NONLINEAR PLASMA DYNAMICS AND EXPERIMENTAL STUDY OF RADIATION FROM A MAGNETIZED PLASM

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has been accepted as satisfactory in partial fulfilment for the degree of Master of Philosophy in Physics and certify that the student demonstrated a sastisfactory knowledge of the field covered by this thesis in an oral examination on 27th April 1995.

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ABSTRACT

Nonlinear wave-particle interaction in unmagnetized and magnetized hot plasmas are studied analytically. Intense circularly polarized electromagnetic wave propagating along field direction is considered. Homogeneous and inhomogeneous plasmas are studied. Electrostatic vortex mode and various types of envelope solitons (cavitons) are found to exist in the considered plasmas. Radiation emitted from a glow discharge plasma is studied. It is found that due to magnetic field the radiation is red shifted. A special type of electromagnet is designed, constructed and calibrated. Furthermore, a mirror type magnetic bottle (mirror machine) is constructed and the mapping of magnetic field lines are done. Problem of plasma confinement in mirror machine is also studied.

CONTENTS

INTRODUCTION

Problems concerning the interaction of large amplitude waves with magnetized and unmagnetized plasmas are presently receiving increased interest as the results can be applicable in laser fusion studies, high frequency wave heating of laboratory plasmas and as well as in astrophysics. Practically, studies of the wave-particle interaction in a magnetized plasma are very important in laser-pellet interaction, the r.f. plugging for plasma confinement, as also some wave driven particle acclerator such as beat wave particle acclerator etc. However, for large field intensities, non- linear effects[1-6] such as the relativistic electron mass variation[1-4] and the ponderomotive force[6,7] can lead to a downshift of the local electron plasma frequency.

STENFLO and TSINTSADZE[8] have studied the dispersion relation for a large amplitude circularly polarized wave which propagates along the external magnetic field $B_0\hat{z}$ in a cold electron-ion plasma. In the ultra relativistic limit it was demonstrated that a completely new type of wave, with frequency proportional to amplitude can appear.

AKCHIEZER and POLOVIN[1] presented a general theory of non linear wave motion in electron plasma. Taking into account the electron mass variation as well as second order density perturbation effects, they derived a set of non-linear equations and also the corresponding dispersion relation of the waves. It is shown that the intense electromagnetic wave can drive electrons of relativestic velocities and produce a strong nonlinear current in the longitudinal direction, thus leading to coupling between the transverse and the parallel momentum. Hence electromagnetic waves are coupled to the Langmuir waves. Nonlinear propagation of intense electromagnetic radiation in magnetised plasma has been studied by MOFIZ et. al. and others[1-5]. DECOSTER[5] reviewed the recent progress in the study of wave motion including relativistic effects.

It is well known that a high frequency wave, which propagates in plasma, can be responsible for a ponderomotive force, which causes density modulations as well as soliton formation or collapse[9].

Non-linear behavior in plasma may be classified with the intensity of the electromagnetic field, which determines the process of interaction. There are two types of nonlinearities: weak and strong. Weak nonlinearities are those, which may be described with the help of first term in the field amplitude expansion. It is well known that the electron velocity in a given electric field is $V_e = eE_0/m_e\ \omega_0$. As a character velocity of plasma particle one may consider the tharmal speed of electron $V_{te} = \sqrt{KT_e/m_e}$. The conditions for weak nonlinearity and strong nonlinearity are $v_e/v_{te} << 1$ and $v_e/v_{te} >> 1$ respectively. The collective process in plasma may cause the instabilities which increase potential for the electric field in plasma. Linear approximation may describe only the initial state of plasma instability. However, in the case of exponential growth of the perturbation, the linear approximation become invalid. Only through the linear analysis one may get the level of saturation in which plasma attains a quasistatic state. The formation of instability in plasma may cause the change of plasma parameters (density, temperature etc.) which can lead the plasma in the turbulent state.

The relativistic nonlinearity and the ponderomotive nonlinearity are the two factors which influence greatly the evoluation of an intense electromagnetic wave propagating in plasma. The relativistic nonlinearity arises due to the electron mass variation in the intense field and it has some consequences[10-12]. TSINTSADZE[10] demonstrated that the electron mass variation nonlinearity can parametrically excite both high frequency and low frequency modes in plasma. Later LIN and TSINTSADZE[11] carried out computer simulations of the same effect and found that the relativistic modulational instability can complete with the oscillating two-stream instability of NISHIKAWA[12]. Suppose an electron plasma wave arising from noise is superimposed on the pump wave and produces a field pattern in the form of sine wave: it happens due to relativistic mass variation effect, the electrons are lighter in the trough region and heavier in crest region. As a result the electrons in the troughs lead to plasma oscillation faster than those in the crests. The results include enhanced density bunching and produces the relativistic instability. This is the physical picture of relativistic modulational instability as described in ref. [11]. Relativistic mass variation also cause wave breaking phenomenon. The physical mechanism responsible for the relativistic wave breaking has been given by DRAKE and LEE[13]. In the underdense region in plasma where the local plasma frequency is larger than the radiation frequency, the plasma responds very weakly to the driver because of the large mismatch between driver and the normal mode frequencies. However, as one moves down the density gradient, the frequency mismatch become smaller and hence the driver has much larger amplitude. Around the critical point ($\omega_p > \omega$), the electron mass increase locally and it reduces the electron plasma frequency to balance the initial mismatch. Accordingly a resonance between ω and the new normal mode is established. Consequently, the plasma wave reaches a much larger amplitude on the overdense side of the critical layer than above it. A sharp discontinuity in the wave structure results and causes the wave to break.

MOFIZ et. al[14,15] have investigated the space time evolution of the relativistic modulational instability of intense wave propagating in uniform plasma. The later is described by the nonlinear Schrodinger equation and admits periodic and as well as localized solitons[16]. Envolope solitons arise when the relativistic nonlinearity is balanced by the dispersion of the waves. Small amplitude as well as finite amplitude one dimensional electromagnetic solitary waves have been found[17]. The propagation of three-dimensional, localized circularly polarized electromagnetic wave has been studied by GERSTEN and TZOAR[18] including relativistic mass variation nonlinearity. Soliton propagation occurs in both cases below and above the plasma frequency (ω_p).

Besides the electron mass variation nonlinearity, the self interaction of large amplitude waves also leads to a time averaged low frequency nonlinear force (ponderomative force)[7]. The later acts mainly on the electrons (ion motions are neglected due to its heavy mass) expelling from regions of neglected due to its heavy mass) expelling from regions of high field intensity. The ambipolar field thus created pulls away the ions. The local plasma density is, therefore, reduced by ponderomotive force[19]. The combined effects of ponderomotive force and the relativistic mass variation of the wave has been investigated[20-25].

Using the quasineutrality assumption TSINTSADZE and TSKHAKAYA[20] discussed small amplitude laser pulses, where as SHUKLA and SPATSCHEK[21] demonstrated the existance of finite amplitude rarefaction and compressional solitons with subsonic and super sonic velocities in hot plasma.

Nonlinear processes caused by ultra relativistic effects were studied by TSINTSADZE et. al[26,27] whose results show that the relativistic electron motion can lead to the formation of shock waves. If the RF pressure force exceeds the gas pressure, the solitary wave is a compressional wave. Observation of these effects in the laboratory requires a laser power density of 10^{16} - 10^{17} watt/sq.cm. or more for laser fusion, where the power density required in the microwave range is several magnitude lower. In space, fields of these type may be produced by intense redio sources.

Heat transfer in fluids is known as convection. An example of convection is the formation Benard convection cells[28] in which steady-state vertical motion of a liquid occurs. Analogous effects can occur in a plasma confined by a magnetic field.

The theory of convective transfer in a plasma dates back to the discovery of drift waves and drift instabilities[29]. The growth of drift waves is analogous to a convective instability of an inhomogeneous liquid. It was found that the drift instability can be suppressed by the shear of the confining magnetic field[30]. Subsequent experiment in a large-shear devices, however, revealed anomalously high thermal conductivity of the plasma. In an effort to overcome the difficulties which arose from this, a nonlinear theory was derived with out the assumptions that the phases are random. Within the framework of this theory it has been found that there is spontaneous localization of drift waves. It turns out that even if their amplitude is small these waves undergo self-organization into a set of solitary vortices. Like solitons, these vortices can exist in a manner independent of each other with interactions only in collisions. Because of their comperatively small dimensions, they are insensitive to shear, so that they can cause collective mixing of plasma even in large-shear devices.

The theory of nonlinear waves is based on the model equations in which the most important effects are described explicitly, while quantitative corrections of secondary importance are ignored. The best known model equation is the korteweg-de Vries equations, which escaped notice for decades, untill its usual properties are discovered[31] in 1965. It was only in 1976, when the soliton solution were found[32], that this equation started to gain widespread acceptance in the theory of synoptic vortices in meteorology[33].

A new entity has found a place in research in nonlinear plasma theory; LARICHEV and REZNIK solitary vortices, which were originally studied in connection with the problem of solitary Rossby waves in a rotating liquid[34].

A glow discharge plasma is a simple experimental device to study plasma properties. It may also be a simpler device to study wave-particle interaction as well as plasma radiation, plasma behaviour in fusion system and space plasmas[35-39] etc. The properties of the radiations that are emitted by glow discharge plasma have been studied[40,41] with imposed magnetic field on plasma.

The plasma confinement with the help of a combination of r.f. electric field and open ended magnetic trap under r.f. electric field and open ended magnetic trap under condition of ion-cyclotron resonance have been investigated theoretically and experimentally[42-44].

The aim of this dissertation is to study the nonlinear behavior of wave-particle interactions in both unmagnetized and magnetized plasmas. Since intense electromagnetic waves are related to fusion and space research, relativistic nonlinear interactions is considered in this work. Furthermore for tapping of plasma particles vortex motion are considered for electrostatic mode. Experimental facilities are built up to study the effects of external magnetic field on plasma radiations. The magnetic mirror machine is built up to trap the plasma particle by r.f. plugging.

In Chapter-2: macroscopic behaviour of wave plasma interaction is treated using two fluid model (MHD theory). The dispersion relation and the expression for the ponderomotive force is obtained. The complete set of governing equations accounting for relativistic and ponderomotive nonlinearity are obtained. There are four sections in this chapter. In section 2.1 we consider electron-ion unmagnetized plasma. Here wave particle interactions are studied in the two fluid model. It is found that in the unmagnetized cold plasma, intense envelope solitons are generated. It is also found that both amplitude and the width of the soliton decrease with increase of plasma density. In section 2.2; plasma is considered as inhomogeneous and dispersive media. The propagation of intense electromagnetic radiation in such media produces in the envelope solitons a little change i.e. hump in tail. In section 2.3 ion motions are taken into account. This study shows that double humped solitons are formed in this case. In section 2.4(a) relativistic nonlinear effect in wave plasma interaction is studied using two fluid model. For a better understanding of wave-plasma intraction, initially the classical non relativistic case is investigated. It is found that in the strongly magnetized high temperature plasma, intense envelope solitons may be generated. The soliton width is not influenced by the magnetic field, but it is determined by the phase shift and by the plasma density. Isolated super thermal envelope solitons may exist in this case. The weak relativistic case, nonlinearity modify the existing super thermal solitons. In section 2.4(b) the case of ultra relativistic nonlinear interaction is considered. It is found that the two types of waves may exist in this situation. One is the "high frequency wave" and the other is the "low frequency wave". The nonlinear propagation of these waves as well as their modulational and filamentational instabilities are studied in this Chapter.

In Chapter-3. vortex motion of plasma particle in a homogeneous plasma are studied. In a strong magnetic field the plasma particles are trapped. For the simplicity electrosatic drift vortices in plasma are considered. Vortex motion is related to the vector nonlinarity which arises due to drift motion in the direction perpendicular to the wave propagation. Perpendicular drift plasma motion produces vortices.

In experimental studies we determine the wavelengths of plasma radiation and setup of a magnetic mirror machine for confinement of plasma particles. We get the plasma source[45] from glow discharge plasma. The positive column of a glow discharge plasma is the ideal source of plasma radiation. The wavelengths of radiation emitted from positive column are determined for two cases: (a) without applying external magnetic field; (b) with the application of a moderately strong magnetic field. A special type of electro-magnet is designed, developed and calibrated, which gives us a field of several kilo-gauss. The effect of external magnetic field on plasma radiation is studied for two types of plasmas: nitrogen plasma and hydrogen plasma. We also designed and developed a magnetic mirror machine for plasma confinement. Analytical and experimental values of the mirror ratios are obtained.

FORMATION OF SOLITONS

2.1 SOLITONS IN WAVE-PLASMA INTERACTION:

We consider the plasma in two fluid model. One is electron fluid and the other is ion fluid. The relativistic fluid equations are given by

$$\partial_t n_j + \vec{\nabla}. (n_j \vec{v}_j) = 0 \quad (2.1)$$

and $$\partial_t \vec{P}_j + (\vec{v}_j. \vec{\nabla})\vec{P}_j = \vec{e}_j \{\vec{E} + \{1/c\}\vec{v}_j \times \vec{B}\} - (1/n_j)\vec{\nabla} p_j \quad (2.2)$$

where equation (1) and equation (2) are the equation of continuity and the equation of motion respectively.

The electric field ($\vec{E}$) and the magnetic field ($\vec{B}$) of the wave are governed by the Maxwell's equations,

$$\vec{\nabla} \times \vec{E} = -\{1/c\}\partial_t \vec{B} \quad (2.3)$$

$$\vec{\nabla} \times \vec{B} = -\{4\pi/c\}\vec{J} + (1/c)\partial_t \vec{E} \quad (2.4)$$

$$\vec{\nabla} . \vec{E} = - 4\pi \rho \quad (2.5)$$

and $$\vec{\nabla} . \vec{B} = 0 \quad (2.6)$$

where, $\vec{J} = \sum_j n_j q_j \vec{v}_j$ is the current density, and $\rho = \sum_j n_j q_j$ is the charge density.

The subscript j indicates the species, namely j = i(ions) or j = e(electrons); n_j is the number density, $\vec{v}_j$ is the velocity of the fluid considered, q_j is the charge of the species; $\vec{P}_j$ is the momentum of the fluid = $m_{0j} \gamma_j \vec{v}_j$; m_{0j} is the rest mass of the particle which constitutes the fluid; $\gamma_j = (1 - \frac{v_j}{c^2})^{-\frac{1}{2}}$; c is the velocity of light in space and the pressure of fluid $p_j = n_j T_j$ for isothermal case, where T_j temperature of the fluid.

The high frequency transverse oscillation of homogeneous plasma are considered. Let a right handed circularly polarized (RHCP) electromagnetic wave propagates along the z-direction, we can write

$$P_{jx} + iP_{jy} = \tilde{P}_j (z,t) = P_j(z,t)e^{-i\omega t+ikz} \qquad (2.7)$$

and $E_x + iE_y = \tilde{E} (z,t) = E(z,t)\ e^{-i\omega t+ikz}$, (2.8)

where, we assume $P_j(z,t)$ and $E(z,t)$ are the complex amplitude with slowly varying functions depends on the coordinate (z) and time(t).

The transverse oscillations are described by the following equation

$$(\partial_t + v_{jz}\ \partial_z)\ \tilde{P}_j = e_j\ [\tilde{E} +(i/c)v_{jz}\ \tilde{B}] \qquad (2.9)$$

For slowly varying amplitudes of the transverse oscillation, we have

$$P_j(z,t)= - \frac{e_j[1-(k/\omega)\ v_{jz}]}{i(\omega - kv_{jz})}\ E(z,t) \qquad (2.10)$$

From Maxwell's equation [i.e. from 2.3] we have

$$\vec{\nabla} \times \vec{\nabla} \times \vec{E} = -(1/c)\partial_t \, (\vec{\nabla} \times \vec{B})$$

Now we are neglecting the ion current due to its heavy mass ($m_i \gg m_e$) and also consider the phase velocity of the wave is much higher than the longitudinal plasma motion (i.e. $|\omega/k| \gg v_{jz}$)

Then,

$$(c^2 \, \partial_z^{\,2} - \partial_t^{\,2})\vec{E} = -\, 4\pi e^2 \, n_0 \partial_t \, (\frac{P_e}{m_{0e} \, \gamma_e})\vec{E} \qquad (2.11)$$

where n_0 is the background density.

From (2.11) we obtain,

$$(c^2\partial^2_z - \partial^2_t + \frac{i \, \omega_{peo}^2}{\omega} \partial_t) \, \vec{E} = \frac{i\lambda}{\omega} \, \omega^2_{peo} \, |\vec{E}|^2 \, \partial_t \, \vec{E} \qquad (2.12)$$

where, $\gamma_e = 1 - \frac{\lambda}{2} \, |\vec{E}|^2; \; \lambda = \frac{e^2}{m^2_{0e} \, c^2 \, \omega^2}$

and electron plasma frequency $\omega^2_{peo} = \frac{4\pi e^2 n_e}{m_{0e}}$

Now, the dispersion relation is

$$\omega^2 = k^2c^2 + \omega^2_{peo}/\gamma_e \qquad (2.13)$$

From which we get the transverse di-electric constant by the following equation.

$$\epsilon_\perp = \frac{k^2 \, c^2}{\omega^2} = 1 - \frac{\omega^2_{peo}}{\omega^2 \, \gamma_e} \qquad (2.14)$$

An explicit form of the ponderomotive force is given by[7]

$$fp = \frac{1}{16\pi}\,[(\epsilon_{\perp}-1)\partial_z|\vec{E}|^2 + \frac{k}{\omega^2}\,\frac{\partial}{\partial\omega}\{\omega^2(\epsilon_{\perp}-1)\}\partial_t|\vec{E}|^2] \qquad (2.15)$$

which gives

$$fp = \frac{1}{16\pi}\,\{\,\frac{\omega^2_{peo}}{\omega^2}\,(1-\frac{\lambda}{2}\,|\vec{E}|^2)\}\partial_z|\vec{E}|^2 \qquad (2.16)$$

For slow plasma response, the equation of continuity, Poission equation and the equation of motion are

$$\partial_t\, n_e + \partial_z\, n_e\, v_{ez} = 0 \qquad (2.17)$$

$$\partial_z\, E = 4\pi e\,(n_i - n_e) \qquad (2.18)$$

and $\partial_t\, P_{ez} = e\,E_z + \frac{fp}{n_e} - \frac{T_e}{n_e}\,\partial_z\, n_e$ (2.19)

In our case, let $n_e = n_0 + \delta n_e$ and $n_i = n_0$ where δn_e is the density fluctuation.

From equation (2.17) ~ (2.19), we can write

$\partial^2_t\ \delta n_e + n_0\ \partial_z\ \partial_t\ v_{ez} = 0$; which gives

$$(\partial^2_t - v_{te}^2\,\partial^2_z)\,\frac{\delta n_e}{n_0} = \frac{\omega^2_{peo}}{16\pi n_0\omega^2 m_{0e}}\,(1-\frac{\lambda}{4}\,|\vec{E}|^2)\,\partial_z|\vec{E}|^2 \qquad (2.20)$$

where $v^2_{te} = \frac{T_e}{m_{0e}}$ is the thermal velocity of electrons.

Let us introduce a new coordinate system as

$$\xi = z - v_0\, t \qquad (2.21)$$

where v_0 is a constant of velocity. With the help of equation (2.21) and (2.20), we get

$$\frac{\delta n_e}{n_0} = \frac{\omega^2_{peo}(1 - \frac{\lambda}{4}|\vec{E}|^2)|\vec{E}|^2}{16\pi m_{0e}\, n_0\, \omega^2(v^2_0 - v^2_{te})} \tag{2.22}$$

Now from equation (2.3) we have

$$[c^2\, \partial_z{}^2 - 2i(\omega\partial_t + kc^2\, \partial_z) + \omega^2_{peo}\, \frac{\partial n_e}{n_0}\, (1 - \frac{\lambda}{2}\, |\vec{E}|^2)]E_{(z,t)} = 0 \tag{2.23}$$

Now using $E_{(z,t)} = |\vec{E}|\; e^{-i\theta t + i\chi z}$

where θ is the frequency shift and χ is the wave number shift.

we obtain

$$c^2\, \partial^2_z\, |\vec{E}| - (c^2\chi^2 + 2k\chi c^2 - 2\omega\theta)|\vec{E}| - 2i(c^2\chi\partial_z + \omega\partial_t + kc^2\, \partial_z)|\vec{E}| + [\frac{\omega^2_{peo}\{1-(\lambda/4)|\vec{E}|^2\}\{1-(\lambda/2)|\vec{E}|^2\}}{16\pi\, m_{0e}\, n_0\, \omega^2(v^2_0 - v^2_{te})}]|\vec{E}|^3 = 0 \tag{2.24}$$

From the imaginary part of equation (2.24), we get

$$v_0 = \frac{kc^2}{\omega}\, (\, 1 + \frac{\chi}{k}) \tag{2.25}$$

Introducing some dimensionless variables in the real part of equation (2.24), such as

$$\sigma = \frac{e\, |\vec{E}|}{m_{0e}\, c\omega} \text{ and } \eta = \frac{\omega_{peo}}{c}\, \xi\ ,$$

We obtain

$$\partial^2_\eta\, \sigma - a\sigma + b\sigma^3 = 0, \tag{2.26}$$

$$\text{where} \quad a = \frac{c^2}{\omega^2_{peo}} \left(\chi^2 + 2k\chi - \frac{2\omega\theta}{c^2} \right. \qquad (2.27(a))$$

$$\text{and} \quad b = \frac{c^2 (1 - \lambda/4)}{4 (v^2_0 - v^2_{te})} \quad \ldots\ldots\ldots\ldots\ldots \quad 2.27(b)$$

The solution of equation (2.26) is given by

$$\sigma = g \operatorname{sech} (g\mu\eta) \qquad (2.28)$$

where $g = (\frac{2a}{b})^{\frac{1}{2}}$ and $\mu = (\frac{b}{2})^{\frac{1}{2}}$

The equation (2.28) represents the envelope soliton of the wave whose amplitude is

$$\sigma^2_0 = g^2 = \frac{2a}{b} = \frac{8(\chi^2 + 2k\chi - 2\omega\theta/c^2)(v^2_0 - v^2_{te})}{\omega^2_{peo} (1 - \lambda/4)} \qquad (2.29)$$

If θ, χ is zero, i.e, there is no phase shift $\sigma^2_0 = 0$. On the other hand soliton is formed due to phase shift. Here

$$\sigma_0 \; \alpha \; \frac{1}{\sqrt{n_0}} \text{ and } \sigma^2_0 > 0; \text{ i.e, } v^2_0 > v^2_{te}$$

which means that only super thermal soliton may exist.

The width of the soliton is

$$\delta = \frac{1}{g\mu} = \sqrt{a} = \sqrt{\left(\frac{c^2}{\omega^2_{peo}} (\chi^2 + 2k\chi - \frac{2\omega\theta}{c^2})\right)}$$

It is also noticable that $\delta \; \alpha \; \frac{1}{\sqrt{n_0}}$

i.e, both the amplitude and the width of the soliton decreases with increase in plasma density.

For particular case if $b < 0$, then the solution of equation (2.26) is

$$\sigma = g' \operatorname{cosech}(g' \mu' \eta) \tag{2.30}$$

where $g' = \sqrt{\left\{\frac{2a}{|b|}\right\}}$ and $\mu' = \sqrt{\left\{\frac{|b|}{2}\right\}}$

The equation (2.30) represent an isolated spiky solitons in a plasma.

2.2 EFFECT OF INHOMOGINITY IN SOLITON DYNAMICS

The propagation of electromagnetic waves in an inhomogeneous medium like plasma, which is non-linear in nature, soliton is formed with some special characteristics. For example we consider electron-ion plasma as in nature (burning stars) where density changes with space. In this case we also consider plasma as two fluid model as before. The continuity equation and the equation of motion are

$$\partial_t n_j + \vec{\nabla} \cdot (\vec{v}_j n_j) = 0 \qquad (2.31)$$

and

$$(\partial_t + \vec{v}_j \cdot \vec{\nabla}) \vec{P}_j = e_j (\vec{E} + \vec{v}_j \times \vec{B}) - (1/n_j) \vec{\nabla} p_j \qquad (2.32)$$

where j is electrtons (e) or ions (i), n is the number density of the spices, $\vec{v}$ is the velocity of the fluid, $\vec{P}$ is the momentum of the fluid p is the pressure of the fluid, $\vec{E}$ is the electric field and $\vec{B}$ is the magnetic field. Let us define the electric field and the magnetic field by

$$\vec{B} = \vec{\nabla} \times \vec{A} \text{ and } \vec{E} = -(1/c)\partial_t \vec{A} - \vec{\nabla}\phi$$

where $\vec{A}$ is the vector potential and ϕ is the scalar potential. Now the perpendicular motion of the fluid is

$$P_{j\perp} = \frac{-e_j(kv_{jz} - \omega)A_\perp}{c(kv_{jz} - \omega + \Omega_c)} \qquad (2.33)$$

where Ω_c is called the cyclotron frequency which is

$$\Omega_c = \frac{e_j B_0}{m_{0j} \gamma_j c}$$; m_{0j} is the rest mass of the spices and

$$\gamma_j = \frac{1}{\sqrt{\{1-v_j^2/c^2\}}}$$; v_j → velocity of the spices &

c is the velocity of light.

The transverse motion is

$$(\partial_t + v_{jz} \partial_z) P_{jz} = e_j \partial_z \phi - \frac{e_j}{c} (v_{jy} \partial_z A_z + v_{jz} \partial_z A_y) - \frac{T_j}{n_j} \partial_z n_j \tag{2.34}$$

which gives

$$c^2 \partial^2_z \tilde{A} - \partial^2_t \tilde{A} = 4\pi \sum_j \frac{e^2 n_j}{m_{0j}} \left(\frac{kv_{jz} - \omega}{kv_{jz} - \omega + \Omega_c}\right) \tilde{A} \tag{2.35}$$

Let $\tilde{A} = A(z,t)\, e^{-i\omega t + ikz}$

The dispersion relation is

$$\omega^2 = k^2 c^2 + \frac{4\pi e^2 n_j}{m_{0j}} \left(\frac{kv_{jz} - \omega}{kv_{jz} - \omega + \Omega_c}\right) \tag{2.36}$$

and the transverse di-electric constant

$$\epsilon_\perp = \frac{k^2 c^2}{\omega^2} = 1 - \frac{4\pi e^2 n_j}{m_{0j} \omega^2} \left(\frac{\omega}{\omega - \omega_c}\right) \tag{2.37}$$

In our investigation, we consider non relativistic plasma, i.e, $\gamma = 1$. So $\Omega_c = \omega_c$; we neglect the ion motion due to its heavy mass.

We consider $v_{jz} << |\frac{\omega - \omega_c}{k}|$. Then the electron density is

$n_e = n_0\,(z) + \delta n_e$ and $\nabla \rightarrow \partial_z$.

An explicit form of ponderomotive force is

$$fp = \frac{1}{16\pi}\left[\nabla\left\{|\vec{E}|^2\left(\frac{\partial\epsilon_\perp}{\partial\rho}\right)\rho\right\} - |\vec{E}|^2\,\nabla\epsilon_1 + \vec{B}\times\left\{\vec{\nabla}\times\frac{\partial\epsilon_\perp}{\partial\vec{B}}\,|\vec{E}|^2\right\}\right]$$

$$+ \frac{1}{16\pi}\,\frac{k}{\omega}\,[\partial\,\{\omega^2\,(\epsilon_\perp - 1)\}/\omega\partial\omega]\,\partial_t\,|\vec{E}|^2 \qquad (2.38)$$

In our case $\rho = n_{0(z)}$ and $\vec{B} = 0$, i.e, unmagnetized plasma. Therefore.

$$fp = -\frac{e^2 n_{0(z)}}{4 m_{0e}\,\omega^2}\left(\frac{\omega}{\omega-\omega_c}\right)\partial_z|\vec{E}|^2 - \frac{k\,\omega_c}{(\omega-\omega_c)}\,\partial_t|\vec{E}|^2 \qquad (2.39)$$

From continuity equation (2.31), we obtain

$$\partial^2_t\,\delta n_e + \partial_z\,n_{0(z)}\,\partial_t\,v_{ez} = 0 \qquad (2.40)$$

From eqn(2.32), we get

$$\partial_t\,v_{ez} = \frac{fp}{m_{0e} n_0(z)} - \frac{v^2_{te}}{n_{0(z)}}\,\partial_z\,n_{0(z)} - \frac{v^2_{te}}{n_0(z)}\partial_z\,\delta_{ne}$$

where $v^2_{te} = \frac{T_e}{n_0(z)}$ thermal velocity of electron along $\vec{B} \sim B_0\,\hat{z}$, which gives

$$(\partial^2_t - v_{te}\,\partial^2_z)\delta n_e = -\frac{1}{m_{0e}}\,\partial_z\,fp \qquad (2.41)$$

Now, let $n_{0(z)} = \alpha z + \beta$

where $\alpha = \dfrac{n_0(max) - n_0(min)}{L}$ and $\beta = n_0(min)$, where L is the characteristic length of the system.

Let $\nu = \dfrac{\omega}{\omega - \omega_c} + \dfrac{kv_0\, \omega_c}{(\omega - \omega_c)^2}$,

$$a_1 = \frac{e^2\, \beta\, \nu}{4m^2_{0e}\, \omega^2 (v^2_0 - v^2_{te})} \; ; \; a_2 = \frac{e^2\, \alpha\, \nu}{4m_{0e}^2\, \omega^2 (v_0^2 - v_{te}^2)}$$

Also introducting new coordinate system

$\xi = z - v_0 t$ i.e, $\partial^2_\xi = \partial^2_z$ and $\partial_\xi^2 = \dfrac{1}{v_0^2}\, \partial^2_t$

So, the purtarbatiion of the electron density from equation (2-41)

is $\delta n_s = -(a_1 + a_2 z)\, |\vec{E}|^2 + a_1\, |\vec{E}_0|^2$

which gives

$$\frac{\delta n_s}{n_0(z)} = \frac{\omega^2}{n_0(z) c^2} \{ a_1 |\tilde{A}|^2 - (a_1 + a_2 z) |\tilde{A}|^2 \} \qquad (2.42)$$

as $|\vec{E}|^2 = \dfrac{\omega^2}{c^2} |\tilde{A}|^2$

From equation (2.35) we obtain

$$c^2\, \partial^2_z\, \tilde{A} - \partial^2_t\, \tilde{A} = \frac{4\pi e^2 n_0(z)}{m_{0e}} \left(\frac{\omega}{\omega - \omega_c}\right) \left(1 + \frac{\delta n_s}{n_0(z)}\right) \tilde{A} \qquad (2.43)$$

Let

$$\tilde{A} = |\tilde{A}| \ e^{-i\theta t + i\chi z}$$

where $|\tilde{A}|$ is the complex amplitude and θ, χ are the phase shift.

Eqn. (2.43) is now

$$[i(\partial_t + \frac{kc^2}{\omega}\partial_z + \frac{\chi c^2}{\omega}\partial_z) + \{\theta - \frac{k\chi c^2}{\omega} - \frac{\chi^2 c^2}{2\omega} + \frac{c^2}{2\omega}\partial^2_z$$

$$+ \frac{\omega^2_{peo}\ \omega^2}{2n_0(z)\ c^2(\omega-\omega_c)}([a_1 + a_2\ Z]|\tilde{A}|^2 - a_1|\tilde{A}_0|^2)\}]|\tilde{A}| = 0 \qquad (2.44)$$

From the imaginary part of eqn. (2.44) we obtain

$$v_0 = \frac{kc^2}{\omega}(1 + \frac{\chi}{k}) \qquad (2.45)$$

From the real part of eqn. (2.44) we get

$$\partial^2_z\ |\tilde{A}| - (\chi + 2k\chi - \frac{2\omega\theta}{c^2} + \frac{\omega^2_{peo}\ \omega^3\ a_1|A_0|^2}{n_0(z)\ c^4(\omega - \omega_c)})\ |A|$$

$$+ \frac{\omega^2_{peo}\ \omega^3\ (a_1 + a_2\ z)}{n_0(z)\ c^4(\omega - \omega_c)}\ |\tilde{A}|^3 = 0 \qquad (2.46)$$

Now, for simplification introducing some dimensionless quantities as

$$\sigma = \frac{e|\tilde{A}|}{m_{0e}\ c^2} \text{ and } \eta = \frac{\omega_{peo}\ z}{c}$$

The equation (2.46) is

$$\partial^2_\eta \sigma - a\sigma + b\sigma^3 = 0 \tag{2.47}$$

where $a = \dfrac{c^2}{\omega^2_{peo}} [\chi^2 + 2k\chi - \dfrac{2\omega\theta}{c^2} + \dfrac{\omega^2_{peo}\, \omega^3\, a_1\, |\tilde{A}_0|^2}{n_0(z)\, c^4(\omega - \omega_c)}]$

and $b = \dfrac{c^2\, m^2_{0e}\, \omega^2}{n_0(z)\, e^2\, (\omega-\omega_c)} (a_1 + a_2\, c\, \eta/\omega_{peo})$

Let $b = b_0 + b_1$ and $a = a_0$, where

$$b_0 = \frac{c^2\, m^2_{0e}\, \omega^2\, a_1}{n_0(z)\, e^2\, (\omega-\omega_c)} \text{ and } b_1 = \frac{c^3\, m^2_{0e}\, \omega^3\, a_2\, \eta}{n_0(z)\, e^2\, \omega_{peo}(\omega-\omega_c)}$$

Now eqn. (2.47) may take the form

$$\sigma' \; [\; \sigma'' = a_0\, \sigma - b_0\, \sigma^3 - b_1\, \eta\, \sigma^3 \;]$$

$$\Rightarrow \sigma'^2 - a_0\, \sigma^2 + \frac{b_0}{2} \sigma^4 + C' = - \int \eta d\, (\frac{b_1}{2} \sigma^4) \tag{2.48}$$

where C' is the constant of integration and is equal to zero under certain boundary condition:

i.e, at ∞, $\tilde{A} = 0$, $\sigma = 0$, $C' = 0$;

then eqn. (2.48) is

$$\sigma'^2 - a_0\, \sigma^2 + \frac{b_0}{2} \sigma^4 = - \eta \frac{b_1}{2} \sigma^4 + \int \frac{b_1}{2} \sigma^4\, d\eta \tag{2.49}$$

Let us suppose,

$$Y = \sigma^2 = \sigma^2_H + \sigma^2_1$$

where $\sigma^2_H >> \sigma_1^2$ (σ_1 - inhomogeneous part)

i.e, $Y = Y_H + Y_1$ and $Y_H >> Y_1$

i,e. eqn (2.49) may be written as

$$\sigma'^2 - a_0 \sigma^2 + (b_0/2)\sigma^4 = -\eta \frac{b_1}{2} Y^2_H + \int \frac{b_1}{2} Y^2_H \, d\eta \quad (2.50)$$

For homogeneous plasma, we have

$$\sigma'^2_H - a\sigma^2_H + \frac{b_0}{2}\sigma^4_H = 0 \quad (2.51)$$

whose solution is

$$\sigma_H = \sigma_0 \operatorname{sech}(\sigma_0 \mu \eta) \quad (2.52)$$

where $\sigma_0 = \sqrt{\{2a_0/b_0\}}$ and $\mu = \sqrt{\{2/b_0\}}$

$$Y_H = \sigma^2_H = \sigma^2_0 \operatorname{seeh}^2(\sigma_0 \mu \eta) \quad (2.53)$$

From (2.50) and (2.53), one can obtain

$$\sigma'^2 - a_0 \sigma^2 + \frac{b_0}{2}\sigma^4 = -\eta \frac{b_1}{2}\sigma^4_0 \operatorname{sech}^4 z + \frac{b_1}{2\mu\sigma_0}\int \sigma^4_0 \operatorname{sech}^4 z dz \quad (2.54)$$

where $z = \sigma_0 \mu \eta$

$$\Rightarrow \sigma'^2 - a_0\sigma^2 + \frac{b_0}{2}\sigma^4 = -\eta\, d_1 \operatorname{sech}^4 z + d_2 \operatorname{th} z - d_3 \operatorname{th}^3 z \quad (2.55)$$

where $d_1 = \frac{b_1}{2}\sigma^4_0$; $d_2 = \frac{b_1}{2\mu}\sigma^3_0$ and $d_3 = \frac{b_1}{6\mu}\sigma^3_0$

Another form of eqn. (2.55) is

$$(\ln \sigma_H)' \ (\sigma^2_1)' - a_0 \sigma_1^2 + b_0 \sigma^2_H \sigma^2_1 = \eta \ d_1 \operatorname{sech}^4 z + d_2 \operatorname{th} z - d_3 \operatorname{th}^3 z$$

$$\text{Or,} \quad (\sigma^2_1)' + \frac{1}{(\ln \sigma_H)'} (b_0 \sigma^2_H - a_0) \sigma^2_1 = \frac{1}{(\ln\sigma_H)'} . \{-\eta d_1 \operatorname{sech}^4 z + d_2 \operatorname{th} z - d_3 \operatorname{th}^3 z\} \tag{2.56}$$

The solution of eqn. (2.56) is

$$\sigma^2_1 = \tfrac{1}{2} (1 + \sigma^2_0 b_0) \mu^2 \eta^3 b_0 b_1 \sigma^8_0 \tag{2.57}$$

So, finally the total solution is

$$\sigma^2 = \sigma^2_H + \sigma^2_1 = \sigma^2_0 \operatorname{sech}^2 (\sigma_0 \mu \eta) + \tfrac{1}{2} (1+\sigma^2_0 b_0) \mu^2 b_0 b_1 \eta^3 \sigma^8 \tag{2.58}$$

where $\mu = \surd\{2/b_0\} = \surd\{\dfrac{8(\omega-\omega_c)^3 (v^2_0 - v^2_{te})}{c^2 \omega (\omega^2 - \omega\omega_c + kv_0 \omega_c)}\}$

$$b_0 = \frac{c^2 \omega(\omega^2 - \omega\omega_c + kv_0 \omega_c)}{4(\omega-\omega_c)^3 (v^2_0 - v^2_{te})}$$

and $b_1 = \dfrac{c^3 \omega(n_{omax} - n_{omin}) (\omega^2 - \omega\omega_c + kv_0\omega_c)}{4n_{omin} L \omega_{peo} (\omega - \omega_c)^3 (v^2_0 - v^2_{te})}$

2.3 EFFECTS OF ELECTROSTATIC CHARGE SEPERATION IN SOLITONS

We use a two fluid MHD model to describe the electron, ion plasma. The relevent two fluid equations together with the Maxwell's equations are:

$$\partial_t n_j + \nabla \cdot (n_j \vec{v}_j) = 0 \qquad (2.59)$$

$$(\partial_t + \vec{v}_j \cdot \nabla)\, \vec{P}_j = c\, [\vec{E} + (1/c)\vec{v}_j \times \vec{B}] - (1/n_j)\nabla p_j \qquad (2.60)$$

$$\nabla \times \vec{E} = -(1/c)\partial_t \vec{B} \qquad (2.61)$$

$$\nabla \times \vec{B} = \frac{4\pi}{c} \vec{J} + \frac{1}{c} \partial_t \vec{E} \qquad (2.62)$$

$$\nabla \cdot \vec{E} = 4\pi\rho \qquad (2.63)$$

$$\nabla \cdot \vec{B} = 0 \qquad (2.64)$$

where n_j, $\vec{v}_j$, p_j are the number density, the fluid velocity and the pressure of the spicies j = i(ions) or e(electrons), respectively;

$\vec{P}_j = m_{j0} \vec{v}_j (1 - v_j^2/c^2)^{-1/2}$ denotes relativestic momentum; m_{j0} is the rest mass of the species and c is the speed of light. The wave field $\vec{E}$ and $\vec{B}$ can be defined in terms of the vector potential ($\vec{A}$) and scalar potential (ϕ) as

$$\vec{E} = -(1/c)\partial_t \vec{A} - \nabla \phi \qquad (2.65)$$

$$\vec{B} = \nabla \times \vec{A} \qquad (2.66)$$

We consider the wave propagation along the ambient magnetic field i.e. $\vec{B} = B_0\hat{z}$; and the vector potential $\vec{A} = (\hat{i}_x A_x, \hat{i}_y A_y, o)$ and the number density $n_j = n_0 + \delta n_j$ where $n_0 >> \delta n_j$ and $\nabla = \hat{i}_z \partial_z$ and non relativestic case $\gamma = \dfrac{1}{(1 - v_j^2 / c^2)^{\frac{1}{2}}} = 1.$

For transverse motion and transverse velocity we get the following equations

$$\vec{P}_{j\perp} = - \frac{e_j (kv_{jz} - \omega)}{c(kv_{jz} - \omega + \omega_c)} \vec{A}_\perp \tag{2.67}$$

and
$$\vec{v}_{j\perp} = - \frac{e_j (kv_{jz} - \omega)}{m_{j0} c(kv_{jz} - \omega + \omega_c)} \vec{A}_\perp \tag{2.68}$$

Now, the equation for the transverse current is (from 2.62 & 2.66)

$$(1/c^2)\, \partial^2_t \vec{A}_\perp - \partial^2_z \vec{A}_\perp = \frac{4\pi}{c} \sum_j e_j\, n_j\, \vec{v}_{j\perp} \tag{2.69}$$

where $\vec{J}_\perp = \sum_j e_j n_j \vec{v}_{j\perp}$ is the transverse current density.

From equation (2.68) and eqn. (2.69) we obtain

$$c^2\, \partial^2_z \vec{A}_\perp - \partial^2_t \vec{A}_\perp = \sum_j \omega^2_{pj} \left(\frac{kv_{jz} - \omega}{kv_{jz} - \omega + \omega_c} \right) \vec{A}_\perp \tag{2.70}$$

Let $\vec{A}_\perp = \vec{A}(z,t)\, e^{-i\omega t + ikz}$, where $\vec{A}(z,t)$ showing the varying amplitude of the field.

From which we get the dispersion relation

$$\omega^2 = k^2c^2 + \sum_j \omega^2_{pj} \left(\frac{kv_{jz} - \omega}{kv_{jz} - \omega + \omega_c}\right) \tag{2.71}$$

From (2.71), we may write the transverse dielectric constant which is defined as

$$\epsilon_\perp = \frac{k^2c^2}{\omega^2} = 1 - \sum_j \frac{\omega^2_{pj}}{\omega^2} \left(\frac{kv_{jz} - \omega}{kv_{jz} - \omega + \omega_c}\right) \tag{2.72}$$

An explicit form of ponderomotive force is

$$f_p = (1/16\pi) \left[\nabla\{|\vec{E}|^2(\partial_\rho \epsilon_\perp)\rho\} - |\vec{E}|^2 \nabla\epsilon_\perp + \vec{B} \times \{ \nabla \times \partial_{\vec{\Omega}} \epsilon_\perp |\vec{E}|^2\}\right]$$

$$+(1/16\pi)(k/\omega^2)[\partial_\omega \{\omega^2 (\epsilon_\perp - 1)\}] \partial_t |\vec{E}|^2 \tag{2.73}$$

In our case $\rho = n_0$; So ponderomotive force is

$$fp = -\frac{1}{16\pi} \left[\sum_j \frac{\omega^2_{pj}}{\omega^2} \left\{\left(\frac{(kv_{jz} - \omega)}{kv_{jz} - \omega + \omega_c}\right)\partial_z |E|^2\right\} - \frac{k\omega_c}{(kv_{jz} - \omega + \omega_c)^2} \partial_t |E|^2\right] \tag{2.74}$$

The continuity eqn. (2.59) and the equation of motion (2.60) along longitudinal direction are

$$\partial_t (n_0 + \delta n_j) + \partial_z (n_0 + \delta n_j)v_{jz} = 0 \tag{2.75}$$

and

$$\partial_t P_{jz} = e_j\left(-\frac{1}{c} \partial_t A_z - \partial_z \phi\right) + \frac{fp}{n_0} - \frac{T_j}{n_j} \partial_z (n_0 + \delta n_j) \tag{2.76}$$

$$\text{Or, } \partial_t v_{jz} = - \frac{e_j}{m_{j0}} \partial_z \phi + \frac{fp}{m_{j0}\, n_0} - \frac{T_j}{m_{0j}\, n_0} \partial_z(n_0 + \delta n_j) \qquad (2.77)$$

From equation (2.74), (2.75), (2.76) and (2.77) one can obtain

$$(\partial^2_t - v^2_{tj}\, \partial^2_z)\, \delta_{nj} = \frac{e_j\, n_0}{m_{j0}} \partial^2_z \phi - \frac{1}{m_{j0}} \partial_z\, fp \qquad (2.78)$$

where, we consider $v_{jz} << (\omega/k) = v_p$,

Again, we introduce new coordinate system as

$\xi = z - v_{0t}$, i.e. $\partial^2_t = v^2_0 \partial^2_\xi$ and $\partial^2_z = \partial^2_\xi$

One can easily obtain the following expression

$$\frac{\delta n_j}{n_0} = \frac{e_j\, \phi}{m_{j0}(v^2_0 - v^2_{tj})} + \frac{e^2_j}{4\omega^2 m^2_{j0}(v^2_0 - v^2_{tj})} \left\{ \frac{\omega^2 - \omega\omega_c + kv_0\omega_c}{(\omega - \omega_c)^2} \right\} |E|^2$$

(2.79)

From (2.68) and (2.69), we may obtain

$$c^2\, \partial^2_z\, \vec{A}_\perp - \partial^2_t\, \vec{A}_\perp = \sum_j \frac{4\pi e^2_j}{m_{j0}} \left(\frac{\omega}{\omega - \omega_c}\right)(n_0 + \delta n_j)\vec{A}_\perp \qquad (2.80)$$

we have already consider $\vec{A}_\perp = \tilde{A}(z,t)e^{-i\omega t + ikz}$

So, equation (2.80) can be written as

$$c^2\partial^2_z\, \tilde{A} + 2i\, (kc^2\partial_z + \omega\partial_t)\, \tilde{A} = \sum_j \omega^2_{pj} \left(\frac{\omega}{\omega - \omega_c}\right) \frac{\delta n_j}{n_0} \qquad (2.81)$$

Again let $\tilde{A} = |\tilde{A}|\ e^{-i\theta t + i\chi z}$ (where θ & χ are the small phase shift) and from equation (2.79)

we obtain

$$c^2\partial^2_z\ |\tilde{A}|\ +\ 2i\ (\chi c^2\partial_z + kc^2\partial_z + \omega\partial_t)|\tilde{A}| -$$

$$[c^2\ \chi^2\ +\ 2k\chi\ c^2\ -\ 2\ \omega\theta\ +\ \sum_j \omega^2_{pj}\ (\frac{\omega}{(\omega-\omega_c)})\ \frac{e\phi}{m_{j0}(v^2_0\ -\ v^2_{tj})}]|\tilde{A}| + \quad (2.82)$$

$$+\ \sum_j \frac{\omega\omega^2_{pj}\ e^2(\omega^2-\omega\omega_c\ +kv_0\ \omega_c)}{4m^2_{j0}\ c^2(v^2_{tj}\ -\ v^2_0)(\omega-\omega_c)}|\tilde{A}|^3\ =\ 0 \qquad (2.82)$$

From the imaginary part of eqn. (2.82), we get

$$v_0\ =\ \frac{kc^2}{\omega}\ (1\ +\ \frac{\chi}{k}) \qquad (2.83)$$

In real part of equation (2.82) introducing some dimensionless variables as

$$\sigma\ =\ \frac{e|\tilde{A}|}{m_{e0}\ c^2},\ \Phi\ =\ \frac{e\phi}{m_{e0}\ c^2}\ \text{and}\ \eta\ =\ \frac{\omega_{pe}\ z}{c}$$

and considering j = e as $m^2_i >> m^2_e$, we get

$$\partial^2_\eta\ \sigma\ -\ a_0\sigma\ -\ a_2\ \Phi\ \sigma\ +\ a_1\ \sigma^3\ =\ 0 \qquad (2.84)$$

where $\quad a_0\ =\ \frac{c^2}{\omega^2_{pe}}\ (\chi^2\ +\ 2k\chi\ -\ \frac{2\omega\theta}{c^2})$

$$a_1 = \frac{c^2 \omega(\omega^2 - \omega\omega_c + kv_0\omega_c)}{4(v^2_{te} - v^2_0)(\omega-\omega_c)}$$

and $$a_2 = \frac{c^2 \omega}{(\omega-\omega_c)(v^2_0 - v^2_{te})}$$

From poission equation, one can write

$$\partial^2_z \phi = 4\pi e n_0 \left(\frac{\delta n_e}{n_0} - \frac{\delta n_i}{n_0}\right) \tag{2.85}$$

With eqn. (2.79) and assume $m_i >> m_e$, we get

$$\partial^2_\xi \Phi - \frac{c^2 \Phi}{v^2_{te} - v^2_0} + \frac{c^2(\omega^2-\omega\omega_c + kv_0\omega_c)}{4(\omega-\omega_c)^2(v^2_{te} - v^2_0)} \sigma^2 = 0 \tag{2.86}$$

From which, we get

$$\Phi' \; |\Phi'' - b_0 \Phi + b_1 \sigma^2 = 0 \; | \tag{2.87}$$

where $b_0 = \dfrac{c^2}{v^2_{te}-v^2_0}$ and $b_1 = \dfrac{c^2(\omega^2 - \omega\omega_c + kv_0 \omega_c)}{4(\omega-\omega_c)^2(v_{te}^2 - v^2_0)}$

From eqn. (2.84) we can write

$$\sigma' \; |\sigma'' - a_0 \sigma + a_1 \sigma^3 - a_2 \Phi \sigma = 0 \; | \tag{2.88}$$

From (2.87) and (2.88) we can easily obtain the following equation:

$$\sigma'^2 - \lambda \Phi'^2 = a_0 \sigma^2 -(a_1/2)\sigma^4 - \lambda b_0 \Phi^2 + a_2 \Phi \sigma^2, \tag{2.89}$$

where $\lambda = \dfrac{a_2}{2b_1}$

Let us define

$$\sigma'' = a_0\sigma - a_1\sigma^3 + a_2\Phi\sigma = F(\sigma, \Phi),$$

$$\Phi'' = b_0\Phi - b_1{}^2 = G(\sigma,\Phi),$$

and $\sigma'^2 - \lambda\Phi' = a_0\sigma^2 - (a_1/2)\sigma^4 - \lambda b_0\Phi^2 + a_2\Phi\sigma^2 = H(\sigma,\Phi).$

Let $\Phi = \Phi(\sigma)$. We get the following relation:

$$H\Phi - \lambda F\Phi^3 + \lambda G\Phi^2 + F\Phi - G = 0 \qquad (2.90)$$

Again $\Phi = \sum_n C_n\sigma^n = C_1\sigma + C_2\sigma^2 + C_3\sigma^3 + \dots\dots\dots$

$$\dots . + \text{higher order terms} \qquad (2.91)$$

where n is integer and C_n is a constant.

$$\Phi = C_1 + 2C_2\sigma + 3C_3\sigma^2,$$

$$\Phi' = 2C_2 + 6C_3\sigma,$$

$$(\Phi)^2 = C^2{}_1 + 4C_1C_2\sigma + (4C^2{}_2 + 6C_1C_3)\sigma^2 + 2C_2C_3\sigma^3 + 9C^3{}_3\sigma^4,$$

$$(\Phi)^3 = C^3{}_1 + 6C^2{}_1 C_2\sigma + (9C^2{}_1C_3 + 6C_1C^2{}_2)\sigma^2 + (8C^3{}_2 + 36C_1C_2C_3)\sigma^3 + (27C_1C^2{}_3 + 36C^2{}_2C_3)\sigma^4 + 54C_2C^2{}_3\sigma^5 + 27C_3{}^3\sigma^4.$$

Putting these values in (2.91) and equating the coefficients of σ, σ^2, σ^3 etc., we get the values of constants C_1, C_2, C_3 as

$$C_1 = \pm\sqrt{\left\{\frac{a_0 - b_0}{a_0a_1 - a_2b_2}\right\}};\quad C_2 = \frac{a^2{}_2C^4{}_1 - a_2b_1C^2{}_1 - 2b^2{}_1}{2a_0b_1 + 3a_2b_0C^2{}_1 - 6a_0a_2C^2{}_1 + 4a_0b_1 - 2b_0b_1}$$

and

$$C_3 = \frac{7a^2{}_1C^3{}_1C_3 - 4a_2b_0C_1C^2{}_2 - a_1a_2C^3{}_1 + 12a_0a_2C_1C^2{}_2 - 2a_2b_1C_1C_2 + 2a_1b_1C_1C_2 - 4a_1b_1C_1C_2}{18a_0b_1 + a_2b_0C^2{}_1 - 9a_0a_2C^2{}_1 - 2b_0b_1}$$

Case (1):

For $v_0^2 > v_{te}^2$; $C_1^2 > 0$ and small phase shift (χ, θ is small)

$1 \gg C_1 \gg C_2 \gg C_3$ ....

So, $\Phi = C_1 \sigma + C_2 \sigma^2 + C_3 \sigma^3 + \ldots\ldots\ldots$.

$$\Phi - C_1\sigma. \tag{2.92}$$

Putting the value of Φ in (2.84) and after simplification we get:

$$\sigma'^2 = \alpha_1 \sigma^2 + \alpha_2 \sigma^3 + \alpha_3 \sigma_4, \tag{2.93}$$

where $\alpha_1 = a_0$; $\alpha_2 = \dfrac{2a_2 C_1}{3}$, $\alpha_3 = \dfrac{a_2C_2 - a_1}{2}$.

The solution of eqn. (2.93) is

$$\sigma = \sigma_{12} = \frac{\beta_{1,2}\, \beta_{2,1}\, \text{Sech}^2(\chi' \eta)}{\beta_{1,2} - \beta_{2,1} \tanh^2(\chi' \eta)}, \tag{2.94}$$

where $\chi' = \frac{1}{2} \sqrt{\{\alpha_1\}}$; $\beta_{1,2} = \dfrac{1}{2\alpha_3} [-\alpha_2 \mp (\alpha_2^2 - 4\alpha_1\alpha_3)^{\frac{1}{2}}]$.

Then $\Phi = \Phi_{1,2} = C_1 \sigma_{1,2} + C_2 \sigma^2_{1,2} + C_3 \sigma^3_{1,2} + \ldots\ldots\ldots$.

Case-I $\quad \sigma_1 = \dfrac{\beta_1\, \beta_2\, \text{Sech}^2\, (\chi' \eta)}{\beta_1 - \beta_2 \tanh^2(\chi' \eta)}$

i.e. $\Phi_1 = C_1 \sigma_1 + C_2 \sigma^2_1 + C_3 \sigma^3_1 + \ldots\ldots\ldots$.

Case-II $\quad \sigma_2 = \dfrac{\beta_2\, \beta_1\, \text{Sech}^2\, (\chi' \eta)}{\beta_2 - \beta_1 \tanh^2(\chi' \eta)}$.

2.4 RELATIVISTIC EFFECTS ON SOLITONS

We are considering relativistic electron ion plasma immersed in ambient magnetic field. Ions are immobile and neutralize the average charge density. Electrons undergo non-linear interaction with the wave and hence modulate the latter. We shall investigate the nonlinear evolution of circularly polarized waves propagating along the magnetic field. In this case the governing equations are

$$\partial_t n_j + \nabla \cdot (n_j \vec{v}_j) = 0 \; ; \tag{2.95}$$

$$(\partial_t + \vec{v}_j \cdot \nabla) \vec{P}_j = e_j \, [\vec{E} + (1/c)\vec{v}_j \times \vec{B}] - (1/n_j) \nabla p_j, \tag{2.96}$$

$\vec{E}$ and $\vec{B}$ related by the Maxwell's equations and the other terms are discussed earlier. In present case the basic assumptions are:

$$\vec{B} = B_0 \hat{z}, \; j = e(\text{electrons}), \quad \vec{E} = (\hat{i}_x E_x, \; \hat{i}_y E_y, \; 0),$$

$$\nabla = \hat{i}_z \, \partial_z \text{ and } n_j = n_0 + \delta_{ne} \; .$$

Electron motion is related to the electric field by the following equation:

$$\vec{P}_e = \frac{e(1 - k v_{ez}/\omega)}{i \, (\omega - k v_{ez} - \Omega_c)} \vec{E} \; . \tag{2.97}$$

Since phase velocity ($v_p = \omega/k$) >> v_{ez} i,e. electron velocity along field $\vec{B}$, we can neglect v_{ez}, thus

$$\vec{P}_0 = \frac{e\vec{E}}{(\omega - \Omega_c)} , \tag{2.98}$$

where $\Omega_c \rightarrow$ cyclotran frequency $\left(= \frac{\omega_c}{\gamma_e} \right)$,

Here $\gamma_e = \sqrt{\{1 + \lambda_e|\vec{E}|^2} \simeq 1 + (\lambda_e/2)|\vec{E}|^2,$ (2.99)

where $\lambda_e = e^2 / m_{0e}^2 \, c^2 \, \omega^2.$

From Maxwell's equation $(\nabla \times \vec{E}) = -(1/c)\partial_t\vec{B}$, one can obtain the following relation:

$$\partial^2_t \vec{E} - c^2 \partial^2_z \vec{E} = 4\pi e n_0 \partial_t \vec{v}_e . \tag{2.100}$$

From which we can write

$$(\partial^2_t - c^2\partial^2_z - \frac{\omega^2_{pe}}{i\omega} \partial_t)\vec{E} = - \frac{\lambda_e \, \omega^2_{pe}}{2i\omega} \partial_t \, (|\vec{E}|^2 \, \vec{E}). \tag{2.101}$$

Let us suppose

$$\vec{E} = \tilde{E} \, (z,t) = |\tilde{E}| e^{-i\omega t + ikz}. \tag{2.102}$$

The dispersion relation of the wave become

$$\omega^2 = k^2c^2 + \omega^2_{pe}(1- \frac{\lambda_e|\tilde{E}|^2}{2}). \tag{2.103}$$

From which the transverse di-electric constant is

$$\epsilon_{\perp} = \frac{k^2 c^2}{\omega^2} = 1 - \frac{\omega^2_{pe}}{\omega^2} (1 - \frac{\lambda_e |\tilde{E}|^2}{2}). \qquad (2.104)$$

Putting this value in equation (2.101) one can find the value of ponderomotive force

$$fp = - \frac{1}{16\pi} \frac{\omega^2_{pe}}{\omega^2} (1 - \frac{\lambda_e |\tilde{E}|^2}{2}) \partial_z |\tilde{E}|^2. \qquad (2.105)$$

From continuity equation and the equation of motion we can write

$$\partial^2_t \delta n_e + n_0 \partial_z \partial_t v_{ez} = 0, \qquad (2.106)$$

and

$$\partial_t v_{ez} = \frac{v_{ez}}{2\gamma_e} \partial_t |\tilde{E}|^2 + \frac{1}{m_{e0} \gamma_e} [-eE_z + \frac{fp}{n_0} - \frac{1}{n_0} \partial_z(n_0 + \delta n_e)T_e]. \qquad (2.107)$$

From (2.106) and (2.107) we get

$$\partial^2_t \delta n_e + \frac{n_0}{2} \partial_z \{ v_{ez} (1- \frac{\lambda_e}{2} |\tilde{E}|^2)\partial_t |\tilde{E}|^2 - \frac{en_0}{m_{e0}} \partial_t (1- \frac{\lambda_e}{2} |\tilde{E}|^2) E_z$$

$$+ \frac{1}{m_{e0}} \partial_z fp - \frac{1}{m_{e0}} \partial^2_z \delta n_e T_e = 0. \qquad (2.108)$$

2.4(a) FOR WEAK RELATIVISTIC CASE

We consider

$$1 \gg \frac{\lambda_e}{2} |\tilde{E}|^2; \; v_p = \frac{\omega}{k} \gg v_{ez}$$

$\phi = 0$, and $E_z = 0$.

With these assumptions and introducing new coordinates as

$\xi = z - v_0 t$ as earlier, we get

$$\frac{\delta n_e}{n_0} = \frac{\omega^2_{pe}}{16\pi\ m_{e0}\ n_0\ \omega^2(v^2_0 - v^2_{te})}(1 - \frac{\lambda_e}{4}\ |\tilde{E}|^2)|\tilde{E}|^2. \qquad (2.109)$$

Again from Maxwell's equation $[\nabla \times \vec{E} = -(1/c)\partial_t \vec{B}]$ and applying the relation $\vec{E} = \tilde{E}\ (z,t) = |\tilde{E}|e^{-i\omega t + ikz}$, one can get the following relation:

$$[c^2\partial^2_z - 2i(\omega\partial_t + kc^2\partial_z) + \omega^2_{pe}\frac{\delta n_e}{n_0}(1 - \frac{\lambda_e}{2}\ |\tilde{E}|^2)]\tilde{E} = 0. \qquad (2.110)$$

From (2.109) and (2.110) we get

$$[c^2\partial^2_z - 2i(\omega\partial_t + kc^2\partial_z) + \frac{\omega_{pe}^4}{16\pi m_{e0} n_0 \omega^2(v^2_0 - v^2_{te})}\{|\tilde{E}|^2 - (4/3)\lambda_e|\tilde{E}|^4 + \frac{\lambda_e}{8}\ |\tilde{E}|^6\}]\tilde{E} = 0. \qquad (2.111)$$

Again $\tilde{E} = |\tilde{E}|\ e^{-i\theta t + i\chi z}$, equation (2.111) is

$$[2i(\chi c^2\partial_z - \omega\partial_t - kc^2\partial_z) + c^2\partial_z^2 - \chi^2c^2 - 2\omega\theta + 2\ k\chi c^2 + \frac{\omega^4_{pe}}{16\pi m_{e0}\ n_0\omega^2(v^2_0 - v^2_{te})}\{|\tilde{E}|^2 - (3/4)\lambda_e|\tilde{E}|^4 + \frac{\lambda^2_e}{8}\ |\tilde{E}|^6\}]|\tilde{E}| = 0.$$

(2.112)

From imaginary part we get the velocity of the frame

$$v_0 = \frac{kc^2}{\omega}\ (1 - \frac{\chi}{k}). \qquad (2.113)$$

From the real part of equation (2.112), we get

$$\partial_{\eta} \sigma - a \sigma + b\sigma^{3} - d\sigma^{5} + g \sigma^{7} = 0, \tag{2.114}$$

where $\sigma = \dfrac{e|\tilde{E}|}{m_{e0} \omega c}$; $\eta = \dfrac{\omega_{pe}}{c} \xi$,

$$a = \frac{1}{\omega^{2}_{pe}} (\chi^{2}c^{2} + 2\omega\theta - k\chi c^{2}) = C_{1} \text{ (say)},$$

$$b = \frac{\omega^{2}_{pe} m_{e0} c^{2}}{16\pi e^{2} n_{0}(v_{0}^{2}-v_{te}^{2})} = - C_{2}\text{(say)},$$

$$d = \frac{3\lambda_{e} m^{3}_{e0} \omega^{2}_{pe} \omega^{2} c^{4}}{64\pi e^{4} n_{0}(v_{0}^{2}-v_{te}^{2})} = C_{3}\text{(say)},$$

and $$g = \frac{\lambda^{2}_{e} m^{5}_{e0} \omega^{2}_{pe} \omega^{4} c^{6}}{128\pi e^{6} n_{0}(v^{2}_{0}-v^{2}_{te})} = - C_{4}\text{(say)}.$$

From equation (2.114) we can write

$$\sigma'^{2} = C_{1} \sigma^{2} + C_{2} \sigma^{4} + C_{3} \sigma^{6} + C_{4} \sigma^{8}. \tag{2.115}$$

The solution of equation (2.115) is

$$\sigma^{2} = \frac{4C_{1}}{e^{-\mu\eta}-(4C_{1}C_{3}-C^{2}_{2})e^{\mu\eta} - 2C_{2}}. \tag{2.116}$$

2.4(b) FOR ULTRA RELATIVESTIC CASE

Assumptions are as before, in this case $\lambda_e|\tilde{E}|^2 >> 1$.

So, $\gamma = \sqrt{\{1+\lambda_e|\tilde{E}|^2\}} = \sqrt{\lambda_e}|\tilde{E}| = \dfrac{|e||\tilde{E}|}{m_{e0}\ \omega c}$

Electrons motion is

$$\tilde{p}_e = \frac{e\ \tilde{E}}{i(\omega + \Omega_c)}.$$

From Maxwell's equation $[\nabla \times \vec{E} = -(1/c)\ \partial_t \vec{B}]$, one can get

$$\partial^2_t\ \tilde{E} - c^2\partial^2_z\ \tilde{E} = -\frac{i\omega^2_{pe}\ m_{e0}\ c}{|e|}\ \partial_t\ \frac{\tilde{E}}{|\tilde{E}|}; \qquad (2.117)$$

Now applying boundary conditions that at ∞ , $|\tilde{E}| = |\tilde{E}_0|$, we get the dispersion relation

$$\omega^2 = k^2c^2 + \frac{\omega^2_{pe}\ m_{e0}\ c}{|e||\tilde{E}_0|}.$$

$$\text{So, } \epsilon_1 = \frac{k^2c^2}{\omega^2} = 1 - \frac{\omega^2_{pe}\ m_{e0}\ \omega c}{|e||\tilde{E}_0|}.$$

The magnitude of the ponderomotive force is

$$fp = -\frac{1}{16\pi}\frac{\omega^2_{pe}\, m_{e0}\, c}{\omega|e||\tilde{E}_0|}[\partial_z\, |\tilde{E}|^2)+(k/\omega)\partial_t|\tilde{E}|^2]. \qquad (2.118)$$

From the equation of continuity and that of motion one can obtain the following relation

$$\frac{\delta n_e}{n_0} = \mu\,\frac{|\tilde{E}|^2}{|\tilde{E}_0|^2}, \qquad (2.119)$$

$$\text{where } \mu = \frac{\omega^2_{pe}\, m_{e0}\, c^2\, (1-kv_0/\omega)}{16\pi\, e^2\, n_0\, (v^2_0 - v^2_{te}\,\frac{m_{e0}\,\omega c}{|e||\tilde{E}_0|})}, \qquad (2.120)$$

Again from Maxwell's equation, the form of the wave equation is

$$(\partial^2_t - c^2\partial^2_z)\tilde{E} = -\frac{i\omega^2_{pe}\, m_{e0}c}{|e|}\,\partial_t\,\frac{\tilde{E}}{|\tilde{E}_0|}, \qquad (2.121)$$

where $\tilde{E} = |\tilde{E}|\; e^{-i\theta t+i\chi t}$.

Finally one can get the following relation

$$[c^2\partial^2_z + 2\omega\theta - c^2\chi^2 - 2k\chi c^2 + \frac{\omega^2_{pe}\, m_{e0}\, \omega c}{|e||\tilde{E}_0|}]|\tilde{E}|$$

$$+\, 2i(\chi c^2 + \omega\partial_t + kc^2)\; |\tilde{E}| \; +$$

$$\frac{\omega^2_{pe}\ m_{e0}\ \omega c}{|e||E_0|}\ [\ \frac{\omega^2_{pe}\ m_{e0}c^2(1-kv_0/\omega)}{16\pi\ e^2 n_0(v^2_0\ -\ v^2_{te})\dfrac{m_{e0}\ \omega c}{|e||\tilde{E}_0|}}].\ \frac{|E|^3}{|\tilde{E}_0|^2} = 0. \quad (2.122)$$

From imaginary part of equation (2.122) we get

$$v_0 = \frac{kc^2}{\omega}(1 + \frac{\chi}{k})$$

and from the real part of equation (2.122), introducing

$\sigma\ (\ = \dfrac{|e||\tilde{E}_0|}{m_{e0}\ \omega c})$ and $\eta (= \dfrac{\omega_{pe}}{c}\ \xi\ )$ as before,

we get the following relation:

$$\partial^2_\eta\ \sigma\ -\ a\ \sigma\ +\ (\frac{2\sigma}{2\sigma^2 + 1})\ \sigma\ b_1\ \sigma^4\ =\ 0, \qquad (2.123)$$

where $\qquad a = \dfrac{c^2}{\omega^2_{pe}}\ (\chi^2\ +\ 2k\chi\ -\ \dfrac{2\omega\theta}{c^2})$ and

$$b_1 = \frac{c^2(1\ -\ kv_0/\omega)}{4(v^2_0\ -\ v^2_{te}/\sigma)\sigma^2_0}.$$

Let us consider $v^2_0 >> v^2_{te}/\sigma$ i.e. b_1 is positive,

Equation (2.123) may be written as

$$\tfrac{1}{2}\ \sigma'^2\ -(a/2)\sigma^2\ +\ \sigma\ +\ 1/(2\sigma)\ +(b_1/5)\ \sigma^5\ =\ C_1, \qquad (2.124)$$

where C_1 is constant. Applying boundary condition

that at $\eta \to \infty$, $\sigma \to \sigma_0$

So, $-(a/2)\sigma_0^2 + \sigma_0 + 1/(2\sigma_0) + (b_1/5)\sigma_0^5 = C_1$. (2.125)

From eq. (2.124)

$\sigma^2 [\frac{1}{2} \sigma'^2 = C_1 + (a/2)\sigma^2 - \sigma^2 - \frac{1}{2} \sigma^2 - (b/5) \sigma^3]$

or, $(\partial_\eta \sigma^2)^2 + V(\sigma) = 0$, (2.126)

$$\text{where } V(\sigma) = \frac{4b_1}{5} \sigma^7 - 2a\sigma^4 + 4\sigma^3 - 4C_1 \sigma^2 + 2\sigma. \quad (2.127)$$

Putting c_1 from (2.125), equation (2.127) is

$$V(\sigma) = \frac{(1-kv_0/\omega)c^2 \sigma^2(\sigma^5 - \sigma_0^5)}{5v_0^2 \sigma_0^2} - (2a-4)\sigma^2(\sigma^2 - \sigma_0^2)$$

$$- \frac{2\sigma}{\sigma_0} (\sigma - \sigma_0). \quad (2.128)$$

Let us suppose $\sigma - \sigma_0 = X$, i.e. $\sigma = \sigma_0 + X$

So, eqn. (2.128) will be

$V(X) = A_1X + A_2X^2 + A_3X^3 + A_4X^4 + A_5X^5 + A_6X^6 + A_7X^7$, (2.129)

where

$$A_1 = 5b\sigma_0^4 - 4a\sigma_0^3 + 4\sigma_0^2 - 2$$

$$A_2 = 15b\sigma_0^3 - 10a\sigma_0^2 + 8\sigma_0 - 2/\sigma_0$$

$$A_3 = 35b\sigma_0^2 - 8a\sigma_0 + 4$$

$$A_4 = 35b\sigma_0 - 2a$$

$$A_5 = 21b$$

$$A_6 = \frac{7b}{\sigma_0}$$

$$A_7 = \frac{2b}{\sigma_0^2}$$

Here $b = \dfrac{c^2(1-kv_0/\omega)}{5v_0^2}$ and b>o

From (2.126) $(\partial_\eta \sigma^2)^2 = |V(\sigma)|$

i.e $\eta = \pm \int \dfrac{2\sigma\partial\sigma}{\surd\{|V(\sigma)|}$, (2.130)

neglecting the higher order terms, we can write

$v(\sigma) = A_0X^0 + A_1 X + A_2 X^2$

here $A_0 = 0$. The solution of equation (2.130) is

$$X = \frac{e^{\mu\eta} - A_1^2\ e^{-\mu\eta} - A_1}{2A_2} \qquad (2.131)$$

In this case $\mu = \surd(A_2/2\sigma_0)$.

CHAPTER-3

VORTICES IN PLASMAS

ELECTROSTATIC VORTEX MODE: CLASSICAL CASE:

Vortex motion is a non-linear phenomenon in plasma. It is related to the vector nonlinearity and this nonlinearity arises due to plasma drift motion in the direction perpendicular to the wave propagation. For simplicity we consider the electrostatic drift motion.

Let us consider a cold electron-ion plasma in presence of an external magnetic field $\vec{B} = B_0\hat{z}$. The dynamics of low frequency electrostatic motion in such a plasma is governed by the fluid equation

$$(\partial_t + \nabla \cdot \vec{v}_j)n_j = 0; \tag{3.1}$$

$$\vec{v}_{j\perp} = (c/B_0)[z \times \nabla_\perp \phi - 1/(\omega_{cj});\{\partial_t + (c/B_0)(z \times \nabla_\perp \phi)\} \nabla_\perp \phi]; \tag{3.2}$$

$$\partial_t v_{jz} + \vec{v}_j \cdot \nabla v_{jz} = -(e_j/m_j)\, \partial_z \phi; \tag{3.3}$$

$$\nabla^2 \phi = 4\pi e(n_e - n_i); \tag{3.4}$$

where ϕ is the electrostatic potentials, n_j and v_j are the number density and the velocity of particle species j(=e and i for the electron and the ions respectively), i.e, $e_j = \mp e$.

From the continuity equation, we get

$$\partial_t (n_e - n_i) + \nabla_\perp \cdot (n_e \vec{v}_{e\perp} - n_i \vec{v}_{i\perp}) + \nabla_z \cdot (n_e \vec{v}_{ez} - n_i \vec{v}_{iz}) = 0 \tag{3.5}$$

From equation (3.3) we have

$$\partial_t u + (c/B_0)(\hat{Z} X \nabla_\perp \phi \cdot \nabla_\perp) u = e^2 \left(\frac{m_i + m_e}{m_i m_e}\right) \partial_z \phi, \tag{3.6}$$

$$\text{where } u = e(v_{ez} - v_{iz}) \tag{3.7}$$

Now, we consider a moving frame of cylindrical coordinate

$$X = r - r_0$$

$$\xi = r\theta + \alpha z - vt,$$

where α and v are constants; i.e. the frame is moving with a speed v in a spiral manner with a radius r_0 about the external field direction. One can easily show that equation (3.6) is satisfied by

$$u = -\frac{e^2 \alpha^2}{v}(1/m_e + 1/m_i)\, \phi. \tag{3.8}$$

Now the equation (3.5) can be re-written as

$$\partial_t \nabla^2_\perp \phi + \mu \nabla_\perp \cdot n_0 \{\partial_t + (c/B_0)(z x \nabla_\perp \phi) \cdot \nabla_\perp\} \nabla_\perp \phi + 4\pi n_0 \partial_z u = 0, \tag{3.9}$$

where $\mu = \dfrac{4\pi c^2}{B_0^2}(m_i + m_e)$

In the given cylindrical coordinate system the equation (3.9) is re-written as

$$\partial_\xi (\nabla^2_\perp + D)\phi = \frac{c \mu n_0}{B_0 v (1+\mu n_0)} \nabla_\perp \cdot \{(z \times \nabla_\perp \phi) \cdot \nabla_\perp\} \nabla_\perp \phi, \tag{3.10}$$

where $D = \dfrac{4\pi n_0 \alpha e^2}{v^2 (1+\mu n_0)} \left(\dfrac{1}{m_i} + \dfrac{1}{m_e}\right)$

It can be shown that

$$\nabla_\perp^2 \phi + D\phi = C_2 X \tag{3.11}$$

is the solution of equation (3.10),

where C_2 is related with D by the following relation:

$$D = \frac{cC_2 \mu n_0}{B_0 v(1+\mu n_0)}. \tag{3.12}$$

Now, we are interested to find a localized vortex solution, that means at $r \to \infty$; $\phi = 0$, this condition implies $C_2 = 0$. Therefore the outer solution

$$\nabla_\perp \phi + D^0 \phi = 0 , \tag{3.13}$$

which gives the following solution:

$$\phi^0(r,\theta) = \phi^0 J_\perp (\sqrt{D^0}\, r) \cos \theta , \tag{3.14}$$

where $J_\perp$ is first order Bessel function.

For $r < r_0$; we have the internal solution of the equation

$$\nabla_\perp^2 \phi + D^i \phi = C^i{}_2 X . \tag{3.15}$$

In this case, the solution is

$$\Phi^i(r,\theta) = [\Phi^i J_1 (r\sqrt{D^i}) + \bar{C}_2 r \sqrt{D^i}] \cos \theta \tag{3.16}$$

where $\bar{C}_2 = \dfrac{C^i{}_2}{\sqrt{D^3{}_i}}$.

These solutions (3.14 and 3.16) are obtained by using the standard method developed by Lerichev and Reznik 1976. The constants of the solutions i.e. α, v and r_0 can be obtained from the continuity of ϕ, $\nabla\phi$ and $\nabla^2 \phi$ of the outer and the inner solution at $r = r_0$.

CHAPTER 4

4.1 PLASMA SPECTROSCOPY:

The aim of the experiment:

(a) To study the glow discharge plasma radiation by a spectrometer: without and with applying moderately strong external magnetic field.

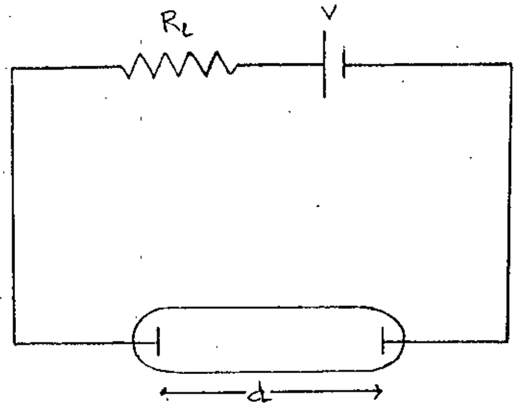
(b) Design, development and calibration of an electro-magnet to study magnetic field effects on plasma column.

(c) Design, development of a magnetic mirror for plasma confinement and determination of its mirror ratio.

GENERAL INTRODUCTION:

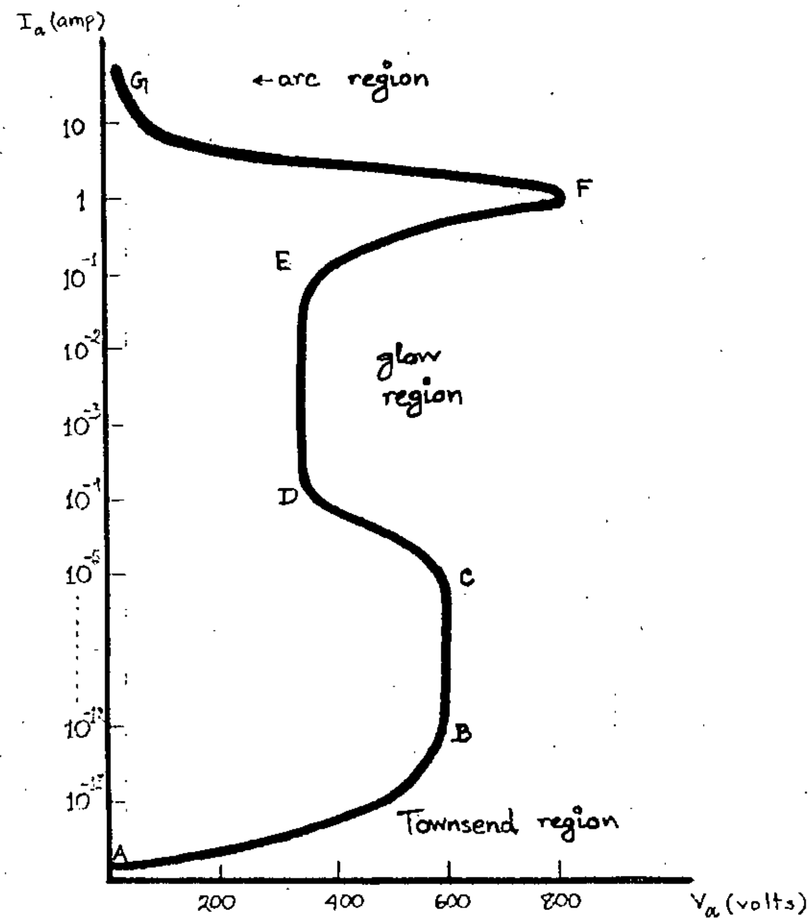
Electrical discharge in gases

The essential parts of a gas discharge experiment is shown in fig. I. A pair of electrodes, at a seperation d, are enclosed in a vacuum vessel which has been evacuted and filled with the working gas at a fixed pressure P. The electrode seperation d, the pressure P, the applied voltage V and the working gas have significant effects on the appearance and on the properties of the plasma produced. A typical I-V characteristics of the electrical gas discharge is shown in



R_L
V
d

Fig. I



I_a (amp)
G
← arc region
F
E
glow region
D
C
B
A
Townsend region
10
1
200
400
600
800
V_a (volts)

Fig. II

fig.II. As V_d (the potential diff. between the two electrode) increases I_d (current through the discharge tube) does not increase; it is of the order of 10^{-12} Amp. Practically the tube is an insulator of high impedance. However, the situation changes when break down voltage V_B is reached. The conductivity of the discharge tube switches suddenly from that of an insulator to become a conductor.

It can be seen that after break down, the current increases from 10^{-12} Amp. to 10^{-6} Amp. while the tube voltage stays approximately constant at V_B (BC). Beyond C, as the current is further increased, the voltage across the discharge tube begins to drop and finally, a point is reached (D), where the voltage remains constant again on further increase of the current. This is the glow discharge region (DE), which is the region of interest in our experiment. Further increase of current leads to abnormal glow region (EF) and eventually to the arc region (beyond G).

Some characteristics of the glow discharge:

The glow of a gas discharge is usually broken-up into several regions of which the more important ones are shown in fig. III(a). The relative length of these regions depends on the tube pressure (P), the applied voltage (V_a) as well as the seperation between the electrodes (d). Usually it is the

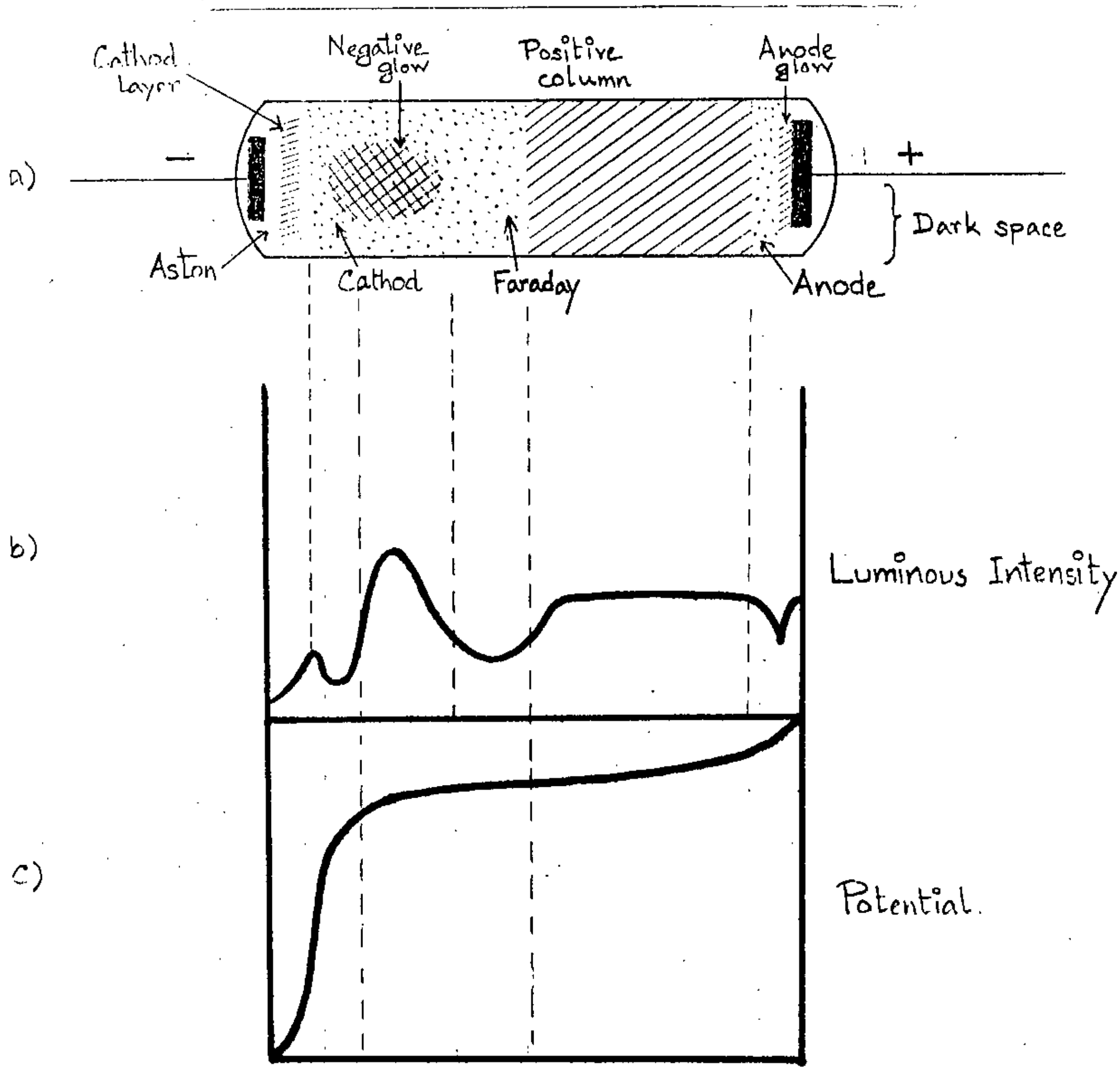

a)
b)
c)
Cathod layer
Negative glow
Positive column
Anode glow
−
+
Dark space
Aston
Cathod
Faraday
Anode
Luminous Intensity
Potential.

Fig. III

positive column that will vary in length when these parameters are varied. The regions that we are interested in are the two large bright regions, namely the negative glow and the positive column. These are the regions where the plasmas of the glow discharge are situated. The colours of these bright regions are characteristics of the gas used to fill the discharge tube. The characteristic colours for the gases that are used in our glow discharge experiment are shown in table below:

Gas	Colour of the negative glow	Colour of the positive column
Air(Nitrogen)	Blue	Red
Hydrogen (H)	Pale-Blue	Pink.

Another important characteristic of the glow discharge is the variation of potential accross the tube from cathode to anode. In most glow discharge experiment, the cathode is fixed at ground potentials. The potential across the tube is found to be as shown in fig. III(c).

In positive column, the potential is found to fall gradually from the anode and becomes constant in the negative glow region. This constant potential is called the plasma potential V_d.

Physical appearance of the glow discharge. If a glow discharge is maintained between two electrodes in a long glass tube, several bright and dark regions will be observed as shown in fig. III(a). It is seen that the negative glow region is the brightest region, the positive column constitutes a uniform large bright region and the other regions are usually difficult to observe experimentally. The distribution of luminous intensity are shown in fig. III(b).

The positive column is the ideal source of steady plasma radiation and has bright and uniform luminous intensity. We have studied the spectrum of plasma radiations from the positive column.

4.2 EXPERIMENTAL STUDIES OF PLASMA RADIATION

INTRODUCTION:

Plasma Radiation:

In plasma the radiation is produced largely as a result of various types of collisions between the plasma particles. These are: (a) Bremsstrahlung, (b) Recombination radiation, (c) Radiation emitted by excited atoms and ions and (d) Cyclotron emission from plasma.

(a) Breamsstrahlung:

It is the energy lost in the form of electromagnetic radiation by a charged particle, moving with a very high velocity, is accelerated or decelerated. When a free electron moving with a very high velocity (v = c) passes through the electric field of an ion or an atom, its path will be bent a little, which is due to deceleration of the electron under the influence of the electric field. This gives rise to the emission of radiation and represents energy loss for the electron. The emitted radiation, which has continuous frequencies between zero and W_e/h, where W_e is the initial K.E. of the electron and h is Plank's constant.

b) Recombination radiation:

The phenomenon of recombination involves the capture of free electrons by ions. The energy which is liberated in this processes is equal to the sum of kinetic energy of the free electron and its binding energy.

(c) Radiation emitted by excited atoms and ions:

The origin of the atomic radiation is well known. The spectrum of this radiation consists of discrete lines corresponding to various transitions from the excited energy states of atoms and ions, which depends on the electron temperature (t_e). At low t_e it will remain a neutral atom or will lose one of its most weakly bound electron.

(d) Cyclotron emission from plasma:

A new source of radiation comes into play when plasma is placed in a magnetic field. The electron now execute circular motion in the plane perpendicular to the magnetic field, and since the motion is accelerated (towards the guiding center) it should, in accordance with the laws of electro-dynamics, give rise to the emission of radiation. For single electron the intensity of this radiation is proportional to the square of the magnetic field and is also a function of the velocity of electron executing the larmor rotation. Radiation of this kind is usually known as cyclotron radiation having frequency $\omega_c = eB/mc$ and is of major importance in accelerator technology.

Measurement of radiation wave length:

Radiation emitted from glow discharge experiment was measured by the usual manner with a spectrometer and a plane diffraction grating. A spectrometer was leveled optically (Schuster's method) with a prism and a monochromatic source. The sources was then replaced by the positive column of a glow discharge and the prism was replaced by a plane diffraction grating (15,000 lines per inch). The wave lengths of the observed spectral lines were measured by using the relation

$$\lambda = \frac{\sin\theta}{nN}$$

where λ is the wave length of the spectral line, n is the order of the spectrum, N is the grating constant and θ is the diffraction angle.

Effects of magnetic field on plasma radiation:

The origin of plasma radiation was discussed earlier. In our experimental studies, we measured the different wave lengths of the radiation that were emitted by the excited ions and atoms. If external magnetic field acting on electrons and ions in the plasma produce the so called cyclotron rotation. Due to this rotations the charged particles emit cyclotron radiation. On the other hand, magnetic field effects also the quantum mechanical phenomenon of excitation of atoms. These two effects change the nature of radiation of the excited atoms in the presence of the magnetic field as compared to the case where the magnetic field is absent. Thus this radiation differs from that of unmagnetized atoms. In our experiment we measured the radiation from the magnetized plasma as tabulated in table 2 and table 3.

Design, Development and Calibration of a Horse-shoe Magnet:

To apply an external magnetic field perpendicular to the positive column, we designed and developed a special electromagnet (horse - shoe type)

A bar of soft-iron (25cm x 5 cm x 5 cm) was wounded with 18 gauge enamelled copper wire of 2700 turns. Two bars of soft-iron (12cm x 5cm x 5cm) were connected at the two ends of the winded bar by the screws of the same material for conducting the magnetic flux lines. Finally, another two bars (8cm x 5 cm x 5cm) of the same material were connected to the free ends of the bars (12cm x 5 cm x 5 cm) as shown in the fig.11. There is a field gap of 9 cm. in between the two free ends of the bars (8cm x 5 cm x 5 cm) which form the magnetic poles. By varying the magnitude of the direct current through the coil the magnetic field can be changed from 0 to 2 kiloGauss. The I-B characteristics are shown in Fig. 8. The magnet was fabricated in the BUET Central Workshop. The calibration chart of the electromagnet is shown in the Table-1.

4.3 EXPERIMENTAL SET-UP:

The experimental set-up for glow-discharge experiment consists of the following components:

1. The Vacuum Vessel

The vacuum vessel used in this experiment is a cylindrical glass bulb (T-glass chamber) with three parts attached to it, which are shown in fig. 12. Length of the tube is L=19.1 cm and its diameter 8.2 cm. The tube consists of three parts:

i) One for mounting the anode

ii) One for mounting the cathode

iii) One for connection to the vacuum pump and gauge.

2. The Vacuum System:

The cylindrical bulb vessel is pumped by rotary pump which is capable of pumping to a pressure of 30 millibar. However, the minimum pressure that can be achieved in the system is mainly limited by the vacuum sealing capability of the system itself. The pump is operated at the voltage of $V_{r.p}$ = 110 volts and current of $I_{r.p}$ = 5.6 amp.

3. The electrodes:

Two circular brass electrodes with circular perspex holders are used in this system. The electrodes are fixed at a distance of 8 cm apart. The diameter and the thickness of the electrodes are 3 cm and 0.15 cm respectively.

4. The D.C. Power Supply:

For glow discharge experiment one d.c. power supply is needed. The power supply used to operate the discharge tube can supply a maximum voltage of 1200 volts and maximum current of 1 ampere and it is sweping voltage in the order of 400 V, 600 V, 800 V, 1000 V and 1200 V respectively.

5. The current-limiting resistance R_8:

This is a variable resistance comprising of a chain of 22 KΩ resistors connected in series. By setting R_8 to large value, it is possible to limit the current to the range of 10^{-6} amp. after break down has occurred.

6. Meters:

To study the I-V characteristic of the discharge, an ammeter is used covering the range of 10^{-6} amp to 0.999 amp. along with a voltmeter to measure the voltage. Both are connected after the break down has occurred. Under such condition, the reading on the ammeter (I_a) should remain unchanged when the voltmeter is connected (the earth wire only).

One of the most important sources of information about the properties of the plasma is the spectral composition of the radiation emitted by plasma. Such study may be carried out with various spectrographs, that is normal prism instruments with glass or quartz optics, which are suitable for the analysis of the visible and near ultraviolet radiation or vacuum spectrographs with diffraction gratings, which are suitable for short wave. Radiation enters the slits of the spectrograph through a window in the vessel containing the plasma. The intensity of the spectral lines emitted by plasma depends on the concentration of electrons and ions and on the electron temperature. With increasing electron temperature, the spectrum changes. New lines, corresponding to a higher degree of excitation of the atoms, appear. Spectroscopic determination of plasma concentration is very difficult because it involve the accurate measurement of the absolute intensities of spectral lines. The temperature of the atoms is related to the width of the spectral lines emitted by them. In plasma, the atoms and ions move at random mean velocities proportional to $\sqrt{T}$. Therefore different particles emit slightly different wavelengths, with the result that these spectral lines may be used to determine the temperature of the particles which are responsible for the emission of the radiation.

Measurement of wavelengths of magnetized plasma:

The positive column of a glow discharge experiment was magnetized with the horse shoe magnet. In our experiment we studied two types of plasma: nitrogen and hydrogen plasma. The wave lengths of the different spectral lines were determined by spectrometer: first without any magnetic field and then in the presence of a magnetic field. The results are listed in Table 2 and Table 3.

4.4 DESIGN AND DEVELOPMENT OF A MAGNETIC MIRROR FOR PLASMA CONFINEMENT:

A convenient method of confining plasma is the use of a magnetic field. A uniform magnetic field restricts the perpendicular motion of the charged particles. There are two confinement configurations for plasma viz.:

(i) Closed line - Stellarator, Toroidal Theta pinch and Tokamak.

(ii) Open line - Magnetic mirror, Z-pinch, Plasma focus, Screw pinch and Theta pinch.

Confinement of plasma particles (high density) is a difficult problem. However, for low density, plasma confinement is possible in small scale by mirror confinement. The mirror confinement of plasma is based on the fact that the magnetic moment and the total energy of the charged particle in a magnetic field is constant along the orbit, provided the field does not change much in one gyroperiod.

The magnetic mirrors are linear devices with axial magnetic fields to keep the particles away from the walls. The magnetic mirrors at the ends reduce the number of particle escaping at each end. We designed and developed two coaxial magnetic mirrors shown in figure below: which was fabricated in plasma lab., INST, AERE, Saver, Dhaka.

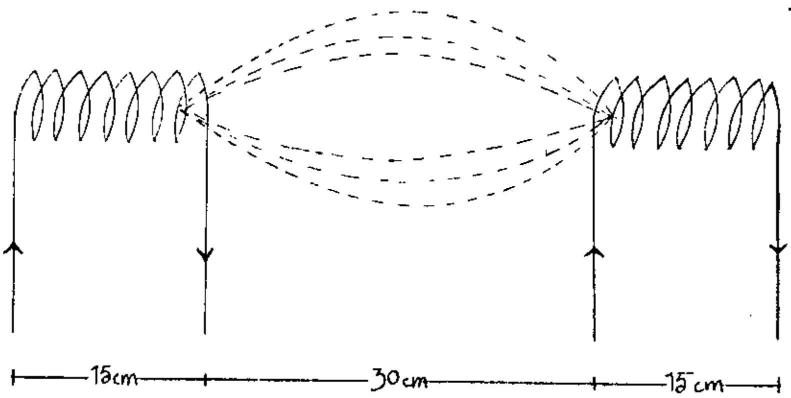


Fig. IV:

The charged particles may be reflected by the magnetic mirrors and travel back and forth in the space between them and thus get trapped. This trapping region has been named as magnetic bottle.

The effectiveness of the co-axial magnetic mirror system in trapping charged particles can be measured by the mirror ratio B_m / B_0, where B_m is the intensity of the magnetic field at the point of reflection and B_0 is the same at the center of the magnetic bottle.

Two coils (solenoids) are constructed for this purpose, which are 15cm in lengths and 10cm. in internal diameters. The coils are placed 30 cm. apart. Experimental results are listed in Table 4.

Ion-cyclotron resonant/Electron cyclotron resonant in plugging of plasma:

The end loss of a cusp plasma is reasonably suppressed by plugging radio frequency electric field. The frequency of the applied field is chosen to be an eigen frequency of the ion-cyclotron/electron-cyclotron wave. The field is applied perpendicular to the static magnetic field. Such radiofrequency plugging is due to the quasipotential ψ, which is written as

$$\psi = \frac{q_s^2}{4m_s} \frac{E_{rf}^2}{\omega^2 - \Omega^2}$$

for the case of a single particle, where q and m are the charge and mass respectively of the spices "s", and E_{rf} is the radiofrequency electric field strength, ω is the angular frequency of the applied radiofrequency field and Ω_s is the local cyclotron frequency of particle.

The relative ion/electron loss flux (α_s) is defined as the ratio of the ion/electron radiofrequency plugging relative to that without radiofrequency plugging, can be written as

$$\alpha_s = \exp(- \psi/T_s)$$

where T_s(eV) is the ion/electron temperature. ($1-\alpha_s$) is called the radiofrequency plugging effectiveness.

The experimental setup for rf. plugging to determine the plugging co-efficient α_s is developed for the first time in our laboratory. Further experiments with the present setup will be possible in near future.

RESULTS AND DISCUSSION:

Analytically, we studied the wave-particle interaction in unmagnetized and magnetized hot plasmas. Circularly polarized intense electromagnetic wave propagation along field direction is considered in both homogeneous and inhomogeneous plasmas. We also considered the case of electrostatic drift. In our experimental studies, we investigated the effects of magnetic field on plasma radiation. For this purpose we designed, developed and callibrated a horse-shoe magnet. We also constructed and developed a magnetic mirror machine and measured its mirror ratio experimentally.

In section 2.1, we consider electron-ion homogeneous plasma. Circularly polarized intense electromagnetic wave propagates along ambient field direction. Under some boundary conditions envelope soliton of the wave is formed: figure 1 (equation 2.28). If the thermal velocity of electrons (v_{te}) is higher than the frame velocity (V_0), we get isolated spiky solitons figure-2 (Equation 2.30).

In section 2.2, we consider electron-ion inhomogeneous plasma, where density of plasma increased with space (length). Soliton is also formed but there is a little hump in its tail figure-3 (Equation 2.58). This hump is due to the inhomogenity.

In section 2.3; ion motions are taken into account, whereas in the other sections we neglect ion motion due to its heavy mass as compared to that of an electron. Two types of solitons are formed for the two forms of equation (2.94) as shown in figure-4 (Case-1) and figure-5 (Case-II).

In section 2.4(a); for weak relativistic case soliton is formed as shown in figure 6 (equation 2.116).

In section 2.4(b) for ultra relativestic case the wave takes a different form as shown in figure-7(Equation 2.13).

In section 3: We study a non linear phenomenon of plasma called vortex motion. Here we consider the case of electrostatic drift. Using two fluid MHD equations for electron-ion plasma and also using some standard method we find new kind of nonlinearity that exist in electron-ion plasma which is a vortex. It is caused by the perpendicular drift motion of plasma particles. These vortices are very much important to study transport phenomenon in plasmas.

In chapter-4; We investigated experimentally the effect of magnetic field on plasma radiation. We obtained the spectral lines of plasma radiation from two types plasmas: nitrogen and hydrogen plasma respectively. The wavelengths of plasma radiation is measured by spectrometry. The effects of magnetic field on these radiation is investigated experimentally. It is found that due to magnetic field the radiation is red shifted. We have designed, constructed and callibrated a horse-shoe electro-magnet, which can give a field of the order of two kilo-gauss. With this magnet, we can apply suitable field on a plasma column. Secondly, we designed and constructed a magnetic mirror mahine (magnetic bottle) to study the r.f. confinement of plasma. We have determined the mirror ratio experimentally. The results obtained are shown in Fig. 9 and Table-4. This opened up a new possibility for further experiments to be done in the future and will have wider application in fusion energy research and in plasma astrophysics.

FIGURE CAPTION

Fig-1: Pumpwave envolope soliton for sup thermal case ($V^2_{te} < V^2_0$) in electron-ion plasma with the parameters g=1 and μ = 10 (Eqn 2.28).

Fig-2: Isolated spiky soliton for sub thermal case ($V^2_{te} > V^2_0$) in electron-ion plasma with the parameter g=1 and μ = 10 (Eqn 2.30).

Fig-3: Pump wave envelope soliton in electron-ion inhomogeneous plasma with the parameters σ_0 = 1, μ=10, b_0 = 0.01 and b_1 = 0.001 (Equation 2.58).

Fig-4: Pumpwave envolve soliton for electro-static change seperation (eqn 2.94 case-1) with the parameters β_1=0.9, β_2=0.8 and x'=10.

Fig-5: Cusp solitons for electro-static charge seperation (Eqn 2.94 - Case-II) with the parameters β_1=0.9, β_2=0.8 and x'=10.

Fig-6: Pump wave envolope spiky soliton in a weak-relativestic plasma with the parameters C_1=1, C_2 = -4.85; C_3=10 and μ=10 (Equation 2.116).

Fig-7: Shape of the pump wave amplitude in ultra relativestic plasma with the parameters A_1=10, A_2=1 and μ=10 (Equation 2.131).

Fig-8: Callibration curve of the horse shoe magnet (Table-1): magnetic field strength (B) vs. current (I).

Fig-9: Graphs of mirror field (Table-4) distance between the coils d=30 cm field through the axis (a) for curren I_1=0.5 amp and (b) for current I_2=0.9 amp.

Fig-10: Schematic diagram of experimental setup.

Fig-11: Photograph of the electro-magnet.

Fig-12: Photograph of the setup for radiation measurement.

Fig-13: Photograph of mirror machine.

Figure 1

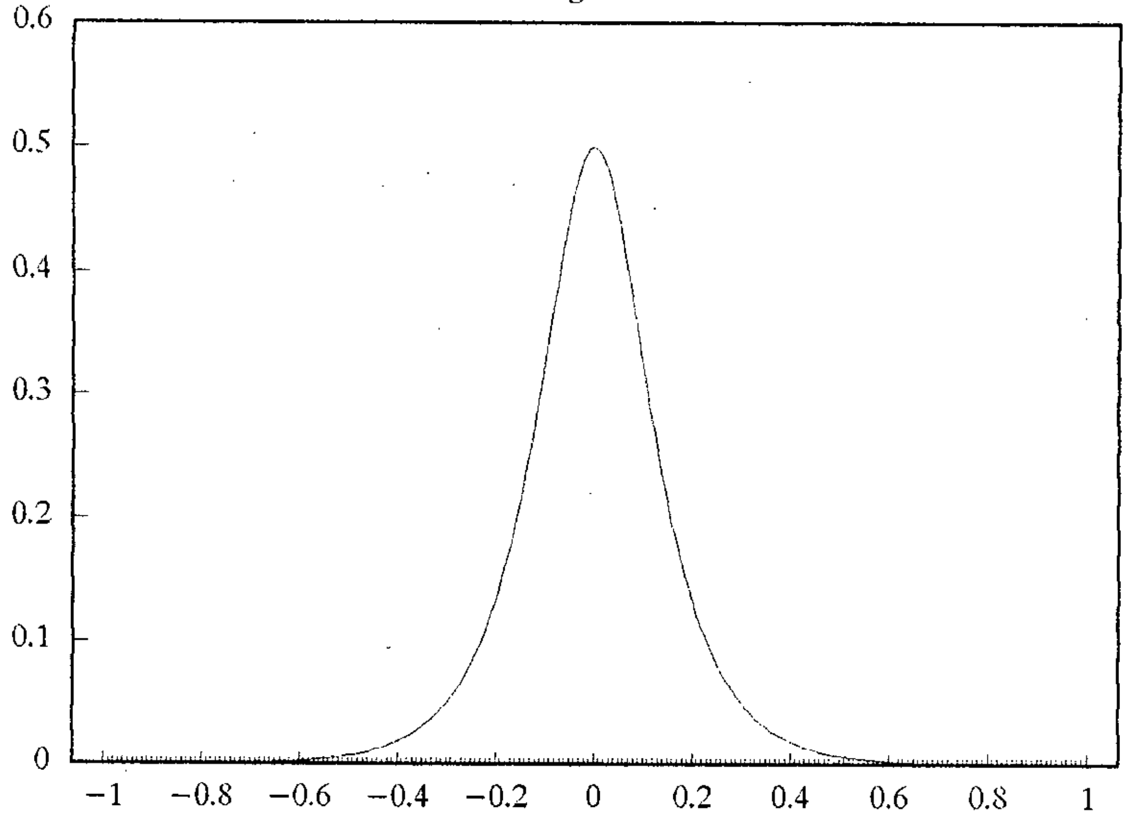


Figure 2

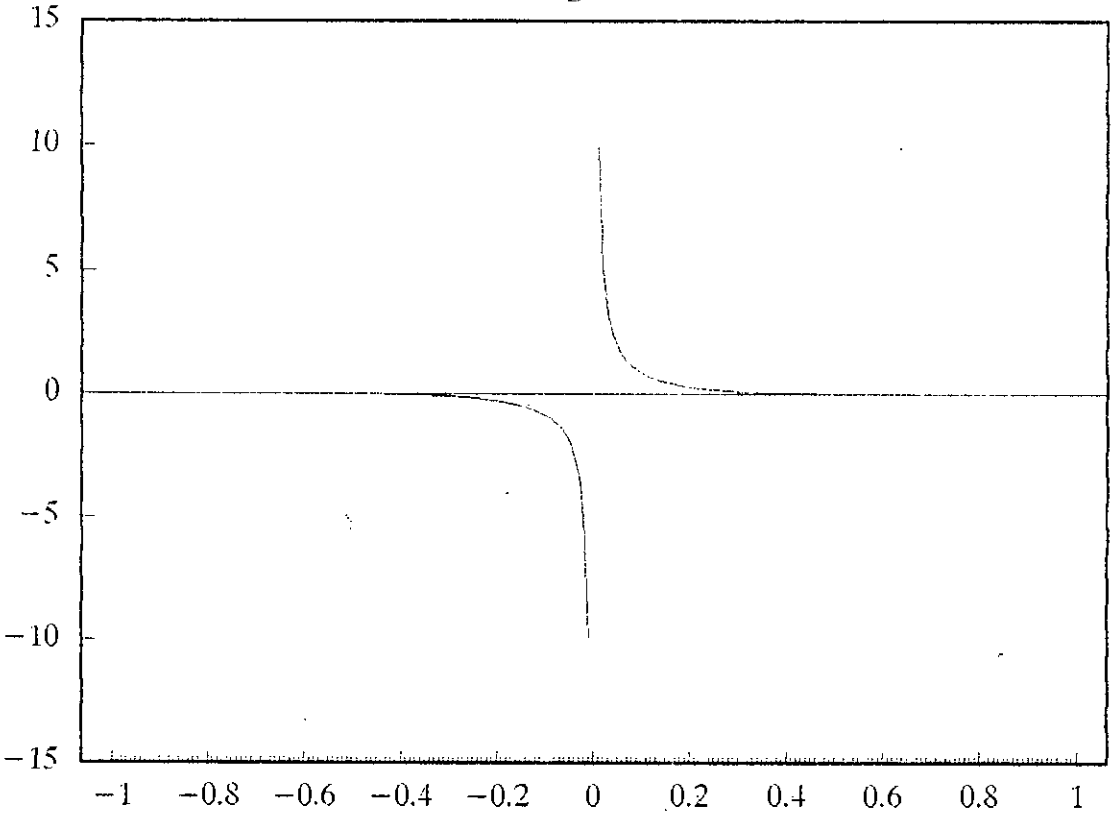


Figure 3

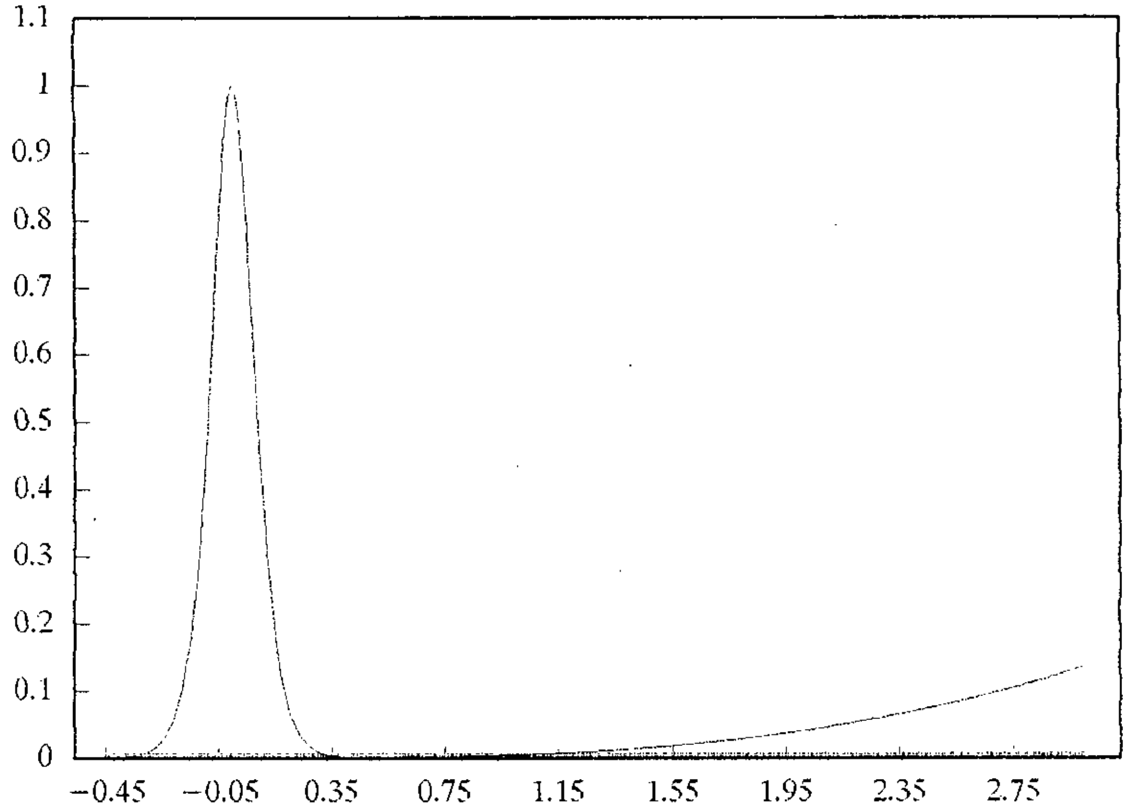


Figure 4

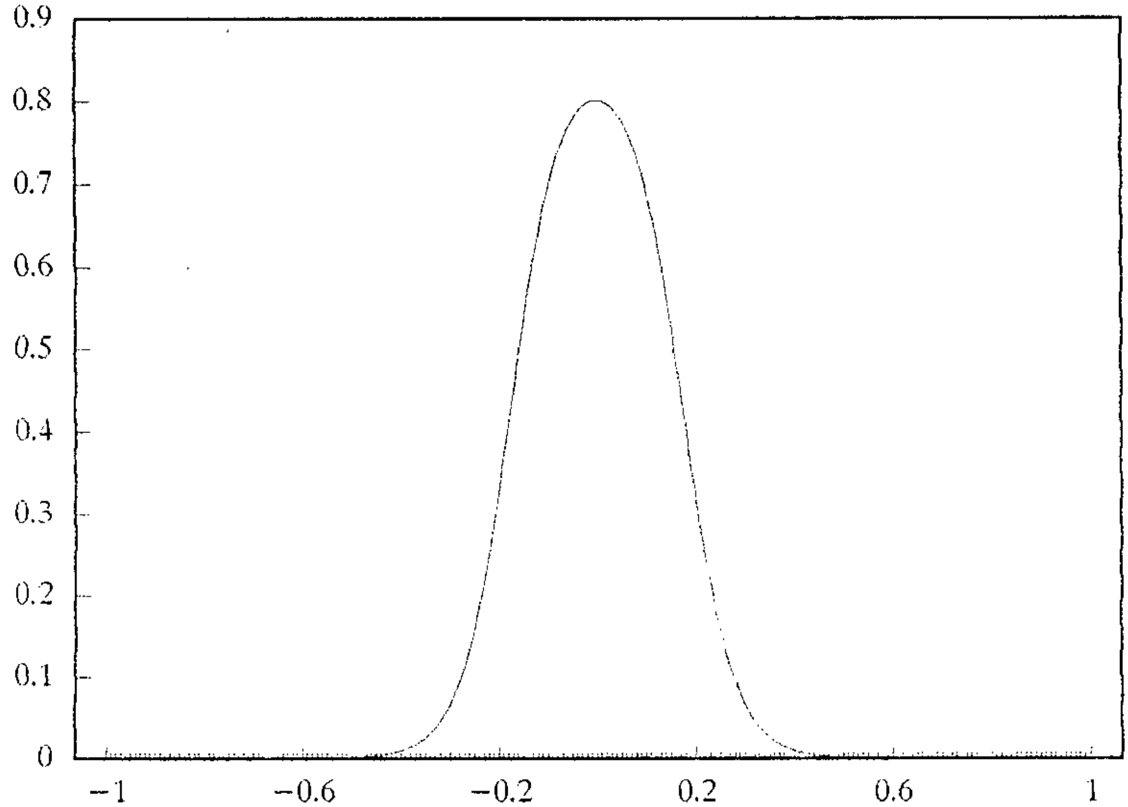


Figure 5

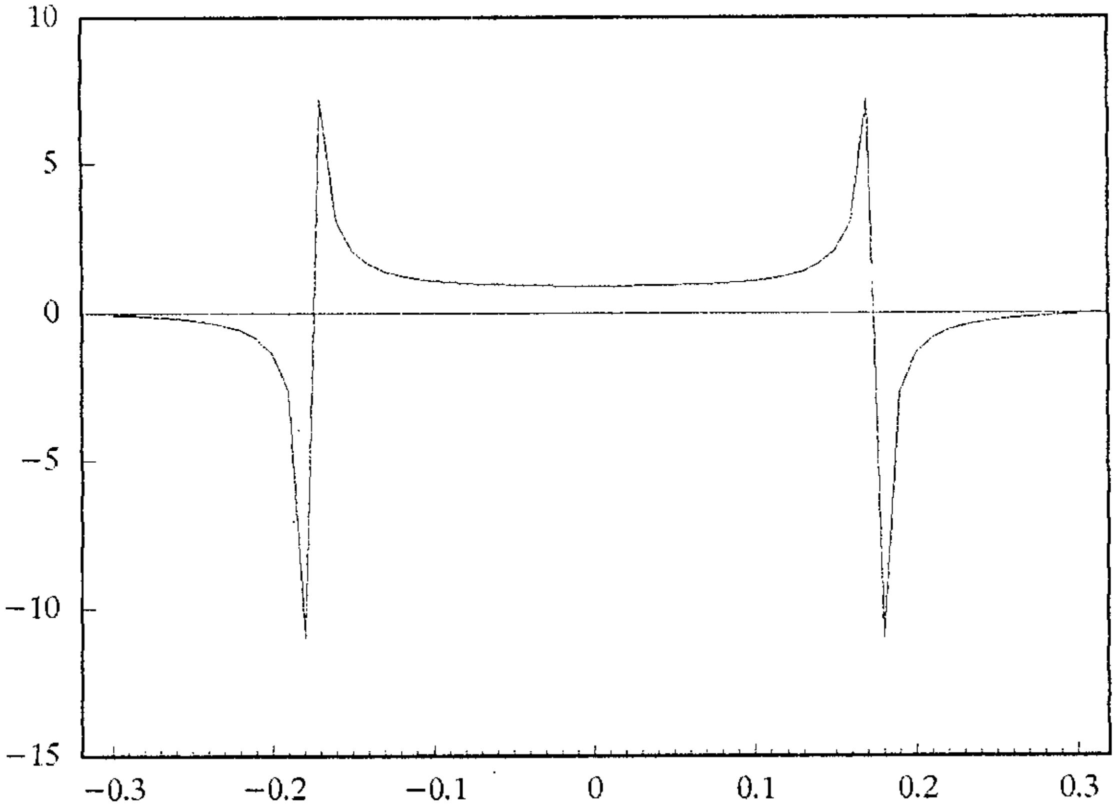


Figure 6

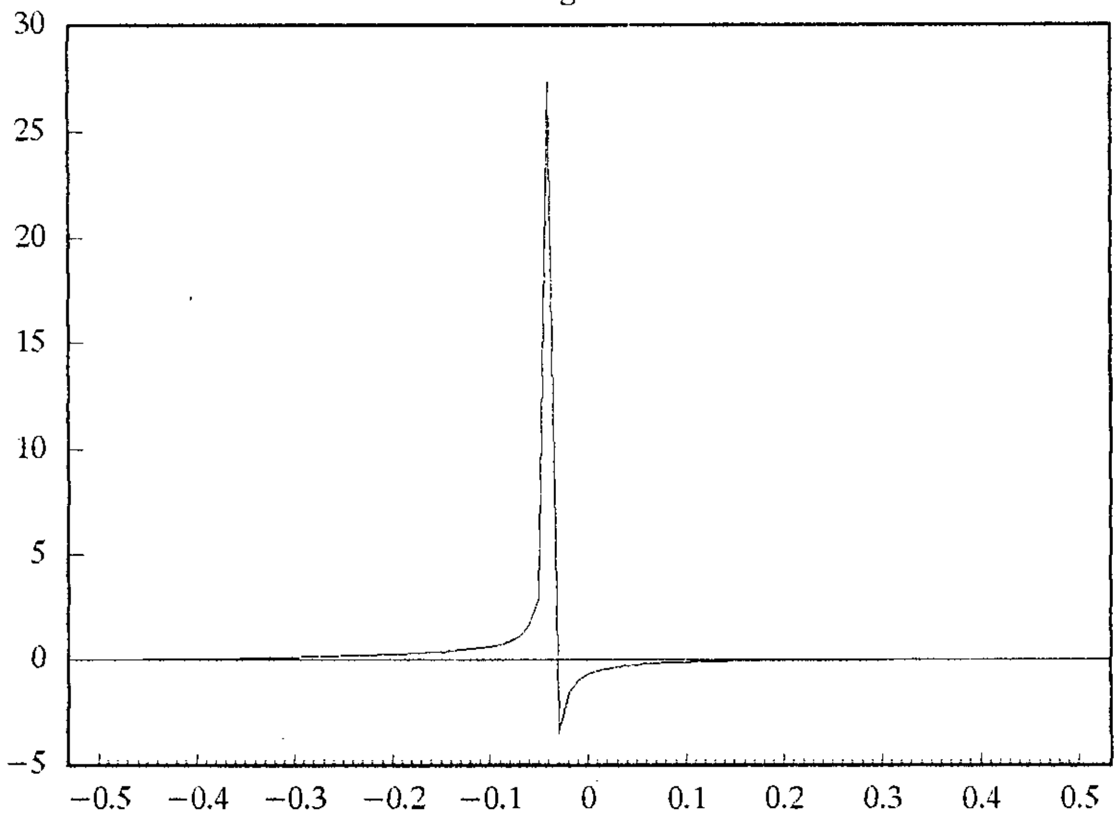
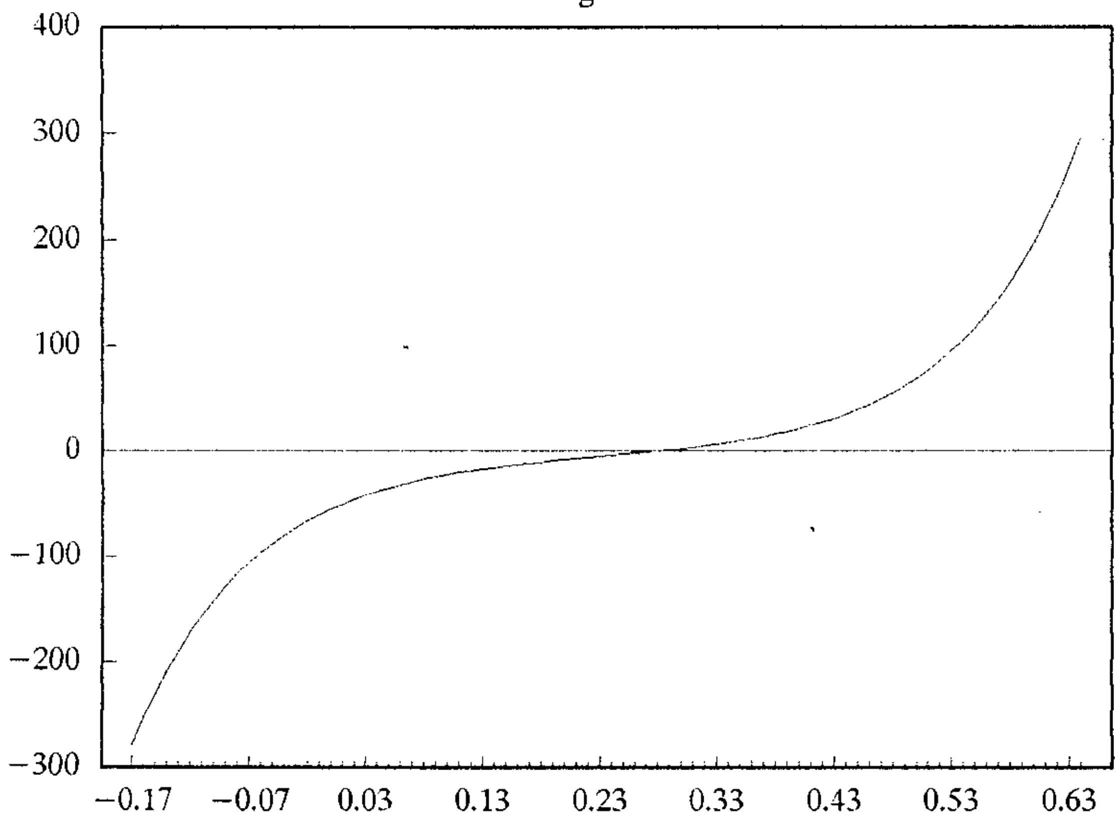


Figure 7



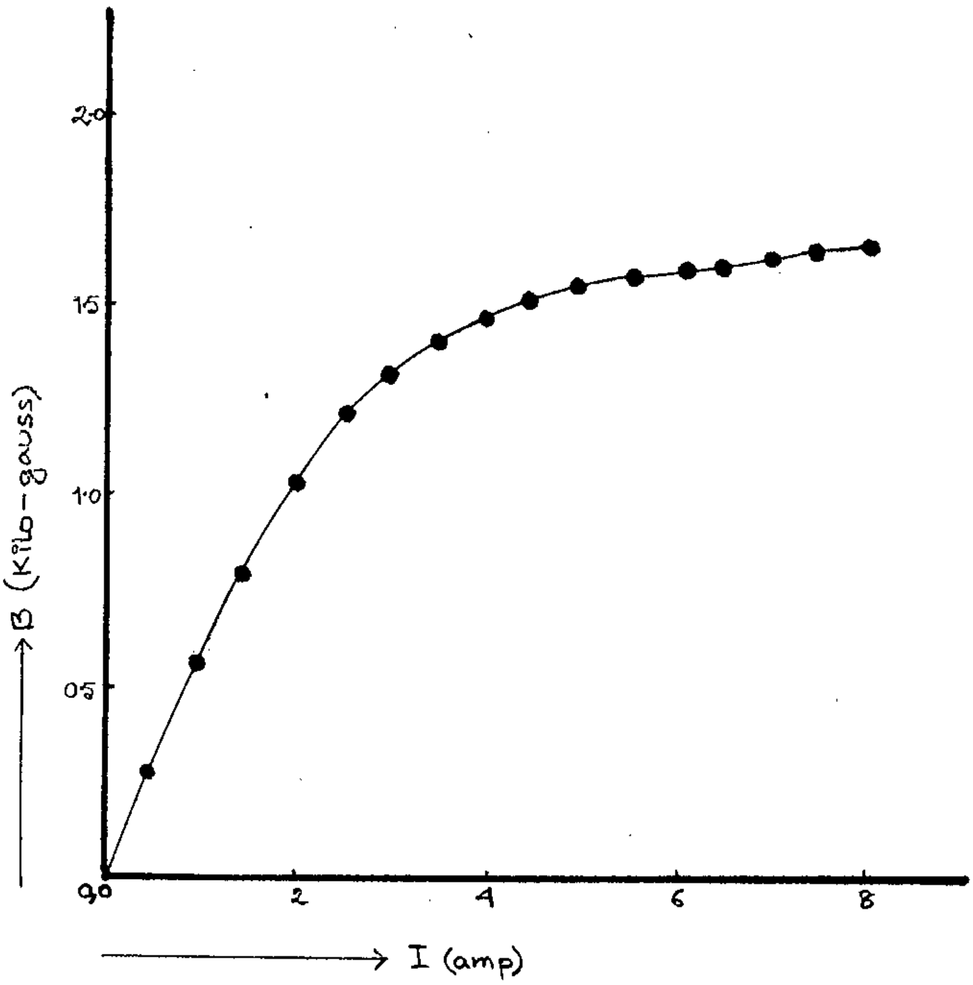
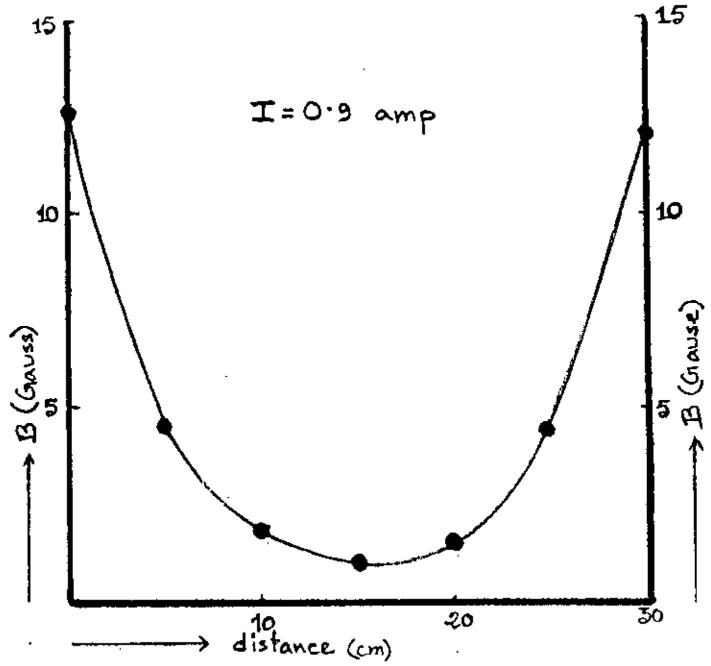
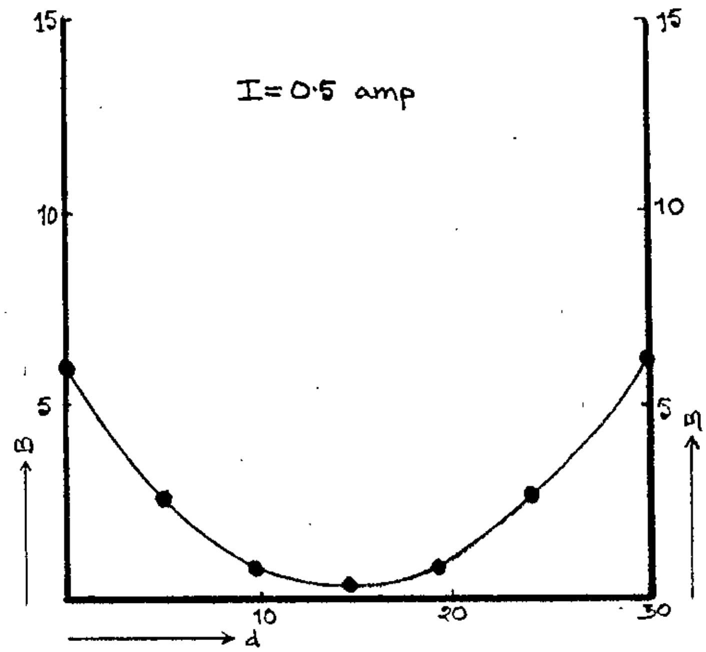


Fig. 8



I = 0·9 amp
B (Gauss)
B (Gause)
distance (cm)
15
10
5
10
20
30

(a)



I = 0·5 amp
B
B
d
15
10
5
10
20
30

(b)

Fig. 9

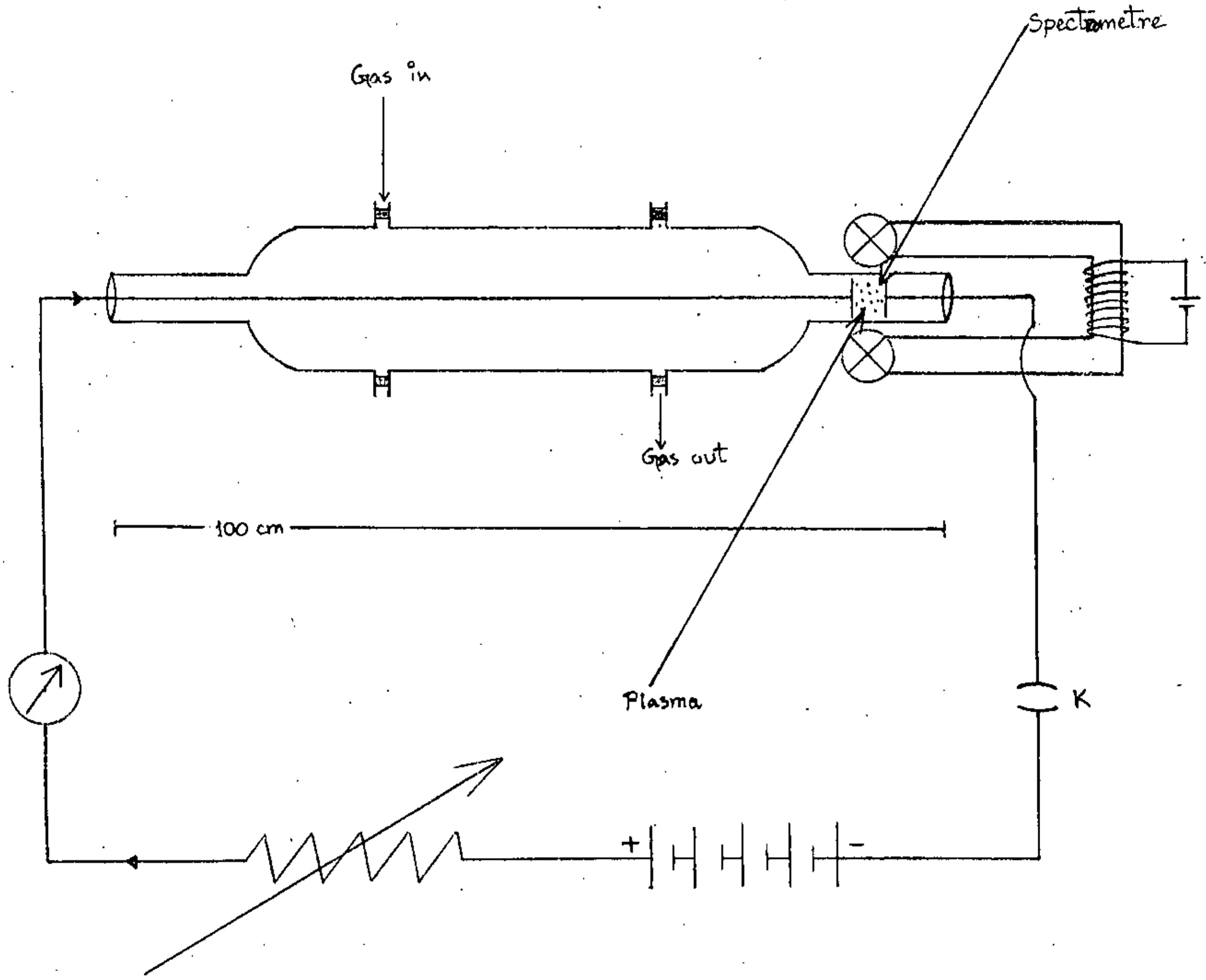


Fig. 10

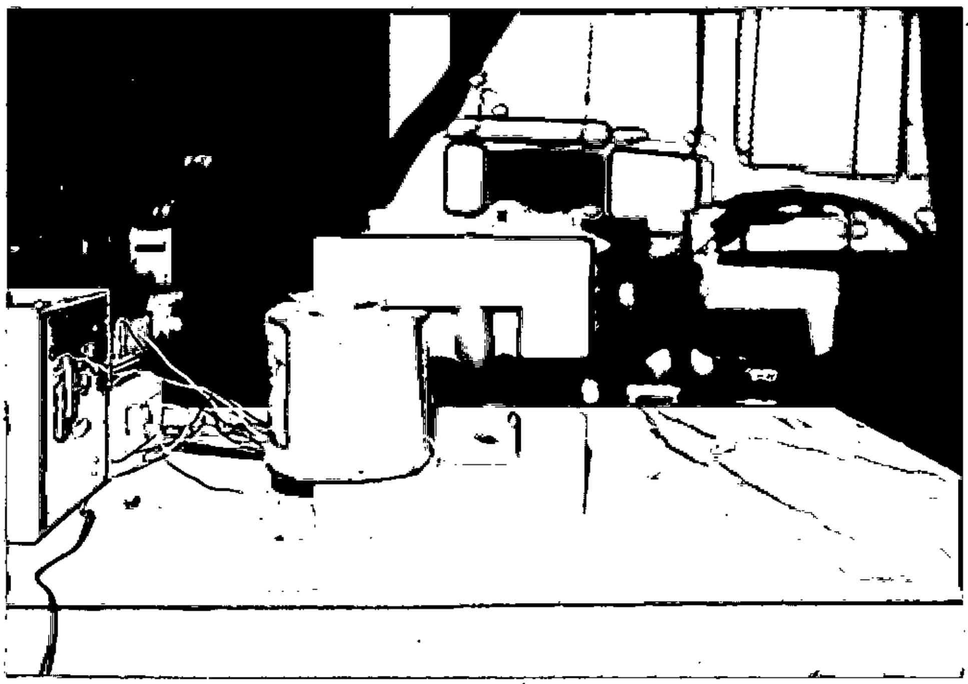


Fig. 11

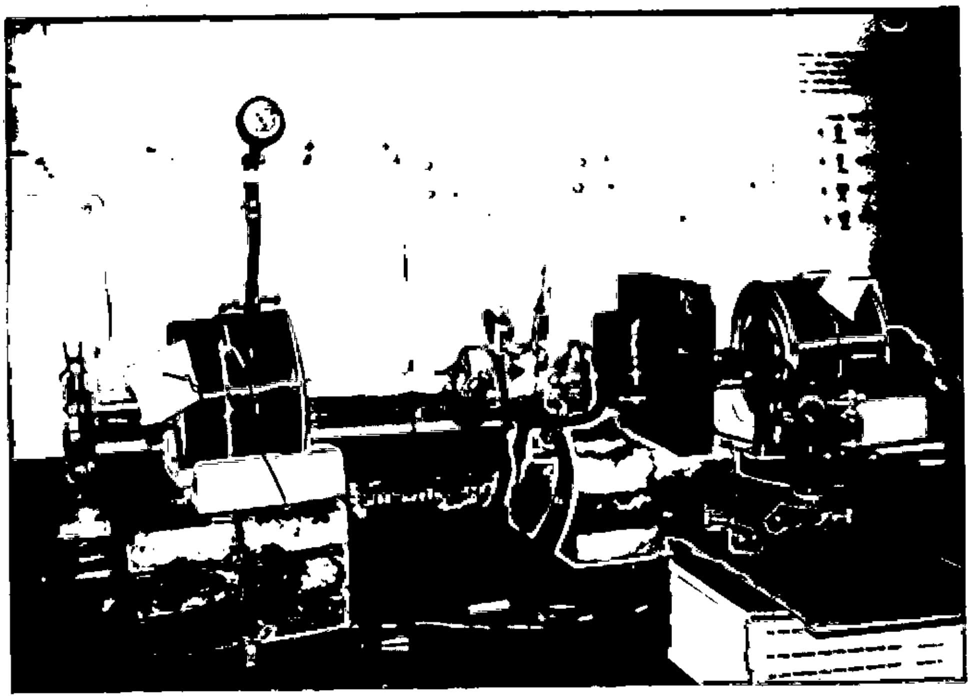


Fig. 12

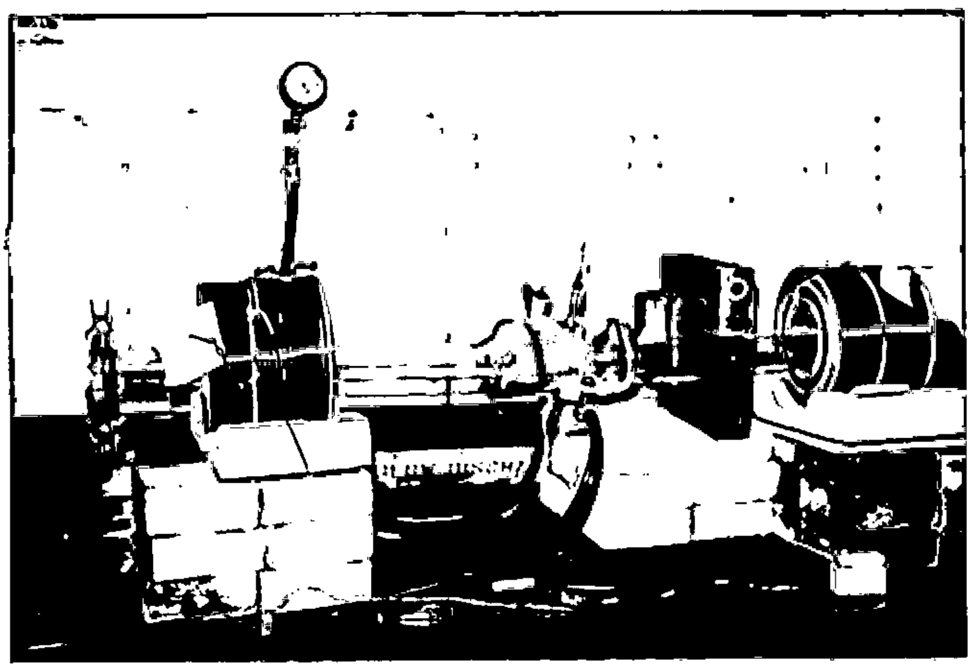


Fig. 13

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TABLE 1

I (A)	0.0	0.5	1.0	1.5	2.0	2.5	3.0
B (KG)	0.000	0.294	0.571	0.824	1.059	1.224	1.328

3.5	4.0	4.5	5.0	5.5	6.0	6.5	7.0	7.5	8.0
1.403	1.462	1.513	1.551	1.582	1.609	1.630	1.654	1.674	1.692

TABLE 2
Gas used in discharge tube: Nitrogen (air))

Grating line N/cm	Diffrec- tion order n	Order of Spectral Line	Magnetized (B = 1.224KG)		Unmagnetized (B=0)	
			θ in deg.	λ in A	θ in deg.	λ in A
		1	13.1011	3837.97	12.8678	3770.78
		2	13.4178	3929.06	13.3678	3914.69
		3	14.1679	4144.34	14.1846	4149.12
		4	14.3994	4210.63	14.3679	4201.62
		5	14.9180	4358.90	14.8512	4339.77
5906	01	6	15.2013	4439.75	15.1680	4430.25
		7	15.4680	4515.75	15.5180	4529.99
		8	15.7513	4596.39	15.6847	4577.44
		9	16.1181	4700.62	16.0180	4672.87
		10	16.4181	4785.72	16.3681	4771.55
		11	17.5178	5096.55	17.3466	5048.28

TABLE 3

Gas used in discharge tube: Hydrogen (air))

Grating line N/cm	Diffrec-tion order n	Order of Spectral Line	Magnetized (B = 1.224KG)		Unmagnetized (B=0)	
			θ in deg.	λ in A	θ in deg.	λ in A
		1	12.8678	3770.79	12.6678	3713.14
		2	13.4511	3938.69	12.8760	3773.15
		3	14.1512	4139.55	13.0345	3818.79
		4	14.3679	4201.62	13.2178	3871.55
		5	14.8347	4335.11	13.1678	3943.16
		6	15.2013	4439.75	13.6267	3989.08
		7	15.6180	4558.46	13.9679	4086.01
		9	16.0198	4672.70	14.5346	4249.32
		10	16.7220	4871.80	14.8347	4335.11
		11	17.2515	5021.45	15.3347	4477.78

TABLE 4

Experimental results for mirror field: Distance between two coils are 30cm, measured from the left. Number of terms per meter 2400.

d(cm)	I_1(A)	B_1(G)	I_2(A)	B_2(G)
0	0.5	6.07	0.9	12.62
5	0.5	2.54	0.9	4.91
10	0.5	0.97	0.9	1.96
15	0.5	0.70	0.9	1.39
20	0.5	0.98	0.9	1.98
25	0.5	2.63	0.9	4.82
30	0.5	6.21	0.9	12.46

RESULTS

I(A)	Mirror ratio(B_m/B_o)
0.5	8.97
0.9	9.08