A NUMERICAL STUDY ON FLOW SEPARATION AND SHOCK WAVE INTERACTION OVER A FLAT PLATE WITH FACING STEP

BY

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RECOMMENDATION OF THE BOARD OF EXAMINERS

The board of examiners hereby recommends to the Department of Mechanical Engineering, Bangladesh University of Engineering and Technology, Dhaka, the acceptance of this thesis "**A NUMERICAL STUDY ON FLOW SEPARATION AND SHOCK WAVE INTERACTION OVER A FLAT PLATE WITH FACING STEP**", submitted by Konica Sarker, in partial fulfillment of the requirements for the degree of Master of Science in Mechanical Engineering.

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ABSTRACT

Results of a numerical investigation on a transonic compressible flow over a flat plate with facing steps are presented. The study has been performed by solving Two-Dimensional Navier-Stokes equations. The system of governing equations has been solved, using an explicit Harten-Yee Non- MUSCL Modified flux type TVD scheme and a zero-equation algebraic turbulence model to calculate the eddy viscosity coefficient. Simulations for backward and forward facing step have been done over a range of Mach numbers from 0.8 to 1.2 for four different step heights (0.025, 0.05, 0.075 and 0.10). The results show the details on pressure, temperature and velocity field, together with recirculation length, expansion shock, reattached shock, oblique shock, detached shock and interaction among shock waves of flow field. The flow field characteristics with the change of facing step, Mach numbers and heights of the step are also reported. The main objective of this study is to disclose the effect of forward and backward facing step, Mach numbers and heights of the step on the flow pattern of a transonic compressible flow over a flat plate which would be helpful for the subsequent construction in reality.

The various parameters of the calculation are (i) the type of the facing step (ii) the height of the step and (iii) Mach number of the incoming flow. Due to the variation of type of step, four different shocks appear in the flow field; corner expansion shock and reattached shock appear in backward facing step, on the other hand leading edge oblique shock and detached shock appear in forward facing step. In both cases, recirculation is found on the flow field which is located behind the backward facing step and in front the forward facing step, respectively**.** It can be found that the change of step heights and Mach numbers changes the feature of different shocks, the dynamic behavior of the flow field, affects the temperature and pressure field, and modifies the length and strength of recirculation regions.

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NOMENCLATURE

Symbol	Meaning	Unit
c	Sound speed	m/s
C_p	Specific heat at constant pressure	J/(kg.K)
E	Total energy	J/m^3
$\hat{F}$	Transform flux vector in x-direction	
G	Flux vector in y-direction	
$\hat{G}$	Transformed flux vector in ¶ -direction	
h	Height of the step	mm
H	Total height of the calculation domain	m
J	Transformation Jacobian	
J	Number of grid point in x-direction	
JJ	Maximum number of grid point in x-direction	
K	Number of grid point in y-direction	
KK	Maximum number of grid point in y-direction	
L	Total length of the calculation domain	m
M	Mach number	
P	Pressure	Pa
P_{in}	Inlet Pressure	Pa
R	Universal gas constant	J/(kg.mol.K)
U	Incoming Stream	m/s
U	Vector in conservative variables	
$\hat{U}$	Transformed vector in conservative variables	
u	Horizontal velocity	m/s
V	Contravariant velocity in ¶ direction	
v	Vertical velocity	m/s
x	Horizontal Cartesian co-ordinate	
y	Vertical Cartesian co-ordinate	
¶	Transformed coordinate in horizontal direction	
¦	Transformed coordinate in vertical direction	
°	Mass density	kg/ m^3
$\pm_{x,y}$	Normal stress	Pa
²	Shear stress	Pa
«	Coefficient of dynamic viscosity	Kg/(m.s)
©	Thermal conductivity	W/(m.K)
¬	Viscous term	

CHAPTER-I

INTRODUCTION

1.1 General

Numerical simulation is an important drive for the design and analysis of more complicated system. The advancement of computing urges engineers to include high fidelity in computational fluid dynamics (CFD) in the design and testing tools of new technological products and processes. Numerical simulation is now recognized to be a part of the computer aided engineering (CAE) spectrum of tools used extensively today in all engineering field.

The separation and reattachment of flows occur in many practical engineering applications, both in internal flow systems such as diffusers, combustors channel with sudden expansion, and in external flows like flows around airfoils and buildings. In these situations, the flow experiences an adverse pressure gradient, i.e, the pressure increases in the direction of flow, which causes the boundary layer to separate from the solid surface. The flow subsequently reattaches downstream forming a recirculation bubble. In some application such as combustors, the presence of the recirculation and turbulence due to separation can help enhance the mixing of fuel and air. On the other hand, separation in the pipe and duct flow causes loss of available energy. Thus understanding the flow separation and reattachment phenomena is important in engineering design.

Transition of boundary layer, separation and reattachment of flow are strongly influenced by any imperfections that exit on aerodynamic surfaces. These imperfections, or roughness elements, can occur in various forms and are of different sizes. They may include regions of waviness, bulges, steps, gaps at junctions, surface contamination from insect debris, ice and dirt particles of various sorts. Although, modern manufacturing and maintenance procedures make it possible to provide reasonable good operational surfaces, some imperfections are unavoidable, particularly those arising from such things as the installation of imperfection panels. They often arise in the form of sharp-edged steps. Supersonic compressible flow over a flat plate with steps (both forward facing and backward facing) is very important in many engineering applications. In present works a high speed flow field on flat plate with facing steps is considered. The physics of flow separation, flow transition from subsonic to supersonic and dynamical behavior of the flow field are addressed for different Mach number and step height. It is expected that the results of this investigation would be a useful guide to the effect of different Mach number and step size on various flow field characteristics.

1.2 Background

Both experimental and numerical investigations have been performed to analyze the different flow field characteristic of high speed compressible flow over a flat plate with facing step. A comprehensive review of the current understanding flow has been published [1-31]. A number of works which have been done on back ward facing step are discussed first.

Chen et al. [1] and Scherberg [2] did an experimental study on a supersonic laminar flow over a backward facing step. They investigated the flow field for different Mach number with a fixed step height. Chen et al. showed the flow structures, including supersonic laminar boundary layer, separation, reattachment, redeveloping turbulent boundary layer, expansion wave fan and reattachment shock in the transient flow fields. Again Scherberg presented that the pressure changes from free stream pressure to base pressure were linear functions of free stream pressure for each Mach speed and step height; the corner pressure expansion is not the linear Prandtl-Meyer fan; the separation shock emerging from the separated shear layer may not be neglected in such flows; base pressure influence the flow up steam of the corner. Halupovich et al. [3] and Yang et al. [4] numerically investigated the flow field over a backward facing step. Yulia indicated that the separation point was positioned on the step face, below the corner and a lip shock was formed to match the flow conditions at the step corner. On the other hand, due to the damping effect introducing by the explicit artificial dissipation, the lip shock was not observed in Yang's investigation [4]. Popusco and Panait [5] conducted an experimental study to analyze the field velocity of a fully developed turbulent incompressible flow behind a backward facing step with a curve nose shape. They compared and analyzed the numerical simulation and of the experimental study. In their investigation both the numerical and experimental results showed the existence of four interacting zones: separated free shear layer, the recirculation region under the shear layer, the reattachment region and the attached/recovery region. Armaly et al. [6] found additional regions of flow separation downstream of the step and on the both side of the channel test section. Al-Maaitah et al. [7] investigated the effectiveness of wall suction on the stabilization of subsonic (up to Mach number 0.8) flows around 2-dimensional backward facing step. The results showed that continuous suction stabilized the flow outside the separation bubble, but is destabilized the the flow inside it. For the same suction flow rate, properly distributed suction strips stabilized the flow more than continuous suction. Furthermore, the size of suction bubble and hence its effect on the instability can be considerable reduced by placing strips with high suction velocities in the separation region. Savel'ev [8] numerically investigated the pressures at the boundary layer separation and reattachment points in case of supersonic laminar flow past a two-dimensional "flat-plate/wedge". The results presented in the form of generalized Mach-number-dependences of the theoretical pressure on the wedge surface initiating boundary layer separation and the pressure at the boundary layer reattachment point

Now discuss the paper related to forward facing step. Hassan et al. [9] and Leite and Santos [10] investigated numerically the effect of the frontal face height of the step on flow field characteristics. They showed the pressure profile, temperature, the length of the separated region, location of detached shock, the wall skin friction and heat transfer coefficients in front of the step for different cases. Leite and Santos [10] also showed that the peak values for the heat transfer coefficient on the frontal face surface were at least one order of magnitude larger than the maximum value observed for a smooth surface, i.e., a flat plate without step. Czarnecki [11], Abu-Mulaweh et al. [12], Zukoski [13] and Saravanan et al. [14] performed an experimental investigation on forward facing step. Czarnecki and Jackson [11] tested the pressure distribution at Mach numbers of 1.61 and 2.00 over a step height range from 0.0005 to 2.54 cm. The analyzed data indicated that the existence of a transverse vortex in the separate flow region which weaken with increase in Renolds and Mach number and significantly affected the pressure distribution. Abu-Mulaweh et al. [12] showed that the inlet velocity and the step height significantly affected the flow and thermal field. The local heat transfer coefficient was found to increase as the inlet velocity increased and step height decreased. On the other hand, the length of recirculation regions upstream and downstream of the step were found to increase as the inlet velocity and step height increased. Zukoski [13] presented that the pressure rise in the separation region expressed in normalized form is independent of Mach and Renolds number and that the scale for the separation phenomena is the boundary layer thickness. They also showed, the induced side force increases linearly with Mach number. Saravanan et al. [14] addressed the effect of forward facing cavity on heat transfer and aerodynamics coefficient. They carried out the test at a hypersonic Mach number of 8.0. The important result of this study was the smaller cavity diameter had the highest lift-to-drag ratio, whereas the medium cavity had the highest heat flux reduction. Amaha et al. [15], Goodrich et al. [16] and Sidik [17] had been carried out a computational study to analyze the supersonic shock wave turbulent boundary layer interaction in 2-D compression corner for supersonic flow. Amaha [15] had been done for 20ºcorner angle. The result showed the surface pressure distribution, wall shear stress, stream wise velocity distribution and velocity contour. Goodrich et al. [16] predicted the location of dividing streamline reattachment, pressure distribution and characteristic corner flow behavior. Sidik [17] showed the effect of Mach number and corner angle on peak pressure and recirculation region. Lee [18] dealt with laminar boundary layer/shock-wave interaction in which the pressure rise generated in an external supersonic, inviscid flow was communicated upstream through the boundary layer and thereby introduced flow separation. In order to understand these phenomena approximately, including the subsequent reattachment flow, he utilized an integral or moment method in which the first moment of momentum was employed, in addition to the usual momentum integral. Klineberg [19] investigated the fluid mechanic problem in which the pressure distribution was determined by the interaction between an external, supersonic inviscid flow and an inner laminar viscous layer. The boundary layer approximations were assumed to remain valid throughout the viscous region and the integral or momentum method of Lee and Reeves, extended to include flows with heat transfer. Lewis [20] performed an experimental investigation of boundary layer separation associates with compression corner. The surface pressure distribution for the cold wall was compared with adiabatic configuration of laminar interaction and dependence on Renolds number for both laminar and transactional interaction was observed.

Suxum [21] performed an experimental investigation of hypersonic flow over a set of rectangular cylinders. All tests were carried out at Mach 5 under turbulent conditions. The results indicated that the aspect ratio (H/W) is important parameter for the classification of the flow field. Hung and MacCormach [22] developed an efficient time splitting, second-order accurate, numerical scheme which was sued to solve the complete Navier-Stokes equation for supersonic and hypersonic laminar flow over a two-dimensional compressible corner. Favorable comparisons with previous calculations and with experimental results were made. The pressure profile is neither constant across the boundary layer nor constant along simple straight characteristic lines, as has been assumed is some previous analyses. Carter [23] obtained the numerical solution of the Navier-stokes equations for laminar flow past a compression corner at low Mach number. He used Brailovskaya difference scheme, which was first order accurate in time and second order accurate in space. He computed the result for supersonic flow over a 10º wedge with an adiabatic wall. Hattori and Nagano [24] investigated the detail turbulent structure of boundary layer over a forward facing step. They showed the effect of step height and inlet boundary layer thickness on turbulence structure. They conduct their investigation by analyzing counter gradient diffusion, turbulence energy, frictional coefficient, Reynolds shear stress etc.

Some researchers did their investigation on both forward and backward facing step. Saleel et al. [25] were facilitated a numerical investigation of the laminar forced fluid flow over a forward-backward facing steps through a 2-D channel. They presented the effect of Reynolds numbers and different sizes of the step on various flow field characteristics. Crouch et al. [26] reported on the boundary layer displacement thickness due to stepping. They investigated the step effects on transition in boundary layer under favorable or adverse pressure gradients. In the results they showed for step heights up to 1.5 times the local boundary-layer displacement thickness. Wang and Gaster [27] did an experimental study on the effect of a sharp-edged step on boundary layer transition and presented the effect of steps (both forward and backward) on transition positions, transition Renolds number, spectra distribution etc. Redford et al. [28] did a numerical study on roughness element. Redford found a correlation based on roughness height, Renolds and Mach numbers and wall temperature to separate laminar from transitional. Nguyyen et al. [29] numerically study the flow field in which the flow was turned through 20 deg compression and 20 deg expansion successively. Here some models of simulations were employed in the computations. The result illustrated and discussed the detail comparisons with measured wall pressure, skin friction, velocity profiles and Renolds stress profiles downstream of the interaction region. Moreover, Rgab et al. [30] determined the credibility of interacting boundary layers in predicting compressible subsonic flows over smooth surface imperfections and the results showed that the interacting boundary-layer formulation produced accurate mean flows that yield accurate linear stability characteristics, such as growth rates and amplification factors. (Hattori and Nagano did both numerical and experimental study on) Arnal et al. [31] did both numerical and experimental study on effect of different types of surface imperfections. In result they showed that the upstream transition movement is significantly more abrupt with forward steps than with backward steps.

1.3 Objectives

A simplified representation of surface irregularities is often accomplished by considering backward facing steps or forward facing steps. The physical mechanisms underlying this evidence are not yet clear and the results available in literature are limited. So for high speed flow over a facing step, the characteristic of the boundary layer and boundary layer separation, dynamic behavior of the flow field, effect of facing step on flow separation, effect of Mach number etc. are the major parameters of investigation. The objectives of this present study are:

1. To study the effects of step heights (0.025, 0.050, 0.075 and 0.100) on the various flow characteristics

2. To analysis the flow field with the change of Mach number from 0.8 to 1.2.

3. To study the effect of forward and backward facing step on separation of boundary layer.

CHAPTER-II

FLOW FIELD MODEL

2.1 Governing Equations

The flow field is governed by the unsteady, two-dimensional full Navier-Stokes and species continuity equations. The body forces are neglected. With the conservation-law form, these equations can be expressed by

$$\frac{\partial U}{\partial t}+\frac{\partial F}{\partial x}+\frac{\partial G}{\partial y}=\frac{\partial F_v}{\partial x}+\frac{\partial G_v}{\partial y} \text{ .... } (2.1)$$

Where

$$U=\begin{pmatrix}\rho\\ \rho u\\ \rho v\\ E\\ \rho Y_i\end{pmatrix},\qquad F=\begin{pmatrix}\rho u\\ \rho u^2+p\\ \rho uv\\ (E+p)u\\ \rho Y_i u\end{pmatrix},\qquad G=\begin{pmatrix}\rho u\\ \rho uv\\ \rho u^2+p\\ (E+p)v\\ \rho Y_i v\end{pmatrix},$$

$$F_v=\begin{pmatrix}0\\ \sigma_x\\ \tau_{xy}\\ \sigma_x u+\tau_{yx}v-q_x\end{pmatrix},\qquad G_v=\begin{pmatrix}0\\ \tau_{yx}\\ \sigma_y\\ \tau_{xy}u+\sigma_{yv}-q_y\end{pmatrix}$$

$$P=\sum_{i=1}^{ns}\rho_i R_i T=\sum_{i=1}^{ns}\rho_i\frac{R}{W_i}T \text{ ....(2.2)}$$

$$E=\sum_{i=1}^{ns}\rho_i h_i-\sum_{i=1}^{ns}\rho_i\frac{R}{W_i}T+\frac{\rho}{2}(u^2+v^2)$$

$$= \sum_{i=1}^{ns} \rho_i C_{pi} T - \sum_{i=1}^{ns} \rho_i \frac{R}{W_i} T + \frac{\rho}{2}\left(u^2 + v^2\right) \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots..(2.3)$$

$$\sigma_x = \lambda\left(\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y}\right) + 2\mu\left(\frac{\partial u}{\partial x}\right), \qquad \sigma_y = \lambda\left(\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y}\right) + 2\mu\left(\frac{\partial v}{\partial y}\right),$$

$$\tau_{xy} = \tau_{yx} = \mu\left(\frac{\partial u}{\partial y} + \frac{\partial v}{\partial x}\right) \qquad \lambda = -\frac{2}{3}\mu$$

2.2 Calculation of Temperature

Temperature at various grid points is calculated by Newton-Raphson method. By rearranging Eq. (2.3), a relation for temperature can be expressed as

$$F(T) = \sum_{i-1}^{ns} \rho_i h_i - \sum_{i-1}^{ns} \rho_i \frac{R}{W_i} T + \frac{\rho}{2}(u^2 + v^2) - E \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots..(2.4)$$

The value of h_i is determined from the polynomial curve fitting developed by Moss [32]. Which is as follows:

$$\frac{h_i}{R_i T} = a_{1i} + \frac{a_{2i}T}{2} + \frac{a_{3i}T^2}{3} + \frac{a_{4i}T^3}{4} + \frac{a_{5i}T^4}{5} + \frac{a_{6i}}{T} \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots..(2.5)$$

The values of the coefficient are available in table-1 of which one for temperature range 0~1000K, the other for set for 1000~5000K.

Table 1 Co-efficients of Thermodynamic Polynomials

Temperature range	Temperature range from 0 ~ 1000 K	Temperature range from 1000 ~ 5000 K
Coefficients	N_2	N_2
a_1	0.03744177E+01	0.28532899E+01
a_2	-0.14218753E-02	0.16022128E-02
a_3	0.28670392E-05	-0.62936893E-06
a_4	-0.12028885E-08	0.11441022E-09
a_5	-0.13954677E-13	-0.78057465E-14
a_6	-0.10640795E+04	-0.89008093E+04

Substituting the value of h_i , Eq. (2.4) can be written as

$$F(T) = b_0 + b_1 T + b_2 T^2 + b_3 T^3 + b_4 T^4 + b_5 T^5 \text{...}(2.6)$$

Where the coefficients are

$$b_0 = \sum_{i-1}^{ns} \circ_i R_i a_{6i} + \frac{\circ}{2}(u^2 + v^2) - E \qquad b_1 = \sum_{i-1}^{ns} \circ_i R_i a_{6i} - \sum_{i-1}^{ns} \circ_i R_i$$

$$b_2 = \frac{1}{2}\sum_{i-1}^{ns} \circ_i R_i a_{2i} \qquad b_3 = \frac{1}{3}\sum_{i-1}^{ns} \circ_i R_i a_{3i}$$

$$b_4 = \frac{1}{4}\sum_{i-1}^{ns} \circ_i R_i a_{4i} \qquad b_5 = \frac{1}{5}\sum_{i-1}^{ns} \circ_i R_i a_{5i}$$

Differentiating Eq. (2.6) with respect to T, we get

$$F'(T) = b_1 + 2b_2 T + 3b_3 T^2 + 4b_4 T^3 + 5b_5 T^4$$

Then the temperature is calculated by the following equation:

$$T_{new} = T_{old} - \frac{F(T_{old})}{F'(T_{old})} \text{..}(2.7)$$

The calculation of Eq. (2.7) is repeated until it fulfills the criterions for the temperature, T_{new}. The criterion for this calculation is $\left|(T_{new} - T_{old})\right| < 1.0$

2.3 Numerical Scheme

The system of governing equations has been solved, using an explicit Harten-Yee Non- MUSCL Modified flux type TVD scheme. The scheme is second order accurate in time and space. The two-dimensional, rectangular physical coordinate system (x,y) is transformed into the computational coordinate system (ζ,η)in order to solve the problem on uniform grids. After applying the transformation, the Eq. (2.1) can be expressed as

$$\frac{\partial U}{\partial t} \quad \frac{\hat{\partial}\hat{F}}{\partial \P} \quad \frac{\partial \hat{G}}{\partial \text{¦}} \quad \frac{\partial \hat{F}_v}{\partial \P} \quad \frac{\partial \hat{G}_v}{\partial \text{¦}}$$

$$\hat{U} = J^{-1}U$$
$$\hat{F} = J^{-1}(\P_x F_v + \P_y G)$$
$$\hat{G} = J^{-1}(\text{¦}_x F + \text{¦}_y G_v)$$
$$\hat{F}_v = J^{-1}(\P_x F_v + \P_y G_v)$$
$$\hat{G}_v = J^{-1}(\text{¦}_x F_v + \text{¦}_y G_v)$$

$$J^{-1} = X_{\P} Y_{\text{¦}} - X_{\text{¦}} Y_{\P},$$

$$\P_x = Jy_{\text{¦}}, \P_y = Jx_{\text{¦}},$$
$$\text{¦}_x = -JY_{\P}, \text{¦}_y = -Jx_{\P}$$

$$\Delta t = \frac{CFL}{\max\{|U| + |V| + a(\P_x^{\,2} + \P^{\,2}_{\,y})^{1/2} + c(\text{¦}_x^{\,2} + \text{¦}^{\,2}_{\,y})^{1/2}\}}$$

$$U = \P_x u + \P_y v, V = \text{¦}_x u + \text{¦}_y v$$

$$\left(\partial T / \partial n\right)_w = 0$$

boundary the variables are determined by first-order extrapolation due to supersonic character of flow. Throughout the present study, the following convergence criterion has been set on the variation of density:

$$\sqrt{\frac{\sum_{J=1,K=1}^{JJ,KK}\left(\frac{\rho_{new}-\rho_{old}}{\rho_{old}}\right)^2}{JJ.KK}} \leq 10^{-5} \quad \text{...(2.11)}$$

Where

JJ and KK are the total numbers of nodes in the horizontal and vertical directions respectively

2.5 Use of turbulence Model

Throughout the all investigation the facing step makes the flow field turbulent at the present Mach number. Particularly, the recirculation in both upstream and downstream of the step of incoming flow leads me to use a turbulence model. Therefore, to calculate eddy viscosity an algebraic turbulence model proposed by Bladwin and Lomax [33] is selected. The primary advantage of using this model is that it does not need to calculate the boundary layer thickness; rather it calculates the eddy viscosity coefficient μ_t based on the local vorticity, ω. Secondly, this model can successfully calculate the separated flows both over a flat plate and a compression corner. According to Reference 33, the eddy viscosity μ_t is defined as

$$\mu_t = \begin{cases} (\mu_t)_{inner} & y \leq y_{crossover} \\ (\mu_t)_{outer} & y > y_{crossover} \end{cases}$$

Where y is the normal distance from the wall and $y_{crossover}$ is the smallest value of y at which the value of viscosity in the outer region becomes less than or equal to the value of viscosity in the inner region.

CHAPTER-III

RESULTS AND DISCUSSION

3.1 Introduction

A numerical study of transonic flow over a flat plate with facing steps has been performed by solving Two-Dimensional Navier-Stokes equations. An explicit Harten-Yee Non-MUSCL Modified flux type TVD scheme has been used to solve the system of equations, and a zero equation algebraic turbulence model to calculate the eddy viscosity coefficient. The main objective of this study is to investigate the flow field characteristics of a transonic compressible flow over a flat plate with forward and backward facing step of different heights and different Mach numbers of flow. The investigation has been done by varying (i) the type of the facing step (ii) the height of the step and (iii) Mach number of the incoming flow.

3.2 Code validation

The code is verified with the data published by Hassan et al. [9]. For validation the forward facing step of height 0.075 and Mach number 1.2 is considered. It can be point out that [9] performed the numerical investigation considering the flow of a laminar supersonic viscous flow over forward facing step mounted on a flat plate.

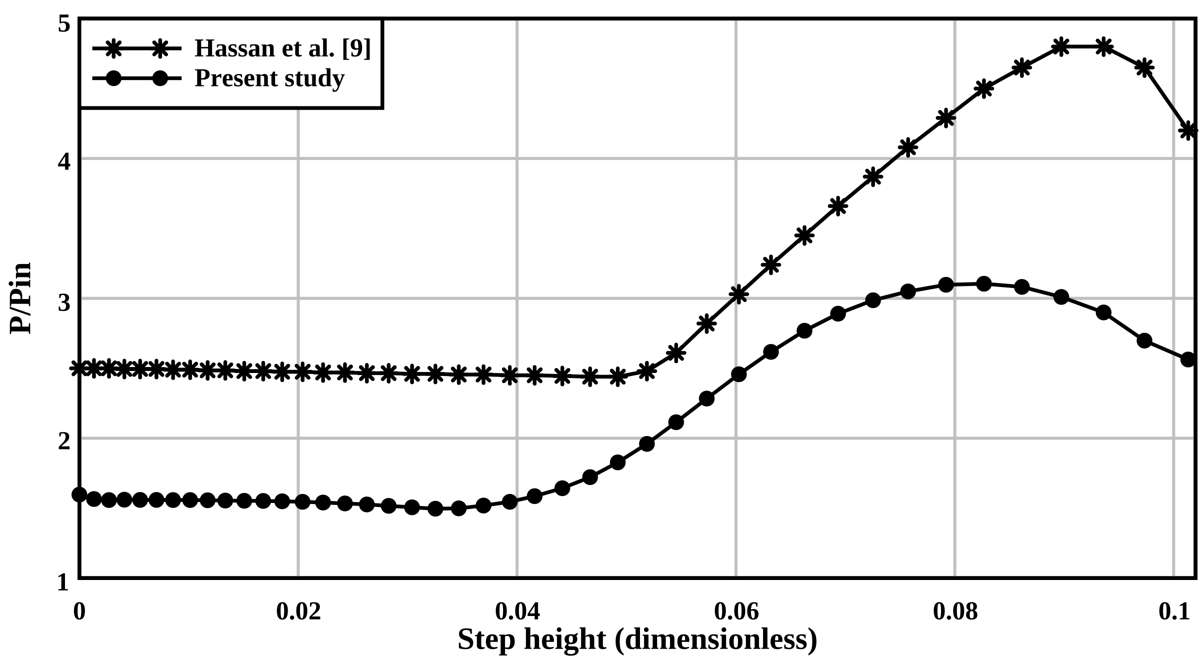

Fig. 1 Pressure distribution along the step height

Fig. 1 shows the comparison of pressure distribution along the vertical face of the step height. From the qualitative comparison of this two investigations (present study and Hassan et al. [9]), it can be observed that there is a little deviation at the tip of the step. It might be due to the out flow boundary conditions.

3.2 Flow Field Description and Problem Statement

The geometric configuration of the calculation domain is shown in Fig. 2 & 3. The domain dimensions in the stream wise horizontal and vertical directions are L= 10 cm and H= 5 cm, respectively. Two facing steps are considered: (i) Backward facing step, and (ii) Forward facing step. Backward facing step is located at the inlet (left) boundary of the calculation domain as shown in Fig. 2, on the other hand forward facing step is located at the exit (right) boundary as shown in Fig. 3. Throughout the study, the grid system consists of 194 nodes in the longitudinal direction and 121 nodes in the transverse direction. The grids are clustered near the wall. The flow field is non-reacting in nature. The heights of the facing steps are considered as 2.5, 5.0, 7.5 and 10 mm. The Mach numbers are varied as 0.8, 1.0 and 1.2. The whole computational runs are summarized in Table-2. Each computational run takes one specific step height with a Mach number as calculation parameter. In the analysis, all the calculations make dimensionless on the basis of total length of the domain. The results and discussion are presented under two headings: Backward facing step and Forward facing step. Results with varying parameters are to be analyzed and discussed under the following contexts: (i) Characteristics of different shocks (ii) Dynamical behavior of the flow field and (iii) Effect of pressure and temperature.

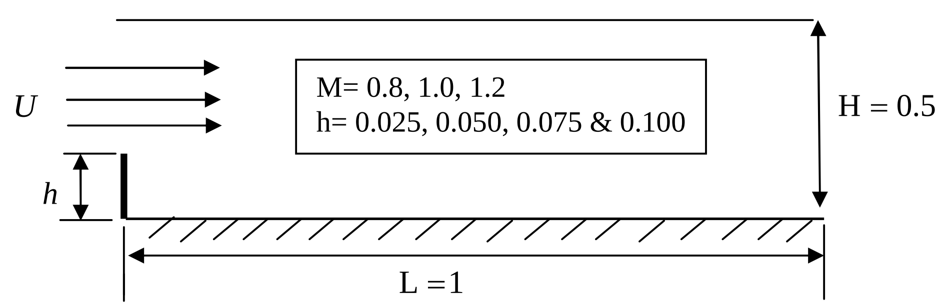

Fig. 2 Calculation domain of backward facing step

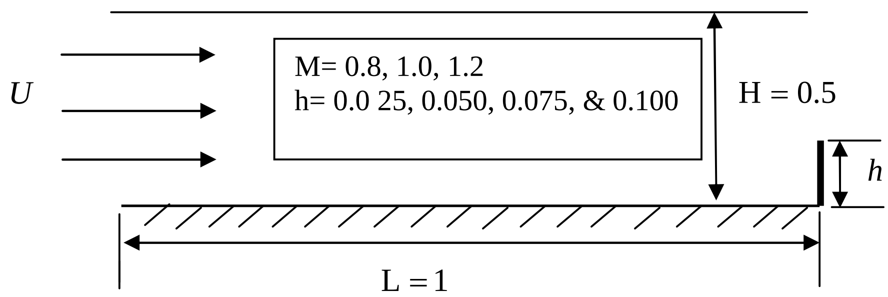

Fig. 3 Calculation domain of forward facing step

Table-2: Summary of all computational runs

Arrangement of Facing Step	Step height	Mach Number		
Forward facing step	0.025	0.8 Case-1	1.0 Case-2	1.2 Case-3
	0.050	0.8 Case-4	1.0 Case-5	1.2 Case-6
	0.075	0.8 Case-7	1.0 Case-8	1.2 Case-9
	0.100	0.8 Case-10	1.0 Case-11	1.2 Case-12
Backward facing step	0.025	0.8 Case-13	1.0 Case-14	1.2 Case-15
	0.050	0.8 Case-16	1.0 Case-17	1.2 Case-18
	0.075	0.8 Case-19	1.0 Case-20	1.2 Case-21
	0.100	0.8 Case-22	1.0 Case-23	1.2 Case-24

3.3 Results of backward facing step

The schematic diagram of calculation domain considering backward facing step is shown in Fig. 2, where the facing step is varied as 0.025, 0.050, 0.075 and 0.100 and the Mach number is varied as 0.8, 1.0 and 1.2. The results are analyzed and discussed, firstly considering the effect of step heights with Mach numbers, secondly the effect of Mach numbers with step heights.

3.3.1 Effect of step heights with Mach numbers

In this section the results are analyzed by varying the height of the step as 0.025, 0.050, 0.075 and 0.100 with specific Mach number.

3.3.1.1 *Characteristic of different shocks*

Figures 4(a-d) illustrate the Mach contour for different step heights at Mack number 0.8. Mack contours are characterized by the presence of shocks in the flow. The two shock regions are visible in the flow field, namely corner expansion shock and reattachment shock. Expansion shock appears to emanate from the vicinity of the top of the step and reattached shock appears at the reattachment region, which is located below the expansion shock region. By analyzing the figures, it is found that, the expansion shock rotates clockwise, moves downward and the angle of shock with main flow direction decreases with the increase of step height. Due to the change of step height, the width of the expansion shock and the degree of expansion also increase. An interaction of two shocks (expansion shock and reattachment shock) is found in the flow field and its position also rotates with the increase of step height. And so the area of reattach zone decreases.

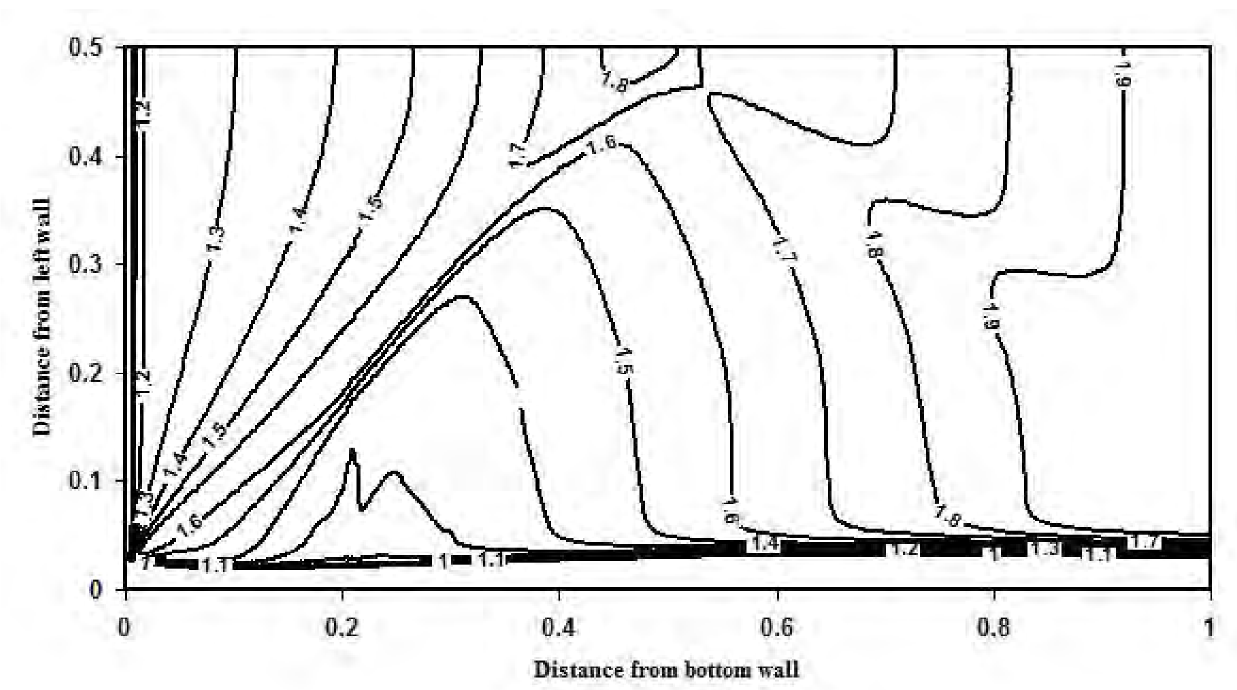

Fig. 4(a) Mach contour for h= 0.025 (M=0.8)

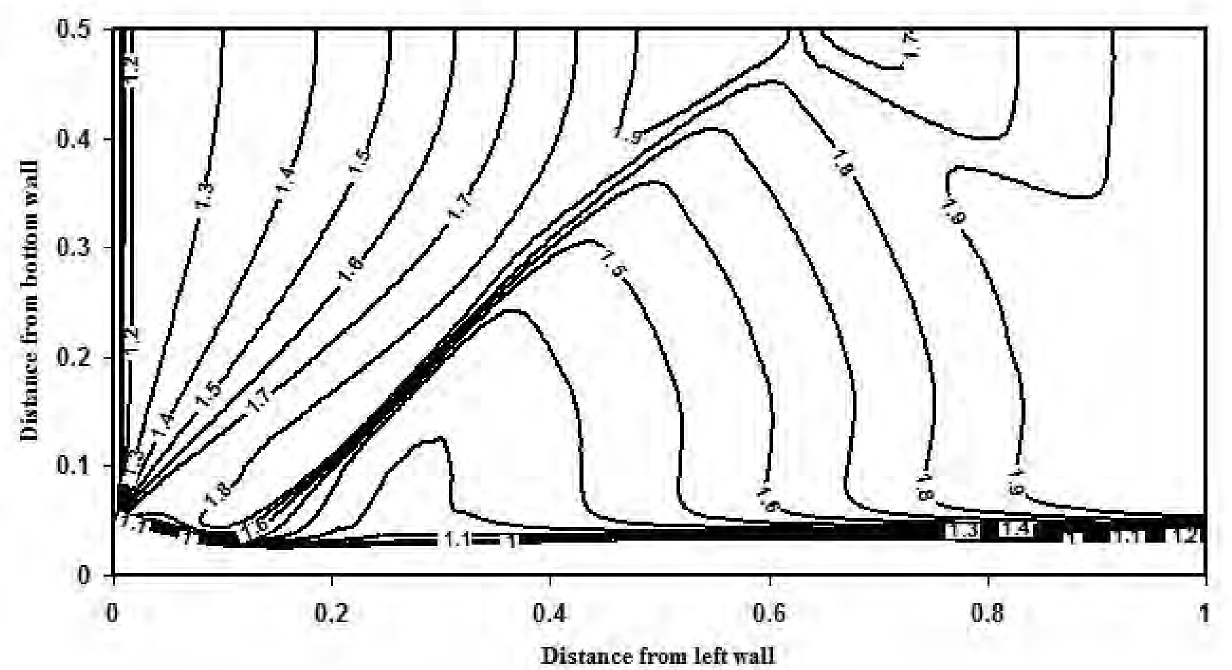

Fig. 4(b) Mach contour for h= 0.050 (M=0.8)

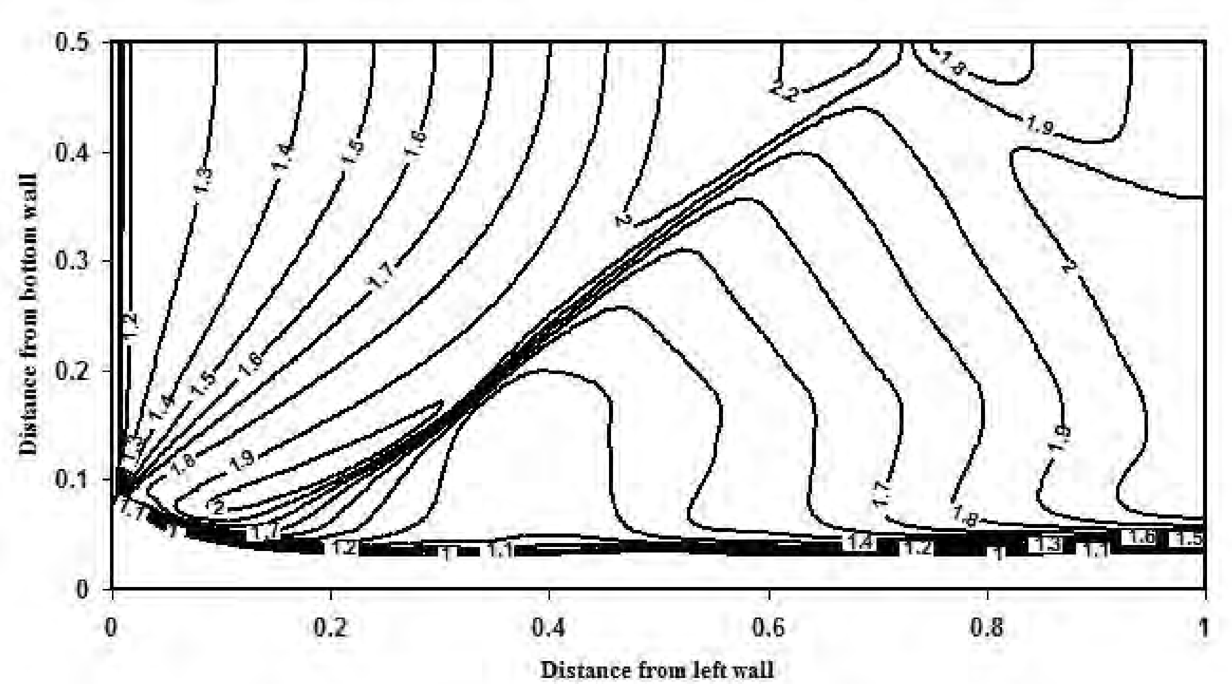

Fig. 4(c) Mach contour for h= 0.075 (M=0.8)

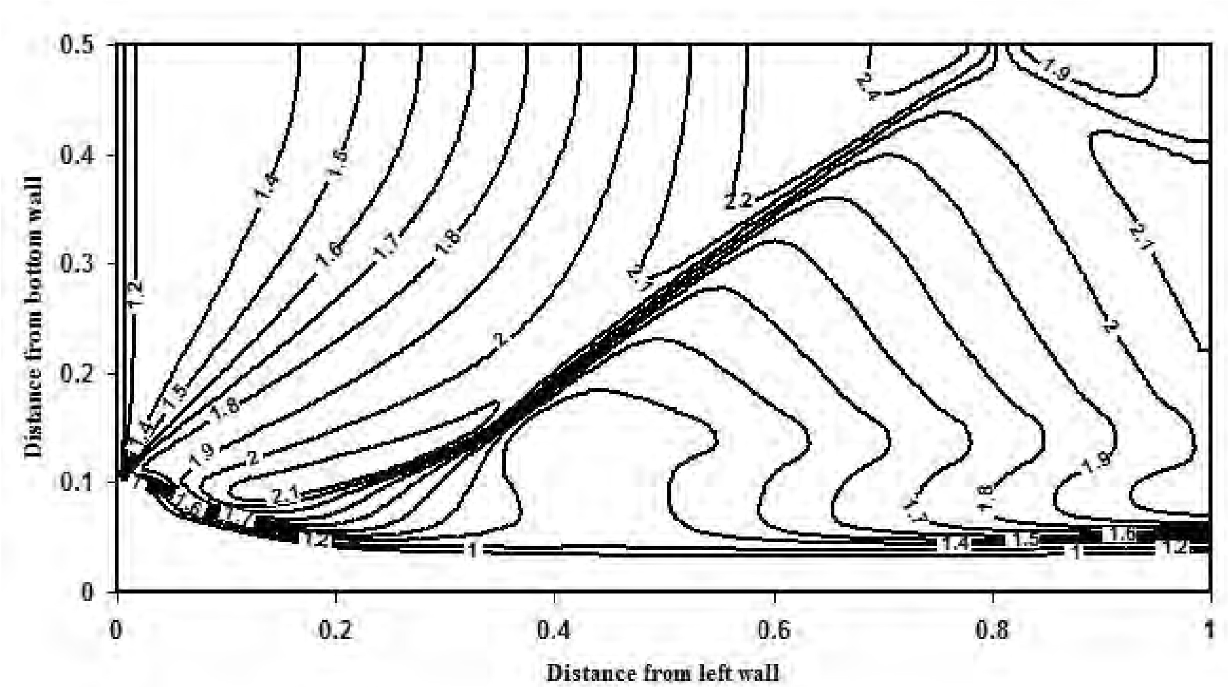

Fig. 4(d) Mach contour for h= 0.100 (M=0.8)

3.3.1.2 *Dynamical behavior of the flow field*

Figure 5 (a-d) shows the velocity contour for different step heights. The velocity contours are characterized by the sudden rise of velocity in the expansion shock region. The velocity is low at reattached shock region. In the flow field, it is found that after the interaction of two shocks (expansion shock and reattached shock) a part of flow is reflected to the downward direction which exists up to the exit of the boundary. The velocity at expansion shock region as well as exit of the boundary increases, with the increase of step height. At the left bottom corner of the velocity field recirculation appears and the zero value of the velocity contour is considered as recirculation zone. The length of this zone along the bottom wall is considered as recirculation length. Figure 6 (a-d) shows the recirculation zone for different step heights at Mach number 0.8. To observe closely the figures are magnified and shown partly by height of 0.10 (from bottom wall) and length of 0.45 (from left boundary). With the increase of step height, the length of recirculation increases. The strength of recirculation also increases, as the step height increases. The stream wise vorticity/recirculation is stronger near the step and reduces in strength further downstream. The maximum strength of recirculation is found at about one-fourth of the recirculation length.

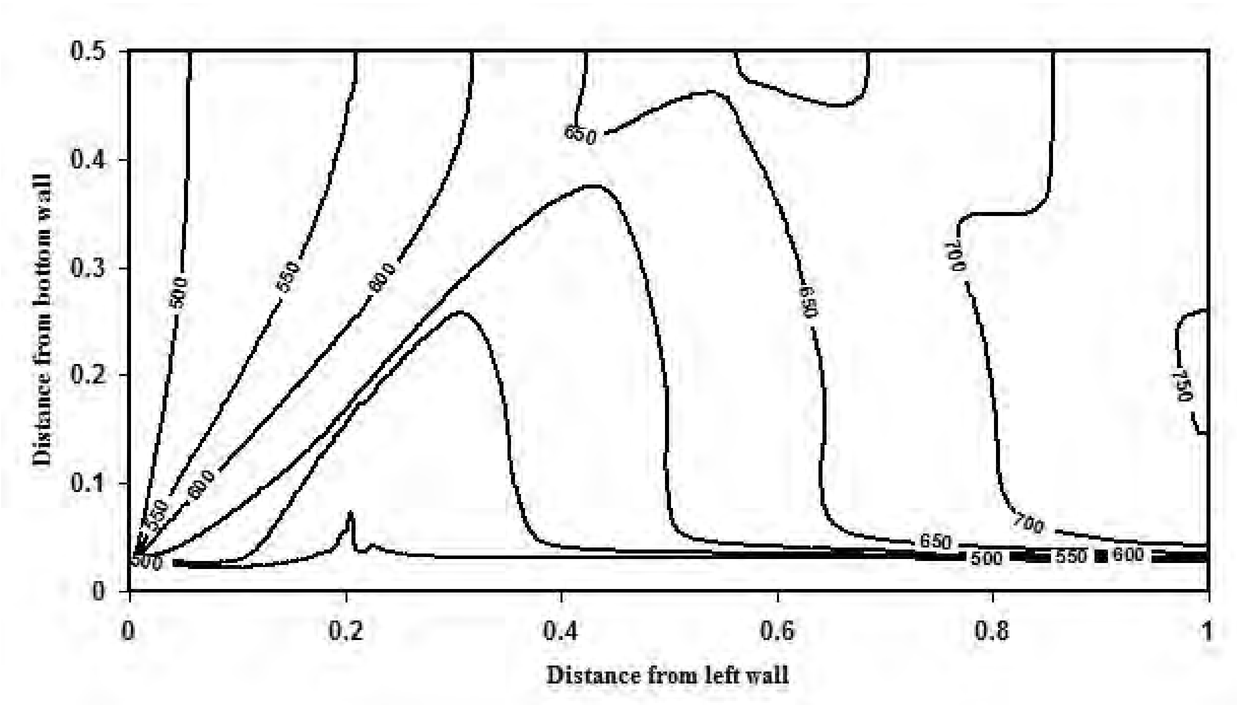

Fig. 5(a) Stream wise velocity contour for h=0.025 (M=1.0)

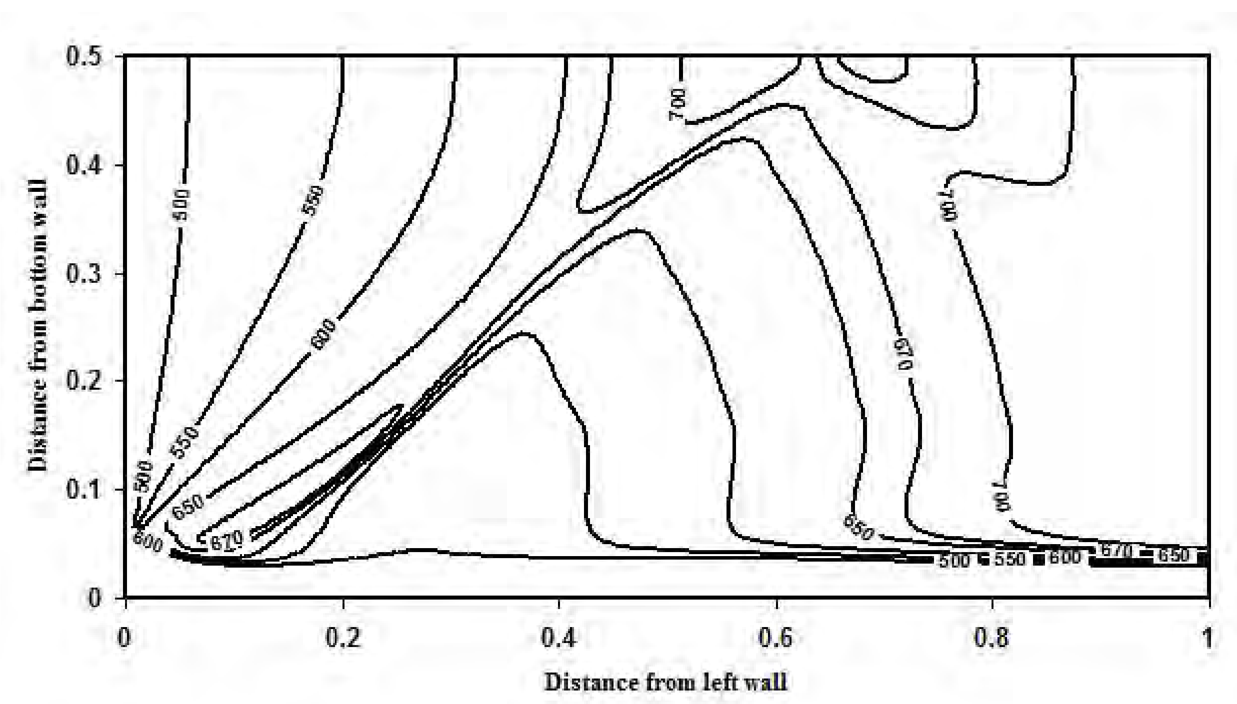

Fig. 5(b) Stream wise velocity contour for h= 0.050 (M=1.0)

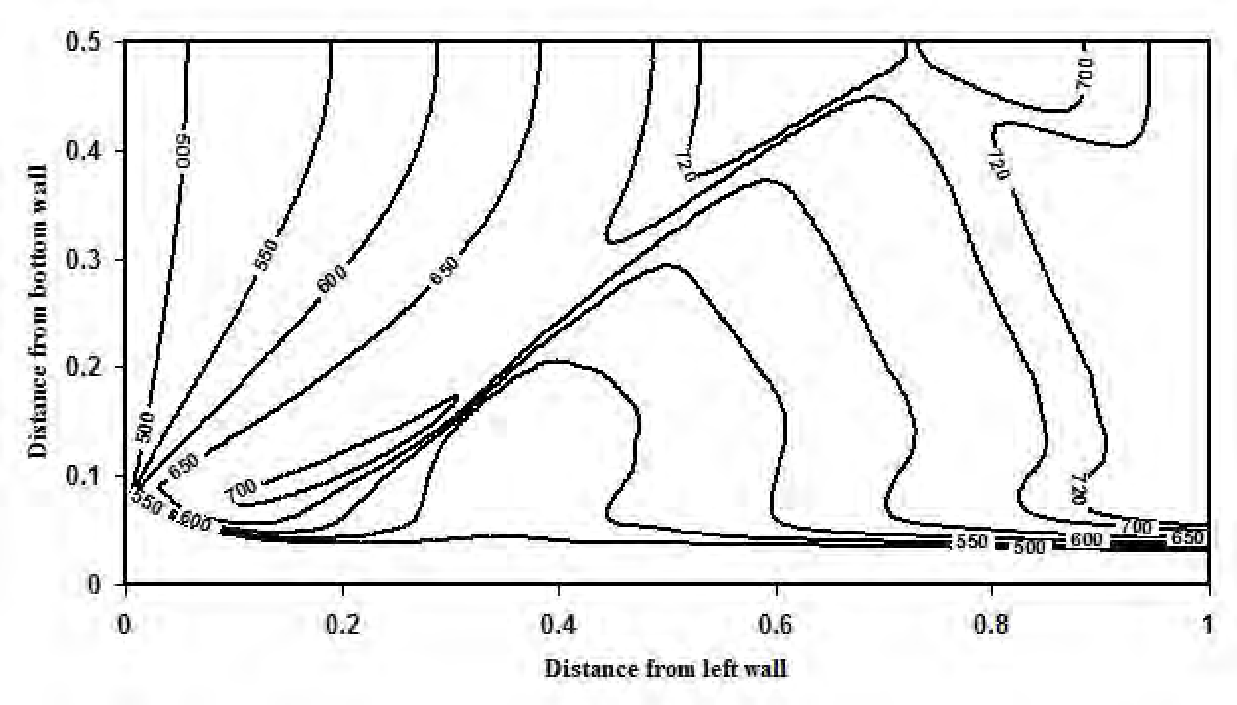

Fig. 5(c) Stream wise velocity contour for h= 0.075 (M=1.0)

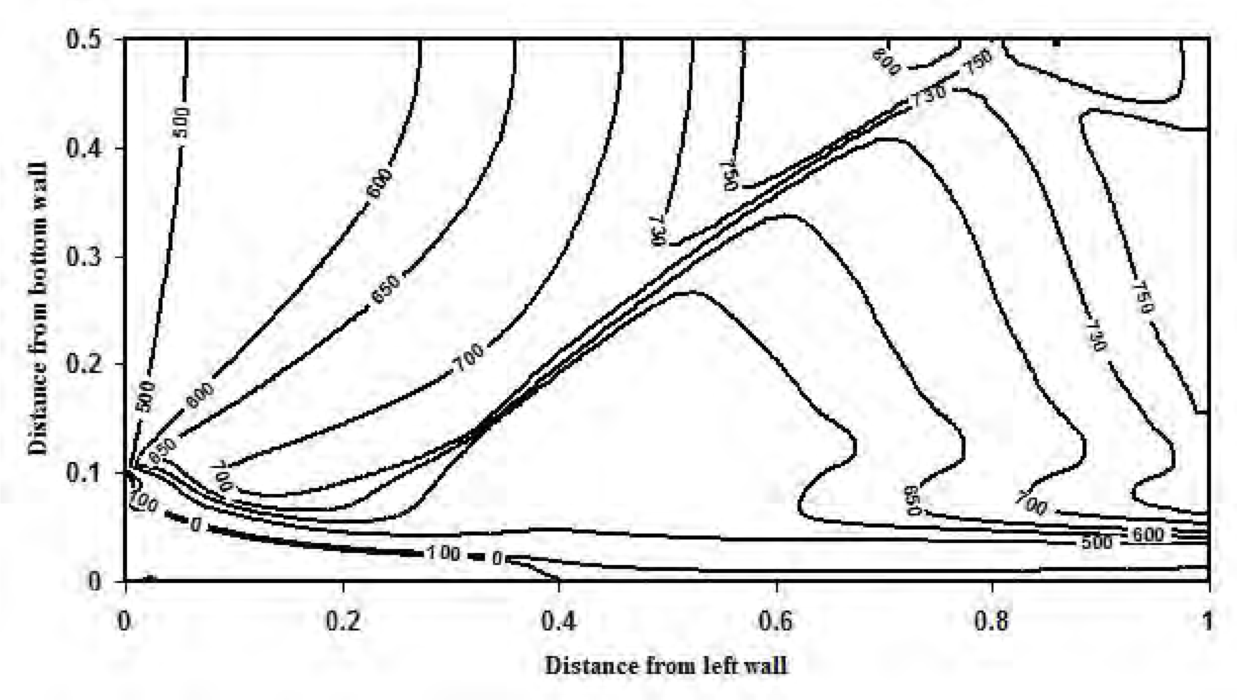

Fig. 5(d) Stream wise velocity contour for h= 0.100 (M=1.0)

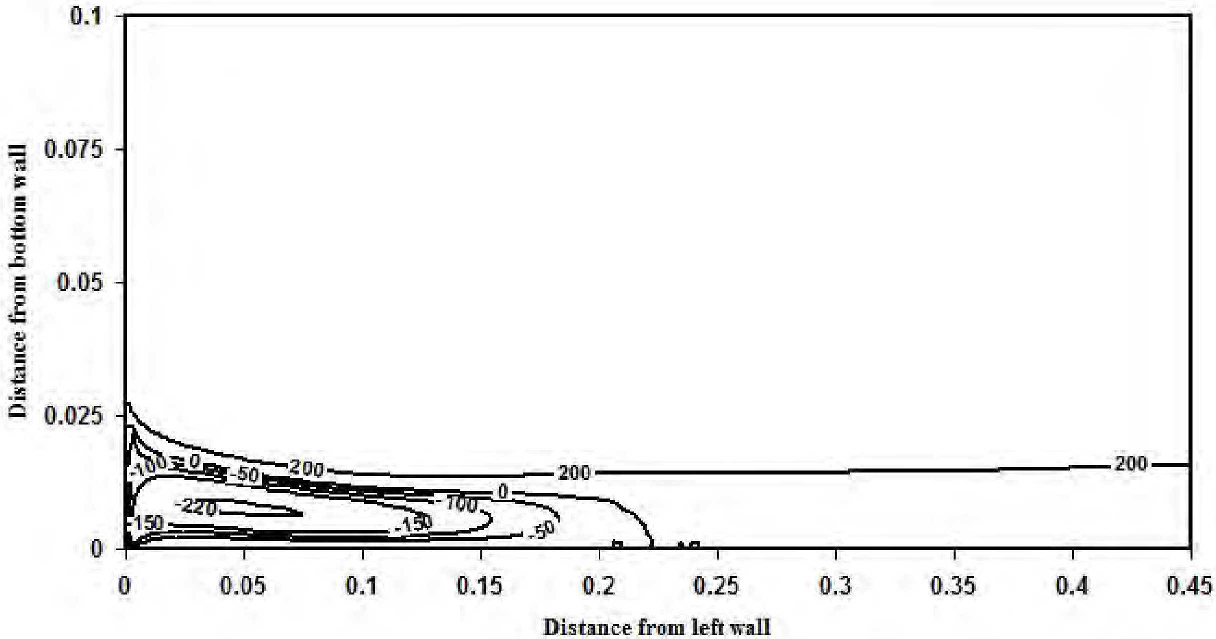

Fig. 6 (a) Zone of recirculation for h= 0.025 (M=0.8)

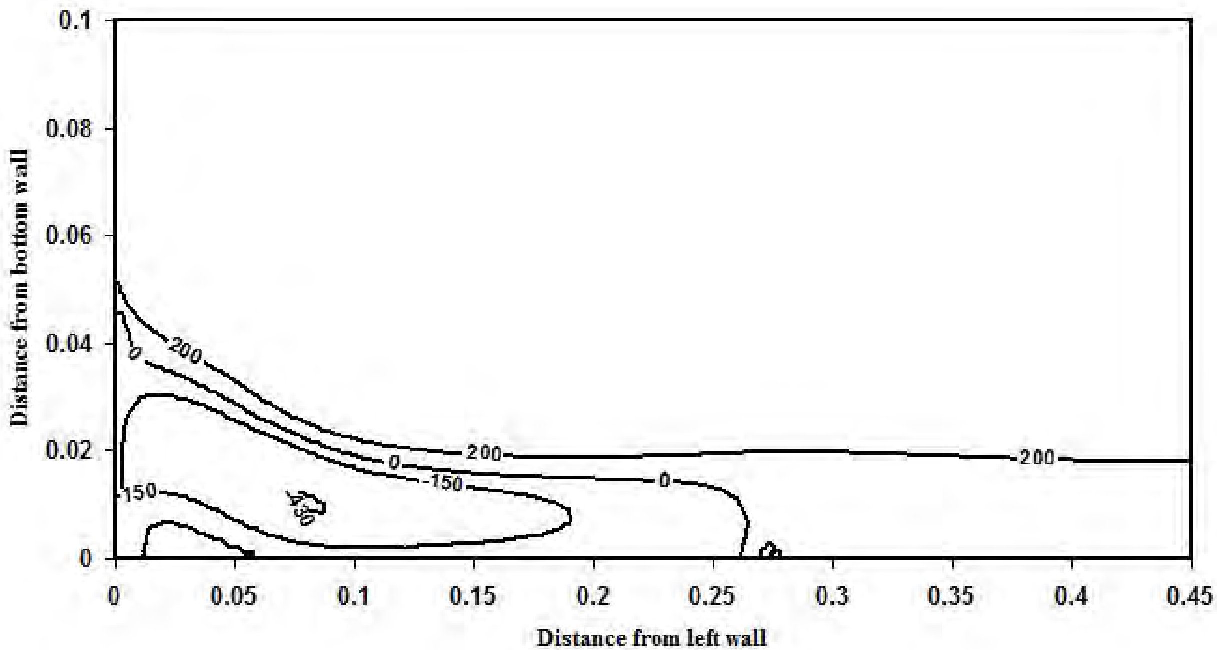

Fig. 6 (b) Zone of recirculation for h= 0.050 (M=0.8)

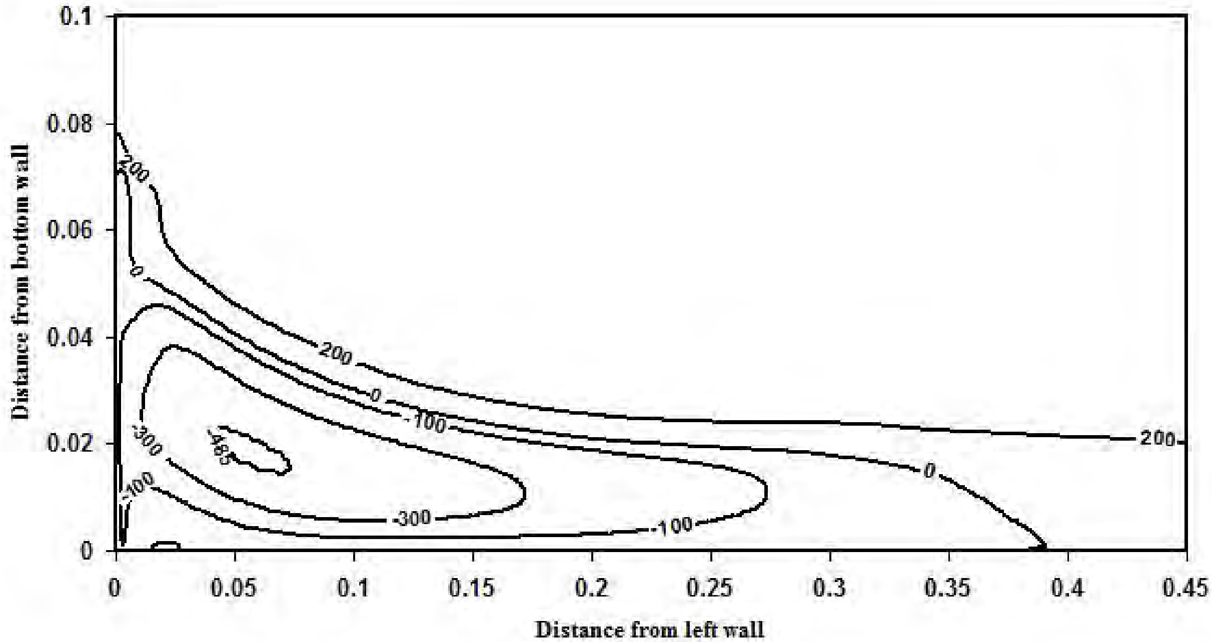

Fig. 6 (c) Zone of recirculation for h= 0.075 (M=0.8)

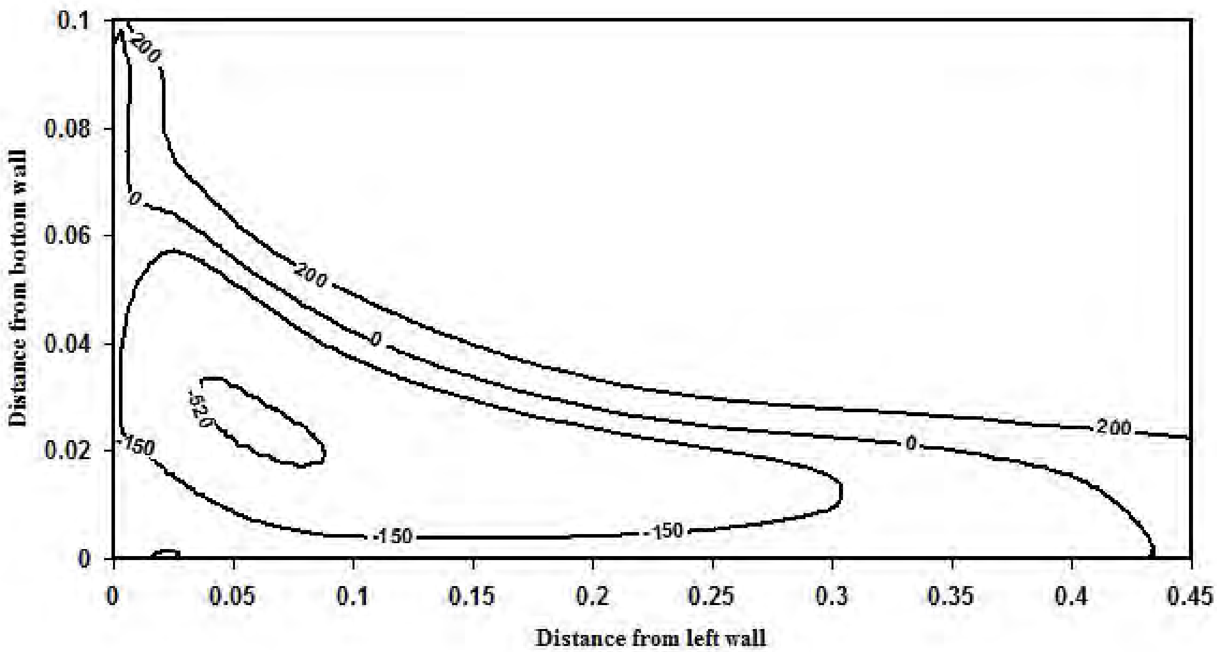

Fig. 6 (d) Zone of recirculation for h= 0.100 (M=0.8)

3.3.1.3 Effect on Pressure and Temperature in the flow Field

Figure 7 (a-d) represents pressure contour for different step heights at Mach number 1.0. The pressure contours show a sudden drop of pressure in the expansion shock region and immediate behind the expansion shock, the reattachment shock evolves where the pressure again increases. The minimum pressure is found in the recirculation zone. Actually this low pressure zone triggers the flow to generate recirculation. In the expansion shock region, with the increase of step height, the pressure gradually decreases due to the result of overexpansion. The pressure developed along the step height, recirculation and reattach zone also decreases with the increase of step height. The pressure of striking becomes weaker and moves to the right with the increase of step height which is found in figure 8. For clear observation figure 8 is magnified and shown partly by length of 0.60 along the plate. From the temperature contour at Mach number 1.2 is shown in Fig. 9(a-c), it is found that there is a sudden drop of temperature in the expansion shock region and again the temperature gradually increases at reattachment shock region. In the expansion shock region, it is found that the temperature decreases, as the height of the step increases. There is a sudden increase of temperature near the step because of recirculation zone, which tends to accumulate the recirculated particles near the left-bottom corner causing the increasing of temperature. The maximum temperature in this region can be found around 940K, which is close to the plate. From the Fig. 10, it is found that the temperature along the plate (excluding recirculation length) is almost equal, but there is a little fall of temperature at the point of striking/at the end of recirculation.

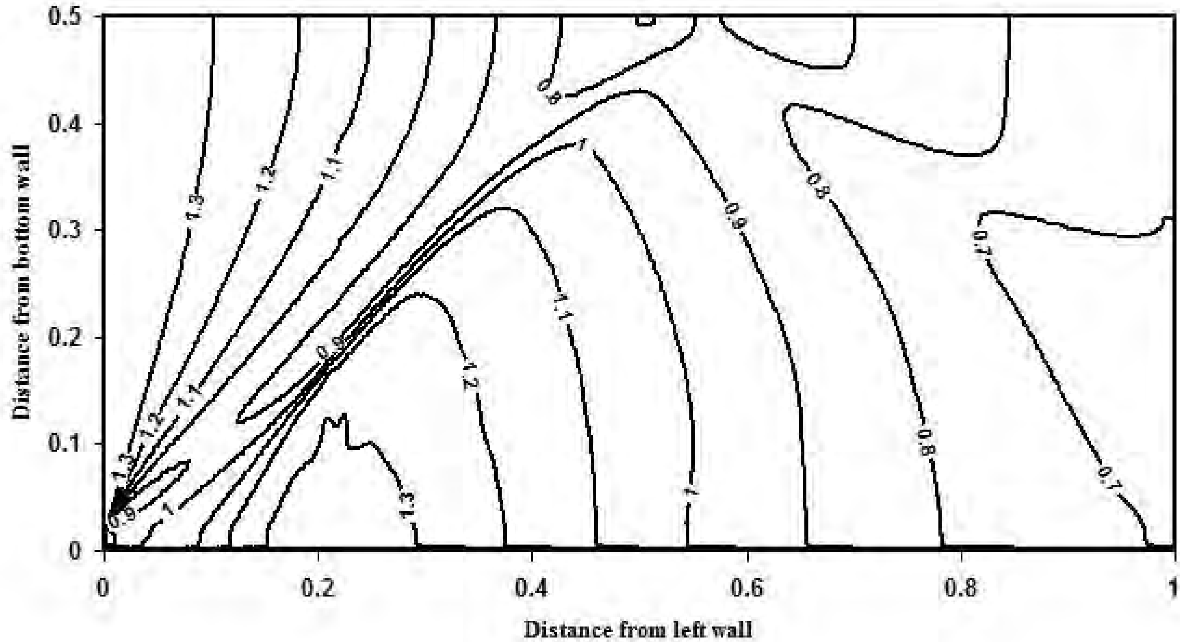

Fig. 7 (a) P/P_{in} contour for h= 0.025 (M= 1.0)

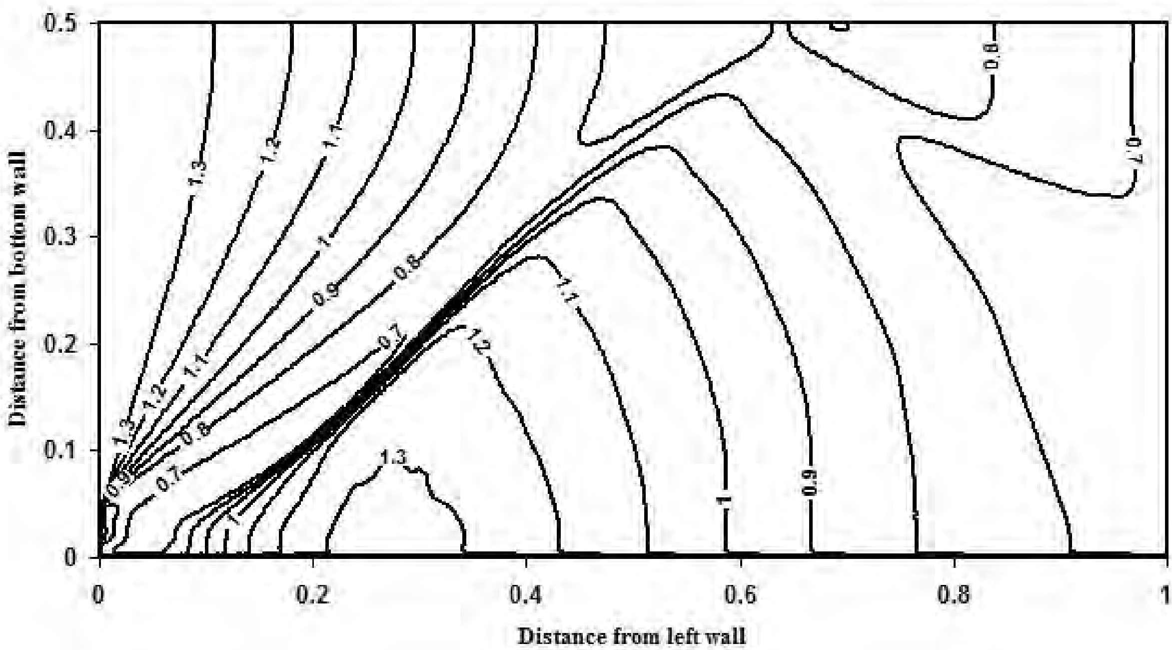

Fig.7 (b) P/P_{in} contour for h=0.050 (M= 1.0)

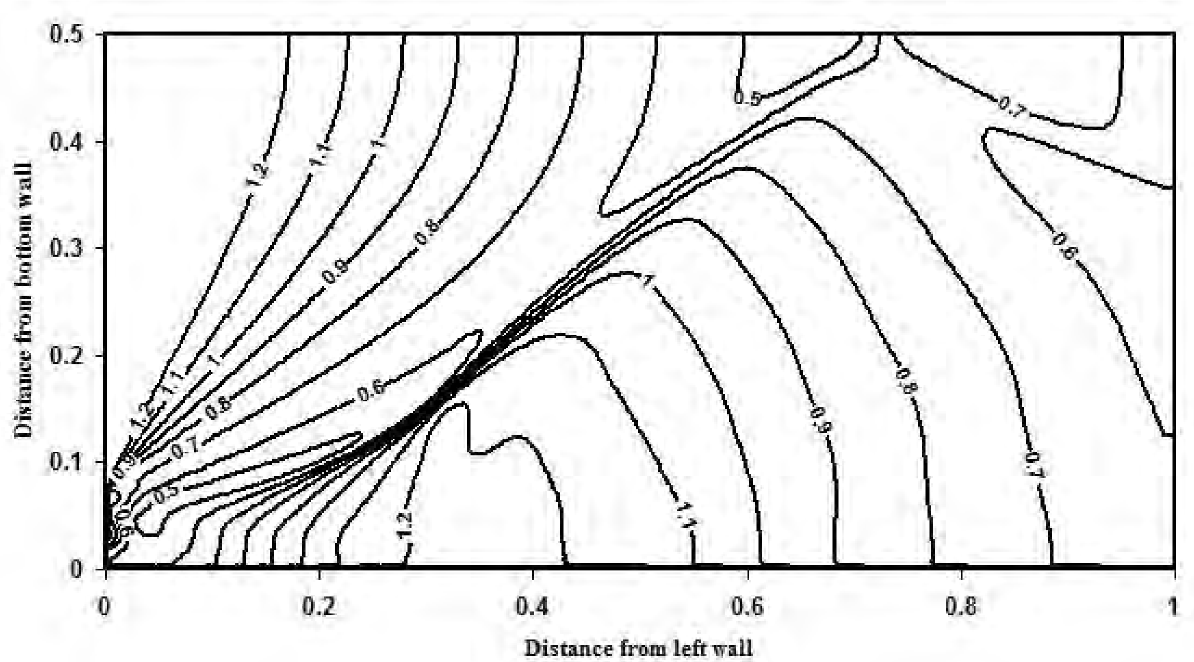

Fig. 7 (c) P / P_{in} contour for h=0.075 (M= 1.0)

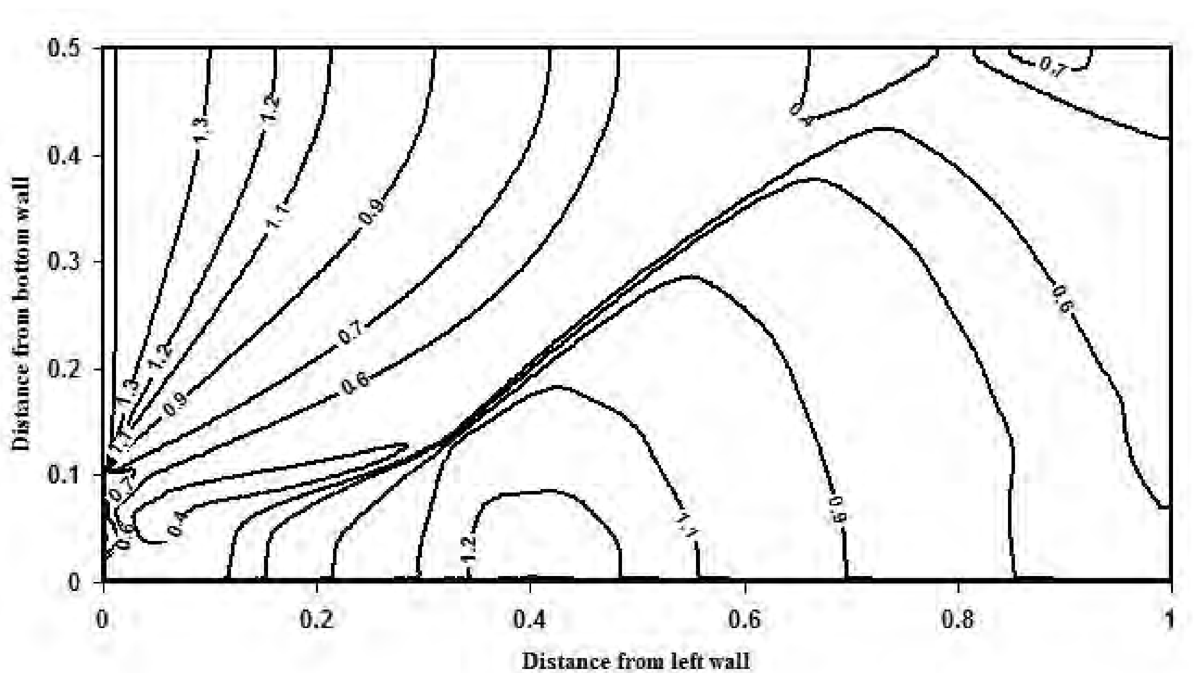

Fig. 7 (d) P / P_{in} contour for h=0.100(M= 1.0)

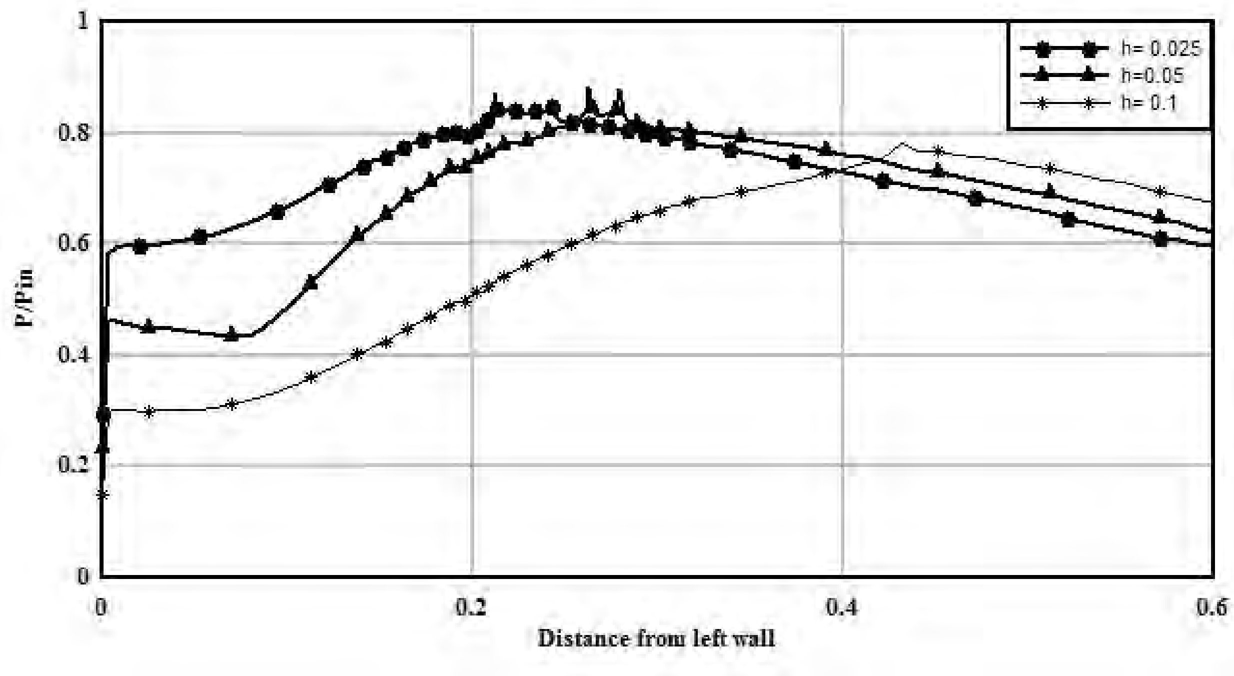

Fig. 8 Wall pressure variation for different step heights (M= 0.8)

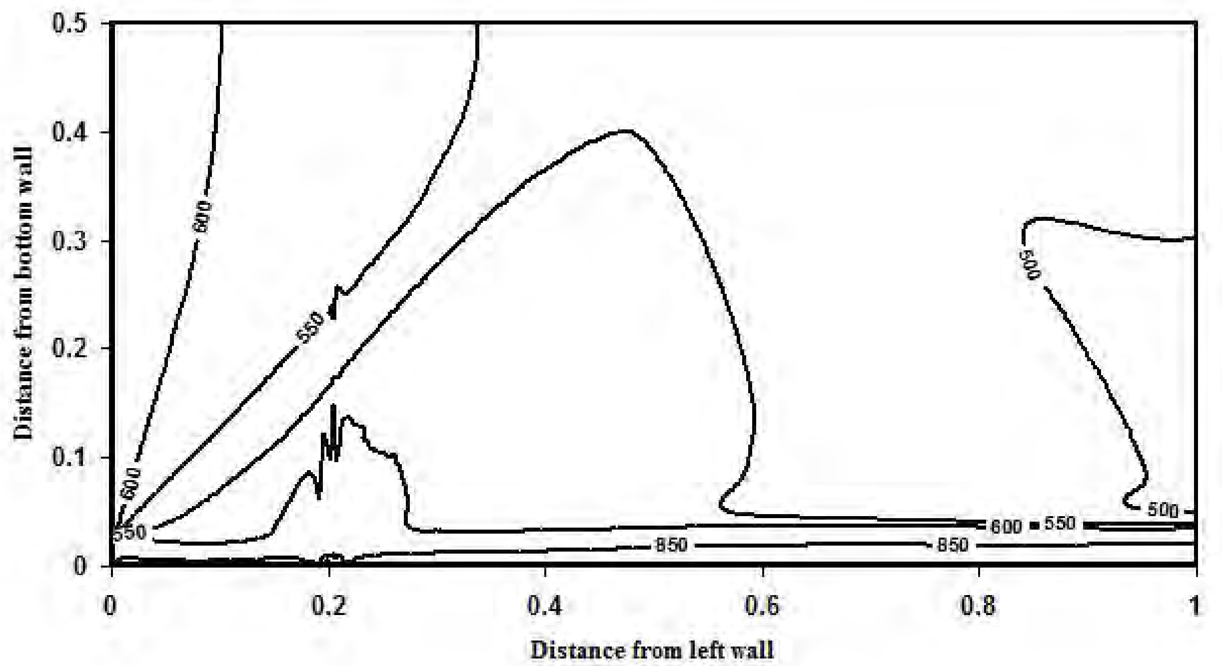

Fig. 9 (a) Temperature contour for h=0.025 (M=1.2)

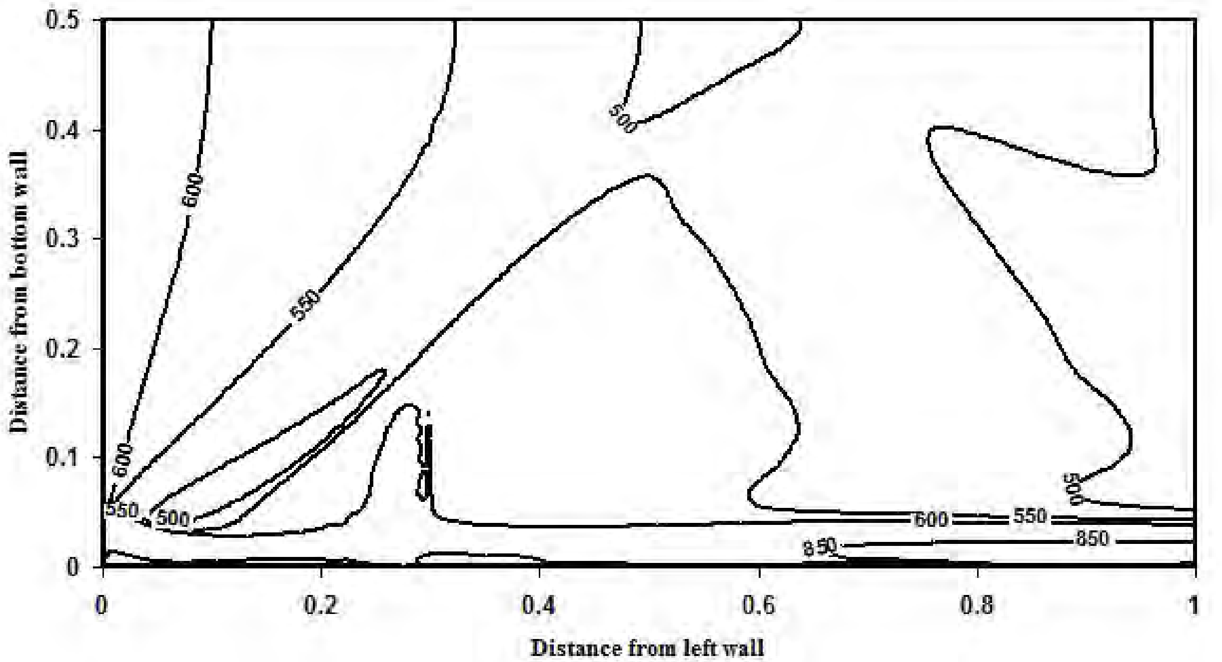

Fig. 9 (b) Temperature contour for h= 0.050(M=1.2)

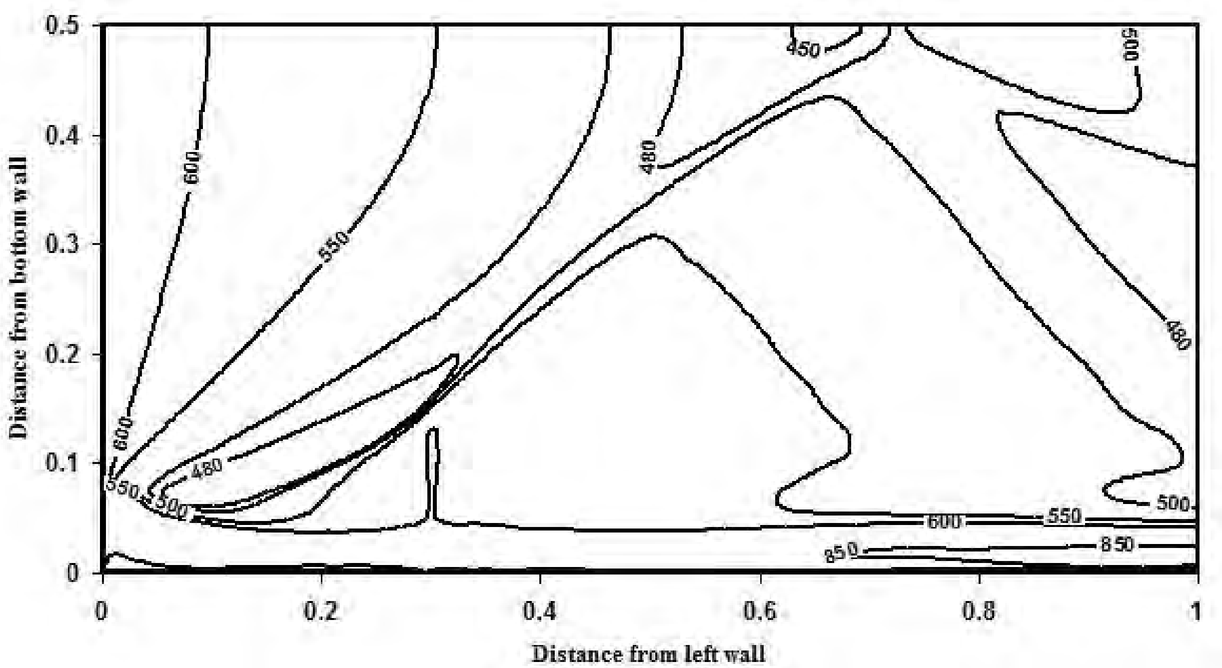

Fig. 9 (c) Temperature contour for h=0.075 (M=1.2)

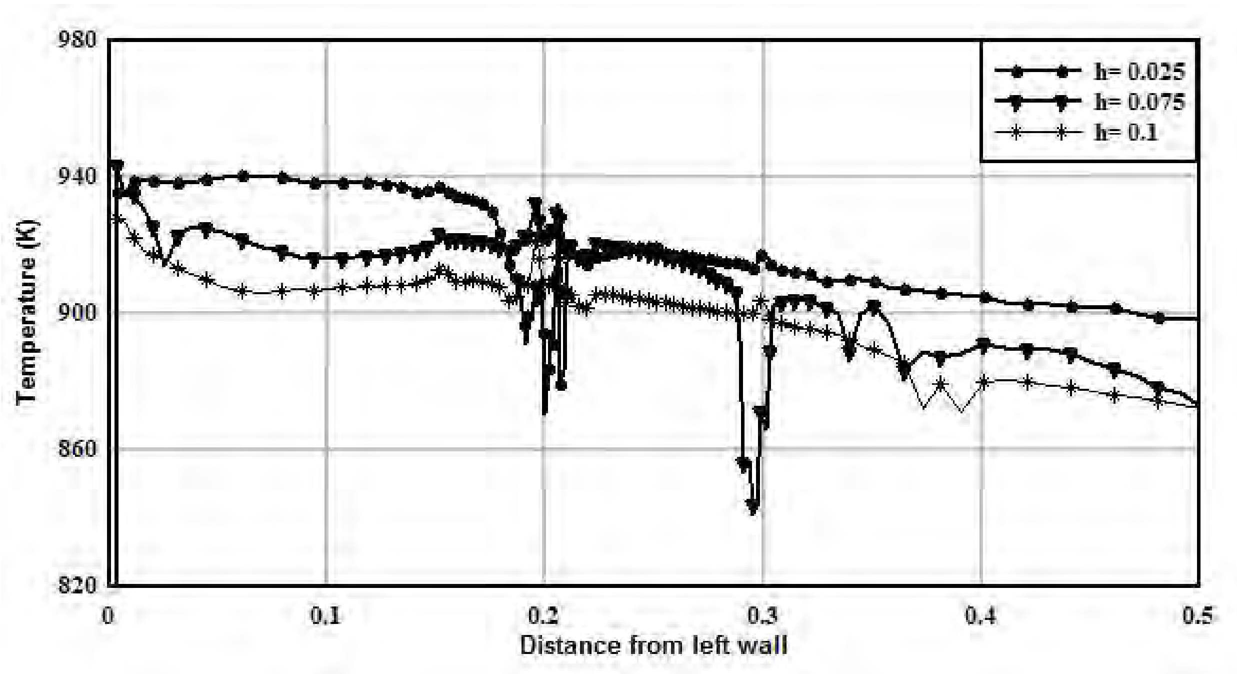

Fig. 10 Wall temperature variation for different step height (M= 1.2)

3.3.2 Effect of Mach numbers with step heights

In this section results are analyzed by varying Mach numbers as 0.8, 1.0 and 1.2 with specific step heights.

3.3.2.1 Characteristic of different shocks:

Figures 11(a-c) illustrate M / M_{in} contour for different Mach numbers at step height 0.075. By analyzing the figures, it is found that, the corner expansion shock rotates clockwise with the increase of Mach number. Due to the change of Mach number, the width of the corner expansion shock and the degree of expansion decreases. The position of the interaction of two shocks also rotates with the increase of Mach number. With the increase of Mach number, the position of reattachment shock moves to the right and the area of the reattachment shock reduce.

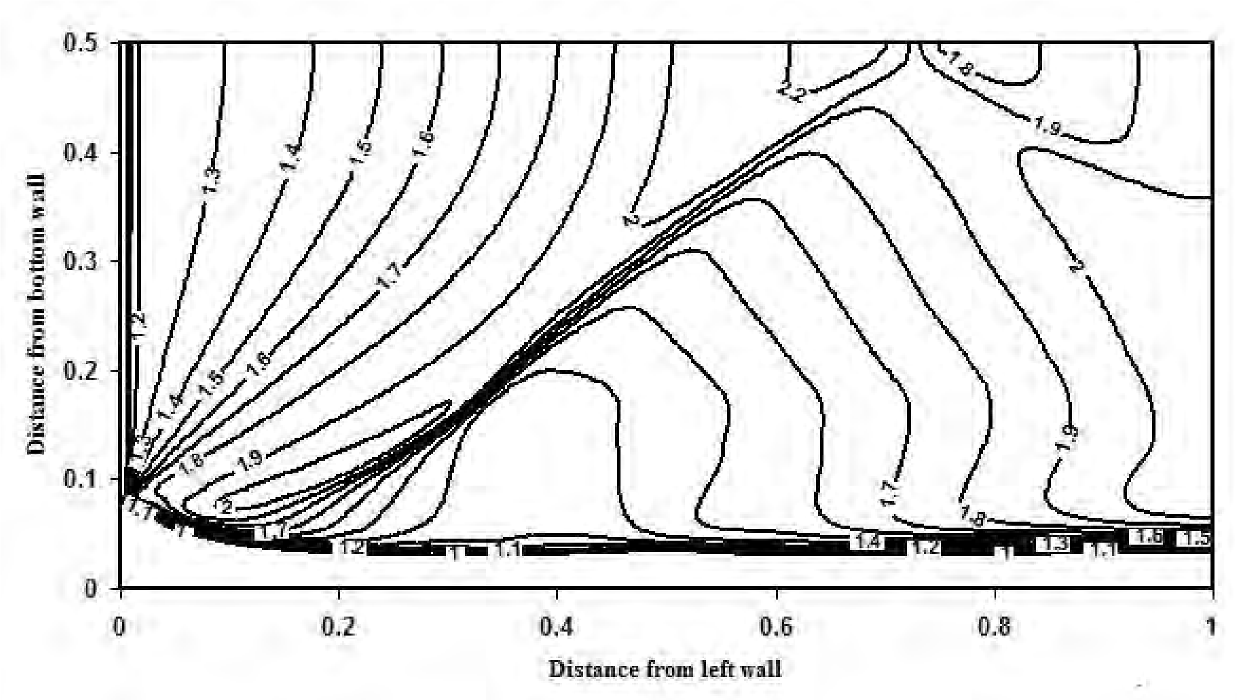

Fig. 11 (b) M / M_{in} contour for M=0.8 (h=0.075)

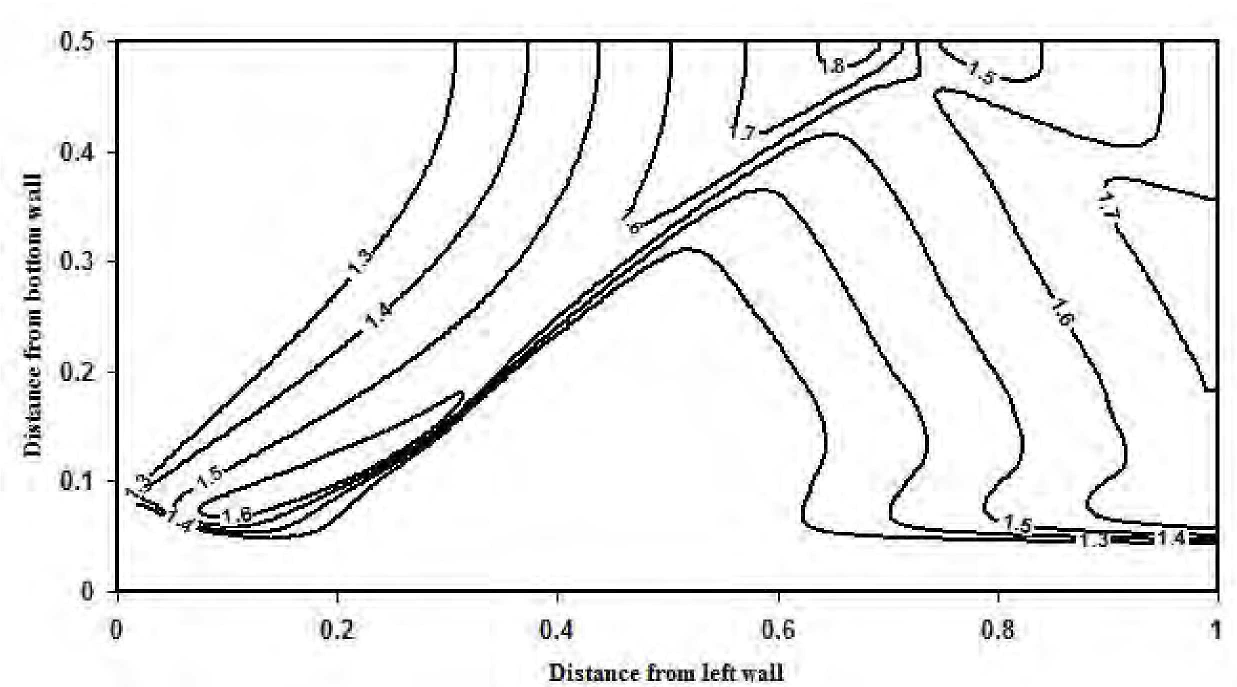

Fig. 11 (b) M / M_{in} contour for M=1.0 (h=0.075)

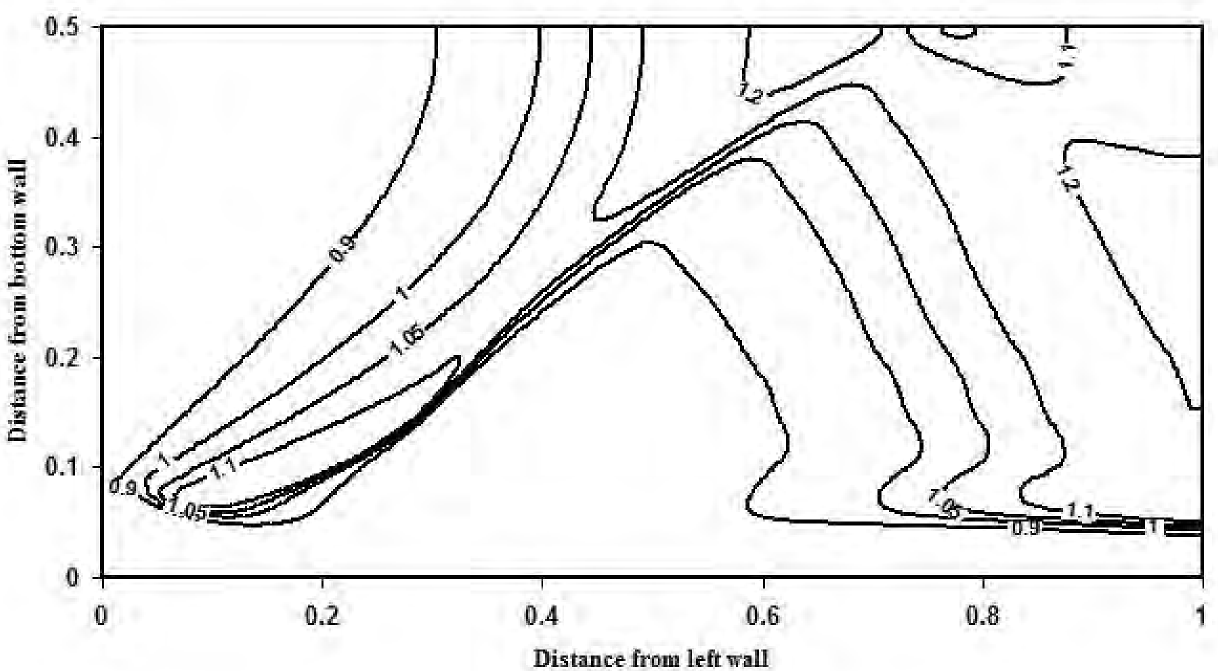

Fig. 11 (c) M / M_{in} contour for M=1.2 (h=0.075)

3.3.2.2 Dynamical behavior of the flow field

Figure 12(a-c) shows recirculation zone for different Mach number at step height 0.075. The figures are magnified and shown partly by height 0.10 (from bottom wall) and 0.45 (from the left boundary). The length of recirculation reduces with the increase of Mach number. By analyzing the data, it is found that for small step height the change of recirculation length is smaller. But in case of high step height the change is larger. From Fig. 6(a-d) and Fig. 12 (a-c), it is found that recirculation/separation length is sensitive to step height, at least in the range studied here. The strength of recirculation increases, as the Mach number increases.

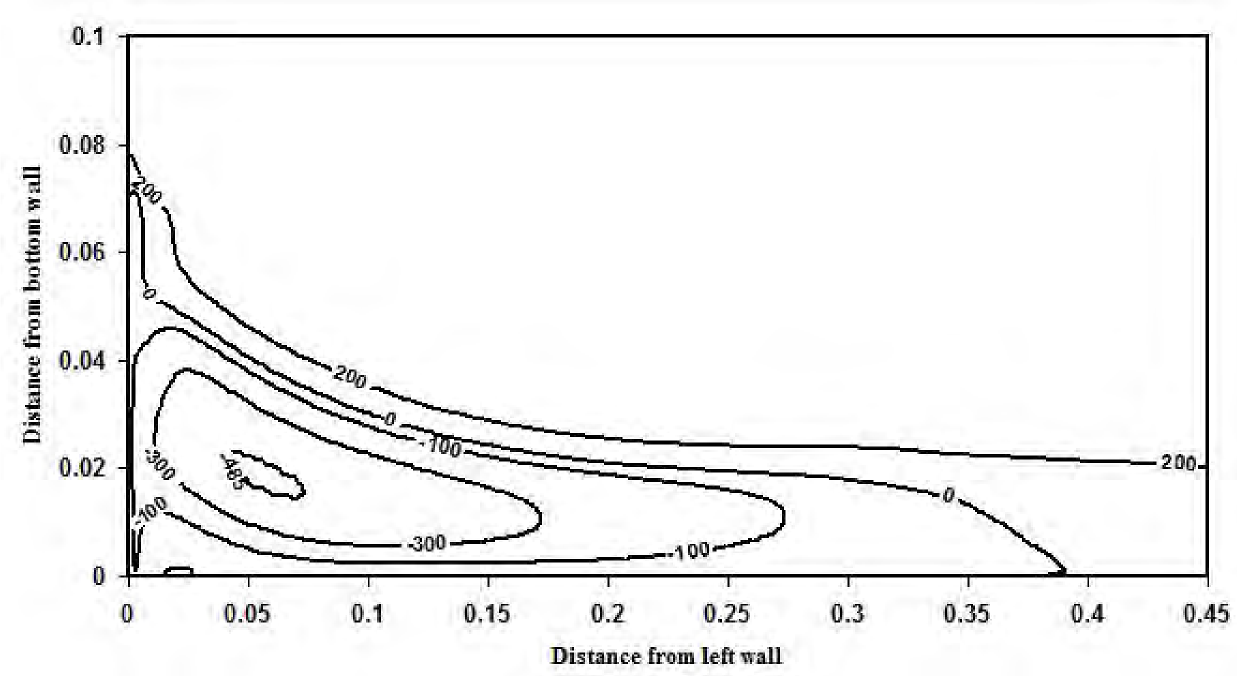

Fig. 12 (a) Zone of recirculation for Mach number 0.8 (h=0.075)

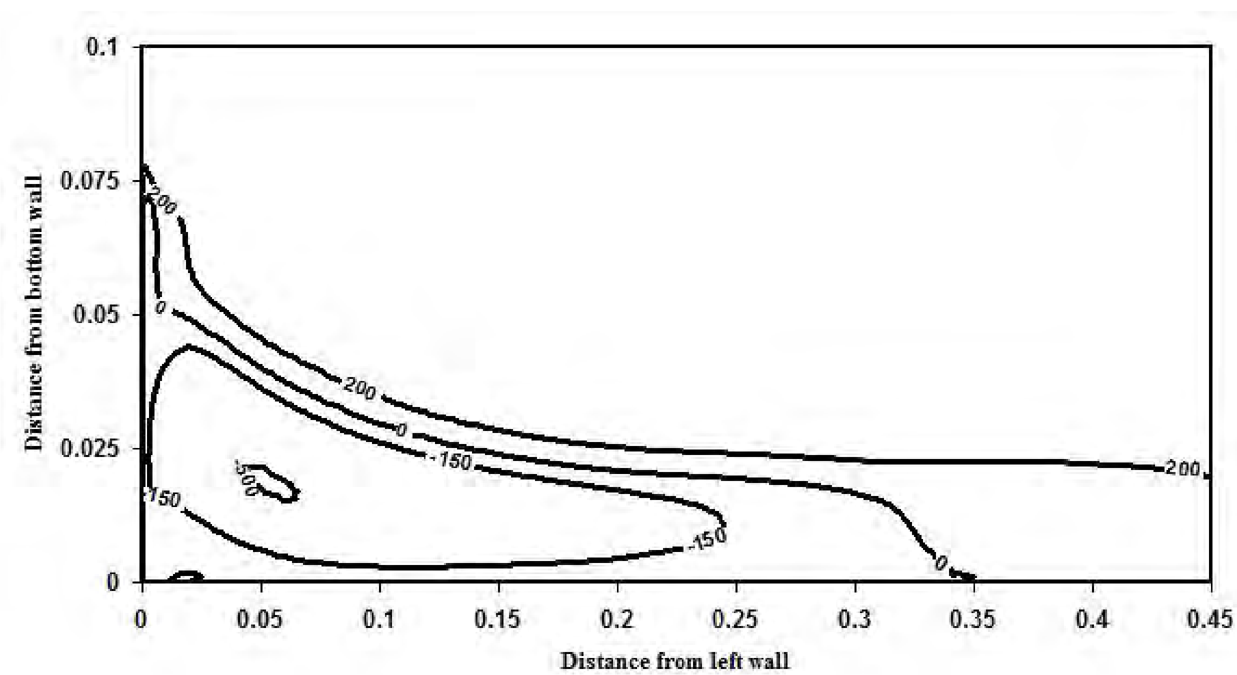

Fig. 12 (b) Zone of recirculation for Mach number 1.0 (h=0.075)

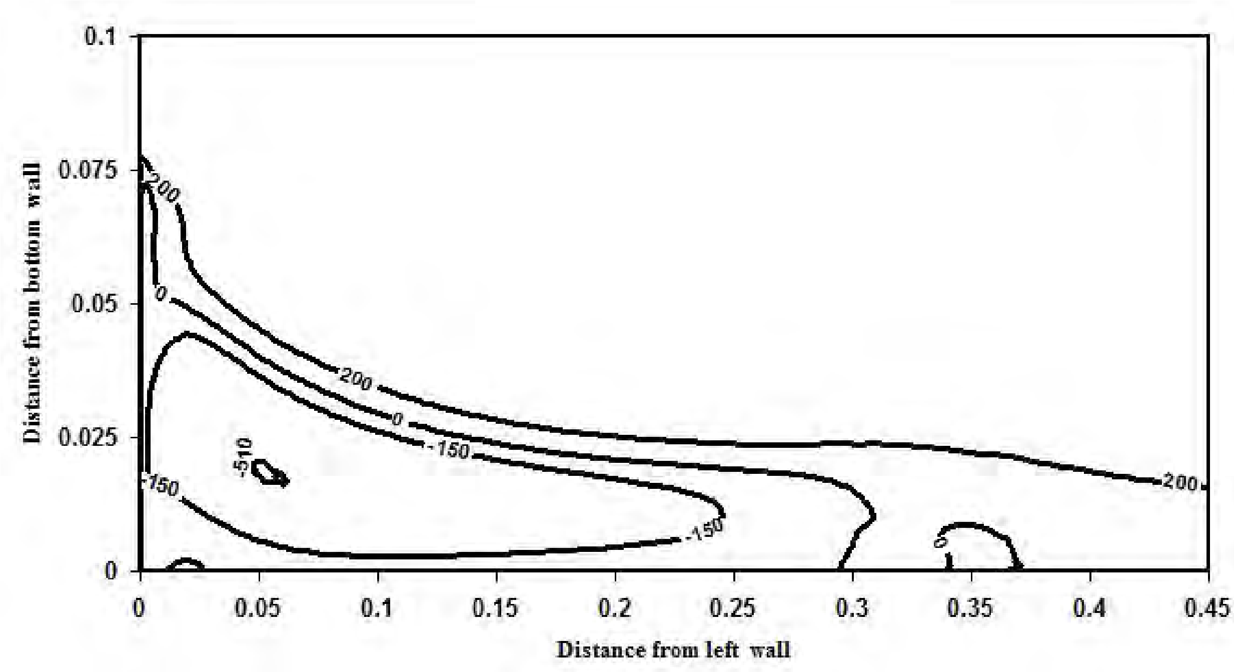

Fig. 12 (c) Zone of recirculation for Mach number 1.2 (h=0.075)

3.3.2.3 Effect on Pressure and Temperature in the flow Field

Figure 13(a-c) shows pressure contours for different Mach number at step height 0.100. A little increase of pressure is in the expansion shock region, reattached shock region, along the step height and recirculation zone, as the Mach number increases. There is no remarkable change of pressure and temperature in the flow field, as the change of Mach number. The striking pressure increases, with the increase of Mach numbers (Fig. 14) and the position moves to the left.

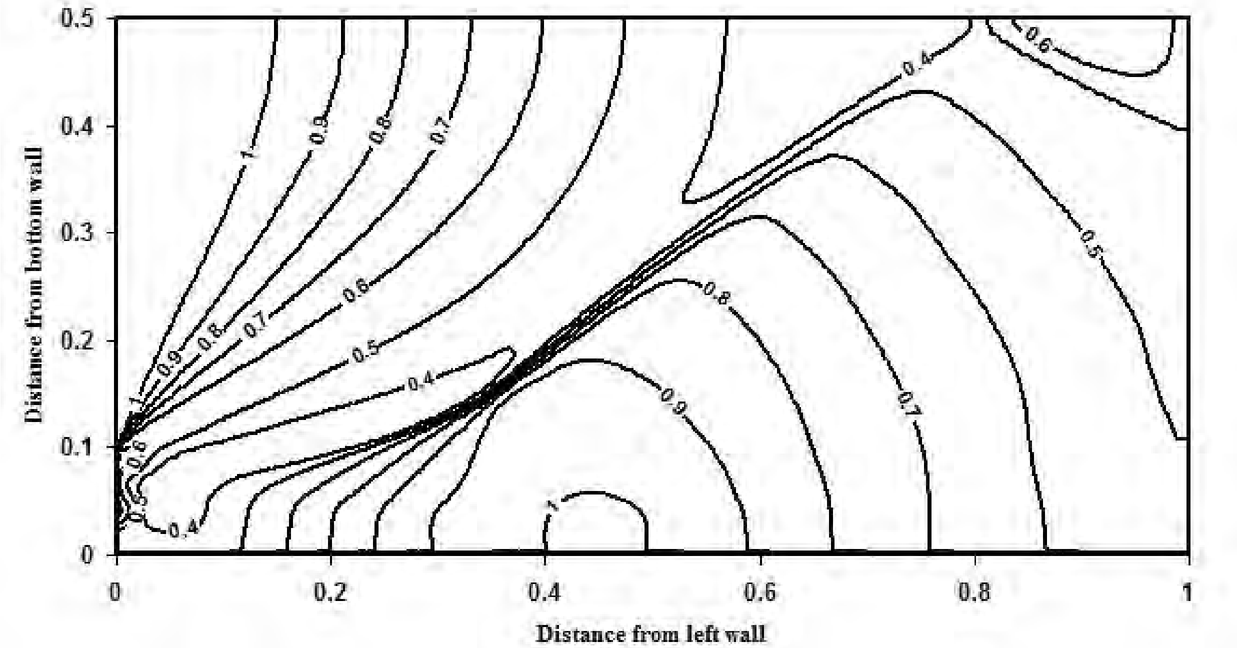

Fig. 13 (a) P/P_{in} contour for Mach number 0.8 (h=0.100)

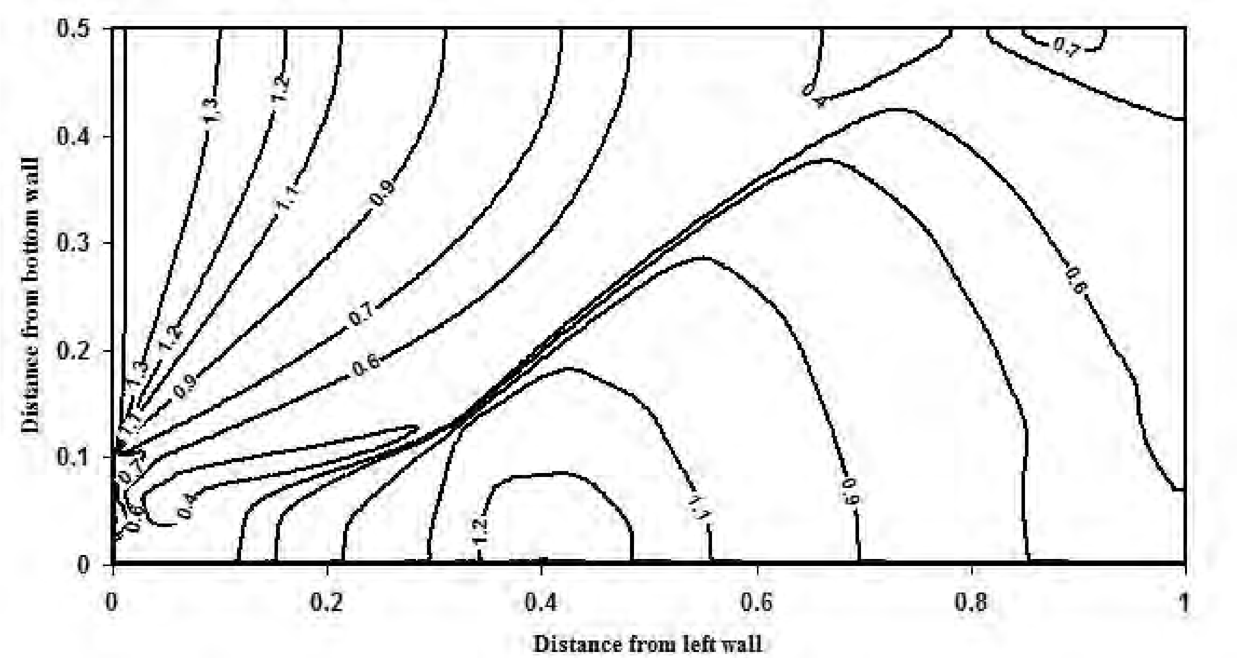

Fig. 13 (b) P/P_{in} contour for Mach number 1.0 (h=0.100)

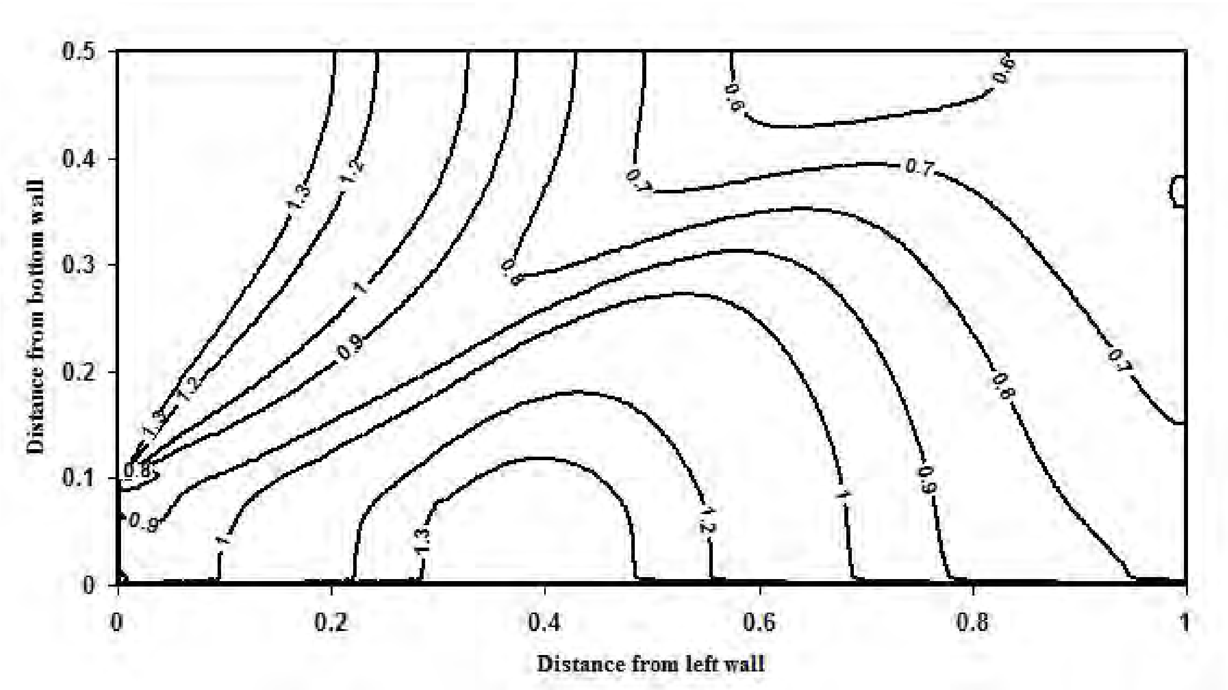

Fig. 13 (c) P / P_{in} contour for Mach number 1.2 (h= 0.100)

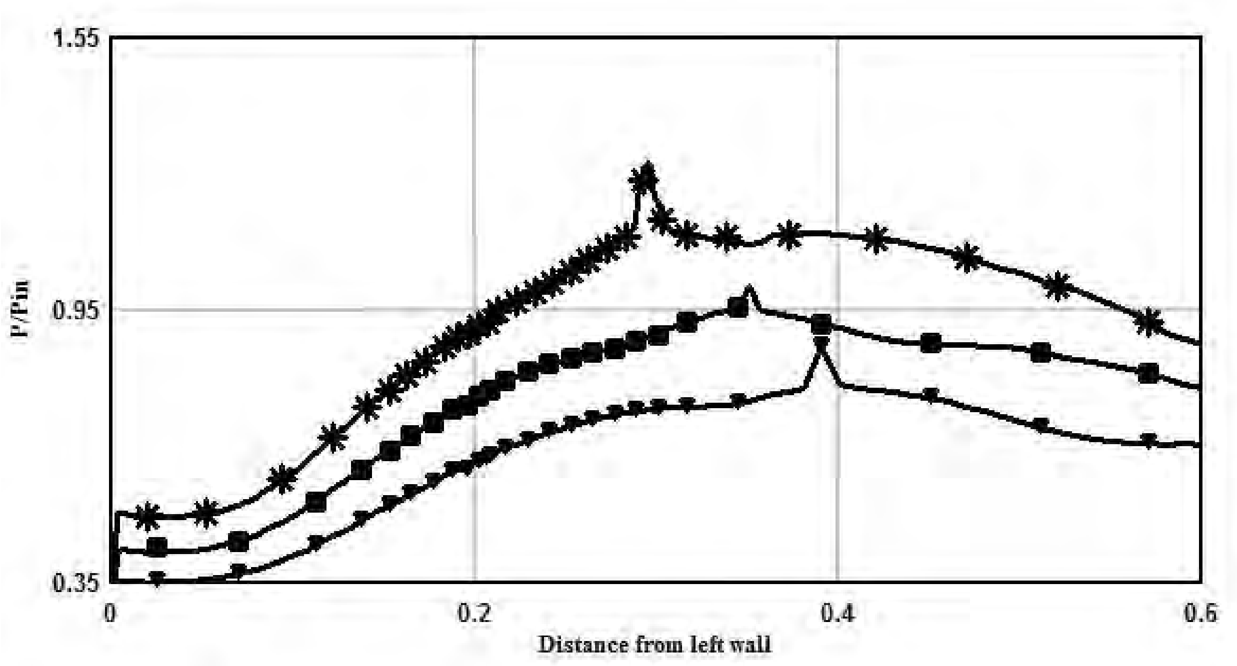

Fig. 14 Wall pressure variation for different Mach numbers (h=0.075)

3.4 Results and discussion on forward facing step

The schematic diagram of calculation domain considering forward facing step is shown in fig. 3, where the facing step is varied as 0.025, 0.05, 0.075 and 0.10 and the Mach number is varied as 0.8, 1.0 and 1.2. Like backward facing step, the results are analyzed and discussed, firstly considering the effect of step heights with Mach numbers, secondly the effect of Mach numbers with step heights.

3.4.1 Effect of step heights with Mach numbers

In this section the results are analyzing by varying the height of the step as 0.025, 0.050, 0.075 and 0.100 with specific Mach numbers.

3.4.1.1 Characteristic of different shock

Figure 15 (a-d) shows Mach contours for different step heights. Here the Mach number of incoming flow is 0.8. From Mach contours, two shock regions are visible in the flow field, namely the leading edge oblique shock and detached shock in front of the step. Mach contours are characterized by a group of contour line near to the plate (it looks like a band and this group of lines is named as band in further discussion) which is starting from the leading edge of the plate. This band has an important significance; it demarcates the supersonic and subsonic regions in the flow field. The Mach number of the region above the band is high and below the band is low. The detached shock is located left side of the right-bottom corner and developed due to separation. With the increase of step height, steepness of the band increases and the area of detached shock region increases.

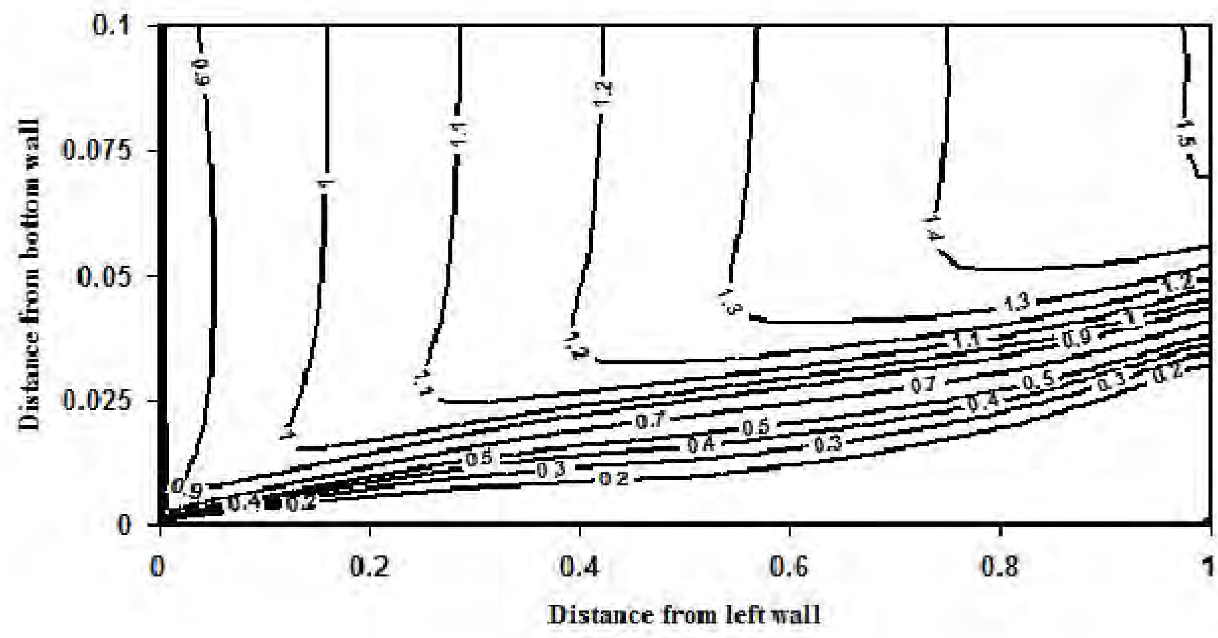

Fig. 15 (a) Mach contour for h= 0.025 (M=0.8)

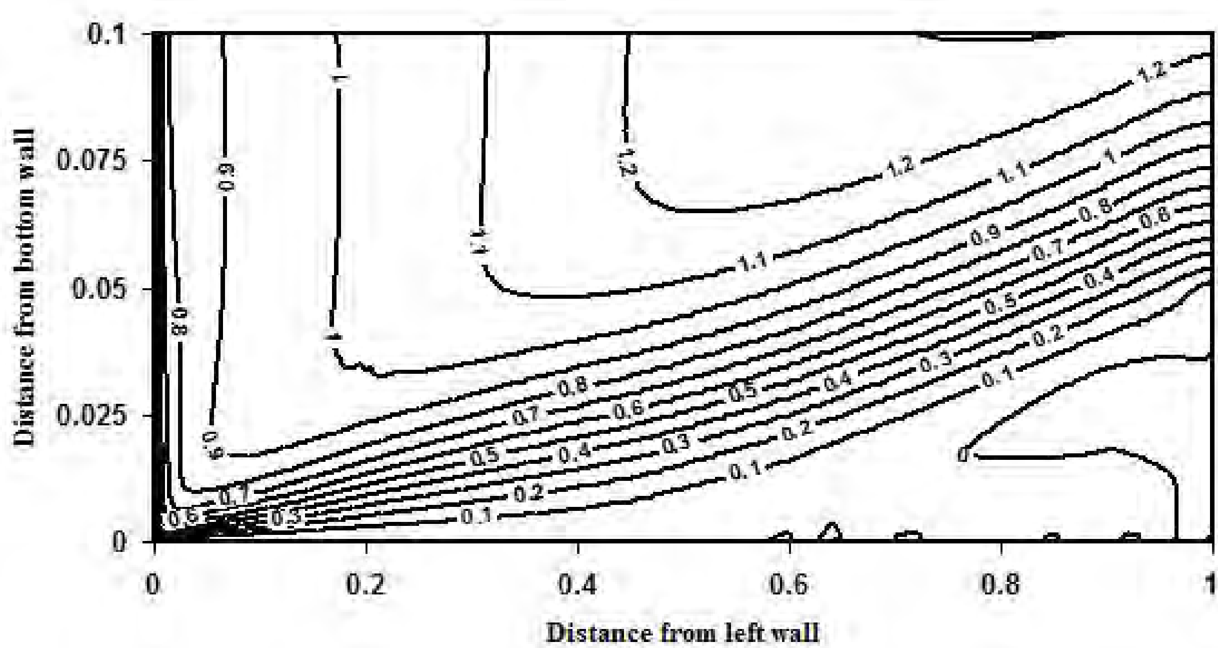

Fig. 15 (b) Mach contour for h= 0.050 (M=0.8)

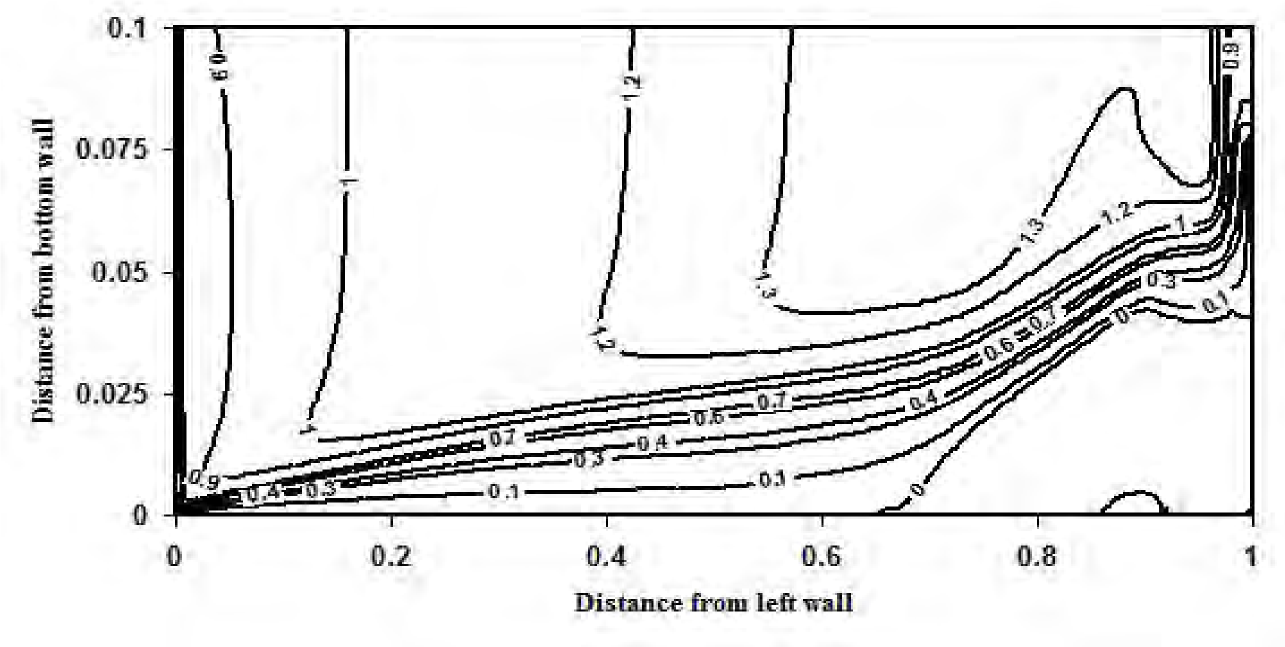

Fig. 15 (c) Mach contour for h= 0.075 (M=0.8)

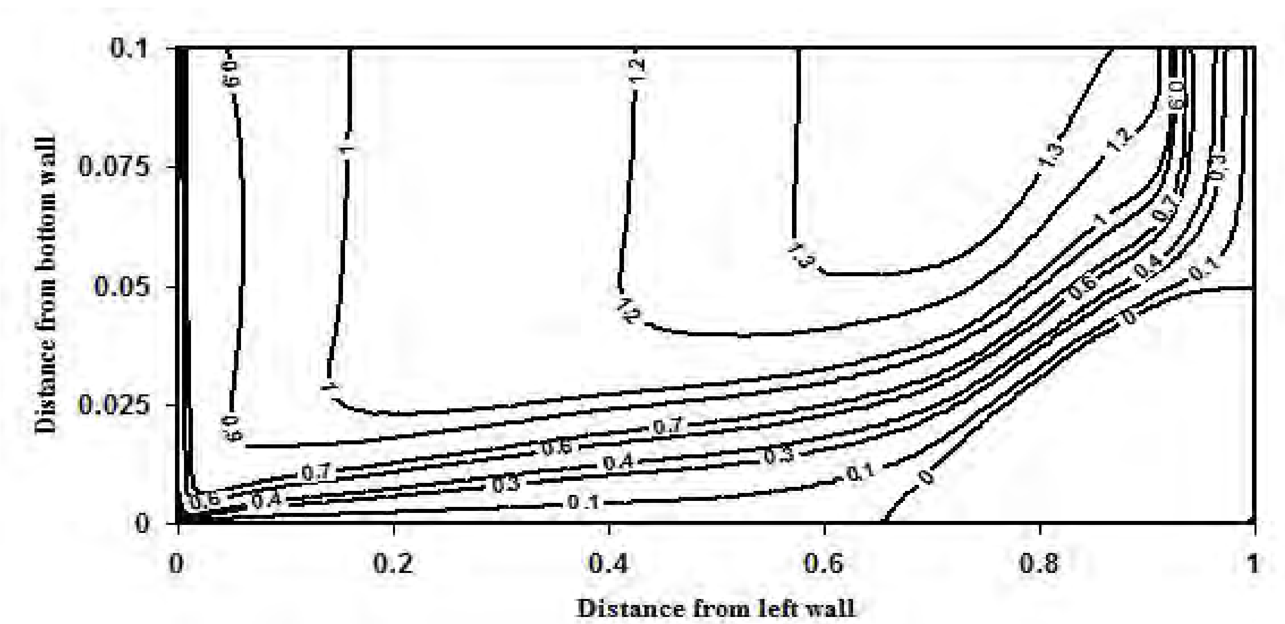

Fig. 15 (d) Mach contour for h= 0.100 (M=0.8)

3.4.1.2 Dynamical behavior of the flow field

Figure 16 shows the stream wise velocity along the plate for step heights at Mach number 1.0. From this Fig. 16 it is found that there is a .sudden increase of velocity in leading edge oblique shock region. On the other hand, the velocity is low in detached shock region. Recirculation appears at the right bottom corner of the velocity field. Figure 17 (a-d) shows the recirculation zone for different step height at M=1.0. The figures are magnified and are shown partly by height of 0.10 (from bottom wall) and length of 0.60 (from right boundary). From the Fig. 17 (b-d), it is found that the recirculation is located at about a half to two thirds of the step height. For step height 0.025, the recirculation zone is negligible. With the increase of step height, the length of recirculation increases. The strength of recirculation also increases, as the step height increases. The strength of recirculation becomes stronger as move towards the wall. The velocity finds in leading edge oblique shock region is not change with the change of step height (Fig. 16).

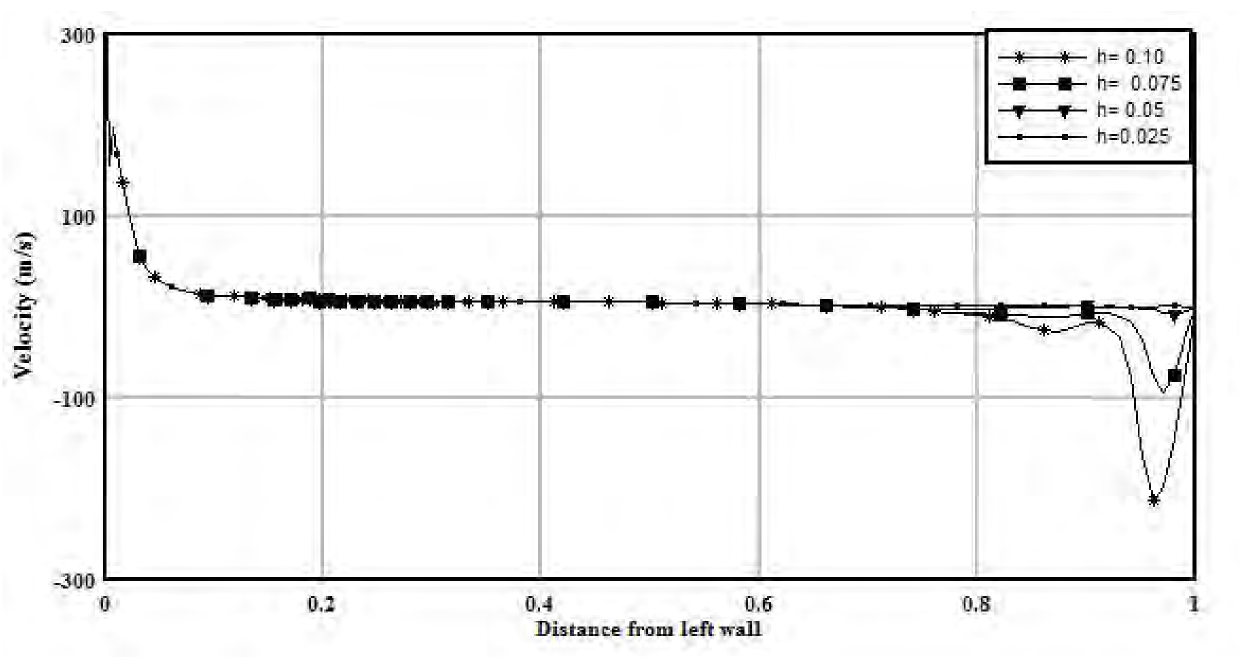

Fig. 16 Stream wise velocity along the plate for different step heights (M=1.0)

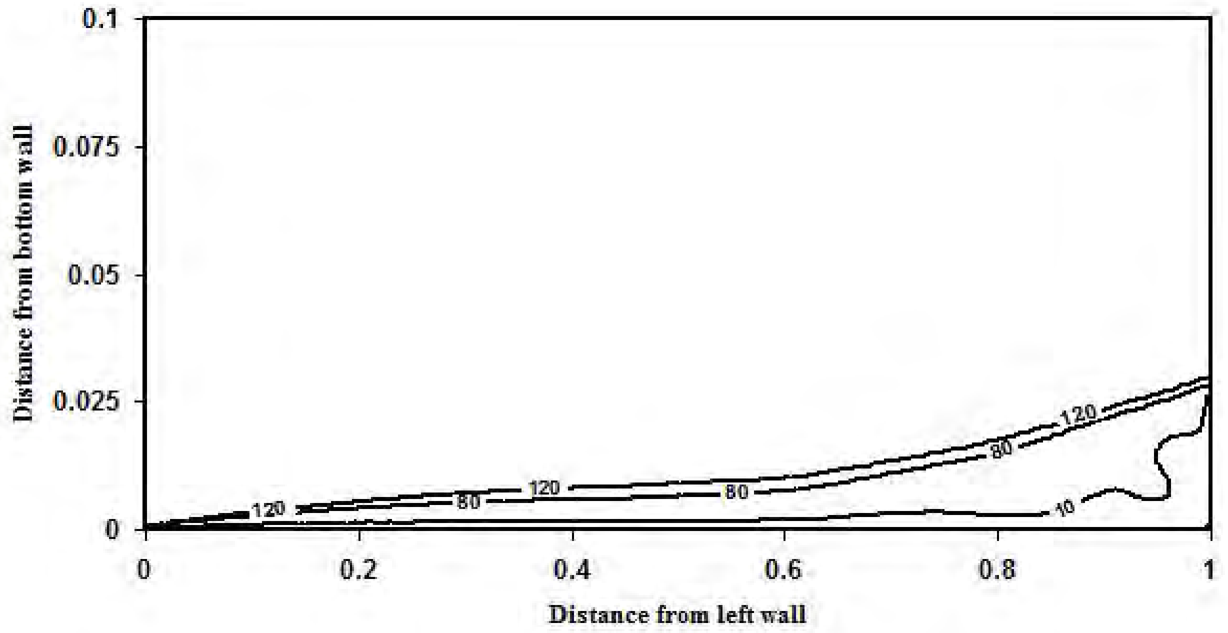

Fig. 17 (a) Zone of recirculation for h=0.025 (M=1.0)

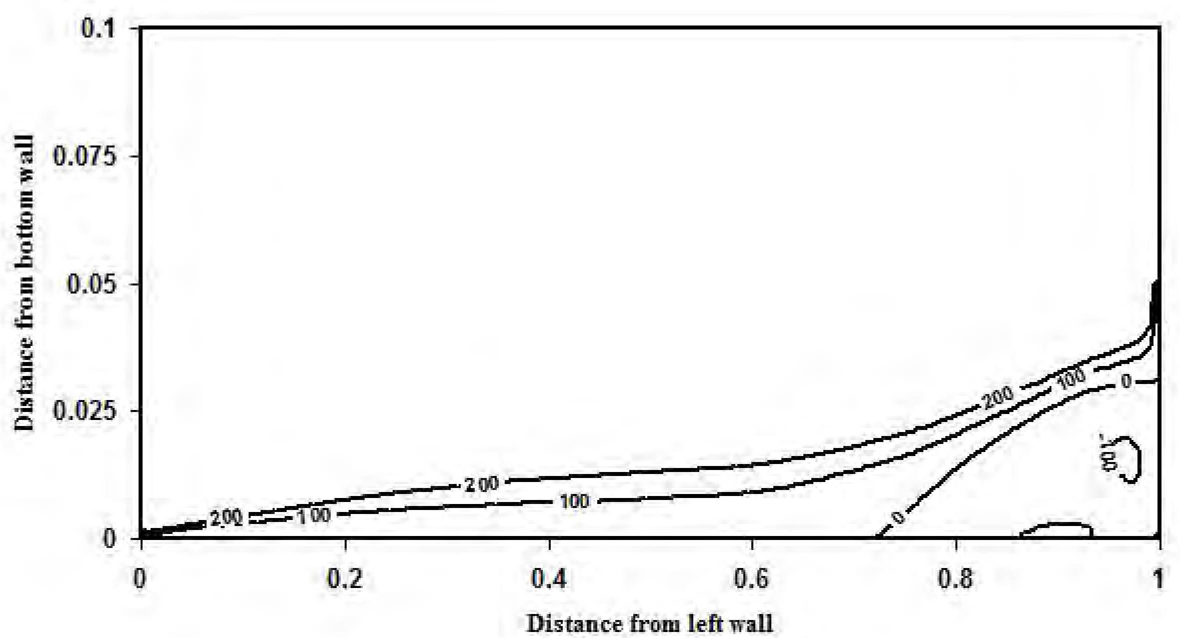

Fig. 17 (b) Zone of recirculation for h=0.05 (M=1.0)

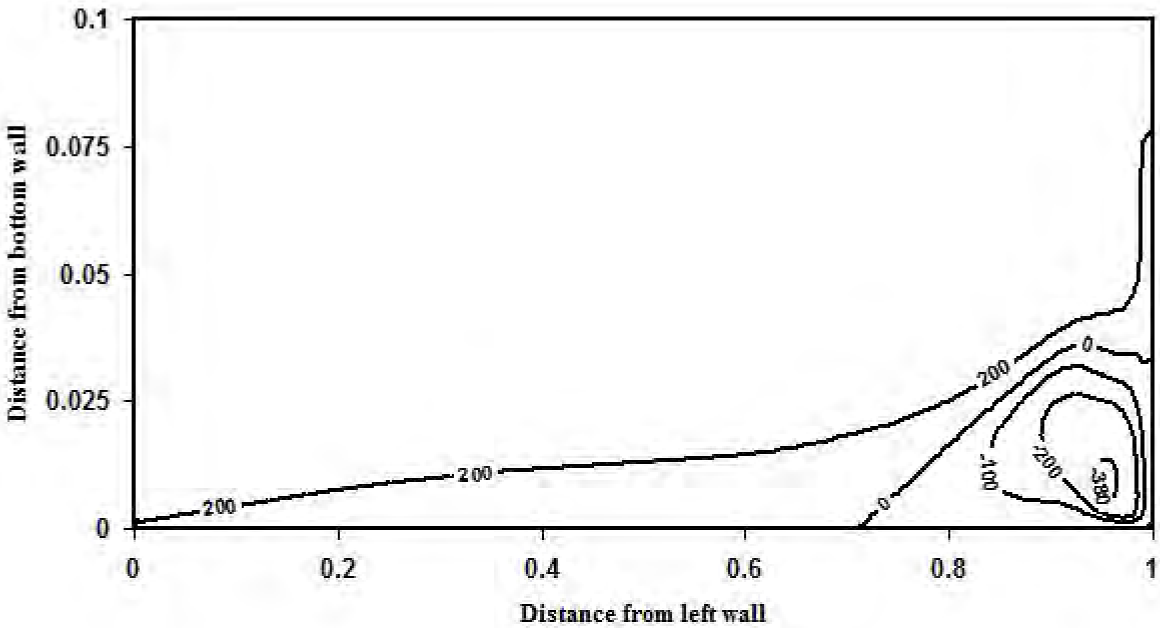

Fig. 17 (c) Zone of recirculation for h=0.075 (M=1.0)

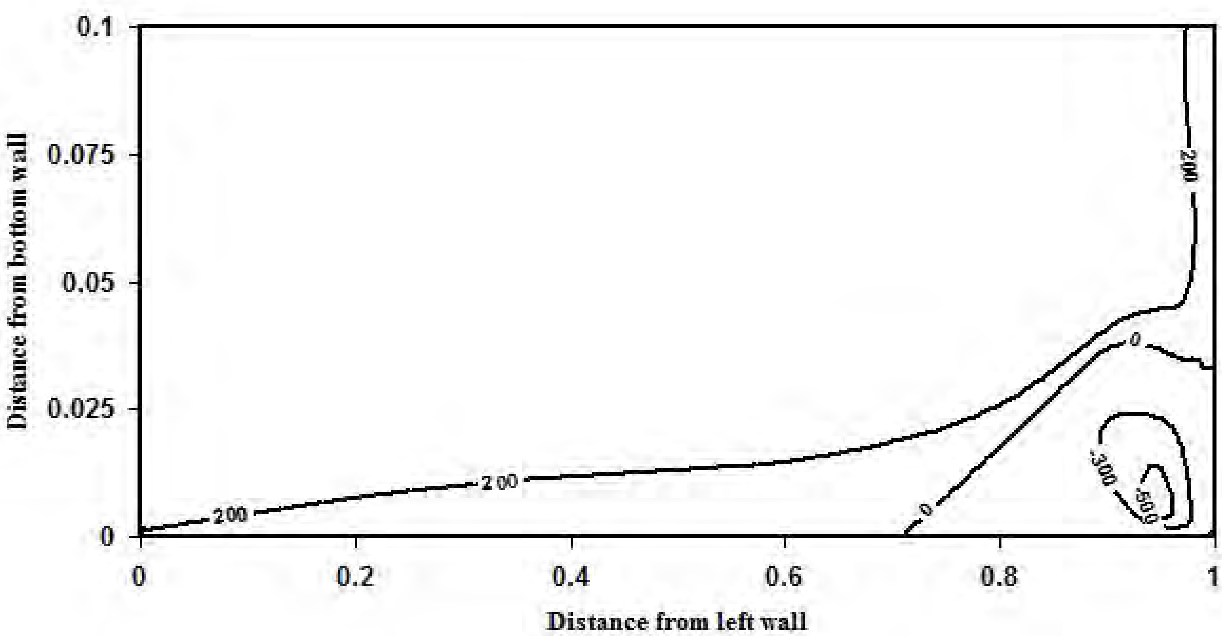

Fig. 17 (d) Zone of recirculation for h=0.10 (M=1.0)

3.4.1.3 Effect on Pressure and Temperature in the flow Field

Figure 18 shows the pressure along the plat for different step height at Mach number 1.2. Due to the formation of leading edge oblique shock, there is a rapid rise of pressure at the starting of the plate then it reduces continuously along the plate. For detached shock, there is a little increases of the pressure in front of the step. The maximum pressure of the flow field is found at the point where the oblique shock forms. It is same for different step heights. Above the step height the pressure gradually decreases due to high Mach number. With the increase of step height, the pressure at the recirculation zone also increases. From the fig 19, it is found that along the vertical face of the step the pressure first attains a minimum, then gradually attains a maximum just before the tip of the step, and then falls again at the tip. The pattern of pressure distribution is similar for different step heights. But with the increase of step height, the pressure distribution along the step body increases.

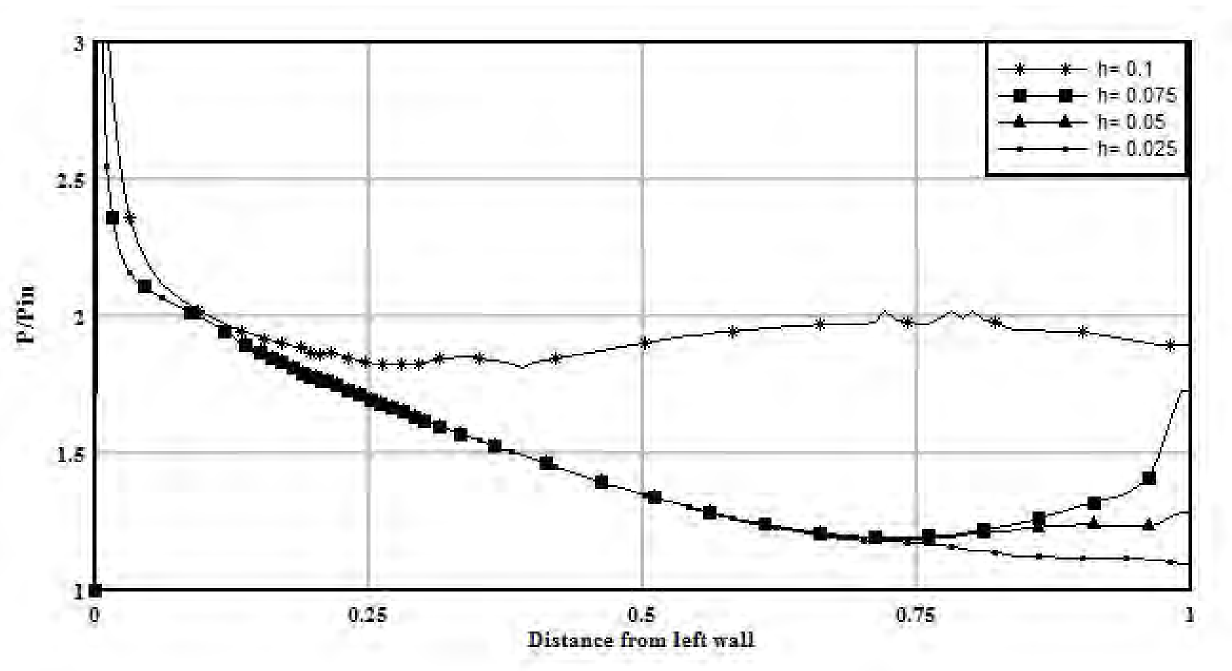

Fig. 18 The pressure along the plat for different step height (M=1.2)

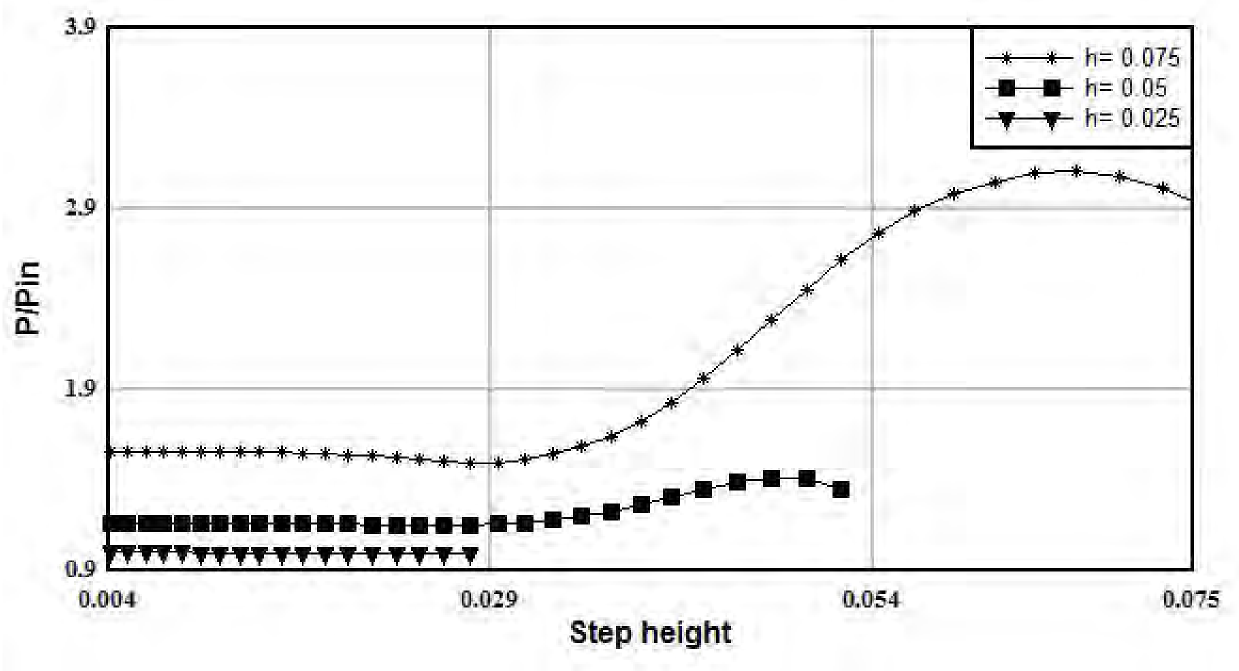

Fig. 19 Pressure distribution along the vertical surface of step height (M=1.2)

From the Fig. 20 temperature along the plate at Mach number 1.0, it is found that initially the temperature rises in oblique shock region and it gradually decreases along the plate. The temperature again increases at the start of recirculation zone and it continues along the recirculation length. The maximum temperature of the flow field is found in recirculation zone which is at about two-third to one-fourth of the recirculation length. The recirculation temperature increases with the increase of step height, which is found 961K, 983K, 1024K and 1046K, respectively.

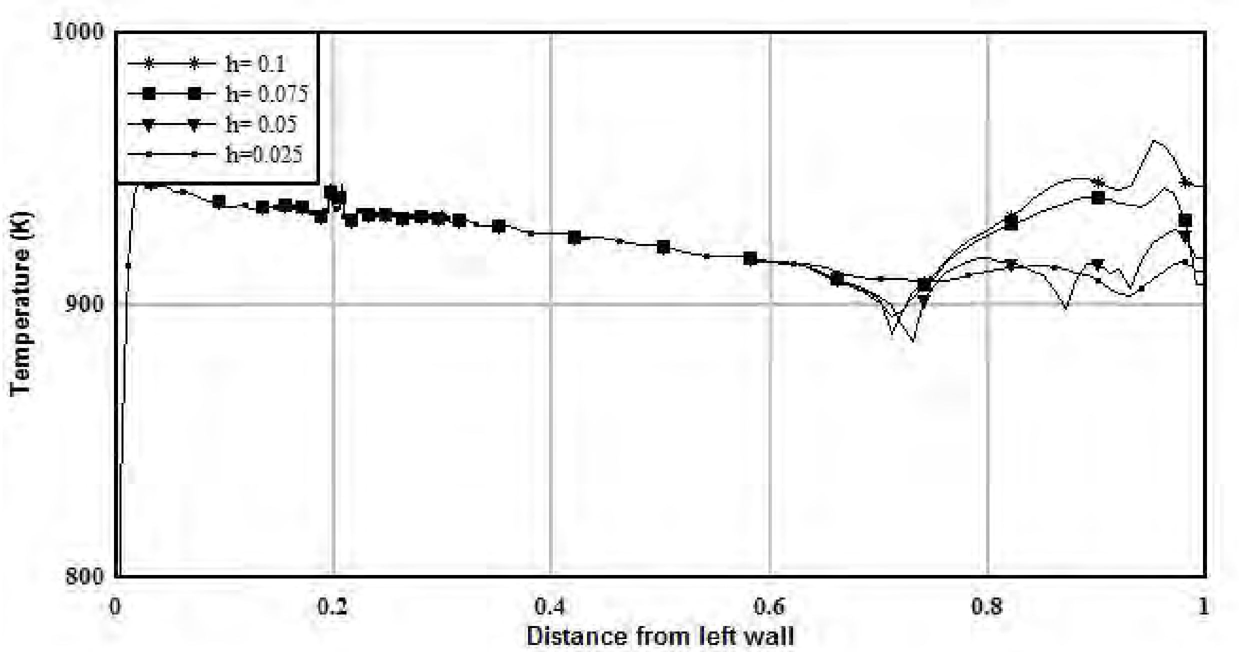

Fig. 20 Temperature along the plate for different step heights (M=1)

3.4.2 Effect of Mach numbers with step heights

In this section the results are analyzing by varying the Mach number as 0.8, 1.0 and 1.2 at specified step heights.

3.4.2.1 Characteristic of different shock

From the fig.21 it is found that, the position of formation of oblique shock does not change, due to the change of Mach numbers as the range of study. With the increase of Mach number, the angle between the flat plate and the band finding in the oblique shock region decreases i.e. the steepness of the band reduces which is found in Fig. 22 (a-c). Fig. 22(a-c) are magnified by height 0.10 from bottom wall. As the Mach number is increased, the detached shock moves to the right as well as reduces the area of this zone.

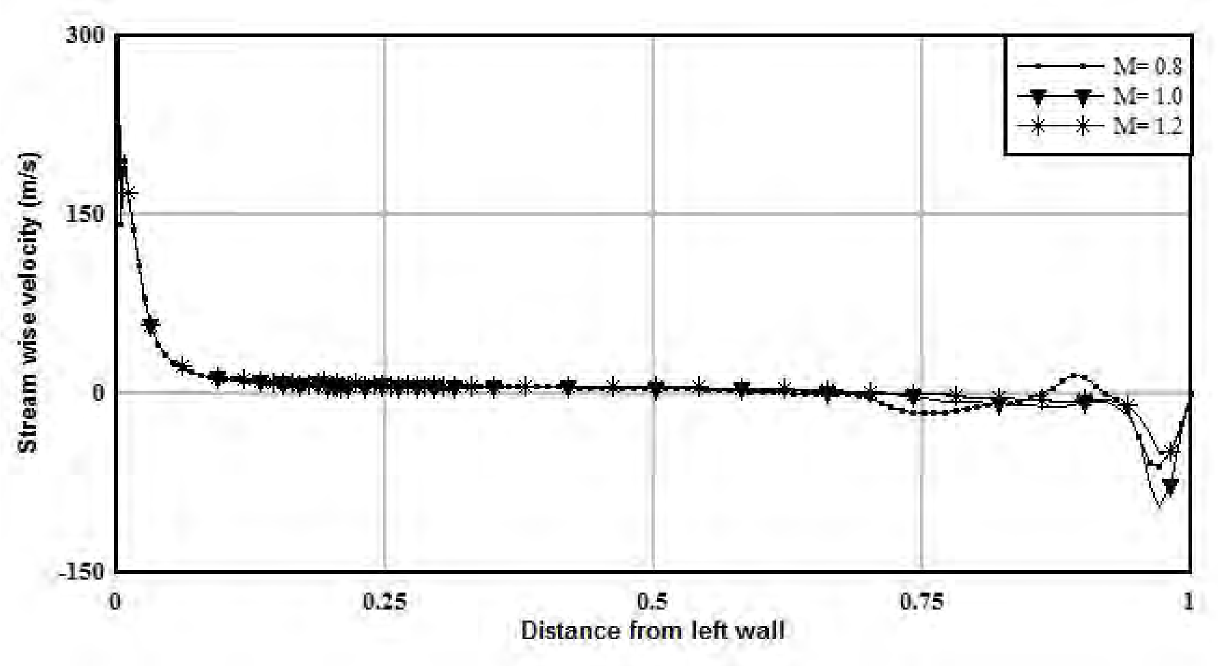

Fig. 21 Velocity along the plate for different Mach numbers (h=0.075)

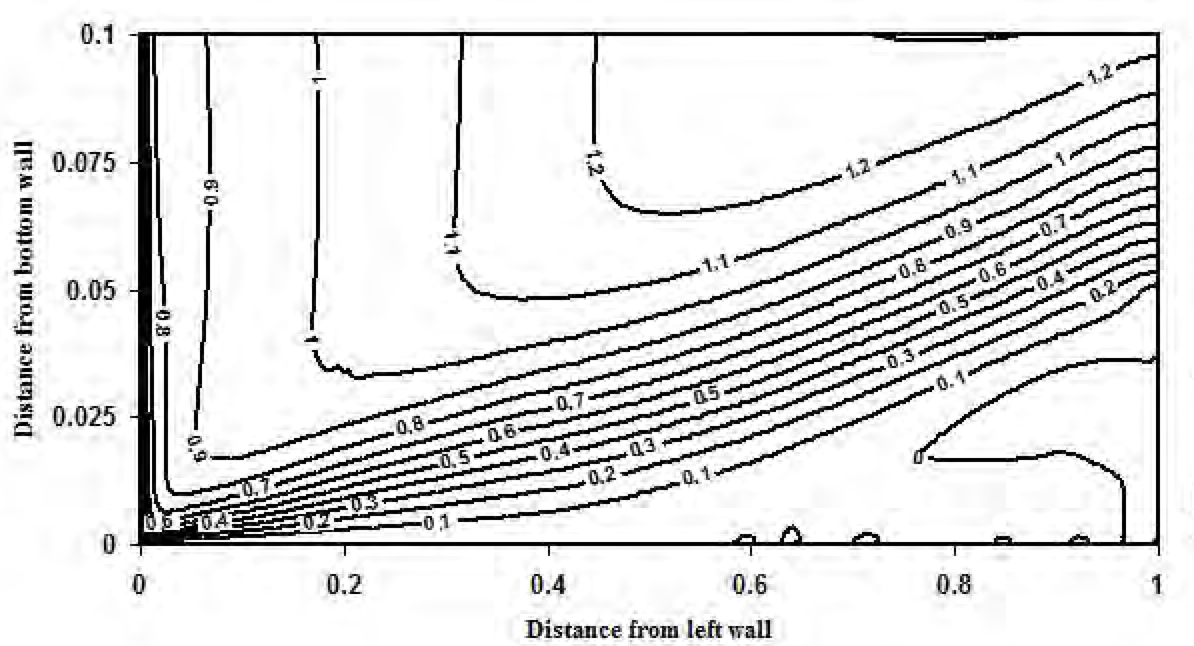

Fig. 22 (a) Mach contour for M=0.8 (h=0.05)

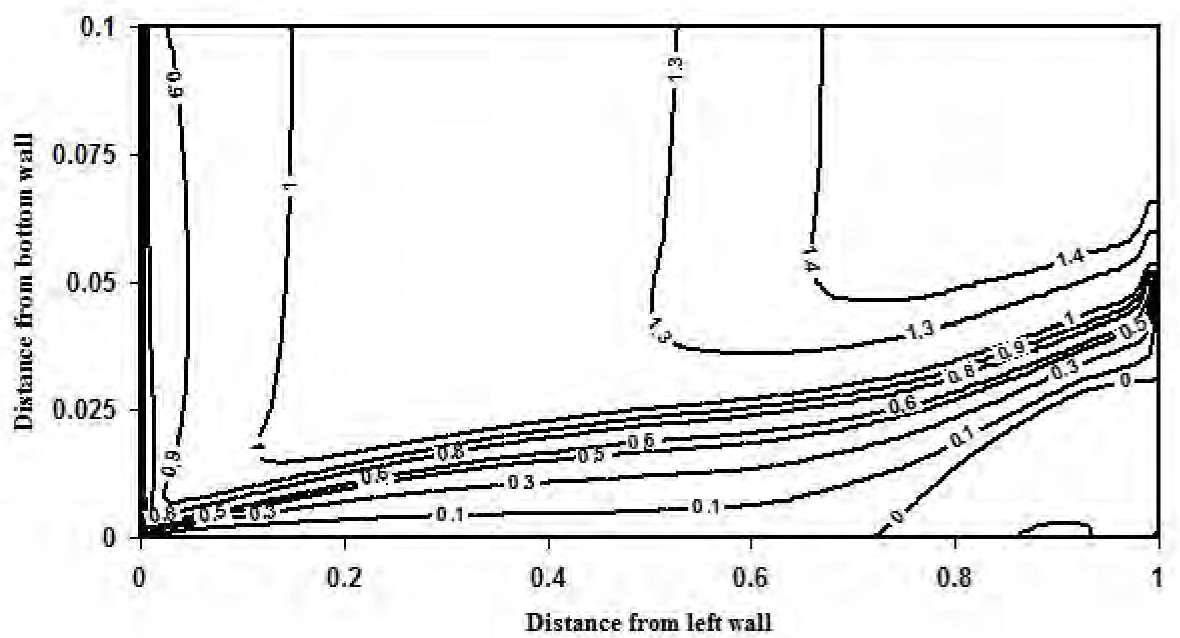

Fig. 22 (b) Mach contour for M=1.0 (h=0.05)

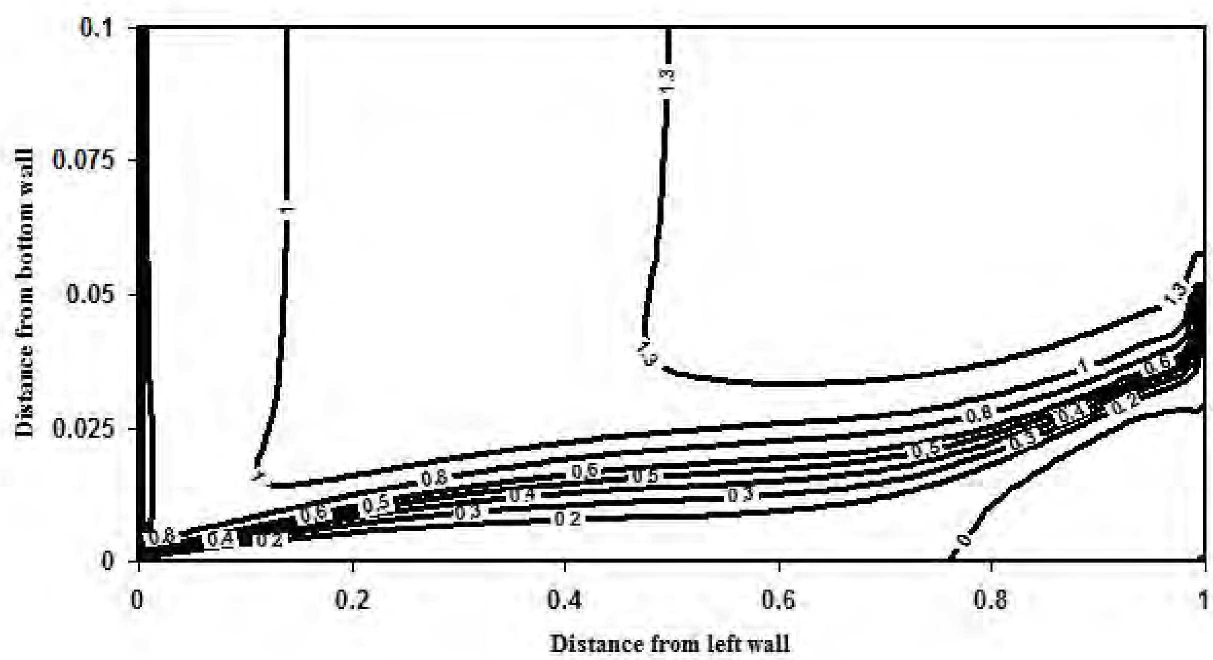

Fig. 22 (c) Mach contour for M=1.2 (h= 0.05)

3.4.2.2 Dynamical behavior of the flow field

Figure 23 (a-c) shows the recirculation zone for different Mach numbers for step height 0.075. In this investigation, due to the compression of flow it is found that the height of recirculation decreases, with the increase of Mack number. The length of recirculation is not so responsive in case of small step height. For high step height, recirculation length decreases with the increase of Mach number. As the Mach number of the flow increases, the zone of recirculation moves to the right and the recirculation becomes weaker.

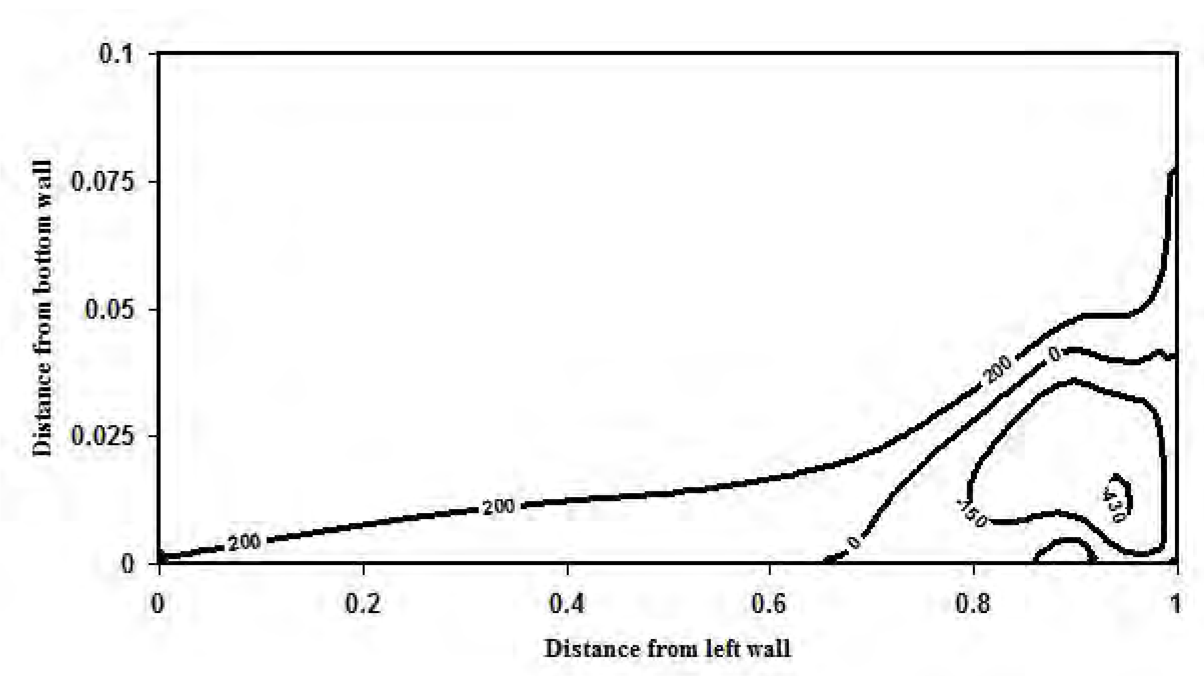

Fig. 23 (a) Zone of recirculation for M=0.8 (h=0.075)

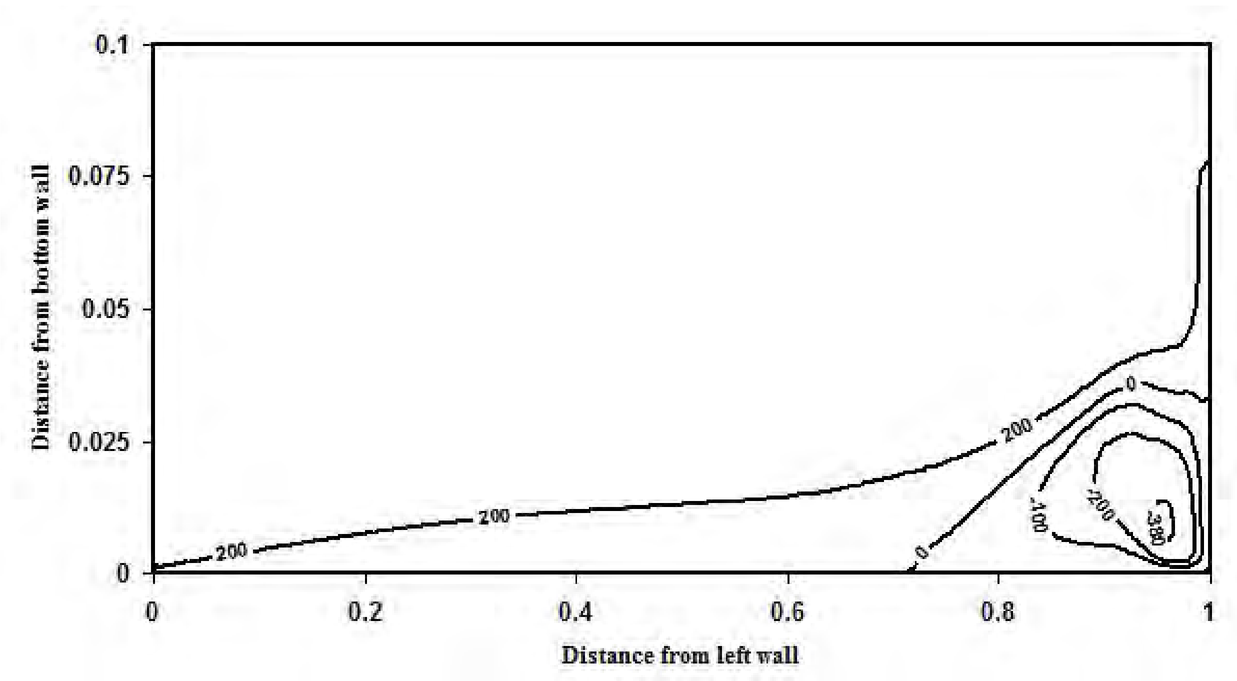

Fig. 23 (b) Zone of recirculation for M=1.0 (h=0.075)

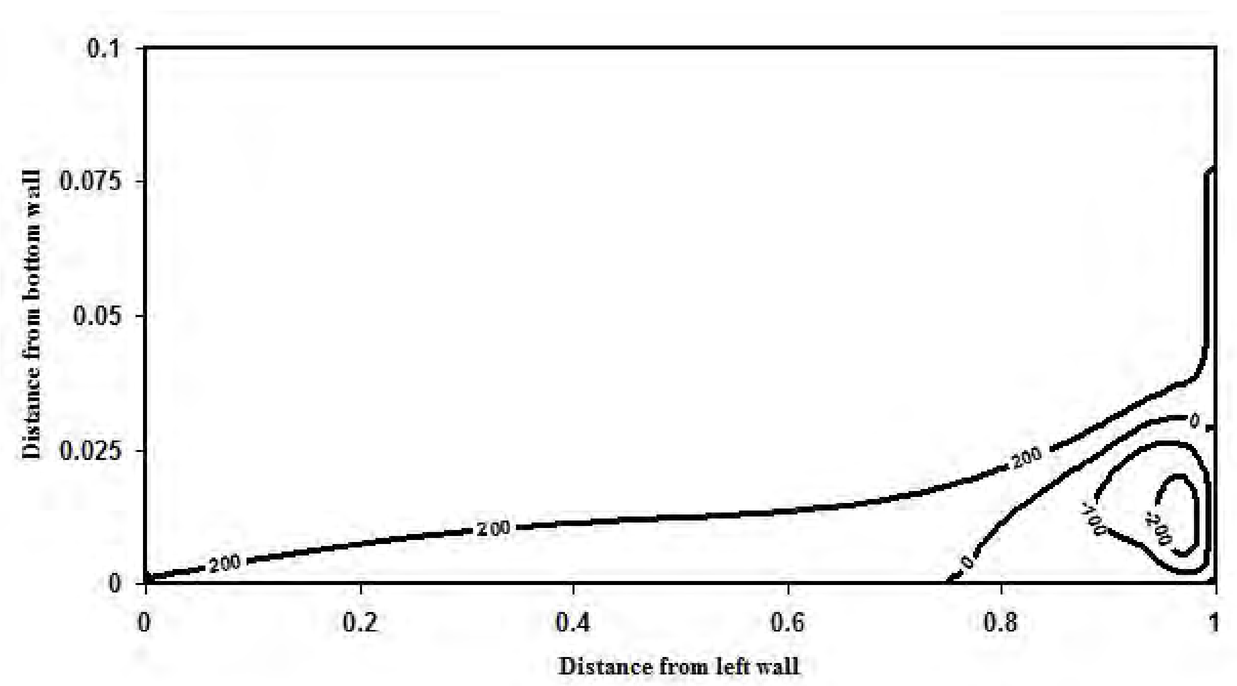

Fig. 23 (c) Zone of recirculation for M=1.2 (h=0.075)

3.4.2.3 Effect on Pressure and Temperature in the flow Field

With the increase of Mach numbers the pressure in oblique shock region increases, which is found in Fig. 24. With the increases of Mach number the pressure in the recirculation zone and along the vertical face of the step also increases. Maximum pressure of the recirculation zone is found just before the step. The minimum pressure of the flow field is found at exit of the boundary. No remarkable change is found in exit pressure. The pressure distribution along the step increases, as the Mach number changes from 0.8 to1.2 (in Fig.25).

There is no remarkable effect of Mach number on the temperature of the flow field. The increase of temperature at recirculation zone is about 720K. The temperature at the tip of the step increases, as the Mach number increases. The temperature, above the tip of the step changes rapidly. In a very short distance it falls continuously then it changes slowly.

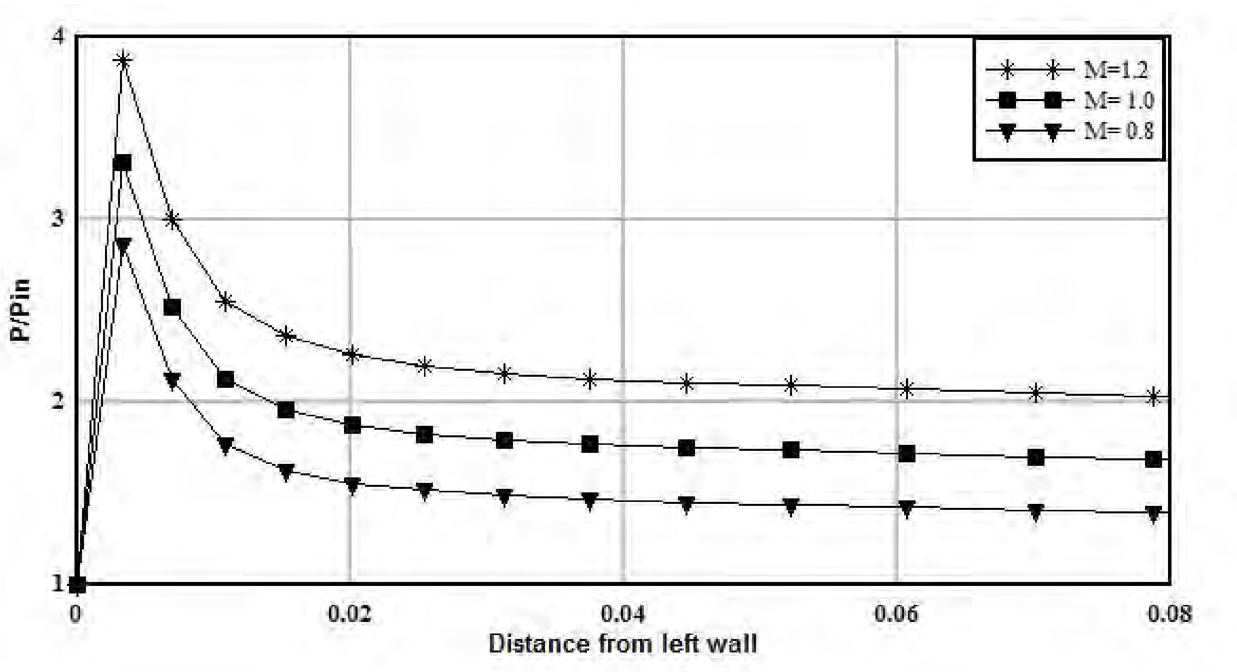

Fig. 24 Change of pressure along the plate for different Mach numbers (h=0.075)

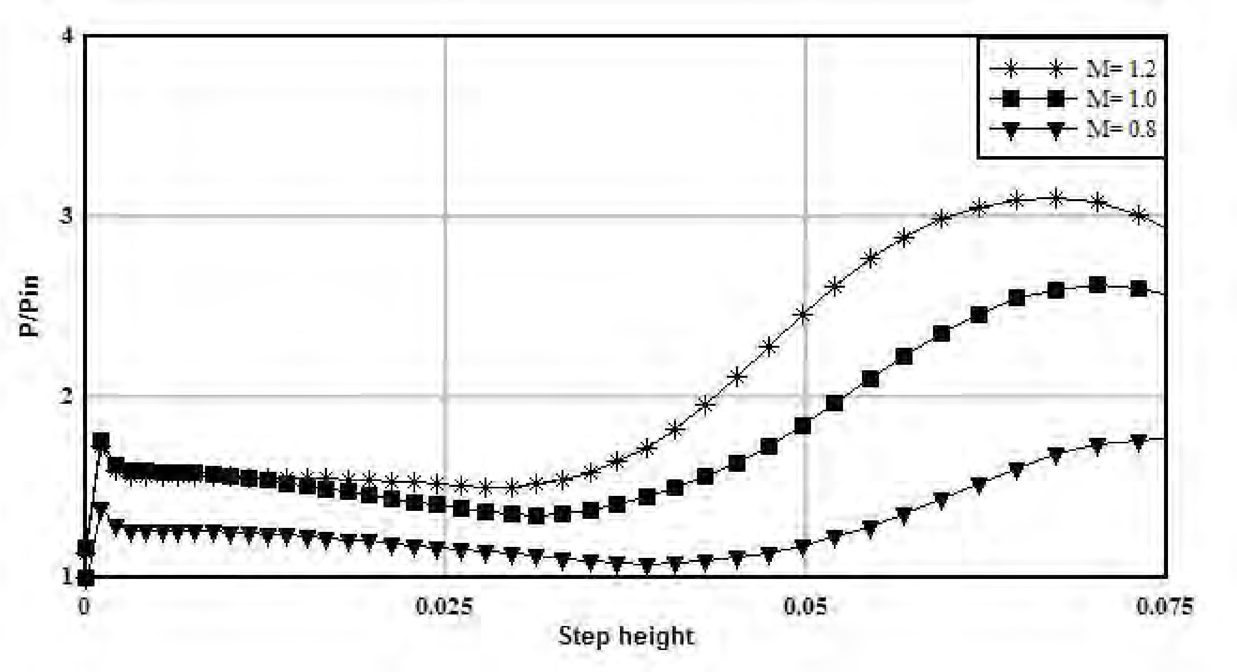

Fig. 25 Pressure distribution along the step height for different Mach numbers (h=0.075)

3.5 Comparison between backward and forward facing step

3.5.1 *Characteristic of different shock*

In backward facing step two types of shock are visible in the flow field, namely corner expansion shock and reattached shock. On the other hand, in forward facing step the leading edge oblique shock and detached shock appear. In corner expansion shock, the pressure and temperature is low, but the velocity is high and in reattached shock region the pressure and temperature is high but velocity is low. Again the pressure, temperature and velocity are high in oblique shock region; and a small increase of pressure and temperature in detached shock region. Out of this four types of shock, the maximum increase of velocity in expansion shock region, maximum rise of pressure in oblique shock region.

3.5.2 Dynamical behavior of the flow field

For same Mach number and same height, from the Fig. 26 & 27, it is observed that, for same Mach number and step height the length of recirculation is longer in case of backward facing step. The strength of recirculation i.e. the turbulence excited in backward facing step is stronger than forward facing step. In forward and backward facing step, the length of recirculation is more responsive to step height than Mach number. In both cases the length of recirculation increases with the increases of step height and decreases with the increase of Mach number. Figure 28(a-d) & 29(a-d) show the transverse velocity contours for Mach number, 1.2 and different step heights. Apart from the inlet, the maximum and minimum value of transverse wise velocity is located immediately before (in forward facing step) and after the tips of the step (in backward facing step). The minimum value of the transverse velocity immediately after the tip physically demands that the flow needs some stream wise distance to adjust and become fully developed. Due to the change of step heights, there occurs an increase in area of realm where maximum and minimum cross-stream velocity demanding more stream wise distance to get fully developed. On the other hand, with the increase of Mach number it is found that there is a small increase in area of realm in backward facing step and decrease in area in forward facing step.

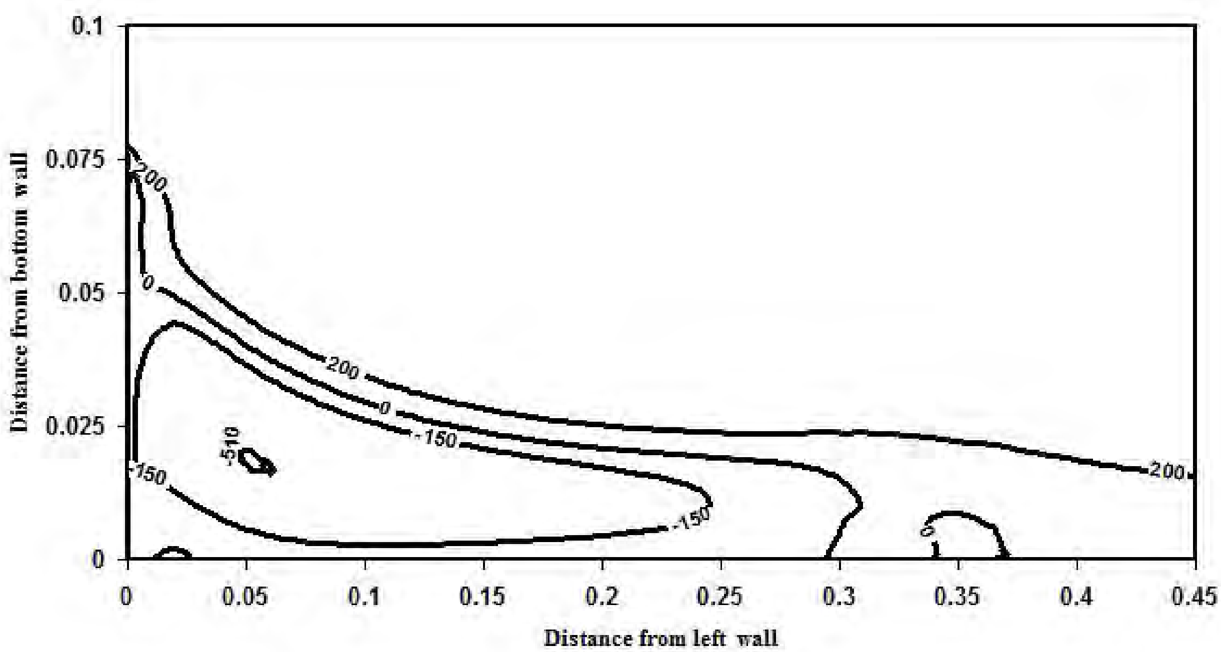

Fig. 26 Recirculation zone for backward facing step (M=1.2 & h= 0.075)

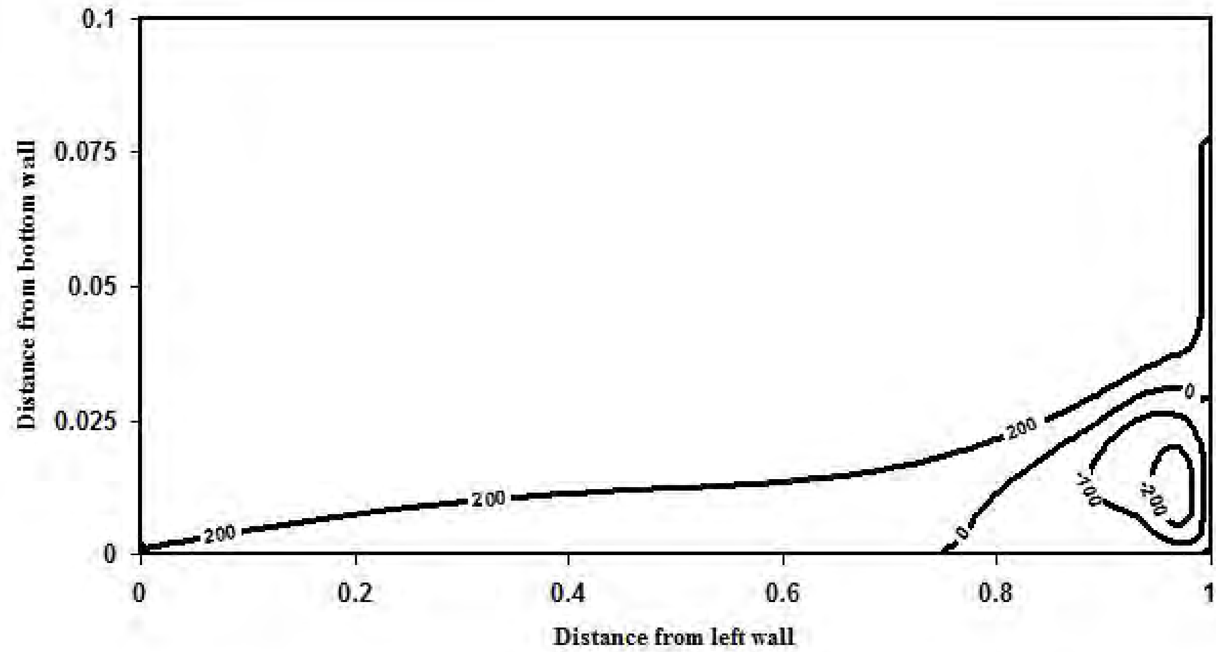

Fig. 27 Recirculation zone for forward facing step (M=1.2 & h= 0.075)

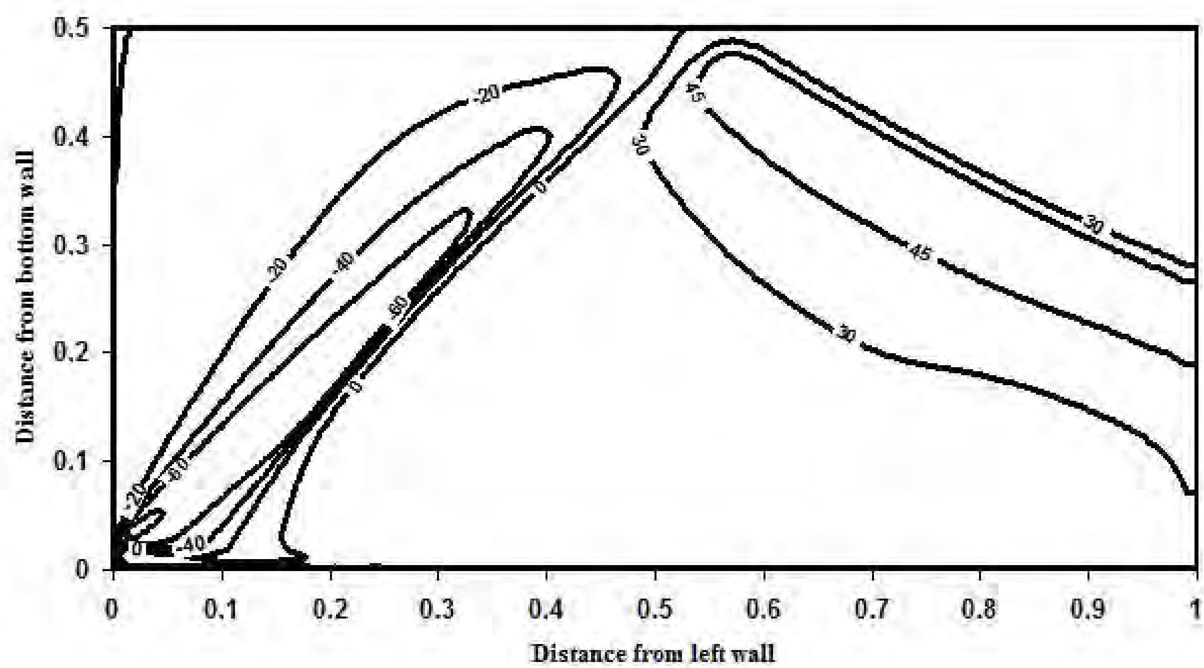

Fig. 28(a) Transverse velocity contour for backward facing step (M=1.2 & h=0.075)

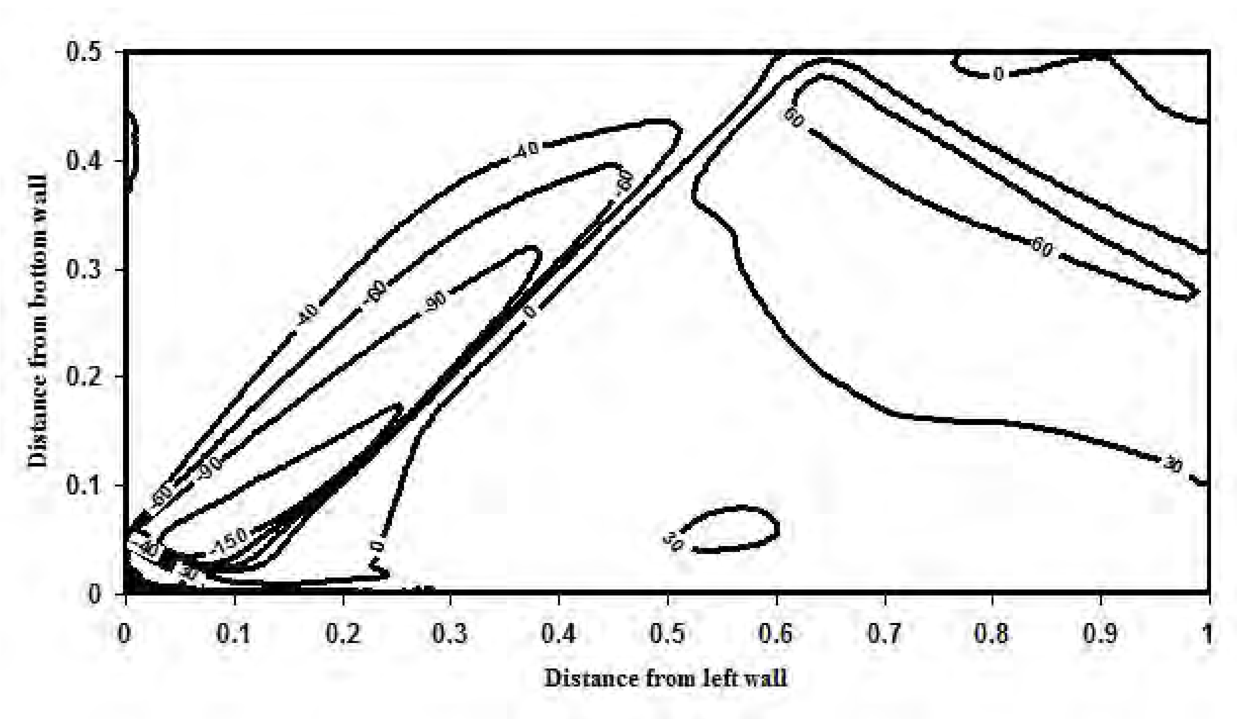

Fig. 28(b) Transverse velocity contour for backward facing step (M=1.2 & h=0.075)

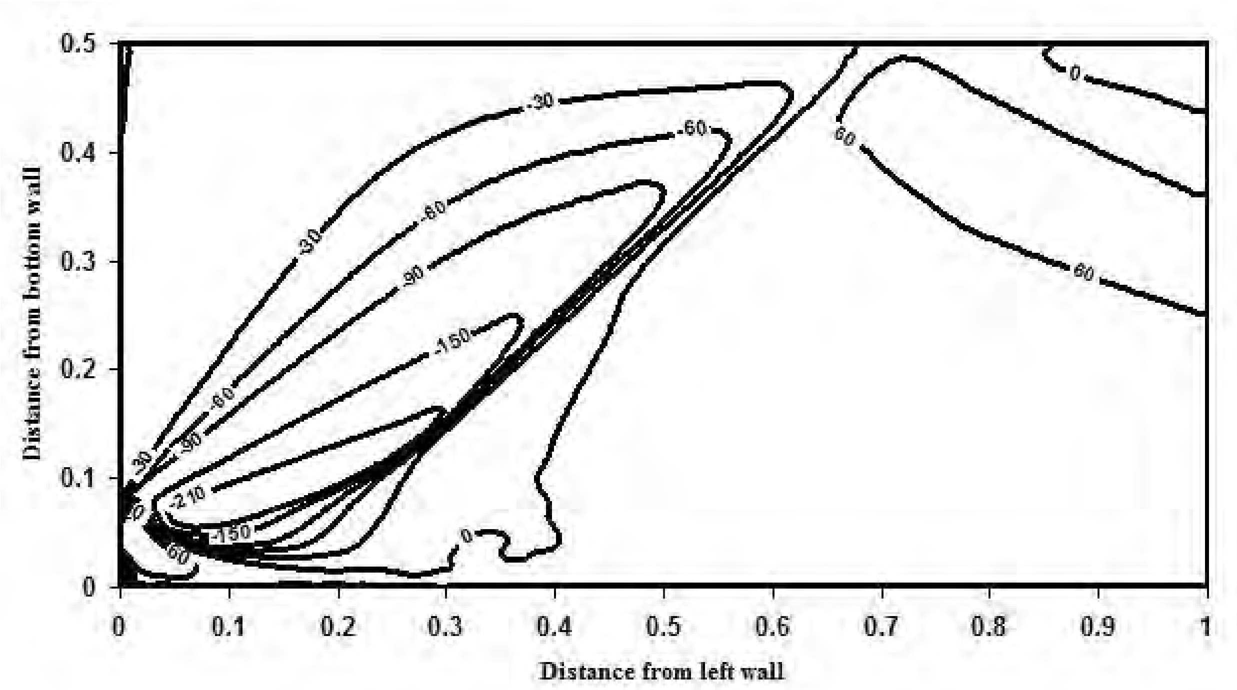

Fig. 28(c) Transverse velocity contour for backward facing step (M=1.2 & h=0.075)

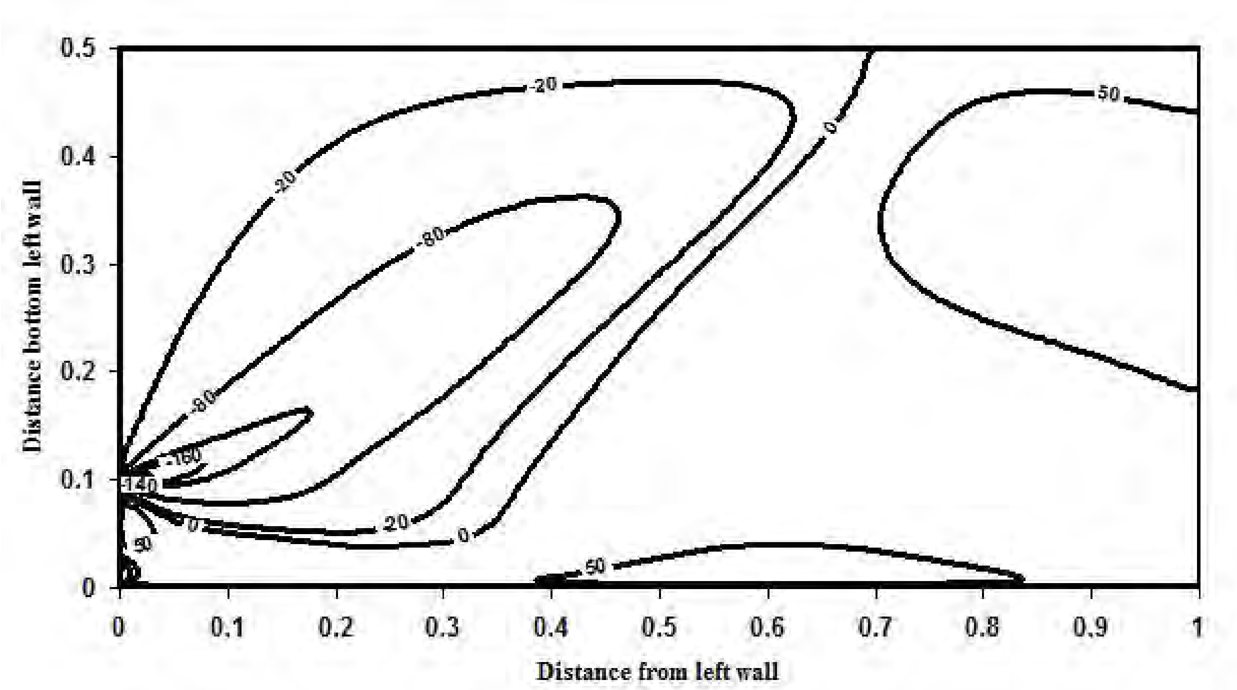

Fig. 28(d) Transverse velocity contour for backward facing step (M=1.2 & h=0.10)

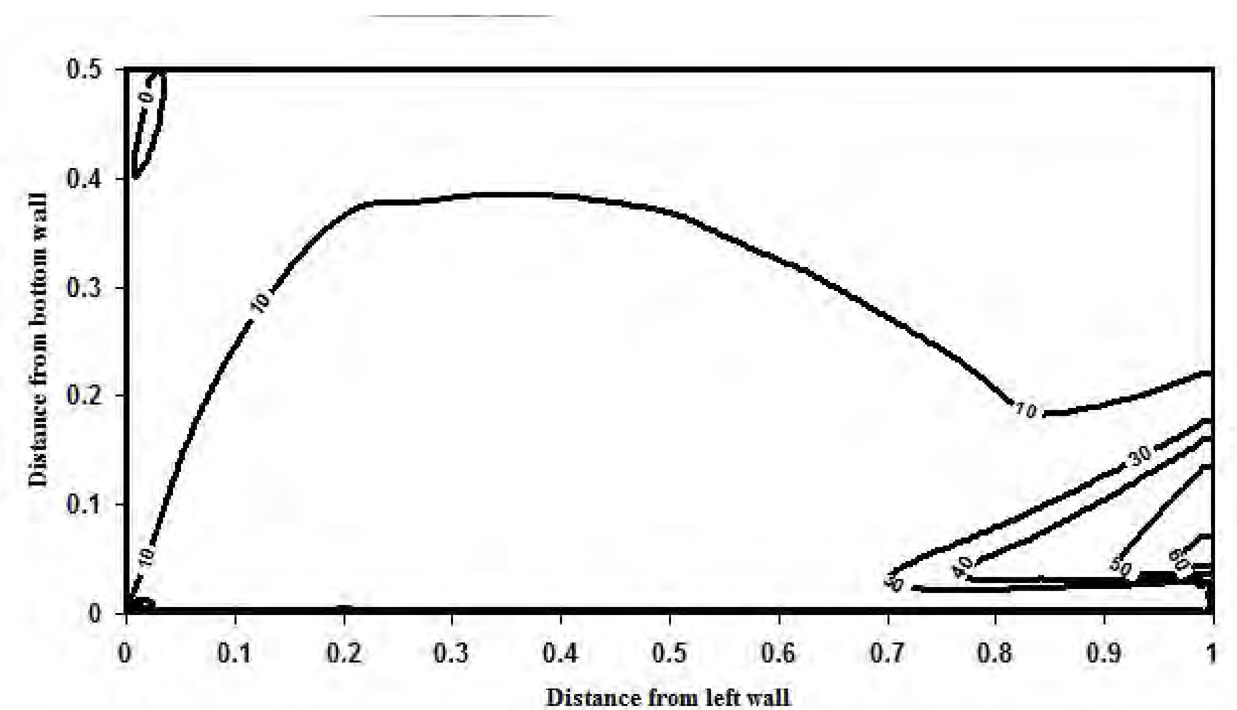

Fig. 29(a) Transverse velocity contour for forward facing step (M=1.2 & h=0.025)

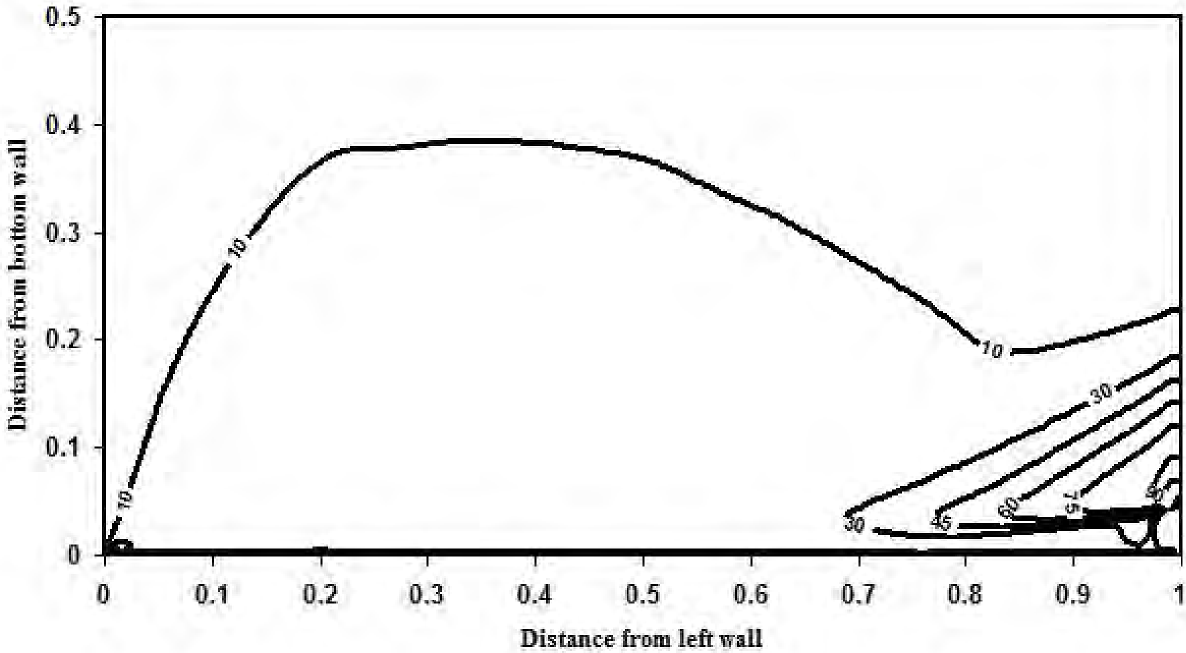

Fig. 29(b) Transverse velocity contour for forward facing step (M=1.2 & h=0.05)

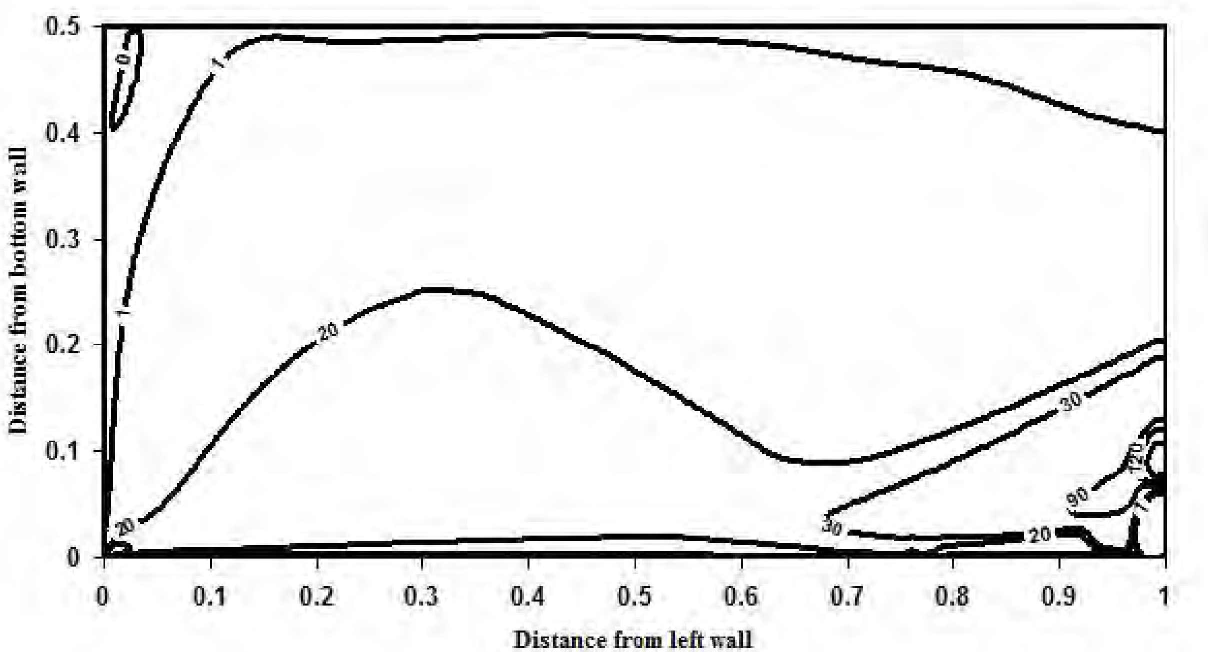

Fig. 29(c) Transverse velocity contour for forward facing step (M=1.2 & h=0.075)

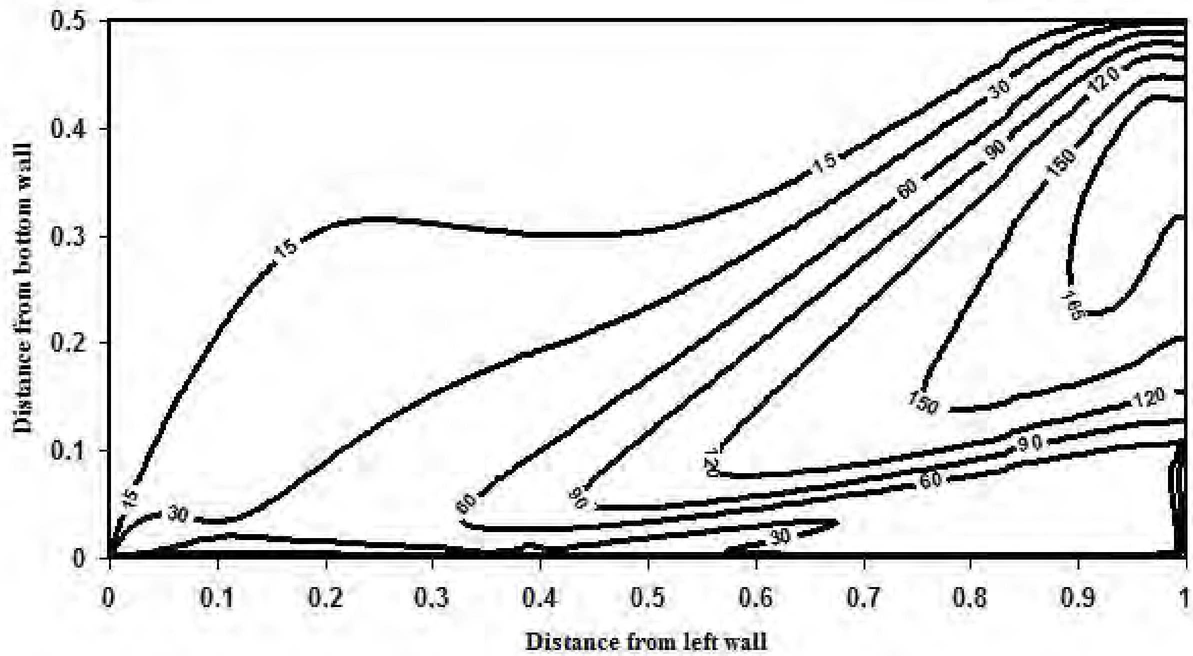

Fig. 29(d) Transverse velocity contour for forward facing step (M=1.2 & h=0.10)

3.5.3 Effect on Pressure and Temperature in the flow field

For same Mach number and step height, from the Fig. 30 it is found that the maximum pressure exerted by the incoming flow on the bottom wall is greater in forward facing step. It is about twice times greater than backward facing. In forward facing step the maximum wall pressure is found in front of the wall (the position of the formation of oblique shock) and in case of backward facing step it is found after the length of recirculation. In forward facing step higher pressure is developed at recirculation zone than backward facing step. The pattern of pressure distribution along the step height (Fig. 31) is not same. It is found that for backward facing step the pressure increases, and reaches to the maximum at about the half of the height then it falls before the corner. On the other hand, in forward facing step initially the pressure falls, then gradually increases and reaches to the maximum, then falls just before the corner.

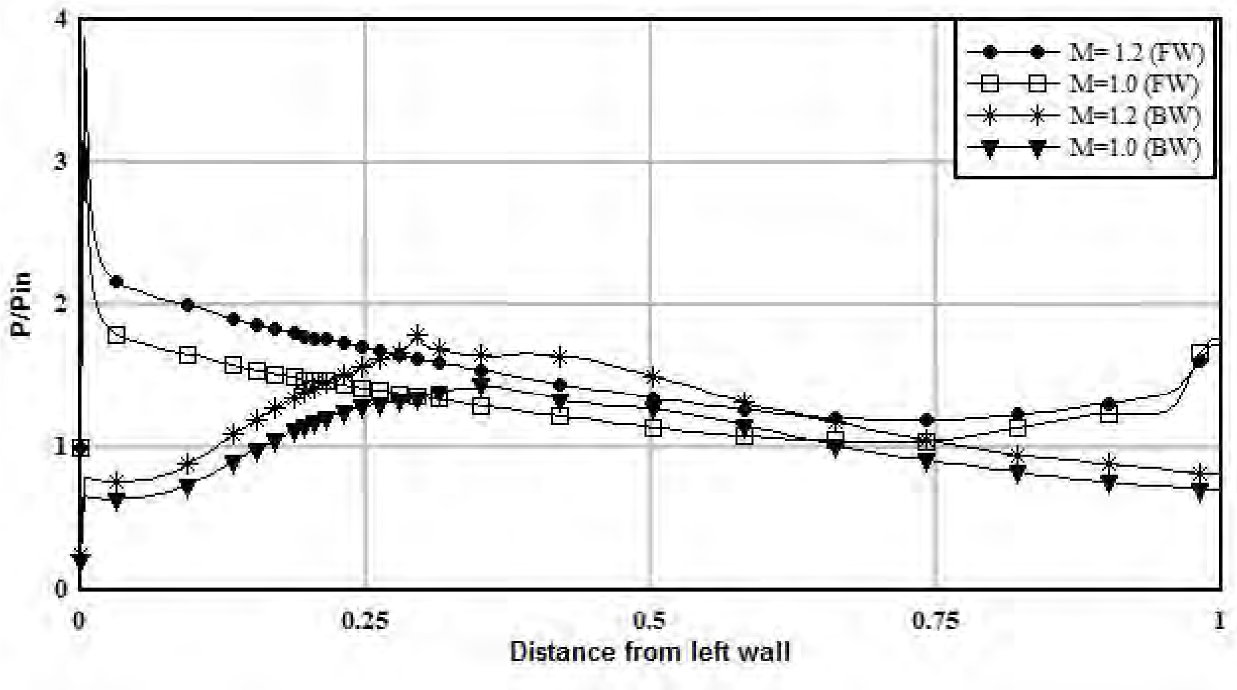

Fig. 30 Pressure distribution along the plate for backward and forward facing step (h=0.075)

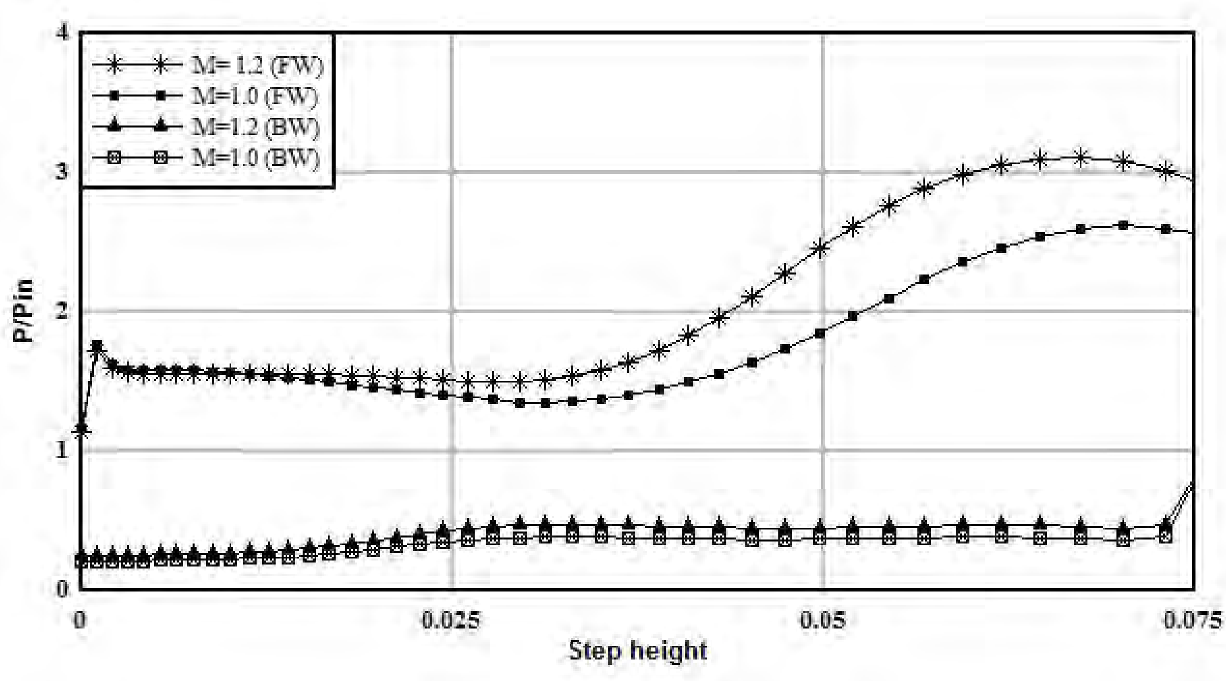

Fig. 31 Pressure distribution along the step height for backward and forward facing step (h=0.075)

In both cases (backward and forward), the maximum temperature of the flow field can be found in recirculation zone. Temperature raise in forward facing step is greater than backward facing step. The pattern of temperature along the step height is almost similar in both cases. At the bottom of the step the temperature is high then it falls continuously along the height of the step.

Nature of Recirculation length with Mach no:

From fig. 32 it is found that, for all cases, recirculation length increases linearly with the increase of step height. However, the increasing great of recirculation length for lowest Mach number. For higher Mach number the increasing rate is almost same.

This is caused by the high expansion of inlet flow for higher Mach number.

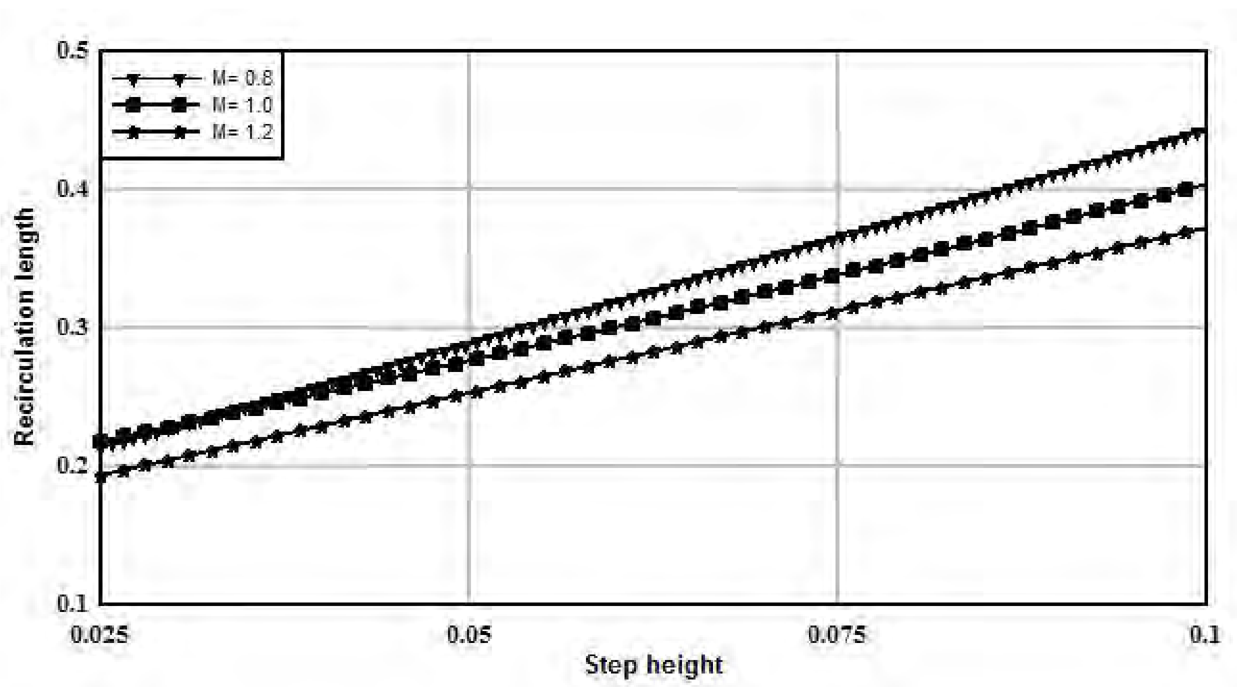

Fig. 32 Recirculation length for different Mach Numbers in case of Backward facing step

In general, for all cases the rate of recirculation is highest for lowest step height and for medium step height (0.05-0.075) increment is small and for highest step height the recirculation length decreases from the previous length. Highest recirculation length can be found in lowest Mach number and the lowest recirculation length can be found for highest Mach number. The reason of lowest recirculation length is due to compression.

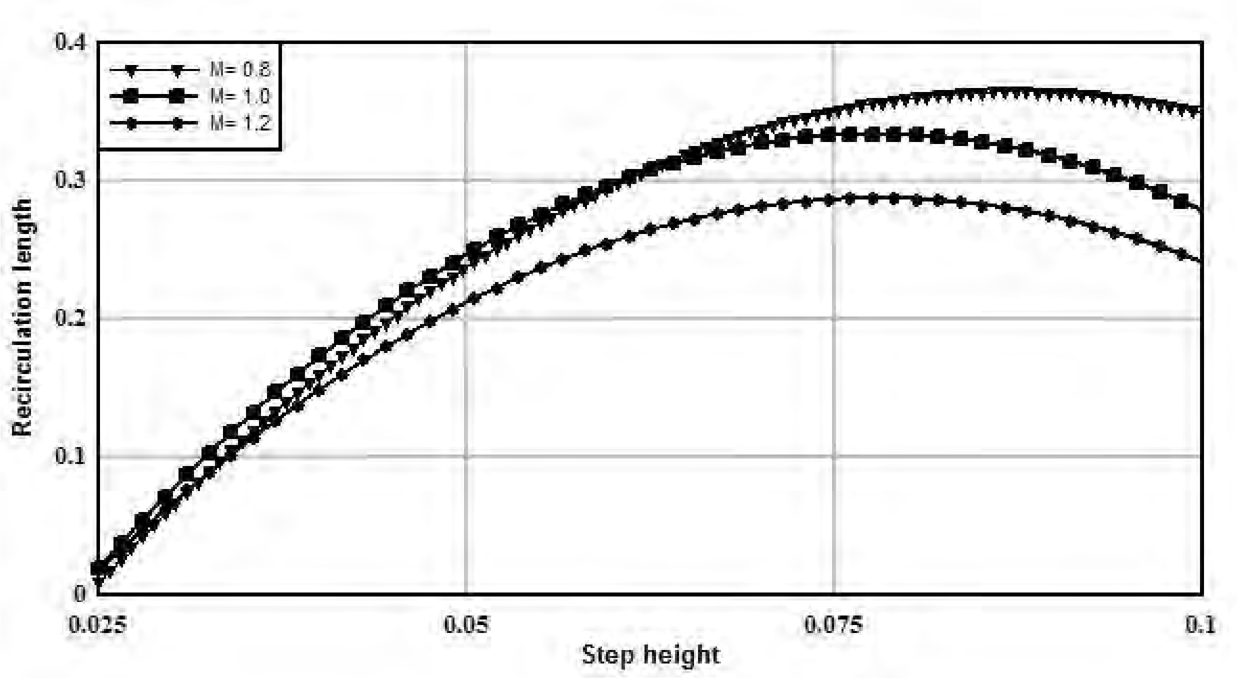

Fig. 33 Recirculation length for different Mach Numbers in case of Forward facing step

CHAPTER-IV

CONCLUSIONS

4.1 Summary of the study

A study of transonic flow over a backward facing step is carried out by using an explicit Harten-Yee Non- MUSCL Modified flux type TVD scheme. The goal of this study is to investigate the physics of flow phenomena of a high speed compressible flow over a flat plate with facing steps and the effects of step arrangement (backward and forward), Mach numbers as well as height of the step on this flow pattern.

High speed flow over a facing step occurs in many practical applications of aerodynamic/fluid dynamics. A considerable number of researchers carried out their researches, related to this topic. Corner expansion shock, reattached shock, leading edge oblique shock, detached shock, recirculation zone, interaction of shocks and their characteristics are the major findings of the flow field addressed herein.

In case of backward facing step, a corner expansion shock, emanate from the vicinity of the top of the step; reattached shock, below the expansion shock region; and interaction of this two shocks appear. A recirculation zone is found immediate behind the step. Sudden drop of pressure and temperature in expansion shock region can be observed. At the end of the recirculation, the reattachment shock follows the expansion shock where both the pressure and temperature again increase. Maximum temperature and minimum pressure of the flow field are found in recirculation zone. The maximum temperature in this region can be found around 940K.

- Corner expansion shock and the interaction of two shocks rotate clockwise, move downward and the angle of shock with main flow direction decreases with the increase of step height. The degree of expansion at expansion shock region, the width of the expansion shock and the length as well as strength of recirculation have increased, with the increase of step height. The increment rate of recirculation length increases with step height. Maximum temperature and minimum pressure of the flow field are found in recirculation zone, which is almost same for different step heights. Ac The striking pressure of the incoming stream becomes weaker, as the height increases.

- With the increase of Mach number, the corner expansion shock and the interaction position of two shocks rotate clockwise, move downward and the angle of shock with main flow direction decreases. The degree of expansion at expansion shock region, the width of the expansion shock and length of recirculation decrease, with the increase of Mach number. But the strength of recirculation increases in this condition. The striking pressure of incoming stream becomes stronger as the Mach number increases. The position of striking moves to the left.

In case of forward facing step, leading edge oblique shock appears at the start of the plate; recirculation zone in front of the step, and detached shock close to the recirculation zone. The pressure, temperature and velocity are high in oblique shock region. But the pressure, temperature and velocity is low in detached shock region. The maximum temperature of the flow field is around 1000K, which is observed in recirculation zone. The pressure along the vertical face of the step increases gradually, attains a maximum just before the tip of the step and then falls again at the tip.

- The increase of step height makes the different contour lines near the step steeper and increases the area of the detached shock zone. But the rate of increment is decreasing. The length and strength of recirculation increases with the increase of step height. But the rate of increment is decreasing. The pressure and temperature of recirculation zone increases with the increase of height. Though the pattern of pressure distribution along the vertical face of the step is similar, the increase of step height increases the pressure along the length of the step.

- With the increase of Mach number, the steepness of the band decreases. The area of detached shock, the recirculation length and strength also reduce. The pressure at oblique shock region, recirculation zone and along the step increases with the increase of Mach number. The temperature at the tip of the step increases.

In comparison between forward and backward facing step two sets of different shocks can be observed, namely corner expansion shock, reattached shock (in case of backward facing step) and leading edge oblique shock, detached shock (in case of forward facing step). The pressure and temperature is very high in oblique shock region. On the other hand, pressure and temperature is low in corner expansion shock region. In case of forward facing step, higher pressure is exerted by the incoming stream on the plate than backward facing step. Due to the flow orientation of the domain, the height and length of recirculation is longer in backward facing step than forward facing step. Weaker recirculation is formed in forward facing step. Pressure developed in front of the step is greater than the pressure developed behind the step. In backward facing step lesser temperature is developed along the step body as well as recirculation zone as compare to forward facing step. It is to be noted that the existence of flow separation caused by a sudden expansion or compression in flow geometry, such as backward-forward facing steps greatly influences the by pressure and temperature.

4.2 Recommendation for future study:

In this present investigation although some significant results are investigated, more investigation along the plate is desired. More simulations in the future are needed to extend the present parameter range, for example for different step heights, Mach numbers etc. The area of calculation domain can be increased. An experimental study is also recommended.

In this thesis, zero equation turbulence model is used. But there are some limitations of this model. This model takes no account of convection and diffusion of turbulence. A two-equation turbulence model is suggested.

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List of Publication (Related to this Thesis)

- Sarker, K., Ali, M. and Islam, Q., "A Numerical Study on the Physics of Flow Over a Flat Plate with Backward Facing Step", Accepted by 10th ICME, 20-22 December, 2013.