

BANGLADESH UNIVERSITY OF ENGINEERING AND TECHNOLOGY, DHAKA

L-2/T-2 B.Sc. Engineering Examinations 2012-2013

Sub : **EEE 269** (Electrical Drives and Instrumentation)

Full Marks : 210

Time : 3 Hours

The figures in the margin indicate full marks.

USE SEPARATE SCRIPTS FOR EACH SECTIONSECTION – AThere are **FOUR** questions in this Section. Answer any **THREE**.

If you think any data is missing in a problem, make a reasonable assumption and state it in your solution.

1. (a) Explain why a transformer is an integral part of a regulated power supply. Discuss the role of open circuit test and short circuit test in determining transformer parameters. Why is the supply typically connected to the LT side for the open circuit test and to the HT side for the short circuit test? (15)

- (b) A 20 kVA 8000/277-V distribution transformer has the following resistance and reactance parameters: (20)

$$R_p = 32 \Omega \quad R_s = 0.05 \Omega$$

$$X_p = 45 \Omega \quad X_s = 0.06 \Omega$$

$$R_c = 250 \text{ k}\Omega \quad X_m = 30 \text{ k}\Omega$$

The  $R_c$  and  $X_m$  parameters are provided referred to the high tension side.

- (i) Find the equivalent circuits of this transformer referred to the high tension side.  
 (ii) Assume that the transformer is supplying rated load at 277 V and 0.8 PF lagging. What is this transformer's input voltage? Calculate its voltage regulation.  
 (iii) What is the transformer's efficiency under the conditions of part (ii)?

2. (a) With diagrams, explain why reversal of conductor current is required in a dc motor to produce continuous rotation. How is this achieved in a commercial dc motor? Name three factors which can be used to regulate the speed of a dc motor. Do you need to take any precaution while manipulating them? (17)

- (b) A 220 V dc shunt motor has armature and field resistances of  $0.2 \Omega$  and  $220 \Omega$ , respectively. The load torque of the motor follows the relationship  $T_L \propto n^2$ , where  $n$  is the running speed. Initially, the motor is running at 1000 rpm drawing 10 A current from the supply. Calculate the new speed and armature current if an external armature resistance of  $5 \Omega$  is inserted in the armature circuit. Neglect armature reaction and saturation. (18)

3. (a) Derive the relationship between line current and phase voltage for a balanced delta-wye three phase system. Why a balanced three-phase circuit can be analyzed by an equivalent single-phase circuit? Draw the delta-wye system's per-phase equivalent circuit and justify it. (18)

**EEE 269**

**Contd ... Q. No. 3**

(b) A balanced three-phase source is supplying power to a three-phase load. The specifications of the loads are given below:

Load 1: 30 kVA at 0.8 pf lagging

Load 2: 24 kW at 0.6 pf leading

Load 3: 30 unknown

If the line voltage and total complex power at the load are 208 V rms and  $60\angle 0^\circ$  kVA, respectively, find the rating of the unknown load. (17)

4. (a) Two wattmeters are employed to measure the power of a three-phase system as shown below: (17)

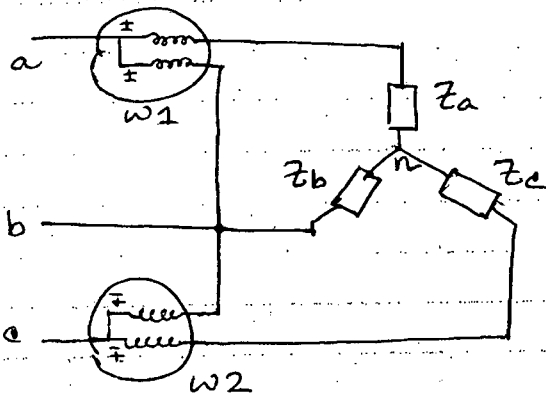


Fig. for Q. 4(a).

For this particular system,

$$V_{an} = 220 \angle 0^\circ \text{ V}, V_{bn} = 220 \angle -120^\circ \text{ V}, V_{cn} = 220 \angle 120^\circ \text{ V}.$$

$$I_{an} = 20 \angle -20^\circ \text{ A}, I_{bn} = 20 \angle -140^\circ \text{ A}, I_{cn} = 20 \angle 100^\circ \text{ A}.$$

Find the expected readings of wattmeters  $w_1$  and  $w_2$ . If one of them gives a downscale deflection, what can be done to reverse the situation?

(b) For the three-phase network shown below, find out the currents  $I_a$ ,  $I_b$ , and  $I_c$ . Also calculate the power delivered to the line and load components. Assume, (18)

$$Z_\Delta = 12 - 15j, Z_Y = 4 + 6j \Omega, \text{ and } Z_l = 2 \Omega.$$

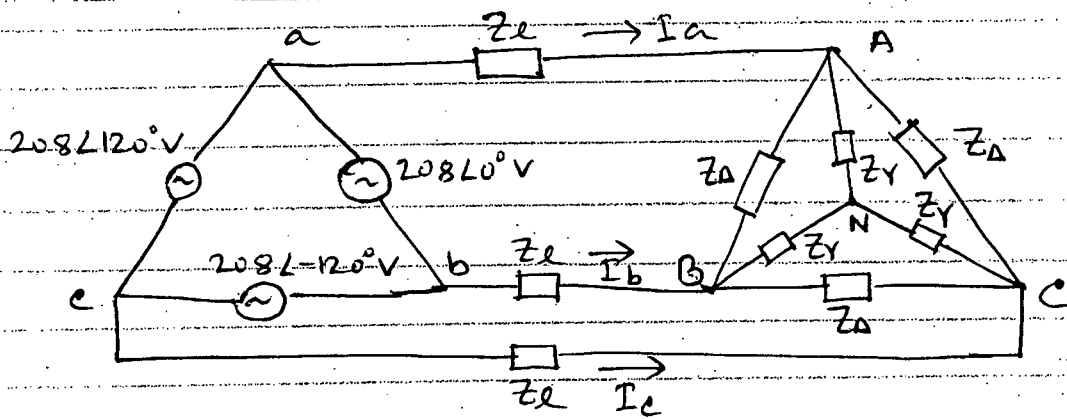


Fig. for Q. 4(b).

**EEE 269**

**SECTION – B**

There are **FOUR** questions in this Section. Answer any **THREE**.

5. (a) A 460 V, 25 HP, 60 Hz, four pole Y-connected induction motor has the following impedances in the rotor:- (20)

$R_r = 0.332 \Omega/\text{phase}$ ,  $X_{BR} = 0.464 \Omega/\text{phase}$ . Given, at full load friction and windage losses are 600 W, coreloss 500 W, shaft speed 1760 rpm and blocked rotor voltage per phase is 244 V. At rated load find-

- (i) Slip speed
- (ii) Slip
- (iii) Rotor frequency
- (iv) Rotor impedance
- (v) Rotor current
- (vi) Output power,  $P_{out}$
- (vii) load torque,  $\tau_{load}$
- (viii) Induced torque,  $\tau_{ind}$

- (b) Draw the per phase equivalent circuit of a 3-phase synchronous motor. (5+10)

A 2300 V, 1000 HP, 0.8 pf leading, 60 Hz, two pole Y- connected synchronous motor has synchronous reactance of 2.8  $\Omega$  and armature resistance of 0.4  $\Omega$  per phase. At rated load it draws 165 A current at 0.8 leading power factor from line. At rated condition find

- (i) Internally generated voltage ( $E_A$ ) per phase
- (ii) Torque angle,  $\delta$
- (iii) Total stator copper loss in 3-phase.

6. (a) For a synchronous generator show that induced torque, (17)

$$\tau_{ind} = \frac{3V_{\phi} E_A \sin \delta}{\omega_m X_s},$$

here all symbol represents usual meaning.

- (b) Draw the power flow diagram of a synchronous generator. (3+15)

A 480 V, 60 Hz,  $\Delta$ -connected 4-pole synchronous generator has a synchronous reactance of 0.1  $\Omega$  and armature resistance of 0.015  $\Omega$  per phase. At full load, the machine supplies 1200 A at 0.8 PF lagging. Under full load condition friction and windage losses are 40 kW and core losses are 30 kW. Ignore field circuit losses. Find -

- (i) Speed of rotation
- (ii) When the generator is supplying to the rated full load what will be the  $E_A$  per phase? ( $E_A$  stands for internally generated voltage)
- (iii) Rated input power,  $p_{in}$
- (iv) Rated output power,  $P_{out}$
- (v) Efficiency,  $\eta$
- (vi) Terminal voltage ( $V_T$ ) if load is suddenly disconnected.

**EEE 269**

**Contd ... Q. No. 7**

7. (a) A thermistor has a resistance of  $3980 \Omega$  at the ice point ( $0^\circ\text{C}$ ) and  $794 \Omega$  at  $50^\circ\text{C}$ . The resistance temperature relationship is given by  $R_T = aR_0 \exp (b/T)$ . Calculate the constants  $a$  and  $b$ . Also calculate the range of resistance to be measured in case the temperature varies from  $40^\circ\text{C}$  to  $100^\circ\text{C}$ . (10)

(b) Show that, for a piezo-electric transducer, output voltage  $E_0 = gtp$ . Here,  $g$  = voltage sensitivity,  $p$  = pressure, and  $t$  = thickness of the piezo crystal. (10)

(c) A barium titanate piezo-crystal has the dimensions of  $5 \text{ mm} \times 5 \text{ mm} \times 5 \text{ mm}$ . The force acting on it is  $5 \text{ N}$ . The charge sensitivity of barium titanate is  $150 \text{ pC/N}$  and its permittivity is  $12.5 \times 10^{-9} \text{ F/m}$ . If the modulus of elasticity of barium titanate is  $12 \times 10^6 \text{ N/m}^2$ , calculate the strain. Also calculate the charge and the capacitance. (15)

8. (a) With the help of phasor diagram and necessary calculation show that the transformation ratio of a current transformer (C.T.) (17)

$$R \approx n + \frac{I_o}{I_s} \sin(\delta + \alpha)$$

Here,

$n$  = Turns ratio,  $I_o$  = exciting current,  $I_s$  = secondary current

$\delta$  = Angle between secondary induced voltage and current

$\alpha$  = Angle between exciting and magnetizing current

(b) Draw the schematic diagram of a 'Chopper Stabilized Amplifier' and briefly explain its operation. (10)

(c) What are the important characteristics of an instrumentation amplifier? In an instrumentation amplifier  $R = 25 \text{ k} \Omega$  and  $aR = 50 \Omega$ . Calculate the voltage gain. Here, symbols represent usual meaning. (3+5)

-----

**SECTION – A**There are **FOUR** questions in this Section. Answer any **THREE**.

1. (a) Give formal definition of pushdown automata. Construct a pushdown automaton that recognizes the same language as recognized by the following context free grammar: (10)

$$\begin{aligned} S &\rightarrow BS \mid A \\ A &\rightarrow 0A \mid \epsilon \\ B &\rightarrow BB1 \mid 2 \end{aligned}$$

- (b) Prove that a language is context free language (CFL) if and only if there exists a pushdown automaton (PDA) accepting it. (13 1/3)

2. (a) What is the difference between Turing recognizable and decidable languages? Explain. The halting problem can be described by the following language:  $L = \{ \langle M, w \rangle : M \text{ is a Turing machine and } M \text{ halts on input string } w \}$ . Show that the language  $L$  is Turing-acceptable. (10)

- (b) Give a formal definition of pumping lemma for context free language (CFL). Use the pumping lemma for CFL to show that the language  $L = \{ a^n b^n c^n \mid n \geq 0 \}$  is not CFL. (13 1/3)

3. (a) Prove that every multi-tape Turing machine has an equivalent single-tape Turing machine. (8)

- (b) Let  $\Sigma = \{ -, a, b \}$  and  $L = \{ w-w-w \mid w \in \{a, b\}^* \}$ . Give a high level description of a Turing machine  $M$  that decides  $L$ . Draw a state diagram for  $M$ . You may assume  $M$  has a doubly infinite tape. (7)

- (c) Give the state diagram of a Turing machine  $M$  that does the following on input string '#w' where  $w \in \{0, 1\}^*$ . Let  $n = |w|$  (the length of the string  $w$ ). If  $n$  is even, then  $M$  converts string '#w' to the string '#0^n' (symbol 0 is repeated  $n$  times). If  $n$  is odd, then  $M$  converts '#w' to the string '#1^n' (symbol 1 is repeated  $n$  times). The machine should enter the accepting state after the conversion. Do not care where the head should be at the end of conversion. Do not show the reject state and the corresponding transitions to the reject state. (8 1/3)

4. (a) Construct the state diagram of a pushdown automaton that recognizes the following language  $L = \{ a^i b^j c^k : i + j = k \}$ . (8)

**CSE 211**

**Contd ... Q. No. 4**

(b) Show that every nondeterministic Turing machine has an equivalent deterministic Turing machine. (7)

(c) Construct the state diagram of a pushdown automaton that accepts the following language:  $L = \{ w : w \in \{a, b\}^* \text{ and length of } w \text{ is odd with middle symbol 'a'} \}$ .

Few example strings in the language  $L$  are:  $aaa, bbabb, baaba$ . (8 1/3)

**SECTION - B**

There are **FOUR** questions in this Section. Answer any **THREE**.

Assume reasonable any missing data. Symbols carry their usual meaning.

5. (a) Consider the grammar  $E \rightarrow E + E | E * E | (E) | a$ . Is this grammar ambiguous? Explain with respect of  $a + a * a$ . (8)

(b) Prove that Language  $\{0^n \mid n \text{ is a perfect square}\}$  is not regular. (5 1/3)

(c) Define regular language. Prove that the class of regular language is closed under the following operations: (10)

(i) Star operation

(ii) Concatenation operation

6. (a) Let  $L = \{a^n b^n c^m \mid n \geq 1, m \geq 1\} \cup \{a^n b^m c^m \mid n \geq 1, m \geq 1\}$  be a context free language. Explain that all grammars for  $L$  are ambiguous. (8)

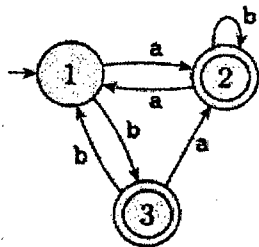
(b) Is there any technique to test whether a given language  $L$  is regular or not? State and prove it. (12 1/3)

(c) Write regular expressions for the following language: (3)

String of 0's and 1's having even parity, that is, an even number of 1's.

7. (a) Design a context free grammar  $G$  to generate the language of regular expression  $0^*1(0 + 1)^*$ . Using  $G$ , give leftmost and rightmost derivations of the string 00101. (8)

(b) Convert the following DFA into Regular expression (7 1/3)



(c) Design a DFA to accept the language:

$L = \{w \mid w \text{ is a binary string having a decimal value that is multiple of five}\}$ . (8)

8. (a) Give three differences between DFA and NFA with examples. (5)

(b) Prove that a set is regular if and only if it can be described by a regular expression. (12 1/3)

(c) Prove that  $\{0^n 10^n \mid n > 1\}$  is not a regular language. (6)

**SECTION - A**

There are **FOUR** questions in this Section. Answer any **THREE**.

1. (a) Does the Dijkstra's algorithm terminate, if it is applied on a graph  $G$  that contains a negative-weight cycle? Why or why not? Justify your answer. (5)
- (b) What does  $d[v]$  represent at the end of the execution of Algorithm 1, if the algorithm is applied on an unweighted directed acyclic graph (DAG)  $G$  with a source  $s$ ? What is the runtime complexity in terms of  $|V|$  and  $|E|$  of Algorithm 1? (6+6)

**Algorithm 1**

compute a topological sorting of  $G$

for each vertex  $v$  in  $V[G]$

$d[v] \leftarrow 0$

for each  $u$  taken in topologically sorted order

for each  $v$  in  $adj(u)$

if  $d[v] < d[u] + 1$

$d[v] \leftarrow d[u] + 1$

- (c) In Figure 1, the shortest path estimate of each vertex appears within the vertex and the distance to traverse from a vertex to another appears as the weight of the corresponding directed edge. Show the updated shortest path estimates of vertices after relaxing the following edges. (6)

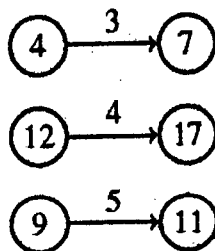


Figure 1

- (d) State and prove the convergence property of the shortest paths. (12)
2. (a) Write the Bellman-Ford algorithm. Analyze the time complexity and correctness of the Bellman-Ford algorithm. (20)
- (b) Apply the Transitive-Closure algorithm on the graph given in Figure 2. Show the matrices computed at all stages of the execution of the Transitive-Closure algorithm. (15)

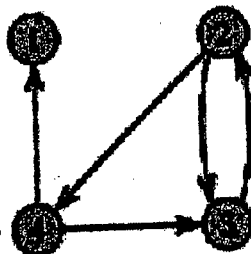


Figure 2

**CSE 207**

3. (a) State the parenthesis theorem for the depth-first search. (6)
- (b) Assume that a breath-first tree has already been computed with BFS on a graph  $G$  with the source vertex  $s$ . Write an algorithm to point out the vertices on a shortest path from  $s$  to  $v$ . (8)
- (c) Prove that the component graph is a DAG. (8)
- (d) Write a linear time algorithm to compute the strongly connected components of a directed graph  $G = (V, E)$  using two depth-first searches. Give an example to show every step of the algorithm. (8+5)
4. (a) Explain the asymptotic upper, lower and tight bounds of an algorithm. (9)
- (b) Use a recursion tree to determine a good asymptotic upper bound on the recurrence  $T(n) = T(n/4) + T(n/2) + n^2$ . (10)
- (c) If prim's algorithm is applied on the graph in Fig. 3, which will be the fifth edge to be added onto the spanning tree being formed. Start from vertex "a". (10)

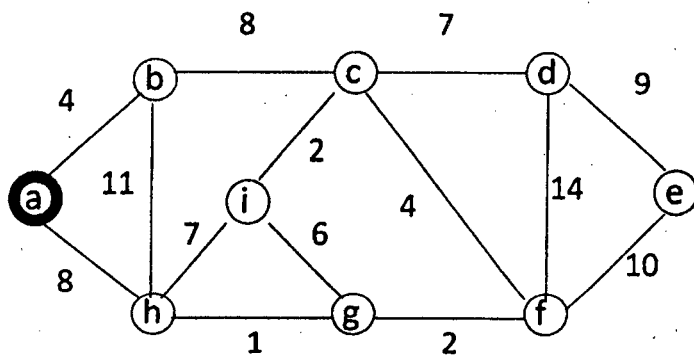


Figure 3

- (d) Write pseudocode of FIND-SET algorithm that works using the path compression heuristic. FIND-SET( $x$ ) procedure returns a pointer to the representative of the set containing  $x$ . (6)

**SECTION - B**

There are **FOUR** questions in this Section. Answer any **THREE**.

5. (a) Describe the pseudocode of a *divide and conquer* sorting algorithm that runs in  $O(n^2)$  time in the worst case. (15)
- (b) Deduce the best case running time of your algorithm in Q. 5(a). (7)
- (c) An inversion in an array  $A[1..n]$  is a pair of indices  $(i, j)$  such that  $i < j$  and  $A[i] > A[j]$ . The number of inversions in an  $n$ -element array is between 0 (if the array is sorted) and  $\frac{n}{2}$  (if the array is sorted backward). Describe and analyze an algorithm to count the number of inversions in an  $n$ -element array in  $O(n \log n)$  time (hints: modify *mergesort* algorithm). (13)



**CSE 207**

6. (a) Suppose we are given a sequence of integers  $A[1..n]$  and we want to find the longest subsequence whose elements are in decreasing order. More concretely, we want to find the longest sequence of indices  $1 < i_1 < i_2 < \dots < i_k < n$  such that  $A[i_j] > A[i_{j+1}]$  for all  $j$ . Describe the pseudocode of a backtrack algorithm that solves this problem in  $O(2^n)$  time. (15)
- (b) In a previous life, you worked as a cashier in the ancient city of Egypt, spending the better part of your day giving change to your customers. Because paper was a very rare and valuable resource at the time, cashiers were required by law to use the fewest notes possible whenever they gave change. The currency of Egypt at that time, called Dream Dollars, was available in the following denominations: \$1, \$4, \$7, \$13, \$28, \$52, \$91, \$365. (20)
- (i) A greedy change algorithm repeatedly takes the largest note that does not exceed the target amount. For example, to make \$122 using the greedy algorithm, we first take a \$91 note, then a \$28 note, and finally three \$1 notes. Give an example where this greedy algorithm uses more Dream Dollar notes than the minimum possible.
- (ii) Describe a dynamic programming algorithm that computes, given an integer  $k$ , the minimum number of notes needed to make  $k$  Dream Dollars. Write down the pseudocodes for both memorized approach and bottom-up interactive approach. Analyze the running-times of both approaches.
7. (a) Suppose you are given  $n$  tasks with start times  $S[1..n]$  and finish times  $F[1..n]$ . Your job is to schedule as many tasks as possible (in one room) so that no two scheduled tasks overlap. We say that two tasks  $i$  and  $j$  do not overlap if  $S[i] \geq F[j]$  or  $S[j] \geq F[i]$ . (15)
- (i) Describe a greedy algorithm that solves the above problem i.e., finds the maximum number of non-overlapping tasks.
- (ii) Prove the correctness of your greedy algorithm.
- (b) Show that the decision version of vertex cover optimization problem is *NP-Complete* using a reduction from Independent Set decision problem. (12)
- (c) Prove that the optimization version of Independent set problem is *NP-Hard*. (8)
8. (a) On an auspicious day, while walking alone in a forest, assume that you find  $n$  magic boxes kept in a row. Each box is numbered from 1 to  $n$  sequentially and contains some dollars! While you are about to touch the first box, a giant Genie appears and told you that you can open a box and take the dollars in it. However, you can only open a box (say box  $i$ ) provided that you did not open the previous two boxes (box  $i - 1$  and box  $i - 2$ ). Given the amount of dollars kept in each box  $D[1..n]$ , your task is to find the maximum number of dollars that you can obtain from the  $n$  boxes. Describe a dynamic programming solution to solve this problem. Analyze the running time of your solution. (12)

**CSE 207****Contd ... Q. No. 8**

(b) Consider the following greedy approximation algorithm for the vertex cover optimization problem.

**(15)**

Algorithm <i>GreedyVertexCover</i> ( <i>G</i> )	
1	$C \leftarrow \phi$
2	while $E$ is not empty. // $E$ is the set of edges
3	$u \leftarrow$ vertex in $V$ with maximum degree // $V$ is the set of // vertices
4	$V \leftarrow V - \{u\}$
5	$C \leftarrow C \cup \{u\}$
6	for each edge $(u, v)$ adjacent to $u$
7	$E \leftarrow E - \{(u, v)\}$ // remove all edges from $E$ that are // covered by vertex $u$
8	return $C$ // $C$ is the vertex cover produced by this algorithm

- (i) Give an example graph where the above algorithm may not produce the optimal vertex cover.
- (ii) Show that the approximation ratio of the above greedy algorithm is  $O(\lceil \lg |E| \rceil)$ .
- (c) Give an efficient 2-approximation algorithm for vertex cover optimization problem.

**(8)**

-----

The figures in the margin indicate full marks.

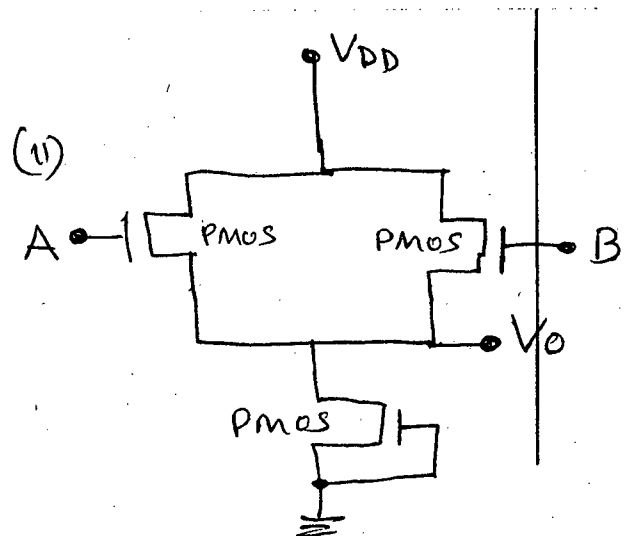
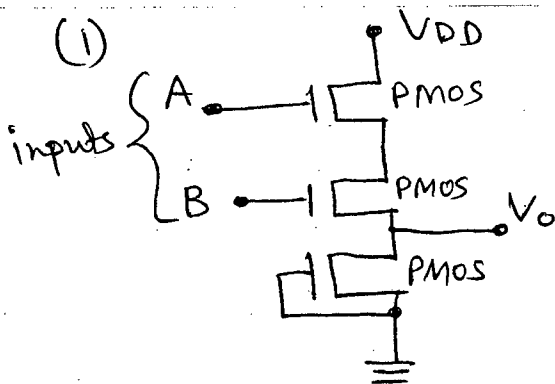
Symbols have their usual meaning.

USE SEPARATE SCRIPTS FOR EACH SECTION

**SECTION – A**

There are **FOUR** questions in this Section. Answer any **THREE**.

1. (a) Draw the basic circuit of a 2-input DTL NAND gate. (5)  
 (b) Discuss fan-out and speed problem of the above gate. (10)  
 (c) Draw the basic circuit of a 2-input TTL NAND gate and describe how TTL overcomes the above-mentioned disadvantages of DTL. (7+13)
  
2. (a) List the various types of ROMs that are available commercially. Describe the internal structure of any two types. (15)  
 (b) Depicting addressing, reading and writing facilities, draw the circuit diagram of a six-transistor static MOS RAM. Explain how the memory cell works. (20)
  
3. (a) Explain why ECL outputs are taken through emitter followers. (13)  
 (b) Verify the logic operation performed by the following circuits: (11+11)



4. (a) Draw the circuit diagram and explain the operation of a 2-input CMOS NAND gate. (12)  
 (b) Explain why MOS/CMOS logic gates are slower than BJT logic gates. (12)

**CSE 209**

**Contd ... Q. No. 4**

(c) The following data are obtained, from datasheet, for a CMOS logic gate: (11)

$I_{iH} = 10 \text{ pA}$	$V_{iHmin} = 3.5 \text{ V}$
$I_{iL} = 10 \text{ pA}$	$V_{iLmax} = 1.5 \text{ V}$
$I_{oH} = -0.5 \text{ mA}$	$V_{oHmin} = 4.99 \text{ V}$
$I_{oL} = 0.4 \text{ mA}$	$V_{oHmax} = 0.01 \text{ V}$

For a TTL gate, following data are found:

$I_{iH} = 100 \text{ }\mu\text{A}$	$V_{iHmin} = 2.8 \text{ V}$
$I_{iL} = -1.6 \text{ mA}$	$V_{iLmax} = 0.8 \text{ V}$
$I_{oH} = -0.4 \text{ mA}$	$V_{oHmin} = 2.4 \text{ V}$
$I_{oL} = 10 \text{ mA}$	$V_{oHmax} = 0.4 \text{ V}$

Can the two gates be interfaced directly? Explain your answer.

**SECTION – B**

There are **FOUR** questions in this Section. Answer any **THREE**.

5. (a) Mathematically show that, for any periodic input waveform, the average level of the steady state output of the circuit in Fig. 5(a) is always zero, independent of the DC level of input. (10)

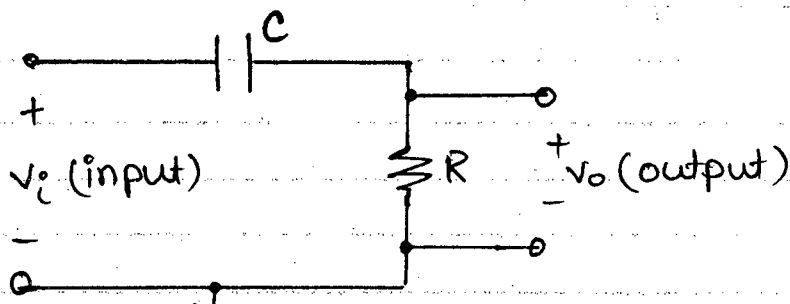


Figure - 5(a)

(b) If the time constant(RC) is very large in comparison with the time required for the input signal to make an appreciable change, show that the ckt in Fig. 5(b) acts as an integrator (10)

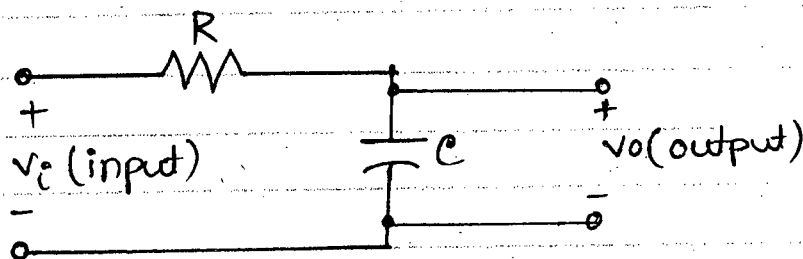


Figure - 5(b)

(c) Consider the clipper circuit show in Fig. 5(c)-A and answer the following questions. (9+3+3=15)

\* Consider piecewise linear model for diode  $v_f = 0.7 \text{ V}$ ,  $R_f = 100 \text{ }\Omega$

(i) Derive the transfer function for the network.

(ii) Find out the ranges of input, in which the diode, D remains in (1) Forward Bias and

(2) Reverse Bias.

**CSE 209**

Contd ... Q. No. 5(c)

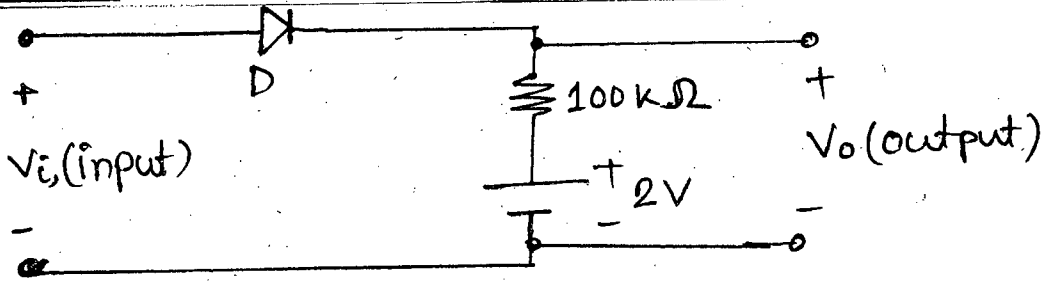


Figure - 5(c)-A

(iii) Draw the output waveform clearly, if the signal in Fig. 5(c)-B is fed to the network as input.

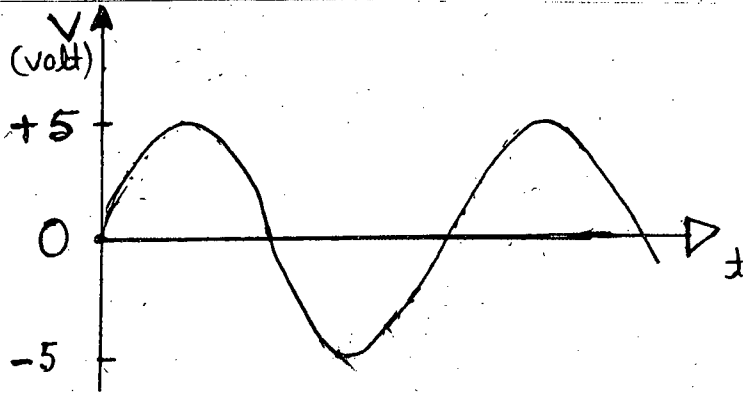


Figure - 5(c)-B

6. (a) Consider the input signal of Fig. 6(a)-A (A 1000 Hz square wave) is fed to a High pass circuit (Fig. 5(a)). If we want to obtain a steady-state output as shown in Fig. 6(a)-B. Calculate the value of RC for the high pass circuit. (10)

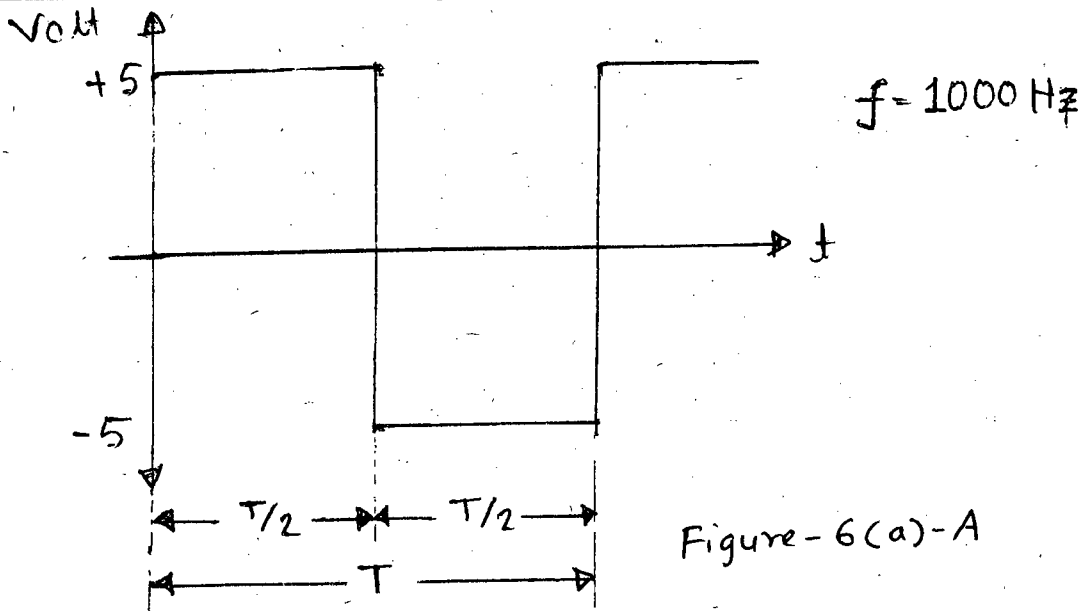


Figure - 6(a)-A

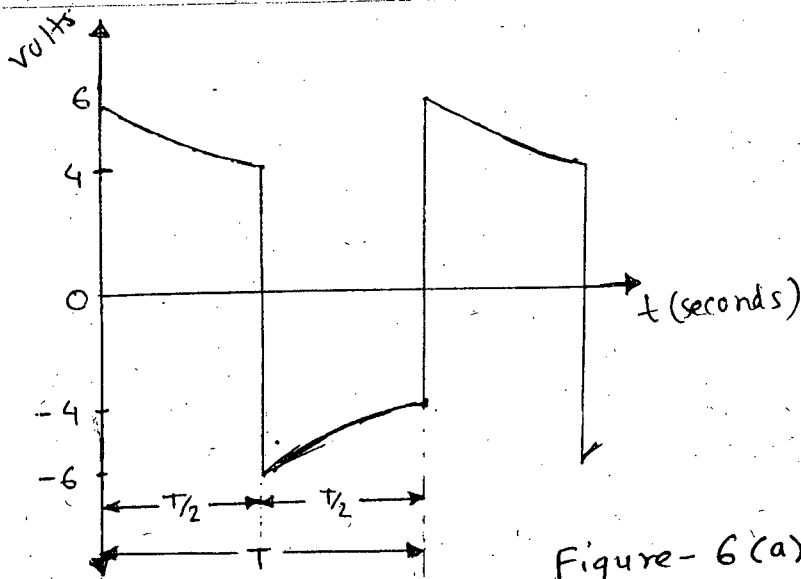


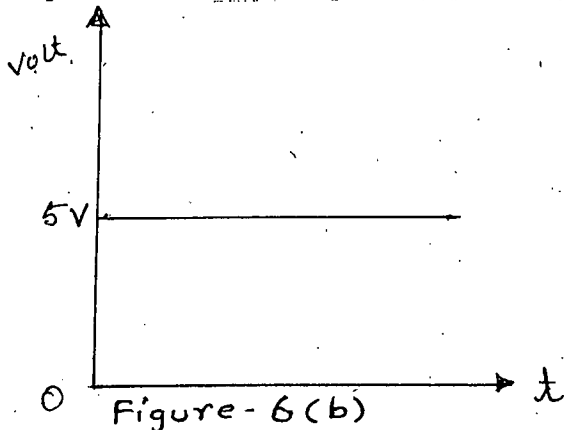
Figure - 6(a)-B

Contd ..... P/4

**CSE 209**

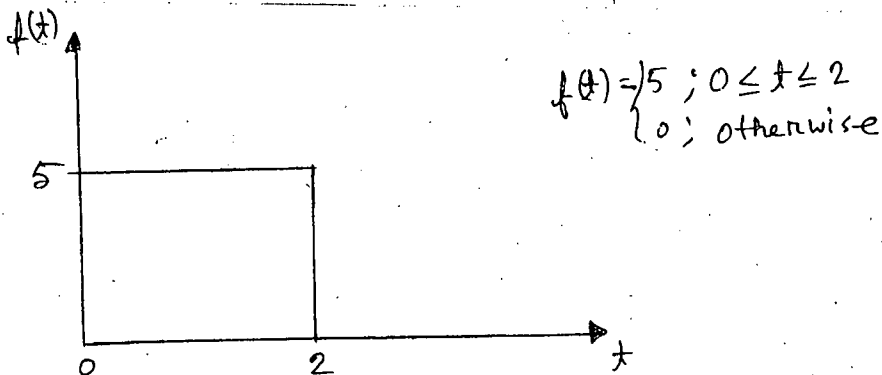
**Contd ... Q. No. 6**

(b) Derive the output equation for the network shown in Fig. 5(b), if the signal in Fig. 6(b) is fed to the network as input. Also draw the output waveform clearly. (10)



(c) Consider the signal  $f(t)$  shown in Fig. 6(c). Draw the following signals: (5×2=10)

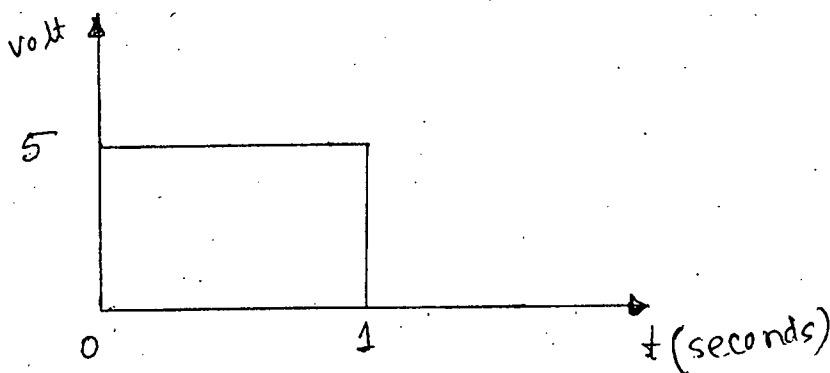
- (i)  $f(t-2)$  (ii)  $f(t+2)$  (iii)  $f(2t)$  (iv)  $f(\frac{1}{2}t)$  (v)  $f(-t)$ .



(d) Consider a 4-bit Digital to Analog Converter (DAC). (i) What is the smallest amount of change possible in output (in percentage) with respect to the total output range? (3+2=5)

(ii) Let, the total output range is 15 volts. What is the smallest amount of change possible in output in volts?

7. (a) The signal shown in Fig. 7(a) is fed to the circuit of Fig. 5(a), as input. Derive the output equation and draw the output waveshape. Use  $R = 1 \text{ k}\Omega$  and  $C = 1 \mu\text{F}$ . (10)



**CSE 209**

**Contd ... Q. No. 7**

- (b) Consider the triangle wave generator in Fig. 7(b)-A. The output is shown in Fig. 7(b)-B. Mathematically derives the values of  $v_{min}$ ,  $v_{max}$ ,  $T_1$  and  $T_2$ . (15)

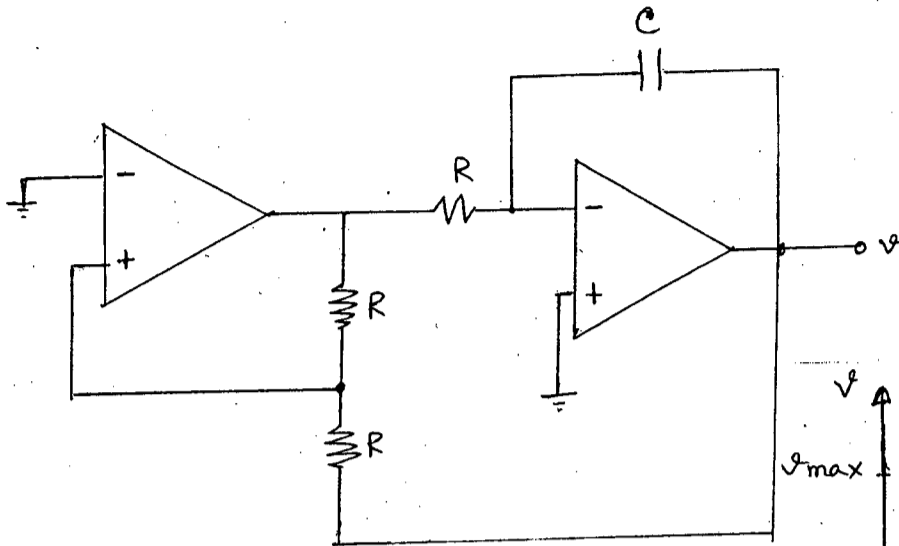


Figure-7(b)-A: Triangulare wave generator

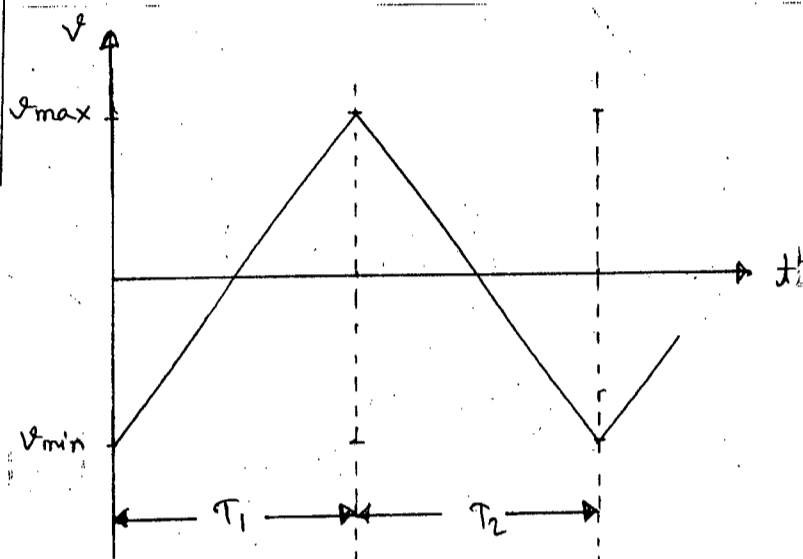


Figure - 7(b)-B

- (c) Briefly discuss 2 ways to improve the accuracy of Analog to Digital Conversion process. (6)
- (d) Briefly state the application/necessity of a sample and hold circuit in an analog to digital converter. (4)
8. (a) Design a 3-bit weighted resistance Digital to Analog Converter (DAC). Clearly draw the circuit diagram and derive the output equation. (12)
- (b) Using the circuit designed in Q. No. # 8(a), design a DAC that shows the input-output relationship shown in Table 8(b). You must use "Adding offset current" method. Derive the output equation and clearly draw the circuit diagram. You do not need to repeat any calculation/derivation, which is already done in Question 8(a). (13)

Input			Output (Volts)
$S_2$	$S_1$	$S_0$	
0	0	0	-3
0	0	1	-2
0	1	0	-1
0	1	1	0
1	0	0	1
1	0	1	2
1	1	0	3
1	1	1	4

Table - 8(b)

**CSE 209**

**Contd ... Q. No. 8**

(c) Design a clamper circuit that produces the output shown in Fig. 8(c)-B from the input shown in Fig. 8(c)-A consider constant voltage drop model for diodes,  $v_f = 0.7 \text{ V}$ .

(10)

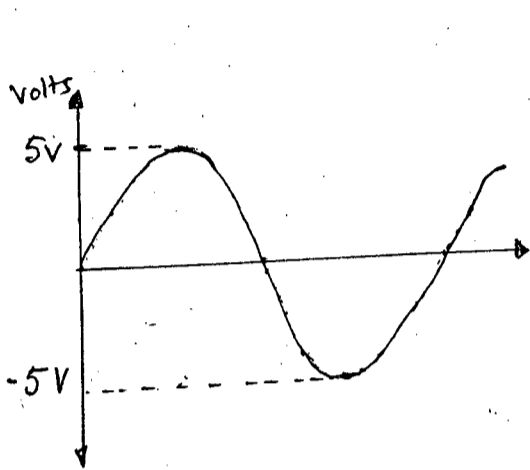


Figure- 8(c)A : Input waveform

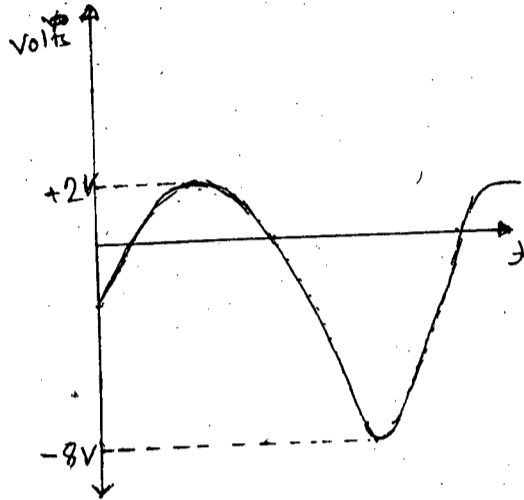


Figure- 8(c)-B : Output waveform

-----



The figures in the margin indicate full marks.

Symbols used have their usual meanings.

USE SEPARATE SCRIPTS FOR EACH SECTION

**SECTION – A**

There are **FOUR** questions in this Section. Answer any **THREE**.

1. (a) Reduce the matrix  $A$  to canonical form and hence find the rank of the matrix, where (23)

$$A = \begin{bmatrix} 1 & 2 & -1 & 2 \\ 3 & 1 & -2 & -1 \\ 4 & -3 & 1 & 1 \\ 3 & -2 & 0 & -1 \end{bmatrix}$$

- (b) Use elementary row transformations to find the inverse of the matrix (23 $\frac{2}{3}$ )

$$A = \begin{bmatrix} 1 & 2 & 3 & 4 \\ 2 & 3 & 4 & 6 \\ 3 & 4 & 5 & 7 \\ 4 & 5 & 5 & 7 \end{bmatrix}$$

2. (a) Reduce the matrix  $A$  to the normal form  $B$  and obtain the non-singular matrices  $P$  and  $Q$  such that  $PAQ = B$ , where (23)

$$A = \begin{bmatrix} 1 & -2 & 1 & 3 \\ 4 & -1 & 5 & 8 \\ 2 & 3 & 3 & 2 \end{bmatrix}$$

- (b) Solve the following system of linear equations (23 $\frac{2}{3}$ )

$$\begin{aligned} x_1 + 2x_2 - x_3 - 2 &= 0 \\ 3x_1 + x_2 + 2x_3 - 11 &= 0 \\ 4x_1 + 4x_2 - 3x_3 - 3 &= 0 \\ 2x_1 - x_2 + 3x_3 - 9 &= 0 \end{aligned}$$

3. (a) State Cayley-Hamilton theorem and verify it for the matrix (16 $\frac{2}{3}$ )

$$A = \begin{bmatrix} 1 & 2 & 3 \\ 1 & 3 & 6 \\ 2 & 6 & 13 \end{bmatrix}$$

Hence find the inverse of  $A$ .

- (b) Let  $W$  be a subspace of  $R^5$  spanned by the following vectors: (15)

$$\begin{aligned} u_1 &= (1, 2, 1, 3, 2), u_2 = (1, 3, 3, 5, 3), u_3 = (3, 8, 7, 13, 8), u_4 = (1, 4, 6, 9, 7), \\ u_5 &= (5, 13, 13, 25, 19). \end{aligned}$$

Find a basis of  $W$  consisting of the original given vectors, and find  $\dim W$ .

**CSE 243**

**Contd ... Q. No. 3**

(c) Let  $F: \mathbb{R}^3 \rightarrow \mathbb{R}^3$  be a linear transformation defined by

$$F(x, y, z) = (x + 2y - 3z, 2x + 5y - 4z, x + 4y + z).$$
 Find a basis and dimension of the

kernel and the image of  $F$ .

(15)

4. (a) The vectors  $u_1 = (1, 2, 0), u_2 = (1, 3, 2), u_3 = (0, 1, 3)$  form a basis  $S$  of  $\mathbb{R}^3$ . Find the coordinate vector of  $v = (2, 7, -4)$  relative to the basis  $S$ .

(16 2/3)

(b) If  $u = x + y + z, v = x^2 + y^2 + z^2$  and  $w = xy + yz + zx$ , find  $[\text{grad } u \text{ grad } v \text{ grad } w]$ .

(15)

(c) If  $\vec{F} = x^2 y \hat{i} - 2xz \hat{j} + 2yz \hat{k}$ , find  $\text{div curl } \vec{F}$ .

(15)

**SECTION - B**

There are **FOUR** questions in this Section. Answer any **THREE**.

5. (a) Evaluate  $\iint_S \underline{F} \cdot \underline{n} \, ds$  where  $\underline{F} = y \hat{i} + 2x \hat{j} - z \hat{k}$  and  $S$  is the part of the plane

$$2x + y + 2z = 6$$
 lying in the first octant.

(20)

(b) Verify the divergence theorem for  $\underline{F} = 2x^2 y \hat{i} - y^2 \hat{j} + 4xz^2 \hat{k}$  taken over the region in the first octant bounded by the cylinder  $y^2 + z^2 = 9$  and the plane  $x = 2$ .

(26 2/3)

6. (a) Find  $L \left\{ \frac{\cos \sqrt{t}}{\sqrt{t}} \right\}$ .

(12)

(b) Prove that  $L\{Si(t)\} = \frac{1}{s} \tan^{-1} \frac{1}{s}$ .

(10)

(c) Use convolution theorem to evaluate  $L^{-1} \left\{ \frac{s}{(s^2 + 1)^2} \right\}$ .

(12)

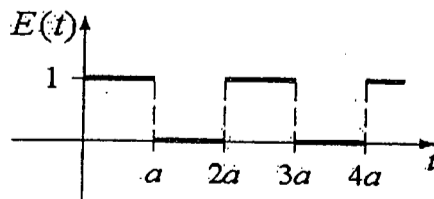
(d) Using Laplace transform show that  $\int_0^\infty \sin x^2 \, dx = \frac{1}{2} \sqrt{\frac{\pi}{2}}$ .

(12 2/3)

7. (a) Solve the differential equation  $L \frac{dI}{dt} + RI(t) = E(t)$  subject to  $I(0) = 0$  where  $L = 1,$

$R = 1$  and  $E(t)$  is given by the following square wave function with amplitude 1 and  $a = 1$ .

(24 2/3)



**CSE 243****Contd ... Q. No. 7**

(b) Use Laplace transform to solve the following system of differential equations:

$$\begin{aligned}x''(t) + y''(t) &= e^{2t} \\ 2x'(t) + y''(t) &= -e^{2t}\end{aligned}$$

subject to  $x(0) = 0$ ,  $y(0) = 0$  and  $x'(0) = 0$ ,  $y'(0) = 0$ .

**(22)**

8. (a) Expand  $f(x)$  in a half-range Fourier sine series, where

$$f(x) = \begin{cases} x, & 0 \leq x < \frac{\pi}{2} \\ \pi - x, & \frac{\pi}{2} \leq x \leq \pi \end{cases}$$

Hence show that  $\frac{\pi^2}{8} = 1 + \frac{1}{3^2} + \frac{1}{5^2} + \frac{1}{7^2} + \dots$ . Also, sketch the graph of  $f(x)$ .

**(15)**

(b) Use Fourier cosine integral formula to prove

$$e^{-\lambda x} = \frac{2\lambda}{\pi} \int_0^{\infty} \frac{\cos \alpha x}{\alpha^2 + \lambda^2} d\alpha, \quad (\lambda > 0, x \geq 0).$$

and hence show that  $(1+x)e^{-x} = \frac{4}{\pi} \int_0^{\infty} \frac{\cos \alpha x}{(1+\alpha^2)^2} d\alpha, x \geq 0$ .

**(16 $\frac{2}{3}$ )**

(c) Find the Fourier transform of  $f(x) = \begin{cases} 1-x^2, & |x| < 1 \\ 0, & |x| > 1 \end{cases}$

and hence show that  $\int_0^{\infty} \frac{\sin x - x \cos x}{x^3} \cos \frac{x}{2} dx = \frac{3\pi}{16}$ .

**(15)**

-----