ANALYSIS OF TALL BUILDINGS
WITH INTERCONNECTED SHEAR WALLS AND FRAMES

A Thesis
by
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### Chapter 3

3.3.3 System "W" ..... 24
3.3.4 System "F" ..... 24
3.3.5 First Stage of Analysis ..... 26
3.3.6 Second Stage of Analysis ..... 28
3.4 Component Stiffness Method ..... 30
3.5 Cardan Method ..... 36
   3.5.1 Assumptions ..... 36
   3.5.2 Governing Equations ..... 36

### Chapter 4

"Exact Analysis of Plane Frame by Stiffness Method and Recursion Procedure"

4.1 Introduction ..... 38
4.2 Types of Frames ..... 39
   4.2A Additional Notation ..... 41
4.3 One Bay Frame without Shear Wall ..... 42
   4.3.1 Stiffness Matrix of Frame ..... 45
   4.3.2 Solution by Recursion Procedure ..... 48
   4.3.3 Slope Deflection Equations ..... 50
4.4 One Bay Frame with Shear Wall ..... 51
4.5 General One Bay Frame Analysis ..... 56
   4.5.1 Stiffness Matrix for General One Bay Frame ..... 57
4.6 Two Bay Frame Analysis with Shear Wall ..... 62
4.7 General Two Bay Frame Analysis ..... 68
   4.7.1 Stiffness Matrix of General Two Bay Frame ..... 69
## CHAPTER 5  COMPUTER PROGRAMS FOR STRUCTURES

<table>
<thead>
<tr>
<th>Section</th>
<th>Title</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>5.1</td>
<td>Introduction</td>
<td>75</td>
</tr>
<tr>
<td>5.2</td>
<td>Outline of Programs</td>
<td>75</td>
</tr>
<tr>
<td>5.2.1</td>
<td>General Outline of Programs</td>
<td>76</td>
</tr>
<tr>
<td>5.2.2</td>
<td>Units</td>
<td>77</td>
</tr>
<tr>
<td>5.3</td>
<td>Computer Storage and Time</td>
<td>77</td>
</tr>
<tr>
<td>5.4</td>
<td>Flow Diagram</td>
<td>78</td>
</tr>
<tr>
<td>5.5</td>
<td>One Bay Frame Program</td>
<td>79</td>
</tr>
<tr>
<td>5.6</td>
<td>General One Bay Frame Program</td>
<td>80</td>
</tr>
<tr>
<td>5.7</td>
<td>Two Bay Frame Program</td>
<td>82</td>
</tr>
<tr>
<td>5.8</td>
<td>General Two Bay Frame Program</td>
<td>84</td>
</tr>
<tr>
<td>5.9</td>
<td>Program for GA for One Bay Frame</td>
<td>87</td>
</tr>
<tr>
<td>5.10</td>
<td>Program for GA for Two Bay Frame</td>
<td>87</td>
</tr>
<tr>
<td>5.11</td>
<td>Program for Approximate Analysis by Heidebrecht and Smith Method</td>
<td>87</td>
</tr>
</tbody>
</table>

## CHAPTER 6  COMPARISON OF EXACT METHOD WITH APPROXIMATE METHOD

<table>
<thead>
<tr>
<th>Section</th>
<th>Title</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>6.1</td>
<td>Introduction</td>
<td>80</td>
</tr>
<tr>
<td>6.2</td>
<td>Comparison of GA Parameter of One and Two Bay Frames</td>
<td>88</td>
</tr>
<tr>
<td>6.3</td>
<td>Comparison of Exact Method with Portal and Cantilever Method</td>
<td>93</td>
</tr>
<tr>
<td>6.3.1</td>
<td>One Bay Frame</td>
<td>94</td>
</tr>
<tr>
<td>6.3.1.1</td>
<td>Discussion on Point of Inflection in Columns</td>
<td>94</td>
</tr>
<tr>
<td>6.3.2</td>
<td>Two Bay Frame</td>
<td>101</td>
</tr>
</tbody>
</table>
CHAPTER 7  TWO BAY FRAME WITH SHEAR WALL

7.1 Introduction ................................................................. 110
7.2 Comparison Between Exact Method and Heidebrecht and Smith Method 110

CHAPTER 8  BUILDING SWAY

8.1 Introduction ................................................................. 119
8.2 Sway Control by Using Heavier Crown Coupling Beam ................. 119
8.2.1 Description of Structures ........................................... 120
8.2.2 Frame with Shear Wall ............................................. 120
8.2.3 Frame without Shear Wall ......................................... 121

CHAPTER 9  LOAD TRANSFER FROM A VERTICALLY LOADED COLUMN

9.1 One Bay Frame ............................................................. 123

CHAPTER 10 CONCLUSIONS AND SUGGESTIONS

10.1 Conclusion ............................................................... 125
10.2 Suggestions for Future Study ......................................... 127

REFERENCES ................................................................. 128

APPENDIX "A" LISTING OF COMPUTER PROGRAMS .......................... 129
This thesis deals with the methods of analysing tall buildings with frames and shear walls subjected to static loads, considering only the linear elastic behaviour.

The design procedures for frame structure and shear wall interconnected with frame structure are discussed. Exact analysis as well as approximate analysis by different authors are presented.

Analysis of structures with or without shear walls is performed by stiffness matrix method and the solution is made by recursion procedure. Computer programs using the above method are developed and written in Fortran II and IV. A program is also developed and written in Fortran II and IV for the approximate analysis of shear wall structures by using the continuous connection method suggested by Heidbrecht and Smith.

The values of lateral stiffness of frames, as expressed by the GA parameter are tabulated both by the 'exact' method and Heidbrecht and Smith method and compared for different structural properties of frame expressed by a non-dimensional parameter.

Frame programs are run for lateral loads and the results are analyzed to find out the validity of assumptions of portal and cantilever methods. The results are presented in Tabular and graphical form. These graphs may be used to find out the point of inflection at columns at different stories.
A comparison of "interactions" of shear walls and frames by Heidebrecht and Smith method is made with that of 'exact' method.

An attempt is also made to evaluate the role of heavier crown coupling beam in reducing sway of one bay frames.

The problem of load transfer from a vertically loaded column to another column depending on the structural properties of different elements of one bay frames is also discussed.
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NOTATION

A = Cross-sectional area
b = length of girder in frame
E = modulus of elasticity
G = shear modulus
H = height of building
h = storey height

I, I_h, I_b = moment of inertia of shear wall, frame column and frame girder respectively

Mo = total bending moment at base of the structure
M_B, M_S = bending moment in flexural and shear components respectively
Q = transverse interactive force at the top of the structure
q = transverse interactive force distribution
S_o = total shearing force at base of the structure
S_F, S_S = shearing force in the flexural and shear components respectively

w = lateral load distribution
x = position of ordinate along the height of the structure, relative to the base

y = lateral deflection

\( \alpha = \sqrt{GA/EI} \)

P = force or load

A = Width of frame

C = Width of column or shear wall

C_1, C_2 = distances from centerline of beams to centerline of columns of shear wall.
D = depth of beam
N = number of stories
\( \triangle \) = deflection
\( \triangle_A \) = top deflection due to column axial deformation
\( \triangle_B \) = top deflection due to bending deformation

\[ \Lambda = \text{ratio} \left( \frac{E_c I_c}{h} \right) \text{ or } \frac{\sum (E_c I_c/h)}{2 \sum (E_b I_b)/\ell} \]

\( K_w \) = stiffness of a shear wall. In general \( K_w \) = lateral point load applied at top of a shear wall to cause unit deflection in its line of action

\( K_f \) = lateral point load applied at top of frame to cause unit deflection in its line of action

\( W \) = total lateral loading


<table>
<thead>
<tr>
<th>Figure</th>
<th>Description</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Qualitative cost per sq. ft. versus number of stories</td>
<td>3</td>
</tr>
<tr>
<td>2</td>
<td>Interaction behavior of shear wall frame</td>
<td>5</td>
</tr>
<tr>
<td></td>
<td>(a) Free frame</td>
<td></td>
</tr>
<tr>
<td></td>
<td>(b) Free wall</td>
<td></td>
</tr>
<tr>
<td></td>
<td>(c) Combined frame and wall</td>
<td></td>
</tr>
<tr>
<td>3</td>
<td>(a) Independence House Lagos, 25 storey</td>
<td>7</td>
</tr>
<tr>
<td>3</td>
<td>(b) Radiation Ltd. Office, Neasden, 13 storey</td>
<td>7</td>
</tr>
<tr>
<td>3</td>
<td>(c) Moorfields Development, 36 storey</td>
<td>7</td>
</tr>
<tr>
<td>3</td>
<td>(d) Typical floor plan of Brunswick Building, U.S.A. - 38 Storey</td>
<td>7</td>
</tr>
<tr>
<td>4</td>
<td>Mathematical model for shear-flexure member</td>
<td>15</td>
</tr>
<tr>
<td></td>
<td>(a) Flexural member</td>
<td></td>
</tr>
<tr>
<td></td>
<td>(b) Shear member</td>
<td></td>
</tr>
<tr>
<td></td>
<td>(c) Shear-flexural member</td>
<td></td>
</tr>
<tr>
<td></td>
<td>(d) Components of shear flexural member</td>
<td></td>
</tr>
<tr>
<td>5</td>
<td>Typical idealized structure</td>
<td>29</td>
</tr>
<tr>
<td></td>
<td>(a) Floor plan</td>
<td></td>
</tr>
<tr>
<td></td>
<td>(b) Idealized structure</td>
<td></td>
</tr>
<tr>
<td></td>
<td>(c) Simplified idealized structure</td>
<td></td>
</tr>
<tr>
<td>6</td>
<td>Idealization of structure for equation c of component stiffness method</td>
<td>33</td>
</tr>
<tr>
<td></td>
<td>(a) Structure</td>
<td></td>
</tr>
<tr>
<td></td>
<td>(b) Interaction at top only</td>
<td></td>
</tr>
<tr>
<td></td>
<td>(c) Frame modeled by spring</td>
<td></td>
</tr>
<tr>
<td>7</td>
<td>Effect of $\lambda$ on frame deformation</td>
<td>95</td>
</tr>
<tr>
<td></td>
<td>(a) Shear mode deformation</td>
<td></td>
</tr>
<tr>
<td></td>
<td>(b) Bending mode deformation</td>
<td></td>
</tr>
</tbody>
</table>
1.1 Introduction

After the Independence of Bangladesh, the social and economic aspects of the urban areas have changed quite considerably, causing the tremendous influx of population to these areas from all over the country. This increase in population in one hand and the limited space in the urban areas on the other hand have necessitated the need for constructing tall office and residential buildings in the urban areas of Bangladesh. As a result several buildings from 10 to 20 stories high are being built in concrete in the major cities; and it is becoming apparent that in the near future a large number will be required.

1.1.1 The Criteria for Tall Buildings

Arbitrary definitions of tall buildings have in some cases included, even a two storey building. From the structural engineer’s point of view, a tall building is one whose structural analysis and design is in some way affected by one or more of the following factors:

(a) Height to width ratio.
(b) Forces and moments in the structural members due to lateral loads.
(c) Sway caused by lateral loads.
(d) Perception of motion under strong wind gusts.
1.1.2 Optimum Building Cost

The structural design of buildings of smaller height are governed by the gravity loads, viz. the dead and live load. The stresses due to lateral loads, viz. wind and seismic in low height buildings remain within the allowable overstress. But for taller buildings the lateral sway as well as lateral load stresses begin to control the design. As a result, the structural elements designed only for gravity loads need to be increased in order to increase the stiffness and strength of the building.

In the past, it was generally considered by the engineers that concrete buildings more than about 15 stories high are neither economic nor feasible. Such opinions were based on the use of traditional beam-column type frame construction.

In fact, if some preliminary design of taller buildings, using only the frame action, are made, it would be found that the estimated cost per sq. ft. of floor would be too high to be considered competitive. On the other hand, if there was no lateral load such as wind or earthquake any tall building could be primarily designed only for gravity loads in which case there would be no premium for height. This is qualitatively shown by the two curves in Fig. 1. Since there is no way of avoiding the gravity loads, it can be concluded that the minimum possible cost (or material) for a building of any number of stories cannot be less than that shown by the lower curve in Fig. 1. Therefore, from the structural point of view the most efficient or optimum building cost should qualitatively correspond to this curve.
The search for a system of tall buildings should therefore, be aimed at creating "optimum" structures that need be designed for the gravity loads only while the stresses due to lateral loads will automatically remain within the normally allowable overstress 33 1/3% in U.S. code and 25% in British code.

1.1.3 Shear Wall Construction

In the past and in the early post war period the systems providing lateral stability to tall buildings were essentially the rigid structural frames. The stability of such frames depends largely on the rigid connections between the columns and beams. The fully rigid connection, in practice, for a reinforced concrete
frame results in a complicated arrangement of reinforcements. As shown by the upper curve of Fig. 1, the traditional rigid frame construction results in excessive structural cost as the building grows taller, thereby making the construction of tall reinforced concrete framed buildings uneconomical. To overcome the problem of lateral stability of tall buildings, the structural engineers have introduced concrete shear walls at suitable locations inside the building.

Shear walls are reinforced concrete walls providing stability to structure against lateral loads. As the wall can take care of most of the lateral shear coming to the structure, that is why it is called shear wall. The primary behavior of a shear wall of normal proportions is however governed by flexural deformations.

The moment resistance capacity of the frame in bending fails to achieve a structural action as efficient as that of a shear wall. The lateral stability of tall building is solved by constructing the walls around the stair and elevator shafts in reinforced concrete. These would act as shafts cantilevering out of the ground and resisting the entire lateral force. By incorporating shear walls in tall buildings, flat-plate type slab construction could be used typically for all floors without any regard to lateral load.

However, to assume that a shear wall resists the entire wind load is an obvious oversimplification. In reality, a shear wall can never act as a truly independent unit but must interact
with the remaining frame elements of the building. Even though the frame part of the structure may be a rather flexible combination of flat plate and columns, as the number of stories increases its interaction with the shear wall becomes more significant which may actually contribute greatly to the lateral load resistance of the building. Therefore, when the frame portion of the building is fairly rigid by itself the interaction between the shear wall and frame can result in a considerably more rigid and efficient design. The interaction behaviour of a shear wall and frame structure can be schematically shown in Fig. 2. Because of the different lateral deflection characteristics of the frame and shear wall, the frame tends to pull back the shear wall in
the upper portion of the building and push it forward in the lower portion. As a result, the frame participates more effectively in the upper portion of the building where the lateral load shears are relatively less and the shear wall carries most of the shear in the lower portion of the building where the frame generally cannot afford to carry high lateral load.

1.1.4 Choice for Shear Walls

The concrete shear wall has come to be accepted as a natural component of a multistorey concrete building. Judicious use of shear walls in a tall building has resulted in their optimum use both as a column element as well as the wind bracing.

Depending on height to width ratio, the use of extra shear wall, other than the walls around the stair and elevator shafts, is a matter of choice for buildings say upto 15 stories. For buildings taller than 15 stories and particularly in the range of 20 - 60 stories, the use of shear walls other than walls around stair and elevator shafts, in one form or other becomes imperative from the point of view of economy. The use of shear walls in several already constructed tall buildings are shown in Fig. 3.
FIG. 3(a) INDEPENDENCE HOUSE
LAGOS, 25 STOREY

FIG. 3(b) RADIATION LTD. OFFICE
NEASDEN, 13 STOREY

FIG. 3(c) MOORFIELDS DEVELOPMENT, 36 STOREY
1.2 Assumptions

The following assumptions are made in the analysis of tall buildings:

1) Shear wall and frames are fixed at its base.
2) All the joints are assumed to be rigid.
3) The floor diaphragms are rigid in their own plane (but have no stiffness normal to this plane) and there produces no rotations of the diaphragms in their plane. Thus, each floor level is constrained to translate without rotation, and each parallel frame is subjected to the same displacement at any given floor level.
(4) Axial deformations of the columns and shear walls are considered in the analysis, but girder axial deformations are neglected. Shear deformations in the columns, walls and girders are neglected.

(5) Plane sections of the wall before bending remain plane after bending so that the moment-curvature relations based on the simple "engineer's theory of bending" (ETB) may be used.

The "Exact" analysis as used in this thesis is based on the above assumptions.

1.3 Scope of the Thesis

The purpose of this work is to investigate methods of analysis of tall buildings with frames and interconnected shear walls. Only the linear elastic behaviour under static loads is considered. The exact analysis of Tall structures with or without shear walls are made and computer programmes are developed and written in Fortran II and IV.

Analysis by Heidebracht and Smith method are compared with exact method in relation to GA parameter of frames and the interaction of shear-wall frame structure.

The validity of portal and cantilever methods are studied, and the transfer of vertical concentrated load from one vertical member to others are examined.

The role of heavier crown coupling beam at roof level in reducing the sway of frame is investigated.
CHAPTER 2
LITERATURE REVIEW

2.1 Introduction

In recent years the number of tall buildings, for both commercial and residential purposes, have increased rapidly throughout the world. This increase has necessitated the need for a greater knowledge of the behavior of these structures, and, in particular, the necessity for producing methods of analysis capable of giving rapid and accurate assessments of their overall strength and stiffness, as well as detailed information about any local stress concentration.

Although normal assumptions of lateral and vertical load distribution is adequate for smaller heights, these assumptions result in serious error in analysis and design for taller buildings. As buildings increase in height, it becomes more important to ensure adequate lateral stiffness to resist loads which may arise due to wind, seismic or even blast effects. In framed structures this stiffness may be obtained by bracing members, by the rigidity of the joints, by complete shear truss assemblies acting in conjunction with the frame, or by infilling the frame with shear-resistant panels. A simplification of the latter is shear wall construction, in which the high in-plane stiffness of the walls, is employed to resist the lateral force. The object of this literature review is to attempt to provide a summary of the current knowledge relating to shear wall structures, by
reviewing briefly the relevant papers. The detail solution of some of the papers along with relevant figures are given in Chapter 3.

2.2 Review of Different Papers

A simplified method of determining the interaction of shear wall and frames was given by Khan and Sbarounis\(^{(1)}\). The method presented can be solved on a slide rule or a calculator. A method of converging approximations was proposed to provide a solution that can be carried to any degree of refinement. The material presented is applicable to analyses for lateral forces due to wind or earthquake. Useful graphs were presented to enable interaction of frame and shear walls to be determined rapidly. The detail solution of this paper along with figures are given in Chapter 3.

McLeod\(^{(2)}\) in his paper on "Shear Wall-Frame Interaction" introduced a so called "component stiffness method" for determining the interaction of frames and shear walls. He used an equation to find the interaction force at top due to point load, or uniformly distributed load or triangular load (due to earthquake). The main assumption is that the frame takes constant shear, that is, the interaction force between the frame and wall can be represented by a concentrated force at the top. If the frame shear is assumed to be constant, the system can be treated as a wall supported at the top by a spring. This method lacks
accuracy if the wall is more flexible than the frame. The detail solution of this paper along with figures are presented in Chapter 3.

Another simple approach to the design of shear wall with frames was published by Gould (3) in 1965. The simplest approach to the design problem is to consider the shear wall as an independent member and thus design it as a vertical cantilever allowing all the lateral loads to be taken by this member. It is very simple but quite conservative in a structure where substantial frame is provided along with shear wall system.

Cardan (4) has given a method of analyzing the shear wall with frame. He shows that it is possible to express the angle of deflection of wall at all points with a second degree differential equation taking the effect of bending and shear. By solving the equation the deflection of the system as well as internal actions can be evaluated. The assumptions and the governing equations of this method are given in Chapter 3.

Heidebrecht and Smith (5) has given an approximate analysis of Tall Wall-Frame building structures. The method is relatively simple and can be applied to both static and dynamic analyses of uniform and non-uniform structures. The structure is assumed to consist of a combination of flexural and shear vertical cantilever beams, deforming either in bending or shear configuration, respectively. The detail solution of this paper along with figures are presented in Chapter 3.
Rosman (6) has presented a paper on "Interaction between shear walls and frames" in the Tall building symposium held at the University of Southampton, U.K. In this paper he derived a governing equation by the energy method to deal with laterally triangular or uniformly distributed loaded systems consisting of walls and frames.

Parme (7) in his paper on "Design of Combined frame and Shear Walls" has presented a procedure which consists of relating the total load at each floor level to the displacements of that floor and the two floors above and below. At each level an equation is written in terms of the relative stiffness of the columns, girders, shear wall and the applied loading. This leads to a series of simultaneous equations equal to the number of floors.

Murachev, Sigalov, and Raikov (8) in their analysis of frame wall systems, neglected the total flexural stiffness of columns where the wall is solid.
3.1 Introduction

With the increasing availability of large scale digital computers, it is now-a-days possible to obtain exact analysis of tall buildings. However, to have the application of analysis of in a design office without the help of computers or to have a preliminary design in rapid hand calculations before sending it to the computer for more exact analysis, numerous methods have come up during the past decade.

In Chapter 2 a brief review of different papers dealing with analysis of tall shear wall frame structure was given. In this Chapter solution of some of the methods are presented. These methods deal on approximate analysis of tall shear wall-frame structures.

3.2 Heidebrecht and Smith Method (5)

3.2.1 Basic Mathematic Model

The structure is considered to consist of a combination of flexural and shear vertical cantilever beams i.e. deforming either in bending or shear configurations, respectively.

The equations governing the behaviour of the two types of beam are, referring to Figures 4a and 4b, are
FIG. 4 MATHEMATICAL MODEL FOR SHEAR-FLEXURAL MEMBER
for flexure, \( EI \frac{d^4 y_B}{dx^4} = W_B(X) \) \hspace{1cm} (3.2.1)

and for shear, \( GA \frac{d^2 y_s}{dx^2} = W_s(X) \) \hspace{1cm} (3.2.2)

in which the subscripts \( B \) and \( s \) refer to the flexural and shear beams respectively; \( W(X) \) is the distributed horizontal load; \( y(X) \) is the horizontal deflection of the beam; \( E \) and \( I \) are the modulus of elasticity and the second moment of area, respectively, of the flexural beam; \( G \) and \( A \) are the shear modulus and cross-sectional area, respectively, of the shear beam.

In addition to the above governing differential equations, the equations for the stress resultants in each case are

\[ S_B(X) = -\frac{d M_B(X)}{dX} \] \hspace{1cm} (3.2.3)

\[ M_B(X) = EI \frac{d^2 y_B}{dx^2} \] \hspace{1cm} (3.2.4)

and \( S_s(X) = \frac{-d M(X)}{dX} = GA \frac{d y_s}{dX} \) \hspace{1cm} (3.2.5)

in which \( S \) and \( M \) are the distributed shearing force and bending moments, respectively.

Now if the shear beam and flexural beam are linked together so that they have identical horizontal deflection at all positions throughout their height, as shown in Fig. 4c. In this case, a distributed horizontal interaction force of magnitude \( q(x) \) and a concentrated top interaction force of magnitude \( Q \) are required to maintain this compatibility, as shown in Fig. 4d. The concen-
trated for \( Q \) is required so that both components are in equilibrium at the top of the structure.

The governing equations of the components of this combination member, referred to as a shear-flexure beam or member, are given by:

\[
\frac{d^4 y}{dx^4} = W_B(X) + q(x) \quad (3.2.6)
\]

\[
- \frac{d^2 y}{dx^2} = W_s(x) - q(x) \quad (3.2.7)
\]

Adding Eqs. (3.2.6) and (3.2.7), and dividing through by \( EI \), yields

\[
\frac{d^4 y}{dx^4} - \alpha^2 \frac{d^2 y}{dx^2} = \frac{W(x)}{EI} \quad (3.2.8)
\]

in which \( W(x) = W_B(x) + W_s(x) \quad (3.2.9) \)

and \( \alpha^2 = \frac{GA}{EI} \quad (3.2.10) \)

The contribution of a single column to the \( GA \) parameter of the equivalent shear-flexure beam is given by

\[
GA = \frac{12EIh}{h^2} \left\{ \frac{1}{2} \frac{I_h}{h} - 1 + \frac{Ib_1 + Ib_2}{h \left( \frac{b_1}{b_1} + \frac{b_2}{b_2} \right)} \right\}
\]

in which \( I_h = \) Second moment of area of the column

\( h = \) Storey height

\( b_1, b_2 = \) total length of adjacent beams
Ib₁, Ib₂ = Second moments of area of corresponding beams

E = Young's modulus of Elasticity.

The total GA contribution of a planar frame is the arithmetic sum of the GA terms for each of the columns in a typical storey of the frame.

The total EI and GA terms of the respective flexure and shear components, can then be substituted in equation (3.2.10) to determine the parameter α² for the structure or for the particular segment of the structure.

For the case of a uniformly distributed load of magnitude W₀ the solution to equation (3.2.8) can be written in the form

\[ y(x) = C₁ + C₂x + C₃ \cosh \alpha x + C₄ \sin \alpha x - \frac{W₀ x^2}{2E₁a^2} \]  (3.2.12)

in which C₁, C₂, C₃ and C₄ are constants in the homogenous part of the solution, to be determined by the boundary conditions.

A tall building can be considered as a vertical cantilever beam, with zero deflection and rotation at the base and free at the top, the corresponding boundary conditions for the shear-flexure cantilever of height H are:

\[ y(0) = 0 \]  (3.2.13a)

\[ \frac{dy}{dx}(0) = 0 \]  (3.2.13b)

\[ M(H) = M_B(H) + M_S(H) = 0 \]  (3.2.13c)

\[ S(H) = S_B(H) + S_S(H) = 0 \]  (3.2.13d)
Since equilibrium requires that the moment is zero at the free and of each of the components of the shear-flexure cantilever, the condition expressed by eqn. (3.2.13) can be replaced by

\[ M_B(H) = EI \frac{d^2y}{dx^2}(H) = 0 \]  
(3.2.14)

Substituting equations (3.2.3) and (3.2.5) in Eq. (3.2.13d) yields

\[ S(H) = GA \frac{dy}{dx}(H) - EI \frac{d^3y}{dx^3}(H) = 0 \]  
(3.2.15)

The constants of equation (3.2.12) can be obtained from the above boundary conditions, leading to the following expression for deflexion:

\[ Y(x) = \frac{W H^4}{EI (\alpha H)^4} \left\{ \frac{\alpha H \sin \alpha H}{\cos \alpha H} \right( \cos \alpha x - 1 \right) - \alpha H \sin \alpha x + (\alpha H)^2 \left[ \frac{x}{H} - \frac{1}{2} \left( \frac{x}{H} \right)^2 \right] \} \]  
(3.2.16)

The expressions for stress resultants and the interacting force are given by

\[ \frac{M_B(x)}{M_0} = \frac{2}{(\alpha H)^2} \left[ \frac{\alpha H \sin \alpha H}{\cos \alpha H} \right] \cos \alpha x - \alpha H \sin \alpha x - 1 \]  
(3.2.17)

\[ \frac{M_s(x)}{M_0} = \left( 1 - \frac{x}{H} \right)^2 - \frac{M_B(x)}{M_0} \]  
(3.2.18)

\[ \frac{S_B(x)}{S_0} = \frac{1}{\alpha H} \left[ \alpha H \cos \alpha x - \frac{\alpha H \sin \alpha H + 1}{\cos \alpha H} \right] \sin \alpha x \]  
(3.2.19)
\[ \frac{s_3(x)}{s_0} = (1 - \frac{x}{H}) - \frac{s_H(x)}{s_0} \]  
\[ \frac{q(x)}{w_0} = \frac{w_s}{w_0} + \frac{(\alpha H)^2}{2} \frac{M_B(x)}{M_0} \]  

Similar expressions for the case of a horizontal concentrated top load and a triangularly distributed load are given in section 3.2.3. These three types of loading can be appropriately combined to simulate the static lateral loadings specified by most of the building codes for wind or earthquake effect.

3.2.2 Design Curves

Design curves for deflection, moment, shearing force and horizontal interacting force constructed from equations (3.2.16) (3.2.17), (3.2.15) and (3.2.21) are constructed and presented by Heidebrecht and Smith (5). The curves can be used for a rapid estimate of the total deflection and the variation with height of the load distribution within the structure, for a range of the parameter \( \alpha H \).
3.2.3 Design Equations for Concentrated Load and Triangular Load Distribution, Uniform Structure

A. CONCENTRATED LOAD P AT TOP OF STRUCTURE

\[
y(x) = \frac{PH^3}{EI} \left\{ \frac{\sinh \alpha H}{(\alpha H)^3 \cosh \alpha H} (\cosh \alpha x - 1) - \frac{\sinh \alpha x}{(\alpha H)^3} \right\}
\]

\[
+ \left( \frac{1}{(\alpha H)^2} \right) \frac{x}{H}
\]

(3.2.22)

\[
\frac{M_B(x)}{M_o} = \frac{1}{\alpha H} \left( \tanh \alpha H \cosh \alpha x - \sinh \alpha x \right)
\]

(3.2.23)

\[
\frac{M_s(x)}{M_o} = \left( 1 - \frac{x}{H} \right) - \frac{M_B(x)}{M_o}
\]

(3.2.24)

\[
M_o = PH
\]

(3.2.25)

\[
\frac{S_B(x)}{p} = \left( \cosh \alpha x - \tanh \alpha H \sinh \alpha x \right)
\]

(3.2.26)

\[
\frac{S_s(x)}{p} = 1 - \frac{S_B(x)}{p}
\]

(3.2.27)

\[
\frac{H_q(x)}{p} = (\alpha H)^2 \frac{M_B(x)}{M_o}
\]

(3.2.28)
B. TRIANGULAR DISTRIBUTION

\[ w(x) = \frac{w_1 x}{H} \]

\[ y(x) = \frac{w_1 H^4}{EI(\alpha H)^2} \left\{ \left( \frac{\sinh \alpha H}{2\alpha H} - \frac{\sinh \alpha H}{(\alpha H)^3} + \frac{1}{(\alpha H)^2} \right) \frac{\cosh \alpha x - 1}{\cosh \alpha H} \right\} 
+ \left( \frac{x}{H} - \frac{\sinh \alpha x}{\alpha H} \right) \left( \frac{1}{(\alpha H)^2} - \frac{1}{6} \left( \frac{x}{H} \right)^3 \right) \]  \hspace{1cm} (3.2.29)

\[ \frac{M_B(x)}{M_o} = \frac{3}{(\alpha H)^3} \left\{ \left( \frac{(\alpha H)^2}{2} \right) \frac{\sinh \alpha H}{\alpha H} - \sinh \alpha H + \alpha H \frac{\cosh \alpha H}{\cosh \alpha H} \right. 
- \left. \left( \frac{(\alpha H)^2}{2} - 1 \right) \sinh \alpha x - \alpha H \left( \frac{x}{H} \right) \right\} \]  \hspace{1cm} (3.2.30)

\[ \frac{M_B(x)}{M_o} = (1 - \frac{3}{2} \left( \frac{x}{H} \right) + \frac{1}{3} \left( \frac{x}{H} \right)^3 - \frac{M_B(x)}{M_o} \]  \hspace{1cm} (3.2.31)

\[ M_o = \frac{w_1 H^2}{3} \]  \hspace{1cm} (3.2.32)

\[ \frac{S_B(x)}{S_o} = -\frac{2}{(\alpha H)^2} \left\{ \left( \frac{(\alpha H)^2}{2} \right) \frac{\sinh \alpha H}{\alpha H} - \sinh \alpha H + \alpha H \frac{\cosh \alpha H}{\cosh \alpha H} 
- \left( \frac{(\alpha H)^2}{2} - 1 \right) \cosh \alpha x - 1 \right\} \]  \hspace{1cm} (3.2.33)

\[ \frac{S_B(x)}{S_o} = 1 - \left( \frac{x}{H} \right)^2 - \frac{S_B(x)}{S_o} \]  \hspace{1cm} (3.2.34)

\[ S_o = \frac{w_1 H}{2} \]  \hspace{1cm} (3.2.35)

\[ q(x) = \frac{w_1}{w_1} \left( \frac{x}{H} + \frac{(\alpha H)^2}{3} \frac{M_B(x)}{M_o} \right) \]  \hspace{1cm} (3.2.36)
3.3 Khan and Sbarounis Method\(^{(1)}\)

3.3.1 Analysis

The interaction of a shear wall and frame is a special case of indeterminacy in which two basically different components are tied together to produce one structure. If the frame alone is considered to take the full lateral load, it would develop moments in columns and beams to resist the total shear at each storey while the effects of overturning would normally be considered secondary, and in most cases, negligible. In resisting all lateral loads, a frame would deflect as in Fig. 2(a). If a shear wall on the other hand, is considered to resist all the lateral loads it would develop moments at each floor equal to the overturning moment at that level and the deflected shape Fig. 2(b), would be that of a cantilever.

If a shear wall and a frame exist in a building, each one will try to obstruct the other from taking its natural free deflected shape, and as a result a redistribution of forces between the two would be expected. As shown in Fig. 2(c), the frame will restrain or pull the shear wall back in the upper stories, while in the lower regions the opposite will occur.

The conflicting physical characteristics of the two systems can be considered if the structure is first divided into two parts, a frame, and a shear wall, and then the two parts are brought together so that all structural laws are fully satisfied.
3.3.2 **Concept and Method of Analysis**

The analysis is performed in two stages. In the first stage of analysis of a structure, shown in plan in Fig. 5(a) it is necessary to determine the deflected shape and the amount of lateral load distributed to the walls and frame, respectively at each story. For this purpose, the structure is separated into two distinct systems Fig. 5(b) as follows:

3.3.3 **System "W"**

This system consists shear wall or combination of shear walls. The moment of inertia of this system at any story equals the sum of the moments of inertia of all the shear walls regardless of their shape and size. Shape and size are considered in computing an average $L_s$, the distance from the N.A. of system "W" to its extreme fibre. Coupled shear walls can often be represented in high multi-storey buildings as a single wall with an equivalent stiffness.

3.3.4 **System "F"**

This system is a one bay frame connected with the system "W" by means of link beams and consists of all other framing elements outside of system "W". This includes all columns, beams, spandrels, and slabs contributing to the lateral stiffness. Member linking the frames with the shear walls ("Link Beams") are also included in system "F". The stiffnesses of the columns, beams, and
"link" beams ($S_c$, $S'_b$, $S''_b$) are simply the sum of the stiffnesses of all such members in the structure. The "link" beam span, $L_b$ of system "F", is an average of the "link" beam spans of the structure when these spans are within the same range of magnitude.

In the first stage of analysis, average rotation and vertical displacement of the frame joints will yield acceptable values of the distribution of lateral forces between the two systems and the computed deflection at each story will be quite satisfactory. The computed distribution of lateral forces in the first stage of analysis will inform the designer of the effectiveness of the shear walls in resisting the lateral forces so that he may adjust the size and stiffness of the shear wall to obtain a more economical structure. The deflected shape is needed in the second stage of computation.

In most cases, a further simplification can be made (Fig. 5(c)) by adding the stiffness of "link" beams to the stiffness of the other flexural members. The two systems are then tied together with members that can transmit only lateral force. The quantities $L_b$ and $L_s$ are no longer needed.

The analysis of the system is performed by the iterative solution. At the completion of the iterative solution the deflections of the combined system comprising the structure are known. It is then possible to analyze each column line as an isolated resisting system. The resisting elements at any column line may form a rigid frame or a combination of rigid frame and shear wall.
The second stage of analysis may be performed by subjecting these isolated bents to the deflection pattern that was derived for the entire structure from the interactive solution. Fixed end moments imposed on the columns and connecting links by this known set of deflections can be balanced rapidly by a moment distribution solution. In this manner, local effects on moments and shears resulting from localized stiffness variations are fully accounted for.

3.3.5 First Stage of Analysis

The conditions required to be satisfied for the equilibrium of total structure are:

a) Deflection in system "W" and system "F" must be the same at corresponding levels.

b) "Link" members connecting system "F" to system "W" must undergo the same rotations and vertical translations as those of system "W" at their points of connection.

c) Horizontal shear, $V_w$, developed in system W plus the horizontal shear, $V_f$, developed in system F must be equal to the external shear $V_t$, at every story.

The steps adopted to achieve the above conditions are as follows:

1. The total computed external loads (wind or seismic) on the idealized structure are applied to system W at each floor.
level and the slopes and deflections of system W at each floor level are determined. The vertical movements of the connecting points with system W are computed by multiplying the slope at each level by the distance from the neutral axis of the wall to the connecting point.

2. This is the first cycle of iteration. For quick convergence, a final deflected shape could be assumed or approximated from the figures as given by Khan and Sbarounis. In the absence of good guess, the final deflected shape is assumed to be the same as the free deflected shape of system W.

System F is forced to undergo the assumed deflections at each floor. This also requires that the connecting members at each floor must have the same rotations and vertical translations as system W at their points of connection with system W. However, if there is a deliberate hinge at these points, only the vertical translation must be considered.

3. Moments induced by "force fitting" can be determined directly by using moment distribution. The force fitted frame has no external forces but only known story deflections and rotations at the connecting points.

4. After "force-fitting" system F to system W, the total shears in each story of system F as well as moments and reactions applied on system W by the connecting links are computed. The shears generated by force fitting can be used directly in the next step.
5. All shears, forces, and moments generated by force-fitting system $F$ are applied to the isolated free system $W$. Negative deflections and rotations of the system $W$ are then calculated. The net deflection with respect to the original unloaded shape of system $W$ would be the algebraic sum of step 1 and step 5. This is the end of one cycle of iteration. For the stable condition the assumed initial deflection at any floor at the beginning of the $n$th cycle must be the same as the end deflections, at the completion of the $n$th cycle.

3.3.6 Second Stage of Analysis

After convergence of the iteration solution has been achieved, the final deflected shape of the structure is used to distribute moments and shears to every member in each bent of the structure. At a column line that contains no shear walls, a set of fixed end column moments obtained from the difference in story deflection can be apportioned to all the members by moment distribution. No sidesway correction is needed because the bent is in its final deflected shape.

If a shear wall is contained in a bent, it can be treated separately from the frame segment. With a known deflected shape and $EI$, the moment at any floor $i$, can be obtained from

$$M_i = \left( \frac{EI}{h_i} \right) \left( \triangle_{i+1} - 2 \triangle_i + \triangle_{i-1} \right)$$  \hspace{1cm} (3.3.1)

in which $M_i$ denotes the moment at floor, $i$, $I$ refers to the moment of inertia of wall at floor, $i$, $h_i$ represents the storey
height; \( \Delta_i \) is the deflection at floor, \( i \), \( \Delta_{i+1} \) describes the deflection at floor, \( i+1 \), \( \Delta_{i-1} \) is the deflection at floor, \( i-1 \).

FIG. 5 TYPICAL IDEALIZED STRUCTURE

(a)-Floor Plan (b)-Idealized structure (c)-Simplified Idealized structure

FIG. 5 TYPICAL IDEALIZED STRUCTURE
3.4 Component Stiffness Method

The main assumption in component stiffness method by McLeod\(^{(2)}\) is that the frame takes constant shear, that is, the interaction force between the frame and wall can be represented by a concentrated force at the top. The method is simple but lacks accuracy if the wall is more flexible than the frame \((K_w/K_f > 1\), see below). Consider the single-bay frame and shear wall loaded in-plane by the uniformly distributed load shown in Fig. 6(a). If the frame shear is assumed to be constant, the system can be treated as a wall supported at the top by a spring (Fig. 6(c)). The spring stiffness \(K_f\) is defined as the lateral point load applied at the top of the frame to cause unit deflection in its line of action. \(K_f\) can be calculated using equations A and B (Table 2). \(K_w\) is defined as the lateral point load required to cause unit deflection at the top of the wall (similarly to \(K_f\)).

Normally

\[
K_w = \frac{3EI}{H^3} \quad (3.4.1)
\]

where \(E\) is the Young's modulus of elasticity, \(I_w\) the moment of inertia of the wall, and \(H\) the total height. This equation can be modified to take account of variations in properties with height and the effect of openings in the walls.

The relationships between \(P/W\), \(y_w\), and \(K_w/K_f\) for different loading cases can be evaluated by using equation C (Table 1). \(P\) is the interaction load at the top of the frame, the constant
shear; \( W \) is the total applied lateral load; \( \gamma_w \) is a dimensionless parameter which relates the rotational stiffness of the wall to that of the foundation, that is

\[
\gamma_w = \frac{K_B H}{4E_w I_w}
\]

(3.4.2)

where \( K_B \) is the rotational stiffness of the shear wall support. If the rotation at the base of the shear wall is to be neglected, the term \( \gamma_w \) in Equation 3.4.2 should be omitted. However, the effect of shear wall base rotation can significantly affect the distribution of load between shear walls and frames, and Equation 3.4.2 can be used as a simple method of assessing this factor.

Problems involving several frames and walls may be reduced to that of a single wall and frame. Alternatively, \( K_f \) and \( K_w \) for each vertical unit may be calculated separately and the results summed. \( \sum K_f \) and \( \sum K_w \) are then used instead of \( K_f \) and \( K_w \) in Equation 3.4.2.

Studies on shear wall-frame interaction normally use three parameters to define behaviour, namely, \( \Lambda \), \( i_w \), and \( I_c \), where

\[
\Lambda = \frac{E_c I_c / h}{E_b I_b / l}
\]

(3.4.3)

with \( h \) as the column height. By using \( K_f \), which is a function of \( \Lambda \) and \( \sum I_c \), behavior can be discussed in terms of only two variables, \( K_f \) and \( K_w \). This simplifies the physical interpretation of
the behavior. Also, the parameter P/W is useful for estimating
the effectiveness of the frame (or frames) in comparison with
the shear wall(s) in resisting lateral load and for assessing
the effect of various assumptions in analysis.

When a frame and wall are interconnected as shown in
Fig. 6(a), maximum shear on the frame tends to occur towards
midheight and in this area Equation C can underestimate maximum
frame shear by as much as 30 percent. Therefore, when calculating
moments in the frame, it is worth while to increase the calculated
value of P by 30%.

If K_w/K_f is less than 1, the use of Equation C is not
recommended and the use of charts as given by Khan and Sbarounis(1)
produces more accurate results.
Link bars

(a) STRUCTURE     (b) INTERACTION AT TOP ONLY     (c) FRAME MODELED BY SPRING

FIG. 6 IDEALIZATION OF STRUCTURE FOR EQUATION C

Table 1. Equation C

<table>
<thead>
<tr>
<th>Load condition</th>
<th>Equation C</th>
</tr>
</thead>
<tbody>
<tr>
<td>Point load at top</td>
<td>( P/W = \frac{1 + \frac{3}{4\gamma_w}}{1 + \frac{3}{4\gamma_w} + \frac{K_w}{K_f}} )</td>
</tr>
<tr>
<td>Uniformly distributed</td>
<td>( P/W = \frac{\frac{1}{6}(1 + \frac{1}{\gamma_w})}{1 + \frac{3}{4\gamma_w} + \frac{K_w}{K_f}} )</td>
</tr>
<tr>
<td>Triangular (earthquake)</td>
<td>( P/W = \frac{\frac{11}{20} + \frac{1}{2\gamma_w}}{1 + \frac{3}{4\gamma_w} + \frac{K_w}{K_f}} )</td>
</tr>
</tbody>
</table>

NOTATION

\( W \) = total applied load
\( P \) = interaction force at top
\( I_w \) = moment of inertia of wall
\( H \) = total height
\( K_f \) = point load at top of frame to cause unit deflection in its line of action

\( K_w = \frac{3EI_w}{H^3} \)  (with constant \( I_w \))

\( K_{BH} \) = rotational stiffness of shear wall support

\( \gamma_w = \frac{K_{BH} H}{4E_{wI_w}} \)

top deflection \( \Delta = \frac{P}{K_f} \)
Table 2. Equation A for Top Deflection of Rigid Regular Frames

EQUATION A- for Axial deformation:

\[
A = \frac{wH^3F}{E\frac{A_n}{n^2}}
\]

where \( \Delta_A \) = deflection at top of frame
due to axial deformation of exterior columns

\( F_n = \) function of \( n \), dependent on the type of loading

\( n = \) ratio \( \frac{\text{Area of exterior column at top of frame}}{\text{Area of exterior column at bottom of frame}} \)

(linear variation of \( A_c \) with height)

\( A_c = \) area of exterior columns at first-story level

\( B = \) total width of frame
Table 2. Equation B for Top Deflection of Rigid Regular Frames

EQUATION B for Bending deformation:

\[
\Delta_B = \frac{w h^2 H}{12 \sum (E_c I_c)} \left[ F_s (1 - \beta_d)^3 + F_g (1 - \beta_c)^3 \lambda \right]
\]

where \( \Delta_B \) = deflection at top of frame due to bending of members
\( W \) = total lateral load
\( h \) = story height
\( H \) = total height
\( E \) = Young's modulus (subscript denotes structural system)
\( I_c \) = sum of moments of inertia of columns at first-story level
\( F_s, F_g \) = functions of \( s \) and \( g \), dependent on the type of loading
\( s = \frac{I_c \text{ at top of frame}}{I_c \text{ at bottom of frame}} \) Linear variation of \( I_c \) and \( I_b \) with height. If \( E \) varies, use \( EI \) instead of \( I \).
\( g = \frac{I_b \text{ at top of frame}}{I_b \text{ at bottom of frame}} \)
\( \beta_d = \frac{D}{h} \), where \( D \) is beam depth
\( \beta_c = \frac{C}{\ell} \), where \( C \) is column width and \( \ell \) is distance between column center lines
\( \lambda = \frac{\sum (E_c I_c / h)}{2 \sum (E_b I_b / \ell)} \), i.e. summation over width of structure at first story level
\( I_b \) = moment of inertia of beam at bottom of structure

TOTAL DEFLECTION \( \Delta = \Delta_B + \Delta_A \)
3.5 Cardan Method

Cardan expressed the angle of deflection of the wall at all points with a second degree differential equation, taking the effect of bending and shear.

3.5.1 Assumption

The assumptions made in Cardan method is as follows:

a) Shear wall and frames are fixed at its base.
b) Columns are monolithic with their bases fixed.
c) The joints of the frames are rigid.
d) Buildings are symmetrical.
e) Girders are infinitely rigid as compared to the walls and frames. All points on the same floor will then have same horizontal deflection.

3.5.2 Governing Equations

The equation governing the effect is

$$\theta = \theta_B + \theta_S$$

(3.5.1)

and by differentiating

$$\frac{d^2\theta}{dx^2} = \frac{d^2\theta_B}{dx^2} + \frac{d^2\theta_S}{dx^2}$$

(3.5.2)

where $\theta_B$ = Slope of deflected wall due to bending only.
$\theta_S$ = Slope of deflected wall due to shear only.
$\theta$ = Slope of the deflected wall due to all forces.
By solving the above equation the value of $\theta$ may easily be determined. Once the value of $\theta_B$ and $\theta_5$ are known, deflection

$$Y = \int_0^x \theta \, dx$$  \hspace{1cm} (3.5.3)$$

For uniform load, $Y/H$ for shear wall coupled with frame is

$$Y/H = \frac{C}{B} \cdot H \cdot (\xi - \frac{1}{2} \xi^2) + (\frac{C}{B} - D) \cdot \frac{H}{\alpha}$$

$$\times \frac{\sin \alpha \cdot (1 - \xi) - \sin \alpha - \frac{1}{\alpha} \cdot (1 - \cos \alpha)}{\cos \alpha}$$  \hspace{1cm} (3.5.4)$$

Where, $C = \left[1 + 3 \left( K_1 - K_2 \right) / AE \right] / \beta EI$  \hspace{1cm} (3.5.5)$$

$D = 3/\beta AE$  \hspace{1cm} (3.5.6)$$

$B = \left[ \beta \left( K_1 - K_2 \right) + K_2 + K_3 \right] / \beta EI$  \hspace{1cm} (3.5.7)$$

$\alpha = H \gamma_B$  \hspace{1cm} (3.5.8)$$

$\beta = 1 + 3K_3 / AE$  \hspace{1cm} (3.5.9)$$

$\xi = X/H$  \hspace{1cm} (3.5.10)$$

$K_1$ and $K_2$ = Constants, defined as

$$m_s = K_1 \theta_B + K_2 \theta_5$$  \hspace{1cm} (3.5.11)$$

$K_3$ = Constant, defined as $V_f = - K_3$

$A =$ Cross-sectional area of shear wall

$m_s =$ Elastic moment reaction per inch of wall height

$Y =$ Horizontal deflection of wall
4.1 Introduction

In Chapter 3 different methods for approximate analysis of shear-wall frame structure was given. In this chapter "exact" analysis of plane frame with or without shear wall is presented. The analysis of a structure is a process in which information given in one form is transformed into another form and the common objective in the analysis is finding the internal forces which result from the application of external forces. The forces and displacements in a structure can be related by using its flexibility and stiffness coefficients. These coefficients are characteristics of a structure and its coordinate system.

The flexibility coefficients characterize the behaviour of the structure by specifying its displacement response to applied forces at the coordinates, and the stiffness coefficients by specifying the forces required to produce given displacements at the coordinates. The stiffness and flexibility coefficients depend on the force displacement properties of the structure and the coordinate system.

A tall structure may be analysed either by flexibility or by stiffness method. But it has been found that the time required for the analysis by stiffness method is much less than
the time required for flexibility method. As such the tall plane frames are generally analysed by stiffness method.

The solution of large building frames by stiffness method uses a special technique by taking advantage of the sparse nature of the stiffness matrices of the frame. This technique was first introduced by Clough, Wilson and King (9). To avoid the inversion of large matrices, the system coordinates of the structure are selected in such a way that the resulting matrix which must be inverted will be tri-diagonal band matrix. Using the band matrix, the analysis is conducted by recursion procedure which requires the inversion of low order matrices.

4.2 Types of Frames

At first a one bay frame without shear wall is analyzed. Storey heights, cross-sections, moments of inertia and Young's modulus of elasticity are assumed to be constant for both columns and beams throughout the full height of the system. The analysis is modified to take care of wall system.

The one bay frame analysis is further modified to take care of variable storey heights, moments of inertia, cross-sections etc. The name of this analysis is given as general one bay frame analysis.
Subsequently a two bay frame with and without shear wall is analyzed. The matrices are formulated in a general form so that they can be used either for frame with wall or without wall. Two Bay frame analysis deals with constant storey heights, cross-sections, moments of inertia etc.

A "general two bay frame" with variable storey heights, cross-sections moments of inertia of vertical and horizontal members throughout the full height is also analyzed.

Computer programs for all of the above analyses are developed and written in Fortran II and Fortran IV.
4.2A ADDITIONAL NOTATION

\[ A_L, A_R = \text{cross-sectional area of left column or shear wall} \]

\[ I_L, I_R = \text{moment of inertia of left column or left} \]

\[ H = \text{storey height} \]

\[ L, X, Y = \text{Beam lengths} \]

\[ B_0, B, C, A = \text{stiffness matrices of the frame} \]

\[ \{ F \}_j = \text{forces at the coordinate level } j \]

\[ \{ U \}_j = \text{displacements at the coordinate level } j \]

\[ S_{ij} = \text{elements of frame stiffness matrix} \]

\[ \theta_A = \text{rotation at end } A \text{ of member } AB \]

\[ \theta_B = \text{rotation at end } B \text{ of member } AB \]

\[ \Delta = \text{displacement of one end of a member width respect to other end} \]

\[ M_{FAB} = \text{fixed end moment at end } A \text{ of member } AB \]

\[ M_{FBA} = \text{fixed end moment at end } B \text{ of member } AB \]

\[ M_{AB} = \text{moment at end } A \text{ of member } AB \]

\[ M_{BA} = \text{moment at end } B \text{ of member } AB \]

\[ L_1, L_2, L_3, L_4 = \text{distance from c.g. of wall to end of wall or from} \]

\[ S_1, S_2, S_3, S_4 = \text{c.g. of wall to beam end} \]

\[ A_M = \text{Gross-sectional area of center column or wall} \]

\[ C_M = \text{Moment of inertia of center column or wall} \]

\[ X_L = \text{Length of left beam} \]

\[ X_R = \text{Length of right beam} \]
4.3 ONE BAY FRAME
(WITH COLUMN AND BEAM)

[Diagram of a one bay frame with numbers from Level 1 to Level 20 marked with arrows between levels, indicating connections and coordinates.]
\[
S_{10.1} = 6EI2/H^2 \\
S_{11} = 12E/H^3 (I_1 + I_2) \\
S_{31} = 6EI1/H^2 \\
S_{51} = 6EI2/H^2 \\
S_{61} = -12E/H^3 (I_1 + I_2) \\
S_{13} = 6EI1/H^2 \\
S_{23} = 6EI3/L^2 \\
S_{33} = 4E (I_1/H + I_3/L) \\
S_{43} = -6EI3/L^2 \\
S_{53} = 2EI3/L \\
S_{63} = -6EI1/H^2 \\
S_{83} = 2EI1/H
\]

\[
S_{10.5} = 2EI2/H \\
S_{15} = 6EI2/H^2 \\
S_{25} = 6EI3/L^2 \\
S_{35} = 2EI3/L \\
S_{45} = -6EI3/L^2 \\
S_{55} = 4E(I_2/H + I_3) \\
S_{65} = -6EI2/H^2
\]
\[ S_{11.6} = -12E/H^3 (I_1 + I_2) \]
\[ S_{13.6} = 6EI_1/H^2 \]
\[ S_{15.6} = 6EI_2/H^2 \]
\[ S_{16} = -12E/H^3 (I_1 + I_2) \]
\[ S_{56} = -6EI_2/H^2 \]
\[ S_{66} = 24E/H^3 (I_1 + I_2) \]

\[ S_{10.8} = 2EI_3/L \]
\[ S_{11.8} = -6EI_1/H^2 \]
\[ S_{13.8} = 2EI_1/H \]
\[ S_{38} = 2EI_1/H \]
\[ S_{78} = 6EI_3/L^2 \]
\[ S_{88} = 4E(2I_1/H + I_3/L) \]
\[ S_{98} = -6EI_3/L^2 \]
\[ S_{18} = 6EI_1/H^2 \]

\[ S_{10.7} = 6EI_3/L^2 \]
\[ S_{12.7} = -EA_1/H \]
\[ S_{27} = -EA_1/H \]
\[ S_{77} = 2EA_1/H + 12EI_3/L^3 \]
\[ S_{87} = 6EI_3/L^2 \]
\[ S_{97} = -12EI_3/L^3 \]

\[ S_{10.9} = -6EI_3/L^2 \]
\[ S_{14.9} = -EA_2/H \]
\[ S_{49} = -EA_2/H \]
\[ S_{79} = -12EI_3/L^2 \]
\[ S_{89} = -6EI_3/L^3 \]
\[ S_{99} = 2EA_2/H + 12EI_3/L^3 \]

\[ S_{11.10} = 6EI_2/H^2 \]
\[ S_{510} = 2EI_2/H \]
\[ S_{710} = 6EI_3/L^2 \]
\[ S_{810} = 2EI_3/L \]
\[ S_{910} = -6EI_3/L^2 \]
\[ S_{10.10} = 4E(2I_2/H + I_3/L) \]
\[ S_{11.10} = -6EI_2/H^2 \]
\[ S_{15.10} = 2EI_2/H \]
4.3.1 STIFFNESS MATRIX OF THE FRAME

\[
[K]
\]

\[
\begin{array}{cccc}
\text{B} & \text{O} & \text{C} \\
\text{A} & \text{B} & \text{C} \\
\text{A} & \text{B} & \text{C} \\
\text{A} & \text{B} & \text{C} \\
\text{A} & \text{B} & \text{C} \\
\text{A} & \text{B} & \text{C} \\
\text{A} & \text{B} & \text{C} \\
\end{array}
\]
\[
\begin{array}{cccccc}
& S_{11} & S_{12} & S_{13} & S_{14} & S_{15} \\
S_1 & S_{21} & S_{22} & S_{23} & S_{24} & S_{25} \\
S_2 & S_{31} & S_{32} & S_{33} & S_{34} & S_{35} \\
S_3 & S_{41} & S_{42} & S_{43} & S_{44} & S_{45} \\
S_4 & S_{51} & S_{52} & S_{53} & S_{54} & S_{55} \\
\end{array}
\]

\[
B_0 = \begin{array}{c|c|c|c|c|c}
\frac{12E}{H^3} (I_1 + I_2) & 0 & \frac{6E_1}{H^2} & 0 & \frac{6E_2}{H^2} \\
\hline
0 & \frac{E A_1 + 12E_1}{H} & \frac{6E_1}{L^2} & \frac{-12E_1}{L^2} & \frac{6E_1}{L^2} \\
\hline
\frac{6E_1}{H^2} & \frac{6E_1}{L^2} & \frac{4E(\frac{l_1}{H} + \frac{l_3}{L})}{L^2} & \frac{-6E_1}{L^2} & \frac{2E_1}{L} \\
\hline
0 & \frac{-12E_1}{L^2} & \frac{-6E_1}{L^2} & \frac{E A_2 + 12E_1}{H} & \frac{-6E_1}{L^2} \\
\hline
\frac{6E_1}{L^2} & \frac{6E_1}{L^2} & \frac{2E_1}{L} & \frac{-6E_1}{L^2} & \frac{4E(\frac{l_2}{H} + \frac{l_3}{L})}{L^2} \\
\end{array}
\]

\[
\begin{array}{cccccc}
S_{66} & S_{67} & S_{68} & S_{69} & S_{6,10} \\
S_{76} & S_{77} & S_{78} & S_{79} & S_{7,10} \\
S_{86} & S_{87} & S_{88} & S_{89} & S_{8,10} \\
S_{96} & S_{97} & S_{98} & S_{99} & S_{9,10} \\
S_{10,6} & S_{10,7} & S_{10,8} & S_{10,9} & S_{10,10} \\
\end{array}
\]

\[
B = \begin{array}{c|c|c|c|c|c}
\frac{24E}{H^3} (I_1 + I_2) & 0 & 0 & 0 & 0 \\
\hline
0 & \frac{2E A_1 + 12E_1}{H} & \frac{6E_1}{L^2} & \frac{-12E_1}{L^2} & \frac{6E_1}{L^2} \\
\hline
0 & \frac{6E_1}{L^2} & \frac{4E(\frac{l_1}{H} + \frac{l_3}{L})}{L^2} & \frac{6E_1}{L^2} & \frac{2E_1}{L} \\
\hline
0 & \frac{-12E_1}{L^2} & \frac{-6E_1}{L^2} & \frac{2E A_2 + 12E_1}{H} & \frac{-6E_1}{L^2} \\
\hline
0 & \frac{6E_1}{L^2} & \frac{2E_1}{L} & \frac{-6E_1}{L^2} & \frac{4E(\frac{l_2}{H} + \frac{l_3}{L})}{L^2} \\
\end{array}
\]
$$A = \begin{bmatrix}
S_{61} & S_{62} & S_{63} & S_{64} & S_{65} \\
S_{71} & S_{72} & S_{73} & S_{74} & S_{75} \\
S_{81} & S_{82} & S_{83} & S_{84} & S_{85} \\
S_{91} & S_{92} & S_{93} & S_{94} & S_{95} \\
S_{101} & S_{102} & S_{103} & S_{104} & S_{105}
\end{bmatrix}$$

$$A = \begin{bmatrix}
-\frac{12E}{H^3} (I_1 + I_2) & 0 & -\frac{6EI_1}{H^2} & 0 & -\frac{6EI_2}{H^2} \\
0 & -\frac{EA_1}{H} & 0 & 0 & 0 \\
\frac{6EI_1}{H^2} & 0 & 2\frac{EI_1}{H} & 0 & 0 \\
0 & 0 & 0 & -\frac{EA_2}{H} & 0 \\
\frac{6EI_2}{H} & 0 & 0 & 0 & 2\frac{EI_2}{H}
\end{bmatrix}$$

$$C = \begin{bmatrix}
S_{16} & S_{17} & S_{18} & S_{19} & S_{1,10} \\
S_{26} & S_{27} & S_{28} & S_{29} & S_{2,10} \\
S_{36} & S_{37} & S_{38} & S_{39} & S_{3,10} \\
S_{46} & S_{47} & S_{48} & S_{49} & S_{4,10} \\
S_{56} & S_{57} & S_{58} & S_{59} & S_{5,10}
\end{bmatrix}$$

$$C = \begin{bmatrix}
-\frac{12E}{H^3} (I_1 + I_2) & 0 & \frac{6EI_1}{H^2} & 0 & \frac{6EI_2}{H^2} \\
0 & -\frac{EA_1}{H} & 0 & 0 & 0 \\
-\frac{6EI_1}{H} & 0 & 2\frac{EI_1}{H} & 0 & 0 \\
0 & 0 & 0 & -\frac{EA_2}{H} & 0 \\
-\frac{6EI_2}{H} & 0 & 0 & 0 & 2\frac{EI_2}{H}
\end{bmatrix}$$
4.3.2 SOLUTION BY RECURSION PROCEDURE

By using the stiffness matrix of the frame the force displacement equation of the frame may be written as

\[
\begin{bmatrix}
\{F\}_1 \\
\{F\}_2 \\
\{F\}_3 \\
\vdots \\
\{F\}_n
\end{bmatrix} =
\begin{bmatrix}
B_0 & C \\
A & B & C \\
A & B & C \\
& & & & A & B
\end{bmatrix}
\begin{bmatrix}
\{U\}_1 \\
\{U\}_2 \\
\{U\}_3 \\
\vdots \\
\{U\}_n
\end{bmatrix}
\]

Where \( \{F\}_1, \{F\}_2, \{F\}_3, \ldots, \{F\}_n \) & \( \{U\}_1, \{U\}_2, \{U\}_3, \ldots, \{U\}_n \) etc. are all \( 5 \times 1 \) column matrix.

Equation (A) can be written as \( n \) separate matrix equations as follows:

\[
\begin{align*}
\{F\}_1 &= B_0 \{U\}_1 + C \{U\}_2 \\
\{F\}_2 &= A \{U\}_1 + B \{U\}_2 + C \{U\}_3 \\
\{F\}_3 &= A \{U\}_2 + B \{U\}_3 + C \{U\}_4 \\
\{F\}_4 &= A \{U\}_3 + B \{U\}_4 + C \{U\}_5 \\
\{F\}_j &= A \{U\}_{j-1} + B \{U\}_j + C \{U\}_{j+1} \\
\{F\}_{n-1} &= A \{U\}_{n-2} + B \{U\}_{n-1} + C \{U\}_n \\
\{F\}_n &= A \{U\}_{n-1} + B \{U\}_n
\end{align*}
\]

From eqn (1) Solve for \( \{U\}_1 

\[
\{U\}_1 = B_0^{-1} \{F\}_1 - B_0^{-1} C \{U\}_2
\]

Substituting eqn (21) into (2) and rearranging yields

\[
\{F\}_2^* = B_1 \{U\}_2 + C \{U\}_3
\]

in which \( \{F\}_2^* = \{F\}_2 - A B_0^{-1} \{F\}_1 \)

\[
B_1 = B - A B_0^{-1} C
\]

From eqn (22a) \( \{U\}_2 = B_1^{-1} \{F\}_2^* - B_1^{-1} C \{U\}_3 \) (22d)

Substituting eqn (22d) in (3) and rearranging yields

\[
\{F\}_3^* = B_2 \{U\}_3 + C \{U\}_4
\]

in which \( \{F\}_3^* = \{F\}_3 - A B_1^{-1} \{F\}_2^* \)

\[
B_2 = B - A B_1^{-1} C
\]
From eqn $23a$, \( \{U\}_3 = B_2^{-1} \{F\}_3^* - B_2^{-1} C \{U\}_4 \)  

Proceeding in this manner the general equations in the recursion process have the following form

\[
\{F\}_j^* = B_{j-1} \{U\}_j + C \{U\}_{j+1} \tag{24a}
\]

in which

\[
\{F\}_j^* = \{F\}_j^* - AB_{j-2} \{F\}_{j-1}^* \tag{24b}
\]

\[
B_{j-1} = B - AB_{j-2} C \tag{24c}
\]

From eqn $24a$ \( \{U\}_j = \{B\}^{-1}_{j-1} \{F\}_j^* - \{B\}_{j-1}^{-1} C \{U\}_{j+1} \)  

The last of the matrix eqn shall be

\[
\{F\}_n^* = B_{n-1} \{U\}_n \tag{25a}
\]

in which

\[
\{F\}_n^* = \{F\}_n^* - AB_{n-2} \{F\}_{n-1}^* \tag{25b}
\]

\[
B_{n-1} = B - AB_{n-2} C \tag{25c}
\]

From eqn $25a$ \( \{U\}_n = B_{n-1}^{-1} \{F\}_n^* \)  

By back substitution we solve for \( \{U\}_j \)

The general equation for \( \{U\}_j \) shall be

\[
\{U\}_j = B_{j-1}^{-1} \left[ \{F\}_j^* - C \{U\}_{j+1} \right]
\]
4.3.3 SLOPE DEFLECTION EQUATIONS

BEAM

Anticlockwise moment = +ve
Clockwise rotation = +ve

\[ M_{AB} = M_{FAB} + \frac{2EI}{L} (-2\theta_A - \theta_B) \]
\[ M_{BA} = M_{FBA} + \frac{2EI}{L} (\theta_A - 2\theta_B) \]

\[ M_{AB} = M_{FAB} + \frac{2EI}{L} (-2\theta_A - \theta_B + \frac{3\Delta}{L}) \]
\[ M_{BA} = M_{FBA} + \frac{2EI}{L} (\theta_A - 2\theta_B + \frac{3\Delta}{L}) \]

COLUMNS

APPLY SAME FORMULAS AS THOSE FOR BEAM
4.4. FRAME ANALYSIS (ONE BAY)
(SHEAR WALL AND BEAM)

Level 1
Level 2
Level 3
Level 4
Level 5
Level 6
Level 7
Level 8
Level 9
Level 10
Level 11
Level 12
Level 13
Level 14
Level 15
Level 16
Level 17
Level 18
Level 19
Level 20

SHEAR WALL FRAME

COORDINATES
\[
\begin{align*}
S_{10.1} &= 6EI_2/H^2 \\
S_{11} &= 12E/H^3 (I_1+I_2) \\
S_{31} &= 6EI_1/H^2 \\
S_{51} &= 6EI_2/H^2 \\
S_{61} &= -12E/H^3 (I_1+I_2) \\
S_{81} &= 6EI_1/H^2 \\
S_{33} &= 6EI_1/H^2 \\
S_{53} &= 2EI_1/H \\
S_{83} &= -2EI_1/H \\
S_{13} &= 6EI_1/H^2 \\
S_{23} &= 6EI_3/L^2 \left[1 + \frac{2L^2}{L}\right] \\
S_{33} &= \frac{4EI_1}{H} + \frac{4EI_3}{L} \left[1 + \frac{3L^2}{L} \left(1 + \frac{L^2}{L}\right)\right] \\
S_{43} &= -6EI_3/L^2 \left[1 + 2L^2/L\right] \\
S_{53} &= \frac{2EI_3}{L} \left[1 + \frac{3}{L} \left(L^2 + L^3 + 2L^2L^3/L\right)\right] \\
S_{35} &= \frac{4EI_2}{H} + \frac{4EI_3}{L} \left[1 + \frac{3L^3}{L} \left(1 + \frac{L^3}{L}\right)\right] \\
S_{55} &= \frac{4EI_2}{H} + \frac{4EI_3}{L} \left[1 + \frac{3L^3}{L} \left(1 + \frac{L^3}{L}\right)\right] \\
S_{55} &= \frac{4EI_2}{H} + \frac{4EI_3}{L} \left[1 + \frac{3L^3}{L} \left(1 + \frac{L^3}{L}\right)\right] \\
S_{65} &= -6EI_2/H^2 \\
\end{align*}
\]
\[ S_{11.6} = -12E|H|^3 (I_1 + I_2) \]

\[ S_{13.6} = 6E11/H^2 \]

\[ S_{15.6} = 6E12/H^2 \]

\[ S_{16} = -12E|H|^3 (I_1 + I_2) \]

\[ S_{36} = -6E11/H^2 \]

\[ S_{10.8} = 2EI3/L[I + 3L (L2 + L3 + 2L2L3/L)] \]

\[ S_{11.8} = -6E11/H^2 \]

\[ S_{13.8} = 2EI1/H \]

\[ S_{18} = 6E11/H^2 \]

\[ S_{38} = 2EI1/H \]

\[ S_{78} = 6EI3/L^2 [1 + 2L2/L] \]

\[ S_{88} = 8EI1/H + 4EI3 [1 + 3L2/L (1 + L2/L)] \]

\[ S_{14.9} = -EA2/H \]

\[ S_{10.9} = 6EI3/L^2 [1 + 2L2/L] \]

\[ S_{49} = -EA2/H \]

\[ S_{79} = -12EI3/L^3 \]

\[ S_{89} = -6EI3/L^2 [1 + 2L2/L] \]

\[ S_{99} = 2EA2/H + 12EI3/L^3 \]

\[ S_{1,10} = 6E12/H^2 \]

\[ S_{5,10} = 2E12/H \]

\[ S_{7,10} = 6EI3/L^2 [1 + 2L2/L] \]

\[ S_{8,10} = 2EI3/L [1 + 3L (L2 + L3 + 2L2L3/L)] \]

\[ S_{9,10} = -6EI3/L^2 [1 + 2L3/L] \]

\[ S_{10,10} = 8E12/H + 4EI3/L [1 + 3L3/L (1 + L3/L)] \]

\[ S_{11,10} = -6E12/H^2 \]

\[ S_{15,10} = 2E12/H \]
Note: All the elements of matrix "C" is same as frame stiffness matrix.

Matrix $A = \text{transpose of matrix } C \text{ OR, } A = C^T$
<table>
<thead>
<tr>
<th>$E\times$</th>
<th>$\frac{24}{H^3} (I_1 + 12)$</th>
<th>0</th>
<th>$611/H^2$</th>
<th>0</th>
<th>$612/H^2$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>$2A_1/H + \frac{1213}{L^3}$</td>
<td>$613/L^2$</td>
<td>$-1213/L^3$</td>
<td>$613/L^2 + 1213L_3/L^3$</td>
<td></td>
</tr>
<tr>
<td>0</td>
<td>$613/L^2$</td>
<td>$\frac{811}{H} + \frac{4I_3}{L} + \frac{1213L/L^2}{2} (1 + \frac{L^2}{L})$</td>
<td>$-613/L^2$</td>
<td>$\frac{213}{L} + \frac{6I_3}{L^2} (L_2 + L_3 + 2L_2L_3/L)$</td>
<td></td>
</tr>
<tr>
<td>0</td>
<td>$-1213/L^3$</td>
<td>$-613/L^2$</td>
<td>$2A_2/H + 1213L_3/L^3$</td>
<td>$-613/L^2$</td>
<td></td>
</tr>
<tr>
<td>0</td>
<td>$613/L^2$</td>
<td>$\frac{213 + 6I_3}{L^2} (L_2 + L_3 + 2L_2L_3/L)$</td>
<td>$-613/L^2$</td>
<td>$\frac{812}{H} + \frac{4I_3}{L}$</td>
<td></td>
</tr>
</tbody>
</table>

or, $B_0 =$

$$
\begin{array}{cccccc}
S_{11} & S_{12} & S_{13} & S_{14} & S_{15} \\
S_{21} & S_{22} & S_{23} & S_{24} & S_{25} \\
S_{31} & S_{32} & S_{33} & S_{34} & S_{35} \\
S_{41} & S_{42} & S_{43} & S_{44} & S_{45} \\
S_{51} & S_{52} & S_{53} & S_{54} & S_{55} \\
\end{array}
$$

<table>
<thead>
<tr>
<th>$E\times$</th>
<th>$\frac{12}{H^3} (I_1 + 12)$</th>
<th>0</th>
<th>$611/H^2$</th>
<th>0</th>
<th>$612/H^2$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>$A_1/H + \frac{1213}{L^3}$</td>
<td>$\frac{613}{L^2} + \frac{1213L_2}{L^3}$</td>
<td>$-1213/L^3$</td>
<td>$613/L^2 + 1213L_3/L^3$</td>
<td></td>
</tr>
<tr>
<td>$611/H^2$</td>
<td>$\frac{613}{L^2} + \frac{1213L_2}{L^3}$</td>
<td>$\frac{4I_1/H + 4I_3}{L} + \frac{1213L_2}{L^2} (1 + \frac{L^2}{L})$</td>
<td>$-613/L^2$</td>
<td>$213/L + 6I_3/L^2 (L_2 + L_3 + 2L_2L_3/L)$</td>
<td></td>
</tr>
<tr>
<td>0</td>
<td>$-1213/L^3$</td>
<td>$-613/L^2$</td>
<td>$\frac{A_2}{H} + \frac{1213}{L^3}$</td>
<td>$-613/L^2 - 1213L_3/L^3$</td>
<td></td>
</tr>
<tr>
<td>$612/H^2$</td>
<td>$\frac{613}{L^2} + \frac{1213L_3}{L^3}$</td>
<td>$\frac{2I_3}{L} + \frac{6I_3}{L^2} (L_2 + L_3 + 2L_2L_3/L)$</td>
<td>$-613/L^2$</td>
<td>$\frac{812}{H} + \frac{4I_3}{L}$</td>
<td></td>
</tr>
</tbody>
</table>

or, $B_0 =$
4.5. GENERAL ONE BAY FRAME ANALYSIS

Moment of Inertia of Left Column or Wall = CL
Moment of Inertia of Right Column or Wall = CR
Area of Left Column or Wall = AL
Area of Right Column or Wall = AR
Moment of Inertia of Beam = BL
Beam Length = X
Story Height = H
Distance from c.g. of left wall to beam end = S2
Distance from c.g. of right wall to beam end = S3

All the above quantities may vary from story to story
### 4.5.1. Stiffness Matrix for General One Bay Frame

\[
[K] = \begin{bmatrix}
[S] & [C] \\
[B_1 C_1] & [B_2 C_2] \\
[B_3 C_3] & [B_4 C_4] \\
[B_5 C_5] & [B_6 C_6] \\
[B_7 C_7] & [B_8 C_8] \\
[B_9 C_9] & [B_{10} C_{10}] \\
[B_{11} C_{11}] & [B_{12} C_{12}]
\end{bmatrix}
\]
The above matrix is a symmetric matrix. Hence the other elements of the matrix can be generated from the above elements.
The elements of matrix \([c]\) are:

\[
\begin{align*}
S_{16} &= -\frac{12E}{H(1)^3} (C(1) + CR(1)); \\
S_{26} &= 0; \\
S_{36} &= \frac{6ECL(1)}{H(1)^2}; \\
S_{46} &= 0; \\
S_{56} &= -\frac{6ECR(1)}{H(1)^2}; \\
S_{17} &= 0; \\
S_{27} &= -\frac{EAL(1)}{H(1)}; \\
S_{37} &= S_{47} = S_{57} = 0; \\
S_{18} &= -\frac{6ECL(1)}{H(1)^2}; \\
S_{28} &= 0; \\
S_{38} &= 2ECR(1); \\
S_{48} &= S_{58} = 0; \\
S_{19} &= S_{29} = S_{39} = 0; \\
S_{49} &= -\frac{EAR(1)}{H(1)}; \\
S_{59} &= 0; \\
S_{1,10} &= \frac{6ECR(1)}{H(1)^2}; \\
S_{2,10} &= S_{3,10} = S_{4,10} = 0; \\
S_{5,10} &= \frac{2ECR(1)}{H(1)}.
\end{align*}
\]

FROM MATRIX \([c]\) GENERATE MATRIX \([c]^T\)

IN GENERAL (IF \(N_i = 1\), WHERE 1 IS STORY LEVEL FROM TOP), THE MATRIX \([c]^T\) IS GIVEN BY: \([TAKING "E" COMMON FOR ALL]

\[
\begin{align*}
C(1,1) &= 12 \cdot (C(N_i) + CR(N_i))/H(N_i) ** 3; \\
C(3,1) &= -6 \cdot C(N_i)/H(N_i) ** 2; \\
C(5,1) &= 6 \cdot CR(N_i)/H(N_i) ** 2; \\
C(2,2) &= -AL(N_i)/H(N_i); \\
C(1,3) &= 6 \cdot CL(N_i)/H(N_i) ** 2; \\
C(3,3) &= 2 \cdot CL(N_i)/H(N_i); \\
C(4,4) &= -AR(N_i)/H(N_i); \\
C(1,5) &= 6 \cdot CR(N_i)/H(N_i) ** 2; \\
C(5,5) &= 2 \cdot CR(N_i)/H(N_i).
\end{align*}
\]

\[
[B]_2 = 
\begin{array}{cccccccc}
S_{66} & S_{67} & S_{68} & S_{69} & S_{6,10} \\
S_{76} & S_{77} & S_{78} & S_{79} & S_{7,10} \\
S_{86} & S_{87} & S_{88} & S_{89} & S_{8,10} \\
S_{96} & S_{97} & S_{98} & S_{99} & S_{9,10} \\
S_{10,6} & S_{10,7} & S_{10,8} & S_{10,9} & S_{10,10}
\end{array}
\]
The above matrix is a symmetric matrix. Therefore all other elements can be generated from the above elements.
In general (if $N_1 = I_1$ where $I$ is story level from top) the matrix $[B]_I$ is given by as follows: where $I > 1$ (Take "E" common)

Let: 

$Z_1 = 1. + S_2 (I) / X(I)$
$Z_2 = 1. + S_3 (I) / X(I)$
$Z_3 = 1. + S_2 (I) / X(I) * (1. + S_2 (I) / X(I))$
$Z_4 = 1. + S_3 (I) / X(I) * (1. + S_3 (I) / X(I))$
$Z_5 = 1. + S_2 (I) + S_3 (I) + 2. * S_2 (I) * S_3 (I) / X(I)$

$B(1, 1) = 12. * (C(I) + E(I)) / H(I) + 3. + C(N_1) + E(N_1) / H(N_1) * * 3$

$B(2, 1) = 0$

$B(3, 1) = 6. * (C(I) / H(I) * * Z - C(N_1) / H(N_1) * * 2$

$B(4, 1) = 0$

$B(5, 1) = 6. * (C(I) / H(I) * * Z - C(N_1) / H(N_1) * * Z$

$B(2, 2) = A_L(N_1) / H(N_1) + A_L(I) / H(I) + 12. * B_L(I) * X(I) * * 3$

$B(3, 2) = 6. * B_L(I) / X(I) * * 2 * Z_1$

$B(4, 2) = -12. * B_L(I) / X(I) * * 3$

$B(5, 2) = 6. * B_L(I) / X(I) * * 2 * Z_2$

$B(3, 3) = 4. * (C(N_1) / H(N_1) + C(I) / H(I) + B_L(I) / X(I) * * Z_3)$

$B(4, 3) = -6. * B_L(I) / X(I) * * 2 * Z_1$

$B(5, 3) = 2. * B_L(I) / X(I) * Z_5$

$B(4, 4) = A_R(N_1) / H(N_1) + A_R(I) / H(I) + 12. * B_L(I) / X(I) * * 3$

$B(5, 4) = -6. * B_L(I) / X(I) * * 2 * Z_2$

$S(5, 5) = 4. * (C(N_1) / H(N_1) + C(I) / H(I) + B_L(I) / X(I) * * Z_4)$
4.6. TWO BAY FRAME ANALYSIS (WITH SHEAR WALL)

Shear wall no. 1

Shear wall no. 2

Shear wall no. 3

Level 1

Level 2

Level 3

Level 4

Level 5

Level 6

Level 7

Level 8

Level n

SHEAR WALL FRAME

COORDINATES
\( S_{15} = 6EW_{12}/H^2 \)
\( S_{25} = (6EB_{11}/L_{12}) (1+2S_{3}/L_{1}) \)
\( S_{35} = (2EB_{11}/L_{1}) [1+3/L_{1} \{ S_{2}+S_{3} +2S_{2}S_{3}/L_{1} \}] \)
\( S_{45} = (6EB_{12}/L_{2}^2)(1+2S_{4}/L_{2}) \)
\( - (6EB_{11}/L_{12})(1+2S_{3}/L_{1}) \)
\( S_{55} = (4EW_{12}/H^2)(4EB_{11}/L_{1}) [1+3S_{3}/L_{1} + (1+S_{3}/L_{1})] + 4EB_{12}L_{2}^2 [1+3S_{4}/L_{2}] \)
\( S_{65} = -(6EB_{12}/L_{2}^2)(1+2S_{4}/L_{2}) \)
\( S_{75} = (2EB_{12}/L_{2}^2)(1+3/L_{2} \{ S_{4}+S_{5} +2S_{4}S_{5}/L_{2} \}] \)
\( S_{85} = -(6EW_{12}/H^2) \)
\( S_{12.5} = (2EW_{12}/H) \)

\( \delta_{\theta} = 1 \)

\( S_{18} = -(12E/H^3)(W_{11}+W_{12}+W_{13}) \)
\( S_{38} = -(6EW_{11}/H^2) \)
\( S_{58} = -(6EW_{12}/H^2) \)
\( S_{78} = -(6EW_{13}/H^2) \)
\( S_{88} = (24E/H^3)(W_{11}+W_{12}+W_{13}) \)
\( S_{15.8} = (12E/H^3)(W_{11}+W_{12}+W_{13}) \)
\( S_{17.8} = (6EW_{11}/H^2) \)
\( S_{19.8} = (6EW_{12}/H^2) \)
\( S_{21.8} = (6EW_{13}/H^2) \)

\( \delta_{\theta} = 1 \)

\( S_{29} = -(EWA_{1}/H) \)
\( S_{99} = (2EWA_{1}/H)(12EB_{11}/L_{1}^3) \)
\( S_{10.9} = (6EB_{11}/L_{1}^2)(1+2S_{2}/L_{1}) \)
\( S_{11.9} = -(12EB_{11}/L_{1}^3) \)
\( S_{12.9} = (6EB_{11}/L_{1}^2)(1+2S_{3}/L_{1}) \)
\( S_{16.9} = -(EWA_{1}/H) \)
S1,10 = (6EWI1/H²)
S3,10 = (2EWI1/H)
S9,10 = (6EBI1/L1²)(1+2S2/L1)
S10,10 = (6EWI1/H)(6EBI1/L1)
[1+3S2/L1 (1+S2/L1)]
S11,10 = -(6EBI1/L1²)(1+2S2/L1)
S12,10 = (2EBI1/L1)(1+3/L1 (S2+S3 +2S2S3/L1))
S15,10 = -(6EWI1/H²)
S17,10 = (2EWI1/H)

S4,11 = -(EWA2/H)
S9,11 = -(12EBI1/L1³)
S10,11 = (6EBI1/L1²)(1+2S2/L1)
S11,11 = (EWA2/H + 12EBI1/L1³ + 12EBI2/L2³)
S12,11 = (6EBI2/L2²)(1+2S4/L2)
S13,11 = -(6EBI1/L1²)(1+ 2S3/L1)
S14,11 = +(6EBI2/L2²)(1+2S5/L2)
S18,11 = -(EWA2/H)

S1,12 = (6EWI2/H²)
S5,12 = (2EWI2/H)
S9,12 = (6EBI1/L1²)(1+2S3/L1)
S10,12 = (2EBI1/L1)(1+3/L1 (S2+S3 +2S2S3/L1))
S11,12 = (6EBI2/L2²)(1+2S4/L2) -(6EBI1/L1²)(1+2S3/L1)
S12,12 = (6EWI2/H)(4EBI1/L1)(1+3S3/L1 (1+S3/L1))(4EBI2/L2)(1+3S4/L2(1+S4/L2)
S13,12 = -(6EBI2/L2²)(1+2S4/L2)
S14,12 = (2EBI2/L2²)(1+3/L2 (S4+S5+2S4S5/L2))
S15,12 = -6EWI2/H²
S19,12 = 2EWI2/H
\[
S_{13} = 1
\]

\[
S_{6,13} = -(EWA_3/H)
\]

\[
S_{11,13} = -(12EBI_2/L^2)
\]

\[
S_{12,13} = -(6EBI_2/L^2)(1 + 2S_4/L)
\]

\[
S_{13,13} = (2EWA_3/H) + (12EBI_2/L^2)
\]

\[
S_{14,13} = -(6EBI_2/L^2)(1 + 2S_5/L)
\]

\[
S_{20,13} = -(EWA_3/H)
\]

\[
S_{14} = 1
\]

\[
S_{11,14} = (6EWI_3/H^2)
\]

\[
S_{12,14} = (2EBI_2/L^2)(1 + 2S_4/L)
\]

\[
S_{13,14} = (2EBI_2/L^2)(1 + 3S_5/L)
\]

\[
S_{14,14} = (6EWI_3/H^2)
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\[
S_{20,14} = (2EWI_3/H)
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4.7 GENERAL TWO BAY FRAME ANALYSIS

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C wall

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no. 2

no. 3

no. 1

no. 2

no. 3
4.7.1 STIFFNESS MATRIX OF GENERAL FRAME

\([K] = \begin{bmatrix}
    \mathbf{[S]}_1 & \mathbf{[S]}_2 & \mathbf{[S]}_3 & \mathbf{[S]}_4 & \mathbf{[S]}_5 & \mathbf{[S]}_6 & \mathbf{[S]}_7 \\
    \mathbf{[S]}_8 & \mathbf{[S]}_9 & \mathbf{[S]}_{10} & \mathbf{[S]}_{11} & \mathbf{[S]}_{12} & \mathbf{[S]}_{13} & \mathbf{[S]}_{14} \\
    \mathbf{[S]}_{15} & \mathbf{[S]}_{16} & \mathbf{[S]}_{17} & \mathbf{[S]}_{18} & \mathbf{[S]}_{19} & \mathbf{[S]}_{20} & \mathbf{[S]}_{21} \\
    \mathbf{[S]}_{22} & \mathbf{[S]}_{23} & \mathbf{[S]}_{24} & \mathbf{[S]}_{25} & \mathbf{[S]}_{26} & \mathbf{[S]}_{27} & \mathbf{[S]}_{28} \\
    \mathbf{[S]}_{29} & \mathbf{[S]}_{30} & \mathbf{[S]}_{31} & \mathbf{[S]}_{32} & \mathbf{[S]}_{33} & \mathbf{[S]}_{34} & \mathbf{[S]}_{35} \\
    \mathbf{[S]}_{36} & \mathbf{[S]}_{37} & \mathbf{[S]}_{38} & \mathbf{[S]}_{39} & \mathbf{[S]}_{40} & \mathbf{[S]}_{41} & \mathbf{[S]}_{42} \\
    \mathbf{[S]}_{43} & \mathbf{[S]}_{44} & \mathbf{[S]}_{45} & \mathbf{[S]}_{46} & \mathbf{[S]}_{47} & \mathbf{[S]}_{48} & \mathbf{[S]}_{49} \\
    \mathbf{[S]}_{50} & \mathbf{[S]}_{51} & \mathbf{[S]}_{52} & \mathbf{[S]}_{53} & \mathbf{[S]}_{54} & \mathbf{[S]}_{55} & \mathbf{[S]}_{56} \\
    \mathbf{[S]}_{57} & \mathbf{[S]}_{58} & \mathbf{[S]}_{59} & \mathbf{[S]}_{60} & \mathbf{[S]}_{61} & \mathbf{[S]}_{62} & \mathbf{[S]}_{63} \\
    \mathbf{[S]}_{64} & \mathbf{[S]}_{65} & \mathbf{[S]}_{66} & \mathbf{[S]}_{67} & \mathbf{[S]}_{68} & \mathbf{[S]}_{69} & \mathbf{[S]}_{70} \\
    \mathbf{[S]}_{71} & \mathbf{[S]}_{72} & \mathbf{[S]}_{73} & \mathbf{[S]}_{74} & \mathbf{[S]}_{75} & \mathbf{[S]}_{76} & \mathbf{[S]}_{77}
\end{bmatrix}
\[
\begin{array}{cccccccc}
S_{18} & S_{19} & S_{1,10} & S_{1,11} & S_{1,12} & S_{1,13} & S_{1,14} \\
S_{28} & S_{29} & S_{2,10} & S_{2,11} & S_{2,12} & S_{2,13} & S_{2,14} \\
S_{38} & S_{39} & S_{3,10} & S_{3,11} & S_{3,12} & S_{3,13} & S_{3,14} \\
S_{48} & S_{49} & S_{4,10} & S_{4,11} & S_{4,12} & S_{4,13} & S_{4,14} \\
S_{58} & S_{59} & S_{5,10} & S_{5,11} & S_{5,12} & S_{5,13} & S_{5,14} \\
S_{68} & S_{69} & S_{6,10} & S_{6,11} & S_{6,12} & S_{6,13} & S_{6,14} \\
S_{78} & S_{79} & S_{7,10} & S_{7,11} & S_{7,12} & S_{7,13} & S_{7,14} \\
\end{array}
\]

\[
\begin{array}{cccccccc}
S_{81} & S_{82} & S_{83} & S_{84} & S_{85} & S_{86} & S_{87} \\
S_{91} & S_{92} & S_{93} & S_{94} & S_{95} & S_{96} & S_{97} \\
S_{10,1} & S_{10,2} & S_{10,3} & S_{10,4} & S_{10,5} & S_{10,6} & S_{10,7} \\
S_{11,1} & S_{11,2} & S_{11,3} & S_{11,4} & S_{11,5} & S_{11,6} & S_{11,7} \\
S_{12,1} & S_{12,2} & S_{12,3} & S_{12,4} & S_{12,5} & S_{12,6} & S_{12,7} \\
S_{13,1} & S_{13,2} & S_{13,3} & S_{13,4} & S_{13,5} & S_{13,6} & S_{13,7} \\
S_{14,1} & S_{14,2} & S_{14,3} & S_{14,4} & S_{14,5} & S_{14,6} & S_{14,7} \\
\end{array}
\]

\[
\begin{array}{cccccccc}
S_{88} & S_{89} & S_{8,10} & S_{8,11} & S_{8,12} & S_{8,13} & S_{8,14} \\
S_{98} & S_{99} & S_{9,10} & S_{9,11} & S_{9,12} & S_{9,13} & S_{9,14} \\
S_{10,8} & S_{10,9} & S_{10,10} & S_{10,11} & S_{10,12} & S_{10,13} & S_{10,14} \\
S_{11,8} & S_{11,9} & S_{11,10} & S_{11,11} & S_{11,12} & S_{11,13} & S_{11,14} \\
S_{12,8} & S_{12,9} & S_{12,10} & S_{12,11} & S_{12,12} & S_{12,13} & S_{12,14} \\
S_{13,8} & S_{13,9} & S_{13,10} & S_{13,11} & S_{13,12} & S_{13,13} & S_{13,14} \\
S_{14,8} & S_{14,9} & S_{14,10} & S_{14,11} & S_{14,12} & S_{14,13} & S_{14,14} \\
\end{array}
\]
\[ [c]^T \]

<table>
<thead>
<tr>
<th>S15, 8</th>
<th>S15, 9</th>
<th>S15, 10</th>
<th>S15, 11</th>
<th>S15, 12</th>
<th>S15, 13</th>
<th>S15, 14</th>
</tr>
</thead>
<tbody>
<tr>
<td>S16, 8</td>
<td>S16, 9</td>
<td>S16, 10</td>
<td>S16, 11</td>
<td>S16, 12</td>
<td>S16, 13</td>
<td>S16, 14</td>
</tr>
<tr>
<td>S17, 8</td>
<td>S17, 9</td>
<td>S17, 10</td>
<td>S17, 11</td>
<td>S17, 12</td>
<td>S17, 13</td>
<td>S17, 14</td>
</tr>
<tr>
<td>S18, 8</td>
<td>S18, 9</td>
<td>S18, 10</td>
<td>S18, 11</td>
<td>S18, 12</td>
<td>S18, 13</td>
<td>S18, 14</td>
</tr>
<tr>
<td>S19, 8</td>
<td>S19, 9</td>
<td>S19, 10</td>
<td>S19, 11</td>
<td>S19, 12</td>
<td>S19, 13</td>
<td>S19, 14</td>
</tr>
<tr>
<td>S20, 8</td>
<td>S20, 9</td>
<td>S20, 10</td>
<td>S20, 11</td>
<td>S20, 12</td>
<td>S20, 13</td>
<td>S20, 14</td>
</tr>
<tr>
<td>S21, 8</td>
<td>S21, 9</td>
<td>S21, 10</td>
<td>S21, 11</td>
<td>S21, 12</td>
<td>S21, 13</td>
<td>S21, 14</td>
</tr>
</tbody>
</table>

\[ \delta_1 = 1 \]

- \( S_{11} = (12E/H_1^3) (C LI(i)) + (CMI(i)) + CRI(i) \)
- \( S_{31} = 6E CLI(i)/H_1^2 \)
- \( S_{51} = 6ECMI(i)/H_1^2 \)
- \( S_{71} = 6ECRI(i)/H_1^2 \)
- \( S_{81} = (12E/H_1^3) (C LI(i)) + (CMI(i)) + CRI(i) \)
- \( S_{10,1} = 6E CLI(i)/H_1^2 \)
- \( S_{12,1} = 6ECMI(i)/H_1^2 \)
- \( S_{14,1} = 6ECRI(i)/H_1^2 \)

\[ \delta_2 = 1 \]

- \( S_{22} = EAL(i)/H_1 + 12EBLI(i)/XL(i)^3 \)
- \( S_{32} = (6EBLI(i))/XL(i)^2 (1 + 2S2(i)/XL(i)) \)
- \( S_{42} = -12EBLI(i)/XL(i)^3 \)
- \( S_{52} = (6EBLI(i))/XL(i)^2 (1 + 2S3(i)/XL(i)) \)
- \( S_{92} = -EAL(i)/H_1 \)
CHAPTER 5

COMPUTER PROGRAMS FOR STRUCTURES

5.1 Introduction

In chapter 4 the stiffness method/recursion procedure of analysis was presented in a form which is suitable for computer programming. In this chapter flow charts of computer programs are presented. For each program, a table containing the identifiers of the program is given to facilitate the reader to identify the names of variables, constants, or other entities which are used in the program. The programs are written in Fortran. The listings of the programs are given in Appendix 'A'.

In the programs, all of the structures to be analyzed are assumed to consist of straight prismatic members. The material properties for a given structure are taken to be constant throughout the structure. Only the effects of loads are considered, and no other influences, such as temperature change, are taken into account.

All the programs are written for twenty storied frame. However the number of storey may be increased to any number depending on the storage capacity of computer.

5.2 Outline of Programs

Since the detailed steps in a computer program are related to the manner in which data are prepared each program is preceded...
by a summary which explains the preparation of the necessary data. In addition, some of the variables in the program will be of integer type while others are of floating-point type. Therefore, a listing of the variables of integer type is given in each identifier table. The variables which are subscripted in the program (and which require blocks of storage in the computer) are given in the dimension statement of each program.

5.2.1 General outline of Programs

The general outline of all the programs is shown in the following five steps.

1. Input and print structure data.
   a) Number of storey, number of storey loaded
   b) Structure parameters and elastic moduli
   c) Member designations, properties and orientations

2. Structures stiffness matrix
   a) Generation of stiffness matrix
   b) Inversion of stiffness matrix

3. Input and print load data
   a) Actions applied at joints
   b) Actions at ends of restrained members due to loads

4. Construction of vectors associated with loads
   a) Equivalent joint loads
   b) Combined joint loads
5) Calculation and output of results
   a) Joint displacement and support reactions
   b) Member and actions.

5.2.2 Units

The units of input data used for the programs are loads in kips, lengths in inches, areas in square inches, modulus of elasticity in kips per square inch, and so on. The units of final results are in kips, inches and radians.

5.3 Computer Storage and Time

IBM 1620 computer of Atomic Energy Center was used in the development and test run of all the programs. After the programs were developed the production run were made in the IBM 360-30 computer available in the Bureau of Statistics. The storage capacity of IBM 1620 is 60K and the existing storage capacity of IBM 360-30 is 65K. Time required by IBM 1620 computer for the compilation and execution of a program is much more than the time required by IBM 360-30 computer. To cite an example: for a two bay frame program, the compilation time required by IBM 1620 is 18 min., while the compilation time required by IBM 360-30 is 3½ min., and the execution time for the same program for twenty storey is 15 min. for IBM 1620 and approximately 48 Sec. for IBM 360-30.

Originally the programs were tested and run with single precision. The results were found to be satisfactory; and the
equilibrium was satisfied at all joints and the whole frame. However in the final production run, when the value of $\lambda$ was varied from 0.5 to 100, it was found that if the value of $\lambda$ goes beyond certain range, then neither the equilibrium of joints nor of the whole frame does satisfy. To overcome this problem, all the programs were subsequently written for double precision. It is found that double precision gives very accurate result for any frame irrespective of the value of $\lambda$.

A chart showing the computer core used and the time required by IBM 360-30 computer for the compilation and execution of each program in double precision is given below:

<table>
<thead>
<tr>
<th>Name of program</th>
<th>Core used (Hexadecimal Unit) (Bytes)</th>
<th>Compilation time (min.)</th>
<th>Execution time for one 20 storey frame (min.)</th>
</tr>
</thead>
<tbody>
<tr>
<td>One bay frame analysis</td>
<td>00A567</td>
<td>03.10</td>
<td>00.24</td>
</tr>
<tr>
<td>General one bay frame analysis</td>
<td>00A2FF</td>
<td>03.20</td>
<td>01.22</td>
</tr>
<tr>
<td>Two bay frame analysis</td>
<td>00C587</td>
<td>03.38</td>
<td>00.48</td>
</tr>
<tr>
<td>General two bay frame analysis</td>
<td>00B27</td>
<td>04.40</td>
<td>02.05</td>
</tr>
<tr>
<td>GA for two bay frame</td>
<td>00B787</td>
<td>03.08</td>
<td>00.28</td>
</tr>
<tr>
<td>GA for one bay frame</td>
<td>009CFF</td>
<td>02.53</td>
<td>00.16</td>
</tr>
</tbody>
</table>

Note: Time : - 03.38 min. means 3 min. 38 sec., 00.24 min. means 0 min. 24 sec. etc.
FIG. 5.1 FLOW DIAGRAM OF THE COMPUTER PROGRAM FOR THE ANALYSIS OF PLANE FRAME
5.5 One Bay Frame Program

One bay frame program was written to analyze frame with or without shear wall. The identifiers used in this program are listed in Table 5A. The input data required in the program are summarized in Table 5B.

**TABLE 5A**

Identifier used in one Bay frame computer programs

Integer type variables are: N, NSLOAD, LS

<table>
<thead>
<tr>
<th>Identifier</th>
<th>Definition</th>
</tr>
</thead>
<tbody>
<tr>
<td>N</td>
<td>Number of story</td>
</tr>
<tr>
<td>NSLOAD</td>
<td>Number of story loaded</td>
</tr>
<tr>
<td>XI1</td>
<td>Moment of Inertia of left column (equal for all floors)</td>
</tr>
<tr>
<td>XI2</td>
<td>Moment of inertia of right column (equal for all floors)</td>
</tr>
<tr>
<td>XI3</td>
<td>Moment of inertia of beam (equal for all floors)</td>
</tr>
<tr>
<td>A1</td>
<td>Area of cross-section - Left column (equal for all floors)</td>
</tr>
<tr>
<td>A2</td>
<td>Area of cross-section - Right column (equal for all floors)</td>
</tr>
<tr>
<td>EM</td>
<td>Young's modulus of Elasticity</td>
</tr>
<tr>
<td>XL</td>
<td>Beam span from center to center of columns or beam length from face to face of shear walls</td>
</tr>
<tr>
<td>H</td>
<td>Storey height (equal for all story)</td>
</tr>
<tr>
<td>XL2</td>
<td>Distance from c.g. of left wall to end of beam. It is zero in case of column.</td>
</tr>
<tr>
<td>XL3</td>
<td>Distance from c.g. of right wall to end of beam. It is zero in case of column.</td>
</tr>
</tbody>
</table>
TABLE 5A (Contd.)

<table>
<thead>
<tr>
<th>Identifier</th>
<th>Definition</th>
</tr>
</thead>
<tbody>
<tr>
<td>LS</td>
<td>Loaded storey level from top</td>
</tr>
<tr>
<td>XP( )</td>
<td>Loads applied at joints of a storey,</td>
</tr>
<tr>
<td>FER( )</td>
<td>Fixed end reactions on storey beams. The order of FER( ) is left F.E. shear, Left F.E.M, Right F.E. shear, Right F.E.M.</td>
</tr>
</tbody>
</table>

TABLE 5B

Preparation of Data for one Bay frame

<table>
<thead>
<tr>
<th>Data</th>
<th>Number of cards</th>
<th>Items on data cards</th>
</tr>
</thead>
<tbody>
<tr>
<td>a) Number of storey of storey loaded, member properties, structure parameters and elastic modulus.</td>
<td>1</td>
<td>N, NSLOAD, XI1, XI2, XI3, A1, A2, EM, XL, H, XL2, XL3</td>
</tr>
<tr>
<td>a) Storey level, actions applied at joints, actions at end of restrained members due to loads.</td>
<td>N</td>
<td>LS, XP( ), FER( )</td>
</tr>
</tbody>
</table>

5.6 General One Bay Frame Program

While one bay frame program can be used for constant storey height, constant moment of inertia of Beams etc., the General one bay frame program can take care of variable storey height, moment of inertia, area etc. Identifiers used for general one bay frame is given in Table 5D and preparation of data is given in Table 5C.
### TABLE 5C

<table>
<thead>
<tr>
<th>Data</th>
<th>Number of cards</th>
<th>Items on data card</th>
</tr>
</thead>
<tbody>
<tr>
<td>a) Number of story,</td>
<td>1</td>
<td>N, NSL, E</td>
</tr>
<tr>
<td>Number of story loaded,</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Modulus of Elasticity</td>
<td></td>
<td></td>
</tr>
<tr>
<td>a) Member properties,</td>
<td>N</td>
<td>CL(I), CR(I), BL(I),</td>
</tr>
<tr>
<td>span, storey height etc.</td>
<td></td>
<td>AL(I), AR(I), X(I), H(I),</td>
</tr>
<tr>
<td></td>
<td></td>
<td>S2(I), S3(I).</td>
</tr>
<tr>
<td>a) Loaded story level,</td>
<td>NSL</td>
<td>LS, XP( ), FER( )</td>
</tr>
<tr>
<td>Actions applied at</td>
<td></td>
<td></td>
</tr>
<tr>
<td>joint, Actions at</td>
<td></td>
<td></td>
</tr>
<tr>
<td>ends of restrained</td>
<td></td>
<td></td>
</tr>
<tr>
<td>members due to loads.</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

### TABLE 5D

**Identifiers used in general one Bay Computer Programs**

<table>
<thead>
<tr>
<th>Identifier</th>
<th>Definition</th>
</tr>
</thead>
<tbody>
<tr>
<td>N</td>
<td>Number of story</td>
</tr>
<tr>
<td>NSL</td>
<td>Number of story loaded</td>
</tr>
<tr>
<td>E</td>
<td>Modulus of Elasticity</td>
</tr>
<tr>
<td>CL( )</td>
<td>Moment of Inertia of left column or wall</td>
</tr>
<tr>
<td>CR( )</td>
<td>Moment of Inertia of Right column or wall</td>
</tr>
<tr>
<td>BL( )</td>
<td>Moment of Inertia of Beam</td>
</tr>
<tr>
<td>AL( )</td>
<td>Area of left column or wall</td>
</tr>
<tr>
<td>AR( )</td>
<td>Area of right column or wall</td>
</tr>
<tr>
<td>X( )</td>
<td>Beam span C/C of column or face to face of wall</td>
</tr>
</tbody>
</table>
### TABLE 5D (Contd...)

<table>
<thead>
<tr>
<th>Identifier</th>
<th>Definition</th>
</tr>
</thead>
<tbody>
<tr>
<td>H( )</td>
<td>Storey height</td>
</tr>
<tr>
<td>S2( )</td>
<td>Distance from c.g. of left wall to Beam end.</td>
</tr>
<tr>
<td>S3( )</td>
<td>Distance from c.g. of right wall to Beam end.</td>
</tr>
<tr>
<td>L5</td>
<td>Loaded storey level counted from top</td>
</tr>
<tr>
<td>XP( )</td>
<td>Actions (Loads) applied at joints on a storey.</td>
</tr>
<tr>
<td>FER( )</td>
<td>Actions at ends of restrained members due to loads. Order of FER( ) is left F.E. shear, Left F.E.M., Right F.E. shear, Right F.E.M.</td>
</tr>
</tbody>
</table>

#### 5.7 Two Bay Frame Program

Two bay frame program was written to analyze frames of columns or walls or combination of both. Storey height, Moment of inertia of left beams, right beams, left vertical members, center vertical members right vertical members etc. have been kept constant throughout the full height. The identifiers used in the program are listed in Table 5E and the input data required are summarized in Table 5F.
### TABLE 5E

**Identifiers used in two Bay Frame Computer Program**

Integer type variables are: - N, NSLOAD, LS

<table>
<thead>
<tr>
<th>Identifier</th>
<th>Definition</th>
</tr>
</thead>
<tbody>
<tr>
<td>N</td>
<td>Number of storey</td>
</tr>
<tr>
<td>NSLOAD</td>
<td>Number of storey loaded</td>
</tr>
<tr>
<td>WI1</td>
<td>Moment of inertia of left column or wall</td>
</tr>
<tr>
<td>WI2</td>
<td>Moment of inertia of center column or wall</td>
</tr>
<tr>
<td>WI3</td>
<td>Moment of inertia of right column or wall</td>
</tr>
<tr>
<td>BI1</td>
<td>Moment of inertia of left beam</td>
</tr>
<tr>
<td>BI2</td>
<td>Moment of inertia of right beam</td>
</tr>
<tr>
<td>WA1</td>
<td>Area of left column or wall</td>
</tr>
<tr>
<td>WA2</td>
<td>Area of center column or wall</td>
</tr>
<tr>
<td>WA3</td>
<td>Area of right column or wall</td>
</tr>
<tr>
<td>E1</td>
<td>Modulus of elasticity</td>
</tr>
<tr>
<td>XL1</td>
<td>Span c/c of columns or face to face of walls of left beam</td>
</tr>
<tr>
<td>XL2</td>
<td>Span c/c of columns or face to face of walls of right beam</td>
</tr>
<tr>
<td>H2</td>
<td>Storey height</td>
</tr>
<tr>
<td>S2</td>
<td>Distance from c.g. of left wall to beam end</td>
</tr>
<tr>
<td>S3</td>
<td>Distance from c.g. of center wall to end of left beam</td>
</tr>
<tr>
<td>S4</td>
<td>Distance from c.g. of center wall to end of right beam</td>
</tr>
<tr>
<td>LS</td>
<td>Loaded storey level from top</td>
</tr>
<tr>
<td>SP(...)</td>
<td>Actions (loads) applied at joints on a storey</td>
</tr>
<tr>
<td>FER(...)</td>
<td>Actions at ends of restrained members due to loads. Order of FER(...) is same as for one bay frame.</td>
</tr>
</tbody>
</table>
### TABLE 5F

**Preparation of Data for two Bay Program**

<table>
<thead>
<tr>
<th>Data</th>
<th>Number of Cards</th>
<th>Items on data cards</th>
</tr>
</thead>
<tbody>
<tr>
<td>a) Number of storey, number of storey loaded, section properties of members etc.</td>
<td>1</td>
<td>N, NSLOAD, Wi1, Wi2, Wi3, Bi1, Bi2, Wa1, Wa2, Wa3.</td>
</tr>
<tr>
<td>b) Elastic modulus, span, storey height, wall widths from c.g. etc.</td>
<td>1</td>
<td>EM, XL1, XL2, H, S2, S3, S4, S5.</td>
</tr>
</tbody>
</table>

a) Storey level, actions applied at joints

b) Actions at ends of restrained members due to loads

5.8 **General two Bay Frame Program**

General two Bay frame program can take care of variable storey height, variable spans and section properties of all members. The identifiers are listed in Table 5G and the input data required are summerized in Table 5H.
# TABLE 5G

Identifiers used in General Two Bay Computer Program

Integer type variables are: - N, NSL, LS

<table>
<thead>
<tr>
<th>Identifier</th>
<th>Definition</th>
</tr>
</thead>
<tbody>
<tr>
<td>N</td>
<td>Number of storey</td>
</tr>
<tr>
<td>NSL</td>
<td>Number of storey loaded</td>
</tr>
<tr>
<td>E</td>
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<tr>
<td>LS</td>
<td>Storey level from top</td>
</tr>
<tr>
<td>SP( )</td>
<td>Actions (loads) applied at joints on a storey</td>
</tr>
<tr>
<td>FER( )</td>
<td>Actions at ends of restrained members due to loads. Order of FER( ) is same as two Bay frame</td>
</tr>
<tr>
<td>H( )</td>
<td>Storey height</td>
</tr>
<tr>
<td>AL( )</td>
<td>Area of left column or wall</td>
</tr>
<tr>
<td>AM( )</td>
<td>Area of center column or wall</td>
</tr>
<tr>
<td>AR( )</td>
<td>Area of right column or wall</td>
</tr>
<tr>
<td>CL( )</td>
<td>Moment of inertia of left column or wall</td>
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<tr>
<td>CM( )</td>
<td>Moment of inertia of right column or wall</td>
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<tr>
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<td>Moment of inertia of left beam</td>
</tr>
<tr>
<td>BR</td>
<td>Moment of inertia of right beam</td>
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<td>X</td>
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<tr>
<td>Y</td>
<td>Span right beam</td>
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S2, S3, S4 and S5 are same as two bay frame
### TABLE 5H

**Preparation of Data for General two Bay frame Program**

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<tr>
<th>Data</th>
<th>Number of cards</th>
<th>Items on data cards</th>
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<td>N, NSL, E</td>
</tr>
<tr>
<td>loaded, Modulus of Elasticity</td>
<td></td>
<td></td>
</tr>
<tr>
<td>a) Loaded storey level, actions applied at joint</td>
<td>NSL</td>
<td>LS, XP( )</td>
</tr>
<tr>
<td>b) Loaded storey level, actions at ends of restrained members due to loads</td>
<td>NSL</td>
<td>LS, FER( )</td>
</tr>
<tr>
<td>a) Storey height, properties of vertical members</td>
<td>N</td>
<td>H(I), AL(I), AM(I)</td>
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<tr>
<td></td>
<td></td>
<td>AR(I), CL(I), CM(I)</td>
</tr>
<tr>
<td></td>
<td></td>
<td>CR(I)</td>
</tr>
<tr>
<td>b) Beam properties, spans, wall depths from c.g.</td>
<td>2N</td>
<td>BL, ER, X, Y, S2, S3, S4, S5</td>
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</table>
5.9 Program for GA for One Bay Frame

Ga parameter of a frame is defined as the concentrated load required at the top of the frame top cause unit lateral displacement. GA for one bay frame was written to compare the actual GA value of a frame evaluated by stiffness method and recursion procedure and the approximate GA value evaluated by Heidebrecht and Smith method (HSM). Identifiers used in this program is same as the identifiers of first data card of one bay frame program. Only one card is required in this program to evaluate the GA value.

5.10 Program for GA for Two Bay Frame

This program was written to find out the GA value by exact as well as HSM for two bay frame. Only two data cards are required for this program. Identifiers used in this program are same as the identifiers of the first two cards of two bay frame except that the identifiers are arranged in different manners in this program.

5.11 Program for Approximate Analysis by Heidebrecht and Smith Method

This program was written to find out the deflection, B.M, shear and the interacting forces in flexural and shear beam by HSM. Identifiers used in this program is same as the identifiers of the first two cards of two bay frame program.
CHAPTER 6

COMPARISON OF EXACT METHOD WITH APPROXIMATE METHOD

6.1 Introduction

Tall building frame is one of the highly redundant structure encountered in Civil Engineering practice. Numerical solution of the exact analysis, without the help of high speed digital computers, is too time consuming and sometimes impossible. Designers with no access to high speed digital computers go for approximate analysis. Here in this chapter an attempt has been made to compare the different parameters or assumptions of approximate method with that of "exact" method.

For academic interest the value of $\lambda$ ($= \frac{I_c \cdot l}{I_b \cdot h}$ or $\sum I_c \cdot l / 2 \sum I_b \cdot h$) has been used between 0.5 and 100. However, the realistic range of $\lambda$ is between 1 and 10.

6.2 Comparison of GA Parameter of One and Two Bay Frames

GA parameter of a frame may be defined as the lateral load required at the top of the frame to cause unit lateral translation of the frame at top. In Chapter 3, GA parameter by Heidebrecht and Smith was given in equation (3.2.11).
To compare the value of GA by Heidebrecht and Smith method (HSM) and the exact method (EM) programs were developed for one bay and two bay frames.

For different values of $\lambda ( = I_c / h / I_d / l)$

GA value by the above two methods were evaluated. The results are given in tabular form.

From the results it is seen that for a two Bay frame, the GA value by Heidebrecht and Smith method may be used in the following range of storeys for different values of $\lambda$.

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<th>$\lambda$</th>
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<th>2 - 3</th>
<th>3 - 4</th>
<th>4 - 5</th>
<th>5 - 6</th>
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Similarly for one Bay frame the GA value may be used in the following the range of storey for different values of $\lambda$.

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<td>9 - 10</td>
<td>10-11</td>
<td>11 - 12</td>
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</table>
GA FOR ONE BAY FRAME

\[
\lambda = \frac{I_c \cdot \ell}{I_b \cdot h}, \text{ where } h = \text{Storey height}
\]

\[
\ell = \text{Span}
\]

\[
I_c = \text{Moment of inertia of column}
\]

\[
I_b = \text{Moment of inertia of Beam}
\]

HSM = Heidebrecht and Smith method
EM = Exact Method

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<td>GA BY EM</td>
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GA FOR ONE BAY FRAME (CONTD.)

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GA VALUE FOR TWO BAY FRAME

\[ \lambda = \frac{\sum I_c \cdot I}{2 \sum I_b \cdot h}, \]

where: 
- \( h \): Storey height
- \( \ell \): Span
- \( \sum I_c \): Sum of moment of inertia of columns
- \( \sum I_b \): Sum of moment of inertia of Beams

HSM = Heidebrecht and Smith Method
EM = Exact method

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6.3 Comparison of "Exact" Method with Portal and Cantilever Methods

Probably the most popular method of analyzing a rigid rectangular frame under lateral load is either the portal method or the cantilever method. These are, in effect, the methods generally used in the preliminary analysis of tall buildings.

The two common assumptions made in portal and cantilever methods are:

1. There is a point of inflection at the center of each girder.

2. There is a point of inflection at the center of each column.

The third assumption for portal method is "The total horizontal shear on each storey is divided between the columns of that storey in a manner such that each interior column carries twice as much shear as each exterior column".

While the third assumption for cantilever method is "The intensity of axial stress in each column of a storey is proportion 1 to the horizontal distance of the column from the center of gravity of all the columns of the storey under consideration".

Several programs for lateral loads with different structure data were run to check the validity of the above simplified assumptions.
6.3.1 One Bay Frame

By varying the value of $\lambda = \frac{l \cdot f}{I_b \cdot h}$ from 0.5 to 100, programs for lateral load for one bay frame were run with a view to assessing the range of $\lambda$ for which the above assumptions of portal and cantilever methods may safely be used in the design office. By analyzing the results it is found that for symmetrical frames the assumptions 1 and 3 of the above methods are valid for all values of $\lambda$, while assumption 2 does not hold good for any value of $\lambda$. Point of inflection of each column is dependent on $\lambda$ and varies from storey to storey for same value of $\lambda$. Point of inflection of column for different storey with different values of $\lambda$ are given in Tabular and graphical form.

6.3.1.1 Discussion on point of inflection in columns

The amount by which the points of inflection in the columns move off-center can be estimated using the tables or graphs as given after this section.

If $\lambda$ is low, then the beams are stiff and the frame deflects purely in a shear mode as shown in Fig. 7(a). Points of inflection are at midheight throughout.

As $\lambda$ increases the beams become less effective until, when they have no bending stiffness, the load is resisted by the columns alone, as in Fig. 7(b). The frame then assumes a bent shape similar to that of a single cantilever under lateral load.
that is, it deflects in a bending mode. There are no points of inflection in the columns.

Neither of the ideal conditions illustrated in Fig. 7 are normally achieved in reality. Beams can never be fully rigid and even a very low beam stiffness can have a significant effect in restraining the frame deflection. What does happen is that within a realistic range of $\lambda$ (say 1 to 10) the points of inflection in the columns (except those in the lower and upper stories) are close to midheight, and shear mode deformation predominates.

If significant axial strains do occur in the columns, bending mode deformations becomes more prominent. From the point of view of stiffness calculations, it is important to note that as $\lambda$ decreases, deflection due to axial deformation of the columns will become more prominent.

![Diagram showing ideal conditions for shear and bending modes of deformation.](image)

(\(\lambda = 0\)) shear mode deformation

(\(\lambda = \infty\)) bending mode deformation

**Fig. 7 Effect of $\lambda$ on Frame Deformation**
### POINT OF INFLECTION AT COLUMN

(ONE BAY FRAME)

\[ \lambda = \frac{I_c \cdot \ell}{I_b \cdot h} \]

- \( h \) = storey height
- \( \ell \) = span
- \( I_c \) = Moment of Inertia of column
- \( I_b \) = Moment of Inertia of Beam

\( X \) = Distance of point of inflection from bottom of column.

NPC = No point of inflection

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NO. OF STOREY = 20

LOCATION OF POINT OF INFLECTION AT COLUMNS
6.3.1 Two Bay Frame

Comparison of exact method with portal and cantilever methods where made for two Bay frame. Two types of data were used for this comparison. In one set the moment of inertia of all the columns were kept equal, while in the other set the moment of inertia of interior column was kept double the moment of inertia of exterior column. The frames are symmetrical and the values of $\lambda (\sum l_c \ell / 2 \sum l_b h)$ varies from 0.5 to 100.0.

From the computer result it is found that for the frames where the moment of inertia of interior column is double the moment of inertia of exterior column, the assumptions 1 and 3 of portal and cantilever methods hold good. The point of inflection at columns varies from storey to storey. For the upper stories it is below the center of columns and for lower stories it is above the center of columns. For both exterior and interior columns the point of inflections are at the same level in a storey.

For the frames where the moment of inertia of all the columns are same, only the assumption 1 of portal method and assumptions 1 and 3 of cantilever method hold good. The point of inflection on column varies from storey to storey and depends on $\lambda$. The point of inflection at exterior and interior columns on a storey do not fall on the same level. Except at the top and bottom storey, the ratio of shear at interior column to shear at exterior columns varies from 1.665 to 2.40 depending on $\lambda$. The ratio increases with the increase of $\lambda$.

Point of inflection at column and ratio of column shear are given in the tabular form.
### POINT OF INFLECTION AT COLUMN

Area and moment of inertia of interior column is double that of exterior column

$$\lambda = \frac{\sum I_c \ell}{2 \sum I_b h}$$

where:
- $\sum I_c = $ Sum of moment of inertia of columns
- $\sum I_b = $ Sum of moment of inertia of beams
- $\ell = $ Span of beams, $h = $ Storey height

$X = $ Distance of point of inflection from bottom of column, NPC = No point of inflection

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POINT OF INFLECTION AT COLUMN (CONT'D.)

Area and moment of inertia of interior column is double that of exterior column

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## Point of Inflection and Ratio of Shear at Exterior and Interior Columns

All columns are of same section

\( \text{SIC/SEC} = \frac{\text{Shear interior column}}{\text{shear exterior column}} \)

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POINT OF INFLECTION AND RATIO OF SHEAR AT INTERIOR AND EXTERIOR COLUMNS

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POINT OF INFLEXION AND RATIO OF SHEAR AT INTERIOR AND EXTERIOR COLUMN

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7.1 Introduction

In Chapter 3 an approximate method of analysing a shear wall-frame by Heidebrecht and Smith method was given. In this method the frame is reduced to a shear beam and tied to the shear wall, which is termed as flexural beam. The stiffness of the "link beam" between the shear beam and flexural beam is assumed to be zero. In this chapter several structures were analysed with different stiffness of the "link beam" to make a comparison between the exact method and the Heidebrecht and Smith method.

7.2 Comparison Between Exact Method and Heidebrecht and Smith Method

Two Bay frame program was modified to calculate the translation, bending moments, shear etc. by exact as well as Heidebrecht and Smith method. The right vertical member was taken to be shear wall and the other two vertical members as columns. Several programs were run by keeping the sectional properties of all members, except the link beams, same for all programs. Moment of inertia of link beam \((I_{lb})\) was varied in such a way that \(h = I_{lb} h / I_{c} b\) varied from 0.0 to 8.0. Computer results obtained are given in Tabular form.
Except for $h = 0.0$, the translation obtained by Heidenbrecht and Smith method (HSM) is found to be approximately 50% more than the exact method while for $h = 0.0$, the translation by exact method is approximately 50% more than HSM. Bending moments at the wall in the middle stories differs considerably from one system to other, while in the top and bottom stories they differ from 10% to 50% depending on $h$ and $\alpha H$. Shear on wall differs considerably from one system to other in the middle stories, while in top and bottom stories they are within the range of 20%.

Note: $I_{lb}$ = moment of inertia of link beam
$h$ = storey height
$I_c$ = moment of inertia of column
$l$ = length of link beam
$\alpha^2$ = $GA/EI$ as given in equation (3.2.10)
$H$ = Structure height
### COMPARISON BETWEEN EXACT METHOD AND HEIDEBRECHT AND SMITH METHOD

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**HSM** = Heidebrecht and Smith method  
**Number of storeys** = 20

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CHAPTER 8
BUILDING SWAY

8.1 Introduction

It is exceedingly difficult to formulate the simple answer to the question "How much can we allow a building to sway in the wind?".

The engineer in his design, establishes the nature of the wind that is the environment of the particular building in question. For a very tall building the engineer would like the structure of the wind to be described in relatively great detail. The engineer must also determine the response of the structure to the turbulent wind environment.

Great steps forward are being made both in describing the nature of the wind and the effects of that wind on a structural system. Deflection and particularly total deflection or sway has little value in an engineering sense. It does not really matter if a building sways 1 inch, 10 inches or 10 feet so long its performance is satisfactory. The three things that do matter are the integrity of the structural system, the integrity of the architectural finishes and the comfort of the building occupants.

8.2 Sway Control by Using Heavier Crown Coupling Beams

For the satisfactory performance of the tall buildings, it is customary for the engineers to try to reduce the sway of the buildings
as much as possible without any substantial increase in cost of the structure. One economic way of reducing the sway is by using heavier crown coupling beam. In this chapter the role of heavier crown coupling beam in reducing the sway of building is investigated.

8.2.1 Description of Structures

General one Bay frame program was run for two different twenty storied frames to evaluate the percentage of sway reduction by using heavier crown coupling beam. Vertical members of one of the frames are shear walls while the vertical number of the other frame are columns. For each frame two sets of data are used. Sectional properties of frame elements for both sets of data were same except the Beam of the top storey. Lateral load is assumed to \(1\) k/ft height. The sections and length of the frame elements as used in the program are as follows:

8.2.2 Frame with Shear Wall

Set:-1 For all stories

Left vertical member = 12" x 30'-0"
Right vertical member = 12" x 10'-0"
Beam section = 12" x 2'-0"
Beam length = 10'-0" Storey height = 10'-0"
Set: 2

For all stories: Same as above except the x-section of the top storey beam, which is $1\text" \times 4\text" - 0\text"$.

The maximum sway reduction occurred for this structure is $2.35\%$. The results of sway (translation) is given in Tabular form.

8.2.3 Frame without Shear Wall

Set: 1 For all Stories

Left and right vertical members = $30\text" \times 40\text"$

Moment of inertia of beam = $32000 \text{ in}^4$

Beam length = $24\text' - 0\text"$  

Storey height = $12\text' - 0\text"$

Set: 2

For all stories: Same as above except the moment of inertia of top storey beam, which is $= 54000 \text{ in}^4$.

The maximum sway reduction occurred for this structure is $0.50\%$. The results of sway (translation) is given in Tabular form.
PERCENTAGE OF SWAY REDUCTION WITH HEAVIER CROWN COUPLING BEAM

Number of storey = 20

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<th>Frame with column</th>
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<td></td>
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9.1 One Bay Frame

Several programs were run for a one-bay frame to find out the percentage of load transfer from a vertically loaded column to another column. The ratio of $V_b/V_c$ varies from 0.0 to 1.0. $V_b$ is the shear stiffness of the beam and is equal to $12EI/L^3$ (assuming equivalent shear stiffness of beam $C_v = 1.0$) and $V_c$ is the axial stiffness of column and is equal to $AE/H$.

For $V_b/V_c$ from 0.1 to 1.0, the maximum percentage of load transfer is 1.0204%, which takes place within the top three stories. The results are given in Tabular form.
PERCENTAGE OF LOAD TRANSFER FROM ONE VERTICALLY LOADED COLUMN TO OTHER COLUMN

\[ V_b = \frac{12EI_b}{L^3} \] = Shear stiffness of the beam
\[ V_c = \frac{AE}{H} \] = Axial stiffness of column

PCL = Percentage of load transfer.

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</table>
10.1 Conclusion

From the comparison of exact analysis with that of approximate analysis of a frame structure the following conclusions may be drawn.

1. Calculation of GA parameter of one bay frame by Heidebrecht and Smith method (HSM) shall be satisfactory provided for different value of $\lambda$, the storey number is limited to the following:

   - For $\lambda = 2$ storey number $= 6$
   - For $\lambda = 3$ storey number $= 7$
   - For $\lambda = 4$ storey number $= 8$
   - For $\lambda = 5$ storey number $= 9$
   - For $\lambda = 7$ storey number $= 10$
   - For $\lambda = 9$ storey number $= 11$
   - For $\lambda = 10$ storey number $= 12$

2. Calculation of GA parameter of two bay frame by HSM shall be satisfactory provided for different value of $\lambda$, the storey number is limited to the following:

   - For $\lambda = 2$ storey number $= 10$
   - For $\lambda = 3$ storey number $= 12$
   - For $\lambda = 4$ storey number $= 13$
   - For $\lambda = 5$ storey number $= 14$
   - For $\lambda = 6$ storey number $= 16$
3. For a one bay symmetrical frame there is a point of inflection at the center of each beam and the total horizontal shear on each storey is equally divided between the columns of that storey. Point of inflection at columns do not fall at the center of each column in each storey. In the upper stories the point of inflection is nearer to the bottom of column and in the lower stories it is nearer to the top of column. As the value of $\lambda$ increases the point of inflection shifts towards the bottom of column in the upper stories and towards the top of column in the lower stories. For $\lambda > 4.0$ certain upper storey/stories or certain upper and lower stories may not have any point of inflection at all.

4. For a two bay symmetrical frame there is a point of inflection at the center of each beam. If the moment of inertia of interior column is double the moment of inertia of exterior column, then the total horizontal shear on each storey is divided between the columns of that storey in such a manner that the shear at interior column is double the shear at exterior column. The behaviour of point of inflections at columns are similar to that of one bay frame. Point of inflection for both interior and exterior columns at a storey fall on same level.

5. For a two bay symmetrical frame if the moment of inertia of all the columns are same, the point of inflection at exterior and interior columns on a storey do not fall on same level. Except at the top and bottom storey, the ratio of shear at interior column to shear at exterior column varies from 1.6 to 2.4 depending on $\lambda$. The ratio increases with the increase of $\lambda$. For $\lambda > 20$, the shear
at exterior columns of top storey acts in the opposite direction to the shear at interior column.

For the most two bay frame structures with interconnected shear wall, the analysis by Heidebrecht and Smith method is quite conservative in relation to sway of the structure. Except for the stiffness of "link beam" is zero, the deflection by HSM is nearly 50% more than the exact value. The B.M. and shear force in the middle stories differs considerably from the exact analysis. Shears at top and bottom stories are within the range of ±20% depending on the value of $\eta$ and $\alpha H$.

10.2 Suggestions for Future Study

The following approximate analysis of tall shear wall-frame structures may be compared with "exact" analysis to draw a conclusion as to which method of approximate analysis is best suited for a particular structure.

1. Khan and Sharounia (1) method
2. McLeod (2) method

Programs may be developed for multibay frames and validity of the assumptions of portal and cantilever methods may be verified for the following:

1. Unsymmetrical one bay frames
2. Unsymmetrical two bay frames
3. Multibay frames.
REFERENCES


6. Rosman, R. "Interaction Between Shear Walls and Frames" Laterally Loaded Systems Consisting of Walls and Frames, Tall Buildings, the Proceedings of a Symposium on Tall Buildings held at University of Southampton, Edited by A. Coull and E. Stafford Smith.


8. Murachev, Sigalov, Baikov, "Analysis of Frame and Brace System".

APPENDIX 'A'

LISTING OF COMPUTER PROGRAMS

1. One bay frame analysis

2. General one bay frame analysis

3. Two bay frame analysis

4. General two bay frame analysis

5. GA for one bay frame

6. GA for two bay frame

7. Approximate analysis of two bay shear-wall frame
   by Heidebrecht and Smith method.
JOB UEG91666 ONE BAY FRAME ANALYSIS  DEC.1977

OPT L'INK, LIST, LOG
EXEC.FORTRAN

THIS IS A PROGRAM FOR ONE BAY FRAME ANALYSIS

PROGRAM MAY BE USED FOR FRAME OF WALLS OR COLUMNS CR OF BOTH
DOUBLE PRECISION BO, B, C, A, FM, FRED, DI SP, D, E, FI, BI, U, FS, P, C, XP, FER,
1X11, X12, X13, A1, A2, XL, H, XL2, XL3, XI, X2, X3, X4, X5, X6, X7, X8, X9, XI0, XI1;
2X12, X13, X14, X15, X16, TL, TR, DEL, XM, XMR, VM, VL, VR, TT, TB, X20, XM, XMB,
3VT, VB, AXIAL, EH

DIMENSION BO(5, 5), BI(5, 5), C(5, 5), A(5, 5), FM(5, 5), FRED(5, 5), DISP(5, 5)
1, DI(5, 5), E(5, 5), FII(5, 1), BII(95, 5), U(105), P(100), Q(80), FS(95)
2X, XP(5), FER(4)

WRITE (3, 111)

111 FORMAT (6X66ONE BAY FRAME ANALYSIS BY STIFFNESS MATRIX, AND RECURS
ION PROCEDURE)

501 READ(1,1,ERR=99,END=95)N,NSSLOAO, XI11, X12, XI3, A1, A2, EM, XL, H, XL2, XL3

WRITE (3, 101)

101 FORMAT (6X17H BEGINNING OF DATA)

IF X1L=0, LEFT WALL IS A COLUMN, IF X13=0, RIGHT WALL IS A COLUMN.

WRITE (3, 111)

111 FORMAT (214, 3IF10.6, 7F6.0)

N1= 5*(N+1)
N2= 5*N
N3= 4*N

DO 4 I=1, N1

U(1)= 0

DO 5 I=1, N2

P(1)= 0

DO 6 I=1, N3

Q(I)= 0

DO 7 I=1, N1

NSLOAO= NUMBER OF STORY LOADED. LS=oaded STORY LEVEL FROM TOP
XP(K)= LOAD AT EACH STORY FER(J)=FIXED END REACTION ON STORY LAN

DO 7 I=1, NSLOAO

READ (1, 3) LS, (XP(K), K=1, 5), (FER(J), J=1, 4)

WRITE (3, 31) LS, (XP(K), K=1, 5), (FER(J), J=1, 4)

DO 8 K=1, 5

I2= 5*(LS-1)+K

8 P(I2)= XP(K)

DO 9 J=1, 4

12= 4*(LS-1)+J

9 Q(I2)= FER(J)

13 CONTINUE

3 CONTINUE

3 FORMAT (13, 5F8.2, 4F8.2)

DO 50 J=1, 5

DO 50 I=1, 5

BO(I, J)= 0

Bi(I, J)= 0.

50 CONTINUE

C(I, J)= 0.
X1= 12.*XI1/H**3
X2= 12.*XI2/H**3
X3= 6.*XI1/H**2
X4= 6.*XI2/H**2
X5= 4.*XI1/H
X6= 4.*XI2/H
X7 = A1/I
X8 = A2/I
X9 = 12 + X13/XL
X10 = 6 + X13/XL
X11 = 4 + X13/XL
X12 = 1 + 2 + XL2/XL
X13 = 1 + 2 + XL2/XL
X14 = 1 + 3 + XL2/XL + (1 + XL2/XL)
X15 = 1 + 3 + XL3/XL + (1 + XL3/XL)
X16 = 1 + 3 + XL*(XL2 + XL3 + 2) + XL2*X3
BO(1, 1) = X1*X2
BO(2, 2) = X7*X6
BO(2, 2) = X10*X12
BO(3, 2) = X11*X16
BO(4, 4) = X8*X9
BO(5, 4) = X10*X12
BO(5, 5) = X6*X11*X15
B(1, 1) = (X1*X2) + X2
B(2, 2) = 2 + X7*X9
B(3, 2) = X10*X12
B(4, 2) = X9
B(5, 2) = X10*X12
B(3, 3) = 2 + X5 + X11*X14
B(4, 3) = X10*X12
B(5, 3) = X11/2 + X16
B(4, 4) = X8*X9
B(5, 4) = X10*X13
B(5, 5) = 2 + X6 + X11*X15
C(1, 1) = (X1*X2)
C(3, 1) = X3
C(5, 1) = X4
C(2, 2) = X7
C(1, 3) = X3
C(3, 3) = X5/2
C(4, 4) = X8
C(1, 5) = X4
C(5, 5) = X6/2
DO 51 J = 1, 4
J = J + 1
DO 52 I = J, 5
BO(I, J) = BO(I, J)
DO 51 CONTINUE
CALL MATTRA (C, A, 5, 5)
CALL MATINV (BO, 5, 5)
DO 666 J = 1, 5
DO 666 I = 1, 5
BI(I, J) = D(I, J)
CALL MATMLY (A, D, E, 5, 5, 5)
DO 9 I = 1, 5
FS(I)=P(I)
9 FRED(I,1)=P(I)
DO 53 1=2,N
I4=5*(1-1)
DO 54 K=1,5
IK=14+K
54 FI(K,1)=P(IK)
CALL MATMLY (E,FRED,FM,5,5,1)
CALL MATSUB (FI,FM,FREO,5,1)
CALL MATMLY (E,C,D,5,5)
CALL MATSUB (B,D,E,5,5)
CALL MATINV(F,5,D)
IF (I-N) 10,11,1C
10 CALL MATMLY (A,D,E,5,5,5)
DO 541 J=1,5
IJ=I4+J
541 FS(IJ)=FRED(J,1)
DO 888 J=1,5
DO 888 L=1,5
IL=14+L
888 BI(IL,J)=O(IL,J)
53 CONTINUE
11 CALL MATMLY (D,FRED,DISP,5,5,5)
K=5*(N-1)
DO 14 J=1,5
KJ=K+J
14 J(KJ)=DISP(J,1)
DO 645 1=2,N
DO 998 J=1,5
DO 998 K=1,5
KJ=5*(N-I)+K
998 D(K,J)=BI(KJ,J)
DO 646 J=1,5
JJ=5*(N-I)+J
646 FI(J,1)=FS(JJ)
CALL MATMLY (C,DISP,FM,5,5,5)
CALL MATSUB (FI,FM,FREO,5,1)
CALL MATMLY (C,FRED,DISP,5,5,5)
DO 647 K=1,5
KK=5*(N-1)+K
647 J(KK)=DISP(K,1)
645 CONTINUE
WRITE (3,777)
777 FORMAT(/1X5HISTORY,9XI11HTRANSLATION,8X12HHT VER.DISP.9X11HRT ROTAT ION,8X12HRT VER.DISP.9X11HRT ROTATION/) DO 20 I=1,N
DO 200 J=1,5
K=5*(I-1)
JL=K+J
200 FRED(J,1)=U(JL)/EM
20 WRITE (3,12) I,(FRED(J,1),J=1,5)
12 FORMAT(2X,14,5D20.1G)
WRITE (3,222)
222 FORMAT(/6X38HBENDING MOMENT AND SHEAR FORCE ON BEAMS)
WRITE (3,88)
SUBROUTINE MATINV(D,N,A)
DOUBLE PRECISION D,A,PIVDT,T
DIMENSION A(N,N),IPVDT(7),INDEX(7,2),PIVOT(7),D(N,N)

DO 21 I = 1,N
K = 5* (I-1)
TL = -U(K+3)
TR = -U(K+5)
DEL = U(K+2)-U(K+4)
XML = Q(K+1+2)-X11/2.* (2.*X14*TL+X16*TR-3.*X12*DELT/X1)
XMR = Q(K+1+4)-X11/2.* (X16*TL+2.*X15*TR-3.*X13*DELT/X1)
VM = (XML+XMR)/(XL+XL2+XL3)
VL = Q(K+1)+VM
VR = Q(K+3)-VM
21 WRITE (3,22) I,XML,XMR,VL,VR
22 FORMAT (2X,14,4020.10)

C CALCULATE INTERNAL FORCES ON LEFT COLUMNS
WRITE (3,233)
333 FORMAT (/6X51HBENDING MOMENT SHEAR AND AXIAL FORCE ON LEFT COLUMN)
WRITE (3,77)
DO 23 I = 1,N
K = 5* (I-1)
TT = -U(K+3)
TB = -U(K+8)
DEL = U(K+1)-U(K+6)
X20 = TT+TB-3.*DEL/H
XMT = -X5/2.*(X20+TT)
XMB = -X5/2.*(X20+TB)
VT = (XMT+XMB)/H
VB = -VT
AXIAL = X7* (U(K+7)-U(K+2))
23 WRITE (3,24) I,XMT,XMB,VT,VB,AXIAL

C CALCULATE INTERNAL FORCES ON RIGHT COLUMNS
WRITE (3,444)
444 FORMAT (/6X51HBENDING MOMENT SHEAR AND AXIAL FORCE ON RIGHT COLUMN)
WRITE (3,77)
DO 25 I = 1,N
K = 5* (I-1)
TT = -U(K+5)
TB = -U(K+10)
DEL = U(K+1)-U(K+6)
X20 = TT+TB-3.*DEL/H
XMT = -X6/2.*(X20+TT)
XMB = -X6/2.*(X20+TB)
VT = (XMT+XMB)/H
VB = -VT
AXIAL = X8* (U(K+9)-U(K+4))
25 WRITE (3,24) I,XMT,XMB,VT,VB,AXIAL
24 FORMAT (2X,14,5D20.10)
GO TO 501

STOP
END
DO 10 J = 1, N
DO 10 J = 1, N
10 A(I, J) = D(I, J)
DO 17 J = 1, N
17 IPVOT(J) = 0
DO 135 I = 1, N
T = 0.
DO 9 J = 1, N
IF(IPVOT(J) = 1) 13, S, 13
13 DO 23 K = 1, N
IF(IPVOT(K) = 1) 43, 23, 81
43 IF(DABS(T) < DABS(A(J, K))) 83, 23, 23
83 IROW = J
ICOL = K
T = A(J, K)
23 CONTINUE
9 CONTINUE
IPVOT(ICOL) = IPVOT(ICOL) + 1
IF(IROW = ICOL) 73, 1C9, 73
73 DO 12 L = 1, N
T = A(IROW, L)
A(IROW, L) = A(ICOL, L)
12 A(ICOL, L) = T
109 INDEX(I, 1) = IROW
INDEX(I, 2) = ICOL
PIVOT(I) = A(ICOL, ICOL)
A(ICOL, ICOL) = 1.
DO 205 L = 1, N
205 A(ICOL, L) = A(ICOL, L) / PIVOT(I)
347 DO 135 L = 1, N
IF(L = ICOL) 21, 135, 21
21 T = A(L, ICOL)
A(L, ICOL) = 0.
DO 89 L = 1, N
89 A(L, L) = A(L, L) - A(ICOL, L) * T
135 CONTINUE
222 DO 3 I = 1, N
L = N - I + 1
IF(INDEX(L, 1) = INDEX(L, 2)) 19, 3, 19
19 JROW = INDEX(L, 1)
JCOL = INDEX(L, 2)
DO 549 K = 1, N
T = A(K, JROW)
A(K, JROW) = A(K, JCOL)
A(K, JCOL) = T
549 CONTINUE
3 CONTINUE
81 RETURN
ENC
SUBROUTINE MATTRA(A, B, M, N)
MATRIX TRANSPOSE A TO B
DOUBLE PRECISION A, B
DIMENSION A(5, 5), B(5, 5)
DO 3 J = 1, N
DO 3 I = 1, M
3 B(J, I) = A(I, J)
SUBROUTINE MATSUB(A, B, C, M, N)

C MATRIX SUBTRACTION A-B=C

DOUBLE PRECISION A, B, C

DIMENSION A(5,5), B(5,5), C(5,5)

DO 3 J = 1, N
  DO 3 I = 1, M
    3 C(I, J) = A(I, J) - B(I, J)

RETURN
END

SUBROUTINE MATMLY(A, B, C, M, L, N)

C MATRIX MULTIPLICATION A*B=C  A=M BY L  B=L BY N

DOUBLE PRECISION A, B, C

DIMENSION A(5,5), B(5,5), C(5,5)

DO 3 J = 1, N
  DO 3 I = 1, M
    C(I, J) = 0.0
    DO 3 K = 1, L
      3 C(I, J) = C(I, J) + A(I, K) * B(K, J)

RETURN
END

/*/ 
// EXEC LNK EDT 
// EXEC 
*/
/*
&
EXEC FORTRAN

C THIS IS A PROGRAM FOR GENERAL ONE BAY FRAME ANALYSIS
C BY STIFFNESS METHOD AND RECURRENCE PROCEDURE
C PROGRAM MAY BE USED FOR FRAME OF WALLS OR OF COLUMNS OR OF BOTH
C PROGRAM CAN TAKE CARE OF VARIABLE HEIGHT, SPAN, MOMENT OF INERTIA.
C DOUBLE PRECISION S, B, C, D, FM, FR, FI, U, FS, Z1, Z2, Z3, Z4, Z5, TL, TR, DEL, XM
C DIMENSION S(5, 5), B(5, 5), C(5, 5, 5), D(5, 5), FM(5, 1), FR(5, 1), FI(5, 1), XP(5, 1), FER(4), UI(105), P(105), Q(105), F(5, 1), E(95), H(1), AL(20), AR(20), CL(20), CR(20), X(20), BL(20), S2(20), S3(20)
C DEFINE FILE 4 (140, 4G, L, ID)
C WRITE (3, 1)

1 FORMAT (1X30, 'GENERAL ONE BAY FRAME ANALYSIS')
C N= NUMBER OF STORY, NSL= NUMBER OF LOADED STORY, E= MODULUS OF ELASTICITY
2 READ (1, 4, ERR=51, END=51) N, NSL, E
ID=1
FIND (4, ID)
WRITE (3, 3) N, NSL, E
3 FORMAT (1X10, 'NO. OF STORY=', I5, ', NO. OF LOADED STORY=', I5, ', E=', F6.0/)
4 FORMAT (12, F6.0)
NN= 5*N+1
DO 5 I= 1, NN
5 U(I)= 0.
NN= 5*N
DO 6 I= 1, NN
6 P(I)= 0.
NN= 4*N
DO 7 I= 1, NN
7 Q(I)= 0.
DO 12 J= 1, 5
DO 12 S(I, J)= 0.
WRITE (3, 102)

102 FORMAT (1X6, 'LT COL., 5X10, 'RT COL., 7X8, 'MOI BEAM, 4X11, 'AREA LT COL., 4X11, 'BEAM LENGTH, 9X6, 'HEIGHT, 7X10, 'WALL DEPTH')
C H= STORY HEIGHT, AL= AREA OF LEFT COLUMN, AR= AREA OF RIGHT COLUMN
C CL= MOMENT OF INERTIA OF LT COLUMN, CR= MOMENT OF INERTIA OF RT COLUMN
C X= BEAM LENGTH, BL= MOMENT OF INERTIA OF BEAM, S2 AND S3 WALL DEPTH
DO 15 I= 1, N
READ (1, 13) CL(I), CR(I), BL(I), AL(I), AR(I), X(I), H(I), S2(I), S3(I)
15 WRITE (3, 14) CL(I), CR(I), BL(I), AL(I), AR(I), X(I), H(I), S2(I), S3(I)
13 FORMAT (3F10.0, F7.0)
14 FORMAT (1X, 3(5X, F10.0), 4(8X, F7.0), 2(5X, F7.0))
WRITE (3, 100)

100 FORMAT (1X5, 'STORY, 7X7, LOG P1, 7X7, LOG P2, 7X7, LOG P3, 7X7, LOG P4')
C LS= LOADED STORY LEVEL FROM TOP
C XP(K)= STORY LOAD, FER(J)= FIXED END REACTION ON STORY BEAM
C ORDER OF FER(J)= LT FESHEAR, LT FEM, RT FES, RT FEM.
DO 11 I= 1, NSL
READ (1, 18) LS, (XP(K), K= 1, 5), (FER(J), J= 1, 4)
WRITE (3, 17) LS, (XP(K), K= 1, 5), (FER(J), J= 1, 4)
CG 9 K = 1, 5
NN = 5*(LS - 1) + K
9 P(NN) = XP(K)

DO 10 J = 1, 4
NN = 4*(LS - 1) + J

10 Q(NN) = FER(J)

11 CONTINUE

8 FORMAT (I3, 9(6X, F12.3))
17 FORMAT (6X, 9(4X, F12.3))

Z1 = 1. + 2.*S2(T1)/X(1)
Z2 = 1. + 2.*S3(T1)/X(1)
Z3 = 1. + 3.*S2(T1)/X(1)*(1. + S2(T1)/X(1))
Z4 = 1. + 3.*S3(T1)/X(1)*((1. + S3(T1)/X(1))
Z5 = 1. + 3./X(1)*((S2(T1) + S3(T1) + 2.*S2(T1) + S3(T1))/X(1))
S(1, 1) = 12.*(C(1) + CR(1))/H(1)**3
S(3, 1) = 6.*C(1)/H(1)**2
S(5, 1) = 6.*CR(1)/H(1)**2

S(2, 2) = AL(1)/H(1) + 12.*BL(1)/X(1)**3
S(3, 2) = 6.*BL(1)/X(1)**2*Z1
S(4, 2) = -12.*BL(1)/X(1)**3
S(5, 2) = 6.*BL(1)/X(1)**2*Z2

S(3, 3) = 4.*(C(1)/H(1) + Z3*B(1)/X(1))
S(4, 3) = -6.*BL(1)/X(1)**2*Z1
S(5, 3) = 2.*BL(1)/X(1)**2*Z5
S(4, 4) = AR(1)/H(1) + 12.*BL(1)/X(1)**3
S(5, 4) = -6.*BL(1)/X(1)**2*Z2
S(5, 5) = 4.*(C(1)/H(1) + BL(1)/X(1)**2*Z4)

DO 16 J = 1, 4
NN = J + 1
16 S(J, 1) = S(1, J)
CALL MAT INV (S, 5, D)
DO 18 I = 1, 5
WRITE (4, 1D) (D(I, J), J = 1, 5)
FS(I) = P(I)
18 FR(I, 1) = P(I)
DO 27 I = 2, N
NN = 5*(I - 1)
17 N(1) = (I - 1)
DO 20 K = 1, 5
KK = NN + K
20 FI(K, 1) = P(KK)
Z1 = 1. + 2.*S2(T1)/X(1)
Z2 = 1. + 2.*S3(T1)/X(1)
Z3 = 1. + 3.*S2(T1)/X(1)*(1. + S2(T1)/X(1))
Z4 = 1. + 3.*S3(T1)/X(1)*((1. + S3(T1)/X(1))
Z5 = 1. + 3./X(1)*((S2(T1) + S3(T1) + 2.*S2(T1) + S3(T1))/X(1))
C(1, 1) = -12.*(C(N) + CR(N))/H(N)**3
C(3, 1) = -6.*C(N)/H(N)**2
C(5, 1) = -6.*CR(N)/H(N)**2
C(2, 2) = -AL(N)/H(N)
C(3, 3) = 2.*C(N)/H(N)
C(4, 4) = -AR(N)/H(N)
C(1, 5) = 6.*CR(N)/H(N)**2
C(5, 5) = 2.*CR(N)/H(N)
CALL MATTRA (C, S, 5, 5)
CALL MATMLY (S, D, B, 5, 5, 5)
CALL MATMLY (B, FR, FM, 5, 5, 5)
CALL MATSUB (F, FM, FR, 5, 5)
CALL MATMLY (B+C, D, 5, 5, 5)

DO 60 J = 1, 5
DO 60 K = 1, 5

60 B(K, J) = 0.
B(1, 1) = 12.*((CL(I)+CR(I))/H(I)**3+(CL(N1)+CR(N1))/H(N1)**3)
B(3, 1) = 6.*(CL(I)/H(I)**2-CL(N1)/H(N1)**2)
B(5, 1) = 6.*(CR(I)/H(I)**2-CR(N1)/H(N1)**2)
B(2, 2) = AL(N1)/H(N1)+AL(I)/H(I)+12.*BL(I)/X(I)**3
B(3, 2) = 6.*BL(I)/X(I)**2*Z1
B(4, 2) = -12.*BL(I)/X(I)**3
B(5, 2) = 6.*BL(I)/X(I)**2*Z2
B(3, 3) = -4.*((CL(N1)/H(N1)+CL(I)/H(I)+BL(I)/X(I)**3)
B(4, 3) = 6.*BL(I)/X(I)**2*Z1
B(5, 3) = 2.*BL(I)/X(I)**2*Z5
B(4, 4) = -AR(N1)/H(N1)+AR(I)/H(I)+12.*BL(I)/X(I)**3
B(5, 4) = -6.*BL(I)/X(I)**2*Z2
B(5, 5) = 4.*((CR(N1)/H(N1)+CR(I)/H(I)+BL(I)/X(I)**4)

DO 22 J = 1, 4
KK = J + 1
DO 22 K = KK, 5

22 B(J, K) = B(K, J)
CALL MATSUB (B, D, S, 5, 5)
CALL MATINV (S, 5, D)
IF (I-N) = 24, 28, 24

24 DO 25 K = 1, 5
WRITE (4, ID) (D(K, J), J = 1, 5)
KK = NN + K
25 FS(KK) = FR(K, 1)
27 CONTINUE
28 CALL MATMLY (D, FR, FM, 5, 5, 1)
K = 5*(N-1)
DO 29 J = 1, 5
KK = K + J
29 J(KK) = FM(J, 1)
DO 32 I = 2, N
N1 = N - I + 1
C(1, 1) = -12.*((CL(N1)+CR(N1))/H(N1)**3
C(3, 1) = 6.*(CL(N1)/H(N1)**2
C(5, 1) = 6.*(CR(N1)/H(N1)**2
C(2, 2) = AL(N1)/H(N1)
C(1, 3) = 6.*(CL(N1)/H(N1)**2
C(3, 3) = 2.*(CL(N1)/H(N1))
C(4, 4) = 2.*(CR(N1)/H(N1))
C(1, 5) = 6.*(CR(N1)/H(N1)**2
C(5, 5) = 2.*(CR(N1)/H(N1))
ID = 5*(N-I)+1
FIN0 (4, ID)

31 DO 32 I = 1, 5
READ (4, ID) (D(J, K), K = 1, 5)
KK = 5*(N-I)+J
31 F1(J, 1) = FS(KK)
CALL MATMLY (C, FM, FR, 5, 5, 1)
CALL MATSUB (FI, FP, FM, I1)
DO 33 J = 1, 5
33 FR(J, I) = FM(J, I)
CALL MATMLY (D, FR, FM, I1, I1)
DO 32 K = 1, 5
KK = 5* (N-I) + K
32 U(KK) = FM(K, I)
WRITE (3, 34)
34 FORMAT (// 1X HISTORY, 9X HISTOR TRANSLATION, 8X 12HRT VER DISP., 9X11HRT ROTA-
ITON, 8X12HRT VER DISP., 9X11HRT ROTA TION)
DO 36 I = 1, N
DO 35 J = 1, 5
KK = 5* (I-1) + J
35 FR(J, I) = U(KK) / E
36 WRITE (3, 37) I, (FR(J, I), J = 1, 5)
37 FORMAT (1X, I5, 5D2G.10)
C CALCULATE INTERNAL FORCES ON BEAMS
WRITE (3, 38)
38 FORMAT (// 1X5HISTORY, 9X5HISTORY, 9X5HISTORY, 9X5HISTORY, 9X5HISTORY, 9X5HISTORY, 9X5HISTORY, 9X5HISTORY, 9X5HISTORY, 9X5HISTORY)
DO 41 I = 1, N
K = 5* (I-1)
KK = 4* (I-1)
Z1 = 1.0 + 2.* S2(I) / X(I)
Z2 = 1.0 + 2.* S3(I) / X(I)
Z3 = 1.0 + 3.* S2(I) / X(I) * ( 1.0 + S2(I) / X(I) )
Z4 = 1.0 + 3.* S3(I) / X(I) * ( 1.0 + S3(I) / X(I) )
Z5 = 1.0 + 3.* X(I) * ( S2(I) + S3(I) + 2.* S2(I) * S3(I) ) / X(I)
TL = -U(K+3)
TR = -U(K+5)
DEL = U(K+2) - U(K+4)
XML = Q(KK+2) - 2.* BL(I) / X(I) * ( 2.* Z3 * TL + Z5 * TR - 3.* Z1 * DEL / X(I) )
XMR = Q(KK+4) - 2.* BL(I) / X(I) * ( 7.5 * TL + 2.* Z4 * TR - 3.* Z2 * DEL / X(I) )
VM = (XML + XMR) / (X(I) + S2(I) + S3(I))
VL = Q(KK+1) + VM
VR = Q(KK+3) - VM
41 WRITE (3, 42) I, XML, XMR, VL, VR
42 FORMAT (1X, I5, 4D18.10)
C CALCULATE INTERNAL FORCES ON LEFT COLUMNS
WRITE (3, 43)
43 FORMAT (// 6X51HISTOR SHEAR AND AXIAL FORCE ON LEFT COLUMN)
WRITE (3, 44)
44 FORMAT (1X, I5, 4D18.10)
K = 5* (I-1)
Z1 = -U(K+3)
Z2 = -U(K+8)
Z3 = U(K+1) - U(K+6)
Z4 = Z1 + Z2 - 3.* Z3 / H(I)
XML = -2.* CL(I) / H(I) * ( Z4 + Z1 )
XMR = -2.* CL(I) / H(I) * ( Z4 + Z2 )
VL = XML + XMR / H(I)
VR = -VL
XMLR = AL(I) / H(I) * ( U(K+7) - U(K+2) )
44 WRITE (3, 50) I, XML, XMR, VL, VR, XMLR
C CALCULATE INTERNAL FORCES ON RIGHT COLUMNS
WRITE (3, 47)
47 FORMAT (/6X52+BENDING MOMENT SHEAR AND AXIAL FORCE ON RIGHT COLUMN) WRITE (3, 48)
48 FORMAT (1X5 HISTORY, 9X7HB, M.TOP, 9X10HB, M.BOTTOM, 9X9HSHEAR TCP, 6X12HAXIAL FORCE)
DO 49 I=1,N
K = 5*(I-1)
Z1 = U(K+5)
Z2 = U(K+10)
Z3 = U(K+1)-U(K+6)
Z4 = Z1+Z2-2.*Z3/H(I)
XML = -2.*CRI/I/H(I)*(Z4+Z1)
XMR = -2.*CRI/I/H(I)*(Z4+Z2)
VL = XML + XMR/H(I)
VR = -VL
XMLR = AR(I)/H(I)*(U(K+9)-U(K+4))
WRITE (3, 50) I, XML, XMR, VL, VR, XMLR
50 FORMAT (IX5HSTORY, 9X7HB, M.TOP, 9X10HB, M.BOTTOM, 9X9HSHEAR TCP, 6X12HAXI )
GO TO 2
51 STOP
SUBROUTINE MATINV (D, N, A)
DOUBLE PRECISION D, A, PIVOT, T
DIMENSION A(N,N), PIVOT(71), INDEX(7,2), PIVOT(71), D(N,N)
DO 10 I = 1,N
DO 10 J = 1,N
10 A(I, J) = D(I, J)
DO 17 J = 1,N
17 IPVOT(J) = 0
DO 135 I = 1,N
135 T = 0.
DO 9 J = 1,N
9 IF (IPVOT(J) = 1) 13, 9, 13
13 DO 23 K = 1,N
23 IF (IPVOT(K) = 1) 43, 23, 81
43 IF (CABS(T) = CABS(A(J, K))) 83, 23, 23
83 IROW = J
ICOL = K
23 CONTINUE
9 CONTINUE
IPVOT(ICOL) = IPVOT(ICOL) + 1
IF (IROW = ICOL) 73, 135, 73
73 DO 12 L = 1,N
12 A(IROW, L) = T
DO 205 A(ICOL, L) = A(ICOL, L)/PIVOT(I)
205 DO 135 L = 1,N
135 IF (L = ICOL) 21, 135, 21
```plaintext
21 T = A(L, I, ICOL)
   A(L, I, ICOL) = 0.
   DO 89 L = 1, N
89   A(L, I, L) = A(L, I, L) - A(ICOL, L) * T
135 CONTINUE

222 DO 3 I = 1, N
   L = N - I + 1
   IF (INDEX(L, 1) - INDEX(L, 2)) 19, 3, 19

19    JROW = INDEX(L, 1)
   JCOL = INDEX(L, 2)
   DO 549 K = 1, N
   T = A(K, JROW)
   A(K, JROW) = A(K, JCOL)
   A(K, JCOL) = T
549 CONTINUE
3   CONTINUE
81 RETURN

ENC
SUBROUTINE MATMLYA, B, C, M, L, N
DOUBLE PRECISION A, B, C
C MATRIX MULTIPLICATION A * B = C  A=M BY L  B=L BY N
DIMENSION A(5, 5), B(5, 5), C(5, 5)
DO 3 J = 1, N
   DO 3 I = 1, M
      C(I, J) = 0.0
   DO 3 K = 1, L
      C(I, J) = C(I, J) + A(I, K) * B(K, J)
   RETURN
END

ENC
SUBROUTINE MATSUB(A, B, C, M, N)
DOUBLE PRECISION A, B, C
C MATRIX SUBTRACTION A - B = C
DIMENSION A(5, 5), B(5, 5), C(5, 5)
DO 3 J = 1, N
   DO 3 I = 1, M
      C(I, J) = A(I, J) - B(I, J)
   RETURN
END

ENC
SUBROUTINE MATTR(A, B, M, N)
DOUBLE PRECISION A, B
C MATRIX TRANSPOSE A TO B
DIMENSION A(5, 5), B(5, 5)
DO 3 J = 1, N
   DO 3 I = 1, M
      B(I, J) = A(I, J)
   RETURN
END

/*
// EXEC LNK EDT
// DLBL UOUT, 'IJSYS01'
// EXTENT SYS001, 1, 0, 40, 20
// EXEC CLRDSK
// UCL B=(K=0, D=40), X='CC', OY
// END
// DLBL IJSYS01
// EXTENT SYS001, 1, 0, 40, 20
```
// EXEC
/*
*/
Program this is a program for two bay frame analysis

by stiffness method and recursion procedure

program may be used for frame of walls or of columns or of both

double precision bo,b,c,a,ff,fred,di sp,d,e,f1,bi,u,fs,p,c,xp,fer,

im1,im2,im3,b11,b12,wa1,wa2,wa3,ml1,ml2,ml3,ml4,ml5,ml6,ml7,ml8,

2x5,x6,x7,x8,x9,x10,x11,x12,x13,x14,x15,x16,x17,x18,x19,x20,x21,x22

3x23,x24,x25,x26,x27,x28,tl,tr,rr,del,delr,x30,xm1,xmr,vl,vl,vr,

4xm1,xmrp,vmr,vlr,vrr,tt,bb,xmt,xmr,vb,axial,em

dimension bo(7,7),bi(7,7),c(7,7),a(7,7),ffred(7,1),disp(7,1),

1,ci(7,7),ei(7,7),bi(133,7),li(147),pi(140),ci(160),fi(133)

write (3,111)

111 format (6x66htwo bay frame analysis by stiffness matrix and recursion

procedure)

c

n=no. of story, wi1=moi lt col., wi2=moi ctr col., wi3=moi wall

c

bi1=moi lt beam, bi2=moi rt beam, wa1=area lt col.,

c

wa2=area ctr col., wa3=area wall, fm=modulus of elasticity

c

xl1=span lt beam, xl2=span rt beam, h=story height

501 read (1,1,err=99,end=99)n,nsload,wi1,wi2,wi3,bi1,bi2,wa1,wa2,wa3

write (3,101)

101 format (/6x17+beginning of data)

write (3,1) n,nsload,wi1,wi2,wi3,bi1,bi2,wa1,wa2,wa3

1 format (2f4.2,i7,i7,i7,i7)

read (1,30) em,xl1,xl2,h,s2,s3,s4,s5

if s2=0.1st wall is cltl. if s3=s4=0. center wall is ccl., and so on

write (3,30) em,xl1,xl2,h,s2,s3,s4,s5

30 format (8f9.2)

n1=7*(n+1)

n2=7*n

n3=8*n

do 4 i=1,n1

4 u(i)=0.

do 5 i=1,n2

5 p(i)=0.

do 6 i=1,n3

6 q(i)=0.

c

nsload=number of story loaded ls=loaded story level from top

c

xp(k)=load at each story, fer(j)=fixed end reaction on story beam

c

order of fer(j)=lt shear,lt fem,rt shear,rt fem. (for both sides)

do 7 i=1,nsload

read (1,3) ls,(xp(k),k=1,7)

write (3,3) ls,(xp(k),k=1,7)

read (1,30) (fer(j),j=1,8)

write (3,30) (fer(j),j=1,8)

do 8 k=1,7

i2=7*(ls-1)+k

8 pi(i2)=xp(k1)

do 13 j=1,8

12=8*(ls-1)+j1

13 qi(k1)=fer(j1)

7 continue

3 format (14,7f10.2)

do 50 j=1,7
DO 50 I = 1, 7
  B0(I, J) = 0.
  B(I, J) = 0.
50 C(I, J) = 0.
  X1 = 12.*W11/H**3
  X2 = 6.*W11/H**2
  X3 = 4.*W11/H
  X4 = W11/H
  X5 = 12.*W12/H**3
  X6 = 6.*W12/H**2
  X7 = 4.*W12/H
  X8 = W12/H
  X9 = 12.*W13/H**3
  X10 = 6.*W13/H**2
  X11 = 4.*W13/H
  X12 = W13/H
  X13 = 12.*B11/XL1**3
  X14 = 6.*B11/XL1**2
  X15 = 4.*B11/XL1
  X16 = 12.*B12/XL2**3
  X17 = 6.*B12/XL2**2
  X18 = 4.*B12/XL2
  X19 = 1.+2.*S2/XL1
  X20 = 1.+2.*S3/XL1
  X21 = 1.+3.*S3/XL1*(1.+S2/XL1)
  X22 = 1.+3.*S4/XL1*(S2+S3+2.*S2*S3/XL1)
  X23 = 1.+2.*S4/XL2
  X24 = 1.+2.*S5/XL2
  X25 = 1.+3.*S5/XL2*(1.+S3/XL1)
  X26 = 1.+3.*S4/XL2*(1.+S4/XL2)
  X27 = 1.+3.*S4/XL2*(S4+S5+2.*S4*S5/XL2)
  X28 = 1.+3.*S5/XL2*(1.+S5/XL2)
  BO(1, 1) = X1+X5+X9
  BO(3, 1) = X2
  BO(5, 1) = X6
  BO(7, 1) = X10
  BO(3, 2) = X14-X19
  BO(4, 2) = -X13
  BO(5, 2) = X14-X20
  BO(3, 3) = X3+X15*X21
  BO(4, 3) = -X14-X19
  BO(5, 3) = X15/2.*X22
  BO(4, 4) = X8+X13+X16
  BO(5, 4) = X17*X23-X14*X20
  BO(6, 4) = -X16
  BO(7, 4) = X17*X24
  BO(5, 5) = X7+X15*X25+X18*X26
  BO(6, 5) = -X17*X23
  BO(7, 5) = X18/2.*X27
  BO(6, 6) = X12+X16
  BO(7, 6) = -X17*X24
  BO(7, 7) = X11+X18*X28
  C(1, 1) = -(X1+X5+X9)
  C(3, 1) = -X2
  C(5, 1) = X6
C(1, 1) = -X10
C(1, 2) = -X4
C(1, 3) = X2
C(3, 3) = X3/2
C(4, 4) = -X8
C(4, 5) = X6
C(5, 5) = X7/2
C(6, 6) = -X12
C(7, 7) = X10

B(1, 1) = 2.5 * (X4 + X5 + X6)
B(2, 2) = 2.5 * X4 * X13
B(3, 2) = X14 * X15
B(4, 2) = -X13
B(5, 2) = X14 * X20
B(1, 3) = 2.5 * X3 + X15 * X21
B(4, 3) = -X14 * X19
B(5, 3) = X15/2 * X22
B(4, 4) = 2 * X8 + X13 + X16
B(5, 4) = X17 * X23 - X14 * X20
B(6, 4) = -X16
B(7, 4) = X17 * X24
B(5, 5) = 2 * X7 + X15 * X25 + X16 * X26
B(6, 5) = -X17 * X23
B(7, 5) = X18/2 * X27
B(6, 6) = 2 * X12 + X16
B(7, 6) = -X17 * X24
B(7, 7) = 2 * X11 + X18 * X28

DO 51 J = 1, 6
J = J + 1
DO 52 I = J, 7
B(I, J) = B(I, J)
51 CONTINUE
CALL MATT (C, A, 7, 7)
CALL MATTINV (B, 7, D)
DO 666 J = 1, 7
DO 666 I = 1, 7
666 B(I, J) = D(I, J)
CALL MATT (A, D, E, 7, 7, 7)
DO 9 I = 1, 7
FS(I) = P(I)
9 FRED(I, 1) = P(I)
DO 53 I = 2, N
I4 = 7 * (I - 1)
DO 54 K = 1, 7
IK = I4 + K
54 FI(K, 1) = P(IK)
CALL MATT (E, FRED, FM, 7, 7, 1)
CALL MATSUB (FI, FM, FRED, 7, 7, 1)
CALL MATT (E, C, D, 7, 7, 7)
CALL MATSUB (B, D, E, 7, 7)
CALL MATTINV (E, 7, D)
IF (I = N) 10, 11, 18
10 CALL MATT (A, D, E, 7, 7, 7)
DO 541 J = 1, 7
\begin{verbatim}
IJ = I4 + J

541 FS(IJ) = FRED(J, 1)
DO 888 J = 1, 7
DO 888 L = 1, 7
IL = I4 + L
888 B1(IL, J) = D(L, J)
53 CONTINUE

11 CALL MATMLY (D, FRED, DISP, 7, 7, 1)
K = 7* (N-1)
DO 14 J = 1, 7
KJ = K + J
14 U(KJ) = DISP(J, 1)
DO 645 I = 2, N
DO 998 J = 1, 7
DO 998 K = 1, 7
KJ = 7* (N-1) + K
998 D(K, J) = B1(KJ, J)
DO 646 J = 1, 7
JJ = 7* (N-1) + J
646 FI(J, I) = FS(IJ)

CALL MATMLY (C, DISP, FM, 7, 7, 1)
CALL MATSUB (FI, FM, FRED, 7, 1)
CALL MATMLY (D, FRED, DISP, 7, 7, 1)
DO 647 K = 1, 7
KK = 7* (N-1) + K
647 J(KK) = DISP(K, 1)
645 CONTINUE

WRITE (3, 777)
777 FORMAT (/XSELECTORY, 6X11HTRANSLATION, 6X11HLT VER DISP, 6X11HLT ROTA
1ITION, 5X12HCTR VER DISP, 5X12HCTR ROTATION, 6X11HT VER DISP, 6X11HT
2ROTATION/) ,
DO 20 I = 1, N
DO 200 J = 1, 7
K = 7* (I-1)
JL = K + J
200 FRED(J, 1) = U(JL) / EM
20 WRITE (3, 12) (I, FRED(J, 1), J = 1, 7)
12 FORMAT (2X, 14, /017.10)
C CALCULATE INTERNAL FORCES ON BEAMS
WRITE (3, 222)
222 FORMAT (/X43X39HBENDING MOMENT AND SHEAR FORCE ON BEAMS)
WRITE (3, 555)
555 FORMAT (26X15HLEFT SIDE BEAMS, 44X16HRIGHT SIDE BEAMS)
WRITE (3, 88)
88 FORMAT (H1H, 5HSTORY, 6XSHB.M. LEFT, 5X10SHB.M. RIGHT, 5X10HSHEAR LEFT, 4
1X11HSHEAR RIGHT, 6X9HB.M. LEFT, 5X10HB.M. RIGHT, 5X10HSHEAR LEFT, 4X11
2HSHEAR RIGHT/) ,
DO 21 I = 1, N
K = 7* (I-1)
K1 = 8* (I-1)
TL = -U(K+3)
TR = -U(K+5)
TRR = -U(K+7)
DEL = U(K+2) - U(K+4)
DELR = U(K+4) - U(K+6)
XML = Q(TK+2)-X1572.*(2.*X21*TL+X22*TR-3.*X19*DEL/XL1)
\end{verbatim}
$XMR = Q(K1 + 4) - X15/2 * (X22 + TL + 2) * X25 * TR - 3 * X20 * DE / XL1)$

$VM = (XML + XMR) / (XL1 + S2 + S3)$

$VL = Q(K1 + 1) + VM$

$VR = Q(K1 + 3) - VM$

$XMLR = Q(K1 + 6) - X18/2 * (2. * X26 + TR + X27 * TR - 3. * X23 * DE / XL1)$

$XMLR = Q(K1 + 8) - X18/2 * (X27 * TR + 2. * X28 * TR - 3. * X24 * DE / XL1)$

$VMR = (XMLR + XMRR) / (XL2 + S4 + S5)$

$VL = Q(K1 + 5) + VMR$

$VRR = Q(K1 + 7) - VMMR$

21 WRITE (3, 22) I, XML, XMR, VL, VR, XMLR, XMRR, VLR, VRR

22 FORMAT (2X, I4, E015.8)

C CALCULATE INTERNAL FORCES ON LEFT COLUMNS

WRITE (3, 223)

333 FORMAT (1/6X51HBENDING MOMENT SHEAR AND AXIAL FORCE ON LEFT COLUMN)

WRITE (3, 77)

DO 23 I = 1, N

K = 7 * (I - 1)

TT = -U(K + 3)

TB = -U(K + 10)

DEL = U(K + 1) - U(K + 8)

X30 = TT + TB - 3. * DEL / H

XMT = -X3/2. * (X30 + TT)

XMLB = -X3/2. * (X30 + TB)

VT = (XMT + XMIB) / H

VB = -VT

AXIAL = X8#(U(K + 9) - U(K + 2))

23 WRITE (3, 24) I, XMT, XMLB, VT, VB, AXIAL

C CALCULATE INTERNAL FORCES ON CENTER COLUMNS

WRITE (3, 890)

890 FORMAT (1/6X52HBENDING MOMENT SHEAR AND AXIAL FORCE ON CENTER COLUMN)

WRITE (3, 77)

DO 26 I = 1, N

K = 7 * (I - 1)

TT = -U(K + 5)

TB = -U(K + 12)

DEL = U(K + 1) - U(K + 8)

X30 = TT + TB - 3. * DEL / H

XMT = -X7/2. * (X30 + TT)

XMLB = -X7/2. * (X30 + TB)

VT = (XMT + XMIB) / H

VB = -VT

AXIAL = X8#(U(K + 11) - U(K + 4))

26 WRITE (3, 24) I, XMT, XMLB, VT, VB, AXIAL

C CALCULATE INTERNAL FORCES ON RIGHT COLUMNS

WRITE (3, 444)

444 FORMAT (1/6X52HBENDING MOMENT SHEAR AND AXIAL FORCE ON RIGHT COLUMN)

WRITE (3, 77)

77 FORMAT (1H, 5HISTORY, 6X8HB.M. TOP, 8X11HB.M. BOTTOM, 8X9HSHEAR TOP, 8X1
12HSHEAR BOTTOM, 6X11HAXIAL FORCE $)

DO 25 I = 1, N

K = 7 * (I - 1)

TT = -U(K + 7)

TB = -U(K + 14)

DEL = U(K + 1) - U(K + 8)

X30 = TT + TB - 3. * DEL / H

XMT = -X11/2. * (X30 + TT)
\[ XMB = -X^{11/2} \times (X^{30} + TB) \]
\[ VT = (XMT + XMB) / T \]
\[ VB = -VT \]
\[ AXIAL = X^{12} \times (U(K + 12) - U(K + 6)) \]

25 WRITE (3, 24) I, XMT, XMB, VT, VB, AXIAL
24 FORMAT (2X, 14, 5D18.10)
GO TO 501

99 STOP

ENC

SUBROUTINE MATINV (D, N, A)
DOUBLE PRECISION D, A, PIVOT, T
DIMENSION A(N, N), PIVOT(N), INDEX(7, 2), PIVOT(7), D(N, N)
DO 10 J = 1, N
DO 10 I = 1, N
10 A(I, J) = D(I, J)
DO 17 J = 1, N
17 IPVOT(J) = 0
DO 135 I = 1, N
T = 0.
DO 9 J = 1, N
IF(IPVOT(J) - 1) 12, 6, 13
12 IPVOT(J) = 0.
DO 23 K = 1, N
IF(IPVOT(K) - 1) 42, 23, 81
43 IF (DABS(T) - DABS(A(J, K))) 83, 23, 23
63 IROW = J
ICOL = K
T = A(J, K)
23 CONTINUE
9 CONTINUE
IPVOT(ICOL) = IPVOT(ICOL) + 1
IF(IROW - ICOL) 73, 1CS, 73
73 DO 12 L = 1, N
T = A(I ROW, L)
A(I ROW, L) = A(ICOL, L)
12 A(ICOL, L) = T
109 INDEX(I, 1) = IROW
INDEX(I, 2) = ICOL
PIVOT(I) = A(ICOL, ICOL)
A(ICOL, ICOL) = 1.
DO 205 L = 1, N
205 A(ICOL, L) = A(ICOL, L) / PIVOT(I)
347 DO 135 L = 1, N
IF(L - ICOL) 21, 135, 21
21 T = A(L, ICOL)
A(L, ICOL) = 0.
DO 89 L = 1, N
89 A(L, L) = A(L, L) - A(ICOL, L) * T
135 CONTINUE
222 DO 3 I = 1, N
L = N - I + 1
IF(INDEX(L, 1) - INDEX(L, 2)) 15, 3, 19
19 JROW = INDEX(L, 1)
JCOL = INDEX(L, 2)
DO 549 K = 1, N
T = A(K, JROW)
A(K, JROW) = A(K, JCOL)
A(K, J COL) = T

549 CONTINUE
3 CONTINUE
81 'RETURN
ENC
SUBROUTINE MATTRA(A, B, M, N)
C MATRIX TRANSPOSE A TO B
DOUBLE PRECISION A, B
DIMENSION A(7, 7), B(7, 7)
DO 3 J = 1, N
DO 3 I = 1, M
3 B(J, I) = A(I, J)
RETURN
ENC
SUBROUTINE MATMULY(A, B, C, M, L, N)
C MATRIX MULTIPLICATION A * B = C  A = M BY L  B = L BY N
DOUBLE PRECISION A, B, C
DIMENSION A(7, 7), B(7, 7), C(7, 7)
DO 3 J = 1, N
DO 3 I = 1, M
C(I, J) = 0.0
DO 3 K = 1, L
3 C(I, J) = C(I, J) + A(I, K) * B(K, J)
RETURN
ENC
SUBROUTINE MATSUB(A, B, C, M, N)
C MATRIX SUBTRACTION A - B = C
DOUBLE PRECISION A, B, C
DIMENSION A(7, 7), B(7, 7), C(7, 7)
DO 3 J = 1, N
DO 3 I = 1, M
3 C(I, J) = A(I, J) - B(I, J)
RETURN
ENC
/
// EXEC LINK EDT
// EXEC
/*
*/
C THIS IS A PROGRAM FOR GENERAL TWO BAY FRAME ANALYSIS
C BY STIFFNESS METHOD AND RECURSION PROCEDURE
C PROGRAM MAY BE USED FOR FRAME OF WALLS OR OF COLUMNS OR OF BOTH
C PROGRAM CAN TAKE CARE OF VARIABLE HEIGHT, SPAN, MOMENT OF INERTIA.
1, CR, E, BL, BR, X, Y, S2, S3, S4, S5, Z1, Z2, Z3, Z4, Z5, Z6, Z7, Z8, Z9, Z10, X11, TL,
2TR, TRR, DEL, DLR, XM, XMR, VM, VR, XM(R), XMRR, VR, VR, VRR
DIMENSION S(7,7), B(7,7), C(7,7), D(7,7), F(7,7), P(140), Q(160), F(133),
2H(20), AL(20), AM(20), AR(20), CL(20), CM(20), CR(20)
DEFINE FILE4 (140, 56, 1, ID)
WRITE (3, 1)
1 FORMAT (1X30=GENERAL TWO BAY FRAME ANALYSIS)
C N=NUMBER OF STORY, NSL=NUMBER OF LOADED STORY, E=MODU. OF ELASTICITY
2 READ (1, 4) ERR, N, NSL, E
   ID=1
   FIND (4, ID)
   WRITE (3, 3) N, NSL, E
3 FORMAT (/1X10HNO. OF STY=, I3, 9X17HNO. OF STY LOADED=, I3, 5X2HE=, F6.0/)
4 FORMAT (2I2, F6.0)
   NN=7*N
   DO 5 I=1, NN
      U(I)=0.
      NN=7*N
   DO 6 I=1, NN
      P(I)=0.
      NN=8*N
   DO 7 I=1, NN
      Q(I)=0.
      C LS=LOADED STORY LEVEL FROM TOP
      C XP(K)=STORY LOAD, FER(J)=FIXED END REACTION ON STORY BEAM
      C ORDER OF FER(J)=LT FESHEAR, LT FEM, RT FES, RT FEM, (FER BOTH BEAMS)
   WRITE (3, 200)
   200 FORMAT (1H, 5H=STORY, 4X7HLOAD P1, 4X7HLOAD P2, 4X7HLOAD P3, 4X7HLOAD P4,
   4X7HLOAD P5, 4X7HLOAD P6, 4X7HLOAD P7)
   DO 9 I=1, NSL
      READ (1, 8) LS, (XP(K), K=1, 7)
      WRITE (3, 201) LS, (XP(K), K=1, 7)
   DO 9 K=1, 7
      NN=7*(LS-1)*K
      P(NN)=XP(K)
   8 FORMAT (13, 7F11.2)
   WRITE (3, 202)
   202 FORMAT (/18X14HLEFT SIDE BEAM, 30X15HRIGHT SIDE BEAM)
   WRITE (3, 203)
   203 FORMAT (1X5H=STORY, 3X6HFES LT, 3X6HFEM LT, 3X6HFES RT, 3X6HFEM RT, 13X6
   1HFES LT, 3X6HFEM LT, 3X6HFES RT, 3X6HFEM RT)
   DO 10 I=1, NSL
      READ (1, 11) LS, (FER(J), J=1, 8)
      WRITE (3, 204) LS, (FER(J), J=1, 8)
   DO 10 J=1, 8
   10 CONTINUE
   WRITE (3, 205) LS, (FER(J), J=1, 8)
5 CONTINUE
NN=-8*(LS-I)+J
10 Q=NN=FET(IJ)
11 FORMAT (13,8F9.1)
204 FORMAT (1X,15,4F5.1,10X,4F9.1)
   DO 12 J=1,7
   DO 12 I=1,7
      S(I,J)=0.
12 C(I,J)=0.
C   NN=STORY HEIGHT, AL=AREA OF LEFT COL., AM=AREA OF CENTER COLUMN
C   AR=AREA OF RIGHT COLUMN, CL=MOM. OF INERTIA OF LT COL.
C   CM=MOM. OF CENTER COLUMN, CR=MOM. OF RIGHT COLUMN
WRITE (3,205)
205 FORMAT (/1X#HEIGHT, 3X11HAREA LT COL,3X12HAREA CTR COL,3X11HAREA RT
   1 COL,4X10HMOI LT COL,3X11HM01 CTR COL,4X10HM01 RT COL)
   DO 15 I=1,N
      READ (1,13) F(I),AL(I),AM(I),AR(I),CL(I),CM(I),CR(I)
15 WRITE (3,206) H(I),AL(I),AM(I),AR(I),CL(I),CM(I),CR(I)
13 FORMAT (F6.1,F6.1,F11.1)
206 FORMAT (1X,F6.1,6G(3X,F11.1))
C   BL AND BR=MOM. I. OF LT AND RT BEAM. X AND Y=SPAN LT AND RT BEAM.
C   S2,S3,S4,S5=DEPTH OF WALL FROM C.G. (FOR COLUMNS THEY ARE ZERO)
WRITE (3,207)
207 FORMAT (/1X#SECTION PROPERTIES OF BEAMS AND WALL DEPTH)
WRITE (3,208)
208 FORMAT (4X11HMOI LT BEAM,3X11HM01 RT BEAM,3X12HS
   1PM RT BEAM,10X42H WALL DEPTH FROM C.G. OF WALL LEFT TO RIGHT)
   READ (1,14) BL,BR,X,Y,S2,S3,S4,S5
   WRITE (3,209) BL,BR,X,Y,S2,S3,S4,S5
14 FORMAT (2F10.1,6F6.1)
209 FORMAT (1X,4F3X,F11.1),10X,4F11.1)
Z1=1.+2.*S2/X
Z2=1.+2.*S3/X
Z3=1.+2.*S4/Y
Z4=1.+2.*S5/Y
ZS=1.+3.*S2/X*(1.+S2/X)
Z6=1.+3.*S3/X*(1.+S3/X)
Z7=1.+3.*S4/Y*(1.+S4/Y)
Z8=1.+3.*S5/Y*(1.+S5/Y)
Z9=1.+3./*S2+S3+2.*S2*S3/X)
Z10=1.+3./*S4+S5+2.*S4+S5/Y)
S(1,1)=12.*(CL(1)+CM(1)+CR(1))/H(1)**3
S(3,1)=6.*(CL(1))/H(1)**2
S(5,1)=6.*(CM(1))/H(1)**2
S(7,1)=6.*(CR(1))/H(1)**2
S(2,2)=AL(1)/H(1)+12.*BL/X**3
S(3,2)=Z1*6.*BL/X**2
S(4,2)=-12.*BL/X**3
S(5,2)=Z2*6.*BL/X**2
S(3,3)=4.*CL(1)/H(1)+2*S4*4.*BL/X
S(4,3)=Z1*6.*BL/X**2
S(5,3)=Z9*2.*BL/X
S(4,4)=AM(1)/H(1)+12.*(BL/X**3+BR/Y**3)
S(5,4)=6.*(Z2*BR/Y**2-Z2*BL/X**2)
S(6,4)=-12.*BR/Y**2
S(7,4)=Z4*6.*BR/Y**3
S(5,5)=4.*CM(1)/H(1)+Z6*4.*BL/X+Z7*4.*BR/Y
S(6,5) = -Z3*6.*BR/Y**2
S(7,5) = Z10*2.*BR/Y
S(6,6) = AR(1)/H(1)+12.*BR/Y**3
S(7,6) = -Z4*6.*BR/Y**2
S(7,7) = 4.*CR(1)/H(1)+Z8*4.*BR/Y

DO 16 J=1,6
NN=J+1
DO 16 I=NN,7
DO 16 S(J, J)=S(I,J)
CALL MATINV (S, 7, D)
DO 18 I=1,7
WRITE (4, ID)(D(I,J), J=1,7)
FS(I)=P(I)
DO 27 I=2, N
READ (1, 14) BL, BR, X, Y, S2, S3, S4, S5
WRITE (3, 209) BL, BR, X, Y, S2, S3, S4, S5
NN=7*(I-1)
N1=(I-1)
DO 20 K=1,7
KK=NN+K

20 F(I, K, 1)=P(KK)
Z1=1.+2.*S2/X
Z2=1.+2.*S3/X
Z3=1.+2.*S4/Y
Z4=1.+2.*S5/Y
Z5=1.+3.*S2/X*(1.+S2/X)
Z6=1.+3.*S3/X*(1.+S3/X)
Z7=1.+3.*S4/Y*(1.+S4/Y)
Z8=1.+3.*S5/Y*(1.+S5/Y)
Z9=1.+3./X*(S2+S3+2.*S2*S3/X)
Z10=1.+3./Y*(S4+S5+2.*S4*S5/Y)
Z11=(CL(1)+CM(1)+CR(1))/H(1)**3+(CM(N1)+CM(N1)+CR(N1))/H(N1)**3
C(1, 1) = -12.*(CL(N1)+CM(N1)+CR(N1))/H(N1)**2
C(3, 1) = -6.*CL(N1)/H(N1)**2
C(5, 1) = -6.*CM(N1)/H(N1)**2
C(7, 1) = -6.*CR(N1)/H(N1)**2
C(2, 2) = -AL(N1)/H(N1)
C(1, 3) = 6.*CL(N1)/H(N1)**2
C(3, 3) = 2.*CL(N1)/H(N1)
C(4, 4) = -AM(N1)/H(N1)
C(1, 5) = 6.*CM(N1)/H(N1)**2
C(5, 5) = 2.*CM(N1)/H(N1)
C(6, 6) = -AR(N1)/H(N1)
C(1, 7) = 6.*CR(N1)/H(N1)**2
C(7, 7) = 2.*CR(N1)/H(N1)
CALL MATTRA (C, S, 7, 7)
CALL MATMLY (S, D, B, 7, 7)
CALL MATMLY (B, FR, FM, 7, 7, 1)
CALL MATSUB (FI, FM, FR, 7, 1)
CALL MATMLY (B, C, D, 7, 7, 7)
DO 100 J=1,7
DO 100 K=1,7

100 B(K, J)=0.
B(1,1)=12.*Z11
B(3,1)=6.*(CL(1)/H(1)**2-CL(N1)/H(N1)**2)
B(5, 1) = 6.* (CM(1)/H(1)**2 - CM(N1)/H(N1)**2)
B(7, 1) = 6.* (CR(1)/H(1)**2 - CR(N1)/H(N1)**2)
B(2, 2) = AL(N1)/H(N1) + AL(1)/H(1) + 12.* BL/X**2
B(3, 2) = Z 1*6.* BL/X**2
B(4, 2) = -12.* BL/X**2
B(5, 2) = Z 2*6.* BL/X**2
B(3, 3) = 4.* (CL(N1)/H(N1) + CL(1)/H(1) + Z 5* BL/X)
B(4, 3) = -Z 1*6.* BL/X**2
B(5, 3) = Z 9*2.* BL/X
B(4, 4) = AM(N1)/H(N1) + AM(1)/H(1) + 12.* (BL/X**3 + BR/Y**3)
B(5, 4) = 6.* (Z 3* BR/Y**2 - Z 2* BL/X**2)
B(6, 4) = -12.* BR/Y**2
B(7, 4) = Z 4*6.* BR/Y**2
B(5, 5) = 4.* (CM(N1)/H(N1) + CM(1)/H(1) + Z 6* BL/X + Z 7* BR/Y)
B(6, 5) = 4.* Z 6* BR/Y**2
B(7, 5) = Z 10*2.* BR/Y
B(6, 6) = AR(N1)/H(N1) + AR(1)/H(1) + 12.* BR/Y**3
B(7, 6) = 4.* (CR(N1)/H(N1) + CR(1)/H(1) + Z 8* BR/Y)
DO 22 J = 1, 7
J = J + 1
DO 22 K = KK, 7
J = J + 1
DO 25 K = 1, 7
WRITE (4, ID) (D(K, J), J = 1, 7)
DO 25 K = 1, 7
WRITE (4, ID) (D(K, J), J = 1, 7)
DO 25 K = 1, 7
WRITE (4, ID) (D(K, J), J = 1, 7)
DO 25 K = 1, 7
WRITE (4, ID) (D(K, J), J = 1, 7)
DO 25 K = 1, 7
WRITE (4, ID) (D(K, J), J = 1, 7)
DO 25 K = 1, 7
WRITE (4, ID) (D(K, J), J = 1, 7)
DO 25 K = 1, 7
WRITE (4, ID) (D(K, J), J = 1, 7)
DO 25 K = 1, 7
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DO 25 K = 1, 7
WRITE (4, ID) (D(K, J), J = 1, 7)
DO 25 K = 1, 7
WRITE (4, ID) (D(K, J), J = 1, 7)
DO 25 K = 1, 7
WRITE (4, ID) (D(K, J), J = 1, 7)
DO 25 K = 1, 7
WRITE (4, ID) (D(K, J), J = 1, 7)
DO 25 K = 1, 7
WRITE (4, ID) (D(K, J), J = 1, 7)
DO 25 K = 1, 7
WRITE (4, ID) (D(K, J), J = 1, 7)
DO 25 K = 1, 7
WRITE (4, ID) (D(K, J), J = 1, 7)
DO 25 K = 1, 7
WRITE (4, ID) (D(K, J), J = 1, 7)
DO 25 K = 1, 7
31 F(J,1)=FS(KK)
CALL MATMLY (C,FM,FR,7,7,1)
CALL MATSUB (FI,FR,FM,7,1)
DO 33 J=1,7
33 FR(J,1)=FM(J,1)
CALL MATMLY (D,FR,FM,7,7,1)
DO 32 K=1,7
KK=7*(N-1)+K
32 U(KK)=FM(K,1)
WRITE (3,34)
34 FORMAT (/1X5-STORY,6X11HTRANSLATION,5X12H LT VER DISP.,6X11H LT ROTATION,4X13HCTR VER DISP.,5X12HCTR ROTATION,5X12HRT VER DISP.,5X11HRT ROTATION/) DO 36 I=1,N DO 35 J=1,7
K=7*(I-1)
KK=K+J
35 FR(J,1)=U(KK)/E
36 WRITE (3,37) I,(FR(J,1),J=1,7)
37 FORMAT (3X,13,7D17.10)
C CALCULATE INTERNAL FORCES ON BEAMS
WRITE (3,38)
38 FORMAT (/21X39HBENDING MOMENT AND SHEAR FORCE ON BEAMS
WRITE (3,21C)
210 FORMAT (20X25HEFT SIDE BEAM,35X27HIGHT SIDE BEAM)
WRITE (3,40)
40 FORMAT (1X5-STORY,8X7HB,M. LT,8X7HB,M. RT,7X8HSHEAR LT,7X8HSHEAR RT,10X7HB,M. LT,8X7HB,M. RT,7X8HSHEAR LT,7X8HSHEAR RT)
DO 41 I=1,N
READ (1,14) BL,BR,X,Y,S2,S3,S4,S5
K=7*(I-1)
KK=8*(I-1)
Z1=1.+2.*S2/X
Z2=1.+2.*S3/X
Z3=1.+2.*S4/Y
Z4=1.+2.*S5/Y
Z5=1.+3.*S2/X/(1.+52/X)
Z6=1.+3.*S3/X/(1.+53/X)
Z7=1.+3.*S4/Y*(1.+54/Y)
Z8=1.+3.*S5/Y*(1.+55/Y)
Z9=1.+3./X*(S2+S3+2.*S2*S3/X)
Z10=1.+3./Y*(S4+S5+2.*S4*S5/Y)
TL=-U(K+3)
TR=-U(K+5)
TRR=-U(K+7)
DEL=U(K+2)-U(K+4)
DELR=U(K+4)-U(K+6)
XML=Q(KK+2)-2.*BL/X*(2.*Z5*TL+Z9*TR-3.*Z1*DEL/X)
XMR=Q(KK+4)-2.*BL/X*(Z9*TL+2.*Z6*TR-3.*Z2*DELR/X)
VM=XML*XMR/(X+S2+S3)
+ VIL=Q(KK+1)+VM
VR=Q(KK+3)-VM
XML=Q(KK+6)-2.*BR/Y*(2.*Z7*TR+Z10*TRR-3.*Z3*DELR/Y)
XMR=Q(KK+8)-2.*BR/Y*(Z10*TR+2.*Z8*TRR-3.*Z4*DELR/Y)
VMR=(XMLR*XMRR)/(Y+S4+S5)
C CALCULATE INTERNAL FORCES ON LEFT COLUMNS
Write (3, 43)

43 Format (/6x51:BENDING MOMENT SHEAR AND AXIAL FORCE ON LEFT COLUMN)
Write (3, 48)

Do 44 I = 1, N
K = 7* (I - 1)
Z1 = -U (K + 3)
Z2 = -U (K + 10)
Z3 = U (K + 1) - U (K + 8)
XML = -2. * CR (I) / H (I) * (Z4 + Z1)
XMR = -2. * CR (I) / H (I) * (Z4 + Z2)
VL = (XML + XMR) / H (I)
VR = -VL
XMLR = AR (I) / H (I) * (U (K + 9) - U (K + 2))

44 Write (3, 50) I, XML, XMR, VL, VR, XMLR
C CALCULATE INTERNAL FORCES ON CENTER COLUMN
Write (3, 45)

45 Format (/6x52:BENDING MOMENT SHEAR AND AXIAL FORCE ON CENTER COLUMN)
Write (3, 48)

Do 46 I = 1, N
K = 7* (I - 1)
Z1 = -U (K + 5)
Z2 = -U (K + 12)
Z3 = U (K + 1) - U (K + 8)
XML = -2. * CM (I) / H (I) * (Z4 + Z1)
XMR = -2. * CM (I) / H (I) * (Z4 + Z2)
VL = (XML + XMR) / H (I)
VR = -VL
XMLR = AM (I) / H (I) * (U (K + 11) - U (K + 4))

46 Write (3, 50) I, XML, XMR, VL, VR, XMLR
C CALCULATE INTERNAL FORCES ON RIGHT COLUMNS
Write (3, 47)

47 Format (/6x52:BENDING MOMENT SHEAR AND AXIAL FORCE ON RIGHT COLUMN)
Write (3, 48)

48 Format (/1x5:STOR Y, 9X7HB. M TOP, 9X1CHB. M BOTTOM, 9X9H SHEAR TCP, 7X12HS
1 HEAR BOTTOM, 7X1) AXIAL FORCE)
Do 49 I = 1, N
K = 7* (I - 1)
Z1 = -U (K + 7)
Z2 = -U (K + 14)
Z3 = U (K + 1) - U (K + 8)
XML = -2. * CR (I) / H (I) * (Z4 + Z1)
XMR = -2. * CR (I) / H (I) * (Z4 + Z2)
VL = (XML + XMR) / H (I)
VR = -VL
XMLR = AR (I) / H (I) * (U (K + 13) - U (K + 6))

49 Write (3, 50) I, XML, XMR, VL, VR, XMLR
50 Format (/3X, 13, 5D18.10)
Go to 2
SUBROUTINE MATINV (D, N, A)
DOUBLE PRECISION D, A, PIVOT, T
DIMENSION A(N, N), IPVOT(7), INDEX(7, 2), PIVOT(7), D(N, N)

DO 10 I = 1, N
DO 10 J = 1, N
10 A(I, J) = D(I, J)
DO 17 J = 1, N
17 IPVOT(J) = 0
DO 135 I = 1, N
T = 0.
DO 9 J = 1, N
IF (IPVOT(J) - 1) 13, 9, 13
13 DO 23 K = 1, N
IF (IPVOT(K) - 1) 43, 23, 81
43 IF (DABS(T) - DABS(A(J, K))) 83, 23, 23
83 IRW = J
ICOL = K
T = A(J, K)
23 CONTINUE
9 CONTINUE
IPVOT(ICOL) = IPVOT(ICOL) + 1
IF (IRW - ICOL) 73, 139, 73
73 DO 12 L = 1, N
T = A(IRW, L)
A(IRW, L) = A(ICOL, L)
A(ICOL, L) = T
12 INDEX(1, 1) = IRW
INDEX(1, 2) = ICOL
PIVOT(I) = A(ICOL, ICOL)
A(ICOL, ICOL) = 1.
DO 205 L = 1, N
205 A(ICOL, L) = A(ICOL, L) / PIVOT(I)
347 DO 135 L = 1, N
IF (L - ICOL) 21, 135, 21
21 T = A(L, ICOL)
A(L, ICOL) = 0.
DO 89 L = 1, N
89 A(L, 1) = A(L, 1) - A(ICOL, L) * T
135 CONTINUE
222 DO 3 I = 1, N
L = N - I + 1
IF (INDEX(L, 1) - INDEX(L, 2)) 19, 3, 19
19 JROW = INDEX(L, 1)
JCOL = INDEX(L, 2)
DO 549 K = 1, N
T = A(K, JROW)
A(K, JROW) = A(K, JCOL)
A(K, JCOL) = T
549 CONTINUE
3 CONTINUE
81 RETURN
END
SUBROUTINE MATTRA(A, B, M, N)
C MATRIX TRANSPOSE A TO B
DOUBLE PRECISION A,B
DIMENSION A(M,N),B(M,N)
DO 3 J=1,N
DO 3 I=1,M
3 B(J,I)=A(I,J)
RETURN
END
SUBROUTINE MATMLY(A,B,C,M,L,N)
DOUBLE PRECISION A,B,C
DIMENSION A(M,L),B(L,N),C(M,N)
DO 3 J=1,N
DO 3 I=1,M
C(I,J)=0.0
DO 3 K=1,L
3 C(I,J)=C(I,J)+A(I,K)*B(K,J)
RETURN
END
SUBROUTINE MATSUB(A,B,C,M,N)
DOUBLE PRECISION A,B,C
DIMENSION A(M,N),B(M,N),C(M,N)
DO 3 J=1,N
DO 3 I=1,M
3 C(I,J)=A(I,J)-B(I,J)
RETURN
END

/*
// EXEC LNK EDT
// DL BL UOUT, 'IJSYS01'
// EXTENT SYS001, 1, 0, 40, 20
// EXEC CLRDSK
// UCL B=(K=0, D=56), X 'CC', DY
// ENC
// DL BL IJSYS01
// EXTENT SYS001, 1, 0, 40, 20
// EXEC
*/
/*
*/
DOUBLE PRECISION B0, B1, B2, C, A, F, E, D, H, X, X1, X2, X3, X4, X5, X6, X7, X8, X9, X10, X11, X12, X13, X14, X15, X16
DIMENSION B0 (5, 5), BI (5, 5), C (5, 5), A (5, 5), F (5, 5), D (5, 5), H (5, 5), XH (5, 5), X (5, 5), X (5, 5), X (5, 5)

WRITE (3, 1)
1 FORMAT (/IX35CALCULATION OF GA FOR ONE BAY FRAME)
501 READ (1, 2, ERR=99, END=99) N, XI1, XI2, XI3, XL, H, A1, A2, EM, X12, X13
WRITE (3, 8) N, XI1, XI2, XI3, XL, H, A1, A2, EM, X12, X13
2 FORMAT (15, 3F8.0, 7F7.1)
8 FORMAT (/IX5, 15, 3F8.0, 7F7.1)

C IF XL 2=0, LEFT WALL IS A COLUMN. IF XL 3=0, RIGHT WALL IS A COLUMN.

X1 = X11 / H * X13 / XL
X2 = 1. + 2. * X11 / X1
X3 = 1. + 2. * X12 / X1
GA = 12. * EM / H**2 * (X11 / X2 + X12 / X3)
WRITE (3, 3) GA
3 FORMAT (/IX41HGA VALUE BY HEIDEBRECHT AND SMITH METH. = D17, 10/)
N1 = 5# (N + 1)
N2 = 5# N
DO 4 I = 1, N1
4 U (I) = 0.
DO 5 I = 1, N2
5 P (I) = 0.
DO 50 J = 1, 5
C (I, J) = 0.
50 X1 = 12. * X11 / H**2
X2 = 12. * X12 / H**2
X3 = 6. * X11 / H**2
X4 = 6. * X12 / H**2
X5 = 4. * X11 / H
X6 = 4. * X12 / H
X7 = A1 / H
X8 = A2 / H
X9 = 12. * X13 / XL**2
X10 = 6. * X13 / XL**2
X11 = 4. * X13 / XL
X12 = 1. + 2. * XL 2 / XL
X13 = 1. + 2. * XL 2 / XL
X14 = 1. + 3. * XL 2 / XL * (1. + XL 2 / XL)
X15 = 1. + 3. * XL 2 / XL * (1. + XL 3 / XL)
X16 = 1. + 3. * XL 2 / XL * (1. + XL 3 / XL)
BO (1, 1) = X1 + X2
BO (3, 1) = X3
BO (5, 1) = X4
BO (2, 2) = X7 + X9
BO (3, 2) = X10 + X12
BO (4, 2) = X9
BO (5, 2) = X10 + X13
DO 54 K = 1, 5
   IK = 14 + K
   CALL MATMLY (E, FRED, FM, 5, 5, 5)
   CALL MATSUB (F1, FM, FRED, 5, 1)
   CALL MATMLY (E, G, D, 5, 5, 5)
   CALL MATSUB (B, D, F, 5, 5)
   CALL MATINV (E, 5, D)
   IF (1 - N) 10, 11, 1C
10 CALL MATMLY (A, D, E, 5, 5, 5)
   DO 541 J = 1, 5
      IJ = 14 + J
   CALL MATMLY (A, D, E, 5, 5, 5)
      DO 888 J = 1, 5
DO 888 L = 1, 5
IL = I4 + L
888 BI(IL, J) = D(L, J).
53 CONTINUE
11 CALL MATMLY (D, FRED, DISP, 5, 5, 1)
   K = 5* (N - 1)
   DO 14 J = 1, 5
   KJ = K + J
14 U(KJ) = DISP(J, 1)
   DO 647 I = 2, N
   DO 998 J = 1, 5
   KJ = K + J
998 D(K, J) = BI(KJ, J)
   DO 646 J = 1, 5
   JJ = 5* (N - 1) + J
646 FI(J, 1) = FS(J, J)
   CALL MATMLY (C, DISP, FM, 5, 5, 1)
   CALL MATSUB (FI, FM, FRED, 5, 1)
   CALL MATMLY (D, FRED, DISP, 5, 5, 1)
   DO 647 K = 1, 5
   KK = 5* (N - 1) + K
647 U(KK) = DISP(K, 1)
   AN = N
   GA = AN* I* FM/U(1)
   WRITE (3, 7) GA
7 FORMAT (17X25F10.10)
777 FORMAT (/***TRANSLATION OF DIFFERENT STORY FROM TCP***)
    DO 20 I = 1, N
    K = 5* (I - 1) + 1
20 Q(I) = U(K) + EM
    WRITE (3, 24) (Q(I), I = 1, N)
24 FORMAT (1X, 7D18.10)
    GO TO 501
99 STOP
END

SUBROUTINE MATINV (D, N, A)
DOUBLE PRECISION D, A, PIVOT, T
DIMENSION A(N, N), IPVOT(7), INDEX(7, 2), PIVOT(7), D(N, N)
DO 10 I = 1, N
   DO 10 J = 1, N
10 A(I, J) = D(I, J)
   DO 17 J = 1, N
17 IPVOT(J) = 0
   DO 135 I = 1, N
   T = 0.
   DO 9 J = 1, N
   IF (IPVOT(J) - 1) 13, 9, 9
9 IF (IPVOT(J) - 1) 43, 23, 81
13 DO 23 K = 1, N
   IF (IPVOT(K) - 1) 43, 23, 81
43 IF (CABS(T) - CABS(A(J, K))) 83, 23, 23
83 IROW = J
   ICOL = K
   T = A(I, K)
23 CONTINUE
CONTINUE
IPVOT(icol) = IPVOT(icol) + 1
IF (icol == icol) 73, 1C9, 73
73 DO 12 L = 1, N
  T = A(icol, L)
  A(icol, L) = A(icol, L)
  12 A(icol, L) = T
109 INDEX(I, 1) = IROW
INDEX(I, 2) = icol
PIVOT(I) = A(icol, icol)
A(icol, icol) = 1.
DO 205 L = 1, N
205 A(icol, L) = A(icol, L) / PIVOT(I)
347 DO 135 L = 1, N
  IF (L - icol) 21, 135, 21
  21 T = A(L, icol)
  A(L, icol) = 0.
  DO 89 L = 1, N
  89 A(L, L) = A(L, L) - A(icol, L) * T
  135 CONTINUE
222 DO 3 I = 1, N
  L = N - I + 1
  IF (INDEX(L, 1) - INDEX(L, 2)) 15, 3, 19
19 IROW = INDEX(L, 1)
  JCOL = INDEX(L, 2)
  DO 549 K = 1, N
    T = A(K, JROW)
    A(K, JROW) = A(K, JCOL)
    A(K, JCOL) = T
  549 CONTINUE
3 CONTINUE
81 RETURN
END
SUBROUTINE MATTRA(A, B, M, N)
MATRIX TRANSPOSE A TO B
DOUBLE PRECISION A, B
DIMENSION A(5, 5), B(5, 5)
DO 3 J = 1, N
  DO 3 I = 1, M
  3 B(J, I) = A(I, J)
RETURN
END
SUBROUTINE MATMLY(A, B, C, M, L, N)
MATRIX MULTIPLICATION A*B = C  A=M BY L  B=L BY N
DOUBLE PRECISION A, B, C
DIMENSION A(5, 5), B(5, 5), C(5, 5)
DO 3 J = 1, N
  DO 3 I = 1, M
    C(I, J) = 0.0
  3 CONTINUE
3 C(I, J) = C(I, J) + A(I, K) * B(K, J)
RETURN
END
SUBROUTINE MATSUB(A, B, C, M, N)
MATRIX SUBTRACTION A - B = C
DOUBLE PRECISION A, B, C
DIMENSION A(5,5), B(5,5), C(5,5)
DO 3 J = 1, N
DO 3 I = 1, M
3 C(I, J) = A(I, J) - B(I, J)
RETURN
END

EXEC LNK EDT
EXEC
/*
DOUBLE PRECISION A0,B,C,A,FM,FRED,DISP,D,E,F1,BI,W,FS,P,C,GA,EM,
1W11,W12,W13,B11,B12,WA1,WA2,WA3,FL1,FL2,H,S2,S3,S4,S5,X1,X2,X3,X4,
2X5,X6,X7,X8,X9,X10,X11,X12,X13,X14,X15,X16,X17,X18,X19,X20,X21,X22
3,X23,X24,X25,X26,X27,X28

DIMENSION BO(7,7),B(7,7),C(7,7),A(7,7),FM(7,1),FRED(7,1),DISP(7,1)
1,D(7,7),E(7,7),F1(7,1),B1(133,7),U(147),P(140),FS(133),Q(20)

WRITE (3,1)
1 FORMAT (/1X35H"CALCULATION OF GA FOR TWO BAY FRAME")
501 READ(1,2,ERR=99,END=99)N,W11,W12,W13,B11,B12,X11,X12,H,EM
2 FORMAT (15,5F8.0)
8 FORMAT (/1X,15,9F8.0)

C
IF S2=0.1ST WALL IS COL.,IF S3=S4=0.CENTER WALL IS COL., AND SO ON
READ (1,3)WA1,WA2,WA3,S2,S3,S4,S5
WRITE (3,3)WA1,WA2,WA3,S2,S3,S4,S5
3 FORMAT (5X,7F8.0)

X1=H*B11/XL1
X2=1.+2.*WI1/X1
X3=H*B12/XL2
X4=1.+2.*WI3/X3
X5=1.+2.*WI2/(XL1*XL2)

GA=12.*EM/H**2*(WI1/X2+W12/X5+W13/X4)
WRITE (3,6)GA
6 FORMAT (/1X41H"GA VALUE BY HEIDEBR ECHT AND SMITH METHOD CD=0.017,10/

N1=7*(N+1)
N2=7*N
4 DO 1 I=1,N1
2 U(I)=0.
4 DO 5 I=1,N2
5 \*P(I)=0.
5 P(I)=1.
50 \*C(I,J)=0.
50 DO 50 J=1,7
50 \*B(I,J)=0.
50 \*C(I,J)=0.

X1=12.*WI1/H**3
X2=6.*WI1/H**2
X3=4.*WI1/H
X4=WA1/H
X5=12.*WI2/H**3
X6=6.*WI2/H**2
X7=4.*WI2/H
X8=WA2/H
X9=12.*WI3/H**3
X10=6.*WI3/H**2
X11=4.*WI3/H
X12=WA3/H
X13=12.*BI1/XL1**3
X14=6.*BI1/XL1**2
X15=4.*BI1/XL1
X16=12.*BI2/XL2**3
X17=6.*BI2/XL2**2
\[ X_{18} = 4 \cdot X_{12} / X_{12} \]
\[ X_{19} = 1 \cdot 2 \cdot X_{2} / X_{12} \]
\[ X_{20} = 1 \cdot 2 \cdot X_{3} / X_{12} \]
\[ X_{21} = 1 \cdot 3 \cdot X_{2} / X_{12} \cdot (1 \cdot S_{2} / X_{11}) \]
\[ X_{22} = 1 \cdot 3 \cdot X_{1} / X_{2} \cdot (S_{2} + S_{3} + 2 \cdot S_{2} / S_{3} / X_{11}) \]
\[ X_{23} = 1 \cdot 2 \cdot X_{4} / X_{2} \]
\[ X_{24} = 1 \cdot 2 \cdot X_{5} / X_{2} \]
\[ X_{25} = 1 \cdot 3 \cdot X_{3} / X_{12} \cdot (1 \cdot S_{2} / X_{12}) \]
\[ X_{26} = 1 \cdot 3 \cdot X_{4} / X_{2} \cdot (1 \cdot S_{4} / X_{2}) \]
\[ X_{27} = 1 \cdot 3 \cdot X_{2} / X_{2} \cdot (S_{4} + S_{5} + 2 \cdot S_{4} \cdot S_{5} / X_{2}) \]
\[ X_{28} = 1 \cdot 3 \cdot X_{3} / X_{12} \cdot (1 \cdot S_{5} / X_{2}) \]

BO(1, 1) = X_{14} \cdot X_{5} + X_{9}
BO(3, 1) = X_{2}
BO(5, 1) = X_{6}
BO(7, 1) = X_{10}
BO(2, 2) = X_{4} + X_{12}
BO(3, 2) = X_{14} \cdot X_{15}
BO(4, 2) = -X_{13}
BO(5, 2) = X_{14} \cdot X_{20}
BO(3, 3) = X_{13} \cdot X_{15} \cdot X_{21}
BO(4, 3) = -X_{14} \cdot X_{15}
BO(5, 3) = X_{15} / 2 \cdot X_{22}
BO(4, 4) = X_{8} \cdot X_{13} \cdot X_{16}
BO(5, 4) = X_{17} \cdot X_{23} - X_{14} \cdot X_{20}
BO(6, 4) = -X_{16}
BO(7, 4) = X_{17} \cdot X_{24}
BO(5, 5) = X_{7} + X_{15} \cdot X_{25} \cdot X_{16} \cdot X_{26}
BO(6, 5) = -X_{17} \cdot X_{23}
BO(7, 5) = X_{18} / 2 \cdot X_{27}
BO(6, 6) = X_{12} \cdot X_{16}
BO(7, 6) = -X_{17} \cdot X_{24}
BO(7, 7) = X_{11} \cdot X_{18} \cdot X_{28}
C(1, 1) = -(X_{1} \cdot X_{5} + X_{9})
C(3, 1) = -X_{2}
C(5, 1) = -X_{6}
C(7, 1) = -X_{10}
C(2, 2) = -X_{4}
C(1, 3) = X_{2}
C(3, 3) = X_{3} / 2.
C(4, 4) = -X_{8}
C(1, 5) = X_{6}
C(5, 5) = X_{7} / 2.
C(6, 6) = -X_{12}
C(1, 7) = X_{10}
C(7, 7) = X_{11} / 2.
B(1, 1) = 2 \cdot (X_{1} \cdot X_{5} + X_{9})
B(2, 2) = 2 \cdot X_{4} \cdot X_{13}
B(3, 2) = X_{14} \cdot X_{15}
B(4, 2) = -X_{13}
B(5, 2) = X_{14} \cdot X_{20}
B(3, 3) = 2 \cdot X_{3} + X_{15} \cdot X_{21}
B(4, 3) = -X_{14} \cdot X_{19}
B(5, 3) = X_{15} / 2 \cdot X_{22}
B(4, 4) = 2 \cdot X_{8} + X_{13} \cdot X_{16}
B(5, 4) = X_{17} \cdot X_{23} - X_{14} \cdot X_{20}
B(6, 4) = -X_{16}
\begin{verbatim}
B(7,4) = X17*X24
B(5,5) = 2*X7+X15*X25+X18*X26
B(6,5) = -X17*X23
B(7,5) = X18/2*X27
B(6,6) = 2*X12+X16
B(7,6) = -X17*X24
B(7,7) = 2*X11+X18*X28
DO 52 J = 1, 6
   J1 = J + 1
   DO 52 I = J1, 7
      B(J, I) = B(I, J)
   DO 52
52 B(J, I) = B(I, J)
   CALL MATTRIA (C, A, 7, 7)
   CALL MAT INV (B0, 7, D)
   DO 666 J = 1, 7
   666 B(J, I) = A(J, I)
   CALL MATML Y (A, D, E, 7, 7, 7)
   CALL MATSUB (E, FRED, FM, 7, 7, 7)
   CALL MATML Y (E, FRED, FM, 7, 7, 7)
   CALL MAT INV (E, 7, 7)
   IF (I-N) > 10, II, IC
   10 CALL MATML Y (A, D, E, 7, 7, 7)
   DO 541 J = 1, 7
      IJ = I + J
   541 B(IJ, I) = FRED(I, J)
   DO 888 J = 1, 7
53 CONTINUE
   11 CALL MATML Y (D, FRED, DISP, 7, 7, 7)
   K = 7*(N-1)
   DO 14 J = 1, 7
      KJ = K + J
14 J(KJ) = DISP(J, I)
   DO 647 I = 2, N
   DO 998 J = 1, 7
   DO 998 K = 1, 7
      KJ = 7*(N-1)+K
998 D(K, J) = B(KJ, J)
   DO 646 J = 1, 7
      JJ = 7*(N-1)+J
646 F(J, I) = FS(IJ)
   CALL MATML Y (C, DISP, FM, 7, 7, 7)
   CALL MATSUB (F, FM, FRED, 7, 7)
\end{verbatim}
CALL MAINLY (I, FRED, DISP, 7, 7, 1)
DO 647 K = 1, 7
KK = 7*(N-1)+K
647 U(KK) = DISP(K, 1)
AN = N
GA = AN*K*EM/U(1)
WRITE (3, 7) GA
7 FORMAT (17X25F6.1) WRITE (3, 777)
777 FORMAT (1X39) TRANSLATION OF DIFFERENT STORY FROM TCP
DO 20 I = 1, N
K = 7*(I-1)+1
20 Q(I) = U(K)/EM
WRITE (3, 24) (Q(I), I = 1, N)
24 FORMAT (1X, 7D18.1C}
GO TO 501
99 STOP
END
SUBROUTINE MATINV (D, N, A)
DOUBLE PRECISION D, A, PIVOT, T
DIMENSION A(N, N), PIVOT(7), INDEX(7, 2), PIVOT(7), D(N, N)
DO 10 I = 1, N
DO 10 J = 1, N
10 A(I, J) = D(I, J)
DO 17 J = 1, N
17 PIVOT(J) = 0
DO 135 I = 1, N
T = 0.
DO 9 J = 1, N
IF (PIVOT(J) - 1) 13, 5, 13
13 DO 23 K = 1, N
IF (PIVOT(K) - 1) 43, 23, 81
43 IF (DABS(T) - DABS(A(J, K))) 83, 23, 23
83 IROW = J
ICOL = K
T = A(J, K)
CONTINUE
23 CONTINUE
9 CONTINUE
PIVOT(ICOL) = PIVOT(ICOL) + 1
IF (IROW - ICOL) 72, 1C5, 73
73 DO 12 L = 1, N
T = A(IROW, L)
A(IROW, L) = A(ICOL, L)
12 A(ICOL, L) = T
INDEX(I, 1) = IROW
INDEX(I, 2) = ICOL
PIVOT(I) = A(ICOL, ICOL)
A(ICOL, ICOL) = 1.
DO 205 L = 1, N
205 A(ICOL, L) = A(ICOL, L)/PIVOT(I)
347 DO 135 L = 1, N
IF (L - ICOL) 21, 135, 21
21 T = A(L, ICOL)
A(L, ICOL) = 0.
DO 89 L = 1, N
89 A(L, L) = A(L, L) - A(ICOL, L) * T
CONTINUE
DO 3 I = 1, N
   L = N - I + 1
   IF (INDEX(L, 1) = INDEX(L, 2)) 19, 3, 19
19 JROW = INDEX(L, 1)
   JCOL = INDEX(L, 2)
   DO 549 K = 1, N
      T = A(K, JROW)
      A(K, JROW) = A(K, JCOL)
      A(K, JCOL) = T
549 CONTINUE
3 CONTINUE
RETURN

SUBROUTINE MATTRA(A, B, M, N)
C MATRIX TRANSPOSE A TO B
DOUBLE PRECISION A, B
DIMENSION A(7, 7), B(7, 7)
DO 3 J = 1, N
   DO 3 I = 1, M
      3 B(J, I) = A(I, J)
   RETURN
END

SUBROUTINE MATMLY(A, B, C, M, L, N)
C MATRIX MULTIPLICATION A*B=C A=M BY L B=L BY N
DOUBLE PRECISION A, B, C
DIMENSION A(7, 7, 7), B(7, 7, 7), C(7, 7)
DO 3 J = 1, N
   DO 3 I = 1, M
      C(I, J) = 0.0
   DO 3 K = 1, L
      3 C(I, J) = C(I, J) + A(I, K) * B(K, J)
   RETURN
END

SUBROUTINE MATSUB(A, B, C, M, N)
C MATRIX SUBTRACTION A-B=C
DOUBLE PRECISION A, B, C
DIMENSION A(7, 7, 7), B(7, 7, 7), C(7, 7)
DO 3 J = 1, N
   DO 3 I = 1, M
      C(I, J) = A(I, J) - B(I, J)
   RETURN
END

/*
// EXEC LINKEDT
// EXEC
/*
*/
EXEC FORTRAN

C APPROXIMATE WALL-FRAME ANALYSIS BY HEIDEBRECHT AND SMITH METHOD
C N=NO. OF STORY, NSL=NO. OF STORY LOADED, WI1=MOI LT COL.,
C W12=MOI CTR COL., WI3=MOI RT WALL
C B11=MOI LT BEAM, B12=MOI RT BEAM, WA1=AREA LT COL.,
C A2=AREA CTR COL., WA3=AREA WALL, EM=MODULUS OF ELASTICITY
C XL1=SPAN LT BEAM, XL2=SPAN RT BEAM, H=STORY HEIGHT
C S2=S3=S4=0. S5=WALL DEPTH FROM C.G. TO BEAM END
C DOUBLE PRECISION WI1, WI2, WI3, B11, B12, WA1, WA2, WA3, EM, XL1, XL2, H, S2,
C S3, S4, S5, Z1, Z2, Z3, Z4, Z5, GA, A SQ, A TH, AH, SINH AH, COSH AH, CONS, XMO, X, Y,
C ZAX, SIN HAX, COS HAX, XMB, XMS, SB, SS, Q
C WRITE (3,1)
1 FORMAT (6X46,FRAME ANALYSIS BY HEIDEBRECHT AND SMITH METHOD/)
C READ (1,4,ERR=15,END=15) N,NSL,W11,W12,W13,B11,B12,WA1,WA2,WA3
C 4 FORMAT (2I4,2F11.0,2F9.0,3F7.0)
C WRITE (3,5) N,NSL,W11,W12,W13,B11,B12,WA1,WA2,WA3
C 5 FORMAT (1X,2I5,2F15.4)
C READ (1,7) EM, XL1, XL2, H, S2, S3, S4, S5
C 7 FORMAT (8F9.2)
C WRITE (3,8) EM, XL1, XL2, H, S2, S3, S4, S5
C 8 FORMAT (11X,8F15.4)
C Z1=H*B11/XL1
C Z2=1.+Z2.*W11/Z1
C Z3=H*B12/XL2
C Z4=1.+Z2.*W12/(Z1+Z3)
C GA=12.*EM/H**2*(W11/2+W12/2)
C ASQ=GA/(EM*W13)
C A=CSQRT(ASQ)
C AN=N
C TH=AN*T
C AH=A**T
C WRITE (3,9) GA, A, AH
C 9 FORMAT (1X3=GA=,D17.10,9X6HALPHA=,D17.10,8X7HALPHA=,D17.10)
C Z1=DEXP(AH)
C Z2=DEXP(-AH)
C SINH AH=(Z1-Z2)/2.
C COS HAH=(Z1+Z2)/2.
C CONS=(1.+AH**2)*SINH AH/COSH AH
C XMO=MOMENT IN. AT BASE DUE TO UNIT LATERAL U.D.L.
C X MO=T H**2/2.
C X=TH
C N1=N+1
C WRITE (3,10)
C 10 FORMAT (7X1I1=STORY LEVEL, 9X1I1=TRANSLATION, 7X13H, W, FLEX. BEAM, 6X14
C 1H, B, M, SHEAR BEAM, 5X15H, SHEAR FLEX. BEAM, 4X16H, SHEAR SHEAR BEAM, 3X17H)
C 2TRACTION FORCE)
C DO 11 I=1, N1
C AX=A*X
C Z1=DEXP(AX)
C Z2=DEXP(-AX)
C SINH AX=(Z1-Z2)/2.
C COS HAX=(Z1+Z2)/2.
C Z3=EM*W13*A**4
C Z4=X/TH**(1.-X/(2.*TH))
\[ Y = 1.0 \times (12.2 \times 2.3) + \text{CONS} \times (2 \times \text{SHA} \times 1.0) - \text{AH} \times \text{SINHA} \times 2^4 \times \text{AH}^2 \]

\[ XMB = 2.0 \times \text{XM} \times \text{AH} \times 2^4 \times (\text{CONS} \times \text{CO} \times \text{SHA} \times \text{AH} \times \text{SINHA}) \]

\[ Z5 = 1.0 \times X/T \]

\[ XMS = XMO + Z5 \times 2^2 - XMB \]

\[ SB = 1.0 \times (12.2 \times 2.3) + (\text{AH} \times \text{CO} \times \text{SHA} \times \text{CONS} \times \text{SINHA} \times X) \]

\[ SS = \text{TH} \times Z5 \times 2^2 - SB \]

\[ Q = XMB \times \text{AH} \times 2^4 \times (\text{XM} \times XMO) + 1 \]

\[ X = X - P \]

11 WRITE (3,12) 11, Y, XMB, XMS, SB, SS, Q

12 FORMAT (1X,14,6D20.10)

WRITE (3,13)

13 FORMAT (1X,18+END OF CALCULATION//)

GO TO 2

15 STOP

END

/ *

/ * EXEC LNK EDT

/ * EXEC