78

NON-EQUILIBRIUM DISTRIBUTION OF SUSPENDED SEDIMENT

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ABSTRACT

Computation of suspended sediment transport is normally based on the assumption of an equilibrium condition. But in practice non-equilibrium condition of suspended sediment distribution is a frequent condition. In the present investigation attempts were made to establish a correct distribution pattern of suspended sediment under non-equilibrium condition. To do this, the two dimensional diffusion equation was solved analytically assuming that -

(a) Fall velocity is a function of the distance of the particle from the channel bed.

(b) Flow profile can be described by a power law equation.

(c) Diffusion co-efficient is a function of prandtl's mixing length.

To verify the analytical analysis experiment was done in a laboratory flume 70' long 2½'wide and 2½' deep. It was found that the new power law equation $\frac{w}{w_o} = (y/h)^{1+n}$ of particle fall velocity gave promising results and gave values similar to the values obtained by the simultaneous solution of Maude and Whitmore and Rouse equation. The power law equation $\frac{u}{U} = (y/h)^n$ for the vertical velocity profile gave encouraging results describing the velocity profile for the entire flow depth in a betterway. The exponent of the power law equation varied little with the change of bed slope but there was a marked increase of the multiplying constant with the increase of the bed slope. The expression derived for the distribution of suspended material under non-equilibrium condition agreed very well with the

experimental results. It was found that the ratio of fall velocity to mean flow velocity (λ) was the governing factor of the distribution equation. The concentration ratio was found to decrease to a great extent due to an increase in the ratio of fall velocity to mean flow velocity.

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NOTATIONS

A = Projected area of particle

A = Constant

B = Constant

B_1 = Constant of diffusion co-efficient equation

c = Concentration of particles

C = Concentration ratio

C_a = Concentration ratio at reference level

$\bar{c}_v$ = Mean value of volume concentration

d = Diameter of particles

D = Depth of flow

D_{50} = Particle size for which 50% of the sediment mixture is finer

F_D = Drag force

g = Acceleration due to gravity

h = Depth of flow

l = Prandtl's mixing length

n = Exponent of velocity distribution equation

P = Roots of the hypergeometric equation

R = Reynold's number

S = Slope of the channel

U = Velocity at a depth y in the x-direction

$\bar{U}$ = Average velocity in the x-direction

U_* = Shear velocity

v = Velocity in the y-direction

v_x = Velocity component in the x-direction

v_y = Velocity component in the y-direction

w = Fall velocity

w_s = Particle fall velocity at a depth of y.

w_o = Fall velocity of a single particle under quiescent conditions

x = Longitudinal distance

X = Non-dimensional longitudinal distance

y = Distance from bottoms

α = Element of hypergeometric equation

β = Element of hypergeometric equation

ϵ_m = Diffusion co-efficient for momentum

ϵ_s = Sediment diffusion co-efficient

ϵ_x, ϵ_y = Diffusion co-efficient in the x and y direction respectively

λ = Ratio of fall velocity to average flow velocity

η = Turbulent mixing co-efficient

κ = Universal constant of von-Karman

μ = Dynamic viscosity

ν = Kinematic viscosity

ρ_s = Density of particle

ρ_f = Density of fluid or water

$\sum$ = Summation

τ = Shear stress

CONTENTS

Subjects Page

Subjects | Page

LIST OF TABLES

LIST OF FIGURES

Subjects		Page

Subjects		Page

CHAPTER - 1

INTRODUCTION

1.1 BACKGROUND:

Sediment transport has been a subject of studies by engineers and others for over two centuries due to its importance in design and operation of hydraulic structures and river regulations. Problems created by sediment in motion are many. Over the head water reaches of rivers large boulders may be transported at times of floods and these may cause serious damage of the hydraulic structures. Similarly, the problems created by the suspended sediment are equally serious. Reference is only made here to industrial and commercial water supply where one of the gravest problem is the removal of suspended particles. The problems caused by the sediments in suspension may be viewed as of the origin of non-equilibrium and equilibrium condition. Water spilling over structures like weir, barrages, dams etc. are bright example of the transformation of non-equilibrium condition into an equilibrium condition. Once spilled the water picks up bed material and creat a non-equilibrium condition which has been a great concern of engineers dealing with the development of water resources. Studies have been made to have an insight into the problems but still many questions remain unanswered.

In the present case a theoretical analysis have been attempted and an experimental study under controlled condition have been taken to have a better idea and knowledge about the non-equilibrium distribution pattern of sediment in suspension.

1.2 OBJECTIVES OF THE STUDY:

In open channel flow non-equilibrium transport of suspended sediment is an important part of the general flow problem of sediment transport. Problems which have been investigated are usually of two types (i) the transition from one equilibrium state to another due to abrupt changes in bed condition and (ii) the dispersion from the mean flow of an initial concentration of suspended sediment. An analytical study of the first type of problem is presented herein with a view to satisfy the following objectives:

a. To find an analytical equation that can be used to draw a concentration profile under non-equilibrium condition.
b. To compare the analytical results with those of the flume results.
c. To draw longitudinal sediment concentration profile within the range of non-equilibrium transport.

1.3 SCOPE OF STUDY:

The hydraulic parameters characteristics of the water course are indicative of the type of sediment transported therein. The sediment in rapid steep rivers or river section is overwhelmingly coarse consisting of large fractions. In slow streams flowing at mild slopes the sediment on the otherhand is finely grained. The hydraulic properties of the water courses are accordingly characteristics of the sediment transport. Moreover, from the previous discussions, it is obvious that a complete description of a water course is impossible, unless the sediment conditions are taken into considerations along with the hydraulic properties. The relationships expected to exist between the hydraulic properties and the sediment conditions of a water course may be applied in particular cases also for the quantitative description of sediment transportation. The fundamental objective of theoretical and experimental research alike is invariably to relate the hydraulic parameters to the quality and quantity of sediment. And as for suspended sediment the present study can be used in the following cases:

a. If the distribution of the concentration along the vertical is known then it will be possible to calculate the total amount of suspended load carried by the streams.

b. The results obtained thus can be further extended for the prediction of deposition or erosion of the stream reaches.

c. The study may help in solving the problems mentioned earlier.

d. At last it may be concluded that the relationship determined either theoretically or experimentally provide the means for a more detailed understanding of the complex phenomenon of sedimentation in alluvial channels.

CHAPTER - 2

SEDIMENT PROPERTIES

2.1 INTRODUCTION:

The dynamic problem of fluid sediment interaction are greatly influenced by the sediment properties. The description of the later, however, is exceedingly complex and one is forced to make many semplifying assumptions. The first of which is the sub-division into cohesive and non-cohesive sediment.

In cohesive sediments the resistance to erosion depends on the strength of the cohesive bonds between the particles which may far outweigh the influence of the physical characteristics of individual particles. But once erosion has taken place cohesive material may become non-cohesive with respect to further transport. Characteristics may also change through chemical or physical reactions. The problem of erosion resistance of cohesive soil is a very complex one and at present our understanding of the physics of it is very incomplete.

The non-cohesive soils generally consists of larger discrete particles than the cohesive soils and the movements of these particle depends on the physical properties of the individual particles. Several important properties of the sediment particles are discussed in the following articles:

2.1.1 Particle Size:

Particle size has a direct effect on the mobility of the grain and it can range from boulders, which are rolled only by mountain torrents to fine clays which once strirred up take days to settle.

Natural sediments are also irregular in shape and therefore the definition of size by a single length dimension is necessarily very incomplete and due to convenience of measurement only. Common definitions in use are:

i. Sieve diameter: Used for sand and fine gravel
ii. Equivalent or sedimentation diameter: Diameter of a sphere of the same density with the same terminal settling velocity in the same fluid at the same temperature as the given particles. It is used for clays, silts and fine sands.
iii. Nominal diameter: Diameter of a sphere of equal volume. Used for larger particles.

2.1.2 Particle Shape:

Apart from size, shape affects the transport of sediment but there is no direct quantitative way to measure shape and its effects. One measure of the shape is the ratio of the surface area of the particle (A) to that of a sphere (As) of the same volume $\Psi = A/A_s$. This is known as the shape factor or sphericty, although another definition of sphericity is "the cube root of the volume of the particle to the volume of its circumscribing sphere". But this does not give information on the actual shape of the particle (46). McNown (46) suggested a shape factor S.F. = $c/\sqrt{ab}$ where 'c' is the shortest of the three perpendicular axes (a,b,c) of the particles.

2.1.3 <u>Particle Density:</u>

Density of the particle is important and must be known. Where the sediment is composed of a variety of minerals the proportions and sizes need be determined. The average density of the sample may change little but the variation from say pumice sand to iron sands or magnetite may be appreciable. Such variations in density affect sediment transport by seggregation. In case of suspended sediment the fall velocity of the particles greatly depends on the particle density.

2.1.3 <u>Fall Velocity:</u>

The fall velocity figures prominently in all sediment transport problems and although the concept is straight forward, its precise evaluation or calculation is not. The literature dealing with the motion of particles of various shapes in ideal and in viscous fluid is extensive. Reference may be made to the survey by Graf (20).

The fall velocity is a function of size, shape, density and viscosity. In addition it depends on the extent of the fluid in which it falls, on the number of particles falling and on the level of turbulence intensity. Turbulent conditions occur when settling takes place in flowing fluid and can also occur when a cluster of particles is settling.

Falling under the influence of gravity, the particle will reach a constant velocity known as the terminal velocity when the drag equals the submerged weight of the particle. Thus, for a spherical particle, writing w for the terminal velocity

$$w^2 = \frac{4}{3} \frac{1}{C_D} g.d. \left(\frac{\rho_s - \rho_f}{\rho_f} \right) \quad \ldots \quad \ldots \quad (2.1)$$

where C_D is the drag co-efficient, ρ_s density of particle, ρ_f is the fluid density and d is the diameter of particle.

For a spherical perticle of diameter 'd' in a viscous fluid of infinite extent, the drag co-efficient (C_D) is fairly well defined. In laminar flow regions, Stokes solution is:

$$F_D = 3 \pi \mu \, dw \quad \ldots \quad \ldots \quad \ldots \quad (2.2)$$

$$\text{and} \quad C_D = \frac{24}{R} \quad \ldots \quad \ldots \quad \ldots \quad (2.3)$$

where R = Reynolds number, μ = co-eff. of dynamic viscosity.

But there are certain limitations to determining the fall velocities on the basis of Stokes law. The limiting conditions are (6):

(a) The settling solid particles should be regular spheres

(b) The particles should be solids having a smooth surface. No slip takes place between the fluid and the particle but friction develops between the water layer adhering to the moving particle and the water at rest.

(c) The particle settles in a fluid space of infinite extent.

(d) The particles are larger enough to permit the fluid to be assumed homogeneous relative to their size.

(e) The resistance to the settling of the particles depends exclusively on the viscosity of the fluid.

Now the most frequent practical cases will be considered.

Condition (a) is usually not satisfied in practice. The effect of particle shapes other than spherical should be taken into consideration. (See for Ref. Graf.(20)).

Condition (b) is usually satisfied in practice. Experiments carried out thus far have shown no slip to take place between the water and the sediment particles (6).

Condition (c) is very simple to meet in practice.

Sediment materials occuring in practice invariably meet condition (d) for even the smallest settleable particle size may be regarded as extremely large relative to the irregularities in the fluid structure.

Studies into the validity of condition (e) have revealed the existence of a limit particle size, the magnitude of which depends on the properties of the fluid and the solid particles, beyond which the fall velocities obtained by the Stoke's law are greater than actual ones, owing to the neglected inertia effects.

However, for higher Reynold's number (R), the theoretical treatment has not yet succeeded in accurately predicting the value of the drag co-efficient. The difficulties arise mainly from the interaction of the turbulence with the particle. Here additional terms arising from inertia have to be considered (46). The two significant ones are shown in the following two equations:

$$\left(\frac{r\,dv_s}{dt} \right) \Big/ v_s^2$$

or, $$\rho \left\{ \frac{dv_s}{dt} - \frac{dv}{dt} \right\} \Big/ (v_s - v)^2 = A_c \quad \ldots \quad \ldots \quad (2.5)$$

and $$\sqrt{(v')^2} \Big/ (v_s - v) = I_p \quad \ldots \quad \ldots \quad \ldots \quad (2.6)$$

where r = radius of sphere

v = velocity in the y-direction

v'= fluctuating component of v

v_s= velocity of solids

But even with these two terms included, the drag co-efficient in turbulent flow is not uniquely defined.

Schiller and Naumann (46) multiplied equation (2.1) by $(d/\nu)^2$ to obtain relationship in non-dimensional form

$$C_D\, R^2 = \frac{4}{3} \cdot g \cdot \frac{\rho_s - \rho_f}{\rho_f} \cdot \frac{d^3}{\nu} \quad \ldots \quad (2.7)$$

where ν is the coefficient of kinematic visocity. With the aid of graph of C_D versus R (Fig.2.1), the Reynolds number (R) is obtained and hence the desired fall velocity.

Attempts have made to develope a relationship for the fall velocity which can be applied to all Reynold's number of practical interest. W.W.Rubey (6) developed the following formula for the fall velocity.

$$w = \sqrt{\frac{2}{3} \cdot \rho_f' g d + \frac{36 \mu^2}{\rho_f^2 d^2}} - \frac{6 \mu}{\rho_f d} \quad \ldots \quad \ldots \quad (2.8)$$

Rubey's formula involves implicity though the relationship of two dimensionless numbers namely the Reynold's number and the Froude number.

So far we have considered a single spherical particle in a fluid of infinite extent. In practice not the single grain but a cloud of grains is the problem encountered. The fall velocity decreases when the same particles are dispersed throughout the fluid in quantity and this accounts for widely varying results in the determination of fall velocity.

In the majority of cases the fall velocity is required in flowing sediment carrying water. This fall velocity is no longer characteristics of the sediment material since it depends obviously on hydraulic factors, such as the flow velocity, turbulence of water and the degree of concentration (Pollution) of the stream.

In early investigations, the fall velocity was found to be influenced to any appreciable extent at extremely high sediment concentration only. Recent research,however, has revealed that the reduction in fall velocity may be substantial even at low sediment concentrations.

The reduction in fall velocity with sediment concentration is shown in figure (2.2) on the basis of data obtained by Schokiltch (6) for quartz particles. In this figure the fall velocity vurses particle diameter relation is shown for clear water and for four different sediment concentrations C (KP/m^3) at a constant temperature of 10^oC.

The ratio of the fall velocity in clear and muddy water has recently been studied by McNown and Pin-Num-Lin (41) who relate the variations in this ratio to the sediment concentration (c) in terms of the sediment fall velocity Reynolds number, $R = \frac{wd}{\nu}$. The results of their theoretical and experimental investigations are shown in figure (2.3). Bogardi (6) noted that the validity of their relation is confined to Reynolds number smaller than 2. However, it should be realized finally that the fall velocity is modified in flowing water as well.

The reduction in fall velocity in flowing water is closely related to the circumstance that the movement of sediment carrying water is turbulent practically without exception. Consequently the reduction offall velocity in flowing water can be attributed to turbulence. Although several experiments have been conducted concerning the reducing effect of turbulence on fall velocity the problem is not fully understood as yet. In earlier practice for instance the fall velocity in water at rest was used in considerations on settling and the retarding effects of turbulence were allowed for a variety of approaches.

Relying on the experimental results of Bestelli, Valinakov and others, Levin (6) relates the reduction in the magnitude of fall velocity under the influence of turbulence to the main velocity of flow and to the depth of water (D). The reduction in fall velocity is accordingly:

$$w_1 (m/sec) = v\eta \quad \ldots \quad \ldots \quad \ldots \quad (2.9)$$

where, $\eta = \frac{0.132}{D}$ ... ... (2.10)

In the above expressions D and v should be substituted in metre and metre/sec. respectively. Thus the actual fall velocity in water of velocity v, according to Levin is:

$$w_t = w - w_1 \quad \ldots \quad \ldots \quad \ldots \quad (2.11)$$

where, $w_1 = 0.0282 \frac{v}{D^{0.2}}$ ... ... (2.12)

Maude and Whitmore (38) propose that the fall velocity affected by concentration hindered settling can be described at all Reynold's number by:

$$w = w_o (1-C)^a \quad \ldots \quad \ldots \quad \ldots \quad (2.13)$$

where, C is the volumetric concentration w_o is the settling velocity of a single particle and a is a function of particle size, shape and Reynold's number. For $R < 1$, $a \simeq 4.65$ and for $R > 10^3$, $a \simeq 2.32$. with an s-curve transition between these values for a log-log plot of a versus R.

From the previous discussions it is now clear that the fall velocity of particles wheather in single or in cluster depends, excluding other parameters, on the velocity of streams and on the concentration of particles in the streams. In other words, the velocity profile and the concentration profile of a particular river section affect the particle fall velocity. Under non-equilibrium condition the sediment distribution profile changes from section to section and as a result the particle fall velocity must also change from section to section. On the other hand at a particular river section the velocity profile and the concentration profile is not constant along the vertical. Hence in this case also, the particle fall velocity at a particular section varies along the vertical distance. In order to find the concentration profile many investigator put forward different equations based on different approaches (4,6,23,29,32,34,42 and 47). But their analysis was based on one or both of the following assumptions which are not correct. The assumptions are:

1. At a particular section the velocity of the stream is constant and is equal to the mean velocity of flow.
2. The fall velocity is always constant and is equal to the particle fall velocity in quiescent water.

It is to be noted that in suspended sediment transportation the particle fall velocity is an important parameter. Its variation with distance (both horizontal and vertical) has considerable effect on concentration profile or velocity profile and must be taken into consideration to obtain a correct equation for the distribution of suspended sediment under non-equilibrium condition. In functional form: the fall velocity at a section can be expressed as:

$$w = \varphi\left[\rho_f, \rho_s, \mu, d, S_p, f, S_r, F, C\right] \quad \cdots \quad (2.14)$$

where, $C = \psi\left[\vartheta, x, y, \epsilon\right] \quad \ldots \quad \ldots \quad \ldots \quad (2.15)$

Where, S_p = particle shape factor

f = frequency of oscillation

F = boyant force

ϵ = diffusion co-efficient

x = horizontal distance from origin

y = vertical distance from origin or bottom

ρ_f = fluid density.

By dimensional analysis and neglecting the surface roughness (S_r) the equation reduces to

$$\varphi_2\left[\frac{\rho_f \omega d}{\mu}, \frac{F}{\rho_f \omega^2 d^2}, S_p, \frac{fd}{\omega}, \frac{\rho_s}{\rho_f}, C\right] = 0 \quad \ldots (2.16)$$

$$\text{where, } C = \Psi_2'\left[\frac{x}{y}, \frac{\vartheta x}{\epsilon}\right] \quad \ldots (2.17)$$

But, $\frac{\rho_f \omega d}{\mu}$ is the Reynolds no. of the particle and therefore

$$\varphi_3\left[R, \frac{F}{\rho_f \omega^2 d^2}, S_p, \frac{fd}{\omega}, \frac{\rho_s}{\rho_f}, C\right] = 0 \quad (2.18)$$

Again, $\frac{\vartheta x}{\epsilon}$ may be taken as the Reynolds number (R_1) in terms of diffusion co-efficient.

$$C = \Psi_3\left[R_1, \frac{x}{y}\right] \quad \ldots \quad \ldots \quad \ldots (2.20)$$

At last neglecting ρ_s/ρ_f and assuming $f = 0$, the equation reduces to

$$\varphi_4\left[R, \frac{F}{\rho_f \omega^2 d^2}, S_p, C\right] \quad \ldots \quad \ldots (2.21)$$

$$\text{for } C = \Psi_4\left[R_1, \frac{x}{y}\right] \quad \ldots \quad \ldots (2.22)$$

$$\varphi_5\left[\mathbb{R}, \frac{F}{\rho_f \omega^2 d^2}, S_p, \mathbb{R}_1, \frac{x}{y}\right] = 0 \qquad \ldots \qquad \ldots (2.23)$$

$$\text{or,} \quad \frac{\rho_f \omega^2 d^2}{F} = \varphi_6\left[\frac{\mathbb{R}}{\mathbb{R}_1}, S_p, \frac{x}{y}\right] \qquad \ldots \qquad \ldots (2.24)$$

$$w^2 = \frac{F}{\rho_f d^2}\varphi_6\left[\frac{\mathbb{R}}{\mathbb{R}_1}, S_p, \frac{x}{y}\right] \qquad \ldots \qquad \ldots (2.25)$$

Therefore, from equation (2.25) it is seen that in case of non-equilibrium distribution of suspended sediment the particle fall velocity can't be taken as constant rather it should be considered as a variable.

CHAPTER - 3

CHARACTERISTICS OF SEDIMENT CARRYING STREAMS

3.1 INTRODUCTION:

Open channel flow over a movable boundary behaves differently from open channel flow through rigid boundary. In alluvial channel rigid boundary relations apply only if there is no movement of the bed and bank materials. Once the general movement of the bed material has started, the flow and the boundary interact in a complex manner. Simon and others (52) discussed in details the salient features that differentiate between flow over movable and rigid boundaries. A summary of their discussions is presented here:

(a) In alluvial channels, the flow and the boundary shape are intertelated. After general movement of the bed has started, the alluvial bed is distorted, giving rise to bed forms. The shape, size and rate of movement of these bed forms vary with flow conditions.

(b) The magnitude of roughness elements as represented by the bed forms can be of the same order of magnitude as the depth of flow. Relative roughness of this magnitude is generally not encountered in rigid boundary systems.

(c) An alluvial bed is not impervious unlike rigid boundaries. Therefore, there is a possibility of flow, however small within the bed. Thus the turbulent fluctuations normal to the flow may not vanish at the boundary.

(d) The alluvial boundary moves at both grain and bed form scales. Grains rolling at the boundary may introduce additional shear by their rotation and thus may change the turbulence level close to the boundary. In addition the movement of the bed forms creats unsteadiness of flow in the vertical due to the changing bed elevation and resultant flow patterns.

(e) At advanced stages of sediment movement, some of the bed material is entrained by the flow and is referred to as suspended materials. The presence of the particles in suspension affects the turbulence characteristics, the specific weight and the apparent viscosity of fluid.

(f) As the bed forms achieve dimensions comparable to the depth the flow is no longer uniform. But the depth and velocity change along and accross the channel.

Since there is no turbulent flow theory for movable beds comparable to theories available for rigid boundary turbulent flows, analysis of flow over alluvial beds must relay to a significant degree on a rigid boundary turbulent flow theories. In order to understand the flow characteristics in alluvial channels, the basic concepts of the flow over rigid boundaries are reviewed and the hydraulics of alluvial channels is introduced.

3.2 VELOCITY DISTRIBUTION FOR UNIFORM FLOW IN OPEN CHANNELS WITH RIGID BED:

Knowledge of the velocity distribution in artificial and natural channels is necessary to solve many engineering problems. The velocity distribution of turbulent flow is different from the velocity distribution of laminar flow. Here only the former type of distribution will be discussed in detail.

3.2.1 Velocity Distribution for Laminar Uniform Flow:

Several approaches can be used to derive the laminar velocity distribution for uniform flow. In a laminar flow three types of forces act on a fluid element and they are: Shear, pressure and weight. For uniform flow to exist these forces must be in equilibrium. Assuming zero velocity at the bed of an open channel, one can obtain a parabolic velocity distribution for laminar flow condition. For detail reference can be made to Simon et al (52).

3.2.2 Turbulent Velocity Distribution for Uniform Flow:

In turbulent flow a precise definition of velocity at a given point and time is not possible. The velocity vector is not constant. Both the velocity and the pressure are fluctuating with time and space. Only the mean values of these fluctuating elements can be computed. For methods of computation reference can be made to Hinze (25).

3.2.2.1 Shear Stress in Turbulent Flow:

In turbulent flow velocity fluctuations cause a continuous interchange of fluid masses between the neighbouring layers, which is accompanied by a transfer of momentum. Such momentum transport due to fluctuations results in developing additional shear stresses of high magnitude between the adjacent layers. In order to determine the magnitude of turbulent shear stress, a number of semi-empirial theories have been developed and three of them are discussed below:

3.2.2.1.1 Boussinesq Approach: In analogy with the expression for the viscous shear J.Boussinesq in 1877 (39) developed an expression for the turbulent shear stress which may be expressed as

$$\tau = \eta \frac{\partial \bar{v}}{\partial y} = \rho_f \cdot \epsilon_m \cdot \frac{\partial \bar{v}}{\partial y} \qquad \ldots \qquad \ldots (3.1)$$

where, η is called the eddy viscosity and ϵ_m is the eddy kinematic viscosity or the transfer co-efficient of momentum. Now, whereas μ is a fluid property and is a function of temperature of fluid alone, η and ϵ_m are mainly the function of the characteristics of flow and may be expected to vary from point to point in the flow. Since the values of η and ϵ_m can not be predicted the Boussinesq hypothesis is, however, of limited use (39).

3.2.2.1.2 Reynold's Approach: In 1886 Reynold's developed an expression for the turbulent shear stress (or the apparent shear stress) due to the exchange of momentum in the turbulent mixing process. The expression for the time average value of shear stress is:

$$\overline{\tau} = \rho_f \overline{\vartheta_x \vartheta_y} \quad \ldots \quad \ldots \quad \ldots (3.2)$$

where v_x = velocity component in the x - direction

v_y = velocity component in the y - direction

3.2.2.1.2.1 Shear Velocity or Friction Velocity:

In equation (3.2) $\overline{v_x v_y}$ is a positive value (63) and thus it is possible to write:

$$\sqrt{\frac{\tau}{\rho_f}} = \sqrt{|\overline{\vartheta_x \vartheta_y}|}$$

The term $\sqrt{\overline{\vartheta_x \vartheta_y}}$ has the dimension of a velocity and is called the friction velocity (U_*). Thus -

$$U_* = \sqrt{|\overline{\vartheta_x \vartheta_y}|}$$

$$\text{or,} \quad U_* = \sqrt{\tau/\rho_f} \quad \ldots \quad \ldots (3.3)$$

3.2.2.1.3 Prandtl's Mixing Length Approach:

In 1925 L.Prandtl made an important advance presenting his mixing length hypothesis by means of which the turbulent shear stress can be expressed in terms of measurable quantities related to the average flow characteristics. In his hypothesis Prandtl indicated that the velocity fluctuation in the x-direction (v_x) and velocity fluction in the y-direction (v_y) may be related to the mixing length (I) by the following expressions:

$$v_x = l \frac{\partial \overline{\vartheta}}{\partial y} \quad \ldots \quad \ldots \quad \ldots (3.4)$$

$$\text{and} \quad v_y = l \frac{\partial \overline{\vartheta}}{\partial y} \quad \ldots \quad \ldots \quad \ldots (3.5)$$

Therefore,

$$\overline{v_x v_y} = l^2 \left(\frac{\partial \vartheta}{\partial y}\right)^2 \quad \ldots \quad \ldots \quad \ldots (3.6)$$

From equation (3.2), the expression for turbulent shear stress is:

$$\overline{\tau} = \rho_f l^2 \left(\frac{\partial \overline{v}}{\partial y}\right)^2 \quad \ldots \quad \ldots \quad \ldots (3.7)$$

Again from equation (3.1) one gets -

$$\overline{\tau} = \rho_f l^2 \left(\frac{\partial v}{\partial y}\right)^2 = \rho_f \epsilon_m \left(\frac{\partial \overline{v}}{\partial y}\right)$$

$$\text{or, } \epsilon_m = l \left(\frac{\partial \overline{v}}{\partial y}\right) \quad \ldots \quad \ldots \quad \ldots (3.8)$$

The advantage of equation (3.7) lies in the fact that it is possible to make suitable assumptions regarding the variation of mixing length. Prandtl's assumption for mixing length can be expressed as:

$$l = \kappa y \quad \ldots \quad \ldots \quad \ldots (3.9)$$

where κ is the universal constant defined by von-Karman.

Using equation (3.7) and (3.9) and after intergation one will get -

$$v = \frac{U_*}{\kappa} \log_e y + Z \quad \ldots \quad \ldots \quad \ldots (3.10)$$

where Z is the constant of integration. Equation (3.10) is known as the von-Karman velocity equation. For the condition that at $y = y'$, the velocity $v = 0$, equation (3.10) reduces to

$$v = \frac{U_*}{\kappa} \log_e (y/y') \quad \ldots \quad \ldots (3.11)$$

Equation (3.11) indicates a zero velocity at a certain distance above the boundary which is in disagreement with the physical fact that the velocity is zero at the boundary. As such equation (3.11) applies only to turbulent flow in the upper region of the channel and it cannot be applied to the regions close to the boundary.

3.2.3 Velocity Distribution in Different Regimes:

The determination of y' as a function of the characteristics of the boundary leads to a definition of velocity distribution related to the characteristics of the boundaries. This idea was developed by Richardson and Simons (49). The velocity distribution in different regimes can be expressed as follows:

(a) Velocity distribution in liminar sublayer is given by -

$$\frac{U_* y}{\nu} = \frac{u}{U_*} \quad \ldots \quad \ldots \quad \ldots \quad (3.12)$$

(b) Velocity distribution of flow in hydraulically smooth boundary is given by -

$$\frac{U}{U_*} = 5.75 \log \frac{U_* y}{\nu} \quad \ldots \quad \ldots \quad (3.13)$$

(c) Velocity distribution over rough boundary is:

$$\frac{U}{U_*} = 5.75 \log \frac{30.1\ y}{K_s} \quad \ldots \quad \ldots \quad (3.14)$$

where K_s is the roughness height.

Several other relationships for describing the turbulent velocity distribution are utilized by various researches. Some of them are:

(a) Einstein's formula (19):

$$\frac{U}{U_*} = 5.75 \log \frac{30.2y}{K_s} \cdot x \quad \ldots \quad \ldots \quad (3.15)$$

where x is a correction factor and K_s is assumed to be equal to D_{65}.

where D_{65} is the particle size for which 65% of the sediment mixture is finer.

(b) Brook's (5) equation·

$$U' = U_{max} + \frac{U_*}{\kappa} \log_e (y/h) \quad \ldots \quad \ldots (3.16)$$

where, U_{max} is the maximum velocity and h is the depth of flow.

(c) Cheng et al (11a) gave the following equation for the velocity defect:

$$\frac{U_{max} - U}{2\, U_*/\kappa} = \log_e \left[\frac{\sqrt{\xi}}{1-\sqrt{1-\xi}} \right] - \sqrt{1-\xi} \quad \ldots (3.17)$$

where $\xi = y/h$

3.3 TURBULENT VELOCITY DISTRIBUTION FOR UNIFORM FLOW OVER A MOVABLE BOUNDARY OF ALLUVIAL CHANNELS:

The formulas of open channel flow discussed thus far refer to channels with an immovable bed. However, the river bed consists of movable sediment and it stands to reason that an interaction will take place between the flow and the bed with the following complications:

i. Change in bed configuration

ii. Change in character of fluid due to suspension of materials

iii. Effect of sediment transport on flow characteristics.

Special studies to investigate the validity of the clean channel flow formulas for sediment laden channels have been undertaken by Liu and Hwang (35) and Vanoni (63). In all these studies it turned out that the formulas for clean water flow are indeed no longer valid. Vanoni and Brooks (53) made special studies to determine the influence of the sediment load upon the discharge. From their studies it turned out that there is no single valued relationship between the velocity and any combination of depth and slope of river. The results of Vanoni and co-workers (61,63) in this regard, however, are purely empirical and no theoretical explanation could be given.

Sayre and Albertson (54) have studied the influence of roughness on velocity distribution by varying the density (spacing) and magnitude of roughness element in the laboratory flume. Their results are plotted in a dimensionless form in Fig. (3.1). The distribution of velocity is discribed as -

$$\frac{v}{\sqrt{\tau_0/\rho}} = 6.06 \log y/x + 2.6 \quad \ldots (3.18)$$

where x = a roughness parameter having the dimension of length representing the magnitude, spacing and shape of the roughness elements.

The distribution of velocities in flow over sand ripples have been studied by Raudkivi (50), whose data are plotted in Fig. (3.2). The velocity distribution was found to follow a logarithmic law as long as the bed was smooth, regardless of wheather the bed was solid or mobile.

Toffaleti (59) from field observation showed that the vertical velocity profile can accurately be described by the equation -

$$U = 1.15\ \bar{U}\ \left(\frac{y}{h}\right)^{0.155} \quad \ldots \quad \ldots \quad \ldots \quad (3.19)$$

where $\bar{U}$ is the average velocity of flow in the mean depth section.

McCutcheon (40) assumed that the open channel flow carrying suspended sediment was analogous to stratified flow. Under this assumption he showed that if the density profile can be approximated as a straight line, then the velocity profile can be expressed as:

$$\frac{U}{U_*} = \frac{1}{k} \log_e \left[\frac{(y/\epsilon)^2 + \frac{g \alpha \beta}{\rho_f U_*^2} k^2 y^2}{1 + \frac{g \alpha \beta}{\rho_f U_*^2} k^2 y^2} \right]^{1/2} \quad \ldots \quad (3.20)$$

where, α = Monin Obukhov power series co-efficient and

β = Constant density gradient.

But in recent years suggestions frequently has been made that the logrithmic velocity distribution should be applied only to the turbulent wall region or inertial sublayer (23). Most of the objections to the use of the logarithmic equation over the entire depth arise primarily from Prandtl's assumption of a constant stress layer, an assumption that can be replaced by a linear stress distribution and a parabolic distribution of eddy diffusivity, but other functional form can be used better to approximate observed distribution.

From an error-fluction model Willis(65) gave the following equation to describe the vertical velocity profile:

$$\frac{U}{U_*} = \frac{\bar{U}}{U_*} - \frac{3}{k\sqrt{\pi}} \left[\frac{1}{2\sqrt{\pi}} e^{-P^2/2} - P(1-m) - \frac{1}{\sqrt{\pi}} \right] \quad \ldots \quad \ldots \quad (3.21)$$

where, m = is the relative distance from bed

p = is a normalized depth variable

O'Brien (22) used the following equation to describe the velocity distribution in Mississippi river -

$$\frac{U_{max} - U}{U_*} = \frac{1}{K}\left[\log_e\left\{1 - \sqrt{1 - y/h}\right\} + \left(\sqrt{1 - y/h}\right)\right] \quad (3.22)$$

Garde (22) has suggested that a logarithmic velocity distribution of the following form could be used in alluvial stream.

$$\frac{U}{U_*} = \frac{2.3}{K}\log\ (y/K'_s) \quad \ldots \quad \ldots \quad \ldots (3.23)$$

Here K'_s is some length parameter. Indeed K'_s is the distance from the bed at which the above equation will give zero velocity.

Chien (12) suggested that the logarithmic velocity distribution equation for clear water is no longer adequate. A new equation for determining the velocity distribution of flow over alluvial beds was developed by Einstein (52) which is given as:

$$\frac{U}{U_*} = 17.66 + \frac{2.3}{K}\log\frac{y}{35.45\ K_s} \quad \ldots \quad \ldots (3.24)$$

where K is given by figure (3.3). This eouation is shown in figure (3.4). Now for $\frac{U}{U_*} = 0$, $\frac{y}{h} = 0.012$, that is for $\frac{y}{h} < 0.012$, equation (3.24) is no longer valid. This considerations results in a new approximate equation of the form of:

$$\frac{U}{U_*} = 5.75\frac{\sqrt{1 + \frac{\rho_s - \rho_f}{\rho_f}\cdot\frac{1}{d}\int_0^h C'_s\,dy}}{\sqrt{1 + \frac{\rho_s - \rho_f}{\rho_f}\cdot C_0}}\log_e\left[A_e\ \frac{y}{k_s}\right] \quad \ldots (3.25)$$

Where A_e is a constant to be determined and Co is the sediment concentration by weight at the top of the bed layer.

From the discussions made so far, it is now very clear that the velocity distribution along the vertical of a sediment carrying river is not yet fully understood. There are different types of logarithmic velocity distribution equation. But the main point is that at the bottom the velocity become infinity since log 0 is equal to infinity. Moreover, the logarithmic velocity distribution equation develops a number of new parameters which are not constant at all. All of them varys with the sediment concentration, depth of flow bed roughness etc. Again some of the equations are semi-empirical and some are based on experimental results.

3.3.1 κ -Value:

Although Prandtl's mixing length theory was modified to determine the velocity distribution of the flow over movable beds, the assumption that

$$l = \kappa y \quad \ldots \quad \ldots \quad \ldots (3.9)$$

is not valid for flow over movable boundaries. Experimental evidence presented by Vanoni and Namicos (63) showed that for the same discharge the average velocity for sediment laden flow is large and the velocity distribution is less uniform than for clear water flow (Figure 3.5). In this case κ is substantially reduced by the suspended sediment. In other words the turbulent intensity is damped. Vanoni (64) sussgested that a reduction of κ means that mixing is less effective and apparently the presence of sediment suppreses or damps the turbulence. Einstein at al (52) suggested the following explanation. The rate of frictional energy spent in supporting the suspended sediment per unit weight of fluid and unit time is given by

$$\sum \frac{\bar{C}\,\omega_s}{\bar{U}S} \cdot \frac{\rho_s - \rho_f}{\rho_f} \qquad \ldots \qquad \ldots \qquad \ldots \; (3.26)$$

where, $\bar{C}$ = average concentration by weight of a given grain size

$\bar{U}$ = mean flow velocity

S = slope

This argument can be correlated with the κ value and is given in Fig. (3.3). Scatter is evident but neverthless a reasonable correlation is obtained with data from flume studies and river measurements. Chien (12) postulates that the main damping effect of the sediment on turbulence takes place near the bed where the concentration is highest. Vanoni and Nomicos (63) followed this idea and used P_s, the power to suspend the sediment in a thin layer near the bed and found that the data fits with less scatter as shown in figure 3.6. Although these enable us to estimate a new average value of κ it does not follow that the velocity distribution of the mixture is necessarily logarithmic or that the average velocity increases proportionally to change in κ .

Now it is to be noted that although it was found that the value indeed decreased with a concentration increase turbulence measurements indicated an increase of the intensity with an increase in concentration. Recently Hino (27) offered a theory to explain this phenomenon. With two foundamental equations, an energy equation for flow with suspended particles and an acceleration balance equation of turbulent motion, the change of the κ values and of the turbulence intensities can be predicted.

3.3.2 DIFFUSION CO-EFFICIENT:

To determine the distribution of suspended sediment concentration some investigators have used diffusion model. Here it is assumed that the rate of particle movement in a particular direction is proportional to the concentration gradient in the same direction. For vertical sediment movement it can be expressed as:

$$P = \epsilon_s \cdot \frac{\partial c}{\partial y} \quad \ldots \quad \ldots \quad \ldots \quad (3.27)$$

where, P = is the rate of sediment transport accross unit area normal to the y-direction

ϵ_s = sediment transfer co-efficient or the diffusion co-efficient for sediment movement or the mass transfer co-efficient.

Various solutions of diffusion equations assume a direct relationship between the turbulent momentum exchange co-efficient (ϵ_m) and the mass transfer co-efficient (ϵ_s). However, the relationship between the diffusivity of solid particles and the one of linear momentum is proportional and not necessarily identical. Some knowledge could be gained if we know how willingly a solid particle followed its liquid environment. At the present state of knowledge it is possible only to answer in a qualitative way.

The relation between ϵ_s and ϵ_m is generally given as:

$$\epsilon_s = \beta \epsilon_m \quad \ldots \quad \ldots \quad \ldots (3.28)$$

$$\beta = \epsilon_s / \epsilon_m \quad \ldots \quad \ldots \quad \ldots (3.29)$$

By analytical reasoning Graf (20) has shown that

1. For ideal fluids

$$\nu = 0, \quad \frac{B}{A} = \frac{3 \rho_f}{2\rho_s + \rho_f} \text{ and } \quad \beta < 1 \quad \ldots \quad \ldots (3.30)$$

2. For real fluids

$$w \to \infty \; , \; \frac{B}{A} = \frac{3 \rho_f}{2 \rho_s + \rho_f} \text{ and } \quad \beta < 1 \quad \ldots \quad \ldots (3.31)$$

$$w \to 0 \; , \; \frac{B}{A} = \beta = 1$$

$$a \to \infty \; , \; \frac{B}{A} = \frac{3 \rho_f}{2 \rho_s + \rho_f}, \quad \ldots \quad \ldots (3.32)$$

$$a \to 0 \; , \; \frac{B}{A} \to 1$$

where $\frac{B}{A}$ = amplitude ratio and is given by Carstens(14)

$$\left(\frac{B}{A}\right)^2 = \frac{\epsilon_s}{\epsilon_m} = \beta$$

w = circular frequency

ν = kinematic viscosity

ρ_s = density of suspended particles

a = radius of solid suspended particles.

Thus for sediment in water it is not at all clear when β is equal to smaller than or possibly even larger than unit or

$$\beta \lesseqgtr 1$$

certainly β depends on the frequency and on the particle size (d) and the exact interrelationship is very complicated. But for practical purposes and if w is about constant, one might conclude that:

For fine particles

$\beta \simeq 1$, or $\epsilon_s \approx \epsilon_m$

For coarse particles

$\beta < 1$, or $\epsilon_s < \epsilon_m$

Further ideas as to how the β value changes may be gotten from Householder et al (26) and Jobson et al (30).

Expressions like C(y) or C(x,y) are dependant on the selection of the value of ϵ_s. Therefore, a direct experimental check on ϵ_s is attractive. However, the determination of ϵ_s from Rouse (48) equation leads to much scatter as the measurements C(y) have to be differentiated. In Fig. 3.7 Coleman's data (13) have been plotted, together with the theoretical expressions of $\frac{\epsilon_s}{U_* h} = f(y/h.)$ Similarly Fig. 3.8 shows the plotting of the Enoree river data carried out by Anderson (4). Comparing theoretical and experimental results shows that on the average $\left[\frac{\epsilon_s}{U_* h}\right]_{Exp} > \left[\frac{\epsilon_s}{U_* h}\right]_{theoretical}$ which indicates that $\beta > 1$ and/or, $\kappa > 0.4$. Moreover, it seems that ϵ_s also depends on w/U_*.

The question now arises as to how ϵ_m varies over flow depth. A number of models have been proposed to approximate the distribution of ϵ_m over the flow depth.

Results of the measurements of the transfer co-efficients are few but for the momentum transfer they do indicate approximately the parabolic form as obtained from logarithmic velocity distribution. The distribution of ϵ_s appears to be a little asymmetric with the maximum being somewhat closer to the bed than predicted. The mass transfer co-efficient (ϵ_m) also indicate the above form at least in the lower part of the depth, but they generally do not go to zero at the surface and their values at times depart appreciably from those of momentum transfer. Here the measurements by Coleman (13) are typical as shown in Fig.3.7 and 3.8. Several other equations expressing diffusion co-efficients or ϵ_s are given below:

(a) Schmidt (61), Dobbins (16), Mei (42)

$$\epsilon_s = \epsilon_m = \text{constant} \quad \ldots \quad \ldots \quad \ldots (3.33)$$

(b) Rouse (48)

$$\epsilon_m = \epsilon_s = U_* \kappa y(1-y/h) \quad \ldots \quad \ldots (3.34)$$

(c) Hunt (22): $\epsilon_s = \beta \epsilon_m = \beta U_* y(1-y/h) \ldots \quad \ldots (3.35)$

(d) Jobson and Sayre (29,30)

$$\frac{\epsilon_s}{hU_*} = \alpha_1 \kappa (1-\xi)\xi + \alpha_2 \left(\frac{1-\xi}{0.9}\right)^{\alpha_3} \quad [\xi \geqslant 0.1]$$

$$\ldots (3.36)$$

$$\frac{\epsilon_s}{hU_*} = \alpha_1 \kappa (1-\xi)\xi + \alpha_2 \left(\frac{\xi}{0.1}\right)^{\alpha_3} \quad [\xi \leq 0.1]$$

where, $\xi = y/h$

(e) Coleman (13)

$$\epsilon_s = \epsilon_{max} = \left[\alpha_1 + \alpha_2 \left(\frac{w}{U_*}\right)^{\alpha_3} U_* h\right], \text{ for } y/h > 0.5$$

$$\epsilon_s = 4\left(\frac{y}{h}\right)(1-y/h)\, \epsilon_{max}, \text{ for } \frac{y}{h} < 0.5$$

where, $\alpha_1 = 0.1$, $\alpha_2 = 0.38$ and $\alpha_3 = 4.31$ for flumes and $\alpha_1 = 0.13$, $\alpha_2 = 0.20$ and $\alpha_3 = 2.12$ for natural channels.

CHAPTER - 4

DISTRIBUTION OF SUSPENDED SEDIMENT (REVIEW OF LITERATURE)

4.1 INTRODUCTION:

At low values of average shear stress at the bed of an alluvial channel, the material moves as contact load or saltation load and the stream will have only clear water flow. With further increase in shear stress some of the bed particles are carried into the main stream and thus loose contact with the bed. These particles will travel with velocity almost equal to the flow velocity and they constitute the suspended load.

4.2 MECHANISM OF SUSPENSION:

One of the most interesting problems in mechanics of suspension is to study the exact method by which sediment particles resting on the bed are carried in suspension. Jeffreys (22) has proposed a theory based on hydrodynamic lift. According to Jeffreys, when the lift on a particle is greater than its submerged weight, the particles moves up into the flow. On the otherhand, it is also supposed by some that the turbulent fluctuation near the bed or boundary are responsible for the entrainment of sediment particles in the flow.

Laursen (36) visualized a somewhat different mechanism of sediment entrainment. When a particle is moving either over the surface of the dunes or over any small irregularity on the bed a stage is reached when the particle loses contact with the bed momentarily. If in such a case the gravitational force is small and the flow pattern and the velocity of the particles are such that it can be taken into the main flow and the particle will move into the main flow. In this connection, Laursen concluded that the rate of bed load will govern the rate of suspended load transport.

The sequence of events leading to entraiment of particles into the main flow is hypothesized by Sutherland (55) to be as follows: As the rounded or oval (assumed) shaped eddies approach the bed, they are distorted and the velocity of the fluid within the eddies increases. Such eddies disrupt the laminar sublayer and impinge on the surface layer of the particles. As a result the local shear stress at the spot increases and causes rolling of the particles at the incipient motion condition. At the incipient motion the eddies impinge at one spot once in a while and hence sediment movement is intermittent. At high rate of sediment transport, when eddies impinge on the surface layer of particles often and at a number of places, they exert considerable drag on the particles and accelerate them. Some particles, because of their position or because of their rolling up and over neighbouring particles, project above the mean bed level. In such a position they are likely to be entrained because of the vertical velocity component of fluid within the eddy. Another factor that can assist suspension is the lift on the particles. If the sediment bed is covered with dunes, the bed features aid the entrainment process, the troughs and upstream slopes of the dunes being the most active region.

Bagnold (62) has advanced the idea that the sediment grains can be suspended in a flow by intergranular collisions alone without the aid of turbulence. Due to vertical velocity profile, there will be forward and downward as well as backward and upward momentum xxxxxxxxxxxxxxxxxxxxxxx. Continuous impact of this kind will develope stresses with vertical and horizontal components that will be transmitted to the bed. The horizontal stress will act as a shear stress at the bed. Because grain collisions are important only at the near bed region, such collisions advanced by Bagnold should not be of any consequence in the suspension process in the main body of streams.

One of the approximate quantitative indicator of the mode of transport is (46) the ratio of w/U_*. When

$6 > \frac{w}{U_*} > 2$, this condition signifies bed load.

$2 > \frac{w}{U_*} > 0.6$, this condition signifies saltation

and $0.85 > \frac{w}{U_*} > 0$, this condition signifies suspended load.

From the discussions made so far, it may be concluded that the physics of the process of suspension of particles denser than fluid is still inadequately explained. According to Bagnold (46) it is reasonable to assume that no solid can remain suspended unless at least some of the turbulent eddies have upward velocity components exceeding the fall velocity of the solid.

4.3 THEORIES ON THE DISTRIBUTION OF SUSPENDED SEDIMENT:

In principle, the turbulent suspension of sediment is an advanced stage of saltation and bed load transport and one ought to be able to describe both by one theory but no such theory is available at present and suspension is conventionally treated as a phenomenon of its own. The majority of the analytical treatments are based on the concept of diffusion. These models being kinematic in nature, describe the distribution of the suspended sediment, provided we know the concentration at a reference level. The theories tell us nothing about the mechanism through which the particles are put in suspension or how much suspended sediment at a given flow can carry.

Although energy considerations had been applied to suspended sediment before the diffusion theories were formulated this approach has received little attention. In the following paragraphs discussions of the diffusion theory for the balanced as well as unbalanced contidion together with the energy and stochastic models (for only balanced condition) are taken up.

4.3.1 Diffusion Equation:

To derive the equation for unsteady non-unifor distribution of suspended sediment in a two dimensional steady uniform flow, an expression is developed which states that in time Δt, the flow of sediment into an element of volume minus the flow out is equal to the change in concentration in the volume. Fig. (4.1) indicates the flow of sediment in time Δt into and out of the element of volume in the x and y directions due to flow of water and diffusion. The flow of sediment due to settling under gravitational attraction is denoted by the two terms containing the settling velocity. Now the differential equation for the concentration can be written by equating the contributions of sediment from the x- and y directions to the increase in concentration within the elementary volume in time Δt as follows:

$$\left[-\frac{\partial}{\partial x}(uc) + \frac{\partial}{\partial x}\left(\epsilon_x \frac{\partial c}{\partial x}\right) - \frac{\partial}{\partial y}(vc) + \frac{\partial}{\partial y}\left(\epsilon_y \frac{\partial c}{\partial y}\right) + \frac{\partial}{\partial y}(wc)\right]\Delta x\,\Delta y\,\Delta t = \frac{\partial c}{\partial t}\cdot\Delta x\,\Delta y\,\Delta t \qquad \ldots \quad \ldots \quad \ldots (4.1)$$

Dividing by $\Delta x\ \Delta y \Delta t$ and noting that $\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0$ equation (4.1) can be written as:

$$-U\frac{\partial c}{\partial x} - v\frac{\partial c}{\partial y} + \epsilon_x \frac{\partial^2 c}{\partial x^2} + \frac{\partial \epsilon_x}{\partial x}\cdot\frac{\partial c}{\partial x} + \epsilon_y \frac{\partial^2 c}{\partial y^2} + \frac{\partial \epsilon_y}{\partial y}\cdot\frac{\partial c}{\partial y} + \frac{\partial (wc)}{\partial y} = \frac{\partial c}{\partial t} \qquad \ldots (4.2)$$

Equation (4.2) is the basic and generalized diffusion equation for two dimensional distribution of suspended sediment.

4.3.1.1 <u>Concentration Distribution for Equilibrium Condition:</u>

Under equilibrium or balanced condition the concentration of the suspended sediment at the same depth is constant with respect to time disregarding the turbulent pulsation. Moreover, if the channel dimensions remain constant, it does not change in successive cross-section along the main direction of flow provided that the same depth is always considered.

4.3.1.1.1 Diffusion Equation for Equilibrium Condition:

For steady $\left(\frac{\partial c}{\partial t} = 0\right)$ and uniform (derivative with respect to x is zero) distribution and when mean flow is horizontal equation (4.2) can be written as:

$$\frac{\partial}{\partial y}\left(\epsilon_y \frac{\partial c}{\partial y}\right) + \frac{\partial}{\partial y}(wc) = 0 \qquad \ldots (4.3)$$

Again $\frac{\partial c}{\partial y} \cdot \frac{\partial \epsilon_y}{\partial y}$ is of second order and when the fall velocity is assumed to be constant equation (4.3) reduces to:

$$wC + \epsilon_y \frac{\partial c}{\partial y} = 0 \qquad \ldots \quad \ldots \quad \ldots (4.4)$$

To understand equation (4.4) it is to be noted that material in suspension is subjected to two actions. The first is the action of the upward and downward turbulent velocity component. The second is the gravitational action which causes the settling of sediment which is heavier than water. Equation (4.4) is known as the diffusion equation for suspended sediment under equilibrium condition. Different investigator solved equation (4.4) in different ways which are discussed in the following paragraphs.

Schmidt (61) assumed that both fall velocity (w) and diffusion co-efficient ϵ_y is constant with respect to y and gave

$$\frac{C}{C_a} = e^{-w(y-a)/\epsilon_y} \qquad \ldots \quad \ldots \quad \ldots (4.5)$$

But as we have discussed in chapter 3 that the momentum transfer co-efficient varies with y, the assumption that ϵ_y is constant is a rather bold one. Moreover, from equation (4.5) it is seen that concentration distribution is independent of the velocity profile, which is not true. Hence further refinement of this assumption is required.

Using equation (3.34) Rouse (48) gave equation for the concentration distribution along the vertical as:

$$\frac{C}{C_a} = \left(\frac{D-y}{y} \cdot \frac{a}{D-a}\right)^{Z_o} \quad \ldots \quad \ldots \quad \ldots (4.6)$$

where, $Z_o = \frac{w}{U_* K}$ is the exponent of the sediment distribution equation. Figure (4.2) shows a graph of Rouse equation for several values of the exponent of Z_o. However, Rouse equation raises a number of questions (31).

(a) The solution given in equation (4.6) gives the relative concentration since a reference concentration (C_a) has to be known. The reference level (a) should not be choosen too close to the bed as the differential equation is not valid there.

(b) The main problem exist near the bed. It can easily be seen that $C(0) \to \infty$ which is physically impossible as the concentration can't be larger than the concentration of the loosely packed sand at the bed (31).

(c) The inconsistency near the bed is due to the fact that $\epsilon_s(0) = 0$ has been introduced which means there is no exchange of sediment at the bed.

(d) Definition of the location of the bed is difficult. This is obvious if ripples or dunes are present, but even in the case of a plane bed the origin of the Z-axis is difficult to define.

Laursen and Lin in their discussions to Ismail's paper (28) argued that $\epsilon_y = \beta \epsilon_m$ and that the power law of velocity distribution holds good for the whole depth. Thus they gave:

$$\log\left(\frac{C}{C_a}\right) = \frac{\omega(1+1/m)^2}{\beta U_m f}\left[I_\eta - I_\alpha\right] \qquad \ldots (4.7)$$

where,

$$I_\eta = \frac{1}{m}\int_0^\eta \frac{d\eta}{(\eta-1)\,\eta^{(m-1)/m}}$$

and $y/h = \eta$, and $a/h = \alpha$

m being the exponent of the velocity distribution equation. From Toffaleti's field observation (59) it can be said that the velocity distribution taken here is of the same form of equation (3.19) and moreover $\epsilon_y = \beta \epsilon_m$ indicates that the equation (4.7) will give a better sediment distribution than the former equations. But the most bold assumption in this case is that the particle velocity is assumed constant which is not true.

On the otherside Hunt (22) has taken into account the space occupied by the particles suspended in the fluid and has obtained the following relationship for the volume concentration.

$$\left(\frac{C}{1-C}\right)\left(\frac{1-C_a}{C_a}\right) = \left\{\frac{\sqrt{1-y/h}}{\sqrt{1-a/h}}\cdot\frac{A-\sqrt{(1-a/h)}}{A-\sqrt{(1-y/h)}}\right\}^{\frac{\omega}{U_* A K_H}} \qquad \ldots (4.8)$$

where A is a constant which has a value slightly smaller than unity and the constant K_H is similar to Karman's universal constant. Equation (4.8) is not used in practice because of its complexity. Moreover Hunt used equation (3.34) for ϵ_y and calculated the velocity gradient from the following equation:

$$\frac{U - U_{max}}{U_*} = \frac{1}{\kappa}\left[\sqrt{1 - y/h} + B \log_e \frac{B - \sqrt{1 - y/h}}{B}\right] \qquad \ldots (4.9)$$

where B is a constant to be determined experimentally. Paintal and Garde (43) concluded that equation (4.8) and Rouse equation (4.6) fitted the flume data equally well and therefore, that equation (4.6) was preferable because of its simplicity.

Einstein and Chien (62) proposed several modifications of the theory on which equation (4.6) is based in an attempt to explain discrepencies between river data and the theory. The modifications all involved changes in the concepts of the details of the turbulent exchange process. This theory is based on the idea of the mixing length which is taken from Prandtl's theory of turbulence as:

$$\overline{v'l} = 2\,\kappa\, U_*\left(\frac{h-y}{h}\right) y \qquad \ldots \qquad \ldots \qquad \ldots (4.10)$$

and

$$l = B_1 \kappa\, y \sqrt{\left(\frac{h-y}{h}\right)} \qquad \ldots \qquad \ldots \qquad \ldots (4.11)$$

where B_1 is a dimensionless factor. Einstein and Chien's differential equation was:

$$wC + \kappa y U_*\left(\frac{h-y}{h}\right)\frac{\partial C}{\partial y} + Nw\kappa y\sqrt{\left(\frac{h-y}{h}\right)}\,\frac{\partial C}{\partial y} = 0 \qquad \ldots (4.12)$$

where, $N = B_1\left(\frac{1}{2} - A_1\right)$

and the solution of the equation is:

$$\frac{C}{C_a} = \left[\frac{1-\sqrt{\frac{h-a}{h}}}{1-\sqrt{\frac{h-y}{h}}}\right]^{\frac{Z_1}{1+N\kappa Z_1}} \left[\frac{1+\sqrt{\frac{h-a}{h}}}{1+\sqrt{\frac{h-y}{h}}}\right]^{\frac{Z_1}{1-N\kappa Z_1}} \left[\frac{\sqrt{\frac{h-a}{h}}+N\kappa Z_1}{\sqrt{\frac{h-y}{h}}+N\kappa Z_1}\right]^{\frac{2Z_1}{N^2\kappa^2 Z_1^2-1}} \qquad \ldots (4.13)$$

where, $Z_1 = \frac{w}{\kappa U_*}$

Here, Einstein and Chien again did the same assumption of Rouse i.e., $\epsilon_y = \beta\,\epsilon_m$. Hence, the limitations of Rouse equation (4.6) are also valid in this case also. Moreover, when y = 0, equation (4.13) gives $C = \infty$, which is not correct. Again when $A_1 = \frac{1}{2}$ or $N = 0$, equation (4.12) then becomes:

$$wC + \kappa y\, U_*\left(\frac{h-y}{h}\right) \text{———} = 0$$

or, $\epsilon_y \cdot \frac{\partial C}{\partial y} + wC = 0$, which is Rouse equation for balanced condition. Therefore it can be said that Einstein and Chien equation is a modified form of Rouse equation or vice-versa.

In conclusion it may be stated that the basic equation of the theory of turbulent sediment transport was derived from the general mass balance equation by introducing certain assumptions. Integration of the basic equation yields an expression for the distribution of concentration. The validity of the results is governed by the assumptions introduced (See figure 4.3). These are as follows:

(a) The process is a steady one

(b) Collisions between the particles do not result in comminution and the particles are of identical size.

(c) The flow may be regarded two dimensional

(d) Variation in concentration occur in the vertical direction only.

(e) The mean value of the vertical component of flow velocity of water is much smaller than the fall velocity and can thus be neglected.

The applicability of the relations derived depends on these assumptions being satisfied.

4.3.1.1.2 <u>The Energy Approach:</u>

Rubey (46) approached the sediment transport by flow as a problem of expenditure of the stream energy and these ideas were further extended by Knapp (34). However, the major development of the concept is due to Bagnold (6,46) although Valinakov (6) proposed his gravitational theory which leads to similar results. Valinakov gave three basic equations for his gravitational theory. They are -

$$\rho g S(1-\bar{C}_v)\bar{v} = (\rho_1-\rho) g \omega \bar{C}_v (1-\bar{C}_v) + \rho \frac{d}{dy}\left[(1-\bar{C}_v)\overline{v'v'}\right]\bar{v} \quad (4.14)$$

where,

$\bar{C}_v$ = mean value of volume concentration

S = relative slope

v = main velocity

V'= fluctuating component of v.

ρ_1 = density of particle

ρ = density of fluid

g = Acceleration due to gravity.

According to him the L.H.S. term of the equation indicates the loss in potential energy over unit length of flow, the second term of the R.H.S. is the work performed by the resistance and the first term representing the work of suspension.

$$\rho_1 g s \bar{C}_v \bar{v} + (\rho_1 - \rho) g w \bar{C}_v (1 - \bar{C}_v) = \rho_1 \frac{d}{dy}\left(\bar{C}_v \overline{v' u'}\right) \bar{v} \quad \ldots \quad (4.15)$$

and

$$g s (1 + a \bar{C}_v) = \frac{d}{dy}\left[(1 + a \bar{C}_v) \overline{v' u'}\right] \quad \ldots \quad \ldots \quad (4.16)$$

Valinakov claims these equation to provide answers to the following two questions:

(a) What is the quantity of a particular sediment material which a fluid stream is capable of carrying.

(b) What is the law controlling the variation of sediment concentration over the depth of flow.

However, the following points are to be noted regarding Valinakovs theory:

(1) For the process to be a steady one, the first term in all equations are zero.

(2) The polydisperse character of the fluid is neglected and the sediment is taken into consideration with the average concentration $\bar{C}_v$ only.

(3) Equation (4.16) is simply the two dimensional momentum transport equation of the combination of water and sediment.

(4) Equation (4.14) and (4.15) are the balanced equation of kinetic energy for the two dimensional cases.

In addition to these several investigators gave objections to the Valinakov's gravitational theory. The objections are summerised by Bogardi (6) in the following way:

Investigators	Limitations or objections to the gravitational theory
1. V.M.Makkareev & A.D.Greshaev	1. Concentration resulting at the surface is zero even in the case of rather high sediment concentration and small particle size.
2. A.D.Greshaev	1. Valikanov's gravitational theory neglects the increment resistance caused by the presence of sediment.
	2. The expression for momentum current and momentum current density in Valikanov's theory was unfounded since the solid and fluid phases pass simultaneously through the area of magnitude equal to the length of the differential element ($\delta x = 1$).
3. A.E.Ivanov	1. Valikanov neglected the relation and the effect of collisions between particles, which may assume significant proportions in suspended sediment transport.

4. E.N. Teverovsky — 1. Valikanov neglected the energy due to turbulence of the streams.

4.3.1.1.3 Statistical Models:

Since suspension is maintain by turbulence which is random by nature it is only natural that distribution of suspended sediment should be subjected to description by probabilistic methods. Again since the equations of diffusion can be put in the form identical with those describing random functions, the present article will give only the references without any detail discussions on the approaches. The studies of suspension which have led to the statistical method started with attempts to relate the particle motion to turbulence. One of the earliest and most quoted theoretical treatment is that by Tchen (Delft Ph.D. Thesis, 1947) (46) which is summerized by Hinze (25). Besides this references for statistical models of approaches can be made to Bayazit (7), Bugliarello and Jackson (8), Chiu and Chen (15), Praded (44), Sarikaya (57), Sayre and Conover (58), Todorovic (60) and Yalin and Sayre (67).

4.4 CONCLUSION:

Wheather diffusional gravitational or statistical, the models are not capable of answering questions like "How much sediment can be suspended in a given flow ?" or what is the mechanism of suspension ? Even within the various theories there are analytical and conceptual problems to overcome. The diffusion co-efficient is a second order tensor, like stress and is not symmetrical in nonisotropic turbulence. The solution depend on the distribution of shear stress and velocity in the flow of mixture and little is known about it. The use of clean water values can hardly be justified from a theoretical point of view because even the mean velocity profile is appreciably changed by the presence of suspended sediment as seen in Chapter-3. Valikanov's gravitational theory also has limitations. Starting from the general dynamic relations G.Karadi (6) succeeded in developing the basic equation of the diffusion and gravitational theories alike and found no substantial difference to exist between the two theories as long as the flow is a steady one considering the fact that diffusion theory better fits the observed data well, the non-equilibrium distribution of suspended sediment phenomena will be discussed in the light of diffusional theory in the following articles.

4.5 REVIEW OF LITERATURE ON NON-EQUILIBRIUM DISTRIBUTION OF SUSPENDED SEDIMENT:

Equation (4.2) is the generalized two dimensional diffusion equation. This is the basic equation used to solve dispersion problems. Sayre (56) discussed in detail the assumptions that go into the derivation and application of this equation. He noted that this equation describes the dispersion process quite well if the dispersent is in solution form or if the solid particles to which it is applied are of low volume concentration so that the volume they occupy is negligible. However applications of equation (4.2) requires simplifying assumptions.

Kalinske (33) neglected the varriation of ϵ_x and ϵ_y with x and y respectively. Also he assumed v = 0, $\frac{\partial^2 c}{\partial x^2} \ll \frac{\partial^2 c}{\partial y^2}$ and $\frac{\partial c}{\partial t} = 0$. Then again he took $\epsilon_s = \epsilon_m$ and obtained the equation as:

$$U \frac{\partial c}{\partial x} = W \frac{\partial c}{\partial y} + \epsilon_s \frac{\partial^2 c}{\partial y^2} \quad \ldots \quad \ldots \quad \ldots (4.17)$$

Equation (4.17) considers the case of steady non-uniform sediment distribution. Kalinske solved this equation for the following boundary conditions:

i. C = 0 at x = 0

ii. C = 0 at y = ∞

and iii. C = constant at y = a = 0

using these boundary conditions Kalinske obtained the solution as:

$$\frac{C}{C_a} = e^{-\omega y/\epsilon_s} - \frac{2}{\pi} \int_0^{\infty} \frac{n}{-B} e^{[B \epsilon_s x/u + \omega y/2\epsilon_s]} \cdot \sin ny \, dy \quad \ldots (4.18)$$

where, $B = -n^2 + \frac{w^2}{4 \epsilon_s^2}$ and x is the downstream distance.

Equation (4.18) reduces to the equilibrium solution when ϵ_s is constant. There is no experimental evidence to verify equation (4.18). Neverthless this equation provides an insight into the factors affecting equilibrium condition.

Dobbins (62) assuming that all derivatives with respect to x in equation (4.2) are zero and that u = 0 and $\epsilon_s = \epsilon_y$ = const. Presented the one dimensional diffusion equation as:

$$\frac{\partial c}{\partial t} = \epsilon_y \frac{\partial^2 c}{\partial y^2} + \omega \frac{\partial c}{\partial y} \quad \ldots \quad \ldots (4.19)$$

Dobbins solved this equation using two boundary conditions:

1. No net transfer of sediment through the water surface i.e., at y = d, $\epsilon_s \frac{\partial c}{\partial y} = -$ Wc and

2. The rate of sediment picks up equals the rate of deposition on the channel bottom. That is at y = a = 0, $\epsilon_s \frac{\partial c}{\partial y} = -$ WC.

The initial condition was obtained from a known consentration distribution. Ultimately he obtained the following solution:

$$\frac{C}{C_a} = e^{-\frac{wy}{\epsilon_s}} + \frac{C_o - C_a}{C_a} \cdot e^{-\frac{wy^2}{\epsilon_s}} \sum \frac{2\alpha_n^2 \omega e^{-\epsilon_s A t}}{\epsilon_s A (Ad + W/\epsilon_s)} \cdot Y_n \quad \ldots (4.20)$$

where, C_o = conc at t = 0, $Y = 0$

$$A = \alpha_n^2 + \frac{w^2}{4\epsilon_s^2}$$

$$Y_n = \cos \alpha_n y + \frac{W}{2\epsilon_s \alpha_n} \sin \alpha_n y$$ and is the same as in the relation:

$$2 \cot d\alpha = \frac{d\alpha}{wd/2\epsilon_s} - \frac{d\alpha/2\epsilon_s}{\alpha}$$

Equation (4.20) reduces after steady state is obtained to $WC + \epsilon_s \frac{\partial c}{\partial y} = 0$

If ϵ_s is constant with $C_o = 0$ equation (4.20) provides a solution for the case of scour in which the original concentration is zero for all values of y. Equation (4.20) has been checked experimentally with good agreement. The experimental set up as used by Dobbins was similar to one used by Rouse (48). Experimental data obtained are compared with values predicted by equation (4.20) and is shown in figure (4.4) and (4.5). From figures (4.4) and (4.5) one may safely conclude the validity of the basic equation

$$\frac{\partial c}{\partial t} + U\frac{\partial c}{\partial x} = w\frac{\partial c}{\partial y} + \epsilon_y \frac{\partial^2 c}{\partial y^2} \quad \ldots \quad \ldots (4.21)$$

and the assumptions regarding the boundary conditions. Dobbins contribution was successfully applied to the design of sedimentation basins by Camp (11).

The approaches discussed so far are based on the uniform turbulence distribution. But in actual streams, the turbulence intensity and thus the diffusivily vary with distance from the bottom. Therefore, in principle two questions must be answered before proceeding to the solution of the fundamental equation of diffusion. These are -

1. Wheather the particle fall velocity is influenced or not by the turbulent fluctuations of the fluid.
2. Wheather the turbulent transfer of discrete particles as well as the transfer of fluid particles is similar or not to the transfer of momentum.

Question (1) is answered in detail in article (2.14) and for question (2) reference should be made to Jobson and Sayre (29,30).

Now, as noted earlier, it is seldom that a river flow experience a sudden change in bed roughness and bed material sands. In such conditions, the following processes take place -

1. The development of internal boundary layer
2. The change in the bed forms of loose boundary
3. The non-equilibrium transport of suspended sediment.

If the first two processes are neglected the problem becomes analogous to the standard heat conduction problem. The basic partial differential equation then is:

$$U\frac{\partial C}{\partial x} = \frac{\partial}{\partial y}\left(\epsilon_y \frac{\partial C}{\partial y}\right) + w\frac{\partial C}{\partial y} \quad \ldots \quad \ldots (4.22)$$

The boundary conditions are as follows:

$C = 0$, for $x \leq 0$, $d > y > a$

$\epsilon_y \cdot \frac{\partial C}{\partial y} + wC = 0$ for, $x > 0$, $y = d$

$C = C_a$ for $x > 0$, $y = a$

Mei (42) obtained an analytical solution to the above equation, assuming that U and ϵ_y are independant of y and constant. The solution is:

$$C(x,y) = e^{-zy/n} - e^{-z^2x/(4Rd - zy/2d)} \sum \frac{e^{q_n x/Rd}}{z/4 - q_n} \cdot \left[\frac{\frac{z}{2}\cdot \sinh\sqrt{q_n}(1 - y/d)\sqrt{q_n}\cosh\sqrt{q_n}(1 - y/d)}{\frac{\partial}{\partial q_n}\left(\frac{z}{2}\cdot\sinh\sqrt{q_n} + \sqrt{q_n}\cosh\sqrt{q_n}\right)}\right] \quad \ldots (4.23)$$

where, $Z = \frac{wd}{D}$, $R = \frac{Vd}{D}$, q_n, $n = 1, 2, 3 \ldots\ldots$ are the roots of the equation

$$-\sqrt{q} = \frac{z}{2}\cdot \tanh\sqrt{q}$$

and D = constant diffusion coeff. The main objections to Meis (42) analysis are:

1. Diffusion co-efficient is assumed to be constant (Art.3.3.2)
2. Velocity variation along the vertical is assumed to be constant which is not true for natural as well as for turbulent flow (Art.3.3).
3. Fall velocity of particles is assumed to be constant which is contrary to fact (Art. 2.1.4).
4. The boundary condition

$$C = 0 \text{ for } x \leqslant 0, \quad d > y > a$$

indicates that values of x may be negetive. But equation (4.23) was not verified experimentally for a negetive x value. Moreover, a negetive x - value is difficult to define physically.

On the otherhand Hjelmfelt and Lenau (23) derived an analytical solution by considering that ϵ_y is given by equation (3.35).However, the boundary condition at the surface is given by:

$$C = 0 \quad \text{at} \quad x > 0, \quad y = d$$

and the solution is:

$$C = \left(\frac{A}{1-A}\right)^{\lambda}\left(\frac{1-Y}{Y}\right)^{\lambda} + 2\sum_{K=1}^{\infty}\frac{\alpha_K \, P(Y,\alpha_K)\, e^{-(\alpha_K^2 - 1/4)X}}{(\alpha_K^2 - 1/4)\ \frac{\partial P(A,\alpha_K)}{\partial \alpha}} \quad \ldots (4.24)$$

where, $X = \frac{\epsilon x}{d}$, $Y = y/d$

$C = c/C_a$, $\lambda = w/\beta K U_*$, $\epsilon = \beta K U_*$ $\beta = \epsilon_y/\epsilon_m$.

$A = a/d$

Similar objections as to Meis (248) analysis can be put forward for Hjelmfelt and Lenau's (23) analysis except that the boundary condition is justified in this case.

Kerssens and et al (32) assumed that $\epsilon_y = \epsilon_m = \kappa U_* d(1-y/d)$ and they adopted Coleman's (13) experimental results on sediment diffusion co-efficient equation 3.37. The concentration profile of their analysis was given by the equation for uniform flow.

$$C_e\ (y) = C_a \left[\frac{A}{1-A}\right]^{\frac{wd}{4\epsilon_{max}}} \cdot e^{-\frac{w}{\epsilon_m}(y-d/2)} \quad \text{for } y/d > 0.5 \qquad \ldots(4.25a)$$

$$C_e\ (y) = C_a \left[\frac{d-y}{y} \cdot \frac{A}{d-A}\right]^{\frac{wd}{4\epsilon_{max}}} \quad \text{for } y/d \leq 0.5 \qquad \ldots(4.25b)$$

Now, equation (4.25b) is similar to Rouse's (48) solution for equilibrium case. This indicates that when $y/d = 0.5$ the suspended sediment distribution is the same as for equilibrium case. Therefore, the same question arises for the validity of equation (4.25b) as was seen in case of Rouse's equation (4.6) in article 4.3.1.1.1. Again, equations (4.23), (4.24),(4.25a) and (4.25b) contains an exponent of the form of the exponent of Rouse's sediment distribution equation. From article (3.3.2) it can be said that the exponent of the equations requires further modifications, to fit correctly the observed data. Therefore, it can be said that to get a true sediment distribution profile. One must consider the following parameters:

1. Velocity as a function of depth (y)
2. Settling velocity as a function of concentration
3. Diffusion co-efficient as a function of Prandtl's mixing length.

CHAPTER - 5

FORMULATION OF THE EQUATION FOR NON-EQUILIBRIUM DISTRIBUTION OF SUSPENDED SEDIMENT

5.1 INTRODUCTION:

From the discussions on the previous chapters it is evident that the existing equations can not reliably give good results. The assumptions made by several authors are sometimes quite contrary to the existing fact. In this chapter several new assumptions are made to solve the generalized diffusion equation.

5.2 FORMULATION:

From equation (4.2) the basic and generalized diffusion equation for the two dimensional distribution of suspended sediment is:

$$-u\frac{\partial c}{\partial x} - v\frac{\partial c}{\partial y} + \epsilon_x \frac{\partial^2 c}{\partial y^2} + \frac{\partial \epsilon_x}{\partial x}\cdot\frac{\partial c}{\partial x} + \epsilon_y\cdot\frac{\partial^2 c}{\partial y^2} + \frac{\partial c}{\partial y}\cdot\frac{\partial \epsilon_y}{\partial y} + \frac{\partial (wc)}{\partial y} = \frac{\partial c}{\partial t}$$

Following Kalinske (33) it is assumed that ϵ_x is very small and can be neglected and also $\frac{\partial^2 c}{\partial x^2} \ll \frac{\partial^2 c}{\partial y^2}$. Therefore equation (4.2) becomes:

$$- U\frac{\partial c}{\partial x} - v\frac{\partial c}{\partial y} + \epsilon_y\frac{\partial^2 c}{\partial y^2} + \frac{\partial \epsilon_y}{\partial y}\cdot\frac{\partial c}{\partial y} + \frac{\partial}{\partial y}(wc) = \frac{\partial c}{\partial t} \qquad \ldots (5.1)$$

For steady case $\frac{\partial c}{\partial t} = 0$ and therefore

$$- U\frac{\partial c}{\partial x} - v\frac{\partial c}{\partial y} + \frac{\partial}{\partial y}\left(\epsilon_y\frac{\partial c}{\partial y}\right) + \frac{\partial}{\partial y}(wc) = 0 \qquad \ldots \qquad \ldots (5.2)$$

or,

$$U\frac{\partial c}{\partial x} = \frac{\partial}{\partial y}\left(\epsilon_y\frac{\partial c}{\partial y}\right) + \frac{\partial}{\partial y}(wc) - v\frac{\partial c}{\partial y} \qquad \ldots \qquad \ldots (5.3)$$

From turbulent theory it can be assumed that v is independant of y and hence:

$$U \frac{\partial c}{\partial x} = \frac{\partial}{\partial y}\left(\epsilon_y \frac{\partial c}{\partial y}\right) + \frac{\partial}{\partial y} (w-v)\, c \qquad \ldots \qquad \ldots (5.4)$$

Now, the term (w-v) gives the net vertical velocity of a particle. If w is greater than v, then the particle will fall at the bottom.

Thus if

$$W_s^{\cdot} = w-v \qquad \ldots \qquad \ldots \qquad \ldots (5.5)$$

then it can be said that equation (5.4) takes care of the turbulent velocity component in the vertical direction, which was not taken into considerations by other investigators. Thus equation (5.4) is

$$U \frac{\partial c}{\partial x} = \frac{\partial}{\partial y}\left(\epsilon_y \cdot \frac{\partial c}{\partial y}\right) + \frac{\partial}{\partial y}(w_s c) \qquad \ldots \qquad \ldots (5.6)$$

To solve equation (5.6) several boundary conditions consistent with the ract are assumed. They are:

1. $C = 0$ at $x > 0$, $y \doteq h$ according to Hjelmfelt and Lenau (23).
2. The boundary condition at channel bottom is less apparent than those at the free surface and along the y-axis. Kalinske (33) and Apman and Rumer (3) assumed that concentration is constant along the channel bottom. The same boundary condition is adopted in this case also except that it is assumed that $C = C_a$ = const. at $y = a$, where a is a constant and greater than zero.

From chapter -3 it is seen that Toffaleti (59) gave a simple equation (Eq. 3.19), which described the flow velocity variation along the vartical quite well. The same form of ecuation is adopted here that is

$$\frac{U}{\bar{U}} = D\,(y/h)^n \qquad \ldots \qquad \ldots (5.7)$$

where, U = velocity at a depth of from the bottom

$\bar{U}$ = Average velocity

h = Depth of flow

n = an exponent

D = a constant

For diffusion co-efficient Raudkivi's (46) analysis is followed here, which is given as:

$$\epsilon_y = l^2 \frac{du}{dy} \qquad \ldots \qquad \ldots \qquad \ldots (5.8)$$

Now, from Prandtl's theory of turbulence (63) the mixing length is given as -

$$l = B_1 \kappa y \sqrt{1-y/h} \qquad \ldots \qquad \ldots (5.9)$$

where, B_1 = a constant. Hence, from equation (5.8) and (5.9)

$$\epsilon_y = l^2 \frac{d}{dy} = B_1^2 \kappa^2 y^2 \,(1-y/h)\, \frac{\partial u}{\partial y} \qquad \ldots \qquad \ldots (5.10)$$

But from equation (5.7)

$$\frac{\partial u}{\partial y} = \frac{nu}{y} \ldots \qquad \ldots \qquad \ldots (5.11)$$

Therefore, $\epsilon_y = nB_1^2 \kappa^2\, y(1-y/h)\, U \qquad \ldots \qquad \ldots (5.12)$

Fig. (7.1) shows the diffusion distribution from equation (5.12).

Almost all of the investigations on the distribution of suspended sediment were carried out under the assumption that fall velocity of particles are constant. But from article 2.14 it is seen that fall velocity is related to the concentration by a power law. This concept has been used in the present investigation in a modified way, as described below:

Let in figure (5.1) at y = h, the fluid is free of suspended sediment. Therefore fall velocity at this section is equal to the fall velocity in clear water. But as y decreases the suspended sediment content in the flow increases and therefore the fall velocity will be reduced corrospondingly. Therefore it can be said that fall velocity is a function of y. Based upon this reasoning here it is assumed that variation of fall velocity along the vertical can be expressed by an equation of the form of

$$\frac{w_s}{w_o} = (y/h)^{1+n} \qquad \ldots \qquad \ldots \qquad \ldots \quad (5.13)$$

where, n is the same as in equation (5.7)

From equation (5.13) when y = 0, W = 0 which indicates that at y = 0, fall velocity is equal to zero which is a fact, because at the bed of channel particle can not have any downward velocity. Again equation (5.13) also shows that fall velocity is equal to clear water fall velocity when y = h, which support the boundary condition that at y = h concentration, is zero.

Now, differentiating equation (5.13) with respect to y,

$$\frac{\partial w_s}{\partial y} = \frac{(1+n)}{h} \cdot (y/h)^n \cdot w_o \qquad \ldots \qquad \ldots (5.14)$$

That is, $$\frac{\partial w_s}{\partial y} = \frac{(1+n)}{h} \cdot \frac{w_o U}{D\,\bar{U}} \qquad \ldots \qquad \ldots \qquad \ldots (5.15)$$

Now, putting the values from equation (5.7), (5.12) and (5.15) in equation (5.6) one will get,

$$\frac{\partial c}{\partial x} = nB_1^2\kappa^2\, y(1-y/h)\frac{\partial^2 c}{\partial y^2} + nB_1^2\kappa^2\left[(1+n) - (2+n)\,y/h\right]\frac{\partial c}{\partial y} + \frac{w_s}{U}\frac{\partial c}{\partial y} + \frac{c}{U}\cdot\frac{(1+n)}{Dh}\cdot\frac{U}{\bar{U}}\cdot w_o \qquad \ldots \qquad \ldots (5.16)$$

That is,

$$\frac{\partial c}{\partial x} - \frac{(1+n)c\,w_o}{Dh\,\bar{U}} = nB_1^2\kappa^2\, y(1-y)\frac{\partial^2 c}{\partial y^2} + nB_1^2\kappa^2\left[(1+n) - (2+n)y/h\right]\frac{\partial c}{\partial y} + \frac{w_s}{U}\frac{\partial c}{\partial y} \qquad \ldots \qquad \ldots (5.17)$$

Let for simplicity, it is assumed that the ratio of w_s/U is constant and is equal to $$\frac{w_s}{\bar{U}} = nB_1^2\kappa^2\lambda = \frac{w_o}{D\bar{U}} \qquad \ldots \qquad \ldots (5.18)$$

Then equation (5.17) becomes:

$$\frac{\partial c}{\partial x} - \frac{(1+n)\,c\,w_o}{Dh\bar{U}} = nB_1^2\kappa^2\left[y(1-y/h)\frac{\partial^2 c}{\partial y^2} + \left\{(1+n+\lambda) - (2+n)\,y/h\right\}\frac{\partial c}{\partial y}\right] \qquad \ldots \qquad \ldots (5.19)$$

To reduce equation (5.19) into a non-dimensional equation let

$$C = c/c_a$$

$$X = nB_1^2\kappa^2\, x/h$$

$$Y = y/h$$

Then equation (5.19) becomes

$$\frac{\partial C}{\partial X} - (1+n)\lambda C = Y(1-Y)\frac{\partial^2 C}{\partial Y^2} + \left[(1+n+\lambda) - (2+n)Y\right]\frac{\partial C}{\partial Y} \quad \ldots (5.20)$$

with the following boundary conditions:

$C = 0$ for $Y = 1$, $X \geqslant 0$

$C = 0$ for $1 \geqslant Y \geqslant \frac{a}{h} = Y_A$. $X = 0$

$C = 1$, for $Y = Y_A$. $X > 0$

Equation (5.20) can be solved by the classical method of seperating variables. Hence assuming

$C = G(X)\, F(Y)$

One can obtain

$$\frac{G'}{G} - (1+n)\lambda = Y(1-Y)\frac{F''}{F} + \left[(1+n+\lambda) - (2+n)Y\right]\frac{F'}{F} = K^2 \text{ (Say)} \quad \ldots (5.21)$$

Therefore, the two resulting equations are -

$$G' - \left[(1+n)\lambda - K^2\right] = 0 \quad \ldots \quad \ldots \quad \ldots (5.22)$$

and, $$Y(1-Y)F'' + \left[(1+n+\lambda) - (2+n)Y\right]F' + K^2 F = 0 \quad \ldots (5.23)$$

Solution of equation (5.22) is:

$$G = B\, e^{-\left[K^2 - \lambda(1+n)\right]X} \quad \ldots \quad \ldots \quad \ldots (5.24)$$

Now to solve equation (5.23), let

$$\gamma = 1 + n + \lambda$$

$$\alpha = \frac{1+n}{2} + P$$

$$\beta = \frac{1+n}{2} - P$$

and $K^2 = -\alpha\beta = \left[P^2 - \left(\frac{1+n}{2}\right)^2\right]$

then equation (5.23) becomes:

$$Y(1-Y)F'' + \left[\gamma - (\alpha + \beta + 1)Y\right] F' - \alpha\beta F = 0 \quad \ldots \quad \ldots (5.25)$$

Equation (5.25) is a type of the hypergeometric differential equation. Hence the general solution of equation (5.25) can be expressed in terms of hypergeometric function of the form:

$$H[\alpha, \beta, \gamma, Y] = \sum_{m=0}^{\infty} \frac{(\alpha)_m (\beta)_m}{(\gamma)_m} \cdot \frac{Y^m}{\underline{|m}}, \qquad |Y| < 1$$

Where, $(\alpha)_m = \alpha(\alpha+1)(\alpha+2)(\alpha+3) \ldots\ldots\ldots (\alpha+m-1)$ for $m \geqslant 1$ and $(\alpha)_0 = 1$

For the interval $0 < Y_A \leqslant Y \leqslant 1$ the general solution of equation (5.25) is:

$$F = \bar{A}\, H[\alpha, \beta, \alpha+\beta+1-\gamma, 1-Y] + B(1-Y)^{\gamma-\alpha-\beta} H[\gamma-\beta, \gamma-\alpha, \gamma-\alpha-\beta+1, 1-Y] \quad (5.26)$$

where $\bar{A}$ and B are arbitrary constants.

Now from the first boundary condition (equation 5.21), it can be seen that value of $\bar{A}$ of equation (5.26) is zero and therefore,

$$F = B(1-Y)^{\gamma-\alpha-\beta} H[\gamma-\beta, \gamma-\alpha, \gamma-\alpha-\beta+1, 1-Y] \qquad (5.27)$$

Combining equations (5.24) and (5.27) the general solution of equation (5.20) can be obtained as -

$$C(X,Y) = B\bar{B}\, e^{-[K^2 - \lambda(1+m)]X} \cdot (1-Y)^{\gamma-\alpha-\beta} H[\gamma-\beta, \gamma-\alpha, \gamma-\alpha-\beta+1, 1-Y] \quad (5.28)$$

Now the interesting point of equation (5.28) is that it can be used for the distribution of suspended load under equilibrium condition also. This is described in the following article.

5.2.1 Equilibrium Condition:

Under equilibrium condition the concentration of suspended sediment at a particular level does not change with distance, that is, $\frac{\partial c}{\partial x} = 0$.

In this case, for equilibrium condition one can write:

$$\frac{\partial G}{\partial X} = 0 \qquad \ldots \qquad \ldots \qquad \ldots (5.29)$$

Equation (5.29) gives: for $\frac{\partial G}{\partial X} = 0$

$$P^2 = \frac{(1+n)^2}{4} + (1+n)\lambda \qquad \ldots (5.30)$$

The 2nd term of the R.H.S. Of equation (5.30) arises due to the variation of fall velocity with depth. Hence in case of equilibrium condition, if the fall velocity is constant, then one should assume that (1+n) is negligible and hence in this case it is neglected and therefore equation (5.30) reduces to:

$$P^2 = \left[\frac{(1+n)}{2}\right]^2$$

$$\text{or} \quad P = \frac{(1+n)}{2} \qquad \ldots \qquad \ldots \qquad \ldots (5.31)$$

Thus the distribution of suspended sediment under equilibrium condition is:

$$C = B\bar{B}\,(1-Y)^{\gamma-\alpha-\beta} \cdot H\left[\gamma-\beta, \gamma-\alpha, \gamma-\alpha-\beta+1, 1-Y\right]$$

$$\text{or, } C(X,Y) = B\bar{B}\,\frac{(1-Y)}{Y^{n+\lambda}} \cdot H\left[-n, 1, 1+\lambda, 1-Y\right] \qquad (5.32)$$

If $a = 1-n$, then one will get

$$C(X,Y) = B\bar{B}\,\frac{(1-Y)^{\lambda}}{Y^{n+\lambda}} \cdot H\left[-n, a+n, 1+\lambda, 1-Y\right] \ldots \qquad \ldots (5.33)$$

Therefore, the distribution of suspended sediment under equilibrium condition can be expressed by the equation:

$$C(X,Y) = B\bar{B}\,\frac{(1-Y)^{\lambda}}{Y^{n+\lambda}} \cdot F_g^{a,1+n}\,(1-Y) \qquad \ldots \qquad \ldots (5.34)$$

Where, $F_g^{a,1+n}\,(1-Y)$ is called the Jacobi Polynomical.

Now, equation (5.34) can be put in form as that of Rouse (48) equation by assuming that at $Y = Y_A$, $C = 1$. Then equation (5.34) reduces to:

$$C(X,Y) = \frac{Y_A^{n+\lambda}}{Y^{n+\lambda}} \cdot \frac{(1-Y)^{\lambda}}{(1-Y_A)^{\lambda}} \cdot \frac{F_g^{a,1+\lambda}\,(1-Y)}{F_g^{a,1+\lambda}\,(1-Y_A)} \qquad \ldots \qquad \ldots (5.35)$$

Neglecting the ratio $\dfrac{F_g^{a,1+\lambda}\,(1-Y)}{F_g^{a,1+\lambda}\,(1-Y_A)}$ one will get

$$C(X,Y) = \frac{Y_A^{\lambda+n}}{Y^{\lambda+n}} \cdot \frac{(1-Y)^{\lambda}}{(1-Y_A)^{\lambda}} \qquad \ldots \qquad \ldots (5.36)$$

If n is taken equal to zero for constant velocity. Then

$$C(X,Y) = \frac{Y_A^{\lambda}}{Y^{\lambda}} \cdot \frac{(1-Y)^{\lambda}}{(1-Y_A)^{\lambda}} \ldots \qquad \ldots \qquad \ldots (5.37)$$

Which is of the form of Rouse (48) equation.

5.2.2 NON-EQUILIBRIUM CONDITION:

The generalized equation for the distribution of suspended sediment under non-equilibrium condition now stands as:

$$C(X,Y) = B\bar{B}\, e^{-[k^2-\lambda(1+n)]X} \cdot H\left[\frac{1+n}{2}+\lambda+P,\ \frac{1+n}{2}+\lambda-P,\ 1+\lambda,\ 1-Y\right] \qquad (5.38)$$

Following Hjelmfelt and and Lenau (23) let equation (5.38) is written as a series of the type of

$$C(X,Y) = \left[\frac{1-Y}{1-Y_A}\right]^{\lambda}\left[\frac{Y_A}{Y}\right]^{n+\lambda}\left[1+\sum_{t=1}^{\infty} e^{-[K_t^2-\lambda(1+n)]X} \cdot H\left[\frac{1+n}{2}+\lambda+P_t,\ \frac{1+n}{2}+\lambda-P_t,\ 1+\lambda,\ 1-y\right]\right] \qquad (5.39)$$

where P_t are the roots of the equation

$$H\left[\frac{1+n}{2}+\lambda+P,\ \frac{1+n}{2}+\lambda-P,\ 1+\lambda,\ 1-Y_A\right] = 0,$$ such that at $Y = Y_A$

$C(X,Y_A) = 1$,. Now let $Q(Y,P_t) = H\left[\frac{1+n}{2}+\lambda+P_t,\ \frac{1+n}{2}+\lambda-P_t,\ 1+\lambda,\ 1-Y\right]$ (5.40)

Therefore equation (5.40) reduces to

$$C(X,Y) = \left[\frac{Y_A}{Y}\right]^{n+\lambda}\left[\frac{1-Y}{1-Y_A}\right]^{\lambda}\left[H\sum_{t=1}^{\infty} A_t\, e^{-[K_t^2-\lambda(1+m)]X} \cdot Q(Y,P_t)\right] \qquad (5.41)$$

Now one of the boundary condition is that at $X = 0$, $C = 0$ and therefore from (5.41) one will get

$$-1 = \sum A_t \cdot Q(Y, P_t) \qquad \ldots (5.42)$$

Multiplying both sides of equation (5.42) by $Y^{n+\lambda}(1-Y)^{-\lambda} Q(Y, P_p)$ one will get

$$\sum A_t \cdot Y^{n+\lambda}(1-Y)^{-\lambda} Q(Y, P_t) Q(Y, P_r) = -Y^{n+\lambda}(1-Y)^{-\lambda} Q(Y, P_r) \qquad \ldots (5.43)$$

Integrating equation (5.43) within the limit Y_A to 1,

$$\int_{Y_A}^{1} A_t \cdot Y^{n+\lambda}(1-Y)^{-\lambda} Q(Y, P_t) Q(Y, P_r)\, dY = -\int_{Y_A}^{1} Y^{n+\lambda}(1-Y)^{-\lambda} Q(Y, P_r)\, dY \qquad \ldots (5.44)$$

In Appendix-I, it has been shown that $Q(Y, P_t)$. $t = 1, 2 \ldots\ldots$ form an orthogonal system with the weight function $Y^{n+\lambda}(1-Y)^{-\lambda}$ and therefore from equation (5.44)

$$A_t = \frac{-\int_{Y_A}^{1} Y^{n+\lambda}(1-Y)^{-\lambda} Q(Y, P_t)\, dY}{\int_{Y_A}^{1} Y^{n+\lambda}(1-Y)^{-\lambda} Q^2(Y, P_t)\, dY}, \quad t = r \ \ldots \qquad \ldots (5.45)$$

From Appendix-II values of the integrals of equation (5.45) are as follows:

$$\int_{Y_A}^{1} Y^{n+\lambda}(1-Y)^{-\lambda} Q(Y, P_t)\, dY = \frac{Y_A^{1+n+\lambda}(1-Y_A)^{1-\lambda} \frac{\partial Q(Y_A, P_t)}{\partial Y}}{\left[P_t^2 - \left(\frac{1-n}{2}\right)^2\right]}$$

and

$$\int_{Y_A}^{1} Y^{n+\lambda}(1-Y)^{-\lambda} Q^2(Y, P_t)\, dY = -\frac{Y_A^{1+n+\lambda}(1-Y_A)^{1-\lambda} \frac{\partial Q(Y_A, P_t)}{\partial P} \cdot \frac{\partial Q(Y_A, P_t)}{\partial Y}}{2 P_t}$$

Therefore, equation (5.45) reduces to

$$A_t = \frac{2P_t}{\left[P_t^{\,2} - \left(\frac{1-n}{2}\right)^2\right] \frac{\partial Q}{\partial P}\,(Y_A, P_t)} \qquad \ldots \qquad \ldots (5.47)$$

Thus the equation for the distribution of suspended sediment under non-equilibrium condition is:

$$C(X,Y) = \left[\frac{1-Y}{1-Y_A}\right]^{\lambda} \left[\frac{Y}{YA}\right]^{n+\lambda} \left[1 + \sum_{t=1}^{\infty} \frac{2\,P_t\, e^{-\left[P_t^2 - \left(\frac{1\pm n}{2}\right)^2 - \lambda(1+m)\right]X}}{\left[P_t^{\,2} - \left(\frac{1-n}{2}\right)^2\right] \frac{\partial Q}{\partial Y}\,(Y_A, P_t)} \cdot Q(Y, P_t)\right]$$

$$\ldots (5.48)$$

CHAPTER - 6

EXPERIMENTAL INVESTIGATION

6.1 INTRODUCTION:

The experimental certification of the analytical solution described in chapter-5 for the distribution of suspended sediment under non-equilibrium condition was made in the River Engineering Laboratory of the Water Resources Engineering Department of BUET. The experimental set up as well as the measuring technique are discussed in the following articles.

6.2 EXPERIMENTAL SET-UP:

Sediment free water was allowed to flow over the 70' x $2\frac{1}{2}$' x $2\frac{1}{2}$' tilting flume bed at the entrance. The sediment free water gradually started picking up sediment from the bed until equilibrium is reached. Water with suspended sediment was transported to a settling tank, where particles in suspension settled down. The clear water (water at the upper level of the tank) was then delivered to the Sump tank attached with the flume for recirculation. Detailed descriptions of the experimental apparatus and auxilliary equipment are made in the following articles with reference to Figure 6.1.

6.2.1 <u>Settling Tank:</u> Two settling tanks of dimensions 40' x 2-5" x 2'-6" and 70'-0" x 7'-6" x 2'-6" were used to collect the water with suspended particles from the flume. The smaller tank may be called an additional settling tank in which small brick baffles of

1' height were used to retard the fluid motion. This tank was located 3' ft above the larger settling tank or the main settling tank. Partially cleared water of the additional tank were transported to the main tank by gravity action through two 8" diameter pipes. Pumps for delivering clear water to the sump tank of the flume were installed at one end of the main settling tank.

6.2.2 Pumps Used: In total four number of pumps were used for the experiment. Two of them were set to pump water into the main channel from the sump tank and the remaining two were used for delivering the clear water from the main settling tank to the sump tank.

6.2.3 Recirculating Tilting Flume: The 70' x 2.5' x 2.5' glass sided tilting flume of the River Engineering Laboratory had been used as the semi-rigid channel. The thickness of the sand filling was 0.5' and this was kept constant for every set of run. To protect the over all movement of the sand bed from the flume a 0.5 ft high steel plate was put at the end of the flume.

6.2.3.1 Discharge and velocity measurements: Flow meters were (one for each line) attached at the pipe lines connecting the flume outlet and the additional settling tank. Thus the discharge through the channel were obtained directly in litre/second. The velocity of different levels of the main channel were determined by a current meter.

6.2.3.2 Determination of concentration: Sampling of water containing suspended particles were made by syphoning method. For this purpose a $\frac{1}{2}$" D glass tube is bent into a L-pattern (See Figure 6.2). The top end of the larger limb of the L-tube was pushed into a rubber tube which terminated into the sampling beaker. Water entered into the lower horizontal limb of the L-tube

was forced to flow upward to be collected at the sampling beaker by creating a vacum in the tube by mouth. The volume of the sample was then measured. Then beaker with the sample in it was put into an oven until all the water had been evaporated leaving behind the particles in the beaker. Then the weight of the particles was taken to find the amount of particle per unit volume of the sample.

6.2.3.3 Bed materials: Sands were used as the bed material of the channel. The thickness of the sand bed was kept 0-5' for every set of investigations. Three types of sands namely Type-I, Type-II and Type-III were used for three sets of investigations. The grain size distribution curves of the bed material sand are shown in Figures 6.3, 6.4 and 6.5.

6.3 PROCEDURE:

Runs were conducted for three different fall velocities of particles (hence three types of bed materials) and for each particular fall velocity investigation again were carried our for three different channel slopes. A single set of investigation involved the following steps:

Step-1: Preparing the channel bed with the desired material at the desired slope.

Step-2: Positioning the sampling tubes (L-tubes) along the centre line of the channel according to the plan shown in Fig.6.6.

Step-3: Starting of the pumps.

Step-4: When the desired condition at the sampling station is reached current meter readings at the sampling sections were taken along the centre line of the channel see Fig.6.6.

Step-5: Collecting samples of sufficient volume by the tubes already installed. Sample from depth of $y/h = 0.1$ was taken by a tube which was not fixed earlier.

Step-6: Recording of temperature and flow meter reading.

CHAPTER - 7

COMPILATION OF DATA AND EVALUATION OF RESULTS

7.1 INTRODUCTION:

The present research was aimed at finding the vertical and longitudinal concentration profiles of suspended sediment under non-equilibrium condition. To do this the two dimensional diffusion equation was solved analytically based on several assumptions. To test the theoretical analysis laboratory experiments were carried out for different set of conditions. The data of the laboratory experiments are given in Table 7-1 to 7-18. The observed velocity and concentration profiles are shown in Figures 7-3 to 7-20. In the following articles detail discussions on the results are made.

7.2 DIFFUSION CO-EFFICIENT:

In this investigation the author developed a new equation to describe the diffusion pattern assuming that the diffusion co-efficient was a function of Prandtl's mixing length l(19). Due to this assumptions, a new constant B_1, had been developed in place of β, which is the ratio of the momentum transfer co-efficient to that of the sediment transfer co-efficient.

The values of the experimental results were put into the authors equation and compared with both the Rouse (3.34) and Coleman equation (3.37) and it was found that the value of B_1 was 1.5. Figure 7.1 shows the plotting between the non-dimensional depth (y/h) against ϵ_s/U_*h drawn on the basis of author's equation (5.12). On this figure, curve drawn on the basis of Rouse and Coleman equation were also drawn for the convenience of comparison with the authors curve. From the figure it could be seen that diffusion co-efficient increased as (y/h) increased and this tendency remained upto $y/h = 0.6$. After this, the value of ϵ_s/U_*h decreased with the increase of (y/h). But curves after colemn's equation showed that ϵ_s/U_*h remains constant for $y/h > 0.5$. Again, when $y/h < 0.5$ Rouse and Coleman's equation give practically the same value of ϵ_s/U_*h.

Figure 7.1 also indicated that for $y/h < 0.6$, the value given by the author's equation was always less than the values obtained by either Rouse or Coleman equation. For example, when y/h is equal to 0.3, the authors equation gives diffusion co-efficient equál to 0.04 which was about 30% less than the values from either Rouse or Coleman equation. But for $y/h > 0.6$, there was practically no difference between the values obtained by the authors equation and the Rouse equation. Since Coleman's equation give a constant ϵ_s/U_*h, the discrepency between the values from authors equation and Coleman equation was much after $y/h > 0.5$. This might be due to the fact that Colemans equation was based solely on the observed data and he did not consider the velocity variation along the vertical. Moreover his flume study was carried out under equilibrium condition of sediment distribution. Regarding the observed diffusion pattern of the Enoree river by him (Figure 3.8), it could be said that the values of the ratio of fall velocity to shear velocity were high and in this cases equilibrium of concentration occured rapidly. Moreover, as the depth of flow of the river was not more than 5.0 ft, it could be said that mixing of the sediment over the flow depth was uniform and resulted in a constant diffusion co-efficient when y/h is greater than 0.5.

7.3 VELOCITY DISTRIBUTION EQUATION:

Following Laursen and Lin (28), Toffaleti (59) and Scheidegger (53) the author also assumed a power law velocity distribution equation to describe the velocity pattern of a sediment laden channel. In Figures 7.3 to 7.11 curves showing the variation of velocity ratio ($U/\overline{U}$) with depth ratio (y/h) were drawn. In these figures theoretical flow profiles were drawn on the basis of the authors equation while the observed profiles were drawn from the test data given in Table 7.1 to 7.9.

From Figures 7.3 to 7.11 it was apprent that velocity varied with the distance and this went on increasing to the maximum value when the depth ratio was 1. The figures also indicated that the observed and the theoretical velocity profiles practically coincided with each other except near the bottom region. This might be due to the fact that the observed profiles were drawn on the basis of the current meter reading obtained at points away from the channel bed. Since it was very difficult to measure the velocity close to the channel bed, hence no measurements were taken at the bed level zone. Still it could be said that the authors equation could describe the velocity profile for the entire depth of flow which was not possible by the logarithmic velocity distribution equation. As for an example, at the bed level (y/h) = 0 the author's equation gave zero velocity which was assumed to be correct for all practical pruposes. But from logarithmic velocity equation the velocity was infinite at the channel bottom. The following table had been compiled using the data of Table 7.1 to 7.9.

Table 7.19

Multiplying factor and exponent obtained from laboratory tests

	BED MATERIAL-I			BED MATERIAL-II			BED MATERIAL-III		
	SLOPE OF BED			SLOPE OF BED			BED SLOPE		
	0.0	0.0025	.00375	0.0	0.0025	0.00375	0.0	.0025	.0037
Exponent of velocity Equation(a)	0.39	0.39	.441	0.395	.405	.417	.40	.432	.45
Multiply constant (D)	1.21	1.296	1.17	1.24	1.15	1.058	1.19	1.11	1.17

From Table 7.19, it could be said that the multiplying constants (D) of the authors equation varied between a narrow range of 1.058 to 1.296 and their average was 1.177. Toffaleti(59) from field observation found that the multiplying constant was 1.15, which was very close to the authors value. The above table also indicates that the exponent of the velocity equation increased as the slope of the channel was increased for a constant bed material. This was in good agreement with Vanoni's (63) investigation. But Toffaleti (59) found that the exponent of the velocity equation was 0.155 which was about 30 to 40 percent less than the authors values. The reasons for this discrepency might be due to the following conditions:

(a) Toffaleti (59) assumed that the velocity at $y/h = 0.5$ was equal to the average flow velocity

(b) His computation was based on data from Mississippi river only

(c) Flow of the Mississippi river was not strictly two dimensional

(d) In field observation shear due to wind play an important role in determining the velocity distribution.

7.4 FALL VELOCITY:

Considering the fall velocity as a function of depth, the author assumed an equation of the form:

$$\frac{w}{w_o} = \left(\frac{y}{h}\right)^{1+n}$$

to describe the fall velocity of particles in a sediment laden channel. Figure 7.2 is a plotting of the authors equation for fall velocity. Concentration ratio obtained from Rouse equation was put into Maude and Whitemore equation (2.13) to get a relationship between the fall velocity and the vertical distance. This relation was shown in Figure 7.2 for comparison with the authors equation. It is indicated in the figure that fall velocity increased with the distance from the channel bottom. Both the equations showed that fall velocity was zero at $y/h = 0$. This was a clear indication of the fact that concentration of particle at the channel bottom was so high that the particle could not have any falling velocity. Furthermore, the equations showed that the fall velocity was equal to the velocity of a single particle falling under a quiescent condition through a fluid of infinite extent. This was due to the fact that at the free surface level of the channel, the concentration of the particle tended to be zero.

A comparison between the two curves of the figure showed that the authors curve nearly coincided with Maude and Whitmore curve for $y/h < 0.5$. But for the zone $0.5 < y/h < 1$, the authors equation gives a greater value of fall velocity than Maude and Whitmore equation. This might be due to the fact that Maude and Whitmore did not consider the coagulation or flocculation effect of the particles. As the suspended particle settled, they coagulated and their size increased resulting in an increase in the fall velocity.

Equation (5.13) can also be written in the following functional form:

$$\frac{W}{W_O} = f(U) \quad \ldots \quad \ldots \quad \ldots \quad (7,1)$$

where the fall velocity is a function of the flow velocity. Levin (6) derived similar type of equation based on average flow velocity. But the above equation is more rational because of incorporation of point velocity. Using data of Run-5. Levin's equation gives net fall velocity equal to 0.0242 fps, whereas the authors equation (at Y = 0.6) gives fall velocity equal to 0.0238 fps, the discrepency being only 1.42%. However this discrepency increases as the depth ratio (y/h) increased or decreased. This is because Levin (6) assumed that both the fall velocity and the flow velocity, remained constant, which was not true.

7.5 CONCENTRATION DISTRIBUTION:

Figures 7.12 to 7.20 show the concentration variation for constant Y(= y/h). Vertical sediment concentration profiles of the sampling sections are also shown in these curves. From these figures, the curves of concentration variation for constant Y are seen to be vertical at the origin, bending until they are parallel to the channel bottom as equilibrium is reached. The relation between the concentration ratio at a particular depth and the longitudinal distance were given in figure 7.21. From this figure the influence of λ, the ratio of particle fall velocity to the main flow velocity could be observed. As λ is increased due to increased fall velocity or decreased average flow velocity the concentration ratio was also decreased and equilibrium was reached more rapidly. Conversely, concentration ratio increased and equilibrium was approached less rapidly as λ was decreased. Hjelmfelt and Lenau (23) also obtained similar results in their theoretical analysis. (See Figures 7.23 and 7.24). For example an 66.67% decrease of λ caused an 150% increase of relative concentration whereas Hjelmpelt and Lenaus (23) analysis gave approximately 143% increase. The distance required to attain

P% of the equilibrium concentration at a depth of y/h = 0.5 was shown in Fig. 7.22. Similar graphs could be drawn for different depth ratios. The maximum value of x/h occured for $\lambda = 0$, as would be expected from the results of Figure 7.21 and x/h decreased as λ increased. Figure 7.22 indicated that for typical values of λ a considerable length of the channel is required to reach any degree of equilibrium. For example if $w/\bar{U} = 0.06$, P = 0.70% and Y = 0.5, then the distance required would be x/h = 3.9 or x = 3.9h. Hjelmfelt and Lenau (23) developed similar graphs for average concentration over the flow depth.

It was also observed during the laboratory experiments that scouring of the bed at the channel entrance was much more than the downstream end (Plate 1 and 2). The explanation of this behavior could be drawn from Figure 7.25. Near the origin most of the suspended sediment was confined to their boundary layer and the diffusion co-efficient within this layer was smaller than the average value (42). As the boundary layer grow in thickness, the average diffusion co-efficient within the layer increased until it exceeded the channel depth average and the relative magnitude of the scour rates were reversed. Shahjahan (54b) also found similar results and explained the behavior from the competency points of views.

It might be concluded from the above that the present investigations and observations could be used in predicting the concentration profile under non-equilibrium condition.

7.6 CONCLUSIONS:

The experimental works presented in this study has given considerable support to the following conclusions:

1. The analysis presented is suitable for the computation of harizontal as well as vertical concentration profiles, under non-equilibrium and equilibrium conditions.

2. The distance required to reach equilibrium concentration is a function of $W/\bar{U}$ and can be obtained using this analysis. The less is the value of $W/\bar{U}$ the more will be the distance required to reach equilibrium and vice versa.

3. A small increase in $W/\bar{U}$ ratio causes a tremendous decrease in relative concentration.

4. The diffusion co-efficient is a function of Prandtl's mixing length, (l) and the authors equation can be used for all practical purposes.

5. A power law velocity distribution equation is suitable to describe the concentration variation

6. Fall velocity of particles is a function of the distance from the channel bottom and the authors equation can be used to represent the concentration profiles.

7. Erosion of the bed material is less under equilibrium condition compared to erosion of the bed material under non-equilibrium condition.

8. The results of the computations depend on the selection of the representative flow velocity, the particle fall diameter and the reference level (Y_A). These parameters should always be verified by means of prototype measurements.

7.7 SCOPE FOR FURTHER STUDY:

Several avenues of additional study have been opened up by this work and they may be summerized as follows:

i. Since the shape and size of the channel geometry depend on the suspended sediment transport experiments need to be taken up to predict the channel geometry from suspended sediment distribution pattern.

ii. The study have been done under non-tidal condition. The mathematical analysis can be extended for tidal regime under well mixed flow condition.

iii. Test need to be carried out to find the value of the total suspended load transport through a particular section by integrating the equation of vertical concentration profile.

iv. Scour rate study should be carried out on the basis of the present theoretical study.

v. The study should be extended for collection of field data of suspended sediment to correlate with the mean flow velocity.

vi. More study should be carried out to obtain a correct pattern of diffusion distribution.

REFERENCES

1. Agnew, R.P., "Diff. Equations", 2nd ed., McGraw Hill Book Company, Inc., Kogakusha Company Ltd., Tokoyo, Japan,1960.

2. Adamczyk, Z & Van De Van, T.G.M., "Particle Transfer to a Plate in Uniform Flow", Chemical Engg. Science, Vol.37, No. 6, 1982. pp. 869-880.

3. Apmann, R.P. & Rumer, R.R.Jr., "Diffusion of Sediment in Developing Flow", Journ. of the Hydraulics Division, ASCE, Vol.96, No. HY1, Proc. Paper 7018, January 1970, pp.109-123.

4. Anderson, A.G., "Distribution of Suspended Sediment in a Natural Stream", Trans. Am. Gephys. Union, Vol.23, 1942.

5. Brooks, N.H., "Boundary Shear Stress in Curved Channels", a discussion, Proc. ASCE. Vol.89, No. HY3, 1963.

6. Bogardi, J., "Sediment Transport in Alluvial Streams",Akademiai Kiado, Budapest, Hungary. 1978.

7. Bayazit, M., "Random Walk Model for Motion of a Solid Particle in Turbulent Open Channel Flow", Journ. of Hydr. Research, ASCE, Vol. 10, no.1, 1972.

8. Pugliarello, G & Jackson, E.D., "Rnadom Walk Study of Convective Diffusion", Proc. ASCE, Vol.90, EM.4, 1950.

9. Chang F.M. & Richard D.L. "Deposition of Sediment in Transient Flow." JHD, Proc. ASCE, Vol. 97, No. HY6, June, 1971.

10. Calabres , R.V., & Middleman, S., "The Dispersion of Discrete Particles in a Turbulent Fluid Field", A.I.Ch.E. Journal Vol. 25, No.6, Nov. 1979.

11.(a)Camp,T.R., "Sedimentation and the Design of Settling Tanks", Transaction ASCE, Vol. III, 1945.

11.(b)Chang, F.M., et al., "Total Bed Material Discharge in Alluvial Channels", Proceedings 12th Congress IAHR, Fort Collins, Colorado. 1967, U.S.A.

12. Chien,N., "The Present Statutes of Research in Sediment Transport", Proceedings ASCE, Vol. 80, 1954.

13. Coleman, N.L., "Flume Studies of Sediment Transport Co-efficient", Water Resources Research, Vol.6, No.3, June, 1970.

14. Carstens, M.R., "Accelerated Motion of Spherical Particles", Trans, Americah Geophysical Union, Vol.33, 1952.

15. Chiu, C.L., and Chen , K.C., "Stochastic Hydrodynamics of Sediment Transport, "Proc. ASCE, No. 95, EM 5,1969.

16. Dobbins, W.E., "Effects of Turbulence on Sedimentation", Transactions, ASCE, Vol. 109, 1943.

17. Dettman, J.W., "Mathematical Methods in Physics & Engineering McGraw Hill Book Company, Inc. U.S.A., 1962.

18. Exton,H., "Hand book of Hypergeometric Integrals". Ellish Horwood Ltd., A division of John Wiley & Sons, Market Cross House, Cooper Street, Sussex, England, 1978.

19(a) Einstein, H.A., "The Bed Load Function for Sediment Transportation in Open Channel Flows", U.S.Dept. Agric.Soil Conservation Serv., T.B. No. 1026, 1950.

19(b) Einstein,H.A., and Abdel-Aal, E.M., "Einstein Bed Load Function at High Sediment Rate", Journ. of Hyd.Div.,ASCE, Vol. 98, No. HY1,Jounary, pp. 137-152, 1972.

20. Graf, W.H., "Hydraulics of Sediment Transport", McGraw Hill Book Company, U.S.A., 1971.

21. Gilluly, J., Waters, A.C., and Woodford, A.O., "Principles of Geology" 3rd edition, W.H. Freeman & Company, San Francisco, U.S.A., 1968.

22. Garde, R.J. and Raju,K.G.R., "Mchanics of Sediment Transportation and Alluvial Stream Problems", Wiley Eastern Limited, New Delhi, India, 1978.

23. Hielmfelt,A.T & Lenau, C.W. "Non-equilibrium Transport of Suspended Sediment", Journ.of Hyd..Div., ASCE, HY7, 1970.

24. Holeman,J.N., "The Sediment Yield of Major Rivers of the World", U.S. Dept.of Agriculture, Hyatts Ville, U.S.A., 1968.

25. Hinze, J.O., "Turbulence, An Introduction to its Mechanism & Theory", McGraw Hill, U.S.A. pp. 355, 1957.

26. Householder, M.K., and Goldschmidit, V.W., "Turbulent Diffusion and Schmidt Number of Particles", Proc. ASCE, Vol. 95, No. EM 6, 1969.

27. Hino, M., "Turbulent Flow with Suspended Particles", Proc. ASCE, Vol. 89, No. HY4, 1963.

28. Ismail, H.M., "Turbulent Transfer Mechanism & Suspended Sediment in Closed Channels", Trans. ASCE, Vol. 117, 1962 (See also the discussions).

29. Jobson, H.E. and Sayre, W.W., "Predicting Concentration Profile in Open Channels", Journ. of Hyd. Div., ASCE, Vol.96 No.HY10, 1970.

30. Jobson, H.E. and Sayre, W.W., "Vertical Transfer in an Open Channel", Journ. of Hyd. Div., ASCE, Vol.96, No.HY3, 1970

31. Jansen, P.Ph. and others, "Principles of River Engineering-The Non-tidal Alluvial Rivers", Pitman Publishing Limited, 39 Parker Street, London WC2B 5PB, 1979.

32. Kerssens, P.J., Prims Ad. and Rijn, C.C., "Model for Suspend Sediment Transport", Journ. of HYD, No. HY5, May 1979.

33. Kalinske, A.A., "Suspended Material Transportation under Non-equilibrium conditions", Transaction, American Geophysi Union, 1940.

34. Knapp, R.T., "Energy Balance in Stream Flows Carrying Suspended Load", Am. Geophys. Union, Trans. pp. 501-505, 1938.

35. Liu,H.K., and Hwang,S.Y., Proc. ASCE, Journ. of Hyd.Div. Vol.65, No. HY11, 1959.

36. Laursen,E.M., "Total Sediment Load of Streams", Journ. of Hydraulics Division, Proc. ASCE, Vol. 84, No. HY-1,Feb.,

37. Mahmood K. & Yevjevich V., "Unsteady Flow in Open Chann Vol. 1, Water Resources Publication, U.S.A., 1975.

38. Maude, A.D. and Whitmore, R.L., "A Generalized Theory of Sedimentation", Brit. Journ. of Appl. Physics", Vol.9, pp. 477-82, 1958.

39. Modi, P.N. and Seth, S.M., "Hydraulics and Fluid Mecha Standard Book House 1705-A, Nai Sarak, Delhi-6, Indian

40. McCutcheon, S.C., "Vertical Velocity Profiles in Stratif Flow", Journ. of Hydraulic Div., ASCE, Vol. 107, No. HY8, Aug. 1981.

41. McKnown, J.S. and Pin-Nam Lin, "Sediment Concentration Fall velocity", Second Midwestern conference on fluid Mechanics, The Ohio State University, U.S.A. 1952.

42. Mei, C.C., "Non-uniform Diffusion of Suspended Sedime Journ. of Hyd. Div., ASCE, Vol. 95, No. HY1, 1969.

43. Paintal, A.S., and Garde, R.J., Discussion of "Sediment Transportation Mechanics: Suspension of Sediment", by the Task Committee on prepareation of sedimentation manual, Committee on Sedimentation of the Hydraulic Division, Vito. A. Vamoni, Chmn., Journal of the Hydraulic Div., ASCE, Vol. 90, No. HY4, Proc. Paper 3980, July, 1964.

44. Pradad, S.N., "Random Walk Model and Longitudinal Dispersion of Suspended Sediment in Channels", Proc. XV the Congress, IAHR, Vol.1, 1973.

45. Rijn, L.C., "Model for Sedimentation Prediction", Delft Hydraulic Laboratory Publication No. 241, Nov.1980.

46. Raudkivi, A.J. "Loose Boundary Hydraulics", Pergamon Press Ltd., England, 1976.

47. Rathbun, R.E., & Guy, H.P., "Effect of Non-equilibrium Flow Condition on Sediment Transport and Bed Roughness in a Laboratory Alluvial Channels", Proceedings 12th Congress of the IAHR Sept. 11-14, 1967, Vol.1, Colorado State University, U.S.A.

48. Rouse,H., "Modern Conception of Mechanics of Fluid Turbulence", Transaction ASCE, Vol.102, paper no. 1965, 1037, pp.463-543.

49. Richardson, E.V., and Simons, D.B., "Resistance to Flow in Sand Channels", Proceedings IAHR 12th Congress, Fort Collins, Colorado pp. 141-150, 1967.

50. Raudkivi, A.J., "Study of Sediment Ripple Formation", ASCE Hyd. Div., HY6, 1963.

51. McCutcheon, S.C., "Vertical Velocity Profile in Stratified Flow", Journ.of HYD, No. HY8, ASCE, August 1981.

52. Simon D.B. & Senturk, F. "Sediment Transport Technology", Water Resources Publication, U.S.A.,1976.

53. Scheidegger, A.E., "Theoretical Geomorphology", Springer-Verlag Berlin, Germany, 1970.

54(a) Sayre, W.W., And Albertson,L., "Roughness Spacing in Rigid open channel", ASCE Trans,Vol.128, 1963.

54(b) Shahjahan,M., "Factors Controlling the Geometry of Stream Meanders", Ph.D.Thesis submitted to the Dept.of Civil Engg. of Univ. of Strathclyde, U.K. 1970.

55. Sutherland, A.J., "Proposed Mechanism of Sediment Entrainment by Turbulent Flow", Journ. of Geophysical Research, Vol.72, No.24, Dec. 1967.

56. Sayre,W.W., "Dispersion of Mass in Open Channel Flow", Hydraulic Paper, Colorado State University. No.3,73, p.,1968.

57. Sarikaya, H.Z.,"Numerical Model For discrete Settling", Journ.of Hydr. Div., Vol. 103, No. HY8, 1977.

58. Sayre,W.W. and Conover,W.J., "General Two Dimensional Stochastics Model for the Transport and Dispersion of Bed Material Sediment Particle", Proc. 12th Congress IAHR,Fort Collins,1967.

59. Toffaleti, F.B., "Deep River Velocity and Sediment Profiles and the Suspended load", Proc. of the Fedral Inter-Agency Sedimentation Conference 1963, U.S. Dept. of Agriculture, 1965.

60. Todorovic, P., "A Stochastic Model of Dispersion of Sediment Particle Released from a continuous Source", Water Resources Research, Vol. 11, 1975.

61. Vanoni, V.A. "Transportation of Suspended Sediment by Water", Trans. ASCE, Vol. III, 1946.

62. Vanoni, V.A., "Sedimentation Engineering", Prepared by the ASCE Committee for the preparation of the manual on sedimentation of the sedimentation committee of the Hyd. Div., ASCE, 1975.

63. Vanoni, V.A. and Namicos, G.N., "Resistance Properties of Sediment Laden Streams", Trans. ASCE, pp. 3055, 1960.

64. Vanoni, V.A., "Sediment Transportation Mechanics: Suspension of Sediment (Progress Report of Task Committee), Proc. Am. Aoc. of Civil Engrs., Vol. 89, No. HY5, 1963.

65. Willis, J.C., "Suspended Load from Error Function Model", Journal of the Hydraulic Division, HY7, July 1979, ASCE.,

66. Willis, J.C., "An Error Function Discription of the Vertical Suspended Sediment Distribution", Water Resources Research, Vol. 5, No. 6, 1969.

67. Yang, C.T. and Sayre, W.W., "Stochastic Model for Sand Dispersion", Journ. of Hyd. Div., ASCE, Vol. 97, No. HY2, 1971.

TABLE - 7.1

MEASUREMENT FOR VELOCITY PROFILE

Expt.No.1
Date:9.9.83

AVERAGE VELOCITY = 0.70 fps
DISCHARGE = 2.70 cfs
SLOPE = 0.0025
TYPE OF BED MATERIAL = TYPE-I (D_{50} = 0.196 mm)

DIST. FROM Ch.ENTRANCE(ft)	HEIGHT ABOVE BOTTOM(y/h)	VELOCITY (U)(fps)	VELOCITY RATIO ($U/\bar{U}$)
5	0.106	0.34	0.485
	0.320	0.52	0.742
	0.640	0.73	1.040
	0.810	0.76	1.080
10	0.106	0.35	0.500
	0.320	0.59	0.840
	0.640	0.80	1.140
	0.810	0.78	1.110
20	0.106	0.35	0.500
	0.320	0.64	0.914
	0.640	0.72	1.028
	0.810	0.80	1.140
30	0.106	0.37	0.530
	0.320	0.61	0.870
	0.640	0.76	1.085
	0.810	0.83	1.816
45	0.106	0.36	0.514
	0.320	0.60	0.857
	0.640	0.79	1.128
	0.810	0.79	1.128
65	0.106	0.38	0.540
	0.320	0.62	0.880
	0.640	0.75	1.070
	0.810	0.81	1.160

TABLE - 7.2

MEASUREMENT FOR VELOCITY PROFILE

Expt.No.2
Date:10.9.83

DISCHARGE = 2.72 cfs
AVERAGE VELOCITY $\bar{U}$ = 0.725 fps
SLOPE = 0.00357
TYPE OF BED MATERIAL = TYPE-I (D_{50} = 0.195 mm)

DIST. FROM Ch.ENTRANCE(ft)	HEIGHT ABOVE BOTTOM(y/h)	VELOCITY (U)(fps)	VELOCITY RATIO (U/$\bar{U}$)
5	0.11	0.317	0.437
	0.33	0.510	0.703
	0.67	0.780	1.070
	0.83	0.750	1.034
10	0.11	0.316	0.435
	0.33	0.520	0.717
	0.67	0.770	1.060
	0.83	0.790	1.090
20	0.11	0.308	0.424
	0.33	0.570	0.786
	0.67	0.770	1.062
	0.83	0.790	1.090
30	0.11	0.306	0.422
	0.33	0.540	0.750
	0.67	0.760	1.040
	0.83	0.800	1.100
45	1.11	0.333	0.460
	0.33	0.520	0.720
	0.67	0.720	0.990
	0.83	0.820	1.130
65	0.11	0.306	0.420
	0.33	0.510	0.700
	0.67	0.710	0.980
	0.83	0.840	1.160

TABLE - 7.3

MEASUREMENT FOR VELOCITY PROFILE

Expt.No.3
Date:11.9.83

DISCHARGE = 2.70 cfs
AVERAGE VELOCITY = 0.632 fps
SLOPE = 0-0
TYPE OF BED MATERIAL = TYPE-I, (D_{50} = 0.195 mm)

DIST. FROM Ch. ENTRANCE(ft)	HEIGHT ABOVE BOTTOM (y/h)	VELOCITY (U)(fps)	VELOCITY RATIO ($U/\overline{U}$)
5	0.097	0.333	0.53
	0.292	0.487	0.77
	0.585	0.641	1.01
	0.810	0.641	1.01
10	0.097	0.320	0.50
	0.292	0.450	0.71
	0.585	0.660	1.04
	0.810	0.620	0.98
20	0.097	0.300	0.47
	0.292	0.470	0.74
	0.585	0.660	1.04
	0.810	0.670	1.06
30	0.097	0.300	0.47
	0.292	0.480	0.76
	0.585	0.650	1.02
	0.810	0.680	1.08
45	0.097	0.290	0.46
	0.292	0.460	0.73
	0.585	0.560	0.89
	0.810	0.730	1.15
65	0.097	0.310	0.49
	0.292	0.430	0.68
	0.585	0.650	1.02
	0.810	0.760	1.20

TABLE - 7.4

MEASUREMENT FOR VELOCITY PROFILE

Expt.No.4
Date:12.9.83

DISCHARGE = 2.70 cfs
AVERAGE VELOCITY = 0.7045 fps
SLOPE = 0.0025
TYPE OF BED MATERIAL = TYPE-II (D_{50} = 0.235)

DIST. FROM Ch.ENTRANCE(ft)	HEIGHT ABOVE BOTTOM (y/h)	VELOCITY (U)(fps)	VELOCITY RATIO ($U/\bar{U}$)
5	0.108	0.32	0.454
	0.324	0.52	0.740
	0.650	0.73	1.037
	0.810	0.70	0.990
10	0.108	0.39	0.550
	0.324	0.55	0.780
	0.650	0.76	1.070
	0.810	0.82	1.160
20	0.108	0.38	0.540
	0.324	0.58	0.820
	0.650	0.77	1.090
	0.810	0.78	1.100
30	0.108	0.36	0.511
	0.324	0.61	0.860
	0.650	0.73	1.040
	0.810	0.79	1.120
45	0.108	0.36	0.511
	0.324	0.57	0.809
	0.650	0.76	1.080
	0.810	0.78	1.100
65	0.108	0.35	0.496
	0.324	0.58	0.820
	0.650	0.69	0.980
	0.810	0.80	1.135

TABLE - 7.5

MEASUREMENT FOR VELOCITY PROFILE

Expt.No.5
Date:13.9.83

DISCHARGE = 2.70 cfs
AVERAGE VELOCITY = 0.76 fps
SLOPE = 0.00357
TYPE OF BED MATERIAL = TYPE-II (D_{50}= 0.235)

DIST. FROM Ch. ENTRANCE(ft)	HEIGHT ABOVE BOTTOM (y/h)	VELOCITY (U)(fps)	VELOCITY RATIO ($U/\bar{U}$)
5	0.117	0.39	0.51
	0.350	0.58	0.76
	0.700	0.79	1.04
	0.880	0.76	1.00
10	0.117	0.42	0.55
	0.350	0.61	0.80
	0.700	0.81	1.06
	0.880	0.83	1.09
20	0.117	0.40	0.53
	0.350	0.61	0.80
	0.700	0.81	1.06
	0.880	0.84	1.10
30	0.117	0.43	0.56
	0.350	0.63	0.83
	0.700	0.80	1.05
	0.880	0.82	1.08
45	0.117	0.41	0.58
	0.350	0.64	0.84
	0.700	0.80	1.05
	0.880	0.83	1.09
65	0.117	0.41	0.54
	0.350	0.63	0.83
	0.700	0.79	1.04
	0.880	0.84	1.10

TABLE - 7.6

MEASUREMENT FOR VELOCITY PROFILE

Expt.No.6
Date:13.9.83

DISCHARGE = 2.70 cfs
AVERAGE VELOCITY = 0.648 fps
SLOPE = 0-0
TYPE OF BED MATERIAL = TYPE-II (D_{50}= 0.235)

DIST. FROM Ch. ENTRANCE(ft)	HEIGHT ABOVE BOTTOM(y/h)	VELOCITY (U)(fps)	VELOCITY RATIO ($U/\bar{U}$)
5	0.10	0.29	0.44
	0.30	0.37	0.58
	0.60	0.58	0.90
	0.75	0.68	1.05
10	0.10	0.33	0.51
	0.30	0.49	0.76
	0.60	0.69	1.07
	0.75	0.70	1.08
20	0.10	0.36	0.55
	0.30	0.50	0.77
	0.60	0.67	1.03
	0.75	0.74	1.15
30	0.10	0.34	0.53
	0.30	0.49	0.76
	0.60	0.65	1.00
	0.75	0.64	0.98
45	0.10	0.41	0.64
	0.30	0.50	0.77
	0.60	0.69	1.06
	0.75	0.72	1.11
65	0.10	0.39	0.60
	0.30	0.55	0.84
	0.60	0.68	1.05
	0.75	0.75	1.16

TABLE - 7.7

MEASUREMENT FOR VELOCITY PROFILE

Expt.No.7
Date:14.9.83

DISCHARGE = 2.71 cfs
AVERAGE VELOCITY = 0.632 fps
SLOPE = 0-0
TYPE OF BED MATERIAL = TYPE-III(D_{50} = 0.161 mm)

DIST. FROM Ch. ENTRANCE(ft)	HEIGHT ABOVE BOTTOM (y/h)	VELOCITY (U)(fps)	VELOCITY RATIO ($U/\bar{U}$)
5	0.097	0.34	0.54
	0.292	0.50	0.79
	0.585	0.64	1.01
	0.810	0.65	1.05
10	0.097	0.32	0.50
	0.292	0.46	0.73
	0.585	0.67	1.06
	0.810	0.68	1.08
20	0.097	0.30	0.47
	0.292	0.47	0.74
	0.585	0.64	1.01
	0.810	0.66	1.04
30	0.097	0.31	0.49
	0.292	0.46	0.73
	0.585	0.65	1.02
	0.810	0.69	1.02
45	0.097	0.32	0.50
	0.292	0.46	0.73
	0.585	0.65	1.02
	0.810	0.66	1.09
65	0.097	0.33	0.52
	0.292	0.51	0.80
	0.585	0.66	1.04
	0.810	0.71	1.12

TABLE- 7.8

MEASUREMENT FOR VELOCITY PROFILE

Expt.No.8

Date:15.9.83

DISCHARGE = 2.70 cfs

AVERAGE VELOCITY = 0.7045 fps

SLOPE = 0.0025

TYPE OF BED MATERIAL = TYPE-III(D_{50}= 0.161 mm)

DIST. FROM Ch. ENTRANCE(ft)	HEIGHT ABOVE BOTTOM(y/h)	VELOCITY (U)(fps)	VELOCITY RATIO ($U/\bar{U}$)
5	0.108	0.32	0.45
	0.324	0.50	0.71
	0.650	0.74	1.05
	0.810	0.71	1.00
10	0.108	0.32	0.45
	0.324	0.55	0.78
	0.650	0.75	1.06
	0.810	0.76	1.07
20	0.108	0.33	0.46
	0.324	0.56	0.79
	0.650	0.75	1.06
	0.810	0.74	1.05
30	0.108	0.30	0.42
	0.324	0.56	0.79
	0.650	0.74	1.05
	0.810	0.73	1.03
45	0.108	0.31	0.44
	0.324	0.57	0.81
	0.650	0.75	1.06
	0.810	0.76	1.08
65	0.108	0.32	0.45
	0.324	0.56	0.79
	0.650	0.75	1.06
	0.810	0.75	1.06

TABLE - 7.9

MEASUREMENT FOR VELOCITY PROFILE

Expt.No.9
Date:15.9.83

DISCHARGE = 2.70 cfs
AVERAGE VELOCITY = 0.76 fps
SLOPE = 0.00357
TYPE OF BED MATERIAL = TYPE-III(D_{50}= 0.161 mm)

DIST. FROM Ch. ENTRANCE(ft)	HEIGHT ABOVE BOTTOM (y/h)	VELOCITY (U)(fps)	VELOCITY RATIO ($U/\bar{U}$)
5	0.117	0.42	0.55
	0.350	0.58	0.76
	0.700	0.84	1.10
	0.880	0.78	1.02
10	0.117	0.41	0.54
	0.350	0.64	0.84
	0.700	0.86	1.13
	0.880	0.89	1.17
20	0.117	0.40	0.53
	0.350	0.63	0.83
	0.700	0.79	1.04
	0.880	0.86	1.13
30	0.117	0.39	0.51
	0.350	0.64	0.84
	0.700	0.86	1.13
	0.88	0.87	1.14
45	0.117	0.40	0.53
	0.350	0.63	0.83
	0.700	0.85	1.12
	0.880	0.86	1.13
65	0.117	0.41	0.54
	0.350	0.65	0.85
	0.700	0.86	1.13
	0.880	0.90	1.18

TABLE - 7.10

CONCENTRATION MEASUREMENT

Expt.No.1
Date:9.9.83
Temp.29.6°C

DISCHARGE = 2.70 cfs
SLOPE = 0.0025
AVERAGE VELOCITY = 0.70 fps
TYPE OF BED MATERIAL = TYPE-I

$X = nB_1^2 \kappa^2 \frac{x}{h}$
$Y = y/h$
$Y_A = 0.1$
$n = 0.39$
$\lambda = 0.68$

LONGT. DISTANCE(X)	HT.ABOVE BOTTOM(Y)	CONCENTRATION(C)(gm/l)	CONC.RATIO $[C/C_a]$	$\frac{C}{C_a}\left[\frac{Y_A}{Y}\right]^{-(n+\lambda)}$
0.45	0.1	0.8300	1.000	1.00
	0.3	0.1826	0.200	0.70
	0.6	0.0740	0.089	0.50
	0.8	0.0269	0.032	0.30
0.90	0.1	1.6000	1.000	1.00
	0.3	0.3360	0.210	0.68
	0.5	0.1712	0.107	0.60
	0.8	0.0864	0.054	0.50
1.80	0.1	2.6500	1.000	1.00
	0.3	0.6700	0.253	0.82
	0.5	0.3070	0.116	0.65
	0.8	0.1000	0.037	0.35
2.70	0.1	2.9400	1.000	1.00
	0.3	0.7790	0.265	0.86
	0.5	0.3150	0.107	0.60
	0.8	0.1111	0.037	0.35
4.04	0.1	4.1000	1.000	1.00
	0.3	1.0740	0.262	0.85
	0.5	0.4980	0.121	0.68
	0.8	0.1680	0.041	0.38
5.84	0.1	5.6000	1.000	1.00
	0.3	1.5500	0.277	0.90
	0.5	0.6160	0.110	0.62
	0.8	0.2600	0.046	0.43

TABLE - 7.11

CONCENTRATION MEASUREMENT

$n = 0.441$
$\lambda = 0.57$

Expt.No.2
Date:10.9.83
Temp.35.2°C

DISCHARGE = 2.72 cfs
SLOPE = 0.00357
TYPE OF BED MATERIAL = TYPE-I

LONGT. DISTANCE(X)	HT. ABOVE BOTTOM(Y)	CONCENTRATION(C)(gm/l)	CONC.RATIO (C/C_a)	$\frac{C}{C_a}\left[\frac{Y_A}{Y}\right]^{-(n+\lambda)}$
0.529	0.1	0.2890	1.000	1.00
	0.3	0.0664	0.230	0.70
	0.5	0.0312	0.108	0.55
	0.8	0.0107	0.037	0.30
1.050	0.1	0.9090	1.000	1.00
	0.3	0.2545	0.280	0.85
	0.5	0.9270	0.102	0.52
	0.8	0.0389	0.042	0.35
2.110	0.1	1.2500	1.000	1.00
	0.3	0.3375	0.270	0.82
	0.5	0.1837	0.147	0.75
	0.8	0.0625	0.050	0.41
3.175	0.1	1.4350	1.000	1.00
	0.3	0.4104	0.286	0.87
	0.5	0.2100	0.147	0.73
	0.8	0.0738	0.051	0.42
4.760	0.1	2.3600	1.000	1.00
	0.3	0.7220	0.306	0.93
	0.5	0.3510	0.149	0.76
	0.8	0.1290	0.055	0.45
6.870	0.1	2.8600	1.000	1.00
	0.3	0.8950	0.313	0.95
	0.5	0.4500	0.157	0.80
	0.8	0.1610	0.056	0.46

TABLE - 7.12

CONCENTRATION MEASUREMENT

Expt.No.3
Date:11.9.83
Temp. 34.9°C

DISCHARGE = 2.70
AVERAGE VELOCITY = 0.632 fps
SLOPE = 0-0
TYPE OF BED MATERIAL = TYPE-I

$n = 0.39$
$\lambda = 0.72$

LONGT. DISTANCE(X)	HT. ABOVE BOTTOM(Y)	CONCENTRATION(C)(gm/l)	CONC.RATIO (C/C_a)	$\frac{C}{C_a}\left[\frac{Y_A}{Y}\right]^{-(n+\lambda)}$
0.421	0.1	0.128	1.00	1.00
	0.3	0.027	0.21	0.72
	0.5	0.010	0.08	0.51
	0.8	0.003	0.02	0.25
0.842	0.1	0.500	1.00	1.00
	0.3	0.118	0.23	0.80
	0.5	0.049	0.09	0.60
	0.8	0.016	0.03	0.32
1.682	0.1	0.680	1.00	1.00
	0.3	0.162	0.23	0.81
	0.5	0.068	0.10	0.62
	0.8	0.022	0.03	0.34
2.520	0.1	0.766	1.00	1.00
	0.3	0.185	0.24	0.82
	0.5	0.074	0.09	0.59
	0.8	0.026	0.03	0.35
3.800	0.1	0.780	1.00	1.00
	0.3	0.191	0.24	0.83
	0.5	0.080	0.10	0.63
	0.8	0.027	0.03	0.36
5.470	0.1	1.070	1.00	1.00
	0.3	0.284	0.26	0.90
	0.5	0.114	0.10	0.65
	0.8	0.045	0.04	0.42

TABLE 7.13

CONCENTRATION MEASUREMENT

$n = 0.405$
$\lambda = 0.48$

Expt.No.4
Date: 12.9.83
Temp.33.3°C

DISCHARGE = 2.70 cfs
SLOPE = 0.0025
AVERAGE VELOCITY = 0.7045 fps
TYPE OF BED MATERIAL = TYPE-II

LONGT. DISTANCE(X)	HT. ABOVE BOTTOM(Y)	CONCENTRATION(C)(gm/l)	CONC. RATIO (C/C_a)	$\frac{C}{C_a}\left[\frac{Y_A}{Y}\right]^{-(n+\lambda)}$
0.53	0.1	0.1300	1.000	1.00
	0.3	0.0343	0.264	0.70
	0.5	0.0174	0.134	0.56
	0.8	0.0071	0.055	0.35
1.06	0.1	0.1470	1.000	1,00
	0.3	0.0433	0.295	0.78
	0.5	0.0212	0.144	0.60
	0.8	0.0086	0.058	0.37
2.12	0.1	0.3300	1.000	1.00
	0.3	0.1023	0.310	0.82
	0.5	0.0594	0.180	0.75
	0.8	0.0236	0.071	0.45
3.18	0.1	0.6700	1.000	1.00
	0.3	0.2360	0.352	0.93
	0.5	0.1260	0.188	0.78
	0.8	0.0531	0.079	0.50
4.76	0.1	1.2200	1.000	1.00
	0.3	0.4510	0.370	0.98
	0.5	0.2020	0.106	0.69
	0.8	0.0990	0.081	0.51
6.89	0.1	2.1300	1.000	1.00
	0.3	0.7700	0.363	0.96
	0.5	0.3750	0.176	0.73
	0.8	0.1860	0.087	0.55

TABLE 7.14

CONCENTRATION MEASUREMENT

Expt.No.5
Date:13.9.83
Temp. 31.6°C

n = 0.417
λ = 0.43

DISCHARGE = 2.70
SLOPE = 0.00357
AVERAGE VELOCITY = 0.76 fps
TYPE OF BED MATERIAL = TYPE-II

LONGT. DISTANCE(X)	HT. ABOVE BOTTOM(Y)	CONCENTRATION(C)(gm/l)	CONC.RATIO (C/C_a)	$\frac{C}{C_a}\left[\frac{Y_A}{Y}\right]^{-(n+\lambda)}$
0.472	0.10	0.445	1.000	1.00
	0.30	0.114	0.256	0.65
	0.50	0.017	0.102	0.40
	0.80	0.008	0.050	0.29
0.945	0.10	0.700	1.000	1.00
	0.30	0.198	0.284	0.72
	0.50	0.116	0.166	0.65
	0.80	0.047	0.067	0.39
1.890	0.10	0.800	1.000	1.00
	0.30	0.245	0.307	0.78
	0.50	0.139	0.174	0.68
	0.80	0.059	0.074	0.43
2.830	0.10	1.100	1.000	1.00
	0.30	0.368	0.335	0.85
	0.50	0.205	0.187	0.73
	0.80	0.084	0.077	0.45
4.250	0.10	1.300	1.000	1.00
	0.30	0.446	0.343	0.87
	0.50	0.249	0.192	0.75
	0.80	0.113	0.087	0.51
6.140	0.10	1.600	1.000	1.00
	0.30	0.576	0.360	0.91
	0.50	0.132	0.202	0.79
	0.80	0.149	0.093	0,54

TABLE - 7.15

CONCENTRATION MEASUREMENT

Expt.No.6

Date: 13.9.83

Temp. 31.5°C

DISCHARGE = 2.70

SLOPE = 0.00

TYPE OF BED MATERIAL = TYPE-II

$n = 0.395$

$\lambda = 0.534$

LONGT. DISTANCE(X)	HT. ABOVE BOTTOM(Y)	CONCENTRATION(C)(gm/l)	CONC. RATIO (C/C_a)	$\frac{C}{C_a}\left[\frac{Y_A}{Y}\right]^{-(n+\lambda)}$
0.42	0.1	0.104	1.000	1.00
	0.3	0.024	0.216	0.60
	0.5	0.012	0.117	0.52
	0.8	0.004	0.046	0.32
0.85	0.1	0.300	1.000	1.00
	0.3	0.082	0.274	0.76
	0.5	0.041	0.138	0.61
	0.8	0.159	0.053	0.37
1.70	0.1	1.750	1.000	1.00
	0.3	0.525	0.300	0.83
	0.5	0.287	0.164	0.73
	0.8	0.104	0.058	0.41
2.56	0.1	2.100	1.000	1.00
	0.3	0.657	0.313	0.87
	0.5	0.344	0.164	0.73
	0.8	0.130	0.620	0.43
3.84	0.1	2.920	1.000	1.00
	0.3	0.934	0.320	0.89
	0.5	0.467	0.960	0.71
	0.8	0.172	0.059	0.41
5.54	0.1	3.330	1.000	1.00
	0.3	1.020	0.306	0.85
	0.5	0.571	0.171	0.76
	0.8	0.227	0.068	0.47

TABLE - 7.16

CONCENTRATION MEASUREMENT

Expt.No.7
Date: 14.9.83
Temp. 30.6°C

DISCHARGE = 2.70
SLOPE = 0-0
AVERAGE VELOCITY = 0.632 fps
TYPE OF BED MATERIAL = TYPE-III

n = 0.400
λ = 0.324

LONGT. DISTANCE(X)	HT. ABOVE BOTTOM(Y)	CONCENTRATION(C)(gm/l)	CONC.RATIO (C/C_a)	$\frac{C}{C_a}\left[\frac{Y_A}{Y}\right]^{-(n+\lambda)}$
0.421	0.1	0.1250	1.000	1.00
	0.3	0.0310	0.248	0.55
	0.5	0.0175	0.140	0.45
	0.8	0.0069	0.055	0.25
0.840	0.1	0.3120	1.000	1.00
	0.3	0.0958	0.307	0.68
	0.5	0.0535	0.171	0.55
	0.8	0.0263	0.084	0.38
1.680	0.1	0.4800	1.000	1.00
	0.3	0.1620	0.338	0.75
	0.5	0.0970	0.203	0.65
	0.8	0.0400	0.084	0.45
2.520	0.1	0.7930	1.000	1.00
	0.3	0.3045	0.384	0.85
	0.5	0.1800	0.227	0.73
	0.8	0.0912	0.115	0.52
3.800	0.1	1.1200	1.000	1.00
	0.3	0.4540	0.406	0.90
	0.5	0.2720	0.243	0.78
	0.8	0.1187	0.106	0.48
5.480	0.1	2.3500	1.000	1.00
	0.3	0.8810	0.375	0.83
	0.5	0.6016	0.250	0.82
	0.8	0.3290	0.140	0.63

TABLE 7.17

CONCENTRATION MEASUREMENT

$n = 0.432$
$\lambda = 0.27$

Expt.No.8
Date: 15.9.83
Temp. 33.6°C

DISCHARGE = 2.70
SLOPE = 0.0025
AVERAGE VELOCITY = .7035 fps
TYPE OF BED MATERIAL = TYPE-III

LONGT. DISTANCE(X)	HT. ABOVE BOTTOM(Y)	CONCENTRATION (C)(gm/l)	CONC.RATIO (C/C_a)	$\frac{C}{C_a}\left[\frac{Y_A}{Y}\right]^{-(n+\lambda)}$
0.50	0.1	0.145	1.000	1.00
	0.3	0.047	0.323	0.70
	0.5	0.010	0.025	0.55
	0.8	0.013	0.093	0.40
1.00	0.1	0.335	1.000	1.00
	0.3	0.124	0.370	0.80
	0.5	0.073	0.219	0.68
	0.8	0.039	0.116	0.50
2.02	0.1	0.600	1.000	1.00
	0.3	0.250	0,416	0.90
	0.5	0.140	0.233	0.72
	0.8	0.840	0.139	0.60
3.03	0.1	0.920	1.000	1.00
	0.3	0.383	0.416	0.90
	0.5	0.238	0.259	0.80
	0.8	0.135	0.146	0.63
4.54	0.1	0.875	1.000	1.00
	0.3	0.384	0.439	0.95
	0.5	0.240	0.275	0.85
	0.8	0.132	0.150	0.65
6.56	0.1	1.680	1.000	1.00
	0.3	0.691	0.411	0.89
	0.5	0.472	0.281	0.87
	0.8	0.254	0.150	0.65

TABLE 7.18

CONCENTRATION MEASUREMENT

Expt.No.9
Date: 15.9.83
Temp. 32.9°C

DISCHARGE = 2.70 cfs
SLOPE = 0.00357
TYPE OF BED MATERIAL = TYPE-II

$n = 0.45$
$\lambda = 0.24$

LONGT. DISTANCE(X)	HT. ABOVE BOTTOM(Y)	CONCENTRATION(C)(gm/l)	CONC.RATIO (C/C_a)	$\frac{C}{C_a}\left[\frac{Y_A}{Y}\right]^{-(n+\lambda)}$
0.57	0.1	0.163	1.000	1.00
	0.3	0.046	0.281	0.60
	0.5	0.026	0.164	0.50
	0.8	0.015	0.095	0.40
1.14	0.1	0.470	1.000	1.00
	0.3	0.158	0.337	0.72
	0.5	0.092	0.196	0.59
	0.8	0.050	0.107	0.45
2.28	0.1	0.870	1.000	1.00
	0.3	0.346	0.398	0.85
	0.5	0.197	0.227	0.69
	0.8	0.124	0.143	0.60
3.43	0.1	1.19	1.000	1.00
	0.3	0.502	0.422	0.90
	0.5	0.313	0.263	0.80
	0.8	0.172	0.145	0.61
5.14	0.1	1.580	1.000	1.00
	0.3	0.690	0.435	0.93
	0.5	0.447	0.283	0.86
	0.8	0.225	0.142	0.60
7.43	0.1	3.380	1.000	1.00
	0.3	1.480	0.440	0.94
	0.5	0.912	0.270	0.82
	0.8	0.588	0.174	0.73

EXPERIMENTAL DATA

① AFTER SCHILLER

THEORETICAL FORMULAS

② STOKES

③ GOLDSTEIN

④ PROUDMAN et al.

EMPIRICAL FORMULAS

⑤ SCHILLER et al.

⑥ DALLAVALLE

⑦ LANGMUIR et al.

⑧ OLSON

QUASITHEORETICAL FORMULA

⑨ RUBEY

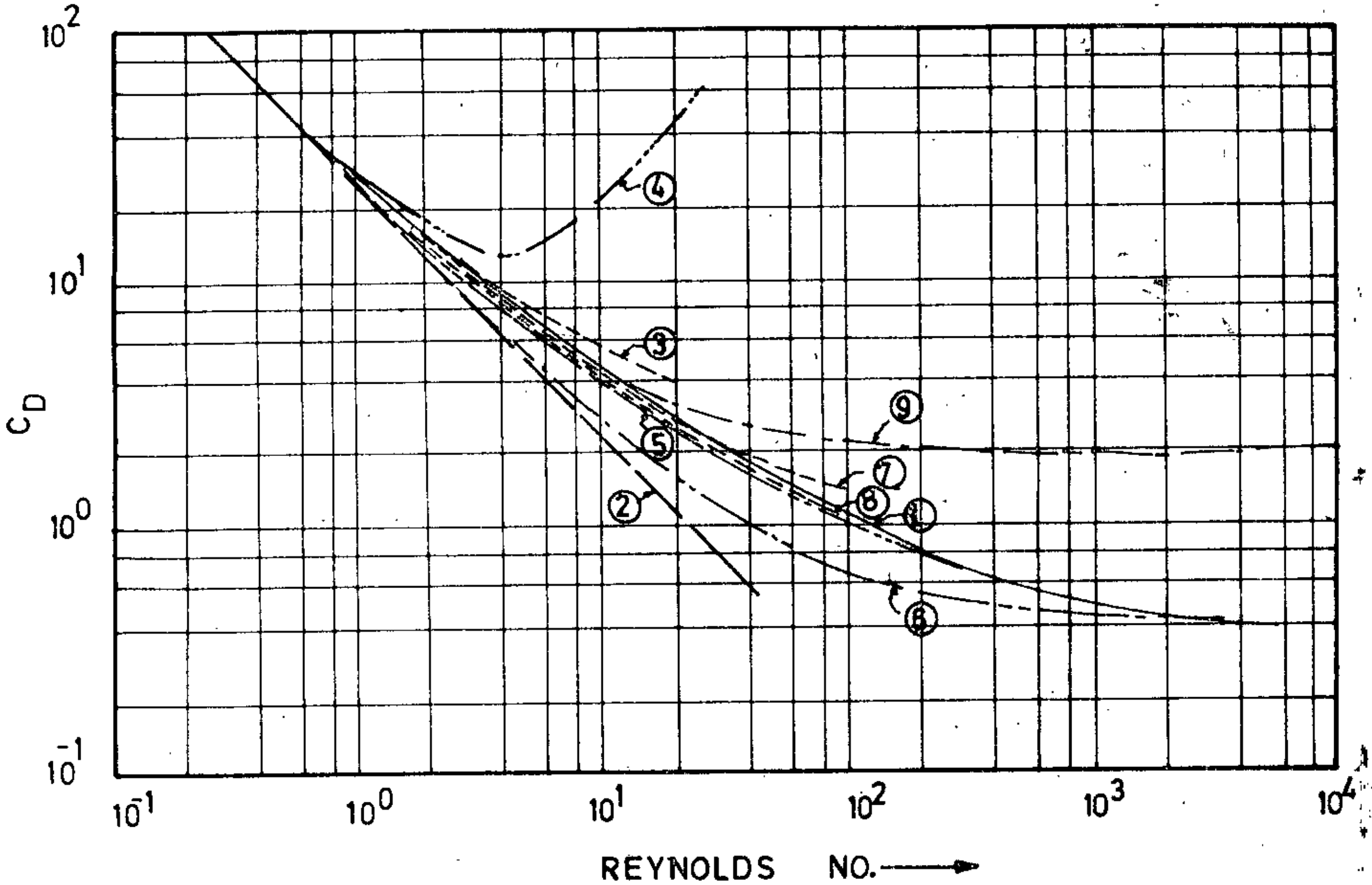

FIG. 2.1 DRAG COEFFICIENT VS. REYNOLDS NUMBER FOR SPHERE; EXPERIMENTAL DATA COMPARED FORMULAS [After Graf et al. (20)]

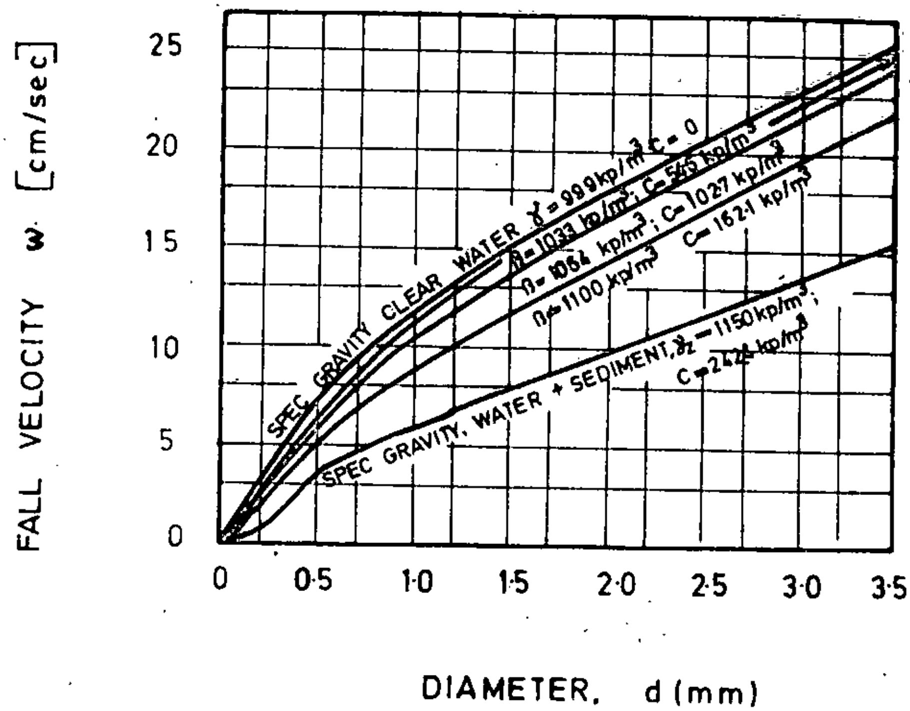

FIG. 2·2 FALL VELOCITY OF QUARTZ PARTICLES IN SEDIMENT CARRYING WATER. [After Schoklitsch (6)]

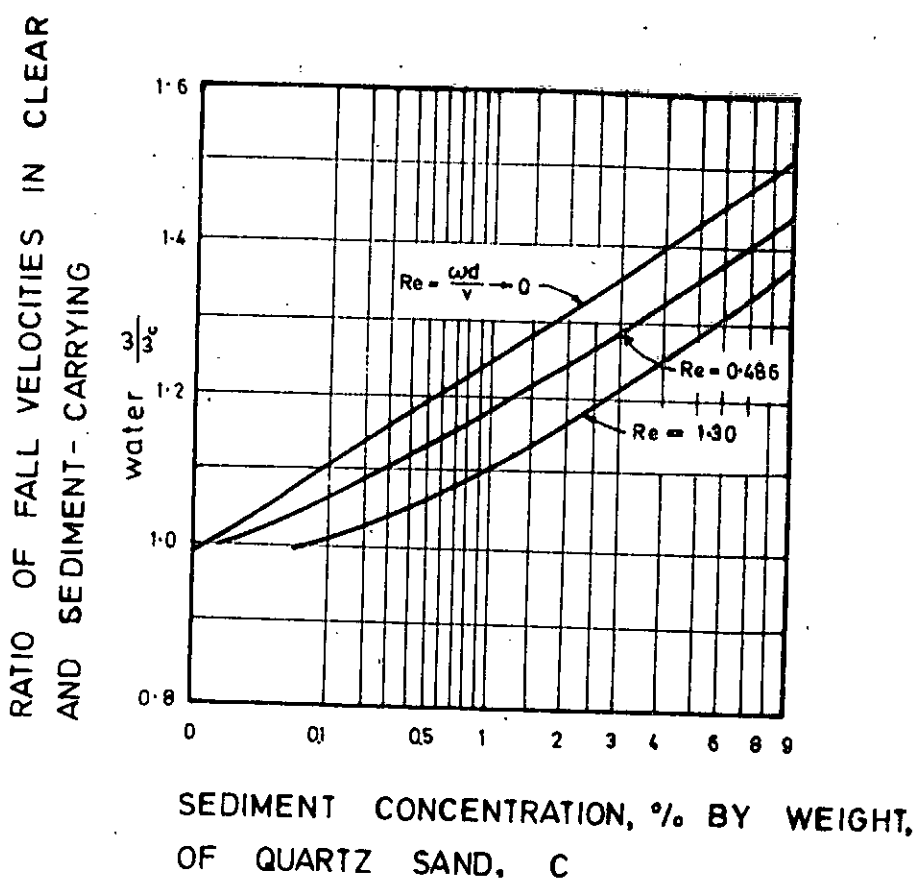

FIG. 2.3 RATIO OF FALL VELOCITIES IN CLEAR AND SEDIMENT CARRYING WATER VS. SEDIMENT CONCENTRATION, WITH THE FALL VELOCITY REYNOLDS NUMBER AS PARAMETER [After McNown and Lin (41)]

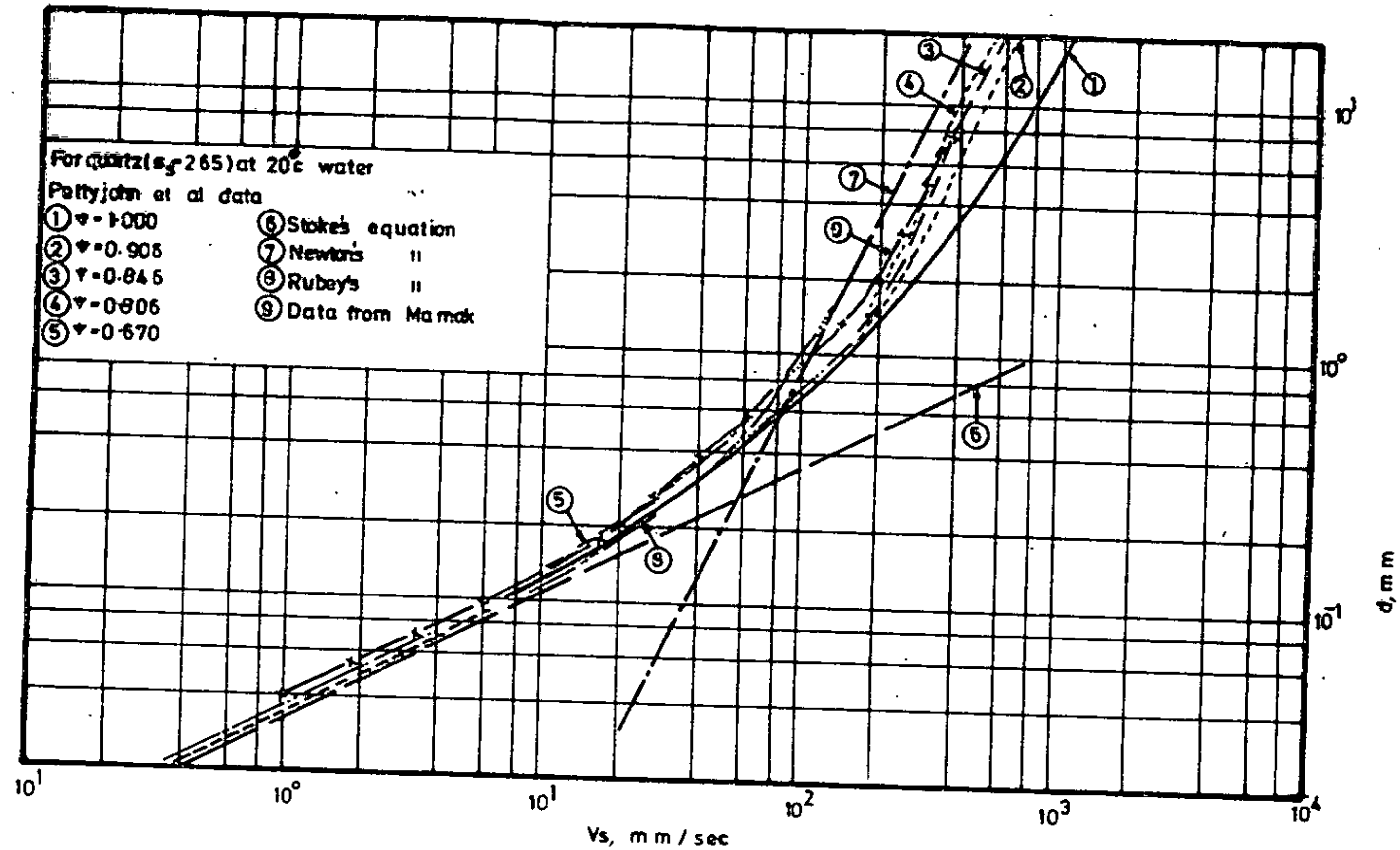

FIG. 2·4. SETTLING VELOCITY VS PARTICLE DIAMETER; VARIOUS EQUATIONS AND SHAPE FACTOR PARAMETERS Ψ. [After GRAF et al. (20)]

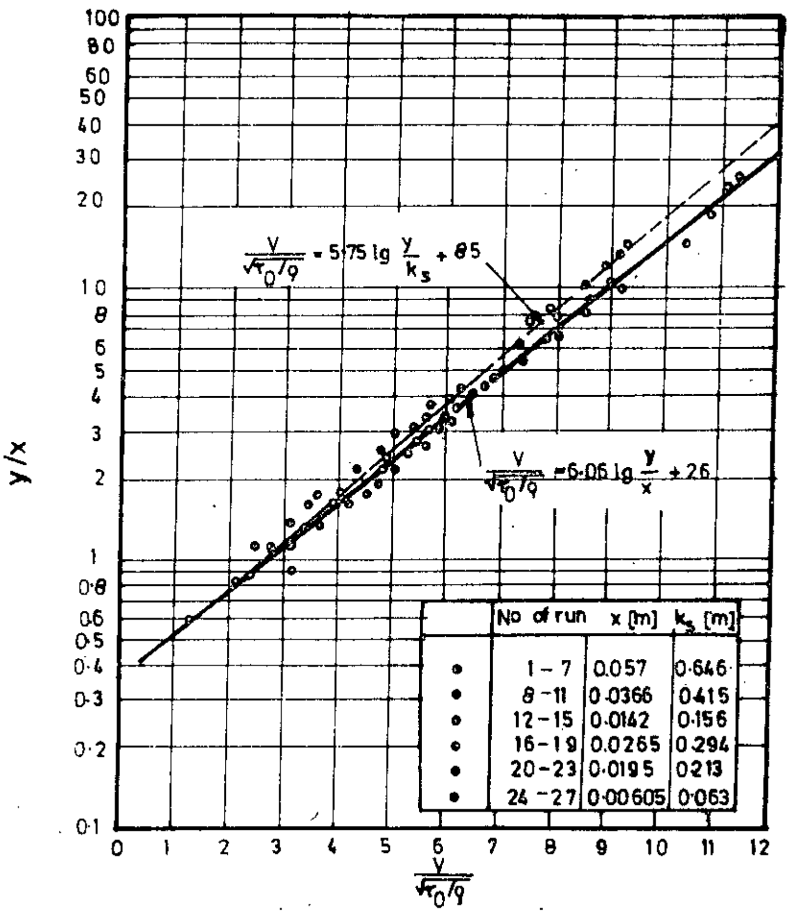

FIG. 3·1 VELOCITY DISTRIBUTION. [After Sayre and Albertson (54)]

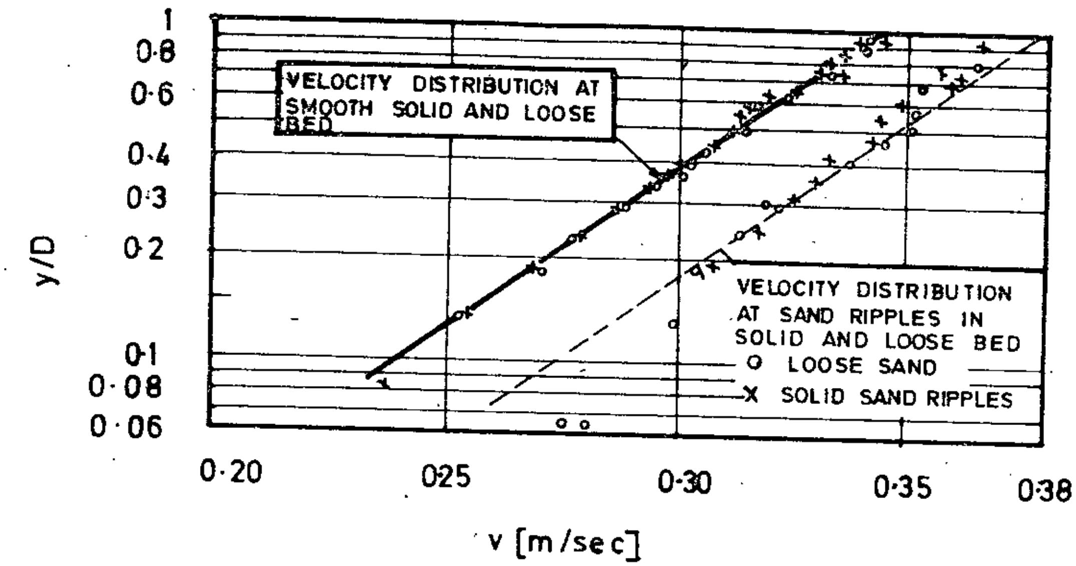

FIG. 3·2 VELOCITY DISTRIBUTION FOR DIFFERENT BED CONFIGURATIONS FOR SOLID AND LOOSE BED [After Raudkivi (50)]

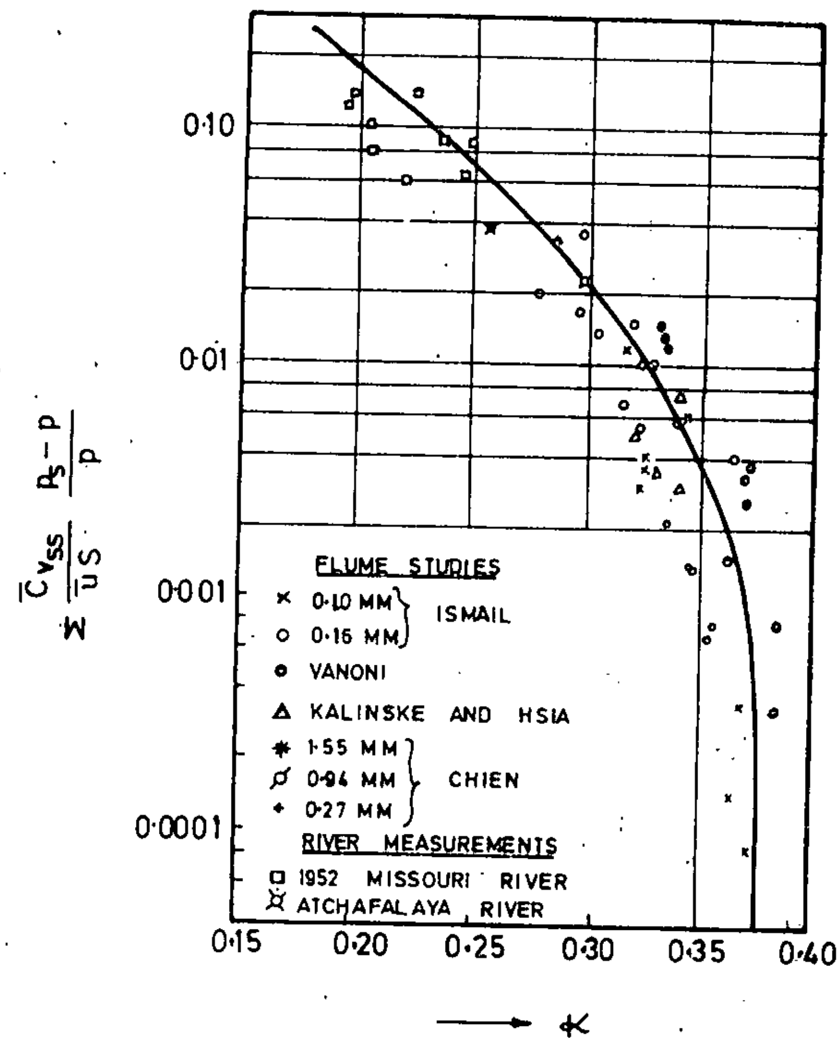

FIG. 3·3 EFFECT OF SUSPENDED LOAD ON THE K VALUE. [After Einstein et. al.(52)]

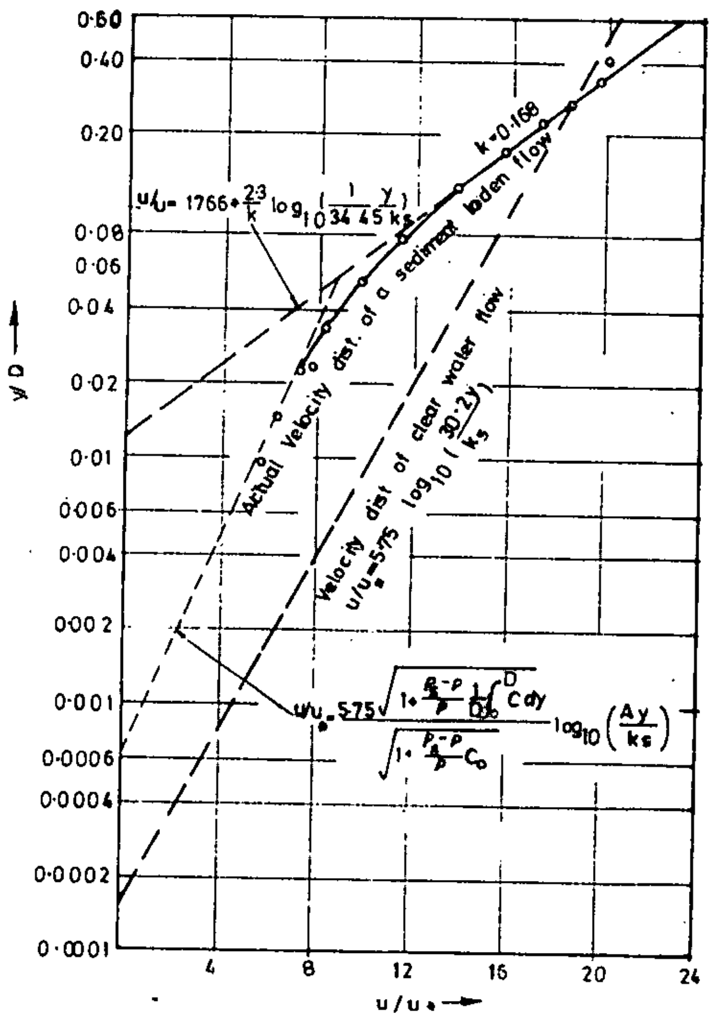

FIG. 3.4. VELOCITY DISTRIBUTION OF CLEAR-WATER AND SEDIMENT LADEN FLOW. [After EINSTEIN et al. (19 b)]

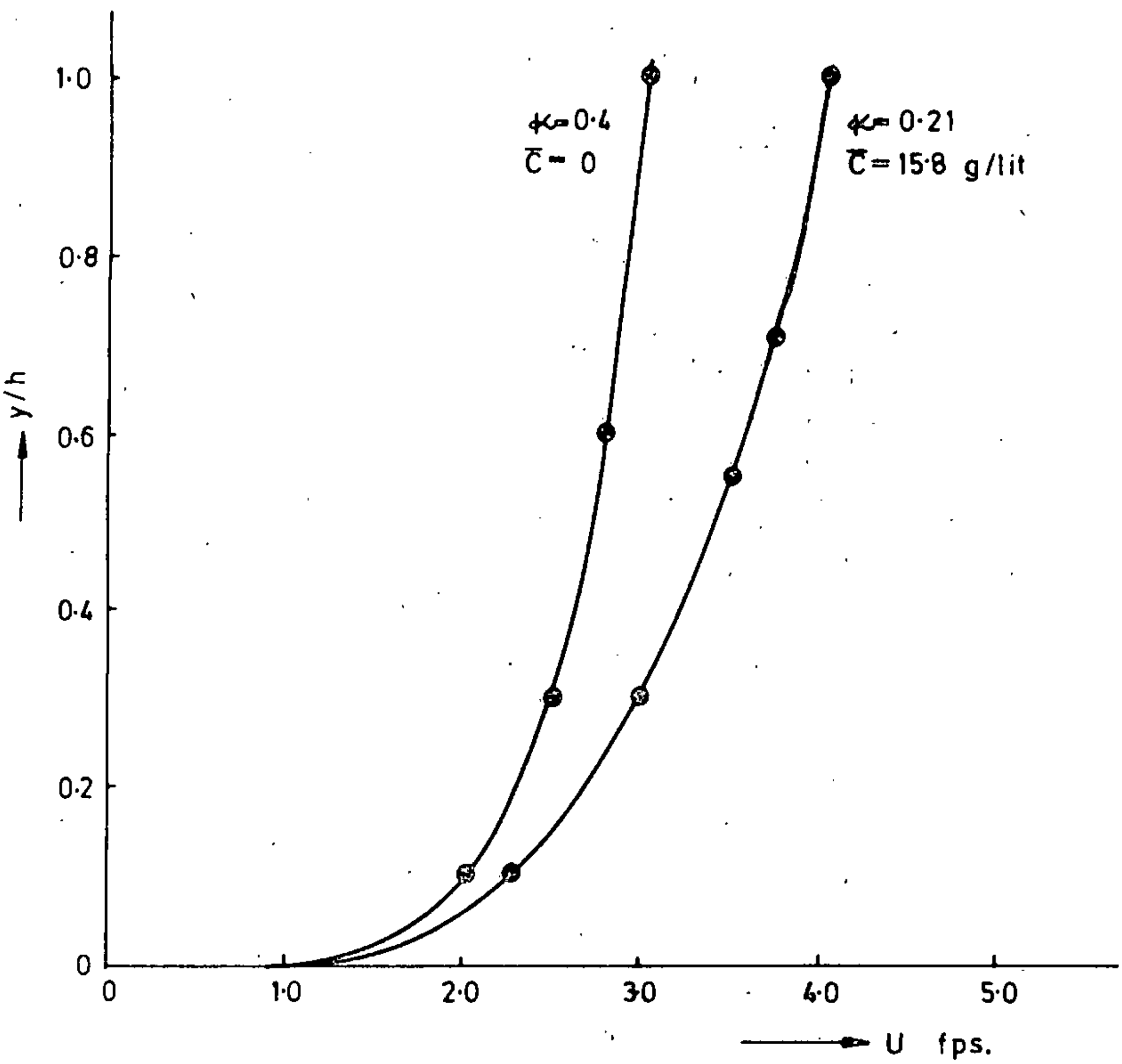

FIG. 3·5 VELOCITY PROFILES FOR CLEAR WATER AND SEDIMENT LADEN FLOW. [After Vanoni et al. (63)]

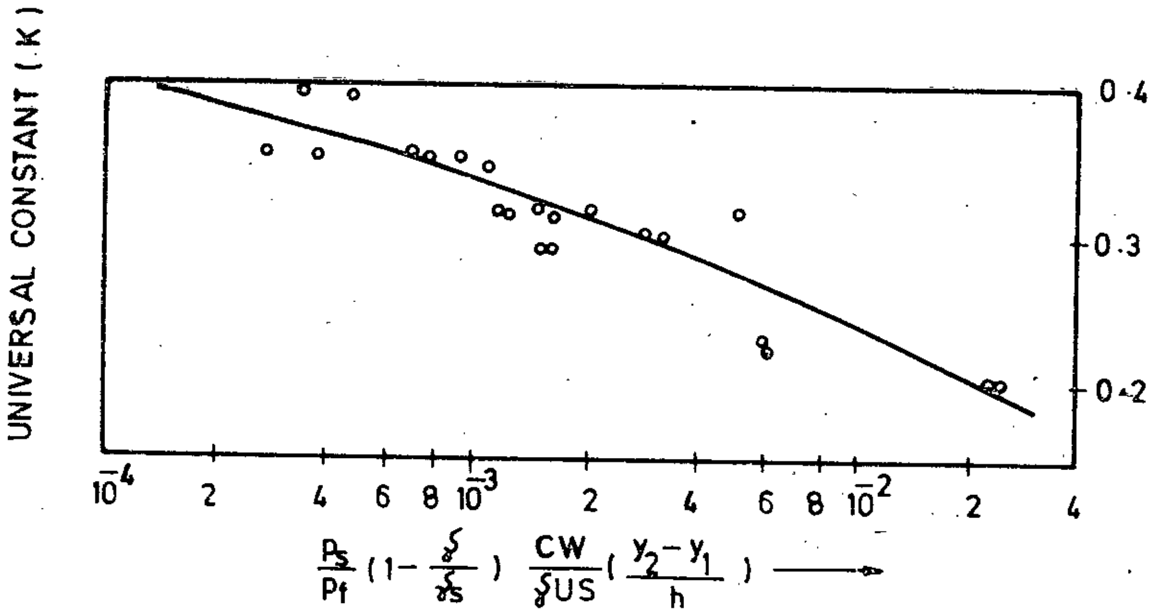

FIG. 3.6 REDUCTION OF von KARMAN CONSTANT IN SEDIMENT LADEN FLOW [Coleman (13)]

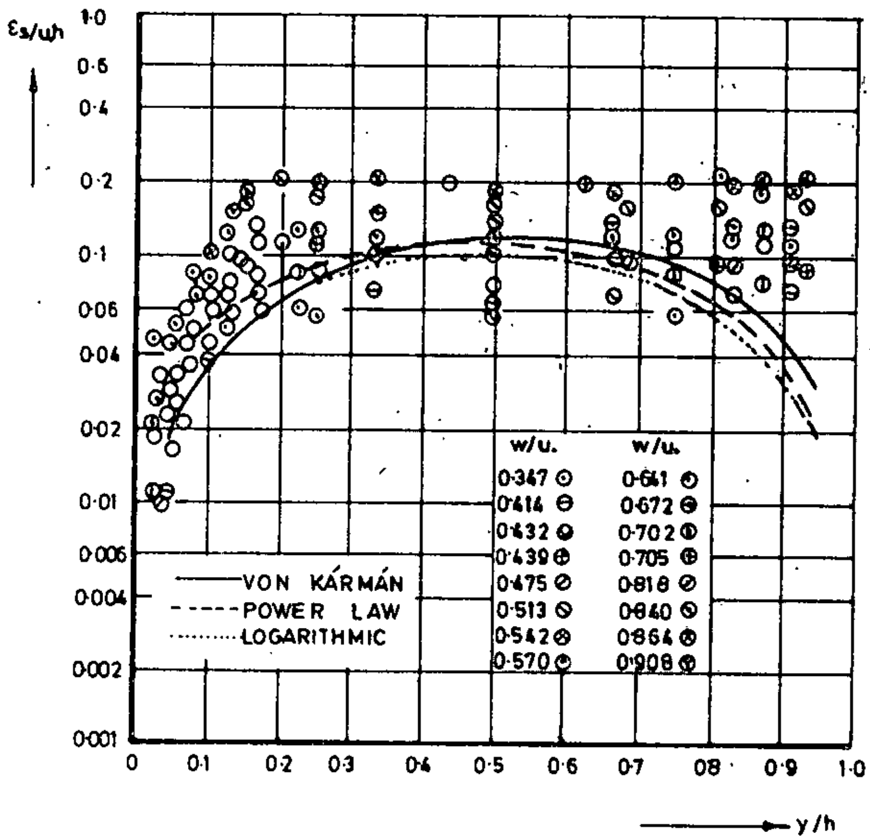

FIG. 3·7. RELATION OF $\epsilon_s/(hu)$ and y/h (flume experiments)
[After Coleman (13)]

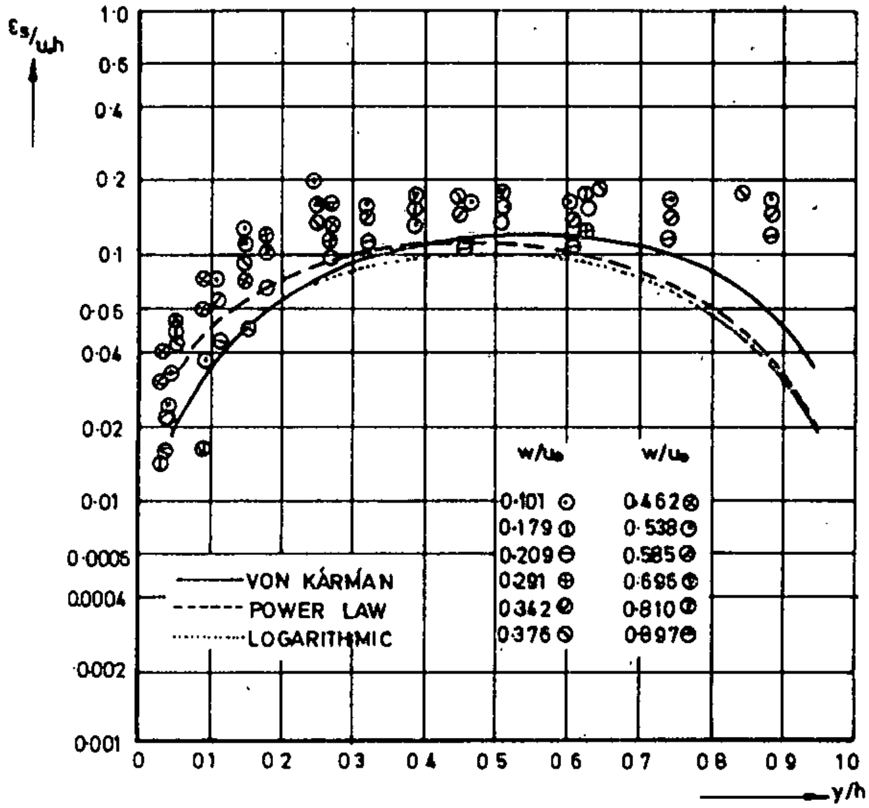

FIG. 3·8· RELATION OF $\epsilon/(hu)$ and y/h (Enoree River)
[After Coleman (13)]

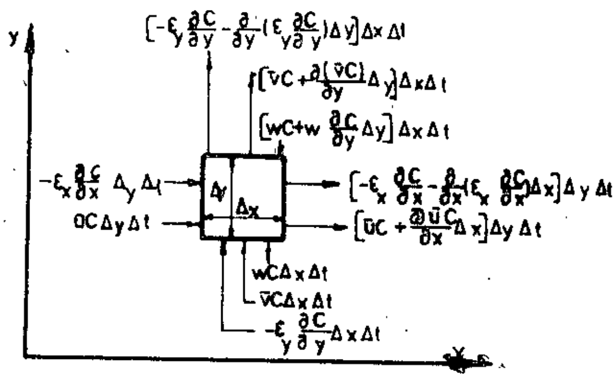

FIG. 4.1. UNSTEADY AND NONUNIFORM DISTRIBUTION OF SUSPENDED SEDIMENT (Two-dimensional flow).

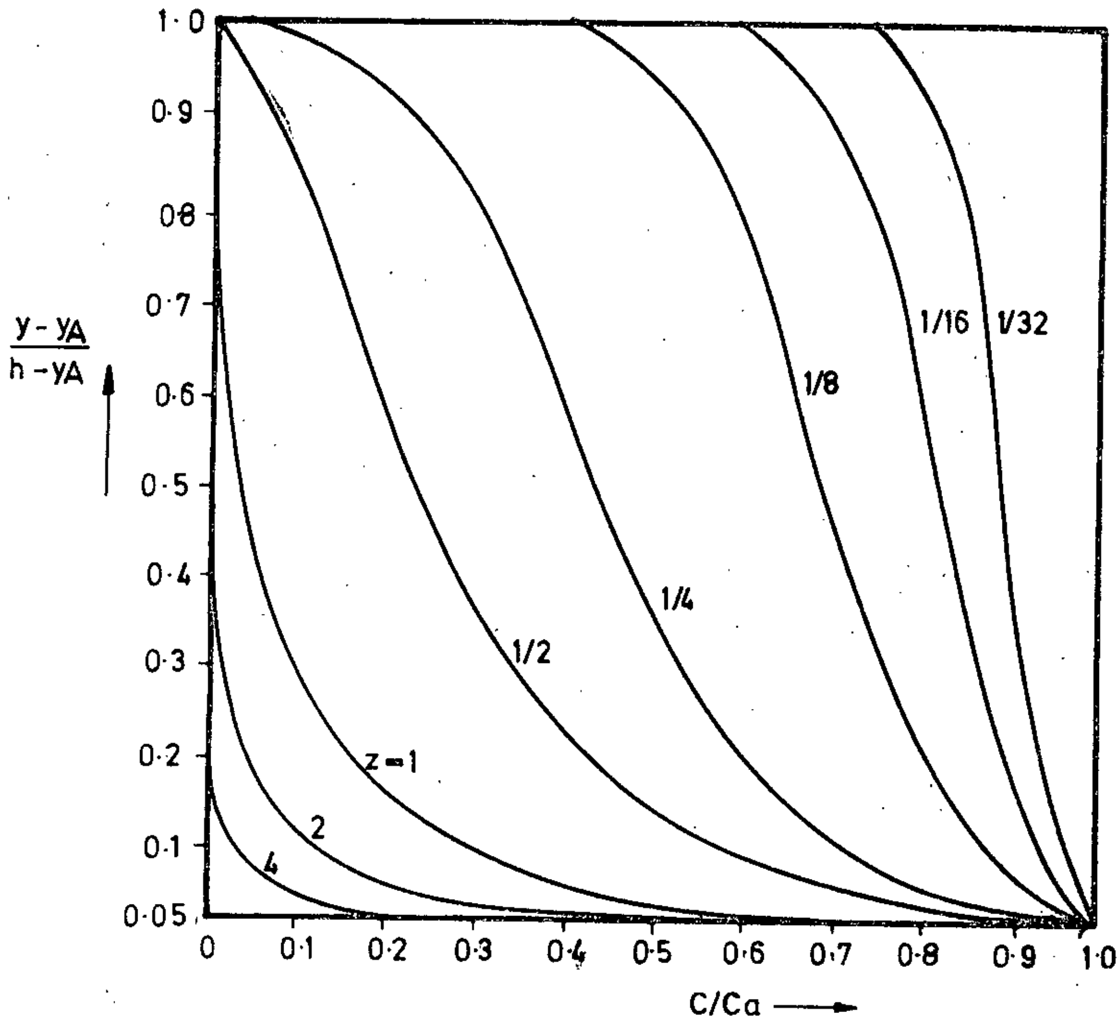

FIG. 4·2 DISTRIBUTION OF SUSPENDED LOAD IN FLOW FOR EQUILIBRIUM CONDITION

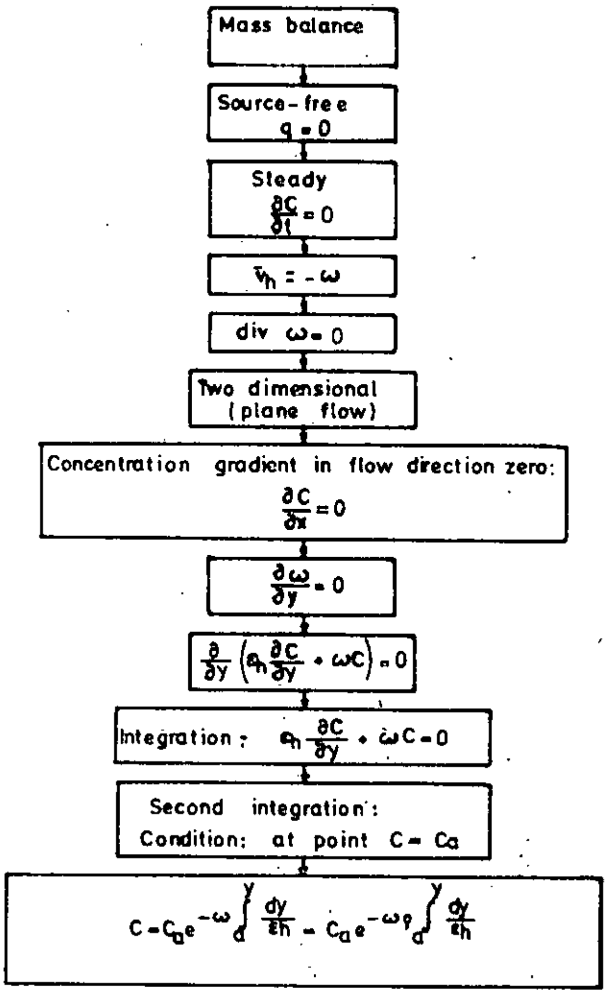

FIG. 4·3 DERIVATION OF THE FUNDAMENTAL EQUATION OF TURBULENT SEDIMENT TRANSPORT THEORY [After Bogardi (6)]

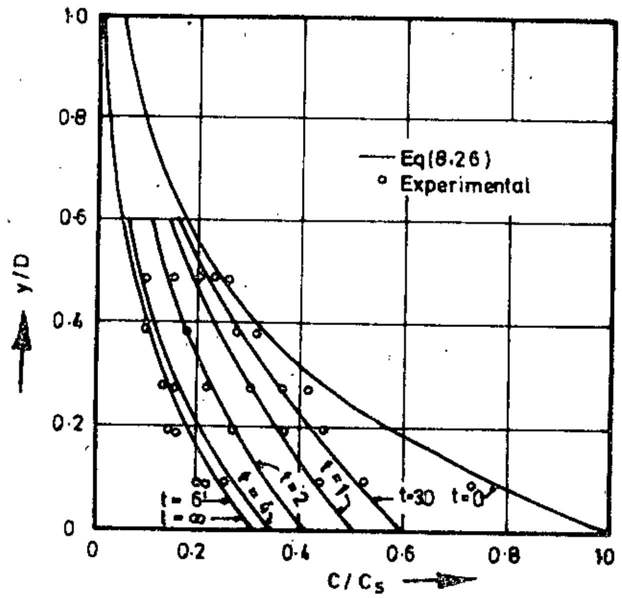

FIG. 4·4. CONCENTRATION CHANGES WITH CONSTANT RATE OF PICKUP; ω/ϵ_s = 0·0638 cm^{-1}; D= 45·2 cm. [After DOBBINS (16)]

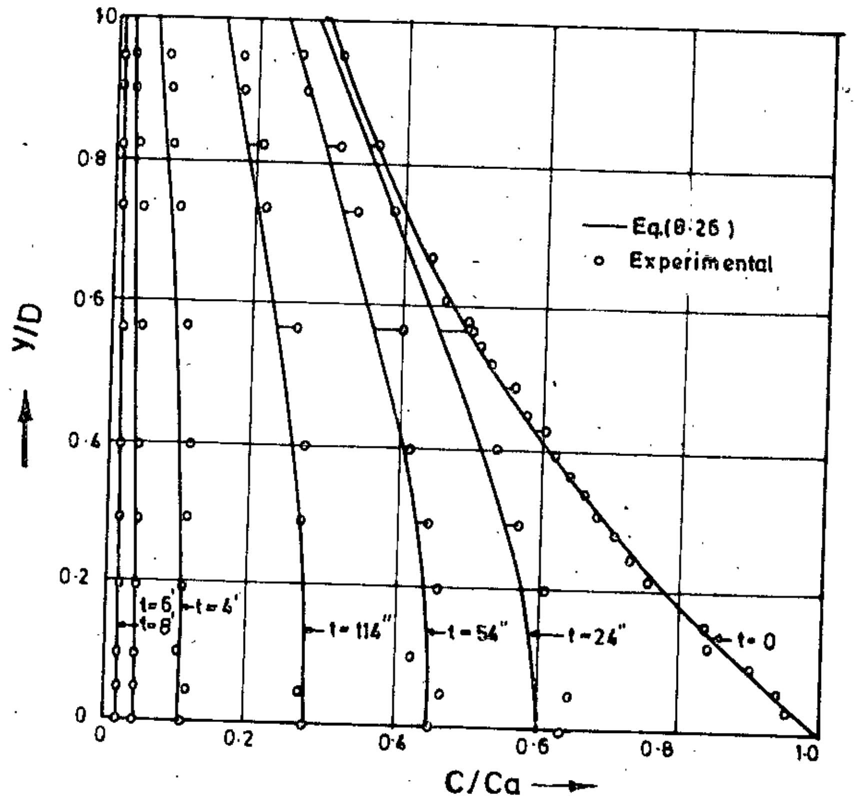

FIG. 4.5. CONCENTRATION CHANGES WITH NO PICKUP; $w/\epsilon_s = 0.030\ cm^{-1}$ D= 416 cm.
[After DOBBINS (16)]

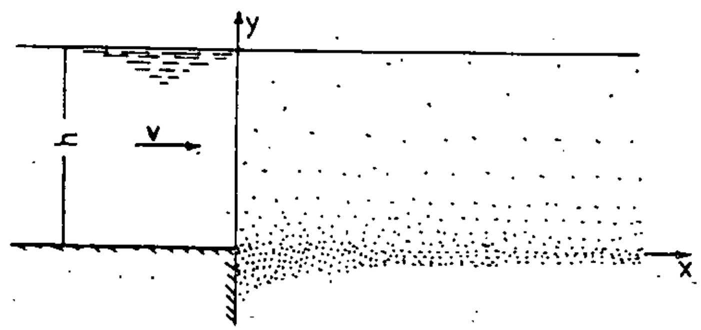

FIG. 5·1. PROBLEM DEFINITION

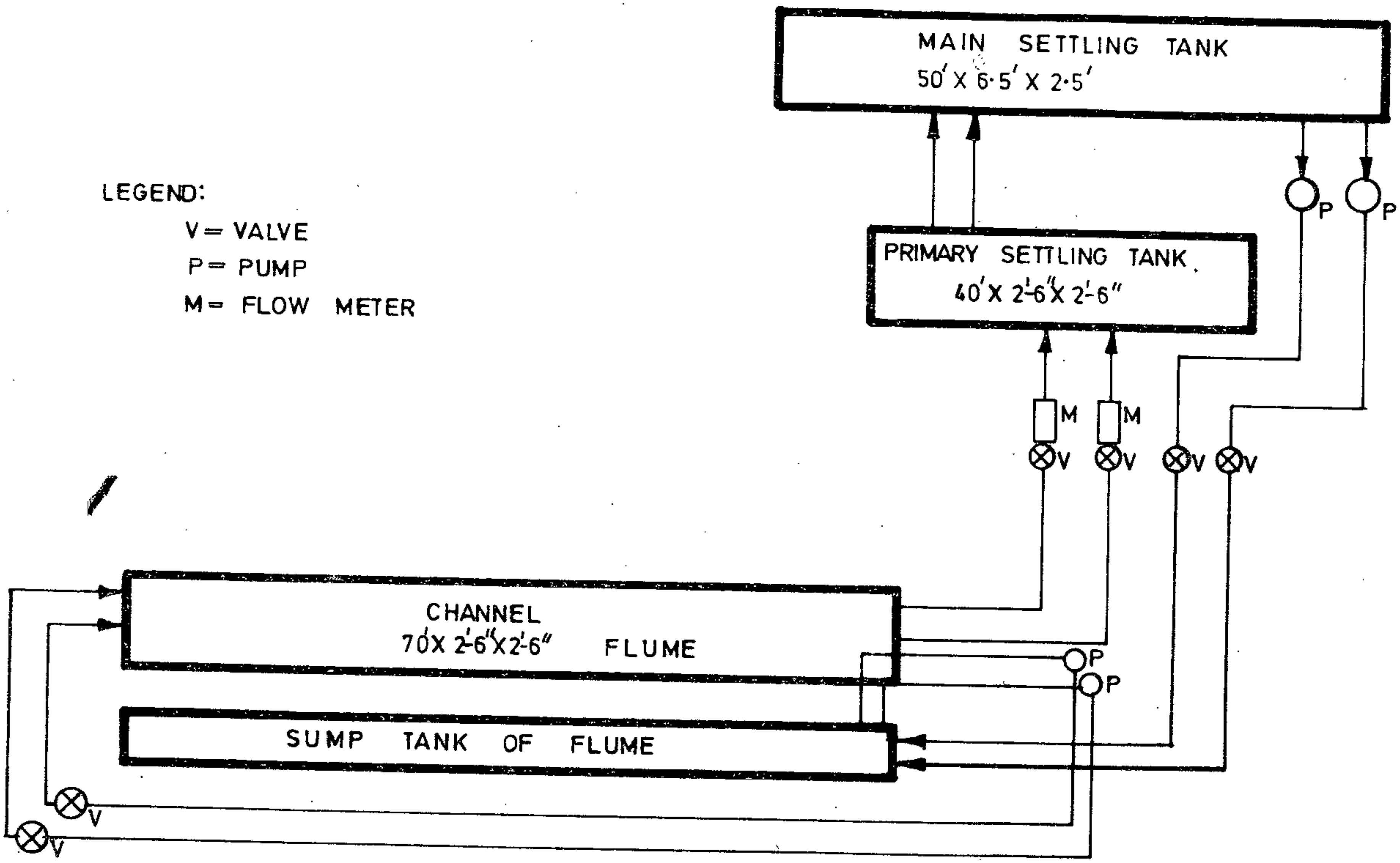

FIG. 6·1. LAYOUT OF EXPERIMENTAL SETUP

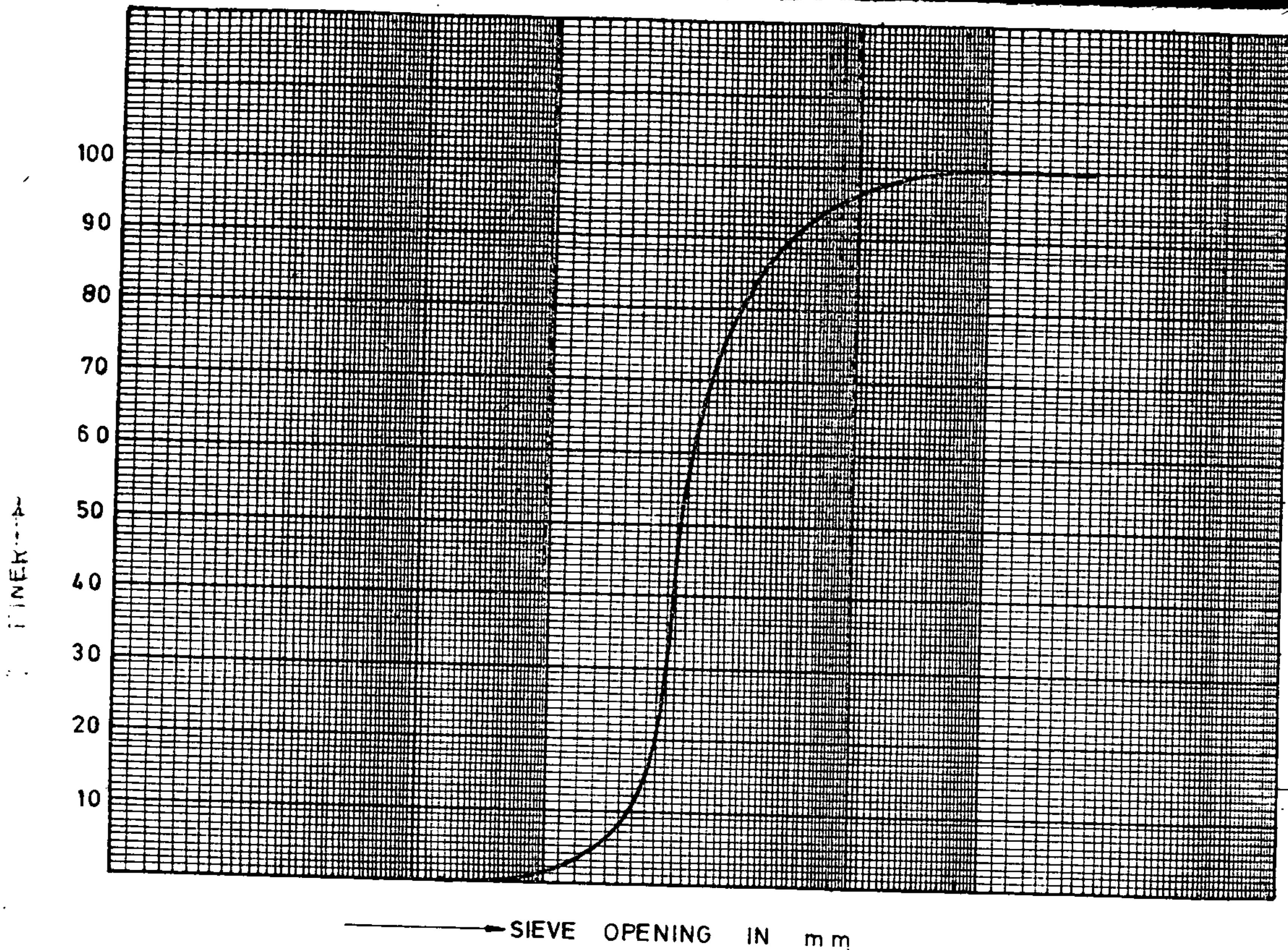

FIG. 6·3 GRAIN SIZE DISTRIBUTION OF BED MATERIAL TYPE – I

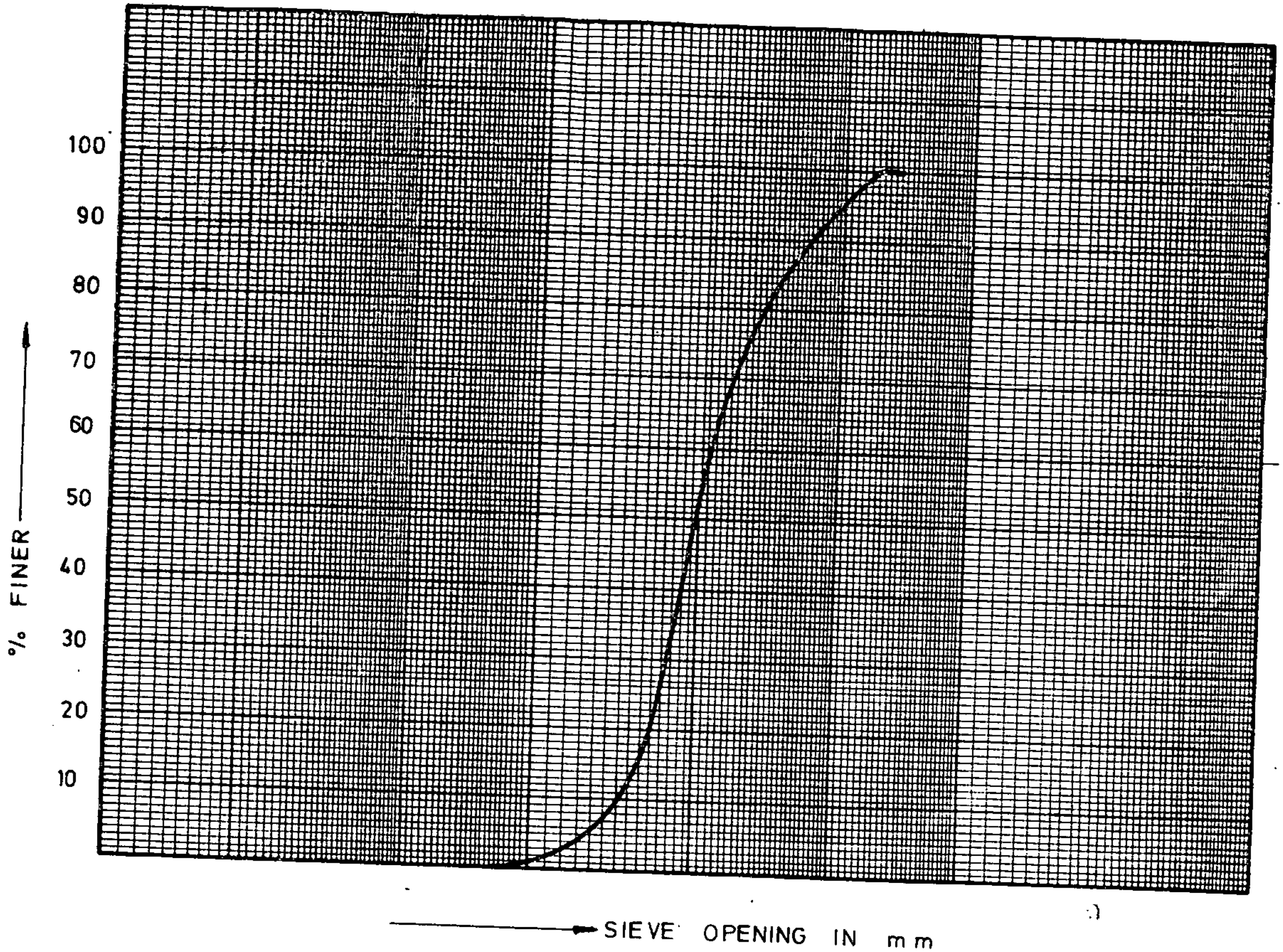

FIG. 6·4. GRAIN SIZE DISTRIBUTION OF BED MATERIAL TYPE - II

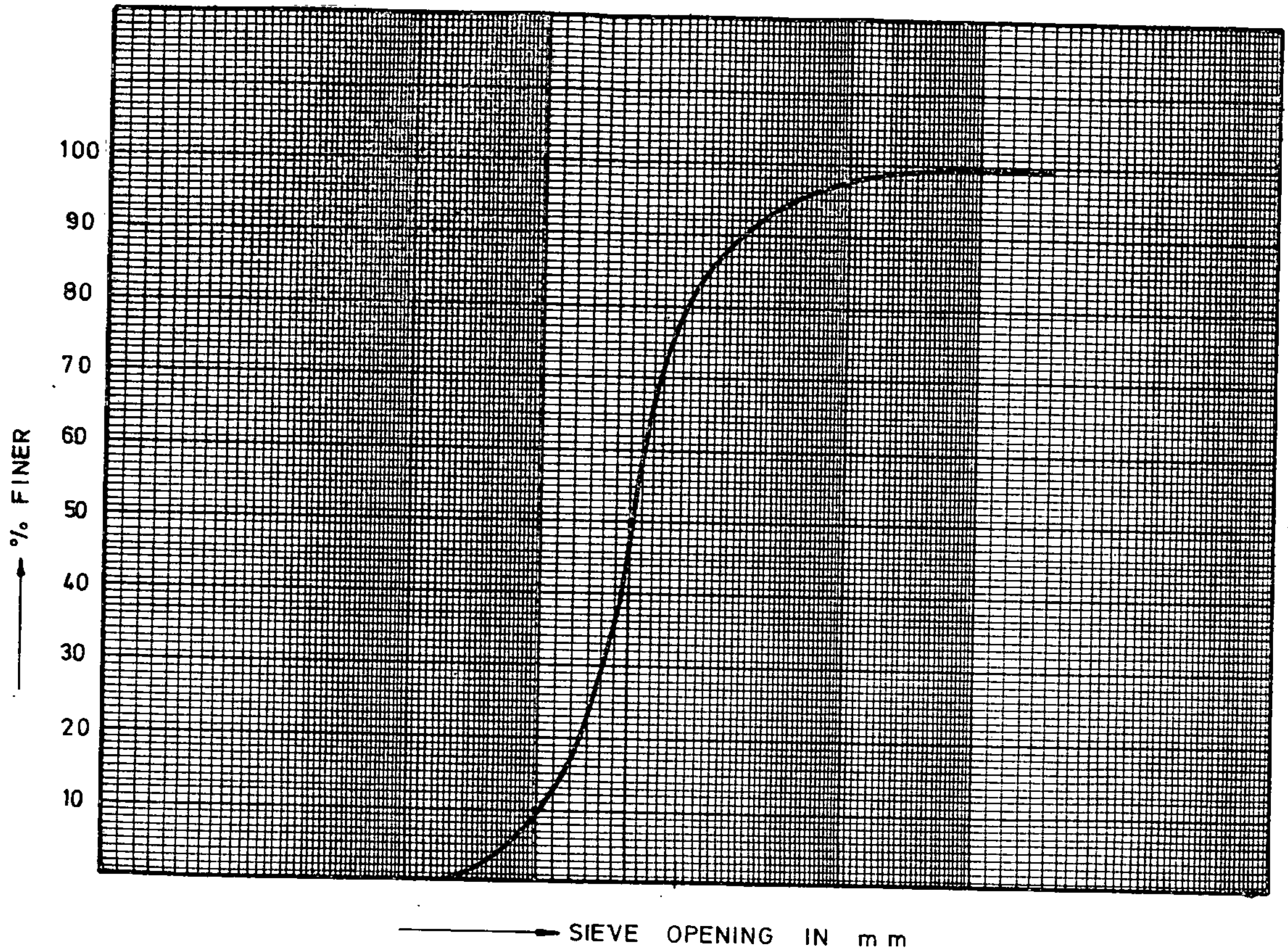

FIG. 6·5. GRAIN SIZE DISTRIBUTION OF BED MATERIAL TYPE III

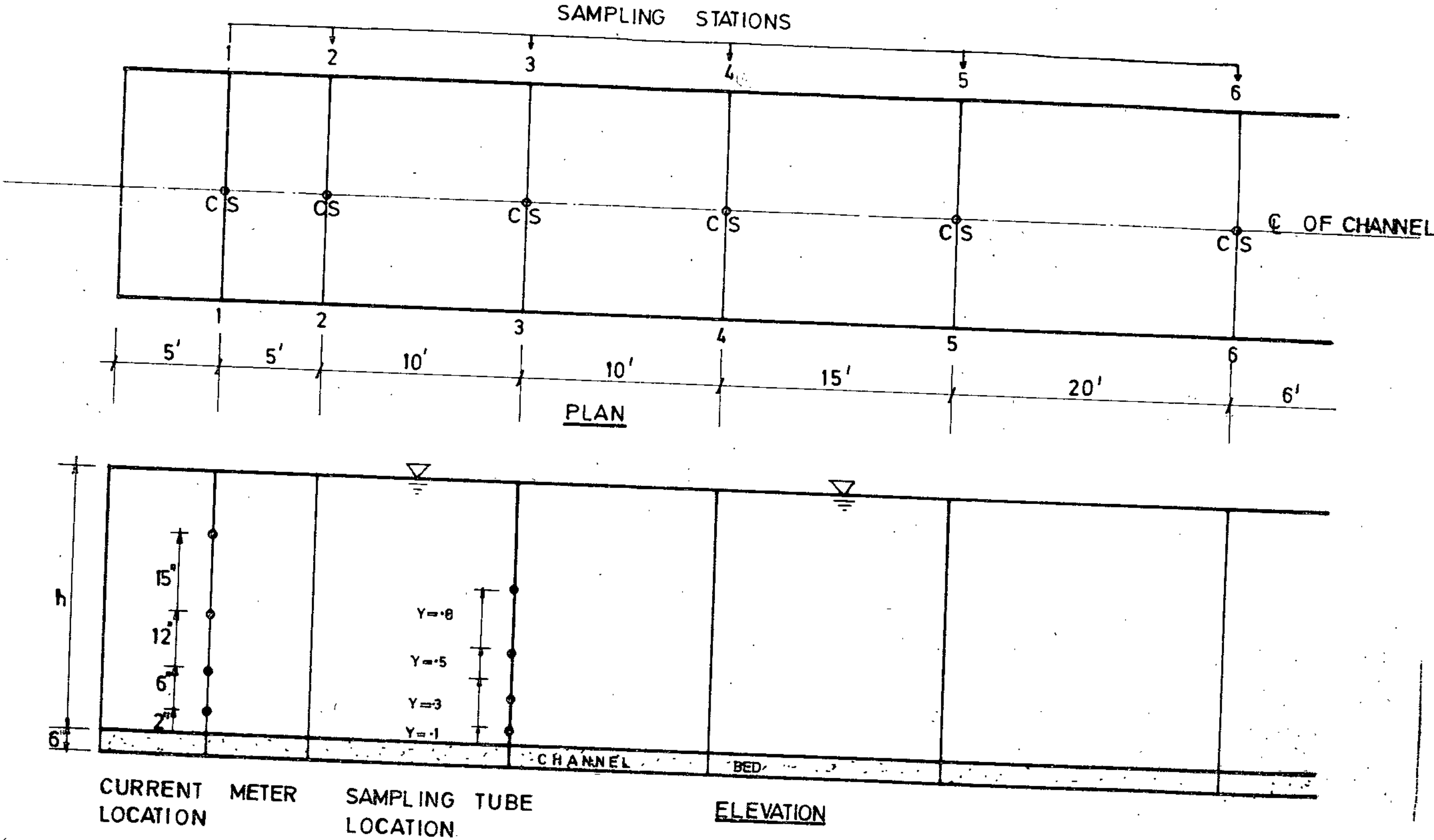

FIG. 6. 6. LOCATION OF SAMPLING TUBES (S) AND CURRENT METER (C)

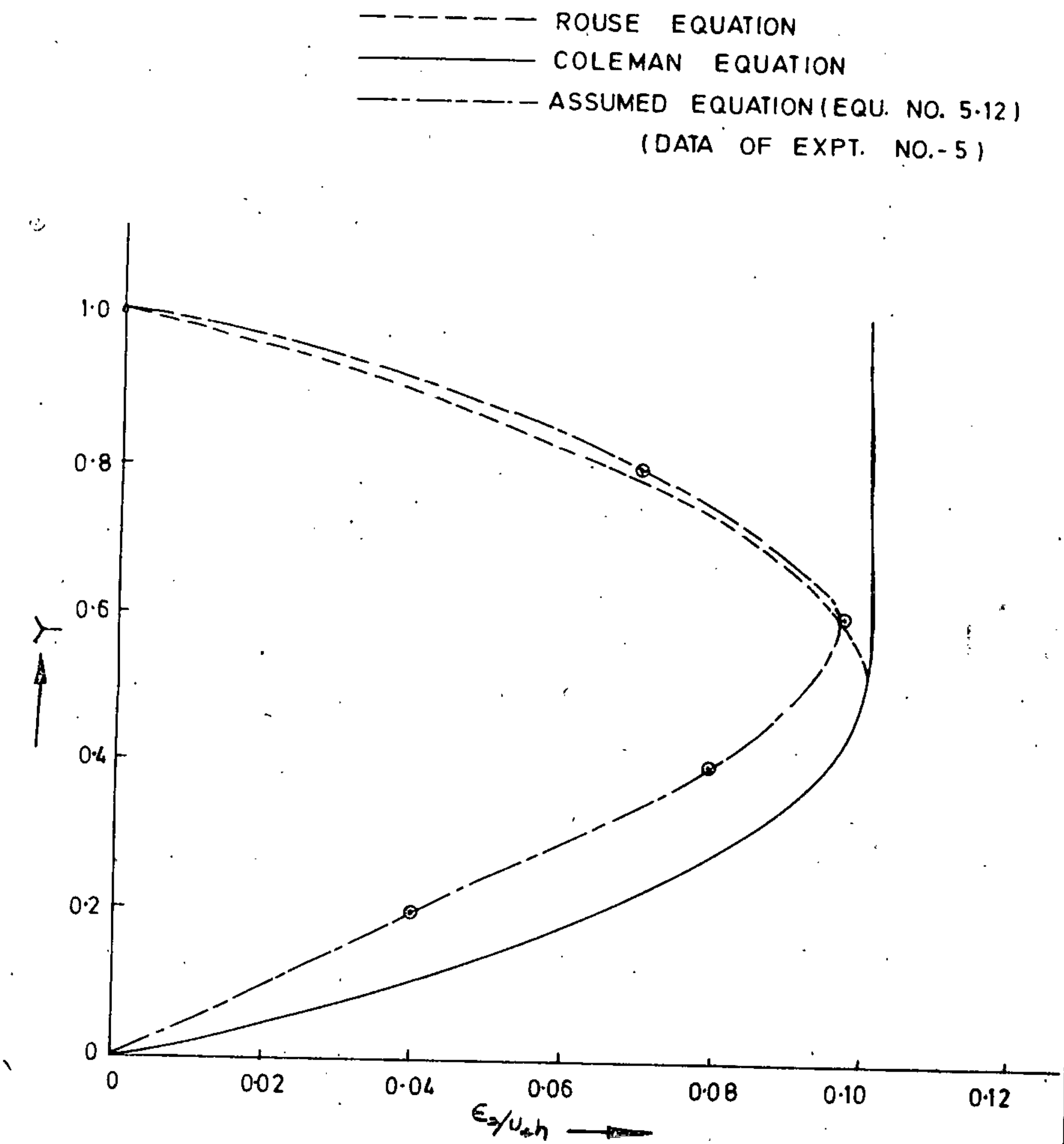

FIG. 7·1. VARIATION OF DIFFUSION CO-EFFICIENT.

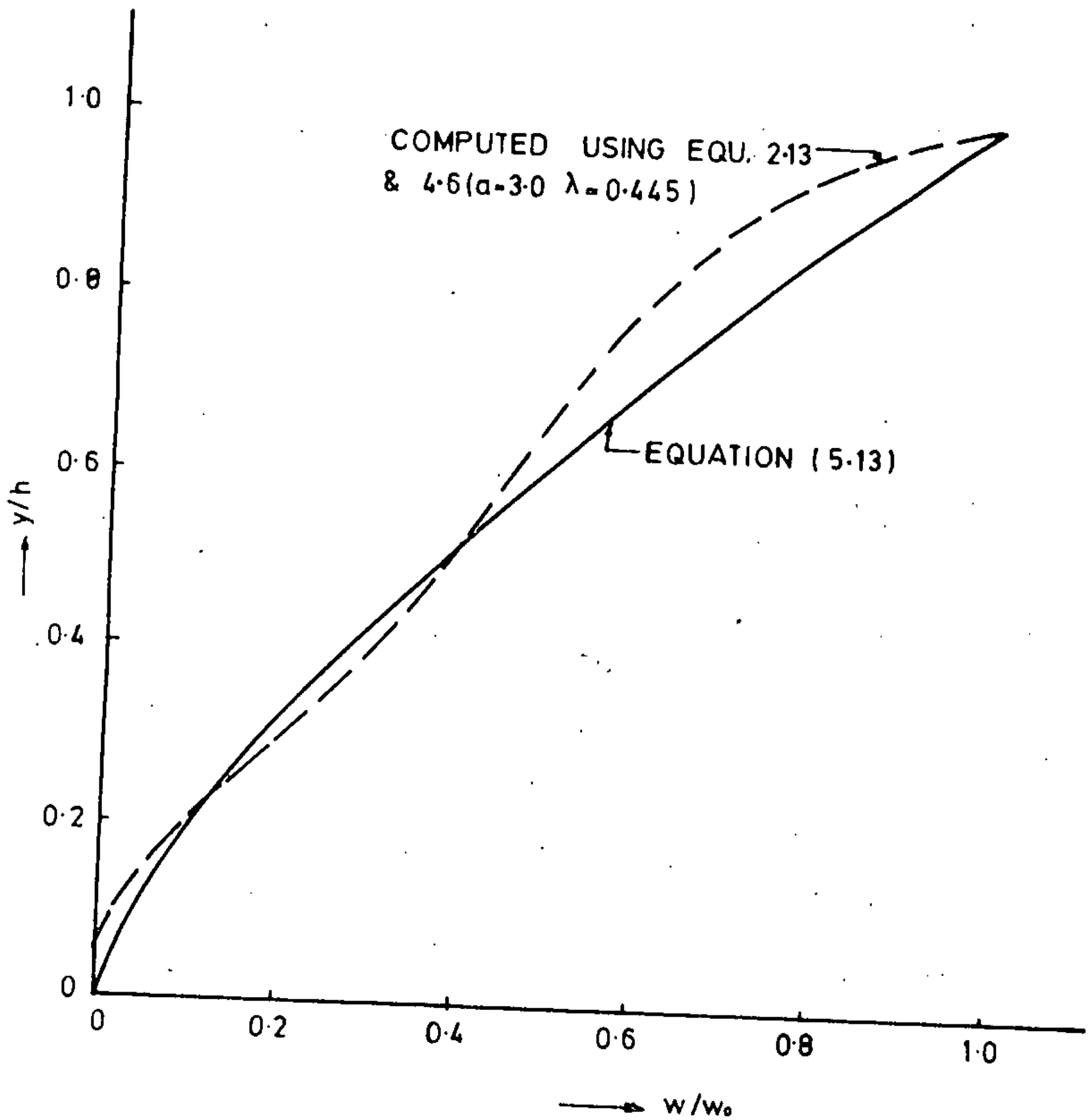

FIG. 7·2. RELATION BETWEEN FALL VELOCITY AND VERTICAL DISTANCE

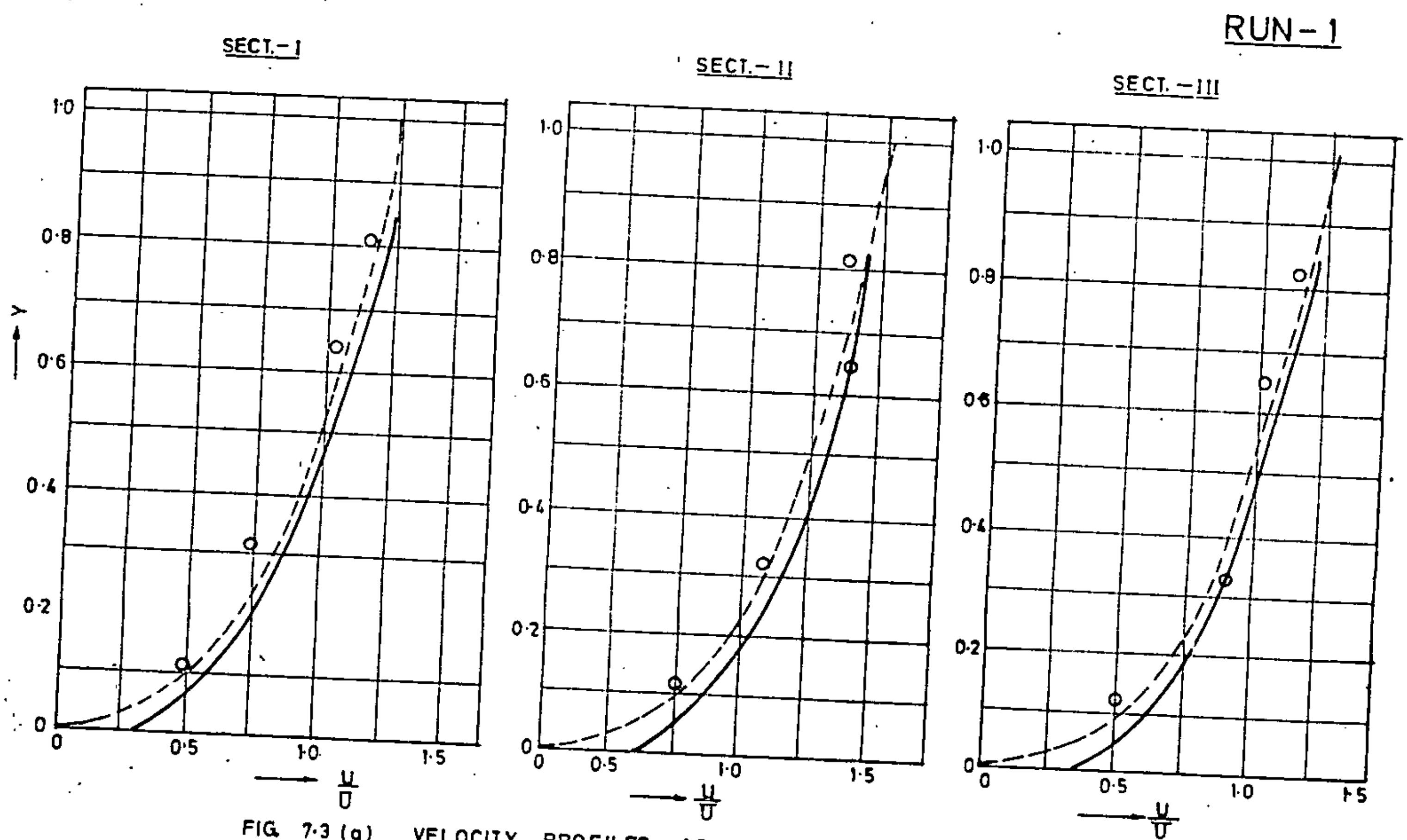

FIG. 7·3 (a) VELOCITY PROFILES AT DIFFERENT SECTIONS

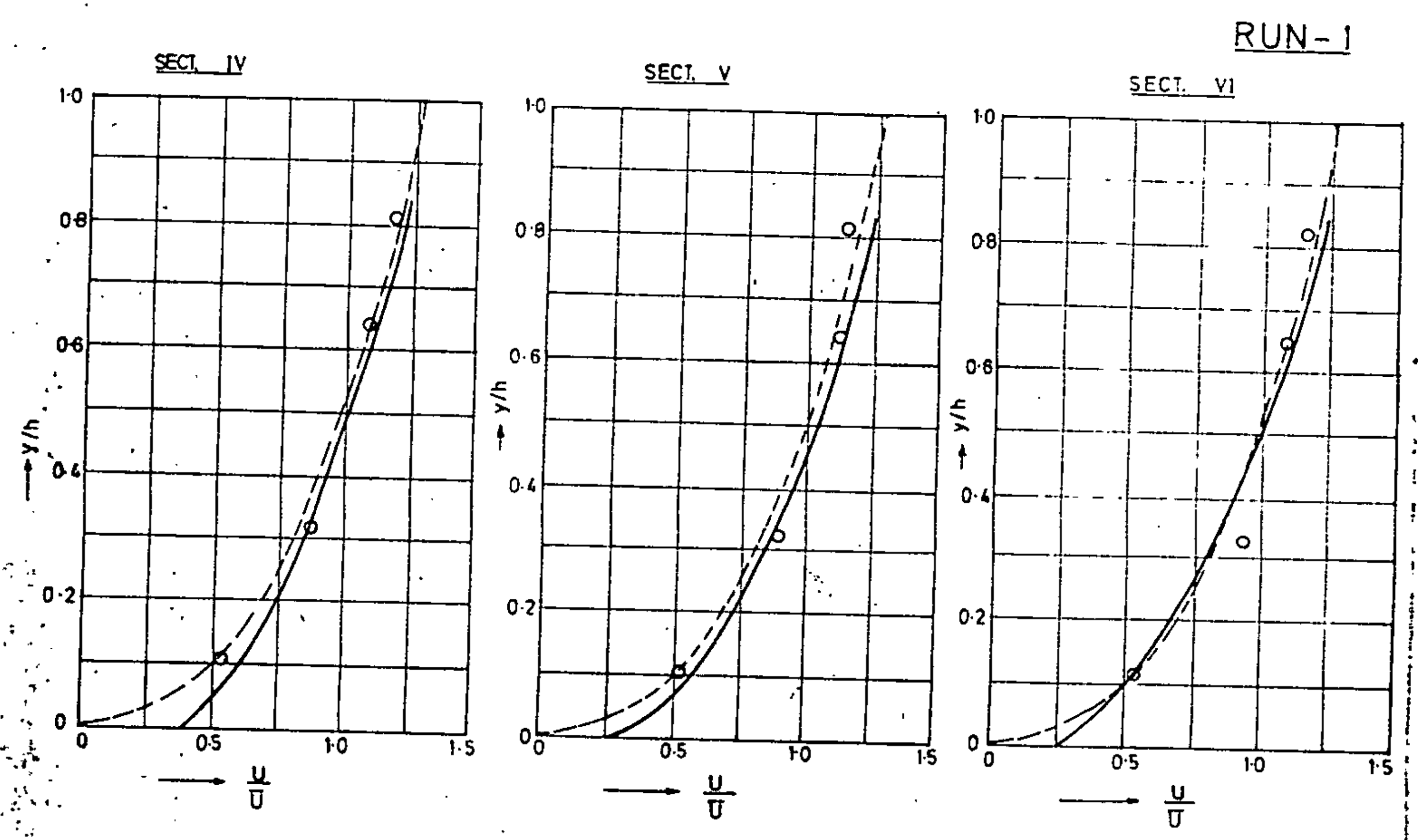

FIG. 7·3 (b) VELOCITY PROFILES AT DIFFERENT SECTIONS

RUN– 2

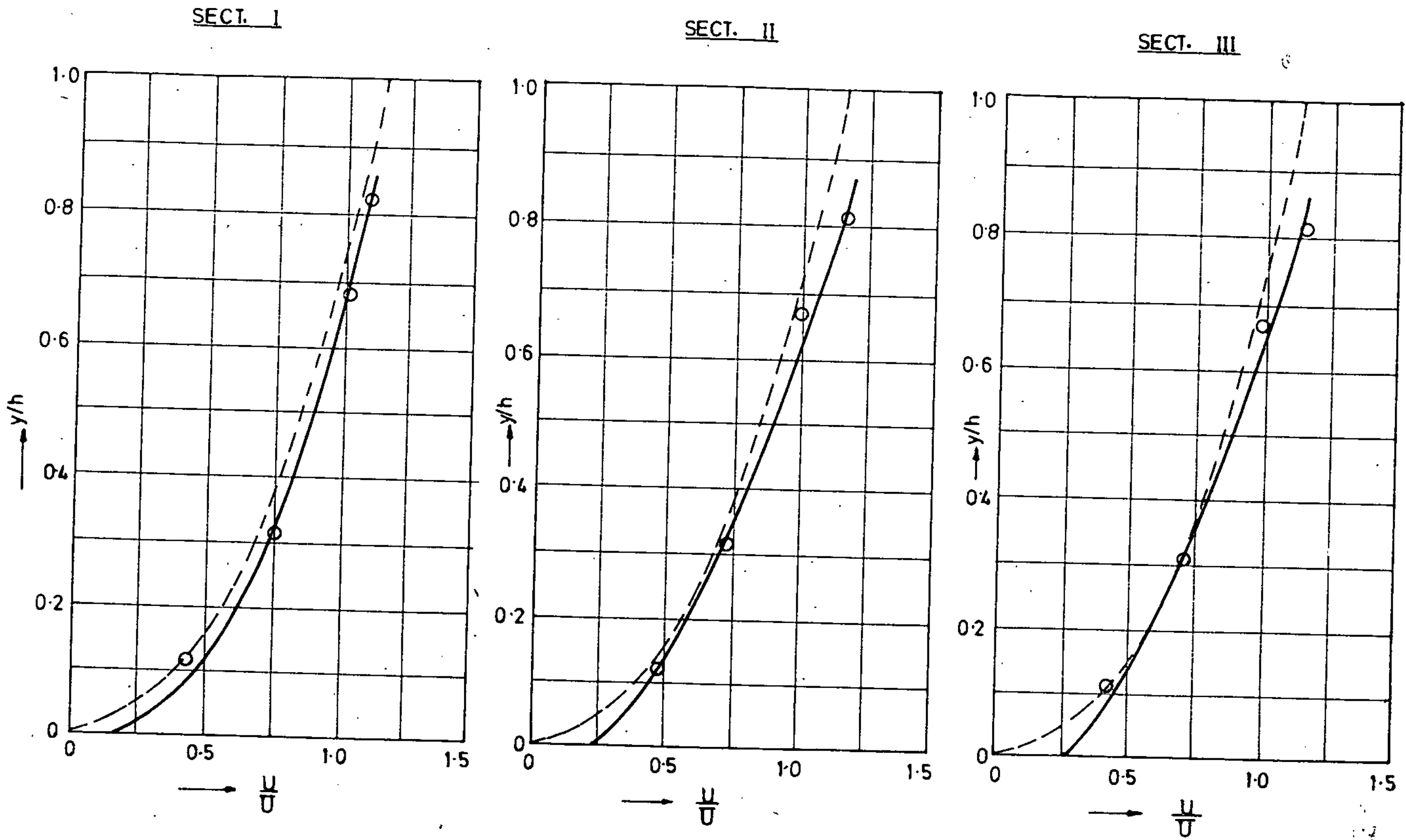

FIG. 7·4(a) VELOCITY PROFILES AT DIFFERENT SECTIONS

RUN-2

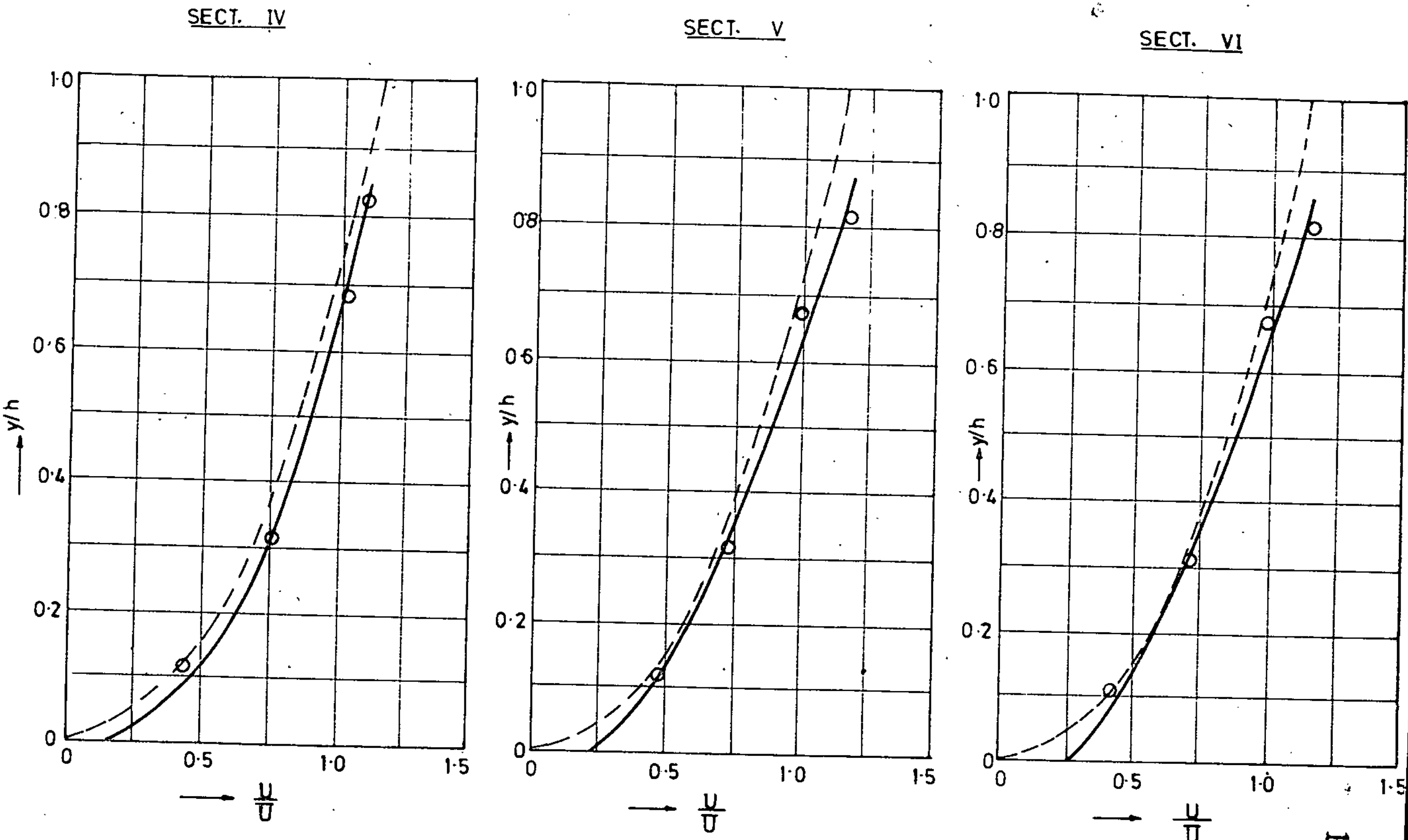

FIG. 7·4 (b) VELOCITY PROFILES AT DIFFERENT SECTIONS

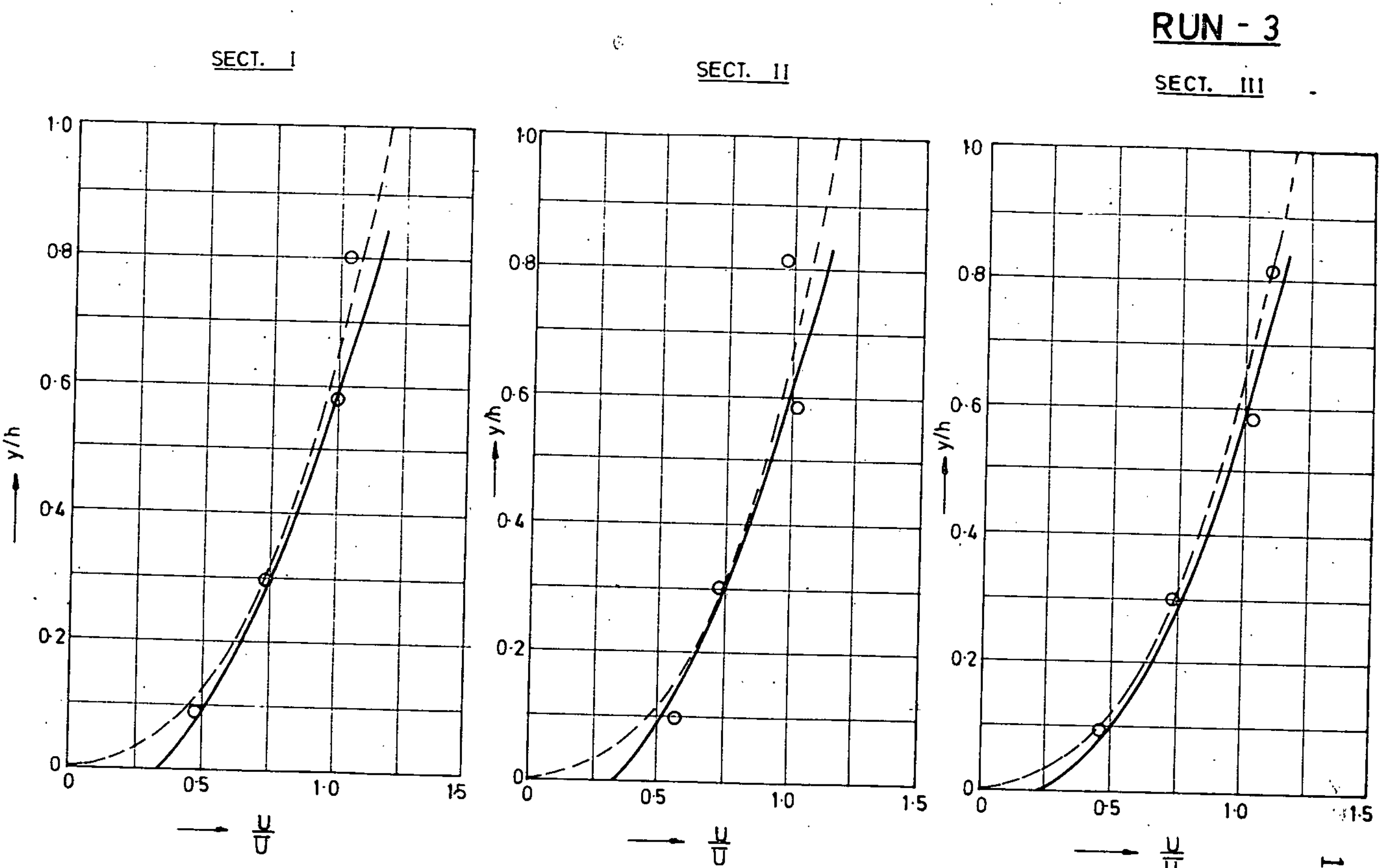

FIG. 7·5 (a) VELOCITY PROFILES AT DIFFERENT SECTIONS

RUN – 3

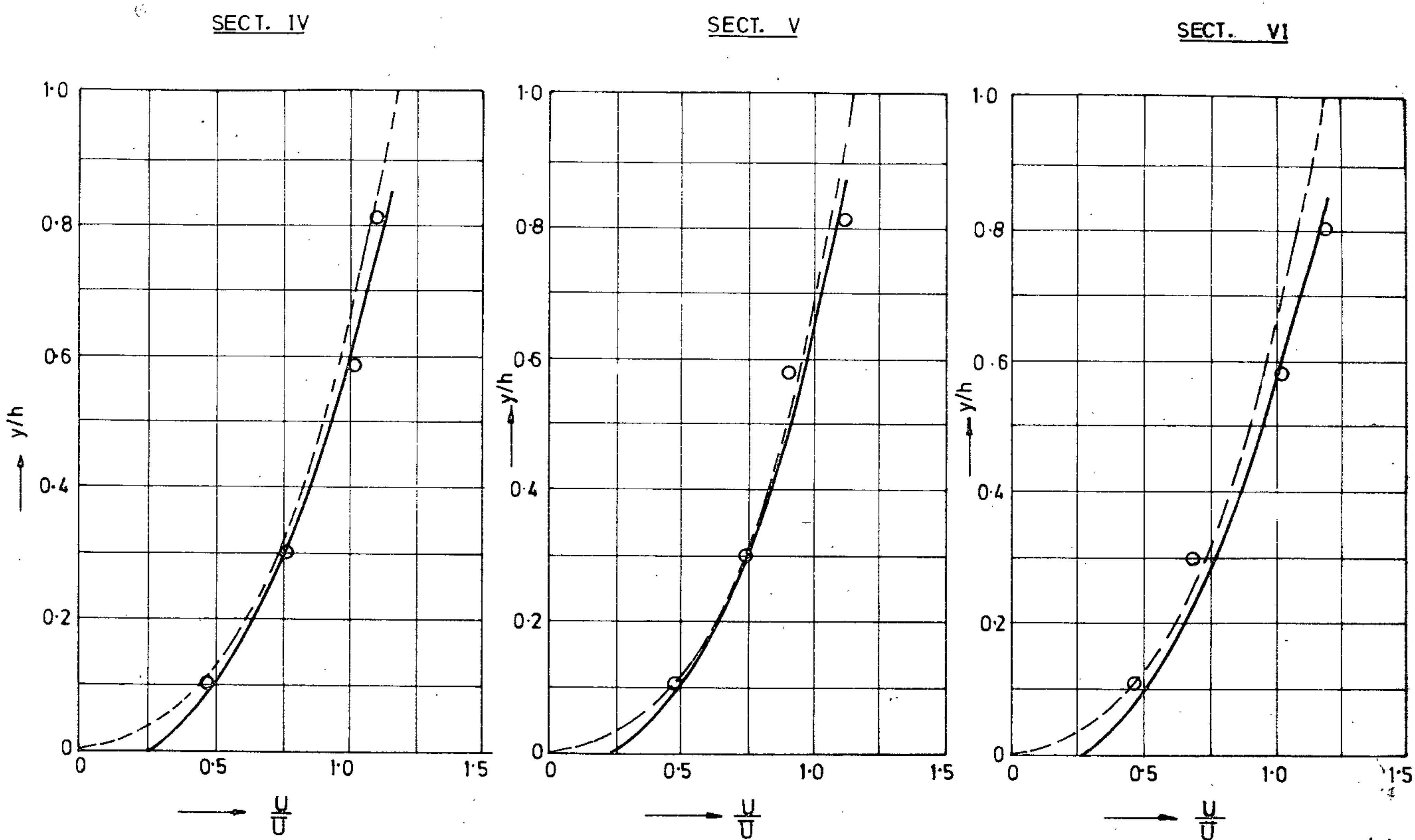

FIG. 7·5(b) VELOCITY PROFILES AT DIFFERENT SECTIONS

RUN - 4

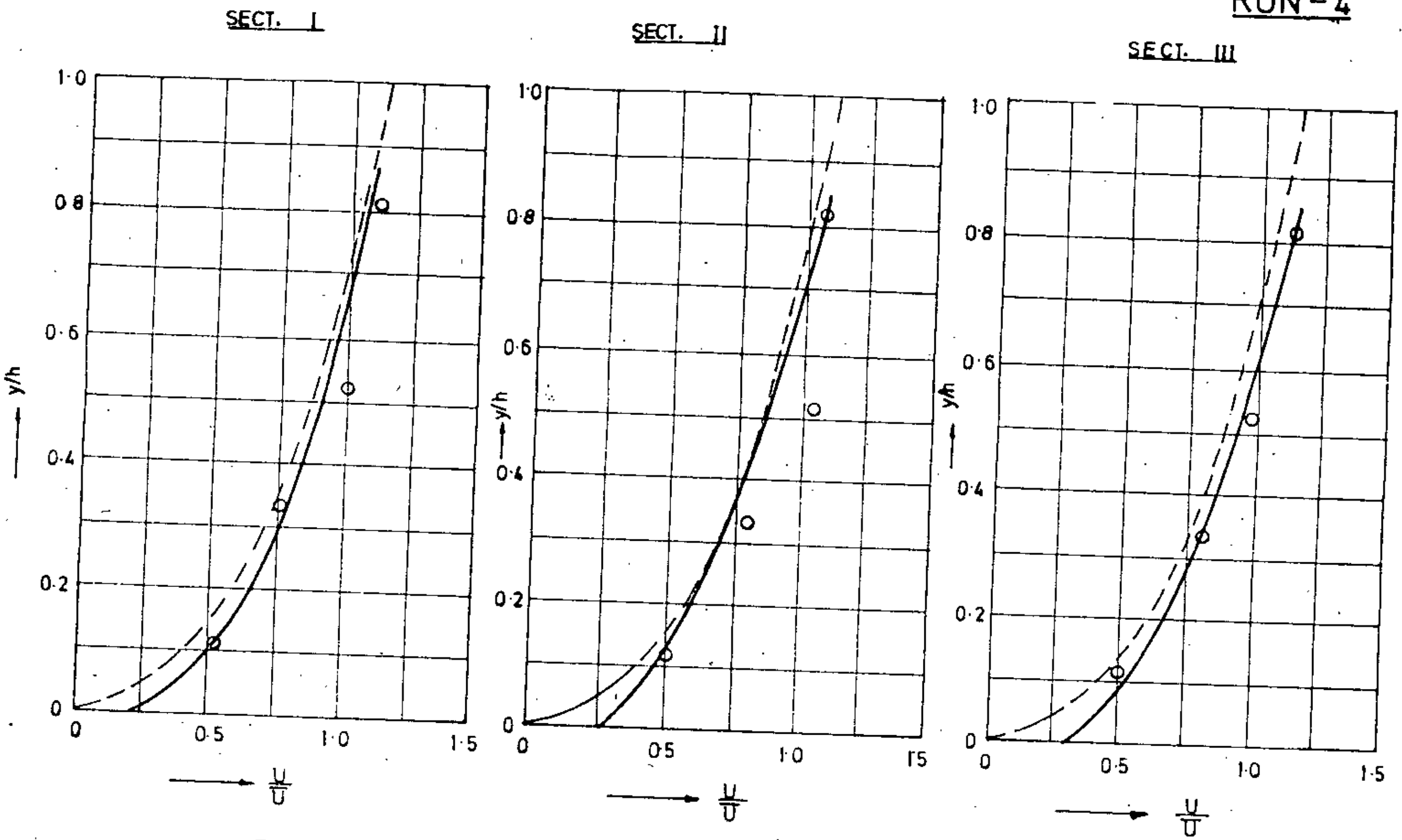

FIG. 7·6 (a): VELOCITY PROFILES AT DIFFERENT SECTIONS

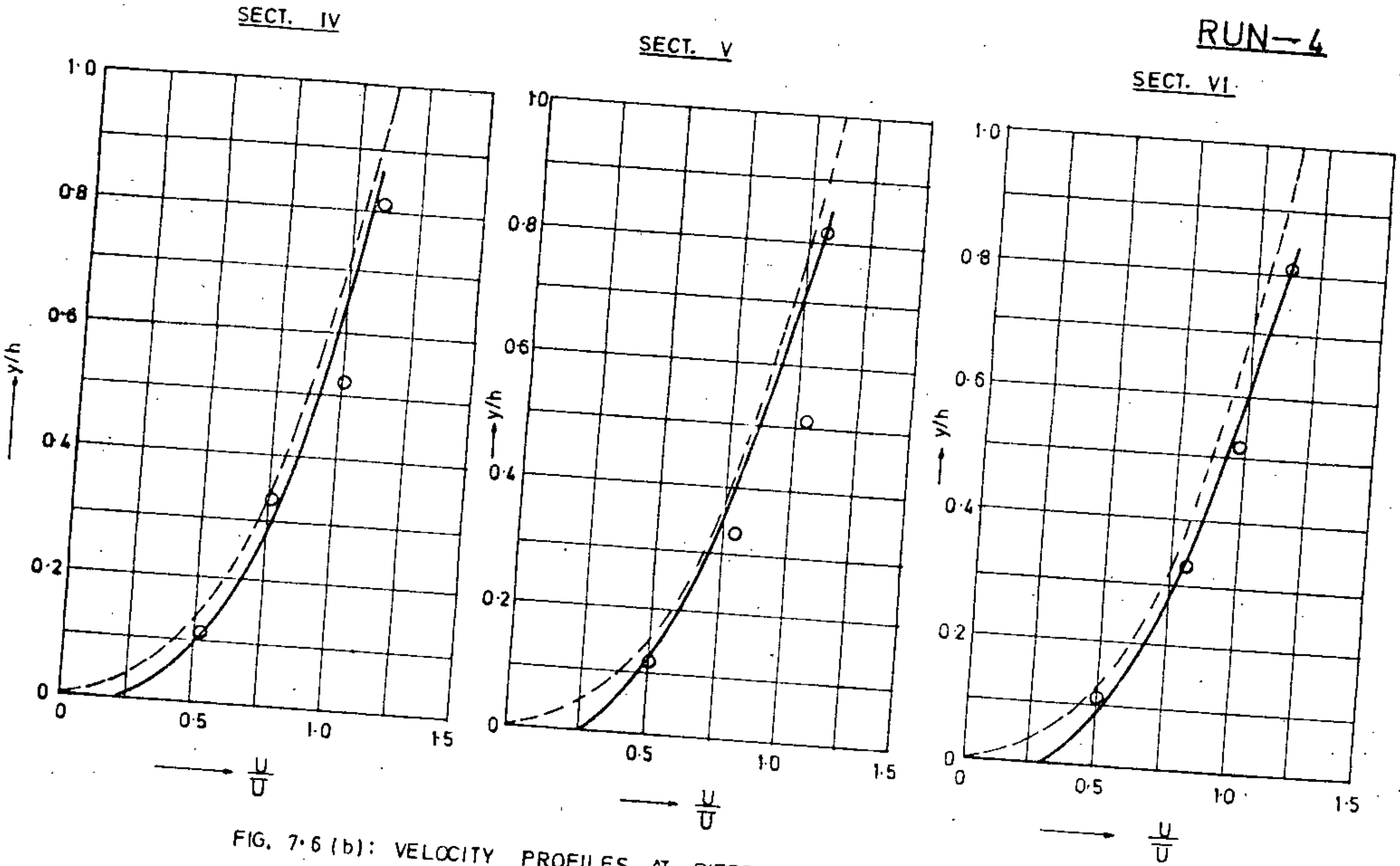

FIG. 7·6 (b): VELOCITY PROFILES AT DIFFERENT SECTIONS

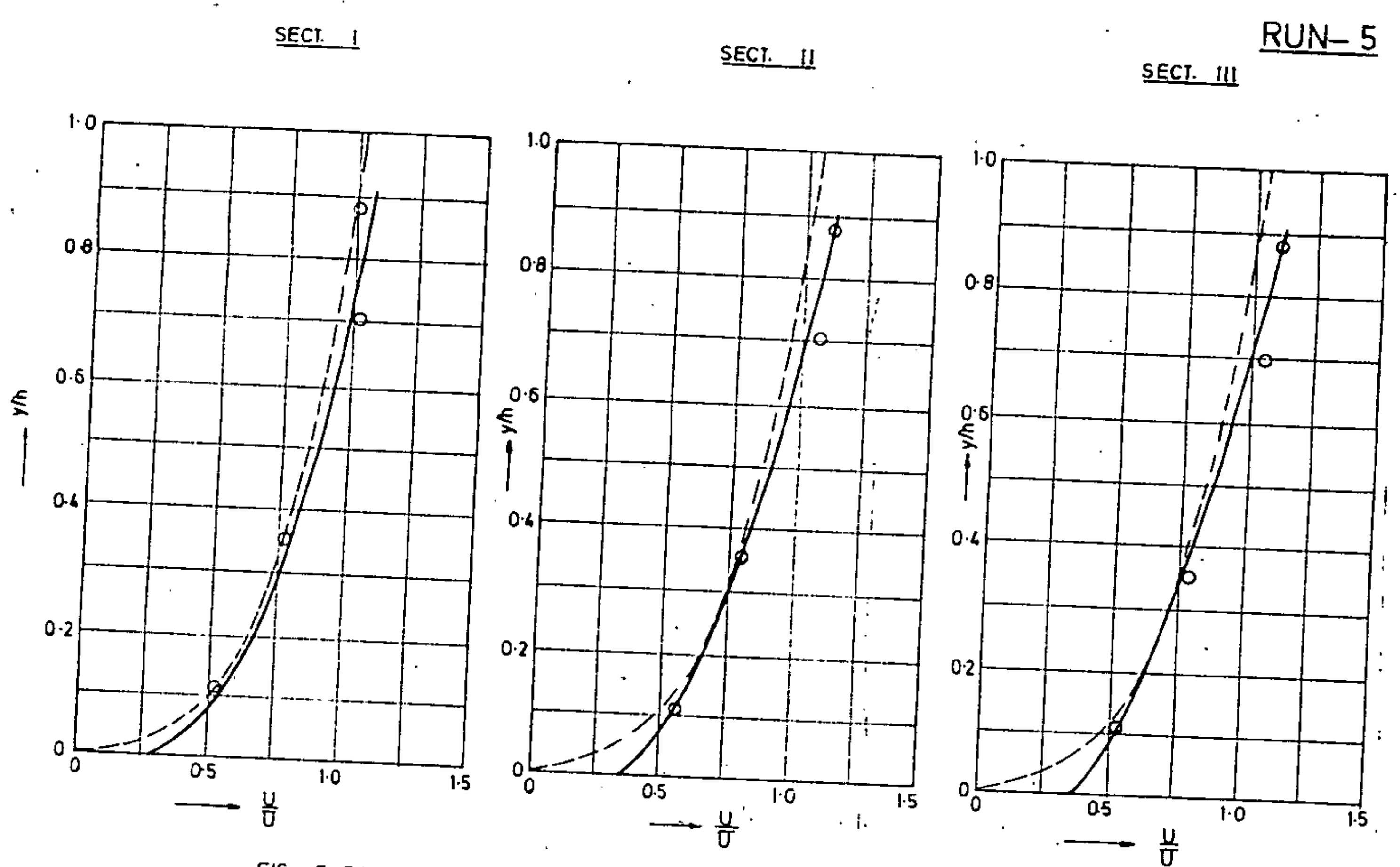

FIG. 7·7(a): VELOCITY PROFILES AT DIFFERENT SECTIONS

RUN - 5

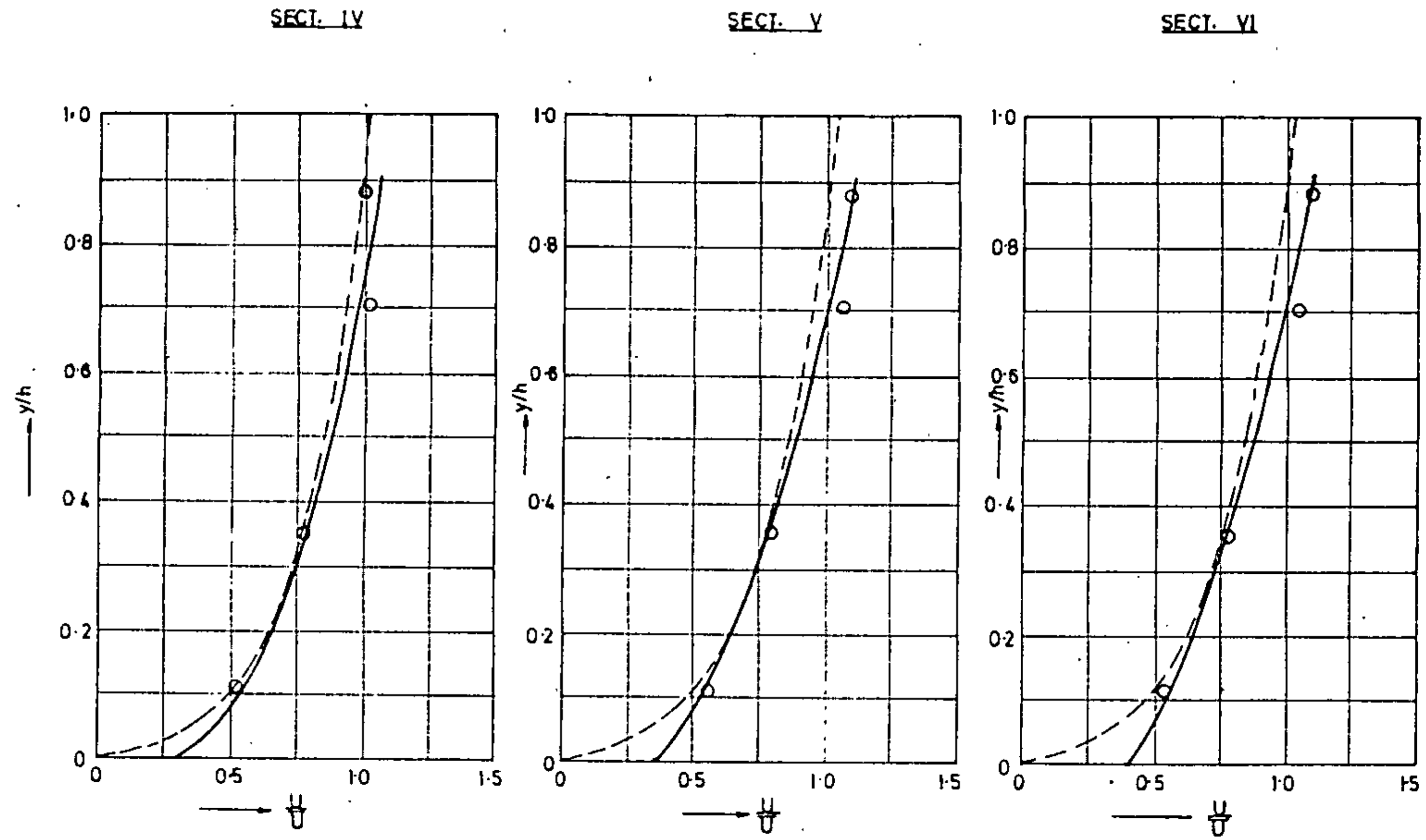

FIG. 7·7(b): VELOCITY PROFILES AT DIFFERENT SECTIONS

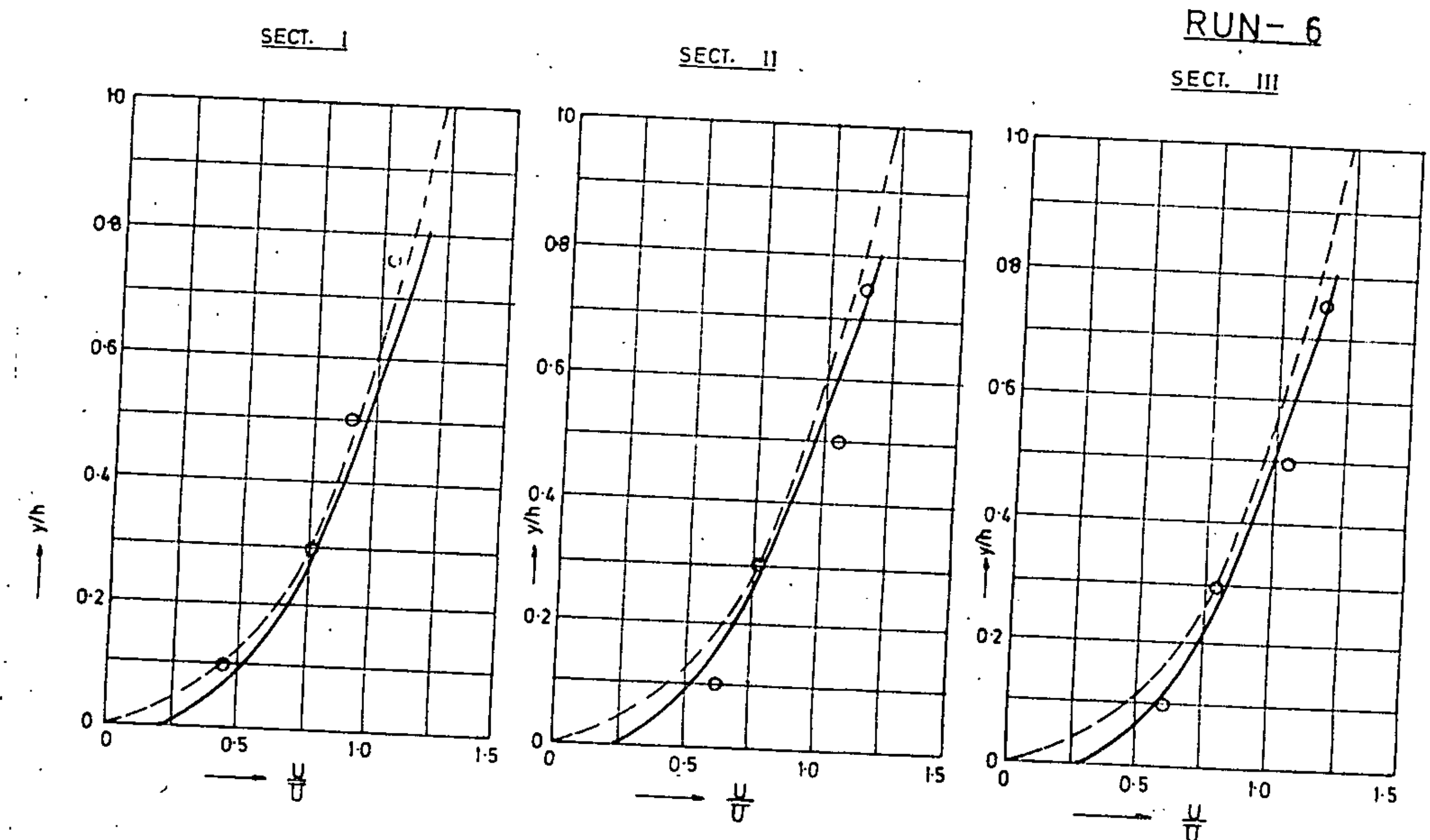

FIG. 7·8(a): VELOCITY PROFILES AT DIFFERENT SECTIONS

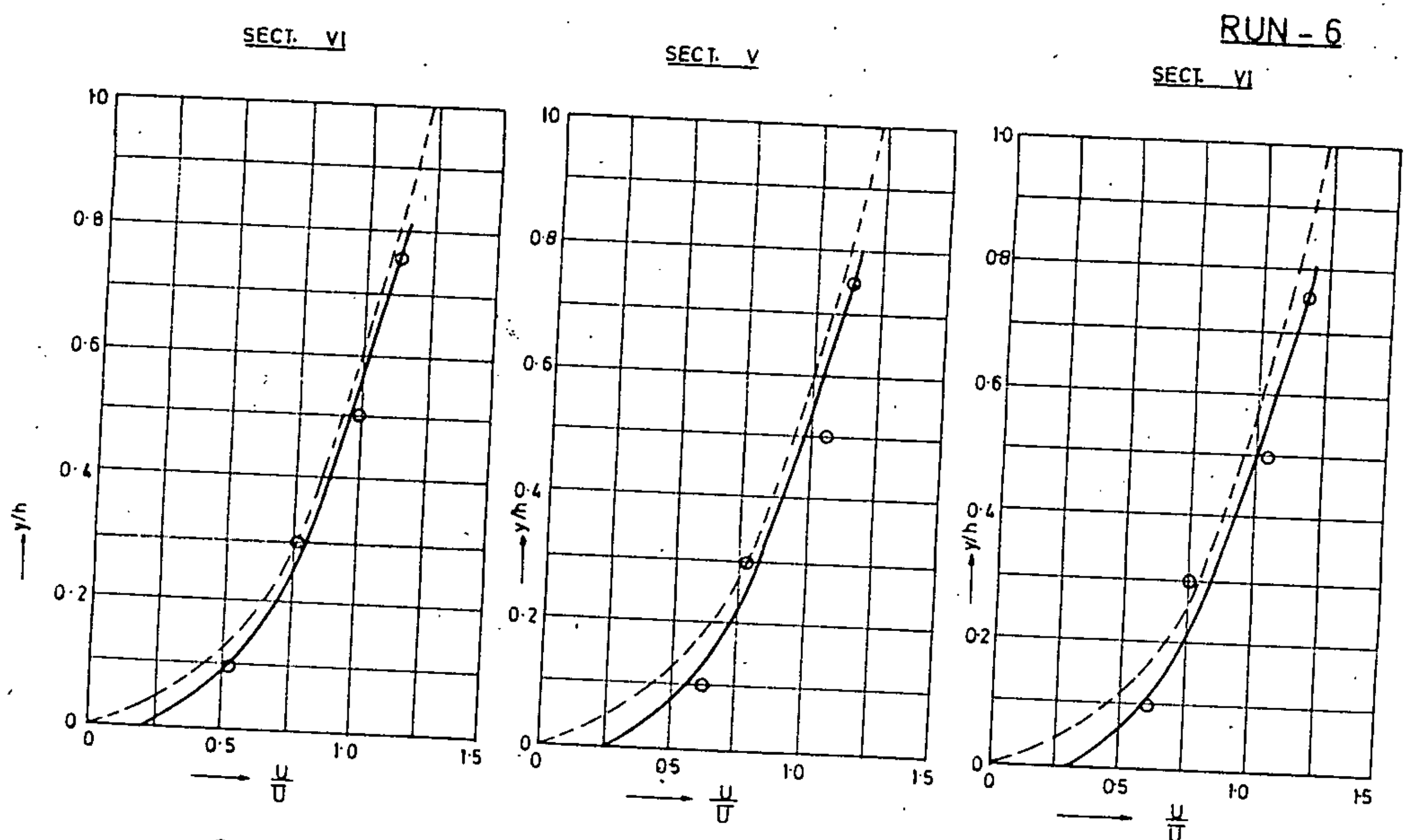

FIG. 7·8(b): VELOCITY PROFILES AT DIFFERENT SECTIONS

RUN - 7

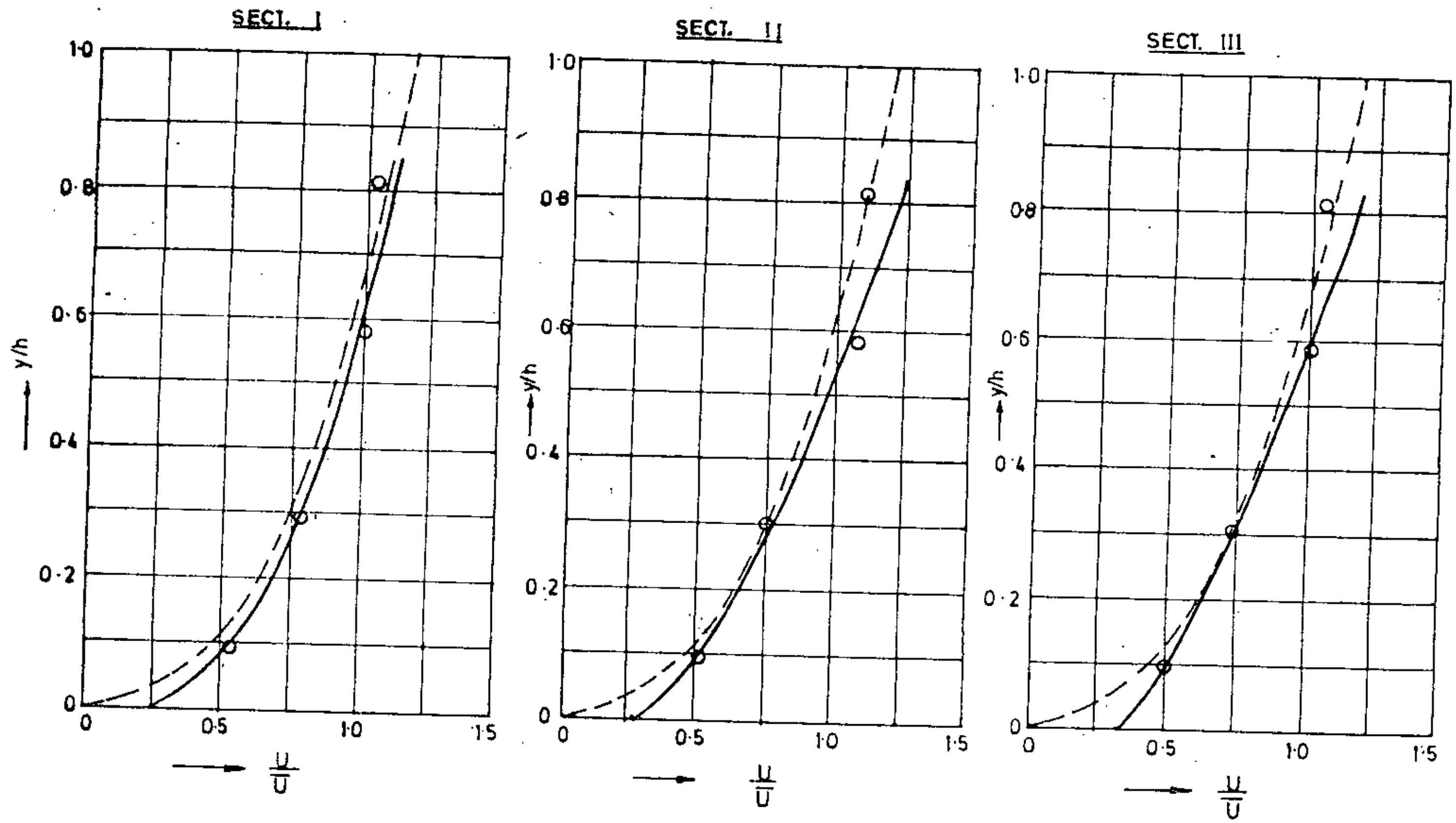

FIG. 7·9(a): VELOCITY PROFILES AT DIFFERENT SECTIONS

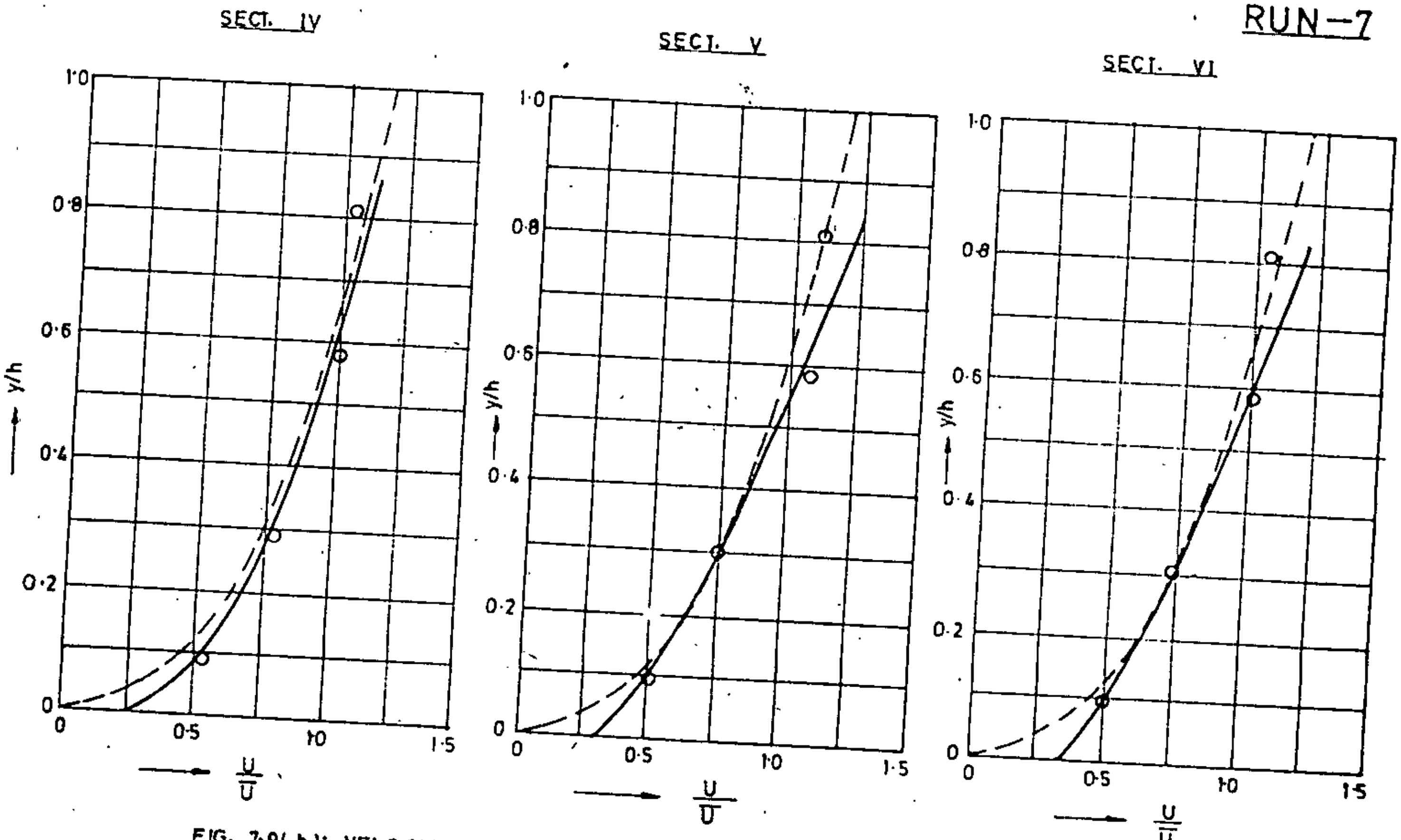

FIG. 7·9(b): VELOCITY PROFILES AT DIFFERENT SECTIONS

RUN - 8

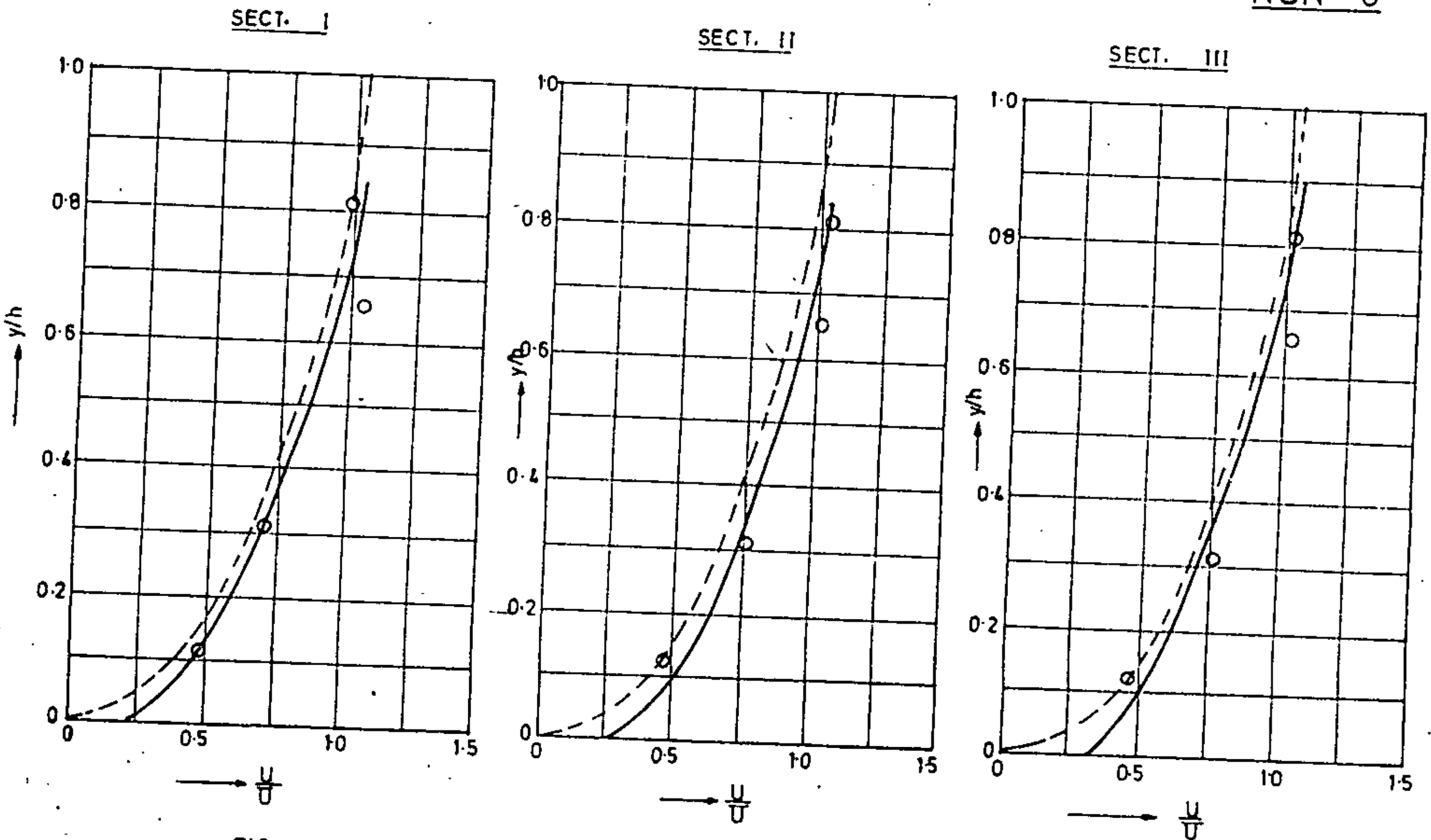

FIG. 7·10(a): VELOCITY PROFILES AT DIFFERENT SECTIONS

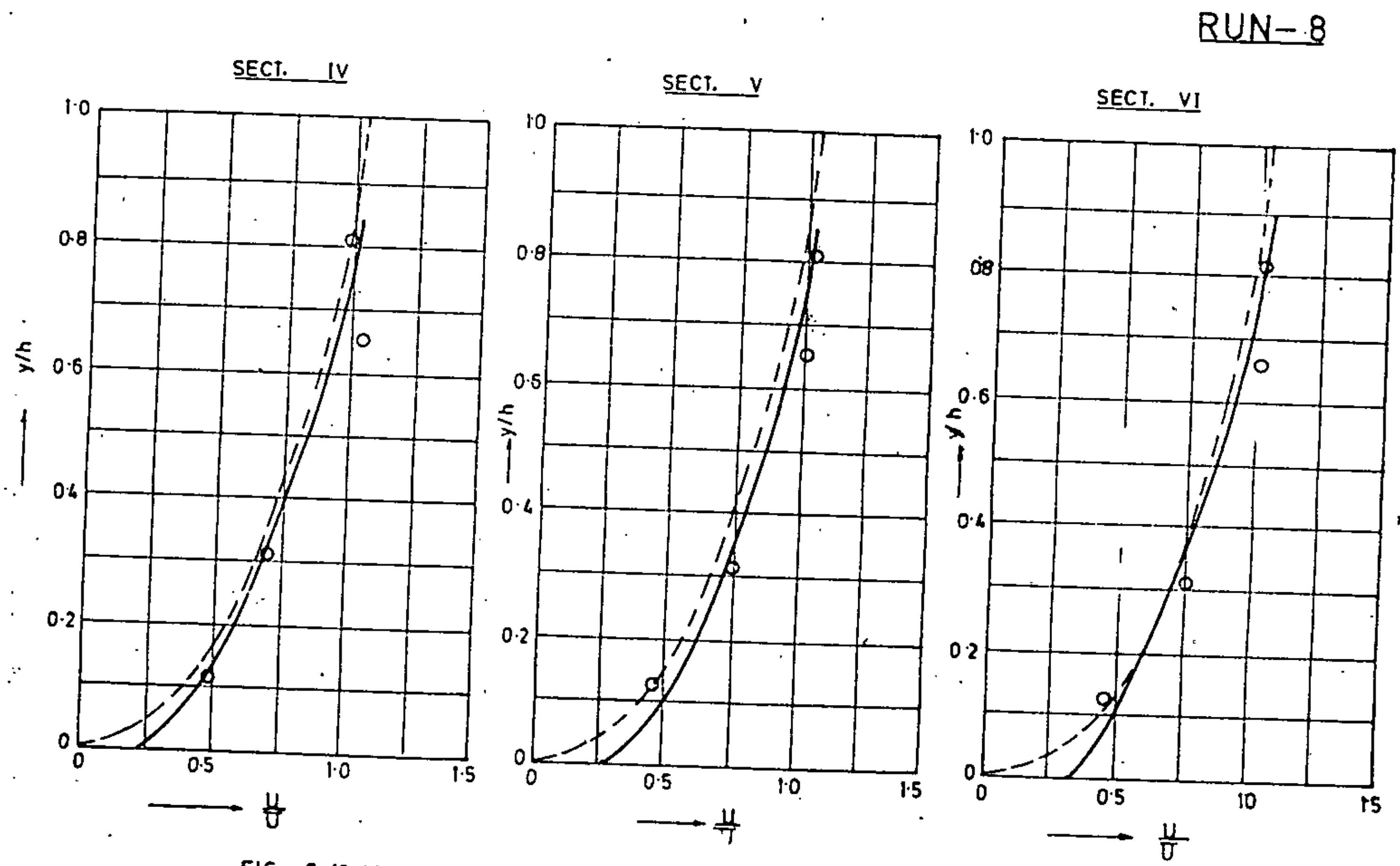

FIG. 7·10 (b): VELOCITY PROFILES AT DIFFERENT SECTIONS

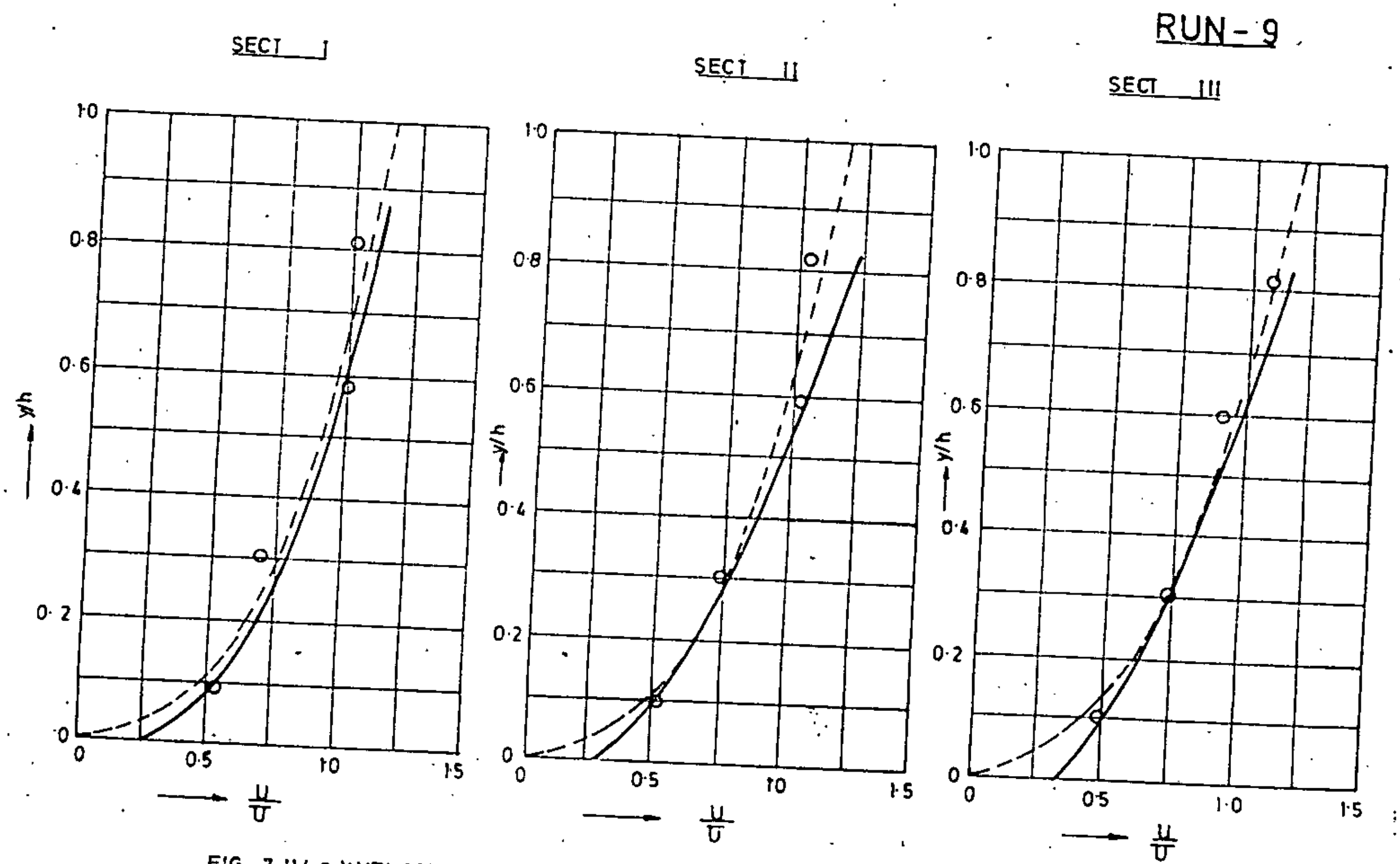

FIG. 7·11 (a): VELOCITY PROFILES AT DIFFERENT SECTIONS

RUN- 9

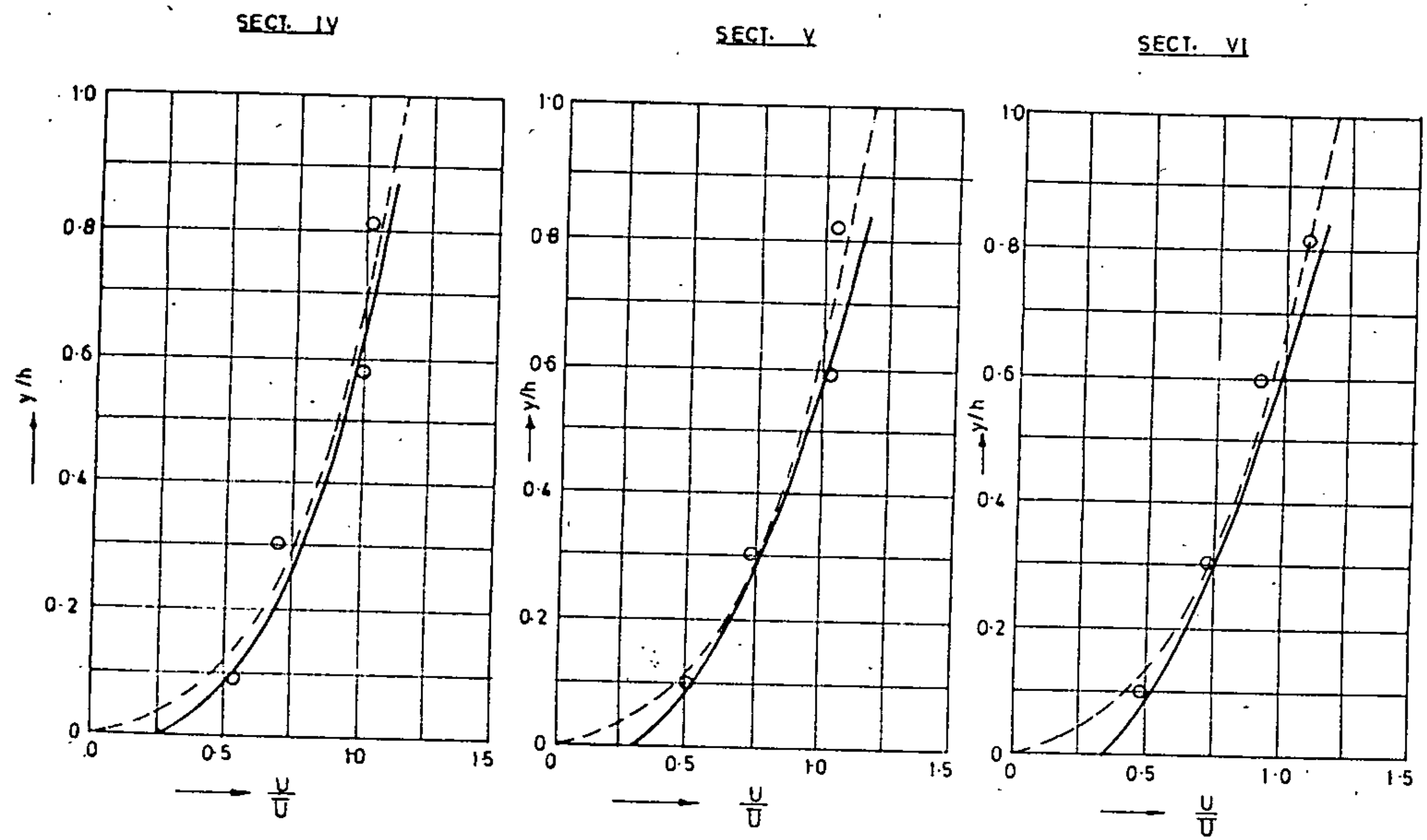

FIG. 7·11(b):VELOCITY PROFILES AT DIFFERENT SECTIONS

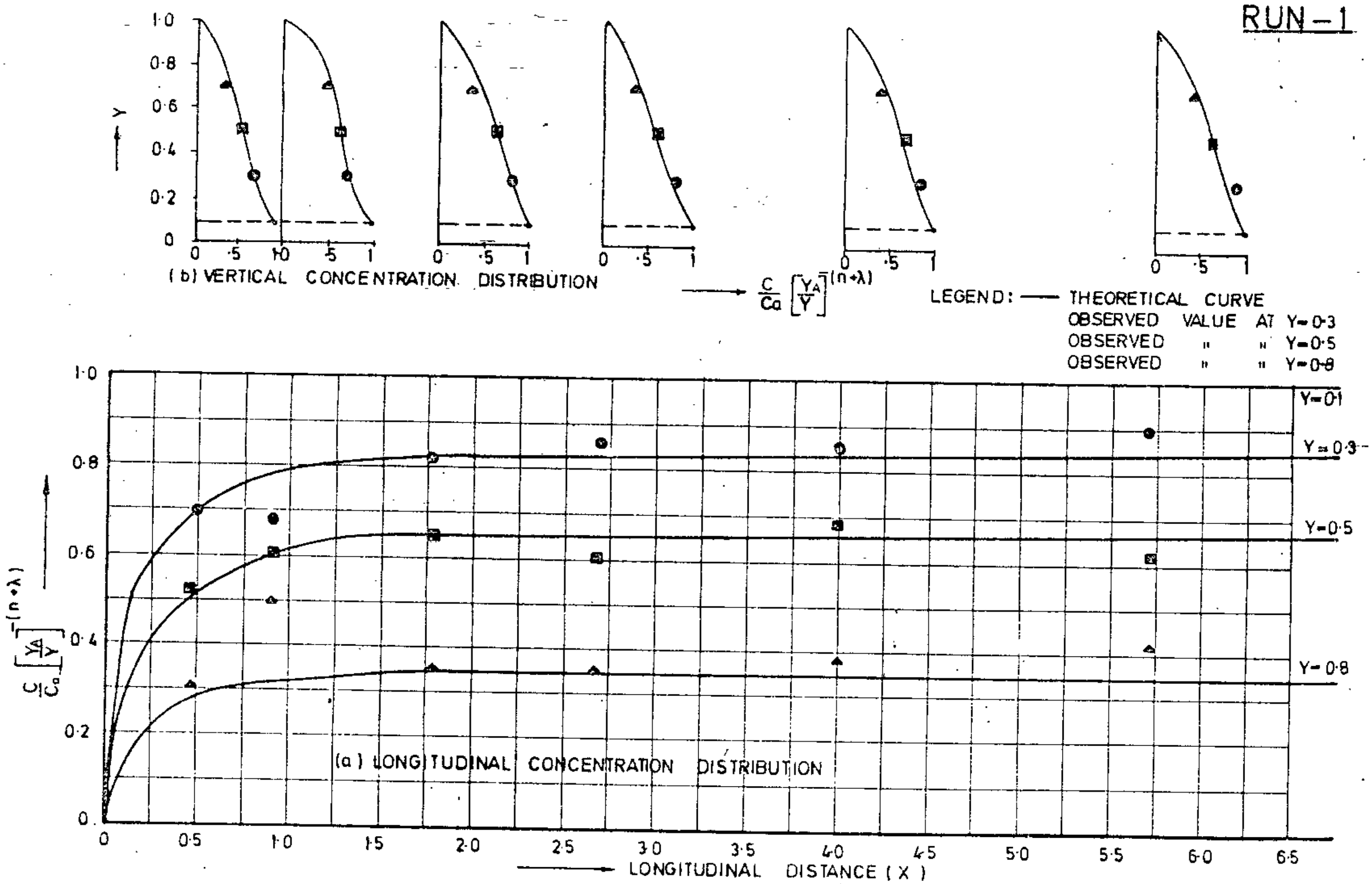

FIG. 7·12. LONGITUDINAL AND VERTICAL CONCENTRATION DISTRIBUTION

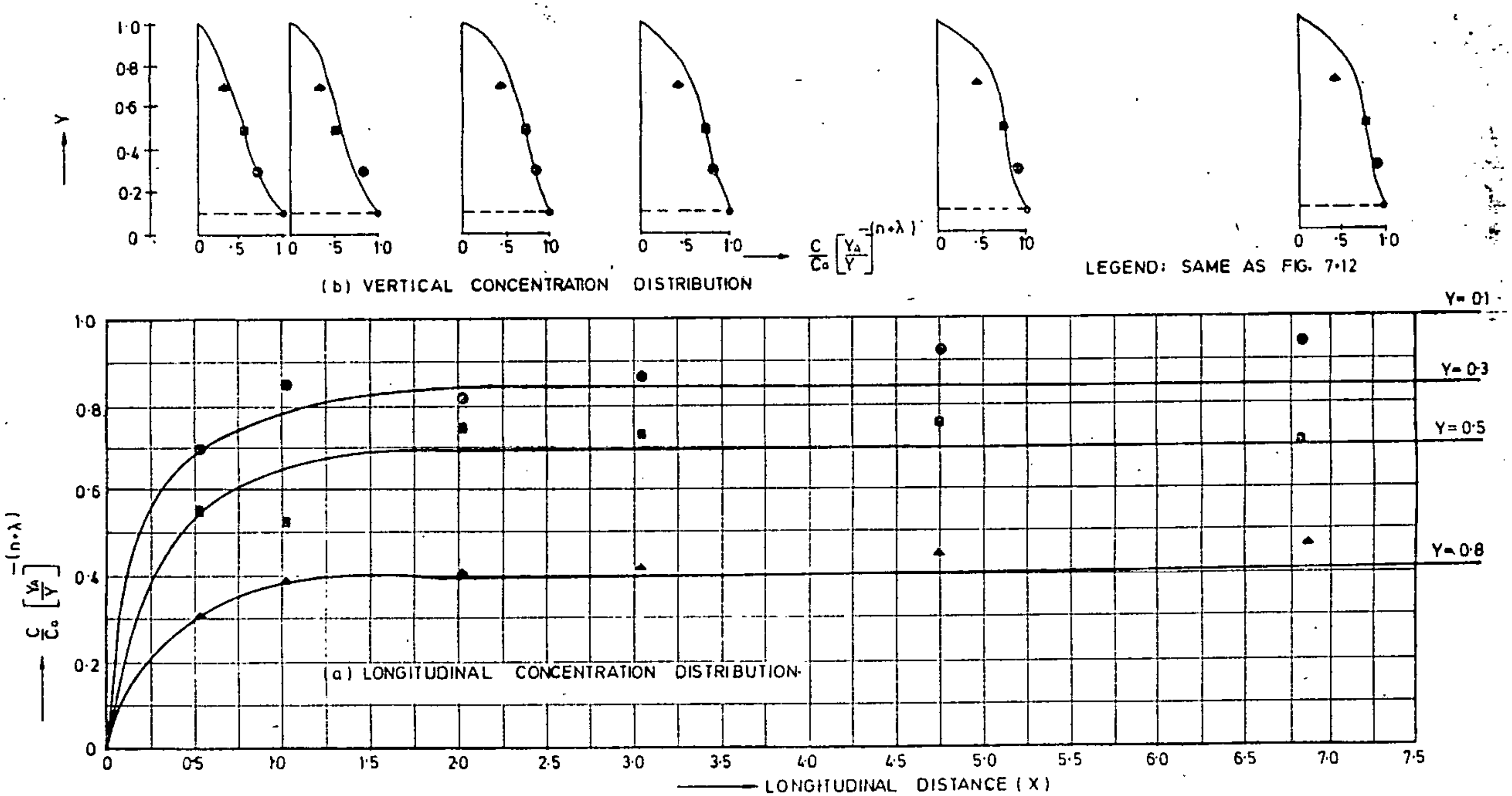

FIG. 7·13. LONGITUDINAL AND VERTICAL CONCENTRATION DISTRIBUTION

RUN – 3

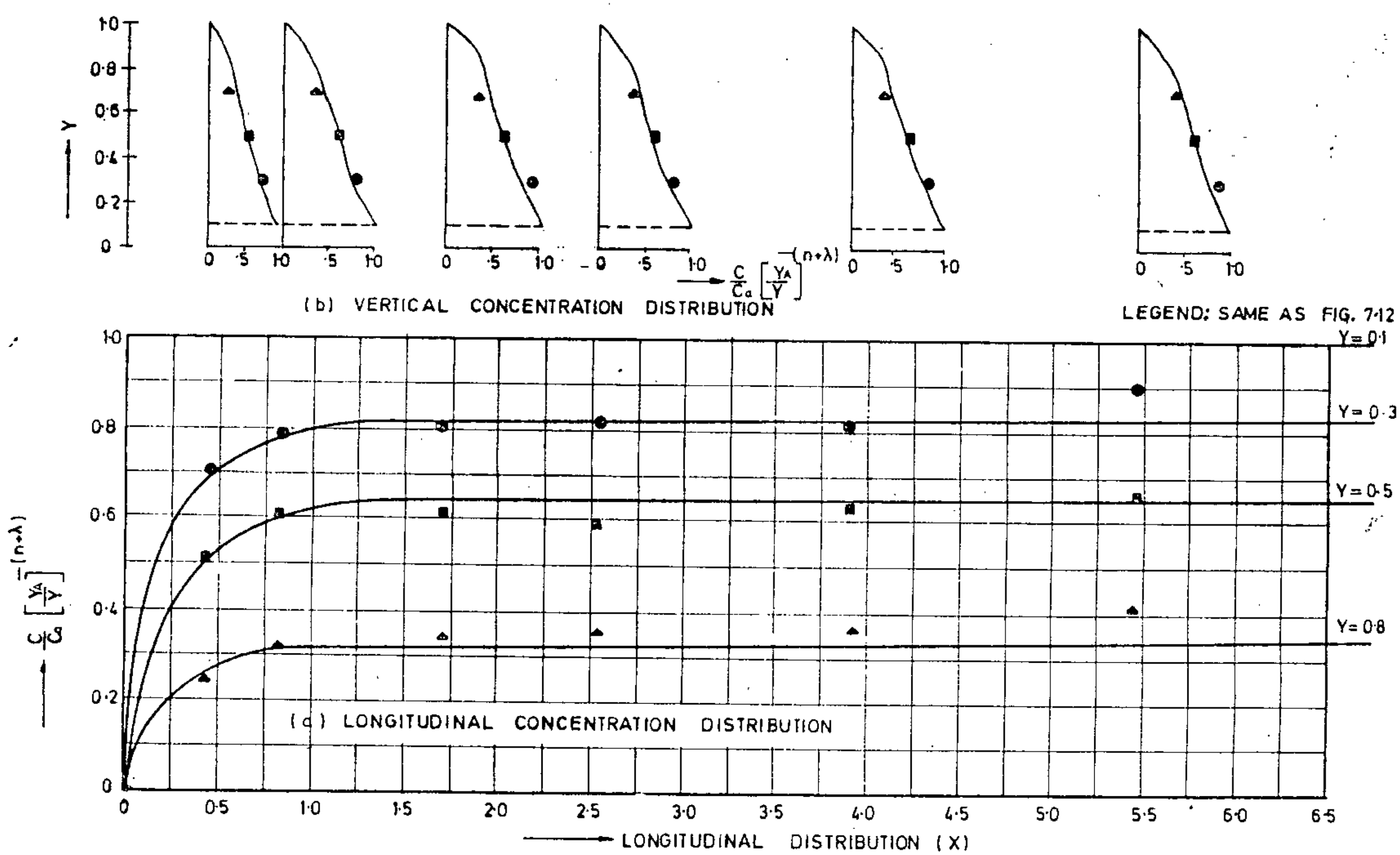

FIG. 7·14. LONGITUDINAL AND VERTICAL CONCENTRATION DISTRIBUTION

RUN– 4

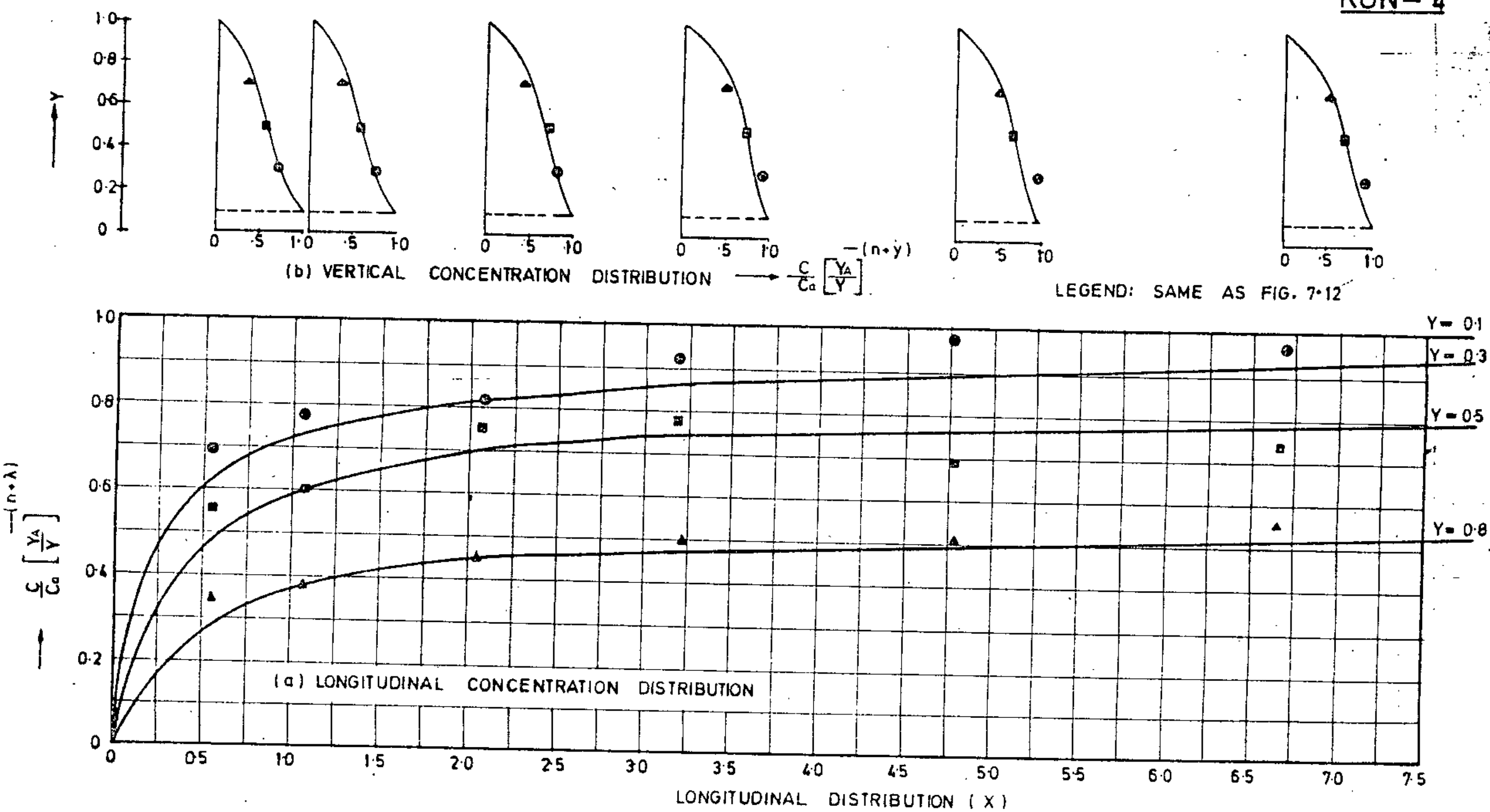

FIG. 7·15. LONGITUDINAL AND VERTICAL CONCENTRATION DISTRIBUTION

RUN – 5

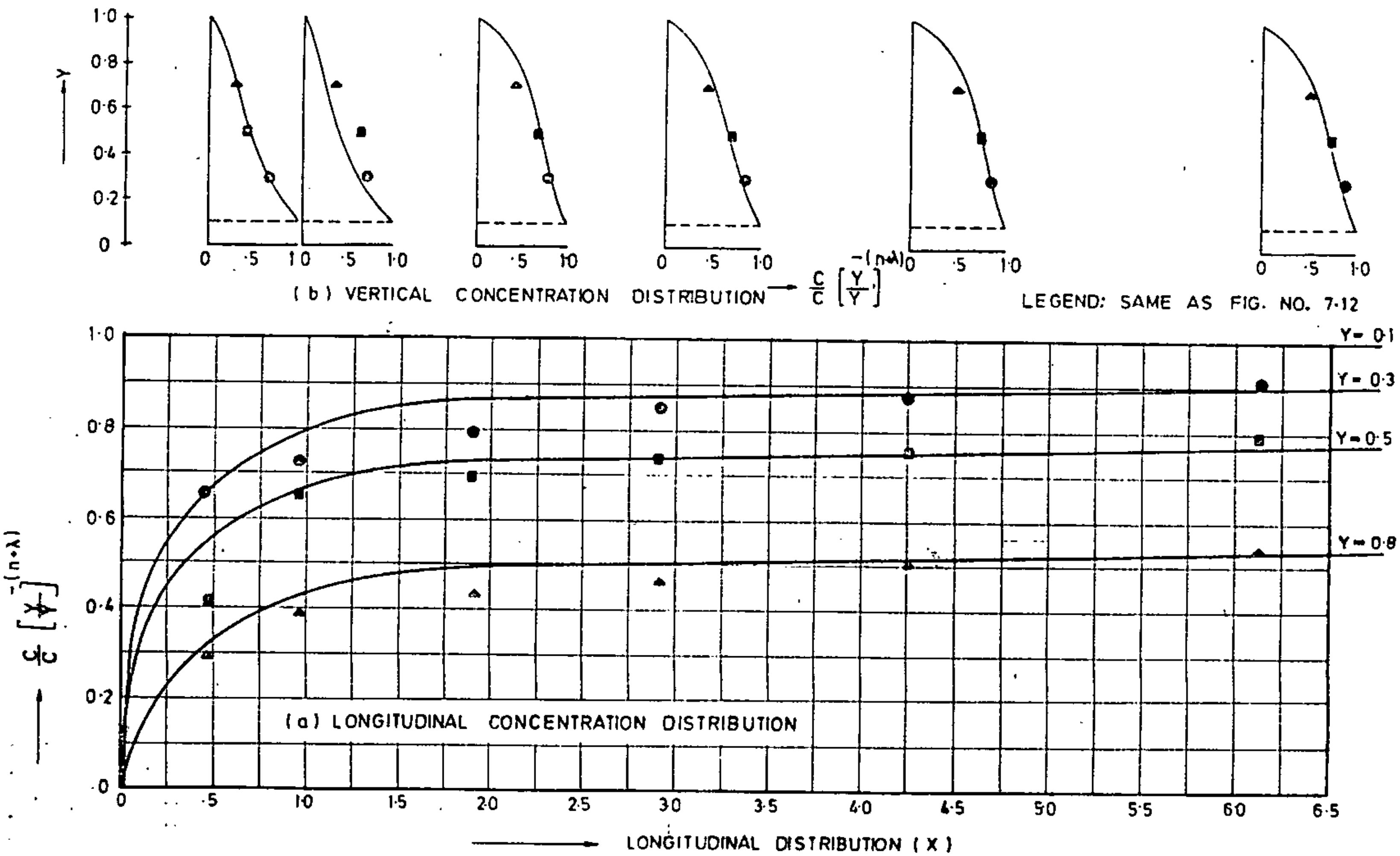

FIG. 7·16. LONGITUDINAL AND VERTICAL CONCENTRATION DISTRIBUTION

RUN - 6

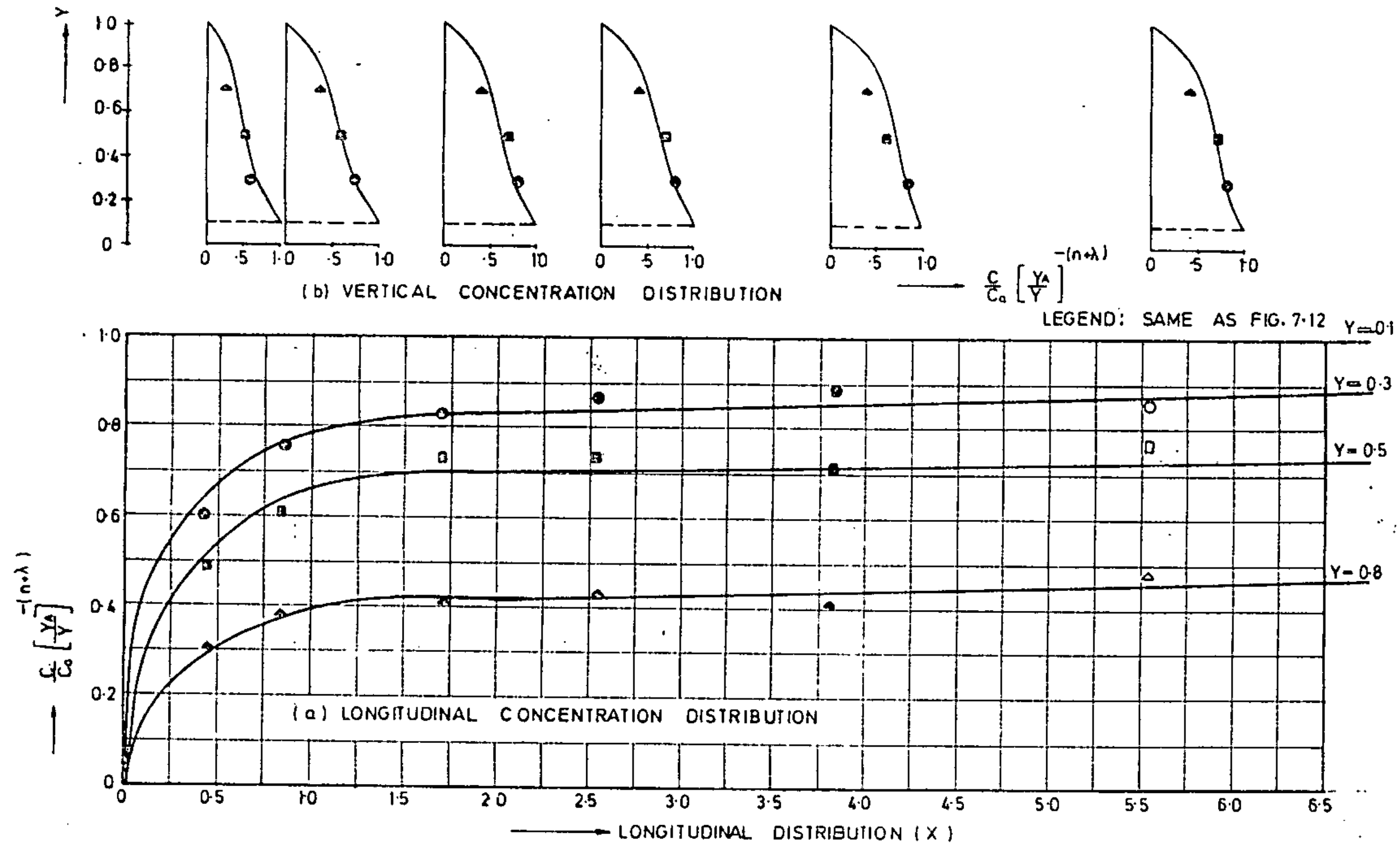

FIG. 7·17. LONGITUDINAL AND VERTICAL CONCENTRATION DISTRIBUTION

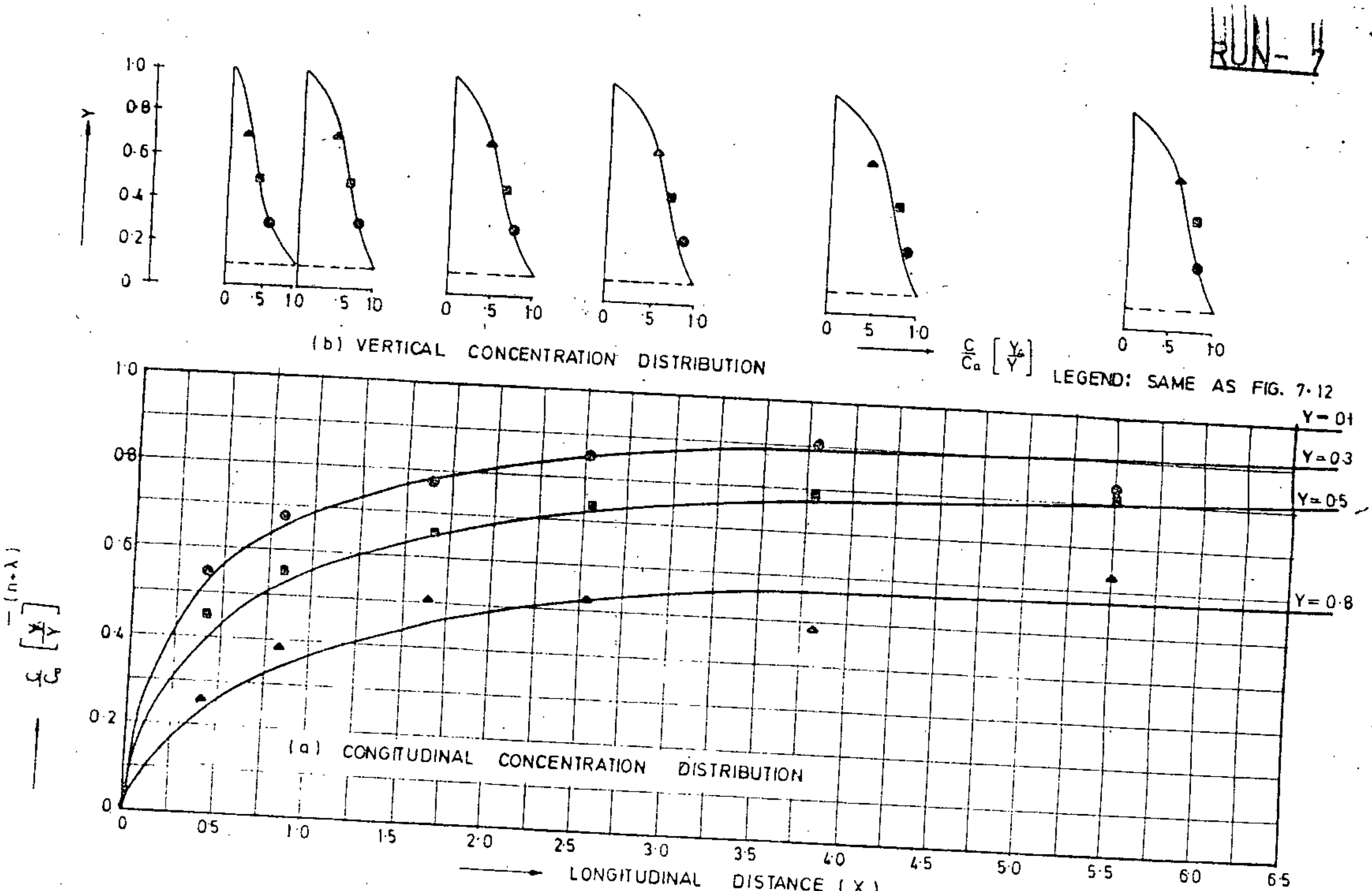

FIG. 7·18. LONGITUDINAL AND VERTICAL CONCENTRATION DISTRIBUTION

RUN - 8

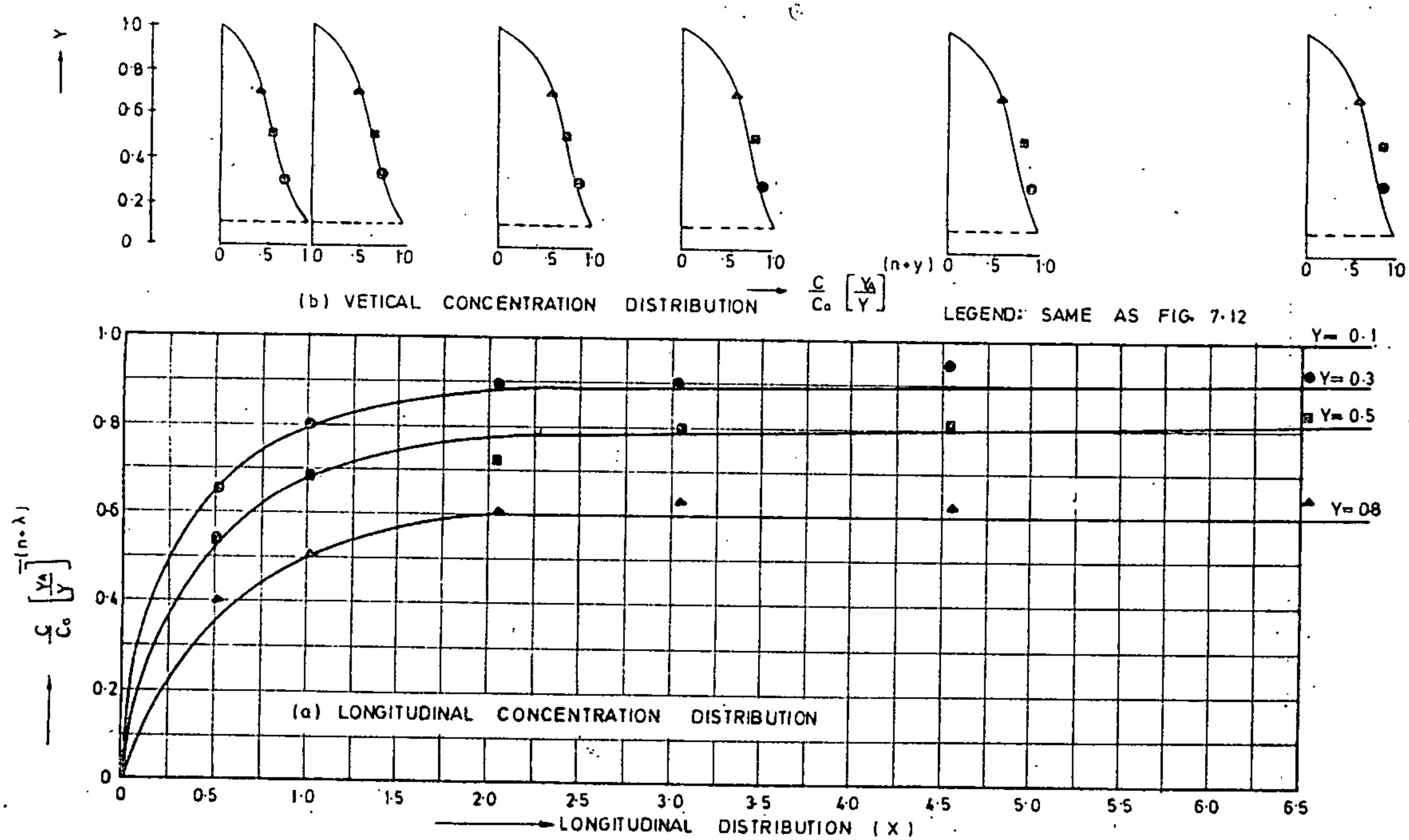

FIG. 7·19. LONGITUDINAL AND VERTICAL CONCENTRATION DISTRIBUTION

RUN — 9

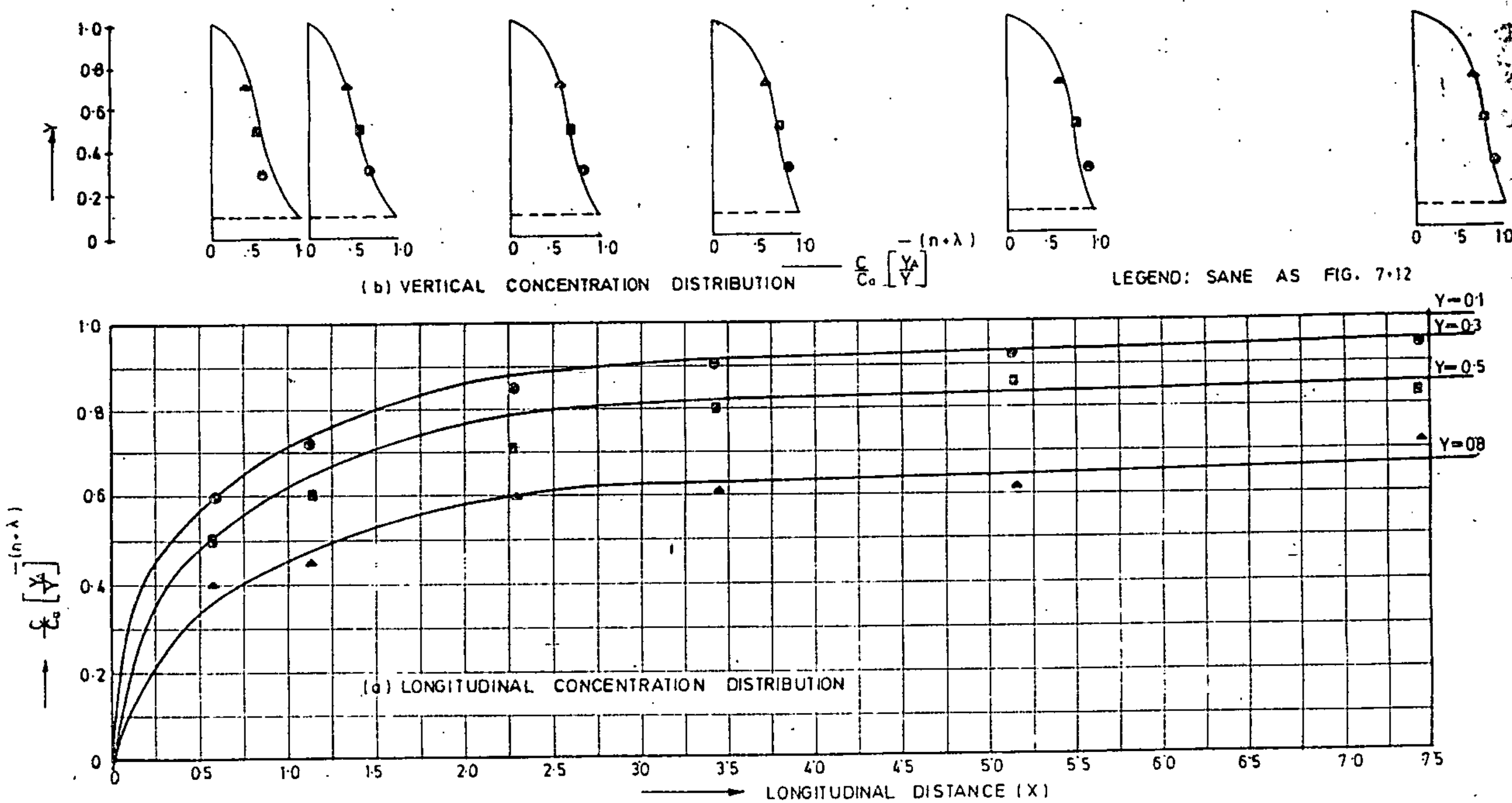

FIG. 7·20. LONGITUDINAL AND VERTICAL CONCENTRATION DISTRIBUTION

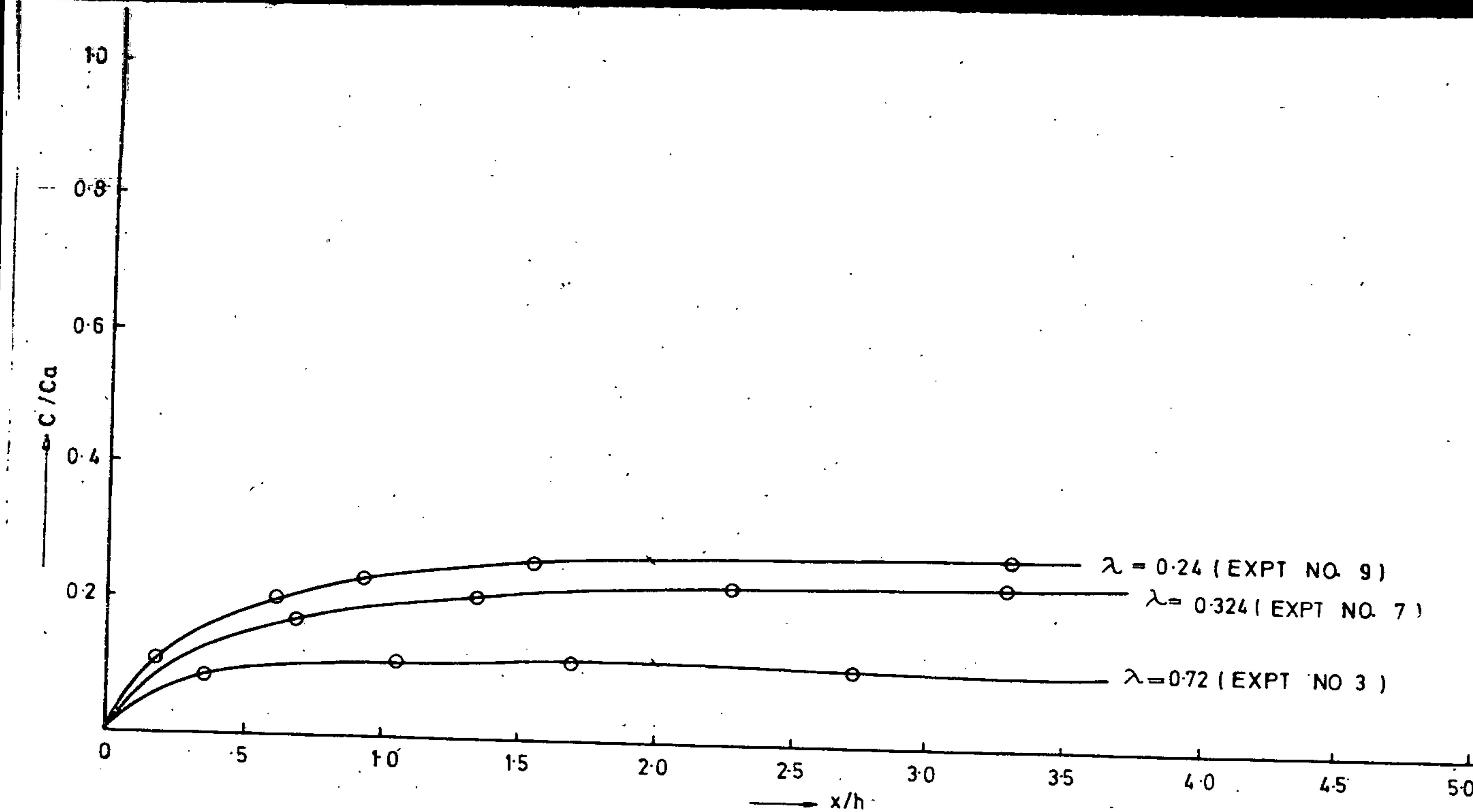

FIG. 7·21. CONCENTRATION RATIO VS DISTANCE FOR DIFFERENT VALUES OF λ

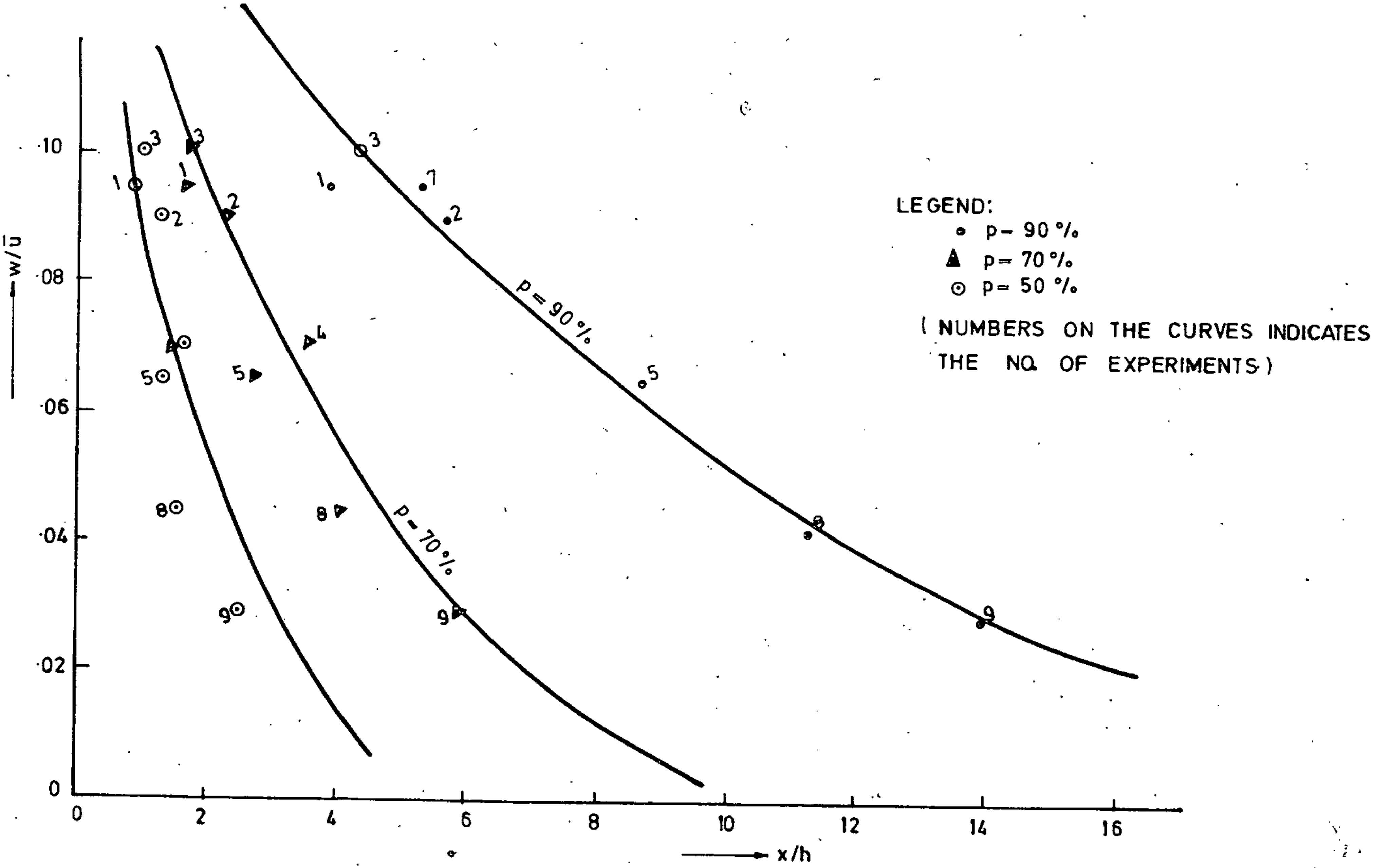

FIG. 7.22. DISTANCE REQD. TO REACH P % OF EQUILIBRIUM AT Y = 0·5

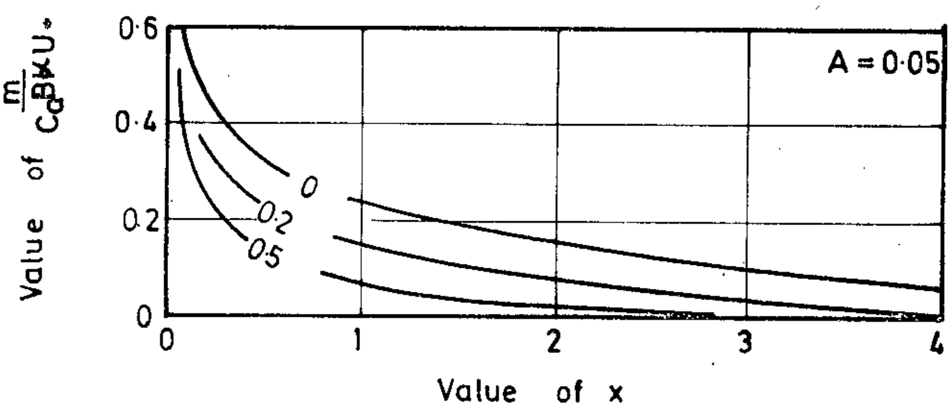

FIG. 7·25. SCOUR RATE VERSUS X

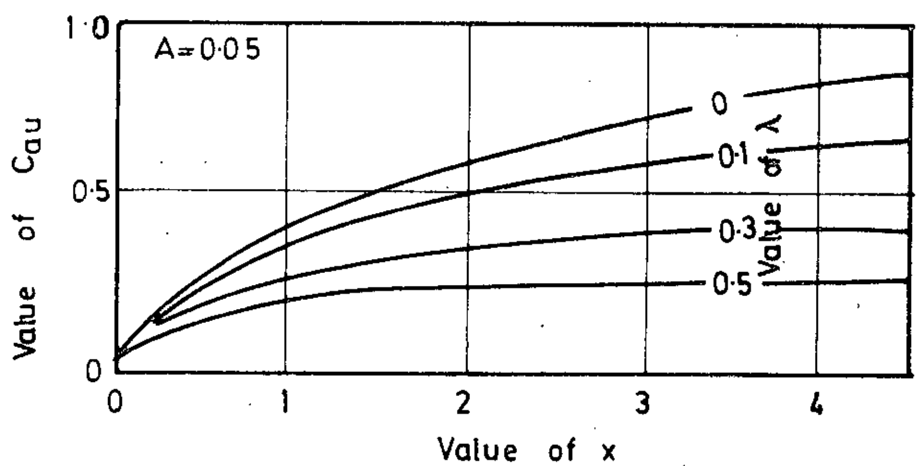

FIG. 7·23. AVERAGE CONCENTRATION VERSUS X
[After Hjelmfelt & Lenau (23)]

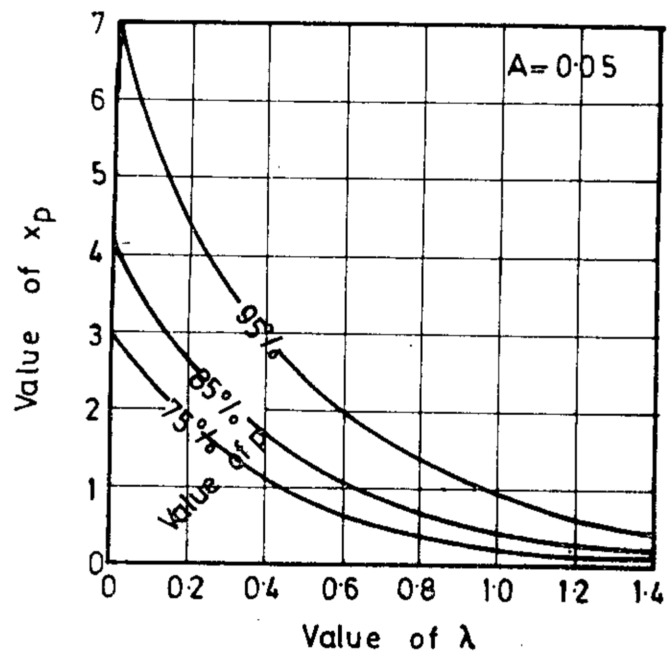

FIG. 7.24, DISTANCE REQUIRED TO REACH p % OF EQUILIBRIUM

APPENDIX - I

ORTHOGONAL SYSTEM Q(Y.P)

It is shown herein that $Q(Y,P_t)$, $t = 1,2,3.......$forms an orthogonal system with respect to the weight function $Y^{n+\lambda}(1-Y)^{-\lambda}$. The methods used and the results obtained can be found in many works on applied mathematics such as Refs. (17), but are included here for the convenience of understanding.

Now, from equation (5.23) if F(Y,P) is substituted by Q(Y,P). Then one can write:

$$Y(1-Y)\frac{\partial^2 Q(Y,P)}{\partial Y^2} + \left[(1+n+\lambda) - (2+n)Y\right]\frac{\partial Q(Y,P)}{\partial Y} + K^2 Q(Y,P) = 0 \quad .....(A\text{-}I\text{-}1)$$

or, $$\frac{\partial^2 Q(Y,P)}{\partial Y^2} + \left[\frac{1+n+\lambda}{Y(1-Y)} - \frac{2+n}{1-Y}\right]\frac{\partial Q(Y,P)}{\partial Y} + \frac{\left[P^2 - \left(\frac{1+n}{2}\right)^2\right]Q(Y,P)}{Y(1-Y)} = 0 \quad ...(A\text{-}I\text{-}2)$$

Multiplying equation (A-I-2) by $\left(\frac{Y}{1-Y}\right)^{n+\lambda+1}(1-Y)^{1-\lambda}$, one will get

$$\frac{\partial}{\partial Y}\left[Y^{1+n+\lambda}(1-Y)^{1-\lambda}\frac{\partial Q(Y,P)}{\partial Y}\right] + \left[P^2 - \left(\frac{1+n}{2}\right)^2\right]Y^{n+\lambda}(1-Y)^{-\lambda}Q(Y,P) = 0 \quad ......(A\text{-}I\text{-}3)$$

Now, let P_t and P_r be any two distinct values with corrosponding eigen functions Y_t and Y_r. Then

$$\frac{\partial}{\partial Y}\left[Y^{1+n+\lambda}(1-Y)^{1-\lambda}\frac{\partial Q(Y,P_r)}{\partial Y}\right] = -\left[P_r^2 - \left(\frac{1+n}{2}\right)^2\right]Y^{n+\lambda}(1-Y)^{-\lambda}Q(Y,P_r) \quad .....(A\text{-}I\text{-}4)$$

and

$$\frac{\partial}{\partial Y}\left[Y^{1+n+\lambda}(1-Y)^{1-\lambda}\frac{\partial Q(Y,P_t)}{\partial Y}\right] = -\left[P_t^2 - \left(\frac{1+n}{2}\right)^2\right] Y^{n+\lambda}(1-Y)^{-\lambda} Q(Y,P_t)$$

......(A-I-5)

Multiplying equation (A-I-4) by $Q(Y,P_t)$ and equation (A-I-5) by $Q(Y,P_r)$ and then substracting it is possible to obtain -

$$(P_t^2 - P_r^2)\, Y^{n+\lambda}(1-Y)^{-\lambda} Q(Y,P_r)\, Q(Y,P_t) =$$

$$\frac{\partial}{\partial Y}\left[Y^{1+n+\lambda}(1-Y)^{1-\lambda}Q(Y,P_t)\frac{\partial Q(Y,P_r)}{\partial Y} - Y^{1+n+\lambda}(1-Y)^{1-\lambda}Q(Y,P_r)\frac{\partial Q(Y,P_t)}{\partial Y}\right]$$

.......(A-I-6)

Integrating equation (A-I-6) between limits Y_A to 1,

$$(P_t^2 - P_r^2)\int_{Y_A}^{1} Y^{n+\lambda}(1-Y_A)^{-\lambda} Q(Y,P_t)\, Q(Y,P_r)\, dY$$

$$= \left[Y^{1+n+\lambda}(1-Y)^{1-\lambda}\left\{Q(Y,P_t)\frac{\partial Q(Y,P_r)}{\partial Y} - Q(Y,P_r)\frac{\partial Q(Y,P_t)}{\partial Y}\right\}\right]_{Y_A}^{1}$$

.....(A-I-7)

Now, the conditions are such that when $Y = 1$, $(1-Y) = 0$ and when $Y = Y_A$, $Q(Y_A, P_t) = 0$, $t = 1,2,3$ ....Therefore if the values are put in equation (A-I-7) it will give:

$$(P_t^2 - P_r^2)\int_{Y_A}^{1} Y^{n+\lambda}(1-Y)^{-\lambda} Q(Y,P_r)\, Q(Y,P_t)\, dY = 0 \quad .... \qquad \text{(A-I-8)}$$

Moreover, since $r \neq t$ one can write

$$\int_{Y_A}^{1} Y^{n+\lambda}(1-Y)^{-\lambda} Q(Y,P_t)\ Q(Y,P_r)\ dY = 0 \quad \ldots \qquad \text{(A-I-9)}$$

Equation (A-I-9) shows that $Q(Y,P_t)$, $t = 1,2,3$ .....form an orthogonal system with the weight function $Y^{n+\lambda} \cdot (1-Y)^{-\lambda}$

APPENDIX - II

INTEGRATION OF $Y^{n+\lambda}(1-Y)^{-\lambda}Q(Y,P_t)$ and $Y^{n+\lambda}(1-Y)^{-\lambda}Q^2(Y,P_t)$

From equation (A-I-7) it is possible to write.

$$(P_t^2 - P^2)\int_{Y_A}^{1} Y^{n+\lambda}(1-Y)^{-\lambda}Q(Y,P_t)\ Q(Y,P_r)\ dY =$$

$$Y_A^{1+n+\lambda}(1-Y_A)^{1-\lambda}\left\{Q(Y_A,P)\ \frac{\partial Q(Y_A,P_t)}{\partial Y} - Q(Y_A,P_t)\ \frac{\partial Q(Y_A,P)}{\partial Y}\right\} \quad \text{(A-II-1)}$$

That is,

$$\int_{Y_A}^{1} Y^{n+\lambda}(1-Y)^{-\lambda}Q(Y,P_t)\ Q(Y,P)\ dY = \frac{Y_A^{1+n+\lambda}\ (1-Y_A)^{1-\lambda}\ Q(Y_A,P)\ \frac{\partial Q(Y_A,P_t)}{\partial Y}}{(P_t^2 - P^2)} \quad \text{(A-II-2)}$$

Now, as $P \longrightarrow P_t$ one can get using L'Hospitals priciple

$$\int_{Y_A}^{1} Y^{n+\lambda}\ (1-Y)^{-\lambda}Q^2(Y,P_t)\ dY = -\frac{Y_A^{1+n+\lambda}\ (1-Y_A)^{1-\lambda}\ \frac{\partial Q(Y_A,P)}{\partial Y} \cdot \frac{\partial Q(Y_A,P_t)}{\partial Y}}{2P_t} \quad \text{(A-II-3)}$$

Again integrating equation (A-I-3) term by term over the interval Y_A to 1, one will get

$$\int_{Y_A}^{1} Y^{n+\lambda}(1-Y)^{-\lambda}Q(Y,P_t)\ dY = \frac{Y_A^{1+n+\lambda}\ (1-Y_A)^{1-\lambda}\ \frac{\partial Q(Y_A,P_t)}{\partial Y}}{\left[P_t^2 - (\frac{1+n}{2})^2\right]} \quad \text{(A-II-4)}$$

APPENDIX - III

LOCATION OF ROOTS P_t

From mathematics the hypergeometric solution of the hyprogeometric equation is:

$$F(\alpha,\beta,\gamma,x) = \frac{\Gamma(\gamma)\,\Gamma(\gamma-\alpha-\beta)}{\Gamma(\gamma-\alpha)\,\Gamma(\gamma-\beta)}\, H\left[\alpha,\beta,\alpha+\beta-\gamma+1,1-x\right] + \frac{\Gamma(\gamma)\,\Gamma(\alpha+\beta-\gamma)}{\Gamma(\alpha)\,\Gamma(\beta)}$$

$$(1-x)^{\gamma-\alpha-\beta}\, H\left[\gamma-\beta,\ \gamma-\alpha,\ \gamma-\alpha-\beta+1,\ 1-x\right] \quad \ldots\ldots(A\text{-}III\text{-}1)$$

But in this case:

$$\alpha = \frac{1+n}{2} + \lambda + P. \quad \beta = \frac{1+n}{2} + \lambda - P, \quad \gamma = 1+\lambda, \quad x = 1-Y_A$$

Therefore one can write by identify:

$$H\left[\frac{1+n}{2}+\lambda+P,\ \frac{1+n}{2}+\lambda-P,\ 1+\lambda,\ 1-Y_A\right] = \frac{\Gamma(1+\lambda)\,\Gamma(-\lambda-n)}{\Gamma\left(\frac{1-n}{2}+P\right)\Gamma\left(\frac{1-n}{2}-P\right)}$$

$$H\left[\frac{1+n}{2}+\lambda+P,\ \frac{1+n}{2}+\lambda-P,\ 1+n+\lambda,\ Y_A\right] + \frac{\Gamma(1+\lambda)\,\Gamma(\lambda+n)}{\Gamma\left(\frac{1+n}{2}+\lambda+P\right)\Gamma\left(\frac{1+n}{2}+\lambda-P\right)}$$

$$Y_A^{-(n+\lambda)}\, H\left[\frac{1-n}{2}+P,\ \frac{1-n}{2}-P,\ 1-n-\lambda,\ Y_A\right] \quad \ldots \quad (A\text{-}III\text{-}2)$$

Since Y_A is very small, then one will get,

$$\frac{\Gamma(1+\lambda)\,\Gamma(-\lambda-n)}{\Gamma\left(\frac{1-n}{2}-P\right)\Gamma\left(\frac{1-n}{2}+P\right)} + Y_A^{-(n+\lambda)}\, \frac{\Gamma(1+\lambda)\,\Gamma(\lambda+n)}{\Gamma\left(\frac{1+n}{2}+\lambda+P\right)\Gamma\left(\frac{1+n}{2}+\lambda-P\right)} = 0$$

$$\ldots \quad (A\text{-}III\text{-}3)$$

Now, for $(n+\lambda) > 0$ the second term in equation (A-III-3) will dominate the first. Hence the positive zeros of equations (A-III-3) are approximately the poles of $\Gamma\left(\frac{1+n}{2}+\lambda-P\right)$

Hence,

$$P_t = t + \lambda + \frac{n}{2} - \frac{1}{2} \quad , \quad t = 1, 2, 3, \ldots\ldots$$

APPENDIX - IV

EVALUATION OF $\frac{\partial Q}{\partial P}(Y_A, P_t)$

It was assumed that

$$Q(Y,P) = H\left[\frac{1+n}{2} + \lambda + P,\ \frac{1+n}{2} + \lambda - P,\ 1+\lambda,\ 1-Y\right] \quad \ldots \quad \text{(A-IV-1)}$$

or

$$Q(Y_A,P) = H\left[\frac{1+n}{2} + \lambda + P,\ \frac{1+n}{2} + \lambda - P,\ 1+\lambda,\ 1-Y_A\right] \ldots \quad \text{(A-IV-2)}$$

Differentiating equation (A-IV-2) term by term, one will get -

$$\frac{\partial Q}{\partial P}(Y_A,P) = -2P\left[\frac{(1-Y_A)}{1+\lambda} + \frac{\left[\left(\frac{3+n}{2}+\lambda\right)^2 - P^2\right](1-Y_A)^2}{(1+\lambda)\ (2+\lambda)\ \lfloor 2} + \frac{\left[\left(\frac{1+n}{2}+\lambda\right)^2 - P^2\right](1-Y_A)^2}{(1+\lambda)(2+\lambda)\lfloor 2} + \ldots\ldots \quad \ldots\right] \quad \text{(A-IV-3)}$$

Therefore,

$$\frac{\partial Q}{\partial P}(Y_A,P) = -2P\sum_{r=1}^{\infty}\frac{(n/2+\lambda+P+\frac{1}{2})_r\ (n/2+\ \ -P+\frac{1}{2})_r\ (1-Y_A)^r}{(1+\lambda)_r\ \lfloor r}$$

$$\sum_{r=1}^{r}\frac{1}{(\frac{n}{2}+\lambda+P+r-\frac{1}{2})\ (n/2+\lambda-P+r-\frac{1}{2})} \quad \ldots \quad \text{(A-IV-4)}$$