ANALYSIS OF A FREE SPACE OPTICAL (FSO) LINK UNDER THE INFLUENCE OF SPATIAL WIDENING EFFECT FROM PROPAGATION THROUGH CLOUDS

A thesis submitted in partial fulfillment of the requirements for the degree of

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By

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Approval

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Dedication

To my dearest Parents & younger Sister

Acknowledgement

The endless support, relentless encouragement and valuable suggestions from my respected supervisor Prof. Satya Prasad Majumder is one of the most inspirational, notable and worth mentioned to accomplish my research work. I am whole-heartedly grateful to him for all of those hard works even when he came to BUET during holidays to guide me.

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List of Abbreviations

SNR	**Signal to Noise Ratio**
BER	**Bit Error Rate**
FSO	**Free Space Optical**
IM/DD	**Intensity Modulation Direct Detection**
LED	**Light Emitting Diode**
LASER	**Light Amplification by Stimulated Emission of Radiation.**
MIMO	**Multiple Input Multiple Output**
MISO	**Multiple Input Single Output**
OFC	**Optical Fiber Communication**
SISO	**Single Input Single Output**
SINR	**Signal to Noise and Interference Ratio**

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Abstract

Free Space Optical communication is a line of sight technology that transmits light between two points through atmosphere as the channel of propagation. Clouds as a part of the communication channel cause signal power attenuation and spatial widening due to the inter symbol interference. These effects decrease the received signal power and increase the bit error rate (BER). The main objective of this thesis is to find out the bit error rate (BER) and signal to interference and noise ratio (SINR) in presence of cloud in different system parameters including cloud thickness, bit rate and wavelength. The transfer function of cloud is used to determine the spatial widening due to the inter symbol interference effect.

The results are presented in terms of signal to interference and noise ratio (SINR), bit error rate (BER) and power penalty using the intensity modulation direct detection (IM/DD) system. It is noticed that cloud has a great effect on signal transmission due to the spatial widening. Analytical results show that the shorter wavelength region of 0.532 μm is the preferred wavelength region for optical communication according to criteria of least power transmission and least spatial widening. It is also found that with the increase of bit rate, the spatial widening increases and consequently BER increases.

Chapter 1

Introduction

Chapter: 1

Introduction

1.1 Communication System

A Communication System is defined as the transfer of information from one point to another over the transmission medium, whether separated by a few kilometers or by transoceanic distances. Within a Communication system the information transfer is frequently achieved by modulating the information on to an electromagnetic wave which acts as a carrier for the information signal [16]. The modulated carrier is then transmitted to the required destination where it is received and the original information signal is obtained by demodulation. The transmission medium can consists of a pair of cables, a coaxial cable or a radio link through free space down which the signal is transmitted to the receiver where it is demodulated before being passed to the destination. It may be mentioned that in any transmission medium the signal is attenuated, or suffers loss, and is subject to degradations imposed by medium itself [18]. That’s why in any communication system there is a maximum permitted distance between the transmitter and the receiver.

1.2 Historical Perspective of Optical Communication

The use of visible optical carrier waves or light for communication has been common for many years. Simple systems such as signal fires, reflecting mirrors and more recently, signaling lamps have provided successful information transfer [19]. The same idea was used up to the end of the eighteenth century. The idea was extended further, following a suggestion of Claude Chappe in 1792 to transmit mechanically coded message over long distances by the use of intermediate relay stations, acting as repeaters in modern-day language [18].

The invention of the telegraph by Samuel F. B. Morse in 1838 began a new era in electrical communications. In the ensuing years, an increasingly larger portion of the electromagnetic spectrum was utilized for conveying information from one place to another. The first successful transatlantic telegraph cable went into operation in 1866. The invention of telephone in 1876 brought a major change in as much as electric signals were transmitted in analog form through a continuously varying electric current [18].

The invention of the laser in 1960 paved the way to another solution, in the form of *optical telecommunication based on optical fibers*, which offers a quasi-unlimited line capacity. The almost simultaneous development in 1970−1971 of low attenuation optical fibers, and semiconductor lasers emitting in continuous mode at room temperature, led to the explosion of wired optical telecommunications. Glass is the transmission medium for photons, and glass fibers can extend over distances of several thousand kilometers. *Wired optics* unquestionably currently dominates the fields of submarine transmissions, long-distance transmissions and interurban transmissions. It has become an integral and indispensable part of the Information Superhighway System.

Around from 1965, the beam guide system which arranged the lens in a pipe, and the space propagation system that emits light to free space is studied to use laser for free space optical communication. In 1979, an indoor optical wireless communication system has been presented by F. R. Gfeller and U. Bapst [16]. In their system, diffuse optical radiation in the near infrared region was utilized as signal carrier to interconnect a cluster of terminals located in the same room to a common cluster controller. However, the invention of semiconductor laser has moved the mainstream of the research to optical fiber communication. The invention of the optical fiber amplifier at the 1980's and the invention of the in-fiber Bragg grating at 1990's has significantly contributed to the emergence of optical communication in modern days.

Originally developed by the military and NASA, FSO has been used for more than three decades in various forms to provide fast communication links in remote locations.

1.3 Review of Previous Work

Free space optical communications is an optical communication technology that uses visible or infrared light to transmit data between two points through atmosphere as the channel of propagation [1]-[4], [6]. The advantages of optical communication are information bandwidth, transmitter power, directionality, and immunity to jamming [4]. FSO finds a great potential in space communications especially between a low earth orbit (LEO) satellite, a geostationary earth orbit (GEO) satellite, an air plane, and ground station [3]. In most of the scenarios, part of the optical channel goes through the earth's atmosphere, which occasionally contains water clouds. Clouds as part of the communication channel cause signal power attenuation and spatial widening. These effects decrease the received signal power and increase the bit error rate (BER). When optical radiation propagates through physically and optically thin clouds, most of the radiation power passes through the clouds but is widened in the spatial domain through scattering processes [5], [7]. The purpose of this work is to derive a simple model to analyze the spatial widening of optical communication beams by clouds using the transfer function of Cloud [8] and to suggest overcoming distortions that arise from propagation through clouds.

1.4 Objective With Specific Aims And Possible Outcome

The main objective of this thesis is to analyze the spatial widening performance of FSO communication through clouds. The main purposes of this thesis are:

I. To carry out the BER performance analysis of a FSO link with spatial widening effect due to Propagation through Clouds considering intensity modulation with direct detection receiver.

II. To develop the expression for the SNR at the receiver output considering the scattering of light during propagation through Clouds.

III. To evaluate the SNR and BER for the performance results for a FSO link with above system impairments.

IV. To evaluate the performance degradation due to scattering and spatial widening effect due to clouds on the system performance.

1.5 Outline of Methodology/ Experimental Design

Analysis has been carried out for a Free Space Optical Link taking into consideration the spatial widening effect of the cloud. The Transfer Function of the Cloud has been used to find the spatial widening of the output pulses. The amount of ISI due to widening effect has been determined and the expression of the Signal to Noise plus Interference ratio has been derived and the results have been evaluated numerically. Performance degradation due to effect of cloud has been determined for different system parameters including cloud thickness.

1.6 Organization of this Thesis

This thesis consists of five chapters. Chapter-1 is concentrated on the brief history of Optical Communication, discusses on the previous work and the main objective of this thesis.

In Chapter-2, a brief overview of Free Space Optical Communication, different components of the FSO system and the impact of atmospheric condition on FSO communication link is discussed.

In Chapter-3, the techniques and technological details of Intensity Modulation Direct Direction scheme through atmospheric turbulence channel considering Cloud with the transfer function of Cloud is discussed analytically and the SINR and BER analytical analysis is presented.

In Chapter-4, numerical results are presented with varying different parameters plotting SNR and BER.

In Chapter-5, finally, conclusions and some proposals for future work are provided.

Chapter 2

Free Space Optical Communication System

Chapter: 2

Free Space Optical Communication System

2.1 Introduction

The objective of this chapter is to show how free-space optics, which is already commonly used for information transfer, is also taking off as a telecommunication technique, and is becoming an integral and essential part of data-processing architecture and telecommunications due its numerous advantages (flow, low cost, mobility of materials, safety, etc). The basics of free space optical communication along with few terminologies will be described in brief.

2.2 Free Space Optical Communication System

Free Space Optical (FSO) Communication is an optical communication technology that uses light propagating in free space to send out information between two points. The technology plays a significant role where the physical connections by the means of fiber optic cables are impractical due to high costs or other considerations. If we have to consider a technology that offers full-duplex Gigabit Ethernet throughput, that can be installed license-free worldwide, can be installed in less than a day, that offers a fast, high ROI then the technology is free-space optical Communication (FSO).

The line-of-sight technology approach uses invisible beams of light to provide optical bandwidth connections. It's capable of sending up to 1.25 Gbps of data, voice, and video communications simultaneously through the air enabling fiber-optic connectivity without requiring physical fiber-optic cable [16]. It enables optical communications at the speed of light. Over the last two decades free-space optical communication (FSO) has become more and more interesting as an adjunct or alternative to radio frequency communication.

Free-space optical communication (FSO) systems (in space and inside the atmosphere) have developed in response to a growing need for high-speed and tap-proof communication systems. Links involving satellites, deep-space probes, ground stations, unmanned aerial vehicles (UAVs), high altitude platforms (HAPs), aircraft, and other nomadic communication partners are of practical interest. Moreover, all links can be used in both military and civilian contexts. FSO is the next frontier for net-centric connectivity, as bandwidth, spectrum and security issues favor its adoption as an adjunct to radio frequency (RF) communications

While fixed FSO links between buildings have long been established and today form a separate commercial product segment in local and metropolitan area networks, the mobile and long-range applications of this technology are aggravated by extreme requirements for pointing and tracking accuracy because of the small optical beam divergences involved. This challenge has to be addressed to fully exploit the benefits of optical links. Furthermore, long-haul optical links through the atmosphere suffer from strong fading as a result of index-of-refraction turbulence (IRT) and link blockage by obscuration such as clouds, snow, fog and rain.

2.3 Application of FSO Communication System

FSO provides vastly improved EMI behavior using light instead of microwaves. The light beam can be very narrow, which makes FSO hard to intercept, improving security. In any case, it is comparatively easy to encrypt any data traveling across the FSO connection for additional security. Some of the typical scenarios where FSO can play a vital role are shown below:

- LAN-to-LAN connections on campuses at fast FE or GE speeds.
- LAN-to-LAN connections in a city, for *example in Metropolitan Area Network.*
- To cross a public road or other barriers which the sender and receiver do not own.
- Speedy service delivery of high-bandwidth access to optical fiber networks.

- Converged Voice-Data-Connection.
- Temporary network installation (for events or other purposes).
- Reestablish high-speed connection quickly (disaster recovery).
- As an alternative or upgrade add-on to existing wireless technologies.
- As a safety add-on for important fiber connections (redundancy).
- For communications between spacecraft, including elements of a satellite constellation.
- For inter- and intra chip communication.

2.4 Advantages of FSO Communication System

One of the big demands in today's modern world is the growing need for high bandwidth in different services. FSO can play a vital role achieving the high bandwidth. FSO has the following advantages:

- Ease of deployment
- Requires no RF spectrum licensing. So FSO is a License-free long-range operation (in contrast with radio communication)
- High bit rates
- Low bit error rates
- Immunity to electromagnetic interference
- Full duplex operation
- Protocol transparency
- Very secure due to the high directionality and narrowness of the beam(s)
- No Fresnel zone necessary

- Easily upgradeable, and its open interfaces support equipment from a variety of vendors, which helps enterprises and service providers protect their investment in embedded telecommunications infrastructures.
- Requires no security software upgrades.
- Can be deployed behind windows, eliminating the need for costly rooftop rights.

Considering the above advantages, presently FSO communication systems are becoming more and more popular as the interest and requirement in high capacity and long distance space communications grow.

2.5 Main Challenges in FSO Communication System

While fiber-optic cable and FSO technology share many of the same attributes, they face different challenges due to the way they transmit information. While fiber is subject to outside disturbances from wayward construction backhoes, gnawing rodents, and even sharks when deployed under sea, FSO technology is subject to its own potential outside disturbances. Optical wireless networks based on FSO technology must be designed to combat changes in the atmosphere, which can affect FSO system performance capacity [19]. And because FSO is a line-of-sight technology, the interconnecting points must be free from physical obstruction and able to "see" each other.

All potential disturbances can be addressed through thorough and appropriate network design and planning. Among the issues to be considered when deploying FSO-based optical wireless systems:

2.5.1 FOG

The primary challenge to FSO-based communications is dense fog. Rain and snow have little effect on FSO technology, but fog is different. Fog is vapor composed of water droplets, which are only a few hundred microns in diameter but can modify light characteristics or completely hinder the passage of light through a combination of absorption, scattering, and reflection. The primary answer to counter fog when deploying

FSO-based optical wireless products is through a network design that shortens FSO link distances and adds network redundancies. FSO installations in extremely foggy cities such as San Francisco have successfully achieved carrier-class reliability.

2.5.2 Absorption

Absorption occurs when suspended water molecules in the terrestrial atmosphere extinguish photons. This causes a decrease in the power density (attenuation) of the FSO beam and directly affects the availability of a system. Absorption occurs more readily at some wavelengths than others. However, the use of appropriate power, based on atmospheric conditions, and use of spatial diversity (multiple beams within an FSO-based unit) helps maintain the required level of network availability.

2.5.3 Scattering

Scattering is caused when the wavelength collides with the scatterer. The physical size of the scatterer determines the type of scattering. When the scatterer is smaller than the wavelength, this is known as Rayleigh scattering. When the scatterer is of comparable size to the wavelength, this is known as Mie scattering. When the scatterer is much larger than the wavelength, this is known as non-selective scattering. In scattering — unlike absorption — there is no loss of energy, only a directional redistribution of energy that may have significant reduction in beam intensity for longer distances.

2.5.4 Physical Obstructions

Flying birds or construction cranes can temporarily block a single-beam FSO system, but this tends to cause only short interruptions, and transmissions are easily and automatically resumed. Multi-beam systems (spatial diversity) are used to address temporary obstructions, as well as other atmospheric conditions, to provide for greater availability.

2.5.5 Building Sway/Seismic Activity

The movement of buildings can upset receiver and transmitter alignment. FSO-based optical wireless offerings use a divergent beam to maintain connectivity. When combined with tracking, multiple beam FSO-based systems provide even greater performance and enhanced installation simplicity.

2.5.6 Scintillation

Heated air rising from the earth or man-made devices such as heating ducts creates temperature variations among different air pockets. This can cause fluctuations in signal amplitude which leads to "image dancing" at the FSO-based receiver end. Unique multi-beam system is designed to address the effects of this scintillation. Called "Refractive turbulence," this causes two primary effects on optical beams.

- **Beam Wander:** Beam wander is caused by turbulent eddies that are larger than the beam.
- **Beam Spreading:** Beam spreading — long-term and short-term — is the spread of an optical beam as it propagates through the atmosphere.

2.5.7 Safety

To those unfamiliar with FSO technology, safety can be a concern because the technology uses lasers for transmission. The proper use and safety of lasers have been discussed since FSO devices first appeared in laboratories more than three decades ago. The two major concerns involve eye exposure to light beams and high voltages within the light systems and their power supplies. Strict international standards have been set for safety and performance.

2.5.8 Cloud

Clouds as part of the communication channel cause signal power attenuation and spatial widening. These effects decrease the received signal power and increase the bit error rate (BER). When optical radiation propagates through physically and optically thin clouds, most of the radiation power passes through the clouds but is widened in the spatial domain through scattering processes. So design of FSO requires special attention considering Cloud as a barrier.

These above factors cause an attenuated receiver signal and lead to higher bit error ratio (BER). To overcome these issues, different solutions are adapted, like multi-beam or multi-path architectures, which use more than one sender and more than one receiver. Some state-of-the-art devices also have larger fade margin (extra power, reserved for rain, smog, fog). To keep an eye-safe environment, good FSO systems have a limited laser power density and support laser classes 1 or 1M. Atmospheric and fog attenuation, which are exponential in nature, limit the practical range of FSO devices to several kilometers.

2.6 Block Diagram of an FSO Communication System

A Free Space Optical link basically consists of a transmitter, Atmospheric channel as the medium of transmission and a receiver to reproduce the transmitted signal. The Block diagram of the basic FSO system is shown in Fig 2.1

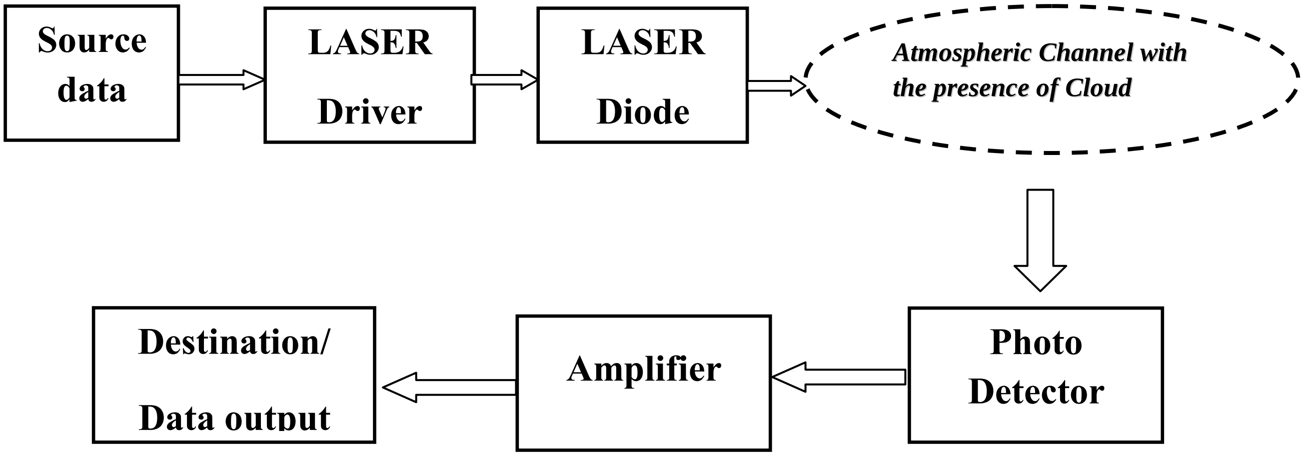


Fig. 2.1: FSO System Block Diagram

2.6.1 The Transmitter

First of all, a light source is required to convert the electrical signal into optical signal. The optical source along with modulation and other supporting electronics taken together are called a transmitter, which is used to send an optical signal through the atmospheric channel. A transmitter consists of a light source, coupling optics and electronics. Basically LED or LASER is used in FSO technology.

The properties of thermal radiation sources do not correspond to what we expect and what we think is needed and useful from light transmitters in terms of brightness, radiation characteristic and spectrum. Most of the light sources which are used are quantum devices, and most commonly lasers. Lasers are the brightest sources, among them: solid lasers (YAG-Nd or ruby); gas lasers (HeNe, metal vapors excited by electronic discharge); lasers with amplifying optical fibers doped with rare earth ions and pumped optically; and semiconductor electroluminescent lasers. Electroluminescent diodes and semiconductor junction lasers have a great advantage since their radiant powers and optical frequencies can be directly modulated by the injection current. Modulation frequencies up to several tens of GHz can be reached; these performances depend on the material and the internal structure of the components

- **LED**

LEDs have been around for more than 30 years. They have found application in nearly every consumer-electronic device: TV sets, VCRs, telephones, car electronics, and many others. They are used in optical communications, mostly because of their small size and long life. However, their low intensity, poor beam focus, low modulation bandwidth, and incoherent radiation-in comparison with laser diodes.

A light-emitting diode (LED) is a semiconductor light source. When a light-emitting diode is forward biased (switched on), electrons are able to recombine with electron holes within the device, releasing energy in the form of photons. This effect is called electroluminescence and the color of the light (corresponding to the energy of the photon) is determined by the energy gap of the semiconductor. The basic serface emission is shown in Fig 2.2.

LED-based systems have a number of advantages, the most obvious being expenditure and dimension. Not only is the optical sub-system design far less expensive, but the driving electronics are also more simplified. The result is that system cost per milliwatt for an LED based system is much lower, than for a laser diode design. The problem is the emitted light by LED is incoherent and unfocused.

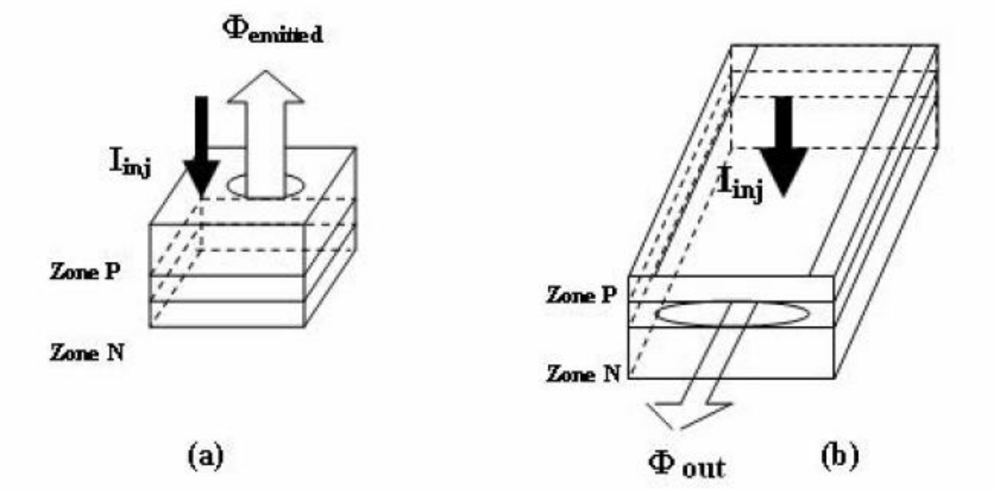


Figure 2.2: Examples of electroluminescent diode sources of incoherent light waves: (a) surface emission, (b) emission through the edge facet (edge-emitter)

➢ LASER Diodes (LDs)

Semiconductor Laser diodes, developed in the 1970s, have found vast commercial applications in compact-disc (CD) players. LD radiation properties as brightness, directivity, narrow spectral width, and coherence made LD the best light sources for optical communication links. Laser diodes are electrically pumped semiconductor lasers in which the gain is generated by an electrical current flowing through a p–n junction or (more frequently) a p–i–n structure. In such a heterostructure, electrons and holes can recombine, releasing the energy portions as photons. This process can be spontaneous, but can also be stimulated by incident photons, in effect leading to optical amplification, and with optical feedback in a laser resonator to laser oscillation. Laser diodes are small in size and power efficient.

Long range, very high speed (gigabit or more) point-to-point FSO systems require laser diodes. The wavelength of the laser diodes chosen for FSO system is 850 nm or 1550 nm. The 1550 nm light source is chosen for longer distance communication. The 1550 is also chosen as more photons per watt power arrive for longer wavelengths and therefore more photo current is produced per watt of incident power for same efficient devices.

Laser diodes are planar dielectric optical waveguides with rectangular section along which the carriers recombine, generating photons; and inside which the created photons can propagate and be amplified. These guides lie inside resonant cavities between two parallel facets oriented along natural crystal planes in the case of ―Fabry–Pérot" type lasers (Figure 2.3). The resonant structure can also be a Bragg grating, periodic corrugation along the axis of the guide, whose function is to select only one single mode in the case of distributed feedback lasers (DFBs).

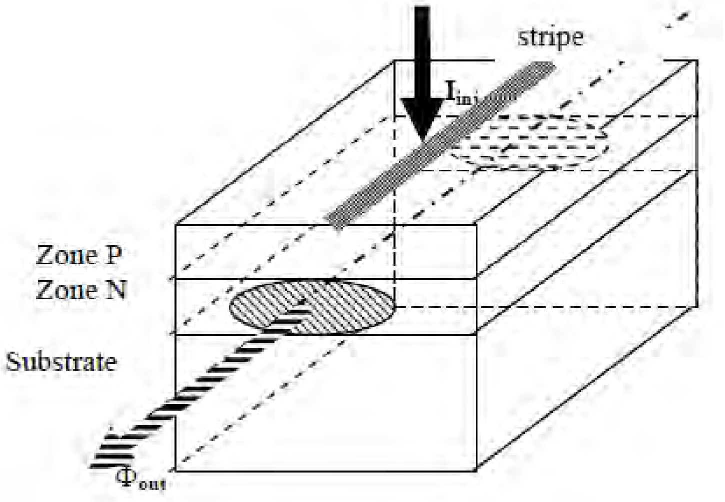


Figure 2.3: Laser structure

2.6.2 The Receiver

Once the transmitter coverts an electrical signal into optical signal and passes the signal through the atmosphere, it must be collected by the receiver which will convert the optical signal back into electrical signal. The light signal collected from the air is firstly converted into electrical signal by the photo detector. Then it is amplified and passes to the demodulator. Both coherent and non-coherent detection schemes are possible in the receiver end. But considering the complexity and cost, non-coherent or direct detection scheme is preferred. For this thesis Intensity Modulated Direct Detection (IM/DD) system is considered. The quality of reception is measured by the probability of error, expressed in terms of bit error rate (BER).

Photo-detectors, particularly those made of semiconductors, are either photo resistances, or current or voltage generators. When they are illuminated, the values of the resistance, the current or the voltage depend on the incident radiant power. In the case of semiconductor devices, a part of the incident radiant power is absorbed in the detector volume: each photon whose energy is higher or equal to the energy band gap is absorbed and dissociates a pair of carriers. An electron and a hole are therefore released in the conduction and the valence bands respectively.

In non-coherent optical signal detection, the incident photons are absorbed by the detectors and free-carriers are generated accordingly. Though it is possible for a photon to pass through the photo detector without generating any free-carriers, in a well-designed photo detector, the probability of an incident photon causing the free carrier is high.

In the case of free-space optical links, the photo-detectors receive images of the near fields of the transmitters after propagation along trajectories, optical beams through the atmosphere and some optics elements. A first approach consists of using detectors with both sensitive areas and apertures large enough to capture these images whatever the sizes and displacements of the beams under conditions of maximum illumination. A second approach, technically more sophisticated, consists of using the photo-detectors adapted to optic fiber links and fixing them in the image plane of an auto-focusing optical system. Under these conditions the incident field remains inside the perimeter of the detector sensitive area and the incident radiant power is close to maximum.

For the optical communication, the most popular photo detectors are p-i-n photodiode and avalanche photodiode.

➢ P-i-n Photodiodes

A photodiode is a type of photo detector capable of converting light into either current or voltage, depending upon the mode of operation. A PIN diode is a diode with a wide, lightly doped 'near' intrinsic semiconductor region between a p-type semiconductor and an n-type semiconductor region [Fig 2.4].

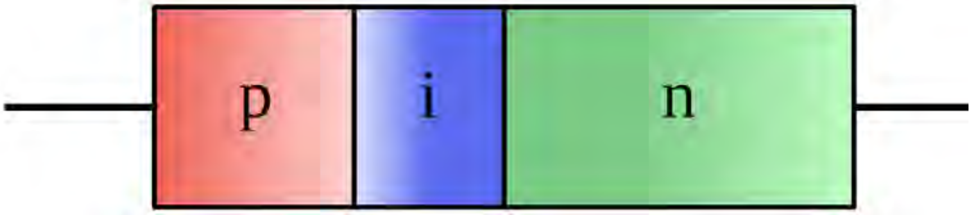


Fig. 2.4: P-i-N Photodiode

A PIN diode functions under high-level injection. In other words, the intrinsic "i" region is flooded with charge carriers from the "p" and "n" regions. When the diode is forward biased, the injected carrier concentration is typically several orders of magnitude higher than the intrinsic level carrier concentration. Due to this high level injection, which in turn is due to the depletion process, the electric field extends deeply (almost the entire length) into the region. This electric field helps in speeding up of the transport of charge carriers from P to N region, which results in faster operation of the diode, making it a suitable device for high frequency operations.

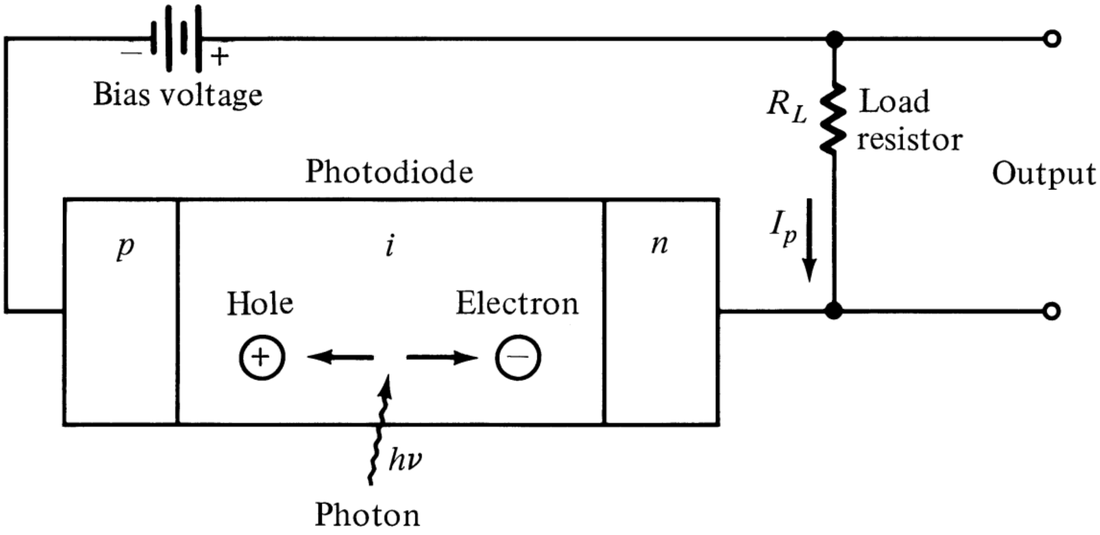


Fig 2.5: Basic p-i-n photodiode circuit

Incident photons trigger a photocurrent I_p in the external circuitry by pumping energy Photocurrent which is proportional to the incident Optical Power [Fig 2.5].

A p-i-n photodiode is the most commonly used light detector in today's optical communication systems because of its ease in fabrication, high reliability, low noise, low voltage and relatively high bandwidth.

➢ Avalanche Photodiodes

An avalanche photodiode (APD) is a highly sensitive semiconductor electronic device that exploits the photoelectric effect to convert light to electricity. APDs can be thought of as photo-detectors that provide a built-in first stage of gain through avalanche multiplication. The internal gain of the APD is obtained by having a high electric field that energizes photo-generated electrons and holes. APD has high gain due to self multiplying mechanism, used in high end systems. But APD's are costly and need high reverse bias voltage.

APD can amplify the photo current without an external amplifier and therefore, without the noise associated with the circuitry. APD applicability and usefulness depends on many parameters. Two of the larger factors are: quantum efficiency, which indicates how well incident optical photons are absorbed and then used to generate primary charge carriers; and total leakage current, which is the sum of the dark current and photocurrent and noise. Electronic dark noise components are series and parallel noise. Series noise, which is the effect of shot noise, is basically proportional to the APD capacitance while the parallel noise is associated with the fluctuations of the APD bulk and surface dark currents. Another noise source is the excess noise factor, *F*. It describes the statistical noise that is inherent with the stochastic APD multiplication process. Figure 2.6 shows the reach-through structure of APD which offers the best available combination of high speed, low noise and capacitance and extended red response.

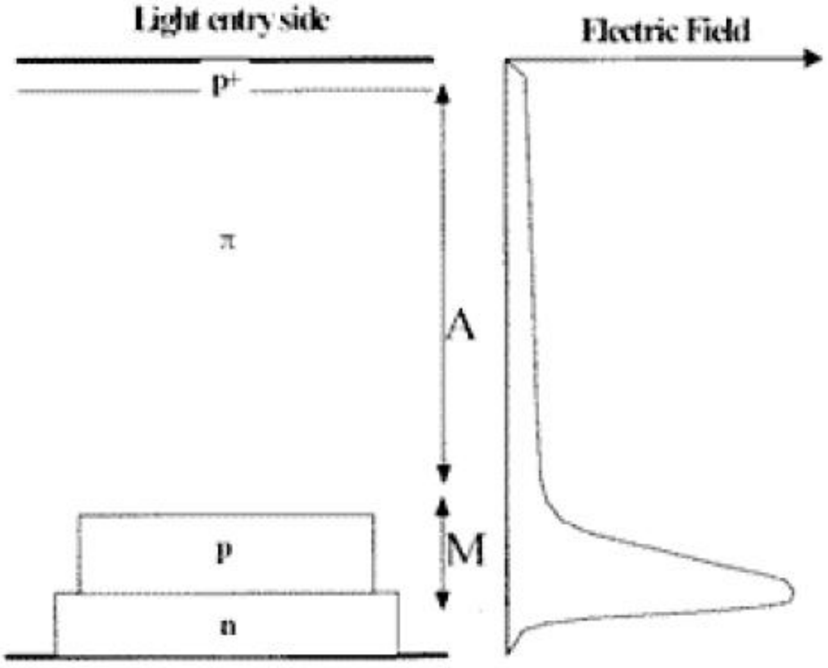


Figure 2.6: APD Structure

2.6.3 FSO Transmitting Medium

In FSO communication, the transmitter and the receiver are separated by the propagation channel, which is the atmosphere. Free-Space Optical (FSO) links involve the transmission, absorption and scattering of light by the Earth's atmosphere. The atmosphere interacts with light due to the composition of the atmosphere. Under normal conditions, the atmosphere consists of a variety of different molecules and small suspended particles called aerosols. This interaction produces a wide variety of optical phenomena as follows:

- Attenuation of radiation that propagates through atmosphere.
- Absorption at specific optical wavelengths due to the molecules.
- Generation by scattering (the sky blue color, the red sunset, etc.) or by radioactive emission of an optical beam comparable to noise at the source of perception contrasts loss. This loss is all the more important since the distance is large.
- Scintillation due to the variation of the air's refractive index under the effect of Temperature.

The construction of an opto-electronic arrangement, made up of a transmitter and a receiver in free atmosphere, requires a good quality of knowledge of specific optical properties of the atmosphere. Let us describe the effects of the atmosphere on light propagation.

2.6.3.1 Atmospheric Absorption

Atmospheric absorption results from the interaction between the photons of the radiation and the atoms or molecules of the medium. Due to atmospheric absorption the incident photon disappears. Let us consider a light beam of wavelength λ which passes through an absorbing medium of thickness *dx*.

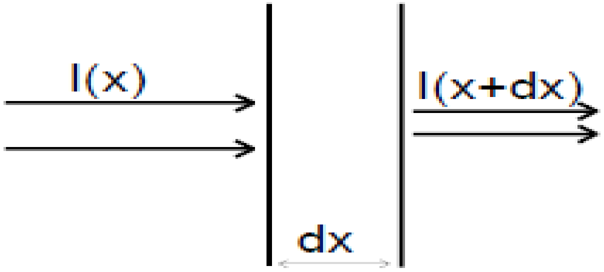


Fig. 2.7 Light absorption by atmosphere for a thickness of dx.

Due to absorptions of the medium, the number of photons in the radiation is reduced throughout the length of its path. The intensity of the radiation, measured at $x + dx$ (Figure 2.2), in relation to the intensity measured at x , is written as:

$$I(\lambda,x+dx)= I(\lambda,x)- dI_a(\lambda,x)\ldots\ldots(2.1)$$

The quantity $dI_a(\lambda, x)$ corresponds to the intensity of the light absorbed by the absorbing medium, this latter being proportional to the incident intensity $I(\lambda, x)$, to dx and to a spectral parameter that represents the absorption of the medium $\alpha(\lambda, x)$ at this wavelength-

$$dI_a(\lambda,x)= \alpha(\lambda,x)\, I(\lambda,x)dx\ldots\ldots(2.2)$$

2.6.3.2 Atmospheric Scattering

Atmospheric scattering results from the contact of a part of the light with the atoms and/or the molecules in the propagation medium, which causes an angular redistribution of this part of the radiation with or without modification of the wavelength.

To calculate the transmission of a scattering medium we proceed from the preceding paragraph to write:

$$\tau_d(\lambda,x)=\frac{I(\lambda,x)}{I(\lambda,0)}=\exp\left[-\int_0^X \boldsymbol{\beta}(\lambda,x)dx\right]\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\text{(2.3)}$$

where $\beta(\lambda, x)$ is the specific spectral scattering coefficient. In the case of scattering, the scattered light does not disappear locally as with absorption.

2.6.3.3 Atmospheric Turbulence due to Aerosols

Aerosols are extremely fine particles (solid or liquid) suspended in the atmosphere with a very low fall speed caused by gravity. Their size generally lies between 0.01 μm and 100 μm. Due to the action of terrestrial gravity, the biggest sized particles ($r > 0.2$ μm) are in the vicinity of the ground. Fog and mist are liquid aerosols, salt crystals and sand grains are solid aerosols. The presence of aerosols may cause severe disturbance to the propagation of optical and infrared waves, since their dimensions are very close to the wavelengths of these frequencies. It is not the same in the range for instance of centimeter and millimeter waves where the wavelength is much longer than the size of the aerosols.

Although rain and snow can hamper light propagation, the most harmful environmental conditions are fog and haze. This is due to their high concentration of scattering particles with radii on the same order of magnitude as the laser wavelength.

The following figure shows the atmospheric effect on the transmitted signal-

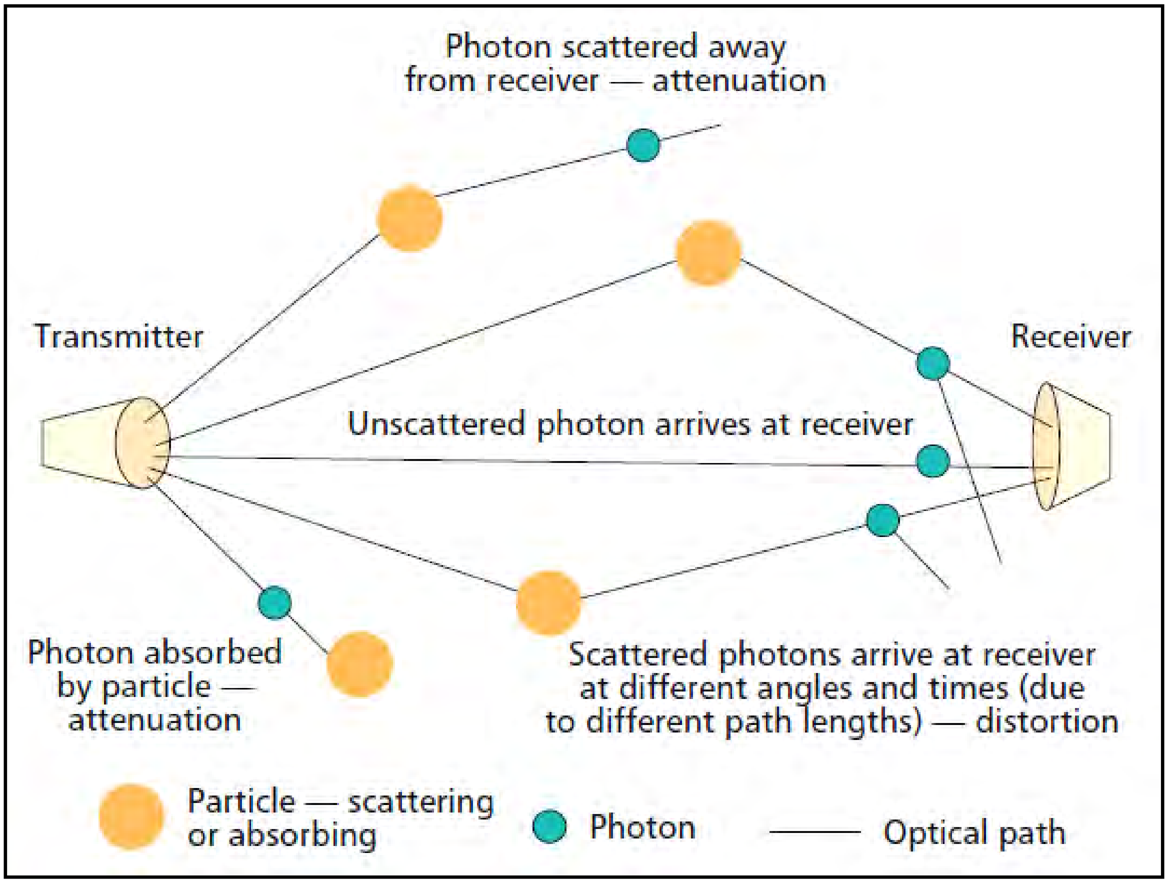


Fig. 2.8 Multiple scattering showing spatial and angular spreading and variations in path length. Absorption of a photon by a particle is also shown.

In the above discussion it is observed that the transmitted optical beam passing through the atmosphere can be absorbed, scattered or displaced depending on the atmospheric condition. This is the fundamental limits of FSO systems. Compared with the RF system, FSO is less affected by snow and rain, but can be severely affected by the atmospheric turbulence and fog.

2.7 Intensity Modulation Direct Detection (IM/DD) System

In optical communications, intensity modulation (IM) is a form of modulation in which the optical power output of a source is varied in accordance with some characteristic of the modulating signal. The reception and regeneration of the modulating signal is done by Direct Detection technique. In the free-space, optical Intensity Modulated Direct Detection (IM/DD) is frequently used. Most practical wireless optical channels use light emitting diodes or Laser diodes as transmitters and photodiodes as detectors, as shown below. These devices modulate and detect solely the intensity of the carrier, not its phase, which implies that all transmitted signal intensities are nonnegative.

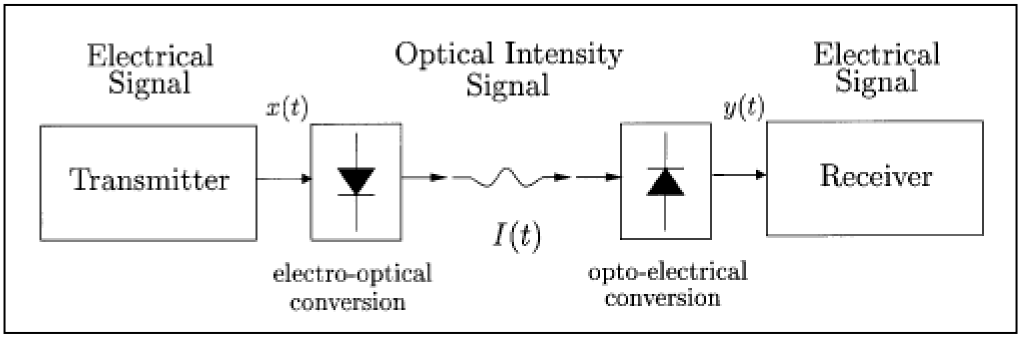


Fig.2.9 A simplified block diagram of an optical intensity direct detection communications system.

2.8 Noise

Intrinsic noise, random and uncorrelated fluctuations of signals, is a fundamental ingredient in any measuring process. Two major electrical noises is considered as thermal noise and shot noise. Thermal noise is an energy equilibrium fluctuation phenomenon whereas shot noise involves current fluctuations, which deliver power to the system in question. Both are inherent noise that is always present in a real electrical system and represent fundamental limitations and difficulties in making sensitive electrical measurements.

Thermal noise arises from the thermal fluctuations in the electron density within a conductor. Thermal noise is shown as

$$<I_{th}^{2}>= 4KTB/R_L$$

where, R is the resistance of the conductor, k is Boltzmann's constant, T is the absolute temperature, B is the bandwidth and R_L is the load resistance.

Another random noise signal is due to fluctuations in current. Temperature limited vacuum diodes, zener diodes, heated resistors, and gas discharge tubes can generate these fluctuations. The instantaneous anode current deviates from the average value due to the discreteness of the electron's charge, and this fluctuation in the anode current is called shot noise. The mean squared value of the noise current, according to Schottky formula is

$$<I_{sh}^{2}>= 2eBI_{shot}$$

where, e is the charge of the electron, I_{shot} is the average dc diode current, and B is the bandwidth. Shot noise generates due to the signal current, due to background light and due to leakage current.

Total noise is the addition of thermal noise and shot noise.

2.9 Signal to Noise Ratio (SNR)

Signal-to-noise ratio (often abbreviated SNR or S/N) is a measure used in science and engineering to quantify how much a signal has been corrupted by noise. It is defined as the ratio of signal power to the noise power corrupting the signal.

$$SNR = \frac{\text{Signal power from photocurrent}}{\text{Detector Noise + Amplifier Noise}}$$

To achieve high SNR, the Photo detector must have large quantum efficiency to generate large signal current. Also Detector and amplifier noise must be low. One important thing is SNR cannot be improved by amplification.

2.10 Bit Error Rate (BER)

BER is the ratio of erroneous bits to correct bit. Estimate of BER often needed to estimate the performance of a communication link. BER depends on the signal and noise power i.e. Signal to Noise Ratio (SNR). The bit error rate or bit error ratio (BER) is the number of bit errors divided by the total number of transferred bits during a studied time interval. Too high a BER may indicate that a slower data rate would actually improve overall transmission time for a given amount of transmitted data since the BER might be reduced, lowering the number of packets that had to be resent.

2.11 Summary

This chapter discusses the basics of Free Space Optical Communication System along with its application, advantages, and main challenges in FSO and the effect of atmospheric turbulence on the FSO communication link.

Chapter 3

Analysis of FSO link from Propagation through Clouds

Chapter: 3

Analysis of FSO link from Propagation through Clouds

3.1 Introduction

Space optical communication from satellite to earth (ground or airplane) occasionally involves clouds as part of the optical channel. In this chapter, based upon Monte Carlo simulations, mathematical models are developed for the temporal characteristics of optical pulse propagation through clouds. These include temporal impulse response, transfer function, bandwidth, received energy and bode analysis [8]. The simulation results strongly support the use of the double gamma function model to best describe optical pulse spread through clouds.

In this chapter also the details of FSO link through atmospheric turbulence channel of Cloud considering Intensity Modulation Direct Detection scheme is discussed analytically. When a signal is passed through Clouds, Clouds as part of the communication channel cause signal power attenuation and spatial widening. These effects decrease the received signal power and increase the bit error rate (BER). When optical radiation propagates through physically and optically thin clouds, most of the radiation power passes through the clouds but is widened in the spatial domain through scattering processes. Basically an analytical approach is described in this chapter.

3.2 System Model

A Free Space Optical link basically consists of a transmitter, Atmospheric channel as the medium of transmission which may include Cloud and a receiver to reproduce the transmitted signal. The Block diagram of the basic FSO system is shown in Fig 3.1

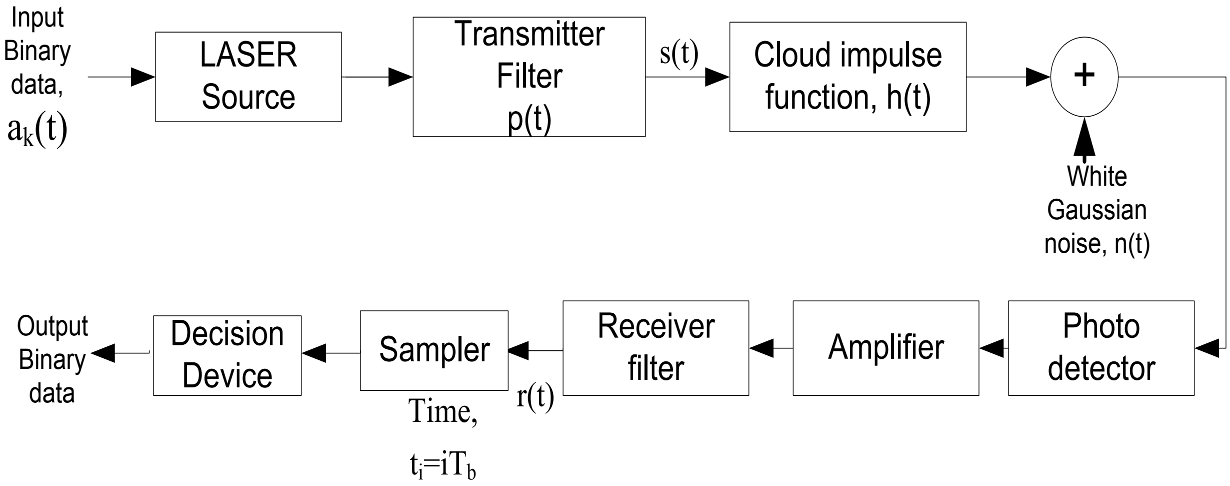


Fig 3.1 Basic FSO System Model

Intensity modulation (IM) is a form of modulation in which the optical power output of a source is varied in accordance with some characteristic of the modulating signal. The reception and regeneration of the modulating signal is done by direct detection technique. In the free-space, optical intensity-modulated direct detection (IM/DD) is frequently used. Most practical wireless optical channels use light emitting diodes or Laser diodes as transmitters and photodiodes as detectors, as shown below. These devices modulate and detect solely the intensity of the carrier, not its phase, which implies that all transmitted signal intensities are nonnegative.

The input binary data is used to modulate the LASER using intensity modulation and thus pass through the transmitter filter and then it is passed through the atmosphere where the impulse response of cloud has a greater impact. The optical signal is detected by a photo detector and received by the receiver circuit. The Sampler and Decision device is used to determine the output binary data.

3.3 Cumulus Cloud Model and Gamma Constant

Cumulus clouds are generally located at elevations between 200 and 20000 feet. This elevation range is very relevant for space communication involving planes. Low lever stratus clouds are usually between 1000 and 2000 feet elevation and as such are unsuitable for aircraft communication. Since particulate scatter close to the receiver is of more serious consequences than scatter far away from it, we will concentrate here on cumulus clouds [8].

A cumulus cloud contains water droplets, which are of various radii. The droplets' size distribution has been found to obey the modified gamma distribution:

$$n(r)=2.604r^{3}\exp(-0.5r) \tag{3.1}$$

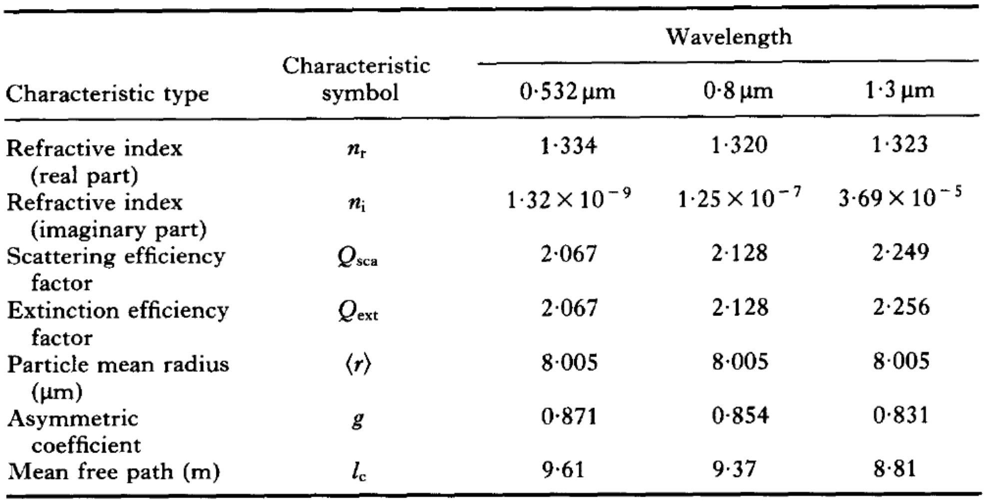
where r is the particle radius (μm) and n(r) is the particle size distribution .

The total particulate density number in a cumulus cloud is about 250 particulates cm^{-3} [8].The average cloud base height above ground level is about 3400m. The refractive indices of water at the three wavelengths were estimated from the literature. Mie equations were solved for the poly dispersion case in order to calculate the scattering and absorption cross sections [8] :

$$S[\lambda, n(x)]=\pi k^{-3}\int_0 x^2 n(x) Q(x) dx \tag{3.2}$$

where S is either the scattering or absorption cross section of the poly dispersion

The integral in equation (3.2) was evaluated numerically. Mie phase functions were also calculated for the three wavelengths, with 200 point resolution. In addition, the asymmetric coefficient was calculated, as was the mean free path between successive collisions. All the single scattering characteristics of the cumulus cloud at the three wavelengths are summarized in table 1.



Characteristic type	Characteristic symbol	Wavelength		
		0·532 μm	0·8 μm	1·3 μm
Refractive index (real part)	n_r	1·334	1·320	1·323
Refractive index (imaginary part)	n_i	$1{\cdot}32\times10^{-9}$	$1{\cdot}25\times10^{-7}$	$3{\cdot}69\times10^{-5}$
Scattering efficiency factor	Q_{sca}	2·067	2·128	2·249
Extinction efficiency factor	Q_{ext}	2·067	2·128	2·256
Particle mean radius (μm)	$\langle r\rangle$	8·005	8·005	8·005
Asymmetric coefficient	g	0·871	0·854	0·831
Mean free path (m)	l_c	9·61	9·37	8·81

Table 3.1 Cumulus single scattering parameters

It is assumed that light is propagating in a medium that contains vacuum and particulates with known density. The shape of the medium is cylindrical. The cylinder radius is five units, and its height is one unit. The receiver is placed at the bottom centre, and the transmitter at the top centre. Figure 4 describes the geometry of the propagation model. The light is monochromatic with frequency f and photon energy hf. Photon emission is assumed to be a delta function to both spatial and angular coordinates, i .e. it contains neither spatial not angular spread . Emitted photons move in space until colliding with a particulate. The new direction of the photon, if it does not escape or is absorbed or received, derives from three elements: the rotation angle 0 with uniform distribution, the polar angle 0 with distribution according to the Mie phase diagram, and the distance r to the next collision with an exponential distribution. The probability of a photon being absorbed or scattered depends on the relation between the scattering and absorption cross sections.

The photon tracing ends when one of the following happens: (i) the photon is absorbed, (ii) the photon is received by the detector, i .e . it arrives at the bottom cylinder cover,

(iii) the photon is back scattered, i .e . it is returned to the upper cylinder cover, or (iv) the photon escapes, i .e . it arrives at the side cylinder cover .

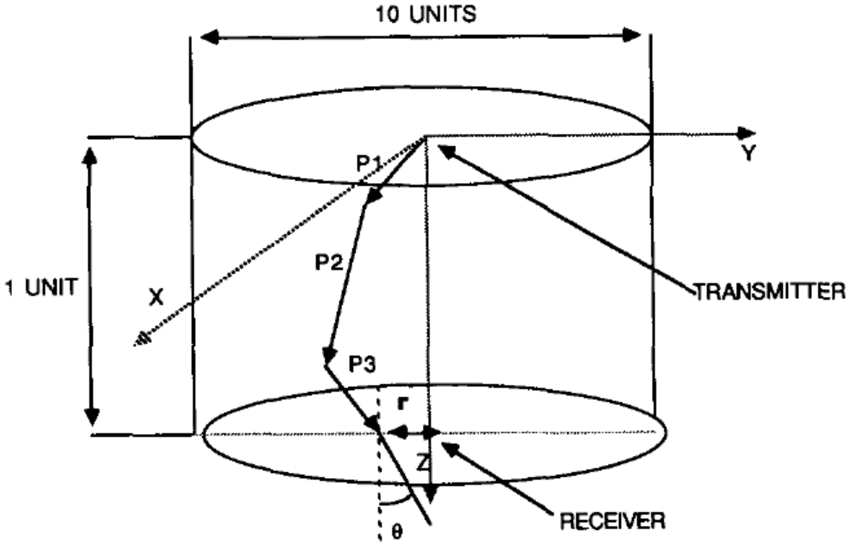
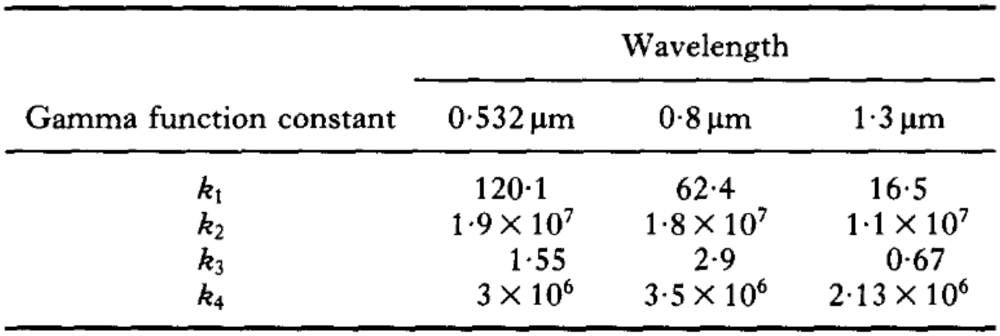


Fig 3.2 Geometry of the simulation propagation model

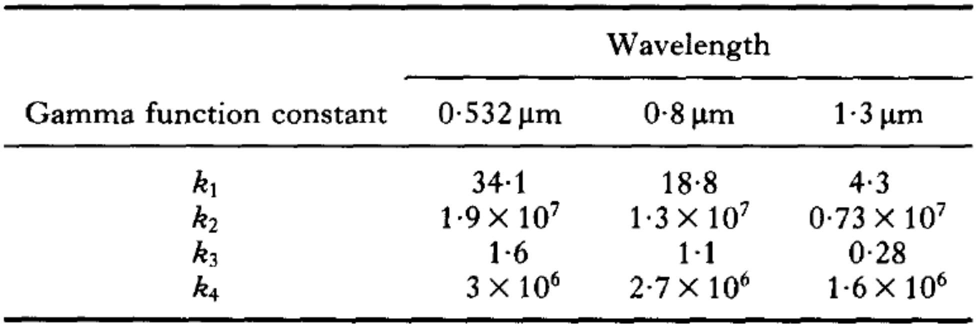
For practical reasons, all the scattered light characteristics refer only to the central part of the beam near the optical axis. This is relevant to conventional detector sizes and derives from calculation of the scattered light spatial distribution for each simulation. The simulations only considered photons from the central part of the beam constituting 5% of the total transmitted energy. Only photons from this part of the beam are relevant when dealing with optical communication or other practical remote-sensing applications, since the rest of the photons arrive at large transverse distances, irrelevant for any practical receiver. It is assumed that the scatter at the beam centre upon the bottom cylinder cover is uniform. Calculations indicate that the unscattered component of the radiation comprises only exp (- 20) of the total radiation incident on the bottom cylinder cover. Receiver field-of-view is unrestricted in order to collect as many signal photons as possible and in this way to minimize transmitter power.

The scattered light spatial distribution is calculated for each simulation. Only photons from the central part of the beam are considered which constitute 5% of the total transmitted energy. The geometrical path lengths of all the scattered photons are calculated. These are the geometrical paths illustrated in figure 3.3 designated as P_i. They are divided by the speed of light c to evaluate the temporal impulse response. The distance between each photon on the bottom cylinder cover to the centre was calculated, together with the mean radius, defining an average blocking circle. The temporal impulse response is normalized by this area, representing the receiver area, constituting the final result of the normalized cloud impulse response. This procedure is repeated for the three wavelengths 0.532 μm, 0.8 μm and 1.3 μm, and for different cloud geometrical thicknesses from 200 to 275 m, in steps of 25 m. Using least mean square fits between simulation results and the double gamma function model, the constants k(c,) were evaluated for each cloud thickness and at each radiation wavelength; these are summarized in Tables 3.2-3.5



	Wavelength		
Gamma function constant	0·532 μm	0·8 μm	1·3 μm
k_1	120·1	62·4	16·5
k_2	$1{\cdot}9 \times 10^7$	$1{\cdot}8 \times 10^7$	$1{\cdot}1 \times 10^7$
k_3	1·55	2·9	0·67
k_4	3×10^6	$3{\cdot}5 \times 10^6$	$2{\cdot}13 \times 10^6$

Table 3.2 Double gamma function constants: cloud thickness = 200 m.



	Wavelength		
Gamma function constant	0·532 μm	0·8 μm	1·3 μm
k_1	34·1	18·8	4·3
k_2	$1{\cdot}9 \times 10^7$	$1{\cdot}3 \times 10^7$	$0{\cdot}73 \times 10^7$
k_3	1·6	1·1	0·28
k_4	3×10^6	$2{\cdot}7 \times 10^6$	$1{\cdot}6 \times 10^6$

Table 3.3 Double gamma function constants: cloud thickness = 225 m.

Gamma function constant	Wavelength		
	0·532 μm	0·8 μm	1·3 μm
k_1	12·4	5·2	2
k_2	$1{\cdot}1 \times 10^7$	$0{\cdot}83 \times 10^7$	$0{\cdot}71 \times 10^7$
k_3	0·66	0·41	0·3
k_4	$2{\cdot}4 \times 10^6$	$1{\cdot}9 \times 10^6$	$1{\cdot}8 \times 10^6$

Table 3.4 Double gamma function constants: cloud thickness = 250 m.

Gamma function constant	Wavelength		
	0·532 μm	0·8 μm	1·3 μm
k_1	5·1	2·3	0·53
k_2	$0{\cdot}8 \times 10^7$	$0{\cdot}65 \times 10^7$	$0{\cdot}75 \times 10^7$
k_3	0·28	0·2	0·31
k_4	$1{\cdot}8 \times 10^6$	$1{\cdot}5 \times 10^6$	2×10^6

Table 3.5 Double gamma function constants: cloud thickness = 275 m.

3.4 Transfer Function of Cloud

Optical radiation propagating through clouds experiences temporal distortions. A function that describes well the temporal impulse response is the double gamma function [4-6]:

$$h(t)=\{k_1(c_1)t\exp[-k_2(c_1)t]+k_3(c_1)t\exp[-k_4(c_1)t]\}U(t) \qquad (3.3)$$

where h(t) is in m $^{-2}$, cl is a parameter defining the physical characteristics of the optical channel such as particulate size distribution, particulate refractive index, geometrical cloud thickness, and radiation wavelength, kl-k4 are the gamma function constants depending on cl, and U(t) is a unit step function .

The optical power density at the receiver is a function of this temporal broadening, the transmitted power and the receiver aperture, and is given by

$$P_R(c_1, t, D_R, P_T)= \{k_1(c_1)t\exp[-k_2(c_1)t]+k_3(c_1)t\exp[-k_4(c_1)t]\}\frac{1}{4}D_R^2\pi P_T U(t) \qquad (3.4)$$

where P_R the optical received power (W), D_R is the receiver aperture diameter (m)

and P_T the transmitted power (W).

Broadening of the signal's optical pulses arriving at the receiver due to the scattering cloud causes narrowing of the optical system's frequency bandwidth . The temporal frequency transfer function can be evaluated by Fourier transforming the temporal impulse response :

$$H(f)=\int_{\_} h(t)\exp(-j2\pi ft)dt \qquad (3.5)$$

where f is the temporal frequency (Hz) . Substituting (3.1) into (3.3) yields

$$H(f)=\left\{\frac{k_1(c_1)}{[k_2(c_1+j2\pi f)]^2}+\frac{k_3(c_1)}{[k_4(c_1+j2\pi f)]^2}\right\} \qquad (3.6)$$

where H(f) is in m $^{-2}$.

According to equation (3.5), the cloud's transfer function can be represented in terms of multiplications of poles and zeros

$$H(f)=G\left[1+j\left(\frac{f-b}{f_3}\right)\right]\left[1+j\left(\frac{f+b}{f_3}\right)\right]\Big/\left[1+j\left(\frac{f}{f_1}\right)\right]^2\left[1+j\left(\frac{f}{f_2}\right)\right]^2 \qquad (3.7)$$

where fl -f3, b and G are given by

$$f_1=\frac{k_2}{2\pi}$$

$$f_2=\frac{k_4}{2\pi}$$

$$f_3=\frac{(k_1k_4+k_3k_2)}{2\pi(k_1+k_3)}$$

$$b=\frac{4\pi^2(k_1+k_3)}{(k_2k_4)^2}f_3^2$$

$$G=\frac{4\pi^2(k_1+k_3)}{(k_2k_4)^2}k_3^2$$

Equation (3.7) is the general transfer function of the cloud. It contains two double poles at the frequencies f_1 and f_2. Every double pole changes the slope of the transfer function's absolute value by-40 db/decade, and changes the phase of the transfer function asymptotically by - 180 degrees, and by 90 degrees at the knee frequency. The transfer function also contains two zeros at the frequency f_3 shifted by b and - b. Every zero changes the slope of the transfer function's absolute value by 20 db/decade, and changes the phase of the transfer function asymptotically by 90 degrees, and by 45 degrees at the knee frequency. The total slope of the transfer function's absolute value and its total phase, for frequencies larger than f_1, f_2 and f_3 + b, are - 40 db/decade and - 180 degrees, respectively.

Bode analysis of the cloud transfer function is a very practical mathematical tool that can be used to characterize the cloud channel as part of the more general communication channel (including transmitter and receiver electronics). It can also be used to design adaptive filters at the receiver as mentioned previously, in order to minimize the effects of clouds on bit error rate. One way is by using a match-filter technique. Assuming the transmitted pulse is a temporal delta function, the received pulse shape after propagating through the cloud channel is in the form of the cloud's transfer function. By using an adaptive match filter [2], according to the clouds transfer function as in equation (3.6) or equation (3.7), the received signal-to-noise ratio can be improved significantly.

3.5 Analysis of SINR and BER

The transmitted optical signal is given by

$$s(t)=\sqrt{2p_T}\sum_{k=-\infty}^{\infty} a_k p\,(t-kT_b)\exp\,(j\omega_c t) \qquad (3.8)$$

where P_T is the transmitted optical power, a_k is the k-th information bit whose value is 1 and 0, p(t) is the optical pulse shape of bit duration T_b and carrier frequency of f_c. The received optical signal is given by

$$r(t)=s(t)\otimes p(t) + n_b(t) \qquad (3.9)$$

where h(t) is the impulse response of cloud as described in section 3.4 and $n_b(t)$ is the total background noise component.

The received optical signal is expressed as

$$r(t)=\sqrt{2P_s}\sum_{k=-} a_k p\,(t-kT_b)\otimes h(t)\,\exp\,(j\omega_c t)) + n_b(t)$$

$$=\sqrt{2P_s}\sum_{k=-} a_k g\,(t-kT_b)\exp\,(j\omega_c t) + n_b(t) \qquad (3.10)$$

where, P_s is the received optical power and $g(t)=h(t)\otimes p(t)$ is the received optical pulse shape which overlaps over a number of bits and produces Inter Symbol Interference (ISI).

The photodiode current can be expressed as

$$i(t)=|r(t)|^2 R_d$$

$$=2R_d P_s|\sum_{k=-} a_k g\,(t-kT_b)|^2 + i_n(t) \qquad (3.11)$$

where, R_d is the responsivity of the detector and $i_n(t)$ is the noise current due to photodiode and receiver noise which can be expressed as

$$i_n(t) = i_{sh}(t) + i_{th}(t) \qquad (3.12)$$

From equation (3.3) is expressed as,

$$r(t)=2R_d p_s|a_0|^2.|g(t)|^2+\sum_{\substack{k\neq 0\\ k=-}} 2R_d p_s|a_k g(t-kT_b)|^2 + i_n(t)$$

$$=i_s(t) + i_{isi}(t) + i_n(t) \qquad (3.13)$$

where, $i_s(t)$= Signal Current = $2R_d P_s|a_0|^2.|g(t)|^2$

$i_{isi}(t)$= ISI Current= $\sum_{\substack{k\neq 0\\ k=-}} 2R_d p_s|a_k g(t-kT_b)|^2$

Now the mean signal current and mean ISI current can be expressed as

$$\text{Signal Current } I_s = 2R_d p_s|a_o|^2 \frac{1}{T_b}\int_0^{T_b}|g(t)|^2 dt \qquad (3.14)$$

$$\text{Mean ISI Current } I_{isi}=2R_d p_s \frac{1}{T_b}\int_0^{T_b}|a_k g(t-kT_b)|^2 dt \qquad (3.15)$$

SINR can be defined as the ratio of signal power to noise power

$$\text{SINR}=\frac{\text{Signal Power}}{\text{oise Power}}$$

$$=\left[\frac{{I_s}^2}{\sigma_n^2+\sigma_{isi}^2}\right] \qquad (3.16)$$

where,

$$\sigma_n^2 = \sigma_{shot}^2 + \sigma_{th}^2$$

$$\sigma_{shot}^2 = 2eBI_{shot}$$

$$\sigma_{th}^2 = 4KTB/R_L$$

From Appendix A, the expression of BER for Intensity Modulation Direct detection (IM/DD) can be expressed as,

$$\text{BER} = 0.5\,\text{erfc}\left(\frac{\sqrt{\text{SINR}}}{2\sqrt{2}}\right) \qquad (3.17)$$

3.6 Summary

In this chapter theoretical analysis of Signal to noise ratio and Bit error rate calculation using the Intensity Modulation Direct Detection is described with the presence of Cloud. The cloud transfer function is used to analyze the effect of Cloud on FSO.

Chapter 4

Results and Discussion

Chapter-4

Results and Discussion

4.1 Introduction

Following the theoretical analysis, the performance of an FSO link considering the effect of Cloud is evaluated. The analytical results are presented and discussed in this chapter. Here the performance has been shown in terms of Signal to Interference and Noise Ratio (SINR) and Bit Error Rate (BER) for an FSO communication system in presence of Cloud with varying different parameters.

4.2 Results and Discussion

In this thesis the performance of FSO communication through atmospheric channel in presence of Cloud using Intensity Modulation Direct Direction (IM/DD) is analyzed. In this thesis the computation is carried out considering without Cloud effect first and gradually moving forwards to Cloud effect in different parameters including bit rate, cloud thickness and wavelength. The transfer function of cloud is used to simulate the effects of cloud. A comparison with and without cloud is also presented.

The numerical computations are carried out by using MATLAB software package. The parameters used for computation in this chapter are shown in the Table 4.1.

Parameter Name	**Value**
Bit Rate	0.01Gbps, 0.1Gbps, 0.5Gbps,1 Gbps,10Gbps
Responsivity of the Detector	0.85
Band width of the optical filter	0.01GHz, 0.1GHz, 0.5GHz,1 GHz,10GHz
Load resistance	50Ω
Temperature	300K

Gamma function constant	**Wavelengths at Cloud Thickness of 200m**		
	0.532μm	***0.8μm***	***1.3μm***
K_1	120.1	62.4	16.5
K_2	$1.9\text{x}10^7$	$1.8\text{x}10^7$	$1.1\text{x}10^7$
K_3	1.55	2.9	0.67
K_4	$3\text{x}10^6$	$3.5\text{x}10^6$	$2.13\text{x}10^6$

Table 4.1 Nominal values used in FSO communication link in Presence of Cloud

Although in most cases Cloud thickness of 200m is used but actually variable cloud thickness of 225m, 250m and 275m are also used.

4.3 Performance Analysis of a FSO System without Cloud Effect

Fig 4.1 shows the plots of SNR versus Received power, Ps for an FSO link without the effect of cloud at a bit rate of 1Gbps. It is noticed that SNR increases with the increase in signal power Ps which is also shown in tabular form in Table 4.2.

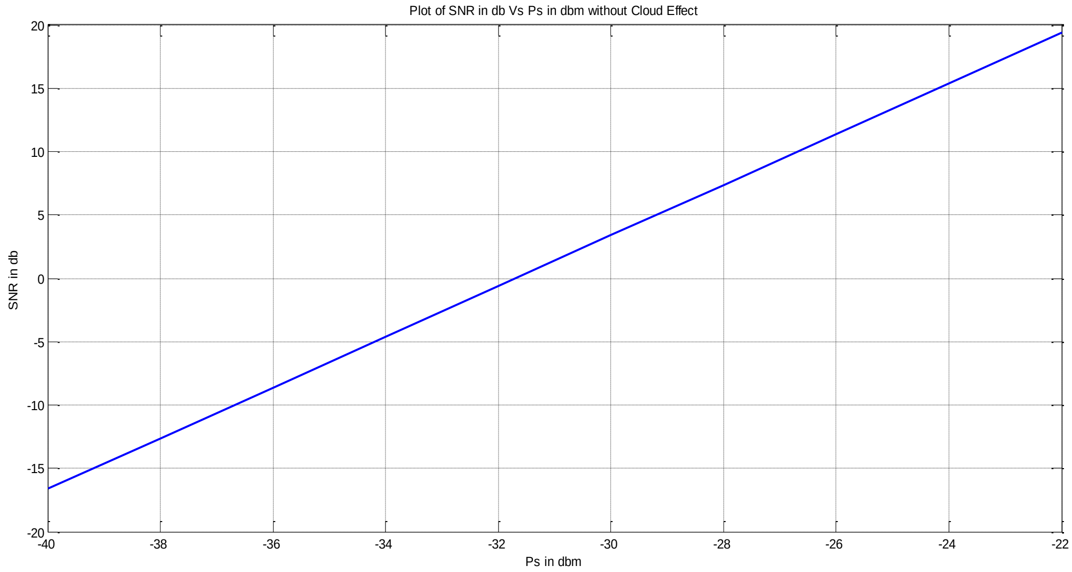


Fig 4.1 Plot of SNR Vs received signal power without Cloud Effect at a bit rate of 1Gbps

Signal Power in dbm	SNR in db
-40	-16.61
-32	-0.6125
-28	+7.4
-24	+15.4

Table 4.2: Numerical values of SNR Vs Received signal power in dbm

Fig 4.2 shows the plots of BER versus received power, Ps for an FSO link without the effect of cloud at a bit rate of 1Gbps. It is noticed that BER decreases with the increase in signal power Ps since in Fig 4.1 it is shown that with increase in signal power SNR increases. If a certain BER of 10^{-9} is to be maintained then the receiver sensitivity should be -25dbm which is shown in tabular form in Table 4.3

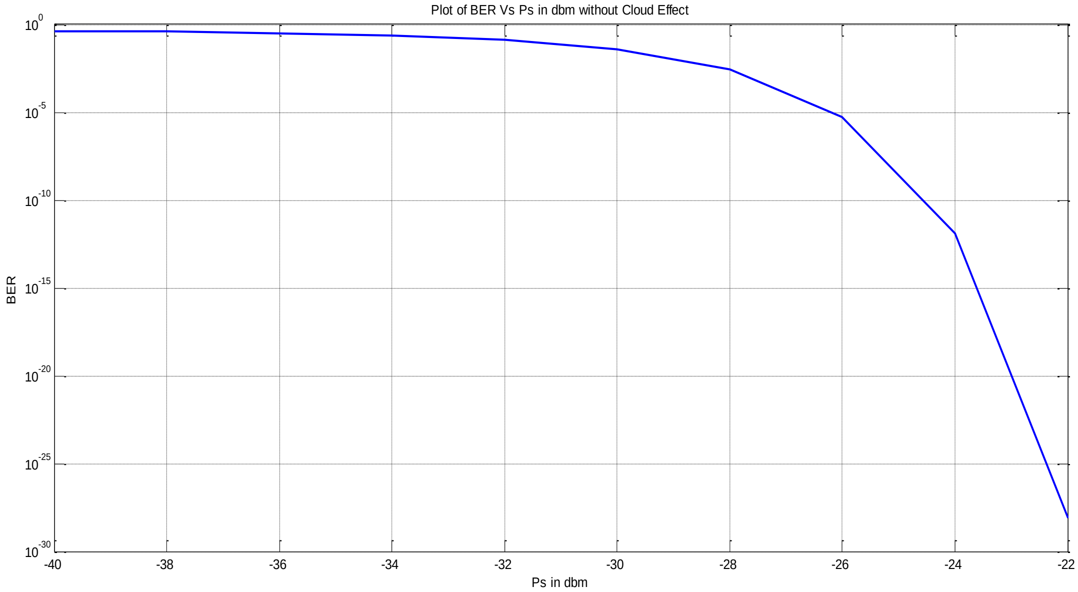


Fig 4.2 Plot of BER Vs received signal power without Cloud Effect at a bit rate of 1Gbps

BER	Receiver Sensitivity, Ps in dbm
10^{-6}	-25.8
10^{-9}	-25
10^{-12}	-24

Table 4.3: Numerical analysis of BER Vs received signal power in dbm

Fig 4.3 shows the plots of SNR versus received power, Ps for an FSO link without the effect of cloud with bit rate as a parameter. It is noticed that SNR decreases significantly with the increase in bit rate R_b which is also shown in tabular form in Table 4.4.

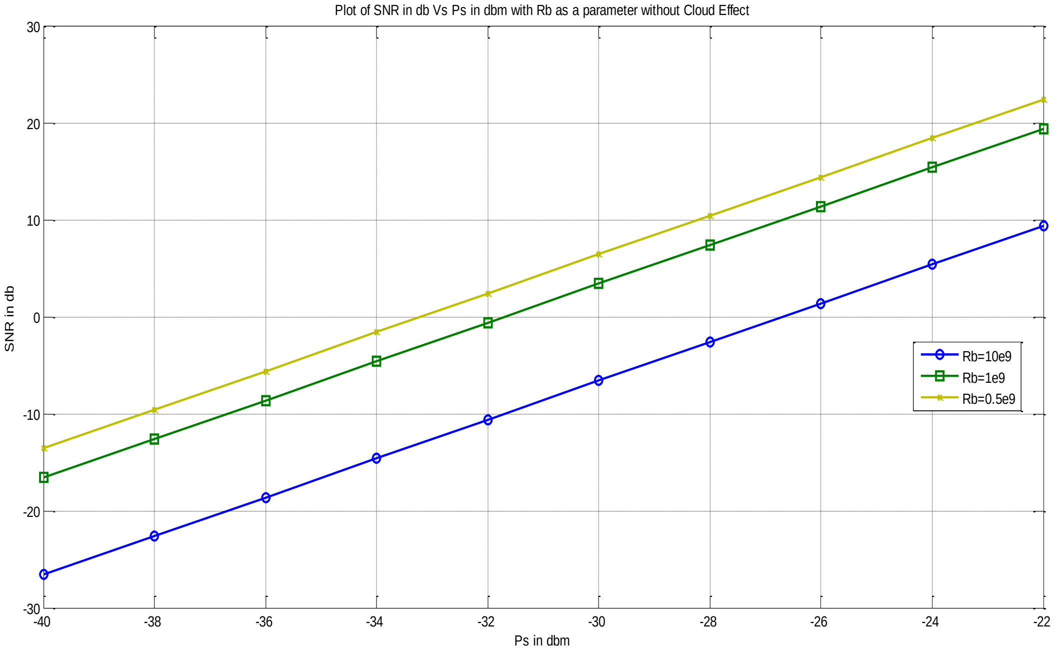


Fig 4.3 Plots of SNR Vs received signal power with R_b as a Parameter without Cloud Effect

Signal power in dbm	**SNR without Cloud Effect with R_b as a parameter**		
	$R_b = 0.5x10^9$	**$R_b = 10^9$**	**$R_b = 10x10^9$**
-40	-13.6	-16.61	-26.61
-32	2.4	-0.6125	-10.61
-24	18.4	15.4	5.4

Table 4.4: Numerical analysis of SNR Vs received signal power in dbm with R_b as a parameter

Fig 4.4 shows the plots of BER versus received power, Ps for an FSO link without the effect of cloud with bit rate as a parameter. It is noticed that BER increases significantly with the increase in bit rate R_b which is also shown in tabular form in Table 4.5. From Table 4.5 it is evident that to achieve BER of 10^{-9}, if the bit rate is $0.5x10^9$ then receiver sensitivity requirement is -26.3dbm where as if the bit rate is 10^9 then receiver sensitivity requirement is -25dbm.

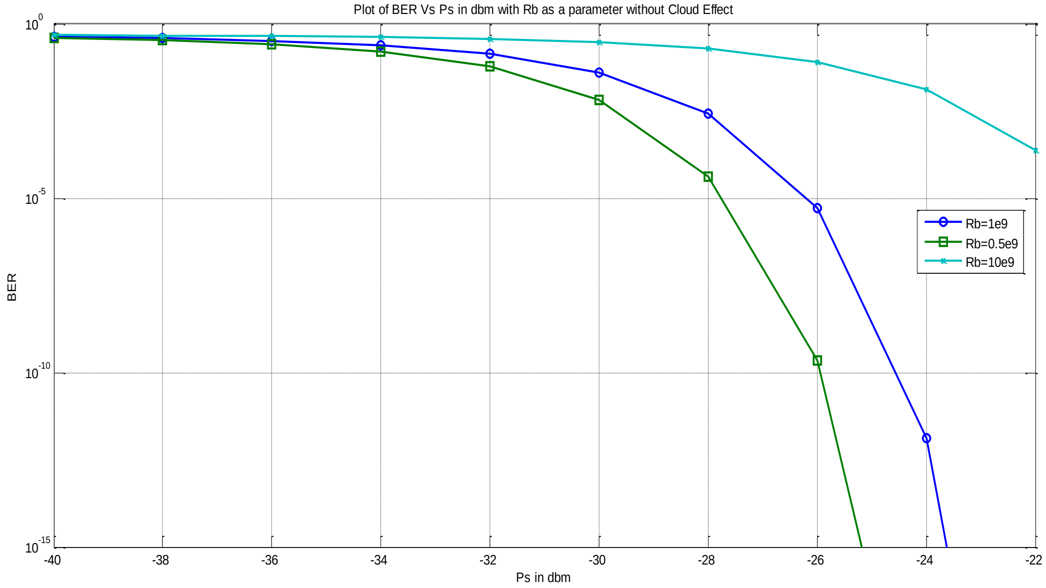


Fig 4.4 Plots of BER Vs received signal power with Rb as a Parameter without Cloud Effect

BER	**Receiver Sensitivity, P_s in dbm at different R_b**	
	R_b =0.5x10^9	**R_b =10^9**
10^{-6}	-27.4	-25.8
10^{-9}	-26.3	-25
10^{-12}	-25.7	-24

Table 4.5: Numerical values of BER Vs received signal power in dbm with R_b as a parameter

4.4 Performance Analysis of Impulse Response of Cloud

Fig 4.5 depicts the plot of the impulse response of a cloud as a function of time where the cloud thickness is 200m. It is noticed that the impulse response continues over a number of bits and thus causes inter symbol interference (ISI) which actually lessen the BER performance.

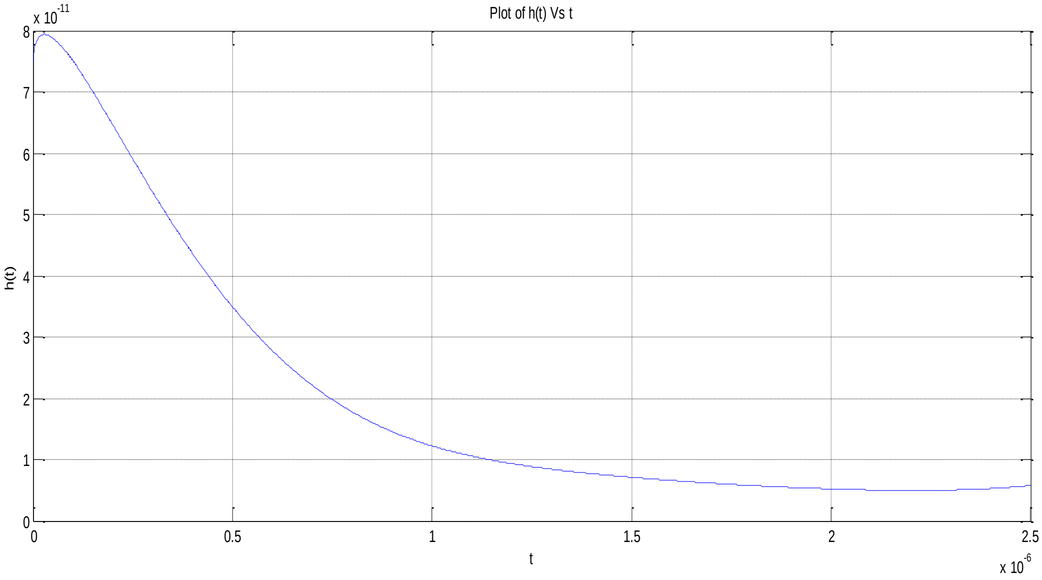


Fig 4.5 Plot of impulse response of Cloud h(t) Vs time t at a Cloud thickness of 200m

Fig 4.6 depicts the plot of the impulse response of a cloud as a function of wavelength where the cloud thickness is 200m. Three wavelengths of 0.532µm, 0.8µm and 1.3µm are considered and found that in 0.532 µm the amplitude of the impulse response is higher. In the shorter wavelength region of 0.532 μm is the preferred wavelength region for optical communication according to criteria of least power transmission and least spatial widening. This is because of low absorption at this wavelength and greater concentration of scattered light at small angles in the forward direction. In high wavelength BER increases with SNR decreases and thus in shorter IR wavelengths such as 0.532 μm is the best preferred for FSO which is shown in Fig 4.6

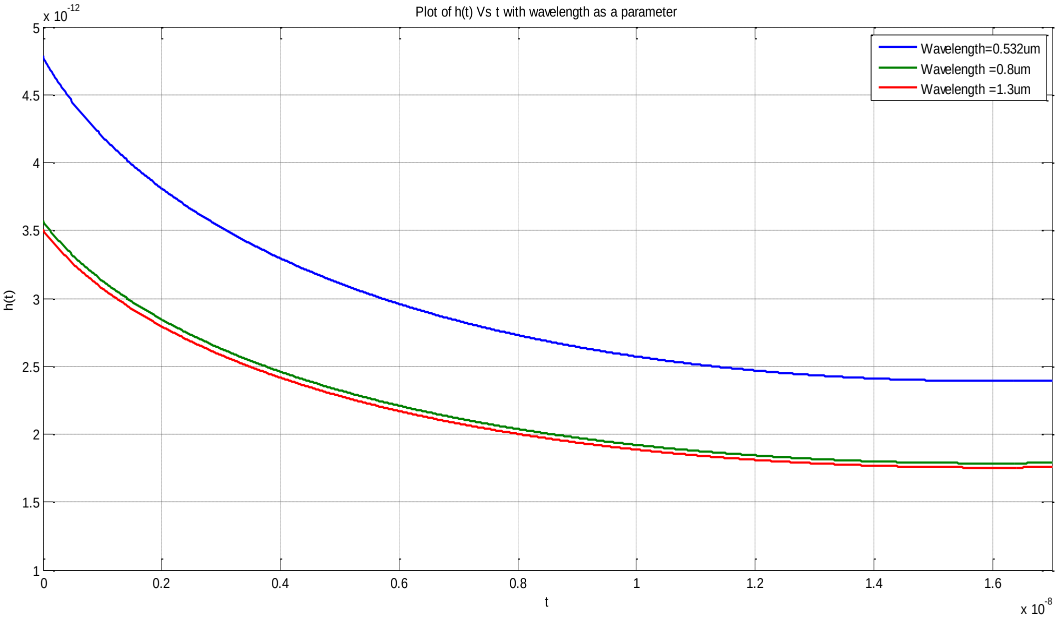


Fig 4.6 Plots of impulse response of Cloud h(t) Vs t with wavelength as a parameter at Cloud thickness of 200m

4.5 Performance Analysis of FSO System with and without Cloud Effect

Fig 4.7 shows the SNR versus Received power, Ps with and without the cloud effect. Due to the effect of Cloud SNR reduces significantly which is depicted in fig 4.7 and in tabular form in Table 4.6. SNR with cloud thickness of 200m is used for computation. A bit rate of 1Gbps is used to find out the cloud effect.

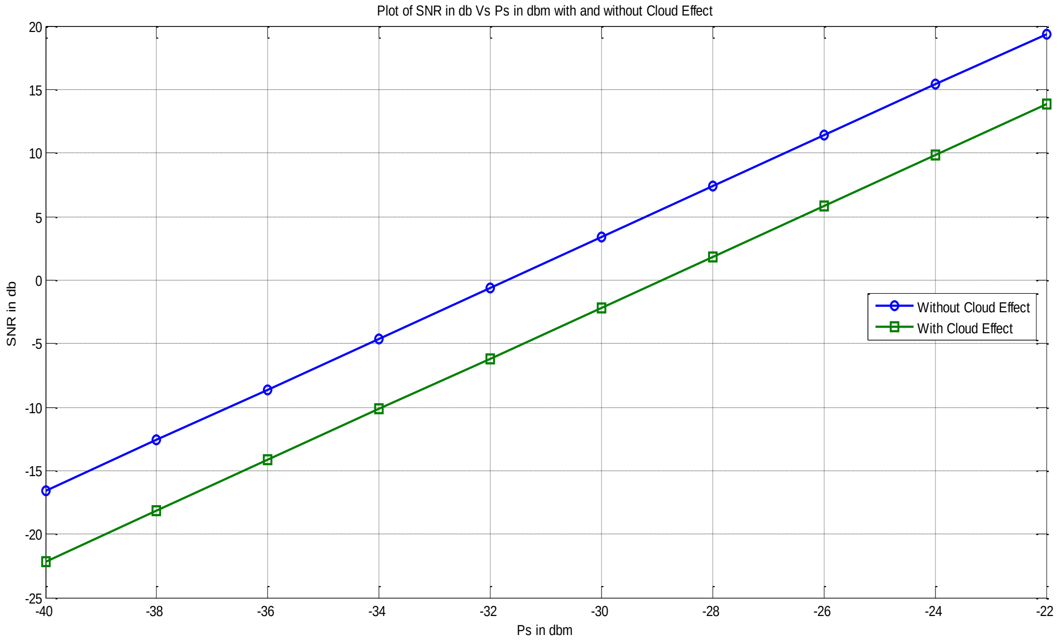


Fig 4.7 Plots of SNR Vs received signal power with and without Cloud Effect at a bit rate of 1 Gbps

Power in dbm	**SNR without Cloud Effect**	**SNR with Cloud Effect**
-40	-16.61	-22.17
-32	-0.6125	-6.2
-24	15.39	9.83

Table 4.6: Numerical values of SNR Vs Signal power with and without Cloud Effect

Fig 4.8 shows the BER versus Received power, Ps with and without the cloud effect. Due to the effect of Cloud BER increases significantly that is depicted in Fig 4.8 and in tabular form in Table 4.7. BER with cloud thickness of 200m is used for computation. A bit rate of 1Gbps is used to find out the cloud effect. From Table 4.7 it is evident that to achieve BER of 10^{-6}, the receiver sensitivity requirement is -25.8dbm without cloud effect whereas considering cloud effect, the receiver sensitivity requirement is -23dbm.

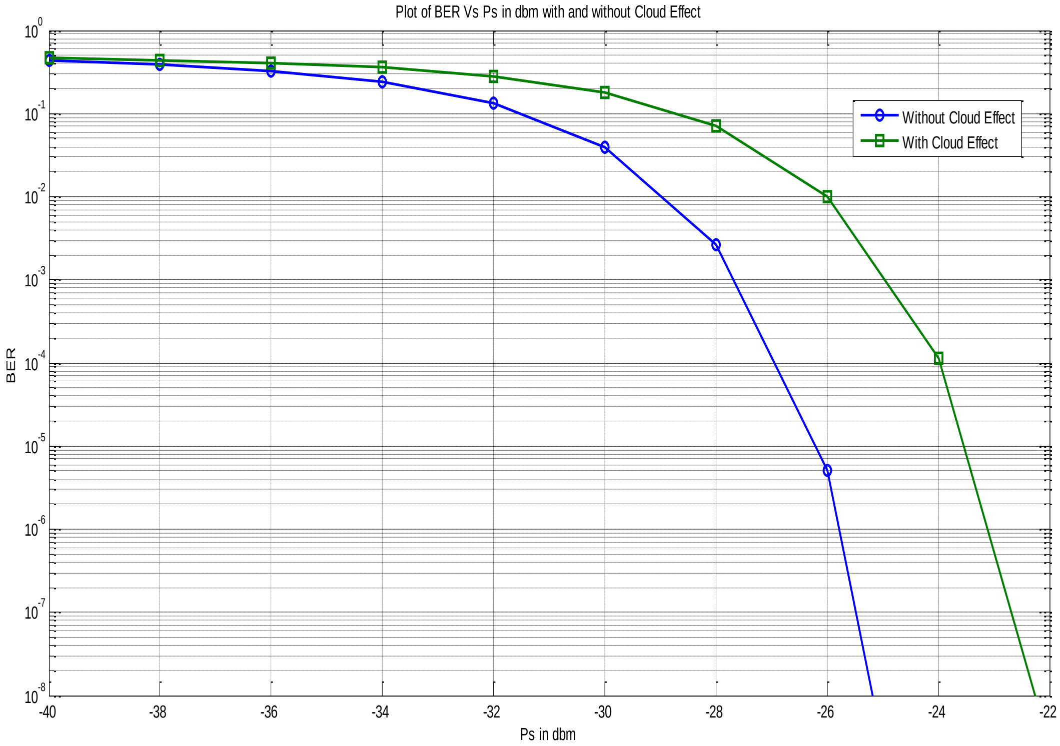


Fig 4.8 Plots of BER Vs received signal power with and without Cloud Effect at a bit rate of 1 Gbps

BER	**Receiver Sensitivity, P_s in dbm**	
	Without Cloud Effect	**With Cloud Effect**
10^{-4}	-27	-24
10^{-6}	-25.8	-23
10^{-8}	-25	-22.2

Table 4.7: Numerical values of BER Vs Signal power with and without Cloud Effect

4.6 Performance Analysis of FSO System considering the Effect of Cloud with varying different Parameters

Fig 4.9 shows the plots of SINR versus Received power, Ps for an FSO link with the effect of cloud with bit rate as a parameter. It is noticed that SINR decreases significantly with the increase in bit rate R_b which is also shown in tabular form in Table 4.8. For computation R_b is taken as 0.001Gbps, 0.1Gbps, 0.5Gbps, 1Gbps and 10Gbps and cloud thickness is taken as 200m.

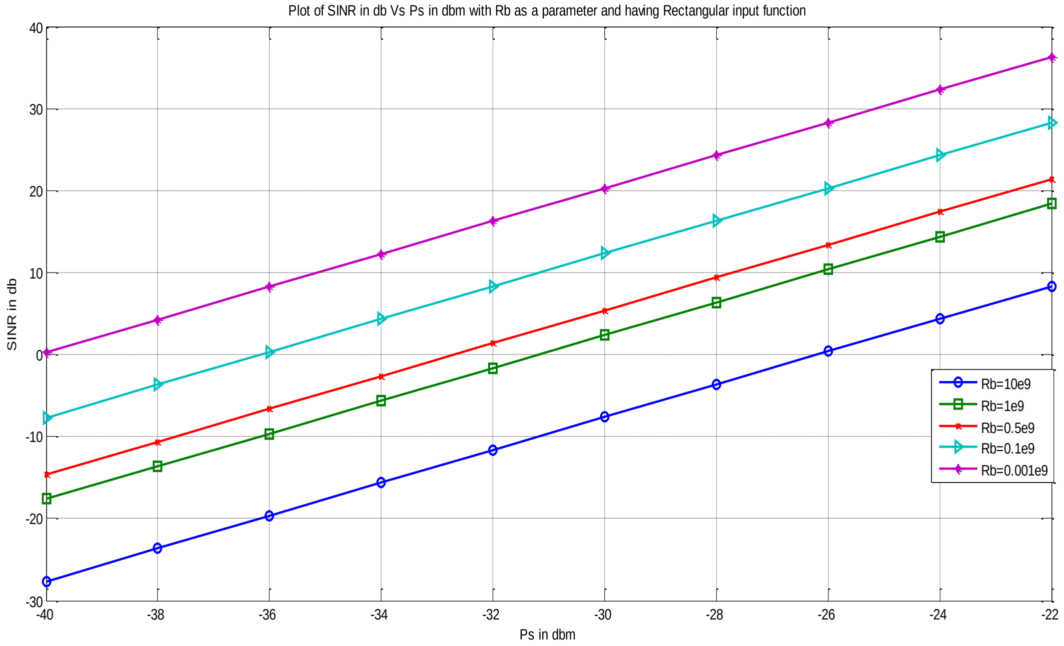


Fig 4.9 Plots of SINR Vs received signal power with Rb as a Parameter at a cloud thickness of 200m and wavelength of 0.532µm

Signal power in dbm	**SINR with the Cloud Effect with R_b as a parameter**				
	$R_b = 0.01x10^9$	**$R_b = 0.1x10^9$**	**$R_b = 0.5x10^9$**	**$R_b = 10^9$**	**$R_b = 10x10^9$**
-40	0.2896	-7.7	-14.62	-17.6	-27.6
-32	16.3	8.3	1.4	-1.6	-11.6
-24	32.3	24.3	17.4	14.4	4.4

Table 4.8: Numerical values of SINR Vs Signal power with the Cloud Effect with Rb as a parameter

Fig 4.10 shows the plots of BER versus Received power, Ps for an FSO link with the effect of cloud with bit rate as a parameter. It is noticed that BER increases significantly with the increase in bit rate R_b which is also shown in tabular form in Table 4.9. For computation R_b is taken as 0.001Gbps, 0.1Gbps, 0.5Gbps, 1Gbps and 10Gbps and cloud thickness is taken as 200m. From Table 4.9 it is evident that to achieve BER of 10^{-9}, the receiver sensitivity requirement is -33.6 dbm, -29.4 dbm, -25.9dbm and -24.6 dbm for bit rate of 0.01Gbps, 0.1Gbps, 0.5Gbps and 1Gbps respectively.

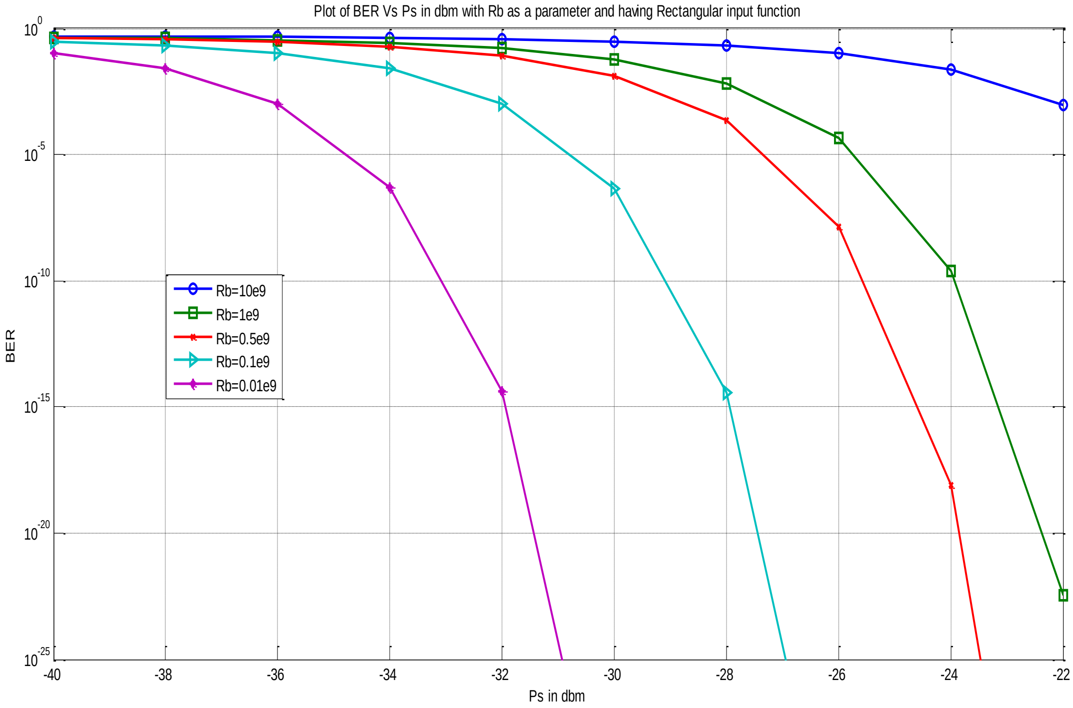


Fig 4.10 Plots of BER Vs received signal power with Rb as a Parameter at a cloud thickness of 200m and wavelength of 0.532µm

BER	**Receiver Sensitivity, P_s in dbm**			
	R_b =0.01x10^9	**R_b =0.1x10^9**	**R_b =0.5x10^9**	**R_b =10^9**
10^{-6}	-34.2	-30.2	-26.9	-25.4
10^{-9}	-33.6	-29.4	-25.9	-24.6
10^{-12}	-32.8	-28.8	-25.2	-23.7

Table 4.9: Numerical values of BER Vs received signal power with the Cloud Effect with Rb as a parameter

Fig 4.11 shows the plots of SINR versus Received power, Ps for an FSO link with cloud thickness as a parameter. It is noticed that SINR increases significantly with the increase in cloud thickness which is also shown in tabular form in Table 4.10. Cloud thickness is taken as 200m, 225m, 250m, 275m and Bit rate is taken as 1Gbps for computation.

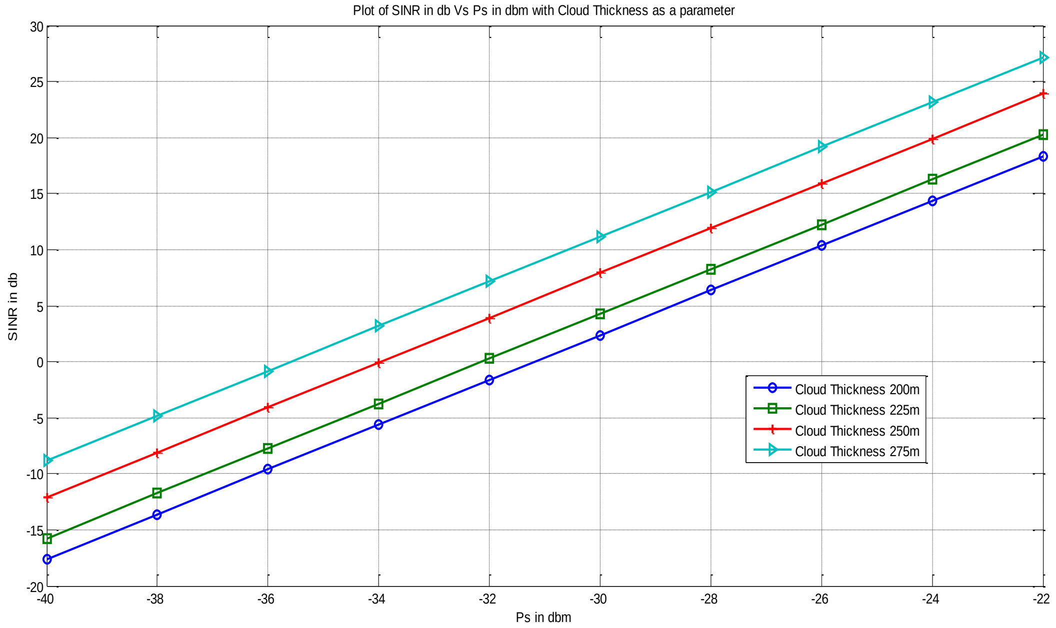


Fig 4.11 Plots of SINR Vs received signal power with Cloud Thickness as a Parameter at a bit rate of 1Gbps and wavelength of 0.532µm

Signal power in dbm	**SINR with Cloud Thickness as a parameter**			
	Thickness=200m	**Thickness=225m**	**Thickness=250m**	**Thickness=275m**
-36	-9.63	-7.8	-4.1	-0.82
-32	-1.63	0.252	3.9	7.2
-28	6.4	8.25	11.9	15.2

Table 4.10: Numerical values of SINR Vs received signal power with Cloud Thickness as a parameter

Fig 4.12 shows the plots of BER versus Received power, Ps for an FSO link with cloud thickness as a parameter. It is noticed that BER decreases significantly with the increase in cloud thickness which is also shown in Table 4.11. Cloud thickness is taken as 200m, 225m, 250m, 275m and Bit rate is taken as 1Gbps for computation. From Table 4.11 it is evident that to achieve BER of 10^{-9}, the receiver sensitivity requirement is -24.5 dbm, -25.7 dbm, -27.1 dbm and -28.8 dbm for cloud thickness of 200m, 225m, 250m and 275m respectively.

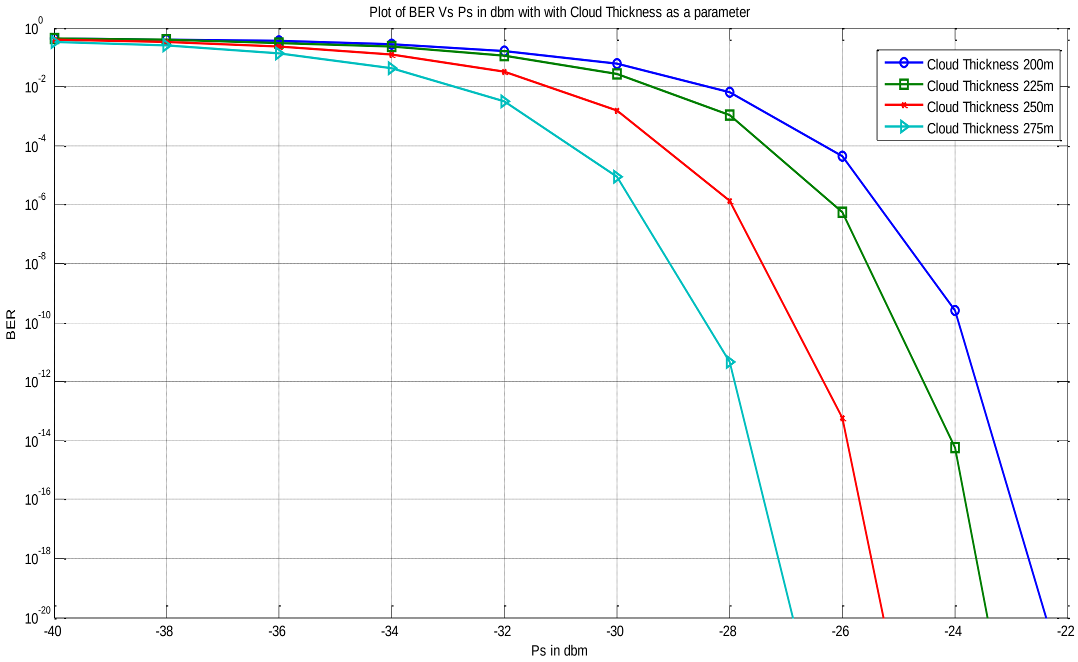


Fig 4.12 Plots of BER Vs received signal power with Cloud Thickness as a Parameter at a bit rate of 1Gbps and wavelength of 0.532µm

BER	**Receiver Sensitivity, P_s in dbm**			
	Thickness=200m	**Thickness=225m**	**Thickness=250m**	**Thickness=275m**
10^{-6}	-25.3	-26.3	-27.95	-29.7
10^{-9}	-24.5	-25.7	-27.1	-28.8
10^{-12}	-23.8	-25.2	-26.7	-27.9

Table 4.11: Numerical values of BER Vs Signal power with Cloud Thickness as a parameter

Fig 4.13 shows the plots of SINR versus Received power, Ps for an FSO link with wavelength as a parameter. It is noticed that SINR decreases significantly with the increase in wavelength which is also shown in tabular form in Table 4.12. Wavelength is taken as 0.532µm, 0.8 µm, 1.3 µm and Bit rate is taken as 1Gbps for computation.

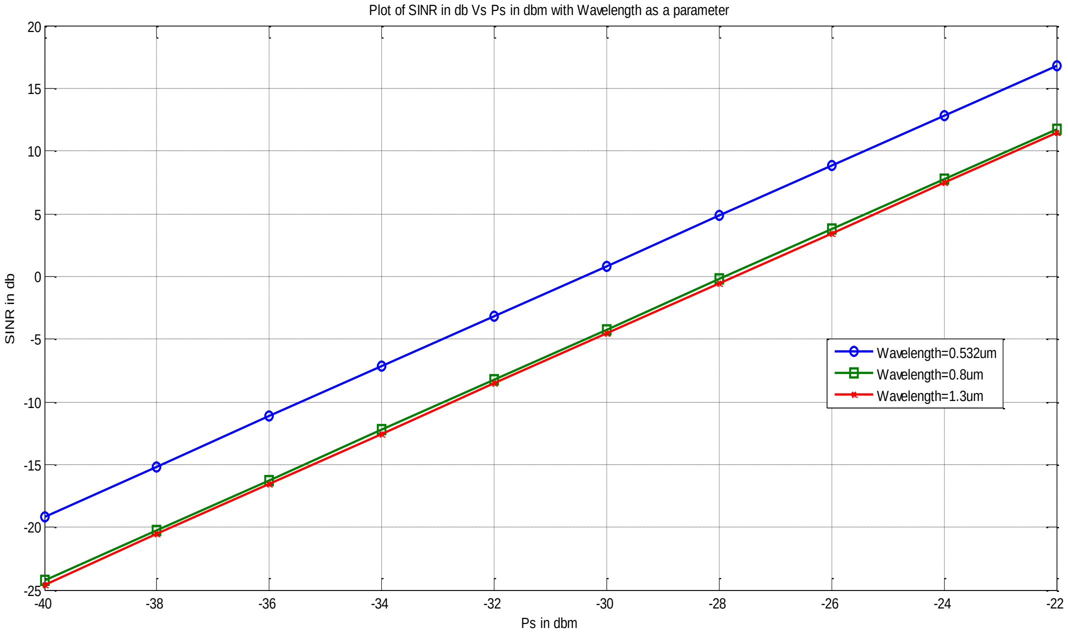


Fig 4.13 Plots of SINR Vs received signal power with Wavelength as a Parameter at a bit rate of 1Gbps and cloud thickness of 225m

Signal power in dbm	**SINR with Wavelength as a parameter**		
	Wavelength=0.532um	**Wavelength=0.8um**	**Wavelength=1.3um**
-36	-11.16	-16.24	-16.55
-32	-3.2	-8.23	-8.6
-24	12.83	7.76	7.45

Table 4.12 Numerical values of SINR Vs received signal power in dbm with Wavelength as a parameter

Fig 4.14 shows the plots of BER versus Received power, Ps for an FSO link with wavelength as a parameter. It is noticed that BER increases significantly with the increase in wavelength which is also shown in tabular form in Table 4.13. Wavelength is taken as 0.532µm, 0.8 µm, 1.3 µm and Bit rate is taken as 1Gbps for computation. From Table 4.13 it is evident that to achieve BER of 10^{-5}, the receiver sensitivity requirement is -25 dbm, -22.7 dbm and -22.5 dbm for wavelength of 0.532µm, 0.8µm and 1.3µm respectively.

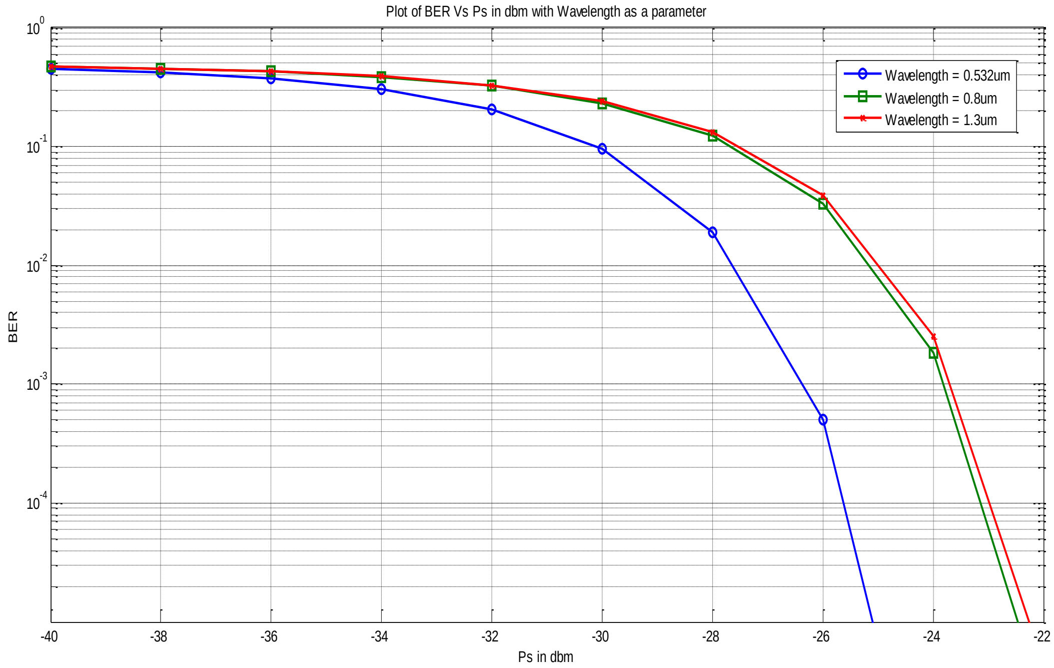


Fig 4.14 Plots of BER Vs received signal power with Wavelength as a Parameter at a bit rate of 1Gbps and cloud thickness of 225m

BER	**Receiver Sensitivity, P_s in dbm**		
	Wavelength=0.532um	**Wavelength=0.8um**	**Wavelength=1.3um**
10^{-2}	-27.8	-25.2	-25
10^{-4}	-25.7	-23.5	-23.2
10^{-5}	-25	-22.7	-22.5

Table 4.13 Numerical values of BER Vs received signal power in dbm with Wavelength as a parameter

Fig 4.15 shows the plots of Power penalty versus Cloud thickness for an FSO link with the effect of Cloud to achieve BER of 10^{-10}. It is noticed that Power penalty increases significantly with the increase in bit rate, R_b. For computation, cloud thickness of 200m, 225m, 250m and 275m is considered at a wavelength of 0.532µm with bit rate of 0.5Gbps, 0.8Gbps and 1Gbps.

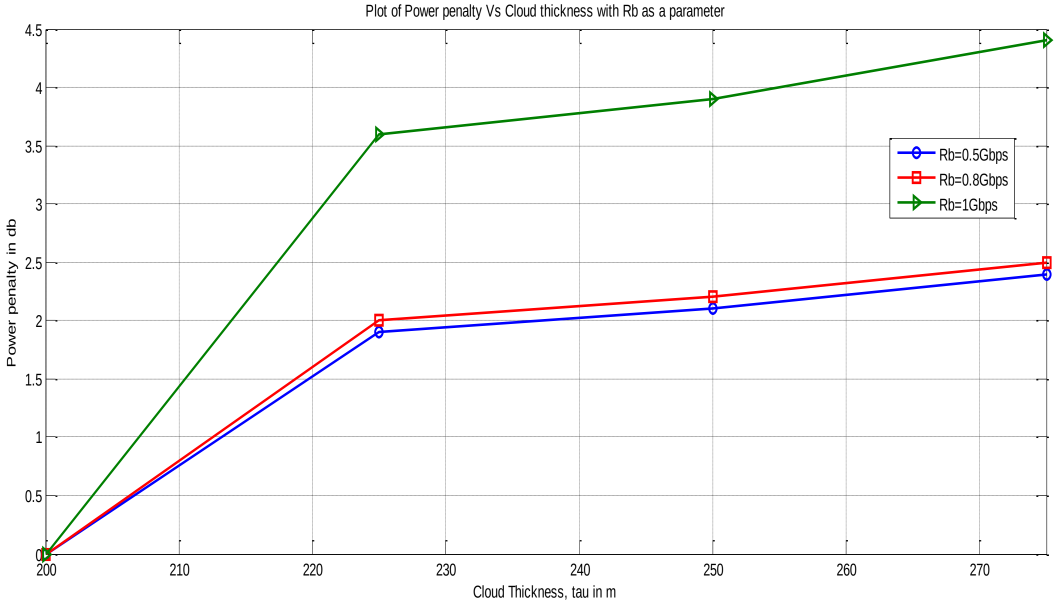


Fig 4.15 Plots of Power Penalty Vs Cloud thickness with bit rate as a Parameter at a wavelength of 0.532µm

4.7 Summary:

In this chapter analytical results are presented in the graphical and tabular format considering cloud effect and without considering Cloud effect. SINR and BER analysis is carried out in different parameters. From the overall analysis one thing is clear that in higher bit rates with having cloud effect, BER increases which is an important parameter for transmission. Another thing is that in low wavelength 0.532μm the BER becomes lowest whereas in high wavelength 1.3 μm BER is highest.

Chapter 5

Conclusion and Future Work

Chapter-5

Conclusion and Future Work

5.1 Conclusion

An analytical approach is developed to evaluate the bit error rate performance of an optical free space link considering the effect of cloud. Analysis is carried out to find the expression of the signal to noise and interference ratio at the receiver output considering the effect of inter symbol interference caused by the finite duration impulse response of the cloud. The intensity modulation and direct detection (IM/DD) is considered for analysis. Performance results are evaluated at various bit rates in terms of signal to noise and interference ratio and bit error rate for several system parameters like cloud thickness and wavelength of transmission.

From the numerical performance results it is revealed that the bit error rate (BER) of an optical free space link is highly degraded due to the effect of the inter symbol interference effect of cloud. It is noticed that there is significant increase in BER due to the presence of cloud as the effect of is more pronounced at higher bit rates. For example, to achieve BER of 10^{-8}at bit rate of 1Gbps, the required received optical power is -25dbm without the effect of the cloud and is -22.2 dbm with the effect of cloud with a cloud thickness of 200m. At a bit rate of 10Gbps there is further deterioration in bit error rate performance. It is found that for a cloud thickness of 200m, to achieve BER of 10^{-9} the received sensitivity is -33.6dbm, -29.4 dbm, -25.9 dbm and -24.6 dbm corresponding to bit rate of 0.01 Gbps, 0.1 Gbps, 0.5 Gbps and 1 Gbps. The system thus suffers penalty due to the effect of clouds at a given BER. However it is noticed that the impulse response of the cloud has a smaller duration at longer cloud thickness and consequently has less effect of inter symbol interference on the system performance. It is found that a penalty due to cloud is then less at higher cloud thickness. For example, the receiver sensitivity is -24.5 dbm, -25.7 dbm, -27.1 dbm and -28.8 dbm at a 1 Gbps corresponding to cloud thickness of 200m, 225m, 250m and 275m respectively at a bit error rate of 10^{-9}.

5.2 Scope of Future Research Work

In the free space optical communication, the atmospheric channel is prone to different atmospheric conditions. So actual parameter need to be selected for maintaining desired specific BER. In future, adaptive transmitter can be used to improve the BER where as variable bit rate also can be used. In receiver end, adaptive circuit can be used too for improving BER performance.

Extensive work can be carried out on different modulation scheme and different coding technique like Convolution code, Turbo code, Low Density Parity Check Code (LDPC), Space Time Block Code (STBC) can also be used to minimize the effect of atmospheric scintillation and spatial widening in presence of cloud. Also the performance of the transmitted signal can be improved by using spatial diversity. In addition, considering the atmospheric turbulence along with Cloud effect can be analyzed in future.

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Appendix A

BER Calculation for Intensity Modulation Direct Detection (IM/DD) system

In the presence of noise the signal at receiver is not well defined although the transmitted signal consists of two well defined light levels [Figure 3.2].

I_1= output current for ―1”

I_0= output current for ―0”

n(t)=thermal noise+ shot noise=AWGN

when 1 is transmitted,

$$i_1(t)=i_{j1}(t)+n_1(t) \tag{A.1}$$

when 0 is transmitted

$$i_0(t)=i_{j0}(t)+n_0(t) \tag{A.2}$$

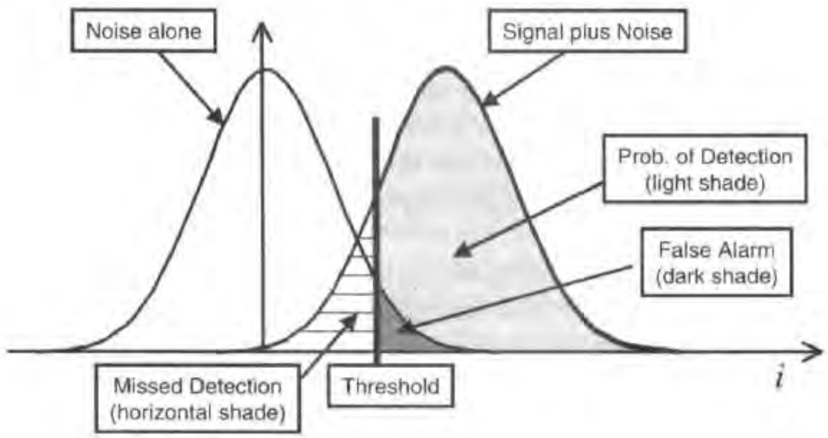

Figure: A.1: Probability of detection and false alarm in IM/DD system.

σ_1^2= output noise variance when 1 is received

$$=2eBI_1+4kTB/R_L \tag{A.3}$$

σ_0^2= output noise variance when 0 is received

$$=2eBI_0+4kTB/R_L \tag{A.4}$$

Here $\sigma_0^2 << \sigma_1^2$

Let us consider $P_r\{1\}$ and $P_r\{0\}$ be the probabilities of transmission for binary ones and zeros. Also consider the probability that a signal is transmitted as 1 but received as 0 is $P_r\{0|1\}$ and the probability that a signal is transmitted as 0 but received as 1 is $P_r\{1|0\}$. If a decision threshold i_{th} is set between the two signals states where signals greater than i_{th} is registered as a one and those less than i_{th} is a zero.

BER= Probability of Bit Error

$$= P_r\{1\}.\ P_r\{0|1\}+ P_r\{0\}\ P_r\{1|0\}$$

$$=0.5[P_r\{0|1\}+ P_r\{1|0\}]$$

$$=0.5[P_r\{i<i_{th}|'1'\}+ P_r\{i>i_{th}|'0'\}] \tag{A.5}$$

$P(n_0)$= Gaussian $(0, \sigma_0^2)$

$P(n_1)$= Gaussian $(0, \sigma_1^2)$

$P(i_0)$= Gaussian (I_0, σ_0^2)

$$=\frac{1}{\sqrt{2\pi\sigma_0^2}} \exp\left(\frac{-(i-I_0)}{2\sigma_0^2}\right) \tag{A.6}$$

$P(i_1)$= Gaussian (I_1, σ_1^2)

$$=\frac{1}{\sqrt{2\pi\sigma_1^2}} \exp\left(\frac{-(i-I_1)}{2\sigma_1^2}\right) \tag{A.7}$$

$$P_r\{1|0\}=\int_{I_{th}}^{\infty} P(i_0)di=\int_{I_{th}}^{\infty}\left(\frac{1}{\sqrt{2\pi\sigma_0^2}}\exp\left(\frac{-(i-I_0)^2}{2\sigma_0^2}\right)\right)di$$

$$=\frac{1}{\sqrt{\pi}}\int_{I_{th}}^{\infty}\exp(-x^2)dx \quad \text{(A.8)}$$

$$P_r\{0|1\}=\int_{I_{-\infty}}^{I_{th}} P(i_1)di=\int_{I_{-\infty}}^{I_{th}}\left(\frac{1}{\sqrt{2\pi\sigma_1^2}}\exp\left(\frac{-(i-I_1)^2}{2\sigma_1^2}\right)\right)di$$

$$=\frac{1}{\sqrt{\pi}}\int_{-\infty}^{\infty}\exp(-y^2)dy \quad \text{(A.9)}$$

Where,

$x=\frac{(i-I_0)}{\sqrt{2}\sigma_0}$ and $dx=\frac{(di)}{\sqrt{2}\sigma_0}$

$y=\frac{(i-I_1)}{\sqrt{2}\sigma_1}$ and $dy=\frac{(di)}{\sqrt{2}\sigma_1}$

$$P_r\{1|0\}=\frac{1}{2}\frac{2}{\sqrt{\pi}}\int_{\frac{I_{th}-I_0}{\sqrt{2}\sigma_0}}^{\infty}\exp(-x^2)dx$$

$$=\frac{1}{2}erfc\left(\frac{I_{th}-I_0}{\sqrt{2}\sigma_0}\right) \quad \text{(A.10)}$$

Similarly,

$$P_r\{0|1\}=\frac{1}{2}erfc\left(\frac{I_1-I_{th}}{\sqrt{2}\sigma_1}\right) \quad \text{(A.11)}$$

For optimized threshold. I_{th} should be at middle.

Now,

$$BER=0.5\left[\frac{1}{2}erfc\left(\frac{I_{th}-I_0}{\sqrt{2}\sigma_0}\right)+\frac{1}{2}erfc\left(\frac{I_1-I_{th}}{\sqrt{2}\sigma_1}\right)\right] \quad \text{(A.12)}$$

The BER will be minimum, when $P_r\{0|1\}=P_r\{1|0\}$

So, $\frac{I_{th}-I_0}{\sqrt{2}\sigma_0}=\frac{I_1-I_{th}}{\sqrt{2}\sigma_1}$

Or, $I_{th}=\frac{I_1\sigma_0+I_0\sigma_1}{\sigma_0+\sigma_1}$

If it is considered that $\sigma_0 = \sigma_1$,
i.$e. shot\ noise\ is\ negligible\ and\ thermal\ noise\ is\ dominated$ then

$I_{th}=\frac{I_1+I_0}{2}$

BER=0.5 erfc$\left(\frac{(I_1-I_0)}{2\sqrt{2}(\sigma_1+\sigma_0)}\right)$

And so, BER =0.5erfc$\left(\frac{\sqrt{SNR}}{2\sqrt{2}}\right)$ (A.13)

So calculating the SNR we can find out the BER performance of IM/DD system.