1. (a) When does a firm emerge as a monopolist?
   (b) Why is there no unique supply curve for the monopolist derived from his marginal cost curve? Explain graphically.
   (c) Explain the short-run equilibrium of a firm under monopoly.

2. (a) What are the assumptions of a perfectly competitive market? Explain them.
   (b) How would you derive the short-run supply curve of a firm under perfect competition?
   (c) Graphically explain the shut-down point of a firm under perfect competition.

3. (a) How would you derive the long-run average cost curve of a firm from its short-run average cost curves?
   (b) A manufacturer has a fixed cost of $120,000 and a variable cost of $20 per unit made and sold. Selling price is $50 per unit.
      (i) Find the revenue, cost and profit functions using q for number of units.
      (ii) Compute profit if 10,000 units are made and sold.
      (iii) Find the break-even quantity.
      (iv) Find the break-even dollar volume of sales.
      (v) Construct the break-even chart. Label the cost and revenue lines, the fixed cost line, and the break-even point.

4. (a) Explain producer's equilibrium.
   (b) From the following functions, calculate the amount of labour and capital that maximizes output. What is the maximum amount of output?
      \[
      Q = 500 L^{0.6} K^{0.4} \\
      3000 = 50 L + 70 K
      \]
      Here \( Q \) = Output or quantity
      \( L \) = Labour
      \( K \) = Capital
There are **FOUR** questions in this Section. Answer any **THREE**.

5. (a) Describe the reasons behind "shift of demand" and show graphically.  
(b) What is "paradox of bumper harvest"? What policies can be addressed to avoid the losses of farmers result from the bumper harvest? 
(c) Write short note on "minimum wage determination".

6. (a) Explain the basic problems that an economy has to face. How can these problems be solved? 
(b) Given the demand and supply equations: 
\[ Q = \frac{16777216}{p} \quad \text{and} \quad Q = P^\dagger \]  
Find out the equilibrium price and quantity and then show them graphically. 
(c) Write the determinants which affect the changes in supply of an individual.

7. (a) Discuss in detail price elasticity of demand, cross elasticity of demand and income elasticity of demand.  
(b) Let the price of coffee rise from Tk. 9 per hundred grams to Tk. 10 per hundred grams. As a result, the consumers' demand for tea increases from 120 hundred grams to 140 hundred grams. Find the cross elasticity of demand of tea for coffee. 
(c) Applying the knowledge of price elasticity of demand show that "the more elastic the demand, the more tax burden on a producer and more inelastic the demand, the more tax burden on a consumer".

8. (a) What is indifference curve? Mention the characteristics of an indifference curve. 
(b) How can a consumer attain his/her equilibrium with the reference to the law of equi-marginal utility?  
(c) There are two commodities 'A' and 'B' on which a consumer spends his entire income. His daily income equals Tk. 1000. He has utility function \( U = \sqrt{AB} \). The prices of 'A' and 'B' are Tk. 10 and Tk. 4 respectively. Find out the optimal quantities of 'A' and 'B'.
1. (a) Water is flowing in a trapezoidal channel at a rate of \( Q = 20 \text{ m}^3/\text{s} \). The critical depth \( y \) for such a channel must satisfy the equation

\[
1 - \frac{Q^2}{gA_c} B = 0
\]

where \( g = 9.81 \text{ m/s}^2 \), \( A_c = \text{cross sectional area (m}^2) = 3y + \frac{y^2}{2} \), \( B = \text{width of the channel at surface (m)} = 3 + y \)

(i) Solve for the critical depth using both bisection method and regula falsi method. In both methods, use initial guesses of \( X_L = 0.5 \) and \( X_U = 2.5 \), and iterate until the approximate error falls below 1% or the number of iterations exceed 10.

(ii) Which method performs better for the given stopping criteria and why?

(b) Give a graphical depiction of convergence and divergence of simple fixed-point iteration. What is the general condition for convergence of simple fixed-point iteration?

2. (a) Use least-squares regression to fit a saturation-growth-rate equation \( y = a \frac{x}{b + x} \) for the given data:

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<tr>
<th>( x )</th>
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<td>42</td>
<td>41</td>
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</tbody>
</table>

(b) Find to four places of decimal, the smallest root of the equation \( e^{-x} = \sin x \) using Newton-Raphson method.

3. (a) Solve the following equation numerically over the interval from \( x = 0 \) to 1.

\[
\frac{dy}{dx} = (1 + 2x)\sqrt{y} \quad y(0) = 1
\]

Use the classical fourth order Runge-Kutta method with \( h = 0.5 \). Also compare the results with the analytical solution.
(b) Write down the set of linear equations (in matrix form) resulting from the finite difference approximation of the following boundary value problem: (Use \( \Delta x = 1 \))

\[
\frac{d^2T}{dx^2} = 0.15 T = 0 \quad \begin{cases} T(0) = 240 \\ T(10) = 150 \end{cases}
\]

4. (a) Why do we need to use numerical computing techniques to solve differential equations?

(b) What is an initial value problem? How is it different from a boundary value problem? Given examples.

(c) "Heun's method is an improved version of Euler's method" — Comment.

(d) The differential equation governing the displacement of a body attached to a spring over time \( t \) is given by

\[
\frac{d^2y}{dt^2} + 2 \frac{dy}{dt} + 5y = 0
\]

Find the displacement \( y \) at time \( t = 1.5 \) given that \( y(0) = 2 \) and \( y'(0) = -4 \). Use Heun's method with a stepsize of 0.5.

SECTION B

There are FOUR questions in this Section. Answer any THREE.

5. (a) Write short notes on:

   (i) Pivotal Condensation and Back Substitution

   (ii) Matrix Inversion using Gauss-Jordan method.

(b) Fit a polynomial equation passing through the points provided in the following table and use it to find the interpolated value for \( x = 3.0 \).

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6. (a) Derive the general expression of \( I = \int_a^b f(x) \, dx \), using Simpson's Rule.

(b) Use the following data to get the integral between the limits of \( x = 1.6 \) and \( x = 3.8 \) using Romberg's Quadrature.

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</tr>
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</table>

Contd ........ P/3
7. (a) Derive the final expression of Gregory-Newton Interpolation formula.

(b) Solve the following system using Crout's method:

\[
\begin{align*}
2x_1 + x_2 - x_3 &= 2 \\
3x_1 - x_2 + 2x_3 &= 4 \\
2x_1 - 2x_2 + x_3 &= 6
\end{align*}
\]

8. (a) Evaluate \( I = \int_0^\pi \sin x \, dx \) for \( n = 3 \) using Gauss Quadrature technique.

(b) Experimentally observed values of deflections of a beam are shown in the following figure. Estimate slope, bending moment and shear force at all points.

Given \( E = 30 \times 10^6 \) psi; \( I = 1500 \) in\(^4\).
SECTION – A

There are FOUR questions in this Section. Answer any THREE.

1. (a) Water is flowing in a trapezoidal channel at a rate of \( Q = 20 \, \text{m}^3/\text{s} \). The critical depth \( y \) for such a channel must satisfy the equation

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T(0) &= 240 \\
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SECTION - B

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\[2x_1 + x_2 - x_3 = 2\]
\[3x_1 - x_2 + 2x_3 = 4\]
\[2x_1 - 2x_2 + x_3 = 6\]

8. (a) Evaluate \( I = \frac{\int \sin x \, dx}{\delta} \) for \( n = 3 \) using Gauss Quadrature technique.

(b) Experimentally observed values of deflections of a beam are shown in the following figure. Estimate slope, bending moment and shear force at all points.

Given \( E = 30 \times 10^6 \text{ psi} \); \( I = 1500 \text{ in}^4 \).
1. (a) Express the following periodic function in a Fourier Series.

![Periodic Function](image)

(b) Consider the driven mechanical oscillator shown in Fig. 1, governed by the differential equation of motion

\[ m\ddot{x} + kx = F(t) \]

where \( m \) and \( k \) are the mass and spring stiffness, respectively. Let \( m = 1 \) kg and \( k = 15 \) kg/sec\(^2\), and let \( F(t) \) (in newtons) be as shown in Fig. 2. Determine the response of the system using Fourier Integral. Suppose the system is initially at rest.

![Driven Oscillator](image)

2. (a) Given: \( f(x) = e^{-2x} \left( \frac{1}{6} e^{-3x} \right) \), evaluate its transform \( \hat{f}(w) \).

(b) Solve the following differential equation using Fourier Transform.

\[ u'' + \beta^2 u = H(x) e^{-ax}, \quad (a > 0) \]

Here, \( H(x) \) is Heaviside Step Function.

(c) The fuel consumption of a certain type of vehicle is approximately normal, with standard deviation 3 miles per gallon. How large a sample of vehicles is needed if we wish to be 95% confident that the population mean will be within 0.5 miles per gallon of the sample mean? Use Table 2.
3. (a) The life time $T$ of electric bulbs (e.g. number of hours they operate before they fail) has an exponential distribution with cumulative distribution function

$$F_T(t) = 1 - e^{-\frac{t}{1000}}, \quad \text{for } t \geq 0 \text{ (in hours)}$$

Suppose, you have used a bulb for 500 hours without failure. Find the probability that the bulbs will last at least 500 hours more.

Hint: Use $P(A \mid B) = P(A \cap B) / P(B)$, with appropriately defined $A$ and $B$.

Also, $F_T(t) = P(T \leq t) = 1 - P[T > t]$.

(b) You went to see a doctor for high fever. The doctor selects you at random to have blood tests for 'Bird Flu', which is currently suspected to affect 1 in 10,000 people in Bangladesh. The test is 99% accurate, in the sense that probability of false positive is 1%. The probability of false negative is zero. You test positive. What is the probability that you have Bird Flu?

Now imagine you traveled to country 'A' recently. It is known that 1 in 200 people who visited country 'A' came back with Bird Flu. Given the same test characteristics as above, what should the revised estimate be for the probability you have the disease?

Hint: Use Bayes' principle with appropriately defined events.

4. (a) A rope with 100 strands supports a weight of 2100 pounds. If the breaking strength of each strand is random, with mean equal to 20 pounds, and standard deviation 4 pounds, and if the breaking strength of the rope is the sum of the independent breaking strengths of its strands, determine the probability that the rope will not fail under the load. (Assume there is no individual strand breakage before rope failure.) Table 2.

(b) According to new emission control requirements, mean emission for vehicle engines should be below 19 ppm. We have tested 10 engines and found the following values: 17.6, 16.2, 22.5, 20.5, 16.3, 19.4, 15.9, 17.5, 12.7, and 13.9.

Does the data supply sufficient evidence to conclude that this type of engine meets new requirements? Assume we are willing to risk a Type I error with probability 0.05. Use Table 3.

5. (a) Distinguish between ordinary differential equation and partial differential equation.

(b) A glider flies after jumping from an airplane that was cruising at a velocity, $v$. Explain the phenomena and identify the systems affecting this flight. In this course write down the differential equations for this attained acceleration. Comment on the equation and the order of the equation.
6. (a) A tank in Figure 3 contains 1000 gal. of water in which 200 lb of salt are dissolved. Fifty gallons of brine each containing \((1 + \cos t)\) lb of dissolved salt run into the tank per minute. The mixture, kept uniform by stirring, runs out at the same rate. Find out the amount of salt \(y(t)\) in the tank at any time, \(t\).

In this process write down the standard solution steps that you will logically consider.

(b) If the occurrences of earthquakes and high winds are unrelated, and if, at a particular location, probability of a "high" wind occurring though out any single minute is \(10^{-5}\) and probability of a "moderate" earthquake during any single minute is \(10^{-8}\):

(i) Find the probability of the joint occurrence of the two events during any minute. Building codes do not require the engineer to design the building for the combined effect of these loads. Is this reasonable?

(ii) Find the probability of occurrence of one or the other or both during any minute. For rare events, i. e. events with small probabilities of occurrence, the engineer frequently assumes: \(P(A \cup B) = P(A) + P(B)\). Give your comments.

(iii) If the events in successive minutes are mutually independent, what is the probability that there will be no moderate earthquake in a year in this location? What will be the probability in 10 years?

7. (a) What are the cases when you will consider "Power Series Method" in solving a differential equation?

(b) Apply appropriate method to solve the following differential equation.

\[(1 + x)y' = y\]

Write down the solution.

(c) What is "Remainder"?

8. (a) Write down the conditions that are necessary to confirm the existence of power series solution of a differential equation?

(b) Write a short note on "Orthogonality" of Legendre Polynomials.
Contd ... Q. No. 8

(c) A set of earthquake occurrence times (in years since the beginning of recording at $t = 0$) is given in Table 1. The mean recurrence rate ($\lambda$) may be estimated as total number of events divided by the observation period (in this case, $\lambda = 50/101.74$).

(i) Construct a histogram of the earthquake interarrival time and compare it with the exponential PDF with parameter, $\lambda$.

(ii) Find the empirical distribution of $N$ = number of earthquakes in $T = 4$ years and compare it with Poisson Distribution with $\lambda T$.

(iii) Would you reasonably conclude that the occurrence of earthquake follows a Poisson Point Process?

(d) The delay time $x$ (in minutes) of a vehicle waiting at a traffic signal has the following density function.

$$
    f_x(x) = \begin{cases} 
    7e^{-2x}, & \text{for } x \geq 0 \\
    0, & \text{elsewhere}
    \end{cases}
$$

Determine the average delay time.

---

Table (1)

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**Two-Tailed Probabilities**

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**PERCENTAGE POINTS OF THE T DISTRIBUTION**

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## Appendix

### Table of Fourier Transforms

<table>
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<th>( f(x) )</th>
<th>( \tilde{f}(x) = \int_{-\infty}^{\infty} f(x) e^{-iwx} , dx )</th>
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<tr>
<td>[ f(x) = \frac{1}{a^2 + x^2} \quad (a &gt; 0) ]</td>
<td>[ \tilde{f}(x) = \frac{e^{-i\gamma}}{a} ]</td>
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<tr>
<td>[ f(x) = H(x) e^{-ax} \quad (Re , a &gt; 0) ]</td>
<td>[ \tilde{f}(x) = \frac{1}{a + iw} ]</td>
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<td>[ f(x) = H(-x) e^{ax} \quad (Re , a &gt; 0) ]</td>
<td>[ \tilde{f}(x) = \frac{1}{a - iw} ]</td>
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<td>[ f(x) = e^{-ax} \quad (a &gt; 0) ]</td>
<td>[ \tilde{f}(x) = \frac{2a}{\sqrt{\pi} e^{-ax^2}} ]</td>
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<tr>
<td>[ f(x) = e^{-\frac{1}{2a^2}e^{-(ax)^2}} \quad (a &gt; 0) ]</td>
<td>[ \tilde{f}(x) = e^{-ax^2} ]</td>
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<td>[ f(x) = H(x) e^{-ax} \quad (Re , a &gt; 0) ]</td>
<td>[ \tilde{f}(x) = \sqrt{\frac{2a}{\pi}} e^{-ax} ]</td>
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<td>[ f(x) = e^{-i\alpha \sqrt{\pi} \sin \left( \frac{\alpha}{\sqrt{2}}</td>
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<td>[ f(x) = H(a + \alpha) - H(\alpha - a) ]</td>
<td>[ \tilde{f}(x) = e^{-i\omega \alpha} ]</td>
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<td>[ f(x) = \delta(x - a) ]</td>
<td>[ \tilde{f}(x) = \frac{1}{a} \cdot e^{i\omega a} f \left( \frac{\omega}{a} \right) ]</td>
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<td>[ f(x) = f(x + b) \quad (a &gt; 0) ]</td>
<td>[ \tilde{f}(x) = \tilde{f}(\omega + b) ]</td>
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<tr>
<td>[ f(x) = \frac{1}{a} e^{-\frac{\pi}{a}} f \left( \frac{x}{a} \right) \quad (a &gt; 0, \text{ real}) ]</td>
<td>[ \tilde{f}(x) = \frac{1}{2a} \left[ f \left( \frac{\omega - \epsilon}{a} \right) + f \left( \frac{\omega + \epsilon}{a} \right) \right] ]</td>
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<td>[ f(x) = \cos \alpha x \quad (a &gt; 0, \text{ real}) ]</td>
<td>[ \tilde{f}(x) = \frac{1}{2a} \left[ f \left( \frac{\omega - \epsilon}{a} \right) - f \left( \frac{\omega + \epsilon}{a} \right) \right] ]</td>
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<td>[ f(x) = \sin \alpha x \quad (a &gt; 0, \text{ real}) ]</td>
<td>[ \tilde{f}(x) = \frac{2 \sin \alpha \omega}{\omega} ]</td>
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<td>[ f(x) = f(x + c) + f(x - c) \quad (\text{ real}) ]</td>
<td>[ \tilde{f}(x) = 2 \tilde{f}(\omega) \sin \omega c ]</td>
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<tr>
<td>[ f(x) = f(x + c) - f(x - c) \quad (\text{ real}) ]</td>
<td>[ \tilde{f}(x) = 2 \tilde{f}(\omega) \sin \omega c ]</td>
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<tr>
<td>[ f(x) = x^n f(x) \quad (n = 1, 2, \ldots) ]</td>
<td>[ \tilde{f}(x) = \frac{d^n}{dx^n} \tilde{f}(\omega) ]</td>
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<td>[ f^{(n)}(x) ]</td>
<td>[ (\omega)^n \tilde{f}(\omega) ]</td>
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<td>[ f(x) = \int_{-\infty}^{\infty} g(x) , dx ]</td>
<td>[ \tilde{f}(\omega) = \frac{1}{i\omega} \tilde{g}(\omega) ]</td>
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### Linearity of Transform and Inverse:

\[ \alpha f(x) + \beta g(x) \Rightarrow \alpha \tilde{f}(\omega) + \beta \tilde{g}(\omega) \]

### Transform of Derivative:

\[ f^{(n)}(x) \Rightarrow (\omega)^n \tilde{f}(\omega) \]

### Transform of Integral:

\[ f(x) = \int_{-\infty}^{\infty} g(\xi) \, d\xi \]  

where \( f(x) \to 0 \) as \( x \to \infty \)

### Fourier Convolution Theorem:

\[ (f * g)(x) = \int_{-\infty}^{\infty} f(x - \xi) g(\xi) \, d\xi \]  

\[ \tilde{f}(\omega) \tilde{g}(\omega) \]
1. (a) Derive the expression for friction loss with laminar flow.
(b) Determine the diameter of steel pipe (\(e = 0.045 \text{ mm}\)) to carry 30 l/s of water if the permissible head loss per meter of the pipe length is 0.05 m. Use moody diagram. Take \(\nu = 1 \times 10^{-6} \text{ m}^2/\text{s}\).
(c) Name the different types of losses in pipe flow. A pipe 60 m long and 15 cm in diameter is connected to a water tank at one end and flows freely into the atmosphere at the other end. The height of the water level in the tank is 2.8 m above the center of the pipe. The pipe is horizontal with \(f = 0.04\). Determine the discharge through the pipe.

2. (a) What is viscosity of fluid? Derive the equation of viscosity of fluid. In this respect differentiate between Newtonian and non-Newtonian, ideal plastic and elastic solid with examples. Support your answer with proper figures.
(b) A space of 25 mm width between two large plane surfaces is filled with SAE 30 western lubricating oil at 25°C (\(\mu = 2.1 \times 10^{-1} \text{ N.s/m}^2\)). What force is required to drag a very thin plate of 0.35 m² area between the surfaces at a speed of 0.1 m/s.
   (i) If this plate is equally spaced between the two surfaces?
   (ii) If it is at a distance of 8.5 mm from one surface?
(c) Differentiate between
   (i) Adhesion and cohesion.
   (ii) Laminar and turbulent flow.
   (iii) Ideal and real fluid.
   (iv) Pathline and streamline.
   (v) Steady and unsteady flow.

3. (a) Calculate the rate of flow of water from the reservoir A to B for the system shown in Figure 1. Pipe dimensions are as follows:
   - \(L_1 = 400 \text{ m}\), \(D_1 = 600 \text{ mm}\), \(e_1 = 2 \text{ mm}\)
   - \(L_2 = 300 \text{ m}\), \(D_2 = 1000 \text{ mm}\), \(e_2 = 0.6 \text{ mm}\)
Consider minor losses, and
\(\nu = 2 \times 10^{-6} \text{ m}^2/\text{s}\). Use Moody diagram.
(b) Initial distribution of flows through a pipe network is shown in Figure 2. Taking \( n = 2 \) for all pipes, obtain flows in each pipe after applying correction twice. Discharge is in l/s.  

(c) Write short notes on

(i) Viscous sublayer
(ii) Uniform and non-uniform flow
(iii) Compressible and in-compressible fluid
(iv) Solid and fluid.

4. (a) For the pipes connected in parallel as shown in Figure 3, the pipe dimensions and friction factors are as follows:

\[
\begin{align*}
L_1 &= 900 \text{ m}, & D_1 &= 0.3 \text{ m}, & f_1 &= 0.021 \\
L_2 &= 600 \text{ m}, & D_2 &= 0.2 \text{ m}, & f_2 &= 0.018 \\
L_3 &= 1200 \text{ m}, & D_3 &= 0.4 \text{ m}, & f_3 &= 0.019
\end{align*}
\]

For a total discharge of 0.34 m\(^3\)/s, determine the flow through each pipe and head loss from A to B.

(b) The head loss in 60 m of 15 cm diameter pipe is known to be 8 m when oil \((S = 0.90)\) of viscosity 0.04 N.s/m\(^2\) flows at 0.06 m\(^3\)/s. Determine the centerline velocity, the shear stress at the wall of the pipe, and the velocity at 5 cm from the centerline.

(c) Derive the expression for steady compressible flow within fixed boundaries. A compressible fluid \((\gamma = 6.3 \text{ N/m}^3)\) flows at 15 l/s through a circular pipe (section A) into a conically converging nozzle as shown in Figure 4. At section B, some distance apart the fluid weighs 10 N/m\(^3\). Determine the average velocity of flow at section A and B.

SECTION - B

There are FOUR questions in this Section. Answer any THREE.

Assume any reasonable value, if missing. The symbols have their usual meanings.

5. (a) Derive the general equation which gives the variation of pressure in a static fluid in vertical direction.

(b) Suppose the atmospheric pressure is 91 kPa, abs. Calculate the vapor pressure of the liquid and the gage reading as shown in Figure 5.

(c) A closed tank contains immiscible fluids as shown in Figure 6. For a gage reading of -17.30 kPa at A, determine the (i) elevation of the liquid surface in the open piezometer columns B and C and (ii) manometer reading. Neglect the weight of the air.
6. (a) Define buoyancy. Write down the criteria for stability of a floating body with a neat sketch.
(2+6=8)

(b) The tank shown in Figure 7 is 50 cm wide perpendicular to the plane of paper. Compute the hydrostatic force on lower panel BC and on upper panel AD.
(12)

(c) A 2 m diameter cylinder supported as shown in Figure 8 retains water on one side. The length of the cylinder is 3 m perpendicular to the plane of paper. If the cylinder weighs 150 kN, make calculations for the vertical reaction at A and the horizontal reaction at B. Ignore the frictional effects.
(15)

7. (a) State Bernoulli's equation and mention its limitations.
(2+3=5)

(b) A pipe line of length L connects two reservoirs A and B. Draw the qualitative HGL and EL for the following cases:
(i) The diameter for the 1st L/2 pipe line is D whereas that for the remaining portion is 2 D, i.e., a sudden expansion.
(ii) A gradual converging pipe for 1st L/2 length followed by gradual diverging pipe for the remaining portion.
(6+6=12)

(c) A closed tank is partly filled with water and the air space above it is under pressure. A 5 cm hose connected to the tank discharges water to the atmosphere on to the roof of a building 3.0 m above the level of water in the tank as shown in Figure 9. If the frictional losses are 1.2 m of water, what air pressure must be maintained in the tank to deliver 20 l/s to the roof?
(8)

(d) A liquid (S = 0.86) with a vapor pressure of 26 kN/m², abs flows through the horizontal constriction as shown in Figure 10. Atmospheric pressure is 70 cm mercury. Find the maximum theoretical flow rate (i.e. at what discharge does cavitation occur?)
(10)

8. (a) Explain why a momentum correction factor is introduced.
(4)

(b) A curved pipe section of length 12.0 m is attached to the straight pipe section as shown in Figure 11. Determine the resultant force on the curved pipe and find the horizontal component of the jet reaction. Also show the pressure diagram of the fluid system. Required data is given in the figure. Assume an ideal fluid with \( \gamma = 8.64 \text{ kN/m}^3 \).
(14)

(c) A reaction turbine has \( r_1 = 1.5 \text{ m}, r_2 = 1.05 \text{ m}, \beta_1 = 60^\circ, \beta_2 = 140^\circ \) and a thickness of 0.3 m parallel to the axis of rotation. With a guide vane angle of 15° and a flow rate of 15 m³/s, calculate the required speed of the runner for smooth flow at inlet. For this condition also calculate:
(i) the torque exerted on the runner
(ii) the power developed
(iii) the energy extracted from each Newton of fluid
(12)

(d) What are the relations between absolute and relative velocities?
(5)

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Contd... P/4
Figure 1 for Q: 3(a)

Figure 2 for Q: 3(b)

Figure 3 for Q: 4(a)

Figure 4 for Q: 4(c)

Contd... P/5
Fig: 5 for Q. No. 5 (a)

Fig: 5 for Q. No. 5 (b)

Fig: 7 for Q. No. 6 (b)

Fig: 8 for Q. No. 6 (c)

Fig: 9 for Q. No. 7 (c)

Fig: 10 for Q. No. 7 (d)
Figure 1. Friction factor for pipes (Moody diagram).