BANGLADESH UNIVERSITY OF ENGINEERING AND TECHNOLOGY, DHAKA

L-4/T-1   B. Sc. Engineering Examinations 2010-2011

Sub : CHE 401 (Reaction Engineering)

Full Marks: 210   Time : 3 Hours

The figures in the margin indicate full marks.

USE SEPARATE SCRIPTS FOR EACH SECTION

SECTION – A

There are FOUR questions in this Section. Answer any THREE.

1. (a) Discuss briefly how the concepts of other chemical engineering courses are related to chemical reaction engineering taking a gas-liquid-solid reactive system as an example. **(12)**

(b) What are the different ways to define reaction rate? Define mathematically reaction rate in the each different way. For each different way, write down the condition(s) at which it is normally used to define the reaction rate. **(3+4+3)**

(c) How does an open sequence reaction differ from a chain reaction? **(4)**

(d) Write down the kinetic parameters which are required to develop a reactor for a homogeneous reactive process. **(3)**

(e) Write down the basic reaction rate equation in terms of concentration, order, and rate constant for the following cases: **(3+3)**

    (i)   Second-order reaction opposed by first-order reaction.

    (ii) Irreversible first-order simple parallel reactions.

2. (a) A reaction having very small conversion of limiting reagent (A) has the following overall rate expression: **(8)**

$$r = \frac{k_1\, c_A}{1 + k_1 + k_2\, c_A}$$

Describe the determination of k_1 and k_2 briefly choosing a suitable method from the methods available for determining kinetic parameters.

(b) Dyashkovkii and Shilov studied the kinetics of the reaction between ethyl-lithium and ethyl-iodide in decalin solution. **(27)**

$$C_2H_5L_i + C_2H_5I \rightarrow 2\,C_2H_5 + L_iI$$
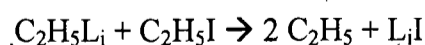
The following data are typical of those observed by these authors at 20°C. They correspond to initial ethyl-lithium and ethyl-iodide concentrations fo 2.0 and 1.0 kmoles/m³, respectively.
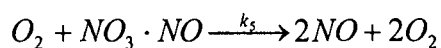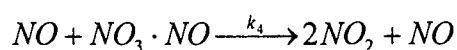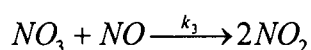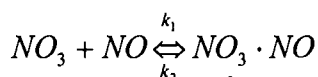
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Contd ... Q. No. 2(b)

Time, t (sec)	C_{Lil} (kmoles/m^3)
600	0.167
1200	0.306
1800	0.412
2400	0.498
3000	0.569

What rate expression is consistent with these data? What is the reaction rate constant at 20°C? You are allowed to make all the reasonable assumption(s) which are necessary to solve this problem.

3. (a) What do you understand by reaction mechanism? Write down five guidelines which are used in postulating a mechanism. What are the two crucial criteria with which a proposed mechanism must be consistent? **(4+5+3)**

(b) The following mechanism has been proposed for the oxidation of nitric oxide to nitrogen di-oxide (NO + $\frac{1}{2}$O$_2$ → NO$_2$). **(23)**

$$NO + O_2 \Leftrightarrow NO_3; \text{ equilibrium constant } K_c$$

$$NO_3 + NO \underset{k_2}{\overset{k_1}{\Leftrightarrow}} NO_3 \cdot NO$$

$$NO_3 + NO \xrightarrow{k_3} 2NO_2$$

$$NO + NO_3 \cdot NO \xrightarrow{k_4} 2NO_2 + NO$$

$$O_2 + NO_3 \cdot NO \xrightarrow{k_5} 2NO + 2O_2$$
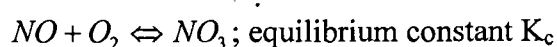
Derive a rate expression that is consistent with this mechanism. Theacy and Daniels determined that the orders of the reaction with respect to oxygen and nitric oxide are one and two, respectively, at high pressures and less than one and greater than two at low pressures. Is the proposed mechanism consistent with this data?

4. (a) A consecutive first order reaction sequence is given below:

$$A \xrightarrow{k_1} B \xrightarrow{k_2} F \xrightarrow{k_3} P$$
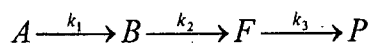
(i) Derive expressions for the maximum concentration of B and the time at which the concentration of species B reaches at its maximum value. **(5)**

(ii) Show graphically how the concentrations of A, B, and P change with time provided that $k_2 = 2.25 k_1$ and $B_0 = 0.5 A_0$. (There is no need to use separate graph paper) **(5)**

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(iii) Briefly discuss a method to determine the individual value of k_1 and k_2 for such a consecutive reaction. **(8)**

(b) Write a short note on the relaxation technique which is used to study rapid reversible reaction. **(7)**

SECTION – B

There are **FOUR** questions in this Section. Answer any **THREE**.

5. (a) Discuss the assumptions and limitations of Langmuir Adsorption Isotherm. The BET approach is essentially an extension of the Langmuir approach — Explain the statement. **(10)**

(b) What do you understand by the term physical and chemical characterization of heterogeneous catalyst? Write a short note on components of industrial catalyst. **(10)**

(c) Some investigators have studied the decomposition of ammonia over a headed platinum filament at 1000°C. The reaction stoichiometry is **(15)**

$$2\,NH_3 \rightarrow N_2 + 3\,H_2$$

Initially, pure NH_3 was present in the reaction vessel. The data shown below are representative of the kinetics of the reaction. The reactor volume is constant.

Time, t (sec)	Total pressure (kPa)
0	26.7
60	34.1
240	38.5
720	42.7

It has been postulated that the rate expression for the reaction is of the form

$$r = k\frac{P_{NH_3}}{P_{H_2}}$$

Is the experimental data consistent with this rate expression? If so, what is the value of the rate constant? What type of Hougen-Watson model gives rise to this form for the rate expression?

6. (a) What is the set of information necessary to start reactor design? **(10)**

(b) Discuss the advantages and disadvantages of Plug Flow and Stirred Tank reactors. **(10)**

(c) Explain the graphical approach to the analysis of Batteries of Stirred Tank Reactors operating at steady state. **(15)**

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7. (a) Derive an expression for Recycle reactor. **(17)**

(b) Two gaseous streams are available for use in carrying out a chemical reaction. The first contains pure A and is produced at a rate of 400 ft³/min. The second contains 50% B (remainder is an inert material) and has a flow rate of 200 ft³/min. These streams are mixed instantaneously and fed to a flow reactor. Both streams are at the same temperature (80°C) and pressure (1 atm) and these quantities remain unchanged during the instantaneous mixing process. The gases behave ideally. A and B react to form an addition product. **(18)**

$$A + B \rightarrow C$$
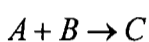
with $r_c = k\, C_A\, C_B$

The reaction is carried out isothermally in two flow reactors (plug flow and stirred tank). Both reactors operate at a constant total pressure of 1 atm. If the reactor is a plug flow reactor with a volume of 600 ft³ and 90% of the B is converted to C, determine

   (i) The space velocity in the reactor

   (ii) The effluent volumetric flow rate

   (iii) The rate constant k

   (iv) The average holding time in the reactor.

8. (a) Discuss the advantages and disadvantages of fixed bed reactor design models. **(15)**

(b) Point out the problems encountered in deriving the equation applicable for calculation of effectiveness factors for non-isothermal catalyst pellets. **(11)**

(c) Write short note on the followings: **(9)**

   (i) Effectiveness factor and Effective diffusivity

   (ii) Knudsen diffusion and Combined diffusivity

   (iii) Global and Intrinsic reaction rates.

BANGLADESH UNIVERSITY OF ENGINEERING AND TECHNOLOGY, DHAKA

L-4/T-1   B. Sc. Engineering Examinations 2010-2011

Sub : CHE 403 (Process Control)

Full Marks : 210   Time : 3 Hours

The figures in the margin indicate full marks.

USE SEPARATE SCRIPTS FOR EACH SECTION

SECTION – A

There are **FOUR** questions in this section. Answer any **THREE**.

1. (a) Describe the direct synthesis method of designing a PID controller for a second-order plus time delay (SOPTD) process.   **(18)**

   (b) An experiment has been performed to determine the steady-state power delivered by a gas turbine generator as a function of fuel flow rate. N data points for two variables fuel flow rate (u) and power output (y) have been recorded during the experiment. Describe the least square method to estimate a quadratic model of the form: $y = \beta_0 + \beta_1 u + \beta_2 u^2$.   **(17)**

2. (a) Describe the effect of pole locations in the s-plane on the stability of a system.   **(10)**

   (b) IMC controller tuning method does not work well for lag dominant processes. Suggest two methods for overcoming this limitation.   **(6)**

   (c) Describe combined feedback-feedforward control strategy for controlling the level of a boiler drum. Express such a control system using a general block diagram and find the expression of a dynamic feedforward controller using block diagram algebra.   **(7+6+6)**

3. (a) What is a DCS? Compare briefly DCS and PLC.   **(7)**

   (b) For Figure 3(b), find the closed loop response relation between Y and Y_{sp}.   **(20)**

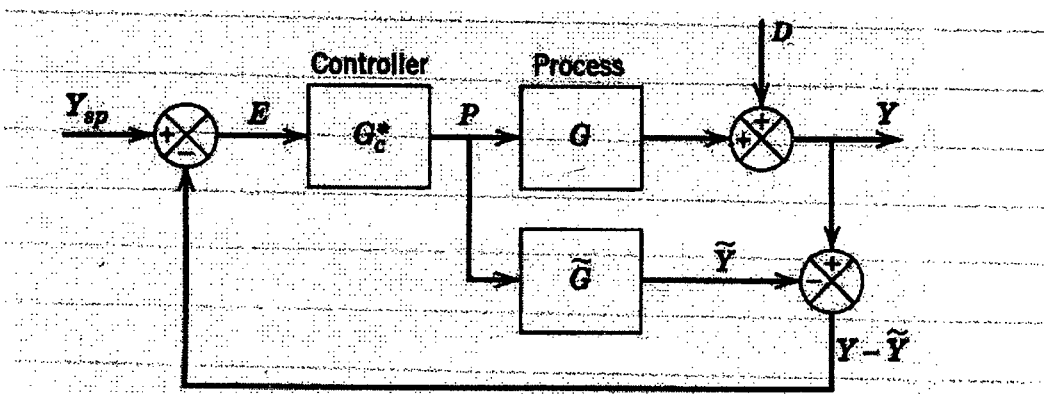Fig. 3(b): Figure for Q 3(b)

For $G_c^* = K_c$, $G = \dfrac{2}{(2s+1)(4s+1)}$ and $\tilde{G} = \dfrac{1}{(2s+1)}$,

using Routh array, find the maximum values of K_c for which the system remains stable.

(c) Describe typical layers of safety protection used in modern plants.   **(8)**

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4. (a) What is a Bode diagram? Describe the concept of gain margin and phase margin using Bode diagram. How gain margin and phase margin are useful in designing a controller. **(15)**

(b) Find the expressions for Amplitude ratio and phase lag of a PI controller. Sketch qualitatively these expressions as a function of frequency. **(10)**

(c) Write down five criteria for selecting a sensor. **(5)**

(d) What are the performance criteria for a closed loop control system? **(5)**

SECTION – B

There are **FOUR** questions in this section. Answer any **THREE**.

5. Show that the step response of a second order underdamped process can be expressed as **(20+15=35)**

$$y(t) = KM\left\{1 - e^{-\zeta t/\tau}\left[\cos\left(\frac{\sqrt{1-\zeta^2}}{\tau}t\right) + \frac{\zeta}{\sqrt{1-\zeta^2}}\sin\left(\frac{\sqrt{1-\zeta^2}}{\tau}t\right)\right]\right\}$$

where the symbol carries their usual meaning. Also find the expression for Decay Ratio.

6. (a) Discuss the advantages and disadvantages of feedback and feedforward control loops. **(10)**

(b) A surge tank shown in the Figure 6(b) is designed so that the outflow rate, ω, is proportional to the liquid level to the 1.5 power, that is **(25)**

$$\omega = Rh^{1.5}$$

where R is a constant. If a single stream enters the tank with flow rate ω_i, find the transfer function $H'(s)/\omega_i'(s)$. Identify the gain and time constant and show their units. The cross sectional area of the tank is A, and density is constant.
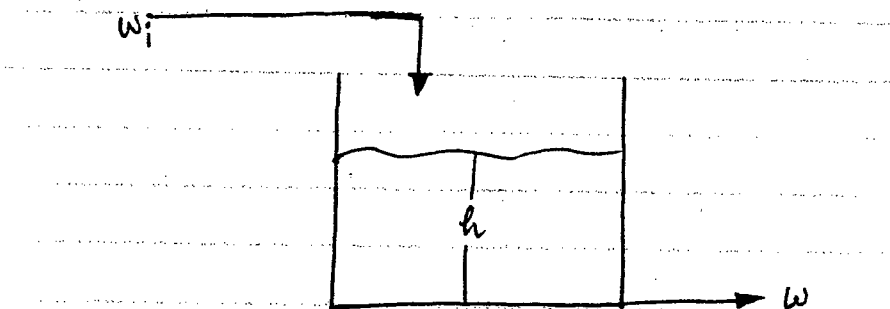

Figure for Question 6(b)

7. (a) Explain the terms: **(9)**

    (i) Reset windup

    (ii) Derivative kick

    (iii) Bumpless transfer

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Contd ... Q. No. 7

(b) Show that, for a first order process, a time interval of τ, (t = τ) is required to complete 63.2% of the process response. Also determine the time needed to complete 99% of the response. **(16)**

(c) Consider the following transfer function **(10)**

$$G(s) = \frac{K\,(3-s)\,e^{-2.5s}}{(12s+1)(5.2s+1)(0.2s+1)(0.05s+1)}$$

Use Skogestad's method to derive two approximate models:

   (i) A first-order-plus-time-delay model

   (ii) A second-order-plus-time-delay model in the form

$$\widetilde{G}(s) = \frac{Ke^{-\theta s}}{(\tau_1 s+1)(\tau_2 s+1)}$$

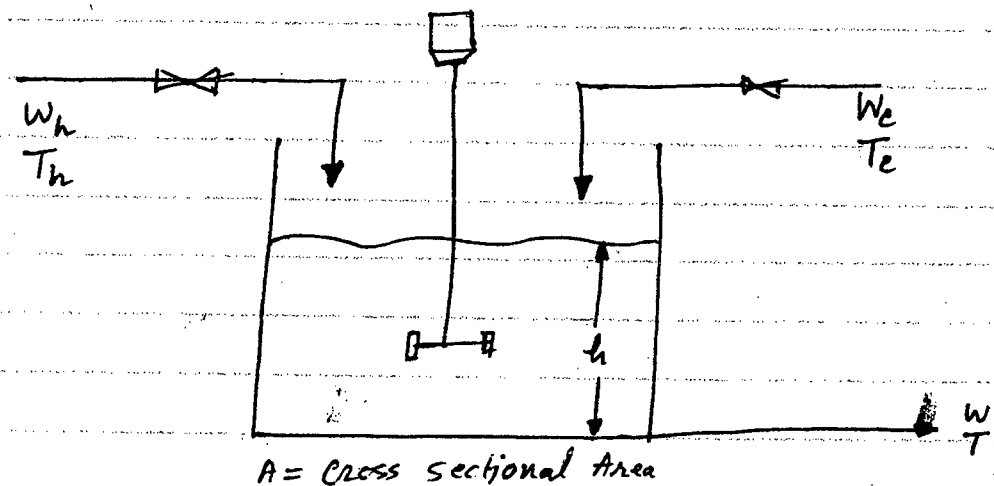8. A MIMO mixing process is shown in the Figure. **(35)**

$A = Cross\ sectional\ Area$

Figure for Question 8.

The level, h and the temperature, T are to be controlled by manipulating mass flow rates w_h and w_c. The temperatures of the inlet stream T_h and T_c are considered as disturbance variables. The outlet flow-rate w can be assumed constant. Derive a state-space model of the system in terms of deviation variables.
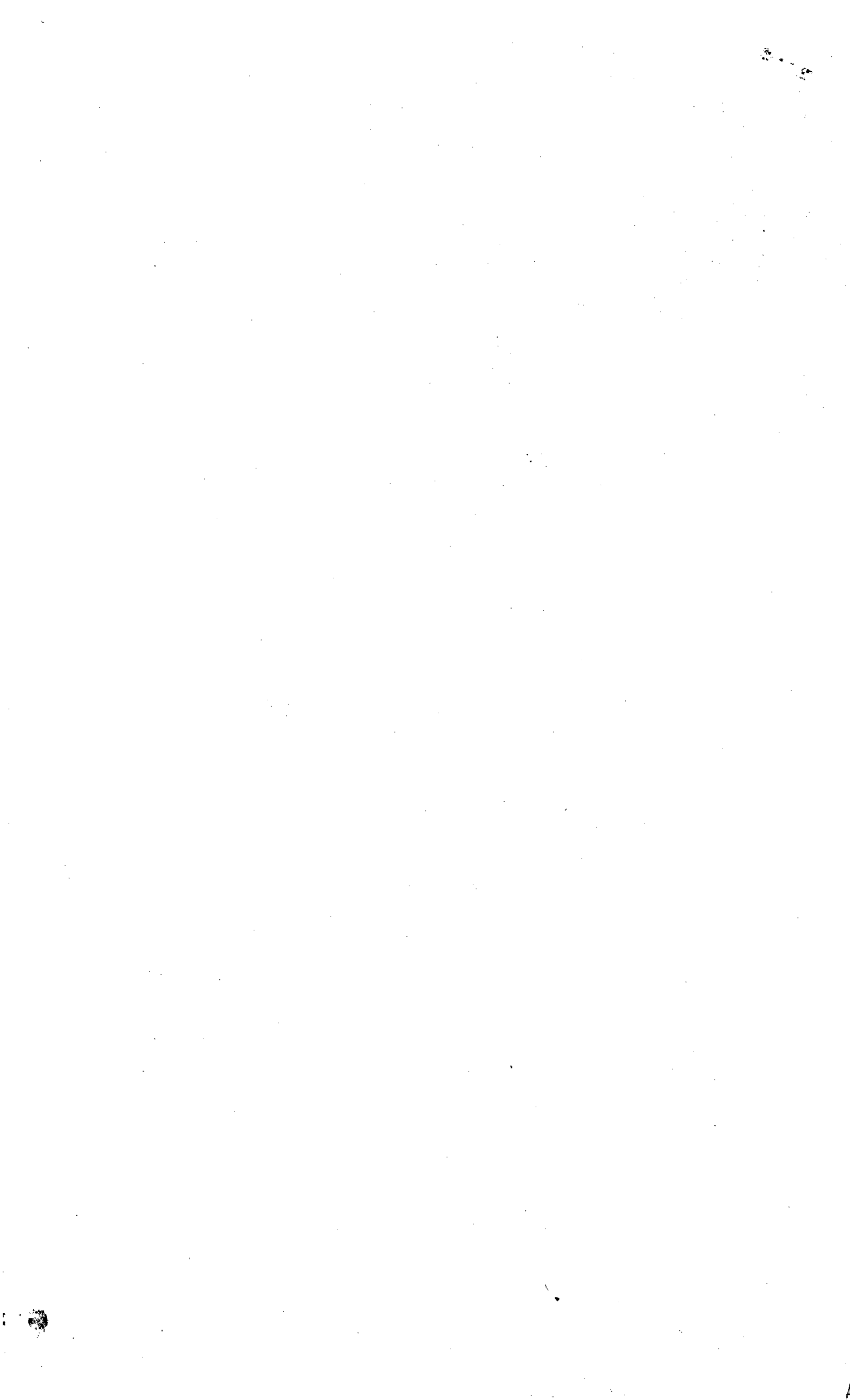
BANGLADESH UNIVERSITY OF ENGINEERING AND TECHNOLOGY, DHAKA

L-4/T-1   B. Sc. Engineering Examinations 2010-2011

Sub : **CHE 405** (Process Design I)

Full Marks: 210   Time : 3 Hours

The figures in the margin indicate full marks.

USE SEPARATE SCRIPTS FOR EACH SECTION

SECTION – A

There are **FOUR** questions in this Section. Answer any **THREE.**

1. (a) Make a study estimate of the fixed capital investment for a process plant if the purchased-equipment cost is Taka one crore. Use the ranges of process-plant component cost outlined in given Table for a process plant handling both solids and fluids with a high degree of automatic controls and essentially outdoor operation. Do not include Land.   **(17)**

    (b) If the above chemical process plant is erected in Mountain area for a fixed-capital investment as calculated in 1(a) in 1988, estimate the fixed-capital investment in 1999 for a similar process plant located in New England with three times the process capacity but with an equal number of process units. Use the power factor method (direct + indirect plant cost) to evaluate the new fixed-capital investment, and assume the factors given in supplied Table apply.   **(9)**

    (c) The purchased cost of a 8 m^2 single atmospheric drum dryer was $12,000 in 1992. Estimate the purchased cost of a similar 10 m^2 dryer in 2000. Use the annual average "Chemical Engineering Plant Cost Index" to update the purchase cost of the dryer.   **(9)**

2. (a) A process for making a single product involves reacting two liquids in a continuously agitated reactor and distilling the resulting mixture. Unused reactants are recovered as overhead and are recycled. The product is obtained in sufficiently pure form as bottoms from the distribution tower.   distillation   **(20)**

    (i)   Prepare a qualitative flowsheet for the process showing all pieces of equipment.

    (ii) Tabulate the information needed concerning chemicals and the process, in order to design the agitated reactor.

    (b) Write briefly on (i) Control of exposure hazards (ii) Patent search, and, (iii) Safety factor in equipment sizing.   **(15)**

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3. A process flowsheet diagram is supplied. In it, (H) denotes a heat exchanger where the stream temperature is increased and (C), where stream temperature is lowered. Since the reboiler temperature is too high for heat exchange with any of the process streams, it becomes an independent heat exchange problem. A hot oil utility stream available at 320°C and cooled to 310°C is to be used for heating the reboiler, as well as any process heating loads not met by process-process exchange. The cost of the hot oil is $2.25/GJ. The condenser temperature is in a range that could be used to heat some of the process stream; however, for brevity purpose it will not be included in the present analysis. The cooling utility is cooling water available at 10°C with an allowable temperature rise of 10°C and a cost of $0.25/GJ.

For your convenience the "Initial Temperature Interval Table" and "Composite diagram with 33°C approach temperature" are provided. Using the above mentioned Table and Figure as the starting point for your solution.

   (a) Construct the revised temperature interval table. **(12)**

   (b) Reconstruct the composite diagram to achieve a ΔT_{min} of 20°C. **(8)**

   (c) Construct the balanced composite diagram for the process with a ΔT_{min} of 20°C, and find the minimum utility duties and the annual cost for the utilities after the heat integration. **(10)**

   (d) Compare the cost with case where only hot and cold utilities are used. **(5)**

4. Air must be cooled from 120°C to 80 °C. Cooling water is available at 50 °C. It is advised to use a floating heat Shell and Tube heat exchanger for this operation. **(35)**

   Determine the tube length, number of tubes and installed cost for the optimum exchanger which will handle 9000 kg air/hr.

   Use data from the supplied Tables for properties of air and water. Other information are given as below:

   (a) Exchanger Specifications:
       (i) Steel Shell and Tube exchanger with cross flow baffling
       (ii) Cooling water passes through shell side
       (iii) One tube pass and counter-current flow
       (iv) Tube OD = 1.0 in; Tube ID = 0.782 inch
       (v) $\frac{15}{16}$ inch triangular pitch. Tubes are staggered.

   (b) Costs:
       (i) Purchased cost: per square foot outside heat transfer area = $34.
       (ii) Installation cost = 15% of purchased cost
       (iii) Annual-fixed cost including maintenance = 20% of installed cost
       (iv) Cost for cooling water (not including pumping cost) = $0.009/450 kg
       (v) Cost for energy supplied to force the cooling water and the gas through the exchanger (including effect of pump and motor efficiency and cost is $0.04/kWh).

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Contd ... Q. No. 4

   (c) <u>General:</u>

      (i) Average absolute pressure of gas in exchanger is 10 atm.

      (ii) Unit operates 7000 hr/year

      (iii) In the friction relations, take

          $B_i = 1.2$

          B_0 = number of crosses

      (iv) Safety factor for outside film co-efficient, $F_s = 1.3$

      (v) Fouling co-efficient for cooling water = 3000 kcal/(hr. m^2.$^\circ$C)

          Fouling co-efficient for air = 4000 kcal/(hr.m^2.$^\circ$C)

      (vi) Optimum flow conditions are turbulent both for shell and tube side.

      (vii) $\dfrac{N_r N_c}{N_f} = 1$

      (vii) ψ_i (see attached file)

      (viii) ψ_0 (see attached file)

SECTION – B

There are **FOUR** questions in this Section. Answer any **THREE**.

5. A distillation column is separating a feed that is 50 mol% n-hexane and 50 mole% n-heptane. Feed is a saturated liquid. Average column pressure is 1 atm. Distillate composition is $X_D = 0.99$ (mole fraction of n-hexane) and $X_B = 0.001$. Feed rate is 1000 lb moles/hr. Internal reflux ratio $L/V = 0.8$. The column has a total reboiler and a total condenser. Determine the diameter and packing height of the column if random packing is used. Physical properties for pure n-hexane at 69°C are given below: **(35)**

      Liquid specific gravity = 0.659

      Surface tension $\sigma = 13.2$ dynes/cm   *relative volatility = 2.5*

      Saturated vapor density = 0.1917 lb/ft^3

      Following design equations are also provided

$$V_{nf} = 0.38\left(\frac{\sigma}{20}\right)^{0.2}\left(\frac{\rho_L - \rho_V}{\rho_v}\right)^{0.5}$$

For random packing:

$$\text{HETP} = \begin{cases} D & \text{for } D \le 0.5\,\text{m} \\ 0.5\,D^{0.3} & \text{for } D > 0.5\,\text{m} \\ D^{0.3} & \text{for absorption columns with } D > 0.5\,\text{m} \end{cases}$$

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6. (a) Write down the general aspects that need to be evaluated in selecting piping materials. **(10)**

(b) Liquid benzene at 38°C with a vapor pressure of 26.4 kN/m^2 and a density of 860 kg/m^3 is to be pumped at a rate of 0.0025 m^3/s from a storage tank to a discharge location 3 m above the liquid level in the tank. The pump with a mechanical efficiency of 60 percent is 1 m above the liquid level. The storage tank is at atmospheric pressure. The pressure at the end of the discharge line is 445 kPa absolute. The inside diameter of the pipe used to transfer the benzene is 0.0409 m. The frictional pressure drops in the suction line and the discharge time are 3.45 and 37.9 kN/m^2, respectively. Determine the head developed by the pump and the total power requirement. If the pump manufacturer specifies a required net positive suction head of 0.3 m, will such a pump be applicable for this service. **(25)**

7. (a) In order to design a batch distillation column the designer needs to estimate the quantity of liquid that need to be distilled. In a given batch distillation of a binary mixture the concentration of the liquid need to be changed from x_{i1} to x_{i2} (in mole fraction of the more volatile component). A single batch will be operated to produce W kmol residual liquid. If the relative volatility of the components in the mixture is. α_{ij}, estimate the amount of initial liquid to be charged in the batch. **(17)**

(b) A centrifugal pump is tested with water and found at 1800 rpm to deliver 200 gal/min at a pressure rise of 50 psi. Determine optimum economic pipe diameter for installation of the pump system. **(18)**

8. A gas fired furnace is shown in the figure for question 8. The hot combustion gases pass through a heat exchanger to heat fresh air for space heating. The gas flow is controlled by an electric solenoid value connected to a thermostat. The gas is ignited by a pilot light flame. A high temperature switch sheets off all gas in the of high temperature in the air space. **(35)**

Perform a HAZOP study on the furnace control system. Note that failure of the system will lead to excessive heating of the air space and possible fire. Besides, failure of the pilot light will lead to combustible gas in the furnace, heat exchanger and chimney which may lead to explosion.

Typical percentages of fixed-capital investment values for direct and indirect cost segments for multipurpose plants or large additions to existing facilities

Component	Range of FCI, %
Direct costs	
Purchased equipment	15–40
Purchased-equipment installation	6–14
Instrumentation and controls (installed)	2–12
Piping (installed)	4–17
Electrical systems (installed)	2–10
Buildings (including services)	2–18
Yard improvements	2–5
Service facilities (installed)	8–30
Land	1–2
Indirect costs	
Engineering and supervision	4–20
Construction expenses	4–17
Legal expenses	1–3
Contractor's fee	2–6
Contingency	5–15

Cost indexes as annual averages

Year	Marshall and Swift installed-equipment index (1926=100), All industries	Process industries				Nelson-Farrar refinery construction	Chemical Engineering plant cost index, 1957-59=100
1987	814	830	4406	956	410	1121.5	324
1988	852	859.3	4519	980	421	1164.5	343
1989	895	905.6	4615	1001	430	1195.9	355
1990	915.1	929.3	4732	1026	441	1225.7	357.6
1991	930.6	949.9	4835	1049	450	1252.9	361.3
1992	943.1	957.9	4985	1081	464	1277.3	358.2
1993	964.2	971.4	5210	1130	485	1310.8	359.2
1994	993.4	992.8	5408	1173	504	1349.7	368.1
1995	1027.5	1029.0	5471	1187	509	1392.1	381.1
1996	1039.1	1048.5	5620	1219	523	1418.9	381.7
1997	1056.8	1063.7	5825	1264	542	1449.2	386.5
1998	1061.9	1077.1	5920	1284	551	1477.6	389.5
1999	1068.3	1081.9	6060	1315	564	1497.2	390.6
2000	1089.0	1097.7	6221	1350	579	1542.7	394.1
2001	1093.9	1106.9	6342	1376	591	1579.7	394.3
2002	1102.5[‡]	1116.9[‡]	6490[‡]	1408[‡]	604[‡]	1599.2[‡]	390.4[†,§]

[†]All costs presented in this text and in the McGraw-Hill website are based on this value for January 2002, obtained from the *Chemical Engineering* index unless otherwise indicated. The website provides the corresponding mathematical cost relationships for all the graphical cost data presented in the text.

[‡]Projected.

[§]Calculated with revised index; see *Chem. Eng.*, 109: 62 (2002).

Relative labor rate and productivity indexes in chemical and allied products industries for the United States (1999)[†]

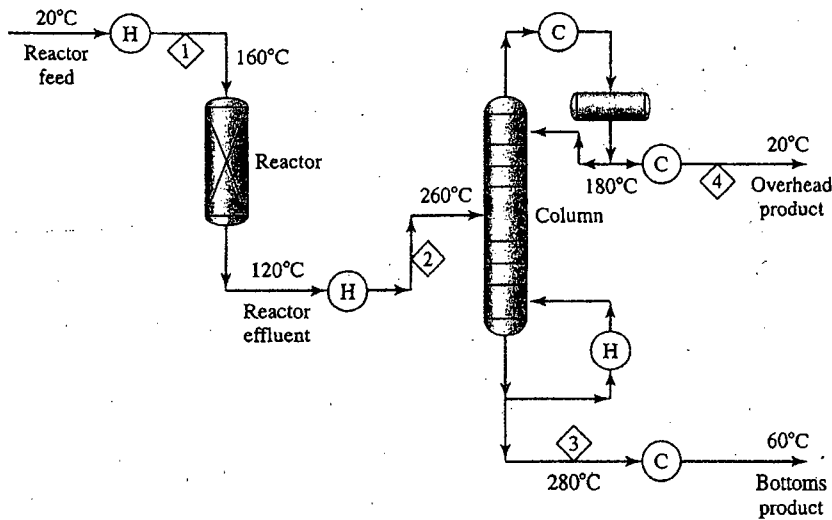| Region	Relative labor rate	Relative productivity factor
New England	1.14	0.95
Middle Atlantic	1.06	0.96
South Atlantic	0.84	0.91
Midwest	1.03	1.06
Gulf	0.95	1.22
Southwest	0.88	1.04
Mountain	0.88	0.97
Pacific Coast	1.22	0.89

[†]Adapted from J. M. Winton, *Chem. Week*, 121(24): 49 (1977), and updated with data from M. Kiley, ed., *National Construction Estimator*, 37th ed., Craftsman Book Company of America, Carlsbad, CA, 1989. Productivity, as considered here, is an economic term that gives the value added (products minus raw materials) per dollar of total payroll cost. Relative values were determined by taking the average of Kiley's weighted state values in each region divided by the weighted-average value of all the regions. See also Tables 6-14 and 6-15.

Fon Q. 1 (C)

Typical exponents for equipment cost as a function of capacity

Equipment	Size range	Exponent
Blender, double cone rotary, carbon steel (c.s.)	1.4–7.1 m³ (50–250 ft³)	0.49
Blower, centrifugal	0.5–4.7 m³/s (10³–10⁴ ft³/min)	0.59
Centrifuge, solid bowl, c.s.	7.5–75 kW (10–10² hp) drive	0.67
Crystallizer, vacuum batch, c.s.	15–200 m³ (500–7000 ft³)	0.37
Compressor, reciprocating, air-cooled, two-stage, 1035-kPa discharge	0.005–0.19 m³ (10–400 ft³/min)	0.69
Compressor, rotary, single-stage, sliding vane, 1035-kPa discharge	0.05–0.5 m³/s (10²–10³ ft³/min)	0.79
Dryer, drum, single vacuum	1–10 m² (10–10² ft²)	0.76
Dryer, drum, single atmospheric	1–10 m² (10–10² ft²)	0.40
Evaporator (installed), horizontal tank	10–1000 m² (10²–10⁴ ft²)	0.54
Fan, centrifugal	0.5–5 m³/s (10³–10⁴ ft³/min)	0.44
Fan, centrifugal	10–35 m³/s (2×10⁴–7×10⁴ ft³/min)	1.17
Heat exchanger, shell-and-tube, floating head, c.s.	10–40 m² (100–400 ft²)	0.60
Heat exchanger, shell-and-tube, fixed sheet, c.s.	10–40 m² (100–400 ft²)	0.44
Kettle, cast-iron, jacketed	1–3 m³ (250–800 gal)	0.27
Kettle, glass-lined, jacketed	0.8–3 m³ (200–800 gal)	0.31
Motor, squirrel cage, induction, 440-V, explosion-proof	4–15 kW (5–20 hp)	0.69
Motor, squirrel cage, induction, 440-V, explosion-proof	15–150 kW (20–200 hp)	0.99
Pump, reciprocating, horizontal cast-iron (includes motor)	1×10⁻⁴–6×10⁻³ m³/s (2–100 gpm)	0.34
Pump, centrifugal, horizontal, cast steel (includes motor)	4–40 m³/s·kPa (10⁴–10⁵ gpm·psi)	0.33
Reactor, glass-lined, jacketed (without drive)	0.2–2.2 m³ (50–600 gal)	0.54
Reactor, stainless steel, 2070-kPa	0.4–4.0 m³ (10²–10³ gal)	0.56
Separator, centrifugal, c.s.	1.5–7 m³ (50–250 ft³)	0.49
Tank, flat head, c.s.	0.4–40 m³ (10²–10⁴ gal)	0.57
Tank, c.s., glass-lined	0.4–4.0 m³ (10²–10³ gal)	0.49
Tower, c.s.	5×10²–10⁶ kg (10³–2×10⁶ lb)	0.62
Tray, bubble cap, c.s.	1–3 m (3–10 ft) diameter	1.20
Tray, sieve, c.s.	1–3 m (3–10 ft) diameter	0.86

Pnocess FLowsheet fon Q. 3

For Q.3

Initial temperature interval table

Stream number*	Temperature at interval ends, °C		Heat capacity flow rate, kW/°C	Enthalpy change, kW	Cascade, kW		Initial enthalpy cascade, kW
	In	Out					
Streams to be heated							
1	20	120	50	5,000	5,000		20
1 & 2	120	160	105	4,200	10,000		120
2	160	260	55	5,500	14,200		160
				Total 14,700	19,700		260
Stream to be cooled					15,000		280
3	280	180	30	-3,000	12,000		180
3 & 4	180	60	70	-8,400	3,600		60
4	60	20	40	-1,600	2,000		20
				Total-13,000			
Utilities							
Hot oil	320	310		-3,900			
Cooling water	10	20		2,200			

*See Figure ▓▓ to identify these streams

For Q. 3
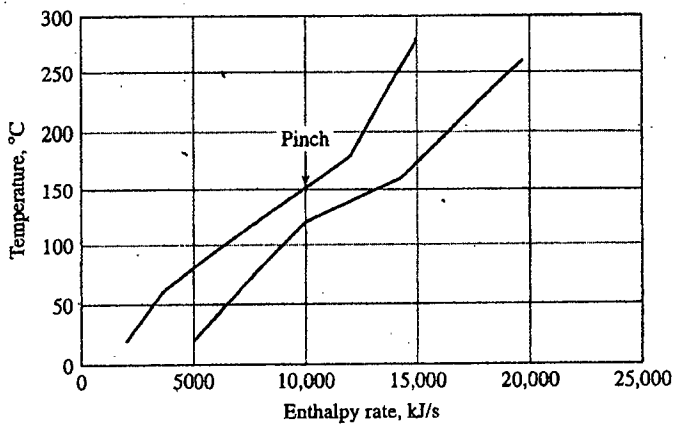
Composite diagram with 33°C approach temperature

For Q.4

$$\psi_i = B_i \left[\frac{12{,}200 D_i^{1.5} \mu_i^{1.83} \left(\mu_{w_i} / \mu_i \right)^{0.63}}{g_c D_o \rho_i^2 k_i^{2.33} c_{p_i}^{1.17}} \right]$$

$$\psi_o = \frac{B_o}{n_b} \frac{N_r N_c}{N_t} \left(\frac{2 b_o D_c D_o^{0.75} F_s^{4.75} \mu_{fo}^{1.42}}{\pi a_o^{4.75} g_c \rho_o^2 k_{fo}^{3.17} c_{p_{fo}}^{1.58}} \right)$$

Table A-5 | Properties of air at atmospheric pressure.[†]

The values of μ, k, c_p, and Pr are not strongly pressure-dependent and may be used over a fairly wide range of pressures

| T, K	ρ kg/m³	c_v kJ/kg·°C	$\mu \times 10^5$ kg/m·s	$\nu \times 10^6$ m²/s	k W/m·°C	$\alpha \times 10^4$ m²/s	Pr
100	3.6010	1.0266	0.6924	1.923	0.009246	0.02501	0.770
150	2.3675	1.0099	1.0283	4.343	0.013735	0.05745	0.753
200	1.7684	1.0061	1.3289	7.490	0.01809	0.10165	0.739
250	1.4128	1.0053	1.5990	11.31	0.02227	0.15675	0.722
300	1.1774	1.0057	1.8462	15.69	0.02624	0.22160	0.708
350	0.9980	1.0090	2.075	20.76	0.03003	0.2983	0.697
400	0.8826	1.0140	2.286	25.90	0.03365	0.3760	0.689
450	0.7833	1.0207	2.484	31.71	0.03707	0.4222	0.683
500	0.7048	1.0295	2.671	37.90	0.04038	0.5564	0.680
550	0.6423	1.0392	2.848	44.34	0.04360	0.6532	0.680
600	0.5879	1.0551	3.018	51.34	0.04659	0.7512	0.680
650	0.5430	1.0635	3.177	58.51	0.04953	0.8578	0.682
700	0.5030	1.0752	3.332	66.25	0.05230	0.9672	0.684
750	0.4709	1.0856	3.481	73.91	0.05509	1.0774	0.686
800	0.4405	1.0978	3.625	82.29	0.05779	1.1951	0.689
850	0.4149	1.1095	3.765	90.75	0.06028	1.3097	0.692
900	0.3925	1.1212	3.899	99.3	0.06279	1.4271	0.696
950	0.3716	1.1321	4.023	108.2	0.06525	1.5510	0.699
1000	0.3524	1.1417	4.152	117.8	0.06752	1.6779	0.702
1100	0.3204	1.160	4.44	138.6	0.0732	1.969	0.704
1200	0.2947	1.179	4.69	159.1	0.0782	2.251	0.707
1300	0.2707	1.197	4.93	182.1	0.0837	2.583	0.705
1400	0.2515	1.214	5.17	205.5	0.0891	2.920	0.705
1500	0.2355	1.230	5.40	229.1	0.0946	3.262	0.705
1600	0.2211	1.248	5.63	254.5	0.100	3.609	0.705
1700	0.2082	1.267	5.85	280.5	0.105	3.977	0.705
1800	0.1970	1.287	6.07	308.1	0.111	4.379	0.704
1900	0.1858	1.309	6.29	338.5	0.117	4.811	0.704
2000	0.1762	1.338	6.50	369.0	0.124	5.260	0.702
2100	0.1682	1.372	6.72	399.6	0.131	5.715	0.700
2200	0.1602	1.419	6.93	432.6	0.139	6.120	0.707
2300	0.1538	1.482	7.14	464.0	0.149	6.540	0.710
2400	0.1458	1.574	7.35	504.0	0.161	7.020	0.718
2500	0.1394	1.688	7.57	543.5	0.175	7.441	0.730

[†]From Natl. Bur. Stand. (U.S.) Circ. 564, 1955.

$$V_{nf} = C_{sb} \left(\frac{6}{20}\right)^{0.2} \left(\frac{\rho_L - \rho_V}{\rho_V}\right)^{0.5}$$
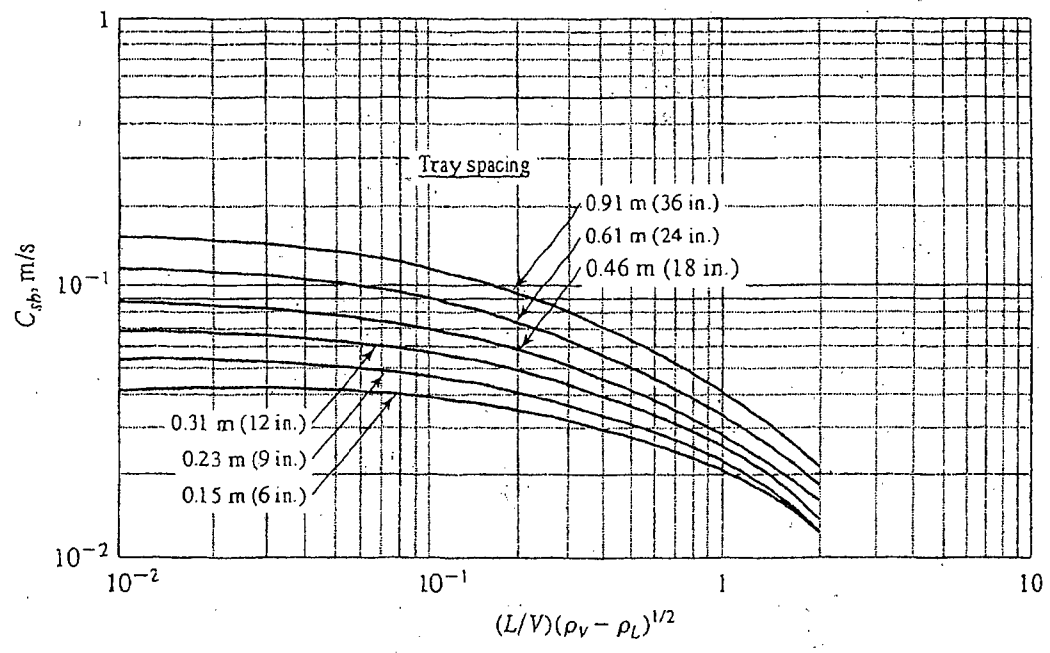
Figure
Chart for estimating values of C_{sb} (± 10 percent)   [*Adapted from J. R. Fair, Petro/Chem. Eng., 33(10): 45 (1961) with permission.*]

Figure for Question No. 5
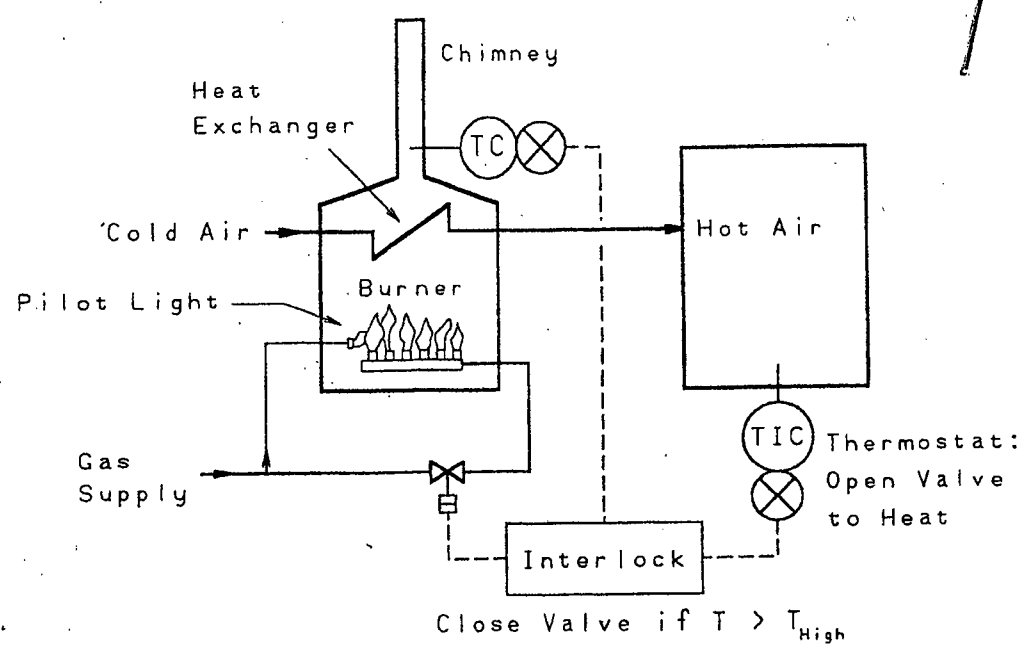
Figure   Furnace control system.

Figure for Question No. 8

Table A-9 | Properties of water (saturated liquid).[t]

Note: $Gr_x Pr = \left(\dfrac{g\beta\rho^2 c_p}{\mu k}\right) x^3 \Delta T$

°F	°C	c_p kJ/kg·°C	ρ kg/m³	μ kg/m·s	k W/m·°C	Pr	$\dfrac{g\beta\rho^2 c_p}{\mu k}$ 1/m³·°C
32	0	4.225	999.8	1.79×10^{-3}	0.566	13.25	
40	4.44	4.208	999.8	1.55	0.575	11.35	1.91×10^{9}
50	10	4.195	999.2	1.31	0.585	9.40	6.34×10^{9}
60	15.56	4.186	998.6	1.12	0.595	7.88	1.08×10^{10}
70	21.11	4.179	997.4	9.8×10^{-4}	0.604	6.78	1.46×10^{10}
80	26.67	4.179	995.8	8.6	0.614	5.85	1.91×10^{10}
90	32.22	4.174	994.9	7.65	0.623	5.12	2.48×10^{10}
100	37.78	4.174	993.0	6.82	0.630	4.53	3.3×10^{10}
110	43.33	4.174	990.6	6.16	0.637	4.04	4.19×10^{10}
120	48.89	4.174	988.8	5.62	0.644	3.64	4.89×10^{10}
130	54.44	4.179	985.7	5.13	0.649	3.30	5.66×10^{10}
140	60	4.179	983.3	4.71	0.654	3.01	6.48×10^{10}
150	65.55	4.183	980.3	4.3	0.659	2.73	7.62×10^{10}
160	71.11	4.186	977.3	4.01	0.665	2.53	8.84×10^{10}
170	76.67	4.191	973.7	3.72	0.668	2.33	9.85×10^{10}
180	82.22	4.195	970.2	3.47	0.673	2.16	1.09×10^{11}
190	87.78	4.199	966.7	3.27	0.675	2.03	
200	93.33	4.204	963.2	3.06	0.678	1.90	
220	104.4	4.216	955.1	2.67	0.684	1.66	
240	115.6	4.229	946.7	2.44	0.685	1.51	
260	126.7	4.250	937.2	2.19	0.685	1.36	
280	137.8	4.271	928.1	1.98	0.685	1.24	
300	148.9	4.296	918.0	1.86	0.684	1.17	
350	176.7	4.371	890.4	1.57	0.677	1.02	
400	204.4	4.467	859.4	1.36	0.665	1.00	
450	232.2	4.585	825.7	1.20	0.646	0.85	
500	260	4.731	785.2	1.07	0.616	0.83	
550	287.7	5.024	735.5	9.51×10^{-5}			
600	315.6	5.703	678.7	8.68			

[t] Adapted to SI units from A. I. Brown and S. M. Marco, *Introduction to Heat Transfer*, 3d ed. New York: McGraw-Hill, 1958.

BANGLADESH UNIVERSITY OF ENGINEERING AND TECHNOLOGY, DHAKA

L-4/T-1   B. Sc. Engineering Examinations 2010-2011

Sub : **CHE 453** (Transport Phenomena)

Full Marks : 210   Time : 3 Hours

The figures in the margin indicate full marks.

USE SEPARATE SCRIPTS FOR EACH SECTION

SECTION – A

There are **FOUR** questions in this section. Answer any **THREE**.

1. (a) Explain Reynolds analogy and Taylor-Prandtl analogy and show that $j_H = \frac{1}{2} C_f$. **(20)**

   (b) The following equation gives the asymptotic temperature profile for heating a fluid of constant ρ, μ, C_p and k in a long tube with constant heat flux at the wall. **(15)**

$$T = T_0 - \frac{4zq_1}{\rho C_p v_{z,max} R} + \frac{q_1 R}{k}\left[-\xi^2 + \frac{1}{4}\xi^4 + \frac{7}{24}\right]$$

   where $\xi = \frac{r}{R}$

   Use this temperature profile to show that the limiting Nusselt number for these conditions is $\frac{48}{11}$.

2. (a) A solid body occupying the space from $y = 0$ to $y = \infty$ is initially at temperature T_0. At time $t = 0$, the surface at $y = 0$ is suddenly raised to temperature T_1 and maintain at that temperature for $t > 0$. Find:

   (i) The time-dependant temperature profiles $T(y, t)$. **(20)**

   (ii) The expression for thermal penetration thickness. **(3)**

   (iii) The expression for wall heat flux. **(5)**

   (b) Prove that

   (i) $v - v^* = \omega_A(v_A - v_A^*) + \omega_B(v_B - v^*)$ **(3)**

   (ii) $N_A = J_A^* + C_A v^*$ **(4)**

3. Consider a catalytic reactor shown in Fig. for Q. 3, in which the dimerization reaction $2A \rightarrow A_2$ is being carried out. Find:

   (i) The expression for concentration profile **(15)**

   (ii) The expression for molar mass flux through the film **(5)**

   (iii) Consider the above system when the reaction $2A \rightarrow A_2$ is not instantaneous at the catalytic surface at $(z = \delta)$ and assume that the rate at which A disappears at the surface is proportional to the concentration: $N_{AZ} = K_{C_A}$ where K is a rate constant. **(15)**

CHE 453

4. (a) Consider porous catalyst particles in the shape of thin disks, such that the surface area of the edge of the disk is small in comparison with that of the two circular faces. Obtain the expression for the effectiveness factor. **(25)**

(b) Consider the leaching of a substance A from solid particles by a solvent B. Obtain a differential equation for C_A as a function of z by making a man balance on A over a thin slab of thickness Δz. Show the rate of leaching is given by $N_{AZ} = \dfrac{D_{AB}(C_{A0} - C_{A\delta})}{\delta}$. **(10)**

SECTION – B

There are **FOUR** questions in this section. Answer Q. No. 5 and any **TWO** from the rest.

Q. No. 5 is compulsory.

The symbols have their usual meaning.

COMPULSORY

5. (a) Transform the x-component of equation of motion given below into Navier-Stokes equation and Euler equation by making necessary assumptions and simplifications: **(12)**

$$\frac{\partial}{\partial t}(\rho vx) = -\left(\frac{\partial}{\partial x}\rho v_x v_x + \frac{\partial}{\partial y}\rho v_y v_x + \frac{\partial}{\partial z}\rho v_z v_x\right)$$
$$-\left(\frac{\partial}{\partial x}\tau_{xx} + \frac{\partial}{\partial y}\tau_{yx} + \frac{\partial}{\partial z}\tau_{zx}\right) - \frac{\partial p}{\partial x} + \rho g_x$$

(b) Neglecting the viscous dissipation terms make equation (A) of table 10.2.3 dimensionless. **(8)**

(c) What are the usual boundary conditions encountered in fluid flow problems? **(3)**

(d) Give comparison between forced and free convection in non-isothermal systems. **(6)**

(e) Explain the physical significance and mention application area for the dimensionless groups: **(6)**

$$\left(\rho g b^3 \Delta p / \mu^2\right); \left[\mu v^2 / k(T_b - T_0)\right); (hb/2k)$$

6. (a) Derive the equation of continuity in cylindrical co-ordinates. **(20)**

(b) Determine $v_\theta(r)$ between two coaxial cylinders of radii R and kR rotating at angular velocities Ω_0 and Ω_i, respectively. Assume the space between cylinders is filled with an incompressible isothermal fluid in laminar flow. **(15)**

CHE 453

7. (a) Find the bulk temperature (mixing-cup temperature) for the fluid being heated inside a tube for the following velocity and temperature profiles: **(10)**

$$v_z = V_{z, max} [1 - r^2/R^2]$$
$$T = T_{centre} + A [r^2/R^2 - (1/4) (r^4/R^4)$$

where, A is constant.

(b) A standard schedule 40 two-inch steel pipe (inside diameter 2.067 in. and wall thickness 0.154 in.) carrying steam is insulated with 2 in. of 85 percent magnesia covered in turn with 2 in. of cork. Estimate the heat loss per hour per foot of pipe if the inner surface of the pipe is at 250°F and the outer surface of the cork is at 90°F. The thermal conductivities of the substances concerned are: **(18)**

Steel : 26.1 Btu hr^{-1} ft^{-1} $°F^{-1}$

85 percent magnesia : 0.04 Btu hr^{-1} ft^{-1} $°F^{-1}$

Cork : 0.03 Btu hr^{-1} ft^{-1} $°F^{-1}$

(c) A thermocouple is inserted into a gas stream (Figure 7c). Estimate the true temperature of the gas stream if: **(7)**

$T_1 = 525°F$

T_w = wall temperature = 360°F

h = 120 Btu hr^{-1} ft^2 $(°F)^{-1}$

k = 60 Btu hr^{-1} ft^{-1} $(°F)^{-1}$

B = thickness of well wall = 0.08 in.

L = length of wall = 0.2 ft

[Hints: The temperature profile of a fin is expressed as: $\theta = \dfrac{\cosh N (1 - \zeta)}{\cosh N}$ all the components of the equation have the conventional meanings.]

8. (a) A viscous fluid is in laminar flow in a horizontal slit formed by two parallel walls a distance 2B apart, where y is the distance from the mid-plane and x is the distance in the direction of flow. The fluid is under pressure gradient. Carry out shell momentum balance to final an expression for velocity profile and make a sketch of the profile. Sketch the system and list the assumptions made. **(18)**

(b) Derive an expression for the temperature distribution $T(\infty)$ in a viscous fluid in steady laminar flow between large flat parallel plates, as described in question 8(a). Both plates are maintained at constant temperature T_0. Take into account explicitly the heat generated by viscous dissipation. Neglect the temperature dependence of μ and k. **(17)**
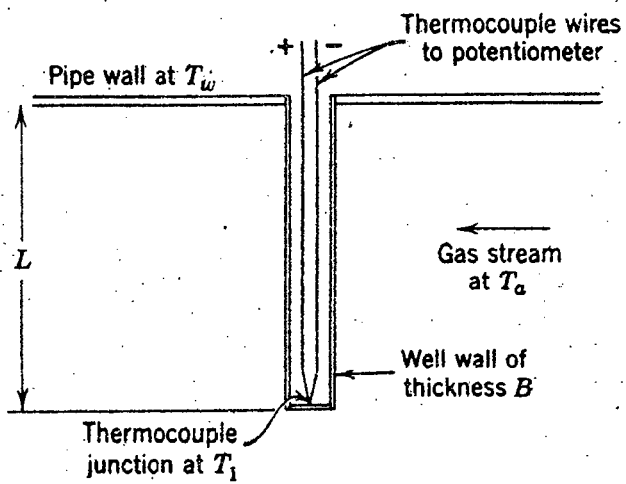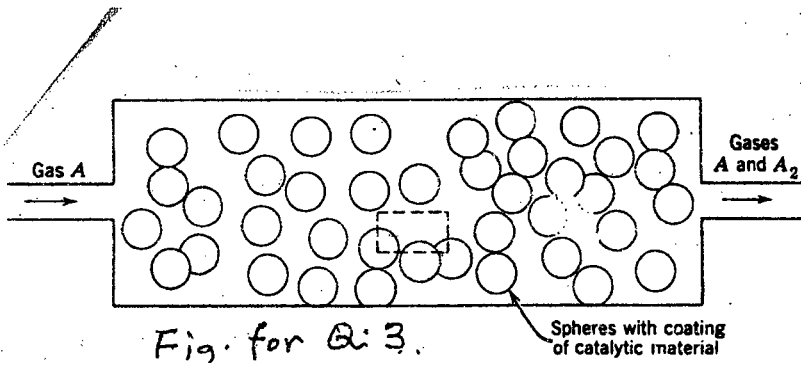

Fig. for Q. 3.

Gas A →

Gases A and A_2 →

Spheres with coating of catalytic material

Thermocouple wires to potentiometer

Pipe wall at T_w

+ −

Gas stream at T_a

L

Well wall of thickness B

Thermocouple junction at T_1

Fig for Q. 7C: Thermocouple in cylindrical wall

TABLE 3.4-1
THE EQUATION OF CONTINUITY IN SEVERAL COORDINATE SYSTEMS

Rectangular coordinates (x, y, z):

$$\frac{\partial \rho}{\partial t} + \frac{\partial}{\partial x}(\rho v_x) + \frac{\partial}{\partial y}(\rho v_y) + \frac{\partial}{\partial z}(\rho v_z) = 0 \qquad (A)$$

Cylindrical coordinates (r, θ, z):

$$\frac{\partial \rho}{\partial t} + \frac{1}{r}\frac{\partial}{\partial r}(\rho r v_r) + \frac{1}{r}\frac{\partial}{\partial \theta}(\rho v_\theta) + \frac{\partial}{\partial z}(\rho v_z) = 0 \qquad (B)$$

Spherical coordinates (r, θ, ϕ):

$$\frac{\partial \rho}{\partial t} + \frac{1}{r^2}\frac{\partial}{\partial r}(\rho r^2 v_r) + \frac{1}{r \sin \theta}\frac{\partial}{\partial \theta}(\rho v_\theta \sin \theta) + \frac{1}{r \sin \theta}\frac{\partial}{\partial \phi}(\rho v_\phi) = 0 \qquad (C)$$

TABLE 3.4-2
THE EQUATION OF MOTION IN RECTANGULAR COORDINATES (x, y, z)

In terms of τ:

x-component
$$\rho\left(\frac{\partial v_x}{\partial t} + v_x\frac{\partial v_x}{\partial x} + v_y\frac{\partial v_x}{\partial y} + v_z\frac{\partial v_x}{\partial z}\right) = -\frac{\partial p}{\partial x}$$
$$-\left(\frac{\partial \tau_{xx}}{\partial x} + \frac{\partial \tau_{yx}}{\partial y} + \frac{\partial \tau_{zx}}{\partial z}\right) + \rho g_x \qquad (A)$$

y-component
$$\rho\left(\frac{\partial v_y}{\partial t} + v_x\frac{\partial v_y}{\partial x} + v_y\frac{\partial v_y}{\partial y} + v_z\frac{\partial v_y}{\partial z}\right) = -\frac{\partial p}{\partial y}$$
$$-\left(\frac{\partial \tau_{xy}}{\partial x} + \frac{\partial \tau_{yy}}{\partial y} + \frac{\partial \tau_{zy}}{\partial z}\right) + \rho g_y \qquad (B)$$

z-component
$$\rho\left(\frac{\partial v_z}{\partial t} + v_x\frac{\partial v_z}{\partial x} + v_y\frac{\partial v_z}{\partial y} + v_z\frac{\partial v_z}{\partial z}\right) = -\frac{\partial p}{\partial z}$$
$$-\left(\frac{\partial \tau_{xz}}{\partial x} + \frac{\partial \tau_{yz}}{\partial y} + \frac{\partial \tau_{zz}}{\partial z}\right) + \rho g_z \qquad (C)$$

In terms of velocity gradients for a Newtonian fluid with constant ρ and μ:

x-component
$$\rho\left(\frac{\partial v_x}{\partial t} + v_x\frac{\partial v_x}{\partial x} + v_y\frac{\partial v_x}{\partial y} + v_z\frac{\partial v_x}{\partial z}\right) = -\frac{\partial p}{\partial x}$$
$$+ \mu\left(\frac{\partial^2 v_x}{\partial x^2} + \frac{\partial^2 v_x}{\partial y^2} + \frac{\partial^2 v_x}{\partial z^2}\right) + \rho g_x \qquad (D)$$

y-component
$$\rho\left(\frac{\partial v_y}{\partial t} + v_x\frac{\partial v_y}{\partial x} + v_y\frac{\partial v_y}{\partial y} + v_z\frac{\partial v_y}{\partial z}\right) = -\frac{\partial p}{\partial y}$$
$$+ \mu\left(\frac{\partial^2 v_y}{\partial x^2} + \frac{\partial^2 v_y}{\partial y^2} + \frac{\partial^2 v_y}{\partial z^2}\right) + \rho g_y \qquad (E)$$

z-component
$$\rho\left(\frac{\partial v_z}{\partial t} + v_x\frac{\partial v_z}{\partial x} + v_y\frac{\partial v_z}{\partial y} + v_z\frac{\partial v_z}{\partial z}\right) = -\frac{\partial p}{\partial z}$$
$$+ \mu\left(\frac{\partial^2 v_z}{\partial x^2} + \frac{\partial^2 v_z}{\partial y^2} + \frac{\partial^2 v_z}{\partial z^2}\right) + \rho g_z \qquad (F)$$

TABLE 3.4-3

THE EQUATION OF MOTION IN CYLINDRICAL COORDINATES (r, θ, z)

In terms of τ:

r-component[a]

$$\rho\left(\frac{\partial v_r}{\partial t} + v_r \frac{\partial v_r}{\partial r} + \frac{v_\theta}{r}\frac{\partial v_r}{\partial \theta} - \frac{v_\theta^2}{r} + v_z \frac{\partial v_r}{\partial z}\right) = -\frac{\partial p}{\partial r}$$

$$-\left(\frac{1}{r}\frac{\partial}{\partial r}(r\tau_{rr}) + \frac{1}{r}\frac{\partial \tau_{r\theta}}{\partial \theta} - \frac{\tau_{\theta\theta}}{r} + \frac{\partial \tau_{rz}}{\partial z}\right) + \rho g_r \quad (A)$$

θ-component[b]

$$\rho\left(\frac{\partial v_\theta}{\partial t} + v_r \frac{\partial v_\theta}{\partial r} + \frac{v_\theta}{r}\frac{\partial v_\theta}{\partial \theta} + \frac{v_r v_\theta}{r} + v_z \frac{\partial v_\theta}{\partial z}\right) = -\frac{1}{r}\frac{\partial p}{\partial \theta}$$

$$-\left(\frac{1}{r^2}\frac{\partial}{\partial r}(r^2\tau_{r\theta}) + \frac{1}{r}\frac{\partial \tau_{\theta\theta}}{\partial \theta} + \frac{\partial \tau_{\theta z}}{\partial z}\right) + \rho g_\theta \quad (B)$$

z-component

$$\rho\left(\frac{\partial v_z}{\partial t} + v_r \frac{\partial v_z}{\partial r} + \frac{v_\theta}{r}\frac{\partial v_z}{\partial \theta} + v_z \frac{\partial v_z}{\partial z}\right) = -\frac{\partial p}{\partial z}$$

$$-\left(\frac{1}{r}\frac{\partial}{\partial r}(r\tau_{rz}) + \frac{1}{r}\frac{\partial \tau_{\theta z}}{\partial \theta} + \frac{\partial \tau_{zz}}{\partial z}\right) + \rho g_z \quad (C)$$

In terms of velocity gradients for a Newtonian fluid with constant ρ and μ:

r-component[a]

$$\rho\left(\frac{\partial v_r}{\partial t} + v_r \frac{\partial v_r}{\partial r} + \frac{v_\theta}{r}\frac{\partial v_r}{\partial \theta} - \frac{v_\theta^2}{r} + v_z \frac{\partial v_r}{\partial z}\right) = -\frac{\partial p}{\partial r}$$

$$+ \mu\left[\frac{\partial}{\partial r}\left(\frac{1}{r}\frac{\partial}{\partial r}(rv_r)\right) + \frac{1}{r^2}\frac{\partial^2 v_r}{\partial \theta^2} - \frac{2}{r^2}\frac{\partial v_\theta}{\partial \theta} + \frac{\partial^2 v_r}{\partial z^2}\right] + \rho g_r \quad (D)$$

θ-component[b]

$$\rho\left(\frac{\partial v_\theta}{\partial t} + v_r \frac{\partial v_\theta}{\partial r} + \frac{v_\theta}{r}\frac{\partial v_\theta}{\partial \theta} + \frac{v_r v_\theta}{r} + v_z \frac{\partial v_\theta}{\partial z}\right) = -\frac{1}{r}\frac{\partial p}{\partial \theta}$$

$$+ \mu\left[\frac{\partial}{\partial r}\left(\frac{1}{r}\frac{\partial}{\partial r}(rv_\theta)\right) + \frac{1}{r^2}\frac{\partial^2 v_\theta}{\partial \theta^2} + \frac{2}{r^2}\frac{\partial v_r}{\partial \theta} + \frac{\partial^2 v_\theta}{\partial z^2}\right] + \rho g_\theta \quad (E)$$

z-component

$$\rho\left(\frac{\partial v_z}{\partial t} + v_r \frac{\partial v_z}{\partial r} + \frac{v_\theta}{r}\frac{\partial v_z}{\partial \theta} + v_z \frac{\partial v_z}{\partial z}\right) = -\frac{\partial p}{\partial z}$$

$$+ \mu\left[\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial v_z}{\partial r}\right) + \frac{1}{r^2}\frac{\partial^2 v_z}{\partial \theta^2} + \frac{\partial^2 v_z}{\partial z^2}\right] + \rho g_z \quad (F)$$

---

[a] The term $\rho v_\theta^2/r$ is the *centrifugal force*. It gives the effective force in the r-direction resulting from fluid motion in the θ-direction. This term arises automatically on transformation from rectangular to cylindrical coordinates; it does not have to be added on physical grounds. Two problems in which this term arises are discussed in Examples 3.5-1 and 3.5-2.

[b] The term $\rho v_r v_\theta/r$ is the *Coriolis force*. It is an effective force in the θ-direction when there is flow in both the r- and θ-directions. This term also arises automatically in the coordinate transformation. The Coriolis force arises in the problem of flow near a rotating disk (see, for example, H. Schlichting, *Boundary-Layer Theory*, McGraw-Hill, New York (1955), Chapter 5, §10.

TABLE 3.4-4
THE EQUATION OF MOTION IN SPHERICAL COORDINATES (r, θ, ϕ)

In terms of τ:

r-component
$$\rho\left(\frac{\partial v_r}{\partial t} + v_r\frac{\partial v_r}{\partial r} + \frac{v_\theta}{r}\frac{\partial v_r}{\partial \theta} + \frac{v_\phi}{r\sin\theta}\frac{\partial v_r}{\partial \phi} - \frac{v_\theta^2 + v_\phi^2}{r}\right)$$

$$= -\frac{\partial p}{\partial r} - \left(\frac{1}{r^2}\frac{\partial}{\partial r}(r^2\tau_{rr}) + \frac{1}{r\sin\theta}\frac{\partial}{\partial \theta}(\tau_{r\theta}\sin\theta)\right.$$

$$\left. + \frac{1}{r\sin\theta}\frac{\partial\tau_{r\phi}}{\partial\phi} - \frac{\tau_{\theta\theta} + \tau_{\phi\phi}}{r}\right) + \rho g_r \qquad (A)$$

θ-component
$$\rho\left(\frac{\partial v_\theta}{\partial t} + v_r\frac{\partial v_\theta}{\partial r} + \frac{v_\theta}{r}\frac{\partial v_\theta}{\partial \theta} + \frac{v_\phi}{r\sin\theta}\frac{\partial v_\theta}{\partial \phi} + \frac{v_r v_\theta}{r} - \frac{v_\phi^2\cot\theta}{r}\right)$$

$$= -\frac{1}{r}\frac{\partial p}{\partial\theta} - \left(\frac{1}{r^2}\frac{\partial}{\partial r}(r^2\tau_{r\theta}) + \frac{1}{r\sin\theta}\frac{\partial}{\partial\theta}(\tau_{\theta\theta}\sin\theta) + \frac{1}{r\sin\theta}\frac{\partial\tau_{\theta\phi}}{\partial\phi}\right.$$

$$\left. + \frac{\tau_{r\theta}}{r} - \frac{\cot\theta}{r}\tau_{\phi\phi}\right) + \rho g_\theta \qquad (B)$$

ϕ-component
$$\rho\left(\frac{\partial v_\phi}{\partial t} + v_r\frac{\partial v_\phi}{\partial r} + \frac{v_\theta}{r}\frac{\partial v_\phi}{\partial \theta} + \frac{v_\phi}{r\sin\theta}\frac{\partial v_\phi}{\partial \phi} + \frac{v_\phi v_r}{r} + \frac{v_\theta v_\phi}{r}\cot\theta\right)$$

$$= -\frac{1}{r\sin\theta}\frac{\partial p}{\partial\phi} - \left(\frac{1}{r^2}\frac{\partial}{\partial r}(r^2\tau_{r\phi}) + \frac{1}{r}\frac{\partial\tau_{\theta\phi}}{\partial\theta} + \frac{1}{r\sin\theta}\frac{\partial\tau_{\phi\phi}}{\partial\phi}\right.$$

$$\left. + \frac{\tau_{r\phi}}{r} + \frac{2\cot\theta}{r}\tau_{\theta\phi}\right) + \rho g_\phi \qquad (C)$$

TABLE 3.4-4 (contd.)

In terms of velocity gradients for a Newtonian fluid with constant ρ and μ:

r-component
$$\rho\left(\frac{\partial v_r}{\partial t} + v_r\frac{\partial v_r}{\partial r} + \frac{v_\theta}{r}\frac{\partial v_r}{\partial \theta} + \frac{v_\phi}{r\sin\theta}\frac{\partial v_r}{\partial \phi} - \frac{v_\theta^2 + v_\phi^2}{r}\right)$$

$$= -\frac{\partial p}{\partial r} + \mu\left(\frac{1}{r^2}\frac{\partial^2}{\partial r^2}(r^2 v_r) + \frac{1}{r^2\sin\theta}\frac{\partial}{\partial\theta}\left(\sin\theta\frac{\partial v_r}{\partial\theta}\right) + \frac{1}{r^2\sin^2\theta}\frac{\partial^2 v_r}{\partial\phi^2}\right)$$

$$+ \rho g_r \qquad (D)$$

θ-component
$$\rho\left(\frac{\partial v_\theta}{\partial t} + v_r\frac{\partial v_\theta}{\partial r} + \frac{v_\theta}{r}\frac{\partial v_\theta}{\partial \theta} + \frac{v_\phi}{r\sin\theta}\frac{\partial v_\theta}{\partial \phi} + \frac{v_r v_\theta}{r} - \frac{v_\phi^2\cot\theta}{r}\right)$$

$$= -\frac{1}{r}\frac{\partial p}{\partial\theta} + \mu\left(\frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial v_\theta}{\partial r}\right) + \frac{1}{r^2}\frac{\partial}{\partial\theta}\left(\frac{1}{\sin\theta}\frac{\partial}{\partial\theta}(v_\theta\sin\theta)\right) + \frac{1}{r^2\sin^2\theta}\frac{\partial^2 v_\theta}{\partial\phi^2}\right.$$

$$\left. + \frac{2}{r^2}\frac{\partial v_r}{\partial\theta} - \frac{2\cos\theta}{r^2\sin^2\theta}\frac{\partial v_\phi}{\partial\phi}\right) + \rho g_\theta$$

$$(E)$$

ϕ-component
$$\rho\left(\frac{\partial v_\phi}{\partial t} + v_r\frac{\partial v_\phi}{\partial r} + \frac{v_\theta}{r}\frac{\partial v_\phi}{\partial \theta} + \frac{v_\phi}{r\sin\theta}\frac{\partial v_\phi}{\partial \phi} + \frac{v_\phi v_r}{r} + \frac{v_\theta v_\phi}{r}\cot\theta\right)$$

$$= -\frac{1}{r\sin\theta}\frac{\partial p}{\partial\phi} + \mu\left(\frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial v_\phi}{\partial r}\right) + \frac{1}{r^2}\frac{\partial}{\partial\theta}\left(\frac{1}{\sin\theta}\frac{\partial}{\partial\theta}(v_\phi\sin\theta)\right) + \frac{1}{r^2\sin^2\theta}\frac{\partial^2 v_\phi}{\partial\phi^2}\right.$$

$$\left. + \frac{2}{r^2\sin\theta}\frac{\partial v_r}{\partial\phi} + \frac{2\cos\theta}{r^2\sin^2\theta}\frac{\partial v_\theta}{\partial\phi}\right) + \rho g_\phi \qquad (F)$$

$$\tau_{xx} = -\mu\left[2\frac{\partial v_x}{\partial x} - \tfrac{2}{3}(\nabla \cdot v)\right] \qquad (A)$$

$$\tau_{yy} = -\mu\left[2\frac{\partial v_y}{\partial y} - \tfrac{2}{3}(\nabla \cdot v)\right] \qquad (B)$$

$$\tau_{zz} = -\mu\left[2\frac{\partial v_z}{\partial z} - \tfrac{2}{3}(\nabla \cdot v)\right] \qquad (C)$$

$$\tau_{xy} = \tau_{yx} = -\mu\left[\frac{\partial v_x}{\partial y} + \frac{\partial v_y}{\partial x}\right] \qquad (D)$$

$$\tau_{yz} = \tau_{zy} = -\mu\left[\frac{\partial v_y}{\partial z} + \frac{\partial v_z}{\partial y}\right] \qquad (E)$$

$$\tau_{zx} = \tau_{xz} = -\mu\left[\frac{\partial v_z}{\partial x} + \frac{\partial v_x}{\partial z}\right] \qquad (F)$$

$$(\nabla \cdot v) = \frac{\partial v_x}{\partial x} + \frac{\partial v_y}{\partial y} + \frac{\partial v_z}{\partial z} \qquad (G)$$

TABLE 3.4-6
COMPONENTS OF THE STRESS TENSOR FOR NEWTONIAN FLUIDS
IN CYLINDRICAL COORDINATES (r, θ, z)

$$\tau_{rr} = -\mu\left[2\frac{\partial v_r}{\partial r} - \tfrac{2}{3}(\nabla \cdot v)\right] \qquad (A)$$

$$\tau_{\theta\theta} = -\mu\left[2\left(\frac{1}{r}\frac{\partial v_\theta}{\partial \theta} + \frac{v_r}{r}\right) - \tfrac{2}{3}(\nabla \cdot v)\right] \qquad (B)$$

$$\tau_{zz} = -\mu\left[2\frac{\partial v_z}{\partial z} - \tfrac{2}{3}(\nabla \cdot v)\right] \qquad (C)$$

$$\tau_{r\theta} = \tau_{\theta r} = -\mu\left[r\frac{\partial}{\partial r}\left(\frac{v_\theta}{r}\right) + \frac{1}{r}\frac{\partial v_r}{\partial \theta}\right] \qquad (D)$$

$$\tau_{\theta z} = \tau_{z\theta} = -\mu\left[\frac{\partial v_\theta}{\partial z} + \frac{1}{r}\frac{\partial v_z}{\partial \theta}\right] \qquad (E)$$

$$\tau_{zr} = \tau_{rz} = -\mu\left[\frac{\partial v_z}{\partial r} + \frac{\partial v_r}{\partial z}\right] \qquad (F)$$

$$(\nabla \cdot v) = \frac{1}{r}\frac{\partial}{\partial r}(rv_r) + \frac{1}{r}\frac{\partial v_\theta}{\partial \theta} + \frac{\partial v_z}{\partial z} \qquad (G)$$

TABLE 3.4-7
COMPONENTS OF THE STRESS TENSOR FOR NEWTONIAN FLUIDS
IN SPHERICAL COORDINATES (r, θ, ϕ)

$$\tau_{rr} = -\mu\left[2\frac{\partial v_r}{\partial r} - \tfrac{2}{3}(\nabla \cdot v)\right] \qquad (A)$$

$$\tau_{\theta\theta} = -\mu\left[2\left(\frac{1}{r}\frac{\partial v_\theta}{\partial \theta} + \frac{v_r}{r}\right) - \tfrac{2}{3}(\nabla \cdot v)\right] \qquad (B)$$

$$\tau_{\phi\phi} = -\mu\left[2\left(\frac{1}{r \sin \theta}\frac{\partial v_\phi}{\partial \phi} + \frac{v_r}{r} + \frac{v_\theta \cot \theta}{r}\right) - \tfrac{2}{3}(\nabla \cdot v)\right] \qquad (C)$$

$$\tau_{r\theta} = \tau_{\theta r} = -\mu\left[r\frac{\partial}{\partial r}\left(\frac{v_\theta}{r}\right) + \frac{1}{r}\frac{\partial v_r}{\partial \theta}\right] \qquad (D)$$

$$\tau_{\theta\phi} = \tau_{\phi\theta} = -\mu\left[\frac{\sin \theta}{r}\frac{\partial}{\partial \theta}\left(\frac{v_\phi}{\sin \theta}\right) + \frac{1}{r \sin \theta}\frac{\partial v_\theta}{\partial \phi}\right] \qquad (E)$$

$$\tau_{\phi r} = \tau_{r\phi} = -\mu\left[\frac{1}{r \sin \theta}\frac{\partial v_r}{\partial \phi} + r\frac{\partial}{\partial r}\left(\frac{v_\phi}{r}\right)\right] \qquad (F)$$

$$(\nabla \cdot v) = \frac{1}{r^2}\frac{\partial}{\partial r}(r^2 v_r) + \frac{1}{r \sin \theta}\frac{\partial}{\partial \theta}(v_\theta \sin \theta) + \frac{1}{r \sin \theta}\frac{\partial v_\phi}{\partial \phi} \qquad (G)$$

TABLE 10.2-1
COMPONENTS OF THE ENERGY FLUX q

Rectangular	Cylindrical	Spherical
$q_x = -k \dfrac{\partial T}{\partial x}$ (A)	$q_r = -k \dfrac{\partial T}{\partial r}$ (D)	$q_r = -k \dfrac{\partial T}{\partial r}$ (G)
$q_y = -k \dfrac{\partial T}{\partial y}$ (B)	$q_\theta = -k \dfrac{1}{r} \dfrac{\partial T}{\partial \theta}$ (E)	$q_\theta = -k \dfrac{1}{r} \dfrac{\partial T}{\partial \theta}$ (H)
$q_z = -k \dfrac{\partial T}{\partial z}$ (C)	$q_z = -k \dfrac{\partial T}{\partial z}$ (F)	$q_\phi = -k \dfrac{1}{r \sin \theta} \dfrac{\partial T}{\partial \phi}$ (I)

TABLE 10.2-2
THE EQUATION OF ENERGY IN TERMS OF ENERGY AND MOMENTUM FLUXES
(Eq. 10.1-19)

Rectangular coordinates:

$$\rho \hat{C}_v \left(\frac{\partial T}{\partial t} + v_x \frac{\partial T}{\partial x} + v_y \frac{\partial T}{\partial y} + v_z \frac{\partial T}{\partial z} \right) = -\left[\frac{\partial q_x}{\partial x} + \frac{\partial q_y}{\partial y} + \frac{\partial q_z}{\partial z} \right]$$

$$- T \left(\frac{\partial p}{\partial T} \right)_\rho \left(\frac{\partial v_x}{\partial x} + \frac{\partial v_y}{\partial y} + \frac{\partial v_z}{\partial z} \right) - \left\{ \tau_{xx} \frac{\partial v_x}{\partial x} + \tau_{yy} \frac{\partial v_y}{\partial y} + \tau_{zz} \frac{\partial v_z}{\partial z} \right\}$$

$$- \left\{ \tau_{xy} \left(\frac{\partial v_x}{\partial y} + \frac{\partial v_y}{\partial x} \right) + \tau_{xz} \left(\frac{\partial v_x}{\partial z} + \frac{\partial v_z}{\partial x} \right) + \tau_{yz} \left(\frac{\partial v_y}{\partial z} + \frac{\partial v_z}{\partial y} \right) \right\} \quad (A)$$

Cylindrical coordinates:

$$\rho \hat{C}_v \left(\frac{\partial T}{\partial t} + v_r \frac{\partial T}{\partial r} + \frac{v_\theta}{r} \frac{\partial T}{\partial \theta} + v_z \frac{\partial T}{\partial z} \right) = -\left[\frac{1}{r} \frac{\partial}{\partial r} (r q_r) + \frac{1}{r} \frac{\partial q_\theta}{\partial \theta} + \frac{\partial q_z}{\partial z} \right]$$

$$- T \left(\frac{\partial p}{\partial T} \right)_\rho \left(\frac{1}{r} \frac{\partial}{\partial r} (r v_r) + \frac{1}{r} \frac{\partial v_\theta}{\partial \theta} + \frac{\partial v_z}{\partial z} \right) - \left\{ \tau_{rr} \frac{\partial v_r}{\partial r} + \tau_{\theta\theta} \frac{1}{r} \left(\frac{\partial v_\theta}{\partial \theta} + v_r \right) \right.$$

$$\left. + \tau_{zz} \frac{\partial v_z}{\partial z} \right\} - \left\{ \tau_{r\theta} \left[r \frac{\partial}{\partial r} \left(\frac{v_\theta}{r} \right) + \frac{1}{r} \frac{\partial v_r}{\partial \theta} \right] + \tau_{rz} \left(\frac{\partial v_z}{\partial r} + \frac{\partial v_r}{\partial z} \right) \right.$$

$$\left. + \tau_{\theta z} \left(\frac{1}{r} \frac{\partial v_z}{\partial \theta} + \frac{\partial v_\theta}{\partial z} \right) \right\} \quad (B)$$

Spherical coordinates:

$$\rho \hat{C}_v \left(\frac{\partial T}{\partial t} + v_r \frac{\partial T}{\partial r} + \frac{v_\theta}{r} \frac{\partial T}{\partial \theta} + \frac{v_\phi}{r \sin \theta} \frac{\partial T}{\partial \phi} \right) = -\left[\frac{1}{r^2} \frac{\partial}{\partial r} (r^2 q_r) \right.$$

$$\left. + \frac{1}{r \sin \theta} \frac{\partial}{\partial \theta} (q_\theta \sin \theta) + \frac{1}{r \sin \theta} \frac{\partial q_\phi}{\partial \phi} \right] - T \left(\frac{\partial p}{\partial T} \right)_\rho \left(\frac{1}{r^2} \frac{\partial}{\partial r} (r^2 v_r) \right.$$

$$\left. + \frac{1}{r \sin \theta} \frac{\partial}{\partial \theta} (v_\theta \sin \theta) + \frac{1}{r \sin \theta} \frac{\partial v_\phi}{\partial \phi} \right) - \left\{ \tau_{rr} \frac{\partial v_r}{\partial r} + \tau_{\theta\theta} \left(\frac{1}{r} \frac{\partial v_\theta}{\partial \theta} + \frac{v_r}{r} \right) \right.$$

$$\left. + \tau_{\phi\phi} \left(\frac{1}{r \sin \theta} \frac{\partial v_\phi}{\partial \phi} + \frac{v_r}{r} + \frac{v_\theta \cot \theta}{r} \right) \right\} - \left\{ \tau_{r\theta} \left(\frac{\partial v_\theta}{\partial r} + \frac{1}{r} \frac{\partial v_r}{\partial \theta} - \frac{v_\theta}{r} \right) \right.$$

$$\left. + \tau_{r\phi} \left(\frac{\partial v_\phi}{\partial r} + \frac{1}{r \sin \theta} \frac{\partial v_r}{\partial \phi} - \frac{v_\phi}{r} \right) + \tau_{\theta\phi} \left(\frac{1}{r} \frac{\partial v_\phi}{\partial \theta} + \frac{1}{r \sin \theta} \frac{\partial v_\theta}{\partial \phi} - \frac{\cot \theta}{r} v_\phi \right) \right\}$$

$$(C)$$

Note: The terms contained in braces { } are associated with viscous dissipation and may usually be neglected, except for systems with large velocity gradients.

TABLE 3.4-8

THE FUNCTION $-(\tau : \nabla v) = \mu\Phi_v$ FOR NEWTONIAN FLUIDS[a]

Rectangular

$$\Phi_v = 2\left[\left(\frac{\partial v_x}{\partial x}\right)^2 + \left(\frac{\partial v_y}{\partial y}\right)^2 + \left(\frac{\partial v_z}{\partial z}\right)^2\right]$$

$$+ \left[\frac{\partial v_y}{\partial x} + \frac{\partial v_x}{\partial y}\right]^2 + \left[\frac{\partial v_z}{\partial y} + \frac{\partial v_y}{\partial z}\right]^2 + \left[\frac{\partial v_x}{\partial z} + \frac{\partial v_z}{\partial x}\right]^2$$

$$- \frac{2}{3}\left[\frac{\partial v_x}{\partial x} + \frac{\partial v_y}{\partial y} + \frac{\partial v_z}{\partial z}\right]^2 \tag{A}$$

Cylindrical

$$\Phi_v = 2\left[\left(\frac{\partial v_r}{\partial r}\right)^2 + \left(\frac{1}{r}\frac{\partial v_\theta}{\partial \theta} + \frac{v_r}{r}\right)^2 + \left(\frac{\partial v_z}{\partial z}\right)^2\right]$$

$$+ \left[r\frac{\partial}{\partial r}\left(\frac{v_\theta}{r}\right) + \frac{1}{r}\frac{\partial v_r}{\partial \theta}\right]^2 + \left[\frac{1}{r}\frac{\partial v_z}{\partial \theta} + \frac{\partial v_\theta}{\partial z}\right]^2$$

$$+ \left[\frac{\partial v_r}{\partial z} + \frac{\partial v_z}{\partial r}\right]^2$$

$$- \frac{2}{3}\left[\frac{1}{r}\frac{\partial}{\partial r}(rv_r) + \frac{1}{r}\frac{\partial v_\theta}{\partial \theta} + \frac{\partial v_z}{\partial z}\right]^2 \tag{B}$$

Spherical

$$\Phi_v = 2\left[\left(\frac{\partial v_r}{\partial r}\right)^2 + \left(\frac{1}{r}\frac{\partial v_\theta}{\partial \theta} + \frac{v_r}{r}\right)^2\right.$$

$$\left. + \left(\frac{1}{r\sin\theta}\frac{\partial v_\phi}{\partial \phi} + \frac{v_r}{r} + \frac{v_\theta\cot\theta}{r}\right)^2\right]$$

$$+ \left[r\frac{\partial}{\partial r}\left(\frac{v_\theta}{r}\right) + \frac{1}{r}\frac{\partial v_r}{\partial \theta}\right]^2$$

$$+ \left[\frac{\sin\theta}{r}\frac{\partial}{\partial \theta}\left(\frac{v_\phi}{\sin\theta}\right) + \frac{1}{r\sin\theta}\frac{\partial v_\theta}{\partial \phi}\right]^2$$

$$+ \left[\frac{1}{r\sin\theta}\frac{\partial v_r}{\partial \phi} + r\frac{\partial}{\partial r}\left(\frac{v_\phi}{r}\right)\right]^2$$

$$- \frac{2}{3}\left[\frac{1}{r^2}\frac{\partial}{\partial r}(r^2 v_r) + \frac{1}{r\sin\theta}\frac{\partial}{\partial \theta}(v_\theta\sin\theta) + \frac{1}{r\sin\theta}\frac{\partial v_\phi}{\partial \phi}\right]^2 \tag{C}$$

[a] These expressions are obtained by inserting the components of τ from Tables 3.4–5,6,7 into the expression for $(\tau : \nabla v)$ given in Appendix A. (See Tables A.7–1, 2, and 3.)

TABLE 10.2–3

THE EQUATION OF ENERGY IN TERMS OF THE TRANSPORT PROPERTIES

(for Newtonian fluids of constant ρ and k)

(Eq. 10.1–25 with viscous dissipation terms included)

Rectangular coordinates:

$$\rho \hat{C}_p \left(\frac{\partial T}{\partial t} + v_x \frac{\partial T}{\partial x} + v_y \frac{\partial T}{\partial y} + v_z \frac{\partial T}{\partial z} \right) = k \left[\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} + \frac{\partial^2 T}{\partial z^2} \right]$$

$$+ 2\mu \left\{ \left(\frac{\partial v_x}{\partial x} \right)^2 + \left(\frac{\partial v_y}{\partial y} \right)^2 + \left(\frac{\partial v_z}{\partial z} \right)^2 \right\} + \mu \left\{ \left(\frac{\partial v_x}{\partial y} + \frac{\partial v_y}{\partial x} \right)^2 \right.$$

$$\left. + \left(\frac{\partial v_x}{\partial z} + \frac{\partial v_z}{\partial x} \right)^2 + \left(\frac{\partial v_y}{\partial z} + \frac{\partial v_z}{\partial y} \right)^2 \right\} \qquad (A)$$

Cylindrical coordinates:

$$\rho \hat{C}_p \left(\frac{\partial T}{\partial t} + v_r \frac{\partial T}{\partial r} + \frac{v_\theta}{r} \frac{\partial T}{\partial \theta} + v_z \frac{\partial T}{\partial z} \right) = k \left[\frac{1}{r} \frac{\partial}{\partial r} \left(r \frac{\partial T}{\partial r} \right) + \frac{1}{r^2} \frac{\partial^2 T}{\partial \theta^2} + \frac{\partial^2 T}{\partial z^2} \right]$$

$$+ 2\mu \left\{ \left(\frac{\partial v_r}{\partial r} \right)^2 + \left[\frac{1}{r} \left(\frac{\partial v_\theta}{\partial \theta} + v_r \right) \right]^2 + \left(\frac{\partial v_z}{\partial z} \right)^2 \right\} + \mu \left\{ \left(\frac{\partial v_\theta}{\partial z} + \frac{1}{r} \frac{\partial v_z}{\partial \theta} \right)^2 \right.$$

$$\left. + \left(\frac{\partial v_z}{\partial r} + \frac{\partial v_r}{\partial z} \right)^2 + \left[\frac{1}{r} \frac{\partial v_r}{\partial \theta} + r \frac{\partial}{\partial r} \left(\frac{v_\theta}{r} \right) \right]^2 \right\} \qquad (B)$$

Spherical coordinates:

$$\rho \hat{C}_p \left(\frac{\partial T}{\partial t} + v_r \frac{\partial T}{\partial r} + \frac{v_\theta}{r} \frac{\partial T}{\partial \theta} + \frac{v_\phi}{r \sin \theta} \frac{\partial T}{\partial \phi} \right) = k \left[\frac{1}{r^2} \frac{\partial}{\partial r} \left(r^2 \frac{\partial T}{\partial r} \right) \right.$$

$$\left. + \frac{1}{r^2 \sin \theta} \frac{\partial}{\partial \theta} \left(\sin \theta \frac{\partial T}{\partial \theta} \right) + \frac{1}{r^2 \sin^2 \theta} \frac{\partial^2 T}{\partial \phi^2} \right] + 2\mu \left\{ \left(\frac{\partial v_r}{\partial r} \right)^2 \right.$$

$$+ \left(\frac{1}{r} \frac{\partial v_\theta}{\partial \theta} + \frac{v_r}{r} \right)^2 + \left(\frac{1}{r \sin \theta} \frac{\partial v_\phi}{\partial \phi} + \frac{v_r}{r} + \frac{v_\theta \cot \theta}{r} \right)^2 \right\}$$

$$+ \mu \left\{ \left[r \frac{\partial}{\partial r} \left(\frac{v_\theta}{r} \right) + \frac{1}{r} \frac{\partial v_r}{\partial \theta} \right]^2 + \left[\frac{1}{r \sin \theta} \frac{\partial v_r}{\partial \phi} + r \frac{\partial}{\partial r} \left(\frac{v_\phi}{r} \right) \right]^2 \right.$$

$$\left. + \left[\frac{\sin \theta}{r} \frac{\partial}{\partial \theta} \left(\frac{v_\phi}{\sin \theta} \right) + \frac{1}{r \sin \theta} \frac{\partial v_\theta}{\partial \phi} \right]^2 \right\} \qquad (C)$$

Note: The terms contained in braces { } are associated with viscous dissipation and may usually be neglected, except for systems with large velocity gradients.

BANGLADESH UNIVERSITY OF ENGINEERING AND TECHNOLOGY, DHAKA

L-4/T-1   B. Sc. Engineering Examinations 2010-2011

Sub : **CHE 455** (Mathematical Modelling)

Full Marks: 210   Time : 3 Hours

The figures in the margin indicate full marks.

USE SEPARATE SCRIPTS FOR EACH SECTION

SECTION – A

There are **FOUR** questions in this Section. Answer any **THREE**.

1.   (a) Derive the general solution of Riccati Equation   **(15)**

Riccati Equation: $y_{n+1}y_n + Ay_{n+1} + By_n + C = 0$

(b) It is desired to find the required number of ideal stages in the extraction cascade in
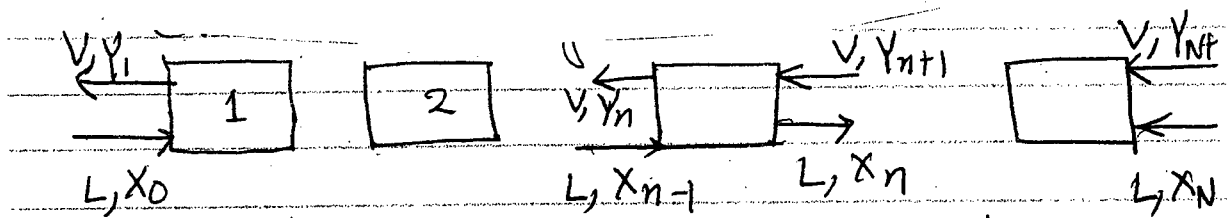Figure below   **(10)**

Figure : Continuous countercurrent cascade

Here,   feed-to-solvent ratio L/V = 1

Equilibrium distribution constant K = 1

Pure solvent is assumed.

Feed composition X_0 = 1 kg solute/kg carrier. It is desired to produce a rich extract
product such that

Y_1 = 0.9 kg solute/kg solvent

Use the following formula–

$$N+1 = \log \frac{\left[1 - \frac{Y_{N+1}}{Y_1}(1-\beta) - \frac{KX_0}{Y_1}\beta\right]}{\left[1 - \frac{KX_0}{Y_1}\right]} \cdot \frac{1}{\log \beta}$$

where   N = number of stages

$\beta = \frac{L}{kV}$

(c) Find the particular solution using Inverse Operator Method   **(10)**

$y_{n+2} - 2y_{n+1} - 8y_n = e^n$

2.   (a) Show that Backward Euler Method is always stable.   **(15)**

(b) What do we mean explicit and implicit method?   **(10)**

(c) What is the role of step size control for integration method?   **(10)**

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3. (a) What are the types of mathematical model in chemical engineering? Develop a model for simple flash and analyse in terms of variables, degrees of freedom, no of equations. **(15)**

   (b) Consider a reaction **(20)**

   A → B carried out in a batch reactor. The differential equation for species A is

   $$\frac{dC_A}{dt} = -kC_A$$
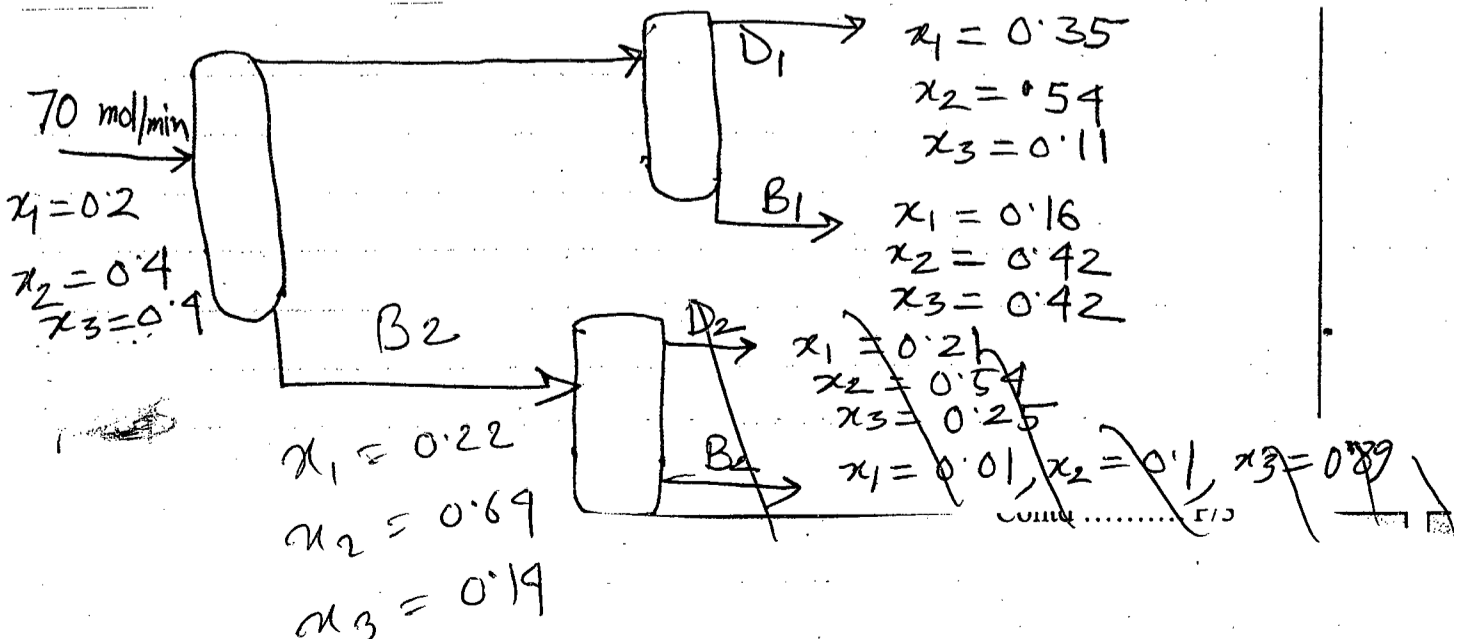
   The initial condition is: at t = 0, C_A = 1 mol/m^3.

   The rate constant of the reaction is 1 s^{-1}. Using the Runge-kutta fourth order method, determine the concentration of A at 3 s. Assume step size = 0.1 s.

4. (a) What are the uses of Mathematical Model. Consider the following flowsheet **(12)**

Consider the following flowsheet
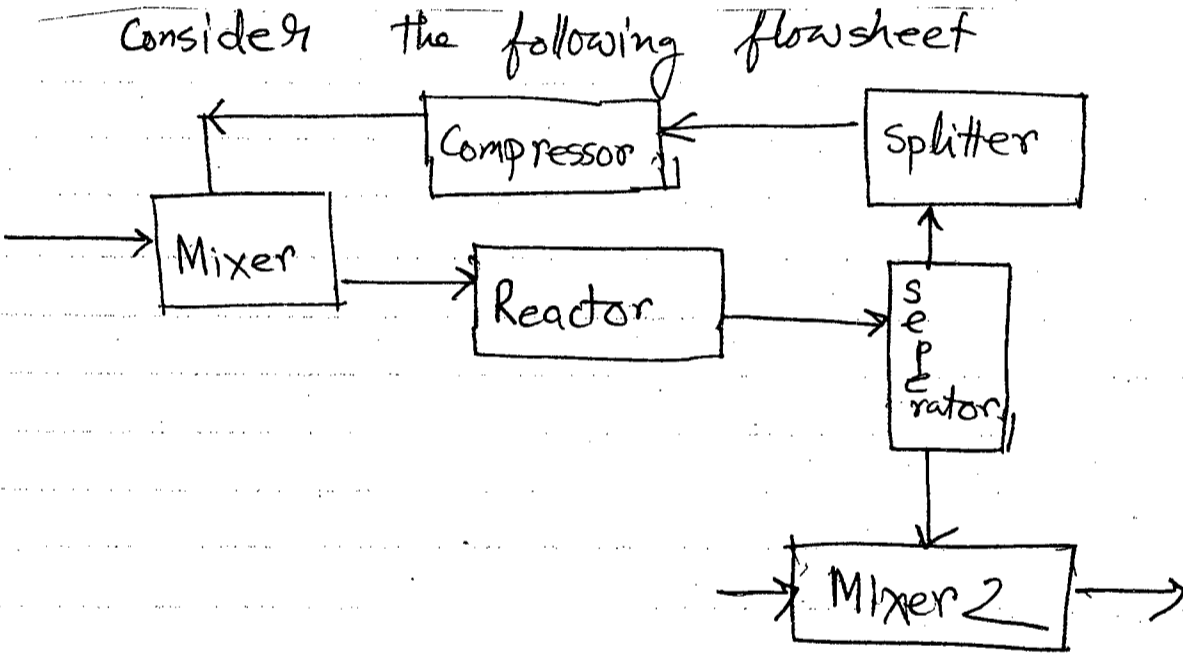
Calculate the degrees of Freedom according to Westerberg et al (CAPD).

   (b) Benzene(1), toluene(2), styrene(3) are to be separated in the sequence of distillation column in Figure below. Determine molar flow rates of D_1, B_1, D_2 and B_2 using Gauss-Seidel method. **(23)**

70 mol/min

$x_1 = 0.2$
$x_2 = 0.4$
$x_3 = 0.4$

D_1 $x_1 = 0.35$
$x_2 = 0.54$
$x_3 = 0.11$

B_1 $x_1 = 0.16$
$x_2 = 0.42$
$x_3 = 0.42$

B_2

D_2 $x_1 = 0.2$
$x_2 = 0.54$
$x_3 = 0.25$

$x_1 = 0.22$
$x_2 = 0.69$
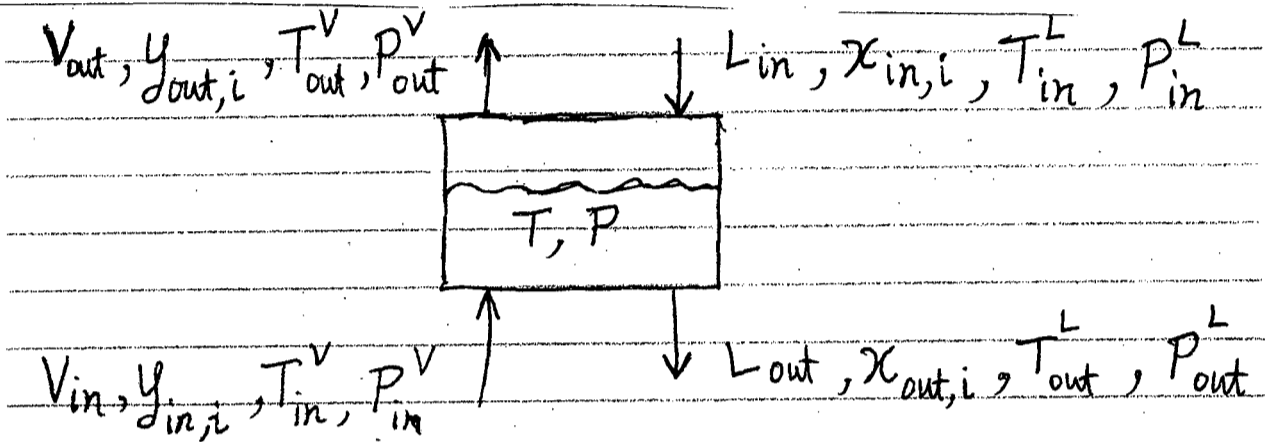$x_3 = 0.19$

B_2 $x_1 = 0.01$, $x_2 = 0.1$, $x_3 = 0.89$

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SECTION – B

There are **FOUR** questions in this Section. Answer any **THREE**.

5. Figure for Question No. 5 shows a single vapor-liquid equilibrium stage. The liquid and the vapor streams pot of the stage are in equilibrium.

The model to calculate the stage temperature can be written as:

$$y_{out,i} = k_i x_{out,i} \quad \text{and} \quad \sum y_{out,i} = 1.0$$

$k_i = \dfrac{p_i^v}{P}$ where, p_i^v is the pure component vapor pressure in mm Hg and P is the total pressure in mm Hg.

Where, $Log_{10}(p_i^v) = A_i - \left(\dfrac{B_i}{C_i + T}\right)$, T is the stage temperature in °C.

Given: P = 760 mm Hg; and there are two components and $x_{out,1} = 0.5$ and $x_{out,2} = 0.5$

(a) Write the model equations in function form. **(10)**

(b) Convert the set of non-linear equations to a set of linear equations. **(9)**

(c) Write the Jacobian Matrix of the model. **(6)**

(d) Write a Matlab code to solve the model. **(10)**

6. (a) Write down the steps that should be followed to solve an optimization problem. **(5)**

(b) What are the major difficulties experienced in formulating optimization problems? **(6)**

(c) Find the point on the curve $f = 3x^2 + 5x + 2$ nearest the origin. **(9)**

(d) The dependent variable y is related with the independent variable x by $y = a.x.\exp(-bx)$. You are supplied with ten experimental data points for y vs x. Show how would you use least-squares method to find a and b. **(15)**

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7. (a) Does the following constraint set form a convex region? **(6)**

$$x_1^2 + x_2^2 - 9 = 0$$

$$x_1 + x_2^2 \leq 1$$

$$x_1 + x_2 \leq 1$$

(b) Find the <u>maximum</u> of the function $f(x) = 1 - 8x + 2x^2 - \dfrac{10}{3}x^3 + \dfrac{1}{4}x^4 + \dfrac{4}{5}x^5 - \dfrac{1}{6}x^6$

   (i) Analytically [Hint: $f'(x) = (1 + x)^2(2 - x)^3$] **(6)**

   (ii) By Newton's method (two iterations will suffice). Start at $x = -2$ **(10)**

   (iii) By quadratic interpolation (three iterations will suffice). Start from $x = -2, 0$ and 1. **(13)**

8. A farmer is raising pigs for market, and he wishes to determine the quantities of the available types of feed that should be given to each pig to meet certain nutritional requirements at a minimum cost. The numbers of each type of basic nutritional ingredient contained within a kg of each feed type is given in the following table, along with the daily nutritional requirements and the feed costs:

Nutritional ingredient	Corn	Slaughterhouse		Minimum daily requirement
		Waste	Oil cake	
Carbohydrate	9	2	4	20
Protein	3	8	6	18
Vitamin	1	2	6	15
Cost, Tk/kg	21	18	15	

(a) Formulate the linear programming model for the problem. **(8)**

(b) Construct the initial simplex tableau, introducing artificial variables and so forth as needed for applying the simplex method. **(5)**

(c) Solve the problem by simplex method. **(12)**

(d) Write a Matlab code to solve the problem by linear programming. **(10)**

BANGLADESH UNIVERSITY OF ENGINEERING AND TECHNOLOGY, DHAKA

L-4/T-1   B. Sc. Engineering Examinations 20010-2011

Sub : **CHE 485** (Industrial Pollution Control)

Full Marks : 210   Time : 3 Hours

USE SEPARATE SCRIPTS FOR EACH SECTION

The figures in the margin indicate full marks.

SECTION – A

There are **FOUR** questions in this Section. Answer any **THREE**.

1. (a) What are the basic components in any Environmental Management system? Write a short note on : ISO series related to Envrionmental Management System.   **(10+10)**

    (b) How environmental auditing help to any control environmental hazard of industry?   **(10)**

    (c) Compose different types of auditing.   **(5)**

2. (a) Describe the factors affecting dispersion of air pollutants. What is Gaussian dispersion model? Write down the types of air monitoring instrumentation.   **(8+3)**

    (b) Why Air/Fuel ratio is important in combustion process?   **(8)**

    (c) Write down the mechanism of FGD for SO_2 gas removal.   **(8)**

    (d) On a clear summer afternoon with a wind speed of 3.20 m/s, the TSP concentration was found to be 2,000 glm^3 at a point 3 km downwind and 0.5 km perpendicular to the plume centerline from a coal-fired power plant. Given the following parameters and conditions, determine the TSP emission rate f' power plant: **(8)**

Stack parameters:	Atmospheric Conditions:-
Height = 75.0 m	
Diameter = 1.5 m	
Exit velocity = 12.0 m/s	Pressure = 100 kPa
Temperature = 322°C	Temperature = 28°C

See attached figures 2(d) (i) and 2d (ii) and Table 2(d) for additional data.

3. (a) Write a short note on different water quality parameters: **(4+4)**

    (i) Turbidity (ii) BOD

    (b) How water is purified in natural "water cycle"? What is "acid rain"?   **(4+3)**

    (c) Draw and describe the water filtration plant?   **(6)**

    (d) What are the parameters that affect the colloid stability? Show the reaction mechanism of any coagulant.   **(9)**

    (e) What are the pH values of lime soda and $Mg(OH)_2$ for softening process? Name three disinfectant used in water treatment process.   **(2+3)**

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4. (a) Write a short notes on: (5×3)

    (i) Primary clarifier (ii) Dechlorination (iii) the role of ultraviolet light in wastewater treatment plant.

(b) Create a wastewater treatment model to cleanwater from textile process. (Here (i) draw the block diagram (ii) Describe primary and secondary wastewater treatment)   (10)

(c) What are the common microbes in wastewater system and their operation ranges? Briefly describe the role of "Digester" for removing microbes from wastewater?   (5+5)

SECTION – B
There are **FOUR** questions in this Section. Answer any **THREE**.

5. (a) Compare the main features of the Environment Conservation Acts adopted in Bangladesh.   (10)

(b) Write down the requirements of Environmental clearance for four categories of industries.   (20)

(c) What do you understand by the term-"EIA"?   (5)

6. (a) Draw a simplified flow chart of a Nitrogenous Fertilizer Production Unit and point out the origins of pollutants.   (15)

(b) Suggest some NO_x and SO_2 reduction methods suitable for Ammonia - Urea plants.   (15)

(c) Write a short note on Environmental Management System of a fertilizer plant.   (5)

7. (a) Discuss the environmental impact of Petroleum Industries.   (25)

(b) Suggest some remedies for reduction of air pollution level of a Refinery.   (10)

8. (a) Draw a block diagram of a Leather industry pointing the sources of air and water pollution components.   (10)

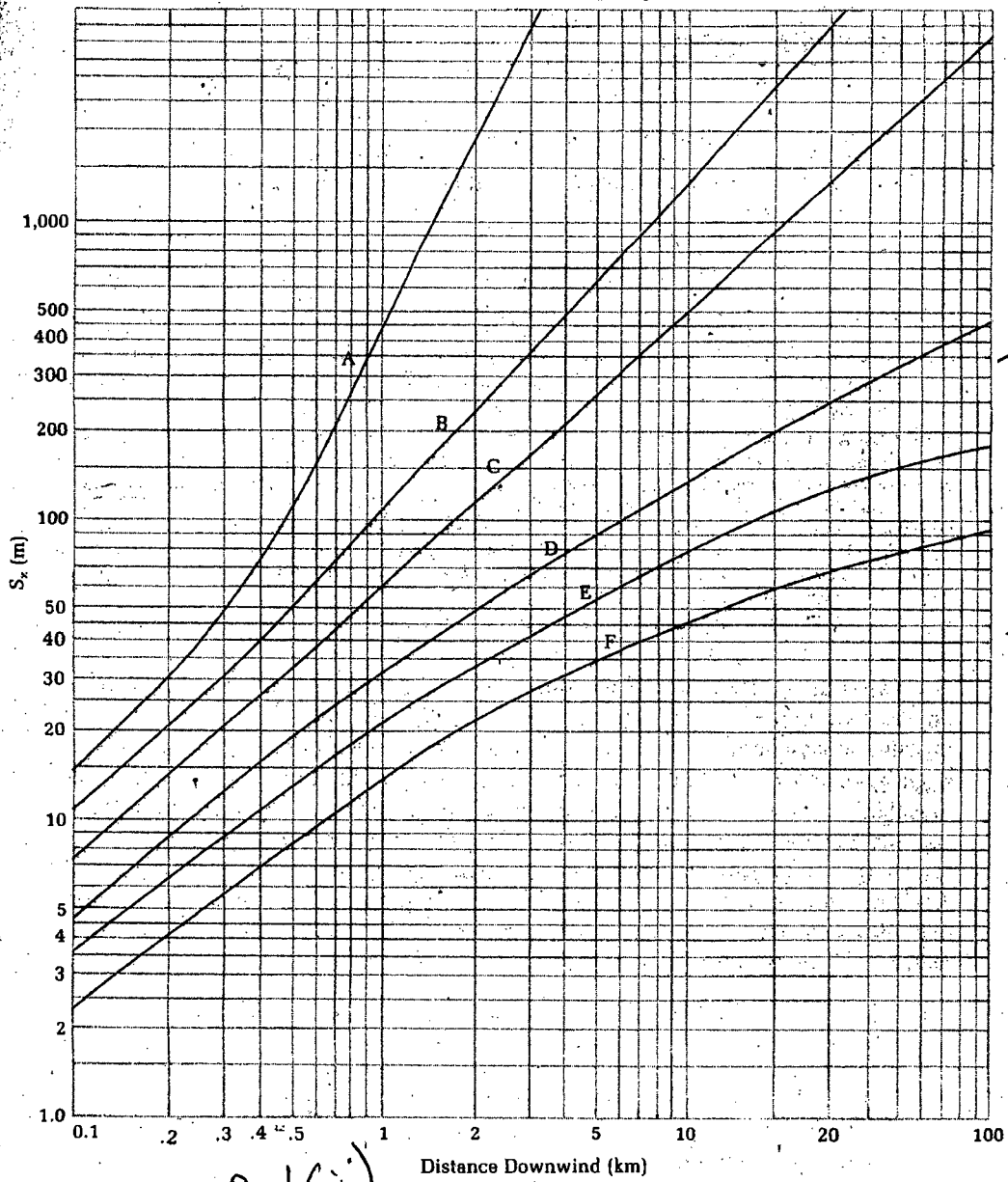(b) Explain the steps that are involved in Effluent Treatment Process of Leather and Textile industries of Bangladesh.   (25)

FIGURE 6.21 2 d (ii)
Vertical dispersion coefficient. (*Source:* Turner, *Workbook of Atmospheric Dispersion Estimates.*)

TABLE 6.2 *(2d)*
Key to stability categories

Table for 2(d)

Surface Wind speed (at 10 m) (m/s)	Day[a]			Night[a]	
	Incoming solar radiation			Thinly overcast or ≥ 4/8 Low cloud	≤ 3/8 Cloud
	Strong	Moderate	Slight		
<2	A	A–B	B		
2–3	A–B	B	C	E	F
3–5	B	B–C	C	D	E
5–6	C	C–D	D	D	D
>6	C	D	D	D	D

[a] The neutral class, D, should be assumed for overcast conditions during day or night. Note that "thinly overcast" is not equivalent to "overcast."

Notes: Class A is the most unstable and class F is the most stable class considered here. Night refers to the period from one hour before sunset to one hour after sunrise. Note that the neutral class, D, can be assumed for overcast conditions during day or night, regardless of wind speed.

"Strong" incoming solar radiation corresponds to a solar altitude greater than 60° with clear skies; "slight" insolation corresponds to a solar altitude from 15° to 35° with clear skies. Table 170, Solar Altitude and Azimuth, in the Smithsonian Meteorological Tables, can be used in determining solar radiation. Incoming radiation that would be strong with clear skies can be expected to be reduced to moderate with broken (5/8 to 7/8 cloud cover) middle clouds and to slight with broken low clouds.

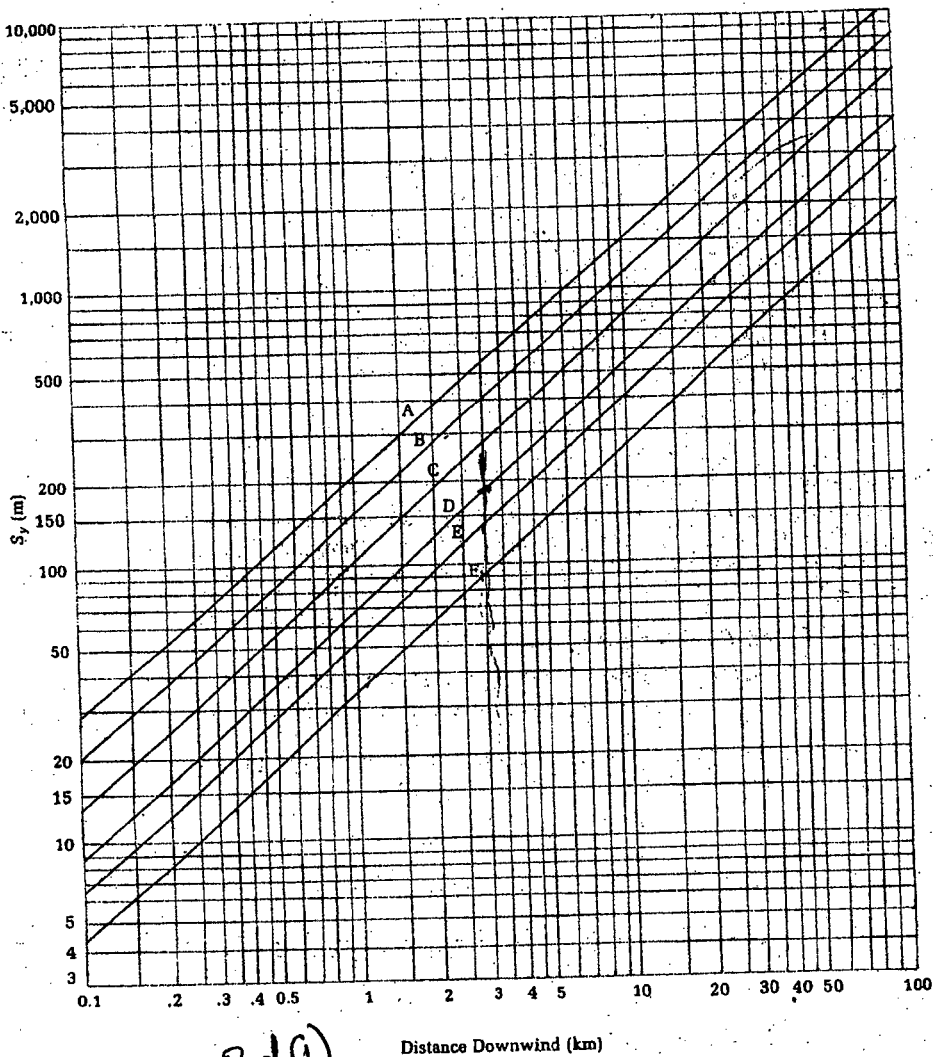*Source:* D. Bruce Turner, *Workbook of Atmospheric Dispersion Estimates.*

Figure 2d (i)

FIGURE 6.20 *2d(i)*
Horizontal dispersion coefficient. [*Source:* Turner, *Workbook of Atmospheric Dispersion Estimates* (U.S. Department of Health, Education and Welfare, Public Health Service, National Center for Air Pollution Control Publication No. 999-AP-28), Washington, DC: U.S. Government Printing Office, 1967.]

BANGLADESH UNIVERSITY OF ENGINEERING AND TECHNOLOGY, DHAKA

L-4/T-1   B. Sc. Engineering Examinations 2010-2011

Sub : **CHE 491** (Natural Gas Engineering)

Full Marks: 280   Time : 3 Hours

The figures in the margin indicate full marks.

USE SEPARATE SCRIPTS FOR EACH SECTION

SECTION – A

There are **FOUR** questions in this Section. Answer any **THREE**.

1. (a) A sample natural gas has the following composition   **(20)**

Component	Mol wt	Mole frac
N_2	28.02	0.0062
CO_2	44.01	0.0084
H_2S	32.06	0.0068
C_1	16.04	0.8668
C_2	30.07	0.0391
C_3	44.09	0.0280
nC_4	58.12	0.0224
nC_5	72.15	0.0224

Assuming real gas behavior. Calculate molecular weight specific gravity, pseudocritical pressure and temperature, and density at 1000 Psia and 100° F.

(b) Show that the isothermal compressibility of an ideal gas at any pressure is inverse of that pressure.   **(5)**

(c) Draw and explain Z vs P graph   **(10)**

2. (a) What are the phase diagrams? What are the uses of phase diagrams?   **(5)**

(b) Draw the followings: **(5×5=25)**

– Density – Temperature diagram of a single component system

– PV diagram of a two components system

– Composite P-T diagram of a two components system

– Pressure – composition diagram for a two components system at temperature between the critical temperature of the two pure components.

– PV diagram of a multicomponents system

(c) State the law of rectilinear diameter. What are the uses of a density – temperature diagram of a single component system?   **(2+3=5)**

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3. (a) Write short notes on the followings-   (3×5=15)

    (i)   Dew point depression

    (ii)   Partial pressure approach for water content calculation.

    (iii)   Conditions promoting hydrate formation

(b) List three methods for preventing hydrate formation at wellsites   (5)

(c) Justify the following statement: (5)

    – Natural gas hydrates can be a future source of energy.

(d) A gas at 1000 psia has the following analysis: $N_2 = 1.7\%$, $CO_2 = = 0.3\%$, $C_1 = 65.5\%$, $C_2 = 16.61\%$, $C_3 = 8.8\%$, $n\text{-}C_4 = 3\%$, $i\text{-}C_4 = 1.6\%$, $i\text{-}C_5 = 0.9\%$, $n\text{-}C_5 = 0.9\%$, $C_6 = 0.5\%$, $C_7 + = 0.2\%$.

What is the water content of this gas at 120°F? Use all the methods applicable and compare the results.   (10)

4. (a) What are the criteria for the selection of a dehydration plant?   (5+10+5=20)

List the properties of a good solid and liquid dessicant.

Name some widely used solid and liquid dessicants.

(b) What are the available processes for NGL extraction?   (5)

(c) Draw natural gas cycle.   (5)

(d) Write a short note on natural gas transportation   (5)

SECTION – B

There are **FOUR** questions in this Section. Answer any **THREE**.

5. (a) With the help of illustrations explain the reasons why regular flow after flow back pressure test evolved to modified isochronal test?   (18)

(b) Explain the variation in the index(n) and co-efficient (c) of the back pressure equations   (5)

$q = c\,(p^2 - p_{wf}^2)^n$ with time and flow.

(c) Starting from Darcy's equation for a spherical flow system, derive an expression for pressure p at any radius r for gas.   (12)

6. (a) Estimate AOF from the modified isochronal test data provided in Table 1. (20)

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Contd ... Q. No. 6(a)

Table 1 : Modified isochronal Test Data.

Tes	Duration (hrs)	P_{wf} or P_{ws} (psia)	q_g (MMscf/D)
Pretest shut – in	20	1,948	—
First flow	12	1,784	4.50
First shut in	12	1927	—
Second flow	12	1680	5.60
Second shut in	12	1911	—
Third flow	12	1,546	6.85
Third shut	12	1887	—
Fourth flow	12	1355	8.25
Extended flow (stabilized)	81	1233	8.00
Final shut-in	120	1948	—

(b) Suggest 5 measures to improve energy management in Bangladesh **(10)**

(c) What are acid gases? Why are they needed to be removed from the produced gas? **(5)**

7. (a) What is vertical separator? Write down its advantage and disadvantage. Draw neat sketch of vertical separator and level it. When vertical separator is preferable over horizontal separator? **(3+4+5+3=15)**

(b) Explain the Low Temperature Separation (LTS) process plant in Titas gas field with flow diagram starting from wellhead to sales point. **(10)**

(c) In the tree format, name all the gas sweetening processes. Which would you prefer for Bangladeshi gas field. **(10)**

8. (a) Derive an expression for the size of the smallest droplet that can be removed by a centrifuge? **(10)**

(b) What happens to separation quality and the separator when **(10)**

(i) the gas flow rate exceeds the allowable rate through the separator.

(ii) the liquid flow rate exceeds the allowable rate through the separator.

(c) What is gas cleaning? Why it is necessary? Write down the different gas cleaning methods. **(5)**

(d) In designing a separator, describe the gas capacity and liquid capacity of a separator. **(5)**

(e) List at least three basic factors that must be considered in designing separator. **(5)**

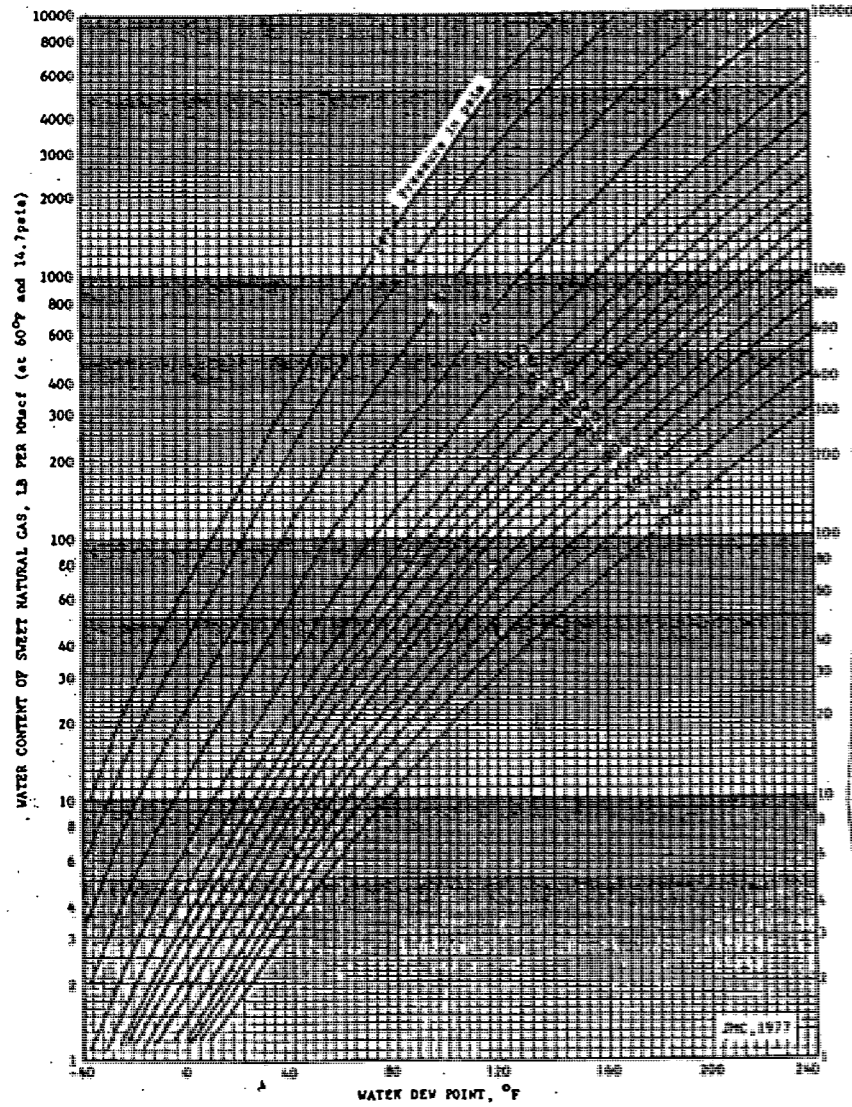---

Fig 4   Campbell's correlation for water content of sweet gases. (After Campbell, 1984a; courtesy of Campbell Petroleum Series.)
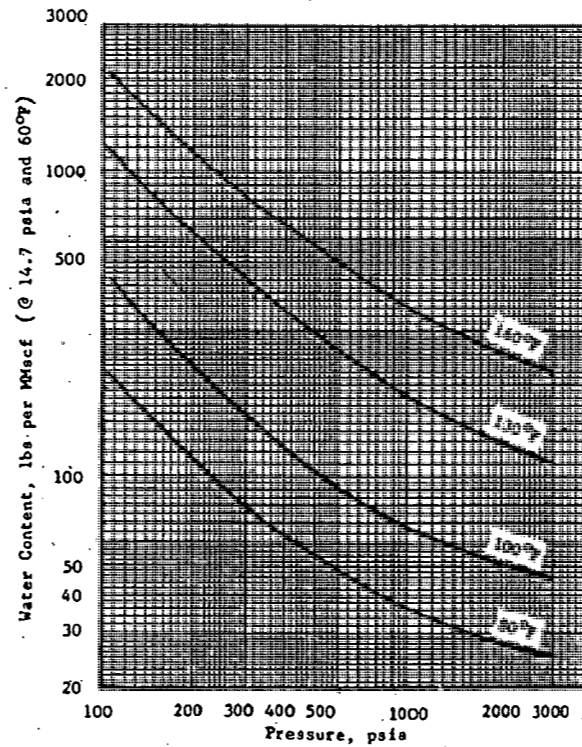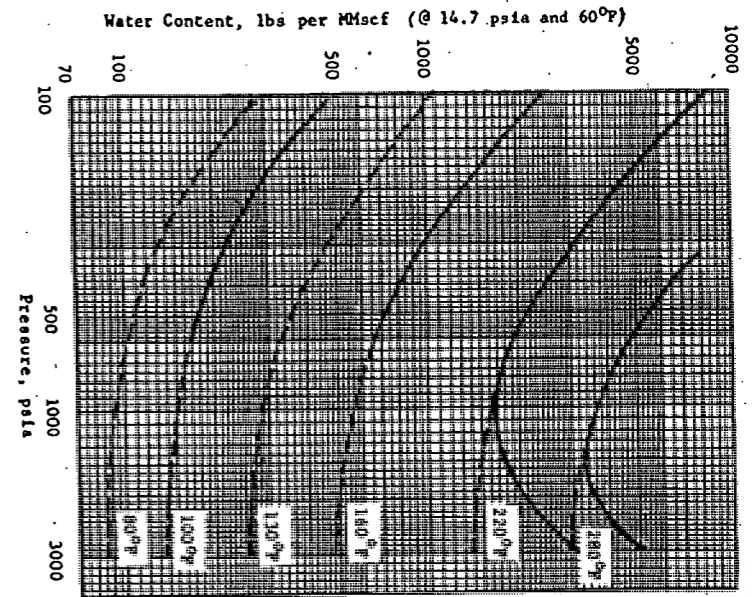
Fig for Q 3(d)

Fig-5   Water content of CO₂ in saturated natural gas mixtures. (After Campbell, 1984a; courtesy of Campbell Petroleum Series.)

Fig 6.   Water content of H₂S in saturated natural gas mixtures. Note: Solid curved lines are for pure H₂S only. (After Campbell, 1984a; courtesy of Campbell Petroleum Series.)
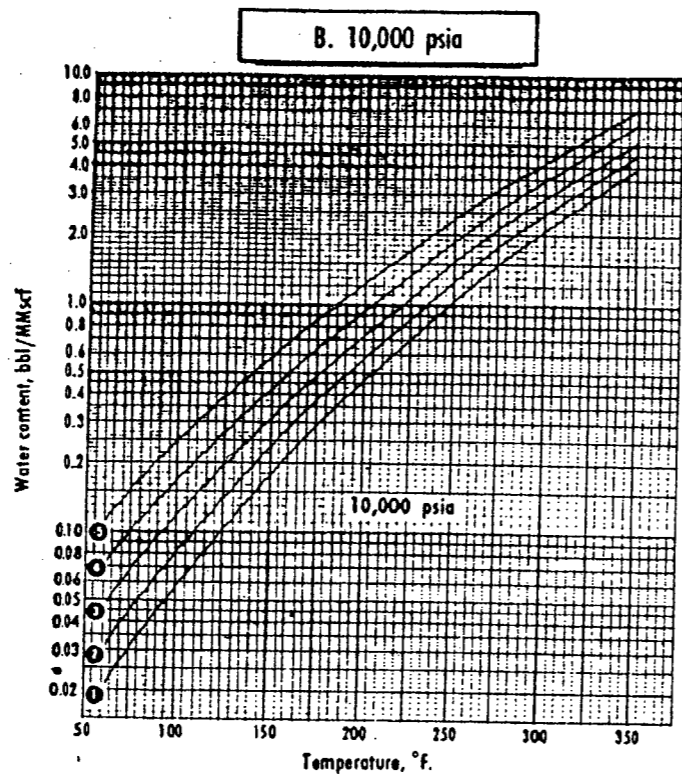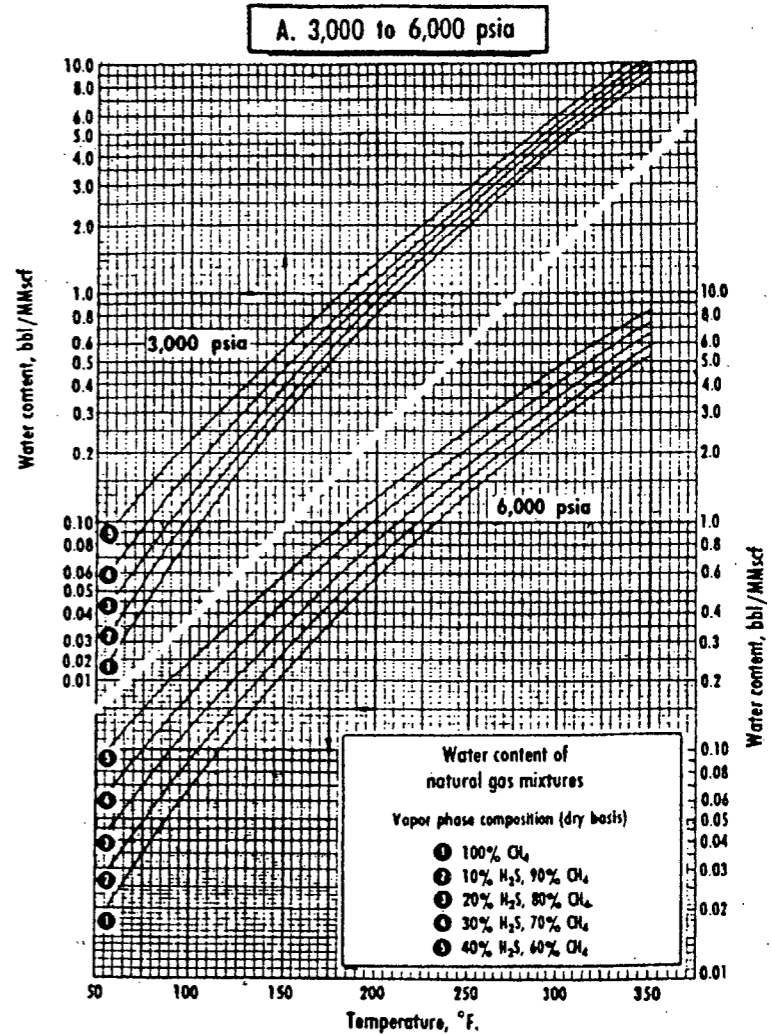
Fig for Q 3(d)

Fig 3   Continued.

Fig   for   Q   3 (d)

Fig 3   Robinson et al. correlation for water content of sour gases in the
3,000–6,000 psia pressure range, and at 10,000 psia. (After Robinson et al.,
1978; courtesy of *Oil & Gas Journal*.)

Fig   for   Q   3 (d)
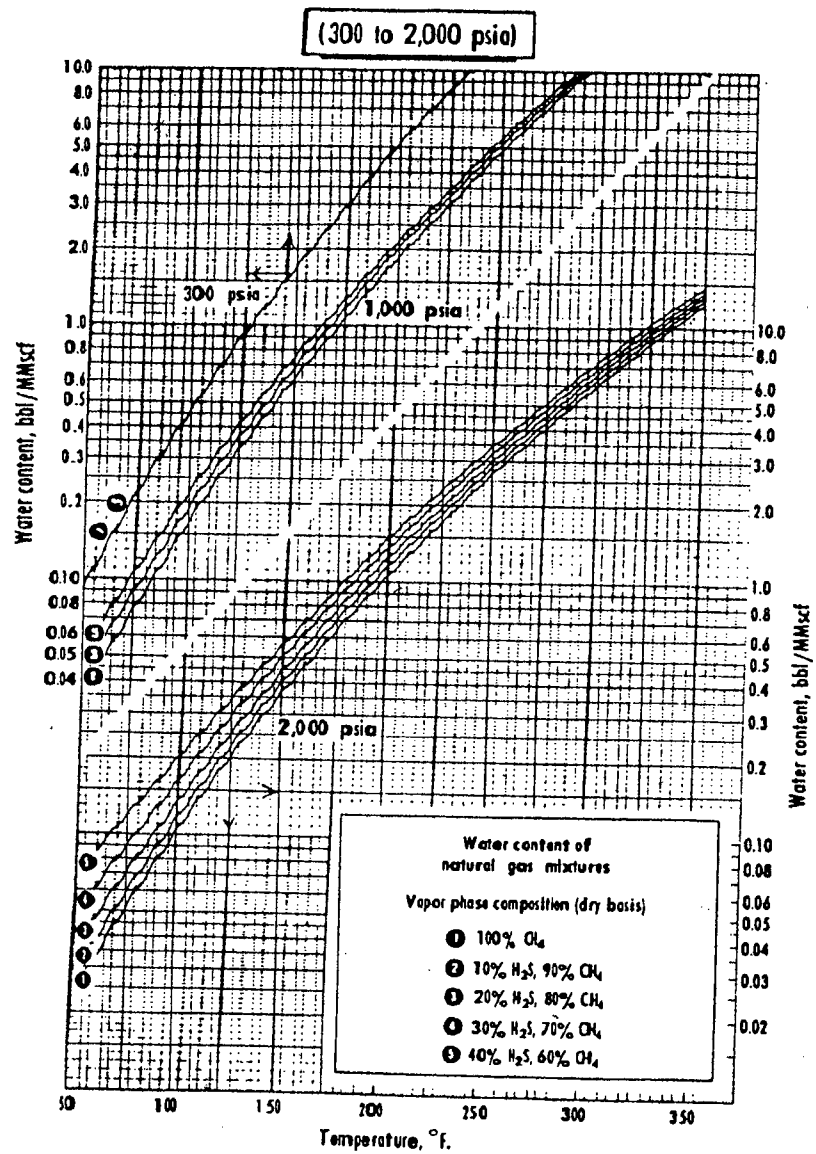
Fig-2   Robinson et al. correlation for water content of sour gases in the 300–2,000 psia pressure range. (After Robinson et al., 1978; courtesy of *Oil & Gas Journal.*)
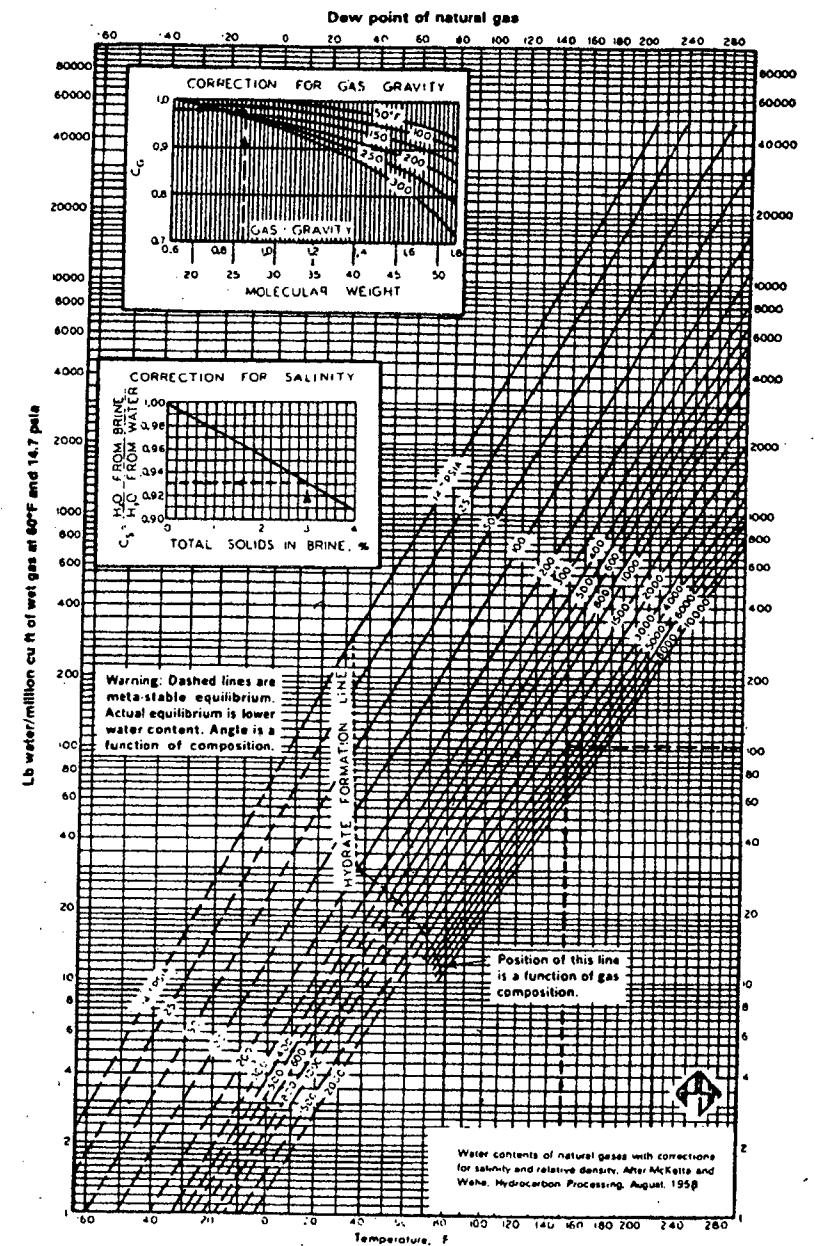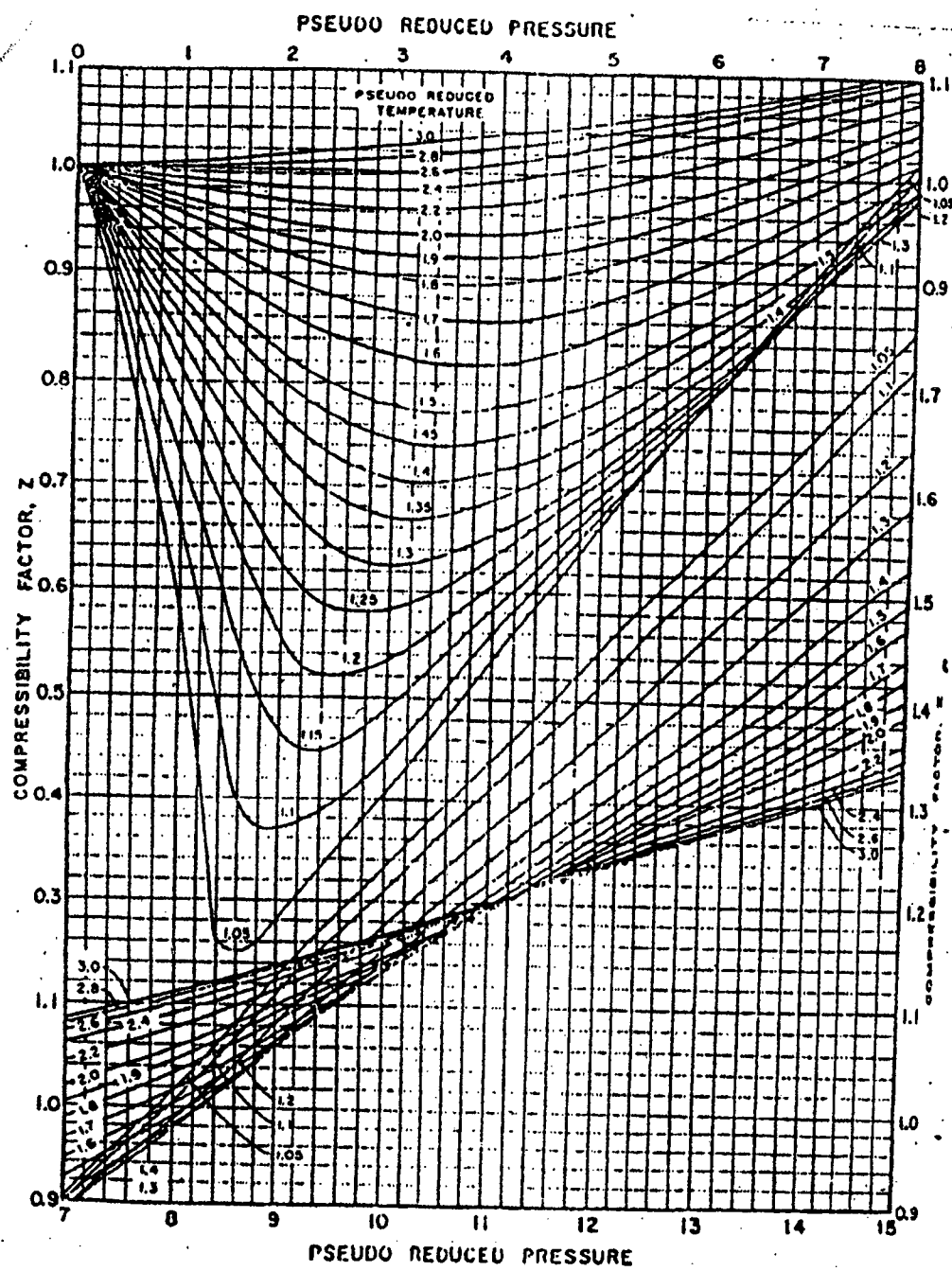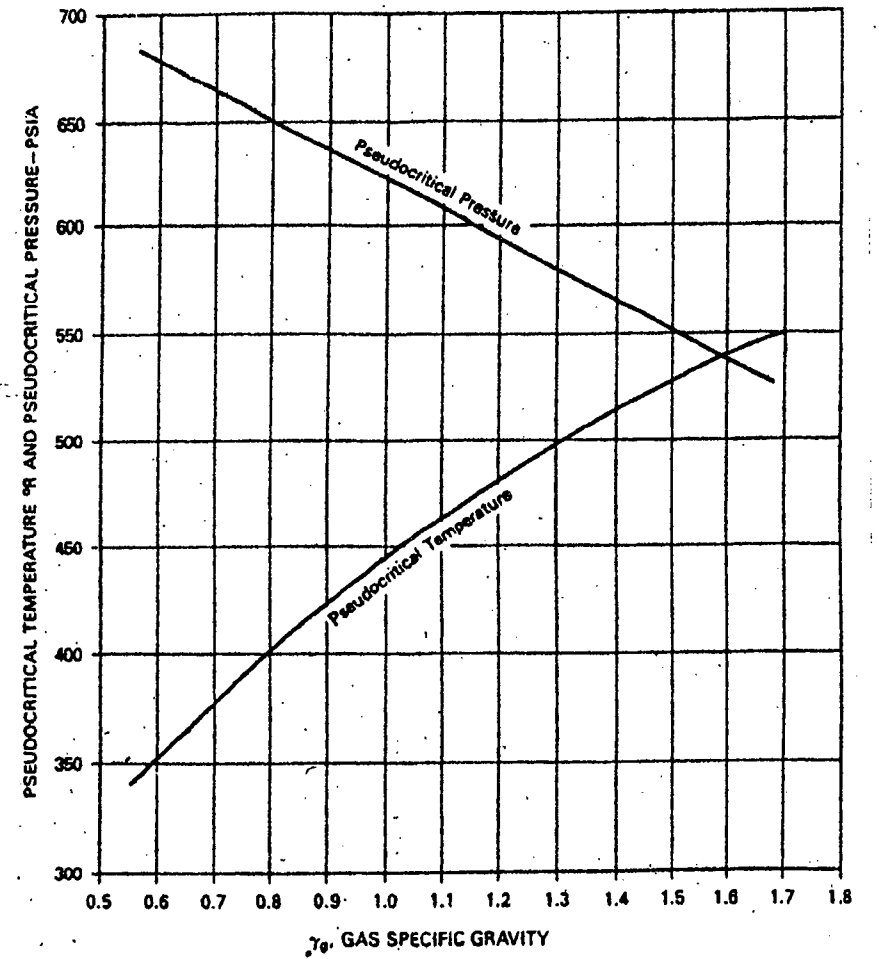
Fig for Q 3(d)

Fig - 1   The McKetta-Wehe correlation for water content of natural gases, with corrections for water salinity and gas gravity. (After McKetta and Wehe, 1958; reprinted from *Engineering Data Book*, 1981; courtesy of GPSA.)
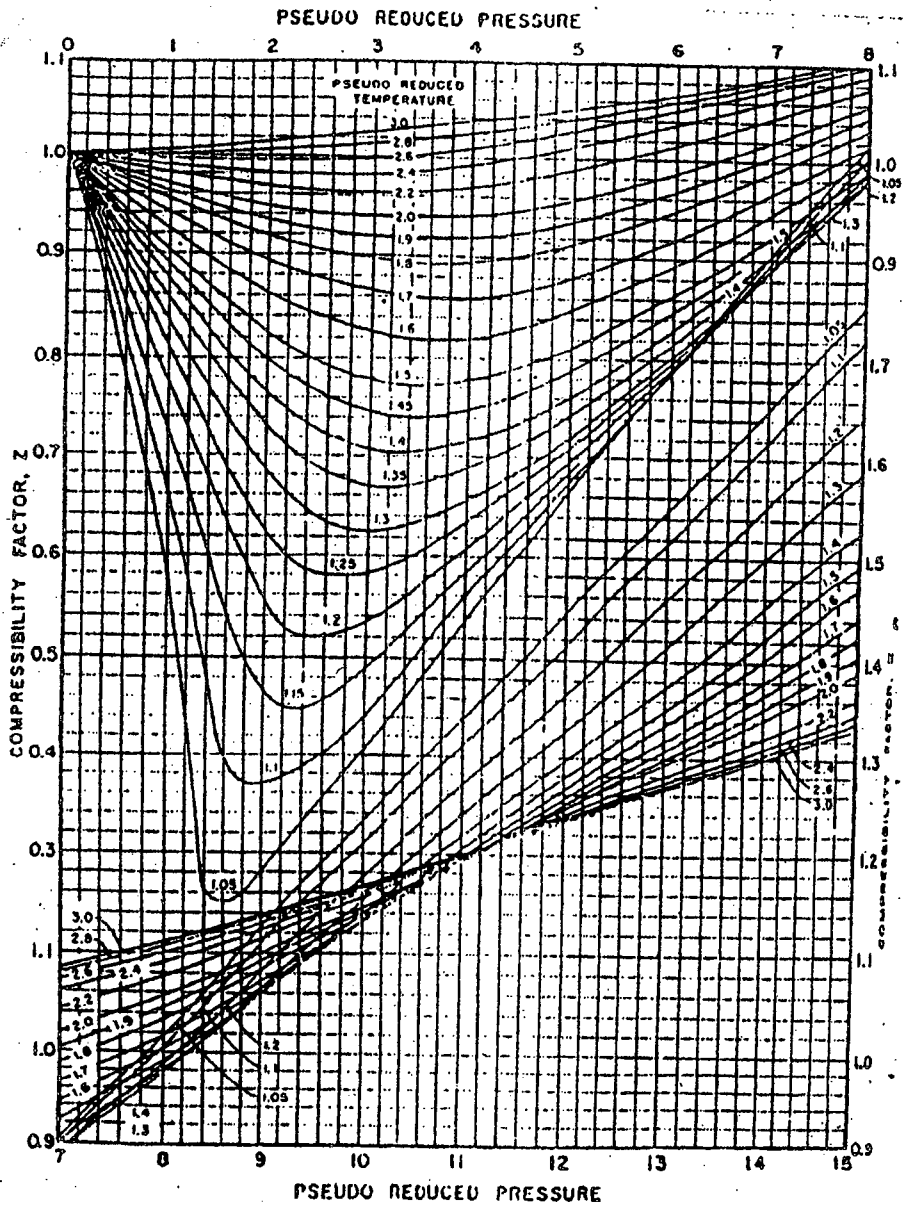
Fig for Q 3(d)

Compressibility factors for natural gases. (*After* Standing and Katz, *Trans.*
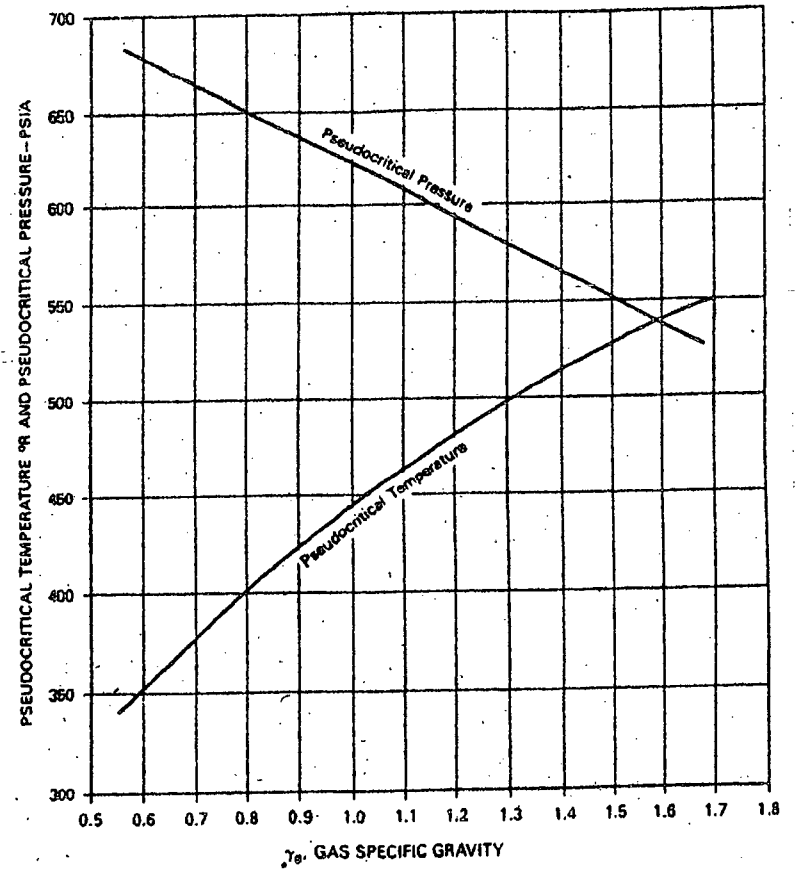
For Q no 1(a)

Pseudocritical properties of natural gases. (*After* Sutton.[32])

For Q no. 1(a)

Compressibility factors for natural gases. (*After* Standing and Katz, *Trans.*

Fon Q no 1(a)

Pseudocritical properties of natural gases. (*After* Sutton.[32])

Fon Q no. 1(a)