Base Transit Time for All Levels of Injection of a Bipolar Junction Transistor with Gaussian Base Doping Profile

A thesis submitted to the Department of Electrical and Electronic Engineering (EEE)
of
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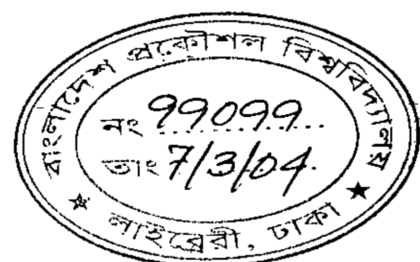
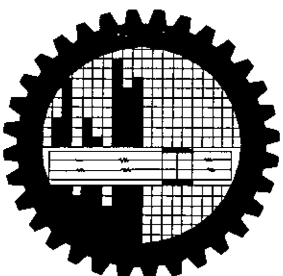
in partial fulfillment of the requirement for the degree of
MASTER OF SCIENCE IN ELECTRICAL AND ELECTRONIC ENGINEERING



by
Touhidur Rahman



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The thesis titled "**Base Transit Time for All Levels of Injection of a Bipolar Junction Transistor with Gaussian Base Doping Profile**" submitted by Touhidur Rahman, Roll No.: 040206203P, Session: April 2002 has been accepted as satisfactory in partial fulfillment of the requirement for the degree of MASTER OF SCIENCE IN ELECTRICAL AND ELECTRONIC ENGINEERING on January 22, 2004.

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I hereby declare that this thesis or any part of it has not been submitted elsewhere for the award of any degree or diploma.

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____

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DEDICATION

To My Parents

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LIST OF SYMBOLS

Symbols	Description
D_n	Diffusion co-efficient for electron
D_P	Diffusion co-efficient for hole
E	Electric field
L_n	Diffusion length for electron
I_B	Base current
I_C	Collector current
I_E	Emitter current
J_P	Current density for hole
J_{nh}	Current density for high injection region
J_{nl}	Current density for low injection region
J_n	Current density for all levels of injection
n_{ie}	Effective intrinsic carrier concentration
$p(x)$	Hole concentration in the base
$n_l(x)$	Injected carrier concentration for low injection
$n_h(x)$	Injected carrier concentration for high injection
$N_A(0)$	Base peak doping concentration in the base
$n(0)$	Injected electron at base-emitter junction
Q_{hb}	Stored base charge per unit area for high injection
Q_{lb}	Stored base charge per unit area for low injection

Q_{bn}	Stored base charge density for all levels of injection
W_b	Base width
v_s	Saturation velocity
V_T	Thermal voltage
Δn_C	The minority carrier concentrations at the base side of base-collector junction
Δn_E	The minority carrier concentrations at the base side of base-emitter junction
γ_1	0.42
γ_2	0.69
τ_{hb}	Base transit time for high injection
τ_{lb}	Base transit time for low injection
τ_B	Base transit time for all levels of injection

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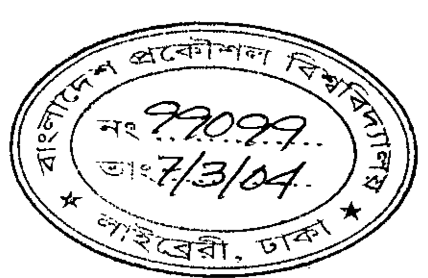
ABSTRACT

Base transit time of a bipolar junction transistor with Gaussian doped base is studied. Analytical expressions for the base transit time for low and high levels of injection are obtained incorporating doping dependence of mobility, band gap narrowing effect and velocity saturation effect of electron at collector-base junction. For the intermediate region of injection the differential equation is not analytically tractable. By using the asymptotic behaviors in the low and high injection regions an empirical relation is obtained. The empirical expression is not only applicable for intermediate region of operation but also applicable for all levels of injection. For both low and high levels of injection the base transit time is found to be independent of base-emitter voltage. In the intermediate region it is an increasing function of base-emitter voltage. The transit time for low injection is found to be smaller than that of the high injection. The results obtained by using the empirical relations are compared with the results available in the literature and are found in good agreement.

CHAPTER ONE



INTRODUCTION

1.1 BIPOLAR JUNCTION TRANSISTOR

The bipolar junction transistor (BJT) was the first solid-state amplifier element and started the solid-state electronics revolution. Bardeen, Brattain and Shockley at the Bell Laboratories invented it in 1948 as part of a post-war effort to replace vacuum tubes with solid-state devices. Solid-state rectifiers were already in use at the time and were preferred over vacuum diodes because of their smaller size, lower weight and higher reliability. A solid-state replacement for a vacuum triode was expected to yield similar advantages. The work at Bell Laboratories was highly successful and culminated in Bardeen, Brattain and Shockley receiving the Nobel Prize in 1956.

Although CMOS has acquired an ever-increasing role, the bipolar transistor retains its position as a premier technology for high-speed circuits, ultra-high-speed discrete logic circuits such as emitter coupled logic (ECL), power amplifiers, power-switching applications, mixed-signal and precision analog components, microwave power amplifiers and in other applications. The speed advantage, together with transconductance (i.e., current drive), high self-gain, and low $1/f$ noise continue to make the bipolar transistor the device of choice for many demanding applications.

A bipolar junction transistor consists of two back-to-back p-n junctions, who share a thin common region. When the common region is doped with acceptor atoms and the other two regions are doped with donor atoms the transistor is called an n-p-n bipolar junction transistor. For opposite types of doping in these regions the transistor becomes a p-n-p bipolar junction transistor. Metal contacts are made to all three regions. The operation of a bipolar junction transistor depends on the forward and the reverse current of the two back-to-back p-n junctions. The forward-biased junction, which injects electrons into the center p region, is called base-emitter junction, and the reversed-biased

junction, which collects the injected electrons, is called the base-collector junction. The region, which serves as the source of injected electrons, is called the emitter and the n region into which electrons are swept by the reverse-biased junction is called the collector. The center region is called the base. For practical interest doping concentration of emitter is made very high. Thus it forms an n+-p-n structure. Fig 1.1 shows an n+-p-n bipolar junction transistor.

To have a good n+-p-n transistor, we would prefer that almost all the electrons injected by the emitter into the base be collected. Thus the p-type base region should be narrow. This requirement is summed up by specifying neutral base width should be less than the diffusion length of electron in the base. With this requirement satisfied, an average electron injected at the emitter junction will diffuse to the depletion region of the base-collector junction without recombination in the base. A second requirement is that the emitter current, I_E crossing the emitter junction should be composed almost entirely of electrons injected into the base, rather than holes crossing from base to emitter [1].

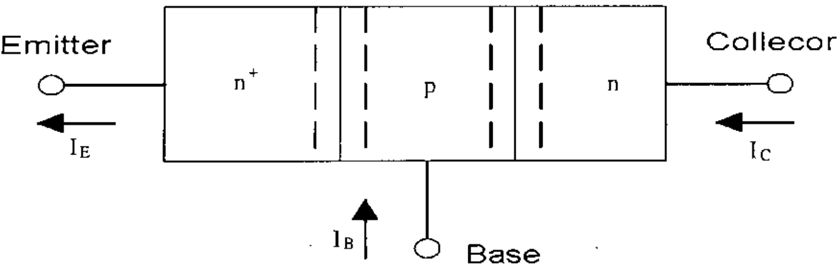


Fig. 1.1 An n$^+$-p-n bipolar junction transistor

When base-emitter junction is forward biased emitter injects electron into the base. If injected electron density is less than the doping density, the expressions for emitter current, I_E, collector current, I_C and base current I_B can be written as [1]

$$I_E = qA\frac{D_n}{L_n}\left(\Delta n_E \coth\left(\frac{W_b}{L_n}\right) - \Delta n_C \operatorname{csc}h\left(\frac{W_b}{L_n}\right)\right) \tag{1.1a}$$

$$I_C = qA\frac{D_n}{L_n}\left(\Delta n_E \operatorname{csc}h\left(\frac{W_b}{L_n}\right) - \Delta n_C \coth\left(\frac{W_b}{L_n}\right)\right) \tag{1.1b}$$

and,

$$I_B = I_E - I_C = qA\frac{D_n}{L_n}\left((\Delta n_E + \Delta n_E)\tanh\left(\frac{W_b}{2L_n}\right)\right) \tag{1.1c}$$

where, D_n is the diffusion coefficient for electron, L_n is diffusion length for electron, q is the charge of electron, W_b is base width, A is the cross sectional area in the perpendicular direction of current flow, Δn_E and Δn_C are excess minority carrier concentrations at the base side of base-emitter and base-collector junctions respectively.

One of the important figures of merit of BJTs is the cut-off frequency, f_T. Bipolar transistors have been used mainly due to their speed advantage and driving capability. Major applications today are RF front-end as well as fiber-optic circuits, in which the transistors are operated at high frequency. Among different delay terms, the base transit time τ_B is one of the dominant factor in deciding $f_T(1/2\pi\tau)$, where τ is the total delay time. It is of great importance to obtain an accurate, yet relatively simple analytical model of base transit time and collector current density for high frequency bipolar transistors for efficient device design. Thus, developing an accurate analytical model for the base transit time is useful. Several transit time models have been proposed in the literature. Unfortunately, in many cases, the model equations were complicated and difficult for integration to obtain the stored charge in the base Q_{bn}. Most of the models considered low injection. But modern BJT is subjected to high injection.

1.2 BASE TRANSIT TIME

When emitter-base junction of n^+-p-n transistor is forward biased, electrons from the emitter are injected in the base. Injected electrons are called minority carrier in p type base. The average time taken by minority carriers to diffuse across the quasi-neutral base or the average time that an electron spends in the base is called the base transit time, τ_B.

The base transit time can be written as [1]

$$\tau_B = \int_0^{W_b} \frac{qn(x)}{J_n(x)}dx \tag{1.2}$$

If the recombination within the base is neglected then electron current within the base is constant and eqn. (1.2) can be written as

$$\tau_B = \frac{1}{J_n}\int_0^{W_b} qn(x)dx = \frac{Q_{bn}}{J_n} \tag{1.3}$$

where, J_n is the current density.

and $$Q_{bn} = q\int_0^{W_b} n(x)dx \tag{1.4}$$

where, Q_{bn} is the stored base charge per unit area and $n(x)$ is the minority carrier distribution within the base.

1.3 REVIEW OF RECENT WORKS ON BASE TRANSIT TIME

Some works have been reported on base transit time for low and high levels of injection in the base. Moll and Ross [2] formulated a double integral relation of transit time for a homojunction bipolar transistor. The proposed expression is given below

$$\tau_B = \frac{1}{D_n}\int_B \frac{1}{N_A}\left[\int_x^{W_b} N_A dy\right]dx \tag{1.5}$$

The transit time depends on the acceptor distribution. But the above equaion for transit time is applicable for low level of injection.

In 1986 J. J. H. Van den Biesen [3] studied the base transit time using regional analysis. He used an expression for base transit time based on perturbation theory and that was written as [3]

$$\tau_B = q\int_{x_{eb}}^{x_{bc}} \left|dn/dJ_c\right|_{v_{ce}=0} dx \tag{1.6}$$

M. M. S. Hassan and A. H. Khondoker [4] derived mathematical expressions for electron current density and base transit time for uniform base doping profile for all levels of injection. In this work base doping density dependence of mobility was considered but the carrier saturation velocity was neglected.

K. Suzuki [5] in 1993 derived expressions for electron current density and base transit time for uniformly doped base for high level of injection considering carrier velocity

saturation at base-collector junction. But in practical transistor base is not uniformly doped.

The works on uniformly doped base have shown i) for both low and high levels of injection τ_B is independent of base-emitter voltage V_{be} for a given doping profile N_A ii) for intermediate values of V_{be}, τ_B decreases with V_{be} iii) τ_B increases with increase of N_A and W_b.

K. Suzuki [6] in 1991 showed the dependency of transit time on built-in electric field, mobility and band gap narrowing. But carrier velocity saturation at base-collector junction was neglected in this work.

Later, K. Suzuki and N. Nakayama [7] in 1992 derived an analytical expression for the base transit time considering carrier velocity saturation at base-collector junction. The derived equation is

$$\tau_B = \int_0^{W_b} \frac{n_{ie}^2(x)}{N_A(x)} \left\{ \int_x^{W_b} \frac{1}{D_n(y)} \frac{N_A(y)}{n_{ie}^2(y)} dy \right\} dx + \frac{1}{v_s} \frac{N_A(W_b)}{n_{ie}^2(W_b)} \int_0^{W_b} \frac{n_{ie}^2(x)}{N_A(x)} dx = \frac{W_b^2}{\eta_1 D_n(0)} + \frac{W_b}{\eta_2 v_s} \quad (1.6)$$

$$\text{where, } \eta_1 = \frac{W_b^2}{\int_0^{W_b} \frac{n_{ie}^2(x)}{N_A(x)} \int_x^{W_b} \frac{1}{D_n(y)} \frac{N_A(y)}{n_{ie}^2(y)} dy dx} \quad (1.7)$$

$$\eta_2 = \frac{W_b}{\int_0^{W_b} \frac{N_A(W_b)}{n_{ie}^2(W_b)} \frac{n_{ie}^2(x)}{N_A(x)} dx} \quad (1.8)$$

But the analysis was done only for low injection region.

K. Suzuki [8] proposed another model for base transit time for arbitrary doped bases. The model was valid in all injection regions before the onset of the Kirk effect. The high injection and bandgap narrowing were considered in the work. But the analysis was based on perturbation theory.

J. S. Yuan [9] in 1994 reported a work on base transit time for all levels of injection. He solved the equations numerically to show the effect of base profile on base transit time.

P. Ma, L. Zhang and Y Wang [10] developed an expression for τ_B based on iterative techniques. They considered the velocity saturation of carrier and electric field dependence of mobility. As it was based on iterative techniques the equations for electron

current density and base transit time are not concise and they are inconvenient for us to understand the device physics.

In practical transistor base doping follows Gaussian distribution. Recently Li G., Neugroschel A., Sah C. T., Hemmenway D., Rivoli T., Maddux J. [11] used a method for a quantitative charge-control analysis of bipolar base-junction transistors with a Gaussian dopant impurity-concentration profile. Analytical expression for the base transit time is given for Gaussian impurity-concentration. In this analysis Gaussian profile was approximated by simple exponential profile. Exponential profile distribution gives a constant electric field in the base [12]. On the other hand Gaussian profile gives linearly increasing electric field. In the work, doping dependence of mobility was not considered.

For arbitrary doped base transit time is also independent of V_{be} for low and high levels of injection. But for intermediate region of injection τ_B increases with V_{be}. This is opposite to the observations for a uniformly doped base.

1.4 OBJECTIVE OF THE THESIS

The analytical work on base transit time with Gaussian base distribution was carried by approximating the profile with an exponential distribution [11]. There is no analytical work for base transit time for Gaussian base distribution. The main objective of this work is to obtain an empirical expression for base transit time of an n^+-p-n bipolar transistor with Gaussian base doping profile. The expression will be applicable for all levels of injection. The injected minority carrier, collector current density, stored base charge, electric field in the base will also be studied. Knowing the mathematical expression for base transit time we can determine the dependence of base transit time on different device parameters. Analytical results will be compared with numerical data available in the literature.

1.5 SUMMARY OF THE THESIS

In this work the expression for the base transit time of a bipolar junction transistor is derived. In chapter one previous works on base transit time have been reviewed. Mathematical analyses are given in chapter two. In chapter three the dependence of current and transit time on device parameters are studied. The transit time obtained from the derived equation is compared to that of numerical analysis in order to demonstrate the validity of the assumptions made in deriving expression for base transit time. This paper ends in chapter four containing salient features of this work and possible future field of studies.

CHAPTER TWO

MATHEMATICAL ANALYSIS

2.1 INTRODUCTION

It is of great importance to obtain an accurate, yet relatively simple analytical model for base transit time and collector current density for modern bipolar junction transistors for efficient device design. Within the framework of drift and diffusion approaches, a successful analytical model should consider the following important effects: (a) Gaussian doping profiles; (b) heavy doping effect; (c) finite carrier velocity at the collector edge; (d) dependence of mobility on doping concentration; (e) high injection effects. In this chapter an analytical expression for base transit time for Gaussian base doping is derived considering all the above-mentioned effects.

2.2 DERIVATION OF THE MODEL EQUATIONS

An n^+-p-n transistor is shown in Fig. 2.1. The electron current density J_n and hole current density J_p with arbitrary base doping concentration $N_A(x)$ are given by [9]

$$-J_n(x) = q\,D_n(x)\frac{dn(x)}{dx} + q\,\mu_n(x)n(x)E(x) \tag{2.1a}$$

$$J_p(x) = -q\,D_p(x)\frac{dp(x)}{dx} + q\,\mu_p(x)p(x)E(x) \tag{2.1b}$$

where, x is the distance of a point in the base from the base-emitter junction, $D_n(x)$ and $D_p(x)$ are the diffusion co-efficient for electron and hole respectively, $\mu_n(x)$ and $\mu_p(x)$ are the electron and hole mobility respectively, $n(x)$ and $p(x)$ are electron and hole concentration respectively, q is the charge of electron and $E(x)$ is the electric field at a

point x in the base. We define the direction of J_n in eqn. (2.1a) so that it has a positive value.

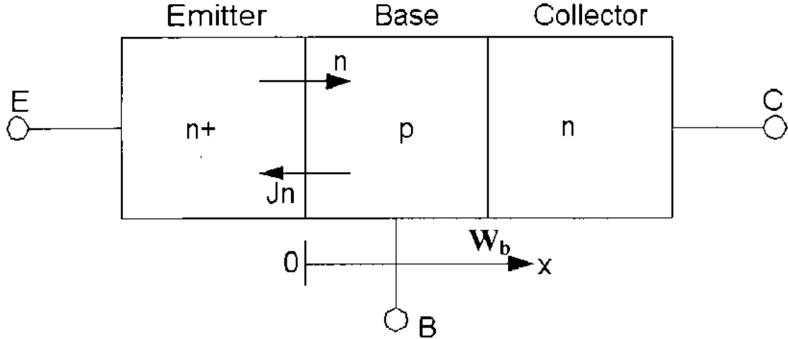


Fig. 2.1 One-dimensional view of an n^+-p-n bipolar junction transistor

The total current density is the sum of the electron and hole current density.

$$J_C(x) = J_n(x) + J_p(x) \tag{2.2}$$

The base width of a modern bipolar transistor is very thin. The carrier recombination within the base can be neglected [13]. We can write,

$$J_p(x) = 0 \tag{2.3a}$$

and,
$$J_C(x) \cong J_n(x) \tag{2.3b}$$

Therefore, the minority carrier current density J_n within the base becomes constant and is equal to collector current density J_C.

The electric field considering band gap narrowing effect and neglecting recombination in the base can be given by [14]

$$E(x) = V_T \frac{d}{dx}\left[\ln\left(\frac{n(x) + N_A(x)}{n_{ie}^2(x)}\right)\right] \tag{2.4}$$

where, $n_{ie}(x)$ is the effective intrinsic carrier concentration in the base.

From eqn. (2.4) we can write [10]

$$E(x) = V_T \frac{1}{2n(x) + N_A(x)}\left[\frac{d N_A(x)}{dx} - \frac{n(x) + N_A(x)}{n_{ie}^2(x)} \frac{d n_{ie}^2(x)}{dx} - \frac{J_n}{qD_n(x)}\right] \tag{2.5}$$

The first term in eqn. (2.5) represents electric field due to concentration gradient and the second term in eqn. (2.5) represents the quasi-field due to nonuniform band gap narrowing.

If the electric field from eqn. (2.4) is substituted in eqn (2.1a) the collector current density can be written as [15]

$$-J_n = q\,D_n(x)\frac{n_{ie}^2(x)}{n(x)+N_A(x)}\frac{d}{dx}\left[\frac{n(x)[n(x)+N_A(x)]}{n_{ie}^2(x)}\right] \tag{2.6}$$

Eqn. (2.6) should be solved to find the current density J_n and stored base charge per unit area Q_{bn}.

The coefficient of electron and intrinsic carrier concentration are given by [16]

$$D_n(x) = D_{no}\left(\frac{N_A(x)}{N_r}\right)^{-\gamma 1} \tag{2.7}$$

$$n_{ie}^2(x) = n_{i0}^2\left(\frac{N_A(x)}{N_r}\right)^{\gamma 2} \tag{2.8}$$

where, $D_{no} = 20.72cm^2/s$, $N_r = 10^{17}cm^{-3}$, $n_{io} = 1.4\times10^{10}cm^{-3}$, $\gamma_1 = 0.42$ and $\gamma_2 = 0.69$.

In this work, a Gaussian doped base is considered. Gaussian distribution is given by [17]

$$N_A(x) = N_A(0)e^{-x^2/2\sigma^2} \tag{2.9}$$

where, $$\sigma = \frac{W_b}{\sqrt{2\ln[N_A(0)/N_A(W_b)]}} \tag{2.10}$$

$N_A(0)$ is the peak impurity concentration at $x = 0$ and $N_A(W_b)$ is the doping density at $x = W_b$.

We need to find a mathematical expression for $n(x)$ and J_n in order to derive an expression for base transit time τ_B.

When the emitter-base junction is forward biased injected electron density depends on the externally applied voltage across base-emitter junction. The base will be subjected to three types of injection. At low level of injection injected electron density will be much less than that of the doping in the base, i.e. $N_A(x) >> n(x)$. At high level of injection injected electron density will be much higher than that of the doping in the base, i.e. $N_A(x) << n(x)$. But for intermediate level of injection the injected carrier density is

comparable to that of the base doping. In this chapter we have obtained mathematical expressions for $n(x)$, J_n, Q_{bn} and τ_B.

2.2.1 LOW INJECTION

For low injection, the quasi-neutral condition becomes

$$N_A(x)+n_l(x)\approx N_A(x) \tag{2.11}$$

The subscript 'l' stands for low injection value of the associated parameter hereafter.
From eqn. (2.5) electric field for low injection

$$E(x)=V_T\frac{1}{N_A(x)}\left[\frac{d\,N_A(x)}{dx}-\frac{N_A(x)}{n_{ie}^2(x)}\frac{d\,n_{ie}^2(x)}{dx}-\frac{J_{nl}}{qD_n(x)}\right] \tag{2.12}$$

Using the expressions of $D_n(x)$, $n_{ie}(x)$ and $N_A(x)$ in eqn. (2.12)

$$E(x)=V_T\left[-\frac{1-\gamma_2}{\sigma^2}x-\frac{J_{nl}}{q\,N_A(x)D_n(x)}\right] \tag{2.13}$$

Using (2.11), eqn. (2.6) becomes,

$$-J_{nl}=q\,D_n(x)\frac{n_{ie}^2(x)}{N_A(x)}\frac{d}{dx}\left[\frac{n(x)N_A(x)}{n_{ie}^2(x)}\right] \tag{2.14}$$

Using the expressions of $D_n(x)$, $n_{ie}(x)$ and $N_A(x)$ in eqn. (2.14) gives,

$$n_l(x)e^{-(1-\gamma_2)x^2/2\sigma^2}-n_l(0)=-\frac{J_{nl}}{q\,D_{no}}\left(\frac{N_A(0)}{N_r}\right)^{\gamma_1}\int_0^x e^{-ax^2/2\sigma^2}dx \tag{2.15}$$

where, $\qquad a=1+\gamma_1-\gamma_2 \qquad$ (2.16)

From (2.16) we obtain (see Appendix)

$$n_l(x)=n_l(0)e^{(1-\gamma_2)x^2/2\sigma^2}-\frac{J_{nl}}{q\,D_{n0}}\left(\frac{N_A(0)}{N_r}\right)^{\gamma_1}\sigma\sqrt{\frac{\pi}{2a}}\left[e^{(1-\gamma_2)x^2/2\sigma^2}-e^{-\gamma_1x^2/2\sigma^2}\right] \tag{2.17}$$

Assuming that the electron velocity in the base-collector depletion region saturates at v_s, the electron current density J_{nl} at $x=W_b$ is given by

$$J_{nl}=n_l(W_b)q\,v_s \tag{2.18}$$

Substituting this value in (2.17) and rearranging it, J_{nl} for low injection can be expressed as

$$J_{nl} = q\, v_s B_l\, n_l(0) \tag{2.19}$$

where,

$$B_l = \frac{e^{(1-\gamma_2)W_b^2/2\sigma^2}}{1+\sigma\sqrt{\frac{\pi}{2a}}\frac{v_s}{D_{n0}}\left(\frac{N_A(0)}{N_r}\right)^{\gamma 1}\left[e^{(1-\gamma_2)W_b^2/2\sigma^2} - e^{-\gamma_1 W_b^2/2\sigma^2}\right]} \tag{2.20}$$

Integrating $n_l(x)$ of (2.17) gives

$$Q_{bnl} = qn(0)F_l \tag{2.21}$$

$$\begin{aligned} F_l &= \sigma\sqrt{\frac{\pi}{2(1-\gamma_2)}}\left[\sqrt{e^{(1-\gamma_2)W_b^2/\sigma^2}} - 1\right] \\ &-\sigma^2\frac{\pi}{2}\sqrt{\frac{1}{a}}\frac{B\, v_s}{D_{n0}}\left(\frac{N_A(0)}{N_r}\right)^{\gamma 1}\left[\sqrt{\frac{1}{\gamma_1}}\left(e^{-\gamma_1 W_b^2/2\sigma^2} - 1\right) + \sqrt{\frac{1}{1-\gamma_2}}\sqrt{e^{(1-\gamma_2)W_b^2/\sigma^2}} - 1\right] \end{aligned} \tag{2.22}$$

Using (2.17) and (2.19) the base transit time can be obtained as

$$\tau_{Bl} = \frac{F_l}{v_s B_l} \tag{2.23}$$

Here B_l and F_l are the function of $N_A(0)$ and W_b. Therefore, the base transit time depends upon $N_A(0)$ and W_b. As long as low level of injection condition prevails in the base, τ_{bl} is independent of $n_l(0)$ which is a function of base-emitter junction voltage V_{be} [17]. For uniformly and exponentially doped base, transit time was also found independent of V_{be} [12,15,18].

2.2.2 HIGH INJECTION

For high injection, the quasi-neutral condition becomes

$$N_A(x) + n_h(x) \approx n_h(x) \tag{2.24}$$

The subscript 'h' stands for high injection value of the associated parameter hereafter.

From eqn. (2.5) electric field for high injection

$$E(x) = V_T \frac{1}{2n_h(x)}\left[-N_A(x)\frac{x}{\sigma^2} + n_h(x)\frac{x\gamma_2}{\sigma^2} - \frac{J_{nh}}{qD_n(x)}\right] \tag{2.25}$$

Using (2.24), eqn. (2.6) becomes,

$$-J_{nh} = q\,D_n(x)\frac{n_{ie}^2(x)}{n_h(x)}\frac{d}{dx}\left[\frac{n_h^2(x)}{n_{ie}^2(x)}\right] \tag{2.26}$$

Using the equation of $D_n(x)$, $n_{ie}(x)$ and $N_A(x)$ in (2.26) and then integrating (see appendix), we get

$$n_h(x) = n_h(0)e^{-\gamma_2 x^2/4\sigma^2} - \frac{J_{nh}}{q}\left[\frac{1}{2D_{n0}}\left(\frac{N_A(0)}{N_r}\right)^{\gamma 1}\sigma\sqrt{\frac{\pi}{2\left(\gamma_1 - \frac{\gamma_2}{2}\right)}}\left(e^{-\gamma_2 x^2/4\sigma^2} - e^{-\gamma_1 x^2/2\sigma^2}\right)\right] \tag{2.27}$$

Assuming that the electron velocity in the base-collector depletion region saturates at v_s, the electron current density J_{nh} at $x = W_b$ is given by

$$J_{nh} = n_h(W_b)q\,v_s \tag{2.28}$$

Substituting this value in (2.27) and rearranging it, J_{nh} for low injection can be expressed as

$$J_{nh} = q\,v_s B_h n_h(0) \tag{2.29}$$

where,

$$B_h = \frac{e^{-\gamma_2 W_b^2/4\sigma^2}}{1+\sigma\sqrt{\frac{\pi}{2\left(\gamma_1 - \frac{\gamma_2}{2}\right)}}\frac{v_s}{2D_{no}}\left(\frac{N_A(0)}{N_r}\right)^{\gamma 1}\left(e^{-\gamma_2 W_b^2/4\sigma^2} - e^{-\gamma_1 W_b^2/2\sigma^2}\right)} \tag{2.30}$$

Integrating $n_h(x)$ of (2.27) gives

$$Q_{bnh} = q\,n_h(0)\,F_h \tag{2.31}$$

$$F_h = \sigma\sqrt{\frac{\pi}{\gamma_2}}\left[1 - e^{-\gamma_2 W_b^2/4\sigma^2}\right]$$

$$-\sigma^2\frac{\pi}{2}\sqrt{\frac{1}{\gamma_1 - \frac{\gamma_2}{2}}}\frac{B_h v_s}{2D_{n0}}\left(\frac{N_A(0)}{N_r}\right)^{\gamma 1}\left[\sqrt{\frac{1}{\gamma_1}}\left(e^{-\gamma_1 W_b^2/2\sigma^2} - 1\right) + \sqrt{\frac{2}{\gamma_2}}\left(1 - e^{-\gamma_2 W_b^2/4\sigma^2}\right)\right] \tag{2.32}$$

Using (2.29) and (2.31) the base transit time can be obtained as

$$\tau_{bh} = \frac{F_h}{v_s B_h} \tag{2.33}$$

Here B_h and F_h are the function of $N_A(0)$ and W_b. Therefore, the base transit time depends upon $N_A(0)$ and W_b. As long as high level of injection condition prevails in the base, τ_{bh} is independent of $n(0)$ which is a function of emitter-base junction voltage V_{be} [17]. For uniformly and exponentially doped base transit time was also found independent of V_{be} [12,15,18].

2.2.3 General Formulation

We have solved eqn. (2.6) for two cases, low and high injection. But for intermediate region, where $n(x)$ is comparable to $N_A(x)$, eqn. (2.6) is not analytically tractable. However, a general expression for J_n and Q_{bn} is possible to obtain by exploring their asymptotic behaviors in low and high injection.

Electron current densities for low and high injection regions for different peak doping concentrations $N_A(0)$ are sketched in Fig. 2.2 (a-d) as function of base-emitter voltage, using eqns. (2.19) and (2.29). Stored base charge for low and high injection regions for different peak doping concentrations $N_A(0)$ are sketched in Fig. 2.3 (a-d) as function of base-emitter voltage, using eqns. (2.21) and (2.31). In the intermediate range between low and high injection, eqn. (2.6) has no analytical solution. However, there is a fit function that satisfies the variation of current density in all level of injection.

According to (2.19) current density for low injection is

$$J_{nl} = q v_s B_l n_l(0)$$

According to (2.29) current density for high injection is

$$J_{nh} = q v_s B_h n_h(0)$$

In general case we can write

$$J_n = q v_s H n(0) \tag{2.34}$$

Comparing eqn. (2.34) with eqn. (2.19) and (2.29), we can write

$$H = B_l \qquad n(0) << N_A(0)$$

$$H = B_h \qquad n(0) >> N_A(0)$$

A best fit for all levels of injection is to be,

$$H = \frac{B_l + B_h n(0)/N_A(0)}{1 + n(0)/N_A(0)}$$

The empirical expression for current density then becomes,

$$J_n = q\, v_s \frac{B_l + B_h n(0)/N_A(0)}{1 + n(0)/N_A(0)} n(0) \tag{2.35}$$

Fig. 2.2 shows current density for low, high and all levels of injection.

According to (2.21) current density for low injection is

$$Q_{bnl} = q\, F_l\, n_l(0)$$

According to (2.31) current density for high injection is

$$Q_{bnh} = q\, F_h\, n_h(0)$$

In general case we can write

$$Q_{bn} = qGn(0) \tag{2.36}$$

Comparing eqn. (2.36) with eqn. (2.21) and (2.31), we can write

$$G = F_l \qquad n(0) << N_A(0)$$

$$G = F_h \qquad n(0) >> N_A(0)$$

A best fit form all levels injection is to be,

$$G = \frac{F_l + F_h n(0)/N_A(0)}{1 + n(0)/N_A(0)}$$

The empirical expression for current density then becomes,

$$Q_{bn} = q \frac{F_l + F_h n(0)/N_A(0)}{1 + n(0)/N_A(0)} n(0) \tag{2.37}$$

Fig. 2.3 shows stored base charge for low, high and all levels of injection.

The base transit time, τ_B from the eqn. (2.6) is given by

$$\tau_B = \frac{F_l + F_h n(0)/N_A(0)}{v_s\left(B_l + B_h n(0)/N_A(0)\right)} \tag{2.38}$$

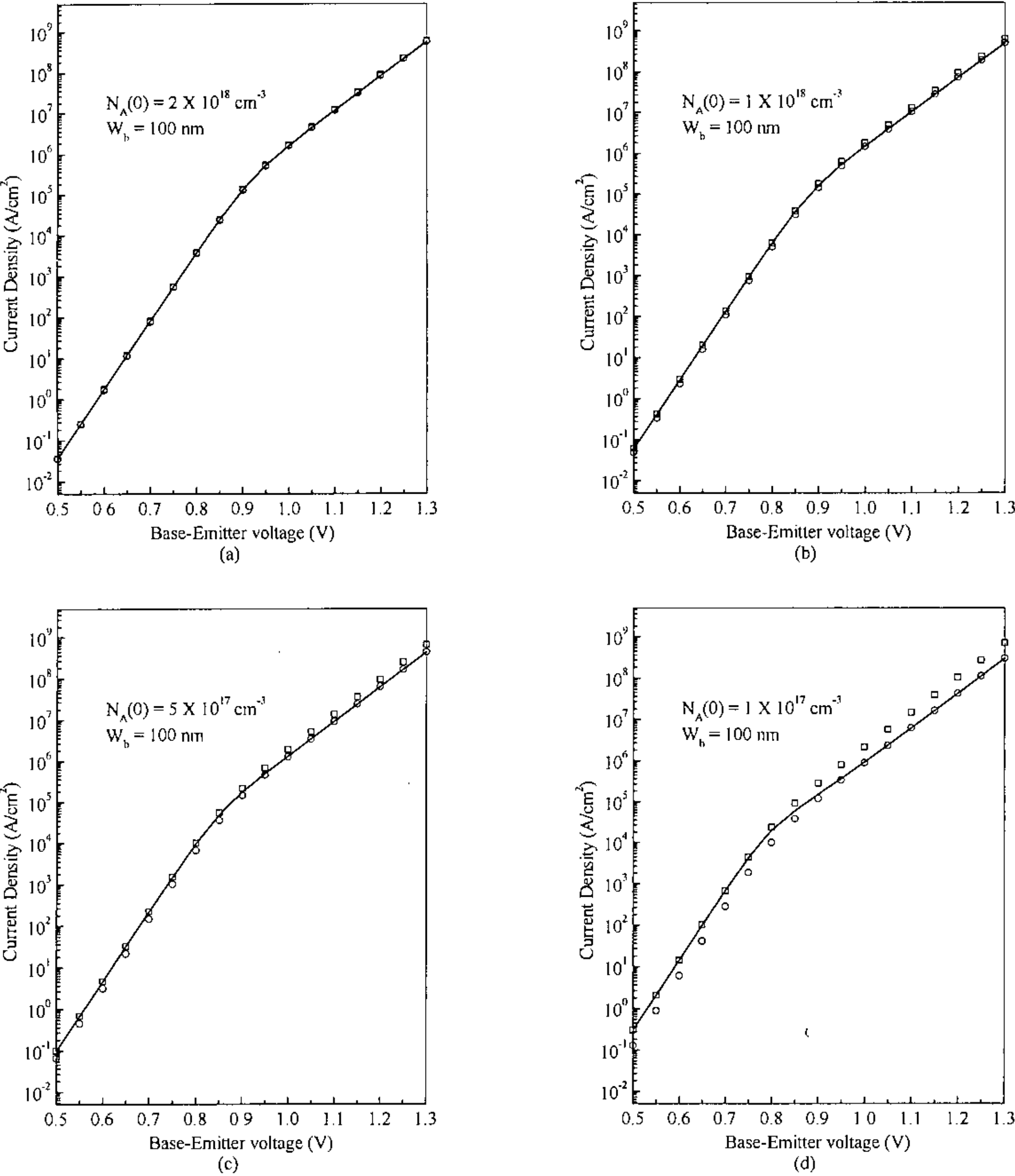


Fig. 2.2 Current density as a function of base –emitter voltage. Square and circle represent current density for low and high levels of injection respectively and the line represents the current density for all levels of injection as expressed by empirical expression.

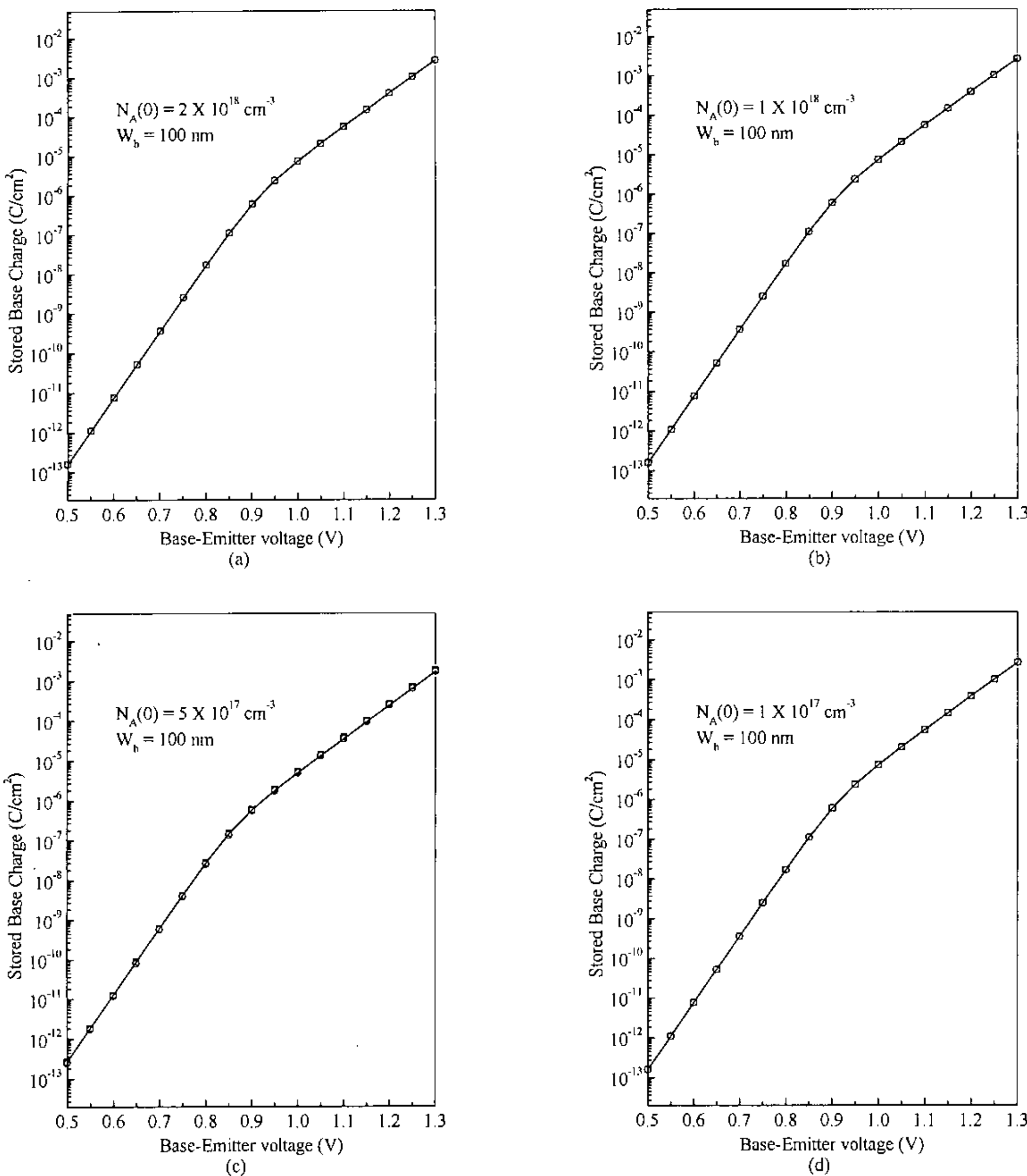


Fig. 2.3 Stored base charge as a function of base –emitter voltage. Square and circle represent current density for low and high levels of injection respectively and the line represents the current density for all levels of injection as expressed by empirical expression.

2.3 OPTIMIZATION WITH CONSTANT BASE SHEET RESISTANCE

The concept of sheet resistance is used to characterize wafers as thin-doped layers, since it is typically easier to measure the sheet resistance rather than the resistivity of the material. The sheet resistance of a layer with resisitivity, ρ, and thickness, T, is given by their ratio:

$$R_S = \frac{\rho}{T}\Omega/\square \tag{2.39}$$

While strictly speaking the unit of the sheet resistance is Ohms, one refers to it as being in Ohms per square. This nomenclature comes in handy when the resistance of a rectangular piece of material with length, L, and width W must be obtained. It equals the product of the sheet resistance and the number of squares or:

$$R = R_S \frac{L}{W}\Omega \tag{2.40}$$

where the number of squares equals the length divided by the width.

The intrinsic base sheet resistance is given by [6]

$$R_B = \frac{1}{q\int_0^{W_b} N_A(x)\mu_p(x)dx} \tag{2.41}$$

and, $$\mu_p = \mu_{p1} + \frac{\mu_{p0}}{1+\frac{N_A}{N_{Pref}}} \tag{2.42}$$

where, $\mu_{p0} = 407cm^2/V.s$, $\mu_{p1} = 54.3cm^2/V.s$ and $N_{Pref} = 2.67\times10^{17}cm^{-3}$.

Eqn. (2.41) can be well approximated by following expression

$$D_p(x) = D_{p0}\left(\frac{N_A(x)}{N_{pr}}\right)^{-\gamma_3} \tag{2.43}$$

where, $D_{p0} = 5.2510cm^2/V.s$, $\gamma_3 = 0.3479$ and $N_{pr} = 4\times10^{17}cm^{-3}$

Eqn. (2.42) and (2.43) are drawn in fig. 2.4 and are found to match well within the range from $5\times10^{16}cm^{-3}$ to $1\times10^{19}cm^{-3}$.

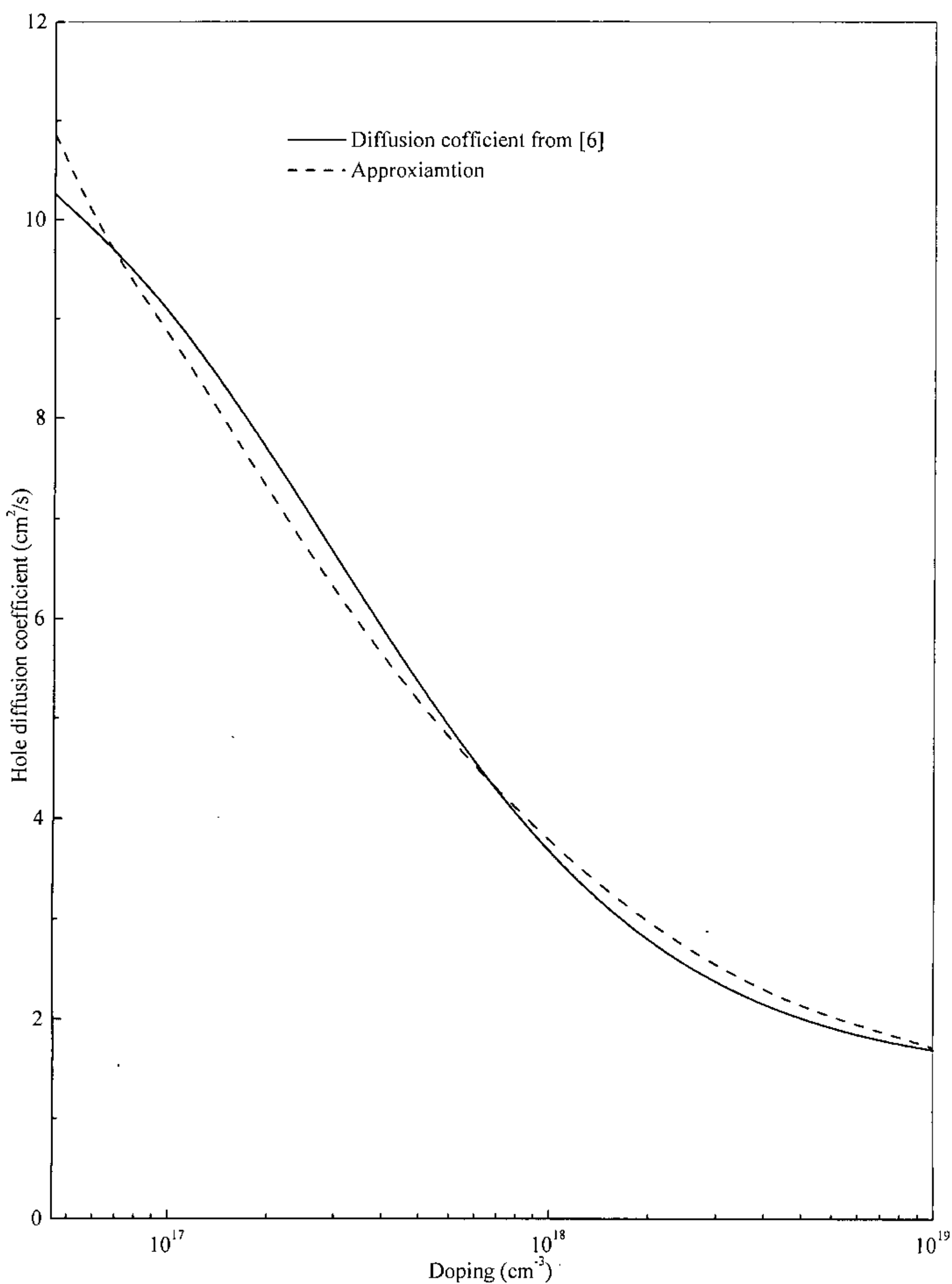


Fig. 2.4 Comparison of diffusion coefficient of hole between eqn. (2.42) [6] and (2.43).

From Einstein's relation eqn. (2.41) can be written as

$$R_B = \frac{V_T}{q\int_0^{W_b} N_A(x)D_p(x)dx} \tag{2.44}$$

Optimization can done in two way keeping the base sheet resistance to a fixed value

i) optimization for peak doping concentration;

ii) optimization for base width.

In the following sections optimization is done for box and Gaussian base doping profile.

2.3.1 Uniform base doping

For a given R_B and W_b, optimum base doping density N_{AU} can be found by solving eqn. (2.44) and expressed as

$$N_{AU} = \left(\frac{V_T}{qR_B D_{p0} N_{rp}^{\gamma 3} W_b}\right)^{\frac{1}{1-\gamma 3}} \tag{2.45}$$

Similarly for a given R_B and N_A, optimum base width W_{bU} can be found by solving eqn. (2.44) and expressed as

$$W_{bU} = \frac{V_T}{qR_B N_A^{1-\gamma 3} D_{p0} N_{rp}^{\gamma 3}} \tag{2.46}$$

The base transit time for uniformly doped base is given by [5]

$$\tau_B = \left(\frac{W_b^2}{2D_n} + \frac{W_b}{v_s}\left[\frac{3}{2} - \frac{N_A}{2n(0)}\ln\left(1+\frac{n(0)}{N_A}\right)\right]\right) \Big/ \left[2 - \frac{N_A}{n(0)}\ln\left(1+\frac{n(0)}{N_A}\right)\right] \tag{2.47}$$

2.3.2 Gaussian base doping

For a given R_B and W_b, optimum peak base doping density N_{AG} can be found by solving eqn. (2.44) and expressed as

$$N_{AG} = \left(\frac{V_T}{qR_B \sigma D_{p0} N_{rp}^{\gamma 3}}\sqrt{\frac{2(1-\gamma_3)}{\pi}}\right)^{\frac{1}{1-\gamma 3}} \left(1 - e^{-(1-\gamma 3)W_b^2/2\sigma^2}\right) \tag{2.48}$$

Similarly for a given R_B and $N_A(0)$, optimum base width W_{bU} can be found by solving eqn. (2.44) and expressed as

$$W_{bG} = \frac{2\sigma^2}{\gamma_3 - 1} \ln\left(1 - \frac{V_T}{qR_B\sigma(N_A(0))^{1-\gamma_3} D_{p0} N_{rp}^{\gamma_3}} \sqrt{\frac{2(1-\gamma_3)}{\pi}}\right) \quad (2.49)$$

The base transit time for Gaussian doped base is given by eqn. (2.38)

2.4 CONCLUSION

The analytical expressions of collector current density and stored base charge density within the base are obtained for low and high levels of injection. For intermediate level of injection analytical expression is not possible. So empirical expressions for current and charge are established by considering the behavior of those in low and high injections. The expression for base transit time is obtained using the empirical expressions for electron current and stored base charge. In the next chapter results obtained from the derived equations are plotted and the transit time is compared with that of numerical results available in the literature.

CHAPTER THREE

RESULTS & DISCUSSION

3.1 INTRODUCTION

The mathematical expressions related to this work have been derived in the previous chapter. A computer program is developed based on these derived equations to generate numerical data. Those data are plotted in this chapter to study the effects of various parameters on base transit time.

3.2 RESULTS AND DISCUSSIONS

The base transit time τ_B of a bipolar junction transistor has been calculated in low injection and high injection conditions through eqn. (2.23) and (2.33) respectively. The dependence of τ_B on arbitrary doping profiles, heavy doping effect, finite carrier velocity at the collector edge in the base and dependence of mobility on doping concentration have been incorporated in the derivations. Analytical expressions for $n(x)$, J_n, Q_{bn} are obtained assuming a number of approximations. The validities of the assumptions have been verified by comparing the values obtained from the derived expressions with the results found in the literatures.

3.2.1 Distribution of minority carrier within the base

The distributions of minority carrier $n(x)$ within the base for low and high levels of injection for different base emitter voltage are shown in Fig. 3.1. The electron concentration profile is not linear due to the built-in electric field caused by the Gaussian doping concentration. When the base-emitter voltage increases more carriers are injected

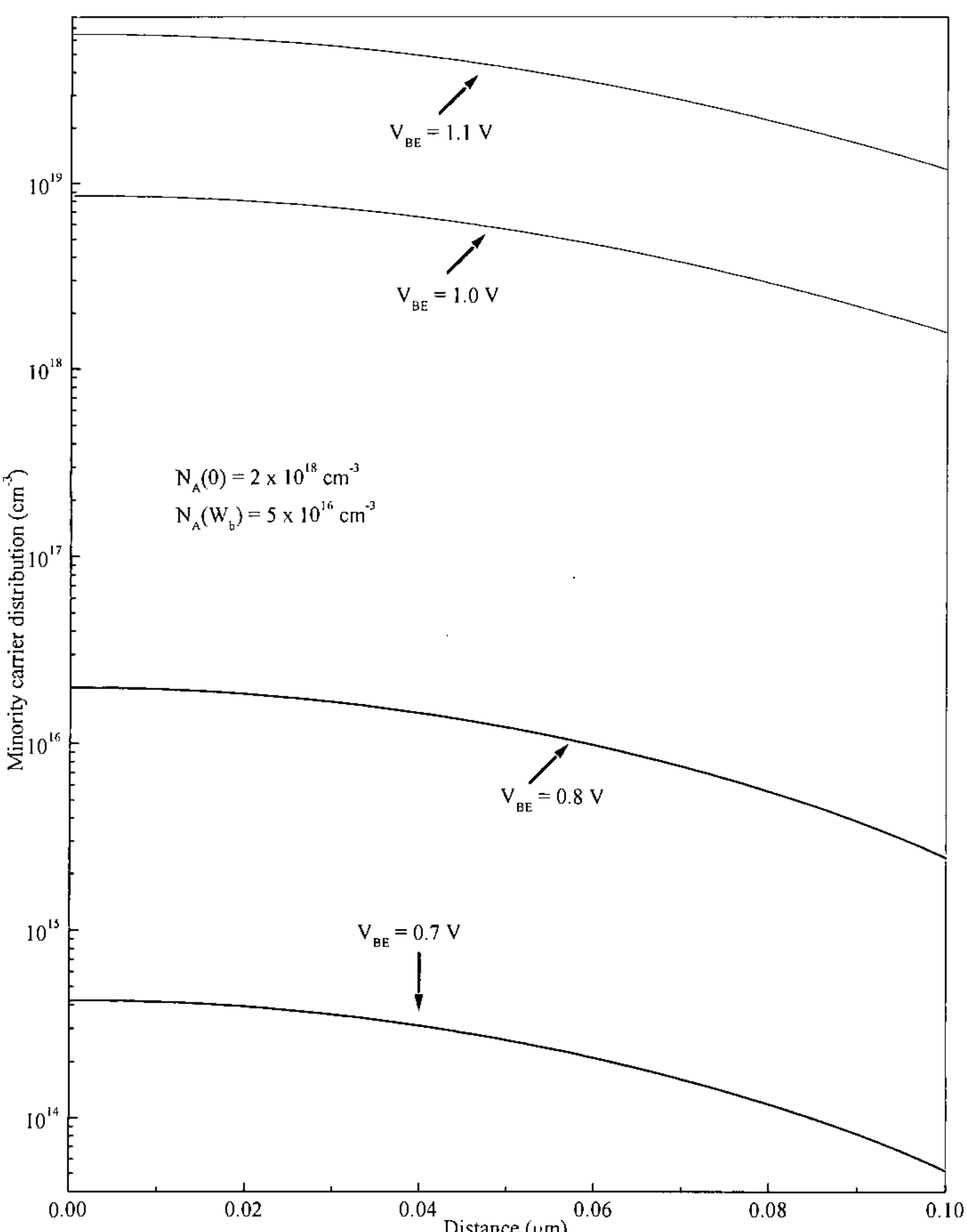


Fig. 3.1 Minority carrier distribution within the base.

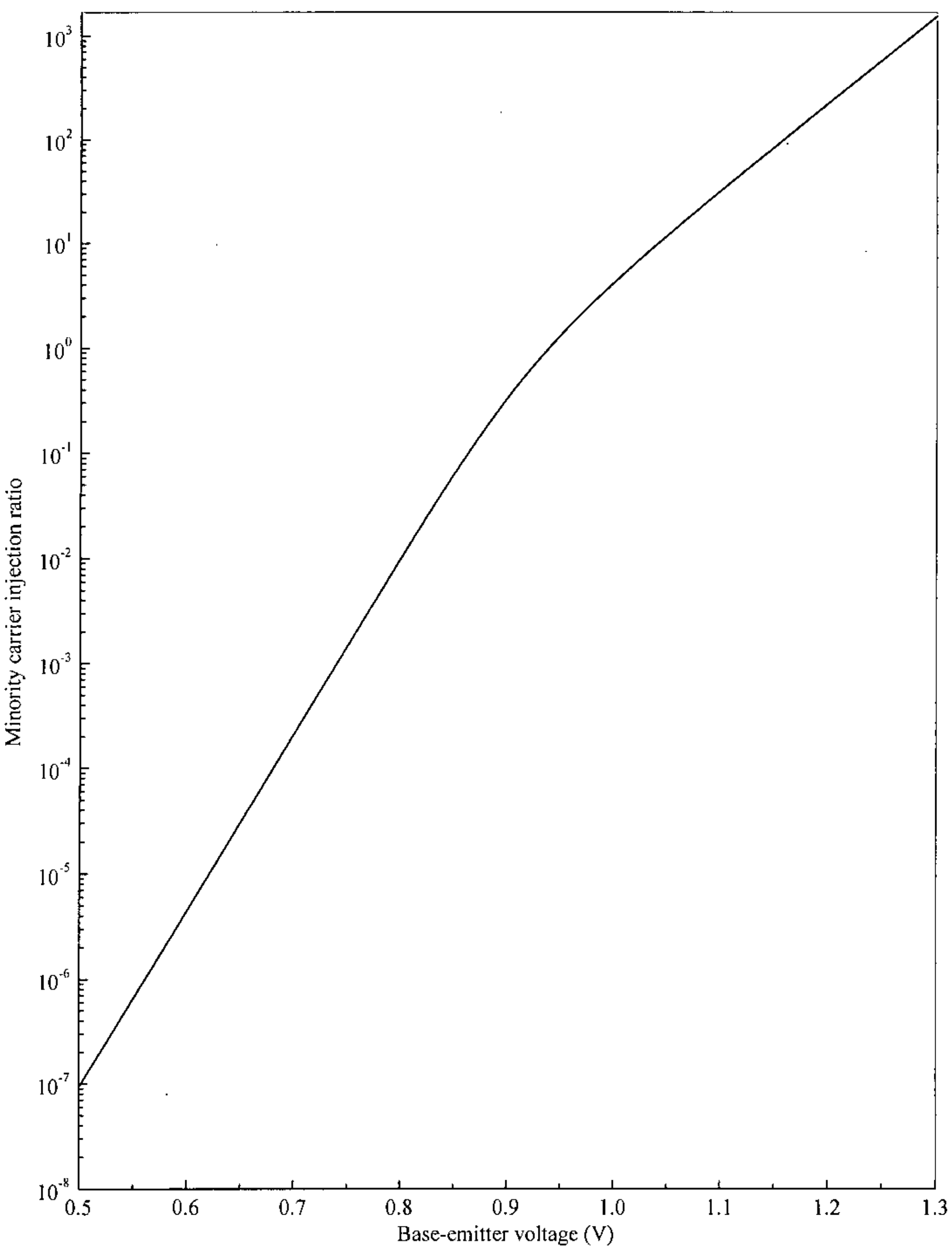


Fig. 3.2 Minority carrier injection ratio as a function of base-emitter voltage.

from emitter to base. From Fig. 3.1, we also see that the minority carrier has maximum value at the base-emitter junction and minimum value at the collector-base junction. In this work, we consider velocity saturation of electron at the collector-base junction. So at collector-base junction the minority carrier concentrations for low and high levels of injection are not zero.

The variation of minority carrier injection ratio, $n(0)/N_A(0)$ with base–emitter voltage is shown in Fig. 3.2. From the Fig. 3.2 we see that the minority carrier injection ratio is an increasing function of base-emitter voltage.

3.2.2 Electric field distribution within the base

The distributions of electric field, $E(x)$ within the base for different base-emitter voltage are shown in Fig. 3.3. The curves show that the aiding electric field within the base increases linearly for small base-emitter voltage, but for high base-emitter voltage it decreases from the emitter junction and then increases at the collector junction. Furthermore electric field in high injection deviates from linearity near base-collector junction. The change of aiding electric field in the base is due to modulation of electron concentration as well as electron concentration gradient at high injection. Obviously, the decrease of aiding electric field in the base slows down electron flow and increases base transit time. The increase of aiding electric field in the base near collector junction affects base transit time less because electrons in that region are already approaching the collector-base junction.

3.2.3 Variation of base transit time with minority carrier injection ratio

The variations of base transit time, τ_B with minority carrier injection ratio $n(0)/N_A(0)$ for various peak base doping are shown in Fig. 3.4. From Fig. 3.4 we see that transit time is independent of injection ratio for both low and high of injection. But the value of base transit time is larger for high injection region than that of low injection region. This is due to the reduction of aiding field in the Gaussian base when the level of injection

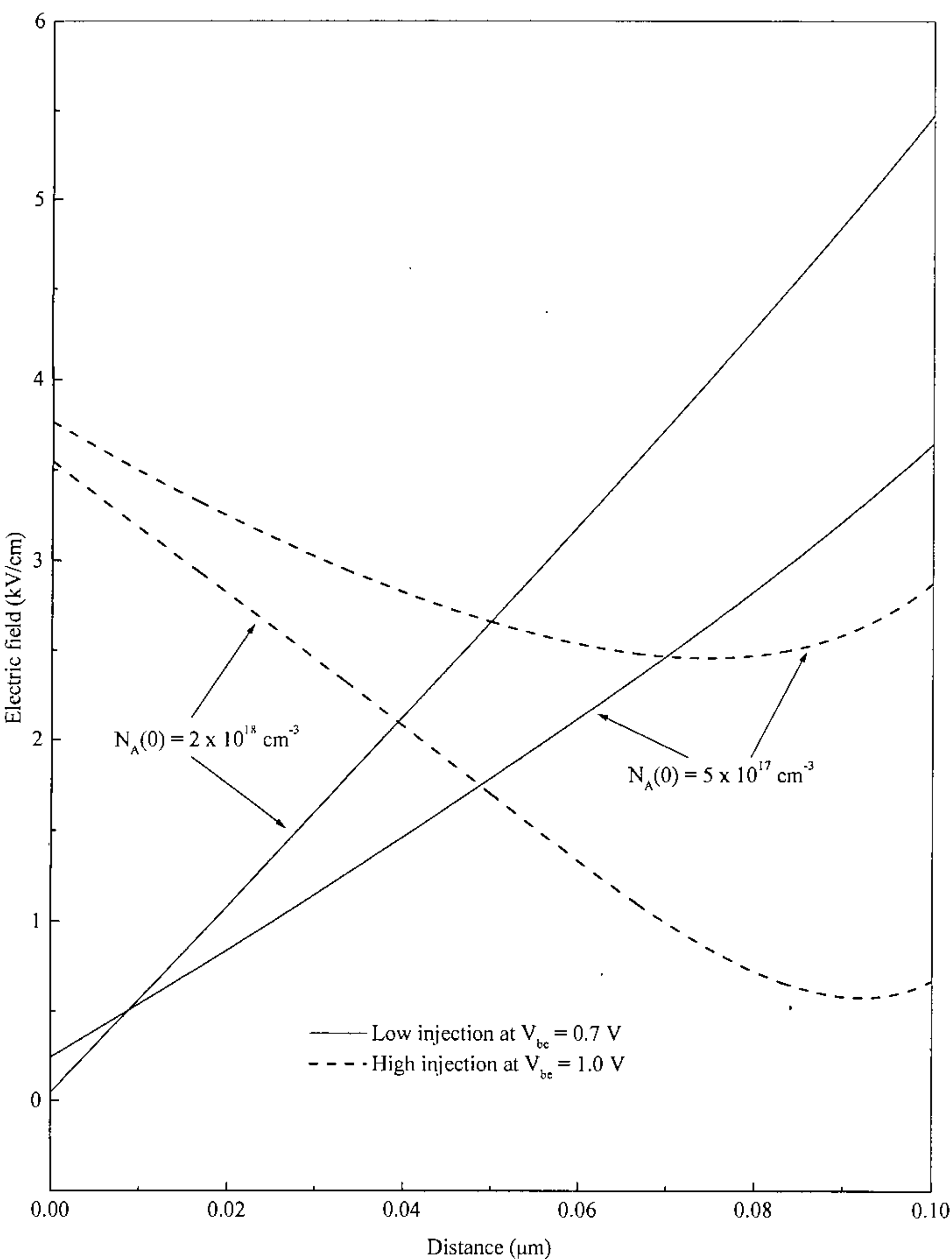


Fig 3.3 Electric field distribution within the base for low and high level of injection.

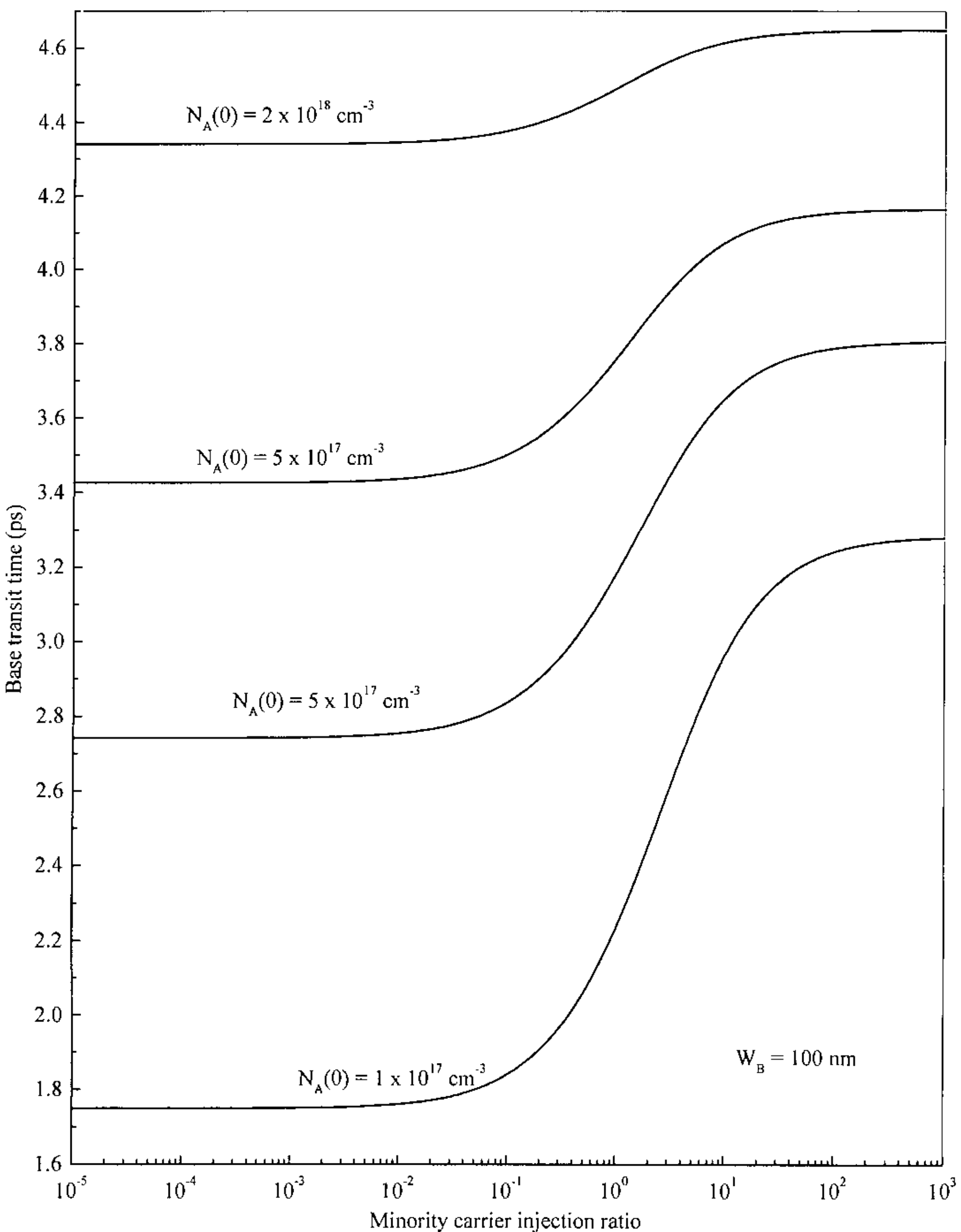


Fig. 3.4 Base transit time as a function of minority carrier injection ratio.

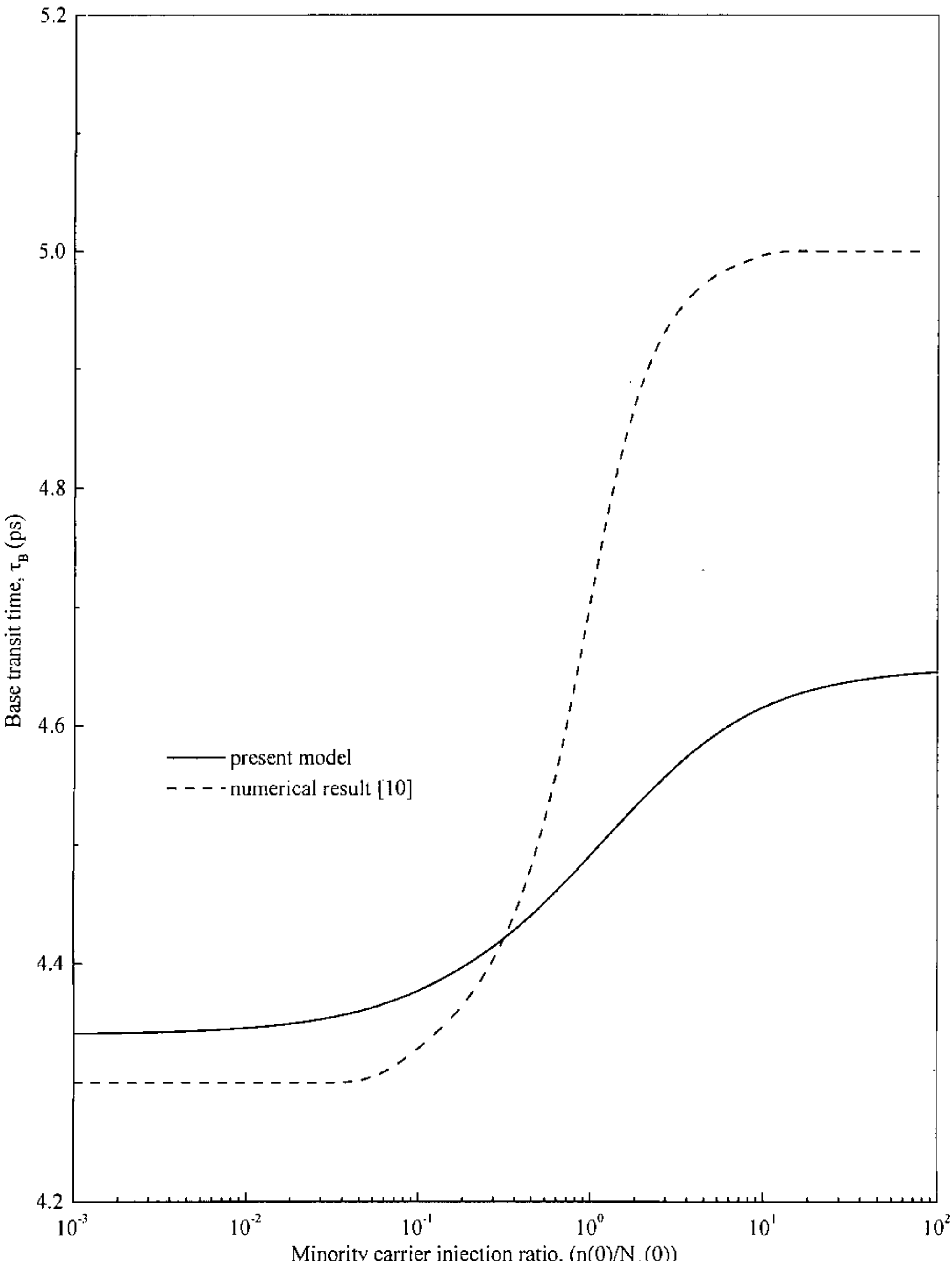


Fig. 3.5 Base transit time as a function of minority carrier injection ratio. The dashed line represents base transit time of [10] and the other line represents present model.

increases. Due to this effect electrons slow down and hence transit time increases.

In Fig. 3.5 the variation of base transit time with minority carrier injection ration obtained by the proposed empirical expression is compared with the result obtained in [10]. The two curves show similar variation with minority carrier injection ratio. But maximum values of base transit time are different. Results obtained by the proposed model are slightly smaller than that of [10] for high injection region.

3.2.4 Variation of base transit time with base-emitter voltage

The variations of base transit time with base-emitter voltage for various peak base concentrations are shown in Fig. 3.6. From the Fig. 3.6, we see that the variation of the base transit time with base-emitter voltage has the same pattern as that with minority carrier injection ratio. As discussed earlier, the minority carrier injection ratio is an increasing function of base-emitter voltage. The increase in the base-emitter voltage reduces the aiding field in the base as we see from Fig. 3.4. So with the increase of base-emitter voltage the base transit time increases.

3.2.5 Dependence of base transit time upon base width

The dependence of base transit time upon base width for various peak base concentrations is shown in Fig. 3.7. From the figures we see that the base transit time is an increasing function of base width. For the same base width and $N_A(x)$, the base transit time is larger for high injection region. With the increase of base width the stored base charge increases and electron current density deceases. As the base transit time is define as the ratio of the stored base charge to the electron current density the base transit time increases with base width.

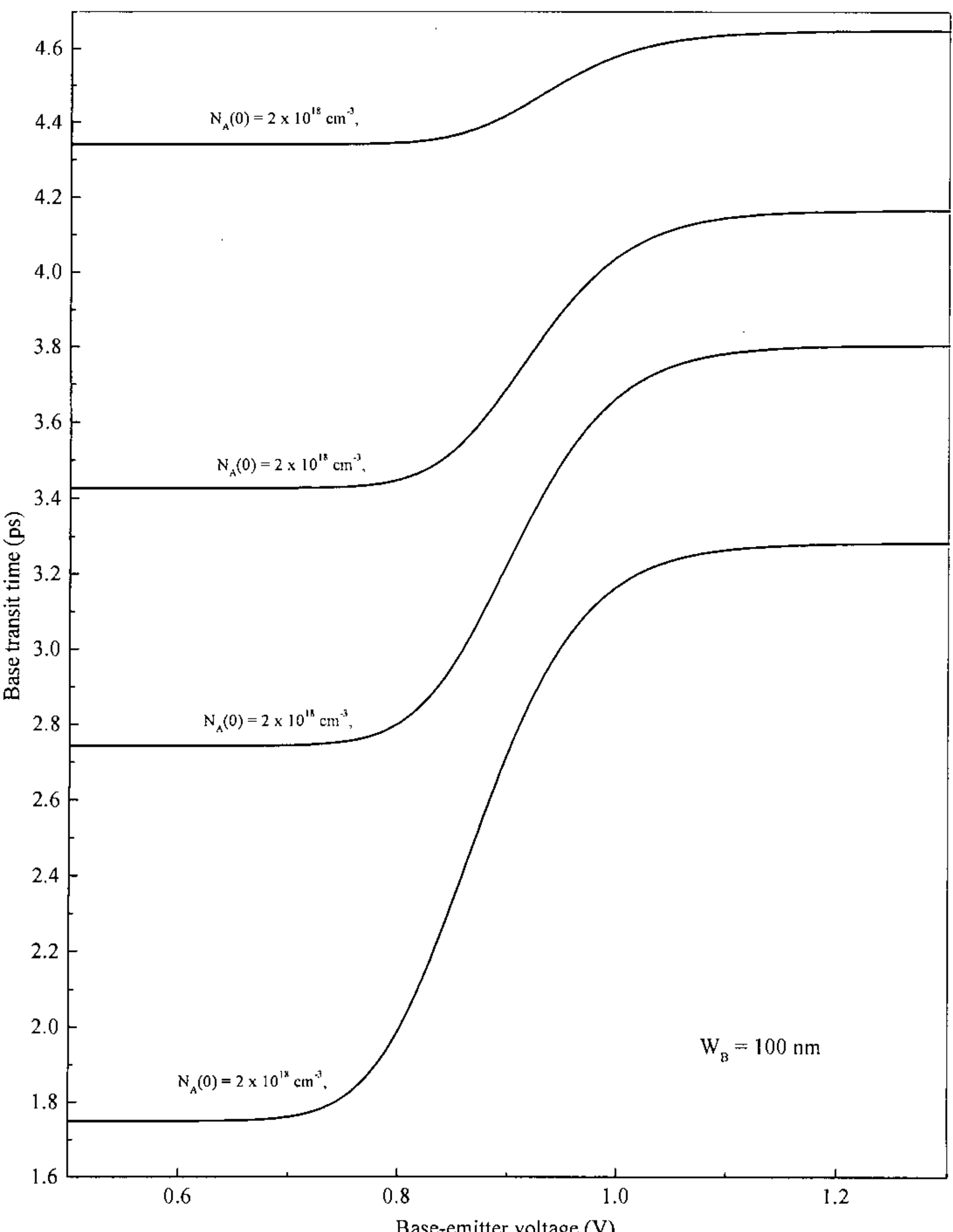


Fig. 3.6 Base transit time as a function of base-emitter voltage.

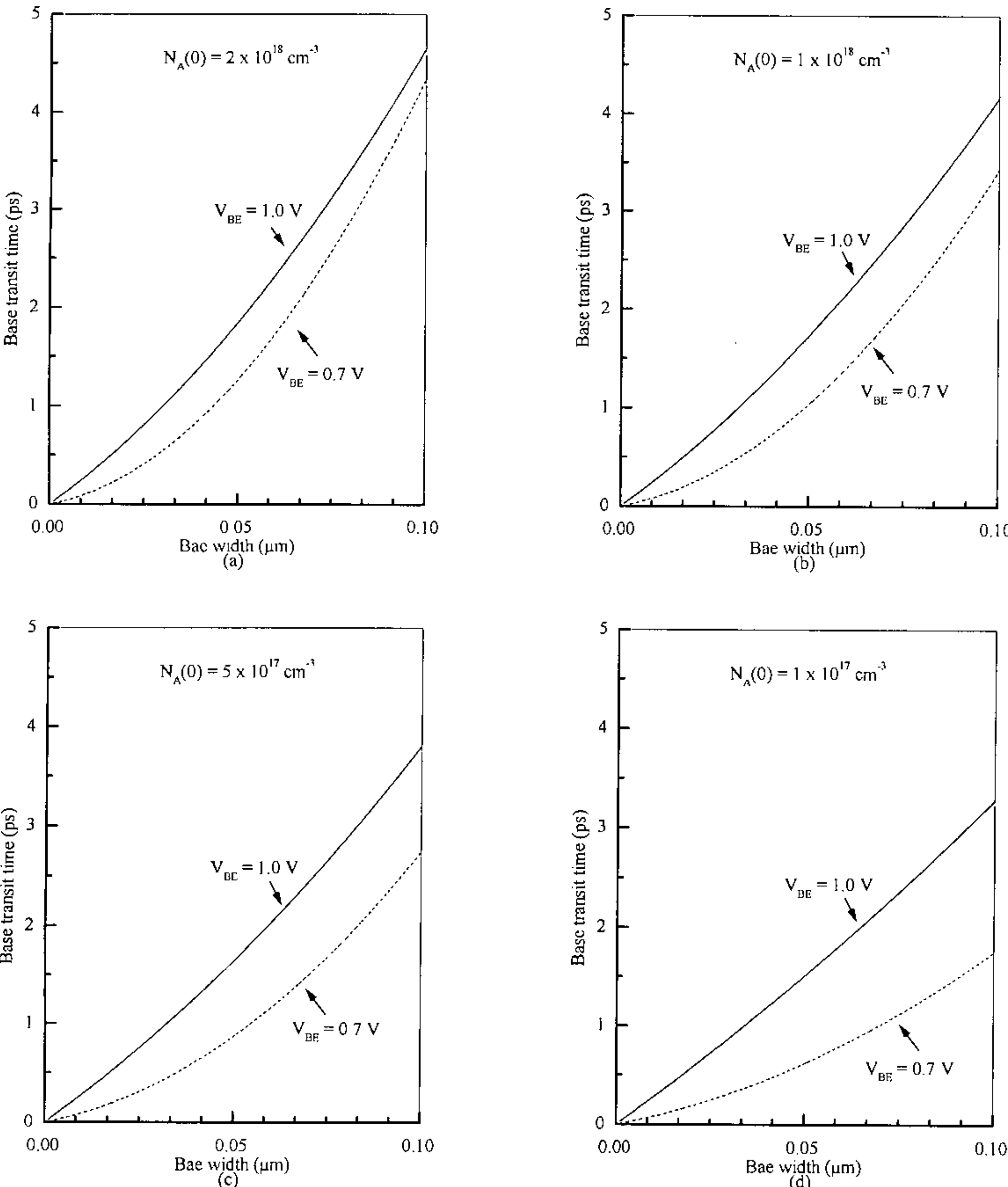


Fig. 3.7 Base transit time as function of base width.

3.2.6 Dependence of base transit time upon peak base doping concentration

The dependence of base transit time upon base width for various peak base concentration is shown in Fig. 3.8. From the figures we see that the base transit increases with peak base concentration. Eqn. (2.7) shows that $D_n(\mu_n)$ decreases with $N_A(0)$. For low level of injection $E(x)$ is given by eqn. (2.13). It shows that $E(x)$ is independent of $N_A(0)$. Due to the decrease of $D_n(\mu_n)$ the stored base charge increases resulting in increase of τ_B. For high level of injection $E(x)$ is given by eqn. (2.25). The first term decreases with $n(0)$ and the aiding electric field decreases. Both electric field and mobility reduction contribute to increase of τ_B.

3.2.7 Optimization with respect to base width and peak doping concentration for constant base sheet resistance

For a given base width and constant base sheet resistance, Gaussian profile gives higher peak density $N_A(0)$ than doping density of uniformly doped base. For constant base sheet resistance and constant peak doping density the base width of the uniformly doped base is smaller than that of Gaussian doped base.

Fig 3.9 shows the transit time as a function of base-emitter voltage for optimized $N_A(0)$ with constant base sheet resistance and constant base width. The base transit time in low injection for uniform doping is greater than that of Gaussian profile. But the transit time for uniform doping is smaller than that of Gaussian profile in high injection. For Gaussian profile the aiding electric field decreases with increase of injection. On the other hand for the uniform doping there is no electric field in the base in the low injection. The field builds up within the base when the injection is high for uniform base doping and it accelerates the electron motion.

Fig 3.10 show the transit time as a function of base-emitter voltage for optimized W_b with constant base sheet resistance and constant peak base doping concentration. The base transit time for uniform doping is always smaller than that of Gaussian profile. The base transit time strongly depends on base width (Fig. 3.7). As a result the base transit

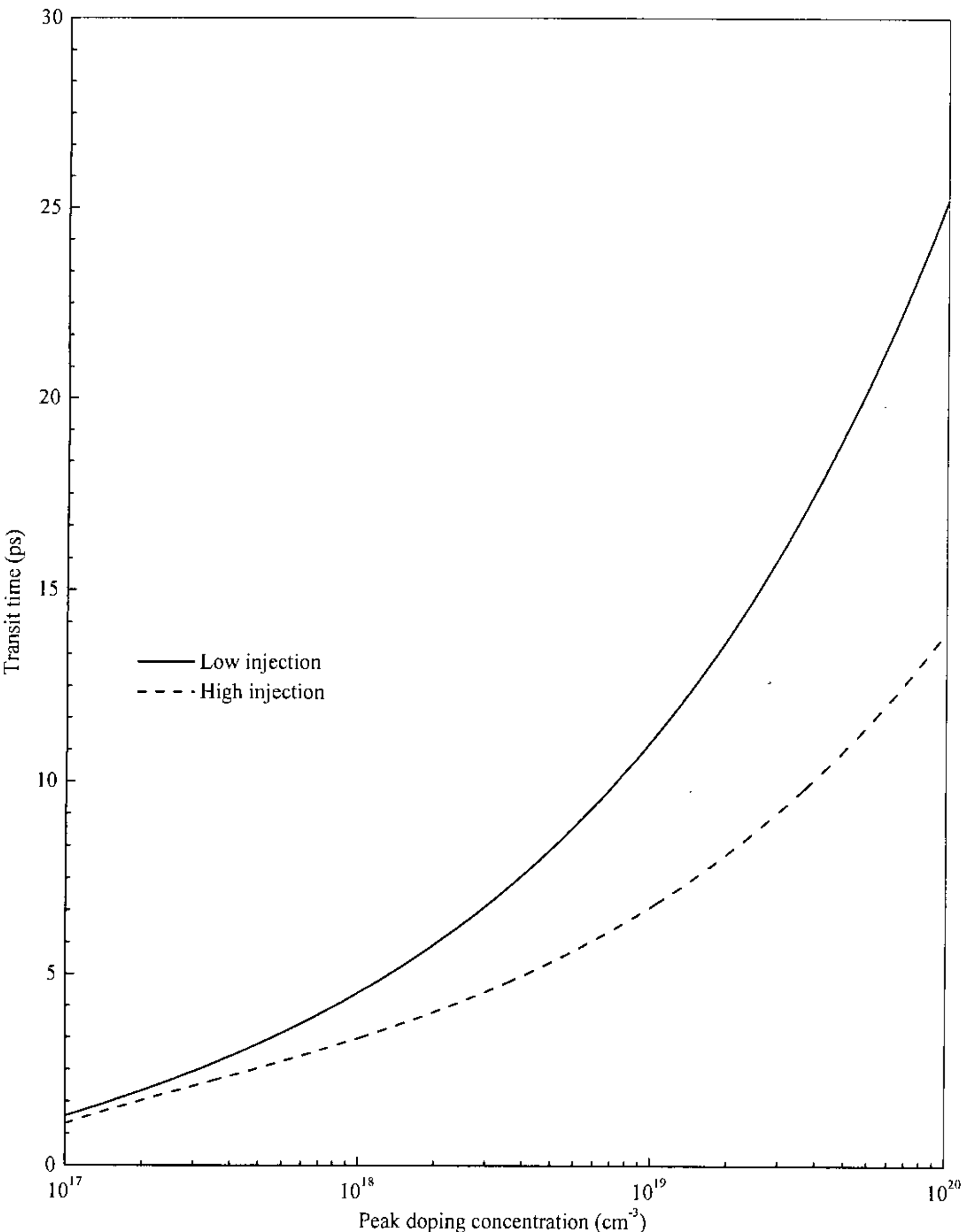


Fig. 3.8 Base transit time as a function of peak base doping concentration.

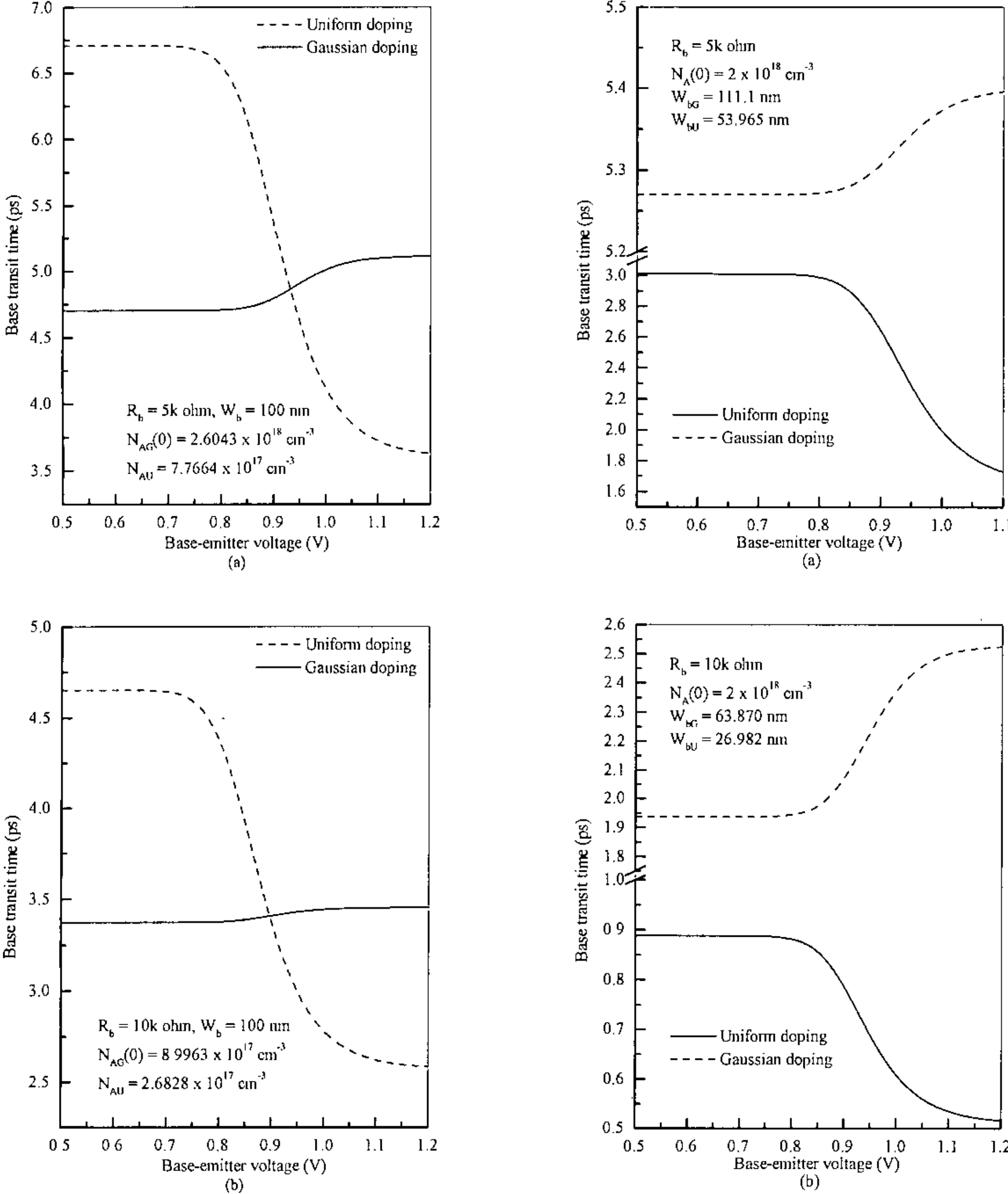


Fig 3.9 Transit time as function of base-emitter voltage for optimized peak doping for constant base sheet resistance and constant base width.

Fig 3.10 Transit time as function of base-emitter voltage for optimized base width for constant base sheet resistance and constant peak base doping.

time of Gaussian profile greater at any level of injection than that of uniform doping.

3.3 CONCLUSION

The empirical expression obtained in chapter two is used to determine the dependence of base transit time on different parameters of BJT. The results show that base transit time increase with base-emitter voltage, minority carrier injection ratio, base width and peak base doping concentration. The results obtained are compared with the results available in the literature. The comparisons show that the proposed empirical expression of base transit time is in good agreement with the available results.

CHAPTER FOUR

CONCLUSION AND SUGGESTIONS

4.1 CONCLUSION

The base transit time of a bipolar transistor with Gaussian doped base is studied. For low and high levels of injection closed form expressions are obtained. But for intermediate level of injection analytical solution cannot be obtained. For this an empirical expression is derived which is applicable for all levels of injection. The analysis shows that the base transit time increases with minority carrier injection, peak base doping concentration, base width and base-emitter voltage.

4.2 SUGGESTIONS FOR FUTURE WORK

In this work, only the doping dependence of mobility is considered. But mobility also depends on electric field. In future, an expression for transit time considering field dependence of mobility may be carried out.

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APPENDIX

The integration of a Gaussian function for low level of injection $f(x)=e^{-ax^2/2\sigma^2}$ is given by

$$I=\int_0^x f(x)dx$$

Here a and σ can be any positive number.

$$I=\int_0^\infty f(x)dx-\int_x^\infty f(x)dx=I_1-I_2 \qquad \text{(A1)}$$

where, $I_1=\int_0^\infty f(x)dx$ and $I_2=\int_x^\infty f(x)dx$.

Now we can write

$$I_2^2=\int_0^x\int_0^y e^{-a\{x^2+y^2\}/2\sigma^2}dydx \qquad \text{(A2)}$$

Let, $\sqrt{\frac{a}{2\sigma^2}}x=u$ and $\sqrt{\frac{a}{2\sigma^2}}y=v$.

After changing the variables,

$$I_2^2=\frac{2\sigma^2}{a}\int_0^u\int_0^v e^{-(u^2+v^2)}dudv \qquad \text{(A3)}$$

Transformation to polar coordinate gives

$$I_2^2=\frac{2\sigma^2}{a}\int_0^{\frac{\pi}{2}}\int_0^r e^{-r^2}rdrd\theta \qquad \text{(A4)}$$

Integration of (A4) gives

$$I_2=\sqrt{\frac{\pi}{2a}}\sigma\, e^{-ax^2/2\sigma^2} \qquad \text{(A5)}$$

Following the same procedure we can obtain

$$I_1=\sqrt{\frac{\pi}{2a}}\sigma \qquad \text{(A6)}$$

Using (A5) and (A6) in (A1) gives

$$I=\sqrt{\frac{\pi}{2a}}\sigma\left(1-e^{-ax^2/2\sigma^2}\right) \qquad \text{(A7)}$$

The integration of a Gaussian function for high level of injection can be obtained from the previous derivation by replacing $a = \gamma_1 - \frac{\gamma_2}{2}$. It can be written as

$$I = \sqrt{\frac{\pi}{2\left(\gamma_1 - \frac{\gamma_2}{2}\right)}}\sigma\left(1 - e^{-\left(\gamma_1 - \frac{\gamma_2}{2}\right)x^2/2\sigma^2}\right) \quad \text{(A8)}$$