

SECTION – A

There are **FOUR** questions in this section. Answer any **THREE**.

ChE 111/201 Data Booklet is supplied.

1. n-Hexane is used to extract oil from soybeans. The solid residue from the extraction unit, which contains 0.75 kg liquid hexane/kg dry solids, is contacted in a dryer with nitrogen that enters at 85°C. The solids leave the dryer containing 0.05 kg liquid hexane/kg dry solids, and the gas leaves the dryer at 80°C and 1.0 atm with a relative saturation of 72%. The gas is then fed to a condenser in which it is compressed to 5.0 atm and cooled to 28°C, enabling most of the hexane to be recovered as condensate.
 - (a) Calculate the fractional recovery of hexane (kg condensed/kg hexane fed in wet solids). (18)
 - (b) A proposal has been made to split the gas stream leaving the condenser, combining 90% of it with fresh make-up nitrogen, heating the combined stream to 85°C, and recycling the heated stream to the dryer inlet. What fraction of fresh nitrogen required in part(a) would be saved by introducing the recycle? (17)

2. (a) Powdered milk produced in a spray dryer 6 m in diameter by 6 m High. Air enters at the bottom of the dryer at 167°C and –40 cm H₂O gauge pressure. Liquid milk is sprayed from the top of the dryer contains 70% water by mass, all of which evaporates and leaves the dryer as wet gas. This wet gas contains 12% (mole) water and leaves from the top of the dryer at 83°C and 1 atm (absolute) at a rate of 311 m³/min. Calculate the production rate of dried milk and the volumetric flow rate of the inlet air. Estimate the upward velocity of air (m/s) at the bottom of the dryer. (20)
 - (b) A mixture of liquid, 60 mole% n-hexane and 40 mole% n-heptane, is in equilibrium with its associated vapor at 1 atm. Calculate the value of T_{bp} and the equilibrium vapor composition. State assumptions, if any. (15)

3. Methanol is synthesized from carbon monoxide and hydrogen in a catalytic reactor. The fresh feed to the process contains 32 mole% CO, 64 mole% H₂ and the balance N₂. This

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Contd... Q. No. 3

stream is mixed with a recycle stream in a ratio 5 mole recycle/1 mole fresh feed to produce the feed to the reactor, which contains 13% N₂. A low single pass conversion is attained in the reactor. The reactor effluent goes to a condenser from which two streams emerge: a liquid product stream containing essentially all the methanol formed in the reactor, and a gas stream containing all the CO, H₂, and N₂ leaving the reactor. The gas stream is split into two fractions: one is removed as a purge stream and the other is recycle stream that combines with the fresh feed to the reactor. Calculate the production rate of methanol (mol/h), the molar flow-rate and composition of the purges gas, and the overall and single pass conversion. What are the reasons for using recycle and purge streams?

(35)

4. (a) Ethane is burned with 30% excess air. The percentage conversion of the ethane is 85%. Of the ethane burned 30% reacts to form CO and the balance reacts to form CO₂. Calculate the molar composition of the stack gas on a dry basis, and the mole ratio of water to dry stack gas.

(17)

(b) The air in a building is to be maintained at 25°C and 55% relative humidity by passing outside air through a water spray. The air enters the spray chamber at 32°C and 75% relative humidity, leaves the chamber cooled and saturated with water vapor, and is then reheated to 25°C for delivering it into the building. Estimate the temperature of the air leaving the spray chamber and the water (kg) added to or removed from each kg of dry air processed.

(18)

SECTION – B

There are **FOUR** questions in this section. Answer any **THREE**.

5. (a) An air conditioner cools 226 m³/min of humid air at 36°C and 98% relative humidity to 10°C. Calculate the rate of water removal in the unit and the cooling duty in tons (1 ton = 12,000 Btu/h)

(20)

(b) An aqueous ammonia solution contains 30 wt% NH₃.

(15)

(i) Use the enthalpy-concentration diagram for the ammonia-water system at 1 atm to determine the mass fraction of NH₃ in the vapor that would be in equilibrium with this solution in a closed flask at 1 atm and the corresponding system temperature (see Figure for Question 5(b)).

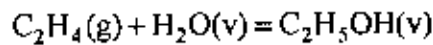
(ii) If the liquid in (i) accounts for 90% of the total system mass, calculate the overall system composition and specific enthalpy using balances.

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6. Methane at 25°C is burned in a boiler furnace with 10% excess air preheated to 100°C. Ninety percent of the methane fed is consumed, the product gas contains 10 mol CO₂/mol CO, and the combustion products leave the furnace at 400°C. (35)

- (i) Calculate the heat transferred from the furnace, $-\dot{Q}$ (kw), for a basis of 1000 mol CH₄ fed/s.
- (ii) Would the following changes increase or decrease the rate of steam production? (Assume the fuel feed rate and the fractional conversion of the CH₄ remain constant)
 - Increasing inlet air temperature.
 - Increasing the stack gas temperature
 - Increasing the mole ratio of CO₂ to CO formation in the furnace.

7. Ethanol is produced commercially by the hydration of ethylene: (35)



Some of the product is converted to diethyl ether in the undesirable side reaction



The combined feed to the reactor contains 53.7 mol% C₂H₄, 36.7% H₂O and the balance N₂ which enters the reactor at 310°C. The reactor operates isothermally at 310°C. An ethylene conversion of 5% is achieved and the yield of ethanol (moles ethanol produced/mole ethylene consumed) is 0.900.

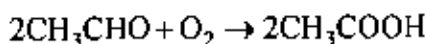
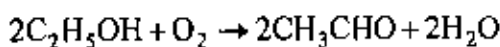
Data for Diethyl Ether:

$$\Delta \hat{H}_f^\circ = -272.8 \text{ kJ/mol for the liquid}$$

$$\Delta \hat{H}_v = 26.05 \text{ kJ/mol (independent of T)}$$

$$C_p [\text{kJ}/(\text{mol} \cdot ^\circ\text{C})] = 0.08945 + 40.33 \times 10^{-5} T(^{\circ}\text{C}) - 2.244 \times 10^{-7} T^2$$

- (i) Calculate the reactor heating or cooling requirement in kJ/mol feed
 - (ii) Why would the reactor be designed to yield such a low conversion of ethylene? What processing steps would probably follow the reaction in a commercial process?
8. (a) Ethyl alcohol can be bacterially oxidized to acetic acid in the following two step fermentation sequence: (17)



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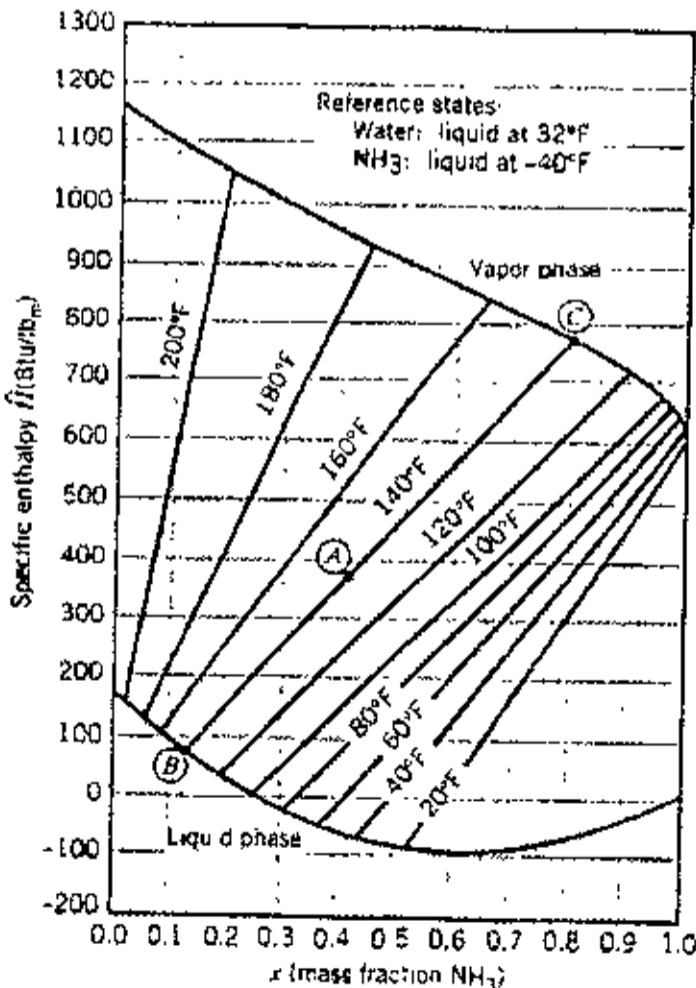
Contd... Q. No. 8(a)

If the alcohol - containing feed stock is derived from malt solution, the resulting solution is vinegar. An aqueous solution containing ethyl alcohol in water is fermented to produce dilute acetic acid. The feed mixture (the ethanol solution) and air are fed at a temperature T_0 . The product solution contains ethanol, acetaldehyde (CH_3CHO), acetic acid and water. All liquid and gaseous effluents are at temperature T . Perform a degrees of freedom analysis for the process and specify the design variables that must be known to solve the problem completely.

(b) Methanol is added to a storage tank at a rate of 1200 kg/h and is simultaneously withdrawn at a rate $\dot{m}_w(t)$ kg/h that increases linearly with time. At $t = 0$ the tank contains 750 kg of the liquid and $\dot{m}_w = 750$ kg/h. Five hours later \dot{m}_w equals 1000 kg/h. Calculate an expression for $\dot{m}_w(t)$ and incorporate it into a differential mass balance equation for the system. Find the expression for mass of water in the tank at any time t . Calculate how long it will take for the mass of methanol in the tank to reach its maximum value z and calculate that value. When will the tank completely drain out?

(18)

Figure for Question 5. (b).



Enthalpy-concentration diagram for the ammonia-water system at 1 atm. (From G. G. Brown et al., *Unit Operations*, ©1950, Figure 531. Reprinted by permission of John Wiley & Sons)

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13.12.14

L-2/T-1/CHE

Date : 13/12/2014

BANGLADESH UNIVERSITY OF ENGINEERING AND TECHNOLOGY, DHAKA

L-2/T-1 B. Sc. Engineering Examinations 2013-2014

Sub : **CHE 203** (Chemical Engineering Thermodynamics-I)

Full Marks: 210

Time : 3 Hours

USE SEPARATE SCRIPTS FOR EACH SECTION

The figures in the margin indicate full marks.

SECTION - A

There are **FOUR** questions in this section. Answer any **THREE**.

Notations indicate their usual meanings.

1. (a) Outside temperature of a cold country is -5°C . For comfortable living the inside of a house is to be maintained at 27°C . One kilowatt electric power is available for room heating. What is the heating potential of this electric power? (12)
- (b) With neat sketches describe the working of an ideal vapor-compression refrigeration cycle. (13)
- (c) Discuss the differences between the ideal and the actual vapor-compression cycles. (10)
2. (a) Derive the Maxwell Relations. (9)
- (b) Show that $C_p - C_v = T \left(\frac{\partial P}{\partial T} \right)_v \left(\frac{\partial V}{\partial T} \right)_p$. (16)
- (c) Write a short note on Joule-Thomson co-efficient (include inversion temperature and related terms in your answer). (10)
3. (a) Discuss the different ways of improving the efficiency of a simple steam power plant. (15)
- (b) After using all the ways of improving the efficiency of a simple steam power plant (as discussed in part (a) of this question), the efficiency still remains much below 100%. Discuss the underlying factors responsible for this. (8)
- (c) Describe the operation of a dual-cycle power plant. Properly explain why the dual-cycle efficiency is higher than the simple cycle ones. (12)
4. (a) Derive the expression for thermal efficiency of air-standard diesel cycle. (20)
- (b) Derive the expression for thermal efficiency of a simple gas-turbine cycle. (15)

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SECTION - B

There are **FOUR** questions in this section. Answer any **THREE**.

Property tables are provided.

5. (a) Write short notes on (3×4=12)
- (i) control mass and control volume
 - (ii) extensive and intensive properties
 - (iii) zeroth law of thermodynamics
- (b) Draw and explain P-V and P-T diagrams for a typical pure substance. (12)
- (c) What do you understand by the quality of a vapour-liquid mixture? (3)
- (d) A 20.0-m³ tank contains nitrogen at 23°C and 600 kPa. Some nitrogen is allowed to escape until the pressure in the tank drops to 400 kPa. If the temperature at this temperature at this point is 20°C, determine the amount of nitrogen that has escaped. (8)
6. (a) Derive the energy equation for a steady-flow system. (15)
- (b) An adiabatic air compressor is to be powered by a direct-coupled adiabatic steam turbine that is also driving a generator. Steam enters the turbine at 12.5 MPa and 500°C at a rate of 25 kg/s and exits at 10 kPa and a quality of 0.92. Air enters the compressor at 98 kPa and 295 K at a rate of 10 kg/s and exits at 1 MPa and 620 K. Determine the net power delivered to the generator by the turbine. [Property tables are provided] (20)
7. (a) Write down the "Kelvin-Planck" and "Clausius" statements of the second law of thermodynamics. Show that both the statements are equivalent. (2+2+6=10)
- (b) An inventor claims to have developed a resistance heater that supplies 1.2 kWh of energy to a room for kWh of electricity it consumes. Is this a reasonable claim? (5)
- (c) Define reversible and irreversible processes? What are the irreversibilities we experience in an irreversible process? (4+6=10)
- (d) An inventor claims to have developed a heat engine that receives 700 kJ of heat from a source at 500 K and produces 300 kJ of net work while rejecting the waste heat to a sink at 290 K. is this a reasonable claim? (10)
8. (a) How can you save work by using multistage compression with intercooling? Explain graphically. (10)
- (b) Draw and explain typical T-S and h-S diagram for a pure substance. (10)
- (c) A 50-kg iron block ($C_p = 0.45$ kJ/kg. °C) and a 20-kg copper block ($C_p = 0.386$ kJ/kg. °C), both initially at 80°C, are dropped into a large lake at 15°C. Thermal equilibrium is established after a while as a result of heat transfer between the blocks and the lake water. Determine the total entropy change for this process. (15)
-

For Q 6(b)

TABLE A-17

Ideal-gas properties of air

T K	h kJ/kg	P _r	u kJ/kg	v _r	s° kJ/kg · K	T K	h kJ/kg	P _r	u kJ/kg	v _r	s° kJ/kg · K
200	199.97	0.3363	142.56	1707.0	1.29559	580	586.04	14.38	419.55	115.7	2.37348
210	209.97	0.3987	149.69	1512.0	1.34444	590	596.52	15.31	427.15	110.6	2.39140
220	219.97	0.4690	156.82	1346.0	1.39105	600	607.02	16.28	434.78	105.8	2.40902
230	230.02	0.5477	164.00	1205.0	1.43557	610	617.53	17.30	442.42	101.2	2.42644
240	240.02	0.6355	171.13	1084.0	1.47824	620	628.07	18.36	450.09	96.92	2.44356
250	250.05	0.7329	178.28	979.0	1.51917	630	638.63	19.44	457.78	92.84	2.46048
260	260.09	0.8405	185.45	887.8	1.55848	640	649.22	20.64	465.50	88.99	2.47716
270	270.11	0.9590	192.60	808.0	1.59634	650	659.84	21.86	473.25	85.34	2.49364
280	280.13	1.0889	199.75	738.0	1.63279	660	670.47	23.13	481.01	81.89	2.50985
285	285.14	1.1584	203.33	706.1	1.65055	670	681.14	24.46	488.81	78.61	2.52589
290	290.16	1.2311	206.91	676.1	1.66802	680	691.82	25.85	496.62	75.50	2.54175
295	295.17	1.3068	210.49	647.9	1.68515	690	702.52	27.29	504.45	72.56	2.55731
298	298.18	1.3543	212.64	631.9	1.69528	700	713.27	28.80	512.33	69.76	2.57277
300	300.19	1.3860	214.07	621.2	1.70203	710	724.04	30.38	520.23	67.07	2.58810
305	305.22	1.4686	217.67	596.0	1.71865	720	734.82	32.02	528.14	64.53	2.60319

For Q 6(b)

TABLE A-5

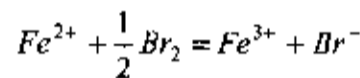
Saturated water—Pressure table

Press., P kPa	Sat. temp., T _{sat} °C	Specific volume, m ³ /kg		Internal energy, kJ/kg			Enthalpy, kJ/kg			Entropy, kJ/kg · K		
		Sat. liquid, v _f	Sat. vapor, v _g	Sat. liquid, u _f	Evap., u _{fg}	Sat. vapor, u _g	Sat. liquid, h _f	Evap., h _{fg}	Sat. vapor, h _g	Sat. liquid, s _f	Evap., s _{fg}	Sat. vapor, s _g
1.0	6.97	0.001000	129.19	29.302	2355.2	2384.5	29.303	2484.4	2513.7	0.1059	8.8690	8.9749
1.5	13.02	0.001001	87.964	54.686	2338.1	2392.8	54.688	2470.1	2524.7	0.1956	8.6314	8.8270
2.0	17.50	0.001001	66.990	73.431	2325.5	2398.9	73.433	2459.5	2532.9	0.2606	8.4621	8.7227
2.5	21.08	0.001002	54.242	88.422	2315.4	2403.8	88.424	2451.0	2539.4	0.3118	8.3302	8.6421
3.0	24.08	0.001003	45.654	100.98	2306.9	2407.9	100.98	2443.9	2544.8	0.3543	8.2222	8.5765
4.0	28.96	0.001004	34.791	121.39	2293.1	2414.5	121.39	2432.3	2553.7	0.4224	8.0510	8.4734
5.0	32.87	0.001005	28.185	137.75	2282.1	2419.8	137.75	2423.0	2560.7	0.4762	7.9176	8.3938
7.5	40.29	0.001008	19.233	168.74	2261.1	2429.8	168.75	2405.3	2574.0	0.5763	7.6738	8.2501
10	45.81	0.001010	14.670	191.79	2245.4	2437.2	191.81	2392.1	2583.9	0.6492	7.4996	8.1488
15	53.97	0.001014	10.020	225.93	2222.1	2448.0	225.94	2372.3	2598.3	0.7549	7.2522	8.0071
20	60.06	0.001017	7.6481	251.40	2204.6	2456.0	251.42	2357.5	2608.9	0.8320	7.0752	7.9073
26	64.96	0.001020	6.2034	271.93	2190.4	2462.4	271.96	2345.5	2617.5	0.8932	6.9370	7.8302
30	69.09	0.001022	5.2287	289.24	2178.5	2467.7	289.27	2335.3	2624.6	0.9441	6.8234	7.7675
40	75.86	0.001026	3.9933	317.58	2158.8	2476.3	317.62	2318.4	2636.1	1.0261	6.6430	7.6691
50	81.32	0.001030	3.2403	340.49	2142.7	2483.2	340.54	2304.7	2645.2	1.0912	6.5019	7.5931

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SECTION - AThere are **FOUR** questions in this section. Answer any **THREE**.

1. (a) What are reversible and irreversible cells? Explain with suitable examples. (6)
 (b) Derive an expression for the variation of emf of a cell with temperature. (14)
 (c) Show the half cell reactions and overall reactions for the following cells: (6)
 (i) $\text{Pb}, \text{PbCl}_2(\text{s}) ; \text{KCl}(\text{aq}) | \text{AgCl}(\text{s}), \text{Ag}$
 (ii) $\text{Pt} | \text{Cu}^+, \text{Cu}^{2+} || \text{Cu}^+ | \text{Cu}$
 (d) Formulate a cell for which the following reaction could occur: (9)



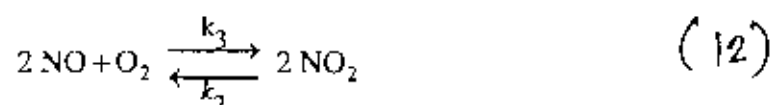
Determine E^0 , ΔG^0 and K for the reaction at 25°C (Given: $E_{\text{Pt}|\text{Fe}^{2+}, \text{Fe}^{3+}}^0 = -0.77$ volt and $E_{\text{Pt}, \text{Br}_2(\text{l})|\text{Br}^-}^0 = 1.065$ volt).

2. (a) What do you mean by the term "concentration cell"? Explain with a suitable example. (8)
 (b) Discuss how you can determine the activity coefficient of an electrolyte from emf measurement of a cell. (17)
 (c) Calculate the mean activity coefficient of 0.1 m hydrochloric acid at 25°C . Given that the emf of the cell. (10)
 $\text{H}_2(1 \text{ atm}) | \text{HCl}(\text{aq}) | \text{AgCl}(\text{s}) | \text{Ag}$
 is 0.3524 volt at 25°C and the standard electrode potential of Ag-AgCl is 0.2224 volt at 25°C .
3. (a) State and explain the Ostwald dilution law. How can you determine the values of K and Λ_c for weak electrolytes with the help of this law. (The symbols bear usual meaning.) (16)
 (b) What is meant by salting-in effect? Show that the solubility of a sparingly soluble salt increases with the increasing concentration of an electrolyte having no common ion. (12)
 (c) The solubility of silver iodate in pure water at 25°C is $1.77 \times 10^{-4} \text{ mol L}^{-1}$. Calculate the solubility in the presence of $0.3252 \times 10^{-2} \text{ mol L}^{-1}$ of potassium nitrate. (7)
4. (a) State the phase rule. Explain the terms present in it and derive it. (12)
 (b) Draw the phase diagram of a system of water and a salt which gives two or more hydrates. Briefly discuss the important information that can be obtained from the diagram. (15)
 (c) Define the following terms: (i) Triplice point (ii) Eutectic point, (iii) Congruent melting point and (iv) Peritectic change. (8)

CHEM 235/CHE**SECTION – B**

There are **FOUR** questions in this section. Answer any **THREE**.

5. (a) What is a zero order reaction? Show that for a zero order reaction the concentration varies linearly with time. (2+6=8)
- (b) For the following reaction the forward reaction is third order and the reverse reaction is second order. Derive the rate expression and show how the rate expression can be used to determine the value of rate constants k_2 and k_3



- (c) Mn^{2+} is produced in the reaction of potassium permanganate with oxalic acid in presence of sulphuric acid acts as a catalyst. Show the mechanism of the catalysis. (7)
- (d) Show the steps in the heterogeneous catalysis according to adsorption theory of catalysis. (8)
6. (a) What are the origins of adsorption spectra of inorganic compounds during the analysis in UV-Visible spectrophotometer. (12)
- (b) What is an auxochrome? What are the main functions of auxochrome? (3+6=9)
- (c) What is the rigid rotor model of a vibrating diatomic molecule? What happens when the molecule do not obey the ideal rigid rotor model? (8+6=14)
7. (a) What is multilayer adsorption? Why solid surface follow multilayer adsorption rather than monolayer? (3+5=8)
- (b) Derive an expression which follow Langmuir adsorption isotherm for a dynamic adsorption-desorption process. How the equation can be applied for the determination of surface area? (10)
- (c) Write the mathematical expression of phase rule for condensed system with brief justification. (4)
- (d) Discuss the thermal analysis method for the determination of equilibrium conditions between solid and liquid phases with suitable diagrams. (13)
8. (a) Discuss the chronological development of the theories of electrolytic conduction. Which law supports the modern theory of electrolytic conduction? Briefly describe it. (14)
- (b) Describe a method by which you can determine the conductance of an electrolytic solution. What is cell constant? (8+2=10)
- (c) Describe in details the applications of conductance measurement of electrolytic solution. (11)

SECTION - A

There are **FOUR** questions in this section. Answer any **THREE**.

1. (a) Evaluate the voltage V_A and current I_A in the circuit shown in Fig. 1(a)

(15)

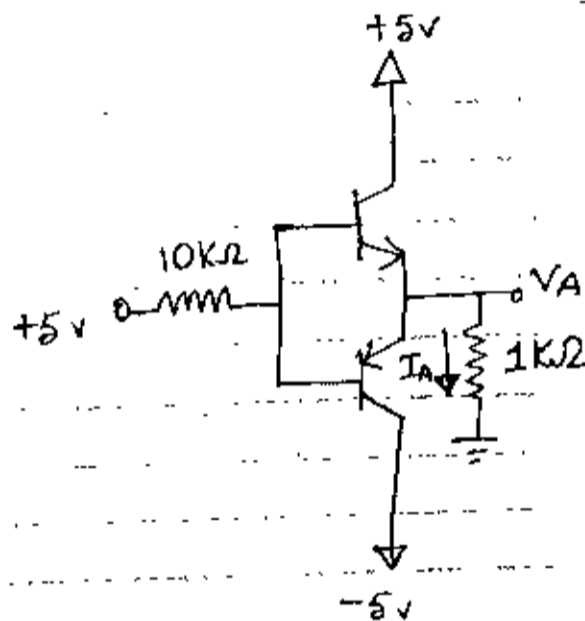


Fig. for Q. 1(a)

- (b) Determine the voltage gain and I/O impedance of the circuit shown in Fig. 1(b).

(20)

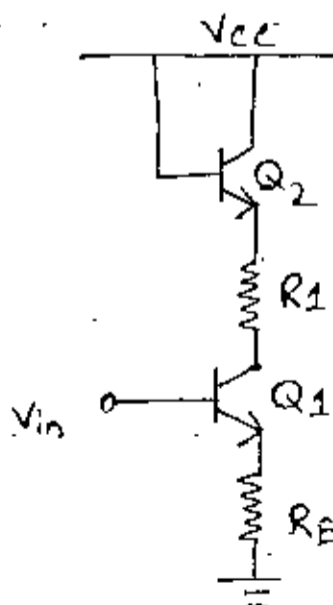


Fig. for Q. 1(b)

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2. (a) Design an analog computer to solve the following differential equation. (20)

$$\frac{d^2 v_0}{dt^2} + 3 \frac{dv_0}{dt} + v_0 = 0, \quad t > 0$$

Subject to $v_0(0) = 4$ and $v_0'(0) = 0$.

- (b) The triangular wave form of Fig. 2(a) is applied to the input of the op-Amp differentiator in Fig. 2(b). Plot the output. (15)

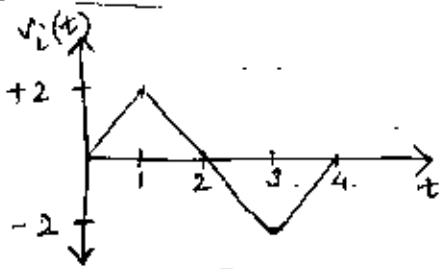


Fig. for Q. 2(a)

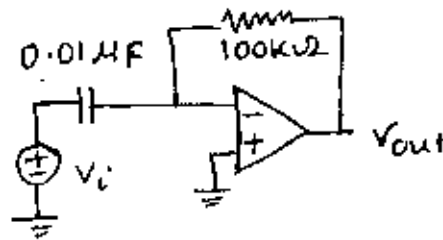


Fig. for Q. 2(b)

3. (a) Design a diode circuit within the "Black Box" in Fig. 3(a). (17)

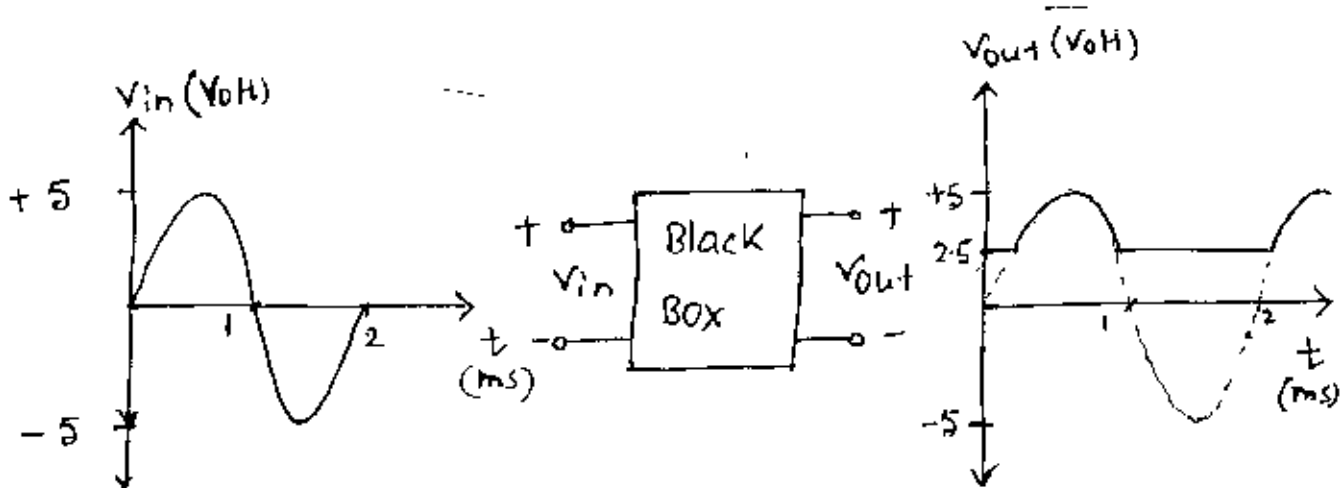


Fig. for Q. 3(a)

- (b) Design a diode circuit to implement the following I/O relation: $\frac{V_{out}}{|V_{in}|} = 1$. (18)

4. (a) Define "Transducer". Discuss the working principle of capacitive transducer. (10)

- (b) Define "Piezoelectricity". Show how it can be employed to measure temperature and force. (15)

- (c) Draw V_{out} for the circuit in Fig 4(c). (10)

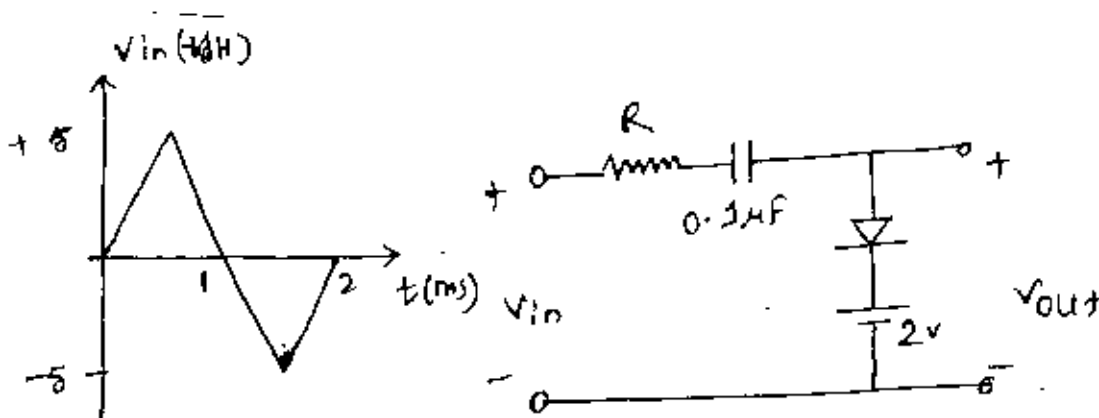


Fig. for Q. 4(c)

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SECTION – B

There are **FOUR** questions in this section. Answer any **THREE**.

5. (a) A three phase, balanced Y-connected motor takes 10 kVA at 0.7 power-factor lagging from a source of 220 volts. It is paralleled with a balanced three phase delta load having impedance of $(15-j10)\Omega$ in each phase. Find (i) the total kVA (ii) line current (iii) power-factor of the combination. (20)

(b) Prove that, single phase system uses 33 percent more material than three phase system to deliver the same amount of power with fixed line loss for the same line voltage. (15)

6. (a) What is armature reaction? How does it affect the operation of DC machines? Explain with appropriate diagrams. (12)

(b) What are compensating windings? Briefly explain how they can solve the problem of commutation in DC machines. (11)

(c) A 200-kVA, 480-V, 50 Hz, Y-connected synchronous generator with rated field current of 5 A was tested. The following data were taken: (12)

(i) Open circuit terminal voltage, $V_{T,OC} = 540$ V at $I_f = 5$ A.

(ii) Short circuit current, $I_{L,SC} = 300$ A at $I_f = 5$ A.

(iii) When a DC voltage of 10 V was applied to two of the terminals, a current of 20 A was measured.

Find the values of armature resistance and synchronous reactance.

7. (a) 'Open circuit test minimizes copper loss' — explain. (5)

(b) Draw phasor diagrams of a transformer operating at different power factor of load. How does the power factor of load affect the voltage regulation of a transformer? (10)

(c) A 75-kVA, 4600-230 V, 60 Hz single phase transformer has been tested to determine its equivalent circuit. Test results are: (20)

<u>Open Circuit Test</u>	<u>Short Circuit Test</u>
$V_{OC} = 230$ V	$V_{SC} = 160.8$ V
$I_{OC} = 13.04$ A	$I_{SC} = 16.3$ A
$P_{OC} = 521$ Watt	$P_{SC} = 1200$ Watt

(i) Find the impedance of the approximate equivalent circuit referred to the high side, and sketch the circuit.

(ii) Find the voltage regulation of the transformer at rated conditions at 0.75 power factor lagging.

(iii) Find the efficiency of the transformer at rated conditions at 0.75 power factor lagging.

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8. (a) Derive the torque-speed characteristic of a series DC motor. (12)

(b) Briefly explain how speed of a shunt DC motor can be varied by changing the armature voltage. (6)

(c) A 100-hp, 250 V, 1200 rpm shunt DC motor with compensating windings has an armature resistance of 0.03Ω and field resistance of 41.67Ω . The motor is assumed to be driving a load with a line current of 126 A and an initial speed of 1103 rpm. Assume that change in field resistance does not affect the armature current. The magnetizing curve for this motor, taken at a speed of 1200 rpm is given in tabular form below: (17)

E_A, V	83	125	180	220	250	270	280	284
I_F, A	1	2	3	4	5	6	7	8

- (i) Find the armature current and internal generated voltage of the motor.
 - (ii) What will be the value of motor speed if the field resistance is increased to 50Ω ?
 - (iii) Determine the value of field resistance for the motor to run at its rated speed.
-

SECTION - AThere are **FOUR** questions in this section. Answer any **THREE**.

1. (a) The line that joins one vertex of a parallelogram to the middle of an opposite side trisects the diagonal – Prove vectorially. (20)
- (b) A force of 6 units acts through the point $P(3, -2, 5)$ in the direction of the vector $(7, 6, -1)$. Find its moment about the point $A(1, -3, \frac{5}{2})$ and the moment about axes through that point parallel to the coordinate axes. (16 $\frac{2}{3}$)
- (c) Give the geometrical interpretation of the scalar triple product. (10)
2. (a) State and prove Frenet-Serret formulae. (21 $\frac{2}{3}$)
- (b) If $P = A\cos kt + B\sin kt$, where A and B are constant vectors and k , a constant scalar, show that $\frac{d}{dt}\left(P \times \frac{dP}{dt}\right) = 0$. (10)
- (c) Find $\nabla^2(r^n)$ where r is the position vector. (15)
3. (a) Find the angle of intersection at the point $(-3, 0, -5)$ of the spheres $x^2 + y^2 + z^2 + 6x - 5y + 2z + 27 = 0$ and $x^2 + y^2 + z^2 - 29 = 0$. (15)
- (b) Show that the gradient of a scalar function f is a vector along the normal to the level surface whose magnitude is the greatest rate of change of f . (16 $\frac{2}{3}$)
- (c) Find curl of F where $F = (x^2 - y^2 + 2xz) i + (xz - xy + yz) j + (z^2 + x^2) k$. Also show that the vectors given by curl F at the points $P(1, 2, -3)$ and $Q(2, 3, 12)$ are orthogonal. (15)
4. (a) State and verify Green's theorem in the plane for $\int_C (2x - y^3) dx - xy dy$ where C is the boundary of the region enclosed by $x^2 + y^2 = 1$ and $x^2 + y^2 = 4$. (26 $\frac{2}{3}$)
- (b) Evaluate $\iiint_V (2x + y) dV$ where V is the closed region bounded by the cylinder $z = 4 - x^2$ and the planes $x = 0, y = 0, y = 2$ and $z = 0$. (20)



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SECTION – B

There are **FOUR** questions in this section. Answer any **THREE**.

5. (a) Reduce $A = \begin{bmatrix} 1 & -2 & 1 & 3 \\ 4 & -1 & 5 & 8 \\ 2 & 3 & 3 & 2 \end{bmatrix}$ (20)

to the normal form B and compute the matrices P and Q such the PAQ = B, where A and B are equivalent matrices.

(b) Give $A = \begin{bmatrix} 1 & 2 & 3 \\ 0 & 3 & 2 \\ 1 & 5 & 4 \end{bmatrix}$, (16)

find A^{-1} by algebraic method.

(c) Find the matrix A satisfying the following matrix equation

$\begin{bmatrix} 2 & 1 \\ 3 & 2 \end{bmatrix} A \begin{bmatrix} -3 & 2 \\ 5 & -3 \end{bmatrix} = \begin{bmatrix} -2 & 4 \\ 3 & -1 \end{bmatrix}$. (10½)

6. (a) For what values of λ and μ , the following system of equations (15)

$$\begin{aligned} 2x + 3y + 5z &= 9 \\ 7x + 3y - 2z &= 8 \\ 2x + 3y + \lambda z &= \mu \end{aligned}$$

will have (i) unique solution (ii) no solution.

(b) Show that the quadratic form $q = x_1^2 + 5x_2^2 + 6x_3^2 + 4x_1x_2 - 8x_2x_3 - 6x_3x_1$ is indefinite and find two sets of values of x_1, x_2, x_3 for which the form assumes positive and negative values. (15)

(c) State and prove Cayley's Hamilton theorem and also verify the above theorem for the matrix $A = \begin{bmatrix} 1 & 2 \\ 3 & 2 \end{bmatrix}$ and hence find A^{-1} . (16½)

7. (a) Evaluate (8+8+8=24)

(i) $L\left\{\int_0^t f(u) du\right\}$

(ii) $L\left\{\frac{f(t)}{t}\right\}$

(iii) $L\{J_0(at)\}$

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Contd... Q. No. 7

(b) Using laplace transformation, show that $\int_0^{\infty} \frac{e^{-xt}}{\sqrt{t}(1+t)} dt = \pi e^{-x} \operatorname{erfc}(\sqrt{x})$. (16 $\frac{2}{3}$)

(c) Find $L^{-1}\left\{\frac{1}{(s^2 + 2s + 5)^{\frac{3}{2}}}\right\}$. (6)

8. (a) Using laplace transform, evaluate $\frac{1}{\pi} \int_0^{\pi} \cos(t \cos \theta) d\theta$. (16)

(b) Solve (using laplace transform) (20 $\frac{2}{3}$)

$$\begin{cases} 2x(t) - y(t) - y'(t) = 4(1 - e^{-t}) \\ 2x(t) + y(t) = 2(1 + 3e^{-2t}) \\ x(0) = y(0) = 0 \end{cases}$$

(c) Solve: $t y''(t) + y'(t) + 4t y(t) = 0$; $y(0) = 3$, $y'(0) = 0$ using laplace transformation. (10)
