DEVELOPMENT OF A MODEL FOR REPAIRABLE SPARE PARTS PROVISIONING

A Thesis

by

MD. JAMAL UDDIN

Submitted to the Department of Industrial and Production Engineering, Bangladesh University of Engineering & Technology, Dhaka in partial fulfilment of the requirements for the degree of

MASTER OF SCIENCE IN INDUSTRIAL AND PRODUCTION ENGINEERING

September, 1983

BANGLADESH UNIVERSITY OF ENGINEERING & TECHNOLOGY, DHAKA.

CERTIFICATE

This is to certify that this work has been done by me and it has not been submitted elsewhere for the award of any degree or diploma.

Countersigned

4.9.83

(Supervisor)

4.9.83

Signature of the Candidate

DEVELOPMENT OF A MODEL FOR REPAIRABLE SPARE PARTS PROVISIONING

A THESIS

BY

MD. JAMAL UDDIN

Approved as to style and content by:

(Dr. A.B.M. Zohrul Kabir)
Asstt. Professor,
Dept. of IPE, BUET, Dhaka.
: Chairman

(Dr. A.F.M. Anwarul Hoque)
Professor and Head,
Dept. of IPE, BUET, Dhaka.
: Member

(Dr. M. Anwarul Azim)
Professor,
Dept. of IPE, BUET, Dhaka.
: Member

(Dr. M. Mizanur Rahman)
Associate Professor,
Dept. of IPE, BUET, Dhaka.
: Member

(Abdur Rashid Talukder)
Director (RQC),
BJMC, Dhaka.
: Member (External)

September, 1983

BANGLADESH UNIVERSITY OF ENGINEERING & TECHNOLOGY, DHAKA.

ALL PRAISES ARE FOR ALLAH, THE ALMIGHTY

ACKNOWLEDGEMENTS

The author wishes to express his heartfelt gratitude and sincerest indebtedness to Mr. A.B.M. Zohrul Kabir, Asstt. Professor, Dept. of IPE, BUET,Dhaka for his expert guidance, suggestions, encouragement, interest, contribution to new ideas and constant supervision at all stages of the research.

Sincere gratitudes are due to Dr. M. Anwarul Azim, Dean, Faculty of Mechanical Engineering, BUET for his kind interest and encouragement.

Sincere gratitudes are due to Dr. A.F.M. Anwarul Hoque, Professor and Head, Dept. of IPE, BUET for his kind suggestions at different times.

The author expresses his gratitude to Dr. M. Mizanur Rahman, Associate Professor, Dept. of IPE, BUET for his kind interest in the research work.

Special debt of gratitudes are due to Mr. Ghulam Kabir Beg, former Lecturer of Mathematics, BUET and presently a Research Assistant, UPM, Dhahran, Saudi Arabia for his generous help and heartiest interest.

Due thanks are for the friends who helped in many ways.

The author is very grateful to the following:

BJMC authority for arranging this M.Sc.Engg. course on deputation.

BUET authority for awarding fellowship and necessary expenses for this research.

Due thanks are for operators of the computer centre for their cooperation and also for Mr. Abdur Rahman, Draftsman for his nice drafting.

Due thanks are for Mr. Abdul Malek for his patient, accurate, neat and smooth typing.

ABSTRACT

10

A general model for repairable spare parts provisioning has been developed considering a group of M identical units in operation, served by an on-site store containing N spares. As the units fail, they are replaced by spares from the store. The failed units are sent away for repair and then returned to the store. Should the units fail in quick succession, the stock of spares may be exhausted causing certain amount of delay. In the long run, each of the M units will experience an unavailability U due to occasional stock-outs. Using state transition diagram, mathematical relationships have been derived between quantities M,N,U, unit failure rate in operation, λ; unit failure rate in storage, λ_s; and repair rate μ. Tables and graphs have been prepared in order to determine minimum number of spares needed to keep unavailability of plant units within certain limits. Cost-optimal inventory policy has also been considered.

CONTENTS

Page

LIST OF TABLES

LIST OF FIGURES

NOTATIONS

M	= Number of working units served by an on-site store
N	= Number of spares carried by store
U	= Unavailability of a particular working unit owing to lack of spares
D	= Mean time spent by unit out of service following an unsatisfied demand owing to stock exhaustion
T	= Mean time spent by unit away from site ($T = 1/\mu$) in hour.
μ	= Repair rate of unit in hour
λ	= Failure rate of unit when working, in hour
λ_s	= Failure rate of unit in storage, in hour
ϕ	= Mean time between unsatisfied demands for spares as applied to a particular unit.

CHAPTER 1

INTRODUCTION

CHAPTER 1

INTRODUCTION

1.1 General Introduction

Over the past few decades, a substantial number of mathematical models have been developed relating to the various aspects of inventory management. All these inventory modellings are for maintaining, for a given financial investment, an adequate supply of spares, raw materials, finished goods etc. in order to meet an expected distribution or pattern of demand.

The inventory of spares is utilised in replacing the failed or going to fail units in breakdown maintenance or in preventive maintenance work to maintain an efficient continuity of operation of equipment to minimize down-time and so to minimize cost due to down-time. The models are applied to determine the optimum number of spares to minimize the overall cost of inventory which includes ordering cost, holding cost, stockout cost etc. By such modelling, reorder level, reorder period etc. can be calculated.

To manufacturing management, the word 'inventory' immediately connotes finished goods inventories, the stocks of the company's end product available for distribution or sale to the company's customers. These modellings included the costs of acquiring inventory, holding it and failing to supply consumers, to arrive at inventory policies which will minimize the total of these costs or maximize profit. In those modellings of spare parts, considerations were paid to the consumable items i.e. spares once failed are not recoverable and are replaced only by new spares.

There are situations where spares are seldom consumed. When equipment fails, the failed unit is repairable i.e. item subject to repair when it fails. The recoverable items are typically expensive. The failed unit is sent for repair to workshop or depot and it is replaced by a good one from the onsite store and the failed unit after repairing from depot returns to the store. In such situations, the typical inventory modelling for spares so far developed, can not be applied to determine the optimum quantity of spares required to keep unavailability of plant units due to occasional exhaustion of spares within certain limits.

One typical case of such repairable spare situation is the 'Power Pack' in Drilling Rig Machine. While in operation, the 'Power Pack' fails and causes breakdown of the drilling rig machine. To minimize down time of the machine, a good spare (power pack) is used from the store. The failed 'power pack' is sent to workshop for repair and then returned to the store. The unit may also fail in the store and when detected, it is repaired and stored again.

1.2 Objective and methodology of the research

The objective of this research is to develop a general model considering a group of 'M' identical units served by a store containing 'N' spares. Possibility of spare failure in store is also incorporated into the model. The objectives of the problem are to determine the mininum number of spares 'N' in order to keep unavailability of 'M' plant units due to occasional exhaustion of stocks, within certain limits and to determine the optimal number of spares in order to minimize total cost, which may include all inventory costs and cost due to lost production.

For the construction of the generalized mathematical model and for the solution of the model, the following procedures are adopted.

i) Formation of a state transition diagram following the events of failure of an operational unit, failure of spare in storage and repair of the failed item in workshop.

ii) Derivation of formulae for transition from one state to another.

iii) Derivation of a set of simultaneous differential equations from state transition rate diagram and thereupon formation of transition matrix for calculation of the individual state probabilities.

iv) Derivation of the formulae for calculating steady state unavailability of 'M' operational units, probability of stock exhaustion and other associated parameters.

v) Development of a computer programme in FORTRAN-IV (language) to generate the coefficients of individual state probabilities, and matrix solution procedure for the model.

vi) Development of a model to determine optimal inventory of spares to minimize cost. Formulation can be used on variable costs only, which can affect inventory decision.

vii) Presentation of results in tabular as well as graphical form, showing the interrelationship of various parameters affecting the number of spares 'N' in store.

Although the objective set above concern a model formulation of spare parts provisioning, an application of the model has been made in the thesis. An example showing its application has been given on an engineering unit, which is a power pack used in a deep tube-well sinking drilling rig. The original data has been collected at various times. The computer model was developed using the data. The mathematical model formulation and its computerization have been given in Chapter 3 and 4.

CHAPTER 2

LITERATURE REVIEW

CHAPTER 2

LITERATURE REVIEW

Over the last few decades, literatures involving the inventory management have grown extensively. The firm establishment of reliability engineering disciplines has helped the development of many inventory models. Moreover, the practical need for subtle and delicate maintenance policies has stimulated theoretical interest and led to the development of models that possess theoretical novelty as well as practical importance.

Organizations maintain inventories e.g. spare parts, raw materials, finished goods etc. Spare parts inventories serve to keep the equipment in operation (within certain allowable unavailability); raw material inventories serve as inputs to the production process and finished goods inventories are used to satisfy customer needs. Since these inventories often amount to a considerable investment, decisions regarding the amount of inventory are important. The basic inventory system and models given by different authors are presented here.

Fifty years ago, "Wilfredo Pareto" pointed out that many variables have a 'vital few' and a 'trival many'. Very often a small number of important items dominate the results while at the other end of the line, a large number of items whose volume is so small that they have little effect on results. So it should be the primary aim in stock control, to control the 'fast-moving/expensive' items, since, by doing so, greater potential savings are possible than by controlling inexpensive items, the usage of which is small.

Ray Wild[1], F.G. Moore[2], William Vories[3], Mayer, R.M.[4], Moore, P.G.[5], Voris, William[6] and others discussed in their writings that, one of the ubiquitous phenomena of business is expressed by the so-called '80/20 law'. In relation to the inventory stock, the law reads as follows:

'Eighty per cent of the firm's total inventory cost is caused by only twenty per cent of all items'. In other-words the twenty per cent high-cost/high usage items account for eighty per cent of total inventory costs. This law or relationship is expressed by the ABC or pareto curve, and often such curves are used by companies to divide stock items into three classes, A,B and C, i.e. those accounting for 80, 13 and 7 per cent of total inventory costs respectively.

Such a classification, once achieved, enables appropriate stock control 'rules' for each type of product or item to be implemented. Thus a comprehensive and regular stock procedure should be designated for items of type A. Less rigorous control is necessary for type B, whereas for C a simple procedure is probably sufficient.

Ray Wild, Shamblin, Churchman, Sasieni, Spriegel[1,7,8,9,10] discussed in their writings few deterministic inventory models which provide an economic order quantity.

Model-1 Purchasing Model: No shortage:-

This model is one of the simplest inventory models. It makes the following assumptions.

(a) Demand is at a constant rate.

(b) Replacement is instantaneous.

(c) All cost coefficients; i.e.

(i) Cost per unit

(ii) Cost of making one purchase

(iii) Cost of holding one unit in inventory

are constant.

Model-2 Manufacturing Model: No shortages:-

In this model, the assumptions are same as model-1 except that the replenishment rate is finite and greater than the demand rate.

Model-3 Purchasing Model with shortage:-

This model has the same assumptions as Model-1 except that shortages are allowed.

Model-4 Manufacturing Model with shortage:-

In this model, the assumptions are same as model-2 except that shortages are allowed.

All the above models have related to the ordering of a quantity of a single product, but in practice many occasions exist when ordering decision and ordering cost covers quantities of more than one type of product. Ray Wild[1] discussed in his writings such a situation in which there are 'N' different types of product to be ordered and for an optimum ordering policy, all ordering cycles should be of equal length and orders for different products should be made at the same time.

It was discussed in the literatures[1,2,3,6] that the variations in either or both, lead time and demand, might (unless buffer stock is used) result in a stock-out situation. Lead time usage for any product can be determined by an examination of stock records, Lead time usage will not be constant, consequently a probability distribution might be obtained.

In practice, the probability distribution of lead time usage often conforms to the lognormal distribution; however, the normal distribution can often be used as a close approximation and, of course, is computationally more convenient. To set the reorder level at a value which represents the absolute maximum lead time usage may be an unduly costly policy. Setting the reorder point to a lower level will mean stock-outs will occur, even so, the cost over a given period of a few stock-outs may be less than the additional holding costs that would be incurred if stock-outs were to be avoided.

Ultimately, it is the magnitude of these costs, stock-out and holding, that will determine the reorder level and hence the buffer stock level.

Shamblin[7] in his writings discussed that there are two widely used inventory systems, fixed order size system and fixed order interval system. He designated the fixed order size system as the 'Q' system and the fixed order interval system as the 'P' system. When the inventory system is deterministic and the demand rate is constant, there is little difference between the 'Q' and 'P' systems. Difference between the two systems occurs when demand, lead time or both become probabilistic.

One approach to handling probabilistic inventory system is to superimpose an inventory model on a safety stock. The safety stock serves as a cushion that absorbs variations in demand and lead time. It also serves as a means of adjusting the number of shortages. But the 'Q' system requires less safety stock than the 'P' system. In general, the order quantity for both 'Q' and 'P' inventory systems is given by:

Order Quantity = optimum 'Q' + safety stock - inventory on hand - units in order + average demand over lead time.

E.S. Buffa[11] established two ways of classifying production inventory system:

(a) Continuous versus intermittent system.

(b) System that produce for inventories versus those that do not.

An operations management's point of view requires not only to look at the inventory and the material flow of processes within the manufacturing phase, but to examine the over-all system of flow from suppliers of raw materials through manufacturing and distribution and ultimate delivery to users.

When the total system is not viewed as an integrated whole, organizational fragmentation can occur which is likely to lead to each organizational unit establishing policies and practices which may not be the best ones for the system as a whole. The concepts of 'Integrated material flow management' also require to look at not just the production unit separately or the distribution of products separately but also the combined production distribution system which is in effect. The objective in such a situation is to minimize total system costs rather than certain costs taken separately.

Churchman, Folts[12,13] discussed in their writings an inventory model in which the unit manufacturing or purchase cost is variable. This situation is quite typical for purchased parts which are subject to quantity discounts.

All the above models are valid for single use items, i.e. items once consumed are not repairable. So the above models are not applicable in the situations where the items are recoverable.

John A. Muckstadt(14) described a mathematical model for the control of multi-item inventory system for recoverable items. This model, called MOD-METRIC, permits the explicit consideration of a hierarchical parts structure. For example, a major assembly may consist of a casing and several components. The components are subordinate to the assembly in the parts hierarchy. The objectives of the model are to describe the logistics relationship between the components and the final assembly, and to compute base and depot spare stock levels for all items with explicit consideration of this logistics relationship.

G.G. Pullum and H. Grayson(15) in their paper gave a model for repairable spare parts provisioning, considering parameters like:

failure rate of the item in operation,
repair rate in the workshop, and failure rate in the storage.

They have given a general state transition matrix and a matrix solution procedure to determine:

unavailability per unit,
downtime per unit,
cycle length between stock-outs, and
probability of stock-outs with or without all units in operation.

From their models, a particular number of spares required to keep unavailability down to a certain limits, can be obtained. But the authors did not go to the extent of finding a cost optimal policy by trading off the cost of lost production with the inventory cost.

CHAPTER 3

FORMULATION OF THE MATHEMATICAL MODEL

CHAPTER 3

FORMULATION OF THE MATHEMATICAL MODEL

3.1 Introduction to Model Development

Let a group of 'M' identical units be served by an on-site store, each unit having a constant failure rate 'λ' and let the store contain 'N' spares each with a failure rate 'λ_s'. It is supposed that when a unit fails in service, it is sent to a repair depot, and after having been repaired, it is returned to the store where it arrives at time 'T'. Thus 'T' is the total transit time spent by a unit away from the site where the 'M' units and 'N' spares are located. The repairing rate is 1/T which is denoted by 'μ'. The spares failing in storage are also sent to repair depot and then sent back to the store after repair. The repair cycle is shown in Fig. 3.1.1.

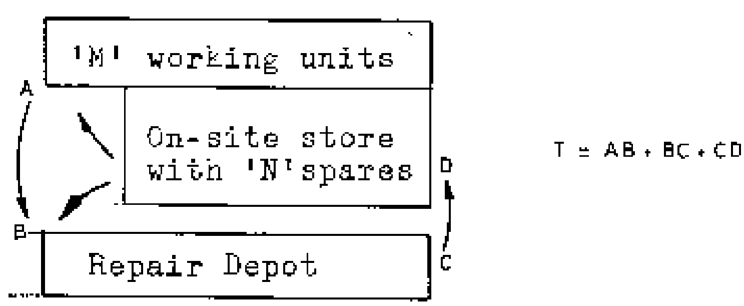


Fig. 3.1.1 Unit failure-repair-storage cycle.

It is assumed that no preferential treatment is given to any of the 'M' units/'N' spares so that in the long run each unit will experience the same delay on average as any other. Let 'D' be the mean time spent by a unit out of service following an unsatisfied demand due to the exhaustion of stock of spares, and let 'ϕ' be the mean time between such unsatisfied demand for spares as applied to a particular unit. Thus the steady state unavailability, 'U' of a particular working unit owing to lack of spare is given by $U = D/\phi$, provided 'D' is much less than ϕ. Thus the problem can be formulated to find

(a) Unavailability of a particular working unit owing to lack of spares.

(b) Mean time spent by unit out of service due to lack of spare.

(c) Mean time between unsatisfied demand for spare as applied to a particular unit.

(d) Probability of stock exhaustion while 'M' units still working.

(e) Probability of stock exhaustion regardless of number working, and

(f) Probability of failing to meet requests for spares, from the parameters $M, N, \lambda, \lambda_s, \mu$.

In otherwords, from a given or allowable value of U or D or ϕ, the optimum number of spares required can be formulated from the other parameters.

3.2 Underlying Principles of the Model

The behaviour of the system as failure occurs and repair carriedout, is described by means of the transition rate diagram illustrated in Fig. 3.2.1. At any instant in time, the system is regarded as being represented by one of the many states drawn in the diagram. As failure occurs, the system moves from a state (it was in when the failure occured) to another appropriate state in the next column (level) on the right. As item arrives at the store from the repair depot, the corresponding movement is from right to left.

3.2.1 Failure Transition of the Model

In the diagram 3.2.1, the transitions shown by dashed lines are those which arise when a unit selected at random from store is found failed and hence returned to the repair depot. Let (t) denote unit on transit and (u) denote unit of unknown failure.

The following assumptions are made for the model:

i) The parameters 'failure rate of unit when working', 'failure rate of unit when in storage', 'repair rate of unit' are constants with respect to time.

ii) Defective items are despatched for repair as soon as they are discovered and repaired items are returned to the store as they are repaired.

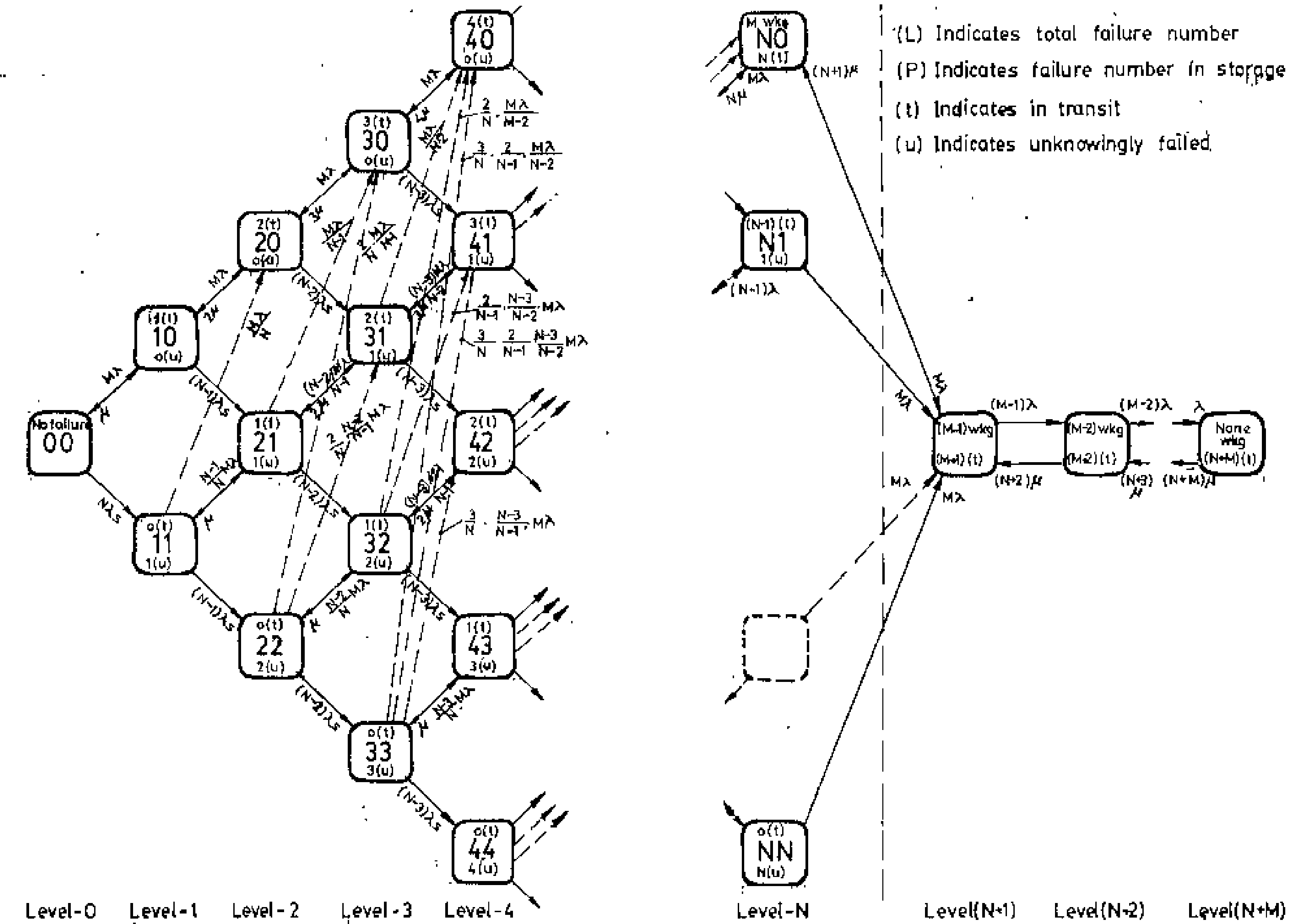


Fig3.2.1, Transition rate diagram

iii) There are available as many repair men as there are units needing repair.

iv) Only units discovered as failed and units on transit can be repaired.

First, a situation is considered in which the system state is such that all 'M' units and all 'N' sapres are in a nonfailed state. This state is denoted by state (00) in the transition rate diagram. In the notations of states, the first digit denotes the total number of failures which also indicates the 'level' number, and the second digit denotes the number of spares failed in store unknowingly. Now if a unit fails at state (00), the system can enter any one of the two states (10) or (11).

Let it be supposed that an operational unit fails. In order to keep the 'M' units working, the failed unit is replaced by a good spare from store out of 'N' spares and the failed unit is sent for repair, that is on transit. This state is denoted by state (10). In this state, there is one unit on transit, 1(t) and no unknown failure, 0(u). Since failure rate of operational unit is 'λ' and there are 'M' units in total, the probability that the system will move from state (00) to state (10) is '$M\lambda$', that is, the transition rate between state (00) to state (10) is '$M\lambda$'. In this situation there are (N-1) good spares in the store and there is no failed spare.

There may be another situation denoted by state (11). This represents the system when one of the spares fails unknowingly in the store. Since, the failure rate of spare in store is 'λ_s' and the number of spares is 'N', the probability that the system will move from state (00) to state (11) is '$N \lambda_s$'. In this situation, there are (N-1) good spares, 1 unknowingly failed spare and a total of N spares in the store. If the system is in state (10) and another operational unit fails, then the state (20) is reached. To keep 'M' units in operation, the failed unit is replaced by a good spare from the store and the failed one is sent on transit. Now there are two failed units. The probability of transition from state (10) to state (20) is '$M\lambda$'. In state (20), there are (N-2) good spares in the store and no failed spare in the store.

If the system is in state (10) and if instead of an operational unit failing, a spare fails unknowingly, the system moves from state (10) to state (21). The probability of this transition is $(N-1)\lambda_s$. In state (21), there are (N-2) good spares, one unknowingly failed spare and a total of (N-1) spares in the store. If the system is in state (11) and an operational unit fails, then the system may move to either state (21) or to state (20). When the failed operational unit is replaced by a good spare from store out of (N-1) good spares, state (21) is reached. The failed one is sent on transit. The probability of operational unit failing is '$M\lambda$' and the probability of replacing the failed unit by a good spare is $(N-1)/N$. So the probability of transition from state (11) to state (21) is $(N-1)(M\lambda)/N$. In state (21) there are $(N-2)$ good spares, one unknowingly failed spare and a total of $(N-1)$ spares in the store.

The transition from state (11) to state (20) occurs, if upon failure of an operational unit, the replacement spare picked up from the store is the one, which has already failed unknowingly in the store. The operational failed unit is sent on transit. Having been detected, the unknowingly failed spare is also sent on transit and is shown by dashed line in the figure. Now there are two items on transit and the failed unit is replaced by a good spare from the store. The probability of coming out the unknowingly failed unit from state (11) is $1/N$ and the probability of operational unit failing from state (11) is '$M\lambda$'. So the probability of the system coming to state (20) from state (11) is $M\lambda/N$. At state (20) there are $(N-2)$ good spares, and no failed spare in the store.

From state (11), if another spare fails in store unknowingly, the state (22) is reached. This is the state where there are $(N-2)$ good spares, two unknowingly failed spares, $2(U)$; and total N spares in the store. The probability of the system coming to state (22) from state (11) is $(N-1)\lambda_s$.

By similar logic, the transition rates from various states of one level to the various states of the succeeding level can be established upto level 'N'. For example:-

Preceeding state	Succeeding state	Transition probability
20	30	$M\lambda$
	31	$(N-2)\lambda_s$
21	31	$(N-2)(M\lambda)/(N-1)$
	30	$M\lambda/(N-1)$
	32	$(N-2)\lambda_s$
22	32	$M\lambda(N-2)/N$
	31	$(2/N)M\lambda(N-2)/(N-1)$
	30	$(2/N)M\lambda(N-1)$
	33	$(N-2)\lambda_s$
30	40	$M\lambda$
	41	$(N-3)\lambda_s$
31	41	$M\lambda(N-3)/(N-2)$
	40	$M\lambda/(N-2)$
	42	$(N-3)\lambda_s$
32	42	$M\lambda(N-3)/(N-1)$
	41	$((N-3)/(N-2))2M\lambda/(N-1)$
	40	$(2/(N-1))M\lambda/(N-2)$
	43	$(N-3)\lambda_s$

Preceeding state	Succeeding state	Transition probability
33	43	$M\lambda(N-3)/N$
	42	$(3/N)((N-3)/(N-1))M\lambda$
	41	$(3/N)(2/(N-1))((N-3)/(N-2))M\lambda$
	40	$(3/N)(2/(N-1))(M\lambda/(N-2))$
	44	$(N-3)\lambda_s$

At level 'N', when the system is in state (NO), all the 'N' spares are on transit and no good spare lies in store and so there lies no unknowingly failed spare in the store. When the system is at state (N1), there are (N-1) spares on transit, one spare is left in store unknowingly failed and no good spare is lying in store. At state (NN) there is no spare on transit; all the 'N' spares in store have failed unknowingly and obviously, there is no good spare left in store.

From any state of level 'N', if another operational unit fails, there will remain no more spare available to keep 'M' units in operation and so (M-1) units will be in operation and the system will cross the dotted line in the transition rate diagram and will enter the state (N+1). The probability of failing an operational unit from any state of level 'N' is '$M\lambda$'. The transition rate diagram thus 'collapses' to a series of single states (one state per column) as soon as the dotted line is crossed. Now there are (N+1) units in transit.

If the system is at level (N+1) and if another operational unit fails, one more operational unit will become idle, that is, (M-2) operational units will be working, the probability of which is (M-1)λ, now (N+2) spares are in transit. Similarly if the system is in state (N+M-1) and another operational unit fails, there will remain no unit in operation and all the (M+N) items will be in transit. The probability of the system to move from state (N+M-1) to state (N+M) is 'λ' and no other states are assumed to be possible.

3.2.2 Repair Transition of the Model

In the repair transition, it is evident that movements of states will be from right to left as repair of the failed units are carried out. For example, if a repaired unit is received while the system is in state (20), the system will return to state (10). Similarly from state (21), after receiving a repaired item, the system will return to state (11). As the repair rate of item is 'μ', at state (20) there are two units in transit for repair, so repair transition rate from state (20) to state (10) is 2μ. Also at state (21), there is one unit in transit for repair, so the repair transition rate from state (21) to state (11) is 'μ'. When the system is in state (N+1), there are (N+1) units in transit, the repair transition rate from state (N+1) to state (N0) is (N+1)μ . In the same way, the repair transition rates for other levels can also be established.

3.3 Derivation of Mathematical Formulations From Transition Rate Diagram

Let at any instant, $P_{ij}(t)$ denote the probability of the system being in (ij) state, where 'i' varies from '0' to 'N+M' and 'j' varies from '0' to 'N'. The rate of change of $P_{ij}(t)$, which is denoted by $\dot{P}_{ij}(t)$, is given by

$\dot{P}_{ij}(t)$ = Summation of the products of the probability rates of coming from different possible states to 'ij' state and probability of the system being in those respective states — summation of the products of the probability rates of leaving 'ij' state and the 'ij' state.

$\dot{P}_{ij}(t)$ = Σ[(probability of possible entering states)x (associated transition rates)] — Σ[(probability of possible leaving states)x (associated transition rates)].

Thus from the transition rate diagram shown in Fig.3.2.1 at any instant of time, the movement between adjacent states in the diagram can be described by the following set of simultaneous differential equations which can be written as:-

$$
\begin{aligned}
\dot{P}_{00}(t) &= \mu P_{10}(t) - (M\lambda + N\lambda_s)P_{00}(t) \\
\dot{P}_{10}(t) &= 2\mu P_{20}(t) + M\lambda P_{00}(t) - [(N-1)\lambda_s + \\
&\qquad M\lambda + \mu P_{10}(t) \\
&\vdots \\
\dot{P}_{N0}(t) &= M\lambda P_{N-1\,0}(t) + \tfrac{1}{2} M\lambda P_{N-1\,1}(t) + \\
&\qquad \tfrac{1}{3} M\lambda P_{N-1\,2}(t) + \ldots + \tfrac{1}{N} M\lambda P_{N-1\,N-1} + \\
&\qquad (N+1)\mu P_{N+1}(t) - (N\mu + M\lambda)P_{N0}(t) \\
&\vdots \\
\dot{P}_{NN}(t) &= \lambda_s P_{N-1\,N-1}(t) - M\lambda P_{NN}(t)
\end{aligned}
\qquad (3.3.1)
$$

$$
\begin{aligned}
\dot{P}_{N+1}(t) &= M\lambda(P_{N0}(t) + P_{N1}(t) + \ldots + P_{NN}(t)) + \\
&\qquad (N+2)\mu P_{N+2}(t) \\
&\qquad - ((N+1)\mu + (M-1)\lambda)P_{N+1}(t)
\end{aligned}
\qquad (3.3.2)
$$

$$
\begin{aligned}
\dot{P}_{N+2}(t) &= (M-1)\lambda P_{N+1}(t) + (N+3)\mu P_{N+3}(t) \\
&\qquad - ((N+2)\mu + (M-2)\lambda)P_{N+2}(t) \\
&\vdots \\
\dot{P}_{N+M-1}(t) &= 2\lambda P_{N+M-2}(t) + (N+M)\mu P_{N+M}(t) \\
&\qquad - (\lambda + (N+M-1)\mu)P_{N+M-1}(t) \\
\dot{P}_{N+M}(t) &= \lambda P_{N+M-1}(t) - \mu(N+M)P_{N+M}(t)
\end{aligned}
\qquad (3.3.3)
$$

When the statistical equilibrium has been reached, the left hand sides of all the equations become zero and the quantities $P_{00}(t)$, $P_{10}(t)$ .... $P_{N+M}(t)$ become constant with respect to time and so are written as P_{00}, P_{10} ... P_{N+M}.

From the transition rate diagram (Fig. 3.2.1) the unavailability of any particular working unit is given by,

$$U = \frac{1}{M}(P_{N+1} + 2P_{N+2} + \dots + MP_{N+M}) \qquad (3.3.4)$$

Let, D = average downtime/(hour) owing to lack of spare due to exhaustion, and

ϕ = average time/(hour) between such down time (ie. average cycle length), then the following expressions can be deduced for 'D' and 'ϕ'

$$D = U/(\Sigma P_N - U)\lambda \quad \text{(Ref. 15)} \qquad (3.3.5)$$

[but, when $\mu \gg \lambda$, approximate value of D is given by

$$D = \frac{1}{(N+1)\mu}$$]

and $\phi = (1 - U)/(\Sigma P_N - U)\lambda$ (Ref. 15) (3.3.6)

where ΣP_N is the probability of stock exhaustion regardless of number working.

3.4 Determination of Optimal Number of Spares to Minimize Cost

Expected total variable cost = Expected cost due to production loss + expected variable cost due to inventory + total item cost.

(a) Expected cost due to production loss per unit time (hour)

Let V = production loss of a single operational unit per unit time.

P_{N+1} = probability of one operational unit not working due to spare spare stock exhaustion.

P_{N+M} = probability of 'M' operational unit not working due to spare stock exhaustion.

Therefore, expected cost due to production loss per hour

$$= VP_{N+1} + 2VP_{N+2} + \dots + MVP_{N+M}$$

$$= V \sum_{i=1}^{M} iP_{N+i}$$

$$= MVU$$

Since $\sum_{i=1}^{M} iP_{N+i} = MU$ (from eqn. 3.3.4)

b) Expected variable cost due to inventory

The only variable inventory cost in this particular situation is the holding cost of 'N' spares. This cost can be expressed as $i_c bN$ where

i_c = holding cost as percentage of item value

b = item value

c) Total item cost = bN

So, expected total variable cost

$$T_c = MVU + i_c bN + bN$$

$$= MVU + bN(1+i_c)$$

$$= \left(\frac{MVU}{(1+i_c)b} + N\right)(1+i_c)b \,.$$

Denoting by R the relative cost of lost production in terms of holding cost and item cost, T_c can be expressed as

$$T_c = (MRU + N)(1 + i_c)b$$

Dividing throughout by $(1 + i_c)b$, the scaled expected total variable cost in multiples of $(1 + i_c)b$ is given by -

$$T_{cs} = MRU + N$$

From this equation, optimal number of spares can be determined by minimizing T_{cs}.

CHAPTER 4

SOLUTION PROCEDURE FOR MATHEMATICAL MODEL

CHAPTER 4

SOLUTION PROCEDURE FOR MATHEMATICAL MODEL

4.1 Matrix Solution for Individual Probabilities upto Level 'N'

The equations 3.3.1 to 3.3.3 in Chapter-3 are such that any one variable (individual probability) can be obtained from the remainder in terms of another one. To solve for the various values of P_{ij}, another equation is required. As the sum of the probabilities of all the states is equal to unity,

$$P_{00} + P_{10} + \dots + P_{N0} + P_{N1} + \dots P_{NN} + P_{N+1} + \dots + P_{N+M} = 1 \tag{4.1.1}$$

Dividing the equation (4.1.1) throughout by P_{N+1}, the new equation is obtained as follows:

$$\frac{P_{00}}{P_{N+1}} + \dots + \frac{P_{NN}}{P_{N+1}} + \frac{P_{N+1}}{P_{N+1}} + \dots + \frac{P_{N+M}}{P_{N+1}} = \frac{1}{P_{N+1}} \tag{4.1.2}$$

Dividing the equations of (3.3.1) throughout by P_{N+1}, another new set of equations is obtained whose variables become

$$\frac{P_{00}}{P_{N+1}}, \quad \frac{P_{10}}{P_{N+1}}, \dots\dots \text{ upto } \frac{P_{NN}}{P_{N+1}}$$

Let these new variables be denoted by

X_{00}, X_{10}, ... X_{NN} and

Let $$X_{00} + X_{10} + \dots + X_{NN} = X \tag{4.1.3}$$

This new set of simultaneous equations is solved by "Gauss Jordan Elimination Method" to obtain the values of the variables (X_{00}, X_{10} ... upto X_{NN}), i.e to obtain the value of 'X'. For this solution procedure, number of equations must be equal to the number of unknown variables. For this particular problem,

Number of variables = Number of equations

$$= (N+1)(N+2)/2 = NO \text{ (say)}$$

Let the augmented matrix be 'A'. Therefore, the dimension of the augmented matrix becomes

NO x (NO + 1) or (NO x NOO)

where NOO = NO +1

The elements of the last column of this matrix are zero except the element of the row (N(N+1)/2) + 1= IJ (say) which is from the equation, equal to - (N+1) μ= H(say).

From the Fig. 3.2.1, let the total number of failure of units be denoted by 'L' and the number of unknowingly failed spares be denoted by 'P'. Let (L P) denote the state of the system at any instant of time. The (L P) state equation gives the coefficients of the Ith row of the matrix where

I = (L(L+1)/2) + P + 1

The coefficients of the variables ($X_{JL\ JP}$) are stored in the Jth columns of the Ith row where

J = (JL(JL+1)/2) + JP + 1

(JL JP) is the state from which the system moves to the (L P) state.

From the notations so far used, the following can be written.

N - L = Total number of good spares in store.

P = Total failed spare in store.

L - P = Total units in transit

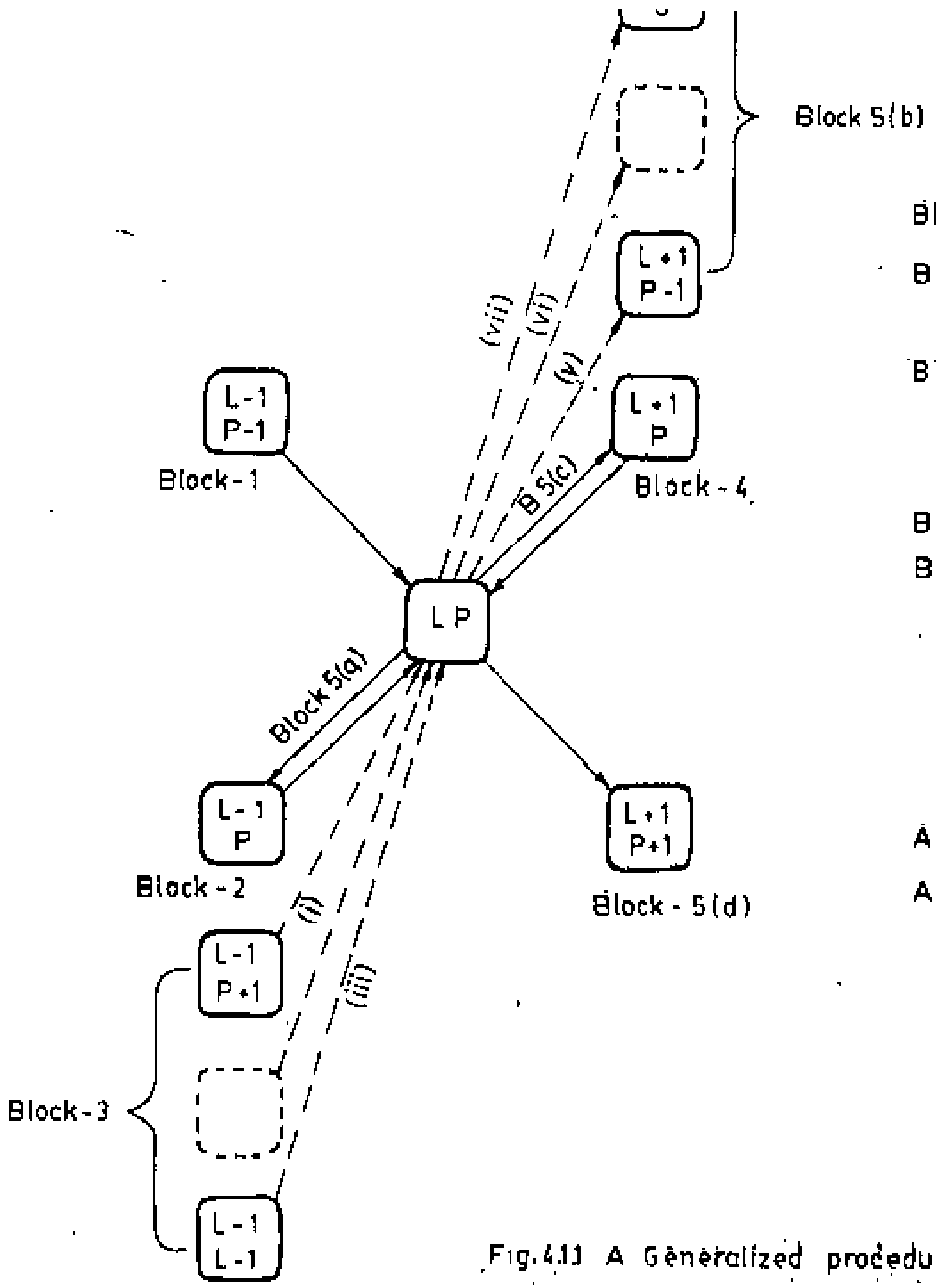
N - L + P = Total stock in store.

To generate the coefficient matrix, of Fig. 3.2.1, a generalized procedure is shown in Fig. 4.1.1, which is very simple to programme by computer.

The generalized procedure (Fig. 4.1.1) is explained as follows:

Block-1, There are (N-L+1) good spares in store, so the probability of failing spares unknowingly is $(N-L+1)\lambda_s$, which is the probability of moving to state (L P) from state (L-1 P-1).

Block-2, When the system is in state (L-1 P), there are (N-L+1) good spares and total (N-L+1+P) units in the store. When an operational unit fails, the probability of replacing it by good spare is $((N-L+1)/(N-L+1+P))M\lambda$ which is the probability of the system moving from state (L-1 P) to state (L P).



Block-1. $(N-L+1)\lambda S$, but not valid when $p = o$

Block-2 $\frac{N-L+1}{N-L+1+P} M\lambda = \frac{X}{Y} M\lambda$ where $X = N-L+1$, $Y = N-L+1+P$

Block-3 (i) $\frac{X}{Y} \cdot \frac{P+1}{Y+1} M\lambda$

(iii) $\frac{X}{Y} \cdot \frac{P+1}{Y+1} \cdots\cdots \frac{L-1}{Y+L-P-1} M\lambda$

Not valid when $P = L$

Block-4. $(L-P+1)\mu$

Block-5 a) $(L-P)\mu$, Not valid when $P = L$

b) $\frac{P}{Q}\left(1-\frac{P+1}{Q-1}\right)M\lambda + \cdots + \frac{P}{Q} \cdot \frac{P+1}{Q-1} \cdots \frac{1}{Q-P+1} M\lambda$

where $Q = N-L+P$

c) $\left(1-\frac{P}{Q}\right)M\lambda$

d) $(N-L)\lambda S$

$A(I,J) = [(B-1)+(B-2)+(B-3)+(B-4)]$

$A(I,I) = -[5(a)+5(b)+5(c)+5(d)]$

Fig. 4.11 A Generalized procedure to generate coefficient matrix upto level N

Block-3 When the system is in state (L-1 P+1), there are (P+1) failed spares in the store. If the system moves to state (L P) by failing an operational unit, the probability of replacing that one by a good spare is

$$\frac{X}{Y} \quad \frac{P+1}{Y+1} \quad M\lambda \text{ . where } \quad N - L + 1 = X$$
$$N - L + 1 + P = Y$$

Similarly when the system is in state (L-1 L-1), there are (L-1) unknowingly failed spares in the store. If the system moves to state (L P) by the failure of an operational unit, the probability of replacing it by a good one is

$$\frac{X}{Y} \quad \frac{P+1}{Y+1} \quad \cdots \quad \frac{L-1}{Y+L-1-P} M\lambda.$$

Block-4 When the system is in state (L+1 P), there are (L+1-P) units in transit, so the probability of the system coming to state (L P) after repair is $(L+1-P)\mu$.

Block-5a There are (L -P) spares in transit while the system is in state (L P), so the probability of the system returning to state (L-1 P) after repair is $(L-P)\mu$, which is also the probability of leaving the state (L P).

Block-5b When the system is in state (L P) there are 'P' units failed unknowingly in the store. On failure of an operational unit, the probability of replacing it by a good one (first coming out a failed spare and then replacing by a good spare) is

$$\frac{P}{N-L+P} \times \frac{N-L}{N-L+P-1} \; M\lambda.$$

$$= (P/Q)\left(1-\frac{P-1}{Q-1}\right)M\lambda \text{ where } Q = N-L+P$$

which is the probability of leaving state (L P) to state (L+1 P-1)

Similarly, the probability of the system leaving state (L P) to (L+1 0) is

$$\frac{P}{Q} \quad \frac{P-1}{Q-1} \quad \cdots\cdots \quad \frac{1}{Q-P+1} \; M\lambda.$$

Block 5c When the system is in state (L P) there are (N-L) good spares out of (N-L+P) units in the store. The probability of system leaving (L P) state to (L+1 P) state is $((N-L)/(N-L+P))M\lambda$.

Block 5d When the system is in state (L P) there are (N-L) good spares in the store. The probability of failing any spare unknowingly in the stre is $(N-L)\lambda_s$.

This coefficient matrix is now solved by the use of subroutine incorporating Gauss Jordan Elimination Method (flow chart of the programme is given in the Appendix-A). The solutions are kept in the last column after calling the subroutine.

Now equation 4.1.2 can be written as

$$X_{00} + X_{10} + \dots + X_{NN} + \frac{\Sigma P_{N+1}}{P_{N+1}} = 1/(P_{N+1})$$

or $$X + \frac{\Sigma P_{N+1}}{P_{N+1}} = \frac{1}{P_{N+1}} \qquad (4.1.4)$$

The probability of stock exhaustion while 'M' units are still working is given by 'P_N', which is from the solution

$$P_N = (X_{00} + X_{10} + \dots + X_{NN}) x P_{N+1} = X \times P_{N+1} \qquad (4.1.5)$$

4.2 Solution for Individual probabilities Beyond Level 'N' by Recursive Formulation

When the system has attained steady state condition, transition rate equations of (3.3.3) can be rearranged as -

$$\left.\begin{aligned} P_{N+M} &= (\lambda/\mu)(P_{N+M-1}/(N+M)) \\ P_{N+M-1} &= (\lambda/\mu)((2/(N+M-1))\ P_{N+M-2} \\ &\vdots \\ P_{N+3} &= (\lambda/\mu)((M-2)/(N+3))\ P_{N+2} \\ P_{N+2} &= (\lambda/\mu)((M-1)/(N+2))\ P_{N+1} \end{aligned}\right] \qquad (4.2.1)$$

It is evident from the above equations that there is a relationship between the adjacent probabilities of the equations of (4.2.1).

Denoting by ΣP_{N+1}, the sum of all the probabilities from level 'N+1' to level 'N+M', and putting $\lambda/u = \lambda T$, the following equation can be obtained from equations (4.2.1).

$$\frac{\Sigma P_{N+1}}{P_{N+1}} = 1 + (\lambda T)(M-1)/(N+2) + \ldots + (\lambda T)\frac{(M-1)}{N+2}\,\lambda T\,\frac{M-2}{N+3}\ \ldots\ \frac{\lambda T}{(N+M)} = Z \text{ (say)} \quad (4.2.2)$$

Now equation (4.1.4) becomes

$$X + Z = 1/P_{N+1}$$

or $$P_{N+1} = 1/(X+Z) \quad (4.2.3)$$

Putting equation (4.2.3) in equation (4.2.1), the values of P_{N+2}, P_{N+3} ... upto P_{N+M} i.e P_{N+i}'s can be found.

as $$\frac{\Sigma P_{N+1}}{P_{N+1}} = Z,$$

$$\Sigma P_{N+1} = Z \times P_{N+1} \quad (4.2.4)$$

which is the probability of failing to meet requests for spares.

Also, $$\Sigma P_N = P_N + \Sigma P_{N+1} \quad (4.2.5)$$

Putting values of 'P_N' and ΣP_{N+1} from eqn. (4.1.5) and (4.2.4) respectively, in eqn. (4.2.5), the value of ΣP_N can be found which is the probability of stock exhaustion regardless of number working.

4.3 Development of Computer Programme in Fortran Language

A generalized computer programme has been written in Fortran language, the listing of which is given in Appendix-B.

The programme first generates the coefficient matrix from the input variables M,N,λ, λ_s,μ. Then the matrix is solved by Gauss Jordan Elimination Procedure as mentioned before (subroutine of the programme). Having calculated the individual probabilities, respective equations are solved for P_N, ΣP_N, ΣP_{N+1}, D, ϕ and U.

The cost equations are also solved to determine the cost-optimal spares level.

CHAPTER 5

RESULTS AND DISCUSSION

CHAPTER 5

RESULTS AND DISCUSSION

From the given parameters (M, N, λ, λ_s and μ) the followings are calculated by computer programming:

a) The steady state unavailability of a unit, (U)

b) The probability of stock exhaustion while 'M' units still working, (P_N)

c) The probability of stock exhaustion regardless of number of unit working (ΣP_N)

d) The probability of failing to meet requests for spares, (ΣP_{N+1})

e) Downtime due to stock exhaustion (D)

f) Meantime between such stock-out situation (ϕ).

g) Optimum number of spares to minimize cost.

The results are given in tabular form (Tables 5.1-5.6) and in semi-log graphs also (Figs. 5.1-5.3). For the results in Table 5.1, the input data was for power pack failure and this was collected from 'The Milnars Tube Wells Ltd., Dhaka. The values of different parameters are as follows:

Failure rate of operational unit (λ) = 0.800×10^{-2}/hour.

Failure rate of spares in storage (λ_s) = 0.1×10^{-2}/hour.

Repair rate of units (μ) = 0.25/hour.

For calculating results, the number of operational units were varied from 1 to 10 (i.e. M = 1,10). The number of spares in the store were also varied from 1 to 8 (i.e. N = 1,8). From the results, it is evident that the unavailability (U) and the downtime (D) of the operational units decrease with the increase in number of spares, for any value of M. Using results of Table 5.1, graphs are plotted in Fig. 5.1 for U vs. N for values of M = 1,2,3,4,5

In Table 5.2, the results of the cost-optimal policy have been shown. The values of λ, λ_s and μ are same as in Table 5.1. Only the value of cost ratio (R), which is the ratio of the cost due to lost production to the variable inventory cost has been varied. For values of M = 1-10, and for values $R = 1x10^3 - 6x10^3$, the optimum number of spares have been calculated. The corresponding optimal variable cost (T'_{CS}) in multiples of $(1+i_c)b$, down time in hours, unavailability and mean time between stockouts have also been shown. For example when M = 3 and $R = 6x10^3$, the resulting policy will be:

Optimum number of spares (N_{opt}) = 3

Unavailability/unit (U) = $0.276x10^{-4}$

Down-time/unit = 1.01 hours

Meantime between stockouts (ϕ) = $0.366x10^5$ hours.

It is quite possible that there are many equipments which do not fail in the store at all. In that case $\lambda_s = 0$. Tables 5.3 and 5.4 show the results (the values of other parameters are same as before). For comparison, the cost optimal policy for the above example with $\lambda_s = 0$, is shown below.

When $M = 3$, $R = 6\times10^3$

Optimum number of spares (N_{opt}) = 2

Unavailability/unit (U) $= 0.461\times10^{-4}$

Downtime/unit (D) = 1.333 hours

Meantime between stockouts (ϕ) $= 0.292\times10^5$ hours.

From Table 5.1, fixing N = 2, and varying 'M', it is seen that as 'M' increases, 'U' starts to decrease at first and continues to do so until M = 3. However for M = 9 and greater, 'U' increases, suggesting that optimum value of 'M' lies between 3 and 9.

Sensitivity analysis:

To see the effects of variations in repair rates and failure rates on the number of spares, unavailability, downtime and cycle length, tables (5.5 and 5.6) and graphs (Fig. 5.2 and 5.3) have been prepared. In Table 5.5, the data input was

$M = 2, \lambda = 0.667\times10^{-2}$/hour, $\lambda_s = .667\times10^{-3}$/hour.

$M = 3, \lambda = 0.400\times10^{-2}$/hour $\lambda_s = 0.400\times10^{-3}$/hour.

The values of repair rates per hour were taken as

μ = 0.250, 0.125, 0.083, 0.620, 0.050, 0.040.

It can be seen that with the increase in number of spares 'N', unavailability and downtime decrease and cycle length increases. The nature of the curves has been shown in Fig. 5.2. The results are shown for two sets of values, which are as follows:

From Table 5.6,

$$M = 2, \mu = 0.125, \quad \lambda_s = 0.667 \times 10^{-3}$$

$$M = 3, \mu = 0.125, \quad \lambda_s = 0.667 \times 10^{-3}$$

Curves have been plotted in Fig. 5.3. The nature of the curve is interesting because for higher values of λ and N, unavailability decreases sharply.

TABLES SHOWING RESULTS FOR U, D, ϕ ETC.
BY VARYING 'M' WITH λ_s AS NONZERO VALUE
AND WITH $\lambda_s = 0$; INCLUDING COST OPTIMAL
POLICY

TABLE- 5.1

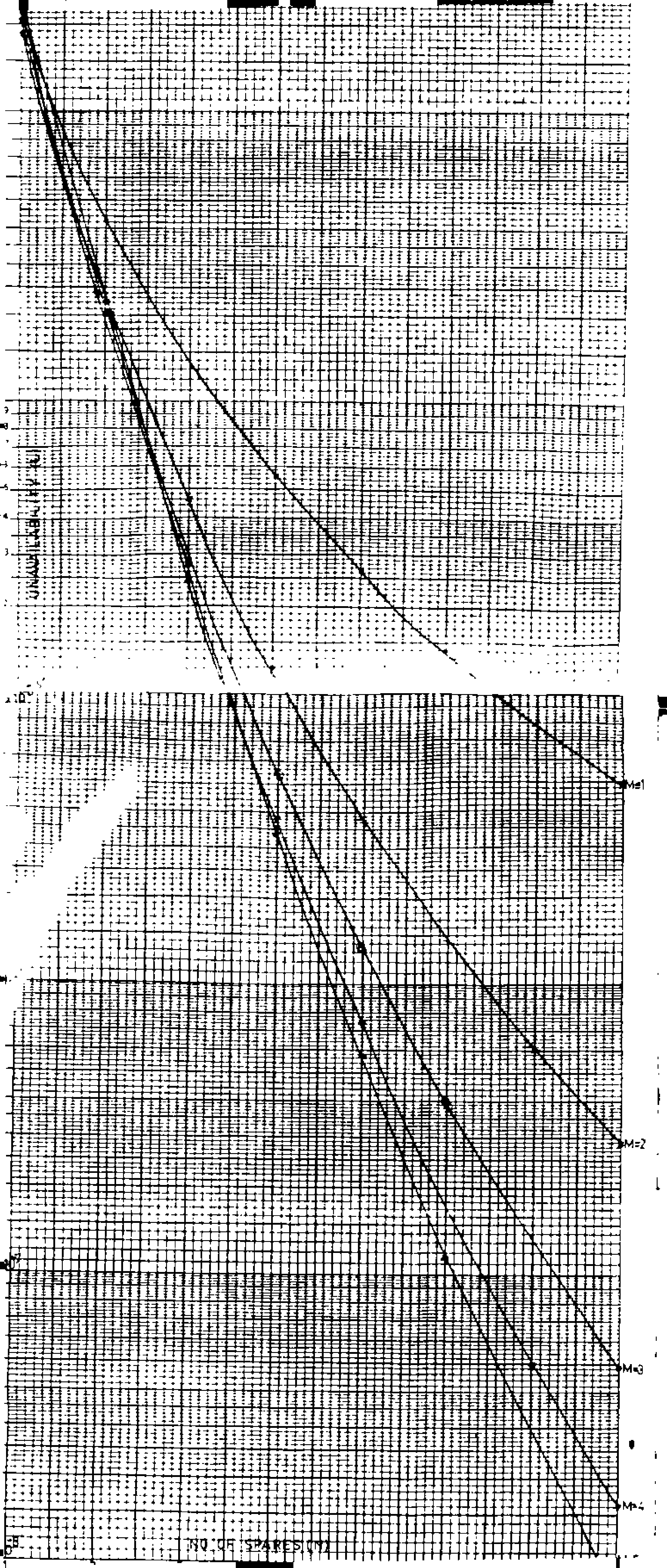
MED= 0.230E 03 LEMDA= 0.800E-02 LEMDA-SP= 0.100E-02

M	N	U	PN	SUMP(N)	SUMP(N+1)	D	PHAI
1	1	0.221E-02	0.138	0.141	0.221E-02	2.000	301.
1	2	0.432E-03	0.405E-01	0.410E-01	0.432E-03	1.333	0.308E 04
1	3	0.136E-03	0.170E-01	0.171E-01	0.136E-03	1.000	0.735E 04
1	4	0.552E-04	0.862E-02	0.868E-02	0.552E-04	0.800	0.145E 05
1	5	0.262E-04	0.492E-02	0.494E-02	0.262E-04	0.667	0.254E 05
1	6	0.139E-04	0.304E-02	0.306E-02	0.139E-04	0.572	0.411E 05
1	7	0.799E-05	0.200E-02	0.201E-02	0.799E-05	0.500	0.626E 05
1	8	0.489E-05	0.127E-02	0.138E-02	0.489E-05	0.445	0.910E 05
2	1	0.188E-02	0.115	0.119	0.572E-02	2.010	0.107E 04
2	2	0.221E-03	0.204E-01	0.209E-01	0.439E-03	1.340	0.605E 04
2	3	0.433E-04	0.534E-02	0.543E-02	0.860E-04	1.005	0.232E 05
2	4	0.116E-04	0.179E-02	0.181E-02	0.230E-04	0.803	0.694E 05
2	5	0.381E-05	0.708E-03	0.716E-03	0.759E-05	0.670	0.176E 06
2	6	0.146E-05	0.316E-03	0.319E-03	0.291E-05	0.574	0.394E 06
2	7	0.624E-06	0.154E-03	0.156E-03	0.124E-05	0.503	0.806E 06
2	8	0.291E-06	0.811E-04	0.817E-04	0.581E-06	0.447	0.154E 07
3	1	0.206E-02	0.123	0.129	0.605E-02	2.021	980.
3	2	0.193E-03	0.175E-01	0.181E-01	0.569E-03	1.347	0.699E 04
3	3	0.276E-04	0.336E-02	0.344E-02	0.817E-04	1.010	0.366E 05
3	4	0.543E-05	0.830E-03	0.846E-03	0.161E-04	0.807	0.149E 06
3	5	0.134E-05	0.247E-03	0.251E-03	0.399E-05	0.672	0.501E 06
3	6	0.395E-06	0.847E-04	0.858E-04	0.118E-05	0.579	0.146E 07
3	7	0.133E-06	0.325E-04	0.329E-04	0.397E-06	0.509	0.382E 07
3	8	0.496E-07	0.136E-04	0.138E-04	0.148E-06	0.452	0.912E 07

M	N	U	PN	SUMP(N)	SUMP(N+1)	D	PHAI
4	1	0.237E-02	0.135	0.148	0.519E-02	2.031	855.
4	2	0.205E-03	0.184E-01	0.192E-01	0.662E-03	1.355	0.659E 04
4	3	0.243E-04	0.293E-02	0.302E-02	0.554E-04	1.014	0.417E 05
4	4	0.383E-05	0.579E-03	0.594E-03	0.151E-04	0.810	0.212E 06
4	5	0.756E-06	0.138E-03	0.141E-03	0.296E-05	0.675	0.892E 06
4	6	0.179E-06	0.381E-04	0.388E-04	0.709E-06	0.580	0.323E 07
4	7	0.501E-07	0.119E-04	0.121E-04	0.198E-06	0.519	0.104E 08
4	8	0.161E-07	0.412E-05	0.419E-05	0.636E-07	0.481	0.330E 08
5	1	0.273E-02	0.157	0.170	0.131E-01	2.042	743.
5	2	0.237E-03	0.209E-01	0.220E-01	0.115E-02	1.362	0.574E 04
5	3	0.253E-04	0.300E-02	0.312E-02	0.123E-03	1.019	0.434E 05
5	4	0.341E-05	0.510E-03	0.526E-03	0.167E-04	0.814	0.239E 06
5	5	0.568E-06	0.102E-03	0.105E-03	0.279E-05	0.682	0.120E 07
5	6	0.114E-06	0.238E-04	0.243E-04	0.561E-06	0.599	0.516E 07
5	7	0.267E-07	0.626E-05	0.639E-05	0.132E-06	0.525	0.196E 08
5	8	0.737E-08	0.184E-05	0.187E-05	0.364E-07	0.494	0.670E 08
6	1	0.311E-02	0.175	0.193	0.177E-01	2.052	657.
6	2	0.281E-03	0.244E-01	0.260E-01	0.162E-02	1.369	0.486E 04
6	3	0.285E-04	0.334E-02	0.351E-02	0.166E-03	1.024	0.369E 05
6	4	0.347E-05	0.514E-03	0.534E-03	0.203E-04	0.917	0.236E 06
6	5	0.508E-06	0.908E-04	0.938E-04	0.296E-05	0.680	0.134E 07
6	6	0.886E-07	0.184E-04	0.189E-04	0.521E-06	0.589	0.665E 07
6	7	0.189E-07	0.420E-05	0.431E-05	0.111E-06	0.549	0.291E 08
6	8	0.485E-08	0.107E-05	0.110E-05	0.267E-07	0.554	0.114E 09

M	N	U	PN	SUMP(N)	SUMP(N+1)	D	PHAI
7	1	0.353E-02	0.193	0.216	0.23[illegible]E-01	2.363	387.
7	2	0.336E-03	0.286E-01	0.308E-01	0.224E-02	1.570	0.416E 04
7	3	0.336E-04	0.389E-02	0.411E-02	0.226E-03	1.029	0.307E 05
7	4	0.383E-05	0.562E-03	0.588E-03	0.260E-04	0.821	0.214E 06
7	5	0.512E-06	0.905E-04	0.940E-04	0.349E-05	0.684	0.134E 07
7	6	0.799E-07	0.164E-04	0.170E-04	0.546E-06	0.591	0.740E 07
7	7	0.142E-07	0.334E-05	0.343E-05	0.576E-07	0.521	0.365E 08
7	8	0.278E-08	0.752E-05	0.771E-06	0.191E-07	0.453	0.163E 09
8	1	0.390E-02	0.210	0.239	0.250E-01	2.073	530.
8	2	0.398E-03	0.334E-01	0.364E-01	0.301E-02	1.383	0.347E 04
8	3	0.403E-04	0.460E-02	0.491E-02	0.308E-03	1.034	0.257E 05
8	4	0.444E-05	0.643E-03	0.678E-03	0.342E-04	0.824	0.186E 06
8	5	0.552E-06	0.972E-04	0.102E-03	0.428E-05	0.684	0.124E 07
8	6	0.786E-07	0.162E-04	0.168E-04	0.611E-06	0.5[illegible]	0.746E 07
8	7	0.121E-07	0.299E-05	0.309E-05	0.942E-07	0.491	0.407E 08
8	8	0.965E-09	0.600E-06	0.607E-06	0.755E-08	0.199	0.206E 09
9	1	0.429E-02	0.226	0.251	0.355E-01	2.084	484.
9	2	0.469E-03	0.387E-01	0.426E-01	0.396E-02	1.393	0.296E 04
9	3	0.486E-04	0.548E-02	0.590E-02	0.416E-03	1.039	0.214E 05
9	4	0.528E-05	0.757E-03	0.803E-03	0.455E-04	0.827	0.157E 06
9	5	0.630E-06	0.110E-03	0.115E-03	0.546E-05	0.687	0.109E 07
9	6	0.846E-07	0.172E-04	0.179E-04	0.738E-06	0.593	0.700E 07
9	7	0.130E-07	0.295E-05	0.306E-05	0.114E-06	0.534	0.410E 08
9	8	0.152E-08	0.546E-05	0.560E-06	0.134E-07	0.341	0.224E 09

N	U	PN	SUMP(N)	SUMP(N+1)	D	PHAI
1	0.467E-02	0.241	0.284	0.425E-01	2.094	446.
2	0.547E-03	0.444E-01	0.495E-01	0.509E-02	1.398	0.255E 04
3	0.586E-04	0.652E-02	0.708E-02	0.553E-03	1.044	0.178E 05
4	0.637E-05	0.904E-03	0.964E-03	0.607E-04	0.531	0.150E 06
5	0.739E-06	0.128E-03	0.135E-03	0.709E-05	0.689	0.933E 06
6	0.943E-07	0.191E-04	0.200E-04	0.510E-06	0.591	0.627E 07
7	0.128E-07	0.308E-05	0.320E-05	0.124E-06	0.502	0.392E 08
9	0.117E-08	0.530E-06	0.541E-06	0.114E-07	-0.271	0.232E 09



NO. OF SPARES (N)
UNAVAILABILITY (U)
M=1
M=2
M=3
M=4

TABLE 5.2

COST OPTIMAL POLICY
****

M	R	N(OPT)	TCS	U	L	[illegible]
1	1	2	2.432	0.547E-03	1.398	0.255E 04
1	2	2	2.363	0.547E-03	1.398	0.255E 04
1	3	2	3.257	0.547E-03	1.398	0.255E 04
1	4	3	3.344	0.586E-04	1.044	0.178E 05
1	5	3	5.060	0.586E-04	1.044	0.178E 05
1	6	3	3.316	0.586E-04	1.044	0.178E 05
2	1	2	2.443	0.547E-03	1.398	0.255E 04
2	2	2	2.965	0.547E-03	1.390	0.255E 04
2	3	3	3.260	0.586E-04	1.044	0.178E 05
2	4	3	3.346	0.586E-04	1.044	0.178E 05
2	5	3	3.433	0.586E-04	1.044	0.178E 05
2	6	3	3.520	0.586E-04	1.044	0.178E 05
3	1	2	2.573	0.547E-03	1.398	0.255E 04
3	2	2	3.156	0.547E-03	1.398	0.255E 04
3	3	3	3.243	0.586E-04	1.044	0.178E 05
3	4	3	2.331	0.586E-04	1.044	0.178E 05
3	5	3	3.414	0.586E-04	1.044	0.178E 05
3	6	3	3.497	0.586E-04	1.044	0.178E 05
4	1	2	2.922	0.547E-03	1.398	0.255E 04
4	2	3	3.195	0.586E-04	1.044	0.178E 05
4	3	3	3.292	0.586E-04	1.044	0.178E 05
4	4	3	3.389	0.586E-04	1.044	0.178E 05
4	5	3	3.485	0.586E-04	1.044	0.178E 05
4	6	3	3.524	0.586E-04	1.044	0.178E 05

N	K	N(GPT)	TCS	U	D	PHAI
5	1	3	3.126	0.586E-04	1.044	0.178E 05
5	2	3	3.253	0.586E-04	1.044	0.178E 05
5	3	3	3.377	0.586E-04	1.044	0.178E 05
5	4	3	3.503	0.586E-04	1.044	0.178E 05
5	5	3	3.631	0.586E-04	1.044	0.178E 05
5	6	3	3.754	0.586E-04	1.044	0.178E 05
6	1	3	3.171	0.586E-04	1.044	0.178E 05
6	2	3	3.342	0.586E-04	1.044	0.178E 05
6	3	3	3.513	0.586E-04	1.044	0.178E 05
6	4	3	3.684	0.586E-04	1.044	0.178E 05
6	5	3	3.855	0.586E-04	1.044	0.178E 05
6	6	3	4.027	0.586E-04	1.044	0.178E 05
7	1	3	3.235	0.586E-04	1.044	0.178E 05
7	2	3	3.470	0.586E-04	1.044	0.178E 05
7	3	3	3.705	0.586E-04	1.044	0.178E 05
7	4	3	3.940	0.586E-04	1.044	0.178E 05
7	5	4	4.134	0.637E-05	0.631	0.130E 06
7	6	4	4.161	0.637E-05	0.631	0.130E 06
8	1	3	3.322	0.586E-04	1.044	0.178E 05
8	2	3	3.644	0.586E-04	1.044	0.178E 05
8	3	3	3.966	0.586E-04	1.044	0.178E 05
8	4	4	4.142	0.637E-05	0.631	0.130E 06
8	5	4	4.177	0.637E-05	0.631	0.130E 06
8	6	4	4.213	0.637E-05	0.631	0.130E 06

N	F	N(OPT)	TCS	U	D	PHAI
9	1	3	3.437	0.586E-04	1.044	0.178E 05
9	2	3	3.675	0.586E-04	1.044	0.178E 05
9	3	4	4.143	0.637E-05	0.831	0.130E 06
9	4	4	4.190	0.637E-05	0.831	0.130E 06
9	5	4	4.233	0.637E-05	0.831	0.130E 06
9	6	4	4.265	0.637E-05	0.831	0.130E 06
10	1	3	3.586	0.586E-04	1.044	0.178E 05
10	2	4	4.127	0.637E-05	0.831	0.130E 06
10	3	4	4.191	0.637E-05	0.831	0.130E 06
10	4	4	4.255	0.637E-05	0.831	0.130E 06
10	5	4	4.318	0.637E-05	0.831	0.130E 06
10	6	4	4.382	0.637E-05	0.831	0.130E 06

TABLE- 5.3
****** *****

MED= 0.250E 00   LEMDA= 0.8000E-02   LEMDA-S= 0.0

M	N	U	PN	SUMP(N)	SUMP(N+1)	D	PHAI
1	1	0.496E-03	0.310E-01	0.315E-01	0.496E-03	2.000	0.403E 04
1	2	0.529E-05	0.496E-03	0.501E-03	0.529E-05	1.333	0.252E 06
1	3	0.448E-07	0.529E-05	0.534E-05	0.448E-07	1.000	0.236E 08
1	4	0.223E-08	0.448E-07	0.470E-07	0.223E-08	0.800	0.279E 10
1	5	0.163E-08	0.223E-08	0.387E-08	0.163E-08	0.667	0.560E 11
1	6	0.140E-08	0.163E-08	0.303E-08	0.140E-08	0.571	0.765E 11
1	7	0.122E-08	0.140E-08	0.262E-08	0.122E-08	0.500	0.894E 11
1	8	0.109E-08	0.122E-08	0.231E-08	0.109E-08	0.444	0.102E 12
2	1	0.981E-03	0.600E-01	0.620E-01	0.194E-02	2.000	0.205E 04
2	2	0.208E-04	0.192E-02	0.196E-02	0.413E-04	1.333	0.644E 05
2	3	0.332E-06	0.410E-04	0.416E-04	0.663E-06	1.000	0.303E 07
2	4	0.536E-08	0.658E-06	0.669E-06	0.107E-07	0.800	0.188E 09
2	5	0.969E-09	0.106E-07	0.125E-07	0.193E-08	0.667	0.108E 11
2	6	0.790E-09	0.192E-08	0.349E-08	0.157E-08	0.571	0.462E 11
2	7	0.689E-09	0.157E-08	0.294E-08	0.137E-08	0.500	0.555E 11
2	8	0.611E-09	0.137E-08	0.259E-08	0.122E-08	0.444	0.633E 11
3	1	0.146E-02	0.872E-01	0.915E-01	0.426E-02	2.000	0.139E 04
3	2	0.461E-04	0.419E-02	0.432E-02	0.136E-03	1.333	0.292E 05
3	3	0.110E-05	0.134E-03	0.137E-03	0.326E-05	1.000	0.918E 06
3	4	0.227E-07	0.322E-05	0.329E-05	0.674E-07	0.800	0.383E 08
3	5	0.172E-08	0.667E-07	0.718E-07	0.513E-08	0.667	0.178E 10
3	6	0.119E-08	0.508E-08	0.862E-08	0.354E-08	0.571	0.168E 11
3	7	0.103E-08	0.351E-08	0.658E-08	0.307E-08	0.500	0.225E 11
3	8	0.914E-09	0.305E-08	0.578E-08	0.272E-08	0.444	0.257E 11

M	N	U	PN	SUMP(N)	SUMP(N+1)	D	PHAI
4	1	0.192E-02	0.113	0.120	0.744E-02	2.000	0.106E 04
4	2	0.806E-04	0.721E-02	0.752E-02	0.315E-03	1.333	0.168E 05
4	3	0.256E-05	0.308E-03	0.318E-03	0.100E-04	1.000	0.397E 06
4	4	0.654E-07	0.984E-05	0.101E-04	0.258E-06	0.800	0.125E 08
4	5	0.171E-08	0.253E-06	0.260E-06	0.675E-08	0.667	0.484E 09
4	6	0.305E-09	0.666E-08	0.786E-08	0.121E-08	0.571	0.165E 11
4	7	0.244E-09	0.119E-08	0.216E-08	0.566E-09	0.500	0.653E 11
4	8	0.216E-09	0.956E-09	0.181E-08	0.354E-09	0.444	0.784E 11
5	1	0.237E-02	0.136	0.148	0.114E-01	2.000	858.
5	2	0.124E-03	0.109E-01	0.115E-01	0.501E-03	1.333	0.110E 05
5	3	0.490E-05	0.582E-03	0.606E-03	0.239E-04	1.000	0.208E 06
5	4	0.156E-06	0.233E-04	0.240E-04	0.765E-06	0.800	0.523E 07
5	5	0.485E-08	0.749E-06	0.773E-06	0.238E-07	0.667	0.163E 09
5	6	0.715E-09	0.234E-07	0.269E-07	0.352E-08	0.571	0.477E 10
5	7	0.541E-09	0.346E-08	0.613E-08	0.267E-08	0.500	0.224E 11
5	8	0.477E-09	0.263E-08	0.498E-08	0.235E-08	0.444	0.277E 11
6	1	0.282E-02	0.158	0.175	0.161E-01	2.000	726.
6	2	0.176E-03	0.152E-01	0.162E-01	0.101E-02	1.333	0.779E 04
6	3	0.830E-05	0.974E-03	0.102E-02	0.483E-04	1.000	0.123E 06
6	4	0.317E-06	0.467E-04	0.486E-04	0.185E-05	0.800	0.259E 07
6	5	0.111E-07	0.180E-05	0.187E-05	0.651E-07	0.667	0.673E 08
6	6	0.119E-08	0.636E-07	0.707E-07	0.701E-08	0.571	0.180E 10
6	7	0.803E-09	0.687E-08	0.113E-07	0.473E-08	0.500	0.116E 11
6	8	0.703E-09	0.465E-08	0.880E-08	0.415E-08	0.444	0.154E 11

M	N	U	PN	SUMP(N)	SUMP(N+1)	D	PHAI
7	1	0.326E-02	0.179	0.200	0.214E-01	2.000	632.
7	2	0.235E-03	0.201E-01	0.216E-01	0.157E-02	1.333	0.584E 04
7	3	0.129E-04	0.150E-02	0.158E-02	0.672E-04	1.000	0.795E 05
7	4	0.573E-06	0.839E-04	0.877E-04	0.388E-05	0.800	0.143E 07
7	5	0.214E-07	0.376E-05	0.390E-05	0.146E-06	0.667	0.322E 08
7	6	0.908E-09	0.142E-06	0.149E-06	0.621E-08	0.571	0.848E 09
7	7	0.222E-09	0.606E-08	0.758E-08	0.152E-08	0.500	0.170E 11
7	8	0.180E-09	0.149E-08	0.273E-06	0.124E-08	0.444	0.491E 11
8	1	0.368E-02	0.158	0.226	0.274E-01	2.000	561.
8	2	0.303E-03	0.254E-01	0.277E-01	0.229E-02	1.333	0.457E 04
8	3	0.189E-04	0.216E-02	0.231E-02	0.145E-03	1.000	0.546E 05
8	4	0.957E-06	0.139E-03	0.146E-03	0.738E-05	0.800	0.862E 06
8	5	0.416E-07	0.710E-05	0.743E-05	0.322E-06	0.667	0.169E 08
8	6	0.254E-08	0.312E-06	0.332E-06	0.197E-07	0.571	0.380E 09
8	7	0.972E-09	0.192E-07	0.268E-07	0.759E-08	0.500	0.485E 10
8	8	0.816E-09	0.740E-08	0.138E-07	0.639E-08	0.444	0.964E 10
9	1	0.410E-02	0.216	0.250	0.339E-01	2.000	506.
9	2	0.377E-03	0.311E-01	0.343E-01	0.319E-02	1.333	0.369E 04
9	3	0.265E-04	0.299E-02	0.321E-02	0.226E-03	1.000	0.392E 05
9	4	0.150E-05	0.215E-03	0.228E-03	0.129E-04	0.800	0.552E 06
9	5	0.723E-07	0.124E-04	0.130E-04	0.628E-06	0.667	0.965E 07
9	6	0.396E-08	0.605E-06	0.639E-06	0.345E-07	0.571	0.197E 09
9	7	0.102E-08	0.334E-07	0.423E-07	0.890E-08	0.500	0.303E 10
9	8	0.806E-09	0.864E-08	0.157E-07	0.707E-08	0.444	0.838E 10

M	N	U	PN	SUMP(N)	SUMP(N+1)	D	PHAI
-0	1	0.451E-02	0.232	0.273	0.410E-01	2.000	463.
-0	2	0.456E-03	0.372E-01	0.414E-01	0.427E-02	1.333	0.305E 04
0	3	0.356E-04	0.397E-02	0.430E-02	0.336E-03	1.000	0.293E 05
-0	4	0.224E-05	0.317E-03	0.339E-03	0.213E-04	0.800	0.372E 06
0	5	0.119E-06	0.203E-04	0.215E-04	0.114E-05	0.667	0.586E 07
0	6	0.634E-08	0.109E-05	0.116E-05	0.511E-07	0.571	0.109E 09
-0	7	0.109E-08	0.590E-07	0.695E-07	0.105E-07	0.500	0.183E 10
-0	8	0.776E-09	0.102E-07	0.177E-07	0.754E-08	0.444	0.738E 10

TABLE 5.4

COST OPTIMAL POLICY
******* **** *******

M	R	N(OPT)	TCS	U	D	FMAX
1	1	1	1.455	0.451E-02	2.000	0.463E 03
1	2	1	1.592	0.451E-02	2.000	0.463E 03
1	3	2	2.016	0.458E-03	1.333	0.305E 04
1	4	2	2.021	0.458E-03	1.333	0.305E 04
1	5	2	2.025	0.458E-03	1.333	0.305E 04
1	6	2	2.032	0.458E-03	1.333	0.305E 04
2	1	2	2.042	0.458E-03	1.333	0.305E 04
2	2	2	2.083	0.458E-03	1.333	0.305E 04
2	3	2	2.125	0.458E-03	1.333	0.305E 04
2	4	2	2.167	0.458E-03	1.333	0.305E 04
2	5	2	2.209	0.458E-03	1.333	0.305E 04
2	6	2	2.250	0.458E-03	1.333	0.305E 04
3	1	2	2.139	0.458E-03	1.333	0.305E 04
3	2	2	2.277	0.458E-03	1.333	0.305E 04
3	3	2	2.415	0.458E-03	1.333	0.305E 04
3	4	2	2.553	0.458E-03	1.333	0.305E 04
3	5	2	2.691	0.458E-03	1.333	0.305E 04
3	6	2	2.830	0.458E-03	1.333	0.305E 04
4	1	2	2.323	0.458E-03	1.333	0.305E 04
4	2	2	2.645	0.458E-03	1.333	0.305E 04
4	3	2	2.967	0.458E-03	1.333	0.305E 04
4	4	3	3.041	0.356E-04	1.000	0.293E 05
4	5	3	3.061	0.356E-04	1.000	0.293E 05
4	6	3	3.081	0.356E-04	1.000	0.293E 05

N	R	N(OPT)	TCS	U	D	PHAI
5	1	2	2.620	0.456E-03	1.333	0.305E 04
5	2	3	3.049	0.356E-04	1.000	0.293E 05
5	3	3	3.073	0.356E-04	1.000	0.293E 05
5	4	3	3.098	0.356E-04	1.000	0.293E 05
5	5	3	3.122	0.356E-04	1.000	0.293E 05
5	6	3	3.147	0.356E-04	1.000	0.293E 05
6	1	3	3.050	0.356E-04	1.000	0.293E 05
6	2	3	3.100	0.356E-04	1.000	0.293E 05
6	3	3	3.149	0.356E-04	1.000	0.293E 05
6	4	3	3.199	0.356E-04	1.000	0.293E 05
6	5	3	3.249	0.356E-04	1.000	0.293E 05
6	6	3	3.299	0.356E-04	1.000	0.293E 05
7	1	3	3.091	0.356E-04	1.000	0.293E 05
7	2	3	3.161	0.356E-04	1.000	0.293E 05
7	3	3	3.272	0.356E-04	1.000	0.293E 05
7	4	3	3.362	0.356E-04	1.000	0.293E 05
7	5	3	3.453	0.356E-04	1.000	0.293E 05
7	6	3	3.543	0.356E-04	1.000	0.293E 05
8	1	3	3.152	0.356E-04	1.000	0.293E 05
8	2	3	3.303	0.356E-04	1.000	0.293E 05
8	3	3	3.455	0.356E-04	1.000	0.293E 05
8	4	3	3.606	0.356E-04	1.000	0.293E 05
8	5	3	3.758	0.356E-04	1.000	0.293E 05
8	6	3	3.909	0.356E-04	1.000	0.293E 05

N	K	N(CPT)	TCS	U	D	PHAI
9	1	3	3.235	0.356E-04	1.000	0.293E 05
9	2	3	3.475	0.356E-04	1.000	0.293E 05
9	3	3	3.715	0.356E-04	1.000	0.293E 05
9	4	3	3.953	0.356E-04	1.000	0.293E 05
9	5	4	4.063	0.224E-05	0.600	0.372E 06
9	6	4	4.081	0.224E-05	0.600	0.372E 06
10	1	3	3.356	0.356E-04	1.000	0.293E 05
10	2	3	3.713	0.356E-04	1.000	0.293E 05
10	3	4	4.067	0.224E-05	0.600	0.372E 06
10	4	4	4.090	0.224E-05	0.600	0.372E 06
10	5	4	4.112	0.224E-05	0.600	0.372E 06
10	6	4	4.134	0.224E-05	0.600	0.372E 06

TABLES SHOWING RESULTS FOR SENSITIVITY ANALYSIS BY VARYING μ AND λ

TABLE-- 5.5

M= 2 LEMDA= 0.667E-02 LEMDA-S= 0.667E-03

MEQ	N	U	UN	SUMP(N)	SUMP(N+1)	D	PHAI
0.250	1	0.128E-02	0.956E-01	0.[illegible]E-01	0.255E-02	1.974	0.155E 04
0.250	2	0.127E-03	0.143E-01	0.145E-01	0.254E-03	1.322	0.104E 05
0.250	3	0.213E-04	0.319E-02	0.323E-02	0.426E-04	0.993	0.467E 05
0.250	4	0.455E-05	0.927E-03	0.937E-03	0.989E-05	0.755	0.161E 06
0.250	5	0.143E-05	0.322E-03	0.325E-03	0.286E-05	0.664	0.464E 06
0.250	6	0.487E-06	0.127E-03	0.128E-03	0.973E-06	0.571	0.117E 07
0.250	7	0.197E-06	0.554E-04	0.560E-04	0.374E-06	0.503	0.269E 07
0.250	8	0.782E-07	0.263E-04	0.265E-04	0.156E-06	0.444	0.568E 07
0.125	1	0.369E-02	0.136	0.140	0.739E-02	3.855	0.105E 04
0.125	2	0.400E-03	0.225E-01	0.233E-01	0.806E-03	2.620	0.655E 04
0.125	3	0.659E-04	0.494E-02	0.507E-02	0.132E-03	1.974	0.300E 05
0.125	4	0.148E-04	0.138E-02	0.141E-02	0.295E-04	1.583	0.107E 06
0.125	5	0.412E-05	0.453E-03	0.471E-03	0.824E-05	1.322	0.321E 06
0.125	6	0.135E-05	0.177E-03	0.180E-03	0.271E-05	1.138	0.840E 06
0.125	7	0.505E-06	0.753E-04	0.763E-04	0.101E-05	1.000	0.198E 07
0.125	8	0.206E-06	0.347E-04	0.351E-04	0.411E-06	0.882	0.429E 07
0.083	1	0.711E-02	0.177	0.191	0.142E-01	5.791	809.
0.083	2	0.360E-03	0.321E-01	0.338E-01	0.172E-02	3.911	0.454E 04
0.083	3	0.144E-03	0.716E-02	0.745E-02	0.288E-03	2.953	0.205E 05
0.083	4	0.318E-04	0.198E-02	0.204E-02	0.635E-04	2.372	0.746E 05
0.083	5	0.867E-05	0.647E-03	0.664E-03	0.173E-04	1.982	0.229E 06
0.083	6	0.273E-05	0.242E-03	0.247E-03	0.556E-05	1.703	0.613E 06
0.083	7	0.101E-05	0.100E-03	0.102E-03	0.202E-05	1.495	0.148E 07
0.083	8	0.403E-06	0.454E-04	0.462E-04	0.807E-06	1.321	0.328E 07

WEC	N	U	PN	SUMF(N)	SUMP(N+1)	D	PHAI
0.062	1	0.114E-01	0.211	0.234	0.227E-01	7.053	666.
0.062	2	0.154E-02	0.429E-01	0.460E-01	0.306E-02	5.193	0.337E 04
0.062	3	0.266E-03	0.939E-02	0.104E-01	0.532E-03	3.927	0.148E 05
0.062	4	0.537E-04	0.273E-02	0.284E-02	0.117E-03	3.158	0.538E 05
0.062	5	0.158E-04	0.881E-03	0.915E-03	0.316E-04	2.639	0.167E 06
0.062	6	0.497E-05	0.324E-03	0.334E-03	0.995E-05	2.263	0.456E 06
0.062	7	0.178E-05	0.132E-03	0.136E-03	0.355E-05	1.983	0.112E 07
0.062	8	0.596E-06	0.587E-04	0.601E-04	0.139E-05	1.758	0.252E 07
0.050	1	0.160E-01	0.246	0.272	0.320E-01	9.373	576.
0.050	2	0.240E-02	0.546E-01	0.586E-01	0.490E-02	6.383	0.265E 04
0.050	3	0.431E-03	0.129E-01	0.138E-01	0.862E-03	4.839	0.112E 05
0.050	4	0.959E-04	0.359E-02	0.379E-02	0.192E-03	3.895	0.406E 05
0.050	5	0.257E-04	0.116E-02	0.121E-02	0.514E-04	3.263	0.127E 06
0.050	6	0.800E-05	0.420E-03	0.436E-03	0.160E-04	2.802	0.350E 06
0.050	7	0.282E-05	0.169E-03	0.175E-03	0.563E-05	2.455	0.872E 06
0.050	8	0.109E-05	0.741E-04	0.763E-04	0.219E-05	2.179	0.199E 07
0.040	1	0.227E-01	0.273	0.318	0.455E-01	11.533	496.
0.040	2	0.364E-02	0.691E-01	0.766E-01	0.759E-02	7.855	0.205E 04
0.040	3	0.730E-03	0.175E-01	0.190E-01	0.146E-02	6.003	0.822E 04
0.040	4	0.165E-03	0.496E-02	0.525E-02	0.331E-03	4.839	0.243E 05
0.040	5	0.443E-04	0.159E-02	0.166E-02	0.886E-04	4.055	0.915E 05
0.040	6	0.137E-04	0.575E-03	0.602E-03	0.274E-04	3.490	0.255E 06
0.040	7	0.477E-05	0.229E-03	0.238E-03	0.955E-05	3.068	0.643E 06
0.040	8	0.184E-05	0.938E-04	0.102E-03	0.367E-05	2.734	0.149E 07

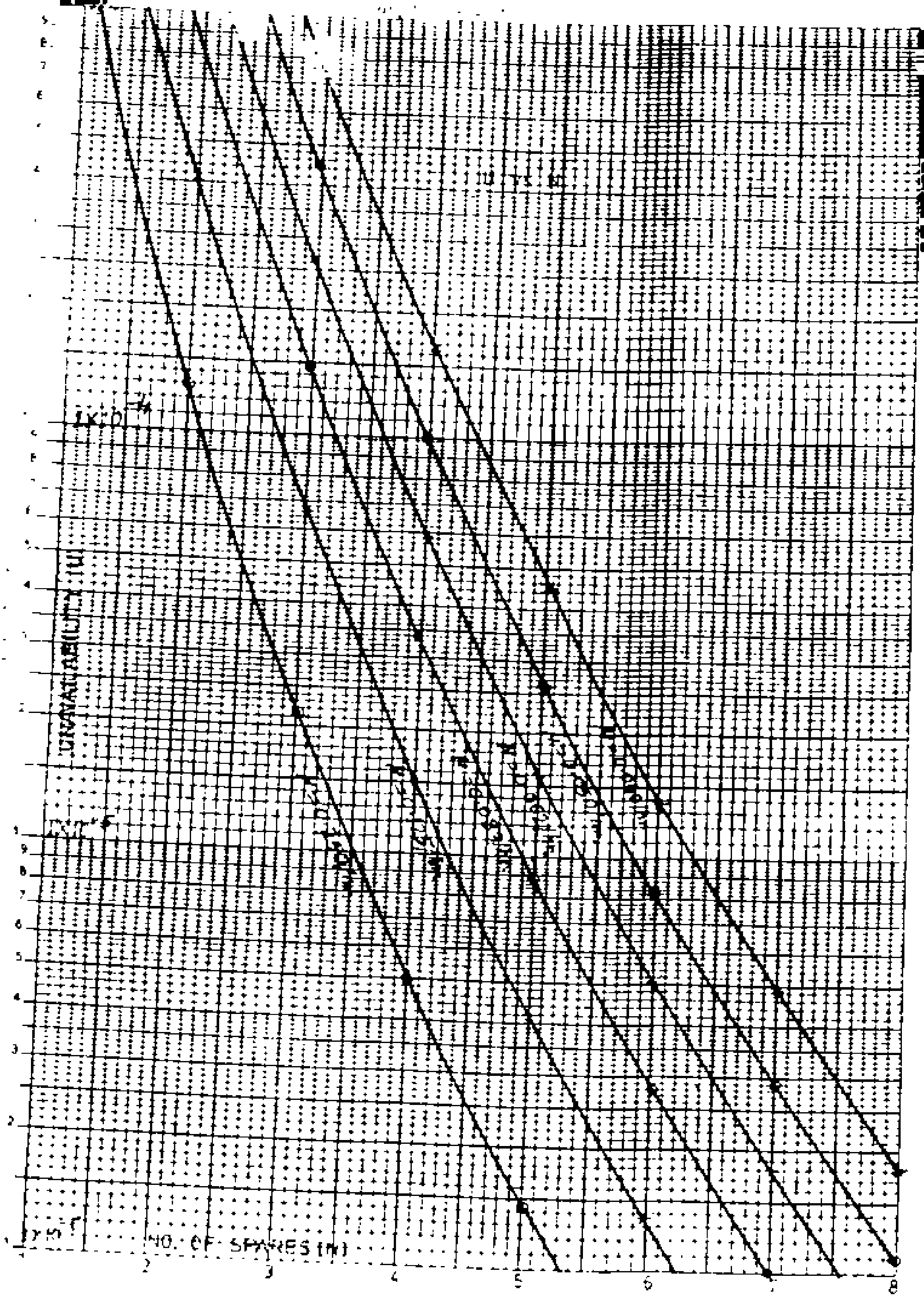


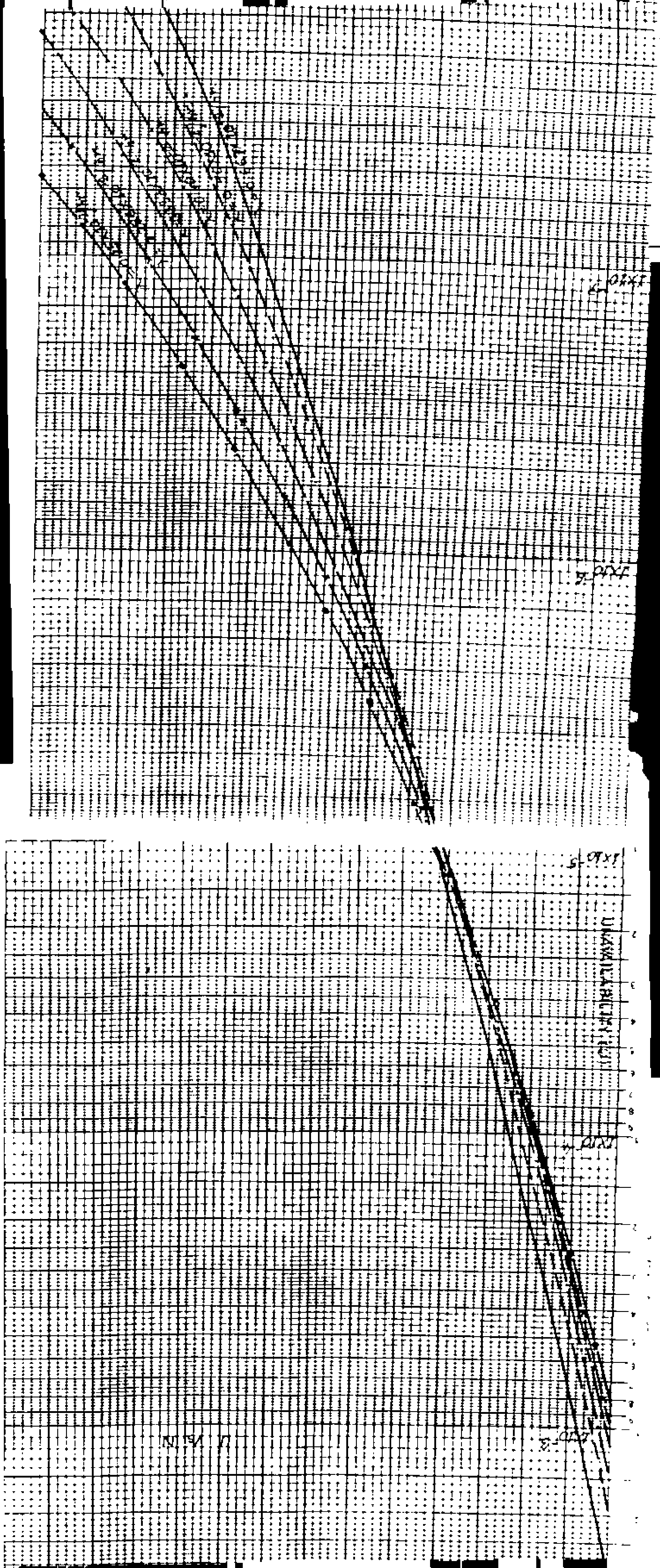
FIG-52

TABLE-- 5.6

M= 2 MED= 0.125 LEMDA-C= 0.250E-02

LEMDA	N	U	PI	SUM(N)	SUM(N-1)	D	PHAI
0.667E-02	1	0.300E-02	0.112	0.116	0.300E-02	3.896	0.130E 04
0.667E-02	2	0.192E-03	0.108E-01	0.112E-01	0.334E-03	2.620	0.137E 05
0.667E-02	3	0.157E-04	0.113E-02	0.121E-02	0.314E-04	1.574	0.126E 06
0.667E-02	4	0.167E-05	0.157E-03	0.160E-03	0.335E-05	1.531	0.944E 06
0.667E-02	5	0.225E-06	0.252E-04	0.257E-04	0.451E-06	1.329	0.589E 07
0.667E-02	6	0.377E-07	0.476E-05	0.484E-05	0.734E-07	1.179	0.312E 08
0.667E-02	7	0.803E-08	0.103E-05	0.105E-05	0.161E-07	1.158	0.144E 09
0.667E-02	8	0.214E-08	0.251E-06	0.255E-06	0.429E-08	1.271	0.592E 09
0.500E-02	1	0.193E-02	0.963E-01	0.100	0.385E-02	3.922	0.203E 04
0.500E-02	2	0.127E-03	0.554E-02	0.579E-02	0.254E-03	2.631	0.207E 05
0.500E-02	3	0.124E-04	0.124E-02	0.126E-02	0.247E-04	1.980	0.160E 06
0.500E-02	4	0.165E-05	0.206E-03	0.210E-03	0.330E-05	1.587	0.961E 06
0.500E-02	5	0.283E-06	0.422E-04	0.422E-04	0.567E-06	1.332	0.470E 07
0.500E-02	6	0.603E-07	0.102E-04	0.103E-04	0.121E-06	1.179	0.196E 08
0.500E-02	7	0.156E-07	0.279E-05	0.282E-05	0.311E-07	1.110	0.713E 08
0.500E-02	8	0.415E-08	0.850E-06	0.858E-06	0.830E-08	0.972	0.234E 09
.400E-02	1	0.141E-02	0.884E-01	0.912E-01	0.283E-02	3.937	0.278E 04
.400E-02	2	0.102E-03	0.956E-02	0.976E-02	0.204E-03	2.638	0.259E 05
.400E-02	3	0.117E-04	0.147E-02	0.149E-02	0.235E-04	1.984	0.169E 06
.400E-02	4	0.189E-05	0.296E-03	0.300E-03	0.379E-05	1.590	0.840E 06
.400E-02	5	0.390E-06	0.729E-04	0.737E-04	0.780E-06	1.330	0.341E 07
.400E-02	6	0.968E-07	0.210E-04	0.212E-04	0.194E-06	1.148	0.119E 08
400E-02	7	0.289E-07	0.681E-05	0.687E-05	0.377E-07	1.055	0.365E 08
400E-02	8	0.826E-08	0.244E-05	0.246E-05	0.165E-07	0.843	0.102E 09

LEMDA	N	U	PN	SUMP(N)	SUMP(N+1)	D	PHAI
0.333E-02	1	0.113E-02	0.647E-01	0.676E-01	0.226E-02	3.947	0.349E 04
0.333E-02	2	0.505E-04	0.102E-01	0.104E-01	0.191E-03	2.643	0.292E 05
0.333E-02	3	0.121E-04	0.162E-02	0.184E-02	0.242E-04	1.537	0.164E 06
0.333E-02	4	0.228E-05	0.423E-03	0.432E-03	0.456E-05	1.592	0.699E 06
0.333E-02	5	0.543E-06	0.122E-03	0.123E-03	0.109E-05	1.331	0.245E 07
0.333E-02	6	0.154E-06	0.402E-04	0.405E-04	0.308E-06	1.145	0.744E 07
0.333E-02	7	0.495E-07	0.148E-04	0.145E-04	0.936E-07	1.006	0.202E 08
0.333E-02	8	0.179E-07	0.599E-05	0.603E-05	0.335E-07	0.896	0.500E 08
0.286E-02	1	0.557E-03	0.636E-01	0.656E-01	0.191E-02	3.955	0.413E 04
0.286E-02	2	0.653E-04	0.112E-01	0.114E-01	0.171E-03	2.646	0.310E 05
0.286E-02	3	0.130E-04	0.227E-02	0.229E-02	0.260E-04	1.989	0.153E 06
0.286E-02	4	0.276E-05	0.602E-03	0.606E-03	0.551E-05	1.593	0.578E 06
0.286E-02	5	0.735E-06	0.192E-03	0.194E-03	0.147E-05	1.330	0.181E 07
0.286E-02	6	0.232E-06	0.705E-04	0.716E-04	0.463E-06	1.144	0.494E 07
0.286E-02	7	0.821E-07	0.287E-04	0.286E-04	0.164E-06	0.997	0.122E 08
0.286E-02	8	0.323E-07	0.127E-04	0.126E-04	0.646E-07	0.887	0.275E 08
0.250E-02	1	0.841E-03	0.641E-01	0.658E-01	0.166E-02	3.960	0.470E 04
0.250E-02	2	0.830E-04	0.125E-01	0.126E-01	0.136E-03	2.649	0.319E 05
0.250E-02	3	0.141E-04	0.282E-02	0.285E-02	0.292E-04	1.990	0.141E 06
0.250E-02	4	0.333E-05	0.631E-03	0.636E-03	0.665E-05	1.594	0.479E 06
0.250E-02	5	0.975E-06	0.292E-03	0.294E-03	0.195E-05	1.331	0.127E 07
0.250E-02	6	0.334E-06	0.117E-03	0.117E-03	0.669E-06	1.143	0.342E 07
0.250E-02	7	0.129E-06	0.514E-04	0.517E-04	0.257E-06	0.998	0.776E 07
0.250E-02	8	0.537E-07	0.245E-04	0.246E-04	0.107E-06	0.876	0.163E 08


UNAVAILABILITY (U)

CHAPTER 6

CONCLUSIONS

CHAPTER 6

CONCLUSIONS

From the study on 'repairable spare parts provisioning rules', the following conclusions can be made.

(1) The mathematical model which has been developed in this study, is sufficiently general incorporating failure rate of component in operation, failure rate of component in storege, repair rate etc. The only limitation comes from the assumptionsoof constant failure rates and constant repair rate.

(2) From the results, it is apparent that for a fixed number of operational units, unavailability and down-time of the operational units decrease with the increase in number of spare parts, but mean time between stockout situations increase correspondingly. The plot of 'U' vs. 'N' for fixed 'M', shows the decrease of 'U' as negative exponential as 'N' is increased.

(3) From the results of cost optimal policy model, it is observed that the optimal number of spares increase in step function for the increase of cost ratio 'R'. This indicates that for a certain range of 'R', the optimal number of spares is a fixed quantity.

CHAPTER 7
SCOPE FOR FURTHER WORK

CHAPTER 7

SCOPE FOR FURTHER WORK

The following list may provide a helpful guide to additional work, the need for which has become apparent during the course of this research.

i) Consideration should be given to the unit, the failure rate of which is not constant. Various failure distributions should be considered.

ii) Consideration may be given to the situations where repair rates are not constant.

iii) Consideration may also be given to the situations whe-re there is no sufficient repair facilities i.e. items are to wait for repair.

iv) Consideration may be given to the situation where units failed in operation or in storage are sent to workshop in a group rather sending individually. Also repaired items should be returned to store in a lot after repair rather than individually.

REFERENCES

REFERENCES

1. RAY WILD,
   'Production Operations Management' - Principles and Techniques'
   Holt, Rinehart and Winston Ltd.
   London 1979.

2. F.G. MOORE
   'Manufacturing Management'
   Georgetown, Ontario
   U.S.A. 1973.

3. WILLIAM VORIS,
   'Production Control Text & Cases'
   Richard D. Irwin INC,
   U.S.A 1961.

4. MAYER, R.M.
   'Production and operation Management'
   McGraw-Hill Book Company
   1975.

5. MOORE, P.G.
   'Basic Operational Research'
   Sir Isaac Pitman and Sons.
   London 1968.

6. VORIS, WILLIAM
   'The Management of Production'
   Ronald Press Company,
   New York 1960.

7. J.E. SHAMBLIN & G.T. STEVENS. Jr.
   'Operation Research Fundamental Approach'
   U.S.A. 1974.

8. CHURCHMAN, C. WEST & OTHERS
   'Introduction to Operations Research'
   John Wiley and Sons
   1963.

9. SASIENI, MAURICE & OTHERS
   'Operations Research'
   John Willey &Sons
   1967.

10. SPRIEGEL, WILLIAM
    'Industrial Management'
    1970.

11. ELWOOD S. BUFFA
'Basic Production Management'
John Willey & Sons Inc.
New York 1971.

12. CHURCHMAN, C. WEST
'Introduction to Operations Research'
Toppen Printing Company
Japan 1959.

13. FOLTS, FRANKLINE
'Introduction to Industrial Management' -
Text, Cases & Problems.
McGraw-Hill Book Company
1954.

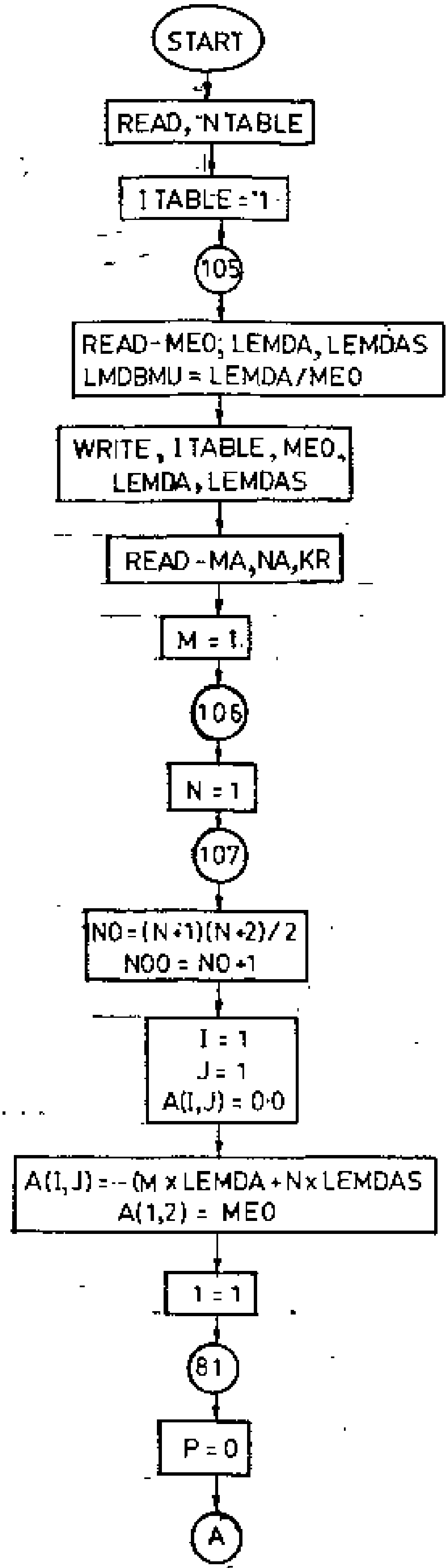
14. JOHN A. MUCKSTADT
'Management Science'
Vol. 20, No.4, December 1973
Part-1 Printed in U.S.A.

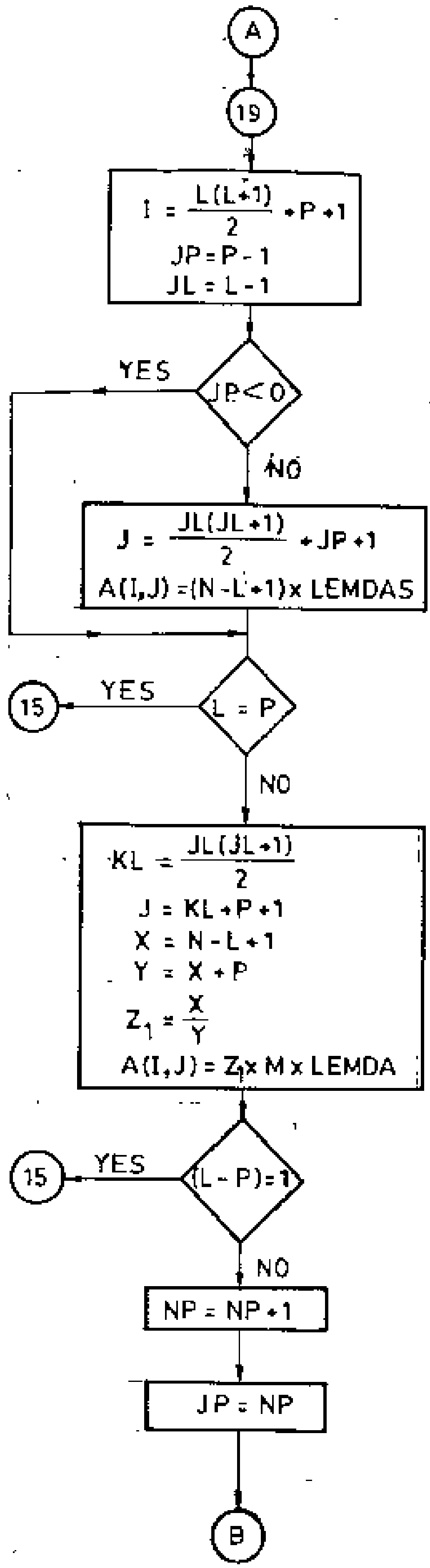
15. G.G. PULLUM, H. GRAYSON
'Second National Reliability Conference'
U.K., March 1979.

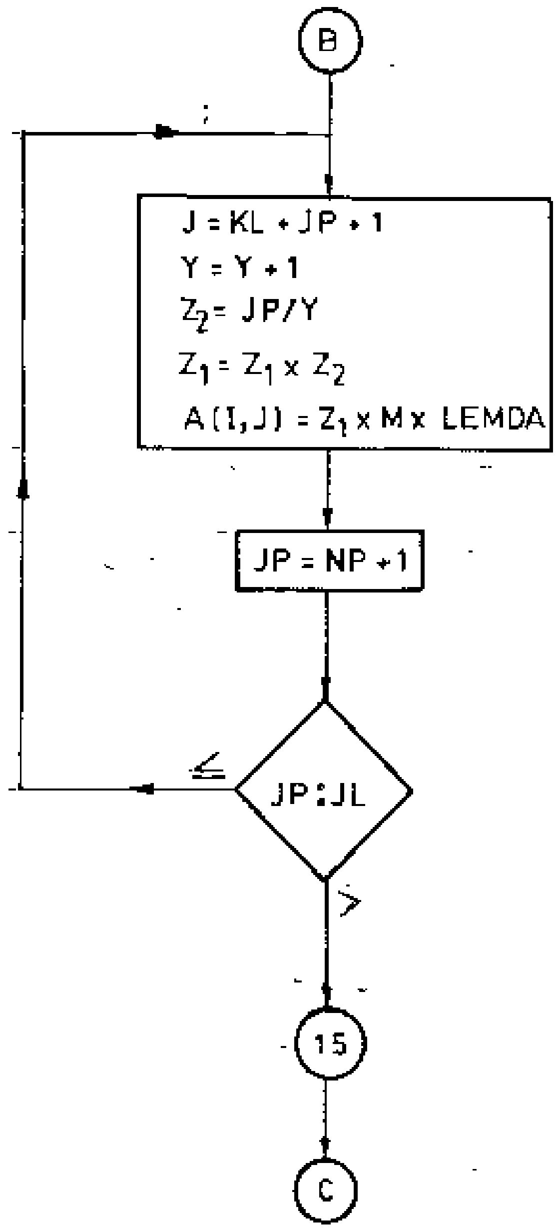
APPENDIX-A

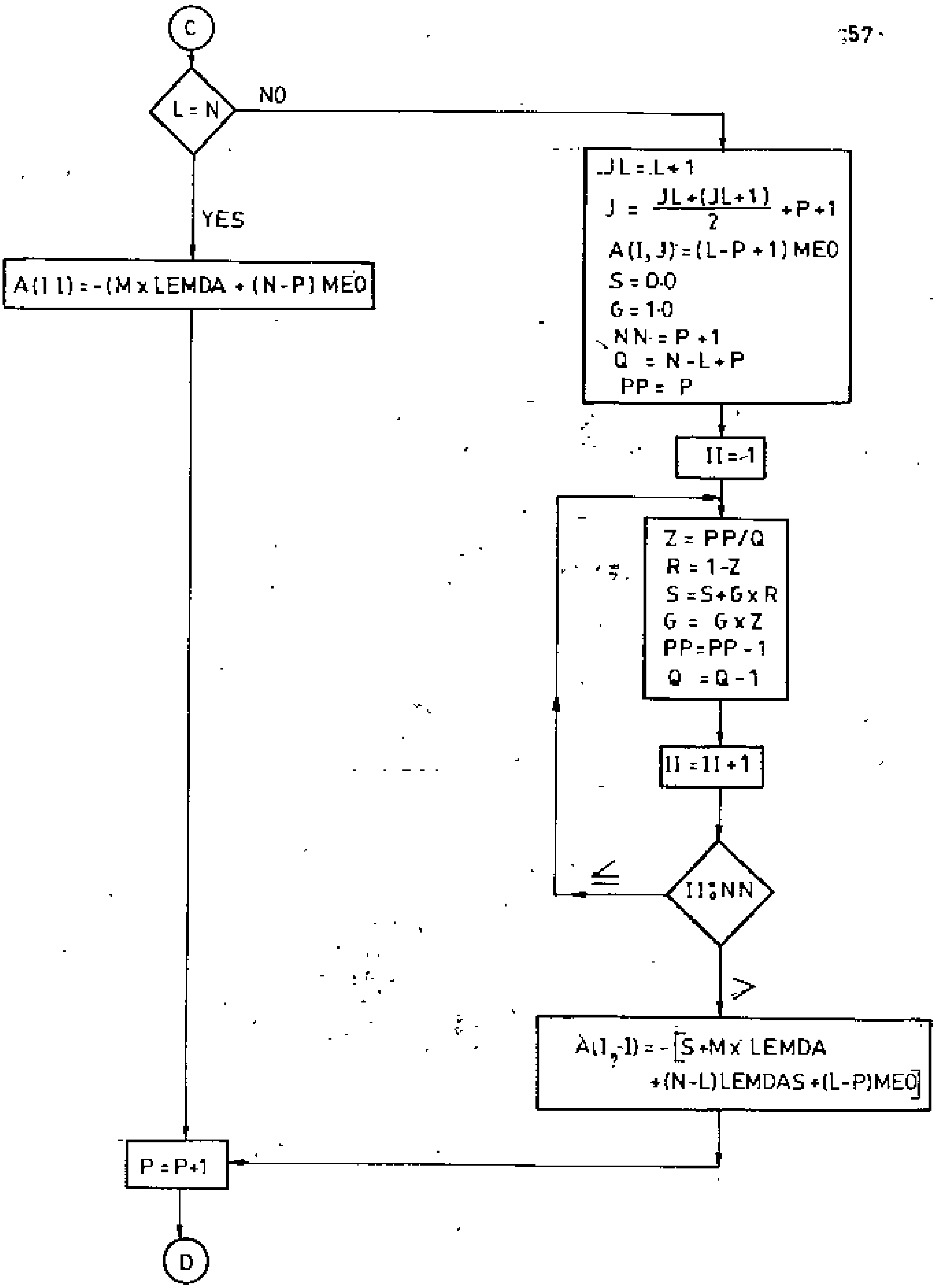
FLOW CHART FOR THE PROGRAMME WITH COST OPTIMAL POLICY

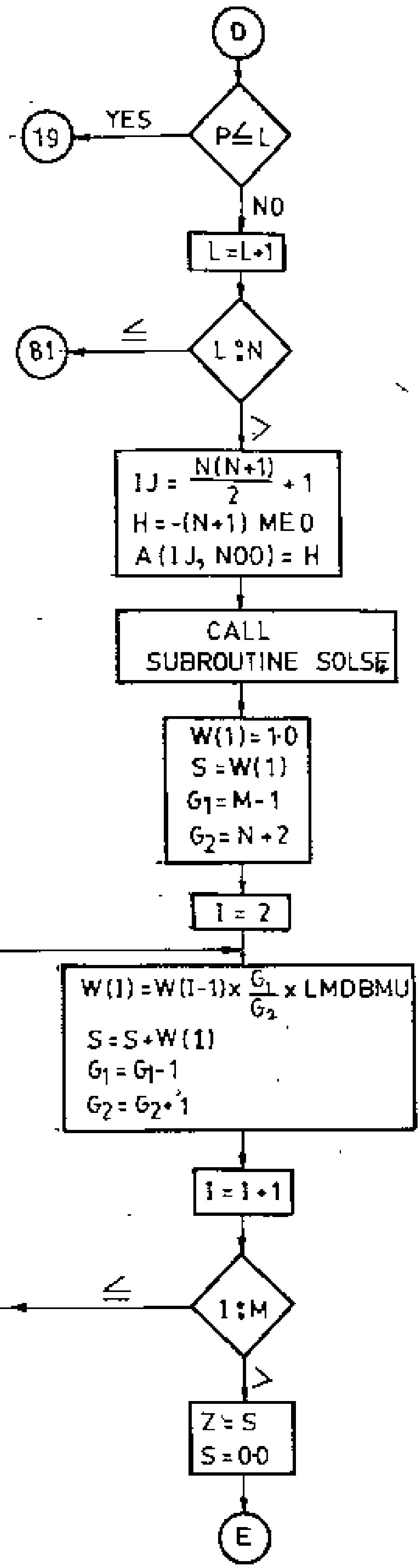
FLOW CHART FOR THE PROGRAMME

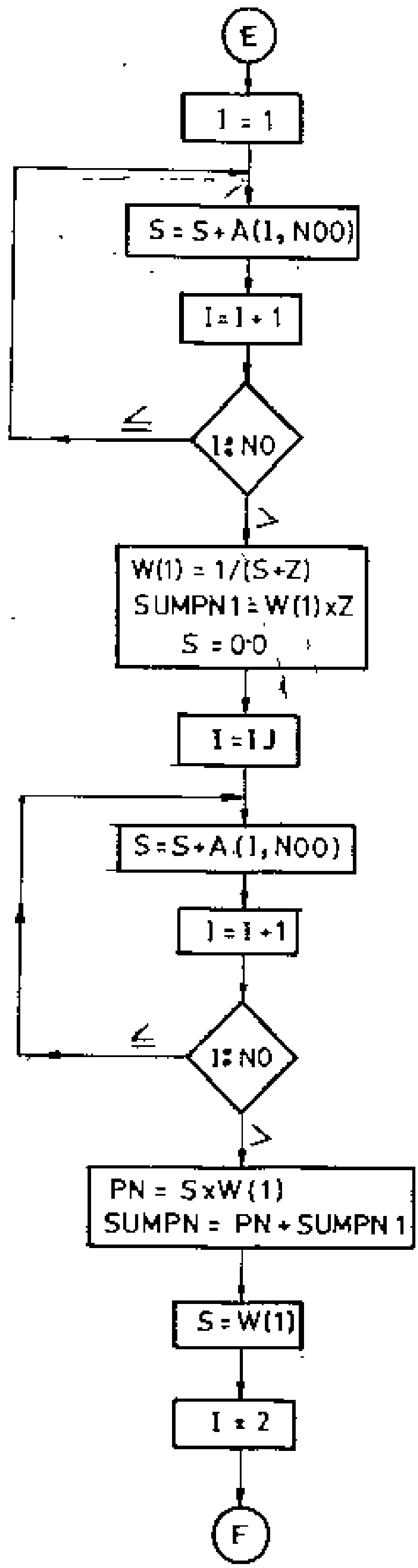


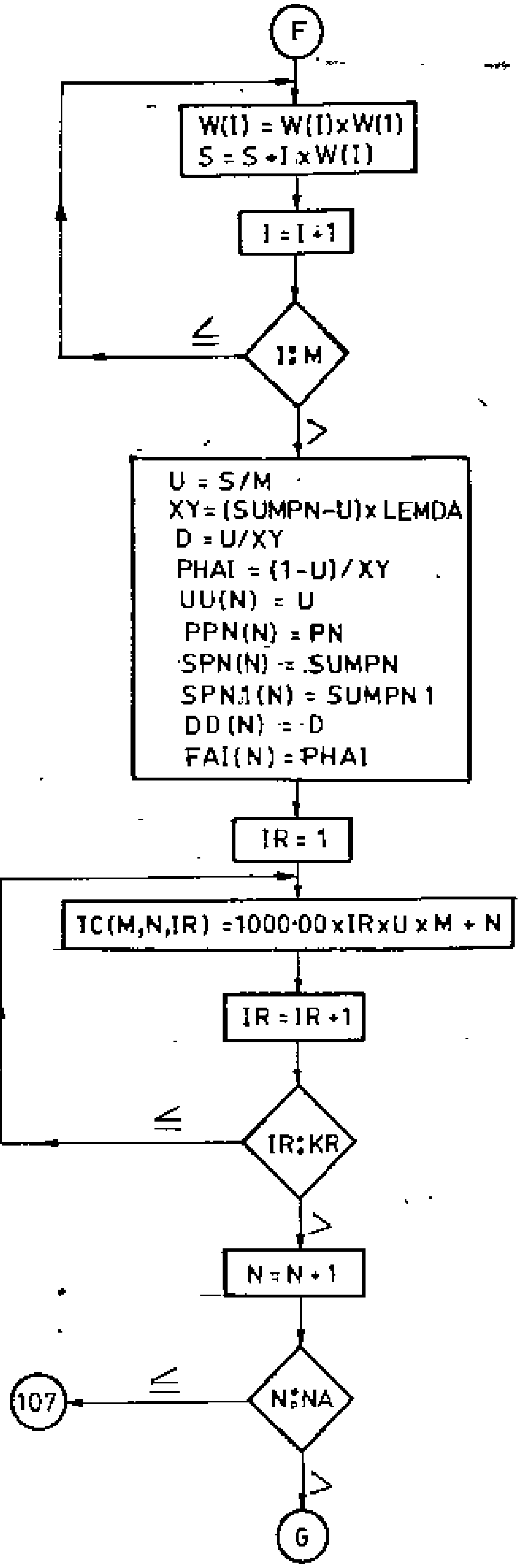

A
19
$I = \frac{L(L+1)}{2} + P + 1$
JP = P - 1
JL = L - 1
YES
JP < 0
NO
$J = \frac{JL(JL+1)}{2} + JP + 1$
A(I,J) = (N - L + 1) x LEMDAS
15
YES
L = P
NO
$KL = \frac{JL(JL+1)}{2}$
J = KL + P + 1
X = N - L + 1
Y = X + P
$Z_1 = \frac{X}{Y}$
$A(I,J) = Z_1 \times M \times LEMDA$
15
YES
(L - P) = 1
NO
NP = NP + 1
JP = NP
B

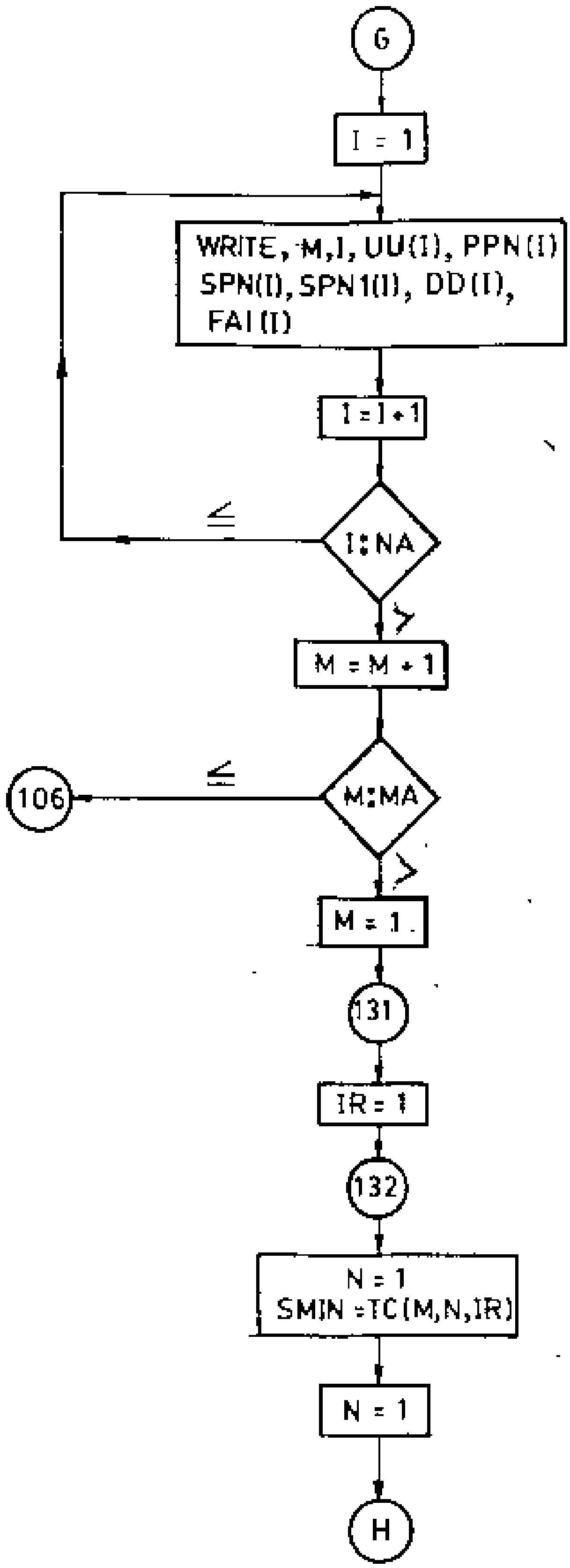

B
J = KL + JP + 1
Y = Y + 1
Z2 = JP/Y
Z1 = Z1 x Z2
A (I,J) = Z1 x M x LEMDA
JP = NP + 1
JP : JL
≤
>
15
C


C
L = N
NO
YES
A(I I) = -(M x LEMDA + (N-P) MEO
JL = L+1
J = (JL+(JL+1))/2 +P+1
A(I, J) = (L-P+1) MEO
S = 0.0
G = 1.0
NN = P+1
Q = N-L+P
PP = P
II = 1
Z = PP/Q
R = 1-Z
S = S+G x R
G = G x Z
PP = PP-1
Q = Q-1
II = II+1
II : NN
≤
>
A(I, I) = -[S+M x LEMDA +(N-L)LEMDAS+(L-P)MEO]
P = P+1
D

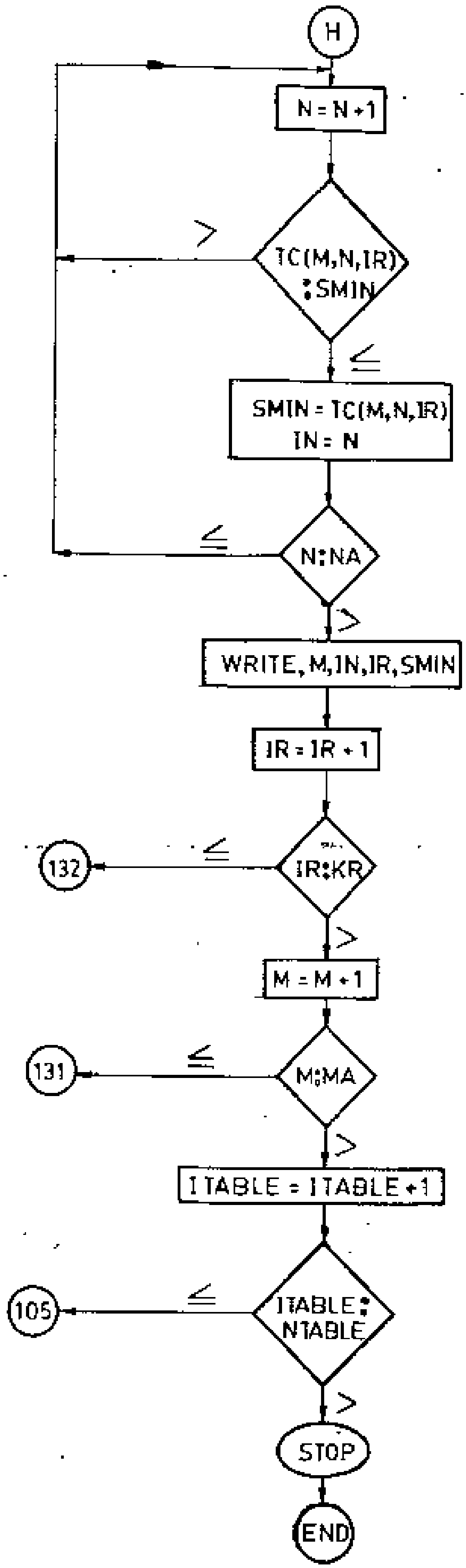

D
P ≦ L
YES
19
NO
L = L+1
L : N
≦
81
>
IJ = N(N+1)/2 + 1
H = -(N+1) ME0
A(IJ, N00) = H
CALL
SUBROUTINE SOLSE
W(1) = 1·0
S = W(1)
G1 = M - 1
G2 = N + 2
I = 2
W(I) = W(I-1) x G1/G2 x LMDBMU
S = S + W(I)
G1 = G1 - 1
G2 = G2 + 1
I = I + 1
I : M
≦
>
Z = S
S = 0·0
E


E
I = 1
S = S + A(I, NOO)
I = I + 1
≤
I : NO
>
W(1) = 1/(S+Z)
SUMPN1 = W(1) x Z
S = 0·0
I = IJ
S = S + A(I, NOO)
I = I + 1
≤
I : NO
>
PN = S x W(1)
SUMPN = PN + SUMPN1
S = W(1)
I = 2
F

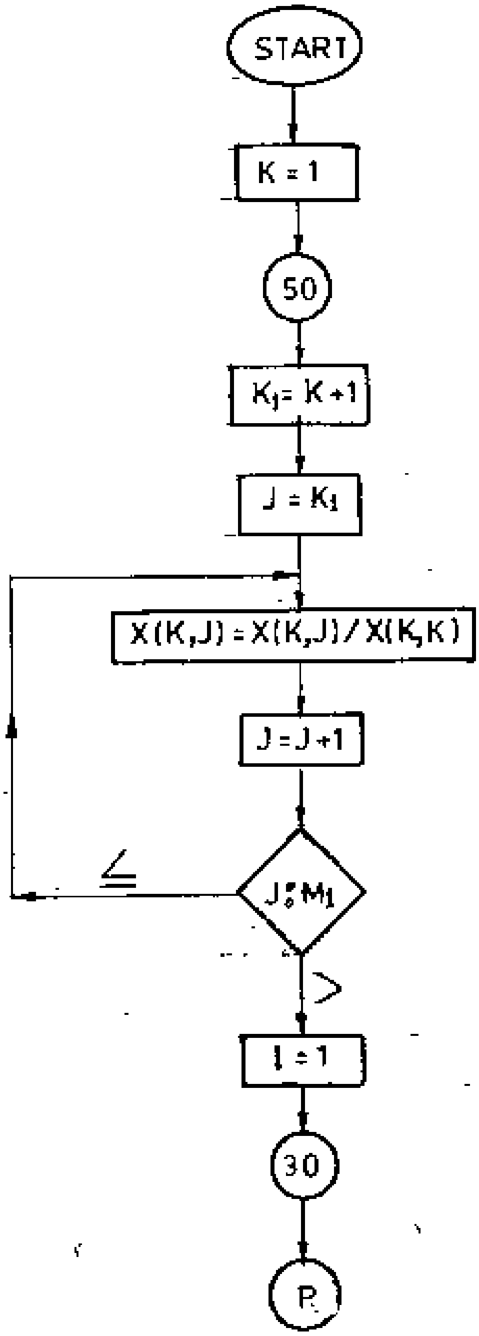

F
W(I) = W(I)xW(1)
S = S + I x W(I)
I = I + 1
≦
I : M
>
U = S/M
XY = (SUMPN-U) x LEMDA
D = U/XY
PHAI = (1-U)/XY
UU(N) = U
PPN(N) = PN
SPN(N) = SUMPN
SPN1(N) = SUMPN 1
DD(N) = D
FAI(N) = PHAI
IR = 1
IC(M,N,IR) = 1000·00 x IR x U x M + N
IR = IR + 1
≦
IR : KR
>
N = N + 1
107
≦
N : NA
>
G



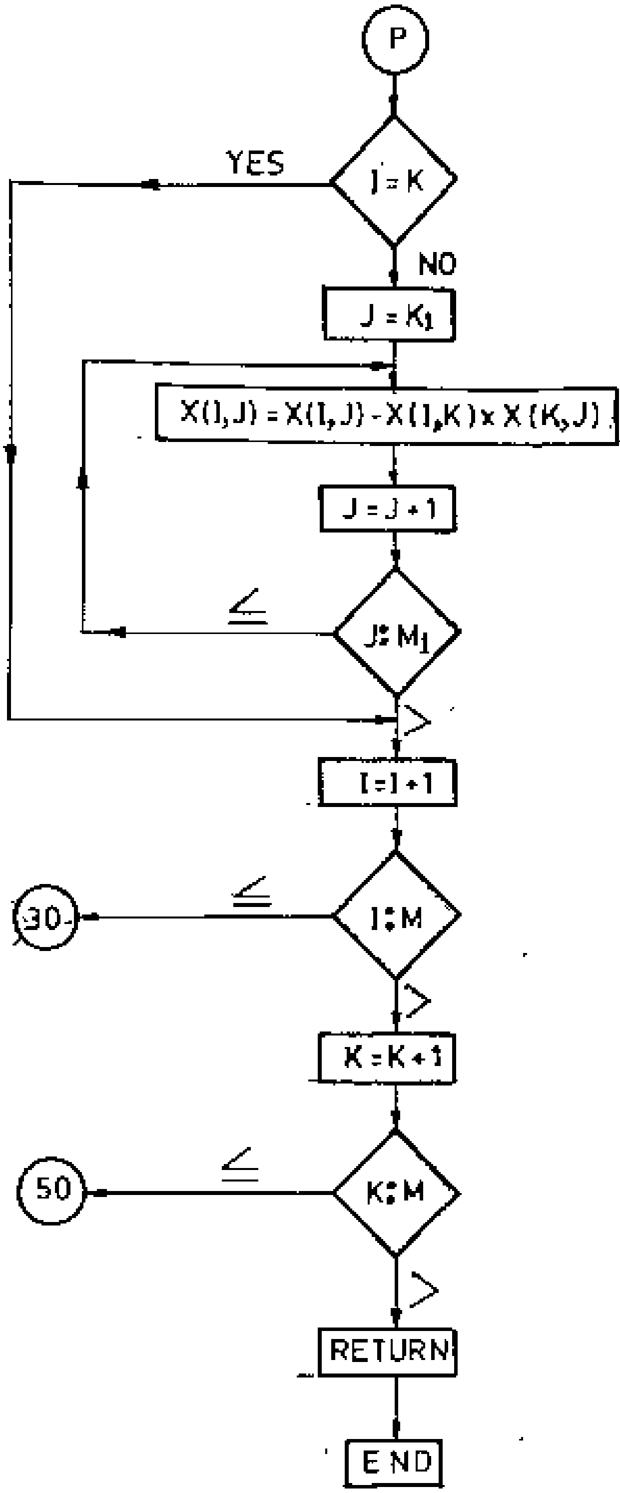
G
I = 1
WRITE, M, I, UU(I), PPN(I) SPN(I), SPN1(I), DD(I), FAI(I)
I = I + 1
I : NA
≤
>
M = M + 1
M : MA
≤
106
>
M = 1
131
IR = 1
132
N = 1
SMIN = TC(M,N,IR)
N = 1
H


H
N = N + 1
TC(M,N,IR) : SMIN
>
≦
SMIN = TC(M,N,IR)
IN = N
N : NA
≦
>
WRITE, M,IN,IR,SMIN
IR = IR + 1
IR : KR
≦
132
>
M = M + 1
M : MA
≦
131
>
ITABLE = ITABLE + 1
ITABLE : NTABLE
≦
105
>
STOP
END

FLOW CHART FOR SUBROUTINE



55811


P
YES
J = K
NO
J = K1
X(I,J) = X(I,J) - X(I,K) x X(K,J)
J = J + 1
≤
J : M1
>
I = I + 1
30
≤
I : M
>
K = K + 1
50
≤
K : M
>
RETURN
END

APPENDIX-B

PROGRAMME WITH COST OPTIMAL POLICY

```
C         S T E P   (1)
C       TO FIND COEFFICIENTS OF THE MATRIX OF ORDER(70*70)
        DIMENSION A(70,70),W(10),UL(10),PPN(10),SPN(10),
       1SPNI(10),DD(10),FAI(10),TC(10,10,10)
        INTEGER P,COUNTR,LPTCNT
        REAL LEMDA,LEMDAS,MEL,LMDBMU
        READ(1,101)NTABLE
  101   FORMAT(I2)
        COUNTR=0
        DO 105 ITABLE=1,NTABLE
        READ(1,102)MEU,LEMDA,LEMDAS
  102   FORMAT(3E10.3)
        LMDBMU=LEMDA/MEU
        WRITE(3,91)ITABLE,MEL,LEMDA,LEMDAS
        READ(1,95)MA,NA,KR
   95   FORMAT(3I2)
        DO 106 M=1,MA
        DO 107 N=1,NA
   11   NO=(N+1)*(N+2)/2
        NOC=NO+1
        DO 80 I=1,NO
        DO 80 J=1,NOO
   80   A(I,J)=0.0
C         FOR L E V E L (0)
        A(1,1)=-(M*LEMDA+N*LEMDAS)
        A(1,2)=MEU
        DO 81 L=1,N
C           FROM B L O C K (1)
        P=0
   19   I=L*(L+1)/2+P+1
        JP=P-1
        JL=L-1
        IF(JP.LT.0)GO TO 14
C         COEFFICIENTS OF BLOCK (1)
        J=JL*(JL+1)/2+JP+1
        A(I,J)=(N-L+1)*LEMDAS
C         B L O C K (2)
   14   IF(L.EQ.P)GO TO 15
        KL=JL*(JL+1)/2
        J=KL+P+1
        X=N-L+1
        Y=X+P
        Z1=X/Y
        A(I,J)=Z1*M*LEMDA
        IF((L-P).EQ.1)GO TO 15
C       B L O C K (3)
        NP=P+1
        DO 82 JP=NP,JL
        J=KL+JP+1
        Y=Y+1
        Z2=JP/Y
        Z1=Z1*Z2
   82   A(I,J)=Z1*M*LEMDA
   15   IF(L.EQ.N)GO TO 17
C       B L O C K (4)
        JL=L+1
        J=JL*(JL+1)/2+P+1
        A(I,J)=(L-P+1)*MEL
C         B L O C K (5)
        S=0.0
        G=1.0
        NN=P+1
        G=N-L+P
        PP=P
        DO 83 II=1,NN
        Z=PP/G
        R=1-Z
        S=S+G*R
        G=G*Z
        PP=PP-1
   83   G=G-1
        A(I,I)=-(S*M*LEMDA+(N-L)*LEMDAS+(L-P)*MEU)
        GO TO 84
   17   A(I,I)=-(P*LEMDA+(N-P)*MEU)
   84   P=P+1
        IF(P.LE.L)GO TO 19
   81   CONTINUE
C       P IS THE COEFFICIENT OF P(N+1) WITH NEGATIVE SIGN.
```

```
      IJ=N*(N+1)/2+1
      H=-(N+1)*MEU
      A(IJ,NCJ)=H
      CALL SCLSE(A,NO,NEQ)
C                  S T E P  (2)-
C     (TO FIND THE VALUE OF --Z--)
      W(1)=1.
      S=W(1)
      G1=M-1
      G2=N+2
      DO 30 I=2,M
      W(I)=W(I-1)*G1/G2*LKDBKU
      S=S+W(I)
      G1=G1-1
      G2=G2+1
   30 CONTINUE
      Z=S
C               S T E P  (3)
C     (SUMMATION OF THE MATRIX SOLUTION)
      S=0.0
      DO 33 I=1,NO
      S=S+A(I,NOJ)
   33 CONTINUE
C               S T E P  (4)
C     (TO FIND THE VALUE OF --PN+1--)
      W(1)=1/(S+Z)
C               S T E P  (5)
C     (TO FIND THE VALUE OF--SUMPN+1--)
      SUMPN1=W(1)*Z
C     S T E P  (6)
C     (TO FIND THE VALUE OF.PN..
      S=0.0
      DO 35 I=IJ,NO
      S=S+A(I,NCJ)
   35 CONTINUE
      PN=S*W(1)
C               S T E P   (7)
C     (TO FIND THE VALUE OF--SUMPN--)
      SUMPN=PN+SUMPN1
C               S T E P   (8)
C     (TO FIND THE VALUE OF--U--)
      S=W(1)
      DO 37 I=2,M
      W(I)=W(I)*W(1)
      S=S+I*W(I)
   37 CONTINUE
C      S T E P  (9)
      U=S/M
      XY=(SUMPN-U)*LEMDA
      C=U/XY
      PHAI=(1-U)/XY
      UU(N)=U
      PPN(N)=PN
      SPN(N)=SUMPN
      SPN1(N)=SUMPN1
      CC(N)=C
      FAI(N)=PHAI
      DO 110 IR=1,KK
      TC(M,N,IR)=100.0*IR*U*M+N
  110 CONTINUE
  107 CONTINUE
   91 FORMAT(1H1//////30X,'I A B L E -',I2/30X,I2('*')///
     16X,'MEU=',E10.3,3X,'LEMDA=',G12.4,3X,'LEMDA-B=',G12.4/////4X,
     2'M',2X,'N',5X,'U',9X,'PN',7X,'SUMP(N)',4X,'SUMP(N+1)',
     35X,'U',5X,'PHAI'///)
      DO 92 I=1,NA
   92 WRITE(3,93)M,I,UU(I),PPN(I),SPN(I),SPN1(I),CC(I),FAI(I)
   93 FORMAT(3X,I2,1X,I2,4(1X,G10.3),2X,F5.3,1X,G10.3/)
      WRITE(3,98)
   98 FORMAT(//)
      COUNTR=COUNTR+1
      IF(COUNTR.EQ.3)GO TO 510
      GO TO 106
  510 COUNTR=0
      WRITE(3,600)
      GO TO 106
  600 FORMAT(1H1//////////////////4X,'M',2X,'N',5X,'U',9X,'PN',
     17X,'SUMP(N)',4X,'SUMP(N+1)',5X,'U',6X,'PHAI'///)
```

```
106 CONTINUE
    WRITE(3,141)
141 FORMAT(1H1//////25X,'COST OPTIMAL POLICY'/25X,
   119('*')///10X,'M',3X,'R',1X,'N(OPT)',4X,'TCS',
   27X,'U',10X,'D',7X,'PHAI'///)
    OPTCNT=0
    DO 131 M=1,MA
    DO 132 IR=1,KR
    N=1
    SMIN=TC(M,N,IR)
    DO 133 N=1,NA
    IF(TC(M,N,IR).GT.SMIN)GO TO 133
        SMIN=TC(M,N,IR)
    IN=N
133 CONTINUE
    WRITE(3,142)M,IR,IN,SMIN,UL(IN),DC(IN),FAI(IN)
142 FORMAT(/9X,I2,2X,I2,2X,I2,4X,F7.3,
   12X,E10.3,2X,F6.3,2X,E10.3)
132 CONTINUE
    WRITE(3,801)
801 FORMAT(//)
    OPTCNT=OPTCNT+1
    IF(OPTCNT.EQ.4)GO TO 710
    GO TO 131
710 OPTCNT=0
    WRITE(3,802)
    GO TO 131
802 FORMAT(1H1//////////10X,'M',3X,'R',1X,'N(OPT)',
   14X,'TCS',7X,'U',10X,'D',7X,'PHAI'///)
131 CONTINUE
105 CONTINUE
    STOP
    END
```

```
      SUBROUTINE GJLSE(X,M,M1)
C     (GAUSS JORDAN ELIMINATION METHOD)
C     M=NUMBER OF EQUATION  &  K=PIVOT ROW.
      DIMENSION X(70,70)
      DO 50 K=1,M
      K1=K+1
      DO 20 J=K1,M1
      X(K,J)=X(K,J)/X(K,K)
   20 CONTINUE
      DO 30 I=1,M
      IF(I.EQ.K)GO TO 30
      DO 40 J=K1,M1
      X(I,J)=X(I,J)-X(I,K)*X(K,J)
   40 CONTINUE
   30 CONTINUE
   50 CONTINUE
      RETURN
      END
```

APPENDIX-C

PROGRAMME FOR SENSITIVITY ANALYSIS VARYING μ&λ

```
C                 S T E P  (1)
C        TO FIND THE COEFFICIENTS OF THE MATRIX OF ORDER 70*70
         DIMENSION A(70,70),W(10),UL(10),PPN(10),SPN(10),
        1SPN1(10),UD(10),FAI(10)
         INTEGER P,COUNTR
         REAL LEMDA,LEMDAS ,MEU,LMDBMU
         READ(1,101)NTABLE
   101   FORMAT(I2)
         COUNTR=0
         DO 103 ITABLE=1,NTABLE
         READ(1,102)M,LEMDA,LEMDAS,NSET
   102   FORMAT(I2,2E10.3,I2)
         LMDBMU=LEMDA/MEU
         WRITE(3,91)ITABLE,M,LEMDA,LEMDAS
         DO 193 ISET=1,NSET
         READ(1,95)MEU,NA
    95   FORMAT(F6.3,I2)
         DO 107 N=1,NA
    11   NO=(N+1)*(N+2)/2
         NOO=NO+1
         DO 80 I=1,NO
         DO 80 J=1,NOO
    80   A(I,J)=0.0
C             FOR L E V E L  (0)
         A(1,1)=-(M*LEMDA+N*LEMDAS)
         A(1,2)=MEU
         DO 81 L=1,N
C                 FROM B L O C K (1)
         P=0
    19   I=L*(L+1)/2+P+1
         JP=P-1
         JL=L-1
         IF(JP.LT.0)GO TO 14
C                 COEFFICIENTS OF BLOCK (1)
         J=JL*(JL+1)/2+JP+1
         A(I,J)=(N-L+1)*LEMDAS
C                      B L O C K  (2)
    14   IF(L.EQ.P)GO TO 15
         KL=JL*(JL+1)/2
         J=KL+P+1
         X=N-L+1
         Y=X+P
         Z1=X/Y
         A(I,J)=Z1*M*LEMDA
         IF((L-P).EQ.1)GO TO 15
C                 B L O C K  (3)
         NP=P+1
         DO 82 JP=NP,JL
         J=KL+JP+1
         Y=Y+1
         Z2=JP/Y
         Z1=Z1*Z2
    82   A(I,J)=Z1*M*LEMDA
    15   IF(L.EQ.N)GO TO 17
C             B L O C K (4)
         JL=L+1
         J=JL*(JL+1)/2+P+1
         A(I,J)=(L-P+1)*MEU
C                      B L O C K  (5)
         S=0.0
         C=1.0
         NN=P+1
         G=N-L+P
         PP=P
         DO 83 II=1,NN
         Z=PP/G
         R=1-Z
         S=S+G*R
         C=G*Z
         PP=PP-1
    83   G=G-1
         A(I,I)=-(S*M*LEMDA+(N-L)*LEMDAS +(L-P)*MEU)
         GO TO 84
    17   A(I,I)=-(M*LEMDA+(N-P)*MEU)
    84   P=P+1
         IF(P.LE.L)GO TO 19
    81   CONTINUE
C        JJ INDICATES EQUATION/  NOW THE RIGHT HAND SIDE
```

```
C      OF WHICH IS THE COEFFICIENT OF  P(N+1)
C      H IS THE COEFFICIENT OF P(N+1) WITH NEGATIVE SIGN.
       IJ=N*(N+1)/2+1
       H=-(N+1)*MEQ
       A(IJ,NCQ)=H
       CALL SCLSC(A,NQ,NQQ)
C                    S T E P  (2)-
C     (TO FIND THE VALUE OF--Z--)
       W(1)=1.0
       S=W(1)
       G1=M-1
       G2=N+2
       DO 30 I=2,M
       W(I)=W(I-1)*G1/G2*LAMDAC
       S=S+W(I)
       G1=G1-1
       G2=G2+1
   30 CONTINUE
       Z=S
C               S T E P  (3)
C      (SUMMATION OF THE MATRIX SOLUTION)
       S=0.0
       DO 33 I=1,NQ
       S=S+A(I,NQQ)
   33 CONTINUE
C               S T E P  (4)
C      (TO FIND THE VALUE OF  --PN+1--)
       W(1)=1/(S+Z)
C               S T E P  (5)
C      (TO FIND THE VALUE OF--SUMPN+1--)
       SUMPN1=W(1)*Z
C                    S T E P  (6)
       S=0.0
       DO 35 I=IJ,NQ
       S=S+A(I,NQQ)
   35 CONTINUE
       PN=S*W(1)
C               S T E P   (7)
C      (TO FIND THE VALUE OF--SUMPN--)
       SUMPN=PN+SUMPN1
C                 S T E P   (8)
C      (TO FIND THE VALUE OF--U--)
       S=W(1)
       DO 37 I=2,M
       W(I)=W(I)*W(1)
       S=S+I*W(I)
   37 CONTINUE
C       S T E P  (9)
       U=S/M
       XY=(SUMPN-U)*LEMDA
       D=U/XY
       PHAI=(1-U)/XY
       UU(N)=U
       PPN(N)=PN
       SPN(N)=SUMPN
       SPN1(N)=SUMPN1
       DD(N)=D
       FAI(N)=PHAI
  107 CONTINUE
   91 FORMAT(1H1/////30X,'TABLE--',I2/30X,12('*')///
     16X,'M=',I2,4X,'LEMDA=',G10.3,4X,'LEMDA-S=',G10.3,
     2/////6X,'MEQ',
     33X,'N',6X,'U',9X,'PN',7X,' SUMP(N)',3X,'SUMP(N+1)',
     44X,'D',6X,'PHAI'///)
       DO 92 I=1,NA
   92 WRITE(3,93)MEQ,I,UU(I),PPN(I),SPN(I),SPN1(I),DD(I),FAI(I)
   93 FORMAT(4X,F6.3,1X,I2,4(1X,G10.3),1X,F6.3,1X,G10.3/)
       WRITE(3,98)
   98 FORMAT(//)
       COUNTR=COUNTR+1
       IF(COUNTR.EQ.3)GO TO 510
       GO TO 106
  510 COUNTR=0
       WRITE(3,600)
       GO TO 106
  600 FORMAT(1H1////////////////6X,'MEQ',3X,'N',6X,
     1'U',9X,'PN',7X,'SUMP(N)',3X,'SUMP(N+1)',4X,'D',
     26X,'PHAI'///)
```

```
0124      106 CONTINUE
0125      105 CONTINUE
0126          STOP
0127          END
```

```
      SUBROUTINE SOLSE(X,M,M1)
      DIMENSION X(70,70)
      DO 50 K=1,M
      K1=K+1
      DO 20 J=K1,M1
      X(K,J)=X(K,J)/X(K,K)
   20 CONTINUE
      DO 30 I=1,M
      IF(I.EQ.K)GO TO 30
      DO 40 J=K1,M1
      X(I,J)=X(I,J)-X(I,K)*X(K,J)
   40 CONTINUE
   30 CONTINUE
   50 CONTINUE
      RETURN
      END
```