

BANGLADESH UNIVERSITY OF ENGINEERING AND TECHNOLOGY, DHAKA

L-2/T-2 B. Sc. Engineering Examinations 2013-2014

Sub : **CE 213** (Mechanics of Solids-II)

Full Marks : 210

Time : 3 Hours

The figures in the margin indicate full marks.

USE SEPARATE SCRIPTS FOR EACH SECTION

SECTION – A

There are **FIVE** questions in this Section. Answer any **FOUR**.

1. (a) An inclined tensile force F is applied to an aluminum alloy bar such that its line of action goes through the centroid of the bar, as shown in Fig.-1 (all dimensions are in mm) What is the magnitude of force F if it causes a longitudinal strain of $+ 20 \times 10^{-6}$ in the gage at A? Assume that the bar behaves as a linearly elastic material and let $E = 70$ GPa. (16 ¼)
 (b) Determine the kern of a rectangular section with cross-sectional dimensions $b \times h$. (10)
2. (a) Using direct integration method, determine the equation of the elastic curve for the cantilever beam shown in the Fig.-2 due to the applied loading. EI for the beam is constant (10 ¼)
 (b) A beam fixed at both ends supports a uniformly distributed downward load of w_0 , as shown in Fig.-3. Using direct integration method, determine the equation of the elastic curve for the beam. EI for the beam is constant. (16)
3. (a) Using the moment-area method, determine the deflection and the slope of the elastic curve at point A, due to the applied load for the beam shown in Fig.-4. Specify the direction of deflection and of rotation for the calculated quantities. Let $E = 200$ GPa. (10 ¼)
 (b) Find the moments at the supports for the fixed-end beam of span L , loaded with a uniformly distributed load of w_0 N/m as shown in Fig.-3. (16 ¼)
4. (a) Given the state of stress shown in Fig.-5, using Mohr's circle transform it: (i) into the principal stresses and (ii) into the maximum shear stresses and the associated normal stresses. Shown the results for both cases on properly oriented elements. (16 ¼)
 (b) Using general equations for transformation of stresses, transform the stresses shown in Fig.-6 into stresses acting on the plane at an angle of $22\frac{1}{2}$ degrees with the vertical axis. (10)
5. (a) A uniform shaft of length L is fixed at one end and subjected to a torque T at the free end. Using strain energy method calculate the angle of twist of the shaft at the free end. (13 ¼)
 (b) A cantilever beam of span L is subjected to a downward concentrated load of P at the free end. Using strain energy method calculate the deflection at the free end . EI for the beam is constant. (13)

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SECTION – B

There are **FIVE** questions in this Section. Answer any **FOUR**.

Assume any reasonable value for any missing data.

6. Determine the force 'P' needed to hold the cable in position (Fig. 7) so that segment BC remains horizontal. Also, compute the sag y_B and the tension in all the segments of the cable. Which segment is subjected to maximum tension? (26 $\frac{1}{4}$)

7. A plate is attached to the frame of a machine by two fillet welds as shown in Fig. 8. Determine the size of the welds to resist a vertical load of 50 kN. Given : the allowable shearing stress of weld is 150 MPa. (26 $\frac{1}{4}$)

8. An aluminum strut 1.8 m long has a rectangular section 20 mm \times 50 mm. A bolt through each end secures the strut so that it acts as a hinged column about an axis perpendicular to the long (50 mm) dimension and as a fixed column about an axis perpendicular to the 20 mm dimension. Determine the safe strut load, using a factor of safety of 2.0. Given: $E = 72$ GPa and $\sigma_{prop.} = 120$ MPa. (26 $\frac{1}{4}$)

9. Two plates are joined by four rivets 25 mm in diameter as shown in Fig. 9. Find the allowable load that can be transmitted safely and the efficiency of the joint if the working stresses are $\tau = 85$ MPa, $\sigma_t = 95$ MPa and $\sigma_b = 125$ MPa. (26 $\frac{1}{4}$)

10. (a) Name the different failure theories. Describe the maximum shearing stress theory (Coulomb's theory) and hence derive the Tresca yield conditions. Provide a sketch of yield lines. (12)
(b) Derive the equation of maximum cable tension and cable shape for a cable subjected to uniform load. The cable chord is not horizontal. (14 $\frac{1}{4}$)

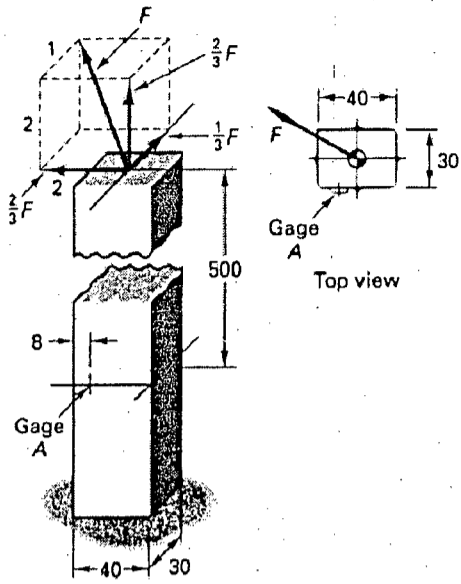


Fig-1

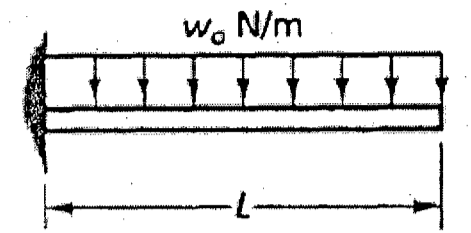


Fig-2

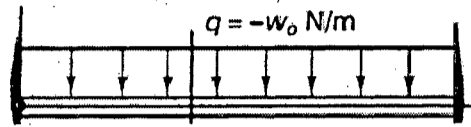


Fig-3

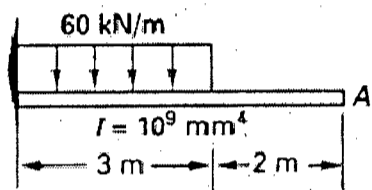


Fig-4

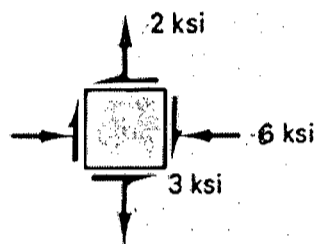


Fig-5

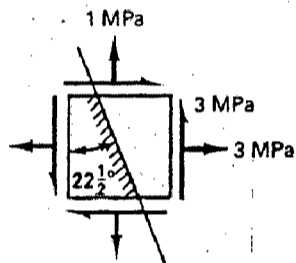


Fig-6

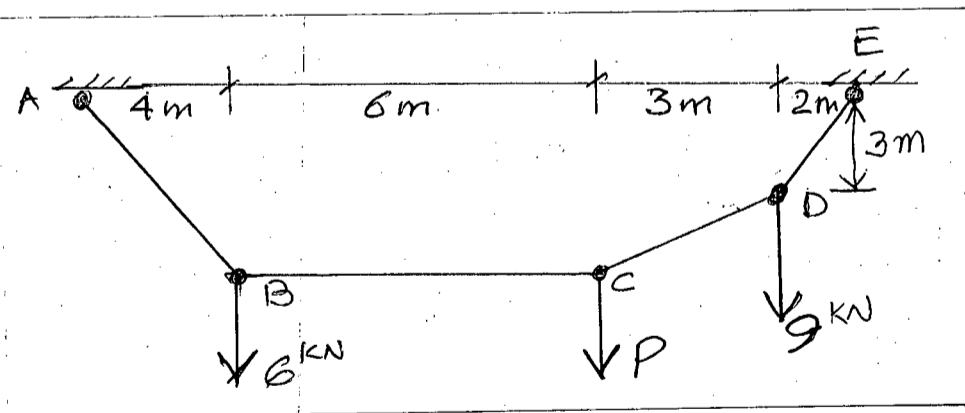


Fig-7

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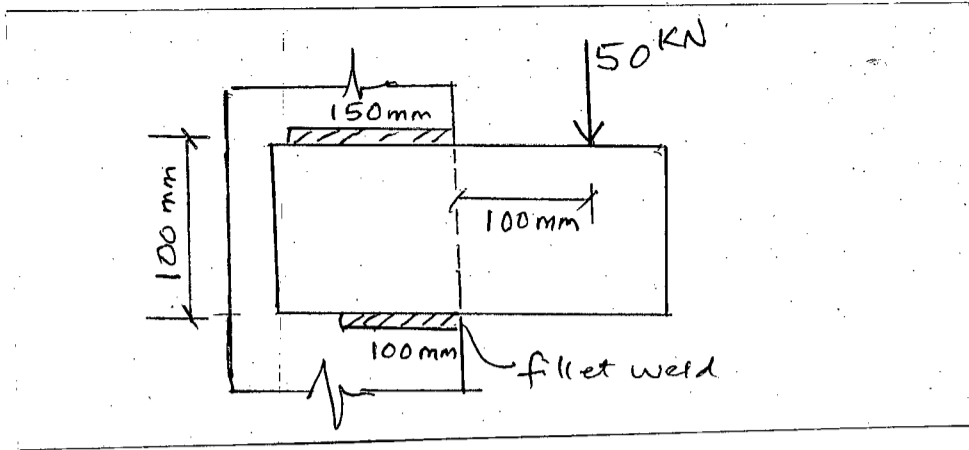


Fig. 8.

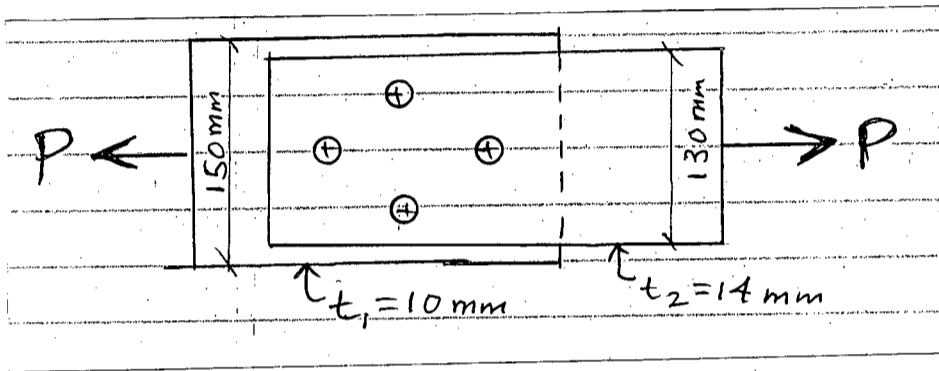


Fig. 9.

SECTION – AThere are **FOUR** questions in this Section. Answer any **THREE**.

1. (a) Use multiple linear regression to fit the equation: $y = axe^{bx}$ to the following dataset: (14)

x	0.1	0.2	0.4	0.6	0.9	1.3	1.5	1.7	1.8
y	0.75	1.25	1.45	1.25	0.85	0.55	0.35	0.28	0.18

- (b) What is the convergence criteria in fixed point iteration method? Give two examples with diagrams where the solution using fixed point iteration diverges. (6 1/3)

- (c) Give an example where the method of false position can converge at a slower rate than the bisection method. (3)

2. (a) Determine the root of the function $f(x) = 8e^{-x} \sin x - 1$ using

(i) Newton Raphson method using an initial guess of 0.3 (perform 3 iterations)

(ii) Secant method using initial guesses 0.5 and 0.4 (perform 5 iterations)

Determine the approximate % error for each iteration. (13)

- (b) Use zero to third order Taylor series expansions to predict $f(3)$ for $f(x) = 25x^3 - 6x^2 + 7x - 88$ using a base point at $x = 1$. Compute the true percent relative error for each approximation. (7 1/3)

- (c) How does the local truncation error affect the global error in Euler's method? (3)

3. (a) Calculate the goodness of fit (R^2 value) of parabola $y = -0.016x^2 + 1.377x + 11.76$ to the following dataset: (10)

x	5	10	15	20	25	30	35	40	45	50
y	17	24	31	33	37	37	40	40	42	41

- (b) Write down the finite difference equations (in matrix form) to solve the following differential equation: (10)

$$\frac{d^2u}{dx^2} + 6\frac{du}{dx} - u = 2 \quad u(0) = 10 \text{ and } u(2) = 1$$

Use a step size of 0.2

- (c) What is the difference between round-off and truncation error? (3 1/3)

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4. (a) Solve the following equation for $y(0.4)$

$$10 \frac{d^2 y}{dx^2} + \left(\frac{dy}{dx} \right)^2 + 6x = 0 \quad \text{using a step of 0.2 with } y(0) = 1 \text{ and } y(0) = 0. \text{ Use Heun's}$$

methods.

(12)

(b) Solve the following equation for $y(1.0)$ using 4th order Runge-Kutta method and a step size of 0.5.

(11 1/3)

$$\frac{dy}{dx} = (1 + 2x)\sqrt{y} \quad y(0) = 1$$

SECTION - B

There are **FOUR** questions in this Section. Answer any **THREE**.

5. (a) Given the four points (1, 0), (-1, 14), (3, -2) and (-2, 2). Derive the polynomial equation that passes through them.

(13 1/3)

(b) Discuss Gauss quadrature method. Also derive the weighting coefficients and associated points for $n = 3$.

(10)

6. (a) Discuss (i) difference table and (ii) upper triangular matrix

(10)

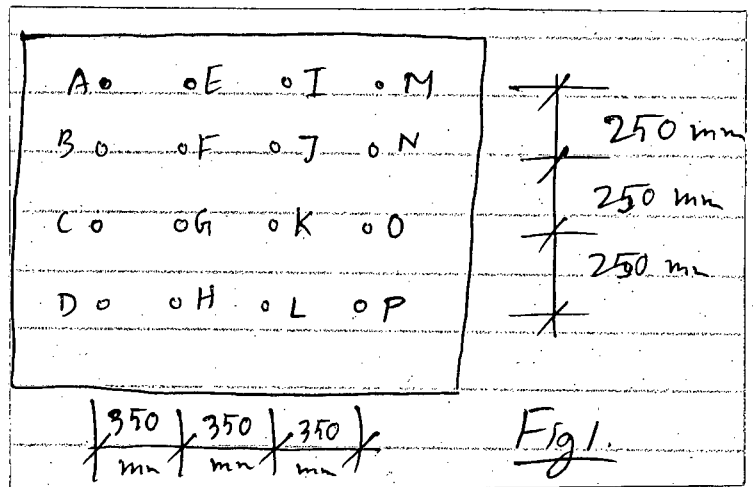
(b) The deflection at various points on a normally loaded plate are shown in Table 1 below.

(13 1/3)

Calculate the bending moment in the plate at points F and G. Points are shown in Fig. 1 below.

Table 1

Point	Deflection (Micron)	Point	Deflection (Micron)
A	7.0	I	9.3
B	11.0	J	11.3
C	10.0	K	10.7
D	8.5	L	9.9
E	7.5	M	8.9
F	11.5	N	9.4
G	10.5	O	10.9
H	9.5	P	10.5



Given:

$$D = 4.5 \times 10^8 \text{ lb-in}, \quad \nu = 0.18$$

$$M_x = -D \left(\frac{\partial^2 z}{dx^2} + \nu \frac{\partial^2 z}{dy^2} \right)$$

$$M_y = -D \left(\frac{\partial^2 z}{dy^2} + \nu \frac{\partial^2 z}{dx^2} \right)$$

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7. (a) Derive the ground expression of $I = \int_a^b f(x)dx$ using Simpson's rule. **(10)**

(b) The following data have been obtained from a tensile test on a steel specimen.

Calculate the elongation corresponding to 1800 lbs by Gregory-Newton method. **(13 1/3)**

Load in lbs	Elongation (inch)
0	0.00
500	0.025
1000	0.058
1500	0.096
2000	0.151
2500	0.212

Also derive the general expression passing through the above five points.

8. (a) Explain Romberg's quadrature. **(3 1/3)**

(b) Solve the following system of linear equation by Gaussian Elimination. **(10)**

$$2x + 5y + 8z = 36$$

$$4x + 7y - 12z = -16$$

$$x + 8y + z = 20$$

(c) Solve the problem in question 8(b) by Crout's method. **(10)**

SECTION – AThere are **FOUR** questions in this Section. Answer any **THREE**.

1. (a) Define supply function. (5)
- (b) What are the factors that influence the shifting of the supply curve? (10)
- (c) Why do demand curves generally slope downward? (8 1/3)
2. (a) How would you measure price elasticity of demand at any point on a straight line demand curve? Explain graphically. (10)
- (b) Show that any straight line supply curve passing through the origin has a unitary elasticity of supply. (3 1/3)
- (c) From the following table calculate elasticity of demand if you move from point A to C and explain what you understand from the result. (10)

POINT	Y	Q
A	5000	180
B	6000	200
C	7000	220

3. (a) What is an indifference curve? Explain the properties of an indifference curve. (15)
- (b) Explain consumer's equilibrium with the help of budget line and indifference curve. (8 1/3)
4. (a) How is price determined in an economy under competition? What will happen to the price and quantity due to change in demand? (10)
- (b) From the following demand and supply functions, calculate equilibrium price and quantity and show the result in a graph. (13 1/3)

$$P = 0.50 Q + 150$$

$$P = - 0.40 Q + 240$$

- (i) What will happen to the equilibrium price and quantity if government imposes a unit tax of Tk. 15 per unit?
- (ii) Describe the change in equilibrium. Show the equilibrium coordinates on the same graph.

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SECTION – B

There are **FOUR** questions in this Section. Answer any **THREE**.

5. (a) What do you know about production function? Define Total Product, Average Product and Marginal Product of a firm. **(8 1/3)**
- (b) As a project engineer you are permitted to use maximum 10 labors to produce a good in your project. By using these labors draw a hypothetical table showing Total Product, Average Product and Marginal Product of your project. Identify the highest level of Marginal Productivity and Average Productivity of your project on this table. **(15)**
6. (a) Suppose a firm faces a cost function of $C = 100 + 10q + q^2$ **(10)**
- (i) What is the firm's fixed cost?
- (ii) Derive an expression for the firm's variable cost and marginal cost.
- (b) Who is Monopolist? How does a monopolist can reach to an equilibrium position under a monopoly market? Explain with a graph. **(13 1/3)**
7. (a) Distinguish between an Isoquant and an Indifference Curve. **(5 1/3)**
- (b) Show a circular flow of national income in a two sector economy. **(6)**
- (c) What are the ways to measure national income of an economy? How can you measure national income by Income Method? Explain in brief. **(12)**
8. (a) What are the causes of Inflation of an economy like Bangladesh? Explain the instruments through which an economist can curb down the inflation rate according to the government instructions. **(15)**
- (b) What are the implications of the inflationary gap and deflationary gap of an economy? Illustrate with graphs. **(8 1/3)**
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The figures in the margin indicate full marks.

Assume reasonable values if needed. Symbols have their usual meanings.

USE SEPARATE SCRIPTS FOR EACH SECTION

SECTION – A

There are **FOUR** questions in this Section. Answer any **THREE**.

1. (a) Define (i) steady flow, (ii) unsteady flow, (iii) uniform flow, (iv) non-uniform flow (5)
 - (b) Write down the general equation for steady flow of any fluid and indicate different terms of it, and hence derive the Bernoulli's equation. (5)
 - (c) The velocity profile at a sudden expansion of a pipe is shown in Figure 1. Compute the energy correction factor. (10)
 - (d) The diameter of a 5 m long horizontal pipe at point A is 90 cm and reduces linearly to 30 cm at point B. The pressure at point A is 70 kPa, Compute the convective acceleration at the middle section of the pipe AB if cavitation just occurs at point B and the head loss is 0.05 m. Use vapor pressure of fluid = 20 kPa, abs., sp. gravity = 0.86. Fluid flows from A to B. (15)
2. (a) What is a streamline? Why is the concept of streamline important in fluid mechanics? (5)
 - (b) Discuss different forms of energy present in fluid flow. (5)
 - (c) A fluid (sp. gravity = 1.26) flows at $10 \text{ m}^3/\text{s}$ in a 150 cm-diameter pipe; the friction loss in a 1000 m length of a pipe is 20 m. Find the increase in temperature assuming no heat enters or leaves the pipe. Also find the rate of energy loss due to pipe friction. [$c = 4200 \text{ N}\cdot\text{m}/(\text{kg}\cdot\text{K})$]. (10)
 - (d) A fire-fighter is spraying water at the second floor of a residential building. The water pumping system of the fire-fighting unit is as follows: A pump draws water from a water tank through a suction pipe of 15 cm diameter and delivers through a 12 cm diameter pipe. A short nozzle of 7.5 cm diameter is attached at the end of the delivery pipe. The elevations of the water surface in the tank, the pump and the nozzle are 10 m, 8 m and 16 m respectively. Find the power of the pump and the force exerted on the fire-fighter if the velocity at the exit of the nozzle is 9 m/s and the fire-fighter holds the nozzle (i) horizontally and (ii) at 45° angle. Neglect head loss. (15)
3. (a) What is a siphon? Briefly describe its working principle. (5)
 - (b) Sketch the typical velocity profiles for an ideal and a real fluid in a circular pipe and explain the difference. (5)

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Contd ... Q. No. 3

- (c) Find the horizontal thrust of water on each meter of width of a sluice gate with a gate opening of 0.6 m. The water depths far upstream and just downstream of the gate are 2 m and 0.35 m respectively. (10)
- (d) The water jet in the Figure 2 moving at 12 m/s is divided by the splitter so that one-third of the water is diverted towards A. Calculate the magnitude and direction of the resultant force on this single blade if the blade is (i) stationary and (ii) moving at 3 m/s to the right. (15)
4. (a) Write short notes on piezometer, pitot tube and Prandtl tube. (5)
- (b) What is a momentum correction factor? Derive an expression for the momentum correction factor. (5)
- (c) Water flows from a 0.6 m diameter tank through an orifice located at 0.3 m above the base of tank. The jet diameter is 7.5 cm. If the static coefficient of friction between the tank and floor is 0.56, determine the minimum height of water in the tank at which the tank will commence to move to the left. The tank itself weighs 450 N. (10)
- (d) In a hydro-electric project, the elevations of a radial flow turbine and reservoir water levels are 3m and 29 m respectively. Water is drawn from the reservoir to the turbine by a 0.5 m diameter pipe and the pressure head just before the pipe enters the turbine is 5 m. The radial turbine has the following dimensions: $r_1 = 0.5$ m, $r_2 = 0.3$ m, and $\beta_1 = 80^\circ$. The width of the flow passage between the two sides of the turbine is 0.25 m. The turbine runs at 300 rpm. Find: (15)
- (i) the blade angle β_2 such that the water exits from the turbine in the radial direction.
 - (ii) the torque exerted by the water on the runner and the power thus developed.
 - (iii) the head utilized by the runner and the power resulting thereof.
- Assume no head loss and the blades are very thin.

SECTION – B

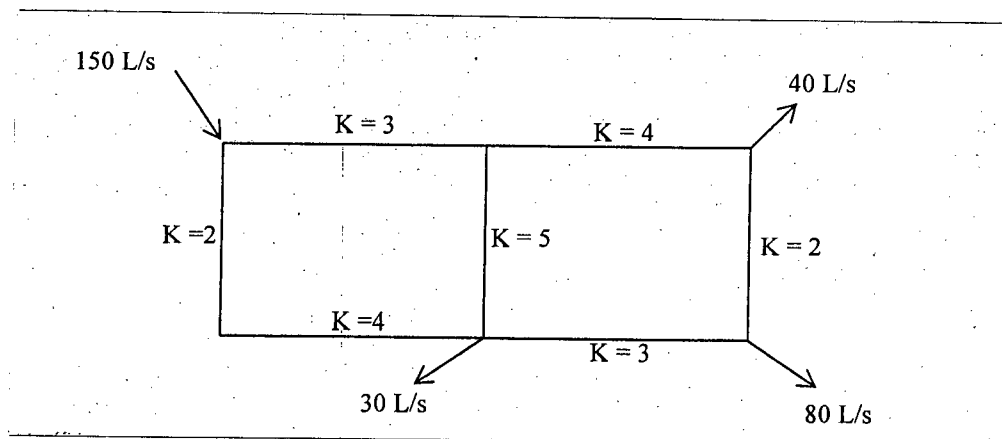
There are **FOUR** questions in this Section. Answer any **THREE**.

5. (a) Write short note on the following items: (10)
- (i) Saturation vapor pressure
 - (ii) Minor losses in pipe flow
 - (iii) Surface tension
 - (iv) Metacentre
- (b) A flat plate of 0.4 m^2 in area moves edgewise through oil between large fixed parallel planes 15 cm apart. If the velocity of the plate is 0.5 m/s and oil has a kinematic viscosity of $0.45 \times 10^{-4} \text{ m}^2/\text{s}$ and specific gravity of 0.85, calculate the drag force when (a) the plate is 5 cm from one of the planes and (b) the plate is equidistant from both the planes. (10)

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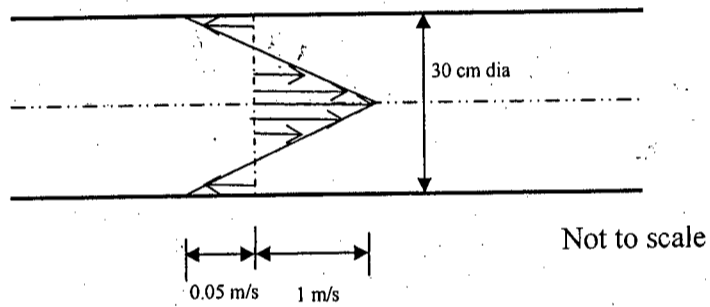
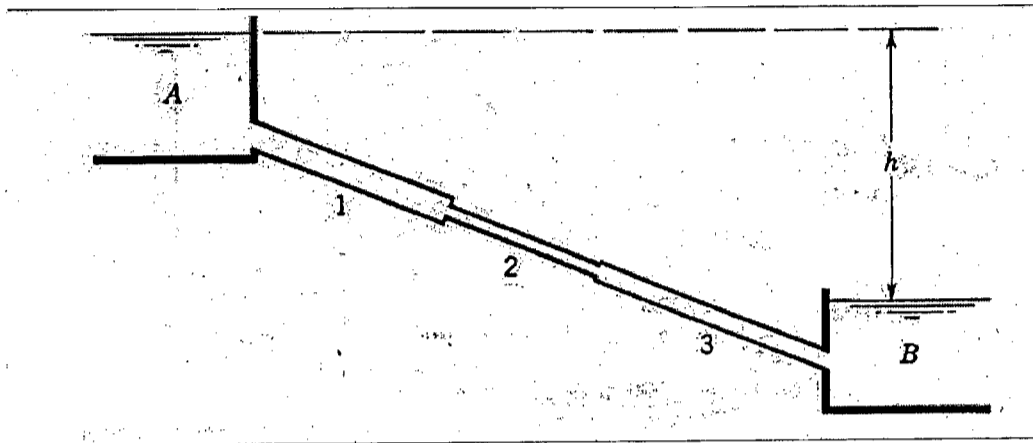
- (c) Prove Newton's equation of viscosity. Explain why the viscosity of all liquids decreases and that of all gases increases with an increase in temperature. (10+5=15)
6. (a) Derive the expression for variation of pressure in the vertical direction. (10)
- (b) A gauge on the suction side of a pump shows a negative pressure of 28.5 kPa. Express this pressure in terms of (i) N/m² absolute and (ii) m of water gauge (iii) m of oil (S = 0.85) absolute and (iv) cm of mercury gauge. Assume the barometric pressure is 70 cm of mercury. (10)
- (c) A glass tube having an internal diameter of 5 mm is dipped into mercury at 20°C. Determine the amount of capillary rise / drop through the tube. Assume a suitable value of the contact angle (θ). Given, $\sigma = 0.51$ N/m and $\rho = 13550$ kg/m³. (10)
- (d) Write down the stability criteria for a floating object. (5)
7. (a) Explain Reynolds experiment to distinguish between laminar and turbulent flow with a sketch. Also define critical Reynolds number. (7+3=10)
- (b) Show the relationship between absolute and gage pressures with a diagram. Explain the relative advantages and disadvantages of piezometer and double column manometers. (10)
- (c) A cylindrical gate of diameter 3 m retains fresh water on one side and saline water (S = 1.05) on the other side as shown in figure 3. Calculate the resultant fluid force acting on unit length of the gate. (15)
8. (a) An open tube is attached to a tank as shown in figure 4. If the water rises to a height of 900 mm in the tube, what are the pressures P_A and P_B of the air above the water? Neglect capillary effects in the tube. (10)
- (b) The flows into and out of a two-loop pipe system and k-values for each pipe are as shown in figure below. Using Hardy Cross method, determine the flow in each pipe. The head loss is given by $h_L = KQ^2$. Show calculation for two trials. (10)



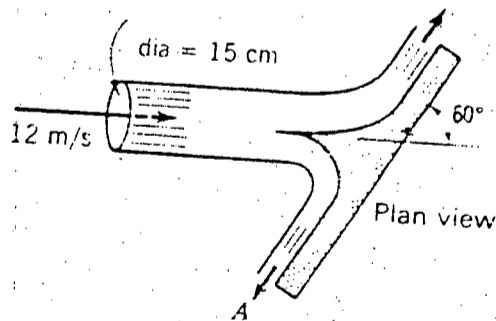
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(c) Suppose in figure below, the pipes 1, 2 and 3 are 400 m of 35 cm diameter, 240 m of 20 cm diameter and 330 m of 30 cm diameter, respectively of new cast iron ($e = 0.25$ mm) and are conveying water. If $h = 20$ m, find the rate of flow from A to B. Use moody diagram for friction factor. Given $\nu = 3 \times 10^{-6} \text{ m}^2/\text{s}$.

(15)



1
Figure for Question No. 1c



2
Figure for Question No. 3d

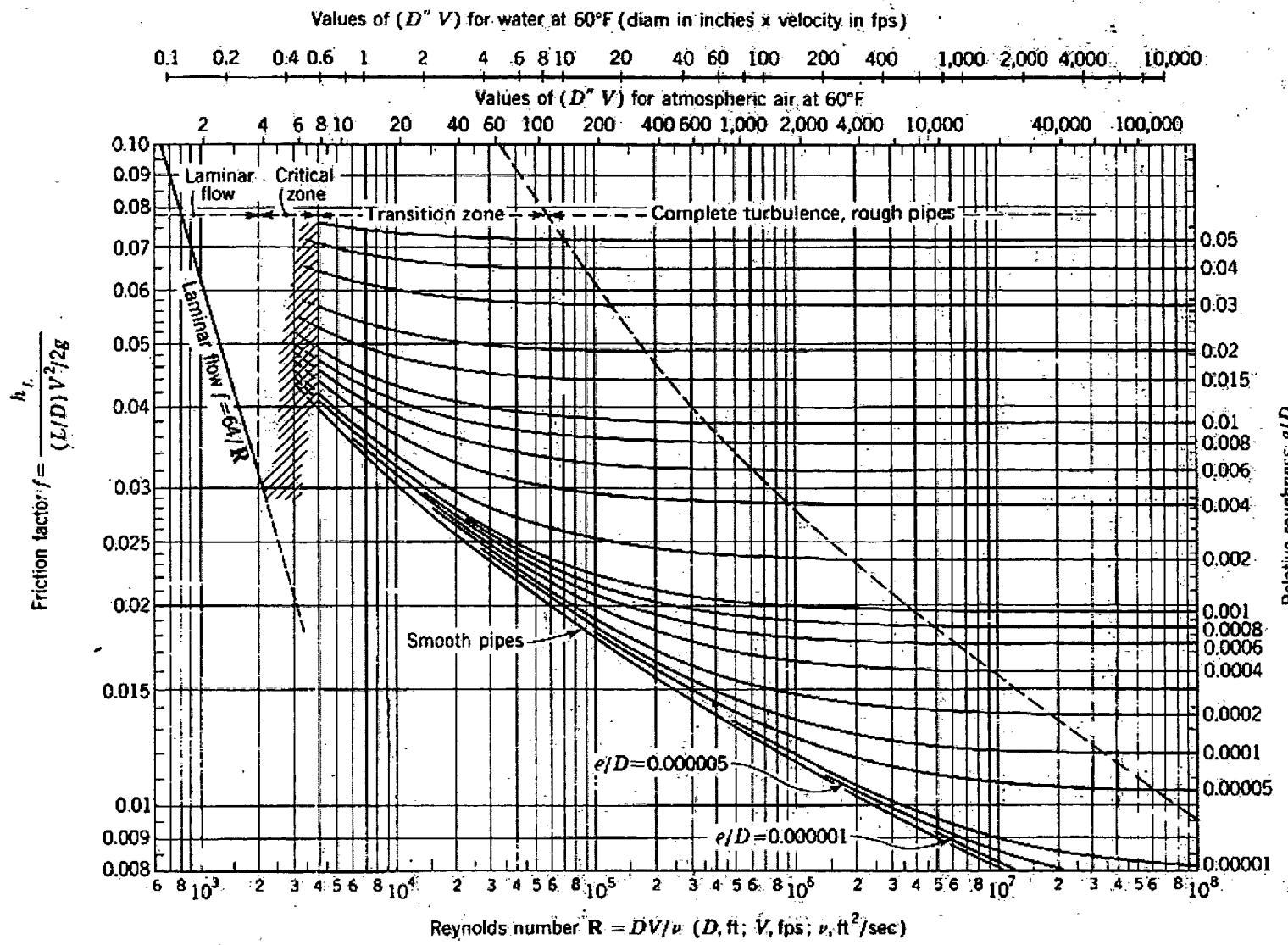


Figure: Moody Diagram for friction factor.

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BANGLADESH UNIVERSITY OF ENGINEERING AND TECHNOLOGY, DHAKA

L-2/T-2 B. Sc. Engineering Examinations 2013-2014

Sub : **CE 207** (Applied Mathematics for Engineers)

Full Marks : 210

Time : 3 Hours

The figures in the margin indicate full marks.

USE SEPARATE SCRIPTS FOR EACH SECTION

SECTION – A

There are **FOUR** questions in this Section. Answer any **THREE**.

Symbols carry their usual meaning. Assume reasonable values missing data, if any.

1. (a) Define the following terms with an example for each: (15)
 - (i) Implicit solution (ii) Explicit solution (iii) Singular solution.
 - (b) Experiments show that a radioactive substance decomposes at a rate proportional to the amount present. Starting with a given amount of substance, say 2 units, at a certain time, $t = 0$, what can be said about the amount available at a later time? Use this concept to plan your own measurement system to measure the flow in a river system. State the limitations of such a plan and write important equations to narrate the phenomena. (20)
2. (a) Define the following terms: (i) Dependent variable, (ii) Independent variable. (6)
 - (b) Name a variable which is independent in most instances. (3)
 - (c) What are the cases when you will consider “Power series Method” in solving a differential equation? (10)
 - (d) Use appropriate method to solve the following differential equation: $y' = 2xy$ (16)
3. (a) Write a short note on “Orthogonality” of Legendre Polynomial. (10)
 - (b) What is “Remainder”? (5)
 - (c) “Bessel’s function is a special form of Frobenius Method” – Explain. (10)
 - (d) “Frobenius method solves the hypergeometric equation whose solutions include many known functions as special cases”. – Explain. (10)
4. (a) A manufacturer of car batteries claims that this batteries will last, on average, 3 years with a variance of 1 year. If 5 of his batteries have lifetimes of 1.9, 2.4, 3.0, 3.5 and 4.2 years, construct a 95% confidence interval for σ^2 and decide if the manufacturer’s claim that $\sigma^2 = 1$ is valid. Assume the population of battery lives to be approximate normally distributed. (15)

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Contd ... Q. No. 4

(b) The following data indicate the relationship between X, the specific gravity of a wood sample, and Y, it's maximum crushing strength in compression parallel to the grain. Xi : 0.41, 0.46, 0.44, 0.47, 0.42, 0.39, 0.41 0.44, 0.43, and 0.44; Yi (psi): 1850, 2620, 2340, 2690, 2160, 1760, 2500, 2750, 2730, and 3120, respectively.

(20)

- (i) Plot a scatter diagram. Does a linear relationship seem reasonable?
- (ii) Estimate the regression coefficients.
- (iii) Predict the maximum crushing strength of a wood sample whose specific gravity is 0.43.

SECTION – B

There are **FOUR** questions in this Section. Answer any **THREE**.

Table of Fourier transform, Table of standard normal distribution, T distribution and $X^2_{\alpha} n$ are given.

5. (a) A very long pile is subjected to a lateral load at its top as shown in Fig. 1. The load can be expressed as $p(x) = p_0 H(x) e^{-ax}$; where $H(x)$ is the Heaviside Step function and 'a' is a constant. Express the load in integral form using Fourier Integral.

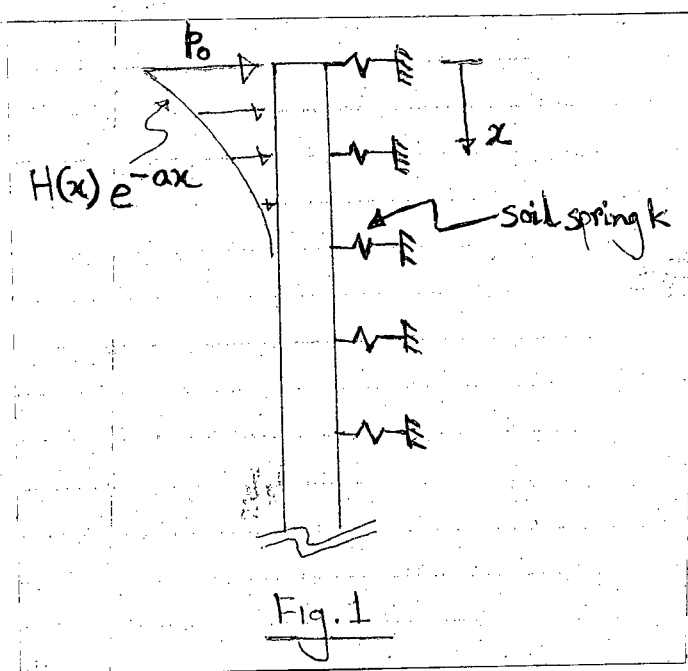
(15)

(b) Express the governing differential equation of lateral deformation of pile in terms of x.

(5)

(c) Solve the differential equation derived in 5(b) for the load of 5(a). Ignore the boundary condition.

(15)



6. (a) Evaluate Fourier Transform of $f(x) = \frac{x-a}{x^2+a^2}$.

(15)

(b) Solve the equation of forced vibration of a damped oscillatory system given as.

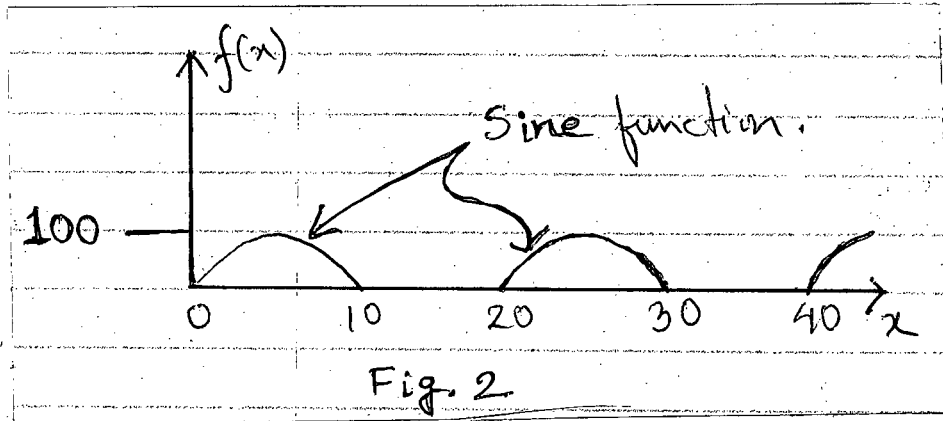
$$m \frac{d^2u}{dt^2} + c \frac{du}{dt} + ku = p(t)$$

Use Fourier Transform.

(20)

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7. (a) What do you understand by periodic expansion of a non-periodic function? Express the function shown in Fig. 2 by Fourier series through periodic expansion of the function. **(14)**



- (b) If X_1, X_2, \dots, X_n are independent Poisson random variables each having mean λ , determine the maximum likelihood estimator of λ . (Derive expression). **(7)**
- (c) If the average number of claims handled daily by an insurance company is 5, what proportions of days have less than 3 claims? What is the probability that there will be 4 claims in exactly 3 of the next 5 days? Assume that the number of claims on different days is independent, and the company insures a large number of clients, each having a small probability of making a claim on any given day. **(14)**
8. (a) Suppose the probability that an item produced by a certain machine will be defective is 0.2. Find the probability that a sample of 9 items will contain at most two defective items. Assume that the quality of successive items is independent. **(10)**
- (b) A public health official claims that the mean home water use is 350 gallons a day. To verify the claim, a study of 20 randomly selected homes was undertaken with the result that the average daily water uses of these 20 homes were as follows: 340, 356, 332, 362, 318, 344, 386, 402, 322, 360, 362, 354, 340, 372, 338, 375, 364, 355, 324, and 370. Do the data contradict the official's claim? **(15)**
- (c) In answering a question on a multiple-choice test, a student either knows the answer or she guesses. Let p be the probability that she knows the answer and $1-p$ be the probability that she guesses. Assume that a student who guesses at the answer will be correct with probability $1/m$, where m is the number of multiple choice alternatives. What is the conditional probability that a student knew the answer to a question given that she answered it correctly? **(10)**

Table of Fourier Transform

$f(x)$	$\hat{f}(\omega) = \int_{-\infty}^{\infty} f(x)e^{-i\omega x} dx$
1. $\frac{1}{x^2 + a^2} \quad (a > 0)$	$\frac{\pi}{a} e^{-a \omega }$
2. $H(x)e^{-ax} \quad (\text{Re } a > 0)$	$\frac{1}{a + i\omega}$
3. $H(-x)e^{ax} \quad (\text{Re } a > 0)$	$\frac{1}{a - i\omega}$
4. $e^{-a x } \quad (a > 0)$	$\frac{2a}{\omega^2 + a^2}$
5. e^{-x^2}	$\sqrt{\pi} e^{-\omega^2/4}$
6. $\frac{1}{2a\sqrt{\pi}} e^{-x^2/(2a)^2} \quad (a > 0)$	$e^{-a^2\omega^2}$
7. $\frac{1}{\sqrt{ x }}$	$\sqrt{\frac{2\pi}{ \omega }}$
8. $e^{-a x /\sqrt{2}} \sin\left(\frac{a}{\sqrt{2}} x + \frac{\pi}{4}\right) \quad (a > 0)$	$\frac{2a^3}{\omega^4 + a^4}$
9. $H(x+a) - H(x-a)$	$\frac{2\sin\omega a}{\omega}$
10. $\delta(x-a)$	$e^{-i\omega a}$
11. $f(ax+b) \quad (a > 0)$	$\frac{1}{a} e^{i\omega b/a} \hat{f}\left(\frac{\omega}{a}\right)$
12. $\frac{1}{a} e^{-ibx/a} f\left(\frac{x}{a}\right) \quad (a > 0, b \text{ real})$	$\hat{f}(a\omega + b)$
13. $f(ax) \cos cx \quad (a > 0, c \text{ real})$	$\frac{1}{2a} \left[\hat{f}\left(\frac{\omega-c}{a}\right) + \hat{f}\left(\frac{\omega+c}{a}\right) \right]$
14. $f(ax) \sin cx \quad (a > 0, c \text{ real})$	$\frac{1}{2ai} \left[\hat{f}\left(\frac{\omega-c}{a}\right) - \hat{f}\left(\frac{\omega+c}{a}\right) \right]$
15. $f(x+c) + f(x-c) \quad (c \text{ real})$	$2\hat{f}(\omega) \cos \omega c$
16. $f(x+c) - f(x-c) \quad (c \text{ real})$	$2i\hat{f}(\omega) \sin \omega c$
17. $x^n f(x) \quad (n = 1, 2, \dots)$	$i^n \frac{d^n}{d\omega^n} \hat{f}(\omega)$

Linearity of transform and inverse:

18. $\alpha f(x) + \beta g(x) \qquad \alpha \hat{f}(\omega) + \beta \hat{g}(\omega)$

Transform of derivative:

19. $f^{(n)}(x) \qquad (i\omega)^n \hat{f}(\omega)$

Transform of integral:

20. $f(x) = \int_{-\infty}^x g(\xi) d\xi,$ $\hat{f}(\omega) = \frac{1}{i\omega} \hat{g}(\omega)$
 where $f(x) \rightarrow 0$ as $x \rightarrow \infty$

Fourier convolution theorem:

21. $(f * g)(x) = \int_{-\infty}^{\infty} f(x-\xi)g(\xi) d\xi \qquad \hat{f}(\omega)\hat{g}(\omega)$

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STANDARD NORMAL DISTRIBUTION: Table Values Represent AREA to the LEFT of the Z score.

Z	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
0.0	.50000	.50399	.50798	.51197	.51595	.51994	.52392	.52790	.53188	.53586
0.1	.53983	.54380	.54776	.55172	.55567	.55962	.56356	.56749	.57142	.57535
0.2	.57926	.58317	.58706	.59095	.59483	.59871	.60257	.60642	.61026	.61409
0.3	.61791	.62172	.62552	.62930	.63307	.63683	.64058	.64431	.64803	.65173
0.4	.65542	.65910	.66276	.66640	.67003	.67364	.67724	.68082	.68439	.68793
0.5	.69146	.69497	.69847	.70194	.70540	.70884	.71226	.71566	.71904	.72240
0.6	.72575	.72907	.73237	.73565	.73891	.74215	.74537	.74857	.75175	.75490
0.7	.75804	.76115	.76424	.76730	.77035	.77337	.77637	.77935	.78230	.78524
0.8	.78814	.79103	.79389	.79673	.79955	.80234	.80511	.80785	.81057	.81327
0.9	.81594	.81859	.82121	.82381	.82639	.82894	.83147	.83398	.83646	.83891
1.0	.84134	.84375	.84614	.84849	.85083	.85314	.85543	.85769	.85993	.86214
1.1	.86433	.86650	.86864	.87076	.87286	.87493	.87698	.87900	.88100	.88298
1.2	.88493	.88686	.88877	.89065	.89251	.89435	.89617	.89796	.89973	.90147
1.3	.90320	.90490	.90658	.90824	.90988	.91149	.91309	.91466	.91621	.91774
1.4	.91924	.92073	.92220	.92364	.92507	.92647	.92785	.92922	.93056	.93189
1.5	.93319	.93448	.93574	.93699	.93822	.93943	.94062	.94179	.94295	.94408
1.6	.94520	.94630	.94738	.94845	.94950	.95053	.95154	.95254	.95352	.95449
1.7	.95543	.95637	.95728	.95818	.95907	.95994	.96080	.96164	.96246	.96327
1.8	.96407	.96485	.96562	.96638	.96712	.96784	.96856	.96926	.96995	.97062
1.9	.97128	.97193	.97257	.97320	.97381	.97441	.97500	.97558	.97615	.97670
2.0	.97725	.97778	.97831	.97882	.97932	.97982	.98030	.98077	.98124	.98169
2.1	.98214	.98257	.98300	.98341	.98382	.98422	.98461	.98500	.98537	.98574
2.2	.98610	.98645	.98679	.98713	.98745	.98778	.98809	.98840	.98870	.98899
2.3	.98928	.98956	.98983	.99010	.99036	.99061	.99086	.99111	.99134	.99158
2.4	.99180	.99202	.99224	.99245	.99266	.99286	.99305	.99324	.99343	.99361
2.5	.99379	.99396	.99413	.99430	.99446	.99461	.99477	.99492	.99506	.99520
2.6	.99534	.99547	.99560	.99573	.99585	.99598	.99609	.99621	.99632	.99643
2.7	.99653	.99664	.99674	.99683	.99693	.99702	.99711	.99720	.99728	.99736
2.8	.99744	.99752	.99760	.99767	.99774	.99781	.99788	.99795	.99801	.99807
2.9	.99813	.99819	.99825	.99831	.99836	.99841	.99846	.99851	.99856	.99861
3.0	.99865	.99869	.99874	.99878	.99882	.99886	.99889	.99893	.99896	.99900
3.1	.99903	.99906	.99910	.99913	.99916	.99918	.99921	.99924	.99926	.99929
3.2	.99931	.99934	.99936	.99938	.99940	.99942	.99944	.99946	.99948	.99950
3.3	.99952	.99953	.99955	.99957	.99958	.99960	.99961	.99962	.99964	.99965
3.4	.99966	.99968	.99969	.99970	.99971	.99972	.99973	.99974	.99975	.99976
3.5	.99977	.99978	.99978	.99979	.99980	.99981	.99981	.99982	.99983	.99983
3.6	.99984	.99983	.99985	.99986	.99986	.99987	.99987	.99988	.99988	.99989
3.7	.99989	.99990	.99990	.99990	.99991	.99991	.99992	.99992	.99992	.99992
3.8	.99993	.99993	.99993	.99994	.99994	.99994	.99994	.99995	.99995	.99995
3.9	.99995	.99995	.99996	.99996	.99996	.99996	.99996	.99996	.99997	.99997

PERCENTAGE POINTS OF THE T DISTRIBUTION

Tail Probabilities		0.10	0.05	0.025	0.01	0.005	0.001	0.0005
One Tail		0.10	0.05	0.025	0.01	0.005	0.001	0.0005
Two Tails		0.20	0.10	0.05	0.02	0.01	0.002	0.001
D	1	3.078	6.314	12.71	31.82	63.66	318.3	637
E	2	1.886	2.920	4.303	6.965	9.925	22.330	31.6
G	3	1.638	2.353	3.182	4.541	5.841	10.210	12.92
R	4	1.533	2.132	2.776	3.747	4.604	7.173	8.610
E	5	1.476	2.015	2.571	3.365	4.032	5.893	6.869
E	6	1.440	1.943	2.447	3.143	3.707	5.208	5.959
E	7	1.415	1.895	2.365	2.998	3.499	4.785	5.408
	8	1.397	1.860	2.306	2.896	3.355	4.501	5.041
O	9	1.383	1.833	2.262	2.821	3.250	4.297	4.781
F	10	1.372	1.812	2.228	2.764	3.169	4.144	4.587
	11	1.363	1.796	2.201	2.718	3.106	4.025	4.437
F	12	1.356	1.782	2.179	2.681	3.055	3.930	4.318
R	13	1.350	1.771	2.160	2.650	3.012	3.852	4.221
E	14	1.345	1.761	2.145	2.624	2.977	3.787	4.140
E	15	1.341	1.753	2.131	2.602	2.947	3.733	4.073
D	16	1.337	1.746	2.120	2.583	2.921	3.686	4.015
O	17	1.333	1.740	2.110	2.567	2.898	3.646	3.965
M	18	1.330	1.734	2.101	2.552	2.878	3.610	3.922
	19	1.328	1.729	2.093	2.539	2.861	3.579	3.883
	20	1.325	1.725	2.086	2.528	2.845	3.552	3.850
	21	1.323	1.721	2.080	2.518	2.831	3.527	3.819
	22	1.321	1.717	2.074	2.508	2.819	3.505	3.792
	23	1.319	1.714	2.069	2.500	2.807	3.485	3.768
	24	1.318	1.711	2.064	2.492	2.797	3.467	3.745
	25	1.316	1.708	2.060	2.485	2.787	3.450	3.725

TABLE A2 Values of $\chi^2_{\alpha, n}$

n	$\alpha = .995$	$\alpha = .99$	$\alpha = .975$	$\alpha = .95$	$\alpha = .05$	$\alpha = .025$	$\alpha = .01$	$\alpha = .005$
1	.0000393	.000157	.000982	.00393	3.841	5.024	6.635	7.879
2	.0100	.0201	.0506	.103	5.991	7.378	9.210	10.597
3	.0717	.115	.216	.352	7.815	9.348	11.345	12.838
4	.207	.297	.484	.711	9.488	11.143	13.277	14.860
5	.412	.554	.831	1.145	11.070	12.832	13.086	16.750
6	.676	.872	1.237	1.635	12.592	14.449	16.812	18.548
7	.989	1.239	1.690	2.167	14.067	16.013	18.475	20.278
8	1.344	1.646	2.180	2.733	15.507	17.535	20.090	21.955
9	1.735	2.088	2.700	3.325	16.919	19.023	21.666	23.589
10	2.156	2.558	3.247	3.940	18.307	20.483	23.209	25.188
11	2.603	3.053	3.816	4.575	19.675	21.920	24.725	26.757
12	3.074	3.571	4.404	5.226	21.026	23.337	26.217	28.300
13	3.565	4.107	5.009	5.892	22.362	24.736	27.688	29.819
14	4.075	4.660	5.629	6.571	23.685	26.119	29.141	31.319
15	4.601	5.229	6.262	7.261	24.996	27.488	30.578	32.801
16	5.142	5.812	6.908	7.962	26.296	28.845	32.000	34.267
17	5.697	6.408	7.564	8.672	27.587	30.191	33.409	35.718
18	6.265	7.015	8.231	9.390	28.869	31.526	34.805	37.156
19	6.844	7.633	8.907	10.117	30.144	32.852	36.191	38.582
20	7.434	8.260	9.591	10.851	31.410	34.170	37.566	39.997
21	8.034	8.897	10.283	11.591	32.671	35.479	38.932	41.401
22	8.643	9.542	10.982	12.338	33.924	36.781	40.289	42.796
23	9.260	10.196	11.689	13.091	35.172	38.076	41.638	44.181
24	9.886	10.856	12.401	13.844	36.415	39.364	42.980	45.558
25	10.520	11.524	13.120	14.611	37.652	40.646	44.314	46.928
26	11.160	12.198	13.844	15.379	38.885	41.923	45.642	48.290
27	11.808	12.879	14.573	16.151	40.113	43.194	46.963	49.645
28	12.461	13.565	15.308	16.928	41.337	44.461	48.278	50.993
29	13.121	14.256	16.047	17.708	42.557	45.772	49.588	52.336
30	13.787	14.953	16.791	18.493	43.773	46.979	50.892	53.672

Other chi-square probabilities:

$\chi^2_{.99} = 4.2$ $P[\chi^2_{16} < 14.3] = .425$ $P[\chi^2_{11} < 17.1875] = .8976$