L-1/T-1/ME
Date: 10/07/2013

BANGLADESH UNIVERSITY OF ENGINEERING AND TECHNOLOGY, DHAKA
L-1/T-1 B. Sc. Engineering Examinations 2012-2013

Sub: MATH 161 (Differential Calculus, Three Dimensional Geometry and Vectors)
Full Marks: 280 Time: 3 Hours

The figures in the margin indicate full marks.
Symbols have their usual meanings.

USE SEPARATE SCRIPTS FOR EACH SECTION

SECTION – A

There are FOUR questions in this Section. Answer any THREE.

1. (a) If the function \( f(x) = \begin{cases} 
1; & \text{when } x < 0 \\
1 + \sin x; & \text{when } 0 \leq x < \frac{\pi}{2} \\
2 + \left(x - \frac{\pi}{2}\right)^2; & \text{when } x \geq \frac{\pi}{2}
\end{cases} \) \( \quad (26\%) \)

Discuss the continuity and differentiability of \( f(x) \) at \( x = \frac{\pi}{2} \). Also sketch the graph.

(b) Evaluate, \( \lim_{x \to 0} \cot x \ln \left(\frac{1-x}{1+x}\right) \) \( \quad (10) \)

(c) Find the \( n \)-th derivative of \( y = \sin^4 x \cos^3 x \). \( \quad (10) \)

2. (a) If \( Y = \sin(m \sin^4 x) \), then show that
\( (1-x^2) Y_{n+2} - (2n+1)x Y_{n+1} + (m^2 - n^2) Y_n = 0 \). Also find \( (Y_n)_0 \). \( \quad (15) \)

(b) Apply Maclaurin's theorem to obtain the terms upto \( x^4 \) in the expansion of \( \ln (1 + \sin^2 x) \). \( \quad (10) \)

(c) State and prove Rolle's theorem. Verify Rolle's theorem for \( f(x) = x^2 - 3x + 2 \) in the interval \( (1, 2) \). \( \quad (21\%) \)

3. (a) Find the equation of the tangent line at the inflection points of the function \( y = x^4 - 6x^3 + 12x^2 - 8x \). \( \quad (16\%) \)

(b) Find the area of the triangle formed by the axes and the tangent to the curve \( \frac{x^2}{2} + \frac{y^3}{2} = a^3 \). \( \quad (15) \)

(c) Find the pedal equation of \( r = a(1 + \cos \theta) \). \( \quad (15) \)

4. (a) Show that the asymptotes of the curve \( (x^2 - y^2)^2 = 2(x^2 + y^2) \) form a square. \( \quad (13\%) \)

(b) If \( u = \ln r \), then show that \( \frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} + \frac{\partial^2 u}{\partial z^2} = \frac{1}{r^2} \),
where \( r^2 = (x-a)^2 + (y-b)^2 + (z-c)^2 \). \( \quad (15) \)

(c) If \( u = \sin^{-1} \frac{x+y}{\sqrt{x} + \sqrt{y}} \), then show that \( \frac{\partial u}{\partial x} + \frac{\partial u}{\partial y} = \frac{1}{2} \tan u \). \( \quad (18) \)

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MATH 161(ME)

SECTION - B

There are FOUR questions in this Section. Answer any THREE.

5. (a) If the edges of a rectangular parallelopiped are \( a, \ b, \ c \), show that the angles between the four diagonals are given by \( \cos^{-1}\left(\frac{\pm a^2 \pm b^2 \pm c^2}{a^2 + b^2 + c^2}\right) \).

(b) A variable plane is at a constant distance \( p \) from the origin \( O \) and meets the axes in \( A, \ B \) and \( C \). Show that the locus of the centroid of the tetrahedron \( OABC \) is \( x^2 + y^2 + z^2 = 16p^2 \).

(c) Find the distance of the point \( (1, -2, 3) \) from the plane \( x - y + z = 5 \) measured parallel to the line \( \frac{x}{2} = \frac{y}{3} = \frac{z}{-6} \).

6. (a) Prove that the straight lines \( \frac{x}{a} = \frac{y}{\beta} = \frac{z}{\gamma}, \ \frac{x}{a} = \frac{y}{b} = \frac{z}{c}, \ \frac{x}{l} = \frac{y}{m} = \frac{z}{n} \) will lie in one plane if \( (b-c)\frac{l}{\alpha} + (c-a)\frac{m}{\beta} + (a-b)\frac{n}{\gamma} = 0 \).

(b) Find the length of the shortest distance between the lines \( \frac{x-3}{1} = \frac{y-5}{-2} = \frac{z-7}{1} \) and \( \frac{x+1}{7} = \frac{y+1}{-6} = \frac{z+1}{1} \). Find also its equation and the points in which it meets the given lines.

(c) Find the equations of the line parallel to \( \frac{x}{2} = \frac{y}{3} = \frac{z}{-6} \) so as to intersect \( 9x + y + z + 4 = 0 \) and \( x + 2y - 3z - 3 = 0 \) at \( 2x - 5y + 3z + 3 \).

7. (a) If \( M \) and \( N \) are the middle points of \( AB \) and \( CD \) of the parallelogram \( ABCD \), prove that \( DM \) and \( BN \) are trisected by \( AC \) and also \( AC \) is trisected by them.

(b) Prove that \( (b \times c) \times (a \times d) + (c \times a) \times (b \times d) + (a \times b) \times (c \times d) = -2[ab \ c]d \).

(c) Ascertain the linear dependence or independence of the three vectors \( a = 2i - j + k, \ b = i + 3j - 2k \) and \( c = -2i + j - 3k \). Find scalars \( x, \ y, \ z \) such that \( d = xa + yb + zc \), where \( d = 3i + 2j + 5k \).

8. (a) Find the value of \( \lambda \) so that the vectors \( 5a + 6b + 7c, \ 7a + \lambda b + 9c \) and \( 3a + 20b + 5c \) are coplanar, where \( a, \ b, \ c \) are three non-coplanar vectors.

(b) If \( a, \ b, \ c \) be unit vectors with \( b \) not parallel to \( c \) such that \( a \times (b \times c) = \frac{1}{2}b \), find the angles \( \alpha \) and \( \beta \) which \( a \) makes with \( b \) and \( c \) respectively.

(c) If the system of vectors \( a', \ b', \ c' \) is reciprocal to the system of vectors \( a, \ b, \ c \), then prove that any vector \( r \) is \( r = (r \cdot a)a' + (r \cdot b)b' + (r \cdot c)c' \).
BANGLADESH UNIVERSITY OF ENGINEERING AND TECHNOLOGY, DHAKA

L-1/T-1 B. Sc. Engineering Examinations 2012-2013

Sub: PHY 105 (Structure of Matter, Electricity and Magnetism and Modern Physics)

Full Marks: 210 Time: 3 Hours

The figures in the margin indicate full marks.

USE SEPARATE SCRIPTS FOR EACH SECTION

SECTION - A

There are FOUR questions in this section. Answer any THREE.

1. (a) Define lattice, basis, unit cell and translation vector. (4x3=12)
   (b) Describe NaCl and CsCl structures. (2x5=10)
   (c) Find the atomic packing fraction of a diamond cubic structure. (8)
   (d) Calculate the number of atoms per unit cell of a Cesium structure with lattice constant 6.045 Å; atomic weight of Cs is 132.9054 (atomic units) and density is 1.90 x 10^3 kg/m^3. (5)

2. (a) Explain the procedure to obtain Miller indices of crystal planes. (8)
   (b) Find the relation between interplanar spacing and Miller indices for a cubic crystal system. (7)
   (c) Show that the ratio d_{100} : d_{110} : d_{111} of a body center cubic lattice is not the same as that of a simple cubic lattice. (10)
   (d) Write down the Bragg's law of X-ray diffraction and explain in short "why normal light cannot be used in diffraction pattern analysis of a crystal?" (2x5=10)

3. (a) Find the cohesive energy of NaCl crystal. (10)
   (b) Explain different type of semiconductors. (15)
   (c) Write short notes on—
      (i) Covalent bond,
      (ii) Dislocations. (2x5=10)

4. (a) Find an expression for the fringe shift in Michelson-Morley experiment and explain each term in it. Calculate the fringe shift when D = 10 m, λ = 5000 Å and the earth speed v = 3 x 10^4 m/s, where the terms have their usual meaning. (25)
   (b) The relativistic equation for the kinetic energy is K = mc^2 - m_0c^2, where the terms have their usual meaning. Find the kinetic energy of the body when it is moving with a very low speed, i.e., v/c << 1. (10)

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5. (a) Explain Compton scattering with necessary diagram. Obtain an expression for Compton shift of a photon undergoing Compton scattering. 

(b) What are the failures of classical wave theory about photoelectric effect?  

(c) An electron has a wavelength of 1 nm. Calculate the kinetic energy of the electron.

6. (a) What is mean life of a radioactive substance? Obtain an expression for the mean life of a radioactive substance.  

(b) Show that it's impossible for an electron to reside inside the nucleus.

(c) Define binding energy of a nucleus. Calculate the binding energy of α-particle. Given that mass of proton = 1.00728 a.m.u.; mass of neutron = 1.00866 a.m.u.; mass of α-particle = 4.00153 a.m.u.

7. (a) Define electric flux and state Gauss' theorem.  

(b) Calculate the electric field intensity at a point (i) outside, (ii) inside and (iii) surface of a uniformly charged spherical shell.

(c) Two charges of +3 μc and −3 μc are placed at the corners of the base of equilateral triangle. The length of a side of the triangle is 0.75 m. Find the electric-field intensity at the apex of the triangle.

8. (a) Derive an expression for Gauss' law from dielectrics.  

(b) Obtain an expression for the growth of charge when a capacitor is charged through a resistance for a constant emf. What is time constant of the circuit?  

(c) A 2-μF capacitor is allowed to discharge through an unknown resistance. If the charge on the capacitor takes 1 minute to drop to half of its original value, what is the value of the resistance?
1. (a) Classify fossil fuels with examples. Arrange 4 types of coal according to their heating value. With simple combustion formula, show the difference between higher heating value and lower heating value of a fuel. (10)

(b) Describe with an open circuit diagram and a sequence Table, various modes of operation of a system having a renewable energy source including a storage battery and a backup generator. (15)

(c) A wind turbine of rotor diameter 10 m extracts 750 Watt power at a wind speed of 4 m/s. Find its power coefficient considering air density 1.1936 Kg/m³. (10)

2. (a) In a locality, wind speeds at 10 m and 50 m heights from the ground level are 5 m/s and 7 m/s respectively. Calculate the "power law index" for that locality. (10)

(b) What type of hills with slopes are the most favorable for wind energy extraction and what type are the most unfavorable? Show these locations with simple sketches including wind profiles at ground levels and top of hills. (15)

(c) Considering the sun as a black body at 5800 K, determine the total radiant power emitted by the sun. If the mean distance of the sun from the earth is $1.5 \times 10^{11}$ m, calculate the solar constant for earth. (10)

(Given that, Stefan Boltzmann constant $\sigma = 5.67 \times 10^{-8}$ W/m²K⁴).

3. (a) Showing all the velocity vectors and angles, draw the combined velocity diagram for a single stage steam turbine blade. (10)

(b) With neat sketches and examples show the differences between impulse and reaction turbines. (15)

(c) With neat sketches show the arrangement of nozzle, fixed and moving blades for a velocity compounded steam turbine. Also show the variation of velocity, pressure and volume along the axis of the turbine. (10)

4. (a) Distinguish among a fan, a blower and a compressor in terms of pressure ratio. (5)

(b) With neat free hand sketches, show the arrangements and their combined H-Q performances for the two centrifugal pumps of the same specifications when they are connected (i) in series and (ii) in parallel. (10)

Contd ......... P/2
(c) (i) Draw and label a centrifugal pump with front and top views. (6)
(ii) Draw the typical H-Q, P-Q and η-Q performance curves for a centrifugal pump (Symbols have their usual meanings). (6)
(iii) Describe with a neat sketch the principle of operation of a reciprocating pump. (8)

SECTION – B

There are FOUR questions in this section. Answer any THREE.

5. (a) specify the essential requirements of a good boiler. (9)
(b) Define "Mountings" and "Accessories" of a boiler. Give three examples of each category. (7)
(c) Distinguish between a "Safety Valve" a "Feed-Check Valve" and a "Stop Valve" of a boiler. (9)
(d) What is a timing belt of an internal combustion engine? Briefly explain how "engine timing" would effect engine operation. (10)

6. (a) A vehicle is advertised as: FWD, SEDAN, 16-valve, MPFI, 1.6 Liter. Briefly explain what do you understand from the specifications. (10)
(b) Define "Compression Ratio". Why do we use higher compression ratio for a Diesel engine? (7)
(c) What are the advantages of EFI system in an SI engine? compare fuel injection in an SI engine and a CI engine. (10)
(d) Why do we need a transmission in a vehicle? How can you identify whether a vehicle has "Manual" or "Automatic" Transmission? (8)

7. (a) Distinguish between a heat pump and a refrigerator. (6)
(b) Briefly explain the configuration and operation of a "Window" and a "Split" air conditioner. What do you understand by a portable air conditioner? (12)
(c) State the desirable properties of a refrigerant. (8)
(d) Which is the most important property of a lubricating oil used in a IC engine? Briefly explain how this property is specified for engine lubricants. (9)

8. (a) Distinguish between OHC and In-block arrangement of camshaft. Briefly explain the advantages of OHC system. (10)
(b) How is a "Jet Engine" different from a "Gas Turbine"? Briefly explain how a Jet plane flies. (11)
(c) Define FCU and AHU of an air-conditioning system. (6)
(d) Briefly explain how a "Cascade Refrigeration" system works. (8)
1. (a) What are the four quantum numbers and how are they originated? Discuss them in identifying an electron in an atom. (12)

(b) What is periodic law? Make a modern periodic table showing different steps of making. Justify the gaps and unequal number of elements present in different periods of the table. (12)

(c) How do you conceive the idea of having electron cloud around the nucleus of an atom? Based on this, discuss the bond formation in hydrogen molecule (H₂), water molecule (H₂O) and ammonia molecule (NH₃). (11)

2. (a) What does Schrödinger wave equation describe? Deduce the equation. Mention the conditions to be applied to obtain practicable solutions for wave function, ψ. Justify for having those conditions. (12)

(b) Discuss the basis of Valence Shell Electron Pair Repulsion (VSEPR) theory for determining the molecular structure. Determine structures of C⁴⁻, H₂O, BrF₃ and ICl₄⁻. (11)

(c) What are paramagnetism and diamagnetism? Discuss the formation of O₂, NO and CO molecules according to molecular orbital theory. Comment on their magnetic property. (12)

3. (a) Discuss the properties of ionic and covalent compounds based on the directional property of the ionic and covalent bond. (10)

(b) Comment on the formation of FCl molecule and H₂O²⁺ ion. (7)

(c) What is a polymerization reaction? Mention the common mechanisms of polymerization reactions. Also mention how the propagation of polymer chain is stopped. (9)

(d) What is Werner’s theory of coordination compounds? How did he develop the theory? Mention important application of complexation. (9)

4. (a) Define 'order of reaction'. Describe how you can determine the order of reaction by using differential rate equation. (12)

(b) Derive an expression for rate constant of a first-order reaction which is opposed by another first-order reaction. (15)

(c) A first-order reaction is 40% complete in one hour. What is the value of the rate constant? In how long will the reaction be 80% complete? (8)
CHEM 109(ME)

SECTION - B

There are FOUR questions in this Section. Answer any THREE.

5. (a) What do you mean by dynamic equilibrium in a saturated solution? Give one evidence of the existence of dynamic equilibrium, "Like dissolves like" - explain the statement. 
(b) Discuss the mechanism of dissolution of solid in liquid and predict how the evolution and absorption of heat arises.
(c) What is the critical solution temperature and why it is necessary to measure? Show that the volume of the gas dissolved in a given volume of the solvent is independent of pressure at constant temperature.

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(c) What is the critical solution temperature and why it is necessary to measure? Show that the volume of the gas dissolved in a given volume of the solvent is independent of pressure at constant temperature.

6. (a) Derive Raoult's law and explain how it deviates from the real solution. How would you calculate the molecular weight of a nonvolatile solute from the depression of freezing point?
(b) What is isotonic solution? Explain how the Avogadro-van't Hoff law for solution is analogous to the Avogadro's law of gasses. Derive an equation relating to the osmotic pressure and molecular weight of the solute.
(c) Calculate the osmotic pressure of the solution obtained by mixing (a) 100 ml of 3.4% urea (MW = 60) and (b) 100 ml 1.6% solution of cane sugar (MW = 342) at 20°C.

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7. (a) What do you mean by the term "transport number"? Discuss the factors which affect the transport number.
(b) Draw and explain the phase diagram of water as a monocomponent system. Calculate the number of components and degrees of freedom in a mixture of O_2(g) and N_2(g).
(c) Define buffer solution. How does buffer solution control the change in its pH upon the addition of small quantities of strong acids or bases?
(d) Calculate the pH of 1 × 10^{-8} M solution of KOH.

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8. (a) Define the following terms:
   (i) Reversible reaction (ii) Thermodynamic equilibrium constant.
(b) Derive an equation relating to the free energy change with the equilibrium constant of a reaction.
(c) A 2000 K, the standard free energy change (ΔG°) for the reaction N_2 + O_2 ⇌ 2 NO is given by ΔG° = 92048 - 100.48 T J. Calculate K_p for the reaction at this temperature.

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SECTION A

There are FOUR questions in this section. Answer any THREE.

1. (a) Prove that, for \( R_L = R_{th} \), maximum power will be transferred to load. Also prove that, the expression of maximum load power is given by,

\[ P_{L,\text{max}} = \frac{V_{th}^2}{4R_{th}} \]

Here, the symbols have their usual meanings.

(b) Find the magnetic flux \( \Phi \) established in the series magnetic circuit of Fig. of Q. 1(b).

2. (a) Find the equivalent resistance \( R_{eq} \) of the following circuit as seen from the pair of the nodes.

[Diagram of the circuit]
(b) Using Ohm's law, KVL and KCL determine $V_1$ and $I_2$ in the following circuit. Here, $I_2$ is the current leaving from 10 V source.

(c) Determine the Thevenin's equivalent of the following circuit as seen from the node pair a and b.

3. (a) Determine the current passing through 26.4 Ω resistor in the given direction in the following circuit with the help of node voltage analysis method.
(b) Determine the Norton's equivalent of the following circuit as seen from the node pair a and b.

![Circuit Diagram](image-url)

Fig. for Q 3(b)

4. (a) Determine the voltage across the 11.5 A current source, $V_{11.5A}$ in the given polarity of the following circuit with the help of mesh current analysis method.

![Circuit Diagram](image-url)

Fig. for Q 4(a)

(b) Determine the value of $R_L$ which will result in transfer of maximum power at $R_L$ of the following circuit. Also determine the value of the maximum power.

![Circuit Diagram](image-url)

Fig. for Q 4(b)
5. (a) If a 50 Hz AC voltage source is applied to an arbitrary combination of resistances, inductances, and capacitances; prove that the frequency of instantaneous power will the 100 Hz.

In your expression of total instantaneous power, separately indicate the real power and reactive power component.

Draw a graph showing the instantaneous real power, reactive power and total power.

(b) A resistance and a reactance are series connected. Voltage across the resistance, \( V_{\text{res}} = 8 \sin(120 \pi t) \) and across reactance, \( V_{\text{react}} = 6 \cos(120 \pi t) \).

What is the voltage amplitude across the series combination?

What is the line frequency?

From the information stated above, can you tell whether the reactive element is an inductor or a capacitor? If not, what more information do you need to answer that question?

6. (a) A power supply is driving two appliances – a bulb and a fan. The instantaneous real power absorbed by the bulb is given in figure 6(a)-(i).

Also the instantaneous voltage in volt unit (not power, be careful) across the fan is given in figure 6(a)-(ii).

The resistance of the bulb and the fan is 10 \( \Omega \) each. Find the total average real power delivered by the power supply.

If you connect a capacitor parallel to the power supply, what will be the change in the real power delivered by the source? Will it increase, or decrease? Why?

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(b) For the following circuit, draw the exact phasor diagram and show the angle between source voltage, \( V_s \) and source current, \( I_s \). Take \( V_s \) as reference. The relative magnitude of various voltage and current quantities necessary to draw your vector diagram are also provided in the figure. Is the circuit leading or lagging?

![Phasor Diagram](image)

For Q. 6(b)

7. (a) If you are asked to solve the following circuit by applying either mesh or node analysis, which of the two methods will you choose? Why?
Find out current and power factor of \( V_1 \) source by applying your preferred method. (1+12+1+6)

\[
V_1 = 10 \sin(100 \pi t) \\
V_2 = 20 \cos(100 \pi t + 60^\circ) \\
L_1 = L_2 = \frac{4}{100\pi} \text{ Henry} \\
C = \frac{1}{100\pi} \text{ Farad} \\
R = 4 \Omega
\]

What is the frequency of the system?
For what value of the frequency, the voltage dropped across the middle branch (consisting of \( C \) and \( L_2 \)) will be zero?

![Circuit Diagram](image)

For Q. 7(a)
(b) A 10 hp single phase motor is operating at 0.5 lagging with a 50 Hz power supply. To improve the power factor, a capacitor bank is connected parallel to the motor. In order to run the motor at 0.95 lagging, what should be the value of the capacitance? For what value of the capacitance, the motor will run at 0.95 leading power factor? Why do we correct the power factor? What type of power factor is more desirable – 0.95 leading or 0.95 lagging? Why?

8. (a) With the help of vector diagram, show that, the power of a three phase balanced circuit can be measured by two wattmeters. Draw a neat diagram showing how to connect the two wattmeters in the three phase circuit. Also show how to measure the power factor from the two wattmeter reading.

In a three phase balanced circuit, only one wattmeter is supposed to be enough to determine the total power (Just measure the power of any phase and multiply it by 3). If so, then why do we need to use two wattmeters (instead of one) for measuring the power of 3 phase balanced circuit?

(b) A three phase, abc sequence, 7 hp, 100 volt balanced Y-connected induction motor has 70% efficiency, and operates at 0.86 pf. It is paralleled with a three phase Δ-connected load; each of its arms is of 36 Ω resistance. Find the KVA required by the combination. Also find the power factor of the supply. Write an advantage that three-phase supply provides over single phase supply.
B - H curve of three different magnetic materials