Multi-Item, Single-Level, Capacitated Lot-Sizing by Heuristic Approach

by

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Declaration

I do hereby declare that this work has been done by me and neither this thesis nor any part of it has been submitted elsewhere for the award of any degree or diploma except for publication.

Countersigned

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List of Symbols

Symbol	Meaning
Δ_i	priority index for item i
$\delta(x_{ij})$	a binary setup variable indicating whether a setup cost must be incurred for item i in period j or not
$AC(T_i)$	average cost per unit time for a lot of item i which will satisfy T_i periods' requirements
AP_j	amount of inventory (in capacity units) resulted from the production of the current period that will be used in period j
C_{inv}	total inventory-holding cost
C_j	the capacity in period j
CR_j	total demand (in capacity units) in period j
C_{set}	total expected setup cost
C_{ss}	total expected safety-stock cost
d_{ij}	equivalent demand for product i in period j
D_{ij}	demand for item i in period j
$d_{\max i}$	maximum periodic demand for the ith item
$d_{\mathrm{rem}\, ij}$	remaining allowable amount that can be produced if x_{ij} is produced at period j for item i
H	the time horizon
h_i	the unit holding cost for item i
$Iend_i$	ending inventory for item i
I_{ij}	the inventory of item i at the end of period j
I'_{ij}	amount of inventory at the end of period j for item i, resulting from only the currently scheduled production in period R
Iin_i	initial inventory for item i
$Irem_i$	remaining initial inventory for item i
k_i	the capacity absorption rate for item i

Symbol	Meaning
N	the number of items
N'	number of total items after meeting the maximum lot-size limitation
Q	amount of production still needed in the current period to eliminate infeasibilities in the later period
S_i	the setup cost for item i
SS_i	safety stock for item i
St_i	setup time for item i
t_c	the earliest period at which the feasibility constraint is not satisfied
T_i	time supply for item i (denotes the integer number of period requirements that this lot will exactly satisfy)
U_i	priority index (the marginal decrease in average costs per unit of capacity absorbed)
x_{ij}	the lot-size of item i in period j
$x_{\max i}$	maximum allowable lot-size for item i which cannot be exceeded in any period

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Abstract

A lot-sizing problem involves decisions to determine the quantity and timing of production for N different items over a horizon of T periods. In the present work, only one machine is available with a fixed capacity in each period. The objective is to minimize the sum of setup and inventory carrying costs for all items without incurring backlogs. In case of a single item production only an optimal solution algorithm exists. But for medium-size and multi-item problems, optimal solution algorithms are not available. It has been proved that even the two-item problem with constant capacity is NP-hard. That is, it is in a class of problems that are extremely difficult to solve in a reasonable amount of time. This has increased the importance of searching good heuristic solutions. In the present research work the Dixon-Silver heuristic for the multi-item, single-level, limited capacity, lot-sizing problem has been implemented in PC based Fortran77. For a multi-item problem, it would be more realistic to consider the setup time, since switching the machine from one item to another would require a setup time. This setup may be independent of item sequences. Moreover, it would still be realistic to set an upper limit on the lot-size per setup for each item, since the machine may not be available for indefinite period for a particular product and the machine may not be able to run continuously. The Dixon-Silver heuristic did not consider these two parameters. The current research work has, therefore, been directed toward the extension of the Dixon-Silver model to incorporate these parameters separately. Based on the extension two programs have been executed on Fortran77 platform and feasible solutions have been obtained.

Chapter 1

Introduction

1.1 Background

It is the age of manufacturing. The manufacturing industries are now facing a time of intense international competition, which will only become more severe in the days to come. The Chinese character for 'crisis' is a combination of the characters for 'danger' and 'opportunity'. For manufacturing companies, the danger lies in lower cost–higher quality producers taking an increasing share of both domestic and foreign markets. The opportunity lies in new technology that can enable a company to improve both productivity and quality and obtain a competitive edge.

The new technology can be divided into two categories: (1) the automation of production activities using computer-aided design and manufacturing, robotics, or flexible manufacturing systems and (2) computer-based production and inventory control.

Computer-based production and inventory control (CBPIC) embodies powerful tools for more effective manufacturing management developed over the last two decades. CBPIC does not simply automate manual systems but, rather, makes possible the use of new and better planning and control concepts and techniques.

1.2 Production and Inventory Control

In manufacturing, production control and inventory control are closely intertwined. The American Production and Inventory Control Society defines production and inventory control as follows [1].

Production control: The function of directing or regulating the movement of goods through the entire manufacturing cycle from the requisitioning of raw materials to the delivery of the finished products.

Inventory control: The activities and techniques of maintaining the stock of items at desired levels, whether they be raw materials, work in process, or finished products.

1.2.1 Forms of Inventory

Inventories exist at many points and in different forms in the procurement, production and distribution chain. The principal role of inventory is to serve as a buffer, decoupling successive stages of production and distribution to achieve greater efficiency. A secondary role is to provide a hedge against price increases and fluctuations in demand. More specifically, inventories can be categorized by the functions they serve, as follows:

Lot-size inventories- These inventories exist because there is some economy of scale in replenishment. It is economical to replenish in large lots or at least at a rate faster than demand. The sources of these economies of scale are setup costs, cost of preparing production or purchase orders etc.

Safety stocks – Inventory control is subject to many uncertainties. Safety stocks protect against failure to fill customer orders or satisfy the needs of manufacturing on time due to these uncertainties.

Anticipation inventories- Inventories may be built up in anticipation of a reduction in supply, an increase in demand, or a price increase.

Pipeline inventories- An inventory system can be regarded as a series of stock points with flows between them.

1.2.2 Role of Inventory

Not long ago, inventory was considered a necessary evil and inventory control a clerical function. Today, however, effective inventory control is recognized as making an important contribution to the overall success of the firm. Inventory control is a subject of concern and decision making for all levels of management, for poor inventory control is one of the principal causes of business failure.

Typically, a manufacturing firm will have one-third of its assets invested in inventory [1]. Inventory control is concerned with the management of this investment. But at the same time in manufacturing firm, production wants efficient operation. This implies large production orders which generate large inventories to reduce machine setup. Production also wants generous quantities of raw materials, components, and work in process so that production will not be interrupted for lack of materials. So the intense international competition in manufacturing has provided a strong incentive to management to seek new, more effective ways of managing production to maintain or achieve a competitive edge. As a result, thousands of companies have implemented CBPIC systems. The most widely adopted systems are called material requirements planning and manufacturing resource planning (MRP II).

1.3 Structure of MRP II

The complete structure of an MRP II system [1,2] can be viewed as presented in Figure 1. This is a generic version of a current, state-of-the-art CBPIC system. This is the model on which most software packages are based and what most professionals are talking about when they refer to MRP II. This system has been applied with little or no modification in a surprisingly large number of manufacturing companies.

Each of the boxes of the MRP II represents a separate subsystem. Each subsystem performs certain functions, is the responsibility of assigned personnel, is supported by computer programs, and interfaces with other subsystems.

The first four subsystems, business planning, marketing planning, production planning, and resource planning, are strategic planning and resource planning, demand management are the input to the production planning. From the following brief description of the MRP II, the position of the lot-sizing has been shown.

1.3.1 Production Planning Function

Production planning is one of the several important functions in a manufacturing organization of today and this would remain so in the organizations of the future. This function is concerned with the overall operation of an organization over a specified time horizon. It is also known by such names as aggregate planning, operation planning, and aggregate scheduling. From forecasts and customer orders, production planning determines the requirement of human and material resources to produce efficiently the outputs demanded. The goal is to effectively allocate the system capacity (plant, equipment, and manpower) over a designated time horizon.

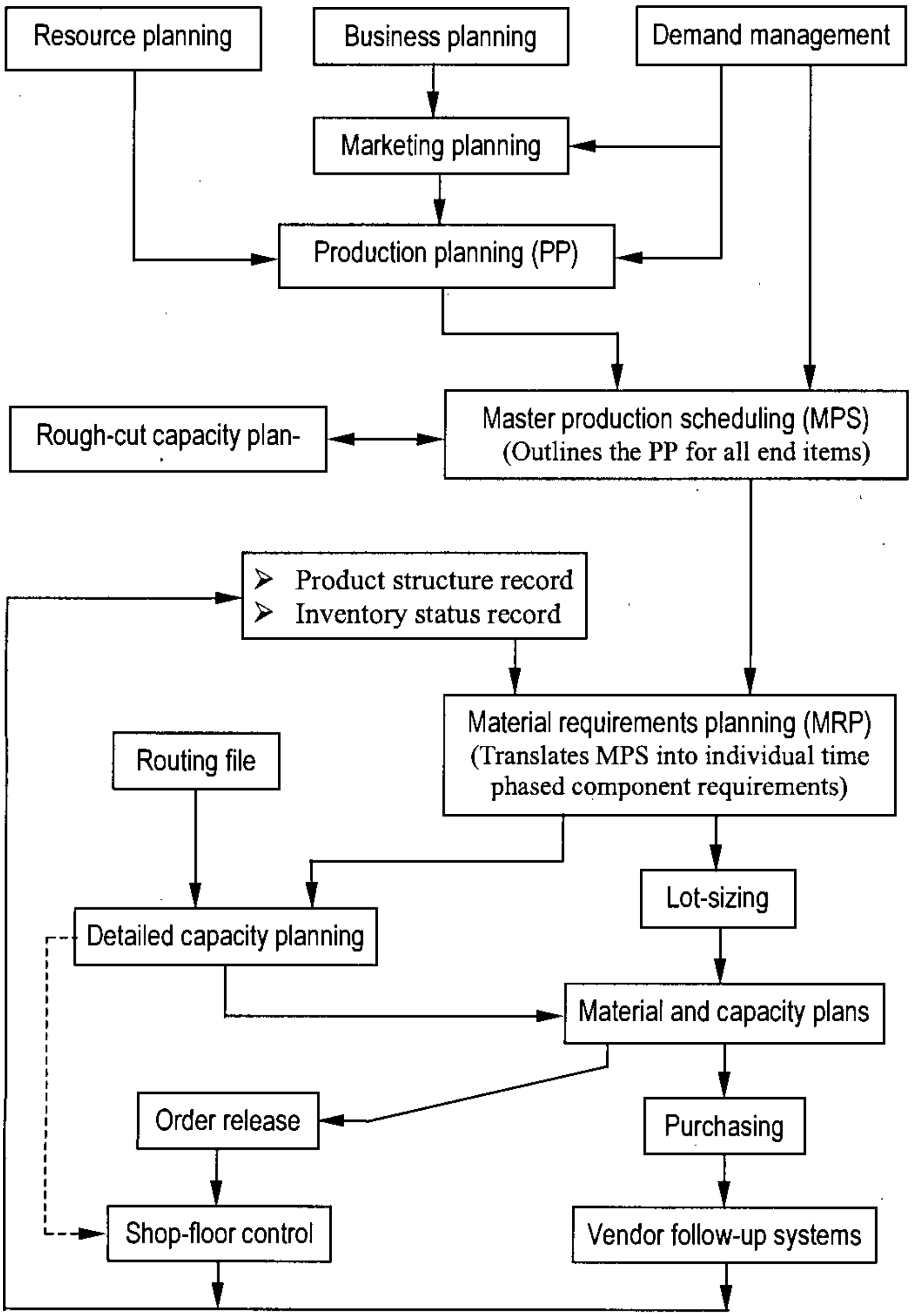

Figure 1.1 Structure of an MRP (II).

Production plan indicates the organization's strategic position in response to the expected demand for its output. A good production plan with the optimal use of resources should yield the results such as (i) be consistent with organizational policy, (ii) meet demand requirements, (iii) be within capacity constraints, and (iv) minimize costs. However, for a constant demand for a product, the planning activity becomes trivial. But with a stochastic demand, the system must have a sound production planning; and the associated planning problem is said to be dynamic. Some major strategic variables associated with production planning for stochastic demand are the production rate, the inventory level, the work force size, etc. These variables could be varied, modified or even kept fixed, or be nonexistent in a given organization, depending on its peculiarities and policies.

1.3.2 Master Production Scheduling

Master Production Scheduling (MPS), is a statement of what end items a company plans to produce by quantity and time period. Production Planning acts as an input to the MPS and it is, therefore, a disaggregation and implementation of the production plan. Thus MPS translates the production plan into specific products or product modules and specifies the time period for their completion.

1.3.3 Material Requirements Planning

Material requirements planning (MRP) is a set of techniques used to plan the production or procurement of subassemblies, components, and raw materials required to support an MPS. MRP is of central importance in manufacturing resource planning for several reasons. Historically, MRP was the first module to be implemented in computer-based production and inventory control (CPBIC) and paved the way for the current comprehensive CBPIC systems. MRP produces plans that are implemented by other modules and it commonly requires most computer processing of any of the MRP II subsystems.

There are two major distinguising features of MRP, (1) requirement for items controlled by MRP are calculated based on schedules for higher-levels items as opposed to being forecast, and (2) plans are time phased in the form of lot-sizing showing order releases and receipts by time periods throughout some planning horizon. So lot-sizing is a significant aspect of the materials requirement planning process and acts as a major component of a balanced MRP operation.

1.4 Scope of the Present Work

Lot-Sizing Problem

In the past few years there have been several activities in computer based production and inventory control dealing with how to select lot-sizes in the face of an essentially deterministic but time-varying demand pattern. Presently, lot-sizing problem has taken its place as one of the most important functions in an industrial enterprise. However, optimizing routines for lot-sizing problems have been shown to be all too demanding from a computing standpoint in both practical as well as research environment. The present work would seek for an efficient means of obtaining an optimal multi-item lot-sizing solution to research problems. This would facilitate development of improved heuristics appropriate for practical settings. Research on the relevant fields has yielded several mathematical and heuristic policies which produce optimal and near optimal results. The ever increasing importance of this issue, therefore, calls for further research and development.

One class of heuristics, initiated by Eisenhut [3]. In Eisenhut's heuristic there is no guarantee that one will find a feasible solution. So Eisenhut's procedure was highly criticized and later extended by Lambrecht and Vandervaken [4], Dixon and Silver [5].

Lambrecht and Vandervaken attempted to incorporate a larger number of steps and the situation became complex.

The Dixon-Silver heuristic guarantees the generation of a feasible solution to a realistic lot-sizing problem. The heuristic is based on a lot-sizing technique and a set of feasibility conditions which should be intuitively appealing to managers. Basic assumptions of the Dixon-Silver model's are`: (i) the requirements for each product are known period by period, out to the end of some common time horizon, (ii) for each product there is a fixed setup cost incurred each period production takes place, (iii) unit production and holding costs are linear, (iv) the time required to set up the machine is assumed to be negligible, (v) all costs and production rates can vary from product to product but not with respect to time, and (vi) in each period there is a finite amount of machine time available that can vary from period to period. The objective is to determine lot-sizes so that (i) costs are minimized, (ii) no backlogging occurs and (iii) capacity is not exceeded.

1.5 Objectives of the Present Work

Switching a machine set up from one item to another incurs a setup time, and is usually independent of the item sequence. In Dixon-Silver heuristic set up time has been neglected. So for a multi-product problem, consideration of the setup time would be more practical. In addition to this it would be more realistic to assume an upper limit, a maximum value of the lot-size from a machine. This restriction may be imposed per setup and this could be a very important parameter from practical point of view for several reasons. Situations like (1) machine's inability to run continuously (2) machine may not be available for indefinite period for a particular product, (3) there may be storage limitation for WIP inventory can be considered in this regard. The current research work has thus been directed toward an extension of the Dixon-Silver model considering the above mentioned situations.

The objectives of the research work have been defined as follows:

i. To modify the Dixon-Silver model and formulate two new models incorporating two parameters such as, (i) setup time, and (ii) maximum limit on the lot-size per setup.

ii. To compare the results obtained using the current model with the results of the Dixon-Silver model.

iii. To make the sensitivity analysis of the current model with respect to various input parameters.

1.6 Organization of the Thesis

This thesis is organized as follows. The Second Chapter discusses the background study of the research work. It includes different lot-sizing techniques and two well-known heuristics. The Third Chapter presents the modified mathematical models and their heuristic methods of solution. This chapter also presents the sample calculations for both modified models. Results are discussed in the Chapter Four. Finally, the Chapter Five concludes with a discussion of the results and future works.

Chapter 2

Literature Survey and Background Study

2.1 Introduction

The importance of lot-sizing in inventory management has been noteworthy over the years, since it is one of the basic features of the MRP. On the other hand the MRP has the central importance in manufacturing resource planning. This has been evident from efforts by researchers from amongst the academics and industries yielding vast literatures containing abstract mathematical approach as well as highly pragmatic techniques. The literatures have been found places in a large number of journals. Some of the lot-sizing techniques are presented in Section 2.1 while the historical background study on the subject is summarized in Section 2.2. Dixon-Silver heuristic used Silver-Meal heuristic, and Wagner-Whitin algorithm. For reasons of self-containedness, brief description of these two heuristics is given in Sections 2.3 and 2.4 followed by Dixon-Silver's work in Section 2.5.

2.2 Lot-Sizing Techniques

The various approaches and techniques of lot-sizing as developed are presented below.

2.2.1 Period Order Quantity

The period order quantity (POQ) uses the same type of economic reasoning as the EOQ (Eonomic Order Quantity which is for a fixed demand or order), but determines the number of periods to be covered by each order rather than the number of units to order. This results in a fixed order cycle as opposed to a fixed quantity as in EOQ. Total cost per period as a function of t, the cycle time in periods is given by

$$C(t) = k/t + h(rt)/2,$$

where

k = order or setup cost

h = inventory holding cost

r = average rate of demand

POQ is an improvement over EOQ as it eliminates remnants, and it performs quite well if demand is relatively stable. However, like EOQ, it does not take full advantage of knowledge of future period-to-period variations in demand. Some other techniques described subsequently outperform POQ when variation in demand is significant [1].

2.2.2 Part-Period Algorithm

The part-period algorithm can determine order sizes under conditions of known, but varying, demand rates. While the algorithm does not ensure optimality, it does approach optimal techniques. It equates the part-period value derived from order and holding costs to the generated part-period value. The generated part-period for an item is the number of parts held in inventory multiplied by the number of time periods over which the parts are held. In calculating the generated number of part-periods, it is assumed that no holding costs are incurred for items consumed in the period in which they arrive.

To express ordering cost and holding cost in part-periods, it is necessary to divide the order cost by the holding cost per part per period. The order cost and holding cost part-periods are referred to as the derived part-period value. The derived part-period value is the number of part-periods it takes to make order cost and holding cost equal. A generated part-period value is obtained by accumulating part-periods over the demand time horizon for one or more periods. When the generated part-period value is first greater than the derived part-period value, an order should be placed. The order quantity will be the accumulated demand up to the time period for the next order [1].

2.2.3 Lot-for-Lot

The simplest lot-sizing technique is lot-for-lot. A lot is scheduled in each period in which a demand occurs for a quantity equal to the net requirement. Lot-for-lot ordering results in a zero inventory balance each period, but does involve many orders. It is most appropriate where the item has a large carrying cost and a small ordering cost, such as large assembles with expensive components. Another situation where lot-for-lot is appropriate is when demand is very sporadic and one or a few units are needed only occasionally. Lot-for-lot also provides a steadier flow of work than other lot-sizing techniques which produce fewer and larger orders [1].

2.2.4 Heuristic techniques

The next three techniques are heuristics. They aim at providing a good, although not necessarily optimal solution with a reasonable amount of computing. All the three techniques use stopping rules. That is, they start from the first period and test prospective orders covering the first period, then the first and second periods, then the first, second, and third periods, and so forth, until a stopping criterion is met. An order is scheduled covering demands in all periods up through the stopping period. Then the process is repeated starting at the next period after the last stopping period.

(a) Least unit cost: The first of these rules is called least unit cost (LUC). The unit costs of orders covering successively greater numbers of periods are calculated. The unit cost for each prospective order is obtained by dividing the sum of the ordering and carrying costs by the number of units on the order. The first time the cost per unit goes up, the prior period becomes the stopping period.

LUC is widely used in industry, and on the surface appears to be a reasonable approach to lot-sizing. However, closer analysis has raised some serious questions concerning the basic logic of the technique [1].

(b) Least Period Cost: The least period cost method was developed by Silver and Meal [2] and is generally referred to as Silver-Meal. The procedure is to determine the total costs of ordering and carrying for lots covering successively greater number of pe-riods into the future and to select the lot with the least total cost per period covered [1].

(c) Least Total Cost: The idea for the Least Total Cost (LTC) method (also called part-period-balancing), was developed by Matties and Mendoza. The concept stems from the fact that in the basic EOQ model, the inventory carrying cost is equal to the ordering cost at the optimum point. In the LTC procedure, lot-sizes covering successively greater number of periods into the future are tested until the largest lot is obtained for which the carrying cost is less than or equal to the ordering cost. Authors presented this method as determining the lot for which the carrying cost was close to the ordering cost. This means that sometimes the carrying cost would be greater than the ordering cost. However, this is not the method presented by the original authors. More-over it did not perform well, because it has a bias toward orders that are too large [1].

2.2.5 Look Ahead/Look Back

Look ahead/look back is a technique used to adjust a schedule of order already obtained using some other technique. It was originally proposed as a refinement of heuristic techniques. However, look ahead/look back can be applied just as well to adjust schedules produced by other heuristics.

Look ahead/look back has the effect of moving orders scheduled for periods of low demand into nearly periods of higher demand. This reduces the number of part-periods and, therefore, the carrying cost. Aucamp and Fogarty have substantially improved and extended the technique. For one thing, their algorithm also takes into account the fact that if an order is moved forward or back to a period in which another order is scheduled, an ordering cost is saved. Their claim is that regardless of what schedule they start with, the end result is virtually optimal.

However, look ahead/look back is not widely used. The reasons are that adding this procedure makes lot-sizing more complex, adds to the amount of computation, and may only improve results marginally if a good lot-sizing procedure has been selected to arrive at the initial lot-sizes [1].

2.3 Dynamic Lot-sizing Problem

The dynamic lot-sizing problem (DLSP) has received considerable attention from both academia and industry during the past two decades. Specifically, the problem is that of determining lot-sizes for a single item when demand is deterministic and time varying. Time is discretized into periods (e.g. days, weeks and months) and production can be initiated only at the start of a period. Each time production is initiated, a setup cost is incurred. A holding cost is incurred for each unit of inventory that is carried from one period to the next. The objective is to minimize the total of setup and holding costs, while ensuring that all demand is satisfied on time. Many optimal and heuristic techniques have been developed for variations of this problem.

2.3.1 Single Item Uncapacitated Lot-sizing Problem

First the concept of single item comes and there is no capacity restriction. Some of the most widely heuristics for lot-sizing for this condition are: Silver-Meal heuristic [6], Least unit cost heuristic [7]. These heuristics are not directly applicable to the present work. The reason is that these heuristics made the following assumptions:

(i) no capacity restrictions,

(ii) only one product to be produced, and

(iii) quantity produced to meet demand in only integer number of periods.

The effective use of the available capacity of plant could not be made in these heuristics. But when capacity constraint is realistically imposed in the scheduling problem, the available capacity use becomes necessary. This part of consideration is an important to the present work.

The Silver-Meal heuristic calculates the lot size as the total demand for an integer number of periods that give the minimum total setup and holding costs per unit time. The least unit cost heuristic calculates the lot-sizes in the same way as the Silver-Meal heuristic. But the exception is that, it minimizes the total costs per unit number of products produced rather than minimizing the total costs per unit time as is done in the Silver-Meal heuristic.

2.3.2 Multi Item Uncapacitated Lot-sizing Problem

Frequently, multiple items are produced on a single machine. This machine has finite capacity and it is usually loaded to or near capacity. Most of the existing methods for the multi-item dynamic lot-sizing problem implicitly assume that capacity is unlimited and hence their use will frequently result in excessive over or under loading in some periods. Therefore, in practice, planned lot-sizes may be split into smaller lots with some demand backlogged. This resuls that orders are not being satisfied on time and the economies of scale of batch production is lost.

2.3.3 Multi Item Capacitated Lot-sizing Problem

The multi-item capacitated lot-sizing problem (CLSP) is found to be NP-hard when the single-item capacitated dynamic lot-sizing problem is already proven to be NP-hard [8-11]. The problem is even harder from practical point of view, since optimal solution methods have failed to solve all but very small problems within reasonable computation times. Moreover, since very few workable techniques have been reported, methods to obtain optimum solutions could not be available easily. It has been found that most methods require extensive computational power, thus, their applicability is rather limited. As a consequence efforts are now being given to develop heuristics for the multi-item capacitated lot-sizing problems. The various heuristics, which have been proposed over the years, are classified into a number of classes. **The first group** of heuristics falling in a class could be called "**common sense**" heuristics. The heuristics belonging to this class can be found in Eisenhut [3], Lambrecht and Vanderveken [4], Dixon-Silver [5] etc. Many different variants have been proposed, for these common-sense heuristics, but they can basically be classified into two categories, such as

(i) the period-by-period heuristics, and

(ii) improvement heuristics.

(i) Period by period heuristic: Heuristics belonging to the period-by-period heuristic work from period 1 to period H. Consider a period t in the process. One certainly has to produce max$\{0, d_{it}, I_{i,t-1}\}$ for all products i in order to avoid stockouts in the current period, where d_{it} is the demand for item i in period t and I_{it} is the inventory of item i at the end of period t. The remaining capacity (if any) can be used to produce demand for some future period, in which case future setup costs may be saved at the expense of added inventory holduingcosts. To indicate the viability of producing demand for a future period in the period under consideration, all heuristics use a priority index. The priority indices used by the heuristics are more sophisticated in that they try to capture the potential savings per time period and per unit demand. Although the exact Priority index may differ from heuristic to heuristic, they all proceed in the same way. Priority indices are calculated for all products and for all future periods. These priority indices are used to include future demands into the current production lot either until no more with a positive index or until the capacity limit is hit.

Besides the difference in using priority index, the period-by-period heuristics also differ in the way in which they ensure feasibility. Infeasibility occurs when the net demand in some period t, i.e. $\sum_{i=1}^{N} \max\left\{0, d_{it} - I_{i,t-1}\right\}$ may exceed available capacity. Two different approaches can be used to overcome this problem. The first one is the feedback mechanism. When an infeasible period is encountered, demand with negative priority indices is shifted from the period to an earlier period. A second approach, look ahead mechanism, however, calculates a priority the required cumulative production up to period t (for all t) such that no infeasibility will arise in period (t + 1). This pure single-pass heuristics require smaller computation time.

(ii) Improvements heuristics : Improvements heuristics start with a solution for the entire horizon and then try to improve this solution in cost effective fashion by going through a set of simple local improvement steps.

The second group of heuristics are all based on optimum seeking **mathematical programming methods** which are truncated in some way to reduce computational effort.

The Mathematical-programming based heuristics are (i) Relaxation heuristics (ii) Branch-and-Bound procedure (iii) Linear programming based heuristics. Heuristics belonging to the class can be found in Wagner-Whitin's algorithm [12] etc. In Wagner-Whitin's algorithm capacity constraints are relaxed i.e. the capacity may be infinite. So the problem decomposes into N number of single-item-uncapacitated-dynamic-lot-sizing problems for which it provides an effective method of solution. The first approach of this type is attributed to Newson [13]. Starting from the Wagner-Whitin solutions for each product, the heuristic proceeds as follows.

(i) Select a period in which capacity is violated. For products with a setup in that period, calculate the next best Wagner-Whitin solution (i.e. the best solution for the problem where production in the violated period is forced to zero).

(ii) Select the next best plan for the product yielding the smallest extra cost per unit capacity absorption, thereby releasing some capacity in the violated period.

(iii) The method proceeds in this way until all infeasibilities are removed.

The above approach has two drawbacks. Firstly, it may end up with no feasible solution at all, and secondly it restricts itself to Wagner-Whitin schedules, whereas the optimal solution may not satisfy the Wagner-Whitin condition $x_{it}I_{i,t-1} = 0$ at all.

Mathematical-programming based heuristics are not considered because these methods may not be very transparent to the casual user and these heuristics limit their regular use in industry. So for the solution of DLSP, **"common sense"** heuristics are applied.

2.4 Wagner-Whitin Algorithm

The "square root formula" for an economic lot-size under the assumption of a steady-state demand rate is well known. The calculation is based on balancing of the costs of holding inventory against the costs of placing an order. When the assumption of a steady-state demand rate is dropped, i.e., when the amounts demanded in each period are known but are different and furthermore, when inventory costs vary from period to period, the square root formula (applied to the overall average demand and costs) no longer assures a minimum cost solution.

The mathematical model may be viewed as a "one-way temporal feasibility" problem, in that it is feasible to order inventory in period t for demand in period $t+k$ but not vice versa. This suggests that the same model also permits an alternative interpretation as the following "one-way technological feasibility" problem.

Mathematical Model

As in the standard lot size formulation, one assumption is that the buying (or manufacturing) cost and selling price of an item are constant throughout all time periods, and consequently only the costs of inventory management are of concern. In the t-th period, $t = 1, 2, \ldots, H$, we let

d_t = amount demanded,

h_t = holding cost per unit of inventory carried forward to period $t + 1$,

s_t = ordering (or setup) cost,

x_t = amount ordered (or manufactured or size of the lot), and

c_t = unit variable cost, which can vary from period to period.

Let all period demands and costs are non-negative. The problem is to find a programme $x_t \geq 0, \quad t = 1, 2, \ldots, H$, such that all demands are met at a minimum total cost; any such program will be termed optimal.

Of course one method of solving the optimization problem is to enumerate 2^{H-1} combinations of either ordering or not ordering in each period (it has been assumed that an order is placed in the first period). A more efficient algorithm evolves from a dynamic programming characterization of an optimal policy.

Let I denote the inventory entering a period and I_o initial inventory; for period t

$$I = I_o + \sum_{j=1}^{t-1} x_j - \sum_{j=1}^{t-1} d_j \geq 0. \tag{2.1}$$

The functional equation representing the minimal cost policy for periods t through H, given incoming inventory I, as

$$f_t(I) = \min \, [h_{t-1}I + \delta(x_t)s_t + f_{t+1}(I + x_t - d_t)], \tag{2.2}$$
$$x_t \geq 0,$$
$$I + x_t \geq d_t,$$

where

$$\delta(x_t) = \begin{cases} 0 & \text{if } x_t = 0 \\ 1 & \text{if } x_t > 0 \end{cases}. \tag{2.3}$$

In period H

$$f_H(I) = \min \, [h_{H-1}I + \delta(x_H)s_H],$$
$$x_H \geq 0, \tag{2.4}$$
$$I + x_H \geq d_H.$$

Thereby obtaining an optimal solution as I for period 1 is specified. Characteristic 2 below establishes that it is permissible to confine consideration to only $H + 2 - t$, $t > 1$, values of I at period t.

By taking cognizance of the special properties of the model, an alternative functional equation has been formulated which has the advantage of potentially requiring less than H periods' data to obtain an optimal program; that is, it may be possible without any loss of optimality to narrow the program commitment to a shorter "planning horizon" than H periods on the sole basis of data for this horizon. Just as one may prove that in a linear programming model it suffices to investigate only basic sets of variables in search of an optimal solution, it is demonstrated that in the model an optimal solution exists among a very simple class of policies.

It is necessary to postulate that $d_i \geq 0$ is demand in period 1 net of starting inventory. Then the fundamental proposition underlying the approach asserts that it is sufficient to consider programs in which at period t one does not both place an order and bring in inventory.

Characteristics:

(1) There exists an optimal program such that $I_{x_t} = 0$ for all t (where I is inventory entering period t).

(2) There exists an optimal program such that for all t, $x_t = 0$ or $x_t = \sum_{j=t}^{k} d_j$ for some k, $t \leq k \leq N$.

(3) There exists an optimal program such that if d_{t^*} is satisfied by some $x_{t^{**}}$, $t^{**} < t^*$, then d_t, $t = t^{**} + 1, \ldots, t^* - 1$, is also satisfied by $x_{t^{**}}$.

For the particular cost structure assumed, it can be shown that an optimal policy has the property that $I_{t-1}x_t = 0$, for $t = 1, 2, \ldots, H$. That is, the requirements in a period are satisfied either entirely from procurement in the period or entirely from procurement in a prior period.

The property of an optimal solution stated above implies that we need to consider only procurement programs where $x_t = 0$, or $x_t = d_t + d_{t-1} + \ldots + d_k$, for some k, $t \leq k \leq H$.

2.5 Silver-Meal Heuristic Model

It is a simple heuristic method for selecting replenishment quantities under conditions of deterministic time-varying demand where replenishment are restricted to the beginning by a period.

It has been wished to select the order quantity Q so as to minimize the costs per unit time over the time period that Q lasts. When there is restriction to replenishments at the beginning of a period the search is restricted to a set of Q's lasting for one, two, three, etc., periods; i.e., searching is on a time variable T which can take on the values of 1, 2, 3, etc.

Symbols

Suppose the following symbols have been designed. $F(j)$ is the demand rate (assumed constant) during the j-th period (where period 1 is the period immediately following the present moment at which a replenishment decision has to be made).

$T = 1, 2, 3, \ldots$ is the decision variable, the time duration that the current replenishment quantity is to last.

R and $G(j)$ are quantities to be used in the algorithm,

S is the ordering cost in the unit of currency,

C is the unit variable cost in the unit of currency per piece,

I is the inventory carrying charge expressed as a decimal fraction per period,

$M = \dfrac{S}{CI}$.

Algorithm

The algorithm is as follows:

Step 1: Initialization

Set $T = 1$,

$R = F(1)$, and

$G(1) = M$.

Step 2

Is $T^2F(T+1) > G(T)$?

NO - go to Step 3

Yes - go to Step 4

Step 3

Set $T = T + 1$

Evaluate $R = R + F(T)$, and

$G(T) = G(T-1) + (T-1)\, F(T)$

go to Step 2

Step 4: Calculation of replenishment quantity

Q = current value of R (because R is defined in such a way that it has accumulated total demand through the end of period T).

$= \sum_{j=1}^{T} F(j)$.

The most complicated operation in the algorithm is seen to be straight multiplication of two terms or the squaring of a number.

2.6 Dixon-Silver Model

One class of "common sense" heuristics considered here was initiated by Eisenhut and could be called period-by-period heuristics. Eisenhuts procedure was later extended by Lambrecht and Vander Vaken, Dixon and Silver. In Eisenhut heuristic there is no guarantee one will find a feasible solution when only positive priority indices are considered, the reason being, that net demand in some period t, i.e., $\sum_{i=1}^{N} \max\{0, d_{it} - I_{i,t-1}\}$ may exceed available capacity.

Lambrecht and Vanderveken, Dixon and Silver both are period-by-period heuristic and based on Wagner-Whitin condition. These period-by-period heuristics have the advantages that their computation time is low. Both heuristics use the priority index which is derived from the well-known Silver-Meal heuristic for the single level uncapacitated dynamic lot-sizing problem.

Lambrecht and Vanderveken use a feedback mechanism (Backtracking) when an infeasible period is encountered, i.e. they try to shift excess demand to leftover capacity in previous periods, taking into consideration setup and holding costs, until the infeasibility in period t is removed.

Dixon and Silver, on the other hand, perform a priority (look ahead) computation of the cumulative production requirements up to period t (for all t) such that no infeasibility will arise in period $(t + 1)$.

From the comparison study of Maes and Van Wassenhove [14], backtracking procedure creates a lot of additional setups whereas in a look-ahead procedure demand to be shifted to earlier periods is incorporated in planned production lots. Indeed, when capacity constraints are tight it may not be possible to shift demand backwards such that it can be added to an already planned production lot. Instead demand may have to be split up and several extra setups may be necessary to fit everything. This explains why rather large differences between Dixon and Silver and the other heuristics occur. On the basis of the results of Maes and Van Wassenhove's [14] comparison study it can be concluded that a look ahead procedure such as the one used by Dixon-Silver should be preferred to a backtracking procedure used by Lambrecht and Vandervaken. However, when a strong trend in demand prevails, one should use a look-ahead procedure to ensure feasibility rather than relying on a backtracking routine as in Lambrecht and Vandervaken. So a good heuristic should have a look ahead mechanism to ensure feasibility at the outset and period-by-period heuristic take advantage when capacities are tight and difference in capacity absorption across products are large.

Considering these points as discussed above the Dixon and Silver heuristic is considered for further improvements in the present work.

Dixon-Silver model determines lot-sizes for a group of products that are produced at a single machine. It is assumed that the requirements for each product are known period by period, out to the end of some common time horizon. For each product there is a fixed setup cost incurred each time production takes place. Unit production and holding costs are assumed linear. The objective of the model is to determine lot-sizes so that the total costs are minimized, with no back-logging and having capacity restriction.

The input to the model would include all the costs and product data for each item, such as inventory holding cost, setup cost, setup time, production rate or capacity absorption rate, safety stock, initial inventory and ending inventory. Forecasted demand would be given for each item in each period. In addition, available capacity would be used period by period as input data. The mathematical model is presented below:

Mathematical model

$$\textit{Minimize } Z(X) = \sum_{i=1}^{N}\sum_{j=1}^{H}(S_i\delta(x_{ij}) + h_i I_{ij})$$

Subject to $I_{ij} = I_{i,j-1} + x_{ij} - D_{ij}$ $\qquad i = 1, 2, \ldots, N$ and $j = 1, 2, \ldots, H$

$I_{i0} = I_{iH} = 0$ $\qquad i = 1, 2, \ldots, N$ and $j = 1, 2, \ldots, H$

$\sum_{i=1}^{N} k_i x_{ij} \le C_j$ $\qquad j = 1, 2, \ldots, H$

$$x_{ij},\ I_{ij} \geq 0 \qquad\qquad i = 1, 2, \ldots, N \text{ and } j = 1, 2, \ldots, H$$

where N = the number of items,

H = the time horizon,

D_{ij} = the given demand for item i in period j,

I_{ij} = the inventory of item i at the end of period j (after period j production and demand satisfied),

x_{ij} = the lot-size of item i in period j,

S_i = the setup cost for item i,

h_i = the unit holding cost for item i,

k_i = the capacity absorption rate for item i,

C_j = the capacity in period j,

$$\delta(x_{ij}) = \begin{cases} 1 & if\, x_{ij} > 0 \\ 0 & if\, x_{ij} = 0 \end{cases}$$

$\delta(x_{ij})$ is a binary setup variable indicating whether a setup cost must be incurred for item i in period j or not.

Chapter 3

Development of the Model

3.1 Introduction

This chapter deals with the modification of the Dixon-Silver model with new parameters: setup time and limited lot-size per setup. The modified models are more attractive than the Dixon-Silver model since the setup time and the limited lot-size per setup would be two important parameters from management point of view. In this regard two models have been formulated. The model with setup time, its heuristic method of solution, and sample output have been presented in Section 3.1. The model with the limited lot-size per setup, its heuristic method of solution, and sample output have been presented in Section 3.2.

3.2 Lot-Size Model with Setup Time

The lot-size model with setup time included is presented below showing the mathematical model, heuristic and sample calculations. The input to the model would include all the costs and product data for each item, such as inventory holding cost, setup cost, setup time, production rate or capacity absorption rate, safety stock, initial inventory and ending inventory. Forecasted demand would be given for each item in each period. In addition, available capacity would be used period by period as input data. The mathematical model is presented below.

Mathematical model

$$Minimize\ Z(X) = \sum_{i=1}^{N}\sum_{j=1}^{H}(S_i\delta(x_{ij}) + h_i I_{ij})$$

Subject to $I_{ij} = I_{i,j-1} + x_{ij} - D_{ij}$ $\quad i = 1, 2, \ldots, N$ and $j = 1, 2, \ldots, H$

$I_{i0} = I_{iH} = 0$ $\quad i = 1, 2, \ldots, N$

$\sum_{i=1}^{N}\left[k_i x_{ij} + St_i.\delta(x_{ij})\right] \le C_j$ $\quad j = 1, 2, \ldots, H$

$x_{ij}, I_{ij} \ge 0$ $\quad i = 1, 2, \ldots, N$ and $j = 1, 2, \ldots, H$

where St_i = setup time for item i.

3.2.1 Heuristic Method of Solution

Several methods have been proposed for a solution of the multi-item constrained dynamic lot-sizing problem (DLSP). Most of these techniques have weakness or limitation that either they can not guarantee the generation of a feasible solution or become computationally prohibitive. It has been proved that even the single-item problem with constant capacity is NP-hard [8-11]. That is, it is in a class of problems that are extremely difficult to solve in a reasonable amount of time. When the setup time would be included, the problem would become strictly NP-hard. Therefore, a simple heuristic has been developed which would guarantee a feasible solution. The heuristic method of solution is presented below in steps.

Step 1 Creation of an equivalent demand matrix:

- Convert the initial demand matrix into equivalent demand matrix with the use of initial inventory, ending inventory and safety stock.
- Use the initial inventory to satisfy as much demand as possible in the first few periods. The net requirements will be that demand not satisfied by the initial inventory. During the calculation of the net demands, the amount of the safety stock should be maintained.

Let Iin_i = initial inventory for item i,

$Iend_i$ = ending inventory for item i,

$Irem_i$ = remaining initial inventory for item i, and

SS_i = safety stock for item i.

d_{ij} = equivalent demand for product i in period j.

Initially set $Irem_i = Iin_i - SS_i$ and period $j = 1$.

Then set $d_{ij} = \begin{cases} 0 & \text{if } Irem_i > D_{ij} \\ D_{ij} - Irem_i & \text{if } Irem_i \leq D_{ij} \end{cases}$.

Compute $Irem_i = Irem_i - D_{ij}$.

Set $j = j + 1$ and recycle till $Irem_i > 0$.

- Since the amount of the safety stock is always maintained, the demand in the last period H would be partially satisfied by the safety stock of the period H-1. If ending inventory is desired, then the requirements in period H should be increased by the desired ending inventory. Then

  $d_{iH} = D_{iH} + Iend_i - SS_i$.
- Compute the net demands for all $i = 1, 2, \ldots, N$.

Step 2 Check the feasibility of the problem:

Feasibility Condition:

$$\sum_{j=1}^{H} CR_j \leq \sum_{j=1}^{H} C_j,$$

where $CR_j = \sum_{i=1}^{N} k_i d_{ij}$,

CR_j = demand in terms of capacity unit for period j,

k_i = capacity absorption rate for product i.

If the feasibility condition is not satisfied, the problem is infeasible i.e. all demands cannot be met with the available capacity.

Step 3 Use the Dixon-Silver heuristic with inclusion of setup time [through steps 3.1 to 3.12]

Step 3.1

- Start at period 1, i.e. set R=1 [R = 1, 2, …, H]. When lot-sizing of period 1 is complete, then lot-sizing is started for period 2 up to period H.

Step 3.2

- Initialize lot-size x_{ij} by equalizing to demand d_{ij}, i.e.,

$$x_{ij} = d_{ij} \qquad i = 1, 2, \ldots, N \text{ and } j = 1, 2, \ldots, H.$$

Step 3.3

- Initially set the value of the time supply to one i.e., $T_i = 1$, where $i = 1, 2, \ldots, N$.

  Time supply (T_i) denotes the integer number of period requirements that this lot will exactly satisfy.

Step 3.4

- Produce $d_{iR} > 0$, in the lot-sizing period R, where $i = 1, 2, \ldots, N$.
- After producing d_{iR} calculate remaining capacity in period R, denoted by RC_R, by

$$RC_R = C_R - \sum_{i=1}^{N} k_i \, d_{iR} \,.$$

- Let I'_{ij} be the amount of inventory at the end of period j for item i, resulting from only the currently scheduled production in period R. Initialize I'_{ij} with zero, i.e.,

$$I'_{ij} = 0, \qquad i = 1, 2, \ldots, N \text{ and } j = 1, 2, \ldots, H.$$

Step 3.5

- Let AP_j be the amount of inventory (in capacity units) resulted from the production of period R that will be used in period j. Then

$$AP_j = \sum_{i=1}^{N} k_i \left(I'_{i,j-1} - I'_{i,j} \right).$$

- Let CR_j be the total demand (in capacity units) in period j. Then

$$CR_j = \sum_{i=1}^{N} k_i d_{ij}.$$

- The production plan for period R is feasible if and only if the following condition is satisfied for $t = 2,\ldots, H$.

$$\sum_{j=R+1}^{R+t-1} AP_j \geq \sum_{j=R+1}^{R+t-1} (CR_j - C_j + St_j).$$

- Determine the earliest period t_c at which the above feasibility constraint is not satisfied, i.e.,

$$t_c = \min\left\{ t \mid \sum_{j=R+1}^{R+t-1} AP_j < \sum_{j=R+1}^{R+t-1} (CR_j - C_j + St_j)\right\}.$$

  To remove infeasibility upto t_c, extra amount is to be produced with the use of remaining capacity RC_R of period R.

  If there is no infeasibility, set $t_c = H + 1$.

Step 3.6

- Consider only items i' which have

  (1) $T_{i'} < t_c$,

  (2) RC_R is sufficient to produce $d_{i',R+T_{i'}}$, and

  (3) $d_{i',R+T_{i'}} > 0$.

  To decide the best item (from a cost standpoint) to be produced in period R, calculate the priority index $U_{i'}$ for all of these items, where

$$U_{i'} = \frac{AC(T_{i'}) - AC(T_{i'}+1)}{k_{i'} d_{i',T_{i'}+1}}, \text{ and} \tag{3.1}$$

$$AC(T_{i'}) = \left\{ S_{i'} + h_{i'} \sum_{j=R}^{R+T_{i'}-1} (j-R) d_{i'j} \right\} / T_{i'}.$$

  Among these find the one, denoted by i, that has the largest U_i.

- U_i is the marginal decrease in average costs per unit of capacity absorbed.

- $AC(T_i)$ is average cost per unit time of a lot of item i which will satisfy T_i periods' requirements. This is from the Silver-Meal model in which future setup cost may be saved at the expense of added inventory holduingcost.

Step 3.7

- Check the value of U_i.

(a) If $U_i > 0$, then it is economic to produce $d_{i,R+T_i}$ in period R.

Increase the value of lot-size x_{iR} and inventory I'_{ij} by $d_{i,R+T_i}$, i.e.,

$$x_{iR} = x_{iR} + d_{i,R+T_i}$$

$$I'_{ij} = I'_{ij} + d_{i,R+T_i} \qquad j = R+1, \ldots, R+T_i.$$

Decrease the value of lot-size $x_{i,R+T_i}$, demand $d_{i,R+T_i}$ and remaining capacity RC_R by $d_{i,R+T_i}$, i.e., set

$$x_{i,R+T_i} = x_{i,R+T_i} - d_{i,R+T_i}$$

$$d_{i,R+T_i} = d_{i,R+T_i} - d_{i,R+T_i} = 0$$

$$RC_R = RC_R - d_{i,R+T_i}.$$

- Set $T_i = T_i + 1$ and continue from Step 3.5.

(b) If $U_i \leq 0$, then it is not economic to increase T_i of any item, because of the increase of the total cost.

- Check the value of t_c.

  (i) If $t_c > H$, then no infeasibilities left and lot-sizing of the current period is complete. Go to Step 3.12.

  (ii) If $t_c < H$, there are infeasibilities and production of one or more item is to be increased and it is done through Steps 3.8 to 3.11.

Step 3.8

- Calculate the value of Q, where

$$Q = \max_{R+t_c-1 \leq t \leq H} \left[\sum_{j=R+1}^{t} \left(CR_j - (C_j - St_j) - AP_j \right) \right].$$

- Q is the amount of production still needed in the current period to eliminate infeasibilities in the later period because the available capacity is not sufficient to meet the demands of those periods.

Step 3.9

- Consider only items i' for which
  i. $T_{i'} < t_c$, and
  ii. $d_{i',R+T_{i'}} > 0$.

  To decide the best item (from a cost standpoint) to be produced in period R, calculate the priority index $\Delta_{i'}$ for all of these items, where

$$\Delta_{i'} = \frac{AC(T_{i'}+1) - AC(T_{i'})}{k_{i'} d_{i',T_{i'}+1}}.$$

- Find the one, denoted by i, that has the smallest Δ_i.

Steps 3.10

- Let $W = k_i d_{i,R+T_i}$.
- Compare the value of Q with W.

(a) If $Q > W$,

Increase the value of lot-size x_{iR}, and inventory I'_{ij} by $d_{i,R+T_i}$, i.e.,

$$x_{iR} = x_{iR} + d_{i,R+T_i}$$

$$I'_{ij} = I'_{ij} + d_{i,R+T_i} \qquad j = R+1, \ldots, R+T_i.$$

Decrease the value of lot-size $x_{i,R+T_i}$, demand $d_{i,R+T_i}$ and remaining capacity RC_R by $d_{i,R+T_i}$, i.e.,

$$x_{i,R+T_i} = x_{i,R+T_i} - d_{i,R+T_i}$$

$$d_{i,R+T_i} = d_{i,R+T_i} - d_{i,R+T_i} = 0$$

$$RC_R = RC_R - d_{i,R+T_i}.$$

Set $Q = Q - W$ and $T_i = T_i + 1$.

Continue from Step 3.9.

(b) If $Q \leq W$,

Set $IQ = \left\lceil \frac{Q}{k_i} \right\rceil$.

Increase the value of lot-size x_{iR} and inventory I'_{ij} by IQ, i.e.,

$$x_{iR} = x_{iR} + IQ$$

$$I'_{ij} = I'_{ij} + IQ.$$

Decrease the value of lot-size $x_{i,R+T_i}$ and demand $d_{i,R+T_i}$ by IQ, i.e.,

$$x_{i,R+T_i} = x_{i,R+T_i} - IQ$$

$$d_{i,R+T_i} = d_{i,R+T_i} - IQ.$$

Step 3.11

- Set $R = R + 1$.
- Check the value of R.
  - (a) If $R < H$, then continue from Step 3.3.
  - (b) If $R > H$, lot-sizing is complete up to period H.

Step 3.12

- Calculate the values of
  - i. Forecasted machine time required/period.
  - ii. Total expected setup cost.
  - iii. Total expected inventory holding cost.
  - iv. Total expected safety stock cost.
- Stop.

The corresponding flowchart has been given in Appendix A.

3.2.2 Sample Output with Setup Time

To illustrate the algorithm a few sample calculations for the period 1 have been shown. The relevant product data are depicted in Table 3.1. Forecasted demand and capacity are depicted in Table 3.2.

Table 3.1 Relevant product data.

Item No (i)	Holding Cost (h_i)	Setup Cost (S_i)	Setup Time (St_i)	Production Rate ($1/k_i$)	Safety Stock (SS_i)	Initial Inventory (Iin_i)	Ending Inventory ($Iend_i$)
01	0.0167	322.0	1.40	524	0	19320	18893
02	0.0167	81.0	2.00	349	10602	200180	124225
03	0.0167	124.0	1.00	245	4577	24460	43294
04	0.0167	124.0	1.50	172	1974	23260	21757
05	0.0167	81.0	0.25	349	7581	55489	92168
06	0.0167	124.0	0.70	245	4861	-2727	44394
07	0.0167	124.0	0.50	172	2026	9659	8466
08	0.0167	105.0	1.20	847	11117	29705	40273
09	0.0167	105.0	0.40	464	9533	11362	84717
10	0.0167	106.0	0.60	575	20417	242944	227344
11	0.0167	105.0	1.00	1261	16634	324215	271627
12	0.0167	105.0	1.30	663	9794	45439	69068

Table 3.2 Forecasted demand and capacity.

Item No	Period											
	1	2	3	4	5	6	7	8	9	10	11	12
01	11456	11456	10501	13365	13365	11456	8592	1909	1909	1909	4773	4773
02	53124	53124	48697	61977	61977	53124	39842	8854	8854	8854	22135	22135
03	18099	18099	16591	21116	21116	18099	13574	3016	3016	3016	7541	7541
04	9250	9250	8480	10792	10792	9250	6938	1542	1542	1542	3854	3854
05	39546	39546	36250	46137	46137	39546	29659	6591	6591	6591	16478	16478
06	18363	18363	16833	21423	21423	18363	13772	3060	3060	3060	7651	7651
07	4976	4976	4562	5806	5806	4976	3732	829	829	829	2074	2074
08	41690	41690	38216	48638	48638	41690	31267	6948	6948	6948	17371	17371
09	32816	32816	30081	38285	38285	32816	24612	5469	5469	5469	13673	13673
10	96745	96745	88683	112868	112868	96745	72559	16124	16124	16124	40310	40310
11	119220	119220	109285	139088	139088	119220	89415	19870	19870	19870	49675	49675
12	27715	27715	25405	32333	32333	27715	20786	4619	4619	4619	11548	11548
	Available Machine Hours											
	706	729	729	706	729	706	729	729	660	729	706	729

Table 3.3 depicts the equivalent demand after considering initial inventory, ending inventory and safety stock.

As for example, consider Item 2.

From Table 3.1, initial inventory, ending inventory and safety stock for Item 2 are

$Iin_2 = 200180$, $Iend_2 = 124225$, and $SS_2 = 10602$.

Initially set $Irem_2 = Iin_2 - SS_2 = 200180 - 10602 = 189578$.

From Table 3.2, $D_{21} = 53124$. Since $Irem_2 > D_{21}$, we have

$d_{21} = 0$.

Then $Irem_2 = Irem_2 - D_{21} = 189578 - 53124 = 136454$.

From Table 3.2, $D_{22} = 53124$. Since $Irem_2 > D_{22}$, we have

$d_{22} = 0$.

Similarly, compute $d_{23} = 0$, and $d_{24} = 27344$.

Finally, $d_{2,12} = D_{2,12} + Iend_2 - SS_2 = 22135 + 124225 - 10602 = 135758$.

Table 3.3 Equivalent demand matrix with the use of initial inventory, ending inventory and safety stock.

Item No	Period											
	1	2	3	4	5	6	7	8	9	10	11	12
01	0	3592	10501	13365	13365	11456	8592	1909	1909	1909	4773	23666
02	0	0	0	27344	61977	53124	39842	8854	8854	8854	2135	135758
03	0	16315	16591	21116	21116	18099	13574	3016	3016	3016	7541	46258
04	0	0	5694	10792	10792	9250	6938	1542	1542	1542	3854	23637
05	0	31184	36250	46137	46137	39546	29659	6591	6591	6591	16478	101065
06	25951	18363	16833	21423	21423	18363	13772	3060	3060	3060	7651	47184
07	0	2319	4562	5806	5806	4976	3732	829	829	829	2074	8514
08	23102	41690	38216	48638	48638	41690	31267	6948	6948	6948	7371	46527
09	30987	32816	30081	38285	38285	32816	24612	5469	5469	5469	13673	88857
10	0	0	59646	112868	112868	96745	72559	16124	16124	6124	0310	247237
11	0	0	40144	139088	139088	119220	89415	19870	19870	19870	9675	304668
12	0	19785	25405	32333	32333	27715	20786	4619	4619	4619	1548	70822

Initialize lot-size by the equivalent demand of Table 3.3.

$x_{ij} = d_{ij}$ for all $1 \le i, j \le 12$.

Let us consider lot-sizing for period 1. Then $R = 1$.

Initially set each item i's time supply at one period, i.e.,

$$T_i = 1 \quad \text{for } i = 1, 2, \cdots, 12.$$

The remaining capacity of period one RC_1 after satisfying the demands of period one is

$$RC_1 = C_1 - \sum_{i=1}^{12} k_i d_{i,1} - \sum_{i=1}^{12} St_i$$

$$= 706 - 199.98 - 2.3$$

$$= 503.72$$

Determine the earliest period t_c at which the feasibility constraint are not satisfied, i.e.,

$$t_c = \min\{\ t \mid \sum_{j=2}^{t} AP_j < \sum_{j=2}^{t} (CR_j - C_j + St_j)\ \}.$$

Set $t = 2, 3, \ldots$, and compute the values of the summations.

t	$\sum_{j=2}^{t} AP_j$	$\sum_{j=2}^{t} (CR_j - C_j + St_j)$
2	0.0	-321.231
3	0.0	-436.586
4	0.0	-129.260
5	0.0	254.301

From the values of the above table, $t_c = 5$ is obtained. Thus the total demand exceeds the capacity available in Period 5. Then some of the requirements of Period 5 must be satisfied by production in the preceding periods.

Consider items i with $T_i < t_c$ and for which RC_1 is sufficient to produce d_{i,T_i+1}.

i	U_i
1	19.1112
3	-1.1147
5	-2.4609
6	-1.2185
7	3.1623
8	-6.0058
9	-3.1321
12	-3.7768

Among these, item 1 has the largest $U_1 = 19.1112$ which is greater than zero.

Then the remaining capacity in period 1 is

$$RC_1 = RC_1 - k_1 d_{1,2} - St_1$$

$$= 503.72 - 1/524 \times 3592 \text{ - } 1.4$$

$$= 495.465$$

The lot-sizes of Periods 1 and 2 are

$$x_{11} = x_{11} + d_{12}$$

$$= 0 + 3592$$

$$= 3592, \text{ and}$$

$$x_{12} = x_{12} \text{ - } d_{12}$$

$$= 3592 - 3592$$

$$= 0.$$

I'_{11} is the inventory at the end of period 1 for item 1 resulting from only the currently scheduled production in period 1. Then

$$I'_{11} = I'_{11} + d_{12}$$

$$= 0 + 3592$$

$$= 3592.$$

Finally, the demand at period 2 for item 1 is set to zero, that is,

$$d_{12} = 0.$$

Set $T_1 = T_1 + 1 = 2$, and go to Step 3 of the algorithm to calculate t_c again.

New $t_c = 5$, and the item i which has the largest U_i is 7, that is, $i = 7$.

Then the remaining capacity in period 1 is

$$RC_1 = RC_1 - k_7 d_{7,2} - St_7$$

$$= 495.465 - 1/172 \times 2319 - 0.5$$

$$= 481.4826$$

Calculate $\quad x_{71} = 2319,$

$$x_{72} = 0,$$

$I'_{71} = 2319$, and

$d_{72} = 0$.

Set $T_7 = T_7 + 1 = 2$, and go to Step 3 of the algorithm to calculate t_c again.

New $t_c = 5$, and there is no item with positive U_i.

Calculate $Q = \max_{t_c \le t \le H}\left[\sum_{j=2}^{t}(CR_j - C_j + St_j - AP_j)\right]$. Then the following table is generated.

t	$\left[\sum_{j=2}^{t}(CR_j - C_j + St_j - AP_j)\right]$
5	232.0636
6	481.3836
7	471.8321
8	-88.0792
9	-578.9907
10	-1138.9020
11	-1439.9390
12	159.8113

Therefore, Q = 481.3836.

Consider items i with $T_i < t_c$ and for which $d_{i,T_i+1} > 0$. The following table is generated.

i	Δ_i
1	3.1559
3	1.1147
5	2.4609
6	1.2185
7	1.1357
8	6.0058
9	3.1321
12	3.7768

Among these, Item 3 has the smallest $\Delta_3 = 1.1147$. Then the capacity needed to produce d_{32} in period 1 is

$$W = k_3 d_{32}$$

$$= 1/245 \times 16315$$

$= 66.592$

The lot-sizes of Periods 1 and 2 are

$$x_{31} = x_{31} + d_{32}$$

$$= 0 + 16315$$

$$= 16315, \text{ and}$$

$$x_{32} = x_{32} - d_{32}$$

$$= 0.$$

Finally, the demand at period 2 for item 3 is set to zero, that is,

$$d_{32} = 0.$$

Set $T_3 = T_3 + 1 = 2$, and

$$Q = Q - W$$

$$= 481.3836 - 66.592$$

$$= 414.792.$$

Since Q > 0, continue from Step 8 of the algorithm.

Q	i	Δ_i	W	T_i	$x_{i,1}$	d_{i,T_i+1}	New $x_{i,1}$	x_{i,T_i+1}
414.792	7	1.13574	26.523	2	2319	4562	6881	0
388.269	6	1.21854	74.951	1	25951	18363	44314	0
313.318	7	1.84818	33.756	3	6881	5806	12687	0
279.562	7	2.11425	33.756	4	12687	5806	18493	0
245.806	3	2.42248	67.718	2	16315	16591	32906	0
178.088	6	2.42687	68.706	2	44314	16833	61147	0
109.381	5	2.46089	89.352	1	0	31184	31184	0
20.029	3	2.94873	20.029	3	32906	21116	37814	16208

In the last row of the above table, the remaining production (in capacity units) required to eliminate all infeasibilities is

Q = 20.029.

Then the remaining production (in lot units) is

$$\left\lceil \frac{Q}{k_3} \right\rceil = \left\lceil \frac{20.029}{1/245} \right\rceil = 4908.$$

Table 3.4 The lot sizes after R=1.

ItemNo	Period											
	1	2	3	4	5	6	7	8	9	10	11	12
01	3592	0	10501	13365	13365	11456	8592	1909	1909	1909	4773	23666
02	0	0	0	27344	61977	53124	39842	8854	8854	8854	22135	135758
03	37814	0	0	16208	21116	18099	13574	3016	3016	3016	7541	46258
04	0	0	5694	10792	10792	9250	6938	1542	1542	1542	3854	23637
05	31184	0	36250	46137	46137	39546	29659	6591	6591	6591	16478	101065
06	61147	0	0	21423	21423	18363	13772	3060	3060	3060	7651	47184
07	18493	0	0	0	0	4976	3732	829	829	829	2074	8514
08	23102	41690	38216	48638	48638	41690	31267	6948	6948	6948	17371	46527
09	30987	32816	30081	38285	38285	32816	24612	5469	5469	5469	13673	88857
10	0	0	59646	112868	112868	96745	72559	16124	16124	16124	40310	247237
11	0	0	40144	139088	139088	119220	89415	19870	19870	19870	49675	304668
12	0	19785	25405	32333	32333	27715	20786	4619	4619	4619	11548	70822

3.3 Model with the Limited Lot-Size Per Setup

The lot-size model with the limited lot-size per setup is presented below showing the mathematical model, heuristic and sample calculations. Like the previous model, the input would include all the cost and product data for each item, such as inventory holding cost, setup cost, the limited lot-size per setup, production rate or capacity absorption rate, safety stock, initial inventory and ending inventory. Forecasted demand would be given for each item in each period. In addition, available capacity would be used period by period as input data. It is to be noted that Dixon-Silver heuristic allows only one setup for each item in each period. But the limitation on lot-size may need more than one setup in a particular period. So should this limitation be incorporated into Dixon-Silver heuristic, each time an item when processed in a new setup is to be considered a new item. This may call for splitting an item into several new items in a particular period. However, the maximum number of the new splitted items will be restricted by the maximum periodical demand of the item. As for example, for the ith item if the maximum periodic demand and the limited lot-size be respectively $d_{\max i}$ and $x_{\max i}$, the number of new items will be $n_i = \left\lceil \frac{d_{\max i}}{x_{\max i}} \right\rceil - 1$. Thus the total number of new items will be $\sum_{i=1}^{N} n_i$, where

N is the number of items. So after meeting the lot-size limitation, the total number of items to be considered in the model should be $N' = N + \sum_{i=1}^{N} n_i$.

In view of the above discussions, the model may now be presented as follows.

Mathematical Model

$$\textit{Minimize } Z(X) = \sum_{i=1}^{N'} \sum_{j=1}^{H} (S_i \delta(x_{ij}) + h_i I_{ij})$$

Subject to $I_{ij} = I_{i,j-1} + x_{ij} - D_{ij}$ $\quad i = 1, 2, \ldots, N'$ and $j = 1, 2, \ldots, H$

$I_{i0} = I_{iH} = 0$ $\quad i = 1, 2, \ldots, N'$

$\sum_{i=1}^{N'} k_i x_{ij} \le C_j$ $\quad j = 1, \ldots, H$

$0 \le x_{ij} \le x_{\max i.}$ $\quad i = 1, 2, \ldots, N'$ and $j = 1, 2, \ldots, H$

$I_{ij} \ge 0$ $\quad i = 1, 2, \ldots, N'$ and $j = 1, 2, \ldots, H$

where N' = number of total items after meeting the maximum lot-size limitation

$= N + \sum_{i=1}^{N} n_i$, $\quad n_i = \left\lceil \frac{d_{\max i}}{x_{\max i}} \right\rceil - 1$. where

$d_{\max i}$ = maximum periodic demand for the ith item.

$x_{\max i}$ = the limited lot-size for item i which cannot be exceeded in any period.

3.3.1 Heuristic Method of Solution

The original two-item problem with constant capacity is NP-hard. In the present work a new constraint on upper limit of the limited lot-size is considered. With this new constraint the problem is also NP-hard. Therefore, a simple heuristic has been developed which guarantees a feasible solution.

Step 1 Creation of an equivalent demand matrix:

- Using the same technique of Step 1 of Section 3.2.1, the given $N \times H$ demand matrix is converted into an equivalent $N \times H$ demand matrix with the use of initial inventory, ending inventory and safety stock.

Step 2 Check the feasibility of the problem:

- The feasibility of the problem for N items is checked using the same formulas of Step 1 of Section 3.1.2.

Step 3 Convert the multi-setup problem into single setup problem [through steps 3.1 and 3.2]

Step 3.1

- Find the maximum demand $d_{\max i}$ for each item i by using the formula
$$d_{\max i} = \max \{d_{ij} \mid j = 1, 2, \ldots, H\}.$$
- Find the number of new items n_i to be considered to satisfy demand $d_{\max i}$ by using the formula
$$n_i = \left\lceil \frac{d_{\max i}}{x_{\max i}} \right\rceil - 1.$$

Then the number of total items after limiting the lot-size is
$$N' = N + \sum_{i=1}^{N} n_i .$$

Item i is splitted into $n_i + 1$ items. Let the new items are $i_0, i_1, \ldots, i_{n_i}$.

Initially set $d_{\text{rem } ij} = d_{ij}$ and $l = 0$.

Then set $d_{i_l, j} = \begin{cases} d_{rem\, ij} & if\ d_{rem\, ij} \le x_{\max i} \\ x_{\max i} & if\ d_{rem\, ij} > x_{\max i} \end{cases}$.

Compute $d_{rem\, ij} = \begin{cases} 0 & if\ d_{rem\, ij} \le x_{\max i} \\ d_{rem\, ij} - x_{\max i} & if\ d_{rem\, ij} > x_{\max i} \end{cases}$.

Set $l = l + 1$ and recycle up to $l = n_i$.

- Now the equivalent demand matrix $N \times H$ is converted into a new demand matrix $N' \times H$.

Step 3.2

- Initialize the values of setup cost, holding cost and capacity absorption rate for the N' new items from that of the N items by using the formulas

$$S_{i_0} = S_{i_1} = \ldots = S_{i_{ni}} = S_i,$$

$$h_{i_0} = h_{i_1} = \ldots = h_{i_{ni}} = h_i,$$

$$k_{i_0} = k_{i_1} = \ldots = k_{i_{ni}} = k_i.$$

Step 4 Apply the Dixon-Silver heuristic with inclusion of the limited lot-size per setup [through Steps 4.1 to 4.13]

Step 4.1

- Start at period 1, i.e. set $R = 1$ [$R = 1, 2, \ldots\ldots, H$]
- After completing the lot-sizing of period 1, the lot-sizing of period 2 is started.

Step 4.2

- Initialize lot-size x_{ij} by equalizing to demand d_{ij}, i.e.,

$$x_{ij} = d_{ij} \qquad i = 1, 2, \ldots, N' \text{ and } j = 1, 2, \ldots, H.$$

- Calculate remaining allowable amount that can be produced by the following equation.

$$x_{\text{rem } ij} = x_{\max i} - x_{ij} \qquad i = 1, 2, \ldots, N' \text{ and } j = 1, 2, \ldots, H.$$

where

$x_{\text{rem } ij}$ = remaining allowable amount that can be produced if x_{ij} is produced at period j for item i.

Step 4.3

- Initially set the value of time supply to one i.e. $T_i = 1$, where $i = 1, 2, \ldots, N'$.

Time supply T_i denote the integer number of periods requirements that this lot will exactly satisfy.

Step 4.4

- For each item i, $i = 1, 2, \ldots, N'$, produce d_{iR} (> 0) in the lot-sizing period R.
- After producing d_{iR} calculate remaining capacity in period R, denoted by RC_R, by

$$RC_R = C_R - \sum_{i=1}^{N'} k_i d_{iR}.$$

- Let I'_{ij} be the amount of inventory at the end of period j for item i, resulting from only the currently scheduled production in period R. Initialize I'_{ij} with zero, i.e.,

$$I'_{ij} = 0, \qquad i = 1, 2, \ldots, N' \text{ and } j = 1, 2, \ldots, H.$$

Step 4.5

- Let AP_j be the amount of inventory (in capacity units) resulted from the production of period R that will be used in period j. Then

$$AP_j = \sum_{i=1}^{N'} k_i \left(I'_{i,j-1} - I'_{i,j}\right).$$

- Let CR_j be the total demand (in capacity units) in period j. Then

$$CR_j = \sum_{i=1}^{N'} k_i d_{ij}.$$

- The production plan for period R is feasible if and only if the following condition is satisfied for $t = 2, \ldots, H$.

$$\sum_{j=R+1}^{R+t-1} AP_j \geq \sum_{j=R+1}^{R+t-1} \left\{ CR_j - C_j \right\}.$$

- Determine the earliest period t_c at which the above feasibility constraint is not satisfied, i.e.,

$$t_c = \min\left\{ t \mid \sum_{j=R+1}^{R+t-1} AP_j < \sum_{j=R+1}^{R+t-1} (CR_j - C_j) \right\}.$$

  To remove infeasibility upto t_c, extra amount is to be produced with the use of remaining capacity RC_R of period R.

  If there is no infeasibility, set $t_c = H + 1$.

Step 4.6

- Consider only items i' which have

(1) $T_{i'} < t_c$,

(2) RC_R is sufficient to produce x_{can},

where $x_{can} = \min\{d_{i',R+T_{i'}}, x_{rem\ i'R}\}$, and

(3) $x_{can} > 0$.

By equation (1) find the item, denoted by i, that has the largest U_i.

Step 4.7

- Check the value of U_i.

(a) If $U_i > 0$, then it is economic to produce x_{can} in period R.

Increase the value of lot-size x_{iR}, inventory I'_{ij} and $x_{\text{rem}\ i,R+Ti}$ by x_{can}, i.e., set

$$x_{iR} = x_{iR} + x_{can}$$

$$I'_{ij} = I'_{ij} + x_{can} \qquad j = R+1, \ldots, R+T_i$$

$$x_{\text{rem}\ i,R+Ti} = x_{\text{rem}\ i,R+Ti} + x_{can}.$$

Decrease the value of lot-size $x_{i,R+T_i}$, demand $d_{i,R+T_i}$, remaining capacity RC_R and $x_{\text{rem}\ iR}$ by x_{can}, i.e., set

$$x_{i,R+T_i} = x_{i,R+T_i} - x_{can}$$

$$d_{i,R+T_i} = d_{i,R+T_i} - x_{can}$$

$$RC_R = RC_R - x_{can}$$

$$x_{\text{rem}\ iR} = x_{\text{rem}\ iR} - x_{can}.$$

- Set $T_i = T_i + 1$ and continue from Step 4.5.

(b) If $U_i \leq 0$, then it is not economic to increase T_i of any item (total cost increases).

- Check the value of t_c.

  (i) If $t_c > H$, then no infeasibilities left and lot-sizing of the current period is complete. Go to Step 4.12.

  (ii) If $t_c < H$, there are infeasibilities and production of one or more item is to be increased and it is done through Steps 4.8 to 4.11.

Step 4.8

- Calculate the value of Q, where

$$Q = \max_{R+t_c-1 \le t \le H} \left[\sum_{j=R+1}^{t} \left(CR_j - C_j - AP_j \right) \right].$$

- Q is the amount of production still needed in the current period to eliminate infeasibilities in the later period because the available capacity is not sufficient to meet the demands of those periods.

Step 4.9

- Consider only items i' for which
  i. $T_{i'} < t_c$,
  ii. RC_R is sufficient to produce x_{can},
  where $x_{can} = \min\{d_{i',R+T_{i'}}, x_{rem\ i'R}\}$, and
  iii. $x_{can} > 0$.

  To decide the best item (from a cost standpoint) to be produced in period R, calculate the priority index $\Delta_{i'}$ for all of these items, where

$$\Delta_{i'} = \frac{AC(T_{i'}+1) - AC(T_{i'})}{k_{i'} d_{i',T_{i'}+1}}.$$

- Among these find the one, denoted by i, that has the smallest Δ_i.

Steps 4.10

- Let $W = k_i x_{can}$.
- Compare the value of Q with W.

(a) If $Q > W$,

Increase the value of lot-size x_{iR}, inventory I'_{ij} and $x_{rem\ i',R+T_i}$ by x_{can}, i.e., set

$x_{iR} = x_{iR} + x_{can}$

$I'_{ij} = I'_{ij} + x_{can}$ $\qquad j = R+1, \ldots, R+T_i$

$x_{\mathrm{rem}\ i,R+Ti} = x_{\mathrm{rem}\ i,R+Ti} + x_{can}$.

Decrease the value of lot-size $x_{i,R+T_i}$, demand $d_{i,R+T_i}$, remaining capacity RC_R and $x_{\mathrm{rem}\ iR}$ by x_{can}, i.e., set

$x_{i,R+T_i} = x_{i,R+T_i} - x_{can}$

$d_{i,R+T_i} = d_{i,R+T_i} - x_{can}$

$RC_R = RC_R - x_{can}$

$x_{\text{rem } iR} = x_{\text{rem } iR} - x_{can}$.

Set $Q = Q - W$ and $T_i = T_i + 1$, and continue from Step 4.9.

(b) If $Q \le W$,

Set $IQ = \left\lceil \frac{Q}{k_i} \right\rceil$.

Increase the value of lot-size x_{iR}, inventory I'_{ij} and $x_{rem\ i',R+T_i}$ by IQ, i.e., set

$x_{iR} = x_{iR} + IQ$

$I'_{ij} = I'_{ij} + IQ$. $\qquad j = R+1, \ldots, R+T_i$

$x_{\text{rem } i,R+Ti} = x_{\text{rem } i,R+Ti} + IQ$.

Decrease the value of lot-size $x_{i,R+T_i}$, demand $d_{i,R+T_i}$ and $x_{rem\ i'R}$ by IQ, i.e., set

$x_{i,R+T_i} = x_{i,R+T_i} - IQ$

$d_{i,R+T_i} = d_{i,R+T_i} - IQ$

$x_{\text{rem } iR} = x_{\text{rem } iR} - IQ$.

Step 4.11

- Set $R = R + 1$.
- Check the value of R.

  (a) If $R < H$, then continue from Step 4.3.

  (b) If $R > H$, lot-sizing is complete up to period H for N' items.

Step 4.12

- Convert the $N' \times H$ lot-sizing matrix into $N \times H$ lot-sizing matrix by applying the formula

  $x_{i,j} = \sum_{l=0}^{n_i} x_{i_l,j}$.

Step 4.13

- Calculate the values of

  i. Forecasted machine time required/period.

ii. Total expected setup cost.

iii. Total expected inventory holduingcost.

iv. Total expected safety stock cost.

- Stop.

The corresponding flowchart has been given in Appendix B.

3.3.2 Sample Output with the Limited Lot-size per Setup

To illustrate the algorithm a few sample calculations for the period 1 have been shown. The relevant product data are depicted in Table 3.5. Forecasted demand and capacity are depicted in Table 3.2. Table 3.3 depicts the demand after considering initial inventory, ending inventory and safety stock.

Table 3.5 Relevant Product data for the limited lot-size per setup.

Item No	Holding Cost	Setup Cost	Maximum Lot-Size	Production Rate	Safety Stock	Initial Inventory	Ending Inventory
01	0.0167	322.0	6000	524	0	19320	18893
02	0.0167	81.0	60000	349	10602	200180	124225
03	0.0167	124.0	68000	245	4577	24460	43294
04	0.0167	124.0	29000	172	1974	23260	21757
05	0.0167	81.0	49000	349	7581	55489	92168
06	0.0167	124.0	68000	245	4861	-2727	44394
07	0.0167	124.0	44000	172	2026	9659	8466
08	0.0167	105.0	41000	847	11117	29705	40273
09	0.0167	105.0	32000	464	9533	11362	84717
10	0.0167	106.0	185000	575	20417	242944	227344
11	0.0167	105.0	150000	1261	16634	324215	271627
12	0.0167	105.0	97000	663	9794	45439	69068

The maximum periodic demand for item 1 is

$$d_{\max 1} = \max \{d_{1j} \mid j = 1, 2, \ldots, H\}$$

$$= \max\{0, 3592, 10501, 13365, 13365, 11456, 8592, 1909, 1909, 1909, 4773, 23666\}$$

$$= 23666$$

The limited lot-size for item 1 is $x_{\max 1} = 6000$.

Then the number of new items to be considered to satisfy demand $d_{\max 1}$ is

$$n_1 = \left\lceil \frac{d_{\max 1}}{x_{\max 1}} \right\rceil - 1 = \left\lceil \frac{23666}{6000} \right\rceil - 1 = 4 - 1 = 3$$

Similarly, the number of new items to be considered to satisfy demands $d_{\max i}$ are

n_1	n_2	n_3	n_4	n_5	n_6	n_7	n_8	n_9	n_{10}	n_{11}	n_{12}
3	2	0	0	2	0	0	1	2	1	2	0

Then the number of total items after limiting the lot-size is

$$N' = N + \sum_{i=1}^{N} n_i = 12 + 13 = 25.$$

Item 1 is splitted into $n_1 + 1 = 4$ items. Let the new items are 1_0, 1_1,..., 1_3.

Let us consider for period $j = 5$.

Then $d_{15} = 13365$, and $x_{\max 1} = 6000$.

Initially set $drem_{1,5} = d_{15} = 13365$ and $l = 0$.

Since $drem_{1,5} > x_{\max 1}$,

set $d_{1_0,5} = x_{\max 1} = 6000$,

$drem_{1,5} = drem_{1,5} - x_{\max 1} = 13365 - 6000 = 7365$,

$l = l + 1 = 0 + 1 = 1$.

Now recycle the same calculation for $l = 1$.

Since $drem_{1,5} > x_{\max 1}$,

set $d_{1_1,5} = x_{\max 1} = 6000$,

$drem_{1,5} = drem_{1,5} - x_{\max 1} = 7365 - 6000 = 1365$,

$l = l + 1 = 1 + 1 = 2$.

Now recycle the same calculation for $l = 2$.

Since $drem_{1,5} < x_{\max 1}$,

set $d_{1_2,5} = drem_{1,5} = 1365$,

$drem_{1,5} = 0$,

$l = l + 1 = 2 + 1 = 3$.

Now recycle the same calculation for $l = 3$.

Since $drem_{1,5} < x_{\max 1}$,

$$\text{set } d_{1_3,5} = drem_{1,5} = 0,$$

$$drem_{1,5} = 0,$$

$$l = l + 1 = 3 + 1 = 4.$$

Since $l > n_1 = 3$, the demand calculation for new items corresponding to the demand d_{15} has been finished.

Continue the same calculation for other demands. From Table 3.3, the new demand matrix for N'=25 items can be obtained as shown in Table 3.6.

Table 3.6 Demand after considering limitation on tne maximum allowable lot-size.

Item No	Period 1	2	3	4	5	6	7	8	9	10	11	12
01$_0$	0	3592	6000	6000	6000	6000	6000	1909	1909	1909	4773	6000
02$_0$	0	0	0	27344	60000	53124	39842	8854	8854	8854	22135	60000
03$_0$	0	16315	16591	21116	21116	18099	13574	3016	3016	3016	7541	46258
04$_0$	0	0	5694	10792	10792	9250	6938	1542	1542	1542	3854	23637
05$_0$	0	31184	36250	46137	46137	39546	29659	6591	6591	6591	16478	49000
06$_0$	25951	18363	16833	21423	21423	18363	13772	3060	3060	3060	7651	47184
07$_0$	0	2319	4562	5806	5806	4976	3732	829	829	829	2074	8514
08$_0$	23102	41000	38216	41000	41000	41000	31267	6948	6948	6948	17371	41000
09$_0$	30987	32000	30081	32000	32000	32000	24612	5469	5469	5469	13673	32000
10$_0$	0	0	59646	112868	112868	96745	72559	16124	16124	16124	40310	185000
11$_0$	0	0	40144	139088	139088	119220	89415	19870	19870	19870	49675	150000
12$_0$	0	19785	25405	32333	32333	27715	20786	4619	4619	4619	11548	70822
01$_1$	0	0	4501	6000	6000	5456	2592	0	0	0	0	6000
01$_2$	0	0	0	1365	1365	0	0	0	0	0	0	6000
01$_3$	0	0	0	0	0	0	0	0	0	0	0	5666
02$_1$	0	0	0	0	1977	0	0	0	0	0	0	60000
02$_2$	0	0	0	0	0	0	0	0	0	0	0	15758
05$_1$	0	0	0	0	0	0	0	0	0	0	0	49000
05$_2$	0	0	0	0	0	0	0	0	0	0	0	3065
08$_1$	0	690	0	7638	7638	690	0	0	0	0	0	5527
09$_1$	0	816	0	6285	6285	816	0	0	0	0	0	32000
09$_2$	0	0	0	0	0	0	0	0	0	0	0	24857
10$_1$	0	0	0	0	0	0	0	0	0	0	0	62237
11$_1$	0	0	0	0	0	0	0	0	0	0	0	150000
11$_2$	0	0	0	0	0	0	0	0	0	0	0	4668

Initialize setup cost, holding cost and production rate for the items 1_0, 1_1, 1_2 and 1_3 from that of the item 1 as follows.

$$S_{1_0} = S_{1_1} = S_{1_2} = S_{1_3} = S_1 = 322.0,$$

$$h_{1_0} = h_{1_1} = h_{1_2} = h_{1_3} = h_1 = 0.0167,$$

$$k_{1_0} = k_{1_1} = k_{1_2} = k_{1_3} = k_1 = 1/524.$$

Similarly set the value of setup cost, holding cost and production rate for the $N' = 25$ new items from those of the N = 12 items.

Item No	Holding Cost	Setup Cost	Production Rate
01_0	0.0167	322.0	524
01_1	0.0167	322.0	524
01_2	0.0167	322.0	524
01_3	0.0167	322.0	524
02_0	0.0167	81.0	349
02_1	0.0167	81.0	349
02_2	0.0167	81.0	349
03_0	0.0167	124.0	245
04_0	0.0167	124.0	172
05_0	0.0167	81.0	349
05_1	0.0167	81.0	349
05_2	0.0167	81.0	349
06_0	0.0167	124.0	245
07_0	0.0167	124.0	172
08_0	0.0167	105.0	847
08_1	0.0167	105.0	847
09_0	0.0167	105.0	464
09_1	0.0167	105.0	464
09_2	0.0167	105.0	464
10_0	0.0167	106.0	575
10_1	0.0167	106.0	575
11_0	0.0167	105.0	1261
11_1	0.0167	105.0	1261
11_2	0.0167	105.0	1261
12_0	0.0167	105.0	663

Now apply the modified Dixon-Silver heuristic with the limited lot-size for 25 items. The lot sizes for the new items are shown in Table 3.7.

Table 3.7 Lot sizes for $N^{'} = 25$ items.

Item No	Period 1	2	3	4	5	6	7	8	9	10	11	12
01_0	6000	6000	6000	6000	3592	0	6000	6000	0	6000	4500	0
02_0	0	0	27344	0	60000	53124	39842	26562	0	60000	22135	0
03_0	32906	0	21116	21472	17743	0	13574	16589	0	0	46258	0
04_0	0	29000	0	0	7528	0	8480	6938	0	0	23637	0
05_0	31184	36250	49000	43274	0	39546	29659	36251	0	0	15866	33134
06_0	44314	38256	0	21423	2041	16322	13772	3060	60955	0	0	0
07_0	12687	0	0	0	10782	0	4561	0	12246	0	0	0
08_0	41000	41000	41000	20318	41000	41000	31267	41000	0	38215	0	0
09_0	32000	32000	32000	32000	32000	29068	24612	32000	0	30080	0	0
10_0	0	59646	7205	105663	112868	96745	72559	88682	0	0	0	185000
11_0	0	40144	150000	128176	0	119220	89415	109285	0	0	150000	0
12_0	45190	0	64666	0	27715	0	25405	20786	0	70822	0	0
01_1	0	6000	6000	6000	3957	0	2592	0	0	6000	0	0
01_2	0	0	2730	0	0	0	0	0	0	6000	0	0
01_3	0	0	0	0	0	0	0	0	0	5666	0	0
02_1	0	0	0	1977	0	0	0	0	0	0	0	60000
02_2	0	0	0	0	0	0	0	0	0	15758	0	0
05_1	0	0	0	0	0	0	0	0	0	0	0	49000
05_2	0	0	0	0	0	0	0	0	0	3065	0	0
08_1	690	0	15966	0	0	0	0	0	0	5527	0	0
09_1	816	0	13386	0	0	0	0	0	0	32000	0	0
09_2	0	0	0	0	0	0	0	0	0	24857	0	0
10_1	0	0	0	0	0	0	0	0	0	48215	14022	0
11_1	0	0	0	0	0	0	0	0	0	0	150000	0
11_2	0	0	0	0	0	0	0	0	0	4668	0	0

Convert the $N^{'} \times H$ lot-sizing matrix into $N \times H$ lot-sizing matrix by applying the formula

$$x_{i,j} = \sum_{l=0}^{n_i} x_{i_l,j}.$$

As an example let us compute $x_{1,5}$.

$$\begin{aligned} x_{1,5} &= \sum_{l=0}^{n_1} x_{1_l,5} \\ &= \sum_{l=0}^{3} x_{1_l,5} \\ &= x_{1_0,5} + x_{1_1,5} + x_{1_2,5} + x_{1_3,5} \\ &= 3592 + 3957 + 0 + 0 \\ &= 7549. \end{aligned}$$

Chapter 4

Results and Discussion

4.1 Introduction

The algorithm developed by Dixon and Silver [5] to generate feasible solution for multi-item single level capacitated lot-sizing problem was tested in PC version with Fortran77 language. Thus a near optimal solution was obtained. The results are detailed in Section 4.2 below. This algorithm has been extended in the present work. The setup time and the upper limit on the lot-size have been included in the original algorithm. Thus the Dixon-Silver algorithm is separately extended with these two new parameters as described in Chapter 3. This chapter presents the results obtained from the modified models using Fortran77 language in PC version. Section 4.3 shows results with setup time consideration, and Section 4.4 shows the results with upper bound on the limited lot-size. Section 4.5 shows the sensitivity of the model with production rate.

4.2 Solution of a Multi-item Single Level Capacitated Lot-sizing Problem

The Dixon-Silver algorithm has been used with hypothetical data. It is assumed that entire production to meet demands is done in the plant and no subcontracting is permissible. Moreover, a further assumption is made that plant capacity could not be increased.

1. Product data

The relevant product data (e.g., holding cost, setup cost, production rate, safety stock, initial inventory and ending inventory) has been depicted in Table 4.1. The problem size has been restricted at 12 products and 12 time periods; each time period corresponds to a month.

Table 4.1 Relevant product data for the hypothetical machine.

Item No (i)	Holding Cost (h_i)	Setup Cost (S_i)	Production Rate ($1/k_i$)	Safety Stock (SS_i)	Initial Inventory (Iin_i)	Ending Inventory ($Iend_i$)
01	0.0167	322.0	524	0	19320	18893
02	0.0167	81.0	349	10602	200180	124225
03	0.0167	124.0	245	4577	24460	43294
04	0.0167	124.0	172	1974	23260	21757
05	0.0167	81.0	349	7581	55489	92168
06	0.0167	124.0	245	4861	-2727	44394
07	0.0167	124.0	172	2026	9659	8466
08	0.0167	105.0	847	11117	29705	40273
09	0.0167	105.0	464	9533	11362	84717
10	0.0167	106.0	575	20417	242944	227344
11	0.0167	105.0	1261	16634	324215	271627
12	0.0167	105.0	663	9794	45439	69068

2. Product demand plant capacity

Product demands are quite seasonal and the same seasonal indices are used for all the products. Forecasted demand and the capacity of the machine are shown in Table 4.2. It has been assumed that the capacity per month is the total number of hours available per month. Two percent of the capacity is reserved as a buffer to guard against uncertainty in the actual production rate. In this hypothetical problem, Period 1 corresponds to the month of June, Period 2 corresponds to the month of July. Thus the machine capacity in Period 1 is 98% of the total hours in June, i.e., $30 \times 24 \times 0.98 = 706$ hours. To be in the safe side, it has been assumed that the number of days in February is 28. Then the machine capacity in Period 9 is $28 \times 24 \times 0.98 = 660$ hours. Similarly the machine capacity for the other periods has been calculated.

Table 4.2 Forecasted demand and capacity of the hypothetical machine.

Item No	Period 1	2	3	4	5	6	7	8	9	10	11	12
01	11456	11456	10501	13365	13365	11456	8592	1909	1909	1909	4773	4773
02	53124	53124	48697	61977	61977	53124	39842	8854	8854	8854	22135	22135
03	18099	18099	16591	21116	21116	18099	13574	3016	3016	3016	7541	7541
04	9250	9250	8480	10792	10792	9250	6938	1542	1542	1542	3854	3854
05	39546	39546	36250	46137	46137	39546	29659	6591	6591	6591	16478	16478
06	18363	18363	16833	21423	21423	18363	13772	3060	3060	3060	7651	7651
07	4976	4976	4562	5806	5806	4976	3732	829	829	829	2074	2074
08	41690	41690	38216	48638	48638	41690	31267	6948	6948	6948	17371	17371
09	32816	32816	30081	38285	38285	32816	24612	5469	5469	5469	13673	13673
10	96745	96745	88683	112868	112868	96745	72559	16124	16124	16124	40310	40310
11	119220	119220	109285	139088	139088	119220	89415	19870	19870	19870	49675	49675
12	27715	27715	25405	32333	32333	27715	20786	4619	4619	4619	11548	11548
	Available Machine Hours											
	706	729	729	706	729	706	729	729	660	729	706	729

3. Equivalent demand schedule

An equivalent demand schedule is generated such that starting and ending inventory are accommodated. In addition, demands are adjusted such that in the heuristic solution, the inventory at the end of any period never drops below the safety stock level. Table 4.3 depicts the equivalent demand after considering initial inventory, ending inventory and safety stock.

Table 4.3 Equivalent demand with the use of initial inventory, ending inventory and safety stock.

Item No	Period 1	2	3	4	5	6	7	8	9	10	11	12
01	0	3592	10501	13365	13365	11456	8592	1909	1909	1909	4773	23666
02	0	0	0	27344	61977	53124	39842	8854	8854	8854	2135	135758
03	0	16315	16591	21116	21116	18099	13574	3016	3016	3016	7541	46258
04	0	0	5694	10792	10792	9250	6938	1542	1542	1542	3854	23637
05	0	31184	36250	46137	46137	39546	29659	6591	6591	6591	16478	101065
06	25951	18363	16833	21423	21423	18363	13772	3060	3060	3060	7651	47184
07	0	2319	4562	5806	5806	4976	3732	829	829	829	2074	8514
08	23102	41690	38216	48638	48638	41690	31267	6948	6948	6948	7371	46527
09	30987	32816	30081	38285	38285	32816	24612	5469	5469	5469	13673	88857
10	0	0	59646	112868	112868	96745	72559	16124	16124	6124	0310	247237
11	0	0	40144	139088	139088	119220	89415	19870	19870	19870	9675	304668
12	0	19785	25405	32333	32333	27715	20786	4619	4619	4619	1548	70822

4. Results

Table 4.4 shows the final lot-sizes and forecasted machine hour requirements for each period, and Table 4.5 shows the inventories at the end of each period for all items.

Table 4.4 Final lot-sizes and forecasted machine time requirements for Dixon-Silver heuristic.

Item No	Period 1	2	3	4	5	6	7	8	9	10	11	12
01	3592	23866	0	13365	11456	0	12410	0	6682	0	23666	0
02	0	0	27344	61977	0	53124	39842	17708	67274	0	99473	0
03	32906	0	21116	21116	18099	0	13574	9048	0	53799	0	0
04	0	27278	0	0	9250	0	8480	6938	0	0	23637	0
05	20637	92934	0	46137	0	39546	29659	13182	124134	0	0	0
06	61147	0	21423	21423	14462	3901	13772	9180	0	54835	0	0
07	18493	0	0	0	4976	0	4561	6141	0	0	6105	0
08	23102	41690	38216	48638	48638	41690	31267	13896	6948	17371	0	46527
09	30987	62897	38285	1747	36538	32816	24612	10938	5469	75079	27451	0
10	0	3243	154554	14717	112868	96745	72559	16124	32248	40310	59995	187242
11	0	0	40144	139088	139088	119220	89415	19870	19870	19870	49675	304668
12	0	45190	32333	0	32333	27715	20786	9238	4619	11548	0	70822
Forecasted Machine Requirements (hours)												
	651.5	729.0	729.0	706.0	729.0	706.0	728.7	336.7	660.0	729.0	706.0	729.0

Table 4.5 Inventories at the end of each period for all items.

Item No	Period 1	2	3	4	5	6	7	8	9	10	11	12
01	11456	23866	13365	13365	11456	0	3818	1909	6682	4773	23666	18893
02	147056	93932	72579	72579	10602	10602	10602	19456	77876	69022	146360	124225
03	39267	21168	25693	25693	22676	4577	4577	10609	7593	58376	50835	43294
04	14010	32038	23558	12766	11224	1974	3516	8912	7370	5828	25611	21757
05	36580	89968	53718	53718	7581	7581	7581	14172	131715	125124	108646	92168
06	40057	21694	26284	26284	19323	4861	4861	10981	7921	59696	52045	44394
07	23176	18200	13638	7832	7002	2026	2855	8167	7338	6509	10540	8466
08	11117	11117	11117	11117	11117	11117	11117	18065	18065	28488	11117	40273
09	9533	39614	47818	11280	9533	9533	9533	15002	15002	84612	98390	84717
10	146199	52697	118568	20417	20417	20417	20417	20417	36541	60727	80412	227344
11	204995	85775	16634	16634	16634	16634	16634	16634	16634	16634	16634	271627
12	17724	35199	42127	9794	9794	9794	9794	14413	14413	21342	9794	69068

Other results are tabulated below:

Total available machine time ($\sum_{t=1}^{H} C_t$)	: 8587.0 hour
Total forecasted machine time	: 8139.8 hour
Total inventory holduingcost, $C_{inv} = \sum_{i=1}^{N} \sum_{t=1}^{H} (I_{it} - SS_i)$	: $ 64674.05
Total expected safety-stock cost, $C_{ss} = \sum_{i=1}^{N} SS_i$	: $ 19862.85
Total expected setup cost, $C_{set} = \sum_{i=1}^{N} n_i S_i$ where n_i is the number of setup for item i.	: $ 11959.00
Total expected cost ($C_{inv} + C_{ss} + C_{set}$)	: $ 96495.90

4.3 Results of Multi-Item Single Level Capacitated Lot-Sizing Problem with Setup Time

In the hypothetical problem in Section 4.1 machine setup time to produce each product item is included. Relevant product data including setup time for each item has been presented in Table 4.6. In the present work setup time is assumed arbitrarily. Use of the randomized values would obviously be a more realistic approach. Selection of the randomized values for the set up time within certain range and running the model would be more acceptable. Forecasted demands and capacities as presented in Table 4.2 are also used in the present case. The equivalent demands after considering initial inventory, ending inventory and safety stock are also same as presented in Table 4.3. The extended heuristic algorithm as developed in chapter 3 has been applied to the problem. Table 4.7 shows the final lot-sizes and forecasted machine hour requirements for each period, and Table 4.8 shows the inventories at the end of each period for all items.

Table 4.6 Relevant product data for the extended heuristic with setup time.

Item No (i)	Holding Cost (h_i)	Setup Cost (S_i)	Setup Time (St_i)	Production Rate ($1/k_i$)	Safety Stock (SS_i)	Initial Inventory (Iin_i)	Ending Inventory ($Iend_i$)
01	0.0167	322.0	1.40	524	0	19320	18893
02	0.0167	81.0	2.00	349	10602	200180	124225
03	0.0167	124.0	1.00	245	4577	24460	43294
04	0.0167	124.0	1.50	172	1974	23260	21757
05	0.0167	81.0	0.25	349	7581	55489	92168
06	0.0167	124.0	0.70	245	4861	-2727	44394
07	0.0167	124.0	0.50	172	2026	9659	8466
08	0.0167	105.0	1.20	847	11117	29705	40273
09	0.0167	105.0	0.40	464	9533	11362	84717
10	0.0167	106.0	0.60	575	20417	242944	227344
11	0.0167	105.0	1.00	1261	16634	324215	271627
12	0.0167	105.0	1.30	663	9794	45439	69068

Table 4.7 Final lot-sizes and forecasted machine time requirements for the extended heuristic with setup time.

Item No	Period 1	2	3	4	5	6	7	8	9	10	11	12
01	3592	23866	0	13365	11456	0	10501	3818	0	28439	0	0
02	0	0	27344	61977	0	53124	39842	17708	68079	0	98668	0
03	37814	0	16208	21116	18099	0	13574	9048	0	53799	0	0
04	0	27278	0	0	9250	0	6938	8480	0	0	23637	0
05	31184	82387	0	46137	0	39546	29659	13182	124134	0	0	0
06	61147	0	21423	21423	16899	1464	13772	9180	0	54835	0	0
07	18493	0	0	0	4976	0	4561	11912	0	0	334	0
08	23102	41690	41862	44992	48638	41690	31267	13896	6948	17371	0	46527
09	30987	62897	38285	11166	27119	32816	24612	10938	5469	45743	56787	0
10	0	14322	158192	0	112868	96745	72559	16124	32248	40310	64595	182642
11	0	0	40144	139088	139088	119220	89415	19870	19870	19870	49675	304668
12	0	45190	32333	0	32333	27715	20786	9238	4619	11548	0	70822
	Forecasted Machine Requirements (hours)											
	705.2	724.7	727.8	704.4	728.3	703.5	727.9	398.3	656.3	727.6	702.2	725.1

Table 4.8 Inventories for the heuristic with setup time.

Item No	Period											
	1	2	3	4	5	6	7	8	9	10	11	12
01	11456	23866	13365	13365	11456	0	1909	3818	1909	28439	23666	18893
02	147056	93932	72579	72579	10602	10602	10602	19456	78681	69827	146360	124225
03	44175	26076	25693	25693	22676	4577	4577	10609	7593	58376	50835	43294
04	14010	32038	23558	12766	11224	1974	1974	8912	7370	5828	25611	21757
05	47127	89968	53718	53718	7581	7581	7581	14172	131715	125124	108646	92168
06	40057	21694	26284	26284	21760	4861	4861	10981	7921	59696	52045	44394
07	23176	18200	13638	7832	7002	2026	2855	13938	13109	12280	10540	8466
08	11117	11117	14763	11117	11117	11117	11117	18065	18065	28488	11117	40273
09	9533	39614	47818	20699	9533	9533	9533	15002	15002	55276	98390	84717
10	146199	63776	133285	20417	20417	20417	20417	20417	36541	60727	85012	227344
11	204995	85775	16634	16634	16634	16634	16634	16634	16634	16634	16634	271627
12	17724	35199	42127	9794	9794	9794	9794	14413	14413	21342	9794	69068

The following results have also been found after applying the heuristic algorithm with setup time.

Total available machine time ($\sum_{t=1}^{H} C_t$)	: 8587.0 hour
Total setup time ($\sum_{i=1}^{N} n_i St_i$) where n_i is the number of setup for item i.	: 93.5 hour
Total forecasted machine time	: 8233.2 hour
Total inventory holduingcost, $C_{inv} = \sum_{i=1}^{N} \sum_{t=1}^{H} (I_{it} - SS_i)$	: \$ 65896.46
Total expected safety-stock cost, $C_{ss} = \sum_{i=1}^{N} SS_i$	: \$ 19862.85
Total expected setup cost, $C_{set} = \sum_{i=1}^{N} n_i S_i$	: \$ 11853.00
Total expected cost ($C_{inv} + C_{ss} + C_{set}$)	: \$ 97612.31

To see the effect of setup time on different parameters, the value of setup time of each item of Table 4.6 has been varied step by step at a 5% interval. With these variations the changes of the used machine time, available machine time, total inventory cost, total setup cost, total safety stock cost and total cost have been determined and shown in Table 4.9. The first column shows the various percentages of the original setup times as tabulated in Table 4.6. The setup time 0% indicates that there is no setup time for each item. This is the same as the Dixon-Silver's original algorithm. The total capacity of the machine is 8587 hours. The available machine time in column three is obtained by subtracting used machine time from the total capacity of the machine.

Table 4.9 Effect of setup time on available machine time and costs.

Setup Time	Used Machine Time	Available Machine Time	Inventory cost	Setup cost	Safety stock cost	Total cost
0%	8139.8	447.2	64674.05	11959	19862.85	96495.9
5%	8144.4	442.6	64413.60	11959	19862.85	95789.5
10%	8149.1	437.9	64405.90	11835	19862.85	96103.8
15%	8153.8	433.2	64474.01	11835	19862.85	96171.9
20%	8158.5	428.5	64542.11	11835	19862.85	96240.0
25%	8163.2	423.8	64610.21	11835	19862.85	96308.1
30%	8167.8	419.2	64678.31	11835	19862.85	96376.2
35%	8172.5	414.5	64746.35	11835	19862.85	96444.2
40%	8176.8	410.2	65268.91	11835	19862.85	96768.8
45%	8181.5	405.5	65337.02	11959	19862.85	96836.9
50%	8186.8	400.2	65330.93	11959	19862.85	97152.8
55%	8191.5	395.5	65399.23	11959	19862.85	97221.1
60%	8196.2	390.8	65467.51	11959	19862.85	97289.4
65%	8200.9	386.1	65518.04	11959	19862.85	97339.9
70%	8205.6	381.4	65568.10	11959	19862.85	97390.0
75%	8210.3	376.7	65618.15	11959	19862.85	97440.0
80%	8215.0	372.0	65677.45	11959	19862.85	97499.3
85%	8219.0	368.0	65727.33	11959	19862.85	97549.2
90%	8223.9	363.1	65783.17	11853	19862.85	97499.0
95%	8228.6	358.4	65839.80	11853	19862.85	97555.7
100%	8233.2	353.8	65896.46	11853	19862.85	97612.3

Figure 4.1 shows the available machine time for various percentage of the setup time. This time decreases linearly with the setup time. The increase in setup time increases the time to produce an item. This increase in production time results in a decrease in the available machine time.

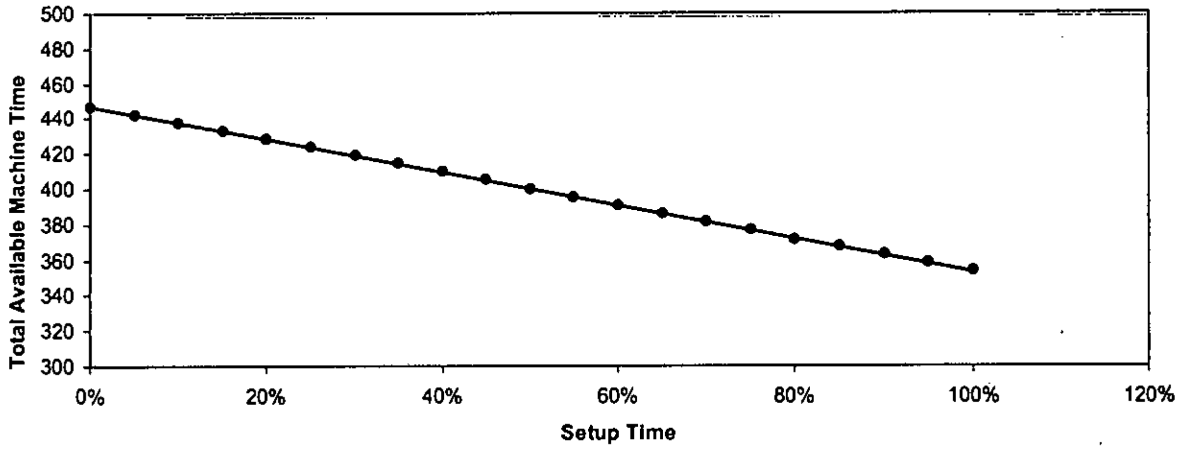

Figure 4.1 Variation of total available machine time with setup time.

Figure 4.2 shows the variation of total inventory holding cost with setup time. With the increase of setup time, total inventory holding cost increases gradually. Since the increase of setup time decreases the available capacity in a period, there could be periods in which total demand exceeds total capacity. To overcome this unbalance situation some inventory will have to be built up in earlier periods with available slack capacity. When setup time increases, number of capacity violating period would increase. Thus the inventory will be more. As a result total inventory holding cost increases with the increase of setup time. These increases are almost linear but at around setup time at 40% there is observed a 'kink jump' increase in inventory cost. The writer reserves any comment on this aspect without a further investigation.

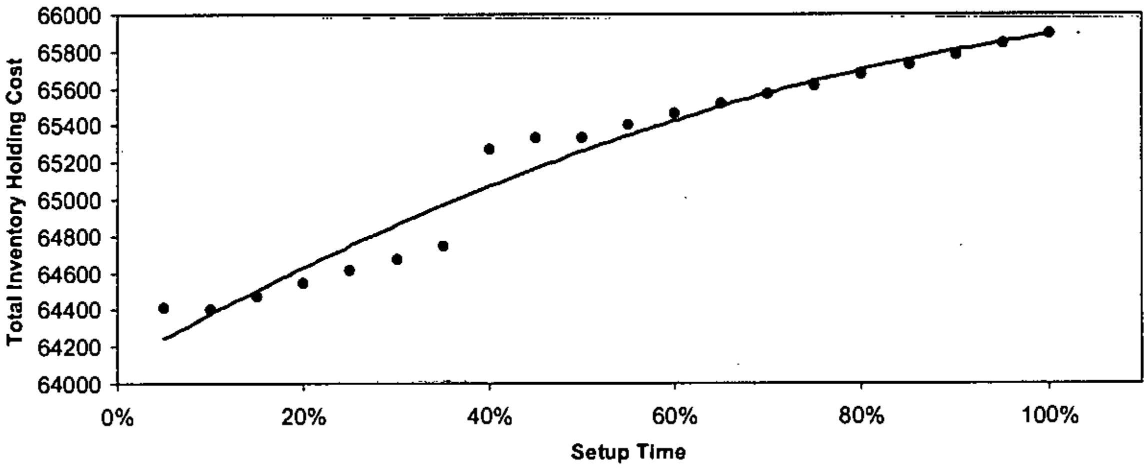

Figure 4.2 Variation of total inventory holding cost with setup time.

The total costs shown in Column 4 for various percentages of the setup times have been calculated using S_i for each item and multiplied by number of setups, n_i. Now, S_i could be estimated as follows,

S_i = (Handling and manipulating cost of the machine for i) +

(Special tool cost for setup) + (Overhead costs).

Where, Handling and manipulating cost = (Handling and manipulating time) × (Cost rate) Now, handling and manipulating time could be reduced and thereby the cost, only by using special type tools (where possible) such as special fixtures and devices with increased tool cost. The objective is to reduce the setup time and thus to increase in machine available time and hence increased production. This is a complex situation where one cost component is decreased but another cost component (namely tool cost) is increased. In the present work for simplicity and lack of available data it is assumed that these two cost components would not vary significantly. Thus, S_i is assumed to remain unaltered. This is a robust assumption which would require further investigation.

Figure 4.3 shows the variation of total cost with setup time. With the increase of setup time, total cost increases, since the inventory holding cost increases, and the setup cost and safety stock cost remains almost unchanged.

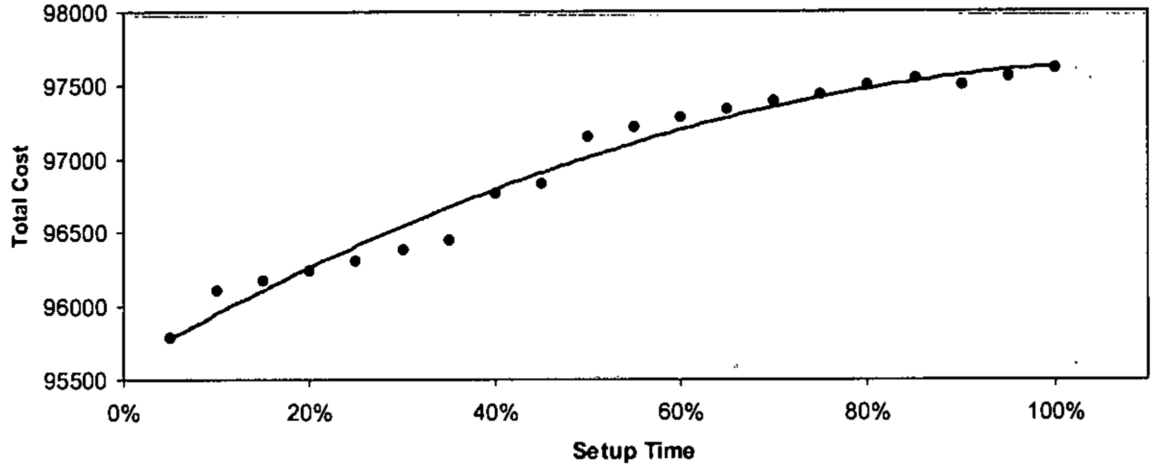

Figure 4.3 Variation of total cost with setup time.

4.4 Results with the Limited Lot-Size per Setup

Relevant product data including the limited lot-size per setup for each item has been depicted in Table 4.10. The limited lot-size per setup for each item has been taken arbitrarily. It will be more realistic if it is taken randomly. The demands and capacities are extracted from Table 4.2. The equivalent demands after considering initial inventory, ending inventory and safety stock are extracted from Table 4.3. Table 4.11 shows the final lot-sizes and forecasted machine hour requirements for each period, and Table 4.12 shows the inventories at the end of each period for all items.

Table 4.10 Relevant Product data for the heuristic with the limited lot-size per setup.

Item No	Holding Cost	Setup Cost	Maximum Lot-Size	Production Rate	Safety Stock	Initial Inventory	Ending Inventory
01	0.0167	322.0	6000	524	0	19320	18893
02	0.0167	81.0	60000	349	10602	200180	124225
03	0.0167	124.0	68000	245	4577	24460	43294
04	0.0167	124.0	29000	172	1974	23260	21757
05	0.0167	81.0	49000	349	7581	55489	92168
06	0.0167	124.0	68000	245	4861	-2727	44394
07	0.0167	124.0	44000	172	2026	9659	8466
08	0.0167	105.0	41000	847	11117	29705	40273
09	0.0167	105.0	32000	464	9533	11362	84717
10	0.0167	106.0	185000	575	20417	242944	227344
11	0.0167	105.0	150000	1261	16634	324215	271627
12	0.0167	105.0	97000	663	9794	45439	69068

Table 4.11 Final lot-sizes and forecasted machine time requirements for the heuristic with the limited lot-size per setup.

Item No	Period											
	1	2	3	4	5	6	7	8	9	10	11	12
01	6000	12000	14730	12000	7549	0	8592	6000	0	23666	4500	0
02	0	0	27344	1977	60000	53124	39842	26562	0	75758	22135	60000
03	32906	0	21116	21472	17743	0	13574	16589	0	0	46258	0
04	0	29000	0	0	7528	0	8480	6938	0	0	23637	0
05	31184	36250	49000	43274	0	39546	29659	36251	0	3065	15866	82134
06	44314	38256	0	21423	2041	16322	13772	3060	60955	0	0	0
07	12687	0	0	0	10782	0	4561	0	12246	0	0	0
08	41690	41000	56966	20318	41000	41000	31267	41000	0	43742	0	0
09	32816	32000	45386	32000	32000	29068	24612	32000	0	86937	0	0
10	0	59646	7205	105663	112868	96745	72559	88682	0	48215	14022	185000
11	0	40144	150000	128176	0	119220	89415	109285	0	4668	300000	0
12	45190	0	64666	0	27715	0	25405	20786	0	70822	0	0
	Forecasted Machine Requirements (hours)											
	677.9	704.5	727.1	706.0	729.0	706.0	728.4	701.6	320.0	704.4	706.0	729.0

Table 4.12 Inventories for the heuristic with the limited lot-size per setup.

Item No	Period											
	1	2	3	4	5	6	7	8	9	10	11	12
01	13864	14408	18637	17272	11456	0	0	4091	2182	23939	23666	18893
02	147056	93932	72579	12579	10602	10602	10602	28310	19456	86360	86360	124225
03	39267	21168	25693	26049	22676	4577	4577	18150	15134	12118	50835	43294
04	14010	33760	25280	14488	11224	1974	3516	8912	7370	5828	25611	21757
05	47127	43831	56581	53718	7581	7581	7581	37241	30650	27124	26512	92168
06	23224	43117	26284	26284	6902	4861	4861	4861	62756	59696	52045	44394
07	17370	12394	7832	2026	7002	2026	2855	2026	13443	12614	10540	8466
08	29705	29015	47765	19445	11807	11117	11117	45169	38221	75015	57644	40273
09	11362	10546	25851	19566	13281	9533	9533	36064	30595	112063	98390	84717
10	146199	109100	27622	20417	20417	20417	20417	92975	76851	108942	82654	227344
11	204995	125919	166634	155722	16634	16634	16634	106049	86179	70977	321302	271627
12	62914	35199	74460	42127	37509	9794	14413	30580	25961	92164	80616	69068

The following results have also been found after applying the heuristic algorithm with the limited lot-size per setup.

Total available machine time ($\sum_{t=1}^{H} C_t$)	: 8587.0 hour
Total setup time ($\sum_{i=1}^{N} n_i St_i$)	: 0 hour
where n_i is the number of setup for item i.	
Total forecasted machine time	: 8139.8 hour
Total inventory holding cost, $C_{inv} = \sum_{i=1}^{N} \sum_{t=1}^{H} (I_{it} - SS_i)$	: \$ 83162.35
Total expected safety-stock cost, $C_{ss} = \sum_{i=1}^{N} SS_i$	: \$ 19862.85
Total expected setup cost, $C_{set} = \sum_{i=1}^{N} n_i S_i$	: \$ 15733.00
Total expected cost ($C_{inv} + C_{ss} + C_{set}$)	: \$ 118758.20

To see the effect of the limited lot-size to different parameters, the first value of the limited lot-size of each item has been chosen as shown below. These values have been chosen so that the number of total items after limiting the lot-size remains unchanged and a little decrease in these values will increase the number of total items.

Item No	01	02	03	04	05	06	07	08	09	10	11	12
Maximum Lot-Size	40000	150000	70000	40000	130000	50000	9000	90000	150000	250000	400000	90000

Next the value of the limited lot-size of each item is reduced step by step. With the variation of the limited lot-size, the change of the values of the number of total items, the machine utilization time, total inventory cost, total setup cost, total safety stock cost and total cost has been shown in Table 4.13. The limited lot-size 100% in Table 4.13 is the same the limited lot-size of each item in the above table. For 80%, the limited lot-size for item 1 is $40000 \times 0.80 = 32000$, for item 2 is $150000 \times 0.80 = 120000$, and so on. If there is no limitation on tne maximum allowable lot-size, then the problem remains same as the Dixon-Silver's original problem.

Table 4.13 Effect of the limited lot-size on number of items and costs.

Maximum Lot-size	Total No of items, N'	Machine Time	Inventory H. Cost	Setup Cost	Safety Stock Cost	Total Cost
100%	12	8139.8	83862.09	9998	19862.85	113722.9
95%	13	8139.8	85869.86	10104	19862.85	115836.7
90%	16	8139.8	85442.45	10558	19862.85	115863.3
85%	16	8139.8	86514.23	10366	19862.85	116743.1
80%	16	8139.8	86385.99	10533	19862.85	116781.8
75%	19	8139.8	84475.54	11374	19862.85	115712.4
70%	19	8139.8	88460.28	11376	19862.85	119699.1
65%	20	8139.8	83607.95	11982	19862.85	115452.8
60%	20	8139.8	86082.63	12088	19862.85	118033.5
55%	23	8139.8	87656.53	12348	19862.85	119867.4
50%	24	8139.8	84621.45	13017	19862.85	117501.3
45%	28	8139.8	86968.52	14177	19862.85	121008.4
40%	28	8139.8	87413.70	15143	19862.85	122419.6
35%	31	8139.8	87122.86	15644	19862.85	122629.7
30%	36	8139.8	84305.80	18652	19862.85	122820.7
25%	43	8139.8	88301.28	21978	19862.85	130142.1
22.5%	47	8139.8	86852.93	23969	19862.85	130684.8
20%	48	8139.8	85690.84	26415	19862.85	131968.7
17.5%	59	8139.8	79265.35	29739	19862.85	128867.2
15%	67	8139.8	82621.71	34858	19862.85	137342.6
12%	80	8139.8	83920.22	41373	19862.85	145156.1
10%	95	8139.8	81328.73	49169	19862.85	150360.6

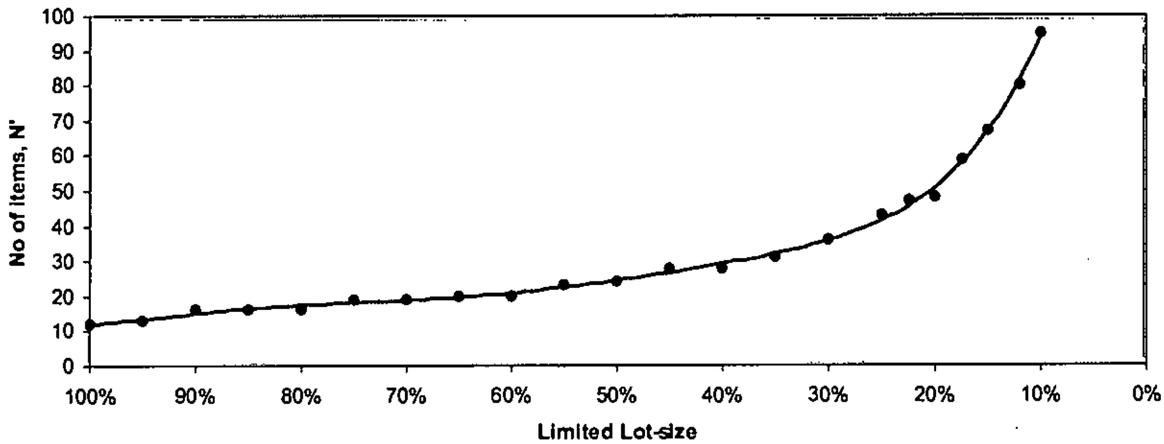

Figure 4.4 The growth rate of number of items with the limited lot-size.

Figure 4.4 shows the growth rate of number of items as a function of the limited lot-size. This growth rate is increasing with the decrease of the limited lot-size. The decrease in the limited lot-size decreases the amount of production quantity per setup of an item. This decrease in production quantity results in an increase in the number of items.

Figure 4.5 shows the variation of setup cost with the limited lot-size. With the decrease of the limited lot-size, the setup cost increases significantly. If the limited lot-size per setup is decreased, then the number of setup needed is increased accordingly. Therefore the setup cost is also increased.

Figure 4.6 shows the variation of total inventory holding cost with the limited lot-size. With the decrease of the limited lot-size, the variation of the total inventory holding cost is fluctuating. This nature of the variation needs to be more investigation.

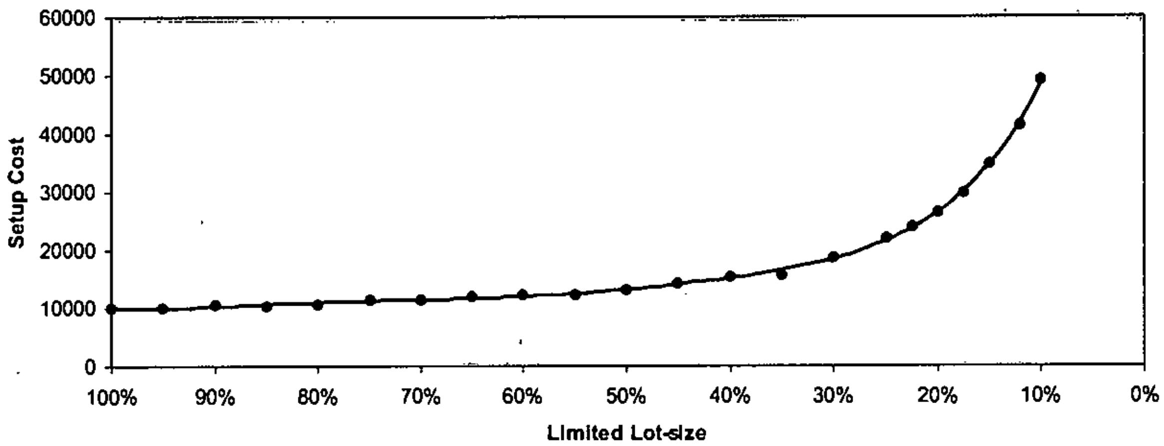

Figure 4.5 The variation of setup cost with the limited lot-size.

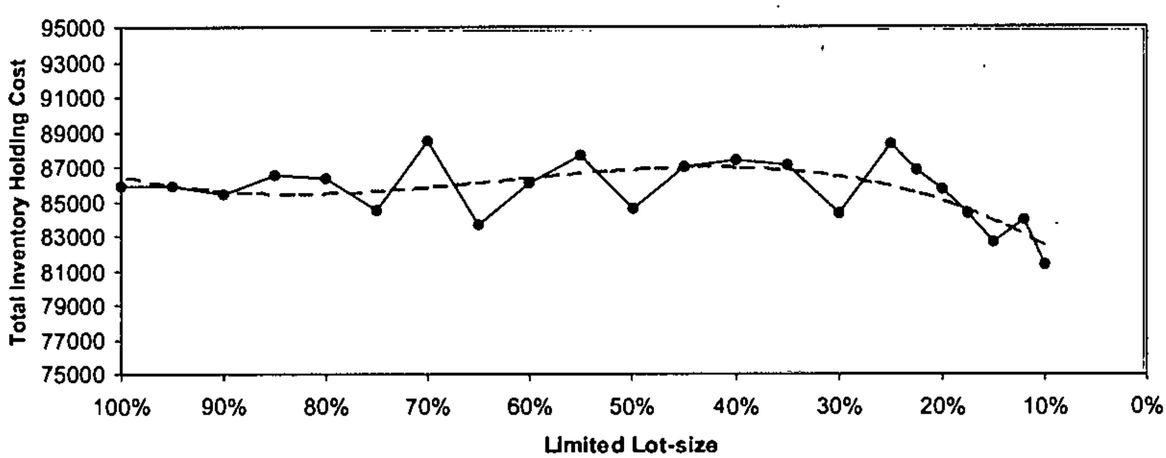

Figure 4.6 The variation of total inventory holding cost with the limited lot-size.

Figure 4.7 shows the variation of total cost with the limited lot-size. With the decrease of the limited lot-size, total cost increases, since the setup cost increases significantly, the inventory holding cost is fluctuating and safety stock cost remains almost unchanged.

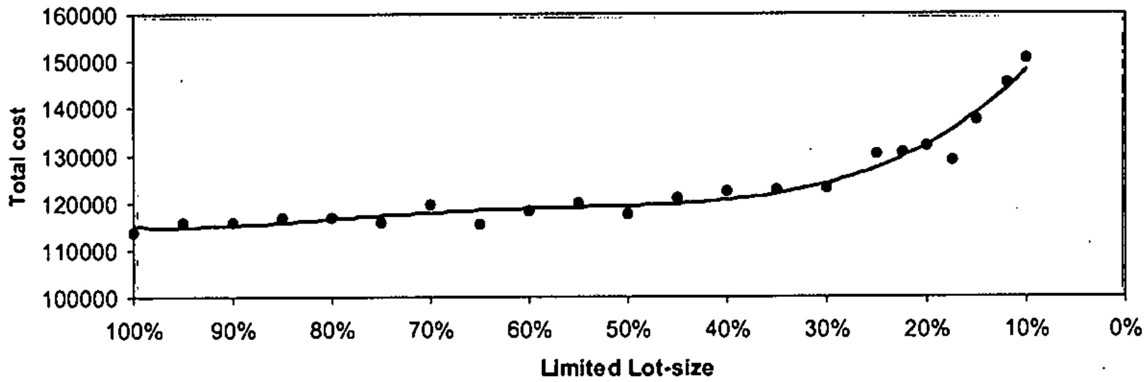

Figure 4.7 The variation of total cost with the limited lot-size.

4.5 Results with Production Rate

The production rate for each item has been varied. Relevant product data, demands and capacities have been extracted from Table 4.10 and Table 4.11, respectively. To see the effect of production rate to different parameters, the value of production rate of each item is increased step by step. With the variation of the production rate, the change of the values of the machine utilization time, total inventory cost, total setup cost, total safety stock cost and total cost has been shown in Table 4.13. The production rate 100% in Table 4.14 represents the production rate of each item in Table 4.10, similarly for other production rates. For example, the production rate for item 1 is 524 × 1.10 = 576.4, for item 2 is 349 × 1.10 = 383.9, and so on. Safety stock cost was calculated for all the items throughout the entire time horizon of 12 periods and was found to be 19862.85.

Table 4.14 Effect of production rate on machine time and costs.

Production Rate	Machine Time	Inventory H. Cost	Setup Cost	Safety Stock Cost	Total cost
99%	8231.9	84942.13	10185	19862.85	114990.0
100%	8139.8	83862.09	9998	19862.85	113722.9
101%	8071.4	87382.74	9682	19862.85	116927.6
102%	7995.3	88687.95	9050	19862.85	117600.8
104%	7844.1	90936.43	8926	19862.85	119725.3
105%	7760.4	95103.63	8568	19862.85	123534.5
106%	7692.6	94004.11	8969	19862.85	122836.0
107%	7612.4	103853.10	8339	19862.85	132055.0
108%	7875.3	132233.50	9118	19862.85	161214.4
110%	7782.5	147691.10	8932	19862.85	176486.0
111%	7714.5	145952.90	8932	19862.85	174747.8
112%	7655.4	147582.40	8851	19862.85	176296.3
113%	7212.7	101805.00	7755	19862.85	129422.9
114%	7153.1	108174.50	7593	19862.85	135630.4
115%	7091.3	117446.40	7259	19862.85	144568.3
116%	7045.0	118701.30	7581	19862.85	146145.2
117%	6988.1	119301.20	7581	19862.85	146745.1
118%	6907.1	106399.80	7445	19862.85	133707.7
119%	7242.7	169146.30	8287	19862.85	197296.2
120%	7107.9	158421.00	8230	19862.85	186513.9
122%	6996.4	165726.60	7891	19862.85	193480.5
123%	7101.6	214511.60	8225	19862.85	242599.5

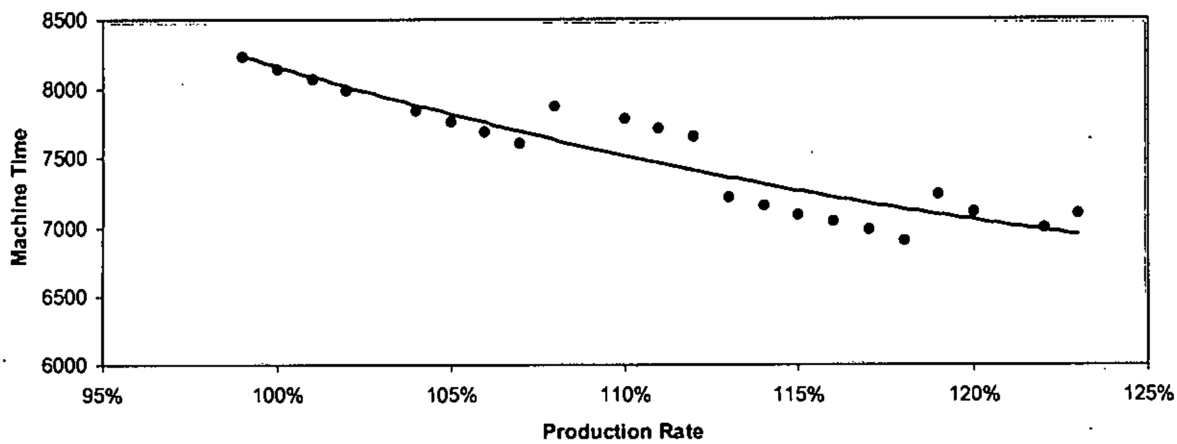

Figure 4.8 Variation of machine utilization time with the production rate.

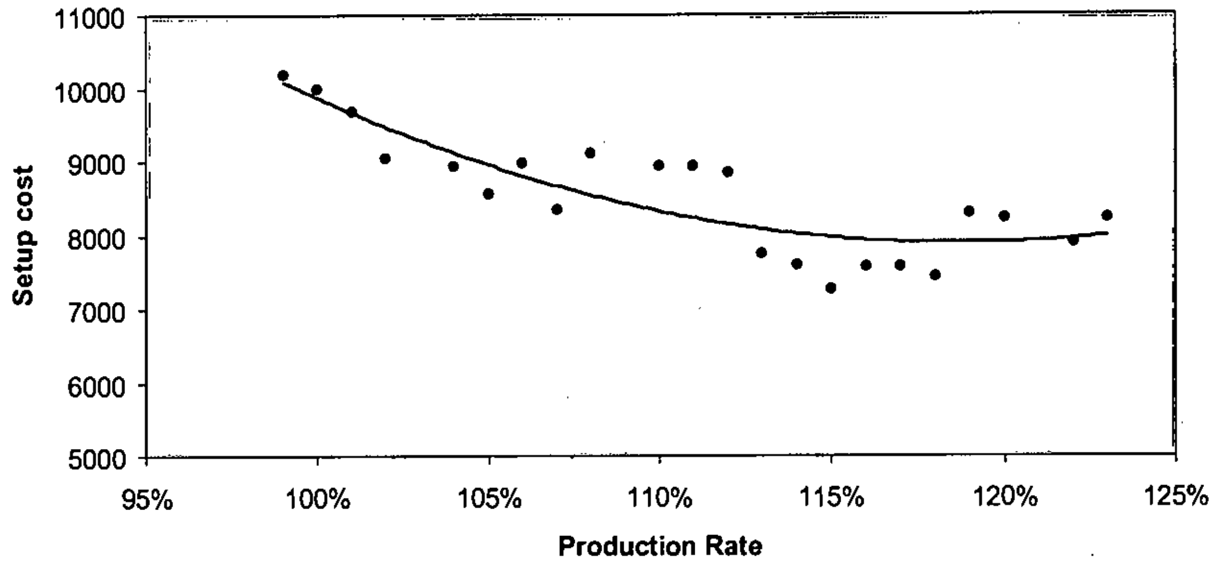

Figure 4.9 Variation of setup cost with the production rate.

Figure 4.8 shows the variation of forecasted machine utilization time with the production rate. With the increase of the production rate, there is a decrease in machine utilization time. The increase in production rate increases the number of item produced per unit time, and hence decreases the time to produce an item. This decrease in per unit production time decreases in the machine utilization time.

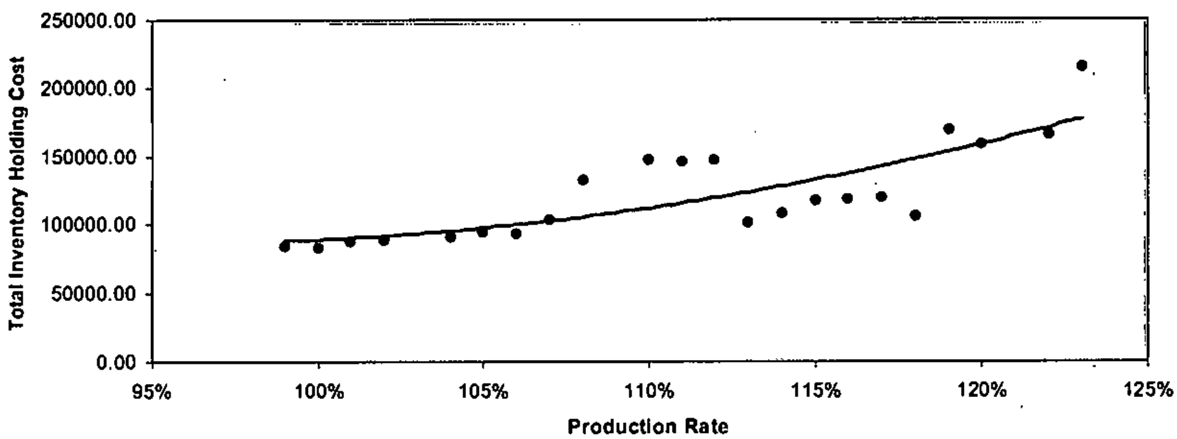

Figure 4.10 Variation of total inventory holding cost with the production rate.

Figures 4.9, 4.10 and 4.11 show the variation of setup cost, inventory holding cost and total cost with production rate. Since the increase of production rate decreases the production time for an item, the number of item produced in a period using the same capacity will increase. Then there could be earlier periods in which more items could be produced. As a result the inventory holding cost increases with the increase of production rate. Since more items could be produced in earlier periods, the number of setup required will be less. This decreasing number of setup decreases the setup cost. With the increase of production rate, the rate increase of inventory holding cost is higher than the rate of

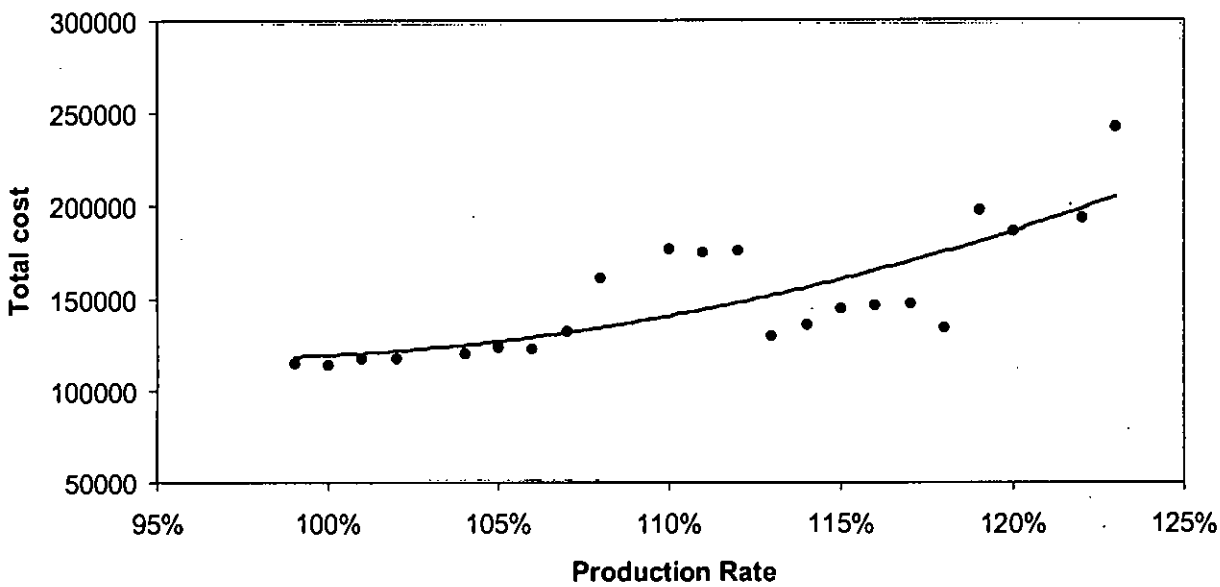

Figure 4.10 Variation of total cost with the production rate.

decrease of setup cost. However, the safety stock cost remains unchanged. Therefore the total cost increases with the increase of production rate.

Chapter 5

Conclusions and Recommendations

5.1 Conclusions

Lot-sizing problem has been recognized to be one of most important functions in industrial units. Thus efforts have been given to develop usable optimizing routines but within limited boundary conditions. Various models have been developed with restricted applications in real-life settings because of their demanding computational enormisity. Thus heuristic models have been evolved. These heuristics produce optimal and near optimal solutions. The Dixon-Silver heuristic was used in the present work. The heuristic was extended to include two very important parameters such as, (i) plant or machine set up time and (ii) maximum limit of production lot-size from a machine. From analysis and results, the present work has demonstrated that feasible solutions could be obtained with competitive computer usage. The results of the two heuristics developed in the present work, have been discussed in Chapter Four. However, the effects and implications arising out of these heuristics have been presented as concluding findings of the present work.

Model with Inclusion of Setup Time

The inclusion of setup time will result in machine occupation time to be increased. This period of increased occupation time of the machine would depend on the actual set up time of the machine. In the present work, the extent of increase was slightly higher than 1%. The consideration of set up time also led to increase in inventory holding cost by about 1.15%. This increase in cost could be attributed to increased inventory held for meeting demand of the later period.

Available machine time, inventory holding cost were found to be highly sensitive to the change in setup time. However, setup cost was not found to be significantly influenced by the setup time.

Model with limitation on lot-size

Effect of the limitation on the lot-size is dependent on the extent of reduction of the lot-size. It is obvious that the smaller the allowable lot-size, the greater will be the number of setup which will eventually lead to more splitted items. Thus when the lot-size was reduced by 90%, the model yielded the total number of splitted items of 95 from the original twelve items. This in turn led to the increase number of required set-ups.

Costs due to implementation of this restriction on lot-size went up quite significantly- the extent of which was found to be more than 23%. Further decrease in lot-size would obviously result in higher costs. But at the lower range of allowable lot-size, there has been a trend of slight increase in setup costs.

It was found that the parameters such as machine time utilization, setup cost, inventory-holding cost and the total cost were highly sensitive to the production rate of the items.

The variation was nonlinear and appeared to have step functions. So careful attention is necessary in selecting the production machineries or their attachments. Selection criteria should not be based only on the production rate as it was found that only enhancement in production rate could not guarantee minimization of costs.

5.2 Recommendations

Though some practical and real-life situations have been incorporated in the Dixon-Silver model, there are plenty of scope of improvement of the model. Following recommendations can be made for further development :

1. The Dixon-Silver model was extended through inclusion of setup time and placing limitation on the maximum allowable lot-size. In the present work these two conditions were considered separately. Further work can be performed combining the two situations to develop a uniform model.
2. A serious restriction of the heuristic lies with number of production stages. Single production stage has been considered in the present work. Development of a heuristic for multiple production stages could be a significant contribution.
3. In the present work setup time was assumed arbitrarily. Use of the randomized values would obviously be a more realistic approach. Selection of the randomized values for the setup times within certain range and running the model would be more realistic and acceptable.
4. Setup costs and setup time have been considered independently. Realistically larger setup time would lead to increased setup costs. Linking of these two parameters in the heuristic would be clearly a more realistic approach.
5. In this heuristic provision has been kept to meet future demand from the lot produced earlier. This situation may not be valid for all products. In the case of

products with shorter shelf-life, the heuristic can be modified by putting some restriction on the period of storage.

6. Back-logging was not considered in this model. Heuristic with back-logging could be developed as further work.
7. In the present work, hypothetical data have been used to run the model implementing the heuristic for a real-life problem could be a challenging work.

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Appendix A

Flow Chart for Setup Time Model

Step 1:

- Convert the initial demand matrix into equivalent demand matrix with the use of initial inventory, ending inventory and safety stock.

Step 2:

Is the problem feasible?
Feasibility condition

$$\sum_{j=1}^{H} CR_j \leq \sum_{j=1}^{H} C_j$$

No → **Stop**

Yes

Step 3 :

Apply the Dixon-Silver heuristic with inclusion of setup time.

Step 3.1:

Start at period 1, i.e. set R = 1

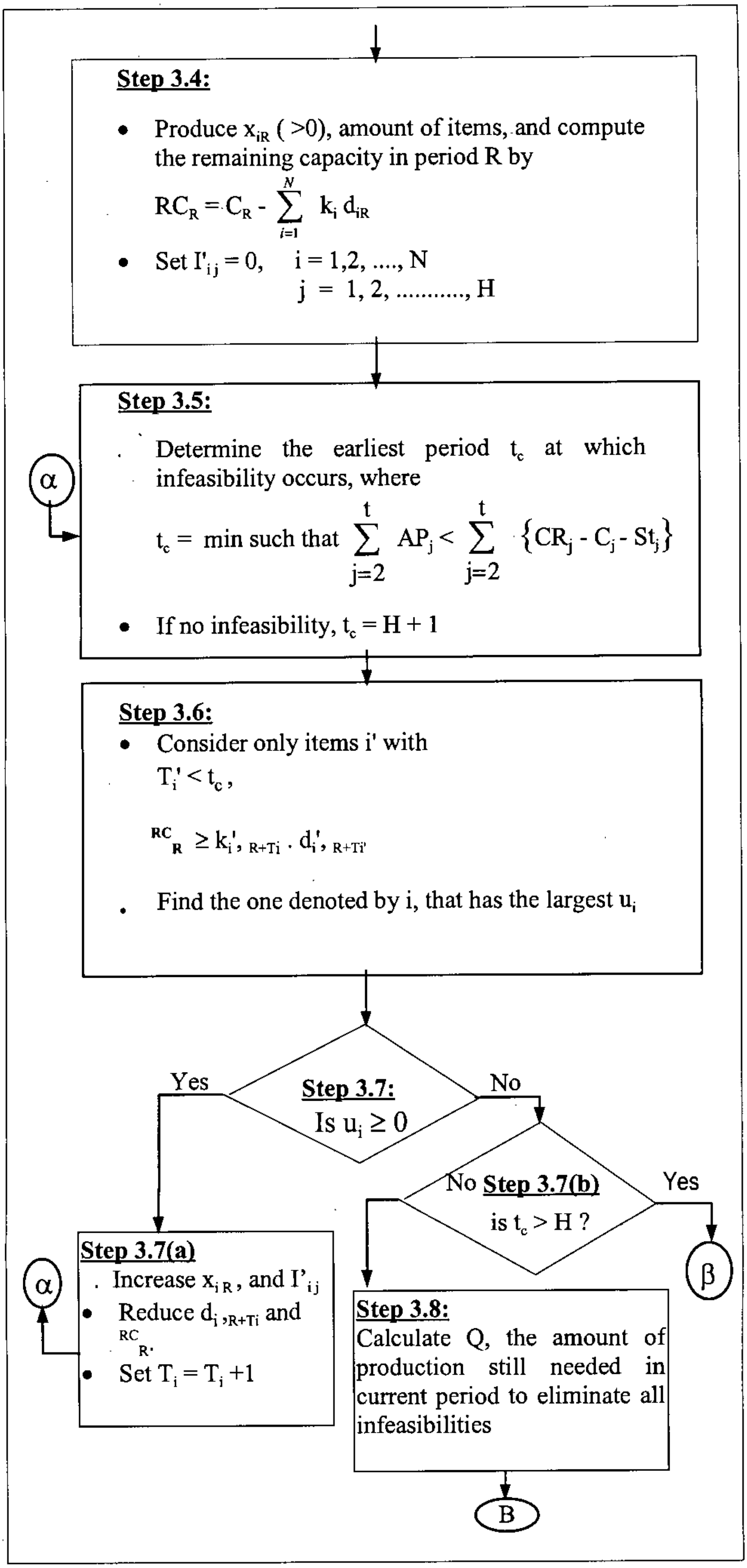
Step 3.2:

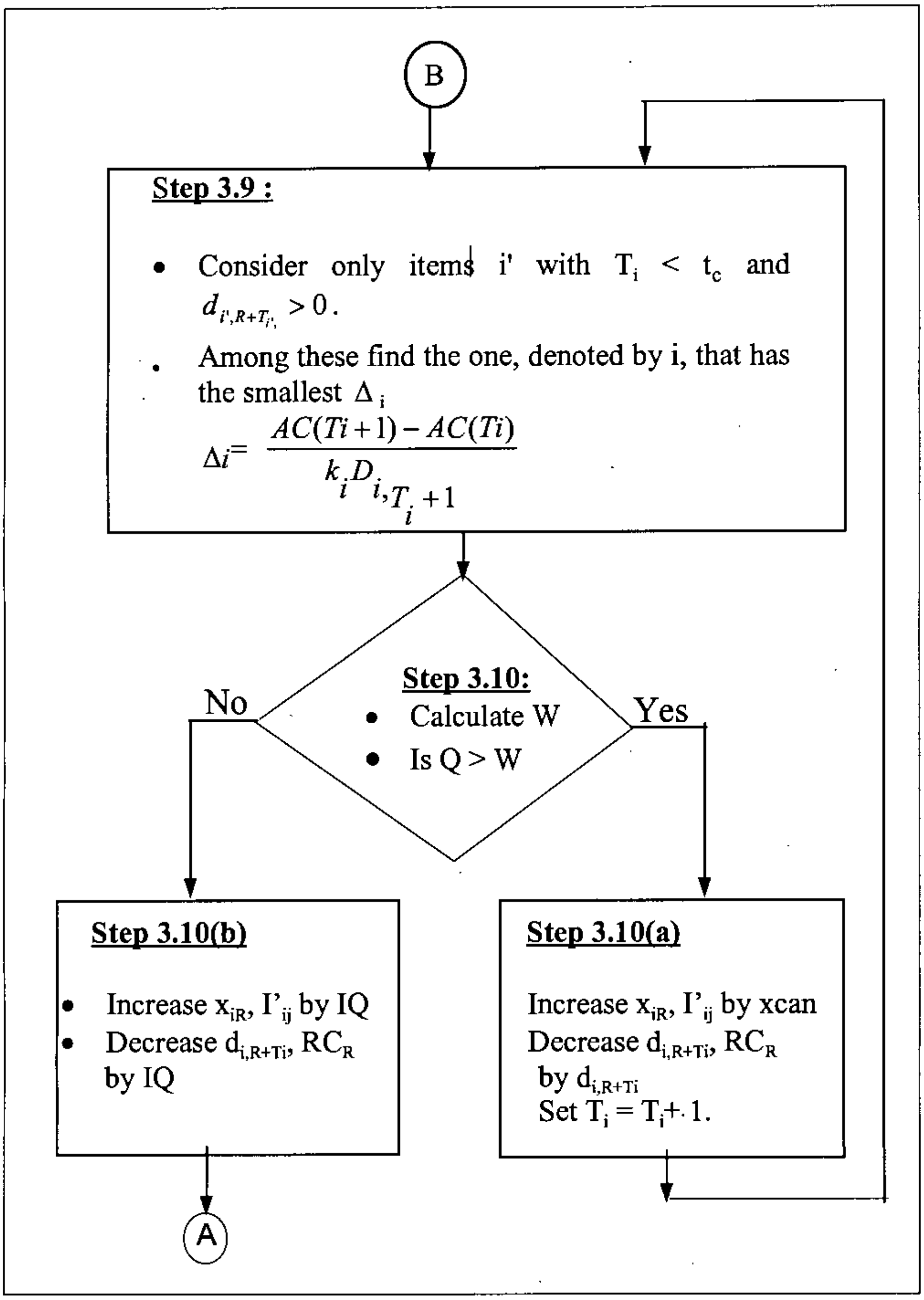
- Initialize lot-size x_{ij} by demand d_{ij}, $i = 1,2, ..., N, j = 1,2, ..., H$.

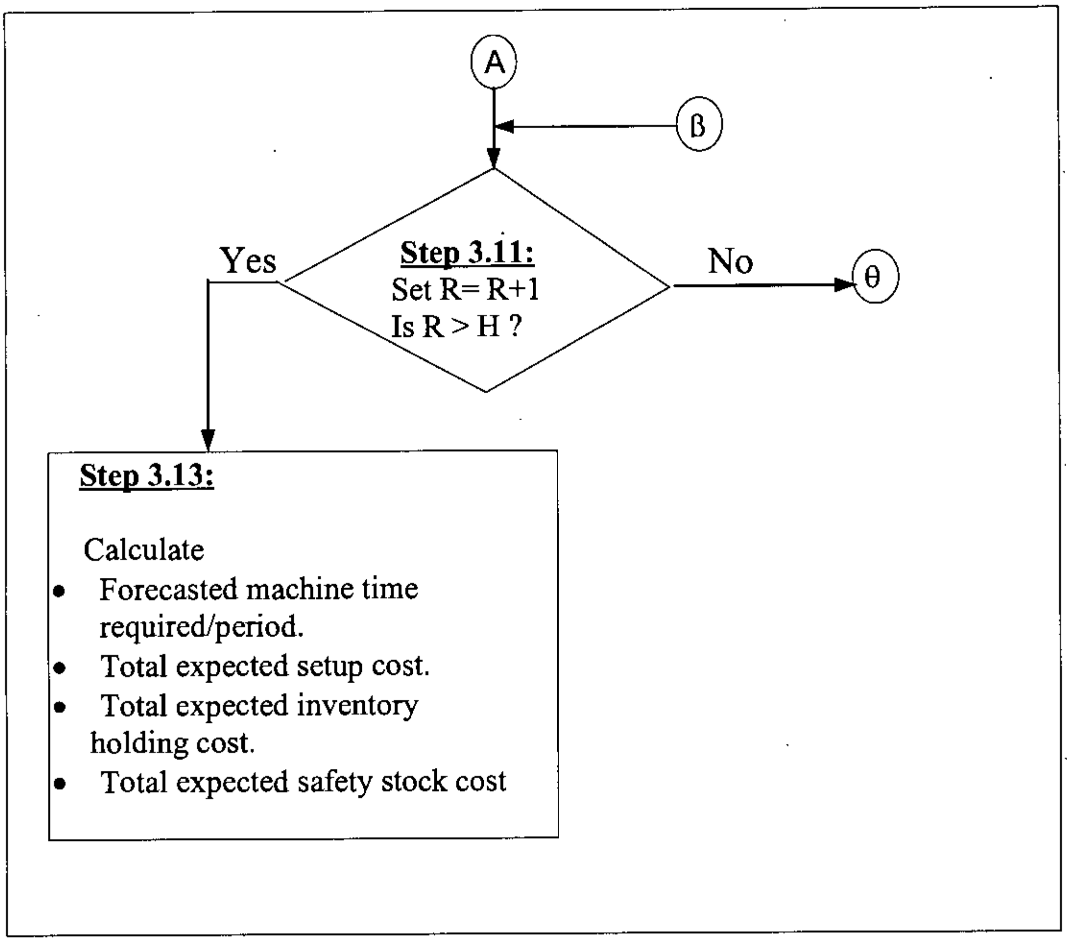
θ

Step 3.3:

Initially Set the value of time supply to one i.e. T_i=1 where $i = 1, 2, \ldots \ldots, N$


Step 3.4:
Produce x_{iR} (>0), amount of items, and compute the remaining capacity in period R by
$RC_R = C_R - \sum_{i=1}^{N} k_i d_{iR}$
Set $I'_{ij} = 0$, $i = 1,2, \ldots, N$; $j = 1, 2, \ldots, H$
Step 3.5:
Determine the earliest period t_c at which infeasibility occurs, where
$t_c =$ min such that $\sum_{j=2}^{t} AP_j < \sum_{j=2}^{t} \{CR_j - C_j - St_j\}$
If no infeasibility, $t_c = H + 1$
α
Step 3.6:
Consider only items i' with
$T_i' < t_c$,
$RC_R \geq k_i'{}_{, R+Ti} \cdot d_i'{}_{, R+Ti'}$
Find the one denoted by i, that has the largest u_i
Yes
Step 3.7:
Is $u_i \geq 0$
No
No
Step 3.7(b)
is $t_c > H$?
Yes
β
Step 3.7(a)
Increase x_{iR}, and I'_{ij}
Reduce $d_{i,R+Ti}$ and RC_R.
Set $T_i = T_i + 1$
α
Step 3.8:
Calculate Q, the amount of production still needed in current period to eliminate all infeasibilities
B


B
Step 3.9 :
• Consider only items i' with $T_i < t_c$ and $d_{i',R+T_{i'}} > 0$.
• Among these find the one, denoted by i, that has the smallest Δ_i
$\Delta i = \frac{AC(Ti+1) - AC(Ti)}{k_i D_{i,T_i+1}}$
Step 3.10:
• Calculate W
• Is Q > W
No
Yes
Step 3.10(b)
• Increase x_{iR}, I'_{ij} by IQ
• Decrease $d_{i,R+Ti}$, RC_R by IQ
Step 3.10(a)
Increase x_{iR}, I'_{ij} by xcan
Decrease $d_{i,R+Ti}$, RC_R by $d_{i,R+Ti}$
Set $T_i = T_i + 1$.
A


A
ß
Step 3.11:
Set R= R+1
Is R > H ?
Yes
No
θ
Step 3.13:
Calculate
Forecasted machine time required/period.
Total expected setup cost.
Total expected inventory holding cost.
Total expected safety stock cost

Appendix B

Flow Chart for the Limited Lot-Size Model

Step 1:

- Convert the initial demand matrix into equivalent demand matrix with the use of initial inventory, ending inventory and safety stock.

Step 2:

Is the problem feasible?
Feasibility condition

$$\sum_{j=1}^{H} CR_j \leq \sum_{j=1}^{H} C_j$$

No → **Stop**

Yes

Step 3 :

- Convert the multi setup problem into single setup problem.

Step 3.1:

- Find the number of setup required for each item per period w.r.t. maximum limited lot-size per setup.
- Convert the equivalent demand matrix N×H into a new demand matrix N'×H considering new items.

Step 3.2:

- Initialize the input arrays for the N' new items from the arrays of the item N.

Step 4 :

Apply the Dixon-Silver heuristic with inclusion of maximum limited lot-size per setup.

Step 4.1:

Start at period 1, i.e. set R = 1

Step 4.2:

- Initialize lot-size x_{ij},by demand d_{ij}, $i = 1,2, \ldots, N', j = 1,2, \ldots, H$.
- Calculate $x_{remi,j}$ (Remaining allowable maximum lot-size) by
- $x_{remi,j} = x_{maxi,j} - x_{ij}$

θ

Step 4.3:

Initially Set the value of time supply to one i.e. $T_i=1$

1 where $i = 1, 2, \ldots \ldots, N'$

Step 4.4:

- Produce x_{iR} (>0), amount of items, and compute the remaining capacity in period R by

$$RC_R = C_R - \sum_{i=1}^{N'} k_i d_{iR}$$

- Set $I'_{ij} = 0$, $i = 1,2, \ldots, N'$
  $j = 1, 2, \ldots\ldots, H$

Step 4.5:

α

- Determine the earliest period t_c at which infeasibility occurs, where

$$t_c = \min \text{ such that } \sum_{j=2}^{t} AP_j < \sum_{j=2}^{t} \{CR_j - C_j\}$$

- If no infeasibility, $t_c = H + 1$

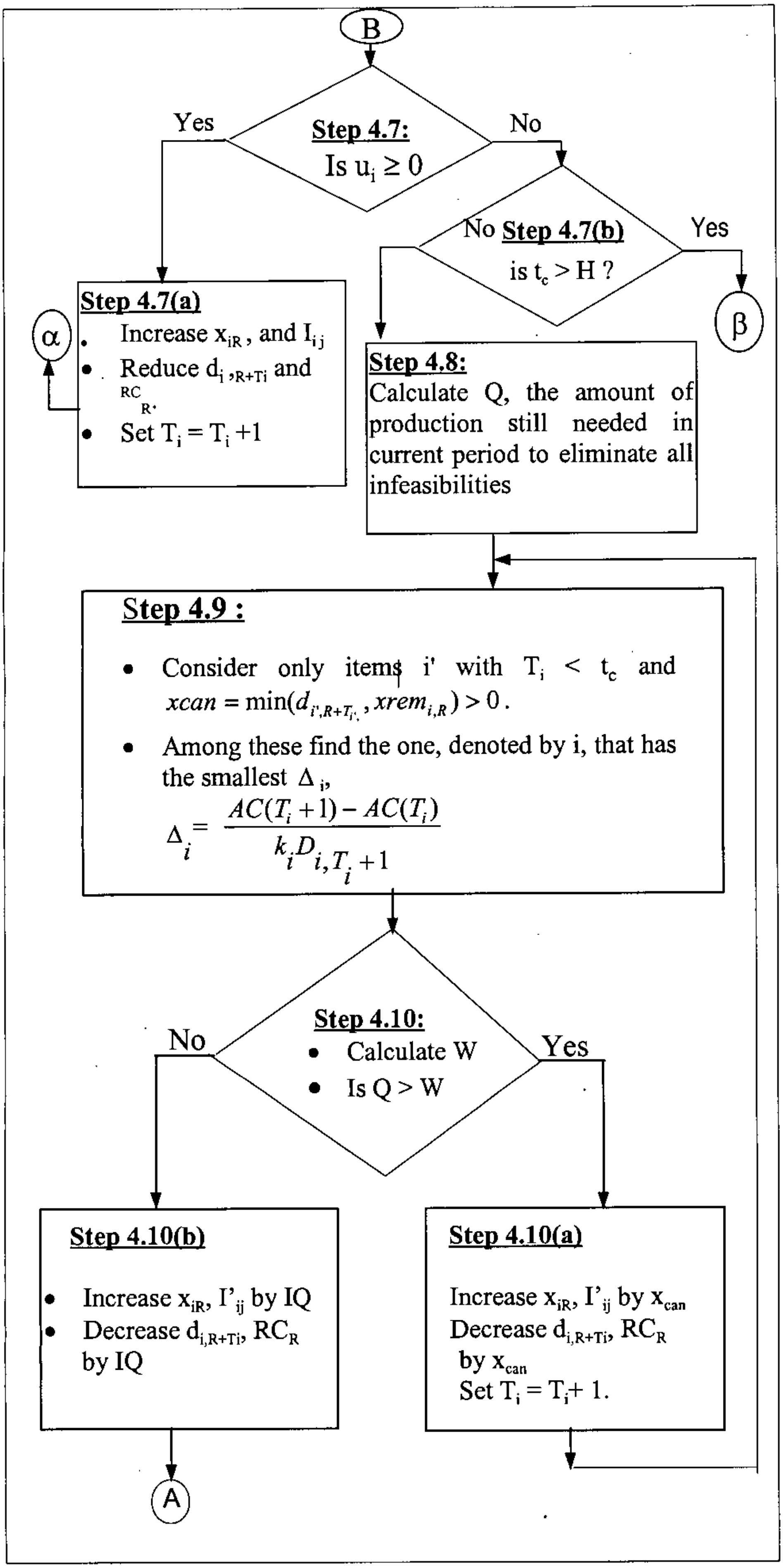
Step 4.6:

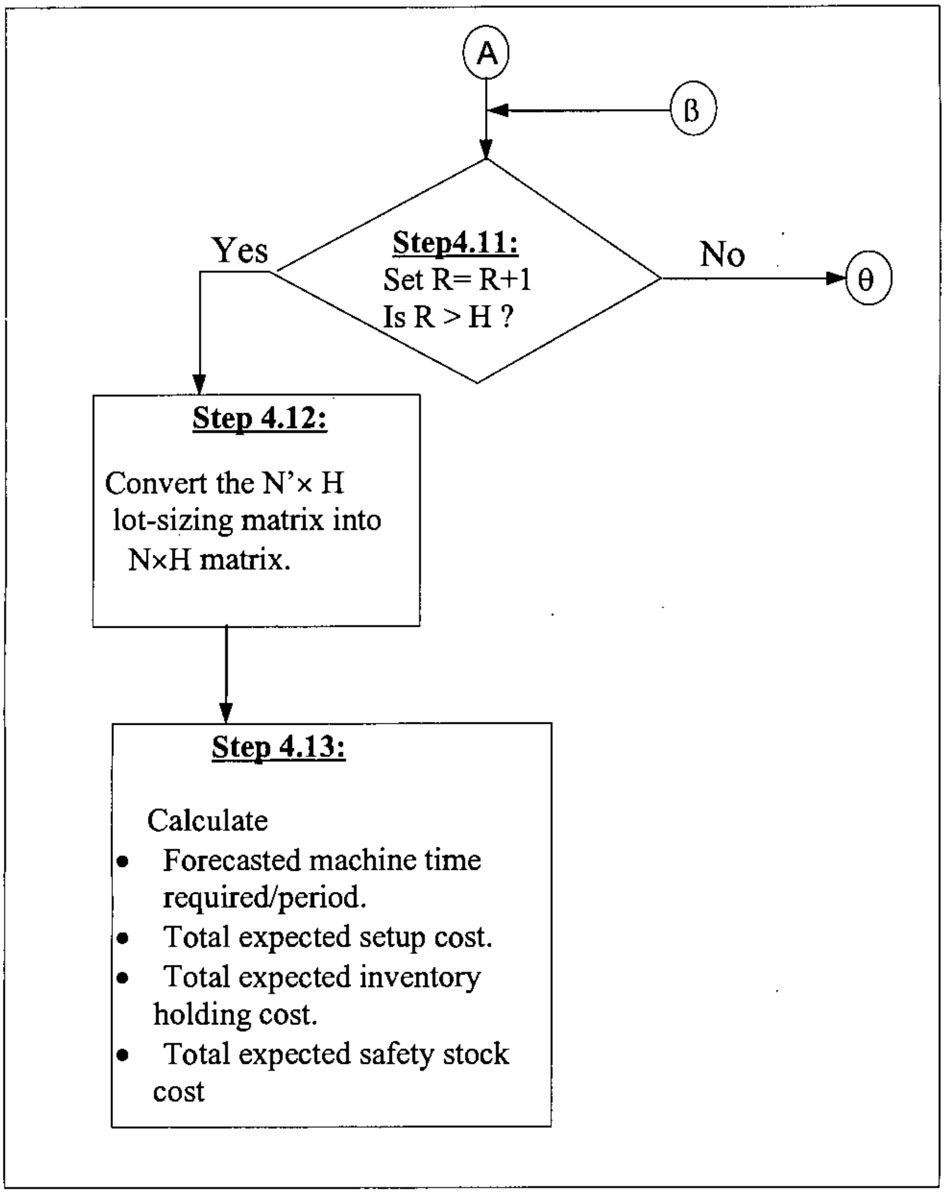
- Consider only items i' with
  $T_i' < t_c$,

  $RC_R \geq k_i', {}_{R+Ti}, x_{can}$, and $x_{can} > 0$,

- where $x_{can} = \min (d_i', {}_{R+Ti'}, x_{rem', iR})$
- Find the one denoted by i, that has the largest u_i

B


B
Yes
Step 4.7:
Is $u_i \geq 0$
No
No Step 4.7(b)
is $t_c > H$?
Yes
β
Step 4.7(a)
α
Increase x_{iR}, and I_{ij}
Reduce $d_{i,R+Ti}$ and RC_R.
Set $T_i = T_i + 1$
Step 4.8:
Calculate Q, the amount of production still needed in current period to eliminate all infeasibilities
Step 4.9 :
Consider only items i' with $T_i < t_c$ and $xcan = \min(d_{i',R+T_{i'}}, xrem_{i,R}) > 0$.
Among these find the one, denoted by i, that has the smallest Δ_i,
$\Delta_i = \frac{AC(T_i+1) - AC(T_i)}{k_i D_{i,T_i+1}}$
Step 4.10:
Calculate W
Is Q > W
No
Yes
Step 4.10(b)
Increase x_{iR}, I'_{ij} by IQ
Decrease $d_{i,R+Ti}$, RC_R by IQ
Step 4.10(a)
Increase x_{iR}, I'_{ij} by x_{can}
Decrease $d_{i,R+Ti}$, RC_R by x_{can}
Set $T_i = T_i + 1$.
A


A
β
Step4.11:
Set R= R+1
Is R > H ?
Yes
No
θ
Step 4.12:
Convert the N'× H lot-sizing matrix into N×H matrix.
Step 4.13:
Calculate
• Forecasted machine time required/period.
• Total expected setup cost.
• Total expected inventory holding cost.
• Total expected safety stock cost