1. (a) Using mesh analysis, calculate the branch currents $I_1$, $I_2$, $I_3$, $I_4$ and $I_5$ for the circuit shown in Fig. for Q. 1(a).

(b) Find $I_0$ using nodal analysis for the circuit shown in Fig. for Q. 1(b).

2. (a) Find the value of $V_o$ using principle of superposition for the circuit shown in Fig. for Q. 2(a).
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Contd ... Q. No. 2

(b) Find the value of $V_x$ using source transformations for the circuit shown in Fig. for Q 2(b).

![Circuit Diagram](image1)

3. (a) Find the value of $V_o$ using Thevenin's theorem for the circuit shown in Fig. for Q. 3(a).

![Circuit Diagram](image2)

(b) The variable resistor ($R_a$) in the circuit shown in Fig. for Q. 3(b) is adjusted until the power dissipated in the resistor ($R_a$) is 250 W. Find the values of $R_a$ which satisfy this condition.

![Circuit Diagram](image3)

4. (a) Define the flux density, permeability and magnetizing force.

(b) Explain the Ampere's Circuital Law.

(c) Determine the value of current $I$ required to establish a flux of $\phi_1 = 1.8 \times 10^{-4}$ Wb in the air gap in Fig. for Q. 4(c).

Contd ........... P/3
There are FOUR questions in this Section. Answer any THREE.

5. (a) Determine a numerical value for each current and voltage \( i_1, v_1 \), etc.) in the circuit Fig. for Q 5(a). Also calculate the power absorbed by each element and verify that the sum to zero.  

(b) Calculate \( R_{ab} \) for each of the networks shown in Fig. for Q. 5(b).

(20)

(15)

6. (a) For the circuit of Fig. for Q. 6(a)

(i) Determine the output voltage \( V_{out} \) and output current \( I_{out} \).

(ii) Determine the voltage gain \( \left| \frac{V_{out}}{V_{in}} \right| \).

(b) Find \( R_{eq} \) and \( I \) in the circuit of Fig. for Q. 6(b).

(20)

7. (a) Obtain expressions for both \( i_1(t) \) and \( i_2(t) \) as labeled in Fig. for Q. 7(a) which are valid for \( t > 0 \).

(b) The switch in Fig. for Q. 7(b) has been closed for a long time before opening at \( t = 0 \). Find

(i) \( i_L(t), t \geq 0 \)

(ii) \( v_L(t), t \geq 0^+ \)

(iii) \( V_L(t), t \geq 0^+ \)

(17)

(18)

8. (a) The current shown in Fig. for Q 8(a) is applied to a 0.5 \( \mu \)F capacitor. The initial voltage on the capacitor is zero.

(i) Find the charge on the capacitor at \( t = 15 \) \( \mu \)s.

(ii) How much energy is stored in the capacitor by this current?

(iii) Sketch \( v(t) \) over the interval \( 0 \leq t \leq 50 \) \( \mu \)s.

(b) Sketch the voltage \( v_c(t) \) as shown in Fig. for Q. 8(b) for the interval \(-0.5 \leq t \leq 40 \) s.

(15)

(20)
Given, Cross-sectional area (throughout), $A = 6 \times 10^{-4} \text{ m}^2$
- $l_{bcde} = 0.2 \text{ m}$
- $l_{efab} = 0.1 \text{ m}$
- $l_{es} = l_{xy} = 2.5 \text{ cm}$
- $l_y = l_{xy} = 0.2 \text{ mm}$

Material: sheet steel.
Fig. from A(c)

= 5 =
Fig. for Q. 8(a)

Fig. for Q. 8(b)

Fig. for Q. 7(b)
1. (a) Create a class called Card that maintains a library category entry. The class stores a book's title, author name, number of copies on-hand. Store the title and author as string and the on-hand as integer. Use a public member function called store() to store the book's information and a public member function called show() to display the information. Indicate a short main function to demonstrate the class. (10)

(b) Explain in details the basic difference between C and C++. (5)

(c) Explain in details the fundamental properties of Object Oriented Programming. (10)

(d) What is the purpose of using namespace in C++? Why is new style header used in C++? (10)

2. (a) What is inline function? What are restrictions of inline function? Which inline functions are called automatic inline functions? (8)

(b) Create a class called line that draws a line on the screen. Store the line in a private variable called len. The line's constructor takes one parameter: the line length. Now add a separate function draw() that actually draw the line from the origin. Add a destructor function that removes the line from the screen. (10)

(c) "A function can be a member function of one class and a friend function of another" – explain with corresponding code. (12)

(d) What are the advantages of using new and delete over malloc() and free()? (5)

3. (a) What are the differences between pointer and reference? (10)

(b) Given the class fragment,

```cpp
class samp
{
    double *p;
public:
    samp (double d)
    {
        p = (double *) malloc (sizeof (double));
        if (!p) exit (1);
        *p=d;
    }
}
```

Contd ......... P/2
4. (a) Write a program that displays individually the values of high and low order bytes of an integer.
(b) Fill in the blanks to complete the following program to read ten integers from an input file and write them to an output file:

```c
#include <stdio.h>

void main (void)
{
    /* Declare file pointers "input" and "output"*/
    ....
    int j, i=0;
    /*open the file "input.dat" (contains ten integers) using input pointer, in read mode*/
    ....
    /* open the file "output.dat" (empty) using output pointer, in write mode*/
    ....
    /* test if either files have not open successfully*/
    if(.............)
    {
        printf("open file error for input.dat or output.dat\n");
        return ;
    }
}
```

What problem is caused by the assignments of `ob1` to `ob2`?
(c) Explain why you must be careful when passing objects to a function or returning objects from a function with complete code fragments.
/* read the integers from input.dat file into j, then write them out to output.dat */

while (i > 10) 
{


++ i;
}

/* close the files */


(c) Suppose we have designed a structure autoPart as follows:

typedef struct
{
    char name [10];
    double price ;
} autoPart;

(i) Write the code to declare an array of data type autoPart, the name of the array is Parts, and Parts has 10 elements.

(ii) Assume that data has already been put into the Parts array. Complete the C function (bellow) by filling in the blanks. You must use the built in C function strcmp to find and then print out the price of a part name "strut". If this part is not in the array, then print "Not found". Assume at most one element in the array Parts has the name "strut".

Void PrintStrutPrice (autoPart Parts [])
{
    int k;  
    for (k=0; k<10; k++)
    {
        if ( 
            printf("%lf
", );
        return;
    }

    printf("Not found \n");
} /* end PrintStrutPrice function */
There are **FOUR** questions in this section. Answer any **THREE**.

5. (a) Is it possible to get out of nested loops using one 'break' statement? Explain your answer.  
(b) Write a program that prints all the leap years since 1762.  
(c) Describe the scope of global and local variables.  
(d) Why do we need function prototypes?  
(e) Create two \((3 \times 3)\) array of doubles, often called as a \((3 \times 3)\) matrix. Then write two functions that  
   (i) Takes the pointer to these arrays and returns a pointer to an \((3 \times 3)\) array that contains the result of the multiplication of these two arrays.  
   (ii) Takes a pointer to a \((3 \times 3)\) matrix and returns the transpose of that array.

6. (a) Write a function that takes a character array as input, converts that array to uppercase and returns it. You are not allowed to use library functions in string.h  
(b) Write a program that takes 3 integers as parameter and then prints the lowest among them.  
(c) What is the output for the following code:  
   ```c
   void int main()
   {  
       int i = 0, j = -1;  
       for (; i+j; j++)  
       {  
           printf("i+j=%d", i+j);  
       }  
   }
   ```

(d) What is the problem with the following code:  
   ```c
   void main()
   {  
       char *p;  
       printf("Enter a string:");  
       gets(p);  
   }
   ```

(e) Discuss about automatic type conversion in C.

Contd ......... P/5
7. (a) Write a C program that takes 10 integers as input from the user, stores them in an array and then arranges these 10 numbers so that the odd numbers are at the beginning of the array in ascending order and the even numbers are in the other part of that array in descending order. You are not allowed to use any other array for temporary storage.

(b) What is wrong with this statement: `char str[10] = "I love programming";`

(c) A magic square is a n×n array of numbers and sum of the numbers in each row, column and diagonal are equal. Your job is to write a function that takes a two dimensional array as input and print whether it is a magic square or not.

(d) What will be the output of the following code?

```c
void main()
{
    double x = 15.5, y = 2.1;
    while (1)
    {
        if(x>10)
        {
            printf("%d", x);
            x=x-y;
            continue;
        }
        if(x<2)
            break;
    }
}
```

8. (a) What is the result of the following expression: `a*b/1.2*3*(b-a)` where the variable a, b, c are all integers and a = -5, b = 3, c = 2.

(b) Will this expression be evaluated as true? `3 && (4%2 || 1 & & 2)`.

(c) Rewrite the following loop using do...while loops only

```c
int x;
scanf("%d", &x);
while(x !=0)
{
    printf("%d", x);
    scanf("%d", &x);
}
```

(d) Write a program that acts as an electronic dictionary. If the user enters a word that is in the dictionary, the program displays its meaning. Use a three dimensional character array to hold the words and their meanings.

(e) What is the advantage of using pointers over array indexing?

(f) Which of these variable names are invalid in C

(i) _abc
(ii) 123abc
(iii) this_is_a_long_name_abcd_edefghijklmnopqrstuvwxyz
(iv) new-abc
(v) more abc

-----------------------------
SECTION - A

1. (a) Write down the fundamental assumptions of the kinetic theory of gases. (8)
(b) Derive the expressions for the work done during expansion of an ideal gas in case of
(i) isothermal process and (ii) adiabatic process. (20)
(c) A motor tyre is pumped to a pressure of 2 atmospheres at 15°C when it suddenly bursts. Calculate the resulting drop in temperature, (given $\gamma = 1.4$) of the air initially enclosed in the tyre. (7)

2. (a) Discuss the main considerations behind the corrections of the ideal gas equation. (8)
(b) Deduce and explain Vander Waals equation for a real gas. (20)
(c) Obtain expressions for the critical pressure, critical temperature and critical volume in terms of the Vander Waals constants and universal gas constant. (7)

3. (a) Explain what do you mean by entropy. (6)
(b) Derive Maxwell thermodynamic relations. (22)
(c) Compute the entropy change of a system considering 2.0 kg of ice at 0°C which melts (reversibly) to water at the same temperature. (Latent heat of melting = 79.6 cal/g) (7)

4. (a) What do you mean by spherical aberration and chromatic aberration of a lens? How does these defects affect the formation of the image by a lens? (15)
(b) What do you mean by the term 'resolving power' of an optical instrument? Show that the smallest detail of the objective can be seen in an optical microscope is about the same size as the wavelength of light being used. (20)

SECTION - B

There are FOUR questions in this Section. Answer any THREE.

5. (a) What is interference of light? How is the law of conservation of energy obeyed in interference? (8)
(b) Explain clearly the theory and the experimental arrangement of Newton's rings experiment. Why the central ring observed by reflected light is dark? (20)
(c) A Newton's rings apparatus is used to determine the radius of curvature of a lens. The radii of the \( n \) th and \( (n + 20) \) th bright rings are measured and found to be 0.162 cm and 0.368 cm, respectively, using a light of wavelength 546 nm. Calculate the radius of curvature of the lower surface of the lens.

6. (a) What is diffraction of light? Yellow light is used in single slit diffraction pattern of slit width 0.6 mm. If yellow light is replaced by smaller, how will diffraction pattern be affected?

(b) What is diffraction grating. A certain diffraction grating has 104 slits with a spacing of \( d = 2.1 \, \mu m = 2100 \, nm \). It is illuminated with yellow sodium light \( (\lambda = 589 \, nm) \). Find the angular position of all the observed principal maxima.

(c) Describe a Nicol prism. Explain its action as a polarizer and as an analyzer.

7. (a) Establish the differential equation of a damped harmonic oscillator. Show that for a damped oscillator the amplitude decays with time exponentially.

(b) A damped harmonic oscillator involves a block of mass 1.0 kg, a spring whose spring constant \( k = 25 \, N/m \) and a damping force \( f = -bv \). Initially it oscillates with an amplitude 20 cm. Because of damping the amplitude falls to \( \frac{1}{e} \) (e \( \rightarrow \) the exponential term) of this initial value after five complete cycles. (i) What is the value of \( b \)? (ii) Calculate the initial energy. How much energy has been lost during these five cycles?

8. (a) Write down the displacement equation of a plane progressive wave traveling in the \( +x \) direction and derive the differentiate equation.

(b) Show that in the case of stationary wave no energy is transferred across any section of the medium.

(c) A simple harmonic wave is propagating through a medium. The displacement equation of the wave is \( y = 0.1 \sin \left( \frac{8 \pi t - 16 \pi x}{\lambda} \right) \) in metres. This wave is reflected from a rigid support and produces four loops.

(i) Write down the equation of the reflected wave.

(ii) Velocity and wavelength of the wave.

(iii) Calculate the length between the incident and reflected points.

(iv) Determine the position of the nodes.
SECTION – A
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Contd .......... P/2
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L-1/T-1/EEE

L-1/T-1
B. Sc. Engineering Examinations 2011-2012

Date : 15/12/2012

BANGLADESH UNIVERSITY OF ENGINEERING AND TECHNOLOGY, DHAKA

Sub: MATH 159 (Calculus II)

Full Marks: 210 Time: 3 Hours

The figures in the margin indicate full marks.
USE SEPARATE SCRIPTS FOR EACH SECTION

SECTION - A

There are FOUR questions in this section. Answer any THREE.
Symbols used have their usual meaning.

1. (a) Describe and graph the region represented by each of the following:
   (i) \(|z+2i|+|z-2i| \leq 6\)
   (ii) \(Im(z^2) \leq 4\).

(b) If \(f(z) = \begin{cases} \frac{(z)^2}{z} & ; \quad z \neq 0 \\ 0 & ; \quad z = 0 \end{cases}\)
   show that Cauchy -Riemann equations are satisfied at \(z = 0\) but \(f(z)\) is not differentiable at \(z = 0\).

(c) Show that \(f(z) = e^{-\theta} \cos(lnr) + ie^{-\theta} \sin(lnr)\); \((r > 0, 0 < \theta < 2\pi)\) is differentiable in the indicated domain of definition and show that \(f'(z) = \frac{i f(z)}{z}\).

(d) Discuss the analyticity of the function \(f(z) = (z^2 - 2) e^{-x}e^{-y}\).

2. (a) Show that the function \(u(x, y) = \frac{1}{2} \log(x^2 + y^2)\) is harmonic in some domain. Find its harmonic conjugate and the corresponding analytic function \(f(z)\) in terms of \(z\).

(b) Expand \(f(z) = \frac{2i}{4+iz}\) in a Taylor series about the point \(z = -3i\) and find the region where the series converges.

(c) Find a Laurent series expansion of \(f(z) = \frac{z}{(z-1)(z-3)}\) in powers of \((z-1)\) and determine the region of convergence of the series.

3. (a) Evaluate the line integral \(\int_C Imz \, dx\) where the contour \(C\) consists of the quarter circle \(z = 3e^{i\theta}; 0 \leq \theta \leq \frac{\pi}{2}\) in positive sense and the parabola \(x = \frac{(y-3)^2}{9}\) from \(3i\) to \(1 + 6i\)

Contd ........... P/2
MATH 159 (EEE)

Contd ... Q. No. 3

(b) Let \( C_R \) denote the upper half of the circle \(|z| = R \ (R > 2)\) taken in counterclockwise direction. Without evaluating the integral show that

\[
\left| \int_{C_R} \frac{2z^2 - 1}{z^4 + 5z^2 + 4} \, dz \right| \leq \frac{\pi R (2R^2 + 1)}{(R^2 - 1)(R^2 - 4)}.
\]

(c) Use Cauchy's integral formula to evaluate the integral

\[
\int_{C_R} \frac{z}{(z + 2)(z - 4i)} \, dz
\]

where \( C \) is the rectangle having sides \( x = \pm 3, \ y = -1, \ y = 5 \) taken in positive sense. (10)

4. Use Cauchy's Residue theorem to evaluate the following integrals where \( C \) is the circle \(|z| = 4\) taken counterclockwise:

(i) \( \int_{C} \frac{\sin z}{z^2(z^2 + 4)} \, dz \) (18)

(ii) \( \int_{C} \frac{8z^2}{(z^2 + 9)^2} \, dz \). (17)

SECTION - B

There are FOUR questions in this Section. Answer any THREE.

5. (a) Show that \( a, b \) and \( c \) are linearly independent. Express \( d \) as a linear combination of \( a, b \) and \( c \), when \( a, b, c \) and \( d \) are \((4, 5, 1), (0, -1, -1), (3, 9, 4) \) and \((-4, 4, 4)\), respectively. Are the end points of the set \( \{a, b, c, d\} \) coplanar? (18)

(b) Find the perpendicular distance of a corner of a unit cube from a diagonal not passing through it. (10)

(c) Prove that \([a \times p \ b \times q \ c \times r] + [a \times q \ b \times r \ c \times p] + [a \times r \ b \times p \ c \times q] = 0\). (7)

6. (a) A particle moves so that its position vector is given by \( r = cos \omega t \, i + sin \omega t \, j \) where \( \omega \) is constant. Show that

(i) the velocity \( v \) is perpendicular to \( r \).

(ii) the acceleration is directed towards the origin.

(iii) \((r \times v)\) is constant.

(b) Find \( T, N, B, \kappa, \tau \) and the equation of osculating plane of the space curve \( x = t, y = 3t^2, z = t^3 \) at the point \( t = 0 \). (13)

(c) Find the equations of the tangent line and normal plane to the curve \( x^2 + y^2 + z^2 = 1 \), \( z + y + z = 1 \) at the point \((0, 0, 1)\). (11)
MATH 159(EEE)

7. (a) Define directional derivative and normal derivative. Find the direction along which the directional derivative of \( \phi = 4xz^3 - 3x^2y^2z \) at the point \((2, -1, 2)\) is the greatest. Determine the greatest value as well.

(b) If \( \mathbf{F} = r \mathbf{r} \) where \( \mathbf{r} = xi + yj + zk \), is there a function \( \phi \) such that \( \mathbf{F} = -\Delta \phi \)? If so, find it. Evaluate \( \int_{C} \mathbf{F} \cdot d\mathbf{r} \) around any simple closed curve \( C \).

(c) Evaluate normal surface integral of \( \mathbf{A} = yi + 2xj - zk \) over the surface of the plane \( 2x + y = 6 \) in the first octant cut off by the plane \( z = 4 \).

8. (a) Evaluate \( \iint_{S} (ax^2 + by^2 + cz^2) \, dS \) over the sphere \( x^2 + y^2 + z^2 = 1 \) using Gauss's divergence theorem.

(b) State Green's theorem for plane and deduce it from Stokes' theorem. Verify Green's theorem for \( \int_{C} (y - \sin x) \, dx + \cos x \, dy \) where \( C \) is the triangle of the adjoining figure.
L-1/T-1/EEE

Date: 17/11/2012

BANGLADESH UNIVERSITY OF ENGINEERING AND TECHNOLOGY, DHAKA

L-1/T-1  B. Sc. Engineering Examinations 2011-2012

Sub: **MATH 157** (Calculus I)

Full Marks: 210  Time: 3 Hours

The figures in the margin indicate full marks.

Symbols used have their usual meaning.

USE SEPARATE SCRIPTS FOR EACH SECTION

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**SECTION - A**

There are **FOUR** questions in this section. Answer any **THREE**.

1. (a) A function \( f(x) \) is defined as follows:

\[
 f(x) = \begin{cases} 
 1, & x < 0 \\
 1 + \sin x, & 0 \leq x < \frac{\pi}{2} \\
 2 + \left( x - \frac{\pi}{2} \right)^2, & x \geq \frac{\pi}{2} 
\end{cases}
\]

Discuss the continuity and differentiability of \( f(x) \) at \( x = \frac{\pi}{2} \). Also sketch the graph of \( f(x) \).

(b) Evaluate the following:

(i) \( \lim_{x \to 0} \left( \tan x \right)^{\frac{1}{x^3}} \)

(ii) \( \lim_{x \to 0} \left( \frac{1}{x} - \frac{1}{x^2} \ln (1 + x) \right) \)

2. (a) If \( y = a \sin(\ln x) + b \cos (\ln x) \), then show that

\[
 x^2 y_{n+2} + (2n + 1) xy_{n+1} + (n^2 + 1) y_n = 0.
\]

(b) Expand \( \frac{1}{1-x} \) in powers of \( x \) with Lagrange's form of remainder after \( n \) terms and find the value of \( \theta \) in \( R_n \).

(c) Applying Mean value theorem prove that, \( \sqrt{\frac{1-x}{1+x}} < \frac{\ln (1+x)}{\sin^{-1} x} < 1 \), when \( 0 < x < 1 \).

3. (a) A closed box with square base is to have volume of 200 \( m^3 \). The material cost for the top and bottom of the box is TK. 3/m\(^2\) and the material cost for sides of the box is TK. 1.5/m\(^2\). Find the minimum material cost of the box.
(b) If \( u = \csc \frac{\sqrt{x} + \sqrt{y}}{\sqrt{x} + \sqrt{y}} \), then show that

\[
\frac{\partial^2 u}{\partial x^2} + 2x \frac{\partial^2 u}{\partial x \partial y} + y^2 \frac{\partial^2 u}{\partial y^2} = \left( \frac{1}{12} \right) \tan u \left( \frac{13}{12} \right) + \left( \frac{1}{12} \right) \tan^2 u.
\]

(c) If \( u = \ln r \), then show that

\[
\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} + \frac{\partial^2 u}{\partial z^2} = \frac{1}{r^2},
\]

where \( r^2 = (x - a)^2 + (y - b)^2 + (z - c)^2 \).

4. (a) Prove that the radius of curvature of the ellipse \( \frac{x^2}{a^2} + \frac{y^2}{b^2} = 1 \), at an end of major axis is half of its latus rectum.

(b) Find the pedal equation of the curve \( x = a \cos^3 \theta, y = a \sin^3 \theta \).

(c) Show that the asymptotes of the cubic \( x^3 - 2y^3 + xy (2x - y) + y (x - y) + 1 = 0 \) cut the curve again at three points which lie on the straight line \( x - y + 1 = 0 \).

SECTION – B

There are FOUR questions in this section. Answer any THREE.

5. (a) Carry out the following:

(i) \( \int \frac{3x - 2}{1 - 6x - 9x^2} \, dx \)

(ii) \( \int (x - 3)^3 \sqrt{6x - x^2} \, dx \)

(b) Prove that, \( \int \frac{x^2 \, dx}{(x \sin x + \cos x)^2} = -\frac{x \sec x}{x \sin x + \cos x} + \tan x + c \).

6. (a) Find a reduction formula for \( \int x^n \cos mx \, dx \) and hence evaluate \( \int x^4 \cos 3x \, dx \).

(b) Evaluate: \( \int_0^{\pi/2} \log (\tan x + \cot x) \, dx \).

(c) Evaluate: \( \int_0^1 \frac{dx}{(1 + x)(2 + x)\sqrt{x(1 - x)}} \).

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7. (a) Prove that, \[ \int_0^1 \frac{x^2 \, dx}{\sqrt{1 - x^4}} \times \int_0^1 \frac{dx}{\sqrt{1 + x^4}} = \frac{\pi}{4\sqrt{2}}. \]
(b) Find the whole area of the curve \( a^2 y^2 = x^3 \) (2a - x).
(c) Find the larger area common to the two curves \( y^2 = ax \) and \( x^2 + y^2 = 4ax \).

8. (a) Find the whole area of the curve \( r^2 = a^2 \cos^2 \theta + b^2 \sin^2 \theta \).
(b) Find the volume of the solid generated by the revolution of the curve \( y (a^2 + x^2) = a^3 \)
about its asymptote.
(c) Find the area of the surface of revolution formed by revolving the curve \( r = 2a \cos \theta \)
about the initial line.