ANALYSIS OF GRID FLOORS

A Thesis

by

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ABSTRACT

The study considers the linear elastic analysis of a grid floor with different boundary conditions and patterns by stiffness method. A generalised computer program is developed for this method in Fortran IV.

Economy in the computer storage requirement is achieved by partitioning method where only the required portion of the overall structure stiffness matrix corresponding to the total degrees of freedom is generated.

Series of analyses are made in this investigation to study the effect of structural parameters such as width and depth of grid member, spacing of grid member, different arrangements of grid member and boundary conditions on moments, shears, deflections etc. The value of the stresses obtained by stiffness method are compared with those obtained by plate method. Finally design charts and tables are also developed for the easy and rapid analysis of grid floor.

Analysis of the results has shown that (i) the maximum moment and shear occur along midspan members for square grids with continuous support, while for corner-supported grid floor these occur along support line (ii) the torsional moment and bending moment change with the change of ρ , the ratio of bending stiffness to torsional stiffness and the change is more rapid in case of torsional moment than the bending moment. (iii) the moments, shears, torsions and deflections of a grid floor can be reduced significantly by decreasing the spacing of grid members (iv) the maximum moment and deflections as obtained by plate method are lower than those obtained by grid analysis (v) the moment and deflections can be reduced by providing skew grids where the maximum moment, shear and torsion occur near the corners.

NOTATIONS

$[A]$	= Action matrix applied at joints (in the direction of structure axes)
$[A_c]$	= Combined action or load matrix (in the direction of structure axes)
$[A_D]$	= Portion of the matrix A_c corresponding to unknown displacement
$[A_E]$	= Equivalent joint load matrix (in the direction of structure axes)
$[A_{ML}]$	= Action matrix at ends of restrained members (in direction of member axes) due to loads
$[A_R]$	= Support reaction matrix
$[A_{RL}]$	= Portion of the matrix A_c corresponding to reaction in the restrained structure due to loads except those that correspond to the unknown displacement
$[A_M]$	= Member end actions
a	= Shorter side of a floor
b	= Longer side of a floor
b/a	= Aspect ratio
b_w	= Width of beam
C_x	= $\cos \alpha$
C_y	= $\sin \alpha$
d	= Depth of beam
$[D]$	= Joint displacement (in the direction of structure Axes)
D_x	= Unit flexural stiffness of a plate in the x-direction
D_y	= Unit flexural stiffness of a plate in the y-direction
E	= Modulus of Elasticity
G	= Shear modulus

h = Thickness of the plate or floor

i = Member designation

I_x = Torsion constant of a member

I_y = Moment of inertia about y-axes

j, k = Designation for two ends of a member i

L = Length of a member

m = Number of members

m_1 = Grid spacing in the longer direction

m_2 = Grid spacing in the shorter direction

m_1/m_2 = Grid's member spacing ratio

M_x = Moment along shorter direction

M_y = Moment along longer direction

n = Number of degrees of freedom

n_j = number of joints

q_o = intensity of the external loading acting on the plate or floor

$[R_T]$ = Rotation transformation matrix

$[S]$ = Over all structure stiffness matrix

$[S_D]$ = The partitioned matrix corresponding to the degrees of freedom

$[S_{RD}]$ = A rectangular submatrix of S that contains action corresponding to the support restraints, due to unit values of displacement corresponding to the degrees of freedom.

$[S_m]$ = Member stiffness matrix for member axes

$[S_{MD}]$ = Member stiffness matrix for structure axes

V_x = Shear along shorter direction

V_y = Shear along longer direction

w = Vertical deflection of a point on the plate or floor

x, y, z = Structure oriented axes

X_m, Y_m, Z_m = Member oriented axes

α = The angle through which member axes are roted from the structure axes

μ = Poisson's ratio

θ_x = Rotation of the joint in the x-sense

θ_y = Rotation of the joint in the y-sense

ρ = Ratio of bending stiffness to torsional stiffness

CONTENTS

CHAPTER 1

INTRODUCTION

1.1 General

A grid floor may be defined as an assemblage of rigidly interconnected beam and slab members. It is useful in framing a large column free space. Whether the beams of a grid are simply supported or fixed there is a two-way distribution of applied load occurs. When a load is applied on a grid system the load is shared by the beams. The stiffer member carries the greater portion of the load. Since the load is shared by the beams in the two directions the two way system i.e grid floor is a structurally efficient system. Increasing the number of grid i.e reducing the spacing, the stresses in the different members are much reduced and hence smaller section of the beams are required.

Grid floor systems are generally found in industrial buildings, sports hall, churches, library room, assembly, exhibition centre and in bridge deck construction. The modern trend in construction is to build grid frame work in developing cities, particularly for office buildings. The beam-column system is often replaced by grid floor system in the more developed areas. The architectural beauty, efficiency and above all the prestige associated with grid floor, have in recent years, increased the rate of construction of grid floor all over the world. The span covered by a plane grid floor system ranges from 30 ft to 100 ft. The pattern of griders and cross beams, familiar in bridge construction is a representation of grid-frame work.

Most solutions of the problem of grid floor analysis are based on the assumption that the torsional resistance of the grid elements can be neglected. Composite grid floor display torsional rigidity owing to the composite action of the slab with the supporting grid beams. The torsional rigidity may be ignored with bridge structures made of steel girders and possessing no reinforced concrete bridge deck. But with monolithic concrete structure in building floor or bridge floor, the rigidity in torsion becomes more important and it should not be neglected.

The structural behaviour of actual grid structures lie in between those of the torsionally stiff case and torsionless case. In the early stage, the analysis of grid problems was based on end-deformation in which the deflections are equated at each node for each of the beams meeting at the node. Later on, an approximate method was developed based on the analogy between a grid system and an orthotropic plate.

In 1946, a solution for orthotropic plates of negligible torsional rigidity was given by Guyon Y.[(4)]. Guyon Y.[(5)] also gave a similar solution for isotropic plates. Later on, Massonet[(9)] derived generally valid governing equation of the orthotropic plate from the principles given by Guyon which include the effect of torsion. This governing equation of the equivalent orthotropic plate was then solved for deflections, moments and shears at the different points by converting the load into Fourier series. The governing equation can also be solved by finite difference method. But these approximate methods are very complicated and time consuming and may not be applied easily for design purpose. Thus for design purposes, these methods are of limited use and applications.

In this study attempts have been made to study the fundamental behavior of grid structures by stiffness method of analysis.

1.2 Types of Grid Floor

Various types of plane grid structures are used in civil engineering practice. Fig. 1.1 illustrates the usual two-way, three-way or four-way grid configurations as shown by Makowski[8].

The most popular is a rectangular grid in which the intersecting elements are perpendicular to each other and to the supporting walls.

The diagonal grid or skew grid consists of beams forming an oblique angle with the walls. This type is often used because of its greater rigidity which leads to a substantial reduction in the deflections.

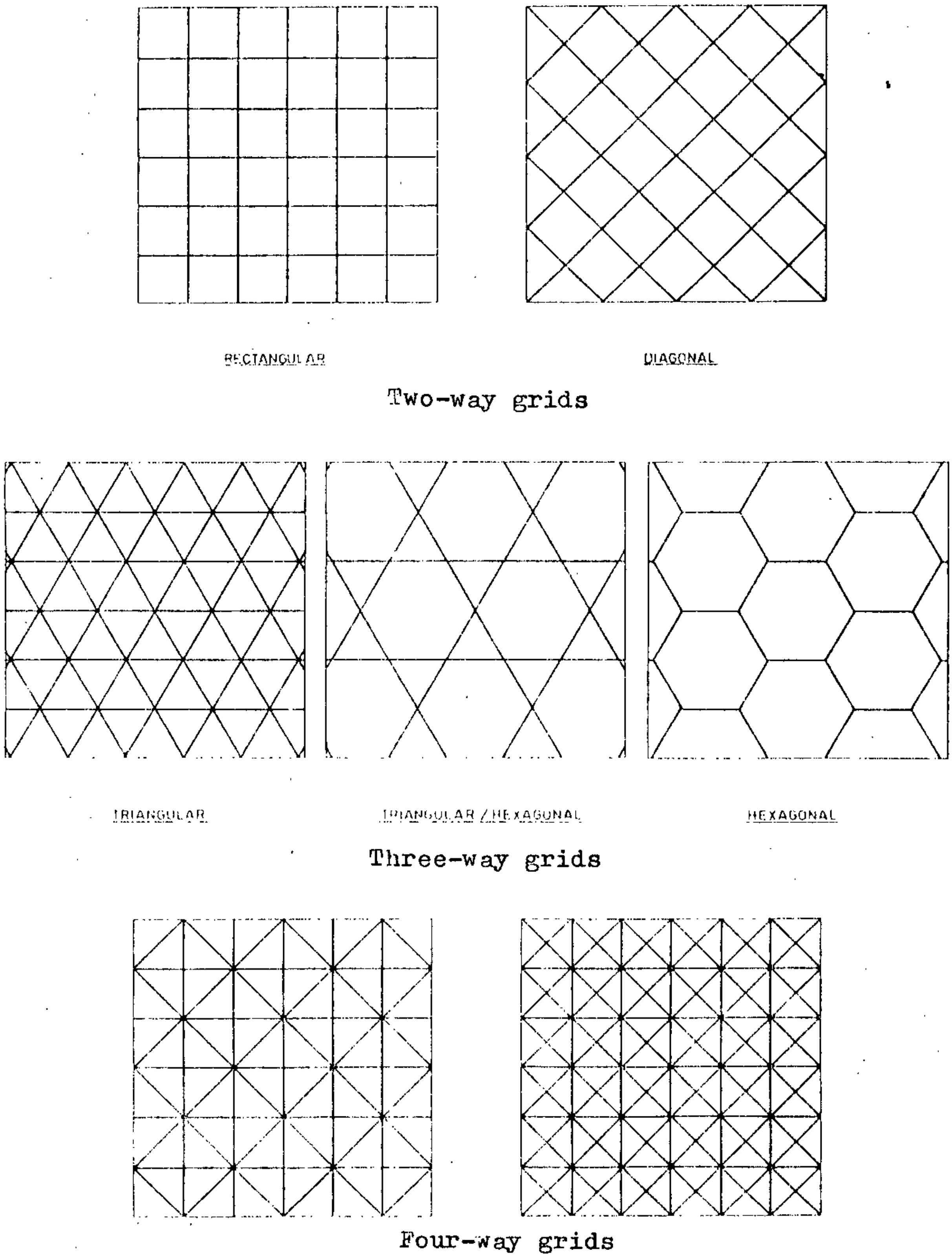


Fig. 1.1 Types of grids.

The fundamental difference between diagonal and rectangular grids is that in the former the beams are of varying length and therefore even if all the beams are of the same cross-sectional dimension and have the same flexural rigidity EI, their relative stiffness EI/L varies very considerably.

As a rule, the three-way grids have been used for larger spans or when large concentrated loads are applied. The stress distribution in three-way grids is more even than in the two-way ones. But as they use more materials, in an attempt to economise, sometimes a mixed system is used in which the normal three-way configuration is simplified by omitting every second member and producing a structure which becomes a combination of equilateral triangles and regular hexagons.

A further simplification will lead to a hexagonal grid. However, this type is rarely used, and then mainly as an architectural feature.

The four-way grids are combinations or superpositions of the rectangular and diagonal grids. There are a few examples of such applications in actual practice.

1.3 Objective of the Work

The objectives of this work are

a) To develop a generalised computer program in Fortran IV for grid structures based on stiffness method.

b) To determine the internal forces such as moment, shear, torsion for a grid floor system by stiffness method.

c) To study the effect of width, depth and spacing of grid members, different arrangements of grid members and boundary conditions on moments, shears, deflections etc.

d) To develop tables, charts and curves for the solution of grid problems.

These results may be applied for rapid analysis and design of grid floors.

CHAPTER 2

REVIEW OF DIFFERENT EXISTING METHODS OF ANALYSIS

2.1 Introduction

Development of an analytical procedure requires appropriate representation of grid floor behavior under load. In this chapter review is made of the existing available methods as used in the analysis of the grid floor structure.

2.2 Deflection Method (No Torsion)

In this method the deflections at all the intersecting points or nodal points are equated at each nodal point of the grid structure. Salvadori and Levy[10] assumed that upper beam systems rest on lower beam systems and both the beam systems are simply supported. The method is described briefly in the following:

Consider a simple grid having a nodal joint at the mid span only (Fig. 2.1). Under the action of the load P,

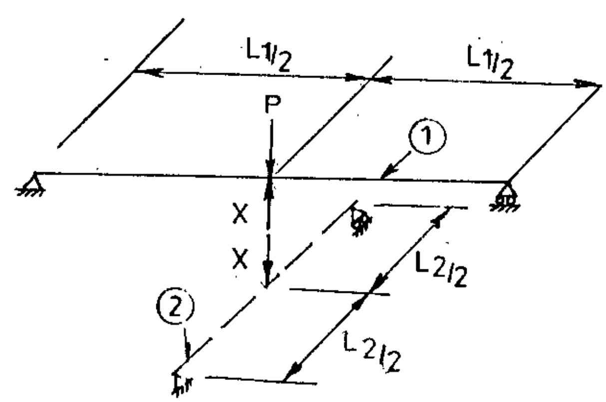


Fig. 2.1

beam 1 deflects and carries down beam 2, which exerts an upward reaction X on beam 1. Hence, beam 2 carries a load X and beam 1 a load of P-X. The midspan deflections w_1 and w_2 of the two beams are equal. Hence indicating the moment of inertia of two beams by I_1 and I_2 and assuming that the beams are made of the same material

$$w_1 = \frac{(P-X)L_1^3}{48EI_1} = \frac{XL_2^3}{48EI_2} = w_2$$

From which

$$X = \frac{P}{1+(L_2/L_1)^3(I_1/I_2)}$$

Now bending moment and shear force of each beam is analysed by conventional method.

The evaluation of the reactions of grid systems is facilitated by Table 2.1, which is taken from Salvadori and Levy[10]. This table gives the deflection w_k at the evenly spaced point k of a simply supported beam due to a unit load applied at one of these points, say the point i. In this table 'n' is the number of subdivisions in the beam. In this respect it must be remembered that the influence coefficient w_k due to a unit load at i is equal to the w_i due to unit load at k.

Table 2.1 Deflection at point "k" due to unit load applied at "i".

n	w at k=	Unit load applied at i =					Factor
		1	2	3	4	5	l^3/EI
2	1	1					1/48
3	1	8					1/486
	2	7	8				
4	1	9					1/768
	2	11	16				
	3	7	11	9			
5	1	32					1/3750
	2	45	72				
	3	40	68	72			
	4	23	40	45	32		
6	1	25					1/3888
	2	38	64				
	3	39	69	81			
	4	31	56	69	64		
	5	17	31	39	38	25	

* Table 2.1 has been taken from Salvadory and Levy[10].

2.3 Orthotropic Plate Method

2.3.1 Governing Equation for Orthotropic Plate

Two-way grids are often of the orthotropic type in which the beams running in two directions at right angles to each other have different elastic properties. Orthotropic plates are a special case of the general anisotropic plate.

The main advantages of this method is that, having determined the general equation of deflected surface of the equivalent plate, it is easy through successive differentations to obtain general experessions for shears and moments for any point denoted by the co-ordinates x and y.

Timoshenko and Woinowsky-Krieger(11) showed that the general differential equation of the deflected surface of an orthotropic plate can be expressed as follows:

$$D_x \frac{\partial^4 w}{\partial x^4} + 2H \frac{\partial^4 w}{\partial x^2 \partial y^2} + D_y \frac{\partial^4 w}{\partial y^4} = q_o \qquad (2.1)$$

where

w = vertical deflection of a point having co-ordinates x, y

q_o = intensity of the external loading acting on the plate at the considered point.

$D_x = \dfrac{E_x h^3}{12(1-\mu^2)}$ = unit flexural stiffness of the plate in the x-direction

$D_y = \dfrac{E_y h^3}{12(1-\mu^2)}$ = unit flexural stiffness of the plate in the y-direction.

$$2H = D'_x + D'_y + 2(D_{xy} + D_{yx})$$

$$D_{xy} = \frac{E_x h^3}{24(1+\mu)} \quad ; \quad D_{yx} = \frac{E_y h^3}{24(1+\mu)}$$

$$D'_x = \mu D_x$$

$$D'_y = \mu D_y$$

E_x, E_y = the moduli of elasticity of the material of the plate along two mutually perpendicular axes x and y.

μ = Poisson's ratio

h = thickness of the plate

A function $w(x,y)$ which satisfies the above governing equation and the boundary conditions for a given load distribution $q_o(x,y)$, is called "a solution of the plate problem". When such a solution for deflection w has been found, through successive differentiations one can obtain the bending moment, torsional moment and shear force per unit length in the x or y directions. To obtain the actual moment or shear in the beams of the grid, one has to multiply these expressions by the appropriate spacing of the grid beams. The bending moment in the x-direction for plate is given by

$$M_x = -\left(D_x \frac{\partial^2 w}{\partial x^2} + D'_x \frac{\partial^2 w}{\partial y^2}\right) \tag{2.2}$$

and in the y-direction

$$M_y = -\left(D_y \frac{\partial^2 w}{\partial y^2} + D'_y \frac{\partial^2 w}{\partial x^2}\right) \tag{2.3}$$

For the torsional moment in the x-direction

$$M_{xy} = 2D_{xy} \frac{\partial^2 w}{\partial x \partial y} \tag{2.4}$$

and in the y-direction

$$M_{yx} = -2D_{yx} \frac{\partial^2 w}{\partial x \partial y} \tag{2.5}$$

For shear forces

$$Q_x = \frac{\partial M_x}{\partial x} - \frac{\partial M_{yx}}{\partial y} \tag{2.6}$$

$$Q_y = \frac{\partial M_y}{\partial y} + \frac{\partial M_{xy}}{\partial x} \tag{2.7}$$

The above governing equation and the expressions for forces can be easily modified to develop equivalent expressions for two-way grids. Fig. 2.2 illustrates a typical layout of a rectangular two-way grid consisting of two

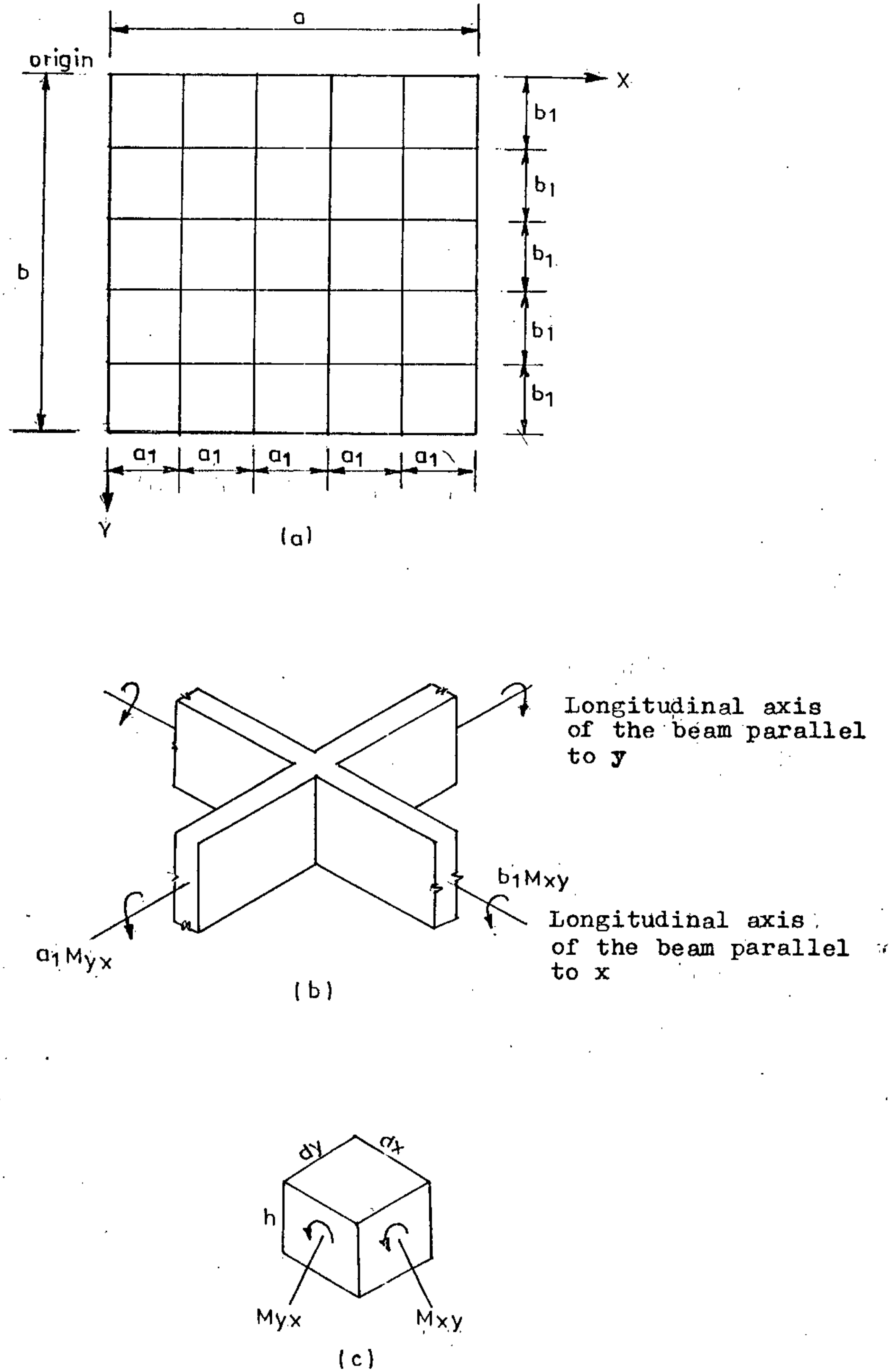


Fig. 2.2 Grid floor idealization.

groups of beams running parallel to the sides of the opening covering an area axb. If the spacing of the beams running in the x-direction is b_1 and their flexural rigidity is EI_x then an equivalent slab replacing the grid would have a unit flexural rigidity

$$D_x = \frac{EI_x}{b_1}$$

Similarly for the y-direction

$$D_y = \frac{EI_y}{a_1}$$

If the beams are rigidly interconnected at their nodes, then the torsional moment in the x-direction for the equivalent slab, collecting over the strip of width b_1 will be $b_1 . M_{xy}$. Similarly the torsional moment resisted by the beam running in the direction parallel to y-axis will be $a_1 M_{yx}$. The torsional moment is related to the angle of twisting of plates by the following equations.

$$M_{xy} b_1 = C_1 \frac{\partial^2 w}{\partial x \, \partial y}$$

and $$M_{yx} a_1 = C_2 \frac{\partial^2 w}{\partial x \, \partial y}$$

where C_1 and C_2 are the torsional rigidities of the beam in the x and y direction respectively.

Therefore, for two-way grids the fundamental equation of the deflected surface becomes

$$\frac{E_x I_x}{b_1} \frac{\partial^4 w}{\partial x^4} + \left(\frac{C_1}{b_1} + \frac{C_2}{a_1}\right) \frac{\partial^4 w}{\partial x^2 \partial y^2} + \frac{E_y I_y}{a_1} \frac{\partial^4 w}{\partial y^4} = q_o \qquad (2.8)$$

where $C_1/b_1 = D_{xy}$ and $C_2/a_1 = D_{yx}$

2.3.2 Methods of Solution of the Plate Equation

(a) The method of Fourier Double series expansion

The most common method for the solution of the governing plate equation is the inverse method. The inverse method relies upon assumed solutions of w which satisfy the governing equation and the boundary conditions. Usually choosing the acceptable series form is laborious and requires a systematic approach. The most powerful such method uses the Fourier series, where, once a solution has been found for sinusoidal loading, any other loading can be handled by infinite series. This approach, originated by Navier, offers as an important advantage the fact that a single expression may apply to the entire surface of the plate.

Navier showed that the vertical deflection w at any point defined by co-ordinates x and y, can be expressed by means of a double trigoncmetrical series.

$$w = \sum_{m=1,3,5..}^{\infty} \sum_{n=1,3,5..}^{\infty} A_{mn} \operatorname{Sin} \frac{m\pi x}{a} \operatorname{Sin} \frac{n\pi y}{b} \qquad (2.9)$$

where a and b are the dimensions of the plate.

In the case of uniformly distributed loading, q_o covering the whole plate, the coefficient

$$A_{mn} = \frac{16q_o}{\pi^6} \quad \frac{1}{mn\left(\frac{m^4}{a^4} D_x + \frac{2m^2 n^2}{a^2 b^2} H + \frac{n^4}{b^4} D_y\right)} \qquad (2.10)$$

For uniformly distributed loading, the convergence of the series is very rapid and for practical applications it is usual to take m=1 and n=1 i.e only the first term of the series.

This allows the use of a very simple expression of deflection, which then becomes

$$w = \frac{16q_o}{\pi^6} \quad \frac{\operatorname{Sin} \frac{\pi x}{a} \operatorname{Sin} \frac{\pi y}{b}}{\frac{D_x}{a^4} + \frac{2H}{a^2 b^2} + \frac{D_y}{b^4}} \qquad (2.11)$$

In practice grids are normally designed under the action of uniformly distributed load and therefore the expressions given in eqn. 2.2 to 2.7 are normally used to find the moments, shears, etc. at the different nodal points of the grid structure.

(b) Finite difference method

The use of Fourier series is only convenient for rectangular grid with simple boundary conditions. When more complex boundary conditions are encountered the method becomes more difficult to apply. The finite difference technique has been used to advantage in solving the above problem.

In this method of analysis, the plate is divided into grids of arbitrary mesh size and the deflection values at the grid points are treated as unknown quantities. The governing equation of the plate and the accompanying boundary conditions are expressed in terms of these unknown grid point deflections as shown by Cusens and Pama[2]. The resulting simultaneous equations are then solved for the unknown deflections. Moments, shears and torsions are then determined from the known deflection pattern.

CHAPTER 3

THEORY OF STIFFNESS METHOD AND ANALYTICAL STUDY FOR GRID FLOORS

3.1 General

The stiffness method gives a relationship between displacement and forces at the ends of a member. Therefore the joint displacements are first of all determined, from which the member forces are found. The relationship between the forces and joint displacement according to the stiffness method is A = SD, where A = generalized force vector, S = structure stiffness matrix and D = displacement vector.

The structure stiffness matrix S is developed by assembling the stiffness matrix of individual elements of the structure. To form the individual member stiffness matrix S_m , unit displacement one at a time is introduced in the joints of the structure in the direction considered to be positive and the holding forces are calculated at every point. The holding forces due to unit displacement form the element stiffness matrix which super-imposing forms the structure stiffness matrix.

3.2 Analytical Study for Grid System(3)

3.2.1 The Grid System

In a grid floor system all the members are in the same plane, the joints are rigid and the loads are perpendicular to the plane. The only difference of grid with other plane structures is the direction of loads that are applied. As the loads are perpendicular to the plane of the structure the members of a grid in general are subjected to torsion, as well as to shear and bending. Since external loads are normal to the plane of structure, the axial deformations are negligible. As a result a free joint of a grid is subjected to a linear displacement and to a rotation in the plane of the structure. The rotation is divided into two components θ_x and θ_y. Displacement of a joint may be represented by (w, θ_x, θ_y) where the plane of the structure is in xy plane.

3.2.2 Generation of Member Stiffness Matrix

The structure lies in the xy plane and all concentrated forces act parallel to the z-axis. Loads in the form of couples have their moment vectors in the xy plane. Fig. 3.1 shows a typical member 'i' framing into joints j and k. The significant displacements of the joints are rotations in the x and y senses and translations in the z direction. The six possible displacements at the ends of member i in the directions of structure axes are shown in Fig. 3.1(c). Fig. 3.1(b) depicts the member i in conjunction with a set of member-oriented axes X_m, Y_m, and Z_m. These axes are rotated from the structure axes about the Z_m axis

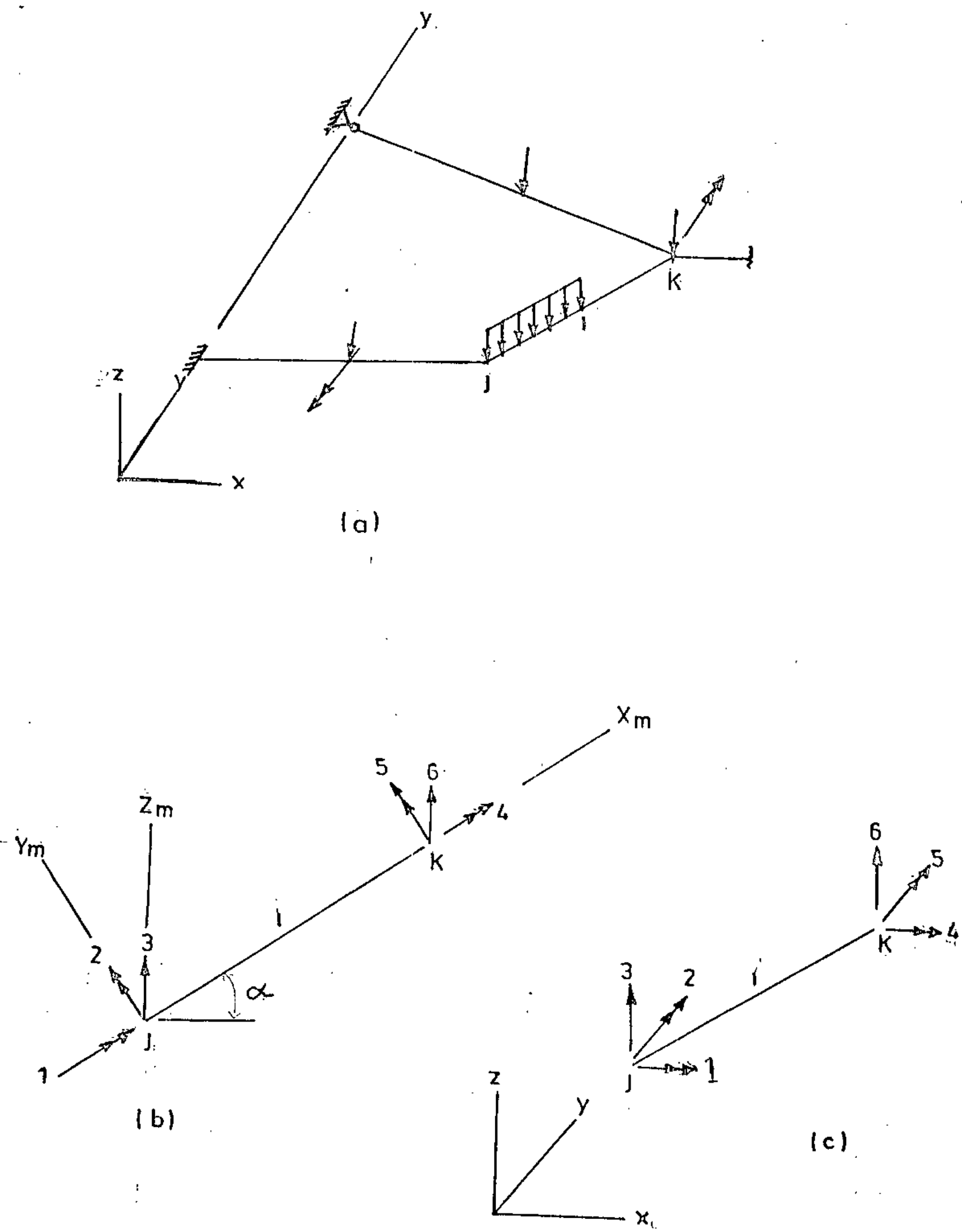


Fig. 3.1 Numbering system for a grid member.

through the angle α. The X_m-Z_m plane for each member in the grid is assumed to be a plane of symmetry. The possible displacements of the ends of member i in the directions of member axes are also indicated in Fig. 3.1(b). The six end-displacement, shown in their positive senses, consist of rotation in the X_m and Y_m senses and a translation in the Z_m direction at the ends j and k, respectively. Unit displacement of these six types may be induced at the ends of the member one at a time for the purpose of developing the member stiffness matrix $[S_m]$ for member axes. The matrix which results is as follows:

$$[S_m] = \begin{bmatrix} \frac{GI_x}{L} & 0 & 0 & \frac{-GI_x}{L} & 0 & 0 \\ 0 & \frac{4EI_y}{L} & \frac{-6EI_y}{L^2} & 0 & \frac{2EI_y}{L} & \frac{6EI_y}{L^2} \\ 0 & \frac{-6EI_y}{L^2} & \frac{12EI_y}{L^3} & 0 & \frac{-6EI_y}{L^2} & -\frac{12EI_y}{L^3} \\ -\frac{GI_x}{L} & 0 & 0 & \frac{GI_x}{L} & 0 & 0 \\ 0 & \frac{2EI_y}{L} & -\frac{6EI_y}{L^2} & 0 & \frac{4EI_y}{L} & \frac{6EI_y}{L^2} \\ 0 & \frac{6EI_y}{L^2} & -\frac{12EI_y}{L^3} & 0 & \frac{6EI_y}{L^2} & \frac{12EI_y}{L^3} \end{bmatrix}$$

Now this member stiffness matrix in local or member axes is transformed to member stiffness matrix in global or structure axes. We need the Rotation Transformation matrix to comvert it structure axes. The rotation transformation matrix for the grid is given by

$$[R_T] = \begin{bmatrix} R & 0 \\ 0 & R \end{bmatrix} \qquad (3.1)$$

where R is a 3x3 rotation matrix expressed in terms of the direction cosines of the member i shown in Fig. 3.1(b).

$$[R] = \begin{bmatrix} \cos\alpha & \sin\alpha & 0 \\ -\sin\alpha & \cos\alpha & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} C_x & C_y & 0 \\ -C_y & C_x & 0 \\ 0 & 0 & 1 \end{bmatrix} \qquad (3.2)$$

Having the rotation transformation matrix on hand, we can calculate the member stiffness matrix for structure axes using the eqn.

$$[S_{MD}] = [R_T]' \, [S_m] \, [R_T] \qquad (3.3)$$

The member stiffness matrix which results from this transformation is given by Gere and Weaver[3] as shown in Table 3.1.

3.2.3 Generation of Structure Stiffness Matrix

The structure stiffness matrix contains the terms for all of the possible joint displacement, including those

$$
[S_{MD}] = \begin{bmatrix}
\frac{GI_x}{L}C_x^2 + \frac{4EI_y}{L}C_Y^2 & \left(\frac{GI_x}{L} - \frac{4EI_y}{L}\right)C_xC_Y & \frac{6EI_Y}{L^2}C_Y & -\frac{GI_x}{L}C_x^2 + \frac{2EI_Y}{L}C_Y^2 & -\left(\frac{GI_x}{L} + \frac{2EI_Y}{L}\right)C_xC_Y & -\frac{6EI_Y}{L^2}C_Y \\[2ex]
\left(\frac{GI_x}{L} - \frac{4EI_y}{L}\right)C_xC_Y & \frac{GI_x}{L}C_Y^2 + \frac{4EI_Y}{L}C_x^2 & -\frac{6EI_Y}{L^2}C_x & -\left(\frac{GI_x}{L} + \frac{2EI_Y}{L}\right)C_xC_Y & -\frac{GI_x}{L}C_Y^2 + \frac{2EI_Y}{L}C_x^2 & \frac{6EI_Y}{L^2}C_x \\[2ex]
\frac{6EI_y}{L^2}C_Y & -\frac{6EI_Y}{L^2}C_xC_Y & \frac{12EI_Y}{L^3} & \frac{6EI_Y}{L^2}C_Y & -\frac{6EI_Y}{L^2}C_x & -\frac{12EI_Y}{L^3} \\[2ex]
-\frac{GI_x}{L}C_x^2 + \frac{2EI_y}{L}C_Y^2 & -\left(\frac{GI_x}{L} + \frac{2EI_y}{L}\right)C_xC_Y & \frac{6EI_Y}{L^2}C_Y & \frac{GI_x}{L}C_x^2 + \frac{4EI_Y}{L}C_Y^2 & \left(\frac{GI_x}{L} - \frac{4EI_Y}{L}\right)C_xC_Y & -\frac{6EI_Y}{L^2}C_Y \\[2ex]
-\left(\frac{GI_x}{L} + \frac{2EI_y}{L}\right)C_xC_y & -\frac{GI_x}{L}C_Y^2 + \frac{2EI_Y}{L}C_x^2 & -\frac{6EI_Y}{L^2}C_x & \left(\frac{GI_x}{L} - \frac{4EI_Y}{L}\right)C_xC_Y & \frac{GI_x}{L}C_Y^2 + \frac{4EI_Y}{L}C_x^2 & \frac{6EI_Y}{L^2}C_x \\[2ex]
-\frac{6EI_y}{L^2}C_Y & \frac{6EI_Y}{L^2}C_x & -\frac{12EI_Y}{L^3} & -\frac{6EI_Y}{L^2}C_Y & \frac{6EI_Y}{L^2}C_x & \frac{12EI_Y}{L^3}
\end{bmatrix}
$$

TABLE 3.1 GRID MEMBER STIFFNESS MATRIX FOR STRUCTURE AXES restrained by supports. As a preliminary step, the members and joints of the structure must be numbered. The joints are numbered consecutively 1 through n_j and the members are membered consecutively 1 through m. The three possible displacements at each joint are the joint rotations in the x and y senses, and the joint translation in the z-direction. Thus, the possible displacement at a joint j may be designated as follows:

$3j-2$ = index for rotation in the x sense

$3j-1$ = index for rotation in the y sense

$3j$ = index for translation in the z-direction.

The number of degree of freedom n in a grid is calculated from the number of joints n_j and the number of restraints n_r by the following expression

$$n = 3n_j - n_r$$

A particular member i in a grid will have joint number j and k at its end (Fig. 3.2). The end displacements at the

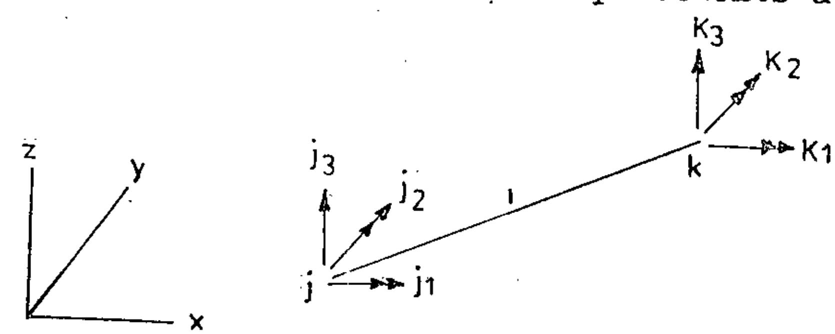


Fig. 3.2 End-displacements for grid member.

joint j and k may be indexed by the following expressions:

$$j1 = 3j - 2 \qquad j2 = 3j - 1 \qquad j3 = 3j$$
$$k1 = 3k - 2 \qquad k2 = 3k - 1 \qquad k3 = 3k \qquad (3.4)$$

These indexes are convenient for the purpose of assessing the contributions of member stiffnesses to the structure stiffness matrix. They are also useful for computing member end-actions due to joint displacements.

In order to construct the structure stiffness matrix in an orderly fashion, first, the 6x6 member stiffness matrix in structure axes, $[S_{MD}]$ is generated for the i-th member in the grid. This technique is already shown in Art. 3.2.2. Member i contributes to the stiffnesses of joint j and k at the ends of the member. Therefore, appropriate elements from the matrix $[S_{MD}]$ for this member may be transferred to the over-all structure stiffness matrix $[S]$ through an organized handling of subscripts as given by Gere and Weaver[3].

Thus the construction of the complete structure stiffness matrix $[S]$ consists of generating and transferring the matrix $[S_{MD}]$ for all members (1 through m) of the structure. After the matrix $[S]$ is generated, it must be rearranged if necessary for partitioning. In the partitioned matrix the left upper part of the matrix constitute the square, symmetric matrix $[S_D]$ corresponding to unknown displacement in the structure i.e to the degreesof freedom. This matrix

$$[S] = \begin{bmatrix} S_D & S_{DR} \\ S_{RD} & S_{RR} \end{bmatrix} \qquad (3.5)$$

is required to find the unknown displacement. The matrix $[S_{RD}]$ is a rectangular submatrix of $[S]$ that contains actions corresponding to the support restraints, due to unit values of displacements corresponding to the degree of freedom. The submatrix $[S_{DR}]$ represents actions corresponding to the degrees of freedom and caused by unit displacements corresponding to the support restraints. It can be seen that $[S_{DR}]$ is the transpose of $[S_{RD}]$. The matrix $[S_{RR}]$ is a square, symmetric submatrix of $[S]$ that contains actions corresponding to the support restraints due to unit displacements corresponding to the same set of restraints. The submatrix $[S_{DR}]$ and $[S_{RR}]$ may be used in analyzing structures having support displacements.

3.2.4 Assembly of Load Data

It is convenient initially to handle the loads at the joints and the loads on member separately. The joint loads are ready for immediate placement into a vector of actions to be used in the solution, but the loads on the members are taken into account by calculating the fixed end actions that they produce. These fixed-end actions may be transformed into equivalent joint loads and combined with the actual joint loads on the structure.

The loads applied at the joints may be listed in a vector $[A]$, which contains the applied load corresponding to all possible joint displacement, including those at support restraints. The elements in $[A]$ are numbered in the same sequence as the joint displacements. The Fig. 3.3 shows the actions which may be imposed at a typical joint k in a grid. The action A_{3k-2} is the x component of a moment vector applied at k, A_{3k-1} is the y-component of the moment vector and A_{3k} represents a force in the z-direction applied at the joint. Thus, the vector A may be formed in the sequence

$$[A] = \Big[A_1, A_2, A_3, \ldots A_{3k-2}, A_{3k-1}, A_{3k}, \ldots A_{3nj-2}, A_{3nj-1}, A_{3nj}\Big] \tag{3.6}$$

Actions $[A_{ML}]$ at the ends of a restrained grid member (due to loads on member) appear in the Fig. 3.4. The end-actions for the i-th member, with respect to member axes, are defined as follows:

$(A_{ML})_{i,1}$ = couple in the X_m sense at the j end

$(A_{ML})_{i,2}$ = couple in the Y_m sense at the j end

$(A_{ML})_{i,3}$ = force in the Z_m direction at the j end

$(A_{ML})_{i,4}$ = couple in the X_m sense at the k end

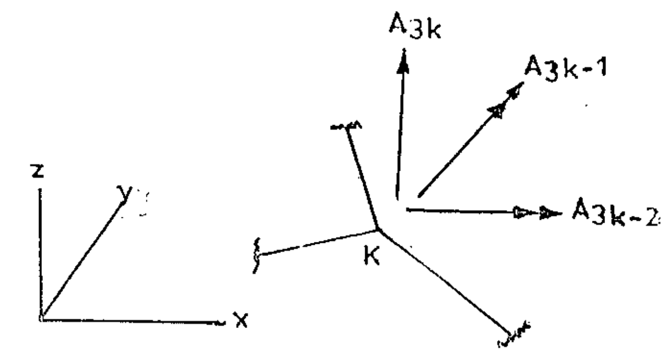


Fig. 3.3 Joint loads for a grid.

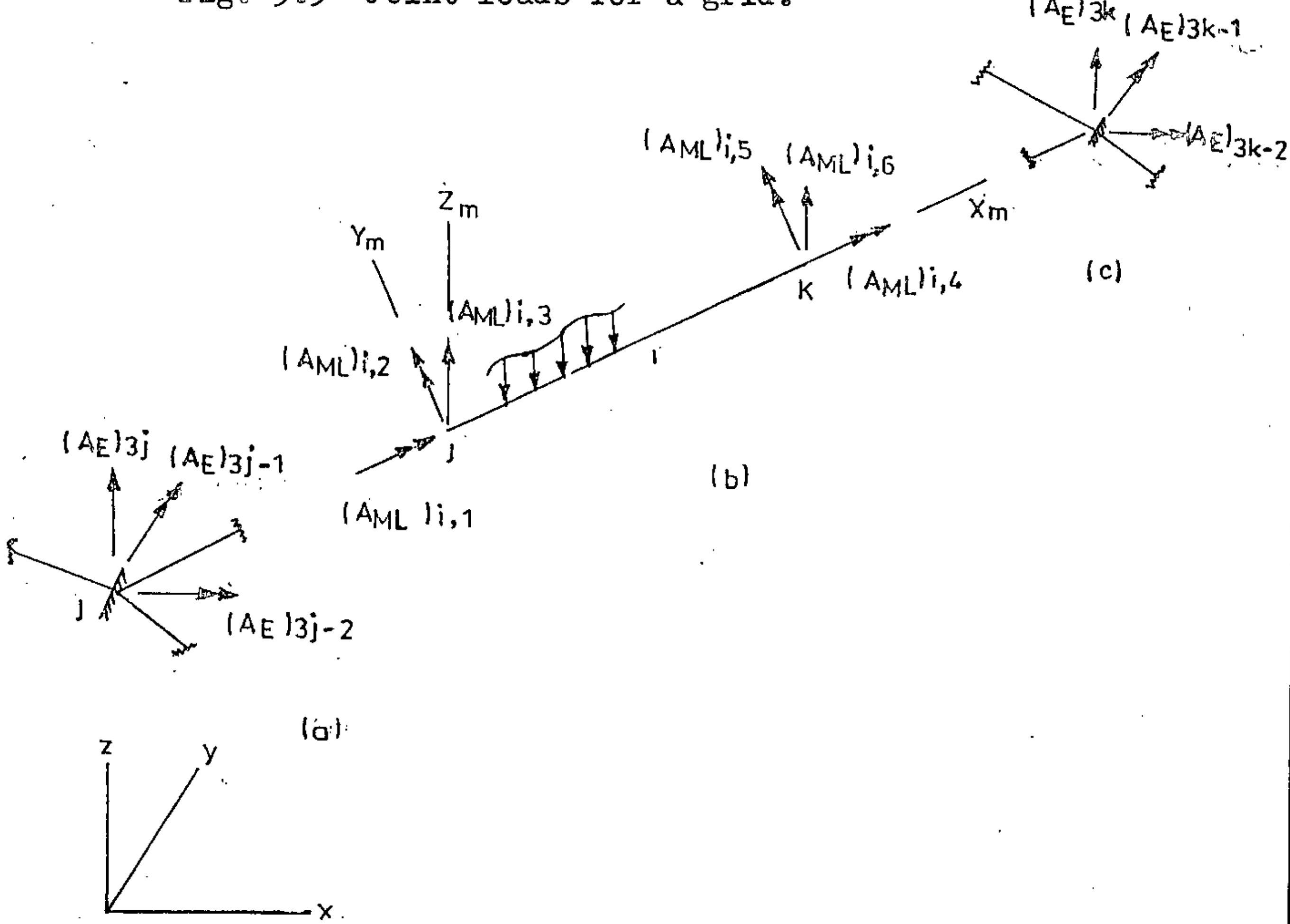


Fig. 3.4 Loads on a grid member.

$(A_{ML})_{i,5}$ = couple in the Y_m sense at the k end

$(A_{ML})_{i,6}$ = force in the Z_m direction at the k end.

The matrix A_{ML} is an array of order mx6 in which each row consist of the elements listed above for a given member i. Thus,

$$[A_{ML}] = \begin{bmatrix} (A_{ML})_{1,1} & (A_{ML})_{1,2} & \cdots & (A_{ML})_{1,6} \\ \cdots & \cdots & \cdots & \cdots \\ (A_{ML})_{i,1} & (A_{ML})_{i,2} & \cdots & (A_{ML})_{i,6} \\ \cdots & \cdots & \cdots & \cdots \\ (A_{ML})_{m,1} & (A_{ML})_{m,2} & \cdots & (A_{ML})_{m,6} \end{bmatrix} \qquad (3.7)$$

The construction of the equivalent load vector $[A_E]$ may be executed by the method rotation of axes. Fig. 3.4(a) and Fig. 3.4(c) show the equivalent loads at j and k which receive contribution from member i. The negatives of these contributions may be evaluated as follows:

$$\left[R_T'\right]_i \left[A_{ML}\right]_i = \begin{bmatrix} C_{xi} & -C_{yi} & 0 & 0 & 0 & 0 \\ C_{yi} & C_{xi} & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & C_{xi} & -C_{yi} & 0 \\ 0 & 0 & 0 & C_{yi} & C_{xi} & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} (A_{ML})_{i,1} \\ (A_{ML})_{i,2} \\ (A_{ML})_{i,3} \\ (A_{ML})_{i,4} \\ (A_{ML})_{i,5} \\ (A_{ML})_{i,6} \end{bmatrix} \qquad (3.8)$$

The expressions which result from this multiplication, with their signs reversed, represent the incremental portions of $[A_E]$ contributed by the i-th member.

Thus the equivalent joint load at a joint is constructed by summing the contributions of all the members meeting at the joint.

Addition of the vector $[A]$ and $[A_E]$ produces the combined load vector $[A_c]$. The vector $[A_c]$ may then be rearranged if necessary in the form

$$[A_c] = \begin{bmatrix} A_D \\ \cdots \\ -A_{RL} \end{bmatrix} \tag{3.9}$$

where

$[A_D]$ = the portion of the matrix corresponding to unknown displacement

$[A_{RL}]$ = portion of the matrix $[A_c]$ corresponding to reactions in the restrained structure due to loads except those that correspond to the unknown displacement.

3.2.5 Calculation of Results

After the generation of the required matrix is accomplished, the solution for joint displacement $[D]$ and support reactions $[A_R]$ are carried out.

$$\left[D\right] = \left[S_D\right]^{-1}\left[A_D\right]$$

$$\left[A_R\right] = \left[A_{RL}\right] + \left[S_{RD}\right]\left[D\right] \qquad (3.10)$$

The joint displacements D is expanded into the vector D_J . Member end-actions in the grid frame is calculated by the following equation

$$\left[A_M\right]_i = \left[A_{ML}\right]_i + \left[S_m\right]_i \left[R_T\right]_i \left[D_J\right]_i \qquad (3.11)$$

in which $\left[R_T\right]_i$ is the 6x6 rotation transformation matrix for member i, $\left[S_m\right]_i$, member stiffness matrix in member axis for member i, and $\left[D_J\right]_i$ joints displacements of the member i.

3.3 Computer Program

In the previous article, the analysis of grids structure by stiffness method have been described. Utilizing this method, a computer program has been written in Fortran IV. A flow diagram showing the sequence of operation of the program is presented in Fig. 3.5.

Informations pertaining to the structure were read. This information includes the number of members, the number of joints, the number of degrees of freedom and the elastic properties of the material. The locations of the joints of the structure were specified by means of geometric coordinates.

In addition, the sectional properties of each member in the structure were read. Finally, the conditions of restraint at the supports of the structure were read.

The member stiffness of each member was generated and then transformed and assembled in the structure stiffness matrix. To reduce the storage requirement, only the required portion of the structure stiffness matrix corresponding to unknown displacements was generated. The structure stiffness matrix was a rectangular array consisting of the matrix $[S_D]$ in the upper portion and the matrix $[S_{RD}]$ in the lower portion.

The joint designations and the actions applied at the joints were read. Similarly the member designations and the actions at the ends of restrained members due to loads were read. Finally the combined joint load vector $[A_c]$ was generated. This load vector was a column matrix consisting of the matrix $[A_D]$ in the upper portion and the matrix $[A_{RL}]$ in the lower portion.

The joint displacement were calculated by the solution of equations by Gaussian elimination method. These joints displacements were utilized to calculate the member end actions and the support reactions.

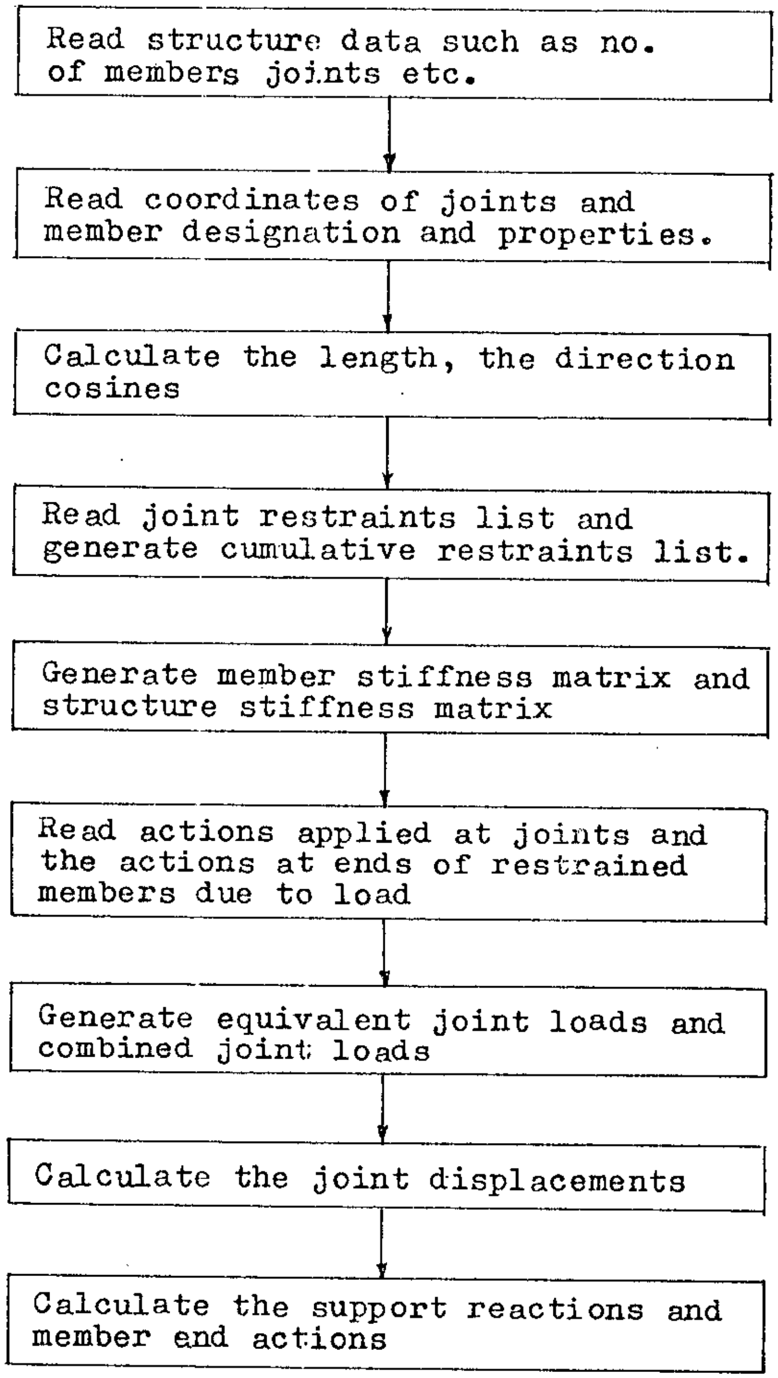


Fig. 3.5 Flow diagram.

CHAPTER 4

ANALYSIS AND DISCUSSION OF RESULTS

4.1 Analytical Procedure

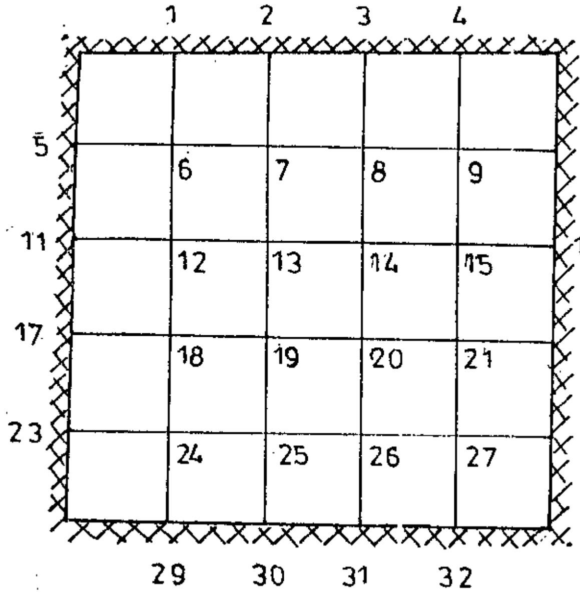
The following steps were followed in the analysis of the grid floor.

4.1.1 Member Idealization and Boundary Conditions

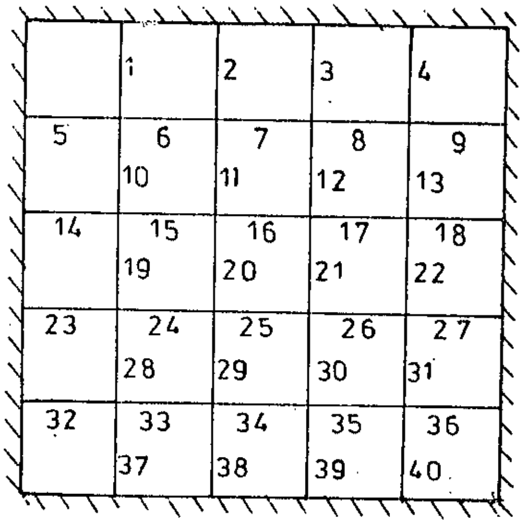
To analyze a grid floor problem by stiffness method, the geometric boundaries and conditions at the boundaries have to be properly defined. In this analysis, the idealization of the grid floor as a grid of stiffness elements is shown in Fig.4.1a. This grid may have a continuous support or it may have only corner support. The support conditions are defined as simply supported or fixed supported. In case of simple support, each node on the support is restrained to vertical displacement only. On the otherhand, the three possible movements of the node i.e rotation along x-axis, rotation along y-axis and translation along z-axis are restrained in the fixed support. The numbering of the joints or nodes and the stiffness members are shown in the Fig. 4.1(a).

4.1.2 Loading Used in the Analysis

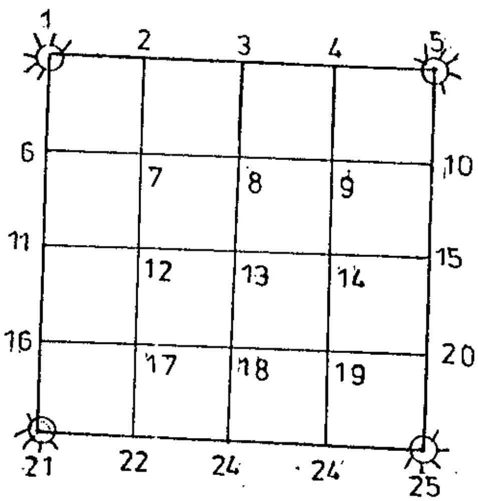
Grid floor structures carry significant live load along with self weight, which produces bending moment,



Fixed grid

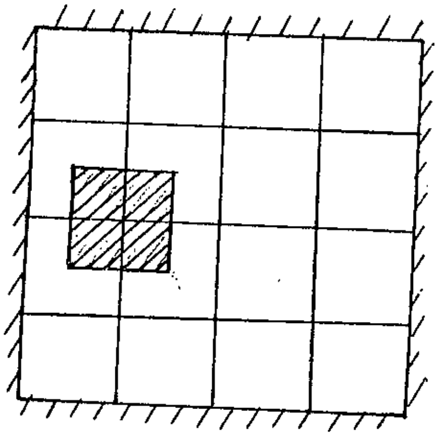


Simply supported grid



Corner supported grid.

(a)



(b)
Hatched area represent loaded area per joint

Fig. 4.1 Grid floor idealization.

shear force and torsion in the structure. A total load of 100 psf including the self weight was used in the analysis. The loads are applied along each intermediate joints except the joints along support line. This does not affect the moment, and torsion but slightly decreases the shear forces. The Fig. 4.1(b) shows the influence area of the load per joint.

4.1.3 Material Properties

For the analysis of the effect of member's width, and depth, spacing of members and support conditions in this study, grid members are assumed to behave linear elastically having a modulus of elasticity of 30×10^3 ksi and Poisson's ratio of 0.25.

But design charts and tables were developed for concrete grid floor. In that case the modulus of elasticity was taken 3.6×10^3 ksi. The Poisson's ratio of concrete was considered zero. The results vary 0-4% only for the Poisson's ratio 0.15, which is negligible.

4.1.4 Analytical Model

In this investigation the parameters studied are the depth and width of grid member, spacing of grid member, different arrangements of the grid member and supporting conditions of the grid floor. The contribution of the slab was neglected in the analysis. The results of the analyses were utilized to produce design chart for different aspect ratio of grid i.e b/a. To achieve this goal a set of computer runs were given by varying the data to incorporate the variations of structural parameters. A list of different data used for the analysis is presented in Table 4.1.

i) To study the variation of moment, shear and torsion, a 60'x60' grid floor with different support conditions was chosen. Material properties were kept constant as described in Art.4.1.3. Moment of inertia and the torsion constant were taken as 1728 in^4 and 2920 in^4 respectively. The results on the analysis are presented in Art. 4.2.

ii) To study the effect of width and depth of grid member, the same 60'x60' grid floor was analyzed. Width of grid member was varied from 12 inch to 36 inches keeping the depth constant. Similarly, the depth of grid member was varied from 12 inch to 36 inch keeping the width constant. Results of analysis are presented in Art. 4.3.

iii) Effects of the spacing of grid member were studied by analysing a 60'x60' grid floor with grid member spacing ratio m_1/m_2 equal to unity. The spacing was varied from 6 ft to 10 ft. The results of the analysis are presented in Art. 4.4.

Table 4.1 List of analyses performed for grid floors

Parameter studied	Size of grid floor in ft.	Spacing of grid member in ft.	Width of member in inch	Depth of member in inch	Moment of inertia in^4
Effect of support conditions	60x60	10x10	12	12	1728
Effect of width and depth of grid member	60x60	10x10	12	12	1728
	"	"	12	18	5832
	"	"	12	24	13824
	"	"	12	30	27000
	"	"	14	12	2016
	"	"	14	18	6804
	"	"	14	24	16128
	"	"	14	30	31500
	"	"	16	12	2304
	"	"	16	18	7776
	"	"	16	24	18432
	"	"	16	30	36000
Effect of spacing of grid member	60x60	10x10	12	12	1728
	"	7.5x7.5	12	12	1728
	"	6x6	12	12	1728
Comparison of stiffness method of analysis with plate method	75x75	12.5x12.5	12	24	13824
	150x100	12.5x12.5	12	24	13824
	150x75	12.5x12.5	12	24	13824

Table 4.1 Contd....

Parameter studied	Size of grid floor in ft.	Spacing of grid member in ft.	Width of member in inch	Depth of member in inch	Moment of inertia in^4
Design charts and tables	75x75	12.5x12.5	12	24	13824
	150x100	"	12	24	13824
	150x75	"	12	24	13824
	100x100	12.5x10.0	12	24	13824
	100x60	"	12	24	13824
	125x60	"	12	24	13824
	100x100	12.5x8.33	12	24	13824
	75x50	"	12	24	13824
	100x50	"	12	24	13824
Effect of angle of skewness of grid member					
30^{o}	60x60		12	12	1728
45^{o}	"		12	12	1728
60^{o}	"		12	12	1728

iv) The stiffness method of grid floor analysis was compared with plate method for grids having aspect ratio b/a 1.0 to 2.0. The results and the comparison of the analysis are presented in Art.4.5.

v) The results of analysis for different boundary conditions, aspect ratios, grid spacings are presented in Art. 4.6.

vi) The effect of angle of skewness of grid member was studied for a simply supported square grid floor of 60'x60'. The angle of skewness was varied from 30^{o} to 60^{o}. The results are presented in Art. 4.7.

4.2 Effect of the Support Conditions on Moment, Shear, Torsion and Deflection for Square Grid

To study the effect of the support conditions on moment, shear, deflection and torsion, a grid floor of 60'x60' was chosen with different boundary conditions such as simple support, fixed support, simple support at corner and fixed support at corner. Tables for moment, shear and torsion are presented to highlight the variation of functions along mid span, one third span, one sixth span and support line. These are presented in Table 4.2 and Table 4.3. The moments are plotted against different span direction which are presented in Fig. 4.2(a) and Fig. 4.3(a) and Fig. 4.3(b).

Table 4.2 Effect of the support conditions on moment, shear, torsion and deflection for square grid

Support condition		Along mid span members						Along one third span members					
		20-21	21-20	21-22	22-21	22-23	23-22	13-14	14-13	14-15	15-14	15-16	16-15
Simple support	Torsion	0	0	0	0	0	0	0	0	-42.1	42.1	-15.0	15.0
	Moment	0	-214.2	150.1	-225.2	195.1	-220.1	0	-176.3	125.1	-199.9	172.9	-197.7
	Shear	21.42						17.63					
	Maximum deflection	2.93						2.58					
Fixed support	Torsion	0	0	0	0	0	0	-15.0	15.0	-20.5	20.5	-8.7	8.7
	Moment	-185.0	-3.51	-19.23	-70.1	52.7	-77.8	-151.7	-4.0	-12.5	-55.3	43.5	-61.4
	Shear	18.84						15.56					
	Maximum deflection	0.604						0.48					

Spacing of grid = 10'

Load per joint = 10 kip

Units:

Moment and torsion in k-ft.

Shear in kips

Deflection in inches

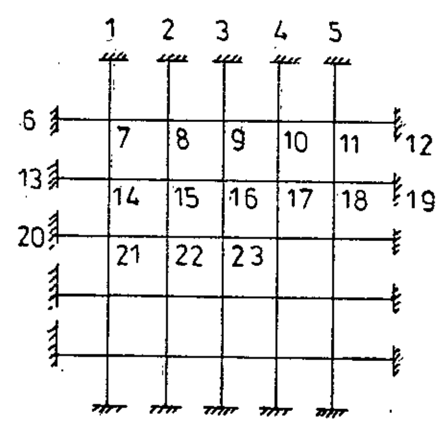


Table 4.2 (Contd..)

Support condition		Along one sixth span members					
		6-7	7-6	7-8	8-7	8-9	9-8
Simple support	Torsion	0	0	-83.2	83.2	-32.0	32.0
	Moment	0	-28.8	112.0	-91.0	133.0	-113.3
	Shear	2.88					
	Maximum deflection	1.55					
Fixed support	Torsion	-22.0	22.0	-27.9	27.9	-11.4	11.4
	Moment	-63.7	1.5	4.3	-16.5	22.0	-22.4
	Shear	6.21					
	Maximum deflection	0.203					

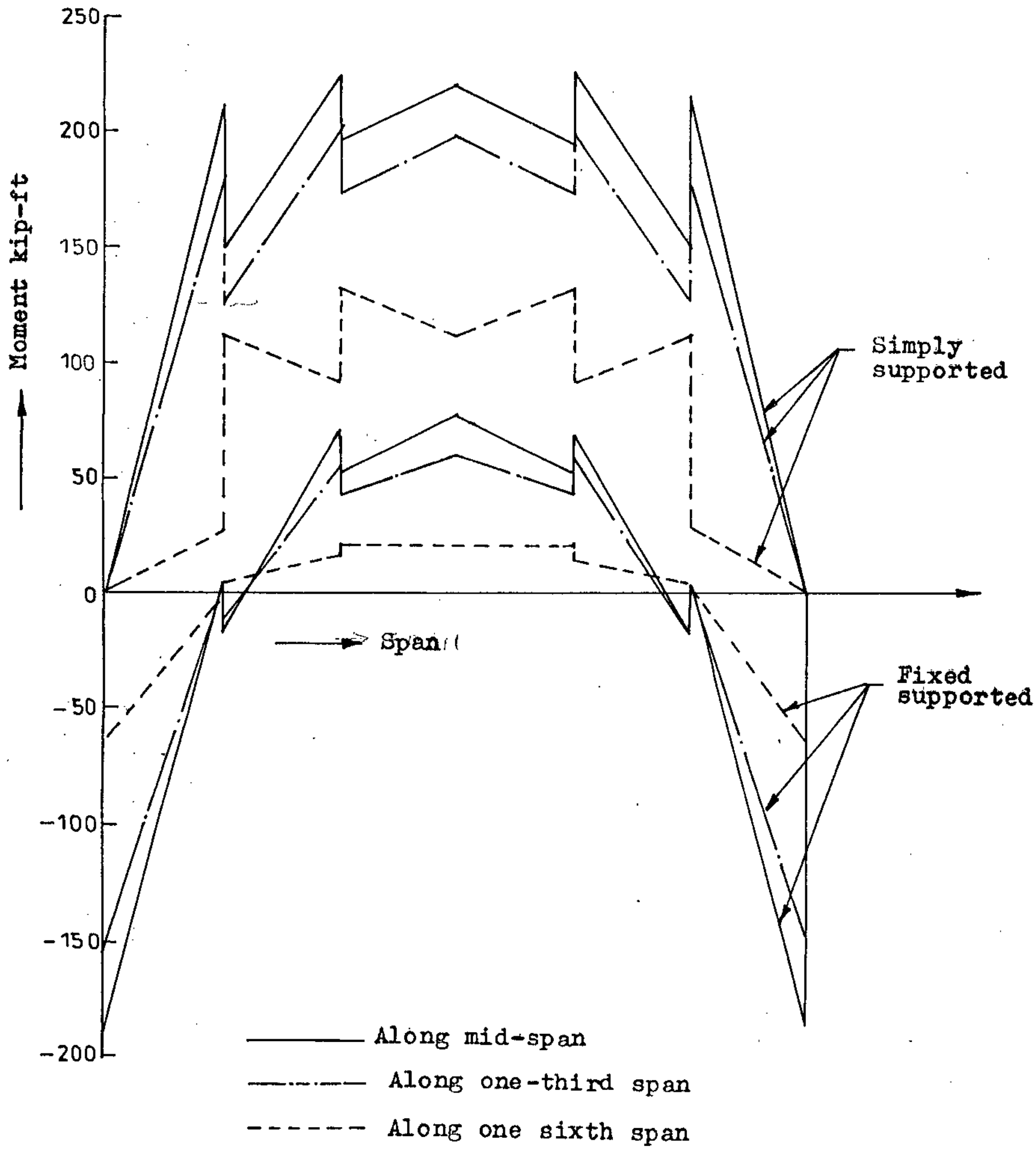


Fig. 4.2(a) Effect of support condition on moment for continuous square grid floor.

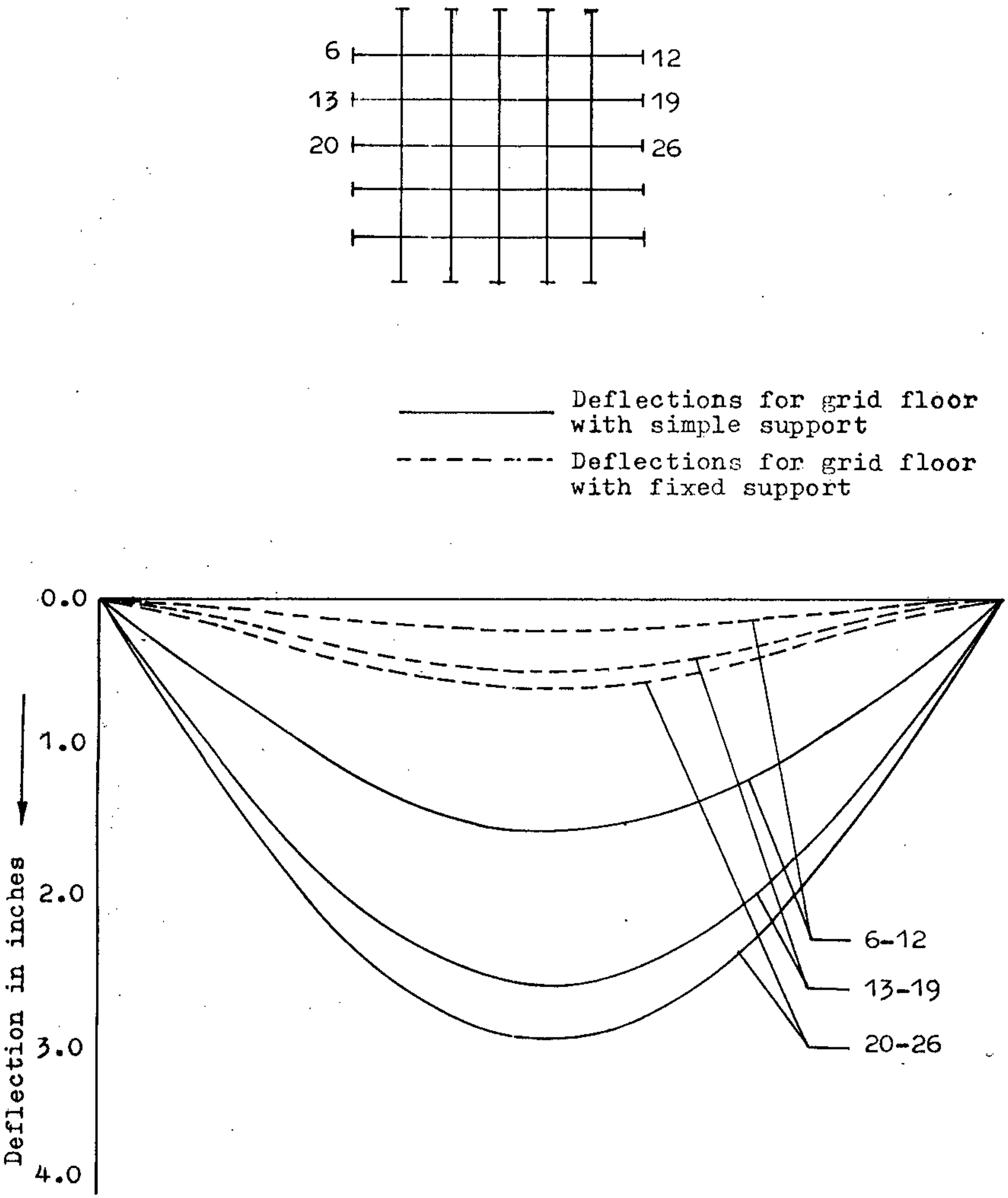


Fig. 4.2(b) Deflection curve for grid floors with simple support and fixed support.

Table 4.3 Effect of the support conditions on moment, shear, torsion and deflection for corner supported grid

Support conditions		Along mid span member						Along one third span member					
		22-23	23-22	23-24	24-23	24-25	25-24	15-16	16-15	16-17	17-16	17-18	18-17
Simple corner support	Torsion	0	0	0	0	0	0	29.8	-29.8	23.4	-23.4	8.4	-8.4
	Moment	31.9	-134.6	163.6	-231.7	248.6	-274.7	29.8	-141.2	168.5	-245.4	259.8	-288.9
	Shear	10.27						11.14					
	Maximum deflection	7.97						7.53					
Fixed corner support	Torsion	0	0	0	0	0	0	-6.2	6.2	2.5	-2.5	2.0	-2.0
	Moment	-14.1	-72.8	72.8	-138.2	142.3	-167.3	-30.6	-68.4	62.7	-142.5	142.9	-172.5
	Shear	8.69						9.9					
	Maximum deflection	3.24						2.98					

Grid spacing = 10'

Load per joint = 10 kip

Units:

Moment and torsion in k-ft.

Shear in kip

Deflection in inches.

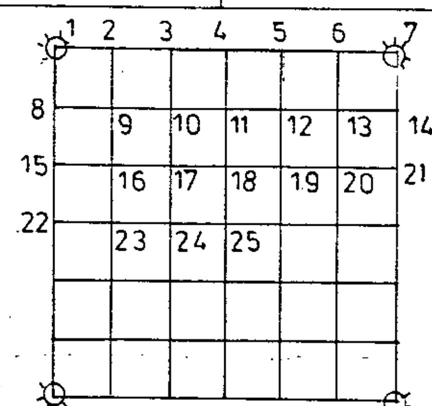


Table 4.3 (Contd..)

Support condition		Along one sixth span members						Along support line member					
		8-9	9-8	9-10	10-9	10-11	11-10	1-2	2-1	2-3	3-2	3-4	4-3
Simple corner support	Torsion	56.8	-56.8	41.8	-41.8	14.4	-14.4	59.6	-59.6	45.8	-45.8	16.0	-16.0
	Moment	13.5	-169.8	184.3	-285.8	292.0	-325.6	-59.7	-252.2	195.6	-355.3	326.0	-378.0
	Shear	15.63						31.20					
	Maximum deflection	6.31						4.62					
Fixed corner support	Torsion	-31.7	31.7	-5.8	5.8	0.0	0.0	-151.2	151.2	-37.8	37.8	-7.1	7.1
	Moment	-113.4	-56.6	30.6	-150.4	141.6	-180.9	-360.2	49.8	-18.0	-123.8	130.1	-173.6
	Shear	17.01						31.04					
	Maximum deflection	2.27						1.32					

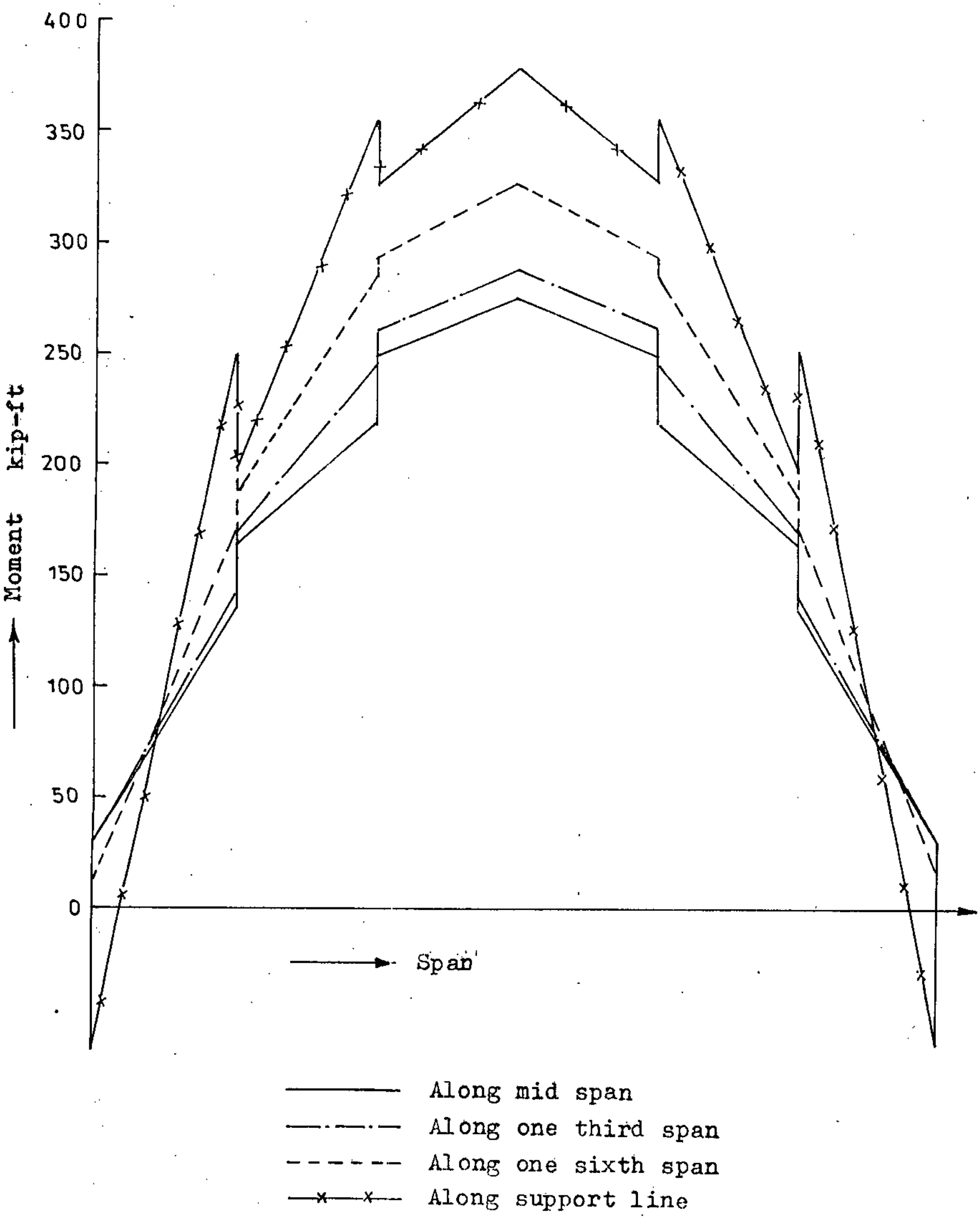


Fig. 4.3(a) Moments for grid floor with simply supported corner.

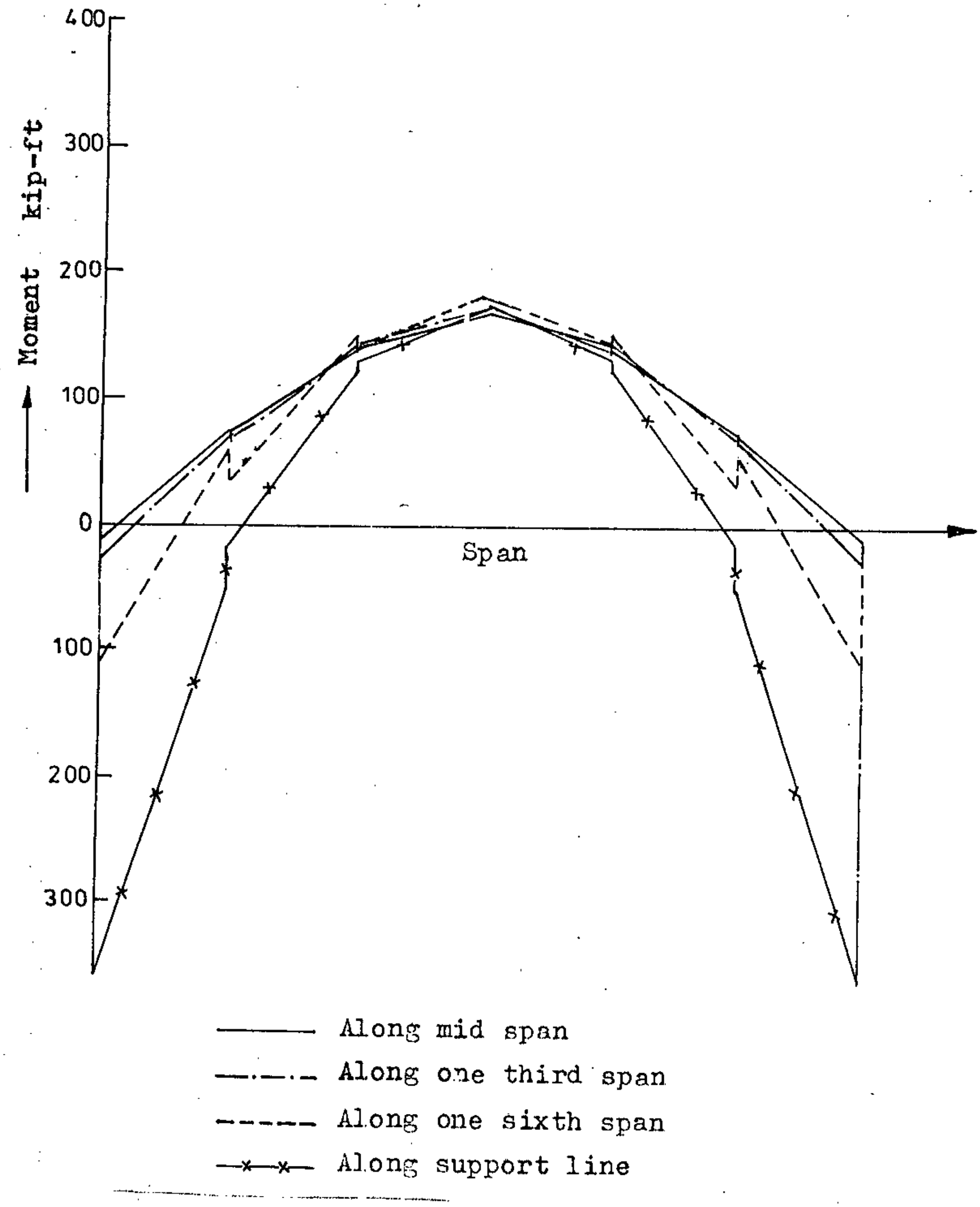


Fig. 4.3(b) Moments for grid floor with fixed supported corner.

In the bending moment diagrams, there are break points at the joints. These break points are due to the difference in moments of the two adjacent member meeting the joint and the difference in moment is equal to the difference in torsional moments of the other two member meeting perpendicularly to the joint.

For the grid floor analysed in this study the sum of the maximum negative and positive moment for fixed square grid is about 20% greater than the maximum positive moment for simply supported grid. This is due to the fact that a greater percent of load is carried by torsional moment in simply supported grid compared to fixed grid.

Maximum absolute moment for fixed square grid is about 17.8% lower than that of simply supported grid and 48.6% lower than that of grid floor with fixed supported corner. This is because deflections in the corner supported grid are higher than those of other grids.

In simply supported grid and fixed grid, the maximum moment and shear occur along central grid member where torsional moment is zero. The moment and shear reduces gradually towards the edge because, in the members located closer to edge such as 6-12, Fig.4.2(b), curvature of deflection curve is smaller than in the central or mid span member 20-26. On the other hand torsional moment increases gradually near the edge and corner, because of the twisting of the grid member. This twisting results in torsional moment which is seen to be most pronounced near the corner.

In the case of corner supported grid floor, the maximum moment occurs along support line. This is because, the load of the floor, is transferred to the support through the member along the support line.

4.3 Effect of the Width and Depth of Grid Member

To ascertain the effect of the width of grid member on moment, shear and torsion, the width of grid beam was varied keeping the depth of grid member constant for a grid floor of 60'x60' at two different boundary conditions (simple support and fixed support). Results of the analysis for various width of grid beam are summarized in Table 4.4. The bending moment diagram along mid span and one third span member are plotted in Fig. 4.4 to Fig. 4.5(b). Similarly to ascertain the effect of depth of grid member the depth of grid beam has been varied keeping the width of grid beam constant for the same grid floor. The result of the analysis are summarized in Table 4.5 and in Fig. 4.6 to Fig. 4.7(b).

It has been found that moment decreases continuously with the increasing width of beam and increases with the increasing depth of beam. On the other hand, torsional moment increases with the increasing width of beam and decreases with the increasing depth of beam. The percentage

Table 4.4 Effect of the width of grid member on moment, shear, deflection and torsion for simply supported square grid

Width and depth		Along mid span member						Along one third span member					
		20-21	21-20	21-22	22-21	22-23	23-22	13-14	14-13	14-15	15-14	15-16	16-15
Width =12" Depth =12"	Torsion	0	0	0	0	0	0	0	0	-42.1	42.1	-15.0	15.0
	Moment	0	-214.2	150.1	-225.2	195.1	-220.1	0	-176.3	125.1	-199.9	172.9	-197.7
	Shear	21.42						17.63					
	Maximum deflection	2.93						2.58					
Width =14" Depth =12"	Torsion	0	0	0	0	0	0	0	0	-45.85	45.85	-16.36	16.36
	Moment	0	-219.2	147.9	-223.0	190.2	-214.97	0	-179.2	123.0	-198.2	168.8	-193.75
	Shear	21.92						17.92					
	Maximum deflection	2.47						2.17					
Width =16" Depth =12"	Torsion	0	0	0	0	0	0	0	0	-48.95	48.95	-17.53	17.53
	Moment	0	-224.12	146.64	-221.2	186.1	-211.1	0	-182.27	121.82	-196.85	165.47	191.01
	Shear	22.41						18.82					
	Maximum deflection	2.138						1.88					

Grid spacing = 10'

Load per joint = 10 kip

Unit:

Moment and torsion in k-ft

Shear in kip

Deflection in inch

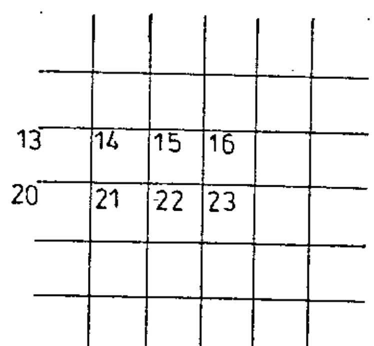


Table 4.4 (Contd...) Effect of the width of grid member on moment, shear, deflection and torsion for fixed grid

Width and depth		Along mid span member						Along one third span member					
		20-21	21-20	21-22	22-21	22-23	23-22	13-14	14-13	14-15	15-14	15-16	16-15
Width =12" Depth =12"	Torsion	0	0	0	0	0	0	-15.0	15.0	-20.5	20.5	-8.7	8.7
	Moment	-185.0	-3.5	-19.2	-70.1	52.7	-77.8	-151.7	-4.0	-12.5	-55.3	43.5	-61.4
	Shear	18.84						15.56					
	Maximum deflection	0.604						0.488					
Width =14" Depth =12"	Torsion	0	0	0	0	0	0	-16.92	16.92	-22.66	22.66	-9.55	9.55
	Moment	-183.21	-5.93	-19.39	-69.92	50.82	-75.88	-150.31	-5.74	-12.67	-55.11	42.0	-60.02
	Shear	18.91						15.61					
	Maximum deflection	0.506						0.410					
Width =16" Depth =12"	Torsion	0	0	0	0	0	0	-18.58	18.58	-24.49	24.49	-10.28	10.28
	Moment	-181.77	-7.99	-19.5	-69.81	49.24	-74.20	-149.16	-7.26	-12.80	-55.05	40.86	-58.73
	Shear	18.98						15.64					
	Maximum deflection	0.434						0.352					

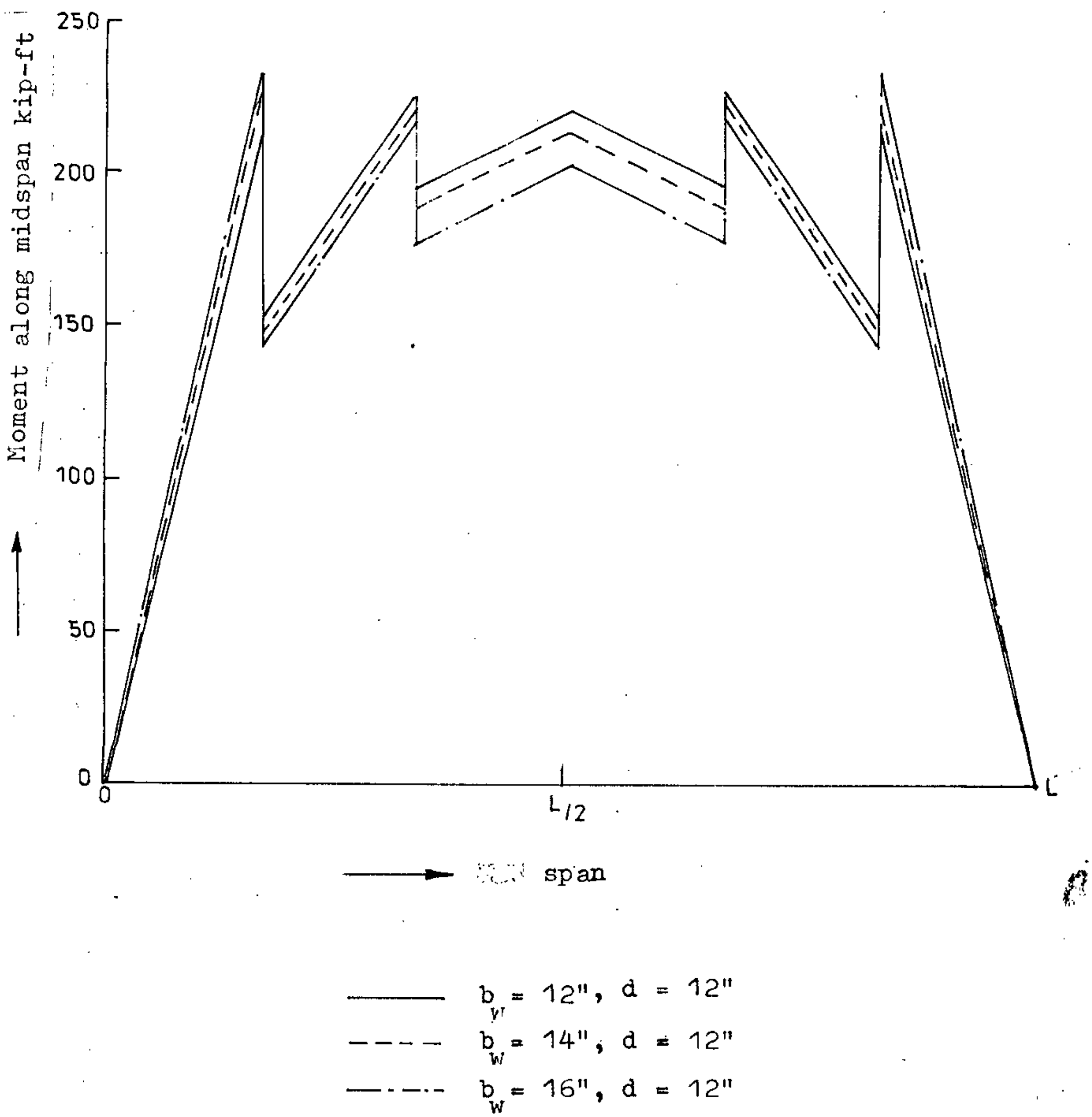


Fig. 4.4 Effect of the width of grid member on moment along mid span for simply supported square grid.

Table 4.5 Effect of the depth of grid member on moment, shear, deflection and torsion for simply supported grid

Depth and width		Along mid span member						Along one third span member					
		20-21	21-20	21-22	22-21	22-23	23-22	13-14	14-13	14-15	15-14	15-16	16-15
Depth =12" Width =12"	Torsion	0	0	0	0	0	0	0	0	-42.1	42.1	-15.0	15.0
	Moment	0	-214.2	150.1	-225.2	195.1	-220.1	0	-176.3	125.1	-199.9	172.9	-197.7
	Shear	21.42						17.63					
	Maximum deflection	2.934						2.58					
Depth =18" Width =12"	Torsion	0	0	0	0	0	0	0	0	-30.0	30.0	-10.7	10.7
	Moment	0	-197.9	155.0	-232.0	210.6	-235.6	0 0	-166.2	130.5	-204.8	185.5	-209.4
	Shear	19.8						16.62					
	Maximum deflection	0.911						0.798					
Depth =24" Width =12"	Torsion	0	0	0	0	0	0	0	0	-21.5	21.5	-7.7	7.7
	Moment	0	-187.5	157.8	-236.5	221.1	-246.2	0	-159.7	134.4	-207.8	194.0	-217.1
	Shear	18.75						15.97					
	Maximum deflection	0.396						0.346					

Grid spacing = 10'

Load per joint = 10 kip

Unit:

Moment and torsion in kip-ft

Shear in kip

Deflection in inch.

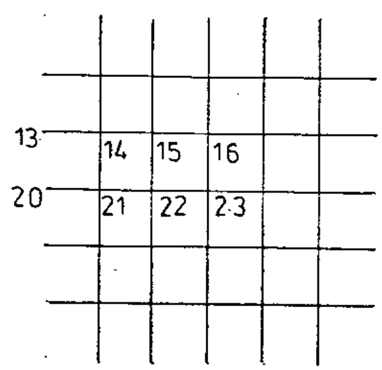


Table 4.5 (Contd...) Effect of the depth of grid member on moment, shear, deflection and torsion for fixed grid

Depth and width		Along mid span member 20-21	21-20	21-22	22-21	22-23	23-22	Along one third span member 13-14	14-13	14-15	15-14	15-16	16-15
Depth =12" Width =12"	Torsion	0	0	0	0	0	0	-15.0	15.0	-20.5	20.5	-8.7	8.7
	Moment	-185.0	-3.5	-19.2	-70.1	52.7	-77.8	-151.7	-4.0	-12.5	-55.3	43.5	-61.4
	Shear	18.84						15.56					
	Maximum deflect-ion	0.604						0.488					
Depth =18" Width =12"	Torsion	0	0	0	0	0	0	-9.7	9.7	-14.0	14.0	-6.0	6.0
	Moment	-189.5	3.7	-19.0	-70.4	58.4	-83.4	-155.5	1.3	-12.2	-55.6	47.7	-65.6
	Shear	18.57						15.42					
	Maximum deflec-tion	0.190						0.153					
Depth =24" Width =12"	Torsion	0	0	0	0	0	0	-6.5	6.5	-9.8	9.8	-4.2	4.2
	Moment	-192.2	8.4	-19.0	-70.5	62.1	-87.1	-157.8	4.5	-12.1	-55.9	50.3	-68.2
	Shear	18.38						15.33					
	Maximum deflec-tion	0.083						0.066					

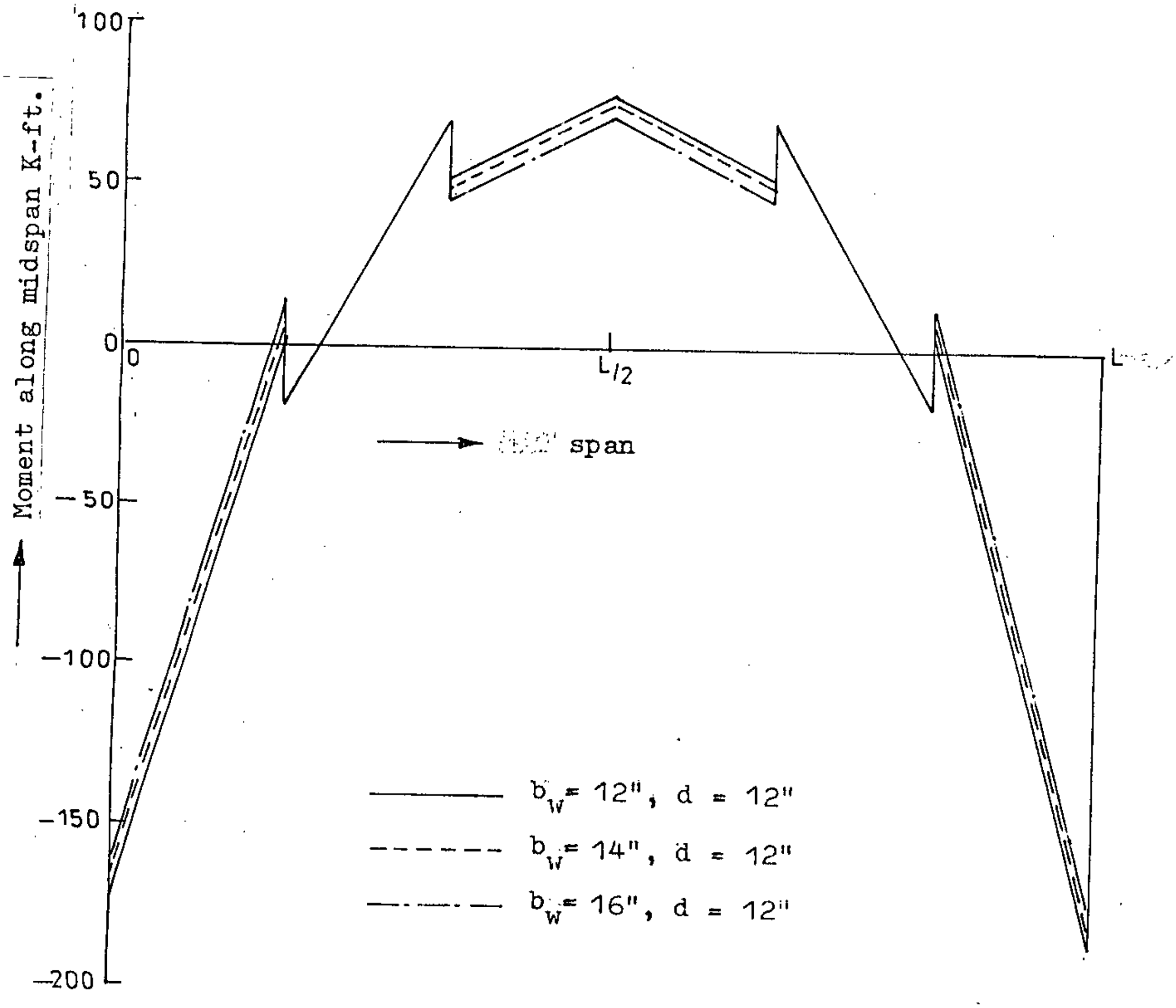


Fig. 4.5(a) Effect of the width of grid member on moment along midspan for fixed square grid.

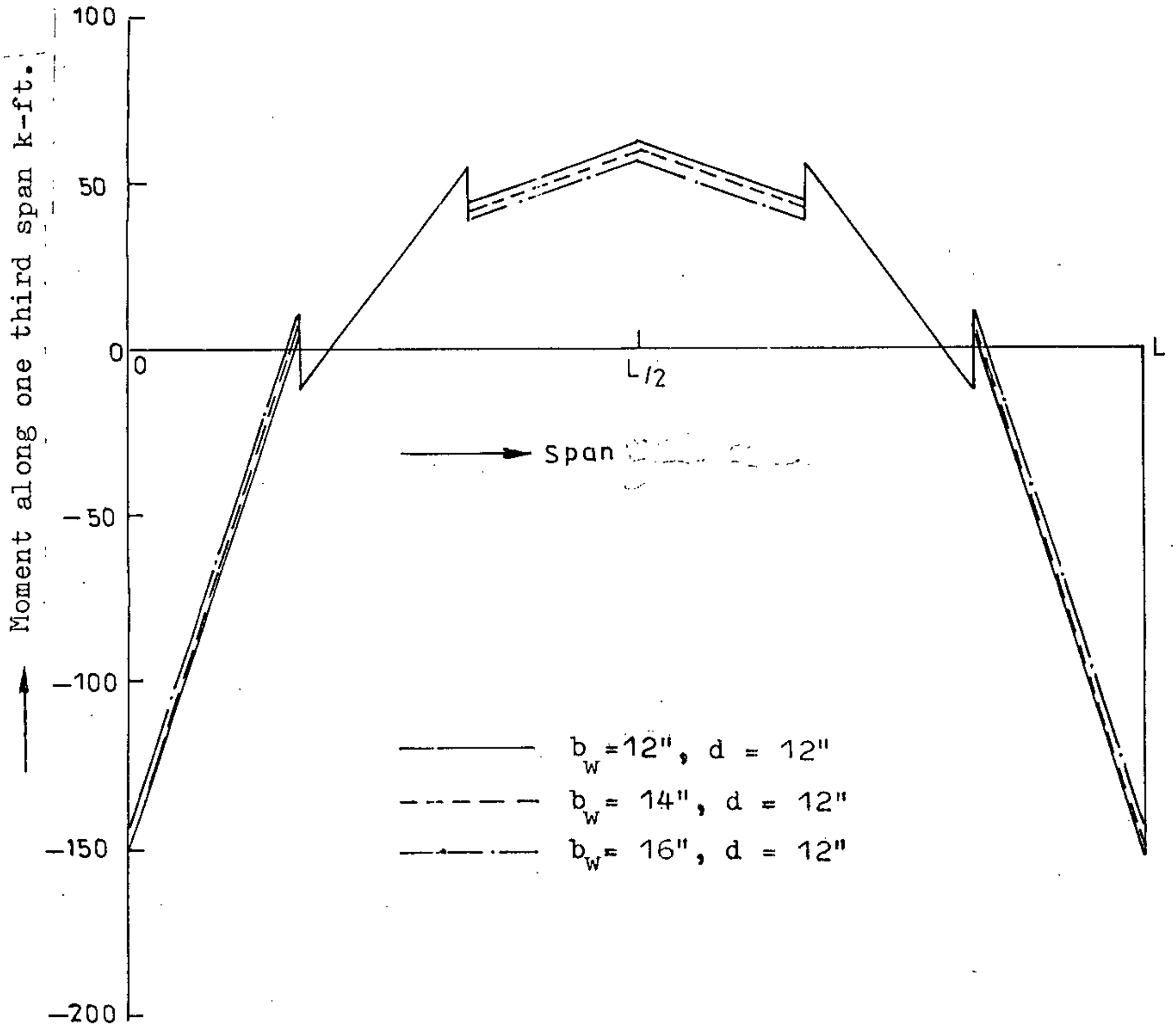


Fig. 4.5(b) Effect of the width of grid member on moment along one third span for fixed square grid.

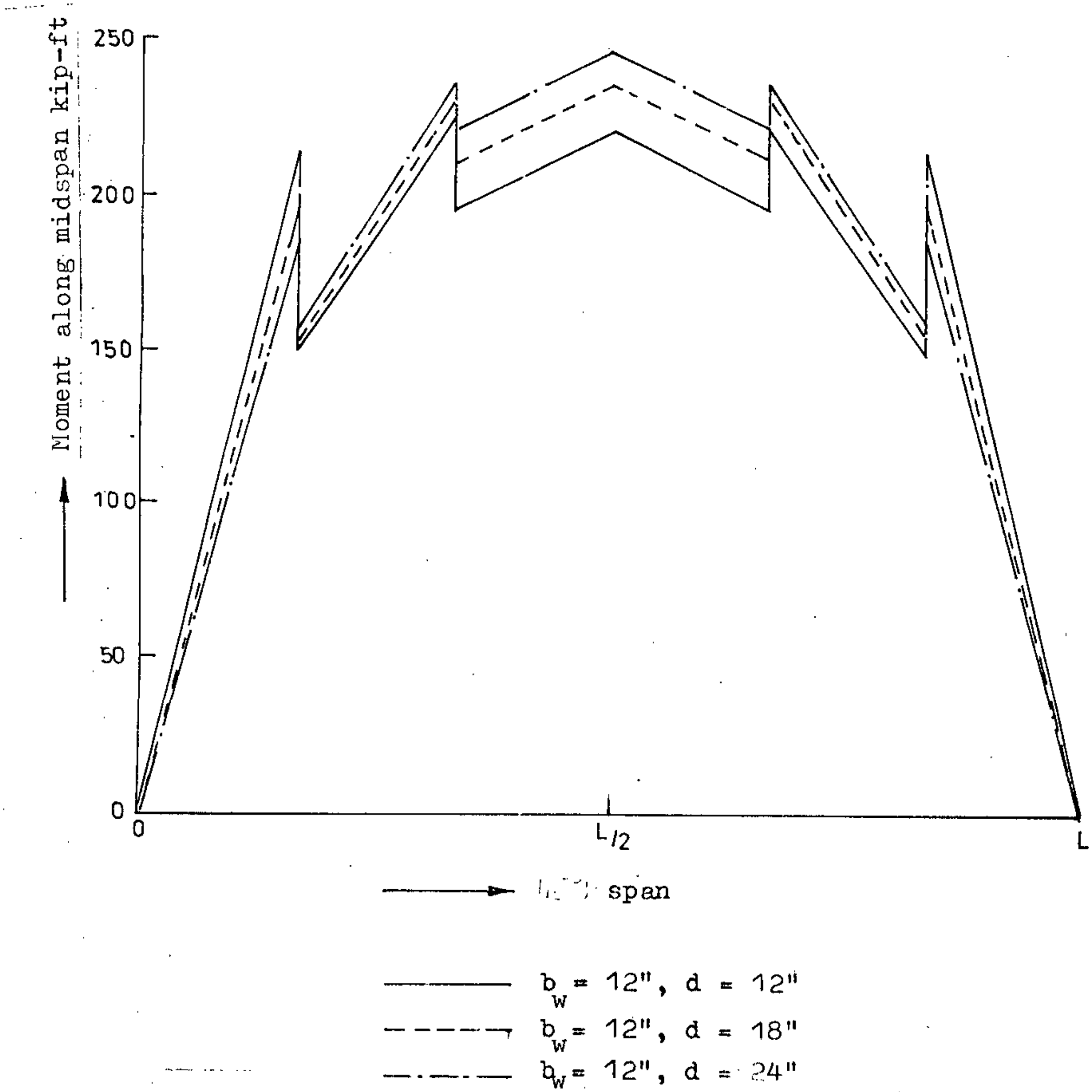


Fig. 4.6 Effect of the depth of grid member on moment along mid span for simply supported square grid.

Table 4.6 Percentage of variation of moment and torsion for various depth and width.
With respect to b_w = 12", d = 12"

Percentage increase in width or depth			% of change in moment at center	% of change in maximum torsion	% of change in deflection at center
Simply supported grid	Width	16%	-2.3	10.3	-15.7
		33%	-4.0	19.2	-27.03
		50%	-5.6	26.0	-35.8
		100%	-8.4	39.7	-52.9
		150%	-10.0	47.5	-62.7
		200%	-11.1	52.6	-69.2
	Depth	50%	7.0	-31.3	-68.9
		100%	11.8	-51.8	-86.5
		150%	14.7	-65.1	-92.9
		200%	16.9	-73.7	-95.8
Fixed grid	Width	16%	-2.4	11.4	-16.2
		33%	-4.6	21.2	-28.1
		50%	-6.3	29.1	-37.1
		100%	-9.6	44.8	-54.3
		150%	-11.5	54.1	-64.2
		200%	-12.8	60.2	-70.5
	Depth	50%	7.2	-33.3	-68.5
		100%	11.8	-54.1	-86.2
		150%	14.7	-67.0	-92.8
		200%	16.7	-75.6	-95.8

* Negative sign indicates decrease in values.

Table 4.7(a) Variation of torsion constant and moment of of inertia with the width and depth of grid member

Width 'b_w' inch	Depth 'd' inch	d/b_w	Torsion constant J	Moment of inertia I	$\rho = \frac{EI}{GJ}$
36	12	0.333	16385	5184	0.790
30	12	0.40	12934	4320	0.835
24	12	0.5	9492	3456	0.910
18	12	0.666	6085	2592	1.064
16	12	0.75	4976	2304	1.157
14	12	0.857	3906	2016	1.290
12	12	1.0	2920	1728	1.479
12	18	1.5	6085	5832	2.396
12	24	2.0	9492	13834	3.643
12	30	2.5	12934	27000	5.218
12	36	3.0	16385	46656	7.118

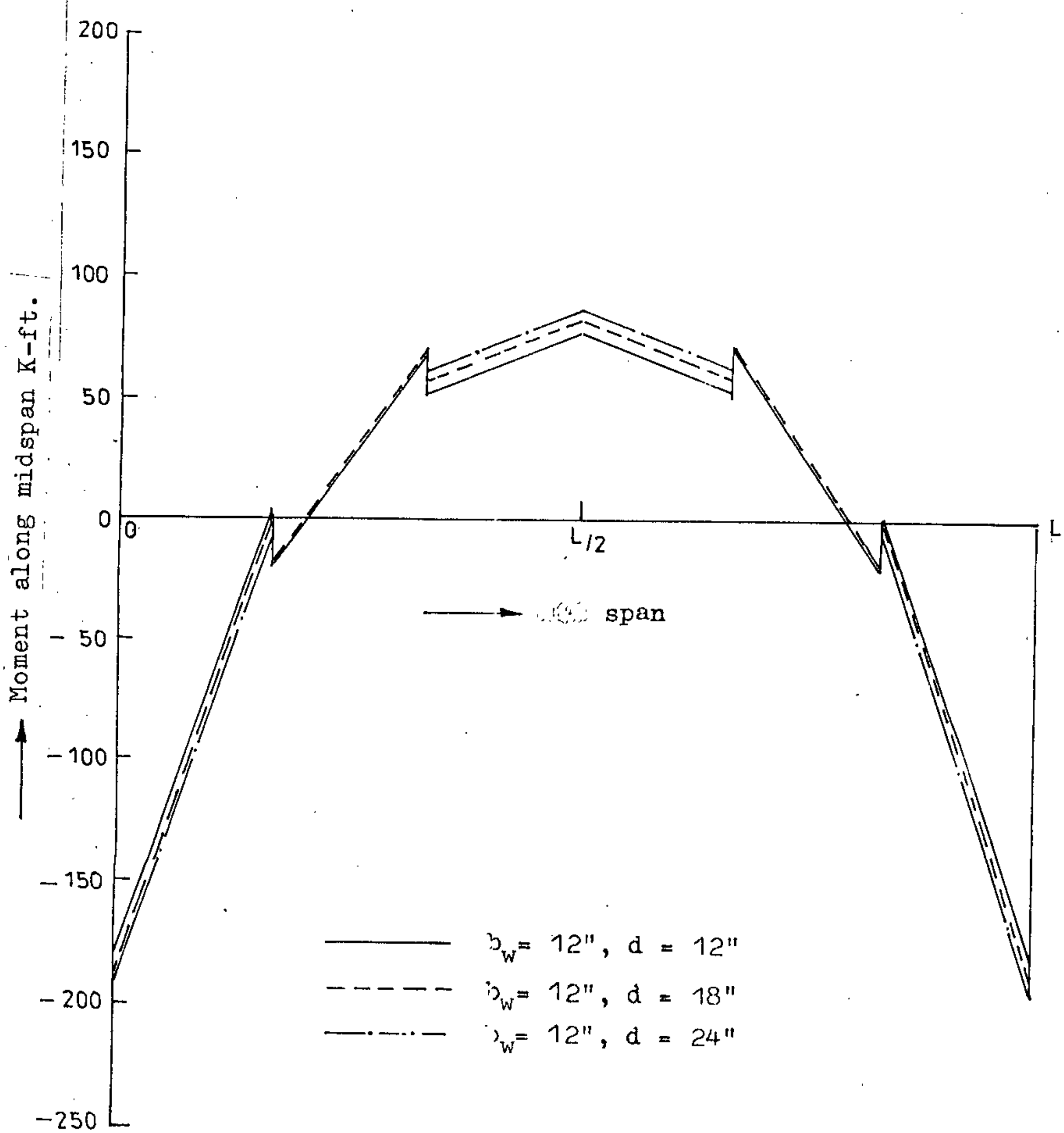


Fig. 4.7(a) Effect of the depth of grid member on moment along midspan for fixed square grid.

Table 4.7(b) Variation of moment and torsion with the ρ, ratio of bending stiffness to torsional stiffness.

$\rho = \frac{EI}{GJ}$	Grid floor with simple support				Grid floor with fixed support					
	maxm.+ve moment at center k-ft	Coefficient	Maximum torsion k-ft	Coefficient	Maxm.-ve moment k-ft	Coefficient	Maxm. +ve moment at center k-ft	Coefficient	Maxm. torsion k-ft	Coeff-cient
0.790	195.6	0.0543	127.0	0.0352	175.5	0.0487	67.8	0.0188	44.7	0.0124
0.835	197.9	0.0549	122.8	0.0341	176.5	0.0490	68.8	0.019	43.0	0.0119
0.910	201.5	0.0559	116.3	0.0323	178.0	0.0494	70.3	0.0195	40.4	0.0112
1.064	207.7	0.0576	104.9	0.0291	180.5	0.0501	72.9	0.020	36.03	0.0100
1.479	220.1	0.0611	83.2	0.0231	184.9	0.0513	77.8	0.0216	27.9	0.00775
2.396	235.6	0.0654	57.1	0.0158	189.5	0.0526	83.4	0.0231	18.6	0.00516
3.643	246.2	0.0683	40.1	0.0111	192.2	0.0533	87.0	0.0241	12.8	0.00355
5.218	252.5	0.0701	29.0	0.0080	193.7	0.0538	89.3	0.0248	9.2	0.00255
7.118	257.3	0.0714	21.8	0.0060	194.8	0.0541	90.8	0.0252	6.8	0.00188

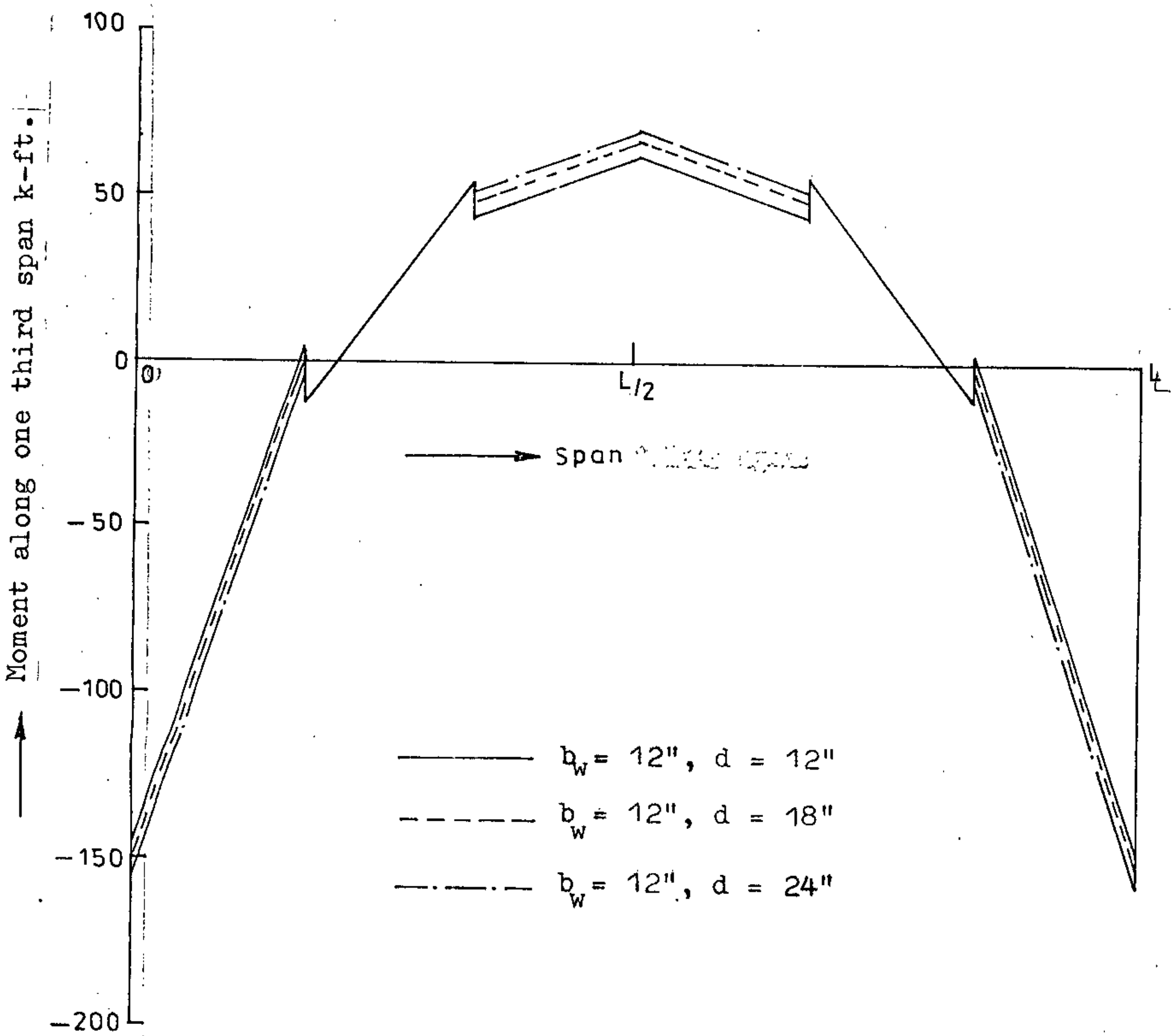


Fig. 4.7(b) Effect of the depth of grid member on moment along one third span for fixed square grid.

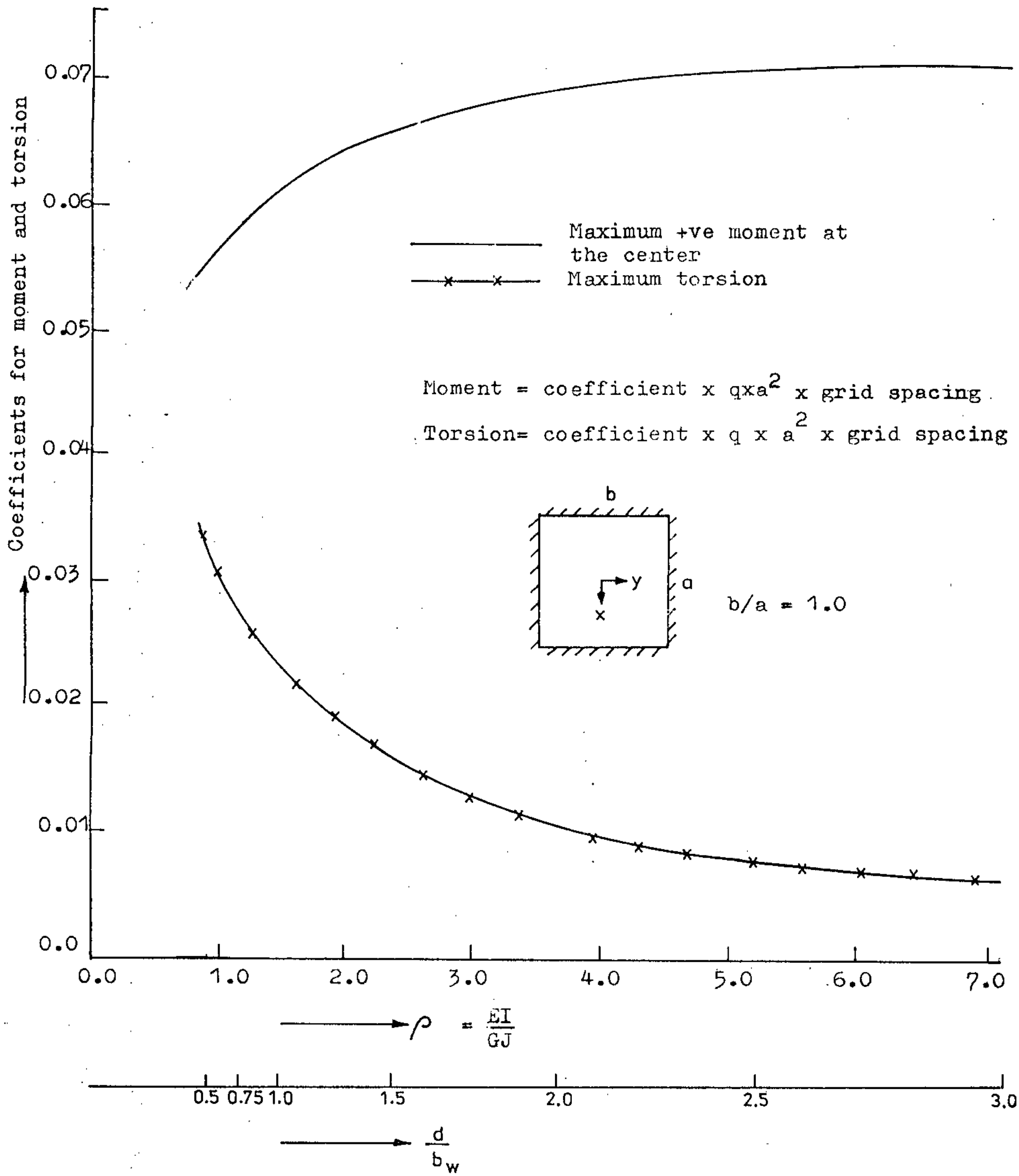


Fig. 4.8(a) Moment and Torsion Vs. ρ for grid floors with simple support.

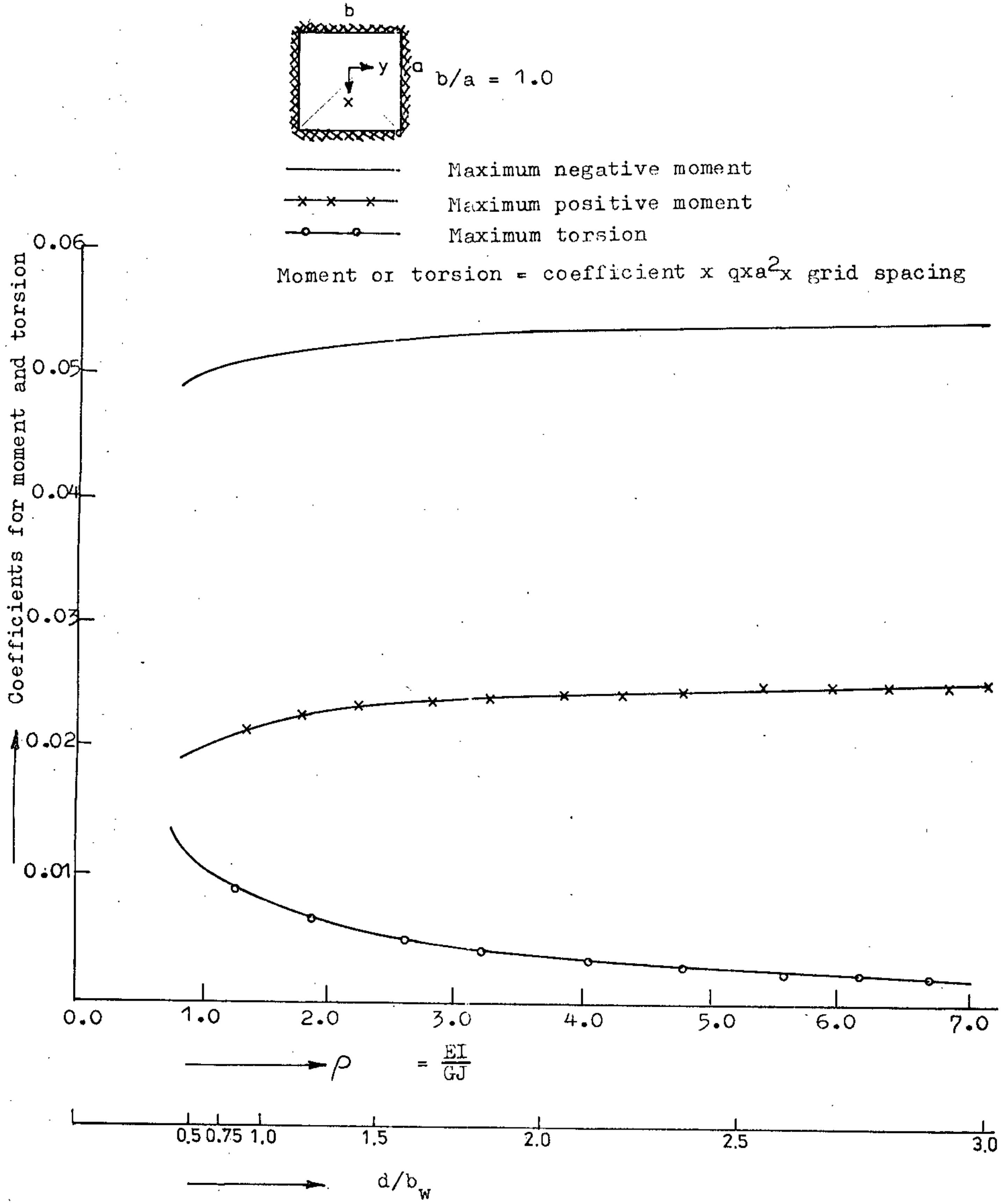


Fig. 4.8(b) Moment and torsion vs. ρ for grid floor with fixed support.

of decrease and increase in moment and torsion is shown in Table 4.6. The deflection decrease significantly with the increase in depth of the member.

The increase in torsional moment with the increase in width of grid member may be attributed to the fact that greater width of beam decreases the parameter ρ , the ratio of the bending stiffness to torsional stiffness. Similarly the increase in bending moment with the increase in depth of beam is due to the fact that the greater depth of beam increases the parameter ρ . The variation of torsion constant, moment of inertia and ρ with the width and depth of grid member is shown in Table 4.7(a).

The variations of torsion and moment with the ρ are given in Table 4.7(b) and in Fig. 4.8(a) and Fig. 4.8(b). It is noted that for a change of ρ , for simply supported square grid floor, from 1.0 to 7.2, the maximum torsional moment decreases by 79.2%, while the maximum bending moment increases by 23.8%. Similarly for fixed square grid floor these values are 81.1% for torsion; 24.5% and 7.9% for positive and negative moments respectively.

The torsion constant and moment of inertia for T-beam sections have been given in Fig. 4.9(a) and Fig. 4.9(b). Although the contribution of the slab in the stiffness formulation was neglected, these tables can be used to study the effect of the slab in the analysis.

4.4 Effect of Spacing of Grid Member

The effect of spacing of grid member on the moment, shear and torsion was studied by varying the spacing of grid member from 6 feet to 10 feet for a square grid floor of 60'x60' with simple and fixed support conditions. The results of the analysis are shown in Table 4.8 and Table 4.9. The moment diagram along midspan members is plotted in the Fig. 4.10 and Fig. 4.11. A comparison on moment and shear as obtained by computer analysis with the result as obtained by proportionate rule is presented in Table 4.10.

It is observed that the percentage of reduction of moment, shear and deflection is approximately equal to the percentage of reduction of the grid beam spacing. The percentage of reduction of torsion is significantly less than the percentage of reduction of grid beam spacing.

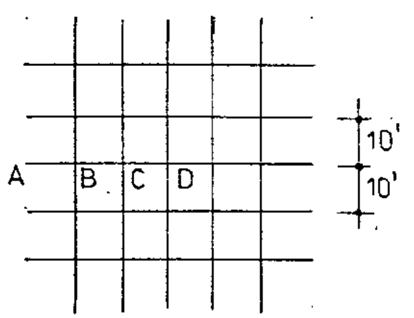
This effect is due to the fact that in the case of smaller spacing of grid members, the total load of the grid floor is carried by a large number of members and hence the load per joint reduces. Another point is that with smaller spacing, the grid floor behaves more like a solid slab having larger torsional stiffness.

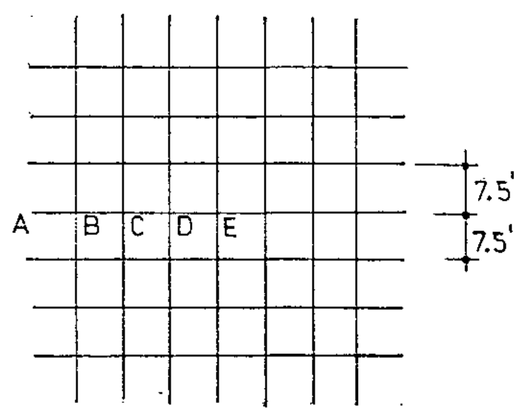
4.5 Comparison of Results of Analysis by Stiffness Method with Plate Method

To compare the results in between stiffness method and plate method, the aspect ratio b/a was varied from 1.0 to 2.0 for both the simply supported grid and fixed grid

Table 4.8 Effect of grid spacing on moment, shear, deflection for simply supported grid

Member		AB	BA	BC	CB	CD	DC	DE	ED	EF	FE	Maximum deflection
Grid spacing=10'	Torsion	0.0	0.0	0.0	0.0	0.0	0.0					
	Moment	0.0	-214.1	150.1	-225.2	195.1	-220.1					2.934
	Shear	21.42										
Grid spacing=7.5'	Torsion	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0			
	Moment	0.0	-131.1	92.5	-150.2	124.3	-155.7	142.9	-153.6			2.069
	Shear	17.48										
Grid spacing=6'	Torsion	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	
	Moment	0.00	-87.7	62.7	-107.9	88.1	-117.0	103.7	-120.3	113.6	-119.1	1.605
	Shear	14.61										





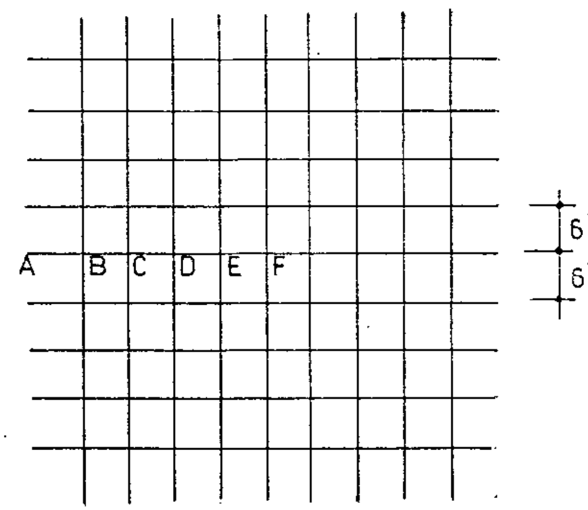


Table 4.9 Effect of grid spacing on moment, shear, deflection for fixed grid.

Member		AB	BA	BC	CB	CD	DC	DE	ED	EF	FE	Maximum deflection
Grid spacing=10'	Torsion	0.0	0.0	0.0	0.0	0.0	0.0	0.0				
	Moment	-185.0	-3.5	-19.2	-70.1	52.8	-77.8					0.604
	Shear	18.84										
Grid spacing= 7.5'	Torsion	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0			
	Moment	-140.5	25.9	-37.6	-31.2	18.2	-53.2	45.3	-55.9			0.449
	Shear	15.28										
Grid spacing=6'	Torsion	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	
	Moment	-113.2	36.4	-43.1	-9.0	0.3	-32.7	25.3	-42.7	38.5	-43.9	0.358
	Shear	12.8										

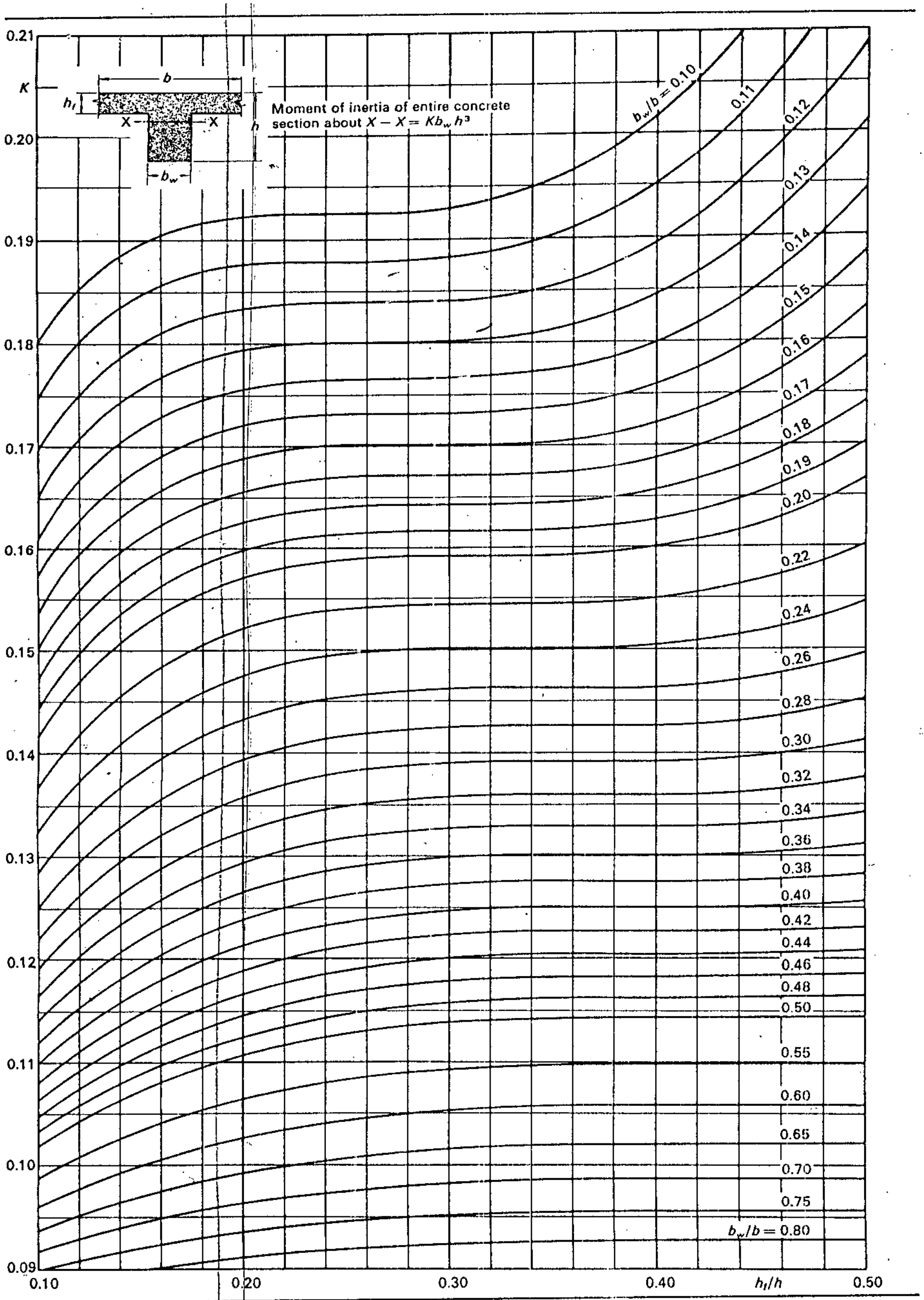


Fig. 4.9(a) Moment of inertia for T-beam. (Source: Reinforced Concrete Designer's Hand book, 8th-ed, by Charles E Renolds & James C Steedman).

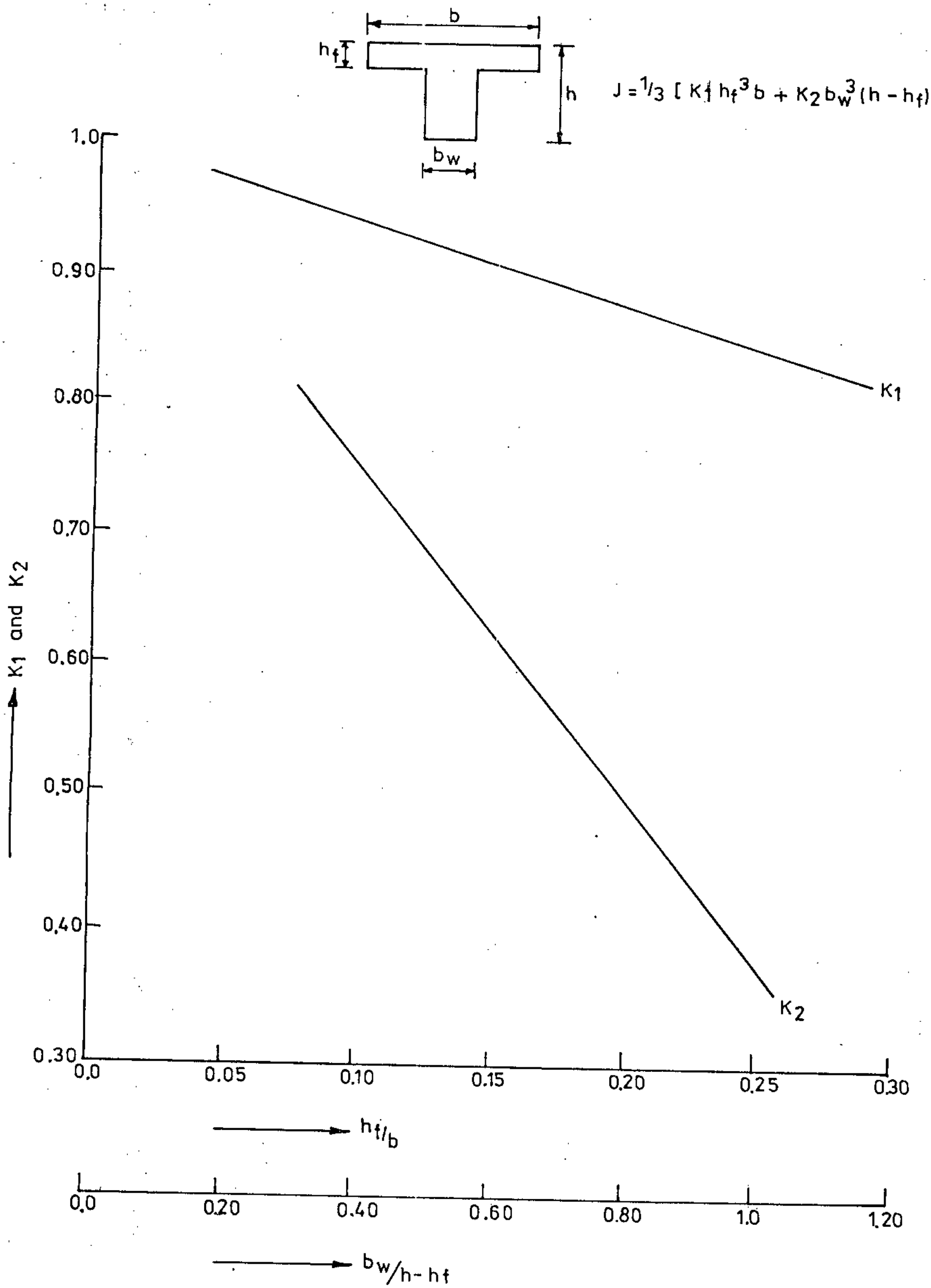


Fig. 4.9(b) Torsion constant for T-beam.

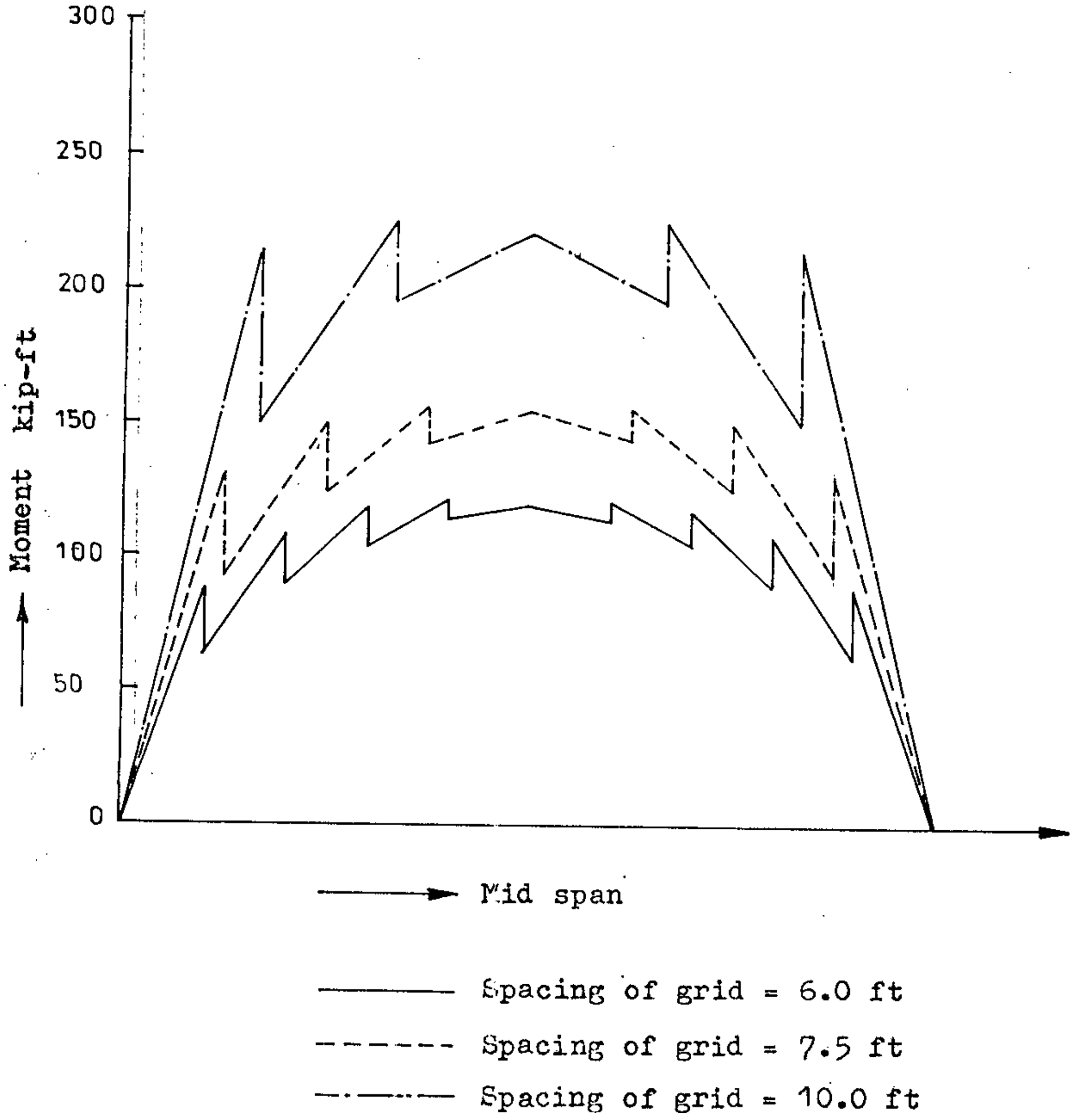


Fig. 4.10 Effect of grid spacing on moment for simply supported grid.

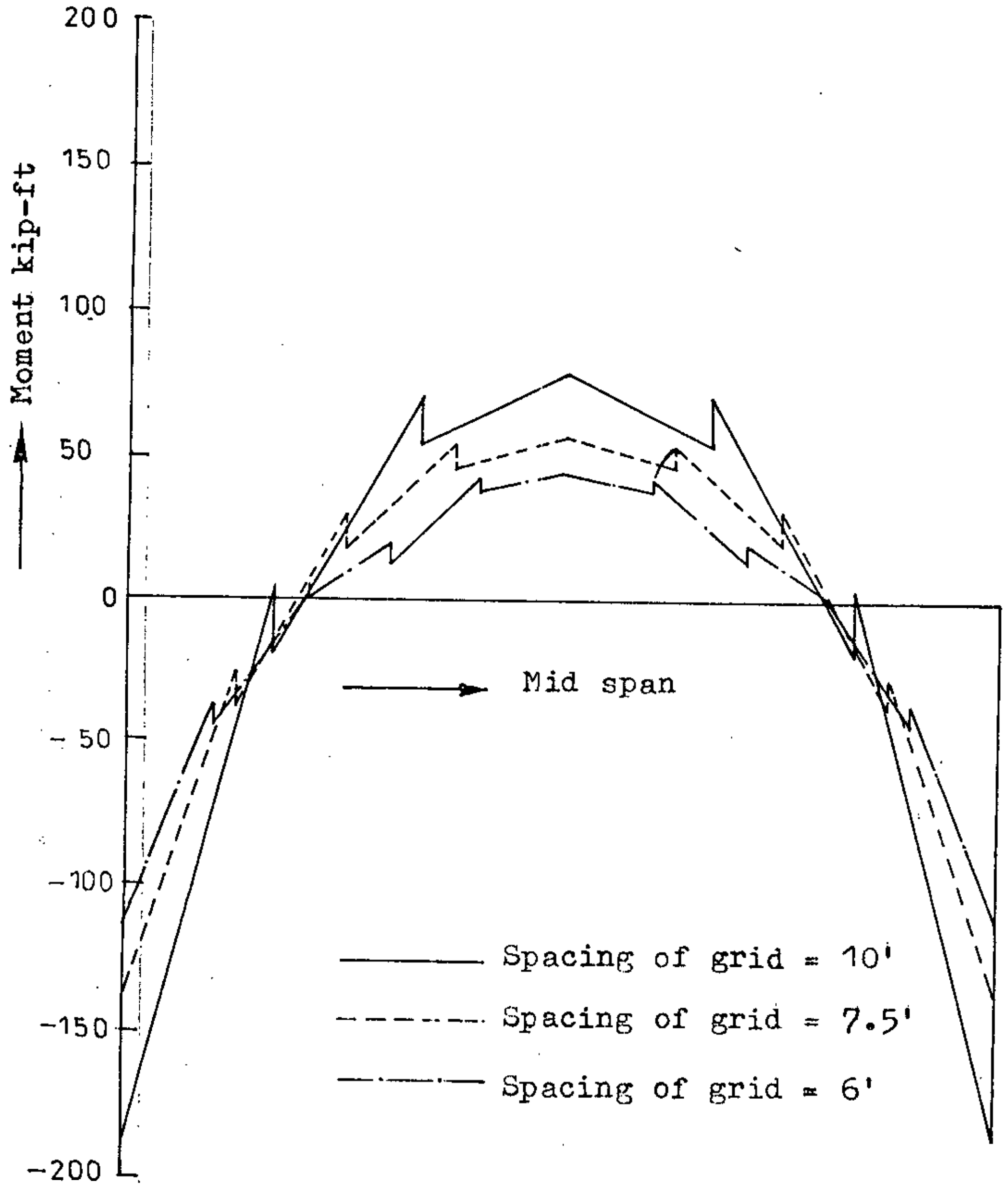


Fig. 4.11 Effect of grid spacing on moment for fixed grid.

Table 4.10 Comparison of moment and shear by stiffness method with the proportionate rule analysis

Support condition	Grid spacing		Analysis by stiffness Method		Analysis by proportionate rule*		Percentage of variation
			Maximum value	Coefficient	Maximum value	Coefficient	
Simple support	7.5	M	153.6	0.0568	165.07	0.0611	-7
		V	17.48	0.388	16.06	0.357	8.6
	6.0	M	119.1	0.0551	132.06	0.0611	-9.8
		V	14.61	0.4058	12.85	0.357	13.6
Fixed support	7.5	M^-	140.5	0.0520	138.75	0.0513	1.3
		M^+	55.9	0.0207	58.35	0.0216	-4.1
		V	15.28	0.339	14.13	0.314	7.9
	6.0	M^-	113.2	0.0524	111.0	0.0513	2.1
		M^+	43.9	0.0203	46.68	0.0216	-6.0
		V	12.80	0.355	11.30	0.314	13.0

b/a = 60/60 = 1.0

Loading = 100 psf

* The coefficients of the proportionate rule analysis are taken for 10' grid spacing, analysed by stiffness method.

Negative sign indicate lower value.

floor. The results of the analyses are shown in Table 4.11 and Table 4.12 and in Fig. 4.12 and Fig. 4.13.

It has been found that in simply supported grid maximum moment as obtained by plate method is 23.4% - 28.5% lower than that of the stiffness method for the above b/a ratio. In the case of fixed grid the maximum negative moment is 2.1% - 3.2% lower than that of stiffness method. Where as the maximum positive moment is 2.5% - 10.6% lower. The reduction of the moment in plate method from that of stiffness method is due to the fact that the plate has a larger torsional rigidity compared to grid floor.

In the case of simply supported grid, the maximum deflection as obtained by plate method is 25% - 43.5% lower than that of stiffness method. Where as in the case of fixed grid it is 14.6% - 24% lower. The difference in deflection between plate method and stiffness method decrease significiantly with the increasing aspect ratio b/a. This indicates that the plate method gives better correlation with the stiffness method for the increasing aspect ratio b/a.

It has been also noted from the analysis that as the aspect ratio b/a increases, the position of the maximum positive moment in the longer direction of the grid floor shifts from the center of the span towards the column strip as shown in Fig. 4.14. In calculating the member end force in stiffness method, deflections of the end joints gives

Table 4.11 Coefficients of moment, shear, deflection for different aspect ratio b/a by stiffness method and plate method (simply supported grid)

Method		b/a = 75/75 = 1.0		b/a = 150/100 = 1.5		b/a = 150/75 = 2.0	
		Maximum value	Coefficient	Maximum value	Coefficient	Maximum value	Coefficient
Stiffness method $m_1=m_2=12.5'$	w	9.93	0.086	51.73	0.143	18.65	0.162
	M_x	470.9	0.066	1422.9	0.1138	933.1	0.132
	V_x	30.12	0.321	58.09	0.464	43.53	0.464
	V_y	30.12	0.321	39.74	0.317	24.95	0.266
Plate method	w	5.607	0.049	33.637	0.093	13.96	0.122
	M_x	336.8	0.0479	1015.0	0.0812	715.1	0.1017
	V_x	31.7	0.338	53.0	0.424	43.59	0.465
	V_y	31.7	0.338	45.37	0.363	34.68	0.370

M = coefficient x q x a^2 x grid spacing

V = coefficient x q x a x grid spacing

w = coefficient x q x a^4/Eh^3

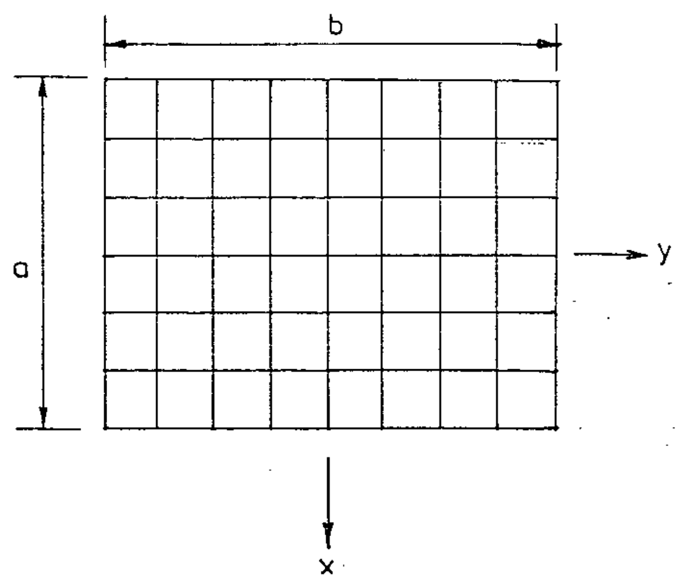
where, q = intensity of load = 100 psf

a = shorter direction span

b = longer direction span

m_1 = grid spacing along longer span = 12.5'

m_2 = grid spacing along shorter span = 12.5'



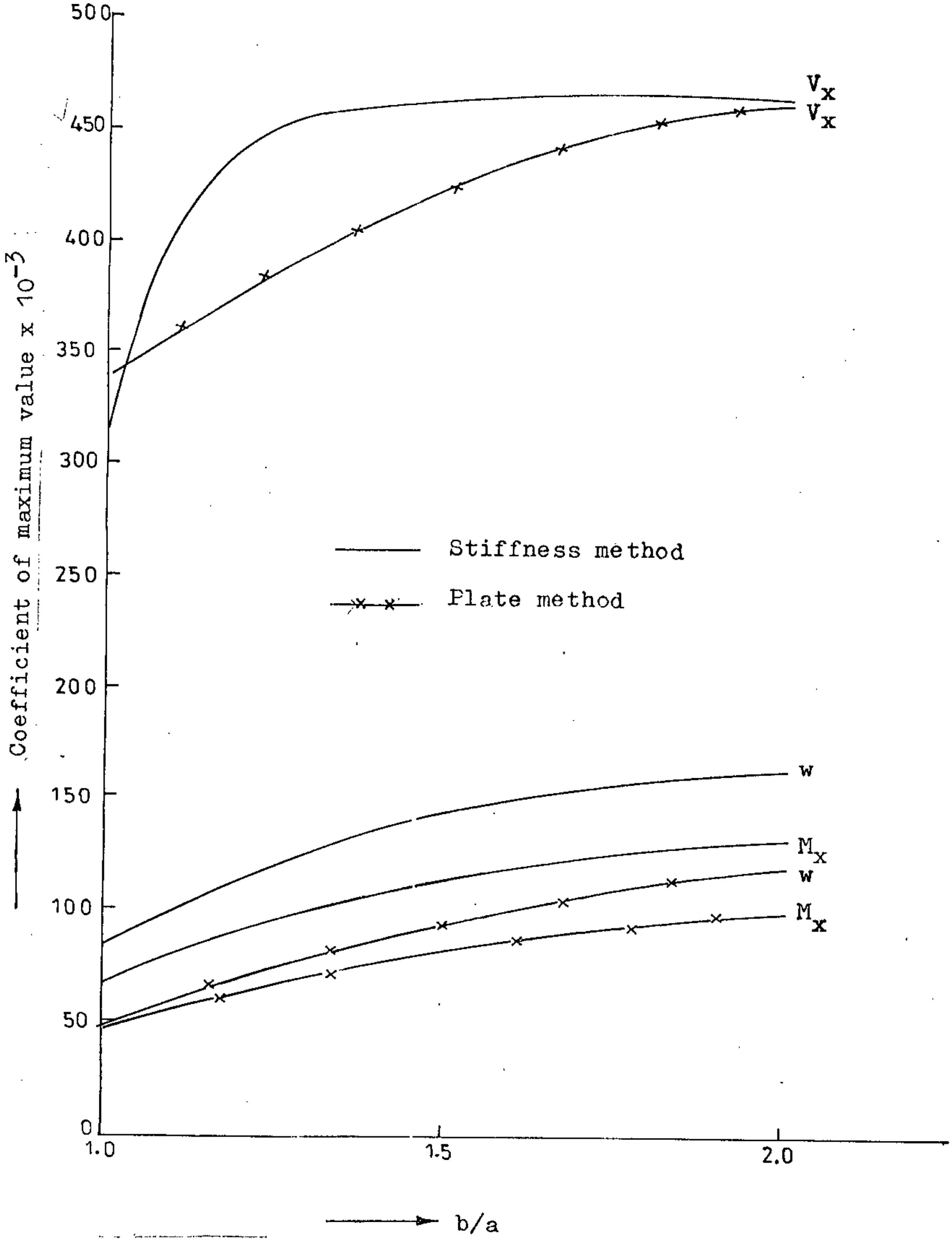


Fig. 4.12 Comparison of coefficients for simply supported grid.

Table 4.12 Coefficients of moment and deflection for different aspect ratio b/a by stiffness method and plate method (Fixed grid)

Method		b/a = 75/75 = 1.0		b/a = 150/100 = 1.5		b/a = 150/75 = 2.0	
		Maximum value	Coefficient	Maximum value	Coefficient	Maximum value	Coefficient
Stiffness method $m_1=m_2=12.5'$	w	2.079	0.0181	10.869	0.030	3.739	0.0326
	M_x^-	372.8	0.053	1009.2	0.0807	595.7	0.0844
	M_x^+	166.6	0.0236	511.41	0.0409	324.3	0.0461
	M_y^-	372.8	0.0530	701.0	0.0560	379.3	0.0539
Plate method	w	1.579	0.0138	8.752	0.0242	3.192	0.0279
	M_x^-	360.7	0.0513	946.2	0.0757	582.9	0.0829
	M_x^+	162.4	0.0231	460.0	0.0368	289.7	0.0412
	M_y^-	360.7	0.0513	712.5	0.0570	401.5	0.0571

M = coefficient x q x a^2 x grid spacing

V = coefficient x q x a x grid spacing

w = coefficient x q x a^4/Eh^3

where, q = intensity of load = 100 psf

a = shorter direction span

b = longer direction span

m_1= grid spacing along longer span = 12.5'

m_2= grid spacing along shorter span = 12.5'

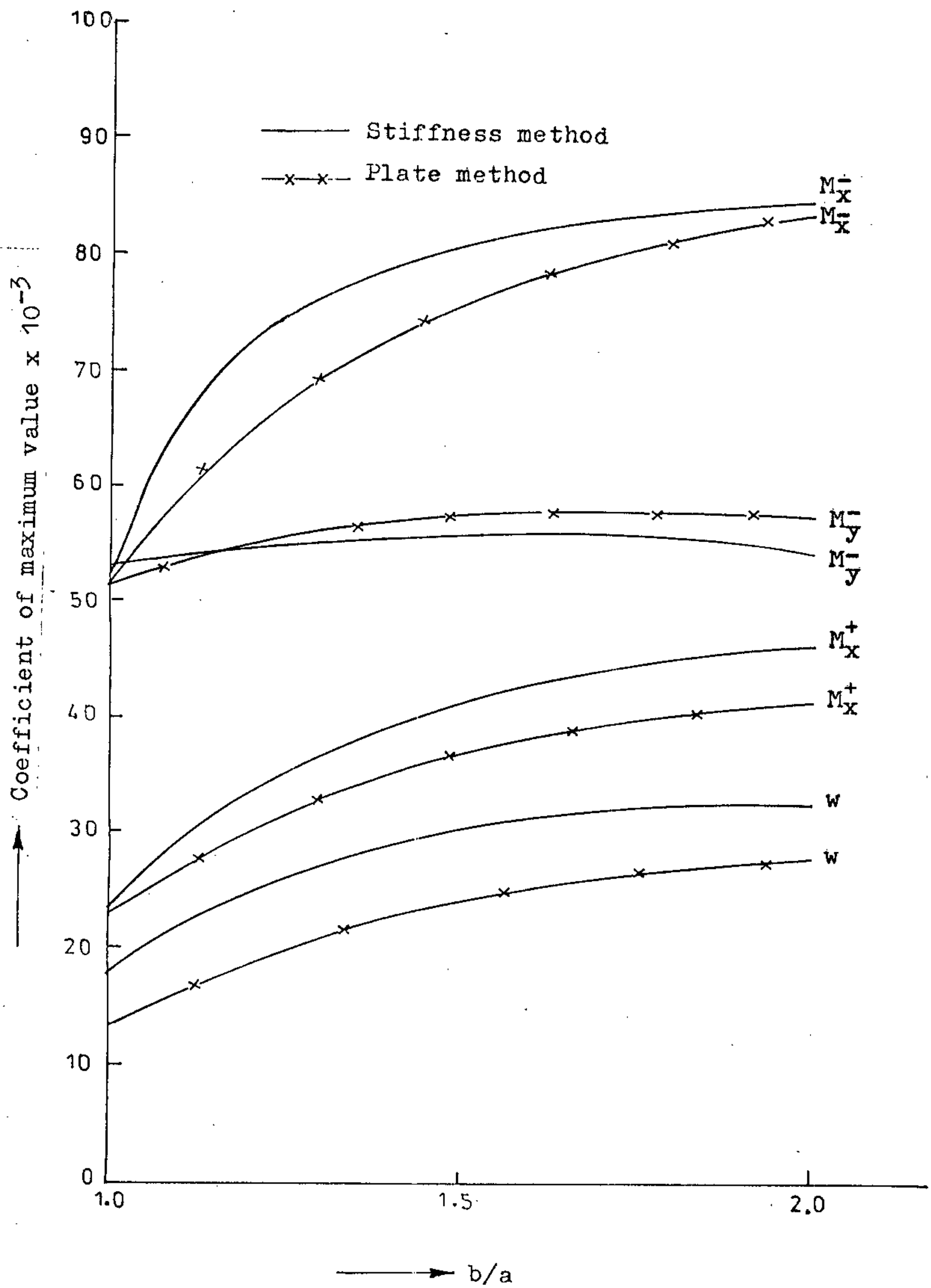


Fig. 4.13 Comparison of coefficient for fixed grid.

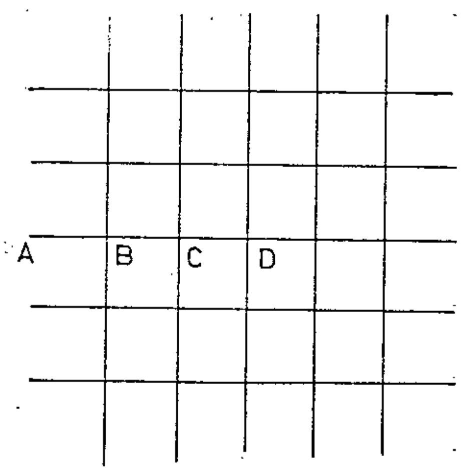
Table 4.13 Percentage of variation of the results by plate method with respect to stiffness method

Support condition	Maximum value	Aspect ratio b/a		
		1.0	1.5	2.0
Simply supported	w	-43.5%	-35.0%	-25.0%
	M_x	-28.5%	-28.6%	-23.4%
	V_x	5.2%	-8.7%	0.0%
Fixed supported	w	-24.0%	-19.5%	-14.6%
	M_x^-	-3.2%	-6.2%	- 2.1%
	M_x^+	-2.5%	-10.0%	-10.6%
	M_y^-	-3.2%	1.6%	5.8%

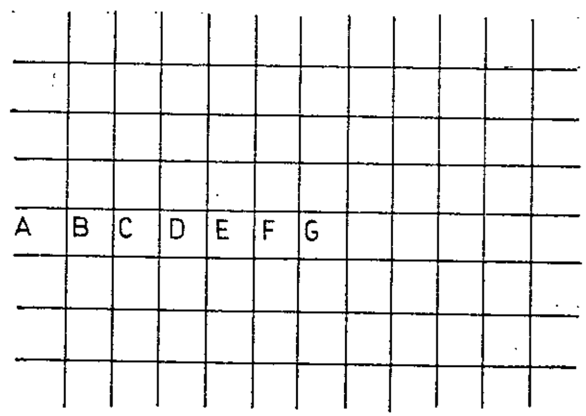
*Negative sign indicate lower value with respect to stiffness method.

Table 4.14 Moment along central grid members (longer direction) for different aspect ratio b/a by stiffness method of analyses

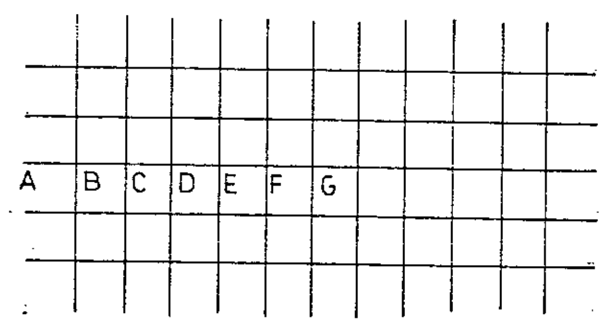
Member	AB	BA	BC	CB	CD	DC	DE	ED	EF	FE	FG	GF
Ratio b/a= $\frac{75}{75}$ = 1.0	-373	12	-37	-137	118	-166						
Ratio b/a= $\frac{150}{100}$ = 1.5	-701	204	-238	-56	12	-158	120	-177	150	-165	152	-153
Ratio b/a= $\frac{150}{75}$ = 2.0	-379	42	-70	-80	51	-90	69	-61	51	-35	31	-24



b/a =1.0



b/a =1.5



b/a = 2.0

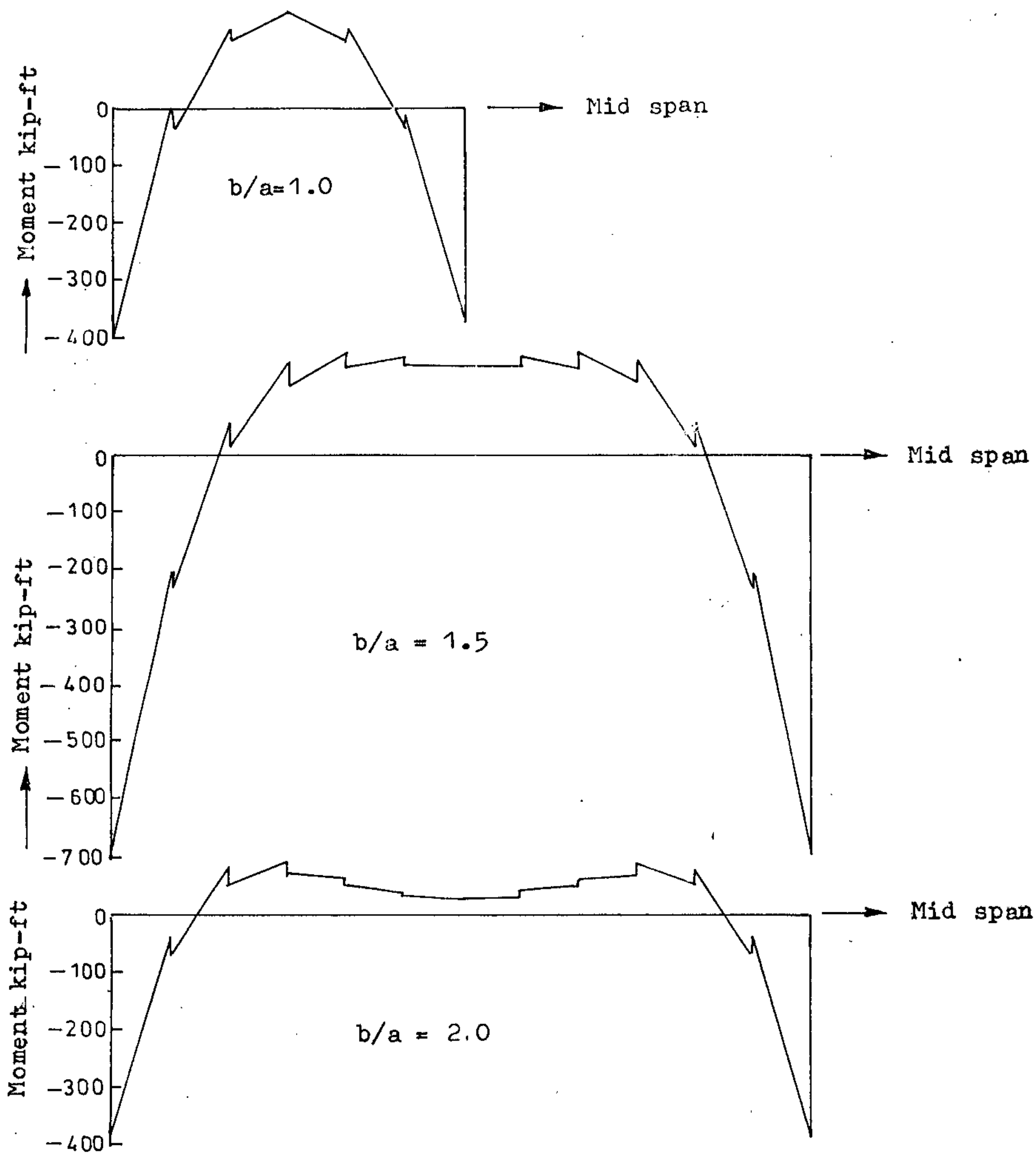


Fig. 4.14 Shifting of the position of maximum positive moment in the long direction towards column strip with the increasing aspect ratio b/a.

positive effect while the rotation of the end joints give negative effect. The fact that the position of maximum positive moment in the longer direction shifts towards column strip for aspect ratio b/a greater than unity, indicates that the effect of rotation of the end of a member on the member end force is less compared to that of deflection of the end joint of that member near the column strip.

4.6 Design Tables and Charts

To develop design tables and charts for square grid the aspect ratio b/a was varied from 1.0 to 2.08. This was done for both the simply supported grid floor and fixed grid floor of concrete. The loading was taken 100 psf and the width and depth of grid beam were chosen 12 inches and 24 inches respectively. Maximum spacing of grid member was limited to 12.5 feet. The coefficients for maximum moment and shear in the shorter direction and the longer direction are plotted against different span directions. Maximum stresses develop along mid span and it decrease gradually towards the edge span. The results are presented in Table 4.15 to Table 4.20 and in Fig. 4.15 to Fig. 4.29.

From the design tables it is found that as the aspect ratio b/a increases, the coefficients for maximum moment and shear in the shorter direction increases. But in the longer direction the coefficients decrease gradually with the

Table 4.15 Coefficient of moment, and shear for fixed grid with different aspect ratio b/a (m_1/m_2 = 1.0)

b/a		Along L/6 span		Along L/4 span		Along L/3 span		Along 3L/8 span		Along L/2 span	
		Maximum value	Coefficient	Maximum value	Coefficient	Maximum value	Coefficient	Maximum value	Coefficient	Maximum value	Coefficient
b/a = 75/75 = 1.0	M_x^-	130.9	0.0186			306.1	0.0435			372.8	0.0530
	M_x^+	47.9	0.0067			130.5	0.0184			166.6	0.0236
	M_y^-	130.9	0.0186			306.1	0.0435			372.8	0.0530
	M_y^+	47.9	0.0067			130.5	0.0184			166.6	0.0236
	V_x	10.32	0.1100			24.0	0.256			28.92	0.3084
	V_y	10.32	0.1100			24.0	0.256			28.92	0.3084

M = coefficient x q x a^2 x grid spacing

V = coefficient x q x a x grid spacing

where,

q = intensity of load = 100 psf

a = shorter direction span

b = longer direction span

m_1 = grid spacing along longer span = 12.5'

m_2 = grid spacing along shorter span = 12.5'

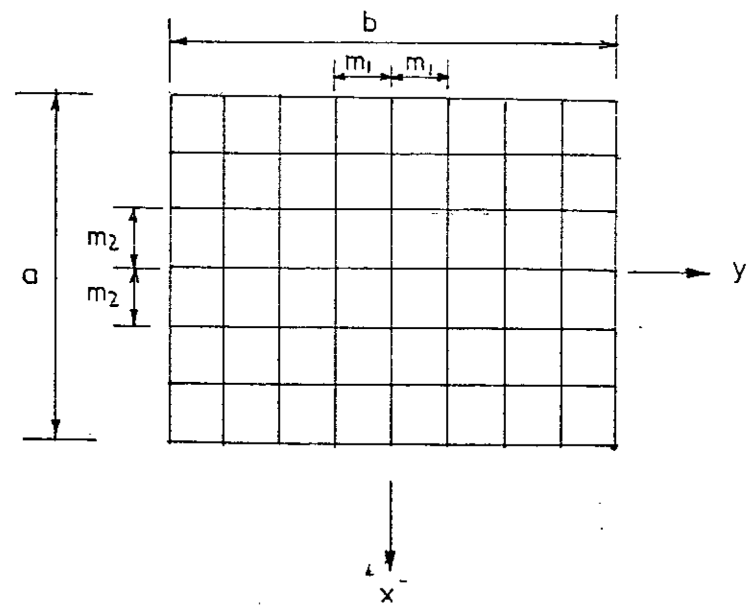
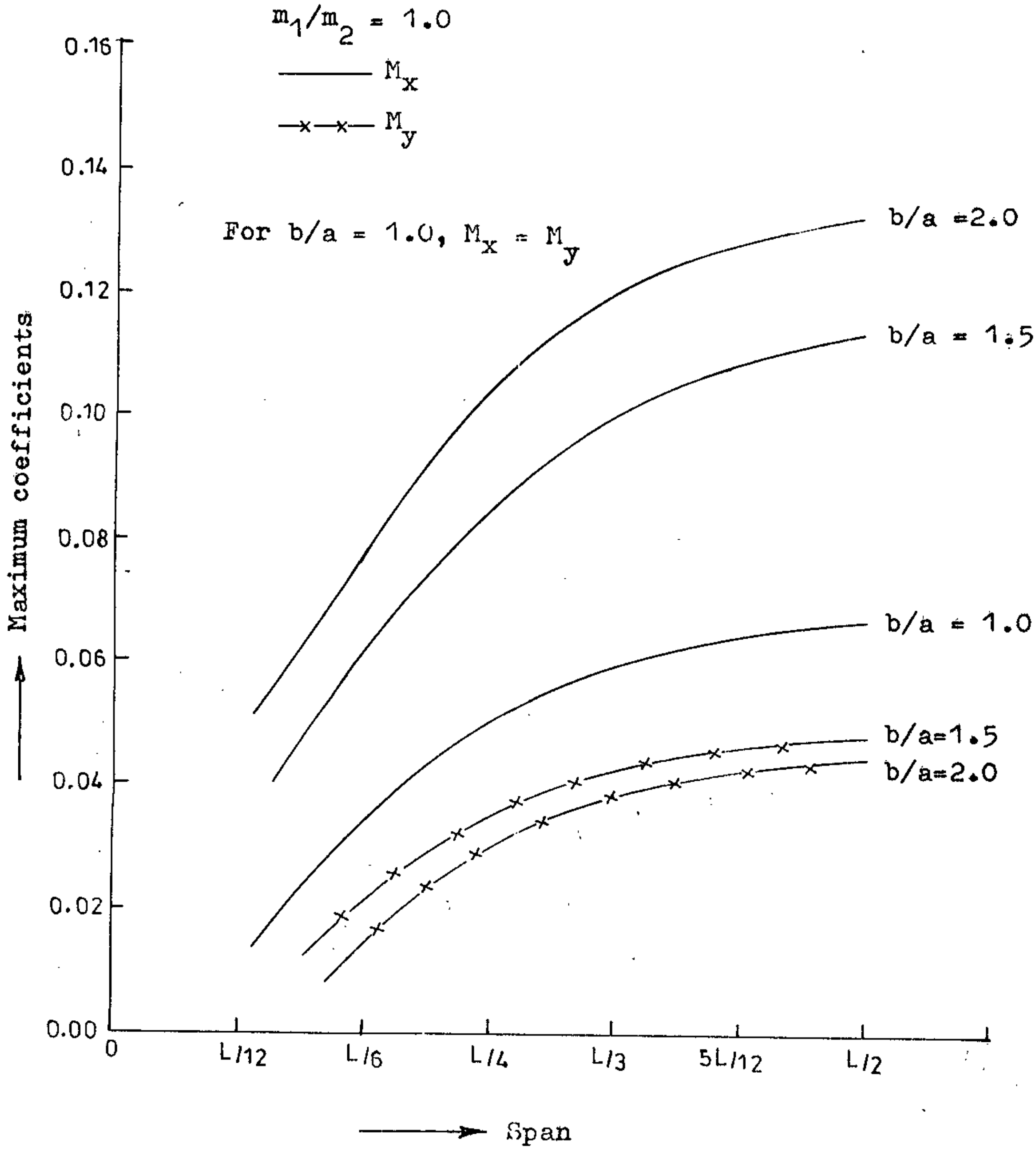


Table 4.15 Contd...

b/a		Along L/6 span Maximum value	Along L/6 span Coefficient	Along L/4 span Maximum value	Along L/4 span Coefficient	Along L/3 span Maximum value	Along L/3 span Coefficient	Along 3L/8 span Maximum value	Along 3L/8 span Coefficient	Along L/2 span Maximum value	Along L/2 span Coefficient
b/a = 150/100=1.5	M_x^-	446.0	0.0356			873.3	0.0698			1009.2	0.0807
	M_x^+	184.6	0.0147			425.3	0.0340			511.4	0.0409
	M_y^-			429.3	0.0342			629.4	0.0502	701.0	0.0560
	M_y^+			91.8	0.0072			153.4	0.0121	176.9	0.0140
	V_x	27.40	0.2192			49.38	0.3950			55.62	0.4449
	V_y			25.70	0.2056			36.21	0.2896	39.73	0.3178
b/a=150/75=2.0	M_x^-	334.1	0.0473			552.3	0.0782			595.7	0.0844
	M_x^+	154.7	0.0219			293.2	0.0416			324.3	0.0461
	M_y^-	130.9	0.0186			309.9	0.0440			379.3	0.0539
	M_y^+	18.7	0.0025			68.3	0.0094			89.6	0.0124
	V_x	24.98	0.2664			38.64	0.4121			40.96	0.4369
	V_y	9.52	0.1015			22.46	0.2395			26.99	0.2878



M = coefficient x q x a^2 x grid spacing

Fig. 4.15 Coefficients of moment for different aspect ratio b/a for simply supported grid.

Table 4.16 Coefficient of moment and shear for simply supported grid with different aspect ratio b/a ($m_1/m_2 = 1.0$)

b/a		Along L/6 span		Along L/4 span		Along L/3 span		Along 3L/8 span		Along L/2 span	
		Maximum value	Coefficient	Maximum value	Coefficient	Maximum value	Coefficient	Maximum value	Coefficient	Maximum value	Coefficient
$b/a = \frac{75}{75} = 1.0$	M_x	239.9	0.0341			417.1	0.059			470.9	0.0669
	M_y	239.9	0.0341			417.1	0.059			470.9	0.0669
	V_x	8.28	0.088			25.44	0.271			30.12	0.321
	V_y	8.28	0.088			25.44	0.271			30.12	0.321
$b/a = \frac{150}{100} = 1.5$	M_x	774.4	0.0619			1257.1	0.100			1422.9	0.1138
	M_y			439.7	0.035			555.9	0.0444	595.7	0.0476
	V_x	31.79	0.253			53.22	0.425			58.09	0.464
	V_y			25.92	0.206			36.73	0.292	39.74	0.317
$b/a = \frac{150}{75} = 2.0$	M_x	557.7	0.0788			846.3	0.1197			933.1	0.132
	M_y	106.2	0.0150			271.7	0.0386			311.8	0.0443
	V_x	27.79	0.2962			41.07	0.4377			43.53	0.464
	V_y	8.45	0.0900			20.64	0.220			24.95	0.266

M = coefficient x q x a^2 x grid spacing

V = coefficient x q x a x grid spacing

where,

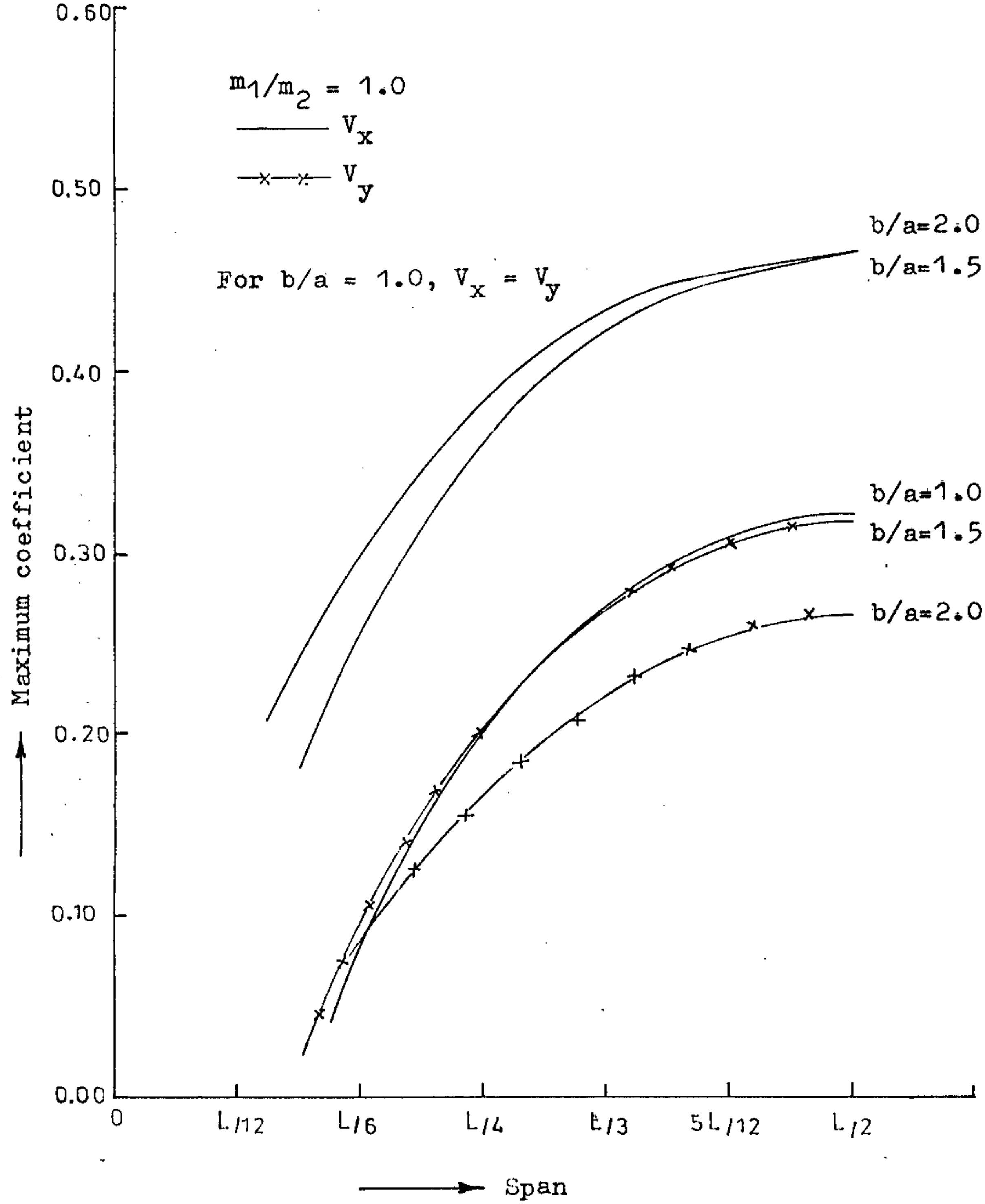
q = intensity of load = 100 psf

a = shorter direction span

b = longer direction span

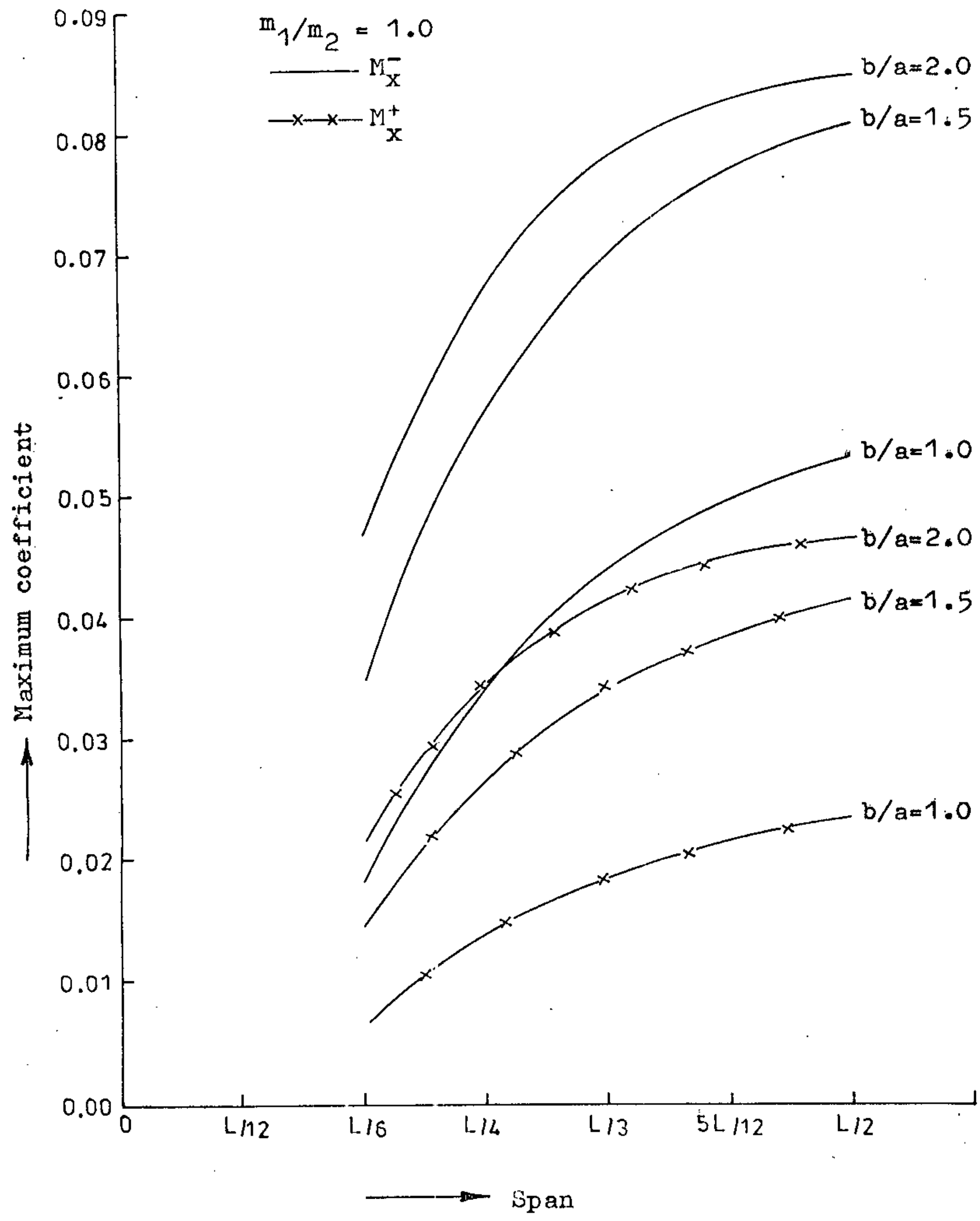
m_1= grid spacing along longer span = 12.5'

m_2= grid spacing along shorter span = 12.5'



V = Coefficient x q x a x grid spacing

Fig. 4.16 Coefficients of shear for different aspect ratio b/a for simply supported grid.



M = coefficient x q x a^2 x grid spacing

Fig. 4.17 Coefficient of moments for different aspect ratio b/a for fixed grid

Table 4.17 (Contd..)

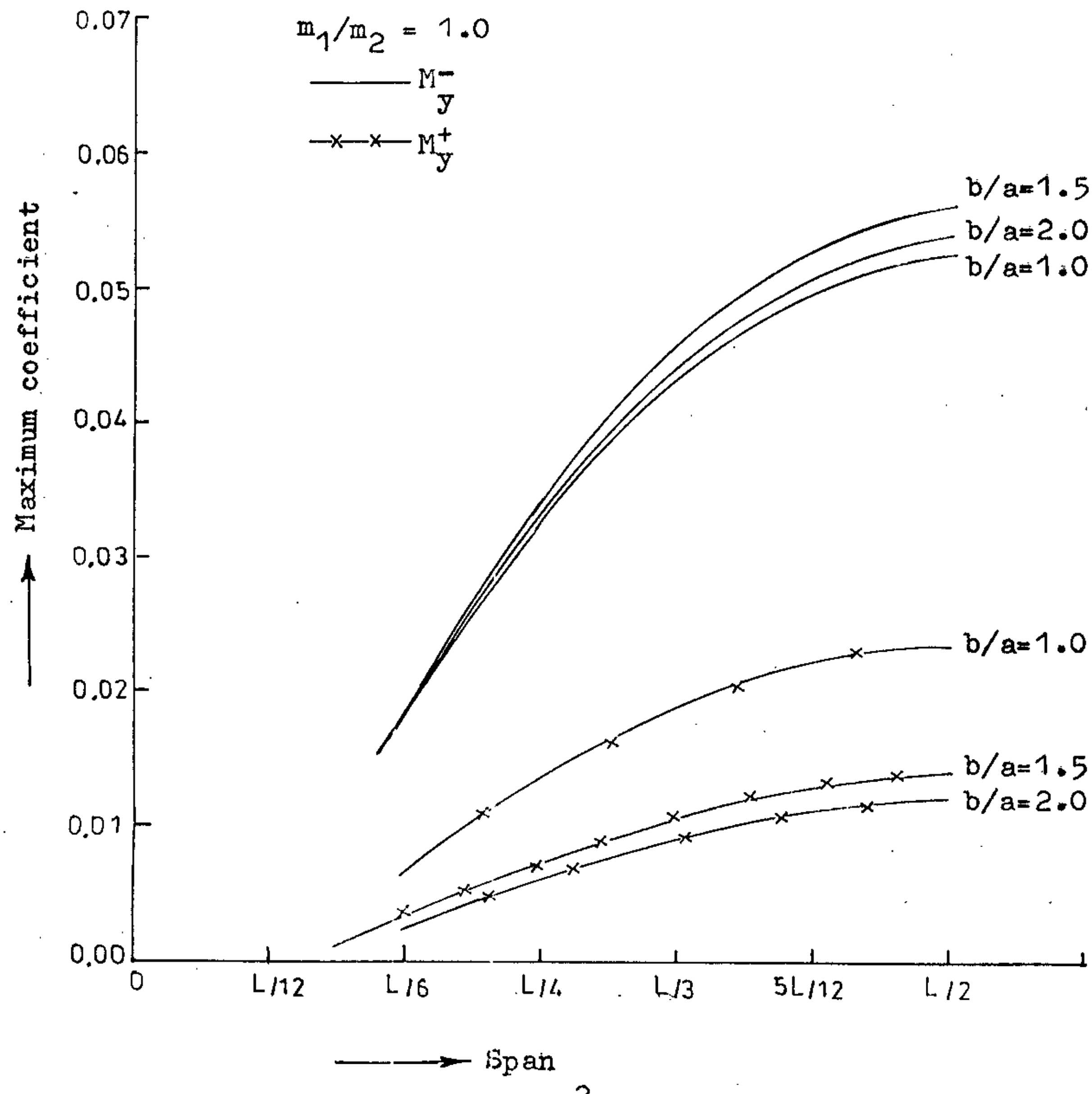
b/a		Along L/6 span		Along 3L/10 span		Along L/3 span		Along 2L/5 span		Along L/2 span	
		Maximum value	Coefficient	Maximum value	Coefficient	Maximum value	Coefficient	Maximum value	Coefficient	Maximum value	Coefficient
$b/a = \frac{125}{60} = 2.08$	M_x^-			335.7	0.0745			371.83	0.0825	381.1	0.0846
	M_x^+			175.55	0.0389			200.65	0.0445	207.4	0.0460
	M_y^-	71.9	0.0199			173.23	0.0480			213.1	0.0591
	M_y^+	11.3	0.0031			42.48	0.0117			54.8	0.0152
	V_x			29.69	0.3958			32.21	0.4294	32.80	0.4373
	V_y	5.76	0.096			14.03	0.2338			17.04	0.2840

Table 4.17 Coefficient of moment and shear for fixed grid with different aspect ratio b/a (m_1/m_2 = 1.25)

b/a		Along L/4 span		Along 3L/10 span		Along 3L/8 span		Along 2L/5 span		Along L/2 span	
		Maximum value	Coefficient	Maximum value	Coefficient	Maximum value	Coefficient	Maximum value	Coefficient	Maximum value	Coefficient
$b/a = \frac{100}{100} = 1.0$	M_x^-	384.4	0.0307			558.4	0.0446			620.2	0.0496
	M_x^+	136.4	0.0108			216.4	0.0172			247.0	0.0197
	M_y^-			441.6	0.0441			547.8	0.0547	584.7	0.0584
	M_y^+			182.5	0.0181			238.7	0.0237	258.8	0.0258
	V_x	26.69	0.2135			37.19	0.2975			40.7	0.3256
	V_y			27.47	0.2747			33.22	0.3322	35.15	0.3515
		Along L/6 span		Along L/4 span		Along L/3 span		Along 3L/8 span		Along L/2 span	
$b/a = \frac{100}{60} = 1.66$	M_x^-			256.8	0.0570			341.0	0.0757	366.8	0.0815
	M_x^+			124.6	0.0276			177.8	0.0395	195.1	0.0433
	M_y^-	72.8	0.0202			175.9	0.0488			216.5	0.0601
	M_y^+	11.7	0.0032			44.1	0.0122			57.1	0.0158
	V_x			23.64	0.3152			30.24	0.4032	32.16	0.4288
	V_y	5.76	0.096			14.16	0.236			17.28	0.2880

m_1 = grid spacing along longer span = 12.5'

m_2 = grid spacing along shorter span = 8.33'



M = coefficient x q x a^2 x grid spacing

Fig. 4.18 Coefficients of moment for different aspect ratio b/a for fixed grid

Table 4.18 Coefficient of moment and shear for simply supported grid with different aspect ratio b/a ($m_1/m_2 = 1.25$)

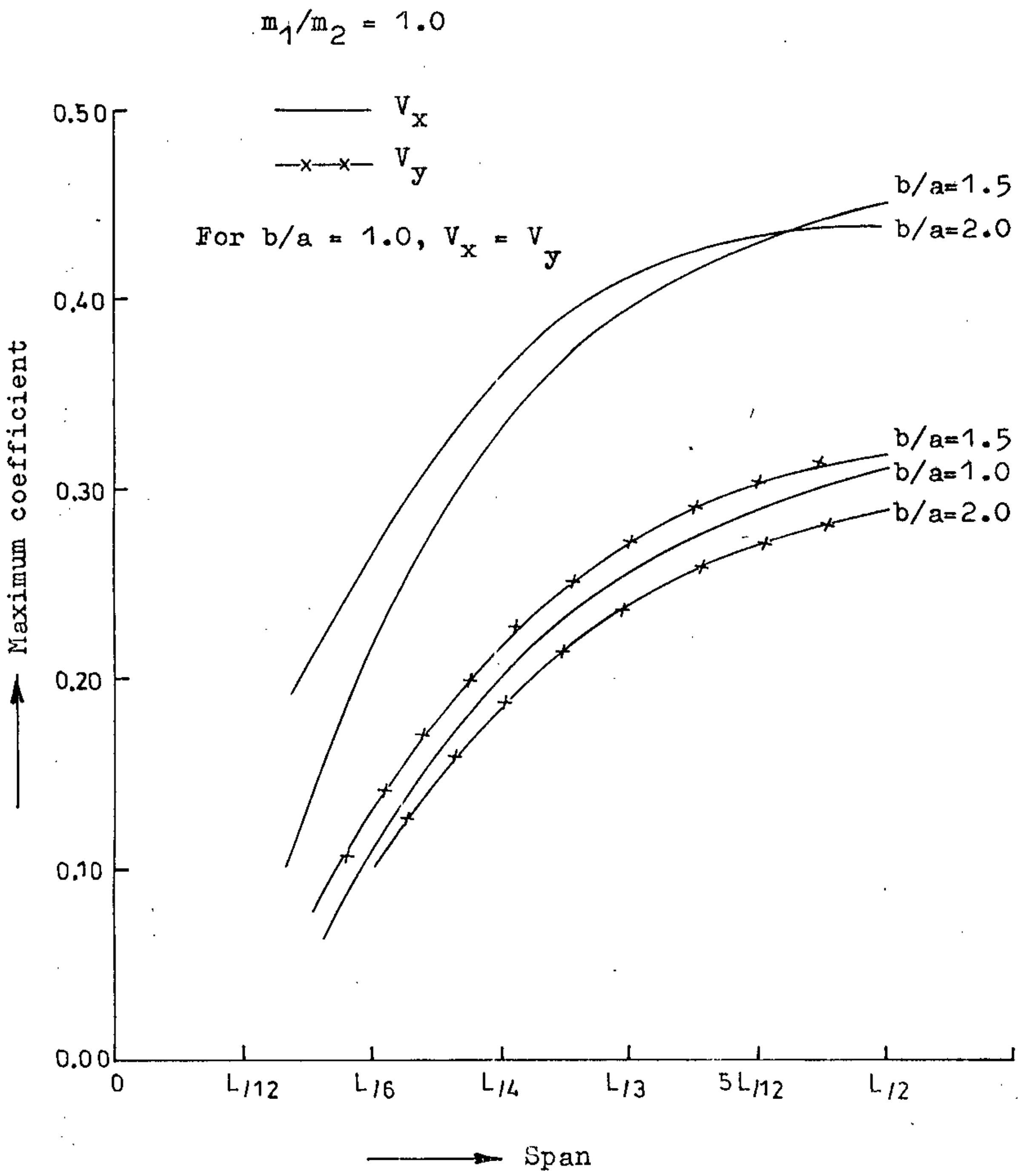
b/a		Along L/4 span		Along 3L/10 span		Along 3L/8 span		Along 2L/5 span		Along L/2 span	
		Maximum value	Coefficient	Maximum value	Coefficient	Maximum value	Coefficient	Maximum value	Coefficient	Maximum value	Coefficient
$b/a = \frac{100}{100} = 1.0$	M_x	512.8	0.0409			658.8	0.0526			708.1	0.0566
	M_y			589.8	0.0589			683.9	0.0683	715.0	0.0715
	V_x	29.22	0.2337			39.78	0.3182			42.69	0.3415
	V_y			30.26	0.3026			35.12	0.3512	36.67	0.3667
		Along L/6 span		Along L/4 span		Along L/3 span		Along 3L/8 span		Along L/2 span	
$b/a = \frac{100}{60} = 1.66$	M_x			410.3	0.0911			509.0	0.1130	543.0	0.1206
	M_y	72.3	0.0200			172.0	0.0477			202.20	0.0561
	V_x			26.16	0.3488			32.52	0.433	34.08	0.4544
	V_y	4.08	0.068			13.44	0.224			16.20	0.270
		Along L/6 span		Along 3L/10 span		Along L/3 span		Along 2L/5 span		Along L/2 span	
$b/a = \frac{125}{60} = 2.08$	M_x			512.6	0.1139			574.45	0.1276	594.0	0.1320
	M_y	60.2	0.0167			158.7	0.0440			191.20	0.0531
	V_x			31.70	0.4226			34.13	0.4550	34.77	0.4636
	V_y	4.97	0.0828			12.69	0.2115			15.30	0.255

m_1 = grid spacing along longer span = 12.5'
m_2 = grid spacing along shorter span = 10.0'

Table 4.19 Coefficient of moment and shear for fixed grid with different aspect ratio b/a ($m_1/m_2 = 1.5$)

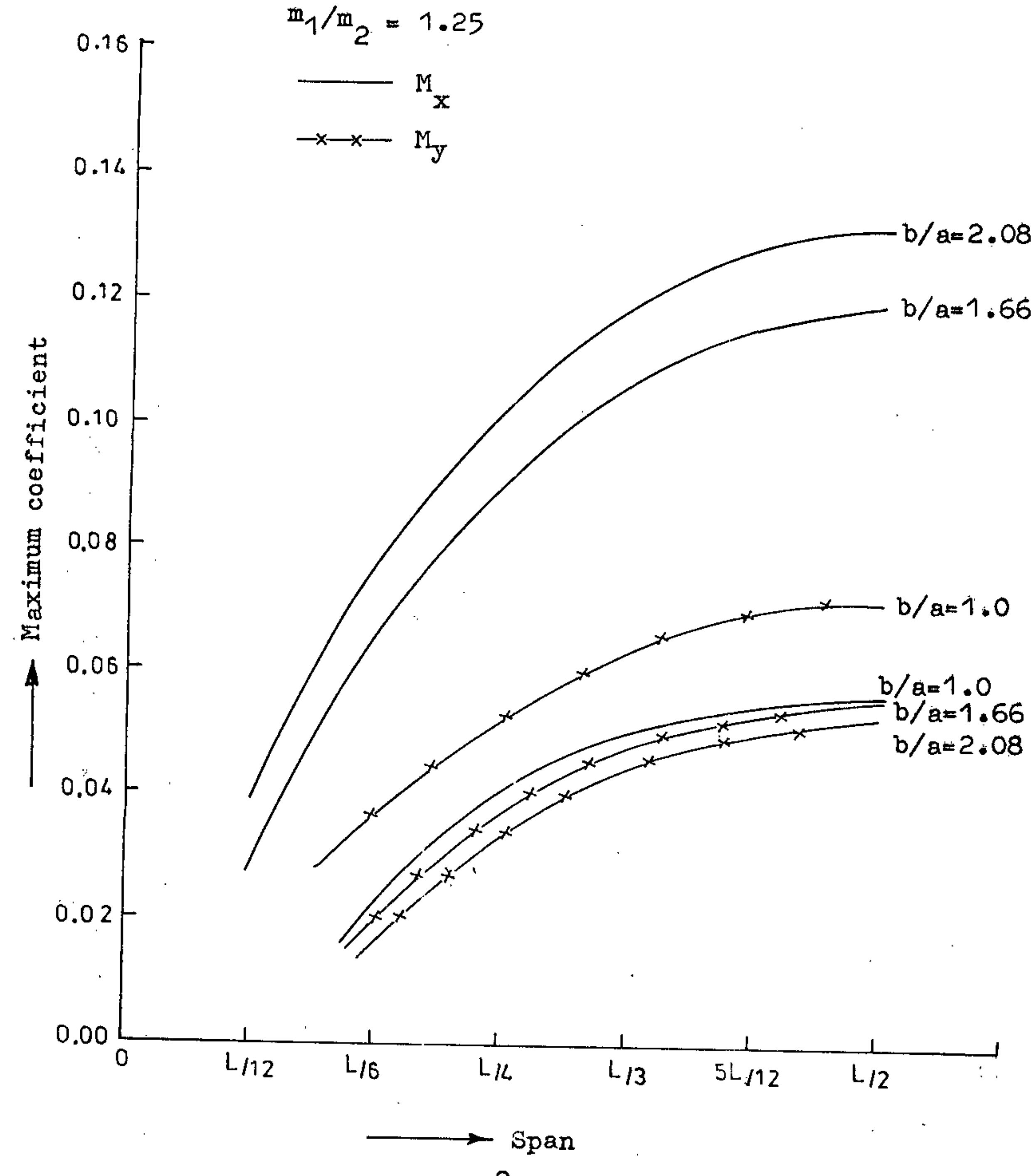
b/a		Along L/6 span		Along L/4 span		Along L/3 span		Along 3L/8 span		Along L/2 span	
		Maximum Value	Coefficient	Maximum value	Coefficient	Maximum value	Coefficient	Maximum value	Coefficient	Maximum value	Coefficient
$b/a = \frac{100}{100} = 1.0$	M_x^-			355.9	0.0284			517.0	0.0413	574.5	0.0459
	M_x^+			118.8	0.0094			188.9	0.0150	215.87	0.0172
	M_y^-	194.8	0.0233			429.2	0.0514			516.5	0.0619
	M_y^+	72.0	0.0086			186.0	0.0222			234.4	0.0281
	V_x			26.22	0.2097			36.36	0.2908	39.74	0.3179
	V_y	12.50	0.2999			25.99	0.6237			30.54	0.7329
$b/a = \frac{75}{50} = 1.5$	M_x^-	94.2	0.0301			199.6	0.0638			235.4	0.0753
	M_x^+	40.3	0.0128			98.67	0.0315			121.1	0.0387
	M_y^-	44.9	0.0215			110.2	0.0528			136.2	0.0653
	M_y^+	10.93	0.0052			34.0	0.0162			44.3	0.0212
	V_x	10.68	0.1708			21.96	0.3513			25.44	0.4070
	V_y	3.84	0.0921			9.84	0.2361			12.00	0.2880
$b/a = \frac{100}{50} = 2.0$	M_x^-			197.9	0.0633			249.4	0.0797	263.2	0.0842
	M_x^+			98.76	0.0315			132.67	0.0424	142.21	0.0455
	M_y^-	43.85	0.0210			107.42	0.0515			132.63	0.0636
	M_y^+	7.16	0.0034			27.60	0.0132			36.12	0.0173
	V_x			21.60	0.3455			26.28	0.4204	27.36	0.4377
	V_y	3.72	0.0892			9.48	0.2274			11.52	0.2764

m_1 = grid spacing along longer spah = 12.5'
m_2 = grid spacing along shorter span = 8.33'



V = Coefficient x q x a x grid spacing

Fig. 4.19 Coefficients of shear for different aspect ratio b/a for fixed grid



M = Coefficient x q x a^2 x grid spacing

Fig. 4.20 Coefficient of moment for different aspect ratio b/a for simply supported grid.

Table 4.20 Coefficient of moment and shear for simply supported grid with different aspect ratio b/a ($m_1/m_2 = 1.5$)

b/a		Along L/6		Along L/4 span		Along L/3 span		Along 3L/8 span		Along L/2 span	
		Maximum value	Coefficient	Maximum value	Coefficient	Maximum value	Coefficient	Maximum value	Coefficient	Maximum value	Coefficient
$b/a = \frac{100}{100} = 1.0$	M_x			450.7	0.0360			580.4	0.0463	624.4	0.0499
	M_y	331.5	0.0397			558.5	0.0670			637.5	0.0765
	V_x			28.77	0.230			38.84	0.3107	41.49	0.3319
	V_y	14.74	0.1768			27.99	0.3358			31.52	0.3782
$b/a = \frac{75}{50} = 1.5$	M_x	180.8	0.0578			298.1	0.0953			334.9	0.1071
	M_y	61.7	0.0295			123.9	0.0594			143.5	0.0688
	V_x	9.24	0.1478			23.76	0.3801			27.12	0.4339
	V_y	3.00	0.0719			9.6	0.2303			11.52	0.2764
$b/a = \frac{100}{50} = 2.0$	M_x			310.6	0.0993			379.7	0.1215	402.5	0.1288
	M_y	38.20	0.0183			107.3	0.0514			128.6	0.0617
	V_x			23.40	0.3743			27.96	0.4473	29.04	0.4646
	V_y	2.88	0.0690			8.64	0.2072			10.32	0.2476

m_1 = grid spacing along longer span = 12.5'

m_2 = grid spacing along shorter span = 8.33'

Table 4.21 Effect of angle of skewness on moment, shear, torsion and deflection

Angle of skewness	No. of members	Maximum deflection inch	Maximum moment k-ft	Maximum shear kip	Maximum torsion kip-ft
30°	252	0.984	140.5	25.72	32.7
45°	144	1.274	165.7	23.7	20.25
60°	84	1.73	235.0	25.81	50.9

increasing aspect ratio b/a. This behaviour in the shorter direction or longer direction is due to the fact that as the aspect ratio b/a increases, the sharpness of the curvature of deflection curve increases in the shorter direction or decreases in the longer direction.

4.7 Effect of Angle of Skewness of the Grid Member on Deflection, Moments

To study the effect of angle of skewness in grids a simply supported square floor 60'x60' was mapped by grids having skewness 30°, 45° and 60°. The results are presented in Fig. 4.30 to Fig. 4.32 and in Table 4.21.

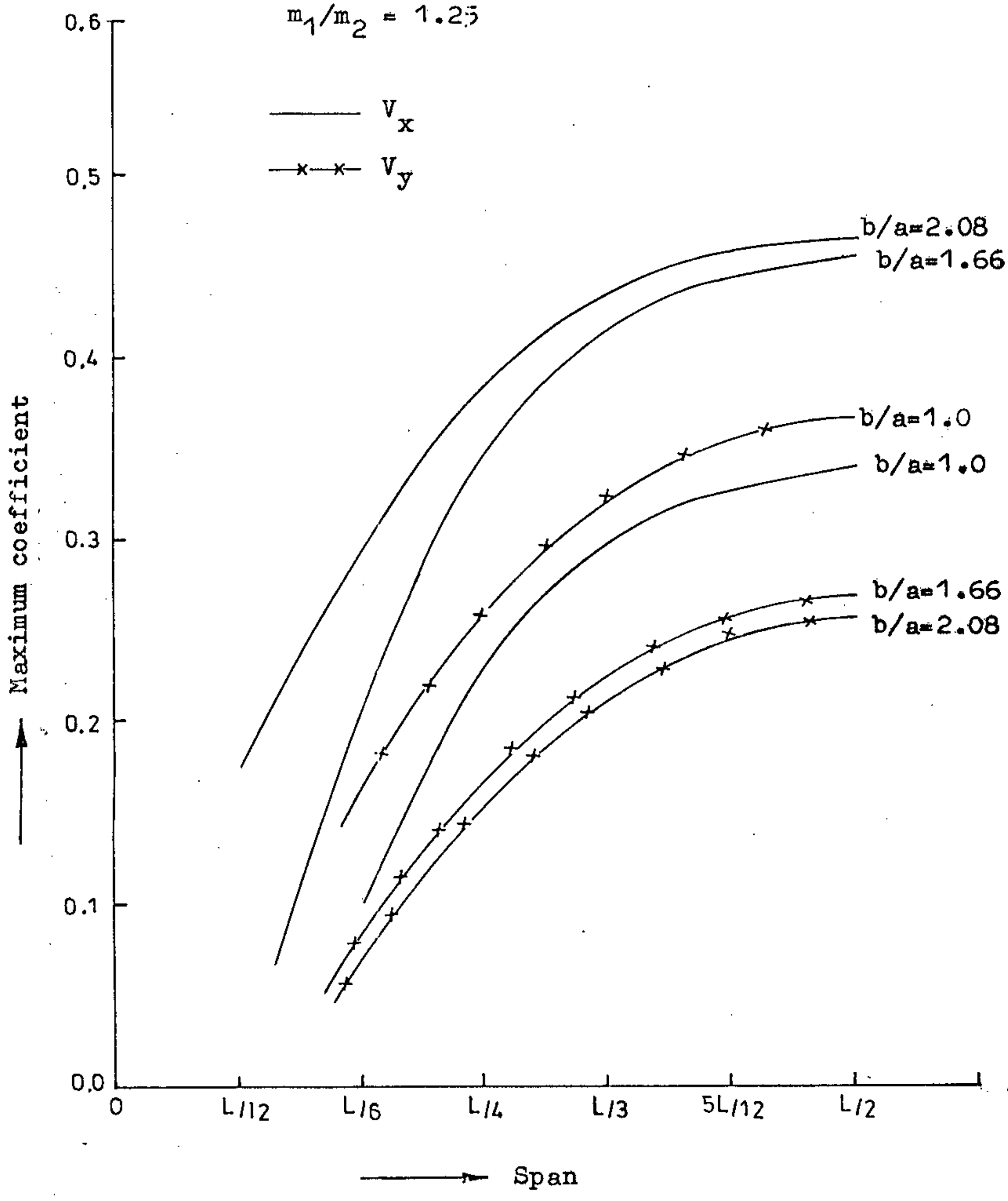
From the analysis, it is found that maximum moment, shear and torsion occur near the corner of the grid floor. This is due to the fact that in the skew grid the shorter corner beams provide intermediate support for the longer diagonal beams and thus carry more loads. This arrangements reduce the midspan bending moments in the diagonal members.

It has been also noted that as the angle of skewness for the same floor increases the moment and deflection increases. This increase in moment and deflection may be attributed to the fact that in a grid with large angle of skewness, the total load is carried by a lower number of nodes. Although the moment can be reduced by decreasing the angle of skewness, the total number of grid members increases with the decreasing angle of skewness, thus sharing the total load to a higher number of members.

To compare the efficiency of 45° and 60° skew grid floors, a comparison is made between these floors and the equivalent (same running foot of skew grid members) square grid floors. The comparison is based on the moment area of the grid members. The results of this comparison is shown in Fig. 4.33 to Fig. 4.36 and in the Table 4.22.

The comparison shows that the area of the moment diagram for skew grids is lower than that of equivalent square grid by 5.7 percent for 60° skew grid and 7.4 percent for 45° skew grid considering the moment variations at the joint. This percentage changes to 8.0 and 14.0 for the same grids considering the maximum moment at the joint. However by using the maximum moment of the member the area of the moment diagram for skew grids increases to 5.5% and 17.5% for 60° and 45° skew grids.

The comparison of the moment of 45° skew grid by stiffness method with those as obtained by the slab analogy is presented in Fig. 4.37. It can be seen that for a 45° skew grid floor, square in plan, the maximum positive moment at the center as obtained by slab analogy method is 7.9% higher than that obtained by stiffness method. While slab analogy method underestimates the positive and negative moments by 36.9% at the corner.



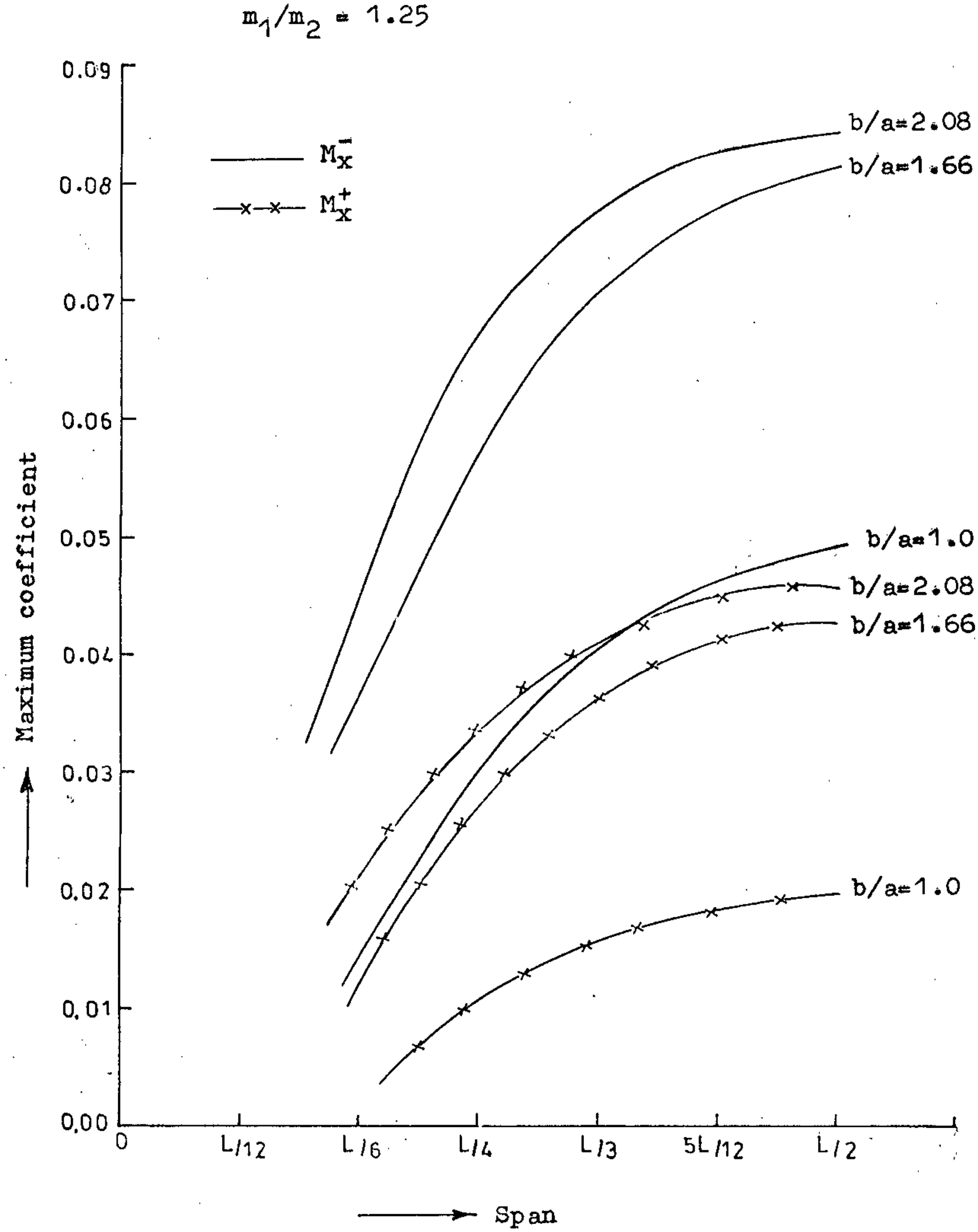
V = Coefficient x q x a x grid spacing

Fig. 4.21 Coefficients of shear for different aspect ratio b/a for simply supported grid.

Table 4.22 Moment area for skew grids and equivalent square grids

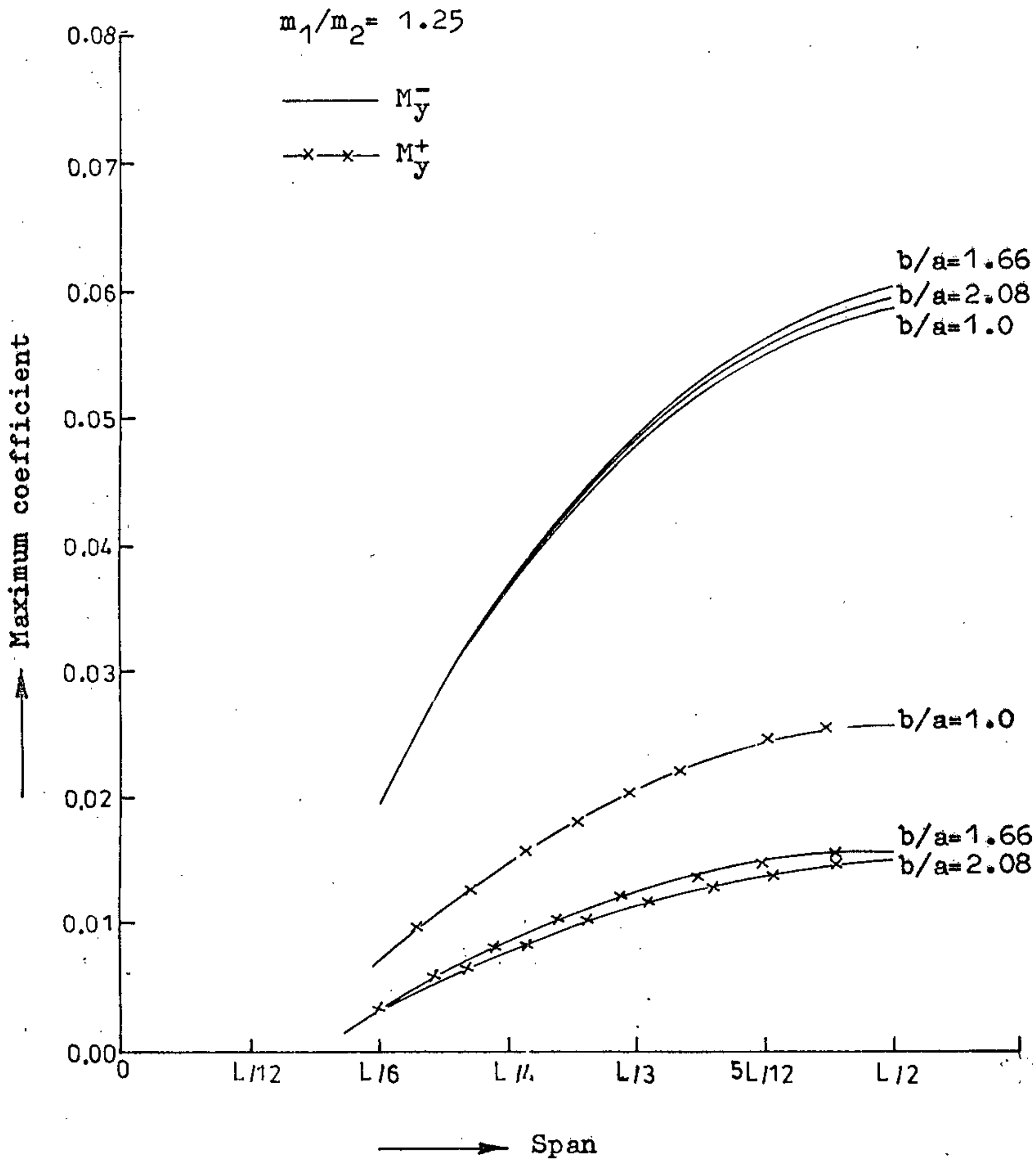
Skew grid floors				Equivalent square grid floor				% of variation of moment area of skew grid w.r.to Equivalent square grid		
Angle of skew-ness	Exact moment area $k\text{-}ft^2$	Moment area using maximum ordinate at the joint $k\text{-}ft^2$	Moment area using maximum ordinate of the member $k\text{-}ft^2$	Spacing of grid ft	Exact moment area $k\text{-}ft^2$	Moment area using maximum ordinate at the joint $k\text{-}ft^2$	Moment area using maximum ordinate of the member $k\text{-}ft^2$	Exact moment area	Moment area using maximum ordinate at the joint	Moment area using maximum ordinate of the member
60°	68122	74800	136422	7.5	72270	81374	102492	-5.7	-8.0	+5.5
45°	66484	68770	123960	6.67	71792	79972	100560	-7.4	-14.0	+17.5

* Negative sign indicate lower value.



M = Coefficient x q x a^2 x grid spacing

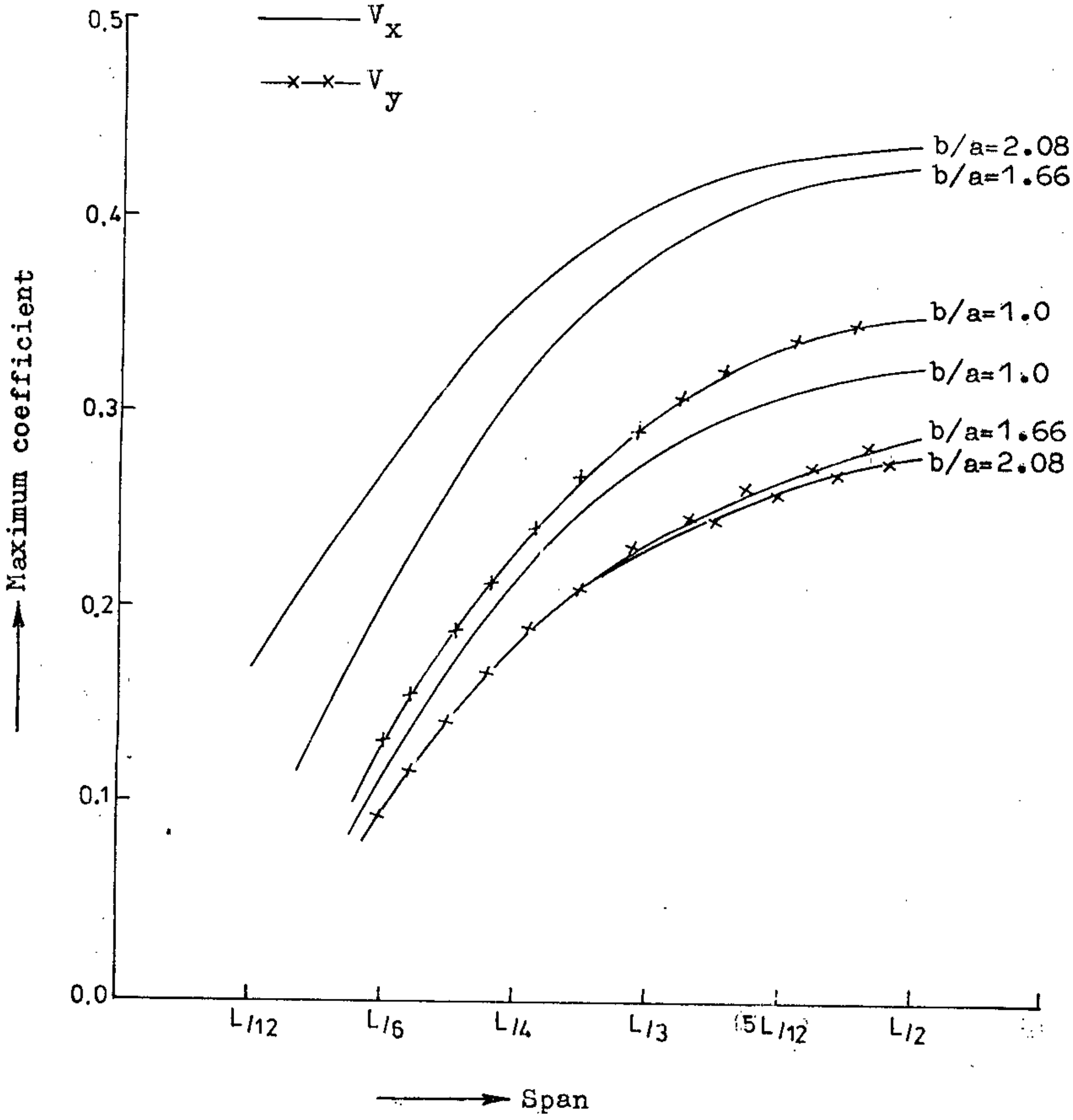
Fig. 4.22 Coefficients of moment for different aspect ratio for fixed grid.



M = Coefficient x q x a^2 x grid spacing

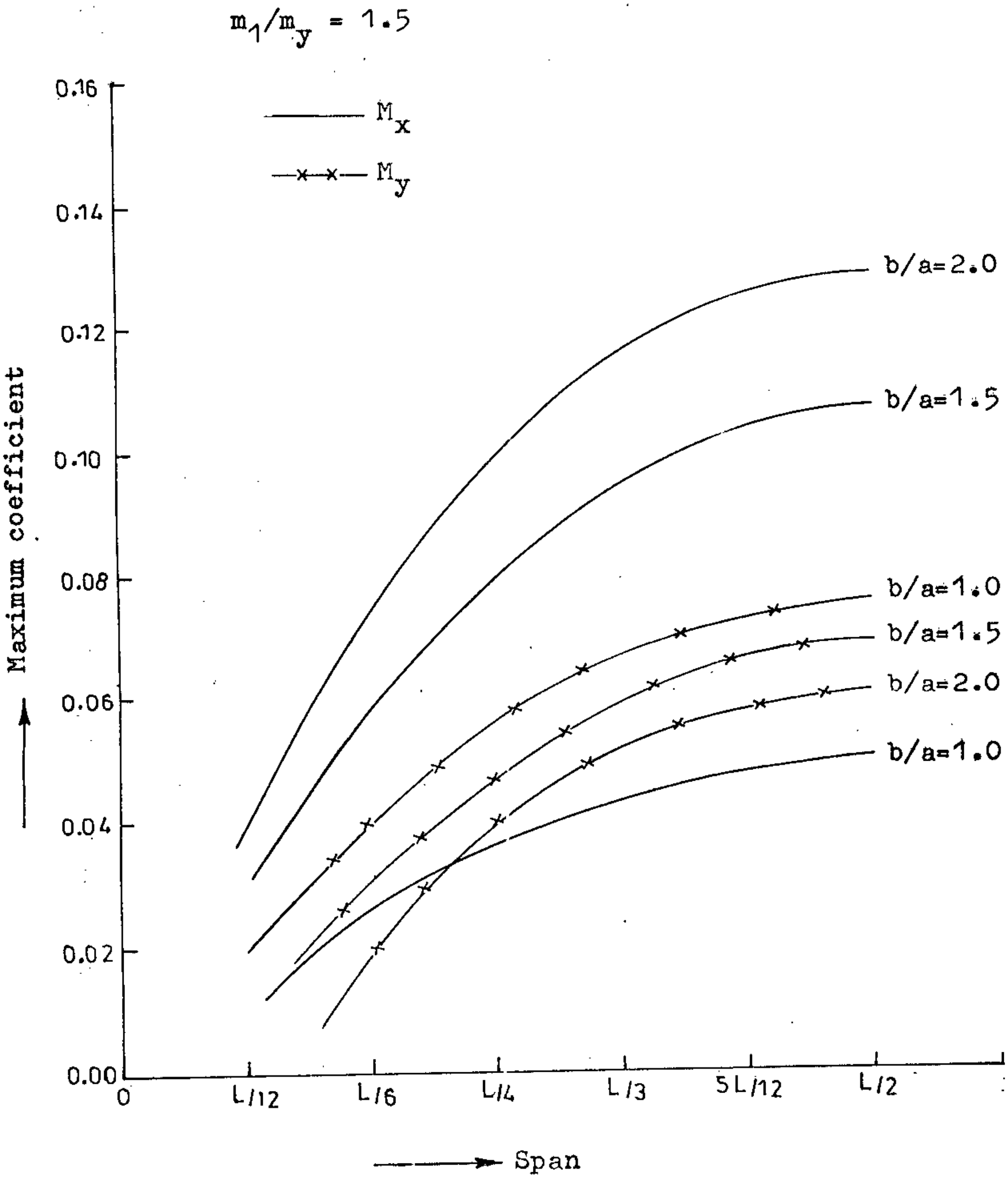
Fig. 4.23 Coefficients of moment for different aspect ratio for fixed grid.

$m_1/m_2 = 1.25$



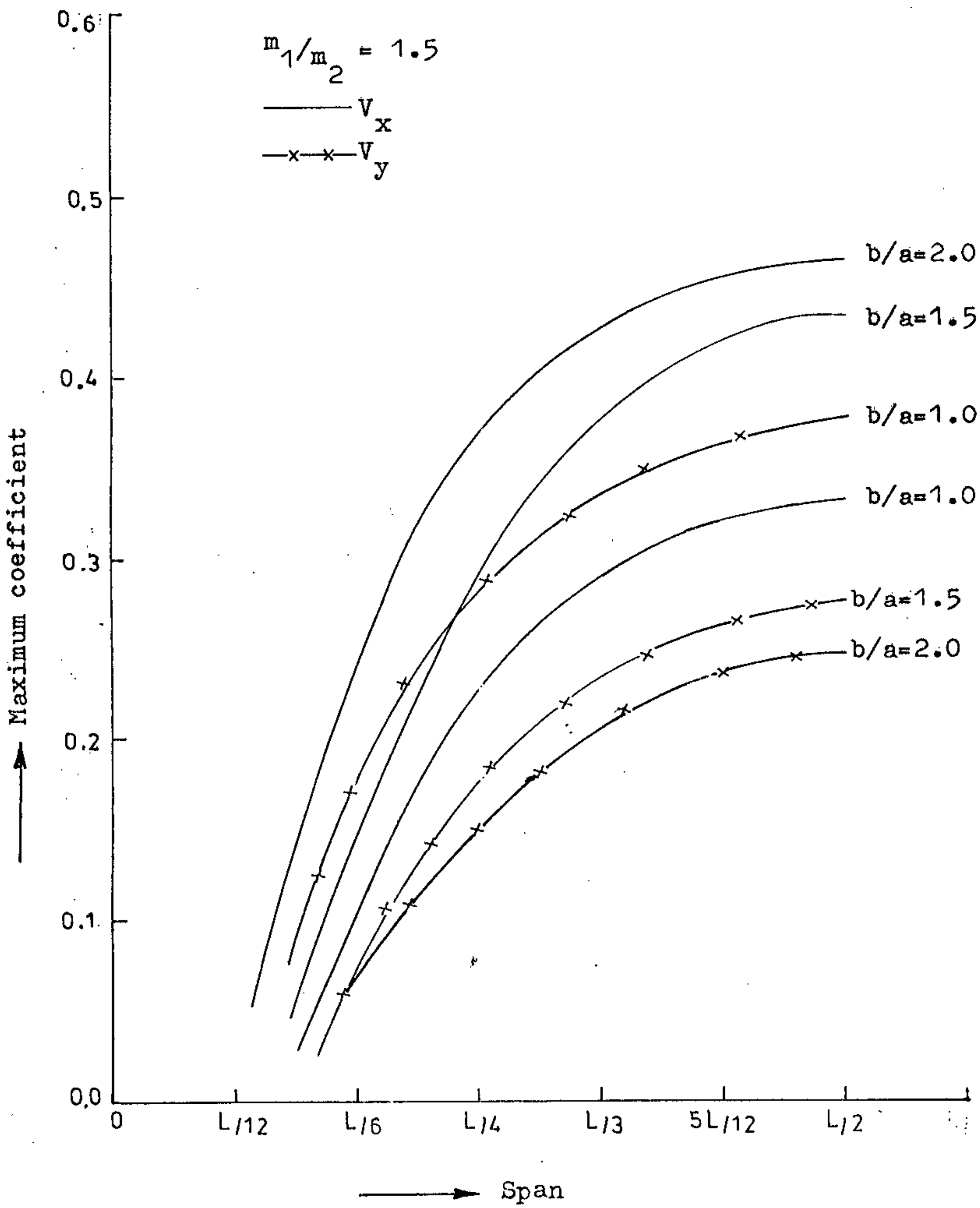
V = Coefficient x q x a x grid spacing

Fig. 4.24 Coefficients of shear for different aspect ratio for fixed grid.



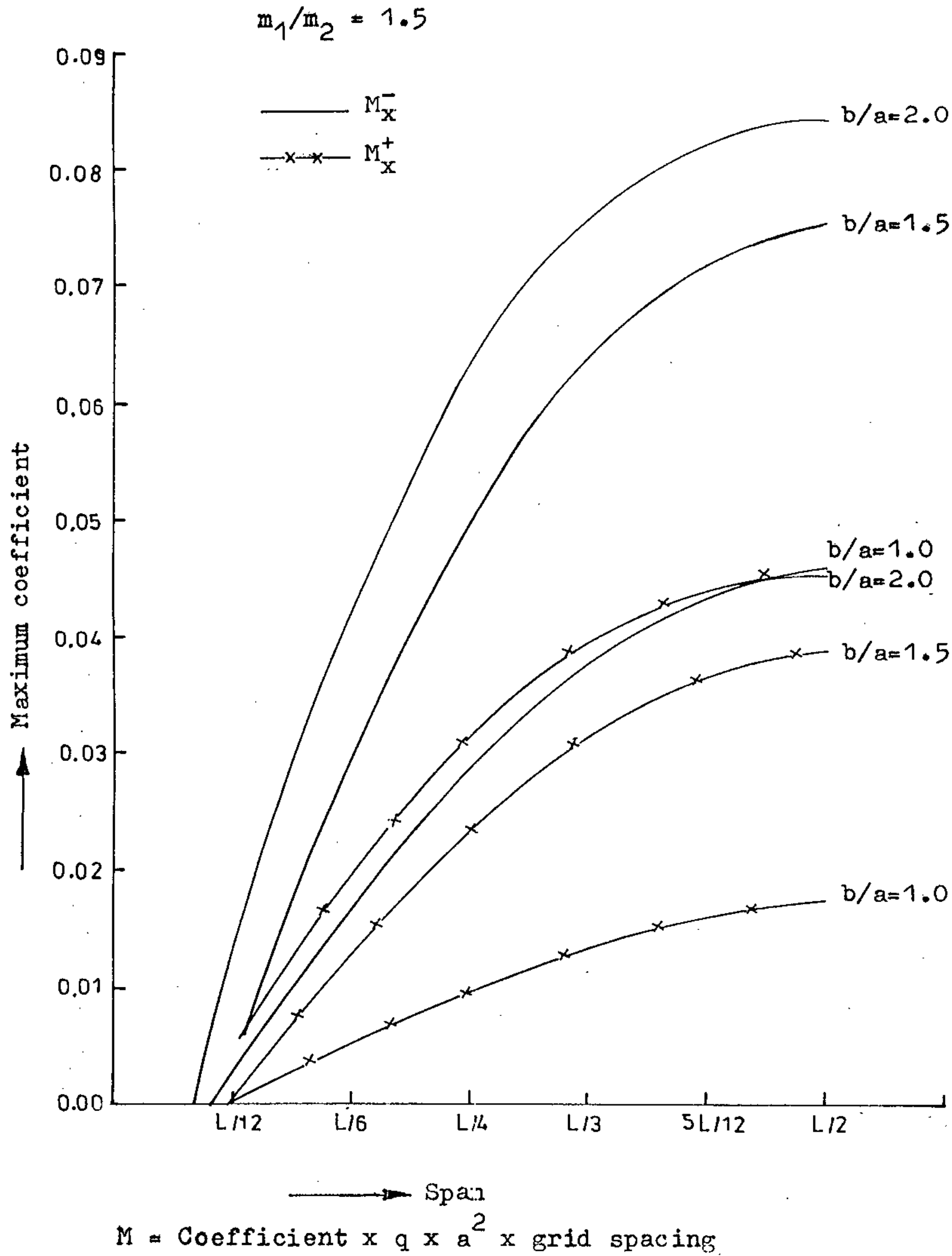
M = Coefficient x q x a^2 x grid spacing

Fig.4.25 Coefficients of moment for different aspect ratio b/a for simly supported grid.



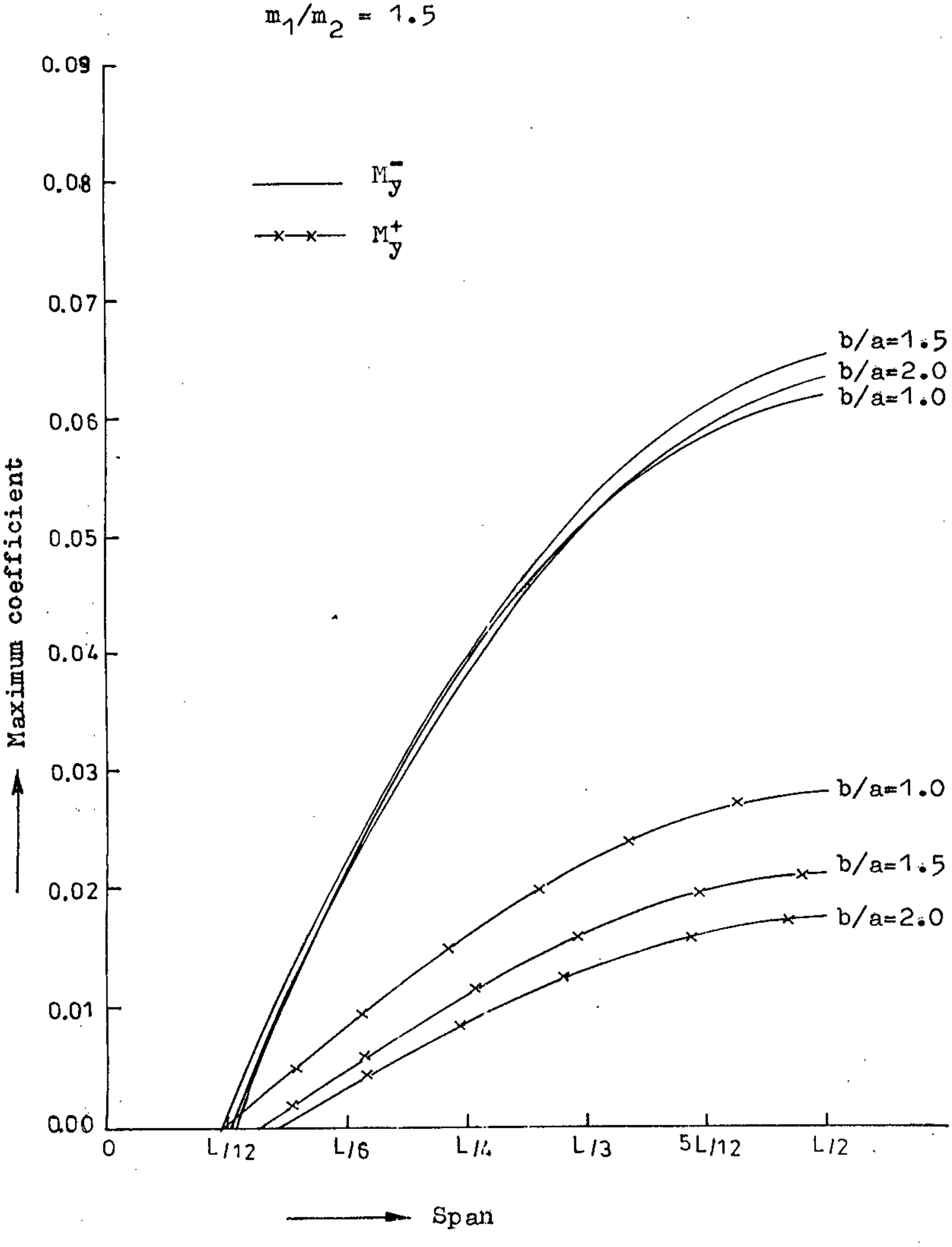
V = Coefficient x q x a x grid spacing

Fig. 4.26 Coefficients of shear for different aspect ratio b/a for simply supported grid.



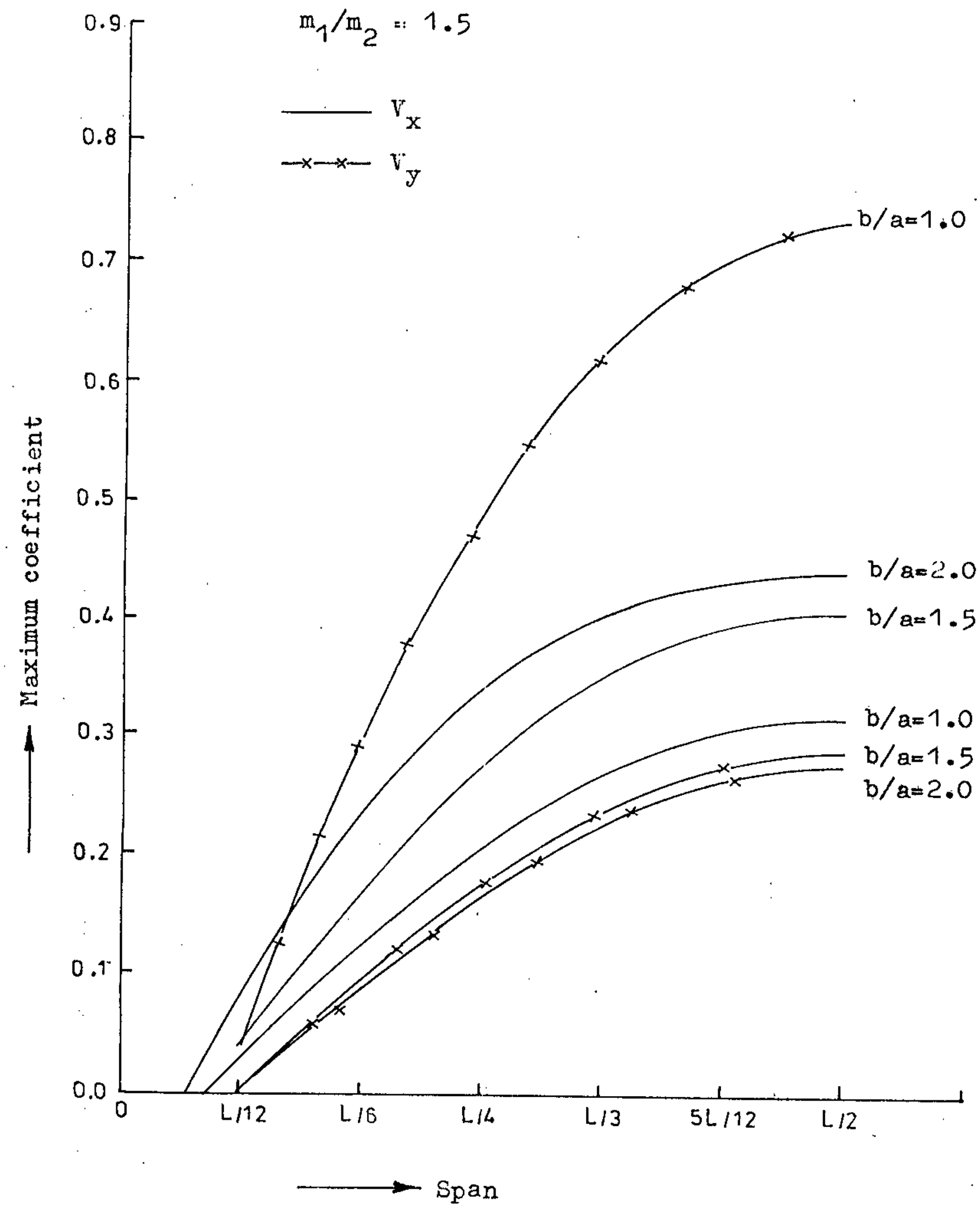
M = Coefficient x q x a^2 x grid spacing

Fig. 4.27 Coefficients of moment for different aspect ratio b/a for fixed grid.



M = Coefficient x q x a^2 x grid spacing

Fig. 4.28 Coefficient of moment for different aspect ratio b/a for fixed grid.



V = Coefficient x q x a x grid spacing

Fig. 4.29 Coefficients of shear for different aspect ratio b/a for fixed grid.

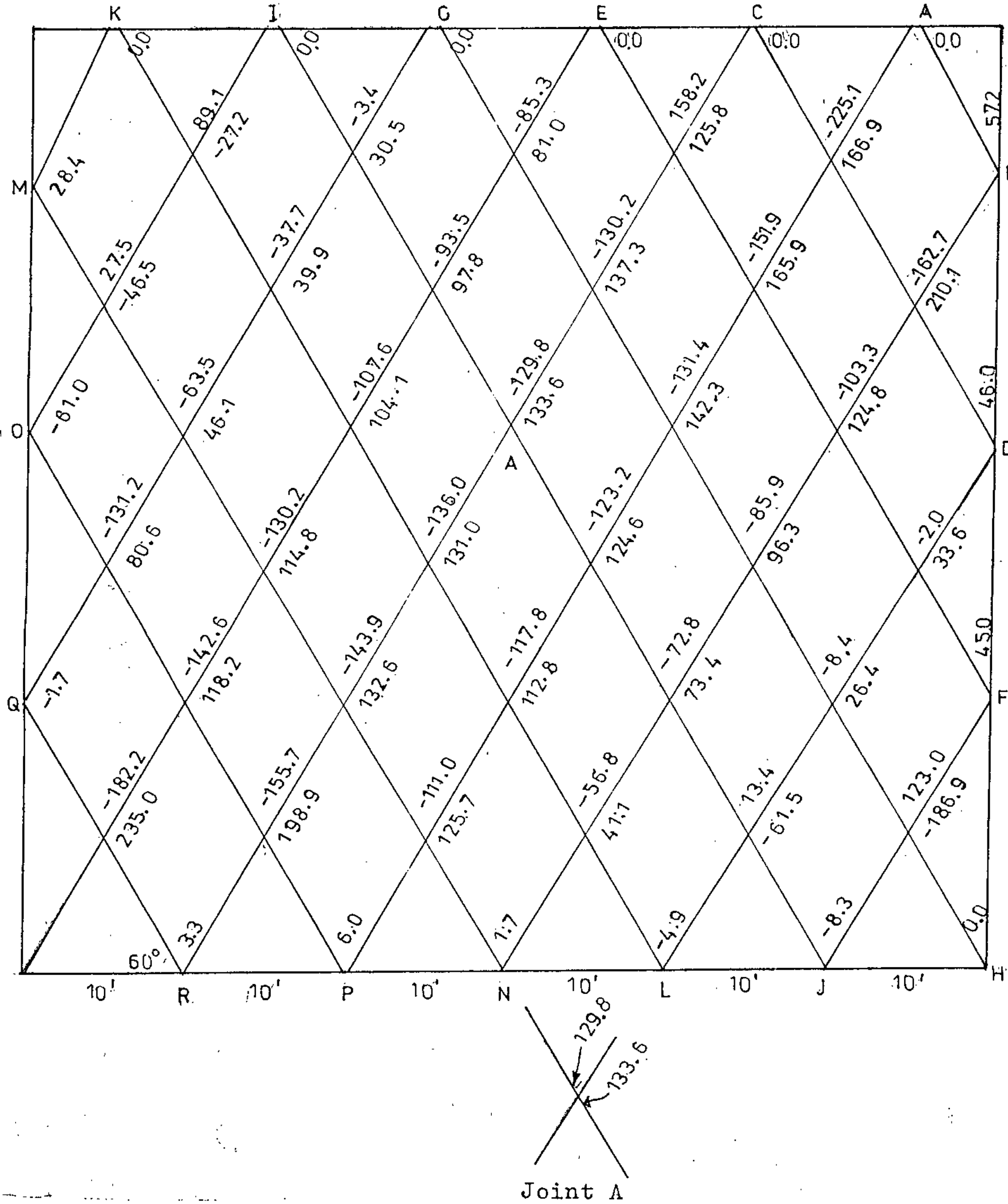


Fig. 4.30 Moments for skew grid with angle of skewness 60°.

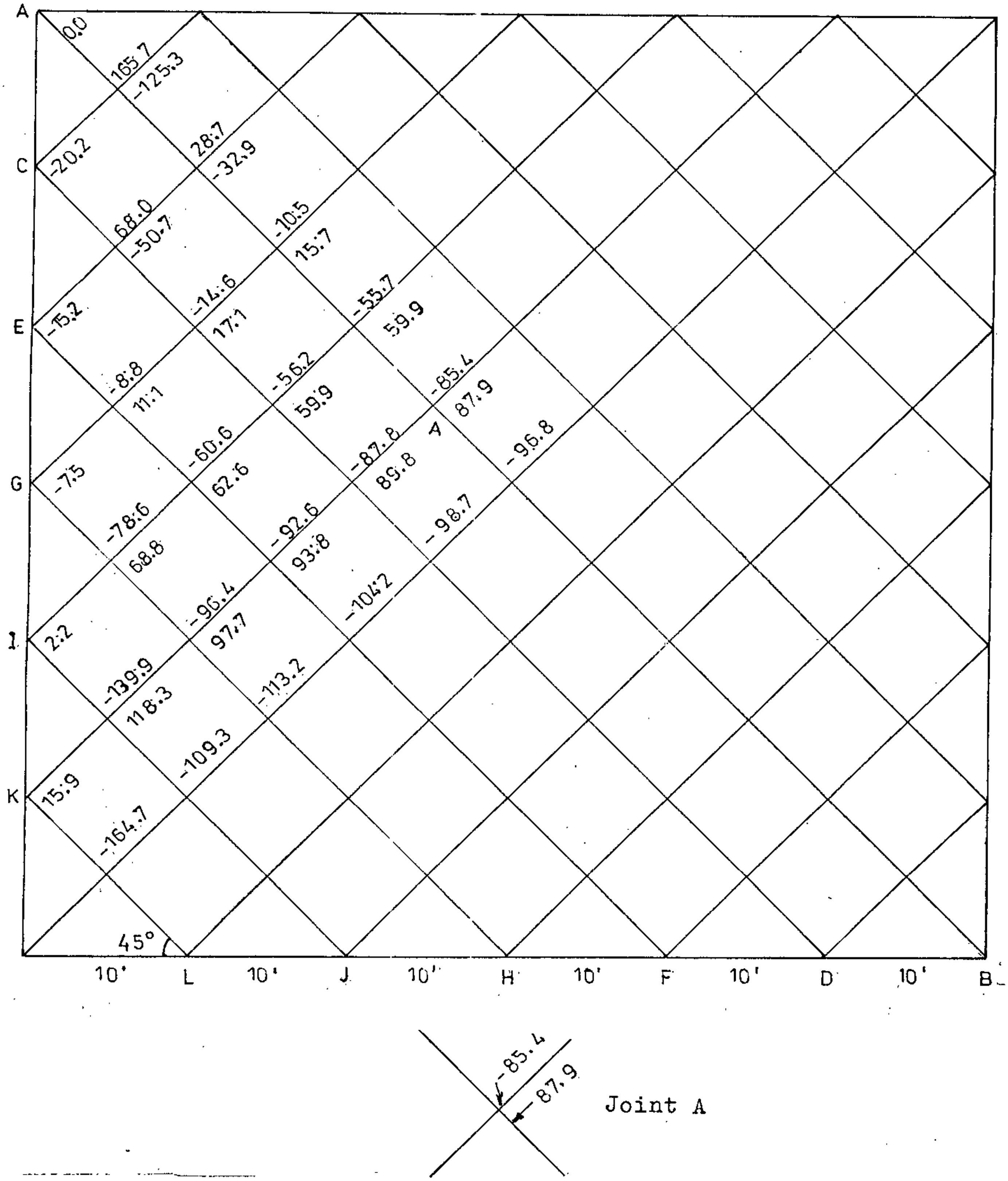


Fig. 4.31 Moments for skew grid with angle of skewness 45°.

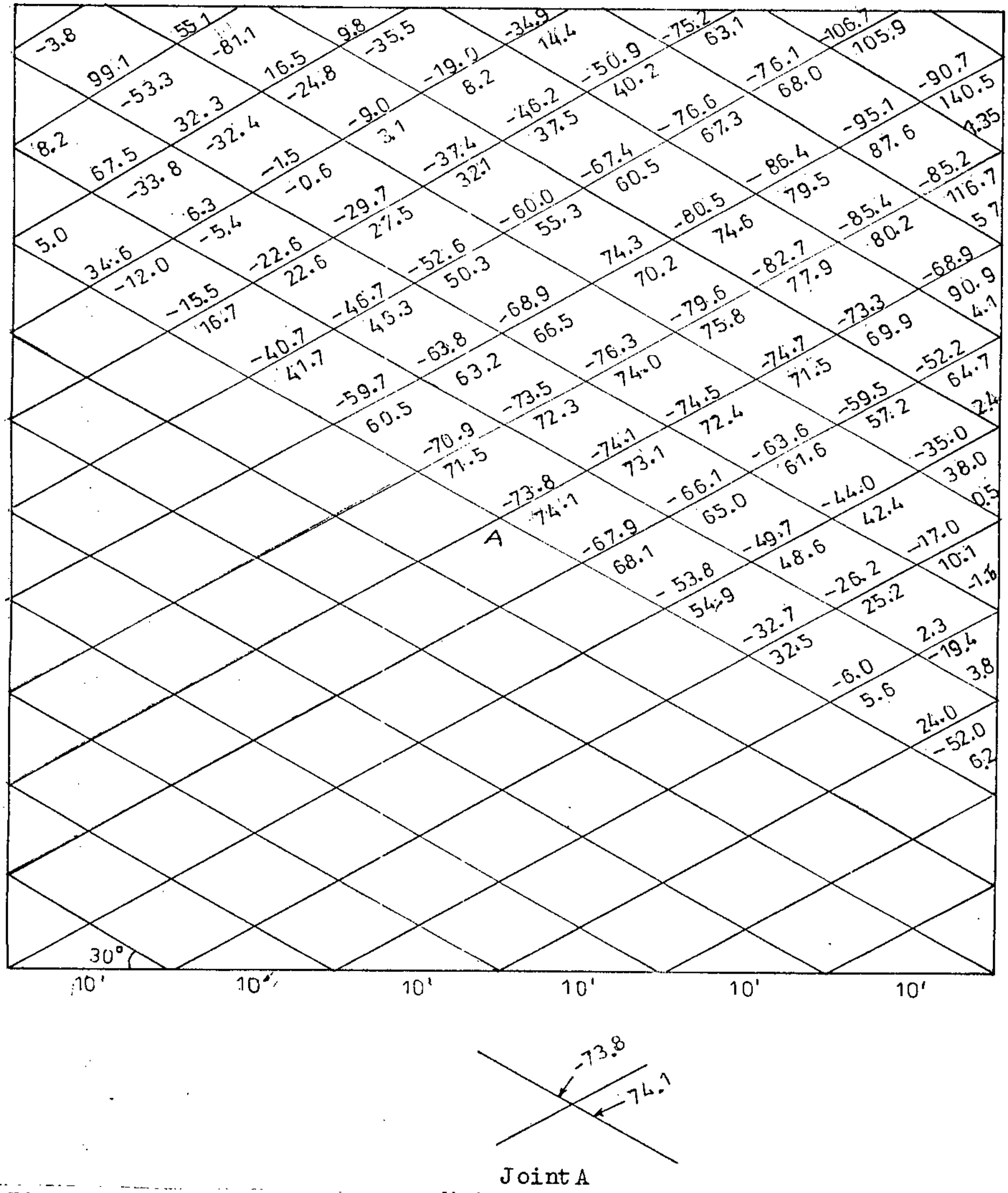
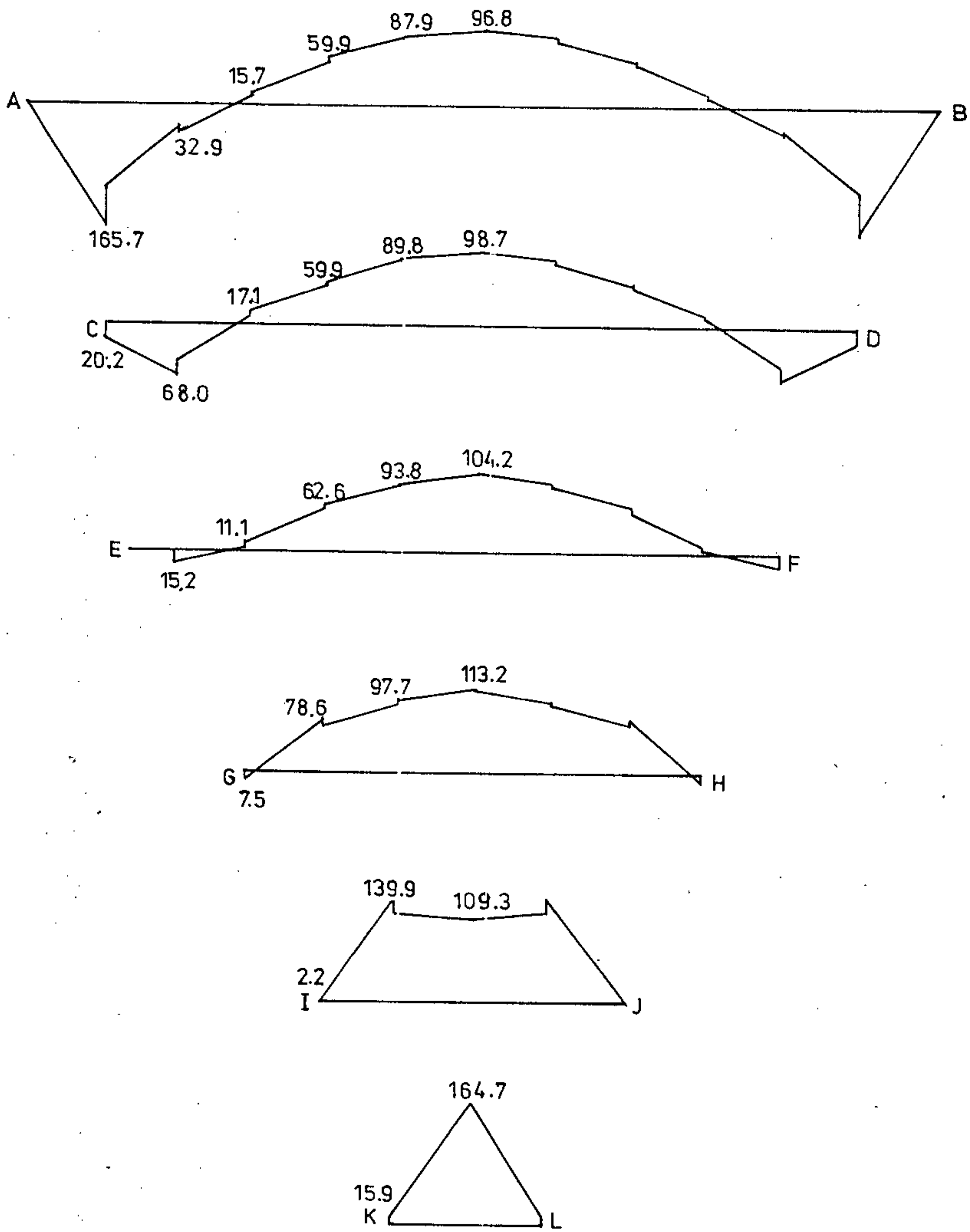


Fig. 4.32 Moments for skew grid with angle of skenwness 30°.



Scale 0.1 inch = 20 k-ft.

Fig. 4.33 Moment diagram of 45° skew grid members.

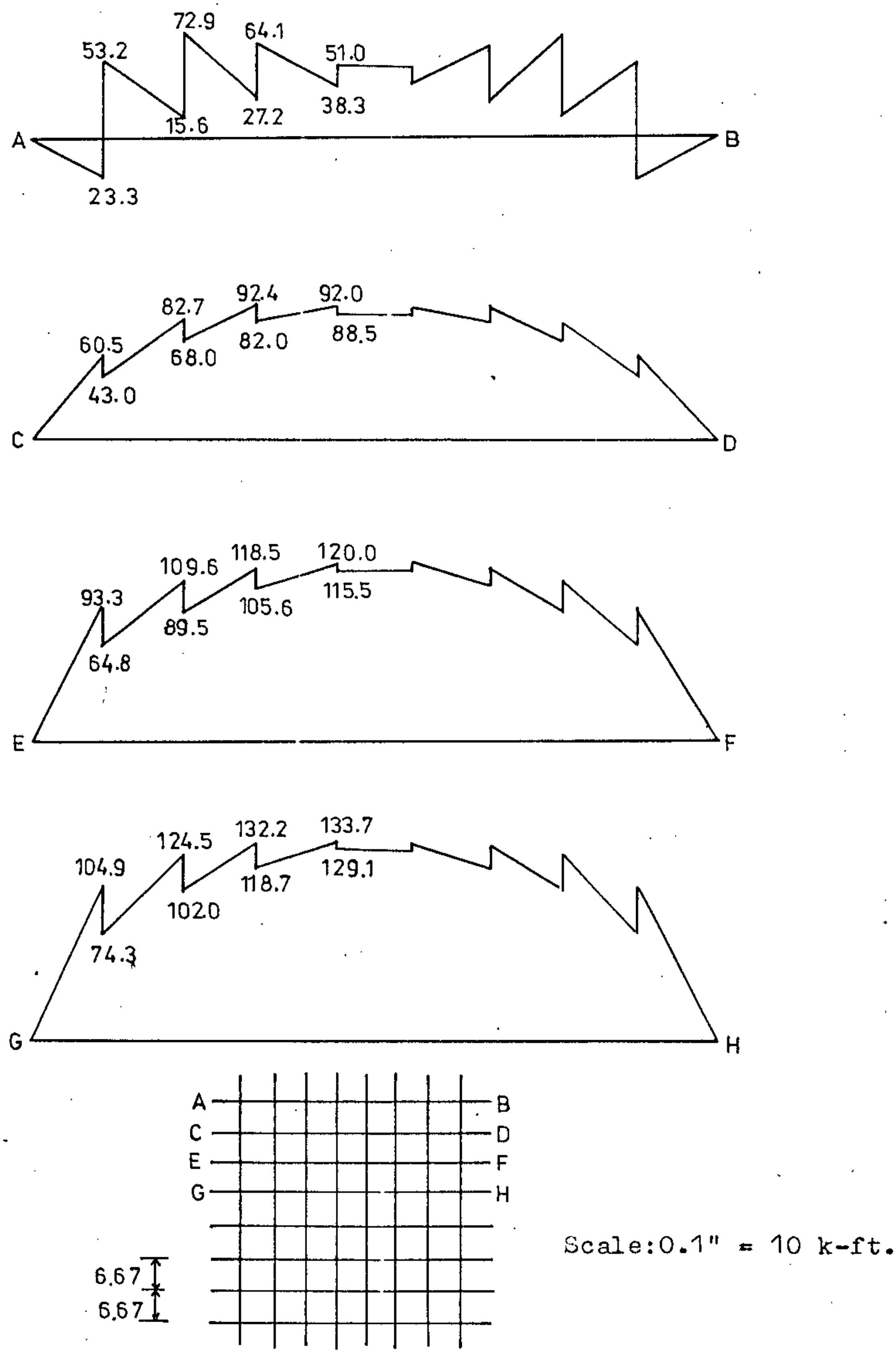


Fig. 4.34 Moment diagram of the equivalent square grid floor member for 45^o skew grid.

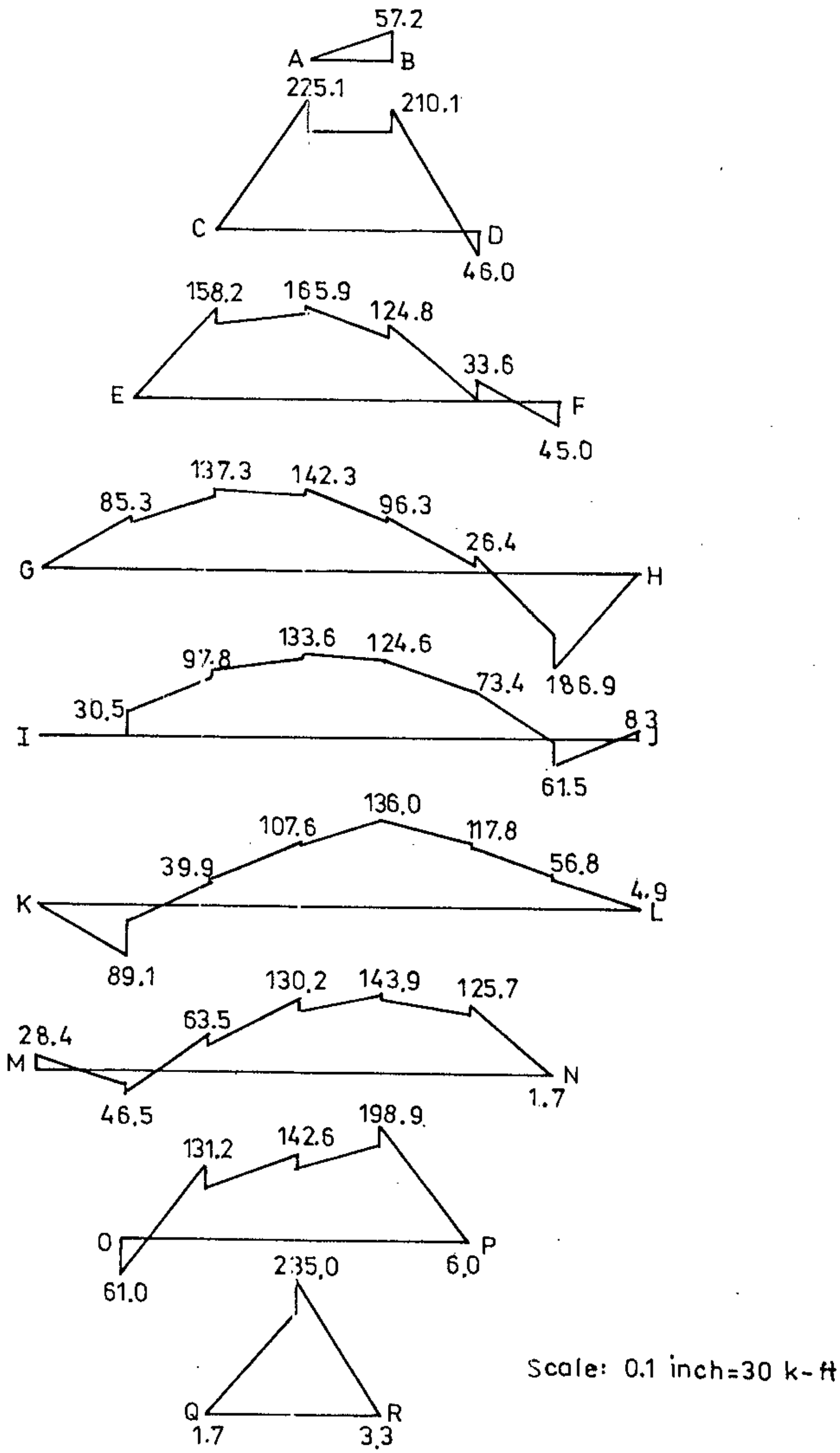


Fig. 4.35 Moment diagram of 60° skew grid members.

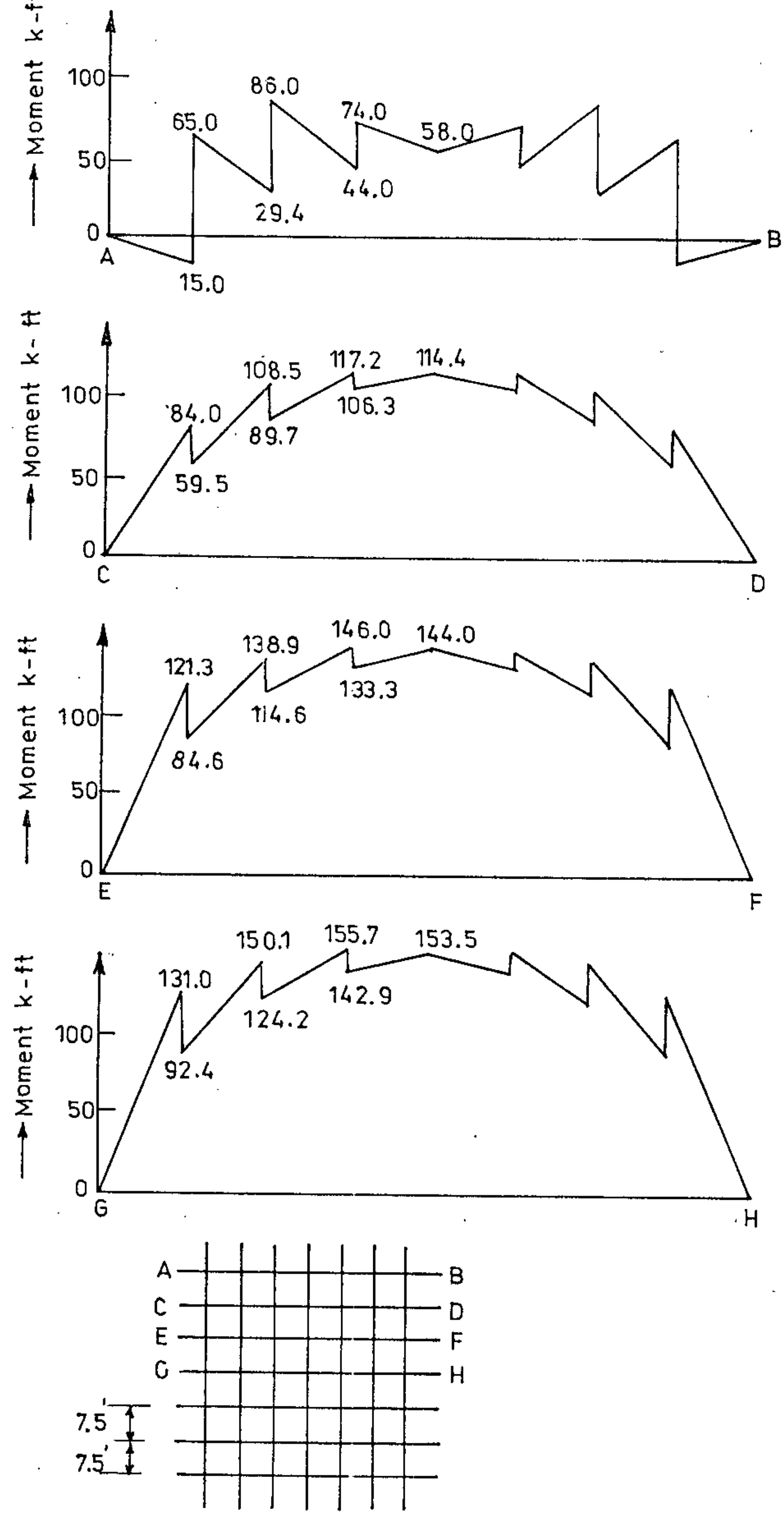


Fig. 4.36 Moment diagram of the equivalent square grid floor member for 60° skew grid.

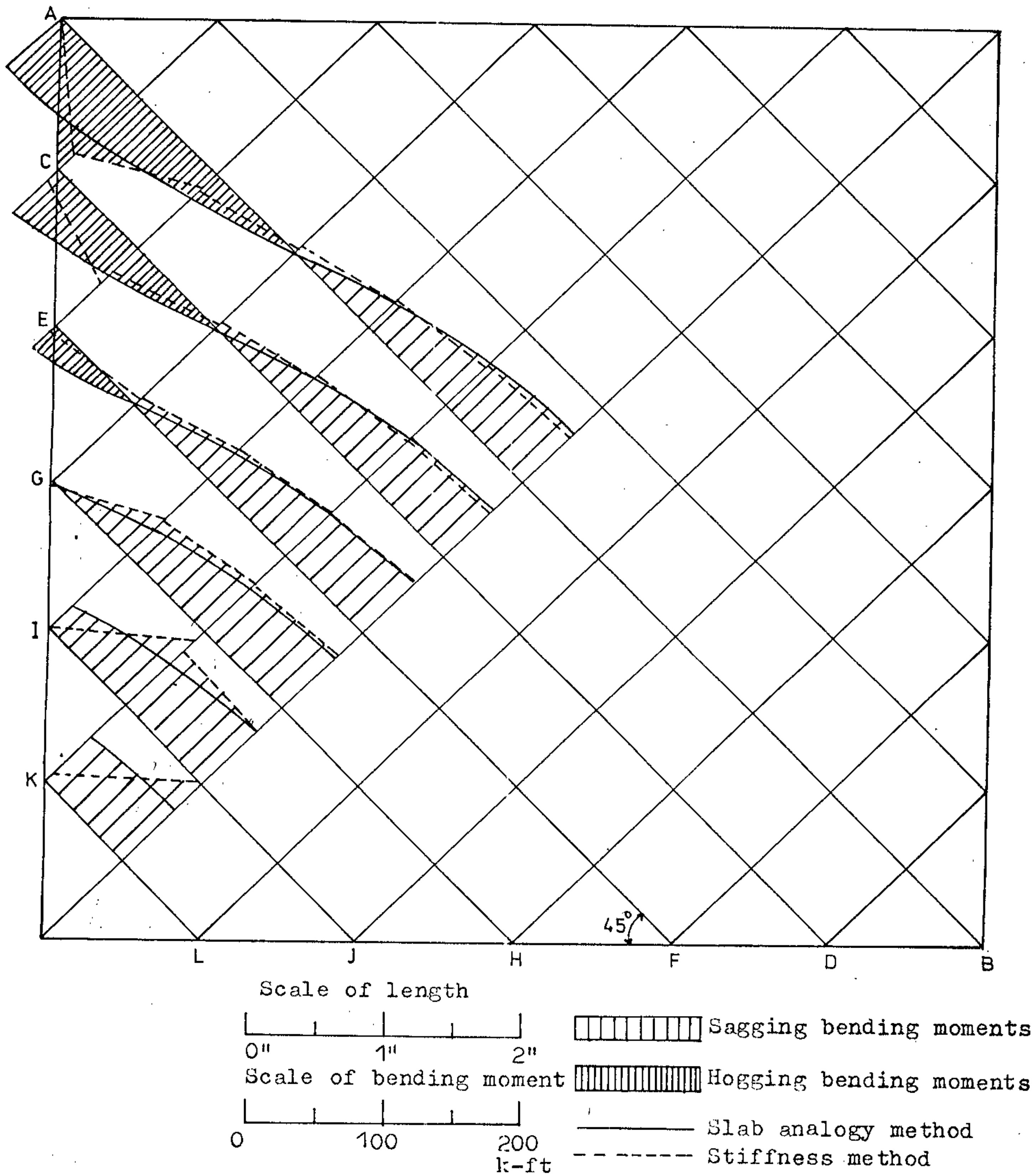


Fig. 4.37 Comparison of bending moment in 45° skew grids under uniformly distributed load 0.1 ksf.

CHAPTER 5

CONCLUSIONS AND RECOMMENDATIONS FOR FURTHER STUDY

5.1 General

Analysis of grid is complicated due to number of interconnected joints. The number of equations to be solved is thus very high. The grid floor can be solved by equivalent orthotropic plate method where the governing equation is solved for deflection, moment and shear at different points by converting the load into Fourier series. This governing equation can also be solved by Finite difference method. But these methods are complicated and time consuming. An exact solution for a grid floor can be easily obtained by stiffness method. For easy and rapid analysis and design of grid floor, the tables and design charts that were developed in this study will be useful.

The saving of the computer storage requirement has been a major aim in the development of the computer program. The large stiffness elements grid analysed in this study required a large computer storage. To reduce this problem partitioning method was followed where only the required portions of the overall structure stiffness matrix corresponding to the total degrees of freedom were generated.

5.2 Conclusions

Parametric study involving various structural parameters such as width, depth, spacing of the grids were made to ascertain its effects on moments, shears and torsions. From the study of the floor made in this investigations the following conclusions may be drawn:

1) The maximum moment and shear occur along mid span members for both the simply supported and fixed supported continuous square grid while in the corner supported grid it occur along support line. Hence critical section for design is at the mid-span for continuous square grid and at the support line for corner supported grid.

2) The torsion is zero at the mid-span and maximum near the edges and corners.

3) For a change of ρ, for simply supported square grid floor, from 1.0 to 7.2, the maximum torsional moment decreases by 79.2%, while the maximum bending moment increases by 23.8%. Similarly for fixed square grid floor these values are 81.1% for torsion, 24.5% and 7.9% for positive and negative moments respectively.

4) The moment, shear, deflection and torsion of a grid floor can be reduced significantly by decreasing the spacing of grid members. The percentage of

reduction for moment, shear and deflection are approximately equal to the percentage of reduction of spacing.

5) For an orthogonal grid floor, the maximum moment and deflection as obtained by plate method is lower than that of stiffness method. This difference in results between stiffness method and plate method is less for fixed grid. For 45^{0} skew grid floor, square in plan, the positive moment at center as obtained by slab analogy method is higher than that obtained by stiffness method, while the slab analogy method underestimates the positive and negative moments at the corners. This difference in results is small at the center and large at the corners.

6) As the aspect ratio b/a increases, the coefficients for maximum moment and shear in the shorter direction increase for both the simply supported and fixed square grid.

7) In the skew grid floor, maximum moment, shear and torsion occur near the corners. So the critical section for design is at the corners.

8) The moment and deflection can be reduced by using skew grids. But skew grid requires higher number of members than the square grid floor.

5.3 Recommendations for Further Study

The program that is developed in this study is suitable for any type of grid structures. However, design tables and charts are developed for right grid only. Future studies should be made to develop charts for skew grids.

In this study linear elastic method of analysis has been used for grid floor. Future studies may be carried out on the nonlinear and post elastic behaviour of grid floor materials.

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