

BANGLADESH UNIVERSITY OF ENGINEERING AND TECHNOLOGY, DHAKA

L-2/T-2 B. Sc. Engineering Examinations 2013-2014

Sub : **EEE 205** (Energy Conversion II)

Full Marks : 210

Time : 3 Hours

The figures in the margin indicate full marks.

All the Symbols and notations have their usual meanings.

USE SEPARATE SCRIPTS FOR EACH SECTION

SECTION – AThere are **FOUR** questions in this Section. Answer any **THREE**.

1. (a) Why commutators are used in DC machines? Illustratively show whether the number of commutator segments has any effect on the average value of the output voltage of a DC generator. (15)
- (b) What are equalizers or equalizing bars? In which type of DC machine they are needed and why? (10)
- (c) In reference to armature reaction, discuss the problems that flux weakening causes in both DC generators and motors. Which machine (motor or generator) is more vulnerable to this problem? (10)
2. (a) Name and describe the features of different types of DC generators. (10)
- (b) What are the possible causes if a shunt generator is started and no voltage builds up? How can this problem be remedied? (18)
- (c) The following data refer to the OCC of a separately excited DC generator at 1000 rpm. (15)
- | | | | | | | | | | | | | |
|----------|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|
| $E_a(V)$ | 5 | 40 | 75 | 100 | 124 | 145 | 162 | 178 | 188 | 195 | 200 | 205 |
| $I_f(A)$ | 0.0 | 0.2 | 0.3 | 0.4 | 0.5 | 0.6 | 0.7 | 0.8 | 0.9 | 1.0 | 1.1 | 1.2 |
- The machine is now connected as a shunt generator with a total field resistance of 200Ω . Estimate graphically: (i) the voltage to which the generator will build up at no load (ii) the armature, field and load currents when the terminal voltage is found to be $150 V$. Neglect the effect of armature reaction and brush drop and assume armature resistance R_A to be 0.8Ω .
3. (a) What happens in a shunt DC motor if its field circuit opens while it is running? How can the speed of a shunt DC motor be controlled? Explain those methods in detail with appropriate diagrams and keeping in mind the power and torque limits of the motor. (18)
- (b) A $250 V$ series DC motor has compensating windings and a total series resistance ($R_A + R_S$) of 0.08Ω . The series field consists of 25 turns per pole, with the magnetization curve shown in the Fig. for Q. No. 3(b), find the speed and induced torque of this motor for when its armature current is $50 A$. (17)

Contd P/2

EEE 205

4. (a) Draw the equivalent circuit of a typical *PV* cell. Show the following effects on the *I-V* characteristics of such a solar cell: (i) effect of resistances (ii) effect of temperature. (11)
- (b) Why should people use renewable energy? Draw the scheme of battery-backed solar power-driven system for DC load. (12)
- (c) A wind farm has 40 wind turbines each generating 0.5 *MW* for 50% of time. If the annual average amount of electricity used by a consumer of the neighboring area is 10 *MWhr*, then determine (i) the number of consumers that the wind farm can supply power to and (ii) the number of such wind farms that cumulatively can generate power equivalent to a 500 *MW* fossil fuel power station. (12)

SECTION – B

There are **FOUR** questions in this Section. Answer any **THREE**.

5. (a) Explain how the synchronous generator model parameters are found from tests. Explain why the short circuit characteristics of a synchronous generator is a straight line. (15)
- (b) A three-phase Y-connected synchronous generator is rated 120 MVA, 13.2 kV, 0.8 PF lagging, and 60 Hz. Its synchronous reactance is 0.9 Ω , and its resistance may be ignored. (20)
- (i) What is its voltage regulation?
- (ii) What would be the voltage and apparent power rating of this generator if it were operated at 50 Hz with the same armature and field losses as it had at 60 Hz?
- (iii) What would be the voltage regulation of this generator at 50 Hz?
6. (a) What conditions are necessary for paralleling synchronous generator to an infinite Bus? (10)
- (b) Why must the oncoming generator on a power system be paralleled at a higher frequency than that of the running system? Explain using house diagram. (7)
- (c) With appropriate house diagram explain the following- (18)
- (i) How can the real power sharing between two generators be controlled without affecting the system's frequency?
- (ii) How can the reactive power sharing between two generators be controlled without affecting the system's terminal voltage?
7. (a) What are the starting problems of a synchronous motor? Explain the methods of starting synchronous motors (i) by changing the frequency of supply and (ii) by using Amortisseur windings. (17)

EEE 205
Contd ... Q. No. 7

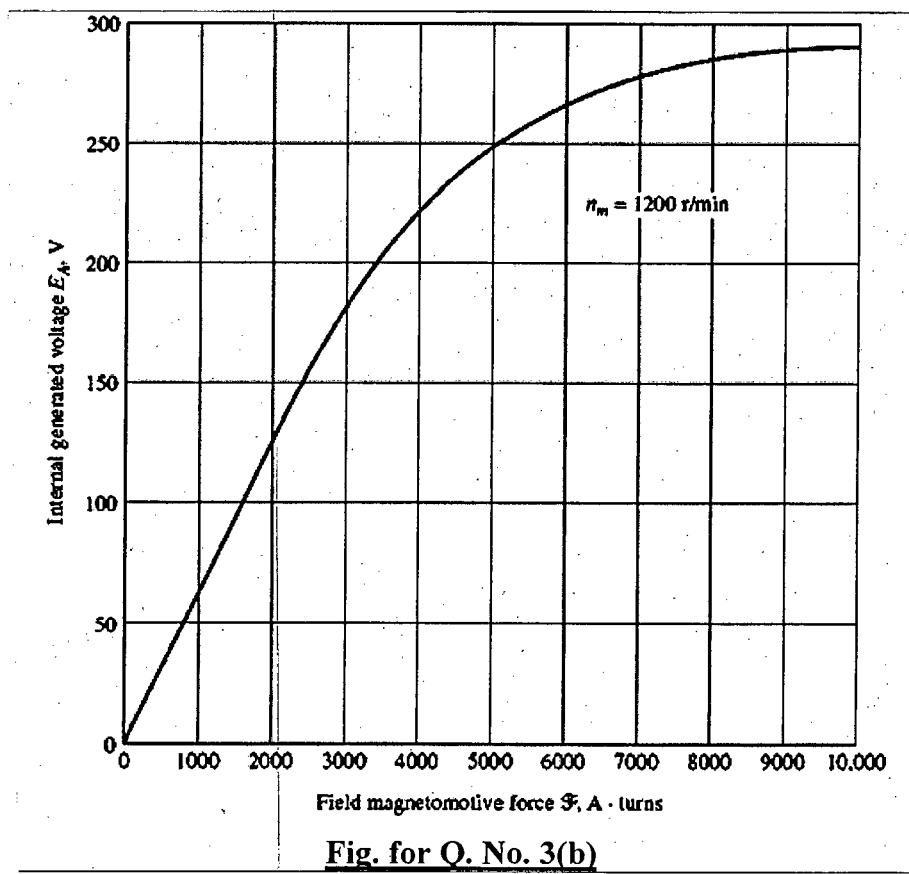
(b) A 480 V, 375-kVA, 0.8 PF-lagging, Y connected synchronous generator has a synchronous reactance of 0.4Ω and a negligible armature resistance. This generator is supplying power to a 480 V, 80-kW, 0.8 PF – leading, Y-connected synchronous motor with a synchronous reactance of 1.1Ω and a negligible armature resistance. The synchronous generator is adjusted to have a terminal voltage of 480 V when the motor is drawing the rated power at unity power factor. (18)

- (i) Calculate the magnitude and angles of E_A for both machines.
- (ii) If the flux of the motor is increased by 10 percent, what happens to the terminal voltage of the power system? What is its new value?
- (iii) What is the power factor of the motor after the increase in motor flux?

8. (a) Is a synchronous motor's field circuit is more vulnerable to overheating when it is operating at a leading or at a lagging power factor? Explain using phasor diagram. (13)

(b) A synchronous motor is operating at a fixed real load and its field current is increased. If the armature current falls, was the motor initially operating at a lagging or a leading power factor? Explain using phasor diagram. (10)

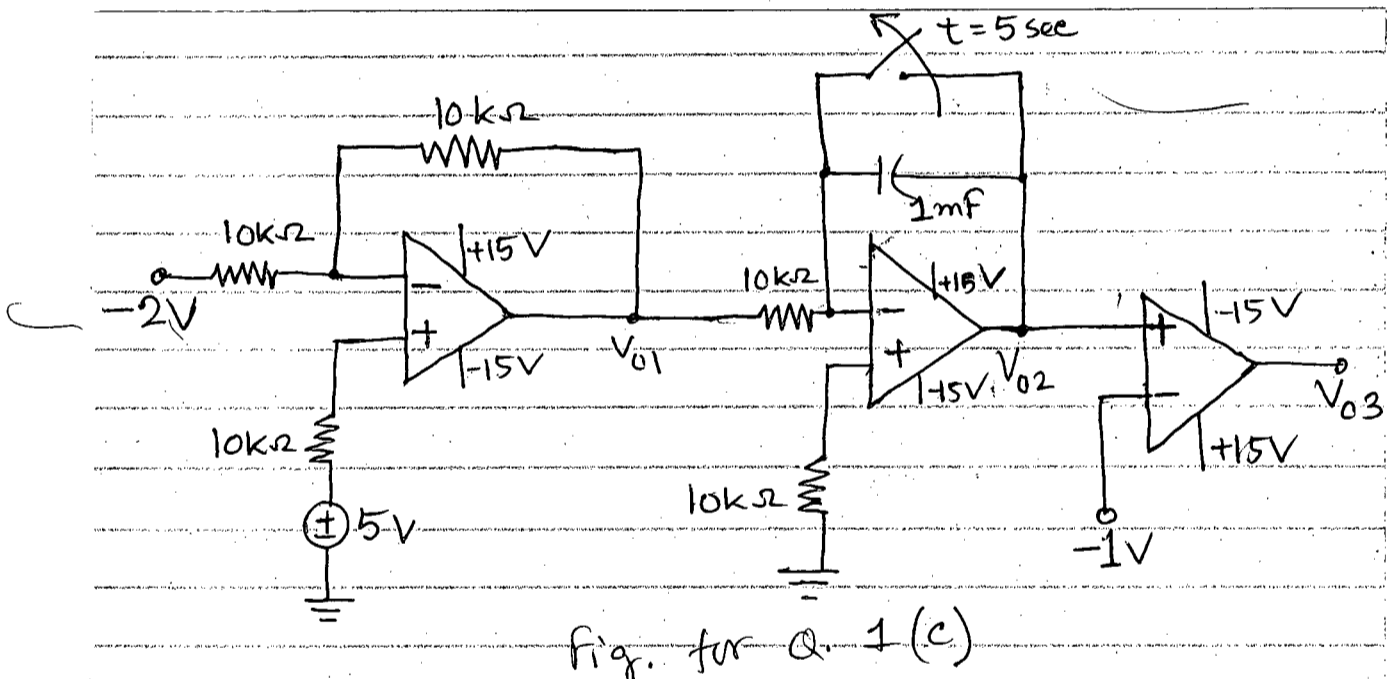
(c) When a synchronous motor would be used even though its constant speed characteristics were not needed? Explain briefly. (12)



SECTION - A

There are **FOUR** questions in this Section. Answer any **THREE**.

1. (a) What are the characteristics of an ideal op-amp? (6)
- (b) Draw the circuit diagram of a differential amplifier using op-amp and show that the output voltage is directly proportional to the difference of its input voltages. (12)
- (c) Draw the wave shapes of V_{o1} , V_{o2} and V_{o3} for the following circuit (Fig. for Q. 1(c)) for a time period of 15 sec ($t = 0$ to $t = 15s$). Consider that the switch is initially closed. It will be opened at $t = 5$ sec and remains open till $t = 10$ sec (i.e., the switch will be closed again at $t = 10$ sec). Assume ideal case. (17)

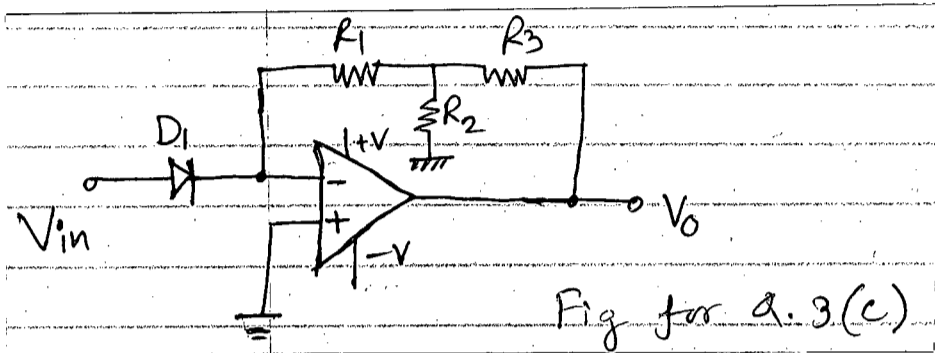


2. (a) Design a 3rd order bandpass filter with resonant frequency of 950 Hz and bandwidth of 2700 Hz. Also, calculate the quality factor of the filter and comment on whether the filter is narrow band or wide band. (15)
- (b) Derive the expression of cut-off frequency for a +40dB/decade Butterworth filter. (15)
- (c) The slew rate for a 741 op-amp is $0.5 \text{ V}/\mu\text{s}$. You can get an undistorted sine wave output voltages $10 \sin 2\pi f_{\text{max}} t$ at maximum frequency, $f = f_{\text{max}}$. Determine the value of maximum frequency. (5)
3. (a) Draw the circuit diagram of a Wein-Bridge Oscillator Using op-amp and other necessary components and derive the expression of oscillation frequency. Also develop the condition for the sustained sinusoidal oscillation. (16)

EEE 207
Contd ... Q. No. 3

(b) Design a Hartley oscillator to produce a 5 kHz output frequency. Use 0.1 μf capacitor and an op-amp with ±10V supply. Make necessary assumptions and consider that the coils are wound on separate core. (12)

(c) Derive the expression of output voltage for the following circuit (Fig. for Q. 3(c)). Consider ideal op-amp. (7)



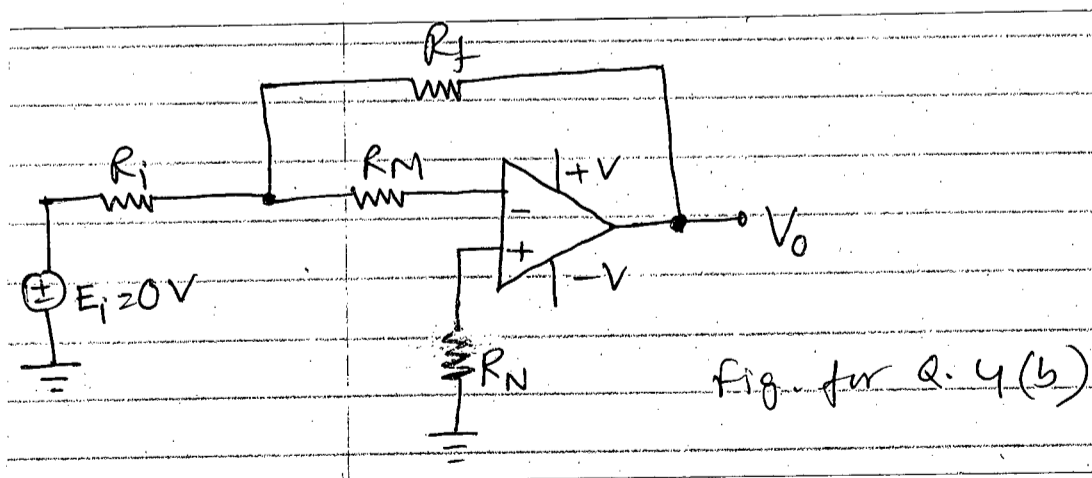
4. (a) Draw the equivalent circuit diagram and derive the expression of the equivalent reactance of a crystal used in crystal oscillator. Also draw the reactance versus frequency curve of the crystal. (10)

(b) (i) Drive the expression of output voltage for the following circuit (Fig. for Q. 4(b)) considering non-ideal effects of input bias currents for the op-amp (i.e. consider the effect of both I_{B+} and I_{B-} simultaneously).

(ii) What is the best value of R_N if the source resistance is 10 Ω to compensate the non-ideal effect of op-amp. $R_f = 1\text{M } \Omega$, $R_M = 10\text{ k } \Omega$ and $R_i = 1\text{ k } \Omega$.

(iii) If input offset voltage, $V_{io} = 2\text{ mV}$ at 25°C calculate the error in the output voltage due to V_{io} only at 25°C.

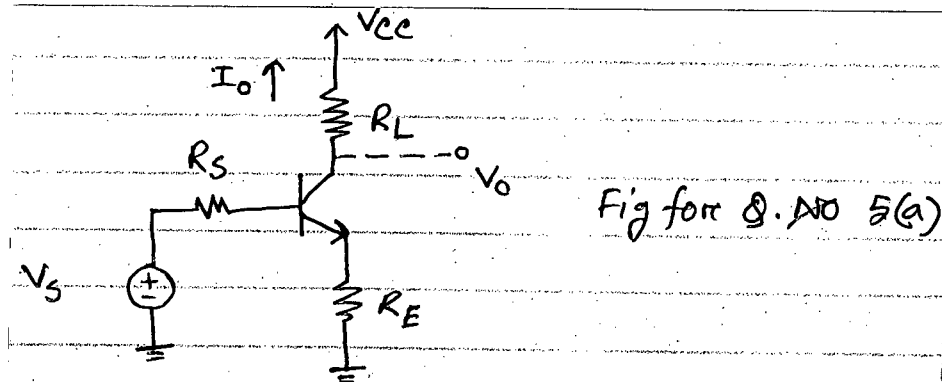
Find the maximum error in output voltage due to drift in V_{io} as temperature changes from 25°C to 75°C. For offset voltage, drift is specified as 20 μV/°C. (25)



SECTION – B

There are **FOUR** questions in this Section. Answer any **THREE**.

5. (a) Identify the feedback topology in the Fig. for Q. 5(a) and derive expressions for input resistance, output resistance, overall transconductance and overall voltage gain with feed back. (18)



- (b) The circuit of Fig. for Q. 5(a) has an overall transconductance of -1mA/V , a voltage gain of -4 and a desensitivity of 50 . If $R_s = 1\text{ k}\Omega$, $h_{fe} = 150$, Find (i) R_E (ii) R_L (iii) R_{if} (iv) R_{of} , (v) R_{of} , where symbols have their usual meaning. (17)

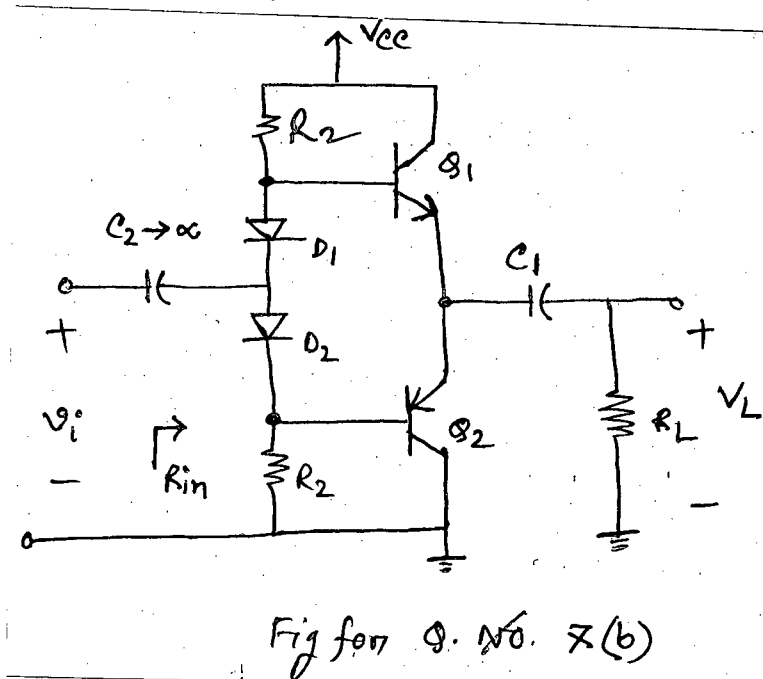
6. (a) Draw the block diagrams of different feedback topologies. For voltage-series feedback show that, $R_{if} > R_i$ and $R_{of} < R_o$, where symbols have their usual meaning. (17)
 (b) Draw the small signal equivalent circuits of current and transconductance amplifiers. (8)
 (c) What are the advantages of $-ve$ feed back in an amplifier? Briefly discuss. (10)

7. (a) For an inductivity coupled class-A amplifier draw the load lines and show that maximum power conversion efficiency is 50% . (10)

- (b) For the Diode compensated class-B Push-Pull amplifier shown in the Fig. for Q. 7(b) prove that maximum power rating of each transistor is, $P_{max} = \frac{V_{cc}^2}{4\pi^2 R_L}$ (10)

- (c) Design the amplifier in Fig. for Q 7(b) to drive a 4Ω load to $\pm 4\text{V}$ for a low-frequency 3-dB point of 50 Hz . Given, $\beta = 100$, $|v_{BE}| = 0.7\text{ V}$, $R_f = 10\Omega$, $V_{cc} = 15\text{ V}$. Calculate- (15)

- (i) R_2 and C_1 (ii) R_{in} (iii) A_i (iv) Power delivered from the supply (v) Power delivered to the load (vi) Power rating of each transistor. All symbols have their usual meaning.



Contd P/4

EEE 207**Contd ... Q. No. 7(c)**

[Hint: Draw the mid-frequency and input equivalent circuit and use,

$$R_2 = \frac{V_{cc} / 2 - v_{BE} - v_{Lp}}{i_{bp}}$$

8. (a) The low frequency response of an amplifier is given by the function-

(10)

$$F_L(s) = \frac{(s + w_{z1})(s + w_{z2})}{(s + w_{p1})(s + w_{p2})}$$

Assuming, dominant pole does not exist show that cutoff frequency,

$$W_L = \sqrt{w_{p1}^2 + w_{p2}^2 - 2w_{z1}^2 - 2w_{z2}^2}$$

- (b) Consider a specific low frequency response function,

(25)

$$F_L(s) = \frac{10^6 s(s + 10^2)}{(s + 10^3)(s + 10^5)}$$

- (i) Sketch the Bode plot for both magnitude and phase on a graph paper.
- (ii) Does dominant pole condition exist?
- (iii) What are the values of approximate and exact cut off frequencies?
- (iv) At approximate cut off frequency calculate the approximate and exact magnitude of the low frequency response function.

SECTION – A

There are **FOUR** questions in this Section. Answer any **THREE**.

Steam table (2 pages) attached.

1. (a) Distinguish between the followings: (12)
 - (i) Control volume and Control mass
 - (ii) Intensive properties and Extensive properties
 - (iii) Open FWH and Closed FWH
- (b) Write down the general form of steady flow energy equation and simplify it for the following device: (5)
 - (i) Turbine
 - (ii) Boiler
- (c) Consider a steam power plant operating under ideal reheat Rankine cycle. Steam enters the high pressure turbine at 12.5 MPa and 600°C and is condensed at a pressure of 10 kPa in the condenser. If the moisture content of the steam at the exit of the low pressure turbine is 11.9 percent, determine (i) the pressure during reheat process, and (ii) the thermal efficiency of the cycle. [Assume the steam is reheated to the inlet temperature of the high pressure turbine] (18)
2. (a) Draw the p-V and T-s diagrams of Otto cycle. Describe how the net work output and the net heat addition can be calculated from these diagrams. (6)
- (b) Deduce the expression for an Otto cycle efficiency and hence draw a plot of efficiency vs compression ratio. (12)
- (c) An air-standard Diesel cycle has a compression ratio of 16 and a cutoff ratio of 2. At the beginning of the compression process, air is at 95 kPa and 27°C. Determine (i) the temperature after the heat addition process, (ii) the thermal efficiency, and (iii) the mean effective pressure. ($c_p = 1.005$ kJ/kg K, $c_v = 0.718$ kJ/kg K). (17)
3. (a) How does an intercooler and a reheater improves the efficiency of a Brayton cycle. Explain with necessary diagrams. (10)
- (b) Draw the block and T-s diagram of a combined gas-vapor power cycle. (5)

ME 267(EEE)

Contd ... Q. No. 3

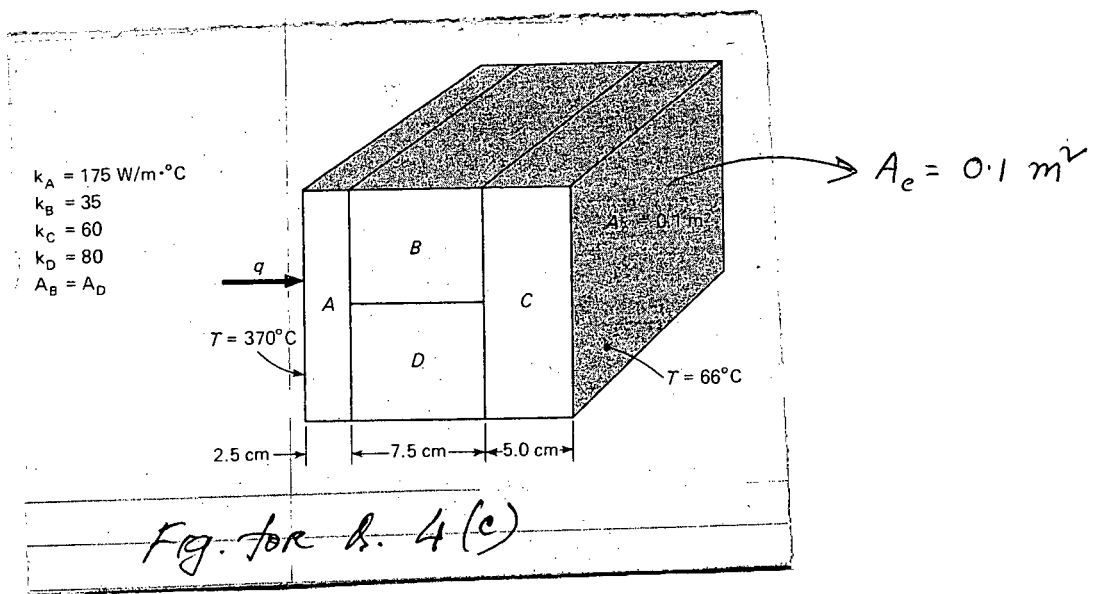
(c) Consider an ideal gas turbine cycle with regeneration and two stages of compression with intercooling. The pressure ratio across each stage of the compressor is 3. The air enters each stage of the compressor at 300 K and the turbine at 1300 K. If the regenerator effectiveness is 80%, determine (i) back work ratio, and (ii) thermal efficiency of the cycle. Consider, $C_p = 1.005 \text{ kJ/kg K}$ and $C_v = 0.718 \text{ kJ/kg K}$ constant. (20)

4. (a) Write short notes on the following: (10)

- (i) Thermal conductivity, (ii) Thermal diffusivity, (iii) Forced convection, and (iv) Emissivity.

(b) Discuss critical thickness of insulation. Deduce the expression of critical radius of insulation. (12)

(c) Find the heat transfer per unit area through the composite wall sketched below. Assume one-dimensional heat flow. (13)



SECTION – B

There are **FOUR** questions in this Section. Answer any **THREE**.

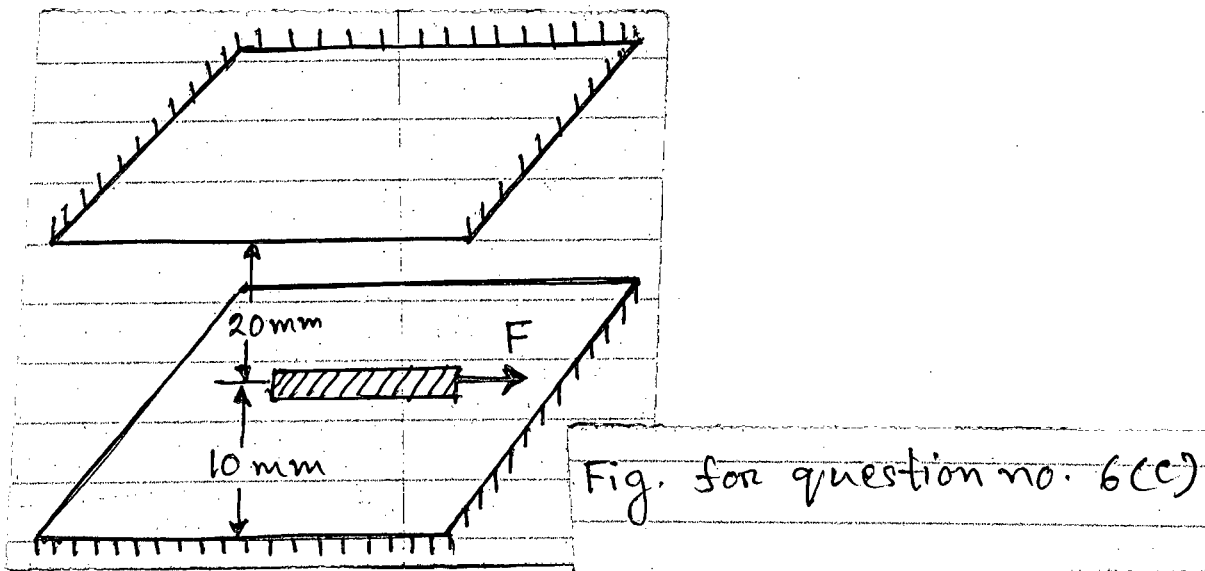
5. (a) Drive Bernoulli equation mentioning the assumptions along a stream line of a flow field. (12)

(b) State the phenomenon of cavitation and its effect on a centrifugal pump. (8)

(c) Draw and explain the performance curve and the system characteristics curve for a centrifugal pumps. Also draw head vs flow rate curves for pumps connected in series and parallel. (15)

ME 267(EEE)

6. (a) Draw the flow chart for fossilized vegetation necessary to extract Anthracitic coal. (5)
- (b) Mention the basic source of renewable energy and describe with schematic diagram the working principle of a hydroelectric power plant. (15)
- (c) Two large horizontal and fixed parallel surfaces are 30 mm apart. A thin flat plate of area 0.6 m^2 is pulled with a velocity of 0.67 m/s parallel to plane surfaces, at a distance of 20 mm from the top surface. The space between the two plane surfaces is filled with oil of viscosity 0.6 Ns/m^2 . Find the force requirement to maintain this motion. (15)



7. (a) For a particular situation, if the outside air is at 22°C DBT and 50% RH, what is the dew point temperature and humidity ratio of this air? If the air is sensibly heated to 30°C , what will be its new relative humidity and density? (8)
- (b) Give a comparative study between window A/C unit and split A/C unit. (10)
- (c) With a net sketch describe the working principle of external gear pump. (10)
- (d) Compare between impulse and reaction turbines. (7)
8. (a) A standard vapor refrigeration cycle developing 50 kW of refrigeration using R-22 operates with a condensing temperature of 36°C and an evaporating temperature of -10°C . Calculate: (15)
- (i) the refrigerating effect kJ/kg, (ii) the circulation rate of refrigerant in kg/sec,
 - (iii) the power required by the compressor in kW, (iv) the COP,
 - (v) the volume flow rate measured at compressor suction and
 - (vi) the compressor discharge temperature.
- (b) Draw a block diagram of ammonia-water vapor absorption refrigeration system and describe its working principle. (15)
- (c) Identify the following items as mountings or accessories and write down the specific function. (5)
- (i) Air preheater (ii) Fusible plug (iii) Economizer (iv) Safety valve (v) water level indicator.

aturated water-Pressure table

Press. kPa <i>P</i>	Sat. temp. °C <i>T_{sat}</i>	Specific volume m ³ /kg		Internal energy kJ/kg			Enthalpy kJ/kg			Entropy kJ/(kg · K)		
		Sat. liquid <i>v_f</i>	Sat. vapor <i>v_g</i>	Sat. liquid <i>u_f</i>	Evap. <i>u_{fg}</i>	Sat. vapor <i>u_g</i>	Sat. liquid <i>h_f</i>	Evap. <i>h_{fg}</i>	Sat. vapor <i>h_g</i>	Sat. liquid <i>s_f</i>	Evap. <i>s_{fg}</i>	Sat. vapor <i>s_g</i>
0.6113	0.01	0.001 000	206.14	0.00	2375.3	2375.3	0.01	2501.3	2501.4	0.0000	9.1562	9.1562
1.0	6.98	0.001 000	129.21	29.30	2355.7	2385.0	29.30	2484.9	2514.2	0.1059	8.8697	8.9756
1.5	13.03	0.001 001	87.98	54.71	2338.6	2393.3	54.71	2470.6	2525.3	0.1957	8.6322	8.8279
2.0	17.50	0.001 001	67.00	73.48	2326.0	2399.5	73.48	2460.0	2533.5	0.2607	8.4629	8.7237
2.5	21.08	0.001 002	54.25	88.48	2315.9	2404.4	88.49	2451.6	2540.0	0.3120	8.3311	8.6432
3.0	24.08	0.001 003	45.67	101.04	2307.5	2408.5	101.05	2444.5	2545.5	0.3545	8.2231	8.5776
4.0	28.96	0.001 004	34.80	121.45	2293.7	2415.2	121.46	2432.9	2554.4	0.4226	8.0520	8.4746
5.0	32.88	0.001 005	28.19	137.81	2282.7	2420.5	137.82	2423.7	2561.5	0.4764	7.9187	8.3951
7.5	40.29	0.001 008	19.24	168.78	2261.7	2430.5	168.79	2406.0	2574.8	0.5764	7.6750	8.2515
10	45.81	0.001 010	14.67	191.82	2246.1	2437.9	191.83	2392.8	2584.7	0.6493	7.5009	8.1502
15	53.97	0.001 014	10.02	225.92	2222.8	2448.7	225.94	2373.1	2599.1	0.7549	7.2536	8.0085
20	60.06	0.001 017	7.649	251.38	2205.4	2456.7	251.40	2358.3	2609.7	0.8320	7.0766	7.9085
25	64.97	0.001 020	6.204	271.90	2191.2	2463.1	271.93	2346.3	2618.2	0.8931	6.9383	7.8314
30	69.10	0.001 022	5.229	289.20	2179.2	2468.4	289.23	2336.1	2625.3	0.9439	6.8247	7.7686
40	75.87	0.001 027	3.993	317.53	2159.5	2477.0	317.58	2319.2	2636.8	1.0259	6.6441	7.6700
50	81.33	0.001 030	3.240	340.44	2143.4	2483.9	340.49	2305.4	2645.9	1.0910	6.5029	7.5939
75	91.78	0.001 037	2.217	384.31	2112.4	2496.7	384.39	2278.6	2663.0	1.2130	6.2434	7.4564
Press. MPa												
0.100	99.63	0.001 043	1.6940	417.36	2088.7	2506.1	417.46	2258.0	2675.5	1.3026	6.0568	7.3594
0.125	105.99	0.001 048	1.3749	444.19	2069.3	2513.5	444.32	2241.0	2685.4	1.3740	5.9104	7.2844
0.150	111.37	0.001 053	1.1593	466.94	2052.7	2519.7	467.11	2226.5	2693.6	1.4336	5.7897	7.2233
0.175	116.06	0.001 057	1.0036	486.80	2038.1	2524.9	486.99	2213.6	2700.6	1.4849	5.6868	7.1717
0.200	120.23	0.001 061	0.8857	504.49	2025.0	2529.5	504.70	2201.9	2706.7	1.5301	5.5970	7.1271
0.225	124.00	0.001 064	0.7933	520.47	2013.1	2533.6	520.72	2191.3	2712.1	1.5706	5.5173	7.0878
0.250	127.44	0.001 067	0.7187	535.10	2002.1	2537.2	535.37	2181.5	2716.9	1.6072	5.4455	7.0527
0.275	130.60	0.001 070	0.6573	548.59	1991.9	2540.5	548.89	2172.4	2721.3	1.6408	5.3801	7.0209
0.300	133.55	0.001 073	0.6058	561.15	1982.4	2543.6	561.47	2163.8	2725.3	1.6718	5.3201	6.9919
0.325	136.30	0.001 076	0.5620	572.90	1973.5	2546.4	573.25	2155.8	2729.0	1.7006	5.2646	6.9652
0.350	138.88	0.001 079	0.5243	583.95	1965.0	2548.9	584.33	2148.1	2732.4	1.7275	5.2130	6.9405
0.375	141.32	0.001 081	0.4914	594.40	1956.9	2551.3	594.81	2140.8	2735.6	1.7528	5.1647	6.9175
0.40	143.63	0.001 084	0.4625	604.31	1949.3	2553.6	604.74	2133.8	2738.6	1.7766	5.1193	6.8959
0.45	147.93	0.001 088	0.4140	622.77	1934.9	2557.6	623.25	2120.7	2743.9	1.8207	5.0359	6.8565
0.50	151.86	0.001 093	0.3749	639.68	1921.6	2561.2	640.23	2108.5	2748.7	1.8607	4.9606	6.8213
0.55	155.48	0.001 097	0.3427	655.32	1909.2	2564.5	665.93	2097.0	2753.0	1.8973	4.8920	6.7893
0.60	158.85	0.001 101	0.3157	669.90	1897.5	2567.4	670.56	2086.3	2756.8	1.9312	4.8288	6.7600
0.65	162.01	0.001 104	0.2927	683.56	1886.5	2570.1	684.28	2076.0	2760.3	1.9627	4.7703	6.7331
0.70	164.97	0.001 108	0.2729	696.44	1876.1	2572.5	697.22	2066.3	2763.5	1.9922	4.7158	6.7080
0.75	167.78	0.001 112	0.2556	708.64	1866.1	2574.7	709.47	2057.0	2766.4	2.0200	4.6647	6.6847
0.80	170.43	0.001 115	0.2404	720.22	1856.6	2576.8	721.11	2048.0	2769.1	2.0462	4.6166	6.6628
0.85	172.96	0.001 118	0.2270	731.27	1847.4	2578.7	732.22	2039.4	2771.6	2.0710	4.5711	6.6421
0.90	175.38	0.001 121	0.2150	741.83	1838.6	2580.5	742.83	2031.1	2773.9	2.0946	4.5280	6.6226
0.95	177.69	0.001 124	0.2042	751.95	1830.2	2582.1	753.02	2023.1	2776.1	2.1172	4.4869	6.6041
1.00	179.91	0.001 127	0.194 44	761.68	1822.0	2583.6	762.81	2015.3	2778.1	2.1387	4.4478	6.5865
1.10	184.09	0.001 133	0.177 53	780.09	1806.3	2586.4	781.34	2000.4	2781.7	2.1792	4.3744	6.5536
1.20	187.99	0.001 139	0.163 33	797.29	1791.5	2588.8	798.65	1986.2	2784.8	2.2166	4.3067	6.5233
1.30	191.64	0.001 144	0.151 25	813.44	1777.5	2591.0	814.93	1972.7	2787.6	2.2515	4.2438	6.4953

Superheated water

T °C	v m ³ /kg	u kJ/kg	h kJ/kg	s kJ/(kg·K)	v m ³ /kg	u kJ/kg	h kJ/kg	s kJ/(kg·K)	v m ³ /kg	u kJ/kg	h kJ/kg	s kJ/(kg·K)		
$P = 4.0 \text{ MPa (250.40}^\circ\text{C)}$					$P = 4.5 \text{ MPa (257.49}^\circ\text{C)}$					$P = 5.0 \text{ MPa (263.99}^\circ\text{C)}$				
Sat.	0.049 78	2602.3	2801.4	6.0701	0.044 06	2600.1	2798.3	6.0198	0.039 44	2597.1	2794.3	5.9734		
275	0.054 57	2667.9	2886.2	6.2285	0.047 30	2650.3	2863.2	6.1401	0.041 41	2631.3	2838.3	6.0544		
300	0.058 84	2725.3	2960.7	6.3615	0.051 35	2712.0	2943.1	6.2828	0.045 32	2698.0	2924.5	6.2084		
350	0.066 45	2826.7	3092.5	6.5821	0.058 40	2817.8	3080.6	6.5131	0.051 94	2808.7	3068.4	6.4493		
400	0.073 41	2919.9	3213.6	6.7690	0.064 75	2913.3	3204.7	6.7047	0.057 81	2906.6	3195.7	6.6459		
450	0.080 02	3010.2	3330.3	6.9363	0.070 74	3005.0	3323.3	6.8746	0.063 30	2999.7	3316.2	6.8186		
500	0.086 43	3099.5	3445.3	7.0901	0.076 51	3095.3	3439.6	7.0301	0.068 57	3091.0	3433.8	6.9759		
600	0.098 85	3279.1	3674.4	7.3688	0.087 65	3276.0	3670.5	7.3110	0.078 69	3273.0	3666.5	7.2589		
700	0.110 95	3462.1	3905.9	7.6198	0.098 47	3459.9	3903.0	7.5631	0.088 49	3457.6	3900.1	7.5122		
800	0.122 87	3650.0	4141.5	7.8502	0.109 11	3648.3	4139.3	7.7942	0.098 11	3646.6	4137.1	7.7440		
900	0.134 69	3843.6	4382.3	8.0647	0.119 65	3842.2	4380.6	8.0091	0.107 62	3840.7	4378.8	7.9593		
1000	0.146 45	4042.9	4628.7	8.2662	0.130 13	4041.6	4627.2	8.2108	0.117 07	4040.4	4625.7	8.1612		
1100	0.158 17	4248.0	4880.6	8.4567	0.140 56	4246.8	4879.3	8.4015	0.126 48	4245.6	4878.0	8.3520		
1200	0.169 87	4458.6	5138.1	8.6376	0.150 98	4457.5	5136.9	8.5825	0.135 87	4456.3	5135.7	8.5331		
1300	0.181 56	4674.3	5400.5	8.8100	0.161 39	4673.1	5399.4	8.7549	0.145 26	4672.0	5398.2	8.7055		
$P = 6.0 \text{ MPa (275.64}^\circ\text{C)}$					$P = 7.0 \text{ MPa (285.88}^\circ\text{C)}$					$P = 8.0 \text{ MPa (295.06}^\circ\text{C)}$				
Sat.	0.032 44	2589.7	2784.3	5.8892	0.027 37	2580.5	2772.1	5.8133	0.023 52	2569.8	2758.0	5.7432		
300	0.036 16	2667.2	2884.2	6.0674	0.029 47	2632.2	2838.4	5.9305	0.024 26	2590.9	2785.0	5.7906		
350	0.042 23	2789.6	3043.0	6.3335	0.035 24	2769.4	3016.0	6.2283	0.029 95	2747.7	2987.3	6.1301		
400	0.047 39	2892.9	3177.2	6.5408	0.039 93	2878.6	3158.1	6.4478	0.034 32	2863.8	3138.3	6.3634		
450	0.052 14	2988.9	3301.8	6.7193	0.044 16	2978.0	3287.1	6.6327	0.038 17	2966.7	3272.0	6.5551		
500	0.056 65	3082.2	3422.2	6.8803	0.048 14	3073.4	3410.3	6.7975	0.041 75	3064.3	3398.3	6.7240		
550	0.061 01	3174.6	3540.6	7.0288	0.051 95	3167.2	3530.9	6.9486	0.045 16	3159.8	3521.0	6.8778		
600	0.065 25	3266.9	3658.4	7.1677	0.055 65	3260.7	3650.3	7.0894	0.048 45	3254.4	3642.0	7.0206		
700	0.073 52	3453.1	3894.2	7.4234	0.062 83	3448.5	3888.3	7.3476	0.054 81	3443.9	3882.4	7.2812		
800	0.081 60	3643.1	4132.7	7.6566	0.069 81	3639.5	4128.2	7.5822	0.060 97	3636.0	4123.8	7.5173		
900	0.089 58	3837.8	4375.3	7.8727	0.076 69	3835.0	4371.8	7.7991	0.067 02	3832.1	4368.3	7.7351		
1000	0.097 49	4037.8	4622.7	8.0751	0.083 50	4035.3	4619.8	8.0020	0.073 01	4032.8	4616.9	7.9384		
1100	0.105 36	4243.3	4875.4	8.2661	0.090 27	4240.9	4872.8	8.1933	0.078 96	4238.6	4870.3	8.1300		
1200	0.113 21	4454.0	5133.3	8.4474	0.097 03	4451.7	5130.9	8.3747	0.084 89	4449.5	5128.5	8.3115		
1300	0.121 06	4669.6	5396.0	8.6199	0.103 77	4667.3	5393.7	8.5475	0.090 80	4665.0	5391.5	8.4842		
$P = 9.0 \text{ MPa (303.40}^\circ\text{C)}$					$P = 10.0 \text{ MPa (311.06}^\circ\text{C)}$					$P = 12.5 \text{ MPa (327.89}^\circ\text{C)}$				
Sat.	0.020 48	2557.8	2742.1	5.6772	0.018 026	2544.4	2724.7	5.6141	0.013 495	2505.1	2673.8	5.4624		
325	0.023 27	2646.6	2856.0	5.8712	0.019 861	2610.4	2809.1	5.7568	0.016 126	2624.6	2826.2	5.7118		
350	0.025 80	2724.4	2956.6	6.0361	0.022 42	2699.2	2923.4	5.9443	0.020 00	2789.3	3039.3	6.0417		
400	0.029 93	2848.4	3117.8	6.2854	0.026 41	2832.4	3096.5	6.2120	0.022 99	2912.5	3199.8	6.2719		
450	0.033 50	2955.2	3256.6	6.4844	0.029 75	2943.4	3240.9	6.4190	0.025 60	3021.7	3341.8	6.4618		
500	0.036 77	3055.2	3386.1	6.6576	0.032 79	3045.8	3373.7	6.5966	0.028 01	3125.0	3475.2	6.6290		
550	0.039 87	3152.2	3511.0	6.8142	0.035 64	3144.6	3500.9	6.7561	0.030 29	3225.4	3604.0	6.7810		
600	0.042 85	3248.1	3633.7	6.9589	0.038 37	3241.7	3625.3	6.9029	0.032 48	3324.4	3730.4	6.9218		
650	0.045 74	3343.6	3755.3	7.0943	0.041 01	3338.2	3748.2	7.0398	0.034 60	3422.9	3855.3	7.0536		
700	0.048 57	3439.3	3876.5	7.2221	0.043 58	3434.7	3870.5	7.1687	0.038 69	3620.0	4103.6	7.2965		
800	0.054 09	3632.5	4119.3	7.4596	0.048 59	3628.9	4114.8	7.4077	0.042 67	3819.1	4352.5	7.5182		
900	0.059 50	3829.2	4364.8	7.6783	0.053 49	3826.3	4361.2	7.6272	0.046 58	4021.6	4603.8	7.7237		
1000	0.064 85	4030.3	4614.0	7.8821	0.058 32	4027.8	4611.0	7.8315	0.050 45	4228.2	4858.8	7.9165		
1100	0.070 16	4236.3	4867.7	8.0740	0.063 12	4234.0	4865.1	8.0237	0.054 30	4439.3	5118.0	8.0937		
1200	0.075 44	4447.2	5126.2	8.2556	0.067 89	4444.9	5123.8	8.2055	0.058 13	4654.8	5381.4	8.2717		
1300	0.080 72	4662.7	5389.2	8.4284	0.072 65	4460.5	5387.0	8.3783						

Refrigerant 22: properties of liquid and saturated vapor⁶

$t, ^\circ\text{C}$	P, kPa	Enthalpy, kJ/kg		Entropy, $\text{kJ/kg} \cdot \text{K}$		Specific volume, L/kg	
		h_f	h_g	s_f	s_g	v_f	v_g
-60	37.48	134.763	379.114	0.73254	1.87886	0.68208	537.152
-55	49.47	139.830	381.529	0.75599	1.86389	0.68856	414.827
-50	64.39	144.959	383.921	0.77919	1.85000	0.69526	324.557
-45	82.71	150.153	386.282	0.80216	1.83708	0.70219	256.990
-40	104.95	155.414	388.609	0.82490	1.82504	0.70936	205.745
-35	131.68	160.742	390.896	0.84743	1.81380	0.71680	166.400
-30	163.48	166.140	393.138	0.86976	1.80329	0.72452	135.844
-28	177.76	168.318	394.021	0.87864	1.79927	0.72769	125.563
-26	192.99	170.507	394.896	0.88748	1.79535	0.73092	116.214
-24	209.22	172.708	395.762	0.89630	1.79152	0.73420	107.701
-22	226.48	174.919	396.619	0.90509	1.78779	0.73753	99.9362
-20	244.83	177.142	397.467	0.91386	1.78415	0.74091	92.8432
-18	264.29	179.376	398.305	0.92259	1.78059	0.74436	86.3546
-16	284.93	181.622	399.133	0.93129	1.77711	0.74786	80.4103
-14	306.78	183.878	399.951	0.93997	1.77371	0.75143	74.9572
-12	329.89	186.147	400.759	0.94862	1.77039	0.75506	69.9478
-10	354.30	188.426	401.555	0.95725	1.76713	0.75876	65.3399
-9	367.01	189.571	401.949	0.96155	1.76553	0.76063	63.1746
-8	380.06	190.718	402.341	0.96585	1.76394	0.76253	61.0958
-7	393.47	191.868	402.729	0.97014	1.76237	0.76444	59.0996
-6	407.23	193.021	403.114	0.97442	1.76082	0.76636	57.1820
-5	421.35	194.176	403.496	0.97870	1.75928	0.76831	55.3394
33	1287.8	240.520	415.207	1.13768	1.70826	0.86101	18.2135
34	1321.0	241.814	415.420	1.14181	1.70701	0.86412	17.7341
35	1354.8	243.114	415.627	1.14594	1.70576	0.86729	17.2686
36	1389.2	244.418	415.828	1.15007	1.70450	0.87051	16.8168
37	1424.3	245.727	416.021	1.15420	1.70325	0.87378	16.3779
38	1460.1	247.041	416.208	1.15833	1.70199	0.87710	15.9517
39	1496.5	248.361	416.388	1.16246	1.70073	0.88048	15.5375

Refrigerant 22: properties of superheated vapor

Temp: t
 L/kg h s
 kJ/kg $\text{kJ/kg} \cdot \text{K}$

Temp	Saturation temperature, 32°C			Saturation temperature, 34°C			Saturation temperature, 36°C		
	v	h	s	v	h	s	v	h	s
35	19.0907	417.648	1.7182	17.8590	416.325	1.7099			
40	19.7093	422.014	1.7322	18.4675	420.792	1.7243			
45	20.3062	426.310	1.7458	19.0526	425.174	1.7382	17.2953	419.483	1.7162
50	20.8847	430.549	1.7591	19.6178	429.487	1.7517	17.8708	423.961	1.7304
55	21.4471	434.743	1.7719	20.1660	433.747	1.7647	18.4247	428.358	1.7442
60	21.9956	438.900	1.7845	20.6994	437.963	1.7775	18.9603	432.690	1.7575
65	22.5318	443.028	1.7968	21.2199	442.143	1.7899	19.4802	436.970	1.7704
70	23.0571	447.133	1.8089	21.7289	446.294	1.8021	19.9865	441.207	1.7830
75	23.5726	451.219	1.8207	22.2278	450.424	1.8141	20.4807	445.410	1.7954
80	24.0794	455.292	1.8323	22.7176	454.535	1.8258	20.9643	449.586	1.8074
							21.4385	453.739	1.8193



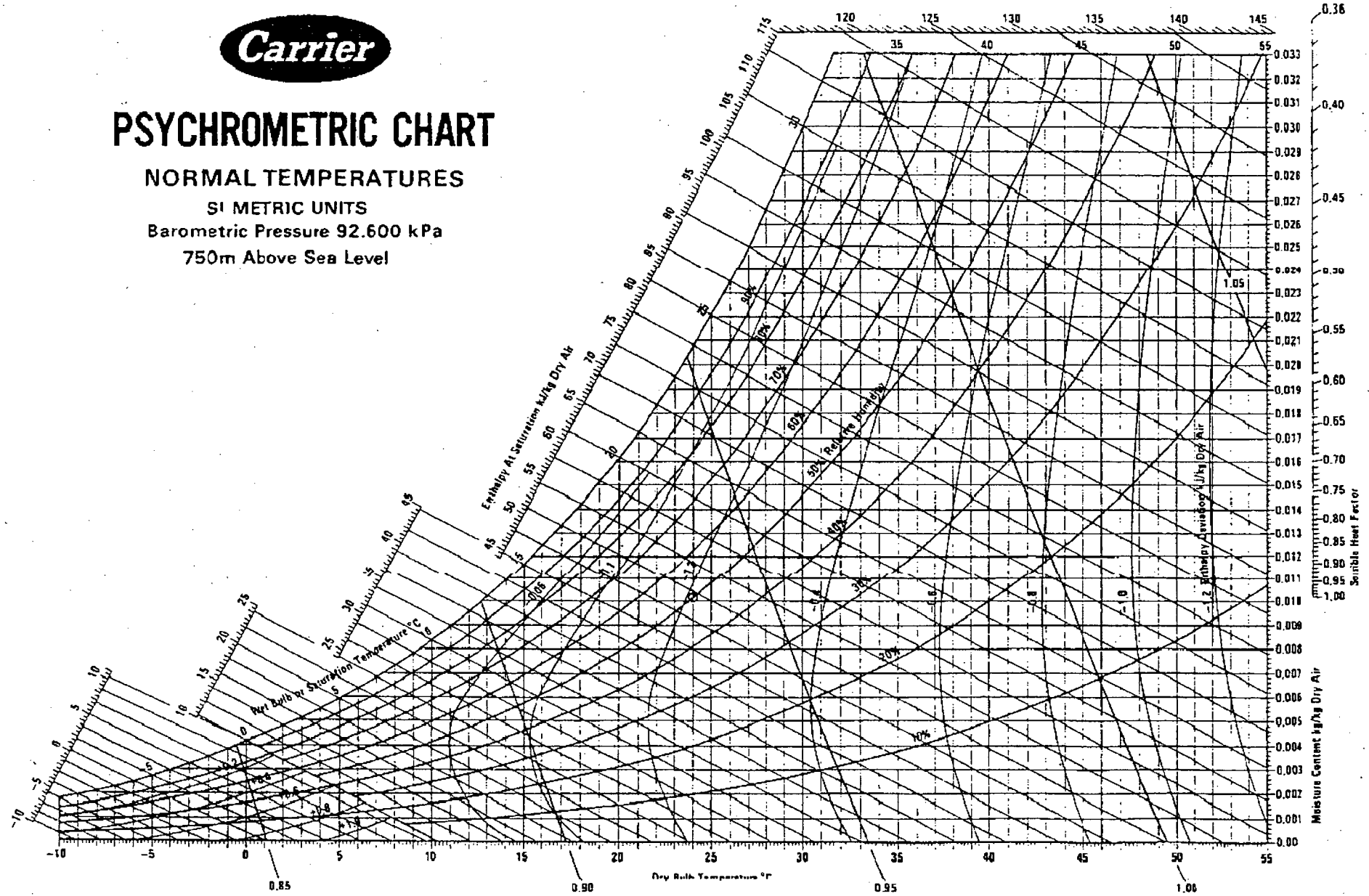
PSYCHROMETRIC CHART

NORMAL TEMPERATURES

SI METRIC UNITS

Barometric Pressure 92.600 kPa

750m Above Sea Level



Below 0°C Properties and Enthalpy Deviation Lines Are for Ice

Copyright ©Carrier Corporation 1975
Cat. No. 794-006 Printed in U.S.A.

SECTION – A

There are **FOUR** questions in this Section. Answer any **THREE**.

1. (a) The following table gives the total unit produced at the beginning of different years.

Represent the data graphically and estimate the mid-year value for 1997 and 2001. (10)

Years:	1995	1996	1997	1998	1999	2000	2001	2002	2003
Unit produced:	20	62	147	300	536	811	1104	1425	1755

- (b) For a distribution, Bowley's coefficient of skewness is -0.48 , third quartile is 10.2 and median is 14.4. What is the quartile coefficient of distribution? (10)

- (c) The shareholder Research Bureau of Bangladesh conducted recently a research study on the price (Tk.) behavior of three leading industrial shares A, B, C for the period 2006 to 2009, the result of which are published as following in the quarterly journal:

Share:	Average Price	Standard Deviation	Current selling price
A:	1800	5.40	36.00
B:	2250	4.50	34.75
C:	2400	6.00	39.00

Which share in your opinion appears to be more stable in value? (15)

2. (a) State Baye's theorem. The probability that an automobile being filled with gasoline will also need an oil change is 0.25; the probability that it needs a new oil filter is 0.40; and the probability that both the oil and filter need changing is 0.14. (18)

(i) If the oil had to be changed, what is the probability that a new oil filter is needed?

(ii) If a new oil filter is needed, what is the probability that the oil has to be changed?

- (b) Pairs of pants are being produced by a particular outlet facility. The pants are "checked" by a group of 10 workers. The workers inspect pairs of pants taken randomly from the production line. Each inspector is assigned a number from 1 through 10. A buyer selects a pair of pants for purchase. Let the random variable X be the inspector number. (17)

(i) Give a reasonable probability mass function for X .

(ii) Plot the cumulative distribution function for X .

MATH 357(EEE)

3. (a) An important system acts in support of a vehicle in our space program. A single crucial component works only 85% of the time. In order to enhance the reliability of the system, it is decided that 3 components will be installed in parallel such that the system fails only if they all fail. Assume the components act independently and that they are equivalent in the sense that all 3 of them have an 85% success rate. Consider the random variable X as the number of components out of 3 that fail. (20)
- (i) What is $E(X)$ (i.e., the mean number of components out of 3 that fail)?
 - (ii) What is $Var(X)$?
 - (iii) What is the probability that the system fails?
 - (v) If the desire is to have the system be successful with probability 0.99, are three components sufficient? If not, how many are required?
- (b) The surface of a circular dart board has a small center circle called the bull's-eye and 20 pie-shaped regions numbered from 1 to 20. Each of the pie-shaped regions is further divided into three parts such that a person throwing a dart that lands on a specified number scores the value of the number, double the number, or triple the number, depending on which of the three parts the dart falls. If a person hits the bull's-eye with probability 0.01 hits a double with probability 0.10, hits a triple with probability 0.05, and misses the dart, board with probability 0.02, what is the probability that 7 throws will result in no bull's-eyes, no triples, a double twice, and a complete miss once? (15)
4. (a) Find mean and variance of Gaussian distribution. (10)
- (b) The life, in years, of a certain type of electrical switch has an exponential distribution with an average life 2. If 100 of these switches are installed in different systems, what is the probability that at most 30 fail during the first year? (10)
- (c) Statistics released by the National Highway Traffic Safety Administration and the National Safety Council show that on an average weekend night, 1 out of every 10 drivers on the road is drunk. If 400 driver are randomly checked next Saturday night, what is the probability that the number of drunk drivers will be (15)
- (i) less than 32?
 - (ii) more than 49?
 - (iii) At least 35 but less than 47?

MATH 357(EEE)

SECTION – B

There are **FOUR** questions in this Section. Answer any **THREE**.

5. (a) Select 30 random samples of 4 students each with replacement using the table given below applying random numbers from Appendix-I. (10)

Diameter (mm)	Frequency
7.247-7.249	2
7.250-7.252	6
7.253-7.255	8
7.256-7.258	15
7.279-7.261	42
7.262-7.264	68
7.265-7.267	49
7.268-7.270	25
7.271-7.273	18
7.274-7.276	12
7.277-7.279	4
7.280-7.282	1
Total	250

- (b) Compute the mean and standard deviation of the sampling distribution of means in part (a). (20)
- (c) Compare the results of part (b) with the theoretical values explaining any discrepancies. (5)
6. (a) What is rank correlation? When do you need it? Derive a formula to calculate the rank correlation coefficient? Calculate the rank correlation coefficient (r_2) from the following data on hourly sales (x) and expenses (y) of the following stores: (15)

x	50	50	55	60	65	65	65	60	60	60
y	11	13	14	16	16	15	15	14	13	13

- (b) To study the tensile strength of a certain type of wire, the following pairs of observations were recorded, where x is the diameter in cm and y is the mass supported in kg/cm. (20)

x	0.6	0.8	1.0	1.2	1.4	1.6	1.8	2.0	2.2	2.4
y	14	26	50	56	42	98	82	88	134	124

- (i) Calculate the least-square regression line $y = b_0 + b_1x$ for above data.
- (ii) Find the predicted tensile strength for a wire with diameter 1.5 cm.
- (iii) Extrapolate the regression line to a diameter of 0.2 cm and explain your result.

MATH 357(EEE)

7. (a) A population consist of the four numbers 3, 7, 11 and 15. Consider all possible samples of size 2 that can be drawn with replacement from this population. Find (i) the population mean, (ii) the population standard deviation, (iii) the mean of the sampling distribution of means. Also find mean and standard deviation of the sampling distribution of means if sampling is without replacement. (20)

(b) A study was conducted to see if increasing the substrate concentration has an appreciable effect on the velocity of a chemical reaction. With a substrate concentration of 1.5 moles per liter, the reaction was run 15 times, with an average velocity of 7.5 micromoles per 30 minutes and a standard deviation of 1.5. With a substrate concentration of 2.0 moles per liter, 12 runs were made, yielding an average velocity of 8.8 micromoles per 30 minutes and a sample standard deviation of 1.2. Is there any reason to believe that this increase in substrate concentration causes an increase in the mean velocity of the reaction of more than 0.5 micromole per 30 minutes? Use a 0.01 level of significance and assume the populations to be approximately normally distributed with equal variances. (Necessary chart is attached.) (15)

8. (a) Five samples of a ferrous-type substance were used to determine if there is a difference between a laboratory chemical analysis and an X-ray fluorescence analysis of the iron content. Each sample was split into two subsamples and the two types of analysis were applied. Following are the coded data showing the iron content analysis: (15)

	Sample				
Analysis	1	2	3	4	5
X-ray	2.0	2.0	2.3	2.1	2.4
Chemical	2.2	1.9	2.5	2.3	2.4

Assuming that the populations are normal, test at the 0.05 level of significance whether the two methods of analysis give, on the average, the same result. (Necessary chart is attached.)

(b) An experiment was conducted to study the effects of temperature and type of oven on the life of a particular component. Four types of ovens and 3 temperature levels were used in the experiment. Twenty-four pieces were assigned randomly, two to each combination of treatments, and the following results recorded. (20)

Temperature (°F)	Oven			
	O ₁	O ₂	O ₃	O ₄
500	227	214	225	260
	221	259	236	229
550	187	181	232	246
	208	179	198	273
600	174	198	178	206
	202	194	213	219

MATH 357(EEE)

Contd ... Q. No. 8(b)

Using a 0.05 level of significance, test the hypothesis that

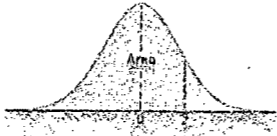
- (i) Different temperatures have no effect on the life of the component;
- (ii) Different ovens have no effect on the life of the component.

(Necessary chart is attached.)

Table A.3 (continued) Areas under the Normal Curve

z	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
0.0	0.5000	0.5040	0.5080	0.5120	0.5160	0.5199	0.5239	0.5279	0.5319	0.5359
0.1	0.5398	0.5138	0.5478	0.5517	0.5557	0.5596	0.5636	0.5675	0.5714	0.5753
0.2	0.5793	0.5832	0.5871	0.5910	0.5948	0.5987	0.6026	0.6064	0.6103	0.6141
0.3	0.6179	0.6217	0.6255	0.6293	0.6331	0.6368	0.6406	0.6443	0.6480	0.6517
0.4	0.6554	0.6591	0.6628	0.6664	0.6700	0.6736	0.6772	0.6808	0.6844	0.6879
0.5	0.6915	0.6950	0.6985	0.7019	0.7054	0.7088	0.7123	0.7157	0.7190	0.7224
0.6	0.7257	0.7291	0.7324	0.7357	0.7389	0.7422	0.7454	0.7486	0.7517	0.7549
0.7	0.7580	0.7611	0.7642	0.7673	0.7701	0.7734	0.7764	0.7794	0.7823	0.7852
0.8	0.7881	0.7910	0.7939	0.7967	0.7995	0.8023	0.8051	0.8078	0.8106	0.8133
0.9	0.8159	0.8186	0.8212	0.8238	0.8264	0.8289	0.8315	0.8340	0.8365	0.8389
1.0	0.8413	0.8438	0.8461	0.8485	0.8508	0.8531	0.8554	0.8577	0.8599	0.8621
1.1	0.8643	0.8665	0.8686	0.8708	0.8729	0.8749	0.8770	0.8790	0.8810	0.8830
1.2	0.8849	0.8869	0.8888	0.8907	0.8925	0.8944	0.8962	0.8980	0.8997	0.9015
1.3	0.9032	0.9049	0.9066	0.9082	0.9099	0.9115	0.9131	0.9147	0.9162	0.9177
1.4	0.9192	0.9207	0.9222	0.9236	0.9251	0.9265	0.9279	0.9292	0.9306	0.9319
1.5	0.9332	0.9345	0.9357	0.9370	0.9382	0.9394	0.9406	0.9418	0.9429	0.9441
1.6	0.9452	0.9463	0.9474	0.9484	0.9495	0.9505	0.9515	0.9525	0.9535	0.9545
1.7	0.9554	0.9564	0.9573	0.9582	0.9591	0.9599	0.9608	0.9616	0.9625	0.9633
1.8	0.9641	0.9649	0.9656	0.9664	0.9671	0.9678	0.9686	0.9693	0.9699	0.9706
1.9	0.9713	0.9719	0.9726	0.9732	0.9738	0.9744	0.9750	0.9756	0.9761	0.9767
2.0	0.9772	0.9778	0.9783	0.9788	0.9793	0.9798	0.9803	0.9808	0.9812	0.9817
2.1	0.9821	0.9826	0.9830	0.9834	0.9838	0.9842	0.9846	0.9850	0.9854	0.9857
2.2	0.9861	0.9864	0.9868	0.9871	0.9875	0.9878	0.9881	0.9884	0.9887	0.9890
2.3	0.9893	0.9896	0.9898	0.9901	0.9904	0.9906	0.9909	0.9911	0.9913	0.9916
2.4	0.9918	0.9920	0.9922	0.9925	0.9927	0.9929	0.9931	0.9932	0.9934	0.9936
2.5	0.9938	0.9940	0.9941	0.9943	0.9945	0.9946	0.9948	0.9949	0.9951	0.9952
2.6	0.9953	0.9955	0.9956	0.9957	0.9959	0.9960	0.9961	0.9962	0.9963	0.9964
2.7	0.9965	0.9966	0.9967	0.9968	0.9969	0.9970	0.9971	0.9972	0.9973	0.9974
2.8	0.9974	0.9975	0.9976	0.9977	0.9977	0.9978	0.9979	0.9979	0.9980	0.9981
2.9	0.9981	0.9982	0.9982	0.9983	0.9984	0.9984	0.9985	0.9985	0.9986	0.9986
3.0	0.9987	0.9987	0.9987	0.9988	0.9988	0.9989	0.9989	0.9989	0.9990	0.9990
3.1	0.9990	0.9991	0.9991	0.9991	0.9992	0.9992	0.9992	0.9992	0.9993	0.9993
3.2	0.9993	0.9993	0.9994	0.9994	0.9994	0.9994	0.9994	0.9995	0.9995	0.9995
3.3	0.9995	0.9995	0.9995	0.9996	0.9996	0.9996	0.9996	0.9996	0.9996	0.9997
3.4	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9998

Chart for question no. 4(c)



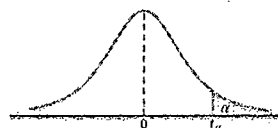
Areas under the Normal Curve

z	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
-3.4	0.0003	0.0003	0.0003	0.0003	0.0003	0.0003	0.0003	0.0003	0.0003	0.0002
-3.3	0.0005	0.0005	0.0005	0.0004	0.0004	0.0004	0.0004	0.0004	0.0004	0.0003
-3.2	0.0007	0.0007	0.0006	0.0006	0.0006	0.0006	0.0006	0.0005	0.0005	0.0005
-3.1	0.0010	0.0009	0.0009	0.0009	0.0008	0.0008	0.0008	0.0008	0.0007	0.0007
-3.0	0.0013	0.0013	0.0013	0.0012	0.0012	0.0011	0.0011	0.0011	0.0010	0.0010
-2.9	0.0019	0.0018	0.0018	0.0017	0.0016	0.0016	0.0015	0.0015	0.0014	0.0014
-2.8	0.0026	0.0025	0.0024	0.0023	0.0023	0.0022	0.0021	0.0021	0.0020	0.0019
-2.7	0.0035	0.0034	0.0033	0.0032	0.0031	0.0030	0.0029	0.0028	0.0027	0.0026
-2.6	0.0047	0.0045	0.0044	0.0043	0.0041	0.0040	0.0039	0.0038	0.0037	0.0036
-2.5	0.0062	0.0060	0.0059	0.0057	0.0055	0.0054	0.0052	0.0051	0.0049	0.0048
-2.4	0.0082	0.0080	0.0078	0.0075	0.0073	0.0071	0.0069	0.0068	0.0066	0.0064
-2.3	0.0107	0.0104	0.0102	0.0099	0.0096	0.0094	0.0091	0.0089	0.0087	0.0084
-2.2	0.0139	0.0136	0.0132	0.0129	0.0125	0.0122	0.0119	0.0116	0.0113	0.0110
-2.1	0.0179	0.0174	0.0170	0.0166	0.0162	0.0158	0.0154	0.0150	0.0146	0.0143
-2.0	0.0228	0.0222	0.0217	0.0212	0.0207	0.0202	0.0197	0.0192	0.0188	0.0183
-1.9	0.0287	0.0281	0.0274	0.0268	0.0262	0.0256	0.0250	0.0244	0.0239	0.0233
-1.8	0.0359	0.0351	0.0344	0.0336	0.0329	0.0322	0.0314	0.0307	0.0301	0.0294
-1.7	0.0446	0.0436	0.0427	0.0418	0.0409	0.0401	0.0392	0.0384	0.0375	0.0367
-1.6	0.0548	0.0537	0.0526	0.0516	0.0505	0.0495	0.0485	0.0475	0.0465	0.0455
-1.5	0.0668	0.0655	0.0643	0.0630	0.0618	0.0606	0.0594	0.0582	0.0571	0.0559
-1.4	0.0808	0.0793	0.0778	0.0764	0.0749	0.0735	0.0721	0.0708	0.0694	0.0681
-1.3	0.0968	0.0951	0.0934	0.0918	0.0901	0.0885	0.0869	0.0853	0.0838	0.0823
-1.2	0.1151	0.1131	0.1112	0.1093	0.1075	0.1056	0.1038	0.1020	0.1003	0.0985
-1.1	0.1357	0.1335	0.1314	0.1292	0.1271	0.1251	0.1230	0.1210	0.1190	0.1170
-1.0	0.1587	0.1562	0.1539	0.1515	0.1492	0.1469	0.1446	0.1423	0.1401	0.1379
-0.9	0.1841	0.1814	0.1788	0.1762	0.1736	0.1711	0.1685	0.1660	0.1635	0.1611
-0.8	0.2119	0.2090	0.2061	0.2033	0.2005	0.1977	0.1949	0.1922	0.1894	0.1867
-0.7	0.2420	0.2389	0.2358	0.2327	0.2296	0.2266	0.2236	0.2206	0.2177	0.2148
-0.6	0.2743	0.2709	0.2676	0.2643	0.2611	0.2578	0.2546	0.2514	0.2483	0.2451
-0.5	0.3085	0.3050	0.3015	0.2981	0.2946	0.2912	0.2877	0.2843	0.2810	0.2776
-0.4	0.3446	0.3409	0.3372	0.3336	0.3300	0.3264	0.3228	0.3192	0.3156	0.3121
-0.3	0.3821	0.3783	0.3745	0.3707	0.3669	0.3632	0.3594	0.3557	0.3520	0.3483
	0.4207	0.4168	0.4129	0.4090	0.4052	0.4013	0.3974	0.3936	0.3897	0.3859
	0.4602	0.4562	0.4522	0.4483	0.4443	0.4404	0.4364	0.4325	0.4286	0.4247
	0.5000	0.4960	0.4920	0.4880	0.4840	0.4801	0.4761	0.4721	0.4681	0.4641

(continued) Areas under the Normal Curve

z	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
0.0	0.5000	0.5040	0.5080	0.5120	0.5160	0.5199	0.5239	0.5279	0.5319	0.5359
0.1	0.5398	0.5438	0.5478	0.5517	0.5557	0.5596	0.5636	0.5675	0.5714	0.5753
0.2	0.5793	0.5832	0.5871	0.5910	0.5948	0.5987	0.6026	0.6064	0.6103	0.6141
0.3	0.6179	0.6217	0.6255	0.6293	0.6331	0.6368	0.6406	0.6443	0.6480	0.6517
0.4	0.6554	0.6591	0.6628	0.6664	0.6700	0.6736	0.6772	0.6808	0.6844	0.6879
0.5	0.6915	0.6950	0.6985	0.7019	0.7054	0.7088	0.7123	0.7157	0.7190	0.7224
0.6	0.7257	0.7291	0.7324	0.7357	0.7389	0.7422	0.7454	0.7486	0.7517	0.7549
0.7	0.7580	0.7611	0.7642	0.7673	0.7704	0.7734	0.7764	0.7794	0.7823	0.7852
0.8	0.7881	0.7910	0.7939	0.7967	0.7995	0.8023	0.8051	0.8078	0.8106	0.8133
0.9	0.8159	0.8186	0.8212	0.8238	0.8264	0.8289	0.8315	0.8340	0.8365	0.8389
1.0	0.8413	0.8438	0.8461	0.8485	0.8508	0.8531	0.8554	0.8577	0.8599	0.8621
1.1	0.8643	0.8665	0.8686	0.8708	0.8729	0.8749	0.8770	0.8790	0.8810	0.8830
1.2	0.8849	0.8869	0.8888	0.8907	0.8925	0.8944	0.8962	0.8980	0.8997	0.9015
1.3	0.9032	0.9049	0.9066	0.9082	0.9099	0.9115	0.9131	0.9147	0.9162	0.9177
1.4	0.9192	0.9207	0.9222	0.9236	0.9251	0.9265	0.9279	0.9292	0.9306	0.9319
1.5	0.9332	0.9345	0.9357	0.9370	0.9382	0.9394	0.9406	0.9418	0.9429	0.9441
1.6	0.9452	0.9463	0.9474	0.9484	0.9495	0.9505	0.9515	0.9525	0.9535	0.9545
1.7	0.9554	0.9564	0.9573	0.9582	0.9591	0.9599	0.9608	0.9616	0.9625	0.9633
1.8	0.9641	0.9649	0.9656	0.9664	0.9671	0.9678	0.9686	0.9693	0.9699	0.9706
1.9	0.9713	0.9719	0.9726	0.9732	0.9738	0.9744	0.9750	0.9756	0.9761	0.9767
2.0	0.9772	0.9778	0.9783	0.9788	0.9793	0.9798	0.9803	0.9808	0.9812	0.9817
2.1	0.9821	0.9826	0.9830	0.9834	0.9838	0.9842	0.9846	0.9850	0.9854	0.9857
2.2	0.9861	0.9864	0.9868	0.9871	0.9875	0.9878	0.9881	0.9884	0.9887	0.9890
2.3	0.9893	0.9896	0.9898	0.9901	0.9904	0.9906	0.9909	0.9911	0.9913	0.9916
2.4	0.9918	0.9920	0.9922	0.9925	0.9927	0.9929	0.9931	0.9932	0.9934	0.9936
2.5	0.9938	0.9940	0.9941	0.9943	0.9945	0.9946	0.9948	0.9949	0.9951	0.9952
2.6	0.9953	0.9955	0.9956	0.9957	0.9959	0.9960	0.9961	0.9962	0.9963	0.9964
2.7	0.9965	0.9966	0.9967	0.9968	0.9969	0.9970	0.9971	0.9972	0.9973	0.9974
2.8	0.9974	0.9975	0.9976	0.9977	0.9977	0.9978	0.9979	0.9979	0.9980	0.9981
2.9	0.9981	0.9982	0.9982	0.9983	0.9984	0.9984	0.9985	0.9985	0.9986	0.9986
3.0	0.9987	0.9987	0.9987	0.9988	0.9988	0.9989	0.9989	0.9989	0.9990	0.9990
3.1	0.9990	0.9991	0.9991	0.9991	0.9992	0.9992	0.9992	0.9992	0.9993	0.9993
3.2	0.9993	0.9993	0.9994	0.9994	0.9994	0.9994	0.9994	0.9995	0.9995	0.9995
3.3	0.9995	0.9995	0.9995	0.9996	0.9996	0.9996	0.9996	0.9996	0.9996	0.9997
3.4	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9998

7

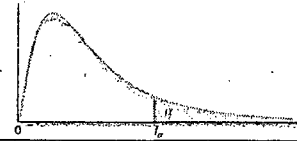


Critical Values of the *t*-Distribution

<i>v</i>	α						
	0.40	0.30	0.20	0.15	0.10	0.05	0.025
1	0.325	0.727	1.376	1.963	3.078	6.314	12.706
2	0.289	0.617	1.061	1.386	1.886	2.920	4.303
3	0.277	0.584	0.978	1.250	1.638	2.353	3.182
4	0.271	0.569	0.941	1.190	1.533	2.132	2.776
5	0.267	0.559	0.920	1.156	1.476	2.015	2.571
6	0.265	0.553	0.906	1.134	1.440	1.943	2.447
7	0.263	0.549	0.896	1.119	1.415	1.895	2.365
8	0.262	0.546	0.889	1.108	1.397	1.860	2.306
9	0.261	0.543	0.883	1.100	1.383	1.833	2.262
10	0.260	0.542	0.879	1.093	1.372	1.812	2.228
11	0.260	0.540	0.876	1.088	1.363	1.796	2.201
12	0.259	0.539	0.873	1.083	1.356	1.782	2.179
13	0.259	0.538	0.870	1.079	1.350	1.771	2.160
14	0.258	0.537	0.868	1.076	1.345	1.761	2.145
15	0.258	0.536	0.866	1.074	1.341	1.753	2.131
16	0.258	0.535	0.865	1.071	1.337	1.746	2.120
17	0.257	0.534	0.863	1.069	1.333	1.740	2.110
18	0.257	0.534	0.862	1.067	1.330	1.734	2.101
19	0.257	0.533	0.861	1.066	1.328	1.729	2.093
20	0.257	0.533	0.860	1.064	1.325	1.725	2.086
21	0.257	0.532	0.859	1.063	1.323	1.721	2.080
22	0.256	0.532	0.858	1.061	1.321	1.717	2.074
23	0.256	0.532	0.858	1.060	1.319	1.714	2.069
24	0.256	0.531	0.857	1.059	1.318	1.711	2.064
25	0.256	0.531	0.856	1.058	1.316	1.708	2.060
26	0.256	0.531	0.856	1.058	1.315	1.706	2.056
27	0.256	0.531	0.855	1.057	1.314	1.703	2.052
28	0.256	0.530	0.855	1.056	1.313	1.701	2.048
29	0.256	0.530	0.854	1.055	1.311	1.699	2.045
30	0.256	0.530	0.854	1.055	1.310	1.697	2.042
	0.255	0.529	0.851	1.050	1.303	1.684	2.021
	0.254	0.527	0.848	1.045	1.296	1.671	2.000
	0.254	0.526	0.845	1.041	1.289	1.658	1.980
	0.253	0.524	0.842	1.036	1.282	1.645	1.960

(continued) Critical Values of the *t*-Distribution

<i>v</i>	α						
	0.02	0.015	0.01	0.0075	0.005	0.0025	0.0005
1	15.894	21.205	31.821	42.433	63.656	127.321	636.578
2	4.849	5.643	6.965	8.073	9.925	14.089	31.600
3	3.482	3.896	4.541	5.047	5.841	7.453	12.924
4	2.999	3.298	3.747	4.088	4.604	5.598	8.610
5	2.757	3.003	3.365	3.634	4.032	4.773	6.869
6	2.612	2.829	3.143	3.372	3.707	4.317	5.959
7	2.517	2.715	2.998	3.203	3.499	4.029	5.408
8	2.449	2.634	2.896	3.085	3.355	3.833	5.041
9	2.398	2.574	2.821	2.998	3.250	3.690	4.781
10	2.359	2.527	2.764	2.932	3.169	3.581	4.587
11	2.328	2.491	2.718	2.879	3.106	3.497	4.437
12	2.303	2.461	2.681	2.836	3.055	3.428	4.318
13	2.282	2.436	2.650	2.801	3.012	3.372	4.221
14	2.264	2.415	2.624	2.771	2.977	3.326	4.140
15	2.249	2.397	2.602	2.746	2.947	3.286	4.073
16	2.235	2.382	2.583	2.724	2.921	3.252	4.015
17	2.224	2.368	2.567	2.706	2.898	3.222	3.965
18	2.214	2.356	2.552	2.689	2.878	3.197	3.922
19	2.205	2.346	2.539	2.674	2.861	3.174	3.883
20	2.197	2.336	2.528	2.661	2.845	3.153	3.850
21	2.189	2.328	2.518	2.649	2.831	3.135	3.819
22	2.183	2.320	2.508	2.639	2.819	3.119	3.792
23	2.177	2.313	2.500	2.629	2.807	3.104	3.768
24	2.172	2.307	2.492	2.620	2.797	3.091	3.745
25	2.167	2.301	2.485	2.612	2.787	3.078	3.725
26	2.162	2.296	2.479	2.605	2.779	3.067	3.707
27	2.158	2.291	2.473	2.598	2.771	3.057	3.689
28	2.154	2.286	2.467	2.592	2.763	3.047	3.674
29	2.150	2.282	2.462	2.586	2.756	3.038	3.660
30	2.147	2.278	2.457	2.581	2.750	3.030	3.646
40	2.123	2.250	2.423	2.542	2.704	2.971	3.551
60	2.099	2.223	2.390	2.504	2.660	2.915	3.460
120	2.076	2.196	2.358	2.468	2.617	2.860	3.373
∞	2.054	2.170	2.326	2.432	2.576	2.807	3.290



Critical Values of the F-Distribution

		$f_{0.05}(v_1, v_2)$								
		v_1								
v_2	1	2	3	4	5	6	7	8	9	
1	161.45	199.50	215.71	224.58	230.16	233.99	236.77	238.88	240.54	
2	18.51	19.00	19.16	19.25	19.30	19.33	19.35	19.37	19.38	
3	10.13	9.55	9.28	9.12	9.01	8.94	8.89	8.85	8.81	
4	7.71	6.94	6.59	6.39	6.26	6.16	6.09	6.04	6.00	
5	6.61	5.79	5.41	5.19	5.05	4.95	4.88	4.82	4.77	
6	5.99	5.14	4.76	4.53	4.39	4.28	4.21	4.15	4.10	
7	5.59	4.74	4.35	4.12	3.97	3.87	3.79	3.73	3.68	
8	5.32	4.46	4.07	3.84	3.69	3.58	3.50	3.44	3.39	
9	5.12	4.26	3.86	3.63	3.48	3.37	3.29	3.23	3.18	
10	4.96	4.10	3.71	3.48	3.33	3.22	3.14	3.07	3.02	
11	4.84	3.98	3.59	3.36	3.20	3.09	3.01	2.95	2.90	
12	4.75	3.89	3.49	3.26	3.11	3.00	2.91	2.85	2.80	
13	4.67	3.81	3.41	3.18	3.03	2.92	2.83	2.77	2.71	
14	4.60	3.74	3.34	3.11	2.96	2.85	2.76	2.70	2.65	
15	4.54	3.68	3.29	3.06	2.90	2.79	2.71	2.64	2.59	
16	4.49	3.63	3.24	3.01	2.85	2.74	2.66	2.59	2.54	
17	4.45	3.59	3.20	2.96	2.81	2.70	2.61	2.55	2.49	
18	4.41	3.55	3.16	2.93	2.77	2.66	2.58	2.51	2.46	
19	4.38	3.52	3.13	2.90	2.74	2.63	2.54	2.48	2.42	
20	4.35	3.49	3.10	2.87	2.71	2.60	2.51	2.45	2.39	
21	4.32	3.47	3.07	2.84	2.68	2.57	2.49	2.42	2.37	
22	4.30	3.44	3.05	2.82	2.66	2.55	2.46	2.40	2.34	
23	4.28	3.42	3.03	2.80	2.64	2.53	2.44	2.37	2.32	
24	4.26	3.40	3.01	2.78	2.62	2.51	2.42	2.36	2.30	
25	4.24	3.39	2.99	2.76	2.60	2.49	2.40	2.34	2.28	
26	4.23	3.37	2.98	2.74	2.59	2.47	2.39	2.32	2.27	
27	4.21	3.35	2.96	2.73	2.57	2.46	2.37	2.31	2.25	
28	4.20	3.34	2.95	2.71	2.56	2.45	2.36	2.29	2.24	
29	4.18	3.33	2.93	2.70	2.55	2.43	2.35	2.28	2.22	
30	4.17	3.32	2.92	2.69	2.53	2.42	2.33	2.27	2.21	
40	4.08	3.23	2.84	2.61	2.45	2.34	2.25	2.18	2.12	
60	4.00	3.15	2.76	2.53	2.37	2.25	2.17	2.10	2.04	
120	3.92	3.07	2.68	2.45	2.29	2.18	2.09	2.02	1.96	
∞	3.84	3.00	2.60	2.37	2.21	2.10	2.01	1.94	1.88	

(continued) Critical Values of the F-Distribution

		$f_{0.05}(v_1, v_2)$									
		v_1									
v_2	10	12	15	20	24	30	40	60	120	∞	
1	241.88	243.91	245.95	248.01	249.05	250.10	251.14	252.20	253.25	254.31	
2	19.40	19.41	19.43	19.45	19.45	19.46	19.47	19.48	19.49	19.50	
3	8.79	8.74	8.70	8.66	8.64	8.62	8.59	8.57	8.55	8.53	
4	5.96	5.91	5.86	5.80	5.77	5.75	5.72	5.69	5.66	5.63	
5	4.74	4.68	4.62	4.56	4.53	4.50	4.46	4.43	4.40	4.36	
6	4.06	4.00	3.94	3.87	3.84	3.81	3.77	3.74	3.70	3.67	
7	3.64	3.57	3.51	3.44	3.41	3.38	3.34	3.30	3.27	3.23	
8	3.35	3.28	3.22	3.15	3.12	3.08	3.04	3.01	2.97	2.93	
9	3.14	3.07	3.01	2.94	2.90	2.86	2.83	2.79	2.75	2.71	
10	2.98	2.91	2.85	2.77	2.74	2.70	2.66	2.62	2.58	2.54	
11	2.85	2.79	2.72	2.65	2.61	2.57	2.53	2.49	2.45	2.40	
12	2.75	2.69	2.62	2.54	2.51	2.47	2.43	2.38	2.34	2.30	
13	2.67	2.60	2.53	2.46	2.42	2.38	2.34	2.30	2.25	2.21	
14	2.60	2.53	2.46	2.39	2.35	2.31	2.27	2.22	2.18	2.13	
15	2.54	2.48	2.40	2.33	2.29	2.25	2.20	2.16	2.11	2.07	
16	2.49	2.42	2.35	2.28	2.24	2.19	2.15	2.11	2.06	2.01	
17	2.45	2.38	2.31	2.23	2.19	2.15	2.10	2.06	2.01	1.96	
18	2.41	2.34	2.27	2.19	2.15	2.11	2.06	2.02	1.97	1.92	
19	2.38	2.31	2.23	2.16	2.11	2.07	2.03	1.98	1.93	1.88	
20	2.35	2.28	2.20	2.12	2.08	2.04	1.99	1.95	1.90	1.84	
21	2.32	2.25	2.18	2.10	2.05	2.01	1.96	1.92	1.87	1.81	
22	2.30	2.23	2.15	2.07	2.03	1.98	1.94	1.89	1.84	1.78	
23	2.27	2.20	2.13	2.05	2.01	1.96	1.91	1.86	1.81	1.76	
24	2.25	2.18	2.11	2.03	1.98	1.94	1.89	1.84	1.79	1.73	
25	2.24	2.16	2.09	2.01	1.96	1.92	1.87	1.82	1.77	1.71	
26	2.22	2.15	2.07	1.99	1.95	1.90	1.85	1.80	1.75	1.69	
27	2.20	2.13	2.06	1.97	1.93	1.88	1.84	1.79	1.73	1.67	
28	2.19	2.12	2.04	1.96	1.91	1.87	1.82	1.77	1.71	1.65	
29	2.18	2.10	2.03	1.94	1.90	1.85	1.81	1.75	1.70	1.64	
30	2.16	2.09	2.01	1.93	1.89	1.84	1.79	1.74	1.68	1.62	
40	2.08	2.00	1.92	1.84	1.79	1.74	1.69	1.64	1.58	1.51	
60	1.99	1.92	1.84	1.75	1.70	1.65	1.59	1.53	1.47	1.39	
120	1.91	1.83	1.75	1.66	1.61	1.55	1.50	1.43	1.35	1.25	
∞	1.83	1.75	1.67	1.57	1.52	1.46	1.39	1.32	1.22	1.00	

Reproduced from Table 18 of *Biometrika Tables for Statisticians*, Vol. I, by permission of E.S. Pearson and the Biometrika Trustees.

(continued) Critical Values of the t-Distribution

		$f_{0.01}(v_1, v_2)$								
		v_1								
v_2	1	2	3	4	5	6	7	8	9	
1	4052.18	4999.50	5403.35	5624.58	5763.65	5858.99	5928.36	5981.07	6022.47	
2	98.50	99.00	99.17	99.25	99.30	99.33	99.36	99.37	99.39	
3	34.12	30.82	29.46	28.71	28.24	27.91	27.67	27.49	27.35	
4	21.20	18.00	16.69	15.98	15.52	15.21	14.98	14.80	14.66	
5	16.26	13.27	12.06	11.39	10.97	10.67	10.46	10.29	10.16	
6	13.75	10.92	9.78	9.15	8.75	8.47	8.26	8.10	7.98	
7	12.25	9.55	8.45	7.85	7.46	7.19	6.99	6.84	6.72	
8	11.26	8.65	7.59	7.01	6.63	6.37	6.18	6.03	5.91	
9	10.56	8.02	6.99	6.42	6.06	5.80	5.61	5.47	5.35	
10	10.04	7.56	6.55	5.99	5.64	5.39	5.20	5.06	4.94	
11	9.65	7.21	6.22	5.67	5.32	5.07	4.89	4.74	4.63	
12	9.33	6.93	5.95	5.41	5.06	4.82	4.64	4.50	4.39	
13	9.07	6.70	5.74	5.21	4.86	4.62	4.44	4.30	4.19	
14	8.86	6.51	5.56	5.04	4.69	4.46	4.28	4.14	4.03	
15	8.68	6.36	5.42	4.89	4.56	4.32	4.14	4.00	3.89	
16	8.53	6.23	5.29	4.77	4.44	4.20	4.03	3.89	3.78	
17	8.40	6.11	5.18	4.67	4.34	4.10	3.93	3.79	3.68	
18	8.29	6.01	5.09	4.58	4.25	4.01	3.84	3.71	3.60	
19	8.18	5.93	5.01	4.50	4.17	3.94	3.77	3.63	3.52	
20	8.10	5.85	4.94	4.43	4.10	3.87	3.70	3.56	3.46	
21	8.02	5.78	4.87	4.37	4.04	3.81	3.64	3.51	3.40	
22	7.95	5.72	4.82	4.31	3.99	3.76	3.59	3.45	3.35	
23	7.88	5.66	4.76	4.26	3.94	3.71	3.54	3.41	3.30	
24	7.82	5.61	4.72	4.22	3.90	3.67	3.50	3.36	3.26	
25	7.77	5.57	4.68	4.18	3.85	3.63	3.46	3.32	3.22	
26	7.72	5.53	4.64	4.14	3.82	3.59	3.42	3.29	3.18	
27	7.68	5.49	4.60	4.11	3.78	3.56	3.39	3.26	3.15	
28	7.64	5.45	4.57	4.07	3.75	3.53	3.36	3.23	3.12	
29	7.60	5.42	4.54	4.04	3.73	3.50	3.33	3.20	3.09	
30	7.56	5.39	4.51	4.02	3.70	3.47	3.30	3.17	3.07	
40	7.31	5.18	4.31	3.83	3.51	3.29	3.12	2.99	2.89	
60	7.08	4.98	4.13	3.65	3.34	3.12	2.95	2.82	2.72	
120	6.85	4.79	3.95	3.48	3.17	2.96	2.79	2.66	2.56	
∞	6.63	4.61	3.78	3.32	3.02	2.80	2.64	2.51	2.41	

(continued) Critical Values of the F-Distribution

		$f_{0.01}(v_1, v_2)$									
		v_1									
v_2	10	12	15	20	24	30	40	60	120	∞	
1	6055.85	6106.32	6157.28	6208.73	6234.63	6260.65	6286.78	6313.03	6339.39	6365.86	
2	99.40	99.42	99.43	99.45	99.46	99.47	99.47	99.48	99.49	99.50	
3	27.23	27.05	26.87	26.69	26.60	26.50	26.41	26.32	26.22	26.13	
4	14.55	14.37	14.20	14.02	13.93	13.84	13.75	13.65	13.56	13.46	
5	10.05	9.89	9.72	9.55	9.47	9.38	9.29	9.20	9.11	9.02	
6	7.87	7.72	7.56	7.40	7.31	7.23	7.14	7.06	6.97	6.88	
7	6.62	6.47	6.31	6.16	6.07	5.99	5.91	5.82	5.74	5.65	
8	5.81	5.67	5.52	5.36	5.28	5.20	5.12	5.03	4.95	4.86	
9	5.26	5.11	4.96	4.81	4.73	4.65	4.57	4.48	4.40	4.31	
10	4.85	4.71	4.56	4.41	4.33	4.25	4.17	4.08	4.00	3.91	
11	4.54	4.40	4.25	4.10	4.02	3.94	3.86	3.78	3.69	3.60	
12	4.30	4.16	4.01	3.86	3.78	3.70	3.62	3.54	3.45	3.36	
13	4.10	3.96	3.82	3.66	3.59	3.51	3.43	3.34	3.25	3.17	
14	3.94	3.80	3.66	3.51	3.43	3.35	3.27	3.18	3.09	3.00	
15	3.80	3.67	3.52	3.37	3.29	3.21	3.13	3.05	2.96	2.87	
16	3.69	3.55	3.41	3.26	3.18	3.10	3.02	2.93	2.84	2.75	
17	3.59	3.46	3.31	3.16	3.08	3.00	2.92	2.83	2.75	2.65	
18	3.51	3.37	3.23	3.08	3.00	2.92	2.84	2.75	2.66	2.57	
19	3.43	3.30	3.15	3.00	2.92	2.84	2.76	2.67	2.58	2.49	
20	3.37	3.23	3.09	2.94	2.86	2.78	2.69	2.61	2.52	2.42	
21	3.31	3.17	3.03	2.88	2.80	2.72	2.64	2.55	2.46	2.36	
22	3.26	3.12	2.98	2.83	2.75	2.67	2.58	2.50	2.40	2.31	
23	3.21	3.07	2.93	2.78	2.70	2.62	2.54	2.45	2.35	2.26	
24	3.17	3.03	2.89	2.74	2.66	2.58	2.49	2.40	2.31	2.21	
25	3.13	2.99	2.85	2.70	2.62	2.54	2.45	2.36	2.27	2.17	
26	3.09	2.96	2.81	2.66	2.58	2.50	2.42	2.33	2.23	2.13	
27	3.06	2.93	2.78	2.63	2.55	2.47	2.38	2.29	2.20	2.10	
28	3.03	2.90	2.75	2.60	2.52	2.44	2.35	2.26	2.17	2.06	
29	3.00	2.87	2.73	2.57	2.49	2.41	2.33	2.23	2.14	2.03	
30	2.98	2.84	2.70	2.55	2.47	2.39	2.30	2.21	2.11	2.01	
40	2.80	2.66	2.52	2.37	2.29	2.20	2.11	2.02	1.92	1.80	
60	2.63	2.50	2.35	2.20	2.12	2.03	1.94	1.84	1.73	1.60	
120	2.47	2.34	2.19	2.03	1.95	1.86	1.76	1.66	1.53	1.38	
∞	2.32	2.18	2.04	1.88	1.79	1.70	1.59	1.47	1.32	1.00	

10

Appendix-I

Random Numbers

51772	74640	42331	29044	46621	62898	93582	04186	19040	87050
24033	23491	83587	06568	21960	21387	76105	10863	97453	80351
45939	60173	52078	25424	11645	55870	56974	37428	93507	94271
30586	02133	75797	45406	31041	86707	12973	17169	88116	42187
03585	79353	81938	82322	96799	85659	36081	50884	14070	74950
64937	03355	95863	20790	65304	55189	00745	65253	11822	15804
15630	64759	51135	98527	62586	41889	25439	88036	24034	67283
09448	56301	57683	30277	94623	85418	68829	06652	41982	49159
21631	91157	77331	60710	52290	16835	48653	71590	16159	14676
91097	17480	29414	06829	87843	28195	27279	47152	35683	47280
50532	25496	95652	42457	73547	76552	50020	24819	52984	76168
07136	40876	79971	54195	25708	51817	36732	72484	94923	75936
27989	64728	10744	08396	56242	90985	28868	99431	50995	20507
85184	73949	36601	46253	00477	25234	09908	36574	72139	70185
54398	21154	97810	36764	32869	11785	55261	59009	38714	38723
65544	34371	09591	07839	58892	92843	72828	91341	84821	63886
08263	65952	85762	64236	39238	18776	84303	99247	46149	03229
39817	67906	48236	16057	81812	15815	63700	85915	19219	45943
62257	04077	79443	95203	02479	30763	92486	54083	23631	05825
53298	90276	62545	21944	16530	03878	07516	95715	02526	33537

BANGLADESH UNIVERSITY OF ENGINEERING AND TECHNOLOGY, DHAKA

L-2/T-2 B. Sc. Engineering Examinations 2013-2014

Sub : **EEE 209** (Engineering Electromagnetics)

Full Marks : 210

Time : 3 Hours

The figures in the margin indicate full marks.

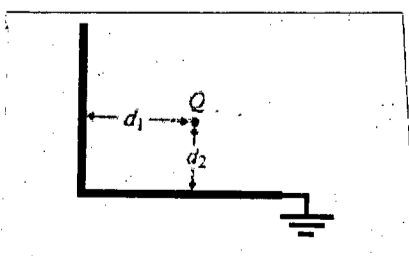
Symbols have their usual meaning.

USE SEPARATE SCRIPTS FOR EACH SECTION

SECTION – AThere are **FOUR** questions in this Section. Answer any **THREE**.

Answer in brief and to the point.

1. (a) Define electric field intensity, \vec{E} . Find \vec{E} at a distance R from a point charge q_1 located at the origin. Calculate the force \vec{F}_{12} exerted on q_2 by \vec{E} due to q_1 , when q_2 is placed at R . Explain the resulting equation. **(2+8=10)**
- (b) Write down the equation of V_{21} , which represents the difference in electric potential energy of a unit charge between point P_2 and point P_1 in an electric field, \vec{E} . Simplify the expression when P_1 is considered at infinity. Define an electric dipole. Plot the two-dimensional sketch of the equipotential and electric field lines of an electric dipole. **(4+6=10)**
- (c) A positive point charge Q is at the center of a spherical dielectric shell of an inner radius R_i and an outer radius R_o . Determine \vec{E} , V , \vec{D} and \vec{P} for $R_i < R < R_o$; assume the dielectric constant of the shell to be ϵ_r . What happens to them if the dielectric shell is replaced by a conducting shell? **(8+7=15)**
2. (a) Define capacitance of a capacitor. Does it depend on Q and V ? Determine the capacitance of a parallel-plate capacitor of area S , separation d and charged to a voltage V_o . The permittivity of the dielectric is ϵ . Also, find the electrostatic energy. **(3+7=10)**
- (b) Derive Poisson equation from the appropriate fundamental equations for electrostatics in a medium. Mention the assumptions used. What is Laplace's equation? Mention its importance in electromagnetics. Using Laplace's equation, determine the potential V at any point between the parallel plates of the capacitor in part (a) above. Also, determine the surface charge densities on the top and bottom plates. **(5+8=13)**
- (c) What is the basic technique of solving electrostatic problems using the "method of images"? A positive point charge Q is located at distances d_1 and d_2 , respectively, from two grounded perpendicular conducting half-planes as shown in Fig. for Q. No. 2(c). Determine the force on Q caused by the charges induced on the planes. **(4+8=12)**



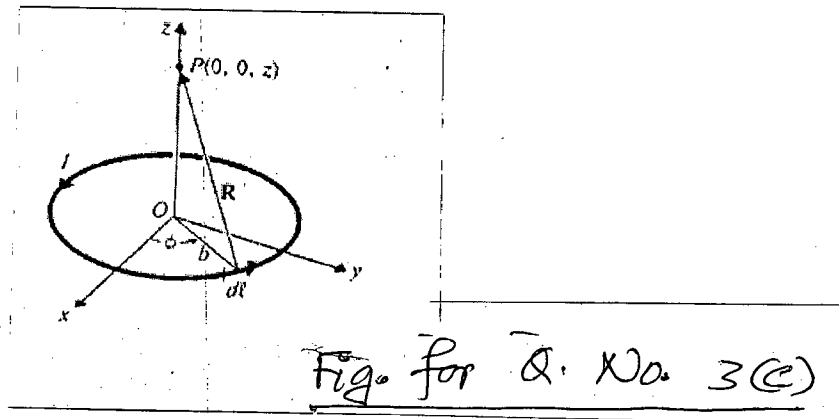
Contd P/2

Fig. for Q. No. 2(c).

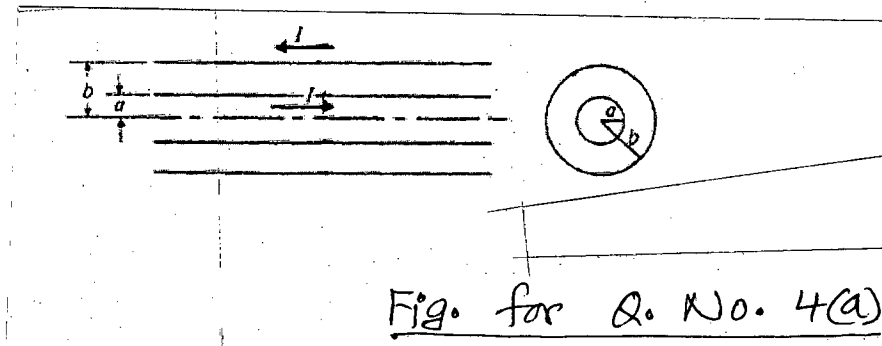
EEE 209

3. (a) Explain the true meaning of $\nabla \cdot \vec{B} = 0$. Considering a pill-box at the boundary between two magnetic media, show that the normal component of \vec{B} is continuous at the boundary, whereas that of \vec{H} is discontinuous. (3+8=11)
- (b) Define vector magnetic potential \vec{A} . Mention its physical significance and practical usage. (4+6=10)
- (c) State Biot-Savart Law. Using this law, find the magnetic flux density \vec{B} at a point on the axis of a circular loop of radius b that carries a direct current I as shown in Fig. for Q. (4+10=14)

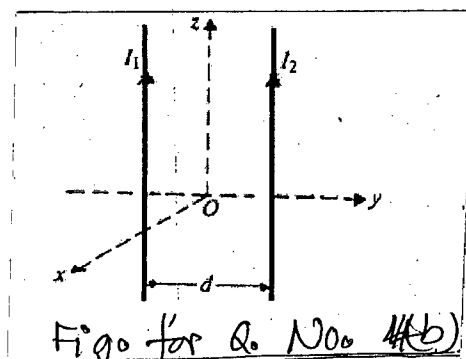
No. 3(c).



4. (a) An air co-axial transmission line has a solid inner conductor of radius a and a very thin outer conductor of inner radius b . (Refer to Fig. for Q. No. 4(a))
- (i) State Ampere's circuital law and using this law determine magnetic flux density \vec{B} , inside the inner conductor and between the inner and outer conductor, (4+6=10)
- (ii) Define self inductance and mutual inductance. Using the necessary information, determine the inductance per unit length of this transmission line. (5+5=10)



- (b) Write down the Lorentz's force equation. Show that $\vec{F}_m = I \oint_C d\vec{l} \times \vec{B}$ (N); where, \vec{F}_m is the magnetic force on a complete circuit of contour C , carrying a steady current I in a magnetic field \vec{B} . Determine the force per unit length between two infinitely long parallel conducting wires carrying currents I_1 and I_2 in the same direction. The wires are separated by a distance d , as shown in Fig. for Q. No. 4(b). (6+9=15)

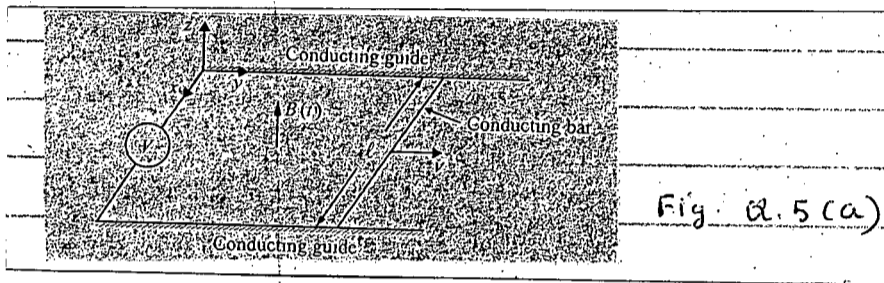


EEE 209

SECTION – B

There are **FOUR** questions in this Section. Answer any **THREE**.

5. (a) The conducting bar of length l is parallel to the x-axis and it moves between two conducting guides in y-direction at constant velocity V in a uniform magnetic Field as shown in the Figure 5(a). The flux density in the z-direction and it varies sinusoidally with time given by $B(t) = B_0 \sin \omega t$. A high resistance voltmeter is connected between the rails at $y = 0$ to complete the loop. Find the voltage induced in the coil. (15)



- (b) Derive Maxwell's equations for a liner homogeneous medium in terms of E and H only assuming the time Factor $e^{-j\omega t}$. Also show that in a source-free region, Maxwell's equation can be reduced to two. (10)
- (c) The conductivity of seawater is approximately 0.4 mS/m and its dielectric constant is 81. Determine the frequency at which the magnitude of the displacement current density is equal to the magnitude of the conduction current density. Comment on the electric behavior of seawater at very low and very high frequencies. Also, justify your comments. (10)
6. (a) Determine the polarization of the wave if the electric field intensity in a region is given by $E = (-j25\hat{a}_x + 25\hat{a}_z) e^{-(0.01+j120)y} \text{ V/m}$. (10)
- (b) A 3(GHz) y-polarized uniform plane wave propagates in the +x direction in a nonmagnetic medium having a dielectric constant 2.5 and a loss tangent 0.05. (25)
- (i) Determine the distance over which the amplitude of the propagating wave will be cut in half.
- (ii) Determine the intrinsic impedance, the wavelength, the phase velocity and the group velocity of the wave in the medium.
- (iii) Assuming $E = \hat{a}_y 50 \sin\left(6\pi 10^9 t + \frac{\pi}{3}\right) \text{ V/m}$ at $x = 0$. Write the instantaneous expression for H for all t and x.
7. (a) Derive instantaneous expression of Poynting's vector and average power density for a source-free conductive region. (15)

EEE 209

Contd ... Q. No. 7

(b) An air-filled co-axial cable has inner conductor of inner radius a and outer conductor of inner radius b . The peak values of the voltage between the conductors is V and the peak value of current in each conductor is I . Voltage and current vary sinusoidally with an angular frequency of ω rad/sec. (i) Determine the condition for the fields to exist within the co-axial cable (ii) compute average power inside the cable and verify Poynting theorem.

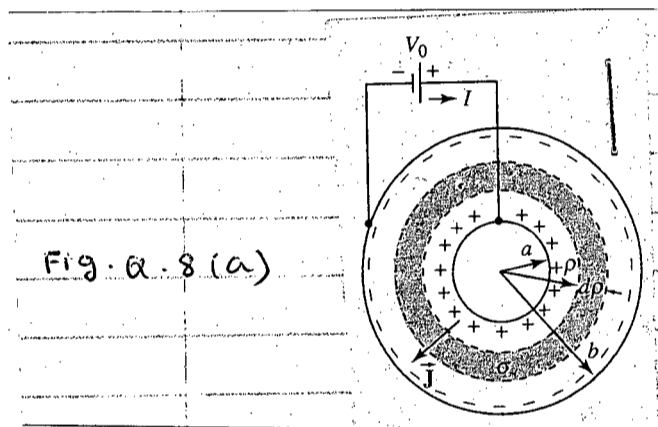
(15)

(c) Derive Lorentz's condition for potential from Maxwell's curl equations of Time varying Electromagnetics.

(5)

8. (a) A material with conductivity $\sigma = m/p + k$, where m and k are constants, fills the space between two concentric, cylindrical conductors of radii a and b as shown in Fig. 8(a). If V_0 is the potential difference between the two conductors and L is the length of each conductor. Obtain expressions for the resistance of the material, the current density and the electric field intensity in the material.

(12)



(b) Derive the continuity equation. Determine the time rate of change of the volume charge density if the current density in the medium is $\vec{J} = \sin(10x)\hat{a}_x + y\hat{a}_y + e^{-3z}\hat{a}_z \text{ A/m}^2$.

(10)

(c) A 50 MHz uniform plane wave travelling in a medium ($\epsilon_r = 16$, $\mu_r = 1$ and $\sigma = 0.02$ S/m) strikes normally to the surface of another medium ($\epsilon_r = 25$, $\mu_r = 1.0$ and $\sigma = 0.2$ S/m). If the amplitude of the incident electric field intensity at the interface is 10 V/m. Determine (i) Reflection co-efficient (ii) Transmission co-efficient (iii) standing wave ratio and (iv) Transmitted E and H Fields. Assume that wave propagating in the z -direction and has only x -component of Electric Field intensity.

(13)