EVAPORATION FROM SURFACES EXPOSED TO A THERMALLY-STRATIFIED ATMOSPHERE

by

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In partial fulfillment of the requirements for the degree of Master of Science in Engineering (Water Resources)

BANGLADESH UNIVERSITY OF ENGINEERING AND TECHNOLOGY

DHAKA

June - 1984.

BANGLADESH UNIVERSITY OF ENGINEERING AND TECHNOLOGY

June 19, 1984

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ABSTRACT

A theoretical method of estimating evaporation from bounded open water surfaces considering varying degrees of atmospheric stability and surface roughness conditions is presented. In this method, evaporation is calculated, using routine climatological measurements, from a numerical solution of the turbulent momentum and diffusion equations. The effects of the atmospheric stability and surface roughness conditions are considered by incorporating into the method the appropriate wind and temperature profile equations and evaporation is obtained from the humidity gradient at the diffusing surface.

The theoretical results indicate that the rate of evaporation depends on the atmospheric stability and the downwind dimension of the surface. The results using the climatological measurements at Lake Hefner indicated a satisfactory overall agreement between the calculated and observed rates of evaporation for all degrees of atmospheric stability and surface roughness conditions. However, when it is necessary to derive short-term e.g. daily evaporation estimates accurately, the present work indicates that very precise measurements of the climatological parameters to determine the atmospheric stability with a sufficient degree of accuracy are necessary.

ACKNOWLEDGEMENT

The author acknowledges his sincere gratitude and thanks to Dr. M.A. Halim, Associate Professor, Department of Water Resources Engineering who introduced the author to the interesting field of Evaporation Studies. The author is really grateful to his supervisor for his constant encouragement and wise guidance at all stages of the study.

The author wishes to express his warm gratitude to other members of his thesis committee, Dr. A. Hannan, Professor and and Head, Department of Water Resources Engineering, Dr. M. Shahjahan, Professor, Department of Water Resources Engineering & Dr. M.A. Hossain, Professor, Department of Mechanical Engineering for their special interest and valuable suggestions on many occasions.

The author also wishes to thank Mr. M. Mofser Ali and Mr. A.K. Azad for their assistance in the typing of the thesis and the drafting of figures respectively.

LIST OF SYMBOLS

This table presents the definitions of the symbols in common use in this thesis. When other symbols have been used they are defined in the text.

A	total area of an evaporating surface
D	molecular diffusivity of water in air
e	vapour pressure
E	evaporation
g	acceleration due to gravity
K	von Karman constant
K_h	turbulent or eddy diffusivity for heat
K_m	turbulent or eddy diffusivity for momentum
K_w	turbulent or eddy diffusivity for water vapour
l	pre-step length
L	Monin-Obukhov 'scale length'
L^*	non-dimensionalised 'scale length'
N_e	evaporation number
P	pressure
q	specific humidity
Q	non-dimensional specific humidity
r	radius of a circular surface area
R_e	Reynolds number
R_i	gradient form of the Richardson number
S_c	laminar Schmidt number

$(S_c)_t$ turbulent Schmidt number

t time

T Temperature

u,v,w mean velocity components in the x,y and z directions respectively

U non-dimensional velocity in the x direction

u_* friction velocity

x,y,z downwind, cross-wind and vertical coordinates respectively

z_0 roughness parameter

Z non-dimensional vertical coordinate

Z_0 non-dimensional roughness parameter

Greek Symbols

Γ dry-adiabatic lapse rate

θ_* scaling temperature

μ dynamic viscosity

ν kinematic viscosity

ρ air density

τ shearing stress

ϕ_h dimensionless temperature gradient

ϕ_m dimensionless velocity gradient

θ potential temperature

Subscripts

o	surface or wall value
s	free-stream value
t	turbulent value
l	laminar value
a	value in the air
d	value at dew point temperature
2,4,8	values at 2,4 and 8 meter levels respectively.

An overbar denotes a time-or ensemble-averaged quantity and a prime denotes the deviation from that average.

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Chapter - 1

INTRODUCTION

Evaporation may be defined as the process by which a liquid or solid is converted into the vapour phase. In terms of the hydrological cycle, it forms the main link in the purification of water and its transfer from the oceans over the continents. The underlying surface from which water evaporates may either be an open water surface, snow, ice, vegetation cover or bare soil.

There is no doubt that evaporation has far - reaching effects on the environment and also upon many fields of human activity, affecting as it does, water resources and supply, agriculture, industry and so on. For these purposes, a reliable estimate of evaporation and a clear understanding of the physical processes involved therein, may be of considerable economic importance especially in arid and semi-arid regions of the world where water is often supplied at great expense and the loss of water due to evaporation is high. A method for the accurate determination of water losses from lakes, reservoirs, watersheds, agricultural lands etc. is of vital importance.

The process by which the vapour is removed is partly molecular and partly a turbulent transfer, the turbulent process being dominant everywhere except very close to the evaporating surface. The physical requirement for evaporation is the existence of a vapour concentration gradient that is favourable for the net transfer of water molecules from the surface to the atmosphere. This gradient is maintained by the supply of thermal energy to the evaporating surface so that the water molecules have sufficient energy to vaporize, and by a continuous turbulent diffusion process that assists in the rapid dispersal of the water molecules in the atmosphere. These mechanisms for maintaining the vapour concentration gradient have resulted in the development of two distinct approaches to the theoretical estimation of evaporation using climatological data. The first approach, known as the mass-transfer approach, considers the mechanics of turbulent diffusion whilst the second, the energy-balance approach, is concerned with the principle of energy conservation.

In the present work, a mass-transfer approach has been considered in which evaporation is estimated, using routine climatological measurements, from a numerical solution of the turbulent momentum and diffusion equations. The objectives of the present study are:

1. to develop a method for estimating evaporation from open water surfaces considering the entire ranges of atmospheric stability and surface roughness conditions;

2. to suggest means by which the proposed method can be applied for estimating evaporation using routine climatological data;

3. to examine the theoretical results and the implications of the proposed evaporation model; and,

4. to determine the validity and limitations of the proposed approach for estimating evaporation from bounded water surfaces.

Chapter - 2

REVIEW OF RELATED LITERATURE

2.1 General

Theoretical studies of evaporation have developed along two distinct approaches. The first approach, known as the mass-transfer approach, considers the mechanics of turbulent diffusion whilst the second, the energy-balance approach ignores the theory of turbulence but is concerned with the principle of conservation of energy. The mass-transfer approach is further sub-divided into (i) the semi-empirical approach, (ii) the diffusion approach, and (iii) the aerodynamic approach.

2.2 The Semi-Empirical Approach

The semi-empirical approach is based upon the classical Dalton (1802) theory where the rate of evaporation from an air-water interface is related to the saturation deficit of the air and an empirically determined function of the horizontal wind speed. The formulae based on the Dalton's law have the general form

$$E = a(1+bu)(e_s-e_d) \qquad \ldots \qquad \ldots \qquad (2.1)$$

where,

E = rate of evaporation per unit area

a,b = empirical constants

u = wind speed at a certain height

e_s = vapour pressure at the water surface

e_d = vapour pressure in the air.

Many variations of the basic Dalton formula exist. The formula given by Rowher (1931), based on the results of very intensive work at Fort Collins, Colorado, at 5000 ft above sea-level, for the daily rate of evaporation from an open water surface 3 ft. square is

$$E = 0.40(1+0.27\ u_o)\ (e_s - e_d) \quad \ldots \quad \ldots \quad (2.2)$$

where E is in mm/day, u_o is wind speed at the ground level in miles per hour and e_s and e_d are in mm of Hg.

Penman (1948) modified the above formula in which he incorporated the wind velocity at 2 meter above the ground instead of the ground wind velocity u_o and recast equation (2.2) as

$$E = 0.40\ (1+0.17\ u_2)(e_s - e_d). \ldots \quad \ldots \quad (2.3)$$

Marciano and Harbeck (1954) introduced a mass-transfer coefficient N and proposed the following formula for the daily rate of evaporation from an open water surface:

$$E = Nu\ (e_s - e_d) \quad \ldots \quad \ldots \quad (2.4)$$

where u is the mean wind velocity at a given height. The best fit with experimental results from Lake Hefner was obtained when equation (2.4) takes the form (Chow,1964)

$$E = 1.77 \times 10^{-3} u_8 (e_s - e_d) \quad \ldots \qquad \ldots \quad (2.5)$$

where u_8 is the wind speed at 8 meter in miles per hour, E is in inches/day and e_s and e_d are in millibars.

Thornthwaite and Holzman (1939) derived the following formula for estimating evaporation which they claim to apply rigorously under adiabatic conditions, but over-estimate under conditions with high wind or with a temperature inversion:

$$E = \frac{17.1 \ (u_2 - u_1) \ (e_1 - e_2)}{T + 459.4} \quad \ldots \qquad \ldots \quad (2.6)$$

where E is in inches/hour, T is in ^{o}F and the differences in vapour pressure and horizontal wind velocity are in inches of Hg. and in miles per hour at two heights above the surface, namely 2 and 28.6 feet, respectively.

There are many other empirical formulae of this type but none of them has been shown to be universally applicable. These formulae are true, at their best, only for the sites where these were first developed and can not be applied to other areas.

2.3 The Diffusion Approach

The diffusion approach involves the solution of the diffusion equation giving evaporation in terms of wind speed and size of the evaporating surface. By definition, mass-transfer by diffusion in the atmosphere involves another physical phenomenon, namely turbulence. The atmosphere is generally turbulent except possibly under calm condition or when there are very light winds. Parcels of turbulent air, or eddies, transport entities, for example heat, momentum, carbon dioxide, water vapour and so on, and allow a rapid spread of these properties through the atmosphere.

The basic diffusion equation in a 3-dimensional Cartesian frame of reference can be written as

$$\frac{dq}{dt} = \frac{\partial}{\partial x}\left(K_x \frac{\partial q}{\partial x}\right) + \frac{\partial}{\partial y}\left(K_y \frac{\partial q}{\partial y}\right) + \frac{\partial}{\partial z}\left(K_z \frac{\partial q}{\partial z}\right) \qquad (2.7)$$

where,

$$\frac{dq}{dt} = \frac{\partial q}{\partial t} + u\frac{\partial q}{\partial x} + v\frac{\partial q}{\partial y} + w\frac{\partial q}{\partial z} . \qquad (2.8)$$

It was not possible to obtain a satisfactory solution to the 3-dimensional diffusion equation (2.7). Attempts were, therefore, being made to solve the 2-dimensional diffusion equation under steady-state conditions in which the lateral and longitudinal components of turbulent diffusivity have been ignored and it was assumed that there was no vertical and lateral components of the wind velocity. Under these conditions, equation (2.7) reduces to

$$u \frac{\partial q}{\partial x} = \frac{\partial}{\partial z} \left(K_z \frac{\partial q}{\partial z} \right). \qquad (2.9)$$

Solution of the above equation for flow over aerodynamically smooth surfaces was considered by Sutton (1934). Assuming that the turbulent diffusivities for momentum and water vapour are identical, Sutton derived the following expression for the turbulent diffusivity of water vapour as

$$K_z = \frac{(\frac{1}{2}\pi K^2)^{1-n}}{(1-n)} \rho\nu^{n} \left\{ \left| \frac{\partial u}{\partial z} \right|^{3} \Big/ \left| \frac{\partial^2 u}{\partial z^2} \right|^{2} \right\}^{1-n} \qquad (2.10)$$

where n is a measure of atmospheric stability having a value of 0.25 under neutral conditions. In deriving equation (2.10), Sutton represented the wind structure near the ground by the equation

$$u = u_1 \left(\frac{z}{z_1}\right)^{\frac{n}{2-n}}, \quad 0 < n < 1 \qquad \ldots \qquad (2.11)$$

where u_1 is the mean wind velocity at a fixed height z_1.

Sutton (1934) found that for a rectangular evaporating surface of dimensions (x_o, y_o) the total evaporation is given by

$$E(x_o, y_o) = A_1 (u_1)^{\frac{2-n}{2+n}} (x_o)^{\frac{2}{2+n}} (y_o) \qquad (2.12)$$

and for a circular area of radius, r,

$$E = A_2 (u_1)^{\frac{2-n}{2+n}} (r)^{\frac{4+n}{2+n}} \qquad (2.13)$$

where the coefficients A_1 and A_2 include the differences in vapour concentrations. For neutral conditions equation (2.12) reduces to

$$E_o = \text{constant } (u_1)^{0.78} x_o^{0.89} \quad \ldots \quad \ldots \quad (2.14)$$

where E_o is the evaporation per unit width across the wind. The rate of evaporation per unit area is given by

$$\frac{\partial E_o}{\partial x} = \text{Constant } (u_1)^{0.78} x_o^{-0.11} \qquad (2.15)$$

The above equation shows that the rate of evaporation is directly proportional to the power of the downwind speed less than unity. This amounts to saying that the local rate of evaporation decreases with the distance from the upwind edge, so that the phrase "evaporation per unit area" is ambiguous unless the complete area of the evaporating surface is specified.

Takhar (1972) stated the physics of evaporation is more complicated than simply the treatment of the diffusion equation alone. It is in fact a continuous interaction of the wind velocity and the concentration or humidity fields. A fully-developed turbulent momentum boundary layer comes into contact with the turbulent diffusion boundary layer. Evaporation is the end product of the interaction between these two fields. Considerations like this led Takhar (1972) to estimate evaporation from a numerical solution of the turbulent momentum and diffusion equations for 2-dimensional flow over aerodynamically smooth surfaces in a neutral atmosphere. He assumed that the physical properties of the fluid are constant, the universal "law of the wall" applies throughout the turbulent boundary layer and the eddy diffusivities for momentum and the water vapour are equal i.e., $K_m = K_w$. He introduced a dimensionless evaporation parameter defined as

$$E_v = D \left(\frac{\partial q}{\partial z} \right)_{z=0} \Big/ (q_s - q_o)\, u_s \qquad (2.16)$$

$$= \frac{1}{S_c \quad U_s} \left[\frac{\partial Q}{\partial Z} \right]_{Z=0} \qquad (2.17)$$

where S_c is the Schmidt number ($= \nu/D$) and Q and Z are the non-dimensional specific humidity and vertical distance given by

$$Q = \frac{q - q_o}{q_s - q_o} \qquad (2.18)$$

and

$$Z = \frac{z u_*}{\nu} \qquad (2.19)$$

respectively. The total evaporation E, from an open water surface of effective width 'w' is then given by

$$E = 'w' \int_1^x E_v \, (q_s - q_o)\, u_s \, dx \qquad (2.20)$$

where l represents the distance between the leading edges of the turbulent momentum and diffusion boundary layers. This theoretical treatment seems to be an improvement upon Sutton's (1934) solution since the coupled effect of the interacting momentum and humidity fields are considered simultaneously.

The diffusion method is only applicable with validity to saturated surfaces (Deacon et al.,1958). When the surface begins to dry out, the mechanism by which water vapour is transferred to the air must be combined with a consideration of the process by which liquid and vapour transfers occur in the subsurface. In other words, the diffusion approach, although soundly based in theory, does have its limitations in practice.

2.4 The Aerodynamic Approach

The aerodynamic approach gives evaporation in terms of the differencesof wind speeds and humidities at two levels above the evaporating surface under the condition that the vapour concentration is independent of the downwind position. Within a study of the aerodynamic properties of the atmosphere another concept needs some consideration, namely the concept of aerodynamic roughness. A physical boundary is aerodynamically rough when fluid flow is turbulent down to the boundary itself.

Over such a boundary the velocity profile and surface drag are independent of the fluid viscosity but are dependent upon a roughness length, usually designated as z_o, which is related to the height and spacing of the roughness elements of the surface. A surface is aerodynamically smooth if a layer adjacent to the surface has a flow which is laminar and in which the velocity profile and surface drag are related to the fluid viscosity. It should be noted that a surface which is aerodynamically smooth at low speeds of flow may become rough at high velocity of flow and vice versa. A surface may therefore be described as rough or smooth only in terms of the associated flow. As Sutton (1949) pointed out, it is now well established that turbulent flow near the surface is aerodynamically rough.

The aerodynamic method is based upon three assumptions, namely that, (i) the wind structure near the surface is represented by the universal logarithmic law given by

$$u_z = \frac{1}{K} \sqrt{\frac{\tau_o}{\rho}} \ \ln \left(\frac{z}{z_o} \right) \tag{2.21}$$

where u_z is the wind speed at the level z, (ii) the eddy diffusivity for water vapour, K_w, is identical with that for momentum, K_m, and, (iii) the shearing stress is constant with height. Based on these assumptions, the simple aerodynamic formula first introduced by Thornthwaite and Holzman (1939) may be stated as

$$E = \frac{\rho K^2 (q_2 - q_8)\ (u_8 - u_2)}{\left[\ln\ \left(\frac{z_8}{z_2}\right)\right]^2} \qquad (2.22)$$

where all values are in CGS units. The above evaporation equation utilizes the 2 and 8 meter climatological measurements.

A similar equation was also given by Sverdrup (1946) as

$$E = \frac{\rho K^2 u_8\ (q_o - q_8)}{\left[\ln\ \left(\frac{z_8}{z_o}\right)\right]^2} \qquad (2.23)$$

where z_o is the surface roughness parameter.

It should be noted in the aerodynamic approach that, the humidity values are observed at two heights and are fundamental to the above formulae. Evaporation is proportional to the difference in vapour concentration between the two reference heights. As the aerodynamic approach is valid under the condition that the vapour concentration is independent of the downwind position, it is only applicable for large evaporating surfaces such as seas, lakes, large reservoirs etc.

2.5 The Energy-Balance Approach

In the energy-balance or energy-budget approach, the energy acting on the unit area of an evaporating surface is balanced against the various sources of its dissipation such that the only unknown quantity is the energy used for evaporation.

Penman (1948) outlined a method for estimating evaporation based on the heat balance equation of Brunt (1939). Using as the unit of energy, the amount required to evaporate 1/10 gm. of water at air temperature (59 cal.), Penman gave the following expression for the heat budget H:

$$H = R_c\,(1-r-\mu) - \sigma\,T_a^4\,(0.56-0.092\,\sqrt{e_d}) \times (1-0.09m) \quad (2.24)$$

where,

R_c = measured short-wave radiation

r = reflection coefficient for the surface

μ = fraction of R_c used in photosynthesis

σ = Boltzmann constant

m = fraction of sky covered with cloud, in tenth

e_d = vapour pressure at dewpoint temperature in mm.Hg.

T_a = temperature in ^{o}K.

The heat budget is used in evaporation, E, heating of the air, K, heating of the test material, S, and heating of the surroundings of the test material, C, i.e.,

$$H = E + K + S + C. \qquad \ldots \qquad \ldots \qquad (2.25)$$

Over a period of several days, and frequently over a single day, the change in the stored heat, S, and the heat conducted through the walls of the test material container, C, are negligible compared with other changes. Thus, equation (2.25) can be safely reduced to

$$H = E + K. \qquad (2.26)$$

The transport of vapour and the transport of heat by eddy diffusion are, in essentials, controlled by the same mechanism, and apart from the differences in the molecular constants, the one is expected to be governed by $(e_s - e_d)$ where the other is governed by $(T_s - T_a)$. To a very good approximation, therefore, it is possible to write down the ratio of K/E in the form

$$\frac{K}{E} = \beta = \gamma \frac{(T_s - T_a)}{(e_s - e_d)} \qquad \ldots \qquad \ldots \qquad (2.27)$$

where β is the Bowen's ratio and γ is the standard constant of the wet and dry bulb hygrometer equation. In $^{\circ}F$ and mm. Hg, $\gamma = 0.27$, Thus, combining equations(2,26) and (2.27) one obtains

$$E = \frac{H}{(1 + \beta)}. \qquad \ldots \qquad \ldots \qquad (2.28)$$

Of the terms on the right-hand side of equation (2.24), the radiation term is rarely directly measurable, but for periods of the order of a month or more it can be estimated from duration of sunshine using the relation

$$R_C = R_A\,(a + bn/N) \qquad \ldots \qquad \ldots \qquad (2.29)$$

where R_A is the Angot value of R_c in a completely transparent sky and n/N is the ratio of actual to possible hours of sunshine and a and b are empirical coefficients.

The value of μ in equation (2.24) is very small ($\simeq 0.005$) and can be neglected. The value of r will vary with season and type of surface. It is obvious that cloud control of long-wave radiation must depend upon cloud type, and as a provisional expedient to make some allowance for this, Penman (1948) proposed to set m/10 = 1-n/N. Equation (2.24) therefore reduces to

$$H = E(1+\beta) = (1-r)\ R_A(a + bn/N) - \sigma T_a^4\ (0.56-0.092\sqrt{e_d})\ x$$
$$(0.01 + 0.90\ n/N) \quad \ldots \quad (2.30)$$

where R_A (a + bn/N) is to be replaced by R_c when this is known. The parameters represented on the right-hand side of equation (2.30) are easily determined. If the water surface temperature is known, equation (2.27) may be used to determine β and evaporation may be obtained from equation (2.30).

Penman (1948) also suggested a mass-transfer-energy-balance combination approach for estimating evaporation which eliminates the surface temperature-the parameter which is measured with considerable difficulty.

From considerations of turbulent transfer, the evaporation rate is given by

$$F = f(u)\ (e_s - e_d) \quad \ldots \quad \ldots \quad (2.31)$$

and sensible heat transfer is given by

$$K = \gamma f(u) \ (T_s - T_a). \quad \ldots \quad \ldots \quad (2.32)$$

This assumes that f(u) is the same for vapour transfer as for heat transfer. It is a function of wind speed at a known height. It is convenient to define a new quantity E_a by

$$E_a = f(u) \ (e_a - e_d) \quad \ldots \quad \ldots \quad (2.33)$$

and also to introduce another parameter

$$\Delta = \frac{de}{dT} \sim \frac{e_s - e_a}{T_s - T_a}. \quad \ldots \quad \ldots \quad (2.34)$$

Normally the surface temperature and vapour pressure are unknown but for open water they are uniquely related and can be eliminated from the above equation. Thus,

$$\frac{E_a}{E} = 1 - \frac{e_s - e_a}{e_s - e_d} = 1 - \psi \ \text{(say)} \quad \ldots \quad (2.35)$$

and we have

$$E = \frac{H}{(1 + \beta)} = \frac{H}{\left[1 + \gamma \dfrac{T_s - T_a}{e_s - e_d}\right]} \ldots \quad (2.36)$$

If we put

$$T_s - T_a = \frac{e_s - e_a}{\Delta}, \quad \ldots \quad \ldots \quad (2.37)$$

$$\frac{H}{E} = 1 + \frac{\gamma}{\Delta}\psi, \quad \ldots \quad \ldots \quad (2.38)$$

therefore

$$E = \frac{H\Delta + \gamma E_a}{\Delta + \gamma} \text{ mm./day.} \quad \dots \quad \dots \quad (2.39)$$

$$e_s = \frac{(e_a - \psi e_d)}{(1 - \psi)} \quad . \quad \dots \quad \dots \quad (2.40)$$

Ultimately we have E_o the evaporation from a hypothetical open water surface as

$$E_o = \frac{H_o \Delta + \gamma E_a}{\Delta + \gamma} \text{ mm./day.} \quad \dots \quad \dots \quad (2.41)$$

The only weather data needed to evaluate E_o to an adequate degree of approximation are the mean air temperature, humidity, wind speed and duration of bright sunshine. Here H_o is expressed in the same manner as H, the albedo being that for open water. E_a also introduces empiricism in the delimitation of $f(u)$. Penman from his experiments at Rothamstead obtained the following expression for E_a:

$$E_a = 0.35\ (1+9.8 \times 10^{-3}\ u_2)\ (e_a-e_d) \text{ mm./day....} \quad (2.42)$$

This expression should ideally be re-evaluated for each site; numerous tests of the formula have however shown that this Penman determination is reasonable for use for most locations, at least in temperate lands.

Although the energy-balance approaches appear to be simple in theory, a very accurate determination of the individual items in the heat-budget equation requires complex and expensive instrumentation and is undesirable for obtaining routine estimates. This approach has been successfully undertaken by Anderson (1954) for determining evaporation losses from Lake Hefner.

Chapter - 3

THE THEORETICAL EVAPORATION MODEL

3.1 General

As stated earlier, Takhar (1972) outlined a method for estimating evaporation from a numerical solution of the turbulent momentum and diffusion equations under steady-state neutral atmospheric conditions for 2-dimensional parallel flow over aerodynamically smooth surfaces. This method is analogous to the approach of Hatton (1964) for determining heat transfer from a semi-infinite horizontal flat plate. It is assumed that the ground vapour concentration is at the lower free-stream value upwind of the diffusing surface followed by a jump in humidity at the onset of the evaporating surface. The universal "law of the wall", as proposed by Spalding(1961), was used by Takhar (1972) to relate the non-dimensional velocity with the non-dimensional vertical height. Experimental investigation of this approach was undertaken by Halim (1982) and Takhar and Liddament (1983). The results of the experimental work indicated that a fairly good agreement between the estimated and observed rates of evaporation is obtained when the daily evaporation losses are less than about 4 mm. For a higher rate of evaporation, the associated high velocity of flow makes the surface so rough that an accurate estimation of the daily evaporation using the "law of the wall" is not possible. It was concluded that for flows over aerodynamically rough surfaces, the effect of the surface roughness on the rate of evaporation has to be taken into consideration.

In the present work, the evaporation model of Takhar (1972) is extended to include the effects of the atmospheric stability and surface roughness on the rate of evaporation. Instead of using the "law of the wall", the stability related wind and temperature profile equations of Businger et al. (1971) are incorporated into the method for estimating evaporation. The "law of the wall" is applicable for flows over aerodynamically smooth surfaces under neutral atmospheric conditions only, whereas, the wind and temperature profile equations of Businger et al. (1971) are applicable for the entire range of atmospheric stability and surface roughness conditions. The proposed method is similar to that developed by Halim (1982) for estimating evaporation from open water surfaces in which the wind and temperature profile equations in the atmosphheric surface layer were obtained from the equations of the turbulent mean and fluctuating motions.

3.2 The Development of the Evaporation Model

The theoretical evaporation model is shown in Fig.3.1. In this method evaporation is considered to take place as a result of the interaction between the turbulent momentum and diffusing boundary layers. The leading edge of the diffusion boundary layer marks the beginning of transition in surface

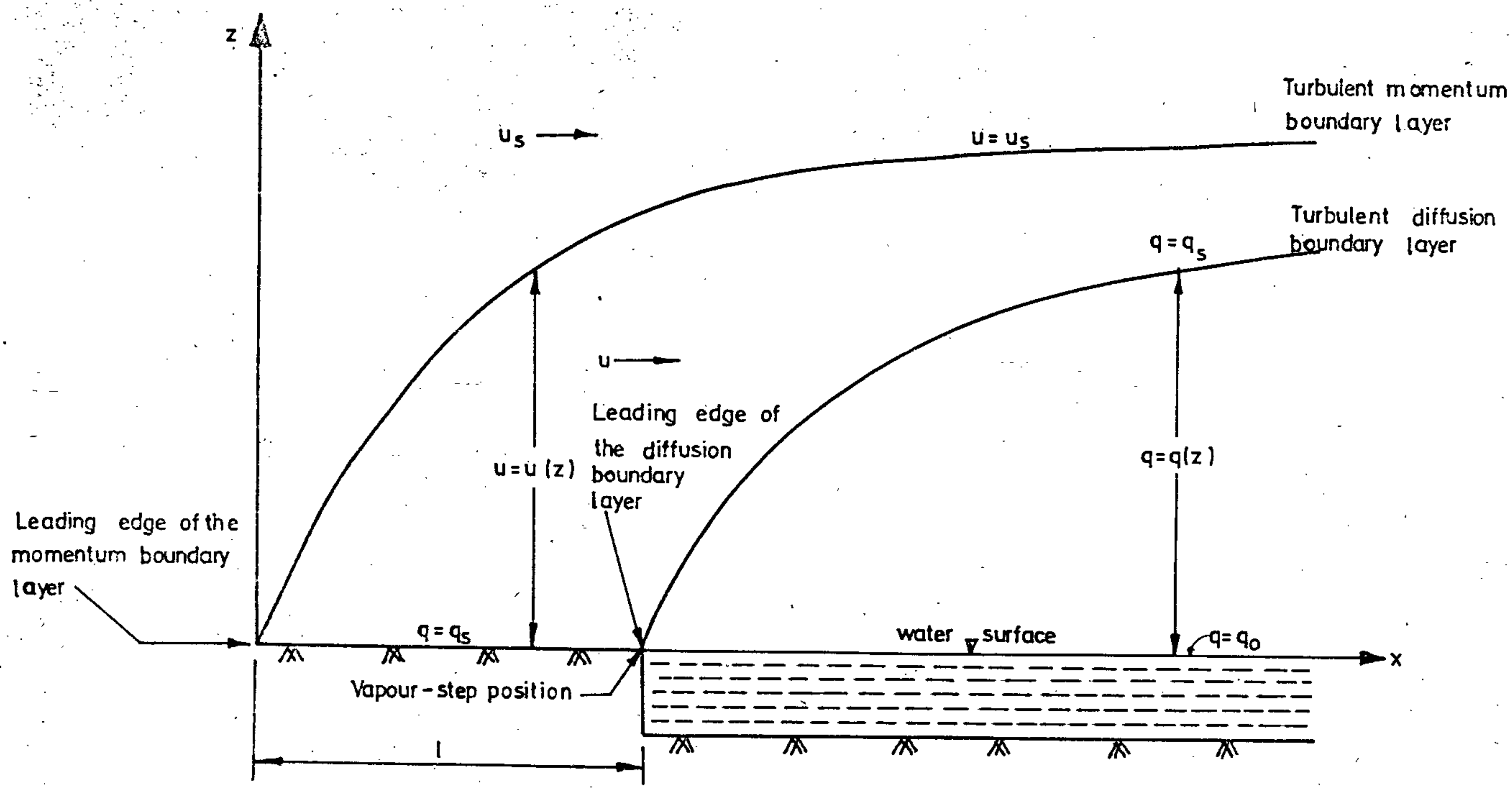

Fig. 3·1 <u>The Theoretical Evaporation Model</u>

vapour concentration from a lower value upwind of the evaporating surface to a higher value over the surface and is commonly known as the "vapour-step position". Also the leading edges of the momentum and diffusion boundary layers (Fig. 3.1) are separated by the distance l which may be called the "separation length".

Taking the x-axis along the mean direction of flow and the z-axis vertically upwards (Fig. 3.1), with associated mean velocities u and w in these directions, the continuity, momentum and diffusion equations for an steady and incompressible turbulent boundary layer in zero pressure gradient may be written respectively as (Takhar, 1972)

$$\frac{\partial u}{\partial x} + \frac{\partial w}{\partial z} = 0 \,, \qquad \ldots \qquad \ldots \qquad (3.1)$$

$$u\frac{\partial u}{\partial x} + w\frac{\partial u}{\partial z} = \frac{\partial}{\partial z}\left[(\nu + K_m)\frac{\partial u}{\partial z}\right] ; \qquad \ldots \qquad (3.2)$$

$$u\frac{\partial q}{\partial x} + w\frac{\partial q}{\partial z} = \frac{\partial}{\partial z}\left[(D + K_w)\frac{\partial q}{\partial z}\right] , \qquad \ldots \qquad (3.3)$$

where ν and D are the kinematic viscosity of air and molecular diffusivity of water in air, K_m and K_w are the eddy diffusivities for momentum and water vapour respectively, and q is the specific humidity or mass concentration of moisture content defined as

$$q = \frac{\text{mass of water vapour}}{\text{mass of moist air}} \, .$$

To obtain equation (3.2), the Boussinesq eddy-diffusivity concept, given by,

$$\frac{\tau}{\rho} = (\nu + K_m)\frac{\partial u}{\partial z} \qquad \ldots \qquad \ldots \qquad (3.4)$$

where τ is the shear stress and ρ is the density of the fluid, has been utilized.

Introducing the stream function, ψ, which defines

$$u = \frac{\partial \psi}{\partial z} \quad \text{and} \quad w = -\frac{\partial \psi}{\partial x},$$

then applying the von Mises transformations and non-dimensionalizing the different variables using the transformations

$$\left.\begin{aligned} U &= \frac{u}{u_*}, \quad Z = \frac{z u_*}{\nu}, \quad Z_o = \frac{z_o\, u_*}{\nu} \\ L^* &= \frac{L\, u_*}{\nu}, \quad R_e = \frac{x\, u_s}{\nu}, \quad Q = \frac{q - q_o}{q_s - q_o} \end{aligned}\right\} \qquad (3.5)$$

where z_o is the surface roughness parameter, u_* ($= \sqrt{\tau_o/\rho}$) is the friction velocity and the subscripts 'o' and 's' refer to the values of an entity at the wall (surface) and in the free-stream respectively, equations (3.2) and (3.3) reduce to (Hatton, 1964 and Takhar, 1972)

$$-U^2 \frac{dU_s}{dR_e} = \frac{1}{\tau_o}\frac{\partial \tau}{\partial Z}, \qquad \ldots \qquad (3.6)$$

and,

$$UU_s \frac{\partial Q}{\partial R_e} = \frac{\partial}{\partial Z}\left[\left(\frac{1}{S_c} + \frac{K_w}{\nu}\right)\frac{\partial Q}{\partial Z}\right]. \qquad (3.7)$$

respectively, where S_c ($= \nu/D$) is the laminar Schmidt number.

Assuming that the functional relationship between U and Z is known (Art. 3.3), equation (3.4) and (3.6) may be used to determine the variation of K_m with Z within the turbulent momentum boundary layer. Then relating K_m to K_w via the turbulent Schmidt number $(S_c)_t$ ($= K_m/K_w$), the variation of K_w can be established. Equation (3.7) can then be solved for Q throughout the turbulent diffusion boundary layer. The humidity gradient at the diffusing surface represented by $Z = Z_o$ can then be determined from which the evaporation loss can be estimated. The boundary conditions for the non-dimensional velocity and humidity are:

$$
\begin{aligned}
U &= 0 , && \text{when } Z = Z_o \\
U &= U(Z) , && \text{when } Z_o < Z < Z_s \\
U &= U_s(R_e) , && \text{when } Z \geqslant Z_s \\
Q &= 0 , && \text{when } Z = 0,\ x > 1 \\
Q &= Q(Z) , && \text{when } 0 < Z < Z_d,\quad x > 1 \\
Q &= 1 , && \text{when } Z \geqslant Z_d,\quad x > 1 \text{ and for all } Z,\ x \leqslant 1
\end{aligned}
\qquad \ldots \quad (3.8)
$$

where Z_d and Z_s are the non-dimensional height of the turbulent diffusion and momentum boundary layers respectively.

Integrating equation (3.6) between the levels Z_o and Z and applying the boundary conditions given in (3.8), we can write

$$-\int_{Z_o}^{Z} U^2 \frac{dU_s}{dR_e} \, dZ = \frac{\tau - \tau_o}{\tau_o} \qquad \ldots \quad (3.9)$$

where τ_o is the shearing stress at the level $Z = Z_o$. Since $\frac{dU_s}{dR_e}$ is not a function of Z we can write

$$-\frac{dU_s}{dR_e} \int_{Z_o}^{Z} U^2 \, dZ = \left(\frac{\tau}{\tau_o} - 1 \right) . \qquad \ldots \quad (3.10)$$

Since at $Z = Z_s$, $\tau = 0$, equation (3.10) gives

$$\frac{dU_s}{dR_e} = \frac{1}{\int_{Z_o}^{Z_s} U^2 \, dZ} , \qquad \ldots \qquad \ldots \quad (3.11)$$

and using equation (3.11) into equation (3.10) we obtain

$$\frac{\tau}{\tau_o} = 1 - \frac{\int_{Z_o}^{Z} U^2 \, dZ}{\int_{Z_o}^{Z_s} U^2 \, dZ} . \qquad \ldots \quad (3.12)$$

Now combining equations (3.4), (3.5) and (3.12), the variation of K_m becomes

$$\frac{K_m}{\nu} = \left[\left(1 - \frac{\int_{Z_o}^{Z} U^2\, dZ}{\int_{Z_o}^{Z_s} U^2\, dZ}\right) \frac{dZ}{dU} - 1 \right] . \qquad \ldots \qquad (3.13)$$

Equation (3.13) may now be used to determine the K_m - variation for any momentum boundary layer height Z_s. In order that the K_m - variation may be determined for the entire momentum boundary layer, the relation between the downwind position, in terms of the Reynolds number, R_e, and Z_s must be determined.

From equation (3.11) we can write

$$\frac{dR_e}{dU_s} = \int_{Z_o}^{Z_s} U^2\, dZ \; . \qquad \ldots \qquad \ldots \qquad (3.14)$$

Integration of equation (3.14) with respect to Z_s between the limits $Z_s = Z_o$ and $Z_s = Z_s$ yields

$$R_e = \int_{Z_o}^{Z_s} \left(\int_{Z_o}^{Z_s} U^2\, dZ \right) \frac{dU_s}{dZ_s}\, dZ_s \quad \ldots \qquad \ldots \qquad (3.15)$$

The right hand side of this equation may be integrated by parts to give

$$R_e = U_s \int_{Z_o}^{Z_s} U^2\, dZ - \int_{Z_o}^{Z_s} U^3\, dZ \; . \qquad \ldots \qquad (3.16)$$

Thus, for a given downwind distance in terms of R_e, equation (3.16) gives the height of the turbulent momentum boundary layer Z_s, and hence, equations (3.13) and (3.16) may be used to determine the K_m-variation throughout the turbulent momentum boundary layer.

The diffusion equation (3.7) may be solved numerically by employing the standard finite-difference techniques. To do so, the area bounded by the diffusion boundary layer and the evaporating surface is to be divided into a rectangular grid system, each rectangle being of dimensions $\Delta R_e \times \Delta Z$ as shown in Fig. A.1. For each step ΔR_e downwind, the value of Q at each grid point may be determined for increments in ΔZ. The procedure for solving the diffusion equation (3.7) using a fully-implicit finite-difference scheme (Spalding, 1977) has been presented in Appendix-A.

The humidity gradient $\partial Q / \partial Z$ at the diffusing surface represented by $Z = Z_o$ may be evaluated by substituting the Q values at the first five grid points near the surface into the five point difference formula (Fox and Mayers, 1968).

The vertical flux of water vapour across the $Z = Z_o$ plane for any position x downwind of the leading edge of the diffusion boundary layer is given by (Sheppard et al., 1972),

$$E_x = -\rho(D + K_w)\left(\frac{dq}{dz}\right)_{z = z_o} \quad \ldots \quad \ldots \quad (3.17)$$

Using the relations given in equation (3.5), it can be shown that

$$E_x = -\rho(q_s - q_o)\, u_s\, N_e \quad \ldots \quad (3.18)$$

where,

$$N_e = \frac{1}{U_s}\left(\frac{1}{S_c} + \frac{K_w}{\nu}\right)\left(\frac{dQ}{dZ}\right)_{Z = Z_o}, \quad \ldots \quad (3.19)$$

is a dimensionless number which may be called the "evaporation number" (Takhar, 1972). By integrating equation (3.18) over the entire surface, the evaporation rate becomes

$$E = \rho\frac{(q_o - q_s)\, u_s}{A}\int_{x = 1}^{x = x_d} b\, N_e\, dx \quad \text{mass/unit area/unit time}, \quad (3.20)$$

where b is the width of an elementary strip of the evaporating surface having a downwind length equal to dx, A is the total area of the evaporating surface and x_d is the total downwind length (Fig.3.2).

3.3 Flux-Profile Relationships in the Atmospheric Surface Layer

It has been stated earlier that the solution of the momentum and diffusion equations requires the functional relationship between U and Z and K_m and K_w. In this work the desired relationships are specified by the following wind-velocity and temperature profile equations (Businger et al., 1971) for a thermally-stratified turbulent flow in the surface layer of the atmosphere:

For an unstable atmosphere ($\zeta < 0$)

$$\varphi_m = (1 - 15\zeta)^{-\frac{1}{4}} \quad \ldots \quad \ldots \quad \ldots \quad (3.21)$$

$$\varphi_h = 0.74\,(1 - 9\zeta)^{-\frac{1}{2}} \quad \ldots \quad \ldots \quad \ldots \quad (3.22)$$

and for a stable atmosphere ($\zeta > 0$)

$$\varphi_m = (1 + 4.7\zeta) \quad \ldots \quad \ldots \quad \ldots \quad (3.23)$$

$$\varphi_h = (0.74 + 4.7\zeta) \quad \ldots \quad \ldots \quad \ldots \quad (3.24)$$

where φ_m and φ_h are the non-dimensional wind velocity and temperature gradients defined by

$$\varphi_m = \frac{Kz}{u_*} \frac{\partial u}{\partial z} \quad \ldots \quad \ldots \quad \ldots \quad (3.25)$$

$$\varphi_h = \frac{z}{\theta_*} \frac{\partial \theta}{\partial z} \quad \ldots \quad \ldots \quad \ldots \quad (3.26)$$

respectively, and

$$\zeta = \frac{z}{L} \quad \ldots \quad \ldots \quad \ldots \quad (3.27)$$

where L is the Monin-Obukhov length given by

$$L = - \frac{T}{g} \frac{u_*^3}{\overline{w' T'}} \frac{1}{K} \quad \ldots \quad \ldots \quad (3.28)$$

and θ_* is the sealing temperature given by

$$\theta_* = \overline{-w' \theta'} / Ku_* . \qquad (3.28a)$$

The length L is a stability parameter which expresses the relative importance between the shear production and the buoyant energy production terms and characterizes the structure of the flow in the atmospheric surface layer.

For a neutral atmosphere, $\zeta = 0$, $L = \infty$, $\varphi_m = 1$ and $\varphi_h = 0.74$. Hence equation (3.25) gives

$$\frac{\partial u}{\partial z} = \frac{u_*}{Kz} \quad \ldots \quad \ldots \quad \ldots \quad (3.29)$$

which leads to the logarithmic profile for $z \gg z_o$. Equations (3.21) and (3.23) may be integrated in conjunction with equation (3.25) to give the following explicit expressions for the wind profiles (Paulson, 1970):

$$\frac{u}{u_*} = \frac{1}{K}\left(\ln \frac{z}{z_o} - A\right) \text{ when } \zeta < 0 \quad \ldots \quad (3.30)$$

where,

$$A = 2 \ln \left[(1+x)/2\right] + \ln \left[(1+x^2)/2\right] - 2 \tan^{-1} x + \pi/2 \quad (3.31)$$

$$x = (1 - 15\zeta)^{\frac{1}{4}} \quad \ldots \quad \ldots \quad (3.32)$$

and

$$\frac{u}{u_*} = \frac{1}{K}\left(\ln \frac{z}{z_o} + 4.7\zeta\right) \text{ when } \zeta > 0. \quad \ldots \quad (3.33)$$

When $\zeta = 0$, integrating equation (3.29) from z_o to z we obtain

$$\frac{u}{u_*} = \frac{1}{K} \ln\left(\frac{z}{z_o}\right) \quad \ldots \quad \ldots \quad (3.34)$$

Non-dimensionalizing equation (3.30), (3.33) and (3.34) using equation (3.5), we can write

$$U = \frac{1}{K}\left(\ln \frac{Z}{Z_o} - A\right) \text{ when } \zeta < 0 \quad \ldots \quad (3.35)$$

$$U = \frac{1}{K}\left(\ln \frac{Z}{Z_o} + 4.7\zeta\right) \text{ when } \zeta > 0 \quad \ldots \quad (3.36)$$

and $U = \frac{1}{K} \ln \frac{Z}{Z_o}$ when $\zeta = 0$ ... ... (3.37)

Also, non-dimensionalization of equation (3.25) using equation (3.5) gives

$$\frac{dU}{dZ} = \frac{\varphi_m}{KZ} \quad \ldots \quad \ldots \quad (3.38)$$

Now, using equations (3.21) and (3.23) into equation (3.38), one obtains

$$\frac{dU}{dZ} = \frac{1}{KZ}\left(1 - 15\zeta\right)^{-\frac{1}{4}} \quad \ldots \quad (3.39)$$

$$\frac{d^2U}{dz^2} = \frac{1}{Kz^2} \; \frac{1}{(1-15\zeta)^{\frac{1}{4}}} \left[\frac{15\zeta}{4(1-15\zeta)} - 1\right]$$

when $\zeta < 0$, and ... ... (3.40)

$$\frac{dU}{dZ} = \frac{1}{KZ}\left(1 + 4.7\zeta\right) \quad \ldots \quad (3.41)$$

$$\frac{d^2U}{dZ^2} = -\frac{1}{KZ^2} \quad \text{when } \zeta > 0 \ . \quad \ldots \quad \ldots \quad (3.42)$$

Under neutral conditions, $\zeta = 0$ and $\varphi_m = 1$. Hence, equation (3.38) gives

$$\frac{dU}{dZ} = \frac{1}{KZ} \quad \ldots \quad \ldots \quad (3.43)$$

and

$$\frac{d^2U}{dZ^2} = -\frac{1}{KZ^2} \quad \ldots \quad \ldots \quad (3.44)$$

For a neutral atmosphere, the value of the free-stream velocity, U_s, can be written, from equation (3.37), as

$$U_s = \frac{1}{K} \ln \frac{Z_s}{Z_o}. \quad \ldots \quad \ldots \quad (3.45)$$

Using equations (3.37) and (3.45) into equation (3.16), it can be shown that, for a neutral atmosphere

$$R_e = \frac{1}{K^3} \left[Z_s \left(\ln \frac{Z_s}{Z_o}\right)^2 - 4Z_s \ln \frac{Z_s}{Z_o} - 2Z_o \ln \frac{Z_s}{Z_o} + 6(Z_s - Z_o)\right]. \quad \ldots \quad (3.46)$$

For the stable and unstable atmospheric conditions, equation (3.16) could not be integrated directly and, therefore, the value of R_e was evaluated numerically.

To solve the diffusion equation (3.7), it is convenient to write it in the form

$$UU_s \frac{\partial Q}{\partial R_e} = \frac{\partial}{\partial Z}\left(M \frac{\partial Q}{\partial Z}\right) = \frac{\partial M}{\partial Z}\frac{\partial Q}{\partial Z} + M \frac{\partial^2 Q}{\partial Z^2} \qquad (3.47)$$

where, $M = \left(\frac{1}{S_c} + \frac{K_w}{\nu}\right)$ $\quad \ldots \quad \ldots \quad (3.48)$

To evaluate the above equation, it is also necessary to specify a relationship between K_m and K_w. Dyer (1967), based on observations from five series of experiments over a freely evaporating surface, concluded that the mechanics of turbulent transport of heat and water vapour are the same and that $K_h = K_w$. Similarly, Webb (1970) concluded that K_w/K_h evidently remains constant, equal to unity, over the whole log-linear range and somewhat beyond. Thus, for the present work, it will be assumed that

$$K_h = K_w. \quad \ldots \quad \ldots \quad (3.49)$$

An immediate implication of the relation (3.49) is that

$$\frac{K_m}{K_h} = \frac{K_m}{K_w} = (S_c)_t \quad \ldots \quad \ldots \quad (3.50)$$

where $(S_c)_t$ is the turbulent Schmidt number. Using equations (3.25), (3.26) and (3.50), it can be shown that

$$(S_c)_t = \frac{\varphi_h}{\varphi_m} \quad \ldots \quad \ldots \quad (3.51)$$

Now, using equations (3.21) to (3.24) into the equation (3.51), we have for an unstable atmosphere ($\zeta < 0$)

$$(S_c)_t = \frac{0.74\,(1-15\zeta)^{\frac{1}{4}}}{(1-9\zeta)^{\frac{1}{2}}}, \quad \ldots \quad \ldots \quad (3.52)$$

for a stable atmosphere ($\zeta > 0$)

$$(S_c)_t = \frac{(0.74 + 4.7\zeta)}{(1+4.7\zeta)} \quad \ldots \quad \ldots \quad (3.53)$$

and for a neutral atmosphere, when $\zeta = 0$, $(S_c)_t = 0.74$.

Using equation (3.50), equation (3.48) can be written as

$$M = \left[\frac{1}{S_c} + \frac{1}{(S_c)_t} \quad \frac{K_m}{\nu}\right] \quad \ldots \quad (3.54)$$

Differentiating the above equation with respect to Z we obtain

$$\frac{\partial M}{\partial Z} = \frac{K_m}{\nu}\frac{\partial}{\partial Z}\left[\frac{1}{(S_c)_t}\right] + \frac{1}{(S_c)_t}\frac{\partial}{\partial Z}\left[\frac{K_m}{\nu}\right] \ldots \quad (3.55)$$

Equations (3.52) and (3.53) in conjunction with equation (3.55) gives for unstable atmosphere ($\zeta < 0$)

$$\frac{\partial M}{\partial Z} = -\frac{1}{(S_c)_t} \frac{U^2}{\int_{Z_o}^{Z_s} U^2\, dZ} \frac{dZ}{dU} + \frac{1}{(S_c)_t}\left(1 - \frac{\int_{Z_o}^{Z} U^2\, dZ}{\int_{Z_o}^{Z_s} U^2\, dZ}\right) \frac{d^2Z}{dU^2}$$

$$\times \frac{dU}{dZ} + \frac{1}{4 \times 0.74\, L} \frac{K_m}{\nu} \frac{(1-9\zeta)^{\frac{1}{2}}}{(1-15\zeta)^{\frac{1}{4}}} \left[\frac{15}{(1-15\zeta)} - \frac{2 \times 9}{(1-9\zeta)}\right], \quad (3.56)$$

for stable atmosphere ($\zeta > 0$)

$$\frac{\partial M}{\partial Z} = -\frac{1}{(S_c)_t} \frac{U^2}{\int_{Z_o}^{Z_s} U^2\, dZ} \frac{dZ}{dU} + \frac{1}{(S_c)_t}\left(1 - \frac{\int_{Z_o}^{Z} U^2\, dZ}{\int_{Z_o}^{Z_s} U^2\, dZ}\right)\frac{d^2Z}{dU^2} \frac{dU}{dZ}$$

$$-\frac{K_m}{\nu} \frac{4.7}{L} (1-0.74) \frac{1}{(0.74 + 4.7\zeta)^2} \quad \ldots \quad (3.57)$$

and for neutral atmosphere ($\zeta = 0$), using equation (3.56) or (3.57)

$$\frac{\partial M}{\partial Z} = -\frac{1}{(S_c)_t} \frac{U^2}{\int_{Z_o}^{Z_s} U^2\, dZ} \frac{dZ}{dU} + \frac{1}{(S_c)_t}\left(1 - \frac{\int_{Z_o}^{Z} U^2\, dZ}{\int_{Z_o}^{Z_s} U^2\, dZ}\right) X$$

$$\frac{d^2Z}{dU^2} \frac{dU}{dZ} \quad . \qquad \ldots \qquad (3.58)$$

Using the values of $\frac{\partial M}{\partial Z}$ from equations (3.56), (3.57) and (3.58), the diffusion equation (3.47) can be solved numerically using finite difference techniques.

3.4 Application of the Evaporation Model for Estimating Evaporation using climatological parameters

In order to apply the proposed method for estimating evaporation, it is first necessary to evaluate R_i, the Richardson number which serves as a useful stability parameter to indicate the atmospheric conditions. Utilizing the temperature and velocity measurements at two discrete levels z_1 and z_2, the gradient form of the Richardson number, R_i, is calculated using the relation

$$R_i = \frac{g}{T_a} \frac{dT/dz + \Gamma}{(du/dz)^2} \qquad \ldots \qquad \ldots \qquad (3.59)$$

$$= \frac{g}{T_a} \frac{\left[(T_2 - T_1) + \Gamma(z_2 - z_1)\right](z_2 - z_1)}{(u_2 - u_1)^2} \qquad (3.60)$$

where, T_a is the absolute ambient air temperature. The sign of R_i depends upon the sign of

$$T_2 - T_1 + \Gamma(z_2 - z_1)$$

where a negative value of the term denotes an unstable atmosphere, a positive value denotes a stable atmosphere and a zero value gives a neutral atmosphere. The dry adiabatic lapse rate, Γ, was taken as 0.01 ^{o}C per meter.

The friction velocity, u_*, can be estimated by integrating equations (3.30), (3.33) and (3.34) between the levels $z = z_1$ and $z = z_2$ to give

$$u_* = \frac{K\,(u_2 - u_1)}{\ln \frac{z_2}{z_1} + A_1 - A_2} \text{ when } \zeta < 0, \quad \ldots \quad (3.61)$$

$$u_* = \frac{K(u_2 - u_1)}{\ln \frac{z_2}{z_1} + 4.7\,(z_2 - z_1)/L} \text{ when } \zeta > 0, \quad \ldots \quad (3.62)$$

and,

$$u_* = \frac{K(u_2 - u_1)}{\ln \frac{z_2}{z_1}} \text{ when } \zeta = 0 \quad \ldots \quad (3.63)$$

where,

$$A_1 = 2 \ln \left[(1+x_1)/2\right] + \ln \left[(1+x_1^2)/2\right] - 2\tan^{-1}x_1 + \pi/2 \quad (3.64)$$

$$A_2 = 2 \ln \left[(1+x_2)/2\right] + \ln \left[(1+x_2^2\ /2\right] - 2\tan^{-1}x_2 + \pi/2 \quad (3.65)$$

$$x_1 = (1-15\ \frac{z_1}{L})^{\frac{1}{4}} \quad \ldots \quad \ldots \quad (3.66)$$

and

$$x_2 = (1 - 15 \frac{z_2}{L})^{\frac{1}{4}} \quad \ldots \quad \ldots \quad (3.67)$$

where u_1 and u_2 are the known wind velocities at the levels z_1 and z_2 respectively.

Integration of equations (3.30), (3.33) and (3.34) between the levels z_o and z_2 gives the value of z_o, the roughness parameter as

$$z_o = \frac{z_2}{\exp\left(\frac{Ku_2}{u_*} + A_2\right)} \quad \text{when } \zeta < 0, \quad \ldots \quad (3.68)$$

$$z_o = \frac{z_2}{\exp\left(\frac{K u_2}{u_*} - \frac{4.7 z_2}{L}\right)} \quad \text{when } \zeta > 0, \quad \ldots \quad (3.69)$$

and when $\zeta = 0$,

$$z_o = \frac{z_2}{\exp\left(\frac{K u_2}{u_*}\right)} \quad . \quad \ldots \quad \ldots \quad (3.70)$$

Thus, to determine the values of u_* and z_o using the above equations, it is necessary to evaluate the value of the Monin-Obukhov Length, L. From equations (3.28), (3.51) and(3.59) it can be shown that

$$R_i = \frac{\zeta \varphi_h}{\varphi_m^2} \quad . \quad \ldots \quad \ldots \quad (3.71)$$

Combining equations (3.21) to (3.24) and (3.71), it is easy to obtain

$$R_i = \frac{0.74\,\zeta\,(1-15\,\zeta)^{\frac{1}{2}}}{(1-9\,\zeta)^{\frac{1}{2}}} \quad \text{when } \zeta < 0, \qquad \ldots \quad (3.72)$$

and,

$$R_i = \frac{\zeta(0.74 + 4.7\,\zeta)}{(1 + 4.7\,\zeta)^2} \quad \text{when } \zeta > 0. \qquad \ldots \quad (3.73)$$

The above two equations may be used to determine the value of L by a trial-and-error approach.

The next set of parameters to be considered here are the mass density, kinematic viscosity and specific humidity of air. The values of these paramters are necessary for evaluating the evaporation equation (3.20). The air density, ρ , was evaluated using the equation of state and taking the molecular weight of dry air as 28.9. At an absolute temperature of 273°K and standard pressure of 1013 millibar, 1 gm. molecular weight of air occupies 22.4×10^3 c.c. Thus, the air density for any temperature T_a and atmospheric pressure P is given by

$$\rho = \frac{28.9 \times 273 \times P}{1013 \times 22.4 \times 10^3\, T_a} = 0.3484 \times 10^{-3}\ \frac{P}{T_a}\ \text{gm/cm}^3. (3.74)$$

The kinematic viscosity of air, ν , could be estimated on the basis of Sutherlands equation of dynamic viscosity (Smithsonian Meteorological Tables, 1966), given by

$$\frac{\mu}{\mu_o} = \frac{T_o + C}{T_a + C} \; (T_a/T_o)^{3/2} \quad . \quad \ldots \quad \ldots \quad (3.75)$$

In the above equation C is the Sutherlands constant, assumed to have the value of 120°C, and μ_o is the known dynamic viscosity at absolute temperature T_o. At $T_o = 296.16°K$, $\mu_o = 1.8325 \times 10^{-4}$ gm^{-1} cm^{-1}sec^{-1} and hence

$$\mu = 1.8325 \times 10^{-4} \; \frac{416.16}{T_a + 120} \left(\frac{T_a}{296.16} \right)^{3/2} . \quad \ldots \quad (3.76)$$

Since, $\nu = \frac{\mu}{\rho}$, combination of equations (3.74) and (3.76) gives

$$\nu = \frac{1.8325}{3.484} \times \frac{T_a}{P} \times \frac{416.16}{(T_a + 120)} \left(\frac{T_a}{(296.16)} \right)^{3/2} . \quad \ldots \quad (3.77)$$

The specific humidity, q, is defined as the ratio of the mass of water vapour to the mass of moist air. However, as the moisture content of the air is small under all atmospheric conditions, the specific humidity has almost the same value as that of humidity mixing ratio, m, defined as the mass of water vapour per unit mass of dry air. By applying the ideal gas law and taking the molecular weight of water to be 18, the vapour pressure e of water vapour at pressure P is given by (Sutton, 1953)

$$e = \frac{m}{0.622 + m} \; P, \quad \ldots \quad \ldots \quad (3.78)$$

from which we obtain

$$m = \frac{0.622e}{P - e} \quad . \quad \ldots \quad \ldots \quad (3.79)$$

Since $m \approx q$ and $P \gg e$ the above equation gives

$$q = \frac{0.622e}{P} \quad \ldots \quad \ldots \quad (3.80)$$

Thus, the evaluation of specific humidity required the determination of vapour pressure, which is given by

$$e = 4.5855 + T_w \left[0.32808 + T_w \left\{0.01172 + T_w \left(1.2793 \times 10^{-4} + 4.1848 \times 10^{-6}\, T_w\right)\right\}\right] - 0.6(T_d - T_w), \quad (3,81)$$

where T_w is positive, and,

$$e = 4.5778 + T_w \left[0.37305 + T_w \left(0.012931 + 1.9309 \times 10^{-4} \times T_w\right)\right] - 0.54(T_d - T_w), \quad \ldots \quad (3.82)$$

where T_w is negative. Here e is the vapour pressure in mm of Hg. and T_d and T_w are the dry and wet bulb temperatures in ^oC respectively. Saturated vapour pressure conditions were assumed to prevail at the water surface and therefore the surface specific humidity was evaluated by substituting $T_d = T_w = T_o$ into equation (3.81) for $T_o > 0$ and into equation (3.82) for $T_o < 0$, where T_o is the surface temperature. The evaluation of the surface specific humidity, q_o, thus requires the measurement of water surface temperature.

The minimum climatological measurements needed to satisfy the requirements of the proposed approach are summarised in Table 3.1.

Table 3.1

Climatological Data Requirements for the Proposed Evaporation Model

Parameter	Function
1. Air (dry bulb) temperature at level z_1 2. Air temperature at level z_2	These are used in conjunction with the wind velocities at levels z_1 & z_2 to evaluate the gradient form of the Richardson number, R_i, using equation(3.60).
3. Wind velocity at level z_1 4. Wind velocity at level z_2	These are used to evaluate the friction velocity, u_*, using equations (3.61),(3.62) & (3.63). Wind velocity at the level z_2 is used to evaluate the surface roughness, z_o, using equations(3.68),(3.69) & (3.70).
5. Wet bulb temperature at level z_2	This is used along with the dry bulb temperature at level z_2 to evaluate the vapour pressure at level z_2 using equation (3.81) or (3.82).
6. Water surface temperature	This is used to estimate the surface vapour pressure using equation (3.81) or (3.82).
7. Atmospheric pressure	This is used in conjunction with the dry bulb, wet bulb and surface temperatures to evaluate the specific humidity, air density and kinematic viscosity using equations (3.80), (3.74) & (3.77) respectively.

In addition to the climatological data, it is also necessary to consider the numerical values of the von Karman constant and the laminar Schmidt number. The value of the von Karman constant, K, was taken as 0.36 in agreement with the observations of Businger et al. (1971). For the normal range of temperature ($-20^{\circ}C$ to $+40^{\circ}C$) and pressure (1000 millibar to 1020 millibar), to be expected in the atmosphere, the numerical value of the laminar Schmidt number varies from 0.594 to 0.598 and an average value of 0.596 was considered to be the most appropriate value for this number to be used in this work.

In order to determine the free-stream value of wind velocity and to satisfy the boundary conditions at $z = z_s$, it is necessary that the leading edge of the turbulent momentum boundary layer with respect to the vapour-step position (Fig.3.1) be located properly so that a fully-developed state of flow is reached before an interaction with the turbulent diffusion boundary layer takes place. The effect of the pre-step length Reynolds number, $(R_e)_1$ $(= \frac{u_s l}{\nu})$, on the rate of evaporation was investigated for various values of $(R_e)_1$. It was observed that for a change of $(R_e)_1$ from 10^6 to 10^7 and from 10^7 to 10^8, the changes in the rate of evaporation were 11% and 6% respectively. As a result, a value of $(R_e)_1$ of 10^8 was taken as being a reasonable indication for the attainment of a fully-developed state. This value of the pre-step length Reynolds number also satisfies the condition of atmospheric flow in that events are governed entirely by inertia and that viscous forces are negligible.

With the value of the pre-step length Reynolds number thus selected, the pre-step length, l, becomes

$$l = \frac{(R_e)_l \nu}{u_s} = \frac{10^8 \nu}{u_s} \quad \ldots \quad \ldots \quad (3.83)$$

The free-stream specific humidity, q_s, was determined by solving the diffusion equation (3.47) for the dimensionless humidity Q at a given level and given downwind position. Using relations (3.5) we have

$$Q_z = \frac{q_z - q_o}{q_s - q_o} \quad \ldots \quad \ldots \quad (3.84)$$

where Q_z is the dimensionless humidity at level z. Rearrangement of the above equation gives

$$q_s = \frac{q_z - q_o}{Q_z} + q_o \quad \ldots \quad \ldots \quad (3.85)$$

The variation of the dimensionless humidity, throughout the diffusion boundary layer, is a unique function of height and downwind position. By measuring the specific humidity, q_z, at the reference level z and known position downwind, equation (3.47) may be used to solve for Q_z at the same level z and same position downwind, and the value of q_s may then be evaluated using equation (3.85).

To evaluate the evaporation equation (3.20), a hypothetical water surface is illustrated in Fig. 3.2 in which A and B represent the leading edges of the turbulent momentum and diffusion boundary layers, separated by the distance l. The water surface is divided into n number of strips of downwind length Δx. Then replacing the integral of the evaporating equation (3.20) by finite increments, the evaporation rate becomes

$$E = \frac{\rho}{A} (q_s - q_o) u_s \sum_{i=1}^{n} b_i (N_e)_i \Delta x_i \qquad (3.86)$$

where b_i refers to the width of the surface at the midpoint of the i'th strip and $(N_e)_i$ is assumed constant for this strip. The Reynolds number, $(R_e)_i$, representing the dimensionless downwind position from the leading edge of the momentum boundary layer is given by

$$(R_e)_i = \left[l + \sum_{1}^{i-1} \Delta x + \frac{\Delta x_i}{2} \right] \frac{u_s}{\nu}. \qquad (3.87)$$

To estimate evaporation from a surface using equation (3.86), it is necessary to approximate the geometry of the surface by a simplified shape so that the effective width, b, for any increment in Δx can be determined. For elongated surfaces, the rectangular approximation is preferable. For this approximation, the width is constant but the dimensions

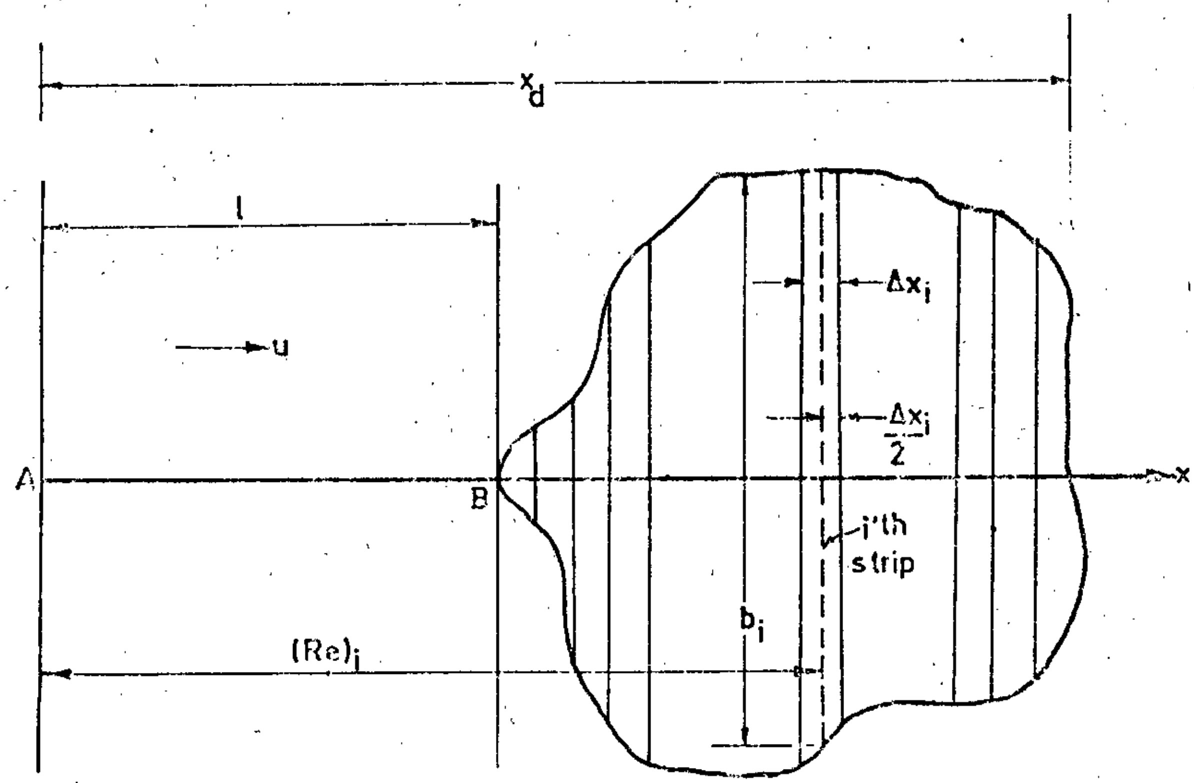

Fig. 3.2 A Hypothetical Open Water Surface Showing the Elementary Strips

of the rectangle now become a function of the wind direction thus necessitating the measurement of wind direction. The circular approximation is advantageous over the rectangular approximation in that the width then becomes independent of the wind direction and the measurement of the wind direction then becomes unnecessary. For circular approximation the width is given by

$$b_i = 2\left[(x + \frac{\Delta x_i}{2})\ (2r - x - \frac{\Delta x_i}{2})\right]^{\frac{1}{2}} \quad \ldots \quad (3.88)$$

$$= 2\left[r^2 - (r - x - \frac{\Delta x_i}{2})^2\right]^{\frac{1}{2}} \quad \ldots \quad (3.89)$$

where, r = radius of the surface and, $x = \sum_{1}^{i-1} \Delta x$.

3.5 Conclusions and Implications of the Theory

In this section the theoretical results and implications of the numerical process are examined. The stability parameter, used to describe the degree of thermal stratification in the lower atmosphere, is taken to be the non-dimensionalised Monin-Obukhov length, given by,

$$L^* = \frac{L\,u_*}{\nu}\ .$$

As $|L^*|$ tends to infinity, the neutral regime, where buoyancy effects are negligible, is approached. When L^* is negative, the atmosphere is unstable and when L^* is positive, the atmosphere is stable. Using this stability parameter, the non-dimensionalised velocity profile laws given by equations (3.35) and (3.36) have been plotted in Fig. 3.3 for an arbitrary value of Z_o of 2.0. As the absolute value of L^* increased, the curves were found to tend to the neutral wind profile given by equation (3.37). Varying the values of Z_o was found merely to add a uniform translation to the curves, as originally suggested by Ellison (1957). The Spalding law of the wall, used by Hatton (1964) and Takhar (1972) is also plotted in the same figure.

The variation of theoretical momentum boundary layer height, Z_s, with Reynolds number, is illustrated in Fig. 3.4 for varying degrees of atmospheric stability. For the stable atmosphere the theoretical boundary layer height was found to be lower than that for the neutral case and as the stability increased , the boundary layer height decreased. Similarly, for increasing atmospheric instability the theoretical moemntum boundary layer height was found to increase. As the absolute value of L^* tended to infinity, the curves representing the stable and unstable atmospheres both tended to the curve representing the neutral atmosphere. This offered some indication that the numerical and analytical methods used to relate R_e to Z_s, for each class of atmospheric stability were correct.

The decrease in theoretical momentum boundary layer height with increasing atmospheric stability is illustrated more clearly in Fig. 3.5 where R_e. was kept constant at an arbitrary value, of the order of 10^9, and Z_s was plotted

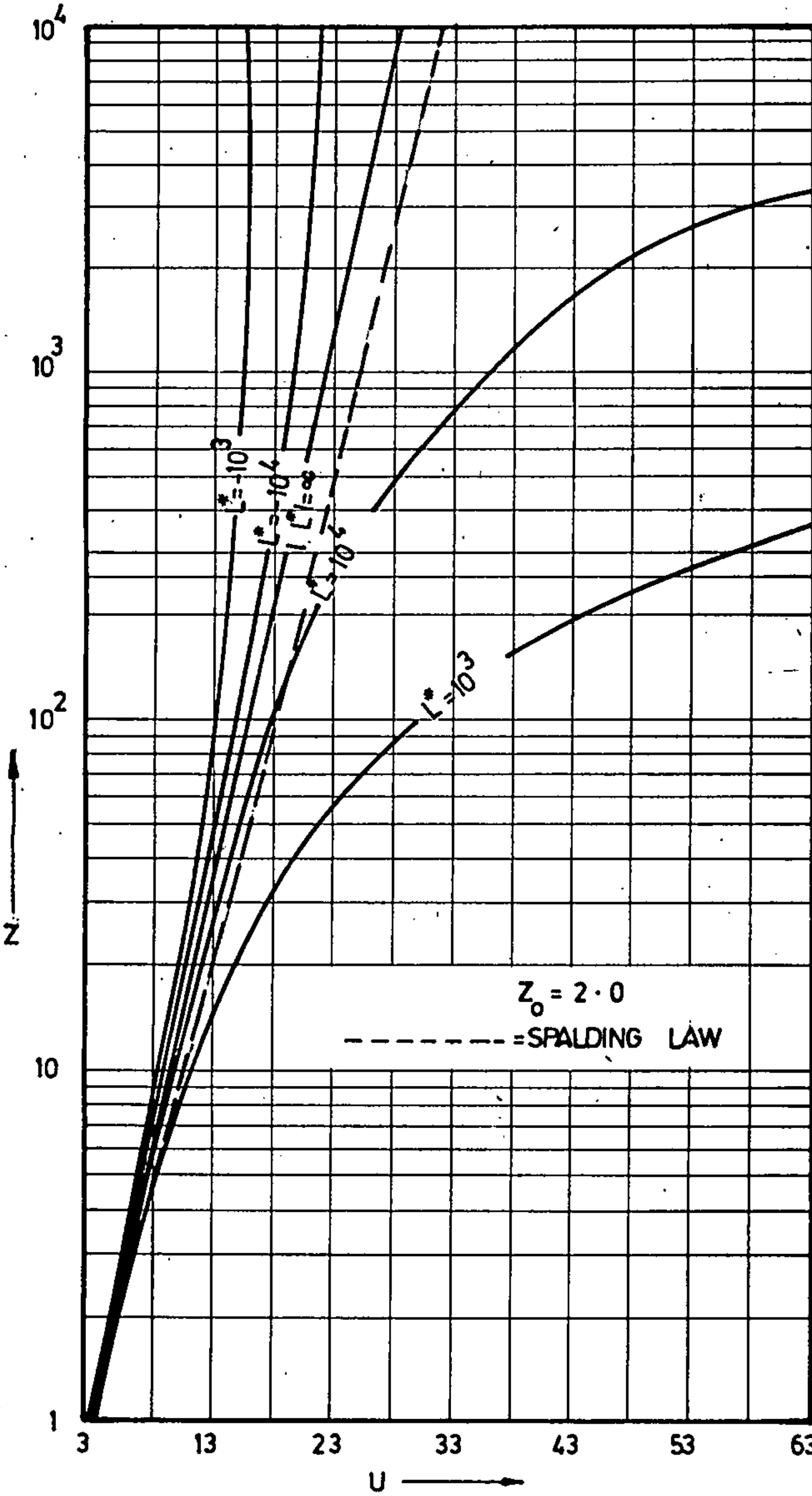

Fig. 3·3 U-Z Relation for Varying Degrees of Atmospheric Stability

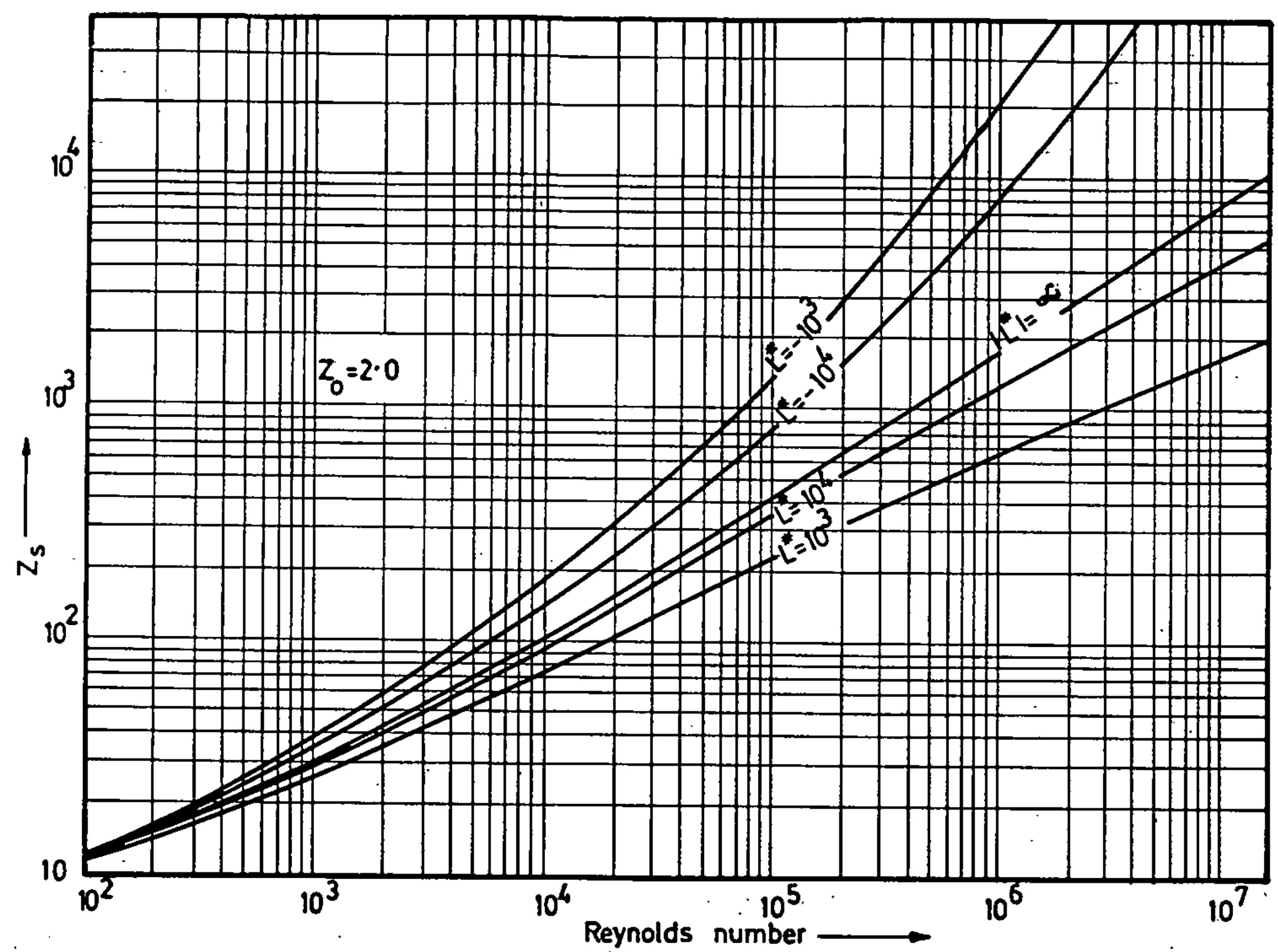

Fig. 3·4 Z_s–R_e Relation for Varying Degrees of Atmospheric Stability

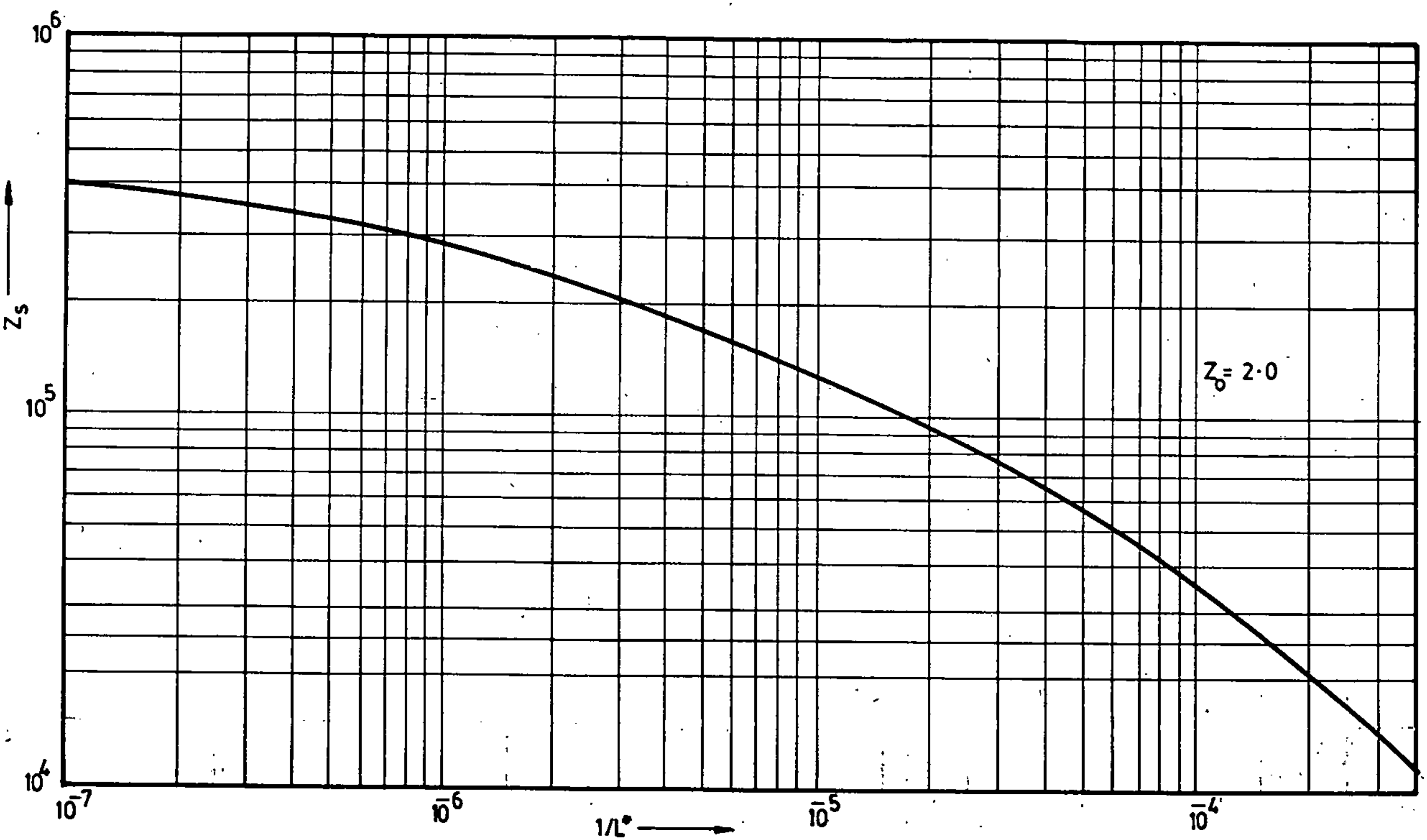

Fig. 3·5 Effect of Stability on Boundary Layer Development against $1/L^*$. The decrease in boundary layer height suggested a weakening of the ground effects on flow with increasing atmospheric stability. In a similar fashion the increase in the momentum boundary layer height with increasing atmospheric instability is illustrated in Fig. 3.6. This increase suggested a strengthening of ground effects due to the increased turbulence generated by buoyancy effects.

The next set of figures illustrates the variation of the eddy diffusivity of momentum throughout the turbulent momentum boundary layer. Fig. 3.7 illustrates the K_m- variation, when the neutral profile law of equation (3.37) was used for varying downwind positions from the leading edge. These curves were found to be almost identical to those obtained by Hetton(1964) when he used the same analysis with the Deissler (1955) and Spalding (1961) laws. It may be seen that the eddy diffusivity variation throughout the boundary layer is only weakly dependent on Reynolds number, with a change in orders of magnitude from 10^6 to 10^{10} having an overall effect on K_m of less than 10%.

The K_m- variation for the neutral state was redrawn as a broken line in Fig. 3.8 and compared with the variations obtained for differing degrees of atmospheric stability. Increased instability was observed to bring about greater values of eddy diffusivity compared with the neutral case, whilst the stable atmosphere brought about considerably damped turbulence, thereby reducing the values of K_m. These variations were very marked compared with those

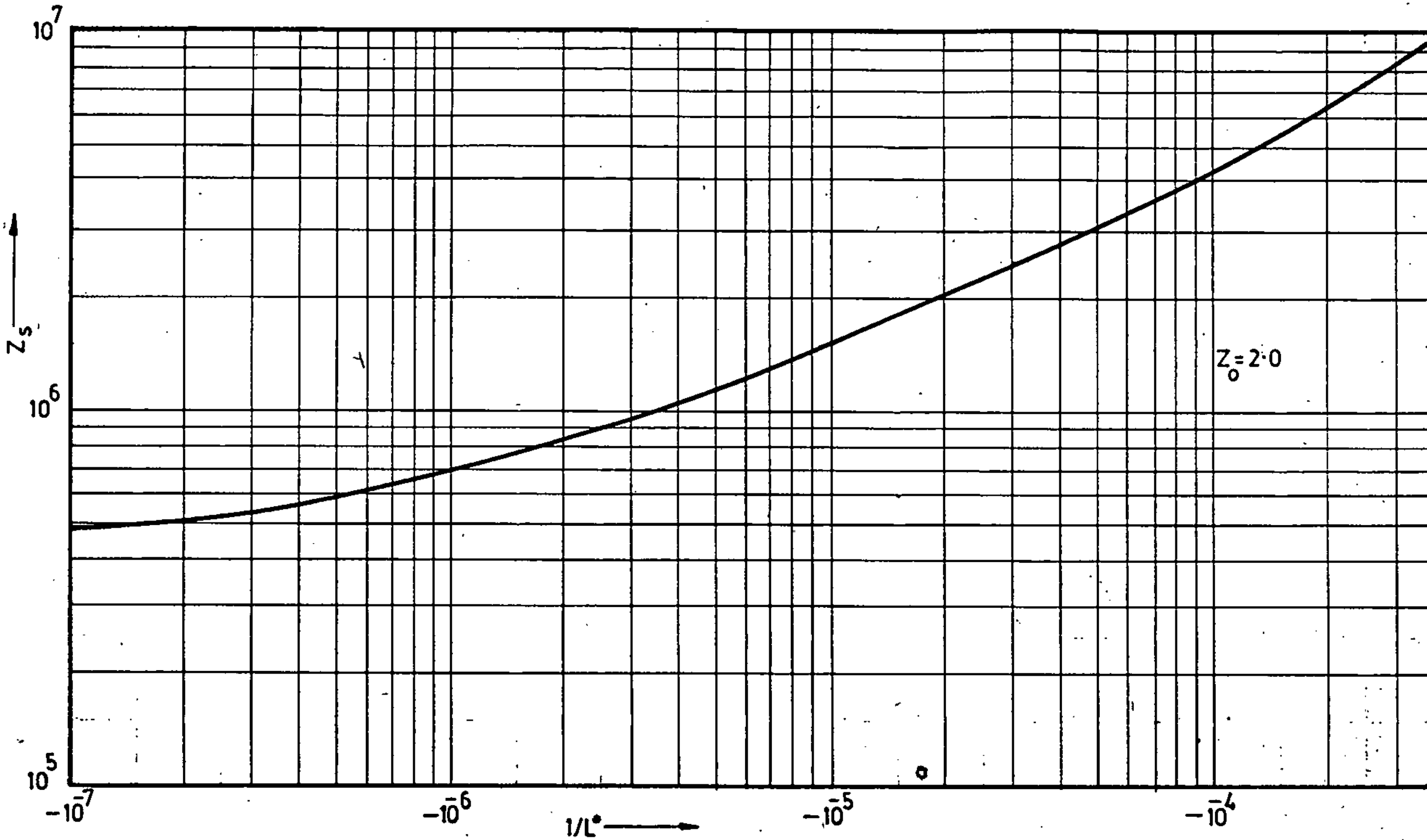

Fig. 3·6 Effect of Instability on Boundary Layer Development

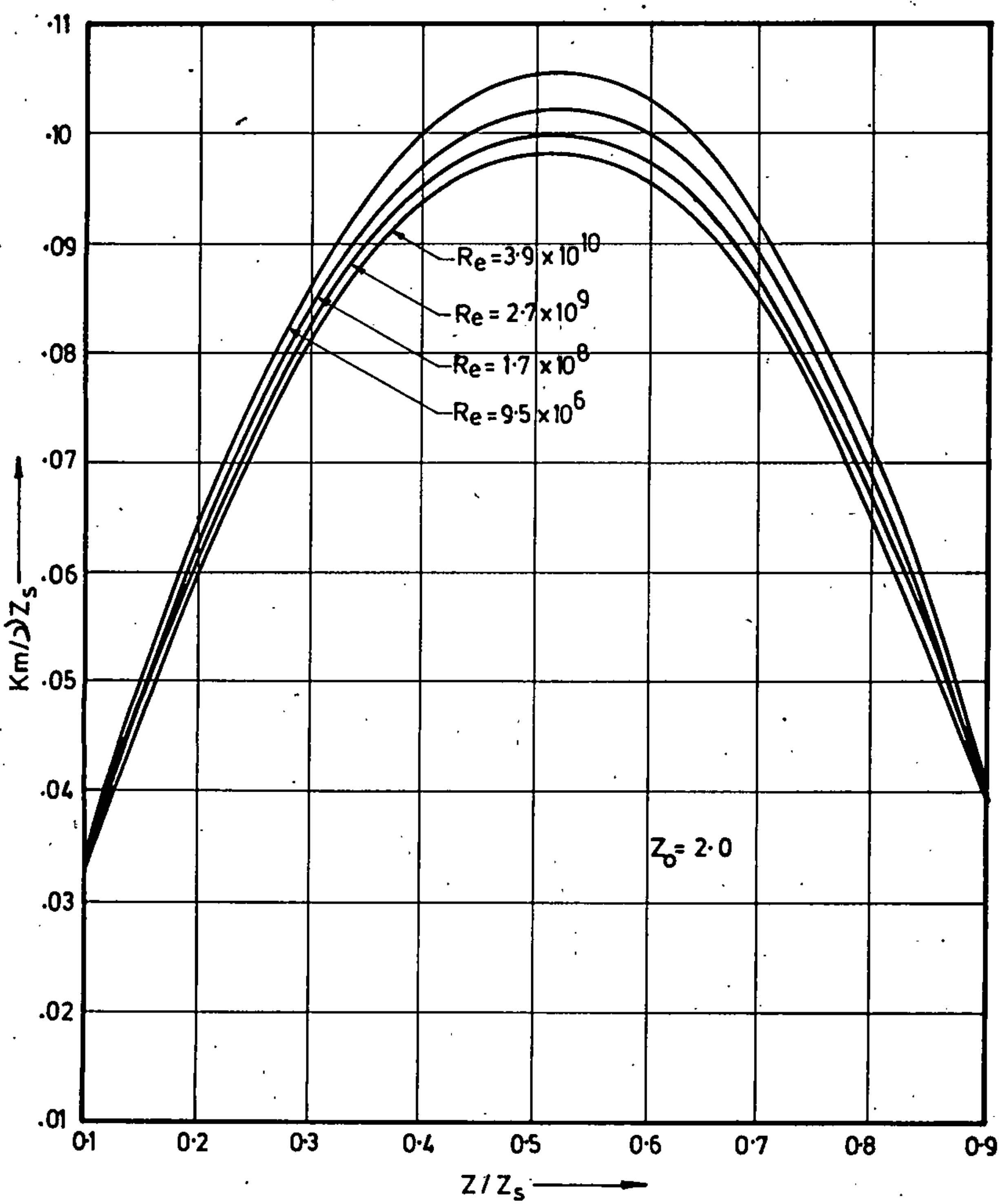

Fig. 3·7 Variation of Km Throughout the Turbulent Diffusion Boundary Layer (Neutral Profile)

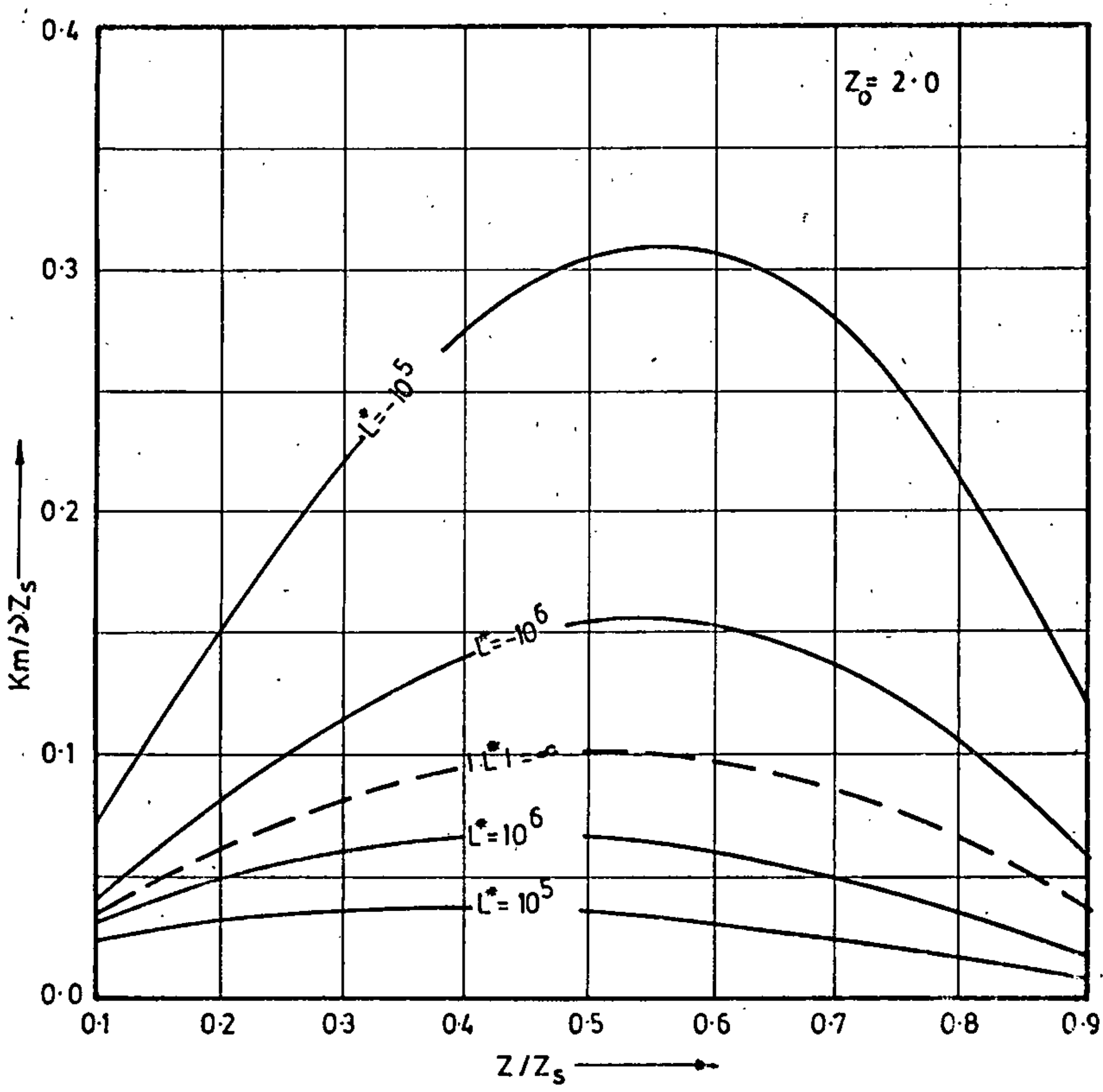

Fig. 3·8 Variation of Km Throughout the Turbulent Diffusion Boundary Layer for Varying Degrees of Atmospheric Stability obtained by changing the Reynolds number, and far outweighed them. Again it was noted that for $|L^*|$ tending to infinity, the results for both atmospheric states tended to a common curve described by the neutral equation.

The next three figures are used to illustrate the results of the finite difference solution of the diffusion equation (3.47). Fig.3.9 shows the variation of the dimensionless humidity, Q, throughout the turbulent diffusion boundary layer, for varying downwind positions, x, over the evaporating surface. The downwind lengths are approximate and were used in place of Reynolds number by selecting a representative wind velocity and kinematic viscosity. The "jump" in humidity at the upwind edge of the water surface was observed to be still evident at 1 meter downwind but within a hundred meters, the humidity profile had taken on a logarithmic form that only altered marginally for further distances downwind. It is to be noted that $\frac{dQ}{dZ} \neq 0$ for lower values of x and the value of $\frac{Z_o}{Z_d}$ decreases with increasing x leading to the decreasing values of $\left(\frac{dQ}{dZ}\right)_{Z = Z_o}$.

Fig. 3.10 illustrates the dimensionless evaporation number-Reynolds number variation of equation (3.19). The results are shown for three positions of "jump step" with respect to the leading edge of the momentum boundary layer. The curves show two important characteristics. Firstly, as the "jump step" is approached the evaporation number tends to infinity. This may be explained by the humidity boundary condition that describes an instantaneous jump in humidity. At this position

$$\left(\frac{dQ}{dZ}\right)_{Z=Z_o} \longrightarrow \infty$$

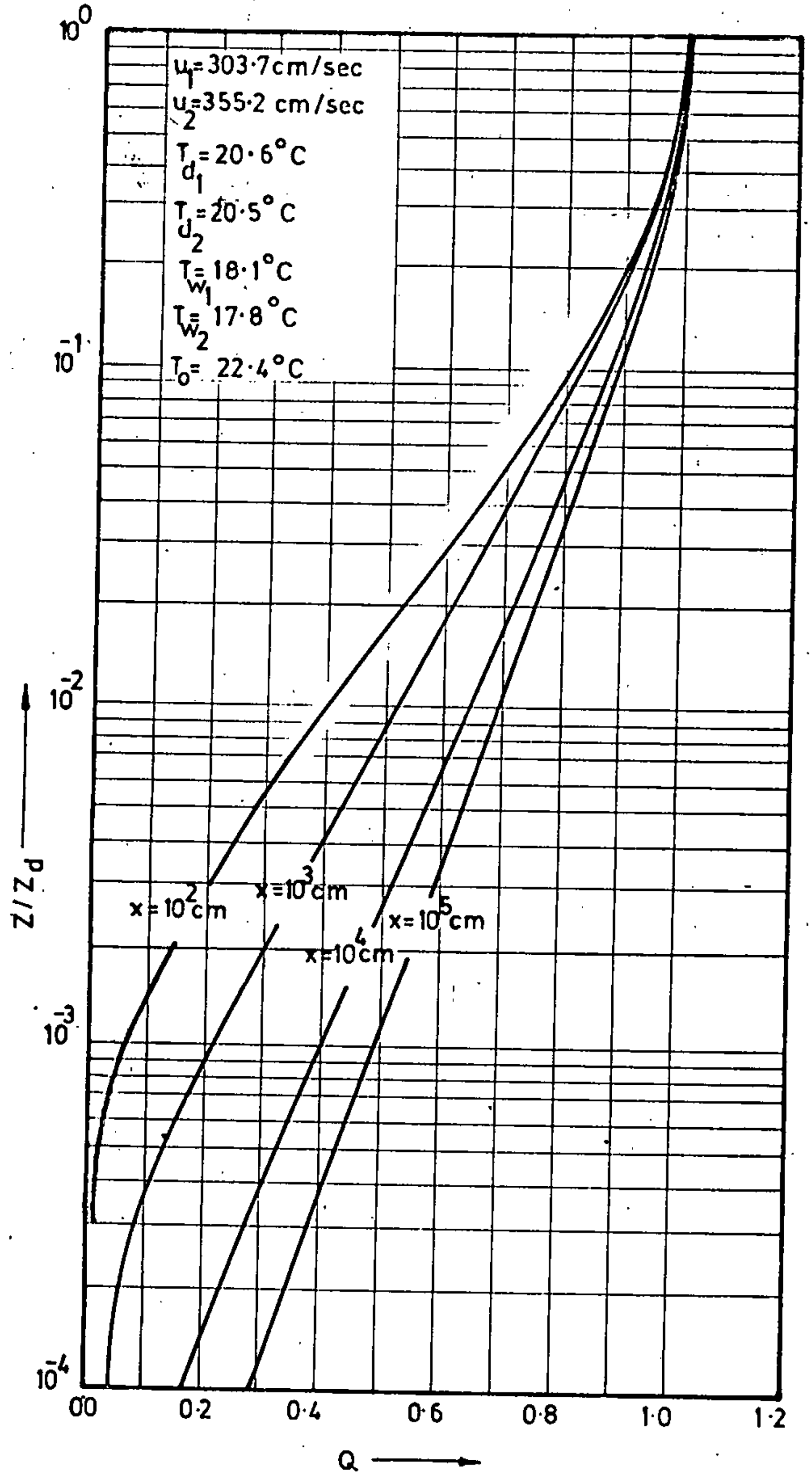

Fig. 3·9 Humidity Profiles for Varying Downwind Lengths

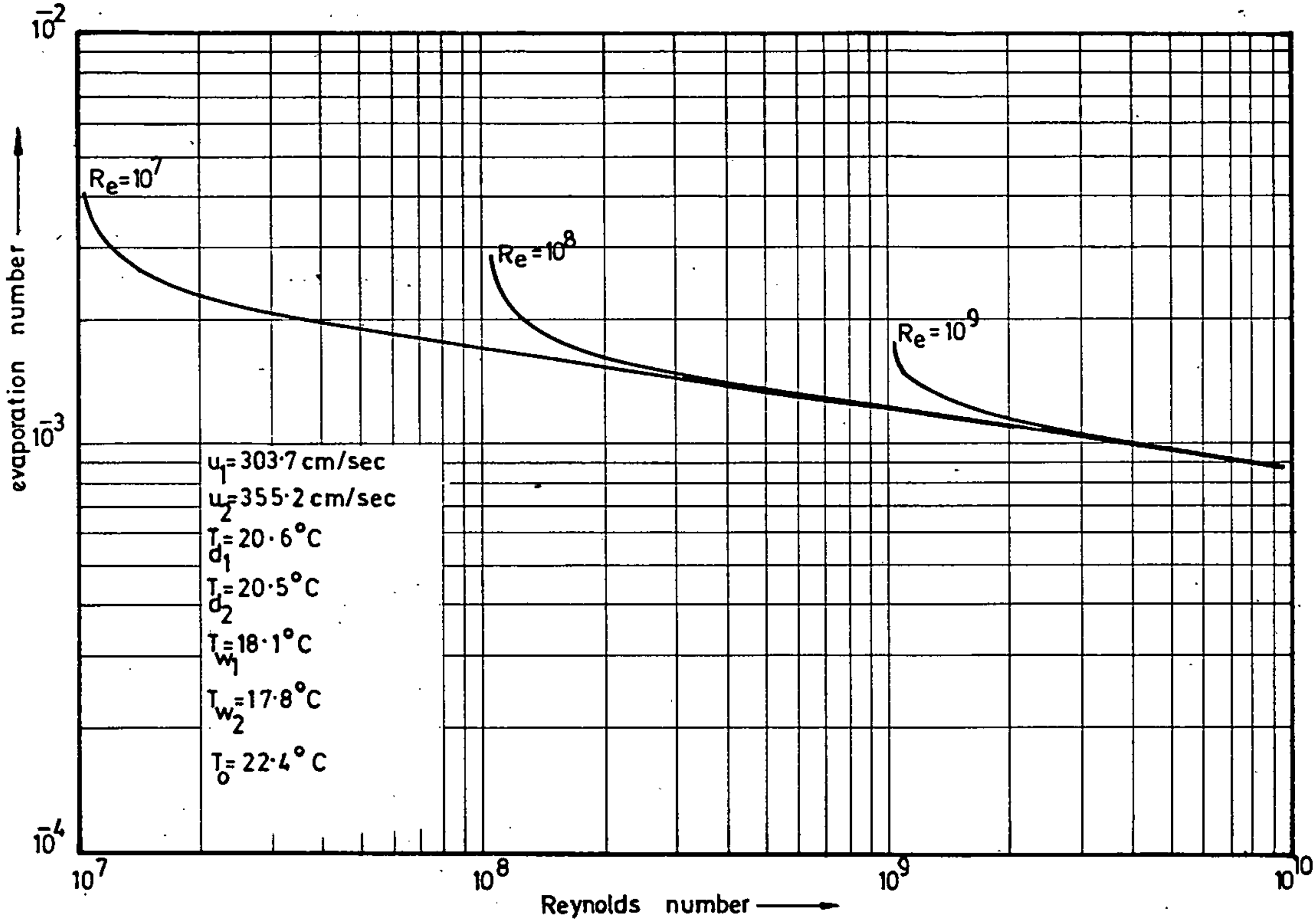

Fig. 3·10 Variation of Evaporation Number With Reynolds Number for Neutral Stability and therefore the evaporation number, N_e, also tends to infinity. Secondly, there is a rapid transition to a common curve, regardless of the "jump step" position. This curve was observed to denote a decreasing value of evaporation number over the evaporating surface, thus implying that the evaporation rate, per unit area of surface, is dependent on the downwind position from the "jump step".

Fig. 3.11 illustrates the evaporation number variation for conditions of thermal stratification. For this situation, equation (3.19) was used to calculate N_e at the level Z_o for a "jump step" Reynolds number of the order of 10^9. The value of N_e was observed to depend not only on downwind position but also on the stability parameter. For increasing instability, the evaporation number, for any value of Reynolds number, was observed to be greater than for the neutral and stable cases. Furthermore, as the instability increased, the evaporation number becomes independent of downwind position and, instead, took on an approximate constant value. Thus the numerical solution suggests that, for very unstable conditions, evaporation will cease to become a function of downwind length and, instead, be constant for any surface area exposed to otherwise identical conditions. For increasing stability, however, the reverse effect was observed, with less evaporation and increasing dependence on downwind position. This may be explained by the

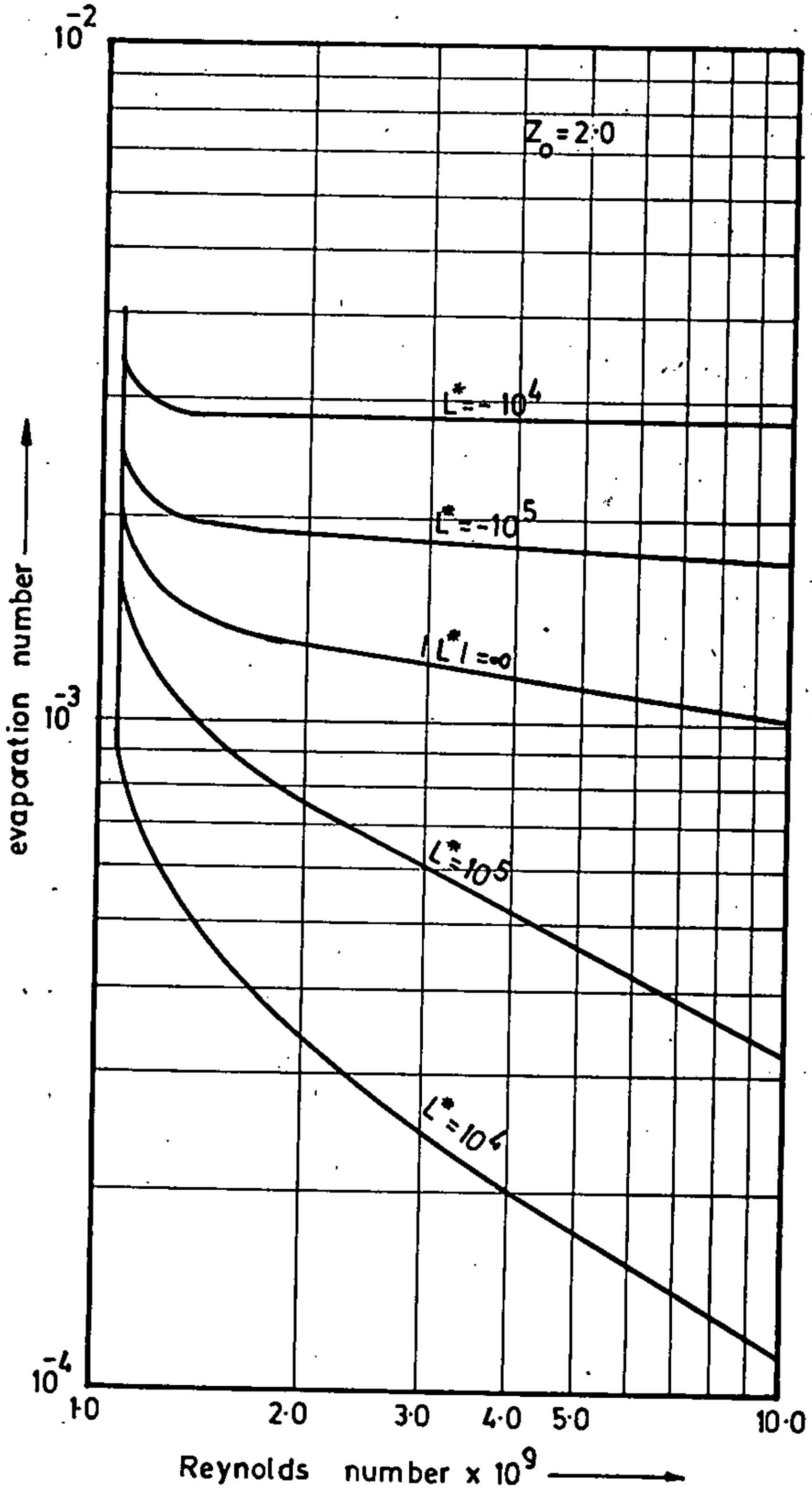

Fig. 3.11 $N_e - R_e$ Relation for Varying Degrees of Atmospheric Stability

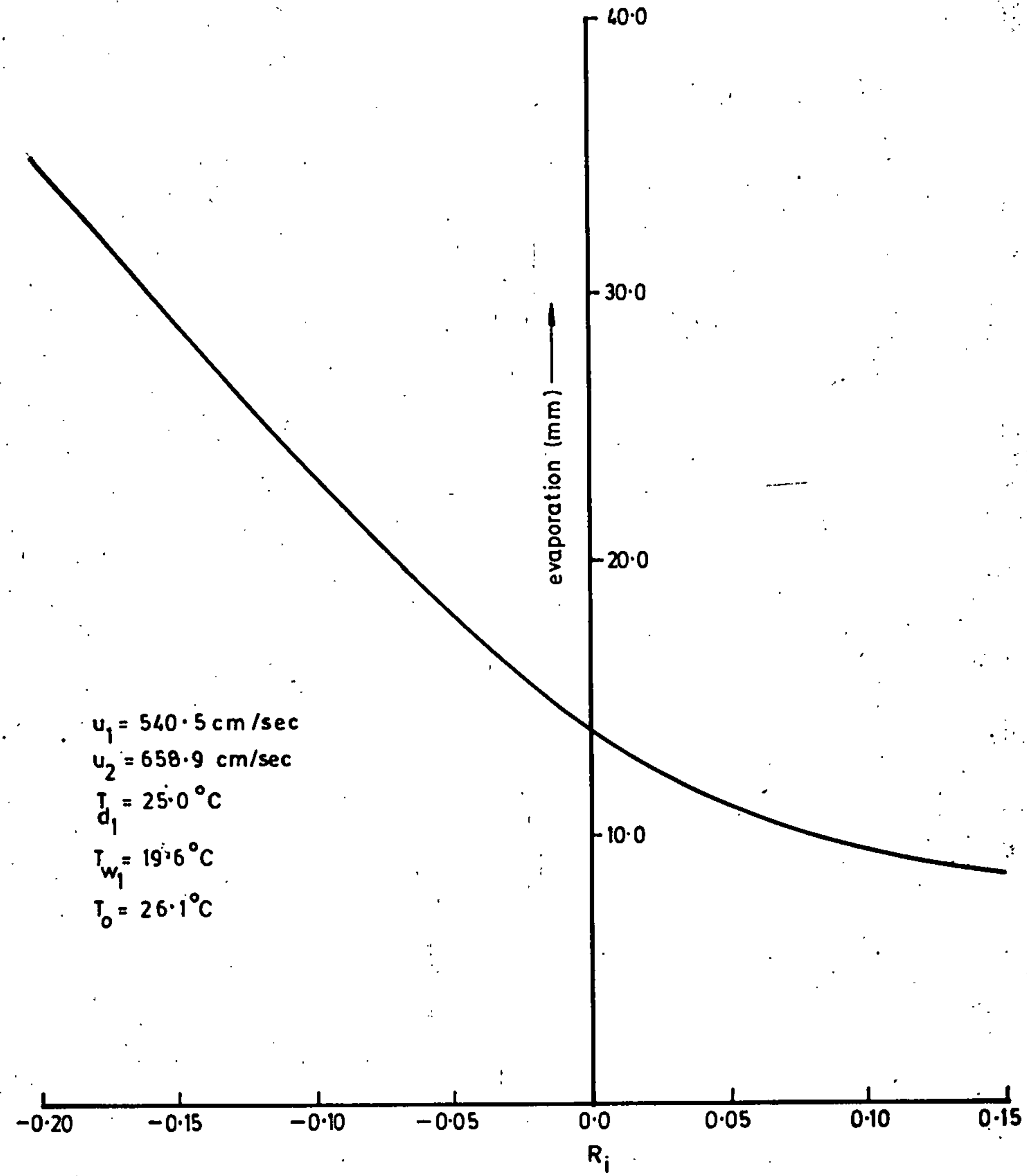

Fig. 3·12 Variation of Evaporation With Atmospheric Stability Condition fact that in the very unstable case, turbulence is so great that the vapour is transported vertically with no lateral movement at all. On the other hand, in the stable regime, the restricted turbulence causes the vapour to drift downwind just above the surface, thereby reducing the vapour concentration gradient.

Finally, Fig. 3.12 shows the above fact more clearly in which the variation of evaporation with atmospheric stability condition for a particular set of climatological measurements is considered. The gradient form of Richardson number, R_i, is used as a measure of the atmospheric stability. It is readily seen that for increasing atmospheric instability, evaporation increases very rapidly, whereas, for increasing stability, evaporation decreases and approaches a constant value under extreme stable conditions.

Chapter - 4

RESULTS USING THE LAKE HEFNER CLIMATOLOGICAL MEASUREMENTS

4.1 The Lake Hefner Experiment

The climatological measurements from Lake Hefner (Lake Hefner Base Data Report, 1954) were used to verify the validity of the proposed method and determine its limitations for estimating evaporation from bounded open water surfaces exposed to a thermally-stratified turbulent atmosphere. The Lake Hefner Project (Harbeck, 1954) was designed for the purpose of developing improved methods of estimating evaporation from large open water surfaces in which various theoretical and empirical approaches were compared with accurate assessments of evaporation by the water budget method. The climate at Lake Hefner was clissified as sub-humid (Marciano and Harbeck, 1954), in which quite a large evaporation rates can be expected. The lake was fairly regular, and approximately circular, in shape and its capacity and surface area at full pool were 75,355 acre-feet ($\simeq 9.295 \times 10^7$ m^3) and 2587 acres ($\simeq 1.047 \times 10^7$m^2) respectively.

Water budget estimates of evaporation were obtained from inflow-outflow data, estimated to within an order of accuracy of 5%, and the measurement of precipitation from 22 rain gauges distributed uniformly around the lake. The water level was monitored at a number of stages sited around the reservoir. An accuracy classification for the water budget estimates was determined from the difference in level recorded at each stage, the differences in precipitation measurement of the rain gauges and the amount of inflow and outflow. The probable errors were expressed by the classification: "A" when the estimated accuracy was within 5 acre-feet or approximately $\pm$ 0.7 mm per day, "B" when the error was upto 10 acre-feet or 1.4 mm per day, "C" for an error of the order of 20 acre-feet perday, and, "D" when the estimated error was more than 20 acre-feet per day.

Climatological measurements were taken at four sites, one of which was located near the center of the lake. At each site, wind velocity, air temperature and wet bulb temperature were recorded at 2,4,8 and 16 meter levels above the surface, along with surface temperature and wind direction. The thermometers and anemometers were rated at $\pm$ 0.1°C and $\pm$ 0.1 knot, respectively. Daily and 3-hourly mean values of these parameters and the daily average surface area for the period July, 1950 to August, 1951 were published in the Lake Hefner Base Data Report (1954).

There were 147 days of records in which "class A", "class B" and "class C" water budget estimates were accompanied by climatological data from the central barge station which satisfied the requirements of the proposed evaporation model formulation. In estimating the theoretical rate of evaporation, the lake was approximated by a circular shape and the radius was calculated from the given surface area. The barge station was assumed to be centrally located so that its downwind position from the vapour-step position was assumed to be constant, equal to the radius of the surface, with respect to the wind direction. The downwind length of the elementrary strips was taken as 30 meters. Thus the total number of strips was about 100. The width of each strip was determined using equation (3.89). The free-stream humidity was evaluated using the barge station 8-meter level specific humidity into equation (3.85). The levels z_1 and z_2 were taken as 2 and 8 meters respectively. The friction velocity, u_*, and the roughness parameter, z_o, were calculated using equations (3.61) to (3.70).

4.2 Results and Discussions

The comparisons between the water budget evaporation estimates and the present theoretical evaporation estimates using the climatological measurements at Lake-Hefner, are illustrated in Fig. 4.1. The results were obtained by using 147 "class A", "class B" and "class C" days. The total theoretical evaporation for these days was 817.82 mm compared to a water budget estimate of 712.68 mm indicating an overestimation of only 14.75% of the actual evaporation by the proposed evaporation model. Thus the overall agreement between the theoretical and water budget estimates of evaporation is satisfactory. Out of the 147 days of observations, there were 16 neutral, 22 unstable and 109 stable days. The theory overestimated the actual evaporation in all the three atmospheric

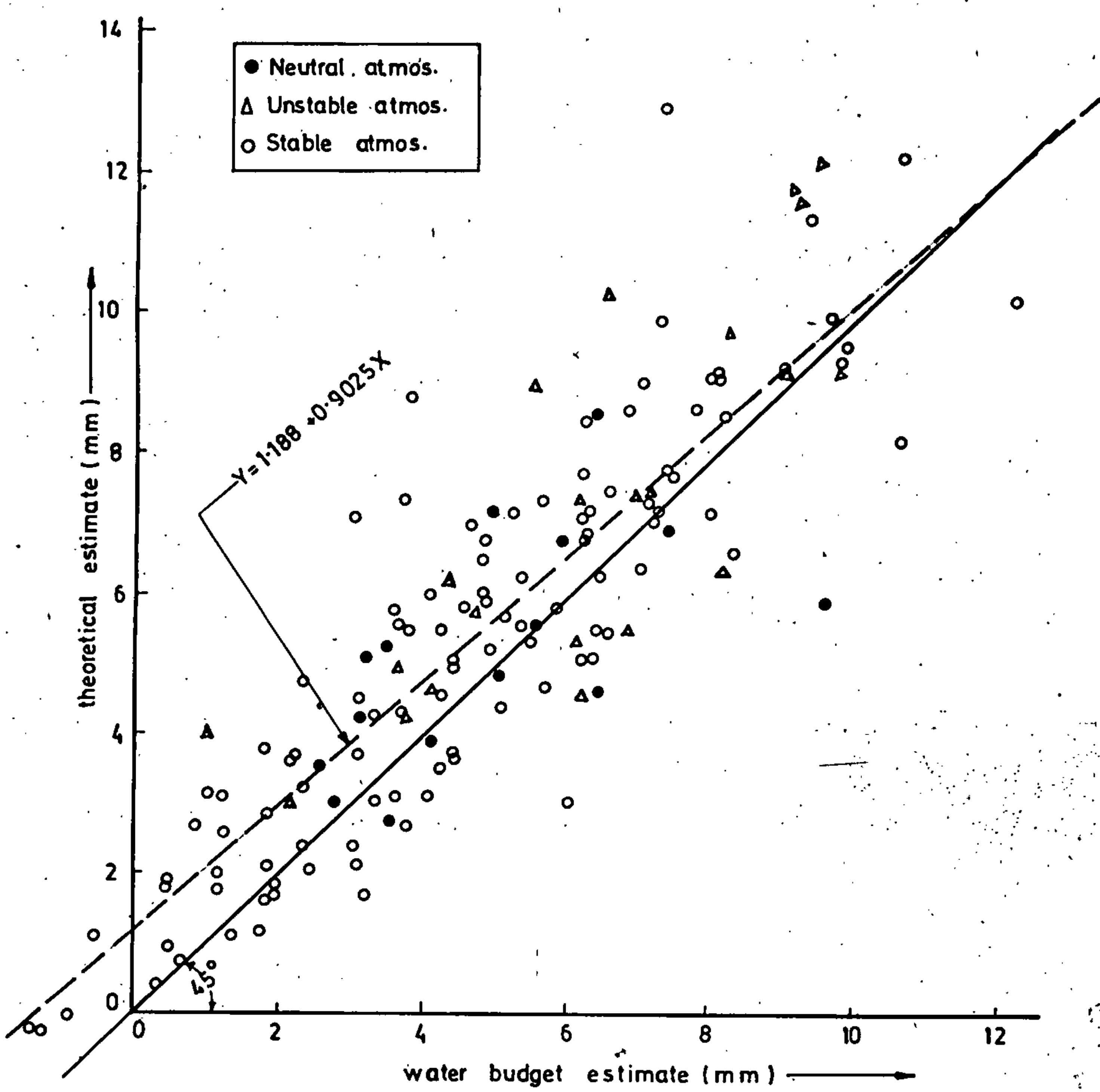

Fig. 4·1 Daily Water Budget and Theoretical Estimates of Evaporation from Lake Hefner (147 Class A, B and C Days) regions, the percentages of overestimation being 8.33, 16.46 and 15.29 under the neutral, unstable and stable conditions respectively. Thus the overall agreement between the theory and observation also appears to be satisfactory in all regions of atmospheric stability conditions.

A linear correlation coefficient of 0.8637 is found to relate the theoretical evaporation, indicating a very good agreement between the two estimates. The equation of the linear regression line passing through the points plotted in Fig. 4.1 is found by a least-square fit to be

$$Y = 1.188 + 0.9025X. \qquad \ldots \qquad \ldots \qquad (4.1)$$

The above equation indicates that the proposed approach over-estimates the actual evaporation for lower rates of evaporation and underestimates for higher rates of evaporation. The 98% confidence limits of the gradient are 1.0044 and 0.8006. The gradient of this linear regression equation suggests that there is a slight departure of the theoretical evaporation estimates from the water budget estimates but that the gradient equal to unity, representing complete agreement between the two distributions, is within the 98% confidence,limits of the regression line.

From Fig. 4.1 it is readily observed that although the overall agreement between the evaporation model and water budget evaporation estimates is close, the results are somewhat scattered and there is often considerable difference between the daily evaporation values given by these two methods. The scatter in the results was thought to be due to the accuracy with which the stability of the atmosphere could be ascertained which in turn was dependent upon the accuracy of the temperature and wind velocity measurements. The accuracies of the temperature and wind velocity measurements at Lake Hefner were $\pm$ 0.1^{o}C and $\pm$ 0.1 knot respectively. The stability of the atmosphere at Lake Hefner was determined by evaluating the gradient form of the Richardson number, R_i, as a mean daily value using equation (3.60) and utilizing the temperature and wind velocity measurements at 2 and 8 meter levels. Since the air temperatures recorded at these levels measured to within $\pm$ 0.1^{o}C, the accuracy of the differences in the measured temperatures at the two levels was within $\pm$ 0.2^{o}C. That this order of accuracy for the difference of temperature measurements is insufficient to meet the requirements of the proposed approach is illustrated in Table 4.1, where the effect of $\pm$ 0.1^{o}C error in the difference of 2 and 8 meter air temperautres on the theoretical rate of evaporation has been shown. A total of 12 days climatological data from Lake Hefner, covering all regions of atmospheric stability conditions, have been considered. The effect of 0.1^{o}C error in the water surface temperature and that of $\pm$ 0.1 knot (= 5.148 cm/sec) in the difference of 2 and 8 meter wind velocities have also been illustrated in the same table.

Table - 4.1

Effects of the Probable Errors of Temperature and Wind Velocity Measurements on the Theoretical Estimates of Evaporation.

Day	Atmospheric Stability	Est.* Evap. (mm)	$\Delta T = \Delta T + 0.1$		$\Delta T = \Delta T - 0.1$		$T_o = T_o + 0.1$		$\Delta U = \Delta U + 0.1$		$\Delta U = \Delta U - 0.1$	
			Estimate (mm)	% Diff.	Estimate (mm)	% Diff.	Estimate (mm)	% Diff.	Estimate (mm)	% Diff.	Estimate (mm)	% Diff.
1	Stable	9.89	9.30	- 5.96	10.67	7.9	10.10	2.12	10.24	3.54	8.84	- 10.62
2	,,	3.80	3.58	- 5.80	4.60	21.05	3.92	3.16	4.39	15.53	3.74	- 1.58
3	,,	1.13	0.83	- 26.55	1.57	38.94	1.15	1.77	1.47	30.10	0.90	- 20.35
4	,,	3.07	2.35	- 23.45	3.98	29.64	3.10	0.98	3.80	23.78	2.48	- 19.22
5	,,	7.25	6.21	- 14.35	8.16	12.55	7.40	2.07	7.76	7.03	6.49	- 10.48
6	,,	7.19	6.13	- 14.74	7.85	9.18	7.34	2.09	7.50	4.31	6.31	- 12.24
7	,,	7.69	6.97	- 9.36	8.36	8.71	7.83	1.82	8.57	11.44	7.09	- 7.80
8	,,	6.26	5.37	- 14.22	7.21	15.18	6.33	1.12	7.22	15.34	5.02	- 19.81
9	Neutral	4.82	4.02	- 16.60	7.37	52.9	4.86	0.83	5.71	18.47	4.02	- 16.60
10	,,	0.44	0.35	- 20.45	0.55	25.0	0.46	4.55	0.54	22.73	0.36	- 18.18
11	Unstable	9.16	8.09	- 11.68	9.94	8.52	9.29	1.42	9.95	8.62	8.66	- 5.46
12	,,	0.76	0.70	- 7.9	0.83	9.21	0.82	7.9	0.94	23.68	0.74	- 2.63

* estimates using the observed climatological parameters at Lake Hefner.

From Table 4.1 it is observed that the proposed evaporation approach for estimating evaporation in a density-stratified atmosphere demands for very precise measurements of air temperatures and wind velocities. Indeed, the stability of the atmosphere, determined by evaluating the Richardson number, R_i, using equation (3.60), and so the rate of evaporation, are found to be highly sensitive to the differences of air temperatures and wind velocities measured at the levels z_1 and z_2 above the surface. The difference of air temperatures recorded at the levels z_1 and z_2 rarely exceeds $\pm$ 0.2°C. As a result, it has been observed that an error of the order of $\pm$ 0.2°C is often sufficient to change the sign of R_i and hence the atmospheric conditions from unstable to stable and vice versa. When the levels of measurements z_1 and z_2 are taken very close or when the measurements of air temperatures are open to errors of still higher magnitudes, the results will be more scattered than those presented in Table 4.1.

It has been observed that a change of $\pm$ 0.2°C in both the temperatures but keeping their difference to remain the same, has a maximum of 3.5% effect upon the rate of evaporation. Similarly, for wind velocities, a change of $\pm$ 0.2 knot affects the evaporation estimates by a maximum of 2.5% and an error of the order of 0.1°C in water surface temperature is found to affect the results at the most by 8%. It is therefore the differences, and not the absolute magnitudes of the air temperatures and wind velocities, which have the most pronounced effect upon the rate of evaporation.

Thus, it is now apparent that the order of accuracy of Lake Hefner climatological measurements was insufficient for the accurate determination of atmospheric stability and is thought to be the main cause of the scatter of the results presented in Fig.4.1. For evaluating the gradient from of the Richardson number accuractely, very precise measurements of air temperature and wind velocity are necessary. The recording instruments have to be capable of resolving a temperature difference of 0.01°C or less so that the adiabatic lapse rate may be detected. The order of accuracy of the measurements of wind velocity should preferably be ± 0.5 cm/sec.

Chapter - 5

CONCLUSIONS AND SUGGESTIONS FOR FURTHER WORK

5.1 Conclusions

The aim of the present work was to develop an evaporation model for estimating evaporation from bounded open water surfaces considering varying degrees of atmospheric stability and surface roughness conditions. It was found by applying climatological data to the proposed approach that there was satisfactory agreement between the total estimated and observed evaporation from Lake Hefner. However, when it was necessary to estimate the daily evaporation accurately, very precise measurement of the climatological parameters to determine the atmospheric stability with a sufficient degree of accuracy, was indicated.

The main advantages of the proposed approach of estimating evaporation are that direct empiricism is not required, the theory appears to be satisfactory regardless of the size and condition of the surface and its climatological location, and unlike the energy budget approach, evaporation may be estimated accurately over short time intervals without any need for considering the heat stored by the water body which is rarely measured as a routine or measured with considerable difficulty.

The main limitation of the proposed approach is that sensitive sensors have to be used to measure the climatological parameters accurately to determine the atmospheric stability with a sufficient degree of accuracy. The results of the experimental work have indicated that the rate of evaporation depends upon the atmospheric stability condition. As a result, when it is necessary to consider an accurate estimation of the daily rates of evaporation, every afford should be made to measure the climatological parameters accurately.

The following conclusions could be drawn from the study of the proposed evaporation model:

a) Provided that the stability of the atmosphere can be determined accurately, the use of the proposed evaporation model seems to be a satisfactory method of estimating evaporation from open water surfaces.

b) The wind velocity and temperature profile equations given by Businger et al. (1971) for atmospheric flow considering varying degrees of atmospheric stability condition, were seems to be satisfactory.

c) The rate of evaporation depends upon the atmospheric stability and surface roughness conditions and on the downwind dimension of the surface.

d) The approximations applied to the evaporation model in determining the free-stream height, free-stream velocity and free-stream specific humidity seems to be satisfactory.

e) The assumption that the turbulent diffusivities for heat and water vapour transport are identical, seems to be satisfactory.

5.2 Further Work

The present approach may be usefully employed to estimate evapotranspiration from vegetated surfaces. This may be achieved by altering the datum level at which the evaporation number N_e is determined. The datum level is specified by the plane at which the humidity gradient (dQ/dZ) is determined. For the neutral case, this gradient was estimated at $Z = 0$ (Takhar, 1972) and for the present case it was estimated at $Z = Z_o$. Using the five-point difference formula the humidity gradient may be evaluated at any level within the turbulent diffusion boundary layer. To estimate the evapotranspiration, this datum may be positioned just clear of the vegetation. The vapour concentration at this level will be due partly to the contribution made by the evaporation of water directly from the ground and partly to the transpiration of vegetation cover. The total evapotranspiration will be given by the vertical flux of vapour across the datum plane. The climatological requirements will be identical to those required for the estimation of evaporation from an open water surface except that the humidity at the wall, q_o, will not be given by the surface temperature but must be evaluated using a psychrometer located at the datum level.

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APPENDIX - A

Finite Difference Representation of the Diffusion Equation

The diffusion equation (3.49) can be written as

$$UU_s \frac{\partial Q}{\partial R_e} = M' \frac{\partial Q}{\partial Z} + M \frac{\partial^2 Q}{\partial Z^2} \quad \ldots \qquad (A-1)$$

where, $M' = \partial M/\partial Z$. The finite difference representation of equation (A-1) may be obtained by integrating each term of this equation over an elementary control volume. Fig.A.1 shows an elementary control volume abcd.

Now, integration of equation (A-1) over the control volume abcd gives

$$\underbrace{\iint_{abcd} UU_s \frac{\partial Q}{\partial R_e} dR_e dZ}_{\mathrm{I}} = \underbrace{\iint_{abcd} M' \frac{\partial Q}{\partial Z} dR_e dZ}_{\mathrm{II}} + M \underbrace{\iint_{abcd} \frac{\partial^2 Q}{\partial Z^2} dR_e dZ}_{\mathrm{III}} \qquad (A-2)$$

The finite difference scheme of equation (A-2) reverts to the fully-explicit form if the upstream Q values are used. On the other hand, when the average of the upstream and downstream Q's are used, the finite difference analogue would give just-stable Crank-Nicholson scheme. Here the downstream Q values are used to give the maximum stability

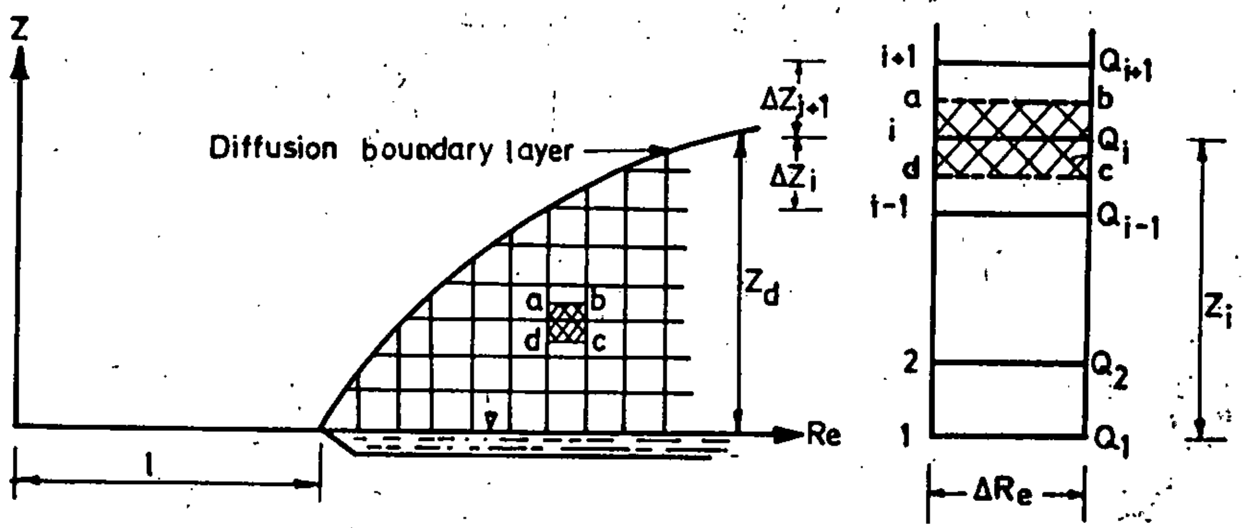

Fig. A-1 The Rectangular Grid System and an Elementary Control Volume and a fully-implicit scheme. Then with the subscripts u and d representing the upstream and downstream values respectively, term I of equation (A-2) can be written as

$$\iint_{abcd} UU_s \frac{\partial Q}{\partial R_e}\, dR_e dZ = U_i U_s \,(Q_{i,d}-Q_{i,u})(Z_{i+\frac{1}{2}}-Z_{i-\frac{1}{2}})$$

$$= \frac{U_i U_s}{2}\,(Q_{i,d}-Q_{i,u})\;(Z_{i+1}-Z_{i-1}) \quad \ldots \qquad \text{(A-3)}$$

Similarly, terms II and III of equation (A-2) may be written respectively as

$$\iint_{abcd} M' \frac{\partial Q}{\partial Z}\, dR_e dZ = \frac{M'_i \,\Delta R_e}{2}\,(Q_{i+1,d}-Q_{i-1,d}) \qquad \text{(A-4)}$$

and

$$\iint_{abcd} M \frac{\partial^2 Q}{\partial Z^2}\, dR_e dZ = M_i \,\Delta R_e \left(\frac{\partial Q}{\partial Z}\right)_{i-\frac{1}{2}}^{i+\frac{1}{2}} \quad . \quad \ldots \qquad \text{(A-5)}$$

Now, $\left(\frac{\partial Q}{\partial Z}\right)_{i+\frac{1}{2}} = \dfrac{Q_{i+1,d} - Q_{i,d}}{Z_{i+1} - Z_i}$

and

$$\left(\frac{\partial Q}{\partial Z}\right)_{i-\frac{1}{2}} = \frac{Q_{i,d} - Q_{i-1,d}}{Z_i - Z_{i-1}} \;.$$

Hence, equation (A-5) becomes

$$\iint_{abcd} M \frac{\partial^2 Q}{\partial Z^2}\, dR_e dZ = M_i \,\Delta R_e \left[\frac{Q_{i+1,d}-Q_{i,d}}{Z_{i+1} - Z_i} - \frac{Q_{i,d}-Q_{i-1,d}}{Z_i-Z_{i-1}}\right] \text{(A-6)}$$

Using equations (A-3), (A-4) and (A-6) into equation (A-2), the finite difference approximation for the diffusion equation (A-1) becomes

$$\frac{U_i U_s}{2}\left(Q_{i,d} - Q_{i,u}\right)\left(\Delta Z_i + \Delta Z_{i+1}\right) = \frac{M'_i \Delta R_e}{2} \left(Q_{i+1,d} - Q_{i-1,d}\right)$$

$$+ M_i \Delta R_e \left[\frac{Q_{i+1,d} - Q_{i,d}}{\Delta Z_{i+1}} - \frac{Q_{i,d} - Q_{i-1,d}}{\Delta Z_i}\right] \quad \ldots \quad (A-7)$$

where,

$$\Delta Z_{i+1} = Z_{i+1} - Z_i \qquad \ldots \qquad \ldots \quad (A-8)$$

and

$$\Delta Z_i = Z_i - Z_{i-1} \qquad \ldots \qquad \ldots \quad (A-9)$$

Equation (A-7) can be written in the form

$$D_i \, Q_{i,d} = A_i \, Q_{i+1,d} + B_i \, Q_{i-1,d} + C_i \qquad \ldots \qquad \ldots \quad (A-10)$$

$$\left.\begin{aligned}
\text{where, } A_i &= \frac{M'_i \Delta Z_i}{2M_i} + \frac{\Delta Z_i}{\Delta Z_{i+1}} \\
B_i &= 1 - \frac{M'_i \Delta Z_i}{2M_i} \\
C_i &= Q_{i,u}\left[\frac{U_i U_s \Delta Z_i}{2M_i \, \Delta R_e}\left(\Delta Z_i + \Delta Z_{i+1}\right)\right] \\
\text{and} \quad D_i &= \frac{U_i U_s \, \Delta Z_i}{2M_i \, \Delta R_e}\left(\Delta Z_i + \Delta Z_{i+1}\right) + \frac{\Delta Z_i}{\Delta Z_{i+1}} + 1.
\end{aligned}\right\} \quad \ldots \quad (A-11)$$

For $i = 2$ control volume, equation (A-10) remains valid but the coefficients A_i, B_i, C_i and D_i become

$$\left.\begin{aligned}
A_2 &= \frac{M_2' Z_2}{2M_2} + \frac{Z_2}{Z_3 - Z_2}\\
B_2 &= 1 - \frac{M_2' Z_2}{2M_2}\\
C_2 &= Q_{2,u}\left[\frac{U_2 U_s Z_2}{2M_2 \, \Delta R_e}\,(Z_2 + Z_3)\right]\\
D_2 &= \frac{U_2 U_s Z_2}{2M_2 \, \Delta R_e}(Z_2 + Z_3) + \frac{Z_2}{Z_3 - Z_2} + 1
\end{aligned}\right\} \qquad \text{(A-12)}$$

The coefficients A_i, B_i, C_i, and D_i in equation (A-10) are known quantities. For N steps in ΔZ there are N-1 equation of the type (A-10) with N-1 unknowns. This system of equations form a tri-diagonal matrix that can be solved using the Gauss elimination method as described by Spalding (1977). The equation which have to be solved for the unknown Q's can be usefully written in the following orderly form:

$$
\begin{aligned}
D_2Q_2 - A_2Q_3 \quad & = C_2+B_2Q_1 \\
-B_3Q_2 + D_3Q_3 - A_3Q_4 \quad & = C_3 \\
-B_4Q_3 + D_4Q_4 - A_4Q_5 \quad & = C_4 \\
\cdots \quad \cdots \quad & \cdots \\
-B_{N-2}Q_{N-3} + D_{N-2}Q_{N-2} - A_{N-2}Q_{N-1} \quad & = C_{N-2} \\
-B_{N-1}\ Q_{N-2} + D_{N-1}\ Q_{N-1} \quad & = C_{N-1}+A_{N-1}Q_N \cdot
\end{aligned} \qquad (A\text{-}13)
$$

The above equations can be converted in succession into:

$$Q_2 = S_2Q_3 + T_2 \qquad \ldots \qquad (A\text{-}14)$$

$$Q_3 = S_3Q_4 + T_3 \qquad \ldots \qquad (A\text{-}15)$$

$$Q_4 = S_4Q_5 + T_4 \qquad \ldots \qquad (A\text{-}16)$$

$$\cdots \quad \cdots \quad \cdots$$

$$Q_{N-1} = S_{N-1}Q_N + T_{N-1} \qquad \ldots \qquad (A\text{-}17)$$

where,

$$S_2 = \frac{A_2}{D_2} \qquad \ldots \qquad (A\text{-}18)$$

$$T_2 = \frac{B_2Q_1 + C_2}{D_2} \qquad \ldots \qquad (A\text{-}19)$$

$$S_3 = \frac{A_3}{D_3 - B_3S_2} \qquad \ldots \qquad (A\text{-}20)$$

$$T_3 = \frac{B_3T_2 + C_3}{D_3 - B_3S_2} \qquad \ldots \qquad (A\text{-}21)$$

$$S_4 = \frac{A_4}{D_4 - B_4\ S_3} \qquad \ldots \qquad \text{(A-22)}$$

$$T_4 = \frac{B_4\ T_3 + C_4}{D_4 - B_4\ S_3} \qquad \ldots \qquad \text{(A-23)}$$

... ...

$$S_{N-1} = \frac{A_{N-1}}{D_{N-1} - B_{N-1}\ S_{N-2}} \qquad \ldots. \qquad \text{(A-24)}$$

and

$$T_{N-1} = \frac{B_{N-1}\ T_{N-2} + C_{N-1}}{D_{N-1} - B_{N-1}\ S_{N-2}} . \qquad \ldots \qquad \text{(A-25)}$$

Thus, proceeding this way, the solution of equation (A-10) is given by the general equation

$$Q_i = S_i\ Q_{i+1} + T_i \qquad \ldots \qquad \text{(A-26)}$$

in which the coefficients S_i and T_i are given by

$$S_i = \frac{A_i}{D_i - B_i\ S_{i-1}} \qquad \ldots \qquad \text{(A-27)}$$

$$T_i = \frac{B_i\ T_{i-1} + C_i}{D_i - B_i\ S_{i-1}} \qquad \ldots \qquad \text{(A-28)}$$

for $3 \leqslant i \leqslant N-1$. The coefficients S_2 and T_2 are to be calculated using equations (A-18) and (A-19).

For each step ΔR_e downwind, Q_i was determined for increments in ΔZ until the difference in $\dot{Q}$ at the i'th and i+1'th levels was less than 10^{-2}. This was assumed to correspond to the top of the turbulent diffusion boundary layer.

APPENDIX - B

THE COMPUTER PROGRAM : A LISTING AND USER INSTRUCTIONS

B.1 Introduction

The purpose of this appendix is to provide a guide for the use of the computer program developed to calculate the daily evaporation using the proposed approach. The program was written in FORTRAN. The computer program, a listing of which is presented at the end of this appendix, was written for the climatological measurements at Lake Hefner and will require a few modifications before it is used for another site.

The program was run on the BUET IBM 370/115/H02 Computer.

B.2 The FORTRAN Symbols

Although adequate comments are included in the computer program that will help to follow the sequence of computation, the symbols used are not defined. These symbols and their meaning are given below. Whenever possible, the mathematical symbols used in the text have been retained.

FORTRAN Symbol	Meaning
A	Coefficient A_1 in equation (A-10)
AREA	Running total of area over which evaporation has been estimated
AREARL	Surface area of the lake
B	Coefficient B_1 in equation(A-10)

FORTRAN Symbol	Meaning
C	Coefficient C_i in equation (A-10)
COUNT	A counter to denote the level at which Z(I) = ZS
D	Coefficient D_i in equation (A-10)
DMDZ	$\partial M / \partial Z$ in equation (3.55)
DUDZ	dU/dZ at the i'th grid point
DRX	ΔR_e
DZ(I)	= Z(I) - Z(I-1)
DZDU(I)	dZ/dU at the i'th grid point
D2UDZ2(I)	d^2U/dZ^2 at the i'th grid point
D2ZDU2(I)	d^2Z/dU^2 at the i'th grid point
EVAP	Evaporation
EVAPRL	Water-budget evaporation
I	Counter to denote the i'th grid point
ICASE	A counter to denote the level at which Z(I) = ZVP
NODAY	Running total of the number of days for which evaporation has been determined
NODRX	Running total of the number of DRX
NODZ	Total number of ΔZ
EN	Evaporation number, N_e
P(I)	S_i of equation (A-26)
PJSTEP	Pre-step length, l
PRESS	Atmospheric pressure
Q(I)	T_i of equation (A-26)
RHO	Air density, ρ
RI	Gradient form of the Richardson number, R_i

FORTRAN Symbol	Meaning
RX	Reynolds number, R_e
RXFIN	Reynolds number corresponding to START
RXRAFT	Downwind Reynolds number corresponding to the center of the lake
RXPJL	The pre-step Reynolds number
RXTEST	A counter to denote the level of T(I) = VPS
RXZS	Reynolds number corresponding to ZS
SC	Laminar Schmidt number
SCT(I)	Turbulent Schmidt number at the i'th grid point
START	Running total of the downwind step lengths
STEP	Downwind step length, Δx
T(I)	$1-Q_i$, where Q_i is the non-dimensional humidity at the i'th grid point.
T2D, T2W	Dry and wet bulb temperatures at 2 meters
T8D, T8W	Dry and wet bulb temperatures at 8 meters
TSURF	Water surface temperature
U(I)	Non-dimensional wind velocity at the i'th grid point
US	Non-dimensional free-stream velocity, U_s
UST	Friction velocity, u_*
U1, U2	Wind velocities at 2 and 8 meter levels (cm/sec) respectively
U2ZOZS	$\int_{Z_o}^{Z_s} U^2 dZ$ at the i'th grid point
U2ZOZ(I)	$\int_{Z_o}^{Z} U^2 dZ$ at the i'th grid point

FORTRAN Symbol	Meaning
USQDZ	U^2 dZ at the i'th grid point
UCUBDZ	U^3 dZ at the i'th grid point
U3ZOZ	$\int_{Z_o}^{Z} U^3$ dZ at the i'th grid point
VPS	Estimated vapour pressure at the free-stream
VPSURF	Vapour pressure at the surface
VP2	Vapour pressure at the 2 meter level
WIDTH	Width of an elementary strip
WINDZS	Estimated wind velocity at the free-stream (cm/sec)
WIND2, WIND8	Wind velocities at 2 and 8 meter levels in knot respectively
XKMNUE	K_m/ν in equation (3.13)
XL	Monin-Obukhov scale length, L
XM	M in equation (3.55)
XK	von Karman constant
XNUE	Kinematic viscosity,
Z(I)	Non-dimensional height, Z_i, of the i'th grid point
ZBYL	z_i/L
ZS	Non-dimensional height of the turbulent momentum boundary layer
ZVP	The non-dimensional 8-meter level. The Known specific humidity at this level is used to evaluate free-stream humidity, q_s, using equation (3.85)
ZO	Surface roughness parameter, z_o

B.3 Subroutines used in the Computer Program

The subroutines used in the computer program and their purpose are given below.

SUBROUTINE VISCOS (T, P, XNUE)

This subroutine evaluates the kinematic visocity XNUE for a given temperature T and a given pressure P using equation (3.77).

SUBROUTINE VAPOUR (T1, T2, VAP)

This subroutine calculates the actual vapour pressure VAP for a given dry bulb temperature T1 and wet bulb temperature T2 using equations (3.81) and (3.82).

The Computer Program

```
 $$ JCB JNM=WTIIALII,CISP=C,CLASS=A
 $$ LST CISP=D
// JOB WTIIALII WTII
// UPSI 11
// CPTION LIST
// EXEC FFCRTRAN
CCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCC     *MAIN*
C                                                                     C     *MAIN*
C EVAPCRATICN FRCM AERCDYNAMICALLY SMCOTH ANC RCUGH SURFACES.EXPOSED C       *MAIN*
C TO A THERMALLY-STRATIFIED ATMCSPHERE (USING LAKE HEFNER DATA).      C     *MAIN*
C                                                                     C     *MAIN*
C      PRCGRAMMED BY MUHAMMEC ALI EHUIYAN, LECTURER, DEPT. CF WATER   C     *MAIN*
C RESOURCES ENGINEERING, BUET, DHAKA.                                 C     *MAIN*
C                                                                     C     *MAIN*
CCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCC     *MAIN*
C                                                                           *MAIN*
      REAL DU(500),DUDZ(500),D2(500),DZDL(500),D2LDZ2(500),D2ZDL2(500)      *MAIN*
     +,P(500),Q(500),SCT(500),T(500),U(500),U2ZDZ(500),Z(500),RX*8,         *MAIN*
     +RXST*8                                                                *MAIN*
C                                                                           *MAIN*
      DATA XK,SC,PRESS,IYEAR,NCDAY/0.36,0.596,1.0E3,1951,0/                 *MAIN*
      DATA ALPHA1,ALPHA2,BETA,GAMMA/15.0,5.0,0.74,4.7/                      *MAIN*
      WRITE(3,560)                                                          *MAIN*
C                                                                           *MAIN*
C     INPUT THE CLIMATOLOGICAL PARAMETERS                                   *MAIN*
50    READ(1,570)IDAY,MCNTH,T2D,T2W,T8D,T8W,TSURF,WIND2,WIND8,AREARL,       *MAIN*
     +EVAPRL                                                                *MAIN*
C                                                                           *MAIN*
C     PERFORM PRELIMINARY CALCULATICNS                                      *MAIN*
      XK2=XK*XK                                                             *MAIN*
      XK3=XK*XK2                                                            *MAIN*
      XK4=XK*XK3                                                            *MAIN*
      PAIE2=1.5708                                                          *MAIN*
C                                                                           *MAIN*
C     SET THE INITIAL VALUES                                                *MAIN*
      SUM=0.0                                                               *MAIN*
      AREA=0.0                                                              *MAIN*
      DRX=100.0                                                             *MAIN*
      NCDRX=1                                                               *MAIN*
      NCD2=500                                                              *MAIN*
      START=3000.0                                                          *MAIN*
      STEP=3000.0                                                           *MAIN*
      AREARL=4046.884*AREARL                                                *MAIN*
      RADIUS=SQRT(AREARL/3.1416)                                            *MAIN*
      RXPJL=1.0E8                                                           *MAIN*
      RXST=RXPJL                                                            *MAIN*
      ICASE=0                                                               *MAIN*
      COUNT=0.0                                                             *MAIN*
      RXTEST=0.0                                                            *MAIN*
C                                                                           *MAIN*
C     OUTPUT THE CLIMATCLOGICAL PARAMETERS                                  *MAIN*
      U1=51.48*WIND2                                                        *MAIN*
      U2=51.48*WIND8                                                        *MAIN*
      Z1=200.0                                                              *MAIN*
      Z2=800.0                                                              *MAIN*
      TD1=T2D                                                               *MAIN*
      TD2=T8D                                                               *MAIN*
      TW1=T2W                                                               *MAIN*
      TW2=T8W                                                               *MAIN*
      WRITE(3,580)IDAY,MCNTH,IYEAR                                          *MAIN*
```

```
      WRITE(3,590)U1,U2,Z1,Z2
      WRITE(3,600)TD1,TD2,TW1,TW2,TSURF
152   IF(U2 .LT. U1) GO TO 150

C     CALCULATE DENSITY,VAPOUR PRESSURE AND KINEMATIC VISCOSITY
155   RHO=0.3484/(TD1+273.0)
      CALL VAPOUR(TD2,TW2,VP2)
      CALL VAPOUR(TSURF,TSURF,VPSURF)
      CALL VISCOS(TD1,PRESS,XNUE)
      WRITE(3,610)VPSURF,VP2,RHO,XNUE


C     CALCULATE THE RICHARDSON NUMBER AND DETERMINE THE ATMOSPHERIC
C     STABILITY
      UDIFF=U2-U1
      ZDIFF=Z2-Z1
      TDIFF=TD2-TD1
      RI=981.0*(TDIFF+0.1)*ZDIFF/(UDIFF**2*(TD1+273.0))
      IF(RI .GT. 0.001)WRITE(3,620)RI
      IF(RI .LT. -0.001)WRITE(3,630)RI
      IF(ABS(RI) .LT. 0.001)WRITE(3,640)RI
      IF(ABS(RI) .LT. 0.001)RI=0.0
      IF(RI .GT. 0.15) GO TO 150

C     DETERMINE THE MONIN-OBUKHOV LENGTH, FRICTION VELOCITY AND
C     ROUGHNESS PARAMETER
      IF(RI)160,180,190


C     UNSTABLE STATE
160   ZBYL=0.0
      XINCR=-1.0
170   ZBYL=ZBYL+XINCR
      RITRI=BETA*ZBYL*((1.0-ALPHA1*ZBYL)/(1.0-ALPHA2*ZBYL))**0.5
      IF(RITRI .GT. RI)GO TO 170
      ZBYL=ZBYL-XINCR
      XINCR=XINCR/10.0
      IF(ABS(XINCR) .GE. 1.0E-4)GO TO 170
      XL=SQRT(Z1*Z2)/ZBYL
      X1=(1.0-ALPHA1*Z1/XL)**0.25
      X2=(1.0-ALPHA1*Z2/XL)**0.25
      A1=2.0*ALOG((1.0+X1)/2.0)+ALOG((1.0+X1*X1)/2.0)-2.0*ATAN(X1)+
     +PAIE2
      A2=2.0*ALOG((1.0+X2)/2.0)+ALOG((1.0+X2*X2)/2.0)-2.0*ATAN(X2)+
     +PAIE2
      UST=XK*UDIFF/(ALOG(Z2/Z1)+A1-A2)
      Z0=Z2/EXP(XK*U2/UST+A2)
      WRITE(3,670)UST,Z0,XL
      GO TO 210


C     NEUTRAL STATE
180   UST = XK* UDIFF/ALOG(Z2/Z1)
      Z0=Z2/EXP(XK*U2/UST)
      WRITE(3,690)UST,Z0
      GO TO 210
```

```
C     STABLE STATE
190   ZBYL=0.0
      XINCR=1.0
200   ZBYL=ZBYL+XINCR
      RITRI=ZBYL*(BETA+GAMMA*ZBYL)/(1.0+GAMMA*ZBYL)**2
      IF(RITRI .LT. RI)GO TO 200
      ZBYL=ZBYL-XINCR
      XINCR=XINCR/10.0
      IF(XINCR .GE. 1.0E-4)GO TO 200
      XL=SQRT(Z1*Z2)/ZBYL
      UST=XK*UDIFF/(ALOG(Z2/Z1)+GAMMA*ZDIFF/XL)
      Z0=Z2/EXP(XK*U2/UST-GAMMA*Z2/XL)
      WRITE(3,670)UST,Z0,XL


C     NON-DIMENSIONALISATION OF MONIN-OBUKHOV LENGTH AND ROUGHNESS
C     PARAMETER
210   IF(RI)220,230,220


C     UNSTABLE AND STABLE STATES
220   Z0=Z0*UST/XNUE
      XL=XL*UST/XNUE
      WRITE(3,700)Z0,XL
      IF(Z0 .LT. 1.0E-6) GO TO 150
      GO TO 270


C     NEUTRAL STATE
230   Z0=Z0*UST/XNUE
      WRITE(3,710)Z0
      IF(Z0 .LT. 1.0E-6) GO TO 150


C     DETERMINE THE PERMANENT FUNCTIONS OF VELOCITY
270   RX=RXST
      IF(RX .GT. RXPJL) GO TO 420
      ZVP=Z2*UST/XNUE
      ZS=0.0
      XINCR=1.0E6
      RXZS=0.0
      U2ZCZ(1)=0.0
      U3ZCZ=0.0
      IF(RI)320,280,320


C     ZS AND US FOR NEUTRAL STATE
280   ZS=ZS+XINCR
      A=ALOG(ZS/Z0)
      RXZS=(A*(ZS*(A-4.0)-2.0*Z0)+6.0*(ZS-Z0))/XK3
      IF(RXZS .LT. RX) GO TO 280
      ZS=ZS-XINCR
      XINCR=XINCR/10.0
      IF(XINCR .GE. 1.0) GO TO 280
      US=A/XK
      U2ZCZS=(ZS*A*(A-2.0)+2.0*(ZS-Z0))/XK2
      GO TO 320


C     FOR STABLE, UNSTABLE AND NEUTRAL STATES
```

```
320   U(1)=0.0
      Z(1)=0.0
      DO 365 I=2,NODZ
      DZ(I)=AMAX1(.05,(Z(I-1)-ZC)/20.0)
      DZ(2)=ZC
      Z(I)=Z(I-1)+DZ(I)
      IF(Z(I) .GE. ZVP .AND. ICASE .EQ. 0) ICASE=I
      A=ALOG(Z(I)/Z0)
      XKMZ=XK*Z(I)
      IF(RI)330,360,370

C     UNSTABLE STATE
330   ZBYL=Z(I)/XL
      X=(1.0-ALPHA1*ZBYL)**0.25
      U(I)=(A-2.0*ALOG((1.0+X)/2.0)-ALOG((1.0+X*X)/2.0)+2.0*ATAN(X)-
     +PAIB2)/XK
      USQDZ=U(I)**2*DZ(I)
      U2ZDZ(I)=U2ZDZ(I-1)+USQDZ
      IF(RXZS .GE. RX) GO TO 340
      UCUBDZ=USQDZ*U(I)
      U3ZDZ=U3ZDZ+UCUBDZ
      RXZS=U(I)*U2ZDZ(I)-U3ZDZ
      IF(COUNT .EQ. 0.0 .AND. RXZS .GE. RX) GO TO 350
340   DZDU(I)=XKMZ*X
      D2UDZ2(I)=(1.25*ALPHA1*ZBYL-1.0)/(XKMZ*Z(I)*X**5)
      SCT(I)=BETA*X/(1.0-ALPHA2*ZBYL)**0.50
      GO TO 365
350   US=U(I)
      ZS=Z(I)
      U2ZDZS=U2ZDZ(I)
      COUNT=1.0
      RXST=RXZS
      RXPJL=RXZS
      RX=RXZS
      GO TO 340

C     NEUTRAL STATE
360   U(I)=A/XK
      DZDU(I)=XKMZ
      D2UDZ2(I)=-1.0/(XKMZ*Z(I))
      U2ZDZ(I)=(Z(I)*A*(A-2.0)+2.0*(Z(I)-ZC))/XK2
      SCT(I)=BETA
      GO TO 365

C     STABLE STATE
370   ZBYL=Z(I)/XL
      B=GAMMA*ZBYL
      U(I)=(A+B)/XK
      USQDZ=U(I)**2*DZ(I)
      U2ZDZ(I)=U2ZDZ(I-1)+USQDZ
      IF(RXZS .GE. RX) GO TO 375
      UCUBDZ=USQDZ*U(I)
      U3ZDZ=U3ZDZ+UCUBDZ
      RXZS=U(I)*U2ZDZ(I)-U3ZDZ
      IF(COUNT .EQ. 0.0 .AND. RXZS .GE. RX) GO TO 378
375   DZDU(I)=XKMZ/(1.0+B)
      D2UDZ2(I)=-1.0/(XKMZ*Z(I))
```

```
      SCT(I)=(BETA+B)/(1.0+B)                                          *MAIN*
      GO TO 380                                                        *MAIN*
378   LS=L(I)                                                          *MAIN*
      ZS=Z(I)                                                          *MAIN*
      U2ZCZS=U2ZCZ(I)                                                  *MAIN*
      COUNT=1.0                                                        *MAIN*
      RXST=RXZS                                                        *MAIN*
      RXPJL=RXZS                                                       *MAIN*
      RX=RXZS                                                          *MAIN*
      GO TO 375                                                        *MAIN*
380   T(I)=0.0                                                         *MAIN*
      DUDZ(I)=1.0/DZDU(I)                                              *MAIN*
      D2ZDU2(I)=-D2UDZ2(I)*DZDU(I)**3                                  *MAIN*
385   CONTINUE                                                         *MAIN*
                                                                       *MAIN*
405   WRITE(3,720)ICASE,US,ZS,RXZS,DZ(2),DZ(3),DZ(4),DZ(5),DZ(6),      *MAIN*
     *DZ(50),RX                                                        *MAIN*
                                                                       *MAIN*
                                                                       *MAIN*
C     DETERMINE THE PRE-STEP LENGTH                                    *MAIN*
410   WINCZS=UST*LS                                                    *MAIN*
      PJSTEP=RXPJL*XNUE/WINCZS                                         *MAIN*
      RXPAFT=(PJSTEP+RADIUS)*WINCZS/XNUE                               *MAIN*
      UST=WINCZS/LS                                                    *MAIN*
420   T(1)=1.0                                                         *MAIN*
      J=1                                                              *MAIN*
430   RXFIN=(START+PJSTEP)*WINCZS/XNUE                                 *MAIN*
      IF(RX .GE. RXFIN) GO TO 540                                      *MAIN*
                                                                       *MAIN*
                                                                       *MAIN*
C     SOLUTION OF TRI-DIAGONAL MATRIX                                  *MAIN*
440   J=J+1                                                            *MAIN*
      SCTINV=1.0/SCT(J)                                                *MAIN*
      CC=0.5*DZ(J)*(DZ(J)+DZ(J+1))/CH)                                 *MAIN*
      RATIO=DZ(J)/DZ(J+1)                                              *MAIN*
      XKMNUE=AMAX1(0.0,(1.0-U2ZCZ(J)/U2ZCZS)*DZDU(J)-1.0)              *MAIN*
      XM=1.0/SC+SCTINV*XKMNUE                                          *MAIN*
      AA=US*U(J)/XM                                                    *MAIN*
      IF( Z(J) .GE. ZS ) AA=US*LS/XM                                   *MAIN*
                                                                       *MAIN*
                                                                       *MAIN*
C     DM/DZ FOR NEUTRAL CASE                                           *MAIN*
      DMDZ=SCTINV*((1.0-U2ZCZ(J)/U2ZCZS)*D2ZDU2(J)*DUDZ(J)-U(J)*U(J)*  *MAIN*
     *DZDU(J)/U2ZCZS)                                                  *MAIN*
      IF(RI .EQ. 0.0) GO TO 480                                        *MAIN*
      ZBYL=Z(J)/XL                                                     *MAIN*
      IF(RI) 450,460,460                                               *MAIN*
                                                                       *MAIN*
                                                                       *MAIN*
C     DM/DZ FOR UNSTABLE STATE                                         *MAIN*
450   TERM1=1.0-ALPHA1*ZBYL                                            *MAIN*
      TERM2=1.0-ALPHA2*ZBYL                                            *MAIN*
      DMDZ=DMDZ+0.25*XKMNUE*SQRT(TERM2)*(ALPHA1/TERM1-2.0*ALPHA2/      *MAIN*
     *TERM2)/(BETA*XL*TERM1**(.25))                                    *MAIN*
      GO TO 480                                                        *MAIN*
                                                                       *MAIN*
                                                                       *MAIN*
C     DM/DZ FOR STABLE STATE                                           *MAIN*
460   GAMMAL=GAMMA/XL                                                  *MAIN*
      DMDZ=DMDZ-XKMNUE*GAMMAL*(1.0-BETA)/(BETA+Z(J)*GAMMAL)**2         *MAIN*
```

```
0    EB=0.5*OMCZ*DZ(J)/XM                                                  *MAIN*
     IF( Z(J) .GE. ZS ) EB=0.0                                             *MAIN*
     A=AMAX1(0.0,EB+RATIO)                                                 *MAIN*
     E=1.0-EB                                                              *MAIN*
     IF(J .EQ. 2) GO TO 490                                                *MAIN*
     C=AA*CC*T(J)                                                          *MAIN*
     D=AA*CC+RATIO+1.0                                                     *MAIN*
     P(J)=1.0/(D-B*P(J-1))                                                 *MAIN*
     Q(J)=P(J)*(E*Q(J-1)+C)                                                *MAIN*
     B(J)=A*P(J)                                                           *MAIN*
     GO TO 500                                                             *MAIN*
90   C=(AA*CC*(2.0*DZ(2)+DZ(3))/(DZ(2)+DZ(3)))*T(J)                        *MAIN*
     D=AA*CC*(2.0*DZ(2)+DZ(3))/(DZ(2)+DZ(3))+RATIO+1.0                     *MAIN*
     P(2)=A/D                                                              *MAIN*
     Q(2)=(E*T(1)+C)/D                                                     *MAIN*
00   TEST=Q(J)/(1.0-P(J))                                                  *MAIN*
     IF(TEST .GT. 1.0E-2) GO TO 440                                        *MAIN*
     T(J)=TEST                                                             *MAIN*
     I=J-1                                                                 *MAIN*
10   T(I)=Q(I)+P(I)*T(I+1)                                                 *MAIN*
     IF((RX .GE. RXRAFT .AND. RXTEST .EQ. 0.0) .AND. ICASE .EQ. 1)         *MAIN*
    +GO TO 520                                                             *MAIN*
     I=I-1                                                                 *MAIN*
     IF(I .GE. 2) GO TO 510                                                *MAIN*
     GO TO 530                                                             *MAIN*
                                                                           *MAIN*
                                                                           *MAIN*
     EVALUATE THE FREE STREAM HUMIDITY                                     *MAIN*
20   VPS=(VP2-VPSURF*T(I))/(1.0-T(I))                                      *MAIN*
     RXTEST=1.0                                                            *MAIN*
     WRITE(3,730)RX, RXRAFT,ICASE,VPS,RXFIN                                *MAIN*
     I=I-1                                                                 *MAIN*
     GO TO 510                                                             *MAIN*
                                                                           *MAIN*
                                                                           *MAIN*
     THE NEXT STEP LENGTH                                                  *MAIN*
30   SX=RX                                                                 *MAIN*
     DRX=AMAX1(100.0,  (SX-RXPJL)/5.0)                                     *MAIN*
     RX=RX+DRX                                                             *MAIN*
     NODRX=NODRX+1                                                         *MAIN*
                                                                           *MAIN*
                                                                           *MAIN*
     THE FREE STREAM VELOCITY FOR THE NEXT STEP                            *MAIN*
     US=US+DRX/U2Z0ZS                                                      *MAIN*
     IF(RI) 532,534,536                                                    *MAIN*
                                                                           *MAIN*
                                                                           *MAIN*
     UNSTABLE STATE                                                        *MAIN*
32   ZSUP=ZS                                                               *MAIN*
     XINCR=1.0E6                                                           *MAIN*
33   ZSUP=ZSUP+XINCR                                                       *MAIN*
     X=(1.0-ALPHA1*ZSUP/XL)**0.25                                          *MAIN*
     A=2.0*ALOG(0.5*(1.0+X))+ALOG(0.5*(1.0+X*X))-2.0*ATAN(X)+PAIE2         *MAIN*
     USTRI=(ALOG(ZSUP/Z0)-A)/XK                                            *MAIN*
     IF(USTRI .LT. US) GO TO 533                                           *MAIN*
     ZSUP=ZSUP-XINCR                                                       *MAIN*
     XINCR=XINCR/10.0                                                      *MAIN*
     IF(XINCR .GE. 1.0) GO TO 533                                          *MAIN*
     U2Z0ZS=U2Z0ZS+(ZSUP-ZS)*US**2                                         *MAIN*
     ZS=ZSUP                                                               *MAIN*
```

```
      GO TO 420

C     NEUTRAL STATE
534   ZS=ZC*EXP(XK*US)
      A=ALOG(ZS/ZC)
      U2ZCZS=(ZS*A*(A-2.C)+2.C*(ZS-ZC))/XK2
      GO TO 420


C     STABLE STATE
536   ZSUP=ZS
      XINCR=1.0E6
537   ZSUP=ZSUP+XINCR
      USTRI=(ALOG(ZSUP/ZC)+GAMMA*ZSUP/XL)/XK
      IF(USTRI .LT. US) GO TO 537
      ZSUP=ZSUP-XINCR
      XINCR=XINCR/10.0
      IF(XINCR .GE. 1.0) GO TO 537
      U2ZCZS=U2ZCZS+(ZSUP-ZS)*US**2
      ZS=ZSUP
      GO TO 420


C     OUTPUT ROUTINE
C     CALCULATION OF WIDTH,AREA,EN,ETC.
540   WIDTH=2.0*SQRT(ABS(START*(2.C*RADIUS-START)))
      ZZ=0.25*(DZ(3)+DZ(4)+DZ(5)+DZ(6))
      SCTINV=1.0/SCT(2)
      XKMNUE=AMAX1(0.0,(1.0-U2Z0Z(2)/U2ZCZS)*DZDU(2)-1.0)
      EN=ABS((25.0*T(2)-48.0*T(3)+36.0*T(4)-16.0*T(5)+3.0*T(6))*
     +(1.0/SC+SCTINV*XKMNUE)/(12.0*ZZ*US))

      SUM=EN*WIDTH*STEP*WINDZS+SUM
      AREA=STEP*WIDTH+AREA
      START=AMIN1(2.0*RADIUS,START+STEP)
      RXST=RX

      IF(START .LT. 2.C*RADIUS) GO TO 270


C     E V A P O R A T I O N
550   DHUM=0.622*(VPSURF-VPS)/PRESS
      EVAP=8.64E5*RHO*DHUM*SUM/AREA
      WRITE(3,750)EN,START,RADIUS,AREA,AREARL,EVAP,EVAPRL,RX
      NODAY=NODAY+1
      IF(NODAY .LT.12) GO TO 150

      STOP




560   FORMAT(1H1)
570   FORMAT(2I3,5F6.2)
580   FORMAT(////,25X,'DATE',5X,2I4,I6)


590   FORMAT(1H0,5X,'U1=',F6.2,10X,'U2=',F7.2,10X,'Z1=',F6.2,10X,
```

```
     +'Z2=',F6.2)

600   FORMAT(1H0,5X,'TD1=',F6.2,10X,'TD2=',F6.2,10X,'TW1=',F6.2,
     +10X,'TW2=',F6.2,10X,'TSURF=',F6.2)

610   FORMAT(1H0,5X,'VPSURF=',F6.2,10X,'VP2=',F6.2,
     +10X,'RHO=',F7.5,10X,'NUE=',F6.4)

620   FORMAT(1H0,25X,'STABLE ATMOSPHERE',10X,'RI=',F9.5)

630   FORMAT(1H0,25X,'UNSTABLE ATMOSPHERE',10X,'RI=',F9.5)

640   FORMAT(1H0,25X,'NEUTRAL ATMOSPHERE',10X,'RI=',F9.5)

670   FORMAT(1H0,5X,'US1=',F8.4,10X,'Z0=',E15.6,10X,'L=',E15.6)

690   FORMAT(1H0,5X,'US1=',F8.4,10X,'Z0=',E15.6,10X,'L=INFINITY')

700   FORMAT(1H0,5X,'Z0+=',E15.6,10X,'L+=',E15.6)
710   FORMAT(1H0,5X,'Z0+=',E15.6,10X,'L+=INFINITY')

720   FORMAT(1H0,5X,'CASE=',I4,5X,'US=',E15.6,5X,'ZS=',E15.6,5X,
     +'RXZS=',E15.6//6X,'DZ=',6F10.2,5X,'FX=',E15.6)

730   FORMAT(1H0,5X,'FX=',E15.6,5X,'RXRAFT=',E15.6,5X,'CASE=',I4,5X,
     +'VPS=',F6.2,5X,'RXFIN=',E15.6)

750   FORMAT(1H0,5X,'EN=',F9.6,5X,'START=',E13.6,5X,'RADIUS=',E10.3,
     +5X,'AREA=',E10.3,5X,'ACTUAL AREA=',E10.3/5X,'EVAPORATION=',F6.2,
     +10X,'ACTUAL EVAPORATION=',F6.2,5X,'RX=',E15.6)

      END
```

```
      SUBROUTINE VAPCUR(T1,T2,VAP)
C     THIS SUBROUTINE CALCULATES THE VAPOUR PRESSURE FOR GIVEN DRY
C     AND WET BULB TEMPERATURES
      IF(T2) 10,20,20
10    SVPTW=4.5776+T2*(0.37305+T2*(0.012931+T2*1.9304E-4))
      VP=SVPTW-0.54*(T1-T2)
      GO TO 30
20    SVPTW=4.5855+T2*(0.32808+T2*(0.01172+T2*(1.279E-4+T2*4.184E-6)))
      VP=SVPTW-0.6*(T1-T2)
30    VAP=1.33*VP
      RETURN
      END


      SUBROUTINE VISCOS(T,P,XNUE)
C     THIS SUBROUTINE CALCULATES THE KINEMATIC VISCOSITY FOR GIVEN
C     TEMPERATURE AND PRESSURE
      A=T+273.16
      XNUE=(1.8375/3.484)*(A/P)*(416.16/(A+120.0))*SQRT((A/296.16)**3)
      RETURN
      END
/*
// EXEC LNKEDT
// EXEC

/*
/&
* $$ EOJ
```