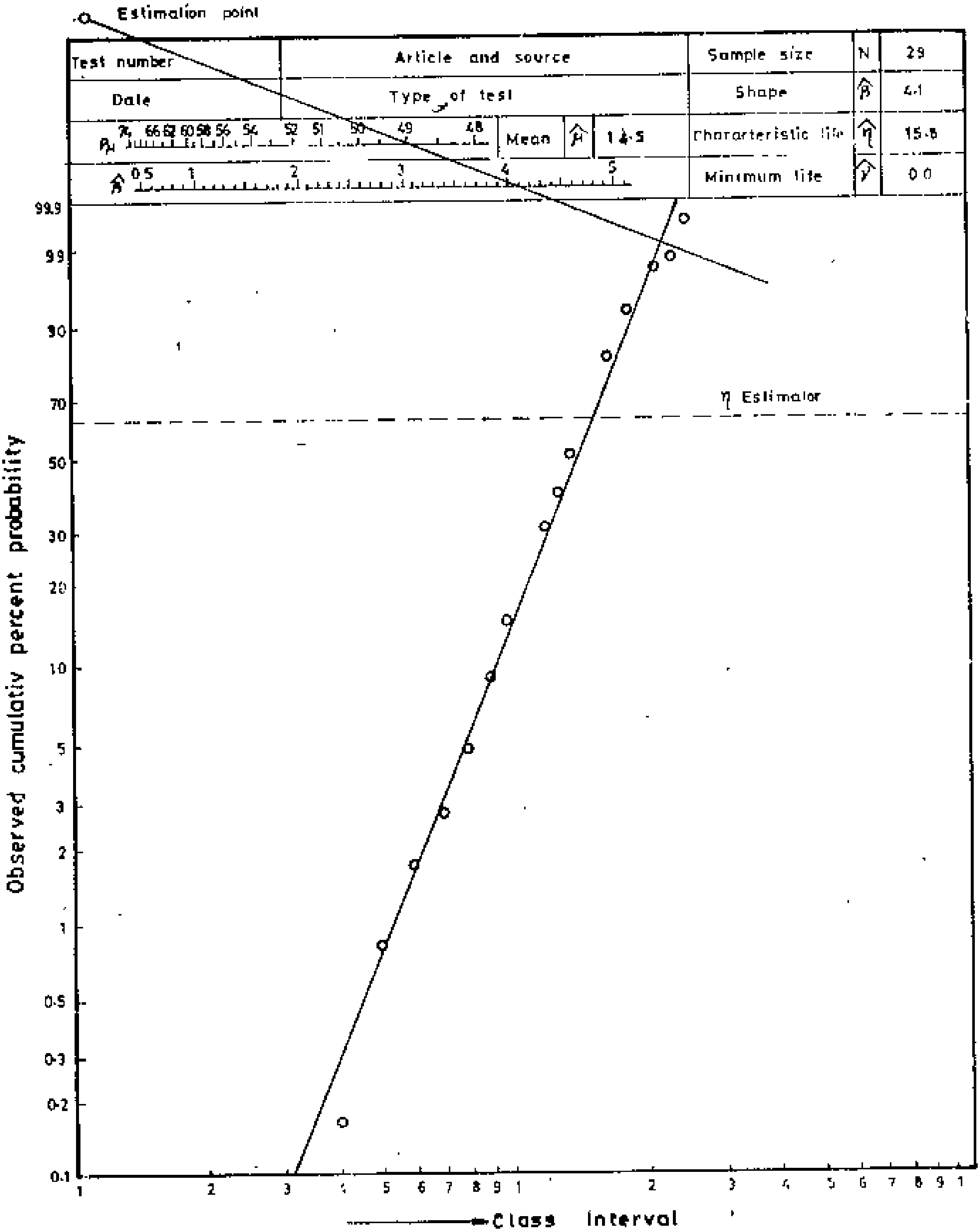


Fig. 3.14 Weibull paper plotting for no of ships in service

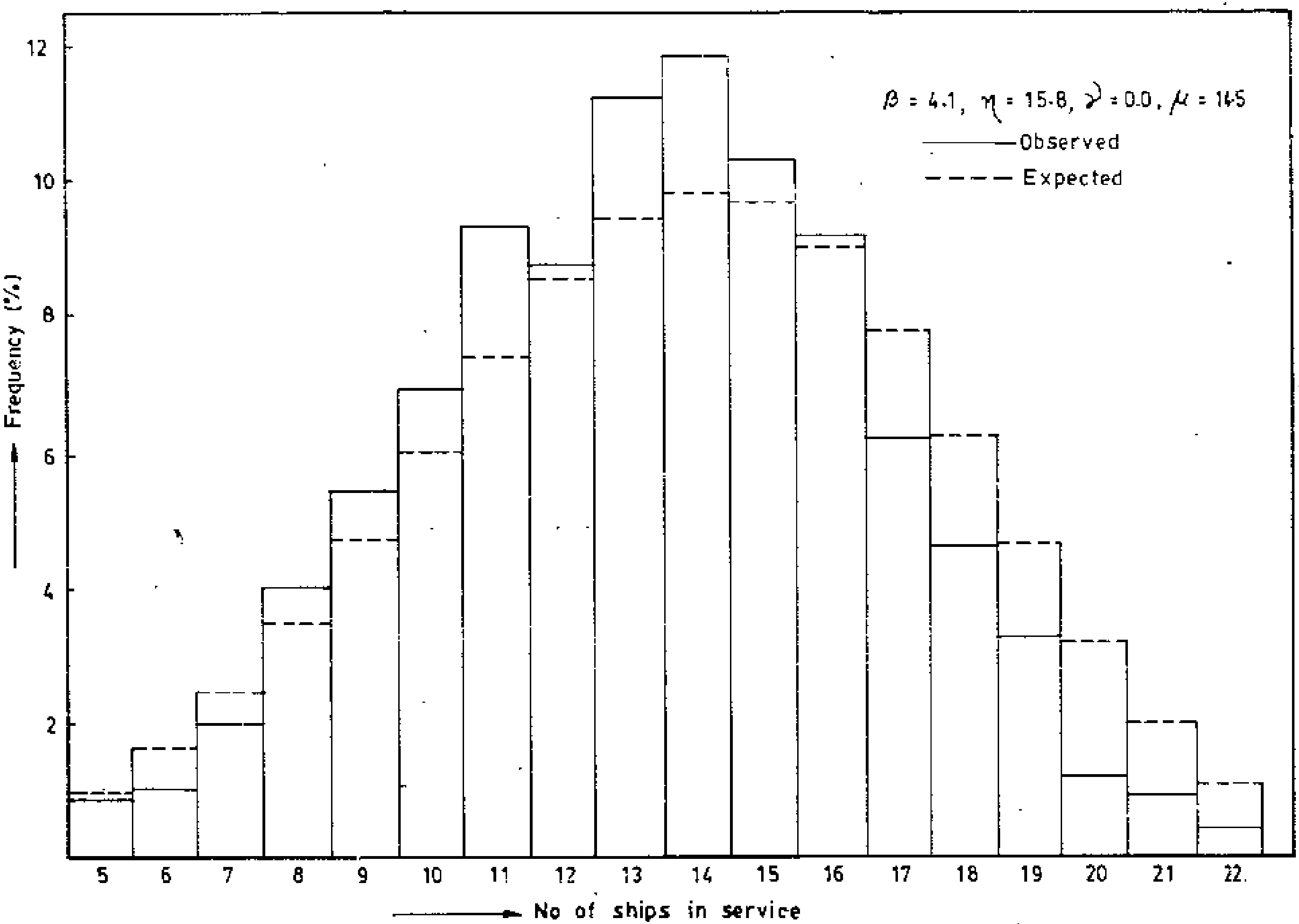


Fig.3.16 Distribution of no of ships in service - observed and expected (weibull)

3.3.7 Analysis of the Number of Ships in System

Normal distribution and Weibull distribution of probabilities are assumed for number of ships in system. Year -wise history of number of ships in system are given in Appendix A-7. The following values are obtained for the number of ships in system using the same procedure as followed in 3.3.1.

Mean number of ships in the system (L) = 18.2519 ships

Variance = 43.32

Standard deviation = 6.58 ships

The Chi-square test shows that the normal distribution of probability does not provides a good fit to the number of ships in the system. The observed and expected frequencies are shown in figure 3.16.

Results from Weibull Probability paper shown in figure 3.17 are as follows:

β = 2.00

η = 14.6 ships

γ = 5.83

μ = 18.83 ships

Calculations for γ are given in Appendix A-7. The (K-S) test shows that the number of ships in the system follows the Weibull distribution of probability with β = 2.0, η = 14.6 & γ = 5.83 at 5% level of significance. The observed and expected frequencies are shown in figure 3.18.

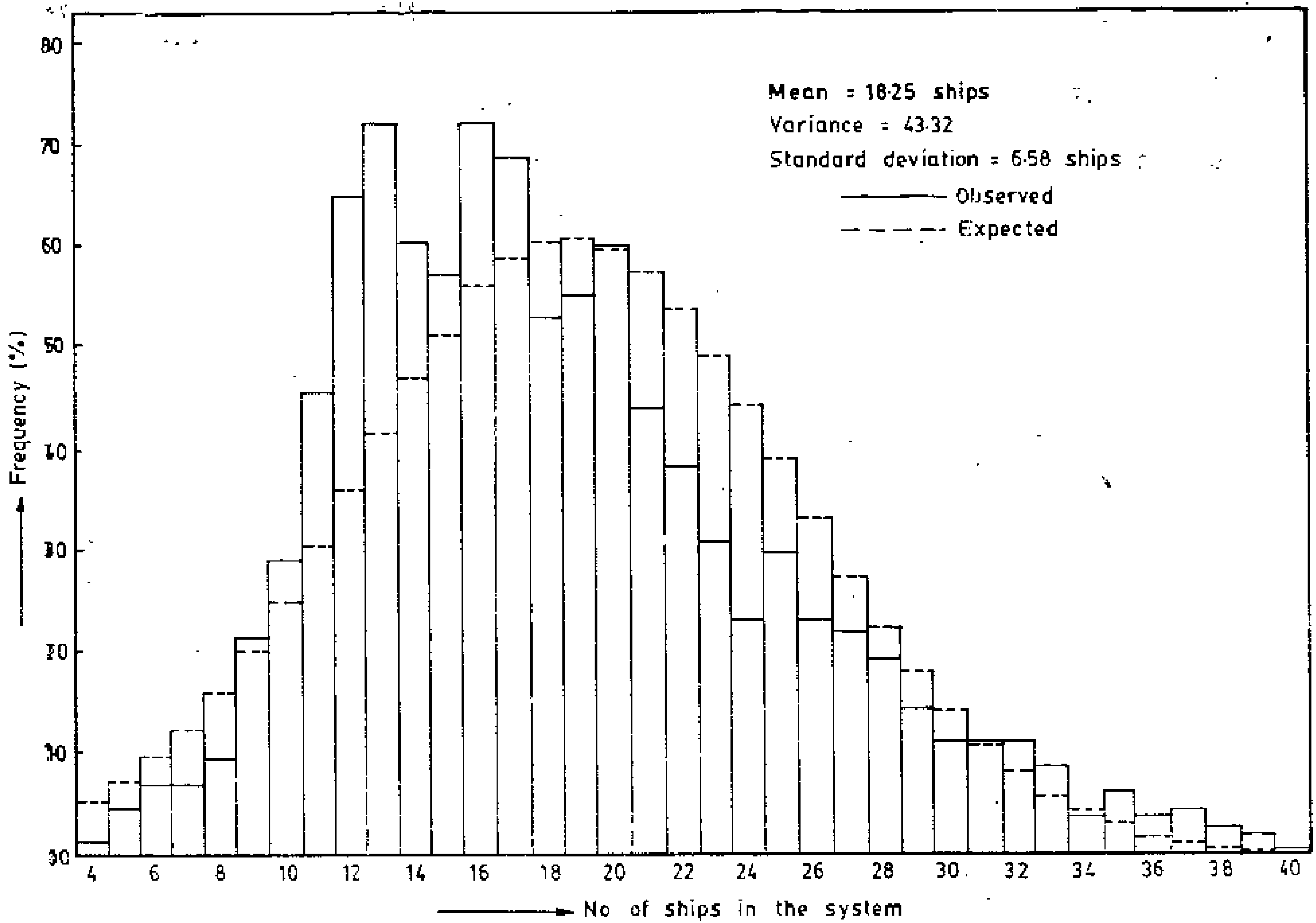


Fig.3.36 Distribution of no of ships in the system - observed expected (normal)

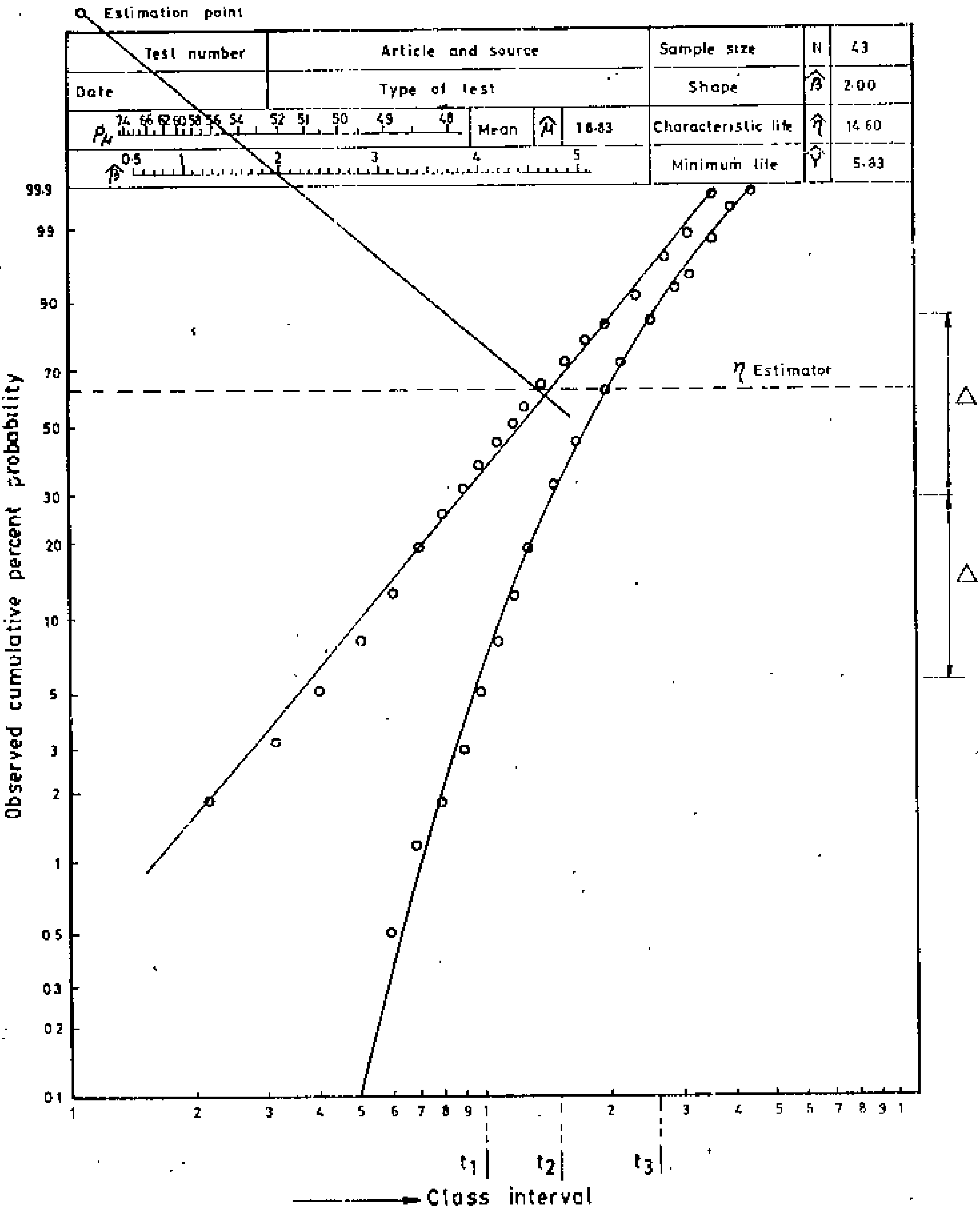


Fig.3.17 Weibull paper plotting for no of ships in the system

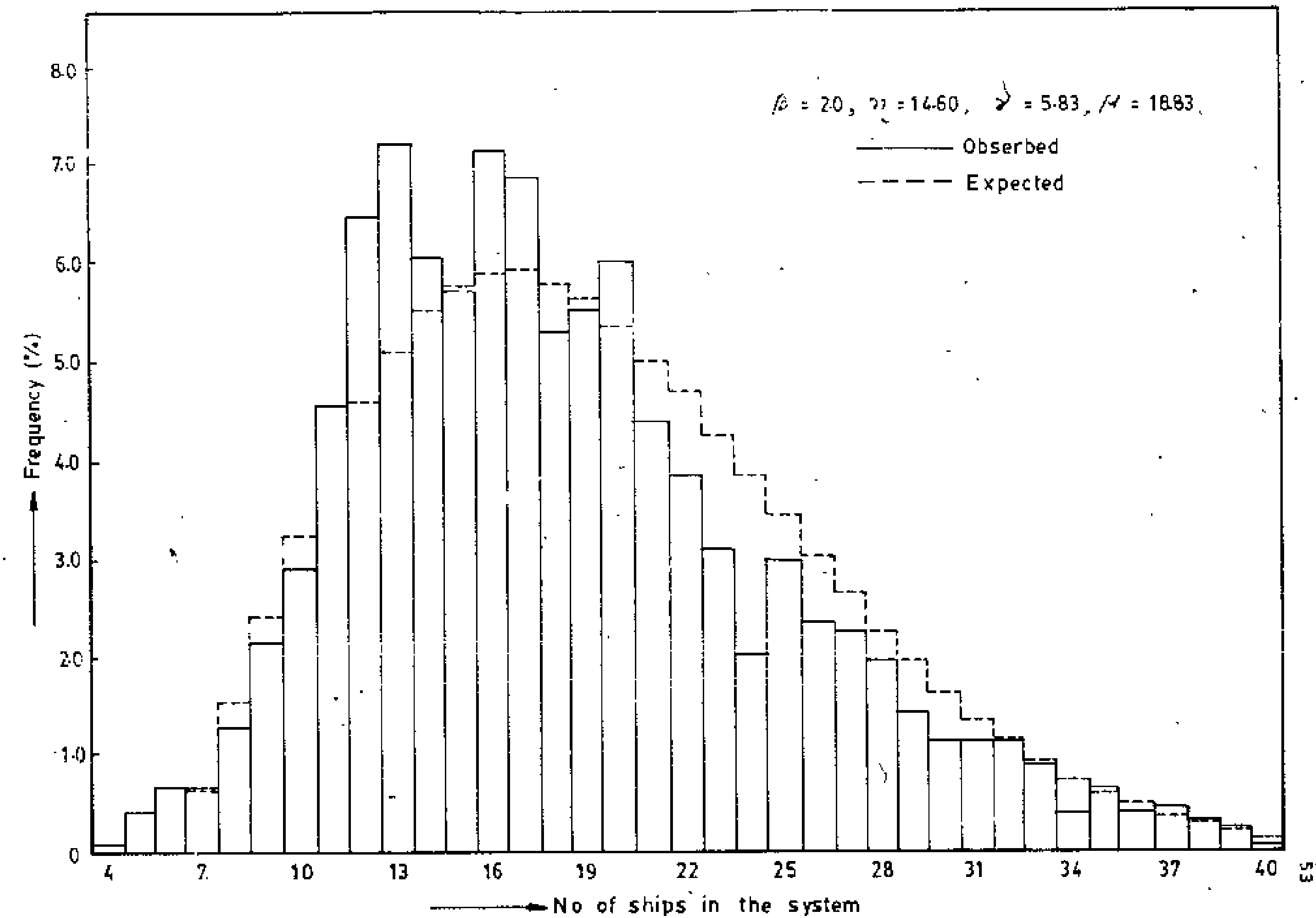


Fig. 3.18 Distribution of the no of ships in the system - observed and expected (weibull)

3.3.8 Analysis of Waiting Time of Ship in the System

The exponential and Weibull distributionsof probabilities are assumed for waiting time in the system. Year-wise history of waiting time in the system are given in Appendix A-8. The following values are obtained for waiting time of ships in the system using the procedure as followed in 3.3.1.

Mean waiting time of ship in the system (W) = 8.7477 days per ship

Variance = 65.0534

Standard deviation = 8.0656 days per ship.

The Chi-square test shows that the exponential distribution of probability does not provide a good fit to the waiting time of ship in system at 5% level of significance. The observed and expected frequencies are shown in figures 3.19, 3.20 Figure 3.19 for discrete values and figure 3.20 for probability density function

Results from Weibull Probability paper, shown in figure 3.21 are as follows:

β = 1.03

η = 9.0 days per ship

γ = 0

μ = 9.0 days per ship

The (K-S) test shows that the Weibull distribution of probability provides a good fit to the waiting time of ships in system with β = 1.03, η = 9 which is actually the exponential distribution of probability. The observed and expected frequencies are shown in figure 3.22.

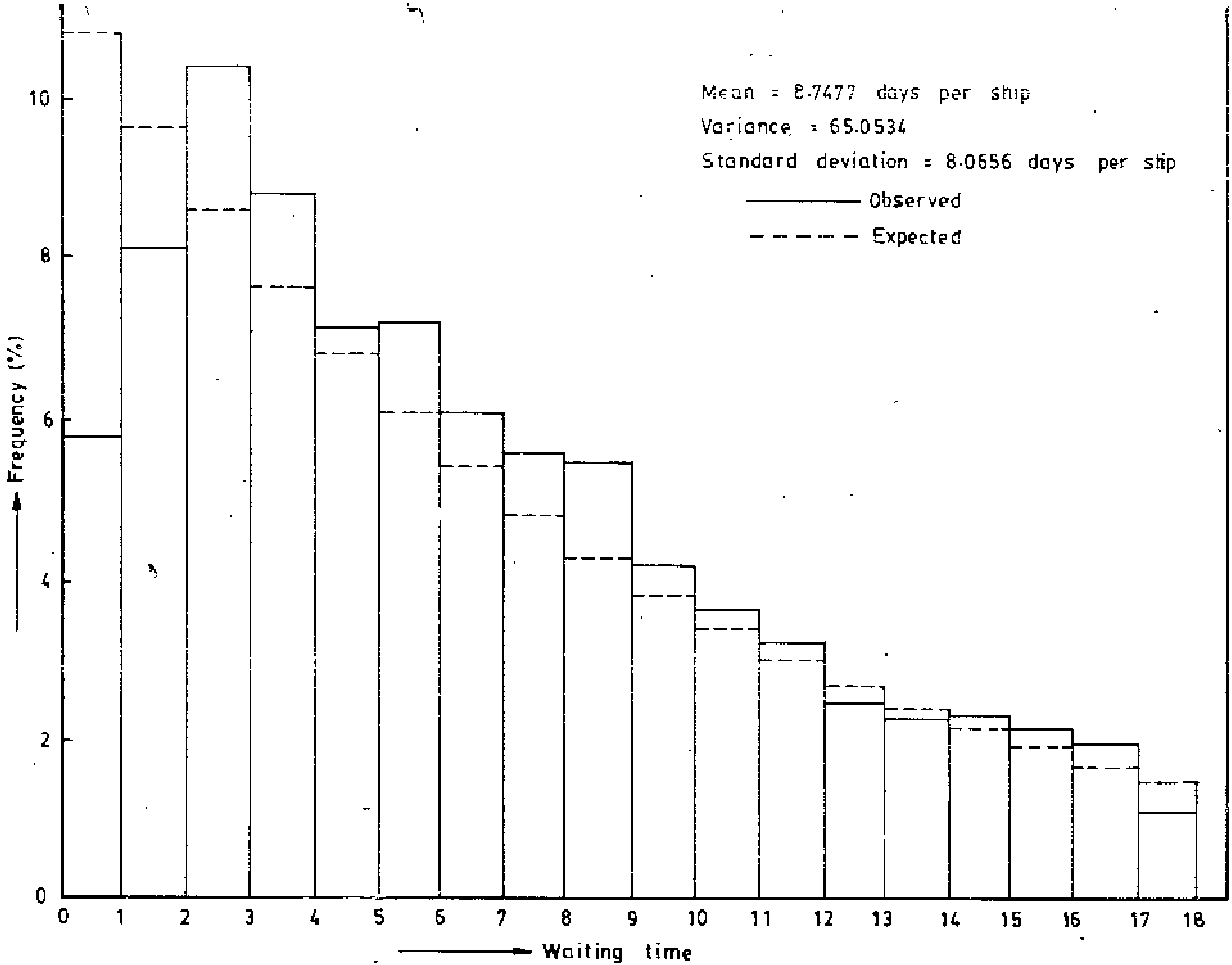


Fig. 3.19 Distribution of waiting time in the system - observed and expected (exponential)

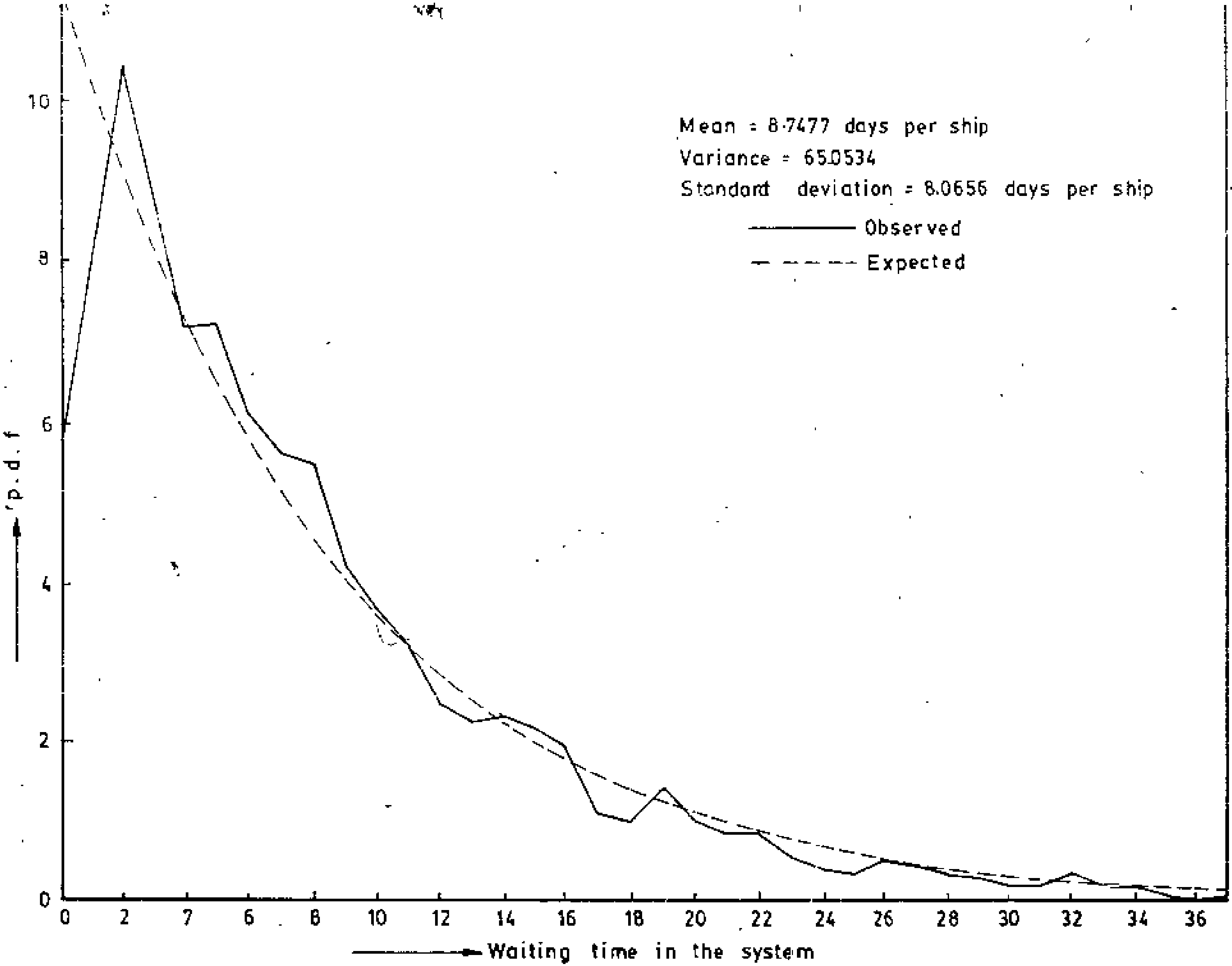


Fig-3.20 p.d.f for waiting time in the system (exponential)

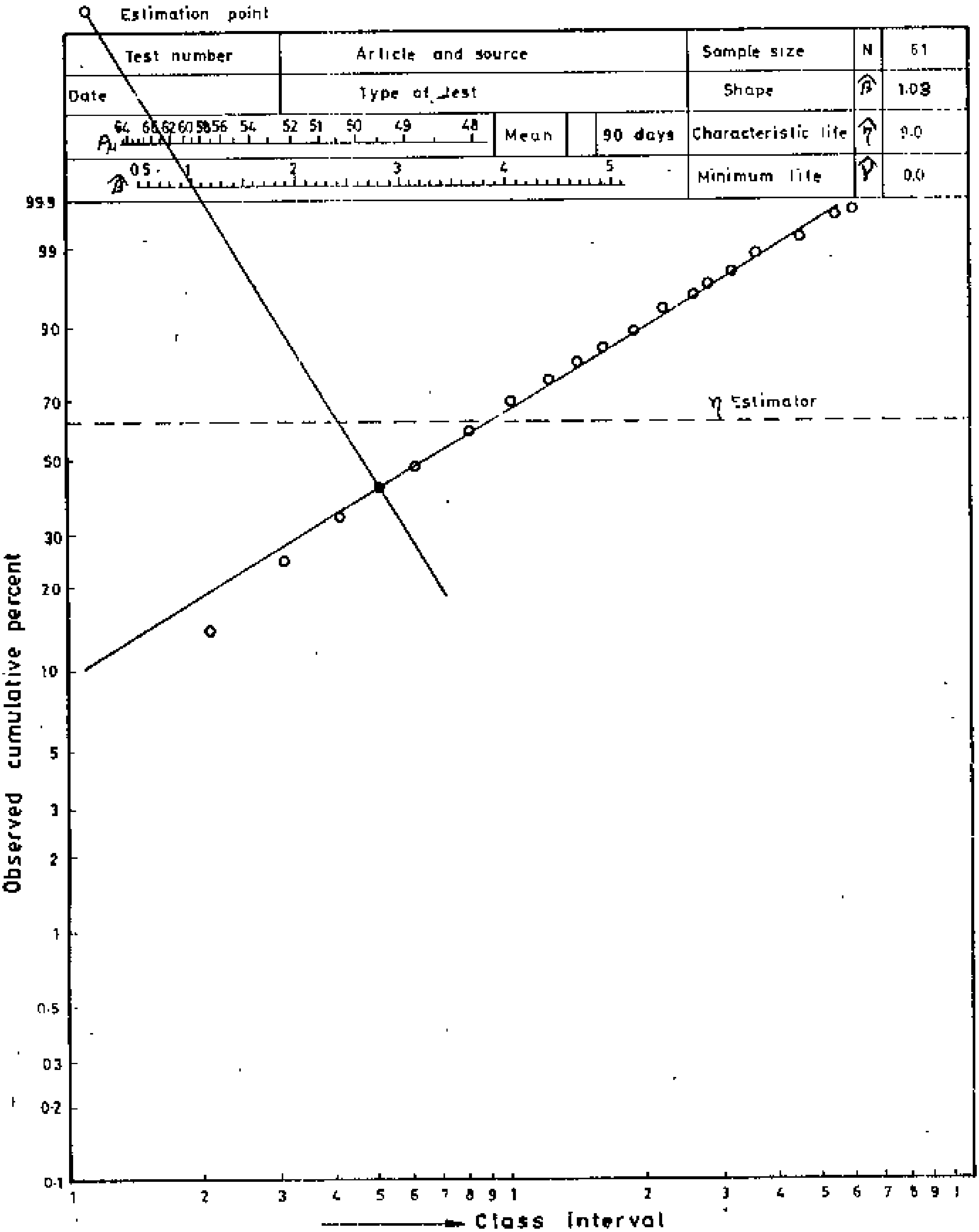


Fig.3.21 Weibull paper plotting for waiting time in the system

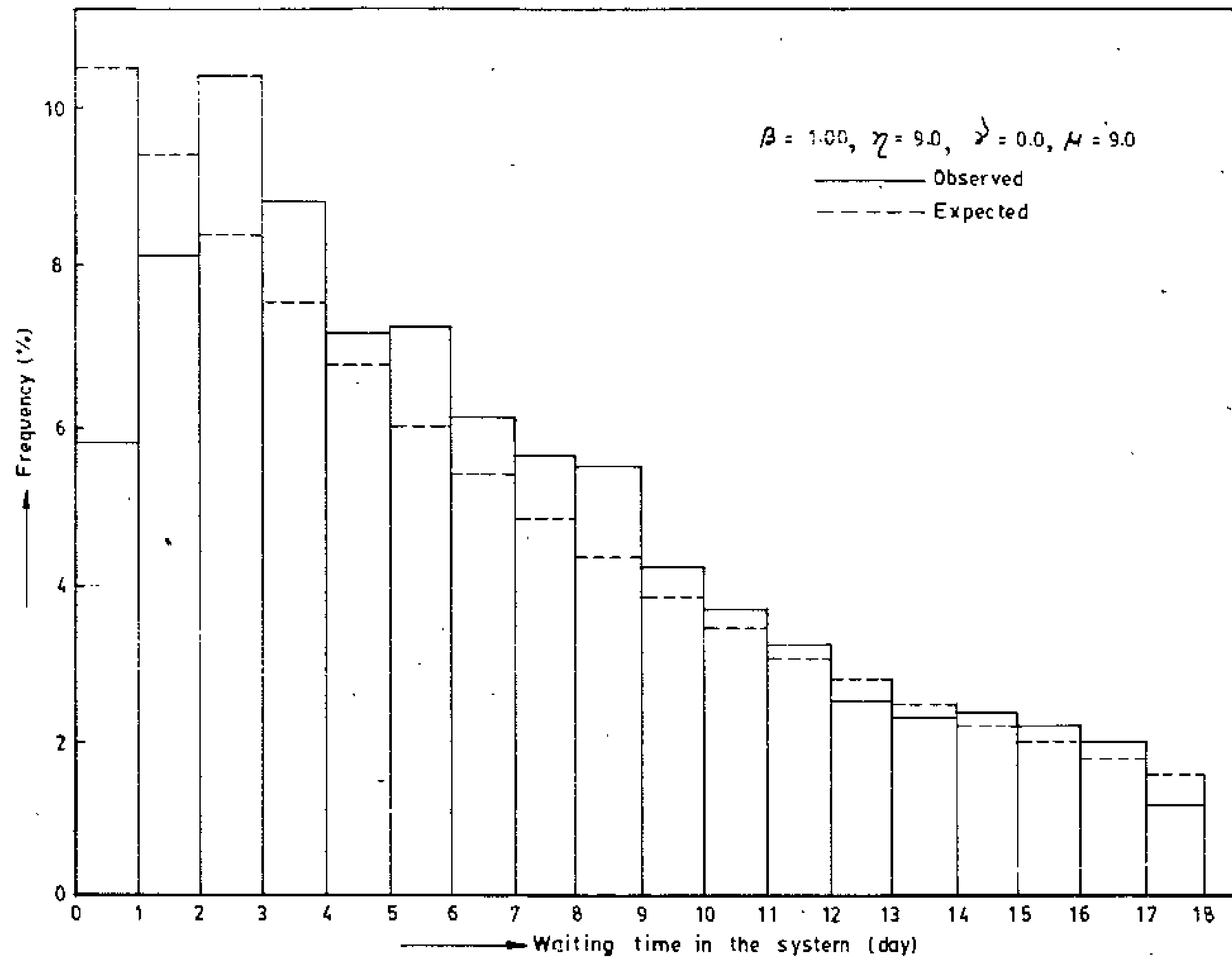


Fig. 3.22 Distribution of waiting time in the system--observed-and-expected (weibull)

3.3.9 Analysis of Cargo Handling at the Port

The Weibull distribution of probability is assumed for the cargo handling in the port. The following values are obtained for cargo handling per ship at the port using the same procedure as followed in 3.3.1.

Mean cargo handling	=	3421 tons per ship
Variance	=	16,010,017
Standard deviation	=	4001 tons per ship

Results from Weibull Probability paper, shown in figure 3.23 are as follows:

β = 0.78

η = 2850 tons per ship

γ = 0

μ = 3400 tons per ship

The (K-S) test shows that the Weibull distribution of probability provides a good fit to the cargo handling in the port with β = 0.78, η = 2850, γ = 0, μ = 3400. The observed and expected probability are shown in the figure 3.24.

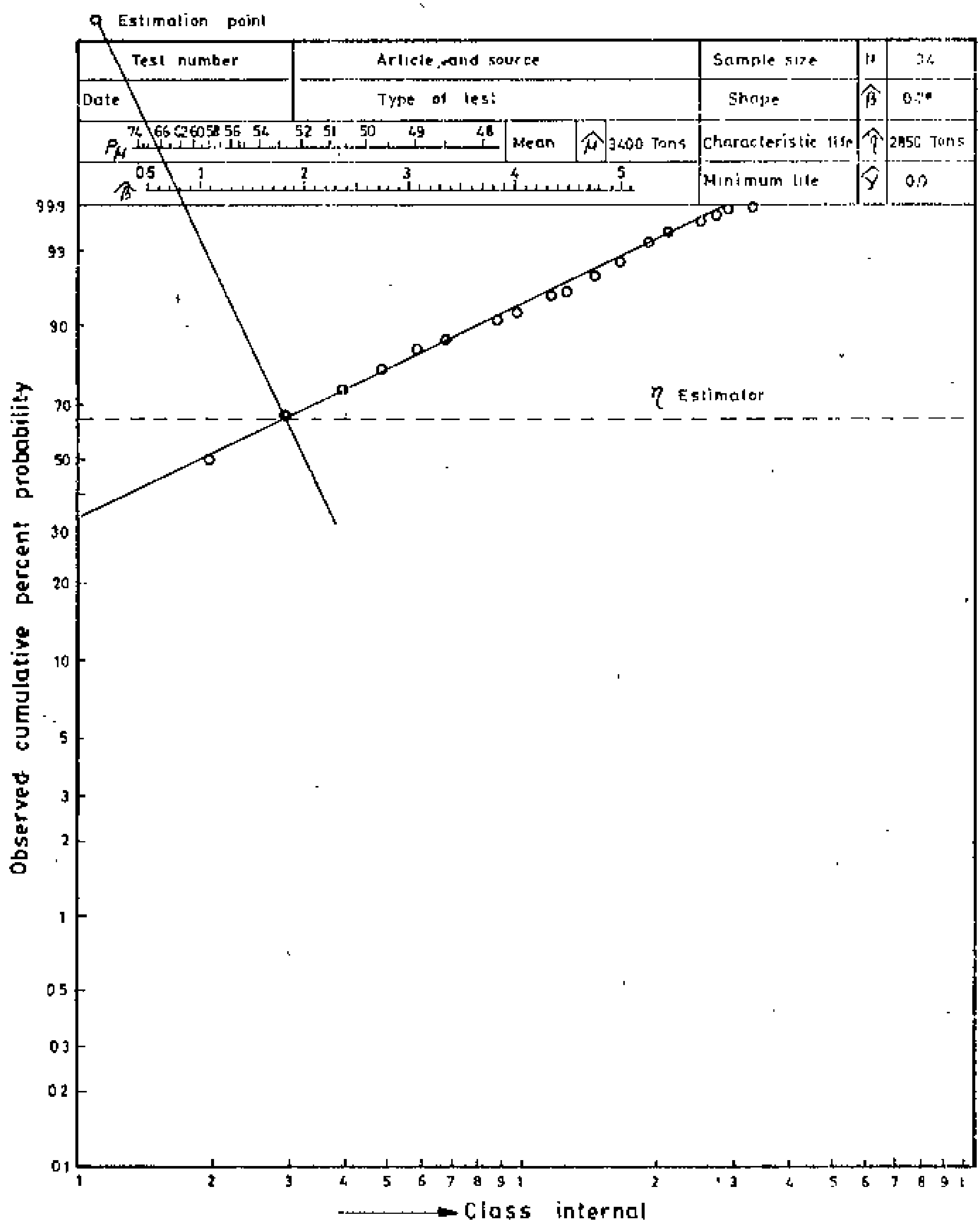


Fig. 3.23 Weibull paper plotting for handling of cargo per ship

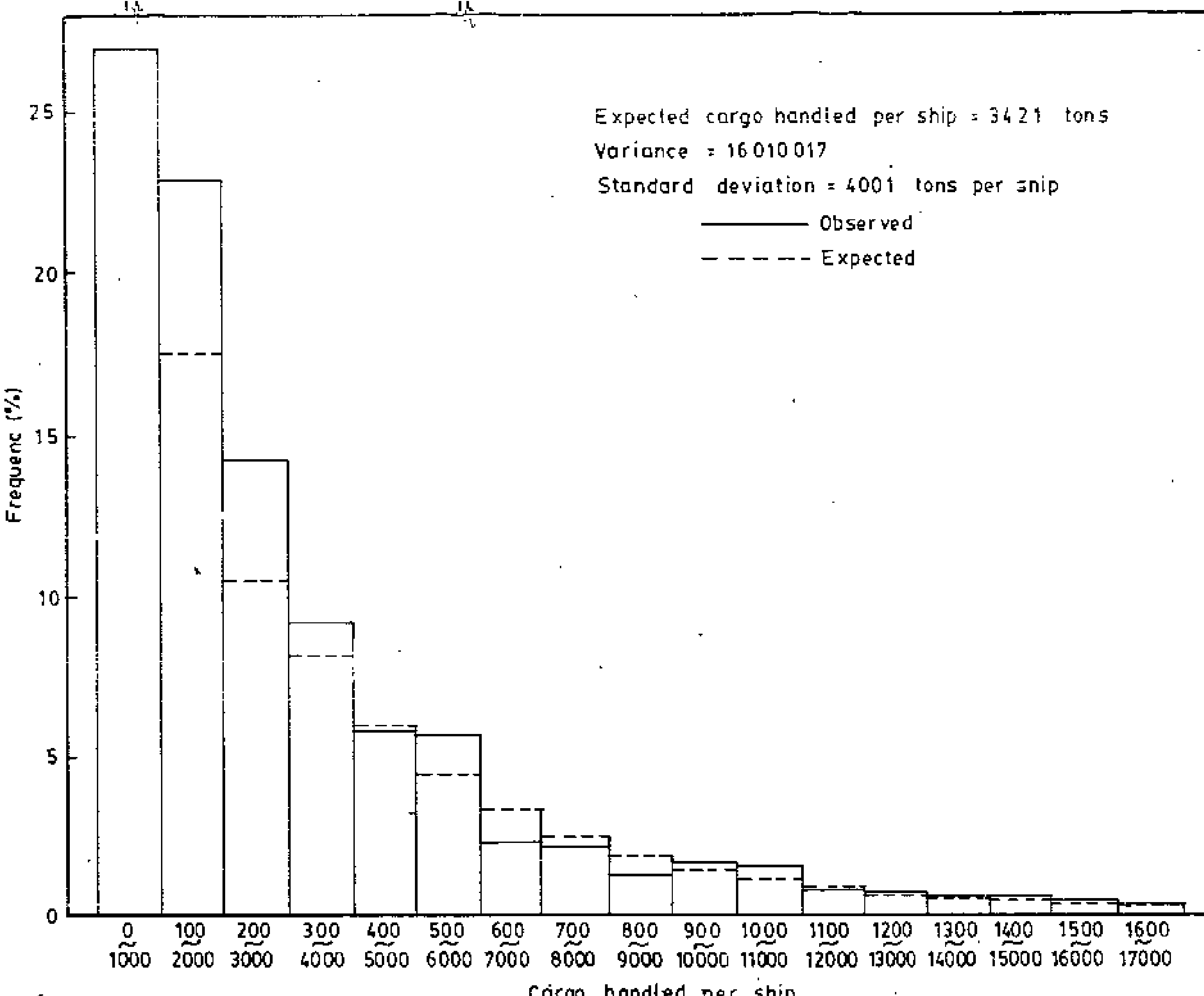


Fig-3.24 Distribution of cargo handled per ship - observed & expected (weibull)

System Parameters	Nature of Distribution	Observed value		Weibull Probability Distribution			
		Mean	Variance	β	η	γ	μ
Arrival rate (ships per day)	Poisson	2.031	2.1398	-	-	-	-
Departure rate (ships per day)	Poisson	2.018	2.135	-	-	-	-
Service time (days per ship)	Exponential	6.46	40.58	1.00	6.80	0.00	6.80
Waiting time in Queue (days/ship)	Weibull	2.869	14.115	0.74	2.35	0.00	2.80
No. of ships in Queue	Weibull	4.825	15.99	1.50	5.75	0.00	5.15
No. of ships in Service	Weibull	13.414	13.643	4.1	15.8	0.00	14.50
No. of ships in the system	Weibull	18.2519	43.32	2.00	14.60	5.83	18.83
Waiting time in the system (days per ship)	Weibull	8.7477	65.053	1.09	9.00	0.00	9.00
Cargo handled (tons/ship)	Weibull	3421	16,010,017	0.78	2850	0.00	3400

Table 3.4 : Summary of results obtained from data analysis

CHAPTER FOUR

DEVELOPMENT OF A MATHEMATICAL QUEUEING MODEL

4.0 Development of a Mathematical Queueing Model

4.1 Introduction

In this chapter an appropriate queueing model is selected on the basis of the statistical analysis carried out in the preceding chapter. The system parameters are then calculated and various aspects of the system are analyzed. Values of system parameters of the model are compared with the observed values to check the effectiveness of the selected model.

4.2 Selection of Queueing Model for the System

The following informations regarding the different characteristics of the system under study have been obtained from system study and data analysis:

i.	Input or arrival distribution	:	Poisson
ii.	Service time distribution	:	Exponential
iii.	Service channels (jetties)	:	15
iv.	Service discipline	:	First come, first served (FCFS)
v.	Maximum number of customers allowed in system	:	Infinite
vi.	Calling source	:	Infinite

It appears from the nature of the six characteristics that the system in question is a multiple servers queueing model with infinite system capacity and infinite calling population having the service discipline of first come first served. The model in Kendall's notation may be represented as

$$M/M/15 \;:\; FCFS/\infty/\infty$$

The model assumes constant mean arrival rate and mean service rate. That is, these rates have been assumed to be independent of the states of the system. This assumption is quite valid for the situation at Chittagong port. The rate diagram of the model has been shown in figure 4.1

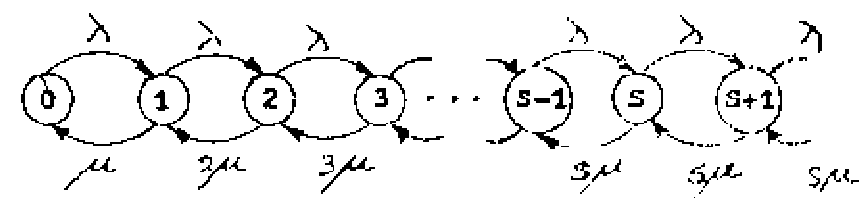
State of the system 

Fig. 4.1 : Rate diagram for M/M/S : FCFS/∞/∞

The different system parameters of the model have been calculated using the following mathematical formulae, under steady state conditions:

$$P_0 = \left[\sum_{n=0}^{S-1} \frac{(\lambda/\mu)^n}{n!} + \frac{(\lambda/\mu)^S}{S!} \cdot \frac{1}{1-\rho} \right]^{-1} \quad \ldots \quad \ldots \quad 4.1$$

$$P_n = \frac{(\lambda/\mu)^n}{n!} \cdot P_0 \quad \text{when} \quad 0 \leq n \leq S \quad \ldots \quad \ldots \quad 4.2$$

$$= \frac{(\frac{\lambda}{\mu})^n}{S!\, S^{n-S}} P_o \quad \text{when} \quad n > S \qquad \ldots \qquad \ldots \qquad 4.3$$

$$Lq = \frac{\rho (\frac{\lambda}{\mu})^S}{S!\, (1-\rho)^2} P_o \qquad \ldots \qquad \ldots \qquad 4.4$$

$$Wq = \frac{Lq}{\lambda} \qquad \ldots \qquad \ldots \qquad 4.5$$

$$W = W_q + \frac{1}{\mu} \qquad \ldots \qquad \ldots \qquad 4.6$$

$$L = \lambda W = \lambda (Wq + \frac{1}{\mu}) = Lq + \frac{\lambda}{\mu} \qquad \ldots \qquad \ldots \qquad 4.7$$

$$X = (1 - P) \times 100 \qquad \ldots \qquad \ldots \qquad \ldots \qquad 4.8$$

where,

- P_n – steady state probabilities of exactly n customers in the system
- S – number of parallel servers
- n – number of customers in the system
- L – mean number of customers in the system
- Lq – mean number of customers in the queue
- W – mean waiting time in the system
- Wq – mean waiting time in the queue
- λ – mean arrival rate (no. of customers arrival per unit time)
- $\frac{1}{\mu}$ – mean service rate per busy server (no. of customers served per unit time)
- $\rho = \frac{\lambda}{S\mu}$ – utilization factor for S service facilities

Calculations of system parameters using the model
$M/M/15 : FCFS/\infty/\infty$

Given that, $S = 15$, $\lambda = 2.03$ ships per day

$\frac{1}{\mu} = 6.46$ days per ship

then $\frac{\lambda}{\mu} = 2.03 \times 6.46 = 13.114$

$$\rho = \frac{\lambda}{3\mu} = \frac{2.03 \times 6.46}{15} = 0.874$$

$$X = (1 - 0.874) \times 100 = 12.6$$

$$P_o = \left[\sum_{n=0}^{14} \frac{(13.114)^n}{n!} + \frac{(13.114)^{15}}{15!} \cdot \frac{1}{1 - 0.874} \right]^{-1}$$

$$= (1 + 13.114 + 85.99 + 375.88 + 1,232.34 + 3.232.17 + 7,064.45 + 13,234.75 + 21,695.06 + 31,612.12 + 41,456.13 + 49,423.24 + 54,011.37 + 54,485.01 + 51,036.88 + 354,125.77)^{-1}$$

$$= \frac{1}{683,085.274}$$

$$= 1.464 \times 10^{-6}$$

$$Lq = \frac{(13.114)^{15} \times 0.874}{15! \ (1 - 0.874)^2} \times 1.464.10^{-6} = 3.6 \text{ ships per day}$$

$$Wq = \frac{3.60}{2.03} = 1.77 \text{ days per ship}$$

$$W = 1.77 + 6.46 = 8.23 \text{ days per ship}$$

$$L = 3.6 + 13.114 = 16.714 \text{ ships per day.}$$

It is very important to mention here that the system under study has two restrictions, such as (i) tide restriction and (ii) draft restriction. As mentioned earlier, the movement of ships (customers) from the outer anchorage to the port is dependent on the tide and draft conditions.

But because of nonavailability of the required data regarding tide and draft the corresponding restrictions could not be incorporated in the model. However, their influences are reflected in the form of higher observed values of some of the system parameters as shown in table 4.1. Fig. 4.1a also demonstrates the same fact. The percentages of deviation between observed values and model values of Lq and Wq as shown in table 4.1 are relatively high. This is quite expected because these parameters are directly affected by the tide and draft conditions.

However, if the expected waiting time in the system (W) and expected waiting time in the queue (Wq) are compared, then it is observed that W is much bigger than Wq, only a fraction of which is the time lost due to the tide and draft restriction. So it may be concluded that the model is not greatly affected by this restriction.

System parameters	Observed values	Model values	% of deviation
Lq	4.825	3.6	25.39
Wq	2.869	1.77	38.30
L-Lq	13.414	14.114	2.236
L	18.252	16.714	8.426
W	8.748	8.23	5.92
P_o	0.00	1.464×10^{-6}	Very negligible

Table 4.1 : Comparison between observed and model values of system parameters.

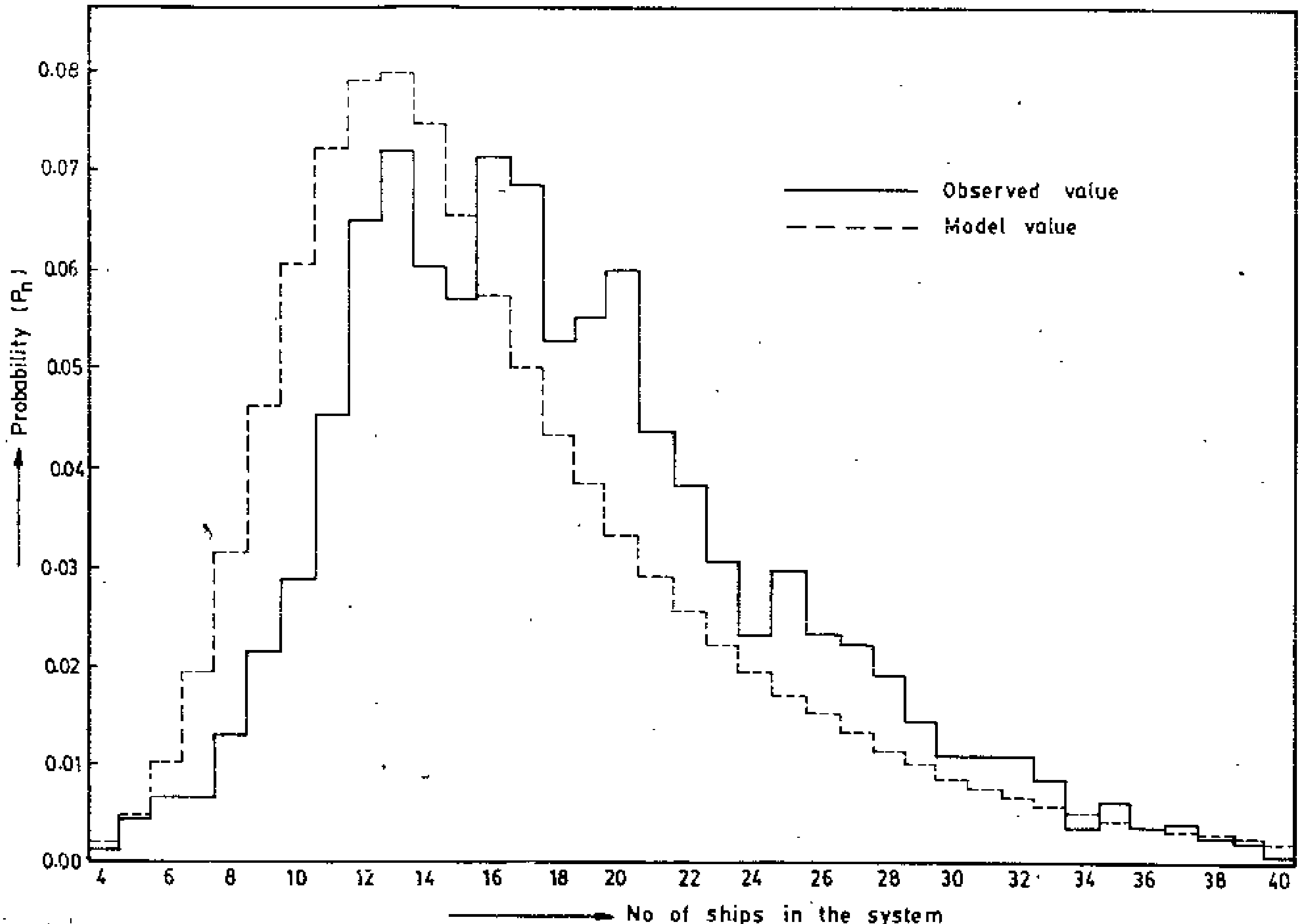


Fig. 4.1a Comparison of P_n

For the model M/M/S : FCFS/∞/∞ the probability density function of waiting time in the system (τ) is as follows:

$$W(\tau) = \mu e^{-\mu\tau} + \frac{\left(\frac{\lambda}{\mu}\right)^{S} \mu e^{-\mu\tau}}{(S-1)!\left(S-1-\frac{\lambda}{\mu}\right)} P_0 \left\{ \frac{1}{S-\frac{\lambda}{\mu}} - e^{-\mu\left(S-1-\frac{\lambda}{\mu}\right)\tau} \right\} \quad \ldots (4.9)$$

$$\text{for } \tau \geq 0$$

The derivation has been given in Appendix A-9.

An idea about the relaibility of W in indicating the actual time customers wait may be obtained by computing the probability that customers will wait more than W; that is,

$$P_r(\tau > W) = 1 - \int_0^W W(\tau)\, d\tau$$

$$= 1 - \int_0^{8.23} W(\tau)\, d\tau \quad \ldots \quad (4.10)$$

W = 8.23 days per ship (model value)

Using the values of system parameters, the probability density function (Equation 4.9) for the system under study is as follows:

$$W(\tau) = 0.24557\, e^{-0.1548\tau} - 0.1712\, e^{-0.29195\tau} \quad \ldots \quad (4.11)$$

Using equation 4.11 in equation 4.10,

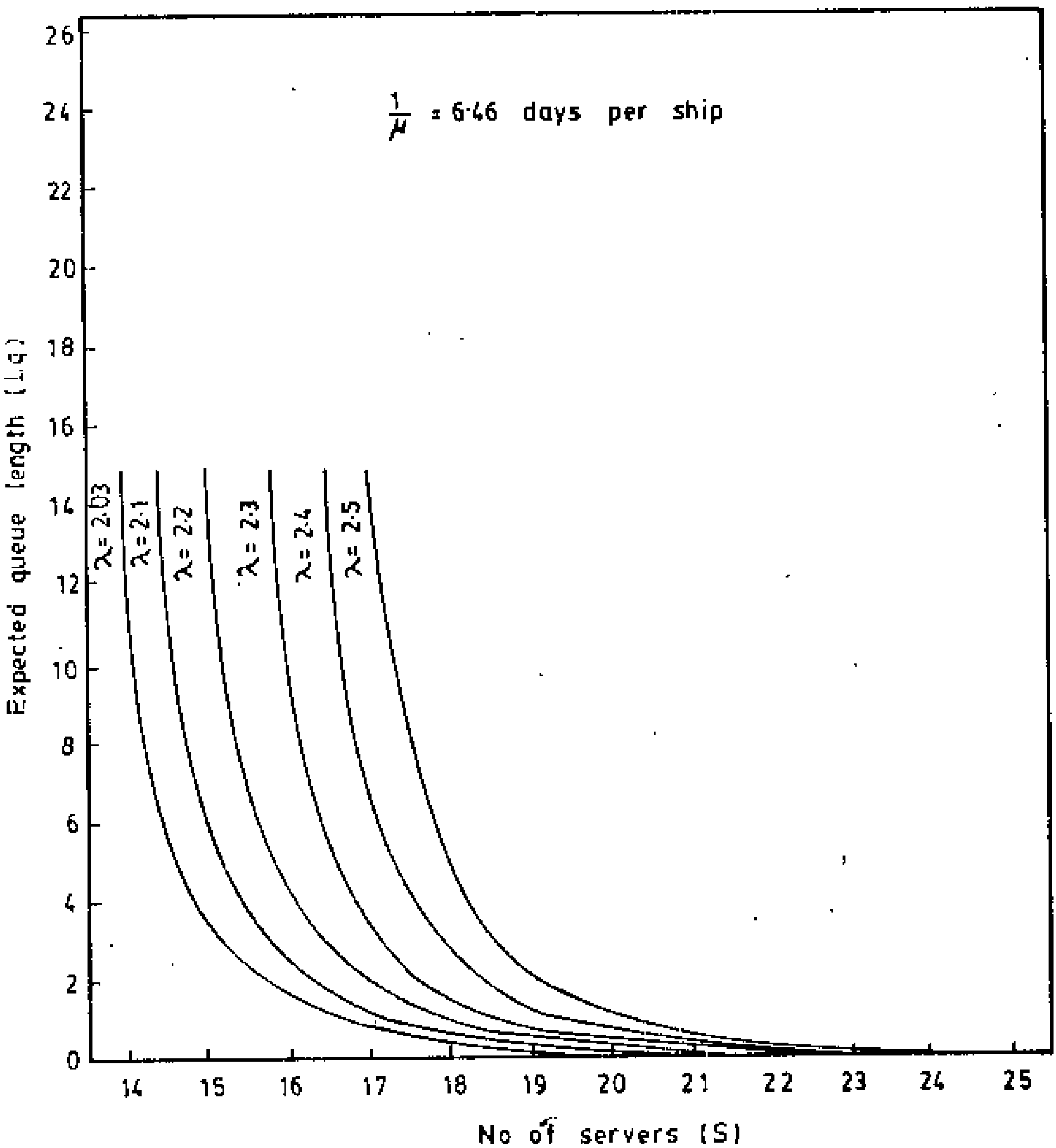
$$P_r(\tau > 8.23) = 1 - \int_0^{8.23} \left[0.24557\, e^{-0.1548\tau} - 0.1712\, e^{-0.29195\tau} \right] d\tau$$

$$= 1 - 0.24557 \left[\frac{e^{-0.1548}}{-0.1548} \right]_0^{8.23} + 0.1712 \left[\frac{e^{-0.29195}}{-0.29195} \right]_0^{8.23}$$

$$= 0.3907$$

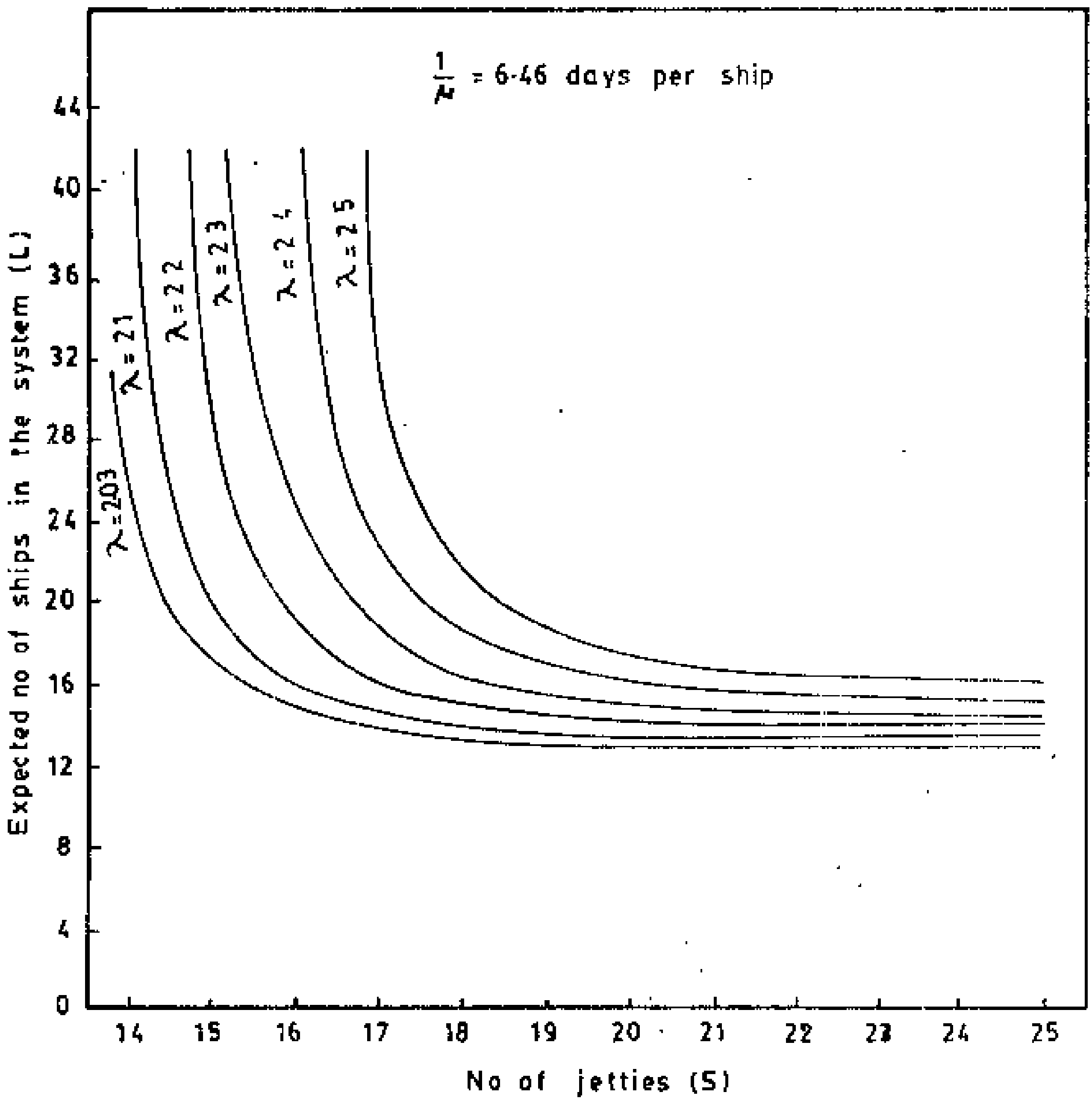
This means that under the service discipline of the stated model 39.07% of the customers will wait more than average waiting time W.

The behaviors of the parameters Lq, L, Wq, W and X are studies for different situations i.e. for different mean arrival rate (λ), different number of servers and constant mean service rate and shown in figures 4.2 to 4.6. When both the mean arrival rate and mean service rate are changed, the parameters are studied for different traffic intensity ($\frac{\lambda}{\mu}$) with different number of servers and shown in figures 4.7 to 4.9.



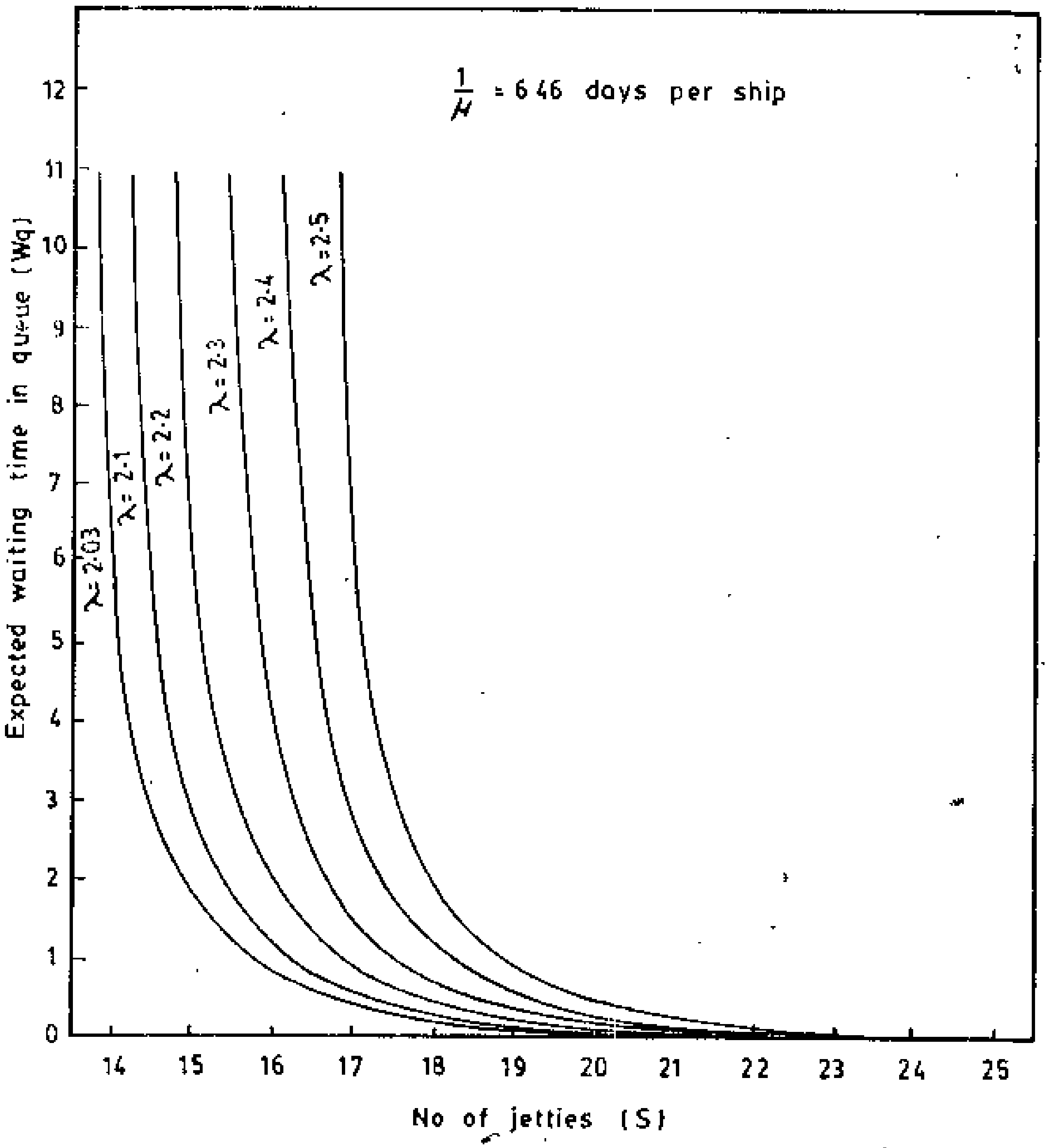
$\frac{1}{\mu}$ = 6·46 days per ship
Expected queue length (Lq)
λ = 2.03
λ = 2·1
λ = 2·2
λ = 2·3
λ = 2·4
λ = 2·5
No of servers (S)

Fig. 4·2



$\frac{1}{\mu}$ = 6·46 days per ship
λ = 203
λ = 21
λ = 22
λ = 23
λ = 24
λ = 25
Expected no of ships in the system (L)
0
4
8
12
16
20
24
28
32
36
40
44
14
15
16
17
18
19
20
21
22
23
24
25
No of jetties (S)

Fig. 4.3



$\frac{1}{\mu}$ = 6.46 days per ship
Expected waiting time in queue (Wq)
No of jetties (S)
λ = 2.03
λ = 2.1
λ = 2.2
λ = 2.3
λ = 2.4
λ = 2.5
0
1
2
3
4
5
6
7
8
9
10
11
12
14
15
16
17
18
19
20
21
22
23
24
25

Fig. 4.4

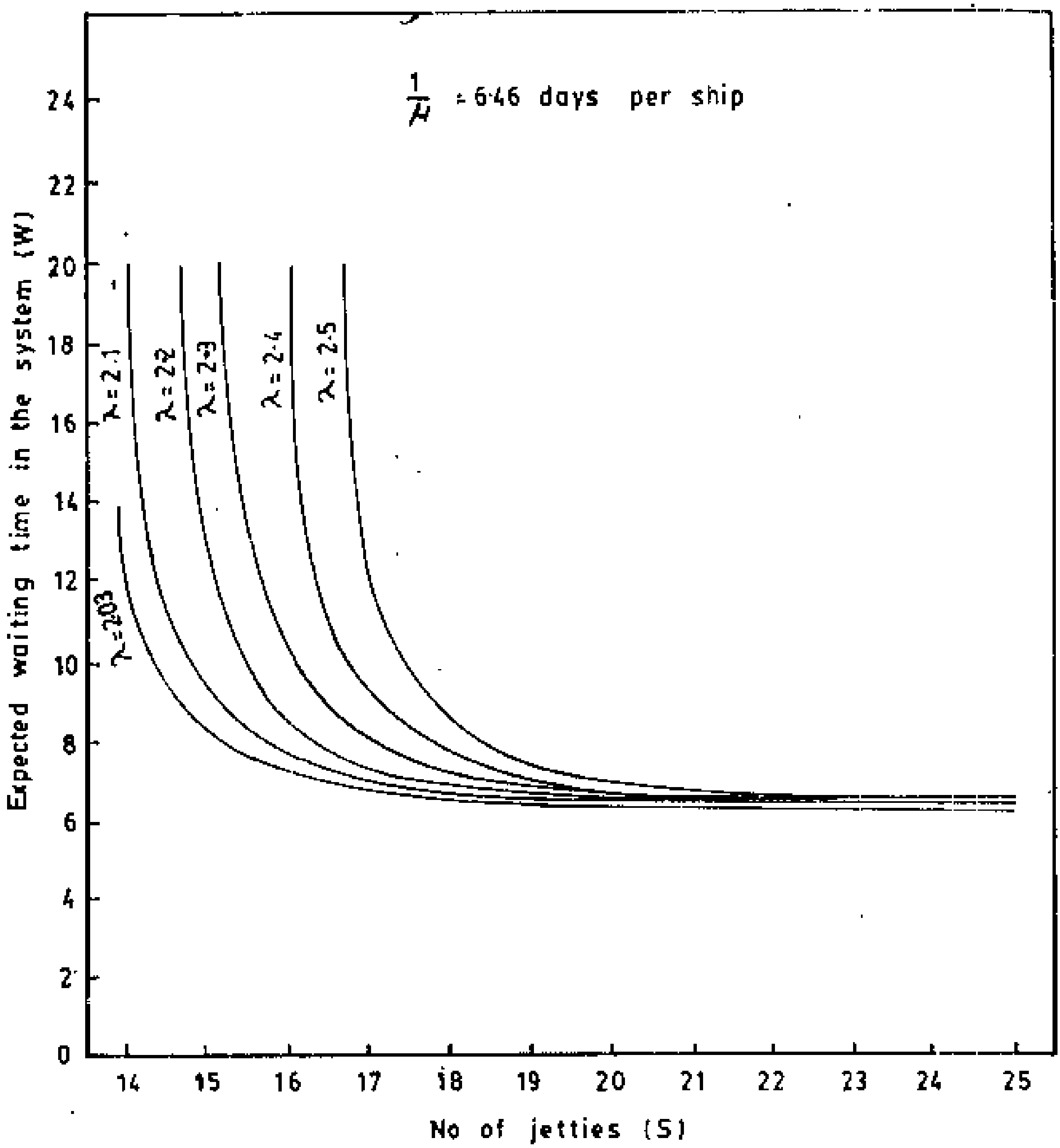
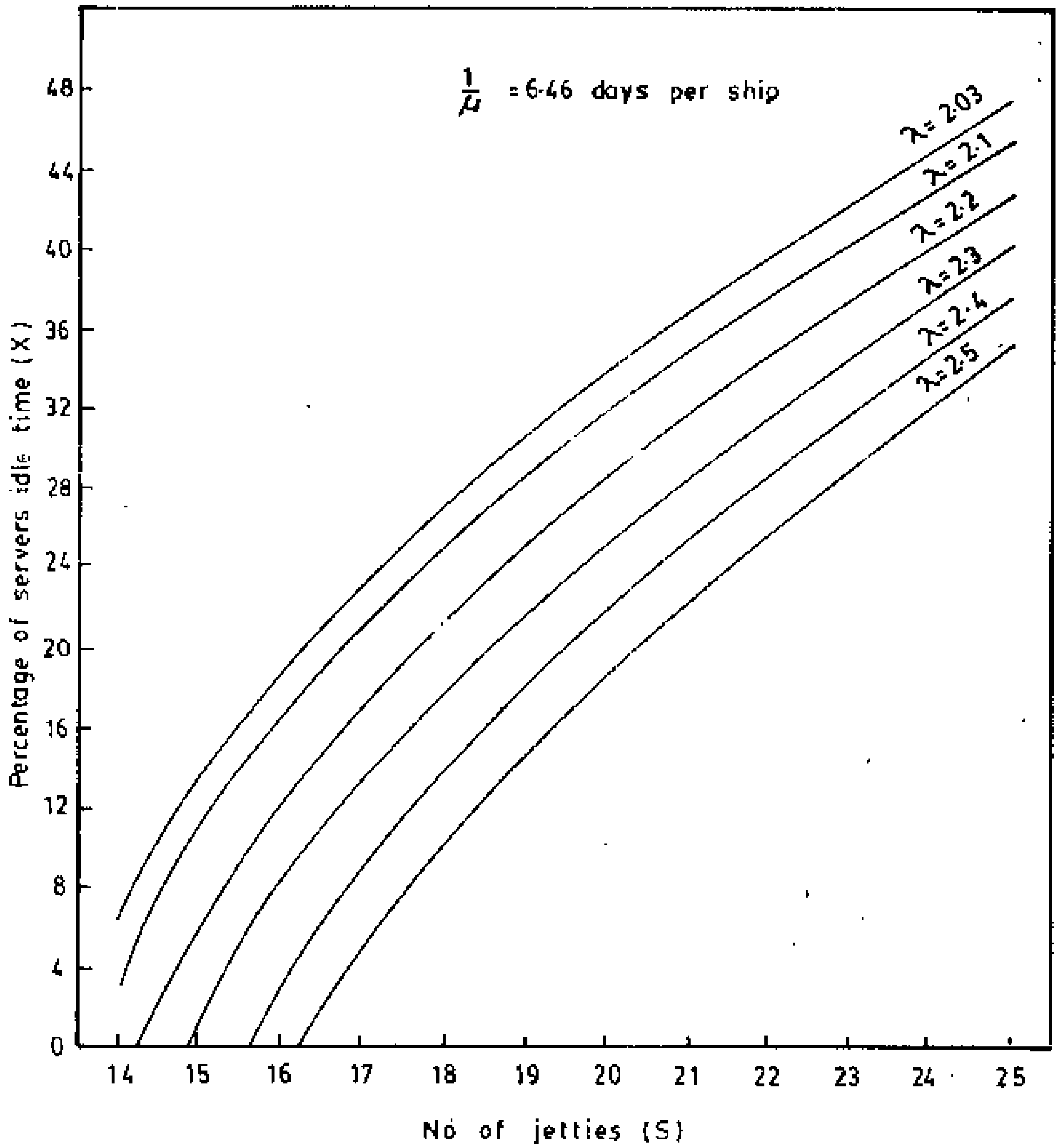
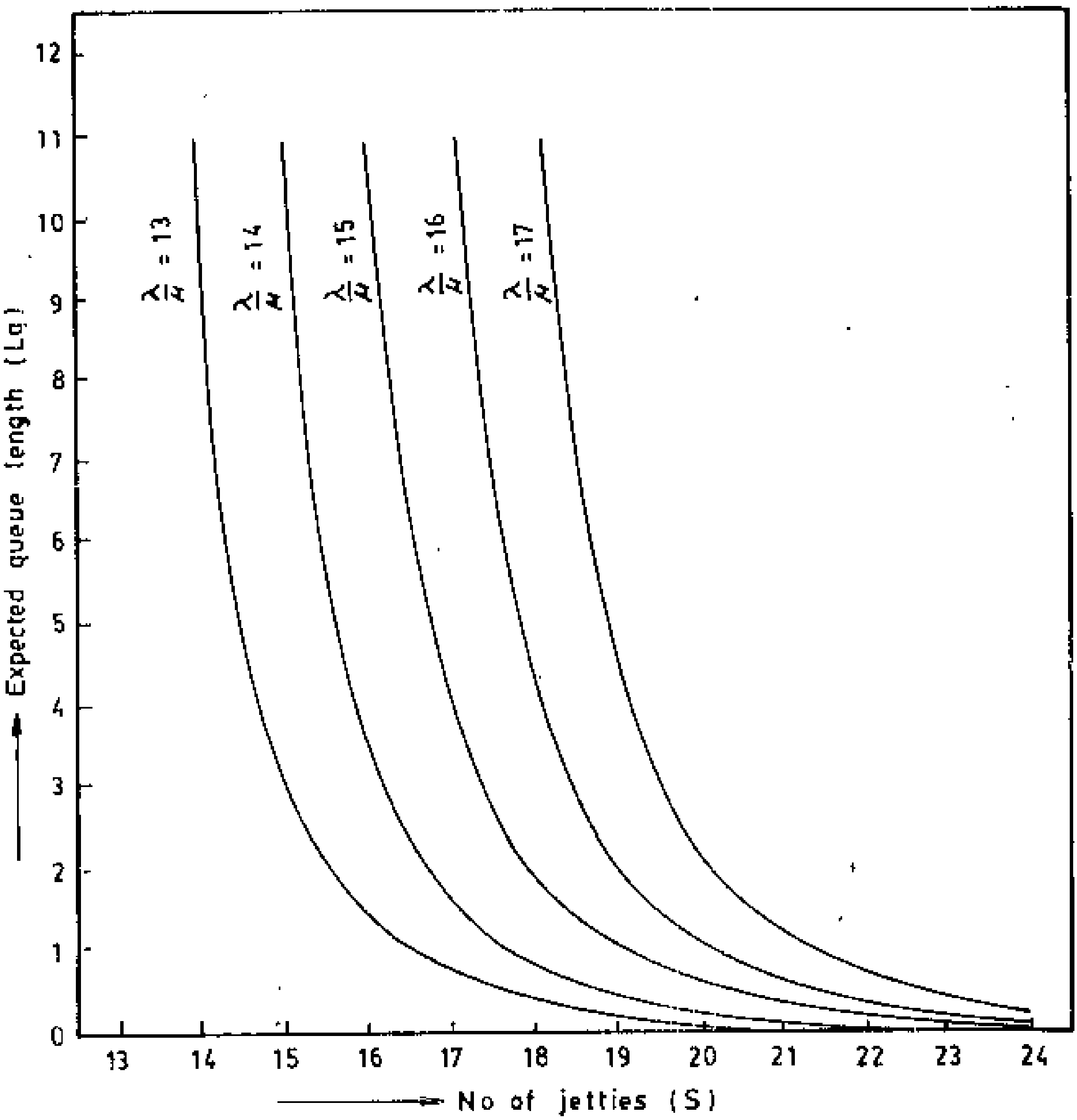

1/μ = 6·46 days per ship
Expected waiting time in the system (W)
λ = 2·1
λ = 2·2
λ = 2·3
λ = 2·4
λ = 2·5
λ = 2·03
0
2
4
6
8
10
12
14
16
18
20
22
24
14
15
16
17
18
19
20
21
22
23
24
25
No of jetties (S)

Fig. 4·5



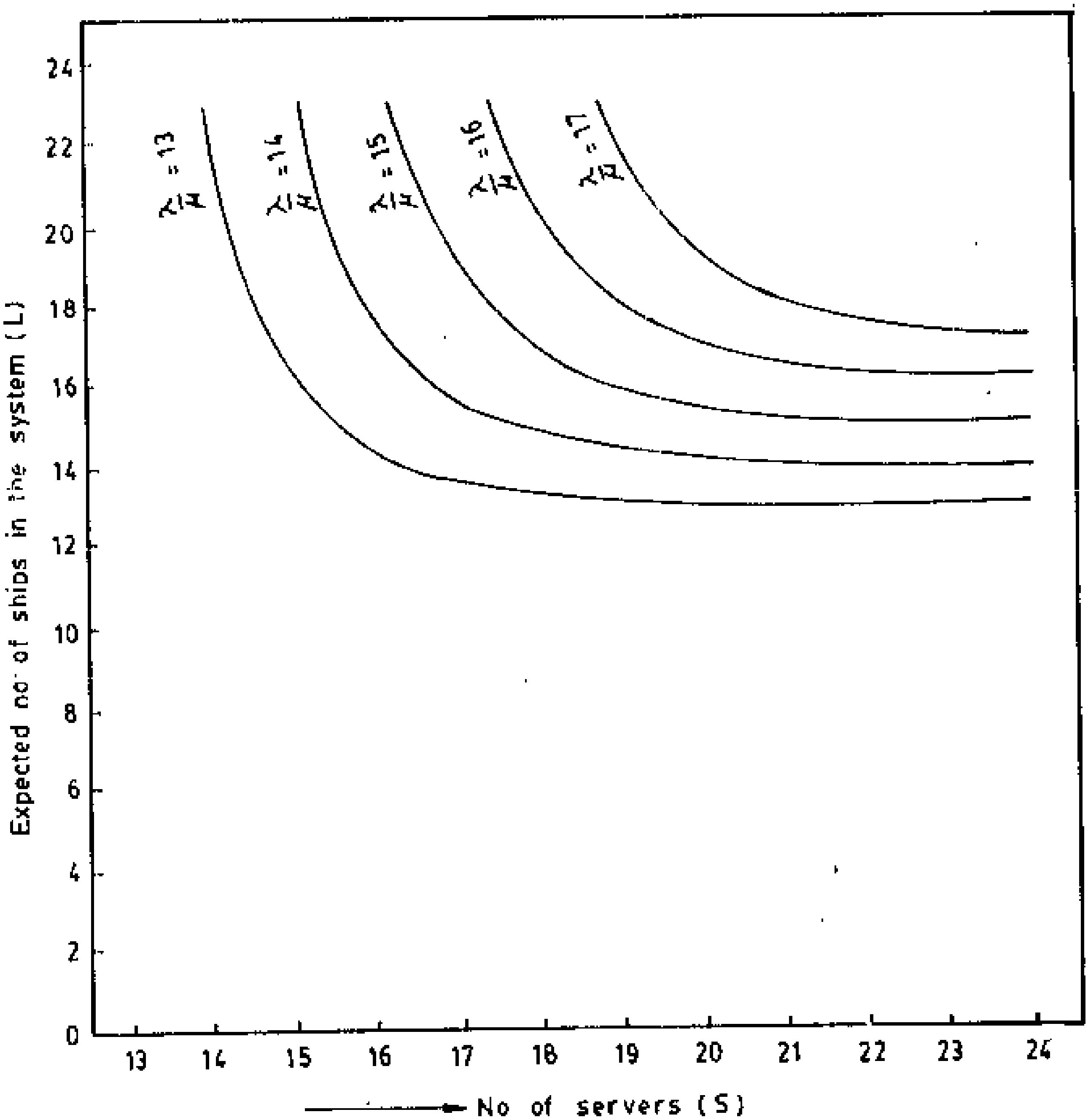
1/μ = 6·46 days per ship
λ = 2·03
λ = 2·1
λ = 2·2
λ = 2·3
λ = 2·4
λ = 2·5
Percentage of servers idle time (X)
0
4
8
12
16
20
24
28
32
36
40
44
48
14
15
16
17
18
19
20
21
22
23
24
25
No of jetties (S)

Fig. 4·6



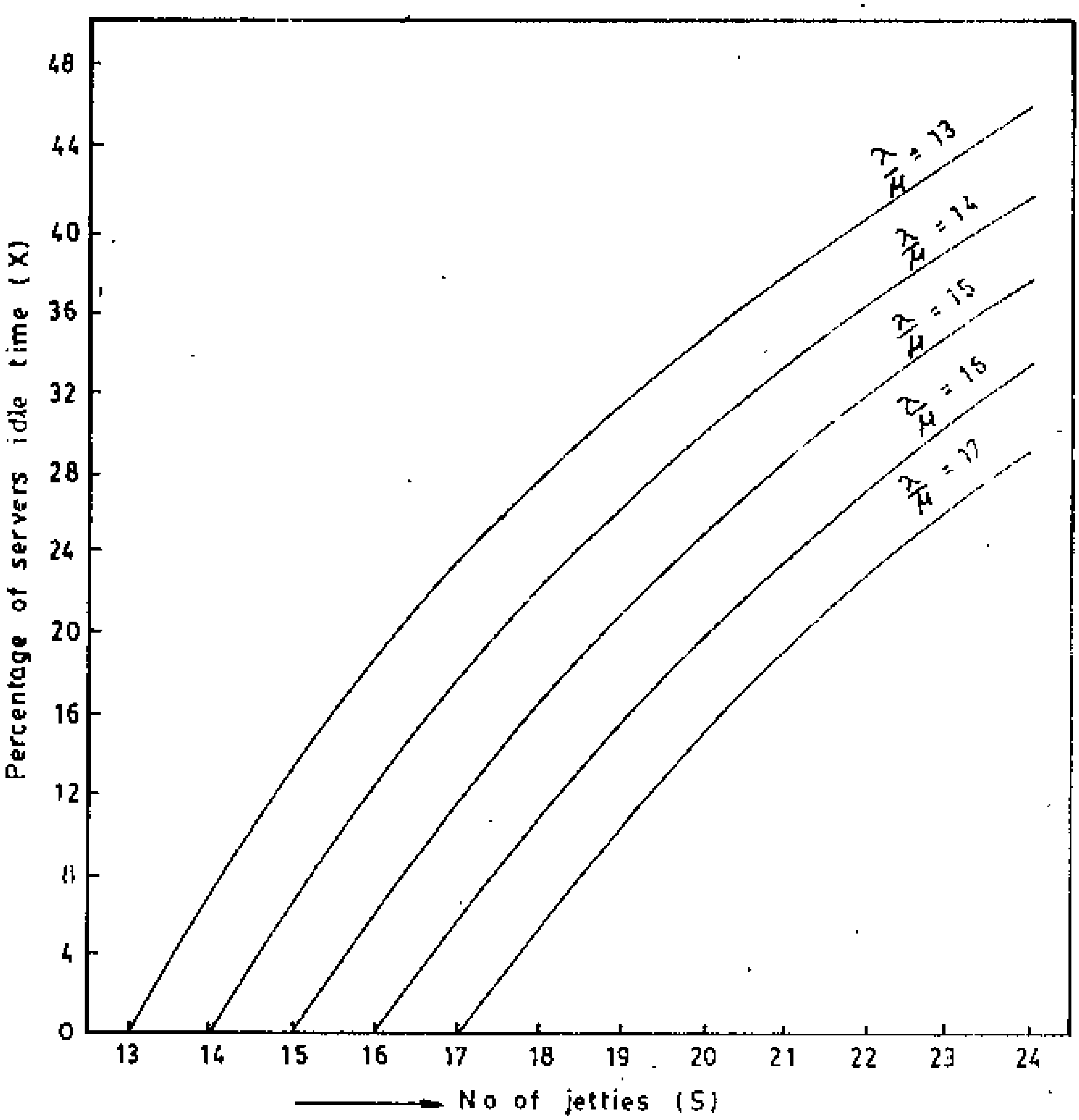
Expected queue length (Lq)
0
1
2
3
4
5
6
7
8
9
10
11
12
λ/μ = 13
λ/μ = 14
λ/μ = 15
λ/μ = 16
λ/μ = 17
13
14
15
16
17
18
19
20
21
22
23
24
No of jetties (S)

Fig. 4..7



Expected no of ships in the system (L)
0
2
4
6
8
10
12
14
16
18
20
22
24
λ/μ = 13
λ/μ = 14
λ/μ = 15
λ/μ = 16
λ/μ = 17
13
14
15
16
17
18
19
20
21
22
23
24
No of servers (S)

Fig. 4.8



48
44
40
36
32
28
24
20
16
12
8
4
0
Percentage of servers idle time (X)
λ/μ = 13
λ/μ = 14
λ/μ = 15
λ/μ = 16
λ/μ = 17
13
14
15
16
17
18
19
20
21
22
23
24
No of jetties (S)

Fig. 4.9

4.3 Aspiration Level Model for the System

The aspiration level model recognizes the difficulty of estimating cost parameters, and hence it is based on a more straight forward analysis. It makes direct use of the operating characteristics of the system in deciding the optimum values of the design parameters. Optimality here is viewed in the sense of satisfying certain aspiration levels set by the decision maker. These aspiration levels are defined as the upper limit on the values of the conflicting measures that the decision maker wishes to balance.

In multiple-servers model where it is required to determine the optimum number of servers(S) the following parameters are the conflicting measures with the server's idle time(X)

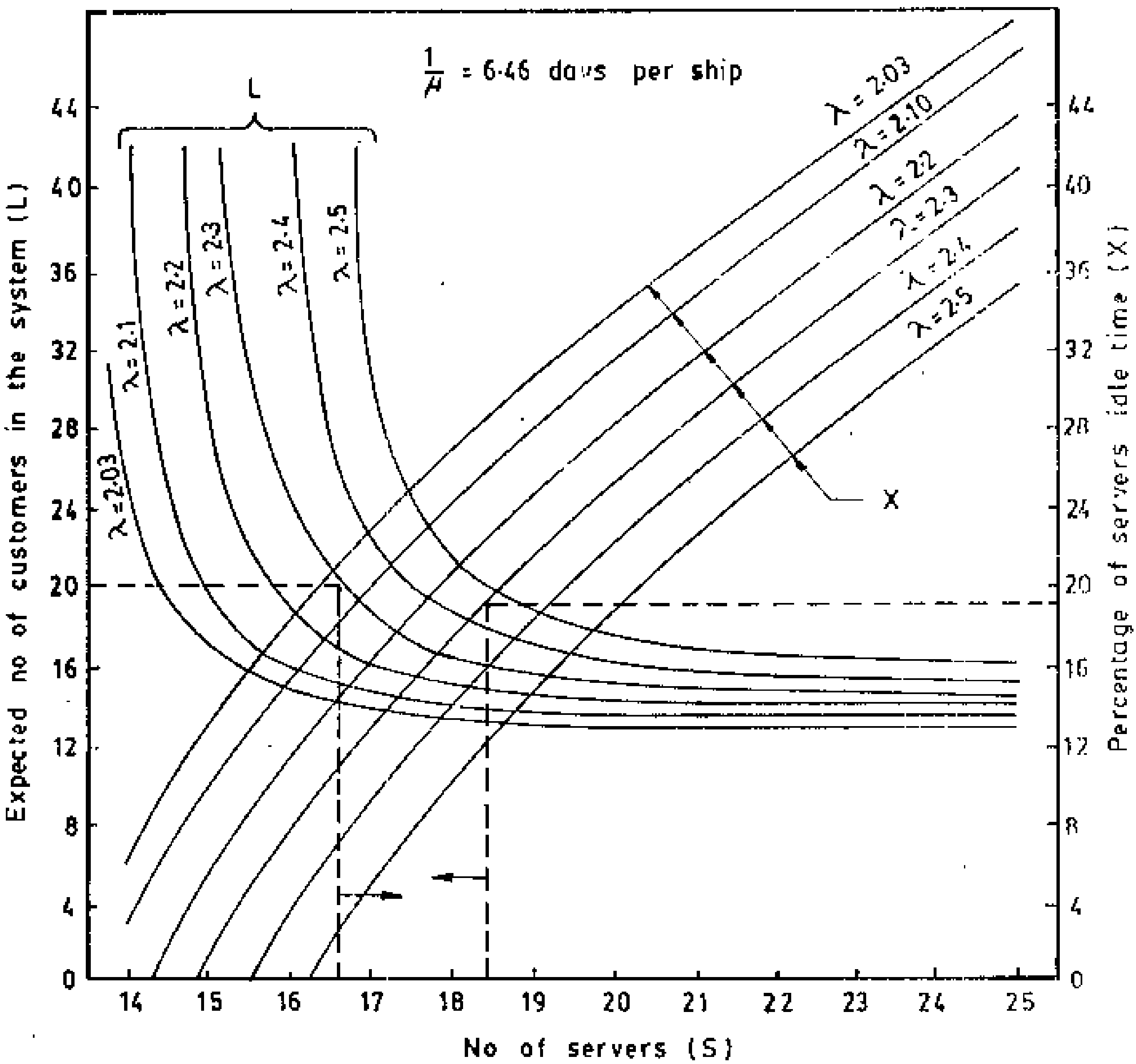
i. Mean number of customers in the system (L)

ii. Mean number of customers in the queue (Lq)

iii. Mean waiting time in the system (W)

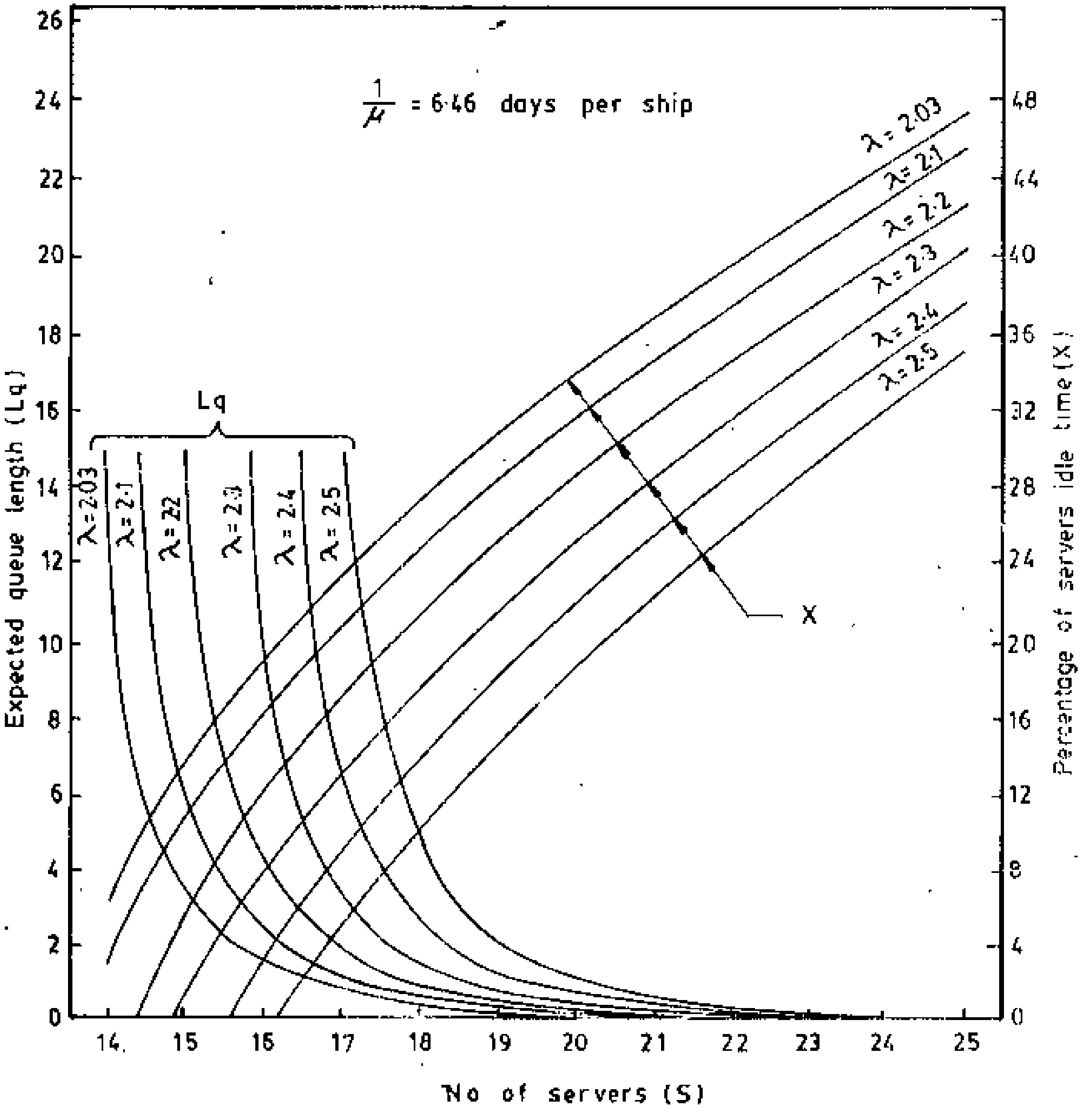
iv. Mean waiting time in the queue (Wq)

For the system under study, a set of curves (figures 4.10 to 4.15) have been drawn in which it is possible to set specified aspiration levels with respect to different conflicting measures (Land X, Lq and X, W and X, Wq and X). In the first four figures, for a constant mean service rate, the behaviors of the conflicting measures with different arrival rates have been shown. When both arrival and service rates are changed, the behaviors of the conflicting measures are studied



1/μ = 6·46 days per ship
L
λ = 2·03
λ = 2·10
λ = 2·2
λ = 2·3
λ = 2·4
λ = 2·5
λ = 2·1
X
Expected no of customers in the system (L)
Percentage of servers idle time (X)
No of servers (S)

Fig. 4.10



1/μ = 6·46 days per ship
Lq
λ=2·03
λ=2·1
λ=2·2
λ=2·3
λ=2·4
λ=2·5
X
Expected queue length (Lq)
Percentage of servers idle time (X)
No of servers (S)

Fig.4.11

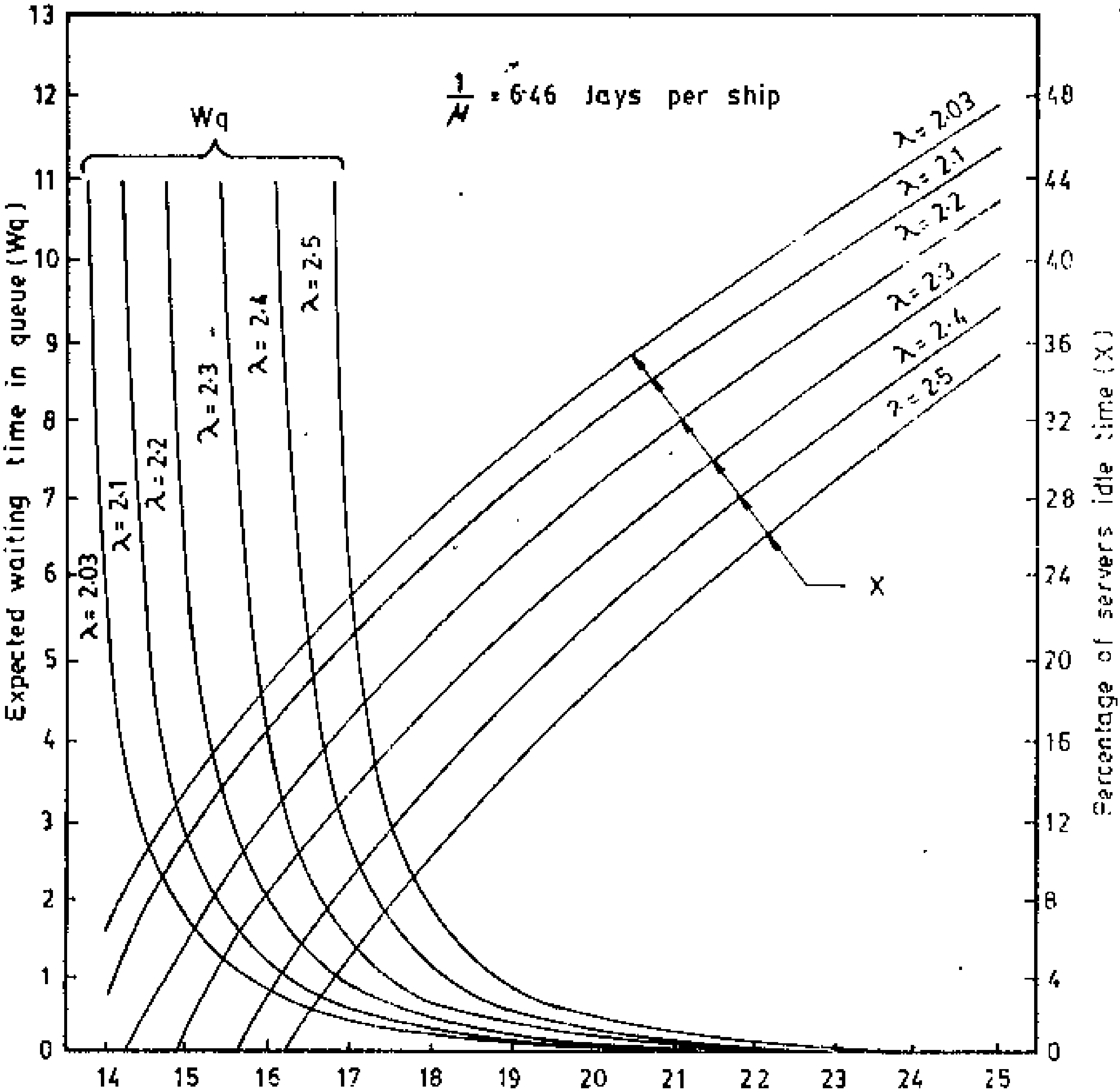

1/μ = 6·46 days per ship
Wq
λ = 2·03
λ = 2·1
λ = 2·2
λ = 2·3
λ = 2·4
λ = 2·5
X
Expected waiting time in queue (Wq)
Percentage of servers idle time (X)
No of servers (S)

Fig. 4.12

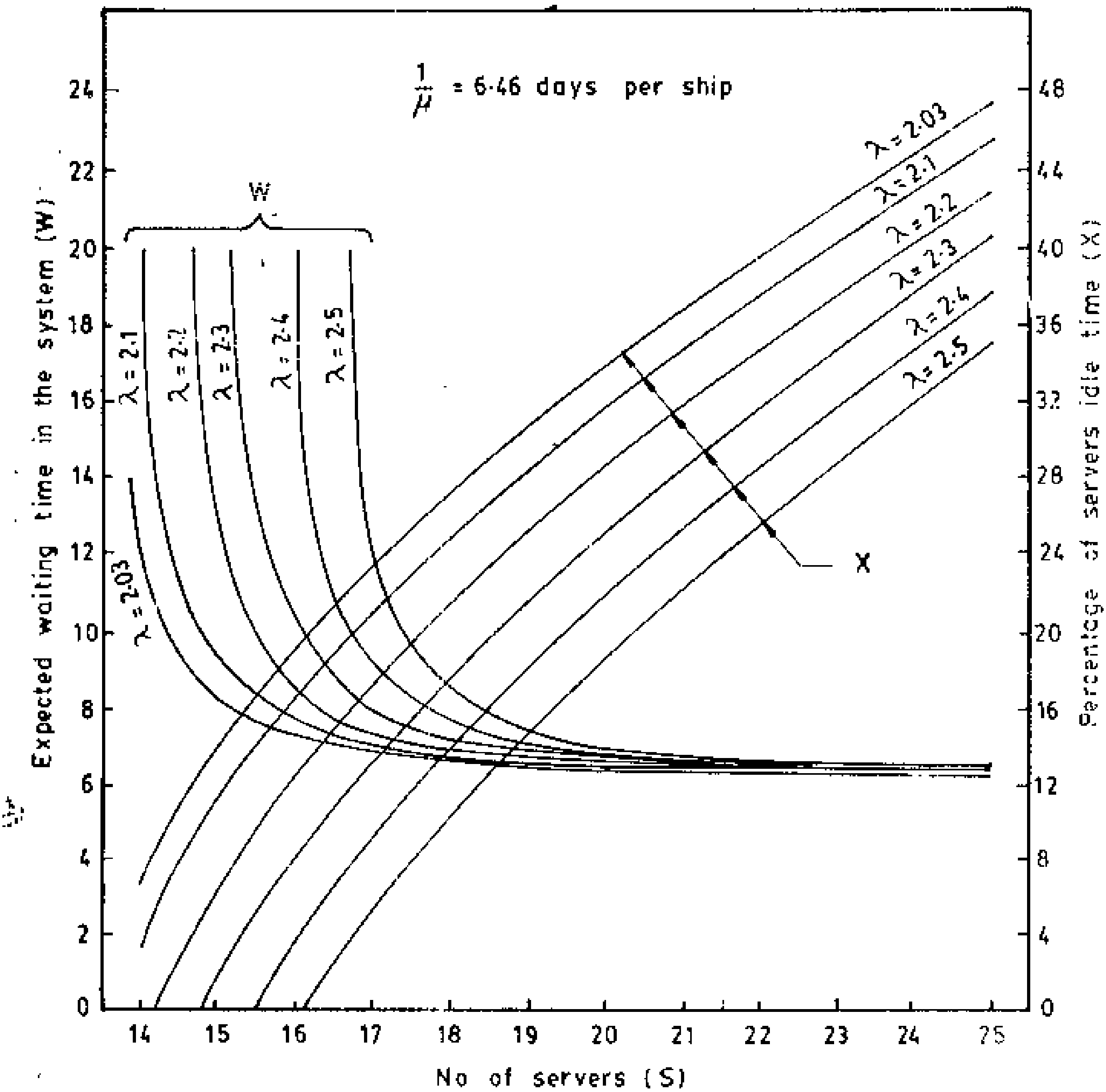
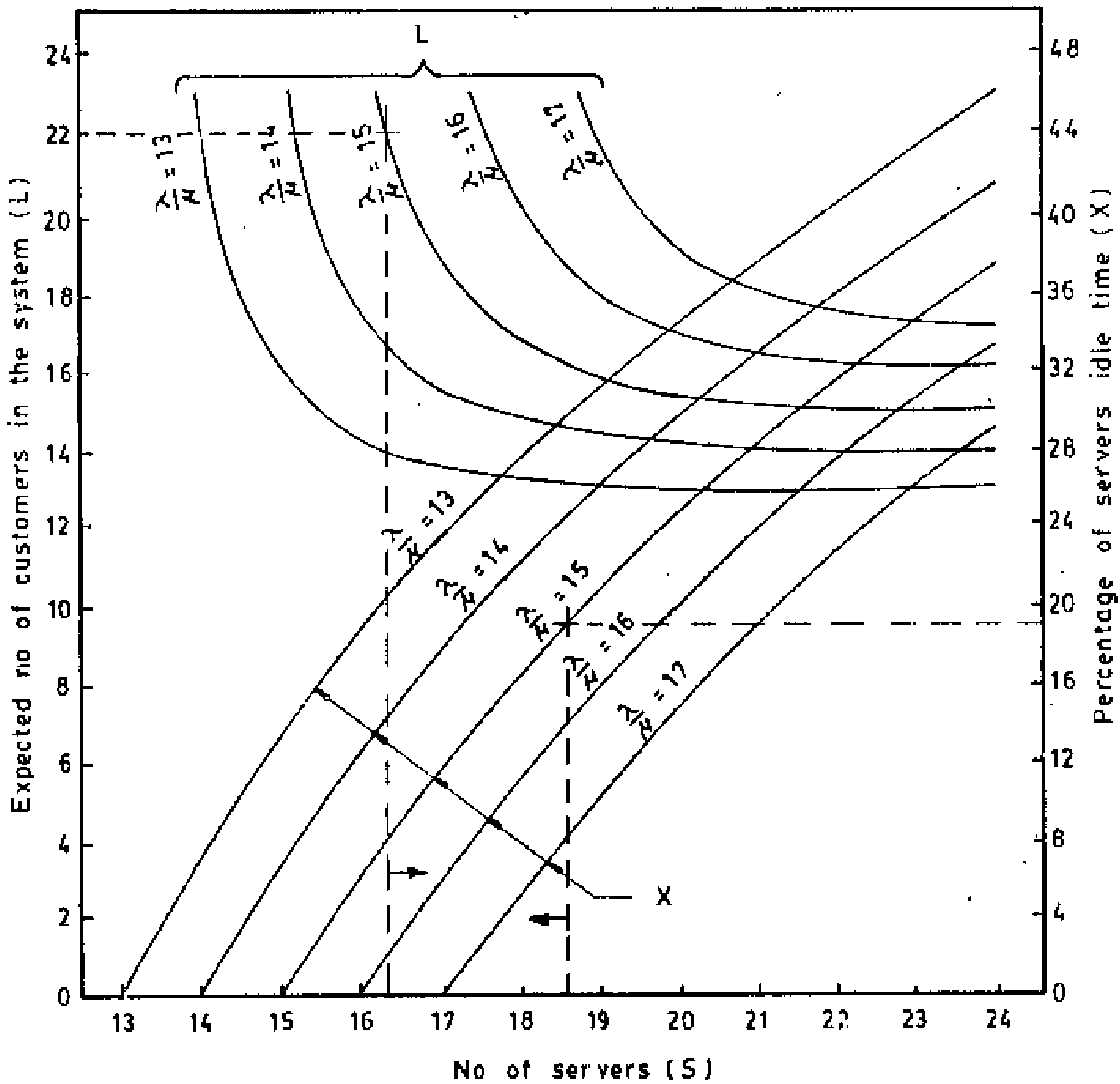

1/μ = 6·46 days per ship
W
X
λ = 2·03
λ = 2·1
λ = 2·2
λ = 2·3
λ = 2·4
λ = 2·5
Expected waiting time in the system (W)
Percentage of servers idle time (X)
No of servers (S)

Fig. 4·13



L
$\lambda/\mu = 13$
$\lambda/\mu = 14$
$\lambda/\mu = 15$
$\lambda/\mu = 16$
$\lambda/\mu = 17$
X
Expected no of customers in the system (L)
Percentage of servers idle time (X)
No of servers (S)
0 2 4 6 8 10 12 14 16 18 20 22 24
0 4 8 12 16 20 24 28 32 36 40 44 48
13 14 15 16 17 18 19 20 21 22 23 24

Fig. 4.14

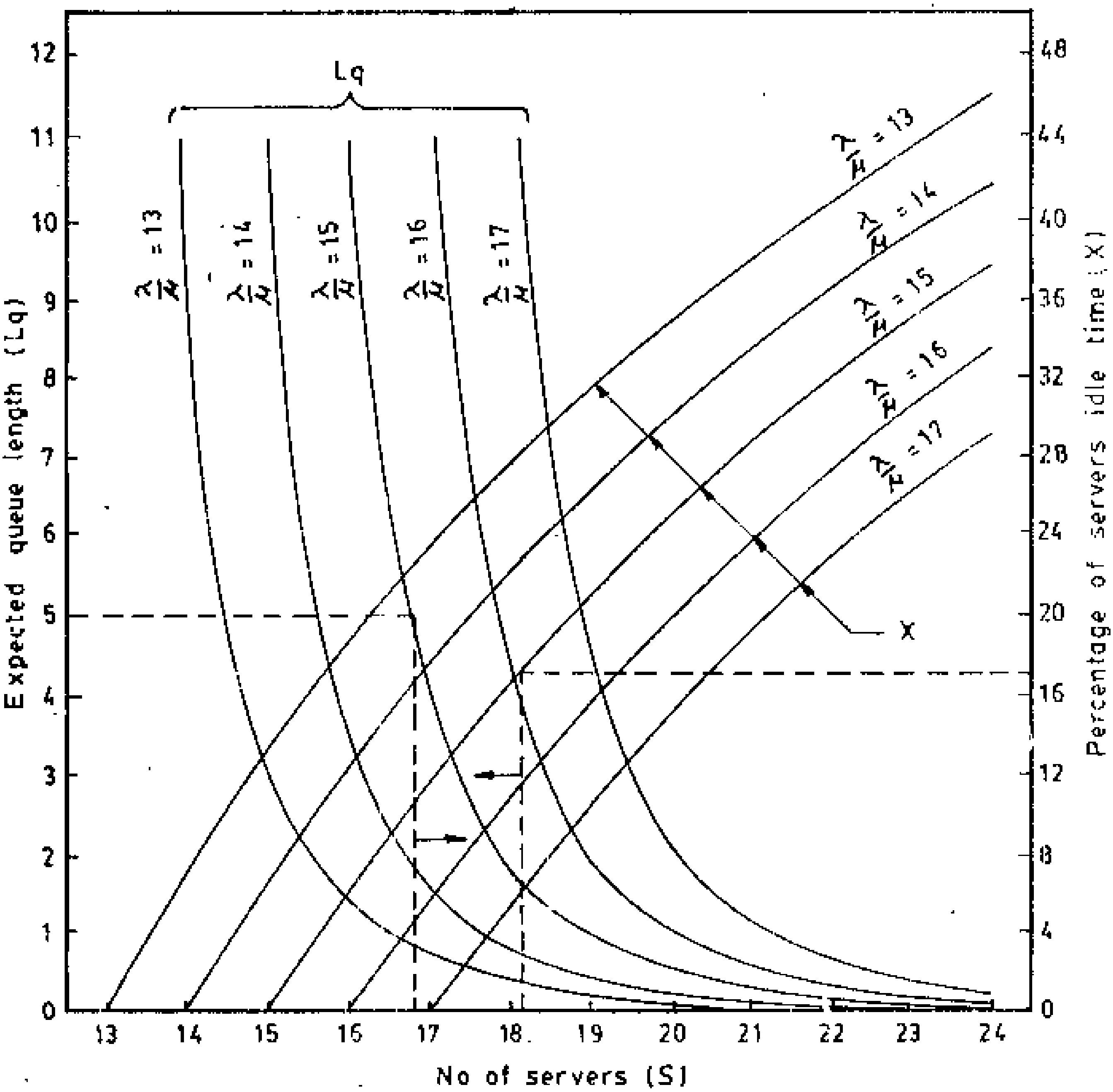

Lq
Expected queue length (Lq)
Percentage of servers idle time (X)
No of servers (S)
λ/μ = 13
λ/μ = 14
λ/μ = 15
λ/μ = 16
λ/μ = 17
X

Fig.4.15

with respect to a single parameter, traffic intensity (figures 4-14 & 4-15). To explain how aspiration levels are set, the fig. 4.10 may be considered as an example.

Suppose, the decision maker wants to determine the expected number of servers such that the expected number of customers in the system remains below 22 and at the same time the servers idle time does not exceed 19% when the mean arrival rate is 2.3 ships per day and mean service time is 6.46 days per ship. From the figure 4.10, it appears that for L (no. of ships in the system) to stay below 22, at least 17 servers (jetties) are required. On the other hand keeping the server's idle time below 19% of the time, requires a maximum of 18 servers. Thus the two aspiration levels are not satisfied simultaneously and so one of the two conditions must be relaxed to get a feasible solution. The choice between S = 17 & S = 18 should be made in view of whether it is worthwhile to reduce the number of customers (ships) in the system from 18 to 16 even though the server's (jetties) idle time will increase from 13% to 17%. In view of the above situation management has to take the final decision about the optimal number of servers.

To assist in making a specific decision in the case of the aspiration level method, it is possible to estimate the range of cost of waiting per unit waiting time per customer resulting from the selection of servers for given aspiration levels. However, in the system under study, this estimate will not provide any worthwhile information because the cost of waiting is not directly incurred by the port authority. In the long run, however, cost of waiting may appear in the form of loss of goodwill and reputation which will ultimately affect revenue earnings.

CHAPTER FIVE

DEVELOPMENT OF DIFFUSION EQUATION BY DIFFUSION APPROXIMATION METHOD

5.1 Introduction

5.2 Diffusion Approximation Method

5.3 Calculation of the System Parameters Using Diffusion Approximation Method

5.4 Application & Validation of the Approximation Method

5.0 Development of Diffusion Equation by Diffusion Approximation Method

5.1 Introduction

In this chapter diffusion equations as developed in chapter-2 by using diffusion approximation method for the system under study have been used to calculate the different system parameters (L, Lq, W, Wq). The values of these parameters are then compared with the corresponding observed values of the system to check the validity of the approximation method.

5.2 Diffusion Approximation Method

The diffusion approximation method assumes both the arrival process and departure process to be renewal processes. However, quality of the approximation deteriorates as the utilization factor becomes small. It is expected that the quality becomes good as utilization factor ($\frac{\lambda}{S\mu}$) approaches unity. On the basis of empirical evidence, it is found that when $\rho > 0.7$, the quality of approximation becomes acceptable, and generally improves as $\rho \rightarrow 1$.[9]

In the diffusion approximation method the percentage absolute errors become, in general higher for $\rho < 0.3$ and $S \leq 2$.[2] In fact, problems arise in the practical field when the queue length is large compared to the number of servers, which generally occurs at higher utilization factor. For $s = 1$ also, good idea about the system performance can be made although errors become comparatively large in some cases.

For the system under study $\rho = 0.874$. So the diffusion approximation method will provide a good approximation to the system parameters of the present system.

As mentioned earlier, the discrete queueing process is replaced by a continuous process in such a way that the characteristics of the original process are not lost. Specifically, the discrete variable N(t), the number of customers in the system at time t is replaced by a continuous variable X(t).

In order to estimate the different system parameters, the distribution functions of the number of customers in the system (E_Q - 2.12 and 2.13) may be recalled:

$$f_1(x) = H_1 \left[\lambda^3 \cdot \sigma_a^2 + x\mu^3 \sigma_s^2\right]^{U-1} \cdot \text{Exp.} \left[-\frac{2x}{\mu^2 \sigma_s^2}\right]$$

for $0 \leq x \leq s$

$$f_2(x) = H_2 \cdot \text{Exp} \left[-2 \left\{\frac{s\mu - \lambda}{s\mu^3 \sigma_s^2 + \lambda^3 \sigma_a^2} \cdot x\right\}\right]$$

where $U = \frac{2\lambda}{\mu^3 \sigma_s^2} \left[\frac{\lambda^2 \sigma_a^2}{\mu^2 \sigma_s^2} + 1\right]$

Expected number of customers in the system L is computed as follows:

$$L = \int_0^s x\, f_1(x).dx + \int_s^{\infty} x\, f_2(x).dx \qquad \ldots \qquad (5.1)$$

Other parameters may be calculated by using standard formulae.

5.3 Calculation of the System Parameters Using Diffusion Approximation Method.

For the system λ = 2.03 ships per day

μ = 0.1548 ships per day

S = 15 jetties

σ_a^2 = 0.24266

σ_s^2 = 40.58

λ, μ, σ_a^2, σ_s^2 are calculated from the observed values.

Now,

$$u = \frac{2\lambda}{\mu^3 \sigma_s^2} \left\{ \frac{\lambda^2 \sigma_a^2}{\mu^2 \sigma_s^2} + 1 \right\}$$

$$= \frac{2 \times 2.03}{(0.1548)^3 \times 40.58} \; \frac{(2.03)^2 \times 0.24266}{(0.1548)^2 \times 40.58} + 1 \quad = 54.7068.$$

$$(\lambda^3 \sigma_a^2 + \mu^3 \sigma_s^2 x) = (2.03)^3 \times 0.24266 + (0.1548)^3 \times 40.58x$$

$$= 2.02995 + 0.15053\,x$$

$$-\frac{2x}{\mu^2 \sigma_s^2} = \frac{2\,x}{(0.1548)^2 \times 40.58} = -\,2.0567\,x$$

$$2\,\frac{s\mu - \lambda}{s\mu^3 \sigma_s^2 + \lambda^3 \sigma_a^2} = 2 \times \frac{15 \times 0.1548 - 2.03}{15(0.1548)^3 \times 40.58 + (2.03)^3 \times 0.24266}$$

$$= 0.1362$$

$$f_1(x) = H_1\,(2.02995 + 0.15053\,x)^{53.7068}\; e^{-2.0567\,x} \quad \ldots \quad (5.2)$$

for $0 \leq x \leq s$

$$f_2(x) = H_2 \, e^{-0.1362 \, x} \qquad \dots \quad \dots \qquad (5.3)$$

for $x \geqslant s$

1. Calculations using continuity criteria:

Using the normalization criteria

$$\int_0^{15} f_1(x).dx + \int_{15}^{\infty} f_2(x)dx = 1$$

and the continuity of probability density function

$$f_2(x=15) = f_1(x=15)$$

the values of H_1 and H_2 are obtained and they are as follows

$H_1 = 1.8242 \times 10^{-22}$

$H_2 = 0.5079$

Hence the probability density functions of the customers in the system become as follows:

$$f_1(x) = 1.8242 \times 10^{-22}(2.02995 + 0.15053 \, x)^{53.7068} \, e^{-2.0567x} \dots \quad (5.4)$$

for $x \leqslant x \leqslant s$

$$f_2(x) = 0.5079 \quad e^{-0.1362 \, x} \qquad \dots \quad \dots \qquad (5.5)$$

for $x \geqslant s$

Now

$$L = \int_0^{15} x f_1(x)\,dx + \int_{15}^{\infty} x f_2(x)\,dx$$

$$= 5.8633 + 10.801 = 16.6643 \text{ ships per day.}$$

$$L_q = L - \frac{\lambda}{\mu} = 16.6643 - \frac{2.03}{0.1548} = 3.55 \text{ ships per day.}$$

$$W = \frac{L}{\lambda} = \frac{16.6643}{2.03} = 8.21 \text{ days per ship.}$$

$$W_q = \frac{L_q}{\lambda} = \frac{3.55}{2.03} = 1.75 \text{ days per ship.}$$

ii. Calculations using discretization criteria:

Using the nromalization criteria and the discretization criteria

$$P_n = \int_{n-0.5}^{n+0.5} f(x)\,dx.$$

Another set of values for H_1 and H_2 may be obtained and they are as follows:

$$H_1 = 1.8257 \times 10^{-22}$$

$$H_2 = 0.5074$$

Hence the probability density functions for the number of customers in the system become as follows:

$$f_1(x) = 1.8257 \times 10^{-22}\,(2.02996 + 0.15053x)^{53.7068}\, e^{-2.0567x} \quad \text{for } 0 \le x \le S \qquad ..(5.6)$$

$$f_2(x) = 0.5074\, e^{-0.1362x} \quad \text{for } x \geq S \qquad ... \qquad ..(5.7)$$

Now

$$L = \sum_{n=0}^{S-1} n\int_{n-0.5}^{n+0.5} f_1(x)\,dx + S\left[\int_{S-0.5}^{S} f_1(x)\,dx + \int_{S}^{S+0.5} f_2(x)\,dx\right] + \sum_{n=S+1}^{\infty} n\int_{n-0.5}^{n+0.5} f_2(x)\,dx .$$

$$= 5.356 + 15(0.03429 + 0.031795) + 10.3103$$

$$= 16.658 \quad \text{ships per day}$$

$$Lq = L - \frac{\lambda}{\mu} = 16.658 - \frac{2.03}{0.1548} = 3.544 \quad \text{ships per day}$$

$$W = \frac{L}{\lambda} = \frac{16.658}{2.03} = 8.206 \quad \text{days per ship}$$

$$Wq = \frac{Lq}{\lambda} = \frac{3.544}{2.03} = 1.746 \quad \text{days per ship}$$

The values of system parameters obtained by diffusion approximation method have been summarized in table 5.1 and compared with their observed values.

System Parameters	Observed values	Approximation Method: Continuous criteria	Approximation Method: descretization criteria	% Deviation
Lq	4.825	3.55	3.544	26.42
Wq	2.869	1.75	1.746	39.00
L-Lq	13.414	13.114	13.114	2.24
L	18.252	16.664	16.664	8.7
W	8.748	8.21	8.206	6.15
P_0	0.00	3.5515×10^{-5}	6.4865×10^{-6}	Very negligible

Tab – 5.1 : Comparison between observed values and those of approximation model

The table shows that the model (approximation) values of the parameters are not much different from their observed values. As discussed in the case of mathematical model, differences may be mainly due to the tide and draft restrictions which are not incorporated in the diffusion approximation model.

It is also observed from the table 5.1 that the values of the system parameters calculated by using continuity criterion and discretization criterion are almost same and for all practical purposes their differences may be ignored.

Aspiration levels for the diffusion approximation model considered in this section have not been determined due to following reasons. If the deciding factor is traffic intensity ($\frac{\lambda}{\mu}$) then it is not possible to set the aspiration level as pdf for the number of customers in the system has no expression with respect to $\frac{\lambda}{\mu}$. To calculate the above p.d.f., independent values of λ and μ must be known. For other cases, it is possible to determine aspiration levels but the calculation becomes too laborious. Moreover as the values of the system parameters of the mathematical model and the diffusion approximation model discussed in this section are almost same, it is expected that aspiration levels for this model will not provide any additional information

5.4 Application & Validation of the Approximation Method

In the preceding section, the approximation method has been used to calculate the parameters of the system having general input and service time distributions. This method can also be successfully applied to E/E/S system (i.e. Erlang input and Erlang service time distribution). The probability density function for the number of customers in the system that may be obtained using diffusion approximation method is given below:

$$f_1(x) = H_1\left(\frac{\lambda}{l} + \frac{\mu x}{k}\right)^{2\frac{\lambda k}{\mu}\left(\frac{k}{l}+1\right)-1} e^{-2kx} \quad \ldots \quad (5.8)$$

for $0 \leq x \leq s$

$$f_2(x) = H_2\, e^{-ax} \quad \ldots \quad (5.9)$$

for $x \geq s$

where

$$a = \frac{2(1-\rho)}{\frac{\rho}{l} + \frac{1}{k}}$$

By taking different values of k and l, the probability density functions of a family of distributions may be obtained.

In order to validate the approximation method, certain system parameters are obtained by this method and their values are compared with those obtained by other models ($M/M/S$, $E_2/M/2$, $H/E_2/2$ & $E_2/E_2/S$). The comparison have been shown in the following tables (5.2 to 5.5). The comparative figures in the different tables amply demonstrate that the diffusion approximation method provides a very good approximation to the system parameters at high utilization factor ($\frac{\lambda}{S\mu}$).

The tabulated values have been shown graphically in figures (5.1 to 5.3).

System Parameters	M/M/15	Diffusion Approximation method
L	16.[illegible]14	16.721
Lq	3.67	3.607
W	8.23	8.237
Wq	1.77	1.777

Table 5.2 : Comparison between M/M/15 and Approximation Method (The values for M/M/15 refer to the mathematical model; other set of values refer to $E_1/E_1/15$ when diffusion approximation method is used).

ρ	L(Model)	L (Diffusion)	% error
0.1	0.180	0.166	7.78
0.2	0.400	0.384	4.00
0.3	0.670	0.627	6.42
0.4	0.920	0.903	5.65
0.5	1.300	1.238	4.77
0.6	1.750	1.687	3.60
0.7	2.400	2.376	1.00
0.8	3.750	3.682	1.80
0.9	7.500	7.482	0.24

Table 5.3 : Mean number of customers in the system in the model $E_2/M/2$ [12] by model and diffusion approximation method.

ρ	L (Model)	L (Diffusion)	% Error
0.1	0.205	0.190	7.32
0.2	0.430	0.424	6.52
0.3	0.680	0.659	6.05
0.4	0.930	0.916	5.76
0.5	[illegible].350	1.228	5.15
0.6	1.700	1.653	4.39
0.7	2.400	2.319	3.93
0.8	3.400	3.603	1.90
0.9	7.800	7.381	1.60

Table 5.4: Mean number of customers in the system in the model $M/E_2/2$ [12] by model and diffusion approximation method

ρ	S = 2		S = 5		S = 10	
	Diffusion	Model	Diffusion	Model	Diffusion	Model
0.1	0.144	0.200	0.472	0.500	0.999	1.000
0.2	0.363	0.403	0.999	1.000	2.001	2.000
0.3	0.592	0.614	1.502	1.501	3.001	3.000
0.4	0.828	0.846	2.005	2.008	4.000	4.000
0.5	1.093	1.118	2.521	2.536	5.002	5.007
0.6	1.426	1.466	3.080	3.120	6.019	6.041
0.7	1.913	1.978	3.765	3.846	7.112	7.181
0.8	2.808	2.928	4.626	4.920	8.517	8.720
0.9	5.261	5.400	7.519	7.425	11.485	11.770

Table 5.5 ; Mean number of of customers in the system in the model $E_2/E_2/S$ by model and diffusion approximation method [2].

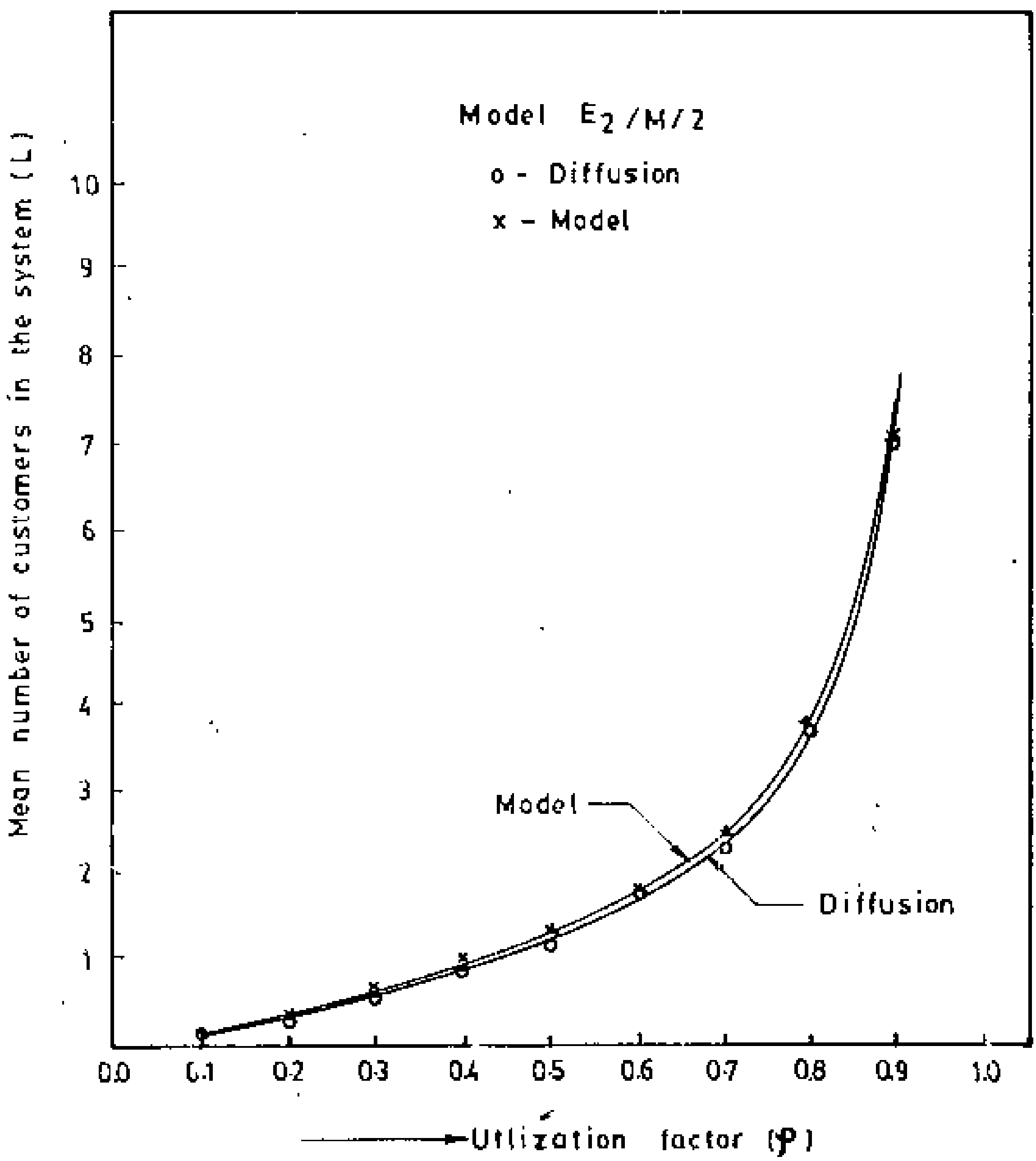

Model $E_2/M/2$
o - Diffusion
x - Model
Mean number of customers in the system (L)
10
9
8
7
6
5
4
3
2
1
Model
Diffusion
0.0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1.0
Utlization factor (ρ)

Fig. 5.1

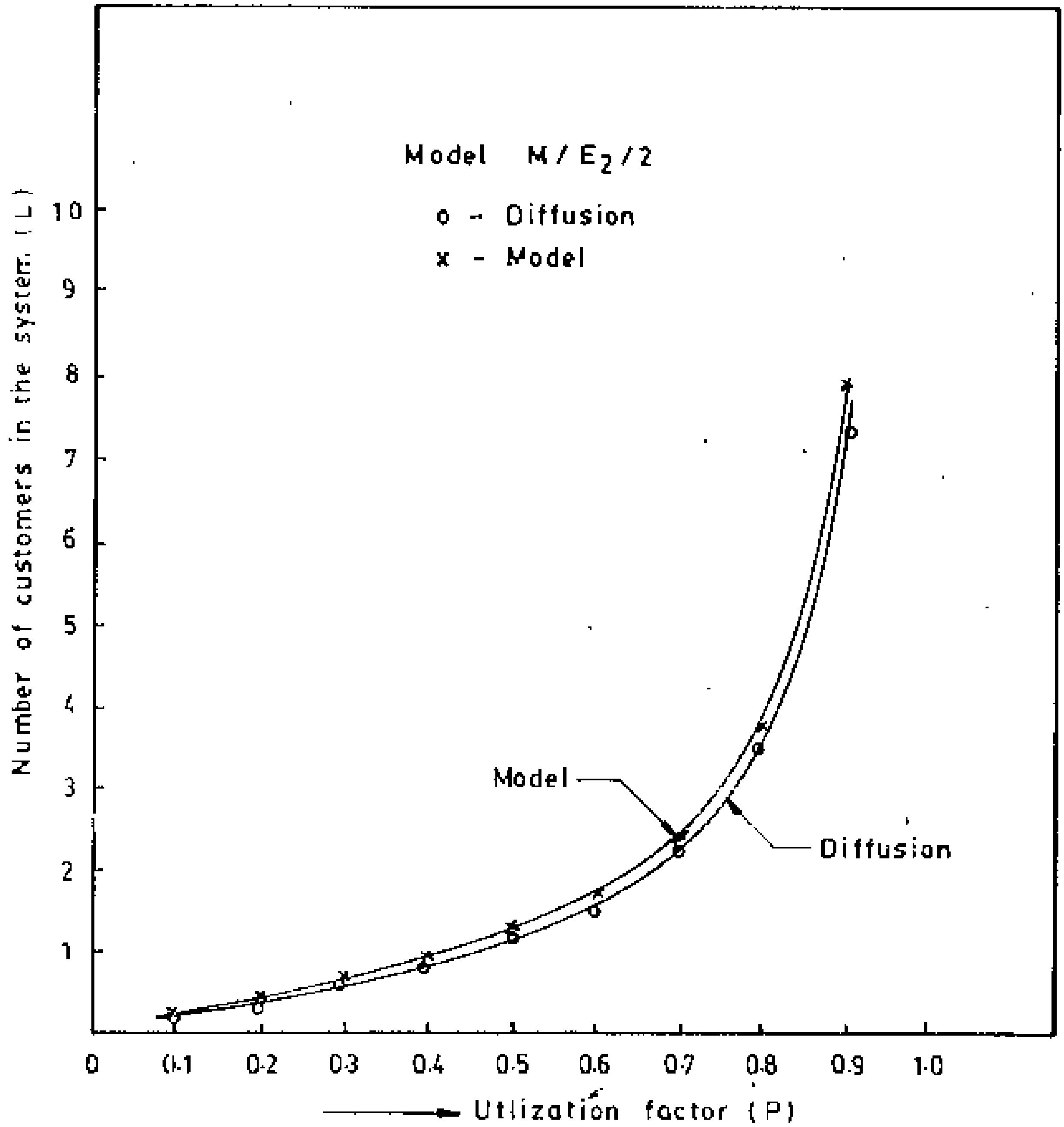
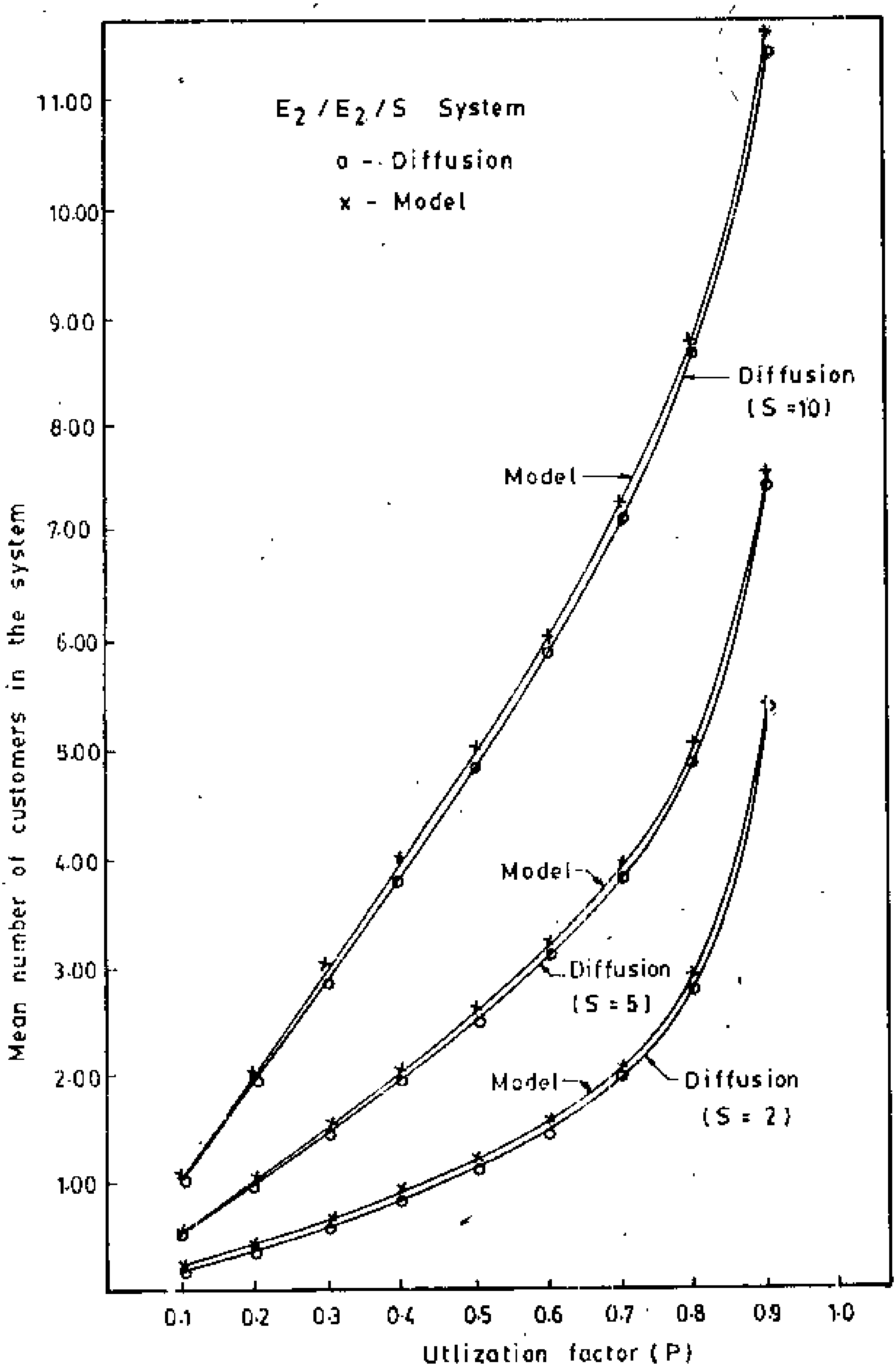

Model M / E2 / 2
o - Diffusion
x - Model
Number of customers in the system (L)
10
9
8
7
6
5
4
3
2
1
0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1.0
Model
Diffusion
Utlization factor (P)

Fig. 5.2



E2 / E2 / S System
o - Diffusion
x - Model
11.00
10.00
9.00
8.00
7.00
6.00
5.00
4.00
3.00
2.00
1.00
Mean number of customers in the system
Diffusion (S = 10)
Model
Model
Diffusion (S = 5)
Model
Diffusion (S = 2)
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1.0
Utlization factor (P)

Fig. 5.3

CHAPTER SIX

DESIGN OF A THEORETICAL EQUIVALENT SYSTEM AND ITS APPLICATION BY SIMULATION

6.0 Design of a Theoretical Equivalent System and Its Application by Simulation

6.1 Introduction

As mentioned earlier that sometimes it is not possible to develop analytical models for queueing systems. This can be due to the characteristics of the input or service mechanisms, the complexity of the system design, the nature of the queue discipline or combinations of the above. Under the situation, it may be necessary to resort to analyses by simulation. In this chapter, a Monte Carlo simulation model will be developed for the system under study and thereby different system Parameters will be calculated using this model.

6.2 Development of Monte Carlo simulation Model

The cumulative probability distributions of arrival of ships and their service time on the basis of historical data are shown in figures 6.1 and 6.2 respectively and the equivalent random numbers are shown in tables 6.1 & 6.2. Figure 6.3 shows the flow chart for computer programming to compute the different system parameters on the basis of tables 6.1 & 6.2 and accordingly a computer program is written in FORTRAN LANGUAGE, given in Appendix A-9a.

According to the flow diagram, first a random number (IRDAY) is selected. Depending on the value of IRDAY the corresponding number of ships arrived (ISHIP) is selected. Then another random number (ISERV) is

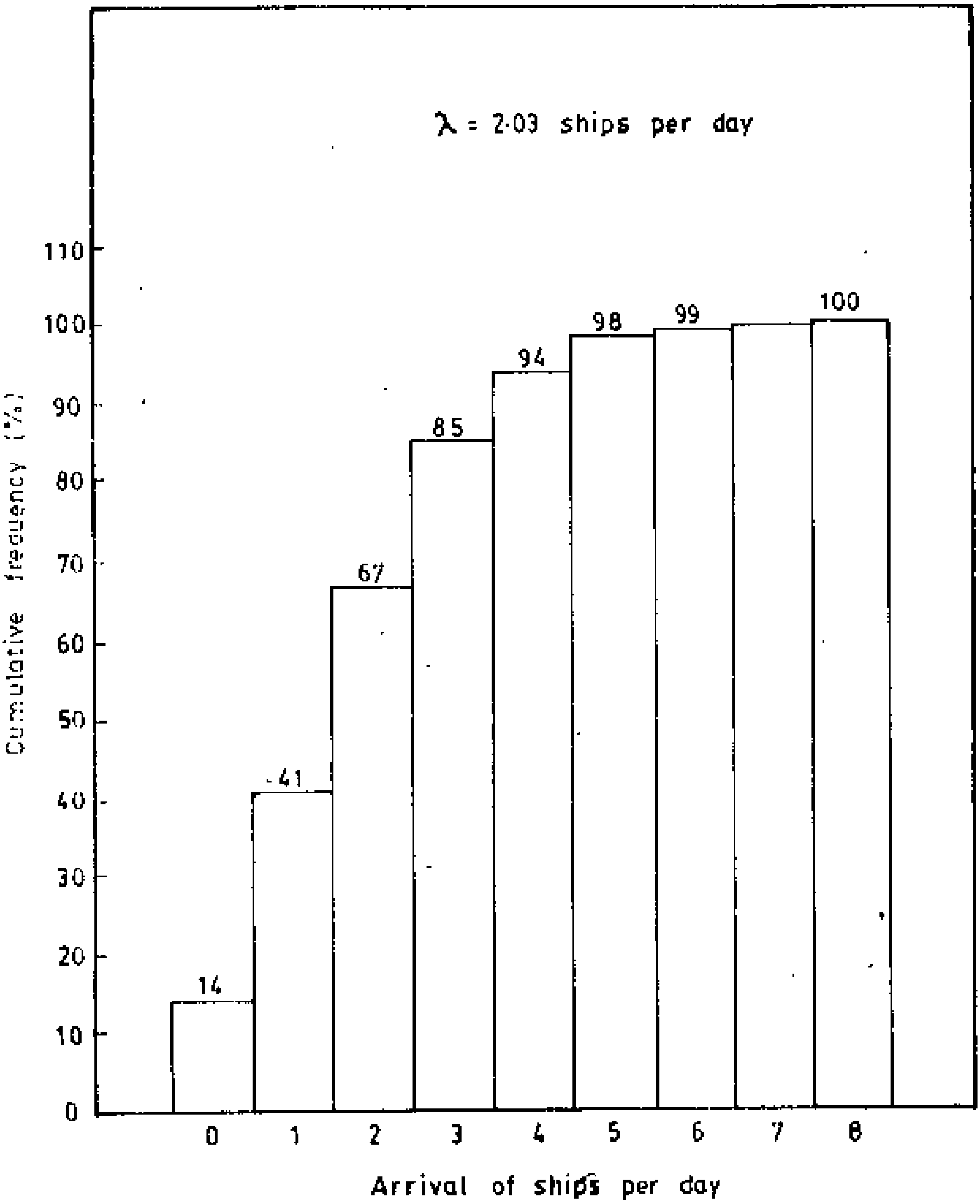


Fig. 6.1 The cumulative frequency distribution for arrival of ship per day

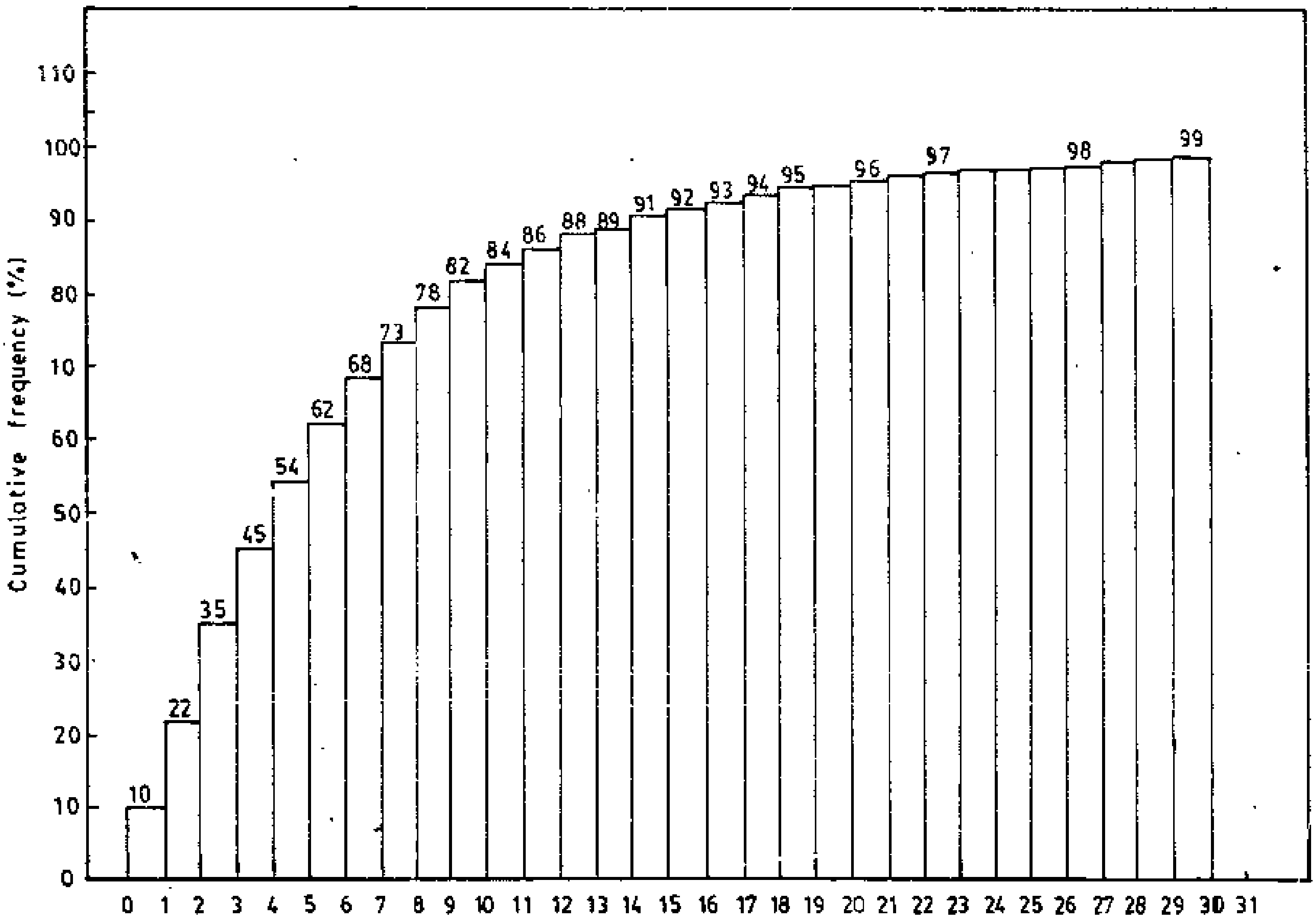


Fig.6.2 The cumulative frequency distribution for service time per ship

Random Numbers	Select No. of ship arrived
0 - 14	0
15 - 41	1
42 - 67	2
68 - 85	3
86 - 94	4
95 - 98	5
99	6

Table 6.1 : Equivalent random numbers for arrival of ship per day

Random Numbers	Select Service time (Day)
0 - 10	0 - 1
11 - 22	1 - 2
23 - 35	2 - 3
36 - 45	3 - 4
46 - 54	4 - 5
55 - 62	5 - 6
63 - 68	6 - 7
69 - 73	7 - 8
74 - 78	8 - 9
70 - 82	9 - 10
83 - 84	10 - 11
85 - 86	11 - 12
87 - 88	12 - 13
89	13 - 14
90 - 91	14 - 15
92	15 - 16
93	16 - 17
94	17 - 18
95	18 - 19
96	20 - 21
97	22 - 23
98	26 - 27
99	29 - 30

Table 6.2: Equivalent random numbers for service time per ship

FLOW CHART TO COMPUTE ARRIVAL OF SHIP, SERVICE TIME, WAITING TIME, NO OF SHIPS IN SERVICE BY SIMULATION

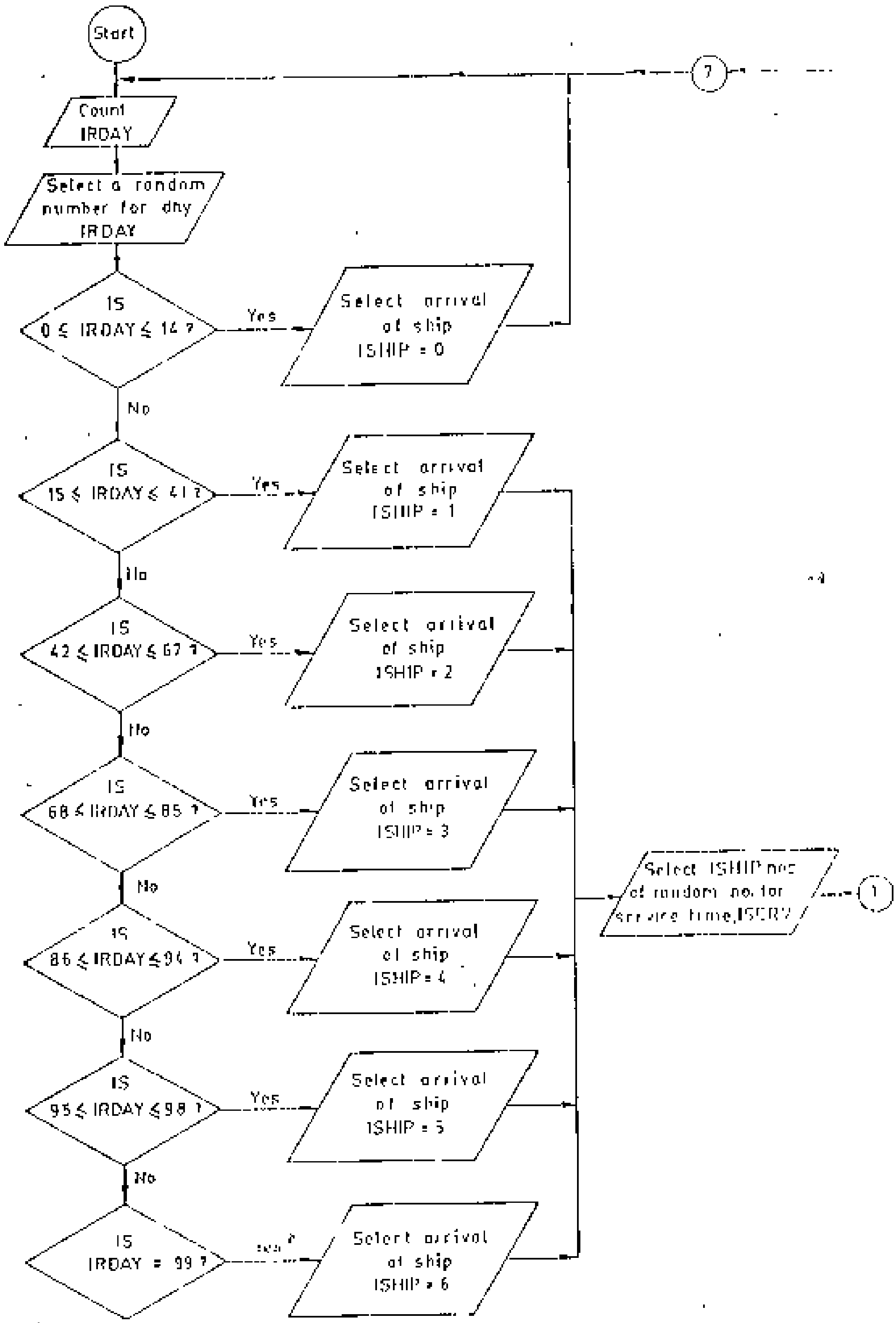


Fig 6.3

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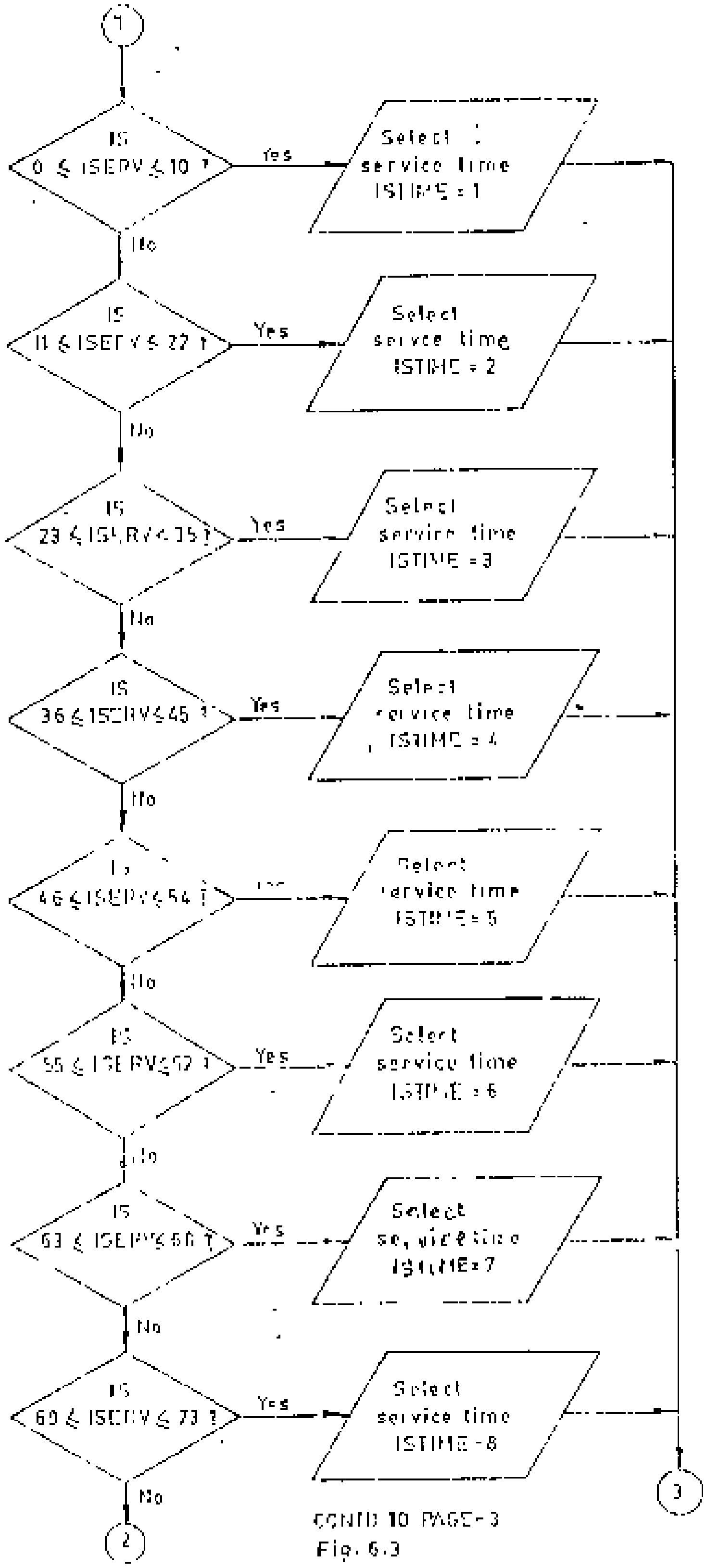

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Fig. 6.3

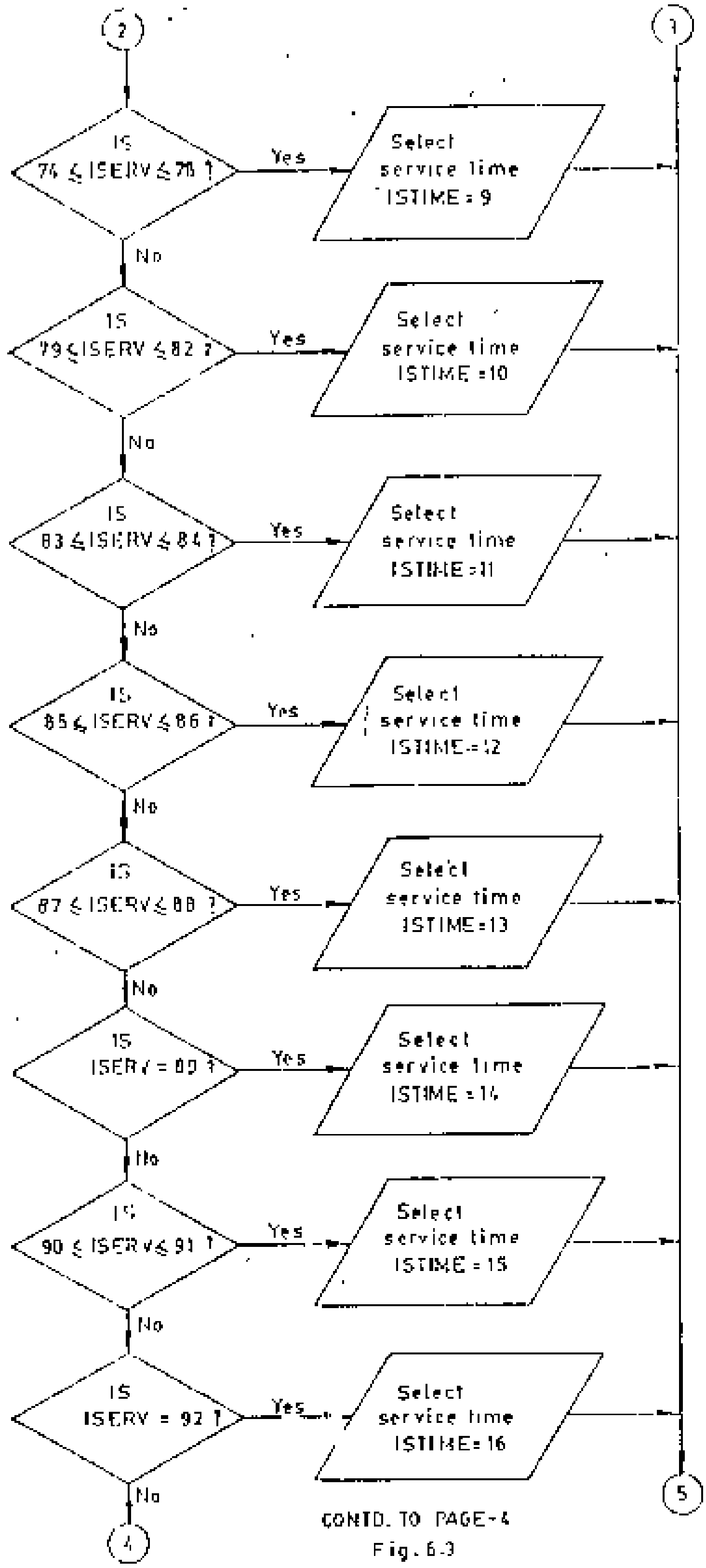

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Fig. 6.3

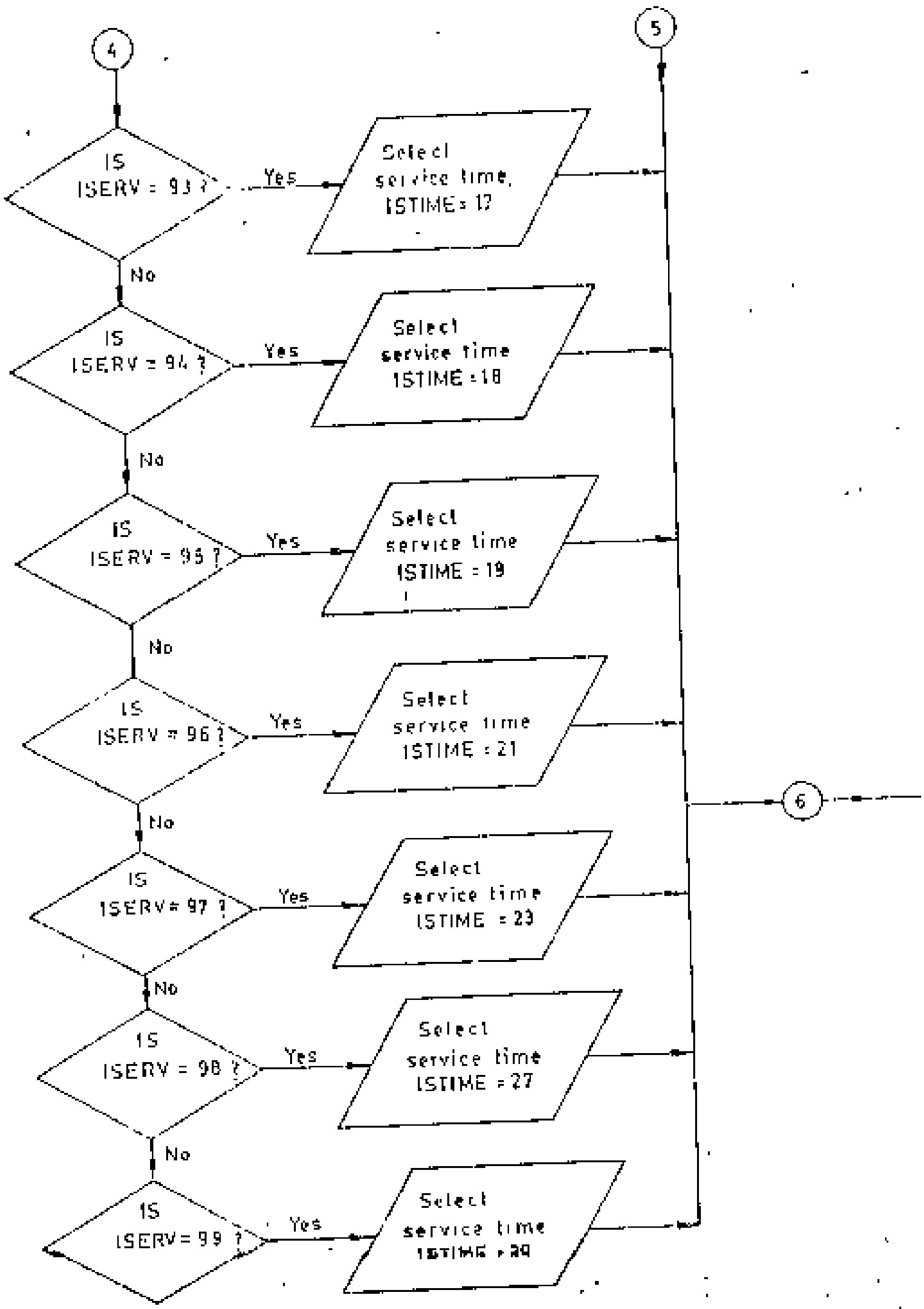

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Fig. 6-3

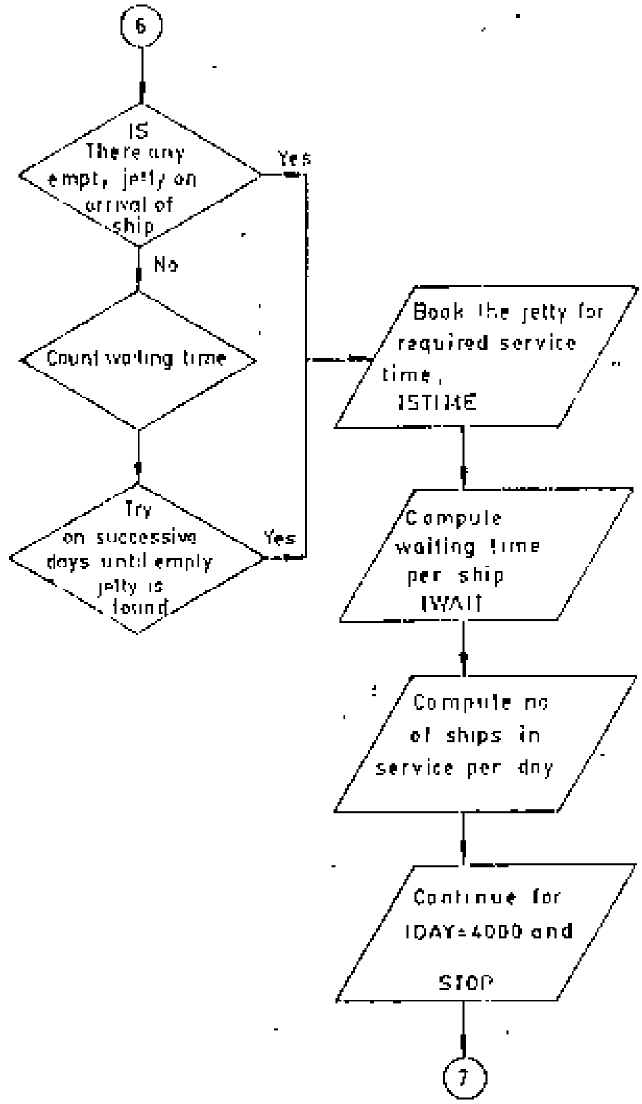

6
IS There any empty jetty on arrival of ship
Yes
No
Count waiting time
Try on successive days until empty jetty is found
Yes
Book the jetty for required service time, ISTIME
Compute waiting time per ship IWAIT
Compute no of ships in service per day
Continue for IDAY=4000 and STOP
7

Fig.6.3

selected for service time. Depending on the value of ISERV the corresponding service time for the ship is selected. When the value of ISHIP is zero it indicates no arrival of ship and ISERV needs not to be selected and it again starts with IRDAY for the next day (IDAY). When the value of ISHIP is more than unity, the equal number of ISERV are selected and corresponding ISTIME are also selected. Then the ship is put into service and the jetty is booked for the period of ISTIME if the jetty is available. If the jetty is not available it will count waiting time until the jetty is available. This way the process is repeated for 4000 days and different system parameters are calculated.

6.3 Results of Simulation Model

Table 6.3 explains the simulation process carried out by computer. As an example, on a particular day, IRDAY is 79. This means the number of ships that arrive is 3 (Table 6.1). Now to simulate service time random numbers are necessary. For the first ship say, ISERV is 69 and this means the service time of the ships is 8 days (table6.2). As the jetty is empty, the ship is put into service and the jetty is booked for 8 days. The above process will be repeated for the second and third ship. As three ships are in service, ISUMJ is 3 and since there is no ship in the queue, IQUE is 0 and the number of ships in the system ISYSM is 3. This process is continued for 4000 days and the following results are obtained for 15 jetties, λ = 2.03 ships per day, $\frac{1}{\mu}$ = 6.46 days per ship.

Day IDAY	Random no for arrival of ship IRDAY	No of ships arrived ISHIP	Random no for service time ISERV	Service time per ship ISTIME	Waiting time in queue IWAIT	Jetties 1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	No of ship in service ISUMJ	No of ship in queue IQUE	No of ship in system ISYSM
1	79	3	69	8	0	1	1	1																		3	0	3
			9	1	0																							
			35	3	0																							
2	30	1	68	7	0	1	1	1																		3	0	3
3	35	1	3	1	0	1	1	1	1																	4	0	4
4	5	0	–	–	–	1	1																			2	0	2
5	9	0	–	–	–	1	1																			2	0	2
6	6	0	–	–	–	1	1																			2	0	2
7	60	2	12	2	0	1	1	1	1																	4	0	4
			30	3	0																							
8	74	3	76	9	0	1	[illegible]	1	1	1	1	1														7	0	7
			92	16	0																							
			69	8	0																							
9	87	4	5	1	0	1	1	1	1	1	1	1	1													8	0	8
			50	5	0																							
			51	5	0																							
			60	6	0																							
10	5	0	–	–	–	1	1	1	1	1	1															6	0	6
11	93	4	11	2	0	1	1	1	1	1	1	1	1	1	1											10	0	10
			37	4	0																							
			21	2	0																							
			95	19	0																							
12	85	3	58	5	0	1	1	1	1	1	1	1	1	1	1	1	1	1								13	0	13
			86	12	0																							
			95	19	0																							

Table 6.3 Simulation process by computer

Total number of ships arrived = 7982

Mean arrival rate = $\frac{7982}{4000}$ = 1.9955 ships per day.

Total ship-days in service = 53,449

Mean service time = $\frac{53449}{7982}$ = 6.6962 days per ship.

Total ship-days in queue = 15,050

Mean queue length = $\frac{15,050}{4000}$ = 3.7625 ships per day.

Mean waiting time in queue = $\frac{15,050}{7982}$ = 1.8855 days per ship

Mean waiting time in the system = $\frac{68499}{7982}$ = 8.5817 days per ship.

Mean number of ship in the system = $\frac{68351}{4000}$ = 17.0878 ships per day.

Following the same procedure, by varying the number of jetties, different sets of system parameters are calculated and the results are entered in table 6.5. Likewise tables (6.6 to 6.10 are constructed considering arrival rates (ships/day) for 2.1, 2.2, 2.3, 2.4 and 2.5 respectively. In these cases, however, arrival distributions are assumed to be poisson. As a sample, the cumulative probability distribution of arrival of ships for λ = 2.1 ships/day is shown in figure 6.4 and the equivalent random numbers are shown in table 6.4 .

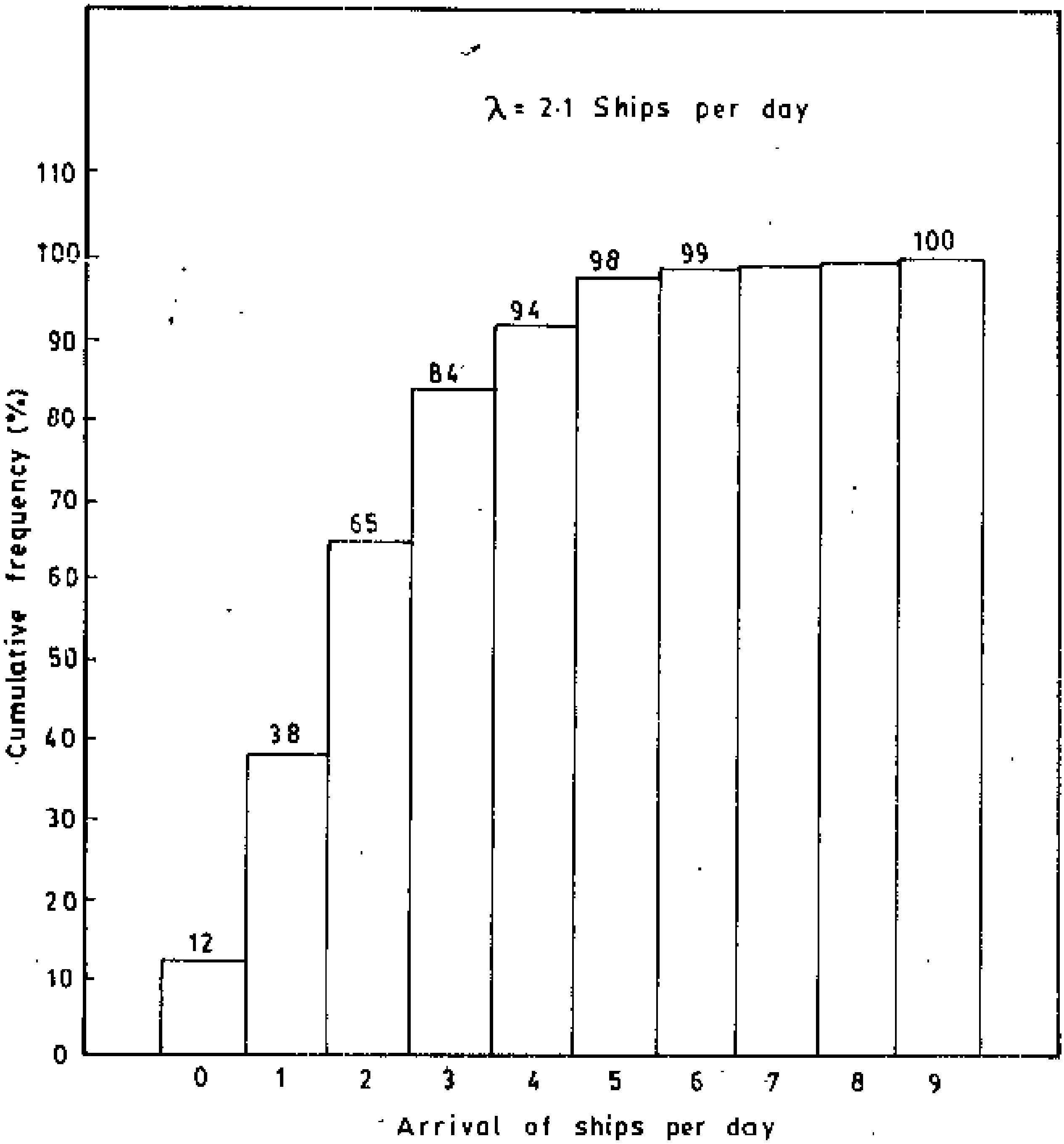


Fig-6.4 The cumulative frequency distribution for arrival of ships per day

Random Numbers	Select number of ships arrived
0 - 12	0
13 - 38	1
39 - 65	2
66 - 84	3
85 - 94	4
95 - 98	5
99	6

Table 6.4 : Equivalent random numbers for arrival of ships (2.1 ships per day)

SYSTEM PARAMETERS	FOR ARRIVAL RATE 2.03 SHIPS PER DAY					
	S = 15	S = 16	S = 17	S = 18	S = 19	S = 20
TOTAL NO. OF SHIP ARRIVED	7982	7982	7982	7982	7982	7982
λ	1.9955	1.9955	1.9955	1.9955	1.9955	1.9955
$S\mu$	2.2401	2.3894	2.5388	2.6881	2.8374	2.9868
λ/μ	13.3623	13.3623	13.3623	13.3623	13.3623	13.3623
$\lambda/\mu S$	0.8908	0.8351	0.7860	0.7424	0.7033	0.6681
SERVERS IDLE TIME (X),%	10.92	16.49	21.60	25.76	29.67	33.19
MEAN SERVICE TIME	6.6962	6.6962	6.6962	6.6962	6.6962	6.6962
MEAN NO. OF SHIP IN SERVICE	13.3253	13.3318	13.3348	13.3368	13.3375	13.3378
(Lq)	3.7625	1.6983	0.8483	0.4493	0.2495	0.1385
(Wq)	1.8855	0.8510	0.4251	0.2251	0.1250	0.0694
W	8.5817	7.5472	7.1213	6.9213	6.8212	6.7656
L	17.0878	15.0301	14.1831	13.7861	13.5870	13.4763

Table 6.5

SYSTEM PARAMETERS	FOR , ARIVAL RATE 2.1 SHIPS PER DAY					
	S = 15	S = 16	S = 17	S = 18	S = 19	S = 20
TOTAL NO. OF SHIP ARRIVED	8303	8303	8303	8303	8303	8303
λ	2.07575	2.07575	2.07575	2.07575	2.07575	2.07575
$S\mu$	2.2382	2.3874	2.5366	2.6858	2.8350	2.9842
λ/μ	13.9115	13.9115	13.9115	13.9115	13.9115	13.9115
$\lambda/S\mu$	0.9274	0.8695	0.8183	0.7729	0.7322	0.6956
SERVERS' IDLE TIME (X),%	7.26	13.05	18.17	22.71	26.78	30.44
MEAN SERVICE TIME	6.7019	6.7019	6.7019	6.7019	6.7019	6.7019
MEAN NO. OF SHIP IN SERVICE	13.8753	13.8838	13.8883	13.8885	13.8885	13.8885
(Lq)	6.0658	2.6745	1.353	0.6633	0.3583	0.1845
(Wq)	2.9222	1.2884	0.6518	0.3195	0.1726	0.0889
(W)	9.6241	7.9903	7.3537	7.0214	6.8745	6.7908
(L)	19.9411	16.5583	15.2413	14.5518	14.2468	14.073

Table 6.6

SYSTEM PARAMETERS	FOR ARRIVAL RATE 2.2 SHIPS PER DAY					
	S = 15	S = 16	S = 17	S = 18	S = 19	S = 20
TOTAL NO. OF SHIP ARRIVED	8710	8710	8710	8710	8710	8710
λ	2.1775	2.1775	2.1775	2.1775	2.1775	2.1775
$S\mu$	2.2311	2.3799	2.5286	2.6774	2.8261	2.9749
λ/μ	14.6393	14.6393	14.6393	14.6393	14.6393	14.6393
$\lambda/S\mu$	0.976	0.915	0.8611	0.8133	0.7705	0.7320
X(%)	2.40	8.50	13.89	18.67	22.95	26.80
MEAN SERVICE TIME	6.723	6.723	6.723	6.723	6.723	6.723
MEAN NO. OF SHIP IN SERVICE	14.5818	14.6278	14.6278	14.6278	14.6278	14.6278
(Lq)	17.5478	5.3315	2.4948	1.251	0.614	0.3158
(Wq)	8.0587	2.4485	1.1457	0.5745	0.282	0.145
W	14.7817	9.1715	7.8687	7.2975	7.0005	6.868
L	32.1296	19.9593	17.1226	15.8788	15.2418	14.9436

Table - 6.7

SYSTEM PARAMETERS	FOR ARRIVAL RATE 2.3 SHIPS PER DAY					
	S = 15	S = 16	S = 17	S = 18	S = 19	S = 20
TOTAL NO. OF SHIP ARRIVED	9093	9093	9093	9093	9093	9093
λ	2.2733	2.2733	2.2733	2.2733	2.2733	2.2733
$S\mu$	2.2353	2.3843	2.5333	2.6824	2.8314	2.9804
λ/μ	15.255	15.255	15.255	15.255	15.255	15.255
$\lambda/S\mu$	1.017	0.9534	0.8974	0.8475	0.8029	0.7628
ρ (%)	-1.7	4.66	10.20	15.25	19.71	23.72
MEAN SERVICE TIME	6.7105	6.7105	6.7105	6.7105	6.7105	6.7105
MEAN NO. OF SHIP IN SERVICE	14.9033	15.1945	15.2113	15.2248	15.2283	15.2288
(Lq)	75.303	9.8028	3.826	1.934	0.942	0.4743
(Wq)	33.13	4.3122	1.6831	0.8508	0.4144	0.2086
(W)	39.84	11.0227	8.3936	7.5613	7.1249	6.9191
(L)	90.2053	24.9973	19.0373	17.1588	16.1703	15.7031

Table - 6.8

SYSTEM PARAMETERS	FOR ARRIVAL RATE 2.4 SHIPS PER DAY					
	S = 15	S = 16	S = 17	S = 18	S = 19	S = 20
TOTAL NO. OF SHIP ARRIVED	9368	9368	9368	9368	9368	9368
λ	2.342	2.342	2.342	2.342	2.342	2.342
$s\mu$	2.214	2.362	2.510	2.657	2.805	2.953
λ/μ	15.855	15.855	15.855	15.855	15.855	15.855
$\lambda/s\mu$	1.057	0.991	0.9327	0.881	0.834	0.793
X (%)	-5.7	0.90	6.73	11.90	16.6	20.7
MEAN SERVICE TIME	6.77	6.77	6.77	6.77	6.77	6.77
MEAN NO. OF SHIP IN SERVICE	14.944	15.705	15.82	15.83	15.83	15.83
(Lq)	291.90	24.15	7.22	3.16	1.58	0.809
(Wq)	124.64	10.31	3.08	1.35	0.68	0.35
(W)	131.41	17.08	9.85	8.12	7.45	7.12
(L)	306.844	39.855	23.04	18.99	17.41	16.64

Table – 6.9

SYSTEM PARAMETERS	FOR ARRIVAL RATE 2.5 SHIPS PER DAY					
	S = 15	S = 16	S = 17	S = 18	S = 19	S = 20
TOTAL NO. OF SHIP ARRIVED	9848	9848	9848	9848	9848	9848
λ	2.462	2.462	2.462	2.462	2.462	2.462
$S\mu$	2.227	2.375	2.524	2.672	2.821	2.969
λ/μ	16.584	16.584	16.584	16.584	16.584	16.584
$\lambda/S\mu$	1.106	1.037	0.976	0.921	0.873	0.829
X (%)	-10.6	-3.7	2.4	7.9	12.7	17.1
MEAN SERVICE TIME	6.736	6.736	6.736	6.736	6.736	6.736
MEAN NO. OF SHIP IN SERVICE	14.974	15.928	16.425	16.556	16.565	16.566
(L_q)	872.67	164.229	19.039	5.635	2.52	1.313
(W_q)	354.456	66.705	7.733	2.289	1.023	0.533
(W)	361.192	73.441	14.469	9.025	7.759	7.269
(L)	887.644	180.157	35.464	22.191	19.085	17.879

Table – 6.10

In table 6.11, the simulation values of the system parameters corresponding to S = 15 are compared with their observed values. The two results seem to be very consistant.

System Parameters	Observed Values	Simulated Values
λ	2.03	1.9955
$1/\mu$	6.46	6.6962
λ/μ	13.114	13.3623
$\lambda/s\mu$	0.874	0.8908
X	12.6%	10.92%
Lq	4.825	3.7625
Wq	2.[illegible]9	1.8855
L-Lq	13.414	13.3253
L	18.252	17.0878
W	8.746	8.5817
P_o	0.00	0.00

Table 6.11 : Comparison between observed and simulated values of system parameters

The aspiration level curves are drawn, shown in figures 6.5 to 6.8 using the values of the system parameters given in tables 6.5 to 6.10. As it is observed that simulated values of μ are found to be different for

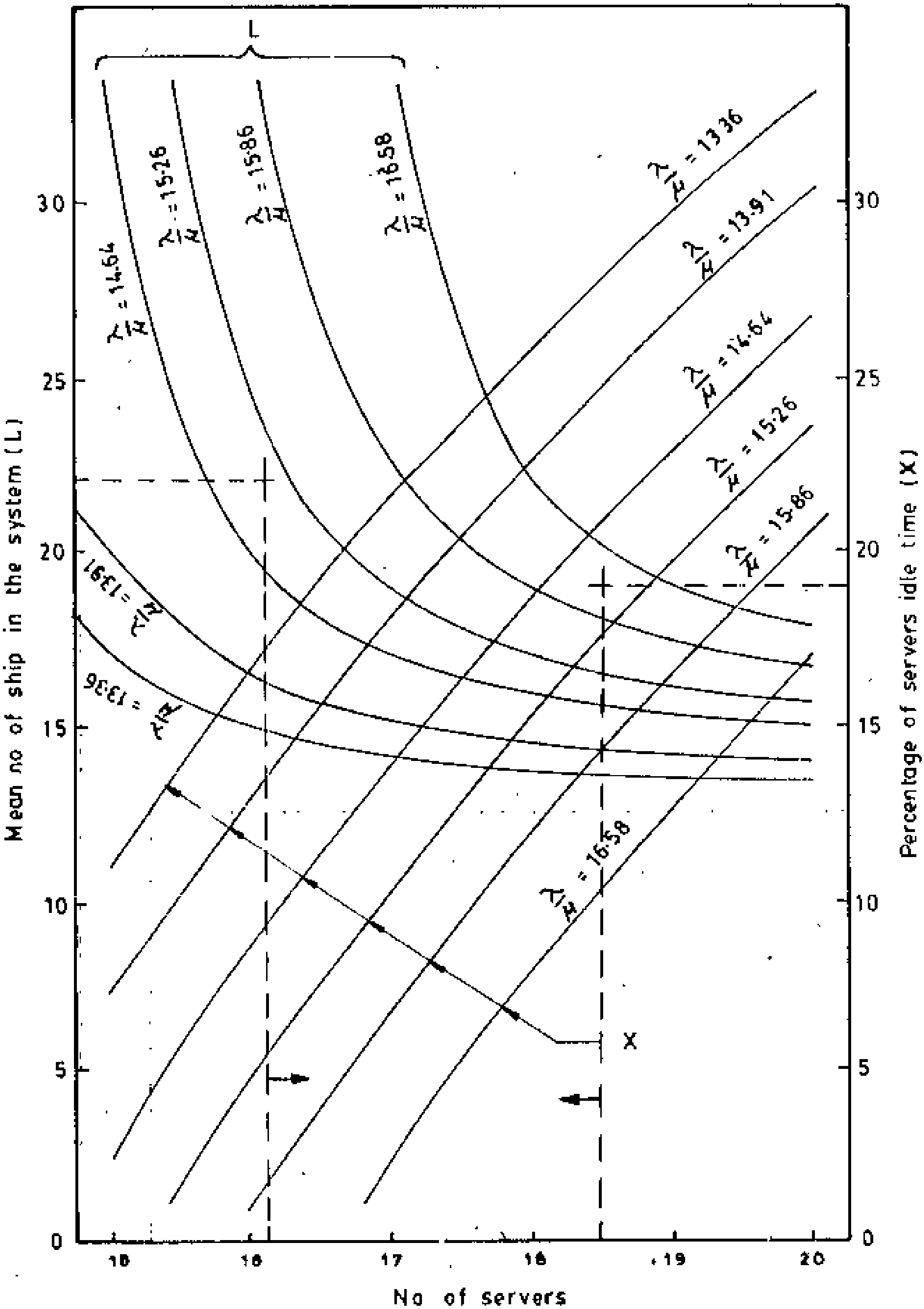

L
λ/μ = 14.64
λ/μ = 15.26
λ/μ = 15.86
λ/μ = 16.58
λ/μ = 13.36
λ/μ = 13.91
λ/μ = 14.64
λ/μ = 15.26
λ/μ = 15.86
λ/μ = 13.91
λ/μ = 13.36
λ/μ = 16.58
X
Mean no of ship in the system (L)
Percentage of servers idle time (X)
No of servers

Fig. 6.5

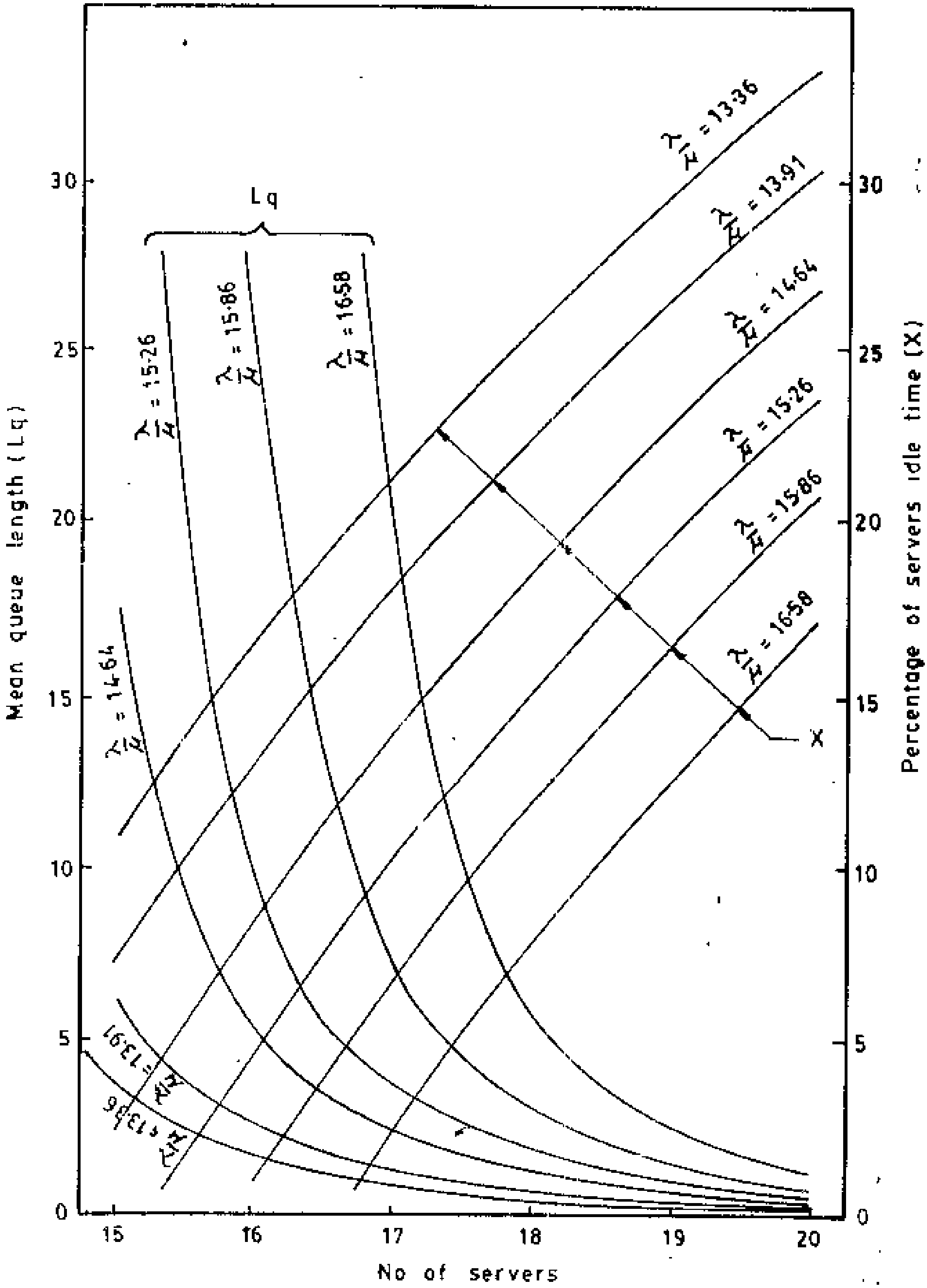

Lq
λ/μ = 15·26
λ/μ = 15·86
λ/μ = 16·58
λ/μ = 13·36
λ/μ = 13·91
λ/μ = 14·64
λ/μ = 15·26
λ/μ = 15·86
λ/μ = 16·58
λ/μ = 14·64
λ/μ = 13·91
λ/μ = 13·36
X
Mean queue length (Lq)
Percentage of servers idle time (X)
0
5
10
15
20
25
30
15
16
17
18
19
20
No of servers

Fig. 6.6

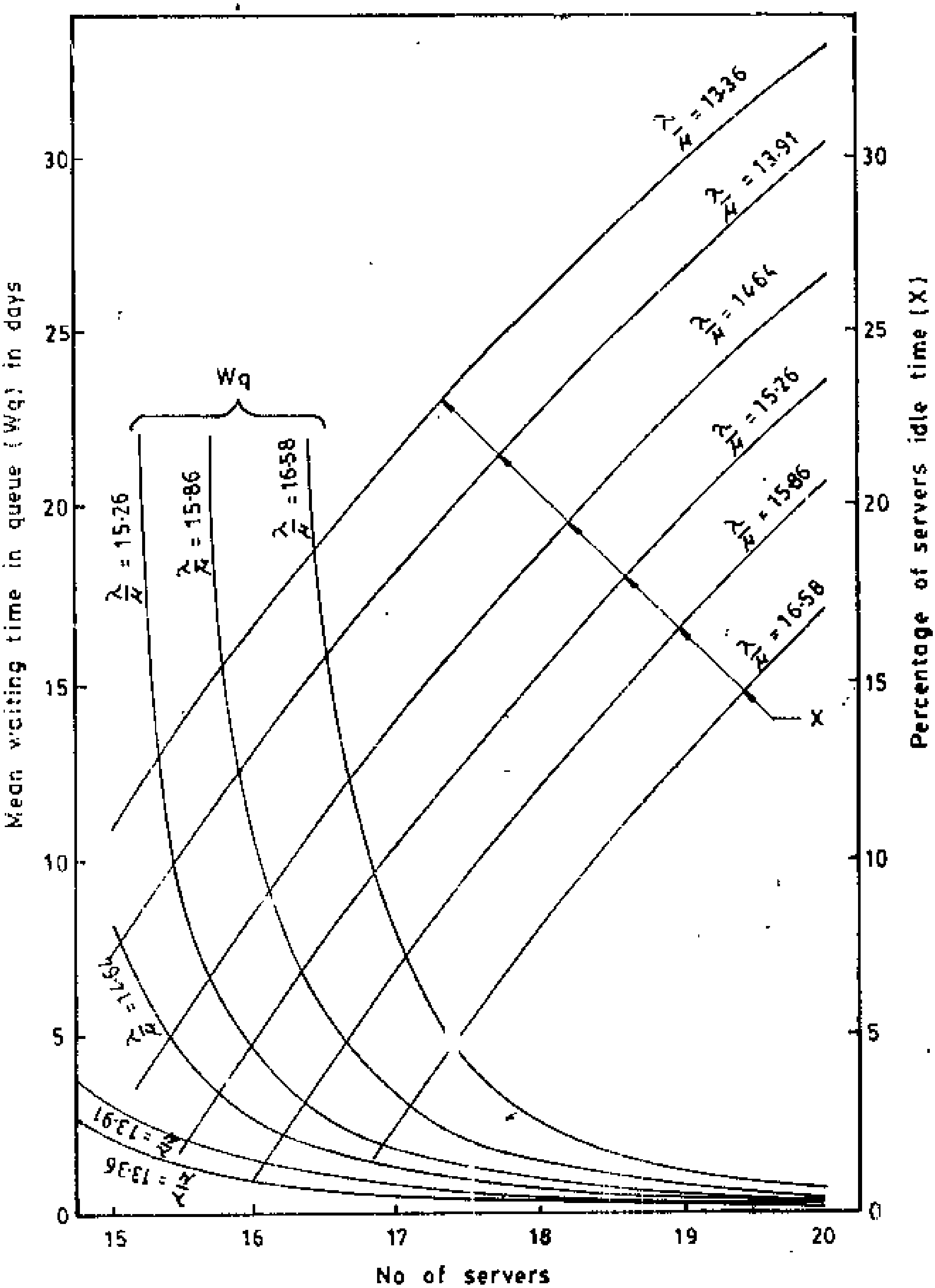

Mean waiting time in queue (Wq) in days
Percentage of servers idle time (X)
No of servers
Wq
X
λ/μ = 13.36
λ/μ = 13.91
λ/μ = 14.64
λ/μ = 15.26
λ/μ = 15.86
λ/μ = 16.58
0
5
10
15
20
25
30
15
16
17
18
19
20

Fig. 6.7

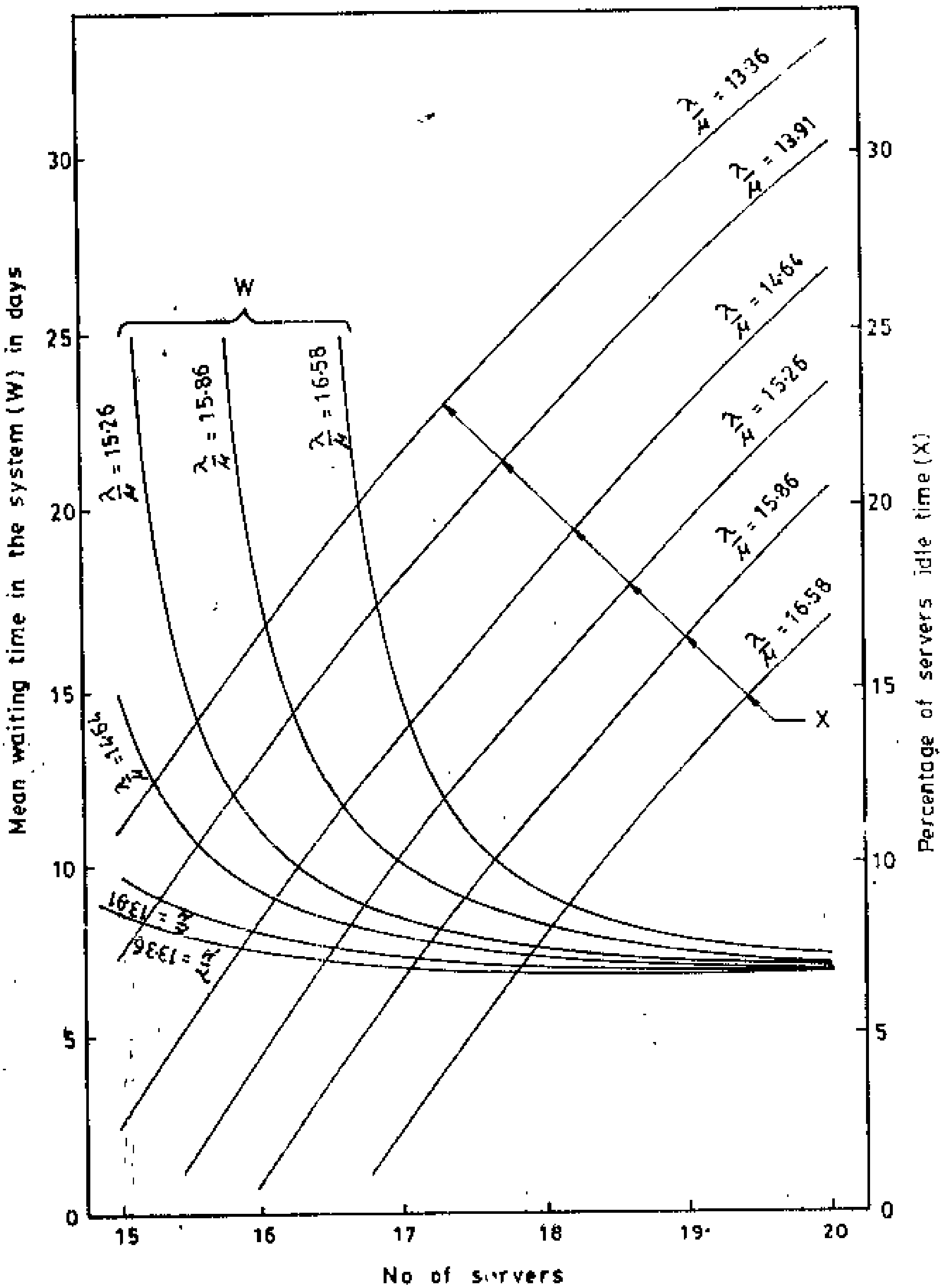

Mean waiting time in the system (W) in days
Percentage of servers idle time (X)
No of servers
W
X
λ/μ = 13·36
λ/μ = 13·91
λ/μ = 14·64
λ/μ = 15·26
λ/μ = 15·86
λ/μ = 16·58
0
5
10
15
20
25
30
15
16
17
18
19
20

Fig. 6.8

different values of λ, all aspiration level curves are drawn on the basis of the traffic intensities ($\frac{\lambda}{\mu}$) not the arrival rates λ.

The aspiration level values of the mathematical model (fig. 4.14 to 4.15) and the simulation model (fig. 6.5 to 6.8) are also found to be quite consistent. For example, if the aspirations levels are set such that number of customers in the system (L) is to remain below 22 and at the same time server's idle time does not exceed 19% when traffic intensity ($\frac{\lambda}{\mu}$) is 15, then the identical decision criterion is to be met both for mathematical model (fig. 4.14) and simulation model (fig. 6.5). For both models, it appears that for L to stay below 22, at least 17 server's are required. On the other hand keeping the server's idle time below 19% of the time, requires a maximum of 18 server's. The choice between S = 17 & S = 18 should be made in view of whether it is worthwhile to reduce the number of customers (ships) in the system (L) from 19 to 17 even though the server's idle time will increase from 12% to 17%.

CHAPTER SEVEN

C O S T   M O D E L

7.1 Introduction

7.2 Revenue and Cost Estimation

7.3 Development of Cost Model

7.4 Results

7.0 COST MODEL

7.1 Introduction

At present the port of Chittagong is rendering services with 15 jetties. As mentioned earlier, with the development of the country, the export-import activities at the port are expected to increase substantially. In order to meet the anticipated traffic demand, management has to find out means to provide adequate amount of facilities which ensure optimum service level. This chapter suggests a cost model to determine the optimal number of jetties with appropriate service levels to get maximum net incremental revenues at different mean arrival rates of the ships. Net incremental revenue refers to the difference between the revenue earned and the cost incurred due to the additional facilities over the existing ones.

7.2 Revenue and Cost Estimation

Data regarding the revenue earned per ship and the cost components of the different port facilities have been collected mainly from the accounts department and the project implementation department of the Chittagong port. Different consulting firms and machinery suppliers have also been contacted to obtain the cost values of certain items.

Revenue:

The main sources of revenue of a port are the service charges and facility charges[24]. Service charges, called P-Bill, refer to the charges for pilotage, Mooring shifting, transport, tug hire, launch hire etc. Facility charges, called J-Bill, include charges due to crane, derrick, cargo handling, light, Berth, water, gangway etc. The revenue earning for each of the 66 ships that arrived at the port in January, 1986 is collected in the following format and given in Appendix A-10.

SL. No.	Amount of P-Bill	Amount of J-Bill

The above data is used to calculate the mean revenue earning per ship. Referring to Appendix A-10.

Total amount of P-Bill for 66 ships = Tk. 12,926,729

Mean amount of P-Bill $= \dfrac{\text{Tk. } 12,926,729}{66}$

$= $ Tk. 195,860 per ship

Total amount of J-Bill for 66 ships = Tk. 18,912,133

Mean amount of J-Bill $= \text{Tk. } \dfrac{18,912,133}{66}$

$= $ Tk. 286,547 per ship

Earnings per ship = Mean amount of P-Bill + Mean amount of J-Bill

= Tk. 195,860 + Tk. 286,547

= Tk. 482,407

Cost:

For the present study, the items considered to estimate the total cost of the port facilities are mainly jetties, jetty-cranes, warehouses, mobile cranes and fork-lifters. The existing facility at the Chittagong Port is as follows:

i. Jetty = 15 numbers each 180 m long and 15 m wide

ii. Jetty crane = 3 per jetty, each having lifting capacity of 3 tons

iii. Warehouse = 1 per jetty, having floor area of 5410 m^2

iv. Mobile crane = 2 per jetty, each having lifting capacity 10 tons

v. Fork lifters = 3 per jetty, each having capacity of 4 tons.

The cost per m^2 floor area of jetty has been estimated to be Tk.26,667

Estimated jetty size = 180 m x 15 m

∴ Cost of an existing jetty = Tk.26,667 x 180 x 15

= Tk. 72,000,900

Cost per m^2 floor area of warehouse = Tk.12,918

Floor area of an existing warehouse = 5410 m^2

∴ Cost of an existing warehouse = Tk.12,918 x 5410

= Tk. 69,886,380

Considering 40 years of life for both jetty and the warehouse, their annual equivalent costs are found to be as follows (Appendix A-11).

Annual equivalent cost of anexisting jetty = Tk.10,840,470.-

Annual equivalent cost of an existing warehouse = Tk.10,522,239.-

It is to be noted that from design point of view, the cost of a jetty changes with the capacity of a jetty crane, because the reaction on the jetty pile will be higher in case of higher capacity cranes. It can be shown that the increase in number of the cranes of same capacity does not increase the reaction on the jetty pile but increase in crane capacity increases the reaction. Calculation of reactions on the jetty pile is shown in appendix (A-12). In this regard, the expert opinion [25] is that the cost of a jetty is increased approximately by 30% for 100% increase in reaction on the jetty pile and the cost varies almost linearly with the change of the reaction. The table 7.1 shows the reactions on the jetty piles for different crane capacities and the corresponding percentage increase in jetty cost. The increases in reaction and cost are calculated with respect to 3 tons crane.

Crane capacity	Reaction on jetty pile	% increase in reaction	% increase in estimated cost
3	6.84	0.0	0.0
3.5	7.98	16.67	5%
4.0	9.12	33.33	10%
4.5	10.26	50.00	15%
5.0	11.40	66.67	20%
5.5	12.54	83.33	25%
6.0	13.68	100.00	30%

Table 7.1 : Reaction on jetty piles and corresponding cost increase

From table 7.1, it is observed that unit increase in crane capacity increases the jetty cost by 10%. Hence the estimated costs of a jetty having cranes of capacities 3 tons or more may be calculated as follows:

$$J(T) = J(3)\left[1 + (T - 3) \times 0.1\right] \quad \dots \quad \dots \quad (7.1)$$

where,

J(T) = Annual equivalent cost of a jetty of T-tons crane

J(3) = Tk.10840470/-

T = Crane capacity of 3 tons or more

The table 7.2 shows the estimated costs of different material handling equipment. This data is obtained from foreign machinery suppliers(19b).

Equipment	Capacity (tons)	Unit Cost (Tk.)
Jetty crane	3.0	8,000,000.-
- do -	3.5	8,000,000.-
- do -	4.0	9,200,000.-
- do -	4.5	9,850,000.-
- do -	5.0	10,500,000.-
- do -	5.5	11,250,000.-
- do -	6.0	12,000,000.-
Mobile crane	10	10,000,000.-
Fork-lifter	4	2,500,000.-

Table 7.2 : Cost of Material handling equipments.

Considering 20 years of life of the material handling equipment, their annual equivalent costs have been calculated (Appendix A-11). In addition to this price of equipment operating cost at the rate of Tk.36000/- per year per equipment is considered in calculating the final annual cost.

7.3 Development of Cost Model

In the preceding section, average revenues earned per ship through different services and the annual equivalent costs of the related port facilities have been estimated. Using these estimates, a cost model is developed below to determine the average net incremental revenue earned per year (NIR). Net incremental revenue, as defined earlier, is the difference between the revenue earned and the cost incurred due to additional facilities over the existing ones. The average net incremental revenue per year (NIR) may be expressed as follows:

$$NIR = RE - AE$$

where, RE - Average additional revenue earned per year for additional facilities

AE - Average additional expenditure incurred per year for additional facilities

$$RE = R.N$$

where, R - Average revenue earned per ship = Tk.402407 per ship

N - Average number of ships served per year by additional facilities

$$AE = E_J + E_W + E_{JC} + E_{MC} + E_{FL}$$

where, E_J = the annual equivalent costs of additional jetties. E_J may be calculated using equation 7.1

E_W = the annual equivalent costs of additional warehouse.

E_{JC} = the annual equivalent costs of additional jetty cranes

E_{MC} = the annual equivalent costs of additional mobile cranes

E_{FL} = the annual equivalent costs of additional fork-lifters.

$$N = \begin{cases} SS - SE & \text{when } SE \leq SS \leq SA \\ SA - SE & \text{when } SE \leq SA \leq SS \\ 0 & \text{otherwise.} \end{cases}$$

where, SS - Average number of ships that can be served per year by proposed facilities

SE - Average number of ships served per year by existing facilities

SA - Average number of ships that arrive per year

$$SS = \frac{365.J}{EST}$$

where, J - the number of jetties

EST - Mean service time for the proposed facilities (days per ship)

$$SE = \frac{365.S}{M}$$

where, S - the existing number of jetties = 15

M - the mean service time for the existing facilities = 6.46 days per ship

$$SA = \lambda.365$$

where λ - the mean arrival rate (ships per day)

Now, Average cargo handling = 3421 tons per ship

Mean service time for the exsiting jetty = 6.46 days per ship

Cargo handling capacity on existing jetty per crane operation - 3 tons

Number of cranes on existing jetty = 3.0 crane

∴ Cargo handling capacity on existing jetty = 9 tons

∴ No. of operations per ship = $\frac{3421}{9}$

∴ No. of operations per day = $\frac{3421}{6.46 \times 9}$ = 58.84

$$EST = \frac{3421.J}{58.84\left[C.T_1.S + T_2.T_3.(J-S)\right]}$$

where, C - the crane capacity on existing jetties

= 3.0 tons

T_1 - the number of cranes on existing jetties

T_2 - the capacity of cranes on proposed jetties

T_3 - the number of cranes on proposed jetties

$$SS = 365.J.\frac{58.84\left[C.T_1.S + T_2.T_3.(J-S)\right]}{3421.J}$$

$$= 6.278\left[C.T_1.S + T_2.T_3(J-S)\right]$$

Hence the final model is

$$
NIR = \begin{cases} \left[6.278\left\{C.T_1.S + T_2.T_3.(J-S)\right\} - \frac{S.365}{M}\right] R - (E_J + E_W + E_{JC} + E_{MC} + E_{PL}) & \text{when } SE \leq SS \leq SA \\ \frac{365.R}{M}(\lambda M - S) - (E_J + E_W + E_{JC} + E_{MC} + E_{FL}) & \text{when } SE \leq SA \leq SS \\ 0 & \text{otherwise} \end{cases}
$$

where, λ = 2.03, 2.04, 2.05 ... ... 4.77 ships per day (at an interval of 0.01)

J = 15, 16, 17, 18, 19, 20

T_1 = 3, 4 for every J

T_2 = 3, 3.5, 4, 4.5, 5, 5.5, 6 for every T_1

T_3 = 1, 2, 3, 4 for every T_2

The above model calculates the average net incremental revenue for a particular value of an arrival rate with different combinations of number of jetties, number of cranes and crane capacities. Existing jetties were constructed for 3-tons crane. Because of thereasons explained in the preceding section, higher capacity cranes can not be installed on these jetties. However, considering the length of the existing jetty (180 m), available cargo handling space and crane movement, this model assumes that one more crane having the same capacity may be installed on each existing jetty. But for additional jetties various combinations of crane capacities and number of cranes have been considered in the model. In view of different practical considerations, the capacity of a crane of each additional jetty has been limited from 3 tons to 6 tons.

7.4 Results

By using all possible combinations of the number of jetties, number of cranes and crane capacities for the specified range of arrival rates, the average net incremental revenues (NIR) have been calculated through the computer programming, given in Appendix A-13. The results are partially shown in table 7.3. The results refer to the optimum values of the average net incremental revenues with respect to different number of jetties and mean arrival rates. The more specific information may be obtained from table 7.4 which gives the optimum net incremental revenues and also the corresponding optimum combinations of crane capacities, number of cranes and number of jetties for different arrival rates.

Figure 7.1 has been drawn using the table 7.3. However, the solid line which refers to the optimum values of the revenues can also be drawn by using table 7.4.

Table 7.4a is an extract from the table 7.4. It is observed that the optimal possible incremental revenue earned by any number of jetties occurs at their maximum capacity. This is quite expected because the

Arrival rate	S = 15	S = 16	S = 17	S = 18	S = 19	S = 20
	Optimum net incremental revenue					
2.43	-	-	-	-	-	-
2.44	1.032	-	-	-	-	-
2.45	2.480	-	-	-	-	-
.						
.						
.						
2.49	9.716	-	-	-	-	-
2.50	11.163	0.346	-	-	-	-
2.51	13.092	1.791	-	-	-	-
.						
.						
.						
2.66	39.625	24.558	-	-	-	-
2.67	41.554	25.523	0.210	-	-	-
2.68	43.002	26.913	1.657	-	-	-
.						
.						
.						
2.84	71.464	40.936	25.448	-	-	-
2.85	72.911	42.383	26.895	1.038	-	-
2.86	74.841	44.313	28.825	2.967	-	-
.						
.						
.						
3.01	101.373	70.845	51.528	25.270	-	-
3.02	102.820	72.292	51.528	26.717	0.419	-
3.03	104.750	74.222	52.862	28.647	2.034	-

Table 7.3 : Net incremental revenue for different mean arrival rate for different jetties

Arrival rate	S = 15	S = 16	S = 17	S = 18	S = 19	S = 20
	Optimum net incremental revenue					
3.19	116.328	102.201	71.563	50.978	24.688	–
3.20	116.328	104.131	73.602	52.908	26.618	1.730
3.21	116.328	106.061	75.533	54.837	28.548	3.659
⋮						
3.26	116.328	114.744	84.216	62.073	37.231	9.920
3.27	116.328	116.674	86.145	62.578	39.161	11.950
3.28	116.328	117.638	87.593	64.025	40.608	13.297
⋮						
3.68	116.328	152.350	151.438	126.043	99.678	74.474
3.69	116.328	152.350	153.368	127.972	101.608	76.404
3.70	116.328	152.350	154.815	129.419	103.055	77.851
⋮						
4.09	116.328	152.350	187.890	187.106	159.987	136.378
4.10	116.328	152.350	187.890	188.554	161.434	137.825
4.11	116.328	152.350	187.890	190.483	163.364	139.755
⋮						
4.51	116.328	152.350	187.890	223.913	220.051	196.145
4.52	116.328	152.350	187.890	223.913	223.981	198.075
4.53	116.328	152.350	187.890	223.913	225.428	199.522
⋮						
4.92	116.328	152.350	187.890	223.913	259.936	258.613
4.93	116.328	152.350	187.890	223.913	259.936	260.060
4.94	116.328	152.350	187.890	223.913	159.936	261.989

Table 7.3 : Net incremental revenue for different mean arrival rate for different jetties

Table - 7.4

Arrival Rate (ships/day)	Optimum net incremental revenue (Million Tk)	COMBINATION OF FACILITIES — Existing Jetty: Crane capacity (ton)	Existing Jetty: Crane No. (Qty)	Additional Jetty: Addl. Jetty	Additional Jetty: Crane capacity (ton)	Additional Jetty: Crane No. (Qty)
2.03		3.0	3.0			
2.04		3.0	3.0			
2.05		3.0	3.0			
2.06		3.0	3.0			
2.07		3.0	3.0			
2.08		3.0	3.0			
2.09		3.0	3.0			
2.10		3.0	3.0			
2.11		3.0	3.0			
2.12		3.0	3.0			
2.13		3.0	3.0			
2.14		3.0	3.0			
2.15		3.0	3.0			
2.16		3.0	3.0			
2.17		3.0	3.0			
2.18		3.0	3.0			
2.19		3.0	3.0			
2.20		3.0	3.0			
2.21		3.0	3.0			
2.22		3.0	3.0			
2.23		3.0	3.0			
2.24		3.0	3.0			
2.25		3.0	3.0			
2.26		3.0	3.0			
2.27		3.0	3.0			
2.28		3.0	3.0			
2.29		3.0	3.0			

Table – 7.4 (Continued)

Arrival Rate (ships/day)	Optimum net incremental revenue (Million Tk)	COMBINATION OF FACILITIES: Existing Jetty: Crane capacity (ton)	Existing Jetty: Crane No. (Qty)	Additional Jetty: Addl. Jetty	Additional Jetty: Crane capacity (ton)	Additional Jetty: Crane No. (Qty)
2.30		3.0	3.0			
2.31		3.0	3.0			
2.32		3.0	3.0			
2.33		3.0	3.0			
2.34		3.0	3.0			
2.35		3.0	3.0			
2.36		3.0	3.0			
2.37		3.0	3.0			
2.38		3.0	3.0			
2.39		3.0	3.0			
2.40		3.0	3.0			
.41		3.0	3.0			
2.42		3.0	3.0			
2.43		3.0	3.0			
2.44	1.032	3.0	4.0			
2.45	2.480	3.0	4.0			
2.46	4.409	3.0	4.0			
2.47	6.309	3.0	4.0			
2.48	7.786	3.0	4.0			
2.49	9.716	3.0	4.0			
2.50	11.645	3.0	4.0			
2.51	13.092	3.0	4.0			
2.52	15.022	3.0	4.0			
2.53	16.469	3.0	4.0			
2.54	18.399	3.0	4.0			
2.55	20.329	3.0	4.0			
2.56	21.776	3.0	4.0			
2.57	23.705	3.0	4.0			

Table - 7.4 (Continued)

Arrival Rate (ships/day)	Optimum net incremental revenue (Million Tk)	COMBINATION OF FACILITIES: Existing Jetty: Crane capacity (ton)	Existing Jetty: Crane No. (Qty)	Additional Jetty: Addle Jetty	Additional Jetty: Crane capacity (ton)	Additional Jetty: Crane No. (Qty)
2.58	25.635	3.0	4.0			
2.59	27.082	3.0	4.0			
2.60	29.012	3.0	4.0			
2.61	30.942	3.0	4.0			
2.62	32.359	3.0	4.0			
2.63	34.318	3.0	4.			
2.64	36.248	3.0	4.0			
2.65	37.655	3.0	4.0			
2.66	39.625	3.0	4.0			
2.67	41.554	3.0	4.0			
2.68	43.002	3.0	4.0			
2.69	44.931	3.0	4.0			
2.70	46.379	3.0	4.0			
2.71	48.308	3.0	4.0			
2.72	50.238	3.0	4.0			
2.73	51.685	3.0	4.0			
2.74	53.615	3.0	4.0			
2.75	55.544	3.0	4.0			
2.76	56.992	3.0	4.0			
2.77	58.921	3.0	4.0			
2.78	60.851	3.0	4.0			
2.79	62.298	3.0	4.0			
2.80	64.228	3.0	4.0			
2.81	66.157	3.0	4.0			
2.82	67.604	3.0	4.0			
2.83	69.534	3.0	4.0			
2.84	71.464	3.0	4.0			

Table 7.4 (Continued)

Arrival Rate (ships/ day)	Optimum net incremental revenue (Million Tk)	COMBINATION OF FACILITIES				
		Existing Jetty		Additional Jetty		
		Crane capacity (ton)	Crane No. (Qty)	Addl. Jetty	Crane capacity (ton)	Crane No. (Qty)
2.85	72.911	3.0	4.0			
2.86	74.841	3.0	4.0			
2.87	76.770	3.0	4.0			
2.88	78.217	3.0	4.0			
2.89	80.147	3.0	4.0			
2.90	81.594	3.0	4.0			
2.91	83.524	3.0	4.0			
2.92	85.454	3.0	4.0			
2.93	86.901	3.0	4.0			
2.94	88.830	3.0	4.0			
2.95	90.760	3.0	4.0			
2.96	92.207	3.0	4.0			
2.97	94.137	3.0	4.0			
2.98	96.066	3.0	4.0			
2.99	97.514	3.0	4.0			
3.00	99.443	3.0	4.0			
3.01	101.373	3.0	4.0			
3.02	102.820	3.0	4.0			
3.03	104.750	3.0	4.0			
3.04	106.679	3.0	4.0			
3.05	108.127	3.0	4.0			
3.06	110.056	3.0	4.0			
3.07	111.986	3.0	4.0			
3.08	113.433	3.0	4.0			
3.09	115.363	3.0	4.0			
3.10	115.378	3.0	4.0			
3.11	116.328	3.0	4.0			
3.12	115.329	3.0	4.0			

Table 7.4 (Continued)

Arrival Rate (Ships/day)	Optimum net incremental revenue (Million Tk)	COMBINATION OF FACILITIES				
		Existing Jetty		Additional Jetty		
		Crane capacity (ton)	Crane No. (Qty)	Addl. Jetty	Crane capacity (ton)	Crane No. (Qty)
3.13	116.328	3.0	4.0			
3.14	115.328	3.0	4.0			
3.15	116.328	3.0	4.0			
3.16	116.328	3.0	4.0			
3.17	116.328	3.0	4.0			
3.18	116.328	3.0	4.0			
3.19	116.328	3.0	4.0			
3.20	116.328	3.0	4.0			
3.21	116.328	3.0	4.0			
3.22	116.328	3.0	4.0			
3.23	116.328	3.0	4.0			
3.24	115.328	3.0	4.0			
3.25	116.328	3.0	4.0			
3.26	116.328	3.0	4.0			
3.27	116.674	3.0	4.0	1	3.5	2.0
3.28	117.638	3.0	4.0	1	3.5	3.0
3.29	119.566	3.0	4.0	1	3.0	4.0
3.30	121.456	3.0	4.0	1	3.0	4.0
3.31	122.017	3.0	4.0	1	3.5	4.0
3.32	123.547	3.0	4.0	1	3.5	4.0
3.33	125.394	3.0	4.0	1	3.5	4.0
3.34	126.841	3.0	4.0	1	3.5	4.0
3.35	128.328	3.0	4.0	1	4.0	4.0
3.36	129.775	3.0	4.0	1	4.0	4.0
3.37	131.705	3.0	4.0	1	4.0	4.0
3.38	132.677	3.0	4.0	1	4.5	4.0
3.39	134.124	3.0	4.0	1	4.5	4.0

Table - 7.4 (Continued)

Arrival Rate (ships/day)	Optimum net incremental revenue (Million Tk)	COMBINATION OF FACILITIES: Existing Jetty — Crane capacity (ton)	Existing Jetty — Crane No. (Qty)	Additional Jetty — Addl. Jetty	Additional Jetty — Crane capacity (ton)	Additional Jetty — Crane No. (Qty)
3.40	135.054	3.0	4.0	1	4.5	4.0
3.41	137.026	3.0	4.0	1	5.0	4.0
3.42	138.473	3.0	4.0	1	5.0	4.0
3.43	140.[illegible]	3.0	4.0	1	5.0	4.0
3.44	142.333	3.0	4.0	1	5.0	4.0
3.45	142.758	3.0	4.0	1	5.5	4.0
3.46	144.688	3.0	4.0	1	5.5	4.0
3.47	146.618	3.0	4.0	1	5.5	4.0
3.48	147.100	3.0	4.0	1	5.5	4.0
3.49	148.973	3.0	4.0	1	6.0	4.0
3.50	150.903	3.0	4.0	1	6.0	4.0
3.51	152.350	3.0	4.0	1	6.0	4.0
3.52	152.350	3.0	4.0	1	6.0	4.0
3.53	152.390	3.0	4.0	1	6.0	4.0
3.54	152.350	3.0	4.0	1	6.0	4.0
3.55	152.350	3.0	4.0	1	6.0	4.0
3.56	152.[illegible]	3.0	4.0	1	6.0	4.0
3.57	152.390	3.0	4.0	1	6.0	4.0
3.58	152.350	3.0	4.0	1	6.0	4.0
3.59	152.350	3.0	4.0	1	6.0	4.0
3.60	152.390	3.0	4.0	1	6.0	4.0
3.61	152.390	3.0	4.0	1	6.0	4.0
3.62	152.350	3.0	4.0	1	6.0	4.0
3.63	152.350	3.0	4.0	1	6.0	4.0
3.64	152.350	3.0	4.0	1	6.0	4.0
3.65	152.350	3.0	4.0	1	6.0	4.0
3.66	152.330	3.0	4.0	1	6.0	4.0
3.67	152.350	3.0	4.0	1	6.0	4.0

Table - 7.4 (Continued)

Arrival Rate (ships/day)	Optimum net incremental revenue (Million Tk)	COMBINATION OF FACILITIES: Existing Jetty — Crane capacity (ton)	Existing Jetty — Crane No. (Qty)	Additional Jetty — Addl. Jetty	Additional Jetty — Crane capacity (ton)	Additional Jetty — Crane No. (Qty)
3.58	152.350	3.0	4.0	1	6.0	4.0
3.69	153.368	3.0	4.0	2	4.5	4.0
3.70	154.815	3.0	4.0	2	4.5	4.0
3.71	156.745	3.0	4.0	2	4.5	4.0
3.72	157.710	3.0	4.0	2	4.5	4.0
3.73	159.207	3.0	4.0	2	5.0	4.0
3.74	160.137	3.0	4.0	2	5.0	4.0
3.75	162.066	3.0	4.0	2	5.0	4.0
3.76	163.513	3.0	4.0	2	5.0	4.0
3.77	165.443	3.0	4.0	2	5.0	4.0
3.78	167.373	3.0	4.0	2	5.0	4.0
3.79	167.855	[illegible]	4.0	2	[illegible]	[illegible]
3.80	169.737	3.0	4.0	2	5.5	4.0
3.81	170.636	3.0	4.0	2	5.5	4.0
3.82	172.084	3.0	4.0	2	5.5	4.0
3.83	174.013	3.0	4.0	2	5.5	4.0
3.84	175.943	3.0	4.0	2	5.5	4.0
3.85	177.390	3.0	4.0	2	5.5	4.0
3.86	177.872	3.0	4.0	2	5.5	4.0
3.87	179.207	3.0	4.0	2	6.0	4.0
3.88	180.654	3.0	4.0	2	6.0	4.0
3.89	182.594	3.0	4.0	2	6.0	4.0
3.90	184.513	3.0	4.0	2	6.0	[illegible]
3.91	185.960	3.0	4.0	2	6.0	4.0
3.92	187.390	3.0	4.0	2	6.0	4.0
3.93	187.390	3.0	4.0	2	6.0	4.0
3.94	187.890	3.0	4.0	2	6.0	4.0

Table - 7.4 (Continued)

Arrival Rate (ships/day)	Optimum net incremental revenue (Million Tk)	COMBINATION OF FACILITIES: Existing Jetty — Crane capacity (ton)	Existing Jetty — Crane No. (Qty)	Additional Jetty — Addl. Jetty	Additional Jetty — Crane capacity (ton)	Additional Jetty — Crane No. (Qty)
3.95	187.890	3.0	4.0	2	6.0	4.0
3.96	187.890	3.0	4.0	2	6.0	4.0
3.97	187.890	3.0	4.0	2	6.0	4.0
3.98	187.890	3.0	4.0	2	6.0	4.0
3.99	187.890	3.0	4.0	2	6.0	4.0
4.00	187.890	3.0	4.0	2	6.0	4.0
4.01	187.890	3.0	4.0	2	6.0	4.0
4.02	187.890	3.0	4.0	2	6.0	4.0
4.03	187.890	3.0	4.0	2	6.0	4.0
4.04	187.890	3.0	4.0	2	6.0	4.0
4.05	187.890	3.0	4.0	2	6.0	4.0
4.06	187.890	3.0	4.0	2	6.0	4.0
4.07	187.890	3.0	4.0	2	6.0	4.0
4.08	187.890	3.0	4.0	2	6.0	4.0
4.09	187.890	3.0	4.0	2	6.0	4.0
4.10	189.336	3.0	4.0	3	5.0	4.0
4.11	190.633	3.0	4.0	3	5.0	4.0
4.12	192.413	3.0	4.0	3	5.0	4.0
4.13	193.850	3.0	4.0	3	5.0	4.0
4.14	193.860	3.0	4.0	3	5.0	4.0
4.15	194.655	3.0	4.0	3	5.5	4.0
4.16	196.103	3.0	4.0	3	5.5	4.0
4.17	198.032	3.0	4.0	3	5.5	4.0
4.18	199.962	3.0	4.0	3	5.5	4.0
4.19	201.409	3.0	4.0	3	5.5	4.0
4.20	203.339	3.0	4.0	3	5.5	4.0
4.21	205.268	3.0	4.0	3	5.5	4.0
4.22	206.715	3.0	4.0	3	5.5	4.0

Table - 7.4 (Continued)

Arrival Rate (Ships/ day)	Optimum net incremental revenue (Million Tk)	COMBINATION OF FACILITIES				
		Existing Jetty		Additional Jetty		
		Crane capacity (ton)	Crane No. (Qty)	Addl. Jetty	Crane capacity (ton)	Crane No. (Qty)
4.23	208.645	3.0	4.0	3	5.5	4.0
4.24	208.645	3.0	4.0	3	5.5	4.0
4.25	208.958	3.0	4.0	3	6.0	4.0
4.26	210.888	3.0	4.0	3	6.0	4.0
4.27	212.817	3.0	4.0	3	6.0	4.0
4.28	214.265	3.0	4.0	3	6.0	4.0
4.29	216.194	3.0	4.0	3	6.0	4.0
4.30	218.124	3.0	4.0	3	6.0	4.0
4.31	219.571	3.0	4.0	3	6.0	4.0
4.32	221.501	3.0	4.0	3	6.0	4.0
4.33	222.948	3.0	4.0	3	6.0	4.0
4.34	223.913	3.0	4.0	3	6.0	4.0
4.35	223.913	3.0	4.0	3	6.0	4.0
4.36	223.913	3.0	4.0	3	6.0	4.0
4.37	223.913	3.0	4.0	3	6.0	4.0
4.38	223.913	3.0	4.0	3	6.0	4.0
4.39	223.913	3.0	4.0	3	6.0	4.0
4.40	223.913	3.0	4.0	3	6.0	4.0
4.41	223.913	3.0	4.0	3	6.0	4.0
4.42	223.913	3.0	4.0	3	6.0	4.0
4.43	223.913	3.0	4.0	3	6.0	4.0
4.44	223.913	3.0	4.0	3	6.0	4.0
4.45	223.913	3.0	4.0	3	6.0	4.0
4.46	223.913	3.0	4.0	3	6.0	4.0
4.47	223.913	3.0	4.0	3	6.0	4.0
4.48	223.913	3.0	4.0	3	6.0	4.0
4.49	223.913	3.0	4.0	3	6.0	4.0

Arrival Rate (ships/day)	Optimum net incremental revenue (Million Tk)	COMBINATION OF FACILITIES				
		Existing Jetty		Additional jetty		
		Crane capacity (ton)	Crane No. (Qty)	Addl. jetty	Crane capacity (ton)	Crane No. (Qty)
4.50	223.913	3.0	4.0	3	6.0	4.0
4.51	223.913	3.0	4.0	3	6.0	4.0
4.52	223.931	3.0	4.0	4	5.5	4.0
4.53	225.425	3.0	4.0	4	5.5	4.0
4.54	227.357	3.0	4.0	4	5.5	4.0
4.55	229.287	3.0	4.0	4	5.5	4.0
4.56	230.754	3.0	4.0	4	5.5	4.0
4.57	232.564	3.0	4.0	4	5.5	4.0
4.58	234.594	3.9	4.0	4	5.5	4.0
4.59	236.041	3.0	4.0	4	5.5	4.0
4.60	237.970	3.0	4.0	4	5.5	4.0
4.61	239.418	3.0	4.0	4	5.5	4.0
4.62	237.418	3.0	4.0	4	5.5	4.0
4.63	239.418	3.0	4.0	4	5.5	4.0
4.64	241.121	3.0	4.0	4	6.0	4.0
4.65	242.569	3.0	4.0	4	6.0	4.0
4.66	244.498	3.0	4.0	4	6.0	4.0
4.67	246.428	3.0	4.0	4	6.0	4.0
4.68	247.875	3.0	4.0	4	6.0	4.0
4.69	249.305	3.0	4.0	4	6.0	4.0
4.70	251.734	3.0	4.0	4	6.0	4.0
4.71	253.131	3.0	4.0	4	6.0	4.0
4.72	255.111	3.0	4.0	4	6.0	4.0
4.73	256.558	3.0	4.0	4	6.0	4.0
4.74	258.436	3.0	4.0	4	6.0	4.0
4.75	259.935	3.0	4.0	4	6.0	4.0
4.76	259.935	3.0	4.0	4	6.0	4.0
4.77	259.935	3.0	4.0	4	6.0	4.0

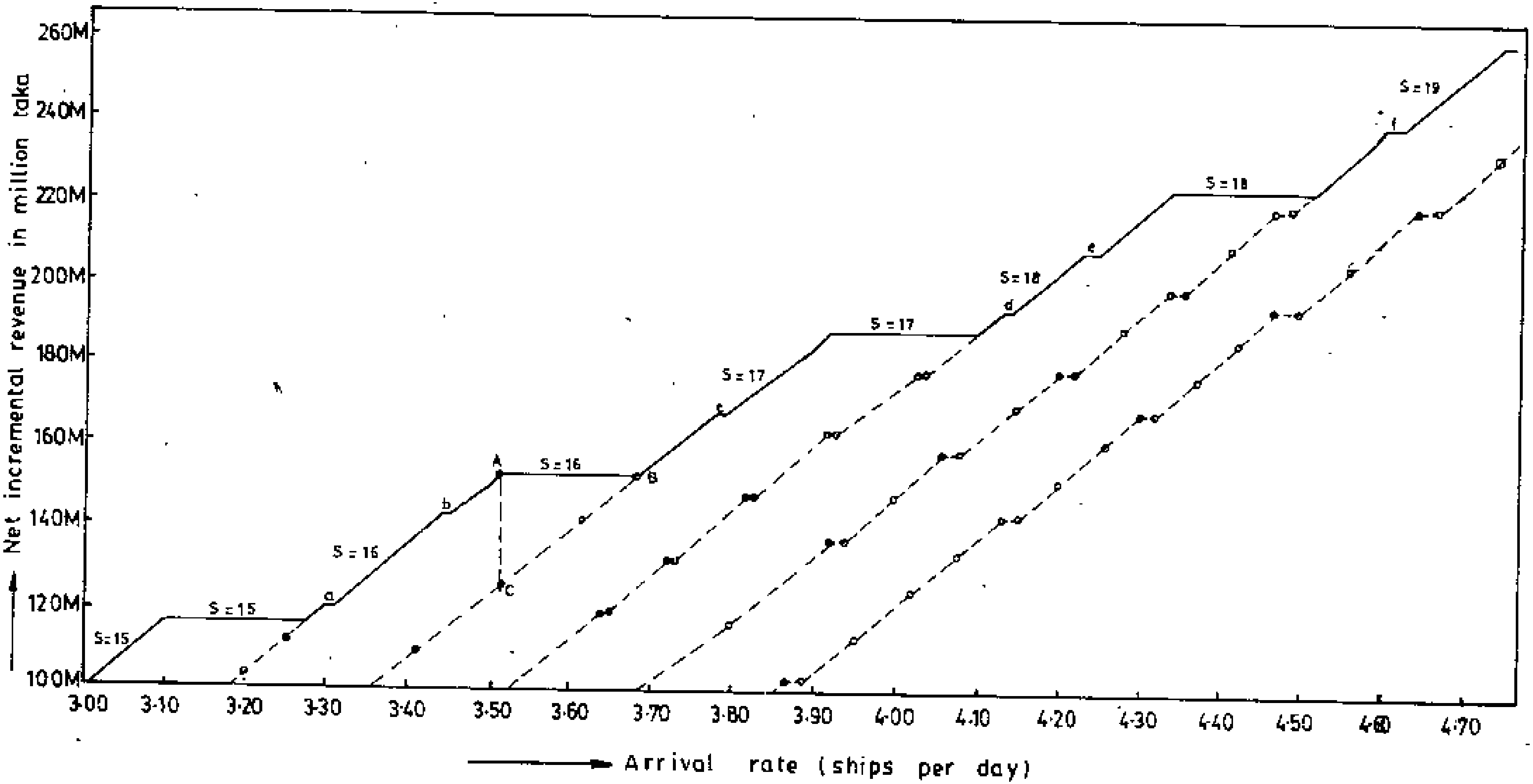

Net incremental revenue in million taka
260M
240M
220M
200M
180M
160M
140M
120M
100M
3·00
3·10
3·20
3·30
3·40
3·50
3·60
3·70
3·80
3·90
4·00
4·10
4·20
4·30
4·40
4·50
4·70
S=15
S = 15
S = 16
S = 16
S = 17
S = 17
S = 18
S = 18
S = 19
a
b
c
d
e
f
A
B
C
Arrival rate (ships per day)

Fig. 7.1

Number of jetty	Optimum net incremental revenue	COMBINATION OF FACILITIES			
		EXISTING JETTY		ADDITIONAL JETTY	
		Crane capacity	Number of crane	Crane capacity	Number of crane
15	116,328,000	3	4	6	4
16	152,350,000	3	4	6	4
17	187,890,000	3	4	6	4
18	223,913,000	3	4	6	4
19	259,935,000	3	4	6	4
20	295,475,000	3	4	6	4

Table 7.4a

cost of a jetty is much more than that of its facilities. That is why it is most unlikely that an additional jetty will be justified before all the facilities of the existing jetties are fully utilized. In fact, the incremental revenue earned by certain number of jetties when they function at their maximum capacity may be much higher than what would be obtained if an additional jetty is added at that point. The fact can be better explained referring to figure 7.1. At point A, the 16 jetties function at their maximum capacity i.e. 4 cranes of 6 tons capacity. The net incremental revenue at this point is Tk.152,350,000.- and the arrival rate is 3.51 ships per day. But at the same arrival rate of 3.51 ships per day, if an additional jetty is added to the system, then the corresponding net incremental revenue falls to Tk.127,146,000.- (point C on the figure). No additional jetty will be justified until the point B is reached where the arrival rate is 3.685 ships per day and the net incremental revenue earnings are same for both 16 and 17 jetties. From A to B, 16 jetties are functioning at the same capacity (maximum capacity) and hence the revenue earnings also remain same.

For a particular arrival rate, the optimum revenue earning is unique and the combination of facilities is also unique. But at different arrival rates, for the same combination the revenue earnings may be different. This is shown in table 7.4b which is an extract from table 7.4. This is because when the capacity of a facility with a particular

Arrival rate	Optimum net incremental revenue	COMBINATION OF FACILITIES				
		EXISTING JETTY		ADDITIONAL JETTY		
		Crane capacity	No. of cranes	Nunmber of additional jetty	Crane capacity	Number of cranes
3.69	153,358,000	3	4	2	4.5	4
3.70	154,815,000	3	4	2	4.5	4
3.71	156,745,000	3	4	2	4.5	4
3.72	157,710,000	3	4	2	4.5	4
3.73	158,207,000	3	4	2	5.0	4

Table 7.4b

combination is higher than the arrival, then this combination needs not be changed until the arrival reaches the maximum capacity. In other words, combination remains same until the facility is fully utilized. That is why, at different arrival rates, for the same combination the revenue earnings may be different. For example, the arrival rates from 3.69 to 3.72 ships per day, combination remains same (4 cranes of 3 ton capacity on each existing jetty and 4. cranes of 4.5 capacity on each additional jetty). But during that period net incremental revenue increases from Tk.153,358,000 to Tk.157,710,000. For an arrival rate of 3.72 ships per day, the above combination reaches its maximum capacity. That is why when the arrival rate is 3.73 ships per day, the combination changes (4 cranes of 3 tons capacity on each existing jetty and 4 cranes of 5 tons capacity on each additional jetty).

For the same jetty at different arrival rates, both the combination and the corresponding revenue earnings may remain constant. This condition is explained by table 7.4c which is an extract from table 7.4. It is observed from the table that for the arrival rates from 4.61 to 4.63 ships

Arrival rate	Optimum net incremental revenue	COMBINATION OF FACILITIES				
		EXISTING JETTY		ADDITIONAL JETTY		
		Crane capacity	No. of cranes	Number of additional jetty	Crane capacity	Number of cranes
4.61	239,418,000	3	4	4	5.5	4
4.62	239,418,000	3	4	4	5.5	4
4.63	239,418,000	3	4	4	5.5	4

Table 7.4c

per day, both the combination (4 cranes of 3 tons capacity on existing jetty and 4 cranes of 5.5 tons capacity on each additional jetty) and the corresponding net incremental revenue are same. This is because the net incremental revenue due to higher arrival can not justify to provide combination of higher capacity. This situation is reflected on figure 7.1 by the horizontal portions (a, b, c, d etc.) of the inclined lines.

CHAPTER EIGHT

CONCLUSIONS AND FUTURE SCOPE OF WORK

8.0 CONCLUSION & FUTURE SCOPE OF WORK

8.1 Conclusion

The objective of the present work was to study the different system parameters of the Chittagong Port, select appropriate queueing models for the system and develop a cost model which could be used to determine the optimum service facilities to be provided at the port in order to maximize the net incremental revenue at varying environmental and service conditions.

By using five years' data (January 1980 to December 1984) obtained from the traffic department of Chittagong Port, different system parameters are estimated and their behaviours (distributions) are identified (Table 3.2).

The mathematical queueing model which has been found appropriate for the existing system is a multiple-servers queueing system with infinite system capacity and calling population and having a service discipline of first come first served. The model has a poisson input process and exponential service time distribution.

Diffusion approximation method which is effective when utilization factor, ρ is at least equal to 0.7 has been successfully applied for the present system for which $\rho = 0.874$.

Simulation model has also been developed.

For the system under study, the mathematical model, diffusion approximation model and the simulation model have been found equally applicable. The values of system parameters obtained by these three models are almost same and slightly differ from the observed values, as shown in tables (4.1, 5.1, 6.11). These variations are mainly due to tide and draft restrictions which are not incorporated in any of the above models.

When the system does not suffer from any limitations and the input process and service time distribution are known the mathematical model will always provide the best result. When the system does not satisfy the general assumptions made for the mathematical model, the diffusion approximation method will provide a very good approximation to the system parameters provided the utilization factor, 0.7. When the system has different limitations and service conditions which can not be incorporated either in the mathematical model or in the diffusion approximation model, simulation will be the best method to calculate the system parameters. With the development of computer technology, Monte Carlo Simulation will finds its wide applicability in queueing problems.

Aspiration level models are also developed for the system for different conflicting measures such as mean waiting time in queue and the server's idle time which the decision maker wishes to balance. These models make direct use of the operating characteristics of the system in deciding the optimum values of the design parameters.

The cost model which is developed on the basis of estimated revenue earned per ship and cost of facilities, suggests that under the present situation of the port, improvement of facilities is not required. However, if the traffic demand which is expected to increase in future the findings of the model (Table 7.4) will serve as a good analytical tool for management to take appropriate decisions about the adequate service facilities to be provided at different arrival rates of the ships and thus solve the possible marine congestion problems at the port.

8.2 Future Scope of Works

i) At present the system has limitations on the movement of ships from outer anchorage to jetty due to tide and draft restrictions. Due to non-availability of required data, these restrictions could not be incorporated in the present models. Variations of tide-height with respect to time are available in the 'tide table'. If in the 'ship movement form', the exact time of arrival of the ship and the time at which cargo handling ends are properly recorded, then the waiting times of the ships both at arrival and departure due to tide and draft restrictions could be easily ascertained. If distributions of the above waiting times are known then the effective arrival rate and service rate (due to tide and draft restrictions) can be easily estimated. Appropriate queueing models can then be applied to get better estimates of the system parameters.

ii) In the present study, the priority discipline is not considered. But in practice, priority is given to the vessels carrying food grains . So the system may be studied considering the above priority.

iii) Apart from rendering services to the ships which are connected with export-import activities, the Chittagong Port occassionally offers services to the vessels carrying cargo within the country. The internal cargo handling is not included in the present work. The system may be studied considering the above service.

iv) During data analysis a study may be carried out to check whether there is any seasonal effect on the arrival of ships at the port. If any such effect is identified the cost model should be modified accordingly. This will help management to take decision in providing appropriate facilities at the port on a monthwise basis.

v) Sensitivity test of the cost model may be carried out to see the effects of changes of values of different cost components on optimum net incremental revenue. This will provide management more flexibility in decision making.

REFERENCES

1. Bhat, U.N., Shalaby, M, and Fischer, M.J.: Approximation techniques in the solution of queueing problems. Naval Research Logistics Quarterly, Vol-26, No.2 (1979), 311-326

2. Biswas, S.K.: Study on the diffusion approximation method for the solutions of multiserver queueing systems, Dr. Engg. Thesis, Kyushu University, Japan, November, 1980

3. Blaker Bolling, W: Queueing model of a hospital emergency room, Industrial Engineering, September, 1972, 26-31

4. Cosmetatos, G.P.: Approximate equilibrium results for the multi-server queue (GI/M/r), Operational Research Quarterly, Vol-25, No-4 (1974), 625-634

4a. Gass, S.I.: Evaluation of Complex Model, Computer Operations Research, 4: 25-37 (1977)

5. Gaver, D.P.Jr: Diffusion Approximation and models for certain congestion problems, Journal of Applied Probability, Vol-5 (1968), 607-623

6. Gelenbe, E: On approximate computer system models. Journal of the Association of Computing Machinery, Vol-22, No.2 (1975), 261-269

7. Gross, D and Harris, M.: Fundamental of Queueing Theory, John Wiley and Sons, 1975

8. Gupta, P.K. and Hira, D.S.: Operation Research - and Introduction, S. Chand & Co. Ltd., New Delhi, 1983

9. Halachmi, B and Franta, W.R.: A diffusion approximation to multi-server queue, Management Science, Vol-24, No.5, (1978), 522-529

REFERENCES

10. Halachmi, B and Franta, W.R.: A diffusion approximate solution to the G/G/K Queueing system, Computers and Operation Research, Vol-4 (1977), 37-46

10a. Halton, J.H.: A Restrospective and Prospective Survey of the Monte Carlo Method, SIAM Review, 12: 1-63 (1970)

10b. Hammersley, J.M., and D.C. Handscomb : Monte Carlo Methods, Methuen, London, 1964

11. Heyman, D.P.: An approximation for the busy period of the M/G/1 Queueing using a diffusion model, Journal of Applied Probability, Vol-11 (1974), 159-169

12. Hiller, F.S. and Lieberman, G.J.: Operations Research, Holden, Day Inc.

13. Iglehart, D.L.: Limiting diffusion approximations for the many server queue and the repairman problem, Journal of Applied Probability, 2(1965), 429-441

14. Kimura, T., Ohno, K and Mine, H.: Diffusion approximation for GI/G/1 queueing systems with finite capacity: II- The stationary behaviour, Journal of the Operations Research Society of Japan, Vol-22, No. 4 (1979), 301-319

14a Kingman, J.F.C.: On queues in heavy traffic. Journal of the Royal Statistical Society, Vol-B 24 (1962), 383-392

14b Kingman, J.F.C : In equalities in the theory of queus. Journal of the Royal Statistical Society, Vol-B 32 (1970) 102-110

15. Kleinrock, L.: Queueing systems Vol-II, Computer applications, John Wiley and Sons (1976), Chapter 2.

R E F E R E N C E S

16. Kobayashi, H.: Application of diffusion approximation of queueing networks I: Equilibrium queue distributions, Journal of the Association of computing Machinery, Vol-21, No. 2 (1974), 310-328

17. Maaloe, E.: Approximation formulae for estimation of waiting time in multiple channel queueing system. Management Science, Vol-19, Applied Series (1973), 703-710

17a. Naylor, T.H. and J.M. Finger: Verification of Computer Simulation Models. Management Science - 14: 92-101 (1967)

18. Newell, G.F.: Queues with time dependent arrival rates I-III. Journal of Applied Probability, Vol-5 (1968) 436-451, 479-606.

19. Newell, G.F.:Applications of queueing theory. Chapman and Hall, London (1971).

19a. Page, E.: Queueing Theory in O.R. Butterworths, London (1972)

19c Schellenberger, R.E.: Criteria of Assessing Model Validity for Managerial Purposes. Decision Science - 5: 644-653 (1974)

19d Shannon, R.E.: Systems Simulation: The Art and Science, Prentice Hall, Englewood Cliffs, N.J., 1975

20 Sen, Pratysh : Optimal Priority assignment in queues: application to Marine congestion problems. MARIT, POL, MGMT, Vol-7, No. 3 (1980) 175-184

21 Sunaga, T, Kondo, E and Biswas, S.K.: An applixation method using continuous models for queueing problems. Journal of the Operations Research Society of Japan, Vol-21, No. 1 (1978), 29-42.

R E F E R E N C E S

22 Taha, H.A.: Operations Research - An Introduction, Macmillan Publishing Co. Inc. 1982

22a Van Horn, R.L.: Validation of Simulation Results, Management Science, 17: 247-258 (1971)

23 Walpole, R.E. and Myers, R.H.: Probability and Statistics for Engineers and Scientists. Macmillan Publishing Co. Inc. 1978

24 Year Book 198384 : The Chittagong Port Authority, Bandar Bhaban, Chittagong.

25 Personal Contact with M/S. Shahidullah & Associates, Dhaka; Chief Engineer, BSEC, Dhaka; Civil Engineering Department, BUET, Dhaka; Foreign Machinery Suppliers at Dhaka.

APPENDIX A-1

Year-wise history of arrival of ship

Arrival of ship per day	Observed Frequency					
	1980	1981	1982	1983	1984	Total
0	60	44	48	52	42	246
1	101	105	96	102	98	502
2	85	101	91	107	92	476
3	64	61	60	62	77	324
4	31	31	46	25	34	167
5	21	14	13	12	12	72
6	2	7	7	4	9	29
7	1	-	3	1	2	7
8	1	2	1	-	-	4

APPENDIX A-2

Year-wise history of departure of ship

Departure of ship per day	Observed Frequency					
	1980	1981	1982	1983	1984	Total
0	54	52	41	68	47	262
1	105	107	88	100	89	489
2	85	87	108	91	103	474
3	62	67	68	55	62	314
4	40	30	38	30	40	178
5	13	15	17	14	16	75
6	4	4	3	6	8	25
7	3	3	2	1	-	9
8	-	-	-	-	1	1

APPENDIX A-3

Year-wise history of service time

Service time (day)	Observed Frequency					
	1980	1981	1982	1983	1984	Total
0 – 1	65	70	86	67	66	354
1 – 2	76	96	117	85	81	455
2 – 3	81	73	120	105	108	487
3 – 4	48	83	92	64	76	363
4 – 5	64	64	59	64	60	311
5 – 6	59	57	60	51	57	284
6 – 7	61	50	33	35	46	225
7 – 8	48	34	34	30	49	195
8 – 9	35	39	34	23	40	171
9 – 10	36	26	17	26	35	140
10 – 11	17	19	13	13	24	86
11 – 12	16	24	10	14	15	79
12 – 13	17	12	15	7	12	63
13 – 14	11	12	10	7	19	59
14 – 15	12	13	10	13	8	56
15 – 16	11	9	6	3	13	42
16 – 17	10	7	6	8	9	40
17 – 18	6	2	7	6	8	29
18 – 19	5	8	4	5	5	27
19 – 20	5	5	4	3	11	28
20 – 21	1	3	5	5	3	17
21 – 22	3	4	3	3	3	16
22 – 23	–	1	2	5	6	14
23 – 24	3	4	3	2	5	17
24 – 25	1	2	2	4	2	11
25 – 26	1	2	2	3	5	13
26 – 27	2	–	2	–	2	6
27 – 28	1	4	3	2	5	15
28 – 29	1	–	–	4	–	5
29 – 30	2	–	1	4	4	11

Contd.

Service time (day)	Observed Frequency					
	1980	1981	1982	1983	1984	Total
30 - 31	2	1	1	4	1	9
31 - 32	1	-	1	2	-	4
32 - 33	-	1	2	1	2	6
33 - 34	1	-	-	1	3	5
34 - 35	1	-	-	2	1	4
35 - 36	-	1	-	-	2	3
36 - 37	1	-	-	-	-	1
37 - 38	1	-	-	-	-	1
38 - 39	-	-	1	-	-	1
39 - 40	-	-	-	-	-	0
40 - 41	-	-	-	1	-	1
41 - 42	-	-	-	-	1	1
42 - 43	-	-	1	-	-	1
43 - 44	1	-	-	-	1	2
44 - 45	-	-	-	-	-	0
45 - 46	-	-	-	-	-	0
46 - 47	-	1	-	-	1	2
47 - 48	-	-	-	-	-	0
48 - 49	-	-	1	-	-	1
49 - 50	-	-	-	1	-	1

APPENDIX A-4

Year-wise history of waiting time in queue

Waiting time per ship in queue (day)	Observed Frequency					
	1980	1981	1982	1983	1984	Total
0 - 1	220	280	340	284	240	1354
1 - 2	173	210	223	171	178	955
2 - 3	76	75	91	58	65	365
3 - 4	56	50	45	29	45	225
4 - 5	30	33	21	17	36	137
5 - 6	37	21	21	21	29	129
6 - 7	28	19	13	13	25	98
7 - 8	20	14	6	12	34	86
8 - 9	24	10	3	8	16	61
9 - 10	16	6	2	15	15	54
10 - 11	4	6	5	9	12	36
11 - 12	7	1	2	11	13	34
12 - 13	8	-	2	4	12	26
13 - 14	3	-	1	4	8	16
14 - 15	3	1	2	3	11	20
15 - 16	4	-	1	1	8	14
16 - 17	4	-	-	1	4	9
17 - 18	5	3	2	2	3	15
18 - 19	2	-	-	1	2	5
19 - 20	3	-	2	3	2	10
20 - 21	-	-	-	1	3	4
21 - 22	4	-	-	2	2	8
22 - 23	1	-	-	2	1	4
23 - 24	1	-	-	-	-	1
24 - 25	1	-	-	-	-	1
25 - 26	-	-	-	1	-	1
26 - 27	1	-	-	-	1	2
27 - 28	-	-	-	1	-	1
28 - 29	-	-	-	-	-	0
29 - 30	-	-	-	1	-	1
30 - 31	-	-	-	1	-	1
31 - 32	-	-	-	-	-	0
32 - 33	1	-	-	-	-	1

APPENDIX A-5

Year-wise history of queue length

Queue Length	Observed Frequency					
	1980	1981	1982	1983	1984	Total
0	12	20	27	31	24	114
1	33	62	61	62	32	250
2	31	70	73	65	30	269
3	50	69	59	49	34	261
4	41	47	43	45	20	196
5	26	41	33	20	18	138
6	30	24	24	9	17	104
7	33	15	23	14	12	
8	24	6	10	8	20	68
9	23	6	6	8	26	69
10	14	4	4	4	25	51
11	13	1	-	6	27	47
12	16	-	2	3	20	41
13	6	-	-	9	22	
14	8	-	-	6	19	33
15	1	-	-	7	7	15
16	3	-	-	10	7	20
17	2	-	-	6	4	12
18	-	-	-	3	1	4
19	-	-	-	-	1	1

APPENDIX A-6

Year-wise history of No. of ships in service

No. of ships in service	Observed Frequency					
	1980	1981	1982	1983	1984	Total
3	-	2	-	1	-	3
4	-	6	-	5	1	12
5	2	3	-	10	1	16
6	3	2	2	10	2	19
7	5	6	7	17	1	36
8	17	4	12	31	10	74
9	16	10	28	30	15	99
10	19	19	40	29	19	126
11	36	30	37	47	20	170
12	41	35	43	22	18	159
13	60	46	49	27	23	205
14	53	31	58	26	49	217
15	33	55	37	19	45	189
16	32	55	19	18	44	168
17	18	31	18	10	36	113
18	11	21	10	15	27	84
19	9	8	3	8	31	59
20	5	1	2	2	10	20
21	1	-	-	6	9	16
22	2	-	-	4	1	7
23	-	-	-	6	-	6
24	2	-	-	8	-	10
25	1	-	-	10	-	11
26	-	-	-	2	-	2
27	-	-	-	1	-	1
28	-	-	-	1	-	1

APPENDIX A-7

Year-wise history of No. of ships in the system

No. of ship in the system	Observed Frequency					
	1980	1981	1982	1983	1984	Total
4	-	1	-	-	1	1
5	-	6	-	2	-	8
6	2	4	1	4	1	12
7	1	1	1	9	-	12
8	3	2	6	11	1	23
9	6	3	5	22	3	39
10	5	4	14	19	11	53
11	8	12	21	32	10	83
12	20	26	20	31	21	118
13	25	21	38	28	19	131
14	19	25	32	23	11	110
15	18	22	35	15	14	104
16	28	32	31	21	18	130
17	32	33	33	19	8	125
18	15	33	23	15	10	96
19	20	31	24	12	8	100
20	21	35	34	5	14	109
21	20	24	9	9	18	80
22	17	17	14	8	14	70
23	21	14	7	4	10	56
24	13	6	7	5	11	42
25	16	6	4	5	23	54
26	14	5	1	7	15	42
27	10	2	-	7	21	40
28	8	-	-	5	22	35
29	8	-	-	4	14	26
30	6	-	-	4	10	20
31	4	-	-	-	16	20

APPENDIX A-7 (Continued)

No. of ships in the system	Observed Frequency					
	1980	1981	1982	1983	1984	Total
32	5	-	-	3	12	20
33	1	-	-	3	12	16
34	-	-	-	3	4	7
35	-	-	-	6	5	11
36	-	-	-	5	2	7
37	-	-	-	6	2	8
38	-	-	-	5	-	5
39	-	-	-	4	-	4
40	-	-	-	1	-	1
41	-	-	-	2	-	2
42	-	-	-	1	-	1

APPENDIX A-7 (Continued)

Calculations for estimating γ

To calculate the minimum life or location parameter, γ , which exists, when data plot gives a curve – usually convex from above, the following equation is used.

$$\gamma = t_2 - \frac{(t_3 - t_2)(t_2 - t_1)}{(t_3 - t_2) - (t_2 - t_1)}$$

t_1, t_2, t_3 are shown in figure 3.17 and their values are as follows:

$t_1 = 10$, $t_2 = 15$, $t_3 = 26$.

$$\therefore \gamma = 15 - \frac{(26 - 15)(15 - 10)}{(26 - 15) - (15 - 10)}$$

$$= 15 - \frac{11 \times 5}{11 - 5}$$

$$= 5.83 \text{ ships per day.}$$

APPENDIX A-8 (Continued)

Waiting time in system (Days)	OBSERVED FREQUENCY					
	1980	1981	1982	1983	1984	Total
0 – 1	52	36	55	39	31	213
1 – 2	48	71	81	55	43	298
2 – 3	61	58	113	85	64	381
3 – 4	45	57	88	71	61	322
4 – 5	47	58	60	46	50	261
5 – 6	50	46	54	62	52	264
6 – 7	43	59	50	33	39	224
7 – 8	39	50	35	35	48	207
8 – 9	42	43	47	28	42	202
9 – 10	41	30	29	21	34	155
10 – 11	27	31	24	25	28	135
11 – 12	32	27	20	17	24	120
12 – 13	18	21	18	18	17	92
13 – 14	19	15	12	15	23	84
14 – 15	21	19	10	13	23	86
15 – 16	24	14	11	12	20	81
16 – 17	22	18	8	11	14	73
17 – 18	7	11	6	7	11	42
18 – 19	5	5	7	7	16	40
19 – 20	13	12	4	6	19	54
20 – 21	8	6	6	4	15	39
21 – 22	9	4	3	8	10	34
22 – 23	12	3	4	5	10	34
23 – 24	6	4	3	4	6	23
24 – 25	3	2	1	4	8	18
25 – 26	4	2	–	3	7	16
26 – 27	5	3	1	5	8	22
27 – 28	5	3	4	2	5	19
28 – 29	3	4	3	1	4	15
29 – 30	4	2	–	–	7	13
30 – 31	1	–	–	4	3	8

Contd.

APPENDIX A-8 (Continued)

Waiting time in system (Days)	OBSERVED FREQUENCY					
	1980	1981	1982	1983	1984	Total
31 – 32	1	–	1	2	5	9
32 – 33	4	1	4	3	4	16
33 – 34	1	–	1	3	3	8
34 – 35	1	1	1	3	2	8
35 – 36	–	–	2	–	–	2
36 – 37	–	–	–	1	–	1
37 – 38	–	–	–	1	1	2
38 – 39	–	–	1	–	3	4
39 – 40	–	–	1	2	1	4
40 – 41	–	–	1	–	–	1
41 – 42	–	–	1	1	–	2
42 – 43	–	–	–	2	–	2
43 – 44	–	–	2	1	1	4
44 – 45	–	–	1	1	1	3
45 – 46	–	–	1	2	2	5
46 – 47	–	–	–	–	–	0
47 – 48	–	–	–	–	2	2
48 – 49	–	–	2	1	–	3
49 – 50	–	–	–	2	1	3
50 – 51	–	–	–	2	–	2
51 – 52	–	–	–	–	–	0
52 – 53	–	–	–	–	–	0
53 – 54	–	1	–	–	1	2
54 – 55	–	–	–	1	–	1
55 – 56	–	–	–	–	–	0
56 – 57	–	–	–	–	–	0
57 – 58	–	–	–	–	–	0
58 – 59	–	1	–	1	–	2
59 – 60	–	–	–	–	–	0
60 – 61	–	–	1	–	–	1

APPENDIX A-9

Derivation of probability density function for the waiting time in the system $W(\tau)$, for the model M/M/S : FCFS/∞/∞

Let $\tau > 0$ be the waiting time in the system given that there are j other customers to be served. Then

$$\tau = \tau_1' + \tau_2 + \tau_3 + \tau_4 + \cdots\cdots + \tau_j + t_j$$

where,

τ_1' - the remaining service time for customer already in service

τ_j - the exponential waiting time in queue with mean

t_j - actual service time for the jth customer which follows exponential with mean

This means that the service time τ is the convolution of the waiting time in queue and the actual service time of the jth customer.

Thus,

$$W(\tau) = W_q(T)\, g(t)$$

where, $g(t) = \mu e^{-\mu t}$ for $t > 0$

$$W(\tau) = W_q(0)\, g(\tau) + \int_0^{\tau} W_q(T)\, g(\tau - T)\, dT$$

$$= \left[1 - \frac{\left(\frac{\lambda}{\mu}\right)^S P_0}{(S-1)!\left(S - \frac{\lambda}{\mu}\right)}\right] \mu e^{-\mu\tau} + \int_0^{\tau} \frac{\mu\left(\frac{\lambda}{\mu}\right)^S e^{-\mu\left(S-\frac{\lambda}{\mu}\right)\tau}}{(S-1)!} P_0 \cdot \mu e^{-\mu(\tau - T)} \cdot dT$$

$$= \left[1 - \frac{\left(\frac{\lambda}{\mu}\right)^s P_0}{(s-1)!\left(s-\frac{\lambda}{\mu}\right)}\right]\mu e^{-\mu\tau} + \frac{\mu^2\left(\frac{\lambda}{\mu}\right)^s P_0 e^{-\mu\tau}}{(s-1)!}\int_0^{\tau} e^{-\mu\left(s-1-\frac{\lambda}{\mu}\right)T}\,dT$$

$$= \left[1 - \frac{\left(\frac{\lambda}{\mu}\right)^s P_0}{(s-1)!\left(s-\frac{\lambda}{\mu}\right)}\right]\mu e^{-\mu\tau} + \frac{\mu^2\left(\frac{\lambda}{\mu}\right)^s P_0 e^{-\mu\tau}}{(s-1)!\,\mu\left(s-1-\frac{\lambda}{\mu}\right)}\left[1 - e^{-\mu\left(s-1-\frac{\lambda}{\mu}\right)\tau}\right]$$

$$= \left[1 - \frac{\left(\frac{\lambda}{\mu}\right)^s P_0}{(s-1)!\left(s-\frac{\lambda}{\mu}\right)}\right]\mu e^{-\mu\tau} + \frac{\mu\left(\frac{\lambda}{\mu}\right)^s P_0 e^{-\mu\tau}}{(s-1)!\left(s-1-\frac{\lambda}{\mu}\right)}\left[1 - e^{-\mu\left(s-1-\frac{\lambda}{\mu}\right)\tau}\right]$$

$$= \mu e^{-\mu\tau} - \frac{\left(\frac{\lambda}{\mu}\right)^s P_0\cdot\mu e^{-\mu\tau}}{(s-1)!\left(s-\frac{\lambda}{\mu}\right)} + \frac{\mu\left(\frac{\lambda}{\mu}\right)^s P_0 e^{-\mu\tau}}{(s-1)!\left(s-1-\frac{\lambda}{\mu}\right)} - \frac{\mu\left(\frac{\lambda}{\mu}\right)^s P_0 e^{-\mu\tau}\cdot e^{-\mu\left(s-1-\frac{\lambda}{\mu}\right)\tau}}{(s-1)!\left(s-1-\frac{\lambda}{\mu}\right)}$$

$$= \mu e^{-\mu\tau} + \frac{\left(\frac{\lambda}{\mu}\right)^s P_0\,\mu e^{-\mu\tau}}{(s-1)!\left(s-1-\frac{\lambda}{\mu}\right)}\left[1 - \frac{s-1-\frac{\lambda}{\mu}}{s-\frac{\lambda}{\mu}} - e^{-\mu\left(s-1-\frac{\lambda}{\mu}\right)\tau}\right]$$

$$= \mu e^{-\mu\tau} + \frac{\left(\frac{\lambda}{\mu}\right)^s P_0\,\mu e^{-\tau\mu}}{(s-1)!\left(s-1-\frac{\lambda}{\mu}\right)}\left[\frac{1}{s-\frac{\lambda}{\mu}} - e^{-\mu\left(s-1-\frac{\lambda}{\mu}\right)\tau}\right]$$

APPENDIX A-9 (Continued)

Hence the probability density function of the waiting time in the system for M/M/S : FCFS/∞/∞

$$W(\tau) = \mu e^{-\mu\tau} + \frac{\left(\frac{\lambda}{\mu}\right)^{s} P_0\, \mu e^{-\mu\tau}}{(s-1)!\left(s-1-\frac{\lambda}{\mu}\right)} \left[\frac{1}{s-\frac{\lambda}{\mu}} - e^{-\mu\left(s-1-\frac{\lambda}{\mu}\right)\tau} \right]$$

for $\tau > 0$

APPENDIX A-9a

```
FILE: PORT      FORTRAN  A1   BUET,COMPUTER CENTRE, DHAKA                    VM/SP

C        PROGRAMME TO SIMULATE THE ARRIVAL OF SHIP
C        SERVICE TIME,WAITING TIME,TOTAL NO OF SHIP
C        IN JETTY PER DAY FOR REAL LIFE SIMULATION
C        FOR 4000 DAYS AND ARRIVAL RATE=2.26 SHIPS PER DAY.
C        THE PROGRAMME IS PREPARED BY ENGR. AKASHA SAHA
C        UNDER THE DIRECT SUPERVISION OF ENGR. ABDUR RAHIM
C        LECTURER,DEPARTMENT OF NAVAL ARCHITECTURE AND
C        MARINE ENGINEERING,B.U.E.T.,DHAKA,BANGLADESH.
C        IDAY=DAY
C        IRDAY=RANDOM NUMBER FOR ARRIVAL OF SHIP
C        ISHIP=NUMBER OF SHIP ARRIVED.
C        ISHIPT=TOTAL NUMBER OF SHIP ARRIVED
C        ISERV= RANDOM NUMBER FOR SERVICE TIME
C        ISTIME=SERVICE TIME
C        ISTIMT=TOTAL SERVICE TIME
C        IWAIT=WAITING TIME
C        ISWAIT=TOTAL WAITING TIME
C        ISBOOK=NUMBER OF SHIP BOOKED
C        ISUMJ=NUMBER OF SHIP IN SERVICE
C        ISUMJT=TOTAL NUMBER OF SHIP IN SERVICE
C        IQQUE =QUEUE LENGTH
C        IQQUET=TOTAL NUMBER OF SHIP IN QUEUE
C        ISYSM=NUMBER OF SHIP IN SYSTEM
C        ISYSMT=TOTAL NUMBER OF SHIP IN SYSTEM
C        C1        FOR DAY (IDAY)
C        C2        FOR RANDOM NUMBER FOR ARRIVAL OF SHIPS (IRDAY)
C        C3        FOR ARRIVAL OF SHIPS (ISHIP)
C        C4        FOR RANDOM NUMBER FOR SERVICE TIME (ISERV)
C        C5        FOR SERVICE TIME (ISTIME)
C        C6        FOR WAITING TIME (IWAIT)
C        C7        FOR NUMBER OF SHIP BOOKED (ISBOOK)
C        J1-J20    FOR JETTY (JETTY STATUS)
C        C8        FOR NUMBER OF SHIP IN SERVICE(ISUMJ)
C        C9        FOR QUEUE LENGTH(IQQUE)
C        C10       FOR NUMBER OF SHIP IN SYSTEM(ISYSM)
C        ERRORS ARE CALCULATED WITH REFERANCE TO THE OBSERVED VALUES AND
C        IN PERCENTAGE.POSITIVE ERRORS INDICATE THAT SIMULATED VALUES ARE
C        SMALLER THAN OBSERVED VALUES.NEGATIVE ERRORS INDICATE THAT
C        SIMULATED VALUES ARE HIGHER THAN OBSERVED VALUES.
         DIMENSION JE(4000,15),JS(4000,15),E(50)
         OPEN(UNIT=3,FILE='SHAH',STATUS='NEW')
C        WRITE(3,5)
5        FORMAT(3X,'C1',4X,'C2',2X,'C3',5X,'C4',2X,'C5',2X,'C6',2X,
        &'C7',3X,'J1',2X,'J2',2X,'J3',2X,'J4',2X,'J5',2X,'J6',2X,'J7',2X,
        &'J8',2X,'J9',2X,'J10',1X,'J11',1X,'J12',1X,'J13',1X,'J14',1X,'J15',
        &1X,'J16',1X,'J17',1X,'J18',1X,'J19',1X,'J20',3X,'C8',3X,'C9',3X,
        &'C10')
         N=15
         NDAY=4000
         IX=25591
         IDAY=0
         ISHIPT=0
         ISTIMT=0
         ISWAIT=0
         ISUMJT=0
```

APPENDIX A-9a (Continued)


```
          INDOKT=0
          IDQUET=0
          ISYSNT=0
          DO 15 I=1,NDAY
          DO 15 J=1,N
          JS(I,J)=0
          JQ(I,J)=0
15        CONTINUE
          DO 19 I=1,50
19        ID(I)=0
25        IDAY=IDAY+1
C         WRITE (3,35)IDAY
35        FORMAT (//,1X,I4)
          IF( IDAY.GT.NDAY ) GO TO 185
45        CALL RANDOM (IX,IY,YFL)
          IX=IY
          K=YFL*100+1
          IF(K.GE.100)   CALL RANDOM(IX,IY,YFL)
          IF(K.GE.100)  IX=IY
          IF(K.GE.100)   K=YFL*100+1
          IF(K.GE.100)   GO TO 45
          IRDAY=K
C         WRITE (3,55)IRDAY
55        FORMAT ('+',8X,I2)
C         TWO FIGURES RANDOM NUMBERS FOR ARRIVAL OF SHIPS
          IF(IRDAY.GE.0.AND.IRDAY.LE.14    )     ISHIP=0
          IF(IRDAY.GE.15   .AND.IRDAY.LE.41   )  ISHIP=1
          IF(IRDAY.GE.42   .AND.IRDAY.LE.67   )  ISHIP=2
          IF(IRDAY.GE.68   .AND.IRDAY.LE.85   )  ISHIP=3
          IF(IRDAY.GE.86   .AND.IRDAY.LE.94   )  ISHIP=4
          IF(IRDAY.GE.95   .AND.IRDAY.LE.99   )  ISHIP=5
          IF(IRDAY.EQ.99   )                     ISHIP=6
C         IF(ISHIP.EQ.0) WRITE (3,65) (JS(IDAY,J),J=1,N)
65        FORMAT ('+',32X,20I4)
          ISHIPT=ISHIPT+ISHIP
C         WRITE (3,75) ISHIP
75        FORMAT ('+',15X,I1)
          IF (ISHIP.EQ.0) GO TO 175
          DO 165 I=1,ISHIP
85        CALL RANDOM (IX,IY,YFL)
          IX=IY
          K=YFL*100+1
          IF(K.GE.100) CALL RANDOM (IX,IY,YFL)
          IF(K.GE.100) IX=IY
          IF(K.GE.100) K=YFL*100+1
          IF(K.GE.100) GO TO 85
          ISERV=K
C         IF (I.EQ.1) WRITE (3,95) ISERV
95        FORMAT ('+',17X,I2)
C         IF (I.NE.1) WRITE (3,105) ISERV
105       FORMAT (/,18X,I2)
C         TWO FIGURES RANDOM NUMBERS FOR SERVICE TIMES
          IF (ISERV.GE.0.AND.ISERV.LE.10  )        ISTIME=1
          IF (ISERV.GE.11  .AND.ISERV.LE.22   )   ISTIME=2
          IF (ISERV.GE.23   .AND.ISERV.LE.35   ) ISTIME=3
```

APPENDIX A-9a (Continued)

FILE: PORT   FORTRAN A1 BUET COMPUTER CENTRE, DHAKA   VM/SP

```
      IF (ISERV.GE.36   .AND.ISERV.LE.45   ) ISTIME=4
      IF (ISERV.GE.46   .AND.ISERV.LE.54   ) ISTIME=5
      IF (ISERV.GE.55   .AND.ISERV.LE.62   ) ISTIME=6
      IF (ISERV.GE.63   .AND.ISERV.LE.68   ) ISTIME=7
      IF (ISERV.GE.69   .AND.ISERV.LE.73   ) ISTIME=8
      IF (ISERV.GE.74   .AND.ISERV.LE.78   ) ISTIME=9
      IF (ISERV.GE.79   .AND.ISERV.LE.82   ) ISTIME=10
      IF (ISERV.GE.83   .AND.ISERV.LE.84   ) ISTIME=11
      IF (ISERV.GE.85   .AND.ISERV.LE.86   ) ISTIME=12
      IF (ISERV.GE.87   .AND.ISERV.LE.88   ) ISTIME=13
      IF (ISERV.EQ.89   )                    ISTIME=14
      IF (ISERV.GE.90   .AND.ISERV.LE.91   ) ISTIME=15
      IF (ISERV.EQ.92   )                    ISTIME=16
      IF (ISERV.EQ.93   )                    ISTIME=17
      IF (ISERV.EQ.94   )                    ISTIME=18
      IF (ISERV.EQ.95   )                    ISTIME=19
      IF (ISERV.EQ.96   )                    ISTIME=21
      IF (ISERV.EQ.97   )                    ISTIME=24
      IF (ISERV.EQ.98   )                    ISTIME=27
      IF (ISERV.EQ.99   )                    ISTIME=31
      ISTIMT=ISTIMT+ISTIME
C     WRITE(3,115) ISTIME
115   FORMAT('+',21X,I2)
      IDTRY=IDAY-1
125   IDTRY=IDTRY+1
      DO 155 ISRCH=1,N
      IFIND=JS(IDTRY,ISRCH)
      IF(IFIND.EQ.1) GO TO 155
      JB(IDTRY,ISRCH)=1
      IBOOK=IDTRY+ISTIME-1
      IWAIT=IDTRY-IDAY
      ISWAIT=ISWAIT+IWAIT
C     WRITE(3,135)IWAIT
135   FORMAT('+',25X,I2)
      DO 145 K=IDTRY,IBOOK
      JS(K,ISRCH)=1
145   CONTINUE
      GO TO 165
155   CONTINUE
      GO TO 125
165   CONTINUE
175   ISUMJ=0
      ISBOOK=0
      DO 185 J=1,N
      ISBOOK=ISBOOK+JB(IDAY,J)
      ISUMJ=ISUMJ+JS(IDAY,J)
185   CONTINUE
      ISUMJT=ISUMJT+ISUMJ
      IBOOKT=IBOOKT+ISBOOK
      IDQUE =ISHIPT-IBOOKT
      IDQUET=IDQUET+IDQUE
      ISYSM=ISUMJ+IDQUE
      ID(ISYSM+1)=ID(ISYSM+1)+1
      ISYSMT=ISYSMT+ISYSM
      A=FLOAT(ISHIPT)/FLOAT(IDAY)
```

APPENDIX A-9a (Continued)

```
FILE: PORT       FORTRAN  A1  BUET COMPUTER CENTRE, DHAKA                VM/SP

      B=FLOAT(ISTIMT)/FLOAT(ISHIPT)
      C=FLOAT(N)/B
      D=A*B
      E=A/C
      X=(1-E)*100
      ALQ=FLOAT(IOQUET)/FLOAT(IDAY)
      WQ=FLOAT(ISWAIT)/FLOAT(ISHIPT)
      F=FLOAT(ISUMJT)/FLOAT(IDAY)
      AL=ALQ+F
      W=B+WQ
C     OBSERVED VALUES OF SYSTEM PARAMETERS
      A1=2.03
      B1=6.46
      C1=2.322
      D1=13.114
      E1=0.574
      X1=12.6
      ALQ1=4.02
      WQ1=2.8564
      F1=13.45
      AL1=18.25
      W1=2.329
C     CALCULATION OF ERRORS OF SIMULATED VALUES OF SYSTEM PARAMETERS
      EA=((A1-A)/A1)*100
      EB=((B1-B)/B1)*100
      EC=((C1-C)/C1)*100
      ED=((D1-D)/D1)*100
      EE=((E1-E)/E1)*100
      EX=((X1-X)/X1)*100
      ELQ=((ALQ1-ALQ)/ALQ1)*100
      EWQ=((WQ1-WQ)/WQ1)*100
      EF=((F1-F)/F1)*100
      EL=((AL1-AL)/AL1)*100
      EW=((W1-W)/W1)*100
C     WRITE (3,195) IDQUE
195   FORMAT ('+',121X,I2)
C     WRITE(3,205)ISYSM
205   FORMAT('+',127X,I2)
C     WRITE(3,225) (JS(IDAY,J),J=1,N )
225   FORMAT('+',32X,20I4)
235   CONTINUE
C     WRITE (3,245) ISUMJ
245   FORMAT('+',115X,I2)
C     WRITE (3,255) ISBOOK
255   FORMAT('+',29X,I2)
      GO TO 25
265   WRITE (3,505) ISHIPT
505   FORMAT (///10X,'THE TOTAL NUMBER OF SHIP ARRIVED =ISHIPT=',I10)
      WRITE (3,515) ISTIMT
515   FORMAT (///10X,'THE TOTAL SERVICE TIME----------- =ISTIMT=',I10)
      WRITE (3,525) ISWAIT
525   FORMAT (///10X,'THE TOTAL WAITING TIME----------  =ISWAIT=',I10)
      WRITE (3,535) IOQUET
535   FORMAT (///10X,'THE TOTAL SHIP IN QUEUE-----------=IOQUET=',I10)
      WRITE (3,545) ISUMJT
```

```
545   FORMAT (///10X,'THE TOTAL SHIP IN SERVICE --------=ISUMJ=',I10)
      WRITE(3,555)ISYSMF
555   FORMAT (///10X,'TOTAL SHIP IN SYSTEM-------------=ISYSMF=',I10)
      WRITE(3,565) A
565   FORMAT(///10X,'THE MEAN ARRIVAL RATE...........=LAMDA=',F8.4)
      WRITE (3,575) B
575   FORMAT(///10X,'THE MEAN SERVICE TIME ...........=E(ST)=',F8.4)
      WRITE(3,585) C
585   FORMAT(///10X,'THE MEAN SERVICE RATE............=MU.=',F8.4)
      WRITE(3,595) D
595   FORMAT(///10X,'THE MEAN TRAFFIC INTENSITY...=LAMDA/MU=',F8.4)
      WRITE (3,605) E
605   FORMAT(///10X,'THE MEAN UTILIZATION FACTOR=LAMDA/S.MU=',F8.4)
      WRITE (3,615)X
615   FORMAT(///10X,'THE PERCENTAGE OF SERVERS IDLE TIME..=X=',F8.4)
      WRITE(3,625) ALQ
625   FORMAT(///10X,'THE MEAN QUEUE LENGTH..............=LQ=',F8.4)
      WRITE(3,635) WQ
635   FORMAT(///10X,'THE MEAN WAITING TIME IN QUEUE.......=WQ=',F8.4)
      WRITE(3,645)F
645   FORMAT(///10X,'THE MEAN NUMBER OF SHIP IN SERVICE...=F=',F8.4)
      WRITE(3,655) AL
655   FORMAT(///10X,'THE MEAN NUMBER OF SHIP IN SYSTEM....=L=',F8.4)
      WRITE(3,665) W
665   FORMAT(///10X,'THE MEAN WAITING TIME IN SYSTEM......=W=',F8.4)
      WRITE(3,675)EA
675   FORMAT(///10X,'ERROR IN ARRIVAL RATE....................=',F7.2)
      WRITE(3,685) EB
685   FORMAT(///10X,'ERROR IN MEAN SERVICE TIME...............=',F7.2)
      WRITE(3,695) EC
695   FORMAT(///10X,'ERROR IN SERVICE RATE....................=',F7.2)
      WRITE(3,705) ED
705   FORMAT(///10X,'ERROR IN TRAFFIC INTENSITY...............=',F7.2)
      WRITE(3,715) EE
715   FORMAT(///10X,'ERROR IN UTILIZATION FACTOR..............=',F7.2)
      WRITE(3,725) EX
725   FORMAT(///10X,'ERROR IN PERCENTAGE OF SERVERS IDLE TIME=',F7.2)
      WRITE(3,735) ELQ
735   FORMAT(///10X,'ERROR IN MEAN QUEUE LENGTH.............. =',F7.2)
      WRITE(3,745) EWQ
745   FORMAT(///10X,'ERROR IN MEAN WAITING TIME IN QUEUE......=',F7.2)
      WRITE(3,755) EF
755   FORMAT(///10X,'ERROR IN MEAN NUMBER OF SHIP IN SERVICE.=',F7.2)
      WRITE(3,765)EL
765   FORMAT(///10X,'ERROR IN MEAN NUMBER OF SHIP IN SYSTEM..=',F7.2)
      WRITE (3,775) EW
775   FORMAT(///10X,'ERROR IN MEAN WAITING TIME IN SYSTEM....=',F7.2)
      WRITE(3,785)
785   FORMAT(///10X,'ERRORS ARE EXPRESSED AS PERCENTAGE OF OBSERVED VALU
     *ES')
      WRITE (3,815)
815   FORMAT (///5X,'ISYSM',5X,'FREQUENCY')
      DO 795 I=1,51
      J=I-1
      WRITE(3,805) J,ID(I)
```

APPENDIX A-9a (Continued)


```
799    CONTINUE
800    FORMAT (////7X,I2,9X,I4)
       STOP
       END
       SUBROUTINE RANDOM (IX,IY,YFL)
       IY=IX*65539
       IF (IY) 5,15,15
5      IY=IY+2147483647+1
15     YFL=IY
       YFL=YFL*0.4656613E-9
       RETURN
       END
```

APPENDIX A-10

History of revenue earning

Sl. No.	Amount of P-Bill	Amount of J-Bill	Sl. No.	Amount of P-Bill	Amount of J-Bill
1	199,183	93,697	34	199,216	240,049
2	196,232	410,471	35	196,756	175,377
3	193,016	459,569	36	209,462	458,593
4	192,968	483,090	37	199,339	350,103
5	200,717	457,358	38	189,093	345,960
6	208,911	376,732	39	186,458	165,894
7	193,445	215,087	40	195,302	150,735
8	192,054	317,088	41	201,188	417,989
9	196,147	134,844	42	200,806	467,449
10	208,785	187,859	43	183,087	125,890
11	190,084	452,703	44	185,836	144,663
12	195,951	207,585	45	205,012	430,943
13	200,975	145,218	46	208,847	365,204
14	193,976	216,729	47	209,643	409,343
15	192,633	395,675	48	195,770	283,927
16	121,889	395,332	49	206,884	298,685
17	195,899	373,147	50	209,522	485,307
18	105,612	195,222	51	204,415	396,562
19	104,783	204,058	52	203,003	384,138
20	188,097	192,533	53	204,854	238,891
21	193,437	192,52[illegible]	54	201,748	201,598
22	104,905	146,570	55	202,450	306,049
23	185,151	463,487	56	200,298	375,515
24	188,768	375,814	57	193,421	166,809
25	172,544	140,780	58	209,879	443,812
26	194,531	100,519	59	205,053	378,526
27	205,657	378,005	60	208,098	370,472
28	201,588	218,577	61	204,809	396,180
29	196,973	340,679	62	209,141	378,376
30	198,798	97,284	63	187,666	130,116
31	197,659	395,813	64	175,043	103,411
32	194,613	248,749	65	190,746	193,775
33	204,600	237,005	66	189,303	182,046
				12,926,729	18,912,133

Calculations of annual equivalent cost of cost item.

By formula, $a = P.\frac{r(1+r)^n}{(1+r)^n - 1}$

where, P - the present value of the cost

r - interest rate (in fraction)

n - the number of year

a - uniform annual rate.

For the present study r = 0.15 is considered.

For jetty for 3 tons $a = Tk.72,000,000 \frac{0.15(1 + 0.15)^{40}}{(1 + 0.15)^{40} - 1}$

$= Tk.72,000,000 \times 0.150562$

$= Tk.10,840,470$

For 3 tons jetty crane

$a = Tk. 8,000,000 \frac{0.15(1 + 0.15)^{20}}{(1 + 0.15)^{20} - 1}$

$= Tk. 8,000,000 \times 0.15976$

$= Tk. 1,278,080$

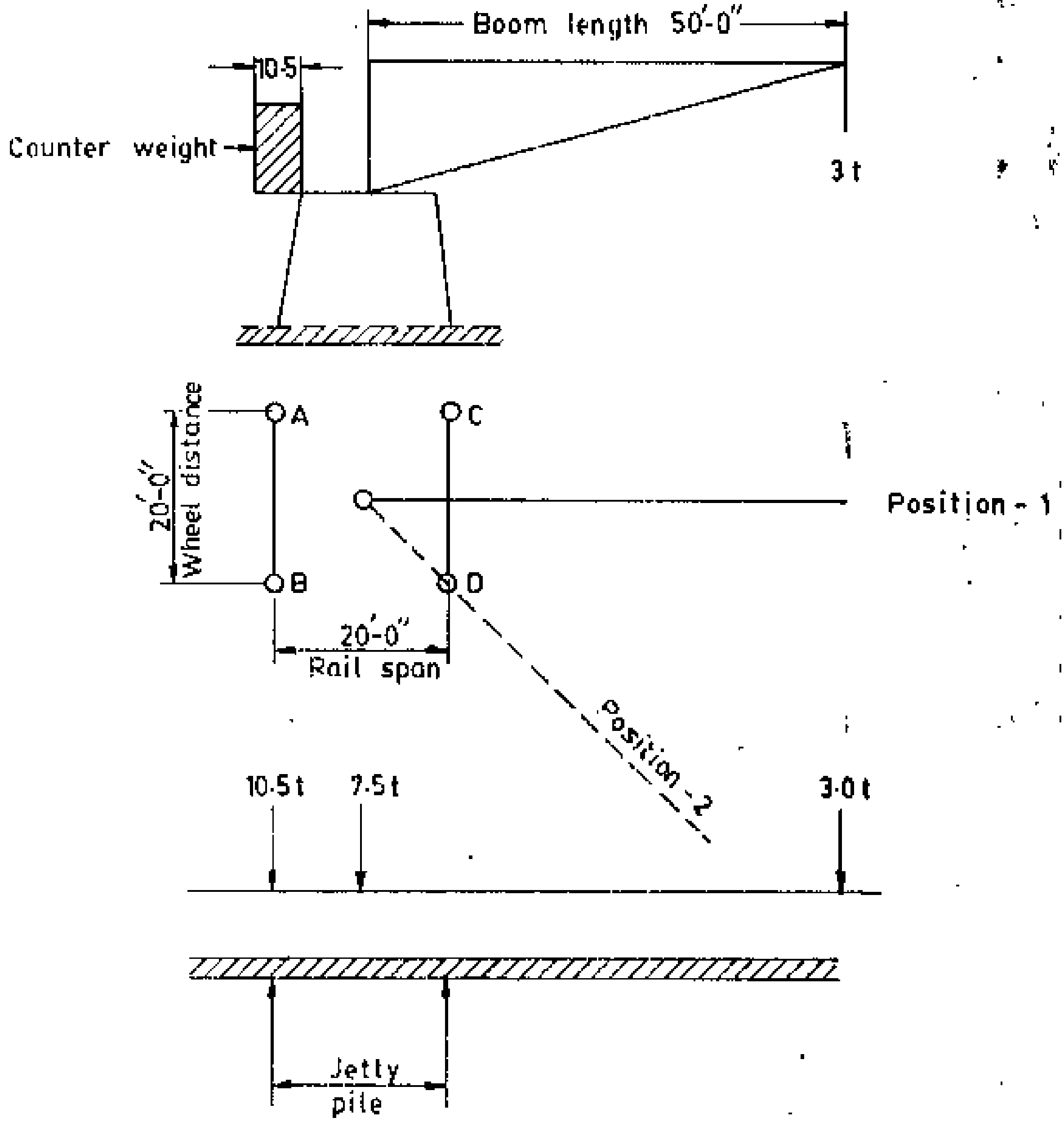
Similarly, annual equivalent cost of other material handling equipment may be calculated,

APPENDIX A-11 (Continued)

For 3.5 tons jetty crane	=	Tk. 1,373,945.-
For 4.0 tons jetty crane	=	Tk. 1,469,801.-
For 4.5 tons jetty crane	=	Tk. 1,573,646.-
For 5.0 tons jetty crane	=	Tk. 1,677,490.-
For 5.5 tons jetty crane	=	Tk. 1,797,311.-
For 6.0 tons jetty crane	=	Tk. 1,917,132.-
Mobile crane 10 tons	=	Tk. 1,597,610.-
For lifters	=	Tk. 399,402.-

APPENDIX A - 12

Calculations of forces on jetty piles for 3 tons crane.



Force $= \frac{P}{A} \pm \frac{Mc}{I}$

$P = 21.0\,t$, $A = 4$, $M = 45$ ton.ft, $I = 400\,ft^4$

For position 1, $C = 10'$-$0''$

$R_A = R_B = \frac{P}{A} - \frac{Mc}{I} = 4.13$ tons

$R_C = R_D = \frac{P}{A} + \frac{Mc}{I} = 6.38$ tons

Force for dead load = 4.5 t

For position 2

$R_B = R_C = \frac{P}{A} = 5.25$ tons

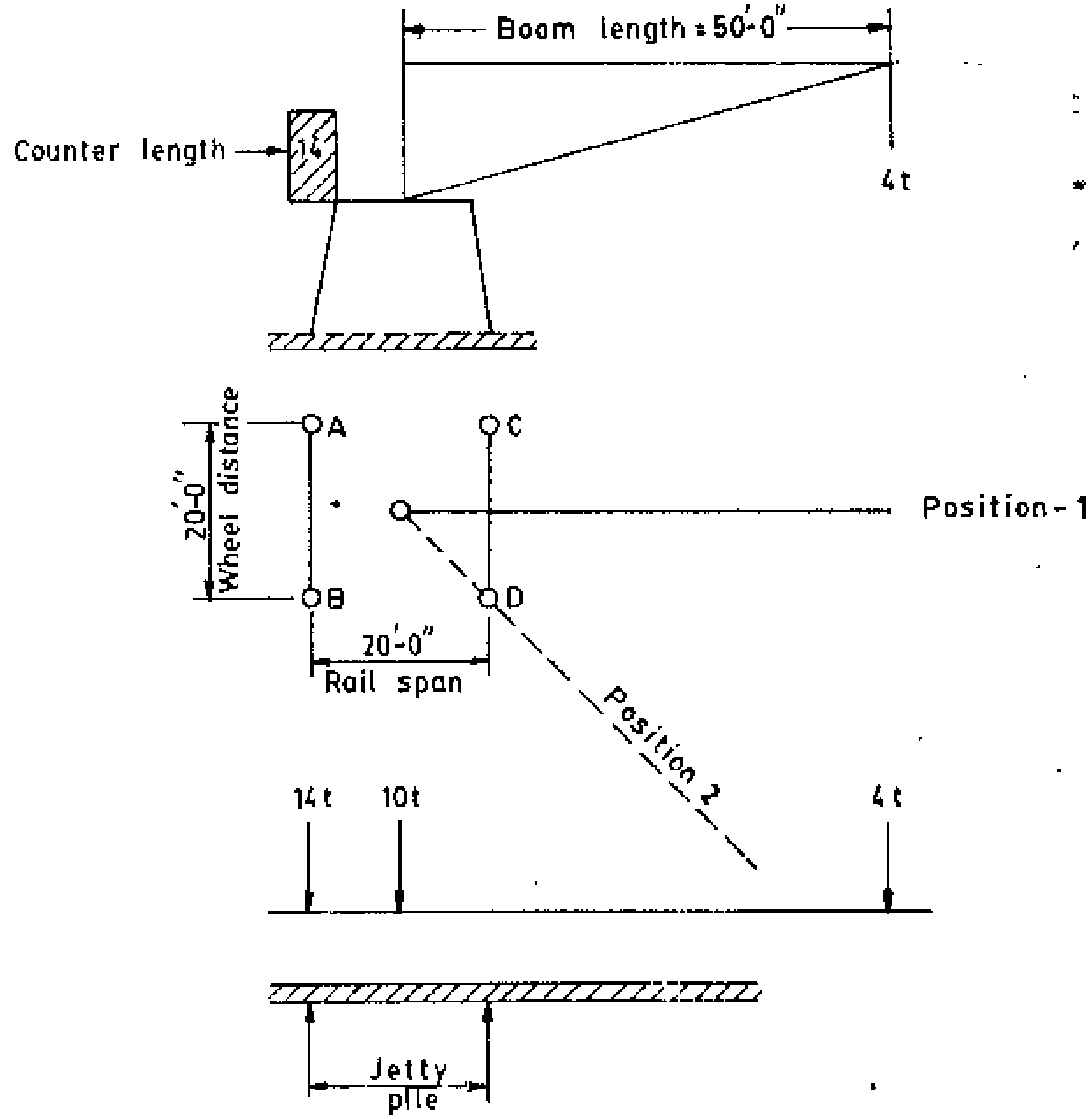
$R_A = \frac{P}{A} - \frac{Mc}{I} = 3.66$ tons

$R_D = \frac{P}{A} + \frac{Mc}{I} = 6.84$ tons

$\therefore$ Max force $R_D = 6.84$ tons in position - 2

APPENDIX A-12

Calculations of forces on jetty piles for 4 tons crane



Reaction $= \frac{P}{A} \pm \frac{MC}{I}$

$P = 28t$, $A = 4$, $M = 60$ ton ft. $I = 400$ ft.^{4}

For position 1, $C = 10'-0"$

$R_A = R_B = \frac{P}{A} - \frac{MC}{I} = 5.5$ tons

$R_C = R_D = \frac{P}{A} + \frac{MC}{I} = 8.5$ tons

Reaction for dead load = 6 tons

For position 2

$R_B = R_C = \frac{P}{A} = 7.0$ tons

$R_A = \frac{P}{A} - \frac{MC}{I} = 4.08$ tons

$R_D = \frac{P}{A} + \frac{MC}{I} = 9.12$ tons

$\therefore$ Max reaction $R_D = 9.12$ tons in position 2

Increase in reaction by 33%

APPENDIX A-13

```
C        PROGRAMME TO COMPUTE OPTIMUM NET REVENUE EARNINGS FROM SHIP
C        AND THE CORRESPONDING NUMBER OF SERVERS.
C        S = EXISTING NUMBER OF JETTIES
C        ECH = MEAN CARGO HANDLED,TONS PER SHIP
C        A0 = CRANE CAPACITY ON  EXISTING JETTY
C        A1 = NUMBER OF CRANES PER EXISTING JETTY
C        A2 = CRANE CAPACITY ON  PROPOSED JETTY
C        A3 = NUMBER OF CRANES PER PROPOSED JETTY
C        O = NUMBER OF OPERATIONS PER CRANE PER DAY
C        PEST = PRESENT MEAN SERVICE TIME,DAYS PER SHIP
C        EST = EXPECTED MEAN SERVICE TIME
C        CJ = COST OF UNIT JETTY
C        CW = COST OF UNIT WAREHOUSE
C        COSTCR = COST OF JETTY CRANES
C        C0 = CRANE CAPACITIES ON EXISTING JETTIES
C        C1 = NUMBER OF CRANES ON EXISTING JETTIES
C        C2 = CRANE CAPACITIES ON PROPOSED JETTIES
C        C3 = NUMBER OF CRANES ON PROPOSED JETTIES
C        ADCST1 = COST OF PROPOSED CRANES ON EXISTING JETTIES
C        ADCST2 = COST OF PROPOSED CRANES ON PROPOSED JETTIES
C        CMC = COST OF MOBILE CRANE
C        CFL = COST OF FORK-LIFTER
C        ARATE = EXPECTED MEAN ARRIVAL RATE OF SHIP,SHIPS PER DAY
C        D = NUMBER OF DAYS IN QUESTION
C        SA = NUMBER OF SHIPS ARRIVED IN D-DAYS
C        IJETTY = NUMBER OF SERVERS
C        ICOMB = COMBINATIONS OF SERVICE FACILITIES
C        SS = NUMBER OF SHIPS SERVED IN D-DAYS
C        S15 = NUMBER OF SHIPS SERVED BY 15 SERVERS WITH
C            PRESENT MEAN SERVICE TIME OF 0.46 DAYS PER SHIP
C        N = ADDITIONAL NUMBER OF SHIPS SERVED BY THE SERVER
C        R = ADDITIONAL REVENUE EARNED
C        E1 = COST OF JETTY
C        E2 = COST OF WAREHOUSE
C        E3 = COST OF JETTY CRANES
C        E4 = COST OF MOBILE CRANES
C        E5 = COST OF FORK-LIFTERS
C        E = TOTAL COST OF SERVERS + SERVICE FACILITIES
C        RE = NET REVENUE EARNED
         DIMENSION  COSTCR(32),C0(32),C1(32),C2(32),C3(32),HOP(32),ARMAX(28
        *),NCOMB(30),OPT(400)
         INTEGER S
         OPEN(UNIT=1,FILE='COST',STATUS='OLD')
         OPEN(UNIT=3,FILE='OUT',STATUS='NEW')
         S=15
         D=58.64
         READ(1,1)(COSTCR(I),I=1,32)
         READ(1,2)(C0(I),I=1,32)
         READ(1,2)(C1(I),I=1,32)
         READ(1,3)(C2(I),I=1,32)
         READ(1,2)(C3(I),I=1,32)
1        FORMAT(8F9.0)
2        FORMAT(16F5.1)
         DO 222  IARRIV=205,350
         ARATE=FLOAT(IARRIV)*0.01
```

APPENDIX A-13 (Continued)

```
      DO 111   IJETTY=15,20
      DO 333 ICOMB=1,32
      A0=C0(ICOMB)
      A1=C1(ICOMB)
      A2=C2(ICOMB)
      A3=C3(ICOMB)
      ECH=3421.0
      EST=ECH*FLOAT(IJETTY)/(D*(A0*A1*FLOAT(S)+A2*A3*FLOAT(IJETTY-S)))
      CJ=10.84000
      CW=10.222239
      CMC=3.159229
      CFL=1.192211
      REST=6.66
      D=365.0
      SA=D*ARATE
      I=IFIX(SA+0.5)
      SIS=(D*S)/REST
      J=IFIX(SIS+0.5)
      SS=(D*IJETTY)/EST
      L=IFIX(SS+0.5)
      IF(L.GT.I)   L=I
      M=L-J
      IF(M.LE.0)  M=0
      R=0.46241779M
      E1=(1.0+(A2-3.0)*0.10)*CJ*(IJETTY-S)
      E2=CW*(IJETTY-S)
      ADCST1=(A1-3)*COSTER(1)*S
      ADCST2=COSTER(ICOMB)*A3*(IJETTY-S)
      E3=ADCST1+ADCST2
      E4=CMC*(IJETTY-S)
      E5=CFL*(IJETTY-S)
      E=E1+E2+E3+E4+E5
      RE=R-E
      HDR(ICOMB)=RE
333   CONTINUE
      REMAX=HDR(1)
      NCOMB=1
      DO   555   ICOMB=2,32
      IF(REMAX.GT.HDR(ICOMB))   GO   TO 555
      REMAX=HDR(ICOMB)
      IF(REMAX.LE.0) REMAX=0.0
      ARMAX(IJETTY)=REMAX
      NCOMB=ICOMB
      MNCOMB(IJETTY)=NCOMB
555   CONTINUE
111   CONTINUE
C     SEARCHING THE  OPTIMUM JETTY NUMBER
      ITT=15
      RMM=ARMAX(15)
      DO 220 IJETTY=16,20
      IF((ARMAX(IJETTY)-ARMAX(IJETTY-1)).GT.0.0)GO TO 75
      GO TO 220
75    CONTINUE
      ITT=IJETTY
      RMM=ARMAX(IJETTY)
```

```
220   CONTINUE
      IO=ITT-5
      IOO=NNCOMB(ITT)
      CB1=C0(IOO)
      CB2=C1(IOO)
      IF(IO.GT.0) GO TO 26
      IF(ABS(REM).GT.0.001)WRITE(3,600)ARATE,REM,CB1,CB2
      IF(ABS(REM).LT.0.001)WRITE(3,601)ARATE,CB1,CB2
      GO TO 222
26    CONTINUE
      CB3=C2(IOO)
      CB4=C3(IOO)
      IF(ABS(REM).GT.0.001)WRITE(3,610)ARATE,REM,CB1,CB2,IO,CB3,CB4
      IF(ABS(REM).LT.0.001)WRITE(3,611)ARATE,CB1,CB2,IO,CB3,CB4
222   CONTINUE
      STOP
600   FORMAT(/6X,F5.2,5X,F7.3,5X,F3.1,5X,F3.1,T90,F6.3)
601   FORMAT(/6X,F5.2,12X,5X,F3.1,5X,F3.1,T90,F6.3)
610   FORMAT(/6X,F5.2,5X,F7.3,5X,F3.1,5X,F3.1,8X,I2,5X,F3.1,5X,F3.1,T90,
     *F6.3)
611   FORMAT(/6X,F5.2,12X,5X,F3.1,5X,F3.1,8X,I2,5X,F3.1,5X,F3.1,T90,F6.3
     *)
      END
```