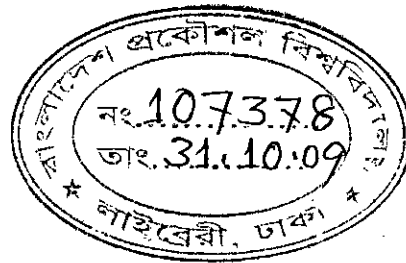


**Base Transit Time of a Nonuniformly Doped Base Heterojunction Bipolar
Transistor (HBT)**

by
Yeasir Arafat

A thesis submitted to the Department of Electrical and Electronic Engineering
Bangladesh University of Engineering & Technology
in partial fulfillment of the requirements
for the degree of

MASTER OF SCIENCE IN ELECTRICAL AND ELECTRONIC ENGINEERING




**DEPARTMENT OF ELECTRICAL AND ELECTRONIC ENGINEERING
BANGLADESH UNIVERSITY OF ENGINEERING & TECHNOLOGY, DHAKA
2009**

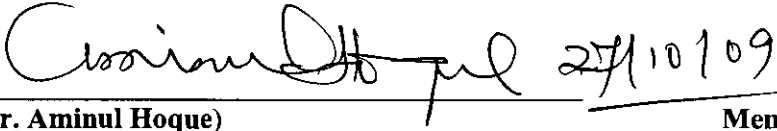
CERTIFICATION

The thesis titled “**BASE TRANSIT TIME OF A NONUNIFORMLY DOPED BASE HETEROJUNCTION BIPOLAR TRANSISTOR (HBT)**” submitted by Yeasir Arafat, Roll No.: 100006204P, Session: October 2000 has been accepted as satisfactory in partial fulfillment of the requirements for the degree of MASTER OF SCIENCE IN ELECTRICAL AND ELECTRONIC ENGINEERING on October 27, 2009.

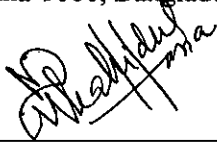
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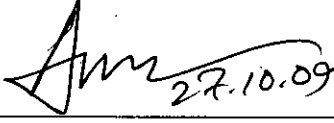
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LIST OF ABBREVIATIONS

<u>ABBREVIATION</u>	<u>ELABORATION</u>
BC	Base-Collector
BiCMOS	Bipolar-Complementary Metal-Oxide-Semiconductor
BJT	Bipolar Junction Transistor
CB	Conduction Band
cf.	Confer (compare or consult)
CMOS	Complementary Metal-Oxide-Semiconductor
CVD	Chemical Vapour Deposition
DHBT	Double Heterojunction Bipolar Transistor
DOS	Density of States
EB	Emitter-Base
ECL	Emitter Coupled Logic
<i>e.g.</i>	<i>(exempli gratia)</i> for example
<i>et al.</i>	<i>(et alii)</i> and co-workers
<i>etc.</i>	et cetera
FET	Field-Effect Transistor
GHz	Giga Hertz
HBT	Heterojunction Bipolar Transistor

<u>ABBREVIATION</u>	<u>ELABORATION</u>
HEMT	High Electron Mobility Transistor
HFET	Heterojunction Field-Effect Transistor
IC	Integrated Circuit
<i>i.e.</i>	<i>(id est)</i> that is
IGBT	Insulated Gate Bipolar Transistor
IGFET	Insulated-Gate Field-Effect Transistor
JFET	Junction Field-Effect Transistor
MBE	Molecular Beam Epitaxy
MOS	Metal-Oxide-Semiconductor
MOSFET	Metal-Oxide-Semiconductor Field-Effect Transistor
Op-Amps	Operational Amplifiers
RF	Radio Frequency
RFIC	Radio Frequency Integrated Circuit
SHBT	Single Heterojunction Bipolar Transistor
SoC	System-On-Chip
SOI	Silicon-On-Insulator
VB	Valence Band
VCO	Voltage Controlled Oscillators
vs.	versus

LIST OF TECHNICAL SYMBOLS AND TERMS

<u>SYMBOL</u>	<u>DESCRIPTION</u>
f	Frequency
f_T	Unity gain cutoff frequency
f_{max}	Maximum oscillation frequency
I_B	Base current
I_C	Collector current
I_E	Emitter current
β	Current gain
J_p	Current density for hole
J_n	Current density for electron (for all levels of injection)
J_{nl}	Electron current density for low injection region
J_{nm}	Current density for low injection considering Webster's effect
J_c	Collector current density
τ_B	Base transit time for all levels of injection
τ_{Bl}	Base transit time for low injection
τ_E	emitter charge storage delay time
τ_{EC}	Total delay time from emitter to collector
x	Distance along the base
y	Ge mole fraction
y_C	Ge fraction at the collector end of the base
y_E	Ge fraction at the emitter end of the base
y_{av}	Average Ge mole fraction
y_D	Ge dose

<u>SYMBOL</u>	<u>DESCRIPTION</u>
a	$=0.7743$
b	$=1+3y_{av}$
c	$=0.342/[0.342+y_{av}(1-y_{av})]$
v_s	Saturation velocity in Si ($=10^7$ cm/s)
v_{sA}	Saturation velocity in SiGe alloy ($=cv_s$)
S	Velocity of electron without saturation
Si	Silicon
Ge	Germanium
SiGe	Silicon-Germanium
$D_n(x)$	Diffusion co-efficient of electron
$D_p(x)$	Diffusion co-efficient of hole
D_{max}	Maximum value of diffusion co-efficient
D_n	Diffusion co-efficient of electron at EB junction
D_{n0}	$=20.72$ cm ² /s
$D_{nSiGe}(x)$	Diffusion co-efficient of electron inside SiGe alloy
D_{nA}	Diffusion co-efficient of electron inside SiGe at EB junction ($=bD_n$)
$\mu_n(x)$	Electron mobility
$\mu_p(x)$	Hole mobility
$\mu_{nSiGe}(x)$	Electron mobility inside SiGe alloy
$E(x)$	Electric field
E_c	Critical electric field
$E_{iSiGe}(x)$	Electric field inside SiGe alloy under low injection
$E_{SiGe}(x)$	Electric field inside SiGe alloy under all level of injection

<u>SYMBOL</u>	<u>DESCRIPTION</u>
n_{i0}	Intrinsic concentration without band gap narrowing ($1.4 \times 10^{10} \text{ cm}^{-3}$)
$n_{ie}(x)$	Effective intrinsic carrier concentration
$n_{ieSi}(x)$	Effective intrinsic carrier concentration for Si
$n_{ieSiGe}(x)$	Effective intrinsic carrier concentration for SiGe
$n(x)$	Injected minority carrier concentration for all levels of injection
$n_l(x)$	Injected minority carrier concentration for low injection
$n_m(x)$	Electron concentration for low injection considering Webster effect
$\delta n(x)$	Change in injected carrier concentration from high to low injection
$n(0)$	Injected electron concentration at EB junction for all injection
$n_l(0)$	Injected electron concentration at EB junction for low injection
$N_B(x)$	Base doping concentration (acceptor)
$N_B(0)$	Peak base doping concentration (acceptor)
N_r	Reference concentration (10^{17} cm^{-3})
$p(x)$	Hole concentration in the base
$p_m(x)$	Hole concentration for low injection considering Webster effect
Q_{Bn}	Stored base charge density for all levels of injection
Q_{Bnl}	Stored base charge per unit area for low injection
W_B	Base width
W_{BC}	Width of base-collector space-charge layer
E_g	Energy gap between conduction and valence band
$\Delta E_{gGe}(x)$	Bandgap narrowing due to the presence of Ge
$\Delta E_{gHD}(x)$	Bandgap narrowing due to heavy doping effect
ΔE_G^C	Difference between conduction band energies

<u>SYMBOL</u>	<u>DESCRIPTION</u>
V_{gGe}	=688 mV
V_{gHD}	=18 mV
V_T	Thermal voltage
V_A	Early voltage
γ	=1~2
γ_1	=0.42
γ_2	= V_{gHD}/V_T =0.69
γ_3	= V_{gGe}/V_T
γ_m	=0.468
γ_r	Ratio of the effective DOS in SiGe to the effective DOS in Si
η	Slope of base doping
η_{Ge}	Gradient in Ge mole fraction (= $y_C - y_E$)
m	Ratio of slope of base doping to base width
m_{Ge}	Ratio of gradient in Ge mole fraction to base width
m_1	= $m\gamma_1$
m_2	= $m\gamma_2$
m_3	= $m_{Ge}\gamma_3$
m_{32}	= $m_3 - m_2$
m_{023}	= $m + m_{32}$
m_{012}	= $\eta(1 + \gamma_1 - \gamma_2)/W_B$
m_{0123}	= $m_{012} + m_3$
C_{EB}	Emitter-base depletion capacitance
C_{BC}	Base-collector depletion capacitance

<u>SYMBOL</u>	<u>DESCRIPTION</u>
R_B	Intrinsic base resistance
R_C	Collector resistance
r_o	Output resistance
g_m	Transconductance
k	Boltzmann constant
T	Absolute temperature
q	Electron charge
AlGaAs	Aluminum Gallium Arsenide
AlGaN	Aluminum Gallium Nitride
GaAs	Gallium Arsenide
GaN	Gallium Nitride
InAlAs	Indium Aluminum Arsenide
InGaAs	Indium Gallium Arsenide
InGaAsN	Indium Gallium Arsenide Nitride
InGaP	Indium Gallium Phosphide
InP	Indium Phosphide
SiC	Silicon Carbide
V_{be}	Base-emitter voltage

ACKNOWLEDGEMENTS

The author believes that writing acknowledgement is the easiest part of a dissertation but it is almost impossible to acknowledge properly all the people who helped him during the course of this research. The author is grateful to Almighty Allah for bringing kindness into being in so many people who provided him with the necessary support he required and giving him the strength to complete this dissertation.

There are always some people who live by their own but care for others and extend their hands to drowning people without waiting for the plea for help from the drowned person. This is especially true for Dr. Md. Ziaur Rahman Khan who came ahead in a very crucial phase of the author's life to supervise this thesis work when the author had lost almost all hopes. From the formative stages of this thesis to the final draft, the author owes an immense debt of gratitude to him for his prompt advice, continuous guidance, suggestion & wholehearted supervision without which this dissertation would not have been possible. He introduced the author to writing paper articles and dissertation in a very attractive way. The author is also very grateful to him for providing with various useful software & computing facilities. From the core of his soul, the author is thankful to his supervisor for revealing generosity and believing in the author's abilities.

And there are some people who have the amazing quality to enlighten dismal souls by their magic wand. This is more than ever true for Professor Dr. Aminul Hoque, the Head of the Department of Electrical and Electronic Engineering, BUET. The author is indebted to him for his incessant encouragement during this research work and providing laboratory, internet and other resources of the department for the purpose of this research. The author is gratified to Prof. Dr. A. Hoque for fortifying his lost confidence conferring plenteous guidance and bountiful insight throughout the past years and very essential recommendations in crucial time. The author saves his deepest respect and admiration for Prof. Dr. A. Hoque.

The author wants to thank Professor Dr. M. M. Shahidul Hassan for tolerating the author so long a time as he had been struggling to understand the physics of heterostructures and visiting Prof. Hassan infrequent for necessary help. The author's reverence for Prof. Hassan grew long ago when he took Prof. Hassan's undergraduate course on semiconductor devices, EEE257. The author is grateful to

Prof. Hassan for acquainting him with the world of advanced research, providing with essential journals and books.

The author wants to thank Prof. Dr. Quazi Deen Mohd Khosru, Dr. Md. Imamul Hassan Bhuiyan, Dr. F. M. Mohammedy, his other colleagues and well wishers for their valuable advices, supports and encouragements. The author also likes to express his gratitude to Dr. Md. Anwarul Abedin for giving him time and opportunity to discuss on this thesis with him. The author is grateful to BUET for providing library and information services support both printed and online forms and for approving the cost of preparation of this dissertation.

The author wants to express his deepest gratitude to his parents and parents-in-law. He is indebted to them for the sacrifices they made to make his life a little bit easier and now taking care of the author's baby with utmost devotion and heed. He dreams to be as good of a parent to his children as they have been to him, to his wife & daughter. This dissertation would be incomplete without a mention of the support given by the author's wife Sharmin Sultana *Liza*. Without her enlivening when this thesis seemed interminable, the author doubts it should ever have been completed. Thank for all the unconditional love, guidance, support and patience as the author went through this journey. And the pretty little girl Shazmin Saiyara *Adeпта*, her smile, cute talking, plays... the author likes most in the world but had to deprive him from these and deprived his daughter from his affection remaining busy with lot of odds. The author wishes to take the privilege of expressing his deep gratitude to both of them as they have been with him every step of the way, through good times and bad.

Finally, the author would like to dedicate this work to his grandfather Late Iman Uddin Gazi; grandmother Late Sayra Begum Gazi and the author's recently departed elder brother Late Elius Alamgir Gazi whose memory will always remain with him and his family. Loss of any soul is irreplaceable yet early departure leaves a family in profound mourning and remembrance. The author feels extremely helpless as he cannot douse the anguish of his father Abul Basher Gazi and mother Aleya Begum Gazi, rather he only prays to Allah "*Oh Lord of Death! Please bestow upon the departed souls your commiseration in the life hereafter. Oh Almighty Lord! Bequeath the left behinds enough strength to bear the affliction*".

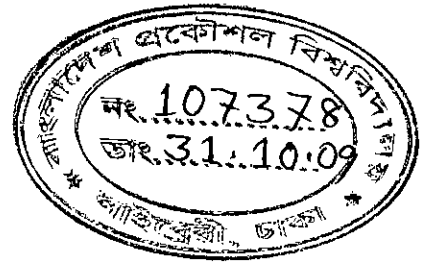
ABSTRACT

Base Transit Time of a Nonuniformly Doped Base Heterojunction Bipolar Transistor (HBT)

Yeasir Arafat

Base transit time of an HBT is more than 70% of its total transit time. So it plays a very crucial role in determining the transistor's high frequency performance. In the present work, base transit time of a nonuniformly doped base $\text{Si}_{1-y}\text{Ge}_y$ HBT with different type of Ge profiles (triangular, trapezoidal and box shape) for all levels of injection is studied to develop an analytical model by first obtaining closed form solution of minority carrier injected into the base under low level of injection. This solution is extended for high level of injection using perturbation theory and Webster effect and the obtained model is applicable for all levels of injection. In doing this, electric field and doping dependent mobility, bandgap narrowing effect due to heavy doping, presence of Ge and change in the density of states, velocity saturation at collector-base junction (modified to address the presence of Ge) etc. are considered to study the injected minority carrier, collector current density and stored base charge. The mathematical expression for base transit time obtained hereby relates its dependence on different device parameters. The results show that base transit time increases with base-emitter voltage, minority carrier injection ratio, base width, peak base doping concentration and if saturation velocity and field dependent mobility is considered. On the other hand base transit time decreases with slope of doping, Ge dose and gradient in Ge profiles. It is found that the transit time decreases with the change in Ge profile from box to triangle. The increase of Ge content for the same profile results in a decrease of transit time for a given Ge gradient. The results obtained by using the analytical formulation are compared with the results available in the literature and are found in good agreement. The analytical expressions derived in the present model can be a useful tool in device design and optimization.

CHAPTER 1



INTRODUCTION

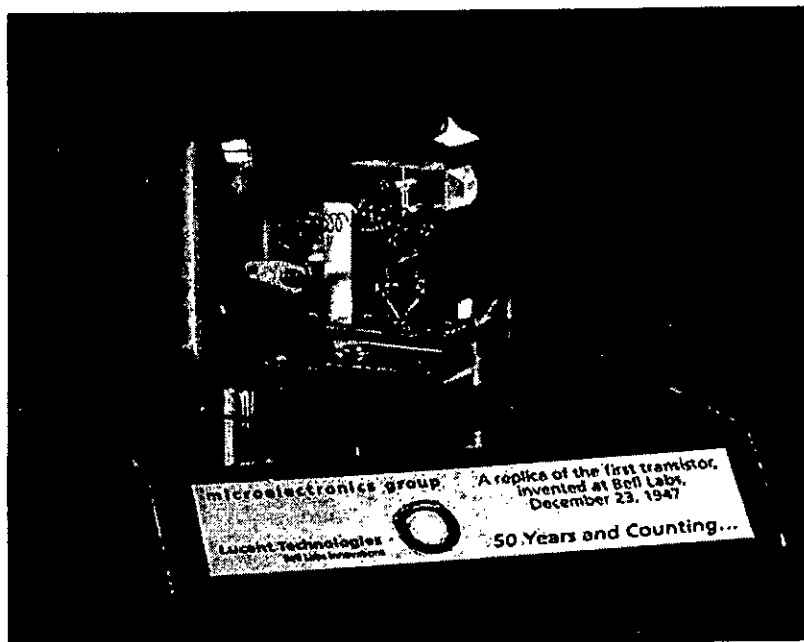
1.1 THE TRANSISTOR

The transistor is often cited as being one of the greatest achievements in the 20th century and some consider it one of the most important technological breakthroughs in human history. The transistor is the fundamental building block of modern electronic devices and is used in radio, television, telephone, computer and other electronic systems [1]. Transistor is a semiconductor device commonly used to amplify or switch electronic signals. It is made of a solid piece of a semiconductor material, with multiple terminals for connection to an external circuit. A voltage or current applied to one pair of the transistor's terminals changes the current flowing through another pair of terminals.

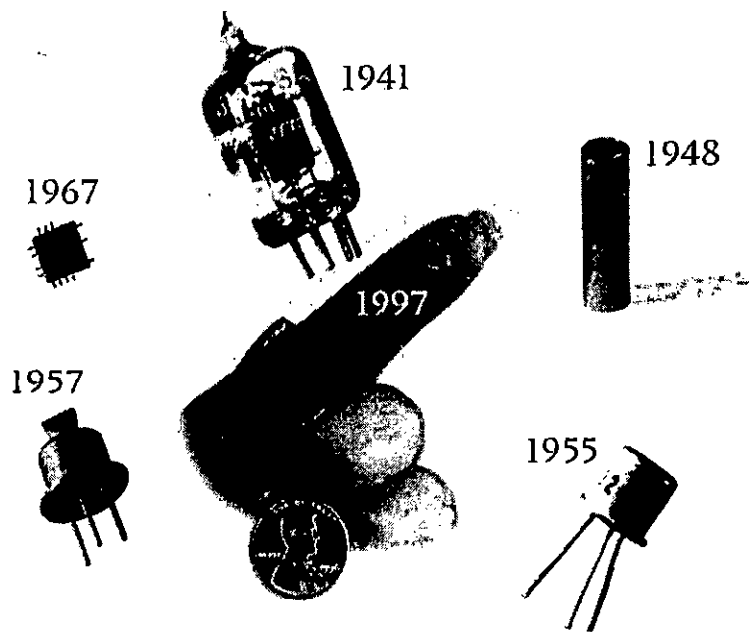
1.1.1 Brief history of Transistor

The first patent for the field-effect transistor principle was filed in Canada by Austrian-Hungarian physicist Julius Edgar Lilienfeld on 22 October 1925 but Lilienfeld did not publish any research articles about his devices [2]. In 1934 German physicist Dr. Oskar Heil patented another field-effect transistor. On 17 November 1947, John Bardeen and Walter Brattain at AT&T Bell Labs observed that when electrical contacts were applied to a crystal of germanium, the output power was larger than the input. William Shockley saw the potential in this invention and as part of a post-war effort to replace vacuum tubes with solid-state devices, worked over the next few months and by December 1947 he greatly succeeded expanding the knowledge of semiconductors. The term "transistor" was coined by John R. Pierce tailoring the words 'transfer' & 'resistor'. According to physicist/historian Robert

Arns, legal papers from the Bell Labs patent show that William Shockley and Gerald Pearson had built operational versions from Lilienfeld's patents, yet they had never



(a)



(b)

Figure 1.1: (a) A replica of the first working transistor (b) Transistor and Integrated Circuits (ICs) chronology [Source: Bell Labs].

referenced this work in any of their research papers or historical articles [2]. Bardeen, Brattain and Shockley were awarded the Nobel Prize in physics in 1956. The first silicon transistor was produced by Texas Instruments in 1954. This was the work of Gordon Teal, an expert in growing crystals of high purity, who had previously worked at Bell Labs. The first MOS transistor actually built was by Kahng and Atalla at Bell Labs in 1960 [3].

1.1.2 Category of Transistors

After the success of Bell Labs, many other organizations came ahead and invested hugely in the research of solid-state electronics. The outcome was tremendous in the next few years as this has instigated a new era called 'electronic age' and till date solid-state electronics research leads the world market based on scientific research. By this long span of time, various forms of transistors have been modeled, patented and experimented. A few of them are categorized here in the following table.

Table 1.1: Different type of Transistors (adapted from [4])

Based on	Category	
Structure	Junction Transistor (Bipolar Transistor)	BJT (Bipolar Junction Transistor) HBT (Heterojunction Bipolar Transistor) Insulated Gate Bipolar Transistor (IGBT)
	Field Effect Transistor (Unipolar Transistor)	Junction Field Effect Transistor (JFET) Metal Oxide Semiconductor FET (MOSFET or IGFET)
Polarity		BJT: <i>npn, pnp</i>
		FET: <i>n-channel, p-channel</i>
Material		Germanium (Ge), Silicon (Si), Gallium Arsenide (GaAs), Indium Phosphide (InP), Silicon Carbide (SiC) etc.
Power rating		Low, medium, high
Frequency		Radio frequency (RF), microwave etc.
Application		Switch, audio, high voltage, super-beta, matched pair etc.
Packaging		Through hole metal or plastic, surface mount, ball grid array etc.

1.2 THE BIPOLAR TRANSISTOR

A bipolar transistor takes shape as the doping of a semiconductor proceeds. The process of adding controlled impurities into the crystal lattice of a semiconductor is known as doping. The amount of impurity (dopant) added to an intrinsic semiconductor varies its level of conductivity. Doped semiconductors are often referred to as extrinsic. By adding impurity to intrinsic semiconductors, the electrical conductivity may be varied not only by the number of impurity atoms but also, by the type of impurity atom and the changes may be thousand folds and million folds. For example- 1 cm^3 of a metal or semiconductor specimen has a number of atoms on the order of 10^{22} . Since every atom in metal donates at least one free electron for conduction in metal, 1 cm^3 of metal contains free electrons on the order of 10^{22} . At the temperature close to 20°C , 1 cm^3 of intrinsic germanium contains about 4.2×10^{22} atoms and 2.5×10^{13} free electrons & 2.5×10^{13} holes. The addition of 0.001% of arsenic donates an extra 10^{17} free electrons in the same volume and the electrical conductivity increases about 10,000 times [5].

1.2.1 The Dopants

Dopants are classified as either electron acceptors or donors. A donor atom donates weakly-bound valence electrons to the material, creating excess negative charge carriers. These weakly-bound electrons can move about in the crystal lattice relatively freely and can facilitate conduction in the presence of an electric field. Conversely, an activated acceptor produces a hole. Semiconductors doped with donor impurities are called *n*-type, while those doped with acceptor impurities are known as *p*-type. The *n* and *p* type designations indicate which charge carrier acts as the material's majority carrier. The opposite carrier is called the minority carrier, which exists due to thermal excitation at a much lower concentration compared to the majority carrier. Intrinsic Si has 4 valence electrons. In Si, the most common dopants are group III & group V elements. Group III (boron(B:2,3), aluminum(Al:2,8,3), gallium(Ga:2,8,18,3), indium(In:2,8,18,18,3) and thallium(Tl:2,8,18,32,18,3)) all contain 3 valence electrons, causing them to function as acceptors when used to dope Si. Group V elements (nitrogen(N:2,5), phosphorus(P:2,8,5), arsenic(As:2,8,18,5),

antimony(Sb:2,8,18,18,5) & bismuth(Bi:2,8,18,32,18,5)) have 5 valence electrons, which allow them to act as a donor.

The number of dopant atoms needed to create a difference in the ability of a semiconductor to conduct is very small. When a comparatively small number of dopant atoms are added on the order of one per 100 million atoms, the doping is said to be low or light. When many more dopant atoms are added on the order of one per ten thousand atoms, the doping is referred to as heavy or high. This is often mentioned as n^+ for n -type doping or p^+ for p -type doping. When a semiconductor is doped to such high level, it acts almost like a conductor and is referred as a degenerate semiconductor [5].

1.2.2 The Doping Process

Some dopants are added as (usually) the silicon boule is grown using the Bridgeman technique or the Czochralski process, giving each wafer an almost uniform initial doping. For desired device properties, selected areas typically controlled by photolithography are further doped by such processes as diffusion and ion implantation. The latter method being more popular in large production runs because of increased controllability. The molecular beam epitaxy (MBE) or chemical vapor deposition (CVD) technologies are also used in transistor manufacturing where precisely controlled doping is required [6].

1.2.3 The n^+p-n Transistor

An $n-p-n$ bipolar junction transistor consists of two back-to-back $p-n$ junctions, which share a thin common region. Metal contacts are made to all three regions. The operation of a bipolar junction transistor depends on the forward and the reverse current of the two back-to-back $p-n$ junctions. The forward-biased junction, which injects electrons into the center p region, is called base-emitter junction and the reversed-biased junction, which collects the injected electrons, is called the base-collector junction. The region, which serves as the source of injected electrons, is called the emitter and the n region into which electrons are swept by the reverse-biased junction is called the collector. The center region is called the base.

For practical interest, doping concentration of emitter is made very high. Thus it forms an n^+p-n structure. Figure 1.2 shows the schematic diagram of an n^+p-n BJT. To have a good n^+p-n transistor, it is preferred that almost all the electrons injected by the emitter into the base be collected. Thus the p -type base region should be narrow. This requirement is summed up by specifying neutral base width should be less than the diffusion length of electron in the base. With this requirement satisfied, an average electron injected at the emitter junction will diffuse to the depletion region of the base-collector junction without recombination in the base. A second requirement is that I_E crossing the emitter junction should be composed almost entirely of electrons injected into the base rather than holes crossing from base to emitter [7].

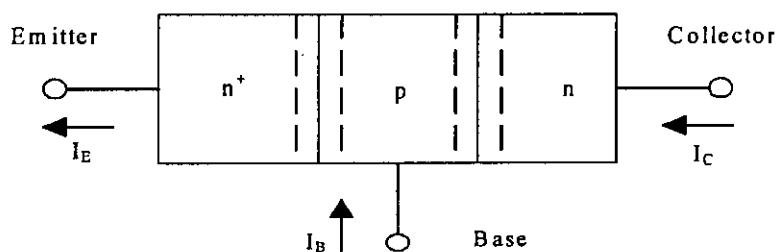


Figure 1.2: The schematic diagram of an n^+p-n bipolar junction transistor.

1.2.4 Suitability of Bipolar Transistor

Although Complimentary Metal Oxide Semiconductor (CMOS) transistor has acquired an ever-increasing role, the bipolar transistor retains its position as a technology of choice for high-speed circuits, ultra-high-frequency discrete logic circuits such as emitter coupled logic (ECL), power amplifiers in cellular phones, mixed-signal & precision analog components, microwave power amplifiers and in other applications like fiber-optic communication. The speed advantage together with trans-conductance (g_m), high current gain (β) and low $1/f$ noise continue to make the bipolar transistor the device of choice for many demanding applications [8]. One of the important figures of merit of BJTs is the transition frequency f_T^* , presently

* It is also called the unity current gain cutoff frequency; the theoretical frequency at which the short circuit current gain of the common-emitter (CE) mode transistor drops to 0 dB.

which ranges in the GHz level [5] and another important high frequency parameter is the maximum oscillation frequency f_{max} [†] which also ranges in the same level.

1.3 HETEROJUNCTION BIPOLAR TRANSISTOR

A heterojunction is the interface that occurs between two layers or regions of dissimilar crystalline semiconductors. These semiconductor materials have unequal band gaps as opposed to a homojunction. It is often advantageous to engineer the energy bands in many solid state devices. The $p-n$ junctions in the Figure 1.2 are assumed to homojunction (same substrate is used for the emitter, base & collector). If different semiconductors were used as substrates for the emitter, base & collector, then the junctions would be termed as hetero junctions. With two hetero junctions, a bipolar transistor is identified as double heterojunction bipolar transistor (DHBT) or simply HBT. With one homojunction and another heterojunction, a bipolar transistor is termed as single heterojunction bipolar transistor (SHBT). Typically the emitter of the SHBT is wide bandgap material which allows heavy doping of the base for reduced base resistance while the emitter is more lightly doped reducing the capacitance and improving the high frequency performance. In the DHBT, the collector and emitter have wide bandgap materials allowing the same advantages of the SHBT, with the additional improvement of increased breakdown voltage and decreased minority carrier injection from the base to the collector in saturation mode [9].

So, HBT is nothing but an improvement of the conventional BJT. Different semiconductor materials in the emitter and base prohibit the injection of electron from emitter into the base region, since the potential barrier in the valence band is higher than in the conduction band. Unlike BJT, HBT technology allows a high doping density to be used in the base, reducing the base resistance while maintaining the gain. The idea of employing a heterojunction is as old as the BJT, dating back to a patent from 1951 [1]. Since then many scientists and engineers have contributed to the improvement of HBT. In 2000, the Nobel Prize in Physics was awarded to H. Kroemer and Z. Alferov for “developing semiconductor heterostructure used in high

[†] Also called the maximum power frequency and it is a function of f_T .

speed and opto-electronics" [10].

Different types of elements and compound semiconductor materials are used to fabricate heterojunction. HBTs are named after the materials used in substrate or in the epitaxial layers. Properties of material used determine the characteristics of HBTs and their costs. Table 1.2 shows a list of such common materials used to fabricate HBTs of different genres.

Table 1.2: Some common materials used for HBT (reproduced from [9])

Material Names	Symbols
Silicon/Silicon Germanium	Si/SiGe
Aluminum Gallium Arsenide/Gallium Arsenide	AlGaAs/GaAs
Indium Phosphide/Indium Gallium Arsenide	InP/InGaAs
Indium Aluminum Arsenide /Indium Gallium Arsenide	InAlAs/InGaAs
Aluminum Gallium Nitride/Gallium Nitride	AlGaN/GaN
Indium Gallium Phosphide/Gallium Arsenide	InGaP/GaAs
Indium Gallium Phosphide/Indium Gallium Arsenide Nitride	InGaP/InGaAsN

1.3.1 The SiGe Device Physics

In SiGe devices, SiGe alloy is introduced into the base of a Si BJT by means of pseudomorphic growth of strained SiGe on Si. This process successfully utilizes bandgap engineering in the Si material system to achieve group III-V like performance while maintaining compatibility with conventional Silicon technology. From a performance perspective, introducing Ge into Si improves speed, current gain, noise characteristics and linearity. However, from a physical perspective, the difference in lattice constants between Ge and Si result in SiGe having a slightly higher lattice constant than Si. Ge has smaller bandgap energy than Si (0.66 eV and 1.12 eV respectively); consequently SiGe has a smaller bandgap than Si facilitating bandgap engineering in Si.

The compressive strain in the SiGe film produces an additional bandgap shrinkage resulting in about 75meV reduction in bandgap for every 10% of Ge introduced! Since this "band offset" occurs primarily in the valence band, *n-p-n* Si BJTs can be tailored to obtain required performance metrics. The compressive strain

also lifts the conduction and valence band degeneracies at the band extremes. This effectively reduces the density of states (DOS) and improves carrier mobilities.

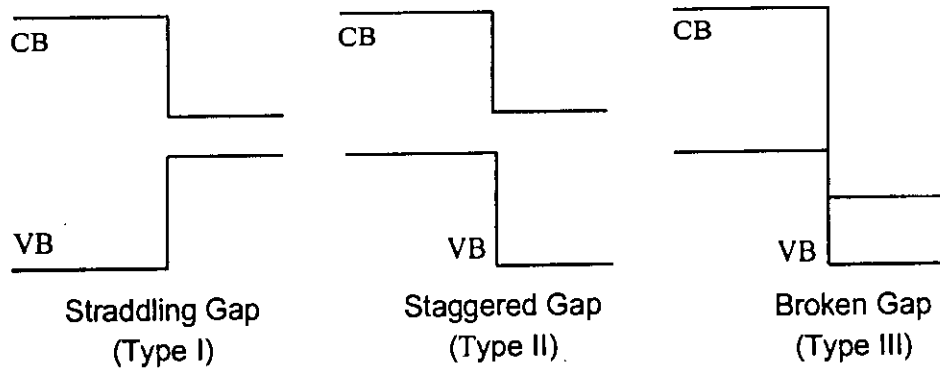


Figure 1.3: The three types of semiconductor heterojunctions organized by band alignment. Here CB stands for conduction band and VB is the valence band.

The presence of Si/SiGe heterojunction in the emitter-base (EB) and base-collector (BC) junctions of the HBT results in a marked performance improvement in both dc and ac characteristics over the Si BJT. The energy band diagrams of a forward biased graded-base SiGe HBT and a Si BJT are shown in Figure 1.4.

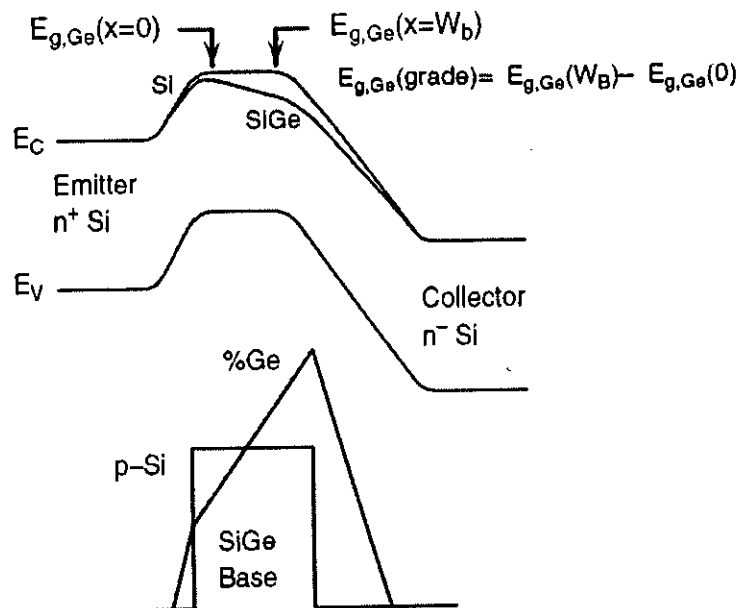


Figure 1.4: Energy band diagram for a graded base SiGe HBT and a Si BJT, biased in forward active mode at low-injection (reproduced from [11]).

The effect of graded Ge content in the base on the band structures is perceptible from the Figure 1.4. A slight reduction in the base bandgap at the EB junction ($\Delta E_{g,Ge}(x=0)$) and a much larger reduction at the BC junction ($\Delta E_{g,Ge}(x=W_B)$) is observed. This grading of Ge mole fraction across the base induces a built-in quasi-drift field ($((\Delta E_{g,Ge}(x=W_B)) - (\Delta E_{g,Ge}(x=0))) / W_B$) in the neutral base region, positively impacting the minority carrier transport. From Figure 1.4, it is seen that the emitter-base potential barrier is reduced in the SiGe HBT with respect to the Si BJT, thereby allowing increased electron injection from emitter to base. The enhanced electron injection leads to a higher collector current and current gain.

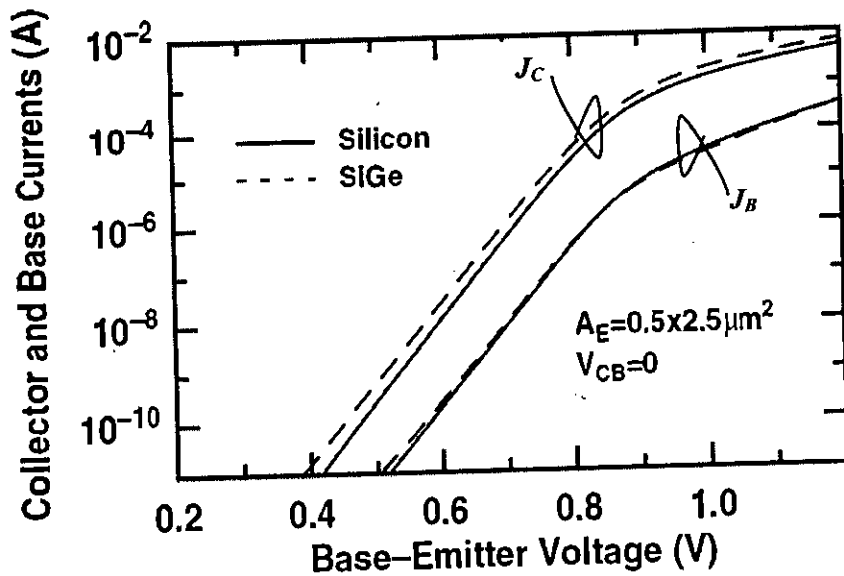


Figure 1.5: Representative Gummel characteristics for a typical SiGe HBT and a similarly constructed Si BJT (reproduced from [11]).

Figure 1.5 depicts the Gummel plot for a SiGe HBT as compared to a Si BJT. As expected, the SiGe HBT exhibits higher collector (J_C) current and approximately the same base current (J_B) as the Si BJT. The increase in J_C for the SiGe HBT in turn leads to an increase in current gain (β). The current gain depends linearly on the band offset due to Ge grading across the base and exponentially on the Ge induced band offset at the EB junction. Therefore, β is dependent on the Ge profile shape and can

be modified for particular circuit applications. The introduction of Ge in the base in effect decouples β from the base doping. This fact implies that the base doping can be increased without degrading β . Note that higher base doping reduces the base resistance which has positive implications in terms of frequency response and broadband noise.

Other two important ac figures-of-merit are the transition frequency (f_T) and the maximum oscillation frequency (f_{max}). Ge content in the base has positive impacts on both these parameters. The transition frequency is given by [12]

$$f_T = \frac{1}{2\pi} \left[\tau_E + \tau_B + \frac{W_{BC}}{2v_s} + \left\{ (C_{EB} + C_{BC}) \frac{1}{g_m} + R_C C_{BC} \right\} \right]^{-1} = \frac{1}{2\pi\tau_{EC}} \quad (1.1)$$

where τ_E is the emitter charge storage delay time, τ_B is the base transit time, W_{BC} is the width of base-collector space-charge layer, v_s is the saturation velocity, C_{EB} and C_{BC} are the emitter-base and base-collector depletion capacitances, g_m is the transconductance, R_C is the collector resistance and τ_{EC} is the total delay time from emitter to collector. The grading of Ge across the base induces a built-in electric field in the neutral base region (directed from collector to emitter). This field accelerates the minority carriers across the base which effectively reduces the base transit time. Due to the inverse relationship between the emitter charge storage delay time (τ_E) and current gain (β), the earlier is reduced as the later is higher for the SiGe HBT. Due to the reduction in τ_B and τ_E , f_T of the SiGe HBT increases over that of a Si BJT.

The maximum oscillation frequency of any bipolar transistor is given by [12]

$$f_{max} = \sqrt{\frac{f_T}{8\pi R_B C_{BC}}} \quad (1.2)$$

where R_B is the ac base resistance which can be reduced by heavy doping in the base region using wide bandgap materials. It is quite evident from equation (1.2) that the maximum oscillation frequency (f_{max}) of the SiGe heterojunction bipolar transistor will experience a two fold increase once for its higher transition frequency and again for its lower base resistance.

1.3.2 The SiGe HBT

In SiGe graded heterojunction transistors, the amount of germanium in the base is graded, making the bandgap narrower at the collector than at the emitter. That tapering of the bandgap leads to a field-assisted transport in the base, which speeds transport through the base and increases frequency response. Compared to silicon BJT, the SiGe HBTs have better device performance parameters (β , g_m and f_T). They satisfy the requirements of RF circuits such as modems, mixers, voltage controlled oscillators (VCOs), power amplifiers etc; mixed signal circuits like fractional N synthesizers & analog to digital converters and in the precision analog circuits such as Op-Amps, band gap references, temperature bias control and current mirrors etc [13]. Further, band gap engineering of SiGe facilitates in reducing forward voltage drop of the emitter-base junction by uniform grading at the emitter-base junction without affecting the other parameters [14] which makes it a best candidate for low voltage applications in wireless phones and other low power battery operated products. In addition to allowing very complex custom designs, high speed and high breakdown voltage SiGe HBTs can be merged with high density CMOS using a mixed signal methodology.

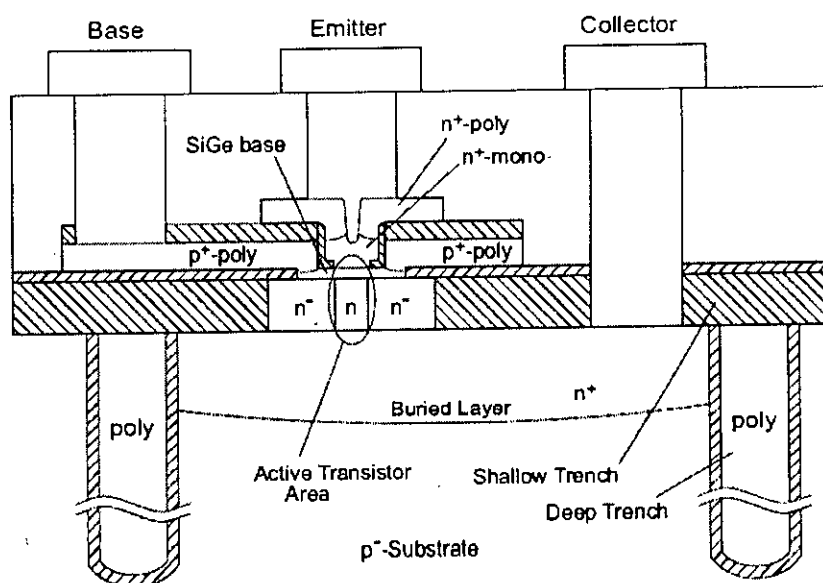


Figure 1.6: The schematic diagram showing the cross section of a SiGe heterojunction bipolar transistor.

Another prime aspect for a SiGe based HBT technology is the ability to merge the high performance SiGe HBT with standard CMOS technology giving rise to a high performance SiGe BiCMOS process without compromising the performance of either the HBT or the CMOS device. The combination of SiGe HBTs with scaled BiCMOS to form SiGe HBT BiCMOS technology presents an exciting possibility for system-on-chip (SoC) solution. Further, the use of SiGe devices allows many new functions to be added onto the silicon chip thus potentially reducing cost and power and increasing speed and yield. Ge ion implantation into Si has been successfully demonstrated to form SiGe [15]. But, it is difficult to obtain shallow junctions with sharp impurity profiles in vertical structures. However, recently it has been shown that this technique can be attractive for lateral SoI (Silicon on Insulator) HBT [16].

Key difference between SiGe HBTs & Si BJTs is the introduction of SiGe epitaxial layer into the base of an otherwise all-Si bipolar transistor. The base region is thus comprised of a Si buffer, boron-doped graded SiGe alloy active layer and a Si cap. Figure 1.6 depicts the schematic cross-section of a SiGe HBT while Figure 1.7 portrays Ge profile for a representative first generation SiGe HBT.

The SiGe HBT technology with 50 GHz (1st-generation) [17] and 120 GHz (2nd-generation) [18] peak cutoff frequency are currently in commercial production worldwide from multiple sources and are being deployed in both the commercial and defense sectors. The announcement of a third-generation SiGe HBT technology with 200 GHz peak cutoff frequency [19] and a fourth-generation SiGe HBT technology with over 300 GHz peak cutoff frequency [20], along with the complementary (*n-p-n* & *p-n-p*) SiGe HBTs with peak f_T values above 180 GHz and 80 GHz, respectively [21], have pushed the upper bound on the speeds achievable in these devices considerably higher than previously believed possible, thus vastly increasing the application options for SiGe HBT technology to encompass a wide variety of analog and RF through millimeter-wave systems [22]. While it might be argued that a peak cutoff frequency in excess of 200 GHz is not needed to support most IC applications, which are currently clustered in the 1-40 GHz range, such extreme levels of performance create a much broader circuit design space where, for instance, a

designer has the option to trade frequency response for dramatic reductions in power consumption (10x reduction in bias current in the third-generation devices over second-generation technology for similar operating speeds) [11].

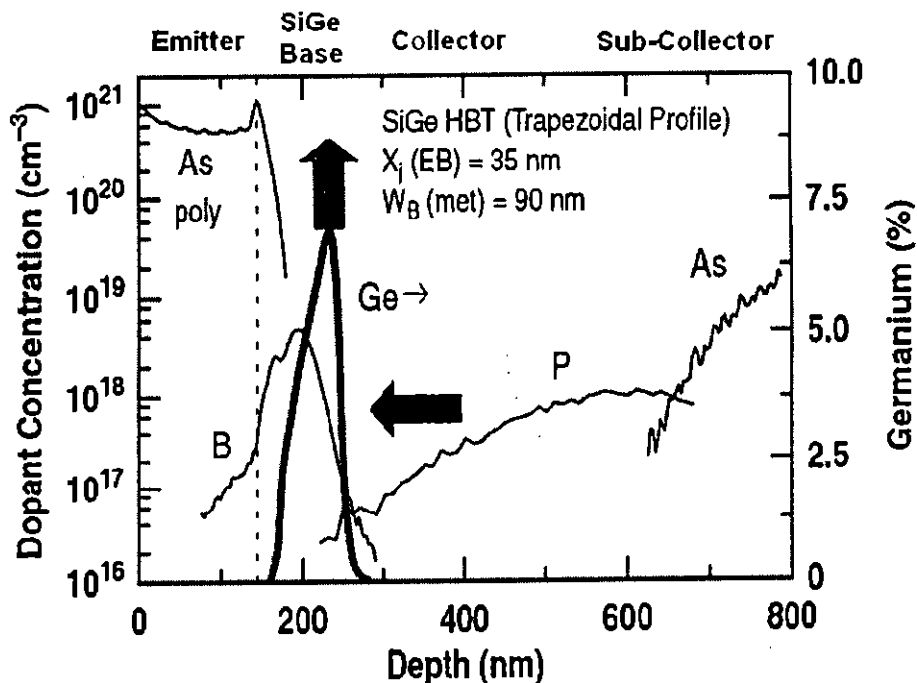


Figure 1.7: Representative base profiles for a first generation SiGe heterojunction bipolar transistor (reproduced from [11]).

1.3.3 RFIC Technology Comparison

Gallium arsenide has historically been the preferred technology for power amplifiers used in RFIC because of its intrinsically higher low-field electron mobility (6 times higher than Si), transition frequency and breakdown voltage. The semi-insulating GaAs substrate also helps in developing high-Q passive elements. Due to higher bandgap energies and mobilities, GaAs power amplifiers have higher gain even at lower operating voltage. Even though GaAs has a number of advantages over Si, it also has some disadvantages which need to be considered. The thermal conductivity of GaAs is about 4 times lower than that of Si. This is a particularly difficult challenge for GaAs ICs as the power which must be dissipated is typically higher than that of Si ICs. Thermal management issue is a major consideration with regards

to the ruggedness of the GaAs power amplifiers. Stability and reliability have also been areas of major concern in GaAs technology [23].

Recent advancements in Si-based RFICs, particularly the advent of SiGe HBTs, offer a significant challenge to GaAs HBT technology. The unity current gain cutoff frequency of SiGe HBT has reached a level comparable to GaAs technology. In fact SiGe HBT has been demonstrated with f_T beyond 500 GHz [24]. GaAs technology still has a significant performance advantage due to the higher low-field mobility which improves the minimum noise figure [25] but this is not a major concern in power amplifier design. SiGe technology also offers a potential cost advantage over GaAs due to compatibility with existing Silicon fabrication infrastructure. Finally, a high level of integration can be achieved in SiGe ICs which in turn reduces the packaging complexity.

A summary of the advantages of SiGe technology over other technologies is given below:

- Size and power consumption are critical for mobile applications. For a given f_T , a SiGe HBT would require only about a third of the collector current as compared to a Si BJT [26]. MOSFETs have lower g_m per mA as compared to BJT/HBT and the current required to obtain the same f_T for MOSFETs is larger [27].
- Breakdown voltages of SiGe HBTs are about twice those of Si BJTs for same f_T . As far as RF CMOS technology is concerned, the breakdown voltage as well as the maximum operating voltage are limited due to thin oxide breakdown and hot-carrier effects [28].
- HBTs have better noise performance since they have higher β and f_T as compared to BJTs. The base resistance of a SiGe HBT is lower than the Si BJT counterpart. CMOS devices have higher minimum noise figure as compared to bipolar devices when biased at the same current density [29].
- By optimizing the Ge profile in the base of a SiGe HBT, much higher Early voltage can be obtained than in BJTs. This results in higher r_o which maximizes gain and also helps in improving stability. The short-channel RF-MOSFETs have

much lower r_o than SiGe HBTs due to increased channel-length modulation effects.

- HBTs have higher current gain than conventional BJTs as a result of which improved linearization by feedback is possible. Cancellation of the base-emitter heterojunction capacitance improves the inter-modulation performance of HBT as compared to MESFET and HEMT. This results in excellent linearity data as reported in [30].
- Due to the advantages of SiGe HBT, such as higher current gain, early voltage, breakdown voltage and transit frequency, reduced base resistance as well as better transport properties of the base, performance approaching GaAs technologies can be achieved. Moreover, SiGe technology is currently being used to develop electronics for space applications due to their excellent analog and radio frequency performance over an extremely wide range of temperatures [31]. Therefore SiGe technology is a very good choice for high frequency and high temperature electronics at a competitive cost.

1.4 THE BASE TRANSIT TIME

When emitter-base junction of n^+p-n transistor is forward biased, electrons from the emitter are injected in the base. Injected electrons are called minority carrier in p -type base. The average time taken by minority carriers to diffuse across the quasi-neutral base or the average time that an electron spends in the base is called the base transit time (τ_B). For an $n-p-n$ HBT, the base transit time is given by [32]

$$\tau_B = \int_0^{W_B} \frac{qn(x)}{J_n(x)} dx \quad (1.3)$$

where, q is the charge of an electron, $n(x)$ is the minority carrier electron concentration within the base, J_n is the electron current density and W_B is the base width. If the recombination within the base is neglected then the electron current density becomes constant and equation (1.3) can be written as

$$\tau_B = \frac{1}{J_n} \int_0^{W_B} qn(x)dx = \frac{Q_{Bn}}{J_n} \quad (1.4)$$

and

$$Q_{Bn} = q \int_0^{W_B} n(x)dx \quad (1.5)$$

where, Q_{Bn} is the stored base charge per unit area.

Among different delay terms in equation (1.1), the base transit time is one of the most dominant factors in deciding f_T . The base transit time is more than 70% of a heterojunction bipolar transistor's total delay time [33]. So, the base transit time plays a very crucial role in determining a transistor's high frequency performance. It is of great importance to obtain an accurate yet relatively simple analytical model of collector current density and base transit time for efficient device design.

The base transit time of a SiGe HBT is influenced by many physical and electrical parameters and various other factors such as

- Base width
- Various doping concentration profiles in the base region (uniform, linear, exponential, Gaussian, retrograde & complementary error function)
- Various Ge profiles in the base region (box, triangular, trapezoidal)
- Velocity saturation effect
- Collector current density
- Built-in electric field in the base region
- Electric field and doping dependent carrier mobility
- Bandgap narrowing
- Base resistance etc.

1.5 REVIEW OF LITERATURE ON BASE TRANSIT TIME

Several analytical works have been reported on the base transit time for low and high levels of injection in the base. Moll and Ross [34] formulated a double integral relation of transit time for a homojunction bipolar transistor. The proposed expression is given below

$$\tau_B = \frac{1}{D_n} \int_0^{W_B} \frac{1}{N_B} \left[\int_x^{W_B} N_B dy \right] dx \quad (1.6)$$

In this model, the base transit time depends on the base doping (here acceptor) concentration (N_B) and the minority carrier (here electron) diffusivity (D_n) and it is only applicable for low level of injection.

J. J. H. Van den Biesen [35] in 1986 studied the base transit time of a bipolar junction transistor using regional analysis by means of a numerical device simulation program for high level of injection. He used an expression for base transit time based on perturbation theory and that was written as

$$\tau_B = q \int_{x_E}^{x_C} \left. \frac{dn}{dJ_c} \right|_{v_{ce}=0} dx \quad (1.7)$$

K. Suzuki and N. Nakayama [36] in 1992 derived an analytical expression for the base transit time considering carrier velocity saturation at base-collector junction. The derived equation is

$$\begin{aligned} \tau_B &= \int_0^{W_B} \frac{n_{ie}^2(x)}{N_B(x)} \left\{ \int_x^{W_B} \frac{1}{D_n(y)} \frac{N_B(y)}{n_{ie}^2(y)} dy \right\} dx + \frac{1}{v_s} \frac{N_B(W_B)}{n_{ie}^2(W_B)} \int_0^{W_B} \frac{n_{ie}^2(x)}{N_B(x)} dx \\ &= \frac{W_B^2}{k_1 D_n(0)} + \frac{W_B}{k_2 v_s} \end{aligned} \quad (1.8)$$

where,

$$k_1 = \frac{W_B^2}{\int_0^{W_B} \frac{n_{ie}^2(x)}{N_B(x)} \int_x^{W_B} \frac{1}{D_n(y)} \frac{N_B(y)}{n_{ie}^2(y)} dy dx} \quad (1.9)$$

$$k_2 = \frac{W_B}{\int_0^{W_B} \frac{N_B(W_B)}{n_{ie}^2(W_B)} \frac{n_{ie}^2(x)}{N_B(x)} dx} \quad (1.10)$$

here n_{ie} is the intrinsic carrier concentration, v_s is the saturation velocity of electron. This analysis was done only for low injection region.

In 1996, M. M. Jahan and A. F. M. Anwar [37] modified the double integral relation of transit time for exponentially doped base, which can be applicable both for homojunction and heterojunction bipolar transistors. This model included doping dependent mobility variation, bandgap narrowing and finite velocity saturation

effects in the calculation of base transit time but ignored the electric field dependency of mobility and avoided high injection region of operation.

In 1997, N. Rinaldi [38] presented an analytical model for base transit time with an exponentially doped base BJT. This model included dependence of the mobility on doping concentration and electric field, heavy-doping effects and finite carrier velocity at the collector edge. This model is also applicable to HBT with a linearly graded Ge composition. Other forms of Ge composition variation (e.g. box, trapezoidal) are not addressed in Rinaldi's model and it is only valid for low level of injection and the analytical expression for base transit time is quite long as given below

$$\tau_B = \left[\tau_{B0}^\gamma + \left(\frac{W_B}{\eta_R v_s} (\eta_R - 1 + e^{-\eta_R}) \right)^\gamma \right]^{1/\gamma} + \frac{W_B}{S} \frac{1 - e^{-\eta_R}}{\eta_R} \quad (1.11a)$$

$$\tau_{B0} = \left(\frac{W_B}{\eta_R} \right)^2 \frac{\eta_R - 1 + e^{-\eta_R}}{D_{\max}} + \left(\frac{W_B}{\eta_R} \right)^2 \frac{(N_b(W_B)/N_r)^{\gamma_m}}{D_{\max} (1 + \gamma_m \eta / \eta_R)} \left[\frac{\eta_R}{\gamma_m \eta} (e^{\gamma_m \eta} - 1) - 1 + e^{-\eta_R} \right] \quad (1.11b)$$

where γ is a constant with value of 1 or 2, v_s is the saturation velocity of electron, S is the velocity of electron (without saturation), D_{\max} is the maximum value of diffusivity, $N_r = 10^{17} \text{ cm}^{-3}$, $\gamma_m = 0.468$, $\eta_R = \eta_2 + \eta_E$, $\eta_2 = \eta(1 - \gamma_2)$, η is the slope of base doping, $\gamma_2 = 0.69$, $\eta_E = (\Delta E_G^C(W_B) - \Delta E_G^C(0))/kT$, ΔE_G^C is the conduction band energy gap.

In 1998, V. S. Patri and M. J. Kumar [39] developed a unified closed form analytical model for base transit time of SiGe HBT's for uniform and exponential base doping with different Ge profiles in the base (e.g. box, trapezoidal, triangular) while taking into consideration doping induced bandgap narrowing and diffusion coefficient. The model avoided high injection region of operation and field dependent mobility. The analytical form is also complicated and difficult to get a complete picture.

K. H. Kwok [40] in 1999 used a regional base transit time model which results in a set of closed-form analytical expressions for an exponentially-doped retrograde base of a SiGe-base HBT with a linearly graded Ge profile. This model considered

the retarding built-in electric field due to the retrograde region, heavy doping effects, the effect of velocity saturation at the base-collector junction, the effects of the doping dependency and the electric-field dependency of the electron diffusivity and the electric field induced by the Ge concentration gradient. However, this model was not valid for high level of injection and the analytical expressions for regional base transit time were very long and complicated.

M. M. S. Hassan and A. H. Khondoker in 2001 [41] derived a new mathematical expressions for electron current density and base transit time for uniform base doping profile of a BJT for all levels of injection. This model did not use Moll's double integral form of base transit time. In this work base doping density dependence of mobility was considered but the carrier saturation, velocity and electric field dependent mobility was neglected. It is also noteworthy that in practical transistor the base is not uniformly doped. In practical transistor base doping follows Gaussian distribution [42]. G. Li *et al.* in their 2001 work [42] showed that a Gaussian profile can be well approximated by a simple exponential profile. M. Z. R. Khan *et al.* [43], F. M. Mohammedy *et al.* [44], M. A. Abedin *et al.* [45], T. Rahman *et al.* [46] and M. W. K. Nomani *et al.* [47] used nonuniformly doped base profile for the modeling of base transit time.

K. H. Kwok and C. R. Selvakumar [48] in 2001 reported a work on base transit time for all levels of injection before onset of Kirk effect. They solved the equations numerically using iterative techniques to show different effects of base doping and Ge profiles on base transit time.

A number of studies have been found in literatures on the base transit time of SiGe base HBTs. S. T. Chang *et al.* [49], S. K. Mandal *et al.* [33] in 2004, M. K. Das *et al.* [50] in 2005, S. Basu [51] in 2008 have reported on transit time and high frequency performance of SiGe HBT but none of these works has modeled the high level of injection. Up to recent time, a very few number of analytical models of base transit time for HBT have addressed the electric field dependency of minority carrier mobility and none of them have dealt with high level of injection.

However, for BJT a number of studies have been found in the literature searching for suitable analytical expressions of base transit time for high level of injection. K. Suzuki [52] in 1994, P. Ma *et al.* [53] in 1998 and M. M. S. Hassan *et al.* [54] in 2006 used perturbation theory to model base transit time for all levels of injection. Suzuki's model ignored field dependent mobility, P. Ma used iterative method and M. M. S. Hassan considered field dependent mobility.

1.6 SCOPE OF THIS DISSERTATION

The main objective of this work is to obtain an analytical expression for base transit time of an n^+p-n $\text{Si}_{1-y}\text{Ge}_y$ base HBT with nonuniform base doping distribution and three different type of Ge profiles (e.g. box, trapezoidal, triangular) for all levels of injection considering electric field dependent mobility. In the proposed work, perturbation theory along with Webster effect will be used to model base transit time. Moreover, effect of doping dependent mobility, bandgap narrowing, carrier velocity saturation etc. will be considered. The injected minority carrier, collector current density, stored base charge, electric field in the base will also be studied. Knowing the mathematical expression for base transit time, its dependence on different device parameters can be determined. It is expected that the derived model will give prompt and precise base transit time for realistic transistors. Analytical results obtained by this model will be compared with numerical data available in the literature.

1.7 LAYOUT OF THE CHAPTERS

In this work the expressions for injected minority carrier concentration profile, collector current density and hence the base transit time of a heterojunction bipolar transistor are derived. In chapter one previous works on base transit time have been reviewed and the justification of carrying out the research is given. Mathematical analyses are given in chapter two. In chapter three the dependence of current and base transit time on various device parameters are studied. The transit time obtained from the derived equation is compared to that of numerical analysis in order to

demonstrate the validity of the assumptions made in deriving expression for base transit time. Finally this dissertation ends in chapter four containing salient features of this work and possible future field of studies.

CHAPTER 2

MATHEMATICAL ANALYSIS

2.1 INTRODUCTION

To obtain an accurate yet simple analytical model for base transit time for $\text{Si}_{1-y}\text{Ge}_y$ base heterojunction bipolar transistor (HBT) is of great importance for efficient device design. Within the framework of drift and diffusion approaches, an outstanding analytical model should consider the following important effects: (a) different Ge profiles with non-uniformly doped base (b) change in Ge dose and gradient (c) heavy doping or band gap narrowing effect (d) finite carrier velocity at the collector edge (e) dependence of mobility on electric field and doping concentration (f) high injection effects. In this chapter an analytical expression for base transit time for exponential base doping $\text{Si}_{1-y}\text{Ge}_y$ n^+p-n HBT is derived considering all the above-mentioned effects. The injected minority carrier concentration profile $n(x)$ and collector current density J_c is modeled and calculated for high level of injection. The derivation of the model equation starts by first considering low level of minority carrier injection within the base. Then the minority carrier profile of low level of injection is extended for finding minority carrier profile for high level of injection using Perturbation Theory [52-54] and Webster effect [55]. An injected minority carrier distribution profile within the SiGe alloy is obtained for low level of injection by solving current continuity equation, electric field equation, expression for field and doping dependent mobility and effective intrinsic carrier concentration for high base doping. It is assumed that for high level of injection, $n(x)$ is perturbed by the electric field from the modified low injection profile only a small amount $\delta n(x)$. Incorporating this small change in the current density equation, the

differential equation for minority carrier concentration $n(x)$ is solved. Once $n(x)$ is known, the analytical expressions for J_n , Q_{Bn} and τ_B can easily be obtained.

2.2 DERIVATION OF THE MODEL EQUATIONS

The Ge mole fraction (y) distribution profiles of an n^+p-n $\text{Si}_{1-y}\text{Ge}_y$ -base HBT is shown in Figure 2.1. For a generalized trapezoidal profile, Ge mole fraction can be expressed as a function of distance along the base

$$y(x) = m_{Ge}x + y_E \quad (2.1)$$

where, x is the distance of a point in the base measured from the emitter-base (EB) junction of the HBT, $m_{Ge} = \eta_{Ge}/W_B$, W_B is the neutral base width, $\eta_{Ge} = y_C - y_E$, y_C and y_E are Ge fractions at the collector and emitter end of the base respectively. For $y_E = 0$, equation (2.1) represents triangular profile, for $y_E = y_C$, equation (2.1) represents box shape Ge profile [56].

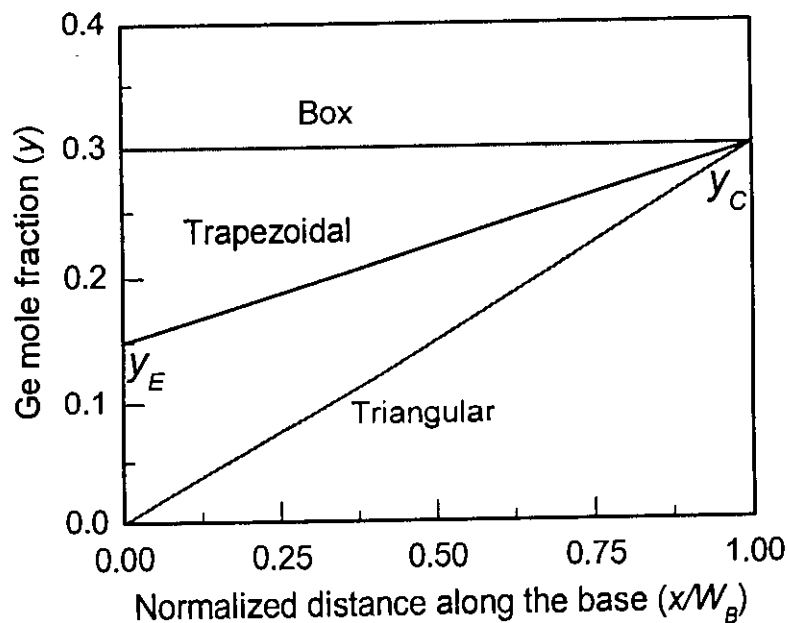


Figure 2.1: Ge mole fraction distribution within the base of a $\text{Si}_{1-y}\text{Ge}_y$ HBT for various geometric profiles.

Total Ge content within the base termed as “Ge dose” is calculated as [48]

$$y_D = y_{av} W_B \quad (2.2)$$

where,

$$y_{av} = (y_C + y_E)/2 \quad (2.3)$$

The schematic representation of an n^+p-n transistor is shown in Figure 2.2. The electron current density J_n and hole current density J_p with arbitrary base doping concentration $N_B(x)$ are given by the transport equations [57, 58]

$$-J_n(x) = q D_n(x) \frac{dn(x)}{dx} + q \mu_n(x) n(x) E(x) \quad (2.4a)$$

$$J_p(x) = -q D_p(x) \frac{dp(x)}{dx} + q \mu_p(x) p(x) E(x) \quad (2.4b)$$

where, $D_n(x)$ & $D_p(x)$ are the diffusion co-efficient (or diffusivity) for electron & hole, $\mu_n(x)$ & $\mu_p(x)$ are the electron & hole mobility, $n(x)$ & $p(x)$ are the electron & hole concentration respectively and $E(x)$ is the electric field at a point x within the base. The direction of J_n in equation (2.4a) is defined such that it has a positive value. The total current density is the sum of the electron and hole current density.

$$J_c(x) = J_n(x) + J_p(x) \quad (2.5)$$

The electron diffusion length in silicon is about 500nm even at doping concentration as high as 10^{20} cm^{-3} [59]. As the base width of a modern high-speed HBT is very thin (less than 100nm even 26nm base width is reported in [48]), the carrier recombination in the base region can safely be neglected. So,

$$J_p(x) = 0 \quad (2.6a)$$

and

$$J_c(x) \cong J_n(x) \quad (2.6b)$$

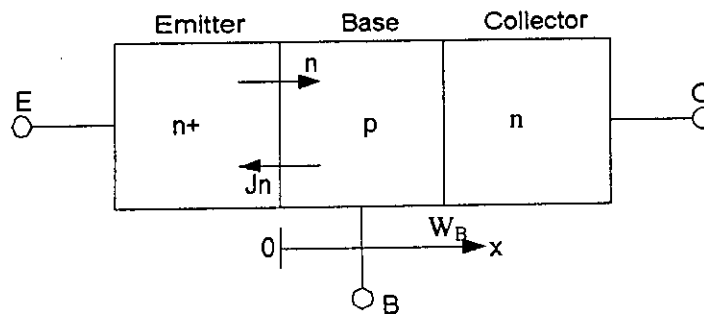


Figure 2.2: One-dimensional view of an n^+p-n SiGe heterojunction bipolar transistor showing flow of injected minority carrier and current density.

Therefore, the minority carrier current density J_n within the base becomes constant and is equal to collector current density J_c . In the present work, minority carrier concentration at thermal equilibrium has been neglected.

The electric field neglecting recombination in the base and considering band gap narrowing effect is [48, 60]

$$E(x) = V_T \frac{d}{dx} \left[\ln \left(\frac{p(x)}{n_{ie}^2(x)} \right) \right] \quad (2.7a)$$

where, V_T is the thermal voltage and is equal to kT/q , k is the Boltzmann constant, T is the temperature in degree Kelvin, $n_{ie}(x)$ is the effective intrinsic carrier concentration in the base.

Equation (2.7a) can be written as follows [57]

$$E(x) = V_T \left(\frac{1}{p(x)} \frac{dp(x)}{dx} - \frac{1}{n_{ie}^2(x)} \frac{dn_{ie}^2(x)}{dx} \right) \quad (2.7b)$$

The first term in equation (2.7b) represents electric field due to concentration gradient and the second term in equation (2.7b) represents the quasi-field due to nonuniform band gap narrowing. Carrier mobility and diffusivity are correlated as described by the Einstein's relation [61]

$$D_n(x) = V_T \mu_n(x) \quad (2.8)$$

The quasi-neutrality of charge within the base of a bipolar transistor is [61]

$$p(x) = n(x) + N_B(x) \quad (2.9)$$

Using equations (2.4a) and (2.7-2.9), the minority carrier current density can be written as

$$-J_n = qD_n(x) \frac{n_{ie}^2(x)}{n(x) + N_B(x)} \frac{d}{dx} \left[\frac{n(x) \{n(x) + N_B(x)\}}{n_{ie}^2(x)} \right] \quad (2.10)$$

Equation (2.10) can be solved for $n(x)$ to find out the current density J_n and stored base charge per unit area Q_{Bn} .

Base doping distribution used in this analysis is assumed to be exponential [58]

$$N_B(x) = N_B(0)e^{-mx} \quad (2.11)$$

where, $N_B(0)$ is the peak base doping concentration, $m=\eta/W_B$, $\eta=\ln(N_B(0)/N_B(W_B))$ is the slope of base doping concentration.

The effective intrinsic carrier concentration $n_{ie}(x)$ is a function of the base doping concentration $N_B(x)$ through bandgap narrowing effect. For large electric field, the carrier mobility turns out to be a function of electric field as well as doping density [54]. The low field doping density dependent electron diffusivity in Si, given in [58] can be rearranged as

$$D_{nSi}(x) = D_n e^{m_1 x} \quad (2.12a)$$

where, $D_n = D_{n0}(N_B(0)/N_r)^{\gamma_1}$, $D_{n0} = 20.72 \text{ cm}^2/\text{s}$, $N_r = 10^{17} \text{ cm}^{-3}$, $m_1 = m\gamma_1$ and $\gamma_1 = 0.42$. Here, γ_1 is a fit value. In the presence of Ge, the electron diffusivity is modified as [51]

$$D_{nSiGe}(x) = bD_{nSi}(x) \quad (2.12b)$$

where, $b = 1 + 3\gamma_{av}$.

The saturation velocity inside the SiGe alloy also differs from that in Si and is [49]

$$v_{sA} = cv_s \quad (2.13)$$

where, $c = 0.342/[0.342 + \gamma_{av}(1 - \gamma_{av})]$ and $v_s = 10^7 \text{ cm/s}$ is the saturation velocity in Si.

The empirical expression for high field and doping dependent mobility in Si is [62]

$$\mu_{nSi}(|E|, x) = \frac{v_s}{a|E(x)| + E_{cSi}(x)} \quad (2.14a)$$

where, $a = 0.7743$. $E_{cSi}(x)$ is the critical electric field and given by

$$E_{cSi}(x) = \frac{v_s}{\mu_{nSi}(x)} \quad (2.14b)$$

The effective intrinsic concentration in SiGe is [51]

$$n_{ieSiGe}^2(x) = n_{ieSi}^2 \gamma_r e^{\frac{\Delta E_{ref}(x)}{kT}} \quad (2.15a)$$

where, $n_{i0Si}=1.4 \times 10^{10} \text{ cm}^{-3}$ is intrinsic carrier concentration in undoped silicon, γ_r is the ratio of the effective density of states in SiGe to the effective density of states in Si as given in [60] and modified following [39]

$$\gamma_r = \exp\left(-\sqrt{5y_{av}}\right) \quad (2.15b)$$

$\Delta E_{g\text{eff}}(x)$ is the effective bandgap reduction in the SiGe base that can be expressed as

$$\Delta E_{g\text{eff}}(x) = \Delta E_{gHD}(x) + \Delta E_{gGe}(x) \quad (2.15c)$$

where $\Delta E_{gGe}(x)$ is the bandgap narrowing due to the presence of Ge which is assumed to have a linear dependence on Ge concentration, $\Delta E_{gHD}(x)$ is the bandgap narrowing due to heavy doping effects and is identical to that of Si [39]. An approximation of the Slotboom–de Graff bandgap narrowing model [63] is used for this term [64]

$$\Delta E_{gHD}(x) = qV_{gHD} \ln\left(\frac{N_B(x)}{N_r}\right) \quad (2.15d)$$

with, $V_{gHD}=18 \text{ mV}$ & the bandgap narrowing due to the presence of Ge is given by [32]

$$\Delta E_{gGe}(x) = qV_{gGe}y(x) \quad (2.15e)$$

with, $V_{gGe}=688 \text{ mV}$. Combining equations (2.15a)-(2.15e), the effective intrinsic concentration in SiGe can be expressed as

$$n_{ieSiGe}^2(x) = n_{ieSiGe}^2(0)e^{m_3x} \quad (2.15f)$$

where, $n_{ieSiGe}^2(0) = n_{i0SiGe}^2(N_B(0)/N_r)^{\gamma_2}$, $\gamma_2 = V_{gHD}/V_T = 0.69$, $n_{i0SiGe}^2 = \gamma_A n_{i0Si}^2$, $\gamma_A = \gamma_r \exp(\gamma_3 \gamma E)$, $\gamma_3 = V_{gGe}/V_T$, $m_2 = m\gamma_2$, $m_3 = m_{Ge}\gamma_3$, $m_{32} = m_3 - m_2$.

It is required to obtain a mathematical expression for minority carrier concentration $n(x)$ and current density (J_n) in order to derive an expression for base transit time (τ_B). When the emitter-base junction is forward biased injected electron concentration depends on the externally applied voltage across base-emitter junction. The base will be subjected to three types of injection. At 'low injection', injected electron concentration will be much less than the doping concentration in the base,

i.e. $n(x) \ll N_B(x)$. At 'strong high' level of injection, injected electron concentration will be much higher than the base doping concentration, i.e. $n(x) \gg N_B(x)$. But for intermediate level of injection (henceforth termed as 'high injection'), the minority carrier concentration is comparable to the base doping concentration. Under this (high injection) condition, neither $n(x)$ nor $N_B(x)$ can be ignored from the differential equation (2.10). As a result, (2.10) appears to be analytically intractable. But using perturbation theory and Webster effect simultaneously, $n(x)$ can be handled analytically a little bit further.

At first, fully closed form analytical expressions for minority carrier electron concentration, minority carrier current density, base stored charge and base transit time are obtained for low level of injection. Then this low-injection model is extended for high level of injection making a realistic assumption that the injected electron concentration $n(x)$ at high injection is perturbed by modulated electric field from its modified low injection solution $n_m(x)$ [see equation (2.27)] only a little amount $\delta_n(x)$. This technique gives a solution for minority carrier electron concentration $n(x)$ which is not only valid for high injection region but also applicable for all levels of electron injection. Applying velocity saturation at the base-collector boundary under reverse bias condition, minority carrier current density J_n can be found from $n(x)$. Base stored charge Q_{Bn} is attained by integrating $n(x)$ over the base width and finally the base transit time can be obtained using equation (1.4).

2.2.1 Low Injection Model

For low injection, the quasi-neutral condition in equation (2.9) becomes [54]

$$p_l(x) = n_l(x) + N_B(x) \approx N_B(x) \quad (2.16)$$

The subscript 'l' stands for low injection value of the associated parameter hereafter. From equation (2.7b) & (2.16), electric field distribution inside the base of a SiGe HBT becomes

$$E_{iSiGe}(x) = V_T \left(\frac{1}{N_B(x)} \frac{dN_B(x)}{dx} - \frac{1}{n_{ieSiGe}^2(x)} \frac{dn_{ieSiGe}^2(x)}{dx} \right) \quad (2.17a)$$

Using equations (2.11), (2.15f) & (2.17a), field for low injection becomes constant

$$E_{iSiGe}(x) = -m_{023}V_T \quad (2.17b)$$

where, $m_{023} = m + m_{32}$.

From equations (2.10) & (2.16), minority carrier current density (J_{nl}) becomes

$$-J_{nl} = qD_{nSiGe}(x) \frac{n_{ieSiGe}^2(x)}{N_B(x)} \frac{d}{dx} \left[\frac{n_i(x)N_B(x)}{n_{ieSiGe}^2(x)} \right] \quad (2.18a)$$

Utilizing equations (2.11), (2.12b), (2.15f) and (2.18a) the following differential equation can be obtained

$$\frac{dn_i(x)}{dx} - m_{023}n_i(x) = -\frac{aJ_{nl}m_{023}}{bqv_{sA}} - \frac{J_{nl}}{qD_{nA}} e^{-m_1x} \quad (2.18b)$$

where, $D_{nA} = bD_n$.

The solution of the differential equation (2.18b) is (cf. APPENDIX A, section A.1)

$$n_i(x) = n_i(0)e^{m_{023}x} - \frac{J_{nl}}{qD_{nA}m_{0123}} (e^{m_{023}x} - e^{-m_1x}) - \frac{aJ_{nl}}{bqv_{sA}} (e^{m_{023}x} - 1) \quad (2.19)$$

where, $n_i(0)$ is the value of electron concentration at emitter-base (EB) junction ($x=0$), $m_{0123} = m_{012} + m_3$ and $m_{012} = \eta(1 + \gamma_1 - \gamma_2)/W_B$.

Assuming that the electron velocity reaching the base-collector (BC) depletion region ($x=W_B$) saturates at v_{sA} , the minority carrier current density becomes

$$J_{nl} = qv_{sA} n_i(W_B) \quad (2.20)$$

Using equations (2.19) and (2.20), electron concentration $n_i(W_B)$ at the BC junction can be written as

$$n_i(W_B) = \frac{n_i(0)e^{\eta_{023}}}{1 + \frac{a}{b}(e^{\eta_{023}} - 1) + \frac{v_{sA}}{D_{nA}} W_\eta} \quad (2.21)$$

where, $W_\eta = (\exp(\eta_{023}) - \exp(-\eta_1))/m_{0123}$, $\eta_{023} = m_{023}W_B$ and $\eta_1 = m_1W_B$.

Integrating equation (2.19) over the base width, base stored charge per unit area can be obtained as

$$Q_{Bnl} = q \int_0^{W_B} n_1(x) dx = J_{nl} \left[r_{l\eta} \frac{W_B^2}{D_{nA}} + r_{lab\eta} \frac{W_B}{v_{sA}} \right] \quad (2.22)$$

where

$$r_{l\eta} = \frac{1}{\eta_{0123}} \left(\frac{1 - e^{-\eta}}{\eta_1} + \frac{1 - e^{-\eta_{023}}}{\eta_{023}} \right) \quad (2.23)$$

$$r_{lab\eta} = \left(\frac{a}{b} - \frac{n_1(0)}{n_1(W_B)} \right) \frac{1 - e^{-\eta_{023}}}{\eta_{023}} + \frac{a}{b}$$

Using equations (1.4) and (2.22), the base transit time of a $\text{Si}_{1-y}\text{Ge}_y$ HBT with exponentially doped base and trapezoidal Ge profile becomes

$$\tau_{Bl} = r_{l\eta} \frac{W_B^2}{D_{nA}} + r_{lab\eta} \frac{W_B}{v_{sA}} \quad (2.24)$$

For uniformly doped base (symbolized by “u”), $\eta=0$ and for box shape Ge profile (symbolized by “square” (\square) as subscript), $\eta_{Ge}=0$. These values can be incorporating into equations (2.20)-(2.24) as follows

$$J_{nl\eta\square} = qv_{s\square} n_{l\eta\square}(W_B) \quad (2.20a)$$

$$n_{l\eta\square}(W_B) = 1 + v_{s\square} W_B / D_{n\square} \quad (2.21a)$$

$$Q_{Bnl\eta\square} = J_{nl\eta\square} \left[\frac{W_B^2}{2D_{n\square}} + \frac{W_B}{v_{s\square}} \right] \quad (2.22a)$$

$$r_{l\eta\square} = -1/2 \quad (2.23a)$$

$$r_{lab\eta\square} = n_{l\eta\square}(0) / n_{l\eta\square}(W_B)$$

$$\tau_{Bl\eta\square} = \frac{W_B^2}{2D_{n\square}} + \frac{W_B}{v_{s\square}} \quad (2.24a)$$

The $\text{Si}_{1-y}\text{Ge}_y$ base HBT becomes a Si BJT if the Ge mole fraction is zero ($y=0$). In this case, the base transit time for a uniformly doped base transistor can be found by putting $D_{n\square}=D_n$ and $v_{s\square}=v_s$ into equation (2.24a)

$$\tau_{Bm} = \frac{W_B^2}{2D_n} + \frac{W_B}{v_s} \quad (2.24b)$$

Base transit time in equation (2.24b) is exactly similar to the expression for τ_B derived in [65] for low level of injection and uniformly doped base transistor. This expression shows that transit time for uniformly doped base BJT is independent of $n_i(0)$ which is a function of base-emitter voltage V_{be} [55, 66]. For exponentially doped $\text{Si}_{1-y}\text{Ge}_y$ base HBT, it is evident from equation (2.23) that $r_{l\eta}$ is independent of $n_i(0)$. Utilizing equation (2.21), $r_{l\eta}$ also becomes independent of $n_i(0)$. As long as low level of injection condition prevails in the base, τ_{Bl} remains independent of base-emitter junction voltage V_{be} . From equations (2.23) and (2.24), it is clear that the base transit time depends upon the base width W_B , saturation velocity (v_{sA}), field dependent mobility (through 'a'), presence of Ge (through 'b'), Ge gradient (η_{Ge}), slope of base doping profile (η) and peak base doping concentration $N_B(0)$ (through ' D_n ').

2.2.2 Model for All Levels of Injection

The injected electron concentration $n(x)$ at high level of injection has been found not significantly changed from $n_i(x)$ for an exponentially-doped base BJT [54]. For the $\text{Si}_{1-y}\text{Ge}_y$ HBT, same thing is assumed and the hypothesis is validated by plotting modified minority carrier electron concentration profile $n_m(x)$ (see equation (2.26a)) for an exponentially doped base HBT and compared with numerically obtained $n(x)$ at high level of injection. The plot is given in Figure 2.3. Numerically obtained $n(x)$ is found not significantly different from $n_m(x)$.

For a given value of the base-emitter junction voltage V_{be} , the initial value of injected minority carrier concentration $n(0)$ is required for the calculation of final boundary value $n(W_B)$ and hence the minority current density J_n . The initial value $n(0)$ is obtained from the following relation [55, 66] with necessary modification for SiGe HBT

$$n(0) = \frac{n_{icSiGe}^2(0)}{N_B(0)} \exp\left(\frac{V_{bc}}{V_T}\right) f_w \quad (2.25a)$$

where,

$$f_w = \frac{1}{0.5 + \sqrt{0.25 + \frac{n_{icSiGe}^2(0)}{N_B^2(0)} \exp(V_{bc}/V_T)}} \quad (2.25b)$$

The modified injected electron concentration and current density, considering Webster effect [55] can be written as

$$n_m(x) = f_w n_i(x) \quad (2.26a)$$

$$J_{nm} = f_w J_{ni} \quad (2.26b)$$

Assume that the injected electron concentration $n(x)$ is perturbed by the modulated electric field from $n_m(x)$ only a little, which is [53]

$$n(x) = n_m(x) + \delta n(x) \quad (2.27a)$$

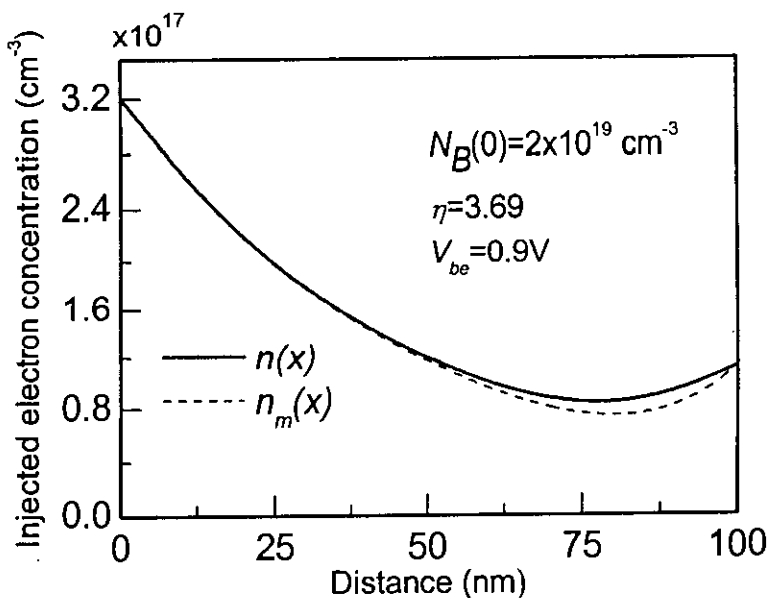


Figure 2.3: Comparison of injected minority carrier electron profiles within the base of a $\text{Si}_{1-y}\text{Ge}_y$ HBT modified from the low injection model ($n_m(x)$) and obtained from numerical solution under high injection ($n(x)$). Ge fractions at boundaries are $y_E=1\%$, $y_C=20\%$. Doping and field dependent mobility is considered.

Using equations (2.9) & (2.27a), the quasi-neutrality of charge equation becomes

$$p(x) = n_m(x) + \delta n(x) + N_B(x) = p_m(x) + \delta n(x) \quad (2.27b)$$

Equating the middle and right parts of equation (2.27b), the following equation can be obtained

$$p_m(x) = n_m(x) + N_B(x) \quad (2.27c)$$

From equations (2.10) & (2.27a-c), minority carrier current density (J_n) becomes

$$-J_n = qD_{nSiGe}(x) \frac{n_{ieSiGe}^2(x)}{p_m(x) + \delta n(x)} \frac{d}{dx} \left[\frac{\{n_m(x) + \delta n(x)\} \{p_m(x) + \delta n(x)\}}{n_{ieSiGe}^2(x)} \right] \quad (2.28a)$$

To ensure the accuracy of the derivation, the magnitude of $\delta n(x)$ must be much smaller than $p_m(x)$ instead of $n_m(x)$. Incorporating this assumption into equation (2.28a), the current density equation reduces to

$$-J_n = qD_{nSiGe}(x) \frac{n_{ieSiGe}^2(x)}{p_m(x)} \frac{d}{dx} \left(\frac{n(x)p_m(x)}{n_{ieSiGe}^2(x)} \right) \quad (2.28b)$$

Using equations (2.11), (2.12a-b), (2.15f), (2.19), (2.26a) & (2.27a-c), the differential equation (2.28b) can be solved for $n(x)$ as (cf. APPENDIX A, section A.2)

$$n(x) = \left(n(0)p_m(0) + \frac{J_n}{qv_{sA}} N(x) \right) \frac{e^{m_{32}x}}{p_m(x)} \quad (2.29)$$

where

$$N(x) = (a/b)N_F(x) + (v_{sA}W_B/D_{nA})N_\eta(x)$$

$$N_F(x) = N_{F1}(x) + N_{F2}(x) + N_{F3}(x) + N_{F4}(x)$$

$$N_\eta(x) = N_{\eta1}(x) + N_{\eta2}(x) + N_{\eta3}(x)$$

Components of $N_F(x)$ and $N_\eta(x)$ can be expressed as

$$N_{F1}(x) = n_{m0\eta a} (e^{nx} - 1)$$

$$N_{F2}(x) = N_B(0) (e^{-m_{023}x} - 1)$$

$$N_{F3}(x) = n_{ma} (e^{-m_{32}x} - 1)$$

$$N_{F4}(x) = n_{m\eta} (1 + \eta_{0123}/\eta_{321}) (e^{-m_{321}x} - 1)$$

$$N_{\eta1}(x) = -(n_{m0\eta a}/\eta_{01}) (e^{m_{01}x} - 1)$$

$$N_{\eta2}(x) = (N_B(0)/\eta_{0123}) (e^{-m_{0123}x} - 1)$$

$$N_{\eta3}(x) = (n_{m\eta}/\eta_{322}) (e^{-m_{322}x} - 1)$$

where, $n_{m0\eta a} = n_m(0) - n_{m\eta} - n_{ma}$, $n_{m\eta} = n_m(W_B)v_{sA}/(D_{nA}m_{0123})$, $n_{ma} = n_m(W_B)a/b$.

Applying velocity saturation at the base-collector depletion region, J_n can be written as

$$J_n = qv_{sA} n(W_B) \quad (2.30)$$

Substituting $x=W_B$ in equation (2.29) and using (2.30), $n(W_B)$ can be expressed as

$$n(W_B) = n(0)r_n \quad (2.31)$$

where

$$r_n = \frac{p_m(0)e^{\eta_{32}}}{p_m(W_B) - N(W_B)e^{\eta_{32}}} \quad (2.32)$$

The constants are given as

$$\begin{aligned} N(W_B) &= (a/b)N_F(W_B) + (v_{sA}W_B/D_{nA})N_\eta(W_B) \\ N_F(W_B) &= N_{F1}(W_B) + N_{F2}(W_B) + N_{F3}(W_B) + N_{F4}(W_B) \\ N_\eta(W_B) &= N_{\eta1}(W_B) + N_{\eta2}(W_B) + N_{\eta3}(W_B) \end{aligned}$$

where

$$\begin{aligned} N_{F1}(W_B) &= n_{m0\eta a} (e^\eta - 1) \\ N_{F2}(W_B) &= N_B(0) (e^{-\eta_{023}} - 1) \\ N_{F3}(W_B) &= n_{ma} (e^{-\eta_{32}} - 1) \\ N_{F4}(W_B) &= n_{m\eta} (1 + \eta_{0123}/\eta_{321}) (e^{-\eta_{321}} - 1) \\ N_{\eta1}(W_B) &= -(n_{m0\eta a}/\eta_{01}) (e^{\eta_{01}} - 1) \\ N_{\eta2}(W_B) &= (N_B(0)/\eta_{0123}) (e^{-\eta_{0123}} - 1) \\ N_{\eta3}(W_B) &= (n_{m\eta}/\eta_{322}) (e^{-\eta_{322}} - 1) \end{aligned}$$

Substituting the value of $n(W_B)$ in equation (2.30), the expression of J_n becomes

$$J_n = qv_{sA}n(0)r_n \quad (2.33)$$

The base stored charge per unit area for all levels of injection is

$$Q_{Bn} = q \int_0^{W_n} n(x) dx \quad (2.34)$$

The integration of the above base stored charge expression in equation (2.34) is not analytically tractable. So, numerical methods have been used to find the base stored charge per unit area.

Now, using equations (2.33) and (2.34) the base transit time of a $\text{Si}_{1-y}\text{Ge}_y$ HBT with exponentially (or uniformly) doped base and trapezoidal (or box/linear) Ge profile can be obtained for all levels of injection as

$$\tau_B = \frac{Q_{Bn}}{J_n} \quad (2.35)$$

2.3 CONCLUSION

In this chapter mathematical expressions for injected electron concentration profile $n(x)$, collector current density (J_n), stored base charge per unit area (Q_{Bn}) and hence the base transit time (τ_B) for an exponentially doped $\text{Si}_{1-y}\text{Ge}_y$ n^+p - n HBT are obtained considering all the effects especially dependence of mobility on electric field. The derivation of the model equation starts by first considering low level of minority carrier injection within the base. Then the minority carrier profile of low level of injection is extended for finding minority carrier profile for high level of injection using perturbation theory and Webster effect. The expression of base transit time thus obtained is applicable for all levels of injection. In the next chapter results obtained from the derived equations are plotted and the transit time is compared with that of numerical results available in the literature.

CHAPTER 3

RESULT AND DISCUSSION

3.1 INTRODUCTION

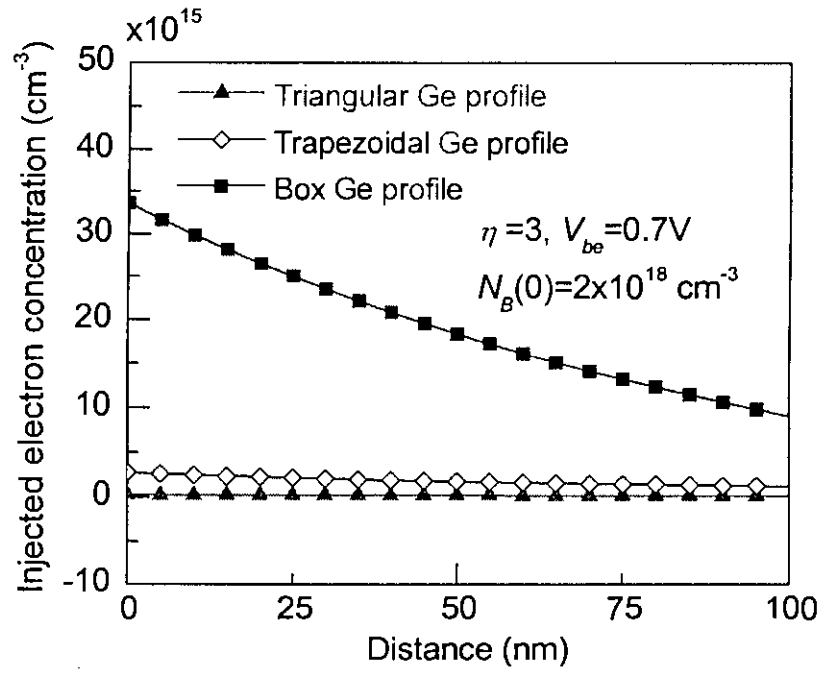
In the previous chapter the mathematical equations related to the analytical modeling of base transit time of a non-uniformly doped $\text{Si}_{1-y}\text{Ge}_y$ base heterojunction bipolar transistor (HBT) have been derived for all levels of injection. Meanwhile necessary computer program has been developed based on the derived equations. Numerical data generated by the program are plotted in this chapter to study the effects of various parameters on the base transit time. The variation of base transit time with different transistor parameters is discussed.

3.2 RESULT AND DISCUSSIONS

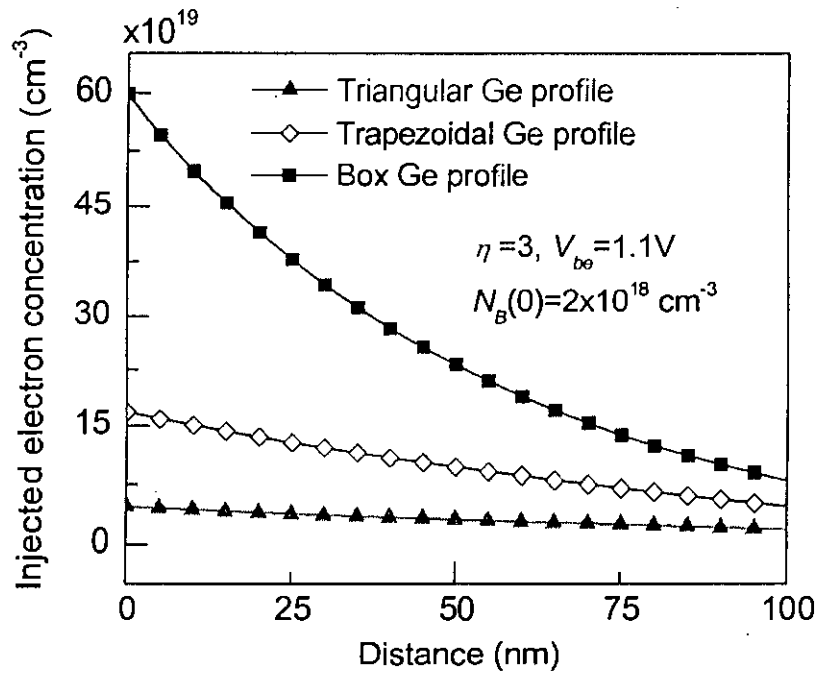
The results obtained by using the derivations of the previous chapter are analyzed and discussed below.

3.2.1 Distribution of Minority Carrier within the Base

Distribution of injected electron concentration $n(x)$ within the base for three different Ge profiles at a given base doping slope (η) & peak base doping ($N_B(0)$) is shown in Figure 3.1 for (a) low and (b) high injection. From Figure 3.1(a)-(b), it is observed that $n(x)$ for triangular Ge profile is lower than that of trapezoidal Ge profile and for box profile, $n(x)$ remains above all other distributions. This is because the transport of electron through the base is assisted by the gradient in Ge profiles [11]. The gradient is the highest for triangular profile, zero for box profile and in-between for trapezoidal profile (referred to Figure 2.1). So for the box Ge profile, electron passes slower through the base to enter the collector and distribution of $n(x)$ remains higher.

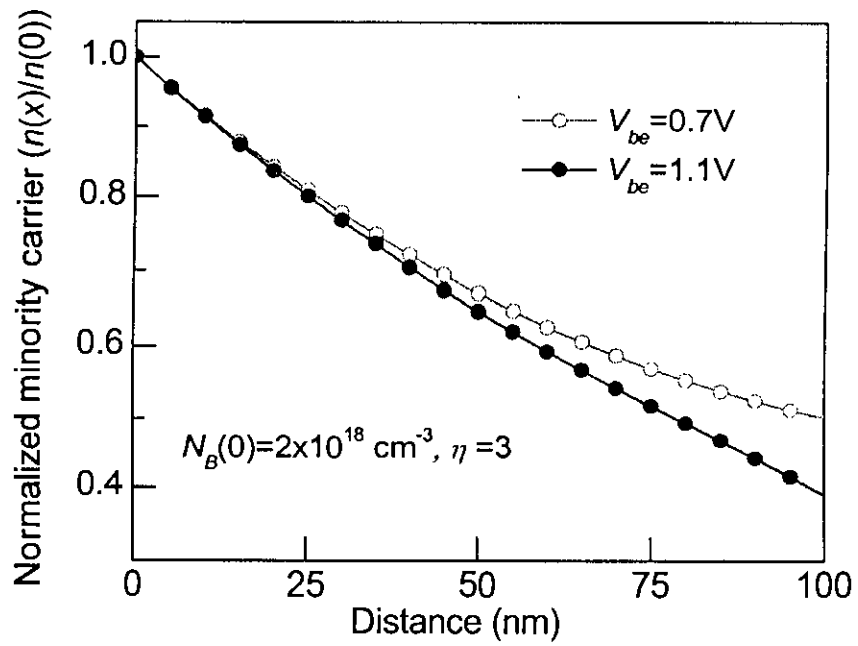


(a)

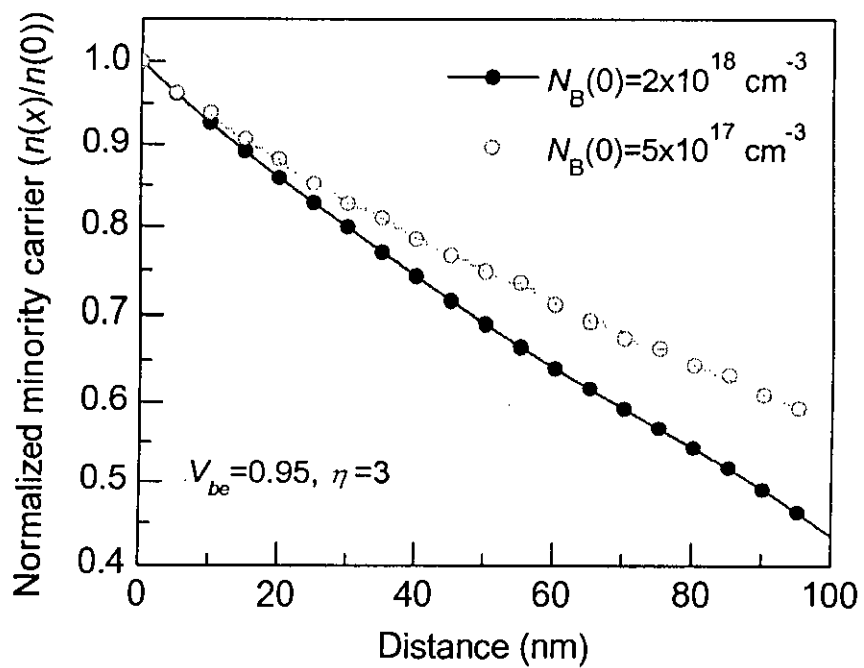


(b)

Figure 3.1: Injected minority electron distribution within the base for different Ge profiles (a) under low injection ($V_{be} = 0.7\text{V}$) (b) under high injection ($V_{be} = 1.1\text{V}$). Ge fractions at boundaries are: for triangular profile $y_E = 0\%$; $y_C = 20\%$, for trapezoidal profile $y_E = 10\%$; $y_C = 20\%$ and for box profile $y_E = y_C = 20\%$.

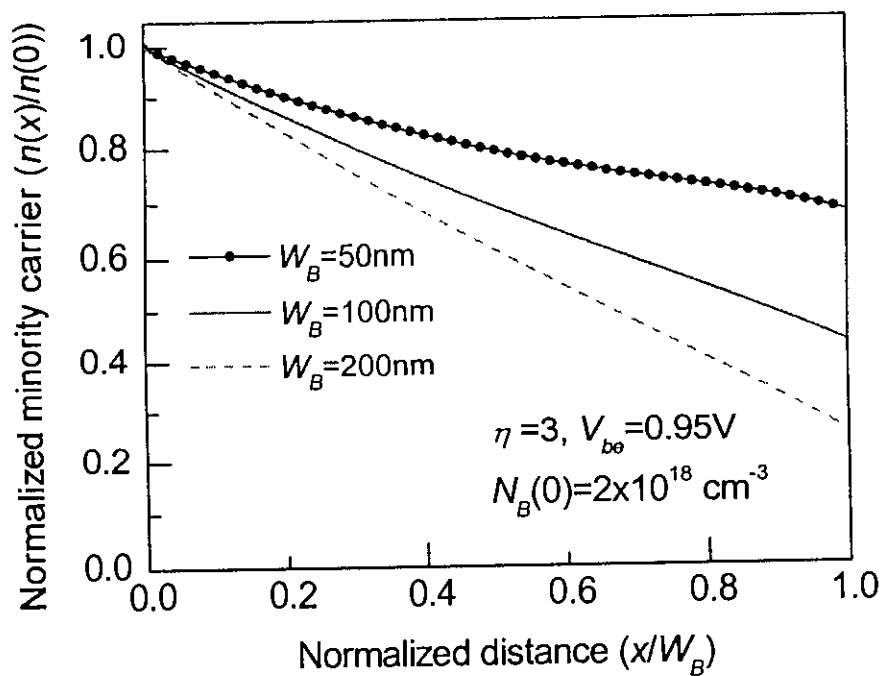


(a)

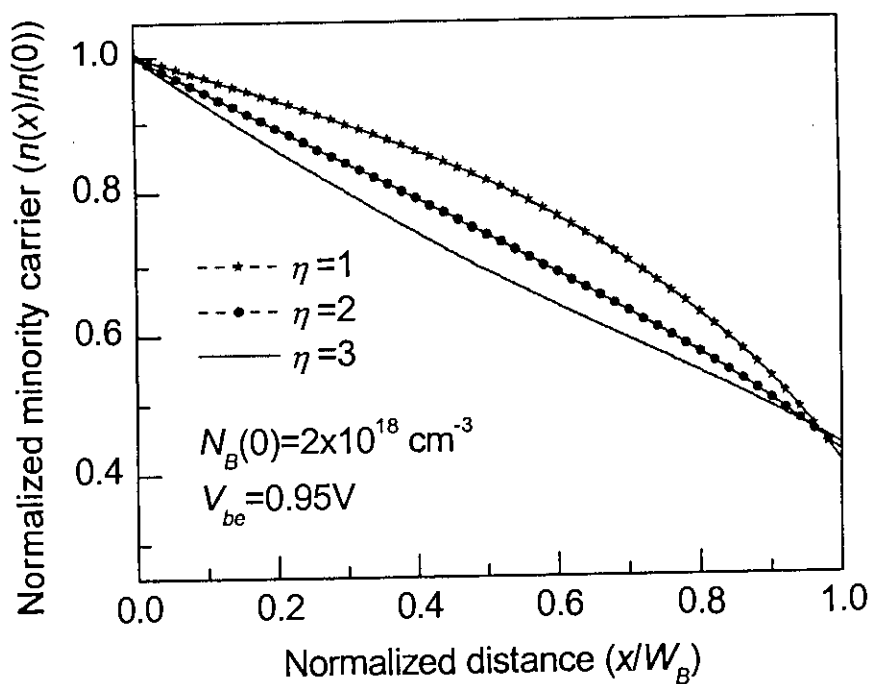


(b)

Figure 3.2: Normalized minority carrier electron distribution within the base (a) for different V_{be} (b) for different $N_B(0)$. Ge fractions at the boundaries are $y_E=1\%$ & $y_C=20\%$.



(a)



(b)

Figure 3.3: Normalized minority carrier distribution within the base (a) for different base widths (b) for different slopes of base doping. Ge mole fractions on the base boundaries are $y_b=1\%$ and $y_c=20\%$.

Normalized injected electron distribution ($n(x)/n(0)$) for different base-emitter voltage (V_{be}) (low and high injections) at a given $N_B(0)$ and η is shown in Figure 3.2(a). It is observed from Figure 3.2(a) that with an increase in V_{be} more electrons are injected from emitter to base. This plot is in good agreement with [67]. In Figure 3.2(b) distribution of ($n(x)/n(0)$) within the base for different $N_B(0)$ at a given V_{be} and η are plotted. This figure shows that when $N_B(0)$ increases, more carriers are injected into the base from the emitter which agrees with [68].

The dependence of normalized injected electron distribution on base width (W_B) is shown in Figure 3.3(a). From this plot it is evident that as W_B decreases, ($n(x)/n(0)$) increases. This happens because for shorter base, injected minority carriers get relatively small time to be diffused compared to longer base. So $n(W_B)/n(0)$ for longer base is lower than that of a shorter base. Normalized injected electron distribution also depends on the slope of base doping as appears in Figure 3.3(b). With the increase in the slope of the base doping, $n(x)/n(0)$ decreases more sharply.

The injected electron concentration profiles shown in Figure 3.1(a)-(b), Figure 3.2(a)-(b) and Figure 3.3(a)-(b) are not linear due to the built-in electric field caused by the exponential base doping and the gradient in Ge profile. It is also found that the minority carrier has maximum value at the base-emitter junction and minimum value at the collector-base junction. It decreases from its peak value as the distance from the emitter increases. In this work, velocity saturation of electron at the collector-base junction is considered. So at collector-base junction the minority carrier concentrations for low and high levels of injection are not zero.

3.2.2 Variation of Minority Carrier Injection Ratio with Base-Emitter Voltage

Variation of minority carrier injection ratio ($n(0)/N_B(0)$) with V_{be} at a given $N_B(0)$, η and W_B is shown in Figure 3.4. The figure shows that ($n(0)/N_B(0)$) is an increasing function of V_{be} . From equation (2.25a), it is found that $n(0)$ is exponentially proportional to V_{be} . So, for a given $N_B(0)$, minority carrier injection ratio will also increase exponentially with V_{be} , but under high injection the exponential slope will

decrease due to Kirk effect at a certain high current [48]. For this reason, the low injection part of Figure 3.4 (which is a logarithmic plot) is linear and at the beginning of high injection, its slope starts to fall.

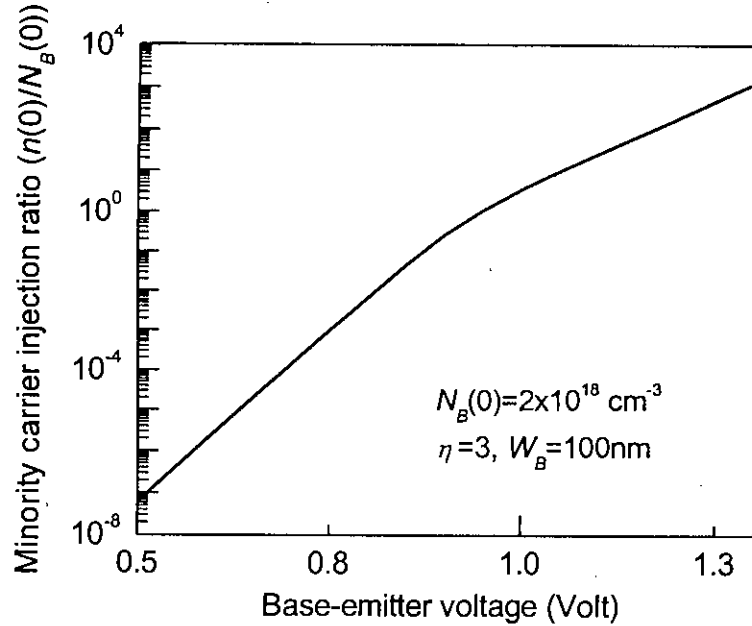
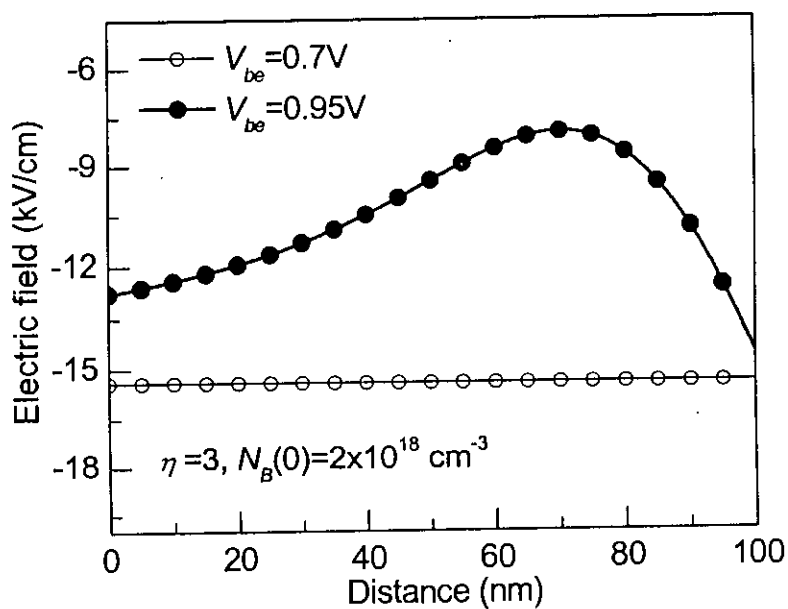


Figure 3.4: Minority carrier injection ratio as a function of base emitter voltage. Ge mole fractions at the emitter and collector side of the base are 1% and 20% respectively.

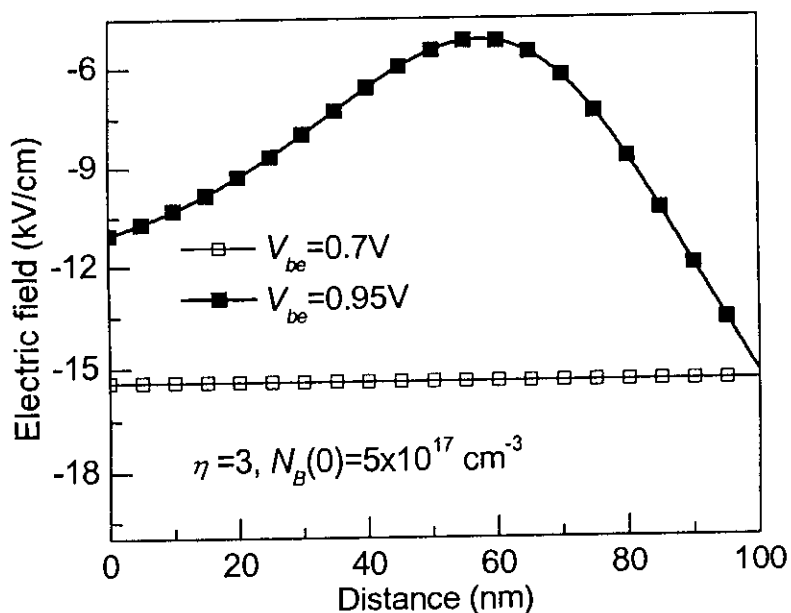
3.2.3 Electric Field Distribution within the Base

Distribution of electric field $E(x)$ within the base for different values of V_{be} and $N_B(0)$ at a given η are shown in Figure 3.5. The curves show that $E(x)$ within the base is negative, i.e., aiding field and constant when base-emitter voltage is low (low level of injection). On the other hand, for higher V_{be} , the field also becomes negative but in magnitude it remains less than the field for low V_{be} . First its magnitude decreases slowly then increases rapidly near the base-collector junction. Comparing two filled (circle & square) lines from Figure 3.5(a) & (b), the influence of $N_B(0)$ on $E(x)$ under high injection can be realized. For heavy doping (Figure 3.5(a)), electric field over the entire base is stronger compared to the field in lightly doped base (Figure 3.5(b)). Under low injection, $N_B(0)$ has no influence on the electric field (hollow circle &

square lines of Figure 3.5(a) & (b)); this is expected as equation (2.17b) is independent of peak base doping concentration.



(a)



(b)

Figure 3.5: Electric field distribution within the base under low (hollow lines) and high (filled lines) injection for (a) heavily doped base (b) lightly doped base. Ge fractions on the base boundaries are $y_b=1\%$ and $y_c=20\%$. Doping dependent mobility is considered.

Distribution of electric field $E(x)$ within the base for different slopes of base doping (η) under low and high injection for a given peak base doping concentration ($N_B(0)$) is illustrated in the Figure 3.5(c). From the figure it is evident that as the slope of the base doping increases, for low injection the aiding electric field also increases. This is because the electric field in equation (2.17b) is proportional to the slope of the base doping under low injection. For high injection, at a higher slope of the base doping, magnitude of the electric field gradually decreases then increases rapidly near the base-collector junction but remains weaker than the electric field for a lower slope. It is also observed from the Figure 3.5(c) that the electric field near the emitter-base and collector-base junctions shows no significant change with slope of the base doping under high injection.

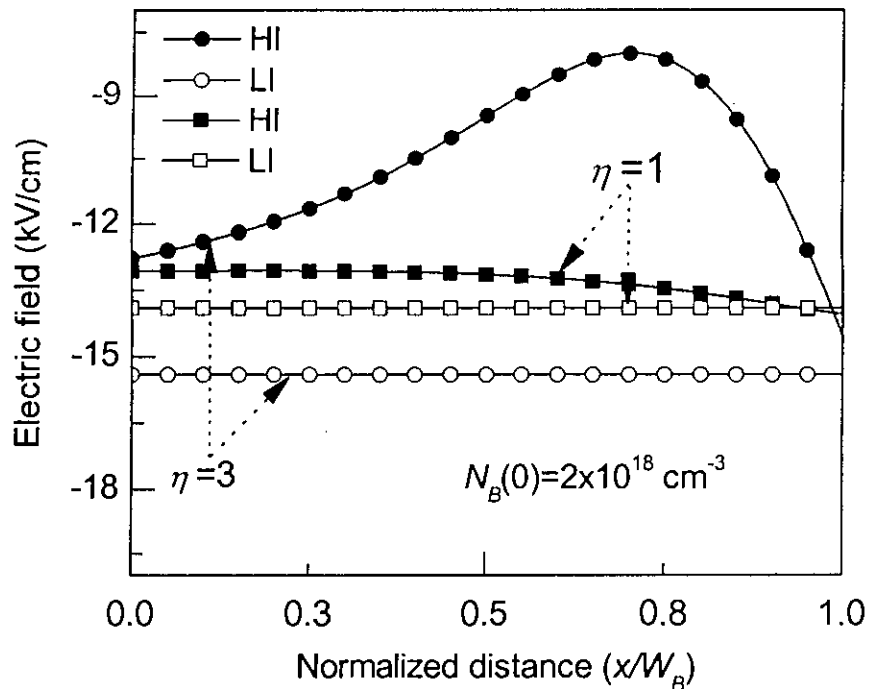


Figure 3.5(c): Electric field distribution within the base for different slopes of base doping profile under low and high injection. Filled (circle & square) lines represent high injection (HI, $V_{be}=0.95\text{V}$) and hollow (circle & square) lines represent low injection (LI, $V_{be}=0.7\text{V}$). Ge fractions at emitter and collector side of the base are 1% and 20% respectively. Doping dependent mobility is considered.

3.2.4 Variation of Collector Current Density with Base-Emitter Voltage

The variation of collector current density (J_c) with base-emitter voltage (V_{be}) for different shape of Ge profiles is shown in Figure 3.6. For low injection, J_c increases with V_{be} maintaining a constant logarithmic slope; with the increase in V_{be} , the slope of the curves decreases. This is because J_c (or J_n) is linearly proportional to $n(0)$ as given in equation (2.33). So the plot J_c as a function of V_{be} will follow the plot of minority carrier injection ratio ($n(0)/N_B(0)$) with the variation of V_{be} as shown in Figure 3.4. J_c for box shape Ge profile is higher than that of triangular profile and J_c for trapezoidal profile remains in-between. This is expected as the injected electron concentration at the collector-base boundary ($n(W_B)$) is the highest for box shape Ge profile as shown in Figure 3.1 and higher $n(W_B)$ contributes higher J_c as given in equation (2.30). This finding agrees well with [50].

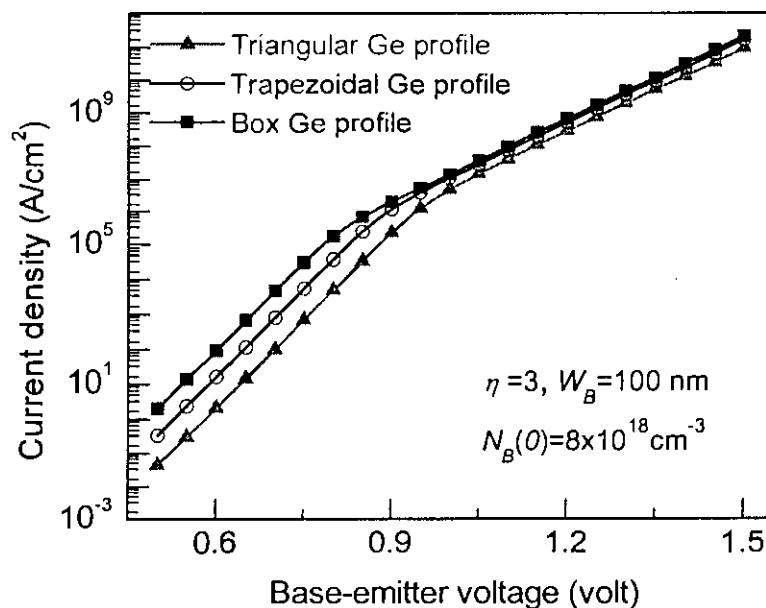


Figure 3.6: Collector current densities for different shape of Ge profiles as a function of base-emitter voltage. Ge mole fractions at boundaries are: for triangular profile $y_E = 0\%$; $y_C = 20\%$, for trapezoidal profile $y_E = 10\%$; $y_C = 20\%$ and for box profile $y_E = y_C = 20\%$.

Figure 3.6 also shows that J_c for the box shape Ge profile declines earlier; later declines that for the trapezoidal profile and J_c for the triangular profile is the last to decline but all the three curves bend at the same current density. This is due the manifestation of Kirk effect at certain high current density irrespective of Ge profiles and is in good agreement with [48].

3.2.5 Variation of Base Transit Time with Base-Emitter Voltage

The variations of base transit time (τ_B) with base-emitter voltage (V_{be}) for various Ge profiles are shown in Figure 3.7. It is observed from Figure 3.7 that the base transit time is independent of base-emitter voltage for both low and strong high levels of injection but the value of base transit time is larger for high injection region than that of low injection region. This is due to the reduction of aiding field in the exponentially doped base when the level of injection increases as shown in Figure 3.5-6. Due to this reduction in aiding field, electrons slow down and hence transit time increases. In the intermediate level of injection, base transit time increases as V_{be} increases.

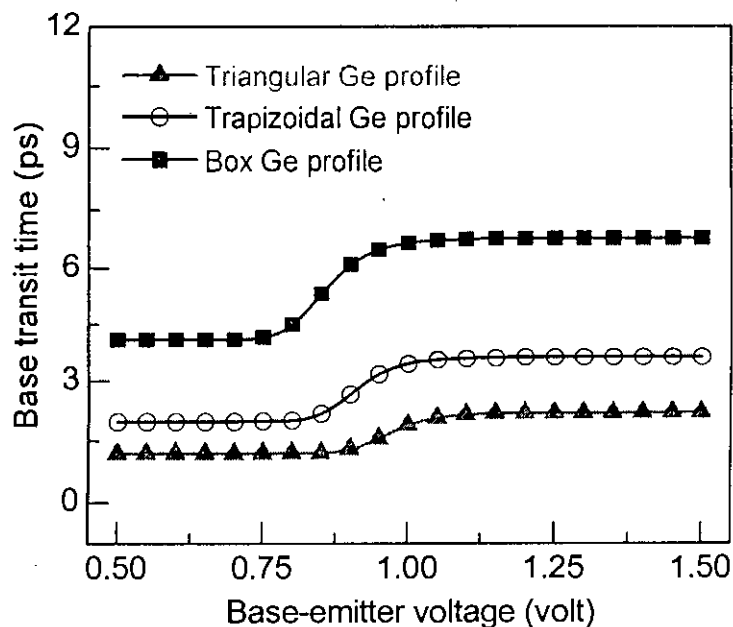


Figure 3.7: Base transit time as a function of base-emitter voltage for different shape of Ge profiles. Ge mole fractions at boundaries are: for triangular profile $y_E = 0\%$; $y_C = 20\%$, for trapezoidal profile $y_E = 10\%$; $y_C = 20\%$ and for box profile $y_E = y_C = 20\%$. Base width and doping profile remain same as Figure 3.6.

From the Figure 3.7, it also can be observed that box shape Ge profile takes the highest time to pass the electron to the collector, triangular Ge profile takes the lowest time and the trapezoidal Ge profile takes in-between time. This is related to the gradient of Ge profiles as discussed in article 3.2.1. This result is also validated by [50] which shows that transit through the base is a minimum when the triangular Ge profile is chosen.

3.2.6 Variation of Base Transit Time with Minority Carrier Injection Ratio

As discussed earlier in article 3.2.2, minority carrier injection ratio is an increasing function of base-emitter voltage. So, the variation of the base transit time with the minority carrier injection ratio has the same pattern as the base transit time varies with base-emitter voltage (Figure 3.7). The variation of base transit time with the minority carrier injection ratio for box shape Ge profile is revealed in Figure 3.8. The increase in the base-emitter voltage reduces the aiding field in the base as shown in Figure 3.5(a)-(b) & Figure 3.5(c).

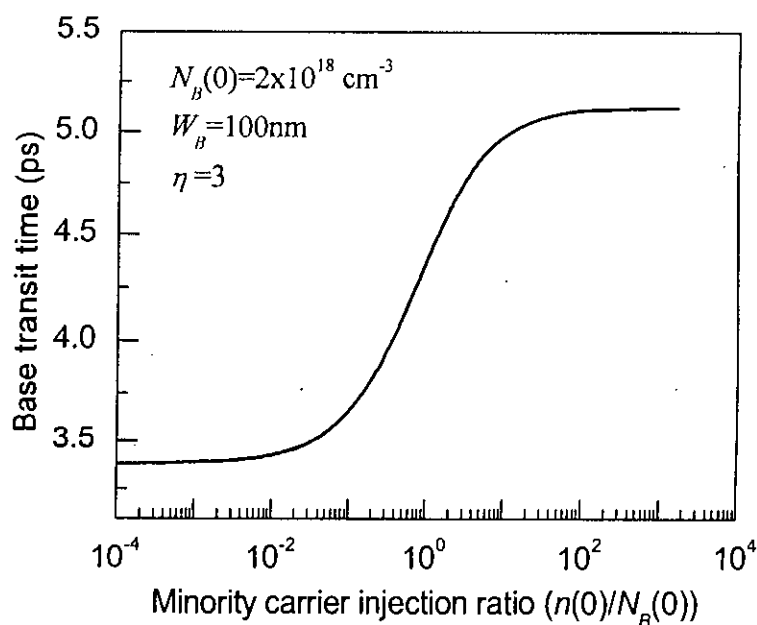
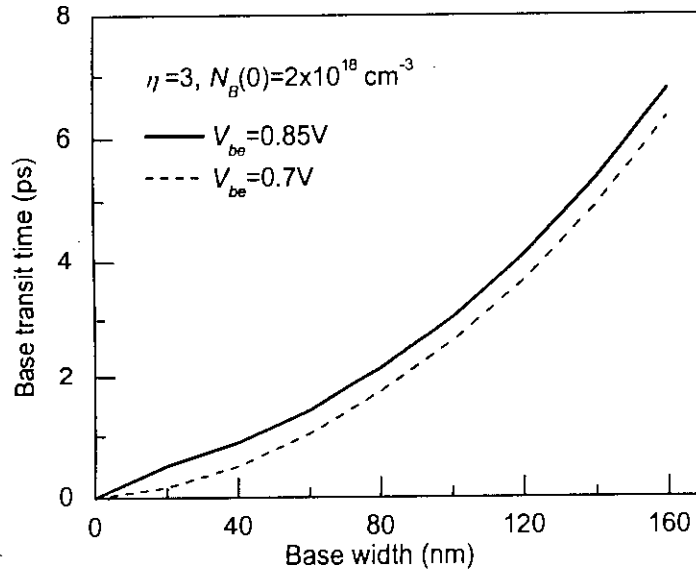


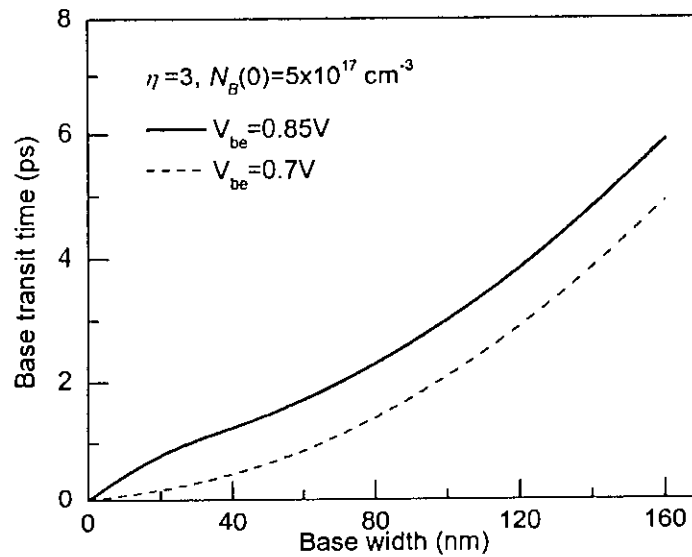
Figure 3.8: Base transit time as a function of minority carrier injection ratio for box shape Ge profile. Doping dependent mobility is considered.

So, with the increase of minority carrier injection ratio, the aiding field reduces, electron slows down and the base transit time increases.

3.2.7 Dependence of Base Transit Time upon base Width



(a)



(b)

Figure 3.9: Base transit time as function of base width (a) for heavily doped base (b) for a comparatively lightly doped base HBT. Box shape Ge profile is considered here.

The dependence of base transit time (τ_B) upon base width (W_B) for different $N_B(0)$ at a given base profile is shown in Figure 3.9(a)-(b). From these figures it is clear that τ_B is an increasing function of W_B as was found in the mathematical analysis. With the increase in W_B , the stored base charge increases and $n(W_B)$ decreases (shown in Figure 3.3(a)); this results in a reduction in electron current density as defined by equation (2.30), eventually the base transit time experiences two fold increase as per equation (2.35). For constant W_B , η and $N_B(0)$, τ_B is smaller for low injection (dashed line) than that of high injection (solid line).

3.2.8 Dependence of Base Transit Time on Peak Base Doping Concentration

The dependence of base transit time upon peak base doping concentrations is shown in Figure 3.10. From the figure it is found that the base transit time increases with peak base concentration. Equation (2.12a) shows that $D_n(\mu_n)$ decreases with $N_B(0)$. For low level of injection $E(x)$ is given by equation (2.17b). It shows that $E(x)$ is independent of $N_B(0)$. Due to the decrease of $D_n(\mu_n)$ the stored base charge increases resulting in an increase of τ_B . For high level of injection $E(x)$ reduces in amplitude. The decrease in both the aiding electric field and the mobility contribute to the increase of τ_B .

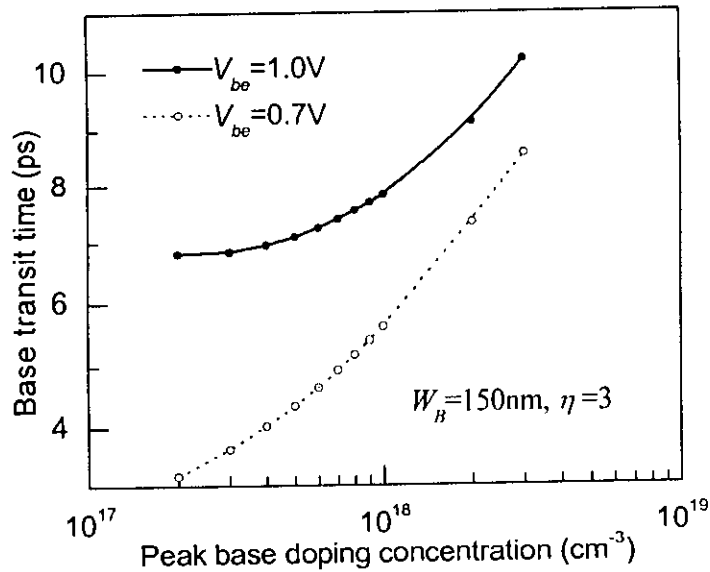


Figure 3.10: Base transit time as a function of peak base doping concentration with a box shape Ge profile for high and low level of injection.

3.2.9 Dependence of Base Transit Time upon Field Dependent Mobility

One of the main objectives of this research work is to obtain base transit time of an exponentially doped base $\text{Si}_{1-y}\text{Ge}_y$ HBT considering electric field dependence of carrier mobility in excess to doping dependent mobility. The effect of electric field dependence of mobility on base transit time was investigated and the result is shown in Figure 3.11. The transit time is found to be higher if the field dependence of mobility is considered. This is because minority carrier mobility is inversely related to the absolute value of electric field equation (2.14a). With the increase in electric field, minority carrier mobility in the base region decreases. So, with the increase in electric field, it decreases the minority carrier transportation through the base region and hence contributes to the increase in base transit time.

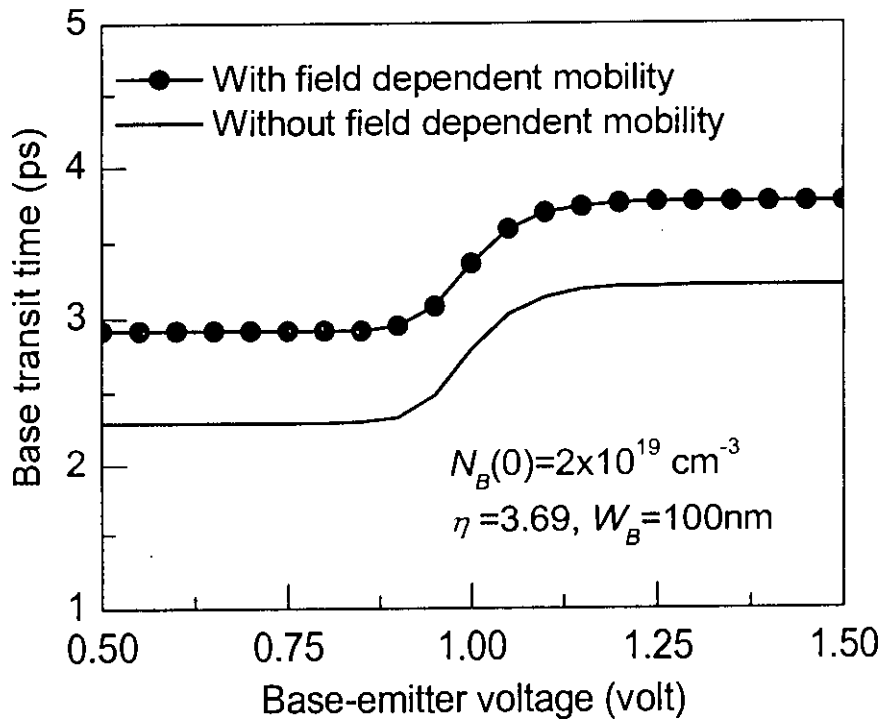


Figure 3.11: Base transit time as a function of base-emitter voltage with and without field dependent electron mobility. Ge mole fractions at boundaries are $y_L=1\%$ and $y_C=20\%$.

3.2.10 Dependence of Base Transit Time upon Slope of Base Doping

Effect of change in the slope of base doping (η) on base transit time (τ_B) is also studied and the results are shown in Figure 3.12. From the Figure 3.12, it is found that the base transit time is a decreasing function of the slope of base doping for both low (dashed line) and high (solid line) level of injection. This is because with the increase in η , the injected electron concentration profile becomes steeper (Figure 3.3(b)) which eventually reduces the stored base charges (area under $n(x)$ profile) and the base transit time decreases. Figure 3.12 also shows that for high level of injection, τ_B is lower than that of low level of injection up to a certain value of η . After that pivotal value of η (>2.5), for high level of injection, τ_B is higher than that of low level of injection. This is because the aiding electric field which reduces τ_B , increases with η in low injection (Figure 3.5(c)), but for high injection the field increases only near the collector-base junction, elsewhere it reduces with increasing η . So, the base transit time increases with increasing η for high injection. This is found in good agreement with [67-69].

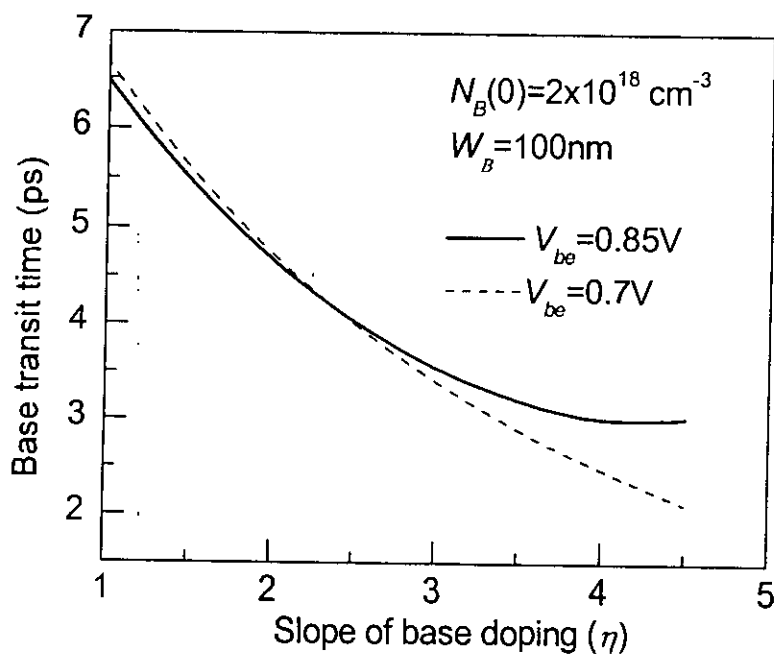


Figure 3.12: Base transit time as a function of slope of base doping (η) with a box shape Ge profile for high and low level of injection.

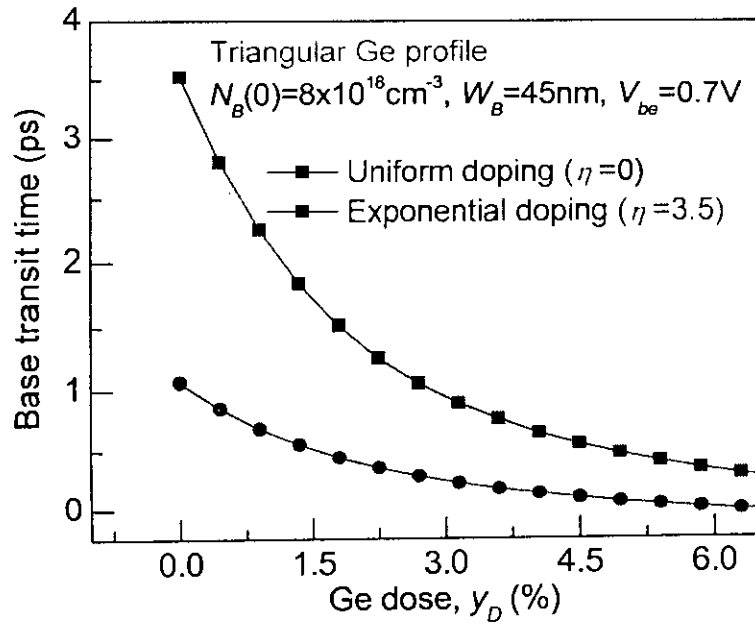
3.2.11 Dependence of Base Transit Time upon Ge Dose (or Average Ge %)

The influence of total Ge content (dose) on τ_B for uniform and exponential doping has been investigated and the results are illustrated in Figure 3.13(a). From the figure, it is found that τ_B for the exponentially doped base is significantly smaller than τ_B for uniformly doped base. For higher dose (here, dose ≥ 4 or an equivalent average Ge content $\geq 10\%$), the difference in the base transit times for the two base doping profiles is insignificant which found to be in good agreement with [39].

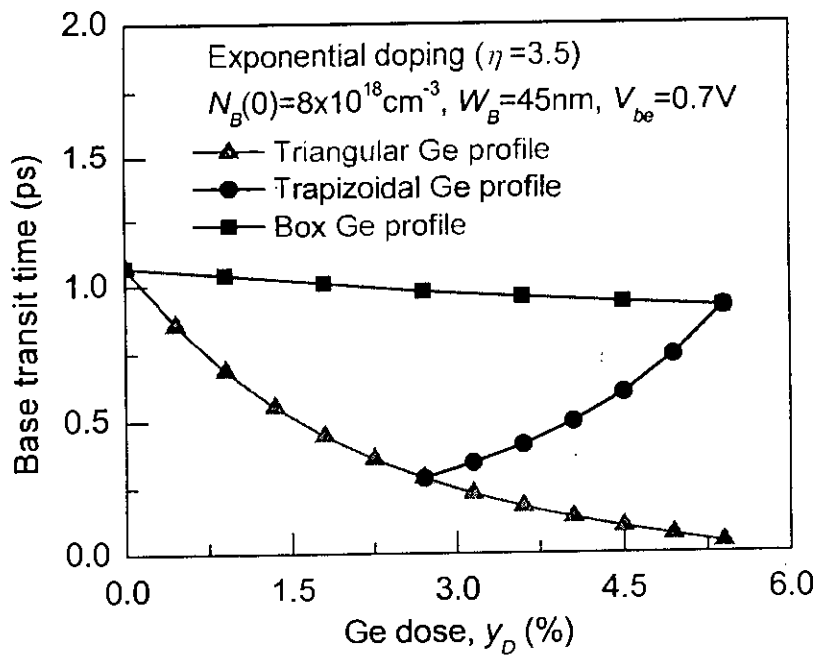
In Figure 3.13(b), the variation of τ_B with Ge dose for different types of Ge profile is given. From Figure 3.13(b), it is realizable that for triangular profile and box profile, high values of Ge dose reduce τ_B but for trapezoidal Ge profile τ_B increases with dose. This is because increased Ge dose for triangular profile keeping $y_E=0$ and gradual increase of y_C , increases the amount of grading η_{Ge} . For box profile, gradual increase in $y_C=y_E$, would not change the amount of grading η_{Ge} ; it remains zero and for the trapezoidal Ge profile, the dose is increased by keeping y_C constant and gradually increasing y_E . This eventually decreases the amount of grading η_{Ge} . Thus the Ge dose for trapezoidal profile can't be zero and τ_B for this profile starts from a non-zero dose point in Figure 3.13(b).

This represents the fact that bandgap grading gives rise to a drift field which aids minority carrier transport through the base with a consequent reduction in τ_B . But this happens effectively only when amount of grading increases (not just the dose). With zero grading, a mere increase in dose would not effectively reduce τ_B unless $N_B(0)$ is light ($\times 10^{17} \text{ cm}^{-3}$) and for heavy doping ($\times 10^{20} \text{ cm}^{-3}$), τ_B will not reduce at all with increasing dose [70].

Figure 3.14 shows the comparison of base transit time obtained by utilizing the present model (solid & dash dot lines) with the base transit time available in [39] (hollow square and circle). Here, 'uni dop' represents uniform doping and 'expo dop' represents exponential base doping ($\eta=3$). Low level of injection ($V_{be}=0.7\text{V}$) and doping dependent mobility is considered. Clearly the result is found to be in good agreement.



(a)



(b)

Figure 3.13: Base transit time as a function of Ge dose for (a) uniform and exponential type of base doping (b) different type Ge profiles with exponentially doped base. Doping dependent mobility is considered.

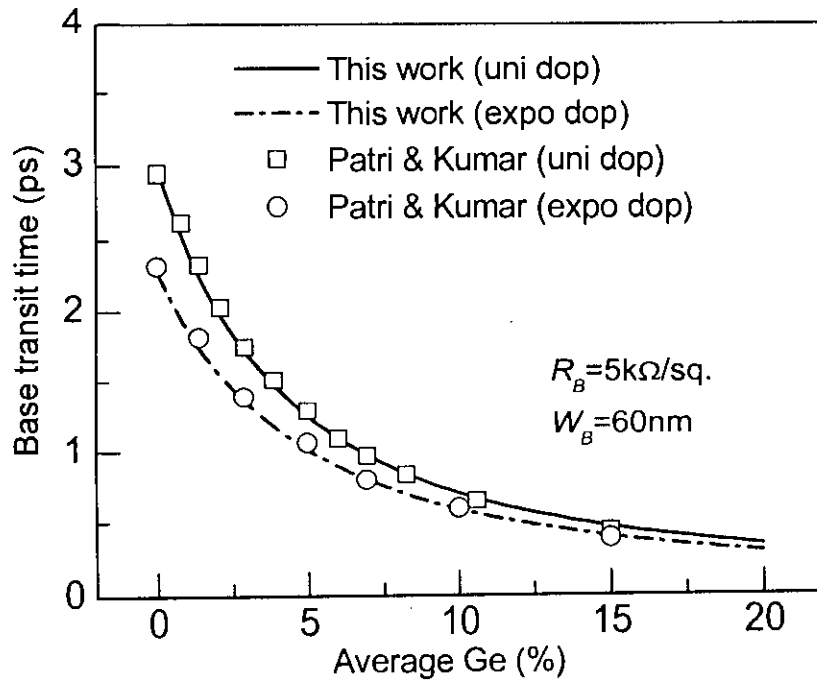


Figure 3.14: Base transit time as a function of average Ge % for triangular Ge profile. The hollow circle & square represent τ_B for [39] and the lines represent the present model.

3.2.12 Dependence of Base Transit Time upon Peak Ge Fraction

The base transit time of a $\text{Si}_{1-y}\text{Ge}_y$ base heterojunction bipolar transistor also depends on peak Ge fraction (y_C). The variation of base transit time with y_C is shown in Figure 3.15. It is observed from the figure that both for uniform ($\eta=0$) and exponential ($\eta=3.5$) base doping with triangular Ge profile in the base, the transit time decreases as the values of the peak Ge fraction increases. This happens because increasing y_C (keeping $y_E=0$), increases both the amount of Ge grading (η_{Ge}) and the Ge dose (y_D). This gives rise to a drift field which consequently decreases the base transit time of the HBT.

The argument stated above is validated by comparing the base transit time obtained in the present work (represented by solid & dash dot lines) with the base transit time availed from [71] (represented by hollow square and circle in Figure

3.15) and the result is found to be in good agreement. In this comparison, doping dependent mobility is considered.

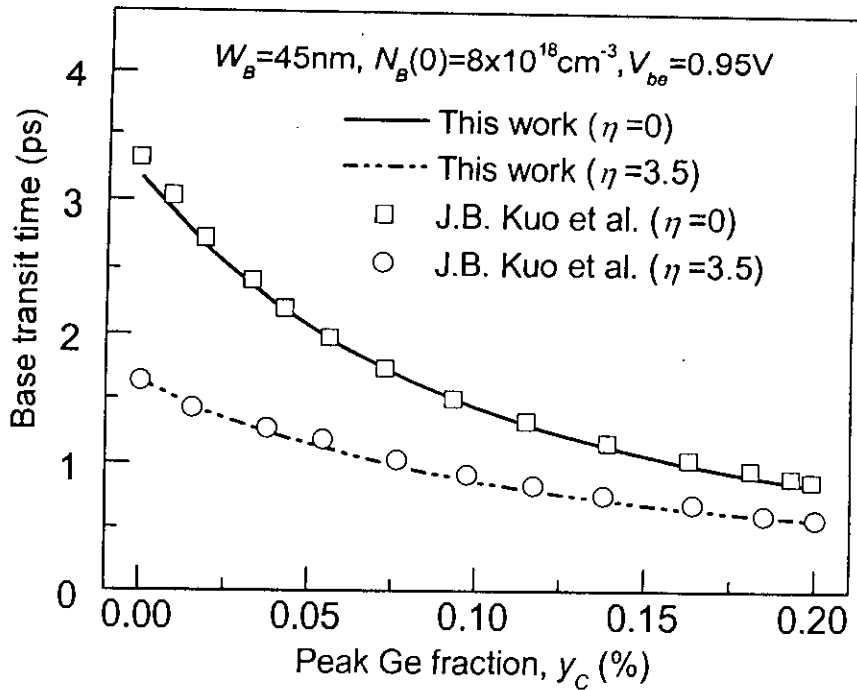


Figure 3.15: Base transit time as a function of peak Ge % (y_c) for triangular Ge profile. The hollow circle & square represent τ_B for [71] and the lines represent the present model.

3.2.13 Dependence of Base Transit Time upon Collector Current Density

The variation of base transit time (τ_B) with collector current density for different Ge profiles is shown in Figure 3.16(a). From the Figure 3.16(a), it is understandable that base transit time is independent of collector current density for both low and strong high level of injection. But the value of base transit time is larger for high injection region than the base transit time for low injection region. This is due to the reduction of aiding electric field in the exponentially doped base when the level of injection increases (Figure 3.5-6). Due to this effect, electrons slow down and transit time increases. In the intermediate level of injection, base transit time increases as J_c increases. From the Figure 3.16(a), it is also perceptible that box shape Ge profile takes higher transit time than triangular Ge profile to pass the electron to the

collector and the trapezoidal Ge profile takes in-between time. This happens due to the difference in gradients of Gc profiles (details in article 3.2.1). It is noticeable that Figure 3.16(a) follows Figure 3.7 (τ_B vs. V_{be}) as collector current is a logarithmic function of V_{be} (Figure 3.6).

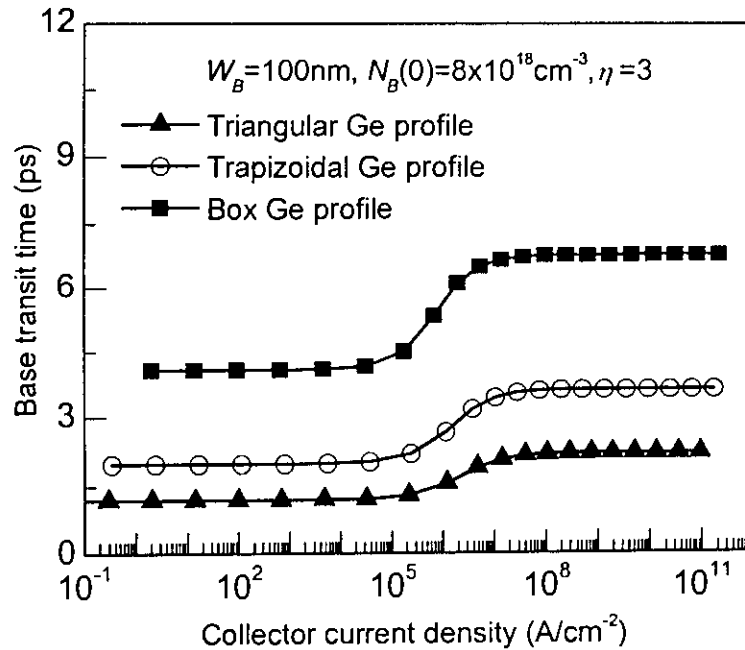


Figure 3.16(a): Base transit time as a function of collector current density for different shape of Ge profiles. Ge mole fractions at boundaries are: for triangular profile $y_E=0\%$; $y_C=20\%$, for trapezoidal profile $y_E=10\%$; $y_C=20\%$ and for box profile $y_E=y_C=20\%$.

The variation of base transit time with collector current density as obtained using the present model is compared with the base transit time excavated from [48] and plotted in Figure 3.16(b). In the figure the hollow circle & square represent the base transit time for [48] and the lines (solid & dash-dot) represent the base transit time availed from this work and they are in good agreement. Unlike Figure 3.14-15, the base transit time in Figure 3.16(b) for uniform doping ($\eta=0$, $W_B=29\text{nm}$ and $y_C=17.24\%$) is smaller than that of exponentially doped ($\eta=4$, $W_B=83\text{nm}$ and $y_C=6.02\%$) base. The reason behind this is the base width of uniform doping has been reduced in comparison with exponential doping to keep the intrinsic base resistance constant to $R_B=5\text{k}\Omega/\square$. With this change in base width, the peak Ge percentage (y_C) has also

been changed to keep the dose fixed (this is a requirement for the stability of SiGe strained layer [72]). Both the changes act to reduce the base transit time for uniform doping, so it becomes smaller compared to the base transit time for exponential doping.

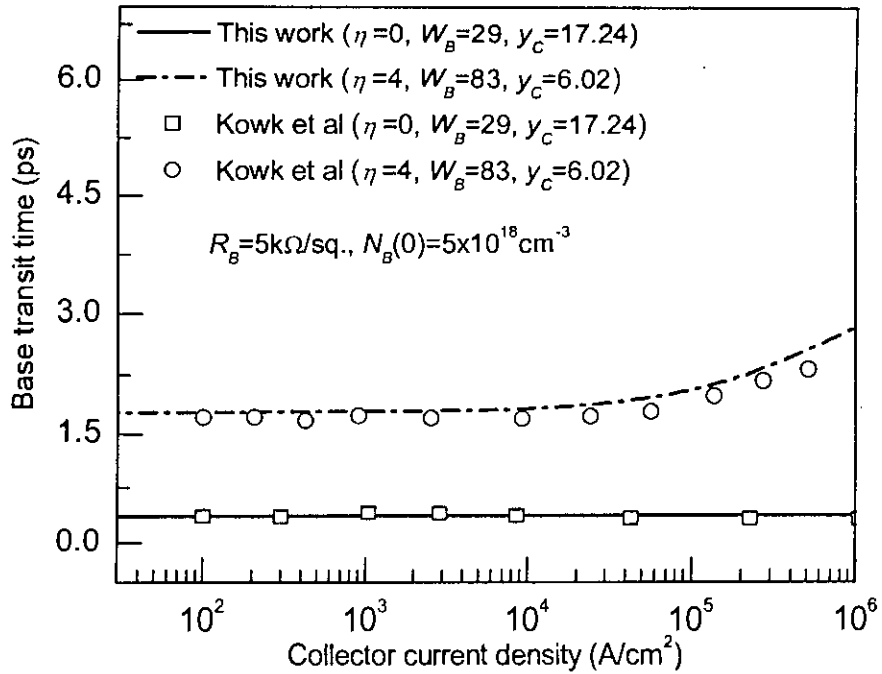


Figure 3.16(b): Base transit time as a function of collector current density with triangular Ge profile. The hollow circle & square represent τ_B for [48] and the lines represent the present model. Here W_B is in nm and y_C is in %.

3.3 CONCLUSION

The analytical expression obtained in chapter two is used to determine the dependence of base transit time on different parameters of HBT. The results show that base transit time increases with base-emitter voltage, minority carrier injection ratio, base width, peak base doping concentration, saturation velocity and field dependent mobility. On the other hand base transit time decreases with Ge dose and gradient in Ge profiles. The results obtained are compared with the results available in the literature. The comparisons show that the proposed analytical expression of base transit time is in good agreement with the available results.

CHAPTER 4

CONCLUSION AND RECOMMENDATION

4.1 CONCLUSION

The base transit time of a nonuniformly doped base $\text{Si}_{1-y}\text{Ge}_y$ HBT with different type of Ge profiles for all levels of injection has been studied to develop an analytical model by first obtaining closed form solution of minority carrier injected into the base under low level of injection. But for high level of injection, differential equation governing the minority carrier concentration is not analytically tractable. For this, the low level analytical solution is extended for high level of injection using perturbation theory and Webster effect and the obtained model is applicable for all levels of injection. In doing this, electric field and doping dependent mobility, bandgap narrowing effect due to heavy doping, presence of Ge and change in the density of states, velocity saturation at collector-base junction (modified to address the presence of Ge) etc. have been considered to study the injected minority carrier, collector current density, stored base charge and hence the base transit time. The analysis shows that the base transit time increases with base-emitter voltage, minority carrier injection ratio, base width and peak base doping concentration & if finite carrier velocity at the collector edge and dependence of mobility on electric field are considered while it decrease with slope of doping, Ge dose and gradient in Ge profiles. It is found that the transit time decreases with the change in Ge profile from box to triangle. The increase of Ge content for the same profile results in a decrease of transit time for a given Ge gradient. The results obtained by using the developed analytical model are compared with the results available in the literature and have been observed to be in good agreement. The analytical expressions derived in the present model can be a useful tool in device design and optimization.

4.2 RECOMMENDATION FOR FURTHER RESEARCH

In this work, three different Ge profiles (box type, triangular or trapezoidal shape as shown in Fig. 2.1) were considered where peak of the Ge mole fraction (y_G) was located on the collector end of the base. Another trapezoidal Ge profile with variable peak position is reported in literature where base transit time is obtained numerically. In future, such a dual slope Ge profile may be considered for analytical treatment.

An exponentially doped base profile was considered for this work. In future other non-uniform base doping profiles like Gaussian, retrograde or epitaxial base doping can be considered for the calculation of base transit time of a SiGe HBT.

A model for all level of injection of a SiGe base HBT has been developed but at strong high level of injection the phenomenon known as Kirk effect will occur which is not addressed here. When the Kirk effect occurs the base transit time will become large and will increase with J_c . In future, an expression for base transit time considering Kirk effect may be carried out.

Like the improvement in base transit time of HBT in comparison with BJT, improvement also are predicted for current gain, Early voltage (hence output resistance), base resistance and cutoff frequency (hence maximum oscillation frequency). These can be investigated in future.

The HBT considered here was a SiGe based heterojunction bipolar transistor. Attempt should be taken to investigate the base transit time using our model for InP, AlGaAs etc based HBT.

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APPENDIX A

DERIVATION OF IMPORTANT EQUATIONS

A.1 Solution of Differential Equation for Low Injection

The differential equation (2.18b) can be written as

$$(D - m_{023})n_i(x) = F_1 + F_2e^{-m_1x} \quad (\text{A.1})$$

where,

$$D = \frac{d}{dx} \quad (\text{A.2a})$$

$$F_1 = -\frac{aJ_{nl}m_{023}}{bqv_{sA}} \quad (\text{A.2b})$$

$$F_2 = -\frac{J_{nl}}{qD_{nA}} \quad (\text{A.2c})$$

For the complementary (homogeneous) solution of (A.1), we write

$$(D - m_{023})n_{ic}(x) = 0 \quad (\text{A.3a})$$

and the solution of (A.3a) is

$$n_{ic}(x) = F_3e^{m_{023}x} \quad (\text{A.3b})$$

where, F_3 is an arbitrary constant.

For the particular solution of (A.1), let

$$n_{ip}(x) = A + Be^{-m_1x} \quad (\text{A.4a})$$

where, A and B are arbitrary constants. Now putting (A.4a) into (A.1), we get

$$-m_1Be^{-m_1x} - m_{023}A - m_{023}Be^{-m_1x} = F_1 + F_2e^{-m_1x} \quad (\text{A.4b})$$

Equating the constant parts from (A.4b)

$$A = -\frac{F_1}{m_{023}} = \frac{aJ_{nl}}{bqv_{sA}} \quad (\text{A.4c})$$

Equating the coefficients of e^{-m_1x} from (A.4b)

$$B = -\frac{F_2}{m_1 + m_{023}} = \frac{J_{nl}}{qD_{nA}m_{0123}} \quad (\text{A.4d})$$

The total solution of (A.1) is the sum of complementary and particular solutions

$$\therefore n_i(x) = n_{ic}(x) + n_{ip}(x) = F_3 e^{m_{123}x} + \frac{aJ_{nl}}{bqv_{sA}} + \frac{J_{nl}}{qD_{nA}m_{0123}} e^{-m_1x} \quad (\text{A.5a})$$

The constant F_3 can be found by putting $x = 0$ in (A.5a)

$$F_3 = n_i(0) - \frac{aJ_{nl}}{bqv_{sA}} - \frac{J_{nl}}{qD_{nA}m_{0123}} \quad (\text{A.5b})$$

Elimination F_3 from (A.5a) gives the complete solution

$$n_i(x) = n_i(0)e^{m_{123}x} - \frac{J_{nl}}{qD_{nA}m_{0123}} (e^{m_{123}x} - e^{-m_1x}) - \frac{aJ_{nl}}{bqv_{sA}} (e^{m_{123}x} - 1) \quad (\text{A.5c})$$

A.2 Solution of Differential Equation for All Level of Injection

Let
$$V_q(x) = \frac{p_m(x)}{n_{ieSiGe}^2(x)} \quad (\text{A.6})$$

With necessary modification, the electric field equation for Si (2.7b) can be rewritten for SiGe alloy as

$$E_{SiGe}(x) = V_T \left(\frac{1}{p(x)} \frac{dp(x)}{dx} - \frac{1}{n_{ieSiGe}^2(x)} \frac{dn_{ieSiGe}^2(x)}{dx} \right) \quad (\text{A.7a})$$

To ensure the accuracy of the derivation, the magnitude of $\delta_n(x)$ must be much smaller than $p_m(x)$ instead of $n_m(x)$. This leads to $p(x) \approx p_m(x)$. Incorporating this assumption into (A.7a), the electric field equation becomes

$$E_{SiGe}(x) = V_T \left(\frac{1}{p_m(x)} \frac{dp_m(x)}{dx} - \frac{1}{n_{ieSiGe}^2(x)} \frac{dn_{ieSiGe}^2(x)}{dx} \right) \quad (\text{A.7b})$$

Using (A.6) and (A.7b), the electric field equation reduces to

$$E_{SiGe}(x) = \frac{V_T}{n_{ieSiGe}^2(x)} \left(\frac{1}{V_q(x)} \frac{d(n_{ieSiGe}^2(x)V_q(x))}{dx} - \frac{dn_{ieSiGe}^2(x)}{dx} \right) = \frac{V_T}{V_q(x)} \frac{dV_q(x)}{dx} \quad (\text{A.7c})$$

From the plots of the electric field (Figure 3.5(a)-(c)), it is found that the equation (A.7c) gives negative value for all levels of injection. This negative sign of $E_{SiGe}(x)$ should be considered during the process of solution for injected minority carrier distribution under all levels of injection. So we write

$$\frac{|E_{SiGe}(x)|}{V_T} = \left| \frac{1}{V_q(x)} \frac{dV_q(x)}{dx} \right| \quad (\text{A.7d})$$

Putting (A.6) into (2.28b), the minority carrier current density J_n of a SiGe HBT becomes

$$-J_n = qD_{nSiGe}(x) \frac{1}{V_q(x)} \frac{d}{dx} (n(x)V_q(x)) \quad (\text{A.8a})$$

and rearranged as

$$\frac{d}{dx} (n(x)V_q(x)) = \frac{-J_n}{q} \frac{V_q(x)}{D_{nSiGe}(x)} \quad (\text{A.8b})$$

Using (2.8), (2.12a) and (2.14a-b), the empirical expression for high electric field and doping dependent mobility in Si is

$$\mu_{nSi}(|E|, x) = \frac{v_s}{a|E(x)| + \frac{v_s}{\mu_n e^{m_1 x}}} \quad (\text{A.9a})$$

Correcting this electric field dependent mobility relation for SiGe alloy using (2.8), (2.12b) and (2.13)

$$\mu_{nSiGe}(|E|, x) = b \frac{v_{sA}}{a|E_{SiGe}(x)| + \frac{v_{sA}}{\mu_n e^{m_1 x}}} \quad (\text{A.9b})$$

and

$$\frac{1}{\mu_{nSiGe}(|E|, x)} = \frac{a|E_{SiGe}(x)|}{bv_{sA}} + \frac{e^{-m_1 x}}{b\mu_n} \quad (\text{A.9c})$$

Now using (2.8) and (A.9c), we can write

$$\frac{1}{D_{nSiGe}(|E|, x)} = \frac{a}{bv_{sA}} \frac{|E_{SiGe}(x)|}{V_T} + \frac{e^{-m_1 x}}{D_{nA}} \quad (\text{A.10})$$

Utilizing (A.7d), (A.8b) and (A.10)

$$\frac{d}{dx} (n(x)V_q(x)) = \frac{-J_n}{q} \left(\frac{-a}{bv_{sA}} \frac{dV_q(x)}{dx} + \frac{1}{D_{nA}} V_q(x) e^{-m_1 x} \right) \quad (\text{A.11})$$

Integrating both sides of (A.11), we get

$$n(x)V_q(x) = \frac{-J_n}{q} \left(-\frac{a}{bv_{sA}} V_q(x) - \frac{aC_1}{bv_{sA}} + \frac{1}{D_{nA}} \int V_q(x) e^{-m_1 x} dx \right) \quad (\text{A.12})$$

where, C_1 is a constant.

Using (A.6) and (2.15f)

$$V_q(x) = \frac{1}{n_{ieSiGe}^2(0)} p_m(x) e^{-m_{32}x} \quad (\text{A.13a})$$

Utilizing (A.13a), (2.11), (2.26a) and (2.27c)

$$V_q(x) = \frac{1}{n_{ieSiGe}^2(0)} (f_w n_l(x) + N_B(0) e^{-m_1 x}) e^{-m_{32}x} \quad (\text{A.13b})$$

Putting the value of $n_l(x)$ from (2.19) into (A.13b), we get

$$V_q(x) = \frac{1}{n_{ieSiGe}^2(0)} \left[n_m(0) e^{m_1 x} - \frac{J_{nm} (e^{m_1 x} - e^{-m_{321}x})}{qD_{nA} m_{0123}} - \frac{aJ_{nm} (e^{m_1 x} - e^{-m_{32}x})}{bqv_{sA}} + N_B(0) e^{-m_{123}x} \right] \quad (\text{A.13c})$$

where, $m_{321} = m_3 - m_{21}$ and $m_{21} = m_2 - m_1$.

Now multiplying (A.13c) by $\exp(-m_1 x)$

$$V_q(x) e^{-m_1 x} = \frac{1}{n_{ieSiGe}^2(0)} \left[n_m(0) e^{m_{01}x} - \frac{J_{nm} (e^{m_{01}x} - e^{-m_{322}x})}{qD_{nA} m_{0123}} - \frac{aJ_{nm} (e^{m_{01}x} - e^{-m_{321}x})}{bqv_{sA}} + N_B(0) e^{-m_{123}x} \right] \quad (\text{A.13d})$$

where, $m_{01} = m - m_1$ and $m_{322} = m_{321} + m_1$.

Integrating (A.13d), we can write

$$\int V_q(x) e^{-m_1 x} dx = \frac{1}{n_{ieSiGe}^2(0)} \left[\frac{n_m(0) e^{m_{01}x}}{m_{01}} - \frac{J_{nm}}{qD_{nA} m_{0123}} \left(\frac{e^{m_{01}x}}{m_{01}} + \frac{e^{-m_{322}x}}{m_{322}} \right) - \frac{aJ_{nm}}{bqv_{sA}} \left(\frac{e^{m_{01}x}}{m_{01}} + \frac{e^{-m_{321}x}}{m_{321}} \right) - \frac{N_B(0) e^{-m_{123}x}}{m_{0123}} + n_{ieSiGe}^2(0) C_2 \right] \quad (\text{A.13e})$$

where, C_2 is a constant.

Utilizing (A.12), (A.13a) and (A.13e), it can be shown that

$$n(x) p_m(x) e^{-m_{32}x} = \frac{-J_n}{qv_{sA}} \left[-\frac{a}{b} \left\{ n_m(0) e^{m_1 x} - \frac{J_{nm} (e^{m_1 x} - e^{-m_{321}x})}{qD_{nA} m_{0123}} - \frac{aJ_{nm} (e^{m_1 x} - e^{-m_{32}x})}{bqv_{sA}} \right\} + N_B(0) e^{-m_{123}x} \right] + \frac{v_{sA} W_B}{D_{nA}} \left[\frac{n_m(0) e^{m_{01}x}}{\eta_{01}} - \frac{J_{nm}}{qD_{nA} m_{0123}} \left(\frac{e^{m_{01}x}}{\eta_{01}} + \frac{e^{-m_{322}x}}{\eta_{322}} \right) - \frac{aJ_{nm}}{bqv_{sA}} \left(\frac{e^{m_{01}x}}{\eta_{01}} + \frac{e^{-m_{321}x}}{\eta_{321}} \right) - \frac{N_B(0) e^{-m_{123}x}}{\eta_{0123}} \right] + C_3 \quad (\text{A.14a})$$

where,
$$C_3 = n_{ieSiGe}^2(0) \left(\frac{v_{sA}}{D_{nA}} C_2 - \frac{a}{b} C_1 \right) \quad (\text{A.14b})$$

Rearranging the right hand side of (A.14a)

$$n(x)P_m(x)e^{-m_{32}x} = \frac{-J_n}{qv_{sA}} \left[\begin{aligned} & -\frac{a}{b} \left(n_m(0) - \frac{J_{nm}}{qD_{nA}m_{0123}} - \frac{aJ_{nm}}{bqv_{sA}} \right) e^{mx} \\ & -\frac{a}{b} N_B(0) e^{-m_{023}x} \\ & -\frac{aJ_{nm}}{bqv_{sA}} e^{-m_{32}x} \\ & -\frac{a}{b} \left(\frac{J_{nm}}{qD_{nA}m_{0123}} + \frac{v_{sA}W_B}{D_{nA}} \frac{J_{nm}}{qv_{sA}} \frac{1}{\eta_{321}} \right) e^{-m_{321}x} \\ & + \frac{v_{sA}W_B}{D_{nA}} \left(n_m(0) - \frac{J_{nm}}{qD_{nA}m_{0123}} - \frac{aJ_{nm}}{bqv_{sA}} \right) \frac{1}{\eta_{01}} e^{m_{01}x} \\ & -\frac{v_{sA}W_B}{D_{nA}} \frac{N_B(0)}{\eta_{0123}} e^{-m_{0123}x} \\ & -\frac{v_{sA}W_B}{D_{nA}} \frac{J_{nm}}{qD_{nA}m_{0123}} \frac{1}{\eta_{322}} e^{-m_{322}x} + C_3 \end{aligned} \right] \quad (\text{A.14c})$$

Simplifying the right hand side of (A.14c)

$$n(x)P_m(x)e^{-m_{32}x} = \frac{-J_n}{qv_{sA}} \left[\begin{aligned} & \left. \begin{aligned} & n_{m0\eta a} e^{mx} \\ & + N_B(0) e^{-m_{023}x} \\ & + n_{ma} e^{-m_{32}x} \\ & + n_{m\eta} \left(1 + \eta_{0123}/\eta_{321} \right) e^{-m_{321}x} \end{aligned} \right\} \\ & + \frac{v_{sA}W_B}{D_{nA}} \left\{ \begin{aligned} & \frac{n_{m0\eta a}}{\eta_{01}} e^{m_{01}x} \\ & - \frac{N_B(0)}{\eta_{0123}} e^{-m_{0123}x} \\ & - \frac{n_{m\eta}}{\eta_{322}} e^{-m_{322}x} \end{aligned} \right\} + C_3 \end{aligned} \right] \quad (\text{A.14d})$$

Putting $x = 0$ into (A.14d)

$$n(0)p_m(0) = \frac{-J_n}{qv_{sA}} \left[\begin{array}{l} \left. \begin{array}{l} a \\ b \end{array} \right\} \begin{array}{l} n_{m0\eta a} \\ +N_B(0) \\ +n_{ma} \\ +n_{m\eta}(1+\eta_{0123}/\eta_{321}) \end{array} \\ + \frac{v_{sA}W_B}{D_{nA}} \left\{ \begin{array}{l} \frac{n_{m0\eta a}}{\eta_{01}} \\ \frac{N_B(0)}{\eta_{0123}} \\ \frac{n_{m\eta}}{\eta_{322}} \end{array} \right\} + C_3 \end{array} \right] \quad (\text{A.14e})$$

Subtracting (A.14e) from (A.14d) will result in

$$n(x)p_m(x)e^{-m_{32}x} - n(0)p_m(0) = \frac{J_n}{qv_{sA}} \left[\begin{array}{l} \left. \begin{array}{l} a \\ b \end{array} \right\} \begin{array}{l} n_{m0\eta a}(e^{m_{32}x} - 1) \\ +N_B(0)(e^{-m_{023}x} - 1) \\ +n_{ma}(e^{-m_{32}x} - 1) \\ +n_{m\eta}(1+\eta_{0123}/\eta_{321})(e^{-m_{323}x} - 1) \end{array} \\ + \frac{v_{sA}W_B}{D_{nA}} \left\{ \begin{array}{l} -\frac{n_{m0\eta a}}{\eta_{01}}(e^{m_{01}x} - 1) \\ +\frac{N_B(0)}{\eta_{0123}}(e^{-m_{0123}x} - 1) \\ +\frac{n_{m\eta}}{\eta_{322}}(e^{-m_{322}x} - 1) \end{array} \right\} \end{array} \right] \quad (\text{A.15a})$$

Equation (A.15a) can be simplified as

$$n(x)p_m(x)e^{-m_{32}x} = n(0)p_m(0) + \frac{J_n}{qv_{sA}} \left[\frac{a}{b} N_F(x) + \frac{v_{sA}W_B}{D_{nA}} N_\eta(x) \right] \quad (\text{A.15b})$$

and (A.15b) can be rearranged as

$$n(x) = \left(n(0)p_m(0) + \frac{J_n}{qv_{sA}} N(x) \right) \frac{e^{m_{32}x}}{p_m(x)} \quad (\text{A.15c})$$

APPENDIX B**LIST OF PUBLICATIONS FROM THIS WORK**

Partial research results of this thesis work are presented in the following papers:

1. **Yeasir Arafat**, M. Z. R. Khan and M. M. S. Hassan, "Base Transit Time of a Heterojunction Bipolar Transistor with Exponentially Doped Base," *Journal of Electrical Engineering, The Institution of Engineers*, Bangladesh, September, 2009 (now in press).
2. **Yeasir Arafat**, M. Z. R. Khan and M. M. S. Hassan, "Analytical Modeling of Base Transit Time for a $\text{Si}_{1-x}\text{Ge}_x$ Heterojunction Bipolar Transistor," *2009 IEEE International Conference on Electron Devices and Solid-State Circuits (EDSSC)*, Xian, China, November 25-27, 2009 (accepted for oral presentation).
3. **Yeasir Arafat**, Md. Ziaur Rahman Khan and M. M. Shahidul Hassan, "Analytical model for base transit time of an exponentially-doped base HBT for all level of injection," *Solid-State Electronics*, 2009 (submitted).

