L-1/T-1/WRE **Date : 12/07/2011**

BANGLADESH UNIVERSITY OF ENGINEERING AND TECHNOLOGY, DHAKA

L-1/T-1 B. Sc. Engineering Examinations 2009-2010

Sub : **MATH 131** (Differential Calculus and Integral Calculus)

Full Marks : 210 Time : 3 Hours

The figures in the margin indicate full marks.

USE SEPARATE SCRIPTS FOR EACH SECTION

SECTION – A

There are **FOUR** questions in this Section. Answer any **THREE**.

Symbols have their usual meanings.

1. (a) A function f(x) is defined as follows: **(17)**

$$f(x)=\begin{cases} 4+x^2 & when\ \ 0<x\le 4 \\ 4 & when\ \ -1\le x\le 0 \\ 1+x & when\ \ -4\le x<-1 \end{cases}$$

Sketch f(x) and discuss the continuity of f(x) at x = –1 and differentiability of the function f(x) at x = 0.

(b) Evaluate the following:

(i) $\underset{x\to 0}{Lim}(\cos x)^{\cos ec^2 x}$ **(8)**

(ii) $\underset{x\to 0}{Lim}\dfrac{e^x+e^{-x}-x^2-2}{\sin^2 x-x^2}$ **(10)**

2. (a) If x = $Sin\left(\dfrac{1}{m}\log_e y\right)$ show that $\left(1-x^2\right)y_{n+2}-(2n+1)xy_{n+1}-\left(n^2+m^2\right)y_n=0$ and find the value of y_n when x = 0. **(13)**

(b) State and prove Rolle's theorem. Verify Rolle's theorem for the function $f(x) = x^2 - 3x + 2$ in the interval (1, 2). **(12+10)**

3. (a) If $u=\dfrac{x}{r^3}$ and $r^2 = x^2 + y^2 + z^2$, find the value of $u_{xx} + u_{yy} + u_{zz}$. **(13)**

(b) Expand $f(x) = e^x \sec x$ in a series of ascending powers of x. **(10)**

(c) If lx + my = 1 is the normal to the parabola $y^2 = 4ax$ then show that $al^3 + 2alm^2 = m^2$. **(12)**

4. (a) Show that the height of the cylinder of maximum volume that can be inscribed in a sphere of radius a is $\dfrac{2\sqrt{3}}{3}a$ **(15)**

(b) Find the pedal equation of the cardioide $r = a(1 - \cos\theta)$. **(10)**

(c) Find the radius of curvature of the curve $x = a(\theta - \sin\theta)$, $y = a(1 - \cos\theta)$. **(10)**

Contd P/2

MATH 131(WRE)

SECTION – B

There are **FOUR** questions in this Section. Answer any **THREE**.
Symbols have their usual meanings.

5. Carry out the following:

(i) $\int \frac{3\sin x}{\sin x + 2\cos x} dx$ **(11)**

(ii) $\int \frac{x^3 \sin^{-1} x}{\sqrt{1-x^2}} dx$ **(14)**

(iii) $\int \frac{xe^x}{(1+x)^2} dx$ **(10)**

6. (a) Obtain a reduction formula for $\int \sin^m x \cos^n x dx$ and hence evaluate $\int \sin^4 x \quad \cos^3 x dx$ **(20)**

(b) $\int_0^{\pi} \frac{xdx}{\left(a^2 \cos^2 x + b^2 \sin^2 x\right)^2}$ **(15)**

7. (a) Evaluate $\int_{-1}^{1} \frac{dx}{x^3}$ **(10)**

(b) Find the value of $\Gamma(\frac{1}{2})$ **(10)**

(c) Determine the area of the loop of the curve $xy^2 + (x + a)^2 (x+ 2a) = 0$ **(15)**

8. (a) Obtain the volume of the solid of revolution of the curve $r^2 = a^2 \cos 2\theta$ about the initial line. **(15)**

(b) Find the Jacobian of $x = r \sin\theta \cos\phi$, $y = r\sin\theta \sin\phi$, $z = r \cos\theta$ with respect to r, θ and ϕ. **(10)**

(c) Evaluate $\int_0^1 \int_0^1 \frac{x-y}{(x+y)^3} dxdy$ and $\int_0^1 \int_0^1 \frac{x-y}{(x+y)^3} dydx$ and comment on your result. **(10)**

L-1/T-1/WRE **Date : 27/06/2011**

BANGLADESH UNIVERSITY OF ENGINEERING AND TECHNOLOGY, DHAKA

L-1/T-1 B. Sc. Engineering Examinations 2010-2011

Sub : **WRE 101** (Analytical Mechanics)

Full Marks: 280 Time : 3 Hours

The figures in the margin indicate full marks.

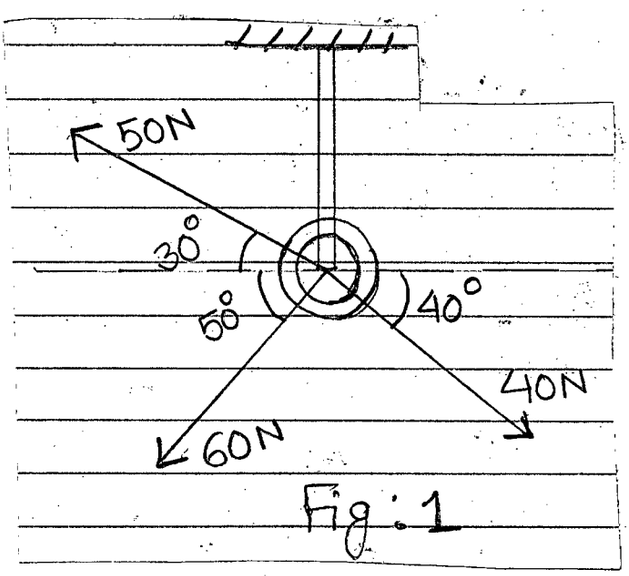
USE SEPARATE SCRIPTS FOR EACH SECTION

SECTION – A

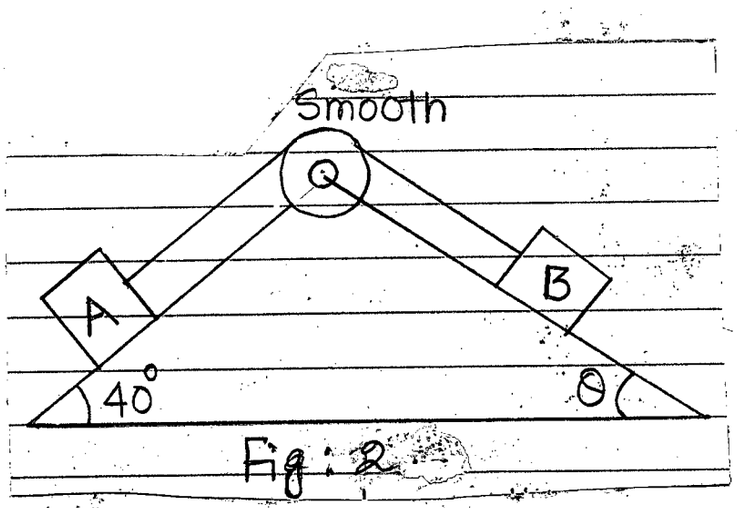
There are **FOUR** questions in this Section. Answer any **THREE**.

Symbols have their usual meanings.

1. (a) Write down Lami's theorem and its application. Determine the resultant of the three forces acting on an eye bolt in Figure 1. **(5+10)**



(b) The bodies $A(W_A = 50\ \text{lb})$ and $B(W_B = 75\ \text{lb})$ are connected by a chord and rest on smooth inclined planes. What is the angle θ if the bodies are in equilibrium? Find also the plane reactions and tension in the chord. See Figure 2. **(15⅔)**



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WRE 101

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(c) Determine support reactions at A and B and bar forces of EE and DC in Figure 3. **(16)**

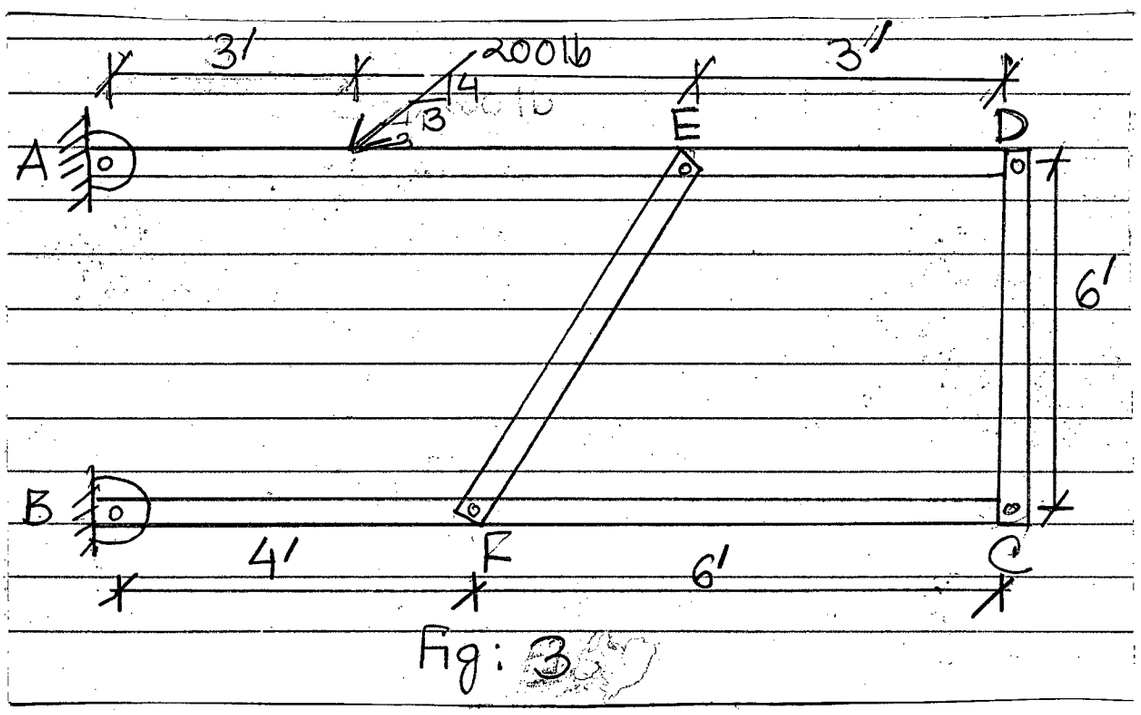

Fig: 3

2. (a) In the system of shaves, What Force F will hold a weight of 800 lb in equilibrium? What will be the tension in cable A, B, C and D? There are no frictional losses at the axes. See Figure 4. **(15)**

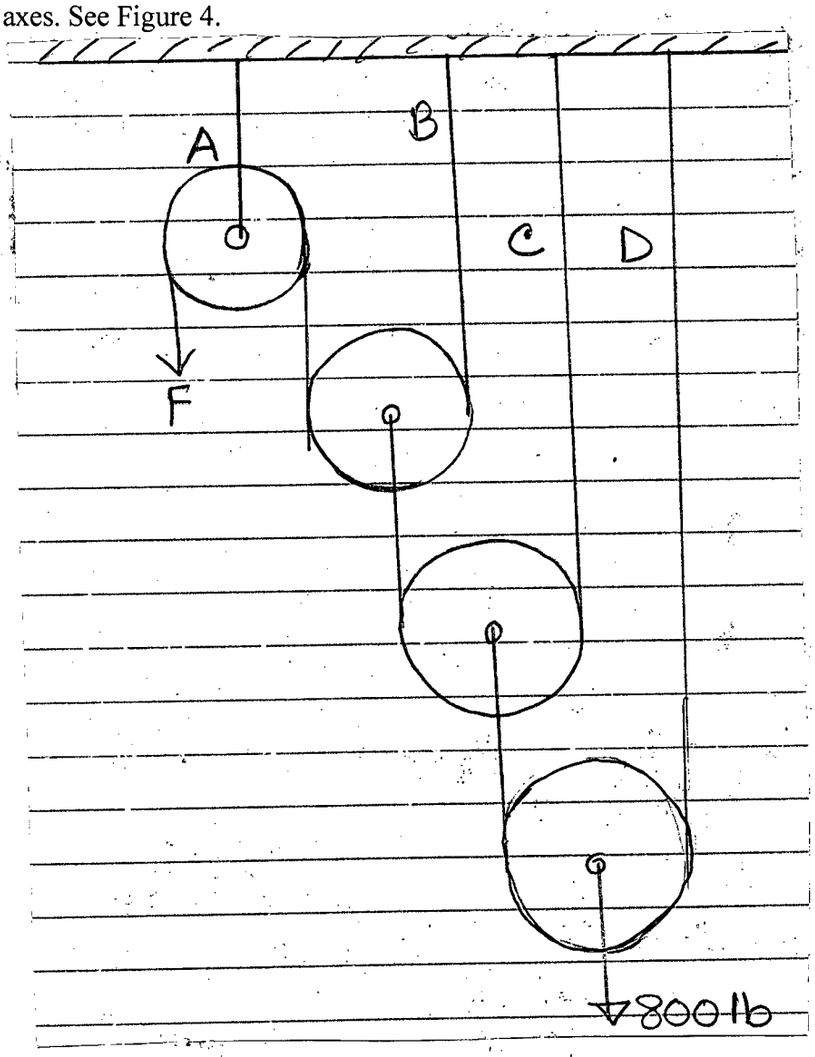

Fig: 4

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WRE 101

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(b) For the truss shown in Fig. 5 determine bar forces of GF and GB. **(16)**

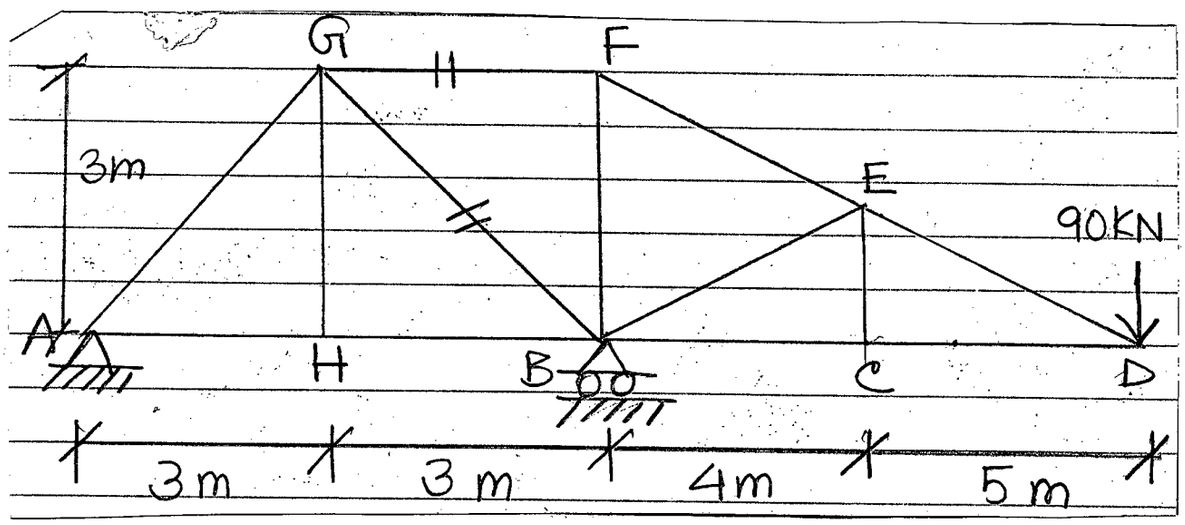

Fig: 5

(c) Given, W_A = 2000 lb, f_A = ⅓ and f_B = 0.15. For impending clockwise motion of the pulley, what is W_B? The bearing at C is smooth. See Figure 6. **(15⅔)**

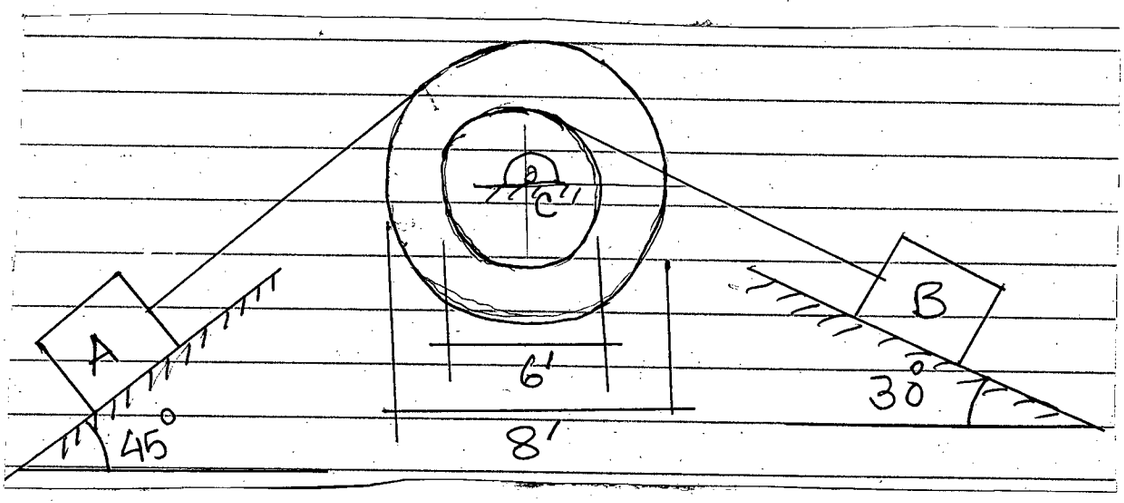

Fig: 6

3. (a) A flexible cable weighing 2.2 lb/ft is strung between two supports. One support is 85 ft lower than the other, and the sag measured from the lower support is 55 ft. The tension in the cable at the upper support is 12600 lb. Calculate the distance between the two supports, total cable length and tension at the lower support. **(15⅔)**

Contd P/4

WRE 101

Contd ... Q. No. 3

(b) In Fig. 7, θ = 30° and the weight of B to be W_B = 5000 lb. If $f = \frac{1}{3}$ for all slipping surfaces, what value of Q cause impending motion of A (weightless is considered) toward the right? **(16)**

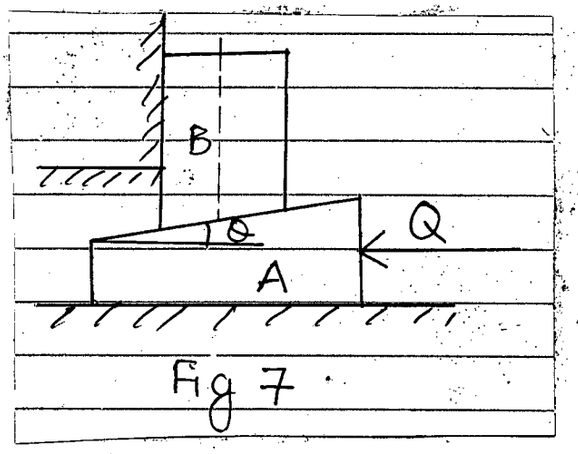

Fig 7.

(c) Find the tensile forces in cables AB, BC, BE and BF. The pulleys E and F are frictionless. See Fig. 8. **(15)**

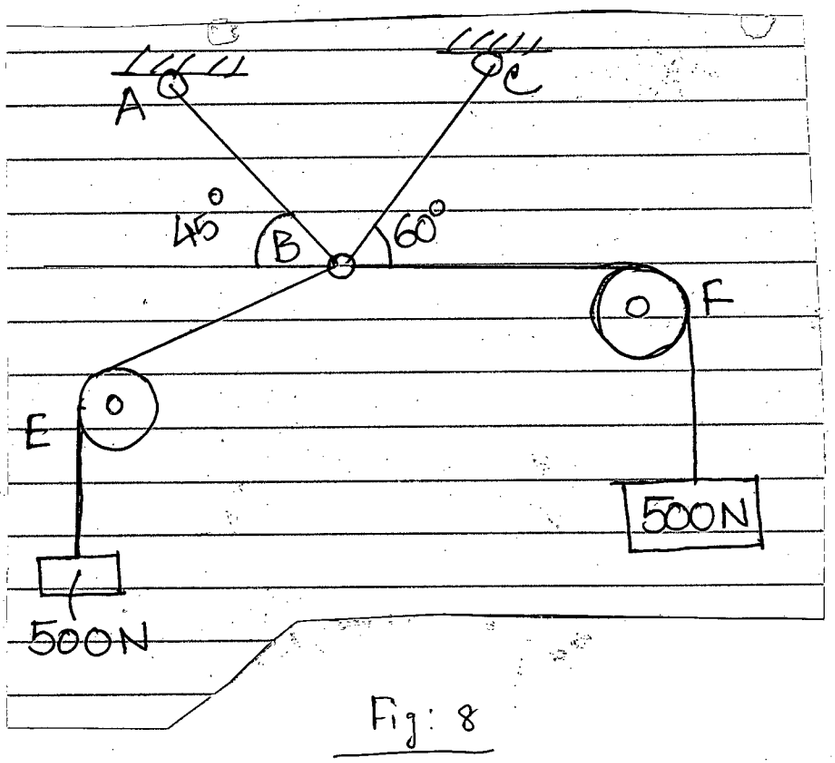

Fig: 8

4. (a) Prove that $y = \frac{k}{2}\left(e^{x/k} + e^{-x/k}\right)$ and $S = \frac{k}{2}\left(e^{x/k} - e^{-x/k}\right)$. **(15)**

(b) A framework is loaded as shown in Fig. 9. Determine the pin reactions at A and B. Given that the self weight of the member is 1200 lb. **(16⅔)**

WRE 101

Contd ... Q. No. 4

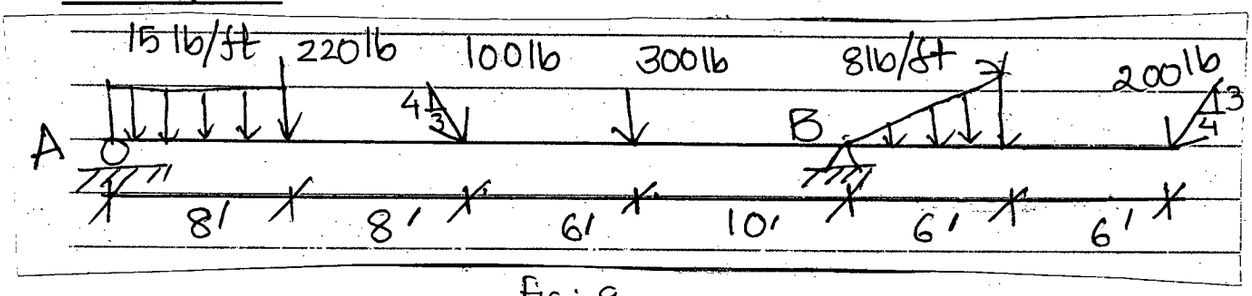

Fig : 9

(c) If a cable in the form of catenary, 200 ft long and weighing 3 lb/ft is suspended from points on a horizontal with a span of 150 ft, find (i) minimum tension; (ii) the maximum tension and (iii) sag. **(15)**

SECTION – B

There are **FOUR** questions in this Section. Answer any **THREE**.

5. (a) Locate the centroid of the wire shown in Figure 10. **(14⅔)**

(b) A 3-ft cylinder A weighing 200 lb (Figure 11) has a central 2 ft groove about which a weightless, inextensible cord is wound. This cord passes over a smooth post C, thence vertically downward to another body B weighing 32.2 lb. The cylinder has a moment of inertia of 6 slug-ft^2. If the frictional force is sufficient to cause the cylinder to roll, then **(16)**

(i) What is the tension in the cord?

(ii) Determine the speed of the center of gravity of A.

(iii) Find the acceleration of B after it has moved 20 ft downward.

(c) Determine the moment of inertia of the shaded area shown in Figure 12 about the x-axis and y-axis. **(16)**

6. (a) Determine the centroid of the area as shown in Figure 13. **(18⅔)**

(b) A body A weighing 50 lb is on a plane inclined at an angle of $\beta = 30^\circ$ [Figure 14]. A cable attached to this body passes over a stationary member C. From the other end of the cable, a body B weighing 100 lb is suspended. Given, $f_A = 0.3$ and $f_C = 0.2$. If the system is released from rest – **(16)**

(i) In what direction does motion occur?

(ii) What are the tensions in the cables?

(iii) What is the acceleration of the bodies?

(c) A jet of water issues from a nozzle (Figure 15) with a velocity of 200 fps and at a absolute rate of 5 lb/sec. It enters a fixed blade of an angle of 120° and passes through the blade with a negligible friction loss. What is the magnitude and direction of the force exerted on the blade? **(12)**

Contd P/6

WRE 101

7. (a) For the Z-section shown in Figure 16, let a = 1 inch, b = 4 inch and c = 8 inch. Determine the maximum and minimum centroidal moments of inertia and minimum radius of gyration. **(20⅔)**

(b) A weight A is supported from a cable which is would about a 4 ft drum (Figure 17). An 8 ft flywheel turns with the drum. The total weight of the rotating part is 1288 lb and the radius of gyration is 2.5 ft. While A travels 80 ft vertically downward, the speed of the rotating part changes from 10 rpm to 120 rpm. The frictional force in the bearings acting tangentially to the 6 inch shaft is 70 lb. What is the weight of A and the tension in the cord? **(16)**

(c) A moving point follows the path of a hyperbola $x^2/36 - y^2/16 = 1$. The x-component of the velocity is constant at 9 fps. At instant when the point is at position $(12, 4\sqrt{3})$, what is the magnitude and direction of velocity and acceleration? **(10)**

8. (a) For the composite solid steel body shown in Figure 18, find the radius of gyration about the geometric axis. Steel weighs 490 lb/ft^3. **(14⅔)**

(b) Two bodies A and B are attached to pulley system as shown in Figure 19. If W_A = 200 lb, W_B = 100 lb, f_A = ¼ and f_B = ⅓, then – **(16)**

(i) How far and in what direction does A travel from rest during 30 seconds?

(ii) What are the tensions in cable C and cable D?

(c) Blocks A and B shown in Figure 20 have a mass of 3 kg and 5 kg respectively. If the system is released from rest, both the blocks travel downward. Determine the velocity of block B after 6 seconds applying impulse and momentum principle. Neglect friction loss in the pulleys. **(16)**

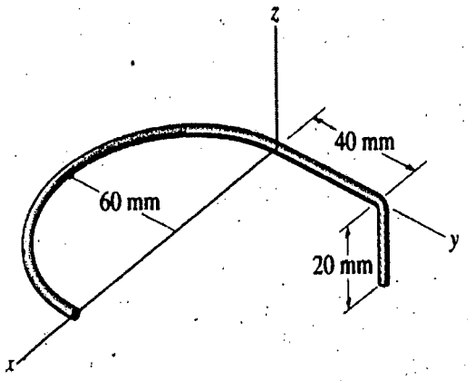

Fig. 10 for Q. No. 5(a)

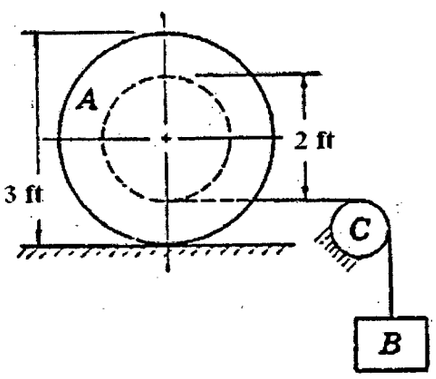

Fig. 11 for Q. No. 5(b)

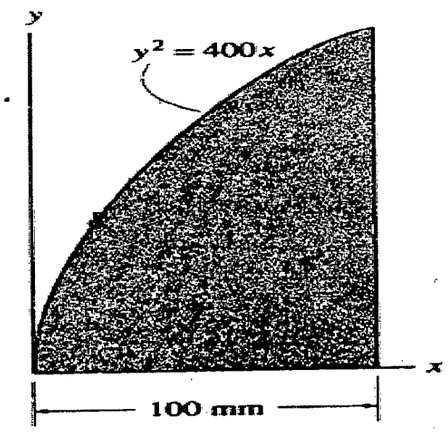

Fig. 12 for Q. No. 5(c)

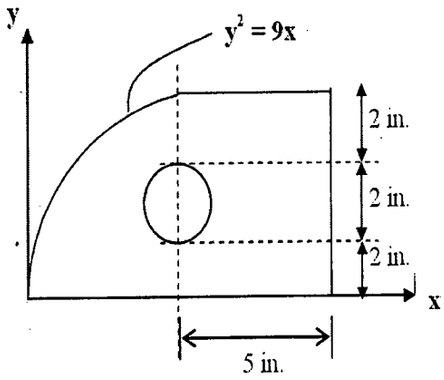

Fig. 13 for Q. No. 6(a)

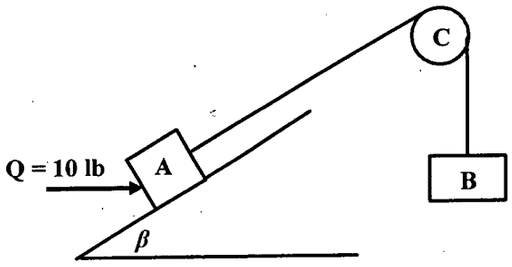

Fig. 14 for Q. No. 6(b)

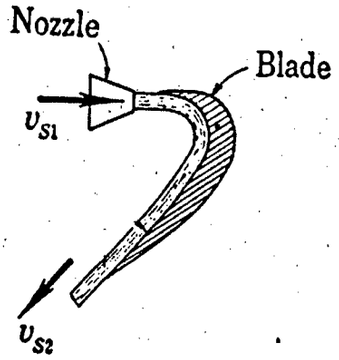

Fig. 15 for Q. No. 6(c)

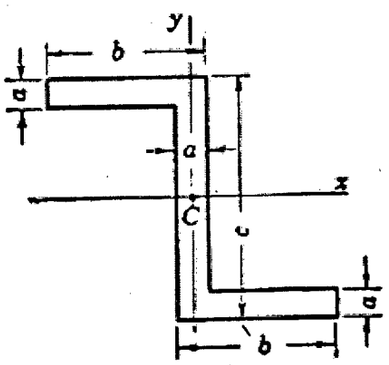

Fig. 16 for Q. No.7(a)

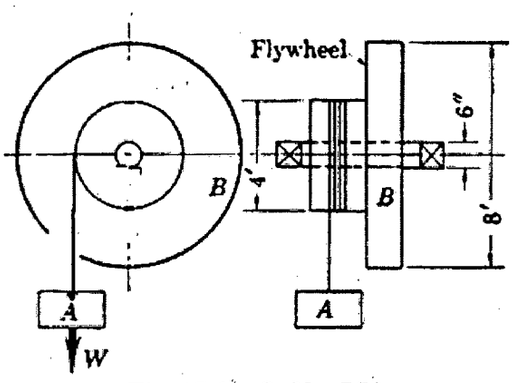

Fig. 17 for Q. No. 7(b)

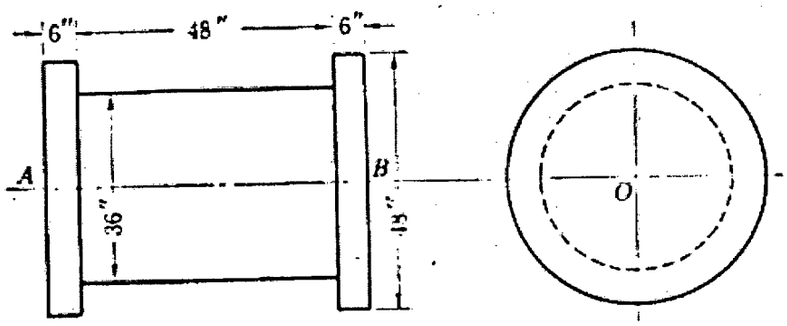

Fig. 18 for Q. No. 8(a)

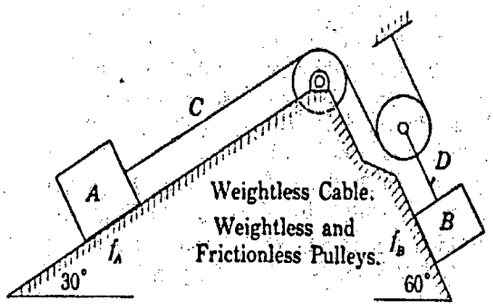

Fig. 19 for Q. No. 8(b)

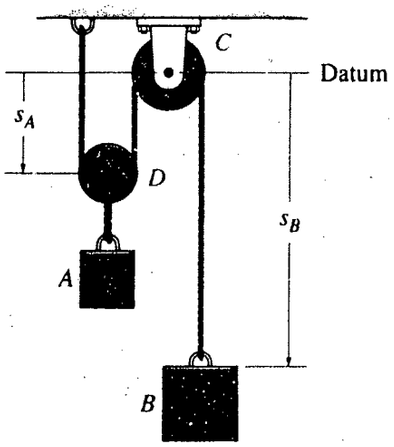

Fig. 20 for Q. No. 8(c)