Sub : CHE 405 (Process Design I)
Full Marks : 210
Time : 3 Hours
The figures in the margin indicate full marks. USE SEPARATE SCRIPTS FOR EACH SECTION

## SECTION-A

There are FOUR questions in this Section. Answer any THREE.

1. (a) This question is on "Fire and Explosion Hazards" -
(i) What is UFL? How can you calculate the UFL of a mixture?
(ii) How are fires classified?
(iii) Briefly explain detonation, deflagration and TNT.
(b) Write a brief essay on PARTICULATE REMOVVAL.
(c) What do you understand by "Economic assessment of Base-Case Design"-explain.
2. (a) A continuous evaporator is operated with a given feed material under conditions in which the concentration of the product remains constant. The feed rate at the start of the cycle after the tubes have been cleaned has been found to be $6000 \mathrm{~kg} / \mathrm{hr}$. After 48 hours of continuous operation, tests have shown that the feed rate decreases to 3000 (ks hr. The reduction in capacity is due to true scale formation. If the downtime per cycle for emptying, cleaning, and recharging is 6.5 hours, how long should the evaporator be operated between cleanings in order to obtain the maximum amount of product per 60 days.
(b) A solvent extraction operation is carried out continuously in a plate column with gravity flow. The unit is operated continuously. The feed rate of $45 \mathrm{~m}^{3} /$ day must be handled 310 day/year. The allowable velocity per square meter of cross-sectional tower area is $12.6 \mathrm{~m}^{3}$ of combined solvent and charge per hour. The annual fixed costs for the installed equipment can be predicted from the following equation-

$$
C_{F}=8700 F_{s f}^{2}-50,500 F_{s f}+105,000 \$ / y r
$$

where $\mathrm{F}_{\mathrm{sf}}$ is the cubic meters of solvent per cubic meter of feed. Operating and other variable costs depend on the amount of solvent that must be recovered, and the costs are $\$ 1.38$ for each cubic meter of solvent passing through the tower. What tower diameter should be used for optimum conditions of minimum total cost per year?
3. (a) Make a study estimate of the fixed capital investment for a process plant if the purchased-equipment cost is Taka 1.65 crore. Use the ranges of process-plant component cost outlined in given Table with very low degree of automatic controls, no yard improvements and an average service facilities. Moreover, the engineering and supervision costs are known to be on the high side and the company policy is that the contingency should be $12 \%$ of FCI .

$$
=2=
$$

## CHE 405

## Contd ... Q. No. 3

(b) If the above chemical process plant is erected in the Pacific Coast area for a fixedcapital investment as calculated in 3(a) in 1990, estimate the fixed-capital investment in 2000 for a similar process plant located in the Gulf area with two times the process capacity but with an equal number of process units. Use the power factor method (direct + indirect plant cost) to evaluate the new fixed - capital investment, and assume the factors given in supplied Tables apply.
(c) Using a suitable example explain how the following 2 methods of estimating Capital

Investment of chemical process plants work-
(i) Method F: Investment Cost per Unit of Capacity
(ii) Method G: Turnover Ratio
4. A P \& ID of a waste kerosene refinery plant is shown in figure for $Q$ No. 4. The refinery will recover kerosene from the waste kerosene solvent returned from auto engine repairers. The main plant items include a distillation column H 3 , gas fired hot oil furnace H 1 , product reboiler H 2 , condenser C 1 and associated pumps, controls and piping. The contaminated waste kerosene is fed to H 3 .
The closed hot oil system uses a heating fluid, which is heated in H1 by a natural gas burner and circulated through H 2 by pump P1. The waste kerosene is boiled in H 2 (shell and tube heat exchanger).

Temperature indicator and controller TIC on H 3 controls the piped natural gas feed valve V1 to the burner in H 1 , to maintain the set temperature in H 3 . The residues in H 3 are maintained at the required level by the pump P3 and valve V12 which is controlled by the level indicator and controller LIC.
Carry out HAZOP studies around the hot oil furnace H 1 , and reboiler H 2 with a target to safeguard the system from building up of high pressure and temperature.

## SECTION - B

There are FOUR questions in this Section. Answer any THREE.
5. (a) Universal Power Ltd needs to design a heat transfer equipment for cooling one of the product $40^{\circ}$ API oil which comes from distillation column at $300^{\circ} \mathrm{F}$. Their production capacity is $3.2 \times 10^{5} \mathrm{lb} /$ day based on eight hour of operation time per day. You need to cool the product stream to $100^{\circ} \mathrm{F}$ by using cooling water of $85^{\circ} \mathrm{F}$, pumping at a rate of $2 \times 10^{5} \mathrm{lb} / \mathrm{hr}$. Pressure drop allowance is 10 psi for both stream and dirt factor of each stream is 0.002 .

## CHE 405

## Contd ... Q. No. 5

You may assume a 2-4 shell and tube heat exchanger and can assume overall heat transfer coefficient from the range for organic-water system of $80-120 \mathrm{Btu} / \mathrm{hr}^{\mathrm{ff}}{ }^{2} .{ }^{\circ} \mathrm{F}$. Table and Graph for required data is attached; assume suitable values if you need additional data. Material of construction for carbon steel is $\$ 30$ per square feet for shell and tube heat exchanger.

Shell side: $25 \%$ vertically cut baffle, Baffle spacing 8 inch.
Tube side: $3 / 4$ inch OD, $16 \mathrm{BWG}, 16 \mathrm{ft}$ long, 1 inch. square pitch
Physical properties for oil at average temperature,
Viscosity $=2.58 \mathrm{lb} / \mathrm{ft} . \mathrm{hr}$.
Thermal conductivity $=0.085 \mathrm{Btu} / \mathrm{hr} . \mathrm{ft} .^{\circ} \mathrm{F}$
Specific heat capacity $=0.58 \mathrm{Btu} / \mathrm{lb} .^{\circ} \mathrm{F}$
Physical properties for water at average temperature,
Viscosity $=1.57 \mathrm{lb} / \mathrm{ft} . \mathrm{hr}$
Thermal conductivity $=0.078 \mathrm{Btu} / \mathrm{hr} . \mathrm{ft} .^{\circ} \mathrm{F}$
Specific heat capacity $=1 \mathrm{Btu} / \mathrm{lb} .{ }^{\circ} \mathrm{F}$
(a) Decide the tube and shell side fluids and explain the reason of your decision.
(b) Find the required surface area, number of tubes and shell ID.
(c) Check if the velocity of tube side flow is in the acceptable range or not?
(d) Check the heat exchanger is suitable for this use or not?
(e) Calculate the \% of overdesign. What is the loss due to overdesign? Is it acceptable?
6. (a) What are the factors you should consider during the selection of a pump?
(b) Which pump do you recommend
(i) If you need to handle a fluid of operating pressure larger than 50 MPa .
(ii) If the fluid is highly viscous.
(c) A reciprocating compressor is used to compress $0.85 \mathrm{~m}^{3} / \mathrm{s}$ of $25^{\circ} \mathrm{C}$ methane gas from 100 to 6000 kPa .

Assume that over the entire temperature range of compression, it has constant Heat capacity $=2430 \mathrm{~J} /(\mathrm{kg} . \mathrm{K})$
$\mathrm{C}_{\mathrm{p}} / \mathrm{C}_{\mathrm{v}}=1.31$
(i) How many stages of compression should be used?
(ii) What is the power requirement if the compressor exhibits an efficiency of $80 \%$ ?
(iii) What is the exit temperature if a single-stage compressor is used?
(iv) If the temperature of the cooling water only increases by $15^{\circ} \mathrm{C}$, how much water is required for intercoolers and aftercooler to ensure that the compressed gas from each stage after cooling is again returned to $25^{\circ} \mathrm{C}$.

$$
=4=
$$

## CHE 405

## Contd... O. No. 7(a)

7. (a) A heat exchanger is used to cool lubricating oil of $180^{\circ} \mathrm{F}$ with a flow rate of 40,000 $\mathrm{lb} / \mathrm{hr}$ by using water entering at $77^{\circ} \mathrm{F}$. Water is pumped at a rate of $56,000 \mathrm{lb} / \mathrm{hr}$. Overall heat transfer coefficient is $110 \mathrm{Btu} / \mathrm{hr} . \mathrm{ft}^{2} .{ }^{\circ} \mathrm{F}$ and total surface area is $1200 \mathrm{ft}^{2}$.

Heat capacity of oil $=0.62 \mathrm{Btu} / \mathrm{lb} .{ }^{\circ} \mathrm{F}$
Heat capacity of water $=1 \mathrm{Btu} / \mathrm{b} .{ }^{\circ} \mathrm{F}$
(i) Find the exit temperature of both stream
(ii) What is the effectiveness of the heat exchanger and NTU for cold stream?
(iii) Find LMTD and correction factor.
(b) A mixture of propane/propylene is to be separated for high purity propylene product. Write down the possible options of separation processes for this mixture. Key properties of Propane and propylene are attached in Table for Q. No. 7(b).
(c) Why do you think diameter evaluation for columns with random packing is different from that of columns with trays?
8. You are asked to design a packed column for the following distillation process.

The distillation column is with a partial reboiler and a total condenser and will be used to separate a mixture of benzene, tolune and cumene. The feed will be 40 mole\% benzene, $30 \mathrm{~mole} \%$ toluene and $30 \mathrm{~mole} \%$ cumene and will be supplied as saturated vapor at the rate of 100 kg moles/hour. It is desired to recover $95 \%$ of the toluene in the distillate and $95 \%$ of the cumene in the bottoms. The distillation will be carried out at 1 atm . A reflux ratio of 2 will be maintained. Equilibrium can be represented as constant relative volatilities. Choosing toluene as the reference component, $\alpha_{\mathrm{BT}}=2.25$ and $\alpha_{C T}=0.21$.
You are supplied with 25 mm plastic pall rings having void fraction 0.9 , surface area per volume, $a_{p}=267$ and approximate HETP 0.4-0.5. Calculate the required packing height and diameter of the column. (Hint: use shortcut method to determine the equilibrium number of stages and a pressure drop of $500 \mathrm{~Pa} / \mathrm{m}$ for the attached figure).


${ }^{\dagger}$ All costs presented in this text and in the McGraw-Hill website are based on this value for January 2002, obtained from the Chemical Engineering index unless otherwise indicated. The website provides the corresponding mathematical cost relationships for all the graph-
ical cost data presented in the text.
${ }^{\ddagger}$ Projected.
${ }^{3}$ Calculated with revised index; see Chem. Eng., 109: 62 (2002).
Relative labor rate and productivity indexes in chemical and allied products
industries for the United States (1999)
${ }^{1}$ Adapted from J. M. Winton, Chem. Week, 121 (24): 49 (1977), and updated with data from M. Kiley, ed., National Construction Estimator, 37th ed., Craftsman Book Company of America, Carlsbad, CA, 1989. Productivity, as considered here, is an economic term that gives the value added (products minus raw materials) per dollar of total payroll cost. Relative values were determined by taking the average of Kiley's weighted state values in each region divided by the weighted-average value of all the regions. See also Tables 6-14 and 6-15


Table 9. Tone-shen: Layouts (Tube Counvis)
Squ ure Pitch


APFENDIX OF C ILCULATION DANA
Tamle 10. Eleat Exchangita and Condenfer Tubr Data


$\log _{9} \frac{3 O_{7}}{4}$


(Shonarde of 2'ubular Exchanger Mfonufacturers Acsaciation, $2 d$ ad.; New Ya:k, 1919.)



Fia, 24. Tubertide liest-Cranster e ITVe. (Adanted from Stadir and Tate.)



5

Ro $\frac{T_{1}-T_{1}}{t_{2}-T_{1}} \quad s=\frac{t_{2}-t_{1}}{T_{1}-t_{1}}$



Figure for Question No. 8



Flooding and pressure drop correlations for packed columns. (From R. E. Treybal, Mass Transport Operations, Sd ed., McGraw-Hill, New York, 1980, p. 195 with
permission of McGraw-HyI.)

Table for Question no: 7(b)


Date : 23/07/2013
BANGLADESH UNIVERSITY OF ENGINEERING AND TECHNOLOGY, DHAKA
L-4/T-1 B. Sc. Engineering Examinations 2011-2012
Sub : CHE 491 (Natural Gas Engineering)
Full Marks: 210
Time: 3 Hours
The figures in the margin indicate full marks.
USE SEPARATE SCRIPTS FOR EACH SECTION

## SECTION-A

There are FOUR questions in this Section. Answer any THREE.

1. (a) Explain organic theory of origin of petroleum.
(b) What are the impurities found in natural gas? Explain with examples.
(c) Write down the physical properties of natural gas.
(d) A gas reservoir has the following composition.

|  | mole frac | Mol wt |
| :--- | :---: | :---: |
| $\mathrm{CO}_{2}$ | 0.02 | 44.1 |
| $\mathrm{~N}_{2}$ | 0.01 | 28.02 |
| $\mathrm{C}_{1}$ | 0.85 | 16.04 |
| $\mathrm{C}_{2}$ | 0.04 | 30.1 |
| $\mathrm{C}_{3}$ | 0.03 | 44.1 |
| $\mathrm{i}-\mathrm{C}_{4}$ | 0.03 | 58.1 |
| $\mathrm{n}-\mathrm{C}_{4}$ | 0.02 |  |

Initial reservoir P and T are 3000 psia and $180^{\circ} \mathrm{F}$ respectively. Assuming real gas behavior, calculate the density of the gas phase under the initial reservoir conditions. Compare the result with that of ideal gas behavior. Comment on your result.
2. (a) Draw P-T diagrams of a single component and a multicomponent system. Explain the main features of the diagrams.
(b) Explain the dissimilarities among the P-V diagrams of single component, two component and multicomponent systems.
(c) What are different types of petroleum reservoirs? What do they produce at surface condition?
(d) Explain isothermal compressibility of natural gas?
3. (a) What are the problems encountered with natural gas with high water content?
(b) What are the methods available for calculation of water content of natural gas? Draw phase equilibrium diagram for a gas water hydrate system.
(c) What is the most widely used dehydration process? Explain with process diagram.
(d) What do you know about gas hydrates?

$$
=2=
$$

## CHE 491

4. (a) Write short notes on

- performance coefficient
-n.
(b) Explain isochronal test with diagrams.
(c) An isochronal test was conducted on a well located in a reservoir that had an average pressure of 1952 psia. The well was flowed on four choke sizes and the flow rate and flowing bottom hole pressure were measured at 3 hr and 6 hr for each choke size. An extended test was conducted for a period of 72 hr at a rate of 6 MMSCFD , at which time $\mathrm{P}_{\mathrm{wt}}$ was measured at 1151 psia . Using the preceding data find the following
(i) stabilized deliverability equation
(ii) AOF

Data:

| $\mathrm{q}_{\mathrm{sc}}$ | $\underline{\mathrm{t}=3 \mathrm{hr}}$ | $\underline{\mathrm{t}=6 \mathrm{hr}}$ |
| :---: | :---: | :---: |
| $(\mathrm{Mscfd})$ | $\mathrm{P}_{\mathrm{wf}}(\mathrm{psia})$ | $\mathrm{P}_{\mathrm{wf}}(\mathrm{psia})$ |
| 2600 | 1793 | 1761 |
| 3300 | 1757 | 1657 |
| 5000 | 1623 | 1510 |
| 6300 | 1505 | 1320 |

## SECTION - B

There are FOUR questions in this Section. Answer any THREE.
5. (a) What is horizontal separator? Draw a neat sketch of horizontal separator and label it.

Write down its advantages and disadvantages.
(3+4+3=10)
(b) What are the minimum physical size requirement in design of a
(i) Vertical Separator.
(ii) Horizontal Separator.
(c) What are the minimum functional requirement of a well designed separator?
(d) A separator, to be operated at 1,000 psia, is required to handle a wellstream with gas flow rate of 7 MMscfd at a $\mathrm{GLR}=40 \mathrm{bbl} / \mathrm{MMscf}$. Determine the separator size required for
(i) A Vertical Separator
(ii) A Horizontal Separator
(iii) A Spherical Separator.

Assume a liquid (oil + water) density of $52 \mathrm{lbm} / \mathrm{ft}^{3}$, ideal gas gravity $=0.75$, an operating temperature equal to $110^{\circ} \mathrm{F}$, a retention time $=3.5 \mathrm{~min}$ and $1 / 2$ full of liquid condition.

$$
=3=
$$

## CHE 491

6. (a) Write down the assumptions of steady-state isothermal flow of gas through a horizontal pipe.
(b) Find an expression of average of a 200 miles long, $36^{\prime \prime}$ pipeline, when it carries a compressible fluid at 350 MMscfd . If the pressure at upstream and downstream of the pipeline are 850 psia and 425 psia , respectively, what is the average pressure of this transmission line?
(c) For a gas flow measuring device, define the following with example
(i) Accuracy (ii) Rangeability (iii) Repeatability
(d) A natural gas pipeline is 20 miles long with diameter $4^{\prime \prime}$. To increase its capacity, the first 7 miles of pipeline is replaced by a $6^{\prime \prime}$ pipeline, provided that the total pressure drop in the pipeline remain same. What is the percentage increase in flow rate?
7. (a) (i) What is NGL? What are the usages of NGL? (ii) Explain Joule-Thomson process of NGL extraction.
(b) Describe Amine Process of sweetening of natural gas with a simplified diagram.
(c) Describe the "Impingement" method of gas cleaning.
8. (a) Describe the modes of natural gas transportation. What are the advantages and disadvantages of each modes.
(b) Write down the name of the companies under "Petrobangla", indicating their role.
(c) Write down the name of the producing gas fields in Bangladesh, along with their operating companies.
(d) What is the " 3 P " natural gas reserve in Bangladesh? Include producing, Nonproducing and production suspended fields.


Pseudocritical properties of natural gases. (After Sutton. ${ }^{32}$ )
Fig for $Q \ldots 1(d)$
$=4=$


Compressibility factors for natural gases. (After Standing and Katz, Trans.
Fig for Q no N (s)

## CHE 491: Natural Gas Engineering

Symbols represent their usual meaning
Gas capacity (MMscfd) of a separator at standard condition
$q_{s c}=\frac{2.40 D^{2} K p\left(\rho_{l}-\rho_{g}\right)^{0.5}}{Z(T+460) \rho_{g}^{0.5}}$

## K values for separators

Vertical separator, $\mathrm{K}=0.167$ with mist extractor
Horizontal Separator, $\mathrm{K}=0.382$ with mist extractor
Spherical Separator, $K=0.35$ with mist extractor

Liquid Capacity $\quad W=\frac{1440 V_{L}}{\cdot t}$
$V_{L}=0.1399 D^{2} h$; For vertical separators
$V_{L}=0.1399 D^{2}(L / 2)$; For horizontal single-tube separators
$V_{L}=0.1399 D^{2} L$; For horizontal double-tube separators
$V_{L}=0.0466 D^{3}(D / 2)^{0.5}$; For spherical separators

For flow in horizontal pipes

$$
\dot{q}_{s c}=5.6353\left(\frac{T_{s c}}{P_{s c}}\right)\left[\frac{\left(p_{1}^{2}-p_{2}^{2}\right) d^{16 / 3}}{\gamma_{g} Z_{a v} T f L}\right]^{0.5}
$$

L-4/T-1/ChE
Date : 23/07/2013
BANGLADESH UNIVERSITY OF ENGINEERING AND TECHNOLOGY, DHAKA
L-4/T-1 B. Sc. Engineering Examinations 2011-2012
Sub : CHE 485 (Industrial Pollution Control)
Full Marks : 210
Time: 3 Hours
The figures in the margin indicate full marks.
USE SEPARATE SCRIPTS FOR EACH SECTION

## SECTION - A

There are FOUR questions in this Section. Answer any THREE.

1. (a) How will you consider the ambient atmosphere to be polluted? What are the general types of particulate pollutants in atmosphere? - Describe their respective physical state, sources, and transportation behavior in atmosphere.
(b) Classify ambient air pollutants according to USEPA and describe briefly each class type with examples.
(c) Enumerate the atmospheric sources of atmospheric $\mathrm{SO}_{\mathrm{x}}, \mathrm{NO}_{\mathrm{x}}, \mathrm{O}_{3}$ and Pb . What are the common and detrimental effects of these pollutants to human health and welfare.
2. (a) How is the air quality of an area affected by atmospheric stability? - Explain the phenomena in terms of temperature gradient and mixing height.
(b) Show the plume nature in atmosphere along with different atmospheric stability and temperature inversions.
(c) Is morning walk in winter (before-sun-rise) good for health? - Justify your answer.
3. (a) What is iso-kinetic sampling for stack particulate measurement? Why is iso-kinetic sampling so important for stack particulate? - Explain.
(b) Describe iso-kinetic sampling procedure for stack particulates step by step.
(c) You are given a task to measure the particulate concentration of a stack emission. The stack is circular and vertical. The internal diameter of the stack is $1^{\prime}$. The only straight section available of the stack is $5^{\prime}$ between the flue gas entry and exit. The flow inside the stack is unsteady. Sketch the stack with dimensions and find the location for sampling point and also find the number of traverses for this case and show them on the stack cross section.
[Note: Figures are provided]
4. (a) What are the chief advantages and disadvantages of thermal oxidation, Catalytic oxidation and GAC adsorption process to remove VOCs from industrial emission stream? -State briefly.
(b) How will you pre-treat the VOCs emission stream before passing it through thermal oxidizer, catalytic oxidizer and GAC bed? - Describe briefly.

## CHE 485

## Contd ... Q. No. 4

(c) (i) How can you reduce thermal NOx and fuel NOx from combustion system? -State briefly.
(ii) How will you remove acid gases and black carbon from emission stream in coal fired power plant? - Describe the process with schematic diagram.

## SECTION - B

There are FOUR questions in this Section. Answer any THREE.
5. (a) What are "Flow Equalization" and "Neutralization" in wastewater treatment system?

Why will the actual volume be always greater than the theoretical Equalization tank? $\quad(\mathbf{7}+\mathbf{3}=\mathbf{1 0})$
(b) What are the factors influence coagulation? Write the values of Relative "Coagulation Power" of following Electrolytes.

$$
\begin{equation*}
\mathrm{Al}_{2}\left(\mathrm{SO}_{4}\right)_{3}, \mathrm{NaCl}, \mathrm{Na}_{2} \mathrm{SO}_{4} \tag{12+3=15}
\end{equation*}
$$

(c) Write a short notes on:

Hydroxide Precipitation, Sulfide Precipitation
6. (a) What is Grid Chamber? Describe its basic operating principle.
(b) Describe Anaerobic Reactor Design and sizing in context of biological treatment option.
(c) What is Hazardous waste? What are the characteristics of Hazardous waste?
(d) What are the types of Hazardous waste according to US EPA. Describe them.
7. (a) Prepare a Table for classifying the general characteristics of Textile Effluents from different process involved in textile industry.
(b) What are the values of following parameters in Textile and Tannery wastewater according to ECR'97? Oil and Grease, $\mathrm{PH}, \mathrm{COD}, \mathrm{BOD} 5, \mathrm{TDS}$.
(c) Write a short notes on Tannery industries in Bangladesh. Describe the wastewater treatment process of Tannery industry in Bangladesh.
8. (a) Prepare a comparison table for different pulp producing technologies.
(b) Describe the wastewater treatment process for recycle paper industry in Bangladesh.
(c) How will you manage the solid waste generated from the ETP of recycle paper industry in Bangladesh?

## L-4/T-1/CHE

Date : 21/09/2013
BANGLADESH UNIVERSITY OF ENGINEERING AND TECHNOLOGY, DHAKA
L-4/T-1 B. Sc. Engineering Examinations 2011-2012
Sub : CHE 403 (Process Control)
Full Marks: 210
Time : 3 Hours
USE SEPARATE SCRIPTS FOR EACH SECTION
The figures in the margin indicate full marks.

## SECTION - A

There are FOUR questions in this section. Answer any THREE. Symbols have their usual meanings.

1. (a) What do you understand by the stability of a system? Write down the Routh stability criterion and the Bode stability criterion? What are the advantages of the Bode stability criterion over the Routh stability criterion?
(b) State five criteria for selecting a sensor.
(c) If a system having a first-order transfer function with gain K and time constant $\tau$ is forced with a sinusoidal input signal $x(t)=A \sin (\omega t)$, find the output response of the system. Given, Laplace transform of $\sin (\omega t)$ is $\omega /\left(s^{2}+\omega^{2}\right)$ :
2. (a) Derive a closed loop transfer function for disturbance changes, i.e., $Y(s) / D(s)$ for the closed-loop system shown in fig. Q. 2.


Figure Q.2: Closed loop system for Q.2(a)
(b) Write down the characteristic equation from part (a) and substitute the following transfer functions. What values of $\mathrm{K}_{\mathrm{c}}$ will result in a stable closed-loop system?

$$
\begin{equation*}
G_{1}(s)=\frac{5}{s+1} \quad G_{2}(s)=\frac{4}{2 s+1} \quad K_{m}=1 \quad G_{3}(s)=\frac{1}{s-1} \tag{15}
\end{equation*}
$$

(c) What is a Nyquist diagram? Describe the concept of gain margin and phase margin using this diagram.

$$
=2=
$$

## CHE 403

3. (a) Mathematically show that a purely integrating process, with $G_{p}(s)=K_{p} / s$ and $\mathrm{G}_{\mathrm{d}}(\mathrm{s})=\mathrm{K}_{\mathrm{d}} /(2 s+1)$, does not require an integral action in the controller for rejecting a steptype disturbance, while a purely integrating disturbance transfer function, with $G_{d}(s)=K_{d} / s$ and $G_{p}(s)=K_{p} /(2 s+1)$, does require an integral action for rejection of steptype disturbances.
(b) Describe the fraction-incomplete method of estimating First Order Plus Time Delay (FOPTD) model from step test data.
(c) State five performance criteria of a closed-loop system.
4. (a) Describe the 'Internal Model Control' method of designing a controller.
(b) Describe two methods for on-line tuning of PID controllers.
(c) Describe the combined feedback-feedforward control strategy for controlling the level of a boiler drum. Express such a control system using a block diagram.

SECTION - B
There are FOUR questions in this section. Answer any THREE.
5. (a) A distillation column shown in Fig. 5(a) is used to separate a binary mixture. Symbols $x, y$, and $z$ denote mole fractions of the more volatile component, while $\mathrm{B}, \mathrm{D}$, R , and F represent molar flow rates. It is desired to control distillate composition y despite disturbances in feed flow rate F and feed composition z . All the flow rates can be measured and manipulate with the exception of $\dot{F}$, which can only be measured. Composition analyzers provide measurements of y , and z .
(i) Propose a feedback control method and draw the P\&I (Process and Instruments) diagram.
(ii) Suggest a feedforward control method and draw the P\&I diagram.
[Note: In the P\&I diagrams show the measuring elements, transmitters, controllers, control valves, etc.]


Fig. 5(a)

$$
=3=
$$

## CHE 403

## Contd... O. No. 5(a)

(b) With appropriate examples describe what is known as "inverse response" in process control. Derive the necessary condition for inverse response for two first order processes acting parallelly.
6. (a) Why do we use Laplace Transform for analyzing process control systems? Elaborate on your answer.
(b) Derive the Laplace transform of unit impulse function.
(c) Liquid flow out of a spherical tank discharging through a valve can be modeled as:

$$
\rho V \frac{d\left(h^{3}\right)}{d t}=\rho q_{i}-C_{v} \sqrt{h}
$$

where the symbols have their usual meanings.
(i) Develop a transfer function relating the liquid level to the volumetric flow of liquid into the tank.
(ii) Convert the above transfer function into state-space model.
7. (a) "Dynamic process models are very useful for process control analysis" - elaborate on it by showing the importance of dynamic models for analysis, design and optimizing control systems.
(b) Describe the steps that are normally followed to develop the theoretical models for process control purpose.
(c) A process is modeled by:

$$
\begin{align*}
& 2 \frac{d y_{1}}{d t}=-2 y_{1}-3 y_{2}+2 u_{1}  \tag{12}\\
& \frac{d y_{2}}{d t}=4 y_{1}-6 y_{2}+u_{1}+4 u_{2}
\end{align*}
$$

Draw the block diagram of the process after converting the above model into transfer function.
8. (a) Discuss with appropriate examples how we can predict the response of a system from the locations of the poles and zeros on the s-plane.
(b) Describe the Skogestad's method for approximating higher-order models with lowerorder models for process control purposes. Give appropriate examples.
(c) Discuss the relative advantages and disadvantages of P, PI, and PID controllers.
(d) Discuss the different modifications that are used on the basic digital PID control algorithm.

BANGLADESH UNIVERSITY OF ENGINEERING AND TECHNOLOGY, DHAKA

## L-4/T-1 $\quad$ B. Sc. Engineering Examinations 2011-2012

Sub : CHE 401 (Reaction Engineering)
Full Marks : 210
Time : 3 Hours
The figures in the margin indicate full marks.
USE SEPARATE SCRIPTS FOR EACH SECTION

## SECTION-A

There are FOUR questions in this section. Answer any THREE.

1. (a) Explain the term - Adsorption isotherm. How adsorption isotherms are being used in the physical characterization of heterogeneous catalysis.
(b) Explain the term "Poisoning and Deactivation of heterogeneous catalysts".
(c) Discuss the Langmuir assumptions and point out their limitations.
(d) Develop expressions for the average pore radius $(\overline{\mathrm{r}})$, the average pore length $(\overline{\mathrm{L}})$, and the number of pores per particle $\left(n_{p}\right)$ in terms of parameters that can be measured in the laboratory [i.e., the apparent particle dimensions, the void volume per gram $\left(\mathrm{V}_{\mathrm{g}}\right)$, and the surface area per gram $\left(\mathrm{S}_{\mathrm{g}}\right)$ considering cylindrical pores without interconnection.
2. (a) Discuss the fundamental concepts used in chemical reactor design.
(b) Distinguish between plug flow and stirred tank reactors. What assumptions have you made in the derivation? What are the limitations of the assumptions made? Derive the equations for a plug flow reactor.
(c) Some investigators have studied the Diebs-Alder reaction of benzoquinone (B) and cyclopentacdiene (C) at $40^{\circ} \mathrm{C}$.



Volume changes on reaction may be neglected. At $40^{\circ} \mathrm{C}$ the reaction rate constant is equal to $9.92 \times 10^{-3} \mathrm{~m}^{3} / \mathrm{kmole} . \mathrm{sec}$. If one employs a well-stirred isothermal batch reactor to carry out this reaction, determine the holding time necessary to achieve $95 \%$ conversion of the limiting reagent using initial concentrations of 0.1 and $0.08 \mathrm{kmole} / \mathrm{m}^{3}$ for cyclopentadiene and benzoquinone, respectively.

$$
=2=
$$

## CHE 401

3. (a) Derive an expression for the concentration of the reactant in the effluent from a series of N equal sized CSTR for first order reaction.
(b) Show that for the same overall conversion, the total volume of a cascade of equalized CSTR's approach the volume of plug flow reactor as the number of reactors is increased.
Assume first order reaction.
(c) Some investigators have studied the reaction of chlorine (A) with 2-ethylhexene-1 (B) in carbon tetrachloride solution. Solutions of these materials were prepared and brought together in a mixing chamber at the inlet to a tabular flow reactor. The following data were reported at $40^{\circ} \mathrm{C}$. The initial concentrations refer to values calculated on the basis of perfect mixing.

| Run | $\mathrm{C}_{\mathrm{AO}}\left(\mathrm{moles} / \mathrm{m}^{3}\right)$ | $\mathrm{C}_{\mathrm{BO}}\left(\mathrm{mole} / \mathrm{m}^{3}\right)$ | $\mathrm{C}_{\mathrm{A}}\left(\mathrm{moles} / \mathrm{m}^{3}\right)$ <br> at exit | $\tau(\mathrm{sec})$ |
| :---: | :---: | :---: | :---: | :---: |
| 1 | 91 | 209 | 23 | 0.600 |
| 2 | 91 | 209 | 32 | 0.376 |
| 3 | 91 | 209 | 45 | 0.284 |
| 4 | 110 | 211 | 34 | 0.525 |
| 5 | 110 | 211 | 46 | 0.324 |
| 6 | 110 | 211 | 59 | 0.232 |

It has been suggested that the rate expression, $r=k C_{A} C_{B}$. Graphically determine if this expression is consistent with the above data, if so, what is the reaction rate constant?
4. (a) Distinguish between intrinsic and global reaction rates. Discuss the steps that are involved in an iterative trial and error procedure employed to determine the global rate of a heterogeneous catalytic reaction.
(b) What is the set of information necessary to start reactor design?
(c) Write short note on the followings:
(i) Knudsen diffusion and Combined diffusivity
(ii) Effectiveness factor and Effective diffusivity
(iii) One-Dimensional and Two-Dimensional Pseudo Homogeneous Model of fixed Bed Reactors.

## SECTION - B

There are FOUR questions in this section. Answer any THREE.
5. (a) Discuss briefly the parameter(s) used to represent reaction progress in opensequence and chain reactions.
(b) State the basic assumptions involved in the derivation of a rate expression from a proposed reaction mechanism.

$$
=3=
$$

## CHE 401

## Contd ... O. No. 5

(c) Distinguish between the elementary and overall reactions considering each crucial criterion.
(d) Houser and Lee have studied the pyrolysis of ethyl nitrate using a stirred flow reactor. They have proposed the following mechanism for the reaction.

Initiation: $\quad \mathrm{C}_{2} \mathrm{H}_{5} \mathrm{ONO}_{2} \xrightarrow{\mathrm{k}_{1}} \mathrm{C}_{2} \mathrm{H}_{5} \mathrm{O} \cdot+\mathrm{NO}_{2}$
Propagation: $\quad \mathrm{C}_{2} \mathrm{H}_{5} \mathrm{O} \xrightarrow{\mathrm{k}_{2}} \mathrm{CH}_{3} \cdot+\mathrm{CH}_{2} \mathrm{O}$

$$
\mathrm{CH}_{3} \cdot+\mathrm{C}_{2} \mathrm{H}_{5} \mathrm{ONO}_{2} \xrightarrow{\mathrm{k}_{3}} \mathrm{CH}_{3} \mathrm{NO}_{2}+\mathrm{C}_{2} \mathrm{H}_{5} \mathrm{O}
$$

Termination: $\quad 2 \mathrm{C}_{2} \mathrm{H}_{5} \mathrm{O} \cdot \xrightarrow{\mathrm{k}_{4}} \mathrm{CH}_{3} \mathrm{CHO}+\mathrm{C}_{2} \mathrm{H}_{5} \mathrm{OH}$
What rate expression is consistent with this mechanism?
6. (a) What is the use and limitation of fractional life approach of reaction rate data analysis? From the expression for first order reaction in constant volume system, derive $\mathrm{t}_{\frac{1}{2}}=\frac{\ln 2}{\mathrm{k}}$.
(b) A reaction with very small conversion of limiting reagent (A) has the following overall rate expression:

$$
r=\frac{k_{1} C_{A}}{1+k_{1} k_{2} C_{A}}
$$

Describe the determination of $k_{1}$ and $k_{2}$ briefly choosing a suitable method from the methods available for determing kinetic parameters.
(c) Name the variables which influence the rate of conversion in a reaction.
(d) Moelwyn-Hughes has tabulated the following values of the rate constant for the reaction $-\mathrm{N}_{2} \mathrm{O}_{5} \rightarrow \mathrm{~N}_{2} \mathrm{O}_{4}+\frac{1}{2} \mathrm{O}_{2}$.

| Temperature $\mathrm{T}\left({ }^{\circ} \mathrm{K}\right)$ | $\mathrm{k}\left(\sec ^{-1}\right)$ |
| :---: | :---: |
| 288.1 | $1.04 \times 10^{-5}$ |
| 298.1 | $3.38 \times 10^{-5}$ |
| 313.1 | $2.47 \times 10^{-4}$ |
| 323.1 | $7.59 \times 10^{-4}$ |

If the rate constant is of the form of $\mathrm{k}=\mathrm{Ae}^{-\mathrm{E} / \mathrm{RT}}$, determine the parameters A and E .
7. (a) Consider the following consecutive first order reactions:

$$
\mathrm{A} \xrightarrow{\mathrm{k}_{1}} \mathrm{~B} \xrightarrow{\mathrm{k}_{2}} \mathrm{C} \xrightarrow{\mathrm{k}_{3}} \mathrm{D}
$$

(i) When will the species $B$ reach the maximum concentration? Derive the equation.
(ii) Draw the concentration profile for $\mathrm{A}, \mathrm{B}$ and D provided that $\mathrm{k}_{2}=2.25 \mathrm{k}_{1}$ and
$\mathrm{B}_{0}=0.5 \mathrm{~A}_{0}$. You are not required to use separate graph paper.

$$
=4=
$$

## CHE 401

## Contd ... Q. No. 7

(b) The reaction stoichiometry for a reactive system can be represented as:
(15)

$$
\begin{aligned}
& \mathrm{A}+\mathrm{B} \xrightarrow[94 \%]{\mathrm{k}_{1}} \mathrm{C} \\
& \mathrm{~A}+\mathrm{B} \xrightarrow[6 \%]{\mathrm{k}_{2}} \mathrm{D}
\end{aligned}
$$

Using equimolal concentrations of reactants $A$ and $B$, the researchers obtained the following data:

| Time, $\mathrm{t}(\mathrm{ksec})$ | $\mathrm{A}\left(\mathrm{kmoles} / \mathrm{m}^{3}\right)$ |
| :---: | :---: |
| 0.0 | 0.181 |
| 21.6 | 0.131 |
| 32.4 | 0.119 |
| 86.4 | 0.0683 |

It may be assumed that the rate expressions for reactions 1 and 2 are identical expect for the values of the rate constants. Determine the rate constant $k_{1}$ and $k_{2}$ provided that reactions 1 and 2 are of first order with respect to reactant $A$ and reactant $B$.
8. (a) Write down the fundamental rate equation in terms of concentration, order and rate constant for the following cases:
(i) Opposing second-order reactions
(ii) Irreversible first-order simple parallel reactions
(iii) Reversible first-order parallel reactions.
(b) List the parameters on which the reaction rate constant depends.
(c) What is the key difference between reaction order and molecularity?
(d) A coupled enzyme assay may be represented as

$$
\begin{equation*}
\mathrm{A} \xrightarrow{\mathrm{k}_{1}} \mathrm{~B} \xrightarrow{\mathrm{k}_{2}} \mathrm{C} \tag{13}
\end{equation*}
$$

The first reaction is zero-order irreversible and the second reaction is first-order in B. Determine the steady-state concentration of $B$ when $\mathrm{k}_{1}=0.833 \mathrm{~mole} / \mathrm{m}^{3}-\mathrm{ksec}$ and $\mathrm{k}_{2}=0.767 \mathrm{sec}^{-1}$.

The figures in the margin indicate full marks. USE SEPARATE SCRIPTS FOR EACH SECTION

## SECTION-A

There are FOUR questions in this section. Answer any THREE.

1. (a) Liquified gases are sometimes stored in well-insulated spherical containers vented to the atmosphere. Develop àn expression, by shell momentum balance, for the steady-state heat transfer rate through the walls of such a container, with the radii of the inner and outer walls being $\mathrm{r}_{0}$ and $\mathrm{r}_{1}$ respectively and the temperatures at the inner and outer walls being $T_{0}$ and $T_{1}$. The thermal conductivity of the insulation varies linearly with temperature from $K_{0}$ at $T_{0}$ to $K_{1}$ at $T_{1}$ (i.e., you may use the expression:

$$
\begin{equation*}
\frac{\mathrm{K}-\mathrm{K}_{0}}{\mathrm{~K}_{1}-\mathrm{K}_{0}}=\frac{\mathrm{T}-\mathrm{T}_{0}}{\mathrm{~T}_{1}-\mathrm{T}_{0}} \equiv \theta(\theta \text { being dimensionless }) \tag{25}
\end{equation*}
$$

Refer to Figure for Question 1.
(b) Given that, in the above problem, $Q_{0}=4 \pi r_{0} r_{1}\left(\frac{K_{0}+K_{1}}{2}\right)\left(\frac{T_{1}-T_{0}}{r_{1}-r_{0}}\right)$, estimate the rate of evaporation of liquid oxygen from a spherical container of 6 ft inside diameter covered with a 1 -ff-thick annular evacuated jacket filled with particulate insulation. The following information is available:
-Temperature at inner surface of insulation: $-183^{\circ} \mathrm{C}$
-Temperature at outer surface of insulation: $0^{\circ} \mathrm{C}$
-Boiling point of $\mathrm{O}_{2}$ : $-183^{\circ} \mathrm{C}$
-Heat of vaporization of $\mathrm{O}_{2}: 1636 \mathrm{Cal} / \mathrm{g}-\mathrm{mol}$
-Thermal conductivity of insulation at $0^{\circ} \mathrm{C}: 9.0 \times 10^{-4} \mathrm{Btu} / \mathrm{hr} . \mathrm{ff} .{ }^{\circ} \mathrm{F}$
-Thermal conductivity of insulation at $-183^{\circ} \mathrm{C}: 7.2 \times 10^{-4} \mathrm{Btu} / \mathrm{hr} . \mathrm{ft} .{ }^{\circ} \mathrm{F}$
Note: $1 \mathrm{Btw} / \mathrm{hr} . \mathrm{ft} .{ }^{\circ} \mathrm{F}=4.136 \times 10^{-3} \mathrm{Cal} / \mathrm{s} . \mathrm{cm} . \mathrm{K}$
2. Refer to figure for Question 2(a). A system with two concentric porous spherical shells of radii kR and R is shown. The inner surface of the outer shell is at temperature $\mathrm{T}_{1}$ and the outer surface of the inner shell is at a lower temperature $T_{k}$. Dry air at temperature $T_{k}$ is blown outward radially from the inner shell into the intervening space and then through the outer shell. Assume steady laminar flow and low gas velocity. Under such circumstances, the equation of motion has very little significance.
(a) Simplify the equations of continuity and energy for this situation, clearly stating the boundary conditions to be used in solving the equations. Take $\omega_{\mathrm{r}}$ as the radial mass flow rate.

$$
=2=
$$

## CHE 453

## Contd... Q. No. 2

(b) Use the equations to derive a second order differential equation.
(c) Given that,

$$
\left.\frac{d T}{d r}\right|_{k R}=\frac{e^{-\frac{M}{k R}+C}}{k^{2} R^{2}}
$$

where, $M=\frac{\omega_{r} C_{p}}{4 \pi k}, \quad$ and $\quad C=\frac{\left(T_{1}-T_{k}\right) M}{e^{-\frac{M}{R}}-e^{-\frac{M}{k R}}}$,
derive an expression for the rate of heat removal by refrigeration.
(d) Refer to figure for Question 2(d). Liquid oxygen is stored in a spherical container surrounded by a porous spherical shell with a thin gas space left between the two. The opening of the porous shell (porous insulation) is stoppered. When in operation, the evaporating oxygen leaves the container properly, moves to the gas space and flows uniformly out of the porous insulation shell. Calculate the rate of heat gain from a tank 1 ft in diameter covered with a porous shell of insulation 6 inch thick with transpiration under following condition:

- Temperature of liquid $\mathrm{O}_{2}:-297^{\circ} \mathrm{F}$
- Temperature of outer surface of insulation: $30^{\circ} \mathrm{F}$
- Effective thermal conductivity of insulation: 0.02 Btu/hr.ft. ${ }^{\circ} \mathrm{F}$
- Heat of evaporation of oxygen: 91.7 Btu/lb.
- Average $\mathrm{C}_{\mathrm{p}}$ of $\mathrm{O}_{2}$ flowing through insulation: $0.22 \mathrm{Btu} / \mathrm{lb}{ }^{\circ} \mathrm{F}$

3. (a) Show that for turbulent flow

$$
\begin{equation*}
\frac{1}{2 \Delta \mathrm{t}} \int_{\mathrm{t}-\Delta \mathrm{t}}^{\mathrm{t}+\Delta \mathrm{t}}(\mathrm{~V} \cdot \nabla \mathrm{~V}) \mathrm{dt}=\overline{\mathrm{V}} \cdot \nabla \overline{\mathrm{~V}}+\overline{\mathrm{V}^{\prime} \cdot \nabla \mathrm{V}^{\prime}} \tag{10}
\end{equation*}
$$

(b) Find the time-smoothed velocity distribution for turbulent flow in a rectangular duct of half-width $b$ (in the $y$-direction) using the Prandtl mixing length relation. The duct is assumed to be very wide in the x -direction so that $\overline{\mathrm{V}}_{\mathrm{z}}=\overline{\mathrm{V}}_{\mathrm{z}}(\mathrm{y})$. The Prandtl mixinglength expression is $\tau_{y z}^{(t)}=-\rho^{2}\left|\frac{d \bar{V}_{z}}{d y}\right| \frac{d \bar{V}_{z}}{d y}$, where the mixing length $l$ is proportional to the distance $y$ from the solid surface.
4. (a) Explain Reynold's Analogy and Colburn Analogy.
(b) Equation of Energy in terms of fluid temperature T is given by

$$
\begin{equation*}
\rho \hat{C_{v}} \frac{D T}{D t}=-(\nabla \cdot q)-T\left(\frac{\partial P}{\partial T}\right)_{\hat{v}}(\nabla \cdot v)-(\tau: \nabla V) \tag{15}
\end{equation*}
$$

## CHE 453

## Contd... O. No. 4(b)

Write simplified expressions of the above equation for
(i) An ideal gas
(ii) A fluid at constant pressure
(iii) A fluid with $\rho$ independent of T .

## SECTION - B

There are FOUR questions in this section. Answer any THREE.
5. (a) In a gas absorption experiment a viscous fluid flows upward through a small circular tube and then downward on the outside as shown in figure for Q . 5(a). Set up a momentum balance over a shell of thickness $\Delta r$ in the film,
(i) Determine the velocity distribution in the film.
(ii) Obtain an expression for the volume rate of flow in the film.
(b) (i) What is Hagen-Poiseulle law? Why is it important? Show that it is dimensionally constant.
(ii) For annular flow, is the surface of zero momentum flux closer to the inner or the outer wall? Why?
6. Determine the velocity and shear stress distributions for the tangential laminar flow of an incompressible fluid between two vertical coaxial cylinders, the outer one of which is rotating with an angular velocity $\Omega_{0}$.
(i) Determine the velocity distribution and shear stress distribution by using equation of continuity and equation of motion.
(ii) Determine the torque required to turn the outer shaft.
7. (a) A semi-infinite body of liquid with constant $\rho$ and $\mu$ is bounded on one side by a flat surface. Initially the fluid and the solid surface are at rest, but at time $t=0$ the solid surface is set in motion in the positive z-direction with a velocity V, as shown in Fig. for Q. 7(a). Determine the velocity as a function of $y$ and $t$.
(b) Show that:
(i) $J_{A}=N_{A}-\omega_{A}\left(N_{A}+\frac{M_{B}}{M_{A}} N_{B}\right)$
(ii) $d \omega_{A}=\frac{M_{A} M_{B} d x_{A}}{\left(x_{A} M_{A}+x_{B} M_{B}\right)^{2}}$

## CHE 453

8. (a) Consider a porous catalyst particles in the shape of thin disks, such that the surface area of the edge of the disk is small in comparison with that of the two circular faces.
(i) Obtain an expression for the concentration profile.
(ii) Determine the expression for the effectiveness factor.
(b) (i) How do forced and pressure diffusion differ?
(ii) What are the mass transfer analogs of the Grashof and Prandtl numbers?

## Figure for Question 1



Figure for Question 2(a)


Transpiration cooling. The Inner sphere is being cooled by means of a refrigeration coil to maintain its temperature at $T_{x^{*}}$. When air is blown outwatd, as shown, less refrigeration is required.

Figure for Question 2(d)


Jse of transpiration to reduce evaporation rate

TABLE 3.4-1
THE EQUATION OF CONTINUITY IN SEVERAL

Rectangular coordinates $(x, y, z)$ :

$$
\begin{equation*}
\frac{\partial \rho}{\partial t}+\frac{\partial}{\partial x}\left(\rho v_{x}\right)+\frac{\partial}{\partial y}\left(\rho v_{y}\right)+\frac{\partial}{\partial z}\left(\rho v_{z}\right)=0 \tag{A}
\end{equation*}
$$

Cylindrical coordinates ( $r, \theta, z$ ):

$$
\begin{equation*}
\frac{\partial \rho}{\partial t}+\frac{1}{r} \frac{\partial}{\partial r}\left(\rho r v_{r}\right)+\frac{1}{r} \frac{\partial}{\partial 0}\left(\rho v_{0}\right)+\frac{\partial}{\partial z}\left(\rho v_{2}\right)=0 \tag{B}
\end{equation*}
$$

Spherical coordinates $(r, \theta, \phi)$ :

$$
\begin{equation*}
\frac{\partial \rho}{\partial t}+\frac{1}{r^{2}} \frac{\partial}{\partial r}\left(\rho r^{2} v_{r}\right)+\frac{1}{r \sin \theta} \frac{\partial}{\partial 0}\left(\rho v_{0} \sin \theta\right)+\frac{1}{r \sin \theta} \frac{\partial}{\partial \phi}\left(\rho v_{\phi}\right)=0 \tag{C}
\end{equation*}
$$

TABLE 3.4-2
THE EQUATION OF MOTION IN RECTANGULAR COORDINATES $(x, y, z)$
In terms of x :
$x$-component $\rho\left(\frac{\partial v_{x}}{\partial t}+v_{x} \frac{\partial v_{x}}{\partial x}+v_{y} \frac{\partial v_{x}}{\partial y}+v_{x} \frac{\partial v_{x}}{\partial z}\right)=-\frac{\partial p}{\partial x}$

$$
\begin{equation*}
-\left(\frac{\partial \tau_{x x}}{\partial x}+\frac{\partial \tau_{y x}}{\partial y}+\frac{\partial \tau_{z x}}{\partial z}\right)+p g_{x} \tag{A}
\end{equation*}
$$

$y$-component $\rho\left(\frac{\partial v_{v}}{\partial t}+v_{x} \frac{\partial v_{v}}{\partial x}+v_{v} \frac{\partial v_{v}}{\partial y}+v_{z} \frac{\partial v_{y}}{\partial z}\right)=-\frac{\partial p}{\partial y}$

$$
\begin{equation*}
-\left(\frac{\partial \tau_{x y}}{\partial x}+\frac{\partial \tau_{v y}}{\partial y}+\frac{\partial \tau_{s y}}{\partial z}\right)+p g_{y} \tag{B}
\end{equation*}
$$

$z$-component $p\left(\frac{\partial v_{z}}{\partial t}+v_{x} \frac{\partial v_{z}}{\partial x}+v_{y} \frac{\partial v_{s}}{\partial y}+v_{z} \frac{\partial v_{z}}{\partial z}\right)=-\frac{\partial p}{\partial z}$

$$
-\left(\frac{\partial \tau_{z i}}{\partial x}+\frac{\partial \tau_{y s}}{\partial y}+\frac{\partial \tau_{z z}}{\partial z}\right)+p g_{z}
$$

In terms of velocity gradients for a Newtonian fluid with constant $p$ and $\mu$ :
$x$-component $p\left(\frac{\partial v_{x}}{\partial t}+v_{x} \frac{\partial v_{x}}{\partial x}+v_{v} \frac{\partial v_{x}}{\partial y}+v_{x} \frac{\partial v_{x}}{\partial z}\right)=-\frac{\partial p}{\partial x}$

$$
\begin{equation*}
+\mu\left(\frac{\partial^{2} v_{x}}{\partial x^{2}}+\frac{\partial^{2} v_{x}}{\partial y^{2}}+\frac{\partial^{2} v_{x}}{\partial z^{2}}\right)+\rho g_{2} \tag{D}
\end{equation*}
$$

$y$-component $\quad \rho\left(\frac{\partial v_{y}}{\partial t}+v_{x} \frac{\partial v_{y}}{\partial x}+v_{v} \frac{\partial v_{y}}{\partial y}+v_{z} \frac{\partial v_{y}}{\partial z}\right)=-\frac{\partial p}{\partial y}$

$$
\begin{equation*}
+\mu\left(\frac{\partial^{2} v_{y}}{\partial x^{2}}+\frac{\partial^{2} v_{v}}{\partial y^{2}}+\frac{\partial^{2} v_{v}}{\partial z^{2}}\right)+p g_{v} \tag{E}
\end{equation*}
$$

z-сомронелt $\rho\left(\frac{\partial v_{s}}{\partial t}+v_{x} \frac{\partial v_{x}}{\partial x}+v_{v} \frac{\partial v_{z}}{\partial y}+v_{x} \frac{\partial v_{s}}{\partial z}\right)=-\frac{\partial p}{\partial z}$

$$
+\mu\left(\frac{\partial^{2} v_{z}}{\partial x^{2}}+\frac{\partial^{2} v_{z}}{\partial y^{2}}+\frac{\partial^{2} v_{z}}{\partial z^{2}}\right)+p_{z}{ }_{z}(F)
$$

TAULE 3.4-3
THE EQUATION OF MOTION IN CYLINDRICAL COORDINATES (r, $\theta, z$ )

In terms of $\tau:$
$\quad r$-componcnt
$\quad \rho\left(\frac{\partial v_{r}}{\partial t}+v_{r} \frac{\partial v_{r}}{\partial r}+\frac{v_{\theta}}{r} \frac{\partial v_{r}}{\partial \theta}-\frac{v_{\theta}^{2}}{r}+v_{i} \frac{\partial v_{r}}{\partial z}\right)=-\frac{\partial \rho}{\partial r}$

$$
\begin{equation*}
-\left(\frac{1}{r} \frac{\partial}{\partial r}\left(r \tau_{r r}^{\prime}\right)+\frac{1}{r} \frac{\partial \tau_{r \theta}}{\partial \theta}-\frac{\tau_{\theta \theta}}{r}+\frac{\partial \tau_{r z}}{\partial z}\right)+p g_{r} \tag{A}
\end{equation*}
$$

$\theta$-component ${ }^{5} \quad \rho\left(\frac{\partial v_{\theta}}{\partial t}+v_{r} \frac{\partial v_{\theta}}{\partial r}+\frac{v_{0}}{r} \frac{\partial v_{0}}{\partial 0}+\frac{v_{r} v_{0}}{r}+v_{2} \frac{\partial v_{\theta}}{\partial z}\right)=-\frac{1}{r} \frac{\partial p}{\partial \theta}$

$$
\begin{equation*}
-\left(\frac{1}{r^{2}} \frac{\partial}{\partial r}\left(r^{2} \tau_{r \theta}\right)+\frac{1}{r} \frac{\partial \tau_{0 \theta}}{\partial \theta}+\frac{\partial \tau_{\theta z}}{\partial z}\right)+\rho g_{\theta} \tag{B}
\end{equation*}
$$

$z$-component $\quad \rho\left(\frac{\partial v_{z}}{\partial t}+v_{r} \frac{\partial v_{z}}{\partial r}+\frac{v_{\theta}}{r} \frac{\partial v_{z}}{\partial \theta}+v_{z} \frac{\partial v_{z}}{\partial z}\right)=-\frac{\partial p}{\partial z}$

$$
\begin{equation*}
-\left(\frac{1}{r} \frac{\partial}{\partial r}\left(r \tau_{r z}\right)+\frac{1}{r} \frac{\partial \tau_{0 z}}{\partial \theta}+\frac{\partial \tau_{z z}}{\partial z}\right)+p g_{z} \tag{C}
\end{equation*}
$$

In terms of velocity gradients for a Newtonian fluid with constant $\rho$ and $\mu$ :

$$
\begin{align*}
& r \text {-component }{ }^{2} \quad \rho\left(\frac{\partial v_{r}}{\partial t}+v_{r} \frac{\partial v_{r}}{\partial r}+\frac{v_{\theta}}{r} \frac{\partial v_{r}}{\partial \theta}-\frac{\dot{v}_{\theta}^{2}}{r}+v_{a} \frac{\partial v_{r}}{\partial z}\right)=-\frac{\partial p}{\partial r} \\
& +\mu\left[\frac{\partial}{\partial r}\left(\frac{1}{r} \frac{\partial}{\partial r}\left(r v_{r}\right)\right)+\frac{1}{r^{2}} \frac{\partial^{2} v_{r}}{\partial 0^{2}}-\frac{2}{r^{2}} \frac{\partial v_{\theta}}{\partial \theta}+\frac{\partial^{2} v_{r}}{\partial z^{2}}\right]+g_{r}  \tag{D}\\
& \theta \text {-componeni }{ }^{\mathrm{D}} \quad \rho\left(\frac{\partial v_{\theta}}{\partial t}+v_{r} \frac{\partial v_{\theta}}{\partial r}+\frac{v_{\theta}}{r} \frac{\partial v_{\theta}}{\partial 0}+\frac{v_{r} v_{\theta}}{r}+v_{\Sigma} \frac{\partial v_{\theta}}{\partial z}\right)=-\frac{1}{r} \frac{\partial \rho}{\partial \theta} \\
& +\mu\left[\frac{\partial}{\partial r}\left(\frac{1}{r} \frac{\partial}{\partial r}\left(r v_{\theta}\right)\right)+\frac{1}{r^{2}} \frac{\partial^{2} v_{\theta}}{\partial \theta^{2}}+\frac{2}{r^{2}} \frac{\partial v_{r}}{\partial \theta}+\frac{\partial^{2} v_{\theta}}{\partial z^{2}}\right]+\rho g_{\theta}  \tag{E}\\
& z \text {-component } \rho\left(\frac{\partial v_{z}}{\partial t}+v_{r} \frac{\partial v_{2}}{\partial r}+\frac{v_{\theta}}{r} \frac{\partial v_{z}}{\partial \theta}+v_{z} \frac{\partial v_{z}}{\partial z}\right)=-\frac{\partial p}{\partial z} \\
& +\mu\left[\frac{1}{r} \frac{\partial}{\partial r}\left(r \frac{\partial v_{z}}{\partial r}\right)+\frac{1}{r^{2}} \frac{\partial^{2} v_{z}}{\partial \theta^{2}}+\frac{\partial^{2} v_{z}}{\partial z^{2}}\right]+f g_{z} \tag{F}
\end{align*}
$$

[^0]TABLE $3.4-4$
the equation of motion in sphericai coordinates (r, $\theta, \phi$ )

$$
\begin{align*}
& \text { In terms of } 2 \text { : } \\
& r \text {-component } \rho\left(\frac{\partial v_{r}}{\partial t}+v_{r} \frac{\partial v_{r}}{\partial r}+\frac{v_{\theta}}{r} \frac{\partial v_{r}}{\partial \theta}+\frac{v_{\phi}}{r \sin \theta} \frac{\partial v_{r}}{\partial \phi}-\frac{v_{\theta}^{2}+v_{\phi}^{2}}{r}\right) \\
& =-\frac{\partial p}{\partial r}-\left(\frac{1}{r^{2}} \frac{\partial}{\partial r}\left(r^{2} \tau_{r r}\right)+\frac{1}{r \sin \theta} \frac{\partial}{\partial \theta}\left(r_{r \theta} \sin \theta\right)\right. \\
& \left.+\frac{1}{r \sin \theta} \frac{\partial \tau_{r \phi}}{\partial \phi}-\frac{\tau_{\theta g}+\tau_{\phi \phi}}{r}\right)+\rho_{\rho} g_{r} \\
& \text { ब-component } \rho\left(\frac{\partial v_{\theta}}{\partial t}+v_{r} \frac{\partial v_{\theta}}{\partial r}+\frac{v_{\theta}}{r} \frac{\partial v_{\theta}}{\partial \theta}+\frac{v_{\phi}}{r \sin \theta} \frac{\partial v_{\theta}}{\partial \phi}+\frac{v_{r} v_{\theta}}{r}-\frac{v_{\phi}{ }^{2} \cot \theta}{r}\right) \\
& =-\frac{1}{r} \frac{\partial p}{\partial \theta}-\left(\frac{1}{r^{2}} \frac{\partial}{\partial r}\left(r^{2} \tau_{r \theta}\right)+\frac{1}{r \sin \theta} \frac{\partial}{\partial \theta}\left(r_{\theta \theta} \sin \theta\right)+\frac{1}{r \sin \theta} \frac{\partial \tau_{\theta \phi}}{\partial \phi}\right. \\
& \left.+\frac{\tau_{\theta \theta}}{r}-\frac{\cot \theta}{r} \tau_{\phi \phi}\right)+\rho g_{\theta}  \tag{B}\\
& \phi \text {-componeni } \rho\left(\frac{\partial v_{\phi}}{\partial t}+v_{r} \frac{\partial v_{\phi}}{\partial r}+\frac{v_{\theta}}{r} \frac{\partial v_{\phi}}{\partial \theta}+\frac{v_{\phi}}{r \sin \theta} \frac{\partial v_{\phi}}{\partial \phi}+\frac{v_{\phi} v_{r}}{r}+\frac{v_{\theta} v_{\phi}}{r} \cot \theta\right) \\
& =-\frac{1}{r \sin \theta} \frac{\partial p}{\partial \phi}-\left(\frac{1}{r^{2}} \frac{\partial}{\partial r}\left(r^{2} \tau_{r \phi}\right)+\frac{1}{r} \frac{\partial \tau_{\theta_{\phi}}}{\partial \theta}+\frac{1}{r \sin \theta} \frac{\partial r_{\phi \phi}}{\partial \phi}\right. \\
& \left.+\frac{T_{r \phi}}{r}+\frac{2 \cot \theta}{r} \tau_{\theta \phi}\right)+P g_{\phi} \tag{C}
\end{align*}
$$

## TABLE 3.44 (concd.)

In terms of velocity gradients for a Newtonian fluid with constant $\rho$ and $\mu$ :

$$
\begin{gather*}
r \text {-component } p\left(\frac{\partial v_{r}}{\partial t}+v_{r} \frac{\partial v_{r}}{\partial r}+\frac{\nu_{\theta}}{r} \frac{\partial v_{r}}{\partial \theta}+\frac{v_{\phi}}{r \sin \theta} \frac{\partial u_{r}}{\partial \phi}-\frac{v_{0}^{2}+v_{d}^{2}}{r}\right) \\
=-\frac{\partial p}{\partial r}+\mu\left(\frac{1}{r^{2}} \frac{\partial^{2}}{\partial r^{2}}\left(r^{2} v_{1}\right)+\frac{1}{r^{2} \sin \theta} \frac{\partial}{\partial \theta}\left(\sin \theta \frac{\partial v_{\rho}}{\partial \theta}\right)+\frac{1}{r^{1} \sin \theta} \frac{\partial^{2} v_{p}}{\partial \phi}\right) \\
+\mu g_{r} \tag{D}
\end{gather*}
$$

$$
\begin{align*}
& \theta \text {-component } \rho\left(\frac{\partial v_{\theta}}{\partial t}+v_{r} \frac{\partial v_{\theta}}{\partial r}+\frac{v_{\theta}}{r} \frac{\partial v_{\theta}}{\partial \theta}+\frac{v_{\phi}}{r \sin \theta} \frac{\partial v_{\theta}}{\partial \phi}+\frac{\nu_{r} v_{\theta}}{r}-\frac{v_{\phi}^{2} \cot \theta}{r}\right) \\
& =-\frac{1}{r} \frac{\partial p}{\partial \theta}+\mu\left(\frac{1}{r^{2}} \frac{\partial}{\partial r}\left(r^{2} \frac{\partial v_{\theta}}{\partial r}\right)+\frac{1}{r^{2}} \frac{\partial}{\partial \theta}\left(\frac{1}{\sin \theta} \frac{\partial}{\partial \theta}\left(v_{\theta} \sin \theta\right)\right)+\frac{1}{r^{2} \sin ^{2} \theta} \frac{\partial: v_{\theta}}{\partial \phi^{\prime}}\right. \\
& \left.\quad+\frac{2}{r^{2}} \frac{\partial v_{r}}{\partial \theta}-\frac{2 \cos \theta}{r^{2} \sin ^{2} \theta} \frac{\partial v_{\phi}}{\partial \phi}\right)+\rho g_{\theta} \tag{E}
\end{align*}
$$

p-comp.onent $\rho\left(\frac{\partial v_{\phi}}{\partial t}+v_{r} \frac{\partial u_{\phi}}{\partial r}+\frac{v_{0}}{r} \frac{\partial v_{\phi}}{\partial \theta}+\frac{v_{\phi}}{r \sin \theta} \frac{\partial v_{\phi}}{\partial \phi}+\frac{v_{\phi} v_{r}}{r}+\frac{v_{\phi} v_{\phi}}{r} \cot \theta\right)$
$=-\frac{1}{r \sin \theta} \frac{\partial p}{\partial \phi}+\mu\left(\frac{1}{r^{2}} \frac{\partial}{\partial r}\left(r^{2} \frac{\partial \nu_{\theta}}{\partial r}\right)+\frac{1}{r} \frac{\partial}{\partial \theta}\left(\frac{1}{\sin \theta} \frac{\partial}{\partial \theta}\left(\nu_{0} \sin \theta\right)\right)+\frac{1}{r^{2} \sin ^{2} \theta} \frac{\partial^{2} \nu_{\theta}}{\partial \phi^{2}}\right.$

$$
\left.+\frac{2}{r^{2} \sin \theta} \frac{\partial \nu_{r}}{\partial \phi}+\frac{2 \cos 0}{r^{2} \sin ^{2} \theta} \frac{\partial \nu_{e}}{\partial \phi}\right)+\rho g_{\phi} \quad(F)
$$

COMPONENTS OF THE :TRESS TENSOR FOR NEWTONIAN tLUILS

in rectangular coordinates $(x, y, x)$

$$
\begin{align*}
& \tau_{z z}=-\mu\left[2 \frac{\partial v_{z}}{\partial x}-f(\nabla \cdot \nabla)\right]  \tag{A}\\
& \tau_{v y}=-\mu\left[2 \frac{\partial v_{y}}{\partial y}-f(\nabla \cdot \nabla)\right]  \tag{B}\\
& \tau_{z z}=-\mu\left[2 \frac{\partial v_{z}}{\partial z}-f(\nabla \cdot \nabla)\right]  \tag{C}\\
& \tau_{v y}=\tau_{v z}=-\mu\left[\frac{\partial v_{x}}{\partial y}+\frac{\partial v_{y}}{\partial x}\right]  \tag{D}\\
& T_{v z}=\tau_{z v}=-\mu\left[\frac{\partial v_{v}}{\partial z}+\frac{\partial v_{s}}{\partial y}\right]  \tag{E}\\
& T_{z v}=\tau_{x \varepsilon}=-\mu\left[\frac{\partial v_{z}}{\partial x}+\frac{\partial v_{z}}{\partial z}\right]  \tag{F}\\
& (\nabla \cdot \theta)=\frac{\partial v_{z}}{\partial x}+\frac{\partial v_{v}}{\partial y}+\frac{\partial v_{z}}{\partial z} \tag{G}
\end{align*}
$$

TABLE 3.46
COMPONENTS OF THE STRESS TENSOR FOR NEWTONIAN FLUIDS
IN CYUNDRICAL COORDINATES $(1, \theta, 2)$

$$
\begin{align*}
& T_{r r}=-\mu\left[2 \frac{\partial v_{r}}{\partial_{r}}-f(\nabla \cdot \sigma)\right] \\
& T_{\theta \theta}=-\mu\left[2\left(\frac{1}{r} \frac{\partial v_{\theta}}{\partial \theta}+\frac{v_{r}}{r}\right)-f(\nabla \cdot \theta)\right]  \tag{B}\\
& \tau_{A x}=-\mu\left[2 \frac{\partial v_{s}}{\partial z}-\frac{z}{\partial}(\nabla \cdot \theta)\right]  \tag{C}\\
& T_{r \theta}=T_{\theta r}=-\mu\left[r \frac{\partial}{\partial r}\left(\frac{v_{\theta}}{r}\right)+\frac{1}{r} \frac{\partial v_{r}}{\partial \theta}\right]  \tag{D}\\
& \tau_{\theta A}=\tau_{1 \theta}=-\mu\left[\frac{\partial v_{0}}{\partial z}+\frac{1}{r} \frac{\partial v_{s}}{\partial \theta}\right]  \tag{E}\\
& r_{t r}=r_{r s}=-\mu\left[\frac{\partial 0_{a}}{\partial r}+\frac{\partial v_{r}}{\partial x}\right]  \tag{F}\\
& (\nabla \cdot \nabla)=\frac{1}{r} \frac{\partial}{\partial r}\left(r v_{r}\right)+\frac{1}{r} \frac{\partial v_{0}}{\partial \theta}+\frac{\partial v_{i}}{\partial z} \tag{G}
\end{align*}
$$

TABLE 3.4-7
Cl COMPONENTS OF THE STRESS TENSOR FOR NEWTONIAN FLUIDS $S$


TABLE 3.4-8
THE FUNCTION - $(\tau: \nabla 0)=\mu \Phi$, FOR NEWTONIAN FLUIDS*

Rectangular

Cylindrical

$$
\begin{align*}
\Phi_{v}=2 & {\left[\left(\frac{\partial v_{x}}{\partial x}\right)^{2}+\left(\frac{\partial v_{y}}{\partial y}\right)^{2}+\left(\frac{\partial v_{z}}{\partial z}\right)^{2}\right] } \\
& +\left[\frac{\partial v_{y}}{\partial x}+\frac{\partial v_{x}}{\partial y}\right]^{2}+\left[\frac{\partial v_{x}}{\partial y}+\frac{\partial v_{y}}{\partial z}\right]^{2}+\left[\frac{\partial v_{x}}{\partial x}+\frac{\partial v_{s}}{\partial x}\right]^{2} \\
& -\frac{2}{3}\left[\frac{\partial v_{x}}{\partial x}+\frac{\partial v_{y}}{\partial y}+\frac{\partial v_{z}}{\partial z}\right]^{2} \tag{A}
\end{align*}
$$

Spherical

$$
\Phi_{v}=2\left[\left(\frac{\partial v_{r}}{\partial r}\right)^{2}+\left(\frac{1}{r} \frac{\partial v_{0}}{\partial \theta}+\frac{v_{r}}{r}\right)^{2}+\left(\frac{\partial v_{z}}{\partial z}\right)^{2}\right]
$$

$$
+\left[r \frac{\partial}{\partial r}\left(\frac{v_{\theta}}{r}\right)+\frac{1}{r} \frac{\partial v_{r}}{\partial 0}\right]^{2}+\left[\frac{1}{r} \frac{\partial v_{3}}{\partial \theta}+\frac{\partial v_{\theta}}{\partial z}\right]^{2}
$$

$$
+\left[\frac{\partial v_{r}}{\partial z}+\frac{\partial v_{3}}{\partial r}\right]^{z}
$$

$$
\begin{equation*}
-\frac{2}{3}\left[\frac{1}{r} \frac{\partial}{\partial r}\left(r v_{r}\right)+\frac{1}{r} \frac{\partial v_{\theta}}{\partial \theta}+\frac{\partial v_{z}}{\partial z}\right]^{2} \tag{B}
\end{equation*}
$$

$$
\Phi_{v}=2\left[\left(\frac{\partial v_{r}}{\partial r}\right)^{2}+\left(\frac{1}{r} \frac{\partial v_{\theta}}{\partial \theta}+\frac{v_{r}}{r}\right)^{2}\right.
$$

$$
\left.+\left(\frac{1}{r \sin \theta} \frac{\partial v_{\phi}}{\partial \phi}+\frac{v_{r}}{r}+\frac{v_{0} \cot \theta}{r}\right)^{2}\right]
$$

$$
+\left[r \frac{\partial}{\partial r}\left(\frac{v_{\theta}}{r}\right)+\frac{i}{r} \frac{\partial v_{r}}{\partial \theta}\right]^{2}
$$

$$
+\left[\frac{\sin 0}{r} \frac{\partial}{\partial \theta}\left(\frac{v_{\phi}}{\sin \theta}\right)+\frac{-1}{r \sin \theta} \frac{\partial v_{0}}{\partial \phi}\right]^{2}
$$

$$
+\left[\frac{1}{r \sin \theta} \frac{\partial v_{r}}{\partial \phi}+r \frac{\partial}{\partial r}\left(\frac{v_{\phi}}{r}\right)\right]^{2}
$$

$$
\begin{equation*}
-\frac{2}{3}\left[\frac{1}{r^{2}} \frac{\partial}{\partial r}\left(r^{2} v_{r}\right)+\frac{1}{r \sin \theta} \frac{\partial}{\partial \theta}\left(v_{0} \sin \theta\right)+\frac{1}{r \sin \theta} \frac{\partial v_{\phi}}{\partial \phi}\right]_{(C)}^{2} \tag{C}
\end{equation*}
$$

- These expressions are obtained by inserting the components of $\tau$ from Tables 3.4-5,6,7 into the expression for ( $\tau: \nabla v$ ) given in Appendix A. (See Tables A.7-1, 2, and 3.)

TABLE 10.2-1
COMPONENTS OF THE ENERGY FLUX $q$

| Rectangular | Cylindrical | Spherical |
| :---: | :---: | :---: |
| $\begin{equation*} q_{x}=-k \frac{\partial T}{\partial x} \tag{A} \end{equation*}$ | $\begin{equation*} q_{r}=-k \frac{\partial T}{\partial r} \tag{D} \end{equation*}$ | $\begin{equation*} q_{r}=-k \frac{\partial T}{\partial r} \tag{G} \end{equation*}$ |
| $\begin{equation*} q_{v}=-k \frac{\partial T}{\partial y} \tag{B} \end{equation*}$ | $\begin{equation*} q_{0}=-k \frac{1}{r} \frac{\partial T}{\partial \theta} \tag{E} \end{equation*}$ | $\begin{equation*} q_{0}=-k \frac{1}{r} \frac{\partial T}{\partial \theta} \tag{H} \end{equation*}$ |
| $\begin{equation*} q_{z}=-k \frac{\partial T}{\partial z} \tag{C} \end{equation*}$ | $\begin{equation*} q_{z}=-k \frac{\partial T}{\partial z} \tag{F} \end{equation*}$ | $q_{\phi}=-k \frac{1}{r \sin \theta} \frac{\partial T}{\partial \phi}(l)$ |

THE EQUATION OF ENERGY IN TERMS OF ENERGY AND MOMENTUM FLUXES

## Rectangular coordinates:

$$
\begin{align*}
& { }_{\rho} C_{v}\left(\frac{\partial T}{\partial l}+v_{x} \frac{\partial T}{\partial x}+v_{v} \frac{\partial T}{\partial y}+v_{z} \frac{\partial T}{\partial z}\right)=-\left[\frac{\partial q_{x}}{\partial x}+\frac{\partial q_{v}}{\partial y}+\frac{\partial q_{z}}{\partial z}\right] \\
& -T\left(\frac{\partial p}{\partial T}\right)_{\rho}\left(\frac{\partial v_{x}}{\partial x}+\frac{\partial v_{v}}{\partial y}+\frac{\partial v_{z}}{\partial z}\right)-\left\{\tau_{i x} \frac{\partial v_{x}}{\partial x}+\tau_{v v} \frac{\partial v_{v}}{\partial y}+\tau_{1 i} \frac{\partial v_{z}}{\partial z}\right\} \\
& -\left\{T_{x v}\left(\frac{\partial v_{z}}{\partial y}+\frac{\partial y_{y}}{\partial x}\right)+t_{z z}\left(\frac{\partial v_{z}}{\partial z}+\frac{\partial v_{z}}{\partial x}\right)+\tau_{y z}\left(\frac{\partial v_{y}}{\partial z}+\frac{\partial v_{z}}{\partial y}\right)\right\} \tag{A}
\end{align*}
$$

$$
\begin{align*}
& \text { Cylindrical coordinates: } \\
& \begin{array}{l}
\rho C_{v}\left(\frac{\partial T}{\partial t}+v_{r} \frac{\partial T}{\partial r}+\frac{v_{0}}{r} \frac{\partial T}{\partial \theta}+v_{z} \frac{\partial T}{\partial z}\right)=-\left[\frac{1}{r} \frac{\partial}{\partial r}\left(r q_{r}\right)+\frac{1}{r} \frac{\partial q_{\theta}}{\partial \theta}+\frac{\partial q_{z}}{\partial z}\right] \\
\\
-T\left(\frac{\partial p}{\partial T}\right)_{\rho}\left(\frac{1}{r} \frac{\partial}{\partial r}\left(r v_{r}\right)+\frac{1}{r} \frac{\partial v_{\theta}}{\partial \theta}+\frac{\partial v_{z}}{\partial z}\right)-\left\{\tau_{r r} \frac{\partial v_{r}}{\partial r}+\tau_{00} \frac{1}{r}\left(\frac{\partial v_{\theta}}{\partial \theta}+v_{r}\right)\right. \\
\left.\quad+\tau_{z z} \frac{\partial v_{z}}{\partial z}\right\}-\left\{\tau_{r \theta}\left[r \frac{\partial}{\partial r}\left(\frac{v_{\theta}}{r}\right)+\frac{1}{r} \frac{\partial v_{r}}{\partial \theta}\right]+\tau_{r s}\left(\frac{\partial v_{z}}{\partial r}+\frac{\partial v_{r}}{\partial z}\right)\right. \\
\left.\quad+\tau_{0_{z}}\left(\frac{1}{r} \frac{\partial v_{z}}{\partial 0}+\frac{\partial v_{0}}{\partial z}\right)\right\}
\end{array}
\end{align*}
$$

$$
\begin{align*}
& \text { Spherical coordinates: } \\
& \begin{aligned}
& \rho C_{v}\left(\frac{\partial T}{\partial r}+v_{r} \frac{\partial T}{\partial r}+\frac{v_{0}}{r} \frac{\partial T}{\partial \theta}+\frac{v_{\phi}}{r \sin \theta} \frac{\partial T}{\partial \phi}\right)=-\left[\frac{1}{r^{2}} \frac{\partial}{\partial r}\left(r^{2} q_{r}\right)\right. \\
&\left.\quad+\frac{1}{r \sin \theta} \frac{\partial}{\partial \theta}\left(q_{0} \sin \theta\right)+\frac{1}{r \sin \theta} \frac{\partial q_{\phi}}{\partial \phi}\right]-T\left(\frac{\partial \rho}{\partial T}\right)_{\rho}\left(\frac{1}{r^{2}} \frac{\partial}{\partial r}\left(r^{2} v_{r}\right)\right. \\
&\left.\quad+\frac{1}{r \sin \theta} \frac{\partial}{\partial \theta}\left(v_{0} \sin \theta\right)+\frac{1}{r \sin \theta} \frac{\partial v_{\phi}}{\partial \phi}\right)-\left\{\tau_{r r} \frac{\partial v_{r}}{\partial r}+\tau_{\theta 0}\left(\frac{1}{r} \frac{\partial v_{\theta}}{\partial \theta}+\frac{v_{r}}{r}\right)\right. \\
&\left.\quad+\tau_{\phi \phi}\left(\frac{1}{r \sin \theta} \frac{\partial v_{\phi}}{\partial \phi}+\frac{v_{r}}{r}+\frac{v_{0} \cot \theta}{r}\right)\right\}-\left\{r_{r 0}\left(\frac{\partial v_{\theta}}{\partial r}+\frac{1}{r} \frac{\partial v_{r}}{\partial \theta}-\frac{v_{\theta}}{r}\right)\right. \\
&\left.\quad+\tau_{r \phi}\left(\frac{\partial v_{\phi}}{\partial r}+\frac{1}{r \sin \theta} \frac{\partial v_{r}}{\partial \phi}-\frac{v_{\phi}}{r}\right)+\tau_{\theta \phi}\left(\frac{1}{r} \frac{\partial v_{\phi}}{\partial \theta}+\frac{1}{r \sin \theta} \frac{\partial v_{\theta}}{\partial \phi}-\frac{\cot \theta}{r} v_{\phi}\right)\right\}
\end{aligned}
\end{align*}
$$

rms contained in braces \{ \} are associated with viscous dissipation and may usually be neglected, except for systems with large velocity gradienis.

TABLE 10.2-3
the equation of energy in terms of the transport properties
( (or Newtonian fluids of constant $p$ and $k$ )
(Eq.10.1-2.5 with viscous dissipation terms included)

## Rectangular coordinates:

$$
\begin{align*}
& \rho C_{p}\left(\frac{\partial T}{\partial t}+v_{x} \frac{\partial T}{\partial x}+v_{v} \frac{\partial T}{\partial y}+v_{z} \frac{\partial T}{\partial z}\right)=k\left[\frac{\partial^{2} T}{\partial x^{2}}+\frac{\partial^{2} T}{\partial y^{2}}+\frac{\partial^{2} T}{\partial z^{2}}\right] \\
& \quad+2 \mu\left(\left(\frac{\partial v_{x}}{\partial x}\right)^{2}+\left(\frac{\partial v_{y}}{\partial y}\right)^{2}+\left(\frac{\partial v_{z}}{\partial z}\right)^{2}\right)+\mu\left(\left(\frac{\partial v_{x}}{\partial y}+\frac{\partial v_{v}}{\partial x}\right)^{2}\right. \\
& \left.\quad+\left(\frac{\partial v_{x}}{\partial z}+\frac{\partial v_{x}}{\partial x}\right)^{2}+\left(\frac{\partial v_{y}}{\partial z}+\frac{\partial v_{z}}{\partial y}\right)^{2}\right\} \tag{A}
\end{align*}
$$

$$
\begin{align*}
& \text { Cylindrical coordinates: } \\
& \rho_{\rho} C_{p}\left(\frac{\partial T}{\partial t}+v_{r} \frac{\partial T}{\partial r}+\frac{v_{\theta}}{r} \frac{\partial T}{\partial \theta}+v_{s} \frac{\partial T}{\partial z}\right)=k\left[\frac{1}{r} \frac{\partial}{\partial r}\left(r \frac{\partial T}{\partial r}\right)+\frac{1}{r^{2}} \frac{\partial^{2} T}{\partial \theta^{2}}+\frac{\partial^{2} T}{\partial z^{2}}\right] \\
& \quad+2 \mu\left(\left(\frac{\partial v_{r}}{\partial r}\right)^{2}+\left[\frac{1}{r}\left(\frac{\partial v_{\theta}}{\partial \theta}+v_{r}\right)\right]^{2}+\left(\frac{\partial v_{s}}{\partial z}\right)^{2}\right\}+\mu\left(\left(\frac{\partial v_{\theta}}{\partial z}+\frac{1}{r} \frac{\partial v_{s}}{\partial \theta}\right)^{2}\right. \\
& \left.\left.\quad+\left(\frac{\partial v_{z}}{\partial r}+\frac{\partial \theta_{r}}{\partial z}\right)^{2}+\left[\frac{1}{r} \frac{\partial v_{r}}{\partial \theta}+r \frac{\partial}{\partial r}\left(\frac{v_{\theta}}{r}\right)\right]\right]^{2}\right\} \tag{B}
\end{align*}
$$

Spherical coordinates:

$$
\begin{align*}
& \rho \mathrm{C}_{p}\left(\frac{\partial T}{\partial t}+v_{r} \frac{\partial T}{\partial r}+\frac{v_{\theta}}{r} \frac{\partial T}{\partial \theta}+\frac{v_{\phi}}{r \sin \theta} \frac{\partial T}{\partial \phi}\right)=k\left[\frac{1}{r^{2}} \frac{\partial}{\partial r}\left(r^{2} \frac{\partial T}{\partial r}\right)\right. \\
&\left.+\frac{1}{r^{2} \sin \theta} \frac{\partial}{\partial \theta}\left(\sin \theta \frac{\partial T}{\partial \theta}\right)+\frac{1}{r^{2} \sin ^{2} \theta} \frac{\partial^{2} T}{\partial \phi^{2}}\right]+2 \mu\left(\left(\frac{\partial v_{r}}{\partial r}\right)^{2}\right. \\
&\left.+\left(\frac{1}{r} \frac{\partial v_{\theta}}{\partial \theta}+\frac{v_{r}}{r}\right)^{2}+\left(\frac{1}{r \sin \theta} \frac{\partial v_{\phi}}{\partial \phi}+\frac{v_{r}}{r}+\frac{v_{\theta} \cot \theta}{r}\right)^{2}\right) \\
&+\mu\left(\left[r \frac{\partial}{\partial r}\left(\frac{v_{\theta}}{r}\right)+\frac{1}{r} \frac{\partial v_{r}}{\partial \theta}\right]^{2}+\left[\frac{1}{r \sin \theta} \frac{\partial v_{r}}{\partial \phi}+r \frac{\partial}{\partial r}\left(\frac{v_{\phi}}{r}\right)\right]^{2}\right. \\
&\left.+\left[\frac{\sin \theta}{r} \frac{\partial}{\partial \theta}\left(\frac{v_{\phi}}{\sin \theta}\right)+\frac{1}{r \sin \theta} \frac{\partial v_{\theta}}{\partial \phi}\right]^{2}\right) \tag{C}
\end{align*}
$$

Nole: The terms contained in braces $\}$ are associated with viscous dissipation and may usually be neglected, except for systems with large velocity gradients.


Fig for. $Q .(6)$

Fig for $Q \cdot 5(a)$


Fig for Q.F(a)

Fig for $Q .8(a)$


## BANGLADESH UNIVERSITY OF ENGINEERING AND TECHNOLOGY, DHAKA

## L-4/T-1 $\quad$ B. Sc. Engineering Examinations 2011-2012

Sub : CHE 455 (Mathematical Models in Chemical Engineering)
Full Marks : 210
Time: 3 Hours
The figures in the margin indicate full marks.
USE SEPARATE SCRIPTS FOR EACH SECTION

## SECTION - A

There are FOUR questions in this section. Answer any THREE.

1. (a) Benzene (1), toluene (2), styrene (3) and xylene (4) are to be sepqrated in the sequence of distillation columns shown in Fig. Q. 1(a). Determine molar flow rates of streams $D_{1}, B_{1}, D_{2}$ and $\mathrm{B}_{2}$ by solving material balance equation using Gauss-Seidel method.

(b) What are the steps to build process model?
(c) Write the names of Fundamental Laws used in process model / First Principle Model in chemical engineering problem.
2. A hot vapor stream containing 0.4 mole fraction ammonia and 0.6 mole fraction water is to be enriched in a distillation column consisting of enriching section and total condenser. The saturated vapor at 6.8 atm pressure ( 100 psia ) is injected at a rate $100 \mathrm{moles} / \mathrm{hour}$ at the bottom of the column. The liquid distillate product withdrawn from the total condenser has a composition 0.9 mole fraction $\mathrm{NH}_{3}$. Part of the distillate is returned as reflux, so that $85 \%$ of the $\mathrm{NH}_{3}$ charged must be recovered as distillate product.
(a) Write the material balance between Feed stage and the nth stage. Find mole fraction $\mathrm{NH}_{3}$ in liquid leaving Nth tray, $\mathrm{x}_{\mathrm{N}}$ and the required reflux ratio, R .
(b) Derive the finite difference equation for $\mathrm{x}_{\mathrm{n}}$ from the material balance equation and vapor-liquid equilibrium relationship [Hint: use expression of relative volatility, $\alpha$ ).
(c) Rearrange the expression in part (b) to show that Riccati equation arises

$$
\begin{equation*}
X_{n} X_{n+1}+A X_{n+1}+B X_{n}+C=0 \tag{10}
\end{equation*}
$$

## CHE 455

3. (a) Write the recursive formula for $y\left(t_{n+1}\right)$ for the Trapezoidal rule.
(b) Show that Trapezoidal method is always stable.
(c) Consider a reaction $\mathrm{A} \rightarrow \mathrm{B}$ carried out in a batch reactor. The differential equation for species $A$ is

$$
\begin{equation*}
\frac{\mathrm{dC}_{\mathrm{A}}}{\mathrm{dt}}=-\mathrm{kC}_{\mathrm{A}}^{2} \tag{20}
\end{equation*}
$$

The initial condition is at: $t=0, \mathrm{C}_{\mathrm{A}}=2 \mathrm{~mol} / \mathrm{m}^{3}$. The rate constant of the reaction is $1 \mathrm{~s}^{-1}$. Using the Runge-Kutta fourth order method determine the concentration of $A$ at 3 s . Use step size $=0.5$.
4. (a) Determine the root of equation using Newton's method.

$$
15 x^{3}-16 x^{2}+95 x-47=0
$$

(b) Derive the Jacobian Matrix for N simultaneous nonlinear algebraic equations for N unknowns. Prove that
where, $\underline{J}=$ is jacobian matrix
$\underline{\delta}=$ is the correction vector
$\underline{\mathrm{f}}=$ is the vector function

## SECTION - B

There are FOUR questions in this section. Answer any THREE.
5. (a) What are the essential features of optimization? Write down the general mathematical formulation of an optimization problem.
(b) A poster is to contain $300 \mathrm{~cm}^{2}$ of printed matter with margins of 6 cm at the top and bottom and 4 cm at each side. Find the overall dimensions that minimize the total area of the poster.
(c) Find the point on the curve $f(x)=x^{2}-1$ nearest the origin. [Hint: Distance between two points is given by $\left.\sqrt{(\text { abscissa })^{2}+(\text { (tordinate })^{2}}\right]$.
(d) Write down at least three major difficulties experienced in formulating optimization problems.
6. (a) Write some names of the methods used for solving NLP problems. State relative merits of SLP, SQP and GRG algorithms.
(b) Solve the following problem via the Lagrange multiplier method:

Find the values of $\mathrm{x}_{1}, \mathrm{x}_{2}$ and $\omega$ that
Minimize: $\quad f(x)=x_{1}{ }^{2}+x_{2}{ }^{2}$
subject to: $h(x)=2 x_{1}+x_{2}-2=0$

## CHE 455

## Contd ... O. No. 6

(c) For the following reactor, the objective function is $f(c, T)=\left(c-c_{r}\right)^{2}+T^{2}$. The
constraint
objective function is subject to the $c=c_{0}+e^{T}$ and also $c_{0}<K$, where $c_{r}$ is the set point for the outlet concentration, a constant and K is also a constant. Develop the Lagrangian function and first order necessary conditions for finding the minimum value of the objective function. (You do not need to solve the problem).

7. (a) A confectioner manufactures two kinds of candy bars: Ergies and Nergies. Ergies sell at a profit of $50 \notin$ per box, and Nergies have a profit of $60 \notin$ per box. The candy is processed in three main operations: blending, cooking and packaging. The table below records the average time in minutes required by each box of candy, for each of the three activities.

|  | Blending | Cooking | Packing |
| :--- | :---: | :---: | :---: |
| Ergies | 1 | 5 | 3 |
| Nergies | 2 | 4 | 1 |

During each production run, the blending equipment is available for a maximum of 14 machine hours, the cooking equipment for at most 14 hours, and the packaging equipment for at most 15 machine hours. If each machine can be allocated to the making of either type of candy at all times that it is available for production, determine how many boxes of each kind of candy the confectioner should make to realize the maximum profit by simplex method.
(b) What are the basic steps of solution of LP problems by simplex method? Why do linear constraints arise?
8. (a) A refinery has available two crude oils that have the yields shown in the following table. Because of equipment and storage limitations, production of gasoline, kerosene and fuel oil must be limited as shown in table 8(a). There are no plant limitations on the production of other products such as gas oils.

$$
=4=
$$

## CHE 455

## Contd ... O. No. 8(a)

The profit on processing crude\#1 is $\$ 1 / \mathrm{bbl}$ and on crude\#2 it is $\$ 0.7 / \mathrm{bbl}$. Find the approximate optimum daily feed rates of the two crudes to this plant via a graphical method.

|  | Volume percent yields |  | Maximum allowable <br> product rate (bbl/day) |
| :--- | :---: | :---: | :---: |
|  | Crude\#1 | Crude\#2 | 6,000 |
| Gasoline | 70 | 31 | 2,400 |
| Kerosene | 6 | 9 | 12,000 |
| Fuel Oil | 24 | 60 |  |

(b) Prepare a graph of the constraints and objective function, and solve the following linear programming problem.

$$
\begin{array}{ll}
\text { Maximize: } & x_{1}+2 x_{2} \\
\text { subject to: } & -x_{1}+3 x_{2}<10 \\
& x_{1}+x_{2} \leq 6 \\
& x_{1}-x_{2} \leq 2 \\
& x_{1}+3 x_{2} \geq 6 \\
& 2 x_{1}+x_{2} \geq 4 ; \quad x_{1} \geq 0 ; \quad x_{2} \geq 0
\end{array}
$$


[^0]:    - The term $\rho \nu_{0}^{2} \beta r$ is the centrifugal force. It gives the effective force in the $r$-direction resulting from fluid motion in the 0 -direction. This term arises automatically on transformation from rectangular to cylindrical coordinates; it does not have to be added on physical grounds. Two problems in which this term arises are discussed in Examples 3.5-1 and 3.5-2.
    b The term $\rho_{r y} v_{0} / \mathrm{r}$ is the Coriolis force. It is an effective force in the $\theta$-direction when there is flow in both the $r$ - and 0 -directions. This term also arises automatically in the coordinate transformation. The Coriolis force arises in the problem of flow near a rotating disk (see, for example, H. Schlichting, Boundary-Layer Theory, McGraw-Hill, New York (1955), Chapter S, §10.

