L-1/T-1/EEE **Date : 10/07/2013**

BANGLADESH UNIVERSITY OF ENGINEERING AND TECHNOLOGY, DHAKA

L-1/T-1 B. Sc. Engineering Examinations 2012-2013

Sub : **EEE 101** (Electrical Circuits I)

Full Marks : 210 Time : 3 Hours

The figures in the margin indicate full marks.

USE SEPARATE SCRIPTS FOR EACH SECTION

SECTION – A

There are **FOUR** questions in this Section. Answer any **THREE**.

1. (a) Find the Thevenin equivalent of the circuit shown in Fig. Q 1(a) at terminal ab. **(21)**

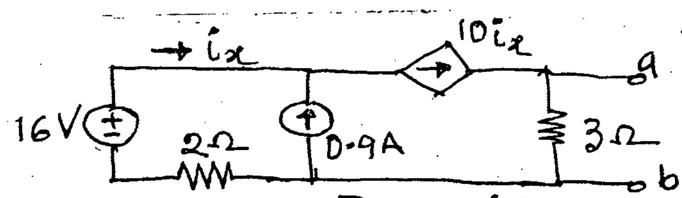

Fig. Q1(a).

(b) Find the capacitor voltage that is associated with the current shown graphically in Fig. Q 1 (b). Given: C = 5 μF. **(14)**

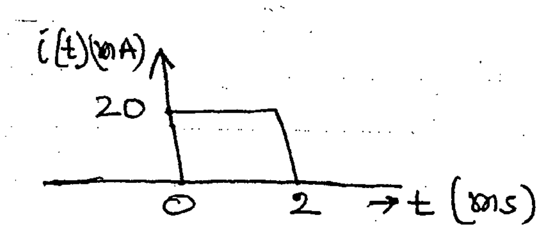

Fig. 1(b).

2. (a) A long time after all connections have been mode in the circuit shown in Fig. Q 2(a), find v_x if (i) a capacitor is present between x and y and (ii) an inductor is present between x and y. **(20)**

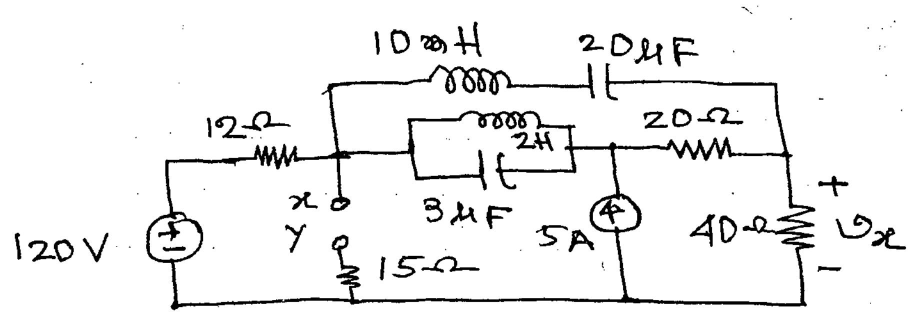

Fig. Q 2(a)

(b) The circuit of Fig. Q 2(b) contains five identical capacitors. Find the value of C. **(15)**

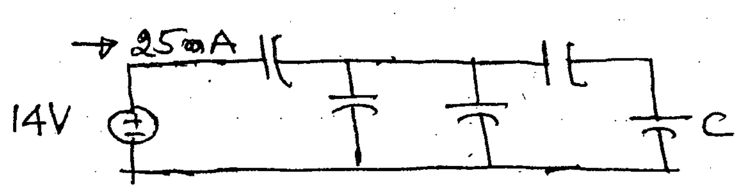

Fig. Q 2(b)

Contd P/2

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3. (a) Find the capacitor voltage v_c (t) and the current i(t) in the 200 Ω resistor of Fig. Q. 3(a). Sketch v_c (t) and i(t). **(20)**

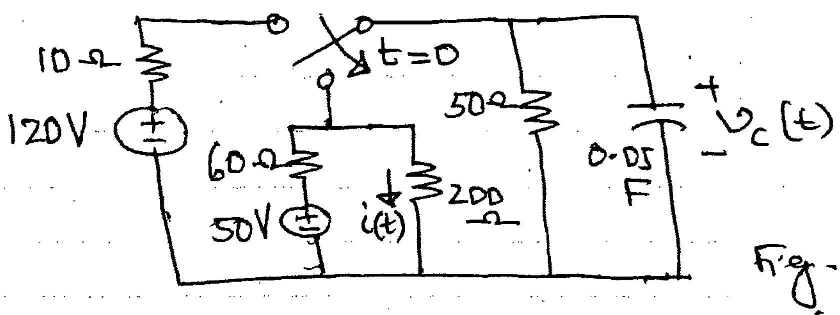

Fig. Q 3(a)

(b) Select values for R_1 and R_2 in the circuit of Fig. Q 3(b) so that $v_R(0^+) = 10$ V and v_R (1 ms) = 5V. **(15)**

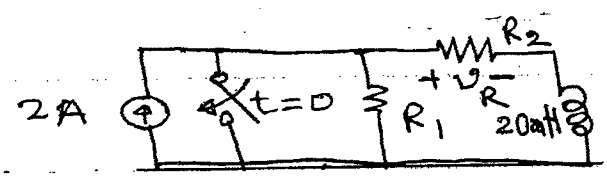

Fig. Q3(b).

4. (a) Find i_x (t) for t ≥ 0 for the circuit shown in Fig. Q 4(a). **(18)**

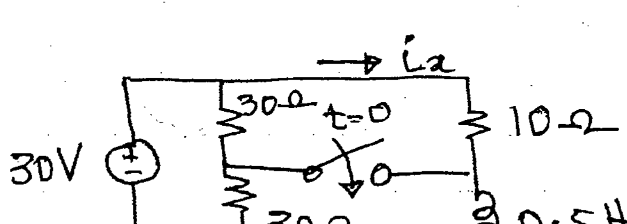

Fig. Q4(a)

(b) Find v_0 (t) for t ≥ 0 in the circuit shown in Fig. Q 4(b). **(17)**

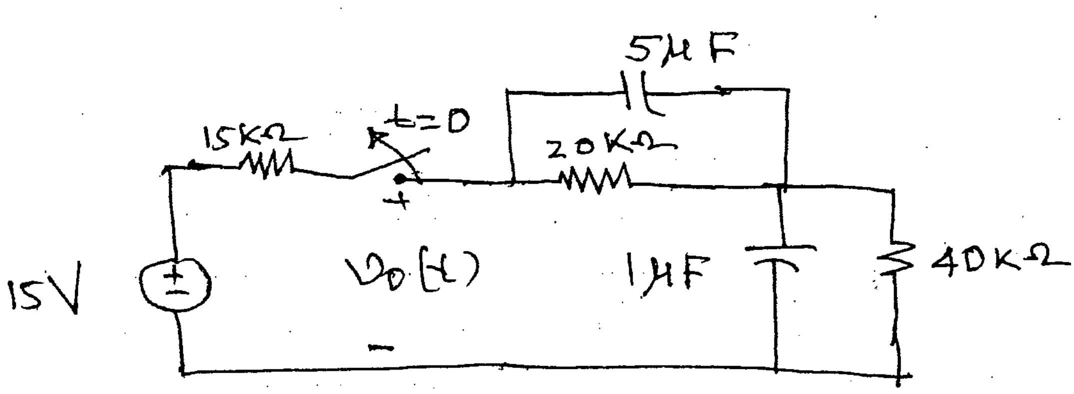

Fig. Q4(b)

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SECTION – B

There are **FOUR** questions in this Section. Answer any **THREE**.

5\. (a) Find the equivalent resistance between terminals 'a' and 'b' in the circuit shown in Fig. for Q. 5(a) **(17)**

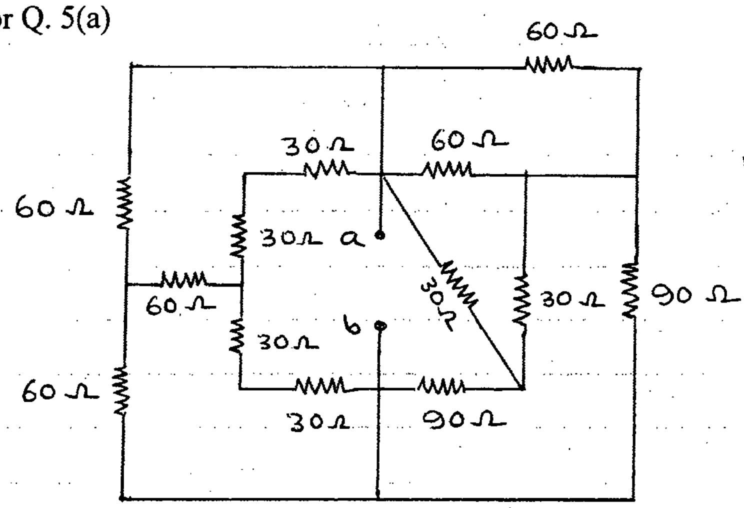

Fig. for Q. 5(a)

(b) Find the value of R in the circuit shown in Fig for Q. 5(b). **(18)**

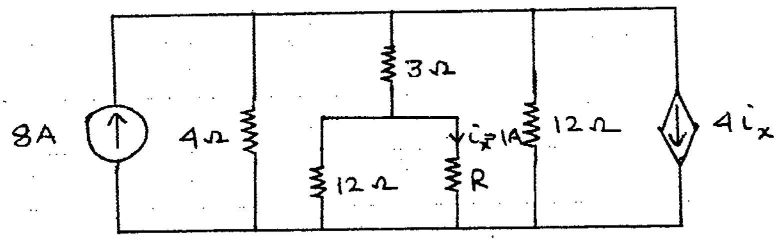

Fig. for Q. 5(b)

6\. (a) Find v_1, v_2 and v_3 using nodal analysis for the circuit shown in Fig. for Q. 6(a). **(17)**

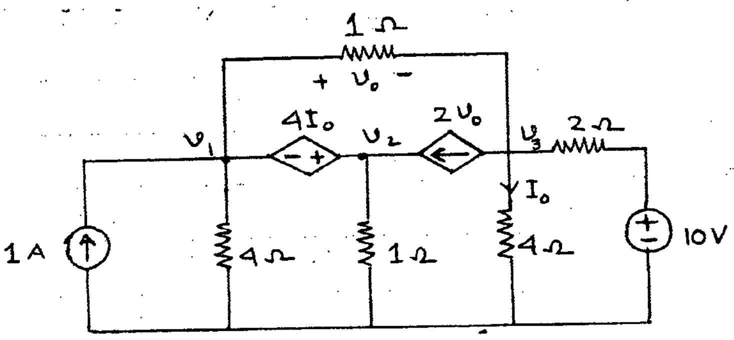

Fig. for Q. 6(a)

(b) Find I_x using mesh analysis for the circuit shown in Fig. for Q 6(b). **(18)**

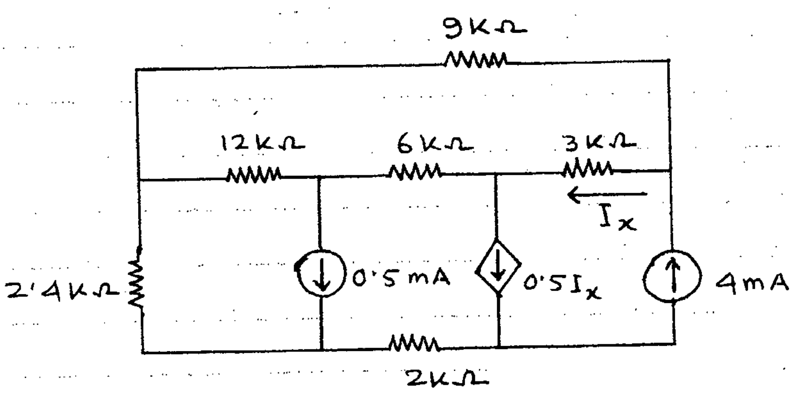

Fig. for Q. 6(b)

EEE 101

7. (a) Apply source transformations and then find v_1 and v_2 by nodal analysis for the circuit shown in Fig. for Q. 7(a). **(17)**

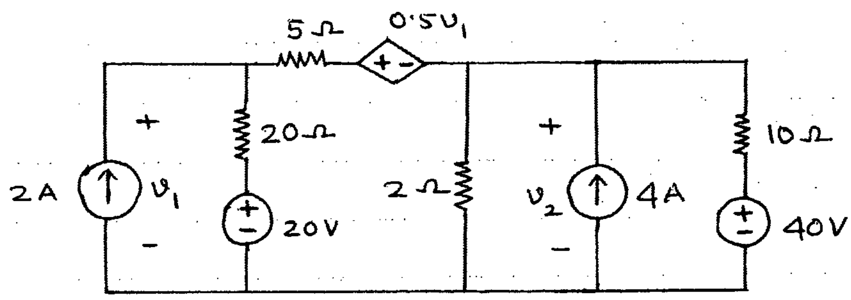

Fig. for Q. 7(a)

(b) Find V_0 using superposition theorem for the circuit shown in Fig. for Q. 7(b) **(18)**

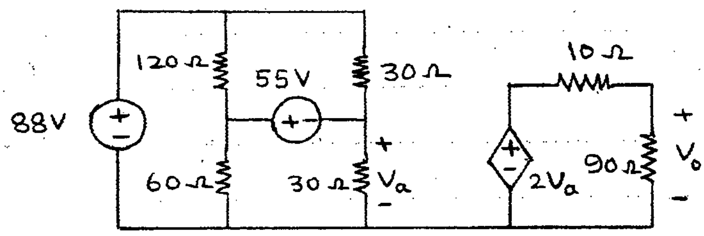

Fig. for Q.7(b)

8. (a) Find I_0 using Norton's theorem for the circuit shown in Fig. for Q. 8(a). **(17)**

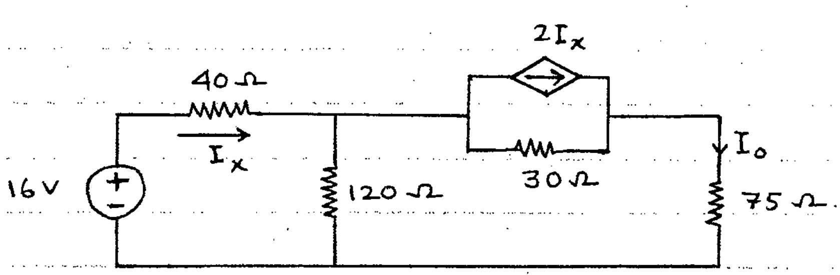

Fig. for Q. 8(a)

(b) Find the value of R_L for maximum power transfer to R_L in the circuit shown in Fig. for Q. 8(b). Also find the value of maximum power delivered to R_L. **(18)**

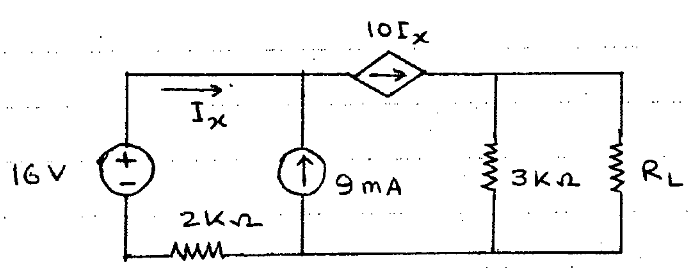

Fig. for Q. 8(b)

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L-1/T-1/EEE **Date : 25/09/2013**

BANGLADESH UNIVERSITY OF ENGINEERING AND TECHNOLOGY, DHAKA

L-1/T-1 B. Sc. Engineering Examinations 2012-2013

Sub : **CSE 109** (Computer Programming)

Full Marks : 210 Time : 3 Hours

The figures in the margin indicate full marks.

USE SEPARATE SCRIPTS FOR EACH SECTION

SECTION – A

There are **FOUR** questions in this section. Answer any **THREE**.

1. (a) A number is called AMAZING if one of the following condition is satisfied: **(10)**

(i) The number is not divisible by 5

(ii) The number is divisible by 3 and not divisible by 21.

Write a program that takes a positive integer as input and prints if it is AMAZING number or not.

For example:
Input: 4
Output: Amazing Number
Input: 5
Output: Not Amazing Number

(b) [Continuation of 1.a] Suppose &,| and ! keys of your keyboard are broken, that is you can not use logical 'and', 'or', 'not' operators. So rewrite your program in 1.a not using any of these three operators. **(13)**

(c) Given ages of 100 persons as input, a person is considered to be in "Class – A" if his/her age is under 18; otherwise she/he is in "Class – B". Write a program to find out the average age of "Class – A" and "Class – B" separately. You may assume that there will be at least one person in each class. **(12)**

2. (a) You are given 100 positive integers as input. All these numbers are in the range of 1 to 100. Find out the numbers in the range of 1 to 100, that are missing in the input. **(10)**

(b) Find out the output of the following program: **(15)**

```
# include <stdio.h>
int u [3], arr [3];
void f (int at, int limit)
{
        int i;
        if (at = = limit)
        {
                for (i = 0; i  < limit; i ++)
                        printf ("%d", arr [i]);
                printf ("\n");
                return;
        }
```

Contd P/2

CSE 109

Contd ... Q. No. 2(b)

```
        for (i = 0; i <limit; i++)
        {
                if (u [i] = = 0)
                {
                        u [i] = 1;
                        arr [at] = i;
                        f(at + 1, limit);
                        u [i] = 0;
                        arr [at] = 0;
                }
        }
}
int main ( )
{
        f(0, 3);
        return 0;
}
```

(c) Write a program that takes a string as input and prints its length. [You can not use any library function to compute length]. **(10)**

3. (a) Explain the use of pointer and reference. Also explain advantages/disadvantages of pointer over reference. **(10)**

(b) Write a program that takes a string as input and reverses the string. Use pointer to solve this problem. [You can not use array indexing. You can not use any function of string.h header file] **(10)**

For example:
input: abcdef
output: fedcba

(c) Given a 2D integer array of size 10 × 10, write a program to sort the 2D array in row major order (That is after sorting the numbers you first fill up the first row from left to right, then next row and so on). For example: **(15)**

input (for 3 × 3 array):
5 6 0
1 2 3
9 6 0
output:
0 0 1
2 3 5
6 6 9

[You may use additional array]

Contd P/3

CSE 109

4. (a) Write a function that takes four parameters, first three of them should be integers and the last one should be a pointer to an integer. The function should set the value of the variable referenced by the pointer to the difference of maximum and minimum of the three integer numbers. **(15)**

(b) Consider the following code snippet: **(10×2=20)**

```
int A [ ] = {2, 6, 5, 1, 3}
int *p = A;
```

Write down the meaning/value of the following expressions:

(i) *p
(ii) *p + 2
(iii) *(p + 2)
(iv) &p
(v) &p + 1
(vi) p
(vii) &A[0]
(viii) &A[0] + 3
(ix) A + 3
(x) p + (*(p + 4) – 3)

SECTION – B

There are **FOUR** questions in this section. Answer any **THREE**.

5. (a) A file "source.c" contains a valid C program with multiline comments in /* and */ structure. Write a C program that copies the C code except all multiline comments to another file named "clean.c". Assume that the source file will not contain comment structure within strings. **(20)**

(b) Define a structure named **Student** in C that contains the following members **(5)**

(i) 80 element character array named **name**
(ii) unsigned int called **id**
(iii) floating point called **gpa**

(c) Write a function in C sort an array of **Student** defined in question 5(b) on the basis of **gpa** in decreasing order, if two **Students** have same **gpa** then sort them according to increasing order of **id**. **(10)**

6. (a) Assume that CSE109 course conductor kept the term final question in a file named "cse109final.txt" in his personal computer. He wants to garble the contents of the file so that the file contents are not comprehensibly by anyone. Write a function in C which swaps the consecutive character pairs of the file. Assume that the **(10)**

(i) file has even number of characters
(ii) file has less than 1000 characters
(iii) function takes the file name as a parameter

You don't have to write the main function.

CSE 109

Contd ... Q. No. 6(a)

For example, If the file content is **CSE109**, then after the function is called the content of the file will be changed to **SC1E90**. So the 1st element is swapped with 2nd element, the 3rd element is swapped with the 4th element and so on.

(b) Arrays in C of N elements have subscripts from 0 to N-1. Other languages (like PASCAL) allows user to specify lower bound of the subscript values. **(3+4+4+4+2=17)**

(i) Define a structure **Array** in C with necessary members which allow other forms of arrays. Each array element will be a floating point value.

(ii) Write a function **set** that takes two integers. The first integer parameter is the lower bound and the second integer parameter is the upper bound of the subscript values. Create the array dynamically with appropriate size. Do not create any array if the upper bound is smaller than the lower bound. Hint: Use **malloc & sizeof** to dynamically create an array.

(iii) Write a function **put** that takes an integer parameters position and a floating point value. The function saves the value in the correct position of the array.

(iv) Write a function **get** that tales an integer parameter position and returns the element in that position of the array.

(v) Write a main function to demonstrate the features.

For example, if the lower bound is –10 and upper bound is 5. Array size will be 16. put(–10, 3.14) will save 3.14 in the 0th element of the array. get(–3) will return 7th element of the array.

Note:

(1) Each array element will be a floating point value

(2) Call by value have no effect on the argument used to call

(c) What are the errors in the following code segment? Give two ways of correcting them. **(8)**

```
# include<iostream.
using namespace std;
class myclass {
        int *a;
public:
        muclass ( )
        {
                this → a = new int;
                cout<<"Constructing\n";
        }
        ~ myclass ( )
        {
                cout<<"Destructing "<<a<<endl;
                delete a;
        }
        void seta (int a)
```

CSE 109

Contd ... Q. No. 6(c)

```
{
        * this->a = a;
}
int geta ( )
{
        return  *a;
}
};
void f (myclass ob)
{
        cout <<ob.geta ( );
}
int main ( )
{
        myclass ob;
        ob.seta (5);
        f (ob);
        return 0;
}
```

7. (a) Define a class **String** that has the following members **(1+1+3+3+2+8+2+2+1+2+4+1=30)**

    (i) Character array **str:** not visible from outside

    (ii) Size of the array **size:** not visible from outside

    (iii) A function that computes the string length

    (iv) Two overloaded version of the function **add** that

    (1) Adds another **String** object to the calling object

    (2) Adds a character to the end of the string

    (v) A function that deletes all the occurrences of a given character in the string

    (vi) Constructors

    (1) Takes a character pointer as parameter

    (2) Takes size as parameter

    (3) Default: initializes **str** as null string

    (4) Takes character array and size as parameters

    (5) Copy constructor

    (vii) Destructor

    You may define additional member functions if needed. No main function is needed. All the member functions of the **String** class must be defined.

    (b) What constraints must be satisfied when overloading operators? **(3)**

    (c) What are the naming conventions of constructor and destructor functions? **(2)**

Contd P/6

CSE 109

8. (a) Define a class **Complex** which overloads the following operators **(4+3×7=25)**

(i) + (adds a **Complex** object with the calling object and returns a **Complex** object)

(ii) + (adds a **double** with the calling object and returns **Complex** object)

(iii) – (subtract a **Complex** object from the calling object and returns a **Complex** object)

(iv) / (divides the calling object by a **Complex** object and returns a **Complex** object)

(v) * (multiplies a **Complex** object with the calling object and returns a **Complex** object)

(vi) ∧ (power: takes an integer parameter **n** and returns a **Complex** object, **n** can be both positive or negative)

(vii) = = (checks if the calling object is equal to a **Complex** object and returns **bool)**

Assume necessary members. All the member functions must be defined.

(b) Consider the following code segment **(2+4+4=10)**

```
Cartesian a, c(3.0, 4.0);
Polar pl, p2 (4.0, 1.0);
c = c + 1.5;
pl = p2 + c;
a = pl;
```

Define the following classes so that the given code segment is valid

(i) class **Coord:** contains two floating point variable x, y

(ii) class **Cartesian:** inherits the class **Coord**

(iii) class **Polar:** inherits the class **Coord**

Declare necessary member functions; no function definition should be given.

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20.09.2013

L-1/T-1/EE **Date : 27/07/2013**

BANGLADESH UNIVERSITY OF ENGINEERING AND TECHNOLOGY, DHAKA

L-1/T-1 B. Sc. Engineering Examinations 2012-2013

Sub : **PHY 121** (Thermal Physics, Optics and Waves and Oscillations)

Full Marks : 210 Time : 3 Hours

The figures in the margin indicate full marks.

USE SEPARATE SCRIPTS FOR EACH SECTION

SECTION – A

There are **FOUR** questions in this section. Answer any **THREE**.

1. (a) Explain the terms: (i) Degrees of freedom and (ii) The law of equipartition of energy of a dynamical system. **(8)**

   (b) Calculate the ratio of the two specific heats for a monoatomic and a diatomic gas. **(20)**

   (c) Calculate the mean free path of a gas molecule in a chamber of 10^{-6} mm of Hg pressure, assuming the molecule diameter to be 0.2 mm. One gram molecule of the gas occupies 22.4 litres at N.T.P. (Avogadro number = 6.023×10^{23} /mole). **(7)**

2. (a) Explain reversible and irreversible processes. **(7)**

   (b) Obtain the expression for the work done for a complete cycle in a Carnot engine and hence find out the efficiency. **(20)**

   (c) A Carnot engine whose low temperature reservoir is at 27 °C has an efficiency of 55%. The efficiency of the engine is to be increased to 70%. By how many degrees should the temperature of the high temperature reservoir be increased? **(8)**

3. (a) State and explain the second law of thermodynamics. **(7)**

   (b) Show that the entropy remains constant in an isolated reversible process and increases in an irreversible process. **(20)**

   (c) At what celcius temperature will oxygen molecules have the same root mean square speed as that of hydrogen molecules at –75 °C? **(8)**

4. (a) What do you mean by the term interference of light? What happens to their energy when two light waves interfere destructively? **(7)**

   (b) Describe in detail Newton's rings experiment for the determination of the wavelength of monochromatic light. Why the central spot in Newton's ring due to reflected light is dark? **(20)**

   (c) A thin film in air is 410 nm thick and is illuminated by white light normal to its surface. Its index of refraction is 1.50. What wavelengths within the visible spectrum will be intensified in the reflected beam? **(8)**

Contd P/2

PHY 121

SECTION – B

There are **FOUR** questions in this section. Answer any **THREE**.

5. (a) What is the resolving power of a diffraction grating? Show that the smallest detail of an object can be resolved in an optical microscope is about the same size as the wavelength of light being used. **(15)**

   (b) Discuss diffraction pattern produced by a circular aperture. **(12)**

   (c) A converging lens 32 mm in diameter has a focal length f of 24 cm. (i) What angular separation must two distant point objects have to satisfy Rayleigh's criterion? Assume that $\lambda = 550$ nm. (ii) How far apart are the centers of the diffraction patterns in the focal plane of the lens? **(8)**

6. (a) What is plane polarized light? Explain the phenomenon of double refraction in a calcite crystal. **(10)**

   (b) Describe the working principle of a Nicol prism. Show how it can be used as a polarizer and as an analyzer. **(15)**

   (c) What do you mean by spherical aberration and chromatic aberration of a lens? How does these defects affect the formation of the image by a lens? **(10)**

7. (a) Define the effective mass of a spring. Show that the motion of a loaded spring is simple harmonic and find the effective mass of the spring during motion. **(18)**

   (b) Find the total energy of a simple harmonic oscillator and show that the average kinetic energy of the oscillator is equal to the half of the total energy. Draw the energy-displacement diagram. **(10)**

   (c) A simple harmonic oscillator consists of a mass 0.47 kg with a spring of force constant 10 N/m has a mechanical energy of 25 mJ. (i) What is the amplitude of the motion? (ii) What is the maximum velocity of the oscillator? and (iii) What is the velocity of the oscillator at a distance of 12 mm from the mean position? **(7)**

8. (a) Define forced vibration with an example. Establish the differential equation of a body executing forced vibration and find its general solution. **(21)**

   (b) Show that the energy density of a plane progressive wave is given by $E = 2\pi^2 \rho n^2 a^2$, where the symbols have their usual meanings. **(8)**

   (c) A 4-kg mass attached to a spring has a force constant 240 N/m and damping constant 0.070 kg/sec. The external periodic force is $F = 2\cos(2\pi t)$, where F is in Newton and t is in sec. Find the amplitude and applied angular frequency of motion. **(6)**

L-1/T-1/EEE Date: 01/10/2013

BANGLADESH UNIVERSITY OF ENGINEERING AND TECHNOLOGY, DHAKA

L-1/T-1 B. Sc. Engineering Examinations 2012-2013

Sub: **MATH 157** (Calculus – I)

Full Marks: 210 Time: 3 Hours

USE SEPARATE SCRIPTS FOR EACH SECTION

The figures in the margin indicate full marks.

SECTION – A

There are **FOUR** questions in this section. Answer any **THREE**.

Symbols used have their usual meaning.

1. (a) Let **(15)**

$$f(x)=\begin{cases} x\sin\frac{1}{x} & , \quad x\neq 0 \\ 0 & , \quad x=0 \end{cases}.$$

Discuss the continuity and differentiability of $f(x)$ at $x = 0$. Also sketch the graph of $f(x)$.

(b) Evaluate

(i) $\lim\limits_{x\to 0}\left\{\frac{1}{x}-\frac{1}{\tan^{-1}x}\right\}$ **(10)**

(ii) $\lim\limits_{x\to 0}\left\{\frac{\tan x}{x}\right\}^{1/x}$. **(10)**

2. (a) If $y=\sinh\left(m\sinh^{-1}x\right)$ then prove that $\left(1+x^2\right)y_{n+2}+(2n+1)xy_{n+1}+\left(n^2-m^2\right)y_n=0$. **(15)**

(b) If $u=\sin^{-1}\frac{x}{y}+\tan^{-1}\frac{y}{x}$, then find the value of $x\frac{\partial u}{\partial x}+y\frac{\partial u}{\partial y}$. **(10)**

(c) Show that $\sin^{-1}x=x+\frac{1.x^3}{2.3}+\frac{1.3.x^5}{2.4.5}+\frac{1.3.5.x^7}{2.4.6.7}+\ldots$. **(10)**

3. (a) If $V=V(x,y), x=r\cos\theta$ and $y=r\sin\theta$, then prove that **(20)**

$$\frac{\partial^2 V}{\partial x^2}+\frac{\partial^2 V}{\partial y^2}=\frac{\partial^2 V}{\partial r^2}+\frac{1}{r}\frac{\partial V}{\partial r}+\frac{1}{r^2}\frac{\partial^2 V}{\partial \theta^2}.$$

(b) A window is in the form of a rectangle surmounted by a semicircle. If the total perimeter be 25 ft., find the dimensions so that the greatest possible amount of light may be admitted. **(15)**

4. (a) Find the pedal equation of the parabola $y^2=4ax$ with regard to the vertex. **(15)**

(b) Find all the asymptotes of $x^3+2x^2y-xy^2-2y^3+4y^2+2xy+y-1=0$. **(10)**

(c) In the curve $x^{m+n}=a^{m-n}y^{2n}$, prove that the m *th* power of the subtangent varies as the n *th* power of the subnormal. **(10)**

Contd P/2

MATH 157

SECTION – B

There are **FOUR** questions in this section. Answer any **THREE**.

5. Integrate the following: **(10+12+13)**

(a) $\int \frac{x^2+1}{x^4+x^2+1}dx$ (b) $\int e^{ax+c}\cos(bx+d)dx$ (c) $\int (x-3)\sqrt{\frac{2x^3-3x+1}{x-1}}\,dx$.

6. (a) Find a reduction formula for $\int x\sin^n x\,dx$. **(12)**

Hence evaluate $\int x\sin^2 x\,dx$.

(b) Evaluate: **(10)**

$$\lim_{n\to\infty}\left[\frac{n^{1/2}}{n^{3/2}}+\frac{n^{1/2}}{(n+3)^{3/2}}+\frac{n^{1/2}}{(n+6)^{3/2}}+\cdots+\frac{n^{1/2}}{\{n+3(n-1)\}^{3/2}}\right].$$

(c) Evaluate: **(13)**

$$\int_0^{\pi}\frac{x\tan x}{\sec x+\tan x}dx.$$

7. (a) Examine for convergence of the integral: **(10)**

$$\int_1^{\infty}\frac{dx}{(1+x)\sqrt{x}}.$$

(b) Prove that, $\beta(m,n)=\frac{\Gamma(m)\Gamma(n)}{\Gamma(m+n)}$. **(12)**

(c) Find the area of the loop of the curve $y^2(a-x)=x^2(a+x)$. **(13)**

8. (a) Find the area enclosed between one arc of the cycloid: **(12)**

$x=a(\theta-\sin\theta),\ y=a(1-\cos\theta)$ and its base.

(b) Find the volume of the solid formed by the revolution of the loop of the curve $y^2(a+x)=x^2(3a-x)$ about the x-axis. **(12)**

(c) The curve $r=a(1+\cos\theta)$ revolves about the initial line. Find the surface area of the figure so formed. **(11)**

Rafra .

L-1/T-1/EEE **Date: 07/10/2013**

BANGLADESH UNIVERSITY OF ENGINEERING AND TECHNOLOGY, DHAKA

L-1/T-1 B. Sc. Engineering Examinations 2012-2013

Sub: **MATH 159** (Calculus II)

Full Marks: 210 Time: 3 Hours

The figures in the margin indicate full marks.

Symbols used have their usual meaning.

USE SEPARATE SCRIPTS FOR EACH SECTION

SECTION – A

There are **FOUR** questions in this section. Answer any **THREE**.

1. (a) Describe and graph the region represented by each of the following:

   (i) $2 < |z - 4 - 5i| < 3$ **(4)**

   (ii) $|z + 1| + |z - 1| \leq 4$ **(4)**

   (b) Show that the function **(9)**

$$f(z) = \begin{cases} \dfrac{x^3(1+i) - y^3(1-i)}{x^2 + y^2}; & z \neq 0 \\ 0 \quad ; & z = 0 \end{cases}$$

   satisfies C-R equations at the point $z = 0$. Is the function f(z) differentiable at $z = 0$? Justify your answer.

   (c) Discuss the analyticity of the function $f(z) = z\bar{z} + \bar{z}^2$ in the complex plane. Also, find the points where f(z) is differentiable but not analytic. **(8)**

   (d) Find the bilinear transformation that maps the points $z_1 = 1$, $z_2 = i$, $z_3 = -1$ onto the points $w_1 = i$, $w_2 = 0$, $w_3 = -i$. Hence find the image of the region $|z| < 1$ under this transformation. **(10)**

2. (a) Show that the function **(18)**

$$u(x, y) = e^{-x}(x \sin y - y \cos y)$$

   is harmonic. Find the harmonic conjugate of u(x, y) and construct the analytic function f(z) whose real part is u(x, y). Also express f(z) in terms of z.

   (b) Evaluate the integral $\int_0^{2+i} (\bar{z})^2 \, dz$ along

   (i) the real axis up to 2 and then vertically up to $2 + i$ **(9)**

   (ii) the line $y = x/2$. **(8)**

Contd P/2

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3. (a) Expand $f(z)=\dfrac{1}{(z-1)(z-3)}$ in a Taylor series about the point $z = 4$ and find the region where the series converges. **(12)**

(b) Find the first four terms of the Laurent series expansion of **(13)**

$$f(z)=\frac{2z+1}{z^3+z^2-2z}$$

in powers of $(z-1)$ and determine the region of convergence of the series.

(c) Use Cauchy's integral formula to evaluate the integral **(10)**

$$\int_C \frac{\cos \pi z^2}{z(z-1)(z-2)(z-3)}\,dz$$

where C is the circle $|z| = \frac{5}{2}$ taken in positive sense.

4. (a) Use Cauchy's Residue theorem to evaluate the following integrals

(i) $\int_C z^4 e^{1/z}\,dz$; C is the circle $|z| = 1$ **(10)**

(ii) $\int_C \frac{\coth z}{z-i}\,dz$; C is the circle $|z| = 2$ **(10)**

(iii) $\int_C \frac{12z-7}{(z-1)^2(2z+3)}\,dz$; C is the circle $|z+i| = 4$, **(15)**

all the circles being taken counterclockwise.

SECTION – B

There are **FOUR** questions in this section. Answer any **THREE**.

5. (a) Prove that the straight line joining the middle points of the diagonals of a trapezium is parallel to parallel sides and half of their difference. **(10)**

(b) Show that any vector **r** can be represented as a linear combination of three non coplanar vectors **a**, **b**, **c**. Hence find a linear relation among the vectors (2, –3, 4), (1, –1, 1), (–1, 1, 1) and (1, 1, 1). **(17)**

(c) Solve the vector equation for **r**: **r** × **b** = **a** × **b**, where **a**, **b** are two given vectors. **(8)**

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6. (a) Given points P(2, 1, 3), Q(1, 2, 1), R(−1, −2, −2) and S(1, −4, 0), find the shortest distance between the lines PQ and RS. **(12)**

(b) Prove that $\nabla \times (\nabla \times \mathbf{A}) = -\nabla^2\mathbf{A} + \nabla(\nabla \cdot \mathbf{A})$. **(11)**

(c) Find the curvature and torsion of the space curve $x = t$, $y = t^2$, $z = \frac{2}{3}t^3$. **(12)**

7. (a) Find the values of the constants a, b, c so that the directional derivative of $\varnothing = axy^2 + byz + cz^2x^3$ at $(1, 2, -1)$ has a maximum magnitude 64 in the direction parallel to z-axis. **(11)**

(b) Prove that $\mathbf{F} = (y^2 \cos x + z^3)\,\mathbf{i} + (2y \sin x - 4)\,\mathbf{j} + (3xz^2 + 2)\,\mathbf{k}$ is a conservative force field. Find the work done in moving an object in this field from $(0, 1, -1)$ to $(\frac{\pi}{2}, -1, 2)$. **(13)**

(c) Evaluate $\int_{(0,0)}^{(\pi,2)} (6xy - y^2)\,dx + (3x^2 - 2xy)\,dy$ along the cycloid $x = \theta - \sin\theta$, $y = 1 - \cos\theta$. **(11)**

8. (a) Verify Gauss's divergence theorem for the function $\mathbf{F} = x^2\mathbf{i} + y^2\mathbf{j} + z^2\mathbf{k}$ over the solid region cut off by the plane $x + y + z = 1$ from the first octant. **(20)**

(b) State Stoke's theorem. Hence evaluate $\iint_S (\nabla \times \mathbf{A}) . \mathbf{n}\, dS$, where

$$\mathbf{A} = (x^2 + y - 4)\mathbf{i} + 3xy\mathbf{j} + (2xz + z^2)\mathbf{k}$$

and S is the surface of the hemisphere $x^2 + y^2 + z^2 = 16$ above the xy plane. **(15)**
