GLOBALLY CONVERGENT VARIABLE TIME-STEP LEVEL SET METHOD FOR FAST AND ROBUST IMAGE SEGMENTATION

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By

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Certification

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It is hereby declared that this thesis or any part of it has not been submitted elsewhere for the award of any degree or diploma and that all sources are acknowledged.

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To my beloved Family

Acknowledgment

It‟s my obligation to reveal the name of all who contributed in many ways to complete my thesis on the selected topic entitled “GLOBALLY CONVERGENT VARIABLE TIME-STEP LEVEL SET METHOD FOR FAST AND ROBUST IMAGE SEGMENTATION”

At first, I am grateful to the Allah for the good health and wellbeing that were necessary to complete this thesis.

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Abstract

Segmentation is an image processing technique that separates foreground object from the background of image. It transforms low-level image data to high-level knowledge and makes many advanced image analysis and understanding tasks to be possible. Image segmentation is a persistent hot research field for a long time and different mechanisms have been proposed to optimize the segmentation process. Among them, active contour method (ACM) is one of the most advanced and widely used methods. Level set is an implicit application of ACM which represents the evolving contours as the zero level set of a higher dimensional function, thus make it possible to handle the numerical estimation with curves and surfaces without using parametric representation. In this way, the level set method is able to represent complex topology and their changes in an efficient manner. Edge-based and region-base approaches are two major ways of realizing level set method in image segmentation. A critical factor that controls the speed of convergence of both edge and region based methods is the time-step parameter. The selection of a larger time-step speeds up the convergence rate but endanger the numerical stability of the algorithm. A smaller time step slows the convergence rate and increases the computational time. In this work, we propose a globally convergent level set method using multiple optimization criteria, such as, edge, region, and neighborhood and formulate a variable time-step parameter for the level set method that maximizes the between-class variance among the foreground and background region in each iteration and thereby ensures numerical stability as well as fast convergence speed. Consequently the computational complexity of the level set method is reduced. The performance of the proposed segmentation technique is tested on synthetic, natural and medical images and is compared to existing state-of-the-art methods. The segmentation results of different techniques is numerically evaluated using Dice criterion, PSNR, Hausdorff distance, and mean sum of square distance (MSSD) with a reference.

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List of Abbreviations

CV	Chan-Vese
DRLSE	Distance Regularized Level Set Evolution
GAC	Geodesic Active Contour
LSE	Level Set Estimation
LSF	Level Set Function
LSM	Level Set Method
OCR	Optical Character Recognition
PDE	Partial Differential Equation

Chapter 1

1 Introduction

A digital image, in its raw form, is essentially an array of numbers that represent brightness and color. The human visual system is naturally capable to segment and identify objects in images. However, Modern medical imaging models generate larger and larger images which simply cannot be examined manually. Moreover performing the same task using a computerized technique is not a trivial task. Therefore it drives the development of more efficient and robust image analysis methods for the problems encountered in medical images. In this thesis, we explore the automatic image segmentation problem, by utilizing various image features such as intensity and edge, and present its applications in synthetic, real and medical images.

1.1 Motivation

Image processing has a great potential for major improvements in science and medicine. Today, it has emerged into one of the most important fields as a result of the rapid and continuing growth of computerized image visualization. The task of image processing is to effectively process and analyze the images in order to effectively extract, quantify, and interpret its information content. Image segmentation is an important task in image analysis, and it is essential in most image processing applications, particularly those related to pattern classification, e.g. medical imaging, remote sensing, security surveillance, military target detection[1-3].

Image segmentation is the step in which computer tries to separate objects from the image background and from each other. It is a process that partitions the image pixels into meaningful groups so that we can achieve a compact representation of the image [4]. Image processing allows the researchers to perform image analysis in order to extract meaningful information from images such as finding shapes, counting objects and measuring object parameters. The aim of this thesis is to develop automatic segmentation methods for various imaging applications. The motivation for this work is to increase patient safety by providing better and more precise data for medical decisions as well as to provide accurate information for other image segmentation application like security surveillance, military target detection etc.

1.1 Mathematical Definition of Image Segmentation

The function of segmentation is to partition an image into its constituent and disjoint sub-regions, which are uniform according to their properties, e.g. intensity, color, and texture. Let R represent the entire spatial region occupied by an image [1]. Any segmentation technique that divides R into n sub-regions, R_1, R_2,.....,R_n, must satisfy the following conditions:

(i) (a) $\bigcup_{i=1}^{n} R_i = R$

(b) R_i is a connected set, i=1,2,......., n.

(c)$R_i \cap R_j = \varnothing$ for all i and j, $i \neq j$.

(d) $Q(R_i)$=TRUE for i =1,2,......., n.

(e) $Q(R_i \bigcup R_j)$=FALSE for any adjacent regions R_i and R_j.

Here, $Q(R_k)$ is a logical predicate defined over the points in set R_k), and Ø is the null set. The symbol ∪ and ∩ represent set union and intersection respectively. Two regions R_i and R_j are said to be adjacent if their uniforms a connected set.

Condition (a) indicates that the segmentation must be complete; that is, every pixel must be in a region. Condition (b) requires that points in a region be connected in some predefined sense [4]. Condition (c) indicates that the, regions must be disjoint. Condition (d) deals with the properties that must be satisfied by the pixels in a segmented region; for example, $Q(R_i)$ = TRUE if all pixels in R_i, have the same intensity level. Finally, condition (e) indicates that two adjacent regions R_i, and R_j, must be different in the sense of predicate Q [4].

Therefore the fundamental problem in segmentation is to partition an image into regions that satisfy the preceding conditions. Segmentation algorithms for monochrome images generally are based on one of two basic categories dealing with properties of intensity values: discontinuity and" similarity. The first category is that boundaries of regions are sufficiently different from each other and from the background to allow boundary detection based on local discontinuities in intensity. Edge-based segmentation is the principal approach used in this category. Region-based segmentation approaches in the second category are based on partitioning an image into regions that are similar according to a set of predefined criteria [4].

1.2 Level Set Segmentation

Level Set Methods (LSM) are a conceptual framework for using level sets as a tool for numerical analysis of surfaces and shapes. The advantage of the level set model is that one can perform numerical computations involving curves and surfaces on a fixed Cartesian grid without having to parameterize objects [5]. It represent the evolving contours as the zero level set of a higher dimensional function, thus make it possible to handle the numerical estimation with curves and surfaces easily without using parametric representation.

The level set method makes it very easy to follow shapes that change topology. The level set method amounts to representing a closed curve Γ (such as the shape boundary) using an auxiliary function φ, called the level set function. Γ is represented as the zero level set of φ by

$$\Gamma = \{(x, y) \mid \varphi (x, y) = 0\}, \tag{1.1}$$

The level set method manipulates Γ implicitly, through the function φ. This function φ is assumed to take positive values inside the region enclosed by the curve Γ and negative values outside [5]. Figure 1.1 shows the evolution of curve of LSM

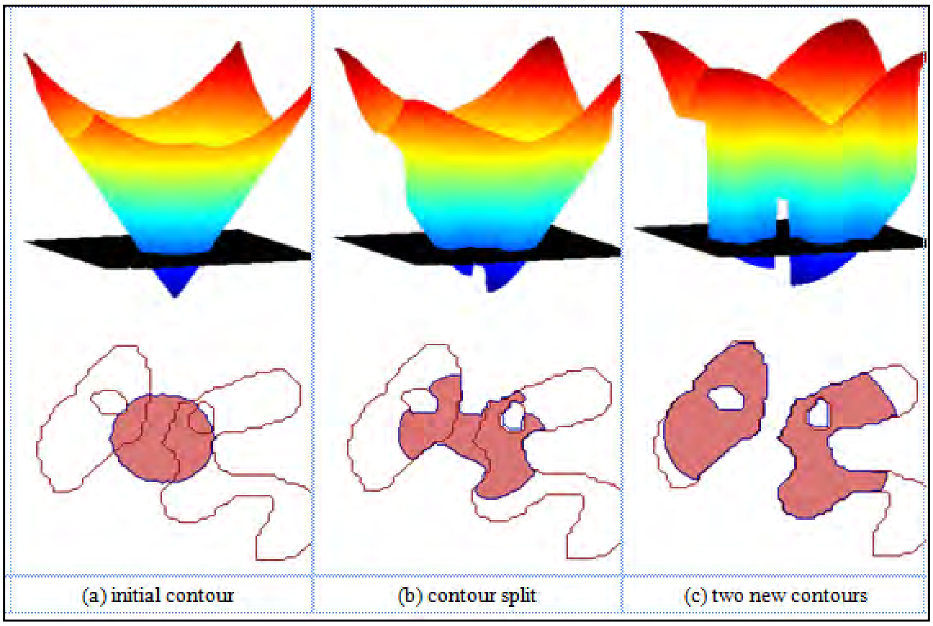


Figure 1.1: The evolution of curve of LSM

From figure 1.1(a) shows the initial contour in a circle shape and the forces applied to it is positive (Γ =1) inside a form, and it is negative (Γ =-1) outside it. Figure 1.1(b) shows how the front split between the two regions. Here, the surfaces Φ also shown to explain how this split is formed. Finally figure 1.1(c) shows the segmented image with two new contours.

1.3 Advantage of Level Set Segmentation

- It work by embedding the propagating front as the zero level set of a higher dimensional
- The resulting techniques are able to handle sharp corners and curves in the propagating solution, as well as topological changes.

- The numerical implementation can be easily adapted to solve any dimensional problems.
- The areas inside and outside of an active contour can be easily determined through level set methods.
- By employing fast narrow band adaptive techniques, the computational labor is the same as other methods.
- It has the advantages of increased accuracy and robust modeling.

1.4 Challenges on Segmentation

Edge based [10-12] and region based [13-16] are two major ways of application of level set in image segmentation. The edge-based methods are designed to pick up the gradients of contour and stop functioning at the object boundary. However edge based method is often criticized to be affected by noise and failure to differentiate boundaries when edge is not prominent. The region based method compute statistical information of inside and outside the contour in order to control its evolution and therefore its application is not affected by noise or in presence of weak or without edge. The Chan-Vase (CV) method which is based on the Mumford–Shah segmentation technique is popular region based approach [17]. Though, its limitations include of being slow and complex in computation. Therefore many hybrid methods combing these two approaches have been tried [18-20]. Most of these hybrids targeted to improve the performance of segmentation but to determine a particular weighted function for both is difficult.

1.5 Contribution of the Thesis

- Development of a novel cost function using both intensity and edge features
- Formulation of variable time-step function
- Narrowband implementation

We propose a noble method of variable time step which aim at optimization of computing and image processing. Therefore a variable time step level set method is proposed which can reduce the iteration so that segmentation become fast and can optimize all the criteria to improve the image segmentation. The proposed level set method will be developed by combining and optimizing different criteria of image segmentation. The region based criteria will attempt to minimize the variance of intensities in the foreground and background region. It will automatically determine the evolution direction of curve so that segmentation result is independent of the initial position of curve. The edge based criteria will pick up the gradient information in the image and will adaptively control the convergence speed. As a result, the curve will evolve with lower speed at boundaries and higher speed in uniform region in order to avoid the boundary leakage. A variable time-step function will be proposed that maximizes the between class variance among the foreground and background region in each iteration. In our proposed method, the level set function will be updated in the neighborhood of the evolving curve and thereby it will reduce the computational complexity.

1.6 Organization of the Thesis

In Chapter 2: The overview of image segmentation methods: This chapter discusses about the basic of segmentation techniques and related work in the area of segmentation applications. In this chapter, all the segmentation techniques are described briefly.

In Chapter 3: Mathematical Formulation of Level set method: In this Chapter major image segmentation algorithms are examined and mathematical formulas are explained in details. Traditional segmentation methods and other existing segmentation algorithms that have been applied to the images are reviewed in Chapter 3. The problems associated applying these methods to images are also discussed. The experiments show that these segmentation algorithms are not good enough for the goals of our application.

In Chapter 4: A new segmentation technique is proposed in this chapter, "globally convergent variable time-step level set method". New algorithm is described in details. How this algorithm makes the program fast and hence reduces computational complexity has been described in this chapter. The characteristic of the algorithm along with advantage of using it has been described in this chapter.

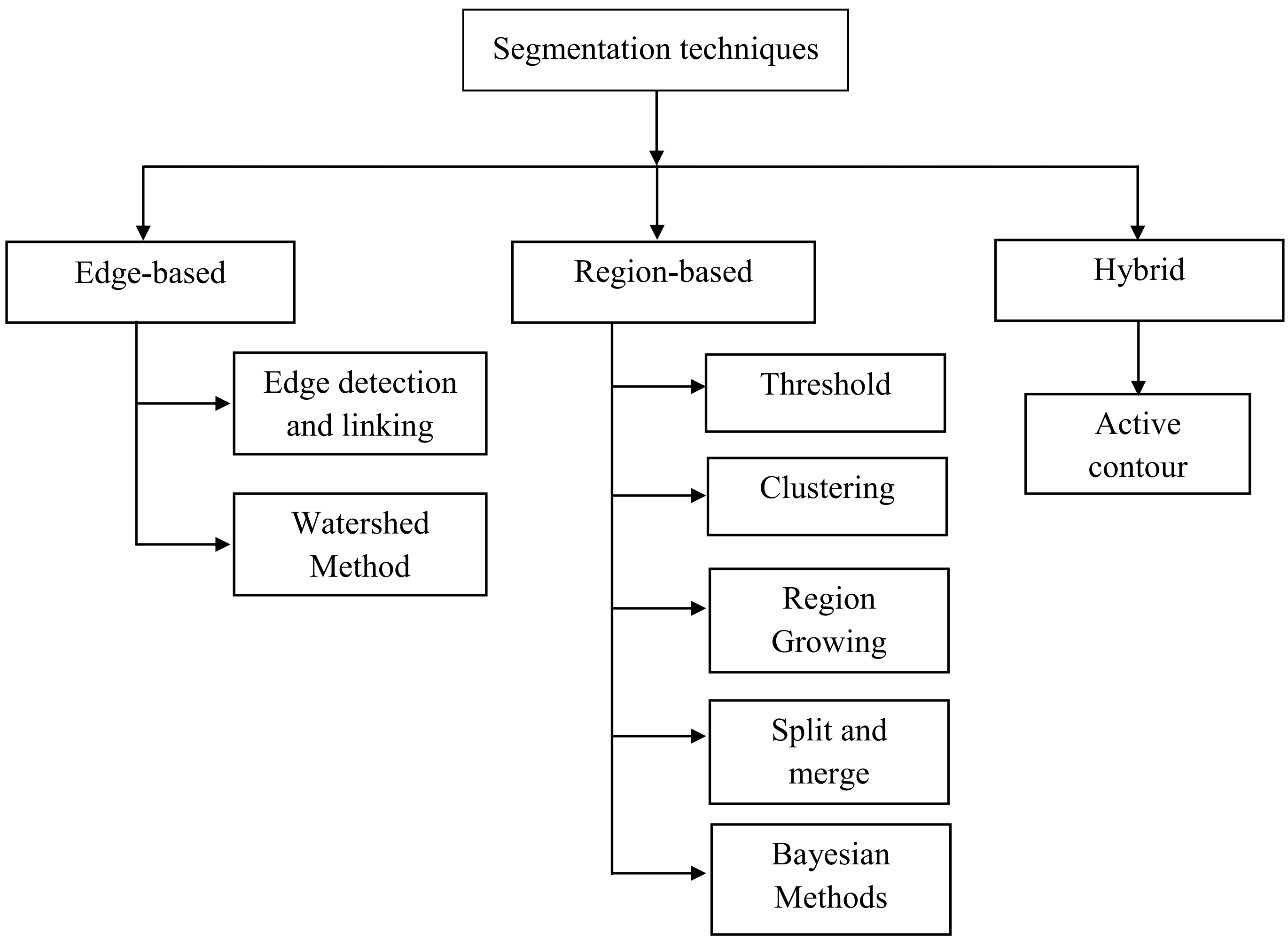
In Chapter 5: Segmentation has been done for different types of images. The entire segmented images have been shown and the results are analyzed in this chapter. Also all the experimental criteria are verified in this chapter. Segmented images are also compared with other existing method including Chan-Vase (CV), DRLSE (Distance Regularized Level Set Evolution) and Level set estimation and bias field correction. Finally, some concluding remarks are stated on the performance of image segmentation algorithm based on the numerical evaluation.

Chapter 2

2 Overview of Image Segmentation Methods

There are three main approaches in image segmentation: edge-based, region-based and hybrid methods [10-16, 22]. Edge-based segmentation partitions an image based on discontinuities among sub-regions, while region-based segmentation does the same function based on the uniformity of a desired property within a sub-region. The hybrid methods utilize both edge and region features to segment images. In this chapter, we briefly discuss existing image segmentation technologies.

2.1 Classification of Segmentation Techniques



2.2 Edge-based segmentation

Edge detection is the most common approach for detecting meaningful discontinuities in gray level. About the edge detection, we discuss approaches for implementing first- and second-order digital derivatives for the detection if edges present in an image as shown in figure 2.1 [4].

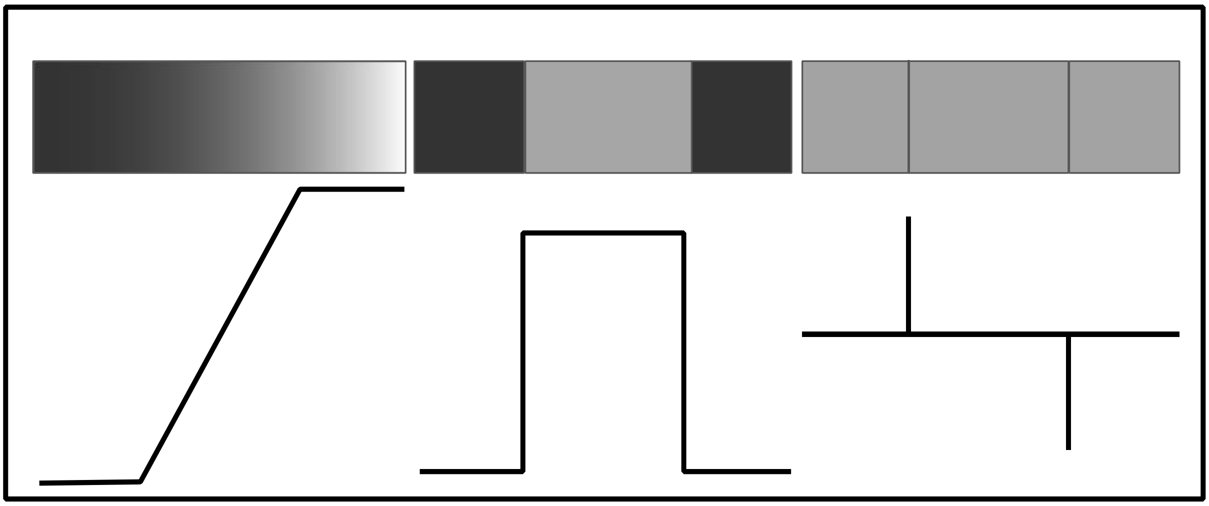


Figure 2.1: Gray-level profile First derivative Second derivative

2.2.1 Edge-detection and edge-linking

Edge-based segmentation looks for discontinuities in the intensity of an image. An edge can be defined as the boundary between two regions with relatively distinct properties. The assumption of edge-based segmentation is that every sub-region in an image is sufficiently uniform so that the transition between two sub-regions can be determined on the basis of discontinuities alone. When this assumption is not valid, region-based segmentation, discussed in the next subsection, usually provides more reasonable segmentation results [4].

Edge detection methods use the first or second derivative to find image areas where one brightness level or color changes to another in a small amount of space. Since the derivative computes rate of change of a function, image regions where pixel values change rapidly have a high spatial derivative. Therefore, when a derivative image is computed, pixels with high values are considered edges.

Edge detection is one of the fundamental steps in image processing, image analysis, image pattern recognition, and computer vision techniques. Edge detection makes use of differential operators to detect changes in the gradients of the grey levels. There are many edge detector operator, among them Canny edge detector is one of them. Canny edge detector have advanced algorithm and has an optimal edge detection technique. It provides good detection, clear response and good localization. It is widely used in current image processing techniques with further improvements. Figure 2.2 shows the edge detection using canny operator.

a) Original Image b) Output Image

Figure 2.2: Edge Detection using canny Operator

2.2.2 Watershed method

The concept of watershed is based on visualizing an image in three dimension two spatial coordinates versus intensity. in such a „topographic" interpretation, we considering three types of points: (a) points belonging to a regional minimum; (b) points at which a drop of water, if placed at the location of any of those points, would fall with certainty to a single minimum; and (c) points at which water would be equally likely to fall to more than one such minimum. For a particular regional minimum, the set of points satisfying condition (b) is called the catchment basin or watershed of that minimum. The points satisfying condition (c) from crest lines on the topographic surface and are termed divide lines or watershed lines.

The principal objective of segmentation algorithms based on these concepts is to find the watershed lines. The basic idea is simple, as the following analogy illustrates in figure 2.3. Suppose that a hole is punched in each regional minimum and that the entire topography is flooded from below by letting water rise through the holes at a uniform rate. When the rising water in distinct catchment basins is about to merge, a dam is built to prevent the merging. The flooding will eventually reach a stage when only the tops of the dams are visible above the water lines. These dam boundaries extracted by watershed segmentation algorithm [4, 22].

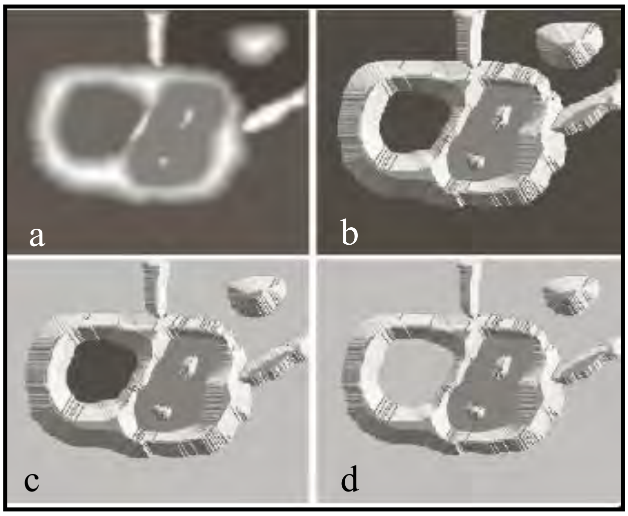


Figure 2.3: Watersheds two stage of flooding

[(a) Ordinal image (b) Topographic view (c)-(d) Two stage of folding]

In figure 2.3(b) is a topographic view, in which the height of the „mountains‟ is proportional to intensity values in the input image. Suppose that a hole is punched in each regional minimum [figure 2.3(b)]. The entire topography is flooded by water. As the water is continued to rise, it will eventually overflow from one catchment basin into another. The first indication is in figure 2.4(f). In figure 2.4(g) shows a dam between the two catchment basins and another dam in the top part of the right basin. The latter dam was built to prevent merging of water from that basin with water from areas corresponding to the background.

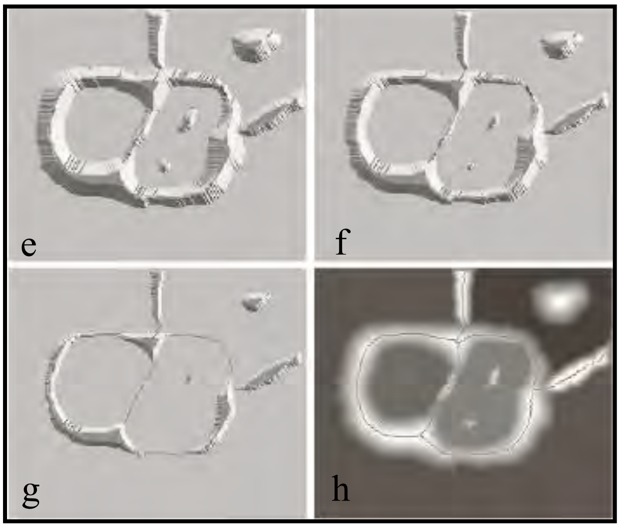


Figure 2.4: Final watershed segmentation

[(e) Result of further folding (f) Beginning of merging of water from two catchment basins (g) Longer dams (h) Final watershed lines]

This process is continued until the maximum level of flooding (corresponding to the highest intensity value in the image) is reached. The final dams corresponding to the water shed lines which are the desired segmentation result. Here the watershed lines form the connected paths thus giving continuous boundaries between regions.

One of the advantages of the watershed transformation is that it always provides closed contours, which is very useful in image segmentation. Another advantage is that the watershed transformation requires low computation times in comparison with other segmentation methods. However, using a standard morphological watershed transformation on the original image or on its gradient, it is usually obtain an over segmented image.

2.3 Region-based segmentation

Region-based segmentation looks for uniformity within a sub-region, based on a desired property, e.g. intensity, color, and texture.

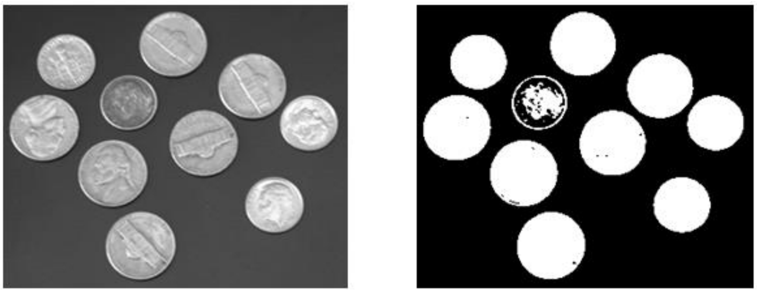
2.3.1 Threshold

Thresholding is an operation that converts a gray-scale image into a binary image where the two levels are assigned to pixels that are below or above the specified threshold value.

Suppose an image f(x, y), [4] composed of light objects on a dark background, in such a way that object and background pixels have intensity values grouped into two dominant modes. One way to extract the objects from the background is to select a threshold T, that separates these modes. Then, any point (x, y) in the image at which f(x, y) > T is called an object point; otherwise, the point is called a background point. Therefore the segmented image, g(x,y) can be written by,

$$g(x, y) = \begin{cases} 1 & \text{if } f(x, y) > T \\ 0 & \text{if } f(x, y) \leq T \end{cases} \qquad (2.1)$$

When T is a constant applicable over an entire image, the process given in this equation is referred to as global thresholding. When the value of T changes over an image, the term is known as variable thresholding. The term local or regional thresholding is used sometimes to denote variable thresholding in which the value of T at any point (x,y) in an image depends on properties of a neighborhood of (x,y). However, Global thresholding methods can fail when the background illumination is uneven. Otsu's threshold clustering algorithm searches for the threshold that minimizes the intra-class variance, defined as a weighted sum of variances of the two classes. Figure 2.5 shows the image segmented by threshoding algorithm.



a) Original Image b) Segmented Image

Figure 2.5: Global image threshold using Otsu's method

2.3.2 Clustering

A cluster is a set of entities, which are similar or homogeneous in type. Clustering may then be defined as the process of organizing objects into groups whose members are similar in some way.

Clusters may also be described as connected regions of a multi-dimensional space containing a relatively high density of points, separated from other such regions by a region containing a relatively low density of points.

Clustering algorithms has been useful in many applications like pattern recognition and data mining. Clustering methods can be divided into two categories: hierarchical and partitional methods. Within each of the types there exist a number of subtypes and different algorithms exist for finding the clusters [23].

2.3.2.1 Hierarchical Clustering

Hierarchical clustering techniques are a method of cluster analysis which tries to build a hierarchy of cluster. The end result is a tree of clusters, called a dendrogram representing the nested grouping of patterns and similarity levels at which groupings change. It advances successively by either merging smaller clusters into larger ones (agglomerative, bottom-up), or by splitting larger clusters (divisive, top-down). A clustering of data item is obtained by cutting the dendrogram at a desired level. This clustering method is divided in a sense into either two small clusters are merged together or a large cluster is split. Cobweb, Cure and Chameleon are some of the hierarchical algorithms. [24].

2.3.2.2 Partitional Clustering

Partitional clustering decomposes a data set into a set of disjoint clusters. Given a data set of N points, a partitioning method constructs K ($N \geq K$) partitions of the data, when each partition representing a cluster. Partitional clustering algorithms try to minimize an objective function to get the best representation. Partitional algorithms are categorized into Partitioning Relocation Algorithms and Density-Based

Partitioning. Partitioning Relocation Algorithms are further categorized into Probabilistic Clustering (SNOB), K-Medoids, and K-Means. The Density-Based Partitioning are called Density-Based Partitioning, which include algorithms such as Dbscan, Optics Dbclasd, Denclue, Gdbscan [25].

A hierarchical clustering is a nested sequence of partitions, whereas a partition clustering is a single partition. Therefore hierarchical clustering is a special sequence of partitional clustering. Hierarchical clustering ultimately produces one cluster tree at the end of the process. . Although hierarchical clustering is only useful on small data sets and it is not yet recommended very usable in image segmentation process.

Partitional clustering techniques such as, K-means clustering has an advantage over the hierarchical clustering techniques. In hierarchical clustering if a data point is assigned to a particular cluster, it cannot be changed. So, if a data point is incorrectly assigned to a particular cluster at an early stage of the process, there is no way to correct the error. However, there is also a disadvantage of the partitional clustering technique, that is different initial partitions or values of K affect outcomes, thus difficult to determine the number of clusters, K.

Clustering methods are global in the sense that they do not retain positional information. The major drawback is its invariance to spatial rearrangement of the pixels, which is an important aspect by the definition of segmentation. Therefore, the resulting segments might not be connected and can be widely scattered. This causes clustering methods sensitive to noise and intensity inhomogenities. This lack of spatial modeling, however, can provide significant advantages for fast computation [25] [26]. K-Means and Fuzzy C-Means [27] are well-known clustering-based image segmentation algorithms.

2.3.3 Region growing

The region-based methods try to identify the areas of images that are uniform and homogeneous with respect to some characteristic, such as gray level, color, or texture. One of the region-based methods is seeded region growing.

2.3.3.1 Seeded Region Growing

Seeded region growing selects some set of seed points based on some user criterion (for example, pixels in a certain grayscale range, pixels evenly spaced on a grid, etc.). The initial region begins as the exact location of these seeds. The regions are then grown from these seed points to adjacent points depending on a region membership criterion. The criterion again could be, for example, pixel intensity, grayscale texture, or color. If the criterion were a pixel intensity threshold value, knowledge of the histogram of the image would be of use, as one could use it to determine a suitable threshold value for the region membership criterion. The growing process is continued until a pixel not sufficiently similar to be combined is obtained. Thus the main goal is to classify the similarity of the image into regions. Important problems of seeded region growing are the selection of initial seeds that properly represent regions and the suitable homogeneity criterion to be used during the growing process [25]. Figure 2.6 shows an image which is segmented by region growing.

a) Original Image   b) Output Image

Figure 2.6: Segmentation result obtained by region growing

2.3.4 Split and Merge

Splitting and merging attempts to divide an image into uniform regions [28]. If this uniformity is not met for some region, which indicates that the region is inhomogeneous and should be split into sub regions. But if it is satisfied of two adjacent regions, then these regions are collectively homogeneous and then it will be merged into a single region. Basic representational structure is pyramidal, i.e. a square region of size m by m at one level of a pyramid has 4 sub-regions of size m/2 by m/2

below it in the pyramid. Usually the algorithm starts from the initial assumption that the entire image is a single region, and then computes the homogeneity criterion to see if it is TRUE. If FALSE, then the square region is split into the four smaller regions. This process is then repeated on each of the sub-regions until no further splitting is necessary. These small square regions are then merged if they are similar to give larger irregular regions. The problem (at least from a programming point of view) is that any two regions may be merged if adjacent and if the larger region satisfies the homogeneity criteria, but there might be regions which are adjacent in image space and may have different parents or be at different levels (i.e. different in size) in the pyramidal structure. Then the process terminates when no further merges are possible.
A quad-tree structure is often used to affect the step of splitting: it is based on the recursive decomposition of the regions that does not need to satisfy the homogeneity criterion into four squared sub regions, starting from the whole image. Therefore, an inverse pyramidal structure is built. The merging step consists of merging the adjacent blocks which represent homogeneous regions but have been divided by the regular decomposition [25].

Split-and-merge method does not suffer from predetermination of number of regions, or any other limitations. However, the main drawback is the artificial blocking effects on the resulting region boundaries. The main advantage of region-based methods is that the regions are obtained directly which are spatially connected.

2.3.5 Bayesian Methods

Bayesian methods are based on probability theory to determine the possibilities of the relative likelihood. In the case of image segmentation, this method is used for the existence of a particular label field realization along with the data. A prior knowledge, which can be exploited to improve the results, is used to regularize the inference of the field along with the given data. Formal optimization techniques are then used to work on the posterior inference [29].
The Bayes rule states that:

$$P(L\backslash X) \propto P(X\backslash L)P(L) \tag{2.2}$$

i.e. the posterior probability $P(L\backslash X)$ of the label field given the data is proportional to the product of the model probability $P(X\backslash L)$ and the prior probability of the label realizations $P(L)$. $P(L)$ is defined using local information about the expected segmentation result (such as shape, etc.) and aims at encouraging spatial connectivity.

2.4 Hybrid Method: Active Contour

The technique of active contours has become quite popular for a variety of applications, particularly image segmentation and motion tracking, during the last decade. This methodology is based upon the utilization of deformable contours which conform to various object shapes and motions [13].

There are two kinds of active contour models according to the force evolving the contours: edge- and region-based. Edge-based active contours use an edge detector, usually based on the image gradient, to find the boundaries of sub-regions and to attract the contours to the detected boundaries. Edge-based approaches are closely related to the edge-based segmentation discussed in Section 2.2. Region-based active contours use the statistical information of image intensity within each subset instead of searching geometrical boundaries. Region-based approaches are also closely related to the region-based segmentation discussed in Section 2.3.

There are two main approaches in active contours based on the mathematic implementation: snakes and level sets. Snakes explicitly move predefined snake points based on an energy minimization scheme, while level set approaches move contours implicitly as a particular level of a function.

2.4.1 Snakes

The Snakes model treat the desired contour as a time evolving curve and the segmentation process as an optimization over time of an adequate energy functional.

A snake is an energy minimizing, deformable spline influenced by constraint and image forces that pull it towards object contours and internal forces that resist deformation. Snakes may be understood as a special case of the general technique of matching a deformable model to an image by means of energy minimization. In two dimensions, the active shape model represents a discrete version of this approach, taking advantage of the point distribution model to restrict the shape range to an explicit domain learned from a training set. Snakes do not solve the entire problem of finding contours in images, since the method requires knowledge of the desired contour shape beforehand. Rather, they depend on other mechanisms such as interaction with a user, interaction with some higher level image understanding process, or information from image data adjacent in time or space.

The curve location *C* is parameterized by a spatial parameter *p* and the iteration time *t* as follows:

$$\boldsymbol{C}(p,t) = (x(p,t),\, y(p,t)) \tag{2.3}$$

The curve is considered to apply forces, reaching equilibrium when the energy is minimized:

1. "External Forces", attracting the contour to its desired location using image features and other knowledge.
2. "Internal forces", keeping the contour regular and smooth.

Therefore, the energy functional E to minimize is generally written as:

$$E(C(p)) = \int_0^1 E_{\text{internal}}(C(p))dp + \int_0^1 E_{\text{external}}(C(p))dp \tag{2.4}$$

Energy functional reaches a local minimum when the energy minimization process achieves equilibrium and that is the optimal solution for this function.

"Image forces", attracting the contour ("Snake") to significant image features from Kass et al. [30] and "External Constraint", added by user to make the Snake attract (or retract) to (from) significant high-knowledge features.
Therefore, the Snake's energy functional is written as:

$$E(C(p)) = \int_0^1 E_{\mathrm{internal}}(C(p))dp + \int_0^1 E_{image}(C(p))dp + E_{\mathrm{constarints}}(C(p)) \tag{2.5}$$

2.4.2 Level set

The following three problems are the major drawbacks of the Snakes algorithm:

- Inability to handle topological changes (split and merges).
- Need to push the snake be close to edge.
- Inability to deal with extension and sharp or thin structures (due to its internal forces).

Level Set is a Partial Differential Equation (PDE) based method that naturally deal with those problems. Level Set was first introduced by Osher and Sethian [9] in fluid dynamics. Applying it to image segmentation was simultaneously suggested by Casseles et al. and Malladi and Sethian [31]. To distinguish it from Snakes it is sometime called implicit (vs. explicit for Snakes) or geometrical (vs. parametrical) Active Contour Models.
Level set method represents the evolving contours as the zero level set of a higher dimensional function, and thus makes it possible to handle the numerical estimation with curves and surfaces easily without using parametric representation. The level set method encodes numerous advantages: it is implicit, parameter free, provides a direct way to estimate the geometric properties of the evolving structure, can change the topology and is intrinsic.

2.5 Conclusion

In this chapter we have overviewed the basic image segmentation methods briefly. Though there are plenty of segmentation techniques available in the literature, they can be broadly classified into edge based and region based techniques. However each of these categories has some drawbacks, therefore hybrid methods combining both edge and region based technique has been introduced. Level set active contour method is popular among hybrid methods which has described in section 1.4.2. In our work, we shall use level set method for image segmentation. In the next chapter the level set methods with mathematical formulation will be described in details.

Chapter 3

3 Mathematical formulation of the level set method

3.1 Level set Function

Osher and Sethian proposed the Level set theory, a formulation to implement active contours; they represented a contour implicitly via a two-dimensional Lipschitz-continuous function [4]

$\Phi(x,y) : \Omega \rightarrow R$, defined on the image plane. The function $\Phi(x, y)$ is called level set function, and a particular level, usually the zero level, of $\Phi(x, y)$ is defined as the contour C, such

$C = \{(x, y) : \Phi(x, y) = 0\}, \forall (x, y) \in \Omega$; Where Φ denotes the entire image plane.

Figure 3.1 shows the evolution of level set function $\Phi(x, y)$, and the propagation of the corresponding contours C.

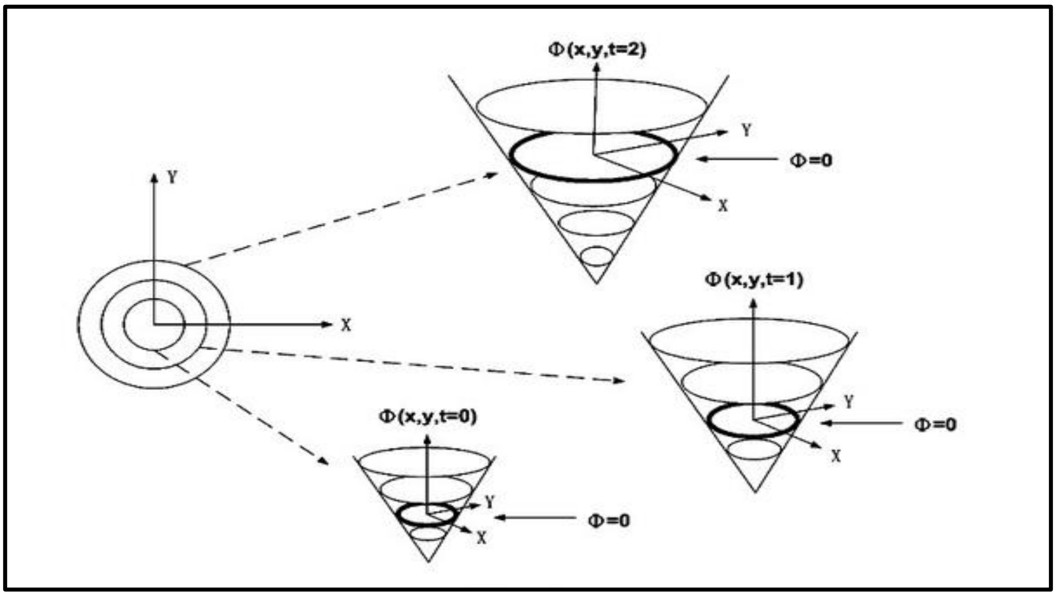


Figure 3.1: Level set evolution and the corresponding contour propagation

As the level set function Φ(x, y) is increased from its initial stage, the corresponding set of contours C propagates toward outside. With this definition, the evolution of the contour is equivalent to the evolution of the level set function,i.e

$$\partial C / \partial t = \partial \phi(x, y) / \partial t \tag{3.1}$$

The advantage of using the zero level is that a contour can be defined as the border between a positive area and a negative area, so the contours can be identified by just checking the sign of Φ(x, y).

Level set movement can be performed explicitly, using forcing functions that depend on the level set function itself or on the outside environment. Additionally, evolution can be driven without explicit forcing functions through the use of energy minimization. This is accomplished by solving differential equations for the change in Φ over time.

3.2 Level set evolution

The level set function (LSF) can be evolved in two different way. The function can evolve

i. In terms of velocity
ii. In terms of Energy

3.2.1 Velocity based Evolution

The first and most straightforward way to evolve a level set function uses a “velocity" function to stretch and move the level set function. This function can be a combination of terms based on both image data and on the level set function itself. Evolution is defined using the equation below, where $\vec{V}$ is the velocity function:

$$\frac{\partial \phi}{\partial t} + \vec{V}.\nabla \phi = 0 \tag{3.2}$$

A commonly used velocity term that is based on the level set function itself is the curvature of the level set function. The equation that defines curvature k in level set functions is:

$$k = \nabla.(\frac{\nabla\phi}{|\partial\phi|}) \tag{3.3}$$

In many cases, the level set function's curvature is used as negative feedback to keep the level set function smooth, rather than allowing it to develop sharp corners and kinks. This is done primarily because most physical systems (and objects in images) are curved rather than sharp and discontinuous. Additionally, sharp corners in the level set translate to high spatial variability in the level set function, which can overwhelm numerical differentiation schemes and produce incorrect results. In this case, the velocity function is normal to the level set contour with magnitude dependent on curvature

A commonly used extrinsic velocity function is the inverse gradient of the image. In image processing, a simple way to find image edges is to calculate the spatial gradient of the image data. Since an edge in an image occurs when color or brightness changes rapidly, edges have high spatial gradients. Conversely, patches of uniform brightness and color have low spatial gradients. The inverse gradient, then, allows the level set function to move rapidly in areas of low spatial gradient (without edges), and to slow dramatically when an edge is encountered. When the level set function is initialized properly, this will cause the level set to converge on edges, which has the effect of segmenting the object to which those edges belong.

3.2.2 Energy based

Rather than using specific velocity functions to evolve the level set function, an energy function can be defined based on image and level set data. The energy function is designed so that its value is minimized at the border of the object to be segmented. Then level set function evolution is driven by attempts to minimize energy.

In [32], Schildkraut, et al. describes energy functions appropriate to lung nodule segmentation and discuss how they are used to drive level set function evolution. In general, the energy terms E is used to take on the form:

$$E = \iint_{\Omega} f(\phi, \phi_x, \phi_y, x, y)\,dxdy \tag{3.4}$$

Where Φ is the level set function, and Φ_x and Φ_y are its partial derivatives with respect to x and y, respectively. Ω is the variable used to denote the problem domain. For image segmentation then, Ω refers to the entire image.

To find the energy minimum, the equation above must be minimized with respect to Φ. This requires the use of the calculus of variations, and results in the following partial differential equation.

$$\frac{\partial \phi(x,y)}{\partial t} = -\left(\frac{\partial f}{\partial \phi} - \frac{\partial}{\partial x}\frac{\partial f}{\partial \phi_x} - \frac{\partial}{\partial y}\frac{\partial f}{\partial \phi_y}\right) \tag{3.5}$$

The use of energy minimization in conjunction with the level set method is known as the variational level set method, because of its reliance on the calculus of variations.

3.3 Edge based Energy Function

In this work the contour C is implicitly indicated by the zero level set, i.e.

$$C(t) = \{x \mid \varphi(t, x) = 0\}. \tag{3.6}$$

For a given image $I(x)$, let Ω represent the whole image domain. Consequently, the zero level set can separate Ω into two regions: Ω_1 for $\varphi(x) > 0$ inside C and Ω_2 for $\varphi(x) < 0$ outside C.

As we know, the general edge-based energy is given by

$$E_{edge}(\phi) = \mu \int_{\Omega} g\partial(\phi)\left|\Delta\phi\right| dx + \nu \int_{\Omega} gH(\phi)\,dx \tag{3.7}$$

Where μ and ν are constants, H (φ) and δ (φ) are the Heaviside function and Dirac delta function respectively, and g, is the edge-detector, can be defined by

$$g = \frac{1}{1+\left|\nabla G_{\sigma} * I\right|^{2}} \tag{3.8}$$

The two terms represent the length of the contour and the area of the region Ω_1 inside C with the weighted factor g respectively, and the choice of parameter v should depend on initial contour position. In other words, when the initial contour is placed outside the object, v should be assigned positive so that the contour can shrink faster, while negative when inside the object so as to speed up the expansion of the contour. The edge-based energy *edge* (φ) will be close to minimal value when the contour locates at large gradient positions. As a result the edge-based energy model can obtain final object contour defined by gradient.

3.4 Region based energy function

The region-based energy model can detect objects whose boundaries do not necessarily have large gradient, and also it is robust to noise. Most widely used region based segmentation is Chan-Vese model based on the Mumford-Shah [33] framework.

3.4.1 Mumford-Shah Segmentation Functional

Mumford and Shah proposed a segmentation method based on a variational framework in their pioneering paper in 1989[33]. Let Ω be a bounded open set of R^N, N =2, 3, and u_0 an initial image data (in 2D or 3D) define on Ω. Segmentation of the image into homogeneous objects is performed via the search for a pair of objects (u, K), where $K \subset \Omega$ is a set of discontinuities (i.e. contours), and u is a piecewise smooth approximation of u_0. The optimal definition for this pair of objects is performed via minimization of an energy functional $F(u,K)$, such that u varies smoothly within the connected components of $\Omega \setminus K$, and rapidly or discontinuously across K. The energy functional is defined in 2D as:

$$F^{MS}(u,K)=\int_{\Omega}|\mu-\mu_0|^2\,dx+\mu\int_{\Omega/k}|\nabla u|^{2dx}+u\int_{k}dH \tag{3.9}$$

Where $\mu>0$, $\upsilon>0$ are fixed parameters. The first term in this functional ensures that u is good approximation of u_0, the second term ensures that u is smooth and the third term ensures that the discontinuities K have minimal length, Here, the edges of the image are represented in the discontinuity set K as the union of a finite set of C embedded curves.

A reduced case of the above model is obtained by restricting the segmented image u to the piecewise constant functions, i.e. $u=$ constant c_i inside each connected component Ω_i. Then the segmentation problem, called the "minimal partition problem", is solved via minimization of the following functional:

$$F_0^{MS}(u,K)=\sum_i\int_{\Omega_i}|u_0-c_i|^2\,dx+\upsilon\int_K dH \tag{3.10}$$

Here υ is a positive parameter, having a scaling role. In this case, for a fixed K, the energy in equation is minimized through computation of the variables c_i by setting c_i is the mean (u_0) in Ω_i.

3.5 Chan-Vese Model

A new energy functional for homogeneity-based segmentation derived from the work of Mumford and Shah (ref) was proposed by Chan and Vese (13). Let us assume that a given image, u_0 is formed by two regions of approximate piecewise constant intensities, of distinct values u_0^i and u_0^o. Let us denote the boundary between the two regions by C_0. Given an initial curve, Defined on the image, the following "fitting energy" can be minimized to segment the two regions:

$$F_1(C)+F_2(C)=\int_{inside(C)}|u_0-c_1|^2\,dx+\int_{outside(C)}|u_0-c_2|^2\,dx \tag{3.11}$$

The parameters c_1 and c_2, correspond to the mean values of the image inside and outside the curve C.

$$\inf_C \left\{F_1(C)+F_2(C)\right\}=0=F_1(C_0)+F_2(C_0) \tag{3.12}$$

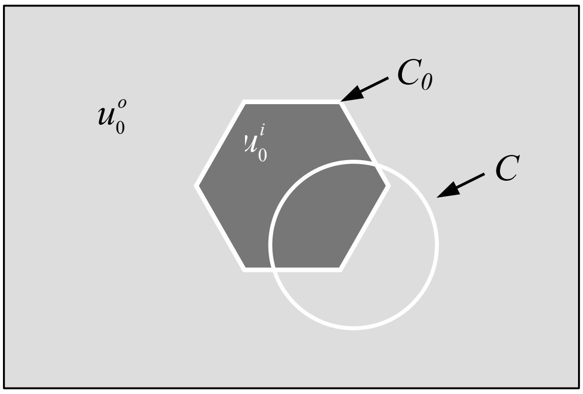


Figure 3.2: Segmentation of a piecewise constant image with an object and a background.

Minimization of the fitting energy is achieved when the curve is on the boundary of the object. Figure 3.2 shows the curve C as an initial curve where the foreground object is C_0.

3.6 Regularization term

Various regularization terms are normally attached to the edge and region based level set function in order to input the segmentation performance. It act as internal force during the segmentation process.

The Energy function ε(φ) incorporating the regularization term is given by :

$$\varepsilon(\phi)=\mu\Re_p(\phi)+\nu L(\phi)+\varepsilon_{ext}(\phi) \tag{3.13}$$

The level set regularization term is $\Re_p(\phi)$ and L (Φ) is defined below

$$\Re_p(\phi)\triangleq\int_\Omega \rho(|\nabla\phi|)dx \tag{3.14}$$

and

$$L(\phi) = \int \left|\nabla H(\phi) dx\right| \tag{3.15}$$

Here L (Φ) computes the arc length of the zero level contour of Φ and therefore serves to smooth the contour by penalizing its arc length.

The regularization term $\Re_p(\phi)$ is called a distance regularization term, which was introduced by Li et al. [12]. The distance regularization effect eliminates the need for reinitialization and thereby avoids its induced numerical errors.

3.7 Implementation of the Level Set Method

In order to minimize the total energy function, gradient descent flow is used, which is defined by

$$\frac{\partial \phi}{\partial t} = -\frac{\partial F}{\partial \phi} \tag{3.16}$$

Then we can figure out the level set evolution formulation, which will drive the motion of the active contour towards the desirable positions.

3.7.1 Edge Based:

Geodesic Active Contours (GAC) model[10] is one of the edge based model which is theoretically combines explicit Active Contour Models (Snakes) with the Implicit Active Contour Models (Level Sets).

The GAC model has the following evolution equation

$$E_{edge}(\phi) = \mu \int_{\Omega} g \partial(\phi) \left|\Delta \phi\right| dx + \nu \int_{\Omega} g H(\phi) dx \tag{3.17}$$

Therefore the corresponding update equation or force equation along with regularization term will be:

$$\frac{\partial \phi}{\partial t} = \delta(\phi)[\mu div(g \frac{\nabla \phi}{|\nabla \phi|}) - \nu g] + \rho[\nabla^2 \phi - div(\frac{\nabla \phi}{|\nabla \phi|})] \tag{3.18}$$

Where, g is the edge indicating function and δ is associated with Dirac function.

3.7.2 Region Based

Chan-Vese method [13] is the region based approach which has already discussed in section 3.7.

As a special case of the Mumford-Shah functional, Chan and Vese proposed an active contour model derived from this energy functional with the addition of two regularizing terms to constrain the length of C and the area inside C:

$$F(C,c_1,c_2) = \mu\big(length(C)\big) + \upsilon\big(area(inside\,C)\big) + \lambda_1 \int_{inside(C)} |u_0 - c_1|^2\,dx + \lambda_2 \int_{inside(C)} |u_0 - c_2|^2\,dx \tag{3.19}$$

Where $\mu \geq 0,\ \upsilon \geq 0,\ \lambda_1, \lambda_2 > 0$ are fixed parameters.

This energy functional can be extended to the segmentation of multiple homogeneous objects in the image by using several curves $\{C_1, C_2, ..., C_i\}$. In the case of two curves we use the following fitting energy.

$$\begin{aligned} F(C_1, C_2, c_{00}, c_{01}, c_{10}, c_{11}) = {} & \lambda_1 \int_{\substack{insideC_1\\ insideC_2}} |u_0 - c_{11}|\,dx + \lambda_2 \int_{\substack{insideC_1\\ outsideC_2}} |u_0 - c_{10}|\,dx + \lambda_3 \int_{\substack{outsideC_1\\ insideC_2}} |u_0 - c_{01}|\,dx + \lambda_4 \int_{\substack{outsideC_1\\ outsideC_2}} |u_0 - c_{00}|\,dx \\ & + \mu_1 length(C_1) + \mu_2 length(C_2) + \upsilon_1 area(insideC_1) + \upsilon_2 area(insideC_2) \end{aligned} \tag{3.20}$$

Minimization of this energy functional deforms simultaneously two curves and identifies four homogeneous areas defined by the intersection of the two curves as illustrated in figure 3.3

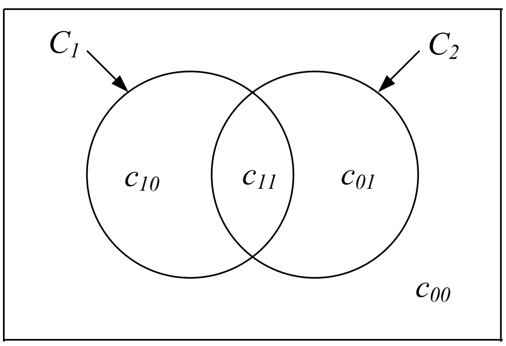


Figure 3.3: Partitioning of an image into four areas with two curves

A given curve C (being now the boundary of an open set $\omega \in \Omega$) is represented implicitly, as the zero level set of a scalar Lipschitz function ϕ : $\Omega \rightarrow R$ (called level set function), such that:

$$\begin{cases} \phi(x,y) < 0 & in\ \omega, \\ \phi(x,y) > 0 & in\ \Omega \setminus \omega, \\ \phi(x,y) = 0 & on\ \partial\omega. \end{cases} \tag{3.21}$$

Given the curve C embedded in a level set function ϕ, its associated Heaviside function H and Dirac function δ are defined respectively as:

$$H(\phi) = \begin{cases} 0, & if\ \phi \geq 0, \\ 1, & if\ \phi < 0. \end{cases} \tag{3.22}$$

$$\delta(\phi) = \frac{d}{d\phi} H(\phi) \tag{3.23}$$

Using these two functions, different components of the functional in Equation[r], parameterized with the contour curve C, can be reformulated with the level function ϕ as:

- Length of the curve C: $Length(C) = Length(\phi = 0) = \int_{\Omega} |\nabla H(\phi)| dx$,
- Area inside the curve C: $Area(C) = Area(\phi < 0) = \int_{\Omega} H(\phi) dx$,
- Average of u_0 inside the curve C: $\int_{\Omega} |u_0 - c_1|^2 H(\phi) dx$,
- Average of u_0 outside the curve C: $\int_{\Omega} |u_0 - c_2|^2 (1 - H(\phi)) dx$.

The single-curve functional in Equation [r] can then be reformulated with the variable ϕ and integrals defined on the entire domain Ω :

$$F(\phi,c_1,c_2) = \lambda_1 \int_{\Omega} |u_0 - c_1|^2 H(\phi)dx + \lambda_2 \int_{\Omega} |u_0 - c_2|^2 \left(1 - H(\phi)\right)dx + \mu \int_{\Omega} \left|\nabla H(\phi)\right| + \upsilon \int_{\Omega} H(\phi)dx \tag{3.24}$$

Finally the mean values c_1 and c_2, are formulated as a function of $H(\phi)$ as:

$$c_1(\phi) = \frac{\int_{\Omega} u_0(x) H(\phi(x))dx}{\int_{\Omega} H(\phi(x))dx}, \quad c_2(\phi) = \frac{\int_{\Omega} u_0(x)\left(1 - H(\phi(x))\right)dx}{\int_{\Omega} \left(1 - H(\phi(x))\right)dx} \tag{3.25}$$

The Euler-Lagrange equation of this system is derived by keeping c_1 and c_2 fixed, and minimizing $F(\phi,c_1,c_2)$ with respect to ϕ as:

$$\frac{\partial \phi}{\partial t} = \delta_\varepsilon(\phi)\left[\mu div\left(\frac{\nabla \phi}{|\nabla \phi|}\right) - \upsilon - \lambda_1 |u_0 - c_1|^2 + \lambda_2 |u_0 - c_2|^2\right] \tag{3.26}$$

3.7.3 Hybrid Method

When both edge based and region based methods are combine together and both criteria are consider then it is known as Hybrid Method. For Hybrid Method the energy Function can be explained as:

$$\varepsilon_{(x,y)} = \varepsilon_{edge} + \varepsilon_{region} + \varepsilon_{regularized} \tag{3.27}$$

And the corresponding update functions:

$$\frac{\partial \phi}{\partial t} = -\frac{\partial E_{edge}}{\partial \phi} - \frac{\partial E_{region}}{\partial \phi} - \frac{\partial E_{regularization}}{\partial \phi} \tag{3.28}$$

Though Hybrid method is useful for image segmentation as it is combining both edge and region criteria but it is difficult to determine a particular weighted function for both. However, in spite of difficulties many hybrid methods are trying to enhance the segmentation result. Among them Shawn Lankton"s method is one which can combine both edge and region criteria by forming a cost based function on a shortest weighted path [35]. The weights at each point along the path are determined from local regions around the curve. The resulting flow is capable of finding significant local minima and partitioning the image without making global assumptions about its makeup.[35]

3.8 Conclusion

In this chapter first we provide the mathematical formulation of level set function. Then we derive the energy function for both edge based and region based. We also describe the regularization term that is necessary to improve the segmentation result. Finally, we discussed about the hybrid method. In the next chapter we will propose our globally convergent level set method in order to overcome the limitations of the state-of-the-art level set method.

Chapter 4

4 Globally convergent variable time-step level set method

4.1 Objective

In order to improve the performance of the level set segmentation method, we focus on the following two vital objectives

- Improving the robustness of the segmentation performance using multiple optimization criteria.
- Ensuring numerical stability as well as fast convergence speed.

4.2 Formulation of a novel hybrid level set method

In this work, we propose a hybrid energy function for the level set method. The method is called hybrid as it uses both edge and region criteria. The region based criterion will attempt to minimize the variance of intensities in the foreground and background region whereas the edge based criterion will stop the evolution process at the object boundary. The method has the following characteristics:

(i) The energy function is globally convergent.

(ii) The evolution process is fast moving in the uniform region but slow in the boundary.

We formulate a composite energy function composed of both region and edge terms that will automatically determine the evolution direction of curve so that segmentation result is independent of the initial position of curve. The edge based criteria will pick up the gradient information in the image and will adaptively control the convergence speed. As a result, the curve will evolve with lower speed at boundaries and higher speed in uniform region in order to avoid the boundary leakage.

The proposed energy function is defined as

$$F(\phi,c_1,c_2)=\lambda_1\int_\Omega g\times|u_0-c_1|^2 H(\phi)dx+\lambda_2\int_\Omega g\times|u_0-c_2|^2\left(1-H(\phi)\right)dx \\ +\mu\int_\Omega\left|\nabla H(\phi)\right|+\upsilon\int_\Omega H(\phi)dx \tag{4.1}$$

The update equation is defined by keeping $c_1$and $c_2$fixed, and minimizing $F(\phi,c_1,c_2)$ with respect to φ as:

$$\frac{\partial\phi}{\partial t}=\delta_\varepsilon(\phi)\left[\mu div\left(\frac{\nabla\phi}{|\nabla\phi|}\right)-\upsilon+(\lambda_1|u_0-c_1|^2-\lambda_2|u_0-c_2|^2)\times g\right] \tag{4.2}$$

$$c_1(\phi)=\frac{\int_\Omega u_0(x)H(\phi(x))dx}{\int_\Omega H(\phi(x))dx} \qquad c_2(\phi)=\frac{\int_\Omega u_0(x)\left(1-H(\phi(x))\right)dx}{\int_\Omega\left(1-H(\phi(x))\right)dx} \tag{4.3}$$

Here, c1 and c2 is the average intensities inside and outside of the contour, and Heaviside function H and Dirac function δ are defined respectively as:

$$H(\phi)=\begin{cases}0, & if\ \phi\geq 0,\\ 1, & if\ \phi<0.\end{cases} \tag{4.4}$$

$$\delta(\phi)=\frac{d}{d\phi}H(\phi) \tag{4.5}$$

Here, „g" represents the edge indicator function defined as:

$$g\triangleq e^{-(\Delta I)} \tag{4.6}$$

where ΔI represent the gradient of given image.

It will help to stop the curve evolution when getting edges define by the gradient of the images. When the region is uniform then the value of g = 1, therefore in uniform region, g is not affecting the update equation, so the curve will normally evolve. However, when it gets any edge or boundary region then the value of g will be insignificant. As a result curve evolution becomes slower at edge. In this way our method is adaptively controlling the convergence speed as well as helping to detect weak boundaries in the images.

4.3 Variable time-step level set method

The selection of a larger time-step speeds up the convergence rate but endanger the numerical stability of the algorithm. A smaller time-step slows the convergence rate and increases the computational time. However in our method we proposed variable time step so that convergence will be done automatically and also reduce computational time. A variable time-step function will be is determined by maximizing the between class variance among the foreground and background region of image in each iteration.

Between Class Variance is defined by

$$\sigma_B^2 = p_1(c_1 - c_g)^2 + p_2(c_2 - c_g)^2 \tag{4.7}$$

Where,

$$p_1 = \frac{N_1}{N_{1+}N_2} \quad \text{And} \quad p_2 = \frac{N_2}{N_{1+}N_2} \tag{4.8}$$

Here N_1 = No. of pixel in foreground image.

N_2 = No. of pixel in background image.

c_g = Mean Value of the image.

c_1 = Average intensity inside the contour

c_2 = Average intensity outside the contour

Actually we are maximizing the between class variance by variable time-step to get the optimum segmentation result for a given force function.

Therefore the update of the LSF is performed as

$$\phi^{t+1} = \phi^{t} + \Delta t[\frac{d\phi}{dt}] \tag{4.9}$$

Where, $$\underbrace{\frac{\partial\phi}{\partial t} = \delta_{\varepsilon}(\phi)\left[\mu div\left(\frac{\nabla\phi}{|\nabla\phi|}\right) - \upsilon + (\lambda_1 |u_0 - c_1|^2 - \lambda_2 |u_0 - c_2|^2) \times g\right]}_{\text{force}}$$

$$\phi^{t+1} = \phi^{t} + \Delta t \times force \tag{4.10}$$

The implementation of variable time-step level set method consists of the following steps:

Step 1: Initialize the LSF Φ and calculate the between class variance
Step 2: Determine the force function Φ_t
Step 3: Update the LSF $\Phi_{t+1=}\Phi_t + \Delta t \times force$
Step 4: Calculate again the Between Class variance
Step 5: Check the difference of old and new Between Class Variance. If the difference is significant then go to step 3
Step 6: Check the difference between old and new Φ. If the difference is significant go to step 2. Otherwise stop the iteration.

Verification of variable time-step:

Now we will show how the variable time step is achieved using the above mentioned scheme.

In every iteration (when the force function is unchanged) the ϕ function is updated.

$$\phi_{n+1} = \phi_n + \Delta t \times force \tag{4.11}$$

$$\phi_{n+2} = \phi_{n+1} + \Delta t \times force \tag{4.12}$$

$$= \phi_n + \Delta t \times force + \Delta t \times force$$
$$= \phi_n + 2\Delta t \times force$$
$$\cdots\cdots\cdots$$

Therefore for the pth iteration the updated function will be like this:

$$\phi_{n+p} = \phi_n + p\Delta t \times force \tag{4.13}$$

In this way time-step become variable and it will maximize the between class variance among the foreground and background region in each iteration.

4.4 Narrowband Approach

In this method level set function will be updated in the neighborhood region of the evolving curve. If there is any noise in the background it does not affect the segmentation process in narrow band approach

In narrowband approach we applied erosion and dilation method for segmentation shown in figure 4.1.The erosion of Φ by a flat structuring element b at any location (x,y) is defined as the minimum value of the image in the region coincident with b when the origin of b is at (x,y) [4]. The erosion at (x,y) of an image f by a structuring element b is:

$$[\varphi \ \Theta \ b] (x, y) = \min_{(s,t)á\ b} \{ \varphi (x + s, y + t)\} \tag{4.14}$$

Similarly the dilation of φ by a flat structuring element b at any location (x, y) is defined as the maximum value of the image is:

$$[\varphi \oplus b](x, y) = \max_{(s,t)\in b} \{ \varphi (x - s, y - t)\} \qquad (4.15)$$

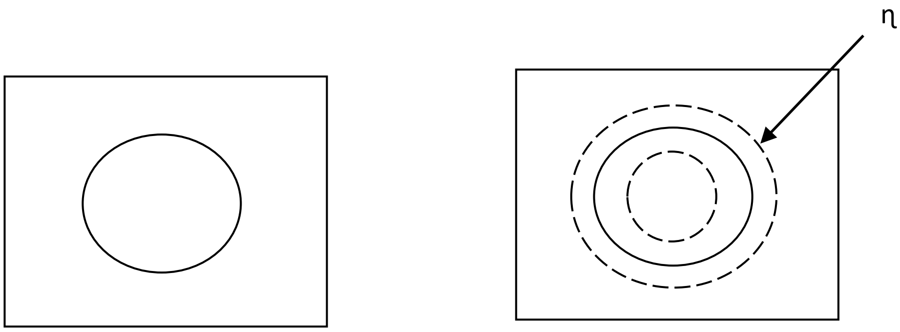


Image with initial curve Erosion and dilation

Figure 4.1: Erosion and dilation determine narrow band of the image

For the narrow band approach force function will be

$$\phi_{n+1} = \phi_n + [\eta] \times \Delta t \times force \qquad (4.16)$$

$\eta = (\Phi \oplus b) - (\Phi \ominus b)$

In this way the evolving curve will evolve in the neighboring region of the initial contour and therefore eliminate unnecessary noise from background and also reduce computational complexity in the program.

4.5 Conclusion

In this chapter we discussed about our novel hybrid model which is globally convergent. We have derived a novel energy function by utilizing both edge and region criteria. Then we discussed about the between class variance to make the time-step variable. Then we discussed the proposed algorithm and formulate all the related mathematical formula to establish our method. Besides this we discussed about a special case which is narrowband approach. Narrowband approach can be use in special case where there is no multiple segmented images and also to reduce noise from background images. We have derived narrowband band approach in a morphological technique where erosion and dilation methods have used. In the next chapter we will analysis our result with multiple optimization criteria and compare the result with the existing methods.

Chapter 5

5 Result and Analysis

In this section, we give details description of our proposed algorithm and our segmentation technique is tasted on synthetic, natural and medical images. We also compared our methods with existing state of the art methods.

5.1 Image segmentation process using the proposed algorithm

In this section, we explain our algorithm step by step. At first we choose an image (figure 5.1) and initialize the level set function (LSF). The initial contour and the initial LSF of the image is given in figure 5.2(a) and figure 5.2(b).

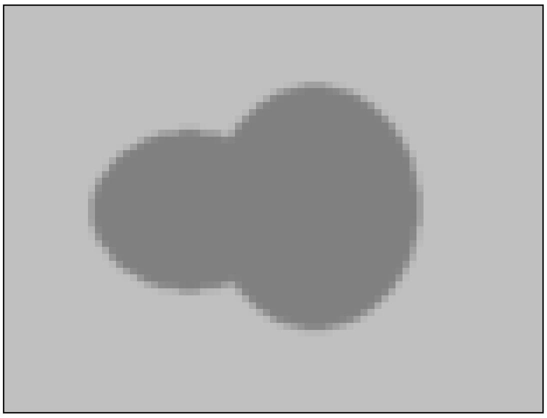
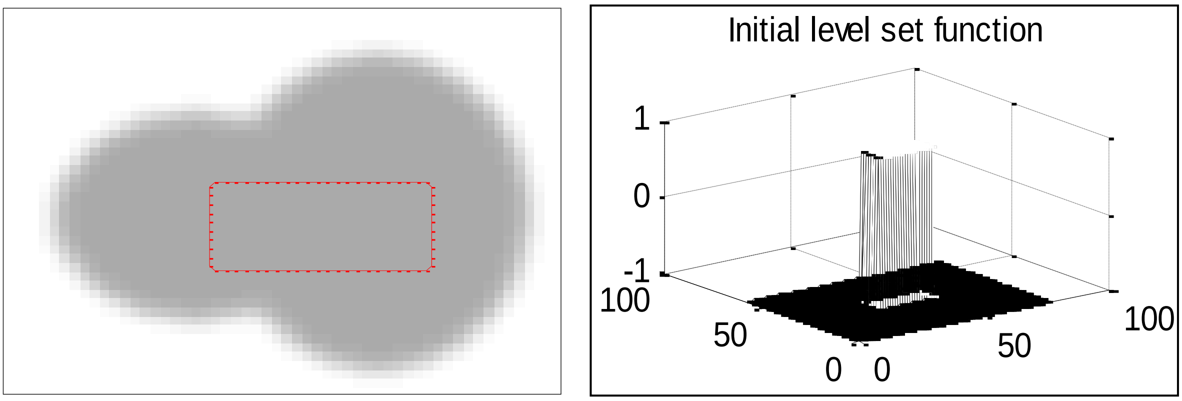


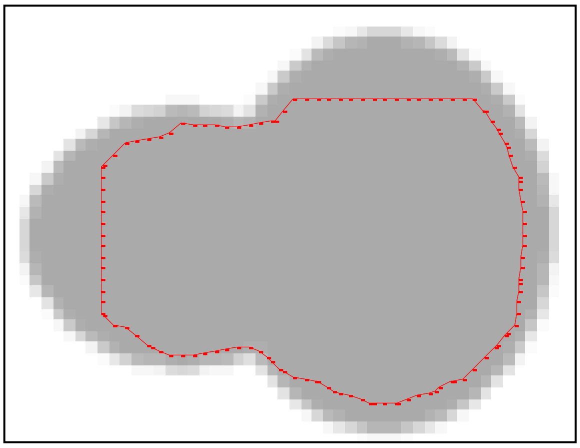
Figure 5.1: Original Image



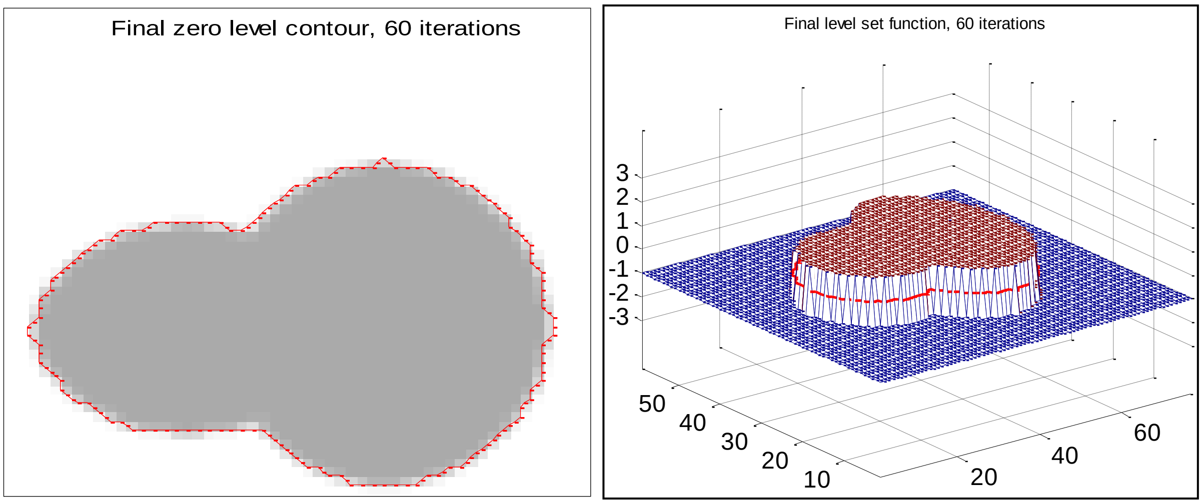
e 5.2(b): Initial level set function

and LSF

In our method we choose the parameter like μ, ϵ, λ a fixed value where μ= 0.1, λ=1 and ϵ= 1. Here Δt is initially 0.5. The LSF will be updated with the iteration of the evolving curve. Initially a force function and a between class variance will be calculated. It will check the between class variance for a given force function and compare it with the initial one. If it gets any changes then it will update the LSF and the contour will increase its area to reach the convergence. The intermediate stage of the initial conto



Finally when t updated anymore. The Final zero level contours after 60 iterations and the final LSF is showed in figure 5.4(a) and figure 5.4(b)



: 5.4(b): Final level set function

Figure 5.4: Final Level Set Function

5.2 Global Convergence Characteristic of the Proposal

Global Convergence means wherever the initial position of the contour is, ultimately it will segment the desire foreground object from the background object. Our method is globally convergence therefore our algorithm is independent of the initial position of the curve. In this section we take four different initial position of the contour and have showed our segmented result in figure 5.5.

Initial Contour | Final Zero Level Contour

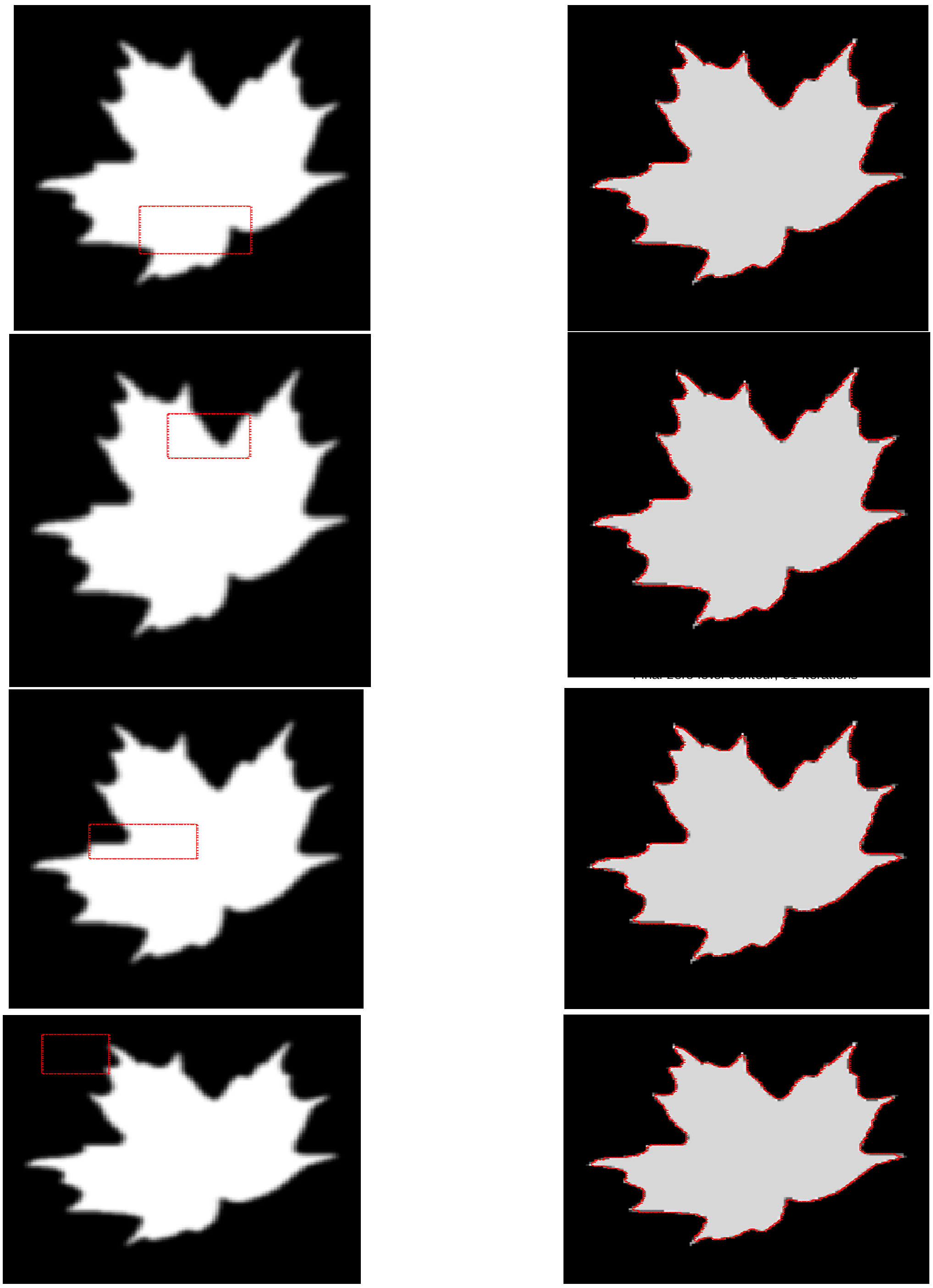


Figure 5.5: Global Convergence Characteristic

5.3 Advantage of incorporating variable time-step

In our proposed method we include variable time step by maximizing the between class variance. The advantage of incorporating the variable time-step is that force function is not be calculated every time in each iteration. Therefore it will speed up the convergence and hence reduce the computational complexity. In figure 17 (a) shows an image segmented in 100 iterating by using Chan-Vese method. Whereas in figure 5.6(b) has shown, our proposed method can obtain the same segmentation result in 30 iterations.

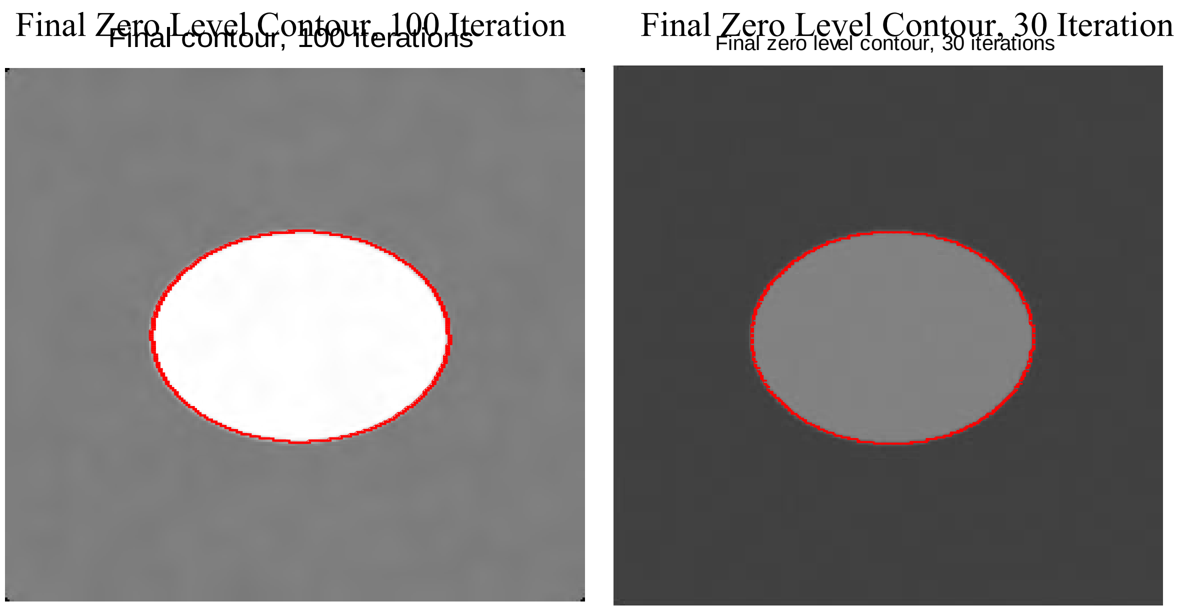


Figure: 5.6 (a) image segmented in 100 iteration

Figure: 5.6 (b) image segmented in 30 iteration

Figure 5.6: Final Zero Level Contour (Incorporate with variable time-step)

5.4 Advantage of implementation of narrow band approach

In our method we propose narrowband approach where the LSF will be updated in the neighborhood of the evolving curve. Therefore if there is any noise present in the background region which is far away from the foreground region will be eliminated. Thus it will also reduce the computational complex city. In figure 5.7(a) shows an image with initial contour, it has some other image in the background. However in figure 5.7(b) shows the segmentation result of final LSF where the initial curve evolve within neighborhood region and give the segmentation result by eliminating the noise from background. Although narrowband approach is not very suitable method when there is multiple segmented images presented in the foreground region. At that time it cannot detect the segmented image properly. Figure 5.7(c) shows the segmentation result compare with Chan-Vese method (figure 5.7(b)) for the same initial zero level contour (figure 5.7(a)).

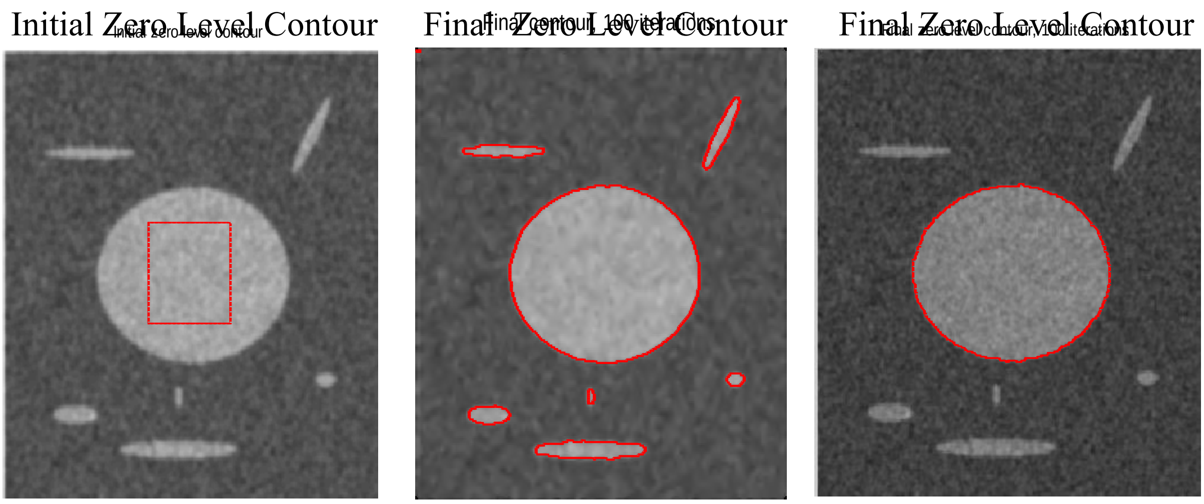


Figure 5.7(a) original image

Figure 5.7(b) Chan-Vese method

Figure 5.7(c) proposed method

Figure 5.7: Narrow band approach

5.5 Robustness of the proposed method in detecting weak boundaries

In our proposed method we include an edge indicator function „g" which can detect weak boundaries or edges in the images. Edge indicator function will pick up the gradient information in the image and the evolving curve will evolve slower speed at boundaries which has already described in chapter four. Therefore „g" is preventing the evolving curve from the leakage at image boundary region. In figure 5.8(a) shows an image where background image is varying from dark to light illumination. As a result it creates weak boundaries in light illumination region. Normal Chan-Vese method cannot detect the weak boundaries as shown in figure 5.8(b) whereas in figure 5.8(c) shows the segmented image by applying our proposed method.

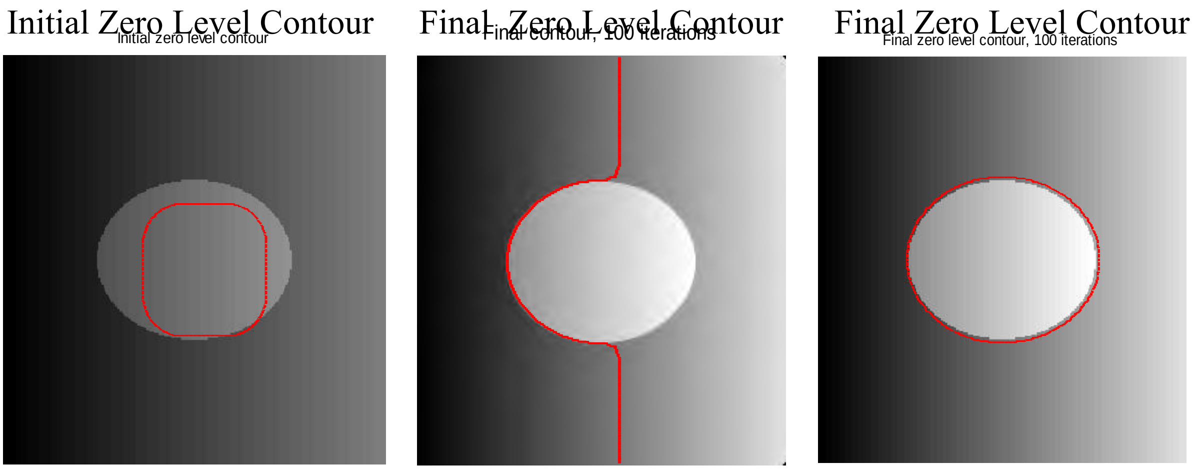


Figure 5.8(a) original image

Figure 5.8(b) Chan-Vese method

Figure 5.8(c) proposed method

Figure 5.8: Detecting Weak Boundaries

5.6 Simulation Result for different types of image

In this section, we evaluate the performance of our proposed segmentation technique on a variety of images, such as, simulation images, medical images, and real images. We also compare our proposed method with Chan-Vese method, DRLSE model, level set evolution (LSE) and bias field estimation model and hybrid method (Lankton‟s method). The visual display of the segmentation results for different images are presented in figures 5.9, 5.10, and 5.11. It is observed that the proposed method provides better segmentation performance in most of the cases.

Input Image Initialization

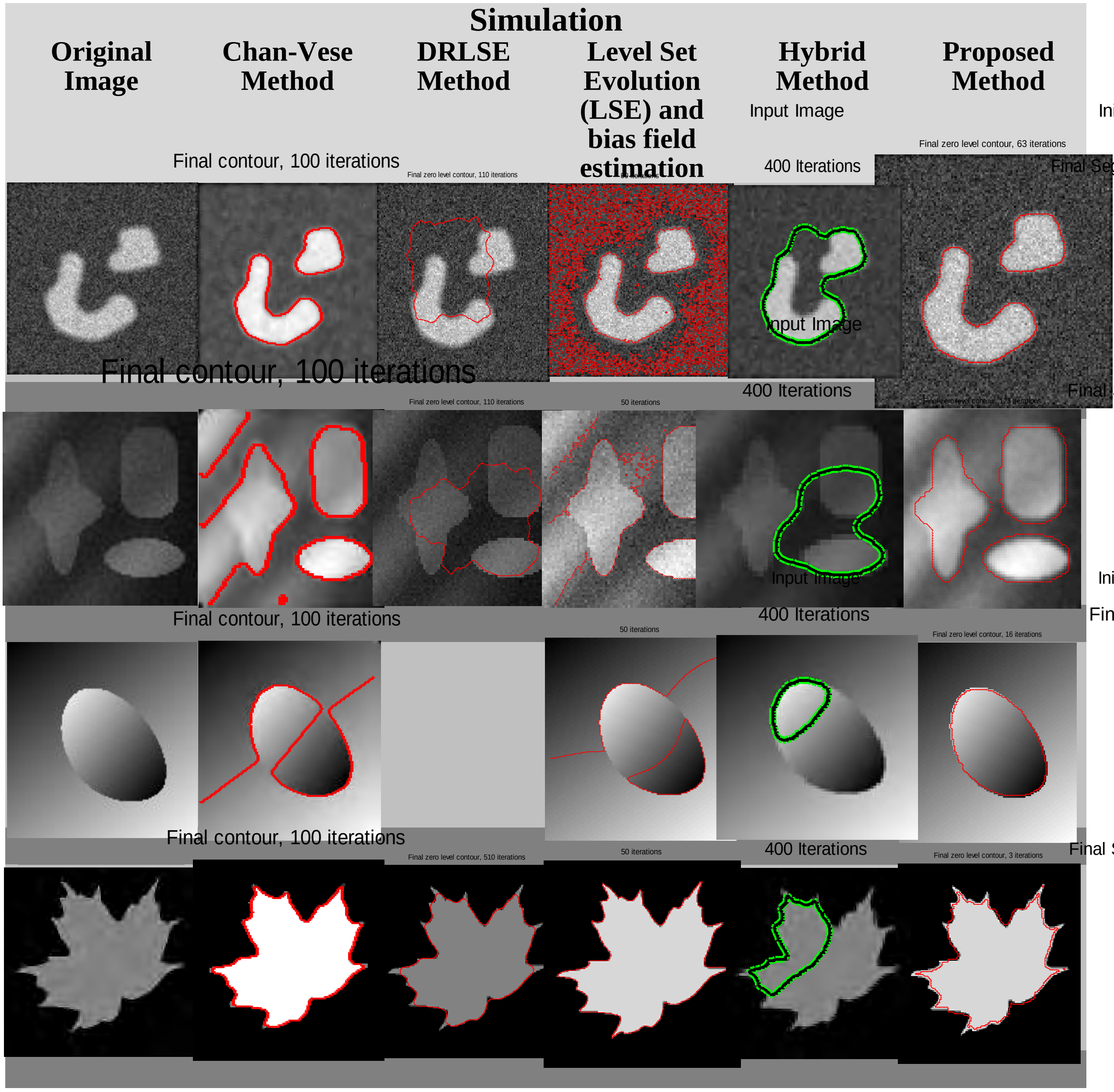


Figure 5.9: Simulation Result on Synthetic Images

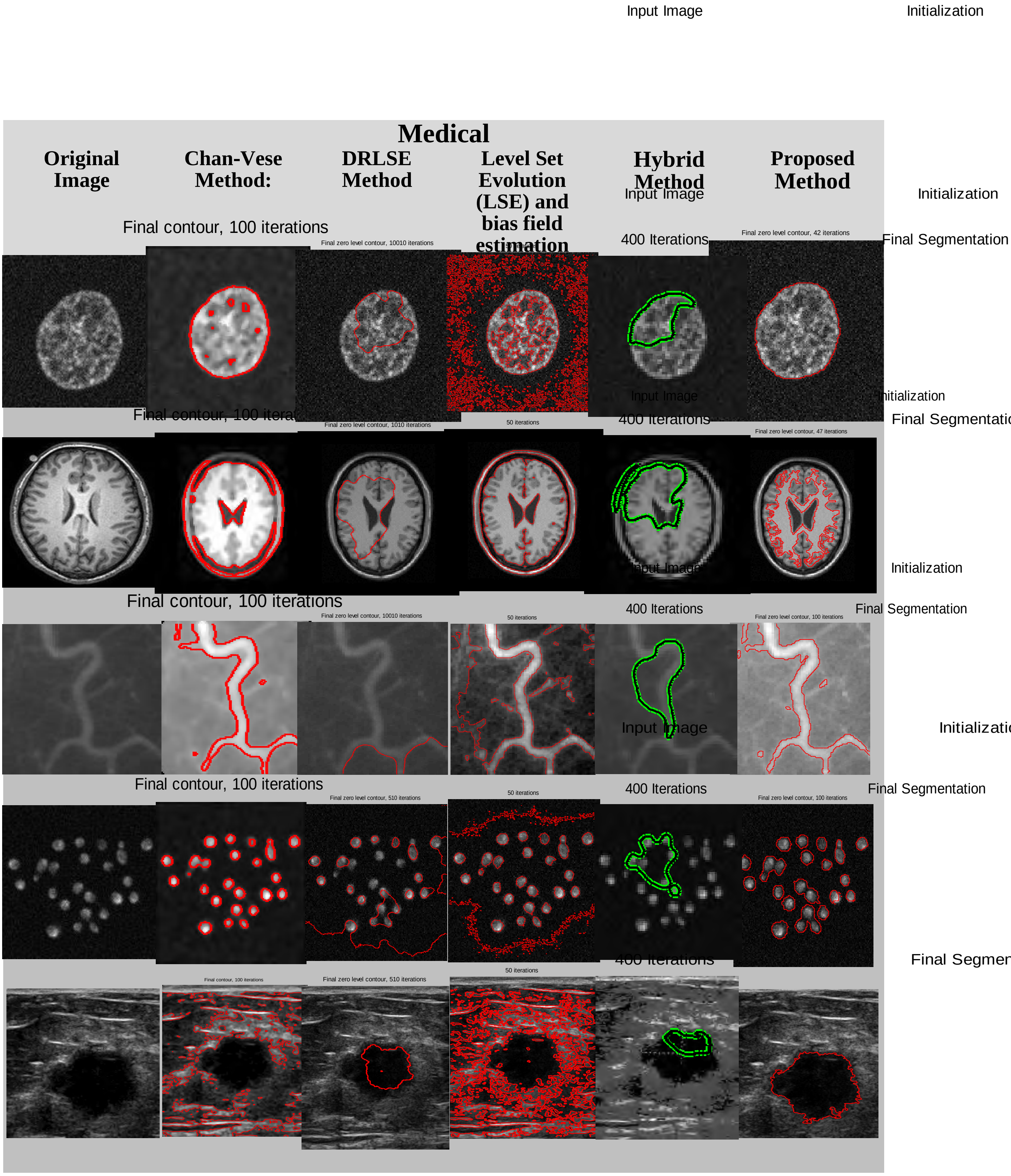


Figure 5.10: Simulation Result on Medical Images

Input Image Initialization

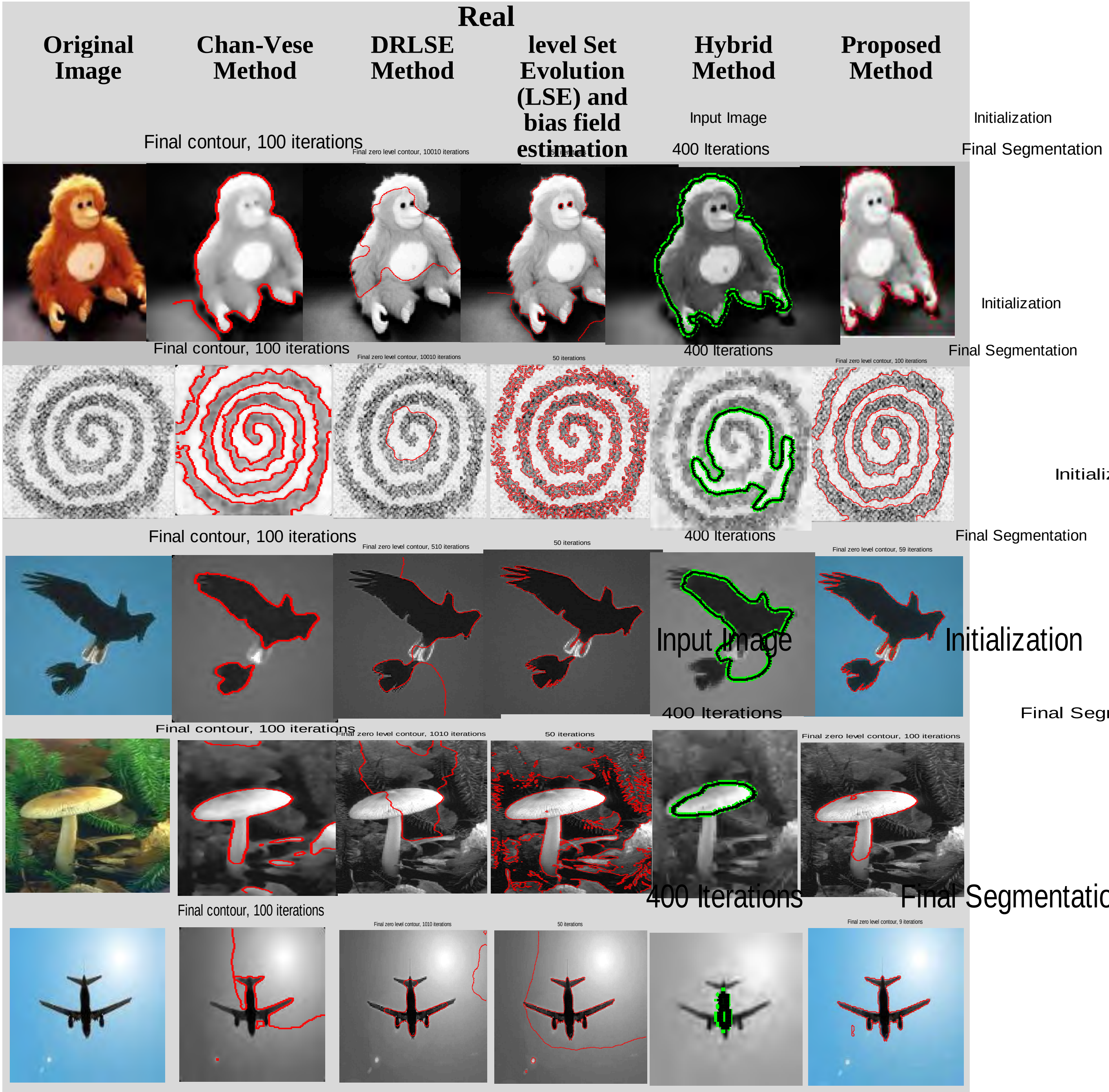


Figure 5.11: Simulation Result on Real Images

5.8 Numerical Evaluation of the segmented Result

The segmentation results of different techniques have numerically evaluated using Dice Coefficient, Hausdorff distance and MSSD with a reference.

5.8.1 Dice's coefficient

Dice's coefficient is a statistic used for comparing the similarity of two samples.

The original formula was intended to be applied to presence or absence data, and is

$$QS = \frac{2C}{A+B} = \frac{2|\mathrm{A} \cap \mathrm{B}|}{|A|+|B|} \tag{5.1}$$

Where A and B are the number of species in samples A and B, respectively, and C is the number of species shared by the two samples; QS is the quotient of similarity and ranges between 0 and 1

It can be viewed as a similarity measure over sets:

$$s = \frac{2|X \cap Y|}{|X|+|Y|} \tag{5.2}$$

If the value of Dice Coefficient is close to 1, then the image has better segmentation result.

5.8.2 Hausdorff Distance

The Hausdorff distance measures how far two subset of a metric space are from each other. It turns the set of non empty compact subsets of a metric space into a metric space in its own right.

Informally, two sets are close in the Hausdorff distance if every point of either set is close to some point of the other set. It is the greatest of all the distances from a point in one set to the closest point in the other set.

Let X and Y be two non-empty subsets of a metric space (M, d). We define their Hausdorff distance $d_H(X, Y)$ by

$$d_H(X,Y) = \max\{\sup_{x\in X}\inf_{y\in Y} d(x,y), \sup_{y\in Y}\inf_{x\in X} d(x,y)\}, \qquad (5.3)$$

Where *sup* represents the supermum and inf is the infimum.
Equivalently

$$d_H(X,Y) = \inf\{\in\geq 0; X \subseteq Y_{\in} \; and \; Y \subseteq X_{\in} \qquad (5.4)$$

where

$$X_{\in} := \bigcup_{x\in X}^{n} \{z \in \mathrm{M}; d(z,x) \leq \in\}, \qquad (5.5)$$

That is, the set of all points within $\in$ of the set X (sometimes called the $\in$-fattening of X or a generalized ball of radius $\in$ around X).

The lower value of Hausdorff distance means the image has better segmentation result.

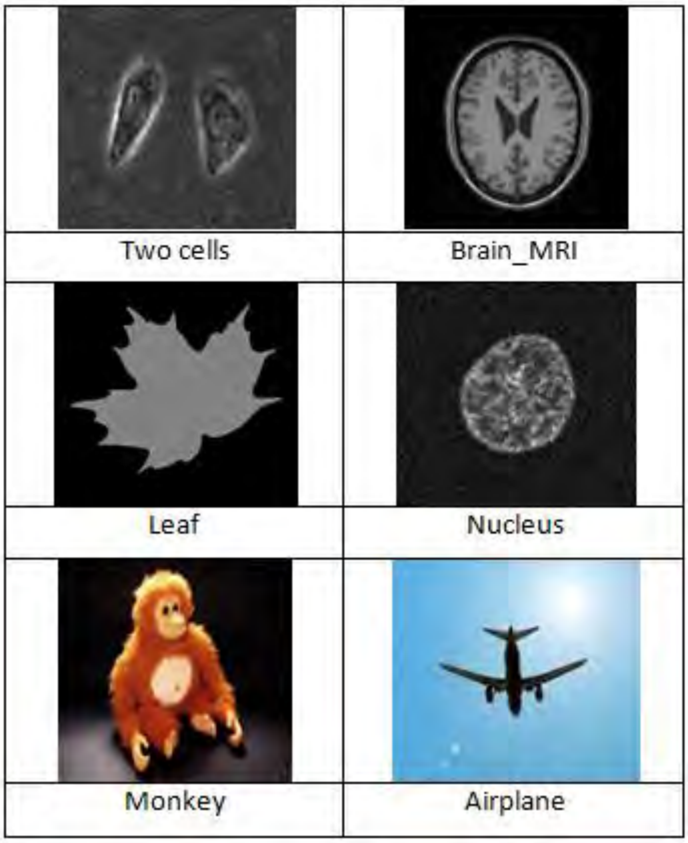


Figure 5.12: Reference Images

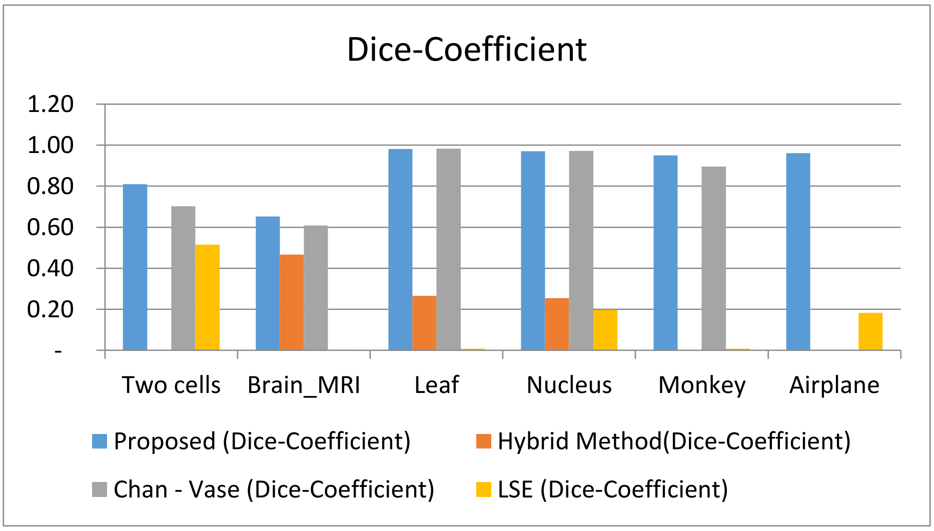
5.8.3 MSSD

MSSD means Mean Sum of Square distance. It is a numerical parameter by which it can be determined that how accurate an image is segmented. The formula of MSSD is given below:

MSSD = $\sum mean(reference\ Image - Segmented\ Image)^2$

The lower value of MSSD indicates the accurate segmentation result.

We take six different images as shown in figure 5.12 and compare our segmentation result through Dice coefficient, Hausdroff Distance and MSSD with the reference images. We also compare our result with the hybrid method, Chan-Vase method and LSE method which is given in figure 5.13, 5.14, 5.15 respectively.

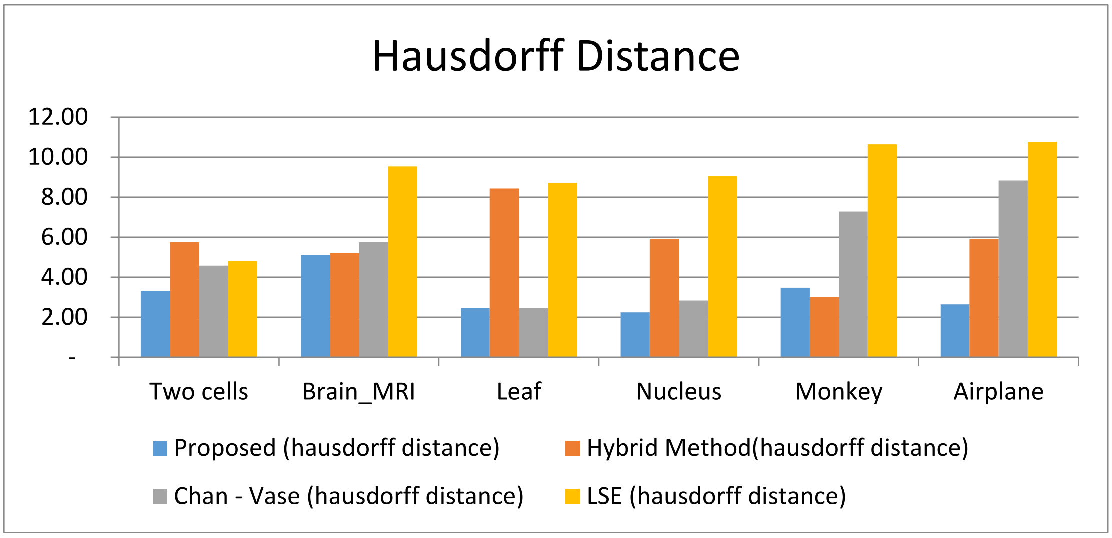


(a) Performance comparison in bar chart

	Proposed (Dice-Coefficient)	Hybrid Method(Dice-Coefficient)	Chan - Vase (Dice-Coefficient)	LSE (Dice-Coefficient)
Average	0.89	0.16	0.69	0.15

(b) Average performance index of different algorithms

Figure 5.13: Performance comparison in terms of Dice-Coefficient

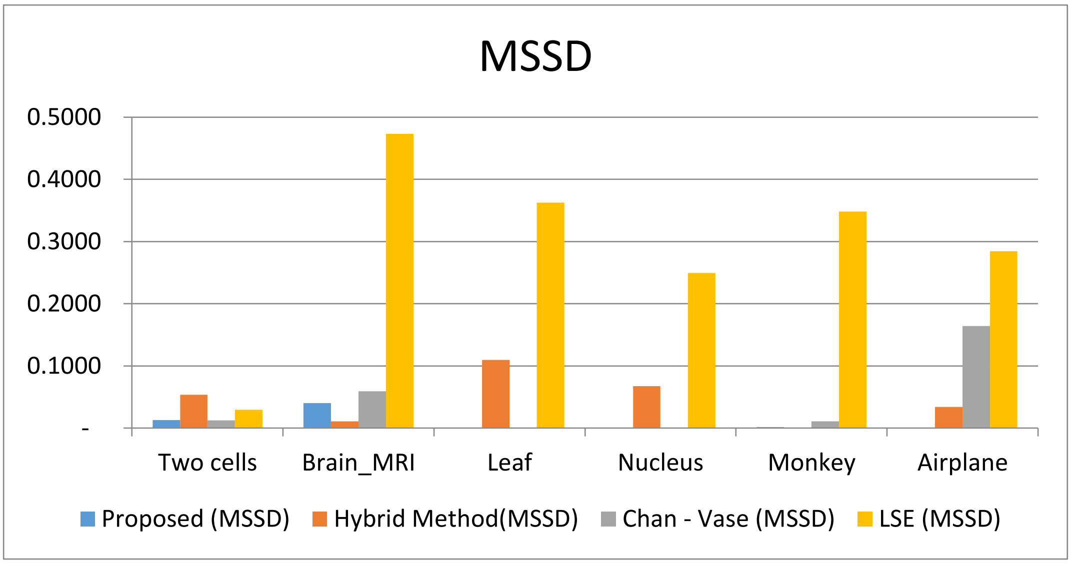


(a) Performance comparison in bar chart

	Proposed (Hausdorff distance)	Hybrid Method (Hausdorff distance)	Chan - Vase (Hausdorff distance)	LSE (Hausdorff distance)
Average	3.20	5.70	5.29	8.92

(b) Average performance index of different algorithms

Figure 5.14: Performance comparison in terms of Hausdorff Distance



(a) Performance comparison in bar chart

	Proposed (MSSD)	Hybrid Method(MSSD)	Chan - Vase (MSSD)	LSE (MSSD)
Average	0.0093	0.0459	0.0411	0.2911

(b) Average performance index of different algorithms

Figure 5.15: Performance comparison in terms of MSSD

5.9 Conclusion

In this chapter we have verified our propose algorithm and analysis our segmentation result. We have verified that our algorithm is independent of the initial position of the curve and thus it is globally convergence. We have included variable time-step by maximizing the between class variance. Therefore it has speeded up the convergence rate. Then we have included a special case which is narrowband approach, where initial curve evolves within neighborhood region and gives the segmented result by eliminating noise from background. Moreover it is also helpful to detect weak boundaries at the edge by the help of edge indicator function. Atlas we have compared our segmented result with other methods including hybrid method, Chan-Vese method, LSE method and DRLSE method. We have shown that among all the methods, our proposed method gives the better segmentation result. Not only that, we also evaluate our method by calculating numerical evaluation using Dice Coefficient, Hausdorff distance, MSSD and compare with other methods. In this time also our segmentation results give better performance than other methods.

6 Conclusion

The main purpose of the thesis is to develop a new robust Segmentation technique which can produce good segmentation result on different images. We have presented an image segmentation technique which combines edges, region and neighborhood criteria to segment an image. Therefore, we are considering the main basic three criteria by which an image could be segmented properly without taking much time. Traditional image segmentation methods only consider either edge or region for segmentation an image. However Hybrid combination is recently a new hot topic, in the fields of research, in image segmentation. Our hybrid method has shown great performance on medical images as well as in realistic ones. Our segmentation algorithm is globally convergence therefore it is independent of the initial curve. Moreover, by including edge indicator function our method can detect weak edges and boundaries, as the evolving curve will go slow at the boundaries and speed up at the uniform region. Not only that, if any image has very weak boundaries or edge ,we proposed a narrowband approach which will only consider the neighboring region and update the LSF in the neighborhood area of the evolving curve. Thus it is become useful to detect weak edges. Another benefit of it is it will remove noises from the background region in the narrowband approach. We also include a variable time-step method by maximizing the between class variance to get the optimum segmentation result. Thus our algorithm become fast and hence reduces computational complexity. Finally, we have compared our results with other traditional segmentation techniques like Chan-Vese method, DRLSE method, Level Set and Bias field correction methods and hybrid method for different types of images including synthetic, natural and medical images. We also compare our segmentation results by numerical evaluation using Dice Criterion, Housdroff distance and MSSD with a reference. It has shown that our technique has a better performance result for those images.

Although the proposed methods produce very robust and promising results, there are still a few aspects could be improved. Since narrowband approach give better performance for weak edge images and remove noise from background region. But it is not very suitable when there are multiple segmented images present in the foreground object. Then narrow band approach could not segment the image properly.

The domain of image segmentation is still improving. Many unsettled problems need to be defined and solved in this area. It is firmly believed that this domain will greatly advance in the future. Merge this technique with some other technique to get the better result for medical image segmentation. It could be used to segment cancer cell and may become a doctors helping hand in near future. For real time applications we may need a fast high performance system to segment any types of images. It can also be seen from the diverse nature of the techniques used that the field has a lot of room for improvement.

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