1. (a) Discuss the continuity and differentiability of the function
\[ f(x) = \begin{cases} x^2 \sin \left( \frac{1}{x} \right), & x < 0 \\ \sqrt{x}, & x \geq 0 \end{cases} \]
at \( x = 0 \). Also sketch the graph of \( f(x) \) near \( x = 0 \).
(b) Evaluate the following:
(i) \( \lim_{x \to 0} \left[ \sqrt{x} - \ln \left( x^2 + 1 \right) \right] \)
(ii) \( \lim_{x \to 0} \left( e^x + x \right)^{\frac{1}{x}} \)

2. (a) State and prove the Mean-Value theorem. Verify the Mean-Value theorem for the function \( f(x) = x^3 + x - 4, \ [-1,2] \).
(b) State Leibnitz's theorem. If \( y = e^{\sin^{-1}x} \), then prove that
\[ (1 - x^2) y_{n+2} - (2n + 1)x y_{n+1} - n^2 y_n = 0 \]
(c) Show that the Cauchy's remainder after \( n \) terms in the expansion of \( \ln(1 + x) \) in powers of \( x \) is \( (-1)^{n-1} \frac{x^n}{1+\theta} \left( 1+\theta \right)^{-n}, \ 0 < \theta < 1 \).

3. (a) If \( f(x, y, z) \) be a homogeneous function in \( x, y, z \) of degree \( n \) having continuous partial derivatives then prove that
\[ \frac{\partial f}{\partial x} + y \frac{\partial f}{\partial y} + z \frac{\partial f}{\partial z} = n f(x, y, z). \]
(b) If \( U = F \left( x^2 + y^2 + z^2 \right) f \left( xy + yz + zx \right) \), then find the value of
\[ \frac{\partial U}{\partial x} + \frac{\partial U}{\partial y} + \frac{\partial U}{\partial z} \]
(c) If the tangent at \( (x_1, y_1) \) to the curve \( x^3 + y^3 = a^3 \) meets the curve again in \( (x_2, y_2) \), then show that \( \frac{x_2}{x_1} + \frac{y_2}{y_1} = -1 \).

4. (a) Discuss the maximum and minimum of the function \( f(x) = (x - a)^2(2x - a)^3 \) for \( a > 0 \).
(b) Find all asymptotes of the curve \( x^3 + x^2y - xy^2 - y^3 + 2xy + 2y^2 - 3x + y = 0 \).
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Contd ..., Q. No. 4

(c) For the polar equation \( r = f(\theta) \), show that the radius of curvature at \((r, \theta)\) is
\[
\rho = \frac{r^2 + r'^2}{r^2 + 2r'^2 - rr''}.
\]
Also find the radius of curvature for the curve \( r = a \sin n\theta \) at the origin.

SECTION - B

There are FOUR questions in this Section. Answer any THREE.

5. (a) Prove by the method of projection that if A, B, C and D be the four points whose coordinates are \((6, -6, 0), (-1, -7, 6), (3, -4, 4), (2, -9, 2)\), AB is perpendicular to CD.
(b) The direction-cosines of two lines are connected by the relations \( l - 5m + 3n = 0 \) and \( 7l^2 + 5m^2 - 3n^2 = 0 \). Find them.
(c) Find the equation of two planes through the points \((0, 4, -3)\) and \((6, -4, 3)\) which cut off from the axes intercepts whose sum is zero.

6. (a) Find the distance of the point \((1, -2, 3)\) from the plane \( x - y + z = 5 \) measured parallel to the line whose direction cosines are proportional to \(2, 3, -6\).
(b) Show that the lines \( \frac{x - 5}{4} = \frac{y - 7}{4} = \frac{z + 3}{5} \) and \( \frac{x - 8}{7} = \frac{y - 4}{1} = \frac{z - 5}{3} \) are coplanar, find their common points and equation of the plane in which they lie.
(c) Find the length and equations of the shortest distance between the lines \( \frac{x - 1}{2} = \frac{y - 2}{3} = \frac{z - 3}{4} \) and \( \frac{x - 2}{3} = \frac{y - 4}{4} = \frac{z - 5}{5} \).

7. (a) Show that \( a = (1, -2, 0), b = (0, 1, 4) \) and \( c = (0, -1, -3) \) are linearly independent. Express \( d = (-1, 2, 3) \) as a linear combination of \(a, b \) and \(c\). Are the terminal points of the set of vectors \(\{d, a, b, c\}\) coplanar?
(b) Show that \( i + j + k, i - k \) and \( i - 2j + k \) are mutually orthogonal. Find \(x, y\) and \(z\) if \( i + j + 2k, -i + zk \) and \(2i + xj + yk \) are mutually orthogonal.
(c) By vector method, obtain the perpendicular distance of the point \((5, 5, 5)\) from the line through points \((3, 4, -1)\) and \((1, 3, 1)\).

8. (a) Show that the four points \(4i + 5j + k, -(j + k), 3i + 9j + 4k\) and \(4(-i + j + k)\) are coplanar.
(b) Obtain a set of vectors reciprocal to the three vectors \(-i + j + k, i - j + k\) and \(i + j - k\).
(c) A force \( F = 4i - j + 3k \) is applied at the point \(P(3, 1, 2)\). The moment of this force about the point \(A(l, 1, 2l)\) is \(6i + 9j - 5k \). Find \(t\).
SECTION – A
There are FOUR questions in this Section. Answer any THREE.

1. (a) Describe different types of pump impellers with neat sketches. (10)
   (b) Explain the working principle of advantages of a mechanical seal in a pump with a neat sketch. (15)
   (c) Give a comparison between a centrifugal pump and a reciprocating pump. (10)

2. (a) What is effective swept volume in a reciprocating compressor. (5)
   (b) Derive the expression for work absorbed by a single stage reciprocating compressor in terms of the effective swept volume, suction and discharge pressures. (20)
   (c) Why are intercoolers used in multistage compressors? Briefly describe the working principle of an intercooler. (10)

3. (a) What are boiler mounting and accessories? Name four devices of each type. (10)
   (b) Draw the schematic diagram of a wet-back-type economic boiler. (10)
   (c) Explain equivalent evaporation capacity of a boiler. (5)
   (d) Classify the methods of assessing boiler efficiency. (10)

4. (a) With a neat sketch describe the blow down valve of a boiler. (10)
   (b) What are the advantages of installing an economizer in a boiler? Write the disadvantages if there are any. (5)
   (c) Classify and explain different forms of energy. (10)
   (d) Briefly describe the impact and future of natural gas on the energy scenario of Bangladesh with relevant data. (10)

Contd ............... P/2
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SECTION – B

There are FOUR questions in this Section. Answer any THREE.
For Question No. 5(a) & 5(b), use the supplied sheet and attach it to answer script.

5. (a) Mention the part names for the components of IC engine as shown in Figure for Q. No. 5(a) (For example F = Inlet valve).
(b) Figure for Q. No. 5(b) shows the actual valve timing diagram for a 4-stroke cycle petrol engine, in this figure show the valve overlap.
(c) From the given information beside the figure of 5(b), determine the valve of minimum and maximum valve overlap.
(d) A 150 c.c motor bike engine has its stroke equal to its bore. Calculate the crank radius of the engine. If the clearance volume is 25 c.c, determine its compression ratio.

6. (a) Draw and label the circuit diagram for a coil ignition system in a 4-stroke 4-cylinder engine.
(b) What is the difference between the magneto ignition system and the coil ignition system? Draw and label only the portion which is different from the coil ignition system. Where is this system used?
(c) Write down the two missing words "i" and "ii" of the following sentences (No need to write the sentences)

The loud pulsating noise heard within the engine cylinder is known as "(i)". It is caused due to the propagation of a high speed pressure wave created by the auto-ignition of unburnt fuel. The scientists all over the world, have concentrated on the design of their IC engines so that the burn gases are completely exhausted from the cylinder before suction starts. The process of removing burnt gases, from the combustion chamber of the engine cylinder is known as "(ii)".

7. (a) A person feels comfortable at 70°F DBT (Dry bulb temperature) with 77% RH (Relative Humidity). If RH is reduced to 30%, determine:
   (i) the new DBT so that the person have the same level of comfort.
   (ii) the degree of Effective Temperature
   (iii) the "percent of subjects feeling comfortable" at these conditions during summer
   [ Use the supplied Comfort Chart for still air ]
(b) With simple sketches, distinguish among a heat engine, a refrigerator and a heat pump working between a source at temperature T₁ and a sink at Temperature T₂ (T₁>T₂). Write down their corresponding expressions of C.O.P.
(c) A refrigerator working between temperatures T₁ and T₂ has its C.O.P = 5.78, what would be the C.O.P of the same device when it is made to operate as the heat pump?
(d) What are the 7 factors upon which the rate of production of human body heat depends?

8. (a) Why are steam turbines compounded? Show with simple sketches a velocity compounded steam turbine. Show also the variation of velocity and pressure along the turbine axis.
(b) With simple sketch, show the necessary components of a closed cycle gas turbine. Show the processes in corresponding P-V and T-S diagrams.
(c) Distinguish between a gas turbine and a steam turbine.
ASHRAE comfort chart for still air.

*Note—Both summer and winter comfort lines apply to inhabitants of the United States only. Application of water comfort line is further limited to rooms heated by central systems of the conventional type. The line does not apply to rooms heated by radiant methods. Application of summer comfort line is limited to homes, offices, and the like, where the occupants become fully adapted to the artificial air conditions. The line does not apply to theaters, department stores, and the like where the exposure is less than 3 hours. The summer comfort line shown pertains to Pittsburgh and to other cities in the northern portion of the United States and Southern Canada, and at altitudes not in excess of 1000 ft above sea level. An increase of one dog FT should be made approximately per 3 deg reduction in north latitudes.

* Drafted portion of winter comfort line was extrapolated beyond test data.
Ans: 5(a): Part names are as follows according to Fig. 5(a)

A =
B =
C =
D =
E =
F = Inlet valve
G =
H =
I =
J =
K =
L =
M =
N =
O =

Ans: 5(b): According to Fig. 5(b)  
Maximum valve overlap =
Minimum valve overlap =
SECTION - A

There are **FOUR** questions in this Section. Answer any **THREE**.

1. (a) Find the equivalent resistance between points A and B for the circuit shown in the Fig. for Q. No. 1(a).

   \[ \text{Fig. for Q. No. 1(a)} \]

(b) The following data are given for the circuit shown in Fig. for Q. No. 1(b).
   \[ I_1 = 5\, \text{A}, \quad I_2 = -3\, \text{A}, \quad I_3 = -6\, \text{A}, \quad I_4 = -2\, \text{A}, \quad I_5 = -3\, \text{A}, \quad I_6 = -7\, \text{A}. \]
   Find the voltage \( V_{ab} \), \( V_{bd} \) and \( V_{ad} \) using KVL and KCL. Blocks in the figure indicate unknown elements.

   \[ \text{Fig. for Q. No. 1(b)} \]

2. (a) In the circuit of Fig. for Q. No. 2(a), the voltage difference between points a and b is 5 V. Find the values of source voltage \( V_S \) and current \( I_S \).

   \[ \text{Fig. for Q. No. 2(a)} \]

Contd ......... P/2
(b) Using mesh analysis, determine the current I supplied by the 30 V DC voltage source in the circuit shown in Fig. for Q. No. 2(b).

3. (a) Using nodal analysis, find the current I through the 25 Ω resistance for the circuit shown in Fig. for Q. No. 3(a). Also determine the total power dissipated in the circuit.

(b) Using nodal analysis, find the voltage V across the 30 Ω resistor in the circuit shown in Fig. for Q. No. 3(b).

4. (a) Find the Thevenin equivalent with respect to the terminals a and b for the circuit shown in Fig. for Q. No. 4(a).
(b) Find the Norton equivalent with respect to the terminals a and b for the circuit shown in Fig. for Q. No. 4(b).

SECTION - B
There are FOUR questions in this Section. Answer any THREE.

5. (a) What is eddy current loss? How can it be reduced? Also using equation show how eddy current loss is dependent on frequency.
(b) Briefly describe the working principle of an elementary DC generator.
(c) Determine the current $I$ required to establish a flux of $2 \times 10^{-4}$ wb in the section of the cast steel core shown in Fig. for Q. No. 5(c). The cross sectional area throughout the core material is 6 cm$^2$.

6. (a) Define form factor and crest factor. Show that the crest factor of a sinusoidal a.c. signal is $\sqrt{2}$.
(b) Calculate the r.m.s. value of the voltage having the waveshape as shown in Fig. for Q. No. 6(b)
(c) Consider a series R-L circuit the current through which is \( i(t) = I_m \cos(\omega t) \). Find the expression of the applied voltage and the average power supplied by the source. Also draw the waveshapes of instantaneous voltage, current and power.

7. (a) For the circuit shown in Fig. for Q. No. 7(a), the voltage across the resistor of Load 3 is 10 V. Consider the current through Load 3 as the reference and draw the phasor diagram indicating all voltages.

(b) Find the equivalent impedance of the circuit shown in Fig. for Q. No. 7(b), for a sinusoidal input voltage of 10 V amplitude and 200 Hz frequency.

(c) Show that both Y and Δ connected loads, the total reactive power supplied by 3 phase source is \( \sqrt{3} \, V_L \, I_L \, \sin \theta \) where \( V_L \) = line voltage, \( I_L \) = line current and \( \theta \) is the phase difference between phase current and corresponding phase voltage.

8. (a) For the circuit shown in Fig. for Q. No. 8(a), the voltage \( v(t) = 10 \sin (1000t + 20^\circ) \) V. Find (i) current \( i(t) \), (ii) voltage \( v(t) \) and (iii) total real reactive powers and apparent powers consumed by Load 2. Here the given voltage is in r.m.s.
(b) Consider the three phase load of Fig. for Q. No. 8(b). Find (i) line currents, (ii) Phase currents and (iii) total real power supplied by source. Here $V_{ab} = 220 \angle 30^\circ$, $V_{bc} = 220 \angle 150^\circ$ and $V_{ca} = 90^\circ$. All these voltages are in r.m.s.
$B-H$ curve for Question no. 5(c)
SECTION – A

There are FOUR questions in this section. Answer any THREE.

1. (a) How does heat evolve or absorb during dissolution of solute in solvent?
   (10)
(b) What types of liquid pairs are steam distilled? How steam distillation is useful in evaluating the molar masses of compounds?
   (6)
(c) An aqueous solution containing 10.0 g of KOH and 90.0 g of H2O has a density of 1.12 kg/dm³. Find (i) W/V% of KOH, (ii) Molality, (iii) Molarity and (iv) Mole fraction of KOH.
   (10)
(d) State how many components, phases and degrees of freedom are present in each of the following systems:
   (i) H₂(g) + N₂(g) mixture
   (ii) I₂(s) in equilibrium with its vapour
   (iii) Saturated solution of NaCl in water
   (9)

2. (a) What are donor and acceptor atoms? Discuss their role in the formation of a coordinate covalent bond. Comment on the formation of H₄O⁺⁺, SO₄²⁻ and O₂Cl⁻ ions.
   (8)
(b) How would you prove that metal atoms have secondary valencies (coordination numbers)?
   (8)
(c) Show the formation of H₂⁺ and He⁺ ions according to molecular orbital theory. Comment on their stability and magnetic properties.
   (8)
(d) Mention the important definitions of acids and bases with their limitations. Describe how relative strength of acids are determined.
   (11)

3. (a) With the help of potential energy diagram discuss the formation of a chemical bond between two atoms.
   (7)
(b) Discuss the properties of ionic and covalent compounds in connection with the directional properties of the bond.
   (8)
(c) Derive thermodynamically Kirchoff's equation of heat of reaction at variable temperature.
   (10)
(d) Calculate the heat of formation of ammonia from its element at 100°C. The heat of formation of NH₃(g) at 25°C is -11.04 kcal/mol. The heat capacities are given as follows:
   \[ \begin{align*}
   \text{N}_2(g) : C_p &= 6.76 \text{ cal.mol}^{-1}.\text{deg}^{-1} \\
   \text{H}_2(g) : C_p &= 6.62 \text{ cal.mol}^{-1}.\text{deg}^{-1} \\
   \text{NH}_3(g) : C_p &= 6.18 \text{ cal.mol}^{-1}.\text{deg}^{-1}
   \end{align*} \]
   (10)

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4. (a) With the help of a diagram, prove that the elevation of boiling point of a solvent produced by a dissolved substance is proportional to the corresponding lowering of vapour pressure and molality of solute in the solvent.

(b) A solution contains 5 g of urea (M = 60.05) per 100 g of water. What will be the vapour pressure of this solution at 25°C? The vapour pressure of pure water at this temperature is 23.76 mm. Find also the boiling point of this solution where $K_b$ is 0.512 °C/m.

(c) Derive Van't Hoff equation for boiling point.

(d) The average osmotic pressure of human bloods is 7.7 atm at 40°C. What should be the total concentration of blood?

SECTION – B

There are FOUR questions in this section. Answer any THREE.

5. (a) Derive the integrated rate equation for the first order reaction

\[ A \rightarrow P \]

How does this differ from second order behaviour? Prove that a first order reaction is never complete.

(b) "Why is it important to know the activation energy of a reaction"? Deduce a mathematical equation to justify the statement.

(c) The rate constant for the decomposition of the compound "P" at 470°C and 510°C are $1.10 \times 10^{-4}$ sec$^{-1}$ and $1.63 \times 10^{-5}$ sec$^{-1}$ respectively. Determine the activation energy of the process. What are half-life values at the above temperatures?

6. (a) What do you understand by equilibrium constant of a chemical reaction? What are the characteristics of such constants?

(b) The equilibrium constant of a homogeneous gaseous reaction is dependent on pressure and temperature change. Explain this with suitable examples and mathematical equation where needed.

(c) The equilibrium constant "$K_p$" for the reaction:

\[ N_2 (g) + 3H_2 (g) \rightleftharpoons 2NH_3 (g) \]

is $1.64 \times 10^{-4}$ atm$^{-2}$ at 400°C and $0.144 \times 10^{-4}$ atm$^{-1}$ at 500°C. Calculate the mean heat of formation of 1 mole of NH$_3$ in this temperature range. Comment on the result.

Contd ............ P/3
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7. (a) Write notes on the following: (4×4\(\frac{1}{2}\) = 18)

(i) Differential method for the determination of order.
(ii) Relationship between Gibb's Free Energy and "K".
(iii) Buffer solution
(iv) Daniel cell

(b) Justify the stability of the nucleus of an atom. (8)

(c) What is Schrödinger wave equation? Deduce the equation and mention the conditions to be applied to obtain practicable solutions for wave functions, \(\psi\). (9)

8. (a) What is photoelectric effect? Based on quantization of energy how Bohr developed the atomic model? Discuss the limitations of Bohr atomic theory. (15)

(b) What is periodic law? How many periods and groups are there in the periodic table? Why are the periods not equal in the periodic table? (8)

(c) Discuss in brief the physical and chemical properties of inert gases. (12)