THESIS
A NUMERICAL MODEL FOR SALINITY INTRUSION IN THE PUSSUR RIVER

Submitted by
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In partial fulfilment of the requirements for the Degree of Master of Science in Engineering (Water Resources)

Bangladesh University of Engineering and Technology
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We hereby recommend that the thesis prepared by LIAQAT ALI KHAN entitled A NUMERICAL MODEL FOR SALINITY INTRUSION IN THE PUSSUR RIVER be accepted as fulfilling this part of the requirements for the degree of Master of Science and Engineering (Water Resources).

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This study concerns with the formulation of a one-dimensional salinity intrusion model for the Pussur estuary, covering a reach of about 120 km from Sundarikota to Khulna. The conventional mass transport equation has been discretized by the generalized six point Stone and Brian scheme, whereas continuity and momentum equations have been discretized by four point implicit Preissmann scheme. The resulting systems of tri-diagonal and quin-diagonal linearized equations were then solved by 'Double Sweep' algorithms. A computer programme has been developed for the numerical solution of these systems of equations. The input data required for the computation are the geometric data of the estuary, upstream inflow and tidal salinity variation at Khulna, downstream tidal hydrograph at Sundarikota, high and low water slack salinities at the downstream model boundary.

The river reach selected for the study has five major tributaries, but their discharge data were not available. The model, therefore neglected the effects of these tributaries. The reach of the channel has been divided into 30 equal sections with 4 km space interval (Δx). The time interval (Δt) used was 30 minutes. The geometric properties at grid points were represented by equivalent rectangular sections. Data availability for comparison with the model results were extremely scarce. Field data for salinity and tidal discharge were available only for one neap tide cycle at Khulna. Reasonable
accuracy has been achieved in simulating tide and salinity at Mongla and Chalna respectively using Manning's roughness coefficient, \( n = 0.02 \) and dispersion coefficient parameters, \( K_1 = 600 \) and \( K_2 = 100 \, \text{m}^2/\text{s} \). For other stations spatial and temporal variation of salinity, tidal elevation and tidal discharge have been predicted and results have been discussed. Finally, suggestions have been put forward for future use of the model and data collection.
ACKNOWLEDGEMENTS

The author gratefully acknowledges his profound gratitude and indebtedness to his supervisor Dr. M.K. Alam, Associate Professor, Department of Water Resources Engineering, for his help, encouragement, guidance and cooperation in making this thesis possible. His active interest in this topic and valuable advice throughout the study were of immense help.

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<td>$A$</td>
<td>cross-sectional area of channel section</td>
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$E_j$  coefficient defined by Eqn. (3.21)

$E'_j$  coefficient defined by Eqn. (3.35)

$E$  estuary number defined by Eqn. (2.11)

$E_i$  densimetric estuary number defined by Eqn. (2.25)

$F_0$  Froude number at the estuary mouth

$F_i$  densimetric Froude number defined by Eqn. (2.27)

$F_j$  coefficient defined by Eqn. (3.22)

$F^j$  coefficient defined by Eqn. (3.36)

$f$  function dependent on $x$ and $t$

$f^n_j$  approximation to function $f$ at $x = j\Delta x$ and $t = n\Delta t$

$G$  rate of tidal energy dissipation per unit mass of fluid

$g$  acceleration due to gravity

$g_1, g_0, g_{-1}$  weighting coefficients in Stone and Brian scheme

$H$  water surface elevation above reference datum

$H^n_j$  approximation to $H$ at $x = j\Delta x$ and $t = n\Delta t$

$\Delta H_j$  change in $H$ at $x = j\Delta x$ in time interval $\Delta t$

$h$  mean depth of flow at the estuary mouth

$h^n_0$  constant defined by Eqn. (4.17)

$J$  rate of gain of potential energy per unit mass of fluid

$K_0, K_1, K_2$  coefficients defined by Eqn. (2.18) & Figs. (2.3), (2.3) & (2.5)

$K_1$  stratification parameter defined in Eqn. (3.4)

$K_2$  parameter in Taylor's dispersion coefficient

$K^j$  coefficient defined by Eqn. (3.23)
$K_j$  \hspace{1cm} \text{coefficient defined by Eqn. (3.23)}

$L$  \hspace{1cm} \text{length of estuary}

$L_j$  \hspace{1cm} \text{wave length}

$M$  \hspace{1cm} \text{length of estuary}

$M_j$  \hspace{1cm} \text{recurring coefficient defined by Eqn. (3.66)}

$M_j$  \hspace{1cm} \text{recurring coefficient defined by Eqn. (3.67)}

$N_j$  \hspace{1cm} \text{recurring coefficient defined by Eqn. (3.68)}

$n$  \hspace{1cm} \text{Manning's roughness coefficient}

$O$  \hspace{1cm} \text{order of truncation error}

$P_T$  \hspace{1cm} \text{tidal prism}

$Q$  \hspace{1cm} \text{instantaneous tidal discharge}

$Q^n_j$  \hspace{1cm} \text{approximation to } Q \text{ at } x = j\Delta x \text{ and } t = n\Delta t

$\Delta Q_j$  \hspace{1cm} \text{change in } Q \text{ at } x = j\Delta x \text{ in time interval } \Delta t

$Q_f$  \hspace{1cm} \text{upstream fresh water inflow}

$q$  \hspace{1cm} \text{lateral inflow per unit length of channel}

$R$  \hspace{1cm} \text{hydraulic radius}

$S$  \hspace{1cm} \text{channel bed slope}

$s$  \hspace{1cm} \text{local salinity concentration}

$s^n_j$  \hspace{1cm} \text{approximation to } s \text{ at } x = j\Delta x \text{ and } t = n\Delta t

$s$  \hspace{1cm} \text{tidal average salinity concentration}

$s_s$  \hspace{1cm} \text{slack tide salinity concentration}

$s_o$  \hspace{1cm} \text{ocean salinity or maximum salinity concentration}

$s_0$  \hspace{1cm} \text{salinity concentration at the estuary mouth}

$s_o$  \hspace{1cm} \text{dimensionless salinity } = s/s_o

$s_HWS$  \hspace{1cm} \text{high water slack salinity}

$s_LWS$  \hspace{1cm} \text{low water slack salinity}
\( s_0^n \) constant defined by Eqn. (4.33)

\( T \) tidal period

\( t \) time

\( \Delta t \) time step

\( t_{HWS} \) time of high water slack

\( t_{LWS} \) time of low water slack

\( u \) cross-sectional average velocity

\( u_o \) maximum flood velocity at estuary mouth

\( u_f \) velocity due to fresh water inflow

\( u_T \) velocity due to tidal movement at the estuary mouth

\( u_0^n \) constant defined by Eqn. (4.18)

\( x \) longitudinal axis of the estuary

\( \Delta x \) distance step

\( x^* \) dimensionless longitudinal distance = \( x/L \)

\( \alpha \) flood number defined by Eqn. (2.19)

\( \beta \) coefficient defined by Eqn. (3.70)

\( \beta_j \) coefficient defined by Eqn. (3.71)

\( \beta_j \) recurring coefficient defined by Eqn. (3.75)

\( \gamma \) velocity factor

\( \gamma_j \) coefficient defined by Eqn. (3.72)

\( \gamma_j \) recurring coefficient defined by Eqn. (3.77)

\( \delta_j \) recurring coefficient defined by Eqn. (3.78)

\( \theta \) weighting coefficient in Preissmann scheme

\( \theta_x \) angle made by channel bed with horizontal

\( \theta_j \) recurring coefficient defined by Eqn. (3.58)
\( \lambda \)  
coefficient defined by Eqn.(4.35)

\( \mu \)  
coefficient defined by Eqn.(4.36)

\( \xi \)  
function defined by Eqn.(4.29)

\( \rho \)  
amplification factor

\( |\rho| \)  
damping factor

\( \rho^R \)  
real part of damping factor

\( \rho^I \)  
imaginary part of damping factor

\( \sigma \)  
wave number defined by Eqn.(4.43)

\( \omega_j \)  
recurring coefficient defined by Eqn.(3.59)
CHAPTER I
INTRODUCTION

The Pussur river is one of the most important water cycle routes in the southwestern zone of Bangladesh. The Mongla port, the Khulna News Print Mills and many other important industries and power plants of the country have come into existence by the sides of this river system. The Sunderban forest which greatly influences the ecological balance of the area is also situated in this region. Recently, greater saline pollution during the lean flow period, between March and May, has been creating adverse effects on agriculture, public health, industries and ecological balance of the area (Govt. of Bangladesh, 1976).

Since 1961 field data are being collected by various agencies of the country, particularly BWDB. However, steps for comprehensive study of the problem to quantify the situation and predict the future behaviour have not been undertaken. The problem has been partially studied by Bangladesh Water Development Board (BWDB), International Engineering Company Inc. (IECO), U.S.A., International Bank for Reconstruction and Development (IBRD) and others (IECO, 1980), and are incomplete in nature. Most of these studies have recommended for proper study of the problem with the help of mathematical models.

For the exact analysis of salinity intrusion a mathematical model of a given estuary should consider all the three space dimensions and time. Governing equations for such models
are available, but several of the solution parameters contained in these equations have not been evaluated by research. Moreover, a three-dimensional model is lengthy in computational time and requires a very large-capacity computer. Therefore, some simplifying assumptions are usually made. The usual approach is to average the mass transport equation over one or two dimensions.

The averaging of mass transport equation in two dimensions results in one-dimensional salinity intrusion models. These models have been applied in many estuaries successfully. The basic assumptions in one-dimensional models are that the velocity and salinity concentration across the estuary cross-section are uniform. A very important feature of these models, over two-dimensional models, is that the data requirements are concise and closely related to information available in the field. Moreover, the results obtained are reasonably good and reliable for most of the engineering purposes.

Mathematical modelling of salinity intrusion is at a very early stage in Bangladesh. Few attempts have been made by the Government of Bangladesh and these studies are not generally available for academic or research purposes. Wasimi (1978) tried to develop a model but due to lack of proper modelling approach and computer facility his study remained incomplete. A mathematical model of saline-water intrusion in the Meghna Delta was developed by Chaudhury (1983) as a part of pre-feasibility study in connection with augmentation problem of the Ganges water. With the availability of IBM 370 computer
in Bangladesh University of Engineering and Technology (BUET), the non-availability of computer facility has been removed. In this context, the present study has been undertaken for a comprehensive study of the problem with a simpler modelling approach. Therefore, the objectives of the present study are as follows:

1) To develop a one-dimensional mathematical model for the study of salinity intrusion in the Pussur estuary.

2) To study the time varying-longitudinal distribution of salinity in the estuary.
CHAPTER II
LITERATURE REVIEW

2.1.0 INTRODUCTION

Salinity intrusion in estuaries is a subject of extensive research because of the major consequences it has on water management. In the last three decades, a large number of salinity intrusion models have been proposed in hydraulic literature. The starting point of all these models is the salt balance equation, which can be written as

\[
\frac{\partial s}{\partial t} + u \frac{\partial s}{\partial x} = \frac{1}{A} \frac{\partial}{\partial x} (AE \frac{\partial s}{\partial x})
\]  

(2.1)

where

\[s = \text{salinity concentration averaged over the channel cross-section},\]
\[u = \text{cross-sectional averaged velocity},\]
\[E = \text{dispersion coefficient},\]
\[A = \text{cross-sectional area}.
\]

The solution of Eqn. (2.1) has been a difficult mathematical problem because of the non-linear nature of the equation. Thus to solve this equation various simplifying assumptions were made, starting from an equilibrium state of the estuary. With the availability of the computer and development of numerical methods, more realistic, but complicated models were developed, which consider the unsteady state of the estuary. The models
for predicting salinity intrusion in estuaries can be broadly categorized into two groups: steady state models and unsteady state models. In the following articles a brief review of the important models are given, which relates with the single channel estuaries.

2.2.0 STEADY STATE MODELS

The steady state models proceed from Eqn. (2.1) with the assumption that the velocity \( u \) is made up of two components: \( u_T \), resulting from the tidal movement and \( u_f \), resulting from the outflow from the river. Both these components are basically functions of \( x \) and \( t \), although on different scales. The tidal component has a period of 12 hours 25 minutes, while the outflow component has a far longer time scale. Assuming that the \( x \)-axis is directed towards the river, it follows that

\[
u = u_T - u_f \quad (2.2)
\]

Substituting Eqn. (2.2) in Eqn. (2.1) gives

\[
\frac{\partial s}{\partial t} + u_T \frac{\partial s}{\partial x} - u_f \frac{\partial s}{\partial x} = \frac{1}{H} \frac{\partial}{\partial x} (A \frac{\partial s}{\partial x}) \quad (2.3)
\]

A further simplification is achieved by averaging the salt content over a tidal period. Since in a state of equilibrium \( u_f \) is constant throughout a tidal period, and further more

\[
\int_0^T \frac{\partial s}{\partial t} \, dt = 0
\]

\[
\int_0^T u_T \frac{\partial s}{\partial t} \, dt = 0,
\]
the following equations are obtained from Eqn. (2.3), after averaging over a tidal period of duration T:

\[
-u_f \, \frac{d\bar{s}}{dx} = \frac{1}{\bar{A}} \, \frac{d}{dx} (\bar{A} \, \bar{E} \, \frac{d\bar{s}}{dx})
\]

or

\[
Q_f \, \frac{d\bar{s}}{dx} = \frac{d}{dx} (\bar{A} \, \bar{E} \, \frac{d\bar{s}}{dx}) \tag{2.4}
\]

where \( \bar{E} \) represents an effective value of the dispersion coefficient over a tidal period, while \( \bar{s} \) is the average salt content and \( \bar{A} \) is the profile average cross-sectional area in a tidal period. Eqns. (2.3) and (2.4) provide the starting point for the steady state models.

2.2.1 ARONS AND STOMMEL MODEL (1951)

Arons and Stommel (1951) proposed a model where each term in Eqn. (2.3) is averaged over a tidal cycle. Averaging Eqn. (2.3) over a tidal period gives

\[
\frac{\partial \bar{s}}{\partial t} + u_f \, \frac{\partial \bar{s}}{\partial x} = \frac{1}{\bar{A}} \, \frac{\partial}{\partial x} (\bar{A} \, \bar{E} \, \frac{\partial \bar{s}}{\partial x}) \tag{2.5}
\]

Arons and Stommel assumed that the longitudinal dispersion coefficient was proportional to the product of the tidal excursion length and the maximum tidal velocity at the entrance. The steady state equation is

\[
u_f \, \frac{\partial \bar{s}}{\partial x} = \frac{\partial}{\partial x} (\bar{E} \, \frac{d\bar{s}}{dx}) \tag{2.6}\]
The dispersion coefficient $E(x)$ is not the same as $E(x,t)$ of Eqn. (2.3) and is not equal to the average value of $E(x,t)$ over a tidal period. In fact, the $E(x)$ distribution must be determined by fitting solutions of Eqn. (2.5). The requirement of a boundary condition on salinity at the ocean end of the estuary is specially difficult using this approach. The application of this model requires the measurements of salinity at the ocean boundary, or statistical predictions derived from such measurements, thereby making the model descriptive rather than predictive in nature.

2.2.2 IPPEN AND HARLEMAN (1961)

Ippen and Herleman (1961) and later Harleman and Abraham (1966) made an analytical solution of Eqn. (2.3) by considering low water slack, e.g. the movement at the end of the ebb flow at which the salinity intrusion into the estuary is at its least. At this moment $\partial \tilde{s}/\partial t = 0$ and $u_f = 0$, so that if one works from the assumption of a constant cross-sectional area in the estuary, Eqn. (2.3) becomes

$$- u_f \frac{d\tilde{s}}{dx} = \frac{d}{dx} \left( E^{\text{LWS}} \frac{d\tilde{s}}{dx} \right) \quad (2.7)$$

Integrating Eqn. (2.7), and bearing in mind that for a large $x$ the values of $d\tilde{s}/dx$ and $\tilde{s}$ approach zero, gives

$$- u_f \tilde{s} = E^{\text{LWS}} \frac{d\tilde{s}}{dx} \quad (2.8)$$
Ippen and Harleman found by analyzing the results of laboratory experiments that the dispersion coefficient is an inverse function of \( x \)

\[
E_{LWS} = \frac{E_0}{x+B} \tag{2.9}
\]

where \( B \) is the distance from the river mouth to the point where the salt content is equal to that of the sea water, while \( E_{LWS} \) represents the dispersion coefficient at the river mouth at low-water slack. Substituting Eqn. (2.9) into Eqn. (2.8) and integrating the resulting equation gives

\[
S_o = \exp \left\{ -\frac{u_f}{2E_0LWS} (x+B)^2 \right\} \tag{2.10}
\]

where \( s_o \) is the salinity concentration of sea water.

Ippen and Harleman carried out fundamental research on salinity intrusion using a flume at the Waterways Experiment Station, Vicksburg, U.S.A. This study showed that the parameter \( E_{LWS} \) and \( B \) could be correlated to 'stratification number', \( G/J \) as defined by the following equation

\[
\frac{G}{J} = \text{rate of energy dissipation per unit mass of fluid} \quad \frac{\text{rate of gain of potential energy per unit mass of fluid}}
\]

The stratification number is, in practice, difficult to determine. Harleman and Abraham (1966) found from reanalysis of the model tests and measurements in the Rotterdam Waterway, Netherlands, that the stratification number correlates well to the
'estuary number', defined as

$$\Psi = \frac{P_T F_o^2}{Q_f T}$$

(2.11)

where

- $P_T$ = tidal prism, defined as the volume of water entering the estuary on the flood tide,
- $F_o$ = Froude number, $= u_o / \sqrt{gh}$,
- $u_o$ = the maximum flood velocity at estuary mouth,
- $h$ = mean depth at estuary mouth,
- $Q_f$ = fresh water discharge,
- $T$ = tidal period.

Harleman and Abraham formulated fresh correlations between $E_{LWS}^O$ and $B$ using this new parameter. It was however, necessary to introduce an additional parameter $h/a$ (depth at the river mouth/tidal amplitude). The correlations found are

$$\frac{E_{LWS}^O}{u_f B} = 0.055 \left( \frac{h}{a} \right)^{2.7} \left( \frac{P_T F_o^2}{Q_f T} \right)^{1.2}$$

(2.12)

$$\frac{2\pi B}{u_0 T} = 0.70 \left( \frac{P_T F_o^2}{Q_f T} \right)^{0.2}$$

(2.13)

The schematic representation of these equations is shown in Fig. 2.1. Using these correlations it was possible to determine the parameters in Eqn. (2.10). The salinity at times other than low-water slack can be found by translating the calculated salinity distribution along the distance covered by a water particle/the desired point in time.
FIG. 2.1 DETERMINING PARAMETERS FOR IPPEN-HARLEMAN-ABRAHAM MODEL
The work of Harleman and Abraham resulted in a practical solution for steady-state salinity intrusion for the case of constant geometry. It seemed reasonable that similar analytical techniques could be attempted for cases of variable area when this variation could be expressed in a simple form, which permitted integration of the salt balance equation. However, study of an exponential area variation, by Enonini in 1968 (Thatcher and Harleman, 1972) did not substantiate a general relationship sufficiently.

2.2.3 O'CONNOR MODEL (1965)

O'Connor (1965) followed a somewhat similar approach and proposed a model which eliminated the tidal motion by considering salinity distributions only at slack tide conditions. The effect of this assumption is to take a 'snap-shot' of the salinity distribution once in every tidal period rather than to consider time-averages over the tidal period. Slack tide conditions were probably chosen, because these were times when salinity data were available. O'Connor based his model on the following equation, which is similar to Eqn. (2.5)

\[
\frac{\partial s_s}{\partial t} + u_f \frac{\partial s_s}{\partial x} = \frac{1}{A_s} (A_s E_s \frac{\partial s_s}{\partial x}) \quad (2.14)
\]

where

- \( s_s \) = slack tide salinity concentration,
- \( A_s \) = slack tide area (not a function of time),
- \( E_s \) = slack tide dispersion coefficient (not a function of time).
O'Connor obtained analytic solution of Eqn. (2.14) for cases where $E_s$ is constant and the area $A_s$ is either constant or a linear, quadratic, or exponential functions of $x$. The basic solutions are for an instantaneous injection of mass, and the solutions for continuous injection follow as an integration of instantaneous injections.

2.2.4 SCHONFELD AND VAN DAM MODEL (1967)

In 1967, Van Dam and Schonfeld (Langeweg and Weerden, 1976) offered in their paper a method for calculating salt distribution in estuaries, in both steady and unsteady states. They worked from a salt content averaged over a tidal cycle. The steady state model was based on Eqn. (2.4). Additionally, an empirical solution of the equation is given for point injection of mass.

In the context of salinity movement, however, the unsteady model is more important. This model too was based on salt content values averaged over a tidal cycle, and should therefore be looked at as a quasi-steady model. The basic equation for the model was obtained by averaging Eqn. (2.1) over a tidal period, which can be written as

$$\bar{A} \frac{\partial \bar{S}}{\partial t} = \frac{\partial}{\partial x} (Q \tau \bar{S}) + \frac{\partial}{\partial x} \left( \bar{A} E \frac{\partial \bar{S}}{\partial x} \right)$$

(2.15)

This equation was solved numerically. The model assumes non-time-dependent effective dispersion coefficients. Taking this starting point means that the model can not yield information...
any more detailed than average values over an average mixing-length in the estuary. This model can not be used if there are steep gradients caused by high resultant velocities, such as will be the case with high river discharge rates. For many applications, it may be sufficient to consider time-averages over one or more tidal periods and this makes it possible, in principle, to apply this model. The model was applied in this way to two Dutch estuaries, the Westerschelde and the Eems. These estuaries have low river discharge rates and tidal capacity, so that a state of equilibrium would seldom be reached. The effective dispersion coefficients in these estuaries were arrived at using actual 'on-site' measurements.

The application of this model to the Westerschelde estuary showed erratic deviations of the results, possibly because of variations in wind and tide and a quite large measure of uncertainty in respect of river discharge data. In the case of Eems estuary, there was a clear distinction between \( E \) values that applied to dry periods and wet periods, as shown in Fig. 2.2. The authors believed that these regular deviations were not ascribable solely to high rates of river discharge.

2.2.5 VEN DER BURGH MODEL (1972)

In 1972, Van der Burgh (Langeweg and Weerden, 1976) considered the average salinity over a tidal period and therefore Eqn. (2.4) formed the basis of his work. He determined
FIG 2-2 EFFECTIVE VALUES OF DISPERSION COEFFICIENT FOR THE EEMS.
the salinity curve at low-water and high-water slacks by translating the average salinity trend over a half-ebb and half-flood distance, as the case may be. He assumed $u_f$ and $s$ to be functions of $x$, and did not try to obtain any analytical solution, but rather solved Eqn. (2.4) numerically. Since in a section of length $\Delta x$ the magnitudes of $\bar{A}$ and $\bar{E}$ were assumed constant, Eqn. (2.4) was replaced by the following difference equation

\[
\frac{\Delta s}{\Delta x} = -\frac{u_f s}{\bar{s}} \tag{2.16}
\]

Van der Burgh arrived, from numerous measurements on site in the Dutch estuaries, at the following empirical relationships for the dispersion coefficient:

\[
\bar{E} = \bar{E}_0 - K_1 \int_0^x u_f \, dx \tag{2.17}
\]

where $\bar{E}_0$ is the dispersion coefficient at the estuary mouth, which can be obtained from the following relation

\[
\bar{E}_0 = K_0 (\alpha g)^{0.5} h^{1.5} \tag{2.18}
\]

In this equation $\alpha$ is the flood number introduced by Canter-Cremers (Langeweg and Weerden, 1976). This can be looked upon as a characteristic for the degree of mixing in the estuary, and is defined by

\[
\alpha = \frac{Q_f T}{p_T} \tag{2.19}
\]
In Eqn. (2.18), \( g \) is the gravitational acceleration, \( h \) is the depth at the estuary mouth and \( K_0 \) and \( K_1 \) are constants. For the salinity at the river mouth \( s_0 \), Van der Burgh found the following empirical relationship

\[
\tilde{s}_0 = s_0 \exp (-K_2 \alpha) \tag{2.20}
\]

He concluded, from prototype measurements in the Rotterdam Waterway, that \( 0.5 < K_2 < 1.0 \), and he used \( K_2 = 0.5 \) in his model. He further used a value of \( K_0 = 25 \) and \( K_1 = 0.9 \) (Figs. 2.3, 2.4 and 2.5).

One important advantage of Van der Burgh's model is that the cross-sectional area can be introduced as a function of \( x \). Against this, there is, however, the fact that this model is based solely on prototype observations, where it is of course not possible to vary the magnitudes in a systematic manner in order to find the constants needed. The constant \( K_0 \) is based on observations for which \( \alpha < 0.5 \), \( K_1 \) for \( u_r < 0.2 \text{ m/s} \) and \( K_2 \) for \( \alpha < 1.0 \). Extrapolations beyond the limit is not practical.

2.3.0 UNSTEADY STATE MODELS

Tidal and salinity movements in an estuary are always unsteady, and virtually steady condition never occurs in any estuary. Unsteady state models make it possible to describe transient salinity movements in estuaries. For the solution of salt balance equation Eqn. (2.1), a knowledge of
Effective Dispersion Coefficient, $E_0$ in $m^2/s$

FIG. 2.3 EFFECTIVE DISPERSION COEFFICIENT IN RIVER MOUTH AS FUNCTION OF FLOOD NUMBER AND DEPTH IN RIVER MOUTH
FIG. 2.4 REDUCTION OF DISPERSION COEFFICIENT IN X-DIRECTION AS FUNCTION OF VELOCITY OF RIVER DISCHARGE

\[ \frac{\Delta E}{\Delta x} \text{ in } \frac{m^2/s}{m} \]

\[ U_f \text{ in m/s} \]

FIG. 2.5 AVERAGE SALINITY IN RIVER MOUTH AS FUNCTION OF FLOOD NUMBER

\[ k_2 \]

\[ S_0 = 19e^{K_d} \]

\[ 1905-1900 \]

\[ + 1956 \]

\[ \times 1966-1971 \]
the time varying tidal discharge and velocity is required. This information is generally obtained from the simultaneous solution of unsteady continuity and momentum equations, which can be written as (Harleman and Lee, 1969)

\[
B \frac{\partial H}{\partial t} + \frac{\partial Q}{\partial x} - q = 0 \tag{2.21}
\]

\[
\frac{\partial Q}{\partial t} + u \frac{\partial Q}{\partial x} + Q \frac{\partial u}{\partial x} + gA \frac{\partial H}{\partial x} + gA \frac{Q}{K^2} = 0 \tag{2.22}
\]

where \(u, H,\) and \(Q\) are the tidal velocity, tidal stage and discharge, \(B\) and \(A\) are the top width and cross-sectional area respectively, \(K\) is the conveyence of the channel section, and \(q\) is the lateral inflow into the river.

The problem defined above requires a numerical approach, as the equations involved can not be solved analytically.

2.3.1 STIGTER AND SIEMONS MODEL (1966)

Stigter and Siemons (1966) developed a model for calculating water movement and salinity intrusion by applying a numerical method that solves Eqs. (2.1), (2.21), and (2.22) simultaneously. Explicit 'leap-frog' method was used in the discretization of Eqns. (2.21) and (2.22). For the solution of Eqn. (2.1) an implicit difference scheme was used, as explicit schemes are very unstable. In this model the water level and average salinity at estuary mouth \(x=0\) and on the river side \(x=L\) have to be specified as boundary conditions.
Using their model Stigter and Siemons provided an insight into the effect of density currents on the size of dispersion coefficient in partially mixed estuaries. Stigter and Siemons subsequently applied their model to the results of the flume study by Ippen and Harleman (1961) and to the Rotterdam Waterway. The size of the dispersion coefficient was found experimentally, starting from the assumption that $E$ is a function of $x$ and not of $t$, with the following form

$$E = E_0 \left(1 - \frac{x}{L}\right)^3 \quad (2.23)$$

By using a trial-and-error procedure $E_0 = 0.25 \ m^2/s$ was found for the flume experiment, while to obtain a match with the measurements taken in the Rotterdam Waterway, $E_0$ had to be taken as $1200 \ m^2/s$. There is, however, no reason to suppose that these values are generally applicable.

The Stigter and Siemons model must thus be regarded as one for analysis, and not for forecasting. To make a model generally applicable for the purpose of providing forecasts, the model would need to develop a method for predicting salinity at the mouth of the estuary and the size of dispersion coefficient. The Thatcher and Harleman (1972) model, which is considered in the next article, goes a substantial way towards doing this.

2.3.2 Thatcher and Harleman Model (1972)

Thatcher and Harleman (1972), like Stigter and Siemons, formulated a finite difference model for the conservation of
mass equation, Eqn. (2.1). The solution includes the tidal motion and allows time and space variations of cross-sectional area, discharge and dispersion coefficient. To solve Eqn. (2.1) the tidal hydraulics are obtained by a finite difference solution of Eqns. (2.21) and (2.22). The finite difference scheme used to solve the conservation of mass equation is the generalized 6-point implicit scheme recommended by Stone and Brian (1963). Fig. 2.6 shows the computational molecule and the weighting factors for the Stone and Brian scheme.

The computational molecule for the salinity intrusion model is superimposed on the computational grid for the tidal hydraulics model by calculating salinity at discharge mesh points, as shown in Fig. 2.7. The grid is staggered in space and time, and was adopted from Harleman and Lee (1969).

Thatcher and Harleman assumed that in the saline part of partially-mixed estuaries, the coefficient of dispersion is closely linked with the longitudinal salinity gradient, \(\partial s/\partial x\). They thus took the dispersion coefficient to be dependent on this gradient, in the following form

\[
E(x,t) = K | \frac{\partial s^0}{\partial x^0} | + E_T \tag{2.24}
\]

where \(s^0 = s/s_o\) and \(x^0 = x/L\), where \(s_o\) is the ocean salinity, \(L\) is the length of the estuary and \(E_T\) is the dispersion coefficient applicable to completely mixed region of the estuary. The term \(K | \partial s^0/\partial x^0 | \) provides for additional dispersion...
FIG. 2.6 COMPUTATIONAL MOLECULE FOR THE STONE AND BRIAN SCHEME

FIG. 2.7 COMPUTATIONAL MOLECULE FOR THE TIDAL HYDRAULICS AND SALINITY MODELS
in the area with saline intrusion. The parameter $K$ is taken to be independent of both $x$ and $t$, and to depend on the degree of stratification in the estuary. Thatcher and Harleman verified this assumption by means of the model test data. The verification of their assumption is shown in Fig. 2.8.

Thatcher and Harleman assumed that the dispersion parameter $K$ is dependent on the degree of stratification in the estuary. According to these authors, the degree of stratification can be indicated better by the internal estuary number, as defined by Harleman and Abraham (1966). The internal estuary number $E_i$ is defined as

$$E_i = \frac{\Delta \rho}{Q_f}$$

where $E_i$ is internal Froude number, defined as

$$F_i = \frac{u_o}{\sqrt{gh \frac{\Delta \rho}{\rho}}}$$

where $\Delta \rho$ is the change in density over the whole length of the estuary. Proceeding from Eqn. (2.5), the dispersion coefficient may be written as

$$\bar{E} = u_f \frac{\frac{\Delta \rho}{\rho}}{ds/dx}$$

Taking into account also Eqn. (2.24), the following equation for $K$ is obtained, for a steady state (salinity averaged
FIG. 2-8 VERIFICATION OF THATCHER AND HARLEMAN MODEL
over a tidal cycle)

\[ K = \frac{u_f L}{s^0} \frac{s^0}{ds^0/dx} \]  

(2.28)

It is seen from Eqn. (2.28) that \( K/u_f L \) gives a dimensionless reproduction of the salinity distribution under the circumstances set out above. Within the tidal movement \( K/\bar{\varphi}_o L \) is a corresponding form, so that this dimensionless parameter correlates to the internal estuary number. Thatcher and Harleman, analyzed numerous test and actual measured data, and deduced the following relationship between the two parameters

\[ \frac{K}{u_f L} = 0.002 \bar{\varphi}_i^{-0.25} \]  

(2.29)

The plot of this correlation is shown in Fig. 2.9.

The boundary conditions for calculating the tidal movement are simple to specify. This is not, however, the case for the boundary value of salinity at the estuary mouth. This is due to the fact that the salinity in estuary mouth is affected by the outflowing water and the salt content of the sea water, which is not always present in the estuary mouth.

Thatcher and Harleman, therefore, divided a tidal cycle into two parts—a flood-flow part and an ebb flow part. During the flood flow, the salinity in the estuary mouth is
FIG. 2.9 CORRELATION OF DISPERSION PARAMETER $K$ WITH INTERNAL ESTUARY NUMBER
assumed to be at the same level as in sea water. During ebb flow, the boundary values are evaluated by mass balancing in the most sea-ward element of the estuary. During ebb flow, salinity in the estuary mouth will fall below the salinity concentration of sea water. When the flood starts, the salt content can not rise immediately to that of sea water, and therefore, a linear interpolation is made over a period of $T/20$ before the turn of the tide, as shown in Fig. 2.10.

One of the difficulties encountered by Thatcher and Harleman in demonstrating the validity of their model was the lack of reliable field data. The authors chose the Delaware, Potomac and Hudson estuaries in U.S.A. for checking the model against prototype, where a series of high water slack salinity measurements were available. Thatcher and Harleman found a satisfactory correspondence between model and prototype, allowance being made for the limited opportunities offered by the field data. As a typical example of this correspondence, the results of the model and field data are shown in Fig. 2.11. This figure provides a comparison between calculated and measured salinities, for a period of 110 tidal cycles, demonstrating the close correspondence already mentioned.

Thatcher and Harleman's way of handling boundary values is likely to present problems if the model is being used for forecasting purposes. This will be the case if the salinity in the estuary mouth during the flood flow is still affected
FIG. 2.10 INTERPOLATION OF BOUNDARY SALINITY AT ESTUARY MOUTH
FIG. 2.11 COMPUTED AND MEASURED SALINITIES AT DIFFERENT STATIONS IN THE DELAWARE ESTUARY
by the water that has reached it from the river. They gave an example of this kind in the Hudson estuary, where it was found impossible to put the boundary value of salinity sufficiently sea-wards. Here, it proved necessary to interpolate linearly from low-water slack over a period of $2T/5$, in order to estimate the actual salinity trend, as shown in Fig. 2.10. It is not clear whether the ebb flow boundary statement is strictly correct in a mathematical sense, however, it seems to work computationally. The likely reason for this is that the ocean boundary salinity is dominated by advection from upstream on ebb tide. Assumption of constant dispersive flux over the boundary segment probably have very little effect on the solution.

2.4.0 PRESENT STATUS IN BANGLADESH

Study of salinity intrusion is in its infancy in Bangladesh. The first data collection started in 1951 (Leedshill Delauw, 1961), but comprehensive methods of data collection and analysis of the collected data are yet to be developed. Chidley, Raikes and Partners (1968) analyzed data collected by FAO and tried to correlate salinity with river discharge, but the study remained incomplete due to the lack of sufficient data. Netherlands Engineering Consultant (NEDECO, 1967) presented informations on tides, currents and channel depths, but little information on salinity. IDA (1972) made a sparse study of salinity intrusion through the coastal belt of Bangladesh. This study mainly dealt with the crop tolerance
of salinity and variations of salinity with tide levels. The Govt. of Bangladesh (1976) published a report where the grave crises caused by upstream withdrawal of fresh water, by India at Farakka, was pointed out. Asian Institute of Technology, Bangkok, Thailand and Bangladesh University of Engineering and Technology, Dhaka, (1977) studied salinity intrusion in the Pussur river. This study was limited to the statistical analysis of tidal fluctuations, upland discharge and the degree and extent of salinity intrusion through the river. The study emphasised the need for thorough study of salinity intrusion and recommended for developing mathematical models.

To date, there have been only a few attempts to study the problem with the help of mathematical models. Wasimi (1979) made a pioneer attempt to formulate a numerical model for Gorai-Modhumati-Sibsa-Pussur river system. He tried to apply the 'Node and Branch' concept of Delft Hydraulics Laboratory, for the simulation of tide and salinity. The extent and volume of work were, however, too ambitious, specially with respect to the available modelling experience, computer facilities and field data. As a result the model could not be developed properly, for the simulation of tide and salinity in the river system. The model developed by Chowdhury (1983) was based on approximate analytical solution of the one-dimensional high water slack salinity equation. He used the methodology developed by Hydraulic Research Station, U.K. The model was capable of simulating HWS salinity
over a period of several days. This type of model is suitable for preliminary studies with limited field data.

2.5.0 SUMMARY

Modelling of salinity intrusion in estuaries has undergone a rapid development in the last two decades, starting from steady state models to unsteady models. Steady state models were developed due to the lack of the understanding of the physical phenomenon and because of the difficulties in solving the non-linear mass balance and tidal dynamics equations. With the availability of computer facilities and the developments of numerical methods, more realistic models were developed, taking into consideration the unsteady tidal condition in the estuaries.

Thatcher and Harleman (1972) model for the prediction of salinity as a function of distance and time represents a definite advance with respect to the models previously used. This model and various other models (Thatcher, Pearson and Mayor-Mora, 1975, Arbhabherama and Wongsisjesemjai, 1978, Thatcher and Harleman, 1981, and others) based on this type of formulations offer a good hope for further developments and analysis of salinity in estuaries.

Unsteady state models that have been developed in recent years are capable of handling wide variations of tidal range in estuary mouth, upstream fresh water inflow, the
complexity of the estuary geometry and stratification conditions, that vary from estuary to estuary and within the same estuary with the change in hydraulic conditions. These models make it possible to study estuaries for which no previous salinity distribution data are available. Thus these models can be used as a tool for prediction of salinity intrusion due to changes in the geometry of estuary produced by dredging or shoaling, withdrawal of upstream inflow of fresh water, and other major works that alter the regimen of the estuary.
CHAPTER III
FORMULATION OF THE MODEL

3.1.0 INTRODUCTION

The numerical modelling of salinity intrusion in an estuary consists of solution of tidal dynamics equations and mass balance equation, commonly known as salt balance equation. Fig. 3.1 illustrates schematically the general functioning of the numerical model. The following articles briefly describe the fundamental equations, principles, assumptions and the logic involved in the formulation and solution of the equations.

3.2.0 TIDAL DYNAMICS EQUATIONS

Mathematical derivation and modelling of tidal propagation in estuary is well documented (Harleman and Lee, 1969, Arbhabhirana and Vongvisessamajj, 1978). According to Harleman and Lee (1969), the one-dimensional unsteady continuity and momentum equations can be written as follows:

Continuity Equation:

$$ B \frac{\partial H}{\partial t} + \frac{\partial Q}{\partial x} - q = 0 \quad (3.1) $$

Momentum Equation:

$$ \frac{\partial Q}{\partial t} + \frac{2Q}{A} \frac{\partial Q}{\partial x} - \frac{Q^2}{A^2} \frac{\partial A}{\partial x} + gA \cdot \frac{\partial H}{\partial x} + gA \cdot \frac{Q}{K^2} \frac{\partial Q}{\partial x} = 0 \quad (3.2) $$
INITIAL INPUT
(SPATIALLY VARYING)
GEOMETRY AND ROUGHNESS
INITIAL CONDITIONS OF:
SALINITY, SURFACE ELEVATION, DISCHARGE

CONTINUING INPUTS
OCEAN SURFACE ELEVATION \( (t) \)
FRESH WATER INFLOWS \( (t) \)

TIDAL DYNAMICS MODEL

OUTPUTS
SURFACE ELEVATIONS \( (x,t) \)
DISCHARGES \( (x,t) \)

SALT BALANCE MODEL

SURFACE ELEVATION \( (x,t) \)
DISCHARGE \( (x,t) \)

SALINITY \( (x,t) \)

FIG. 31 GENERAL FUNCTIONING OF NUMERICAL MODEL
where

\[ H = \text{instantaneous water surface elevation above reference datum,} \]
\[ Q = \text{tidal discharge across the cross-section of river,} \]
\[ q = \text{lateral inflow per unit length of river,} \]
\[ A = \text{cross-sectional area of the river,} \]
\[ B = \text{top width of the river cross-section,} \]
\[ g = \text{acceleration due to gravity,} \]
\[ K = \text{conveyence of the river cross-section,} \]
\[ = \frac{1}{n} AR^{2/3}, \]
\[ R = \text{hydraulic radius of the cross-section,} \]
\[ n = \text{Manning's roughness coefficient.} \]

The definition-sketch of these variables is shown in Fig. 3.2. The quantity \(|Q|\) in the last term of Eqn. (3.2) refers to the absolute magnitude of discharge, consequently the resistance term in Eqn. (3.2) has the same sign as that of \(Q\), which takes into account the reversal of flow in a tidal river.

3.2.1 ASSUMPTIONS AND LIMITATIONS

The formulation of the Eqns. (3.1) and (3.2) is based on certain hypotheses and assumptions. While solving these equations, it should be kept in mind that the results obtained from these equations can not be better than the hypotheses and assumptions on which they are based. Thus, a brief review of these basic assumptions are listed below:
FIG. 3-2 DEFINITION SKETCH FOR TIDAL DYNAMICS EQUATIONS
i) The water surface varies gradually, which is equivalent to stating that the pressure distribution along a vertical is hydrostatic, or that the vertical acceleration is small. Liggett (1975) has shown that this assumption is indeed justified for problems characterized by scales which are much larger in the horizontal than in the vertical. The ratio of a typical depth of flow to the wave length is a measure of this characteristic and 0.55 is taken as the upper limit. Even if the vertical displacement is significant, still the flow can well be described by these equations as long as the vertical acceleration has a negligible effect on pressure distribution.

ii) The average slope of the channel bottom is so small that \( \sin \theta_x \) may be replaced by \( \tan \theta_x \) and \( \cos \theta_x \) is unity, where \( \theta_x \) is the angle made by the channel bed with the horizontal. On the basis of 'first order' and 'second order' wave theories, Liggett (1975) has shown that only 2% error occurs at about \( \theta_x = 16^\circ \) or \( s_0 = 0.29 \), which is an extremely steep slope for an estuary.

iii) The velocity distribution across the cross-section of channel does not substantially affect the wave propagation. The discrepancy caused by velocity variation in the vertical can be accounted for by
introducing momentum correction factor. For turbulent flows in rivers and estuaries, this factor seldom exceeds 1.05 (Liggett, 1975). Consequently, it is a common practice to take the value of momentum correction factor equal to unity.

iv) Frictional loss in unsteady flow is not significantly different from those in the steady flow, and can be accounted for either by Chezy or Manning coefficient. Though these equations were derived for steady uniform flow, they have been successfully applied in unsteady flow (Liggett, 1975). It should be noted that the values of Chezy and Manning coefficients in an estuary will not necessarily be equal to those based on unidirectional flows in an equivalent river. The reason for this is that the flow reversal in the estuaries causes different bed forms from that found in unidirectional rivers (McDowell and O'Connor, 1977). Nevertheless, frictional losses based on these equations generally give acceptable results.

v) The equations are applicable to one-dimensional tidal flows. This implies that velocity is averaged longitudinal value for the schematized flow area. Coriolis effects in the transverse direction are neglected. Coriolis effect is important in wide estuaries and seas in higher latitudes (McDowell and O'Connor, 1977).
vi) Variation of cross-sectional area along the longitudinal axis of the estuary is gradual. Local energy dissipation due to sudden contraction and enlargement or flow around bend or due to sinuosity is negligible. The estuary is considered straight, and the local energy losses are partly taken into consideration by introducing additional friction.

vii) The schematized boundary of the estuary is fixed, so that no erosion or deposition is assumed to take place due to tidal motion. Consequently, there are no changes in the bed forms and the roughness coefficient is constant for a particular reach of the channel. In open channels, a small disturbance at the water surface travels with a celerity $c = \sqrt{gh}$. It is known that the disturbances at the bed level, e.g., ripples and dunes, travel much slower. The water level celerities are normally in the order of 1 to 10 m/s, whereas, ripples and dunes propagate much slower, at a speed ranging from 0.1 to 1.0 m/hour. De Varies (1971) showed that for tidal computations and flood routing with moderate Froude number (subcritical flow) assuming the bed fixed is reasonable.

viii) Variations in the density of water due to the presence of salt in neglected. In well mixed and partially mixed estuaries, vertical density gradient is insignificant. A longitudinal density gradient exists
in these types of estuaries, due to longitudinal density variation ranging from 1000 kg/m$^3$ at the upper tidal limit to 1026 kg/m$^3$ at the sea. This density gradient gives rise to a small landward force (McDowell and O'Conner, 1977). The magnitude of this horizontal force is directly proportional to horizontal density gradient. The variation of density of water from sea to river end is usually of the order of 2%, and this force is much smaller compared to tidal forces. Hence, the effect of density gradient can be easily neglected.

ix) The direct astronomical tidal effects in the estuary are neglected. The tidal motion is assumed to be caused by ocean tide at the estuary mouth.

x) The effect of wind stress on the water surface of the estuary has been neglected. This can be taken into account, provided local field data are available (Harleman and Lee, 1969). The wind effect is negligible unless the estuary is very wide and wind velocity is high.

3.3.0 SALT BALANCE EQUATION

Salinity intrusion in an estuary involves the basic mechanism of mass (salt) transport in the estuary. According to Thatcher and Harleman (1972), the one-dimensional salt-
Transport or salt-balance equation can be written as

\[
\frac{\partial}{\partial t} (As) + \frac{\partial}{\partial x} (Qs) = \frac{\partial}{\partial x} (AE \frac{\partial s}{\partial x}) \tag{3.3}
\]

where

\( s \) = cross-sectional averaged salinity,

\( A \) = cross-sectional area of the channel,

\( Q \) = instantaneous tidal discharge,

\( E \) = longitudinal dispersion coefficient.

The definition sketch of the variables involved in Eqn. (3.3) are shown in Fig. 3.3. The use of one-dimensional salt-balance equation limits the results of the model to quantities which are averaged over the channel cross-section. The salinity concentration is assumed uniform over the cross-section and the longitudinal variation of the density of water is neglected.

3.3.1 SIGNIFICANCE AND CORRELATION OF DISPERSION COEFFICIENT

The physical meaning of the longitudinal dispersion coefficient is difficult to ascertain. Presently, it is being used as a convenient parameter taking into account the various processes involved in the transport of salt. These processes include non-uniformity and unsteadiness of flow, molecular and turbulent diffusions, convective currents associated with tidal motion, fresh water inflow, longitudinal density gradient, non-uniformity of velocity and salinity concentration across the cross-section, non-uniformity of cross-sectional area, etc.
FIG. 3.3 DEFINITION SKETCH FOR SALT BALANCE EQUATION
Many attempts have been made by various investigators (Elata, 1967, Fischer, 1967, Harleman, Lee and Hall, 1968, Holley, 1969, Fischer, 1968, Holley and Cunge, 1975, Gordon and Spaulding, 1975) to quantify these effects but a great deal is yet to be done to develop a mathematically logical method of predicting the magnitude of longitudinal dispersion coefficient for estuaries.

Presently, it is assumed that the dispersion coefficient is closely related to the density induced circulation, in the salinity intrusion region of the estuary (Thatcher and Harleman, 1972). Due to spatial averaging inherent in the one-dimensional models, this circulation is not directly apparent, but its effect should be included in the dispersion coefficient. It is reasonable to assume that this density induced circulation will be greatest in regions of strongest longitudinal salinity gradient, $\partial s / \partial x$, and consequently the dynamic relationship used in this study for the dispersion coefficient is achieved by relating $E(x,t)$ to the absolute value of the local salinity gradient, as proposed by Thatcher and Harleman (1972). The relationship is

$$E(x,t) = K_1 \left| \frac{\partial s^o}{\partial x} \right| + E_T$$  \hspace{1cm} (3.4)

where

$$s^o = s/s_0'$$

$s$ = local salinity,
\( s_0 \) = ocean salinity,
\( x^0 = x/L \),
\( L \) = length of the estuary,
\( K_1 \) = a parameter related to the degree of stratification in the estuary,
\( E_T \) = Taylor's dispersion coefficient.

In Eqn. (3.4), the parameter \( K_1 \) has the dimension of dispersion coefficient, and it is assumed to be independent of \( x \) and \( t \), and to depend upon the degree of stratification which exists in the estuary. The first term in the equation provides additional dispersion in the region of salinity intrusion. The second term \( E_T \) in Eqn. (3.4) is the dispersion coefficient applicable to a completely mixed region, where \( \partial s/\partial x = 0 \), or to the fresh water tidal region, upstream of the limit of salinity intrusion. Harleman, Lee and Hall (1968) have shown that in the fresh water region of the estuary, the dispersion coefficient can be expressed by the modified Taylor's dispersion coefficient

\[
E_T = K_2 nu R^{5/6} \tag{3.5}
\]

where

\( n \) = Manning's roughness coefficient,
\( u \) = cross-sectional averaged velocity,
\( R \) = hydraulic radius,
\( K_2 \) = a calibration coefficient.

In Eqn. (3.5) both \( u \) and \( R \) are functions of \( x \) and \( t \), and the absolute value of \( u \) is used in the computation of \( E_T \).
This formulation of longitudinal dispersion coefficient permits a dynamic calculation of $E(x,t)$, which applies to the entire estuary, in both partially and completely mixed regions.

3.4.0 FORMULATION OF FINITE DIFFERENCE EQUATIONS

The salt balance equation and the tidal dynamics equation are not integrable in closed form, except under extremely simplified cases. The resulting solutions have very little practical value. To overcome this difficulty, numerical methods have been used extensively. In the finite difference method, which has been used in this study, Eqs. (3.1), (3.2) and (3.3) are approximated by three finite difference equations. The estuary is divided into equal reaches of length $\Delta x$ and the time domain in equal intervals $\Delta t$. The basic features of this method are as follows:

i) All the variables are discretized in such a way that they are either constant or changing linearly during time interval $\Delta t$ and along the estuary reach $\Delta x$.

ii) The accuracy of the method depends on the magnitudes and relationship of the finite differences $\Delta t$ and $\Delta x$. The smaller the values of $\Delta t$ and $\Delta x$, the greater is the computational work and in general greater is the accuracy.
iii) Adoption of the values of $\Delta t$ and $\Delta x$ also depends on the accuracy and amount of background data and the initial and boundary conditions.

iv) An iterative procedure is commonly used in determining the dependent variables at the ends of $\Delta t$ and $\Delta x$.

The numerical properties of this method of solution is the subject matter of the next chapter. The rest of this chapter mainly deals with the formulation of the numerical model, the solution of the resulting equations and the initial and boundary conditions required for the solution.

3.4.1 DISCRETIZATION OF TIDAL DYNAMICS EQUATIONS

A large number of numerical schemes have been developed for the numerical solutions of Eqn. (3.1) and (3.2), both explicit and implicit. Implicit schemes were developed because of the limitations imposed on time step $\Delta t$, in using explicit schemes. In this study the Preissmann (SOGREAH) implicit scheme has been used (Liggett and Cunge, 1975 and Samuels, 1977) for the numerical solution of the tidal dynamics equations. Fig. 3.4 and the following relations explain the discretization of the dependent variables and their derivatives,

$$f(x,t) = \frac{0}{2} \left( f_{j+1}^{n+1} + f_j^{n+1} \right) + \frac{1}{2} (1-\theta) \left( f_{j+1}^n - f_j^n \right) (3.6)$$
FIG. 3.4 PREISSMANN FOUR POINT IMPLICIT SCHEME
where $\theta$ is a weighting coefficient, $0 < \theta < 1$, introduced in the time derivative to aid the numerical solution. In these equations $f$ is the dependent variable, which can be either discharge or water level. This scheme and its modifications have been extensively used for the simulation of tides, flood and even rapidly varied flows (Liggett and Cunge, 1975, Samuels, 1977, Arbhabhirama and Wongvisessomjai, 1978, Tingsanchali and Arbhabhirama, 1978, and Cunge, 1975).

Referring to Fig. 3.4, for a small increment of time $\Delta t$, the following linear relationship may be approximated

$$\Delta f_j = f_{j+1}^{n+1} - f_j^n$$ (3.9)

where $\Delta f_j$ is a small increment of $f$ at grid point $j$ in time interval $\Delta t$. These increments are assumed so small that their products, square and their higher powers can be neglected. Using this linearization, Eqns. (3.6), (3.7) and (3.9) can be modified to the following forms

$$f(x,t) = \frac{1}{2} \{ \theta(\Delta f_{j+1} + \Delta f_j) + (f_{j+1}^{n+1} - f_j^n) \}$$ (3.10)

$$\frac{\partial f}{\partial x} = \frac{1}{\Delta x} \{ \theta(\Delta f_{j+1} + \Delta f_j) + (f_{j+1}^{n+1} - f_j^n) \}$$ (3.11)

$$\frac{\partial f}{\partial t} = \frac{1}{2\Delta t} \{ \Delta f_{j+1} + \Delta f_j \}$$ (3.12)
In Eqns. (3.10), (3.11) and (3.12) the functions $f_{j+1}$ and $f_j$ are defined at $t = n$, and the superscript $n$ has been dropped for convenience.

Using the scheme defined by Eqns. (3.10), (3.11) and (3.12) the continuity equation Eqn. (3.1) can be discretized to

$$\frac{\Delta H_{j+1} + \Delta H_j}{2\Delta t} + \frac{2}{\Theta (\Delta B_{j+1} + \Delta B_j) + (B_{j+1} + B_j)}$$

$$= \left\{ 0 \frac{\Delta Q_{j+1} - \Delta Q_j}{\Delta x} + \frac{Q_{j+1} - Q_j}{\Delta x} \right\}$$

(3.13)

where the lateral inflow term in the equation has been neglected. This equation is linearized by developing the denominator of the second term in a power series and keeping only the first order terms, so that

$$\frac{1(\Delta B_{j+1} + \Delta B_j)}{\Theta (\Delta B_{j+1} + \Delta B_j) + (B_{j+1} + B_j)} = \frac{1}{B_{j+1} + B_j} \left\{ 1 - \Theta \frac{\Delta B_{j+1} + \Delta B_j}{B_{j+1} + B_j} \right\}$$

(3.14)

The substitution of Eqn. (3.14) into Eqn. (3.13) and further linearizing according to the principle that $(\Delta H)^2 = (\Delta Q)^2 = (\Delta Q)(\Delta H) = 0$, the following equation is obtained

$$\frac{\Delta H_{j+1} + \Delta H_j}{2\Delta t} + \frac{2}{B_{j+1} + B_j} \left\{ 0 \frac{\Delta Q_{j+1} - \Delta Q_j}{\Delta x} + \frac{Q_{j+1} - Q_j}{\Delta x} \right\}$$

$$- \frac{2\Theta}{(B_{j+1} + B_j)^2} (\Delta B_{j+1} + \Delta B_j) \frac{Q_{j+1} - Q_j}{\Delta x} = 0$$

(3.15)
where

$$\Delta B_j = \frac{dB_j}{dz} \Delta H_j$$  \hspace{1cm} (3.16)

$$\Delta B_{j+1} = \frac{dB_{j+1}}{dz} \Delta H_{j+1}$$  \hspace{1cm} (3.17)

Eqn. (3.15) on simplification can be written as

$$C_j \Delta H_j + D_j \Delta H_{j+1} + E_j \Delta Q_j + F_j \Delta Q_{j+1} + K_j = 0$$  \hspace{1cm} (3.18)

where

$$C_j = 1 - 4 \frac{dt}{Ax} \frac{1}{(B_{j+1} + B_j)^2} (Q_{j+1} - Q_j) \frac{dB_j}{dz}$$  \hspace{1cm} (3.19)

$$D_j = 1 - 4 \frac{dt}{Ax} \frac{1}{(B_{j+1} + B_j)^2} (Q_{j+1} - Q_j) \frac{dB_{j+1}}{dz}$$  \hspace{1cm} (3.20)

$$E_j = - 4 \theta \frac{dt}{Ax} \frac{1}{B_{j+1} + B_j}$$  \hspace{1cm} (3.21)

$$F_j = 4 \theta \frac{dt}{Ax} \frac{1}{B_{j+1} + B_j}$$  \hspace{1cm} (3.22)

$$K_j = 4 \frac{dt}{Ax} \frac{1}{B_{j+1} + B_j} (Q_{j+1} + Q_j)$$  \hspace{1cm} (3.23)

The momentum equation Eqn. (3.2), when discretized, using Eqns. (3.10), (3.11) and (3.12), becomes

$$\frac{\Delta Q_{j+1} + \Delta Q_j}{2\Delta t} + 2 \left[ \frac{Q_{j+1} + \Delta Q_j}{A_{j+1} + \Delta A_j} + \frac{Q_j + \Delta Q_j}{A_j + \Delta A_j} \right] + \frac{1}{2} (1 - \theta) \left[ \frac{Q_{j+1}}{A_{j+1}} + \frac{Q_j}{A_j} \right]$$
\[
\begin{align*}
&\left\{ \frac{\Theta (\Delta q_{j+1} - \Delta q_j)}{\Delta x} + \frac{q_{j+1} - q_j}{\Delta x} \right\} - \frac{\Theta}{2} \left( \frac{(q_{j+1} + \Delta q_{j+1})^2}{(A_{j+1} + \Delta A_{j+1})^2} + \frac{(q_j + \Delta q_j)^2}{(A_j + \Delta A_j)^2} \right) \\
&+ \frac{1}{2} (1 - \Theta) \left\{ \frac{q_{j+1}^2}{A_{j+1}^2} + \frac{q_j^2}{A_j^2} \right\} \left\{ \frac{\Theta (\Delta A_{j+1} - \Delta A_j) + (A_{j+1} - A_j)}{\Delta x} \right\} \\
&+ q \left\{ \Theta (\Delta Q_{j+1} + \Delta A_j) + \frac{1}{2} (A_{j+1} + A_j) \right\} \left\{ \frac{\Theta (\Delta H_{j+1} - \Delta H_j + (H_{j+1} - H_j)}{\Delta x} \right\} \\
&+ q \frac{\Theta}{2} \left\{ \frac{(A_{j+1} + \Delta A_{j+1}) (q_{j+1} + \Delta q_{j+1})}{(K_{j+1} + \Delta K_{j+1})^2} \right\} \\
&+ \frac{1}{2} (1 - \Theta) \left\{ \frac{\Delta Q_{j+1} + \Delta q_j}{K_{j+1}^2} + \frac{\Delta A_j q_j}{K_j^2} \right\} \right. \\
&\left. \right\} (3.24)
\end{align*}
\]

Eqn. (3.23) can be linearized like the continuity equation, first by developing all terms in power series can neglecting the terms of second and higher orders. It should be noted that

\[
\frac{1}{A_j + \Delta A_j} = \frac{1}{A_j} \frac{\Delta A_j}{(1 + \frac{\Delta A_j}{A_j})} = \frac{1}{A_j} \left(1 - \frac{\Delta A_j}{A_j} \right) (3.25)
\]

\[
\frac{1}{(A_j + \Delta A_j)^2} = \frac{1}{A_j^2} \frac{\Delta A_j}{(1 + \frac{\Delta A_j}{A_j})^2} = \frac{1}{A_j^2} \left(1 - 2 \frac{\Delta A_j}{A_j} \right) (3.26)
\]
\[(Q_j + \Delta Q_j)^2 = Q_j^2 + 2Q_j \Delta Q_j \quad (3.27)\]

\[(Q_j + \Delta Q_j) \mid Q_j + \Delta Q_j \mid = Q_j \mid Q_j \mid + 2 \mid Q_j \mid \Delta Q_j \quad (3.28)\]

\[\Delta A_j = \frac{dA_j}{dz} \quad \Delta H_j = B_j \Delta H_j \quad (3.29)\]

\[\Delta B_j = \frac{dB_j}{dz} \quad \Delta H_j \quad (3.30)\]

\[\Delta K_j = \frac{dK_j}{dz} \quad \Delta H_j \quad (3.31)\]

After the linearization, using Eqns. (3.24) through (3.30), the following equation is obtained

\[C_j \Delta H_j + D_j \Delta H_{j+1} + E_j \Delta Q_j + F_j \Delta Q_{j+1} + K_j = 0 \quad (3.32)\]

where

\[C_j = 2 \Omega \frac{\Delta t}{\Delta x} \frac{B_j Q_j}{A_j^2} \left\{ \frac{Q_j}{A_j} (A_{j+1} - A_j) - (Q_{j+1})^2 \right\} + \frac{B_j}{2} \left[ \frac{Q_{j+1}^2}{A_{j+1}^2} + \frac{Q_j^2}{A_j^2} \right]

\[+ \frac{\Omega}{2} \left[ B_j (H_{j+1} - H_j) - (A_{j+1} + A_j) \right] \]

\[+ \frac{\Omega \Delta t}{K_j^2} \quad \left\{ \frac{Q_j}{A_j} \right\} \left\{ B_j - \frac{2}{K_j} A_j \frac{dK_j}{dz} \right\} \quad (3.33)\]

\[D_j = 2 \Omega \frac{\Delta t}{\Delta x} \frac{B_j Q_{j+1}}{A_{j+1}^2} \left\{ \frac{Q_{j+1}}{A_{j+1}} (A_{j+1} - A_j) - (Q_{j+1} - Q_j) \right\}

\[\text{result of (3.32) with } C_j, D_j, E_j, F_j, \text{ and } K_j \text{ determined.}\]
- \frac{1}{2} B_{j+1} \left[ \frac{Q_{j+1}^2}{A_{j+1}^2} + \frac{Q_j^2}{A_j^2} \right] + \frac{1}{2} \left[ B_{j+1} \left( \frac{H_{j+1}}{A_{j+1}} - H_j \right) + (A_{j+1} + A_j) \right]

+ \frac{g \Delta \omega \Delta t}{K_{j+1}^2} Q_{j+1} \left\{ Q_{j+1} - \left( \frac{Q_{j+1}^2}{A_{j+1}} + \frac{Q_j^2}{A_j^2} \right) Q_j \left( A_{j+1} + A_j \right) \right\} (3.34)

E_{j} = 1 + 2 \Theta \frac{\Delta t}{\Delta x} \left\{ \frac{1}{A_{j+1}} (Q_{j+1} - Q_j) - \frac{Q_{j+1}^2}{A_{j+1}^2} - \frac{Q_j^2}{A_j^2} \right\} + 2g \Theta \frac{\Delta \omega}{K_{j+1}^2} A_{j+1} \left| Q_{j+1} \right| (3.35)

F_{j} = 1 + 2 \Theta \frac{\Delta t}{\Delta x} \left\{ \frac{1}{A_{j+1}} (Q_{j+1} - Q_j) + \frac{Q_{j+1}^2}{A_{j+1}^2} + \frac{Q_j^2}{A_j^2} \right\} - \frac{1}{A_{j+1}^2} Q_{j+1} (A_{j+1} - A_j) + 2g \Theta \frac{\Delta \omega}{K_{j+1}^2} A_{j+1} \left| Q_{j+1} \right| (3.36)

K_{j} = 2 \Theta \frac{\Delta t}{\Delta x} \left\{ \frac{Q_{j+1}^2}{A_{j+1}^2} + \frac{Q_j^2}{A_j^2} \left( Q_{j+1} - Q_j \right) - \frac{Q_{j+1}^2}{A_{j+1}^2} - \frac{Q_j^2}{A_j^2} \right\} (A_{j+1} - A_j)

+ \frac{g}{2} (A_{j+1} + A_j) (H_{j+1} - H_j) + g \Delta t \left( \frac{1}{K_{j+1}^2} A_{j+1} \left| Q_{j+1} \right| Q_{j+1} \right)

+ \frac{1}{K_j^2} A_j Q_j \left| Q_j \right| (3.37)

Eqns. (3.18) and (3.32) contain four unknowns, namely \( \Delta H_j, \Delta Q_j, \Delta H_{j+1}, \) and \( \Delta Q_{j+1} \). The coefficients \( C_j, D_j, \ldots, K_j \) and \( C_j^l, D_j^l, \ldots, K_j^l \) as defined by Eqns. (3.19) to (3.23) and Eqns.
(3.33) to (3.37) respectively are functions of known quantities, such as water level, discharge, and geometric parameters at \( t = n \). Thus the coefficients can be evaluated from either the initial condition or from the previous step of computation.

At this stage, it should be noted that Eqns. (3.18) and (3.32) have been linearized by assuming \( \Delta f << f \). When this condition is not satisfied, the solution of the equations, Eqns. (3.18) and (3.32) will not be considered as a good approximation to the solution of the non-linear Eqns. (3.1) and (3.2). However, if \( \Delta f \) is of the same order of magnitude as \( f \), for a certain value of \( \Delta t \), the ratio \( \Delta f/f \) can always be decreased by decreasing the value of \( \Delta t \), and consequently satisfying the requirement that \( \Delta f << f \).

3.4.2 DISCRETIZATION OF SALT-BALANCE EQUATION

In connection to the numerical solution of the diffusion-advection equation, important investigations were done by Stone and Brian (1963) and Price, Cavendish and Varga (1968). Though the starting points of these investigations were different, they arrived at the same conclusion, and proposed the following discretization scheme for the solution of the salt balance equation:

\[
\frac{\Delta f}{\Delta t} = g_{-1} \frac{f_{j+1}^n - f_{j-1}^n}{\Delta t} + g_0 \frac{f_{j+1}^{n+1} - f_j^n}{\Delta t} + g_1 \frac{f_{j+1}^n - f_j^n}{\Delta t} \quad (3.38)
\]
The graphical representation of the scheme is shown in Fig. 3.5.

In Eqn. (3.38), \( g_{-1}, g_0 \) and \( g_1 \) are the weighting coefficients. Stone and Brian (1963) stated that the solution of the linearized diffusion-advection equation as well as the solution of the difference equation, can be written in the form of a Fourier series. For the finite difference solution to be same as the solution of the differential equation, it is required that the modulus of the ratio of terms with the same frequency must have the value one, and the argument of this ratio the value zero (that means no phase-shift). These requirements will be fulfilled as much as possible by choosing \( g_{-1} = g_1 = 1/6 \) and \( g_0 = 2/3 \) (Siemon, 1970), and such that \( g_{-1} + g_0 + g_1 = 1 \). For some cases, the choice of \( g_{-1} = g_1 = 0 \) and \( g_0 = 1 \) may give reasonable results (Arbhabhirama and Vongvisessomjai, 1978), while for most of the cases, this choice will produce rather inaccurate solution. The choice of \( g_{-1} = g_1 = 1/6 \) and \( g_0 = 2/3 \) is in fact a special case of the generalized 6-point implicit scheme, recommended by Stone and Brian (1963).
\[ S_{j-1} \quad Q_{j+1/2} \quad S_{j} \quad Q_{j+1/2} \quad S_{j+1} \]

\( (EA)_{j-1/2} \quad (EA)_{j+1/2} \)

\[ t \]

\[ x \]

**FIG. 3-5 STONE AND BRIAN SIX POINT IMPLICIT SCHEME**
Using Eqns. (3.38), (3.39) and (3.40), the salt balance equation, Eqn. (3.3), can be discretized to the following form (with \( g_{-1} = g_1 = 1/6 \) and \( g_0 = 2/3 \)):

\[
\frac{1}{6\Delta t} \left\{ (As)_{j-1}^{n+1} - (As)_{j-1}^n \right\} + \frac{2}{3\Delta t} \left\{ (As)_{j}^{n+1} - (As)_j^n \right\} + \frac{1}{6\Delta t} \left\{ (As)_{j+1}^{n+1} - (As)_{j+1}^n \right\} + \frac{1}{2\Delta x} \left\{ Q_{j-1/2}^{n+1} (S_{j+1}^{n+1} - S_{j}^{n+1}) \right\} + \frac{1}{2\Delta x} \left\{ Q_{j+1/2}^{n} (S_{j}^{n} - S_{j+1}^{n}) \right\}
\]

\[
= \frac{1}{2\Delta x} \left\{ (AE)_{j+\frac{1}{2}}^{n+1} \frac{S_{j+1}^{n+1} - S_{j}^{n+1}}{\Delta x} - (AE)_{j-\frac{1}{2}}^{n+1} \frac{S_{j}^{n+1} - S_{j-1}^{n+1}}{\Delta x} \right\} + (AE)_{j+\frac{1}{2}}^{n} \frac{S_{j}^{n} - S_{j-1}^{n}}{\Delta x} - (AE)_{j-\frac{1}{2}}^{n} \frac{S_{j-1}^{n} - S_{j}^{n}}{\Delta x} \right\} \right) (3.41)
\]

Simplifying Eqn. (3.41) the following equation results

\[
a_j S_{j-1}^{n+1} + b_j S_{j}^{n+1} + c_j S_{j+1}^{n+1} = d_j \quad (3.42)
\]

where

\[
a_j = \frac{1}{6\Delta t} A_j^{n+1} - \frac{1}{4\Delta t} Q_{j-\frac{1}{2}}^{n+1} - \frac{1}{2\Delta x^2} (AE)_{j-\frac{1}{2}}^{n+1} (3.43)
\]

\[
b_j = \frac{2}{3\Delta t} A_j^{n+1} + \frac{1}{4\Delta x} (Q_{j-\frac{1}{2}}^{n+1} - Q_{j+\frac{1}{2}}^{n+1}) + \frac{1}{2\Delta x^2} (AE)_{j-\frac{1}{2}}^{n+1} + (AE)_{j+\frac{1}{2}}^{n+1} \quad (3.44)
\]

\[
c_j = \frac{1}{6\Delta t} A_j^{n+1} + \frac{1}{4\Delta x} Q_{n+\frac{1}{2}}^{n+1} - \frac{1}{2\Delta x^2} (AE)_{j+\frac{1}{2}}^{n+1} \quad (3.45)
\]
\[ d_j = \left[ \frac{1}{6\Delta t} A^n_{j-1} + \frac{1}{4\Delta x} Q^n_{j-\frac{1}{2}} + \frac{1}{2\Delta x^2} (AE)_{j-\frac{1}{2}}^n \right] S^n_{j-1} \]
\[ + \left( \frac{2}{3\Delta t} A^n_j - \frac{1}{4\Delta x} (Q^n_{j-\frac{1}{2}} - Q^n_{j+\frac{1}{2}}) - \frac{1}{2\Delta x^2} \left[ (AE)_{j-\frac{1}{2}}^n + (AE)_{j+\frac{1}{2}}^n \right] \right) S^n_j \]
\[ + \left( \frac{1}{6\Delta t} A^n_{j+1} - \frac{1}{4\Delta x} Q^n_{j+\frac{1}{2}} + \frac{1}{2\Delta x^2} (AE)_{j+\frac{1}{2}}^n \right) S^n_{j+1} \] (3.46)

The coefficients \( a_j, b_j, c_j \) and \( d_j \) can be evaluated with the discharge values obtained from the tidal dynamics model, and from the geometry of the estuary. The coefficient \( d_j \) requires the initial value of salinity, which can be obtained from initial condition or from the previous step of computation. Thus Eqn. (3.42) can be solved for the next time level.

### 3.5.0 SOLUTIONS OF LINEARIZED EQUATIONS

In solving the linearized tidal dynamics-and-salt-balance equations, these equations are written at every grid point from upstream to downstream and the systems of equations can be solved by any standard method of solution of simultaneous linear equations. However, since the solution of the system of equations is the most time consuming part of computation, any time saving procedure is always preferred. In the present study Double Sweep Algorithm and Thomas Algorithm have been used for the solutions of the linearized tidal dynamics and salt balance equations, respectively. The basic concepts...
and the advantages of these methods over general methods of solution are discussed in the following articles.

3.5.1 DOUBLE SWEEP ALGORITHM

The description of the Double Sweep Algorithm, given in this article, is based on Tingsanchali and Arbhabhirama (1978) and Liggett and Cunge (1975). Referring to Fig. 3.6, the boundary conditions at \( j = 1 \) and \( J \) are given, and thus \( \Delta H_j \) and \( \Delta Q_j \) are known. The Eqns. (3.18) and (3.32) for the first grid point can be written as

\[
D_1 \Delta H_2 + E_1 \Delta Q_1 + F_1 \Delta Q_2 + J = 0 \tag{3.47}
\]

\[
D'_1 \Delta H_2 + E'_1 \Delta Q_1 + F'_1 \Delta Q_2 + J' = 0 \tag{3.48}
\]

where

\[
J = K_1 + C_1 \Delta H_1
\]

\[
J' = K'_1 + C'_1 \Delta H_1
\]

Eliminating \( Q_1 \) between Eqns. (3.47) and (3.48) gives

\[
\Delta Q_2 = \Theta_1 H_2 + \omega_1 \tag{3.49}
\]

where

\[
\Theta_1 = \frac{D_1 E_1 - D_1 E'_1}{F_1 E'_1 - F'_1 E_1} \tag{3.50}
\]

\[
\omega_1 = \frac{J'_1 E_1 - J E'_1}{F'_1 E'_1 - F'_1 E_1} \tag{3.51}
\]
BOUNDARY CONDITION
(H vs t is given)

BOUNDARY CONDITION
(Q vs t is given)

COMPUTATIONAL MOLECULE

FIG. 3-6 RECTANGULAR GRID IN SPACE AND TIME APPLIED TO A SINGLE RIVER REACH
At the second grid point the continuity and momentum equations can be written as

\[ C_2 \Delta H_2 + D_2 \Delta H_3 + E_2 \Delta Q_2 + F_2 \Delta Q_3 + K_2 = 0 \]  
(3.52)

\[ C_1 \Delta H_2 + D_1 \Delta H_3 + E_1 \Delta Q_2 + F_1 \Delta Q_3 + K_1 = 0 \]  
(3.53)

Substituting \( \Delta Q_2 \) from Eqn. (3.49) into Eqns. (3.52) and (3.53) and eliminating \( \Delta H_2 \) from the resulting equation gives

\[ \Delta Q_3 = \theta_2 \Delta H_3 + \omega_2 \]  
(3.54)

where

\[ \theta_2 = \frac{D_2(C_2 + E_2 \theta_1) - D_2(C_1 + E_1 \theta_1)}{F_2(C_2 + E_2 \theta_1) - F_2(C_1 + E_1 \theta_1)} \]  
(3.55)

\[ \omega_2 = \frac{(K_2 + E_2 \omega_1)(C_2 + E_2 \theta_1) - (K_2 + E_2 \omega_1)(C_1 + E_1 \theta_1)}{F_2(C_2 + E_2 \theta_1) - F_2(C_1 + E_1 \theta_1)} \]  
(3.56)

Similarly, the following recurring relationships can be written for the \( j \)-th grid point

\[ \Delta Q_{j+1} = \theta_j \Delta H_{j+1} + \omega_j \]  
(3.57)

where

\[ \theta_j = \frac{D_j(C_j + E_j \theta_{j-1}) - D_j(C_{j-1} + E_{j-1} \theta_{j-1})}{F_j(C_j + E_j \theta_{j-1}) - F_j(C_{j-1} + E_{j-1} \theta_{j-1})} \]  
(3.58)

\[ \omega_j = \frac{(K_j + E_j \omega_{j-1})(C_j + E_j \theta_{j-1}) - (K_j + E_j \omega_{j-1})(C_{j-1} + E_{j-1} \theta_{j-1})}{F_j(C_j + E_j \theta_{j-1}) - F_j(C_{j-1} + E_{j-1} \theta_{j-1})} \]  
(3.59)
Similar equations can be written successively for every grid point from downstream end of the estuary to the upstream end. Therefore, at the last grid point, e.g. at the model boundary, the following equation is obtained

\[ \Delta Q_j = Q_{j-1} \Delta H_j + \omega_{j-1} \]

or \[ \Delta H_j = (\Delta Q_j - \omega_{j-1}) / Q_{j-1} \] (3.60)

In Eqns. (3.60), \( \Delta Q_j \) is known from the boundary condition. The value of \( Q_{j-1} \) and \( \omega_{j-1} \) can be calculated using the values of \( Q_{j-2} \) and \( \omega_{j-2} \) of the previous grid point. Having these parameters known, the value of \( \Delta H_j \) can be calculated directly from Eqn. (3.60).

Referring to Eqs. (3.52) and (3.53), elimination of \( \Delta Q_2 \) gives

\[ \Delta H_2 = L_2 \Delta H_3 + M_2 \Delta Q_3 + N_2 \] (3.61)

where

\[ L_2 = \frac{D_2 E_2 - D_3 E_3}{C_2 E_2 - C_3 E_2} \] (3.62)

\[ M_2 = \frac{F_2 E_2 - F_3 E_3}{C_2 E_2 - C_3 E_2} \] (3.63)

\[ N_2 = \frac{K_2 E_2 - K_3 E_3}{C_2 E_2 - C_3 E_2} \] (3.64)
In a similar way, the following relationships can be written for the j-th grid point

\[ \Delta H_j = L_j \Delta H_{j+1} + M_j \Delta Q_{j+1} + N_j \]  

(3.65)

where

\[ L_j = \frac{D_j^1 E_j - D_j E_j^1}{C_j E_j^1 - C^1_j E_j} \]  

(3.66)

\[ M_j = \frac{F_j^1 E_j - F_j E_j^1}{C_j E_j^1 - C^1_j E_j} \]  

(3.67)

\[ N_j = \frac{K_j^1 E_j - K_j E_j^1}{C_j E_j^1 - C^1_j E_j} \]  

(3.68)

Thus from Eqn. (3.65), \( \Delta H_j \) can be calculated provided that the values of \( \Delta H_{j+1} \) and \( \Delta Q_{j+1} \) are known.

The computation can proceed up to the downstream boundary, at which \( \Delta Q_1 \) is the only unknown. The value of \( \Delta Q_1 \) is computed from the following relation, obtained by eliminating \( \Delta H_1 \) from Eqns. (3.37) and (3.48)

\[ \Delta Q_1 = \alpha \Delta H_2 + \beta \Delta Q_2 + \gamma \]  

(3.69)

where

\[ \alpha = \frac{C_1 D_1^1 - C_1^1 D_1}{C_1^1 E_1 - E_1^1 C_1} \]  

(3.70)

\[ \beta = \frac{C_1 F_1^1 - F_1 C_1^1}{C_1^1 E_1 - E_1^1 C_1} \]  

(3.71)
\[ \gamma = \frac{C_1 D_1 K_1}{C_1 E_1 - E_1 C_1} \]  

(3.72) -

The general computational scheme involves the computations of \( C_j, D_j, E_j, F_j, K_j \) and \( C'_j, D'_j, E'_j, F'_j, K'_j \) from Eqns. (3.19), (3.20), (3.21), (3.22), (3.23) and (3.33), (3.34), (3.35), (3.36), and (3.37) respectively using the initial conditions and the estuary geometry. Using these values and the values of \( \Delta H_j \), \( \Theta_1 \) and \( \omega_1 \) are computed by Eqns. (3.50) and (3.51). Thereafter, \( \Theta_j \) and \( \omega_j \) are computed for all \( j \) on the basis of Eqns. (3.58) and (3.59). The values of \( L_j, M_j \) and \( N_j \) are calculated for \( j = 1, 2, 3, \ldots, J \), by Eqns. (3.62), (3.63) and (3.64). The values of \( \alpha, \beta \) and \( \gamma \) are calculated as defined by Eqns. (3.70), (3.71) and (3.72). At the upstream boundary \( \Delta Q_j \) is known and the value of \( \Delta H_j \) is determined from Eqn. (3.60). Then the values of \( \Delta H_j \) and \( \Delta Q_j \), for \( j = J-1, J-2, \ldots, 2 \) are determined using Eqns. (3.57) and (3.65) respectively. At the downstream boundary \( \Delta Q_1 \) is obtained from Eqn. (3.69).

The practical meaning of the algorithm described above, is that the number of elementary operations and consequently the computer time required to solve the system of equations is proportional to the number of grid points \( J \). This can be compared with the number of operations required by standard method of matrix inversion, which is proportional to \( J^3 \) (Liggett and Cunge, 1975). Thus the use of Double Sweep Algorithm saves a considerable computer time.
3.5.2 THOMAS ALGORITHM

The linearized salt balance equation, Eqn. (3.42), can be written as, for \( j = 2, 3, \ldots, J-1 \):

\[
\begin{align*}
  b_2 s_2 + c_2 s_3 &= d_2 - a_2 s_1 \\
  a_3 s_2 + b_3 s_3 + c_3 s_4 &= d_3 \\
  a_4 s_3 + b_4 s_4 + c_4 s_5 &= d_4 \\
  &\vdots \\
  a_j s_{j-1} + b_j s_j + c_j s_{j+1} &= d_j \\
  &\vdots \\
  a_{j-1} s_{J-2} + b_{j-1} s_{J-1} &= d_{J-1} - c_{J-1} s_J
\end{align*}
\]  

(3.73)

In the system of equations, Eqn. (3.73), the coefficients \( a_j, b_j, c_j \) and \( d_j \) are known from either initial condition or from the previous step of calculation, and the system is to be solved for the unknowns \( s_j, j = 2, 3, \ldots, J-1 \), by an elimination procedure attributed to Thomas (Noye, 1978).

In this method of solution, the first equation in the system of Eqn. (3.73) is used to eliminate \( s_2 \) from the second equation, the new second equation is used to eliminate \( s_3 \) from the third equation and so on, until finally, the new second last equation is used to eliminate \( s_{J-2} \) from the last equation, giving one equation with only one unknown \( s_{J-1} \). The unknowns \( s_{J-2}, s_{J-3}, \ldots, s_3, s_2 \) are then found by back substitution. This method is therefore, a particular example of the Gauss elimination method with back substitution.
Supposing $s_{j-2}$ has been eliminated from the $(j-1)$th equation in the system of Eqn. (3.73), which is now written as

$$\beta_{j-1}s_{j-1} + \gamma_{j-1}s_j = \varphi_{j-1}$$  \hspace{1cm} (3.74)

This equation is now used to eliminate $s_{j-1}$ from the $j$-th equation in Eqn. (3.73) giving

$$(b_j - \frac{a_j \gamma_{j-1}}{\beta_{j-1}})s_j + c_j s_{j+1} = d_j - \frac{a_j \varphi_{j-1}}{\beta_{j-1}}$$  \hspace{1cm} (3.75)

Comparing Eqns. (3.74) and (3.75), shows that the coefficients $\beta_j$, $\gamma_j$ and $\varphi_j$ are given in terms of $\beta_{j-1}$, $\gamma_{j-1}$, $\varphi_{j-1}$ and $a_j$, $b_j$, $c_j$ and $d_j$ by the relations

$$\beta_j = b_j - \frac{a_j \gamma_{j-1}}{\beta_{j-1}}$$  \hspace{1cm} (3.76)

$$\gamma_j = c_j$$  \hspace{1cm} (3.77)

$$\varphi_j = d_j - \frac{a_j \varphi_{j-1}}{\beta_{j-1}}$$  \hspace{1cm} (3.78)

for $j = 3, 4, 5, \ldots J-1$, with $\beta_2 = b_2$, $\gamma_2 = c_2$, and $\varphi_2 = d_2 - a_2 s_1$.

When $j = J-1$, the following is obtained

$$\beta_{J-1} s_{J-1} = \varphi_{J-1}$$

thus

$$s_{J-1} = \frac{\varphi_{J-1}}{\beta_{J-1}}$$  \hspace{1cm} (3.79)
and the remaining values of \( s_j \) are found by substituting into a rearranged form of Eqn. (3.75), that is

\[
\begin{align*}
\frac{\frac{3}{2}j-1 - \gamma j-1}{\beta j-1} \cdot s_j, & \quad j = J-2, J-3, \ldots, 2, \\
\end{align*}
\]

(3.80)

This method of solution of linear simultaneous equations is stable with regard to propagation of round-off errors as long as \(|b_j||a_j| + |c_j|\), for all values of \( j \) (Noye, 1978).

The normal Gauss elimination method requires about \( J^3/3 \) multiplications and divisions to solve a system of \( J \) linear algebraic equations in \( J \) unknowns. The Thomas Algorithm which takes advantage of the tri-diagonal nature of the system, requires only \( 5J-4 \) multiplications and divisions, since the algorithm eliminates all multiplications by zero coefficients of \( s_j \) which takes place when the general Gauss elimination method is applied (Noye, 1978).

3.6.0 BOUNDARY CONDITIONS

When solving the finite difference equations like Eqns. (3.18) and (3.32) of the tidal dynamics model and Eqn. (3.42) of the salt balance model these equations are written for every grid points from upstream to downstream end of the estuary. The total number of equations is equal to \( 2(J-1) \) for the tidal dynamics model and \( J-2 \) for the salt balance model. The total number of unknowns in both the systems of equations exceed the number of available equations by two.
Therefore, two unknowns must be given beforehand, e.g. at each boundary of the model. The number of unknowns are then reduced equal to the number of equations. The unknowns are then solved simultaneously.

3.6.1 TIDAL DYNAMICS MODEL

For the boundary conditions of the tidal model it is necessary to know the water surface elevation as a function of time at the ocean end of the estuary. This is usually obtained from tidal observations or from tide tables. At the upstream end of the estuary, the boundary condition depends upon the type of estuary. For closed end estuary, the specification of zero velocity becomes the appropriate boundary condition. In case of an open end estuary, if the upstream boundary is far upstream of the tidal limit, the specification of the river velocity or discharge hydrograph becomes the appropriate boundary condition. Finally, if the upstream boundary of an open ended estuary is taken within the tidal limit, it is always preferable to use stage hydrograph as the upstream boundary condition.

3.6.2 SALT BALANCE MODEL

There are two possible boundary conditions at the upstream end of the estuary, for the salt balance model. One can specify that the salinity is zero (or near zero) or one
can specify that there is no flux of salt across the upstream boundary. The first boundary condition can be viewed as a free boundary condition as the point of zero salt flux in the upstream direction, and is in fact, a part of the solution. Such boundary condition can lead to complications in the model (Thatcher and Harleman, 1972); however, it has been found that as long as the upstream boundary is specified far upstream so that it is out of the intrusion zone during the study period, its exact location is not important. But this finding becomes useful in saving computer time, because one need not perform the calculations in that part of the estuary, which is upstream of the salinity intrusion region. The second type of boundary condition is applicable to the closed end estuary, where the salinity actually reaches the end of the estuary.

The specification of ocean end boundary condition is a difficult problem, because the salinity concentration does not remain constant over the whole tidal cycle, and because of the non-availability of salinity data over the whole tidal cycle. Ideally, it would be desirable to take the downstream boundary far into the ocean, where the salinity remains constant. Unfortunately, such a schematization is not possible in one-dimensional models, as the ocean can not be represented by one-dimensional model. For all practical purposes, the downstream boundary should be selected just upstream of the ocean-estuary connection. Under such a situation, the boundary condition must
permit relatively unrestricted movement of mass out of the estuary during the ebb tide, however, it must control mass return on the flood tide in order to describe the effects of tidal flushing.

In this study, the ocean end salinity fluctuation is taken sinusoidal during the flood flow, similar to that of the tidal current (Arbhabhirama and Vongvisessanjai, 1978). Taking advantage of this condition, the predicted current or discharge can be used to compute the boundary salinity during flood flow. However, there is still the need to know the maximum and minimum salinity in a tidal cycle. During the flood flow the salinity increases with the tidal flow until the high water slack salinity is reached at the high water slack time. The salinity at any time $t$, during the flood flow may be calculated from the following equation

$$s(t) = \frac{\int_{t_{LWS}}^{t_{HWS}} Q dt}{\int_{t_{HWS}}^{t_{LWS}} Q dt} \left( s_{HWS} - s_{LWS} \right) + s_{LWS}$$

(3.81)

where

$s_{HWS}$ = high water slack salinity,
$s_{LWS}$ = low water slack salinity,
$t_{LWS}$ = time of low water slack salinity,
$t_{HWS}$ = time of high water slack salinity,
$Q$ = tidal discharge at the estuary mouth.
The variations of salinity and tidal discharge during flood flow are shown in Fig. 3.7. The curve \( Q(t) \) is obtained from the tidal dynamics model.

Although, it is possible to approximate the salinity at the ocean boundary by this method during the flood flow, during ebb flow a different means of continuing the solution must be employed, based on continuous mass balance at the most seaward element of the model. This mass balancing is performed on the basis of salt balance equation, Eqn. (3.3). The equation is discretized using a simplified form of discretization scheme, given by Eqs (3.38), (3.39) and (3.40), with weighting coefficients \( g_{-1} = g_1 = 0 \) and \( g_0 = 1 \), so that the salinity at the boundary can be solved explicitly. The scheme used is

\[
\frac{\partial f}{\partial t} = \frac{f_{j}^{n+1} - f_{j}^{n}}{\Delta t} 
\]

\[
\frac{\partial f}{\partial x} = \frac{f_{j+1}^{n} - f_{j}^{n}}{\Delta x} 
\]

\[
\frac{\partial^2 f}{\partial x^2} = \frac{f_{j-1}^{n} - 2f_{j}^{n} + f_{j+1}^{n}}{\Delta x^2} 
\]  

(3.82)  

(3.83)  

(3.84)

In order to write the total flux at the downstream boundary, the advective flux can be evaluated at the boundary grid point by Eqn. (3.83), but the evaluation of the dispersive flux by Eqn. (3.84), can not be expressed in terms of the difference representation of the salinity gradient centered on this grid.
FIG. 3-7 DOWNSTREAM BOUNDARY SALINITY COMPUTATION DURING FLOOD FLOW
In order to continue the finite difference solution, the disperse flux at this grid is approximated by the salinity gradient evaluated at the grid just upstream (Thatcher and Harleman, 1972). Writing the salt balance equation for the estuary mouth, the following equation is obtained.

\[ s_{1}^{n+1} = \frac{A_{1}}{A_{1}} s_{1}^{n} + \frac{1}{A_{1}} \frac{\Delta t}{\Delta x} Q_{1} \frac{1}{2} (s_{1}^{n} - s_{2}^{n}) + \frac{1}{A_{1}} \frac{\Delta t}{\Delta x^{2}} \{(AE)^{n} (s_{1}^{n} - s_{2}^{n}) - (AE)^{n} (s_{2}^{n} - s_{3}^{n})\} \quad (3.85) \]

Eqn. (3.85) provides the downstream boundary condition for salinity during ebb flow. Thus, Eqns. (3.81) and (3.85) combinedly provide the downstream boundary condition over the whole tidal cycle. As the calculation proceeds in time, the discharge at the ocean entrance is continuously tested to see if it reverses the direction of flow, and in this manner the appropriate flood flow or ebb flow boundary treatment is applied.

3.7.0 INITIAL CONDITIONS

3.7.1 TIDAL DYNAMICS MODEL

To begin the computation, it is necessary to supply initial estimates of the water surface elevation \( H_{1}^{1} \), and the discharge \( Q_{1}^{1} \). Usually, the initial conditions are not known, and for practical purposes they need not be. In tidal hydraulic
computations, it is a common practice to make a very crude estimate of the initial conditions and then run the computations for a few tidal cycles prior to the period of interest, to obtain a quasi-steady solution. This is made possible, because the solution of hyperbolic equations (continuity and momentum equations) is essentially independent of the assumed initial conditions. Thus at the start of the computation $H_j$ and $Q_j$ can be set to any arbitrary values, for example to zero and some tidal cycles of calculations will provide convergence to the appropriate values of $H$ and $Q$. In the transient case, the results of quasi-steady computations will provide the initial condition.

3.7.0 SALT BALANCE MODEL

To start the computation, the initial distribution of salinity $s_1$ is required, as in the case of tidal model. If in a particular application the ocean tidal amplitude and fresh water discharge are fixed, then a quasi-steady solution is defined and the calculations will lead to a convergent solution for any arbitrary initial salinity-distribution (Thatcher and Harleman, 1972). For such cases convergence may require as many as 50 to 200 tidal cycles, consequently, it is worthwhile to make a reasonable estimate of the salinity distribution in order to reduce the computational time. Fig. 3.8 shows two possible initial distribution for quasi-steady solutions.
FIG. 3.8 POSSIBLE INITIAL SALINITY DISTRIBUTIONS FOR QUASI-STEADY STATE STUDIES
A transient solution, for salt balance model, is one in which the conditions at the ocean tidal elevation and fresh water discharge may vary over many tidal cycles through the study period. It is assumed that the results of quasi-steady solution will provide reasonable initial salinity distribution.

3.8.0 SUMMARY

A numerical model for the simulation of tide and salinity intrusion has been formulated, using 4-point Preissman implicit scheme for tidal dynamics equations and 6-point Stone and Brian implicit scheme for the salt balance equation. These schemes have been widely used for the discretization of these equations, because of their simulation accuracy. The application of these schemes result in linear quasi-diagonal and tri-diagonal systems of equations, respectively, which can be solved by Double Sweep Algorithm and Thomas Algorithm. These solution procedures are the most efficient methods for solving the respective systems of equations. The downstream boundary condition, for salinity, has been formulated taking advantage of the sinusoidal fluctuations of salinity during flood flow and mass balancing during ebb flow. Thus the model can be used as a tool for prediction of salinity intrusion in estuaries.
CHAPTER IV
PROPERTIES OF LINEARIZED NUMERICAL EQUATIONS

4.1.0 INTRODUCTION

The numerical solutions of differential equations by finite difference method is based on the representation of the continuously defined function \( f(x,t) \) and its derivatives in terms of the values of \( f(x,t) \) defined at discrete points \( f(j,n) \). The resulting numerical model is composed of finite difference analogues (difference equations) of the mathematical model (differential equations). While the differential equation represents the problem in terms of dependent variables defined continuously in the domain of dependent variables \((x,t)\) the difference numerical equation defines the dependent variables at discrete grid points \((j,n)\), within the domain of the differential equation. Thus, the formulation of a 'well poised' numerical model requires the examination of the accuracy of the model and the adequacy of the difference scheme, with respect to its physical representation of the problem. The accuracy and adequacy of the model generally require the analysis of the 'convergency', 'consistency' and 'stability' of the numerical schemes. Moreover, the analysis of the wave propagation characteristics in term of 'damping factor' and 'velocity factor' gives valuable informations about the schemes.

In principle, the method of finite difference can be used for non-linear mathematical models, but their consistency,
convergency and stability are difficult to prove. However, analysis of the linearized equations gives results which are generally transposable directly to the corresponding non-linear equations (Liggett and Cunge, 1975). In the present study, the analysis of the numerical properties of the schemes are thus based on linearized tidal dynamics and salt balance equations.

4.2.0 LINEARIZED EQUATIONS

4.2.1 TIDAL DYNAMICS EQUATIONS

The linearized tidal dynamics equations corresponding to the non-linear equations, Eqns. (3.1) and (3.2) are as follows:

\[
\frac{\partial h}{\partial t} + h \frac{\partial u}{\partial x} = 0 \tag{4.1}
\]

\[
\frac{\partial u}{\partial t} + g \frac{\partial h}{\partial x} = 0 \tag{4.2}
\]

where the coefficients \( H \) and \( g \) are assumed constant and the friction term in the momentum equation has been neglected, for simplicity of analysis. Substituting the finite difference schemes, Eqns. (3.6), (3.7) and (3.8), the Eqns. (4.1) and (4.2) can be discretized to the following equations

\[
\frac{h^n_{j+1} - h^n_j + h^{n+1}_{j+1} - h^n_j}{2\Delta t} + \Theta (u^{n+1}_{j+1} - u^n_j) + (1 - \Theta) (u^n_{j+1} - u^n_j) = 0 \tag{4.3}
\]

\[
\frac{u^{n+1}_{j+1} - u^n_{j+1} + u^{n+1}_j - u^n_j}{2\Delta t} + g \Theta (h^{n+1}_{j+1} - h^n_j) + (1 - \Theta) (h^n_{j+1} - h^n_j) = 0 \tag{4.4}
\]
These two equations will be analyzed in the subsequent articles.

4.2.2 SALT BALANCE EQUATION

The linearized salt balance equation corresponding to the non-linear equation, Eqn. (3.3) is

\[ \frac{\partial s}{\partial t} + u \frac{\partial s}{\partial x} = \varepsilon \frac{\partial^2 s}{\partial x^2} \]  
(4.5)

In Eqn. (4.5), the coefficients \( u \) and \( \varepsilon \) are assumed constant. Using the finite difference schemes, Eqns. (3.3B) to (3.40), Eqn. (4.5) can be discretized to

\[ g_{-1} \left( \frac{s_{j-1}^{n+1} - s_{j-1}^n}{\Delta t} \right) + g_0 \left( \frac{s_{j-1}^{n+1} - s_{j-1}^n}{\Delta t} \right) + g_1 \left( \frac{s_{j+1}^{n+1} - s_{j+1}^n}{\Delta t} \right) 
+ \frac{u}{4} \left( \frac{s_{j+1}^{n+1} - s_{j-1}^n}{\Delta x} \right) \left( \frac{s_{j+1}^{n+1} - s_{j+1}^n}{\Delta x} \right) 
+ \frac{\varepsilon}{4} \left( \frac{s_{j+1}^{n+1} - 2s_{j}^{n+1} + s_{j-1}^{n+1}}{\Delta x^2} \right) \left( \frac{s_{j+1}^{n+1} - 2s_{j}^{n+1} + s_{j-1}^{n+1}}{\Delta x^2} \right) \]  
(4.6)

Eqn. (4.6) will be the basis of the subsequent analysis.

4.3.0 CONVERGENCE

A solution to a finite difference equation which approximates a given differential equation is said to be convergent if at every grid point \((j,n)\) in the solution region \((x,t)\), the finite difference solution approaches the
solution of the differential equation as the grid spacing tends to zero. Convergency thus refers to state of tending to an unique solution (Task Comm., 1982).

The various methods of analyzing convergency are difficult to apply, specially when the equations can not be solved analytically (Jansen et. al., 1979). Noye (1978) has stated that there is an important connection between the consistency of a stable finite difference scheme and the convergency of its solution, which has been enunciated as 'Lax Equivalent Theorem'. This theorem states that 'if a finite difference approximation to well poised linear initial value problem is consistent, then stability is the necessary and sufficient condition for convergence'. This theorem is of considerable practical importance, for while it is relatively easy to show that a finite difference scheme is stable and that it is also consistent with the differential equation, it is usually very difficult to show that the solution of the finite difference equation converges to the solution of the differential equation that it approximates. Lax's theorem bypasses the need to prove convergence, provided the difference equation is consistent and stable. Eqns. (4.3) and (4.4) and Eqn. (4.6) represent a pair of well poised linear, initial and boundary value problems. Thus, there is no need to prove their convergency, provided that the equations are consistent and stable. These two properties of the finite difference equations have been discussed in the articles that follow, which show that the equations are consistent and stable.
4.4.0 CONSISTENCY ANALYSIS

Every difference equation can be written as a differential equation through a term-by-term expansion in a Taylor series. Consistency indicates the quality of a discrete representation behaving as the continuum equations and solution as the discretization is made even more finer (Task Comm., 1982).

4.4.1 TIDAL DYNAMICS EQUATIONS

Expanding the terms in Eqn. (4.3) by Taylor series about grid point \((j,n)\), the following relations are obtained:

\[
\begin{align*}
  h_{j+1}^{n+1} & = h_j^n + \Delta t \frac{\partial h}{\partial t} + \Delta x \frac{\partial h}{\partial x} + \frac{\Delta t^2}{2!} \frac{\partial^2 h}{\partial t^2} + \frac{2\Delta t \Delta x}{2!} \frac{\partial^2 h}{\partial t \partial x} + \frac{\Delta x^2}{2!} \frac{\partial^2 h}{\partial x^2} + \ldots \\
  h_{j+1}^n & = h_j^n + \Delta x \frac{\partial h}{\partial x} + \frac{\Delta x^2}{2!} \frac{\partial^2 h}{\partial x^2} + \ldots \\
  h_{j+1}^n & = h_j^n + \Delta t \frac{\partial h}{\partial t} + \frac{\Delta t^2}{2!} \frac{\partial^2 h}{\partial t^2} + \ldots \\
  u_{j+1}^{n+1} & = u_j^n + \Delta t \frac{\partial u}{\partial t} + \Delta x \frac{\partial u}{\partial x} + \frac{\Delta t^2}{2!} \frac{\partial^2 u}{\partial t^2} + \frac{2\Delta t \Delta x}{2!} \frac{\partial^2 u}{\partial t \partial x} + \frac{\Delta x^2}{2!} \frac{\partial^2 u}{\partial x^2} + \ldots \\
  u_{j+1}^n & = u_j^n + \Delta x \frac{\partial u}{\partial x} + \frac{\Delta x^2}{2!} \frac{\partial^2 u}{\partial x^2} + \ldots \\
  u_{j+1}^n & = u_j^n + \Delta t \frac{\partial u}{\partial t} + \frac{\Delta t^2}{2!} \frac{\partial^2 u}{\partial t^2} + \ldots \\
  u_{j+1}^n & = u_j^n + \Delta t \frac{\partial u}{\partial t} + \frac{\Delta t^2}{2!} \frac{\partial^2 u}{\partial t^2} + \ldots \\

\end{align*}
\]

Substituting these relationships into Eqn. (4.3) and simplifying the following equation is obtained

\[
\frac{\partial h}{\partial t} + H \frac{\partial u}{\partial x} + E_T = 0
\]  
(4.7)
where

\[ E_T = \frac{1}{2} \{ \frac{\partial^2 h}{\partial t^2} + H \Delta x \frac{\partial^2 u}{\partial x^2} \} + \frac{1}{2} \Delta x + \Theta H \Delta t \frac{\partial^2 u}{\partial t \partial x} + O(\Delta x^2, \Delta t^2) \] (4.8)

The term \( E_T \) is known as the truncation error. This term is a measure of the error introduced by the use of finite difference equation, Eqn. (4.3), to solve the differential equation Eqn. (4.1). From Eqn. (4.8), it is evident that as the grid spacing is made finer, the truncation error gets smaller and smaller. Thus, the finite difference equation, Eqn. (4.3), is consistent with the corresponding differential equation, Eqn. (4.1). It is to be noted that the order of truncation error, as indicated by Eqn. (4.8) is \( O(\Delta x, \Delta t) \).

The order of truncation error indicates how rapidly the solution of the difference equation approaches the solution of the differential equation, if the solution is convergent. The order of truncation error is one, both in \( \Delta x \) and \( \Delta t \), which is the lowest possible order of truncation error to make a differene equation convergent to the solution of differential equation.

Expanding the terms in Eqn. (4.4) by Taylor series about grid point (j,n), substituting the resulting relations in Eqn. (4.4) and simplifying, the following equation may be obtained:

\[ \frac{\partial u}{\partial t} + g \frac{\partial h}{\partial x} + E_T = 0 \] (4.9)
where

\[ E_T^{+} = \frac{1}{2} \left( \frac{\partial^2 u}{\partial t^2} + g \frac{\partial^2 h}{\partial x^2} \right) + \frac{1}{2} \frac{\partial}{\partial t} \left( \frac{\partial}{\partial x} \left( \frac{\partial h}{\partial x} \right) \right) + O(\Delta x^2, \Delta t^2) \quad (4.10) \]

The truncation error \( E_T^{+} \) of Eqn. (4.4) indicates that as \( \Delta x, \Delta t \to 0 \), 
\( E_T^{+} \to 0 \). Thus the finite difference equation, Eqn. (4.4), is consistent with the differential equation, Eqn. (4.2), and the associated truncation error is also of first order.

### 4.4.2 SALT BALANCE EQUATION

Expanding the terms in Eqn. (4.6) by Taylor series about grid point \((j, n)\) the following relations are obtained

\[
\begin{align*}
\frac{s_{n+1}^{j}}{s_{j-1}^{n}} &= s_{j}^{n} + \Delta t \frac{\partial s}{\partial t} - \Delta x \frac{\partial s}{\partial x} + \frac{\Delta t^2}{2!} \frac{\partial^2 s}{\partial t^2} - \Delta t \Delta x \frac{\partial^2 s}{\partial t \partial x} + \frac{\Delta x^2}{2!} \frac{\partial^2 s}{\partial x^2} + \ldots \\
\frac{s_{n}^{j-1}}{s_{j}^{n}} &= s_{j}^{n} - \Delta x \frac{\partial s}{\partial x} + \frac{\Delta x^2}{2!} \frac{\partial^2 s}{\partial x^2} + \ldots \\
\frac{s_{n+1}^{j}}{s_{j}^{n}} &= s_{j}^{n} + \Delta t \frac{\partial s}{\partial t} + \Delta x \frac{\partial s}{\partial x} + \frac{\Delta t^2}{2!} \frac{\partial^2 s}{\partial t^2} + \Delta t \Delta x \frac{\partial^2 s}{\partial t \partial x} + \frac{\Delta x^2}{2!} \frac{\partial^2 s}{\partial x^2} + \ldots \\
\frac{s_{n+1}^{j+1}}{s_{j}^{n}} &= s_{j}^{n} + \Delta t \frac{\partial s}{\partial t} + \Delta x \frac{\partial s}{\partial x} + \frac{\Delta t^2}{2!} \frac{\partial^2 s}{\partial t^2} + \frac{2 \Delta t \Delta x}{2!} \frac{\partial^2 s}{\partial t \partial x} + \frac{\Delta x^2}{2!} \frac{\partial^2 s}{\partial x^2} + \ldots \\
\frac{s_{n}^{j+1}}{s_{j}^{n}} &= s_{j}^{n} + \Delta x \frac{\partial s}{\partial x} + \frac{\Delta x^2}{2!} \frac{\partial^2 s}{\partial x^2} + \ldots 
\end{align*}
\]

Substitution of these relations in Eqn. (4.6) and simplification gives

\[
\frac{\partial s}{\partial t} + u \frac{\partial s}{\partial x} = E \frac{\partial^2 s}{\partial x^2} + E_T^{+} \quad (4.11)
\]
where

\[ E_T = - \frac{1}{2} (g_{-1} + g_0 + g_1) \Delta t \frac{\partial^2 s}{\partial t^2} - (g_1 - g_{-1}) \Delta x \frac{\partial^2 s}{\partial t \partial x} - (g_1 + g_{-1}) \frac{\Delta x^2}{2} \frac{\partial^3 s}{\partial x \partial t^2} - (g_1 - g_{-1}) \frac{\Delta x^3}{3!} \frac{\partial^4 s}{\partial t^3 \partial x} - u \frac{\Delta x^2}{3!} \frac{\partial^3 s}{\partial x^3} + 2E \frac{\Delta x^4}{4!} \frac{\partial^4 s}{\partial x^4} + O(\Delta x^4, \Delta t^2) \]  

(4.1)

where \( E_T \) is the truncation error in Eqn. (4.11) and is given by Eqn. (4.12). From Eqn. (4.12) it is clear that as the grid spacing is made smaller the truncation error gets smaller. Thus Eqn. (4.6) is consistent with Eqn. (4.5).

If \( g_{-1} = g_1 = 0 \) and \( g_0 = 1 \), then \( E_T \) becomes

\[ E_T = - \frac{1}{2} \Delta t \frac{\partial^2 s}{\partial t^2} - u \frac{\Delta x^2}{6} \frac{\partial^3 s}{\partial x^3} + E \frac{\Delta x^2}{12} \frac{\partial^4 s}{\partial x^4} + O(\Delta x^4, \Delta t^2) \]  

(4.13)

and if \( g_{-1} = g_1 = 1/6 \) and \( g_0 = 2/3 \), the following expression for \( E_T \) results:

\[ E_T = - \frac{1}{2} \Delta t \frac{\partial^2 s}{\partial t^2} - \frac{\Delta x^2}{6} \frac{\partial^3 s}{\partial t \partial x^2} - u \frac{\Delta x^2}{12} \frac{\partial^3 s}{\partial x^3} + E \frac{\Delta x^2 + \Delta x}{12} \frac{\partial^4 s}{\partial x^4} + O(\Delta x^4, \Delta t^2) \]  

(4.14)

Thus, in both the cases, the order of truncation error is \( O(\Delta x^2, \Delta t^2) \). Comparison of Eqn. (4.13) and (4.14) shows that the difference scheme with a spread time derivative e.g. \( g_{-1} = g_1 = 1/6 \) and \( g_0 = 2/3 \), is the more accurate when (Siemon, 1970)

\[ E \frac{\Delta x^2}{12} \frac{\partial^2 s}{\partial x^2} = 0 \; \text{e.g.} \; E = 0 \]
or somewhat more generally, if

\[ E \frac{\Delta x^2}{12} \frac{\partial^4 s}{\partial x^4} \ll u \cdot \frac{\Delta x^2}{8} \frac{\partial^3 s}{\partial x^3}. \]

Thus the difference scheme with spread time derivatives will give better results in comparison with the non-time spread derivative.

4.5.0 STABILITY ANALYSIS

Stability of a numerical scheme is concerned with the propagation of an error, introduced by inaccurate initial or boundary data or rounding in the numerical calculations. Thus, stability analysis is concerned with the ability of a scheme to control the propagation or growth of small perturbations (Task Comm., 1982). A scheme is unstable if it allows the growth of errors so that it eventually destroys the true solution.

The stability analysis performed in this study is based on the Fourier method, proposed by von Neumann for linearized finite difference equations (Noye, 1978). This is the easier and the most widely used method for such analysis.

4.5.1 TIDAL DYNAMICS EQUATIONS

In the von Neumann method of stability analysis, the growth of an individual Fourier component in the linear solution is investigated and it is usually expressed in the following.
where $\sigma$ is the wave number. Eqns. (4.15) and (4.16) represent sinusoidal wave profiles, in which the phases are absorbed in $h_o$ and $u_o$ and $|h_o|$ and $|u_o|$ represent the amplitudes of the waves represented by Eqs. (4.15) and (4.16) respectively. Generally, $h_o$ and $u_o$ are complex numbers, from which the usual expressions are obtained by taking the real parts (Jansen et al., 1979).

At any instant $t = n\Delta t$ and space $x = j\Delta x$, Eqns. (4.15) and (4.15) can be written as

$$h^n_j = h_o^ne^{ij\sigma j\Delta x}$$ \hspace{1cm} (4.17)

$$u^n_j = u_o^ne^{ij\sigma j\Delta x}$$ \hspace{1cm} (4.18)

Substituting Eqns. (4.17) and (4.18) in Eqns. (4.3) and (4.4) and simplifying, the following equations results

$$\left(h_o^{n+1} - h_o^n\right)(e^{i\sigma \Delta x} + 1) + 2H \frac{\Delta t}{\Delta x} \left[\Theta(u_o^{n+1} - u_o^n) + 1\right] (e^{i\sigma \Delta x} - 1) = 0 \hspace{1cm} (4.19)$$

$$\left(u_o^{n+1} - u_o^n\right)(e^{i\sigma \Delta x} + 1) + 2g \frac{\Delta t}{\Delta x} \left[\Theta(h_o^{n+1} - h_o^n) + 1\right] (e^{i\sigma \Delta x} - 1) = 0. \hspace{1cm} (4.20)$$

In the von Neumann method of stability analysis, amplification
Using Eqns. (4.21) and (4.22), Eqns. (4.19) and (4.20) can be transformed to

\[(\rho - 1)(e^{i\alpha_x} + 1)h^n_o + 2\Delta t \frac{\Delta x}{\Delta t} [0 (\rho - 1) + 1] (e^{i\alpha x} - 1) u^n_o = 0 \tag{4.24}\]

\[(\rho - 1)(e^{i\alpha_x} + 1) u^n_o + 2g \frac{\Delta t}{\Delta x} [0 (\rho - 1) + 1] (e^{i\alpha x} - 1) h^n_o = 0 \tag{4.25}\]

Eqns. (4.24) and (4.25) constitute a pair of homogeneous equations in \(h^n_o\) and \(u^n_o\). For non-trivial solution of the equations, the determinant of the coefficients should be zero, e.g.
\[
\begin{bmatrix}
(ho-1)(e^{i\alpha \Delta x}+1)
& 2H \frac{\Delta t}{\Delta x} [0(\rho-1)+1] (e^{i\sigma \Delta x}-1)

2g \frac{\Delta t}{\Delta x} [\Theta(\rho-1)+1] (e^{i\sigma \Delta x}-1)
& (\rho-1)(e^{i\sigma \Delta x}+1)
\end{bmatrix} = 0
\] (4.26)

Simplifying, the following equation is obtained:

\[
(\rho-1)^2 = 4 \left( \frac{\Delta t}{\Delta x} \right)^2 gH \left\{ \frac{e^{i\sigma \Delta x}-1}{e^{i\sigma \Delta x}+1} \right\}^2 \left( \Theta(\rho-1)+1 \right)^2 = 0
\] (4.27)

Let

\[ C^2 = gH \]
\[ C = \frac{C \Delta t}{\Delta x} \]

where

\[ C = \text{wave celerity}, \]
\[ C = \text{Courant number}. \]

Using the trigonometrical identity

\[ \tan \Theta = -i \frac{e^{i\Theta} - e^{-i\Theta}}{e^{i\Theta} + e^{-i\Theta}} \]

Eqn. (4.27) can be written as

\[
(1 + 40^2 \xi^2)(\rho-1)^2 + 80\xi^2(\rho-1) + 4 \xi = 0
\] (4.28)

where

\[ \xi = C \tan \frac{\sigma \Delta x}{2} \] (4.29)
Eqn. (4.28) is a quadratic equation in \((\rho - 1)\) and its solutions are given by
\[
\rho = 1 - \frac{40\zeta^2}{1 + 40\lambda^2 \zeta^2} \pm i \frac{2\zeta}{1 + 40\lambda^2 \zeta^2} \tag{4.30}
\]

Taking the absolute value of \(\rho\) gives
\[
|\rho| = \frac{\left(4\zeta^2 + (1 - 40(1 - \zeta) \zeta^2)^2\right)^{\frac{1}{2}}}{1 + 40\lambda^2 \zeta^2} \tag{4.31}
\]

For \(\lambda = \frac{1}{2}\), \(|\rho| = 1\), there is neither damping nor amplification of the solution with time. For \(\theta < \frac{1}{2}\), \(|\rho| > 1\), the scheme is always unstable. When \(\frac{1}{2} \leq \theta < 1\), \(|\rho| < 1\), there is an artificial damping of the solutions. On the basis of this analysis, one might be tempted to use the value of \(\theta = \frac{1}{2}\), as giving the best approximation. However, this best approximation is valid only for the amplifications of the solutions and not for their celerities (Liggett and Cunge, 1975). Liggett and Cunge (1975) have described a numerical experiment in which the effects of \(\theta\) on the numerical computations has been discussed. The result of this experiment indicates that parasitic oscillations are found for values of \(\theta\) near \(\frac{1}{2}\) and these oscillations disappear for \(\theta > 0.66\). As \(\theta\) becomes greater than \(\frac{1}{2}\), artificial damping intervenes, which smooth the traces of dispersion. Thus, it is apparent that \(\theta = \frac{1}{2}\), a theoretically 'better' value of the coefficient, is inadequate from the practical point of view. The parasitic oscillation due to the dispersive characteristics of
the scheme can be damped either artificially when \( \theta > \frac{\pi}{2} \), or by natural damping when the equation contains a high resistance term. To avoid excessive damping, it is better to take \( \theta \) near 0.6.

### 4.5.2 SALT BALANCE EQUATION

Let the Fourier series solution of Eqn. (4.5) be

\[
s(x, t) = s_0^n e^{i\sigma x}
\]

Eqn. (4.32) represents a sinusoidal wave, in which the phase is absorbed in the complex number \( s_0 \), where \( |s_0| \) is the amplitude of the wave. At a certain instant \( t = n\Delta t \) and position \( x = j\Delta x \), Eqn. (4.32) can be written as

\[
s_j^n = s_0^n e^{-i\sigma j\Delta x}
\]

Substituting Eqn. (4.33) in Eqn. (4.46) and simplification of the resulting equation gives

\[
( g_{0,1} e^{-i\sigma \Delta x} + g_0 + g_1 e^{i\sigma \Delta x} )( s_{0}^{n+1} - s_{0}^{n} ) + \mu \lambda ( e^{i\sigma \Delta x} - e^{-i\sigma \Delta x} )( s_{0}^{n+1} + s_{0}^{n} )
\]

\[
= \lambda [ e^{i\sigma \Delta x} - 2 + e^{-i\sigma \Delta x} ]( s_{0}^{n+1} - s_{0}^{n} )
\]

where

\[
\mu = \frac{u\Delta t}{4\Delta x}
\]

\[
\lambda = \frac{F\Delta t}{2\Delta x^2}
\]
Defining the amplification factor as
\[ s_0^{n+1} = \rho s_0^n \] (4.37)

Eqn. (4.34) can be simplified to
\[ (g_{-1} e^{-i\sigma \Delta x} + g_0 + g_1 e^{i\sigma \Delta x})(\rho - 1) + \mu (e^{i\sigma \Delta x} - e^{-i\sigma \Delta x})(\rho + 1) \]
\[ = \lambda (e^{i\sigma \Delta x} - 2 + e^{i\sigma \Delta x})(\rho + 1) \] (4.38)

Using the trigonometrical relations
\[ \sin \Theta = -\frac{i}{2} (e^{i\Theta} - e^{-i\Theta}) \]
\[ \cos \Theta = \frac{1}{2} (e^{i\Theta} + e^{-i\Theta}) \]
\[ \cos \Theta = 1 - 2 \sin^2 \frac{\Theta}{2} \]

Eqn. (4.38) can be written as
\[ (g_{-1} e^{-i\sigma \Delta x} + g_0 + g_1 e^{i\sigma \Delta x})(\rho - 1) + 12 \mu \sin \sigma \Delta x (\rho + 1) \]
\[ = 4 \lambda \sin^2 \frac{\sigma \Delta x}{2} (\rho + 1) \] (4.39)

Solving Eqn. (4.39) for \( \rho \) gives
\[ \rho = \frac{g_{-1} e^{-i\sigma \Delta x} + g_0 + g_1 e^{i\sigma \Delta x} - 12 \mu \sin \sigma \Delta x - 4 \lambda \sin^2 \frac{\sigma \Delta x}{2}}{g_{-1} e^{-i\sigma \Delta x} + g_0 + g_1 e^{i\sigma \Delta x} + 12 \mu \sin \sigma \Delta x + 4 \lambda \sin^2 \frac{\sigma \Delta x}{2}} \] (4.40)

Taking the absolute value of \( \rho \) gives
The finite difference equation is unconditionally stable. As in the case of all implicit schemes, it is therefore possible to use large time step. However, it is not always feasible to make use of this property because large time step may adversely affect the accuracy of the scheme.

4.6.0 DAMPING AND VELOCITY FACTORS

It has been shown in the previous articles that the numerical schemes used for the solutions of both the tidal dynamics and salt balance equations are consistent and stable. When a consistent and stable method is used, the computed solution approaches the true solution if the step size tends to zero. Although, this is an important property, it is not really relevant because one does not let the step size approach zero, in a practical case. An additional question is therefore: how large can the error be for some finite value of the step size? For wave propagation problems, this can be estimated by investigating the behaviour of the computed wave for damping and velocity of propagation (Jansen et al., 1979) in terms of damping and velocity factors, respectively. The damping factor is the ratio of the amplitudes of the wave
resulting from the difference and differential equations. The velocity factor represents the relative velocities of propagations of the waves resulting from the difference and differential equations.

4.6.1 TIDAL DYNAMICS EQUATIONS

The absolute value of the amplification factor for the tidal dynamics equations is

\[ |\rho| = \frac{\left\{ 4c_r^2 \tan^2 \frac{\Delta x}{2} \left[ 1 - 4\Theta(1-\Theta)c_r^2 \tan^2 \frac{\Delta x}{2} \right] \right\}^{\frac{1}{2}}}{1 + 4\Theta^2 c_r^2 \tan^2 \frac{\Delta x}{2}} \]  

(4.42)

In terms of wave length \( L \), \( \sigma \) is given by

\[ \sigma = \frac{2\pi}{L} \]  

(4.43)

Using this relation, Eqn. (4.42) can be written as

\[ |\rho| = \frac{\left\{ 4c_r^2 \tan^2 \frac{\pi}{M} + [1-4\Theta(1-\Theta)c_r^2 \tan^2 \frac{\pi}{M}] \right\}^{\frac{1}{2}}}{1 + 4\Theta^2 c_r^2 \tan^2 \frac{\pi}{M}} \]  

(4.44)

where \( M = L/\Delta x \), the number of \( \Delta x \) per wave length, and the modulus \( |\rho| \) is known as the damping factor. Eqn. (4.44) indicates that the damping factor is a function of Courant number and the distance step per wave length. The plot of damping factor as a function of \( c_r \) and \( M \) is shown in Fig. 4.1, for \( \Theta = 0.55 \).

As pointed out by Liggett and Cunge (1975), this scheme shows sign of instability for all values of \( c_r \). The higher the value
FIG. 4.1 DAMPING FACTOR FOR TIDAL DYNAMICS EQUATIONS
of $C_r$ and smaller the value of $M$, the greater is the amplification. As $M$ becomes larger, the numerical solution gradually approaches the true solution. The instability of the scheme, as indicated by Fig. 4.1, can be controlled by introducing friction term in the momentum equation. Thus, in the practical application of the model, where the friction term shows considerable damping effect this instability is generally not encountered.

The velocity factor $\gamma$, as defined by Jansen et al. (1979) is

$$\gamma = - \frac{1}{\sigma C dt} \tan^{-1} \left\{ \frac{\rho^I}{\rho^R} \right\} \quad (4.45)$$

where $\rho^I$ and $\rho^R$ are the imaginary and real parts of the amplification factor $\rho$. Isolating the imaginary and real parts of $\rho$ given by Eqn. (4.31) gives

$$\rho^I = \frac{1}{1+40^2 \xi^2}$$

$$\rho^R = \frac{1-40(1-\Theta) \xi^2}{1+40^2 \xi^2} \quad (4.47)$$

Substituting $\rho^I$ and $\rho^R$ in Eqn. (4.45), the following equation for velocity factor is obtained

$$\gamma = - \frac{M}{2\pi C_r} \tan^{-1} \left\{ \frac{2C_r \tan \frac{\pi}{M}}{1-40(1-\Theta)C_r^2 \tan^2 \frac{\pi}{M}} \right\} \quad (4.48)$$
The plot of $\gamma$ for different values of $C_t$ is shown in Fig. 4.2. This figure indicates that the phase shift of the numerical solution to analytical solution is negligible for all values of $M$ at $C_t = 1$. For higher values of $C_t$, the phase shift is considerable for smaller values of $M$. In practical cases, where $M$ is generally much higher, the solution of the difference equation approaches the true solution and the phase shift is negligible.

4.6.2 SALT BALANCE EQUATION

Substituting the value of $g_{-1} = g_1 = 1/6$ and $g_0 = 2/3$ in Eqn. (4.41) gives

$$|\rho| = \left[ \frac{[1 - (4\lambda + \frac{2}{3})\sin^2 \frac{\sigma\Delta x}{2}]^2 + 4\mu^2 \sin^2 \sigma\Delta x}{[1 + (4\lambda - \frac{2}{3})\sin^2 \frac{\sigma\Delta x}{2}]^2 + 4\mu^2 \sin^2 \sigma x} \right]^{1/2} \quad (4.49)$$

Using Eqn. (4.43), Eqn. (4.49) can be written as

$$|\rho| = \left[ \frac{[1 - (4\lambda + \frac{2}{3})\sin^2 \frac{\pi}{M}]^2 + 4\mu^2 \sin^2 \frac{2\pi}{M}}{[1 + (4\lambda - \frac{2}{3})\sin^2 \frac{\pi}{M}]^2 + 4\mu^2 \sin^2 \frac{2\pi}{M}} \right]^{1/2} \quad (4.50)$$

It is seen from Eqn. (4.50) that the damping factor is a function of the number of distance steps per wavelength and the coefficients $\lambda$ and $\mu$. The plot of the damping factor as a function of $M$ for different combinations of $\lambda$ and $\mu$ is shown in Fig. 4.3. The figure indicates that at lower values of $M$, the damping is excessive. As $M$ increases, the damping of the solution
FIG. 4.2 VELOCITY FACTOR FOR TIDAL DYNAMICS EQUATIONS
Fig. 4.3a: DAMPING FACTOR FOR SALT BALANCE EQUATION
FIG. 4.3b DAMPING FACTOR FOR SALT BALANCE EQUATION
Isolating the imaginary and real parts of $\rho$ in Eqn. (4.52) gives

$$
\rho^I = \frac{4\mu \left( \frac{2}{3}\sin^2 \frac{\pi}{M} - 1 \right) \sin \frac{2\pi}{M}}{\left[ 1 + (4\lambda - \frac{2}{3}) \sin^2 \frac{\pi}{M} + 4\mu^2 \sin^2 \frac{2\pi}{M} \right]^2}
$$

$$
\rho^R = \frac{1 - \left( \frac{4}{3} + (16\lambda^2 - \frac{4}{3}) \sin^2 \frac{\pi}{M} \right) \sin^2 \frac{\pi}{M} - 4\mu^2 \sin^2 \frac{2\pi}{M}}{\left[ 1 + (4\lambda - \frac{2}{3}) \sin^2 \frac{\pi}{M} \right]^2 + 4\mu^2 \sin^2 \frac{2\pi}{M}}
$$

Substituting the values $\rho^I$ and $\rho^R$ from Eqns. (4.53) and (4.54) respectively in Eqn. (4.51), the following expression for velocity factor is obtained

$$
\gamma = -\frac{1}{\sigma u \Delta t} \tan^{-1} \left( \frac{\rho^I}{\rho^R} \right)
$$

(4.55)
The plot of velocity factor as a function of $M$ and for different combinations of $\lambda$ and $\mu$ is shown in Fig. 4.4. These figures indicate that for small values of $M$, the phase shift is very high. For higher values of $M$, the phase shift is negligible and the solution is practically independent of the values of $\lambda$ and $\mu$, as in the case of amplification factor.

6.7.0 SUMMARY

A complete analysis of the numerical properties of the linearized tidal dynamics and salt balance equations have been presented. This analysis shows that the finite difference equations are consistent with first and second order of truncation errors respectively. The tidal dynamics equations in finite difference forms are stable for $\theta = 0.55$ provided the friction term is included in the momentum equation. The pair of problems defined by the tidal dynamics equations and the salt balance equation are well posed initial value and boundary value problems. As the difference equations are consistent and stable, the equations are also convergent. The plots of damping and velocity factors suggest that when there are large number of distance steps per wave length, the computed solutions are very little damped/amplified and dispersed. These figures also indicate that very little accuracy may be gained by refining $\Delta t$ and $\Delta x$ beyond a certain limit.
FIG. 4.4a VELOCITY FACTOR FOR SALT BALANCE EQUATION
FIG. 4.4b VELOCITY FACTOR FOR SALT-BALANCE EQUATION
CHAPTER V

GENERAL FEATURES OF THE PUSSUR RIVER

5.1.0 INTRODUCTION

The Pussur river flows through the southwestern region of Bangladesh, as shown in Fig. 5.1. It is the combined flow of the Bhairab, the Nabaganga and the Atharabanki rivers. The Bhairab river originates from the Ganges river in the name of Mathabhanga river and combines with the Nabaganga upstream of Khulna. The Nabaganga river also gets its flow from the Ganges in the name of Kumer river. On its way it receives the flow from the Gorai river and joins the Bhairab near Khulna. The combined flow takes the name of Rupsa river. At Jalma, the Rupsa river bifurcates to the Kazibacha and the Old Pussur rivers. The main flow of the Rupsa passes through the Kazibacha river. The two rivers combine at Chalna, and the combined flow is known as the Pussur river. On its way to Sibsa Point, it passes through Mongla and Sundarikota and has many tributaries and distributaries. At Sibsa Point, the Pussur river combines with the Sibsa river to form the Kungar river, which finally falls into the Bay of Bengal.

5.2.0 FLOW CONDITION

The Pussur river receives a considerable contribution of flow from the Gorai river. The contributions from the Bhairab and the Nabaganga rivers are mainly from the drainage of the catchment area, due to seasonal rainfalls. Consequently,
FIG. 5.1 THE PUSSUR RIVER SYSTEM
these rivers have very low flows during the dry season. Usually, the driest period occurs during the months of March, April and May.

Fig. 5.2 shows the monthly tidal discharges of the Rupsa at Khulna and the Pussur at Mongla, for 1977 (IECD, 1980). The dry season net flows are so small that they should be interpreted with extreme caution. During this period the net discharge is landward. The net discharge at Mangla shows stronger upstream flow in the dry season and weaker downstream flow in the flood season than at Khulna. Khulna is approximately 40 km. upstream of Mongla. After the dry spell, the discharge increases rapidly from June and reaches its peak in August and again continues decreasing. The average monthly flow varies from thousands to only few cubic meter per seconds. The landward flow during the dry period is due to very low flow contribution from the upstream and the dominating tidal effect at Hiron Point. This dry period is the most critical period for salinity intrusion in the Pussur river.

5.3.0 TIDAL CONDITION

Tides in the Pussur river is due to the water level fluctuations in the Indian Ocean, which is predominantly semi-diurnal in nature. In the Bay of Bengal, where the water level is approximately 150 m. (IECD, 1980), the tide approaches the coastal area at a speed of about 40 m/s, and the wave length is about 500 km.
MONTHLY AVERAGE TIDAL DISCHARGE IN THE PUSSUR RIVER

FIG. 5-2 MONTHLY AVERAGE TIDAL DISCHARGE IN THE PUSSUR RIVER
As the tide approaches the coast, it moves into the shallow water in the Bay of Bengal and then in the estuaries situated in the southern part of Bangladesh. The shallow water affects causes distortion of the tidal wave. At Hiron Point, the tidal wave enters the Pussur river, acquiring the character of progressive wave, but retaining the characteristics of the tidal wave in the Indian Ocean. Fig. 5.3 shows the tidal curves at Hiron Point, Sundarikota, Mongla and Khulna. The tidal curve at Hiron Point is approximately sinusoidal, with equal amplitudes above and below the mean water level.

The propagation of tide upstream of Hiron Point is also influenced by the complexity of the estuary geometry and upland discharge. The gradual convergence of the Pussur river increases the tidal amplitude. Further upstream, the relatively rapid convergence of the river produces continuous reflections and frictional dissipation of energy, eventually leading to a considerable attenuation of amplitude (Khan, 1980). Fig. 5.3 clearly indicates the gradual distortion and attenuation of tidal wave at Sundarikota, Mongla and Khulna, which are about 40 km, 95 km, and 150 km upstream of Hiron Point, respectively. The symmetric tidal wave gradually becomes asymmetric as it travels upstream, tidal range increases and the mean water level becomes higher. The duration of ebb tide also increases. During spring tides in the dry season, tidal range of 2.75 m at Hiron Point, 3.0 m at Mongla and Chlina, and 2.25 m at Khulna are not uncommon (Khan, 1980).
FIG. 5.3 TIDAL STAGE CURVES
FIG. 5-3 'TIDAL STAGE CURVE (CONTINUED)
5.4.0 SALINITY CONDITION

Several months after the cessation of monsoon and recession of flood water, saline water, driven by tidal mixing process begins to intrude the Pussur estuary. The source of saline water is the coastal water of the Bay of Bengal, which itself shows a distinct seasonal pattern (IECO, 1980), as shown in Fig. 5.4. During the high flood period of July, August and September, combined with monsoonal rains across the entire region surrounding Bangladesh, bring low salinities in the Bay of Bengal. During this period salinity falls to about 30,000 ppm. Considering the standard sea water salinity is 36,000 ppm, this is indeed a significant dilution. By March and April, towards the end of rainy season, salinity returns to the range of 32,000 to 33,000 ppm (IECO, 1980), as shown in Fig. 5.4.

According to the degree of mixing, the Pussur estuary may be classified as 'well mixed' estuary. The high sediment load carried by the Pussur has resulted in relatively shallow channels, and the tidal action is quite strong. To date no significant salinity stratification has been found in the Pussur. To illustrate this, the dynamic salinity obtained from BWDB (1972) has been analyzed. The plot of the data at four stations are shown in Fig. 5.5. The percent deviation from the section mean values are also shown in the figure. The extreme deviation is found to be about 6.5%. The method of analysis is
Fig. 5.4 Surface salinity in the Bay of Bengal
FIG. 5.5 VERTICAL DISTRIBUTION OF SALINITY ALONG THE PUSSUR RIVER
is somewhat similar to Thatcher and Harleman (1981). According to these authors, an estuary can be considered as a 'well mixed' one if the salinity over the cross-section does not vary more than 10%.

An important characteristic of a well mixed estuary is the existence of longitudinal salinity gradient. The variation of longitudinal salinity during the flood tide on 7th March 1973 is shown in Fig. 5.6, for the Pussur estuary. The data were obtained from BWDB (1973). The figure indicates that the salinity varied from 500 mmhos at Khulna to 25,000 mmhos at Hiron Point. It is interesting to note that the gradient along the estuary length is not uniform. There is a sharp fall in salinity between Kagaboga and about 15 km. downstream of Mongla. This effect can be attributed to the inflow of fresh water from the Mongla Nala. The data for the same period has been plotted in Fig. 5.7, for Khulna. This figure shows the variation of salinity over the tidal cycle at Khulna. The maximum value of salinity is about 760 mmhos.

Finally, the variations in maximum monthly salinity at Khulna, Mongla and Chalna for 1979-80 are shown in Fig. 5.8, (IECO, 1980), to assess the extent and degree of salinity intrusion through the Pussur estuary. The salinity concentrations at Khulna is considerably less than that at Chalna and Mongla. The variations of salinity at Chalna and Mongla overlap at different months. Though Chalna is upstream of Mongla, salinity concentrations are much higher for a considerable
Date: 7-2-73
Time: 1319 - 1650 Hrs.

FIG. 5-6 SALINITY GRADIENT ALONG PASSUR RIVER
FIG. 5.7 VARIATION OF SALINITY OVER TIDAL CYCLE AT KHULNA

RUPSA RIVER AT KHULNA
DATE: 7-2-73
© SAMPLING POINT

STAGE CURVE

SALINITY

TIME IN HR.

FIG. 5.7 VARIATION OF SALINITY OVER TIDAL CYCLE AT KHULNA
FIG. 5.8 MONTHLY MAXIMUM SALINITY IN THE PASSUR RIVER

YEAR: 1979-80

CHALNA

MONGLA

KHULNA

MONTHS

MONTHS •
period. This is apparently an unusual situation. This might be due to considerable inflow from Mongla Nala and inflow of saline water of the Sibsa river through the Jhapjhapia and Chunkuri Nalas to the Pussur at Chalna.

5.5.0 SUMMARY

The Pussur estuary is a well mixed estuary and the extent and the degree of salinity intrusion depend on the tidal movement at the estuary mouth, upstream fresh water inflow and the geometry of the estuary. The tidal action at the estuary mouth is considerably strong. The upland discharge through the Bhairab, the Atharabanki and the Nabaganga, which discharge into the Pussur at Khulna is about 1225 million cumec annually (BWDB, 1973), of which only a small fraction is available in the dry period. Fresh water is being continuously diverted for various purposes, and this diversion of fresh water is rapidly increasing. As a result, the Pussur river is specially susceptible to salinity intrusion.
CHAPTER VI
APPLICATION OF THE MODEL

6.1.0 INTRODUCTION

The salinity intrusion model developed in the present study has been applied to the Pussur estuary covering the reach of about 120 km from Sundarikota to Khulna. These two locations were taken as model boundary because of the availability of tidal and salinity data and the absence of major tributaries and distributaries within the estuary reach.

6.2.0 DATA COLLECTION

The data required for the application of the model are the channel cross-sections throughout the study reach of the estuary, tidal stage at downstream model boundary, inflow of water at the upstream boundary and high water slack (HWS), and low water slack (LWS) salinities at the downstream boundary.

The channel cross-sectional data in the form of standard low water (SLW) river width and depth were available from the hydrographic charts prepared by Bangladesh Inland Water Transport Authority (1976 and 1981). Tidal gauge records at Sundarikota and Mongla at 1/2 hour interval were also collected from BIWTA (1980). The tidal discharge at Khulna and salinity at Chalna and Khulna were obtained from BWDB (1980). These data were collected in April 1980 by BWDB in cooperation with Delft Hydraulics Laboratory (IECO, 1980). Unfortunately, these
The longitudinal schematization of an estuary's shown—discrete number of longitudinal segments and to assign particular geometric characteristics to the segment. Since the number of segments which can be considered is limited, some degrees of simplification and averaging are necessary.

In one dimensional formulation it is necessary to divide the estuary into a discrete number of longitudinal segments and to assign particular geometric characteristics to the segment. The natural geometry of the estuaries is irregular both longitudinally and transversely. In one dimensional formulation it is necessary to divide the estuary into a discrete number of longitudinal segments and to assign particular geometric characteristics to the segment. Since the number of segments which can be considered is limited, some degrees of simplification and averaging are necessary.

The longitudinal schematization of an estuary is shown in Fig. 6.1a, where the longitudinal segment length is $\Delta x$. The transverse geometry assigned to section 2 is taken as the average of the transverse geometries of section 1b, 1c, 2, 2a and 2b at an interval of 1 km. The spacing $\Delta x$ generally varies from 100 m in small estuaries to as much as 10 km in large tropical deltas. Normally, $\Delta x$ should be several times the width of the channel (Odd, 1982). Thus in the present study $\Delta x = 4$ km has been taken to cover a distance of 120 km in 31 grid points. The schematic representation of the Pussur estuary from Sundarikota to Khulna is shown in Fig. 6.1b. The data are not consistent and most of the data were collected on various dates at different locations. Only one set of simultaneous salinity data were available for Khulna and Chalna on 25th April, 1980. HWS and LWS salinities at Sundarikota were estimated from the salinity measurements on subsequent date.
FIG. 6-1a SPECIFICATION OF AVERAGE GEOMETRIC PROPERTIES TO A GRID POINT
FIG. 6.1b SCHEMATIC REPRESENTATION OF THE PUSSUR RIVER
The figure also shows the positions of tide and salinity gauging stations along the estuary. The gauging stations of Mongla and Chalna did not coincide with the selected grid points and therefore, the values of water level at Mongla and salinity at Chalna were obtained by linear interpolation of the values at adjacent grid points.

The transverse geometric properties assigned to each longitudinal section are generally obtained from hydrographic charts. While using these charts and correlating bed elevations to a common datum it should be remembered that these charts are related to Chart Datum (C.D), which varies along the channel. The Bench Mark values of the gauging stations and SLW along the Pussur estuary were obtained from BIWTA (1976).

For most practical purposes the geometric and hydraulic properties of estuaries can be represented by using dimensions of an equivalent rectangular channel. Chowdhury (1982) has analyzed about 223 cross-sections of the Sibsa-Pussur river system and reported that about 63% of the cross-sections have depth to width ratio smaller than 1/50 and the banks are very steep, so that there is not much loss of accuracy by assuming a constant width of the channel cross-section. Fig. 6.2 shows a cross-section of the Pussur river near Sundarikota below the SLW. The top width is defined as the width at SLW. The cross-sectional area at SLW is obtained by planimetering the area enclosed by water surface at SLW and the natural boundary of the cross-section. The mean depth of the schematized rectangular
FIG. 6-2  TRANSVERSE SCHEMATIZATION OF AN ESTUARY
section is obtained by dividing the cross-sectional area by top width at SLW.

The variations of top width and cross-sectional area at SLW and the schematized depth of flow from PWD datum along the river reach are shown in Figs. 6.3, 6.4 and 6.5 respectively. These figures indicate that the width and cross-sectional area decrease with distance from Sundarikota, but the variation of schematized depth with distance is more irregular. It should be pointed out that the bed generally rises and in the reach between Mongla and Khulna the bed level at some locations are above M.S.L.

6.4.0 CALIBRATION OF THE MODEL

The first problem in the application of the model was the calibration for Manning's roughness coefficient and longitudinal dispersion coefficient, and the conventional trial and error adjustment of the parameters was adopted in the process. During the calibration of roughness coefficient the tide model was decoupled from salinity model. Manning's roughness coefficient was varied from 0.015 to 0.03 until the model reproduced tidal stage at Mongla. The boundary conditions used were tidal stage at Sundarikota and the corresponding tidal discharge at Khulna on the 25th April, 1980, as shown in Fig. 6.6. It was found that for $n = 0.02$ the model gave the best agreement as shown in Fig. 6.7. For the average top width and depth of flow between Sundarikota and Jalma the corresponding value of
FIG. 6.3 Variation of top width at S.I.W. along Pussur River.
FIG. 6.4 VARIATION OF CROSS-SECTIONAL AREA BELOW S.L.W. ALONG PUSSUR RIVER
FIG. 6.5 VARIATION OF SCHEMATIZED BED R.L ALONG THE PUSUR RIVER
FIG. 6.6 BOUNDARY CONDITIONS FOR THE CALIBRATION OF TIDE MODEL
FIG. 6.7 CALIBRATION OF TIDE MODEL

MONGLA
25-4-80

- OBSERVED
- COMPUTED

\( n = 0.02 \)
Chezy's roughness coefficient is about $101 \text{ m}^{2/5}$, which is quite close to that found by Chowdhury (1982) where the coefficient varied from 100 to 111 for different reaches of the estuary. It was observed during calibration that Manning's coefficient did not appreciably affect the stage hydrograph. Lower values of $n$ produced slightly lower peak values and higher values of $n$ slightly increased the peak values of the stage curve with a small phase shift. These variations in stage and phase were not very remarkable.

To start the computation the initial water level was assumed horizontal and equal to MWL at Sundarikota and the discharge was assumed zero. Because of the damping character of the Preissmann scheme the matching of initial discharge with the discharge hydrograph, specified at Khulna, was not necessary. The observed stage and discharge hydrographs were artificially adjusted to obtain cyclic condition. The magnitude of the adjustments were small compared to the range of variations of the stage and discharge hydrographs. The time step ($\Delta t$) used in the numerical computation was 30 minutes. It was found that three iterations were sufficient for convergence with 0.001 m tolerance in stage values between third and fourth computational cycles.

The calibration of $K_1$ and $K_2$ values of the dispersion coefficient were found to 600 and 100 $\text{ m}^2/\text{sec.}$ respectively. Since salinity values at Mongla were not available, Fig. 6.9 shows the comparison of the observed and predicted salinity
FIG. 6.8 BOUNDARY CONDITION AT KHULNA FOR THE CALIBRATION OF
SALINITY MODEL

FIG. 6.9 CALIBRATION OF SALINITY MODEL
at Chalna. The boundary condition used at Khulna is shown in Fig. 6.8. The initial salinity was assumed to vary linearly from Sundarikota to Khulna as discussed in Chapter III. About 12 iterations were necessary for the solution to converge with 0.1 ppt tolerance between the 12th and 13th iterations. Convergence with lower tolerance was found to be very slow. Iterations required for 0.05 ppt tolerance was found to be about 30 computational cycles. Considering the time involved in this extra iterations for only 0.05 ppt more accuracy, it was decided that the use of 0.1 ppt tolerance was justified.

While calibrating the model it was observed that values of $K_1$ and $K_2$ have little effect on salinity distribution in the Pussur. Moreover, $K_2$ has very small effect on the final value of dispersion coefficient compared to $K_1$. Thus $K_2 = 100$ was assumed arbitrarily and $K_1$ was varied to match the model output with prototype data.

It should be pointed out that the validation of the model with a different set of prototype data for the Pussur was not possible due to non-availability of tidal discharge and salinity data at Khulna. The tidal discharge and salinity data collected exclusively by BWDB are classified data and are not available for general use.

6.5.0 RESULTS AND DISCUSSIONS

6.5.1 COMPARISON WITH PROTOTYPE VALUES

As mentioned earlier, Fig. 6.7 represents the tidal variation with time at Mongla. Acceptable agreement has been
obtained in representing the shape and range while a time lag of about 1/2 hour is not unnoticed. This may be attributed to the schematization of the river neglecting the lateral discharges from the tributaries and distributaries. Pussur is actually an important element of the Ganges delta consisting of a large number of tributaries and distributaries. Fig. 6.10a and 6.10b are reproduced from Farleigh (1981 and 1984) showing the main channels joining the Pussur above Mongla and flow condition during a rising tide. It is seen that the exchange of discharges between the tributaries and the Pussur are not negligible as assumed in this study. However, considering the simplification of the problem, the difference in the computed and observed tidal stages at Mongla is not beyond expectation. The reliability of measured tidal discharge at the upstream boundary is also not beyond question and the use of unreliable tidal discharge at Khulna as upstream model boundary has probably introduced an error in the computed tidal stage at Mongla.

Fig. 6.9 compares salinities between computed and observed values at Chalna. Chalna is located at the confluence of Kazibacha river, Nadua nala, Jhapjhapia nala, Chunkuri nala and the Pussur river as shown in Fig. 6.10a. The Chunkuri nala and Jhapjhapia nala which connect the Pussur with the Sibsa river carries relatively more saline water from Sibsa to the Pussur as shown in Fig. 6.11 (IECO, 1980). This figure clearly indicates that the salinity intrusion in the Sibsa river is significantly more than that of the Pussur. This inflow of more
FIG. 6.10a ACTUAL PUSSUR RIVER SYSTEM UPSTREAM OF SUNDARIKOTA
FIG. 6.10b A TYPICAL RISING SPRING TIDE VOLUMES \( \times 10^6 \text{ m}^3 \) IN THE PUSSUR RIVER SYSTEM
FIG. 6-11  MAXIMUM SALINITY CONTOURS IN PUSSUR AND SIBSA RIVERS IN 1980 DRY SEASON
saline water is one of the important factors responsible for the discrepancy between observed and computed salinity. Salinity monitoring station of BWDB at Chalna is located downstream of the confluence on Chunkuri nala (IECO, 1980) and salinity values do not necessarily represent the actual salinity in the main channel, which is expected to be lower than the measured values. Moreover, the simulation of downstream boundary salinity by Eqns. (3.81) and (3.85) is generally applicable to a section very close to the estuary mouth. The selection of downstream boundary at Sundarikota, which is about 40 km upstream of Hiron Point, may have introduced some error in the computed downstream salinity.

6.5.2 PREDICTED RESULTS

Fig. 6.12 indicates the predicted water level at various stations. The tidal range decreases as it moves upstream. At Sundarikota, flood tide is slightly greater than the ebb tide. On the other hand, the duration of flood tide increases appreciably, instead of decreasing at Mongla and Chalna. Moreover, the mean water level at these locations are also higher. This increase of flood tide is mainly due to tidal reflection (Khan, 1980). Fig. 6.13 represents the variation of tidal discharges at these locations. The range of tidal discharge variation is much higher at Sundarikota as in the case of tidal stage. At Sundarikota, flood tide salinity predominates
FIG. 6.12 PREDICTED WATER LEVEL VARIATION AT DIFFERENT STATIONS
FIG. 6.13 PREDICTED DISCHARGE VARIATION AT DIFFERENT STATIONS
as shown in Fig. 14. Contrary to this, the ebb tide salinity occupies the major part of tide at Mongla and Chalna.

Figs. 6.15, 6.16 and 6.17 compare simultaneous variations in tidal stage, discharge and salinity at Sundarikota, Mongla and Chalna respectively. It is seen from these figures that the tidal stage, discharge and velocity at all the locations are approximately in phase. At Sundarikota the tidal discharge slightly lags behind the tidal velocity during ebb tide. The variation of salinity at Sundarikota are in phase with tidal stage and discharge. But at Mongla and Chalna the salinities are not in phase and lag approximately four hours and three hours from the discharge hydrographs. The point to notice in Fig. 6.17 is the large range of tidal velocity at Chalna, which is nearly twice that at Sundarikota and Mongla. This increase in tidal velocity is possibly caused by the rise in bed elevation near Chalna.

The variations in water levels, discharges and salinities along the estuary are shown in Figs. 6.18, 6.19 and 6.20 respectively. Fig. 6.18 indicates that during flood tide at Sundarikota, the water level falls rapidly downstream of Chalna. On the other hand, the water level rises rapidly upstream of Chalna during ebb tide. The longitudinal variations of tidal discharge also shows similar characteristics as shown in Fig. 6.19. The magnitude of variation in tidal discharge gradually decreases in the upstream direction. The salinity profile as
FIG. 6.14 PREDICTED SALINITY VARIATION AT DIFFERENT STATIONS
FIG. 6.15 VARIATIONS OF PREDICTED WATER LEVEL, DISCHARGE, VELOCITY AND SALINITY AT SUNDARIKOTA
FIG. 6.16 VARIATIONS OF PREDICTED WATER LEVEL, DISCHARGE, VELOCITY AND SALINITY AT MONGLA
FIG. 6-17 VARIATIONS OF PREDICTED WATER LEVEL, DISCHARGE VELOCITY AND SALINITY AT CHALNA
FIG. 6.18 PREDICTED LONGITUDINAL WATER LEVEL VARIATION AT DIFFERENT TIME IN TIDAL CYCLE
DATE: 25-4-80

FIG. 6.19 PREDICTED LONGITUDINAL DISCHARGE VARIATION AT DIFFERENT TIME IN TIDAL CYCLE
FIG. 6.20 PREDICTED LONGITUDINAL VARIATION OF SALINITY AT DIFFERENT TIME IN TIDAL CYCLE
shown in Fig. 6.20 indicates rapid change up to about 10 km upstream from Sundarikota and thereafter the profiles decrease with nearly equal gradients. Near Chalna and upstream, the profiles show oscillations and overlap each other which may be attributed to higher advective transport of salinity caused by increased velocity variation upstream of Chalna.

Fig. 6.21 shows the variations of maximum, minimum and mean water levels, discharges and salinities along the estuary in a tidal cycle. An important characteristic of the high and low water levels along the estuary is the existence of dip at high water and a hump at low water near Chalna. High water level falls rapidly and again rises. On the other hand low water level rises rapidly and then falls. This effect is due to the rise in bed elevation above MSL downstream of Chalna as shown in Fig. 6.5. The hump in the bed elevation controls the flow similar to a weir, causing this large variation in water level in a short reach of the river and higher tidal velocity in the vicinity. The variations in flood and ebb discharges are quite normal except that the flood discharge is lower than the ebb discharge in the middle reach of the estuary. The dispersion of maximum and minimum salinities at the lower reach of the estuary is more than that of the other reaches. In the middle and upper reaches the variations of high and low water slack salinities are thus very small.
DATE: 25-4-80
TIME: 0600 TO 1830

FIG. 6.21  LONGITUDINAL VARIATIONS OF HIGH AND LOW WATER LEVEL, DISCHARGE AND SALINITY
6.7.0 SUMMARY

The numerical model formulated in this study was able to simulate both tide and salinity within reasonable limits. The model has been applied on the assumption that the major flow takes place through the Pussur and the flows through the tributaries are negligible, which is an over simplification of the field situation. The calibration of the model is quite reasonable while interesting physical phenomena have been predicted over the reach. However, in absence of more prototype data model results could not be generalized for the estuary.
CHAPTER VII
CONCLUSIONS AND RECOMMENDATIONS

The finding and conclusions of the present numerical model study are as follows:

1) The numerical model developed in this study is capable of predicting the spatial and temporal salinity variations in the Pussur estuary. The model can simultaneously predict water surface elevation, tidal discharge, velocity and salinity. The model is flexible enough and can be extended to estuaries of different geometric configurations and boundary conditions. The model is based on implicit schemes and hence both Δx and Δt can be taken much larger than the values used in this study.

2) Considering the limitations of the model, non-availability of reliable tidal and salinity data and the schematization of the estuary to a single channel, the simulation of both tide and salinity were reasonable. The effect of the hump created by the rise in bed level downstream of Chalna on the water surface elevation and velocity were well established by the model. It was found during the calibration of the model that the dispersion coefficient has little effect on the salinity profile in the estuary and the transport of saline water into the Pussur is thus, mainly due to advective transport.

3) The model developed in this study can solve the governing equations within a channel reach situated between two junctions.
Thus, this model can be further developed to make the model compatible with the requirements of a model capable of handling channel networks. Simultaneous development of the tidal and salinity models will be a big task. Initially the tidal model should be developed. The linking the channels at junction and the formation of junction matrix for tidal model may be done by method similar to Odd (1982). Thereafter, salinity model can be developed by superimposing the salinity model on the tidal model.

4) The data requirements for the model is neither too voluminous nor too costly, the most important step that is needed is the proper planning and execution of the already existing data collection process by BWDB. The present practice of collecting static and dynamic salinity at different stations at different time is not suitable for the calibration and validation of the numerical models. The installation of recording salinographs at strategic locations as recommended by IECO (1980) should be implemented. All the estuaries in Bangladesh are of well mixed type. The variation of salinity over cross-sections is negligible compared to the total salinity and the simplifying process involved in modelling. Moreover, the collection of depth or cross-sectional average salinity is laborious and costly. So recording of surface salinity by salinographs at different locations simultaneously at certain interval is considered sufficient to the present information needs.
5) The tidal data collected and hydrographic charts prepared by BIWTA are related to local chart datum which vary from river to river and also within a river reach. Though these data are considered sufficient for navigational purpose by BIWTA, these data can hardly be used in models without correlating to a common datum. So the data collected by BIWTA should be correlated to PWD datum or Bench Mark values of the chart datum should be indicated. The later recommendation will hardly increase the labour and cost of data preparation, but it will be of immense help for others involved in hydraulic studies of the rivers based on the data collected by BIWTA.
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APPENDIX A

DESCRIPTION OF THE COMPUTER PROGRAMME

The computer programme described herein computes water surface elevation, discharge and salinity as functions of time and of longitudinal distance along an estuary. The programme is written in FORTRAN IV suitable for IBM 370 computer, available at the Computer Center, B.U.E.T.

The computer programme consists of main programme and six subroutines. The names of the main programme and the subroutines are as follows:

<table>
<thead>
<tr>
<th>Name</th>
<th>Programme Type</th>
</tr>
</thead>
<tbody>
<tr>
<td>MAIN</td>
<td>Main Programme</td>
</tr>
<tr>
<td>GEOMET</td>
<td>Subroutine</td>
</tr>
<tr>
<td>COMPHQ</td>
<td>&quot;</td>
</tr>
<tr>
<td>FLOOD</td>
<td>&quot;</td>
</tr>
<tr>
<td>SALT</td>
<td>&quot;</td>
</tr>
<tr>
<td>COMPS</td>
<td>&quot;</td>
</tr>
<tr>
<td>MAXMIN</td>
<td>&quot;</td>
</tr>
</tbody>
</table>

This programme basically consists of two models namely: TIDAL MODEL and SALINITY MODEL. The tidal model can be easily decoupled from the salinity model to study the tidal characteristics of an estuary. A flow chart showing the logic of the computer programme is shown in Appendix B.
The following are the functions of the main programme and the subroutines:

**Main Programme**

The main programme reads the necessary input data for the computation of tide and salinity and finally prints the results of the computations. The input data necessary for the computations are the geometric data of the estuary, upstream and downstream boundary conditions, etc. The initial conditions and the iterations required for convergence of the solutions are also controlled by the main programme.

The main programme calls subroutines GEOMET and COMPHQ for the computation of tidal hydraulics of the estuary. After obtaining quasi-steady tidal condition the main programme calls subroutine FLOOD to determine the flood flow into the estuary. The subroutines SALT and COMPS are then called for the computation of quasi-steady salinity. Finally, the main programme calls subroutine MAXMIN to determine maximum, minimum and mean values of water level, discharge and salinity.

**Subroutine GEOMET**

This subroutine computes the geometric parameters such as cross-sectional area, hydraulic radius, conveyence, etc. at grid points. This subroutine has the provision of terminating the computations if water level falls below the estuary bed level.
Subroutine COMPHQ

This subroutine solves the quin-diagonal system of equations obtained by the discretization of the momentum and continuity equations. The system of equations is solved by the Double Sweep Algorithm. In the forward sweep, the subroutine computes the coefficients of the continuity and momentum equations and the recurring relationships. During the backward sweep, the subroutine computes water level and discharge at the grid points.

Subroutine FLOOD

This subroutine determines the total flood flow and the cumulative flood flows at different time levels during the flood flow period at the downstream model boundary. These informations are used by subroutine SALT to compute the salinity at the downstream boundary.

Subroutine SALT

This subroutine computes the downstream boundary condition for the salinity model. The subroutine checks for flood or ebb flows at the downstream model boundary and computes salinity at new time level from either flood flow or ebb flow conditions.
Subroutine COMPS

This subroutine solves the tri-diagonal system of equations obtained by discretizing the salt balance equation. The system of equations is solved by Thomas Algorithm. In the forward sweep, the subroutine computes the coefficients of the discretized salt balance equation and the recurring relations. In the backward sweep salinity at new time level is calculated at the interior grid points.

Subroutine MAXMIN

After quasi-steady salinity distribution has been obtained, the main programme calls this subroutine. This subroutine computes the maximum, minimum and mean water surface elevation, discharge and salinity at the grid points in a tidal cycle.
APPENDIX-B

FLOW CHARTS
INITIAL DATA

READ NMAX
AND : JMAX
WRITE : NTRANS

COMPUTE : JMX
JMS

SET : ITRANS = 0

READ : LLL, MMM
NNN

READ : CHLEN
AND : DELX
WRITE : DELT

GEOMETRIC DATA

READ : B(J)
AND : DO J
WRITE : FOR ALL J

READ : EMN(J)
AND : FOR ALL J
WRITE

READ : EK1; EK2
AND :
WRITE
BOUNDARY CONDITION

522

READ: INITIAL AND FINAL DATE AND TIME

READ $H(N,1), Q(N,JMAX)$
AND $S(N,JMAX)$
WRITE FOR ALL N

READ SHWS
AND SLWS
WRITE SLWS

INITIAL CONDITION

NO

ITRANS > 0

YES

READ: $H(1,J)$
$Q(1,J)$
$S(1,J)$
FOR ALL J

SET: $H(1,J) = H(NMAX,J)$
$Q(1,J) = Q(NMAX,J)$
$S(1,J) = S(NMAX,J)$
FOR ALL J

SET: $K=1$
$N=1$

COMPUTE: $H, Q$
$K, D$ CON
FOR ALL J
CALL GEOMET

COMPUTE: $H, Q$
FOR ALL J
CALL COMP HQ

$N = N + 1$

NO

YES

$N = NMAX$
WRITE: K > 10

RETAIN: PH(N,J) = H(N,J) FOR ALL N, J

K > 10

YES
WRITE: K > 10

NO

SET: H(1,J) = H(1,JMAX)
Q(1,J) = Q(1,JMAX)

K = K + 1, N = 1

COMPUTE: A, R, K, D, CON
FOR ALL J
CALL GEOMET

FOR ALL J
CALL COMP_HQ

N = N + 1

NO

N = NMAX

YES

NO

PH(N,J) - H(N,J) < 0.001
FOR ALL N, J
WRITE: K > 25

SET: S(1, J) = S(NMAX, J) FOR ALL J

SET: KK = 1, N = 1

CALL SALT

COMPUTE: S(N, J) FOR ALL J

CALL COMPS

COMPUTE: V(N, J) FOR ALL N, J

COMPUTE: QFLOOD AND QF(N) FOR ALL N CALL FLOOD

SET: KK = 1, N = 1

COMPUTE: S(N, 1)

CALL SALI

N = N + 1

NO

N = NMAX

YES

RETAIN: PS(N, J) = S(N, J) FOR ALL N, J

KK > 25

YES

WRITE: K > 25

NO

SET: S(1, J) = S(NMAX, J) FOR ALL J
KK = KK + 1, N = 1

COMPUTE: S(N, 1)
CALL SALT

N = N + 1

NO

N = NMAX

YES

PS(N,J) - S(N,J) ≤ 0.1
FOR ALL N, J

YES

LLL = 1

NO

WRITE: H(N,J) Q(N,J)
S(N,J)
FOR ALL N, J

MMM = 1

NO

YES
WRITE: H(N, J), Q(N, J), S(N, J)
FOR ALL J, N

IF NNN = 1

J = 1

COMPUTE: HMAX, HMIN, HMEAN
QMAX, QMIN, QMEAN
SMAX, SMIN, SMEAN
CALL MAXMIN

WRITE: HMAX, HMIN, HMEAN
QMAX, QMIN, QMEAN
SMAX, SMIN, SMEAN

J = J + 1

IF J = JMAX

ITRANS = ITRANS + 1

IF ITRANS > NTRANS

STOP

NO

YES

522

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SUBROUTINE GEOMET:

START

SET: HH = H(N,J)

HH > DO(J) NO WRITE: HH < DO(J)

YES

J = J + 1

COMPUTE: A(N,J), PERI
R(N,J), CON(J)
DCON(J), DB(J)

NO

J = JMAX

YES

RETURN

STOP
SUBROUTINE CÔMHPQ

START

COMPUTE:
DELH = H(N+1, 1) = H(N, 1)

COMPUTE:
C, D, E, F, PK; CP, DP, EP, PKP

J = J+1

COMPUTE: THETA(J)
OMEGA(J)
ALFA, BETA, GAMA

J ≤ 1

YES

COMPUTE: CL(J)
CM(J), CN(J)

COMPUTE: THETA(J)
OMEGA(J)

NO

J = JMX

YES

NO
J = J \text{MAX}

\text{COMPUTE: DELQ, DELH}\n
H(N+1, JMAX)

J = J - 1

\text{COMPUTE: DELQ}\n
Q(N+1, 1)

\text{RETURN}

J < 1

\text{COMPUTE: DELH}\n
H(N+1, J+1)

\text{COMPUTE: DELQ}\n
Q(N+1, J+1)
SUBROUTINE FLOOD

START

QQ(N) = Q(N, 1)

N = NMAX

N = N + 1

YES

QQ(N) = 0

QQ(N) > 0

NO

QQ(N+1) < 0

NO

QQ(N+1) > 0

YES

COMPUTE:
QFLOOD

N = NMAX

N = N + 1

YES

N = 1

NO

N = N + 1

QQ(N) < 0

NO

QQ(N+1) = 0

YES

II = N

NO

QQ(N) > 0
J = I - NMAX

I = II

J = 1

I ≤ NMAX

QF(J) ≤ 0

QQ(J+1) = 0

QF(J) ≤ 0

COMPUTE
QF

QQQ = QQQ - QFF

QF(J) = QQQ

I = III

RETURN
SUBROUTINE SALT

START

Q(N,1) ≤ 0

NO

COMPUTE: S(N+1,1)

S(N+1,1) > SHWS

NO

RETURN

YES

S(N+1,1) = SHWS

YES

S(N+1,1) = SLWS

NO

COMPUTE: S(N+1,1)

S(N+1,1) < SLWS

RETURN
SUBROUTINE COMPS

START

J = 2

COMPUTE: INTERMEDIATE VALUES OF $A, R, Q, V$

1

COMPUTE:
$EKF = EK1$

$S(N,J) < 0$

NO

COMPUTE:
$EKA = EK1 \cdot EK2$

YES

$S(N,J+1) \neq 0$

NO

COMPUTE:
$EKB = EK1 \cdot EK2$

YES

COMPUTE:
$EKB = EK1$

COMPUTE:
$EKB = EK1 \cdot EK2$

COMPUTE:
$S(N,J+1) = 0$

COMPUTE:
$SA, SB, SC, SD$

$S(N,J) = 0$
COMPUTE: BETA(J)
DELTA(J)

J = J + 1

J > 2

YES

J = JMS

NO

COMPUTE: BETA(J)
DELTA(J)

NO

J = J + 1

COMPUTE: GAMMA(J)
DELTA(J)

YES

COMPUTE: S(N+1,J)

J = JMS

J = J + 1

J ≤ 1

YES

RETURN

NO

COMPUTE: S(N+1,J)
SUBROUTINE MAXMIN

START

XMAX = HQS (1, J)

YES

XMAX ≤ HQS(N, J)

N = NMAX

NO

N = N + 1

XMAX = HQS(N, J)

COMPUTE: XMEAN

RETURN

HMIN = HQS (1, J)

XMIN = HQS(N, J)

YES

N = NMAX

NO

N = N + 1

XMIN = HQS(N, J)

XMIN ≥ HQS(N, J)

NO