EVAPORATION FROM SURFACES EXPOSED TO
A THERMALLY-STRATIFIED ATMOSPHERE

by

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In partial fulfillment of the requirements
for the degree of Master of Science in
Engineering (Water Resources)

BANGLADESH UNIVERSITY OF ENGINEERING AND TECHNOLOGY

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BANGLADESH UNIVERSITY OF ENGINEERING AND TECHNOLOGY

June 19, 1984

WE HEREBY RECOMMEND THAT THE THESIS PREPARED BY

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ENTITLED EVAPORATION FROM SURFACES EXPOSED TO A
THERMALLY-STRATIFIED ATMOSPHERE BE ACCEPTED AS
FULFILLING THIS PART OF THE REQUIREMENTS FOR THE
DEGREE OF MASTER OF SCIENCE IN ENGINEERING (WATER
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ABSTRACT

A theoretical method of estimating evaporation from bounded open water surfaces considering varying degrees of atmospheric stability and surface roughness conditions is presented. In this method, evaporation is calculated, using routine climatological measurements, from a numerical solution of the turbulent-momentum and diffusion equations. The effects of the atmospheric stability and surface roughness conditions are considered by incorporating into the method the appropriate wind and temperature profile equations and evaporation is obtained from the humidity gradient at the diffusing surface.

The theoretical results indicate that the rate of evaporation depends on the atmospheric stability and the downwind dimension of the surface. The results using the climatological measurements at Lake Hefner indicated a satisfactory overall agreement between the calculated and observed rates of evaporation for all degrees of atmospheric stability and surface roughness conditions. However, when it is necessary to derive short-term e.g. daily evaporation estimates accurately, the present work indicates that very precise measurements of the climatological parameters to determine the atmospheric stability with a sufficient degree of accuracy are necessary.
ACKNOWLEDGEMENT

The author acknowledges his sincere gratitude and thanks to Dr. M.A. Halim, Associate Professor, Department of Water Resources Engineering who introduced the author to the interesting field of Evaporation Studies. The author is really grateful to his supervisor for his constant encouragement and wise guidance at all stages of the study.

The author wishes to express his warm gratitude to other members of his thesis committee, Dr. A. Hannan, Professor and Head, Department of Water Resources Engineering, Dr. M. Shahjahan, Professor, Department of Water Resources Engineering, Dr. M.A. Hossain, Professor, Department of Mechanical Engineering for their special interest and valuable suggestions on many occasions.

The author also wishes to thank Mr. M. Mofser Ali and Mr. A.K. Azad for their assistance in the typing of the thesis and the drafting of figures respectively.
LIST OF SYMBOLS

This table presents the definitions of the symbols in common use in this thesis. When other symbols have been used they are defined in the text.

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<thead>
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<th>Definition</th>
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<tr>
<td>A</td>
<td>total area of an evaporating surface</td>
</tr>
<tr>
<td>D</td>
<td>molecular diffusivity of water in air</td>
</tr>
<tr>
<td>e</td>
<td>vapour pressure</td>
</tr>
<tr>
<td>E</td>
<td>evaporation</td>
</tr>
<tr>
<td>g</td>
<td>acceleration due to gravity</td>
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<tr>
<td>K</td>
<td>von Karman constant</td>
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<tr>
<td>K_h</td>
<td>turbulent or eddy diffusivity for heat</td>
</tr>
<tr>
<td>K_m</td>
<td>turbulent or eddy diffusivity for momentum</td>
</tr>
<tr>
<td>K_w</td>
<td>turbulent or eddy diffusivity for water vapour</td>
</tr>
<tr>
<td>l</td>
<td>pre-step length</td>
</tr>
<tr>
<td>L</td>
<td>Monin-Obukhov 'scale length'</td>
</tr>
<tr>
<td>L^*</td>
<td>non-dimensionalised 'scale length'</td>
</tr>
<tr>
<td>N_e</td>
<td>evaporation number</td>
</tr>
<tr>
<td>P</td>
<td>pressure</td>
</tr>
<tr>
<td>q</td>
<td>specific humidity</td>
</tr>
<tr>
<td>Q</td>
<td>non-dimensional specific humidity</td>
</tr>
<tr>
<td>r</td>
<td>radius of a circular surface area</td>
</tr>
<tr>
<td>R_e</td>
<td>Reynolds number</td>
</tr>
<tr>
<td>R_i</td>
<td>gradient form of the Richardson number</td>
</tr>
<tr>
<td>S_c</td>
<td>laminar Schmidt number</td>
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</table>
\( C_t \) turbulent Schmidt number
\( t \) time
\( T \) Temperature
\( u,v,w \) mean velocity components in the \( x,y \) and \( z \) directions respectively
\( U \) non-dimensional velocity in the \( x \) direction
\( u_* \) friction velocity
\( x,y,z \) downwind, cross-wind and vertical coordinates respectively
\( z_0 \) roughness parameter
\( Z \) non-dimensional vertical coordinate
\( Z_0 \) non-dimensional roughness parameter

Greek Symbols
\( \Gamma \) dry-adiabatic lapse rate
\( \Theta_0 \) scaling temperature
\( \mu \) dynamic viscosity
\( \nu \) kinematic viscosity
\( \rho \) air density
\( \tau \) shearing stress
\( \tau_n \) dimensionless temperature gradient
\( \tau_m \) dimensionless velocity gradient
\( \theta \) potential temperature
Subscripts

o  surface or wall value
s  free-stream value
t  turbulent value
l  laminar value
a  value in the air
d  value at dew point temperature

2,4,8 values at 2,4 and 8 meter levels respectively.

An overbar denotes a time-or ensemble-averaged quantity and a prime denotes the deviation from that average.
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Evaporation may be defined as the process by which a liquid or solid is converted into the vapour phase. In terms of the hydrological cycle, it forms the main link in the purification of water and its transfer from the oceans over the continents. The underlying surface from which water evaporates may either be an open water surface, snow, ice, vegetation cover or bare soil.

There is no doubt that evaporation has far-reaching effects on the environment and also upon many fields of human activity; affecting as it does, water resources and supply, agriculture, industry and so on. For these purposes, a reliable estimate of evaporation and a clear understanding of the physical processes involved therein, may be of considerable economic importance especially in arid and semi-arid regions of the world where water is often supplied at great expense and the loss of water due to evaporation is high. A method for the accurate determination of water losses from lakes, reservoirs, watersheds, agricultural lands etc. is of vital importance.
The process by which the vapour is removed is partly molecular and partly a turbulent transfer, the turbulent process being dominant everywhere except very close to the evaporating surface. The physical requirement for evaporation is the existence of a vapour concentration gradient that is favourable for the net transfer of water molecules from the surface to the atmosphere. This gradient is maintained by the supply of thermal energy to the evaporating surface so that the water molecules have sufficient energy to vaporize, and by a continuous turbulent diffusion process that assists in the rapid dispersal of the water molecules in the atmosphere. These mechanisms for maintaining the vapour concentration gradient have resulted in the development of two distinct approaches to the theoretical estimation of evaporation using climatological data. The first approach, known as the mass-transfer approach, considers the mechanics of turbulent diffusion whilst the second, the energy-balance approach, is concerned with the principle of energy conservation.

In the present work, a mass-transfer approach has been considered in which evaporation is estimated, using routine climatological measurements, from a numerical solution of the turbulent momentum and diffusion equations. The objectives of the present study are:
1. to develop a method for estimating evaporation from open water surfaces considering the entire ranges of atmospheric stability and surface roughness conditions;

2. to suggest means by which the proposed method can be applied for estimating evaporation using routine climatological data;

3. to examine the theoretical results and the implications of the proposed evaporation model; and,

4. to determine the validity and limitations of the proposed approach for estimating evaporation from bounded water surfaces.
2.1 General

Theoretical studies of evaporation have developed along two distinct approaches. The first approach, known as the mass-transfer approach, considers the mechanics of turbulent diffusion whilst the second, the energy-balance approach ignores the theory of turbulence but is concerned with the principle of conservation of energy. The mass-transfer approach is further sub-divided into (i) the semi-empirical approach, (ii) the diffusion approach, and (iii) the aerodynamic approach.

2.2 The Semi-Empirical Approach

The semi-empirical approach is based upon the classical Dalton (1802) theory where the rate of evaporation from an air-water interface is related to the saturation deficit of the air and an empirically determined function of the horizontal wind speed. The formulae based on the Dalton's law have the general form

\[ E = a(1+bu)(e_s-e_d) \]  

(2.1)

where,

- \( E \) = rate of evaporation per unit area
- \( a, b \) = empirical constants
- \( u \) = wind speed at a certain height
\[ e_s = \text{vapour pressure at the water surface} \]
\[ e_d = \text{vapour pressure in the air}. \]

Many variations of the basic Dalton formula exist. The formula given by Rowher (1931), based on the results of very intensive work at Fort Collins, Colorado, at 5000 ft above sea-level, for the daily rate of evaporation from an open water surface 3 ft. square is

\[ E = 0.40(1+0.27 u_0)(e_s - e_d) \quad \cdots \quad \cdots \quad (2.2) \]

where \( E \) is in mm/day, \( u_0 \) is wind speed at the ground level in miles per hour and \( e_s \) and \( e_d \) are in mm of Hg.

Penman (1948) modified the above formula in which he incorporated the wind velocity at 2 meter above the ground instead of the ground wind velocity \( u_0 \) and recast equation (2.2) as

\[ E = 0.40 \left(1+0.17 u_2\right)(e_s - e_d) \quad \cdots \quad \cdots \quad (2.3) \]

Marciano and Harbeck (1954) introduced a mass-transfer coefficient \( N \) and proposed the following formula for the daily rate of evaporation from an open water surface:

\[ E = Nu (e_s - e_d) \quad \cdots \quad \cdots \quad (2.4) \]

where \( u \) is the mean wind velocity at a given height. The best fit with experimental results from Lake Heafner was obtained when equation (2.4) takes the form (Chow, 1964)
\[ E = 1.77 \times 10^{-3} \, u_8 \, (e_s - e_d) \] ... \( \ldots \) \( (2.5) \)

where \( u_8 \) is the wind speed at 8 meter in miles per hour, 
\( E \) is in inches/day and \( e_s \) and \( e_d \) are in millibars.

Thornthwaite and Holzman (1939) derived the following formula for estimating evaporation which they claim to apply rigorously under adiabatic conditions, but over-estimate under conditions with high wind or with a temperature inversion:

\[ E = \frac{17.1 \, (u_2 - u_1) \, (e_1 - e_2)}{T + 459.4} \] ... \( \ldots \) \( (2.6) \)

where \( E \) is in inches/hour, \( T \) is in °F and the differences in vapour pressure and horizontal wind velocity are in inches of Hg. and in miles per hour at two heights above the surface, namely 2 and 28.6 feet, respectively.

There are many other empirical formulae of this type but none of them has been shown to be universally applicable. These formulae are true, at their best, only for the sites where these were first developed and can not be applied to other areas.

2.3 The Diffusion Approach

The diffusion approach involves the solution of the diffusion equation giving evaporation in terms of wind
speed and size of the evaporating surface. By definition, mass-transfer by diffusion in the atmosphere involves another physical phenomenon, namely turbulence. The atmosphere is generally turbulent except possibly under calm condition or when there are very light winds. Parcels of turbulent air, or eddies, transport entities, for example heat, momentum, carbon dioxide, water vapour and so on, and allow a rapid spread of these properties through the atmosphere.

The basic diffusion equation in a 3-dimensional Cartesian frame of reference can be written as

\[
\frac{dq}{dt} = \frac{\partial}{\partial x} \left( k_x \frac{dq}{dx} \right) + \frac{\partial}{\partial y} \left( k_y \frac{dq}{dy} \right) + \frac{\partial}{\partial z} \left( k_z \frac{dq}{dz} \right)
\]  
\( (2.7) \)

where,

\[
\frac{dq}{dt} = \frac{\partial q}{\partial t} + u \frac{\partial q}{\partial x} + v \frac{\partial q}{\partial y} + w \frac{\partial q}{\partial z} .
\]  
\( (2.8) \)

It was not possible to obtain a satisfactory solution to the 3-dimensional diffusion equation (2.7). Attempts were, therefore, being made to solve the 2-dimensional diffusion equation under steady-state conditions in which the lateral and longitudinal components of turbulent diffusivity have been ignored and it was assumed that there was no vertical and lateral components of the wind velocity. Under these conditions, equation (2.7) reduces to
\[ u \frac{\partial u}{\partial x} = \frac{\partial}{\partial z} (K_z \frac{\partial u}{\partial z}). \quad (2.9) \]

Solution of the above equation for flow over aerodynamically smooth surfaces was considered by Sutton (1934). Assuming that the turbulent diffusivities for momentum and water vapour are identical, Sutton derived the following expression for the turbulent diffusivity of water vapour as

\[ K_z = \left( \frac{4 \pi n^2}{(1-n)^{1-n}} \right) \rho \left\{ \left| \frac{\partial u}{\partial z} \right|^3 \right\} \left( \frac{\partial^2 u}{\partial z^2} \right)^2 \] \quad (2.10)

where \( n \) is a measure of atmospheric stability having a value of 0.25 under neutral conditions. In deriving equation (2.10), Sutton represented the wind structure near the ground by the equation

\[ u = u_1 \left( \frac{\frac{z}{z_1}}{\frac{z}{z_1}} \right)^{\frac{n}{2-n}}, \quad 0 < n < 1 \quad \ldots \quad (2.11) \]

where \( u_1 \) is the mean wind velocity at a fixed height \( z_1 \).

Sutton (1934) found that for a rectangular evaporating surface of dimensions \((x_0, y_0)\) the total evaporation is given by

\[ E(x_0, y_0) = A_1(u_1)^{\frac{2-n}{2+n}} (x_0)^{\frac{2}{2+n}} (y_0) \quad (2.12) \]

and for a circular area of radius, \( r \).
where the coefficients $A_1$ and $A_2$ include the differences in vapour concentrations. For neutral conditions equation (2.12) reduces to

$$E_0 = \text{constant} \ (u_1)^{0.78} \ x_0^{0.89}$$

(2.14)

where $E_0$ is the evaporation per unit width across the wind. The rate of evaporation per unit area is given by

$$\frac{\partial E_0}{\partial x} = \text{Constant} \ (u_1)^{0.78} \ x_0^{-0.11}$$

(2.15)

The above equation shows that the rate of evaporation is directly proportional to the power of the downwind speed less than unity. This amounts to saying that the local rate of evaporation decreases with the distance from the upwind edge, so that the phrase "evaporation per unit area" is ambiguous unless the complete area of the evaporating surface is specified.

Takhar (1972) stated the physics of evaporation is more complicated than simply the treatment of the diffusion equation alone. It is in fact a continuous interaction of the wind velocity and the concentration or humidity fields. A fully-developed turbulent momentum boundary layer comes into contact
with the turbulent diffusion boundary layer. Evaporation is the end product of the interaction between these two fields. Considerations like this led Takhar (1972) to estimate evaporation from a numerical solution of the turbulent momentum and diffusion equations for 2-dimensional flow over aerodynamically smooth surfaces in a neutral atmosphere. He assumed that the physical properties of the fluid are constant, the universal "law of the wall" applies throughout the turbulent boundary layer and the eddy diffusivities for momentum and the water vapour are equal i.e., $K_m = K_w$. He introduced a dimensionless evaporation parameter defined as

$$E_v = D \left( \frac{\partial q}{\partial z} \right)_{z=0} / (q_s - q_o) u_s \tag{2.16}$$

$$= \frac{1}{S_c u_s} \left[ \frac{\partial q}{\partial z} \right]_{z=0} \tag{2.17}$$

where $S_c$ is the Schmidt number ($= \nu / \sigma$) and $Q$ and $Z$ are the non-dimensional specific humidity and vertical distance given by

$$Q = \frac{q - q_o}{q_s - q_o} \tag{2.18}$$

and

$$Z = \frac{zu}{\nu} \tag{2.19}$$

respectively. The total evaporation $E$, from an open water surface of effective width 'w' is then given by

$$E = 'w' \int_1^x E_v (q_s - q_o) u_s \, dx \tag{2.20}$$
where \( l \) represents the distance between the leading edges of the turbulent momentum and diffusion boundary layers. This theoretical treatment seems to be an improvement upon Sutton's (1934) solution since the coupled effect of the interacting momentum and humidity fields are considered simultaneously.

The diffusion method is only applicable with validity to saturated surfaces (Deacon et al., 1958). When the surface begins to dry out, the mechanism by which water vapour is transferred to the air must be combined with a consideration of the process by which liquid and vapour transfers occur in the subsurface. In other words, the diffusion approach, although soundly based in theory, does have its limitations in practice.

2.4 The Aerodynamic Approach

The aerodynamic approach gives evaporation in terms of the differences of wind speeds and humidities at two levels above the evaporating surface under the condition that the vapour concentration is independent of the downwind position. Within a study of the aerodynamic properties of the atmosphere another concept needs some consideration, namely the concept of aerodynamic roughness. A physical boundary is aerodynamically rough when fluid flow is turbulent down to the boundary itself.
Over such a boundary the velocity profile and surface drag are independent of the fluid viscosity but are dependent upon a roughness length, usually designated as $z_o$, which is related to the height and spacing of the roughness elements of the surface. A surface is aerodynamically smooth if a layer adjacent to the surface has a flow which is laminar and in which the velocity profile and surface drag are related to the fluid viscosity. It should be noted that a surface which is aerodynamically smooth at low speeds of flow may become rough at high velocity of flow and vice versa. A surface may therefore be described as rough or smooth only in terms of the associated flow. As Sutton (1949) pointed out, it is now well established that turbulent flow near the surface is aerodynamically rough.

The aerodynamic method is based upon three assumptions, namely that, (i) the wind structure near the surface is represented by the universal logarithmic law given by

$$u_z = \frac{1}{k} \sqrt{\frac{C_o}{\rho}} \ln \left( \frac{z}{z_o} \right)$$

where $u_z$ is the wind speed at the level $z$, (ii) the eddy diffusivity for water vapour, $K_w$, is identical with that for momentum, $K_m$, and, (iii) the shearing stress is constant with height. Based on these assumptions, the simple aerodynamic
formula first introduced by Thornthwaite and Holzman (1939) may be stated as

$$
E = \frac{\rho K^2 (q_2 - q_8) (u_8 - u_2)}{\ln \left( \frac{z_8}{z_2} \right)^2}
$$

(2.22)

where all values are in GGS units. The above evaporation equation utilizes the 2 and 8 meter climatological measurements.

A similar equation was also given by Sverdrup (1946) as

$$
E = \frac{\rho K^2 u_8 (q_o - q_8)}{\ln \left( \frac{z_8}{z_0} \right)^2}
$$

(2.23)

where $z_0$ is the surface roughness parameter.

It should be noted in the aerodynamic approach that, the humidity values are observed at two heights and are fundamental to the above formulae. Evaporation is proportional to the difference in vapour concentration between the two reference heights. As the aerodynamic approach is valid under the condition that the vapour concentration is independent of the downwind position, it is only applicable for large evaporating surfaces such as seas, lakes, large reservoirs etc.
2.5 The Energy-Balance Approach

In the energy-balance or energy-budget approach, the energy acting on the unit area of an evaporating surface is balanced against the various sources of its dissipation such that the only unknown quantity is the energy used for evaporation.

Penman (1948) outlined a method for estimating evaporation based on the heat balance equation of Brunt (1939). Using as the unit of energy, the amount required to evaporate \( \frac{1}{10} \) gm. of water at air temperature (59 cal.), Penman gave the following expression for the heat budget \( H \):

\[
H = R_c (1 - r - \mu) - \sigma T_a^4 (0.56 - 0.092 \sqrt{e_d}) \times (1 - 0.09m) \quad (2.24)
\]

where,

- \( R_c \) = measured short-wave radiation
- \( r \) = reflection coefficient for the surface
- \( \mu \) = fraction of \( R_c \) used in photosynthesis
- \( \sigma \) = Boltzmann constant
- \( m \) = fraction of sky covered with cloud, in tenth
- \( e_d \) = vapour pressure at dewpoint temperature in mm.Hg.
- \( T_a \) = temperature in \( ^\circ \text{K} \).

The heat budget is used in evaporation, \( E \), heating of the air, \( K \), heating of the test material, \( S \), and heating of the surroundings of the test material, \( C \), i.e.,

\[
H = E + K + S + C. \quad \ldots \quad \ldots \quad (2.25)
\]
Over a period of several days, and frequently over a single day, the change in the stored heat, $S$, and the heat conducted through the walls of the test material container, $C$, are negligible compared with other changes. Thus, equation (2.25) can be safely reduced to

$$N = E + K. \quad (2.26)$$

The transport of vapour and the transport of heat by eddy diffusion are, in essentials, controlled by the same mechanism, and apart from the differences in the molecular constants, the one is expected to be governed by $(e_s - e_d)$ where the other is governed by $(T_s - T_a)$. To a very good approximation, therefore, it is possible to write down the ratio of $K/E$ in the form

$$\frac{K}{E} = \beta = \gamma \frac{(T_s - T_a)}{(e_s - e_d)} \quad \cdots \quad \cdots \quad (2.27)$$

where $\beta$ is the Bowen's ratio and $\gamma$ is the standard constant of the wet and dry bulb hygrometer equation. In °F and mm. Hg, $\gamma = 0.27$. Thus, combining equations (2.26) and (2.27) one obtains

$$E = \frac{H}{(1 + \beta)}. \quad \cdots \quad \cdots \quad (2.28)$$

Of the terms on the right-hand side of equation (2.24), the radiation term is rarely directly measurable, but for periods of the order of a month or more it can be estimated from duration of sunshine using the relation

$$R_c = P_A \left( a + \frac{bn}{N} \right). \quad \cdots \quad \cdots \quad (2.29)$$
where $R_A$ is the Angot value of $R_C$ in a completely transparent sky and $n/N$ is the ratio of actual to possible hours of sunshine and $a$ and $b$ are empirical coefficients.

The value of $\mu$ in equation (2.24) is very small ($\approx 0.005$) and can be neglected. The value of $r$ will vary with season and type of surface. It is obvious that cloud control of long-wave radiation must depend upon cloud type, and as a provisional expedient to make some allowance for this, Penman (1948) proposed to set $m/10 = 1-n/N$. Equation (2.24) therefore reduces to

$$H = E(1+\beta) = (1-r) R_A(a + bn/N) - \sigma T_a^4 (0.56-0.092 \sqrt{e_d}) \times$$

$$(0.01 + 0.90 n/N) \ldots (2.30)$$

where $R_A(a + bn/N)$ is to be replaced by $R_C$ when this is known. The parameters represented on the right-hand side of equation (2.30) are easily determined. If the water surface temperature is known, equation (2.27) may be used to determine $\beta$ and evaporation may be obtained from equation (2.30).

Penman (1948) also suggested a mass-transfer-energy-balance combination approach for estimating evaporation which eliminates the surface temperature—the parameter which is measured with considerable difficulty.

From considerations of turbulent transfer, the evaporation rate is given by

$$F = f(u) (e_s - e_d) \ldots \ldots \ldots (2.31)$$
and sensible heat transfer is given by

\[ K = \gamma f(u) \left( T_s - T_a \right). \]  

(2.32)

This assumes that \( f(u) \) is the same for vapour transfer as for heat transfer. It is a function of wind speed at a known height. It is convenient to define a new quantity \( E_a \) by

\[ E_a = f(u) (e_a - e_d) \]  

(2.33)

and also to introduce another parameter

\[ \Delta = \frac{d e}{d T} \right. \left( \frac{e_s - e_a}{T_s - T_a} \right). \]  

(2.34)

Normally the surface temperature and vapour pressure are unknown but for open water they are uniquely related and can be eliminated from the above equation. Thus,

\[ \frac{E_a}{E} = 1 - \frac{e_s - e_a}{e_s - e_d} = 1 - \gamma \]  

(say)  

(2.35)

and we have

\[ E = \frac{H}{(1 + \beta)} = \left[ 1 + \frac{H}{\gamma} \frac{T_s - T_a}{e_s - e_d} \right]. \]  

(2.36)

If we put

\[ T_s - T_a = \frac{e_s - e_a}{\Delta}, \]  

(2.37)

\[ \frac{H}{E} = 1 + \frac{\gamma}{\Delta} \gamma, \]  

(2.38)
therefore

\[ E = \frac{H \Delta + \gamma E_a}{\Delta + \gamma} \text{ mm./day.} \quad \cdots \quad \cdots \quad (2.39) \]

\[ e_s = \frac{(e_a - \gamma e_d)}{(1 - \gamma)} \quad \cdots \quad \cdots \quad (2.40) \]

Ultimately we have \( E_o \), the evaporation from a hypothetical open water surface as

\[ E_o = \frac{H_o \Delta + \gamma E_a}{\Delta + \gamma} \text{ mm./day.} \quad \cdots \quad \cdots \quad (2.41) \]

The only weather data needed to evaluate \( E_o \) to an adequate degree of approximation are the mean air temperature, humidity, wind speed and duration of bright sunshine. Here \( H_o \) is expressed in the same manner as \( H \), the albedo being that for open water. \( E_a \) also introduces empiricism in the delimitation of \( f(u) \). Penman from his experiments at Rothamstead obtained the following expression for \( E_a \):

\[ E_a = 0.35 \left( 1 + 9.8 \times 10^{-3} u_2 \right) (e_a - e_d) \text{ mm./day.} \quad \cdots \quad (2.42) \]

This expression should ideally be re-evaluated for each site; numerous tests of the formula have however shown that this Penman determination is reasonable for use for most locations, at least in temperate lands.

Although the energy-balance approaches appear to be simple in theory, a very accurate determination of the individual items in the heat-budget equation requires
complex and expensive instrumentation and is undesirable for obtaining routine estimates. This approach has been successfully undertaken by Anderson (1954) for determining evaporation losses from Lake Hefner.
3.1 General

As stated earlier, Takhar (1972) outlined a method for estimating evaporation from a numerical solution of the turbulent momentum and diffusion equations under steady-state neutral atmospheric conditions for 2-dimensional parallel flow over aerodynamically smooth surfaces. This method is analogous to the approach of Hatton (1964) for determining heat transfer from a semi-infinite horizontal flat plate. It is assumed that the ground vapour concentration is at the lower free-stream value upwind of the diffusing surface followed by a jump in humidity at the onset of the evaporating surface. The universal "law of the wall", as proposed by Spalding (1961), was used by Takhar (1972) to relate the non-dimensional velocity with the non-dimensional vertical height. Experimental investigation of this approach was undertaken by Halim (1982) and Takhar and Liddament (1983). The results of the experimental work indicated that a fairly good agreement between the estimated and observed rates of evaporation is obtained when the daily evaporation losses are less than about 4 mm. For a higher rate of evaporation, the associated high velocity of flow makes the surface so rough that an accurate estimation of the daily evaporation using the "law of the wall" is not possible. It was concluded that for flows over aerodynamically rough surfaces,
the effect of the surface roughness on the rate of evaporation has to be taken into consideration.

In the present work, the evaporation model of Takhar (1972) is extended to include the effects of the atmospheric stability and surface roughness on the rate of evaporation. Instead of using the "law of the wall", the stability related wind and temperature profile equations of Businger et al. (1971) are incorporated into the method for estimating evaporation. The "law of the wall" is applicable for flows over aerodynamically smooth surfaces under neutral atmospheric conditions only, whereas, the wind and temperature profile equations of Businger et al. (1971) are applicable for the entire range of atmospheric stability and surface roughness conditions. The proposed method is similar to that developed by Halim (1982) for estimating evaporation from open water surfaces in which the wind and temperature profile equations in the atmospheric surface layer were obtained from the equations of the turbulent mean and fluctuating motions.

3.2 The Development of the Evaporation Model

The theoretical evaporation model is shown in Fig.3.1. In this method evaporation is considered to take place as a result of the interaction between the turbulent momentum and diffusing boundary layers. The leading edge of the diffusion boundary layer marks the beginning of transition in surface
Fig. 3.1 The Theoretical Evaporation Model
vapour concentration from a lower value upwind of the evaporating surface to a higher value over the surface and is commonly known as the "vapour-step position". Also the leading edges of the momentum and diffusion boundary layers (Fig. 3.1) are separated by the distance \( l \) which may be called the "separation length".

Taking the \( x \)-axis along the mean direction of flow and the \( z \)-axis vertically upwards (Fig. 3.1), with associated mean velocities \( u \) and \( w \) in these directions, the continuity, momentum and diffusion equations for an steady and incompressible turbulent boundary layer in zero pressure gradient may be written respectively as (Takhar, 1972)

\[
\frac{\partial u}{\partial x} + \frac{\partial w}{\partial z} = 0 , \quad \ldots \quad \ldots \quad (3.1)
\]

\[
u \frac{\partial u}{\partial x} + w \frac{\partial u}{\partial z} = \frac{\partial}{\partial z} \left[ (v + K_m) \frac{\partial u}{\partial z} \right] , \quad \ldots \quad (3.2)
\]

\[
u \frac{\partial q}{\partial x} + w \frac{\partial q}{\partial z} = \frac{\partial}{\partial z} \left[ (D + K_w) \frac{\partial q}{\partial z} \right] , \quad \ldots \quad (3.3)
\]

where \( v \) and \( D \) are the kinematic viscosity of air and molecular diffusivity of water in air, \( K_m \) and \( K_w \) are the eddy diffusivities for momentum and water vapour respectively, and \( q \) is the specific humidity or mass concentration of moisture content defined as

\[
q = \frac{\text{mass of water vapour}}{\text{mass of moist air}} .
\]
To obtain equation (3.2), the Boussinesq eddy-diffusivity concept, given by,

\[
\frac{\tau}{\rho} = (\nu + k_m) \frac{\partial u}{\partial z} 
\]

where \( \tau \) is the shear stress and \( \rho \) is the density of the fluid, has been utilized.

Introducing the stream function, \( \psi \), which defines

\[
u = \frac{\partial \psi}{\partial z} \quad \text{and} \quad w = -\frac{\partial \psi}{\partial x},
\]

then applying the von Mises transformations and non-dimensionalizing the different variables using the transformations

\[
\begin{align*}
U &= \frac{u}{u_*}, \quad Z = \frac{z u_*}{\nu}, \quad Z_0 = \frac{z_0 u_*}{\nu} \\
L^* &= \frac{L u_*}{\nu}, \quad R_e = \frac{x u_*}{\nu}, \quad Q = \frac{q - q_0}{q_s - q_0}
\end{align*}
\]

where \( z_0 \) is the surface roughness parameter, \( u_* = \sqrt{\tau_o/\rho} \) is the friction velocity and the subscripts 'o' and 's' refer to the values of an entity at the wall (surface) and in the free-stream respectively, equations (3.2) and (3.3) reduce to (Hattan, 1964 and Takhar, 1972)

\[
-U^2 \frac{d U_s}{d R_e} = \frac{1}{\tau_o} \frac{\partial \tau}{\partial Z},
\]

and,

\[
U U_s \frac{\partial Q}{\partial R_e} = \frac{\partial}{\partial Z} \left[ \left( \frac{1}{S_0} + \frac{K_w}{\nu} \right) \frac{\partial Q}{\partial Z} \right]
\]
respectively, where \( S_0 = \frac{\nu}{D} \) is the laminar Schmidt number.

Assuming that the functional relationship between \( U \) and \( Z \) is known (Art. 3.3), equation (3.4) and (3.6) may be used to determine the variation of \( K_m \) with \( Z \) within the turbulent momentum boundary layer. Then relating \( K_m \) to \( K_w \) via the turbulent Schmidt number \( (S_0^t) = \frac{K_m}{K_w} \), the variation of \( K_w \) can be established. Equation (3.7) can then be solved for \( Q \) throughout the turbulent diffusion boundary layer. The humidity gradient at the diffusing surface represented by \( Z = Z_o \) can then be determined from which the evaporation loss can be estimated. The boundary conditions for the non-dimensional velocity and humidity are:

\[
\begin{align*}
U &= 0, & \text{when } Z &= Z_o \\
U &= U(Z), & \text{when } Z_o < Z &< Z_s \\
U &= U_s(Re), & \text{when } Z &\geq Z_s \\
Q &= 0, & \text{when } Z &= 0, \quad x > 1 \\
Q &= Q(Z), & \text{when } 0 < Z &< Z_d, \quad x > 1 \\
Q &= 1, & \text{when } Z &\geq Z_d, \quad x > 1 \text{ and for all } Z, x \leq 1
\end{align*}
\]

where \( Z_d \) and \( Z_s \) are the non-dimensional height of the turbulent diffusion and momentum boundary layers respectively.

Integrating equation (3.6) between the levels \( Z_o \) and \( Z \) and applying the boundary conditions given in (3.8), we can write
\[- \int_{Z_0}^{Z} u^2 \frac{dU_s}{dR_e} dZ = \frac{T - T_o}{T_o} \ldots (3.9)\]

where \(T_o\) is the shearing stress at the level \(Z = Z_0\). Since \(\frac{dU_s}{dR_e}\) is not a function of \(Z\) we can write

\[- \frac{dU_s}{dR_e} \int_{Z_0}^{Z} u^2 dZ = \left( \frac{T}{T_o} - 1 \right) \ldots (3.10)\]

Since at \(Z = Z_s\), \(T = 0\), equation (3.10) gives

\[\frac{dU_s}{dR_e} = \frac{1}{\int_{Z_0}^{Z_s} u^2 dZ} \ldots (3.11)\]

and using equation (3.11) into equation (3.10) we obtain

\[\frac{T}{T_o} = 1 - \frac{\int_{Z_0}^{Z_s} u^2 dZ}{\int_{Z_0}^{Z_s} u^2 dZ} \ldots (3.12)\]

Now combining equations (3.4), (3.5) and (3.12), the variation of \(K_m\) becomes
\[
\frac{K_m}{\gamma} = \left[ \left( 1 - \int_{Z_0}^{Z_s} \frac{U^2 \, dZ}{Z_0} \right) \frac{dZ}{dU} - 1 \right]. \quad \ldots \quad (3.13)
\]

Equation (3.13) may now be used to determine the \( K_m \) - variation for any momentum boundary layer height \( Z_s \). In order that the \( K_m \) - variation may be determined for the entire momentum boundary layer, the relation between the downwind position, in terms of the Reynolds number, \( R_e \), and \( Z_s \) must be determined.

From equation (3.11) we can write

\[
\frac{dR_e}{dU_s} = \int_{Z_0}^{Z_s} U^2 \, dZ. \quad \ldots \quad \ldots \quad (3.14)
\]

Integration of equation (3.14) with respect to \( Z_s \) between the limits \( Z_s = Z_0 \) and \( Z_s = Z_0 \) yields

\[
R_e = \int_{Z_0}^{Z_0} \left( \int_{Z_0}^{Z_s} U^2 \, dZ \right) \frac{dU_s}{dZ_s} \, dZ_s. \quad \ldots \quad \ldots \quad (3.15)
\]

The right hand side of this equation may be integrated by parts to give

\[
R_e = U_s \int_{Z_0}^{Z_0} U^2 \, dZ - \int_{Z_0}^{Z_0} U^3 \, dZ. \quad \ldots \quad (3.16)
\]
Thus, for a given downwind distance in terms of \( R_e \), equation (3.16) gives the height of the turbulent momentum boundary layer \( Z_g \), and hence, equations (3.13) and (3.16) may be used to determine the \( K_m \)-variation throughout the turbulent momentum boundary layer.

The diffusion equation (3.7) may be solved numerically by employing the standard finite-difference techniques. To do so, the area bounded by the diffusion boundary layer and the evaporating surface is to be divided into a rectangular grid system, each rectangle being of dimensions \( \Delta R_e \times \Delta Z \) as shown in Fig. A.1. For each step \( \Delta R_e \) downwind, the value of \( Q \) at each grid point may be determined for increments in \( \Delta Z \). The procedure for solving the diffusion equation (3.7) using a fully-implicit finite-difference scheme (Spalding, 1977) has been presented in Appendix-A.

The humidity gradient \( \partial Q / \partial Z \) at the diffusing surface represented by \( Z = Z_0 \) may be evaluated by substituting the \( Q \) values at the first five grid points near the surface into the five point difference formula (Fox and Mayers, 1968).

The vertical flux of water vapour across the \( Z = Z_0 \) plane for any position \( x \) downwind of the leading edge of the diffusion boundary layer is given by (Sheppard et al., 1972),
\[ E_x = -\rho(D + K_w) \left( \frac{d q}{dz} \right)_{z = z_0} \cdots \cdots (3.17) \]

Using the relations given in equation (3.5), it can be shown that

\[ E_x = -\rho(q_s - q_o) u_s N_e \cdots \cdots (3.18) \]

where,

\[ N_e = \frac{1}{U_b} \left( \frac{1}{S_o} + \frac{K_w}{\gamma} \right) \left( \frac{d q}{dz} \right)_{z = z_0} \cdots \cdots (3.19) \]

is a dimensionless number which may be called the "evaporation number" (Takhar, 1972). By integrating equation (3.18) over the entire surface, the evaporation rate becomes

\[ E = \rho \frac{(q_o - q_s) u_s}{A} \int_{x = 1}^{x = x_d} b N_e \, dx \text{ mass/unit/area/time}, \quad (3.20) \]

where \( b \) is the width of an elementary strip of the evaporating surface having a downwind length equal to \( dx \), \( A \) is the total area of the evaporating surface and \( x_d \) is the total downwind length (Fig.3.2).

3.3 Flux-Profile Relationships in the Atmospheric Surface Layer

It has been stated earlier that the solution of the momentum and diffusion equations requires the functional relationship between \( U \) and \( Z \) and \( K_m \) and \( K_w \). In this work the desired
relationships are specified by the following wind-velocity and temperature profile equations (Businger et al., 1971) for a thermally-stratified turbulent flow in the surface layer of the atmosphere:

For an unstable atmosphere \((\xi < 0)\)

\[
\Phi_m = (1 - 15\xi)^{1/4} \quad \ldots \quad \ldots \quad \ldots \quad (3.21)
\]

\[
\Phi_h = 0.74 \ (1 - 9\xi^{1/4}) \quad \ldots \quad \ldots \quad \ldots \quad (3.22)
\]

and for a stable atmosphere \((\xi > 0)\)

\[
\Phi_m = (1 + 4.7\xi) \quad \ldots \quad \ldots \quad \ldots \quad (3.23)
\]

\[
\Phi_h = (0.74 + 4.7\xi) \quad \ldots \quad \ldots \quad \ldots \quad (3.24)
\]

where \(\Phi_m\) and \(\Phi_h\) are the non-dimensional wind velocity and temperature gradients defined by

\[
\Phi_m = \frac{Kz}{u_*} \frac{\partial u}{\partial z} \quad \ldots \quad \ldots \quad \ldots \quad (3.25)
\]

\[
\Phi_h = \frac{z}{\theta_*} \frac{\partial \theta}{\partial z} \quad \ldots \quad \ldots \quad \ldots \quad (3.26)
\]

respectively, and

\[
\xi = \frac{z}{L} \quad \ldots \quad \ldots \quad \ldots \quad (3.27)
\]

where \(L\) is the Monin-Obukhov length given by

\[
L = - \frac{T}{g} \frac{u^3_*}{w^* T} \frac{1}{K} \quad \ldots \quad \ldots \quad (3.28)
\]

and \(\theta_*\) is the sealing temperature given by

\[
\theta_* = \frac{-w^*}{K u_*} \quad (3.28a)
\]

The length \(L\) is a stability parameter which expresses the relative importance between the shear production and the buoyant energy production terms and characterizes the structure of the flow in the atmospheric surface layer.
For a neutral atmosphere, $\mathcal{S} = 0$, $L = \infty$, $\Phi_m = 1$ and $\Phi_h = 0.74$. Hence equation (3.25) gives

$$\frac{\partial u}{\partial z} = \frac{u_*}{Kz} \quad \ldots \quad \ldots \quad \ldots \quad (3.29)$$

which leads to the logarithmic profile for $z \gg z_0$. Equations (3.21) and (3.23) may be integrated in conjunction with equation (3.25) to give the following explicit expressions for the wind profiles (Paulsen, 1970):

$$\frac{u}{u_*} = \frac{1}{K} \left( \ln \frac{z}{z_0} - A \right) \quad \text{when} \quad \mathcal{S} < 0 \quad \ldots \quad (3.30)$$

where,

$$A = 2 \ln \left[ \frac{(1+x)/2}{(1+x^2)/2} \right] + \ln \left[ \frac{(1+x^2)/2}{2} \tan^{-1} x + \frac{\pi}{2} \right] \quad (3.31)$$

$$x = (1 - 15 \mathcal{S})^{1/4} \quad \ldots \quad \ldots \quad (3.32)$$

and

$$\frac{u}{u_*} = \frac{1}{K} \left( \ln \frac{z}{z_0} + 4.7 \mathcal{S} \right) \quad \text{when} \quad \mathcal{S} > 0 \quad \ldots \quad (3.33)$$

when $\mathcal{S} = 0$, integrating equation (3.29) from $z_0$ to $z$ we obtain

$$\frac{u}{u_*} = \frac{1}{K} \ln \left( \frac{z}{z_0} \right) \quad \ldots \quad \ldots \quad (3.34)$$

Non-dimensionalizing equation (3.30), (3.33) and (3.34) using equation (3.5), we can write

$$U = \frac{1}{K} \left( \ln \frac{z}{z_0} - A \right) \quad \text{when} \quad \mathcal{S} < 0 \quad \ldots \quad (3.35)$$

$$U = \frac{1}{K} \left( \ln \frac{z}{z_0} + 4.7 \mathcal{S} \right) \quad \text{when} \quad \mathcal{S} > 0 \quad \ldots \quad (3.36)$$

and

$$U = \frac{1}{K} \ln \frac{z}{z_0} \quad \text{when} \quad \mathcal{S} = 0 \quad \ldots \quad (3.37)$$
Also, non-dimensionalization of equation (3.25) using equation (3.5) gives
\[
\frac{dU}{d\zeta} = \frac{P_{m}}{KZ} \quad \ldots \quad (3.38)
\]

Now, using equations (3.21) and (3.23) into equation (3.38), one obtains
\[
\frac{dU}{d\zeta} = \frac{1}{KZ} \left( 1 - 15\zeta \right)^{-\frac{1}{2}} \quad \ldots \quad (3.39)
\]
\[
\frac{d^2U}{d\zeta^2} = \frac{1}{K^2Z^2} \left( \frac{1}{1 - 15\zeta} \right)^{-\frac{1}{2}} \left[ \frac{-15\zeta}{4(1 - 15\zeta)^2} - 1 \right] \quad \ldots \quad (3.40)
\]
when \( \zeta < 0 \), and
\[
\frac{dU}{d\zeta} = \frac{1}{KZ} \left( 1 + 4.7\zeta \right) \quad \ldots \quad (3.41)
\]
\[
\frac{d^2U}{d\zeta^2} = -\frac{1}{K^2Z^2} \quad \text{when} \quad \zeta > 0 \quad \ldots \quad (3.42)
\]

Under neutral conditions, \( \zeta = 0 \) and \( P_{m} = 1 \). Hence, equation (3.38) gives
\[
\frac{dU}{d\zeta} = \frac{1}{KZ} \quad \ldots \quad (3.43)
\]
and
\[
\frac{d^2U}{d\zeta^2} = -\frac{1}{K^2Z^2} \quad \ldots \quad (3.44)
\]

For a neutral atmosphere, the value of the free-stream velocity, \( U_{g} \), can be written, from equation (3.37), as
\[
U_{g} = \frac{1}{K} \ln \left( -\frac{Z_{a}}{Z_{0}} \right) \quad \ldots \quad (3.45)
\]
Using equations (3.37) and (3.45) into equation (3.16), it can be shown that, for a neutral atmosphere

\[ R_s = \frac{1}{K^2} \left[ Z_g \left( \ln \frac{Z_g}{Z_0} \right)^2 - 4Z_g \ln \frac{Z_g}{Z_0} - 2Z_0 \ln \frac{Z_g}{Z_0} + 6(Z_g - Z_0) \right]. \quad \ldots \quad (3.46) \]

For the stable and unstable atmospheric conditions, equation (3.16) could not be integrated directly and, therefore, the value of \( R_s \) was evaluated numerically.

To solve the diffusion equation (3.7), it is convenient to write it in the form

\[ \frac{U_C}{S_e} \frac{\partial \theta}{\partial \phi} = \frac{\partial \theta}{\partial z} (M \frac{\partial \theta}{\partial z}) = \frac{\partial M}{\partial z} \frac{\partial \theta}{\partial z} + M \frac{\partial^2 \theta}{\partial z^2} \quad \ldots \quad (3.47) \]

where, \( M = \left( \frac{1}{S_c} + \frac{K_w}{\partial^2} \right) \quad \ldots \quad (3.48) \)

To evaluate the above equation, it is also necessary to specify a relationship between \( K_m \) and \( K_w \). Dyer (1967), based on observations from five series of experiments over a freely evaporating surface, concluded that the mechanics of turbulent transport of heat and water vapour are the same and that \( K_h = K_w \). Similarly, Webb (1970) concluded that \( K_w/K_h \) evidently remains constant, equal to unity, over the whole log-linear range and somewhat beyond. Thus, for the present work, it will be assumed that

\[ K_h = K_w. \quad \ldots \quad (3.49) \]
An immediate implication of the relation (3.49) is that

$$\frac{K_m}{K_h} = \frac{K_m}{K_w} = (S_c)_t \quad \ldots \quad \ldots \quad (3.50)$$

where \((S_c)_t\) is the turbulent Schmidt number. Using equations (3.25), (3.26) and (3.50), it can be shown that

$$\frac{(S_c)_t}{\Phi_h} = \frac{\Phi_h}{\Phi_m} \quad \ldots \quad \ldots \quad (3.51)$$

Now, using equations (3.21) to (3.24) into the equation (3.51), we have for an unstable atmosphere \((S < 0)\)

$$\frac{(S_c)_t}{\Phi_h} = \frac{0.74}{(1-9S)} \quad \ldots \quad \ldots \quad (3.52)$$

for a stable atmosphere \((S > 0)\)

$$\frac{(S_c)_t}{\Phi_h} = \frac{0.74 + 4.7S}{(1 + 4.7S)} \quad \ldots \quad \ldots \quad (3.53)$$

and for a neutral atmosphere, when \(S = 0\), \((S_c)_t = 0.74\).

Using equation (3.50), equation (3.48) can be written as

$$M = \left[ \frac{1}{S_c} + \frac{1}{(S_c)_t} \frac{K_m}{\phi} \right] \quad \ldots \quad (3.54)$$

Differentiating the above equation with respect to \(Z\) we obtain

$$\frac{\partial M}{\partial Z} = \frac{K_m}{\phi} \frac{\partial}{\partial Z} \left[ \frac{1}{(S_c)_t} \right] + \frac{1}{(S_c)_t} \frac{\partial}{\partial Z} \left[ \frac{K_m}{\phi} \right] \quad \ldots \quad (3.55)$$
Equations (3.52) and (3.53) in conjunction with equation (3.55) gives for unstable atmosphere \((S < 0)\)

\[
\frac{\partial M}{\partial z} = - \frac{1}{(S_c)_t} \int_{z_0}^{z} \frac{u^2}{z_s} \frac{dz}{du} + \frac{1}{(S_c)_t} \left( 1 - \frac{z_o}{z_s} \right) \int_{z_0}^{z} \frac{u^2}{dz} \frac{du}{dz}
\]

\[
x \frac{dU}{dz} + \frac{1}{4 \times 0.74 \times L} \frac{K_m}{U} \frac{(1 - 9S)^2}{(1 - 15S)^4} \left[ \frac{15}{(1 - 15S)} - \frac{2 \times 9}{(1 - 9S)} \right]
\]

for stable atmosphere \((S > 0)\)

\[
\frac{\partial M}{\partial z} = - \frac{1}{(S_c)_t} \int_{z_0}^{z} \frac{u^2}{z_s} \frac{dz}{du} + \frac{1}{(S_c)_t} \left( 1 - \frac{z_o}{z_s} \right) \int_{z_0}^{z} \frac{u^2}{dz} \frac{du}{dz}
\]

\[
- \frac{K_m}{U} \frac{4.7}{L} (1 - 0.74) \frac{1}{(0.74 + 4.7S)^2}
\]

and for neutral atmosphere \((S = 0)\), using equation (3.56) or (3.57)
\[
\frac{\partial M}{\partial Z} = - \left( \frac{1}{S_c} \right) \frac{u^2}{z_s} \frac{dz}{du} + \frac{1}{S_c} \left( 1 - \frac{z_s}{z_0} \right) X 
\int_{z_0}^{Z} \frac{u^2}{dz} dz
\]

Using the values of \( \frac{\partial M}{\partial Z} \) from equations (3.56), (3.57) and (3.58), the diffusion equation (3.47) can be solved numerically using finite difference techniques.

3.4 Application of the Evaporation Model for Estimating Evaporation using climatological parameters

In order to apply the proposed method for estimating evaporation, it is first necessary to evaluate \( R_1 \), the Richardson number which serves as a useful stability parameter to indicate the atmospheric conditions. Utilizing the temperature and velocity measurements at two discrete levels \( z_1 \) and \( z_2 \), the gradient form of the Richardson number, \( R_1 \), is calculated using the relation

\[
R_1 = \frac{f}{T_a} \frac{dT/dz + \Gamma}{(du/\bar{d}z)^2} \quad \ldots \quad \ldots \quad (3.59)
\]

\[
= \frac{f}{T_a} \frac{\left[ (T_2 - T_1) + \Gamma (z_2 - z_1) \right]}{(u_2 - u_1)^2} (z_2 - z_1) \quad (3.60)
\]
where, \( T_a \) is the absolute ambient air temperature. The sign of \( R_1 \) depends upon the sign of

\[
T_2 - T_1 + \int (z_2 - z_1)
\]

where a negative value of the term denotes an unstable atmosphere, a positive value denotes a stable atmosphere and a zero value gives a neutral atmosphere. The dry adiabatic lapse rate, \( \Gamma \), was taken as 0.01 °C per meter.

The friction velocity, \( u_* \), can be estimated by integrating equations (3.30), (3.33) and (3.34) between the levels \( z = z_1 \) and \( z = z_2 \) to give

\[
u_* = \frac{K(u_2 - u_1)}{\ln \frac{z_2}{z_1} + A_1 - A_2}
\]

when \( \zeta < 0 \) \( \ldots \) (3.61)

\[
u_* = \frac{K(u_2 - u_1)}{\ln \frac{z_2}{z_1} + 4.7 \frac{(z_2 - z_1)}{L}}
\]

when \( \zeta > 0 \) \( \ldots \) (3.62)

and,

\[
u_* = \frac{K(u_2 - u_1)}{\ln \frac{z_2}{z_1}}
\]

when \( \zeta = 0 \) \( \ldots \) (3.63)

where,

\[
A_1 = 2 \ln \left[ \frac{1+\frac{x_1}{2}}{2} \right] + \ln \left[ \frac{(1+\frac{x_1^2}{2})}{2} \right] - 2\tan^{-1}x_1 + \frac{\pi}{2} \quad (3.64)
\]

\[
A_2 = 2 \ln \left[ \frac{1+\frac{x_2}{2}}{2} \right] + \ln \left[ \frac{(1+\frac{x_2^2}{2})}{2} \right] - 2\tan^{-1}x_2 + \frac{\pi}{2} \quad (3.65)
\]

\[
x_1 = (1-15 \frac{z_1}{L})^{0.44}
\]

\( \ldots \) \( \ldots \) (3.66)
and

\[ x_2 = (1 - 15 \frac{z_2}{L})^{rac{1}{4}} \quad \ldots \quad \ldots \quad (3.67) \]

where \( u_1 \) and \( u_2 \) are the known wind velocities at the levels \( z_1 \) and \( z_2 \) respectively.

Integration of equations (3.30), (3.33) and (3.34) between the levels \( z_0 \) and \( z_2 \) gives the value of \( z_0 \), the roughness parameter as

\[ z_0 = \frac{z_2}{\exp \left( \frac{K u_2}{u_*} + \frac{4.7 z_2}{L} \right)} \quad \text{when} \quad \xi < 0, \quad \ldots \quad (3.68) \]

\[ z_0 = \frac{z_2}{\exp \left( \frac{K u_2}{u_*} - \frac{4.7 z_2}{L} \right)} \quad \text{when} \quad \xi > 0, \quad \ldots \quad (3.69) \]

and when \( \xi = 0, \)

\[ z_0 = \frac{z_2}{\exp \left( \frac{K u_2}{u_*} \right)} \quad \ldots \quad \ldots \quad \ldots \quad (3.70) \]

Thus, to determine the values of \( u_* \) and \( z_0 \) using the above equations, it is necessary to evaluate the value of the Monin-Obukhov Length, \( L \). From equations (3.28), (3.51) and (3.59) it can be shown that

\[ R_1 = \frac{\xi \Phi_h}{\Phi_m^2} \quad \ldots \quad \ldots \quad (3.71) \]
Combining equations (3.21) to (3.24) and (3.71), it is easy to obtain

\[ R_l = \frac{0.74 \xi (1-15 \xi)^{\frac{1}{2}}}{(1-9 \xi)^{\frac{1}{2}}} \quad \text{when } \xi < 0, \quad \ldots \quad (3.72) \]

and,

\[ R_l = \frac{\xi (0.74 + 4.7 \xi)}{(1 + 4.7 \xi)^{2}} \quad \text{when } \xi > 0. \quad \ldots \quad (3.73) \]

The above two equations may be used to determine the value of \( L \) by a trial-and-error approach.

The next set of parameters to be considered here are the mass density, kinematic viscosity and specific humidity of air. The values of these parameters are necessary for evaluating the evaporation equation (3.20). The air density, \( \rho \), was evaluated using the equation of state and taking the molecular weight of dry air as 28.9. At an absolute temperature of 273\(^\circ\)K and standard pressure of 1013 millibar, 1 gm. molecular weight of air occupies 22.4 \( \times 10^3 \) c.c. Thus, the air density for any temperature \( T_a \) and atmospheric pressure \( P \) is given by

\[ \rho = \frac{28.9 \times 273 \times P}{1013 \times 22.4 \times 10^3 \times T_a} = 0.3484 \times 10^{-3} \times \frac{P}{T_a} \text{ gm/cm}^3. \quad (3.74) \]

The kinematic viscosity of air, \( \nu \), could be estimated on the basis of Sutherland's equation of dynamic viscosity (Smithsonian Meteorological Tables, 1966), given by
\[
\frac{\mu}{\mu_0} = \frac{T_0 + C}{T_a + C} \left(\frac{T_a}{T_0}\right)^{3/2}.
\]

(3.75)

In the above equation \(C\) is the Sutherland's constant, assumed to have the value of 120°C, and \(\mu_0\) is the known dynamic viscosity at absolute temperature \(T_0\). At \(T_0 = 296.16°K\),
\[
\mu_0 = 1.8325 \times 10^{-4} \text{ gm}^{-1} \text{ cm}^{-1} \text{ sec}^{-1}
\]
and hence
\[
\mu = 1.8325 \times 10^{-4} \frac{416.16}{T_a + 120} \left(\frac{T_a}{296.16}\right)^{3/2}.
\]
(3.76)

Since, \(\nu = \frac{\mu}{\rho}\), combination of equations (3.74) and (3.76) gives
\[
\nu = \frac{1.8325}{3.484} \times \frac{T_a}{\rho} \times \frac{416.16}{(T_a + 120)} \left(\frac{T_a}{296.16}\right)^{3/2}.
\]
(3.77)

The specific humidity, \(\nu\), is defined as the ratio of the mass of water vapour to the mass of moist air. However, as the moisture content of the air is small under all atmospheric conditions, the specific humidity has almost the same value as that of humidity mixing ratio, \(m\), defined as the mass of water vapour per unit mass of dry air. By applying the ideal gas law and taking the molecular weight of water to be 18, the vapour pressure \(e\) of water vapour at pressure \(P\) is given by (Sutton, 1953)
\[
e = \frac{m}{0.622 + m} P, \quad \cdots \quad \cdots \quad (3.78)
\]
from which we obtain
\[
m = \frac{0.622e}{P - e}, \quad \cdots \quad \cdots \quad (3.79)
\]
Since $m \approx q$ and $P >> e$ the above equation gives

$$q = \frac{0.622e}{p} \quad \ldots \quad \ldots \quad (3.80)$$

Thus, the evaluation of specific humidity required the determination of vapour pressure, which is given by

$$e = 4.5855 + T_w \left[0.32808 + T_w \left(0.01172 + T_w \left(1.2793 \times 10^{-4} + 4.1848 \times 10^{-6} T_w\right)\right)\right] - 0.6(T_d - T_w), \quad \ldots \quad (3.81)$$

where $T_w$ is positive, and,

$$e = 4.5778 + T_w \left[0.37305 + T_w \left(0.012931 + 1.9309 \times 10^{-4} T_w\right)\right] - 0.54(T_d - T_w), \quad \ldots \quad (3.82)$$

where $T_w$ is negative. Here $e$ is the vapour pressure in mm of Hg. and $T_d$ and $T_w$ are the dry and wet bulb temperatures in °C respectively. Saturated vapour pressure conditions were assumed to prevail at the water surface and therefore the surface specific humidity was evaluated by substituting $T_d = T_w = T_0$ into equation (3.81) for $T_0 > 0$ and into equation (3.82) for $T_0 < 0$, where $T_0$ is the surface temperature. The evaluation of the surface specific humidity, $q_o$, thus requires the measurement of water surface temperature.

The minimum climatological measurements needed to satisfy the requirements of the proposed approach are summarised in Table 3.1.
### Table 3.1

**Climatological Data Requirements for the Proposed Evaporation Model**

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Function</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. Air (dry bulb) temperature at level $z_1$</td>
<td>These are used in conjunction with the wind velocities at levels $z_1$ &amp; $z_2$ to evaluate the gradient form of the Richardson number, $R_1$, using equation (3.60).</td>
</tr>
<tr>
<td>2. Air temperature at level $z_2$</td>
<td>These are used to evaluate the friction velocity, $u^<em>_z$, using equations (3.61), (3.62) &amp; (3.63). Wind velocity at the level $z_2$ is used to evaluate the surface roughness, $z_0^</em>$, using equations (3.68), (3.89) &amp; (3.70).</td>
</tr>
<tr>
<td>3. Wind velocity at level $z_1$</td>
<td>This is used along with the dry bulb temperature at level $z_2$ to evaluate the vapour pressure at level $z_2$ using equation (3.81) or (3.82).</td>
</tr>
<tr>
<td>4. Wind velocity at level $z_2$</td>
<td>This is used to estimate the surface vapour pressure using equation (3.81) or (3.82).</td>
</tr>
<tr>
<td>5. Wet bulb temperature at level $z_2$</td>
<td>This is used in conjunction with the dry bulb, wet bulb and surface temperatures to evaluate the specific humidity, air density and kinematic viscosity using equations (3.80), (3.74) &amp; (3.77) respectively.</td>
</tr>
<tr>
<td>6. Water surface temperature</td>
<td></td>
</tr>
<tr>
<td>7. Atmospheric pressure</td>
<td></td>
</tr>
</tbody>
</table>
In addition to the climatological data, it is also necessary to consider the numerical values of the von Karman constant and the laminar Schmidt number. The value of the von Karman constant, $K$, was taken as 0.36 in agreement with the observations of Businger et al. (1971). For the normal range of temperature ($-20^\circ C$ to $+40^\circ C$) and pressure (1000 millibar to 1020 millibar), to be expected in the atmosphere, the numerical value of the laminar Schmidt number varies from 0.594 to 0.598 and an average value of 0.596 was considered to be the most appropriate value for this number to be used in this work.

In order to determine the free-stream value of wind velocity and to satisfy the boundary conditions at $z = z_s$, it is necessary that the leading edge of the turbulent momentum boundary layer with respect to the vapour-step position (Fig. 3.1) be located properly so that a fully-developed state of flow is reached before an interaction with the turbulent diffusion boundary layer takes place. The effect of the pre-step length Reynolds number, $(Re) \left( = \frac{u_1 L}{\nu} \right)$, on the rate of evaporation was investigated for various values of $(Re)$. It was observed that for a change of $(Re)$ from $10^6$ to $10^7$ and from $10^7$ to $10^8$, the changes in the rate of evaporation were 11% and 6% respectively. As a result, a value of $(Re)$ of $10^8$ was taken as being a reasonable indication for the attainment of a fully-developed state. This value of the pre-step length Reynolds number also satisfies the condition of atmospheric flow in that events are governed
entirely by inertia and that viscous forces are negligible.

With the value of the pre-step length Reynolds number thus selected, the pre-step length, \( l \), becomes

\[
\frac{(R_e)^{1/3}}{u_s} = \frac{10^8 \nu}{u_s} \quad \ldots \quad \ldots \quad (3.83)
\]

The free-stream specific humidity, \( q_s \), was determined by solving the diffusion equation (3.47) for the dimensionless humidity \( Q \) at a given level and given downwind position. Using relations (3.5) we have

\[
Q_z = \frac{q_z - q_0}{q_s - q_0} \quad \ldots \quad \ldots \quad (3.84)
\]

where \( Q_z \) is the dimensionless humidity at level \( z \). Rearrangement of the above equation gives

\[
q_s = \frac{q_z - q_0}{Q_z} + q_0 \quad \ldots \quad \ldots \quad (3.85)
\]

The variation of the dimensionless humidity, throughout the diffusion boundary layer, is a unique function of height and downwind position. By measuring the specific humidity, \( q_z \), at the reference level \( z \) and known position downwind, equation (3.47) may be used to solve for \( Q_z \) at the same level \( z \) and same position downwind, and the value of \( q_s \) may then be evaluated using equation (3.85).
To evaluate the evaporation equation (3.20), a hypothetical water surface is illustrated in Fig. 3.2 in which A and B represent the leading edges of the turbulent momentum and diffusion boundary layers, separated by the distance L. The water surface is divided into n number of strips of downwind length \( \Delta x \). Then replacing the integral of the evaporating equation (3.20) by finite increments, the evaporation rate becomes

\[
E = \frac{C}{A} \left( q_s - q_0 \right) u_s \sum_{i=1}^{n} b_i \left( R_e \right)_i \Delta x_i
\]  

(3.86)

where \( b_i \) refers to the width of the surface at the midpoint of the \( i \)th strip and \( \left( R_e \right)_i \) is assumed constant for this strip. The Reynolds number, \( \left( R_e \right)_i \), representing the dimensionless downwind position from the leading edge of the momentum boundary layer is given by

\[
\left( R_e \right)_i = \left[ 1 + \frac{i-1}{\Delta x} \Delta x + \frac{\Delta x_i}{2} \right] \frac{u_s}{2}. 
\] 

(3.87)

To estimate evaporation from a surface using equation (3.86), it is necessary to approximate the geometry of the surface by a simplified shape so that the effective width, \( b \), for any increment in \( \Delta x \) can be determined. For elongated surfaces, the rectangular approximation is preferable. For this approximation, the width is constant but the dimensions
Fig. 3.2 A Hypothetical Open Water Surface Showing the Elementary Strips
of the rectangle now become a function of the wind direction
thus necessitating the measurement of wind direction. The
circular approximation is advantageous over the rectangular
approximation in that the width then becomes independent of
the wind direction and the measurement of the wind direction
then becomes unnecessary. For circular approximation the
width is given by

\[ b_1 = 2 \left[ (x + \frac{\Delta x_i}{2}) \left( 2r - x - \frac{\Delta x_i}{2} \right) \right]^{\frac{1}{2}} \]  \quad \cdots \quad (3.88)  

\[ = 2 \left[ r^2 - \left( r - x - \frac{\Delta x_i}{2} \right)^2 \right]^{\frac{1}{2}} \] \quad \cdots \quad (3.89)  

where, \( r = \) radius of the surface and \( x = \sum_{i=1}^{1-1} \Delta x_i \).

3.5 Conclusions and Implications of the Theory

In this section the theoretical results and implications
of the numerical process are examined. The stability parameter,
used to describe the degree of thermal stratification in the
lower atmosphere, is taken to be the non-dimensionalised Monin-
Obukhov length, given by,

\[ L^* = \frac{L u^*_x}{\gamma} \]  

As \( |L^*| \) tends to infinity, the neutral regime, where buoyancy
effects are negligible, is approached. When \( L^* \) is negative,
the atmosphere is unstable and when \( L^* \) is positive, the
atmosphere is stable. Using this stability parameter, the
non-dimensionalised velocity profile laws given by equations (3.35) and (3.36) have been plotted in Fig. 3.3 for an arbitrary value of $Z_o$ of 2.0. As the absolute value of $L^*$ increased, the curves were found to tend to the neutral wind profile given by equation (3.37). Varying the values of $Z_o$ was found merely to add a uniform translation to the curves, as originally suggested by Ellison (1957). The Spalding law of the wall, used by Hatton (1964) and Takhar (1972) is also plotted in the same figure.

The variation of theoretical momentum boundary layer height, $Z_s$, with Reynolds number, is illustrated in Fig. 3.4 for varying degrees of atmospheric stability. For the stable atmosphere the theoretical boundary layer height was found to be lower than that for the neutral case and as the stability increased, the boundary layer height decreased. Similarly, for increasing atmospheric instability the theoretical momentum boundary layer height was found to increase. As the absolute value of $L^*$ tended to infinity, the curves representing the stable and unstable atmospheres both tended to the curve representing the neutral atmosphere. This offered some indication that the numerical and analytical methods used to relate $R_e$ to $Z_s$, for each class of atmospheric stability were correct.

The decrease in theoretical momentum boundary layer height with increasing atmospheric stability is illustrated more clearly in Fig. 3.5 where $R_e$ was kept constant at an arbitrary value, of the order of $10^9$, and $Z_s$ was plotted
Fig. 3.3 U-Z Relation for Varying Degrees of Atmospheric Stability
Fig. 3.4 $Z_s - R_e$ Relation for Varying Degrees of Atmospheric Stability
Fig. 3.5 Effect of Stability on Boundary Layer Development
against $1/L^*$. The decrease in boundary layer height suggested effects a weakening of the ground/on flow with increasing atmospheric stability. In a similar fashion the increase in the momentum boundary layer height with increasing atmospheric instability is illustrated in Fig. 3.6. This increase suggested a strengthening of ground effects due to the increased turbulence generated by buoyancy effects.

The next set of figures illustrates the variation of the eddy diffusivity of momentum throughout the turbulent momentum boundary layer. Fig. 3.7 illustrates the $K_m$-variation, when the neutral profile law of equation (3.37) was used for varying downwind positions from the leading edge. These curves were found to be almost identical to those obtained by Hetton (1964) when he used the same analysis with the Deissler (1955) and Spalding (1961) laws. It may be seen that the eddy diffusivity variation throughout the boundary layer is only weakly dependent on Reynolds number, with a change in orders of magnitude from $10^6$ to $10^{10}$ having an overall effect on $K_m$ of less than 10%.

The $K_m$-variation for the neutral state was redrawn as a broken line/Fig. 3.8 and compared with the variations obtained for differing degrees of atmospheric stability. Increased instability was observed to bring about greater values of eddy diffusivity compared with the neutral case, whilst the stable atmosphere brought about considerably damped turbulence, thereby reducing the values of $K_m$. These variations were very marked compared with those
Fig. 3-7 Variation of \( K_m \) Throughout the Turbulent Diffusion Boundary Layer (Neutral Profile)
Fig. 3.8  Variation of $K_m$ Throughout the Turbulent Diffusion Boundary Layer for Varying Degrees of Atmospheric Stability
obtained by changing the Reynolds number, and far outweighed them. Again it was noted that for \( |L^*| \) tending to infinity, the results for both atmospheric states tended to a common curve described by the neutral equation.

The next three figures are used to illustrate the results of the finite difference solution of the diffusion equation (3.47). Fig. 3.9 shows the variation of the dimensionless humidity, \( Q \), throughout the turbulent diffusion boundary layer, for varying downwind positions, \( x \), over the evaporating surface. The downwind lengths are approximate and were used in place of Reynolds number by selecting a representative wind velocity and kinematic viscosity. The "jump" in humidity at the upwind edge of the water surface was observed to be still evident at 1 meter downwind but within a hundred meters, the humidity profile had taken on a logarithmic form that only altered marginally for further distances downwind. It is to be noted that \( \frac{dQ}{dZ} \neq 0 \) for lower values of \( x \) and the value of \( \frac{Z_0}{Z} \) decreases with increasing \( x \) leading to the decreasing values of \( \frac{dQ}{dZ} \) at \( Z = Z_0 \).

Fig. 3.10 illustrates the dimensionless evaporation number-Reynolds number variation of equation (3.19). The results are shown for three positions of "jump step" with respect to the leading edge of the momentum boundary layer. The curves show two important characteristics. Firstly, as the "jump step" is approached the evaporation number tends to infinity. This may be explained by the humidity boundary condition that describes an instantaneous jump in humidity. At this position

\[
\left( \frac{dQ}{dZ} \right)_{Z=Z_0} \rightarrow \infty
\]
Fig. 3.9 Humidity Profiles for Varying Downwind Lengths

- $u_1 = 303.7 \text{ cm/sec}$
- $u_2 = 355.2 \text{ cm/sec}$
- $T_1 = 20.6^\circ \text{C}$
- $T_2 = 20.5^\circ \text{C}$
- $T_w_1 = 18.1^\circ \text{C}$
- $T_w_2 = 17.8^\circ \text{C}$
- $T_0 = 22.4^\circ \text{C}$
Fig. 3.10 Variation of Evaporation Number With Reynolds Number for Neutral Stability
and therefore the evaporation number, $N_e$, also tends to infinity. Secondly, there is a rapid transition to a common curve, regardless of the "jump step" position. This curve was observed to denote a decreasing value of evaporation number over the evaporating surface, thus implying that the evaporation rate, per unit area of surface, is dependent on the downwind position from the "jump step".

Fig. 3.11 illustrates the evaporation number variation for conditions of thermal stratification. For this situation, equation (3.19) was used to calculate $N_e$ at the level $Z_0$ for a "jump step" Reynolds number of the order of $10^9$. The value of $N_e$ was observed to depend not only on downwind position but also on the stability parameter. For increasing instability, the evaporation number, for any value of Reynolds number, was observed to be greater than for the neutral and stable cases. Furthermore, as the instability increased, the evaporation number becomes independent of downwind position and, instead, took on an approximate constant value. Thus the numerical solution suggests that, for very unstable conditions, evaporation will cease to become a function of downwind length and, instead, be constant for any surface area exposed to otherwise identical conditions. For increasing stability, however, the reverse effect was observed, with less evaporation and increasing dependence on downwind position. This may be explained by the
Fig. 3.11 \( N_e - R_e \) Relation for Varying Degrees of Atmospheric Stability
Fig. 3.12 Variation of Evaporation With Atmospheric Stability Condition

\[ u_1 = 540.5 \text{ cm/sec} \]
\[ u_2 = 658.9 \text{ cm/sec} \]
\[ T_1 = 25.0^\circ \text{C} \]
\[ \omega_1 \]
\[ T_{w1} = 19.6^\circ \text{C} \]
\[ T_0 = 26.1^\circ \text{C} \]
fact that in the very unstable case, turbulence is so great that the vapour is transported vertically with no lateral movement at all. On the other hand, in the stable regime, the restricted turbulence causes the vapour to drift downwind just above the surface, thereby reducing the vapour concentration gradient.

Finally, Fig. 3.12 shows the above fact more clearly in which the variation of evaporation with atmospheric stability condition for a particular set of climatological measurements is considered. The gradient form of Richardson number, $R_i$, is used as a measure of the atmospheric stability. It is readily seen that for increasing atmospheric instability, evaporation increases very rapidly, whereas, for increasing stability, evaporation decreases and approaches a constant value under extreme stable conditions.
RESULTS USING THE LAKE HEFNER CLIMATOLOGICAL MEASUREMENTS

4.1 The Lake Hefner Experiment

The climatological measurements from Lake Hefner (Lake Hefner Base Data Report, 1954) were used to verify the validity of the proposed method and determine its limitations for estimating evaporation from bounded open water surfaces exposed to a thermally-stratified turbulent atmosphere. The Lake Hefner Project (Harbeck, 1954) was designed for the purpose of developing improved methods of estimating evaporation from large open water surfaces in which various theoretical and empirical approaches were compared with accurate assessments of evaporation by the water budget method. The climate at Lake Hefner was classified as sub-humid (Marciano and Harbeck, 1954), in which quite large evaporation rates can be expected. The lake was fairly regular, and approximately circular, in shape and its capacity and surface area at full pool were 75,355 acre-feet ($\approx 9.295 \times 10^7$ m$^3$) and 2587 acres ($\approx 1.047 \times 10^7$ m$^2$) respectively.

Water budget estimates of evaporation were obtained from inflow-outflow data, estimated to within an order of accuracy of 5%, and the measurement of precipitation from 22 rain gauges distributed uniformly around the lake. The water level was
monitored at a number of stages sited around the reservoir. An accuracy classification for the water budget estimates was determined from the difference in level recorded at each stage, the differences in precipitation measurement of the rain gauges and the amount of inflow and outflow. The probable errors were expressed by the classification: "A" when the estimated accuracy was within 5 acre-feet or approximately \pm 0.7 \text{ mm per day}, "B" when the error was up to 10 acre-feet or 1.4 \text{ mm per day}, "C" for an error of the order of 20 acre-feet per day, and, "D" when the estimated error was more than 20 acre-feet per day.

Climatological measurements were taken at four sites, one of which was located near the center of the lake. At each site, wind velocity, air temperature and wet bulb temperature were recorded at 2, 4, 8 and 16 meter levels above the surface, along with surface temperature and wind direction. The thermometers and anemometers were rated at \pm 0.1^\circ C and \pm 0.1 \text{ knot} respectively. Daily and 3-hourly mean values of these parameters and the daily average surface area for the period July, 1950 to August, 1951 were published in the Lake Hefner Base Data Report (1954).

There were 147 days of records in which "class A", "class B" and "class C" water budget estimates were accompanied by climatological data from the central barge station which satisfied the requirements of the proposed evaporation model.
formulation. In estimating the theoretical rate of evaporation, the lake was approximated by a circular shape and the radius was calculated from the given surface area. The barge station was assumed to be centrally located so that its downwind position from the vapour-step position was assumed to be constant, equal to the radius of the surface, with respect to the wind direction. The downwind length of the elementary strips was taken as 30 meters. Thus the total number of strips was about 100. The width of each strip was determined using equation (3.89). The free-stream humidity was evaluated using the barge station 8-meter level specific humidity into equation (3.85). The levels \( z_1 \) and \( z_2 \) were taken as 2 and 8 meters respectively. The friction velocity, \( u_* \), and the roughness parameter, \( z_0 \), were calculated using equations (3.61) to (3.70).

4.2 Results and Discussions

The comparisons between the water budget evaporation estimates and the present theoretical evaporation estimates using the climatological measurements at Lake-Hefner, are illustrated in Fig. 4.1. The results were obtained by using 147 "class A", "class B" and "class C" days. The total theoretical evaporation for these days was 817.82 mm compared to a water budget estimate of 712.68 mm indicating an overestimation of only 14.75% of the actual evaporation by the proposed evaporation model. Thus the overall agreement between the theoretical and water budget estimates of evaporation is satisfactory. Out of the 147 days of observations, there were 16 neutral, 22 unstable and 109 stable days. The theory overestimated the actual evaporation in all the three atmospheric
Fig. 4.1: Daily Water Budget and Theoretical Estimates of Evaporation from Lake Hefner (147 Class A, B and C Days)
regions, the percentages of overestimation being 8.33, 16.46 and 15.29 under the neutral, unstable and stable conditions respectively. Thus the overall agreement between the theory and observation also appears to be satisfactory in all regions of atmospheric stability conditions.

A linear correlation coefficient of 0.8637 is found to relate the theoretical evaporation, indicating a very good agreement between the two estimates. The equation of the linear regression line passing through the points plotted in Fig. 4.1 is found by a least-square fit to be

\[ y = 1.188 + 0.9025x. \]  

The above equation indicates that the proposed approach overestimates the actual evaporation for lower rates of evaporation and underestimates for higher rates of evaporation. The 98% confidence limits of the gradient are 1.0044 and 0.8006. The gradient of this linear regression equation suggests that there is a slight departure of the theoretical evaporation estimates from the water budget estimates but that the gradient equal to unity, representing complete agreement between the two distributions, is within the 98% confidence limits of the regression line.

From Fig. 4.1 it is readily observed that although the overall agreement between the evaporation model and water budget evaporation estimates is close, the results are somewhat scattered and there
is often considerable difference between the daily evaporation values given by these two methods. The scatter in the results was thought to be due to the accuracy with which the stability of the atmosphere could be ascertained which in turn was dependent upon the accuracy of the temperature and wind velocity measurements. The accuracies of the temperature and wind velocity measurements at Lake Hefner were ± 0.1°C and ± 0.1 knot respectively. The stability of the atmosphere at Lake Hefner was determined by evaluating the gradient form of the Richardson number, $R_i$, as a mean daily value using equation (3.60) and utilizing the temperature and wind velocity measurements at 2 and 8 meter levels. Since the air temperatures recorded at these levels measured to within ± 0.1°C, the accuracy of the differences in the measured temperatures at the two levels was within ± 0.2°C. That this order of accuracy for the difference of temperature measurements is insufficient to meet the requirements of the proposed approach is illustrated in Table 4.1, where the effect of ± 0.1°C error in the difference of 2 and 8 meter air temperatures on the theoretical rate of evaporation has been shown. A total of 12 days climatological data from Lake Hefner, covering all regions of atmospheric stability conditions, have been considered. The effect of 0.1°C error in the water surface temperature and that of ± 0.1 knot (= 5.148 cm/sec) in the difference of 2 and 8 meter wind velocities have also been illustrated in the same table.
Table - 4.1

Effects of the Probable Errors of Temperature and Wind Velocity Measurements on the Theoretical Estimates of Evaporation.

<table>
<thead>
<tr>
<th>Day</th>
<th>Atmospheric Stability</th>
<th>Est.*</th>
<th>(\Delta T = T + 0.1)</th>
<th>% Diff.</th>
<th>Est.</th>
<th>% Diff.</th>
<th>(T_{0} = T_{0} + 0.1)</th>
<th>% Diff.</th>
<th>Est.</th>
<th>% Diff.</th>
<th>(\Delta U = \Delta U + 0.1)</th>
<th>% Diff.</th>
<th>Est.</th>
<th>% Diff.</th>
<th>(\Delta U = \Delta U - 0.1)</th>
<th>% Diff.</th>
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<td>4.39</td>
<td>15.53</td>
<td>3.74</td>
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<td></td>
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<td>5.37</td>
<td>-14.22</td>
<td>7.21</td>
<td>15.18</td>
<td>6.33</td>
<td>1.12</td>
<td>7.22</td>
<td>15.34</td>
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<td>-19.81</td>
<td></td>
<td></td>
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<td></td>
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<tr>
<td>9</td>
<td>Neutral</td>
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<td>4.02</td>
<td>-16.60</td>
<td>7.37</td>
<td>52.9</td>
<td>4.86</td>
<td>0.83</td>
<td>5.71</td>
<td>18.47</td>
<td>4.02</td>
<td>-16.60</td>
<td></td>
<td></td>
<td></td>
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<td>0.54</td>
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<td>0.74</td>
<td>-2.63</td>
<td></td>
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<td></td>
</tr>
</tbody>
</table>

* estimates using the observed climatological parameters at Lake Hefner.
From Table 4.1 it is observed that the proposed evaporation approach for estimating evaporation in a density-stratified atmosphere demands for very precise measurements of air temperatures and wind velocities. Indeed, the stability of the atmosphere, determined by evaluating the Richardson number, \( R_i \), using equation (3.60), and so the rate of evaporation, are found to be highly sensitive to the differences of air temperatures and wind velocities measured at the levels \( z_1 \) and \( z_2 \) above the surface. The difference of air temperatures recorded at the levels \( z_1 \) and \( z_2 \) rarely exceeds ± 0.2°C. As a result, it has been observed that an error of the order of ± 0.2°C is often sufficient to change the sign of \( R_i \) and hence the atmospheric conditions from unstable to stable and vice versa. When the levels of measurements \( z_1 \) and \( z_2 \) are taken very close or when the measurements of air temperatures are open to errors of still higher magnitudes, the results will be more scattered than those presented in Table 4.1.

It has been observed that a change of ± 0.2°C in both the temperatures but keeping their difference to remain the same, has a maximum of 3.5% effect upon the rate of evaporation. Similarly, for wind velocities, a change of ± 0.2 knot affects the evaporation estimates by a maximum of 2.5% and an error of the order of 0.1°C in water surface temperature is found to affect the results at the most by 8%. It is therefore the differences, and not the absolute magnitudes of the air temperatures and wind velocities, which have the most pronounced effect upon the rate of evaporation.
Thus, it is now apparent that the order of accuracy of Lake Hefner climatological measurements was insufficient for the accurate determination of atmospheric stability and is thought to be the main cause of the scatter of the results presented in Fig. 4.1. For evaluating the gradient from of the Richardson number accurately, very precise measurements of air temperature and wind velocity are necessary. The recording instruments have to be capable of resolving a temperature difference of 0.01 °C or less so that the adiabatic lapse rate may be detected. The order of accuracy of the measurements of wind velocity should preferably be ± 0.5 cm/sec.
5.1 Conclusions

The aim of the present work was to develop an evaporation model for estimating evaporation from bounded open water surfaces considering varying degrees of atmospheric stability and surface roughness conditions. It was found by applying climatological data to the proposed approach that there was satisfactory agreement between the total estimated and observed evaporation from Lake Hefner. However, when it was necessary to estimate the daily evaporation accurately, very precise measurement of the climatological parameters to determine the atmospheric stability with a sufficient degree of accuracy, was indicated.

The main advantages of the proposed approach of estimating evaporation are that direct empiricism is not required, the theory appears to be satisfactory regardless of the size and condition of the surface and its climatological location, and unlike the energy budget approach, evaporation may be estimated accurately over short time intervals without any need for considering the heat stored by the water body which is rarely measured as a routine or measured with considerable difficulty.
The main limitation of the proposed approach is that sensitive sensors have to be used to measure the climatological parameters accurately to determine the atmospheric stability with a sufficient degree of accuracy. The results of the experimental work have indicated that the rate of evaporation depends upon the atmospheric stability condition. As a result, when it is necessary to consider an accurate estimation of the daily rates of evaporation, every afford should be made to measure the climatological parameters accurately.

The following conclusions could be drawn from the study of the proposed evaporation model:

a) Provided that the stability of the atmosphere can be determined accurately, the use of the proposed evaporation model seems to be a satisfactory method of estimating evaporation from open water surfaces.

b) The wind velocity and temperature profile equations given by Businger et al. (1971) for atmospheric flow considering varying degrees of atmospheric stability condition, were seems to be satisfactory.

c) The rate of evaporation depends upon the atmospheric stability and surface roughness conditions and on the downwind dimension of the surface.
d) The approximations applied to the evaporation model in determining the free-stream height, free-stream velocity and free-stream specific humidity seems to be satisfactory.

e) The assumption that the turbulent diffusivities for heat and water vapour transport are identical, seems to be satisfactory.

5.2 Further Work

The present approach may be usefully employed to estimate evapotranspiration from vegetated surfaces. This may be achieved by altering the datum level at which the evaporation number $N_e$ is determined. The datum level is specified by the plane at which the humidity gradient $(dQ/dZ)$ is determined. For the neutral case, this gradient was estimated at $Z = 0$ (Takhar, 1972) and for the present case it was estimated at $Z = Z_0$. Using the five-point difference formula the humidity gradient may be evaluated at any level within the turbulent diffusion boundary layer. To estimate the evapotranspiration, this datum may be positioned just clear of the vegetation. The vapour concentration at this level will be due partly to the contribution made by the evaporation of water directly from the ground and partly to the transpiration of vegetation cover. The total evapotranspiration will be given by the vertical flux of vapour across the datum plane. The climatological requirements will be identical to those required for the
estimation of evaporation from an open water surface except that the humidity at the wall, \( q_0 \), will not be given by the surface temperature but must be evaluated using a psychrometer located at the datum level.
REFERENCES


APPENDIX - A

Finite Difference Representation of the Diffusion Equation

The diffusion equation (3.49) can be written as

$$\frac{\partial u}{\partial t} = M' \frac{\partial Q}{\partial Z} + M \frac{\partial^2 Q}{\partial Z^2} \quad \cdots \quad (A-1)$$

where, $M' = \partial M / \partial Z$. The finite difference representation of equation (A-1) may be obtained by integrating each term of this equation over an elementary control volume. Fig. A.1 shows an elementary control volume abcd.

Now, integration of equation (A-1) over the control volume abcd gives

$$\iint_{abcd} \frac{\partial Q}{\partial R_e} dR_e dZ = \int_{abcd} M' \frac{\partial Q}{\partial Z} dR_e dZ + M \int_{abcd} \frac{\partial^2 Q}{\partial Z^2} dR_e dZ \quad (A-2)$$

The finite difference scheme of equation (A-2) reverts to the fully-explicit form if the upstream $Q$ values are used. On the other hand, when the average of the upstream and downstream $Q's$ are used, the finite difference analogue would give just-stable Crank-Nicolson scheme. Here the downstream $Q$ values are used to give the maximum stability
Fig. A1 The Rectangular Grid System and an Elementary Control Volume
and a fully-implicit scheme. Then with the subscripts \( u \) and \( d \) representing the upstream and downstream values respectively, term I of equation (A-2) can be written as

\[
\iint_{abcd} U_s \frac{\partial q}{\partial R_e} dR_e dZ = U_i U_s (q_{i,d} - q_{i-1,u})(Z_{i+\frac{1}{2}} - Z_{i-\frac{1}{2}})
\]

\[
= \frac{U_i U_s}{2} (q_{i,d} - q_{i-1,u})(Z_{i+\frac{1}{2}} - Z_{i-\frac{1}{2}}) \ldots \quad (A-3)
\]

Similarly, terms II and III of equation (A-2) may be written respectively as

\[
\iint_{abcd} M' \frac{\partial q}{\partial Z} dR_e dZ = M'_1 \Delta R_e (q_{i+1,d} - q_{i-1,d}) \quad (A-4)
\]

and

\[
\iint_{abcd} M \frac{\partial^2 q}{\partial Z^2} dR_e dZ = M_1 \Delta R_e \left( \frac{\partial q}{\partial Z} \right)_{1+\frac{1}{2}} \ldots \quad (A-5)
\]

Now, \( \left( \frac{\partial q}{\partial Z} \right)_{1+\frac{1}{2}} = \frac{q_{i+1,d} - q_{i,d}}{Z_{i+1} - Z_1} \)

and

\[
\left( \frac{\partial q}{\partial Z} \right)_{1-\frac{1}{2}} = \frac{q_{i,d} - q_{i-1,d}}{Z_1 - Z_{i-1}}.
\]

Hence, equation (A-5) becomes

\[
\iint_{abcd} M \frac{\partial^2 q}{\partial Z^2} dR_e dZ = M_1 \Delta R_e \left[ \frac{q_{i+1,d} - q_{i,d}}{Z_{i+1} - Z_1} - \frac{q_{i,d} - q_{i-1,d}}{Z_1 - Z_{i-1}} \right] \quad (A-6)
\]
Using equations (A-3), (A-4) and (A-6) into equation (A-2), the finite difference approximation for the diffusion equation (A-1) becomes

\[
\frac{U_1 U_8}{2} (q_{1,d} - q_{1,u}) (\Delta Z_{1} + \Delta Z_{1+1}) = \frac{M_1 \Delta r_0}{2} (q_{1+1,d} - q_{1-1,d}) \\
+ M_1 \Delta r_0 \left[ \frac{q_{1+1,d} - q_{1,d}}{\Delta Z_{1+1}} - \frac{q_{1,d} - q_{1-1,d}}{\Delta Z_{1}} \right] \ldots (A-7)
\]

where,

\[
\Delta Z_{1+1} = Z_{1+1} - Z_{1} \ldots \ldots (A-8)
\]

and

\[
\Delta Z_{1} = Z_{1} - Z_{1-1} \ldots \ldots (A-9)
\]

Equation (A-7) can be written in the form

\[
D_1 \, q_{1,d} = A_1 \, q_{1+1,d} + B_1 \, q_{1-1,d} + C_1 \ldots \ldots (A-10)
\]

where,

\[
A_1 = \frac{M_1 \Delta Z_{1}}{2M_1} + \frac{\Delta Z_{1}}{\Delta Z_{1+1}}
\]

\[
B_1 = 1 - \frac{M_1 \Delta Z_{1}}{2M_1}
\]

\[
C_1 = q_{1,u} \left[ \frac{U_1 U_8 \Delta Z_{1}}{2M_1 \Delta r_0} \left( \Delta Z_{1} + \Delta Z_{1+1} \right) \right] \ldots \ldots (A-11)
\]

and

\[
D_1 = \frac{U_1 U_8 \Delta Z_{1}}{2M_1 \Delta r_0} \left( \Delta Z_{1} + \Delta Z_{1+1} \right) + \frac{\Delta Z_{1}}{\Delta Z_{1+1}} + 1.
\]
For \( i = 2 \) control volume, equation (A-10) remains valid but the coefficients \( A_1, B_1, C_1 \) and \( D_1 \) become

\[
A_2 = \frac{M_2^2 Z_2}{2M_2} + \frac{Z_2}{Z_3 - Z_2}, \\
B_2 = 1 - \frac{M_2^2 Z_2}{2M_2}, \\
C_2 = Q_2, u \left[ \frac{U_2 U_s Z_2}{2M_2 \Delta R_e} (Z_2 + Z_3) \right], \\
D_2 = \frac{U_2 U_s Z_2}{2M_2 \Delta R_e} (Z_2 + Z_3) + \frac{Z_2}{Z_3 - Z_2} + 1.
\]

The coefficients \( A_1, B_1, C_1, \) and \( D_1 \) in equation (A-10) are known quantities. For \( N \) steps in \( \Delta z \) there are \( N-1 \) equation of the type (A-10) with \( N-1 \) unknowns. This system of equations form a tri-diagonal matrix that can be solved using the Gauss elimination method as described by Spalding (1977). The equation which have to be solved for the unknown \( Q' \)'s can be usefully written in the following orderly form:
\[ D_2 Q_2 - A_2 Q_3 \quad = \quad C_2 + B_2 Q_1 \]
\[ B_3 Q_2 + D_2 Q_3 - A_3 Q_4 \quad = \quad C_3 \]
\[ B_4 Q_3 + D_4 Q_4 - A_4 Q_5 \quad = \quad C_4 \]
\[ \ldots \quad \ldots \quad \ldots \]
\[ B_{N-2} Q_{N-3} + D_{N-2} Q_{N-2} - A_{N-2} Q_{N-1} \quad = \quad C_{N-2} \]
\[ B_{N-1} Q_{N-2} + D_{N-1} Q_{N-1} \quad = \quad C_{N-1} + A_{N-1} Q_N \quad (A-13) \]

The above equations can be converted in succession into:

\[ Q_2 = S_2 Q_3 + T_2 \quad \ldots \quad (A-14) \]
\[ Q_3 = S_3 Q_4 + T_3 \quad \ldots \quad (A-15) \]
\[ Q_4 = S_4 Q_5 + T_4 \quad \ldots \quad (A-16) \]
\[ \ldots \quad \ldots \quad \ldots \]
\[ Q_{N-1} = S_{N-1} Q_N + T_{N-1} \quad \ldots \quad (A-17) \]

where,

\[ S_2 = \frac{A_2}{D_2} \quad \ldots \quad (A-18) \]
\[ T_2 = \frac{B_2 Q_1 + C_2}{D_2} \quad \ldots \quad (A-19) \]
\[ S_3 = \frac{A_3}{D_3 - B_3 S_2} \quad \ldots \quad (A-20) \]
\[ T_3 = \frac{B_3 T_2 + C_3}{D_3 - B_3 S_2} \quad \ldots \quad (A-21) \]
Thus, proceeding this way, the solution of equation (A-10) is given by the general equation

\[ Q_1 = S_1 Q_{i+1} + T_1 \quad \cdots \quad (A-26) \]

in which the coefficients \( S_1 \) and \( T_1 \) are given by

\[ S_1 = \frac{A_1}{D_1 - B_1 S_{i-1}} \quad \cdots \quad (A-27) \]
\[ T_1 = \frac{B_1 T_{i-1} + C_1}{D_1 - B_1 S_{i-1}} \quad \cdots \quad (A-28) \]

for \( 3 \leq i \leq N-1 \). The coefficients \( S_2 \) and \( T_2 \) are to be calculated using equations (A-18) and (A-19).
For each step $\Delta R_e$ downwind, $Q_1$ was determined for increments in $\Delta Z$ until the difference in $Q$ at the $i$'th and $i+1$'th levels was less than $10^{-2}$. This was assumed to correspond to the top of the turbulent diffusion boundary layer.
APPENDIX - B

THE COMPUTER PROGRAM : A LISTING AND USER INSTRUCTIONS

P.1 Introduction

The purpose of this appendix is to provide a guide for the use of the computer program developed to calculate the daily evaporation using the proposed approach. The program was written in FORTRAN. The computer program, a listing of which is presented at the end of this appendix, was written for the climatological measurements at Lake Hefner and will require a few modifications before it is used for another site.

The program was run on the IBM 370/155/302 Computer.

B.2 The FORTRAN Symbols

Although adequate comments are included in the computer program that will help to follow the sequence of computation, the symbols used are not defined. These symbols and their meaning are given below. Whenever possible, the mathematical symbols used in the text have been retained.

<table>
<thead>
<tr>
<th>FORTRAN Symbol</th>
<th>Meaning</th>
</tr>
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<tbody>
<tr>
<td>A</td>
<td>Coefficient $A_1$ in equation (A-10)</td>
</tr>
<tr>
<td>AREA</td>
<td>Running total of area over which evaporation has been estimated</td>
</tr>
<tr>
<td>AREAIL</td>
<td>Surface area of the lake</td>
</tr>
<tr>
<td>B</td>
<td>Coefficient $B_1$ in equation (A-10)</td>
</tr>
<tr>
<td>FORTRAN Symbol</td>
<td>Meaning</td>
</tr>
<tr>
<td>----------------</td>
<td>---------</td>
</tr>
<tr>
<td>C</td>
<td>Coefficient $C_i$ in equation (A-10)</td>
</tr>
<tr>
<td>COUNT</td>
<td>A counter to denote the level at which $Z(I) = ZS$</td>
</tr>
<tr>
<td>D</td>
<td>Coefficient $D_i$ in equation (A-10)</td>
</tr>
<tr>
<td>DMdZ</td>
<td>$dM/dZ$ in equation (3.55)</td>
</tr>
<tr>
<td>DUdZ</td>
<td>$dU/dZ$ at the $i$'th grid point</td>
</tr>
<tr>
<td>DEX</td>
<td>$\Delta R_e$</td>
</tr>
<tr>
<td>DZ(I)</td>
<td>$Z(I) - Z(I-1)$</td>
</tr>
<tr>
<td>D2UDU(I)</td>
<td>$dZ/dU$ at the $i$'th grid point</td>
</tr>
<tr>
<td>D2UDZ2(I)</td>
<td>$d^2U/dZ^2$ at the $i$'th grid point</td>
</tr>
<tr>
<td>D2ZDU2(I)</td>
<td>$d^2Z/dU^2$ at the $i$'th grid point</td>
</tr>
<tr>
<td>EVAP</td>
<td>Evaporation</td>
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<tr>
<td>EVAPRL</td>
<td>Water-budget evaporation</td>
</tr>
<tr>
<td>I</td>
<td>Counter to denote the $i$'th grid point</td>
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<td>ICASE</td>
<td>A counter to denote the level at which $Z(I) = ZVP$.</td>
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<td>Running total of the number of days for which evaporation has been determined.</td>
</tr>
<tr>
<td>NODRX</td>
<td>Running total of the number of DRX</td>
</tr>
<tr>
<td>NODZ</td>
<td>Total number of $\Delta Z$</td>
</tr>
<tr>
<td>E</td>
<td>Evaporation number, $N_e$</td>
</tr>
<tr>
<td>P(I)</td>
<td>$S_i$ of equation (A-26)</td>
</tr>
<tr>
<td>PJSTEP</td>
<td>Pre-step length, $l$</td>
</tr>
<tr>
<td>PRESS</td>
<td>Atmospheric pressure</td>
</tr>
<tr>
<td>Q(I)</td>
<td>$T_i$ of equation (A-26)</td>
</tr>
<tr>
<td>RHO</td>
<td>Air density, $\rho$</td>
</tr>
<tr>
<td>RI</td>
<td>Gradient form of the Richardson number, $R_i$</td>
</tr>
<tr>
<td>FORTRAN Symbol</td>
<td>Meaning</td>
</tr>
<tr>
<td>----------------</td>
<td>---------</td>
</tr>
<tr>
<td>RX</td>
<td>Reynolds number, ( \Re )</td>
</tr>
<tr>
<td>RXF5N</td>
<td>Reynolds number corresponding to ( \text{STAR} )</td>
</tr>
<tr>
<td>RXKRAFT</td>
<td>Downwind Reynolds number corresponding to the center of the lake</td>
</tr>
<tr>
<td>RXPJL</td>
<td>The pre-step Reynolds number</td>
</tr>
<tr>
<td>RXTEST</td>
<td>A counter to denote the level of ( T(I) = \text{VPS} )</td>
</tr>
<tr>
<td>RXZS</td>
<td>Reynolds number corresponding to ( \text{ZS} )</td>
</tr>
<tr>
<td>SC</td>
<td>Laminar Schmidt number</td>
</tr>
<tr>
<td>SCT(I)</td>
<td>Turbulent Schmidt number at the ( i^{th} ) grid point</td>
</tr>
<tr>
<td>START</td>
<td>Running total of the downwind step lengths</td>
</tr>
<tr>
<td>STEP</td>
<td>Downwind step length, ( \Delta x )</td>
</tr>
<tr>
<td>( T(I) )</td>
<td>( 1 - Q_i ), where ( Q_i ) is the non-dimensional humidity at the ( i^{th} ) grid point</td>
</tr>
<tr>
<td>T2D, T2W</td>
<td>Dry and wet bulb temperatures at 2 meters</td>
</tr>
<tr>
<td>T8D, T8W</td>
<td>Dry and wet bulb temperatures at 8 meters</td>
</tr>
<tr>
<td>TSURF</td>
<td>Water surface temperature</td>
</tr>
<tr>
<td>( U(I) )</td>
<td>Non-dimensional wind velocity at the ( i^{th} ) grid point</td>
</tr>
<tr>
<td>US</td>
<td>Non-dimensional free-stream velocity, ( \U_s )</td>
</tr>
<tr>
<td>UST</td>
<td>Friction velocity, ( u_* )</td>
</tr>
<tr>
<td>U1, U2</td>
<td>Wind velocities at 2 and 8 meter levels (cm/sec) respectively</td>
</tr>
</tbody>
</table>

\[
\begin{align*}
\text{U2Z0ZS} & = \int_{Z_0}^{Z_s} U^2dz \text{ at the } i^{th} \text{ grid point} \\
\text{U2Z0Z(I)} & = \int_{Z_0}^{Z} U^2dz \text{ at the } i^{th} \text{ grid point}
\end{align*}
\]
<table>
<thead>
<tr>
<th>FORTRAN Symbol</th>
<th>Meaning</th>
</tr>
</thead>
<tbody>
<tr>
<td>USQDZ</td>
<td>$U^2 , dZ$ at the $i$'th grid point</td>
</tr>
<tr>
<td>UCUBDZ</td>
<td>$U^3 , dZ$ at the $i$'th grid point</td>
</tr>
<tr>
<td>U3ZOZ</td>
<td>$\int_{z_o}^{Z} U^3 , dZ$ at the $i$'th grid point</td>
</tr>
<tr>
<td>VPS</td>
<td>Estimated vapour pressure at the free-stream</td>
</tr>
<tr>
<td>VPSURF</td>
<td>Vapour pressure at the surface</td>
</tr>
<tr>
<td>VP2</td>
<td>Vapour pressure at the 2 meter level</td>
</tr>
<tr>
<td>WIDTH</td>
<td>Width of an elementary strip</td>
</tr>
<tr>
<td>WINDZS</td>
<td>Estimated wind velocity at the free-stream (cm/sec)</td>
</tr>
<tr>
<td>WIND2, WIND8</td>
<td>Wind velocities at 2 and 8 meter levels in knot respectively</td>
</tr>
<tr>
<td>XKMNUE</td>
<td>$K_m / \nu$ in equation (3.13)</td>
</tr>
<tr>
<td>XL</td>
<td>Nonin-Obukhov scale length, $L$</td>
</tr>
<tr>
<td>XM</td>
<td>$M$ in equation (3.55)</td>
</tr>
<tr>
<td>XK</td>
<td>von Karman constant</td>
</tr>
<tr>
<td>XNUE</td>
<td>Kinematic viscosity, $\nu$</td>
</tr>
<tr>
<td>Z(I)</td>
<td>Non-dimensional height, $Z_i$, of the $i$'th grid point</td>
</tr>
<tr>
<td>ZBYL</td>
<td>$Z_i / L$</td>
</tr>
<tr>
<td>ZS</td>
<td>Non-dimensional height of the turbulent momentum boundary layer</td>
</tr>
<tr>
<td>ZVP</td>
<td>The non-dimensional 8-meter level. The known specific humidity at this level is used to evaluate free-stream humidity, $q_s$, using equation (3.85)</td>
</tr>
<tr>
<td>ZO</td>
<td>Surface roughness parameter, $z_0$</td>
</tr>
</tbody>
</table>
Subroutines used in the Computer Program

The subroutines used in the computer program and their purpose are given below.

**SUBROUTINE VISCOS (T, P, XNUB)**

This subroutine evaluates the kinematic viscosity \( XNUB \) for a given temperature \( T \) and a given pressure \( P \) using equation (3.77).

**SUBROUTINE VAPOUR (T1, T2, VAP)**

This subroutine calculates the actual vapour pressure \( VAP \) for a given dry bulb temperature \( T1 \) and wet bulb temperature \( T2 \) using equations (3.81) and (3.82).
The Computer Program
**Evaporation from Aerodynamically Smooth and Rough Surfaces Exposed to a Thermally-Stratified Atmosphere (Using Lake Hefner Data)**

Programmed by Muhammad Ali Epilayan, Lecturer, Dept. of Water Resources Engineering, BUET, Dhaka.

```
REAL D(500), DUCZ(500), C(500), C2(500), C2LZ(500), D2LZ(500) +, P(500), C(500), CT(500), L(500), L2Z(500), Z(500), RX*E +, FX*E

DATA XK, SC, FRESS, IYEAR, ACAY/Y.36.C, 5.96C1.0E3.1.9S1.0/ DATA ALPHA, ALPHAGE, BETA, CANNA/1E.C, 5.0.C, 7.4, 4.77 WRITE(3,560)

INPUT THE CLIMATOLOGICAL PARAMETERS
END REAC(1,570) ICAY, MCNTH, TSC, T2, T0, T1, TSLF, INC2, INDC, AEARL, EVAFRL

PERFORM PRELIMINARY CALCULATIONS
XK2=XK*DX
XK3=XK*DX2
XK4=XK*DX3
FA4E2=1.5766

SET THE INITIAL VALUES
SUM=0.0
AREE=0.0
CRX=100.0
NCDFX=1
CC2=500
START=3000.0
STEP=3000.0
AREARL=4C4C.AE41 AEARL
RADIUS=SORT(AREARL/2.141)
FXPL=1.0EE
FXS1=FXPL1
ICASE=0
COUNT=0.0
FXST=0.0

OUTPUT THE CLIMATOLOGICAL PARAMETERS
L1=1.4E*N1ND2
L2=1.4E*N1ND2
21=200.0
22=400.0
TO=T20
TC2=TC2
TW1=TW1
TW2=TW2
WRITE(3,560) DAY, MCNTH, IYEAR
```
WRITE(13,597)U1,U2,Z1,Z2
WRITE(13,600)TD1,TD2,TV1,TV2,TSLFF

152 IF(U2 .LT. L1) GC TC 15C

CALCULATE DENSITY, VAPOUR PRESSURE AND KINEMATIC VISCOSITY
RHC=0.3484/(TC+273.0)
CALL VAPOUR(TD,T2,V2)
CALL VAPOUR(TSURF,TSURF,VPSLF)
CALL VISCCS(TD,FRESS,XALE)
WRITE(3,610)VP4LRF,VPSR,F,RHC,XALE

CALCULATE THE RICHARDSON NUMBER AND DETERMINE THE ATMOSPHERIC
STABILITY
LCIFF=U2-U1
ZCIFF=Z2-Z1
TCIFF=TD2-TC1
RI=581.0*4*(TCIFF+C1)*ZCIFF/(LCIFF+4*(CI+273.0))
IF(RI .GT. C.001) WRITE(3,620)RI
IF(RI .LT. -0.001) WRITE(3,630)RI
IF(ABS(RI) .LT. 0.001) RI=C.0
IF(RI .GT. C.15) GC TC 15C

DETERMINE THE MAIN-CBUK+CE LENGTH, FRICTION VELOCITY AND
ROUGHNESS PARAMETER
IF(RI)160,160,150

UNSTABLE STATE

60 ZBYL=C.0
XINCR=-1.0
70 ZBYL=ZBYL+XINCR
RITRI=BETA*ZBYL*(1.0+ALPHA12*ZBYL)/(1.0+ALPHA24*ZBYL2)*1.05
IF(RITRI .GT. RI) GC TC 15C
ZBYL=ZBYL-XINCR
XINCR=XINCR/15.0
IF(ABS(XINCF) .GE. 1.0E-4) GC TC 15C
XL=SQRT(Z1*22)/2EYL
XI=(1.0-ALPHA12*Z1/XL)*X1
X2=(1.0-ALPHA12*Z2/XL)*X2
A1=0.0*ALCG((1.0+XI)/2.0)+ALCG((1.0+X1)/2.0)-2.0*ATAN(X1)*
+FAIE2
A2=0.0*ALCG((1.0+XI)/2.0)+ALCG((1.0+X2)/2.0)-2.0*ATAN(X2)*
+FAIE2
LST=XK*UCIFF/(ALCG(Z2/2)*1+ALCG(1.0-Z2/2)*ZC)
WRITE(3,670)LST,ZC,XL
GC TC 21C

NEUTRAL STATE

0 LST=XK*UCIFF/ALCG(Z2/2)
ZC=Z2*EXP(XK*U2/UST+A2)
WRITE(3,697)LST,ZC
GC TC 21C
C STAILE STATE
150 ZEYL=0.0
XINCR=1.0
2CC ZEYL=ZBYL+XINCR
RITRI=ZBYL*(BETA+CAMA+ZEYL)/(1.0+CAMA+ZEYL)**2
IF(RITRI .lt. R11) R11=CC TC 2CC
ZBYL=ZBYL-XINCR
XINCR=XINCR/10.0
IF(XINCR .gt. 1.0E-4) R11=CC TC 2CC
XL=SORT(X22)*2ZBYL
UST=XX*UDIFF/(ALCC2.*Z21)*(CAMA+2ZDIFF)**(1.0)
2C=Z22/EXP(XX*U2/UST-CAMA+Z22/XL)
WRITE(3,X70)UST,ZC,XL

C NON-DIMENSIONALISATION OF NON-OBLKFCY LENGTH AND ROUGHNESS
C PARAMETER
110 IF(R1)220,230,220

C UNSTABLE AND STAILE STATES
220 ZQ=ZQ*UST/XALE
XL=XL*LIST/XALE
WRITE(3,X70)ZQ,XL
IF(ZC .LT. 1.0E-6) GO TO 150
CC TC 270

C NEUTRAL STATE
230 ZQ=ZQ*UST/XALE
WRITE(3,X70)ZQ
IF(ZQ .LT. 1.0E-6) GO TO 150
CC TC 150

C DETERMINE THE PERMANENT FRACTIONS OF VELOCITY
270 FX=FXST
IF(FX .gt. 1.0E-4) GC TC 42C
2VP=Z2*UST/XNUE
2S=CC
XINCR=1.0E-6
FXS=0.0
U22CZ(1)=C.C
U33CZ=0.0
IF(R1)120,2E0,320

C ZS AND US FOR NEUTRAL STATE
320 ZS=ZS+XINCR
=ALCC2*(ZS/2C)
RXS=(A*(ZS*(A-A-2C)-2.02C)+C4*(ZS-2C))/XX2
IF(FXZS .lt. RX) GC TC 2EC
ZS=ZS-XINCR
XINCR=XINCR/10.0
IF(XINCR .le. 1.0E-6) GC TC 2EC
US=A/XX
U22CZS=(ZS***(A-2C)+2.0*(ZS-2C))/XX2
GO TC 320

C FOR STAILE, UNSTABLE AND NEUTRAL STATES
C
UNSTABLE STATE
320  U(I)=0.0
   Z(I)=0.0
   CC 3EE I=2, NCDZ
   DZ(I)=AMAX1(0.05, (2(I-1)-2C)/2C)
   DZ(I)=2C
   Z(I)=Z(I-1)+DZ(I)
   IF(Z(I) > ZV) AND (CASE = EC (I)) CASE=1
   A=ALOG(Z(I)/Z0)
   XKMZ=XK*Z(I)
   IF(RL)230, 360, 370

C
STABLE STATE
230  ZBYL=Z(I)/XL
    X=(1.0-ALPH*2*ZEYL)*0.25
    U(I)=(A-2.0*ALOG((1.0+X)/2.0)-ALOG((1.0+X)/2.0)+2.5*TAN(X)-
       FAIE2)/XK
    USQCZ=U(I)*42*DZ(I)
    U2(CZ(I))=U2(CZ(I)+1)+USQCZ
    IF(FXZS * GE * RX) CC TC 34C
    UCUECZ=USQ0D2*U(I)
    UCUECZ=USQ0D2*U(I)
    UCUECZ=USQ0D2+UCLED2
    FXZS=U(I)+U2CZ(I)-L2CZ2
    IF(CGUNT * EC .0 .0 .AND. FXZS .GE . RX) CC TC 350
340  DZCLE(I)=XKM*2X
    DZULZ(I)=XKM*2X
    DZULZ(I)=XKM*2X
    CTZ(I)=BETA*X/(1.0-ALPH*2*ZEYL)*C5
    CC TC 36C
36C  US=U(I)
    2S=2(I)
    U2CZS=U2CZ(I)
    COLT=1.0
    FXST=FXZS
    FXST=FXZS
    FXST=FXZS
    CC TC 34C

C
NEUTRAL STATE
36C  U(I)=A/XK
   DZDU(I)=XKMZ
   DZLZ(I)=-1.0/(XKMZ2Z(I))
   U2CZ(I)=Z(I)*AMAX1(A-2.0*G+2.0*(Z(I)-2C)/2C)
   SCT(I)=BETA
   CC TC 36C

C
STABLE STATE
370  ZBYL=Z(I)/XL
    B=AMAX1(ZBYL
    U(I)=(A+B)/XK
    USQCZ=U(I)*42*DZ(I)
    U2CZ(I)=U2CZ(I)+USQCZ
    IF(FXZS * EC * RX) CC TC 375
    UCUECZ=USQ0D2*U(I)
    UCUECZ=USQ0D2*U(I)
    UCUECZ=USQ0D2+UCLED2
    FXZS=U(I)+U2CZ(I)-L2CZ2
    IF(CGUNT * EC .0 .0 .AND. RX*EC . RX) CC TC 378
375  DZCLE(I)=XKMZ/(1.0*4E)
    CTZ(I)=XKMZ/(1.0*4E)
    CC TC 378
SCT(1)=(BETA+B)/(1.0+B)
CG TC 360
376 LS=L(11)
ZS=Z(11)
UZ2CZS=UZ2CZ1(1)
CC=U=1.0
FXST=RXZS
FXPL=RXZS
FX=RXZS
CC TC 375
380 T(I)=0.0
CUDZ(I)=1.0/D2D(I)
C2ZC2U(2)=-(C2UDZ2(I)*D2CU(1)+2
385 CC=CONTINUE
405 WRITE(3,720)CASE,LS,ZS,RZ2S,OZ(2),OZ(3),OZ(4),OZ(5),OZ(6),OZ(50),FX
C
C DETERMINE THE PRE-STEP LENGTH
410 WINCZS=UST*LS
PJSTEP=RPJL*XNUE*WINCZS
RXPAFT=(PJSTEP+RAI0LS)*WINCZS/XNUE
UST=WINCZS/LS
420 T(I)=1.0
J=1
430 IF(XFIN=(STAR1+PJSTEP)*WINCZS/XNUE)
C IF(X .GE. XFIN) GO TC 440
C
SOLUTION OF TRI-DIAGONAL MATRIX
440 J=J+1
445 SCTINV=1.0/SCT(J)
CC=0.5*OZ(J)+DZ(J)+DZ(J+1)/CH
RAT1=DZ(J)/DZ(J+1)
XKMMNE=MAX(1.0,0.01,(1.0-UZ2CZ(J)/L2CZS)*C2CU(J)-1.0)
XN=1.0/SC+SCTINV*XKMMNE
AA=US*U(J)/XN
IF(Z(J) .GE. ZS) AA=US*LS/XN
C
C CM/CZ FOR NEUTRAL CASE
CMD2=SCTINV*(((1.0-UZ2CZ(J)/L2CZS)*C2ZCU(J)+DUCZ(J)-U(J))U(J)+
+C2DU(J)/UZ2CZS)
455 IF(R1 .GE. C.01) CC TC 460
ZBYL=Z(J)/XL
460 IF(R1) 450,460,460
C
C CM/CZ FOR UNSTABLE STATE
450 TERN1=1.0-ALPHA1*Z2YL
TERM2=1.0-ALPHA2*Z2YL
CMD2=OMI(Z)*0.25*XKMMNE*SCT(1)*TERM1*(ALPHA1/TERM1-2.0)*ALPHA2/
*TERM2/(BETA1*YL*TERM1*C.025)
CC TC 460
C
C CM/CZ FOR STABLE STATE
460 CAMP=GMMA*X2
CMD2=OMI(Z)-XKMMNE*GMMA*(((1.0-CET/3)/(CET12(J)*GMMA))*2
C
MAIN*
THE FREE STREAM VELOCITY FOR THE NEXT STEP

IF(FL) GE .32, .34, .36

UNSTABLE STATE

ZSUP = ZS
XINCR = 1.0E6

ZSUP = ZSUP + XINCR
X = (1.0 - ALPHAI) * ZSUP / XL * 1.5 * 25
A = 2.0 * ALPHAI * (1.0 + X) + ALPHAI / (C - 5 * (1.0 + X) + 2.0) * ATAN(X) * FAIE2

USTRI = (ALPHAI * ZSUP / XL - A) / XK
IF(USTRI LT US) GC TC E33

ZSUP = ZSUP - XINCR
XINCR = XINCR / 10.0

IF(XINCR GE 1.0) GC TC E33

U22C5 = U22GZS * (ZSLP - 25) * .64 * 2

ZS = ZSUP
C  NEUTRAL STATE
E24  ZS=2C*EXP(XK*US)
     A=ALOG(ZS/2C)
     U2ZGZS=[ZS*A+(A-2*C)+2*C*(ZS-ZC)]/XK
CG TC 420

C  STAELE STATE
E36  ZSUF=ZS
     XINC=1.0E6
E37  ZSUF=2SUP+XINC
     USFE=(ALOG(ZSUF/2C)+GAMNAX2SUP/XL)/XK
     IF(LSTHG.LT.US) GC TC E37
     ZSUF=ZSUP+XINC
     XINC=XINC/10.0
     IF(XINC*.GE.1.0) GC TC E37
     U2ZGZS=U2ZGZS+(ZSUF-2S)*LS*/2
     ZS=ZSUP
CG TC 420

C  CUTOUT ROUTINE
E46  WIDTH=2.0*SCRT(ABS(STAR)*(2*C+RADIUS)-STAF11))
     ZC=0.0+CZ(3)+CZ(4)+DZ(5)+DZ(6))
     SCTINV=1.0/SCRT(ZC)
     XMNU=NMAX(0,0,(1.0-U2ZGZS(2)/U2ZGZS(3)*C2U(2)-1.0))
     RN=AUS(25.0+T(2)+48.0+T(3)+55.0+T(4)-15.0+T(5)+3.0+T(6))+(1.0/5C+SCTINV)*XMNU
     SUM=RN*WIDTH*STEP*INDIZ+SUP
     AREA=STEP*WIDTH*AREA
     STAR=NMIN(2.0+RADIUS,START+STEP)
     RXST=RX
     IF(STAR.LT.2.0+RADIUS) GC TC 27C

C  EVAPORATION
E5C  CHUN=0.622*(VPSURF-VPS)/PRESS
     EVAPE=6.65E5+(RHO4DPLM*SUM/AREA)
     WRITE(2,750)XK,START,RADIUS,AREA,AREAFL,EVAPE,EVAPFL,RX
     NCAY=NGCAY+1
     IF(NCAY.LT.12) GC TO 15C
     STCF

E5C  FORMAT(1H1)
E7C  FORMAT(2I3,5F6.2)
E6C  FORMAT(/PAY,2La,6ATE*,5Y,214,1C)
E6C  FORMAT(1HC,EX,2La,2L@,2F6.2,10X,2L=,2F7.2,10X,21=,2F6.2,10X)
**FORTRAN**

```

610 FORMAT (IHC, EX, *VPSURF=*F6.2, ICX, *VF2=*F6.2,
+10X, *RHC=*F7.5, ICX, *RUE=*F6.4)

620 FORMAT (IHC, EX, *STABLE ATMOSPHERE, ICX, *FI=*F9.5)

630 FORMAT (IHC, EX, *UNSTABLE ATMOSPHERE, ICX, *RI=*F9.5)

640 FORMAT (IHC, EX, *NEUTRAL ATMOSPHERE, ICX, *FI=*F9.5)

650 FORMAT (IHC, EX, *STABLE ATMOSPHERE, ICX, *FI=*F9.5)

700 FORMAT (IHC, EX, *VPSURF=*F6.2, ICX, *L=*E15.6)

710 FORMAT (IHC, EX, *VPSURF=*F6.2, ICX, *L=*INF.

720 FORMAT (IHC, EX, *CASE=*I4, EX, *VPS=*E15.6, EX, *2S=*E15.6, 5X,
+*R*X2S=*E15.6/**EX, *CZ=*E15.6, F1(*2, EX, *FX=*E15.6)

730 FORMAT (IHC, EX, *FX=*E15.6, EX, *YXFAF1=*E15.6, 5X, *CASE=*I4, 5X,
+*VPS=*F6.2, 5X, *FXFAF1=*E15.6)

750 FORMAT (IHC, EX, *SI=*F9.5, EX, *START=*E15.6, EX, *FADIUS=*E10.3,

END
```
SUBROUTINE VAPCR(T1,T2,VP)
C
THIS SUBROUTINE CALCULATES THE VAPCR PRESSURE FOR GIVEN CRY.
C AND WET DULE TEMPERATURES
IF(T2) 10.XZ,A20,
10 .SVTPW=4.5776*T2*(C.3723E+12*(C.C12531+121.9364E-4))
VP=SVTPH-0.54*(T1-T2)
CC TO 30
20 .SVTPW=4.5855+T2*(C.32EC+T2*(C.C1171+121.279E-4+T2*4.184E-6))
VP=SVTPW-0.54*(T1-T2)
30 .VP=1.32*VP
RETURN
END

SUBROUTINE VISCGS(T,P,XNUE)
C
THIS SUBROUTINE CALCULATES THE KINEMATIC VISCOSITY FOR GIVEN
C TEMPERATURE AND PRESSURE
A=7+73.16
XNUE=(1.8375/3.4641*(A/P)+4.1616/A+120.*C)*Sqrt((A/296.16)+13)
RETURN
END

// EXEC LAKEDT
// EXEC
//
// ECJ